Frequentist and Bayesian stochastic frontier models in Stata

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Summary

Objectives of the paper

This paper focuses on stochastic frontier models

- for both cross-section and longitudinal data
- with a parametric approach to estimation

Novel features: the newly available STATA command will

- be the first bayesian estimator of frontier parameters
- be comprehensive of most used and state-of-art frequentist estimators
- make extensive use of MATA functions

General framework -1-

- Starting from seminal study by Aigner, Lovell and Schmidt (1977), theoretical literature on stochastic frontier has grown vastly.
- The range of applications of the techniques described is huge.
- The economic meaning of a frontier is to represent the best-practice technology in a production process or in a particular economic sector.
- Cost frontiers describe the minimum level of cost given a certain output level and certain input prices.
- Production frontiers represent the maximum amount of output that can be obtained from a given level of inputs.
- The gap between the actual and the maximum output is a measure of inefficiency and an important issue in many application fields, such as production studies.

General framework -2-

- A general stochastic frontier model may be written as

$$y_i = \mathbf{x}_i^{'}\beta + u_i + v_i \tag{1}$$

  where y_i is the performance of firm i (output, profits, costs), β is the vector of technology parameters, v_i is the classical symmetric disturb, while u_i is the inefficiency.
- As well as the functional assumption on the form of the frontier, we must make some assumptions on the distribution and on the relations between the two errors in order to complete the statistical model.
- The typical assumptions in this model are
  1. The independence between v e u.
  2. $v_i \sim N(0, \sigma^2)$.
  3. $u_i \sim F$, where $F(x)$ is a generic family of distributions with $x \in \mathbb{R}_+$
- Objectives: in the first step we estimate the vector of technology parameters β and in the second the efficiency of each producer.

Cross-section -1-

In a cross-sectional setting, we present two different models: the normal-truncated normal and the normal-gamma. The former one is based on the following set of assumptions

$$
\begin{aligned}
v_i &\sim \mathcal{N}(0, \sigma_{v_i}^2) \\
u_i &\sim \mathcal{N}^+(\mu_{it}, \sigma_{u_i}^2) \\
\mu_i &= \mathbf{q}_{it}\phi \\
\sigma_{v_i}^2 &= \exp(\mathbf{w}_i\delta_i) \\
\sigma_{u_i}^2 &= \exp(\mathbf{t}_i\gamma_i)
\end{aligned}
$$

The log-likelihood function for $i = 1, ..., N$ firms is

$$
\begin{aligned}
\ln \mathscr{L} &= -\frac{1}{2}\sum_i \ln\left[\exp(\mathbf{w}_i\delta_i) + \exp(\mathbf{t}_i\gamma_i)\right] - N\ln\Phi\left(-\frac{\mu_i}{\sigma_u}\right) \\
&+ \sum_i \ln\Phi\left(\frac{\mu_i}{\sigma_i\lambda} - \frac{\varepsilon_i\lambda_i}{\sigma_i}\right) - \frac{1}{2}\sum_i\left(\frac{\varepsilon_i + \mu_i}{\sigma_i}\right)^2 \qquad (2)
\end{aligned}
$$

Cross-section -2-

In the normal-gamma model $u_i \sim iid\Gamma(m)$. This formulation introduced and developed by Greene generalizes the one-parameter exponential distribution. The corresponding log-likelihood function can be written as the likelihood for the normal-exponential model plus a term which has complicated the analysis to date

$$\begin{aligned}\ln \mathscr{L} &= N\left(\frac{\sigma_v^2}{2\sigma_u^2}\right) + \sum_i \frac{\varepsilon_i}{\sigma_u} + \sum_i \ln \Phi\left[-\frac{(\varepsilon_i + \sigma_v^2/\sigma_u)}{\sigma_v}\right] \\ &+ N[(m+1)\ln\sigma_u - \ln\Gamma(m+1)] + \sum_i \ln h(m,\varepsilon_i) \\ &= \ln \mathscr{L}_{EXP} + N[(m+1)\ln\sigma_u - \ln\Gamma(m+1)] + \sum_i \ln h(m,\varepsilon_i) \qquad (3)\end{aligned}$$

where $\sum_i \ln h(m,\varepsilon_i) = E[z^r | z \geq 0]$ and $z \sim \mathscr{N}[\mu_i, \sigma_v^2]$
We estimate $h(m,\varepsilon_i)$ by using the mean of a sample of draws from a normal distribution with underlying mean μ_i and variance σ_v^2 truncated at zero.

Cross-section -3-

After technology parameters, the second step is to obtain an estimate of efficiency. For the truncated normal model we get both Jondrow, Lovell, Materov and Schmidt (1982) and Battese and Coelli (1988) estimators of technical efficiency, respectively

$$TE_i = \exp(-E\{u_i|\varepsilon_i\}) \tag{4}$$
$$TE_i = E(\exp\{-u_i\}|\varepsilon_i) \tag{5}$$

Bera and Sharma (1996) provide the formulas to get confidence intervals for these point estimators.
While for the gamma model we numerically approximate the following expression

$$E\left(u_i|\varepsilon_i\right) = \frac{h(m+1,\varepsilon_i)}{h(m,\varepsilon_i)} \tag{6}$$

where m is the shape parameter of the gamma distribution

Panel -1-

- Panel data estimation has received great coverage in the literature.
- Access to panel data enables one to avoid either strong distributional assumptions or the equally strong independence assumption.
- Latest developments in research community try to disentangle pure inefficiency from what is to be considered unobserved heterogeneity.
- Here we show the Greene (2005) “true” random effect model, the newest random effects formulations.

Panel -2-

- In its “true” random effects formulation Greene (2005) extends the conventional maximum likelihood estimation of random effects models

$$y_{it} = \alpha + \beta' \mathbf{x}_{it} + w_i + v_{it} \pm u_{it} \quad (7)$$

where w_i is the random firm specific effect and v_{it} and u_{it} are the symmetric and one sided components.

- It is necessary to integrate the common term out of the likelihood function in order to estimate this random effects model by maximum likelihood.
- Since there is no closed form for the density of the compound disturbance in this model, we integrate and simulate the log-likelihood

$$\ln \mathscr{L}_S(\beta, \lambda, \sigma, \vartheta) = \sum_{i=1}^{N} \ln \frac{1}{R} \sum_{r=1}^{R} \left[\prod_{t=1}^{T} \frac{2}{\sigma} \phi \left(\frac{\varepsilon_{it} | w_{ir}}{\sigma} \right) \Phi \left(\frac{\lambda \varepsilon_{it} | w_{ir}}{\sigma} \right) \right] \quad (8)$$

where ϑ_i are the parameters in the distribution of w_i and w_{ir} is the r-th simulated draw for observation i.

Historical notes on Bayesian estimation

- The Bayesian inference in this context was proposed by van den Broeck et al. (1994). In this work, the authors computed Bayes factors between a series of parametric models.
- Koop et al. (1997) developed Bayesian inferential procedures to be applied to panel data, distinguishing between fixed and random effects models.
- There is only one existing work (Griffin and Steel (2004, JoE)) which adopts the semiparametric Bayesian inference.
- In this work, we consider two distributions: (i) an exponential and (ii) a flexible gamma (not just an Erlang) for the vector of inefficiencies $\mathbf{u}$

Priors -1-

In order to build a Bayesian regression model, we have to define a set of priors on the unknown vector of parameters $\boldsymbol{\eta} = (\boldsymbol{\beta}, \sigma^2, v, \lambda)$. We assume the following prior structure

$$\begin{aligned}\pi(\boldsymbol{\eta}) &= \pi(\boldsymbol{\beta}, \sigma^2, v, \lambda) \\ &= \pi(\boldsymbol{\beta}|\sigma^2)\pi(\sigma^2)\pi(v)\pi(\lambda)\end{aligned}$$

where all distributions on the right-hand side will be proper, ensuring us to have a proper posterior distribution. In the exponential case $\pi(v) = 1$.

Priors -2-

- **Prior on $\boldsymbol{\beta}$**

$$\pi(\boldsymbol{\beta}|\sigma^2) \sim N_k(\beta_0, \sigma^2 W)$$

where $\beta_0 = \mathbf{0}$ and $W = d_0 I_k$. The tuning of the hyperparameter d_0 does not represent a critical point and, as reference value, we set $d_0 = 10^4$. Moreover the choice of a different reasonable large value for the d_0 should not produce a significative effect on the posterior inference.

- **Prior on σ^2**

Analogously to the previous case, we elicit the variance with the most common informative solution: an Inverse Gamma prior

$$\pi(\sigma^2) \sim IG(a_0/2, b_0/2).$$

In panel data model (Fernandez *et al.*, JoE 1997), we can relax this choice and use a non informative priors on $(\boldsymbol{\beta}, \sigma^2)$.

Priors -3-

- **Prior on** v and λ

In these two cases we choose a Gamma distribution as a prior.
In particular for λ^{-1}, if we define efficiency as $r_i = exp(-u_i)$, and adopt the prior distribution

$$\pi(\lambda^{-1}|\phi) = Ga(\phi, -\ln(r^*)),$$

then r^* is the implied prior median efficiency. We can fix $\phi = 1$ or we shall complete the prior for the general gamma inefficiency distribution by $\phi \sim Ga(1, 1)$ which is centered through the prior mean over the value leading to the exponential distribution, and has a reasonable prior variance for ϕ of unity.

Likelihood

Since the joint density of $\mathbf{y} = (y_i, ..., y_n)$ and $\mathbf{u} = (u_1, ..., u_n n)$ is given by

$$f(\mathbf{y}, \mathbf{u}) = \prod_{1=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ \frac{(y_i - x_i^{'}\beta - u_i)^2}{2\sigma^2} \right\} \times \times \frac{\lambda^{-\nu}}{\Gamma(\nu)} \cdot u_i^{\nu-1} \exp\left\{ -\frac{u_i}{\lambda} \right\} \quad (9)$$

After marginalizing over u the relation (9) the likelihood function can be expressed as

$$L(\boldsymbol{\eta}|y) \propto \prod_{1=1}^{n} \frac{\lambda^{-\nu}}{\Gamma(\nu)} \exp\left\{ \frac{\sigma^2}{2\lambda^2} + \lambda^{-1}(y_i - x_i^{'}\beta) \right\} \times \times \int_0^{+\infty} u_i^{\nu-1} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ \frac{(y_i - m_i)^2}{2\sigma^2} \right\} du_i, \quad (10)$$

where $m_i = y_i - x_i^{'}\beta - \lambda^{-1} \cdot \sigma^2$.

Posterior distribution

The posterior distribution is proportional to the product of the priors $\pi(\boldsymbol{\eta})$ and the likelihood $\pi(y|\boldsymbol{\eta})$, i.e.

$$\begin{aligned} \pi(\boldsymbol{\eta}|\mathbf{y}) &\propto \prod_{1=1}^{n} \frac{\lambda^{-\nu}}{\Gamma(\nu)} \exp\left\{ \frac{\sigma^2}{2\lambda^2} + \lambda^{-1}(y_i - h(\beta, x_i)) \right\} \times \\ &\times \int_0^{+\infty} u_i^{\nu-1} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ \frac{(y_i - m_i)^2}{2\sigma^2} \right\} du_i \\ &\times \pi(\boldsymbol{\beta}|\sigma^2)\pi(\sigma^2)\pi(\nu)\pi(\lambda). \end{aligned}$$

- The posterior is analytically intractable
- We construct a Markov chain defined by conditional distributions of parameters.
- In this Markov chain, a Gibbs sampler, the random draws are made from each full-conditional posterior distribution.
- we apply a data augmentation scheme (Tanner and Wong 1987) to our model treating the latent random vector u as an unknown parameter vector to be estimated.

We provide four new Stata commands:

- `sfcross` and `sfpanel` fit frequentist cross-sectional and panel stochastic frontier models, improving already existing commands `frontier` and `xtfrontier`.
- `bsfcross` and `bsfpanel` fit bayesian cross-sectional and panel stochastic frontier models. They are the first bayesian estimators within Stata which do not make use of WinBugs interface and the first general purpose bayesian estimators of stochastic frontier models.

The general syntax of these commands is as follows

```
sfcross  depvar  [indepvars]  [if]  [in]  [,options]
sfpanel  depvar  [indepvars]  [if]  [in]  [,options]
bsfcross  depvar  [indepvars]  [if]  [in]  [,options]
bsfpanel  depvar  [indepvars]  [if]  [in]  [,options]
```

We use Italian hospitals' data coming from Lazio region. From the Lazio Public Health Agency we got Hospital Discharge Records that were used to build output measures. From the Italian ministry of Health we received input variables such as number of beds, physicians, etc. We limit our analysis to

- Acute care hospitals, since rehabilitation care and long-term care serve very different production functions
- Public and not-for-profit hospitals. While for private hospitals we study only their activity which is publicly financed.
- Years between 2000 and 2005, which represents an interesting period to assess the effect of DRG system.
- Overall we have a weakly balanced panel of 625 observations

```
. d ‘MainVariables’

              storage  display     value
variable name   type   format      label      variable label
-----------------------------------------------------------------------------------------
lnorm_weighta   float  %9.0g                  Sum of DRG weights in acute care
alpha1          float  %9.0g                  # beds
alpha2          float  %9.0g                  # physicians
alpha3          float  %9.0g                  # nurses
alpha4          float  %9.0g                  # other workers
alpha11         float  %9.0g                  Squared # beds
alpha22         float  %9.0g                  Squared # physicians
alpha33         float  %9.0g                  Squared # nurses
alpha44         float  %9.0g                  Squared # other workers
alpha12         float  %9.0g                  Interaction # beds - # physicians
alpha13         float  %9.0g                  Interaction # beds - # nurses
alpha14         float  %9.0g                  Interaction # beds - # other workers
alpha23         float  %9.0g                  Interaction # physicians - # nurses
alpha24         float  %9.0g                  Interaction # physicians - # other workers
alpha34         float  %9.0g                  Interaction # nurses - # other workers
dyear2001       byte   %9.0g                  Time dummy: 2001
dyear2002       byte   %9.0g                  Time dummy: 2002
dyear2003       byte   %9.0g                  Time dummy: 2003
dyear2004       byte   %9.0g                  Time dummy: 2004
dyear2005       byte   %9.0g                  Time dummy: 2005
```

Frequentist paradigm: Cross-section truncated-normal model

Conditional mean model with explanatory variables for idiosyncratic error variance function

```
#delimit; . sfcross $Y $Xlin $Xsq $Xint $dY, d(tn) mu(private1 equip, nocons)
                                 v(lnorm_beds) technique(nr) nolog;
#delimit cr

Truncated-normal distribution of u                     Number of obs    =        625
                                                       Wald chi2(19)    =    4714.45
Log-likelihood =      -293.3616                        Prob > chi2      =     0.0000

------------------------------------------------------------------------------
lnorm_wei~ta |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Frontier     |
      alpha1 |   .7589718   .0462538    16.41   0.000     .6683162    .8496275
      alpha2 |   .2464217   .0429955     5.73   0.000     .1621521    .3306914
      alpha3 |   .0131866   .0509452     0.26   0.796    -.0866642    .1130374
      alpha4 |  -.0281444   .0411581    -0.68   0.494    -.1088128    .0525241
     alpha11 |   .2855433   .0689453     4.14   0.000      .150413    .4206735
     alpha22 |   .1145137   .0385944     2.97   0.003     .0388701    .1901574
     alpha33 |    .066102   .0572529     1.15   0.248    -.0461116    .1783155
     alpha44 |  -.0317559   .0288189    -1.10   0.270    -.0882398     .024728
     alpha12 |  -.1387485   .0655815    -2.12   0.034    -.2672858   -.0102112
```

Frequentist paradigm: Cross-section truncated-normal model

Conditional mean model with explanatory variables for idiosyncratic error variance function

```
     alpha13 |   .0718528   .0605002     1.19   0.235    -.0467254    .1904311
     alpha14 |  -.1007611   .0421255    -2.39   0.017    -.1833255   -.0181966
     alpha23 |  -.0491069   .0352093    -1.39   0.163    -.1181159    .0199021
     alpha24 |    .080828   .0493739     1.64   0.102    -.0159431    .1775992
     alpha34 |   -.063915   .0421846    -1.52   0.130    -.1465954    .0187653
   dyear2001 |   .0455823   .0475237     0.96   0.337    -.0475625     .138727
   dyear2002 |   .1053177   .0472129     2.23   0.026     .0127822    .1978532
   dyear2003 |   .1838898   .0474158     3.88   0.000     .0909565     .276823
   dyear2004 |   .2339058   .0484147     4.83   0.000     .1390147     .328797
   dyear2005 |   .3354143   .0503873     6.66   0.000     .2366571    .4341715
       _cons |   .5071762   .0386438    13.12   0.000     .4314357    .5829166
-------------+----------------------------------------------------------------
MU           |
    private1 |   .0240676   .1303645     0.18   0.854    -.2314421    .2795773
       equip |  -2.355163   .6548962    -3.60   0.000    -3.638736    -1.07159
-------------+----------------------------------------------------------------
Usigma       |
       _cons |  -1.053533   .1170353    -9.00   0.000    -1.282918   -.8241479
-------------+----------------------------------------------------------------
Vsigma       |
  lnorm_beds |  -.5168447   .0965719    -5.35   0.000    -.7061222   -.3275671
       _cons |  -2.813833   .1221077   -23.04   0.000     -3.05316   -2.574506
------------------------------------------------------------------------------
H0: No inefficiency component:          z = -36.593         Prob<=z = 0.000
------------------------------------------------------------------------------
```

Frequentist paradigm: Cross-section gamma model

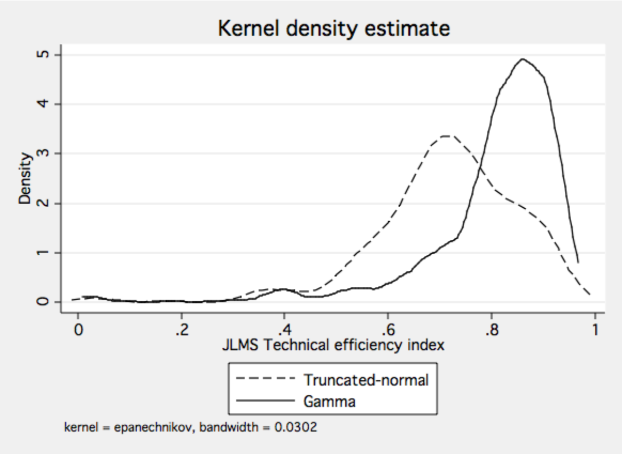
```
sfcross $Y $Xlin $Xsq $Xint $dY, d(g) nsim(100) simtype(3) base(7) technique(bhhh)

Gamma distribution of u                              Number of obs    =       625
                                                     Wald chi2(19)    =   6893.78
Simulated Log-likelihood    =     -235.3336          Prob > chi2      =    0.0000

Number of Randomized Halton Sequences =       100
Base for Randomized Halton Sequences  =         7
------------------------------------------------------------------------------
lnorm_wei~ta |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Frontier     |
      alpha1 |   .8352786   .0393152    21.25   0.000     .7582222     .912335
      alpha2 |   .2293403   .0352901     6.50   0.000      .160173    .2985076
      alpha3 |  -.0501789    .034228    -1.47   0.143    -.1172644    .0169067
      alpha4 |    .011842   .0320268     0.37   0.712    -.0509294    .0746134
     alpha11 |   .3175812   .0516937     6.14   0.000     .2162635    .4188989
     alpha22 |    .133669   .0285461     4.68   0.000     .0777197    .1896183
     alpha33 |  -.0231349   .0315887    -0.73   0.464    -.0850476    .0387778
     alpha44 |  -.0272225   .0195063    -1.40   0.163    -.0654542    .0110093
```

(*Continued on next page*)

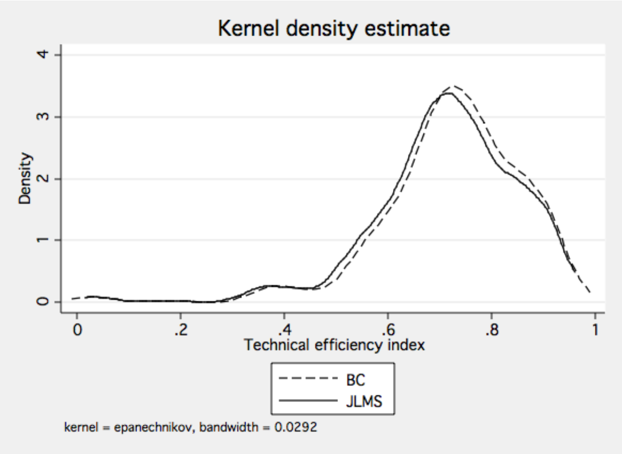
Frequentist paradigm: Cross-section gamma model

```
     alpha12 |  -.1943535   .0452281    -4.30   0.000    -.2829988   -.1057081
     alpha13 |   .1351881   .0378178     3.57   0.000     .0610665    .2093096
     alpha14 |  -.1121965    .030874    -3.63   0.000    -.1727085   -.0516846
     alpha23 |   -.016129   .0214855    -0.75   0.453    -.0582398    .0259819
     alpha24 |   .0785029   .0325432     2.41   0.016     .0147194    .1422864
     alpha34 |  -.0598163   .0262488    -2.28   0.023     -.111263   -.0083697
   dyear2001 |   .0482466    .044134     1.09   0.274    -.0382544    .1347477
   dyear2002 |   .1244262   .0443071     2.81   0.005     .0375858    .2112665
   dyear2003 |   .1814734   .0438916     4.13   0.000     .0954474    .2674993
   dyear2004 |   .2517909    .044257     5.69   0.000     .1650489     .338533
   dyear2005 |   .3423928   .0455804     7.51   0.000     .2530568    .4317288
       _cons |   .3063856   .0339053     9.04   0.000     .2399323    .3728388
-------------+----------------------------------------------------------------
Theta        |
       theta |   2.425863   .2213349    10.96   0.000     1.992055    2.859672
-------------+----------------------------------------------------------------
Vsigma2      |
     sigma2v |   .0614984   .0050543    12.17   0.000     .0515921    .0714047
-------------+----------------------------------------------------------------
Shape        |
           m |   .5028515   .0427102    11.77   0.000     .4191411    .5865619
------------------------------------------------------------------------------
H0: No inefficiency component:            z = -36.593          Prob<=z = 0.000
------------------------------------------------------------------------------
```

Gamma vs Truncated-normal JLMS technical efficiency estimates



Truncated-normal technical efficiency estimates: JLMS vs BC estimator



Bayesian paradigm: Cross-section exponential model

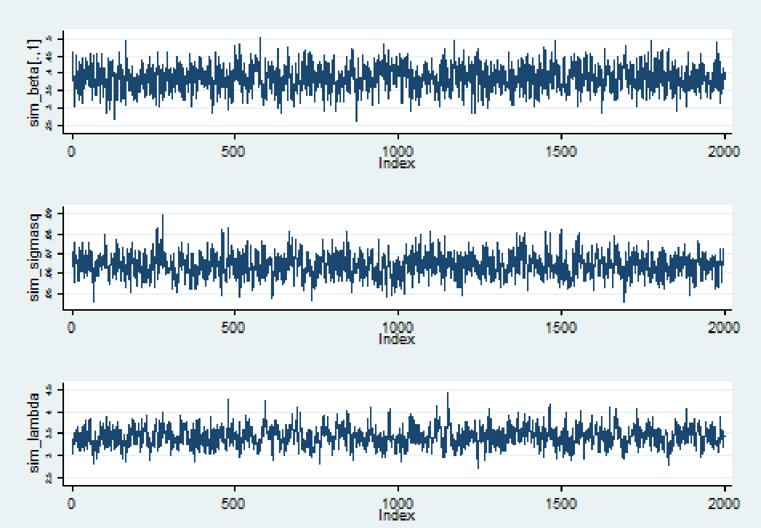
```
. bsfcross $Y $Xlin $Xsq $Xint $dY, d(exp) iteration(2000) burnin(200) thin(2) pred(4)

Bayesian Stochastic frontier - Exponential distribution of u

Prior hyperparameters: Sigma2--> a: 1 b: 1
                       Lambda--> a: 1 b: .2231436

Settings: Iterations: 2000 Burnin:     200 Thinning:   2

------------------------------------------------------------------------------
  lnorm_weighta |       Mean     Std.Dev. |       p25      Median        p75
----------------+-------------------------+-----------------------------------+
         alpha1 |   .3860493    .0368384 |   .3614341    .3854457    .4103777
         alpha2 |    .832221    .0399031 |   .8047699    .8319192     .859648
         alpha3 |    .232736    .0373776 |    .206227    .2330651    .2585723
         alpha4 |  -.0420527    .0398248 |  -.0688815   -.0420544   -.0145645
```

(*Continued on next page*)

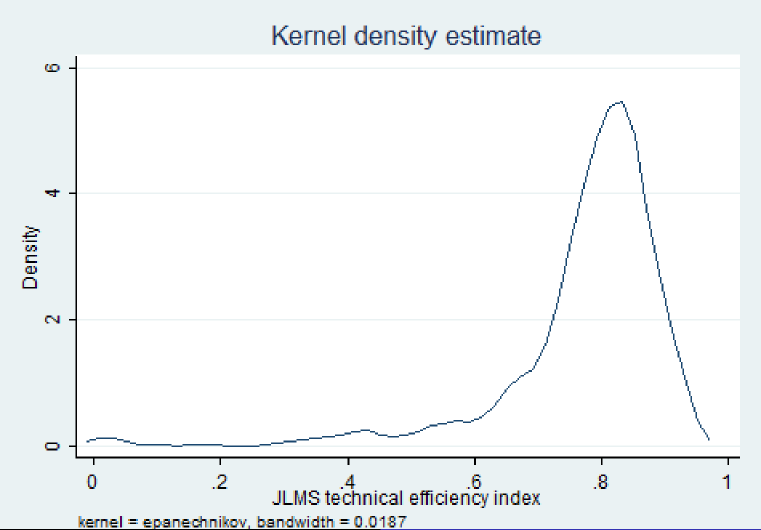
Bayesian paradigm: Cross-section exponential model

alpha11	.0047307	.0368207	-.0205384	.0045468	.0295354
alpha22	.32764	.0576184	.2887665	.3281395	.3641831
alpha33	.1367023	.0348657	.1131869	.1363166	.1605194
alpha44	-.0106984	.046727	-.0407787	-.0111242	.0191945
alpha12	-.0215921	.0257557	-.0391617	-.0232286	-.0057145
alpha13	-.1965854	.0542004	-.2336578	-.1963845	-.15888
alpha14	.1346051	.0508769	.1011163	.1350112	.168994
alpha23	-.114953	.0387588	-.1402465	-.1144225	-.0881108
alpha24	-.0207061	.0330627	-.042861	-.0210401	.0012065
alpha34	.0802777	.0441296	.0502374	.0792411	.1107116
dyear2001	-.0660719	.0359892	-.0899417	-.0658088	-.0412263
dyear2002	.050062	.0475268	.0179117	.0521054	.0816535
dyear2003	.1228649	.0469407	.0919335	.1234543	.1539608
dyear2004	.1850392	.0473779	.1528647	.1856285	.2175337
dyear2005	.2561303	.0467022	.2240115	.2562613	.2885856
_cons	.3529473	.0478826	.3218152	.3519741	.3846548
sigma2	.0638855	.0058361	.0597791	.063804	.0677512
lambda	3.432143	.2247785	3.287601	3.425935	3.576674

Parameters' simulation



Bayesian cross-section estimate of technical efficiency: “mean” JLMS



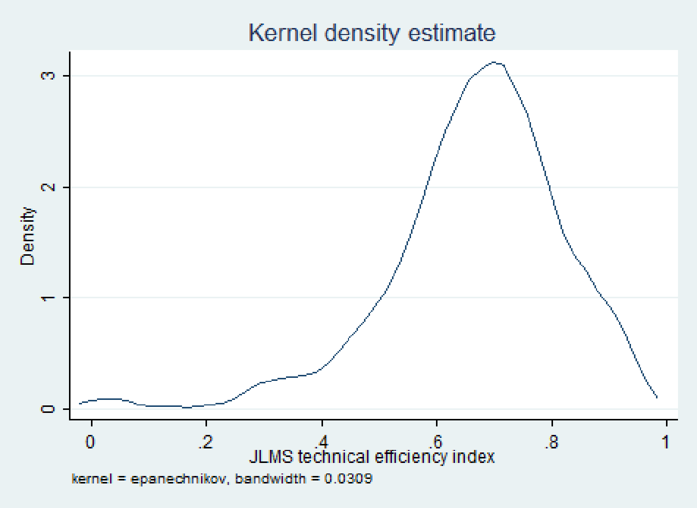
Frequentist paradigm: “True” RE model

```
#delimit; sfpanel $Y $Xlin $Xsq $Xint $dY, model(tre) id(irc_id)
    time(year) simtype(3) base(37) nsim(10) technique(dfp) nolog;
#delimit cr

True Random Effects model (Half-Normal)              Number of obs    =       625
Group variable: irc_id                               Number of groups =       113
                                                   Obs per group: min =         1
                                                                  avg =       5.5
                                                                  max =         6

Simulated Log-likelihood               = -327.7370
Number of Randomized Halton Sequences =        10
Base for Randomized Halton Sequences  =        37
------------------------------------------------------------------------------
             |                Standard
lnorm_wei~ta |      Coef.   Std. Err.       t     P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Frontier     |
      alpha1 |    .770435   .0495812    15.54   0.000     .6730184    .8678515
      alpha2 |   .2372707   .0455114     5.21   0.000     .1478504     .326691
      alpha3 |  -.0276846    .045677    -0.61   0.545    -.1174301    .0620609
      alpha4 |   .0393227   .0448968     0.88   0.382      -.04889    .1275355
     alpha11 |   .3370867   .0611261     5.51   0.000     .2169868    .4571865
```

Frequentist paradigm: “True” RE model

```
     alpha22 |   .1252176    .039408     3.18   0.002     .0477893    .2026459
     alpha33 |   .0520339   .0543918     0.96   0.339    -.0548344    .1589021
     alpha44 |  -.0196035    .031324    -0.63   0.532    -.0811486    .0419415
     alpha12 |  -.1892795   .0601836    -3.15   0.002    -.3075275   -.0710315
     alpha13 |   .1283317   .0527173     2.43   0.015     .0247534    .2319099
     alpha14 |  -.1229131   .0416444    -2.95   0.003    -.2047355   -.0410906
     alpha23 |  -.0560559   .0346994    -1.62   0.107    -.1242329    .0121211
     alpha24 |    .112425   .0459849     2.44   0.015     .0220744    .2027756
     alpha34 |  -.0855912   .0394955    -2.17   0.031    -.1631915   -.0079908
   dyear2001 |   .0556283   .0520666     1.07   0.286    -.0466715     .157928
   dyear2002 |   .1235663   .0515067     2.40   0.017     .0223665    .2247661
   dyear2003 |   .1855624   .0517561     3.59   0.000     .0838727    .2872522
   dyear2004 |   .2702399   .0530714     5.09   0.000     .1659659    .3745138
   dyear2005 |    .385845   .0563583     6.85   0.000      .275113     .496577
       _cons |   .5347613   .0404681    13.21   0.000       .45525    .6142726
-------------+----------------------------------------------------------------
Lambda       |
      lambda |   3.070895    .295635    10.39   0.000     2.490035    3.651755
-------------+----------------------------------------------------------------
Sigma        |
       sigma |   .6469433   .0232561    27.82   0.000     .6012499    .6926367
-------------+----------------------------------------------------------------
Theta        |
       theta |  -.1203508   .0187606    -6.42   0.000    -.1572113   -.0834902
------------------------------------------------------------------------------
```

True RE technical efficiency estimates: JLMS



Bayesian paradigm: Panel exponential model

```
#delimit;
. bsfpanel $Y $Xlin $Xsq $Xint $dY, id(irc_id) time(year) d(exp)
    iteration(2000) thin(2) vid pred(5);
#delimit cr

Bayesian Stochastic frontier - Exponential distribution of u

Prior hyperparameters:
Sigma2--> a: 1 b: 1
Lambda--> a: 1 b: .2231436

Settings:
Iterations: 2000
Burnin:     200
Thinning:   2

-------------------------------------------------------------------------------
  lnorm_weighta |       Mean     Std.Dev. |        p25       Median         p75
----------------+-------------------------+------------------------------------
         alpha1 |   .6160194     .0505232 |   .5817509     .6153182    .6498718
         alpha2 |    .592604     .0557423 |   .5555525     .5935681    .6283623
         alpha3 |   .1546137     .0452847 |   .1226226     .1546886    .1850767
         alpha4 |   .0814004     .0451653 |   .0509097     .0804234    .1114784
```

Bayesian paradigm: Panel exponential model

alpha11	.0525024	.0323081	.030402	.0533914	.0748135
alpha22	.4272994	.0613689	.3867003	.4296765	.4691825
alpha33	.1586183	.0362113	.1348455	.1582763	.1820672
alpha44	.0877115	.0377077	.061082	.0867714	.1136633
alpha12	.0115459	.0181532	-.0010353	.0113679	.0236347
alpha13	-.1984756	.0557347	-.2370119	-.197949	-.1616747
alpha14	.0472381	.0496899	.0143411	.0472077	.0805553
alpha23	-.0555072	.0316713	-.0761171	-.0553523	-.0346504
alpha24	-.0805221	.0245477	-.096861	-.0801524	-.0645422
alpha34	.0525304	.0350739	.0296966	.0515689	.0756384
dyear2001	-.0443351	.0292965	-.0635751	-.0437169	-.0251153
dyear2002	.0661587	.0306075	.0449388	.0666169	.0872127
dyear2003	.1066004	.0305289	.0853737	.106302	.1277608
dyear2004	.1792789	.0319992	.1564392	.1788834	.2013099
dyear2005	.2402727	.0316348	.218862	.2404739	.2618161
_cons	.2884428	.0324821	.2671985	.2879481	.3103192
sigma2	.0450103	.0030748	.0428612	.0448632	.0469912
lambda	1.869713	.2346999	1.712583	1.854712	2.023131

Bayesian panel estimate of technical efficiency: "mean" JLMS