Title

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bayes — Bayesian regression models using the bayes prefix

Description

The bayes prefix fits Bayesian regression models. It provides Bayesian support for many likelihood-based estimation commands. The bayes prefix uses default or user-supplied priors for model parameters and estimates parameters using MCMC by drawing simulation samples from the corresponding posterior model. Also see [BAYES] **bayesmh** and [BAYES] **bayesmh evaluators** for fitting more general Bayesian models.

Quick start

Bayesian linear regression of y on x, using default normal priors for the regression coefficients and an inverse-gamma prior for the variance

```
bayes: regress y x
```

As above, but use a standard deviation of 10 instead of 100 for the default normal priors and shape of 2 and scale of 1 instead of values of 0.01 for the default inverse-gamma prior

```
bayes, normalprior(10) igammaprior(2 1): regress y x
```

Bayesian logistic regression of y on x1 and x2, showing model summary without performing estimation

```
bayes, dryrun: logit y x1 x2
```

As above, but estimate model parameters and use uniform priors for all regression coefficients

```
bayes, prior({y: x1 x2 _cons}, uniform(-10,10)): logit y x1 x2
```

As above, but use a shortcut notation to refer to all regression coefficients

```
bayes, prior({y:}, uniform(-10,10)): logit y x1 x2
```

As above, but report odds ratios and use uniform priors for the slopes and a normal prior for the intercept

```
bayes, prior({y: x1 x2}, uniform(-10,10)) ///
       prior({y:_cons}, normal(0,10)) or: logit y x1 x2
```

Report odds ratios for the logit model on replay

```
bayes, or
```

Bayesian ordered logit regression of y on x1 and x2, saving simulation results to simdata.dta and using a random-number seed for reproducibility

```
bayes, saving(simdata) rseed(123): ologit y x1 x2 x3
```

Bayesian multinomial regression of y on x1 and x2, specifying 20,000 MCMC samples, setting length of the burn-in period to 5,000, and requesting that a dot be displayed every 500 simulations

```
bayes, mcmcsize(20000) burnin(5000) dots(500): mlogit y x1 x2
```

Bayesian Poisson regression of y on x1 and x2, putting regression slopes in separate blocks and showing block summary

```
bayes, block({y:x1}) block({y:x2}) blocksummary: poisson y x1 x2
```

Bayesian multivariate regression of y1 and y2 on x1, x2, and x3, using Gibbs sampling and requesting 90% HPD credible interval instead of the default 95% equal-tailed credible interval

```
bayes, gibbs clevel(90) hpd: mvreg y1 y2 = x1 x2 x3
```

As above, but use mvreg's option level() instead of bayes's option clevel()

```
bayes, gibbs hpd: mvreg y1 y2 = x1 x2 x3, level(90)
```

Suppress estimates of the covariance matrix from the output

```
bayes, noshow(Sigma, matrix)
```

Bayesian Weibull regression of stset survival-time outcome on x1 and x2, specifying starting values of 1 for {y:x1} and of 2 for {y:x2}

```
bayes, initial({y:x1} 1 {y:x2} 2): streg x1 x2, distribution(weibull)
```

Bayesian two-level linear regression of y on x1 and x2 with random intercepts by id

```
bayes: mixed y x1 x2 || id:
```

Menu

Statistics > Bayesian analysis > Regression models > *estimation_command*

Syntax

bayes [, *bayesopts*] : *estimation_command* [, *estopts*]

estimation_command is a likelihood-based estimation command, and *estopts* are command-specific estimation options; see [BAYES] **bayesian estimation** for a list of supported commands, and see the command-specific entries for the supported estimation options, *estopts*.

bayesopts	Description
Priors	
*`gibbs`	specify Gibbs sampling; available only with `regress` or `mvreg` for certain prior combinations
`normalprior(`#*`)`	specify standard deviation of default normal priors for regression coefficients and other real scalar parameters; default is `normalprior(100)`
`igammaprior(`# #*`)`	specify shape and scale of default inverse-gamma prior for variances; default is `igammaprior(0.01 0.01)`
`iwishartprior(`#* [...]`)`	specify degrees of freedom and, optionally, scale matrix of default inverse-Wishart prior for unstructured random-effects covariance
`prior(`*priorspec*`)`	prior for model parameters; this option may be repeated
`dryrun`	show model summary without estimation
Simulation	
`mcmcsize(`*#*`)`	MCMC sample size; default is `mcmcsize(10000)`
`burnin(`*#*`)`	burn-in period; default is `burnin(2500)`
`thinning(`*#*`)`	thinning interval; default is `thinning(1)`
`rseed(`*#*`)`	random-number seed
`exclude(`*paramref*`)`	specify model parameters to be excluded from the simulation results
`restubs(`*restub1 restub2 ...*`)`	specify stubs for random-effects parameters for all levels; allowed only with multilevel models
Blocking	
`blocksize(`#*`)`	maximum block size; default is `blocksize(50)`
`block(`*paramref* [, *blockopts*]`)`	specify a block of model parameters; this option may be repeated
`blocksummary`	display block summary
*`noblocking`	do not block parameters by default
Initialization	
`initial(`*initspec*`)`	initial values for model parameters
`nomleinitial`	suppress the use of maximum likelihood estimates as starting values
`initrandom`	specify random initial values
`initsummary`	display initial values used for simulation
*`noisily`	display output from the estimation command during initialization
Adaptation	
`adaptation(`*adaptopts*`)`	control the adaptive MCMC procedure
`scale(`*#*`)`	initial multiplier for scale factor; default is `scale(2.38)`
`covariance(`*cov*`)`	initial proposal covariance; default is the identity matrix

Reporting	
clevel(#)	set credible interval level; default is clevel(95)
hpd	display HPD credible intervals instead of the default equal-tailed credible intervals
eform_option	display coefficient table in exponentiated form
remargl	compute log marginal likelihood
batch(#)	specify length of block for batch-means calculations; default is batch(0)
saving(*filename*[, replace])	save simulation results to *filename*.dta
nomodelsummary	suppress model summary
nomesummary	suppress multilevel-structure summary; allowed only with multilevel models
[no]dots	suppress dots or display dots every 100 iterations and iteration numbers every 1,000 iterations; default is command-specific
dots(#[, every(#)])	display dots as simulation is performed
[no]show(*paramref*)	specify model parameters to be excluded from or included in the output
showreffects[(*reref*)]	specify that all or a subset of random-effects parameters be included in the output; allowed only with multilevel commands
melabel	display estimation table using the same row labels as *estimation_command*; allowed only with multilevel commands
nogroup	suppress table summarizing groups; allowed only with multilevel models
notable	suppress estimation table
noheader	suppress output header
title(*string*)	display *string* as title above the table of parameter estimates
display_options	control spacing, line width, and base and empty cells
Advanced	
search(*search_options*)	control the search for feasible initial values
corrlag(#)	specify maximum autocorrelation lag; default varies
corrtol(#)	specify autocorrelation tolerance; default is corrtol(0.01)

*Starred options are specific to the bayes prefix; other options are common between bayes and bayesmh.

The full specification of iwishartprior() is iwishartprior(# [*matname*] [, relevel(*levelvar*)]).

Options prior() and block() can be repeated.

priorspec and *paramref* are defined in [BAYES] **bayesmh**.

paramref may contain factor variables; see **[U] 11.4.3 Factor variables**.

See **[U] 20 Estimation and postestimation commands** for more capabilities of estimation commands.

Options

Priors

gibbs specifies that Gibbs sampling be used to simulate model parameters instead of the default adaptive Metropolis–Hastings sampling. This option is allowed only with the regress and mvreg estimation commands. It is available only with certain prior combinations such as normal prior for regression coefficients and an inverse-gamma prior for the variance. Specifying the gibbs option is equivalent to specifying block()'s gibbs suboption for all default blocks of parameters. If you use the `block()` option to define your own blocks of parameters, the `gibbs` option will have no effect on those blocks, and an MH algorithm will be used to update parameters in those blocks unless you also specify `block()`'s `gibbs` suboption.

`normalprior(`*#*`)` specifies the standard deviation of the default normal priors. The default is `normalprior(100)`. The normal priors are used for scalar parameters defined on the whole real line; see *Default priors* for details.

`igammaprior(`*# #*`)` specifies the shape and scale parameters of the default inverse-gamma priors. The default is `igammaprior(0.01 0.01)`. The inverse-gamma priors are used for positive scalar parameters such as a variance; see *Default priors* for details. Instead of a number #, you can specify a missing value (`.`) to refer to the default value of 0.01.

`iwishartprior(`*#* [*matname*] [`, relevel(`*levelvar*`)`]`)` specifies the degrees of freedom and, optionally, the scale matrix *matname* of the default inverse-Wishart priors used for unstructured covariances of random effects with multilevel models. The degrees of freedom # is a positive real scalar with the default value of $d+1$, where d is the number of random-effects terms at the level of hierarchy *levelvar*. Instead of a number #, you can specify a missing value (`.`) to refer to the default value. Matrix name *matname* is the name of a positive-definite Stata matrix with the default of $I(d)$, the identity matrix of dimension d. If `relevel(`*levelvar*`)` is omitted, the specified parameters are used for inverse-Wishart priors for all levels with unstructured random-effects covariances. Otherwise, they are used only for the prior for the specified level *levelvar*. See *Default priors* for details.

`prior(`*priorspec*`)` specifies a prior distribution for model parameters. This option may be repeated. A prior may be specified for any of the model parameters, except the random-effects parameters in multilevel models. Model parameters with the same prior specifications are placed in a separate block. Model parameters that are not included in prior specifications are assigned default priors; see *Default priors* for details. Model parameters may be scalars or matrices, but both types may not be combined in one prior statement. If multiple scalar parameters are assigned a single univariate prior, they are considered independent, and the specified prior is used for each parameter. You may assign a multivariate prior of dimension d to d scalar parameters. Also see *Referring to model parameters* in [BAYES] **bayesmh**.

All `prior()` distributions are allowed, but they are not guaranteed to correspond to proper posterior distributions for all likelihood models. You need to think carefully about the model you are building and evaluate its convergence thoroughly.

`dryrun` specifies to show the summary of the model that would be fit without actually fitting the model. This option is recommended for checking specifications of the model before fitting the model. The model summary reports the information about the likelihood model and about priors for all model parameters.

Simulation

`mcmcsize(`*#*`)` specifies the target MCMC sample size. The default MCMC sample size is `mcmcsize(10000)`. The total number of iterations for the MH algorithm equals the sum of the burn-in iterations and the MCMC sample size in the absence of thinning. If thinning is present, the total number of MCMC iterations is computed as `burnin()` $+$ $($`mcmcsize()` $-\ 1)$ $\times$ `thinning()` $+\ 1$. Computation time of the MH algorithm is proportional to the total number of iterations. The MCMC sample size determines the precision of posterior summaries, which may be different for different model parameters and will depend on the efficiency of the Markov chain. Also see *Burn-in period and MCMC sample size* in [BAYES] **bayesmh**.

`burnin(`*#*`)` specifies the number of iterations for the burn-in period of MCMC. The values of parameters simulated during burn-in are used for adaptation purposes only and are not used for estimation.

The default is `burnin(2500)`. Typically, burn-in is chosen to be as long as or longer than the adaptation period. The burn-in period may need to be larger for multilevel models because these models introduce high-dimensional random-effects parameters and thus require longer adaptation period. Also see *Burn-in period and MCMC sample size* in [BAYES] **bayesmh** and *Convergence of MCMC* in [BAYES] **bayesmh**.

`thinning(`*#*`)` specifies the thinning interval. Only simulated values from every $(1+k\times\#)$th iteration for $k=0,1,2,\ldots$ are saved in the final MCMC sample; all other simulated values are discarded. The default is `thinning(1)`; that is, all simulation values are saved. Thinning greater than one is typically used for decreasing the autocorrelation of the simulated MCMC sample.

`rseed(`*#*`)` sets the random-number seed. This option can be used to reproduce results. `rseed(`*#*`)` is equivalent to typing `set seed` *#* prior to calling the `bayes` prefix; see [R] **set seed** and *Reproducing results* in [BAYES] **bayesmh**.

`exclude(`*paramref*`)` specifies which model parameters should be excluded from the final MCMC sample. These model parameters will not appear in the estimation table, and postestimation features for these parameters and log marginal likelihood will not be available. This option is useful for suppressing nuisance model parameters. For example, if you have a factor predictor variable with many levels but you are only interested in the variability of the coefficients associated with its levels, not their actual values, then you may wish to exclude this factor variable from the simulation results. If you simply want to omit some model parameters from the output, see the `noshow()` option. *paramref* can include individual random-effects parameters.

`restubs(`*restub1 restub2* ...`)` specifies the stubs for the names of random-effects parameters. You must specify stubs for all levels—one stub per level. This option overrides the default random-effects stubs. See *Likelihood model* for details about the default names of random-effects parameters.

Blocking

`blocksize(`*#*`)` specifies the maximum block size for the model parameters; default is `blocksize(50)`. This option does not apply to random-effects parameters. Each group of random-effects parameters is placed in one block, regardless of the number of random-effects parameters in that group.

`block(`*paramref* [, *blockopts*]`)` specifies a group of model parameters for the blocked MH algorithm. By default, model parameters, except the random-effects parameters, are sampled as independent blocks of 50 parameters or of the size specified in option `blocksize()`. Regression coefficients from different equations are placed in separate blocks. Auxiliary parameters such as variances and correlations are sampled as individual separate blocks, whereas the cutpoint parameters of the ordinal-outcome regressions are sampled as one separate block. With multilevel models, each group of random-effects parameters is placed in a separate block, and the `block()` option is not allowed with random-effects parameters. The `block()` option may be repeated to define multiple blocks. Different types of model parameters, such as scalars and matrices, may not be specified in one `block()`. Parameters within one block are updated simultaneously, and each block of parameters is updated in the order it is specified; the first specified block is updated first, the second is updated second, and so on. See *Improving efficiency of the MH algorithm—blocking of parameters* in [BAYES] **bayesmh**.

blockopts include `gibbs`, `split`, `scale()`, `covariance()`, and `adaptation()`.

`gibbs` specifies to use Gibbs sampling to update parameters in the block. This option is allowed only for hyperparameters and only for specific combinations of prior and hyperprior distributions; see *Gibbs sampling for some likelihood-prior and prior-hyperprior configurations* in [BAYES] **bayesmh**. For more information, see *Gibbs and hybrid MH sampling* in [BAYES] **bayesmh**. `gibbs` may not be combined with `scale()`, `covariance()`, or `adaptation()`.

`split` specifies that all parameters in a block are treated as separate blocks. This may be useful for levels of factor variables.

`scale(`*#*`)` specifies an initial multiplier for the scale factor corresponding to the specified block. The initial scale factor is computed as $\#/\sqrt{n_p}$ for continuous parameters and as $\#/n_p$ for discrete parameters, where n_p is the number of parameters in the block. The default is `scale(2.38)`. If specified, this option overrides the respective setting from the `scale()` option specified with the command. `scale()` may not be combined with `gibbs`.

`covariance(`*matname*`)` specifies a scale matrix *matname* to be used to compute an initial proposal covariance matrix corresponding to the specified block. The initial proposal covariance is computed as *rho*×*Sigma*, where *rho* is a scale factor and *Sigma* = *matname*. By default, *Sigma* is the identity matrix. If specified, this option overrides the respective setting from the `covariance()` option specified with the command. `covariance()` may not be combined with `gibbs`.

`adaptation(tarate())` and `adaptation(tolerance())` specify block-specific TAR and acceptance tolerance. If specified, they override the respective settings from the `adaptation()` option specified with the command. `adaptation()` may not be combined with `gibbs`.

`blocksummary` displays the summary of the specified blocks. This option is useful when `block()` is specified.

`noblocking` requests that no default blocking is applied to model parameters. By default, model parameters are sampled as independent blocks of 50 parameters or of the size specified in option `blocksize()`. For multilevel models, this option has no effect on random-effects parameters; blocking is always applied to them.

Initialization

`initial(`*initspec*`)` specifies initial values for the model parameters to be used in the simulation. You can specify a parameter name, its initial value, another parameter name, its initial value, and so on. For example, to initialize a scalar parameter `alpha` to 0.5 and a 2x2 matrix `Sigma` to the identity matrix `I(2)`, you can type

```
bayes, initial({alpha} 0.5 {Sigma,m} I(2)) : ...
```

You can also specify a list of parameters using any of the specifications described in *Referring to model parameters* in [BAYES] **bayesmh**. For example, to initialize all regression coefficients from equations `y1` and `y2` to zero, you can type

```
bayes, initial({y1:} {y2:} 0) : ...
```

The general specification of *initspec* is

paramref # [*paramref* # [...]]

Curly braces may be omitted for scalar parameters but must be specified for matrix parameters. Initial values declared using this option override the default initial values or any initial values declared during parameter specification in the `likelihood()` option. See *Specifying initial values* in [BAYES] **bayesmh** for details.

`nomleinitial` suppresses using maximum likelihood estimates (MLEs) starting values for model parameters. By default, when no initial values are specified, MLE values from *estimation_command* are used as initial values. For multilevel commands, MLE estimates are used only for regression coefficients. Random effects are assigned zero values, and random-effects variances and covariances are initialized with ones and zeros, respectively. If `nomleinitial` is specified and no initial values are provided, the command uses ones for positive scalar parameters, zeros for other scalar parameters, and identity matrices for matrix parameters. `nomleinitial` may be useful for providing an alternative starting state when checking convergence of MCMC. This option cannot be combined with `initrandom`.

`initrandom` specifies that the model parameters be initialized randomly. Random initial values are generated from the prior distributions of the model parameters. If you want to use fixed initial values for some of the parameters, you can specify them in the `initial()` option or during parameter declarations in the `likelihood()` option. Random initial values are not available for parameters with `flat`, `density()`, `logdensity()`, and `jeffreys()` priors; you must provide fixed initial values for such parameters. This option cannot be combined with `nomleinitial`.

`initsummary` specifies that the initial values used for simulation be displayed.

`noisily` specifies that the output from the estimation command be shown during initialization. The estimation command is executed once to set up the model and calculate initial values for model parameters.

Adaptation

`adaptation(`*adaptopts*`)` controls adaptation of the MCMC procedure. Adaptation takes place every prespecified number of MCMC iterations and consists of tuning the proposal scale factor and proposal covariance for each block of model parameters. Adaptation is used to improve sampling efficiency. Provided defaults are based on theoretical results and may not be sufficient for all applications. See *Adaptation of the MH algorithm* in [BAYES] **bayesmh** for details about adaptation and its parameters.

adaptopts are any of the following options:

`every(`*#*`)` specifies that adaptation be attempted every #th iteration. The default is `every(100)`. To determine the adaptation interval, you need to consider the maximum block size specified in your model. The update of a block with k model parameters requires the estimation of a $k \times k$ covariance matrix. If the adaptation interval is not sufficient for estimating the $k(k+1)/2$ elements of this matrix, the adaptation may be insufficient.

`maxiter(`*#*`)` specifies the maximum number of adaptive iterations. Adaptation includes tuning of the proposal covariance and of the scale factor for each block of model parameters. Once the TAR is achieved within the specified tolerance, the adaptation stops. However, no more than # adaptation steps will be performed. The default is variable and is computed as $\max\{25, \texttt{floor(burnin()/adaptation(every()))}\}$.

`maxiter()` is usually chosen to be no greater than $(\texttt{mcmcsize()} + \texttt{burnin()})/$ `adaptation(every())`.

`miniter(`*#*`)` specifies the minimum number of adaptive iterations to be performed regardless of whether the TAR has been achieved. The default is `miniter(5)`. If the specified `miniter()` is greater than `maxiter()`, then `miniter()` is reset to `maxiter()`. Thus, if you specify `maxiter(0)`, then no adaptation will be performed.

`alpha(`*#*`)` specifies a parameter controlling the adaptation of the AR. `alpha()` should be in $[0, 1]$. The default is `alpha(0.75)`.

`beta(`*#*`)` specifies a parameter controlling the adaptation of the proposal covariance matrix. `beta()` must be in [0,1]. The closer `beta()` is to zero, the less adaptive the proposal covariance. When `beta()` is zero, the same proposal covariance will be used in all MCMC iterations. The default is `beta(0.8)`.

`gamma(#)` specifies a parameter controlling the adaptation rate of the proposal covariance matrix. `gamma()` must be in [0,1]. The larger the value of `gamma()`, the less adaptive the proposal covariance. The default is `gamma(0)`.

`tarate(#)` specifies the TAR for all blocks of model parameters; this is rarely used. `tarate()` must be in (0,1). The default AR is 0.234 for blocks containing continuous multiple parameters, 0.44 for blocks with one continuous parameter, and 1/*n_maxlev* for blocks with discrete parameters, where *n_maxlev* is the maximum number of levels for a discrete parameter in the block.

`tolerance(#)` specifies the tolerance criterion for adaptation based on the TAR. `tolerance()` should be in (0,1). Adaptation stops whenever the absolute difference between the current AR and TAR is less than `tolerance()`. The default is `tolerance(0.01)`.

`scale(#)` specifies an initial multiplier for the scale factor for all blocks. The initial scale factor is computed as $\#/\sqrt{n_p}$ for continuous parameters and $\#/n_p$ for discrete parameters, where n_p is the number of parameters in the block. The default is `scale(2.38)`.

`covariance(`*cov*`)` specifies a scale matrix *cov* to be used to compute an initial proposal covariance matrix. The initial proposal covariance is computed as $\rho \times \Sigma$, where ρ is a scale factor and $\Sigma = matname$. By default, Σ is the identity matrix. Partial specification of Σ is also allowed. The rows and columns of *cov* should be named after some or all model parameters. According to some theoretical results, the optimal proposal covariance is the posterior covariance matrix of model parameters, which is usually unknown. This option does not apply to the blocks containing random-effects parameters.

Reporting

`clevel(#)` specifies the credible level, as a percentage, for equal-tailed and HPD credible intervals. The default is `clevel(95)` or as set by [BAYES] **set clevel**.

`hpd` specifies the display of HPD credible intervals instead of the default equal-tailed credible intervals.

eform_option causes the coefficient table to be displayed in exponentiated form; see [R] ***eform_option***. The estimation command determines which *eform_option* is allowed (`eform(`*string*`)` and `eform` are always allowed).

`remargl` specifies to compute the log marginal likelihood for multilevel models. It is not reported by default for multilevel models. Bayesian multilevel models contain many parameters because, in addition to regression coefficients and variance components, they also estimate individual random effects. The computation of the log marginal likelihood involves the inverse of the determinant of the sample covariance matrix of all parameters and loses its accuracy as the number of parameters grows. For high-dimensional models such as multilevel models, the computation of the log marginal likelihood can be time consuming, and its accuracy may become unacceptably low. Because it is difficult to access the levels of accuracy of the computation for all multilevel models, the log marginal likelihood is not reported by default. For multilevel models containing a small number of random effects, you can use the `remargl` option to compute and display the log marginal likelihood.

`batch(#)` specifies the length of the block for calculating batch means, batch standard deviation, and MCSE using batch means. The default is `batch(0)`, which means no batch calculations. When `batch()` is not specified, MCSE is computed using effective sample sizes instead of batch means. Option `batch()` may not be combined with `corrlag()` or `corrtol()`.

`saving(`*filename*[`, replace`]`)` saves simulation results in *filename*`.dta`. The `replace` option specifies to overwrite *filename*`.dta` if it exists. If the `saving()` option is not specified, the `bayes` prefix saves simulation results in a temporary file for later access by postestimation commands.

This temporary file will be overridden every time the `bayes` prefix is run and will also be erased if the current estimation results are cleared. `saving()` may be specified during estimation or on replay.

The saved dataset has the following structure. Variance `_index` records iteration numbers. The `bayes` prefix saves only states (sets of parameter values) that are different from one iteration to another and the frequency of each state in variable `_frequency`. (Some states may be repeated for discrete parameters.) As such, `_index` may not necessarily contain consecutive integers. Remember to use `_frequency` as a frequency weight if you need to obtain any summaries of this dataset. Values for each parameter are saved in a separate variable in the dataset. Variables containing values of parameters without equation names are named as `eq0_p`*#*, following the order in which parameters are declared in the `bayes` prefix. Variables containing values of parameters with equation names are named as `eq`*#*`_p`*#*, again following the order in which parameters are defined. Parameters with the same equation names will have the same variable prefix `eq`*#*. For example,

```
    . bayes, saving(mcmc): ...
```

will create a dataset, `mcmc.dta`, with variable names `eq1_p1` for `{y:x1}`, `eq1_p2` for `{y:_cons}`, and `eq0_p1` for `{var}`. Also see macros `e(parnames)` and `e(varnames)` for the correspondence between parameter names and variable names.

In addition, the `bayes` prefix saves variable `_loglikelihood` to contain values of the log likelihood from each iteration and variable `_logposterior` to contain values of the log posterior from each iteration.

`nomodelsummary` suppresses the detailed summary of the specified model. The model summary is reported by default.

`nomesummary` suppresses the summary about the multilevel structure of the model. This summary is reported by default for multilevel commands.

`nodots`, `dots`, and `dots(`*#*`)` specify to suppress or display dots during simulation. `dots(`*#*`)` displays a dot every # iterations. During the adaptation period, a symbol `a` is displayed instead of a dot. If `dots(..., every(`*#*`))` is specified, then an iteration number is displayed every #th iteration instead of a dot or `a`. `dots(, every(`*#*`))` is equivalent to `dots(1, every(`*#*`))`. `dots` displays dots every 100 iterations and iteration numbers every 1,000 iterations; it is a synonym for `dots(100), every(1000)`. `dots` is the default with multilevel commands, and `nodots` is the default with other commands.

`show(`*paramref*`)` or `noshow(`*paramref*`)` specifies a list of model parameters to be included in the output or excluded from the output, respectively. By default, all model parameters (except random-effects parameters with multilevel models) are displayed. Do not confuse `noshow()` with `exclude()`, which excludes the specified parameters from the MCMC sample. When the `noshow()` option is specified, for computational efficiency, MCMC summaries of the specified parameters are not computed or stored in `e()`. *paramref* can include individual random-effects parameters.

`showreffects` and `showreffects(`*reref*`)` are used with multilevel commands and specify that all or a list *reref* of random-effects parameters be included in the output in addition to other model parameters. By default, all random-effects parameters are excluded from the output as if you have specified the `noshow()` option. This option computes, displays, and stores in `e()` MCMC summaries for the first $\#_{\text{matsize}} - \#_{\text{npar}}$ random-effects parameters, where $\#_{\text{matsize}}$ is the maximum number of variables as determined by `matsize` (see [R] **matsize**) and $\#_{\text{npar}}$ is the number of other model parameters displayed. If you want to obtain MCMC summaries and display other random-effects parameters, you can use the `show()` option or use `bayesstats summary` (see [BAYES] **bayesstats summary**).

melabel specifies that the bayes prefix use the same row labels as *estimation_command* in the estimation table. This option is allowed only with multilevel commands. It is useful to match the estimation table output of bayes: *mecmd* with that of *mecmd*. This option implies nomesummary and nomodelsummary.

nogroup suppresses the display of group summary information (number of groups, average group size, minimum, and maximum) from the output header. This option is for use with multilevel commands.

notable suppresses the estimation table from the output. By default, a summary table is displayed containing all model parameters except those listed in the exclude() and noshow() options. Regression model parameters are grouped by equation names. The table includes six columns and reports the following statistics using the MCMC simulation results: posterior mean, posterior standard deviation, MCMC standard error or MCSE, posterior median, and credible intervals.

noheader suppresses the output header either at estimation or upon replay.

title(*string*) specifies an optional title for the command that is displayed above the table of the parameter estimates. The default title is specific to the specified likelihood model.

display_options: vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(*#*), fvwrapon(*style*), and nolstretch; see [R] **estimation options**.

Advanced

search(*search_options*) searches for feasible initial values. *search_options* are on, repeat(*#*), and off.

search(on) is equivalent to search(repeat(500)). This is the default.

search(repeat(*k*)), $k > 0$, specifies the number of random attempts to be made to find a feasible initial-value vector, or initial state. The default is repeat(500). An initial-value vector is feasible if it corresponds to a state with positive posterior probability. If feasible initial values are not found after *k* attempts, an error will be issued. repeat(0) (rarely used) specifies that no random attempts be made to find a feasible starting point. In this case, if the specified initial vector does not correspond to a feasible state, an error will be issued.

search(off) prevents the command from searching for feasible initial values. We do not recommend specifying this option.

corrlag(*#*) specifies the maximum autocorrelation lag used for calculating effective sample sizes. The default is min{500, mcmcsize()/2}. The total autocorrelation is computed as the sum of all lag-k autocorrelation values for k from 0 to either corrlag() or the index at which the autocorrelation becomes less than corrtol() if the latter is less than corrlag(). Options corrlag() and batch() may not be combined.

corrtol(*#*) specifies the autocorrelation tolerance used for calculating effective sample sizes. The default is corrtol(0.01). For a given model parameter, if the absolute value of the lag-k autocorrelation is less than corrtol(), then all autocorrelation lags beyond the kth lag are discarded. Options corrtol() and batch() may not be combined.

Remarks and examples

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Remarks and examples are presented under the following headings:

For a general introduction to Bayesian analysis, see [BAYES] **intro**. For a general introduction to Bayesian estimation using adaptive MH and Gibbs algorithms, see [BAYES] **bayesmh**. See [BAYES] **bayesian estimation** for a list of supported estimation commands. For a quick overview example of all Bayesian commands, see *Overview example* in [BAYES] **bayesian commands**.

Using the bayes prefix

The `bayes` prefix provides Bayesian estimation for many likelihood-based regression models. Simply prefix your estimation command with `bayes` to get Bayesian estimates—`bayes:` *estimation_command*; see [BAYES] **bayesian estimation** for a list of supported commands. Also see [BAYES] **bayesmh** for other Bayesian models.

Similarly to the `bayesmh` command, the `bayes` prefix sets up a Bayesian posterior model, uses MCMC to simulate parameters of this model, and summarizes and reports results. The process of specifying a Bayesian model is similar to that described in *Setting up a posterior model* in [BAYES] **bayesmh**, except the likelihood model is now determined by the specified *estimation_command* and default priors are used for model parameters. The `bayes` prefix and the `bayesmh` command share the same methodology of MCMC simulation and the same summarization and reporting of simulation results; see [BAYES] **bayesmh** for details. In the following sections, we provide information specific to the `bayes` prefix.

Likelihood model

With the `bayes` prefix, the likelihood component of the Bayesian model is determined by the prefixed estimation command, and all posterior model parameters are defined by the likelihood model. For example, the parameters of the model

```
. bayes: streg age smoking, distribution(lognormal)
```

are the regression coefficients and auxiliary parameters you see when you fit

```
. streg age smoking, distribution(lognormal)
```

All estimation commands have regression coefficients as their model parameters. Some commands have additional parameters such as variances and correlation coefficients.

The bayes prefix typically uses the likelihood parameterization and the naming convention of the estimation command to define model parameters, but there are exceptions. For example, the truncreg command uses the standard deviation parameter {sigma} to parameterize the likelihood, whereas bayes: truncreg uses the variance parameter {sigma2}.

Most model parameters are scalar parameters supported on the whole real line such as regression coefficients, log-transformed positive parameters, and atanh-transformed correlation coefficients. For example, positive scalar parameters are the variance parameters in bayes: regress, bayes: tobit, and bayes: truncreg, and matrix parameters are the covariance matrix {Sigma, matrix} in bayes: mvreg and covariances of random effects in multilevel commands such as bayes: meglm.

The names of model parameters are provided in the model summary displayed by the bayes prefix. Knowing these names is useful when specifying the prior distributions, although the bayes prefix does provide default priors; see *Default priors*. You can use the dryrun option with the bayes prefix to see the names of model parameters prior to the estimation. In general, the names of regression coefficients are formed as {*depvar*:*indepvar*}, where *depvar* is the name of the specified dependent variable and *indepvar* is the name of an independent variable. There are exceptions such as bayes: streg, for which *depvar* is replaced with _t. Variance parameters are named {sigma2}, log-variance parameters are named {lnsigma2}, atanh-transformed correlation parameters are named {athrho}, and the covariance matrix of bayes: mvreg is named {Sigma, matrix} (or {Sigma, m} for short).

For multilevel models such as bayes: meglm, in addition to regression coefficients and variance components, the bayes prefix also estimates random-effects parameters. This is different from the corresponding frequentist commands, such as meglm, in which random effects are integrated out and thus are not among the final model parameters. (They can be predicted after estimation.) As such, the bayes prefix has its own naming convention for model parameters of multilevel commands. Before moving on, you should be familiar with the syntax of the multilevel commands; see, for example, *Syntax* in [ME] **meglm**.

The regression coefficients are labeled as usual, {*depvar*:*indepvar*}. Random-effects parameters are labeled as outlined in tables 1 and 2. You can change the default names by specifying the restubs() option. The common syntax of {*rename*} is {*restub#*}, where *restub* is a capital letter, U for the level specified first, or a sequence of capital letters that is unique to each random-effects level, and # refers to the group of random effects at that level: 0 for random intercepts, 1 for random coefficients associated with the variable specified first in the random-effects equation, 2 for random coefficients associated with the variable specified second, and so on. The full syntax of {*rename*}, {*fullrename*}, is {*restub#*[*levelvar*]}, where *levelvar* is the variable identifying the level of hierarchy and is often omitted from the specification for brevity. Random effects at the observation level or crossed effects, specified as _all: R.*varname* with multilevel commands, are labeled as {U0}, {V0}, {W0}, and so on. Random effects at nesting levels, or nested effects, are labeled using a sequence of capital letters starting with the letter corresponding to the top level. For example, the multilevel model

```
. bayes: melogit y x1 x2 || id1: x1 x2 || id2: x1 || id3:
```

will have random-effects parameters {U0}, {U1}, and {U2} to represent, respectively, random intercepts, random coefficients for x1, and random coefficients for x2 at the id1 level; parameters {UU0} and {UU1} for random intercepts and random coefficients for x1 at the id2 level; and random intercepts {UUU0} at the id3 level. See *Multilevel models* for more examples. Also see *Different ways of specifying model parameters* for how to refer to individual random effects during postestimation.

Table 1. Random effects at nesting levels of hierarchy (nested effects)

Hierarchy	Random effects	{*rename*}
lev1	Random intercepts	{U0}
	Random coefficients	{U1}, {U2}, etc.
lev1>*lev2*	Random intercepts	{UU0}
	Random coefficients	{UU1}, {UU2}, etc.
lev1>*lev2*>*lev3*	Random intercepts	{UUU0}
	Random coefficients	{UUU1}, {UUU2}, etc.
...		

Table 2. Random effects at the observation level, `_all` (crossed effects)

Hierarchy	Random effects	{*rename*}
lev1	Random intercepts	{U0}
lev2	Random intercepts	{V0}
lev3	Random intercepts	{W0}
...		

Variance components for independent random effects are labeled as `{`*rename*`:sigma2}`. In the above example, there are six variance components: `{U0:sigma2}`, `{U1:sigma2}`, `{U2:sigma2}`, `{UU0:sigma2}`, `{UU1:sigma2}`, and `{UUU0:sigma2}`.

Covariance matrices of correlated random effects are labeled as `{`*restub*`:Sigma,matrix}` (or `{`*restub*`:Sigma,m}` for short), where *restub* is the letter stub corresponding to the level at which random effects are defined. For example, if we specify an unstructured covariance for the random effects at the `id1` and `id2` levels (with `cov(un)` short for `covariance(unstructured)`)

```
. bayes: melogit y x1 x2 || id1: x1 x2, cov(un) || id2: x1, cov(un) || id3:
```

we will have two covariance matrix parameters, a 3×3 covariance `{U:Sigma,m}` at the `id1` level and a 2×2 covariance `{UU:Sigma,m}` at the `id2` level, and the variance component `{UUU0:sigma2}` at the `id3` level.

For Gaussian multilevel models such as `bayes: mixed`, the error variance component is labeled as `{e.`*depvar*`:sigma2}`.

Also see command-specific entries for the naming convention of additional parameters such as cutpoints with ordinal models or overdispersion parameters with negative binomial models.

Default priors

For convenience, the `bayes` prefix provides default priors for model parameters. The priors are chosen to be general across models and are fairly uninformative for a typical combination of a likelihood model and dataset. However, the default priors may not always be appropriate. You should always inspect their soundness and, if needed, override the prior specification for some or all model parameters using the `prior()` option.

All scalar parameters supported on the whole real line, such as regression coefficients and log-transformed positive parameters, are assigned a normal distribution with zero mean and variance σ^2_{prior}, $N(0, \sigma^2_{\text{prior}})$, where σ_{prior} is given by the `normalprior()` option. The default value for

σ_{prior} is 100, and thus the default priors for these parameters are $N(0, 10000)$. These priors are fairly uninformative for parameters of moderate size but may become informative for large-scale parameters. See the *Linear regression: A case of informative default priors* example below.

All positive scalar parameters, such as the variance parameters in `bayes: regress` and `bayes: tobit`, are assigned an inverse-gamma prior with shape parameter α and scale parameter β, InvGamma(α, β). The default values for α and β are 0.01, and thus the default prior for these parameters is InvGamma$(0.01, 0.01)$.

All cutpoint parameters of ordinal-outcome models, such as `bayes: ologit` and `bayes: oprobit` are assigned flat priors, improper uniform priors with a constant density of 1, equivalent to specifying the `flat` prior option. The reason for this choice is that the cutpoint parameters are sensitive to the range of the outcome variables, which is usually unknown a priori.

For multilevel models with `independent` and `identity` random-effects covariance structures, variances of random effects are assigned inverse-gamma priors, InvGamma$(0.01, 0.01)$. For `unstructured` random-effects covariances, covariance matrix parameters are assigned fairly uninformative inverse-Wishart priors, InvWishart$(d + 1, I(d))$, where d is the dimension of the random-effects covariance matrix and $I(d)$ is the identity matrix of dimension d. Setting the degrees-of-freedom parameter of the inverse-Wishart prior to $d + 1$ is equivalent to specifying uniform on $(-1, 1)$ distributions for the individual correlation parameters.

The model summary displayed by the `bayes` prefix describes the chosen default priors, which you can see prior to estimation if you specify `bayes`'s `dryrun` option. You can use the `prior()` option repeatedly to override the default prior specifications for some or all model parameters.

Initial values

By default, the `bayes` prefix uses the ML estimates from the prefixed estimation command as initial values for all scalar model parameters.

For example, the specification

```
. bayes: logit y x
```

will use the ML estimates from

```
. logit y x
```

as default initial values for the regression coefficients.

You can override the default initial values by using the `initial()` option; see *Specifying initial values* in [BAYES] **bayesmh**.

If the `nomleinitial` option is specified, instead of using the estimates from the prefixed command, all scalar model parameters are initialized with zeros, except for the variance parameters, which are initialized with ones.

The covariance matrix parameter `{Sigma, matrix}` of `bayes: mvreg` is always initialized with the identity matrix.

For multilevel models, regression coefficients are initialized using the ML estimates from the corresponding model without random effects, variances of random effects are initialized with ones, covariances of random effects are initialized with zeros, and random effects themselves are initialized with zeros.

Command-specific options

Not all command-specific options, that is, options specified with the estimation command, are applicable within the Bayesian framework. One example is the group of maximum-likelihood optimization options such as `technique()` and `gradient`. For a list of supported options, refer to the entry specific to each command; see [BAYES] **bayesian estimation** for a list of commands.

Some of the command-specific reporting options, such as *eform_option* and display options, can be specified either with *estimation_command* or with the `bayes` prefix. For example, to obtain estimates of odds ratios instead of coefficients after the logit model, you can specify the `or` option with the command

```
. bayes: logit y x, or
```

or with the `bayes` prefix

```
. bayes, or: logit y x
```

You can also specify this option on replay with the `bayes` prefix

```
. bayes: logit y x
. bayes, or
```

Introductory example

We start with a simple linear regression model applied to `womenwage.dta`, which contains income data for a sample of working women.

```
. use http://www.stata-press.com/data/r15/womenwage
(Wages of women)
```

Suppose we want to regress women's yearly income, represented by the `wage` variable, on their age, represented by the `age` variable. We can fit this model using the `regress` command.

```
. regress wage age
      Source |       SS           df       MS      Number of obs   =       488
-------------+----------------------------------   F(1, 486)       =     43.53
       Model |  3939.49247         1  3939.49247   Prob > F        =    0.0000
    Residual |  43984.4891       486   90.503064   R-squared       =    0.0822
-------------+----------------------------------   Adj R-squared   =    0.0803
       Total |  47923.9816       487   98.406533   Root MSE        =    9.5133

------------------------------------------------------------------------------
        wage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         age |    .399348   .0605289     6.60   0.000     .2804173    .5182787
       _cons |   6.033077   1.791497     3.37   0.001     2.513041    9.553112
------------------------------------------------------------------------------
```

▷ Example 1: Bayesian simple linear regression

We can fit a corresponding Bayesian regression model by simply adding `bayes:` in front of the `regress` command. Because the `bayes` prefix is simulation based, we set a random-number seed to get reproducible results.

```
. set seed 15

. bayes: regress wage age
Burn-in ...
Simulation ...

Model summary
------------------------------------------------------------------------------
Likelihood:
  wage ~ regress(xb_wage,{sigma2})

Priors:
  {wage:age _cons} ~ normal(0,10000)                                       (1)
          {sigma2} ~ igamma(.01,.01)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_wage.

Bayesian linear regression                       MCMC iterations  =     12,500
Random-walk Metropolis-Hastings sampling         Burn-in          =      2,500
                                                 MCMC sample size =     10,000
                                                 Number of obs    =        488
                                                 Acceptance rate  =      .3739
                                                 Efficiency:  min =      .1411
                                                              avg =      .1766
Log marginal likelihood = -1810.1432                          max =      .2271

------------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
wage         |
         age |  .4008591   .0595579   .001586   .4005088   .2798807   .5183574
       _cons |  5.969069   1.737247   .043218   5.997571    2.60753   9.396475
-------------+----------------------------------------------------------------
      sigma2 |  90.76252   5.891887   .123626   90.43802   79.71145   102.8558
------------------------------------------------------------------------------
Note: Default priors are used for model parameters.
```

The Bayesian model has two regression coefficient parameters, `{wage:age}` and `{wage:_cons}`, and a positive scalar parameter, `{sigma2}`, representing the variance of the error term. The model summary shows the default priors used for the model parameters: `normal(0, 10000)` for the regression coefficients and `igamma(0.01, 0.01)` for the variance parameter. The default priors are provided for convenience and should be used with caution. These priors are fairly uninformative in this example, but this may not always be the case; see the example in *Linear regression: A case of informative default priors*.

The first two columns of the `bayes` prefix's estimation table report the posterior means and standard deviations of the model parameters. We observe that for the regression coefficients `{wage:age}` and `{wage:_cons}`, the posterior means and standard deviations are very similar to the least-square estimates and their standard errors as reported by the `regress` command. The posterior mean estimate for `{sigma2}`, 90.76, is close to the residual mean squared estimate, 90.50, listed in the ANOVA table of the `regress` command. The estimation table of the `bayes` prefix also reports Monte Carlo standard errors (MCSEs), medians, and equal-tailed credible intervals.

The Bayesian estimates are stochastic in nature and, by default, are based on an MCMC sample of size 10,000. It is important to verify that the MCMC simulation has converged; otherwise, the Bayesian estimates cannot be trusted. The simulation efficiencies reported in the header of the estimation table can serve as useful initial indicators of convergence problems. The minimum efficiency in our example is about 0.14, and the average efficiency is about 0.17. These numbers are typical for the MH sampling algorithm used by `bayes` and do not indicate convergence problems; see *Convergence of MCMC* in [BAYES] **bayesmh** for more rigorous convergence diagnostics.

◁

▷ Example 2: Predictions

There are several postestimation commands available after the bayes prefix; see [BAYES] **bayesian postestimation**. Among them is the bayesstats summary command, which we can use to compute simple predictions. Suppose that we want to predict the expected wage of a 40-year-old woman conditional on the above fitted posterior model. Based on our model, this expected wage corresponds to the linear combination $\{\texttt{wage}:\texttt{\_cons}\} + \{\texttt{wage}:\texttt{age}\} \times 40$. We name this expression wage40 and supply it to the bayesstats summary command.

```
. bayesstats summary (wage40: {wage:_cons} + {wage:age}*40)

Posterior summary statistics                        MCMC sample size =    10,000

      wage40 : {wage:_cons} + {wage:age}*40

------------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
      wage40 |  22.00343     .81679   .024045   21.99231   20.39435    23.6718
------------------------------------------------------------------------------
```

The posterior mean estimate for the expected wage is about 22 with a 95% credible interval between 20.39 and 23.67.

◁

▷ Example 3: Gibbs sampling

The bayes prefix uses adaptive MH as its default sampling algorithm. However, in the special case of linear regression, a more efficient Gibbs sampling is available. We can request Gibbs sampling by specifying the gibbs option.

```
. set seed 15

. bayes, gibbs: regress wage age
Burn-in ...
Simulation ...

Model summary
------------------------------------------------------------------------------
Likelihood:
  wage ~ normal(xb_wage,{sigma2})

Priors:
  {wage:age _cons} ~ normal(0,10000)                                       (1)
          {sigma2} ~ igamma(.01,.01)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_wage.

Bayesian linear regression                       MCMC iterations  =     12,500
Gibbs sampling                                   Burn-in          =      2,500
                                                 MCMC sample size =     10,000
                                                 Number of obs    =        488
                                                 Acceptance rate  =          1
                                                 Efficiency:  min =          1
                                                              avg =          1
Log marginal likelihood =  -1810.087                          max =          1

------------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
wage         |
         age |  .3999669   .0611328   .000611   .4005838   .2787908    .518693
       _cons |  6.012074   1.804246   .018042   6.000808   2.488816   9.549921
-------------+----------------------------------------------------------------
      sigma2 |  90.84221   5.939535   .059395   90.54834    79.8132   103.0164
------------------------------------------------------------------------------
Note: Default priors are used for model parameters.
```

The posterior summary results obtained by Gibbs sampling and MH sampling are very close except for the MCSEs. The Gibbs sampler reports substantially lower MCSEs than the default sampler because of its higher efficiency. In fact, in this example, the Gibbs sampler achieves the highest possible efficiency of 1.

◁

Linear regression: A case of informative default priors

Our example in *Introductory example* used the default priors, which were fairly uninformative for those data and that model. This may not always be true. Consider a linear regression model using the familiar `auto.dta`. Let us regress the response variable `price` on the covariate `length` and factor variable `foreign`.

```
. use http://www.stata-press.com/data/r15/auto, clear
(1978 Automobile Data)

. regress price length i.foreign

      Source |       SS           df       MS      Number of obs   =        74
-------------+----------------------------------   F(2, 71)        =     16.35
       Model |   200288930         2   100144465   Prob > F        =    0.0000
    Residual |   434776467        71  6123612.21   R-squared       =    0.3154
-------------+----------------------------------   Adj R-squared   =    0.2961
       Total |   635065396        73  8699525.97   Root MSE        =    2474.6

------------------------------------------------------------------------------
       price |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      length |   90.21239   15.83368     5.70   0.000     58.64092    121.7839
             |
     foreign |
    Foreign  |   2801.143    766.117     3.66   0.000     1273.549    4328.737
       _cons |  -11621.35   3124.436    -3.72   0.000     -17851.3   -5391.401
------------------------------------------------------------------------------
```

▷ Example 4: Default priors

We first fit a Bayesian regression model using the `bayes` prefix with default priors. Because the range of the outcome variable `price` is at least an order of magnitude larger than the range of the predictor variables `length` and `foreign`, we anticipate that some of the model parameters may have large scale, and longer adaptation may be necessary for the MCMC algorithm to reach optimal sampling for these parameters. We allow for longer adaptation by increasing the burn-in period from the default value of 2,500 to 5,000.

```
. set seed 15

. bayes, burnin(5000): regress price length i.foreign
Burn-in ...
Simulation ...

Model summary
------------------------------------------------------------------------------
Likelihood:
  price ~ regress(xb_price,{sigma2})

Priors:
  {price:length 1.foreign _cons} ~ normal(0,10000)                         (1)
                        {sigma2} ~ igamma(.01,.01)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_price.
```

```
Bayesian linear regression                       MCMC iterations  =     15,000
Random-walk Metropolis-Hastings sampling         Burn-in          =      5,000
                                                 MCMC sample size =     10,000
                                                 Number of obs    =         74
                                                 Acceptance rate  =      .3272
                                                 Efficiency:  min =     .05887
                                                              avg =      .1093
Log marginal likelihood = -699.23257                          max =      .1958

------------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
price        |
      length |  33.03301    1.80186   .060848   33.07952   29.36325   36.41022
             |
     foreign |
    Foreign  |  32.77011   98.97104   4.07922    34.3237  -164.1978   222.0855
       _cons | -8.063175   102.9479   3.34161  -9.110308  -205.9497   196.9341
-------------+----------------------------------------------------------------
      sigma2 |   7538628    1297955   29334.9    7414320    5379756   1.04e+07
------------------------------------------------------------------------------
Note: Default priors are used for model parameters.
```

The posterior mean estimates of the regression coefficients are smaller (in absolute value) than the corresponding estimates from the `regress` command, because the default prior for the coefficients, `normal(0, 10000)`, is informative and has a strong shrinkage effect. For example, the least-square estimate of the constant term from `regress` is about −11,621, and its scale is much larger than the default prior standard deviation of 100. As a result, the default prior shrinks the estimate of the constant toward 0 and, specifically, to −8.06.

You should be aware that the default priors are provided for convenience and are not guaranteed to be uninformative in all cases. They are designed to have little effect on model parameters, the maximum likelihood estimates of which are of moderate size, say, less than 100 in absolute value. For large-scale parameters, as in this example, the default priors can become informative.

◁

▷ Example 5: Flat priors

Continuing with example 4, we can override the default priors using the `prior()` option. We can, for example, apply the completely uninformative `flat` prior, a prior with the density of 1, for the coefficient parameters.

```
. set seed 15
. bayes, prior({price:}, flat) burnin(5000): regress price length i.foreign
Burn-in ...
Simulation ...
Model summary
------------------------------------------------------------------------------
Likelihood:
  price ~ regress(xb_price,{sigma2})
Priors:
  {price:length 1.foreign _cons} ~ 1 (flat)                                (1)
                         {sigma2} ~ igamma(.01,.01)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_price.
```

```
Bayesian linear regression                       MCMC iterations  =     15,000
Random-walk Metropolis-Hastings sampling         Burn-in          =      5,000
                                                 MCMC sample size =     10,000
                                                 Number of obs    =         74
                                                 Acceptance rate  =      .3404
                                                 Efficiency:  min =     .07704
                                                              avg =      .1086
Log marginal likelihood = -669.62603                          max =      .1898

------------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
price        |
      length |  89.51576   16.27187   .586237   89.60969   57.96996   122.7961
             |
     foreign |
     Foreign |  2795.683   770.6359   26.0589   2787.139   1305.773   4298.785
       _cons | -11478.83   3202.027   113.271  -11504.65  -17845.87  -5244.189
-------------+----------------------------------------------------------------
      sigma2 |   6270294    1089331   25002.1    6147758    4504695    8803268
------------------------------------------------------------------------------
Note: Default priors are used for some model parameters.
```

The posterior mean estimates for the coefficient parameters are now close to the least-square estimates from `regress`. For example, the posterior mean estimate for `{price:_cons}` is about −11,479, whereas the least-square estimate is −11,621.

However, the `flat` priors should be used with caution. Flat priors are improper and may result in improper posterior distributions for which Bayesian inference cannot be carried out. You should thus choose the priors carefully, accounting for the properties of the likelihood model.

◁

▷ Example 6: Zellner's g-prior

A type of prior specific to the normal linear regression model is Zellner's g-prior. We can apply it to our example using the `zellnersg0()` prior. For this prior, we need to specify the dimension of the prior, which is the number of regression coefficients (3), a degree of freedom (50) and the variance parameter of the error term in the regression model, `{sigma2}`; the mean parameter is assumed to be 0 by `zellnersg0()`. See example 9 in [BAYES] **bayesmh** for more details about Zellner's g-prior.

```
. set seed 15

. bayes, prior({price:}, zellnersg0(3, 50, {sigma2})) burnin(5000):
> regress price length i.foreign
Burn-in ...
Simulation ...

Model summary
------------------------------------------------------------------------------
Likelihood:
  price ~ regress(xb_price,{sigma2})

Priors:
  {price:length 1.foreign _cons} ~ zellnersg(3,50,0,{sigma2})              (1)
                        {sigma2} ~ igamma(.01,.01)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_price.
```

```
Bayesian linear regression                      MCMC iterations  =     15,000
Random-walk Metropolis-Hastings sampling        Burn-in          =      5,000
                                                MCMC sample size =     10,000
                                                Number of obs    =         74
                                                Acceptance rate  =      .3019
                                                Efficiency:  min =     .06402
                                                             avg =       .105
Log marginal likelihood = -697.84862                         max =      .1944

------------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
price        |
      length |  87.53039   16.24762   .569888   87.72965    55.5177   119.9915
             |
     foreign |
    Foreign  |  2759.267    794.043   31.3829   2793.241   1096.567   4202.283
       _cons | -11223.95   3211.553    113.34  -11308.39  -17534.25  -4898.139
-------------+----------------------------------------------------------------
      sigma2 |   6845242    1159035   26286.9    6716739    4978729    9521252
------------------------------------------------------------------------------
Note: Default priors are used for some model parameters.
```

We see that using this Zellner's g-prior has little effect on the coefficient parameters, and the simulated posterior mean estimates are close to the least-square estimates from `regress`.

◁

Logistic regression with perfect predictors

Let's revisit the example in *Logistic regression model: A case of nonidentifiable parameters* of [BAYES] **bayesmh**. The example uses `heartswitz.dta` to model the binary outcome `disease`, the presence of a heart disease, using the predictor variables `restecg`, `isfbs`, `age`, and `male`. The dataset is a sample from Switzerland.

```
. use http://www.stata-press.com/data/r15/heartswitz, clear
(Subset of Switzerland heart disease data from UCI Machine Learning Repository)
```

▷ Example 7: Perfect prediction

The logistic regression model for these data is

```
. logit disease restecg isfbs age male
  (output omitted )
```

To fit a Bayesian logistic regression, we prefix the `logit` command with `bayes`. We also specify the `noisily` option to show the estimation output of the `logit` command, which is run by the `bayes` prefix to set up the model and compute starting values for the parameters.

```
. set seed 15
. bayes, noisily: logit disease restecg isfbs age male
note: restecg != 0 predicts success perfectly
      restecg dropped and 17 obs not used
note: isfbs != 0 predicts success perfectly
      isfbs dropped and 3 obs not used
note: male != 1 predicts success perfectly
      male dropped and 2 obs not used
Iteration 0:   log likelihood = -4.2386144
Iteration 1:   log likelihood = -4.2358116
Iteration 2:   log likelihood = -4.2358076
Iteration 3:   log likelihood = -4.2358076
Logistic regression                             Number of obs     =         26
                                                LR chi2(1)        =       0.01
                                                Prob > chi2       =     0.9403
Log likelihood = -4.2358076                     Pseudo R2         =     0.0007

------------------------------------------------------------------------------
     disease |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     restecg |          0  (omitted)
       isfbs |          0  (omitted)
         age |  -.0097846   .1313502    -0.07   0.941    -.2672263    .2476572
        male |          0  (omitted)
       _cons |   3.763893   7.423076     0.51   0.612    -10.78507    18.31285
------------------------------------------------------------------------------
Burn-in ...
Simulation ...
Model summary
------------------------------------------------------------------------------
Likelihood:
  disease ~ logit(xb_disease)
Prior:
  {disease:age _cons} ~ normal(0,10000)                                    (1)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_disease.
Bayesian logistic regression                     MCMC iterations  =     12,500
Random-walk Metropolis-Hastings sampling         Burn-in          =      2,500
                                                 MCMC sample size =     10,000
                                                 Number of obs    =         26
                                                 Acceptance rate  =      .2337
                                                 Efficiency:  min =      .1076
                                                              avg =      .1113
Log marginal likelihood = -14.795726                          max =       .115

------------------------------------------------------------------------------
                                                             Equal-tailed
     disease |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
     restecg |  (omitted)
       isfbs |  (omitted)
         age | -.0405907    .1650514   .004868  -.0328198  -.4005246   .2592641
        male |  (omitted)
       _cons |  6.616447    9.516872   .290075   5.491008  -8.852858   28.99392
------------------------------------------------------------------------------
Note: Default priors are used for model parameters.
```

As evident from the output of the `logit` command, the covariates `restecg`, `isfbs`, and `male` are dropped because of perfect prediction. Although these predictors cannot be identified using the likelihood alone, they can be identified, potentially, in a posterior model with an informative prior. The default prior `normal(0, 10000)`, used by the `bayes` prefix for the regression coefficients, is not informative enough to resolve the perfect prediction, and we must override it with a more informative prior.

◁

▷ Example 8: Informative prior

In the example in *Logistic regression model: A case of nonidentifiable parameters* of [BAYES] **bayesmh**, we use information from another similar dataset, `hearthungary.dta`, to come up with informative priors for the regression coefficients. We use the same priors with the `bayes` prefix. We specify the `asis` option with the `logit` command to prevent dropping the perfect predictors from the model. We also specify the `nomleinitial` option to prevent the `bayes` prefix from trying to obtain ML estimates to use as starting values; reliable ML estimates cannot be provided by the `logit` command when the perfect predictors are retained.

```
. set seed 15

. bayes, prior({disease:restecg age}, normal(0,10))
> prior({disease:isfbs male}, normal(1,10))
> prior({disease:_cons}, normal(-4,10)) nomleinitial:
> logit disease restecg isfbs age male, asis
Burn-in ...
Simulation ...

Model summary
------------------------------------------------------------------------------
Likelihood:
  disease ~ logit(xb_disease)

Priors:
  {disease:restecg age} ~ normal(0,10)                                     (1)
   {disease:isfbs male} ~ normal(1,10)                                     (1)
        {disease:_cons} ~ normal(-4,10)                                    (1)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_disease.

Bayesian logistic regression                     MCMC iterations  =     12,500
Random-walk Metropolis-Hastings sampling         Burn-in          =      2,500
                                                 MCMC sample size =     10,000
                                                 Number of obs    =         48
                                                 Acceptance rate  =      .2121
                                                 Efficiency:  min =     .01885
                                                              avg =     .04328
Log marginal likelihood = -11.006071                          max =     .06184

------------------------------------------------------------------------------
             |                                                Equal-tailed
     disease |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
     restecg |  1.965122   2.315475   .115615   1.655961  -2.029873   6.789415
       isfbs |  1.708631   2.726071   .113734   1.607439  -3.306837   7.334592
         age |  .1258811   .0707431   .003621   .1245266  -.0016807   .2719748
        male |  .2671381   2.237349   .162967   .3318061  -4.106425   4.609955
       _cons | -2.441911   2.750613   .110611  -2.538183  -7.596747   3.185172
------------------------------------------------------------------------------
```

For this posterior model with informative priors, we successfully estimate all regression parameters in the logistic regression model.

The informative prior in this example is based on information from an independent dataset, `hearthungary.dta`, which is a sample of observations on the same heart condition and predictor attributes as `heartswitz.dta` but sampled from Hungary's population. Borrowing information from independent datasets to construct informative priors is justified only when the datasets are compatible with the currently analyzed data.

◁

Multinomial logistic regression

Consider the health insurance dataset, sysdsn1.dta, to model the insurance outcome, insure, which takes the values Indemnity, Prepaid, and Uninsure, using the predictor variables age, male, nonwhite, and site. This model is considered in more detail in example 4 in [R] **mlogit**.

```
. use http://www.stata-press.com/data/r15/sysdsn1, clear
(Health insurance data)
```

First, we use the mlogit command to fit the model

```
. mlogit insure age male nonwhite i.site, nolog
Multinomial logistic regression                 Number of obs     =        615
                                                LR chi2(10)       =      42.99
                                                Prob > chi2       =     0.0000
Log likelihood = -534.36165                     Pseudo R2         =     0.0387

------------------------------------------------------------------------------
      insure |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Indemnity    |  (base outcome)
-------------+----------------------------------------------------------------
Prepaid      |
         age |   -.011745   .0061946    -1.90   0.058    -.0238862    .0003962
        male |   .5616934   .2027465     2.77   0.006     .1643175    .9590693
    nonwhite |   .9747768   .2363213     4.12   0.000     .5115955    1.437958
             |
        site |
          2  |   .1130359   .2101903     0.54   0.591    -.2989296    .5250013
          3  |  -.5879879   .2279351    -2.58   0.010    -1.034733   -.1412433
             |
       _cons |   .2697127   .3284422     0.82   0.412    -.3740222    .9134476
-------------+----------------------------------------------------------------
Uninsure     |
         age |  -.0077961   .0114418    -0.68   0.496    -.0302217    .0146294
        male |   .4518496   .3674867     1.23   0.219     -.268411     1.17211
    nonwhite |   .2170589   .4256361     0.51   0.610    -.6171725     1.05129
             |
        site |
          2  |  -1.211563   .4705127    -2.57   0.010    -2.133751   -.2893747
          3  |  -.2078123   .3662926    -0.57   0.570    -.9257327     .510108
             |
       _cons |  -1.286943   .5923219    -2.17   0.030    -2.447872   -.1260134
------------------------------------------------------------------------------
```

Next, we use the bayes prefix to perform Bayesian estimation of the same multinomial logistic regression model.

```
. set seed 15
. bayes: mlogit insure age male nonwhite i.site
Burn-in ...
Simulation ...
Model summary
------------------------------------------------------------------------------
Likelihood:
  Prepaid Uninsure ~ mlogit(xb_Prepaid,xb_Uninsure)
Priors:
   {Prepaid:age male nonwhite i.site _cons} ~ normal(0,10000)             (1)
  {Uninsure:age male nonwhite i.site _cons} ~ normal(0,10000)             (2)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_Prepaid.
(2) Parameters are elements of the linear form xb_Uninsure.
```

```
Bayesian multinomial logistic regression         MCMC iterations  =     12,500
Random-walk Metropolis-Hastings sampling         Burn-in          =      2,500
                                                 MCMC sample size =     10,000
Base outcome: Indemnity                          Number of obs    =        615
                                                 Acceptance rate  =      .2442
                                                 Efficiency:  min =     .01992
                                                              avg =     .03086
Log marginal likelihood = -614.49286                          max =     .05659

------------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
Prepaid      |
         age | -.0125521    .006247   .000396  -.0125871   -.024602  -.0005809
        male |  .5462718   .2086422   .012818   .5573004   .1263754   .9271802
    nonwhite |  .9796293   .2275709   .015746   .9737777     .53642   1.401076
             |
        site |
          2  |   .098451    .214039   .012887   .0994476  -.3172914   .5260208
          3  | -.6043961   .2348319   .011596  -.6072807  -1.045069  -.1323191
             |
       _cons |  .3183984   .3309283   .021325   .3219128  -.3423583    .956505
-------------+----------------------------------------------------------------
Uninsure     |
         age |  -.008377   .0118479   .000581  -.0082922  -.0323571   .0140366
        male |  .4687524   .3537416    .02376   .4748359  -.2495656   1.147333
    nonwhite |  .1755361     .42708   .022566    .198253  -.7214481    .938098
             |
        site |
          2  | -1.298562   .4746333   .033628   -1.27997  -2.258622  -.4149035
          3  | -.2057122   .3533365   .020695  -.2009649   -.904768   .4924401
             |
       _cons | -1.305083   .5830491    .02451  -1.296332  -2.463954  -.1758435
------------------------------------------------------------------------------
Note: Default priors are used for model parameters.
```

For this model and these data, the default prior specification of the `bayes` prefix is fairly uninformative and, as a result, the posterior mean estimates for the parameters are close to the ML estimates obtained with `mlogit`.

We can report posterior summaries for the relative-risk ratios instead of the regression coefficients. This is equivalent to applying an exponential transformation, $\exp(b)$, to the simulated values of each of the regression coefficients, b, and then summarizing them. We can obtain relative-risk ratio summaries by replaying the `bayes` command with the `rrr` option specified. We use the already available simulation results from the last estimation and do not refit the model. We could have also specified the `rrr` option during the estimation.

```
. bayes, rrr
Model summary
------------------------------------------------------------------------------
Likelihood:
  Prepaid Uninsure ~ mlogit(xb_Prepaid,xb_Uninsure)

Priors:
   {Prepaid:age male nonwhite i.site _cons} ~ normal(0,10000)              (1)
  {Uninsure:age male nonwhite i.site _cons} ~ normal(0,10000)              (2)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_Prepaid.
(2) Parameters are elements of the linear form xb_Uninsure.
```

```
Bayesian multinomial logistic regression         MCMC iterations  =     12,500
Random-walk Metropolis-Hastings sampling         Burn-in          =      2,500
                                                 MCMC sample size =     10,000
Base outcome: Indemnity                          Number of obs    =        615
                                                 Acceptance rate  =      .2442
                                                 Efficiency:  min =     .02149
                                                              avg =     .03181
Log marginal likelihood = -614.49286                          max =     .06007

                                                             Equal-tailed
             |      RRR   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
Prepaid      |
         age |  .9875456   .0061686   .000391   .9874918   .9756982   .9994192
        male |  1.764212   .3634348   .022268   1.745953   1.134708   2.527372
    nonwhite |  2.732931   .6240495   .042568   2.647929   1.709875   4.059566
             |
        site |
          2  |  1.129077   .2450092   .015242   1.104561   .7281185   1.692189
          3  |  .5617084   .1338774    .00665   .5448304   .3516675   .8760614
             |
       _cons |  1.451983   .4904589   .029972   1.379764   .7100938    2.60259
-------------+----------------------------------------------------------------
Uninsure     |
         age |  .9917276   .0117452   .000575    .991742   .9681608   1.014136
        male |  1.699605   .6045513   .040763    1.60775   .7791391   3.149782
    nonwhite |  1.301138   .5448086   .027742   1.219271   .4860479   2.555117
             |
        site |
          2  |  .3045686   .1461615   .009698   .2780457   .1044944   .6604046
          3  |  .8663719   .3155926    .01806   .8179411   .4046357   1.636304
             |
       _cons |  .3203309   .1976203   .008063   .2735332   .0850978   .8387492
------------------------------------------------------------------------------
Note: _cons estimates baseline relative risk for each outcome.
Note: Default priors are used for model parameters.
```

Generalized linear model

Consider the insecticide experiment dataset, `beetle.dta`, to model the number of beetles killed, `r`, on the number of subjected beetles, `n`; the type of beetles, `beetle`; and the log-dose of insecticide, `ldose`. More details can be found in example 2 of [R] **glm**.

```
. use http://www.stata-press.com/data/r15/beetle, clear
```

Consider a generalized linear model with a binomial family and a complementary log-log link function for these data.

```
. glm r i.beetle ldose, family(binomial n) link(cloglog) nolog
Generalized linear models                        No. of obs      =         24
Optimization     : ML                            Residual df     =         20
                                                 Scale parameter =          1
Deviance         =  73.76505595                  (1/df) Deviance =   3.688253
Pearson          =   71.8901173                  (1/df) Pearson  =   3.594506

Variance function: V(u) = u*(1-u/n)              [Binomial]
Link function    : g(u) = ln(-ln(1-u/n))         [Complementary log-log]

                                                 AIC             =    6.74547
Log likelihood   = -76.94564525                  BIC             =   10.20398

------------------------------------------------------------------------------
             |                 OIM
           r |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      beetle |
   Red flour |  -.0910396   .1076132    -0.85   0.398    -.3019576    .1198783
    Mealworm |  -1.836058   .1307125   -14.05   0.000     -2.09225   -1.579867
             |
       ldose |   19.41558   .9954265    19.50   0.000     17.46458    21.36658
       _cons |  -34.84602    1.79333   -19.43   0.000    -38.36089   -31.33116
------------------------------------------------------------------------------
```

To fit a Bayesian generalized linear model with default priors, we type

```
. set seed 15
. bayes: glm r i.beetle ldose, family(binomial n) link(cloglog)
Burn-in ...
Simulation ...
Model summary
------------------------------------------------------------------------------
Likelihood:
  r ~ glm(xb_r)

Prior:
  {r:i.beetle ldose _cons} ~ normal(0,10000)                              (1)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_r.
Bayesian generalized linear models              MCMC iterations  =     12,500
Random-walk Metropolis-Hastings sampling        Burn-in          =      2,500
                                                MCMC sample size =     10,000
Family : binomial n                             Number of obs    =         24
Link   : complementary log-log                  Scale parameter  =          1
                                                Acceptance rate  =      .2003
                                                Efficiency:  min =     .03414
                                                             avg =     .05094
Log marginal likelihood =  -102.9776                         max =     .08012

------------------------------------------------------------------------------
             |                                                Equal-tailed
           r |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
      beetle |
   Red flour | -.0903569    .106067   .004527  -.093614  -.2964984    .112506
    Mealworm | -1.843952    .130297   .004603 -1.848374  -2.091816  -1.594582
             |
       ldose |  19.52814   .9997765   .054106  19.52709    17.6146    21.6217
       _cons | -35.04832   1.800461   .096777  -35.0574  -38.81427  -31.61378
------------------------------------------------------------------------------
Note: Default priors are used for model parameters.
```

The posterior mean estimates of the regression parameters are not that different from the ML estimates obtained with `glm`.

If desired, we can request highest posterior density intervals be reported instead of default equal-tailed credible intervals by specifying the `hpd` option. We can also change the credible-interval level; for example, to request 90% credible intervals, we specify the `clevel(90)` option. We also could specify these options during estimation.

```
. bayes, clevel(90) hpd
Model summary
------------------------------------------------------------------------------
Likelihood:
  r ~ glm(xb_r)

Prior:
  {r:i.beetle ldose _cons} ~ normal(0,10000)                               (1)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_r.

Bayesian generalized linear models               MCMC iterations  =     12,500
Random-walk Metropolis-Hastings sampling         Burn-in          =      2,500
                                                 MCMC sample size =     10,000
Family : binomial n                              Number of obs    =         24
Link   : complementary log-log                   Scale parameter  =          1
                                                 Acceptance rate  =      .2003
                                                 Efficiency:  min =     .03414
                                                              avg =     .05094
Log marginal likelihood =  -102.9776                          max =     .08012

------------------------------------------------------------------------------
             |                                                HPD
           r |      Mean   Std. Dev.     MCSE     Median  [90% Cred. Interval]
-------------+----------------------------------------------------------------
      beetle |
  Red flour  | -.0903569    .106067   .004527   -.093614  -.2444412   .1020305
   Mealworm  | -1.843952    .130297   .004603  -1.848374   -2.03979  -1.620806
             |
       ldose |  19.52814   .9997765   .054106   19.52709   17.86148   21.16389
       _cons | -35.04832   1.800461   .096777   -35.0574  -37.96057  -32.00411
------------------------------------------------------------------------------
Note: Default priors are used for model parameters.
```

Truncated Poisson regression

The semiconductor manufacturing dataset, `probe.dta`, contains observational data of failure rates, `failure`, of silicon wafers with width, `width`, and depth, `depth`, tested at four different probes, `probe`. A wafer is rejected if more than 10 failures are detected. See example 2 in [R] **tpoisson**.

```
. use http://www.stata-press.com/data/r15/probe, clear
```

We fit a truncated Poisson regression model with a truncation point of 10. We suppress the constant regression term from the likelihood equation using the `noconstant` option to retain all four probe levels by including `ibn.probe` in the list of covariates, which declares `probe` to be a factor variable with no base level.

```
. tpoisson failures ibn.probe depth width, noconstant ll(10) nolog
Truncated Poisson regression
Limits:   lower =          10                   Number of obs     =         88
          upper =        +inf                   Wald chi2(6)      =   11340.50
Log likelihood = -239.35746                     Prob > chi2       =     0.0000

------------------------------------------------------------------------------
    failures |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       probe |
          1  |   2.714025   .0752617    36.06   0.000     2.566515    2.861536
          2  |   2.602722   .0692732    37.57   0.000     2.466949    2.738495
          3  |   2.725459   .0721299    37.79   0.000     2.584087    2.866831
          4  |   3.139437   .0377137    83.24   0.000     3.065519    3.213354
             |
       depth |  -.0005034   .0033375    -0.15   0.880    -.0070447     .006038
       width |   .0330225    .015573     2.12   0.034     .0025001     .063545
------------------------------------------------------------------------------
```

▷ Example 9: Default priors

We first apply the bayes prefix with default priors to perform Bayesian estimation of the model. The estimation takes a little longer, so we specify the dots option to see the progress.

```
. set seed 15
. bayes, dots: tpoisson failures ibn.probe depth width, noconstant ll(10)
Burn-in 2500 aaaaaaaaa1000.........2000..... done
Simulation 10000 .........1000.........2000.........3000.........4000.........
> 5000.........6000.........7000.........8000.........9000.........10000 done
Model summary
------------------------------------------------------------------------------
Likelihood:
  failures ~ tpoisson(xb_failures)
Prior:
  {failures:i.probe depth width} ~ normal(0,10000)                        (1)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_failures.
Bayesian truncated Poisson regression            MCMC iterations  =     12,500
Random-walk Metropolis-Hastings sampling         Burn-in          =      2,500
                                                 MCMC sample size =     10,000
Limits: lower =          10                      Number of obs    =         88
        upper =        +inf                      Acceptance rate  =      .1383
                                                 Efficiency:  min =    .004447
                                                              avg =     .01322
Log marginal likelihood = -288.22663                          max =     .04082

------------------------------------------------------------------------------
             |                                                Equal-tailed
    failures |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
       probe |
          1  |  2.689072   .0696122   .008596   2.688881   2.557394   2.833737
          2  |  2.581567   .0644141    .00966   2.588534   2.436973   2.701187
          3  |  2.712054   .0695932   .006415   2.717959    2.55837   2.844429
          4  |   3.13308   .0397521   .004592   3.133433   3.055979   3.208954
             |
       depth |  -.000404   .0033313   .000165  -.000504  -.0067928   .0061168
       width |   .036127   .0165308   .001821   .0360637    .001239    .067552
------------------------------------------------------------------------------
Note: Default priors are used for model parameters.
Note: There is a high autocorrelation after 500 lags.
```

With the default prior specification, the posterior mean estimates for the regression parameters are similar to the ML estimates obtained with the tpoisson command. However, the bayes prefix issues a high autocorrelation warning note and reports a minimum efficiency of only 0.004. The posterior model with default priors seems to be somewhat challenging for the MH sampler. We could allow for longer burn-in and increase the MCMC sample size to improve the MCMC convergence and increase the estimation precision. Instead, we will provide an alternative prior specification that will increase the model flexibility and improve its fit to the data.

◁

▷ Example 10: Hyperpriors

We now assume that the four probe coefficients, {failures:ibn.probe}, have a normal prior distribution with mean parameter {probe_mean} and a variance of 10,000. It is reasonable to assume that all four probes have positive failure rates and that {probe_mean} is a positive hyperparameter. We decide to assign {probe_mean} a gamma(2, 1) hyperprior, which is a distribution with a positive domain and a mean of 2. We use this prior for the purpose of illustration; this prior is not informative for this model and these data. We initialize {probe_mean} with 1 to give it a starting value compatible with its hyperprior.

```
. set seed 15

. bayes, prior({failures:ibn.probe}, normal({probe_mean}, 10000))
> prior({probe_mean}, gamma(2, 1)) initial({probe_mean} 1) dots:
> tpoisson failures ibn.probe depth width, noconstant ll(10)
Burn-in 2500 aaaaaaaaa1000aaaaaaaaa2000aaaaa done
Simulation 10000 .........1000.........2000.........3000.........4000.........
> 5000.........6000.........7000.........8000.........9000.........10000 done

Model summary
------------------------------------------------------------------------------
Likelihood:
  failures ~ tpoisson(xb_failures)

Priors:
      {failures:i.probe} ~ normal({probe_mean},10000)                      (1)
  {failures:depth width} ~ normal(0,10000)                                 (1)

Hyperprior:
  {probe_mean} ~ gamma(2,1)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_failures.
```

```
Bayesian truncated Poisson regression           MCMC iterations  =     12,500
Random-walk Metropolis-Hastings sampling        Burn-in          =      2,500
                                                MCMC sample size =     10,000
Limits: lower =          10                     Number of obs    =         88
        upper =        +inf                     Acceptance rate  =       .304
                                                Efficiency:  min =     .04208
                                                             avg =      .0775
Log marginal likelihood = -287.91504                         max =       .127

                                                             Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
failures     |
       probe |
          1  |  2.703599   .0770656   .003757   2.704613   2.551404   2.848774
          2  |  2.592738   .0711972   .002796   2.594628   2.446274   2.728821
          3  |  2.716223   .0755001   .003549   2.719622   2.568376   2.863064
          4  |  3.137069   .0388127   .001317   3.136773   3.062074   3.211616
             |
       depth |  -.000461   .0033562   .000109  -.0004457  -.0067607   .0062698
       width |  .0337508   .0152654   .000532   .0337798    .003008   .0622191
-------------+----------------------------------------------------------------
  probe_mean |  2.051072   1.462867   .041051    1.71286   .2211973   5.809428
------------------------------------------------------------------------------
Note: Default priors are used for some model parameters.
```

The MCMC simulation achieves an average efficiency of about 8% with no indication of convergence problems. Not only are the posterior mean estimates for the regression parameters similar to the ML estimates, but the MCMC standard errors are much lower than those achieved by the previous model with default priors. By introducing the hyperparameter {probe_mean}, we have improved the goodness of fit of the model.

◁

Zero-inflated negative binomial model

In this example, we consider a Bayesian model using zero-inflated negative binomial likelihood. We revisit example 1 in [R] **zinb**, which models the number of fish caught by visitors to a national park. The probability that a particular visitor fished is assumed to depend on the variables `child` and `camper`, which are supplied as covariates to the `inflate()` option of `zinb`.

```
. use http://www.stata-press.com/data/r15/fish, clear
. zinb count persons livebait, inflate(child camper) nolog
Zero-inflated negative binomial regression      Number of obs     =        250
                                                Nonzero obs       =        108
                                                Zero obs          =        142

Inflation model = logit                         LR chi2(2)        =      82.23
Log likelihood  = -401.5478                     Prob > chi2       =     0.0000

------------------------------------------------------------------------------
       count |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
count        |
     persons |   .9742984   .1034938     9.41   0.000     .7714543    1.177142
    livebait |   1.557523   .4124424     3.78   0.000     .7491503    2.365895
       _cons |  -2.730064    .476953    -5.72   0.000    -3.664874   -1.795253
-------------+----------------------------------------------------------------
inflate      |
       child |   3.185999   .7468551     4.27   0.000      1.72219    4.649808
      camper |  -2.020951    .872054    -2.32   0.020    -3.730146   -.3117567
       _cons |  -2.695385   .8929071    -3.02   0.003     -4.44545   -.9453189
-------------+----------------------------------------------------------------
    /lnalpha |   .5110429   .1816816     2.81   0.005     .1549535    .8671323
-------------+----------------------------------------------------------------
       alpha |   1.667029   .3028685                      1.167604    2.380076
------------------------------------------------------------------------------
```

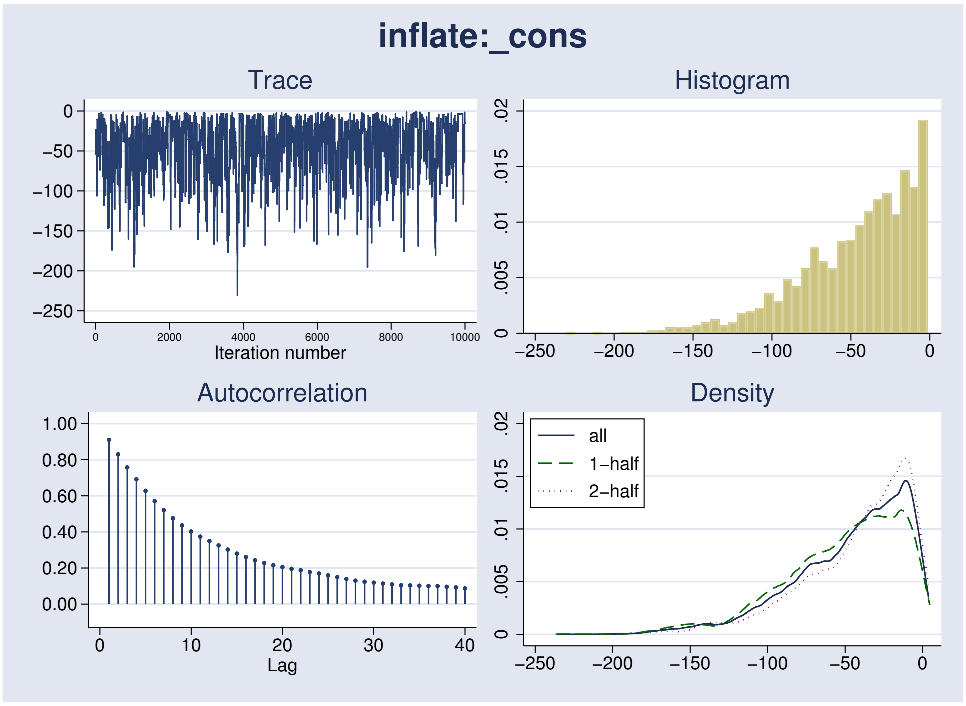
Let’s fit a Bayesian model with default normal prior distributions.

```
. set seed 15
. bayes, dots: zinb count persons livebait, inflate(child camper)
Burn-in 2500 aaaaaaaaa1000aaaaaaaaa2000aaaaa done
Simulation 10000 .........1000.........2000.........3000.........4000.........
> 5000.........6000.........7000.........8000.........9000.........10000 done
Model summary
------------------------------------------------------------------------------
Likelihood:
  count ~ zinb(xb_count,xb_inflate,{lnalpha})

Priors:
  {count:persons livebait _cons} ~ normal(0,10000)                         (1)
    {inflate:child camper _cons} ~ normal(0,10000)                         (2)
                       {lnalpha} ~ normal(0,10000)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_count.
(2) Parameters are elements of the linear form xb_inflate.
```

```
Bayesian zero-inflated negative binomial model   MCMC iterations  =     12,500
Random-walk Metropolis-Hastings sampling         Burn-in          =      2,500
                                                 MCMC sample size =     10,000
Inflation model: logit                           Number of obs    =        250
                                                 Acceptance rate  =      .3084
                                                 Efficiency:  min =     .03716
                                                              avg =      .0791
Log marginal likelihood = -438.47876                          max =      .1613

------------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
count        |
     persons |  .9851217   .1084239   .003601    .985452   .7641609   1.203561
    livebait |  1.536074   .4083865   .013509   1.515838    .753823     2.3539
       _cons | -2.805915   .4700702   .014974  -2.795244   -3.73847   -1.89491
-------------+----------------------------------------------------------------
inflate      |
       child |  46.95902   36.33974   1.87977   38.77997   3.612863   138.3652
      camper |   -46.123   36.34857   1.88567  -37.66796  -137.4568  -2.544566
       _cons | -46.62439   36.36232   1.88355   -38.5171  -137.5522  -3.272469
-------------+----------------------------------------------------------------
     lnalpha |  .7055935   .1591234   .003962   .7048862   .3959316   1.025356
------------------------------------------------------------------------------
Note: Default priors are used for model parameters.
```

The posterior mean estimates for the main regression coefficients `{count:persons}`, `{count:livebait}`, and `{count:_cons}` are relatively close to the ML estimates from the `zinb` command, but the inflation coefficients, `{inflate:child}`, `{inflate:camper}`, and `{inflate:_cons}`, are quite different. For example, `zinb` estimates `{inflate:_cons}` are about -2.7, whereas the corresponding posterior mean estimate is about -46.6. To explain this large discrepancy, we draw the diagnostic plot of `{inflate:_cons}`.

```
. bayesgraph diagnostic {inflate:_cons}
```



The marginal posterior distribution of `{inflate:_cons}` is highly skewed to the left, and it is apparent that its posterior mean is much smaller than its posterior mode. In large samples, under proper noninformative priors, the posterior mode estimator and the ML estimator are equivalent. Therefore, it is not surprising that the posterior mean of `{inflate:_cons}` is much smaller than its ML estimate. We can obtain a rough estimate of the posterior mode in this example.

First, we need to save the simulation results in a dataset, say, `sim_zinb.dta`. You can do this during estimation or on replay by specifying the `saving()` option with the `bayes` prefix.

```
. bayes, saving(sim_zinb)
note: file sim_zinb.dta saved
```

Next, we load the dataset and identify the variable that represents the parameter {inflate:_cons}.

```
. use sim_zinb, clear
. describe
Contains data from sim_zinb.dta
  obs:         6,874
 vars:            11                          8 Feb 2017 13:27
 size:       604,912
-------------------------------------------------------------------------------
              storage   display    value
variable name   type    format     label      variable label
-------------------------------------------------------------------------------
_index          double  %10.0g
_loglikelihood  double  %10.0g
_logposterior   double  %10.0g
eq1_p1          double  %10.0g
eq1_p2          double  %10.0g
eq1_p3          double  %10.0g
eq2_p1          double  %10.0g
eq2_p2          double  %10.0g
eq2_p3          double  %10.0g
eq0_p1          double  %10.0g
_frequency      double  %10.0g
-------------------------------------------------------------------------------
Sorted by:
```

Because {inflate:_cons} is the third parameter in the second equation, its corresponding simulation variable is eq2_p3.

Finally, we use the egen's mode() function to generate a constant variable, mode, which contains the mode estimate for {inflate:_cons}.

```
. egen mode = mode(eq2_p3)
. display mode[1]
-3.417458
```

The mode estimate for {inflate:_cons} is about −3.42, and it is indeed much closer to the ML estimate of −2.70 than its posterior mean estimate.

The inflation parameter α in the likelihood of the zero-inflated negative binomial model is log-transformed, and it is represented by {lnalpha} in our posterior model. To summarize the simulation result for α directly, we can use the bayesstats summary command to exponentiate {lnalpha}.

```
. bayesstats summary (alpha: exp({lnalpha}))
Posterior summary statistics                      MCMC sample size =    10,000
       alpha : exp({lnalpha})
------------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
       alpha |  2.050889   .3292052   .008191   2.023616   1.485768   2.788087
------------------------------------------------------------------------------
```

Parametric survival model

Consider example 7 in [ST] **streg**, which analyzes the effect of a hip-protection device, age, and sex on the risk of hip fractures in patients. The survival dataset is hip3.dta with time to event variable time1 and failure variable fracture. The data are already stset.

```
. use http://www.stata-press.com/data/r15/hip3, clear
(hip fracture study)

. stset
-> stset time1, id(id) failure(fracture) time0(time0)

                id:  id
     failure event:  fracture != 0 & fracture < .
obs. time interval:  (time0, time1]
 exit on or before:  failure

------------------------------------------------------------------------------
        206  total observations
          0  exclusions
------------------------------------------------------------------------------
        206  observations remaining, representing
        148  subjects
         37  failures in single-failure-per-subject data
      1,703  total analysis time at risk and under observation
                                                at risk from t =         0
                                     earliest observed entry t =         0
                                          last observed exit t =        39
```

It is assumed that the hazard curves for men and women have different shapes. We use the `streg` command to fit a model with Weibull survival distribution and the ancillary variable `male` to account for the difference between men and women.

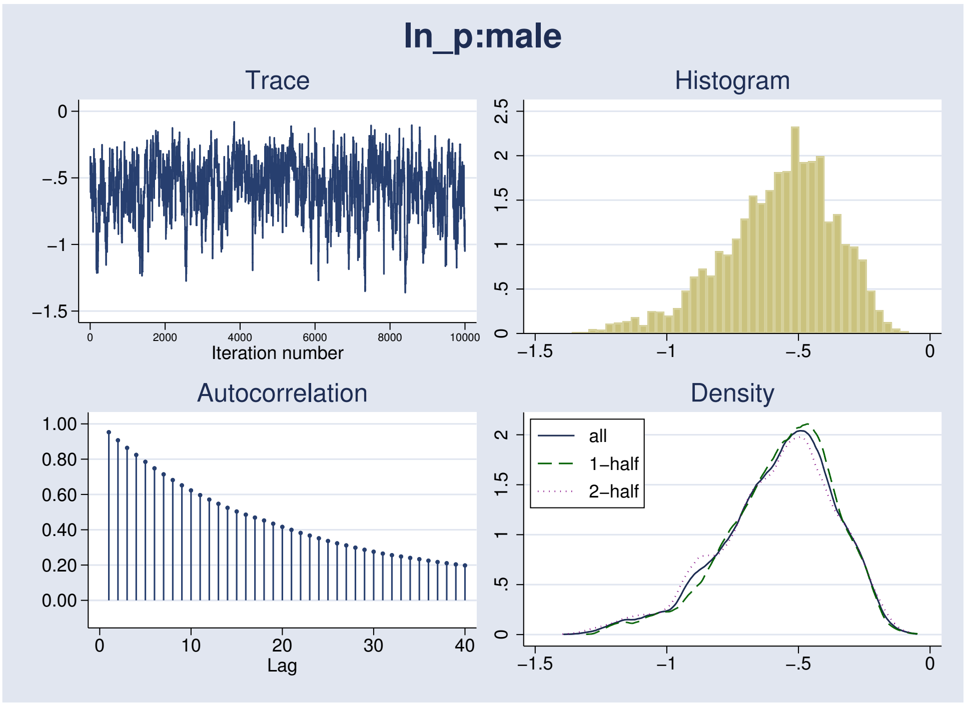
```
. streg protect age, distribution(weibull) ancillary(male) nolog

         failure _d:  fracture
   analysis time _t:  time1
                 id:  id

Weibull PH regression

No. of subjects =          148                  Number of obs    =        206
No. of failures =           37
Time at risk    =         1703
                                                LR chi2(2)       =      39.80
Log likelihood  =   -69.323532                  Prob > chi2      =     0.0000

------------------------------------------------------------------------------
          _t |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
_t           |
     protect |  -2.130058   .3567005    -5.97   0.000    -2.829178   -1.430938
         age |   .0939131   .0341107     2.75   0.006     .0270573    .1607689
       _cons |  -10.17575   2.551821    -3.99   0.000    -15.17722   -5.174269
-------------+----------------------------------------------------------------
ln_p         |
        male |  -.4887189    .185608    -2.63   0.008    -.8525039   -.1249339
       _cons |   .4540139   .1157915     3.92   0.000     .2270667    .6809611
------------------------------------------------------------------------------
```

We then perform Bayesian analysis of the same model using the `bayes` prefix. We apply more conservative normal priors, `normal(0, 100)`, by specifying the `normalprior(10)` option. To allow for longer adaptation of the MCMC sampler, we increase the burn-in period to 5,000, `burnin(5000)`.

```
. set seed 15

. bayes, normalprior(10) burnin(5000) dots:
> streg protect age, distribution(weibull) ancillary(male)

         failure _d:  fracture
   analysis time _t:  time1
                id:  id
Burn-in 5000 aaaaaaaaa1000aaaaaaaaa2000aaaaaaaaa3000aaaaaaaaa4000aaaaaaaaa5000
> done
Simulation 10000 .........1000.........2000.........3000.........4000.........
> 5000.........6000.........7000.........8000.........9000.........10000 done

Model summary
------------------------------------------------------------------------------
Likelihood:
  _t ~ streg_weibull(xb__t,xb_ln_p)

Priors:
  {_t:protect age _cons} ~ normal(0,100)                                   (1)
       {ln_p:male _cons} ~ normal(0,100)                                   (2)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb__t.
(2) Parameters are elements of the linear form xb_ln_p.

Bayesian Weibull PH regression                   MCMC iterations  =     15,000
Random-walk Metropolis-Hastings sampling         Burn-in          =      5,000
                                                 MCMC sample size =     10,000
No. of subjects =        148                     Number of obs    =        206
No. of failures =         37
No. at risk     =       1703
                                                 Acceptance rate  =      .3418
                                                 Efficiency:  min =        .01
                                                              avg =     .03421
Log marginal likelihood = -91.348814                          max =     .05481

------------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
_t           |
     protect | -2.114715   .3486032   .017409  -2.105721  -2.818483   -1.46224
         age |  .0859305   .0328396   .001403   .0862394   .0210016   .1518009
       _cons |  -9.57056   2.457818   .117851  -9.551418  -14.49808   -4.78585
-------------+----------------------------------------------------------------
ln_p         |
        male | -.5753907   .2139477   .014224  -.5468488   -1.07102  -.2317242
       _cons |  .4290642     .11786   .011786   .4242712    .203933   .6548229
------------------------------------------------------------------------------
Note: Default priors are used for model parameters.
```

The posterior mean estimates for the regression parameters {_t:protect}, {_t:age}, and {_t:_cons} are close to the estimates reported by the streg command. However, the estimate for {ln_p:male} is somewhat different. If we inspect the diagnostic plot for {ln_p:male}, we will see that the reason for this is the asymmetrical shape of its marginal posterior distribution.

```
. bayesgraph diagnostic {ln_p:male}
```



As evident from the density plot, the posterior distribution of `{ln_p:male}` is skewed to the left, so the posterior mean estimate, −0.58, is expected to be smaller than the ML estimate, −0.49, given that we used fairly uninformative priors; see *Zero-inflated negative binomial model* for the comparison of posterior mean, posterior mode, and ML estimates for highly skewed posterior distributions.

Heckman selection model

▷ Example 11

A representative example of a Heckman selection model is provided by `wagenwk.dta`, which contains observations on the income of women who choose to work. See example 1 in [R] **heckman**.

```
. use http://www.stata-press.com/data/r15/womenwk, clear
```

The women's income (`wage`) is assumed to depend on their education (`educ`) and their age (`age`). In addition, the selection decision, or the choice of a woman to work, is assumed to depend on their marital status (`married`), number of children (`children`), education, and age. We fit this selection model using the `heckman` command.

```
. heckman wage educ age, select(married children educ age) nolog
Heckman selection model                         Number of obs     =      2,000
(regression model with sample selection)              Selected    =      1,343
                                                      Nonselected =        657

                                                Wald chi2(2)      =     508.44
Log likelihood = -5178.304                      Prob > chi2       =     0.0000

------------------------------------------------------------------------------
        wage |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
wage         |
   education |   .9899537   .0532565    18.59   0.000     .8855729    1.094334
         age |   .2131294   .0206031    10.34   0.000     .1727481    .2535108
       _cons |   .4857752   1.077037     0.45   0.652    -1.625179     2.59673
-------------+----------------------------------------------------------------
select       |
     married |   .4451721   .0673954     6.61   0.000     .3130794    .5772647
    children |   .4387068   .0277828    15.79   0.000     .3842534    .4931601
   education |   .0557318   .0107349     5.19   0.000     .0346917    .0767718
         age |   .0365098   .0041533     8.79   0.000     .0283694    .0446502
       _cons |  -2.491015   .1893402   -13.16   0.000    -2.862115   -2.119915
-------------+----------------------------------------------------------------
     /athrho |   .8742086   .1014225     8.62   0.000     .6754241    1.072993
    /lnsigma |   1.792559    .027598    64.95   0.000     1.738468     1.84665
-------------+----------------------------------------------------------------
         rho |   .7035061   .0512264                      .5885365    .7905862
       sigma |   6.004797   .1657202                       5.68862    6.338548
      lambda |   4.224412   .3992265                      3.441942    5.006881
------------------------------------------------------------------------------
LR test of indep. eqns. (rho = 0):   chi2(1) =    61.20   Prob > chi2 = 0.0000
```

We then apply the bayes prefix to perform Bayesian estimation of the Heckman selection model.

```
. set seed 15
. bayes, dots: heckman wage educ age, select(married children educ age)
Burn-in 2500 aaaaaaaaa1000aaaaaaaaa2000aaaaa done
Simulation 10000 .........1000.........2000.........3000.........4000.........
> 5000.........6000.........7000.........8000.........9000.........10000 done
Model summary
------------------------------------------------------------------------------
Likelihood:
  wage ~ heckman(xb_wage,xb_select,{athrho} {lnsigma})
Priors:
                     {wage:education age _cons} ~ normal(0,10000)          (1)
  {select:married children education age _cons} ~ normal(0,10000)          (2)
                               {athrho lnsigma} ~ normal(0,10000)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_wage.
(2) Parameters are elements of the linear form xb_select.
```

```
Bayesian Heckman selection model                MCMC iterations  =     12,500
Random-walk Metropolis-Hastings sampling        Burn-in          =      2,500
                                                MCMC sample size =     10,000
                                                Number of obs    =      2,000
                                                      Selected   =      1,343
                                                      Nonselected =       657
                                                Acceptance rate  =      .3484
                                                Efficiency:  min =     .02314
                                                             avg =     .03657
Log marginal likelihood = -5260.2024                         max =     .05013

                                                           Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
wage         |
   education |  .9919131    .051865   .002609   .9931531   .8884407   1.090137
         age |  .2131372   .0209631   .001071   .2132548   .1720535   .2550835
       _cons |  .4696264   1.089225     .0716   .4406188  -1.612032    2.65116
-------------+----------------------------------------------------------------
select       |
     married |  .4461775   .0681721   .003045   .4456493   .3178532   .5785857
    children |  .4401305   .0255465   .001156   .4402145   .3911135   .4903804
   education |  .0559983   .0104231   .000484   .0556755   .0360289    .076662
         age |  .0364752   .0042497   .000248   .0362858   .0280584   .0449843
       _cons | -2.494424     .18976   .011327  -2.498414  -2.861266  -2.114334
-------------+----------------------------------------------------------------
      athrho |   .868392    .099374   .005961   .8699977   .6785641   1.062718
     lnsigma |  1.793428   .0269513   .001457   1.793226   1.740569   1.846779
------------------------------------------------------------------------------
Note: Default priors are used for model parameters.
```

The posterior mean estimates for the Bayesian model with default normal priors are similar to the ML estimates obtained with the `heckman` command.

We can calculate posterior summaries for the correlation parameter, ρ, and the standard error, σ, in their natural scale by inverse-transforming the model parameters `{athrho}` and `{lnsigma}` using the `bayesstats summary` command. We also include posterior summaries for the selectivity effect $\lambda = \rho\sigma$.

```
. bayesstats summary (rho:1-2/(exp(2*{athrho})+1))
> (sigma:exp({lnsigma}))
> (lambda:exp({lnsigma})*(1-2/(exp(2*{athrho})+1)))
Posterior summary statistics                      MCMC sample size =    10,000

         rho : 1-2/(exp(2*{athrho})+1)
       sigma : exp({lnsigma})
      lambda : exp({lnsigma})*(1-2/(exp(2*{athrho})+1))

                                                           Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
         rho |  .6970522   .0510145   .003071    .701373   .5905851   .7867018
       sigma |  6.012205   .1621422   .008761   6.008807   5.700587   6.339366
      lambda |  4.196646   .3937209   .024351   4.212609   3.411479   4.946325
------------------------------------------------------------------------------
```

Again, the posterior mean estimates of ρ, σ, and λ agree with the ML estimates reported by `heckman`.

◁

Multilevel models

The bayes prefix supports several multilevel commands such as mixed and meglm; see [BAYES] **bayesian estimation**. Multilevel models introduce effects at different levels of hierarchy such as hospital effects and doctor-nested-within-hospital effects, which are often high-dimensional. These effects are commonly referred to as random effects in frequentist models. Bayesian multilevel models estimate random effects together with other model parameters. In contrast, frequentist multilevel models integrate random effects out, but provide ways to predict them after estimation, conditional on other estimated model parameters. Thus, in addition to regression coefficients and variance components (variances and covariances of random effects), Bayesian multilevel models include random effects themselves as model parameters. With a slight abuse of the terminology, we will sometimes refer to regression coefficients as fixed effects, keeping in mind that they are still random quantities from a Bayesian perspective.

Multilevel models are more difficult to simulate from because of the existence of high-dimensional random-effects parameters. They typically require longer burn-in periods to achieve convergence and larger MCMC sample sizes to obtain precise estimates of random effects and variance components.

Prior specification is particularly important for multilevel models. Using noninformative priors for all model parameters will likely result in nonconvergence or high autocorrelation of the MCMC sample, especially with small datasets. The default priors provided by the bayes prefix are chosen to be fairly uninformative, which may often lead to low simulation efficiencies for model parameters and, especially, for variance components; see *Default priors*. So, do not be surprised to see high autocorrelation with default priors, and be prepared to investigate various prior specifications during your analysis. For example, you may need to use the iwishartprior() option to increase the degrees of freedom and to specify a different scale matrix of the inverse-Wishart prior distribution used for the covariance matrices of random effects.

To change the default priors, you will need to know the names of the model parameters. See *Likelihood model* to learn how the bayes prefix labels the parameters. You can specify your own name stubs for the groups of random-effects parameters using the restubs() option. After simulation, see *Different ways of specifying model parameters* for how to refer to individual random effects to evaluate MCMC convergence or to obtain their MCMC summaries.

By default, the bayes prefix does not compute or display MCMC summaries of individual random effects to conserve computation time and space. You can specify the showreffects() or show() option to compute and display them for chosen groups of random effects. You cannot compute or display more random effects than the current value of set matsize minus other parameters in your model. You can also compute MCMC summaries of random effects after simulation by using [BAYES] **bayesstats summary**.

Also, the bayes prefix does not compute the log marginal likelihood by default for multilevel models. The computation involves the inverse of the determinant of the sample covariance matrix of all parameters and loses accuracy as the number of parameters grows. For high-dimensional models such as multilevel models, the computation can be time consuming, and its accuracy may become unacceptably low. Because it is difficult to access the levels of accuracy of the computation for all multilevel models, the log marginal likelihood is not computed by default. For multilevel models containing a small number of random effects, you can use the remargl option to compute and display it.

Two-level models

Consider example 1 from [ME] **mixed** that analyzed the weight gain of 48 pigs over 9 successive weeks. Detailed Bayesian analysis of these data using `bayesmh` are presented in *Panel-data and multilevel models* in [BAYES] **bayesmh**. Here, we use `bayes: mixed` to fit Bayesian two-level random-intercept and random-coefficient models to these data.

```
. use http://www.stata-press.com/data/r15/pig
(Longitudinal analysis of pig weights)
```

▷ Example 12: Random-intercept model, using option melabel

We first consider a simple random-intercept model of dependent variable `weight` on covariate `week` with variable `id` identifying pigs. The random-intercept model assumes that all pigs share a common growth rate but have different initial weight.

For comparison purposes, we first use the `mixed` command to fit this model by maximum likelihood.

```
. mixed weight week || id:
Performing EM optimization:
Performing gradient-based optimization:
Iteration 0:   log likelihood = -1014.9268
Iteration 1:   log likelihood = -1014.9268
Computing standard errors:
Mixed-effects ML regression                     Number of obs     =        432
Group variable: id                              Number of groups  =         48

                                                Obs per group:
                                                              min =          9
                                                              avg =        9.0
                                                              max =          9

                                                Wald chi2(1)      =   25337.49
Log likelihood = -1014.9268                     Prob > chi2       =     0.0000

------------------------------------------------------------------------------
      weight |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        week |   6.209896   .0390124   159.18   0.000     6.133433    6.286359
       _cons |   19.35561   .5974059    32.40   0.000     18.18472    20.52651
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
id: Identity                 |
                  var(_cons) |   14.81751   3.124226      9.801716    22.40002
-----------------------------+------------------------------------------------
               var(Residual) |   4.383264   .3163348      3.805112     5.04926
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 472.65        Prob >= chibar2 = 0.0000
```

To fit a Bayesian analog of this model, we simply prefix the mixed command with bayes. We also specify the melabel option with bayes to label model parameters in the output table as mixed does.

```
. set seed 15

. bayes, melabel: mixed weight week || id:
note: Gibbs sampling is used for regression coefficients and variance
      components
Burn-in 2500 aaaaaaaaa1000aaaaaaaaa2000aaaaa done
Simulation 10000 .........1000.........2000.........3000.........4000.........
> 5000.........6000.........7000.........8000.........9000.........10000 done

Bayesian multilevel regression                  MCMC iterations  =     12,500
Metropolis-Hastings and Gibbs sampling          Burn-in          =      2,500
                                                MCMC sample size =     10,000
Group variable: id                              Number of groups =         48

                                                Obs per group:
                                                             min =          9
                                                             avg =        9.0
                                                             max =          9

                                                Number of obs    =        432
                                                Acceptance rate  =      .8112
                                                Efficiency:  min =    .007005
                                                             avg =      .5064
Log marginal likelihood                                      max =          1

------------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
weight       |
        week |  6.209734   .0390718   .000391   6.209354   6.133233   6.285611
       _cons |  19.46511   .6239712    .07455   19.48275    18.2534   20.67396
-------------+----------------------------------------------------------------
id           |
  var(_cons) |   15.7247   3.436893   .049048   15.26104   10.31182   23.60471
-------------+----------------------------------------------------------------
var(Residual)|  4.411155   .3193582   .004397   4.396044   3.834341   5.080979
------------------------------------------------------------------------------
Note: Default priors are used for model parameters.
```

The estimates of posterior means and posterior standard deviations are similar to the ML estimates and standard errors from mixed. The results are also close to those from bayesmh in example 23 in [BAYES] **bayesmh**.

The average efficiency of the simulation is about 51% and there is no indication of any immediate convergence problems, but we should investigate convergence more thoroughly; see, for example, example 5 in [BAYES] **bayesian commands** and, more generally, *Convergence of MCMC* in [BAYES] **bayesmh**.

Because Bayesian multilevel models are generally slower than other commands, the bayes prefix displays dots by default with multilevel commands. You can specify the nodots option to suppress them.

Also, as we described in *Multilevel models*, the log marginal likelihood is not computed for multilevel models by default because of the high dimensionality of the models. This is also described in the help file that appears when you click on Log marginal likelihood in the output header in the Results window. For models with a small number of random effects, you can specify the remargl option to compute the log marginal likelihood.

An important note about bayes: mixed is the default simulation method. Most bayes prefix commands use an adaptive MH algorithm to sample model parameters. The high-dimensional nature of multilevel models greatly decreases the simulation efficiency of this algorithm. For Gaussian multilevel models, such as bayes: mixed, model parameters can be sampled using a more efficient, albeit slower, Gibbs algorithm under certain prior distributions. The default priors used for regression coefficients and variance components allow the bayes prefix to use Gibbs sampling for these parameters with the mixed command. If you change the prior distributions or the default blocking structure for some parameters, Gibbs sampling may not be available for those parameters and an adaptive MH sampling will be used instead.

◁

▷ Example 13: Random-intercept model, default output

When we specified the melabel option with bayes in example 12, we intentionally suppressed some of the essential output from bayes: mixed. Here is what we would have seen had we not specified melabel.

```
. bayes
Multilevel structure
------------------------------------------------------------------------------
id
    {U0}: random intercepts
------------------------------------------------------------------------------

Model summary
------------------------------------------------------------------------------
Likelihood:
  weight ~ normal(xb_weight,{e.weight:sigma2})

Priors:
  {weight:week _cons} ~ normal(0,10000)                                  (1)
                 {U0} ~ normal(0,{U0:sigma2})                            (1)
    {e.weight:sigma2} ~ igamma(.01,.01)

Hyperprior:
  {U0:sigma2} ~ igamma(.01,.01)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_weight.
```

```
Bayesian multilevel regression                  MCMC iterations  =     12,500
Metropolis-Hastings and Gibbs sampling          Burn-in          =      2,500
                                                MCMC sample size =     10,000
Group variable: id                              Number of groups =         48

                                                Obs per group:
                                                             min =          9
                                                             avg =        9.0
                                                             max =          9

                                                Number of obs    =        432
                                                Acceptance rate  =      .8112
                                                Efficiency:  min =    .007005
                                                             avg =      .5064
Log marginal likelihood                                      max =          1

-----------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+---------------------------------------------------------------
weight       |
        week |  6.209734   .0390718   .000391   6.209354   6.133233   6.285611
       _cons |  19.46511   .6239712    .07455   19.48275    18.2534   20.67396
-------------+---------------------------------------------------------------
id           |
   U0:sigma2 |   15.7247   3.436893   .049048   15.26104   10.31182   23.60471
-------------+---------------------------------------------------------------
e.weight     |
      sigma2 |  4.411155   .3193582   .004397   4.396044   3.834341   5.080979
-----------------------------------------------------------------------------
Note: Default priors are used for model parameters.
```

Let's go over the default output in detail, starting with the model summary. For multilevel models, in addition to the model summary, which describes the likelihood model and prior distributions, the `bayes` prefix displays information about the multilevel structure of the model.

```
Multilevel structure
-----------------------------------------------------------------------------
id
    {U0}: random intercepts
-----------------------------------------------------------------------------
```

Our multilevel model has one set of random effects, labeled as `U0`, which represent random intercepts at the `id` level. Recall that in Bayesian models, random effects are not integrated out but estimated together with other model parameters. So, `{U0}`, or using its full name `{U0[id]}`, represent random-effects parameters in our model. See *Likelihood model* to learn about the default naming convention for random-effects parameters.

According to the model summary below, the likelihood of the model is a normal linear regression with the linear predictor containing regression parameters {weight:week} and {weight:_cons} and random-effects parameters {U0}, and with the error variance labeled as {e.weight:sigma2}. Regression coefficients {weight:week} and {weight:_cons} have default normal priors with zero means and variances of 10,000. The random intercepts {U0} are normally distributed with mean zero and variance {U0:sigma2}. The variance components, error variance {e.weight:sigma2}, and random-intercept variance {U0:sigma2} have default inverse-gamma priors, InvGamma(0.01, 0.01). The random-intercept variance is a hyperparameter in our model.

```
Model summary
------------------------------------------------------------------------------
Likelihood:
  weight ~ normal(xb_weight,{e.weight:sigma2})

Priors:
  {weight:week _cons} ~ normal(0,10000)                                    (1)
                 {U0} ~ normal(0,{U0:sigma2})                              (1)
    {e.weight:sigma2} ~ igamma(.01,.01)

Hyperprior:
  {U0:sigma2} ~ igamma(.01,.01)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_weight.
```

The default output table of bayes: mixed uses the names of model parameters as they are defined by the bayes prefix.

```
------------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
weight       |
        week |  6.209734   .0390718   .000391   6.209354   6.133233   6.285611
       _cons |  19.46511   .6239712    .07455   19.48275    18.2534   20.67396
-------------+----------------------------------------------------------------
id           |
   U0:sigma2 |   15.7247   3.436893   .049048   15.26104   10.31182   23.60471
-------------+----------------------------------------------------------------
e.weight     |
      sigma2 |  4.411155   .3193582   .004397   4.396044   3.834341   5.080979
------------------------------------------------------------------------------
Note: Default priors are used for model parameters.
```

Becoming familiar with the native parameter names of the bayes prefix is important for prior specification and for later postestimation. The melabel option is provided for easier comparison of the results between the bayes prefix and the corresponding frequentist multilevel command.

◁

▷ Example 14: Displaying random effects

By default, the bayes prefix does not compute or display MCMC summaries for the random-effects parameters to conserve space and computational time. You can specify the showreffects option to display all random effects or the showreffects() or show() option to display specific random effects. For example, continuing example 13, we can display the random-effects estimates for the first five pigs as follows.

```
. bayes, show({U0[1/5]}) noheader

------------------------------------------------------------------------------
             |                                                Equal-tailed
      U0[id] |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
           1 | -1.778442   .8873077   .074832  -1.761984  -3.542545   .0062218
           2 |  .7831408   .8775376   .071421   .7961802  -.9547035   2.491798
           3 | -2.052634   .9038672   .072325  -2.061559  -3.822966  -.3246834
           4 | -1.891103    .878177   .075611  -1.858056  -3.642227  -.1028766
           5 | -3.316584   .8894319   .074946  -3.320502    -5.0469  -1.568927
------------------------------------------------------------------------------
```

These posterior mean estimates of random-effects parameters should be comparable with those predicted by `predict, reffects` after `mixed`. Posterior standard deviations, however, will generally be larger than the corresponding standard errors of random effects predicted after `mixed`, because the latter do not incorporate the uncertainty about the estimated model parameters.

You can also use [BAYES] **bayesstats summary** to obtain MCMC summaries of random-effects parameters after estimation:

```
. bayesstats summary {U0[1/5]}
  (output omitted)
```

If you decide to use the `showreffects` option to display all random-effects parameters, beware of the increased computation time for models with many random effects. Then, the `bayes` prefix will compute and display the MCMC summaries for only the first M random-effects parameters, where M is the maximum number of variables as determined by `matsize` minus the other model parameters. You can specify the `show()` option with `bayes` or use `bayesstats summary` to obtain results for other random-effects parameters.

◁

▷ Example 15: Random-coefficient model

Continuing example 13, let's consider a random-coefficient model that allows the growth rate to vary among pigs.

Following `mixed`'s specification, we include the random slope for `week` at the `id` level by specifying the `week` variable in the random-effects equation.

```
. set seed 15
. bayes: mixed weight week || id: week
note: Gibbs sampling is used for regression coefficients and variance
      components
Burn-in 2500 aaaaaaaaa1000aaaaaaaaa2000aaaaa done
Simulation 10000 .........1000.........2000.........3000.........4000.........
> 5000.........6000.........7000.........8000.........9000.........10000 done
Multilevel structure
------------------------------------------------------------------------------
id
    {U0}: random intercepts
    {U1}: random coefficients for week
------------------------------------------------------------------------------

Model summary
------------------------------------------------------------------------------
Likelihood:
  weight ~ normal(xb_weight,{e.weight:sigma2})

Priors:
  {weight:week _cons} ~ normal(0,10000)                                    (1)
                 {U0} ~ normal(0,{U0:sigma2})                              (1)
                 {U1} ~ normal(0,{U1:sigma2})                              (1)
    {e.weight:sigma2} ~ igamma(.01,.01)

Hyperpriors:
  {U0:sigma2} ~ igamma(.01,.01)
  {U1:sigma2} ~ igamma(.01,.01)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_weight.

Bayesian multilevel regression                   MCMC iterations  =     12,500
Metropolis-Hastings and Gibbs sampling           Burn-in          =      2,500
                                                 MCMC sample size =     10,000
Group variable: id                               Number of groups =         48

                                                 Obs per group:
                                                              min =          9
                                                              avg =        9.0
                                                              max =          9

                                                 Number of obs    =        432
                                                 Acceptance rate  =      .7473
                                                 Efficiency:  min =    .003057
                                                              avg =     .07487
Log marginal likelihood                                       max =      .1503

------------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
weight       |
        week |  6.233977   .0801192    .01449   6.237648    6.05268   6.387741
       _cons |  19.44135   .3426786   .044377   19.44532   18.76211   20.11843
-------------+----------------------------------------------------------------
id           |
   U0:sigma2 |  7.055525   1.649394   .050935   6.844225   4.466329   10.91587
   U1:sigma2 |  .3941786   .0901945   .002717   .3825387   .2526756   .6044887
-------------+----------------------------------------------------------------
e.weight     |
      sigma2 |  1.613775   .1261213   .003254   1.609296   1.386427   1.880891
------------------------------------------------------------------------------
Note: Default priors are used for model parameters.
Note: There is a high autocorrelation after 500 lags.
```

In addition to random intercepts {U0}, we now have random coefficients for week, labeled as {U1}, with the corresponding variance parameter {U1:sigma2}. Compared with the random-intercept model, by capturing the variability of slopes on week, we reduced the estimates of the error variance and the random-intercept variance.

The average simulation efficiency decreased to only 7%, and we now see a note about a high autocorrelation after 500 lags. We can use, for example, bayesgraph diagnostics to verify that the high autocorrelation in this example is not an indication of nonconvergence but rather of a slow mixing of our MCMC sample. If we use bayesstats ess, we will see that the coefficient on weight and the constant term have the lowest efficiency, which suggests that these parameters are likely to be correlated with some of the random-effects estimates. If we want to reduce the autocorrelation and improve precision of the estimates for these parameters, we can increase the MCMC sample size by specifying the mcmcsize() option or thin the MCMC chain by specifying the thinning() option.

◁

▷ Example 16: Random-coefficient model, unstructured covariance

In example 15, we assumed independence between random intercepts {U0} and random slopes on week, {U1}. We relax this assumption here by specifying an unstructured covariance matrix.

Before we proceed with estimation, let's review our model summary first by specifying the dryrun option.

```
. bayes, dryrun: mixed weight week || id: week, covariance(unstructured)
Multilevel structure
------------------------------------------------------------------------------
id
    {U0}: random intercepts
    {U1}: random coefficients for week
------------------------------------------------------------------------------

Model summary
------------------------------------------------------------------------------
Likelihood:
  weight ~ normal(xb_weight,{e.weight:sigma2})

Priors:
  {weight:week _cons} ~ normal(0,10000)                                    (1)
            {U0}{U1} ~ mvnormal(2,{U:Sigma,m})                             (1)
    {e.weight:sigma2} ~ igamma(.01,.01)

Hyperprior:
  {U:Sigma,m} ~ iwishart(2,3,I(2))
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_weight.
```

The prior distributions for random effects {U0} and {U1} are no longer independent. Instead, they have a joint prior—a bivariate normal distribution with covariance matrix parameter {U:Sigma,m}, which is short for {U:Sigma,matrix}. The random-effects stub U is used to label the covariance matrix. The covariance matrix {U:Sigma,m} is assigned a fairly uninformative inverse-Wishart prior with three degrees of freedom and an identity scale matrix; see *Default priors* for details.

Let's now fit the model but suppress the model summary for brevity.

```
. set seed 15

. bayes, nomodelsummary: mixed weight week || id: week, covariance(unstructured)
note: Gibbs sampling is used for regression coefficients and variance
      components
Burn-in 2500 aaaaaaaaa1000aaaaaaaaa2000aaaaa done
Simulation 10000 .........1000.........2000.........3000.........4000.........
> 5000.........6000.........7000.........8000.........9000.........10000 done

Multilevel structure
------------------------------------------------------------------------------
id
    {U0}: random intercepts
    {U1}: random coefficients for week
------------------------------------------------------------------------------

Bayesian multilevel regression                   MCMC iterations  =     12,500
Metropolis-Hastings and Gibbs sampling           Burn-in          =      2,500
                                                 MCMC sample size =     10,000
Group variable: id                               Number of groups =         48

                                                 Obs per group:
                                                              min =          9
                                                              avg =        9.0
                                                              max =          9

                                                 Number of obs    =        432
                                                 Acceptance rate  =      .7009
                                                 Efficiency:  min =    .003683
                                                              avg =     .07461
Log marginal likelihood                                       max =      .1602

------------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
weight       |
        week |  6.207086   .0878022   .014469   6.204974   6.041093   6.384891
       _cons |  19.39551   .4077822   .050353   19.40187   18.53869    20.1993
-------------+----------------------------------------------------------------
id           |
 U:Sigma_1_1 |  6.872161   1.627769   .061568   6.673481   4.282284   10.62194
 U:Sigma_2_1 | -.0866373   .2702822   .009861  -.0796118   -.645439   .4341423
 U:Sigma_2_2 |   .399525   .0904532   .002488   .3885861   .2575883   .6104775
-------------+----------------------------------------------------------------
e.weight     |
      sigma2 |  1.611889   .1263131   .003155   1.605368   1.381651   1.872563
------------------------------------------------------------------------------
Note: Default priors are used for model parameters.
Note: There is a high autocorrelation after 500 lags.
```

◁

The 95% credible interval for the covariance between {U0} and {U1}, labeled as {U:Sigma_2_1} in the output, is $[-.65, .43]$, which suggests independence between {U0} and {U1}.

Crossed-effects model

Let's revisit example 4 from [ME] **meglm**, which analyzes salamander cross-breeding data. Two populations of salamanders are considered: whiteside males and females (variables wsm and wsf) and roughbutt males and females (variables rbm and rbf). Male and female identifiers are recorded in the male and female variables. The outcome binary variable y indicates breeding success or failure.

In example 4 of [ME] **meglm**, we fit a crossed-effects logistic regression for successful mating, in which the effects of `male` and `female` were crossed. For the purpose of illustration, we will fit a crossed-effects probit regression here using `meglm` with the probit link.

```
. use http://www.stata-press.com/data/r15/salamander

. meglm y wsm##wsf || _all: R.male || female:, family(bernoulli) link(probit)
note: crossed random-effects model specified; option intmethod(laplace)
implied
  (iteration log omitted)
Mixed-effects GLM                               Number of obs     =        360
Family:                 Bernoulli
Link:                      probit

-------------------------------------------------------------
                |     No. of       Observations per Group
 Group Variable |     Groups    Minimum    Average    Maximum
----------------+--------------------------------------------
           _all |          1        360      360.0        360
         female |         60          6        6.0          6
-------------------------------------------------------------

Integration method:     laplace
                                                Wald chi2(3)      =      41.50
Log likelihood = -208.11182                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       1.wsm |  -.4121977   .2735675    -1.51   0.132    -.9483802    .1239847
       1.wsf |  -1.720323   .3223052    -5.34   0.000     -2.35203   -1.088617
             |
     wsm#wsf |
        1 1  |   2.121115   .3611665     5.87   0.000     1.413242    2.828989
             |
       _cons |   .5950942   .2350714     2.53   0.011     .1343628    1.055826
-------------+----------------------------------------------------------------
_all>male    |
   var(_cons)|   .3867491   .1789793                       .156139      .95796
-------------+----------------------------------------------------------------
female       |
   var(_cons)|   .4464111   .1976024                      .1874794    1.062959
------------------------------------------------------------------------------
LR test vs. probit model: chi2(2) = 29.35                 Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

To fit the corresponding Bayesian model, we prefix the above command with `bayes:`.

```
. set seed 15

. bayes: meglm y wsm##wsf || _all: R.male || female:, family(bernoulli)
> link(probit)
Burn-in 2500 aaaaaaaaa1000aaaaaaaaa2000aaaaa done
Simulation 10000 .........1000.........2000.........3000.........4000.........
> 5000.........6000.........7000.........8000.........9000.........10000 done

Multilevel structure
------------------------------------------------------------------------------
male
    {U0}: random intercepts

female
    {V0}: random intercepts
------------------------------------------------------------------------------
```

```
Model summary
------------------------------------------------------------------------------
Likelihood:
  y ~ meglm(xb_y)

Priors:
  {y:1.wsm 1.wsf 1.wsm#1.wsf _cons} ~ normal(0,10000)                      (1)
                               {U0} ~ normal(0,{U0:sigma2})                (1)
                               {V0} ~ normal(0,{V0:sigma2})                (1)

Hyperpriors:
  {U0:sigma2} ~ igamma(.01,.01)
  {V0:sigma2} ~ igamma(.01,.01)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_y.

Bayesian multilevel GLM                          MCMC iterations  =     12,500
Random-walk Metropolis-Hastings sampling         Burn-in          =      2,500
                                                 MCMC sample size =     10,000

--------------------------------------------------------------
                |     No. of       Observations per Group
 Group Variable |     Groups    Minimum    Average    Maximum
----------------+---------------------------------------------
           _all |          1        360      360.0        360
         female |         60          6        6.0          6
--------------------------------------------------------------

Family : Bernoulli                               Number of obs    =        360
Link   : probit                                  Acceptance rate  =      .3223
                                                 Efficiency:  min =    .008356
                                                              avg =     .02043
Log marginal likelihood                                       max =     .02773

------------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
y            |
       1.wsm |  -.411886     .28122   .016889  -.4158334  -.9645049    .156521
       1.wsf | -1.722195   .3329918   .023312  -1.713574  -2.381169  -1.094443
             |
     wsm#wsf |
        1 1  |  2.110366   .3671998   .022643    2.09234   1.443113   2.831923
             |
       _cons |  .5858733   .2512646   .015407   .5906893   .0812177   1.077352
-------------+----------------------------------------------------------------
male         |
   U0:sigma2 |  .4291858   .2195246   .024015   .3876708   .1347684   .9648611
-------------+----------------------------------------------------------------
female       |
   V0:sigma2 |  .4928416   .2189307   .019043   .4576824   .1648551   1.003193
------------------------------------------------------------------------------
Note: Default priors are used for model parameters.
```

The variance components for `male` and `female`, `{U0:sigma2}` and `{V0:sigma2}`, are slightly higher than the corresponding ML estimates, but the regression coefficients are similar.

Video examples

Introduction to Bayesian analysis, part 1: The basic concepts

Introduction to Bayesian analysis, part 2: MCMC and the Metropolis–Hastings algorithm

Stored results

In addition to the results stored by `bayesmh`, the `bayes` prefix stores the following in `e()`:

Scalars	
`e(priorsigma)`	standard deviation of default normal priors
`e(priorshape)`	shape of default inverse-gamma priors
`e(priorscale)`	scale of default inverse-gamma priors
`e(blocksize)`	maximum size for blocks of model parameters
Macros	
`e(prefix)`	`bayes`
`e(cmdname)`	command name from *estimation_command*
`e(cmd)`	same as `e(cmdname)`
`e(command)`	estimation command line

Methods and formulas

See *Methods and formulas* in [BAYES] **bayesmh**.

Also see

[BAYES] **bayesian estimation** — Bayesian estimation commands

[BAYES] **bayesmh** — Bayesian models using Metropolis–Hastings algorithm

[BAYES] **bayesian postestimation** — Postestimation tools for bayesmh and the bayes prefix

[BAYES] **bayesian commands** — Introduction to commands for Bayesian analysis

[BAYES] **intro** — Introduction to Bayesian analysis

[BAYES] **Glossary**