Estimation of pre– and post–treatment Average Treatment Effects (ATEs) with binary time-varying treatment using Stata

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Outline

1. Motivations
2. Our contribution
3. The econometric set up
4. Testing for the parallel trend assumption
5. the Stata syntax of `ddid`
6. An application on simulated data
7. Further developments

Motivations (1)

- Main question: are public policy programs effective?
- If yes how long and to what extent?
- Fundamental problem: treated individuals not randomly selected but rather self-selected
- (possible) solution: recovering the Average Treatment Effect (ATE) from panel data, Diff-in-Diff.

Our contribution

THE AIM OF THE WORK IS:

- to provide a Stata routine, `ddid`, which implements a generalization of the Difference-In-Differences (DID) estimator
- to provide a user friendly Stata routine to estimate the pre– and post–intervention effects
- to implement diagnostic tests for the parallel trend assumption
- to facilitate provide useful means for plotting the results in a easy-to-read graphical representation

Istat

The econometric set up (1)

Let us consider a binary treatment indicator

$$D_{it} = \begin{cases} 1 \text{ if unit } i \text{ is treated at time } t \\ 0 \text{ if unit } i \text{ is treated at time } t \end{cases}$$

and an outcome equation with **contemporaneous** treatment plus **lags** and **leads**

$$Y_{it} = \mu_{it} + \beta_{-1} D_{it-1} + \beta_0 D_{it} + \beta_{+1} D_{it+1} + \gamma \mathbf{x}_{it} + u_{it} \tag{1}$$

the β_{+1} coefficient measures the impact of the treatment one period before the treatment occurred and β_{-1} measures the impact of treatment one period after the treatment occurred.

The econometric set up (2)

let us assume that treatment can occur only once over the interval $[t-1, t+1]$ so that we can define the following sequences of possible treatments:

$$\{w^j\} = \{D_{it-1}, D_{it}, D_{it+1}\} = \begin{cases} w^1 = (0,0,0) \\ w^2 = (1,0,0) \\ w^3 = (0,1,0) \\ w^4 = (0,0,1) \end{cases}$$

The sequence w^1 is the usual benchmark of no–treatment.

The generic treatment sequence is indicated by w^j (with $j = 1, \cdots, 4$) and the associated potential outcome as $Y(w^j)$.

The "Average Treatment Effect between two potential outcomes, w^j and w^k $Y(w^j)$ and $Y(w^k)$" is defined as:

$$\mathrm{ATE}_{jk} = \mathrm{E}[Y_{it}(w^j) - Y_{it}(w^k)] \quad \forall\ (i, t) \tag{2}$$

The econometric set up (3)

with treatment occurring only in one period out of three, and one lag and one lead we can define six possible ATEs:

$$\begin{bmatrix} & w_1 & w_2 & w_3 & w_4 \\ w_1 & - & & & \\ w_2 & \text{ATE}_{21} & - & & \\ w_3 & \text{ATE}_{31} & \text{ATE}_{32} & - & \\ w_4 & \text{ATE}_{41} & \text{ATE}_{42} & \text{ATE}_{43} & - \end{bmatrix}$$

The generic ATE_{ij} represents the ATE of the sequence i against the counterfactual sequence j. Obviously $ATE_{ij} = -ATE_{ji}$.

Using equation (1) and the definition of w^j, with $j = 1, \ldots, 4$, it is possible to rewrite the ATEs

$$\begin{aligned}
\mathrm{ATE}_{21} &= \mathrm{E}(Y_{it}|w_2) - \mathrm{E}(Y_{it}|w_1)] = (\bar{\mu} + \beta_{-1} + \gamma\bar{\mathbf{x}}) - (\bar{\mu} + \gamma\bar{\mathbf{x}}) = \beta_{-1} \\
\mathrm{ATE}_{31} &= \mathrm{E}(Y_{it}|w_3) - \mathrm{E}(Y_{it}|w_1)] = \beta_0 \\
\mathrm{ATE}_{41} &= \mathrm{E}(Y_{it}|w_4) - \mathrm{E}(Y_{it}|w_1)] = \beta_{+1} \\
\mathrm{ATE}_{32} &= \mathrm{E}(Y_{it}|w_3) - \mathrm{E}(Y_{it}|w_2)] = \beta_0 - \beta_{-1} \\
\mathrm{ATE}_{42} &= \mathrm{E}(Y_{it}|w_4) - \mathrm{E}(Y_{it}|w_2)] = \beta_{+1} - \beta_{-1} \\
\mathrm{ATE}_{43} &= \mathrm{E}(Y_{it}|w_4) - \mathrm{E}(Y_{it}|w_3)] = \beta_{+1} - \beta_0
\end{aligned}$$

The ATEs have a straightforward interpretation:

The econometric set up (5)

- $\beta_{+1} \neq 0$. Treatment delivered at t affects the outcome at $t-1$. Current treatment has an effect on past outcome (anticipatory effect). Therefore, the pre-treatment period is affected by the current treatment.
- $\beta_0 \neq 0$. Treatment delivered at t affects the outcome at t, simultaneous effect.
- $\beta_{-1} \neq 0$. Treatment delivered at t affects the outcome at $t+1$. Current treatment has an effect on future outcomes (lagged effect). Therefore, the post–treatment period is affected by current treatment.

Parallel trend assumption: Test 1

In the spirit of Granger (1969) if D_{it} causes Y_{it} ==>, $\beta_{+j} = 0$ for $j = 1, \ldots, J$ in an equation like (1). **NO anticipatory effects**

$$H_0 : \beta_{+1} = \beta_{+2} = \cdots = \beta_{+J} = 0 \tag{3}$$

BEWARE: rejecting H_0 would invalidate the causal interpretation of the estimates, but ...
not rejecting H_0 implies only that a necessary condition for the parallel trend assumption holds.
The necessary and sufficient condition still remains untestable being formulated on counterfactual unobservable quantities.

parallel trend assumption: Test 2

Another way to test for the necessary condition of the parallel trend ass.tion

Drop lags and leads from equation (1) and augment it with the time trend variable t, and the interaction between D_{it} and t.

If the coefficient of the interaction term turns out to be statistically equal to zero, one can reasonably expect the parallel trend to hold.

See Angrist and Pischke (2009, pp. 238–239)

parallel trend assumption: Test 2

Proof: let us write down the following potential outcome model:

$$\begin{cases} Y_{0,it} = \mu_0 + \lambda_0 t + \gamma \mathbf{x}_{it} + \theta_i + u_{0,it} \\ Y_{1,it} = \mu_1 + \lambda_1 t + \gamma \mathbf{x}_{it} + \theta_i + u_{1,it} \\ Y_{it} = Y_{0,it} + D_{it} \left(Y_{1,it} - Y_{0,it} \right) \end{cases}$$

By substituting the first two equations into the third, we obtain:

$$Y_{it} = \mu_0 + \lambda_0 t + \gamma \mathbf{x}_{it} + D_{it}(\mu_1 - \mu_0) + D_{it} t(\lambda_1 - \lambda_0) + \theta_i + \eta_{it}$$

with $\eta_{it} = [u_{0,it} + D_{it} \left(u_{1,it} - u_{0,it} \right)]$.

parallel trend assumption: Test 2

in a more compact form:

$$Y_{it} = \mu_0 + \lambda_0 t + \gamma \mathbf{x}_{it} + D_{it}\mu + D_{it} t \cdot \lambda + \theta_i + \eta_{it} \tag{4}$$

estimable by FE, and the following test can be performed:

$H_0 : \lambda = 0$

if H_0 is accepted, we can reasonably hold that the (necessary condition for the) parallel trend assumption is satisfied.
This test can be generalized assuming also quadratic or cubic time trend.

The Stata syntax of ddid (1)

ddid *outcome treatment* [*varlist*] [*if*] [*in*] [*weight*], **model**(*modeltype*) **pre**(#) **post**(#) [**test_tt graph save_graph**(*graphname*) **vce**(*vcetype*)]

*fweight*s, *iweight*s, and *pweight*s are allowed;
where:

- *outcome*: is the target variable over which measuring the impact of the treatment.
- *treatment*: is the binary treatment variable taking 1 for treated, and 0 for untreated units.
- *varlist*: is the set of pre-treatment (or observable confounding) variables.

The Stata syntax of ddid (2)

Options

- **model**(*modeltype*) specifies the estimation model, where modeltype must be one out of these two alternatives: "fe" (fixed effects), or "ols" (ordinary least squares). It is always required to specify one model.
- **pre**(#) allows to specify the number (#) of pre-treatment periods.
- **post**(#) allows to specify the number (#) of post-treatment periods.
- **test_tt** allows for performing the parallel–trend test using the time–trend approach. The default is to use the leads.
- **graph** allows for a graphical representation of results. It uses the coefplot command implemented by Jann (2014).
- **save_graph**(*graphname*) permits to save the graph as *graphname*.
- **vce**(*vcetype*) allows for robust and clustered regression standard errors in model's estimates.

ddid creates a number of variables:

_D_L1,...,**_D_Lm**: are the lags of the treatment variable, with *m* equal to # in the **post**(#) option.

_D_F1,...,**_D_Fp**: are the leads of the treatment variable, with *p* equal to # in the **pre**(#) option

and returns the following scalars:

`e(N)` is the total number of (used) observations.
`e(N1)` is the number of (used) treated units.
`e(N0)` is the number of (used) untreated units.
`e(ate)` is the value of the (contemporaneous) ATE.

REMEMBER: (i) the treatment has to be a 0/1 binary variable;
(ii) before running **ddid**, one has to install the **coefplot** user–written Stata command (Jann, 2014).

An application on simulated data (1)

```
. clear
. set obs 5
. set seed 10101
. gen id=_n
. expand 50
. drop in 1/5
. bys id: gen time=_n+1999
. gen D=rbinomial(1,0.4)
. gen x1=rnormal(1,7)
. tsset id time
  forvalues i=1/6{
  gen L`i'_x=L`i'.x1
  }
```

An application on simulated data (2)

```
bys id: gen y0=5+1*x+ rnormal()
bys id: gen y1=100+5*x+90*L1_x+90*L2_x+120*L3_x+100*L4_x+ ///
90*L5_x+90*L6_x+rnormal()

gen A=6*x+rnormal()
replace D=1 if A>=15
replace D=0 if A<15
gen y=y0+D*(y1-y0)

tsset id time
xi: ddid y D x, model(fe) pre(6) post(6) vce(robust) graph test_tt
```

An application on simulated data (3)

```
     between = 0.3825                                         avg =       37.0
     overall = 0.3865                                         max =         37

                                                F(4,4)            =          .
corr(u_i, Xb)  = -0.1395                        Prob > F          =          .

                                   (Std. Err. adjusted for 5 clusters in id)
------------------------------------------------------------------------------
             |               Robust
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        _D_F6 |  -72.46167   180.2637    -0.40   0.708    -572.9538    428.0305
        _D_F5 |   60.09743   63.70172     0.94   0.399    -116.7669    236.9618
        _D_F4 |   22.75842   278.3109     0.08   0.939    -749.9566    795.4735
        _D_F3 |  -143.4906   158.6749    -0.90   0.417    -584.0427    297.0614
        _D_F2 |   395.8175   257.9226     1.53   0.200    -320.2905    1111.926
        _D_F1 |  -87.25493   186.4611    -0.47   0.664    -604.9539     430.444
            D |   805.2605   215.0375     3.74   0.020     208.2206      1402.3
        _D_L1 |   323.9564   123.6558     2.62   0.059    -19.36713    667.2799
        _D_L2 |   595.6533   183.0706     3.25   0.031     87.36797    1103.939
        _D_L3 |   494.4453   123.5111     4.00   0.016     151.5236     837.367
        _D_L4 |   446.2026   156.2224     2.86   0.046     12.45967    879.9456
        _D_L5 |   499.1779   211.6438     2.36   0.078    -88.43944    1086.795
        _D_L6 |    301.861   73.76806     4.09   0.015     97.04799    506.6739
           x1 |  -9.918519   18.52512    -0.54   0.621    -61.35249    41.51545
        _cons |  -1107.346   307.6728    -3.60   0.023    -1961.582   -253.1091
-------------+----------------------------------------------------------------
      sigma_u |  161.27232
      sigma_e |  935.12866
          rho |   .0288834   (fraction of variance due to u_i)
------------------------------------------------------------------------------
```

An application on simulated data (4)

```
****************************************************************
******* Test for 'parallel trend' using the 'leads' ***********
****************************************************************
 ( 1)  _D_F6 = 0
 ( 2)  _D_F5 = 0
 ( 3)  _D_F4 = 0
 ( 4)  _D_F3 = 0
 ( 5)  _D_F2 = 0
 ( 6)  _D_F1 = 0
       Constraint 2 dropped
       Constraint 6 dropped

       F(  4,     4) =    0.42
            Prob > F =    0.7875

RESULT: 'Parallel-trend' passed
****************************************************************
```

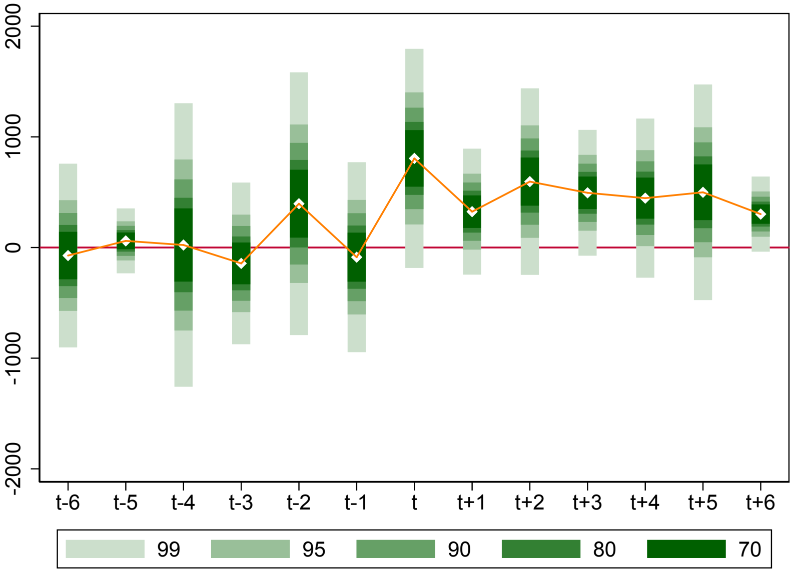
An application on simulated data (5)

```
******************************************************
******* Test for 'parallel trend' using the 'time-trend' *****
******************************************************
 ( 1)  _DT = 0

       F(  1,     4) =    1.44
            Prob > F =    0.2961

RESULT: 'Parallel-trend' passed
******************************************************
```

An application on simulated data (6)



An application on simulated data (7)

The option `graph` provides a graphical representation of the results plotting the lags and leads coefficients with 99, 95, 90, 80, and 70 confidence intervals.

The pre–treatment pattern lays around zero

The post–treatment pattern shows the positive effect of the (simulated) policy with a value laying around 500.

Assuming the sufficient condition of parallel trend to hold, one can conclude that this policy has generated positive effects.

Further developments

1. non binary treatment;
2. more than one treatment over the sequence w^j, with $j = 1, \ldots, 4$;