Title

> **estat sbcusum —** Cumulative sum test for parameter stability

Description

estat sbcusum performs a test of whether the coefficients in a time-series regression are stable over time. The test statistic is constructed from the cumulative sum of either the recursive residuals or the ordinary least-squares (OLS) residuals.

estat sbcusum requires that the current estimation results be from regress.

Quick start

Test stability of parameters based on the cumulative sum of recursive residuals and plot the cumulative sum with 95% confidence bands

    estat sbcusum

As above, but use OLS residuals

    estat sbcusum, ols

Menu for estat

Statistics > Postestimation

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Syntax

> estat sbcusum [, *options*]

options	Description
<u>rec</u>ursive	use cumulative sum of recursive residuals; the default
ols	use cumulative sum of OLS residuals
<u>g</u>enerate(*newvar*)	create *newvar* containing the cumulative sum of the residuals
level(*#*)	set confidence level; default is level(95)
Plot	
recast(*plottype*)	plot cumulative sum statistic using *plottype*
cline_options	affect rendition of the line plotting the cumulative sum statistic
Confidence bands	
cbopts(*area_options*)	affect rendition of the confidence bands
Add plots	
addplot(*plot*)	add other plots to the graph of the cumulative sum
Y axis, X axis, Titles, Legend, Overall	
twoway_options	any options other than by() documented in [G-3] *twoway_options*

You must tsset your data before using estat sbcusum; see [TS] tsset.

Options

<u>rec</u>ursive computes the cumulative sum (cusum) test statistic and draws a cusum plot using the recursive residuals. This is the default.

ols computes the cusum test statistic and draws a cusum plot using the OLS residuals.

<u>g</u>enerate(*newvar*) creates a new variable containing the cusum of the residuals at each time period.

level(*#*) specifies the default confidence level, as a percentage, for confidence bands, when they are reported. The default is level(95) or as set by set level; see [U] **20.8 Specifying the width of confidence intervals**.

> Plot

recast(*plottype*) specifies that the cusum statistic be plotted using *plottype*. *plottype* may be scatter, line, connected, area, bar, spike, or dropline; see [G-2] **graph twoway**.

cline_options affect the rendition of the plotted cusum statistic. *cline_options* are as described in [G-3] *cline_options*. When recast(*plottype*) is specified, the plot options are whatever is appropriate for the specified *plottype*.

> Confidence bands

cbopts(*area_options*) affects the rendition of the confidence bands for the cusum statistic. *area_options* are as described in [G-3] *area_options*.

┌─ Add plots ───

addplot(*plot*) allows adding more graph twoway plots to the graph; see [G-3] *addplot_option*.

┌─ Y axis, X axis, Titles, Legend, Overall ──────────────────────────

twoway_options are any of the options documented in [G-3] *twoway_options*, except by(). These include options for titling the graph (see [G-3] *title_options*) and options for saving the graph to disk (see [G-3] *saving_option*).

Remarks and examples   stata.com

Remarks are presented under the following headings:

> *Introduction*
> *Cusum of recursive residuals*
> *Cusum of OLS residuals*

Introduction

estat sbcusum performs a test of parameter stability for linear regression coefficients. For instance, estat sbcusum can test for structural breaks due to changes in regression coefficients over time. The test can be performed using the cusum from recursive residuals (the default) or using the cusum from OLS residuals.

In addition to reporting a test statistic, estat sbcusum plots the cusum of residuals and corresponding confidence bands over the sample period.

Both cusum of residuals tests have the null hypothesis that all parameters are stable or constant over time, but these tests do not require a specific alternative hypothesis. We reject this null hypothesis based on the test statistic being larger than a critical value or based on the plotted cusum being outside the confidence bands.

In their simulation study, Ploberger and Krämer (1992) show that the cusum test based on recursive residuals has better power to detect parameter instability occurring early in the sample than the test based on OLS residuals. By contrast, the OLS cusum statistic has better power to detect parameter instabilities for breaks occurring later in the sample than the test based on recursive residuals.

Cusum of recursive residuals

The test statistic based on the cusum of recursive residuals was introduced in Brown, Durbin, and Evans (1975). Under the null hypothesis, the recursive residuals are shown to be independent and identically distributed as normal with 0 mean and constant variance. The cusum of the recursive residuals also has a mean of 0 under the null hypothesis. If the coefficients change after a certain time period, the plot of the recursive cusum process will drift away from the expected value of 0.

The limiting distribution of the sequence of the recursive cusum statistic is approximated by a Brownian motion. The bounds of the Brownian process at a specified significance level are approximated by a linear function. The sample cusum process crossing the theoretical bounds at any time point signifies rejection of the null hypothesis. This is easily identified from a graph that plots the recursive cusum statistic and the bounds. Alternatively, a test statistic based on the maximum of the recursive cusum statistic can be used to test the null hypothesis.

▷ Example 1: Recursive cusum test

wpi1.dta contains quarterly data on the log of the U.S. Wholesale Price Index (WPI) from the first quarter of 1960 to the fourth quarter of 1990 (Enders 2004). We want to know whether the growth rate in the WPI has changed over time. To do this, we take the first differences of ln_wpi; see [U] **11.4.4 Time-series varlists**.

The graph below shows a time-series plot of the growth rate of the WPI.

```
. use http://www.stata-press.com/data/r15/wpi1
(Data from Enders (2004))

. tsline D.ln_wpi
```
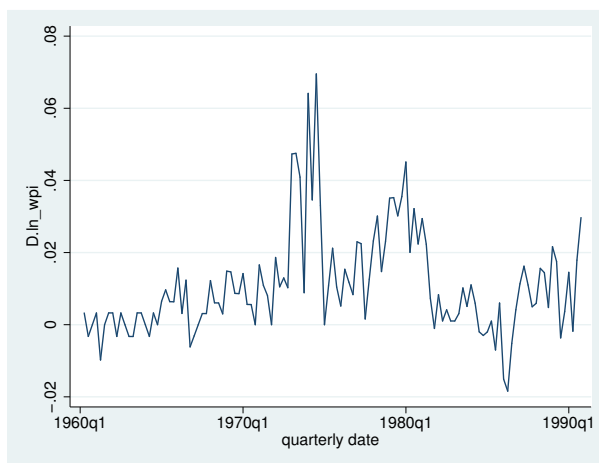

The growth rate changes in the middle of the sample, which leads us to believe that the mean of the series may not be stable during the sample period. Because we are only interested in the average growth rate of the series, we fit a constant-only linear regression model. We fit the model with the quietly command because we are not interested in the coefficient estimates themselves. We then use estat sbcusum to check the stability of the mean in our sample.

```
. quietly regress D.ln_wpi

. estat sbcusum, level(99)
Cumulative sum test for parameter stability
Sample: 1960q2 - 1990q4                          Number of obs =          123
Ho: No structural break
```

Statistic	Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
recursive	1.9030	1.1430	0.9479	0.850

We can interpret the results by comparing the absolute value of the modified cusum statistic with critical values; see *Methods and formulas*. We reject the null hypothesis of a constant mean at the 1% level because the test statistic value of 1.9030 exceeds the 1% critical level of 1.1430.

Recursive cusum plot of D.ln_wpi
with 99% confidence bands around the null
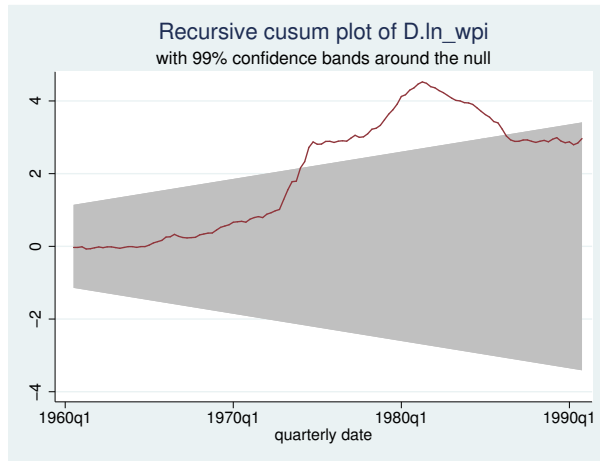
We can also examine the cusum plot. From the graph above, we see that the plot of the recursive cusum process crosses the 99% confidence bands, which implies that the mean of the regression model is not stable at the 1%, 5%, or 10% significance levels.

Moreover, the graph also provides information on the instability of the coefficients in the regression model. Note that under the null hypothesis of no parameter instability, the expected value of the cusum of recursive residuals is 0. In the graph above, the cusum of the recursive residuals starts crossing the 99% confidence bands in the middle of the sample, which also indicates that the parameters of the regression model become unstable during that time.

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Cusum of OLS residuals

A similar test statistic is based on the cusum of OLS residuals. Under the null hypothesis, the OLS residuals are correlated and heteroskedastic. The OLS residuals have an expected value of 0 in the presence of an intercept or if the underlying process has a mean of 0 without an intercept. Their cumulative sum always returns to 0, unlike the recursive cusum process, which will drift away from 0 after a structural break.

The limiting distribution of the OLS cusum statistic under the null hypothesis is approximated by a Brownian bridge process (Ploberger and Krämer 1992). If there is a structural break at a certain time period, the absolute value of the OLS cusum statistic peaks at that time and then returns to the expected value of 0.

A plot with a constant upper bound and lower bound for a specified significance level is used as a test for parameter stability. If the cusum statistic goes outside of the bounds, we reject the null hypothesis of parameter stability. The plot also provides additional information on the timing of the structural break. Alternatively, the absolute value of the OLS cusum statistic may also be used to test the null hypothesis at a specified significance level.

▷ Example 2: OLS cusum test

Continuing with example 1, we also test the null hypothesis of parameter stability by using the cusum of OLS residuals.

```
. estat sbcusum, ols level(99)

Cumulative sum test for parameter stability

Sample: 1960q2 - 1990q4                         Number of obs  =         123
Ho: No structural break
```

Statistic	Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
ols	1.9854	1.6276	1.3581	1.224

We reject the null hypothesis of parameter stability at the 1%, 5%, and 10% levels. The graph below plots the OLS cusum process crossing the 99% band, which also implies rejection at the 1%, 5%, and 10% levels, confirming our results from the table.

OLS cusum plot of D.ln_wpi
with 99% confidence bands around the null

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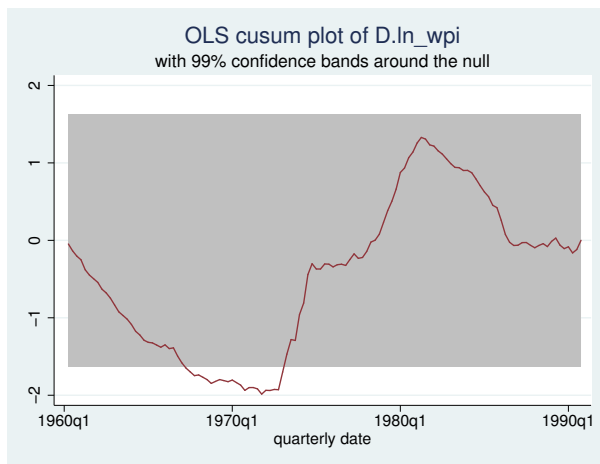
Stored results

estat sbcusum stores the following in r():

Scalars
    r(cusum)   value of the test statistic
    r(level)   confidence level of confidence intervals
    r(df)   degrees of freedom

Macros
    r(statistic) name of the statistic used; recursive or ols
    r(tmins)   formatted minimum time
    r(tmaxs)   formatted maximum time

Matrices
    r(cvalues)   vector of critical values

Methods and formulas

Consider a linear regression model with k variables,

$$y_t = \mathbf{x}_t'\boldsymbol{\beta}_t + e_t \qquad t = 1, \dots, T$$

where y_t is the dependent variable and $\mathbf{x}_t$ is a vector of covariates that may contain lags of the dependent variable. $\boldsymbol{\beta}_t$ is a vector of time-varying parameters and e_t is an independent and identically distributed error term.

The cusum test based on recursive residuals is constructed using the one-step-ahead standardized forecast error

$$e_t^{\text{rec}} = \frac{y_t - \mathbf{x}_t'\widehat{\boldsymbol{\beta}}_{t-1}}{\sqrt{1 + \mathbf{x}_t'\left(\mathbf{X}_{t-1}'\mathbf{X}_{t-1}\right)\mathbf{x}_t}} \qquad t = k+1, \dots, T$$

where $\widehat{\boldsymbol{\beta}}_{t-1}$ is the estimate of $\boldsymbol{\beta}$ for the first $t-1$ observations and $\mathbf{X}_{t-1}' = (\mathbf{x}_1, \dots, \mathbf{x}_{t-1})$. The null hypothesis for the test of parameter stability is $H_0 : \boldsymbol{\beta}_t = \boldsymbol{\beta}$, which implies constant parameters $\boldsymbol{\beta}$. Under the null hypothesis, the sequence $\{e_t^{\text{rec}}\}$ is independent and identically distributed $N(0, \sigma^2)$. The cusum statistic is formed as

$$C_t^{\text{rec}} = \frac{1}{\widehat{\sigma}} \sum_{j=k+1}^{j=t} e_j^{\text{rec}}$$

where $\widehat{\sigma}^2 = \{1/(T-k)\} \sum_{t=k+1}^{T}(e_t^{\text{rec}} - \overline{e}_t^{\text{rec}})^2$. The limiting distribution of the sequence $\{C_t^{\text{rec}}\}$ is a Brownian motion starting from 0 at time $t = k+1$ with an expected value of 0 and variance $t - k$.

Inference is based on the path of the sequence $\{C_t^{\text{rec}}\}$ crossing a theoretical boundary of a Brownian motion with probability α. Brown, Durbin, and Evans (1975) approximate the nonlinear boundaries by a pair of linear functions $\pm c\{1 + 2(t-k)/(T-k)\}$, where the value of c is obtained by solving the following equation:

$$\Phi(3c) + \exp(-4c^2)\{1 - \Phi(c)\} = 0.5\alpha$$

The test statistic is formed as

$$\max_{k+1 \leq t \leq T} \frac{|C_t^{\text{rec}}|}{1 + 2\frac{t-k}{T-k}}$$

The cusum statistic based on OLS residuals is constructed as

$$C_t^{\text{ols}} = \frac{1}{\widetilde{\sigma}\sqrt{T}} \sum_{j=1}^{j=T} e_j^{\text{ols}}$$

where e_t^{ols} is the OLS residual and $\widetilde{\sigma}^2 = \{1/(T-k)\} \sum_{t=k+1}^{T}(e_t^{\text{ols}} - \overline{e}_t^{\text{ols}})^2$. Ploberger and Krämer (1992) show the limiting distribution of the sequence $\{C_t^{\text{ols}}\}$ to be a Brownian bridge process that starts at 0 at time $t = k+1$ and ends at 0 at time T. The boundaries for the cusum statistic are approximated by a pair of parallel lines with the probability of a sample process crossing the boundaries being α.

The critical value c is obtained by solving the following equation:

$$2 \sum_{j=1}^{\infty}(-1)^{j+1} \exp(-2j^2 c^2) = \alpha$$

The test statistic is formed as the maximum of $|C_t^{\text{ols}}|$.

References

Brown, R. L., J. Durbin, and J. M. Evans. 1975. Techniques for testing the constancy of regression relationships over time. *Journal of the Royal Statistical Society, Series B* 37: 149–192.

Enders, W. 2004. *Applied Econometric Time Series.* 2nd ed. New York: Wiley.

Ploberger, W., and W. Krämer. 1992. The CUSUM test with OLS residuals. *Econometrica* 60: 271–285.

Also see

[R] **regress** — Linear regression

[TS] **estat sbknown** — Test for a structural break with a known break date

[TS] **estat sbsingle** — Test for a structural break with an unknown break date

[TS] **tsset** — Declare data to be time-series data