

STATA BASE REFERENCE MANUAL

RELEASE 19



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Cross-referencing the documentation

When reading this manual, you will find references to other Stata manuals, for example, [U] [27 Overview of Stata estimation commands](#); [R] [regress](#); and [D] [reshape](#). The first example is a reference to chapter 27, *Overview of Stata estimation commands*, in the *User's Guide*; the second is a reference to the [regress](#) entry in the *Base Reference Manual*; and the third is a reference to the [reshape](#) entry in the *Data Management Reference Manual*.

All the manuals in the Stata Documentation have a shorthand notation:

[GSM]	<i>Getting Started with Stata for Mac</i>
[GSU]	<i>Getting Started with Stata for Unix</i>
[GSW]	<i>Getting Started with Stata for Windows</i>
[U]	<i>Stata User's Guide</i>
[R]	<i>Stata Base Reference Manual</i>
[ADAPT]	<i>Stata Adaptive Designs: Group Sequential Trials Reference Manual</i>
[BAYES]	<i>Stata Bayesian Analysis Reference Manual</i>
[BMA]	<i>Stata Bayesian Model Averaging Reference Manual</i>
[CAUSAL]	<i>Stata Causal Inference and Treatment-Effects Estimation Reference Manual</i>
[CM]	<i>Stata Choice Models Reference Manual</i>
[D]	<i>Stata Data Management Reference Manual</i>
[DSGE]	<i>Stata Dynamic Stochastic General Equilibrium Models Reference Manual</i>
[ERM]	<i>Stata Extended Regression Models Reference Manual</i>
[FMM]	<i>Stata Finite Mixture Models Reference Manual</i>
[FN]	<i>Stata Functions Reference Manual</i>
[G]	<i>Stata Graphics Reference Manual</i>
[H2OML]	<i>Machine Learning in Stata Using H2O: Ensemble Decision Trees Reference Manual</i>
[IRT]	<i>Stata Item Response Theory Reference Manual</i>
[LASSO]	<i>Stata Lasso Reference Manual</i>
[XT]	<i>Stata Longitudinal-Data/Panel-Data Reference Manual</i>
[META]	<i>Stata Meta-Analysis Reference Manual</i>
[ME]	<i>Stata Multilevel Mixed-Effects Reference Manual</i>
[MI]	<i>Stata Multiple-Imputation Reference Manual</i>
[MV]	<i>Stata Multivariate Statistics Reference Manual</i>
[PSS]	<i>Stata Power, Precision, and Sample-Size Reference Manual</i>
[P]	<i>Stata Programming Reference Manual</i>
[RPT]	<i>Stata Reporting Reference Manual</i>
[SP]	<i>Stata Spatial Autoregressive Models Reference Manual</i>
[SEM]	<i>Stata Structural Equation Modeling Reference Manual</i>
[SVY]	<i>Stata Survey Data Reference Manual</i>
[ST]	<i>Stata Survival Analysis Reference Manual</i>
[TABLES]	<i>Stata Customizable Tables and Collected Results Reference Manual</i>
[TS]	<i>Stata Time-Series Reference Manual</i>
[I]	<i>Stata Index</i>
[M]	<i>Mata Reference Manual</i>

Description

This entry describes the organization of the reference manuals.

Remarks and examples

The complete list of reference manuals is as follows:

[R]	<i>Stata Base Reference Manual</i>
[ADAPT]	<i>Stata Adaptive Designs: Group Sequential Trials Reference Manual</i>
[BAYES]	<i>Stata Bayesian Analysis Reference Manual</i>
[BMA]	<i>Stata Bayesian Model Averaging Reference Manual</i>
[CAUSAL]	<i>Stata Causal Inference and Treatment-Effects Estimation Reference Manual</i>
[CM]	<i>Stata Choice Models Reference Manual</i>
[D]	<i>Stata Data Management Reference Manual</i>
[DSGE]	<i>Stata Dynamic Stochastic General Equilibrium Models Reference Manual</i>
[ERM]	<i>Stata Extended Regression Models Reference Manual</i>
[FMM]	<i>Stata Finite Mixture Models Reference Manual</i>
[FN]	<i>Stata Functions Reference Manual</i>
[G]	<i>Stata Graphics Reference Manual</i>
[IRT]	<i>Stata Item Response Theory Reference Manual</i>
[LASSO]	<i>Stata Lasso Reference Manual</i>
[XT]	<i>Stata Longitudinal-Data/Panel-Data Reference Manual</i>
[META]	<i>Stata Meta-Analysis Reference Manual</i>
[ME]	<i>Stata Multilevel Mixed-Effects Reference Manual</i>
[MI]	<i>Stata Multiple-Imputation Reference Manual</i>
[MV]	<i>Stata Multivariate Statistics Reference Manual</i>
[PSS]	<i>Stata Power, Precision, and Sample-Size Reference Manual</i>
[P]	<i>Stata Programming Reference Manual</i>
[RPT]	<i>Stata Reporting Reference Manual</i>
[SP]	<i>Stata Spatial Autoregressive Models Reference Manual</i>
[SEM]	<i>Stata Structural Equation Modeling Reference Manual</i>
[SVY]	<i>Stata Survey Data Reference Manual</i>
[ST]	<i>Stata Survival Analysis Reference Manual</i>
[TABLES]	<i>Stata Customizable Tables and Collected Results Reference Manual</i>
[TS]	<i>Stata Time-Series Reference Manual</i>
[M]	<i>Mata Reference Manual</i>

When we refer to “reference manuals”, we mean all manuals listed above.

When we refer to the specialty manuals, we mean all the manuals listed above except [R].

Arrangement of the reference manuals

Each manual contains the following sections:

- Contents.
A table of contents can be found at the beginning of each manual.
- Cross-referencing the documentation.
This entry lists all the manuals and explains how they are cross-referenced.
- Introduction.
This entry—usually called *intro*—provides an overview of the manual. In the specialty manuals, this introduction suggests entries that you might want to read first and provides information about new features.
Each specialty manual contains an overview of the commands described in it.
- Entries.
Entries are arranged in alphabetical order. Most entries describe Stata commands, but some entries discuss concepts, and others provide overviews.
Entries that describe estimation commands are followed by an entry discussing postestimation commands that are available for use after the estimation command. For example, the **xtlogit** entry in the [XT] manual is followed by the **xtlogit postestimation** entry.
- Glossary.
A glossary is contained in all the specialty manuals.

To find information and commands quickly, use Stata's `search` command; see [\[R\] search](#) (see the entry `search` in the [R] manual).

Each reference manual does not contain its own author or subject index. A [combined author index](#) and a [combined subject index](#) for all reference manuals can be found in the *Stata Index*, [1]. This manual also contains a [combined subject table of contents](#) for all reference manuals and the *User's Guide*, an [acronym glossary](#), and a [vignette index](#).

Arrangement of each entry

Entries in most of the Stata reference manuals contain the following sections, which are explained below:

Description
Quick start
Menu
Syntax
Options
Remarks and examples
Stored results
Methods and formulas
Acknowledgments
References
Also see

Description

The purpose of the command is briefly described here.

Quick start

A quick start lists common uses of the command and the corresponding syntax for each.

For details on the syntax elements shown in the *Quick start* examples and to further customize the examples shown, see the *Syntax* and *Options* sections of the entry.

If you prefer to use the GUI, see the *Menu* section of the entry.

For applied examples of the syntax, see *Remarks and examples*.

Menu

A menu indicates how the dialog box for the command may be accessed using the menu system.

Syntax

A command's syntax diagram shows how to type the command, indicates all possible options, and gives the minimal allowed abbreviations for all the items in the command. For instance, the syntax diagram for the `summarize` command is

```
summarize [ varlist ] [ if ] [ in ] [ weight ] [ , options ]
```

<i>options</i>	Description
<code>detail</code>	display additional statistics
<code>meanonly</code>	suppress the display; calculate only the mean; programmer's option
<code>format</code>	use variable's display format
<code>separator(#)</code>	draw separator line after every # variables; default is <code>separator(5)</code>
<code>display_options</code>	control spacing and base and empty cells

Main

varlist may contain factor variables; see [U] 11.4.3 **Factor variables**.

varlist may contain time-series operators; see [U] 11.4.4 **Time-series varlists**.

`by`, `collect`, `rolling`, and `statsby` are allowed; see [U] 11.1.10 **Prefix commands**.

`aweight`s, `fweight`s, and `iweight`s are allowed. However, `iweight`s may not be used with the `detail` option; see [U] 11.1.6 **weight**.

Items in the typewriter-style font should be typed exactly as they appear in the diagram, although they may be abbreviated. Underlining indicates the shortest abbreviations where abbreviations are allowed. For instance, `summarize` may be abbreviated `su`, `sum`, `summ`, etc., or it may be spelled out completely. Items in the typewriter font that are not underlined may not be abbreviated.

Square brackets denote optional items. In the syntax diagram above, *varlist*, *if*, *in*, *weight*, and the *options* are optional.

The *options* are listed in a table immediately following the diagram, along with a brief description of each.

Items typed in *italics* represent arguments for which you are to substitute variable names, observation numbers, and the like.

The diagrams use the following symbols:

#	Indicates a literal number, for example, 5; see [U] 12.2 Numbers.
[]	Anything enclosed in brackets is optional.
{ }	At least one of the items enclosed in braces must appear.
	The vertical bar separates alternatives.
%fmt	Any Stata format, for example, %8.2f; see [U] 12.5 Formats: Controlling how data are displayed.
depvar	The dependent variable in an estimation command; see [U] 20 Estimation and postestimation commands.
exp	Any algebraic expression, for example, (5+myvar)/2; see [U] 13 Functions and expressions.
filename	Any filename; see [U] 11.6 Filenaming conventions.
indepvars	The independent variables in an estimation command; see [U] 20 Estimation and postestimation commands.
newvar	A variable that will be created by the current command; see [U] 11.4.2 Lists of new variables.
numlist	A list of numbers; see [U] 11.1.8 numlist.
oldvar	A previously created variable; see [U] 11.4.1 Lists of existing variables.
options	A list of options; see [U] 11.1.7 options.
range	An observation range, for example, 5/20; see [U] 11.1.4 in range.
"string"	Any string of characters enclosed in double quotes; see [U] 12.4 Strings.
varlist	A list of variable names; see [U] 11.4 varname and varlists. If <i>varlist</i> allows factor variables, a note to that effect will be shown below the syntax diagram; see [U] 11.4.3 Factor variables. If <i>varlist</i> allows time-series operators, a note to that effect will be shown below the syntax diagram; see [U] 11.4.4 Time-series varlists.
varname	A variable name; see [U] 11.3 Naming conventions.
weight	A [<i>wgtype=exp</i>] modifier; see [U] 11.1.6 weight and [U] 20.24 Weighted estimation.
xvar	The variable to be displayed on the horizontal axis.
yvar	The variable to be displayed on the vertical axis.

The *Syntax* section will indicate whether factor variables or time-series operators may be used with a command. *summarize* allows factor variables and time-series operators.

If a command allows prefix commands, this will be indicated immediately following the table of options. *summarize* allows by.

If a command allows weights, the types of weights allowed will be specified, with the default weight listed first. *summarize* allows *aweight*s, *fweight*s, and *iweight*s, and if the type of weight is not specified, the default is *aweight*s.

Options

If the command allows any options, they are explained here, and for dialog users the location of the options in the dialog is indicated. For instance, in the **logistic** entry in this manual, the *Options* section looks like this:



Remarks and examples

The explanations under *Description* and *Options* are exceedingly brief and technical; they are designed to provide a quick summary. The remarks explain in English what the preceding technical jargon means. Examples are used to illustrate the command.

Links to video examples posted on [Stata's YouTube channel](#) are provided at the end of this section.

Stored results

Commands are classified as e-class, r-class, s-class, or n-class, according to whether they store calculated results in `e()`, `r()`, `s()`, or not at all. These results can then be used in subroutines by other programs (ado-files). Such stored results are documented here; see [U] [18.8 Accessing results calculated by other programs](#) and [U] [18.9 Accessing results calculated by estimation commands](#).

Methods and formulas

The techniques and formulas used in obtaining the results are described here as tersely and technically as possible.

Acknowledgments

Some Stata commands began as community-contributed commands, or they were enhanced after suggestions by a Stata user. Here we acknowledge these contributions.

References

Published sources are listed that either were directly referenced in the preceding text or might be of interest.

Also see

Other manual entries relating to this entry are listed that might also interest you.

[Elizabeth L. \(“Betty”\) Scott](#) (1917–1988) was an astronomer and mathematician trained at the University of California at Berkeley. She published her first paper when she was just 22 years old, and her work was focused on comets for much of her early academic career.

During World War II, Scott began working at the statistical laboratory at Berkeley, which had recently been established by Jerzy Neyman, sparking what would be a long and fruitful collaboration with him. After the war, she shifted her focus toward mathematics and statistics, partly because of limited career opportunities as an astronomer, though she still applied her research to astronomical topics. For example, in 1949 she published a paper using statistical techniques to analyze the distribution of binary star systems. She also published papers examining the distribution of galaxies, and she is the name behind the “Scott effect”, which helps determine the distances to galaxies. Later in her career, Scott applied her statistical knowledge to problems associated with ozone depletion and its effects on the incidence of skin cancer as well as weather modification. She was also a champion of equality for women graduate students and faculty.

Among Scott’s many awards and accomplishments, she was elected an honorary fellow of the Royal Statistical Society and was a fellow of the American Association for the Advancement of Science. In 1992, the Committee of Presidents of Statistical Societies established the Elizabeth L. Scott Award, a biannual award to recognize those who have strived to enhance the status of women within the statistics profession.

[Janet Lippe Norwood](#) (1923–2015) was born in Newark, New Jersey. She obtained her PhD from Tufts University and taught political science at Wellesley College. Norwood made significant contributions while she was the first female commissioner for the Bureau of Labor Statistics (BLS). She accomplished the goal of conducting the Consumer Expenditure Survey annually, a long-time goal of the BLS, and saved the National Longitudinal Survey from termination. As commissioner, she would present data on national unemployment before the Joint Economic Committee on a monthly basis, a duty that she performed with unwavering impartiality. Under her direction, the statistical quality of reported indicators improved, as did the cooperation of the BLS with the Census Bureau and the National Center for Health Statistics.

After her role as commissioner, she was the Chair of the Advisory Council on Unemployment Compensation, appointed by presidents George H. W. Bush and Bill Clinton. She also held a variety of other leadership positions, including president of the American Statistical Association, board member of the American Economic Association, and chair of a statistical committee for the OECD. Norwood was a trailblazer. In her honor, the University of Alabama at Birmingham created the Janet L. Norwood Award to recognize women in statistics.

Also see

[U] [1.1 Getting started with Stata](#)

Description

`about` displays information about your version of Stata.

Menu

Help > About Stata

Syntax

`about`

Remarks and examples

If you are running Stata for Windows, information about memory is also displayed:

```
. about
StataNow/MP 19.5 for Windows (64-bit x86-64)
Revision date
Copyright 1985-2025 StataCorp LLC
Total usable memory: 8388608 KB
Stata license: 10-user 32-core network perpetual
Serial number: 19.5
Licensed to: Stata Developer
             StataCorp LLC
```

Also see

[R] [which](#) — Display location of an ado-file

[U] [3 Resources for learning and using Stata](#)

[U] [5 Editions of Stata](#)

[Description](#)[Quick start](#)[Syntax](#)[Options](#)[Remarks and examples](#)[Stored results](#)[Also see](#)

Description

`ado update` checks for available updates to community-contributed packages. To update packages, use `ado update`, `update`. By default, only packages in the PLUS directory are checked.

Quick start

List available updates for community-contributed packages

```
ado update
```

Install updates for community-contributed packages

```
ado update, update
```

Install updates from Statistical Software Components (SSC) Archive only

```
ado update, update ssconly
```

Syntax

```
ado update [pkglist] [ , options ]
```

<i>options</i>	Description
<code>update</code>	perform update; default is to list packages that have updates, but not to update them
<code>all</code>	include packages that might have updates; default is to list or update only packages that are known to have updates
<code>ssconly</code>	check only packages obtained from SSC Archive; default is to check all installed packages
<code>dir(<i>dir</i>)</code>	check packages installed in <i>dir</i> ; default is to check those installed in PLUS
<code>verbose</code>	provide output to assist in debugging network problems

Options

`update` specifies that packages with updates be updated. The default is simply to list the packages that could be updated without actually performing the update.

The first time you `ado update`, do not specify this option. Once you see `ado update` work, you will be more comfortable with it. Then type

```
. ado update, update
```

The packages that can be updated will be listed and updated.

`all` is rarely specified. Sometimes, `ado update` cannot determine whether a package you previously installed has been updated. `ado update` can determine that the package is still available over the web but is unsure whether the package has changed. Usually, the package has not changed, but if you want to be certain that you are using the latest version, reinstall from the source.

Specifying `all` does this. Typing

```
. ado update, all
```

adds such packages to the displayed list as needing updating but does not update them. Typing

```
. ado update, update all
```

lists such packages and updates them.

`ssconly` specifies that `ado update` check only packages obtained from the Statistical Software Components (SSC) Archive at Boston College, which is provided at <http://repec.org>. See [R] `ssc` for more information on the SSC Archive.

`dir` (*dir*) specifies which installed packages be checked. The default is `dir(PLUS)`, and that is probably correct. If you are responsible for maintaining a large system, however, you may have previously installed packages in `dir(SITE)`, where they are shared across users. See [P] `sysdir` for an explanation of these directory codewords. You may also specify an actual directory name, such as `C:\mydir`.

`verbose` is specified when you suspect network problems. It provides more detailed output that may help you diagnose the problem.

Remarks and examples

Community-contributed additions to Stata are called packages and can add remarkable abilities to Stata. Community-contributed packages are updated by their developers, just as official Stata software is updated by StataCorp.

Do not confuse `ado update` with `update`. Use `ado update` to update community-contributed files. Use `update` to update the components (including `ado`-files) of the official Stata software. To use either command, you must be connected to the internet.

Although Stata checks for updates automatically and can even be set to update automatically in Stata for Windows and Stata for Mac, you must remember to type `ado update`. Doing this regularly can help prevent errors that occur when accidentally running older versions of community-contributed packages.

Remarks are presented under the following headings:

Using ado update
Notes for developers

Using ado update

The first time you try `ado update`, type

```
. ado update
```

`ado update` without the `update` option produces a report but does not update any files. The first time you run `ado update`, you may see messages such as

```
. ado update
note: ado update updates community-contributed files; type update to check for
      updates to official Stata.
```

```
Checking status of installed packages...
```

```
[1] sjlatex at http://svn.stata.com/svn/press/press/production:
     installed package is up to date
```

(output omitted)

Having the same packages installed multiple times can lead to confusion; `ado update` cleans that up.

To update all of your community-contributed packages that need updating, type

```
. ado update, update
```

You can also update a subset of your packages. You can specify one or many packages after the `ado update` command. You can even use wildcards such as `st*` to mean all packages that start with `st` or `st*8` to mean all packages that start with `st` and end with `8`. For example, if the report indicated package `st0008` had an update available, type the following to update that one package:

```
. ado update st0008, update
```

Notes for developers

`ado update` reports whether an installed package is up to date by comparing its distribution date with that of the package available over the web.

If you are distributing software, include the line

```
d Distribution-Date: date
```

somewhere in your `.pkg` file. The capitalization of `Distribution-Date` does not matter, but include the hyphen and the colon as shown. Code the date in either of two formats:

all numeric:	<code>yyyymmdd</code> , for example, 20240701
Stata standard:	<code>ddMONyyyy</code> , for example, 01jul2024

Stored results

ado update stores the following in `r()`:

Macros

`r(pkglist)` a space-separated list of package names that need updating (`update` not specified) or that were updated (`update` specified)

Also see

[R] [net](#) — Install and manage community-contributed additions from the internet

[R] [search](#) — Search Stata documentation and other resources

[R] [ssc](#) — Install and uninstall packages from SSC

[R] [update](#) — Check for official updates

ameans — Arithmetic, geometric, and harmonic means

[Description](#)

[Options](#)

[Acknowledgments](#)

[Quick start](#)

[Remarks and examples](#)

[References](#)

[Menu](#)

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[Syntax](#)

[Methods and formulas](#)

Description

`ameans` computes the arithmetic, geometric, and harmonic means, with their corresponding confidence intervals, for each variable in *varlist* or for all the variables in the data if *varlist* is not specified.

`gmeans` and `hmeans` are synonyms for `ameans`.

Quick start

Arithmetic, geometric, and harmonic means of variable *v1*

```
ameans v1
```

Same as above, but for variables *v1*, *v2*, and *v3*

```
ameans v1 v2 v3
```

Means for all variables in the dataset

```
ameans
```

Add *n* to each observation before calculating means

```
ameans v1, add(n)
```

Add *n* to each observation only for variables with at least 1 nonpositive value

```
ameans v1 v2 v3, add(n) only
```

Request 99% confidence intervals

```
ameans v1, level(99)
```

Menu

Statistics > Summaries, tables, and tests > Summary and descriptive statistics > Arith./geometric/harmonic means

Syntax

```
ameans [varlist] [if] [in] [weight] [, options]
```

<i>options</i>	Description
----------------	-------------

Main

<code>add(#)</code>	add # to each variable in <i>varlist</i>
<code>only</code>	add # only to variables with nonpositive values
<code>level(#)</code>	set confidence level; default is level(95)

`by` and `collect` are allowed; see [D] [by](#).

`aweight`s and `fweight`s are allowed; see [U] [11.1.6 weight](#).

Options

Main

`add(#)` adds the value # to each variable in *varlist* before computing the means and confidence intervals. This option is useful when analyzing variables with nonpositive values.

`only` modifies the action of the `add(#)` option so that it adds # only to variables with at least one nonpositive value.

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is level(95) or as set by `set level`; see [U] [20.8 Specifying the width of confidence intervals](#).

Remarks and examples

► Example 1

We have a dataset containing 8 observations on a variable named `x`. The eight values are 5, 4, -4, -5, 0, 0, *missing*, and 7.

```
. amean x
```

Variable	Type	Obs	Mean	[95% conf. interval]	
x	Arithmetic	7	1	-3.204405	5.204405
	Geometric	3	5.192494	2.57899	10.45448
	Harmonic	3	5.060241	3.023008	15.5179

```
. amean x, add(5)
```

Variable	Type	Obs	Mean	[95% conf. interval]	
x	Arithmetic	7	6	1.795595	10.2044*
	Geometric	6	5.477226	2.1096	14.22071*
	Harmonic	6	3.540984	.	.*

* 5 was added to the variables prior to calculating the results.

Note: Missing values in confidence intervals for harmonic mean indicate that confidence interval is undefined for corresponding variables.

The number of observations displayed for the arithmetic mean is the number of nonmissing observations. The number of observations displayed for the geometric and harmonic means is the number of nonmissing, positive observations. Specifying the `add(5)` option produces 3 more positive observations. The confidence interval for the harmonic mean is not reported; see *Methods and formulas* below.



Video example

[Descriptive statistics in Stata](#)

Stored results

`ameans` stores the following in `r()`:

Scalars

<code>r(N)</code>	number of nonmissing observations; used for arithmetic mean
<code>r(N_pos)</code>	number of nonmissing positive observations; used for geometric and harmonic means
<code>r(mean)</code>	arithmetic mean
<code>r(lb)</code>	lower bound of confidence interval for arithmetic mean
<code>r(ub)</code>	upper bound of confidence interval for arithmetic mean
<code>r(Var)</code>	variance of untransformed data
<code>r(mean_g)</code>	geometric mean
<code>r(lb_g)</code>	lower bound of confidence interval for geometric mean
<code>r(ub_g)</code>	upper bound of confidence interval for geometric mean
<code>r(Var_g)</code>	variance of $\ln x_i$
<code>r(mean_h)</code>	harmonic mean
<code>r(lb_h)</code>	lower bound of confidence interval for harmonic mean
<code>r(ub_h)</code>	upper bound of confidence interval for harmonic mean
<code>r(Var_h)</code>	variance of $1/x_i$
<code>r(level)</code>	confidence level of confidence interval

Methods and formulas

See [Armitage, Berry, and Matthews \(2002\)](#) or [Snedecor and Cochran \(1989\)](#). For a history of the concept of the mean, see [Plackett \(1958\)](#).

When restricted to the same set of values (that is, to positive values), the arithmetic mean (\bar{x}) is greater than or equal to the geometric mean, which in turn is greater than or equal to the harmonic mean. Equality holds only if all values within a sample are equal to a positive constant.

The arithmetic mean and its confidence interval are identical to those provided by `ci`; see [\[R\] ci](#).

To compute the geometric mean, `ameans` first creates $u_j = \ln x_j$ for all positive x_j . The arithmetic mean of the u_j and its confidence interval are then computed as in `ci`. Let \bar{u} be the resulting mean, and let $[L, U]$ be the corresponding confidence interval. The geometric mean is then $\exp(\bar{u})$, and its confidence interval is $[\exp(L), \exp(U)]$.

The same procedure is followed for the harmonic mean, except that then $u_j = 1/x_j$. The harmonic mean is then $1/\bar{u}$, and its confidence interval is $[1/U, 1/L]$ if L is greater than zero. If L is not greater than zero, this confidence interval is not defined, and missing values are reported.

When weights are specified, `ameans` applies the weights to the transformed values, $u_j = \ln x_j$ and $u_j = 1/x_j$, respectively, when computing the geometric and harmonic means. For details on how the weights are used to compute the mean and variance of the u_j , see [R] [summarize](#). Without weights, the formula for the geometric mean reduces to

$$\exp\left\{\frac{1}{n} \sum_j \ln(x_j)\right\}$$

Without weights, the formula for the harmonic mean is

$$\frac{n}{\sum_j \frac{1}{x_j}}$$

Acknowledgments

This improved version of `ameans` is based on the `gmci` command (Carlin, Vidmar, and Ramalheira 1998) and was written by John Carlin of the Murdoch Children’s Research Institute and the University of Melbourne; Suzanna Vidmar of the University of Melbourne; and Carlos Ramalheira of Coimbra University Hospital, Portugal.

References

- Armitage, P., G. Berry, and J. N. S. Matthews. 2002. *Statistical Methods in Medical Research*. 4th ed. Oxford: Blackwell.
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Also see

- [R] [ci](#) — Confidence intervals for means, proportions, and variances
- [R] [mean](#) — Estimate means
- [R] [summarize](#) — Summary statistics
- [SVY] [svy estimation](#) — Estimation commands for survey data

Description
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Description

The `anova` command fits analysis-of-variance (ANOVA) and analysis-of-covariance (ANCOVA) models for balanced and unbalanced designs, including designs with missing cells; for repeated-measures ANOVA; and for factorial, nested, or mixed designs.

Quick start

One-way ANOVA model of y for factor a

```
anova y a
```

Two-way full-factorial ANOVA for factors a and b

```
anova y a b a#b
```

Same as above

```
anova y a##b
```

ANCOVA model including continuous variable x

```
anova y a##b c . x
```

Factor b nested within a

```
anova y a / b | a /
```

Repeated-measures ANOVA with repeated variable $rvar$

```
anova y a rvar , repeated(rvar)
```

Repeated-measures ANOVA with subjects, $idvar$, observed at each level of $rvar$

```
anova y a / idvar | a rvar rvar#a , repeated(rvar)
```

Menu

Statistics > Linear models and related > ANOVA/MANOVA > Analysis of variance and covariance

Syntax

```
anova varname [termlist] [if] [in] [weight] [, options]
```

where *termlist* is a factor-variable list (see [U] 11.4.3 Factor variables) with the following additional features:

- Variables are assumed to be categorical; use the `c.` factor-variable operator to override this.
- The `|` symbol (indicating nesting) may be used in place of the `#` symbol (indicating interaction).
- The `/` symbol is allowed after a term and indicates that the following term is the error term for the preceding terms.

<i>options</i>	Description
Model	
<code>repeated(<i>varlist</i>)</code>	variables in <i>terms</i> that are repeated-measures variables
<code>partial</code>	use partial (or marginal) sums of squares
<code>sequential</code>	use sequential sums of squares
<code>noconstant</code>	suppress constant term
<code>dropemptycells</code>	drop empty cells from the design matrix
Adv. model	
<code>bse(<i>term</i>)</code>	between-subjects error term in repeated-measures ANOVA
<code>bseunit(<i>varname</i>)</code>	variable representing lowest unit in the between-subjects error term
<code>grouping(<i>varname</i>)</code>	grouping variable for computing pooled covariance matrix

`bootstrap`, `by`, `collect`, `fp`, `jackknife`, and `statsby` are allowed; see [U] 11.1.10 Prefix commands.

Weights are not allowed with the `bootstrap` prefix; see [R] `bootstrap`.

`aweights` are not allowed with the `jackknife` prefix; see [R] `jackknife`.

`aweights` and `fweights` are allowed; see [U] 11.1.6 `weight`.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

Options

Model

`repeated(varlist)` indicates the names of the categorical variables in the *terms* that are to be treated as repeated-measures variables in a repeated-measures ANOVA or ANCOVA.

`partial` presents the ANOVA table using partial (or marginal) sums of squares. This setting is the default. Also see the `sequential` option.

`sequential` presents the ANOVA table using sequential sums of squares.

`noconstant` suppresses the constant term (intercept) from the ANOVA or regression model.

`dropemptycells` drops empty cells from the design matrix. If `c(emptycells)` is set to `keep` (see [R] `set emptycells`), this option temporarily resets it to `drop` before running the ANOVA model. If `c(emptycells)` is already set to `drop`, this option does nothing.

`bse(term)` indicates the between-subjects error term in a repeated-measures ANOVA. This option is needed only in the rare case when the `anova` command cannot automatically determine the between-subjects error term.

`bseunit(varname)` indicates the variable representing the lowest unit in the between-subjects error term in a repeated-measures ANOVA. This option is rarely needed because the `anova` command automatically selects the first variable listed in the between-subjects error term as the default for this option.

`grouping(varname)` indicates a variable that determines which observations are grouped together in computing the covariance matrices that will be pooled and used in a repeated-measures ANOVA. This option is rarely needed because the `anova` command automatically selects the combination of all variables except the first (or as specified in the `bseunit()` option) in the between-subjects error term as the default for grouping observations.

Remarks and examples

Remarks are presented under the following headings:

- [Introduction](#)
- [One-way ANOVA](#)
- [Two-way ANOVA](#)
- [N-way ANOVA](#)
- [Weighted data](#)
- [ANCOVA](#)
- [Nested designs](#)
- [Mixed designs](#)
- [Latin-square designs](#)
- [Repeated-measures ANOVA](#)
- [Video examples](#)

Introduction

`anova` uses least squares to fit the linear models known as ANOVA or ANCOVA (henceforth referred to simply as ANOVA models).

If you want to fit one-way ANOVA models, you may find the `oneway` or `loneway` command more convenient; see [\[R\] oneway](#) and [\[R\] loneway](#). If you are interested in MANOVA or MANCOVA, see [\[MV\] manova](#).

Structural equation modeling provides a more general framework for fitting ANOVA models; see the [Stata Structural Equation Modeling Reference Manual](#).

ANOVA was pioneered by Fisher. It features prominently in his texts on statistical methods and his design of experiments (1925, 1935). Many books discuss ANOVA; see, for instance, [Altman \(1991\)](#); [van Belle et al. \(2004\)](#); [Cobb \(1998\)](#); [Snedecor and Cochran \(1989\)](#); or [Winer, Brown, and Michels \(1991\)](#). For a classic source, see [Scheffé \(1959\)](#). [Kennedy and Gentle \(1980\)](#) discuss ANOVA's computing problems. [Edwards \(1985\)](#) is concerned primarily with the relationship between multiple regression and ANOVA. [Acock \(2023, chap. 9\)](#) and [Baldwin \(2019, chap. 5 and 6\)](#) illustrate their discussion with Stata output. Repeated-measures ANOVA is discussed in [Winer, Brown, and Michels \(1991\)](#) and [Milliken and Johnson \(2009\)](#). Pioneering work in repeated-measures ANOVA can be found in [Box \(1954\)](#); [Geisser and Greenhouse \(1958\)](#); [Huynh and Feldt \(1976\)](#); and [Huynh \(1978\)](#). For a Stata-specific discussion of ANOVA contrasts, see [Mitchell \(2021, chap. 7–9; 2015, chap. 4–9\)](#).

One-way ANOVA

`anova`, entered without options, performs and reports standard ANOVA. For instance, to perform a one-way layout of a variable called `endog` on `exog`, you would type `anova endog exog`.

▷ Example 1: One-way ANOVA

We run an experiment varying the amount of fertilizer used in growing apple trees. We test four concentrations, using each concentration in three groves of 12 trees each. Later in the year, we measure the average weight of the fruit.

If all had gone well, we would have had 3 observations on the average weight for each of the four concentrations. Instead, two of the groves were mistakenly leveled by a confused man on a large bulldozer. We are left with the following data:

```
. use https://www.stata-press.com/data/r19/apple
(Apple trees)
. list, abbrev(10) sepby(treatment)
```

	treatment	weight
1.	1	117.5
2.	1	113.8
3.	1	104.4
4.	2	48.9
5.	2	50.4
6.	2	58.9
7.	3	70.4
8.	3	86.9
9.	4	87.7
10.	4	67.3

To obtain one-way ANOVA results, we type

```
. anova weight treatment
```

	Number of obs =	10	R-squared =	0.9147	
	Root MSE =	9.07002	Adj R-squared =	0.8721	
Source	Partial SS	df	MS	F	Prob>F
Model	5295.5443	3	1765.1814	21.46	0.0013
treatment	5295.5443	3	1765.1814	21.46	0.0013
Residual	493.59167	6	82.265278		
Total	5789.136	9	643.23733		

We find significant (at better than the 1% level) differences among the four concentrations.

Although the output is a usual ANOVA table, let's run through it anyway. Above the table is a summary of the underlying regression. The model was fit on 10 observations, and the root mean squared error (Root MSE) is 9.07. The R^2 for the model is 0.9147, and the adjusted R^2 is 0.8721.

The first line of the table summarizes the model. The sum of squares (Partial SS) for the model is 5295.5 with 3 degrees of freedom (df). This line results in a mean square (MS) of $5295.5/3 \approx 1765.2$. The corresponding F statistic is 21.46 and has a significance level of 0.0013. Thus, the model appears to be significant at the 0.13% level.

The next line summarizes the first (and only) term in the model, `treatment`. Because there is only one term, the line is identical to that for the overall model.

The third line summarizes the residual. The residual sum of squares is 493.59 with 6 degrees of freedom, resulting in a mean squared error of 82.27. The square root of this latter number is reported as the Root MSE.

The model plus the residual sum of squares equals the total sum of squares, which is reported as 5789.1 in the last line of the table. This is the total sum of squares of weight after removal of the mean. Similarly, the model plus the residual degrees of freedom sum to the total degrees of freedom, 9. Remember that there are 10 observations. Subtracting 1 for the mean, we are left with 9 total degrees of freedom.

◀

□ Technical note

Rather than using the `anova` command, we could have performed this analysis by using the `oneway` command. [Example 1](#) in [\[R\] oneway](#) repeats this same analysis. You may wish to compare the output. □

The `regress` command (see [\[R\] regress](#)) is used to fit the underlying regression model corresponding to an ANOVA model fit using the `anova` command. Type `regress` after `anova` to see the coefficients, standard errors, etc., of the regression model for the last run of `anova`.

▷ Example 2: Regression table from a one-way ANOVA

Returning to the apple tree experiment, we found that the fertilizer concentration appears to significantly affect the average weight of the fruit. Although that finding is interesting, we next want to know which concentration appears to grow the heaviest fruit. One way to find out is by examining the underlying regression coefficients.

```
. regress, baselevels
```

Source	SS	df	MS	Number of obs	=	10
Model	5295.54433	3	1765.18144	F(3, 6)	=	21.46
Residual	493.591667	6	82.2652778	Prob > F	=	0.0013
				R-squared	=	0.9147
				Adj R-squared	=	0.8721
Total	5789.136	9	643.237333	Root MSE	=	9.07

weight	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
treatment						
1	0 (base)					
2	-59.16667	7.405641	-7.99	0.000	-77.28762	-41.04572
3	-33.25	8.279758	-4.02	0.007	-53.50984	-12.99016
4	-34.4	8.279758	-4.15	0.006	-54.65984	-14.14016
_cons	111.9	5.236579	21.37	0.000	99.08655	124.7134

See [R] [regress](#) for an explanation of how to read this table. The `baselevels` option of `regress` displays a row indicating the base category for our categorical variable, `treatment`. In summary, we find that concentration 1, the base (omitted) group, produces significantly heavier fruits than concentration 2, 3, and 4; concentration 2 produces the lightest fruits; and concentrations 3 and 4 appear to be roughly equivalent.



▷ Example 3: ANOVA replay

We previously typed `anova weight treatment` to produce and display the ANOVA table for our apple tree experiment. Typing `regress` displays the regression coefficients. We can redisplay the ANOVA table by typing `anova` without arguments:

```
. anova
```

	Number of obs =	10	R-squared =	0.9147	
	Root MSE =	9.07002	Adj R-squared =	0.8721	
Source	Partial SS	df	MS	F	Prob>F
Model	5295.5443	3	1765.1814	21.46	0.0013
treatment	5295.5443	3	1765.1814	21.46	0.0013
Residual	493.59167	6	82.265278		
Total	5789.136	9	643.23733		



Two-way ANOVA

You can include multiple explanatory variables with the `anova` command, and you can specify interactions by placing ‘#’ between the variable names. For instance, typing `anova y a b` performs a two-way layout of `y` on `a` and `b`. Typing `anova y a b a#b` performs a full two-way factorial layout. The shorthand `anova y a##b` does the same.

With the default partial sums of squares, when you specify interacted terms, the order of the terms does not matter. Typing `anova y a b a#b` is the same as typing `anova y b a b#a`.

▷ Example 4: Two-way factorial ANOVA

The classic two-way factorial ANOVA problem, at least as far as computer manuals are concerned, is a two-way ANOVA design from [Afifi and Azen \(1979\)](#).

Fifty-eight patients, each suffering from one of three different diseases, were randomly assigned to one of four different drug treatments, and the change in their systolic blood pressure was recorded. Here are the data:

	Disease 1	Disease 2	Disease 3
Drug 1	42, 44, 36 13, 19, 22	33, 26, 33 21	31, -3, 25 25, 24
Drug 2	28, 23, 34 42, 13	34, 33, 31 36	3, 26, 28 32, 4, 16
Drug 3	1, 29, 19	11, 9, 7 1, -6	21, 1, 9 3
Drug 4	24, 9, 22 -2, 15	27, 12, 12 -5, 16, 15	22, 7, 25 5, 12

Let's assume that we have entered these data into Stata and stored the data as `systolic.dta`. Below we use the data, list the first 10 observations, summarize the variables, and tabulate the control variables:

```
. use https://www.stata-press.com/data/r19/systolic
(Systolic blood pressure data)
. list in 1/10
```

	drug	disease	systolic
1.	1	1	42
2.	1	1	44
3.	1	1	36
4.	1	1	13
5.	1	1	19
6.	1	1	22
7.	1	2	33
8.	1	2	26
9.	1	2	33
10.	1	2	21

```
. summarize
```

Variable	Obs	Mean	Std. dev.	Min	Max
drug	58	2.5	1.158493	1	4
disease	58	2.017241	.8269873	1	3
systolic	58	18.87931	12.80087	-6	44

```
. tabulate drug disease
```

Drug used	Patient's disease			Total
	1	2	3	
1	6	4	5	15
2	5	4	6	15
3	3	5	4	12
4	5	6	5	16
Total	19	19	20	58

Each observation in our data corresponds to one patient, and for each patient we record drug, disease, and the increase in the systolic blood pressure, `systolic`. The tabulation reveals that the data are not balanced—there are not equal numbers of patients in each drug–disease cell. Stata does not require that the data be balanced. We can perform a two-way factorial ANOVA by typing

```
. anova systolic drug disease drug#disease
```

	Number of obs =	58	R-squared =	0.4560	
	Root MSE =	10.5096	Adj R-squared =	0.3259	
Source	Partial SS	df	MS	F	Prob>F
Model	4259.3385	11	387.21259	3.51	0.0013
drug	2997.4719	3	999.15729	9.05	0.0001
disease	415.87305	2	207.93652	1.88	0.1637
drug#disease	707.26626	6	117.87771	1.07	0.3958
Residual	5080.8167	46	110.45254		
Total	9340.1552	57	163.86237		

Although Stata's `table` command does not perform ANOVA, it can produce useful summary tables of your data (see [R] [table](#)):

```
. table drug disease, statistic(mean systolic) nformat(%8.2f)
```

	Patient's disease			
	1	2	3	Total
Drug used				
1	29.33	28.25	20.40	26.07
2	28.00	33.50	18.17	25.53
3	16.33	4.40	8.50	8.75
4	13.60	12.83	14.20	13.50
Total	22.79	18.21	15.80	18.88

These are simple means and are not influenced by our `anova` model. More useful is the `margins` command (see [R] [margins](#)) that provides marginal means and adjusted predictions. Because `drug` is the only significant factor in our ANOVA, we now examine the adjusted marginal means for `drug`.

```
. margins drug, asbalanced
```

```
Adjusted predictions                                Number of obs = 58
Expression: Linear prediction, predict()
At: disease   (asbalanced)
```

drug	Delta-method		t	P> t	[95% conf. interval]	
	Margin	std. err.				
1	25.99444	2.751008	9.45	0.000	20.45695	31.53194
2	26.55556	2.751008	9.65	0.000	21.01806	32.09305
3	9.744444	3.100558	3.14	0.003	3.503344	15.98554
4	13.54444	2.637123	5.14	0.000	8.236191	18.8527

These adjusted marginal predictions are not equal to the simple `drug` means (see the totals from the `table` command); they are based upon predictions from our ANOVA model. The `asbalanced` option of `margins` corresponds with the interpretation of the F statistic produced by ANOVA—each cell is given equal weight regardless of its sample size (see the following three technical notes). You can omit the `asbalanced` option and obtain predictive margins that take into account the unequal sample sizes of the cells.

```
. margins drug
```

```
Predictive margins                                Number of obs = 58
Expression: Linear prediction, predict()
```

drug	Delta-method		t	P> t	[95% conf. interval]	
	Margin	std. err.				
1	25.89799	2.750533	9.42	0.000	20.36145	31.43452
2	26.41092	2.742762	9.63	0.000	20.89003	31.93181
3	9.722989	3.099185	3.14	0.003	3.484652	15.96132
4	13.55575	2.640602	5.13	0.000	8.24049	18.871

□ Technical note

How do you interpret the significance of terms like `drug` and `disease` in unbalanced data? If you are familiar with SAS, the sums of squares and the F statistic reported by Stata correspond to SAS type III sums of squares. (Stata can also calculate sequential sums of squares, but we will postpone that topic for now.)

Let's think in terms of the following table:

	Disease 1	Disease 2	Disease 3	
Drug 1	μ_{11}	μ_{12}	μ_{13}	$\mu_{1.}$
Drug 2	μ_{21}	μ_{22}	μ_{23}	$\mu_{2.}$
Drug 3	μ_{31}	μ_{32}	μ_{33}	$\mu_{3.}$
Drug 4	μ_{41}	μ_{42}	μ_{43}	$\mu_{4.}$
	$\mu_{.1}$	$\mu_{.2}$	$\mu_{.3}$	$\mu_{..}$

In this table, μ_{ij} is the mean increase in systolic blood pressure associated with drug i and disease j , while $\mu_{i.}$ is the mean for drug i , $\mu_{.j}$ is the mean for disease j , and $\mu_{..}$ is the overall mean.

If the data are balanced, meaning that there are equal numbers of observations going into the calculation of each mean μ_{ij} , the row means, $\mu_{i.}$, are given by

$$\mu_{i.} = \frac{\mu_{i1} + \mu_{i2} + \mu_{i3}}{3}$$

In our case, the data are not balanced, but we define the $\mu_{i.}$ according to that formula anyway. The test for the main effect of drug is the test that

$$\mu_{1.} = \mu_{2.} = \mu_{3.} = \mu_{4.}$$

To be absolutely clear, the F test of the term `drug`, called the *main effect* of drug, is formally equivalent to the test of the three constraints:

$$\begin{aligned} \frac{\mu_{11} + \mu_{12} + \mu_{13}}{3} &= \frac{\mu_{21} + \mu_{22} + \mu_{23}}{3} \\ \frac{\mu_{11} + \mu_{12} + \mu_{13}}{3} &= \frac{\mu_{31} + \mu_{32} + \mu_{33}}{3} \\ \frac{\mu_{11} + \mu_{12} + \mu_{13}}{3} &= \frac{\mu_{41} + \mu_{42} + \mu_{43}}{3} \end{aligned}$$

In our data, we obtain a significant F statistic of 9.05 and thus reject those constraints. □

□ Technical note

Stata can display the symbolic form underlying the test statistics it presents, as well as display other test statistics and their symbolic forms; see [Obtaining symbolic forms in \[R\] anova postestimation](#). Here is the result of requesting the symbolic form for the main effect of `drug` in our data:

```

. test drug, symbolic
drug
      1  -(r2+r3+r4)
      2   r2
      3   r3
      4   r4
disease
      1  0
      2  0
      3  0
drug#disease
      1  1  -1/3 (r2+r3+r4)
      1  2  -1/3 (r2+r3+r4)
      1  3  -1/3 (r2+r3+r4)
      2  1   1/3 r2
      2  2   1/3 r2
      2  3   1/3 r2
      3  1   1/3 r3
      3  2   1/3 r3
      3  3   1/3 r3
      4  1   1/3 r4
      4  2   1/3 r4
      4  3   1/3 r4
_cons      0

```

This says exactly what we said in the previous technical note.

□

□ Technical note

Saying that there is no main effect of a variable is not the same as saying that it has no effect at all. Stata's ability to perform ANOVA on unbalanced data can easily be put to ill use.

For example, consider the following table of the probability of surviving a bout with one of two diseases according to the drug administered to you:

	Disease 1	Disease 2
Drug 1	1	0
Drug 2	0	1

If you have disease 1 and are administered drug 1, you live. If you have disease 2 and are administered drug 2, you live. In all other cases, you die.

This table has no main effects of either drug or disease, although there is a large interaction effect. You might now be tempted to reason that because there is only an interaction effect, you would be indifferent between the two drugs in the absence of knowledge about which disease infects you. Given an equal chance of having either disease, you reason that it does not matter which drug is administered to you—either way, your chances of surviving are 0.5.

You may not, however, have an equal chance of having either disease. If you knew that disease 1 was 100 times more likely to occur in the population, and if you knew that you had one of the two diseases, you would express a strong preference for receiving drug 1.

When you calculate the significance of main effects on unbalanced data, you must ask yourself why the data are unbalanced. If the data are unbalanced for random reasons and you are making predictions for a balanced population, the test of the main effect makes perfect sense. If, however, the data are unbalanced because the underlying populations are unbalanced and you are making predictions for such unbalanced populations, the test of the main effect may be practically—if not statistically—meaningless. □

▷ Example 5: ANOVA with missing cells

Stata can perform ANOVA not only on unbalanced populations, but also on populations that are so unbalanced that entire cells are missing. For instance, using our systolic blood pressure data, let's refit the model eliminating the drug 1–disease 1 cell. Because `anova` follows the same syntax as all other Stata commands, we can explicitly specify the data to be used by typing the `if` qualifier at the end of the `anova` command. Here we want to use the data that are not for drug 1 and disease 1:

```
. anova systolic drug##disease if !(drug==1 & disease==1)
```

	Number of obs =	52	R-squared =	0.4545	
	Root MSE =	10.1615	Adj R-squared =	0.3215	
Source	Partial SS	df	MS	F	Prob>F
Model	3527.959	10	352.7959	3.42	0.0025
drug	2686.5783	3	895.52611	8.67	0.0001
disease	327.7926	2	163.8963	1.59	0.2168
drug#disease	703.0076	5	140.60152	1.36	0.2586
Residual	4233.4833	41	103.25569		
Total	7761.4423	51	152.18514		

Here we used `drug##disease` as a shorthand for `drug disease drug#disease`. ◀

□ Technical note

The test of the main effect of drug in the presence of missing cells is more complicated than that for unbalanced data. Our underlying tableau now has the following form:

	Disease 1	Disease 2	Disease 3	
Drug 1		μ_{12}	μ_{13}	
Drug 2	μ_{21}	μ_{22}	μ_{23}	$\mu_{2.}$
Drug 3	μ_{31}	μ_{32}	μ_{33}	$\mu_{3.}$
Drug 4	μ_{41}	μ_{42}	μ_{43}	$\mu_{4.}$
		$\mu_{.2}$	$\mu_{.3}$	

The hole in the drug 1–disease 1 cell indicates that the mean is unobserved. Considering the main effect of drug, the test is unchanged for the rows in which all the cells are defined:

$$\mu_{2.} = \mu_{3.} = \mu_{4.}$$

The first row, however, requires special attention. Here we want the average outcome for drug 1, which is averaged only over diseases 2 and 3, to be equal to the average values of all other drugs averaged over those same two diseases:

$$\frac{\mu_{12} + \mu_{13}}{2} = \frac{(\mu_{22} + \mu_{23})/2 + (\mu_{32} + \mu_{33})/2 + (\mu_{42} + \mu_{43})/2}{3}$$

Thus, the test contains three constraints:

$$\begin{aligned}\frac{\mu_{21} + \mu_{22} + \mu_{23}}{3} &= \frac{\mu_{31} + \mu_{32} + \mu_{33}}{3} \\ \frac{\mu_{21} + \mu_{22} + \mu_{23}}{3} &= \frac{\mu_{41} + \mu_{42} + \mu_{43}}{3} \\ \frac{\mu_{12} + \mu_{13}}{2} &= \frac{\mu_{22} + \mu_{23} + \mu_{32} + \mu_{33} + \mu_{42} + \mu_{43}}{6}\end{aligned}$$

□

Stata can calculate two types of sums of squares, *partial* and *sequential*. If you do not specify which sums of squares to calculate, Stata calculates partial sums of squares. The technical notes above have gone into great detail about the definition and use of partial sums of squares. Use the `sequential` option to obtain sequential sums of squares.

□ Technical note

Before we illustrate sequential sums of squares, consider one more feature of the partial sums. If you know how such things are calculated, you may worry that the terms must be specified in some particular order, that Stata would balk or, even worse, produce different results if you typed, say, `anova systolic drug#disease drug disease` rather than `anova systolic drug disease drug#disease`. We assure you that is not the case.

When you type a model, Stata internally reorganizes the terms, forms the cross-product matrix, inverts it, converts the result to an upper-Hermite form, and then performs the hypothesis tests. As a final touch, Stata reports the results in the same order that you typed the terms.

□

▷ Example 6: Sequential sums of squares

We wish to estimate the effects on systolic blood pressure of drug and disease by using sequential sums of squares. We want to introduce disease first, then drug, and finally, the interaction of drug and disease:

```
. anova systolic disease drug disease#drug, sequential
```

	Number of obs =	58	R-squared =	0.4560	
	Root MSE =	10.5096	Adj R-squared =	0.3259	
Source	Seq. SS	df	MS	F	Prob>F
Model	4259.3385	11	387.21259	3.51	0.0013
disease	488.63938	2	244.31969	2.21	0.1210
drug	3063.4329	3	1021.1443	9.25	0.0001
disease#drug	707.26626	6	117.87771	1.07	0.3958
Residual	5080.8167	46	110.45254		
Total	9340.1552	57	163.86237		

The F statistic on disease is now 2.21. When we fit this same model by using partial sums of squares, the statistic was 1.88.

◀

N-way ANOVA

You may include high-order interaction terms, such as a third-order interaction between the variables A, B, and C, by typing A#B#C.

▷ Example 7: Three-way factorial ANOVA

We wish to determine the operating conditions that maximize yield for a manufacturing process. There are three temperature settings, two chemical supply companies, and two mixing methods under investigation. Three observations are obtained for each combination of these three factors.

```
. use https://www.stata-press.com/data/r19/manuf
(Manufacturing process data)
```

```
. describe
```

```
Contains data from https://www.stata-press.com/data/r19/manuf.dta
Observations:      36      Manufacturing process data
Variables:         4       2 Jan 2024 13:28
```

Variable name	Storage type	Display format	Value label	Variable label
temperature	byte	%9.0g	temp	Machine temperature setting
chemical	byte	%9.0g	supplier	Chemical supplier
method	byte	%9.0g	meth	Mixing method
yield	byte	%9.0g		Product yield

Sorted by:

We wish to perform a three-way factorial ANOVA. We could type

```
. anova yield temp chem temp#chem meth temp#meth chem#meth temp#chem#meth
```

but prefer to use the ## factor-variable operator for brevity.

```
. anova yield temp##chem##meth
```

	Number of obs =	36	R-squared =	0.5474	
	Root MSE =	2.62996	Adj R-squared =	0.3399	
Source	Partial SS	df	MS	F	Prob>F
Model	200.75	11	18.25	2.64	0.0227
temperature	30.5	2	15.25	2.20	0.1321
chemical	12.25	1	12.25	1.77	0.1958
temperature#chemical	24.5	2	12.25	1.77	0.1917
method	42.25	1	42.25	6.11	0.0209
temperature#method	87.5	2	43.75	6.33	0.0062
chemical#method	.25	1	.25	0.04	0.8508
temperature#chemical#method	3.5	2	1.75	0.25	0.7785
Residual	166	24	6.9166667		
Total	366.75	35	10.478571		

The interaction between temperature and method appears to be the important story in these data. A table of means for this interaction is given below.

```
. table method temp, statistic(mean yield) nformat(%8.2f)
```

	Machine temperature setting			
	Low	Medium	High	Total
Mixing method				
Stir	7.50	6.00	6.00	6.50
Fold	5.50	9.00	11.50	8.67
Total	6.50	7.50	8.75	7.58

Here our ANOVA is balanced (each cell has the same number of observations), and we obtain the same values as in the table above (but with additional information such as confidence intervals) by using the `margins` command. Because our ANOVA is balanced, using the `asbalanced` option with `margins` would not produce different results. We request the predictive margins for the two terms that appear significant in our ANOVA: `temperature#method` and `method`.

```
. margins temperature#method method
```

```
Predictive margins                                Number of obs = 36
```

```
Expression: Linear prediction, predict()
```

	Delta-method				
	Margin	std. err.	t	P> t	[95% conf. interval]
temperature# method					
Low#Stir	7.5	1.073675	6.99	0.000	5.284044 9.715956
Low#Fold	5.5	1.073675	5.12	0.000	3.284044 7.715956
Medium#Stir	6	1.073675	5.59	0.000	3.784044 8.215956
Medium#Fold	9	1.073675	8.38	0.000	6.784044 11.21596
High#Stir	6	1.073675	5.59	0.000	3.784044 8.215956
High#Fold	11.5	1.073675	10.71	0.000	9.284044 13.71596
method					
Stir	6.5	.6198865	10.49	0.000	5.220617 7.779383
Fold	8.666667	.6198865	13.98	0.000	7.387284 9.946049

We decide to use the folding method of mixing and a high temperature in our manufacturing process.

◀

Weighted data

Like all estimation commands, `anova` can produce estimates on weighted data. See [U] 11.1.6 [weight](#) for details on specifying the weight.

► Example 8: Three-way factorial ANOVA on grouped data

We wish to investigate the prevalence of byssinosis, a form of pneumoconiosis that can afflict workers exposed to cotton dust. We have data on 5,419 workers in a large cotton mill. We know whether each worker smokes, his or her race, and the dustiness of the work area. The variables are

smokes smoker or nonsmoker in the last five years
 race white or other
 workplace 1 (most dusty), 2 (less dusty), 3 (least dusty)

We wish to fit an ANOVA model explaining the prevalence of byssinosis according to a full factorial model of smokes, race, and workplace.

The data are unbalanced. Moreover, although we have data on 5,419 workers, the data are grouped according to the explanatory variables, along with some other variables, resulting in 72 observations. For each observation, we know the number of workers in the group (pop), the prevalence of byssinosis (prob), and the values of the three explanatory variables. Thus, we wish to fit a three-way factorial model on grouped data.

We begin by showing a bit of the data, which are from [Higgins and Koch \(1977\)](#).

```
. use https://www.stata-press.com/data/r19/byssin
(Byssinosis incidence)
. describe
Contains data from https://www.stata-press.com/data/r19/byssin.dta
Observations:      72                Byssinosis incidence
Variables:         5                  19 Dec 2024 07:04
```

Variable name	Storage type	Display format	Value label	Variable label
smokes	byte	%8.0g	smokes	Smokes
race	byte	%8.0g	race	Race
workplace	byte	%8.0g	workplace	Dustiness of workplace
pop	int	%8.0g		Population size
prob	float	%9.0g		Prevalence of byssinosis

Sorted by:

```
. list in 1/5, abbrev(10) divider
```

	smokes	race	workplace	pop	prob
1.	Yes	White	Most	40	.075
2.	Yes	White	Less	74	0
3.	Yes	White	Least	260	.0076923
4.	Yes	Other	Most	164	.152439
5.	Yes	Other	Less	88	0

The first observation in the data represents a group of 40 white workers who smoke and work in a “most” dusty work area. Of those 40 workers, 7.5% have byssinosis. The second observation represents a group of 74 white workers who also smoke but who work in a “less” dusty environment. None of those workers has byssinosis.

Almost every Stata command allows weights. Here we want to weight the data by pop. We can, for instance, make a table of the number of workers by their smoking status and race:

```
. tabulate smokes race [fw=pop]
```

Smokes	Race		Total
	Other	White	
No	799	1,431	2,230
Yes	1,104	2,085	3,189
Total	1,903	3,516	5,419

The [fw=pop] at the end of the tabulate command tells Stata to count each observation as representing pop persons. When making the tally, tabulate treats the first observation as representing 40 workers, the second as representing 74 workers, and so on.

Similarly, we can make a table of the dustiness of the workplace:

```
. tabulate workplace [fw=pop]
```

Dustiness of workplace	Freq.	Percent	Cum.
Least	3,450	63.66	63.66
Less	1,300	23.99	87.65
Most	669	12.35	100.00
Total	5,419	100.00	

We can discover the average incidence of byssinosis among these workers by typing

```
. summarize prob [fw=pop]
```

Variable	Obs	Mean	Std. dev.	Min	Max
prob	5,419	.0304484	.0567373	0	.287037

We discover that 3.04% of these workers have byssinosis. Across all cells, the byssinosis rates vary from 0 to 28.7%. Just to prove that there might be something here, let's obtain the average incidence rates according to the dustiness of the workplace:

```
. table (workplace) (race smokes) [fw=pop], statistic(mean prob) nototals
```

Dustiness of workplace	Race			
	Other Smokes		White Smokes	
	No	Yes	No	Yes
Least	.0107527	.0101523	.0081549	.0162774
Less	.02	.0081633	.0136612	.0143149
Most	.0820896	.1679105	.0833333	.2295082

Let's now fit the ANOVA model.

```
. anova prob workplace smokes race workplace#smokes workplace#race smokes#race
> workplace#smokes#race [aweight=pop]
(sum of wgt is 5,419)
```

Source	Partial SS	df	MS	F	Prob>F
Model	.17364654	11	.01578605	23.53	0.0000
workplace	.09762518	2	.04881259	72.76	0.0000
smokes	.01303081	1	.01303081	19.42	0.0001
race	.00109472	1	.00109472	1.63	0.2070
workplace#smokes	.01969034	2	.00984517	14.67	0.0000
workplace#race	.00135252	2	.00067626	1.01	0.3718
smokes#race	.00166287	1	.00166287	2.48	0.1214
workplace#smokes#race	.00095084	2	.00047542	0.71	0.4969
Residual	.03555777	53	.0006709		
Total	.2092043	64	.00326882		

Of course, if we want to see the underlying regression, we could type `regress`.

Above, we examined simple means of the cells of `workplace#smokes#race`. Our ANOVA shows `workplace`, `smokes`, and their interaction as being the only significant factors in our model. We now examine the predictive marginal mean byssinosis rates for these terms.

```
. margins workplace#smokes workplace smokes
Predictive margins                                Number of obs = 65
Expression: Linear prediction, predict()
```

	Delta-method				
	Margin	std. err.	t	P> t	[95% conf. interval]
workplace# smokes					
Least#No	.0090672	.0062319	1.45	0.152	-.0034323 .0215667
Least#Yes	.0141264	.0053231	2.65	0.010	.0034497 .0248032
Less#No	.0158872	.009941	1.60	0.116	-.0040518 .0358263
Less#Yes	.0121546	.0087353	1.39	0.170	-.0053662 .0296755
Most#No	.0828966	.0182151	4.55	0.000	.0463617 .1194314
Most#Yes	.2078768	.012426	16.73	0.000	.1829533 .2328003
workplace					
Least	.0120701	.0040471	2.98	0.004	.0039526 .0201875
Less	.0137273	.0065685	2.09	0.041	.0005526 .0269019
Most	.1566225	.0104602	14.97	0.000	.1356419 .177603
smokes					
No	.0196915	.0050298	3.91	0.000	.0096029 .02978
Yes	.0358626	.0041949	8.55	0.000	.0274488 .0442765

Smoking combined with the most dusty workplace produces the highest byssinosis rates.

[Ronald Aylmer Fisher](#) (1890–1962) (Sir Ronald from 1952) studied mathematics at Cambridge. Even before he finished his studies, he had published on statistics. He worked as a statistician at Rothamsted Experimental Station (1919–1933), as professor of eugenics at University College London (1933–1943), as professor of genetics at Cambridge (1943–1957), and in retirement at the CSIRO Division of Mathematical Statistics in Adelaide. His many fundamental and applied contributions to statistics and genetics mark him as one of the greatest statisticians of all time, including original work on tests of significance, distribution theory, theory of estimation, fiducial inference, and design of experiments.

ANCOVA

You can include multiple explanatory variables with the `anova` command, but unless you explicitly state otherwise by using the `c.` factor-variable operator, all the variables are interpreted as *categorical variables*. Using the `c.` operator, you can designate variables as *continuous* and thus perform ANCOVA.

▷ Example 9: ANCOVA (ANOVA with a continuous covariate)

We have census data recording the deathrate (`drate`) and median age (`age`) for each state. The dataset also includes the region of the country in which each state is located (`region`):

```
. use https://www.stata-press.com/data/r19/census2
(1980 Census data by state)
. summarize drate age region
```

Variable	Obs	Mean	Std. dev.	Min	Max
drate	50	84.3	13.07318	40	107
age	50	29.5	1.752549	24	35
region	50	2.66	1.061574	1	4

`age` is coded in integral years from 24 to 35, and `region` is coded from 1 to 4, with 1 standing for the Northeast, 2 for the North Central, 3 for the South, and 4 for the West.

When we examine the data more closely, we discover large differences in the deathrate across regions of the country:

```
. tabulate region, summarize(drate)
```

Census region	Summary of Deathrate		Freq.
	Mean	Std. dev.	
NE	93.444444	7.0553368	9
N Cntrl	88.916667	5.5833899	12
South	88.3125	8.5457104	16
West	68.769231	13.342625	13
Total	84.3	13.073185	50

Naturally, we wonder if these differences might not be explained by differences in the median ages of the populations. To find out, we fit a regression model (via `anova`) of `drate` on `region` and `age`. In the `anova` example below, we treat `age` as a categorical variable.

```
. anova drate region age
```

	Number of obs =	50	R-squared =	0.7927	
	Root MSE =	6.7583	Adj R-squared =	0.7328	
Source	Partial SS	df	MS	F	Prob>F
Model	6638.8653	11	603.53321	13.21	0.0000
region	1320.0097	3	440.00324	9.63	0.0001
age	2237.2494	8	279.65617	6.12	0.0000
Residual	1735.6347	38	45.674598		
Total	8374.5	49	170.90816		

We have the answer to our question: differences in median ages do not eliminate the differences in deathrates across the four regions. The ANOVA table summarizes the two terms in the model, `region` and `age`. The `region` term contains 3 degrees of freedom, and the `age` term contains 8 degrees of freedom. Both are significant at better than the 1% level.

The `age` term contains 8 degrees of freedom. Because we did not explicitly indicate that `age` was to be treated as a continuous variable, it was treated as *categorical*, meaning that unique coefficients were estimated for each level of `age`. The only clue of this labeling is that the number of degrees of freedom associated with the `age` term exceeds 1. The labeling becomes more obvious if we review the regression coefficients:

```
. regress, baselevels
```

Source	SS	df	MS	Number of obs =	50
Model	6638.86529	11	603.533208	F(11, 38) =	13.21
Residual	1735.63471	38	45.6745977	Prob > F =	0.0000
Total	8374.5	49	170.908163	R-squared =	0.7927
				Adj R-squared =	0.7328
				Root MSE =	6.7583

drate	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
region						
NE	0	(base)				
N Cntrl	.4428387	3.983664	0.11	0.912	-7.621668	8.507345
South	-.2964637	3.934766	-0.08	0.940	-8.261981	7.669054
West	-13.37147	4.195344	-3.19	0.003	-21.8645	-4.878439
age						
24	0	(base)				
26	-15	9.557677	-1.57	0.125	-34.34851	4.348506
27	14.30833	7.857378	1.82	0.076	-1.598099	30.21476
28	12.66011	7.495513	1.69	0.099	-2.51376	27.83399
29	18.861	7.28918	2.59	0.014	4.104825	33.61717
30	20.87003	7.210148	2.89	0.006	6.273847	35.46621
31	29.91307	8.242741	3.63	0.001	13.22652	46.59963
32	27.02853	8.509432	3.18	0.003	9.802089	44.25498
35	38.925	9.944825	3.91	0.000	18.79275	59.05724
_cons	68.37147	7.95459	8.60	0.000	52.26824	84.47469

The `regress` command displayed the anova model as a regression table. We used the `baselevels` option to display the dropped level (or base) for each term.

If we want to treat age as a continuous variable, we must prepend `c.` to `age` in our `anova`.

```
. anova drate region c.age
```

	Number of obs =	50	R-squared =	0.7203	
	Root MSE =	7.21483	Adj R-squared =	0.6954	
Source	Partial SS	df	MS	F	Prob>F
Model	6032.0825	4	1508.0206	28.97	0.0000
region	1645.6623	3	548.55409	10.54	0.0000
age	1630.4666	1	1630.4666	31.32	0.0000
Residual	2342.4175	45	52.053721		
Total	8374.5	49	170.90816		

The `age` term now has 1 degree of freedom. The regression coefficients are

```
. regress, baselevels
```

Source	SS	df	MS	Number of obs =	50
Model	6032.08254	4	1508.02064	F(4, 45) =	28.97
Residual	2342.41746	45	52.0537213	Prob > F =	0.0000
Total	8374.5	49	170.908163	R-squared =	0.7203
				Adj R-squared =	0.6954
				Root MSE =	7.2148

drate	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
region	0 (base)					
NE						
N Cntrl	1.792526	3.375925	0.53	0.598	-5.006935	8.591988
South	.6979912	3.18154	0.22	0.827	-5.70996	7.105942
West	-13.37578	3.723447	-3.59	0.001	-20.87519	-5.876377
age	3.922947	.7009425	5.60	0.000	2.511177	5.334718
_cons	-28.60281	21.93931	-1.30	0.199	-72.79085	15.58524

Although we started analyzing these data to explain the regional differences in deathrate, let’s focus on the effect of `age` for a moment. In our first model, each level of `age` had a unique deathrate associated with it. For instance, the predicted deathrate in a north central state with a median `age` of 28 was

$$0.44 + 12.66 + 68.37 \approx 81.47$$

whereas the predicted deathrate from our current model is

$$1.79 + 3.92 \times 28 - 28.60 \approx 82.95$$

Our previous model had an R^2 of 0.7927, whereas our current model has an R^2 of 0.7203. This “small” loss of predictive power accompanies a gain of 7 degrees of freedom, so we suspect that the continuous-age model is as good as the discrete-age model.



□ Technical note

There is enough information in the two ANOVA tables to attach a statistical significance to our suspicion that the loss of predictive power is offset by the savings in degrees of freedom. Because the continuous-age model is nested within the discrete-age model, we can perform a standard Chow test. For those of us who know such formulas off the top of our heads, the F statistic is

$$\frac{(2342.41746 - 1735.63471)/7}{45.6745977} = 1.90$$

There is, however, a better way.

We can find out whether our continuous model is as good as our discrete model by putting age in the model twice: once as a continuous variable and once as a categorical variable. The categorical variable will then measure deviations around the straight line implied by the continuous variable, and the F test for the significance of the categorical variable will test whether those deviations are jointly zero.

```
. anova drate region c.age age
```

	Number of obs =	50	R-squared =	0.7927	
	Root MSE =	6.7583	Adj R-squared =	0.7328	
Source	Partial SS	df	MS	F	Prob>F
Model	6638.8653	11	603.53321	13.21	0.0000
region	1320.0097	3	440.00324	9.63	0.0001
age	699.74137	1	699.74137	15.32	0.0004
age	606.78275	7	86.68325	1.90	0.0970
Residual	1735.6347	38	45.674598		
Total	8374.5	49	170.90816		

We find that the F test for the significance of the (categorical) age variable is 1.90, just as we calculated above. It is significant at the 9.7% level. If we hold to a 5% significance level, we cannot reject the null hypothesis that the effect of age is linear. □

► Example 10: Interaction of continuous and categorical variables

In our census data, we still find significant differences across the regions after controlling for the median age of the population. We might now wonder whether the regional differences are differences in level—*independent of age*—or are instead differences in the regional effects of age. Just as we can interact categorical variables with other categorical variables, we can interact categorical variables with continuous variables.

```
. anova drate region c.age region#c.age
```

	Number of obs =	50	R-squared =	0.7365	
	Root MSE =	7.24852	Adj R-squared =	0.6926	
Source	Partial SS	df	MS	F	Prob>F
Model	6167.7737	7	881.11053	16.77	0.0000
region	188.7136	3	62.904534	1.20	0.3225
age	873.4256	1	873.4256	16.62	0.0002
region#age	135.69116	3	45.230387	0.86	0.4689
Residual	2206.7263	42	52.541102		
Total	8374.5	49	170.90816		

The `region#c.age` term in our model measures the differences in slopes across the regions. We cannot reject the null hypothesis that there are no such differences. The region effect is now “insignificant”. This status does not mean that there are no regional differences in deathrates because each test is a

marginal or *partial* test. Here, with `region#c.age` included in the model, `region` is being tested at the point where `age` is zero. Apart from this value not existing in the dataset, it is also a long way from the mean value of `age`, so the test of `region` at this point is meaningless (although it is valid if you acknowledge what is being tested).

To obtain a more sensible test of `region`, we can subtract the mean from the `age` variable and use this in the model.

```
. quietly summarize age
. generate mage = age - r(mean)
. anova drate region c.mage region#c.mage
```

Source	Partial SS	df	MS	F	Prob>F
Model	6167.7737	7	881.11053	16.77	0.0000
region	1166.1473	3	388.71578	7.40	0.0004
mage	873.4256	1	873.4256	16.62	0.0002
region#mage	135.69116	3	45.230387	0.86	0.4689
Residual	2206.7263	42	52.541102		
Total	8374.5	49	170.90816		

`region` is significant when tested at the mean of the `age` variable.



Remember that we can specify interactions by typing `varname#varname`. We have seen examples of interacting categorical variables with categorical variables and, in the examples above, a categorical variable (`region`) with a continuous variable (`age` or `mage`).

We can also interact continuous variables with continuous variables. To include an age^2 term in our model, we could type `c.age#c.age`. If we also wanted to interact the categorical variable `region` with the age^2 term, we could type `region#c.age#c.age` (or even `c.age#region#c.age`).

Nested designs

In addition to specifying interaction terms, nested terms can also be specified in an ANOVA. A vertical bar is used to indicate nesting: `A|B` is read as `A` nested within `B`. `A|B|C` is read as `A` nested within `B`, which is nested within `C`. `A|B#C` is read as `A` is nested within the interaction of `B` and `C`. `A#B|C` is read as the interaction of `A` and `B`, which is nested within `C`.

Different error terms can be specified for different parts of the model. The forward slash is used to indicate that the next term in the model is the error term for what precedes it. For instance, `anova y A / B|A` indicates that the F test for `A` is to be tested by using the mean square from `B|A` in the denominator. Error terms (terms following the slash) are generally not tested unless they are themselves followed by a slash. Residual error is the default error term.

For example, consider $A / B / C$, where A , B , and C may be arbitrarily complex terms. Then, `anova` will report A tested by B and B tested by C . If we add one more slash on the end to form $A / B / C /$, then `anova` will also report C tested by the residual error.

▷ Example 11: Simple nested ANOVA

We have collected data from a manufacturer that is evaluating which of five different brands of machinery to buy to perform a particular function in an assembly line. Twenty assembly-line employees were selected at random for training on these machines, with four employees assigned to learn a particular machine. The output from each employee (operator) on the brand of machine for which he trained was measured during four trial periods. In this example, the operator is nested within machine. Because of sickness and employee resignations, the final data are not balanced. The following table gives the mean output and sample size for each machine and operator combination.

```
. use https://www.stata-press.com/data/r19/machine, clear
(Machine data)
. table machine operator, statistic(mean output) statistic(freq)
> totals(machine) nformat(%8.2f mean)
```

		Operator nested in machine				
		1	2	3	4	Total
Five brands of machine						
1	Mean	9.15	9.48	8.27	8.20	8.75
	Frequency	2	4	3	4	13
2	Mean	15.03	11.55	11.45	11.53	12.47
	Frequency	3	2	2	4	11
3	Mean	11.27	10.13	11.13		10.84
	Frequency	3	3	3		9
4	Mean	16.10	18.97	15.35	16.60	16.65
	Frequency	3	3	4	3	13
5	Mean	15.30	14.35	10.43		13.63
	Frequency	4	4	3		11

Assuming that operator is random (that is, we wish to infer to the larger population of possible operators) and machine is fixed (that is, only these five machines are of interest), the typical test for machine uses operator nested within machine as the error term. operator nested within machine can be tested by residual error. Our earlier warning concerning designs with either unplanned missing cells or unbalanced cell sizes, or both, also applies to interpreting the ANOVA results from this unbalanced nested example.

```

. anova output machine / operator|machine /
              Number of obs =          57    R-squared      = 0.8661
              Root MSE   =    1.47089    Adj R-squared = 0.8077

```

Source	Partial SS	df	MS	F	Prob>F
Model	545.82229	17	32.107193	14.84	0.0000
machine	430.98079	4	107.7452	13.82	0.0001
operator machine	101.3538	13	7.7964465		
operator machine	101.3538	13	7.7964465	3.60	0.0009
Residual	84.376658	39	2.1635041		
Total	630.19895	56	11.253553		

operator|machine is preceded by a slash, indicating that it is the error term for the terms before it (here machine). operator|machine is also followed by a slash that indicates it should be tested with residual error. The output lists the operator|machine term twice, once as the error term for machine and again as a term tested by residual error. A line is placed in the ANOVA table to separate the two. In general, a dividing line is placed in the output to separate the terms into groups that are tested with the same error term. The overall model is tested by residual error and is separated from the rest of the table by a blank line at the top of the table.

The results indicate that the machines are not all equal and that there are significant differences between operators.

◀

▷ Example 12: ANOVA with multiple levels of nesting

Your company builds and operates sewage treatment facilities. You want to compare two particulate solutions during the particulate reduction step of the sewage treatment process. For each solution, two area managers are randomly selected to implement and oversee the change to the new treatment process in two of their randomly chosen facilities. Two workers at each of these facilities are trained to operate the new process. A measure of particulate reduction is recorded at various times during the month at each facility for each worker. The data are described below.

```

. use https://www.stata-press.com/data/r19/sewage
(Sewage treatment)
. describe
Contains data from https://www.stata-press.com/data/r19/sewage.dta
Observations:      64      Sewage treatment
Variables:         5      9 May 2024 12:43

```

Variable name	Storage type	Display format	Value label	Variable label
particulate	byte	%9.0g		Particulate reduction
solution	byte	%9.0g		2 particulate solutions
manager	byte	%9.0g		2 managers per solution
facility	byte	%9.0g		2 facilities per manager
worker	byte	%9.0g		2 workers per facility

```

Sorted by: solution manager facility worker

```

You want to determine if the two particulate solutions provide significantly different particulate reduction. You would also like to know if `manager`, `facility`, and `worker` are significant effects. `solution` is a fixed factor, whereas `manager`, `facility`, and `worker` are random factors.

In the following `anova` command, we use abbreviations for the variable names, which can sometimes make long ANOVA model statements easier to read.

```
. anova particulate s / m|s / f|m|s / w|f|m|s /, dropemptycells
```

Source	Partial SS	df	MS	F	Prob>F
Model	13493.609	15	899.57396	5.54	0.0000
solution	7203.7656	1	7203.7656	17.19	0.0536
manager solution	838.28125	2	419.14063		
manager solution	838.28125	2	419.14063	0.55	0.6166
facility manager solution	3064.9375	4	766.23438		
facility manager solution	3064.9375	4	766.23438	2.57	0.1193
worker facility manager solution	2386.625	8	298.32813		
worker facility manager solution	2386.625	8	298.32813	1.84	0.0931
Residual	7796.25	48	162.42188		
Total	21289.859	63	337.93428		

While `solution` is not declared significant at the 5% significance level, it is near enough to that threshold to warrant further investigation (see [example 3](#) in [\[R\] anova postestimation](#) for a continuation of the analysis of these data).

◀

□ Technical note

Why did we use the `dropemptycells` option with the previous `anova`? By default, Stata retains empty cells when building the design matrix and currently treats `|` and `#` the same in how it determines the possible number of cells. Retaining empty cells in an ANOVA with nested terms can cause your design matrix to become too large. In [example 12](#), there are $1024 = 2 \times 4 \times 8 \times 16$ cells that are considered possible for the `worker|facility|manager|solution` term because the `worker`, `facility`, and `manager` variables are uniquely numbered. With the `dropemptycells` option, the `worker|facility|manager|solution` term requires just 16 columns in the design matrix (corresponding to the 16 unique workers).

Why did we not use the `dropemptycells` option in [example 11](#), where `operator` is nested in `machine`? If you look at the table presented at the beginning of that example, you will see that `operator` is compactly instead of uniquely numbered (you need both `operator` number and `machine` number to determine the operator). Here the `dropemptycells` option would have only reduced our design matrix from 26 columns down to 24 columns (because there were only 3 operators instead of 4 for machines 3 and 5).

We suggest that you specify `dropemptycells` when there are nested terms in your ANOVA. You could also use the `set emptycells drop` command to accomplish the same thing; see [R] [set](#). □

Mixed designs

An ANOVA can consist of both nested and crossed terms. A split-plot ANOVA design provides an example.

▷ Example 13: Split-plot ANOVA

Two reading programs and three skill-enhancement techniques are under investigation. Ten classes of first-grade students were randomly assigned so that five classes were taught with one reading program and another five classes were taught with the other. The 30 students in each class were divided into six groups with 5 students each. Within each class, the six groups were divided randomly so that each of the three skill-enhancement techniques was taught to two of the groups within each class. At the end of the school year, a reading assessment test was administered to all the students. In this split-plot ANOVA, the whole-plot treatment is the two reading programs, and the split-plot treatment is the three skill-enhancement techniques.

```
. use https://www.stata-press.com/data/r19/reading
(Reading experiment data)
. describe
Contains data from https://www.stata-press.com/data/r19/reading.dta
Observations:      300      Reading experiment data
Variables:         5        9 Mar 2024 18:57
                        (_dta has notes)
```

Variable name	Storage type	Display format	Value label	Variable label
score	byte	%9.0g		Reading score
program	byte	%9.0g		Reading program
class	byte	%9.0g		Class nested in program
skill	byte	%9.0g		Skill enhancement technique
group	byte	%9.0g		Group nested in class and skill

Sorted by:

In this split-plot ANOVA, the error term for `program` is `class` nested within `program`. The error term for `skill` and the `program` by `skill` interaction is the `class` by `skill` interaction nested within `program`. Other terms are also involved in the model and can be seen below.

Our `anova` command is too long to fit on one line of this manual. Where we have chosen to break the command into multiple lines is arbitrary. If we were typing this command into Stata, we would just type along and let Stata automatically wrap across lines, as necessary.

```

. anova score prog / class|prog skill prog#skill / class#skill|prog /
> group|class#skill|prog /, dropemptycells

```

Source	Partial SS	df	MS	F	Prob>F
Model	30656.517	59	519.60198	2.43	0.0000
program	4493.07	1	4493.07	8.73	0.0183
class program	4116.6133	8	514.57667		
skill	1122.6467	2	561.32333	1.54	0.2450
program#skill	5694.62	2	2847.31	7.80	0.0043
class#skill program	5841.4667	16	365.09167		
class#skill program	5841.4667	16	365.09167	1.17	0.3463
group class#skill					
program	9388.1	30	312.93667		
group class#skill					
program	9388.1	30	312.93667	1.46	0.0636
Residual	51346.4	240	213.94333		
Total	82002.917	299	274.25725		

The program#skill term is significant, as is the program term. Let's look at the predictive margins for these two terms and at a marginsplot for the first term.

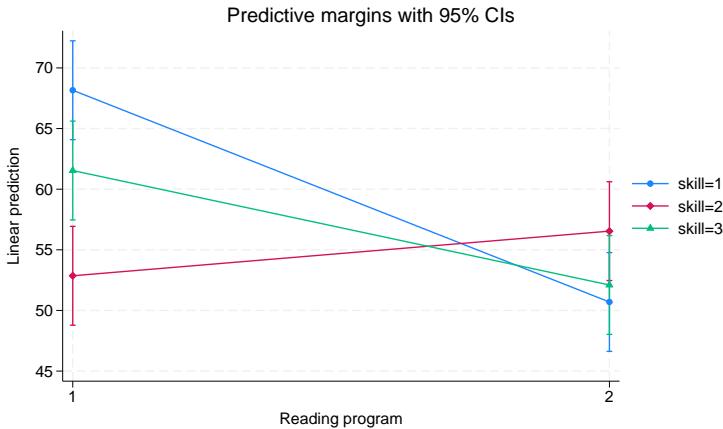
```

. margins, within(program skill)
Predictive margins                                Number of obs = 300
Expression: Linear prediction, predict()
Within:      program skill
Empty cells: reweight

```

	Delta-method		t	P> t	[95% conf. interval]	
	Margin	std. err.				
program#skill						
1 1	68.16	2.068542	32.95	0.000	64.08518	72.23482
1 2	52.86	2.068542	25.55	0.000	48.78518	56.93482
1 3	61.54	2.068542	29.75	0.000	57.46518	65.61482
2 1	50.7	2.068542	24.51	0.000	46.62518	54.77482
2 2	56.54	2.068542	27.33	0.000	52.46518	60.61482
2 3	52.1	2.068542	25.19	0.000	48.02518	56.17482

```
. marginsplot, plot2opts(m(D)) plot3opts(m(T))
Variables that uniquely identify margins: program skill
```



```
. margins, within(program)
Predictive margins                                     Number of obs = 300
Expression: Linear prediction, predict()
Within:      program
Empty cells: reweight
```

program	Delta-method				[95% conf. interval]	
	Margin	std. err.	t	P> t		
1	60.85333	1.194273	50.95	0.000	58.50074	63.20593
2	53.11333	1.194273	44.47	0.000	50.76074	55.46593

Because our ANOVA involves nested terms, we used the `within()` option of `margins`; see [R] [margins](#).

`skill 2` produces a low score when combined with `program 1` and a high score when combined with `program 2`, demonstrating the interaction between the reading program and the skill-enhancement technique. You might conclude that the first reading program and the first skill-enhancement technique perform best when combined. However, notice the overlapping confidence interval for the first reading program and the third skill-enhancement technique.

◀

□ Technical note

There are several valid ways to write complicated anova terms. In the reading experiment example (example 13), we had a term `group|class#skill|program`. This term can be read as `group` nested within both `class` and `skill` and further nested within `program`. You can also write this term as `group|class#skill#program` or `group|program#class#skill` or `group|skill#class|program`, etc. All variations will produce the same result. Some people prefer having only one ‘|’ in a term and would use `group|class#skill#program`, which is read as `group` nested within `class`, `skill`, and `program`.

□

Gertrude Mary Cox (1900–1978) was born on a farm near Dayton, Iowa. Initially intending to become superintendent of an orphanage, she enrolled at Iowa State College. There she majored in mathematics and attained the college’s first Master’s degree in statistics. After working on her PhD in psychological statistics for two years at the University of California–Berkeley, she decided to go back to Iowa State to work with George W. Snedecor. There she pursued her interest in and taught a course in design of experiments. That work led to her collaboration with W. G. Cochran, which produced a classic text. In 1940, when Snedecor shared with her his list of men he was nominating to head the statistics department at North Carolina State College, she wanted to know why she had not been included. He added her name, she won the position, and she built an outstanding department at North Carolina State. Cox retired early so she could work at the Research Triangle Institute in North Carolina. She consulted widely, served as editor of *Biometrics*, and was elected to the National Academy of Sciences.

Latin-square designs

You can use `anova` to analyze a Latin-square design. Consider the following example, published in [Snedecor and Cochran \(1989\)](#).

▷ Example 14: Latin-square ANOVA

Data from a Latin-square design are as follows:

Row	Column 1	Column 2	Column 3	Column 4	Column 5
1	257(B)	230(E)	279(A)	287(C)	202(D)
2	245(D)	283(A)	245(E)	280(B)	260(C)
3	182(E)	252(B)	280(C)	246(D)	250(A)
4	203(A)	204(C)	227(D)	193(E)	259(B)
5	231(C)	271(D)	266(B)	334(A)	338(E)

In Stata, the data might appear as follows:

```
. use https://www.stata-press.com/data/r19/latinsq
. list
```

	row	c1	c2	c3	c4	c5
1.	1	257	230	279	287	202
2.	2	245	283	245	280	260
3.	3	182	252	280	246	250
4.	4	203	204	227	193	259
5.	5	231	271	266	334	338

Before `anova` can be used on these data, the data must be organized so that the outcome measurement is in one column. `reshape` is inadequate for this task because there is information about the treatments in the sequence of these observations. `pkshape` is designed to reshape this type of data; see [\[R\] pkshape](#).

```
. pkshape row row c1-c5, order(beacd daebc ebcda acdeb cdbae)
. list
```

	sequence	outcome	treat	carry	period
1.	beacd	257	b	0	1
2.	daebc	245	d	0	1
3.	ebcda	182	e	0	1
4.	acdeb	203	a	0	1
5.	cdbae	231	c	0	1
6.	beacd	230	e	b	2
7.	daebc	283	a	d	2
8.	ebcda	252	b	e	2
9.	acdeb	204	c	a	2
10.	cdbae	271	d	c	2
11.	beacd	279	a	e	3
12.	daebc	245	e	a	3
13.	ebcda	280	c	b	3
14.	acdeb	227	d	c	3
15.	cdbae	266	b	d	3
16.	beacd	287	c	a	4
17.	daebc	280	b	e	4
18.	ebcda	246	d	c	4
19.	acdeb	193	e	d	4
20.	cdbae	334	a	b	4
21.	beacd	202	d	c	5
22.	daebc	260	c	b	5
23.	ebcda	250	a	d	5
24.	acdeb	259	b	e	5
25.	cdbae	338	e	a	5

```
. anova outcome sequence period treat
```

Number of obs = 25 R-squared = 0.6536
 Root MSE = 32.4901 Adj R-squared = 0.3073

Source	Partial SS	df	MS	F	Prob>F
Model	23904.08	12	1992.0067	1.89	0.1426
sequence	13601.36	4	3400.34	3.22	0.0516
period	6146.16	4	1536.54	1.46	0.2758
treat	4156.56	4	1039.14	0.98	0.4523
Residual	12667.28	12	1055.6067		
Total	36571.36	24	1523.8067		



These methods will work with any type of Latin-square design, including those with replicated measurements. For more information, see [R] [pk](#), [R] [pkcross](#), and [R] [pkshape](#).

Repeated-measures ANOVA

One approach for analyzing repeated-measures data is to use multivariate ANOVA (MANOVA); see [MV] [manova](#). In this approach, the data are placed in wide form (see [D] [reshape](#)), and the repeated measures enter the MANOVA as dependent variables.

A second approach for analyzing repeated measures is to use `anova`. However, one of the underlying assumptions for the F tests in ANOVA is independence of observations. In a repeated-measures design, this assumption is almost certainly violated. In a repeated-measures ANOVA, the subjects (or whatever the experimental units are called) are observed for each level of one or more of the other categorical variables in the model. These variables are called the repeated-measure variables. Observations from the same subject are likely to be correlated, though this is only a problem if the observations violate compound symmetry or the sphericity condition.

The approach used in repeated-measures ANOVA to correct for violation of compound symmetry or sphericity is to apply correction to the degrees of freedom of the F test for terms in the model that involve repeated measures. This correction factor, ϵ , lies between the reciprocal of the degrees of freedom for the repeated term and 1. [Box \(1954\)](#) provided the pioneering work in this area. [Milliken and Johnson \(2009\)](#) refer to the lower bound of this correction factor as Box’s conservative correction factor. [Winer, Brown, and Michels \(1991\)](#) call it simply the conservative correction factor.

[Geisser and Greenhouse \(1958\)](#) provide an estimate for the correction factor called the Greenhouse–Geisser ϵ . This value is estimated from the data. [Huynh and Feldt \(1976\)](#) show that the Greenhouse–Geisser ϵ tends to be conservatively biased. They provide a revised correction factor called the Huynh–Feldt ϵ . When the Huynh–Feldt ϵ exceeds 1, it is set to 1. Thus, there is a natural ordering for these correction factors:

$$\text{Box's conservative } \epsilon \leq \text{Greenhouse-Geisser } \epsilon \leq \text{Huynh-Feldt } \epsilon \leq 1$$

A correction factor of 1 is the same as no correction.

`anova` with the `repeated()` option computes these correction factors and displays the revised test results in a table that follows the standard ANOVA table. In the resulting table, H-F stands for Huynh–Feldt, G-G stands for Greenhouse–Geisser, and Box stands for Box’s conservative ϵ .

▷ Example 15: Repeated-measures ANOVA

This example is taken from table 4.3 of [Winer, Brown, and Michels \(1991\)](#). The reaction time for five subjects each tested with four drugs was recorded in the variable `score`. Here is a table of the data (see [P] [tabdisp](#) if you are unfamiliar with `tabdisp`):

```
. use https://www.stata-press.com/data/r19/t43, clear
(T4.3 -- Winer, Brown, Michels)
. tabdisp person drug, cellvar(score)
```

Person	Drug			
	1	2	3	4
1	30	28	16	34
2	14	18	10	22
3	24	20	18	30
4	38	34	20	44
5	26	28	14	30

drug is the repeated variable in this simple repeated-measures ANOVA example. The ANOVA is specified as follows:

```
. anova score person drug, repeated(drug)
      Number of obs =          20      R-squared      = 0.9244
      Root MSE      =    3.06594      Adj R-squared = 0.8803

      Source | Partial SS      df      MS      F      Prob>F
-----+-----+-----+-----+-----+-----
      Model |          1379      7      197      20.96  0.0000
      person |          680.8      4      170.2    18.11  0.0001
      drug   |          698.2      3    232.73333    24.76  0.0000
      Residual |          112.8     12      9.4
-----+-----+-----+-----+-----
      Total |         1491.8     19    78.515789

Between-subjects error term: person
      Levels: 5      (4 df)
      Lowest b.s.e. variable: person

Repeated variable: drug
```

```
      Huynh-Feldt epsilon      = 1.0789
      *Huynh-Feldt epsilon reset to 1.0000
      Greenhouse-Geisser epsilon = 0.6049
      Box's conservative epsilon = 0.3333
```

Source	df	F	Prob > F			
			Regular	H-F	G-G	Box
drug	3	24.76	0.0000	0.0000	0.0006	0.0076
Residual	12					

Here the Huynh–Feldt ϵ is 1.0789, which is larger than 1. It is reset to 1, which is the same as making no adjustment to the standard test computed in the main ANOVA table. The Greenhouse–Geisser ϵ is 0.6049, and its associated p -value is computed from an F ratio of 24.76 using 1.8147 ($= 3\epsilon$) and 7.2588 ($= 12\epsilon$) degrees of freedom. Box’s conservative ϵ is set equal to the reciprocal of the degrees of freedom for the repeated term. Here it is $1/3$, so Box’s conservative test is computed using 1 and 4 degrees of freedom for the observed F ratio of 24.76.

Even for Box’s conservative ϵ , drug is significant with a p -value of 0.0076. The following table gives the predictive marginal mean score (that is, response time) for each of the four drugs:

```
. margins drug
Predictive margins                                Number of obs = 20
Expression: Linear prediction, predict()
```

drug	Delta-method		t	P> t	[95% conf. interval]	
	Margin	std. err.				
1	26.4	1.371131	19.25	0.000	23.41256	29.38744
2	25.6	1.371131	18.67	0.000	22.61256	28.58744
3	15.6	1.371131	11.38	0.000	12.61256	18.58744
4	32	1.371131	23.34	0.000	29.01256	34.98744

The ANOVA table for this example provides an F test for person, but you should ignore it. An appropriate test for person would require replication (that is, multiple measurements for person and drug combinations). Also, without replication there is no test available for investigating the interaction between person and drug.

◀

▷ Example 16: Repeated-measures ANOVA with nesting

Table 7.7 of [Winer, Brown, and Michels \(1991\)](#) provides another repeated-measures ANOVA example. There are four dial shapes and two methods for calibrating dials. Subjects are nested within calibration method, and an accuracy score is obtained. The data are shown below.

```
. use https://www.stata-press.com/data/r19/t77
(T7.7 -- Winer, Brown, Michels)
. tabdisp shape subject calib, cell(score)
```

4 dial shapes	2 methods for calibrating dials and Subject nested in calib					
	1			2		
	1	2	3	1	2	3
1	0	3	4	4	5	7
2	0	1	3	2	4	5
3	5	5	6	7	6	8
4	3	4	2	8	6	9

The calibration method and dial shapes are fixed factors, whereas subjects are random. The appropriate test for calibration method uses the nested subject term as the error term. Both the dial shape and the interaction between dial shape and calibration method are tested with the dial shape by subject interaction nested within calibration method. Here we drop this term from the `anova` command, and it becomes residual error. The dial shape is the repeated variable because each subject is tested with all four dial shapes. Here is the `anova` command that produces the desired results:

```
. anova score calib / subject|calib shape calib#shape, repeated(shape)
```

Source	Partial SS	df	MS	F	Prob>F
Model	123.125	11	11.193182	9.06	0.0003
calib	51.041667	1	51.041667	11.89	0.0261
subject calib	17.166667	4	4.2916667		
shape	47.458333	3	15.819444	12.80	0.0005
calib#shape	7.4583333	3	2.4861111	2.01	0.1662
Residual	14.833333	12	1.2361111		
Total	137.95833	23	5.9981884		

```

Between-subjects error term: subject|calib
                             Levels: 6      (4 df)
Lowest b.s.e. variable:     subject
Covariance pooled over:    calib      (for repeated variable)
Repeated variable:         shape

Huynh-Feldt epsilon        = 0.8483
Greenhouse-Geisser epsilon = 0.4751
Box's conservative epsilon = 0.3333
    
```

Source	df	F	Prob > F			
			Regular	H-F	G-G	Box
shape	3	12.80	0.0005	0.0011	0.0099	0.0232
calib#shape	3	2.01	0.1662	0.1791	0.2152	0.2291
Residual	12					

The repeated-measure ϵ corrections are applied to any terms that are tested in the main ANOVA table and have the repeated variable in the term. These ϵ corrections are given in a table below the main ANOVA table. Here the repeated-measures tests for shape and calib#shape are presented.

Calibration method is significant, as is dial shape. The interaction between calibration method and dial shape is not significant. The repeated-measure ϵ corrections do not change these conclusions, but they do change the significance level for the tests on shape and calib#shape. Here, though, unlike in the [example 15](#), the Huynh–Feldt ϵ is less than 1.

Here are the predictive marginal mean scores for calibration method and dial shapes. Because the interaction was not significant, we request only the calib and shape predictive margins.

```

. margins, within(calib)
Predictive margins                                Number of obs = 24
Expression:   Linear prediction, predict()
Within:      calib
Empty cells: reweight
    
```

	Delta-method					
	Margin	std. err.	t	P> t	[95% conf. interval]	
calib						
1	3	.3209506	9.35	0.000	2.300709	3.699291
2	5.916667	.3209506	18.43	0.000	5.217375	6.615958

```

. margins, within(shape)
Predictive margins                                Number of obs = 24
Expression:   Linear prediction, predict()
Within:      shape
Empty cells: reweight
    
```

	Delta-method					
	Margin	std. err.	t	P> t	[95% conf. interval]	
shape						
1	3.833333	.4538926	8.45	0.000	2.844386	4.82228
2	2.5	.4538926	5.51	0.000	1.511053	3.488947
3	6.166667	.4538926	13.59	0.000	5.17772	7.155614
4	5.333333	.4538926	11.75	0.000	4.344386	6.32228

□ Technical note

The computation of the Greenhouse–Geisser and Huynh–Feldt epsilons in a repeated-measures ANOVA requires the number of levels and degrees of freedom for the between-subjects error term, as well as a value computed from a pooled covariance matrix. The observations are grouped based on all but the lowest-level variable in the between-subjects error term. The covariance over the repeated variables is computed for each resulting group, and then these covariance matrices are pooled. The dimension of the pooled covariance matrix is the number of levels of the repeated variable (or combination of levels for multiple repeated variables). In [example 16](#), there are four levels of the repeated variable (`shape`), so the resulting covariance matrix is 4×4 .

The `anova` command automatically attempts to determine the between-subjects error term and the lowest-level variable in the between-subjects error term to group the observations for computation of the pooled covariance matrix. `anova` issues an error message indicating that the `bse()` or `bseunit()` option is required when `anova` cannot determine them. You may override the default selections of `anova` by specifying the `bse()`, `bseunit()`, or `grouping()` option. The term specified in the `bse()` option must be a term in the ANOVA model.

The default selection for the between-subjects error term (the `bse()` option) is the interaction of the nonrepeated categorical variables in the ANOVA model. The first variable listed in the between-subjects error term is automatically selected as the lowest-level variable in the between-subjects error term but can be overridden with the `bseunit(varname)` option. `varname` is often a term, such as `subject` or `subsample within subject`, and is most often listed first in the term because of the nesting notation of ANOVA. This term makes sense in most repeated-measures ANOVA designs when the terms of the model are written in standard form. For instance, in [example 16](#), there were three categorical variables (`subject`, `calib`, and `shape`), with `shape` being the repeated variable. Here `anova` looked for a term involving only `subject` and `calib` to determine the between-subjects error term. It found `subject|calib` as the term with six levels and 4 degrees of freedom. `anova` then picked `subject` as the default for the `bseunit()` option (the lowest variable in the between-subjects error term) because it was listed first in the term.

The grouping of observations proceeds, based on the different combinations of values of the variables in the between-subjects error term, excluding the lowest level variable (as found by default or as specified with the `bseunit()` option). You may specify the `grouping()` option to change the default grouping used in computing the pooled covariance matrix.

The between-subjects error term, number of levels, degrees of freedom, lowest variable in the term, and grouping information are presented after the main ANOVA table and before the rest of the repeated-measures output.

□

▷ Example 17: Repeated-measures ANOVA with two repeated variables

Data with two repeated variables are given in table 7.13 of [Winer, Brown, and Michels \(1991\)](#). The accuracy scores of subjects making adjustments to three dials during three different periods are recorded. Three subjects are exposed to a certain noise background level, whereas a different set of three subjects is exposed to a different noise background level. Here is a table of accuracy scores for the `noise`, `subject`, `period`, and `dial` variables:

```
. use https://www.stata-press.com/data/r19/t713
(T7.13 -- Winer, Brown, Michels)
. tabdisp subject dial period, by(noise) cell(score) stubwidth(11)
```

Noise background and Subject nested in noise		10 minute time periods and Type of dial								
		1			2			3		
		1	2	3	1	2	3	1	2	3
1	1	45	53	60	40	52	57	28	37	46
	2	35	41	50	30	37	47	25	32	41
	3	60	65	75	58	54	70	40	47	50
2	1	50	48	61	25	34	51	16	23	35
	2	42	45	55	30	37	43	22	27	37
	3	56	60	77	40	39	57	31	29	46

noise, period, and dial are fixed, whereas subject is random. Both period and dial are repeated variables. The ANOVA for this example is specified next.

```
. anova score noise / subject|noise period noise#period /
> period#subject|noise dial noise#dial /
> dial#subject|noise period#dial noise#period#dial, repeated(period dial)
```

Number of obs = 54 R-squared = 0.9872
Root MSE = 2.81859 Adj R-squared = 0.9576

Source	Partial SS	df	MS	F	Prob>F
Model	9797.7222	37	264.8033	33.33	0.0000
noise	468.16667	1	468.16667	0.75	0.4348
subject noise	2491.1111	4	622.77778		
period	3722.3333	2	1861.1667	63.39	0.0000
noise#period	333	2	166.5	5.67	0.0293
period#subject noise	234.88889	8	29.361111		
dial	2370.3333	2	1185.1667	89.82	0.0000
noise#dial	50.333333	2	25.166667	1.91	0.2102
dial#subject noise	105.55556	8	13.194444		
period#dial	10.666667	4	2.6666667	0.34	0.8499
noise#period#dial	11.333333	4	2.8333333	0.36	0.8357
Residual	127.11111	16	7.9444444		
Total	9924.8333	53	187.26101		

Between-subjects error term: subject|noise
 Levels: 6 (4 df)
 Lowest b.s.e. variable: subject
 Covariance pooled over: noise (for repeated variables)
 Repeated variable: period

Huynh-Feldt epsilon = 1.0668
 *Huynh-Feldt epsilon reset to 1.0000
 Greenhouse-Geisser epsilon = 0.6476
 Box's conservative epsilon = 0.5000

Source	df	F	Prob > F			
			Regular	H-F	G-G	Box
period	2	63.39	0.0000	0.0000	0.0003	0.0013
noise#period	2	5.67	0.0293	0.0293	0.0569	0.0759
period#subject noise	8					

Repeated variable: dial

Huynh-Feldt epsilon = 2.0788
 *Huynh-Feldt epsilon reset to 1.0000
 Greenhouse-Geisser epsilon = 0.9171
 Box's conservative epsilon = 0.5000

Source	df	F	Prob > F			
			Regular	H-F	G-G	Box
dial	2	89.82	0.0000	0.0000	0.0000	0.0007
noise#dial	2	1.91	0.2102	0.2102	0.2152	0.2394
dial#subject noise	8					

Repeated variables: period#dial

Huynh-Feldt epsilon = 1.3258
 *Huynh-Feldt epsilon reset to 1.0000
 Greenhouse-Geisser epsilon = 0.5134
 Box's conservative epsilon = 0.2500

Source	df	F	Prob > F			
			Regular	H-F	G-G	Box
period#dial	4	0.34	0.8499	0.8499	0.7295	0.5934
noise#period#dial	4	0.36	0.8357	0.8357	0.7156	0.5825
Residual	16					

For each repeated variable and for each combination of interactions of repeated variables, there are different ϵ correction values. The anova command produces tables for each applicable combination.

The two most significant factors in this model appear to be dial and period. The noise by period interaction may also be significant, depending on the correction factor you use. Below is a table of predictive margins for the accuracy score for dial, period, and noise by period.

```
. margins, within(dial)
Predictive margins                                Number of obs = 54
Expression:   Linear prediction, predict()
Within:      dial
Empty cells:  reweight
```

	Delta-method				
	Margin	std. err.	t	P> t	[95% conf. interval]
dial					
1	37.38889	.6643478	56.28	0.000	35.98053 38.79724
2	42.22222	.6643478	63.55	0.000	40.81387 43.63058
3	53.22222	.6643478	80.11	0.000	51.81387 54.63058

```
. margins, within(period)
Predictive margins                                Number of obs = 54
Expression:   Linear prediction, predict()
Within:      period
Empty cells:  reweight
```

	Delta-method				
	Margin	std. err.	t	P> t	[95% conf. interval]
period					
1	54.33333	.6643478	81.78	0.000	52.92498 55.74169
2	44.5	.6643478	66.98	0.000	43.09165 45.90835
3	34	.6643478	51.18	0.000	32.59165 35.40835

```
. margins, within(noise period)
Predictive margins                                Number of obs = 54
Expression:   Linear prediction, predict()
Within:      noise period
Empty cells:  reweight
```

	Delta-method				
	Margin	std. err.	t	P> t	[95% conf. interval]
noise#period					
1 1	53.77778	.9395297	57.24	0.000	51.78606 55.76949
1 2	49.44444	.9395297	52.63	0.000	47.45273 51.43616
1 3	38.44444	.9395297	40.92	0.000	36.45273 40.43616
2 1	54.88889	.9395297	58.42	0.000	52.89717 56.8806
2 2	39.55556	.9395297	42.10	0.000	37.56384 41.54727
2 3	29.55556	.9395297	31.46	0.000	27.56384 31.54727

Dial shape 3 produces the highest score, and scores decrease over the periods.



Example 17 had two repeated-measurement variables. Up to four repeated-measurement variables may be specified in the anova command.

Video examples

[Analysis of covariance in Stata](#)

[Two-way ANOVA in Stata](#)

Stored results

anova stores the following in `e()`:

Scalars

<code>e(N)</code>	number of observations
<code>e(mss)</code>	model sum of squares
<code>e(df_m)</code>	model degrees of freedom
<code>e(rss)</code>	residual sum of squares
<code>e(df_r)</code>	residual degrees of freedom
<code>e(r2)</code>	R^2
<code>e(r2_a)</code>	adjusted R^2
<code>e(F)</code>	F statistic
<code>e(rmse)</code>	root mean squared error
<code>e(ll)</code>	log likelihood
<code>e(ll_0)</code>	log likelihood, constant-only model
<code>e(ss_#)</code>	sum of squares for term #
<code>e(df_#)</code>	numerator degrees of freedom for term #
<code>e(ssdenom_#)</code>	denominator sum of squares for term # (when using nonresidual error)
<code>e(dfdenom_#)</code>	denominator degrees of freedom for term # (when using nonresidual error)
<code>e(F_#)</code>	F statistic for term # (if computed)
<code>e(N_bse)</code>	number of levels of the between-subjects error term
<code>e(df_bse)</code>	degrees of freedom for the between-subjects error term
<code>e(box#)</code>	Box's conservative epsilon for a particular combination of repeated variables (<code>repeated()</code> only)
<code>e(gg#)</code>	Greenhouse–Geisser epsilon for a particular combination of repeated variables (<code>repeated()</code> only)
<code>e(hf#)</code>	Huynh–Feldt epsilon for a particular combination of repeated variables (<code>repeated()</code> only)
<code>e(rank)</code>	rank of <code>e(V)</code>

Macros

<code>e(cmd)</code>	anova
<code>e(cmdline)</code>	command as typed
<code>e(depvar)</code>	name of dependent variable
<code>e(varnames)</code>	names of the right-hand-side variables
<code>e(term_#)</code>	term #
<code>e(errorterm_#)</code>	error term for term # (when using nonresidual error)
<code>e(sstype)</code>	type of sum of squares; <code>sequential</code> or <code>partial</code>
<code>e(repvars)</code>	names of repeated variables (<code>repeated()</code> only)
<code>e(repvar#)</code>	names of repeated variables for a particular combination (<code>repeated()</code> only)
<code>e(model)</code>	ols
<code>e(wtype)</code>	weight type
<code>e(wexp)</code>	weight expression
<code>e(properties)</code>	b V
<code>e(estat_cmd)</code>	program used to implement <code>estat</code>
<code>e(predict)</code>	program used to implement <code>predict</code>
<code>e(asbalanced)</code>	factor variables <code>fvset</code> as <code>asbalanced</code>
<code>e(asobserved)</code>	factor variables <code>fvset</code> as <code>asobserved</code>

Matrices

<code>e(b)</code>	coefficient vector
<code>e(V)</code>	variance–covariance matrix of the estimators
<code>e(Srep)</code>	covariance matrix based on repeated measures (<code>repeated()</code> only)

Functions

<code>e(sample)</code>	marks estimation sample
------------------------	-------------------------

In addition to the above, the following is stored in `r()`:

Matrices

<code>r(ANOVA)</code>	matrix containing the sums of squares with their degrees of freedom, mean squares, F statistics, and p -values
<code>r(table)</code>	matrix containing the coefficients with their standard errors, test statistics, p -values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

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Also see

- [R] **anova postestimation** — Postestimation tools for anova
- [R] **contrast** — Contrasts and linear hypothesis tests after estimation
- [R] **icc** — Intraclass correlation coefficients
- [R] **loneway** — Large one-way ANOVA, random effects, and reliability
- [R] **oneway** — One-way analysis of variance
- [R] **regress** — Linear regression
- [MV] **manova** — Multivariate analysis of variance and covariance
- [PSS-2] **power oneway** — Power analysis for one-way analysis of variance
- [PSS-2] **power repeated** — Power analysis for repeated-measures analysis of variance
- [PSS-2] **power twoway** — Power analysis for two-way analysis of variance
- Stata Structural Equation Modeling Reference Manual*
- [U] **20 Estimation and postestimation commands**

Postestimation commands	predict	margins	test
Remarks and examples	References	Also see	

Postestimation commands

The following postestimation commands are of special interest after `anova`:

Command	Description
<code>dfbeta</code>	DFBETA influence statistics
<code>estat hettest</code>	tests for heteroskedasticity
<code>estat imtest</code>	information matrix test
<code>estat ovtest</code>	Ramsey regression specification-error test for omitted variables
<code>estat szroeter</code>	Szroeter's rank test for heteroskedasticity
<code>estat vif</code>	variance inflation factors for the independent variables
<code>estat esize</code>	η^2 and ω^2 effect sizes
<code>rvfplot</code>	residual-versus-fitted plot
<code>avplot</code>	added-variable plot
<code>avplots</code>	all added-variables plots in one image
<code>cprplot</code>	component-plus-residual plot
<code>acprplot</code>	augmented component-plus-residual plot
<code>rvpplot</code>	residual-versus-predictor plot
<code>lvr2plot</code>	leverage-versus-squared-residual plot

The following standard postestimation commands are also available:

Command	Description
<code>contrast</code>	contrasts and ANOVA-style joint tests of parameters
<code>estat ic</code>	Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively)
<code>estat summarize</code>	summary statistics for the estimation sample
<code>estat vce</code>	variance-covariance matrix of the estimators (VCE)
<code>estimates</code>	cataloging estimation results
<code>etable</code>	table of estimation results
<code>forecast</code>	dynamic forecasts and simulations
<code>hausman</code>	Hausman's specification test
<code>lincom</code>	point estimates, standard errors, testing, and inference for linear combinations of parameters
<code>linktest</code>	link test for model specification
<code>lrtest</code>	likelihood-ratio test
<code>margins</code>	marginal means, predictive margins, marginal effects, and average marginal effects
<code>marginsplot</code>	graph the results from margins (profile plots, interaction plots, etc.)
<code>nlcom</code>	point estimates, standard errors, testing, and inference for nonlinear combinations of parameters
<code>predict</code>	predictions and their SEs, leverage statistics, distance statistics, etc.
<code>predictnl</code>	point estimates, standard errors, testing, and inference for generalized predictions
<code>pwcompare</code>	pairwise comparisons of parameters
<code>suest</code>	seemingly unrelated estimation
<code>test</code>	Wald tests of simple and composite linear hypotheses
<code>testnl</code>	Wald tests of nonlinear hypotheses

predict

`predict` after `anova` follows the same syntax as `predict` after `regress` and can provide predictions, residuals, standardized residuals, Studentized residuals, the standard error of the residuals, the standard error of the prediction, the diagonal elements of the projection (\hat{h}) matrix, and Cook's D . See [R] [regress postestimation](#) for details.

margins

`margins` after `anova` follows the same syntax as `margins` after `regress`. See [R] [regress postestimation](#) for details.

`matv1c(matname)`, a programmer's option, saves the variance–covariance matrix of the linear combinations involved in the suite of tests. For the test $\mathbf{Lb} = \mathbf{c}$, what is returned in *matname* is \mathbf{LVL}' , where \mathbf{V} is the estimated variance–covariance matrix of \mathbf{b} .

`showorder` causes `test` to list the definition of each column in the design matrix. `showorder` is not allowed with any other option.

`symbolic` requests the symbolic form of the test rather than the test statistic. When this option is specified with no terms (`test, symbolic`), the symbolic form of the estimable functions is displayed.

Remarks and examples

Remarks are presented under the following headings:

- [Testing effects](#)
- [Obtaining symbolic forms](#)
- [Testing coefficients and contrasts of margins](#)
- [Video example](#)

See examples 4, 7, 8, 13, 15, 16, and 17 in [R] [anova](#) for examples that use the `margins` command.

Testing effects

After fitting a model using `anova`, you can test for the significance of effects in the ANOVA table, as well as for effects that are not reported in the ANOVA table, by using the `test` or `contrast` command. You follow `test` or `contrast` by the list of effects that you wish to test. By default, these commands use the residual mean squared error in the denominator of the F ratio. You can specify other error terms by using the slash notation, just as you would with `anova`. See [R] [contrast](#) for details on this command.

► Example 1: Testing effects

Recall our byssinosis example (example 8) in [R] [anova](#):

```
. anova prob workplace smokes race workplace#smokes workplace#race smokes#race
> workplace#smokes#race [aweight=pop]
(sum of wgt is 5,419)
```

	Number of obs =	65	R-squared =	0.8300	
	Root MSE =	.025902	Adj R-squared =	0.7948	
Source	Partial SS	df	MS	F	Prob>F
Model	.17364654	11	.01578605	23.53	0.0000
workplace	.09762518	2	.04881259	72.76	0.0000
smokes	.01303081	1	.01303081	19.42	0.0001
race	.00109472	1	.00109472	1.63	0.2070
workplace#smokes	.01969034	2	.00984517	14.67	0.0000
workplace#race	.00135252	2	.00067626	1.01	0.3718
smokes#race	.00166287	1	.00166287	2.48	0.1214
workplace#smokes#race	.00095084	2	.00047542	0.71	0.4969
Residual	.03555777	53	.0006709		
Total	.2092043	64	.00326882		

We can easily obtain a test on a particular term from the ANOVA table. Here are two examples:

```
. test smokes
```

Source	Partial SS	df	MS	F	Prob>F
smokes	.01303081	1	.01303081	19.42	0.0001
Residual	.03555777	53	.0006709		

```
. test smokes#race
```

Source	Partial SS	df	MS	F	Prob>F
smokes#race	.00166287	1	.00166287	2.48	0.1214
Residual	.03555777	53	.0006709		

Both of these tests use residual error by default and agree with the ANOVA table produced earlier.

We could have performed these same tests with `contrast`:

```
. contrast smokes
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
smokes	1	19.42	0.0001
Denominator	53		

```
. contrast smokes#race
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
smokes#race	1	2.48	0.1214
Denominator	53		

◀

□ Technical note

After `anova`, you can use the `/` syntax in `test` or `contrast` to perform tests with a variety of non- σ^2 error structures. However, in most unbalanced models, the mean squares are not independent and do not have equal expectations under the null hypothesis. Also, be warned that you assume responsibility for the validity of the test statistic.

□

▷ Example 2: Testing effects with different error terms

We return to the nested ANOVA example (example 11) in [R] `anova`, where five brands of machinery were compared in an assembly line. We can obtain appropriate tests for the nested terms using `test`, even if we had run the `anova` command without initially indicating the proper error terms.

```
. use https://www.stata-press.com/data/r19/machine
(Machine data)
. anova output machine / operator|machine /
```

Source	Partial SS	df	MS	F	Prob>F
Model	545.82229	17	32.107193	14.84	0.0000
machine	430.98079	4	107.7452	13.82	0.0001
operator machine	101.3538	13	7.7964465		
operator machine	101.3538	13	7.7964465	3.60	0.0009
Residual	84.376658	39	2.1635041		
Total	630.19895	56	11.253553		

In this ANOVA table, machine is tested with residual error. With this particular nested design, the appropriate error term for testing machine is operator nested within machine, which is easily obtained from `test`.

```
. test machine / operator|machine
```

Source	Partial SS	df	MS	F	Prob>F
machine	430.98079	4	107.7452	13.82	0.0001
operator machine	101.3538	13	7.7964465		

This result from `test` matches what we obtained from our `anova` command.

▷ Example 3: Pooling terms when testing effects

The other nested ANOVA example (example 12) in [R] anova was based on the sewage data. The ANOVA table is presented here again. As before, we will use abbreviations of variable names in typing the commands.

```
. use https://www.stata-press.com/data/r19/sewage
(Sewage treatment)
. anova particulate s / m|s / f|m|s / w|f|m|s /, dropemptycells
```

Source	Partial SS	df	MS	F	Prob>F
Model	13493.609	15	899.57396	5.54	0.0000
solution	7203.7656	1	7203.7656	17.19	0.0536
manager solution	838.28125	2	419.14063		
manager solution	838.28125	2	419.14063	0.55	0.6166
facility manager solution	3064.9375	4	766.23438		
facility manager solution	3064.9375	4	766.23438	2.57	0.1193
worker facility manager solution	2386.625	8	298.32813		
worker facility manager solution	2386.625	8	298.32813	1.84	0.0931
Residual	7796.25	48	162.42188		
Total	21289.859	63	337.93428		

In practice, it is often beneficial to pool nonsignificant nested terms to increase the power of tests on remaining terms. One rule of thumb is to allow the pooling of a term whose p -value is larger than 0.25. In this sewage example, the p -value for the test of manager is 0.6166. This value indicates that the manager effect is negligible and might be ignored. Currently, solution is tested by manager|solution, which has only 2 degrees of freedom. If we pool the manager and facility terms and use this pooled estimate as the error term for solution, we would have a term with 6 degrees of freedom.

Below are two tests: a test of solution with the pooled manager and facility terms and a test of this pooled term by worker.

```
. test s / m|s f|m|s
```

Source	Partial SS	df	MS	F	Prob>F
solution	7203.7656	1	7203.7656	11.07	0.0159
manager solution					
facility manager solution	3903.2188	6	650.53646		

```
. test m|s f|m|s / w|f|m|s
```

Source	Partial SS	df	MS	F	Prob>F
manager solution					
facility manager solution	3903.2188	6	650.53646	2.18	0.1520
worker facility manager solution	2386.625	8	298.32813		

In the first test, we included two terms after the forward slash (m|s and f|m|s). test after anova allows multiple terms both before and after the slash. The terms before the slash are combined and are then tested by the combined terms that follow the slash (or residual error if no slash is present).

The *p*-value for solution using the pooled term is 0.0159. Originally, it was 0.0536. The increase in the power of the test is due to the increase in degrees of freedom for the pooled error term.

We can get identical results if we drop manager from the anova model. (This dataset has unique numbers for each facility so that there is no confusion of facilities when manager is dropped.)

```
. anova particulate s / f|s / w|f|s /, dropemptycells
```

Source	Partial SS	df	MS	F	Prob>F
Model	13493.609	15	899.57396	5.54	0.0000
solution	7203.7656	1	7203.7656	11.07	0.0159
facility solution	3903.2187	6	650.53646		
facility solution	3903.2187	6	650.53646	2.18	0.1520
worker facility solution	2386.625	8	298.32812		
worker facility solution	2386.625	8	298.32812	1.84	0.0931
Residual	7796.25	48	162.42188		
Total	21289.859	63	337.93428		

This output agrees with our earlier test results.

▷ Example 4: Obtaining overall significance of a term using contrast

In [example 10](#) of [\[R\] anova](#), we fit the model `anova drate region#c.mage region#c.mage`. Now, we use the `contrast` command to test for the overall significance of `region`.

```
. contrast region region#c.mage, overall
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
region	3	7.40	0.0004
region#c.mage	3	0.86	0.4689
Overall	6	5.65	0.0002
Denominator	42		

The overall F statistic associated with the `region` and `region#c.mage` terms is 5.65, and it is significant at the 0.02% level.

In the ANOVA output, the `region` term, by itself, had a sum of squares of 1166.15, which, based on 3 degrees of freedom, yielded an F statistic of 7.40 and a significance level of 0.0004. This is the same test that is reported by `contrast` in the row labeled `region`. Likewise, the test from the ANOVA output for the `region#c.mage` term is reproduced in the second row of the `contrast` output.

◀

Obtaining symbolic forms

`test` can produce the symbolic form of the estimable functions and symbolic forms for particular tests.

▷ Example 5: Symbolic form of the estimable functions

After fitting an ANOVA model, we type `test, symbolic` to obtain the symbolic form of the estimable functions. For instance, returning to our blood pressure data introduced in [example 4](#) of [\[R\] anova](#), let's begin by reestimating systolic on `drug`, `disease`, and `drug#disease`:

```
. use https://www.stata-press.com/data/r19/systolic, clear
(Systolic blood pressure data)
. anova systolic drug disease drug#disease
```

	Number of obs =	58	R-squared =	0.4560	
	Root MSE =	10.5096	Adj R-squared =	0.3259	
Source	Partial SS	df	MS	F	Prob>F
Model	4259.3385	11	387.21259	3.51	0.0013
drug	2997.4719	3	999.15729	9.05	0.0001
disease	415.87305	2	207.93652	1.88	0.1637
drug#disease	707.26626	6	117.87771	1.07	0.3958
Residual	5080.8167	46	110.45254		
Total	9340.1552	57	163.86237		

To obtain the symbolic form of the estimable functions, type

```
. test, symbolic
drug
      1  -(r2+r3+r4-r0)
      2  r2
      3  r3
      4  r4
disease
      1  -(r6+r7-r0)
      2  r6
      3  r7
drug#disease
      1  1  -(r2+r3+r4+r6+r7-r12-r13-r15-r16-r18-r19-r0)
      1  2  r6 - (r12+r15+r18)
      1  3  r7 - (r13+r16+r19)
      2  1  r2 - (r12+r13)
      2  2  r12
      2  3  r13
      3  1  r3 - (r15+r16)
      3  2  r15
      3  3  r16
      4  1  r4 - (r18+r19)
      4  2  r18
      4  3  r19
_cons  r0
```



► Example 6: Symbolic form for a particular test

To obtain the symbolic form for a particular test, we type `test term [term ...], symbolic`. For instance, the symbolic form for the test of the main effect of drug is

```
. test drug, symbolic
drug
      1  -(r2+r3+r4)
      2  r2
      3  r3
      4  r4
disease
      1  0
      2  0
      3  0
drug#disease
      1  1  -1/3 (r2+r3+r4)
      1  2  -1/3 (r2+r3+r4)
      1  3  -1/3 (r2+r3+r4)
      2  1  1/3 r2
      2  2  1/3 r2
      2  3  1/3 r2
      3  1  1/3 r3
      3  2  1/3 r3
      3  3  1/3 r3
      4  1  1/3 r4
      4  2  1/3 r4
      4  3  1/3 r4
_cons  0
```

If we omit the symbolic option, we instead see the result of the test:

```
. test drug
```

Source	Partial SS	df	MS	F	Prob>F
drug	2997.4719	3	999.15729	9.05	0.0001
Residual	5080.8167	46	110.45254		

◀

Testing coefficients and contrasts of margins

`test` allows you to perform tests directly on the coefficients of the underlying regression model. For instance, the coefficient on the third drug and the second disease is referred to as `3.drug#2.disease`. This could also be written as `i3.drug#i2.disease`, or `_b[3.drug#2.disease]`, or even `_coef[i3.drug#i2.disease]`; see [U] 13.5 Accessing coefficients and standard errors.

▷ Example 7: Testing linear combinations of coefficients

Let's begin by testing whether the coefficient on the third drug is equal to the coefficient on the fourth drug in our blood pressure data. We have already fit the model `anova systolic drug##disease` (equivalent to `anova systolic drug disease drug#disease`), and you can see the results of that estimation in [example 5](#). Even though we have performed many tasks since we fit the model, Stata still remembers, and we can perform tests at any time.

```
. test 3.drug = 4.drug
( 1) 3.drug - 4.drug = 0
      F( 1, 46) = 0.13
      Prob > F = 0.7234
```

We find that the two coefficients are not significantly different, at least at any significance level smaller than 73%.

For more complex tests, the `contrast` command often provides a more concise way to specify the tests we are interested in and prevents us from having to write the tests in terms of the regression coefficients. With `contrast`, we instead specify our tests in terms of differences in the marginal means for the levels of a particular factor. For example, if we want to compare the third and fourth drugs, we can test the difference in the mean impact on systolic blood pressure separately for each disease using the `@` operator. We also use the reverse adjacent operator, `ar .`, to compare the fourth level of drug with the previous level.

```
. contrast ar4.drug@disease
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
drug@disease			
(4 vs 3) 1	1	0.13	0.7234
(4 vs 3) 2	1	1.76	0.1917
(4 vs 3) 3	1	0.65	0.4230
Joint	3	0.85	0.4761
Denominator	46		

	Contrast	Std. err.	[95% conf. interval]	
drug@disease				
(4 vs 3) 1	-2.733333	7.675156	-18.18262	12.71595
(4 vs 3) 2	8.433333	6.363903	-4.376539	21.24321
(4 vs 3) 3	5.7	7.050081	-8.491077	19.89108

None of the individual contrasts shows significant differences between the third drug and the fourth drug. Likewise, the overall F statistic is 0.85, which is hardly significant. We cannot reject the hypothesis that the third drug has the same effect as the fourth drug.

◀

□ Technical note

Alternatively, we could have specified these tests based on the coefficients of the underlying regression model using the `test` command. We would have needed to perform tests on the coefficients for drug and for the coefficients on drug interacted with disease to test for differences in the means mentioned above. To do this, we start with our previous `test` command:

```
. test 3.drug = 4.drug
```

Notice that the F statistic for this test is equivalent to the test labeled (4 vs 3) 1 in the contrast output. Let's now add the constraint that the coefficient on the third drug interacted with the third disease is equal to the coefficient on the fourth drug, again interacted with the third disease. We do that by typing the new constraint and adding the `accumulate` option:

```
. test 3.drug#3.disease = 4.drug#3.disease, accumulate
( 1) 3.drug - 4.drug = 0
( 2) 3.drug#3.disease - 4.drug#3.disease = 0
      F( 2, 46) = 0.39
      Prob > F = 0.6791
```

So far, our test includes the equality of the two drug coefficients, along with the equality of the two drug coefficients when interacted with the third disease. Now, we add two more equations, one for each of the remaining two diseases:

```
. test 3.drug#2.disease = 4.drug#2.disease, accumulate
( 1) 3.drug - 4.drug = 0
( 2) 3.drug#3.disease - 4.drug#3.disease = 0
( 3) 3.drug#2.disease - 4.drug#2.disease = 0
      F( 3, 46) = 0.85
      Prob > F = 0.4761

. test 3.drug#1.disease = 4.drug#1.disease, accumulate
( 1) 3.drug - 4.drug = 0
( 2) 3.drug#3.disease - 4.drug#3.disease = 0
( 3) 3.drug#2.disease - 4.drug#2.disease = 0
( 4) 3o.drug#1b.disease - 4o.drug#1b.disease = 0
      Constraint 4 dropped
      F( 3, 46) = 0.85
      Prob > F = 0.4761
```

The overall F statistic reproduces the one from the joint test in the contrast output.

You may notice that we also got the message “Constraint 4 dropped”. For the technically inclined, this constraint was unnecessary, given the normalization of the model. If we specify all the constraints involved in our test or use `contrast`, we need not worry about the normalization because Stata handles this automatically.

□

The `test()` option of `test` provides another alternative for testing coefficients. Instead of spelling out each coefficient involved in the test, a matrix representing the test provides the needed information. `test`, `showorder` shows the order of the terms in the ANOVA corresponding to the order of the columns for the matrix argument of `test()`.

▷ Example 8: Another way to test linear combinations of coefficients

We repeat the last test of [example 7](#) above with the `test()` option. First, we view the definition and order of the columns underlying the ANOVA performed on the systolic data.

```
. test, showorder
Order of columns in the design matrix
 1: (drug==1)
 2: (drug==2)
 3: (drug==3)
 4: (drug==4)
 5: (disease==1)
 6: (disease==2)
 7: (disease==3)
 8: (drug==1)*(disease==1)
 9: (drug==1)*(disease==2)
10: (drug==1)*(disease==3)
11: (drug==2)*(disease==1)
12: (drug==2)*(disease==2)
13: (drug==2)*(disease==3)
14: (drug==3)*(disease==1)
15: (drug==3)*(disease==2)
16: (drug==3)*(disease==3)
17: (drug==4)*(disease==1)
18: (drug==4)*(disease==2)
19: (drug==4)*(disease==3)
20: _cons
```

Columns 1–4 correspond to the four levels of drug. Columns 5–7 correspond to the three levels of disease. Columns 8–19 correspond to the interaction of drug and disease. The last column corresponds to `_cons`, the constant in the model.

We construct the matrix `dr3vs4` with the same four constraints as the last test shown in [example 7](#) and then use the `test(dr3vs4)` option to perform the test.

```
. matrix dr3vs4 = (0,0,1,-1, 0,0,0, 0,0,0,0,0,0,0,0, 0, 0, 0, 0 \
> 0,0,0, 0, 0,0,0, 0,0,0,0,0,0,0,0,1, 0, 0,-1, 0 \
> 0,0,0, 0, 0,0,0, 0,0,0,0,0,0,0,1,0, 0,-1, 0, 0 \
> 0,0,0, 0, 0,0,0, 0,0,0,0,0,0,0,1,0,0,-1, 0, 0, 0)
. test, test(dr3vs4)
( 1) 3.drug - 4.drug = 0
( 2) 3.drug#3.disease - 4.drug#3.disease = 0
( 3) 3.drug#2.disease - 4.drug#2.disease = 0
( 4) 3o.drug#1b.disease - 4o.drug#1b.disease = 0
Constraint 4 dropped
F( 3, 46) = 0.85
Prob > F = 0.4761
```

Here the effort involved with spelling out the coefficients is similar to that of constructing a matrix and using it in the `test()` option. When the test involving coefficients is more complicated, the `test()` option may be more convenient than specifying the coefficients directly in `test`. However, as previously demonstrated, `contrast` may provide an even simpler method for testing the same hypothesis. ◀

After fitting an ANOVA model, various contrasts (1-degree-of-freedom tests comparing different levels of a categorical variable) are often of interest. `contrast` can perform each 1-degree-of-freedom test in addition to the combined test, even in cases in which the contrasts do not correspond to one of the contrast operators.

▷ Example 9: Testing particular contrasts of interest

[Rencher and Schaalje \(2008\)](#) illustrate 1-degree-of-freedom contrasts for an ANOVA comparing the net weight of cans filled by five machines (labeled A–E). The data were originally obtained from [Ostle and Mensing \(1975\)](#). [Rencher and Schaalje](#) use a cell-means ANOVA model approach for this problem. We could do the same by using the `noconstant` option of `anova`; see [\[R\] anova](#). Instead, we obtain the same results by using the standard overparameterized ANOVA approach (that is, we keep the constant in the model).

```
. use https://www.stata-press.com/data/r19/canfill
(Can fill data)
. list, sepby(machine)
```

	machine	weight
1.	A	11.95
2.	A	12.00
3.	A	12.25
4.	A	12.10
5.	B	12.18
6.	B	12.11
7.	C	12.16
8.	C	12.15
9.	C	12.08
10.	D	12.25
11.	D	12.30
12.	D	12.10
13.	E	12.10
14.	E	12.04
15.	E	12.02
16.	E	12.02

```
. anova weight machine
```

	Number of obs =	16	R-squared =	0.4123	
	Root MSE =	.087758	Adj R-squared =	0.1986	
Source	Partial SS	df	MS	F	Prob>F
Model	.05942699	4	.01485675	1.93	0.1757
machine	.05942699	4	.01485675	1.93	0.1757
Residual	.0847167	11	.00770152		
Total	.14414369	15	.00960958		

The four 1-degree-of-freedom tests of interest among the five machines are A and D versus B, C, and E; B and E versus C; A versus D; and B versus E. We can specify these tests as user-defined contrasts by placing the corresponding contrast coefficients into positions related to the five levels of machine as described in *User-defined contrasts* of [R] [contrast](#).

```
. contrast {machine 3 -2 -2 3 -2}
>          {machine 0 1 -2 0 1}
>          {machine 1 0 0 -1 0}
>          {machine 0 1 0 0 -1}, noeffects
```

Contrasts of marginal linear predictions

Margins: asbalanced

	df	F	P>F
machine			
(1)	1	0.75	0.4055
(2)	1	0.31	0.5916
(3)	1	4.47	0.0582
(4)	1	1.73	0.2150
Joint	4	1.93	0.1757
Denominator	11		

`contrast` produces a 1-degree-of-freedom test for each of the specified contrasts as well as a joint test. We included the `noeffects` option so that the table displaying the values of the individual contrasts with their confidence intervals was suppressed.

The significance values above are not adjusted for multiple comparisons. We could have produced the Bonferroni-adjusted significance values by using the `mcompare(bonferroni)` option.

```
. contrast {machine 3 -2 -2 3 -2}
>          {machine 0 1 -2 0 1}
>          {machine 1 0 0 -1 0}
>          {machine 0 1 0 0 -1}, mcompare(bonferroni) noeffects
```

Contrasts of marginal linear predictions

Margins: asbalanced

	df	F	P>F	Bonferroni P>F
machine				
(1)	1	0.75	0.4055	1.0000
(2)	1	0.31	0.5916	1.0000
(3)	1	4.47	0.0582	0.2329
(4)	1	1.73	0.2150	0.8601
Joint	4	1.93	0.1757	
Denominator	11			

Note: Bonferroni-adjusted p-values are reported for tests on individual contrasts only.



▷ Example 10: Linear and quadratic contrasts

Here there are two factors, A and B, each with three levels. The levels are quantitative so that linear and quadratic contrasts are of interest.

```
. use https://www.stata-press.com/data/r19/twowaytrend
. anova Y A B A#B
```

	Number of obs =	36	R-squared =	0.9304	
	Root MSE =	2.6736	Adj R-squared =	0.9097	
Source	Partial SS	df	MS	F	Prob>F
Model	2578.5556	8	322.31944	45.09	0.0000
A	2026.7222	2	1013.3611	141.77	0.0000
B	383.72222	2	191.86111	26.84	0.0000
A#B	168.11111	4	42.027778	5.88	0.0015
Residual	193	27	7.1481481		
Total	2771.5556	35	79.187302		

We can use the p. contrast operator to obtain the 1-degree-of-freedom tests for the linear and quadratic effects of A and B.

```
. contrast p.A p.B, noeffects
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
A			
(linear)	1	212.65	0.0000
(quadratic)	1	70.88	0.0000
Joint	2	141.77	0.0000
B			
(linear)	1	26.17	0.0000
(quadratic)	1	27.51	0.0000
Joint	2	26.84	0.0000
Denominator	27		

All the above tests appear to be significant. In addition to presenting the 1-degree-of-freedom tests, the combined tests for A and B are produced and agree with the original ANOVA results.

Now, we explore the interaction between A and B.

```
. contrast p.A#p1.B, noeffects
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
A#B			
(linear) (linear)	1	17.71	0.0003
(quadratic) (linear)	1	0.07	0.7893
Joint	2	8.89	0.0011
Denominator	27		

The 2-degrees-of-freedom test of the interaction of A with the linear components of B is significant at the 0.0011 level. But, when we examine the two 1-degree-of-freedom tests that compose this result, the significance is due to the linear A by linear B contrast (significance level of 0.0003). A significance value of 0.7893 for the quadratic A by linear B indicates that this factor is not significant for these data.

```
. contrast p.A#p2.B, noeffects
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
A#B			
(linear) (quadratic)	1	2.80	0.1058
(quadratic) (quadratic)	1	2.94	0.0979
Joint	2	2.87	0.0741
Denominator	27		

The test of A with the quadratic components of B does not fall below the 0.05 significance level.



Video example

[Introduction to contrasts in Stata: One-way ANOVA](#)

References

- Mitchell, M. N. 2021. *Interpreting and Visualizing Regression Models Using Stata*. 2nd ed. College Station, TX: Stata Press.
- Ostle, B., and R. W. Mensing. 1975. *Statistics in Research*. 3rd ed. Ames, IA: Iowa State University Press.
- Rencher, A. C., and G. B. Schaalje. 2008. *Linear Models in Statistics*. 2nd ed. New York: Wiley.

Also see

- [R] [anova](#) — Analysis of variance and covariance
- [R] [regress postestimation](#) — Postestimation tools for regress
- [R] [regress postestimation diagnostic plots](#) — Postestimation plots for regress
- [U] [20 Estimation and postestimation commands](#)

areg — Linear regression with many indicator variables

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Description

`areg` fits linear regression absorbing categorical factors. `areg` is designed for datasets with categorical variables that have many groups, but the number of groups for each variable does not increase with the sample size. See the `xtreg`, `fe` command in [XT] `xtreg` for an estimator that handles the case in which one categorical variable (often a panel identifier) has a number of groups that increases with the sample size; additional categorical variables with fixed group sizes may also be included.

Quick start

Linear regression of `y` on `x`, absorbing indicator variables for the levels of `cvar`

```
areg y x, absorb(cvar)
```

Same as above, but add categorical variable `a`

```
areg y x i.a, absorb(cvar)
```

Same as above, but with cluster-robust standard errors

```
areg y x i.a, absorb(cvar) vce(cluster cvar2)
```

Linear regression of `y` on `x`, absorbing indicator variables for the levels of `cvar1` and `cvar2`

```
areg y x, absorb(cvar1 cvar2)
```

Menu

Statistics > Linear models and related > Other > Linear regression absorbing cat. variables

Syntax

```
areg depvar [indepvars] [if] [in] [weight] , absorb(varlist [ , method ]) [options]
```

<i>options</i>	Description
Model	
* <u>absorb</u> (<i>varlist</i> [, <i>method</i>])	specify categorical variables to be absorbed
† <u>dfabsorb</u>	adjust degrees of freedom for collinearity among absorbed variables
<u>noabsorbtest</u>	suppress the <i>F</i> test of absorbed indicators
SE/Robust	
<u>vce</u> (<i>vcetype</i>)	<i>vcetype</i> may be <u>ols</u> , <u>robust</u> , <u>cluster</u> <i>clustvarlist</i> , <u>bootstrap</u> , <u>jackknife</u> , or <u>hc2</u> [<i>clustvar</i>]
Reporting	
<u>level</u> (#)	set confidence level; default is <u>level</u> (95)
<u>clustertable</u>	display table of multiway cluster combinations
<u>display_options</u>	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Optimization	
† <u>tolerance</u> (#)	convergence tolerance for maximum absolute difference; default is <u>tolerance</u> (1e-8)
† <u>iterate</u> (#)	maximum number of iterations for alternating projection method (APM); default is <u>iterate</u> (50)
† <u>nolog</u>	suppress the APM iteration log
<u>coeflegend</u>	display legend instead of statistics

*absorb() is required.

†Ignored if only one absorbed variable is specified.

indepvars may contain factor variables; see [U] 11.4.3 **Factor variables**.

depvar and *indepvars* may contain time-series operators; see [U] 11.4.4 **Time-series varlists**.

bootstrap, by, collect, fp, jackknife, mi estimate, rolling, and statsby are allowed; see [U] 11.1.10 **Prefix commands**.

vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] **mi estimate**.

Weights are not allowed with the bootstrap prefix; see [R] **bootstrap**.

aweights are not allowed with the jackknife prefix; see [R] **jackknife**.

aweights, fweights, and pweights are allowed; see [U] 11.1.6 **weight**.

coeflegend does not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

Options

Model

absorb(*varlist* [, *method*]) specifies the categorical variables to be absorbed. The results are adjusted as if indicator variables for each level of each variable in *varlist* were included in the regression. absorb() is required.

When more than one categorical variable is absorbed, an APM iterative algorithm is used to project the *deprvar* and *indepvars* to absorb these variables. *method* specifies the APM and is one of `halperin` or `cimmino`.

`halperin`, the default, uses the product of the projection matrices.

`cimmino` uses the mean of the projection matrices.

The two methods typically perform similarly. See [Stammann \(2018\)](#) for details.

method is ignored if only one absorbed variable is specified.

`dfabsorb` adjusts the degrees of freedom to account for collinearity among the absorbed variables. The default degrees of freedom assumes all absorbed variables are independent. This option is ignored if only one absorbed variable is specified in `absorb()`.

`noabsorbtest` prevents computation of the F test that all coefficients on indicators for absorbed variables are jointly zero. Computation of this test requires an iterative search. If the F test is of no interest, you may specify `noabsorbtest` and save computational time.

SE/Robust

`vce(vctype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`ols`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvarlist`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [\[R\] vce_option](#).

`vce(ols)`, the default, uses the standard variance estimator for ordinary least-squares regression.

`vce(cluster clustvarlist)` specifies that standard errors allow for intragroup correlation within groups defined by one or more variables in *clustvarlist*, relaxing the usual requirement that the observations be independent. For example, `vce(cluster clustvar1)` produces cluster-robust standard errors that allow for observations that are independent across groups defined by `clustvar1` but not necessarily independent within groups. You could also type `vce(cluster clustvar1 clustvar2 ... clusterp)` to account for correlation within groups formed by p variables (multiway clustering).

`areg` also allows the following:

`vce(hc2 [clustvar] [, dfadjust])` specifies an alternative bias correction for the robust variance calculation. In the unclustered case, `vce(robust)` uses $\hat{\sigma}_j^2 = \{n/(n-k)\}u_j^2$ as an estimate of the variance of the j th observation, where n is the number of observations, k is the number of regressors, u_j is the calculated residual, and $n/(n-k)$ is included to improve the overall estimate's small-sample properties.

`vce(hc2)` instead uses $u_j^2/(1 - h_{jj})$ as the observation's variance estimate, where h_{jj} is the diagonal element of the hat (projection) matrix. This estimate is unbiased if the model really is homoskedastic. `vce(hc2)` tends to produce slightly more conservative confidence intervals. `vce(hc2 clustvar)` produces estimates that allow for intragroup correlation within groups defined by *clustvar*. `dfadjust` computes the [Bell and McCaffrey \(2002\)](#) adjusted degrees of freedom based on *clustvar*. Note that `dfadjust` does not affect multiple-imputation results when the command is used with `mi estimate`. See [Methods and formulas](#) in [\[R\] regress](#) for a description of the computation when *clustvar* is specified.

`vce(hc2)` is not allowed when multiple variables are absorbed.

Reporting

`level(#)`; see [R] [Estimation options](#).

`clustertable` displays a table reporting cluster combinations and the number of clusters per combination. This option is available only when `vce(cluster clustvarlist)` is specified with more than one variable in `clustvarlist` to compute multiway cluster-robust standard errors.

display_options: `nocl`, `nopvalues`, `dfci`, `dfpvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

`dfci` specifies that parameter degrees of freedom and confidence intervals be reported in the coefficient table.

`dfpvalues` specifies that parameter degrees of freedom and *p*-values be reported in the coefficient table.

Optimization

`tolerance(#)` specifies the limit for the maximum absolute difference between iterations for the projected *deprvar* and *indepvars*. The default is `tolerance(1e-8)`.

`iterate(#)` specifies the maximum number of iterations for the APM. The default is `iterate(50)`.

`nolog` specifies that no APM iterative log be displayed.

The following option is available with `areg` but is not shown in the dialog box:

`coeflegend`; see [R] [Estimation options](#).

Remarks and examples

Suppose that we have a linear regression model that includes among predictor variables indicators for the levels of a high-dimensional categorical variable *C*. For instance, the levels of *C* could represent counties, neighborhoods, or streets. We could write the model as

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{d}_1\gamma_1 + \mathbf{d}_2\gamma_2 + \cdots + \mathbf{d}_m\gamma_m + \boldsymbol{\epsilon}$$

Alternatively, if there are *N* observations and *m* categories of variable *C*, we could write this as

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{D}\boldsymbol{\gamma} + \boldsymbol{\epsilon}$$

where **D** is an $N \times m$ indicator matrix for the categories of *C* and

$$\mathbf{D} = (\mathbf{d}_1 \quad \mathbf{d}_2 \quad \cdots \quad \mathbf{d}_m) \quad \boldsymbol{\gamma} = \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_m \end{pmatrix}$$

Each \mathbf{d}_a is a column vector, the indicator variable for category *a* of variable *C*.

One option would be to fit the model with `regress`,

```
. regress y x* i.C
```


Both `regress` and `areg` display the same R^2 values, root mean squared error, and—for `weight` and `gear_ratio`—the same parameter estimates, standard errors, t statistics, significance levels, and confidence intervals. `areg`, however, does not report the coefficients for `rep78`, and, in fact, they are not even calculated. This computational trick makes the problem manageable when m is large. `areg` reports a test that the coefficients associated with `rep78` are jointly zero. Here this test has a significance level of 35.6%. This F test for `rep78` is the same that we would obtain after `regress` if we were to specify `test 1.rep78 2.rep78 3.rep78 4.rep78`; see [R] [test](#).

The model F tests reported by `regress` and `areg` also differ. The `regress` command reports a test that all coefficients except that of the constant are equal to zero; thus, the indicators are included in this test. The `areg` output shows a test that all coefficients excluding the indicators and the constant are equal to zero. This is the same test that can be obtained after `regress` by typing `test weight gear_ratio`. ◀

□ Technical note

`areg` is designed for datasets with many groups, but not a number that grows with the sample size. Consider two different samples from the US population. In the first sample, we have 10,000 individuals and we want to include an indicator for each of the 50 states, whereas in the second sample we have 3 observations on each of 10,000 individuals and we want to include an indicator for each individual. `areg` was designed for datasets similar to the first sample in which we have a fixed number of groups, the 50 states. In the second sample, the number of groups, which is the number of individuals, grows as we include more individuals in the sample. For an estimator designed to handle the case in which the number of groups grows with the sample size, see the `xtreg`, `fe` command in [XT] [xtreg](#).

Although the point estimates produced by `areg` and `xtreg, fe` are the same, the estimated VCEs differ when `vce(cluster clustvarlist)` is specified because the commands make different assumptions about whether the number of groups increases with the sample size. ◻

□ Technical note

The intercept reported by `areg` deserves some explanation because, given m mutually exclusive and exhaustive indicators, it is arbitrary. `areg` identifies the model by choosing the intercept that makes the prediction calculated at the means of the independent variables equal to the mean of the dependent variable: $\bar{y} = \bar{x}\hat{\beta}$.

```
. predict yhat
(option xb assumed; fitted values)
. summarize mpg yhat if rep78 != .
```

Variable	Obs	Mean	Std. dev.	Min	Max
mpg	69	21.28986	5.866408	12	41
yhat	69	21.28986	4.383224	11.58643	28.07367

We had to include `if rep78 != .` in our `summarize` command because we have missing values in our data. `areg` automatically dropped those missing values (as it should) in forming the estimates, but `predict` with the `xb` option will make predictions for cases with missing `rep78` because it does not know that `rep78` is really part of our model.

These predicted values do not include the absorbed effects (that is, the $\mathbf{d}_i\gamma_i$). For predicted values that include these effects, use the `xbd` option of `predict` (see [R] [areg postestimation](#)) or see [XT] [xtreg](#). ◻

► Example 2

`areg, vce(robust)` is a Huberized version of `areg`; see [P] [_robust](#). Just as `areg` is equivalent to using `regress` with indicators, `areg, vce(robust)` is equivalent to using `regress, vce(robust)` with indicators. You can use `areg, vce(robust)` when you expect heteroskedastic or nonnormal errors. `areg, vce(robust)`, like ordinary regression, assumes that the observations are independent, unless the `vce(cluster clustvarlist)` or `vce(hc2 clustvar)` option is specified. If the `vce(cluster clustvarlist)` or `vce(hc2 clustvar)` option is specified, this independence assumption is relaxed and only the clusters identified by equal values of *clustvarlist* or *clustvar* are assumed to be independent.

Assume that we were to collect data by randomly sampling 10,000 doctors (from 100 hospitals) and then sampling 10 patients of each doctor, yielding a total dataset of 100,000 patients in a cluster sample. If in some regression we wished to include effects of the hospitals to which the doctors belonged, we would want to include an indicator variable for each hospital, adding 100 variables to our model. `areg` could fit this model by

```
. areg depvar patient_vars, absorb(hospital) vce(cluster doctor)
```



Stored results

`areg` stores the following in `e()`:

Scalars

<code>e(N)</code>	number of observations
<code>e(k_absorb)</code>	total number of absorbed categories
<code>e(mss)</code>	model sum of squares
<code>e(tss)</code>	total sum of squares
<code>e(df_m)</code>	model degrees of freedom
<code>e(rss)</code>	residual sum of squares
<code>e(df_r)</code>	residual degrees of freedom
<code>e(r2)</code>	R^2
<code>e(r2_a)</code>	adjusted R^2
<code>e(df_a)</code>	degrees of freedom for absorbed effect
<code>e(rmse)</code>	root mean squared error
<code>e(ll)</code>	log likelihood
<code>e(ll_0)</code>	log likelihood, constant-only model
<code>e(N_clust)</code>	number of clusters
<code>e(F)</code>	F statistic
<code>e(F_absorb)</code>	F statistic for absorbed effect (when <code>vce(robust)</code> is not specified)
<code>e(p)</code>	p -value for model F test
<code>e(p_absorb)</code>	p -value for F test of absorbed effect
<code>e(rank)</code>	rank of <code>e(V)</code>
<code>e(converged)</code>	1 if APM converged, 0 otherwise

Macros

<code>e(cmd)</code>	<code>areg</code>
<code>e(cmdline)</code>	command as typed
<code>e(depvar)</code>	name of dependent variable
<code>e(absvar)</code>	names of absorbed variables
<code>e(apm)</code>	alternating projection method
<code>e(wtype)</code>	weight type
<code>e(wexp)</code>	weight expression
<code>e(title)</code>	title in estimation output
<code>e(clustvar)</code>	names of cluster variables
<code>e(cluster#)</code>	cluster combination #
<code>e(vce)</code>	<i>vcetype</i> specified in <code>vce()</code>

e(vctype)	title used to label Std. err.
e(datasignature)	the checksum
e(datasignaturevars)	variables used in calculation of checksum
e(properties)	b V
e(predict)	program used to implement predict
e(footnote)	program used to implement the footnote display
e(marginsnotok)	predictions disallowed by margins
e(asbalanced)	factor variables fvset as asbalanced
e(asobserved)	factor variables fvset as asobserved

Matrices

e(b)	coefficient vector
e(V)	variance–covariance matrix of the estimators
e(V_modelbased)	model-based variance
e(adj_df)	adjusted degrees of freedom when vce(hc2, dfadjust) is specified
e(kcluster)	cluster sizes, multiway clustering
e(kabsorb)	number of levels for each absorbed variable
e(dfabsorb)	adjusted degrees of freedom for each absorbed variable
e(ksingle)	number of singletons for each absorbed variable

Functions

e(sample)	marks estimation sample
-----------	-------------------------

In addition to the above, the following is stored in `r()`:

Matrices

r(table)	matrix containing the coefficients with their standard errors, test statistics, p -values, and confidence intervals
----------	---

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

Methods and formulas

With one absorbed variable, `areg` begins by recalculating *depvar* and *indepvars* to have mean 0 within the groups specified by `absorb()`. The overall mean of each variable is then added back in. The adjusted *depvar* is then regressed on the adjusted *indepvars* with `regress`, yielding the coefficient estimates. This is a direct application of the Frisch–Waugh–Lovell theorem (see Hansen [2022, 82]). The degrees of freedom of the variance–covariance matrix of the coefficients is then adjusted to account for the absorbed variables—this calculation yields the same results (up to numerical roundoff error) as if the matrix had been calculated directly by the formulas given in [R] `regress`.

With multiple absorbed variables, an APM with a conjugate gradient acceleration technique is used to adjust the *depvar* and *indepvars* (Hernández-Ramos, Escalante, and Raydan 2011). Two projection methods are available: Halperin (the default) and Cimmino. You can specify which to use with the `halperin` or `cimmino` suboption within the `absorb()` option. We describe both methods below.

Suppose we have h categorical variables, C_1, \dots, C_h , that we would like to include as controls in our regression. For variable C_k , let m_k be its number of levels and $\mathbf{d}_{k(a)}$ be its $N \times 1$ indicator vector for category a of C_k . Let \mathbf{D}_k denote the $N \times m_k$ matrix of indicators for variable C_k :

$$\mathbf{D}_k = (\mathbf{d}_{k(1)} \quad \mathbf{d}_{k(2)} \quad \dots \quad \mathbf{d}_{k(m_k)})$$

The orthonormal projection matrix for C_k is therefore given by $\mathbf{P}_k = \mathbf{D}_k(\mathbf{D}_k' \mathbf{D}_k)^{-1} \mathbf{D}_k'$. Let \mathbf{y} be the $N \times 1$ vector with the values of the dependent variable in the sample. Similarly, let \mathbf{X} be the matrix with the values of the covariates. Thus, the product $\bar{\mathbf{y}}_k = \mathbf{P}_k \mathbf{y}$ is the projection of the dependent variable onto

the column space of \mathbf{D}_k . That is, $\bar{\mathbf{y}}_k$ is the $N \times 1$ vector containing the (repeated) means of y_i for each level of C_k , in the order that these levels appear in the sample. The same projection is applied to the columns of covariate matrix \mathbf{X} .

The Halperin algorithm first sets $\tilde{\mathbf{y}}_0^{(1)} = \mathbf{y}$ and loops over the h absorbed variables computing projection residuals $\tilde{\mathbf{y}}_k^{(1)} = (\mathbf{I} - \mathbf{P}_k)\tilde{\mathbf{y}}_{k-1}^{(1)}$, for $k = 1, \dots, h$. Then, it repeats the loop with $\tilde{\mathbf{y}}_0^{(j+1)} = \tilde{\mathbf{y}}_h^{(j)}$ until convergence at $j = j$. Convergence is declared when $|\tilde{\mathbf{y}}_h^{(j)} - \tilde{\mathbf{y}}_0^{(j)}|$ is less than the specified tolerance, where the matrix norm $|\cdot|$ is defined as the largest entry in absolute value. The same computations are applied to the columns of covariate matrix \mathbf{X} . On convergence, the overall mean $\bar{\mathbf{y}}$ is added to $\tilde{\mathbf{y}}_h^{(j)}$, and the vector of means $\bar{\mathbf{x}}$ is added to the columns of $\tilde{\mathbf{X}}_h^{(j)}$.

The Cimmino algorithm first sets $\tilde{\mathbf{y}}^{(0)} = \mathbf{y}$ and then iteratively computes

$$\tilde{\mathbf{y}}^{(j+1)} = \left(\mathbf{I} - \frac{1}{h} \sum_{k=1}^h \mathbf{P}_k \right) \tilde{\mathbf{y}}^{(j)}$$

until convergence at $j + 1 = j$. It then repeats the process for covariance matrix \mathbf{X} and adds the overall means, $\bar{\mathbf{y}}$ and $\bar{\mathbf{x}}$, to $\tilde{\mathbf{y}}^{(j)}$ and to the columns of $\tilde{\mathbf{X}}^{(j)}$, respectively.

Efficient computation of the projection $\mathbf{P}\mathbf{y}$ can be done in Mata without generating the $N \times N$ matrix \mathbf{P} directly (see function `panelsum()`, for instance).

If the `dfabsorb` option is specified, an approximation to the absorbed variables, degrees of freedom is computed to account for collinearity among the absorbed variables. By default, `areg` assumes none of the variables in `absorb()` are collinear. Checking for collinearity could be computationally taxing. Thus, when `dfabsorb` is specified, a bipartite graph (Correia 2016) is generated for each pair of absorbed variables. This bipartite graph technique used by `dfabsorb` detects collinearities among pairs of categorical variables specified in the `absorb()` option. However, this technique does not detect collinearities among more than two absorbed variables, which would occur if they are included as factor variables in a regression. Therefore, the absorbed variable degrees of freedom is an upper bound.

More specifically, when `dfabsorb` is specified, the graph for each variable pair is decomposed to a set of subgraphs using Kruskal's greedy search algorithm (Cormen et al. 2022). Consider two absorbed variables a_1 and a_2 with m_1 and m_2 levels, respectively, and assume we generate k subgraphs. The number of estimable absorbed effects is $m_1 + m_2 - k$ (Abowd, Creecy, and Kramarz 2002) and corresponds to the degrees of freedom used when `dfabsorb` is specified. Here k can be considered the number of collinear indicator variables corresponding to the levels of a_1 and a_2 . The default degrees of freedom, on the other hand, is $m_1 + m_2 - 2$, which is the degrees of freedom if a single spanning tree is generated from the bipartite graph.

`areg` with `vce(robust)` or `vce(cluster clustvarlist)` with only one variable specified in `clustvarlist` works similarly, calling `_robust` after `regress` to produce the Huber/White/sandwich estimator of the variance or its clustered version. See [P] `_robust`, particularly *Introduction* and *Methods and formulas*. For p -way clustering, `vce(cluster clustvar1 [clustvar2 [...]])`, `_robust` is called for each of the $2^p - 1$ cluster combinations; details can be found in *Multway clustering* of *Methods and formulas* in [R] `regress`. The model F test uses the robust or cluster-robust variance estimates. There is, however, no simple computational means of obtaining a robust or cluster-robust test of the absorbed indicators. Thus, this test is not displayed when the `vce(robust)`, `vce(cluster clustvarlist)`, or `vce(hc2 [clustvar])` option is specified.

`areg` with `vce(hc2 [clustvar], [dfadjust])` specifies an alternative bias correction for the robust variance calculation. See *Robust calculation for regress* of *Methods and formulas* in [R] `regress` for a description of this VCE and the adjusted degrees of freedom. Computing this alternative VCE after ab-

sorbing indicator variables requires modification of the described procedure because the scores, denoted \mathbf{u}_j , are not invariant to projection unless the absorbed variable is nested within the clusters defined by *clustvar*. Rather than constructing the VCE using the original data, which can be computationally intensive, `areg` computes corrections to the scores using summary information about the absorbed variables. Similar corrections are used to compute the adjusted degrees of freedom if `dfadjust` is specified. When weights are specified, the clustering variable is required to nest the absorbed variable.

The number of groups specified in `absorb()` are included in the degrees of freedom used in the finite-sample adjustment of the cluster-robust VCE estimator. This statement is valid only if the number of groups is small relative to the sample size. (Technically, the number of groups must remain fixed as the sample size grows.) For an estimator that allows the number of groups to grow with the sample size, see the `xtreg`, `fe` command in [XT] `xtreg`.

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Also see

- [R] [areg postestimation](#) — Postestimation tools for areg
- [R] [regress](#) — Linear regression
- [R] [wildbootstrap](#) — Wild cluster bootstrap inference
- [MI] [Estimation](#) — Estimation commands for use with mi estimate
- [XT] [xtreg](#) — Linear models for panel data
- [U] [20 Estimation and postestimation commands](#)

Postestimation commands

The following postestimation commands are available after `areg`:

Command	Description
<code>contrast</code>	contrasts and ANOVA-style joint tests of parameters
<code>estat ic</code>	Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively)
<code>estat summarize</code>	summary statistics for the estimation sample
<code>estat vce</code>	variance–covariance matrix of the estimators (VCE)
<code>estimates</code>	cataloging estimation results
<code>etable</code>	table of estimation results
* <code>forecast</code>	dynamic forecasts and simulations
<code>hausman</code>	Hausman's specification test
<code>lincom</code>	point estimates, standard errors, testing, and inference for linear combinations of parameters
<code>linktest</code>	link test for model specification
<code>lrtest</code>	likelihood-ratio test
<code>margins</code>	marginal means, predictive margins, marginal effects, and average marginal effects
<code>marginsplot</code>	graph the results from margins (profile plots, interaction plots, etc.)
<code>nlcom</code>	point estimates, standard errors, testing, and inference for nonlinear combinations of parameters
<code>predict</code>	predictions and their SEs, residuals, etc.
<code>predictnl</code>	point estimates, standard errors, testing, and inference for generalized predictions
<code>pwcompare</code>	pairwise comparisons of parameters
<code>test</code>	Wald tests of simple and composite linear hypotheses
<code>testnl</code>	Wald tests of nonlinear hypotheses

*`forecast` is not appropriate with `mi` estimation results.

predict

Description for predict

`predict` creates a new variable containing predictions such as fitted values, standard errors, residuals, and the equation-level score.

Menu for predict

Statistics > Postestimation

Syntax for predict

```
predict [type] newvar [if] [in] [ , statistic]
```

where $y_j = \mathbf{x}_j\mathbf{b} + d_{\text{absorbvars}} + e_j$ and *statistic* is

<i>statistic</i>	Description
Main	
<code>xb</code>	$\mathbf{x}_j\mathbf{b}$, fitted values; the default
<code>stdp</code>	standard error of the prediction
<code>dresiduals</code>	$d_{\text{absorbvars}} + e_j = y_j - \mathbf{x}_j\mathbf{b}$
* <code>xbd</code>	$\mathbf{x}_j\mathbf{b} + d_{\text{absorbvars}}$
* <code>d</code>	$d_{\text{absorbvars}}$
* <code>residuals</code>	residual
* <code>score</code>	score; equivalent to <code>residuals</code>

Unstarred statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample. Starred statistics are calculated only for the estimation sample, even when `if e(sample)` is not specified.

Options for predict

Main

`xb`, the default, calculates the prediction of $\mathbf{x}_j\mathbf{b}$, the fitted values, by using the average effect of the absorbed variables. Also see `xbd` below.

`stdp` calculates the standard error of $\mathbf{x}_j\mathbf{b}$.

`dresiduals` calculates $y_j - \mathbf{x}_j\mathbf{b}$, which are the residuals plus the effects of the absorbed variables.

`xbd` calculates $\mathbf{x}_j\mathbf{b} + d_{\text{absorbvars}}$, which are the fitted values including the effects of the absorbed variables.

`d` calculates $d_{\text{absorbvars}}$, the effects of the absorbed variables.

`residuals` calculates the residuals, that is, $y_j - (\mathbf{x}_j\mathbf{b} + d_{\text{absorbvars}})$.

`score` is a synonym for `residuals`.

margins

Description for margins

`margins` estimates margins of response for fitted values.

Menu for margins

Statistics > Postestimation

Syntax for margins

```
margins [marginlist] [, options ]
margins [marginlist] , predict(statistic ...) [options ]
```

<i>statistic</i>	Description
<code>xb</code>	$\mathbf{x}_j\mathbf{b}$, fitted values; the default
<code>stdp</code>	not allowed with <code>margins</code>
<code>dresiduals</code>	not allowed with <code>margins</code>
<code>xbd</code>	not allowed with <code>margins</code>
<code>d</code>	not allowed with <code>margins</code>
<code>residuals</code>	not allowed with <code>margins</code>
<code>score</code>	not allowed with <code>margins</code>

Statistics not allowed with `margins` are functions of stochastic quantities other than $e(b)$.

For the full syntax, see [\[R\] margins](#).

Remarks and examples

▷ Example 1

Continuing with [example 1](#) of [\[R\] areg](#), we refit the model with robust standard errors and then obtain linear predictions and standard errors for those linear predictions.

```
. use https://www.stata-press.com/data/r19/auto2
(1978 automobile data)
. areg mpg weight gear_ratio, absorb(rep78) vce(robust)
(output omitted)
. predict xb_ar
(option xb assumed; fitted values)
. predict stdp_ar, stdp
```

We can obtain the same linear predictions by fitting the model with `xtreg, fe`, but we would first need to specify the panel structure by using `xtset`.

```
. xtset rep78
Panel variable: rep78 (unbalanced)
. xtreg mpg weight gear_ratio, fe vce(robust)
(output omitted)
. predict xb_xt
(option xb assumed; fitted values)
. predict stdp_xt, stdp
. summarize xb_ar xb_xt stdp*
```

Variable	Obs	Mean	Std. dev.	Min	Max
xb_ar	74	21.36805	4.286788	11.58643	28.07367
xb_xt	74	21.36805	4.286788	11.58643	28.07367
stdp_ar	74	.7105649	.1933936	.4270821	1.245179
stdp_xt	74	.8155919	.4826332	.0826999	1.709786

The predicted `xb` values above are the same for `areg` and `xtreg, fe`, but the standard errors for those linear predictions are different. The assumptions for these two estimators lead to different formulations for their standard errors. The robust variance estimates with `areg` are equivalent to the robust variance estimates using `regress`, including the panel dummies. The consistent robust variance estimates with `xtreg` are equivalent to those obtained by specifying `vce(cluster panelvar)` with that estimation command. For a theoretical discussion, see [Wooldridge \(2020\)](#), [Stock and Watson \(2008\)](#), and [Arellano \(2003\)](#); also see the [technical note](#) after example 3 of [XT] `xtreg`.

◀

▶ Example 2

We would like to use `linktest` to check whether the dependent variable for our model is correctly specified:

```
. use https://www.stata-press.com/data/r19/auto2, clear
(1978 automobile data)
. areg mpg weight gear_ratio, absorb(rep78)
(output omitted)
. linktest, absorb(rep78)
```

```
Linear regression, absorbing indicators          Number of obs      =       69
Absorbed variable: rep78                      No. of categories =        5
                                                F(2, 62)           =    46.50
                                                Prob > F           =    0.0000
                                                R-squared          =    0.6939
                                                Adj R-squared     =    0.6643
                                                Root MSE          =    3.3990
```

mpg	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
_hat	-.9305602	.9537856	-0.98	0.333	-2.83715	.9760302
_hatsq	.0462785	.0227219	2.04	0.046	.0008582	.0916989
_cons	19.24899	9.725618	1.98	0.052	-.1922457	38.69022

F test of absorbed indicators: $F(4, 62) = 1.278$

Prob > F = 0.288

The squared fitted values are significant in the regression for `mpg` on the linear and squared fitted values; therefore, the test indicates that our dependent variable does not seem to be well specified. Let's transform the dependent variable into energy consumption, gallons per mile, fit the alternative model, and check the link test again.

```

. generate gpm = 1/mpg
. areg gpm weight gear_ratio, absorb(rep78)
  (output omitted)
. linktest, absorb(rep78)
Linear regression, absorbing indicators          Number of obs   =    69
Absorbed variable: rep78                      No. of categories =    5
                                                F(2, 62)       =   72.60
                                                Prob > F       =  0.0000
                                                R-squared     =  0.7436
                                                Adj R-squared  =  0.7187
                                                Root MSE     =  0.0068

```

	gpm	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
	_hat	.2842582	.7109124	0.40	0.691	-1.136835	1.705352
	_hatsq	6.956965	6.862439	1.01	0.315	-6.760855	20.67478
	_cons	.0175457	.0178251	0.98	0.329	-.0180862	.0531777

F test of absorbed indicators: $F(4, 62) = 0.065$ Prob > F = 0.992

The link test supports the use of the transformed dependent variable.



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Also see

- [R] [areg](#) — Linear regression with many indicator variables
- [U] [20 Estimation and postestimation commands](#)

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Description

The `bayesboot` prefix performs Bayesian bootstrap estimation of specified statistics or expressions for Stata commands or user-written programs. It functions as a wrapper for two consecutive steps: First, `bayesboot` issues the `rwgen bayes` command to generate importance weights for each replication. Second, it issues the `bootstrap` prefix with the `iwweights()` option to compute Bayesian bootstrap estimates.

Quick start

Compute a Bayesian bootstrap estimate of the mean of `v1` returned by `summarize` in `r(mean)`

```
bayesboot mean=r(mean) : summarize v1
```

Same as above, but use the values of variable `x1` as the prior powers for each observation

```
bayesboot mean=r(mean) , priorpowers(x1) : summarize v1
```

Same as above, but use values of both `x2` and `x3` to uniquely identify observations

```
bayesboot mean=r(mean) , priorpowers(x1) id(x2 x3) : summarize v1
```

Compute a Bayesian bootstrap estimate of the statistic `r(mystat)` returned by program `myprog1`

```
bayesboot stat=r(mystat) : myprog1 v1
```

Same as above, but use 100 replications

```
bayesboot stat=r(mystat) , reps(100) : myprog1 v1
```

Same as above, and generate new variables `iw1` through `iw100` to store importance weights

```
bayesboot stat=r(mystat) , reps(100) generate(iw) : myprog1 v1
```

Compute Bayesian bootstrap estimates of the coefficients stored in `e(b)` by `myprog2`

```
bayesboot _b: myprog2 y x1 x2 x3
```

Menu

Statistics > Resampling > Bayesian bootstrap estimation

Syntax

bayesboot *exp_list* [, *options*] : *command*

<i>options</i>	Description
Options	
<code>priorpowers(# <i>varname</i>)</code>	specify prior power for each observation; default is <code>priorpowers(-1)</code>
<code>id(<i>varlist</i>)</code>	specify variables that uniquely identify observations
<code>generate(<i>stub</i>)</code>	generate new variables to store the importance weights
<code>bootstrap_options</code>	options allowed by bootstrap

command is any command that follows standard Stata syntax. *command* syntax must support `iweights`. `collect` is allowed; see [U] 11.1.10 Prefix commands.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

<i>exp_list</i> contains	(<i>name</i> : <i>elist</i>) <i>elist</i> <i>eexp</i>
<i>elist</i> contains	<i>newvar</i> = (<i>exp</i>) (<i>exp</i>)
<i>eexp</i> is	<i>specname</i> [<i>eqno</i>] <i>specname</i>
<i>specname</i> is	<code>_b</code> <code>_b[]</code> <code>_se</code> <code>_se[]</code>
<i>eqno</i> is	<code>##</code> <i>name</i>

exp is a standard Stata expression; see [U] 13 Functions and expressions.

Distinguish between `[]`, which are to be typed, and `[]`, which indicate optional arguments.

Options

Options

`priorpowers(# | varname)` specifies the power value l_i of the prior distribution for the i th observation, $i = 1, 2, \dots, n$. The default is `priorpowers(-1)`, which corresponds to an improper noninformative prior. `priorpowers(#)` specifies the same power `#` for all observations. Alternatively, you can specify different power values in *varname*. See *Methods and formulas* in [R] `rwgen`. The variable specified in `priorpowers()` may not be specified in `id()`.

`id(varlist)` specifies the variables used to uniquely identify each observation. By default, `bayesboot` considers all variables in the dataset to assess uniqueness. Any variables specified in `id()` may not be specified in `priorpowers()`.

`generate(stub)` requests that new variables be generated holding the Bayesian bootstrap weights. For `#` replications, `#` new variables `stub1`, `stub2`, \dots , `stub#` will be created. The importance weights are not stored by default.

bootstrap_options: `reps(#)`, `saving(filename[, suboptions])`, `bca`, `ties`, `mse`, `level(#)`, `notable`, `noheader`, `nolegend`, `verbose`, `nodots`, `dots(#)`, `noisily`, `trace`, `title(text)`, `display_options`, `eform_option`, `nodrop`, `nowarn`, `force`, `reject(exp)`, `rseed(#)`, `jackknifeopts(jkopts)`, and `coeflegend`; see [R] [bootstrap](#).

Remarks and examples

The `bayesboot` prefix performs Bayesian bootstrap estimation of specified statistics or expressions for Stata commands or user-written programs. It implements the technique introduced by [Rubin \(1981\)](#), which modifies the traditional bootstrap technique by adopting a Bayesian approach. Instead of discrete resampling with replacement, it assigns probabilities to each observation, forming an n -vector of weights that follow a specified posterior distribution. This method captures the uncertainty about the representativeness of each observation in the overall distribution.

`bayesboot` is a wrapper for a two-step process: First, it generates replicate weights, and second, it performs Bayesian bootstrap estimation with those weights. Alternatively, you can generate the replicate weights yourself with the `rwgen bayes` command and then specify those weight variables in the `iweights()` option with the `bootstrap` prefix to perform bootstrap estimation. However, `bayesboot` consolidates this all into one step. Alternatively, if you already have replicate weights, you can skip the step of generating them with `rwgen bayes` and simply use `bootstrap` to perform estimation.

All postestimation tools for `bootstrap` are supported after `bayesboot`.

▷ Example 1: Bayesian bootstrap estimation of standard errors

Suppose we want to compute Bayesian bootstrap estimates for the standard errors of the coefficients from the following regression:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)

. regress price mpg weight length
(output omitted)
```

To compute the Bayesian bootstrap estimates, we add the bayesboot prefix to the above regression command. We specify the random-number seed for reproducibility and request 100 replications. We use the default prior for each observation. This way, if all observations are unique, the posterior distribution of weights will follow a Dirichlet distribution, Dirichlet(1,1,...,1). This implies that each data point has the same posterior probability. This setup is comparable with the classical bootstrap, where each observation has a $1/n$ probability of selection in any given draw.

```
. bayesboot, rseed(19) reps(100): regress price mpg weight length
(running regress on estimation sample)

Bayesian bootstrap replications (100): .....10.....20.....30.....
> ..40.....50.....60.....70.....80.....90.....100 done

Bayesian bootstrap
Observation prior: Improper

Linear regression                                Number of obs =          74
                                                Replications =          100
                                                Wald chi2(3) =         35.55
                                                Prob > chi2 =          0.0000
                                                R-squared =            0.3574
                                                Adj R-squared =        0.3298
                                                Root MSE =          2414.5629
```

price	Observed coefficient	Bayesian bootstrap std. err.	z	P> z	Normal-based [95% conf. interval]	
mpg	-86.78928	75.87845	-1.14	0.253	-235.5083	61.92975
weight	4.364798	1.618642	2.70	0.007	1.192318	7.537277
length	-104.8682	52.87436	-1.98	0.047	-208.5	-1.236331
_cons	14542.43	6475.149	2.25	0.025	1851.375	27233.49

First, bayesboot executes `rwgen bayes` to generate 100 importance weights. Then, it executes `bootstrap`; for each replication, it computes a weighted estimate of the parameters, and then it computes the standard deviation of those weighted estimates to report the standard error.

As with the `bootstrap` prefix, the point estimates are not affected when using the bayesboot prefix.



▷ Example 2: Specifying the power values of the prior distribution

By default, bayesboot uses an improper noninformative prior, but we can use the `priorpowers()` option to customize the power values of the prior distribution. Below, we generate a new variable, `priorparm`, based on the values of `trunk` and use the resulting values as the power values. This allows us to account for the varying levels of confidence in our observations when performing our analysis. The prior parameters in `priorparm` control the shape of the posterior Dirichlet distribution that samples the importance weights, where higher values indicate stronger prior beliefs about the data.

```
. generate double priorparm = trunk / 10
. bayesboot, rseed(19) priorpowers(priorparm): regress price mpg weight length
(running regress on estimation sample)
Bayesian bootstrap replications (50): .....10.....20.....30.....
> .40.....50 done

Bayesian bootstrap
Observation prior: priorparm

Linear regression                                Number of obs =          74
                                                Replications =           50
                                                Wald chi2(3) =       157.33
                                                Prob > chi2 =         0.0000
                                                R-squared =          0.3574
                                                Adj R-squared =       0.3298
                                                Root MSE =       2414.5629
```

price	Observed coefficient	Bayesian bootstrap std. err.	z	P> z	Normal-based [95% conf. interval]	
mpg	-86.78928	37.19352	-2.33	0.020	-159.6873	-13.89132
weight	4.364798	.9316533	4.69	0.000	2.538791	6.190805
length	-104.8682	30.15013	-3.48	0.001	-163.9613	-45.77501
_cons	14542.43	3401.082	4.28	0.000	7876.436	21208.43

For more information about the Bayesian bootstrap technique and how weights are generated, see [Methods and formulas](#) in [R] `rwgen`.



▷ Example 3: Determining uniqueness of observations

By default, all variables in the dataset are used to uniquely identify observations, but we can use the `id()` option to specify which variables we want to use.

Suppose we want to compute Bayesian bootstrap estimates for the standard errors of the mean of price, and we want cars from the same car brand (for example, Buick, Cadillac, Dodge) to use the same prior parameters. To begin, we generate a new variable, `brand1`, to store the car brand because the `make` variable stores the make and model.

```
. split make, generate(brand) limit(1)
variable created as string:
brand1
. list make brand1 in 1/5
```

	make	brand1
1.	AMC Concord	AMC
2.	AMC Pacer	AMC
3.	AMC Spirit	AMC
4.	Buick Century	Buick
5.	Buick Electra	Buick

Now we use the `id(brand1)` option with `bayesboot` to specify that we only want to use the values of `brand1` to identify unique observations. `bayesboot` will execute `rwgen bayes` and create temporary variables to store the replicate weights, but we use the `generate()` option to store these variables for later use.

```
. bayesboot _b, rseed(19) id(brand1) generate(iw): mean price
(running mean on estimation sample)
Bayesian bootstrap replications (50): .....10.....20.....30.....
> .40.....50 done
Bayesian bootstrap
Observation prior: Improper
Mean estimation                                Number of obs = 74
                                                Replications = 50
```

	Observed mean	Bayesian bootstrap std. err.	Normal-based [95% conf. interval]	
price	6165.257	281.0588	5614.392	6716.122

Uniqueness ID: brand1

We specified the stub `iw`, which means that we now have variables named `iw1`, `iw2`, etc., in our data. Let's examine the replicate weights for all the Buicks in our data:

```
. list brand1 iw1-iw5 if brand1 == "Buick", separator(0)
```

	brand1	iw1	iw2	iw3	iw4	iw5
4.	Buick	.01236759	.00713592	.01340483	.0216436	.01589735
5.	Buick	.01236759	.00713592	.01340483	.0216436	.01589735
6.	Buick	.01236759	.00713592	.01340483	.0216436	.01589735
7.	Buick	.01236759	.00713592	.01340483	.0216436	.01589735
8.	Buick	.01236759	.00713592	.01340483	.0216436	.01589735
9.	Buick	.01236759	.00713592	.01340483	.0216436	.01589735
10.	Buick	.01236759	.00713592	.01340483	.0216436	.01589735

Note that cars from the same car brand have identical importance weights, as defined by our uniqueness criteria.

Because we stored the replicate weights, we can use them going forward with `bootstrap()`, allowing us to reproduce our analysis without having to re-create the weights or specify the random-number seed. For example, you can use the command below to reproduce the results from `bayesboot` above:

```
. bootstrap, iweights(iw1-iw50): mean price
```



Stored results

`bayesboot` stores the following in `e()`:

Scalars

<code>e(N)</code>	sample size
<code>e(N_reps)</code>	number of complete replications
<code>e(N_misreps)</code>	number of incomplete replications
<code>e(k_eq)</code>	number of equations in <code>e(b)</code>
<code>e(k_exp)</code>	number of standard expressions
<code>e(k_eexp)</code>	number of extended expressions (that is, <code>_b</code>)
<code>e(k_extra)</code>	number of extra equations beyond the original ones from <code>e(b)</code>
<code>e(level)</code>	confidence level for bootstrap CIs
<code>e(bs_version)</code>	version for bootstrap results
<code>e(rank)</code>	rank of <code>e(V)</code>

Macros

<code>e(cmdname)</code>	command name from <i>command</i>
<code>e(cmd)</code>	same as <code>e(cmdname)</code> or <i>bootstrap</i>
<code>e(command)</code>	<i>command</i>
<code>e(cmdline)</code>	command as typed
<code>e(prefix)</code>	<i>bayesboot</i>
<code>e(title)</code>	title in estimation output
<code>e(rngstate)</code>	random-number state used
<code>e(exp#)</code>	expression for the <i>#</i> th statistic
<code>e(ties)</code>	<i>ties</i> , if specified
<code>e(mse)</code>	<i>mse</i> , if specified
<code>e(vce)</code>	<i>bootstrap</i>
<code>e(vctype)</code>	title used to label Std. err.
<code>e(properties)</code>	<i>b V</i>
<code>e(rwb_newvarlist)</code>	generated weight variables
<code>e(rwb_method)</code>	<i>bayes</i>
<code>e(rwb_stub)</code>	<i>stub</i> of new variables

<code>e(rwb_priorpowers)</code>	prior powers
<code>e(rwb_id)</code>	variables for determining uniqueness
Matrices	
<code>e(b)</code>	observed statistics
<code>e(b_bs)</code>	bootstrap estimates
<code>e(reps)</code>	number of nonmissing results
<code>e(bias)</code>	estimated biases
<code>e(se)</code>	estimated standard errors
<code>e(z0)</code>	median biases
<code>e(accel)</code>	estimated accelerations
<code>e(ci_normal)</code>	normal-approximation CIs
<code>e(ci_percentile)</code>	percentile CIs
<code>e(ci_bc)</code>	bias-corrected CIs
<code>e(ci_bca)</code>	bias-corrected and accelerated CIs
<code>e(V)</code>	bootstrap variance-covariance matrix
<code>e(V_modelbased)</code>	model-based variance
Functions	
<code>e(sample)</code>	marks estimation sample

`bayesboot` will carry forward most of the results already in `e()` from `bootstrap`; when `exp_list` is `_b`, `bayesboot` will also carry forward most of the results already in `e()` from `command`.

In addition to the above, the following is stored in `r()`:

Matrices	
<code>r(table)</code>	matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r`-class command is run after the estimation command.

Methods and formulas

The `bayesboot` prefix performs Bayesian bootstrap estimation by combining the `rwgen bayes` command and the `bootstrap` prefix with the `iweights()` option.

The process consists of two sequential steps. First, `bayesboot` executes the `rwgen bayes` command to generate importance weights for each replication. These weights are temporarily stored as variables and automatically removed after execution. You can store these weights by specifying the `generate()` option. For detailed information about the weight-generation process, see [Methods and formulas](#) in [\[R\] rwgen](#).

Second, the command executes the `bootstrap` prefix with the importance weights specified in the `iweights()` option to compute and summarize the final results.

`bayesboot` was primarily developed for nonestimation commands, but it also supports estimation commands. When an estimation command is specified, `bayesboot` first executes the routine to obtain `e(sample)` and then uses this sample to filter observations for subsequent `rwgen bayes` and `bootstrap, iweights()` commands.

Reference

Rubin, D. B. 1981. The Bayesian bootstrap. *Annals of Statistics* 9: 130–134. <https://doi.org/10.1214/aos/1176345338>.

Also see

- [R] **bootstrap** — Bootstrap sampling and estimation
- [R] **bootstrap postestimation** — Postestimation tools for bootstrap
- [R] **jackknife** — Jackknife estimation
- [R] **rwgen** — Generate replicate weights for bootstrap estimation
- [U] **20 Estimation and postestimation commands**

[Description](#)[Options](#)[Acknowledgments](#)[Quick start](#)[Remarks and examples](#)[References](#)[Menu](#)[Stored results](#)[Also see](#)[Syntax](#)[Methods and formulas](#)

Description

`betareg` estimates the parameters of a beta regression model. This model accommodates dependent variables that are greater than 0 and less than 1, such as rates, proportions, and fractional data.

Quick start

Beta regression of y on x_1 and x_2

```
betareg y x1 x2
```

Add categorical variable a using [factor-variable](#) syntax

```
betareg y x1 x2 i.a
```

Add covariates for scale

```
betareg y x1 x2 i.a, scale(x1 z1)
```

Same as above, but use probit link for conditional mean and square-root link for conditional scale

```
betareg y x1 x2 i.a, scale(x1 z1) link(probit) slink(root)
```

Beta regression of y on x_1 and x_2 with robust standard errors

```
betareg y x1 x2, vce(robust)
```

Menu

Statistics > Fractional outcomes > Beta regression

Syntax

```
betareg depvar indepvars [if] [in] [weight] [, options]
```

<i>options</i>	Description
Model	
<code>noconstant</code>	suppress constant term
<code>scale(<i>varlist</i> [, <code>noconstant</code>])</code>	specify independent variables for scale
<code>link(<i>linkname</i>)</code>	specify link function for the conditional mean; default is <code>link(logit)</code>
<code>slink(<i>slinkname</i>)</code>	specify link function for the conditional scale; default is <code>slink(log)</code>
<code>constraints(<i>constraints</i>)</code>	apply specified linear constraints
SE/Robust	
<code>vce(<i>vcetype</i>)</code>	<i>vcetype</i> may be <code>oim</code> , <code>robust</code> , <code>cluster <i>clustvar</i></code> , <code>bootstrap</code> , or <code>jackknife</code>
Reporting	
<code>level(#)</code>	set confidence level; default is <code>level(95)</code>
<code>nocnsreport</code>	do not display constraints
<code>display_options</code>	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Maximization	
<code>maximize_options</code>	control the maximization process; seldom used
<code>coeflegend</code>	display legend instead of statistics

<i>linkname</i>	Description
<code>logit</code>	logit
<code>probit</code>	probit
<code>cloglog</code>	complementary log–log
<code>loglog</code>	log–log

<i>slinkname</i>	Description
<code>log</code>	log
<code>root</code>	square root
<code>identity</code>	identity

indepvars and *varlist* specified in `scale()` may contain factor variables; see [U] 11.4.3 **Factor variables**.

`bayes`, `bayesboot`, `bootstrap`, `by`, `collect`, `fmm`, `fp`, `jackknife`, `nestreg`, `rolling`, `statsby`, `stepwise`, and `svy` are allowed; see [U] 11.1.10 **Prefix commands**. For more details, see [BAYES] `bayes: betareg` and [FMM] `fmm: betareg`.

Weights are not allowed with the `bootstrap` prefix; see [R] **bootstrap**.

`vce()` and weights are not allowed with the `svy` prefix; see [SVY] `svy`.

`fweights`, `iweights`, and `pweights` are allowed; see [U] 11.1.6 **weight**.

`coeflegend` does not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

Options

Model

`noconstant`; see [R] [Estimation options](#).

`scale(varlist[, noconstant])` specifies the independent variables used to model the scale.

`noconstant` suppresses the constant term in the scale model. A constant term is included by default.

`link(linkname)` specifies the link function used for the conditional mean. *linkname* may be `logit`, `probit`, `cloglog`, or `loglog`. The default is `link(logit)`.

`slink(slinkname)` specifies the link function used for the conditional scale. *slinkname* may be `log`, `root`, or `identity`. The default is `slink(log)`.

`constraints(constraints)`; see [R] [Estimation options](#).

SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce_option](#).

Reporting

`level(#)`, `nocnsreport`; see [R] [Estimation options](#).

display_options: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

Maximization

maximize_options: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

The following option is available with `betareg` but is not shown in the dialog box:

`coeflegend`; see [R] [Estimation options](#).

Remarks and examples

Dependent variables such as rates, proportions, and fractional data are frequently greater than 0 and less than 1. There are a variety of methods to model such variables, including beta regression and fractional logistic regression.

Beta regression is widely used because of its flexibility for modeling variables between 0 and 1 and because its predictions are confined to the same range. However, beta regression models are not appropriate for dependent variables with some observations exactly equal to 0 or 1. See [R] [fracreg](#) for models when the dependent variable can equal 0 or 1 that also make predictions over the same range. The predictions from linear regression models are not constrained to the 0 to 1 interval; thus, they are not widely used for these variables.

These models have applications in a variety of disciplines, such as economics, the social sciences, and health science. For example, [Castellani, Pattitoni, and Scorcu \(2012\)](#) use beta regression to estimate Gini index values for the prices of art by famous and nonfamous artists. In political science, [Paolino \(2001\)](#) explores the advantages of beta regression and reviews its applicability for a variety of research topics such as the proportion of minority applicants deemed eligible for the Rural Housing Loans program and the proportion of a state's gay and lesbian population that is covered by antidiscrimination laws. In psychology, [Smithson, Deady, and Gracik \(2007\)](#) analyzed the relationship between how jurors judged the probability of a defendant's guilt and the verdict in a trial. Finally, beta regression has been used to model quality-adjusted life years in health-economics outcome studies ([Hubben et al. \[2008\]](#); [Basu and Manca \[2012\]](#)).

[Ferrari and Cribari-Neto \(2004\)](#) and [Smithson and Verkuilen \(2006\)](#) derived the beta regression estimators implemented in `betareg`. [Basu and Manca \(2012\)](#) also discuss quasimaximum likelihood inference for these estimators. These estimators augment the inherent flexibility of the beta distribution with functional form choices, known as links.

Beta regression is a model of the mean of the dependent variable y conditional on covariates \mathbf{x} , which we denote by $\mu_{\mathbf{x}}$. Because y is in $(0, 1)$, we must ensure that $\mu_{\mathbf{x}}$ is also in $(0, 1)$. We do this by using the link function for the conditional mean, denoted $g(\cdot)$. This is necessary because linear combinations of the covariates are not otherwise restricted to $(0, 1)$.

Algebraically,

$$g(\mu_{\mathbf{x}}) = \mathbf{x}\beta$$

or, equivalently,

$$\mu_{\mathbf{x}} = g^{-1}(\mathbf{x}\beta)$$

where $g^{-1}(\cdot)$ is the inverse function of $g(\cdot)$. For example, the default logit link implies that

$$\ln\{\mu_{\mathbf{x}}/(1 - \mu_{\mathbf{x}})\} = \mathbf{x}\beta$$

and that

$$\mu_{\mathbf{x}} = \exp(\mathbf{x}\beta)/\{1 + \exp(\mathbf{x}\beta)\}$$

Using a link function to keep the conditional-mean model inside an interval is common in the statistical literature; see [\[R\] glm](#) for additional applications of link functions.

The conditional variance of the beta distribution is

$$\text{Var}(y|\mathbf{x}) = \{\mu_{\mathbf{x}}(1 - \mu_{\mathbf{x}})\}/(1 + \psi)$$

The parameter ψ is known as the scale factor because it rescales the conditional variance. We use the scale link to ensure that $\psi > 0$.

► Example 1: Beta regression model of a rate

Suppose we wish to know whether offering a summer instruction program increases a school's pass rate for a mandatory state exam administered to students. The school-wide pass rate must be between 0 and 1. It is unlikely that any school will have either no students pass or all students pass, so we consider estimating the effect of the summer instruction program using beta regression.

The dataset `sprogram` contains fictional data on the pass rate of 1,000 schools (`prate`). We begin by reading in the data and verifying that `prate` contains no 0s or 1s.

```
. use https://www.stata-press.com/data/r19/sprogram
(Fictional summer program data)
. summarize prate
```

Variable	Obs	Mean	Std. dev.	Min	Max
prate	1,000	.8150803	.1233684	.2986041	.9973584

prate ranges from 0.299 to 0.997, so we proceed with our choice of a beta regression model.

We model prate as a function of a binary indicator for whether the school offered voluntary, half-day instruction to students during the past two summers (summer). The summer program should raise the scores of disadvantaged children who otherwise would not have access to programs that maintain their skills through the summer, thereby increasing the total proportion of students who pass the exam the next year.

We include the fraction of students receiving free or reduced-price meals (freemeals) and the sum of parents' monetary donations to the school two years earlier (pdonations) as additional covariates that measure affluence of the students' parents. We estimate the parameters of this model using the default logit link for the conditional mean and log link for the conditional scale.

```
. betareg prate i.summer freemeals pdonations
Initial:      Log likelihood = 781.55846
Rescale:     Log likelihood = 781.55846
Rescale eq:  Log likelihood = 781.55846
(setting technique to bhhh)
Iteration 0:  Log likelihood = 781.55846
Iteration 1:  Log likelihood = 891.57913
Iteration 2:  Log likelihood = 892.99578
Iteration 3:  Log likelihood = 893.02725
Iteration 4:  Log likelihood = 893.0279
Iteration 5:  Log likelihood = 893.02792

Beta regression                               Number of obs   =       1,000
                                                LR chi2(3)      =       164.61
                                                Prob > chi2     =       0.0000

Link function  :  g(u) = log(u/(1-u))         [Logit]
Slink function :  g(u) = log(u)              [Log]
Log likelihood = 893.02792
```

	prate	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
prate							
summer							
Yes		.5560171	.0480307	11.58	0.000	.4618787	.6501555
freemeals		-.4564181	.0834885	-5.47	0.000	-.6200525	-.2927836
pdonations		.0449706	.0097781	4.60	0.000	.025806	.0641353
_cons		1.175013	.0642797	18.28	0.000	1.049027	1.300999
scale							
_cons		2.375433	.0443005	53.62	0.000	2.288606	2.462261

The output table reports the estimated coefficients of the covariates and an estimated scale parameter. The coefficient of the factor variable for summer==1, shown as yes under summer, is significant and positive. Thus we conclude that the summer program was effective at increasing a school's pass rate.

However, we cannot determine the magnitude of the effect from these results. In general, when you use `betareg`, the best way to obtain interpretable effect sizes for the covariates is by using margins. See [example 1](#) in [\[R\] betareg postestimation](#) for more information.

◀

□ Technical note

The results displayed in [example 1](#) can be written concisely in algebraic form. Because we used the default logit link, the estimated conditional mean is

$$\hat{\mu}_x = \exp(\mathbf{x}\hat{\beta}) / \{1 + \exp(\mathbf{x}\hat{\beta})\}$$

where the estimated $\mathbf{x}\hat{\beta}$ is

$$\mathbf{x}\hat{\beta} = -0.456 \times \text{freemeals} + 0.045 \times \text{pdonations} + 0.556 \times (\text{summer}==1) + 1.18$$

The estimated ψ with the default log link for the scale is $\hat{\psi} = \exp(2.38) = 10.80$, which is simply substituted into the formula given [above](#) to express the conditional variance.

See [Methods and formulas](#) for the functional forms of the other links implemented in `betareg` for the conditional mean and scale.

□

▷ Example 2: Modeling conditional variance

Some processes require that we model the scale parameter as a function of covariates. For example, we might believe that the proportion of students with free or reduced meals influences the variance of the estimated mean.

We augment [example 1](#) by modeling the scale parameter as a function of `freemeals`.

```
. betareg prate i.summer freemeals pdonations, scale(freemeals)
(output omitted)
Beta regression          Number of obs   =       1,000
                        LR chi2(4)             =       169.38
                        Prob > chi2            =       0.0000

Link function   :   g(u) = log(u/(1-u))         [Logit]
Slink function  :   g(u) = log(u)               [Log]
Log likelihood = 895.41544
```

prate	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
prate						
summer						
Yes	.5571133	.0480378	11.60	0.000	.4629609	.6512658
freemeals	-.5291892	.0896511	-5.90	0.000	-.7049021	-.3534762
pdonations	.0454228	.0097809	4.64	0.000	.0262527	.0645929
_cons	1.209179	.0649585	18.61	0.000	1.081863	1.336495
scale						
freemeals	-.3598137	.1644214	-2.19	0.029	-.6820737	-.0375536
_cons	2.547047	.0882327	28.87	0.000	2.374114	2.71998

Again, we conclude that having a summer program increases the pass rate. The effect of an increase in the proportion of students receiving free meals on the conditional variance is ambiguous because it is in both equations. We can use margins to estimate the effect of the program on the conditional mean or the conditional variance.

◀

The estimators in `betareg` are consistent and efficient when the model is correctly specified. [Smithson and Verkuilen \(2006\)](#) discuss model selection for beta regression and note that selecting the model that minimizes the Bayesian information criterion (BIC) will select the correct model in large samples. Selecting the model that minimizes the BIC is a general approach to model selection; see [Cameron and Trivedi \(2005\)](#) for more details.

▷ Example 3: Model selection

We fit the models `quietly` and use `estimates store` to store the results under the names `model1`, `model2`, `model3`, and `model4`.

```
. quietly betareg prate i.summer freemeals pdonations, scale(freemeals)
. estimates store model1
. quietly betareg prate i.summer freemeals pdonations, scale(freemeals)
> link(cloglog)
. estimates store model2
. quietly betareg prate i.summer freemeals pdonations, scale(freemeals)
> slink(root)
. estimates store model3
. quietly betareg prate i.summer freemeals pdonations, scale(freemeals)
> link(cloglog) slink(root)
. estimates store model4
```

Next, we use `estimates table` to display the coefficients, standard errors, and the BIC for each model.

```
. estimates table model1 model2 model3 model4, stats(bic) se
```

Variable	model1	model2	model3	model4
prate				
summer				
Yes	.55711332	.27762093	.55719742	.27765283
	.04803785	.02460121	.04803698	.02459953
freemeals	-.52918917	-.25685191	-.5300549	-.25729221
	.08965112	.04308385	.08978883	.04314336
pdonations	.04542281	.02162612	.04542462	.02163225
	.00978086	.00444813	.00978099	.00444817
_cons	1.2091789	.37961457	1.2094991	.37977162
	.06495845	.03212448	.06496946	.03212509
scale				
freemeals	-.35981368	-.36808486	-.59912234	-.61295521
	.16442142	.16448631	.27259725	.27273851
_cons	2.5470469	2.5516626	3.5692049	3.5771444
	.0882327	.08821157	.15204032	.15218042
Statistics				
bic	-1749.3843	-1749.9903	-1749.444	-1750.0511

Legend: b/se

We select `model4`, the model with the complementary log–log link for the conditional mean and the square-root link for the conditional variance, because it has the lowest BIC.

◀

Stored results

`betareg` stores the following in `e()`:

Scalars

<code>e(N)</code>	number of observations
<code>e(k)</code>	number of parameters
<code>e(k_eq)</code>	number of equations in <code>e(b)</code>
<code>e(k_eq_model)</code>	number of equations in overall model test
<code>e(k_dv)</code>	number of dependent variables
<code>e(df_m)</code>	model degrees of freedom
<code>e(ll)</code>	log likelihood
<code>e(ll_0)</code>	log likelihood, constant-only model
<code>e(N_clust)</code>	number of clusters
<code>e(chi2)</code>	χ^2
<code>e(p)</code>	p -value for model test
<code>e(rank)</code>	rank of <code>e(V)</code>
<code>e(ic)</code>	number of iterations
<code>e(rc)</code>	return code
<code>e(converged)</code>	1 if converged, 0 otherwise

Macros

<code>e(cmd)</code>	<code>betareg</code>
<code>e(cmdline)</code>	command as typed
<code>e(depvar)</code>	name of dependent variable
<code>e(wtype)</code>	weight type
<code>e(wexp)</code>	weight expression
<code>e(title)</code>	title in estimation output
<code>e(linkt)</code>	link title in the conditional mean equation
<code>e(linkf)</code>	link function in the conditional mean equation
<code>e(slinkt)</code>	link title in the conditional scale equation
<code>e(slinkf)</code>	link function in the conditional scale equation
<code>e(clustvar)</code>	name of cluster variable
<code>e(chi2type)</code>	Wald or LR; type of model χ^2 test
<code>e(vce)</code>	<code>vce</code> type specified in <code>vce()</code>
<code>e(vcetype)</code>	title used to label Std. err.
<code>e(opt)</code>	type of optimization
<code>e(which)</code>	max or min; whether optimizer is to perform maximization or minimization
<code>e(ml_method)</code>	type of ml method
<code>e(user)</code>	name of likelihood-evaluator program
<code>e(technique)</code>	maximization technique
<code>e(properties)</code>	<code>b V</code>
<code>e(predict)</code>	program used to implement <code>predict</code>
<code>e(marginsok)</code>	predictions allowed by <code>margins</code>
<code>e(marginsnotok)</code>	predictions disallowed by <code>margins</code>
<code>e(asbalanced)</code>	factor variables <code>fvset</code> as <code>asbalanced</code>
<code>e(asobserved)</code>	factor variables <code>fvset</code> as <code>asobserved</code>

Matrices	
e(b)	coefficient vector
e(Cns)	constraints matrix
e(iLog)	iteration log (up to 20 iterations)
e(gradient)	gradient vector
e(V)	variance–covariance matrix of the estimators
e(V_modelbased)	model-based variance
Functions	
e(sample)	marks estimation sample

In addition to the above, the following is stored in `r()`:

Matrices	
r(table)	matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r`-class command is run after the estimation command.

Methods and formulas

Beta regression models were proposed by [Ferrari and Cribari-Neto \(2004\)](#) and extended by [Smithson and Verkuilen \(2006\)](#) to allow the scale parameter to depend on covariates.

Beta regression is appropriate only for a dependent variable that is strictly greater than 0 and strictly less than 1 because the beta distribution has support only on the interval (0, 1). The density of a beta-distributed dependent variable y conditional on covariates \mathbf{x} can be written as

$$f(y; \mu_{\mathbf{x}}, \psi_{\mathbf{x}}) = \frac{\Gamma(\psi_{\mathbf{x}})}{\Gamma(\mu_{\mathbf{x}}\psi_{\mathbf{x}})\Gamma\{(1 - \mu_{\mathbf{x}})\psi_{\mathbf{x}}\}} y^{\mu_{\mathbf{x}}\psi_{\mathbf{x}}-1} (1 - y)^{(1 - \mu_{\mathbf{x}})\psi_{\mathbf{x}}-1}$$

where $\mu_{\mathbf{x}} = \mathbf{E}(y|\mathbf{x})$, $\mu_{\mathbf{x}}$ is linked to the covariates by the link function $g(\mu_{\mathbf{x}}) = \mathbf{x}\boldsymbol{\beta}$, $\psi_{\mathbf{x}}$ scales the conditional variance according to

$$\text{Var}(y|\mathbf{x}) = \mu_{\mathbf{x}}(1 - \mu_{\mathbf{x}})/(1 + \psi_{\mathbf{x}})$$

and $\psi_{\mathbf{x}}$ is linked to the covariates by link function $h(\psi_{\mathbf{x}}) = \mathbf{x}\boldsymbol{\gamma}$.

This parameterization yields a log-likelihood function of

$$\sum_{i=1}^N \omega_i \left(\ln \{\Gamma(\psi_{\mathbf{x},i})\} - \ln \{\Gamma(\mu_{\mathbf{x},i}\psi_{\mathbf{x},i})\} - \ln \{\Gamma\{(1 - \mu_{\mathbf{x},i})\psi_{\mathbf{x},i}\}\} \right. \\ \left. + (\mu_{\mathbf{x},i}\psi_{\mathbf{x},i} - 1) \ln(y_i) + \{(1 - \mu_{\mathbf{x},i})\psi_{\mathbf{x},i} - 1\} \ln(1 - y_i) \right)$$

The definitions of the link functions are

Name	Function
logit	$g(\mu_{\mathbf{x}}) = \ln\{\mu_{\mathbf{x}}/(1 - \mu_{\mathbf{x}})\}$
probit	$g(\mu_{\mathbf{x}}) = \Phi^{-1}(\mu_{\mathbf{x}})$
cloglog	$g(\mu_{\mathbf{x}}) = \ln\{-\ln(1 - \mu_{\mathbf{x}})\}$
loglog	$g(\mu_{\mathbf{x}}) = -\ln\{-\ln(\mu_{\mathbf{x}})\}$

The definitions of the scale-link functions are

Name	Function
log	$h(\psi_{\mathbf{x}}) = \ln(\psi_{\mathbf{x}})$
root	$h(\psi_{\mathbf{x}}) = \sqrt{\psi_{\mathbf{x}}}$
identity	$h(\psi_{\mathbf{x}}) = \psi_{\mathbf{x}}$

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] [_robust](#), particularly *Maximum likelihood estimators* and *Methods and formulas*.

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Also see

- [R] [betareg postestimation](#) — Postestimation tools for betareg
- [R] [fracreg](#) — Fractional response regression
- [R] [glm](#) — Generalized linear models
- [BAYES] [bayes: betareg](#) — Bayesian beta regression
- [FMM] [fmm: betareg](#) — Finite mixtures of beta regression models
- [SVY] [svy estimation](#) — Estimation commands for survey data
- [U] [20 Estimation and postestimation commands](#)

Postestimation commands

The following postestimation commands are available after `betareg`:

Command	Description
<code>contrast</code>	contrasts and ANOVA-style joint tests of parameters
<code>estat ic</code>	Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively)
<code>estat summarize</code>	summary statistics for the estimation sample
<code>estat vce</code>	variance–covariance matrix of the estimators (VCE)
<code>estat (svy)</code>	postestimation statistics for survey data
<code>estimates</code>	cataloging estimation results
<code>etable</code>	table of estimation results
* <code>forecast</code>	dynamic forecasts and simulations
* <code>hausman</code>	Hausman's specification test
<code>lincom</code>	point estimates, standard errors, testing, and inference for linear combinations of parameters
* <code>lrtest</code>	likelihood-ratio test
<code>margins</code>	marginal means, predictive margins, marginal effects, and average marginal effects
<code>marginsplot</code>	graph the results from margins (profile plots, interaction plots, etc.)
<code>nlcom</code>	point estimates, standard errors, testing, and inference for nonlinear combinations of parameters
<code>predict</code>	conditional means and variances, linear predictions, etc.
<code>predictnl</code>	point estimates, standard errors, testing, and inference for generalized predictions
<code>pwcompare</code>	pairwise comparisons of parameters
<code>suest</code>	seemingly unrelated estimation
<code>test</code>	Wald tests of simple and composite linear hypotheses
<code>testnl</code>	Wald tests of nonlinear hypotheses

*`forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results.

predict

Description for predict

`predict` creates a new variable containing predictions such as linear predictions, conditional means, conditional variances, and equation-level scores.

Menu for predict

Statistics > Postestimation

Syntax for predict

```
predict [type] newvar [if] [in] [, statistic]
```

```
predict [type] stub* [if] [in], scores
```

<i>statistic</i>	Description
------------------	-------------

Main

<code>cmean</code>	conditional mean of the dependent variable; the default
<code>cvariance</code>	conditional variance of the dependent variable
<code>xb</code>	linear prediction in the conditional-mean equation
<code>xbscale</code>	linear prediction in the conditional-scale equation
<code>stdp</code>	standard error of the linear prediction

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

Options for predict

Main

`cmean`, the default, calculates the conditional mean of the dependent variable.

`cvariance` calculates the conditional variance of the dependent variable.

`xb` calculates the linear prediction for the conditional-mean equation.

`xbscale` calculates the linear prediction for the conditional-scale equation.

`stdp` calculates the standard error of the linear prediction for the conditional-mean equation.

`scores` calculates equation-level score variables. The first new variable will contain the derivative of the log likelihood with respect to the conditional-mean equation, and the second new variable will contain the derivative of the log likelihood with respect to the conditional-scale equation.

margins

Description for margins

`margins` estimates margins of response for conditional means, conditional variances, and linear predictions.

Menu for margins

Statistics > Postestimation

Syntax for margins

```
margins [marginlist] [, options]
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

<i>statistic</i>	Description
<code>cmean</code>	conditional mean of the dependent variable; the default
<code>cvariance</code>	conditional variance of the dependent variable
<code>xb</code>	linear prediction in the conditional-mean equation
<code>xbscale</code>	linear prediction in the conditional-scale equation
<code>stdp</code>	not allowed with <code>margins</code>
<code>scores</code>	not allowed with <code>margins</code>

Statistics not allowed with `margins` are functions of stochastic quantities other than $e(b)$.

For the full syntax, see [R] [margins](#).

Remarks and examples

▷ Example 1

In [example 3](#) in [R] [betareg](#), we selected a model for school-level data on the fraction of students passing a state-required exam. In that example, we were interested in whether a voluntary summer program increased schools' pass rates. We continue that example to estimate an average treatment effect (ATE) of the program.

After reading in the data and fitting the model, we use `margins` to estimate the ATE.

```
. use https://www.stata-press.com/data/r19/sprogram
(Fictional summer program data)
. betareg prate freemeals pdonations i.summer, scale(freemeals) link(cloglog)
> slink(root) vce(robust)
(output omitted)
```

We specify `vce(robust)` with the estimation command and `vce(unconditional)` with the `margins` command to obtain standard errors for a population ATE instead of a sample ATE.

```
. margins r.summer, contrast(nowald) vce(unconditional)
Contrasts of predictive margins          Number of obs = 1,000
Expression: Conditional mean of prate, predict()
```

	Unconditional			
	Contrast	std. err.	[95% conf. interval]	
summer (Yes vs No)	.0890851	.008626	.0721784	.1059918

The average pass rate would be about 9% higher when all schools offered the program than when no school offered the program.



Also see

[R] [betareg](#) — Beta regression

[U] [20 Estimation and postestimation commands](#)

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

Description

`binreg` fits generalized linear models for the binomial family. It estimates odds ratios, risk ratios, health ratios, and risk differences. The available links are

Option	Implied link	Parameter
<code>or</code>	<code>logit</code>	odds ratios = $\exp(\beta)$
<code>rr</code>	<code>log</code>	risk ratios = $\exp(\beta)$
<code>hr</code>	<code>log complement</code>	health ratios = $\exp(\beta)$
<code>rd</code>	<code>identity</code>	risk differences = β

Estimates of odds, risk, and health ratios are obtained by exponentiating the appropriate coefficients. The `or` option produces the same results as Stata's `logistic` command, and `or coefficients` yields the same results as the `logit` command. When no link is specified, `or` is assumed.

Quick start

Report odds ratios from a model of `y` on `x1` and `x2` using a logit link

```
binreg y x1 x2, or
```

Use the log link and report risk ratios

```
binreg y x1 x2, rr
```

Use the identity link and report risk differences

```
binreg y x1 x2, rd
```

Same as above, but with data stored as the number of successes, `ys`, out of `n` trials

```
binreg ys x1 x2, rd n(n)
```

Menu

Statistics > Generalized linear models > GLM for the binomial family

Syntax

```
binreg depvar [indepvars] [if] [in] [weight] [, options]
```

<i>options</i>	Description
Model	
<u>noconstant</u>	suppress constant term
or	use logit link and report odds ratios
rr	use log link and report risk ratios
hr	use log-complement link and report health ratios
rd	use identity link and report risk differences
n(# <i>varname</i>)	use # or <i>varname</i> for number of trials
<u>exposure</u> (<i>varname</i>)	include ln(<i>varname</i>) in model with coefficient constrained to 1
<u>offset</u> (<i>varname</i>)	include <i>varname</i> in model with coefficient constrained to 1
<u>constraints</u> (<i>constraints</i>)	apply specified linear constraints
mu(<i>varname</i>)	use <i>varname</i> as the initial estimate for the mean of <i>depvar</i>
<u>init</u> (<i>varname</i>)	synonym for mu(<i>varname</i>)
SE/Robust	
vce(<i>vcetype</i>)	<i>vcetype</i> may be <u>eim</u> , <u>robust</u> , <u>cluster</u> <i>clustvar</i> , <u>oim</u> , <u>opg</u> , <u>bootstrap</u> , <u>jackknife</u> , <u>hac</u> <i>kernel</i> , <u>jackknife1</u> , or <u>unbiased</u>
t(<i>varname</i>)	variable name corresponding to time
<u>vfactor</u> (#)	multiply variance matrix by scalar #
disp(#)	quasilikelihood multiplier
<u>scale</u> (x2 dev #)	set the scale parameter; default is scale(1)
Reporting	
<u>level</u> (#)	set confidence level; default is level(95)
<u>coefficients</u>	report nonexponentiated coefficients
<u>nocnsreport</u>	do not display constraints
<u>display_options</u>	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Maximization	
irls	use iterated, reweighted least-squares optimization; the default
ml	use maximum likelihood optimization
<u>maximize_options</u>	control the maximization process; seldom used
fisher(#)	Fisher scoring steps
search	search for good starting values
<u>collinear</u>	keep collinear variables
<u>coeflegend</u>	display legend instead of statistics

indepvars may contain factor variables; see [U] 11.4.3 **Factor variables**.

devar and *indepvars* may contain time-series operators; see [U] 11.4.4 **Time-series varlists**.

bayes, *bayesboot*, *bootstrap*, *by*, *collect*, *fp*, *jackknife*, *mi estimate*, *rolling*, and *statsby* are allowed; see [U] 11.1.10 **Prefix commands**. For more details, see [BAYES] **bayes: binreg**.

vce(bootstrap), *vce(jackknife)*, and *vce(jackknife1)* are not allowed with the *mi estimate* prefix; see [MI] **mi estimate**.

Weights are not allowed with the *bootstrap* prefix; see [R] **bootstrap**.

fweights, *iwweights*, and *pweights* are allowed; see [U] 11.1.6 **weight**.

collinear and *coeflegend* do not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

Options

Model

noconstant; see [R] **Estimation options**.

or requests the logit link and results in odds ratios if *coefficients* is not specified.

rr requests the log link and results in risk ratios if *coefficients* is not specified.

hr requests the log-complement link and results in health ratios if *coefficients* is not specified.

rd requests the identity link and results in risk differences.

n(#|*varname*) specifies either a constant integer to use as the denominator for the binomial family or a variable that holds the denominator for each observation.

exposure(*varname*), *offset*(*varname*), *constraints*(*constraints*); see [R] **Estimation options**. *constraints*(*constraints*) is not allowed with *irls*.

mu(*varname*) specifies *varname* containing an initial estimate for the mean of *devar*. This option can be useful if you encounter convergence difficulties. *init*(*varname*) is a synonym.

SE/Robust

vce(*vcetype*) specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (*robust*), that allow for intragroup correlation (*cluster clustvar*), that are derived from asymptotic theory (*oim*, *opg*), and that use bootstrap or jackknife methods (*bootstrap*, *jackknife*); see [R] **vce_option**.

vce(*eim*), the default, uses the expected information matrix (EIM) for the variance estimator.

binreg also allows the following:

vce(*hac kernel* [#]) specifies that a heteroskedasticity- and autocorrelation-consistent (HAC) variance estimate be used. HAC refers to the general form for combining weighted matrices to form the variance estimate. There are three kernels built into *binreg*. *kernel* is a user-written program or one of

nwest | *gallant* | *anderson*

If # is not specified, $N - 2$ is assumed.

vce(*jackknife1*) specifies that the one-step jackknife estimate of variance be used.

vce(*unbiased*) specifies that the unbiased sandwich estimate of variance be used.

`t(varname)` specifies the variable name corresponding to time; see [TS] [tsset](#). `binreg` does not always need to know `t()`, though it does if `vce(hac...)` is specified. Then you can either specify the time variable with `t()`, or you can `tsset` your data before calling `binreg`. When the time variable is required, `binreg` assumes that the observations are spaced equally over time.

`vfactor(#)` specifies a scalar by which to multiply the resulting variance matrix. This option allows users to match output with other packages, which may apply degrees of freedom or other small-sample corrections to estimates of variance.

`disp(#)` multiplies the variance of *depvar* by `#` and divides the deviance by `#`. The resulting distributions are members of the quasilielihood family. This option is not allowed with option `m1`.

`scale(x2|dev|#)` overrides the default scale parameter. This option is allowed only with Hessian (information matrix) variance estimates.

By default, `scale(1)` is assumed for the discrete distributions (binomial, Poisson, and negative binomial), and `scale(x2)` is assumed for the continuous distributions (Gaussian, gamma, and inverse Gaussian).

`scale(x2)` specifies that the scale parameter be set to the Pearson χ^2 (or generalized χ^2) statistic divided by the residual degrees of freedom, which is recommended by [McCullagh and Nelder \(1989\)](#) as a good general choice for continuous distributions.

`scale(dev)` sets the scale parameter to the deviance divided by the residual degrees of freedom. This option provides an alternative to `scale(x2)` for continuous distributions and overdispersed or underdispersed discrete distributions. This option is not allowed with option `m1`.

`scale(#)` sets the scale parameter to `#`.

Reporting

`level(#)`, `noconstant`; see [R] [Estimation options](#).

`coefficients` displays the nonexponentiated coefficients and corresponding standard errors and confidence intervals. This option has no effect when the `rd` option is specified, because it always presents the nonexponentiated coefficients.

`nocnsreport`; see [R] [Estimation options](#).

display_options: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

Maximization

`irls` requests iterated, reweighted least-squares (IRLS) optimization of the deviance instead of Newton–Raphson optimization of the log likelihood. This option is the default.

`m1` requests that optimization be carried out by using Stata’s `m1` command; see [R] [m1](#).

maximize_options: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

Setting the optimization method to `m1`, with `technique()` set to something other than `BHHH`, changes the *vcetype* to `vce(oim)`. Specifying `technique(bhhh)` changes *vcetype* to `vce(opg)`.

Unless option `m1` is specified, only `maximize_options` `iterate()`, `nolog`, `trace`, and `ltolerance()` are allowed. With IRLS optimization, the convergence criterion is satisfied when the absolute change in deviance from one iteration to the next is less than or equal to `ltolerance()`, where `ltolerance(1e-6)` is the default.

`fisher(#)` specifies the number of Newton–Raphson steps that should use the Fisher scoring Hessian or EIM before switching to the observed information matrix (OIM). This option is available only if `m1` is specified and is useful only for Newton–Raphson optimization.

`search` specifies that the command search for good starting values. This option is available only if `m1` is specified and is useful only for Newton–Raphson optimization.

The following options are available with `binreg` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#). `collinear` is not allowed with `irls`.

Remarks and examples

Wacholder (1986) suggests methods for estimating risk ratios and risk differences from prospective binomial data. These estimates are obtained by selecting the proper link functions in the generalized linear-model framework. (See [Methods and formulas](#) for details; also see [R] [glm](#).)

▷ Example 1

Wacholder (1986) presents an example, using data from Wright et al. (1983), of an investigation of the relationship between alcohol consumption and the risk of a low-birthweight baby. Covariates examined included whether the mother smoked (yes or no), mother’s social class (three levels), and drinking frequency (light, moderate, or heavy). The data for the 18 possible categories determined by the covariates are illustrated below.

Let's first describe the data and list a few observations.

```
. use https://www.stata-press.com/data/r19/binreg
(Alcohol and low-birthweight baby)
. list
```

	category	n_lb_w_s	n_women	alcohol	smokes	social
1.	1	11	84	Heavy	Nonsmoker	1
2.	2	5	79	Moderate	Nonsmoker	1
3.	3	11	169	Light	Nonsmoker	1
4.	4	6	28	Heavy	Smoker	1
5.	5	3	13	Moderate	Smoker	1
6.	6	1	26	Light	Smoker	1
7.	7	4	22	Heavy	Nonsmoker	2
8.	8	3	25	Moderate	Nonsmoker	2
9.	9	12	162	Light	Nonsmoker	2
10.	10	4	17	Heavy	Smoker	2
11.	11	2	7	Moderate	Smoker	2
12.	12	6	38	Light	Smoker	2
13.	13	0	14	Heavy	Nonsmoker	3
14.	14	1	18	Moderate	Nonsmoker	3
15.	15	12	91	Light	Nonsmoker	3
16.	16	7	19	Heavy	Smoker	3
17.	17	2	18	Moderate	Smoker	3
18.	18	8	70	Light	Smoker	3

Each observation corresponds to one of the 18 covariate structures. The number of low-birthweight babies from `n_women` in each category is given by the `n_lb_w_babies` variable.

We begin by estimating risk ratios:

```
. binreg n_lbw_babies i.soc i.alc i.smo, n(n_women) rr
Iteration 1: Deviance = 14.2879
Iteration 2: Deviance = 13.607
Iteration 3: Deviance = 13.60503
Iteration 4: Deviance = 13.60503

Generalized linear models
Optimization : MQL Fisher scoring (IRLS EIM)
Deviance = 13.6050268
Pearson = 11.51517095
Variance function: V(u) = u*(1-u/n_women)
Link function : g(u) = ln(u/n_women)

Number of obs = 18
Residual df = 12
Scale parameter = 1
(1/df) Deviance = 1.133752
(1/df) Pearson = .9595976
[Binomial]
[Log]
BIC = -21.07943
```

n_lbw_babies	EIM		z	P> z	[95% conf. interval]	
	Risk ratio	std. err.				
social						
2	1.340001	.3127382	1.25	0.210	.848098	2.11721
3	1.349487	.3291488	1.23	0.219	.8366715	2.176619
alcohol						
Moderate	1.191157	.3265354	0.64	0.523	.6960276	2.038503
Heavy	1.974078	.4261751	3.15	0.002	1.293011	3.013884
smokes						
Smoker	1.648444	.332875	2.48	0.013	1.109657	2.448836
_cons	.0630341	.0128061	-13.61	0.000	.0423297	.0938656

Note: **_cons** estimates baseline risk.

By default, Stata reports the risk ratios (the exponentiated regression coefficients) estimated by the model. We can see that the risk ratio comparing heavy drinkers with light drinkers, after adjusting for smoking and social class, is 1.974078. That is, mothers who drink heavily during their pregnancy have approximately twice the risk of delivering low-birthweight babies as mothers who are light drinkers.

The nonexponentiated coefficients can be obtained with the coefficients option:

```
. binreg n_lbw_babies i.soc i.alc i.smo, n(n_women) rr coefficients
Iteration 1: Deviance = 14.2879
Iteration 2: Deviance = 13.607
Iteration 3: Deviance = 13.60503
Iteration 4: Deviance = 13.60503

Generalized linear models
Optimization : MQL Fisher scoring          Number of obs = 18
              (IRLS EIM)                  Residual df = 12
Deviance     = 13.6050268                  Scale parameter = 1
Pearson      = 11.51517095                 (1/df) Deviance = 1.133752
Variance function: V(u) = u*(1-u/n_women) [Binomial]
Link function : g(u) = ln(u/n_women)      [Log]
BIC = -21.07943
```

n_lbw_babies	EIM		z	P> z	[95% conf. interval]	
	Coefficient	std. err.				
social						
2	.2926702	.2333866	1.25	0.210	-.1647591	.7500994
3	.2997244	.2439066	1.23	0.219	-.1783238	.7777726
alcohol						
Moderate	.1749248	.274133	0.64	0.523	-.362366	.7122156
Heavy	.6801017	.2158856	3.15	0.002	.2569737	1.10323
smokes						
Smoker	.4998317	.2019329	2.48	0.013	.1040505	.8956129
_cons	-2.764079	.2031606	-13.61	0.000	-3.162266	-2.365891

Risk differences are obtained with the rd option:

```
. binreg n_lbw_babies i.soc i.alc i.smo, n(n_women) rd
Iteration 1: Deviance = 18.67277
Iteration 2: Deviance = 14.94364
Iteration 3: Deviance = 14.9185
Iteration 4: Deviance = 14.91762
Iteration 5: Deviance = 14.91758
Iteration 6: Deviance = 14.91758
Iteration 7: Deviance = 14.91758

Generalized linear models                Number of obs =      18
Optimization   : MQL Fisher scoring      Residual df   =     12
                (IRLS EIM)              Scale parameter =      1
Deviance       = 14.91758277             (1/df) Deviance = 1.243132
Pearson        = 12.60353235             (1/df) Pearson  = 1.050294

Variance function: V(u) = u*(1-u/n_women) [Binomial]
Link function   : g(u) = u/n_women       [Identity]
BIC              = -19.76688
```

n_lbw_babies	EIM					
	Risk diff.	std. err.	z	P> z	[95% conf. interval]	
social						
2	.0263817	.0232124	1.14	0.256	-.0191137	.0718771
3	.0365553	.0268668	1.36	0.174	-.0161026	.0892132
alcohol						
Moderate	.0122539	.0257713	0.48	0.634	-.0382569	.0627647
Heavy	.0801291	.0302878	2.65	0.008	.020766	.1394921
smokes						
Smoker	.0542415	.0270838	2.00	0.045	.0011582	.1073248
_cons	.059028	.0160693	3.67	0.000	.0275327	.0905232

The risk difference between heavy drinkers and light drinkers is 0.0801291. Because the risk differences are obtained directly from the coefficients estimated by using the identity link, the `coefficients` option would have no effect here.

Health ratios are obtained with the `hr` option. The health ratios (exponentiated coefficients for the log-complement link) are reported directly.

```
. binreg n_lbw_babies i.soc i.alc i.smo, n(n_women) hr
Iteration 1: Deviance = 21.15233
Iteration 2: Deviance = 15.16467
Iteration 3: Deviance = 15.13205
Iteration 4: Deviance = 15.13114
Iteration 5: Deviance = 15.13111
Iteration 6: Deviance = 15.13111
Iteration 7: Deviance = 15.13111

Generalized linear models
Optimization      : MQL Fisher scoring
                   (IRLS EIM)
Deviance          = 15.13110545
Pearson           = 12.84203917
Variance function: V(u) = u*(1-u/n_women)
Link function     : g(u) = ln(1-u/n_women)

Number of obs    = 18
Residual df      = 12
Scale parameter  = 1
(1/df) Deviance  = 1.260925
(1/df) Pearson   = 1.07017
[Binomial]
[Log complement]
BIC              = -19.55336
```

n_lbw_babies	EIM				
	Hlth ratio	std. err.	z	P> z	[95% conf. interval]
social					
2	.9720541	.024858	-1.11	0.268	.9245342 1.022017
3	.9597182	.0290412	-1.36	0.174	.9044535 1.01836
alcohol					
Moderate	.9871517	.0278852	-0.46	0.647	.9339831 1.043347
Heavy	.9134243	.0325726	-2.54	0.011	.8517631 .9795493
smokes					
Smoker	.9409983	.0296125	-1.93	0.053	.8847125 1.000865
_cons	.9409945	.0163084	-3.51	0.000	.9095674 .9735075

Note: **_cons** estimates baseline health (probability of no disease).

To see the nonexponentiated coefficients, we could specify the `coefficients` option.

Stored results

`binreg`, `irls` stores the following in `e()`:

Scalars

<code>e(N)</code>	number of observations
<code>e(k)</code>	number of parameters
<code>e(k_eq_model)</code>	number of equations in overall model test
<code>e(df_m)</code>	model degrees of freedom
<code>e(df)</code>	residual degrees of freedom
<code>e(phi)</code>	model scale parameter
<code>e(disp)</code>	dispersion parameter
<code>e(bic)</code>	model BIC
<code>e(N_clust)</code>	number of clusters
<code>e(deviance)</code>	deviance
<code>e(deviance_s)</code>	scaled deviance
<code>e(deviance_p)</code>	Pearson deviance
<code>e(deviance_ps)</code>	scaled Pearson deviance
<code>e(dispers)</code>	dispersion
<code>e(dispers_s)</code>	scaled dispersion
<code>e(dispers_p)</code>	Pearson dispersion
<code>e(dispers_ps)</code>	scaled Pearson dispersion
<code>e(vf)</code>	factor set by <code>vfactor()</code> , 1 if not set
<code>e(rank)</code>	rank of <code>e(V)</code>
<code>e(rc)</code>	return code

Macros

<code>e(cmd)</code>	<code>binreg</code>
<code>e(cmdline)</code>	command as typed
<code>e(depvar)</code>	name of dependent variable
<code>e(eform)</code>	<code>eform()</code> option implied by <code>or</code> , <code>rr</code> , <code>hr</code> , or <code>rd</code>
<code>e(varfunc)</code>	program to calculate variance function
<code>e(varfunct)</code>	variance title
<code>e(varfunctf)</code>	variance function
<code>e(link)</code>	program to calculate link function
<code>e(linkt)</code>	link title
<code>e(linkf)</code>	link function
<code>e(m)</code>	number of binomial trials
<code>e(wtype)</code>	weight type
<code>e(wexp)</code>	weight expression
<code>e(title)</code>	title in estimation output
<code>e(title_fl)</code>	family–link title
<code>e(clustvar)</code>	name of cluster variable
<code>e(offset)</code>	linear offset variable
<code>e(cons)</code>	noconstant or not set
<code>e(hac_kernel)</code>	HAC kernel
<code>e(hac_lag)</code>	HAC lag
<code>e(vce)</code>	<i>vctype</i> specified in <code>vce()</code>
<code>e(vctype)</code>	title used to label Std. err.
<code>e(opt)</code>	type of optimization
<code>e(opt1)</code>	optimization title, line 1
<code>e(opt2)</code>	optimization title, line 2
<code>e(properties)</code>	<code>b V</code>
<code>e(predict)</code>	program used to implement <code>predict</code>
<code>e(marginsok)</code>	predictions allowed by <code>margins</code>
<code>e(marginsnotok)</code>	predictions disallowed by <code>margins</code>
<code>e(asbalanced)</code>	factor variables <code>fvset</code> as <code>asbalanced</code>
<code>e(asobserved)</code>	factor variables <code>fvset</code> as <code>asobserved</code>

Matrices	
e(b)	coefficient vector
e(V)	variance–covariance matrix of the estimators
e(V_modelbased)	model-based variance
Functions	
e(sample)	marks estimation sample

In addition to the above, the following is stored in `r()`:

Matrices	
r(table)	matrix containing the coefficients with their standard errors, test statistics, p -values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

`binreg, ml` stores the following in `e()`:

Scalars	
e(N)	number of observations
e(k)	number of parameters
e(k_eq)	number of equations in <code>e(b)</code>
e(k_eq_model)	number of equations in overall model test
e(k_dv)	number of dependent variables
e(df_m)	model degrees of freedom
e(df)	residual degrees of freedom
e(phi)	model scale parameter
e(aic)	model AIC, if <code>ml</code>
e(bic)	model BIC
e(ll)	log likelihood, if <code>ml</code>
e(N_clust)	number of clusters
e(chi2)	χ^2
e(p)	p -value for model test
e(deviance)	deviance
e(deviance_s)	scaled deviance
e(deviance_p)	Pearson deviance
e(deviance_ps)	scaled Pearson deviance
e(dispers)	dispersion
e(dispers_s)	scaled dispersion
e(dispers_p)	Pearson dispersion
e(dispers_ps)	scaled Pearson dispersion
e(vf)	factor set by <code>vfactor()</code> , 1 if not set
e(rank)	rank of <code>e(V)</code>
e(ic)	number of iterations
e(rc)	return code
e(converged)	1 if converged, 0 otherwise
Macros	
e(cmd)	<code>binreg</code>
e(cmdline)	command as typed
e(depvar)	name of dependent variable
e(eform)	<code>eform()</code> option implied by <code>or</code> , <code>rr</code> , <code>hr</code> , or <code>rd</code>
e(varfunc)	program to calculate variance function
e(varfunct)	variance title
e(varfuncf)	variance function
e(link)	program to calculate link function
e(linkt)	link title
e(linkf)	link function
e(m)	number of binomial trials

e(wtype)	weight type
e(wexp)	weight expression
e(title)	title in estimation output
e(title_f1)	family-link title
e(clustvar)	name of cluster variable
e(offset)	linear offset variable
e(cons)	noconstant or not set
e(hac_kernel)	HAC kernel
e(hac_lag)	HAC lag
e(chi2type)	Wald; type of model χ^2 test
e(vce)	<i>vce</i> type specified in <code>vce()</code>
e(vcetype)	title used to label Std. err.
e(opt)	type of optimization
e(opt1)	optimization title, line 1
e(which)	max or min; whether optimizer is to perform maximization or minimization
e(ml_method)	type of ml method
e(user)	name of likelihood-evaluator program
e(technique)	maximization technique
e(properties)	b V
e(predict)	program used to implement predict
e(marginsok)	predictions allowed by margins
e(marginsnotok)	predictions disallowed by margins
e(asbalanced)	factor variables <code>fvset</code> as <code>asbalanced</code>
e(asobserved)	factor variables <code>fvset</code> as <code>asobserved</code>
Matrices	
e(b)	coefficient vector
e(Cns)	constraints matrix
e(ilog)	iteration log (up to 20 iterations)
e(gradient)	gradient vector
e(V)	variance-covariance matrix of the estimators
e(V_modelbased)	model-based variance
Functions	
e(sample)	marks estimation sample

In addition to the above, the following is stored in `r()`:

Matrices	
r(table)	matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r`-class command is run after the estimation command.

Methods and formulas

Let π_i be the probability of success for the i th observation, $i = 1, \dots, N$, and let $X\beta$ be the linear predictor. The link function relates the covariates of each observation to its respective probability through the linear predictor.

In logistic regression, the logit link is used:

$$\ln\left(\frac{\pi}{1-\pi}\right) = X\beta$$

The regression coefficient β_k represents the change in the logarithm of the odds associated with a one-unit change in the value of the X_k covariate; thus, $\exp(\beta_k)$ is the ratio of the odds associated with a change of one unit in X_k .

For risk differences, the identity link $\pi = X\beta$ is used. The regression coefficient β_k represents the risk difference associated with a change of one unit in X_k . When using the identity link, you can obtain fitted probabilities outside the interval (0, 1). As suggested by Wacholder, at each iteration, fitted probabilities are checked for range conditions (and put back in range if necessary). For example, if the identity link results in a fitted probability that is smaller than $1e-4$, the probability is replaced with $1e-4$ before the link function is calculated.

A similar adjustment is made for the logarithmic link, which is used for estimating the risk ratio, $\ln(\pi) = X\beta$, where $\exp(\beta_k)$ is the risk ratio associated with a change of one unit in X_k , and for the log-complement link used to estimate the probability of no disease or health, where $\exp(\beta_k)$ represents the “health ratio” associated with a change of one unit in X_k .

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] [_robust](#), particularly *Maximum likelihood estimators* and *Methods and formulas*.

References

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Also see

- [R] [binreg postestimation](#) — Postestimation tools for binreg
- [R] [glm](#) — Generalized linear models
- [BAYES] [bayses: binreg](#) — Bayesian generalized linear models: Extensions to the binomial family
- [ME] [mecloglog](#) — Multilevel mixed-effects complementary log–log regression
- [ME] [meglm](#) — Multilevel mixed-effects generalized linear models
- [ME] [melogit](#) — Multilevel mixed-effects logistic regression
- [ME] [meprobit](#) — Multilevel mixed-effects probit regression
- [MI] [Estimation](#) — Estimation commands for use with mi estimate
- [U] [20 Estimation and postestimation commands](#)

Postestimation commands

The following postestimation commands are available after `binreg`:

Command	Description
<code>contrast</code>	contrasts and ANOVA-style joint tests of parameters
* <code>estat ic</code>	Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively)
<code>estat summarize</code>	summary statistics for the estimation sample
<code>estat vce</code>	variance–covariance matrix of the estimators (VCE)
<code>estimates</code>	cataloging estimation results
<code>etable</code>	table of estimation results
† <code>forecast</code>	dynamic forecasts and simulations
<code>hausman</code>	Hausman's specification test
<code>lincom</code>	point estimates, standard errors, testing, and inference for linear combinations of parameters
<code>linktest</code>	link test for model specification
* <code>lrtest</code>	likelihood-ratio test
<code>margins</code>	marginal means, predictive margins, marginal effects, and average marginal effects
<code>marginsplot</code>	graph the results from margins (profile plots, interaction plots, etc.)
<code>nlcom</code>	point estimates, standard errors, testing, and inference for nonlinear combinations of parameters
<code>predict</code>	predictions, residuals, influence statistics, and other diagnostic measures
<code>predictnl</code>	point estimates, standard errors, testing, and inference for generalized predictions
<code>pwcompare</code>	pairwise comparisons of parameters
<code>suest</code>	seemingly unrelated estimation
<code>test</code>	Wald tests of simple and composite linear hypotheses
<code>testnl</code>	Wald tests of nonlinear hypotheses

* `estat ic` and `lrtest` are not appropriate after `binreg`, `irls`.

† `forecast` is not appropriate with `mi` estimation results.

predict

Description for predict

`predict` creates a new variable containing predictions such as expected values, linear predictions, standard errors, residuals, Cook’s distance, diagonals, weighted averages, differences, and first derivatives.

Menu for predict

Statistics > Postestimation

Syntax for predict

```
predict [type] newvar [if] [in] [ , statistic options ]
```

<i>statistic</i>	Description
Main	
<code>mu</code>	expected value of y ; the default
<code>xb</code>	linear prediction $\eta = \mathbf{x}\hat{\beta}$
<code>eta</code>	synonym for <code>xb</code>
<code>stdp</code>	standard error of the linear prediction
<code>anscombe</code>	Anscombe (1953) residuals
<code>cooksd</code>	Cook’s distance
<code>deviance</code>	deviance residuals
<code>hat</code>	diagonals of the “hat” matrix
<code>likelihood</code>	weighted average of the standardized deviance and standard Pearson residuals
<code>pearson</code>	Pearson residuals
<code>response</code>	differences between the observed and fitted outcomes
<code>score</code>	first derivative of the log likelihood with respect to $\mathbf{x}_j\beta$
<code>working</code>	working residuals

<i>options</i>	Description
Options	
<code>nooffset</code>	modify calculations to ignore the offset variable
<code>adjusted</code>	adjust deviance residual to speed up convergence
<code>standardized</code>	multiply residual by the factor $(1 - h)^{1/2}$
<code>studentized</code>	multiply residual by one over the square root of the estimated scale parameter
<code>modified</code>	modify denominator of residual to be a reasonable estimate of the variance of <i>depvar</i>

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

Options for predict

Main

`mu`, the default, specifies that `predict` calculate the expected value of y , equal to $g^{-1}(\mathbf{x}\hat{\boldsymbol{\beta}})$ [$ng^{-1}(\mathbf{x}\hat{\boldsymbol{\beta}})$ for the binomial family].

`xb` calculates the linear prediction $\eta = \mathbf{x}\hat{\boldsymbol{\beta}}$.

`eta` is a synonym for `xb`.

`stdp` calculates the standard error of the linear prediction.

`anscombe` calculates the [Anscombe \(1953\)](#) residuals to produce residuals that closely follow a normal distribution.

`cooks` calculates Cook's distance, which measures the aggregate change in the estimated coefficients when each observation is left out of the estimation.

`deviance` calculates the deviance residuals, which are recommended by [McCullagh and Nelder \(1989\)](#) and others as having the best properties for examining goodness of fit of a GLM. They are approximately normally distributed if the model is correct and may be plotted against the fitted values or against a covariate to inspect the model's fit. Also see the `pearson` option below.

`hat` calculates the diagonals of the "hat" matrix, analogous to linear regression.

`likelihood` calculates a weighted average of the standardized deviance and standardized Pearson (described below) residuals.

`pearson` calculates the Pearson residuals, which often have markedly skewed distributions for nonnormal family distributions. Also see the `deviance` option above.

`response` calculates the differences between the observed and fitted outcomes.

`score` calculates the equation-level score, $\partial \ln L / \partial (\mathbf{x}_j \boldsymbol{\beta})$.

`working` calculates the working residuals, which are response residuals weighted according to the derivative of the link function.

Options

`nooffset` is relevant only if you specified `offset(varname)` for `binreg`. It modifies the calculations made by `predict` so that they ignore the offset variable; the linear prediction is treated as $\mathbf{x}_j \mathbf{b}$ rather than as $\mathbf{x}_j \mathbf{b} + \text{offset}_j$.

`adjusted` adjusts the deviance residual to make the convergence to the limiting normal distribution faster. The adjustment deals with adding to the deviance residual a higher-order term depending on the variance function family. This option is allowed only when `deviance` is specified.

`standardized` requests that the residual be multiplied by the factor $(1 - h)^{-1/2}$, where h is the diagonal of the hat matrix. This step is done to take into account the correlation between `deprvar` and its predicted value.

`studentized` requests that the residual be multiplied by one over the square root of the estimated scale parameter.

`modified` requests that the denominator of the residual be modified to be a reasonable estimate of the variance of `deprvar`. The base residual is multiplied by the factor $(k/w)^{-1/2}$, where k is either one or the user-specified dispersion parameter and w is the specified weight (or one if left unspecified).

margins

Description for margins

`margins` estimates margins of response for expected values and linear predictions.

Menu for margins

Statistics > Postestimation

Syntax for margins

```
margins [marginlist] [, options]  
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

<i>statistic</i>	Description
<code>mu</code>	expected value of y ; the default
<code>xb</code>	linear prediction $\eta = \mathbf{x}\hat{\beta}$
<code>eta</code>	synonym for <code>xb</code>
<code>stdp</code>	not allowed with <code>margins</code>
<code>anscombe</code>	not allowed with <code>margins</code>
<code>cooksd</code>	not allowed with <code>margins</code>
<code>deviance</code>	not allowed with <code>margins</code>
<code>hat</code>	not allowed with <code>margins</code>
<code>likelihood</code>	not allowed with <code>margins</code>
<code>pearson</code>	not allowed with <code>margins</code>
<code>response</code>	not allowed with <code>margins</code>
<code>score</code>	not allowed with <code>margins</code>
<code>working</code>	not allowed with <code>margins</code>

Statistics not allowed with `margins` are functions of stochastic quantities other than $e(b)$.

For the full syntax, see [R] [margins](#).

References

- Anscombe, F. J. 1953. Contribution of discussion paper by H. Hotelling “New light on the correlation coefficient and its transforms”. *Journal of the Royal Statistical Society, B ser.*, 15: 229–230. <https://doi.org/10.1111/j.2517-6161.1953.tb00136.x>.
- McCullagh, P., and J. A. Nelder. 1989. *Generalized Linear Models*. 2nd ed. London: Chapman and Hall/CRC.

Also see

- [R] [binreg](#) — Generalized linear models: Extensions to the binomial family
- [U] [20 Estimation and postestimation commands](#)

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

Description

`biprobit` fits maximum-likelihood two-equation probit models—either a bivariate probit or a seemingly unrelated probit (limited to two equations).

Quick start

Bivariate probit regression of y_1 and y_2 on x_1

```
biprobit y1 y2 x1
```

Bivariate probit regression of y_1 and y_2 on x_1 , x_2 , and x_3

```
biprobit y1 y2 x1 x2 x3
```

Constrain the coefficients for x_1 to equality in both equations

```
constraint define 1 _b[y1:x1] = _b[y2:x1]  
biprobit y1 y2 x1 x2 x3, constraints(1)
```

Seemingly unrelated bivariate probit regression

```
biprobit (y1 = x1 x2 x3) (y2 = x1 x2)
```

With robust standard errors

```
biprobit (y1 = x1 x2 x3) (y2 = x1 x2), vce(robust)
```

Poirier partial observability model with `difficult` option

```
biprobit (y1 = x1 x2) (y2 = x2 x3), partial difficult
```

Menu

biprobit

Statistics > Binary outcomes > Bivariate probit regression

Seemingly unrelated biprobit

Statistics > Binary outcomes > Seemingly unrelated bivariate probit regression

Syntax

Bivariate probit regression

```
biprobit depvar1 depvar2 [indepvars] [if] [in] [weight] [, options]
```

Seemingly unrelated bivariate probit regression

```
biprobit equation1 equation2 [if] [in] [weight] [, su_options]
```

where *equation*₁ and *equation*₂ are specified as

```
( [eqname: ] depvar [=] [indepvars] [, noconstant offset(varname) ] )
```

<i>options</i>	Description
Model	
<u>noconstant</u>	suppress constant term
<u>partial</u>	fit partial observability model
<u>offset1</u> (<i>varname</i>)	offset variable for first equation
<u>offset2</u> (<i>varname</i>)	offset variable for second equation
<u>constraints</u> (<i>constraints</i>)	apply specified linear constraints
SE/Robust	
<u>vce</u> (<i>vcetype</i>)	<i>vcetype</i> may be <u>oim</u> , <u>robust</u> , <u>cluster</u> <i>clustvar</i> , <u>opg</u> , <u>bootstrap</u> , or <u>jackknife</u>
Reporting	
<u>level</u> (#)	set confidence level; default is level(95)
<u>lrmodel</u>	perform the likelihood-ratio model test instead of the default Wald test
<u>nocnsreport</u>	do not display constraints
<u>display_options</u>	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Maximization	
<u>maximize_options</u>	control the maximization process; seldom used
<u>collinear</u>	keep collinear variables
<u>coeflegend</u>	display legend instead of statistics

<i>su_options</i>	Description
Model	
<code>partial</code>	fit partial observability model
<code>constraints</code> (<i>constraints</i>)	apply specified linear constraints
SE/Robust	
<code>vce</code> (<i>vcetype</i>)	<i>vcetype</i> may be <code>oim</code> , <code>robust</code> , <code>cluster</code> <i>clustvar</i> , <code>opg</code> , <code>bootstrap</code> , or <code>jackknife</code>
Reporting	
<code>level</code> (#)	set confidence level; default is <code>level</code> (95)
<code>lrmodel</code>	perform the likelihood-ratio model test instead of the default Wald test
<code>nocnsreport</code>	do not display constraints
<code>display_options</code>	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Maximization	
<code>maximize_options</code>	control the maximization process; seldom used
<code>collinear</code>	keep collinear variables
<code>coeflegend</code>	display legend instead of statistics

indepvars may contain factor variables; see [U] 11.4.3 Factor variables.

*depvar*₁, *depvar*₂, *indepvars*, and *depvar* may contain time-series operators; see [U] 11.4.4 Time-series varlists.

`bayes`, `bayesboot`, `bootstrap`, `by`, `collect`, `fp`, `jackknife`, `rolling`, `statsby`, and `svy` are allowed; see [U] 11.1.10 Prefix commands. For more details, see [BAYES] bayes: biprobit.

Weights are not allowed with the `bootstrap` prefix; see [R] bootstrap.

`vce()`, `lrmodel`, and weights are not allowed with the `svy` prefix; see [SVY] svy.

`pweights`, `fweights`, and `iweights` are allowed; see [U] 11.1.6 weight.

`collinear` and `coeflegend` do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

Options

Model

`noconstant`; see [R] Estimation options.

`partial` specifies that the partial observability model be fit. This particular model commonly has poor convergence properties, so we recommend that you use the `difficult` option if you want to fit the Poirier partial observability model; see [R] Maximize.

This model computes the product of the two dependent variables so that you do not have to replace each with the product.

`offset1` (*varname*), `offset2` (*varname*), `constraints` (*constraints*); see [R] Estimation options.

SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce_option](#).

Reporting

`level(#)`, `lrmodel`, `nocnsreport`; see [R] [Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

Setting the optimization type to `technique(bhhh)` resets the default `vcetype` to `vce(opg)`.

The following options are available with `biprobit` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

Remarks and examples

For a good introduction to the bivariate probit models, see [Greene \(2018, sec. 17.9\)](#) and [Pindyck and Rubinfeld \(1998\)](#). [Poirier \(1980\)](#) explains the partial observability model. [Van de Ven and Van Pragg \(1981\)](#) explain the probit model with sample selection; see [R] [heckprobit](#) for details.

▷ Example 1

We use the data from [Pindyck and Rubinfeld \(1998, 332\)](#). In this dataset, the variables are whether children attend private school (`private`), number of years the family has been at the present residence (`years`), log of property tax (`logptax`), log of income (`loginc`), and whether the head of the household voted for an increase in property taxes (`vote`).

We wish to model the bivariate outcomes of whether children attend private school and whether the head of the household voted for an increase in property tax based on the other covariates.

```

. use https://www.stata-press.com/data/r19/school
. biprobit private vote years logptax loginc
Fitting comparison equation 1:
Iteration 0:  Log likelihood = -31.967097
Iteration 1:  Log likelihood = -31.452424
Iteration 2:  Log likelihood = -31.448958
Iteration 3:  Log likelihood = -31.448958
Fitting comparison equation 2:
Iteration 0:  Log likelihood = -63.036914
Iteration 1:  Log likelihood = -58.534843
Iteration 2:  Log likelihood = -58.497292
Iteration 3:  Log likelihood = -58.497288
Comparison:   Log likelihood = -89.946246
Fitting full model:
Iteration 0:  Log likelihood = -89.946246
Iteration 1:  Log likelihood = -89.258897
Iteration 2:  Log likelihood = -89.254028
Iteration 3:  Log likelihood = -89.254028
Bivariate probit regression
Log likelihood = -89.254028
Number of obs = 95
Wald chi2(6) = 9.59
Prob > chi2 = 0.1431

```

	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
private						
years	-.0118884	.0256778	-0.46	0.643	-.0622159	.0384391
logptax	-.1066962	.6669782	-0.16	0.873	-1.413949	1.200557
loginc	.3762037	.5306484	0.71	0.478	-.663848	1.416255
_cons	-4.184694	4.837817	-0.86	0.387	-13.66664	5.297253
vote						
years	-.0168561	.0147834	-1.14	0.254	-.0458309	.0121188
logptax	-1.288707	.5752266	-2.24	0.025	-2.416131	-.1612839
loginc	.998286	.4403565	2.27	0.023	.1352031	1.861369
_cons	-.5360573	4.068509	-0.13	0.895	-8.510188	7.438073
/athrho	-.2764525	.2412099	-1.15	0.252	-.7492153	.1963102
rho	-.2696186	.2236753			-.6346806	.1938267

```

LR test of rho=0: chi2(1) = 1.38444
Prob > chi2 = 0.2393

```

The output shows several iteration logs. The first iteration log corresponds to running the univariate probit model for the first equation, and the second log corresponds to running the univariate probit for the second model. If $\rho = 0$, the sum of the log likelihoods from these two models will equal the log likelihood of the bivariate probit model; this sum is printed in the iteration log as the comparison log likelihood.

The final iteration log is for fitting the full bivariate probit model. A likelihood-ratio test of the log likelihood for this model and the comparison log likelihood is presented at the end of the output. If we had specified the `vce(robust)` option, this test would be presented as a Wald test instead of as a likelihood-ratio test.

We could have fit the same model by using the seemingly unrelated syntax as

```

. biprobit (private=years logptax loginc) (vote=years logptax loginc)

```

Stored results

biprobit stores the following in `e()`:

Scalars

<code>e(N)</code>	number of observations
<code>e(k)</code>	number of parameters
<code>e(k_eq)</code>	number of equations in <code>e(b)</code>
<code>e(k_aux)</code>	number of auxiliary parameters
<code>e(k_eq_model)</code>	number of equations in overall model test
<code>e(k_dv)</code>	number of dependent variables
<code>e(df_m)</code>	model degrees of freedom
<code>e(ll)</code>	log likelihood
<code>e(ll_0)</code>	log likelihood, constant-only model (<code>lrm</code> only)
<code>e(ll_c)</code>	log likelihood, comparison model
<code>e(N_clust)</code>	number of clusters
<code>e(chi2)</code>	χ^2
<code>e(chi2_c)</code>	χ^2 for comparison test
<code>e(p)</code>	p -value for model test
<code>e(rho)</code>	ρ
<code>e(rank)</code>	rank of <code>e(V)</code>
<code>e(rank0)</code>	rank of <code>e(V)</code> for constant-only model
<code>e(ic)</code>	number of iterations
<code>e(rc)</code>	return code
<code>e(converged)</code>	1 if converged, 0 otherwise

Macros

<code>e(cmd)</code>	<code>biprobit</code>
<code>e(cmdline)</code>	command as typed
<code>e(depvar)</code>	names of dependent variables
<code>e(wtype)</code>	weight type
<code>e(wexp)</code>	weight expression
<code>e(title)</code>	title in estimation output
<code>e(clustvar)</code>	name of cluster variable
<code>e(offset1)</code>	offset for first equation
<code>e(offset2)</code>	offset for second equation
<code>e(chi2type)</code>	Wald or LR; type of model χ^2 test
<code>e(chi2_ct)</code>	Wald or LR; type of model χ^2 test corresponding to <code>e(chi2_c)</code>
<code>e(vce)</code>	<i>vce</i> type specified in <code>vce()</code>
<code>e(vctype)</code>	title used to label Std. err.
<code>e(opt)</code>	type of optimization
<code>e(which)</code>	max or min; whether optimizer is to perform maximization or minimization
<code>e(ml_method)</code>	type of ml method
<code>e(user)</code>	name of likelihood-evaluator program
<code>e(technique)</code>	maximization technique
<code>e(properties)</code>	<code>b V</code>
<code>d(predict)</code>	program used to implement <code>predict</code>
<code>e(marginsok)</code>	predictions allowed by <code>margins</code>
<code>e(marginsnotok)</code>	predictions disallowed by <code>margins</code>
<code>e(asbalanced)</code>	factor variables <code>fvset</code> as <code>asbalanced</code>
<code>e(asobserved)</code>	factor variables <code>fvset</code> as <code>asobserved</code>

Matrices

<code>e(b)</code>	coefficient vector
<code>e(Cns)</code>	constraints matrix
<code>e(ilog)</code>	iteration log (up to 20 iterations)
<code>e(gradient)</code>	gradient vector
<code>e(V)</code>	variance-covariance matrix of the estimators
<code>e(V_modelbased)</code>	model-based variance

Functions
`e(sample)` marks estimation sample

In addition to the above, the following is stored in `r()`:

Matrices
`r(table)` matrix containing the coefficients with their standard errors, test statistics, *p*-values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

Methods and formulas

The log likelihood, $\ln L$, is given by

$$\begin{aligned} \xi_j^\beta &= x_j \beta + \text{offset}_j^\beta \\ \xi_j^\gamma &= z_j \gamma + \text{offset}_j^\gamma \\ q_{1j} &= \begin{cases} 1 & \text{if } y_{1j} \neq 0 \\ -1 & \text{otherwise} \end{cases} \\ q_{2j} &= \begin{cases} 1 & \text{if } y_{2j} \neq 0 \\ -1 & \text{otherwise} \end{cases} \\ \rho_j^* &= q_{1j} q_{2j} \rho \\ \ln L &= \sum_{j=1}^n w_j \ln \Phi_2 \left(q_{1j} \xi_j^\beta, q_{2j} \xi_j^\gamma, \rho_j^* \right) \end{aligned}$$

where $\Phi_2(\cdot)$ is the cumulative bivariate normal distribution function (with mean $[0 \ 0]'$) and w_j is an optional weight for observation j . This derivation assumes that

$$\begin{aligned} y_{1j}^* &= x_j \beta + \epsilon_{1j} + \text{offset}_j^\beta \\ y_{2j}^* &= z_j \gamma + \epsilon_{2j} + \text{offset}_j^\gamma \\ E(\epsilon_1) &= E(\epsilon_2) = 0 \\ \text{Var}(\epsilon_1) &= \text{Var}(\epsilon_2) = 1 \\ \text{Cov}(\epsilon_1, \epsilon_2) &= \rho \end{aligned}$$

where y_{1j}^* and y_{2j}^* are the unobserved latent variables; instead, we observe only $y_{ij} = 1$ if $y_{ij}^* > 0$ and $y_{ij} = 0$ otherwise (for $i = 1, 2$).

In the maximum likelihood estimation, ρ is not directly estimated, but $\text{atanh } \rho$ is

$$\text{atanh } \rho = \frac{1}{2} \ln \left(\frac{1 + \rho}{1 - \rho} \right)$$

From the form of the likelihood, if $\rho = 0$, then the log likelihood for the bivariate probit models is equal to the sum of the log likelihoods of the two univariate probit models. A likelihood-ratio test may therefore be performed by comparing the likelihood of the full bivariate model with the sum of the log likelihoods for the univariate probit models.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] [_robust](#), particularly *Maximum likelihood estimators* and *Methods and formulas*.

`biprobit` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] [Variance estimation](#).

References

- De Luca, G. 2008. SNP and SML estimation of univariate and bivariate binary-choice models. *Stata Journal* 8: 190–220.
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Also see

[R] [biprobit postestimation](#) — Postestimation tools for `biprobit`

[R] [mprobit](#) — Multinomial probit regression

[R] [probit](#) — Probit regression

[BAYES] [bayes: biprobit](#) — Bayesian bivariate probit regression

[SVY] [svy estimation](#) — Estimation commands for survey data

[U] [20 Estimation and postestimation commands](#)

Postestimation commands

The following postestimation commands are available after `biprobit`:

Command	Description
<code>contrast</code>	contrasts and ANOVA-style joint tests of parameters
<code>estat ic</code>	Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively)
<code>estat summarize</code>	summary statistics for the estimation sample
<code>estat vce</code>	variance–covariance matrix of the estimators (VCE)
<code>estat (svy)</code>	postestimation statistics for survey data
<code>estimates</code>	cataloging estimation results
<code>etable</code>	table of estimation results
* <code>hausman</code>	Hausman's specification test
<code>lincom</code>	point estimates, standard errors, testing, and inference for linear combinations of parameters
* <code>lrtest</code>	likelihood-ratio test
<code>margins</code>	marginal means, predictive margins, marginal effects, and average marginal effects
<code>marginsplot</code>	graph the results from margins (profile plots, interaction plots, etc.)
<code>nlcom</code>	point estimates, standard errors, testing, and inference for nonlinear combinations of parameters
<code>predict</code>	probabilities for joint, marginal, and conditional outcomes
<code>predictnl</code>	point estimates, standard errors, testing, and inference for generalized predictions
<code>pwcompare</code>	pairwise comparisons of parameters
<code>suest</code>	seemingly unrelated estimation
<code>test</code>	Wald tests of simple and composite linear hypotheses
<code>testnl</code>	Wald tests of nonlinear hypotheses

*`hausman` and `lrtest` are not appropriate with `svy` estimation results.

predict

Description for predict

`predict` creates a new variable containing predictions such as probabilities, linear predictions, and standard errors.

Menu for predict

Statistics > Postestimation

Syntax for predict

```
predict [type] newvar [if] [in] [ , statistic nooffset ]
```

```
predict [type] stub* [if] [in] , scores
```

<i>statistic</i>	Description
------------------	-------------

Main

<code>p11</code>	$\Phi_2(\mathbf{x}_j\mathbf{b}, \mathbf{z}_j\mathbf{g}, \rho)$, predicted probability $\Pr(y_{1j} = 1, y_{2j} = 1)$; the default
<code>p10</code>	$\Phi_2(\mathbf{x}_j\mathbf{b}, -\mathbf{z}_j\mathbf{g}, -\rho)$, predicted probability $\Pr(y_{1j} = 1, y_{2j} = 0)$
<code>p01</code>	$\Phi_2(-\mathbf{x}_j\mathbf{b}, \mathbf{z}_j\mathbf{g}, -\rho)$, predicted probability $\Pr(y_{1j} = 0, y_{2j} = 1)$
<code>p00</code>	$\Phi_2(-\mathbf{x}_j\mathbf{b}, -\mathbf{z}_j\mathbf{g}, \rho)$, predicted probability $\Pr(y_{1j} = 0, y_{2j} = 0)$
<code>pmarg1</code>	$\Phi(\mathbf{x}_j\mathbf{b})$, marginal success probability for equation 1
<code>pmarg2</code>	$\Phi(\mathbf{z}_j\mathbf{g})$, marginal success probability for equation 2
<code>pcond1</code>	$\Phi_2(\mathbf{x}_j\mathbf{b}, \mathbf{z}_j\mathbf{g}, \rho) / \Phi(\mathbf{z}_j\mathbf{g})$, conditional probability of success for equation 1
<code>pcond2</code>	$\Phi_2(\mathbf{x}_j\mathbf{b}, \mathbf{z}_j\mathbf{g}, \rho) / \Phi(\mathbf{x}_j\mathbf{b})$, conditional probability of success for equation 2
<code>xb1</code>	$\mathbf{x}_j\mathbf{b}$, linear prediction for equation 1
<code>xb2</code>	$\mathbf{z}_j\mathbf{g}$, linear prediction for equation 2
<code>stdp1</code>	standard error of the linear prediction for equation 1
<code>stdp2</code>	standard error of the linear prediction for equation 2

where $\Phi(\cdot)$ is the standard normal-distribution function and $\Phi_2(\cdot)$ is the bivariate standard normal-distribution function.

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

Options for predict

Main

`p11`, the default, calculates the bivariate predicted probability $\Pr(y_{1j} = 1, y_{2j} = 1)$.

`p10` calculates the bivariate predicted probability $\Pr(y_{1j} = 1, y_{2j} = 0)$.

`p01` calculates the bivariate predicted probability $\Pr(y_{1j} = 0, y_{2j} = 1)$.

`p00` calculates the bivariate predicted probability $\Pr(y_{1j} = 0, y_{2j} = 0)$.

`pmarg1` calculates the univariate (marginal) predicted probability of success $\Pr(y_{1j} = 1)$.

`pmarg2` calculates the univariate (marginal) predicted probability of success $\Pr(y_{2j} = 1)$.

`pcond1` calculates the conditional (on success in equation 2) predicted probability of success $\Pr(y_{1j} = 1, y_{2j} = 1) / \Pr(y_{2j} = 1)$.

`pcond2` calculates the conditional (on success in equation 1) predicted probability of success $\Pr(y_{1j} = 1, y_{2j} = 1) / \Pr(y_{1j} = 1)$.

`xb1` calculates the probit linear prediction $\mathbf{x}_j\mathbf{b}$.

`xb2` calculates the probit linear prediction $\mathbf{z}_j\mathbf{g}$.

`stdp1` calculates the standard error of the linear prediction for equation 1.

`stdp2` calculates the standard error of the linear prediction for equation 2.

`nooffset` is relevant only if you specified `offset1(varname)` or `offset2(varname)` for `biprobit`.

It modifies the calculations made by `predict` so that they ignore the offset variables; the linear predictions are treated as $\mathbf{x}_j\mathbf{b}$ rather than as $\mathbf{x}_j\mathbf{b} + \text{offset}_{1j}$ and $\mathbf{z}_j\boldsymbol{\gamma}$ rather than as $\mathbf{z}_j\boldsymbol{\gamma} + \text{offset}_{2j}$.

`scores` calculates equation-level score variables.

The first new variable will contain $\partial \ln L / \partial (\mathbf{x}_j\boldsymbol{\beta})$.

The second new variable will contain $\partial \ln L / \partial (\mathbf{z}_j\boldsymbol{\gamma})$.

The third new variable will contain $\partial \ln L / \partial (\text{atanh } \rho)$.

margins

Description for margins

`margins` estimates margins of response for probabilities and linear predictions.

Menu for margins

Statistics > Postestimation

Syntax for margins

```
margins [marginlist] [, options ]
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options ]
```

<i>statistic</i>	Description
p11	$\Phi_2(\mathbf{x}_j\mathbf{b}, \mathbf{z}_j\mathbf{g}, \rho)$, predicted probability $\Pr(y_{1j} = 1, y_{2j} = 1)$; the default
p10	$\Phi_2(\mathbf{x}_j\mathbf{b}, -\mathbf{z}_j\mathbf{g}, -\rho)$, predicted probability $\Pr(y_{1j} = 1, y_{2j} = 0)$
p01	$\Phi_2(-\mathbf{x}_j\mathbf{b}, \mathbf{z}_j\mathbf{g}, -\rho)$, predicted probability $\Pr(y_{1j} = 0, y_{2j} = 1)$
p00	$\Phi_2(-\mathbf{x}_j\mathbf{b}, -\mathbf{z}_j\mathbf{g}, \rho)$, predicted probability $\Pr(y_{1j} = 0, y_{2j} = 0)$
pmarg1	$\Phi(\mathbf{x}_j\mathbf{b})$, marginal success probability for equation 1
pmarg2	$\Phi(\mathbf{z}_j\mathbf{g})$, marginal success probability for equation 2
pcond1	$\Phi_2(\mathbf{x}_j\mathbf{b}, \mathbf{z}_j\mathbf{g}, \rho) / \Phi(\mathbf{z}_j\mathbf{g})$, conditional probability of success for equation 1
pcond2	$\Phi_2(\mathbf{x}_j\mathbf{b}, \mathbf{z}_j\mathbf{g}, \rho) / \Phi(\mathbf{x}_j\mathbf{b})$, conditional probability of success for equation 2
xb1	$\mathbf{x}_j\mathbf{b}$, linear prediction for equation 1
xb2	$\mathbf{z}_j\mathbf{g}$, linear prediction for equation 2
stdp1	not allowed with <code>margins</code>
stdp2	not allowed with <code>margins</code>

Statistics not allowed with `margins` are functions of stochastic quantities other than $e(b)$.

For the full syntax, see [\[R\] margins](#).

Also see

[\[R\] biprobit](#) — Bivariate probit regression

[\[U\] 20 Estimation and postestimation commands](#)

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Description

`bitest` performs exact hypothesis tests for binomial random variables. The null hypothesis is that the probability of a success on a trial is $\#_p$. The total number of trials is the number of nonmissing values of *varname* (in `bitest`) or $\#_N$ (in `bitesti`). The number of observed successes is the number of 1s in *varname* (in `bitest`) or $\#_{\text{succ}}$ (in `bitesti`). *varname* must contain only 0s, 1s, and missing.

`bitesti` is the immediate form of `bitest`; see [U] 19 **Immediate commands** for a general introduction to immediate commands.

Quick start

Exact test for probability of success ($a = 1$) is 0.4

```
bitest a = .4
```

With additional exact probabilities

```
bitest a = .4, detail
```

Exact test that the probability of success is 0.46, given 22 successes in 74 trials

```
bitesti 74 22 .46
```

Menu

bitest

Statistics > Summaries, tables, and tests > Classical tests of hypotheses > Binomial probability test

bitesti

Statistics > Summaries, tables, and tests > Classical tests of hypotheses > Binomial probability test calculator

Syntax

Binomial probability test

```
bitest varname == #p [if] [in] [weight] [, detail]
```

Immediate form of binomial probability test

```
bitesti #N #succ #p [, detail]
```

by and collect are allowed with bitest; see [U] 11.1.10 Prefix commands.

fweights are allowed with bitest; see [U] 11.1.6 weight.

Option

Advanced

detail shows the probability of the observed number of successes, k_{obs} ; the probability of the number of successes on the opposite tail of the distribution that is used to compute the two-sided p -value, k_{opp} ; and the probability of the point next to k_{opp} . This information can be safely ignored. See the [technical note](#) below for details.

Remarks and examples

Remarks are presented under the following headings:

[bitest](#)
[bitesti](#)

bitest

▷ Example 1

We test 15 university students for high levels of one measure of visual quickness which, from other evidence, we believe is present in 30% of the nonuniversity population. Included in our data is quick, taking on the values 1 (“success”) or 0 (“failure”) depending on the outcome of the test.

```
. use https://www.stata-press.com/data/r19/quick
. bitest quick == 0.3
```

Binomial probability test

Variable	N	Observed k	Expected k	Assumed p	Observed p
quick	15	7	4.5	0.30000	0.46667
Pr(k >= 7)	= 0.131143	(one-sided test)			
Pr(k <= 7)	= 0.949987	(one-sided test)			
Pr(k <= 1 or k >= 7)	= 0.166410	(two-sided test)			

The first part of the output reveals that, assuming a true probability of success of 0.3, the expected number of successes is 4.5 and that we observed seven. Said differently, the assumed frequency under the null hypothesis H_0 is 0.3, and the observed frequency is 0.47.

The first line under the table is a one-sided test; it is the probability of observing seven or more successes conditional on $p = 0.3$. It is a test of $H_0: p = 0.3$ versus the alternative hypothesis $H_A: p > 0.3$. Said in English, the alternative hypothesis is that more than 30% of university students score at high levels on this test of visual quickness. The p -value for this hypothesis test is 0.13.

The second line under the table is a one-sided test of H_0 versus the opposite alternative hypothesis $H_A: p < 0.3$.

The third line is the two-sided test. It is a test of H_0 versus the alternative hypothesis $H_A: p \neq 0.3$. ◀

□ Technical note

The p -value of a hypothesis test is the probability (calculated assuming H_0 is true) of observing any outcome as extreme or more extreme than the observed outcome, with extreme meaning in the direction of the alternative hypothesis. In example 1, the outcomes $k = 8, 9, \dots, 15$ are clearly “more extreme” than the observed outcome $k_{\text{obs}} = 7$ when considering the alternative hypothesis $H_A: p \neq 0.3$. However, outcomes with only a few successes are also in the direction of this alternative hypothesis. For two-sided hypotheses, outcomes with k successes are considered “as extreme or more extreme” than the observed outcome k_{obs} if $\Pr(k) \leq \Pr(k_{\text{obs}})$. Here $\Pr(k = 0)$ and $\Pr(k = 1)$ are both less than $\Pr(k = 7)$, so they are included in the two-sided p -value.

The `detail` option allows you to see the probability (assuming that H_0 is true) of the observed successes ($k = 7$) and the probability of the boundary point ($k = 1$) of the opposite tail used for the two-sided p -value.

```
. bitest quick == 0.3, detail
```

```
Binomial probability test
```

Variable	N	Observed k	Expected k	Assumed p	Observed p
quick	15	7	4.5	0.30000	0.46667
Pr(k >= 7)	= 0.131143	(one-sided test)			
Pr(k <= 7)	= 0.949987	(one-sided test)			
Pr(k <= 1 or k >= 7)	= 0.166410	(two-sided test)			
Pr(k == 7)	= 0.081130	(observed)			
Pr(k == 2)	= 0.091560				
Pr(k == 1)	= 0.030520	(opposite extreme)			

Also shown is the probability of the point next to the boundary point. This probability, namely, $\Pr(k = 2) = 0.092$, is certainly close to the probability of the observed outcome $\Pr(k = 7) = 0.081$, so some people might argue that $k = 2$ should be included in the two-sided p -value. Statisticians (at least some we know) would reply that the p -value is a precisely defined concept and that this is an arbitrary “fuzzification” of its definition. When you compute exact p -values according to the precise definition of a p -value, your type I error is never more than what you say it is—so no one can criticize you for being anticonservative. Including the point $k = 2$ is being overly conservative because it makes the p -value larger yet. But it is your choice; being overly conservative, at least in statistics, is always safe. Know that `bitest` and `bitesti` always keep to the precise definition of a p -value, so if you wish to include this extra point, you must do so by hand or by using the `r()` stored results; see [Stored results](#) below. □

bitesti

▷ Example 2

The binomial test is a function of two statistics and one parameter: N , the number of observations; k_{obs} , the number of observed successes; and p , the assumed probability of a success on a trial. For instance, in a city of $N = 2,500,000$, we observe $k_{\text{obs}} = 36$ cases of a particular disease when the population rate for the disease is $p = 0.00001$.

```
. bitesti 2500000 36 .00001
```

```
Binomial probability test
```

N	Observed k	Expected k	Assumed p	Observed p
2,500,000	36	25	0.00001	0.00001
Pr(k >= 36)		= 0.022458	(one-sided test)	
Pr(k <= 36)		= 0.985448	(one-sided test)	
Pr(k <= 14 or k >= 36)		= 0.034859	(two-sided test)	

◀

Stored results

bitest and bitesti store the following in `r()`:

Scalars

<code>r(N)</code>	number N of trials
<code>r(P_p)</code>	assumed probability p of success
<code>r(k)</code>	observed number k of successes
<code>r(p_l)</code>	lower one-sided p -value
<code>r(p_u)</code>	upper one-sided p -value
<code>r(p)</code>	two-sided p -value
<code>r(k_opp)</code>	opposite extreme k
<code>r(P_k)</code>	probability of observed k (detail only)
<code>r(P_oppk)</code>	probability of opposite extreme k (detail only)
<code>r(k_nopp)</code>	k next to opposite extreme (detail only)
<code>r(P_noppk)</code>	probability of k next to opposite extreme (detail only)

Methods and formulas

Let N , k_{obs} , and p be, respectively, the number of observations, the observed number of successes, and the assumed probability of success on a trial. The expected number of successes is Np , and the observed probability of success on a trial is k_{obs}/N .

bitest and bitesti compute exact p -values based on the binomial distribution. The upper one-sided p -value is

$$\Pr(k \geq k_{\text{obs}}) = \sum_{m=k_{\text{obs}}}^N \binom{N}{m} p^m (1-p)^{N-m}$$

The lower one-sided p -value is

$$\Pr(k \leq k_{\text{obs}}) = \sum_{m=0}^{k_{\text{obs}}} \binom{N}{m} p^m (1-p)^{N-m}$$

If $k_{\text{obs}} \geq Np$, the two-sided p -value is

$$\Pr(k \leq k_{\text{opp}} \text{ or } k \geq k_{\text{obs}})$$

where k_{opp} is the largest number $\leq Np$ such that $\Pr(k = k_{\text{opp}}) \leq \Pr(k = k_{\text{obs}})$. If $k_{\text{obs}} < Np$, the two-sided p -value is

$$\Pr(k \leq k_{\text{obs}} \text{ or } k \geq k_{\text{opp}})$$

where k_{opp} is the smallest number $\geq Np$ such that $\Pr(k = k_{\text{opp}}) \leq \Pr(k = k_{\text{obs}})$.

Reference

Hoel, P. G. 1984. *Introduction to Mathematical Statistics*. 5th ed. New York: Wiley.

Also see

[R] **ci** — Confidence intervals for means, proportions, and variances

[R] **prtest** — Tests of proportions

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

Description

`bootstrap` performs nonparametric bootstrap estimation of specified statistics (or expressions) for a Stata command or a user-written program. Statistics are bootstrapped by resampling the data in memory with replacement. `bootstrap` is designed for use with nonestimation commands, functions of coefficients, or user-written programs. To bootstrap coefficients, we recommend using the `vce(bootstrap)` option when allowed by the estimation command.

Quick start

Bootstrap the mean of `v1` returned by `summarize` in `r(mean)`

```
bootstrap mean=r(mean) : summarize v1
```

Bootstrap the statistic `r(mystat)` returned by program `myprog1`

```
bootstrap stat=r(mystat) : myprog1 v1
```

Same as above, but use 100 replications

```
bootstrap stat=r(mystat), reps(100) : myprog1 v1
```

Same as above, and save the results from each replication in `mydata.dta`

```
bootstrap stat=r(mystat), reps(100) saving(mydata) : myprog1 v1
```

Bootstrap a difference in coefficients estimated by `regress`

```
bootstrap diff=(_b[x2]-_b[x1]) : regress y x1 x2 x3
```

Bootstrap the coefficients stored in `e(b)` by `myprog2`

```
bootstrap _b : myprog2 y x1 x2 x3
```

Same as above, but with bootstrap samples taken independently within strata identified by `svar`

```
bootstrap _b, strata(svar) : myprog2 y x1 x2 x3
```

Resample clusters defined by `cvar` and create `newcvar` identifying resampled clusters

```
bootstrap _b, cluster(cvar) idcluster(newcvar) : myprog2 y x1 x2 x3
```

Menu

Statistics > Resampling > Bootstrap estimation

Syntax

bootstrap *exp_list* [, *options eform_option*] : *command*

<i>options</i>	Description
Main	
<code>reps(#)</code>	perform # bootstrap replications; default is <code>reps(50)</code>
<code>fweights(<i>varlist</i>)</code>	perform bootstrap replications using frequency weight variables
<code>iweights(<i>varlist</i>)</code>	perform bootstrap replications using importance weight variables
Options	
<code>strata(<i>varlist</i>)</code>	specify variables identifying strata
<code>size(#)</code>	draw samples of size #; default is <code>_N</code>
<code>cluster(<i>varlist</i>)</code>	specify variables identifying resampling clusters
<code>idcluster(<i>newvar</i>)</code>	create new cluster ID variable
<code>saving(<i>filename</i>, ...)</code>	save results to <i>filename</i> ; save statistics in double precision; save results to <i>filename</i> every # replications
<code>bca</code>	compute acceleration for BC_a confidence intervals
<code>ties</code>	adjust BC/BCa confidence intervals for ties
<code>mse</code>	use MSE formula for variance estimation
Reporting	
<code>level(#)</code>	set confidence level; default is <code>level(95)</code>
<code>notable</code>	suppress table of results
<code>noheader</code>	suppress table header
<code>nolegend</code>	suppress table legend
<code>verbose</code>	display the full table legend
<code>nodots</code>	suppress replication dots
<code>dots(#)</code>	display dots every # replications
<code>noisily</code>	display any output from <i>command</i>
<code>trace</code>	trace <i>command</i>
<code>title(<i>text</i>)</code>	use <i>text</i> as title for bootstrap results
<code>display_options</code>	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
<code>eform_option</code>	display coefficient table in exponentiated form
Advanced	
<code>nodrop</code>	do not drop observations
<code>nowarn</code>	do not warn when <code>e(sample)</code> is not set
<code>force</code>	do not check for <i>weights</i> or <i>svy</i> commands; seldom used
<code>reject(<i>exp</i>)</code>	identify invalid results
<code>rseed(#)</code>	set random-number seed to #
<code>group(<i>varname</i>)</code>	ID variable for groups within <code>cluster()</code>
<code>jackknifeopts(<i>jkopts</i>)</code>	options for jackknife; see [R] jackknife
<code>coeflegend</code>	display legend instead of statistics

command is any command that follows standard Stata syntax. *weights* are not allowed in *command*.

`collect` and `svy` are allowed; see [U] 11.1.10 Prefix commands.

`group()`, `jackknifeopts()`, and `coeflegend` do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

<i>exp_list</i> contains	(<i>name</i> : <i>elist</i>) <i>elist</i> <i>eexp</i>
<i>elist</i> contains	<i>newvar</i> = (<i>exp</i>) (<i>exp</i>)
<i>eexp</i> is	<i>specname</i> [<i>eqno</i>] <i>specname</i>
<i>specname</i> is	_b _b [] _se _se []
<i>eqno</i> is	# # <i>name</i>

exp is a standard Stata expression; see [U] 13 Functions and expressions.

Distinguish between [], which are to be typed, and [], which indicate optional arguments.

Options

Main

`reps(#)` specifies the number of bootstrap replications to be performed. The default is 50. A total of 50–200 replications are generally adequate for estimates of standard error and thus are adequate for normal-approximation confidence intervals; see Mooney and Duval (1993, 11). Estimates of confidence intervals using the percentile or bias-corrected methods typically require 1,000 or more replications.

`fweights(varlist)` specifies frequency weight variables used to perform each bootstrap replication. These variables are typically generated by `rwgen bsample`. This option cannot be combined with `reps()`, `strata()`, `size()`, `cluster()`, `idcluster()`, or `iweights()`.

`iweights(varlist)` specifies importance weight variables used to perform each bootstrap replication. These variables are typically generated by `rwgen bayes`. This option cannot be combined with `reps()`, `strata()`, `size()`, `cluster()`, `idcluster()`, or `fweights()`.

Options

`strata(varlist)` specifies the variables that identify strata. If this option is specified, bootstrap samples are taken independently within each stratum.

`size(#)` specifies the size of the samples to be drawn. The default is `_N`, meaning to draw samples of the same size as the data. If specified, `#` must be less than or equal to the number of observations within `strata()`.

If `cluster()` is specified, the default size is the number of clusters in the original dataset. For unbalanced clusters, resulting sample sizes will differ from replication to replication. For cluster sampling, `#` must be less than or equal to the number of clusters within `strata()`.

- `cluster(varlist)` specifies the variables that identify resampling clusters. If this option is specified, the sample drawn during each replication is a bootstrap sample of clusters.
- `idcluster(newvar)` creates a new variable containing a unique identifier for each resampled cluster. This option requires that `cluster()` also be specified.
- `saving(filename[, suboptions])` creates a Stata data file (`.dta` file) consisting of (for each statistic in *exp_list*) a variable containing the replicates.
- `double` specifies that the results for each replication be saved as doubles, meaning 8-byte reals. By default, they are saved as floats, meaning 4-byte reals. This option may be used without `saving()` to compute the variance estimates by using double precision.
- `every(#)` specifies that results be written to disk every *#*th replication. `every()` should be specified only in conjunction with `saving()` when *command* takes a long time for each replication. This option will allow recovery of partial results should some other software crash your computer. See [P] [postfile](#).
- `replace` specifies that *filename* be overwritten if it exists. This option does not appear in the dialog box.
- `bca` specifies that bootstrap estimate the acceleration of each statistic in *exp_list*. This estimate is used to construct BC_a confidence intervals. Type `estat bootstrap, bca` to display the BC_a confidence interval generated by the bootstrap command.
- `ties` specifies that bootstrap adjust for ties in the replicate values when computing the median bias used to construct BC and BC_a confidence intervals.
- `mse` specifies that bootstrap compute the variance by using deviations of the replicates from the observed value of the statistics based on the entire dataset. By default, bootstrap computes the variance by using deviations from the average of the replicates.

 Reporting

- `level(#)`; see [R] [Estimation options](#).
- `notable` suppresses the display of the table of results.
- `noheader` suppresses the display of the table header. This option implies `nolegend`. This option may also be specified when replaying estimation results.
- `nolegend` suppresses the display of the table legend. This option may also be specified when replaying estimation results.
- `verbose` specifies that the full table legend be displayed. By default, coefficients and standard errors are not displayed. This option may also be specified when replaying estimation results.
- `nodots` and `dots(#)` specify whether to display replication dots. By default, one dot character is displayed for each successful replication. An “x” is displayed if *command* returns an error or if any value in *exp_list* is missing. You can also control whether dots are displayed using `set dots`; see [R] [set](#).
- `nodots` suppresses display of the replication dots.
- `dots(#)` displays dots every *#* replications. `dots(0)` is a synonym for `nodots`.
- `noisily` specifies that any output from *command* be displayed. This option implies the `nodots` option.
- `trace` causes a trace of the execution of *command* to be displayed. This option implies the `noisily` option.

`title(text)` specifies a title to be displayed above the table of bootstrap results. The default title is the title stored in `e(title)` by an estimation command, or if `e(title)` is not filled in, Bootstrap results is used. `title()` may also be specified when replaying estimation results.

`display_options`: `noci`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

`eform_option` causes the coefficient table to be displayed in exponentiated form; see [R] [eform_option](#).

`command` determines which of the following are allowed (`eform(string)` and `eform` are always allowed):

<i>eform_option</i>	Description
<code>eform(string)</code>	use <i>string</i> for the column title
<code>eform</code>	exponentiated coefficient, <i>string</i> is <code>exp(b)</code>
<code>hr</code>	hazard ratio, <i>string</i> is <code>Haz. ratio</code>
<code>shr</code>	subhazard ratio, <i>string</i> is <code>SHR</code>
<code>irr</code>	incidence-rate ratio, <i>string</i> is <code>IRR</code>
<code>or</code>	odds ratio, <i>string</i> is <code>Odds ratio</code>
<code>rrr</code>	relative-risk ratio, <i>string</i> is <code>RRR</code>

Advanced

`nodrop` prevents observations outside `e(sample)` and the `if` and `in` qualifiers from being dropped before the data are resampled.

`nowarn` suppresses the display of a warning message when `command` does not set `e(sample)`.

`force` suppresses the restriction that `command` not specify weights or be a `svy` command. This is a rarely used option. Use it only if you know what you are doing.

`reject(exp)` identifies an expression that indicates when results should be rejected. When `exp` is true, the resulting values are reset to missing values.

`rseed(#)` sets the random-number seed. Specifying this option is equivalent to typing the following command prior to calling `bootstrap`:

```
. set seed #
```

When either `fweights()` or `iweights()` is specified, `rseed()` is ignored.

The following options are available with `bootstrap` but are not shown in the dialog box:

`group(varname)` re-creates `varname` containing a unique identifier for each group across the resampled clusters. This option requires that `idcluster()` also be specified.

This option is useful for maintaining unique group identifiers when sampling clusters with replacement. Suppose that cluster 1 contains 3 groups. If the `idcluster(newclid)` option is specified and cluster 1 is sampled multiple times, `newclid` uniquely identifies each copy of cluster 1. If `group(newgroupid)` is also specified, `newgroupid` uniquely identifies each copy of each group.

`jackknifeopts(jkopts)` identifies options that are to be passed to `jackknife` when it computes the acceleration values for the BC_a confidence intervals; see [R] [jackknife](#). This option requires the `bca` option and is mostly used for passing the `eclass`, `rclass`, or `n(#)` option to `jackknife`.

`coeflegend`; see [R] [Estimation options](#).

Remarks and examples

Remarks are presented under the following headings:

- Introduction*
- Using bootstrap*
- Regression coefficients*
- Expressions*
- Combining bootstrap datasets*
- A note about macros*
- Achieved significance level*
- Bootstrapping a ratio*
- Warning messages and e(sample)*
- Bootstrapping statistics from data with a complex structure*
- Bootstrapping statistics using frequency or importance weights*

Introduction

With few assumptions, bootstrapping provides a way of estimating standard errors and other measures of statistical precision (Efron 1979; Efron and Stein 1981; Efron 1982; Efron and Tibshirani 1986; Efron and Tibshirani 1993; also see Davison and Hinkley [1997]; Guan [2003]; Mooney and Duval [1993]; Poi [2004]; and Stine [1990]). It provides a way to obtain such measures when no formula is otherwise available or when available formulas make inappropriate assumptions. Cameron and Trivedi (2022, chap. 12) discuss many bootstrapping topics and demonstrate how to do them in Stata.

To illustrate bootstrapping, suppose that you have a dataset containing N observations and an estimator that, when applied to the data, produces certain statistics. You draw, with replacement, N observations from the N -observation dataset. In this random drawing, some of the original observations will appear once, some more than once, and some not at all. Using the resampled dataset, you apply the estimator and collect the statistics. This process is repeated many times; each time, a new random sample is drawn and the statistics are recalculated.

This process builds a dataset of replicated statistics. From these data, you can calculate the standard error by using the standard formula for the sample standard deviation

$$\widehat{\text{se}} = \left\{ \frac{1}{k-1} \sum (\hat{\theta}_i - \bar{\theta})^2 \right\}^{1/2}$$

where $\hat{\theta}_i$ is the statistic calculated using the i th bootstrap sample and k is the number of replications. This formula gives an estimate of the standard error of the statistic, according to Hall and Wilson (1991). Although the average, $\bar{\theta}$, of the bootstrapped estimates is used in calculating the standard deviation, it is not used as the estimated value of the statistic itself. Instead, the original observed value of the statistic, $\hat{\theta}$, is used, meaning the value of the statistic computed using the original N observations.

You might think that $\bar{\theta}$ is a better estimate of the parameter than $\hat{\theta}$, but it is not. If the statistic is biased, bootstrapping exaggerates the bias. In fact, the bias can be estimated as $\bar{\theta} - \hat{\theta}$ (Efron 1982, 33). Knowing this, you might be tempted to subtract this estimate of bias from $\hat{\theta}$ to produce an unbiased statistic. The bootstrap bias estimate has an indeterminate amount of random error, so this unbiased estimator may have greater mean squared error than the biased estimator (Mooney and Duval 1993; Hinkley 1978). Thus, $\hat{\theta}$ is the best point estimate of the statistic.

The logic behind the bootstrap is that all measures of precision come from a statistic's sampling distribution. When the statistic is estimated on a sample of size N from some population, the sampling distribution tells you the relative frequencies of the values of the statistic. The sampling distribution, in turn, is determined by the distribution of the population and the formula used to estimate the statistic.

Sometimes the sampling distribution can be derived analytically. For instance, if the underlying population is distributed normally and you calculate means, the sampling distribution for the mean is also normal but has a smaller variance than that of the population. In other cases, deriving the sampling distribution is difficult, such as when means are calculated from nonnormal populations. Sometimes, as in the case of means, it is not too difficult to derive the sampling distribution as the sample size goes to infinity ($N \rightarrow \infty$). However, such asymptotic distributions may not perform well when applied to finite samples.

If you knew the population distribution, you could obtain the sampling distribution by simulation: you could draw random samples of size N , calculate the statistic, and make a tally. Bootstrapping does precisely this, but it uses the observed distribution of the sample in place of the true population distribution. Thus, the bootstrap procedure hinges on the assumption that the observed distribution is a good estimate of the underlying population distribution. In return, the bootstrap produces an estimate, called the bootstrap distribution, of the sampling distribution. From this, you can estimate the standard error of the statistic, produce confidence intervals, etc.

The accuracy with which the bootstrap distribution estimates the sampling distribution depends on the number of observations in the original sample and the number of replications in the bootstrap. A crudely estimated sampling distribution is adequate if you are going to extract, say, only a standard error. A better estimate is needed if you want to use the 2.5th and 97.5th percentiles of the distribution to produce a 95% confidence interval. To extract many features simultaneously about the distribution, an even better estimate is needed. Generally, replications on the order of 1,000 produce very good estimates, but only 50–200 replications are needed for estimates of standard errors. See [Poi \(2004\)](#) for a method to choose the number of bootstrap replications.

Using bootstrap

Typing

```
. bootstrap exp_list, reps(#): command
```

executes *command* multiple times, bootstrapping the statistics in *exp_list* by resampling observations (with replacement) from the data in memory # times. This method is commonly referred to as the non-parametric bootstrap.

command defines the statistical command to be executed. Most Stata commands and user-written programs can be used with *bootstrap*, as long as they follow standard Stata syntax; see [\[U\] 11 Language syntax](#). If the *bca* option is supplied, *command* must also work with *jackknife*; see [\[R\] jackknife](#). The *by* prefix may not be part of *command*.

exp_list specifies the statistics to be collected from the execution of *command*. If *command* changes the contents in *e(b)*, *exp_list* is optional and defaults to *_b*.

Because bootstrapping is a random process, if you want to be able to reproduce results, set the random-number seed by specifying the *rseed(#)* option or by typing

```
. set seed #
```

where # is a seed of your choosing, before running *bootstrap*; see [\[R\] set seed](#).

Another way to reproduce results is to use frequency weights (*fweights()*) or importance weights (*iweights()*). The weight variables specified in these options are typically generated using either the *rwgen bsample* or *rwgen bayes* command. When either *fweights()* or *iweights()* is specified, *rseed()* is ignored.

Regression coefficients

▷ Example 1

Let's say that we wish to compute bootstrap estimates for the standard errors of the coefficients from the following regression:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. regress mpg weight gear foreign
```

Source	SS	df	MS	Number of obs	=	74
Model	1629.67805	3	543.226016	F(3, 70)	=	46.73
Residual	813.781411	70	11.6254487	Prob > F	=	0.0000
				R-squared	=	0.6670
				Adj R-squared	=	0.6527
Total	2443.45946	73	33.4720474	Root MSE	=	3.4096

mpg	Coefficient	Std. err.	t	P> t	[95% conf. interval]
weight	-.006139	.0007949	-7.72	0.000	-.0077245 -.0045536
gear_ratio	1.457113	1.541286	0.95	0.348	-1.616884 4.53111
foreign	-2.221682	1.234961	-1.80	0.076	-4.684735 .2413715
_cons	36.10135	6.285984	5.74	0.000	23.56435 48.63835

To run the bootstrap, we simply prefix the above regression command with the `bootstrap` command (specifying its options before the colon separator). We must set the random-number seed before calling `bootstrap`.

```
. bootstrap, reps(100) rseed(1): regress mpg weight gear foreign
(running regress on estimation sample)
Bootstrap replications (100): .....10.....20.....30.....40.....
> ...50.....60.....70.....80.....90.....100 done
Linear regression                                Number of obs =    74
                                                Replications   =   100
                                                Wald chi2(3)   = 167.13
                                                Prob > chi2    = 0.0000
                                                R-squared      = 0.6670
                                                Adj R-squared  = 0.6527
                                                Root MSE      = 3.4096
```

mpg	Observed coefficient	Bootstrap std. err.	z	P> z	Normal-based [95% conf. interval]
weight	-.006139	.0006063	-10.13	0.000	-.0073273 -.0049507
gear_ratio	1.457113	1.367917	1.07	0.287	-1.223954 4.138181
foreign	-2.221682	1.169727	-1.90	0.058	-4.514305 .070942
_cons	36.10135	5.20581	6.93	0.000	25.89815 46.30455

The displayed confidence interval is based on the assumption that the sampling (and hence bootstrap) distribution is approximately normal (see [Methods and formulas](#) below). Because this confidence interval is based on the standard error, it is a reasonable estimate if normality is approximately true, even for a few replications. Other types of confidence intervals are available after `bootstrap`; see [\[R\] bootstrap postestimation](#).

We could instead supply names to our expressions when we run `bootstrap`. For example,

```
. bootstrap diff=(_b[weight]-_b[gear]): regress mpg weight gear foreign
```

would bootstrap a statistic, named `diff`, equal to the difference between the coefficients on `weight` and `gear_ratio`.



□ Technical note

`regress`, like many estimation commands, allows the `vce(bootstrap)` option. For any estimation command that allows this option, we recommend using `vce(bootstrap)` over `bootstrap` because the estimation command automatically handles clustering and other model-specific details for you.



Expressions

▷ Example 2

When we use `bootstrap`, the list of statistics can contain complex expressions, as long as each expression is enclosed in parentheses. For example, to bootstrap the range of a variable `x`, we could type

```
. bootstrap range=(r(max)-r(min)), reps(1000): summarize x
```

Of course, we could also bootstrap the minimum and maximum and later compute the range.

```
. bootstrap max=r(max) min=r(min), reps(1000) saving(mybs): summarize x
. use mybs, clear
(bootstrap: summarize)
. generate range = max - min
. bstat range, stat(19.5637501)
```

The difference between the maximum and minimum of `x` in the sample is 19.5637501.

The `stat()` option to `bstat` specifies the observed value of the statistic (`range`) to be summarized. This option is useful when, as shown above, the statistic of ultimate interest is not specified directly to `bootstrap` but instead is calculated by other means.

Here the observed values of `r(max)` and `r(min)` are stored as characteristics of the dataset created by `bootstrap` and are thus available for retrieval by `bstat`; see [R] [bstat](#). The observed range, however, is unknown to `bstat`, so it must be specified.



Combining bootstrap datasets

You can combine two datasets from separate runs of `bootstrap` by using `append` (see [D] [append](#)) and then get the bootstrap statistics for the combined datasets by running `bstat`. The runs must have been performed independently (having different starting random-number seeds), and the original dataset, command, and bootstrap statistics must have been all the same.

A note about macros

In [example 2](#), we executed the command

```
. bootstrap max=r(max) min=r(min), reps(1000) saving(mybs): summarize x
```

We did not enclose `r(max)` and `r(min)` in single quotes, as we would in most other contexts, because it would not produce what was intended:

```
. bootstrap 'r(max)' 'r(min)', reps(1000) saving(mybs): summarize x
```

To understand why, note that `'r(max)'`, like any reference to a local macro, will evaluate to a literal string containing the contents of `r(max)` *before* `bootstrap` is even executed. Typing the command above would appear to Stata as if we had typed

```
. bootstrap 14.5441234 33.4393293, reps(1000) saving(mybs): summarize x
```

Even worse, the current contents of `r(min)` and `r(max)` could be empty, producing an even more confusing result. To avoid this outcome, refer to statistics by name (for example, `r(max)`) and not by value (for example, `'r(max)'`).

Achieved significance level

▷ Example 3

Suppose that we wish to estimate the *achieved significance level* (ASL) of a test statistic by using the bootstrap. ASL is another name for *p*-value. An example is

$$\text{ASL} = \Pr(\hat{\theta}^* \geq \hat{\theta} | H_0)$$

for an upper-tailed, alternative hypothesis, where H_0 denotes the null hypothesis, $\hat{\theta}$ is the observed value of the test statistic, and $\hat{\theta}^*$ is the random variable corresponding to the test statistic, assuming that H_0 is true.

Here we will compare the mean miles per gallon (mpg) between foreign and domestic cars by using the two-sample *t* test with unequal variances. The following results indicate the *p*-value to be 0.0034 for the two-sided test using Satterthwaite's approximation. Thus, assuming that mean mpg is the same for foreign and domestic cars, we would expect to observe a *t* statistic more extreme (in absolute value) than 3.1797 in about 0.3% of all possible samples of the type that we observed. Thus, we have evidence to reject the null hypothesis that the means are equal.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
```

```
. ttest mpg, by(foreign) unequal
```

```
Two-sample t test with unequal variances
```

Group	Obs	Mean	Std. err.	Std. dev.	[95% conf. interval]	
Domestic	52	19.82692	.657777	4.743297	18.50638	21.14747
Foreign	22	24.77273	1.40951	6.611187	21.84149	27.70396
Combined	74	21.2973	.6725511	5.785503	19.9569	22.63769
diff		-4.945804	1.555438		-8.120053	-1.771556

```
diff = mean(Domestic) - mean(Foreign)          t = -3.1797
H0: diff = 0          Satterthwaite's degrees of freedom = 30.5463
Ha: diff < 0          Ha: diff != 0          Ha: diff > 0
Pr(T < t) = 0.0017    Pr(|T| > |t|) = 0.0034    Pr(T > t) = 0.9983
```

We also place the value of the test statistic in a scalar for later use.

```
. scalar tobs = r(t)
```

Efron and Tibshirani (1993, 224) describe an alternative to Satterthwaite's approximation that estimates the ASL by bootstrapping the statistic from the test of equal means. Their idea is to recenter the two samples to the combined sample mean so that the data now conform to the null hypothesis but that the variances within the samples remain unchanged.

```
. summarize mpg, meanonly
. scalar omean = r(mean)
. summarize mpg if foreign==0, meanonly
. replace mpg = mpg - r(mean) + scalar(omean) if foreign==0
variable mpg was int now float
(52 real changes made)
. summarize mpg if foreign==1, meanonly
. replace mpg = mpg - r(mean) + scalar(omean) if foreign==1
(22 real changes made)
. sort foreign
. by foreign: summarize mpg
```

```
-> foreign = Domestic
```

Variable	Obs	Mean	Std. dev.	Min	Max
mpg	52	21.2973	4.743297	13.47037	35.47038

```
-> foreign = Foreign
```

Variable	Obs	Mean	Std. dev.	Min	Max
mpg	22	21.2973	6.611187	10.52457	37.52457

Each sample (foreign and domestic) is a stratum, so the bootstrapped samples must have the same number of foreign and domestic cars as the original dataset. This requirement is facilitated by the `strata()` option to `bootstrap`. By typing the following, we bootstrap the test statistic using the modified dataset and save the values in `bsauto2.dta`:

```
. keep mpg foreign
. set seed 1
. bootstrap t=r(t), rep(1000) strata(foreign) saving(bsauto2) nodots: ttest mpg,
> by(foreign) unequal
```

```
warning: ttest does not set e(sample), so no observations will be excluded
from the resampling because of missing values or other reasons. To
exclude observations, press Break, save the data, drop any
observations that are to be excluded, and rerun bootstrap.
```

Bootstrap results

Number of strata = 2

Number of obs = 74

Replications = 1,000

```
Command: ttest mpg, by(foreign) unequal
t: r(t)
```

	Observed coefficient	Bootstrap std. err.	z	P> z	Normal-based [95% conf. interval]	
t	1.75e-07	1.051867	0.00	1.000	-2.061622	2.061622

We can use the data in `bsauto2.dta` to estimate ASL via the fraction of bootstrap test statistics that are more extreme than 3.1797.

```
. use bsauto2, clear
(bootstrap: ttest)
. generate indicator = abs(t)>=abs(scalar(tobs))
. summarize indicator, meanonly
. display "ASLboot = " r(mean)
ASLboot = .004
```

The result is $ASL_{boot} = 0.004$. Assuming that the mean mpg is the same between foreign and domestic cars, we would expect to observe a t statistic more extreme (in absolute value) than 3.1797 in about 0.4% of all possible samples of the type we observed. This finding is still strong evidence to reject the hypothesis that the means are equal.

Bootstrapping a ratio

▷ Example 4

Suppose that we wish to produce a bootstrap estimate of the ratio of two means. Because `summarize` stores results for only one variable, we must call `summarize` twice to compute the means. Actually, we could use `collapse` to compute the means in one call, but calling `summarize` twice is much faster. Thus, we will have to write a small program that will return the results we want.

We write the program below and save it to a file called `ratio.ado` (see [U] 17 Ado-files). Our program takes two variable names as input and saves them in the local macros `y` (first variable) and `x` (second variable). It then computes one statistic: the mean of ‘`y`’ divided by the mean of ‘`x`’. This value is returned as a scalar in `r(ratio)`. `ratio` also returns the ratio of the number of observations used to the mean for each variable.

```

program myratio, rclass
    version 19.5      // (or version 19 if you do not have StataNow)
    args y x
    confirm var `y'
    confirm var `x'
    tempname ymean
    summarize `y', meanonly
    scalar `ymean' = r(mean)
    return scalar n_`y' = r(N)
    summarize `x', meanonly
    return scalar n_`x' = r(N)
    return scalar ratio = `ymean'/r(mean)
end

```

Remember to test any newly written commands before using them with `bootstrap`.

```

. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
. summarize price

```

Variable	Obs	Mean	Std. dev.	Min	Max
price	74	6165.257	2949.496	3291	15906

```

. scalar mean1=r(mean)
. summarize weight

```

Variable	Obs	Mean	Std. dev.	Min	Max
weight	74	3019.459	777.1936	1760	4840

```

. scalar mean2=r(mean)
. di scalar(mean1)/scalar(mean2)
2.0418412
. myratio price weight
. return list
scalars:
      r(ratio) = 2.041841210168278
    r(n_weight) = 74
    r(n_price) = 74

```

The results of running bootstrap on our program are

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. set seed 1
. bootstrap ratio=r(ratio), reps(1000) nowarn nodots: myratio price weight
Bootstrap results                                     Number of obs =   74
                                                    Replications = 1,000

Command: myratio price weight
ratio: r(ratio)
```

	Observed coefficient	Bootstrap std. err.	z	P> z	Normal-based [95% conf. interval]	
ratio	2.041841	.0953559	21.41	0.000	1.854947	2.228735

As mentioned previously, we should specify the `saving()` option if we wish to save the bootstrap dataset.

◀

Warning messages and `e(sample)`

`bootstrap` works well with weights specified in either `fweights()` or `iweights()`. When neither of these options is specified, `bootstrap` determines the presence of weights by parsing the prefixed command with standard syntax. However, commands like `stcox` and `streg` require that weights be specified in `stset`, and some user commands may allow weights to be specified by using an option instead of the standard syntax. Both cases pose a problem for `bootstrap` because it cannot determine the presence of weights under these circumstances. In these cases, we can only assume that you know what you are doing.

`bootstrap` does not know which variables of the dataset in memory matter to the calculation at hand. You can speed their execution by dropping unnecessary variables because, otherwise, they are included in each bootstrap sample.

You should thus drop observations with missing values. Leaving in missing values causes no problem in one sense because all Stata commands deal with missing values gracefully. It does, however, cause a statistical problem. Bootstrap sampling is defined as drawing, with replacement, samples of size N from a set of N observations. `bootstrap` determines N by counting the number of observations in memory, not counting the number of nonmissing values on the relevant variables. The result is that too many observations are resampled; the resulting bootstrap samples, because they are drawn from a population with missing values, are of unequal sizes.

If the number of missing values relative to the sample size is small, this will make little difference. If you have many missing values, however, you should first drop the observations that contain them.

▷ Example 5

To illustrate, we use the previous example but replace some of the values of price with missing values. The number of values of price used to compute the mean for each bootstrap is not constant. This is the purpose of the warning message.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. replace price = . if inlist(_n,1,3,5,7)
(4 real changes made, 4 to missing)
. set seed 1
. bootstrap ratio=r(ratio) np=r(n_price) nw=r(n_weight), reps(100) nodots:
> myratio price weight
warning: myratio does not set e(sample), so no observations will be excluded
from the resampling because of missing values or other reasons. To
exclude observations, press Break, save the data, drop any
observations that are to be excluded, and rerun bootstrap.
Bootstrap results                                Number of obs = 74
                                                Replications = 100
Command: myratio price weight
ratio: r(ratio)
      np: r(n_price)
      nw: r(n_weight)
```

	Observed coefficient	Bootstrap std. err.	z	P> z	Normal-based [95% conf. interval]	
ratio	2.063051	.0981706	21.01	0.000	1.870641	2.255462
np	70	2.071939	33.78	0.000	65.93908	74.06092
nw	74

◀

Bootstrapping statistics from data with a complex structure

Here we describe how to bootstrap statistics from data with a complex structure, for example, longitudinal or panel data, or matched data. `bootstrap`, however, is not designed to work with complex survey data. It is important to include all necessary information about the structure of the data in the bootstrap syntax to obtain correct bootstrap estimates for standard errors and confidence intervals.

`bootstrap` offers several options identifying the specifics of the data. These options are `strata()`, `cluster()`, `idcluster()`, and `group()`. The usage of `strata()` was described in [example 3](#) above. Below, we demonstrate several examples that require specifying the other three options.

▷ Example 6

Suppose that `auto.dta` in [example 1](#) above are clustered by `rep78`. We want to obtain bootstrap estimates for the standard errors of the difference between the coefficients on `weight` and `gear_ratio`, taking into account clustering.

We supply the `cluster(rep78)` option to `bootstrap` to request resampling from clusters rather than from observations in the dataset.

```

. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
. keep if rep78<.
(5 observations deleted)
. bootstrap diff=(_b[weight]-_b[gear]), rseed(1) cluster(rep78): regress mpg
> weight gear foreign
(running regress on estimation sample)
Bootstrap replications (50): .....10.....20.....30.....40.....
> ..50 done
Linear regression                                Number of obs = 69
                                                Replications = 50

Command: regress mpg weight gear foreign
diff: _b[weight]-_b[gear]

```

(Replications based on 5 clusters in rep78)

	Observed coefficient	Bootstrap std. err.	z	P> z	Normal-based [95% conf. interval]	
diff	-1.910396	2.538558	-0.75	0.452	-6.885879	3.065087

We drop missing values in `rep78` before issuing the command because `bootstrap` does not allow missing values in `cluster()`. See the section above about using `bootstrap` when variables contain missing values.

We can also obtain these same results by using the following syntax:

```

. bootstrap diff=(_b[weight]-_b[gear]), rseed(1): regress mpg weight gear foreign,
> vce(cluster rep78)

```

When only clustered information is provided to the command, `bootstrap` can pick up the `vce(cluster clustvar)` option from the main command and use it to resample from clusters.

◀

▷ Example 7

Suppose now that we have matched data and want to use `bootstrap` to obtain estimates of the standard errors of the exponentiated difference between two coefficients (or, equivalently, the ratio of two odds ratios) estimated by `clogit`. Consider the example of matched case–control data on birthweight of infants described in [example 2](#) of [R] `clogit`.

The infants are paired by being matched on mother’s age. All groups, defined by the `pairid` variable, have 1:1 matching. `clogit` requires that the matching information, `pairid`, be supplied to the `group()` (or, equivalently, `strata()`) option to be used in computing the parameter estimates. Because the data are matched, we need to resample from groups rather than from the whole dataset. However, simply supplying the grouping variable `pairid` in `cluster()` is not enough with `bootstrap`, as it is with clustered data.

```
. use https://www.stata-press.com/data/r19/lowbirth2, clear
(Applied Logistic Regression, Hosmer & Lemeshow)
. bootstrap ratio=exp(_b[smoke]-_b[ptd]), rseed(1) cluster(pairid):
> clogit low lwt smoke ptd ht ui i.race, group(pairid)
(running clogit on estimation sample)
Bootstrap replications (50): .....10.....20.....30.....40.....
> ..50 done

Bootstrap results                                     Number of obs = 112
                                                    Replications = 50

Command: clogit low lwt smoke ptd ht ui i.race, group(pairid)
ratio: exp(_b[smoke]-_b[ptd])

(Replications based on 56 clusters in pairid)
```

	Observed coefficient	Bootstrap std. err.	z	P> z	Normal-based [95% conf. interval]	
ratio	.6654095	2.043274	0.33	0.745	-3.339334	4.670153

For the syntax above, imagine that the first pair was sampled twice during a replication. Then, the bootstrap sample has four subjects with `pairid` equal to one, which clearly violates the original 1:1 matching design. As a result, the estimates of the coefficients obtained from this bootstrap sample will be incorrect.

Therefore, in addition to resampling from groups, we need to ensure that resampled groups are uniquely identified in each of the bootstrap samples. The `idcluster(newcluster)` option is designed for this. It requests that at each replication bootstrap create the new variable, `newcluster`, containing unique identifiers for all resampled groups. Thus, to make sure that the correct matching is preserved during each replication, we need to specify the grouping variable in `cluster()`, supply a variable name to `idcluster()`, and use this variable as the grouping variable with `clogit`, as we demonstrate below.

```
. bootstrap ratio=exp(_b[smoke]-_b[ptd]), rseed(1) cluster(pairid)
> idcluster(newpairid): clogit low lwt smoke ptd ht ui i.race, group(newpairid)
(running clogit on estimation sample)
Bootstrap replications (50): .....10.....20.....30.....40.....
> ..50 done

Bootstrap results                                     Number of obs = 112
                                                    Replications = 50

Command: clogit low lwt smoke ptd ht ui i.race, group(newpairid)
ratio: exp(_b[smoke]-_b[ptd])

(Replications based on 56 clusters in pairid)
```

	Observed coefficient	Bootstrap std. err.	z	P> z	Normal-based [95% conf. interval]	
ratio	.6654095	1.156848	0.58	0.565	-1.601972	2.932791

Note the difference between the estimates of the bootstrap standard error for the two specifications of the bootstrap syntax.

□ Technical note

Similarly, when you have panel (longitudinal) data, all resampled panels must be unique in each of the bootstrap samples to obtain correct bootstrap estimates of statistics. Therefore, both `cluster(panelvar)` and `idcluster(newpanelvar)` must be specified with `bootstrap`, and `i(newpanelvar)` must be used with the main command. Moreover, you must clear the current `xtset` settings by typing `xtset, clear` before calling `bootstrap`. □

▷ Example 8

Continuing with our birthweight data, suppose that we have more information about doctors supervising women's pregnancies. We believe that the data on the pairs of infants from the same doctor may be correlated and want to adjust standard errors for possible correlation among the pairs. `clogit` offers the `vce(cluster clustvar)` option to do this.

Let's add a cluster variable to our dataset. One thing to keep in mind is that to use `vce(cluster clustvar)`, groups in `group()` must be nested within clusters.

```
. use https://www.stata-press.com/data/r19/lowbirth2, clear
(Applied Logistic Regression, Hosmer & Lemeshow)
. set seed 12345
. by pairid, sort: egen byte doctor = total(int(2*runiform()+1)*(_n == 1))
. clogit low lwt smoke ptd ht ui i.race, group(pairid) vce(cluster doctor)
Iteration 0:  Log pseudolikelihood = -26.768693
Iteration 1:  Log pseudolikelihood = -25.810476
Iteration 2:  Log pseudolikelihood = -25.794296
Iteration 3:  Log pseudolikelihood = -25.794271
Iteration 4:  Log pseudolikelihood = -25.794271
Conditional (fixed-effects) logistic regression          Number of obs =   112
Wald chi2(1) = .
Prob > chi2 = .
Pseudo R2 = 0.3355
Log pseudolikelihood = -25.794271
(Std. err. adjusted for 2 clusters in doctor)
```

low	Robust		z	P> z	[95% conf. interval]	
	Coefficient	std. err.				
lwt	-.0183757	.0020314	-9.05	0.000	-.0223571	-.0143942
smoke	1.400656	.3067525	4.57	0.000	.7994322	2.00188
ptd	1.808009	.2092246	8.64	0.000	1.397936	2.218082
ht	2.361152	1.410341	1.67	0.094	-.4030665	5.12537
ui	1.401929	.9406248	1.49	0.136	-.4416617	3.24552
race						
Black	.5713643	.9992656	0.57	0.567	-1.38716	2.529889
Other	-.0253148	.8453206	-0.03	0.976	-1.682113	1.631483

To obtain correct bootstrap standard errors of the exponentiated difference between the two coefficients in this example, we need to make sure that both resampled clusters and groups within resampled clusters are unique in each of the bootstrap samples. To achieve this, `bootstrap` needs the information about clusters in `cluster()`, the variable name of the new identifier for clusters in `idcluster()`, and the information about groups in `group()`. We demonstrate the corresponding syntax of `bootstrap` below.

```

. bootstrap ratio=exp(_b[smoke]-_b[ptd]), rseed(1) cluster(doctor)
> idcluster(uidoctor) group(pairid): clogit low lwt smoke ptd ht ui i.race,
> group(pairid)
(running clogit on estimation sample)
Bootstrap replications (50): .....10.....20.....30.....40.....
> ..50 done

Bootstrap results                                     Number of obs = 112
                                                    Replications = 50

Command: clogit low lwt smoke ptd ht ui i.race, group(pairid)
ratio: exp(_b[smoke]-_b[ptd])

(Replications based on 2 clusters in doctor)

```

	Observed coefficient	Bootstrap std. err.	z	P> z	Normal-based [95% conf. interval]	
ratio	.6654095	.1459234	4.56	0.000	.3794048	.9514142

In the above syntax, although we specify `group(pairid)` with `clogit`, it is not the group identifiers of the original `pairid` variable that are used to compute parameter estimates from bootstrap samples. The way `bootstrap` works is that, at each replication, the clusters defined by `doctor` are resampled and the new variable, `uidoctor`, uniquely identifying resampled clusters is created. After that, another new variable uniquely identifying the (`uidoctor`, `group`) combination is created and renamed to have the same name as the grouping variable, `pairid`. This newly defined grouping variable is then used by `clogit` to obtain the parameter estimates from this bootstrap sample of clusters. After all replications are performed, the original values of the grouping variable are restored.

◀

□ Technical note

The same logic must be used when running `bootstrap` with commands designed for panel (longitudinal) data that allow specifying the `cluster(clustervar)` option. To ensure that the combination of (`clustervar`, `panelvar`) values are unique in each of the bootstrap samples, `cluster(clustervar)`, `idcluster(newclustervar)`, and `group(panelvar)` must be specified with `bootstrap`, and `i(panelvar)` must be used with the main command.

□

□ Technical note

When the `bootstrap` prefix is used with a user-defined program and when the expression list is `_b`, `bootstrap` calls

```
set coeftabresults off
```

before entering the replication loop to prevent Stata from performing unnecessary calculations. This means that, provided option `noisily` is not specified, estimation commands will not build or post the coefficient table matrix `r(table)`.

If your program calls an estimation command and needs `r(table)` to exist to perform properly, then your program will need to call

```
set coeftabresults on
```

before calling other estimation commands.

□

Bootstrapping statistics using frequency or importance weights

We can use frequency weights or importance weights when bootstrapping statistics by specifying the `fweights()` or `iweights()` option with the `bootstrap` prefix.

If your data have a complex structure, for example, if they are clustered or stratified, you can use the `cluster()`, `idcluster()`, and `strata()` options with `rwgen bsample` to generate replicate weights that reflect the complexities of your data. Then, when you specify these replicate weight variables in the `fweights()` option of the `bootstrap` prefix, `bootstrap` will use this information about the structure of your data to produce correct bootstrap estimates.

Clustered and matched case–control data are supported only when using the `fweights()` option of the `bootstrap` prefix; these data structures are not supported when using the `iweights()` option.

When you specify the `fweights()` option, the `bootstrap` prefix calculates the sum of the frequencies for each replication and compares it with the number of observations in the original data used to obtain point estimates. If the numbers do not match, a missing value of the estimate is posted for that replication. To include the results for those replications in the final bootstrap confidence interval calculation, you can specify the `force` option. However, you should use this option only if you understand the implications.

Additionally, suppose you have generated frequency weight variables with `rwgen bsample` by specifying the `cluster()`, `idcluster()`, or `strata()` option. When you specify these variables in the `fweights()` option of the `bootstrap` prefix, `bootstrap` will check whether the number of clusters is consistent with the original dataset. You can ignore such checks by specifying the `force` option.

If the `iweights()` option is specified, the `bootstrap` prefix verifies that the number of missing values is the same as in the original dataset used to obtain point estimates. You can ignore such checks by specifying the `force` option.

In this section, we demonstrate how to generate replicate weights with `rwgen bsample` and how to use them when bootstrapping statistics. For more examples, see [R] [rwgen](#). Also see [R] [bayesboot](#) for examples of Bayesian bootstrap.

▷ Example 9

Consider a variation of the `auto` dataset that contains frequency weights and importance weights. In `wauto.dta`, we have variables `f1–f100`, which are replicate weights generated by random sampling with replacement. In practice, you might obtain these replicate weights from the data source or generate them yourself with the `rwgen bsample` command.

We wish to compute bootstrap estimates for the standard errors of the coefficients from a linear regression using frequency weights. Therefore, we specify the frequency weight variables in the `fweights()` option:

```
. use https://www.stata-press.com/data/r19/wauto, clear
(1978 automobile data with replicate weights)
. bootstrap, fweights(f1-f100): regress mpg weight gear i.foreign
(running regress on estimation sample)
Bootstrap replications (100): .....10.....20.....30.....40.....
> ....50.....60.....70.....80.....90.....100 done
```

```
Linear regression                                Number of obs =    74
                                                Replications =   100
                                                Wald chi2(3) = 130.33
                                                Prob > chi2  = 0.0000
                                                R-squared   = 0.6670
                                                Adj R-squared = 0.6527
                                                Root MSE   = 3.4096
```

mpg	Observed coefficient	Bootstrap std. err.	z	P> z	Normal-based [95% conf. interval]	
weight	-.006139	.0006435	-9.54	0.000	-.0074002	-.0048778
gear_ratio	1.457113	1.495479	0.97	0.330	-1.473972	4.388199
foreign						
Foreign	-2.221682	1.323902	-1.68	0.093	-4.816482	.3731185
_cons	36.10135	5.560168	6.49	0.000	25.20362	46.99908

Because the frequency weights are predetermined, we do not need to specify the number of replications with the `reps()` option or the random-number seed. The command above will issue the `regress` command with `[fw=f1]` for the first replication, `[fw=f2]` for the second replication, and so on up through the 100th replication.

Similarly, if we wanted to apply importance weights when bootstrapping our statistics, we could specify the replicate weights `w1-w100` in the `iweights()` option:

```
. bootstrap, iweights(w1-w100): regress mpg weight gear i.foreign
(running regress on estimation sample)
Bootstrap replications (100): .....10.....20.....30.....40.....
> ....50.....60.....70.....80.....90.....100 done
```

```
Linear regression                                Number of obs =    74
                                                Replications =   100
                                                Wald chi2(3) = 191.58
                                                Prob > chi2  = 0.0000
                                                R-squared   = 0.6670
                                                Adj R-squared = 0.6527
                                                Root MSE   = 3.4096
```

mpg	Observed coefficient	Bootstrap std. err.	z	P> z	Normal-based [95% conf. interval]	
weight	-.006139	.0005861	-10.48	0.000	-.0072877	-.0049904
gear_ratio	1.457113	1.454702	1.00	0.317	-1.39405	4.308276
foreign						
Foreign	-2.221682	1.02959	-2.16	0.031	-4.23964	-.2037225
_cons	36.10135	5.48608	6.58	0.000	25.34883	46.85387

The command above will issue the `regress` command with `[iw=w1]` for the first replication, `[iw=w2]` for the second replication, and so on up through the 100th replication.

▶ Example 10

Below, we continue working with `wauto.dta` and generate a categorical variable based on the values of displacement. We wish to bootstrap the difference in coefficients estimated by `regress` using frequency weights. After generating the categorical variable `dlevel`, we specify this variable as the cluster identifier and `foreign` as the strata identifier when generating frequency weights with `rwgen bsample`. Therefore, for each replication, we get a bootstrap sample of clusters within each stratum of `foreign`. Our frequency weight variables will be named `u1`, `u2`, ..., `u100`.

```
. use https://www.stata-press.com/data/r19/wauto
(1978 automobile data with replicate weights)
. generate dlevel = floor(displacement / 50)
. rwgen bsample u, reps(100) rseed(19) cluster(dlevel) strata(foreign)
```

We can now bootstrap the difference in coefficients estimated by `regress`, using the frequency weights to ensure that we obtain correct bootstrap estimates that account for the complexity of our data.

```
. bootstrap diff=(_b[weight] - _b[gear]), fweights(u1-u100): ///
> regress mpg weight gear
(running regress on estimation sample)
Bootstrap replications (100): .....10.....20.....30.....40.....
> .....50.....60.....70.....80.....90.....100 done

Linear regression
Number of strata = 2                                Number of obs = 74
                                                    Replications = 100

Command: regress mpg weight gear
diff: _b[weight] - _b[gear]

(Replications based on 11 clusters in dlevel)
```

	Observed coefficient	Bootstrap std. err.	z	P> z	Normal-based [95% conf. interval]	
diff	-.1054525	1.38734	-0.08	0.939	-2.824589	2.613684

The estimation results are calculated based on replications that account for the clusters defined by `dlevel` and the strata variable `foreign`. These results are equivalent to the output we would obtain with the following command:

```
bootstrap diff=(_b[weight] - _b[gear]), reps(100) rseed(19) ///
cluster(dlevel) strata(foreign): regress mpg weight gear
```

◀

Bradley Efron (1938–) was born in 1938 in Minnesota and studied mathematics and statistics at Caltech and Stanford; he has lived in northern California since 1960. He has worked on empirical Bayes, survival analysis, exponential families, bootstrap and jackknife methods, and confidence intervals, in conjunction with applied work in biostatistics, astronomy, and physics.

Efron is a member of the US National Academy of Sciences and was awarded the US National Medal of Science in 2005. He is by any standards one of the world's leading statisticians: his work ranges from deep and elegant contributions in theoretical statistics to pathbreaking involvement in a variety of practical applications.

Stored results

`bootstrap` stores the following in `e()`:

Scalars

<code>e(N)</code>	sample size
<code>e(N_reps)</code>	number of complete replications
<code>e(N_misreps)</code>	number of incomplete replications
<code>e(N_strata)</code>	number of strata
<code>e(N_clust)</code>	number of clusters
<code>e(k_eq)</code>	number of equations in <code>e(b)</code>
<code>e(k_exp)</code>	number of standard expressions
<code>e(k_eexp)</code>	number of extended expressions (that is, <code>_b</code>)
<code>e(k_extra)</code>	number of extra equations beyond the original ones from <code>e(b)</code>
<code>e(level)</code>	confidence level for bootstrap CIs
<code>e(bs_version)</code>	version for bootstrap results
<code>e(rank)</code>	rank of <code>e(V)</code>

Macros

<code>e(cmdname)</code>	command name from <i>command</i>
<code>e(cmd)</code>	same as <code>e(cmdname)</code> or <code>bootstrap</code>
<code>e(command)</code>	<i>command</i>
<code>e(cmdline)</code>	command as typed
<code>e(prefix)</code>	<code>bootstrap</code>
<code>e(title)</code>	title in estimation output
<code>e(rwtype)</code>	replicate weight type
<code>e(rwvarlist)</code>	replicate weight variables
<code>e(strata)</code>	strata variables
<code>e(cluster)</code>	cluster variables
<code>e(rngstate)</code>	random-number state used
<code>e(size)</code>	from the <code>size(#)</code> option
<code>e(exp#)</code>	expression for the <code>#</code> th statistic
<code>e(ties)</code>	<code>ties</code> , if specified
<code>e(mse)</code>	<code>mse</code> , if specified
<code>e(vce)</code>	<code>bootstrap</code>
<code>e(vcetype)</code>	title used to label Std. err.
<code>e(properties)</code>	<code>b V</code>

Matrices

<code>e(b)</code>	observed statistics
<code>e(b_bs)</code>	bootstrap estimates
<code>e(reps)</code>	number of nonmissing results
<code>e(bias)</code>	estimated biases
<code>e(se)</code>	estimated standard errors
<code>e(z0)</code>	median biases
<code>e(accel)</code>	estimated accelerations
<code>e(ci_normal)</code>	normal-approximation CIs
<code>e(ci_percentile)</code>	percentile CIs
<code>e(ci_bc)</code>	bias-corrected CIs
<code>e(ci_bca)</code>	bias-corrected and accelerated CIs
<code>e(V)</code>	bootstrap variance–covariance matrix
<code>e(V_modelbased)</code>	model-based variance

Functions

<code>e(sample)</code>	marks estimation sample
------------------------	-------------------------

When `exp_list` is `_b`, `bootstrap` will also carry forward most of the results already in `e()` from *command*.

In addition to the above, the following is stored in `r()`:

Matrices	
<code>r(table)</code>	matrix containing the coefficients with their standard errors, test statistics, p -values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

Methods and formulas

Let $\hat{\theta}$ be the observed value of the statistic, that is, the value of the statistic calculated with the original dataset. Let $i = 1, 2, \dots, k$ denote the bootstrap samples, and let $\hat{\theta}_i$ be the value of the statistic from the i th bootstrap sample.

When the `mse` option is specified, the standard error is estimated as

$$\widehat{\text{se}}_{\text{MSE}} = \left\{ \frac{1}{k} \sum_{i=1}^k (\hat{\theta}_i - \hat{\theta})^2 \right\}^{1/2}$$

Otherwise, the standard error is estimated as

$$\widehat{\text{se}} = \left\{ \frac{1}{k-1} \sum_{i=1}^k (\hat{\theta}_i - \bar{\theta})^2 \right\}^{1/2}$$

where

$$\bar{\theta} = \frac{1}{k} \sum_{i=1}^k \hat{\theta}_i$$

The variance–covariance matrix is similarly computed. The bias is estimated as

$$\widehat{\text{bias}} = \bar{\theta} - \hat{\theta}$$

Confidence intervals with nominal coverage rates $1 - \alpha$ are calculated according to the following formulas. The normal-approximation method yields the confidence intervals

$$[\hat{\theta} - z_{1-\alpha/2} \widehat{\text{se}}, \hat{\theta} + z_{1-\alpha/2} \widehat{\text{se}}]$$

where $z_{1-\alpha/2}$ is the $(1-\alpha/2)$ th quantile of the standard normal distribution. If the `mse` option is specified, `bootstrap` will report the normal confidence interval using $\widehat{\text{se}}_{\text{MSE}}$ instead of $\widehat{\text{se}}$. `estat bootstrap` only uses $\widehat{\text{se}}$ in the normal confidence interval.

The percentile method yields the confidence intervals

$$[\theta_{\alpha/2}^*, \theta_{1-\alpha/2}^*]$$

where θ_p^* is the p th quantile (the 100 p th percentile) of the bootstrap distribution $(\hat{\theta}_1, \dots, \hat{\theta}_k)$.

Let

$$z_0 = \Phi^{-1}\{\#\hat{\theta}_i \leq \hat{\theta}\}/k\}$$

where $\#\hat{\theta}_i \leq \hat{\theta}$ is the number of elements of the bootstrap distribution that are less than or equal to the observed statistic and Φ is the standard cumulative normal. z_0 is known as the median bias of $\hat{\theta}$. When the `ties` option is specified, z_0 is estimated as $\#\hat{\theta}_i < \hat{\theta} + \#\hat{\theta}_i = \hat{\theta}/2$, which is the number of elements of the bootstrap distribution that are less than the observed statistic plus half the number of elements that are equal to the observed statistic.

Let

$$a = \frac{\sum_{i=1}^n (\bar{\theta}_{(\cdot)} - \hat{\theta}_{(i)})^3}{6\{\sum_{i=1}^n (\bar{\theta}_{(\cdot)} - \hat{\theta}_{(i)})^2\}^{3/2}}$$

where $\hat{\theta}_{(i)}$ are the leave-one-out (jackknife) estimates of $\hat{\theta}$ and $\bar{\theta}_{(\cdot)}$ is their mean. This expression is known as the jackknife estimate of acceleration for $\hat{\theta}$. Let

$$p_1 = \Phi \left\{ z_0 + \frac{z_0 - z_{1-\alpha/2}}{1 - a(z_0 - z_{1-\alpha/2})} \right\}$$

$$p_2 = \Phi \left\{ z_0 + \frac{z_0 + z_{1-\alpha/2}}{1 - a(z_0 + z_{1-\alpha/2})} \right\}$$

where $z_{1-\alpha/2}$ is the $(1 - \alpha/2)$ th quantile of the normal distribution. The bias-corrected and accelerated (BC_a) method yields confidence intervals

$$[\theta_{p_1}^*, \theta_{p_2}^*]$$

where θ_p^* is the p th quantile of the bootstrap distribution as defined previously. The bias-corrected (but not accelerated) method is a special case of BC_a with $a = 0$.

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Also see

- [R] **bootstrap postestimation** — Postestimation tools for bootstrap
- [R] **bayesboot** — Bayesian bootstrap estimation
- [R] **jackknife** — Jackknife estimation
- [R] **permute** — Permutation tests
- [R] **rwgen** — Generate replicate weights for bootstrap estimation
- [R] **simulate** — Monte Carlo simulations
- [R] **set rngstream** — Specify the stream for the stream random-number generator
- [SVY] **svy bootstrap** — Bootstrap for survey data
- [U] **13.5 Accessing coefficients and standard errors**
- [U] **13.6 Accessing results from Stata commands**
- [U] **20 Estimation and postestimation commands**

Postestimation commands
estat

predict
Remarks and examples

margins
Also see

Postestimation commands

The following postestimation command is of special interest after `bootstrap`:

Command	Description
<code>estat bootstrap</code>	percentile-based and bias-corrected CI tables

The following standard postestimation commands are also available:

Command	Description
<code>contrast</code>	contrasts and ANOVA-style joint tests of parameters
<code>estat ic</code>	Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively)
<code>estat summarize</code>	summary statistics for the estimation sample
<code>estat vce</code>	variance-covariance matrix of the estimators (VCE)
<code>estimates</code>	cataloging estimation results
<code>etable</code>	table of estimation results
<code>hausman</code>	Hausman's specification test
<code>lincom</code>	point estimates, standard errors, testing, and inference for linear combinations of parameters
<code>margins</code>	marginal means, predictive margins, marginal effects, and average marginal effects
<code>marginsplot</code>	graph the results from margins (profile plots, interaction plots, etc.)
<code>nlcom</code>	point estimates, standard errors, testing, and inference for nonlinear combinations of parameters
<code>predict</code>	predictions, residuals, influence statistics, and other diagnostic measures
<code>predictnl</code>	point estimates, standard errors, testing, and inference for generalized predictions
<code>pwcompare</code>	pairwise comparisons of parameters
<code>test</code>	Wald tests of simple and composite linear hypotheses
<code>testnl</code>	Wald tests of nonlinear hypotheses

The postestimation command is allowed if it may be used after *command*.

predict

The syntax of `predict` (and even if `predict` is allowed) following `bootstrap` depends upon the *command* used with `bootstrap`. If `predict` is not allowed, neither is `predictnl`.

margins

The syntax of `margins` (and even if `margins` is allowed) following `bootstrap` depends upon the *command* used with `bootstrap`.

estat

Description for estat

`estat bootstrap` displays a table of confidence intervals for each statistic from a bootstrap analysis.

Menu for estat

Statistics > Postestimation

Syntax for estat

```
estat bootstrap [ , options ]
```

<i>options</i>	Description
<code>bc</code>	bias-corrected CIs; the default
<code>bca</code>	bias-corrected and accelerated (BC_a) CIs
<code>normal</code>	normal-based CIs
<code>percentile</code>	percentile CIs
<code>all</code>	all available CIs
<code>noheader</code>	suppress table header
<code>nolegend</code>	suppress table legend
<code>verbose</code>	display the full table legend

`bc`, `bca`, `normal`, and `percentile` may be used together.

Options for estat

`bc` is the default and displays bias-corrected confidence intervals.

`bca` displays bias-corrected and accelerated confidence intervals. This option assumes that you also specified the `bca` option on the `bootstrap` prefix command.

`normal` displays normal approximation confidence intervals.

`percentile` displays percentile confidence intervals.

`all` displays all available confidence intervals.

`noheader` suppresses display of the table header. This option implies `nolegend`.

`nolegend` suppresses display of the table legend, which identifies the rows of the table with the expressions they represent.

`verbose` requests that the full table legend be displayed.

Remarks and examples

▷ Example 1

The `estat bootstrap` postestimation command produces a table containing the observed value of the statistic, an estimate of its bias, the bootstrap standard error, and up to four different confidence intervals.

If we were interested merely in getting bootstrap standard errors for the model coefficients, we could use the `bootstrap` prefix with our estimation command. If we were interested in performing a thorough bootstrap analysis of the model coefficients, we could use the `estat bootstrap` postestimation command after fitting the model with the `bootstrap` prefix.

Using [example 1](#) from [\[R\] bootstrap](#), we need many more replications for the confidence interval types other than the normal based, so let's rerun the estimation command. We will reset the random-number seed—in case we wish to reproduce the results—increase the number of replications, and save the bootstrap distribution as a dataset called `bsauto.dta`.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. set seed 1
. bootstrap _b, reps(1000) saving(bsauto) bca: regress mpg weight gear foreign
(output omitted)
. estat bootstrap, all
Linear regression                Number of obs   =          74
                                Replications    =         1000
```

mpg	Observed coefficient	Bias	Bootstrap std. err.	[95% conf. interval]		
weight	-.00613903	.0000686	.00065005	- .0074131	- .004865	(N)
				- .0073115	- .0048083	(P)
				- .0073757	- .0048444	(BC)
				- .0075498	- .0049202	(BCa)
gear_ratio	1.4571134	.0297538	1.4471522	-1.379253	4.29348	(N)
				-1.18779	4.540121	(P)
				-1.185389	4.540121	(BC)
				-1.131393	4.58386	(BCa)
foreign	-2.2216815	.1029615	1.2606565	-4.692523	.2491598	(N)
				-4.513954	.5011647	(P)
				-4.608057	.4208305	(BC)
				-4.614719	.3925043	(BCa)
_cons	36.101353	-.3122698	5.4303717	25.45802	46.74469	(N)
				24.55211	46.0322	(P)
				24.90078	46.05819	(BC)
				24.99072	46.40419	(BCa)

```
Key:  N: Normal
      P: Percentile
      BC: Bias-corrected
      BCa: Bias-corrected and accelerated
```

The estimated standard errors here differ from our previous estimates using only 100 replications by, respectively, 7%, 6%, 8%, and 4%; see [example 1](#) of [\[R\] bootstrap](#). So much for our advice that 50–200 replications are good enough to estimate standard errors. Well, the more replications the better—that advice you should believe.

Which of the methods to compute confidence intervals should we use? If the statistic is unbiased, the percentile (P) and bias-corrected (BC) methods should give similar results. The bias-corrected confidence interval will be the same as the percentile confidence interval when the observed value of the statistic is equal to the median of the bootstrap distribution. Thus, for unbiased statistics, the two methods should give similar results as the number of replications becomes large. For biased statistics, the bias-corrected method should yield confidence intervals with better coverage probability (closer to the nominal value of 95% or whatever was specified) than the percentile method. For statistics with variances that vary as a function of the parameter of interest, the bias-corrected and accelerated method (BC_a) will typically have better coverage probability than the others.

When the bootstrap distribution is approximately normal, all of these methods should give similar confidence intervals as the number of replications becomes large. If we examine the normality of these bootstrap distributions using, say, the `pnorm` command (see [R] [Diagnostic plots](#)), we see that they closely follow a normal distribution. In this case, the normal approximation would also be a valid choice. The chief advantage of the normal-approximation method is that it (supposedly) requires fewer replications than the other methods. Of course, it should be used only when the bootstrap distribution exhibits normality.

We can load `bsauto.dta` containing the bootstrap distributions for these coefficients:

```
. use bsauto
(bootstrap: regress)
. describe *
```

Variable name	Storage type	Display format	Value label	Variable label
<code>_b_weight</code>	float	%9.0g		<code>_b[weight]</code>
<code>_b_gear_ratio</code>	float	%9.0g		<code>_b[gear_ratio]</code>
<code>_b_foreign</code>	float	%9.0g		<code>_b[foreign]</code>
<code>_b_cons</code>	float	%9.0g		<code>_b[_cons]</code>

We can now run other commands, such as `pnorm`, on the bootstrap distributions. As with all standard estimation commands, we can use the `bootstrap` command to replay its output table. The default variable names assigned to the statistics in `exp_list` are `_bs_1`, `_bs_2`, ..., and each variable is labeled with the associated expression. The naming convention for the extended expressions `_b` and `_se` is to prepend `_b_` and `_se_`, respectively, onto the name of each element of the coefficient vector. Here the first coefficient is `_b[weight]`, so `bootstrap` named it `_b_weight`.

◀

Also see

[R] [bootstrap](#) — Bootstrap sampling and estimation

[U] [20 Estimation and postestimation commands](#)

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Description

`boxcox` finds the maximum likelihood estimates of the parameters of the Box–Cox transform, the coefficients on the independent variables, and the standard deviation of the normally distributed errors. Any *depvar* or *indepvars* to be transformed must be strictly positive. Options can be used to control which variables remain untransformed.

Quick start

Box–Cox transform of y in a model of y as a function of $x1$

```
boxcox y x1
```

Same as above

```
boxcox y x1, model(lhsonly)
```

Likelihood-ratio test for each scale-variant parameter

```
boxcox y x1, lrtest
```

Different transform for each side and adding covariates $x2$ and $x3$

```
boxcox y x1 x2 x3, model(theta)
```

Same transform for both sides, and include $x3$ as an untransformed variable transformation

```
boxcox y x1 x2, model(lambda) notrans(x3)
```

Menu

Statistics > Linear models and related > Box–Cox regression

Syntax

```
boxcox depvar [indepvars] [if] [in] [weight] [, options]
```

<i>options</i>	Description
Model	
<code>noconstant</code>	suppress constant term
<code>model(lhsonly)</code>	left-hand-side Box–Cox model; the default
<code>model(rhsonly)</code>	right-hand-side Box–Cox model
<code>model(lambda)</code>	both sides Box–Cox model with same parameter
<code>model(theta)</code>	both sides Box–Cox model with different parameters
<code>notrans(<i>varlist</i>)</code>	do not transform specified independent variables
Reporting	
<code>level(#)</code>	set confidence level; default is <code>level(95)</code>
<code>lrtest</code>	perform likelihood-ratio test
Maximization	
<code>[no]log</code>	suppress all iteration logs
<code>nologlr</code>	suppress restricted-model <code>lrtest</code> iteration log
<code>maximize_options</code>	control the maximization process; seldom used

depvar and *indepvars* may contain time-series operators; see [U] 11.4.4 Time-series varlists.

`bayesboot`, `bootstrap`, `by`, `collect`, `jackknife`, `rolling`, `statsby`, and `xi` are allowed; see [U] 11.1.10 Prefix commands.

Weights are not allowed with the `bootstrap` prefix; see [R] `bootstrap`.

`fweights` and `iwweights` are allowed; see [U] 11.1.6 `weight`.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

Options

Model

`noconstant`; see [R] Estimation options.

`model(lhsonly | rhsonly | lambda | theta)` specifies which of the four models to fit.

`model(lhsonly)` applies the Box–Cox transform to *depvar* only. `model(lhsonly)` is the default.

`model(rhsonly)` applies the transform to the *indepvars* only.

`model(lambda)` applies the transform to both *depvar* and *indepvars*, and they are transformed by the same parameter.

`model(theta)` applies the transform to both *depvar* and *indepvars*, but this time, each side is transformed by a separate parameter.

`notrans(varlist)` specifies that the variables in *varlist* not be transformed when included in the model.

You can specify `notrans(varlist)` with `model(lhsonly)`, but the results will be the same as specifying the variables in *varlist* in *indepvars*.

Reporting

`level(#)`; see [R] Estimation options.

`lrtest` specifies that a likelihood-ratio test of significance be performed and reported for each independent variable.

Maximization

`log` and `nolog` specify whether to display the iteration log. The iteration log is displayed by default unless you used `set iterlog off` to suppress it; see `set iterlog` in [R] *set iter*. These options control the iteration log produced by the full model and, if option `lrtest` is specified, by the fitted restricted models.

`nologlr` suppresses the iteration log when fitting the restricted models required by the `lrtest` option.

maximize_options: `iterate(#)` and `from(init_specs)`; see [R] *Maximize*.

Model	Initial value specification
<code>lhsonly</code>	<code>from(θ_0, copy)</code>
<code>rhsonly</code>	<code>from(λ_0, copy)</code>
<code>lambda</code>	<code>from(λ_0, copy)</code>
<code>theta</code>	<code>from(λ_0 θ_0, copy)</code>

Remarks and examples

Remarks are presented under the following headings:

[Introduction](#)
[Theta model](#)
[Lambda model](#)
[Left-hand-side-only model](#)
[Right-hand-side-only model](#)

Introduction

The Box–Cox transform

$$y^{(\lambda)} = \frac{y^\lambda - 1}{\lambda}$$

has been widely used in applied data analysis. [Box and Cox \(1964\)](#) developed the transformation and argued that the transformation could make the residuals more closely normal and less heteroskedastic. [Cook and Weisberg \(1982\)](#) discuss the transform in this light. Because the transform embeds several popular functional forms, it has received some attention as a method for testing functional forms, in particular,

$$y^{(\lambda)} = \begin{cases} y - 1 & \text{if } \lambda = 1 \\ \ln(y) & \text{if } \lambda = 0 \\ 1 - 1/y & \text{if } \lambda = -1 \end{cases}$$

[Davidson and MacKinnon \(1993\)](#) discuss this use of the transform. [Atkinson \(1985\)](#) also gives a good general treatment.

Theta model

boxcox obtains the maximum likelihood estimates of the parameters for four different models. The most general of the models, the theta model, is

$$y_j^{(\theta)} = \beta_0 + \beta_1 x_{1j}^{(\lambda)} + \beta_2 x_{2j}^{(\lambda)} + \cdots + \beta_k x_{kj}^{(\lambda)} + \gamma_1 z_{1j} + \gamma_2 z_{2j} + \cdots + \gamma_l z_{lj} + \epsilon_j$$

where $\epsilon \sim N(0, \sigma^2)$. Here the dependent variable, y , is subject to a Box–Cox transform with parameter θ . Each of the *indepvars*, x_1, x_2, \dots, x_k , is transformed by a Box–Cox transform with parameter λ . The z_1, z_2, \dots, z_l specified in the `notrans()` option are independent variables that are not transformed.

Box and Cox (1964) argued that this transformation would leave behind residuals that more closely follow a normal distribution than those produced by a simple linear regression model. Bear in mind that the normality of ϵ is assumed and that boxcox obtains maximum likelihood estimates of the $k + l + 4$ parameters under this assumption. boxcox does not choose λ and θ so that the residuals are approximately normally distributed. If you are interested in this type of transformation to normality, see the official Stata commands `lnskew0` and `bcskew0` in [R] [lnskew0](#). However, those commands work on a more restrictive model in which none of the independent variables is transformed.

▷ Example 1

Below, we fit a theta model to a nonrepresentative extract of the Second National Health and Nutrition Examination Survey (NHANES II) dataset discussed in McDowell et al. (1981).

We model individual-level diastolic blood pressure (`bpdiast`) as a function of the transformed variables body mass index (`bmi`) and cholesterol level (`tcresult`) and of the untransformed variables age (`age`) and sex (`sex`).

```
. use https://www.stata-press.com/data/r19/nhanes2
. boxcox bpdiast bmi tcresult, notrans(age sex) model(theta) lrtest
Fitting comparison model
Iteration 0:  Log likelihood = -41178.61
Iteration 1:  Log likelihood = -41032.51
Iteration 2:  Log likelihood = -41032.488
Iteration 3:  Log likelihood = -41032.488
Fitting full model
Iteration 0:  Log likelihood = -39928.606
Iteration 1:  Log likelihood = -39775.026
Iteration 2:  Log likelihood = -39774.987
Iteration 3:  Log likelihood = -39774.987
Fitting comparison models for LR tests
Iteration 0:  Log likelihood = -39947.144
Iteration 1:  Log likelihood = -39934.55
Iteration 2:  Log likelihood = -39934.516
Iteration 3:  Log likelihood = -39934.516
Iteration 0:  Log likelihood = -39906.96
Iteration 1:  Log likelihood = -39896.63
Iteration 2:  Log likelihood = -39896.629
Iteration 0:  Log likelihood = -40464.599
Iteration 1:  Log likelihood = -40459.765 (not concave)
Iteration 2:  Log likelihood = -40459.747
Iteration 3:  Log likelihood = -40459.604
Iteration 4:  Log likelihood = -40459.604
```

Iteration 0: Log likelihood = -39829.859
 Iteration 1: Log likelihood = -39815.576
 Iteration 2: Log likelihood = -39815.575

Log likelihood = -39774.987

Number of obs	=	10,351
LR chi2(5)	=	2515.00
Prob > chi2	=	0.000

bpdiastr	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
/lambda	.6383286	.1577601	4.05	0.000	.3291245	.9475327
/theta	.1988197	.0454088	4.38	0.000	.1098201	.2878193

Estimates of scale-variant parameters

	Coefficient	chi2(df)	P>chi2(df)	df of chi2
Notrans				
age	.003811	319.060	0.000	1
sex	-.1054887	243.284	0.000	1
_cons	5.835555			
Trans				
bmi	.0872041	1369.235	0.000	1
tcresult	.004734	81.177	0.000	1
/sigma	.3348267			

Test H0:	Restricted log likelihood	chi2	Prob > chi2
theta=lambda = -1	-40162.898	775.82	0.000
theta=lambda = 0	-39790.945	31.92	0.000
theta=lambda = 1	-39928.606	307.24	0.000

The output is composed of the iteration logs and three distinct tables. The first table contains a standard header for a maximum likelihood estimator and a standard output table for the Box–Cox transform parameters. The second table contains the estimates of the scale-variant parameters. The third table contains the output from likelihood-ratio tests on three standard functional form specifications.

The right-hand-side and the left-hand-side transformations each add to the regression fit at the 1% significance level and are both positive but less than 1. All the variables have significant impacts on diastolic blood pressure, *bpdiastr*. As expected, the transformed variables—the body mass index, *bmi*, and cholesterol level, *tcresult*—contribute to higher blood pressure. The last output table shows that the linear, multiplicative inverse, and log specifications are strongly rejected.



□ Technical note

Spitzer (1984) showed that the Wald tests of the joint significance of the coefficients of the right-hand-side variables, either transformed or untransformed, are not invariant to changes in the scale of the transformed dependent variable. Davidson and MacKinnon (1993) also discuss this point. This problem demonstrates that Wald statistics can be manipulated in nonlinear models. Lafontaine and White

(1986) analyze this problem numerically, and Phillips and Park (1988) analyze it by using Edgeworth expansions. See Drukker (2000) for a more detailed discussion of this issue. Because the parameter estimates and their Wald tests are not scale invariant, no Wald tests or confidence intervals are reported for these parameters. However, when the `lrtest` option is specified, likelihood-ratio tests are performed and reported. Schlesselman (1971) showed that, if a constant is included in the model, the parameter estimates of the Box–Cox transforms are scale invariant. For this reason, we strongly recommend that you not use the `noconstant` option.

The `lrtest` option does not perform a likelihood-ratio test on the constant, so no value for this statistic is reported. Unless the data are properly scaled, the restricted model does not often converge. For this reason, no likelihood-ratio test on the constant is performed by the `lrtest` option. However, if you have a special interest in performing this test, you can do so by fitting the constrained model separately. If problems with convergence are encountered, rescaling the data by their means may help. □

Lambda model

A less general model than the one above is called the `lambda` model. It specifies that the same parameter be used in both the left-hand-side and right-hand-side transformations. Specifically,

$$y_j^{(\lambda)} = \beta_0 + \beta_1 x_{1j}^{(\lambda)} + \beta_2 x_{2j}^{(\lambda)} + \cdots + \beta_k x_{kj}^{(\lambda)} + \gamma_1 z_{1j} + \gamma_2 z_{2j} + \cdots + \gamma_l z_{lj} + \epsilon_j$$

where $\epsilon \sim N(0, \sigma^2)$. Here the *depvar* variable, y , and each of the *indepvars*, x_1, x_2, \dots, x_k , is transformed by a Box–Cox transform with the common parameter λ . Again, the z_1, z_2, \dots, z_l are independent variables that are not transformed.

Left-hand-side-only model

Even more restrictive than a common transformation parameter is transforming the dependent variable only. Because the dependent variable is on the left-hand side of the equation, this model is known as the `lhsonly` model. Here you are estimating the parameters of the model

$$y_j^{(\theta)} = \beta_0 + \beta_1 x_{1j} + \beta_2 x_{2j} + \cdots + \beta_k x_{kj} + \epsilon_j$$

where $\epsilon \sim N(0, \sigma^2)$. Here only the *depvar*, y , is transformed by a Box–Cox transform with the parameter θ .

▷ Example 2

In this example, we model the transform of diastolic blood pressure as a linear combination of the untransformed body mass index, cholesterol level, age, and sex.

```
. boxcox bpdiastr bmi tcresult age sex, model(lhsonly) lrtest nolog nologlr
Fitting comparison model
Fitting full model
Fitting comparison models for LR tests
Log likelihood = -39777.709
Number of obs = 10,351
LR chi2(4) = 2509.56
Prob > chi2 = 0.000
```

bpdiastr	Coefficient	Std. err.	z	P> z	[95% conf. interval]
/theta	.2073268	.0452895	4.58	0.000	.1185611 .2960926

Estimates of scale-variant parameters

	Coefficient	chi2(df)	P>chi2(df)	df of chi2
Notrans				
bmi	.0272628	1375.841	0.000	1
tcresult	.0006929	82.380	0.000	1
age	.0040141	334.117	0.000	1
sex	-.1122274	263.219	0.000	1
_cons	6.302855			
/sigma	.3476615			

Test H0:	Restricted log likelihood	LR statistic chi2	Prob > chi2
theta = -1	-40146.678	737.94	0.000
theta = 0	-39788.241	21.06	0.000
theta = 1	-39928.606	301.79	0.000

The maximum likelihood estimate of the transformation parameter for this model is positive and significant. Once again, all the scale-variant parameters are significant, and we find a positive impact of body mass index (bmi) and cholesterol levels (tcresult) on the transformed diastolic blood pressure (bpdiastr). This model rejects the linear, multiplicative inverse, and log specifications.



Right-hand-side-only model

The fourth model leaves the *depvar* alone and transforms a subset of the *indepvars* using the parameter λ . This is the *rhsonly* model. In this model, the *depvar*, y , is given by

$$y_j = \beta_0 + \beta_1 x_{1j}^{(\lambda)} + \beta_2 x_{2j}^{(\lambda)} + \dots + \beta_k x_{kj}^{(\lambda)} + \gamma_1 z_{1j} + \gamma_2 z_{2j} + \dots + \gamma_l z_{lj} + \epsilon_j$$

where $\epsilon \sim N(0, \sigma^2)$. Here each of the *indepvars*, x_1, x_2, \dots, x_k , is transformed by a Box–Cox transform with the parameter λ . Again, the z_1, z_2, \dots, z_l are independent variables that are not transformed.

▷ Example 3

Now, we consider a `rhsonly` model in which the regressors `sex` and `age` are not transformed.

```
. boxcox bpdiastr bmi tcresult, notrans(sex age) model(rhsonly) lrtest nolog
> nologlr
Fitting full model
Fitting comparison models for LR tests
Log likelihood = -39928.212
Number of obs = 10,351
LR chi2(5) = 2500.79
Prob > chi2 = 0.000
```

bpdiastr	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
/lambda	.8658841	.1522387	5.69	0.000	.5675018	1.164266

Estimates of scale-variant parameters

	Coefficient	chi2(df)	P>chi2(df)	df of chi2
Notrans				
sex	-3.544042	235.020	0.000	1
age	.128809	311.754	0.000	1
_cons	50.01498			
Trans				
bmi	1.418215	1396.709	0.000	1
tcresult	.0462964	78.500	0.000	1
/sigma	11.4557			

Test H0:	Restricted log likelihood	LR statistic chi2	Prob > chi2
lambda = -1	-39989.331	122.24	0.000
lambda = 0	-39942.945	29.47	0.000
lambda = 1	-39928.606	0.79	0.375

The maximum likelihood estimate of the transformation parameter in this model is positive and significant at the 1% level. The transformed `bmi` coefficient behaves as expected, and the remaining scale-variant parameters are significant at the 1% level. This model rejects the multiplicative inverse and log specifications strongly. However, we cannot reject the hypothesis that the model is linear.



Stored results

boxcox stores the following in `e()`:

Scalars

<code>e(N)</code>	number of observations
<code>e(l1)</code>	log likelihood
<code>e(chi2)</code>	LR statistic of full vs. comparison
<code>e(df_m)</code>	full model degrees of freedom
<code>e(l10)</code>	log likelihood of the restricted model
<code>e(df_r)</code>	restricted model degrees of freedom
<code>e(l1_t1)</code>	log likelihood of model $\lambda = \theta = 1$
<code>e(chi2_t1)</code>	LR of $\lambda = \theta = 1$ vs. full model
<code>e(p_t1)</code>	p -value of $\lambda = \theta = 1$ vs. full model
<code>e(l1_tm1)</code>	log likelihood of model $\lambda = \theta = -1$
<code>e(chi2_tm1)</code>	LR of $\lambda = \theta = -1$ vs. full model
<code>e(p_tm1)</code>	p -value of $\lambda = \theta = -1$ vs. full model
<code>e(l1_t0)</code>	log likelihood of model $\lambda = \theta = 0$
<code>e(chi2_t0)</code>	LR of $\lambda = \theta = 0$ vs. full model
<code>e(p_t0)</code>	p -value of $\lambda = \theta = 0$ vs. full model
<code>e(rank)</code>	rank of <code>e(V)</code>
<code>e(ic)</code>	number of iterations
<code>e(rc)</code>	return code

Macros

<code>e(cmd)</code>	boxcox
<code>e(cmdline)</code>	command as typed
<code>e(depvar)</code>	name of dependent variable
<code>e(model)</code>	lhsonly, rhsonly, lambda, or theta
<code>e(wtype)</code>	weight type
<code>e(wexp)</code>	weight expression
<code>e(ntrans)</code>	yes if untransformed <i>indepvars</i>
<code>e(chi2type)</code>	LR; type of model χ^2 test
<code>e(lrtest)</code>	lrtest, if requested
<code>e(properties)</code>	b V
<code>e(predict)</code>	program used to implement predict
<code>e(marginsnotok)</code>	predictions disallowed by margins

Matrices

<code>e(b)</code>	coefficient vector
<code>e(V)</code>	variance–covariance matrix of the estimators (see note below)
<code>e(pm)</code>	p -values for LR tests on <i>indepvars</i>
<code>e(df)</code>	degrees of freedom of LR tests on <i>indepvars</i>
<code>e(chi2m)</code>	LR statistics for tests on <i>indepvars</i>

Functions

<code>e(sample)</code>	marks estimation sample
------------------------	-------------------------

`e(V)` contains all zeros, except for the elements that correspond to the parameters of the Box–Cox transform.

Methods and formulas

In the internal computations,

$$y^{(\lambda)} = \begin{cases} \frac{y^\lambda - 1}{\lambda} & \text{if } |\lambda| > 10^{-10} \\ \ln(y) & \text{otherwise} \end{cases}$$

The unconcentrated log likelihood for the theta model is

$$\ln L = \left(\frac{-N}{2} \right) \{ \ln(2\pi) + \ln(\sigma^2) \} + (\theta - 1) \sum_{i=1}^N \ln(y_i) - \left(\frac{1}{2\sigma^2} \right) \text{SSR}$$

where

$$\text{SSR} = \sum_{i=1}^N (y_i^{(\theta)} - \beta_0 + \beta_1 x_{i1}^{(\lambda)} + \beta_2 x_{i2}^{(\lambda)} + \cdots + \beta_k x_{ik}^{(\lambda)} + \gamma_1 z_{i1} + \gamma_2 z_{i2} + \cdots + \gamma_l z_{il})^2$$

Writing the SSR in matrix form,

$$\text{SSR} = (\mathbf{y}^{(\theta)} - \mathbf{X}^{(\lambda)} \mathbf{b}' - \mathbf{Z} \mathbf{g}')' (\mathbf{y}^{(\theta)} - \mathbf{X}^{(\lambda)} \mathbf{b}' - \mathbf{Z} \mathbf{g}')$$

where $\mathbf{y}^{(\theta)}$ is an $N \times 1$ vector of elementwise transformed data, $\mathbf{X}^{(\lambda)}$ is an $N \times k$ matrix of elementwise transformed data, \mathbf{Z} is an $N \times l$ matrix of untransformed data, \mathbf{b} is a $1 \times k$ vector of coefficients, and \mathbf{g} is a $1 \times l$ vector of coefficients. Letting

$$\mathbf{W}_\lambda = \left(\mathbf{X}^{(\lambda)} \mathbf{Z} \right)$$

be the horizontal concatenation of $\mathbf{X}^{(\lambda)}$ and \mathbf{Z} and

$$\mathbf{d}' = \begin{pmatrix} \mathbf{b}' \\ \mathbf{g}' \end{pmatrix}$$

be the vertical concatenation of the coefficients yields

$$\text{SSR} = (\mathbf{y}^{(\theta)} - \mathbf{W}_\lambda \mathbf{d}')' (\mathbf{y}^{(\theta)} - \mathbf{W}_\lambda \mathbf{d}')$$

For given values of λ and θ , the solutions for \mathbf{d}' and σ^2 are

$$\hat{\mathbf{d}}' = (W_\lambda' W_\lambda)^{-1} W_\lambda' y^{(\theta)}$$

and

$$\hat{\sigma}^2 = \frac{1}{N} (\mathbf{y}^{(\theta)} - W_\lambda \hat{\mathbf{d}}')' (\mathbf{y}^{(\theta)} - W_\lambda \hat{\mathbf{d}}')$$

Substituting these solutions into the log-likelihood function yields the concentrated log-likelihood function

$$\ln L_c = \left(-\frac{N}{2} \right) \{ \ln(2\pi) + 1 + \ln(\hat{\sigma}^2) \} + (\theta - 1) \sum_{i=1}^N \ln(y_i)$$

Similar calculations yield the concentrated log-likelihood function for the lambda model,

$$\ln L_c = \left(-\frac{N}{2}\right) \{ \ln(2\pi) + 1 + \ln(\hat{\sigma}^2) \} + (\lambda - 1) \sum_{i=1}^N \ln(y_i)$$

the lhsonly model,

$$\ln L_c = \left(-\frac{N}{2}\right) \{ \ln(2\pi) + 1 + \ln(\hat{\sigma}^2) \} + (\theta - 1) \sum_{i=1}^N \ln(y_i)$$

and the rhsonly model,

$$\ln L_c = \left(-\frac{N}{2}\right) \{ \ln(2\pi) + 1 + \ln(\hat{\sigma}^2) \}$$

where $\hat{\sigma}^2$ is specific to each model and is defined analogously to that in the theta model.

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Also see

- [R] **boxcox postestimation** — Postestimation tools for boxcox
- [R] **inskew0** — Find zero-skewness log or Box–Cox transform
- [R] **regress** — Linear regression
- [U] **20 Estimation and postestimation commands**

Postestimation commands
References

[predict](#)
[Also see](#)

[Remarks and examples](#)

[Methods and formulas](#)

Postestimation commands

The following postestimation commands are available after `boxcox`:

Command	Description
<code>estat ic</code>	Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively)
<code>estat summarize</code>	summary statistics for the estimation sample
<code>estat vce</code>	variance–covariance matrix of the estimators (VCE)
<code>estimates</code>	cataloging estimation results
<code>etable</code>	table of estimation results
<code>hausman</code>	Hausman's specification test
* <code>lincom</code>	point estimates, standard errors, testing, and inference for linear combinations of parameters
* <code>nlcom</code>	point estimates, standard errors, testing, and inference for nonlinear combinations of parameters
<code>predict</code>	predictions and residuals
* <code>test</code>	Wald tests of simple and composite linear hypotheses
* <code>testnl</code>	Wald tests of nonlinear hypotheses

*Inference is valid only for hypotheses concerning λ and θ .

predict

Description for predict

predict creates a new variable containing predictions such as predicted values and residuals.

Menu for predict

Statistics > Postestimation

Syntax for predict

```
predict [type] newvar [if] [in] [ , statistic options ]
```

<i>statistic</i>	Description
------------------	-------------

Main

<code>yhat</code>	predicted value of y ; the default
<code>residuals</code>	residuals

<i>options</i>	Description
----------------	-------------

Options

<code>smearing</code>	compute statistic using smearing method; the default
<code>btransform</code>	compute statistic using back-transform method

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

Options for predict

Main

`yhat`, the default, calculates the predicted value of the dependent variable.

`residuals` calculates the residuals, that is, the observed value minus the predicted value.

Options

`smearing` calculates the statistics `yhat` and `residuals` using the smearing method proposed by [Duan \(1983\)](#) (see [Methods and formulas](#) for a description of this method). `smearing` is the default.

`btransform` calculates the statistics `yhat` and `residuals` using the back-transform method (see [Methods and formulas](#) for a description of this method).

Remarks and examples

Below, we present two examples that illustrate how to use the `smearing` and `btransform` options.

▷ Example 1: Predictions with the `smearing` option

In this example, we calculate the predicted values of diastolic blood pressure, `bpdiast`, that arise from the `theta` model calculated in [example 1](#) of [R] `boxcox`.

```
. use https://www.stata-press.com/data/r19/nhanes2
. boxcox bpdiast bmi tcresult, notrans(age sex) model(theta) lrtest
  (output omitted)
. predict yhat
  (statistic yhat and option smearing are assumed)
```

In the expression above, `yhat` is the name we gave to the estimates of the conditional expectation. Given that we did not specify any statistic or option, the corresponding defaults `yhat` and `smearing` were assumed.

As the summary table below illustrates, the mean of the dependent variable is close to the mean of the predicted value `yhat`. This indicates that the `theta` model does a good job approximating the true value of diastolic blood pressure, `bpdiast`.

```
. summarize bpdiast yhat
```

Variable	Obs	Mean	Std. dev.	Min	Max
bpdiast	10,351	81.715	12.92722	35	150
yhat	10,351	81.71406	5.983486	66.93709	110.5283

Similarly, we could have asked that residuals be calculated. Here we again use the default `smearing` option:

```
. predict resid, residuals
  (option smearing assumed to compute residuals)
```



▷ Example 2: Predictions with the `btransform` option

In this example, we illustrate the tradeoffs involved by using the `btransform` option as opposed to the default `smearing` option. Continuing with [example 1](#), we compute the predicted values using the back-transform method.

```
. predict yhatb, btransform
  (statistic yhat assumed)
```

We now compute the predicted values using the `smearing` option and summarize both computations.

```
. predict yhats
  (statistic yhat and option smearing are assumed)
. summarize bpdiast yhats yhatb
```

Variable	Obs	Mean	Std. dev.	Min	Max
bpdiast	10,351	81.715	12.92722	35	150
yhats	10,351	81.71406	5.983486	66.93709	110.5283
yhatb	10,351	81.08018	5.95549	66.37479	109.7671

As can be seen from the mean and the standard deviation of the summary table, the predicted values using the back-transform method give biased estimates but are less variable than those coming from the smearing method. However, the efficiency loss is small compared with the bias reduction.

◀

□ Technical note

boxcox estimates variances only for the λ and θ parameters (see the [technical note](#) in [R] `boxcox`), so the extent to which postestimation commands can be used following `boxcox` is limited. Formulas used in `lincom`, `nlcom`, `test`, and `testnl` are dependent on the estimated variances. Therefore, the use of these commands is limited and generally applicable only to inferences on the λ and θ coefficients.

□

Methods and formulas

The computation of the expected value of the dependent variable conditional on the regressors for the Box–Cox model does not follow the logic of the standard linear regression model because the random disturbance does not vanish from the conditional expectation and must be accounted for. To show this, we will revisit the `lhsonly` model described by

$$y_j^{(\lambda)} = \beta_0 + \beta_1 x_{1j} + \beta_2 x_{2j} + \dots + \beta_{(k-1)} x_{(k-1)j} + \epsilon_j$$

where

$$y^{(\lambda)} = \frac{y^\lambda - 1}{\lambda}$$

and

$$y^{(\lambda)} = \begin{cases} y - 1 & \text{if } \lambda = 1 \\ \ln(y) & \text{if } \lambda = 0 \\ 1 - 1/y & \text{if } \lambda = -1 \end{cases}$$

For the presentation below, let $\mathbf{y}^{(\lambda)}$ be an $N \times 1$ vector of elementwise transformed data, \mathbf{X} be an $N \times k$ matrix of regressors, $\boldsymbol{\beta}$ be a $k \times 1$ vector of parameters, and $\boldsymbol{\iota}$ be an $n \times 1$ vector of ones.

If we were interested in $E(\mathbf{y}^{(\lambda)}|\mathbf{X})$, then the conventional logic would follow, and we would obtain predictions as $y^{(\hat{\lambda})} = \mathbf{X}\hat{\boldsymbol{\beta}}$, where $\hat{\boldsymbol{\beta}}$ is the estimate of $\boldsymbol{\beta}$. However, to estimate the conditional expectation of \mathbf{y} , we need to isolate it on the left-hand side of the model. In the case of the `lhsonly` model, this yields

$$\mathbf{y} = \{\lambda(\mathbf{X}\hat{\boldsymbol{\beta}} + \boldsymbol{\epsilon}) + \boldsymbol{\iota}\}^{1/\lambda}$$

The conditional expectation is then defined by

$$E(\mathbf{y}|\mathbf{X}) = \int \{\lambda(\mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}) + \boldsymbol{\iota}\}^{1/\lambda} dF(\boldsymbol{\epsilon}|\mathbf{X})$$

In the expression above, $dF(\boldsymbol{\epsilon}|\mathbf{X})$ corresponds to the cdf of $\boldsymbol{\epsilon}$ conditional on the regressors. It is also clear that the random disturbance does not vanish.

To address this issue, the default methodology used by `predict` computes this integral using the smearing method proposed by [Duan \(1983\)](#) to implement a two-step estimator, as was suggested by [Abrevaya \(2002\)](#).

In the first step, we get an estimate for ϵ defined as

$$\hat{\epsilon} = \mathbf{y}^{(\hat{\lambda})} - \mathbf{X}\hat{\beta}$$

In the second step, for each j we compute our predicted values as the sum:

$$\hat{y}_j = \frac{1}{N} \sum_{i=1}^N \{ \hat{\lambda}(\mathbf{x}_j \hat{\beta} + \hat{\epsilon}_i) + 1 \}^{1/\hat{\lambda}}$$

In the expression above, \mathbf{x}_j is the j th row of the matrix \mathbf{X} (in other words, the values of the covariates for individual j), and $\hat{\epsilon}_i$ is the residual for individual i . The result of this summation gives us the conditional expectation of the dependent variable for individual j . Given that this operation is performed for each individual j , the methodology is computationally intensive.

The back-transform method can be understood as a naïve estimate that disregards the random disturbance. The predictions using this approach are given by

$$\hat{y}_j = (\hat{\lambda} \mathbf{x}_j \hat{\beta} + 1)^{1/\hat{\lambda}}$$

References

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Also see

- [R] **boxcox** — Box–Cox regression models
- [U] **20 Estimation and postestimation commands**

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Description

`brier` computes the Yates, Sanders, and Murphy decompositions of the Brier mean probability score. The Brier score is a measure of disagreement between the observed outcome and a forecast (prediction).

Quick start

Brier score decomposition for binary outcome `y` and predicted probability `pvar`

```
brier y pvar
```

Same as above, but use 5 groups rather than 10 to compute the decomposition

```
brier y pvar, group(5)
```

Menu

Statistics > Epidemiology and related > Other > Brier score decomposition

Syntax

```
brier outcomevar forecastvar [if] [in] [, ggroup(#)]
```

outcomevar is a binary variable indicating the outcome of the experiment.

forecastvar is the corresponding probability of a positive outcome and must be between 0 and 1.

`by` and `collect` are allowed; see [\[U\] 11.1.10 Prefix commands](#).

Option

Main

`group(#)` specifies the number of groups that will be used to compute the decomposition. `group(10)` is the default.

Remarks and examples

You have a binary (0/1) response and a formula that predicts the corresponding probabilities of having observed a positive outcome (1). If the probabilities were obtained from logistic regression, there are many methods that assess goodness of fit (see, for instance, [\[R\] estat gof](#)). However, the probabilities might be computed from a published formula or from a model fit on another sample, both completely unrelated to the data at hand, or perhaps the forecasts are not from a formula at all. In any case, you now have a *test dataset* consisting of the forecast probabilities and observed outcomes. Your test dataset might, for instance, record predictions made by a meteorologist on the probability of rain along with a variable recording whether it actually rained.

The Brier score is an aggregate measure of disagreement between the observed outcome and a prediction—the average squared error difference. The Brier score decomposition is a partition of the Brier score into components that suggest reasons for discrepancy. These reasons fall roughly into three groups: 1) lack of overall calibration between the average predicted probability and the actual probability of the event in your data, 2) misfit of the data in groups defined within your sample, and 3) inability to match actual 0 and 1 responses.

Problem 1 refers to simply overstating or understating the probabilities.

Problem 2 refers to what is standardly called a goodness-of-fit test: the data are grouped, and the predictions for the group are compared with the outcomes.

Problem 3 refers to an individual-level measure of fit. Imagine that the grouped outcomes are predicted on average correctly but that within the group, the outcomes are poorly predicted.

Using logit or probit analysis to fit your data will guarantee that there is no lack of fit due to problem 1, and a good model fitter will be able to avoid problem 2. Problem 3 is inherent in any prediction exercise.

▷ Example 1

We have data on the outcomes of 20 basketball games (`win`) and the probability of victory predicted by a local pundit (`for`).

```
. use https://www.stata-press.com/data/r19/bball
. summarize win for
```

Variable	Obs	Mean	Std. dev.	Min	Max
win	20	.65	.4893605	0	1
for	20	.4785	.2147526	.15	.9

```
. brier win for, group(5)
```

```
Mean probability of outcome    0.6500
of forecast                    0.4785

Correlation                    0.5907
ROC area                      0.8791  p = 0.0030

Brier score                   0.1828
Spiegelhalter's z-statistic  -0.6339  p = 0.7369
Sanders-modified Brier score  0.1861
Sanders resolution            0.1400
Outcome index variance       0.2275
Murphy resolution            0.0875
Reliability-in-the-small     0.0461
Forecast variance           0.0438
Excess forecast variance     0.0285
Minimum forecast variance    0.0153
Reliability-in-the-large     0.0294
2*Forecast-Outcome-Covar    0.1179
```

The mean probabilities of forecast and outcome are simply the mean of the predicted probabilities and the actual outcomes (wins/losses). The correlation is the product-moment correlation between them.

The Brier score measures the total difference between the event (winning) and the forecast probability of that event as an average squared difference. As a benchmark, a perfect forecaster would have a Brier score of 0, a perfect misforecaster (predicts probability of win is 1 when loses and 0 when wins) would have a Brier score of 1, and a fence sitter (forecasts every game as 50/50) would have a Brier score of 0.25. Our pundit is doing reasonably well.

Spiegelhalter’s z statistic is a standard normal test statistic for testing whether an individual Brier score is extreme. The ROC area is the area under the receiver operating curve, and the associated test is a test of whether it is greater than 0.5. The more accurate the forecast probabilities, the larger the ROC area.

The Sanders-modified Brier score measures the difference between a grouped forecast measure and the event, where the data are grouped by sorting the sample on the forecast and dividing it into approximately equally sized groups. The difference between the modified and the unmodified score is typically minimal. For this and the other statistics that require grouping—the Sanders and Murphy resolutions and reliability-in-the-small—to be well defined, group boundaries are chosen so as not to allocate observations with the same forecast probability to different groups. This task is done by grouping on the forecast using `xtile, n(#)`, with `#` being the number of groups; see [D] [ptile](#).

The Sanders resolution measures error that arises from statistical considerations in evaluating the forecast for a group. A group with all positive or all negative outcomes would have a Sanders resolution of 0; it would most certainly be feasible to predict exactly what happened to each member of the group. If the group had 40% positive responses, on the other hand, a forecast that assigned $p = 0.4$ to each member of the group would be a good one, and yet, there would be “errors” in the squared difference sense. The “error” would be $(1 - 0.4)^2$ or $(0 - 0.4)^2$ for each member. The Sanders resolution is the average across groups of such “expected” errors. The 0.1400 value in our data from an overall Brier score of 0.1828 or 0.1861 suggests that a substantial portion of the “error” in our data is inherent.

Outcome index variance is just the variance of the outcome variable. This is the expected value of the Brier score if all the forecast probabilities were merely the average observed outcome. Remember that a fence sitter has an expected Brier score of 0.25; a smarter fence sitter (who would guess $p = 0.65$ for these data) would have a Brier score of 0.2275.

The Murphy resolution measures the variation in the average outcomes across groups. If all groups have the same frequency of positive outcomes, little information in any forecast is possible, and the Murphy resolution is 0. If groups differ markedly, the Murphy resolution is as large as 0.25. The 0.0875 means that there is some variation but not a lot, and 0.0875 is probably higher than in most real cases. If you had groups in your data that varied between 40% and 60% positive outcomes, the Murphy resolution would be 0.01; between 30% and 70%, it would be 0.04.

Reliability-in-the-small measures the error that comes from the average forecast within group not measuring the average outcome within group—a classical goodness-of-fit measure, with 0 meaning a perfect fit and 1 meaning a complete lack of fit. The calculated value of 0.0461 shows some amount of lack of fit. Remember, the number is squared, and we are saying that probabilities could be just more than $\sqrt{0.0461} = 0.215$ or 21.5% off.

Forecast variance measures the amount of discrimination being attempted—that is, the variation in the forecasted probabilities. A small number indicates a fence sitter making constant predictions. If the forecasts were from a logistic regression model, forecast variance would tend to increase with the amount of information available. Our pundit shows considerable forecast variance of 0.0438 (standard deviation $\sqrt{0.0438} = 0.2093$), which is in line with the reliability-in-the-small, suggesting that the forecaster is attempting as much variation as is available in these data.

Excess forecast variance is the amount of actual forecast variance over a theoretical minimum. The theoretical minimum—called the minimum forecast variance—corresponds to forecasts of p_0 for observations ultimately observed to be negative responses and p_1 for observations ultimately observed to be positive outcomes. Moreover, p_0 and p_1 are set to the average forecasts made for the ultimate negative and positive outcomes. These predictions would be just as good as the predictions the forecaster did

make, and any variation in the actual forecast probabilities above this is useless. If this number is large, above 1%–2%, then the forecaster may be attempting more than is possible. The 0.0285 in our data suggests this possibility.

Reliability-in-the-large measures the discrepancy between the mean forecast and the observed fraction of positive outcomes. This discrepancy will be 0 for forecasts made by most statistical models—at least when measured on the same sample used for estimation—because they, by design, reproduce sample means. For our human pundit, the 0.0294 says that there is a $\sqrt{0.0294}$, or 17-percentage-point, difference. (This difference can also be found by calculating the difference in the averages of the observed outcomes and forecast probabilities: $0.65 - 0.4785 = 0.17$.) That difference, however, is not significant, as we would see if we typed `ttest win=for`; see [R] `ttest`. If these data were larger and the bias persisted, this difference would be a critical shortcoming of the forecast.

Twice the forecast-outcome covariance is a measure of how accurately the forecast corresponds to the outcome. The concept is similar to that of R^2 in linear regression.

◀

Stored results

`brier` stores the following in `r()`:

Scalars

<code>r(p_roc)</code>	one-sided p -value for ROC area test
<code>r(roc_area)</code>	ROC area
<code>r(z)</code>	Spiegelhalter's z statistic
<code>r(p)</code>	one-sided p -value for Spiegelhalter's z test
<code>r(brier)</code>	Brier score
<code>r(brier_s)</code>	Sanders-modified Brier score
<code>r(sanders)</code>	Sanders resolution
<code>r(oiv)</code>	outcome index variance
<code>r(murphy)</code>	Murphy resolution
<code>r(relinism)</code>	reliability-in-the-small
<code>r(Var_f)</code>	forecast variance
<code>r(Var_fex)</code>	excess forecast variance
<code>r(Var_fmin)</code>	minimum forecast variance
<code>r(relinla)</code>	reliability-in-the-large
<code>r(cov_2f)</code>	$2 \times$ forecast-outcome-covariance

Methods and formulas

See [Wilks \(2019\)](#) or [Schmid and Griffith \(2005\)](#) for a discussion of the Brier score.

Let $d_j, j = 1, \dots, N$, be the observed outcomes with $d_j = 0$ or $d_j = 1$, and let f_j be the corresponding forecasted probabilities that d_j is 1, $0 \leq f_j \leq 1$. Assume that the data are ordered so that $f_{j+1} \geq f_j$ (`brier` sorts the data to obtain this order). Divide the data into K nearly equally sized groups, with group 1 containing observations 1 through $j_2 - 1$, group 2 containing observations j_2 through $j_3 - 1$, and so on.

Define

$$\begin{aligned}\bar{f}_0 &= \text{average } f_j \text{ among } d_j = 0 \\ \bar{f}_1 &= \text{average } f_j \text{ among } d_j = 1 \\ \bar{f} &= \text{average } f_j \\ \bar{d} &= \text{average } d_j \\ \tilde{f}_k &= \text{average } f_j \text{ in group } k \\ \tilde{d}_k &= \text{average } d_j \text{ in group } k \\ \tilde{n}_k &= \text{number of observations in group } k\end{aligned}$$

The Brier score is $\sum_j (d_j - f_j)^2 / N$.

The Sanders-modified Brier score is $\sum_j (d_j - \tilde{f}_{k(j)})^2 / N$.

Let p_j denote the true but unknown probability that $d_j = 1$. Under the null hypothesis that $p_j = f_j$ for all j , Spiegelhalter (1986) determined that the expectation and variance of the Brier score is given by the following:

$$\begin{aligned}E(\text{Brier}) &= \frac{1}{N} \sum_{j=1}^N f_j (1 - f_j) \\ \text{Var}(\text{Brier}) &= \frac{1}{N^2} \sum_{j=1}^N f_j (1 - f_j) (1 - 2f_j)^2\end{aligned}$$

Denoting the observed value of the Brier score by $O(\text{Brier})$, Spiegelhalter's z statistic is given by

$$Z = \frac{O(\text{Brier}) - E(\text{Brier})}{\sqrt{\text{Var}(\text{Brier})}}$$

The corresponding p -value is given by the upper-tail probability of Z under the standard normal distribution.

The area under the ROC curve is estimated by applying the trapezoidal rule to the empirical ROC curve. This area is Wilcoxon's test statistic, so the corresponding p -value is just that of a one-sided Wilcoxon test of the null hypothesis that the distribution of predictions is constant across the two outcomes.

The Sanders resolution is $\sum_k \tilde{n}_k \{\tilde{d}_k (1 - \tilde{d}_k)\} / N$.

The outcome index variance is $\bar{d}(1 - \bar{d})$.

The Murphy resolution is $\sum_k \tilde{n}_k (\tilde{d}_k - \bar{d})^2 / N$.

Reliability-in-the-small is $\sum_k \tilde{n}_k (\tilde{d}_k - \tilde{f}_k)^2 / N$.

The forecast variance is $\sum_j (f_j - \bar{f})^2 / N$.

The minimum forecast variance is $\{\sum_{j \in F} (f_j - \bar{f}_0)^2 + \sum_{j \in S} (f_j - \bar{f}_1)^2\} / N$, where F is the set of observations for which $d_j = 0$ and S is the complement.

The excess forecast variance is the difference between the forecast variance and the minimum forecast variance.

Reliability-in-the-large is $(\bar{f} - \bar{d})^2$.

Twice the outcome covariance is $2(\bar{f}_1 - \bar{f}_0)\bar{d}(1 - \bar{d})$.

Glenn Wilson Brier (1913–1998) was an American meteorological statistician who, after obtaining degrees in physics and statistics, was for many years head of meteorological statistics at the US Weather Bureau in Washington, DC. In the latter part of his career, he was associated with Colorado State University. Brier worked especially on verification and evaluation of predictions and forecasts, statistical decision making, the statistical theory of turbulence, the analysis of weather modification experiments, and the application of permutation techniques.

Acknowledgment

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Also see

[R] **logistic** — Logistic regression, reporting odds ratios

[R] **logit** — Logistic regression, reporting coefficients

[R] **probit** — Probit regression

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Description

`bsample` replaces the data in memory with a bootstrap sample (random sample with replacement) drawn from the current dataset. Clusters can be optionally sampled during each replication in place of observations. Bootstrap samples can also be selected within strata.

Quick start

Bootstrap sample with the same number of observations as the current dataset

```
bsample
```

Same as above, but restrict to just 100 observations

```
bsample 100
```

Stratified bootstrap sample of 100 observations at each level of `svar`

```
bsample 100, strata(svar)
```

Bootstrap sample of 10 clusters identified by values of `cvar`

```
bsample 10, cluster(cvar)
```

Same as above, but create a new unique ID code for sampled clusters and store it in `cvar2`

```
bsample 10, cluster(cvar) idcluster(cvar2)
```

Menu

Statistics > Resampling > Draw bootstrap sample

Syntax

```
bsample [exp] [if] [in] [, options]
```

where *exp* is a standard Stata expression specifying the size of the sample; see [\[U\] 13 Functions and expressions](#).

exp must be less than or equal to `_N` (the number of observations; [\[U\] 13.4 System variables \(_variables\)](#)) when neither the `cluster()` nor the `strata()` option is specified. `_N` is the default when *exp* is not specified.

Observations that do not meet the optional `if` and `in` criteria are dropped from the resulting dataset.

<i>options</i>	Description
<code>strata(varlist)</code>	variables identifying strata
<code>cluster(varlist)</code>	variables identifying resampling clusters
<code>idcluster(newvar)</code>	create new cluster ID variable
<code>weight(varname)</code>	replace <i>varname</i> with frequency weights

Options

`strata(varlist)` specifies the variables identifying strata. If `strata()` is specified, bootstrap samples are selected within each stratum, and *exp* must be less than or equal to `_N` within the defined strata.

`cluster(varlist)` specifies the variables identifying resampling clusters. If `cluster()` is specified, the sample drawn during each replication is a bootstrap sample of clusters, and *exp* must be less than or equal to N_c (the number of clusters identified by the `cluster()` option). If `strata()` is also specified, *exp* must be less than or equal to the number of within-strata clusters.

`idcluster(newvar)` creates a new variable containing a unique identifier for each resampled cluster.

`weight(varname)` specifies a variable in which the sampling frequencies will be placed. *varname* must be an existing variable, which will be replaced. After `bsample`, *varname* can be used as an `fweight` in any Stata command that accepts `fweights`, which can speed up resampling for commands like `regress` and `summarize`. This option cannot be combined with `idcluster()`.

By default, `bsample` replaces the data in memory with the sampled observations; however, specifying the `weight()` option causes only the specified *varname* to be changed.

Remarks and examples

Below is a series of examples illustrating how `bsample` is used with various sampling schemes.

▷ Example 1: Bootstrap sampling

We have data on the characteristics of hospital patients and wish to draw a bootstrap sample of 200 patients. We type

```
. use https://www.stata-press.com/data/r19/bsample1
. bsample 200
. count
  200
```

◀

▷ Example 2: Stratified samples with equal sizes

Among the variables in our dataset is `female`, an indicator for the female patients. To get a bootstrap sample of 200 female patients and 200 male patients, we type

```
. use https://www.stata-press.com/data/r19/bsample1, clear
. bsample 200, strata(female)
. tabulate female
```

Indicator for female	Freq.	Percent	Cum.
Male	200	50.00	50.00
Female	200	50.00	100.00
Total	400	100.00	

◀

▷ Example 3: Stratified samples with unequal sizes

To sample 300 females and 200 males, we must generate a variable that is 300 for females and 200 for males and then use this variable in `exp` when we call `bsample`.

```
. use https://www.stata-press.com/data/r19/bsample1, clear
. generate nsamp = cond(female,300,200)
. bsample nsamp, strata(female)
. tabulate female
```

Indicator for female	Freq.	Percent	Cum.
Male	200	40.00	40.00
Female	300	60.00	100.00
Total	500	100.00	

◀

▷ Example 4: Stratified samples with proportional sizes

Our original dataset has 2,392 males and 3,418 females.

```
. use https://www.stata-press.com/data/r19/bsample1, clear
. tabulate female
```

Indicator for female	Freq.	Percent	Cum.
Male	2,392	41.17	41.17
Female	3,418	58.83	100.00
Total	5,810	100.00	

To sample 10% from females and males, we type

```
. bsample round(0.1*_N), strata(female)
```

bsample requires that the specified size of the sample be an integer, so we use the round() function to obtain the nearest integer to 0.1×2392 and 0.1×3418 . Our sample now has 239 males and 342 females:

```
. tabulate female
```

Indicator for female	Freq.	Percent	Cum.
Male	239	41.14	41.14
Female	342	58.86	100.00
Total	581	100.00	

◀

▷ Example 5: Samples satisfying a condition

For a bootstrap sample of 200 female patients, we type

```
. use https://www.stata-press.com/data/r19/bsample1, clear
. bsample 200 if female
. tabulate female
```

Indicator for female	Freq.	Percent	Cum.
Female	200	100.00	100.00
Total	200	100.00	

◀

▷ Example 6: Generating frequency weights

To identify the sampled observations using frequency weights instead of dropping unsampled observations, we use the `weight()` option (we will need to supply it an existing variable name) and type

```
. use https://www.stata-press.com/data/r19/bsample1, clear
. set seed 1234
. generate fw = .
(5,810 missing values generated)
. bsample 200 if female, weight(fw)
. tabulate fw female
```

fw	Indicator for female		Total
	Male	Female	
0	2,392	3,222	5,614
1	0	192	192
2	0	4	4
Total	2,392	3,418	5,810

Note that $(192 \times 1) + (4 \times 2) = 200$.

◀

▷ Example 7: Oversampling observations

`bsample` requires the expression in *exp* to evaluate to a number that is less than or equal to the number of observations. To sample twice as many male and female patients as there are already in memory, we must expand the data before using `bsample`. For example,

```
. use https://www.stata-press.com/data/r19/bsample1, clear
. set seed 1234
. expand 2
(5,810 observations created)
. bsample, strata(female)
. tabulate female
```

Indicator for female	Freq.	Percent	Cum.
Male	4,784	41.17	41.17
Female	6,836	58.83	100.00
Total	11,620	100.00	

◀

▷ Example 8: Stratified oversampling with unequal sizes

To sample twice as many female patients as male patients, we must expand the records for the female patients because there are less than twice as many of them as there are male patients, but first put the number of observed male patients in a local macro. After expanding the female records, we generate a variable that contains the number of observations to sample within the two groups.

```

. use https://www.stata-press.com/data/r19/bsample1, clear
. set seed 1234
. count if !female
  2,392
. local nmale = r(N)
. expand 2 if female
(3,418 observations created)
. generate nsamp = cond(female,2*'nmale','nmale')
. bsample nsamp, strata(female)
. tabulate female

```

Indicator for female	Freq.	Percent	Cum.
Male	2,392	33.33	33.33
Female	4,784	66.67	100.00
Total	7,176	100.00	



▷ Example 9: Oversampling of clusters

For clustered data, sampling more clusters than are present in the original dataset requires more than just expanding the data. To illustrate, suppose we wanted a bootstrap sample of eight clusters from a dataset consisting of five clusters of observations.

```

. use https://www.stata-press.com/data/r19/bsample2, clear
. tabstat x, stat(n mean) by(group)
Summary for variables: x
Group variable: group

```

group	N	Mean
A	15	-.3073028
B	10	-.00984
C	11	.0810985
D	11	-.1989179
E	29	-.095203
Total	76	-.1153269

bsample will complain if we simply expand the dataset.

```

. use https://www.stata-press.com/data/r19/bsample2
. expand 3
(152 observations created)
. bsample 8, cluster(group)
resample size must not be greater than number of clusters
r(498);

```

Expanding the data will only partly solve the problem. We also need a new variable that uniquely identifies the copied clusters. We use the `expandcl` command to accomplish both these tasks; see [D] [expandcl](#).

```
. use https://www.stata-press.com/data/r19/bsample2, clear
. set seed 1234
. expandcl 2, generate(expgroup) cluster(group)
(76 observations created)
. tabstat x, stat(n mean) by(expgroup)
```

Summary for variables: x

Group variable: expgroup (New cluster ID from expandcl)

expgroup	N	Mean
1	15	-.3073028
2	15	-.3073028
3	10	-.00984
4	10	-.00984
5	11	.0810985
6	11	.0810985
7	11	-.1989179
8	11	-.1989179
9	29	-.095203
10	29	-.095203
Total	152	-.1153269

```
. generate fw = .
(152 missing values generated)
. bsample 8, cluster(expgroup) weight(fw)
. tabulate fw group
```

fw	group					Total
	A	B	C	D	E	
0	15	10	22	11	58	116
1	0	0	0	11	0	11
2	15	0	0	0	0	15
5	0	10	0	0	0	10
Total	30	20	22	22	58	152

The results from `tabulate` on the generated frequency weight variable versus the original cluster ID (`group`) show us that the bootstrap sample contains one copy of cluster A, one copy of cluster B, two copies of cluster C, two copies of cluster D, and two copies of cluster E ($1 + 1 + 2 + 2 + 2 = 8$).

◀

▷ Example 10: Stratified oversampling of clusters

Suppose that we have a dataset containing two strata with five clusters in each stratum, but the cluster identifiers are not unique between the strata. To get a stratified bootstrap sample with eight clusters in each stratum, we first use `expandcl` to expand the data and get a new cluster ID variable. We use `cluster(strid group)` in the call to `expandcl`; this action will uniquely identify the $2 * 5 = 10$ clusters across the strata.

```
. use https://www.stata-press.com/data/r19/bsample2, clear
. set seed 1234
. tabulate group strid
```

group	strid		Total
	1	2	
A	7	8	15
B	5	5	10
C	5	6	11
D	5	6	11
E	14	15	29
Total	36	40	76

```
. expandcl 2, generate(expgroup) cluster(strid group)
(76 observations created)
```

Now, we can use `bsample` with the expanded data, stratum ID variable, and new cluster ID variable.

```
. generate fw = .
(152 missing values generated)
. bsample 8, cluster(expgroup) str(strid) weight(fw)
. by strid, sort: tabulate fw group
```

-> strid = 1

fw	group					Total
	A	B	C	D	E	
0	7	0	10	5	14	36
1	0	5	0	5	14	24
2	7	0	0	0	0	7
3	0	5	0	0	0	5
Total	14	10	10	10	28	72

-> strid = 2

fw	group					Total
	A	B	C	D	E	
0	0	0	12	0	15	27
1	16	5	0	12	15	48
2	0	5	0	0	0	5
Total	16	10	12	12	30	80

The results from `by strid: tabulate` on the generated frequency weight variable versus the original cluster ID (`group`) show us how many times each cluster was sampled for each stratum. For stratum 1, the bootstrap sample contains two copies of cluster A, one copy of cluster B, two copies of cluster C, one copy of cluster D, and two copies of cluster E ($2 + 1 + 2 + 1 + 2 = 8$). For stratum 2, the bootstrap sample contains one copy of cluster A, zero copies of cluster B, three copies of cluster C, one copy of cluster D, and three copies of cluster E ($1 + 0 + 3 + 1 + 3 = 8$).

References

- Gould, W. W. 2012a. Using Stata's random-number generators, part 2: Drawing without replacement. *The Stata Blog: Not Elsewhere Classified*. <https://blog.stata.com/2012/08/03/using-statas-random-number-generators-part-2-drawing-without-replacement/>.
- . 2012b. Using Stata's random-number generators, part 3: Drawing with replacement. *The Stata Blog: Not Elsewhere Classified*. <https://blog.stata.com/2012/08/29/using-statas-random-number-generators-part-3-drawing-with-replacement/>.

Also see

- [R] **bootstrap** — Bootstrap sampling and estimation
- [R] **bstat** — Report bootstrap results
- [R] **simulate** — Monte Carlo simulations
- [D] **sample** — Draw random sample
- [D] **splitsample** — Split data into random samples

Description

`bstat` is a programmer's command that computes and displays estimation results from bootstrap statistics. For each variable in *varlist*, `bstat` computes a covariance matrix, estimates bias, and constructs normal confidence intervals (CIs), percentile CIs, bias-corrected (BC) CIs, and bias-corrected and accelerated (BC_a) CIs using a bootstrap dataset in memory or on disk. The computed CIs can be displayed using `estat bootstrap`; see [R] [bootstrap postestimation](#).

`bstat` without *varlist* replays results from the last bootstrap estimation when results are stored in `e()`.

Menu

Statistics > Resampling > Report bootstrap results

Syntax

Bootstrap statistics from variables

```
bstat [varlist] [if] [in] [, options]
```

Bootstrap statistics from file

```
bstat [namelist] [using filename] [if] [in] [, options]
```

options

Description

Main	
* <code>stat</code> (<i>vector</i>)	observed values for each statistic
* <code>accel</code> (<i>vector</i>)	acceleration values for each statistic
* <code>ties</code>	adjust BC/BCa confidence intervals for ties
* <code>mse</code>	use MSE formula for variance estimation
Reporting	
<code>level</code> (#)	set confidence level; default is <code>level(95)</code>
<code>n</code> (#)	# of observations from which bootstrap samples were taken
<code>notable</code>	suppress table of results
<code>noheader</code>	suppress table header
<code>nolegend</code>	suppress table legend
<code>verbose</code>	display the full table legend
<code>title</code> (<i>text</i>)	use <i>text</i> as title for bootstrap results
<code>display_options</code>	control column formats and line width

*Starred options and qualifiers using `if`, `in` and `in` require a bootstrap dataset.

`collect` is allowed; see [U] [11.1.10 Prefix commands](#).

See [U] [20 Estimation and postestimation commands](#) for more capabilities of estimation commands.

Options

Main

`stat` (*vector*) specifies the observed value of each statistic (that is, the value of the statistic using the original dataset).

`accel` (*vector*) specifies the acceleration of each statistic, which is used to construct BC_a CIs.

`ties` specifies that `bstat` adjust for ties in the replicate values when computing the median bias used to construct BC and BC_a CIs.

`mse` specifies that `bstat` compute the variance by using deviations of the replicates from the observed value of the statistics. By default, `bstat` computes the variance by using deviations from the average of the replicates.

Reporting

`level` (*#*); see [R] [Estimation options](#).

`n` (*#*) specifies the number of observations from which bootstrap samples were taken. This value is used in no calculations but improves the table header when this information is not saved in the bootstrap dataset.

`notable` suppresses the display of the output table.

`noheader` suppresses the display of the table header. This option implies `nolegend`.

`nolegend` suppresses the display of the table legend.

`verbose` specifies that the full table legend be displayed. By default, coefficients and standard errors are not displayed.

`title` (*text*) specifies a title to be displayed above the table of bootstrap results; the default title is `Bootstrap results`.

`display_options`: `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no!stretch`; see [R] [Estimation options](#).

Remarks and examples

Remarks are presented under the following headings:

Bootstrap datasets
Creating a bootstrap dataset

Bootstrap datasets

Although `bstat` allows you to specify the observed value and acceleration of each bootstrap statistic via the `stat()` and `accel()` options, programmers may be interested in what `bstat` uses when these options are not supplied.

When working from a bootstrap dataset, `bstat` first checks the data characteristics (see [P] `char`) that it understands:

`_dta[bs_version]` identifies the version of the bootstrap dataset. This characteristic may be empty (not defined), 2, or 3; otherwise, `bstat` will quit and display an error message. This version tells `bstat` which other characteristics to look for in the bootstrap dataset.

`bstat` uses the following characteristics from version 3 bootstrap datasets:

```
_dta[N]
_dta[N_strata]
_dta[N_cluster]
_dta[command]
varname[observed]
varname[acceleration]
varname[expression]
```

`bstat` uses the following characteristics from version 2 bootstrap datasets:

```
_dta[N]
_dta[N_strata]
_dta[N_cluster]
varname[observed]
varname[acceleration]
```

An empty bootstrap dataset version implies that the dataset was created by the `bstrap` command in a version of Stata earlier than Stata 8. Here `bstat` expects `varname[bstrap]` to contain the observed value of the statistic identified by `varname` (`varname[observed]` in version 2). All other characteristics are ignored.

`_dta[N]` is the number of observations in the observed dataset. This characteristic may be overruled by specifying the `n()` option.

`_dta[N_strata]` is the number of strata in the observed dataset.

`_dta[N_cluster]` is the number of clusters in the observed dataset.

`_dta[command]` is the command used to compute the observed values of the statistics.

`varname[observed]` is the observed value of the statistic identified by `varname`. To specify a different value, use the `stat()` option.

`varname[acceleration]` is the estimate of acceleration for the statistic identified by `varname`. To specify a different value, use the `accl()` option.

`varname[expression]` is the expression or label that describes the statistic identified by `varname`.

Creating a bootstrap dataset

Suppose that we are interested in obtaining bootstrap statistics by resampling the residuals from a regression (which is not possible with the `bootstrap` command). After loading some data, we run a regression, save some results relevant to the `bstat` command, and save the residuals in a new variable, `res`.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. regress mpg weight length
```

Source	SS	df	MS	Number of obs	=	74
Model	1616.08062	2	808.040312	F(2, 71)	=	69.34
Residual	827.378835	71	11.653223	Prob > F	=	0.0000
				R-squared	=	0.6614
				Adj R-squared	=	0.6519
Total	2443.45946	73	33.4720474	Root MSE	=	3.4137

mpg	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
weight	-.0038515	.001586	-2.43	0.018	-.0070138	-.0006891
length	-.0795935	.0553577	-1.44	0.155	-.1899736	.0307867
_cons	47.88487	6.08787	7.87	0.000	35.746	60.02374

```
. matrix b = e(b)
. local n = e(N)
. predict res, residuals
```

We can resample the residual values in `res` by generating a random observation ID (`rid`), generate a new response variable (`y`), and run the original regression with the new response variables.

```
. set seed 54321
. generate rid = int(_N*runiform())+1
. matrix score double y = b
. replace y = y + res[rid]
(74 real changes made)
. regress y weight length
```

Source	SS	df	MS	Number of obs	=	74
Model	1695.70314	2	847.851568	F(2, 71)	=	100.11
Residual	601.341031	71	8.46959199	Prob > F	=	0.0000
				R-squared	=	0.7382
				Adj R-squared	=	0.7308
Total	2297.04417	73	31.4663585	Root MSE	=	2.9103

y	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
weight	-.0029676	.0013521	-2.19	0.031	-.0056636	-.0002716
length	-.1158425	.047194	-2.45	0.017	-.2099446	-.0217404
_cons	51.72451	5.190075	9.97	0.000	41.3758	62.07323

Instead of programming this resampling inside a loop, it is much more convenient to write a short program and use the `simulate` command; see [R] [simulate](#). In the following, `mysim_r` requires the user to specify a coefficient vector and a residual variable. `mysim_r` then retrieves the list of predictor variables (removing `_cons` from the list), generates a new temporary response variable with the resampled residuals, and regresses the new response variable on the predictors.

```

program mysim_r
    version 19.5        // (or version 19 if you do not have StataNow)
    syntax name(name=bvector), res(varname)
    tempvar y rid
    local xvars : colnames `bvector'
    local cons _cons
    local xvars : list xvars - cons
    matrix score double `y' = `bvector'
    generate long `rid' = int(_N*runiform()) + 1
    replace `y' = `y' + `res'[_rid]
    regress `y' `xvars'
end
    
```

We can now give `mysim_r` a test run, but we first set the random-number seed (to reproduce results).

```

. set seed 54321
. mysim_r b, res(res)
(74 real changes made)
    
```

Source	SS	df	MS	Number of obs	=	74
Model	1695.70314	2	847.851568	F(2, 71)	=	100.11
Residual	601.341031	71	8.46959199	Prob > F	=	0.0000
				R-squared	=	0.7382
				Adj R-squared	=	0.7308
Total	2297.04417	73	31.4663585	Root MSE	=	2.9103

__000000	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
weight	-.0029676	.0013521	-2.19	0.031	-.0056636	-.0002716
length	-.1158425	.047194	-2.45	0.017	-.2099446	-.0217404
_cons	51.72451	5.190075	9.97	0.000	41.3758	62.07323

Now that we have a program that will compute the results we want, we can use `simulate` to generate a bootstrap dataset and `bstat` to display the results.

```

. set seed 54321
. simulate, reps(200) nodots: mysim_r b, res(res)
    Command: mysim_r b, res(res)
. bstat, stat(b) n(`n')
    
```

Bootstrap results Number of obs = 74
Replications = 200

	Observed coefficient	Bootstrap std. err.	z	P> z	Normal-based [95% conf. interval]	
_b_weight	-.0038515	.0014673	-2.62	0.009	-.0067274	-.0009756
_b_length	-.0795935	.0509772	-1.56	0.118	-.1795069	.0203199
_b_cons	47.88487	5.650947	8.47	0.000	36.80922	58.96053

Finally, we see that `simulate` created some of the data characteristics recognized by `bstat`. All we need to do is correctly specify the version of the bootstrap dataset, and `bstat` will automatically use the relevant data characteristics.

```
. char list
_dta[ringstate]:      XAA000000000000d431c5e5401775ee9b9e24b2604d4885..
_dta[command]:        mysim_r b, res(res)
_b_weight[is_eexp]:   1
_b_weight[colname]:   weight
_b_weight[coleq]:     -
_b_weight[expression]: _b[weight]
_b_length[is_eexp]:   1
_b_length[colname]:   length
_b_length[coleq]:     -
_b_length[expression]: _b[length]
_b_cons[is_eexp]:     1
_b_cons[colname]:     _cons
_b_cons[coleq]:       -
_b_cons[expression]:  _b[_cons]
```

```
. char _dta[bs_version] 3
```

```
. bstat, stat(b) n('n')
```

Bootstrap results Number of obs = 74
Replications = 200

Command: `mysim_r b, res(res)`

	Observed coefficient	Bootstrap std. err.	z	P> z	Normal-based [95% conf. interval]	
weight	-.0038515	.0014673	-2.62	0.009	-.0067274	-.0009756
length	-.0795935	.0509772	-1.56	0.118	-.1795069	.0203199
_cons	47.88487	5.650947	8.47	0.000	36.80922	58.96053

See [Poi \(2004\)](#) for another example of residual resampling.

Stored results

bstat stores the following in `e()`:

Scalars

<code>e(N)</code>	sample size
<code>e(N_reps)</code>	number of complete replications
<code>e(N_misreps)</code>	number of incomplete replications
<code>e(N_strata)</code>	number of strata
<code>e(N_clust)</code>	number of clusters
<code>e(k_aux)</code>	number of auxiliary parameters
<code>e(k_eq)</code>	number of equations in <code>e(b)</code>
<code>e(k_exp)</code>	number of standard expressions
<code>e(k_eexp)</code>	number of extended expressions (i.e., <code>_b</code>)
<code>e(k_extra)</code>	number of extra equations beyond the original ones from <code>e(b)</code>
<code>e(level)</code>	confidence level for bootstrap CIs
<code>e(bs_version)</code>	version for bootstrap results
<code>e(rank)</code>	rank of <code>e(V)</code>

Macros

<code>e(cmd)</code>	<code>bstat</code>
<code>e(command)</code>	from <code>_dta[command]</code>
<code>e(cmdline)</code>	command as typed
<code>e(title)</code>	title in estimation output
<code>e(exp#)</code>	expression for the <code>#</code> th statistic
<code>e(prefix)</code>	<code>bootstrap</code>
<code>e(ties)</code>	<code>ties</code> , if specified
<code>e(mse)</code>	<code>mse</code> , if specified
<code>e(vce)</code>	<code>bootstrap</code>
<code>e(vctype)</code>	title used to label Std. err.
<code>e(properties)</code>	<code>b V</code>

Matrices

<code>e(b)</code>	observed statistics
<code>e(b_bs)</code>	bootstrap estimates
<code>e(reps)</code>	number of nonmissing results
<code>e(bias)</code>	estimated biases
<code>e(se)</code>	estimated standard errors
<code>e(z0)</code>	median biases
<code>e(accel)</code>	estimated accelerations
<code>e(ci_normal)</code>	normal-approximation CIs
<code>e(ci_percentile)</code>	percentile CIs
<code>e(ci_bc)</code>	bias-corrected CIs
<code>e(ci_bca)</code>	bias-corrected and accelerated CIs
<code>e(V)</code>	bootstrap variance–covariance matrix

References

- Ng, E. S.-W., R. Grieve, and J. R. Carpenter. 2013. [Two-stage nonparametric bootstrap sampling with shrinkage correction for clustered data](#). *Stata Journal* 13: 141–164.
- Poi, B. P. 2004. [From the help desk: Some bootstrapping techniques](#). *Stata Journal* 4: 312–328.

Also see

- [R] [bootstrap postestimation](#) — Postestimation tools for bootstrap
- [R] [bootstrap](#) — Bootstrap sampling and estimation
- [R] [bsample](#) — Sampling with replacement

[Description](#)[Options](#)[Acknowledgment](#)[Quick start](#)[Remarks and examples](#)[References](#)[Menu](#)[Stored results](#)[Also see](#)[Syntax](#)[Methods and formulas](#)

Description

`centile` estimates specified centiles and calculates confidence intervals. If no *varlist* is specified, `centile` calculates centiles for all the variables in the dataset. If no centiles are specified, medians are reported.

By default, `centile` uses a binomial method for obtaining confidence intervals that makes no assumptions about the underlying distribution of the variable.

Quick start

50th percentile with 95% confidence intervals for `v1` and `v2`

```
centile v1 v2
```

For all variables in the dataset

```
centile
```

25th, 50th, and 75th percentiles of `v1`

```
centile v1, centile(25 50 75)
```

10th, 20th, 30th, ..., 90th percentiles of `v1`

```
centile v1, centile(10(10)90)
```

Force confidence limits to fall on sample values

```
centile v1 v2, cci
```

Confidence intervals based on standard errors for a normal-distribution quantile

```
centile v1 v2, normal
```

Centile and confidence intervals based on mean and standard deviation

```
centile v1 v2, meansd
```

Replace data in memory with centiles for groups defined by categorical variable `cvar`

```
statsby, by(cvar) clear: centile v1, centile(25 50 75)
```

Menu

Statistics > Summaries, tables, and tests > Summary and descriptive statistics > Centiles with CIs

Syntax

```
centile [varlist] [if] [in] [, options]
```

<i>options</i>	Description
Main	
<code>centile(<i>numlist</i>)</code>	report specified centiles; default is <code>centile(50)</code>
Options	
<code>cci</code>	binomial exact; conservative confidence interval
<code>normal</code>	normal, based on observed centiles
<code>meansd</code>	normal, based on mean and standard deviation
<code>level(#)</code>	set confidence level; default is <code>level(95)</code>

`by`, `collect`, and `statsby` are allowed; see [\[U\] 11.1.10 Prefix commands](#).

Options

Main

`centile(numlist)` specifies the centiles to be reported. The default is to display the 50th centile. Specifying `centile(5)` requests that the fifth centile be reported. Specifying `centile(5 50 95)` requests that the 5th, 50th, and 95th centiles be reported. Specifying `centile(10(10)90)` requests that the 10th, 20th, ..., 90th centiles be reported; see [\[U\] 11.1.8 numlist](#).

Options

`cci` (conservative confidence interval) forces the confidence limits to fall exactly on sample values. Confidence intervals displayed with the `cci` option are slightly wider than those with the default (`nocci`) option.

`normal` causes the confidence interval to be calculated by using a formula for the standard error of a normal-distribution quantile given by [Kendall and Stuart \(1969, 237\)](#). The `normal` option is useful when you want empirical centiles—that is, centiles based on sample order statistics rather than on the mean and standard deviation—and are willing to assume normality.

`meansd` causes the centile and confidence interval to be calculated based on the sample mean and standard deviation, and it assumes normality.

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [\[R\] level](#).

Remarks and examples

The q th centile of a continuous random variable, X , is defined as the value of C_q , which fulfills the condition $\Pr(X \leq C_q) = q/100$. The value of q must be in the range $0 < q < 100$, though q is not necessarily an integer. By default, `centile` estimates C_q for the variables in *varlist* and for the values of q given in `centile(numlist)`. It makes no assumptions about the distribution of X and, if necessary, uses linear interpolation between neighboring sample values. Extreme centiles (for example, the 99th centile in samples smaller than 100) are fixed at the minimum or maximum sample value. An “exact” confidence interval for C_q is also given, using the binomial-based method described below in [Methods and formulas](#) and in [Conover \(1999, 143–148\)](#). Again linear interpolation is used to improve the accuracy of the estimated confidence limits, but extremes are fixed at the minimum or maximum sample value.

You can prevent centile from interpolating when calculating binomial-based confidence intervals by specifying `cci`. The resulting intervals are generally wider than with the default; that is, the coverage (confidence level) tends to be greater than the nominal value (given as usual by `level(#)`, by default 95%).

If the data are believed to be normally distributed (a common case), there are two alternative methods for estimating centiles. If `normal` is specified, C_q is calculated, as just described, but its confidence interval is based on a formula for the standard error (se) of a normal-distribution quantile given by [Kendall and Stuart \(1969, 237\)](#). If `meansd` is alternatively specified, C_q is estimated as $\bar{x} + z_q \times s$, where \bar{x} and s are the sample mean and standard deviation, respectively, and z_q is the q th centile of the standard normal distribution (for example, $z_{95} = 1.645$). The confidence interval is derived from the se of the estimate of C_q .

▷ Example 1

Using `auto.dta`, we estimate the 5th, 50th, and 95th centiles of the price variable:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. format price %8.2fc
. centile price, centile(5 50 95)
```

Variable	Obs	Percentile	Centile	Binom. interp.	
				[95% conf. interval]	
price	74	5	3,727.75	3,291.23	3,914.16
		50	5,006.50	4,593.57	5,717.90
		95	13,498.00	11,061.53	15,865.30

`summarize` produces somewhat different results from `centile`; see [Methods and formulas](#).

```
. summarize price, detail
```

Price					
	Percentiles	Smallest			
1%	3291	3291			
5%	3748	3299			
10%	3895	3667	Obs		74
25%	4195	3748	Sum of wgt.		74
50%	5006.5		Mean		6165.257
		Largest	Std. dev.		2949.496
75%	6342	13466			
90%	11385	13594	Variance		8699526
95%	13466	14500	Skewness		1.653434
99%	15906	15906	Kurtosis		4.819188

The confidence limits produced by using the `cci` option are slightly wider than those produced without this option:

```
. centile price, c(5 50 95) cci
```

Variable	Obs	Percentile	Centile	Binomial exact [95% conf. interval]	
price	74	5	3,727.75	3,291.00	3,955.00
		50	5,006.50	4,589.00	5,719.00
		95	13,498.00	10,372.00	15,906.00

If we are willing to assume that `price` is normally distributed, we could include either the `normal` or the `meansd` option:

```
. centile price, c(5 50 95) normal
```

Variable	Obs	Percentile	Centile	— Normal, based on observed centiles — [95% conf. interval]	
price	74	5	3,727.75	3,211.19	4,244.31
		50	5,006.50	4,096.68	5,916.32
		95	13,498.00	5,426.81	21,569.19

```
. centile price, c(5 50 95) meansd
```

Variable	Obs	Percentile	Centile	— Normal, based on mean and std. dev.— [95% conf. interval]	
price	74	5	1,313.77	278.93	2,348.61
		50	6,165.26	5,493.24	6,837.27
		95	11,016.75	9,981.90	12,051.59

With the `normal` option, the centile estimates are, by definition, the same as before. The confidence intervals for the 5th and 50th centiles are similar to the previous ones, but the interval for the 95th centile is different. The results using the `meansd` option also differ from both previous sets of estimates.

We can use `sktest` (see [R] [sktest](#)) to check the correctness of the normality assumption:

```
. sktest price
```

Skewness and kurtosis tests for normality

Variable	Obs	Pr(skewness)	Pr(kurtosis)	— Joint test —	
				Adj chi2(2)	Prob>chi2
price	74	0.0000	0.0127	21.77	0.0000

`sktest` reveals that `price` is definitely not normally distributed, so the normal assumption is not reasonable, and the `normal` and `meansd` options are not appropriate for these data. We should rely on the results from the default choice, which does not assume normality. If the data are normally distributed, however, the precision of the estimated centiles and their confidence intervals will be ordered (best) `meansd` > `normal` > [default] (worst). The `normal` option is useful when we really do want empirical centiles (that is, centiles based on sample order statistics rather than on the mean and standard deviation) but are willing to assume normality.

Stored results

centile stores the following in `r()`:

Scalars

<code>r(N)</code>	number of observations
<code>r(n_cent)</code>	number of centiles requested
<code>r(c_#)</code>	value of # centile
<code>r(lb_#)</code>	#-requested centile lower confidence bound
<code>r(ub_#)</code>	#-requested centile upper confidence bound

Macros

<code>r(centiles)</code>	centiles requested
--------------------------	--------------------

Methods and formulas

Methods and formulas are presented under the following headings:

Default case

Normal case

meansd case

Default case

The calculation is based on the method of [Mood and Graybill \(1963, 408\)](#). Let $x_1 \leq x_2 \leq \dots \leq x_n$ be a sample of size n arranged in ascending order. Denote the estimated q th centile of the x 's as c_q . We require that $0 < q < 100$. Let $R = (n + 1)q/100$ have integer part r and fractional part f ; that is, $r = \text{int}(R)$ and $f = R - r$. (If R is itself an integer, then $r = R$ and $f = 0$.) Note that $0 \leq r \leq n$. For convenience, define $x_0 = x_1$ and $x_{n+1} = x_n$. C_q is estimated by

$$c_q = x_r + f \times (x_{r+1} - x_r)$$

that is, c_q is a weighted average of x_r and x_{r+1} . Loosely speaking, a (conservative) $p\%$ confidence interval for C_q involves finding the observations ranked t and u , which correspond, respectively, to the $\alpha = (100 - p)/200$ and $1 - \alpha$ quantiles of a binomial distribution with parameters n and $q/100$, that is, $B(n, q/100)$. More precisely, define the i th value ($i = 0, \dots, n$) of the cumulative binomial distribution function as $F_i = \Pr(S \leq i)$, where S has distribution $B(n, q/100)$. For convenience, let $F_{-1} = 0$ and $F_{n+1} = 1$. t is found such that $F_t \leq \alpha$ and $F_{t+1} > \alpha$, and u is found such that $1 - F_u \leq \alpha$ and $1 - F_{u-1} > \alpha$.

With the `cci` option in force, the (conservative) confidence interval is (x_{t+1}, x_{u+1}) , and its actual coverage probability is $F_u - F_t$.

The default case uses linear interpolation on the F_i as follows. Let

$$\begin{aligned} g &= (\alpha - F_t)/(F_{t+1} - F_t) \\ h &= \{\alpha - (1 - F_u)\}/\{(1 - F_{u-1}) - (1 - F_u)\} \\ &= (\alpha - 1 + F_u)/(F_u - F_{u-1}) \end{aligned}$$

The interpolated lower and upper confidence limits (c_{qL}, c_{qU}) for C_q are

$$\begin{aligned} c_{qL} &= x_{t+1} + g \times (x_{t+2} - x_{t+1}) \\ c_{qU} &= x_{u+1} - h \times (x_{u+1} - x_u) \end{aligned}$$

Suppose that we want a 95% confidence interval for the median of a sample of size 13. $n = 13$, $q = 50$, $p = 95$, $\alpha = 0.025$, $R = 14 \times 50/100 = 7$, and $f = 0$. Therefore, the median is the 7th observation. Some example data, x_i , and the values of F_i are as follows:

i	F_i	$1 - F_i$	x_i	i	F_i	$1 - F_i$	x_i
0	0.0001	0.9999	–	7	0.7095	0.2905	33
1	0.0017	0.9983	5	8	0.8666	0.1334	37
2	0.0112	0.9888	7	9	0.9539	0.0461	45
3	0.0461	0.9539	10	10	0.9888	0.0112	59
4	0.1334	0.8666	15	11	0.9983	0.0017	77
5	0.2905	0.7095	23	12	0.9999	0.0001	104
6	0.5000	0.5000	28	13	1.0000	0.0000	211

The median is $x_7 = 33$. Also, $F_2 \leq 0.025$ and $F_3 > 0.025$, so $t = 2$; $1 - F_{10} \leq 0.025$ and $1 - F_9 > 0.025$, so $u = 10$. The conservative confidence interval is therefore

$$(c_{50L}, c_{50U}) = (x_{t+1}, x_{u+1}) = (x_3, x_{11}) = (10, 77)$$

with actual coverage $F_{10} - F_2 = 0.9888 - 0.0112 = 0.9776$ (97.8% confidence). For the interpolation calculation, we have

$$g = (0.025 - 0.0112)/(0.0461 - 0.0112) = 0.395$$

$$h = (0.025 - 1 + 0.9888)/(0.9888 - 0.9539) = 0.395$$

So,

$$c_{50L} = x_3 + 0.395 \times (x_4 - x_3) = 10 + 0.395 \times 5 = 11.98$$

$$c_{50U} = x_{11} - 0.395 \times (x_{11} - x_{10}) = 77 - 0.395 \times 18 = 69.89$$

Normal case

The value of c_q is as above. Its se is given by the formula

$$s_q = \sqrt{q(100 - q)} / \left\{ 100\sqrt{n}Z(c_q; \bar{x}, s) \right\}$$

where \bar{x} and s are the mean and standard deviation of the x_i , and

$$Z(Y; \mu, \sigma) = \left(1/\sqrt{2\pi\sigma^2} \right) e^{-(Y-\mu)^2/2\sigma^2}$$

is the density function of a normally distributed variable Y with mean μ and standard deviation σ . The confidence interval for C_q is $(c_q - z_{100(1-\alpha)}s_q, c_q + z_{100(1-\alpha)}s_q)$.

meansd case

The value of c_q is $\bar{x} + z_q \times s$. Its se is given by the formula

$$s_q^* = s \sqrt{1/n + z_q^2 / (2n - 2)}$$

The confidence interval for C_q is $(c_q - z_{100(1-\alpha)} \times s_q^*, c_q + z_{100(1-\alpha)} \times s_q^*)$.

Acknowledgment

`centile` was written by Patrick Royston of the MRC Clinical Trials Unit, London, and coauthor of the Stata Press book *Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model*.

References

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- Mood, A. M., and F. A. Graybill. 1963. *Introduction to the Theory of Statistics*. 2nd ed. New York: McGraw-Hill.
- Stuart, A., and J. K. Ord. 1994. *Distribution Theory*. Vol. 1 of *Kendall's Advanced Theory of Statistics*, 6th ed. London: Arnold.

Also see

- [R] [ci](#) — Confidence intervals for means, proportions, and variances
- [R] [summarize](#) — Summary statistics
- [D] [pctile](#) — Create variable containing percentiles

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Description

`cfprobit` fits probit models for binary dependent variables with endogenous regressors using control functions. Endogenous variables are first modeled as a function of instruments using linear, probit, fractional probit, or Poisson regression. The residuals, or generalized residuals, from these first-stage regressions are then included in the main equation as control functions to make regression estimates robust to endogeneity.

Quick start

Control function estimates of a probit regression of binary variable y_1 on x and endogenous regressor y_2 that is instrumented by z

```
cfprobit y1 x (y2 = z)
```

Same as above, but with two endogenous regressors and two instruments

```
cfprobit y1 x (y2 y3 = z1 z2)
```

Same as above, but use z_3 as an additional instrument for y_3

```
cfprobit y1 x (y2 = z1 z2) (y3 = z1 z2 z3)
```

Model the first stage for count variable y_4 using Poisson regression

```
cfprobit y1 x (y2 = z1 z2) (y4 = z1 z2 z3, poisson)
```

Include an interaction term between w and the control function of y_2 in the main equation

```
cfprobit y1 x (y2 = z, interact(w))
```

Include an interaction term between the control functions of y_2 and y_3

```
cfprobit y1 x (y2 = z1 z2) (y3 = z1 z2 z3), cfinteract
```

Include w in the main equation for y_1 but not in the first stage

```
cfprobit y1 x (y2 = z), mainonly(w)
```

Include an endogenous interaction term between w and y_2 , and control for its endogeneity by including an interaction term between w and the control function of y_2

```
cfprobit y1 x w (y2 = z, interact(w)), mainonly(c.y2#c.w)
```

Menu

Statistics > Endogenous covariates > Control-function probit regression

Syntax

```
cfprobit depvar [indepvars] (varlisten1 = varlistiv1 [ , cfopts ])
    [ (varlisten2 = varlistiv2 [ , cfopts ]) . . . ] [if] [in] [weight] [ , options ]
```

<i>cfopts</i>	Description
Model	
<u>linear</u>	model the endogenous variables using linear regression; the default
<u>probit</u>	model the endogenous variables using probit regression
<u>fprobit</u>	model the endogenous variables using fractional probit regression
<u>poisson</u>	model the endogenous variables using Poisson regression
<u>interact</u> (<i>varlist</i> _{int})	interact the variables in <i>varlist</i> _{int} with the control functions

Only one of linear, probit, fprobit, or poisson is allowed in each set of parentheses.

<i>options</i>	Description
Model	
<u>mainonly</u> (<i>varlist</i> _m)	include the variables in <i>varlist</i> _m as exogenous variables in the main equation but not in the first-stage equations
<u>cfinteract</u>	include interactions between control functions when there are multiple endogenous variables
<u>noconstant</u>	suppress constant term
<u>asis</u>	retain perfect predictor variables

SE/Robust	
<u>vce</u> (<i>vcetype</i>)	<i>vcetype</i> may be <u>conventional</u> , <u>robust</u> , <u>cluster</u> <i>clustvar</i> , <u>bootstrap</u> , <u>jackknife</u> , or <u>hac</u> <i>hacspec</i>

Reporting	
<u>level</u>	set confidence level; default is level(95)
<u>first</u>	report first-stage regressions
<u>noheader</u>	display only the coefficient table
<u>display_options</u>	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling

Maximization	
<u>maximize_options</u>	control the maximization process
<u>coeflegend</u>	display legend instead of statistics

indepvars, *varlist*_{en}, *varlist*_{iv}, *varlist*_{int}, and *varlist*_m may contain factor variables; see [U] 11.4.3 Factor variables.

depvars, *indepvars*, *varlist*_{en}, *varlist*_{iv}, *varlist*_{int}, and *varlist*_m may contain time-series operators; see [U] 11.4.4 Time-series varlists.

bayesboot, bootstrap, by, collect, jackknife, rolling, and statsby are allowed; see [U] 11.1.10 Prefix commands.

Weights are not allowed with the bootstrap prefix; see [R] bootstrap.

fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.

coeflegend does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

Options

Model

`linear`, `probit`, `fprobit`, and `poisson` specify which regression model is used for the first-stage model. A different model can be specified for each set of parentheses.

`linear`, the default, specifies a linear regression model.

`probit` specifies a probit regression model. Endogenous variables must be coded as 0/1.

`fprobit` specifies a fractional probit regression model. Endogenous variables must take values in $[0, 1]$.

`poisson` specifies a Poisson regression model. Endogenous variables must take nonnegative values.

`interact(varlistint)` includes in the main regression an interaction term between each variable in *varlist_{int}* and the control functions associated with the current set of parentheses. Variables are treated as continuous by default.

`mainonly(varlistm)` includes the variables in *varlist_m* as exogenous variables in the main regression but excludes them from the first-stage regressions.

`cfinteract` specifies that all interactions between control functions be included in the main regression.

If there is only one endogenous regressor, and thus only one control function, the option has no effect.

`noconstant`; see [R] [Estimation options](#).

`asis` requests that all specified variables and observations be retained in the maximization process. This option is typically not used and may introduce numerical instability. Normally, `cfprobit` omits any endogenous or exogenous variables that perfectly predict success or failure in the dependent variable. The associated observations are also excluded. For more information, see [Model identification](#) in [R] [probit](#).

SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce_option](#).

`vce(conventional)`, the default, requests conventional standard errors appropriate under homoskedasticity and no autocorrelation.

`vce(hac hacspec)` requests a heteroskedasticity- and autocorrelation-consistent (HAC) variance-covariance matrix. The full syntax of *hacspec* is one of the following:

`vce(hac kernel [#])` requests a HAC variance-covariance matrix using the specified kernel (see below) with optional # lags. The bandwidth of a kernel is equal to # + 1. If # is not specified, a kernel with $N - 2$ lags is used, where N is the sample size.

`vce(hac kernel opt [#])` requests a HAC variance-covariance matrix using the specified kernel (see below), and the lag order is selected using Newey and West's (1994) optimal lag-selection algorithm. # is an optional tuning parameter that affects the lag order selected; see the [discussion](#) in *Methods and formulas* in [R] [ivregress](#).

kernel may be one of the following:

`bartlett` or `nwest` requests the Bartlett (Newey–West) kernel.

`parzen` or `gallant` requests the Parzen (Gallant 1987) kernel.

`quadraticspectral` or `andrews` requests the quadratic spectral (Andrews 1991) kernel.

Reporting

`level(#)`; see [R] [Estimation options](#).

`first` requests that the results of first-stage regressions be displayed.

`noheader` suppresses the display of the summary statistics at the top of the output, displaying only the coefficient table.

display_options: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

Maximization

maximize_options: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`; see [R] [Maximize](#). These options are seldom used.

The following option is available with `cfprobit` but is not shown in the dialog box:

`coeflegend`; see [R] [Estimation options](#).

Remarks and examples

`cfprobit` fits probit models for binary dependent variables with endogenous regressors by estimating one or more control functions and including them in the main regression equation. These control functions are estimated as the residuals, or generalized residuals, of first-stage regressions.

Control-function methods make use of instruments and are thus related to standard instrumental-variables methods. However, control-function methods allow for more flexibility than comparable instrumental-variables methods. Wooldridge (2015) gives an overview of control-function regression methods.

`cfprobit` fits a model whose main equation has the form

$$P(y_{i0} = 1 | \mathbf{y}_i, \mathbf{x}_i, \mathbf{w}_i) = \Phi(\mathbf{y}_i \boldsymbol{\beta}_1 + \mathbf{x}_i \boldsymbol{\beta}_2 + \mathbf{w}_i \boldsymbol{\beta}_3 + u_i)$$

where y_{i0} is the dependent variable for the i th observation; $\mathbf{y}_i = (y_{i1}, y_{i2}, \dots, y_{ip})'$ is a row vector of p endogenous regressors; \mathbf{x}_i is a row vector of exogenous regressors to be included in the main equation and in first-stage regressions; \mathbf{w}_i is a row vector of exogenous regressors to be included only in the main equation; $\boldsymbol{\beta}_1$, $\boldsymbol{\beta}_2$, and $\boldsymbol{\beta}_3$ are vectors of coefficients; and u_i is an error term that may be correlated with the endogenous regressors.

We assume the existence of a set of exogenous instruments for each endogenous regressor. These sets of instruments can be the same across endogenous regressors, or they can be different. Let \mathbf{z}_i^k be the vector containing the instruments for endogenous regressor y_{ik} , and let $\mathbf{z}_i = (\mathbf{z}_i^1, \mathbf{z}_i^2, \dots, \mathbf{z}_i^p)'$ be the vector containing the instruments for all endogenous regressors in the model.

While the model is similar to those fit by instrumental-variables probit methods, the control-function approach explicitly models the endogeneity in the error term u_i . Specifically, we assume

$$\begin{aligned} P(y_{i0} = 1 | \mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i, \mathbf{w}_i) &= P(y_{i0} = 1 | \boldsymbol{\nu}_i, \mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i, \mathbf{w}_i) \\ &= \Phi(\mathbf{y}_i \boldsymbol{\beta}_1 + \mathbf{x}_i \boldsymbol{\beta}_2 + \mathbf{w}_i \boldsymbol{\beta}_3 + \boldsymbol{\nu}_i \boldsymbol{\rho} + h(\boldsymbol{\nu}_i, \mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i, \mathbf{w}_i)' \boldsymbol{\rho}_h + \epsilon_i) \end{aligned}$$

where ϵ_i is an error term unaffected by endogeneity. Here $\boldsymbol{\nu}_i = (\nu_{i1}, \nu_{i2}, \dots, \nu_{ip})'$ is a row vector of control functions, one for each endogenous variable, and $\boldsymbol{\rho} = (\rho_1, \rho_2, \dots, \rho_p)$ is a vector of parameters. $h(\cdot)$ is a known vector-valued function and can include, for our purposes, interactions among the control functions in $\boldsymbol{\nu}_i$, as well as between the control functions and the exogenous or endogenous variables. $\boldsymbol{\rho}_h$ is a set of associated parameters.

For example, suppose that we have one endogenous variable y_{i1} and two instruments z_{i1}^1 and z_{i2}^1 and that \mathbf{x}_i and \mathbf{w}_i are empty. $h(\boldsymbol{\nu}_i, y_{i1}, \mathbf{z}_i^1)$ might take the form $(\nu_{i1} z_{i1}^1, \nu_{i1} z_{i2}^1)$. We can write

$$P(y_{i0} = 1 | \mathbf{y}_i, \mathbf{x}_i, \mathbf{w}_i) = \Phi(y_{i1} \beta_1 + \nu_{i1} \rho_1 + \nu_{i1} z_{i1}^1 \rho_{h1} + \nu_{i1} z_{i2}^1 \rho_{h2} + \epsilon_i)$$

▷ Example 1: Single endogenous regressor, linear first stage

In practice, control functions are not observed but rather estimated. Specifically, the residuals or generalized residuals produced in first-stage regressions serve as control functions. We can model the endogenous variable y_{i1} by the linear regression

$$y_{i1} = \mathbf{x}_i \boldsymbol{\pi}_{11} + \mathbf{z}_i^1 \boldsymbol{\pi}_{12} + \nu_{i1}$$

and use the estimate $\hat{\nu}_{i1}$ as our control function for y_{i1} .

To illustrate, we revisit [ERM] **Example 3a**, where we used a fictional dataset of university students to investigate the relationship between `graduate`, an indicator for college graduation, and `hsgpa`, a variable for high school grade point average. Rather than modeling the endogenous regressor and the main outcome jointly, here we will model the endogeneity using a control function that enters the main probit model.

A variable for income (`income`) and an indicator for whether a student has roommates (`roommate`) are included as exogenous variables. An index of the competitiveness of a student's high school (`hscomp`) is included as a set of categorical instrumental variables for `hsgpa`, which is thought to be endogenous. The main probit model has the form

$$P(\text{graduate}_i = 1) = \Phi(\beta_0 + \beta_1 \text{hsgpa}_i + \beta_2 \text{1.roommate}_i + \beta_3 \text{income}_i + u_i)$$

To fit this model with `cfprobit`, with a first-stage regression model of `hsgpa` on `income`, `i.hscomp`, and `i.roommate`, we could type

```
. use https://www.stata-press.com/data/r19/class10
. cfprobit graduate income i.roommate (hsgpa = i.hscomp)
```

However, in [ERM] **Example 3a**, robust standard errors are reported, and `i.roommate` appears only in the main equation. We can use the `mainonly(i.roommate)` and `vce(robust)` option to produce similar results as follows:

```
. cfprobit graduate income (hsgpa = i.hscomp), mainonly(i.roommate) vce(robust)
Iteration 0: Log pseudolikelihood = -1670.5207
Iteration 1: Log pseudolikelihood = -1225.1014
Iteration 2: Log pseudolikelihood = -1220.737
Iteration 3: Log pseudolikelihood = -1220.7329
Iteration 4: Log pseudolikelihood = -1220.7329
Control-function probit regression
Number of obs = 2,500
Wald chi2(3) = 366.50
Prob > chi2 = 0.0000
```

Endogenous variable model:
Linear: hsgpa

graduate	Coefficient	Robust std. err.	z	P> z	[95% conf. interval]	
graduate						
hsgpa	1.108869	.4212983	2.63	0.008	.2831396	1.934599
roommate						
Yes	.2835928	.0598045	4.74	0.000	.1663782	.4008074
income	.1712461	.0222423	7.70	0.000	.127652	.2148403
_cons	-3.958217	1.142227	-3.47	0.001	-6.196942	-1.719492
e.graduate						
cf(hsgpa)	1.500675	.4308452	3.48	0.000	.656234	2.345116

Instruments for hsgpa: 2.hscomp 3.hscomp

The estimates here are similar to those in [ERM] **Example 3a** but not identical, as is to be expected. The control-function procedure also gives us an estimate of the coefficient on the control function in the main equation, reported as `cf(hsgpa)`. Here `e.graduate` denotes the model for u_i , the error term in the main probit equation. This error term is modeled as a function of the control functions and, in some cases, other interaction terms involving them. In our example, a test of the hypothesis that the coefficient on `cf(hsgpa)` is different from zero can be interpreted as a test of the endogeneity of `hsgpa`.

cfprobit allows us to specify variables that interact with the control function. One interesting use of this feature, as outlined by Wooldridge (2015) in a linear control-function setting, is to specify a model with a correlated random coefficient on the endogenous variable. To do this, we include an interaction between hsgpa and the control function in the main model:

```
. cfprobit graduate income (hsgpa = i.hscomp, interact(hsgpa)),
> mainonly(i.roommate) vce(robust)

Iteration 0:  Log pseudolikelihood = -1670.5207
Iteration 1:  Log pseudolikelihood = -1227.3099
Iteration 2:  Log pseudolikelihood = -1215.8736
Iteration 3:  Log pseudolikelihood = -1215.6982
Iteration 4:  Log pseudolikelihood = -1215.6981

Control-function probit regression                                Number of obs = 2,500
                                                                Wald chi2(3) = 385.44
                                                                Prob > chi2  = 0.0000

Endogenous variable model:
Linear: hsgpa
```

graduate	Robust		z	P> z	[95% conf. interval]	
	Coefficient	std. err.				
graduate						
hsgpa	1.200042	.4204305	2.85	0.004	.3760133	2.02407
roommate						
Yes	.2894069	.0599019	4.83	0.000	.1720013	.4068124
income	.1729677	.0223577	7.74	0.000	.1291474	.2167881
_cons	-4.289334	1.139105	-3.77	0.000	-6.52194	-2.056729
e.graduate						
cf(hsgpa)	-2.005797	1.372688	-1.46	0.144	-4.696216	.6846215
cf(hsgpa)						
hsgpa	1.217436	.4549708	2.68	0.007	.3257092	2.109162

Instruments for hsgpa: 2.hscomp 3.hscomp

Conveniently, a test of the coefficient on the interaction term `cf(hsgpa)#c.hsgpa` is a valid test of whether the linear coefficient on `hsgpa` is random. The results suggest it is and, thus, that a random coefficient model is appropriate.

► Example 2: Two endogenous regressors

The fictional university student database discussed above includes an indicator variable, `program`, for whether a student participated in a study-skills program. We can include this as an endogenous variable in the regression, instrumenting it using indicator variables `scholar`, for whether the student has a scholarship, and `campus`, for whether the student lived on campus in their first year. We use a probit model for the first-stage regression for `program`.

Because there are two endogenous variables and thus two control functions, we can include the interaction of the two control functions in the main model using the `cfinteract` option.

```
. cfprobit graduate income (hsgpa = i.hsgcomp, interact(hsgpa))
> (program = i.campus i.scholar, probit interact(i.program)),
> mainonly(i.roommate) vce(robust) cfinteract

Iteration 0:  Log pseudolikelihood = -1670.5207
Iteration 1:  Log pseudolikelihood = -1097.3719
Iteration 2:  Log pseudolikelihood = -1077.9706
Iteration 3:  Log pseudolikelihood = -1077.6541
Iteration 4:  Log pseudolikelihood = -1077.6538
Iteration 5:  Log pseudolikelihood = -1077.6538

Control-function probit regression                                Number of obs = 2,500
                                                                Wald chi2(4) = 407.05
                                                                Prob > chi2  = 0.0000

Endogenous variable models:
  Linear: hsgpa
  Probit: 1.program
```

graduate	Coefficient	Robust std. err.	z	P> z	[95% conf. interval]	
graduate						
hsgpa	1.291673	.4500548	2.87	0.004	.4095819	2.173765
1.program	.4903884	.1758995	2.79	0.005	.1456318	.8351451
roommate						
Yes	.3237032	.0633824	5.11	0.000	.1994761	.4479304
income	.2166386	.0250304	8.66	0.000	.1675799	.2656973
_cons	-5.092267	1.222169	-4.17	0.000	-7.487675	-2.696859
e.graduate						
cf(hsgpa)	-2.367223	1.550839	-1.53	0.127	-5.406811	.6723648
cf(1.program)	.3197215	.1520979	2.10	0.036	.021615	.6178279
cf(1.program)						
cf(hsgpa)	.0225537	.1966956	0.11	0.909	-.3629625	.40807
cf(hsgpa)						
hsgpa	1.442845	.519374	2.78	0.005	.4248904	2.460799
cf(1.program)						
program						
0	0	(omitted)				
1	.1701306	.2206631	0.77	0.441	-.2623611	.6026222

```
Instruments for hsgpa:      2.hscomp 3.hscomp
Instruments for 1.program: 1.campus 1.scholar
```

To test the endogeneity of program, we can perform a joint test of the coefficients on cf (program) and its interactions using the postestimation command `estat endogenous`.

```
. estat endogenous program
Tests of endogeneity
H0: Variables are exogenous
( 1) [e.graduate]cf(1.program) - [e.graduate]cf(1.program)#cf(hsgpa) = 0
( 2) [e.graduate]cf(1.program) - [e.graduate]cf(1.program)#0b.program = 0
( 3) [e.graduate]cf(1.program) - [e.graduate]cf(1.program)#1.program = 0
( 4) [e.graduate]cf(1.program) = 0
      Constraint 4 dropped
           chi2( 3) =    12.69
           Prob > chi2 =    0.0054
```

The results suggest that program is indeed endogenous.



Stored results

cfprobit stores the following in `e()`:

Scalars

<code>e(N)</code>	number of observations
<code>e(N_cds)</code>	number of completely determined successes
<code>e(N_cdf)</code>	number of completely determined failures
<code>e(k_endog)</code>	number of endogenous variables
<code>e(df_m)</code>	model degrees of freedom
<code>e(ll)</code>	log likelihood
<code>e(chi2)</code>	χ^2
<code>e(p)</code>	<i>p</i> -value for model test
<code>e(N_clust)</code>	number of clusters
<code>e(hac_lag)</code>	HAC lag
<code>e(rank)</code>	rank of $e(V)$
<code>e(ic)</code>	number of iterations
<code>e(rc)</code>	return code
<code>e(converged)</code>	1 if converged, 0 otherwise

Macros

<code>e(cmd)</code>	cfprobit
<code>e(cmdline)</code>	command as typed
<code>e(depvar)</code>	name of dependent variable
<code>e(endog)</code>	names of endogenous variables
<code>e(exog)</code>	names of exogenous variables
<code>e(exog_main)</code>	names of exogenous variables in main equation only
<code>e(constant)</code>	noconstant, if specified
<code>e(wtype)</code>	weight type
<code>e(wexp)</code>	weight expression
<code>e(modeltypes)</code>	model specification (linear, probit, etc.) for each endogenous regressor
<code>e(cfinteract)</code>	cfinteract, if specified
<code>e(title)</code>	title in estimation output
<code>e(clustvar)</code>	name of cluster variable
<code>e(hac_kernel)</code>	HAC kernel
<code>e(vce)</code>	<i>vce</i> type specified in <code>vce()</code>
<code>e(vcetype)</code>	title used to label Std. err.
<code>e(exogr)</code>	exogenous regressors
<code>e(asis)</code>	asis, if specified
<code>e(method)</code>	ml
<code>e(opt)</code>	type of optimization

e(which)	max or min; whether optimizer is to perform maximization or minimization
e(ml_method)	type of ml method
e(user)	name of likelihood-evaluator program
e(technique)	maximization technique
e(properties)	b V
e(estat_cmd)	program used to implement estat
e(predict)	program used to implement predict
e(footnote)	program used to implement footnote display
e(marginsok)	predictions allowed by margins
e(marginsnotok)	predictions disallowed by margins
e(asbalanced)	factor variables fvset as asbalanced
e(asobserved)	factor variables fvset as asobserved
Matrices	
e(b)	coefficient vector
e(V)	variance-covariance matrix of the estimators
Functions	
e(sample)	marks estimation sample

In addition to the above, the following is stored in `r()`:

Matrices	
<code>r(table)</code>	matrix containing the coefficients with their standard errors, test statistics, p -values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r`-class command is run after the estimation command.

Methods and formulas

The probit model fit by `cfprobit` can be written as

$$P(y_{i0} = 1 | \mathbf{y}_i, \mathbf{x}_i) = \Phi(\mathbf{y}_i \boldsymbol{\beta}_1 + \mathbf{x}_i \boldsymbol{\beta}_2 + u_i)$$

where y_{i0} is the dependent variable for the i th observation; $\mathbf{y}_i = (y_{i1}, y_{i2}, \dots, y_{ip})'$ is a row vector of p endogenous regressors; \mathbf{x}_i is a row vector of exogenous variables; $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$ are vectors of coefficients; and u_i is an error term, which may be correlated with the endogenous regressors \mathbf{y}_i . Here we have omitted the main equation-only variables \mathbf{w}_i for convenience.

We also specify first-stage models for each endogenous regressor y_{ik} as a function of the exogenous regressors \mathbf{x}_i and instruments \mathbf{z}_i^k . These first-stage models, and the associated control functions ν_i and their estimates $\hat{\nu}_i$, are defined in [R] `cfregress`.

Regression estimates are produced using a modified main equation that incorporates the control-function model of the endogeneity in u_i ,

$$P(y_{i0} = 1 | \nu_i, \mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i) = \Phi\{\mathbf{y}_i \boldsymbol{\beta}_1 + \mathbf{x}_i \boldsymbol{\beta}_2 + \nu_i \boldsymbol{\rho} + h(\hat{\nu}_i, \mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i)' \boldsymbol{\rho}_h + \epsilon_i\}$$

where $h(\cdot)$ is known, $\boldsymbol{\rho} = (\rho_1, \rho_2, \dots, \rho_p)$, $\boldsymbol{\rho}_h$ is a vector of coefficients corresponding to the elements of $h(\hat{\nu}_i, \mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i)$, $\mathbf{z}_i = (\mathbf{z}_i^1, \mathbf{z}_i^2, \dots, \mathbf{z}_i^p)'$, and ϵ_i is an error term. This probit model is estimated using maximum likelihood to produce estimates of the coefficients that are appropriately corrected for endogeneity.

Note that control functions enter as estimates that have been computed in a first-stage model. As a result, the standard errors returned by this procedure are incorrect. For this reason, the standard errors are computed as if the model was estimated using generalized method of moments (GMM). The GMM specification used to produce standard errors includes a set of moment conditions for the main equation, as well as a set of moment conditions for each of the first-stage models.

The error function for the dependent variable is

$$\epsilon_i(y_{i0}, \mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \boldsymbol{\beta}_3, \boldsymbol{\rho}, \boldsymbol{\rho}_h) = y_{i0} \frac{\phi(\omega)}{\Phi(\omega)} - (1 - y_{i0}) \frac{\phi(-\omega)}{\Phi(-\omega)}$$

where $\omega = \mathbf{y}_i \boldsymbol{\beta}_1 + \mathbf{x}_i \boldsymbol{\beta}_2 + \nu_i(\mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i) \boldsymbol{\rho} + h(\nu_i(\mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i), \mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i)' \boldsymbol{\rho}_h$.

Note that this error function is equal to the score of the probit log-likelihood function (and thus delivers the same coefficient estimates as a maximum likelihood procedure). It forms a set of moment conditions with associated instruments $\mathbf{y}_i, \mathbf{x}_i, \hat{\nu}_i$, and $h(\hat{\nu}_i, \mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i)$.

Additionally, each of the control functions $\nu_{ki}(\mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i^k)$ is taken as an error function that forms a set of moment conditions with the exogenous variables \mathbf{x}_i and associated instruments \mathbf{z}_i^k .

Together, these moment conditions define an exactly identified model for the purpose of GMM estimation, even if there are more instruments in \mathbf{z}_i than there are endogenous variables (in this sense, it is a method of moments specification). This is because each instrument in the moment conditions is associated with a unique parameter. Because the GMM model is exactly identified, the results are invariant to the choice of the GMM weight matrix.

Acknowledgment

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- Wooldridge, J. M. 2015. Control function methods in applied econometrics. *Journal of Human Resources* 50: 420–445. <https://doi.org/10.3368/jhr.50.2.420>.

Also see

- [R] [cfprobit postestimation](#) — Postestimation tools for cfprobit
- [R] [cfregress](#) — Control-function linear regression
- [R] [ivprobit](#) — Probit model with continuous endogenous covariates
- [R] [probit](#) — Probit regression
- [U] [20 Estimation and postestimation commands](#)

Postestimation commands	predict	margins	estat
Remarks and examples	Stored results	Methods and formulas	Also see

Postestimation commands

The following postestimation command is of special interest after `cfprobit`:

Command	Description
estat endogenous	perform tests of endogeneity

The following postestimation commands are also available:

Command	Description
contrast	contrasts and ANOVA-style joint tests of parameters
estat summarize	summary statistics for the estimation sample
estat vce	variance–covariance matrix of the estimators (VCE)
estimates	cataloging estimation results
etable	table of estimation results
forecast	dynamic forecasts and simulations
hausman	Hausman’s specification test
lincom	point estimates, standard errors, testing, and inference for linear combinations of parameters
margins	marginal means, predictive margins, marginal effects, and average marginal effects
marginsplot	graph the results from margins (profile plots, interaction plots, etc.)
nlcom	point estimates, standard errors, testing, and inference for nonlinear combinations of parameters
predict	predictions and their SEs, residuals, etc.
predictnl	point estimates, standard errors, testing, and inference for generalized predictions
pwcompare	pairwise comparisons of parameters
test	Wald tests of simple and composite linear hypotheses
testnl	Wald tests of nonlinear hypotheses

predict

Description for predict

predict creates a new variable containing predictions such as linear predictions, residuals, and standard errors.

Menu for predict

Statistics > Postestimation

Syntax for predict

```
predict [type] newvar [if] [in] [ , statistic]
```

<i>statistic</i>	Description
------------------	-------------

Main

pr	probability of a positive outcome; the default
xb	linear prediction
xbv	linear prediction that includes control functions
stdp	standard error of the prediction

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

Options for predict

Main

pr calculates the probability of a positive outcome. This has an average structural function interpretation and is conditional on the control functions.

xb calculates the linear prediction for the main equation that does not include control-function terms, that is, $\mathbf{x}_i \mathbf{b}$.

xbv calculates the linear prediction for the main equation that includes the estimated control-function terms with their coefficients, that is, $\mathbf{x}_i \mathbf{b} + \hat{\nu}_i \mathbf{p} + h(\hat{\nu}_i, \mathbf{y}_i, \mathbf{x}_i)' \mathbf{p}_h$.

stdp calculates the standard error of the prediction, which can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. This is also referred to as the standard error of the fitted value.

margins

Description for margins

`margins` estimates margins of response for linear predictions and probabilities.

Menu for margins

Statistics > Postestimation

Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

<i>statistic</i>	Description
<code>pr</code>	probability of a positive outcome; the default
<code>xb</code>	linear prediction
<code>xbv</code>	linear prediction that includes control functions
<code>stdp</code>	not allowed with <code>margins</code>

Statistics not allowed with `margins` are functions of stochastic quantities other than `e(b)`.

For the full syntax, see [\[R\] margins](#).

estat

Description for estat

`estat endogenous` performs tests to determine whether endogenous regressors in the model are in fact exogenous. Tests are performed as Wald tests on the coefficients of relevant control functions and their interactions in the model and account for the type of variance–covariance matrix used.

Menu for estat

Statistics > Postestimation

Syntax for estat

```
estat endogenous [varlist]
```

`collect` is allowed with `estat endogenous`; see [\[U\] 11.1.10 Prefix commands](#).

Remarks and examples

Remarks are presented under the following headings:

Obtaining predicted values
estat endogenous

Obtaining predicted values

`predict`'s `pr` option calculates the probability of a positive outcome as specified by the model, conditional on the control functions. It corresponds to the result of `predict` after `ivprobit` with options `pr` and `asf` and has a similar average structural function interpretation; see [R] [ivprobit postestimation](#).

estat endogenous

Control-function regression lends itself naturally to tests of endogeneity. Under the null hypothesis that an endogenous variable is in fact endogenous, the coefficient on its associated control function, as well as the coefficients on any interactions of the control function, will be zero. Accordingly, a test of these coefficients is a test of the endogeneity of the associated endogenous variable.

`estat endogenous` tests the endogeneity of all endogenous variables jointly if specified without a variable list. Otherwise, only the endogeneity of the listed variables is tested.

For an example of the use of `estat endogenous` after `cfprobit`, see [example 2](#) in [R] [cfprobit](#).

Stored results

`estat endogenous` stores the following in `r()`:

Scalars

<code>r(chi2)</code>	χ^2 statistic
<code>r(df)</code>	degrees of freedom
<code>r(p)</code>	p -value for χ^2 statistic

Methods and formulas

As discussed in [R] [cfprobit](#), the equation estimated by `cfprobit` has the form

$$P(y_{i0} = 1 | \mathbf{y}_i, \mathbf{x}_i, \mathbf{w}_i, \mathbf{z}_i) = \Phi\{\mathbf{y}_i\boldsymbol{\beta}_1 + \mathbf{x}_i\boldsymbol{\beta}_2 + \mathbf{w}_i\boldsymbol{\beta}_3 + \hat{\boldsymbol{\nu}}_i\boldsymbol{\rho} + h(\hat{\boldsymbol{\nu}}_i, \mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i, \mathbf{w}_i)' \boldsymbol{\rho}_h + \epsilon_i\}$$

where $\hat{\boldsymbol{\nu}}_i$ is a set of estimated control functions, one for each of the endogenous variables in \mathbf{y}_i , and $h(\cdot)$ is a known vector-valued function. $h(\cdot)$ can include, for our purposes, interactions between the control functions in $\hat{\boldsymbol{\nu}}_i$, as well as interactions between control functions and the exogenous and endogenous variables in the model.

Methods and formulas are presented under the following headings:

Obtaining predicted values
estat endogenous

Obtaining predicted values

The linear prediction using the `xb` option is computed as $\mathbf{y}_i\widehat{\beta}_1 + \mathbf{x}_i\widehat{\beta}_2 + \mathbf{w}_i\widehat{\beta}_3$. The linear prediction, including control functions using the `xbv` option, is computed as $\mathbf{y}_i\widehat{\beta}_1 + \mathbf{x}_i\widehat{\beta}_2 + \mathbf{w}_i\widehat{\beta}_3 + \widehat{\nu}_i\widehat{\rho} + h(\widehat{\nu}_i, \mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i, \mathbf{w}_i)' \widehat{\rho}_h$. The probability of a positive outcome using option `pr` is computed as the cumulative standard normal distribution function evaluated at the linear prediction including control functions.

estat endogenous

`estat endogenous`, when specified without a variable list, conducts a joint Wald test of $\rho = 0$ and $\rho_h = 0$.

When a variable list is specified, `estat endogenous` conducts a Wald test for the null hypothesis that all the coefficients in ρ and ρ_h , which involve the control functions of the specified variables, are jointly equal to 0. See [R] [test](#) for documentation of Wald tests.

Also see

[R] [cfprobit](#) — Control-function probit regression

[U] [20 Estimation and postestimation commands](#)

[Description](#)[Options](#)[Acknowledgment](#)[Quick start](#)[Remarks and examples](#)[References](#)[Menu](#)[Stored results](#)[Also see](#)[Syntax](#)[Methods and formulas](#)

Description

`cfregress` fits linear models with endogenous regressors using control functions. Endogenous variables are first modeled as a function of instruments using linear, probit, fractional probit, or Poisson regression. The residuals, or generalized residuals, from these first-stage regressions are then included in the main equation as control functions to make regression estimates robust to endogeneity.

Quick start

Control function estimation of a linear regression of y_1 on x and endogenous regressor y_2 that is instrumented by z

```
cfregress y1 x (y2 = z)
```

Same as above, but with two endogenous regressors and two instruments

```
cfregress y1 x (y2 y3 = z1 z2)
```

Same as above, but use z_3 as an additional instrument for y_3

```
cfregress y1 x (y2 = z1 z2) (y3 = z1 z2 z3)
```

Model the first stage for binary endogenous regressor y_4 using probit regression

```
cfregress y1 x (y2 = z1 z2) (y4 = z1 z2 z3, probit)
```

Include an interaction term between w and the control function of y_2 in the main equation

```
cfregress y1 x (y2 = z, interact(w))
```

Include an interaction term between the control functions of y_2 and y_3

```
cfregress y1 x (y2 = z1 z2) (y3 = z1 z2), cfinteract
```

Include w in the main equation for y_1 but not in the first stage

```
cfregress y1 x (y2 = z), mainonly(w)
```

Include an endogenous interaction term between w and y_2 , and control for its endogeneity by including an interaction term between w and the control function of y_2

```
cfregress y1 x w (y2 = z, interact(w)), mainonly(c.y2#c.w)
```

Menu

Statistics > Endogenous covariates > Control-function linear regression

Syntax

```
cfregress depvar [indepvars] (varlisten1 = varlistiv1 [ , cfopts ] )
    [ (varlisten2 = varlistiv2 [ , cfopts ] ) . . . ] [if] [in] [weight] [ , options ]
```

<i>cfopts</i>	Description
<u>linear</u>	model the endogenous variables using linear regression; the default
<u>probit</u>	model the endogenous variables using probit regression
<u>fprobit</u>	model the endogenous variables using fractional probit regression
<u>poisson</u>	model the endogenous variables using Poisson regression
<u>interact</u> (<i>varlist</i> _{int})	interact the variables in <i>varlist</i> _{int} with the control functions

Only one of linear, probit, fprobit, or poisson is allowed in each set of parentheses.

<i>options</i>	Description
<u>mainonly</u> (<i>varlist</i> _m)	include the variables in <i>varlist</i> _m as exogenous variables in the main equation but not in the first-stage equations
<u>cfinteract</u>	include interactions between control functions when there are multiple endogenous variables
<u>noconstant</u>	suppress constant term
<u>hasconst</u>	has user-supplied constant

SE/Robust	
<u>vce</u> (<i>vcetype</i>)	<i>vcetype</i> may be <u>conventional</u> , <u>robust</u> , <u>cluster</u> <i>clustvar</i> , <u>bootstrap</u> , <u>jackknife</u> , or <u>hac</u> <i>hacspec</i>

Reporting	
<u>level</u>	set confidence level; default is level(95)
<u>first</u>	report first-stage regressions
<u>noheader</u>	display only the coefficient table
<u>eform</u> [(<i>string</i>)]	report exponentiated coefficients and, optionally, label as <i>string</i>
<u>display_options</u>	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
<u>coeflegend</u>	display legend instead of statistics

indepvars, *varlist*_{en}, *varlist*_{iv}, *varlist*_{int}, and *varlist*_m may contain factor variables; see [U] 11.4.3 Factor variables.

depvars, *indepvars*, *varlist*_{en}, *varlist*_{iv}, *varlist*_{int}, and *varlist*_m may contain time-series operators; see [U] 11.4.4 Time-series **varlists**.

bayesboot, *bootstrap*, *by*, *collect*, *jackknife*, *rolling*, and *statsby* are allowed; see [U] 11.1.10 Prefix commands.

Weights are not allowed with the *bootstrap* prefix; see [R] **bootstrap**.

*aweight*s are not allowed with the *jackknife* prefix; see [R] **jackknife**.

*aweight*s, *fweight*s, *iweight*s, and *pweight*s are allowed; see [U] 11.1.6 **weight**.

coeflegend does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

Options

Model

`linear`, `probit`, `fprobit`, and `poisson` specify which regression model is used for the first-stage model. A different model can be specified for each set of parentheses.

`linear`, the default, specifies a linear regression model.

`probit` specifies a probit regression model. Endogenous variables must be coded as 0/1.

`fprobit` specifies a fractional probit regression model. Endogenous variables must take values in $[0, 1]$.

`poisson` specifies a Poisson regression model. Endogenous variables must take nonnegative values.

`interact(varlistint)` includes in the main regression an interaction term between each variable in *varlist_{int}* and the control functions associated with the current set of parentheses. Variables are treated as continuous by default.

`mainonly(varlistm)` includes the variables in *varlist_m* as exogenous variables in the main regression but excludes them from the first-stage regressions.

`cfinteract` specifies that all interactions between control functions be included in the main regression.

If there is only one endogenous regressor, and thus only one control function, the option has no effect.

`noconstant`; see [R] [Estimation options](#).

`hascons` indicates that a user-defined constant or its equivalent is specified among the independent variables.

SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce_option](#).

`vce(conventional)`, the default, requests conventional standard errors appropriate under homoskedasticity and no autocorrelation.

`vce(hac hacspec)` requests a heteroskedasticity- and autocorrelation-consistent (HAC) variance-covariance matrix. The full syntax of *hacspec* is one of the following:

`vce(hac kernel [#])` requests a HAC variance-covariance matrix using the specified kernel (see below) with optional # lags. The bandwidth of a kernel is equal to $\# + 1$. If # is not specified, a kernel with $N - 2$ lags is used, where N is the sample size.

`vce(hac kernel opt [#])` requests a HAC variance-covariance matrix using the specified kernel (see below), and the lag order is selected using Newey and West's (1994) optimal lag-selection algorithm. # is an optional tuning parameter that affects the lag order selected; see the [discussion](#) in *Methods and formulas* of [R] [ivregress](#).

kernel may be one of the following:

`bartlett` or `nwest` requests the Bartlett (Newey–West) kernel.

`parzen` or `gallant` requests the Parzen (Gallant 1987) kernel.

`quadraticspectral` or `andrews` requests the quadratic spectral (Andrews 1991) kernel.

Reporting

`level(#)`; see [R] [Estimation options](#).

`first` requests that the results of first-stage regressions be displayed.

`noheader` suppresses the display of the summary statistics at the top of the output, displaying only the coefficient table.

`eform` and `eform(string)` specify that the coefficient table be displayed in exponentiated form and that `exp(b)` and `string`, respectively, be used to label the exponentiated coefficients in the table. Standard errors and confidence intervals are also transformed.

`display_options`: `noci`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

The following option is available with `cfregress` but is not shown in the dialog box:

`coeflegend`; see [R] [Estimation options](#).

Remarks and examples

`cfregress` fits linear models with endogenous regressors by estimating one or more control functions and including them in the main regression equation. These control functions are estimated as the residuals, or generalized residuals, of first-stage regressions.

Control-function methods are closely related to standard instrumental-variables (IV) methods and in the simplest cases produce the same regression estimates. However, control-function methods allow for more flexibility than comparable IV methods. [Wooldridge \(2015\)](#) gives an overview of control-function regression methods.

The main equation in the model fit by `cfregress` is

$$y_{i0} = \mathbf{y}_i\boldsymbol{\beta}_1 + \mathbf{x}_i\boldsymbol{\beta}_2 + \mathbf{w}_i\boldsymbol{\beta}_3 + u_i \quad (1)$$

where y_{i0} is the dependent variable for the i th observation; $\mathbf{y}_i = (y_{i1}, y_{i2}, \dots, y_{ip})'$ is a row vector of p endogenous regressors; \mathbf{x}_i is a row vector of exogenous regressors to be included in the main equation and in first-stage regressions; \mathbf{w}_i is a row vector of exogenous regressors to be included only in the main equation; $\boldsymbol{\beta}_1$, $\boldsymbol{\beta}_2$, and $\boldsymbol{\beta}_3$ are vectors of coefficients; and u_i is an error term whose conditional mean is thought to depend on the endogenous variables \mathbf{y}_i .

We assume the existence of a set of exogenous instruments for each endogenous regressor. These sets of instruments can be the same across endogenous regressors, or they can be different. Let \mathbf{z}_i^k be the vector containing the instruments for endogenous regressor y_{ik} , and let $\mathbf{z}_i = (\mathbf{z}_i^1, \mathbf{z}_i^2, \dots, \mathbf{z}_i^p)'$ be the vector containing the instruments for all endogenous regressors in the model.

The main equation is similar to those fit by linear IV methods. However, the control-function approach imposes additional structure on the model in that the endogeneity in the error term u_i is explicitly modeled. Specifically, we assume

$$\begin{aligned} E(u_i | \mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i, \mathbf{w}_i) &= E(u_i | \boldsymbol{\nu}_i, \mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i, \mathbf{w}_i) \\ &= \boldsymbol{\nu}_i\boldsymbol{\rho} + h(\boldsymbol{\nu}_i, \mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i, \mathbf{w}_i)' \boldsymbol{\rho}_h \end{aligned} \quad (2)$$

Here $\boldsymbol{\nu}_i = (\nu_{i1}, \nu_{i2}, \dots, \nu_{ip})'$ is a row vector of control functions, one for each endogenous variable, and $\boldsymbol{\rho} = (\rho_1, \rho_2, \dots, \rho_p)$ is a vector of coefficients. $h(\cdot)$ is a known vector-valued function and can include, for our purposes, interactions among the control functions in $\boldsymbol{\nu}_i$, as well as between the control functions and the exogenous or endogenous variables. $\boldsymbol{\rho}_h$ is a set of associated parameters.

For example, suppose that we have one endogenous variable y_{i1} and two instruments z_{i1}^1 and z_{i2}^1 and that \mathbf{x}_i and \mathbf{w}_i are empty. $h(\boldsymbol{\nu}_i, y_{i1}, \mathbf{z}_i^1)$ might take the form $(\nu_{i1}z_{i1}^1, \nu_{i1}z_{i2}^1)$. Combining (1) and (2) and specifying that $\epsilon_i = u_i - E(u_i|\mathbf{y}_i, \mathbf{z}_i)$, we can write

$$y_{i0} = y_{i1}\beta_1 + \nu_{i1}\rho_1 + \nu_{i1}z_{i1}^1\rho_{h1} + \nu_{i1}z_{i2}^1\rho_{h2} + \epsilon_i$$

When $h(\cdot) \equiv 0$ and all first-stage models are linear, control-function estimates of the coefficients of the main equation are numerically equivalent to two-stage least-squares IV estimates of the same main equation with the same instruments.

► Example 1: Single endogenous regressor, linear first stage

In practice, control functions are not observed but rather estimated. Specifically, the residuals or generalized residuals produced in first-stage regressions serve as control functions. We can model the endogenous variable y_{i1} by the linear regression

$$y_{i1} = \mathbf{x}_i\boldsymbol{\pi}_{11} + \mathbf{z}_i^1\boldsymbol{\pi}_{12} + \nu_{i1}$$

and use the estimate $\hat{\nu}_{i1}$ as our control function for y_{i1} .

To illustrate, we revisit [example 1](#) in [R] [ivregress](#) using census data on housing. We have state data from the 1980 census on the median home value (`hshngval`) and the median monthly gross rent (`rent`). We can model (`rent`) as a function of `hshngval` and the percentage of the population living in urban areas (`pcturban`),

$$\text{rent}_i = \beta_0 + \beta_1\text{hshngval}_i + \beta_2\text{pcturban}_i + u_i$$

where i indexes states. We believe that `hshngval` is endogenous; thus, we instrument it using the state's median family income (`faminc`) and census region (`region`).

We can re-create the `ivregress 2sls` estimates for this model using `cfregress`. Here, however, we rescale `hsngval` and `faminc` to be in thousands of dollars so that they are on a scale similar to `rent`:

```
. use https://www.stata-press.com/data/r19/hsng
(1980 Census housing data)
. replace hsngval = hsngval/1000
variable hsngval was long now double
(50 real changes made)
. replace faminc = faminc/1000
variable faminc was long now double
(50 real changes made)
. cfregress rent pcturban (hsngval = faminc i.region)
Control-function linear regression
```

Number of obs =	50
Wald chi2(2) =	90.76
Prob > chi2 =	0.0000
R-squared =	0.5989
Root MSE =	22.1656

Endogenous variable model:
Linear: `hsngval`

rent	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
rent						
hsngval	2.239833	.3284392	6.82	0.000	1.596104	2.883562
pcturban	.081516	.2987652	0.27	0.785	-.504053	.667085
_cons	120.7065	15.22839	7.93	0.000	90.85942	150.5536
e.rent						
cf(hsngval)	-1.588908	.4333422	-3.67	0.000	-2.438243	-.7395726

Instruments for `hsngval`: `faminc 2.region 3.region 4.region`

Accounting for scaling, these estimates are identical to comparable estimates produced by `ivregress 2sls`, but `cfregress` also includes an estimate of the coefficient on the control function, reported as `cf(hsngval)`. Here `e.rent` denotes the model used for u_i , the error term in the main equation for the dependent variable, `rent`. This error term is modeled as a function of the control functions and, in some cases, other interaction terms involving them. In our example, a test of the hypothesis that the coefficient on `cf(hsngval)` is different from zero can be interpreted as a test of the endogeneity of `hsngval`.

We may suspect, however, that our model for u_i in the previous example is misspecified. We can add an interaction term between the control function and faminc to this model by using the `interact(faminc)` option:

```
. cfregress rent pcturban (hsngval = faminc i.region, interact(faminc))
Control-function linear regression      Number of obs =      50
                                       Wald chi2(2) =    95.16
                                       Prob > chi2 =   0.0000
                                       R-squared =    0.5945
                                       Root MSE =   22.2851
```

```
Endogenous variable model:
  Linear: hsngval
```

rent	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
rent						
hsngval	2.155381	.3437284	6.27	0.000	1.481686	2.829076
pcturban	.4794597	.2362242	2.03	0.042	.0164688	.9424506
_cons	98.15909	13.86958	7.08	0.000	70.97521	125.343
e.rent						
cf(hsngval)	10.66765	3.619442	2.95	0.003	3.573673	17.76163
cf(hsngval) faminc	-.5610651	.1743049	-3.22	0.001	-.9026965	-.2194338

```
Instruments for hsngval: faminc 2.region 3.region 4.region
```

The coefficient on the endogenous variable, `hsngval`, is slightly different but not substantially changed. However, the coefficient on `pcturban` is now noticeably larger, and there is evidence it is different from zero. The coefficient on the control function `cf(hsngval)` has changed sign, and there is evidence that the coefficient on the interaction term is also relevant in the model, suggesting that including it in the model for the error term is appropriate. In this case, a joint test of `cf(hsngval)` and `cf(hsngval)#faminc` is equivalent to a test of the endogeneity of `hsngval`.

We can perform this test using the postestimation command `estat endogenous`.

```
. estat endogenous
Tests of endogeneity
HO: Variables are exogenous
( 1) [e.rent]cf(hsngval) = 0
( 2) [e.rent]cf(hsngval)#c.faminc = 0
      chi2( 2) =    15.30
      Prob > chi2 =    0.0005
```

We note here that as long as the instruments are valid, misspecification of the endogeneity in the error term, such as by using a two-stage least-squares IV estimator when the data-generating process has $h(u_i|y_i, x_i, z_i, w_i) \neq 0$, will not affect the consistency of the regression estimates. However, it may lead to biased estimates or invalid inference in small samples. [Kim and Petrin \(2011\)](#) discuss issues related to the specification of the endogeneity in the error term. ◀

▷ Example 2: Endogenous variables entering as interactions

Oftentimes, we have a model with a single endogenous regressor, y_{i1} , that appears in the main equation interacted with an exogenous variable x_{i1} ,

$$y_{i0} = y_{i1}\beta_1 + y_{i1}x_{i1}\beta_2 + \mathbf{x}_i\beta_3 + u_i$$

In these cases, it is natural to model u_i as a linear function of the control function for y_{i1} , ν_1 , and the interaction term $\nu_1 x_{i1}$,

$$E(u_i | y_{i1}, \mathbf{x}_i, \mathbf{z}_i^1) = \rho_1 \nu_1 + \rho_2 \nu_1 x_{i1}$$

We can add an interaction term to the main equation and, at the same time, model u_i using the `mainonly()` and `interact()` options.

Returning to the housing value model in the previous example, suppose that we want to include `faminc` as an exogenous variable and to include an interaction term between it and the endogenous regressor `hsngval` in the main equation for `rent`. At the same time, we want to control for endogeneity by including the interaction term between the control function of `hsngval` and `faminc`. We can type

```
. cfregress rent pcturban faminc (hsngval = i.region, interact(faminc)),
> mainonly(c.hsngval#c.faminc)
Control-function linear regression                               Number of obs =      50
                                                                Wald chi2(4) = 169.37
                                                                Prob > chi2   = 0.0000
                                                                R-squared    = 0.7694
                                                                Root MSE    = 16.8055
```

```
Endogenous variable model:
Linear: hsngval
```

rent	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
rent						
hsngval	-.1299135	1.66665	-0.08	0.938	-3.396487	3.13666
c.hsngval#						
c.faminc	.0744869	.078061	0.95	0.340	-.0785099	.2274836
pcturban	.405831	.2078664	1.95	0.051	-.0015798	.8132417
faminc	.7928874	3.834329	0.21	0.836	-6.722259	8.308034
_cons	125.9643	73.74242	1.71	0.088	-18.56818	270.4968
e.rent						
cf(hsngval)	7.225214	2.793552	2.59	0.010	1.749952	12.70048
cf(hsngval)						
faminc	-.374434	.1321422	-2.83	0.005	-.633428	-.1154401

```
Instruments for hsngval: 2.region 3.region 4.region
```

By specifying `mainonly(c.hsngval#c.faminc)`, we request that the interaction term is included in the main equation but not in the first-stage regression for `hsngval`. By specifying `interact(faminc)`, we request that the interaction term `cf(hsngval)#faminc` be included to control for endogeneity.



► Example 3: Binary endogenous regressor, probit first stage

Because control-function methods involve explicitly specifying first-stage models for the endogenous variables, the first stage need not be restricted to linear regression. `cfregress` allows for probit, fractional probit, and Poisson regression in the first-stage models for the endogenous variables. This flexibility means that a surprising variety of models can be estimated using `cfregress`.

For instance, [example 2](#) in [\[CAUSAL\] etfregress](#) uses a control-function estimator to estimate the effect of having health insurance (`ins`) on the log of prescription drug expenditure (`lndrug`). The endogenous binary treatment variable, `ins`, is instrumented using marital status (`married`) and employment status (`work`). We can reproduce the estimates in the example using `cfregress` with the `probit`, `interact()`, and `mainonly()` options:

```
. use https://www.stata-press.com/data/r19/drugexp
(Prescription drug expenditures)

. cfregress lndrug age lninc (ins = i.married i.work, probit interact(i.ins)),
> mainonly(i.chron) vce(robust)

Control-function linear regression                                Number of obs =    6,000
                                                                Wald chi2(4) = 1973.78
                                                                Prob > chi2 = 0.0000
                                                                R-squared = 0.2432
                                                                Root MSE = 1.2172

Endogenous variable model:
  Probit: 1.ins
```

lndrug	Coefficient	Robust std. err.	z	P> z	[95% conf. interval]	
lndrug						
1.ins	-.8598836	.3483648	-2.47	0.014	-1.542666	-.1771011
1.chron	.4671725	.0319731	14.61	0.000	.4045064	.5298387
age	.1021359	.00292	34.98	0.000	.0964128	.1078589
lninc	.0550672	.0225036	2.45	0.014	.0109609	.0991735
_cons	1.665539	.2527527	6.59	0.000	1.170153	2.160925
e.lndrug						
cf(1.ins)	.5252243	.226367	2.32	0.020	.0815532	.9688954
cf(1.ins)#ins						
0	0	(omitted)				
1	.2702095	.2585099	1.05	0.296	-.2364605	.7768796

Instruments for 1.ins: 1.married 1.work

Here the first stage is specified as a probit model and an interaction term is included so that the main error term depends on the value of the treatment variable. We specify the covariate `chron` (whether the individual has a chronic health condition) as present in the main equation but not in the treatment model by using the `mainonly()` option. The main equation, accounting for the specification of endogeneity, thus takes the form

$$\text{lndrug} = \beta_0 + \beta_1 \text{ins} + \beta_2 \text{age} + \beta_3 \text{lninc} + \beta_4 \text{chron} + \rho_1 \text{cf}(1.\text{ins}) + \rho_{h1} \text{cf}(1.\text{ins})\#\text{ins} + \epsilon$$

The first-stage model is a standard probit model with `ins` as the left-hand side variable and `married`, `work`, `age`, and `lninc` as right-hand side variables. The estimated control function `cf(1.ins)` is the generalized error from this first-stage probit regression, as defined in [Methods and formulas](#).

The output allows us to assess the way the endogeneity of `ins` has implicitly been specified by the treatment-regression model. The model allows the main error term to be correlated with the treatment model error conditional on the value of the treatment variable, which implies that the conditional mean of the main error depends on the interaction of `cf(1.ins)` and `ins`. The estimated coefficient on `cf(1.ins)#ins` does not, as it turns out, give evidence that this is the case.

Given this absence of evidence for a treatment-specific error term, we may wish to estimate a regression that does not include the interaction. This is not possible with `etregress`, but in `cfregress` we can simply drop the `interact()` option.

```
. cfregress lndrug age lninc (ins = i.married i.work, probit),
> mainonly(i.chron) vce(robust)
Control-function linear regression                                Number of obs =    6,000
                                                                Wald chi2(4) = 2833.77
                                                                Prob > chi2   =  0.0000
                                                                R-squared    =  0.2393
                                                                Root MSE    =  1.2203

Endogenous variable model:
  Probit: 1.ins
```

	Coefficient	Robust std. err.	z	P> z	[95% conf. interval]	
lndrug						
1.ins	-.8992025	.3399829	-2.64	0.008	-1.565557	-.2328483
1.chron	.4675479	.0319717	14.62	0.000	.4048845	.5302113
age	.1011597	.0027163	37.24	0.000	.0958359	.1064836
lninc	.0505756	.0217621	2.32	0.020	.0079228	.0932285
_cons	1.827957	.1784883	10.24	0.000	1.478126	2.177787
e.lndrug						
cf(1.ins)	.6157838	.1991464	3.09	0.002	.225464	1.006104

Instruments for 1.ins: 1.married 1.work

Here we find a slightly more extreme main effect of `ins` and slightly more precise estimates of each of the coefficients after dropping the interaction term.



Because we have explicit control of the model for the conditional expectation of the error term, we can not only treat other kinds of models as special cases but also customize these special cases for our setting, as in the preceding binary treatment example. Similarly, control function regression can be used to flexibly model correlated random coefficients in linear models, as outlined in [Wooldridge \(2015\)](#).

`cfregress` also allows for multiple endogenous regressors, each of which can have a different first-stage model and set of instruments.

Stored results

`cfregress` stores the following in `e()`:

Scalars

<code>e(N)</code>	number of observations
<code>e(k_endog)</code>	number of endogenous variables
<code>e(df_m)</code>	model degrees of freedom
<code>e(rmse)</code>	root mean squared error
<code>e(r2)</code>	R^2
<code>e(chi2)</code>	χ^2
<code>e(p)</code>	p -value for model test
<code>e(N_clust)</code>	number of clusters
<code>e(hac_lag)</code>	HAC lag
<code>e(rank)</code>	rank of $e(V)$

Macros

<code>e(cmd)</code>	<code>cfregress</code>
<code>e(cmdline)</code>	command as typed
<code>e(depvar)</code>	name of dependent variable
<code>e(endog)</code>	names of endogenous variables
<code>e(exog)</code>	names of exogenous variables
<code>e(exog_main)</code>	names of exogenous variables in main equation only
<code>e(constant)</code>	noconstant or hasconstant, if specified
<code>e(wtype)</code>	weight type
<code>e(wexp)</code>	weight expression
<code>e(modeltypes)</code>	model specification (linear, probit, etc.) for each endogenous regressor
<code>e(cfinteract)</code>	<code>cfinteract</code> , if specified
<code>e(title)</code>	title in estimation output
<code>e(clustvar)</code>	name of cluster variable
<code>e(hac_kernel)</code>	HAC kernel
<code>e(vce)</code>	<i>vctype</i> specified in <code>vce()</code>
<code>e(vcetype)</code>	title used to label Std. err.
<code>e(exogr)</code>	exogenous regressors
<code>e(properties)</code>	b V
<code>e(estat_cmd)</code>	program used to implement <code>estat</code>
<code>e(predict)</code>	program used to implement <code>predict</code>
<code>e(footnote)</code>	program used to implement footnote display
<code>e(marginsok)</code>	predictions allowed by <code>margins</code>
<code>e(marginsnotok)</code>	predictions disallowed by <code>margins</code>
<code>e(asbalanced)</code>	factor variables <code>fvset</code> as <code>asbalanced</code>
<code>e(asobserved)</code>	factor variables <code>fvset</code> as <code>asobserved</code>

Matrices

<code>e(b)</code>	coefficient vector
<code>e(V)</code>	variance–covariance matrix of the estimators

Functions

<code>e(sample)</code>	marks estimation sample
------------------------	-------------------------

In addition to the above, the following is stored in `r()`:

Matrices

<code>r(table)</code>	matrix containing the coefficients with their standard errors, test statistics, p -values, and confidence intervals
-----------------------	---

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

Methods and formulas

As discussed in *Remarks and examples*, the main equation estimated by `cfregress` has the form

$$y_{i0} = \mathbf{y}_i \boldsymbol{\beta}_1 + \mathbf{x}_i \boldsymbol{\beta}_2 + \mathbf{w}_i \boldsymbol{\beta}_3 + u_i$$

where y_{i0} is the dependent variable for the i th observation; $\mathbf{y}_i = (y_{i1}, y_{i2}, \dots, y_{ip})'$ is a row vector of p endogenous regressors; \mathbf{x}_i is a row vector of exogenous regressors to be included in the main equation and in first-stage regressions; \mathbf{w}_i is a row vector of exogenous regressors to be included only in the main equation; $\boldsymbol{\beta}_1$, $\boldsymbol{\beta}_2$, and $\boldsymbol{\beta}_3$ are vectors of coefficients; and u_i is an error term whose conditional mean is thought to depend on the endogenous variables \mathbf{y}_i .

We also specify first-stage models for each endogenous regressor y_{ik} as a function of the exogenous regressors \mathbf{x}_i and instruments \mathbf{z}_i^k . If a linear model is specified for y_{ik} , either using the `linear` option or by default, its first-stage equation has the form

$$y_{ik} = \mathbf{x}_i \boldsymbol{\pi}_{k,1} + \mathbf{z}_i^k \boldsymbol{\pi}_{k,2} + \nu_{ik}$$

where $\boldsymbol{\pi}_{k,1}$ and $\boldsymbol{\pi}_{k,2}$ are coefficients and ν_{ik} is an error term.

If a probit model is specified using the `probit` option, the first-stage model for y_{ik} has the form

$$P(y_{ik} = 1 | \mathbf{x}_i, \mathbf{z}_i^k) = \Phi(\mathbf{x}_i \boldsymbol{\pi}_{k,1} + \mathbf{z}_i^k \boldsymbol{\pi}_{k,2})$$

If a fractional probit model is specified using the `fprobit` option, the first-stage model for the conditional mean of y_{ik} can be written as

$$E(y_{ik} | \mathbf{x}_i, \mathbf{z}_i^k) = \Phi(\mathbf{x}_i \boldsymbol{\pi}_{k,1} + \mathbf{z}_i^k \boldsymbol{\pi}_{k,2})$$

Finally, if a Poisson model is specified using the `poisson` option, the first-stage model for the conditional mean of y_{ik} can be written as

$$E(y_{ik} | \mathbf{x}_i, \mathbf{z}_i^k) = \exp(\mathbf{x}_i \boldsymbol{\pi}_{k,1} + \mathbf{z}_i^k \boldsymbol{\pi}_{k,2})$$

For each endogenous variable y_{ik} , a control function $\nu_{ik}(y_{ik}, \mathbf{x}_i, \mathbf{z}_i^k)$ is estimated. In the linear case, this is simply the estimate of the linear error term.

For probit and fractional probit models, it is an estimate at the optimum of the “generalized error”, which is equal to the first derivative of the probit log likelihood with respect to $\mathbf{x}_i \boldsymbol{\pi}_{k,1} + \mathbf{z}_i^k \boldsymbol{\pi}_{k,2}$,

$$\nu_{ik}(y_{ik}, \mathbf{x}_i, \mathbf{z}_i^k) = y_{ik} \frac{\phi(\mathbf{x}_i \boldsymbol{\pi}_{k,1} + \mathbf{z}_i^k \boldsymbol{\pi}_{k,2})}{\Phi(\mathbf{x}_i \boldsymbol{\pi}_{k,1} + \mathbf{z}_i^k \boldsymbol{\pi}_{k,2})} - (1 - y_{ik}) \frac{\phi\{-(\mathbf{x}_i \boldsymbol{\pi}_{k,1} + \mathbf{z}_i^k \boldsymbol{\pi}_{k,2})\}}{\Phi\{-(\mathbf{x}_i \boldsymbol{\pi}_{k,1} + \mathbf{z}_i^k \boldsymbol{\pi}_{k,2})\}}$$

For a Poisson first-stage model, $\nu_{ik}(y_{ik}, \mathbf{x}_i, \mathbf{z}_i^k)$ is equal to the first derivative of the Poisson log likelihood with respect to $\mathbf{x}_i \boldsymbol{\pi}_{k,1} + \mathbf{z}_i^k \boldsymbol{\pi}_{k,2}$,

$$\nu_{ik}(y_{ik}, \mathbf{x}_i, \mathbf{z}_i^k) = y_{ik} - \exp(\mathbf{x}_i \boldsymbol{\pi}_{k,1} + \mathbf{z}_i^k \boldsymbol{\pi}_{k,2})$$

We also assume a known form for the endogeneity in u_i . Specifically,

$$\begin{aligned} E(u_i | \mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i, \mathbf{w}_i) &= E(u_i | \boldsymbol{\nu}_i, \mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i, \mathbf{w}_i) \\ &= \boldsymbol{\nu}_i \boldsymbol{\rho} + h(\boldsymbol{\nu}_i, \mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i, \mathbf{w}_i)' \boldsymbol{\rho}_h \end{aligned}$$

where $\nu_i = \{\nu_{i1}(y_{i1}, \mathbf{x}_i, \mathbf{z}_i^1), \dots, \nu_{ip}(y_{ip}, \mathbf{x}_i, \mathbf{z}_i^p)\}'$, $h(\cdot)$ is known, $\rho = (\rho_1, \rho_2, \dots, \rho_p)$, ρ_h is a vector of coefficients associated with the elements of $h(\nu_i, \mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i, \mathbf{w}_i)$, and $\mathbf{z}_i = (\mathbf{z}_i^1, \mathbf{z}_i^2, \dots, \mathbf{z}_i^p)'$. Accordingly, we produce estimates of our regression coefficients using a modified main equation of the form

$$y_{i0} = \mathbf{y}_i\beta_1 + \mathbf{x}_i\beta_2 + \mathbf{w}_i\beta_3 + \hat{\nu}_i\rho + h(\hat{\nu}_i, \mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i, \mathbf{w}_i)'\rho_h + \epsilon_i$$

where $\hat{\nu}_i$ is an estimate of ν_i computed in first-stage regressions and ϵ_i is an error term. This equation is estimated by ordinary least squares to produce estimates of the coefficients that are appropriately corrected for endogeneity.

Similarly to two-stage least-squares instrumental-variables estimation, the standard errors returned by this two-stage procedure will be incorrect, because ϵ_i will be incorrectly taken as the overall error term, rather than as a component of the true overall error term $u_i = \nu_i\rho + h(\nu_i, \mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i, \mathbf{w}_i)'\rho_h + \epsilon_i$.

To correct this, the standard errors are computed as if the model was estimated using generalized method of moments (GMM). The GMM specification used to produce standard errors includes a set of moment conditions for the main equation, as well as a set of moment conditions for each of the first-stage models.

The error function for the dependent variable is

$$\begin{aligned} \epsilon_i(\mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i, \mathbf{w}_i, \beta_1, \beta_2, \beta_3, \rho, \rho_h) = & y_{i0} - \mathbf{y}_i\beta_1 - \mathbf{x}_i\beta_2 - \mathbf{w}_i\beta_3 - \nu_i(\mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i)\rho \\ & - h(\nu_i(\mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i), \mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i, \mathbf{w}_i)'\rho_h \end{aligned}$$

It forms a set of moment conditions with associated instruments $\mathbf{y}_i, \mathbf{x}_i, \mathbf{w}_i, \hat{\nu}_i$, and $h(\hat{\nu}_i, \mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i, \mathbf{w}_i)$.

Additionally, each of the control functions $\nu_{ik}(\mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i^k)$ is taken as an error function that forms a set of moment conditions with the exogenous variables \mathbf{x}_i and associated instruments \mathbf{z}_i^k .

Together, these moment conditions define an exactly identified model for the purpose of GMM estimation, even if there are more instruments in \mathbf{z}_i than there are endogenous variables (in this sense, it is a method of moments specification). This is because each instrument in the moment conditions is associated with a unique parameter. Because the GMM model is exactly identified, the results are invariant to the choice of the GMM weight matrix.

Acknowledgment

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Also see

[R] **cfregress postestimation** — Postestimation tools for cfregress

[R] **cfprobit** — Control-function probit regression

[R] **ivregress** — Single-equation instrumental-variables regression

[R] **regress** — Linear regression

[CAUSAL] **eteffects** — Endogenous treatment-effects estimation

[CAUSAL] **etregress** — Linear regression with endogenous treatment effects

[U] **20 Estimation and postestimation commands**

Postestimation commands	predict	margins	estat
Remarks and examples	Stored results	Methods and formulas	Reference
Also see			

Postestimation commands

The following postestimation command is of special interest after `cfregress`:

Command	Description
estat endogenous	perform tests of endogeneity

The following postestimation commands are also available:

Command	Description
contrast	contrasts and ANOVA-style joint tests of parameters
estat summarize	summary statistics for the estimation sample
estat vce	variance–covariance matrix of the estimators (VCE)
estimates	cataloging estimation results
etable	table of estimation results
forecast	dynamic forecasts and simulations
hausman	Hausman’s specification test
lincom	point estimates, standard errors, testing, and inference for linear combinations of parameters
margins	marginal means, predictive margins, marginal effects, and average marginal effects
marginsplot	graph the results from margins (profile plots, interaction plots, etc.)
nlcom	point estimates, standard errors, testing, and inference for nonlinear combinations of parameters
predict	predictions and their SEs, residuals, etc.
predictnl	point estimates, standard errors, testing, and inference for generalized predictions
pwcompare	pairwise comparisons of parameters
test	Wald tests of simple and composite linear hypotheses
testnl	Wald tests of nonlinear hypotheses

predict

Description for predict

`predict` creates a new variable containing predictions such as linear predictions, residuals, and standard errors.

Menu for predict

Statistics > Postestimation

Syntax for predict

```
predict [type] newvar [if] [in] [ , statistic]
```

<i>statistic</i>	Description
------------------	-------------

Main

<code>xb</code>	linear prediction; the default
<code>xbv</code>	linear prediction that includes control functions
<code>e</code>	residuals
<code>ve</code>	residuals that includes control functions
<code>stdp</code>	standard error of the prediction
<code>stdf</code>	standard error of the forecast

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

Options for predict

Main

`xb`, the default, calculates the linear prediction for the main equation that does not include control-function terms, that is, $\mathbf{x}_i \mathbf{b}$.

`xbv` calculates the linear prediction for the main equation that includes the estimated control-function terms with their coefficients, that is, $\mathbf{x}_i \mathbf{b} + \hat{\nu}_i \mathbf{p} + h(\hat{\nu}_i, \mathbf{y}_i, \mathbf{x}_i)' \mathbf{p}_h$.

`e` calculates the residuals after removing the control functions, that is, $y_i - \mathbf{x}_i \mathbf{b} - \hat{\nu}_i \mathbf{p} - h(\hat{\nu}_i, \mathbf{y}_i, \mathbf{x}_i)' \mathbf{p}_h$.

`ve` calculates the residuals that include the control functions, that is, $y_i - \mathbf{x}_i \mathbf{b}$.

`stdp` calculates the standard error of the prediction, which can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. This is also referred to as the standard error of the fitted value.

`stdf` calculates the standard error of the forecast, which is the standard error of the point prediction for one observation. It is commonly referred to as the standard error of the future or forecast value. By construction, the standard errors produced by `stdf` are always larger than those produced by `stdp`; see [Methods and formulas](#) in [R] [regress postestimation](#).

margins

Description for margins

`margins` estimates margins of response for linear predictions.

Menu for margins

Statistics > Postestimation

Syntax for margins

```
margins [marginlist] [, options ]
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options ]
```

<i>statistic</i>	Description
<code>xb</code>	linear prediction; the default
<code>xbv</code>	linear prediction that includes control functions
<code>e</code>	not allowed with <code>margins</code>
<code>ve</code>	not allowed with <code>margins</code>
<code>stdp</code>	not allowed with <code>margins</code>
<code>stdf</code>	not allowed with <code>margins</code>

Statistics not allowed with `margins` are functions of stochastic quantities other than $e(b)$.

For the full syntax, see [R] [margins](#).

estat

Description for estat

`estat endogenous` performs tests to determine whether endogenous regressors in the model are in fact exogenous. Tests are performed as Wald tests on the coefficients of relevant control functions and their interactions in the model and account for the type of variance–covariance matrix used. This method of testing for endogeneity gives different results from that of `estat endogenous` after `ivregress` in finite samples, even when models are identical. See [Hansen \(2022\)](#) for a discussion.

Menu for estat

Statistics > Postestimation

Syntax for estat

```
estat endogenous [varlist ]
```

`collect` is allowed with `estat endogenous`; see [U] [11.1.10 Prefix commands](#).

Remarks and examples

Control-function regression lends itself naturally to tests of endogeneity. Under the null hypothesis that an endogenous variable is in fact endogenous, the coefficient on its associated control function, as well as the coefficients on any interactions of the control function, will be zero. Accordingly, a test of these coefficients is a test of the endogeneity of the associated endogenous variable.

`estat endogenous` tests the endogeneity of all endogenous variables jointly if specified without a variable list. Otherwise, only the endogeneity of the listed variables is tested.

For an example of the use of `estat endogenous` after `cfregress`, see [example 1](#) in [R] `cfregress`.

Stored results

`estat endogenous` stores the following in `r()`:

Scalars

<code>r(chi2)</code>	χ^2 statistic
<code>r(df)</code>	degrees of freedom
<code>r(p)</code>	p -value for χ^2 statistic

Methods and formulas

As discussed in [R] `cfregress`, the equation estimated by `cfregress` has the form

$$y_{i0} = \mathbf{y}_i\boldsymbol{\beta}_1 + \mathbf{x}_i\boldsymbol{\beta}_2 + \mathbf{w}_i\boldsymbol{\beta}_3 + \hat{\boldsymbol{\nu}}_i\boldsymbol{\rho} + h(\hat{\boldsymbol{\nu}}_i, \mathbf{y}_i, \mathbf{x}_i, \mathbf{z}_i, \mathbf{w}_i)' \boldsymbol{\rho}_h + \epsilon_i$$

where $\hat{\boldsymbol{\nu}}_i$ is a set of estimated control functions, one for each of the endogenous variables in \mathbf{y}_i , and $h(\cdot)$ is a known vector-valued function. $h(\cdot)$ can include, for our purposes, interactions between the control functions in $\hat{\boldsymbol{\nu}}_i$, as well as interactions between control functions and the exogenous and endogenous variables in the model.

`estat endogenous`, when specified without a variable list, conducts a joint Wald test of $\boldsymbol{\rho} = 0$ and $\boldsymbol{\rho}_h = 0$.

When a variable list is specified, `estat endogenous` conducts a Wald test for the null hypothesis that all the coefficients in $\boldsymbol{\rho}$ and $\boldsymbol{\rho}_h$, which involve the control functions of the specified variables, are jointly equal to 0.

Reference

Hansen, B. E. 2022. *Econometrics*. Princeton, NJ: Princeton University Press.

Also see

[R] `cfregress` — Control-function linear regression

[U] [20 Estimation and postestimation commands](#)

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

Description

`churdle` fits a linear or exponential hurdle model for a bounded dependent variable. The hurdle model combines a selection model that determines the boundary points of the dependent variable with an outcome model that determines its nonbounded values. Separate independent covariates are permitted for each model.

Quick start

Linear hurdle model of y_1 on x_1 and x_2 , specifying that y_1 is truncated at 0 with x_1 and x_3 predicting selection

```
churdle linear y1 x1 x2, select(x1 x3) ll(0)
```

Add an upper truncation limit of 40

```
churdle linear y1 x1 x2, select(x1 x3) ll(0) ul(40)
```

Same as above, with the upper truncation limit specified in `trunc`

```
churdle linear y1 x1 x2, select(x1 x3) ll(0) ul(trunc)
```

Same as above, and use x_3 to model the variance of the selection model

```
churdle linear y1 x1 x2, select(x1 x3, het(x3)) ll(0) ul(trunc)
```

Same as above, and use x_4 to model the variance of the outcome model

```
churdle linear y1 x1 x2, select(x1 x3, het(x3)) ll(0) ///  
ul(trunc) het(x4)
```

Exponential hurdle model of y_2 on x_1 and x_2 , specifying that y_2 is truncated at 4 with x_1 and x_3 predicting selection

```
churdle exponential y2 x1 x2, select(x1 x3) ll(4)
```

Menu

Statistics > Linear models and related > Hurdle regression

Syntax

Basic syntax

```
churdle linear depvar, select(varlists) { ll(...) | ul(...) }
```

```
churdle exponential depvar, select(varlists) ll(...)
```

Full syntax for churdle linear

```
churdle linear depvar [indepvars] [if] [in] [weight],  
  select(varlists [, noconstant het(varlisto)] )  
  { ll(#|varname) | ul(#|varname) } [options]
```

Full syntax for churdle exponential

```
churdle exponential depvar [indepvars] [if] [in] [weight],  
  select(varlists [, noconstant het(varlisto)] ) ll(#|varname) [options]
```

<i>options</i>	Description
Model	
* <u>select</u> ()	specify independent variables and options for selection model
‡ <u>ll</u> (# <i>varname</i>)	lower truncation limit
‡ <u>ul</u> (# <i>varname</i>)	upper truncation limit
<u>noconstant</u>	suppress constant term
<u>constraints</u> (<i>constraints</i>)	apply specified linear constraints
<u>het</u> (<i>varlist</i>)	specify variables to model the variance
SE/Robust	
<u>vce</u> (<i>vcetype</i>)	<i>vcetype</i> may be <u>oim</u> , <u>robust</u> , <u>cluster</u> <i>clustvar</i> , <u>bootstrap</u> , or <u>jackknife</u>
Reporting	
<u>level</u> (#)	set confidence level; default is <u>level</u> (95)
<u>nocnsreport</u>	do not display constraints
<u>display_options</u>	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Maximization	
<u>maximize_options</u>	control the maximization process; seldom used
<u>coeflegend</u>	display legend instead of statistics

*`select()` is required.

The full specification is `select(varlists [, noconstant het(varlisto)])`.

`noconstant` specifies that the constant be excluded from the selection model.

`het(varlisto)` specifies the variables in the error-variance function of the selection model.

‡ You must specify at least one of `ul(#|varname)` or `ll(#|varname)` for the linear model and must specify `ll(#|varname)` for the exponential model.

`indepvars`, `varlists`, and `varlisto` may contain factor variables; see [U] 11.4.3 Factor variables.

`bayesboot`, `bootstrap`, `by`, `collect`, `fp`, `jackknife`, `rolling`, `statsby`, and `svy` are allowed; see [U] 11.1.10 Prefix commands.

Weights are not allowed with the `bootstrap` prefix; see [R] bootstrap.

`vce()` and weights are not allowed with the `svy` prefix; see [SVY] svy.

`fweights`, `iwweights`, and `pweights` are allowed; see [U] 11.1.6 weight.

`coeflegend` does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

Options

Model

`select(varlists [, noconstant het(varlisto)])` specifies the variables and options for the selection model. `select()` is required.

`ll(#|varname)` and `ul(#|varname)` indicate the lower and upper limits, respectively, for the dependent variable. You must specify one or both for the linear model and must specify a lower limit for the exponential model. Observations with $depvar \leq ll()$ have a lower bound; observations with $depvar \geq ul()$ have an upper bound; and the remaining observations are in the continuous region.

`noconstant`, `constraints(constraints)`; see [R] Estimation options.

`het(varlist)` specifies the variables in the error-variance function of the outcome model.

SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster`, `clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] vce_option.

Reporting

`level(#)`, `nocnsreport`; see [R] Estimation options.

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] Estimation options.

Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] Maximize. These options are seldom used.

The following option is available with `churdle` but is not shown in the dialog box:

`coeflegend`; see [R] [Estimation options](#).

Remarks and examples

`churdle` fits a linear or an exponential hurdle model. It combines a selection model that determines the boundary points of the dependent variable with an outcome model that determines its nonbounded values. Hurdle models treat these boundary values as observed instead of censored. That is to say, observations where the dependent variable is equal to one of the boundary values are not the result of our inability to observe the distribution above or below a certain point; see [Wooldridge \(2010\)](#) chapter 17 for a thorough discussion of this point.

These models were proposed by [Cragg \(1971\)](#) to explain the demand for durable goods. In the Cragg model, individuals purchase zero or a positive amount of the durable good, with different factors determining each of these choices. This may be generalized to other individual decisions, such as money donated to charity, cigarette consumption, and time spent volunteering.

Hurdle models are characterized by the relationship $y_i = s_i h_i^*$, where y_i is the observed value of the dependent variable.

The selection variable, s_i , is 1 if the dependent variable is not bounded and 0 otherwise. In the Cragg model, the lower limit that binds the dependent variable is 0 so the selection model is

$$s_i = \begin{cases} 1 & \text{if } \mathbf{z}_i \boldsymbol{\gamma} + \epsilon_i > 0 \\ 0 & \text{otherwise} \end{cases}$$

where \mathbf{z}_i is a vector of explanatory variables, $\boldsymbol{\gamma}$ is a vector of coefficients, and ϵ_i is a standard normal error term. `churdle` allows a different lower limit to be specified in `ll()` and, for the linear model, an upper limit in `ul()`. Conditional heteroskedasticity of the random error ϵ_i is allowed if the `het()` suboption is specified in `select()`.

The continuous latent variable h_i^* is observed only if $s_i = 1$. The outcome model can be either the linear model or the exponential model, as proposed in [Cragg \(1971\)](#):

$$\begin{aligned} h_i^* &= \mathbf{x}_i \boldsymbol{\beta} + \nu_i && \text{(linear)} \\ h_i^* &= \exp(\mathbf{x}_i \boldsymbol{\beta} + \nu_i) && \text{(exponential)} \end{aligned}$$

where \mathbf{x}_i is a vector of explanatory variables, $\boldsymbol{\beta}$ is a vector of coefficients, and ν_i is an error term.

For the linear model, ν_i has a truncated normal distribution with lower truncation point $-\mathbf{x}_i \boldsymbol{\beta}$. For the exponential model, ν_i has a normal distribution. `churdle` extends the Cragg hurdle models to allow for conditional heteroskedasticity of the random error ν_i if the user specifies the `het()` option.

The parameters and regressors in the models for h_i^* and for s_i may differ.

▷ Example 1: Linear hurdle model

Consider a dataset that contains the number of hours an individual exercises per day (`hours`), their age (`age`), whether they are single (`single`), hours they work per day (`whours`), whether they smoke (`smoke`), their weight in kilograms (`weight`), their distance from the nearest gym (`distance`), and their average commute from work (`commute`).

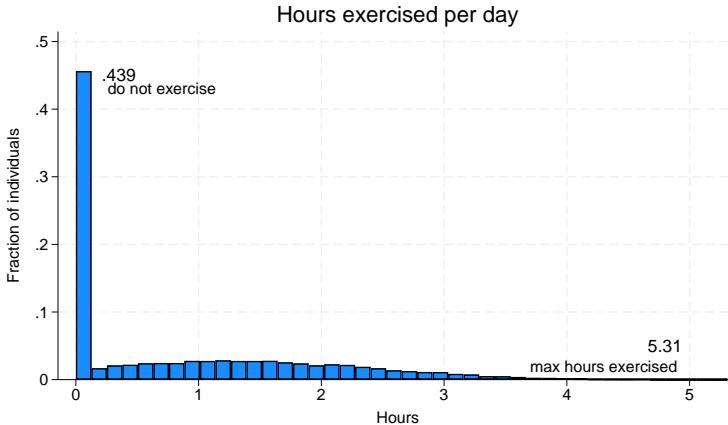


Figure 1.

Figure 1 shows that 43.9% of the individuals in the sample do not exercise and that the hours exercised varies among individuals that decide to exercise.

We model the decision to exercise or not as a function of `commute`, `whours`, and `age`. These variables are written in `select()`. Once a decision to exercise is made, the time an individual exercises is modeled as a linear function of `age`, `smoke`, `distance`, and `single`.

```
. use https://www.stata-press.com/data/r19/fitness
(Fictional fitness data)
. churdle linear hours age i.smoke distance i.single,
> select(commute whours age) ll(0)
```

```
Iteration 0: Log likelihood = -23657.236
Iteration 1: Log likelihood = -23344.182
Iteration 2: Log likelihood = -23340.051
Iteration 3: Log likelihood = -23340.044
Iteration 4: Log likelihood = -23340.044
```

Cragg hurdle regression

```
Number of obs = 19,831
LR chi2(4) = 9059.26
Prob > chi2 = 0.0000
Pseudo R2 = 0.1625
```

Log likelihood = -23340.044

	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
hours						
age	.0015116	.000763	1.98	0.048	.0000162	.003007
smoke						
Smoking	-1.06646	.0460578	-23.15	0.000	-1.156731	-.9761879
distance	-.1333868	.0126344	-10.56	0.000	-.1581497	-.1086238
single						
Single	.9940893	.0258775	38.42	0.000	.9433703	1.044808
_cons	.9138855	.0396227	23.06	0.000	.8362264	.9915447
selection_ll						
commute	-.2953345	.0624665	-4.73	0.000	-.4177666	-.1729024
whours	.0022974	.0069306	0.33	0.740	-.0112864	.0158811
age	-.0485347	.0006501	-74.65	0.000	-.049809	-.0472604
_cons	2.649945	.0499795	53.02	0.000	2.551987	2.747903
lnsigma						
_cons	.0083199	.0099648	0.83	0.404	-.0112107	.0278506
/sigma						
	1.008355	.010048			.9888519	1.028242

The coefficients in the outcome model for the latent variable appear under `hours`. Because we only specified a lower limit to bind the dependent variable, the output shows parameter estimates for a single selection model under `selection_ll`. Information about the estimated standard deviation of the error term in the outcome model appears under `lnsigma` and `/sigma`.

The coefficient estimates are not directly interpretable. To obtain the effect of a covariate on the model, we need to use the `margins` command; see [\[R\] churdle postestimation](#). Consider the effect of `age`:

```
. margins, dydx(age)
Average marginal effects                               Number of obs = 19,831
Model VCE: OIM
Expression: Conditional mean estimates of dependent variable, predict()
dy/dx wrt: age
```

	Delta-method				
	dy/dx	std. err.	z	P> z	[95% conf. interval]
age	-.0216855	.000289	-75.03	0.000	-.022252 - .021119

Each additional year of age is associated with about -0.02 fewer hours, or 1.2 minutes, of exercise.

▷ Example 2: Linear hurdle with models for the outcome and selection variances

In this example, we illustrate the possibility of fitting a heteroskedastic probit for the selection and latent model. In both cases, this is done by specifying `age` and `single` as the variables that affect the conditional variance. As in [example 1](#), we have separate parameters for the outcome model and lower-limit selection model.

```
. churdle linear hours age i.smoke distance i.single,
> select(commute whours age, het(age single)) ll(0) het(age single) nolog

Cragg hurdle regression                               Number of obs = 19,831
LR chi2(4) = 9060.63
Prob > chi2 = 0.000
Pseudo R2 = 0.1626

Log likelihood = -23339.355
```

	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
hours						
age	.0012559	.0008198	1.53	0.126	-.0003508	.0028626
smoke						
Smoking	-1.065564	.0457657	-23.28	0.000	-1.155263	-.9758649
distance	-.1332939	.0126102	-10.57	0.000	-.1580094	-.1085783
single						
Single	1.002511	.032535	30.81	0.000	.9387436	1.066278
_cons	.9166356	.0388318	23.61	0.000	.8405268	.9927445
selection_ll						
commute	-.2959986	.0641594	-4.61	0.000	-.4217488	-.1702484
whours	.0024514	.0069769	0.35	0.725	-.0112231	.0161259
age	-.048886	.0021405	-22.84	0.000	-.0530814	-.0446906
_cons	2.669613	.1139478	23.43	0.000	2.44628	2.892947
lnsigma						
age	.0003537	.0004026	0.88	0.380	-.0004354	.0011427
single	-.0080667	.019253	-0.42	0.675	-.0458019	.0296685
lnsigma_ll						
age	-.0002035	.0008424	-0.24	0.809	-.0018546	.0014475
single	.0268271	.0270133	0.99	0.321	-.0261179	.0797721

The coefficients on age and single have no effect on the conditional variance of the outcome model or on the conditional variance of the selection model. Thus, there is no evidence that the variance depends on age and marital status.



▷ Example 3: Exponential hurdle model

Returning to [example 1](#), if we believe that the conditional mean of the latent variable has an exponential form instead of a linear form, we use `churdle exponential`.

```
. churdle exponential hours age i.smoke distance i.single,
> select(commute whours age) ll(0) nolog
```

```
Cragg hurdle regression                                Number of obs = 19,831
                                                       LR chi2(4)      = 8663.21
                                                       Prob > chi2     = 0.0000
                                                       Pseudo R2      = 0.2166

Log likelihood = -15666.195
```

hours	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
hours						
age	.0008368	.0005341	1.57	0.117	-.00021	.0018836
smoke						
Smoking	-.6431348	.0258509	-24.88	0.000	-.6938016	-.592468
distance	-.0772879	.0079132	-9.77	0.000	-.0927976	-.0617783
single						
Single	.5975111	.016108	37.09	0.000	.5659401	.6290821
_cons	-.0770619	.0254833	-3.02	0.002	-.1270082	-.0271157
selection_ll						
commute	-.2953345	.0624665	-4.73	0.000	-.4177666	-.1729024
whours	.0022974	.0069306	0.33	0.740	-.0112864	.0158811
age	-.0485347	.0006501	-74.65	0.000	-.049809	-.0472604
_cons	2.649945	.0499795	53.02	0.000	2.551987	2.747903
lnsigma						
_cons	-.186917	.0067067	-27.87	0.000	-.200062	-.1737721
/sigma	.8295126	.0055633			.81868	.8404884

What was said previously regarding the interpretation of the effects of the different regressors also holds true for `churdle exponential`. We again use margins to estimate the effect of age on time spent exercising.

```
. margins, dydx(age)
Average marginal effects                                Number of obs = 19,831
Model VCE: OIM
Expression: Conditional mean estimates of dependent variable, predict()
dy/dx wrt: age
```

	Delta-method				[95% conf. interval]	
	dy/dx	std. err.	z	P> z		
age	-.0245582	.0004805	-51.11	0.000	-.0255	-.0236164

With the exponential outcome model of the latent variable, our estimate is that each additional year of age decreases exercise time by about 0.025 hours, or 1.5 minutes.



Stored results

`churdle` stores the following in `e()`:

Scalars

<code>e(N)</code>	number of observations
<code>e(k_eq_model)</code>	number of equations in overall model test
<code>e(df_m)</code>	model degrees of freedom
<code>e(r2_p)</code>	pseudo- R^2
<code>e(chi2)</code>	χ^2
<code>e(ll)</code>	log likelihood
<code>e(ll_0)</code>	log likelihood, constant-only model
<code>e(N_clust)</code>	number of clusters
<code>e(p)</code>	p -value for model test
<code>e(rank)</code>	rank of <code>e(v)</code>
<code>e(converged)</code>	1 if converged, 0 otherwise

Macros

<code>e(cmd)</code>	<code>churdle</code>
<code>e(cmdline)</code>	command as typed
<code>e(depvar)</code>	name of dependent variable
<code>e(estimator)</code>	linear or exponential
<code>e(model)</code>	Linear or Exponential
<code>e(wtype)</code>	weight type
<code>e(wexp)</code>	weight expression
<code>e(title)</code>	title in estimation output
<code>e(clustvar)</code>	name of cluster variable
<code>e(chi2type)</code>	Wald or LR; type of model χ^2 test
<code>e(vce)</code>	<code>vce</code> type specified in <code>vce()</code>
<code>e(vcetype)</code>	title used to label Std. err.
<code>e(opt)</code>	type of optimization
<code>e(which)</code>	max or min; whether optimizer is to perform maximization or minimization
<code>e(technique)</code>	maximization technique
<code>e(properties)</code>	b V
<code>e(predict)</code>	program used to implement <code>predict</code>
<code>e(marginsnotok)</code>	predictions disallowed by <code>margins</code>
<code>e(asbalanced)</code>	factor variables <code>fvset</code> as <code>asbalanced</code>
<code>e(asobserved)</code>	factor variables <code>fvset</code> as <code>asobserved</code>

Matrices

<code>e(b)</code>	coefficient vector
<code>e(Cns)</code>	constraints matrix
<code>e(ilog)</code>	iteration log
<code>e(V)</code>	variance-covariance matrix of the estimators
<code>e(V_modelbased)</code>	model-based variance

Functions

<code>e(sample)</code>	marks estimation sample
------------------------	-------------------------

In addition to the above, the following is stored in `r()`:

Matrices

<code>r(table)</code>	matrix containing the coefficients with their standard errors, test statistics, p -values, and confidence intervals
-----------------------	---

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

Methods and formulas

Let $\ell\ell$ refer to the lower limit and ul to the upper limit. Also let the probabilities of being at these limits be given by

$$\begin{aligned}\Pr(y_i = \ell\ell | \mathbf{z}_i) &= \Phi(\ell\ell - \mathbf{z}'_i \boldsymbol{\gamma}_{\ell\ell}) \\ \Pr(y_i = ul | \mathbf{z}_i) &= \Phi(\mathbf{z}'_i \boldsymbol{\gamma}_{ul} - ul)\end{aligned}$$

where \mathbf{z}_i are the covariates of the selection model for individual i , which may be distinct from the covariates \mathbf{x}_i for the latent model; Φ corresponds to the standard normal cumulative distribution function; $\boldsymbol{\gamma}_{\ell\ell}$ is the parameter vector for the lower-limit selection model; and $\boldsymbol{\gamma}_{ul}$ is the parameter vector for the upper-limit selection model.

Under the assumptions that ν_i has a truncated normal distribution with lower truncation point $\ell\ell - \mathbf{x}'_i \boldsymbol{\beta}$ and upper truncation point $ul - \mathbf{x}'_i \boldsymbol{\beta}$ and has a homoskedastic variance, the log-likelihood function is given by

$$\begin{aligned}\ln \mathbf{L} &= \sum_{i=1}^n (y_i \leq \ell\ell) \log \Phi(\ell\ell - \mathbf{z}'_i \boldsymbol{\gamma}_{\ell\ell}) + (y_i \geq ul) \log \{1 - \Phi(ul - \mathbf{z}'_i \boldsymbol{\gamma}_{ul})\} \\ &\quad + (ul > y_i > \ell\ell) \left[\log \left\{ \Phi(ul - \mathbf{z}'_i \boldsymbol{\gamma}_{ul}) - \Phi(\ell\ell - \mathbf{z}'_i \boldsymbol{\gamma}_{\ell\ell}) \right\} \right] \\ &\quad - (ul > y_i > \ell\ell) \left[\log \left\{ \Phi\left(\frac{ul - \mathbf{x}'_i \boldsymbol{\beta}}{\sigma}\right) - \Phi\left(\frac{\ell\ell - \mathbf{x}'_i \boldsymbol{\beta}}{\sigma}\right) \right\} \right] \\ &\quad + (ul > y_i > \ell\ell) \left[\log \left\{ \phi\left(\frac{y_i - \mathbf{x}'_i \boldsymbol{\beta}}{\sigma}\right) \right\} - \log(\sigma) \right]\end{aligned}$$

Without the homoskedasticity assumption, the heteroskedasticity can be modeled using the form $\sigma^2(\mathbf{w}_i) = \exp(2\mathbf{w}'_i \boldsymbol{\theta})$, where \mathbf{w}_i are the variables that affect the conditional variance of ν_i . The log-likelihood function is obtained by replacing σ with $\exp(\mathbf{w}'_i \boldsymbol{\theta})$.

The log-likelihood function for the exponential model is given by

$$\begin{aligned}\ln \mathbf{L} &= \sum_{i=1}^n (y_i \leq \ell\ell) \log \Phi(\ell\ell - \mathbf{z}'_i \boldsymbol{\gamma}) + (y_i > \ell\ell) \left[\log \{1 - \Phi(\ell\ell - \mathbf{z}'_i \boldsymbol{\gamma})\} \right] \\ &\quad + (y_i > \ell\ell) \left\{ \log \left\{ \phi \left[\log(y_i - \ell\ell) - \mathbf{x}'_i \boldsymbol{\beta} / \sigma \right] \right\} - \log(\sigma) - \log(y_i - \ell\ell) \right\}\end{aligned}$$

Analogous to the linear case, we can model heteroskedasticity by $\sigma^2(\mathbf{w}_i) = \exp(2\mathbf{w}'_i \boldsymbol{\theta})$.

Estimation of both of the aforementioned likelihood functions is done by maximum likelihood.

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Also see

- [R] **churdle postestimation** — Postestimation tools for churdle
- [R] **intreg** — Interval regression
- [R] **tobit** — Tobit regression
- [SVY] **svy estimation** — Estimation commands for survey data
- [U] **20 Estimation and postestimation commands**

Postestimation commands
Methods and formulas

predict
Also see

margins

Remarks and examples

Postestimation commands

The following standard postestimation commands are available after `churdle`:

Command	Description
<code>contrast</code>	contrasts and ANOVA-style joint tests of parameters
<code>estat ic</code>	Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively)
<code>estat summarize</code>	summary statistics for the estimation sample
<code>estat vce</code>	variance–covariance matrix of the estimators (VCE)
<code>estat (svy)</code>	postestimation statistics for survey data
<code>estimates</code>	cataloging estimation results
<code>etable</code>	table of estimation results
* <code>forecast</code>	dynamic forecasts and simulations
* <code>hausman</code>	Hausman's specification test
<code>lincom</code>	point estimates, standard errors, testing, and inference for linear combinations of parameters
* <code>lrtest</code>	likelihood-ratio test
<code>margins</code>	marginal means, predictive margins, marginal effects, and average marginal effects
<code>marginsplot</code>	graph the results from margins (profile plots, interaction plots, etc.)
<code>nlcom</code>	point estimates, standard errors, testing, and inference for nonlinear combinations of parameters
<code>predict</code>	conditional means, residuals, etc.
<code>predictnl</code>	point estimates, standard errors, testing, and inference for generalized predictions
<code>pwcompare</code>	pairwise comparisons of parameters
<code>suest</code>	seemingly unrelated estimation
<code>test</code>	Wald tests of simple and composite linear hypotheses
<code>testnl</code>	Wald tests of nonlinear hypotheses

*`forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results.

predict

Description for predict

`predict` creates a new variable containing predictions such as conditional expectation of *depvar*, residuals, linear predictions, standard errors, and probabilities.

Menu for predict

Statistics > Postestimation

Syntax for predict

```
predict [type] newvar [if] [in] [ , statistic equation(eqno) ]
```

```
predict [type] stub* [if] [in] , scores
```

<i>statistic</i>	Description
------------------	-------------

Main

<code>ystar</code>	conditional expectation of <i>depvar</i> ; the default
<code>residuals</code>	residuals
<code>ystar(a,b)</code>	$E(y_j^*), y_j^* = \max\{a, \min(y_j, b)\}$
<code>xb</code>	linear prediction
<code>stdp</code>	standard error of the linear prediction
<code>pr(a,b)</code>	$\Pr(a < y_j < b)$
<code>e(a,b)</code>	$E(y_j a < y_j < b)$

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

where *a* and *b* may be numbers or variables; *a* missing ($a \geq .$) means $-\infty$, and *b* missing ($b \geq .$) means $+\infty$; see [U] 12.2.1 Missing values. For `churdle exponential`, *b* is . (missing).

Options for predict

Main

`ystar`, the default, calculates the conditional expectation of the dependent variable.

`residuals` calculates the residuals.

`ystar(a,b)` calculates $E(y_j^*)$. *a* and *b* are specified as they are for `pr()`. If *a* and *b* are equal to the lower and upper bounds specified in `churdle`, then $E(y_j^*)$ is equivalent to the predicted value of the dependent variable `ystar`.

`xb` calculates the linear prediction.

`stdp` calculates the standard error of the linear prediction.

`pr(a, b)` calculates $\Pr(a < y_j < b)$, the probability that $y_j | \mathbf{x}_i$ would be observed in the interval (a, b) .

a and b may be specified as numbers or variable names; lb and ub are variable names;

`pr(20, 30)` calculates $\Pr(20 < y_j < 30)$; `pr(lb, ub)` calculates $\Pr(lb < y_j < ub)$;

and `pr(20, ub)` calculates $\Pr(20 < y_j < ub)$.

a missing ($a \geq .$) means 11; `pr(., 30)` calculates $\Pr(11 < y_j < 30)$;

`pr(lb, 30)` calculates $\Pr(11 < y_j < 30)$ in observations for which $lb \geq .$ and calculates $\Pr(lb < y_j < 30)$ elsewhere.

b missing ($b \geq .$) means ∞ ; `pr(20, .)` calculates $\Pr(\infty > y_j > 20)$;

`pr(20, ub)` calculates $\Pr(\infty > y_j > 20)$ in observations for which $ub \geq .$ and calculates $\Pr(ub > y_j > 20)$ elsewhere. For `churdle linear`, ul is ∞ .

`e(a, b)` calculates $E(y_j | a < y_j < b)$, the expected value of $y_j | \mathbf{x}_j$ conditional on $y_j | \mathbf{x}_j$ being in the interval (a, b) , meaning that $y_j | \mathbf{x}_j$ is bounded. a and b are specified as they are for `pr()`.

`equation(eqno)` specifies the equation for which predictions are to be made for the `xb` and `stdp` options.

`equation()` should contain either one equation name or one of #1, #2, ... with #1 meaning the first equation, #2 meaning the second equation, etc.

If you do not specify `equation()`, the results are the same as if you specified `equation(#1)`.

`scores` calculates the equation-level score variables. If you specify one new variable, the scores for the latent-variable equation are computed. If you specify a variable list, the scores for each equation are calculated. The following scores may be obtained:

the first new variable will contain $\partial \ln L / \partial (\mathbf{x}_j \boldsymbol{\beta})$,

the second new variable will contain $\partial \ln L / \partial (\mathbf{z}_j \boldsymbol{\gamma}_{ul})$,

the third new variable will contain $\partial \ln L / \partial (\mathbf{z}_j \boldsymbol{\gamma}_{ul})$,

the fourth new variable will contain $\partial \ln L / \partial (\boldsymbol{\sigma})$,

the fifth new variable will contain $\partial \ln L / \partial (\boldsymbol{\sigma}_{ul})$, and

the sixth new variable will contain $\partial \ln L / \partial (\boldsymbol{\sigma}_{ul})$.

margins

Description for margins

`margins` estimates margins of response for conditional expectations, linear predictions, and probabilities.

Menu for margins

Statistics > Postestimation

Syntax for margins

```
margins [marginlist] [, options ]
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options ]
```

<i>statistic</i>	Description
Main	
<code>ystar</code>	conditional expectation of <i>depvar</i> ; the default
<code>ystar(<i>a</i>, <i>b</i>)</code>	$E(y_j^*), y_j^* = \max\{a, \min(y_j, b)\}$; for churdle exponential <i>b</i> is .
<code>xb</code>	linear prediction
<code>pr(<i>a</i>, <i>b</i>)</code>	$\Pr(a < y_j < b)$; for churdle exponential <i>b</i> is .
<code>e(<i>a</i>, <i>b</i>)</code>	$E(y_j a < y_j < b)$; for churdle exponential <i>b</i> is .
<code>residuals</code>	not allowed with margins
<code>stdp</code>	not allowed with margins

Statistics not allowed with margins are functions of stochastic quantities other than `e(b)`.

For the full syntax, see [R] [margins](#).

Remarks and examples

▷ Example 1: Predictions for depvar

Below, we use the parameters estimated in [example 1](#) of [R] [churdle](#) to calculate the average hours exercised given the covariates.

```
. use https://www.stata-press.com/data/r19/fitness
(Fictional fitness data)
. churdle linear hours age i.smoke distance i.single,
> select(commute whours age) ll(0)
(output omitted)
. predict hourshat
(statistic ystar assumed)
```

We might also be interested in estimating the average number of hours exercised given that an individual exercises. Below we estimate this quantity and compare it with the predicted and true values of the dependent variable for all the observations.

```
. predict exercises, e(0,.)
. summarize hours hourshat exercises
```

Variable	Obs	Mean	Std. dev.	Min	Max
hours	19,831	.8800172	1.051221	0	5.308835
hourshat	19,831	.8786302	.4915214	.0754708	1.904694
exercises	19,831	1.580729	.3998335	.5630298	2.079012

As expected, we observe that the sample-average predictions are higher for those who exercise.



▷ Example 2: Marginal effects

Suppose we want to study whether single individuals exercise more on average than married individuals. Below, we use margins to estimate the average effect of being single on hours spent exercising in the population.

```
. margins, dydx(1.single)
Average marginal effects
Model VCE: OIM
Expression: Conditional mean estimates of dependent variable, predict()
dy/dx wrt: 1.single
```

	Delta-method				
	dy/dx	std. err.	z	P> z	[95% conf. interval]
single	.3858462	.0091398	42.22	0.000	.3679324 .4037599

Note: dy/dx for factor levels is the discrete change from the base level.

The average effect of moving each individual from not single to single is an increase in exercise of 0.39 hours, or about 23 minutes.



The statistics $pr(a, b)$, $e(a, b)$, and $ystar(a, b)$ produce counterfactual predictions.

These statistics predict what would be observed if the limits ll and ul were the specified a and b and the estimated parameters did not change, even though the parameters of the model were estimated using the limits ll and ul .

For example, suppose we model contributions to a retirement plan in a world where the government requires a minimum contribution of 2% so $ll = 2$. After estimating the model parameters, we could predict the average contribution, given the covariates, when the government raises the minimum contribution to 3% with the statistic $ystar(3, .)$.

Methods and formulas

Let $\ell\ell$ refer to the lower limit and ul to the upper limit. Also let the probabilities of being at these limits be given by

$$\begin{aligned}\Pr(y_i = \ell\ell | \mathbf{z}_i) &= \Phi(\ell\ell - \mathbf{z}'_i \boldsymbol{\gamma}_{\ell\ell}) \\ \Pr(y_i = ul | \mathbf{z}_i) &= \Phi(\mathbf{z}'_i \boldsymbol{\gamma}_{ul} - ul)\end{aligned}$$

where \mathbf{z}_i are the covariates of the selection model for individual i , which may be distinct from the covariates \mathbf{x}_i for the outcome model; Φ is the standard normal cumulative distribution function; $\boldsymbol{\gamma}_{\ell\ell}$ is the parameter vector for the lower-limit selection model; and $\boldsymbol{\gamma}_{ul}$ is the parameter vector for the upper-limit selection model.

We will limit the exposition below to the case with a lower and an upper limit.

In `churdle linear`, `ystar` is equivalent to $E(y_i | \mathbf{x}_i)$ and is given by

$$\begin{aligned}E(y_i | \mathbf{x}_i, \mathbf{z}_i) &= \Phi(\mathbf{z}'_i \boldsymbol{\gamma}_{ul} - ul) ul + \Phi(\ell\ell - \mathbf{z}'_i \boldsymbol{\gamma}_{\ell\ell}) \ell\ell \\ &+ \{ \Phi(ul - \mathbf{z}'_i \boldsymbol{\gamma}) - \Phi(\ell\ell - \mathbf{z}'_i \boldsymbol{\gamma}) \} \left\{ \mathbf{x}'_i \boldsymbol{\beta} + \sigma \frac{\phi\left(\frac{\ell\ell - \mathbf{x}'_i \boldsymbol{\beta}}{\sigma}\right) - \phi\left(\frac{ul - \mathbf{x}'_i \boldsymbol{\beta}}{\sigma}\right)}{\Phi\left(\frac{ul - \mathbf{x}'_i \boldsymbol{\beta}}{\sigma}\right) - \Phi\left(\frac{\ell\ell - \mathbf{x}'_i \boldsymbol{\beta}}{\sigma}\right)} \right\}\end{aligned}$$

`pr(a, b)` is given by

$$\Pr(a < y_i < b | \mathbf{z}_i) = \Phi(b - \mathbf{z}'_i \boldsymbol{\beta}) - \Phi(a - \mathbf{z}'_i \boldsymbol{\beta})$$

`e(a, b)` is given by

$$E(a < y_i < b | \mathbf{x}_i) = \mathbf{x}'_i \boldsymbol{\beta} + \sigma \frac{\phi\left(\frac{a - \mathbf{x}'_i \boldsymbol{\beta}}{\sigma}\right) - \phi\left(\frac{b - \mathbf{x}'_i \boldsymbol{\beta}}{\sigma}\right)}{\Phi\left(\frac{b - \mathbf{x}'_i \boldsymbol{\beta}}{\sigma}\right) - \Phi\left(\frac{a - \mathbf{x}'_i \boldsymbol{\beta}}{\sigma}\right)}$$

and `ystar(a, b)` is given by

$$\begin{aligned}E(y^*) &= \Phi(\mathbf{z}'_i \boldsymbol{\gamma}_{ul} - b) b + \Phi(a - \mathbf{z}'_i \boldsymbol{\gamma}_{\ell\ell}) a \\ &+ \Pr(a < y_i < b | \mathbf{x}_i) E(a < y_i < b | \mathbf{x}_i)\end{aligned}$$

For churdle exponential, ystar is equivalent to

$$E(y_i|\mathbf{x}_i) = \Phi(\ell\ell - \mathbf{z}'_i\gamma_{\ell\ell})\ell\ell + \{1 - \Phi(\ell\ell - \mathbf{z}'_i\gamma)\} \exp(\mathbf{x}'_i\beta + \sigma^2/2) \left[\frac{1 - \Phi\left\{\frac{\ln(\ell\ell) - \mathbf{x}'_i\beta}{\sigma} - \sigma\right\}}{1 - \Phi\left\{\frac{\ln(\ell\ell) - \mathbf{x}'_i\beta}{\sigma}\right\}} \right]$$

$p(a, \cdot)$ is given by

$$\Pr(a < y_i|\mathbf{x}_i) = 1 - \Phi(a - \mathbf{z}'_i\gamma_{\ell\ell})$$

$e(a, \cdot)$ is given by

$$E(a < y_i|\mathbf{x}_i) = \exp(\mathbf{x}'_i\beta + \sigma^2/2) \left[\frac{1 - \Phi\left\{\frac{\ln(a) - \mathbf{x}'_i\beta}{\sigma} - \sigma\right\}}{1 - \Phi\left\{\frac{\ln(a) - \mathbf{x}'_i\beta}{\sigma}\right\}} \right]$$

and ystar(a, \cdot) is given by

$$E(y^*) = a\Phi(a - \mathbf{z}'_i\gamma_{\ell\ell}) + \Pr(a < y_i|\mathbf{x}_i) E(a < y_i|\mathbf{x}_i)$$

Also see

[R] **churdle** — Cragg hurdle regression

[U] **20 Estimation and postestimation commands**

[Description](#)[Options](#)[Acknowledgment](#)[Quick start](#)[Remarks and examples](#)[References](#)[Menu](#)[Stored results](#)[Also see](#)[Syntax](#)[Methods and formulas](#)

Description

`ci` computes confidence intervals for population means, proportions, variances, and standard deviations.

`cii` is the immediate form of `ci`; see [\[U\] 19 Immediate commands](#) for a general discussion of immediate commands.

Quick start

Confidence intervals for means of normally distributed variables `v1`, `v2`, and `v3`

```
ci means v1-v3
```

Confidence interval for mean of Poisson-distributed variable `v4`

```
ci means v4, poisson
```

Confidence interval for rate of `v4` with total exposure recorded in `v5`

```
ci means v4, poisson exposure(v5)
```

Confidence interval for proportion of binary variable `v6`

```
ci proportions v6
```

Confidence intervals for variances of `v1`, `v2`, and `v3`

```
ci variances v1-v3
```

Same as above, but Bonett confidence intervals are produced

```
ci variances v1-v3, bonett
```

90% Bonett confidence intervals for standard deviations of `v1`, `v2`, and `v3`

```
ci variances v1-v3, sd bonett level(90)
```

Confidence interval for a mean based on a sample with 85 observations, a sample mean of 10, and a standard deviation of 3

```
cii means 85 10 3
```

90% confidence interval for rate from a sample with 4,379 deaths over 11,394 person-years

```
cii means 11394 4379, poisson level(90)
```

Agresti–Coull confidence interval for proportion based on a sample with 2,377 observations and 136 successes

```
cii proportions 2377 136, agresti
```

Bonett confidence interval for variance based on a sample with 20 observations, sample variance of 9, and estimated kurtosis of 1.8

```
cii variances 20 9 1.8, bonett
```

Same as above, but with confidence interval for standard deviation

```
cii variances 20 3 1.8, sd bonett
```

Menu

ci

Statistics > Summaries, tables, and tests > Summary and descriptive statistics > Confidence intervals

cii for a normal mean

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cii for a Poisson mean

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cii for a proportion

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cii for a variance

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cii for a standard deviation

Statistics > Summaries, tables, and tests > Summary and descriptive statistics > Standard deviation CI calculator

Syntax

Confidence intervals for means, normal distribution

```
ci means [varlist] [if] [in] [weight] [, options]
```

```
cii means #obs #mean #sd [, level(#)]
```

Confidence intervals for means, Poisson distribution

```
ci means [varlist] [if] [in] [weight], poisson [exposure(varname) options]
```

```
cii means #exposure #events, poisson [level(#)]
```

Confidence intervals for proportions

```
ci proportions [varlist] [if] [in] [weight] [, prop_options options]
```

```
cii proportions #obs #succ [, prop_options level(#)]
```

Confidence intervals for variances

```
ci variances [varlist] [if] [in] [weight] [, bonett options]
```

```
cii variances #obs #variance [, level(#)]
```

```
cii variances #obs #variance #kurtosis, bonett [level(#)]
```

Confidence intervals for standard deviations

```
ci variances [varlist] [if] [in] [weight], sd [bonett options]
```

```
cii variances #obs #sd, sd [level(#)]
```

```
cii variances #obs #sd #kurtosis, sd bonett [level(#)]
```

#_{obs} must be a positive integer. #_{exposure}, #_{sd}, and #_{variance} must be a positive number. #_{succ} and #_{events} must be a nonnegative integer or between 0 and 1. If the number is between 0 and 1, Stata interprets it as the fraction of successes or events and converts it to an integer number representing the number of successes or events. The computation then proceeds as if two integers had been specified. If the bonett option is specified, you must additionally specify #_{kurtosis} with cii variances.

<i>prop_options</i>	Description
<code>exact</code>	calculate exact confidence intervals; the default
<code>wald</code>	calculate Wald confidence intervals
<code>wilson</code>	calculate Wilson confidence intervals
<code>agresti</code>	calculate Agresti–Coull confidence intervals
<code>jeffreys</code>	calculate Jeffreys confidence intervals

<i>options</i>	Description
<code>level(#)</code>	set confidence level; default is <code>level(95)</code>
<code>separator(#)</code>	draw separator line after every # variables; default is <code>separator(5)</code>
<code>total</code>	add output for all groups combined (for use with <code>by</code> only)

`by`, `collect`, and `statsby` are allowed with `ci`, and `collect` is allowed with `cii`; see [U] 11.1.10 Prefix commands. `aweight`s are allowed with `ci means` for normal data, and `fweight`s are allowed with all `ci` subcommands; see [U] 11.1.6 weight.

Options

Options are presented under the following headings:

Options for ci and cii means
Options for ci and cii proportions
Options for ci and cii variances

Options for ci and cii means

Main

`poisson` specifies that the variables (or numbers for `cii`) are Poisson-distributed counts; exact Poisson confidence intervals will be calculated. By default, confidence intervals for means are calculated based on a normal distribution.

`exposure(varname)` is used only with `poisson`. You do not need to specify `poisson` if you specify `exposure()`; `poisson` is assumed. *varname* contains the total exposure (typically a time or an area) during which the number of events recorded in *varlist* was observed.

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [R] level.

`separator(#)` specifies how often separation lines should be inserted into the output. The default is `separator(5)`, meaning that a line is drawn after every five variables. `separator(10)` would draw the line after every 10 variables. `separator(0)` suppresses the separation line.

`total` is used with the `by` prefix. It requests that in addition to output for each `by`-group, output be added for all groups combined.

Options for ci and cii proportions

Main

`exact`, `wald`, `wilson`, `agresti`, and `jeffreys` specify how binomial confidence intervals are to be calculated.

`exact` is the default and specifies exact (also known in the literature as Clopper–Pearson [1934]) binomial confidence intervals.

`wald` specifies calculation of Wald confidence intervals.

`wilson` specifies calculation of Wilson confidence intervals.

`agresti` specifies calculation of Agresti–Coull confidence intervals.

`jeffreys` specifies calculation of Jeffreys confidence intervals.

See [Brown, Cai, and DasGupta \(2001\)](#) for a discussion and comparison of the different binomial confidence intervals.

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [\[R\] level](#).

`separator(#)` specifies how often separation lines should be inserted into the output. The default is `separator(5)`, meaning that a line is drawn after every five variables. `separator(10)` would draw the line after every 10 variables. `separator(0)` suppresses the separation line.

`total` is used with the `by` prefix. It requests that in addition to output for each by-group, output be added for all groups combined.

Options for ci and cii variances

Main

`sd` specifies that confidence intervals for standard deviations be calculated. The default is to compute confidence intervals for variances.

`bonett` specifies that Bonett confidence intervals be calculated. The default is to compute normal-based confidence intervals, which assume normality for the data.

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [\[R\] level](#).

`separator(#)` specifies how often separation lines should be inserted into the output. The default is `separator(5)`, meaning that a line is drawn after every five variables. `separator(10)` would draw the line after every 10 variables. `separator(0)` suppresses the separation line.

`total` is used with the `by` prefix. It requests that in addition to output for each by-group, output be added for all groups combined.

Remarks and examples

Remarks are presented under the following headings:

Confidence intervals for means
Normal-based confidence intervals
Poisson confidence intervals
Confidence intervals for proportions
Confidence intervals for variances
Immediate form

Confidence intervals for means

`ci means` computes a confidence interval for the population mean for each of the variables in *varlist*.

Normal-based confidence intervals

▷ Example 1: Normal-based confidence intervals

Without the `poisson` option, `ci means` produces normal-based confidence intervals that are correct if the variable is normally distributed and asymptotically correct for all other distributions satisfying the conditions of the central limit theorem.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. ci means mpg price
```

Variable	Obs	Mean	Std. err.	[95% conf. interval]	
mpg	74	21.2973	.6725511	19.9569	22.63769
price	74	6165.257	342.8719	5481.914	6848.6

The standard error of the mean of `mpg` is 0.67, and the 95% confidence interval is [19.96, 22.64]. We can obtain wider confidence intervals, 99%, by typing

```
. ci means mpg price, level(99)
```

Variable	Obs	Mean	Std. err.	[99% conf. interval]	
mpg	74	21.2973	.6725511	19.51849	23.07611
price	74	6165.257	342.8719	5258.405	7072.108

◀

▷ Example 2: The `by` prefix

The `by` prefix breaks out the confidence intervals according to `by-group`; `total` adds an overall summary. For instance,

```
. by foreign: ci means mpg, total
```

```
-> foreign = Domestic
```

Variable	Obs	Mean	Std. err.	[95% conf. interval]	
mpg	52	19.82692	.657777	18.50638	21.14747

```
-> foreign = Foreign
```

Variable	Obs	Mean	Std. err.	[95% conf. interval]	
mpg	22	24.77273	1.40951	21.84149	27.70396

```
-> Total
```

Variable	Obs	Mean	Std. err.	[95% conf. interval]	
mpg	74	21.2973	.6725511	19.9569	22.63769

◀

▷ Example 3: Controlling the format

You can control the formatting of the numbers in the output by specifying a display format for the variable; see [U] 12.5 **Formats: Controlling how data are displayed**. For instance,

```
. format mpg %9.2f
```

```
. ci means mpg
```

Variable	Obs	Mean	Std. err.	[95% conf. interval]	
mpg	74	21.30	0.67	19.96	22.64

◀

Poisson confidence intervals

If you specify the `poisson` option, `ci means` assumes count data and computes exact Poisson confidence intervals.

▷ Example 4: Poisson confidence intervals

We have data on the number of bacterial colonies on a Petri dish. The dish has been divided into 36 small squares, and the number of colonies in each square has been counted. Each observation in our dataset represents a square on the dish. The variable `count` records the number of colonies in each square counted, which varies from 0 to 5.

```
. use https://www.stata-press.com/data/r19/petri, clear
```

```
. ci means count, poisson
```

Variable	Exposure	Mean	Std. err.	Poisson exact [95% conf. interval]	
count	36	2.333333	.2545875	1.861158	2.888825

`ci` reports that the average number of colonies per square is 2.33. If the expected number of colonies per square were as low as 1.86, the probability of observing 2.33 or more colonies per square would be 2.5%. If the expected number were as large as 2.89, the probability of observing 2.33 or fewer colonies per square would be 2.5%.

◀

▷ Example 5: Option exposure()

The number of “observations”—how finely the Petri dish is divided—makes no difference. The Poisson distribution is a function only of the count. In [example 4](#), we observed a total of $2.33 \times 36 = 84$ colonies and a confidence interval of $[1.86 \times 36, 2.89 \times 36] = [67, 104]$. We would obtain the same $[67, 104]$ confidence interval if our dish were divided into, say, 49 squares rather than 36.

For the counts, it is not even important that all the squares be of the same size. For rates, however, such differences do matter but in an easy-to-calculate way. Rates are obtained from counts by dividing by exposure, which is typically a number multiplied by either time or an area. For our Petri dishes, we divide by an area to obtain a rate, but if our example were cast in terms of being infected by a disease, we might divide by person-years to obtain the rate. Rates are convenient because they are easier to compare: we might have 2.3 colonies per square inch or 0.0005 infections per person-year.

So let’s assume that we wish to obtain the number of colonies per square inch and, moreover, that not all the “squares” on our dish are of equal size. We have a variable called `area` that records the area of each square:

```
. ci means count, exposure(area)
```

Variable	Exposure	Mean	Std. err.	Poisson exact	
				[95% conf. interval]	
count	3	28	3.055051	22.3339	34.66591

The rates are now in more familiar terms. In our sample, there are 28 colonies per square inch, and the 95% confidence interval is $[22.3, 34.7]$. When we did not specify `exposure()`, `ci means` with the `poisson` option assumed that each observation contributed 1 to exposure.

◀

□ Technical note

If there were no colonies on our dish, `ci means` with the `poisson` option would calculate a one-sided confidence interval:

```
. use https://www.stata-press.com/data/r19/petrinone
. ci means count, poisson
```

Variable	Exposure	Mean	Std. err.	Poisson exact	
				[95% conf. interval]	
count	36	0	0	0	.1024689*

(*) one-sided, 97.5% confidence interval

□

Confidence intervals for proportions

The `ci proportions` command assumes binary (0/1) data and computes binomial confidence intervals.

► Example 6: Exact binomial (Clopper–Pearson) confidence interval

We have data on employees, including a variable marking whether the employee was promoted last year.

```
. use https://www.stata-press.com/data/r19/promo
. ci proportions promoted
```

Variable	Obs	Proportion	Std. err.	Binomial exact [95% conf. interval]	
promoted	20	.1	.067082	.0123485	.3169827

The exact binomial, also known as the Clopper–Pearson (1934) interval, is computed by default.

Nominally, the interpretation of a 95% confidence interval is that under repeated samples or experiments, 95% of the resultant intervals would contain the unknown parameter in question. However, for binomial data, the actual coverage probability, regardless of method, usually differs from that interpretation. This result occurs because of the discreteness of the binomial distribution, which produces only a finite set of outcomes, meaning that coverage probabilities are subject to discrete jumps and that the exact nominal level cannot always be achieved. Therefore, the term “exact confidence interval” refers to its being derived from the binomial distribution, the distribution exactly generating the data, rather than resulting in exactly the nominal coverage.

For the Clopper–Pearson interval, the actual coverage probability is guaranteed to be greater than or equal to the nominal confidence level, here 95%. Because of the way it is calculated—see *Methods and formulas*—it may also be interpreted as follows: If the true probability of being promoted were 0.012, the chances of observing a result as extreme or more extreme than the result observed ($20 \times 0.1 = 2$ or more promotions) would be 2.5%. If the true probability of being promoted were 0.317, the chances of observing a result as extreme or more extreme than the result observed (two or fewer promotions) would be 2.5%.

◀

► Example 7: Other confidence intervals

The Clopper–Pearson interval is desirable because it guarantees nominal coverage; however, by dropping this restriction, you may obtain accurate intervals that are not as conservative. In this vein, you might opt for the Wilson (1927) interval,

```
. ci proportions promoted, wilson
```

Variable	Obs	Proportion	Std. err.	Wilson [95% conf. interval]	
promoted	20	.1	.067082	.0278665	.3010336

the Agresti–Coull (1998) interval,

```
. ci proportions promoted, agresti
```

Variable	Obs	Proportion	Std. err.	Agresti–Coull [95% conf. interval]	
promoted	20	.1	.067082	.0156562	.3132439

or the Bayesian-derived Jeffreys interval (Brown, Cai, and DasGupta 2001),

```
. ci proportions promoted, jeffreys
```

Variable	Obs	Proportion	Std. err.	Jeffreys [95% conf. interval]	
promoted	20	.1	.067082	.0213725	.2838533

Picking the best interval is a matter of balancing accuracy (coverage) against precision (average interval length) and depends on sample size and success probability. Brown, Cai, and DasGupta (2001) recommend the Wilson or Jeffreys interval for small sample sizes (≤ 40) yet favor the Agresti–Coull interval for its simplicity, decent performance for sample sizes less than or equal to 40, and performance comparable to Wilson or Jeffreys for sample sizes greater than 40. They also deem the Clopper–Pearson interval to be “wastefully conservative and [...] not a good choice for practical use”, unless of course one requires, at a minimum, the nominal coverage level.

Finally, the binomial Wald confidence interval is obtained by specifying the `wald` option. The Wald interval is the one taught in most introductory statistics courses and, for the above, is simply, for level $1 - \alpha$, $\text{Proportion} \pm z_{\alpha/2}(\text{Std. err.})$, where $z_{\alpha/2}$ is the $1 - \alpha/2$ quantile of the standard normal. Because its overall poor performance makes it impractical, the Wald interval is available mainly for pedagogical purposes. The binomial Wald interval is also similar to the interval produced by treating binary data as normal data and using `ci means`, with two exceptions. First, the calculation of the standard error in `ci proportions` uses denominator n rather than $n - 1$, used for normal data in `ci means`. Second, confidence intervals for normal data are based on the t distribution rather than the standard normal. Of course, both discrepancies vanish as sample size increases.

◀

□ Technical note

Let’s repeat [example 6](#), but this time with data in which there are no promotions over the observed period:

```
. use https://www.stata-press.com/data/r19/promonone
. ci proportions promoted
```

Variable	Obs	Proportion	Std. err.	Binomial exact [95% conf. interval]	
promoted	20	0	0	0	.1684335*

(*) one-sided, 97.5% confidence interval

The confidence interval is $[0, 0.168]$, and this is the confidence interval that most books publish. It is not, however, a true 95% confidence interval because the lower tail has vanished. As Stata notes, it is a one-sided, 97.5% confidence interval. If you wanted to put 5% in the right tail, you could type `ci proportions promoted, level(90)`.

□

□ Technical note

`ci proportions` ignores any variables that do not take on the values 0 and 1 exclusively. For instance, with our automobile dataset,

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. ci proportions mpg foreign
```

Variable	Obs	Proportion	Std. err.	Binomial exact [95% conf. interval]	
foreign	74	.2972973	.0531331	.196584	.4148353

Note: The results are produced only for binary (0/1) variables.

We also requested the confidence interval for `mpg`, but Stata ignored us. It does that so you can type `ci proportions` and obtain correct confidence intervals for all the variables that are 0/1 in your data. □

Confidence intervals for variances

The `ci variances` command computes confidence intervals for the variances or, if the `sd` option is specified, for the standard deviations. The default is a normal-based confidence interval that assumes the data are normal and uses a χ^2 distribution to construct the confidence intervals. If normality is suspect, you may consider using the `bonett` option to compute [Bonett \(2006\)](#) confidence intervals, which are more robust to nonnormality.

▷ Example 8: Normal-based confidence intervals

So far, we have restricted our attention to confidence intervals for means and proportions. Typically, when people think of statistical inference, they usually have in mind inferences concerning population means. However, the population parameter of interest will vary from one situation to another. In many scenarios, the population variance is as important as the population mean. For example, in a quality control study, a machine that fills 16-ounce canned peas is investigated at regular time intervals. A random sample of $n = 8$ containers is selected every hour. Ideally, the amount of peas in a can should vary only slightly about the 16-ounce value. If the variance was large, then a large proportion of cans would be either underfilled, thus cheating the customer, or overfilled, thus resulting in economic loss to the manufacturing company. Suppose that the weights of 16-ounce cans filled by the machine are normally distributed. The acceptable variability in the weights is expected to be 0.09 with the respective standard deviation of 0.3 ounces. To monitor the machine's performance, we can compute confidence intervals for the variance of the weights of cans:

```
. use https://www.stata-press.com/data/r19/peas_normdist
(Weights of canned peas, normal distribution)
. ci variances weight
```

Variable	Obs	Variance	[95% conf. interval]	
weight	8	.3888409	.1699823	1.610708

The command reports the sample estimate of the variance of 0.39 with the 95% confidence interval of [0.17, 1.61].

Instead of the variance, we may be interested in confidence intervals for the standard deviation. We can specify the `sd` option to compute such confidence intervals.

```
. ci variances weight, sd
```

Variable	Obs	Std. dev.	[95% conf. interval]	
weight	8	.6235711	.4122891	1.269137

The 95% confidence interval for the standard deviation of the weights is [0.41, 1.27]. Because the desired value for the standard deviation, 0.3 ounces, falls outside the interval, the machine may require some tuning.

◀

Confidence intervals in [example 8](#) are based on the assumption that the random sample is selected from a population having a normal distribution. Nonnormality of the population distribution, in the form of skewness or heavy tails, can have a drastic impact on the asymptotic coverage probability of the normal-based confidence intervals. This is the case even for distributions that are similar to normal. [Scheffé \(1959, 336\)](#) showed that the normal-based interval has an asymptotic coverage probability of about 0.76, 0.63, 0.60, and 0.51 for the logistic, t with seven degrees of freedom, Laplace, and t with five degrees of freedom distributions, respectively. [Miller \(1997, 264\)](#) describes this situation as “catastrophic” because these distributions are symmetric and not easily distinguishable from a normal distribution unless the sample size is large. Hence, it is judicious to evaluate the normality of the data prior to constructing the normal-based confidence intervals for variances or standard deviations.

[Bonett \(2006\)](#) proposed a confidence interval that performs well in small samples under moderate departures from normality. His interval performs only slightly worse than the exact normal-based confidence interval when sampling from a normal distribution. A larger sample size provides Bonett confidence intervals with greater protection against nonnormality.

▶ Example 9: Bonett confidence interval for normal data

We will repeat [example 8](#) and construct a Bonett confidence interval for the standard deviation by specifying the `bonett` option. The results are similar, and both examples lead to the same inferential conclusion.

```
. ci variances weight, sd bonett
```

Variable	Obs	Std. dev.	Bonett	
			[95% conf. interval]	
weight	8	.6235711	.3997041	1.288498

The Bonett confidence interval is wider than the normal-based confidence interval in [example 8](#). For normal data, [Bonett \(2006\)](#) suggested that if Bonett confidence interval is used for a sample of size $n + 3$, then its average width will be about the same as the average width of the normal-based confidence interval from a sample size of n . Sampling three more observations may be a small price to pay because Bonett confidence intervals perform substantially better than the normal-based confidence intervals for nonnormal data.

◀

▶ Example 10: Bonett confidence interval for nonnormal data

The following data have been generated from a t distribution with five degrees of freedom to illustrate the effect of wrongfully using the normal-based confidence interval when the data-generating process is not normal.

```
. use https://www.stata-press.com/data/r19/peas_tdist
(Weights of canned peas, t distribution)
. ci variances weight, sd
```

Variable	Obs	Std. dev.	[95% conf. interval]	
weight	8	2.226558	1.472143	4.531652

The standard deviation of a t distribution with five degrees of freedom is $\sqrt{5/3} \approx 1.29$ and falls outside the confidence interval limits. If we suspect that data may not be normal, the Bonett confidence interval is typically a better choice:

```
. ci variances weight, sd bonett
```

Variable	Obs	Std. dev.	Bonett	
			[95% conf. interval]	
weight	8	2.226558	1.137505	5.772519

The value 1.29 is within the limits of the Bonett confidence interval [1.14, 5.77]



Immediate form

So far, we computed confidence intervals for various parameters using data in memory. We can also compute confidence intervals using only data summaries, without any data in memory. Each of the considered `ci` commands has an immediate `cii` version that computes the respective confidence intervals using data summaries.

▶ Example 11: Confidence interval for a normal mean

We are reading a soon-to-be-published paper by a colleague. In it is a table showing the number of observations, mean, and standard deviation of the 1980 median family income for the Northeast and West. We correctly think that the paper would be much improved if it included the confidence intervals. The paper claims that for 166 cities in the Northeast, the average of median family income is \$19,509 with a standard deviation of \$4,379:

For the Northeast:

```
. cii means 166 19509 4379
```

Variable	Obs	Mean	Std. err.	[95% conf. interval]	
	166	19509	339.8763	18837.93	20180.07

For the West:

```
. cii means 256 22557 5003
```

Variable	Obs	Mean	Std. err.	[95% conf. interval]	
	256	22557	312.6875	21941.22	23172.78



▷ Example 12: Confidence interval for a Poisson mean

The number of reported traffic accidents in Santa Monica over a 24-hour period is 27. We need know nothing else to compute a confidence interval for the mean number of accidents for a day:

```
. cii means 1 27, poisson
```

Variable	Exposure	Mean	Std. err.	Poisson exact [95% conf. interval]	
	1	27	5.196152	17.79317	39.28358



▷ Example 13: Confidence interval for a proportion

We flip a coin 10 times, and it comes up heads only once. We are shocked and decide to obtain a 99% confidence interval for this coin:

```
. cii proportions 10 1, level(99)
```

Variable	Obs	Proportion	Std. err.	Binomial exact [99% conf. interval]	
	10	.1	.0948683	.0005011	.5442871



▷ Example 14: Confidence interval for a variance

A company fills 32-ounce tomato juice jars with a quantity of juice having a normal distribution with a claimed variance not exceeding 0.2. A random sample of 15 jars is collected to evaluate this claim. The sample variance is 0.5:

```
. cii variances 15 0.5
```

Variable	Obs	Variance	[95% conf. interval]	
	15	.5	.2680047	1.243621

Because the advertised value of 0.2 does not fall inside the confidence interval, the company is allowing too much variation in the amount of tomato juice per jar.



▷ Example 15: Confidence interval for a standard deviation

Suppose the director of statistical development at a statistical software company is a big soccer fan and requires all developers to play on the company team in the city's local soccer league. Ten developers are randomly selected to participate in the game. To ensure an advantage over other teams, the director requires each of the 10 developers to cover 6 miles on average each game. Being merciful, she will tolerate a standard deviation of 0.3 miles across different players, arguing that this will keep the team's performance consistent. The distance covered by each player is measured using a pedometer. At the end of the game, the sample standard deviation of the distances covered by the 10 players was 0.56 miles:

```
. cii variances 10 0.56, sd
```

Variable	Obs	Std. dev.	[95% conf. interval]	
	10	.56	.3851877	1.022342

Because the confidence interval does not include the designated value for the standard deviation, 0.3 miles, it is clear the team is not meeting standards, and an unpleasant meeting is planned. ◀

▷ Example 16: Confidence interval for a standard deviation of nonnormal data

Continuing with [example 15](#), a clever statistician points out that distances covered by company players in a soccer match do not follow the normal distribution because some players, mostly econometricians, walk on the field, while others, mostly statisticians, do all the running. Therefore, the normal-based confidence interval (which assumes normality) is not valid. Instead, we should use the Bonett confidence interval, which additionally requires an estimate of kurtosis; see [Methods and formulas](#). If kurtosis is estimated to be 5, we would obtain the following:

```
. cii variances 10 0.56 5, sd bonett
```

Variable	Obs	Std. dev.	Bonett	
			[95% conf. interval]	
	10	.56	.2689449	1.45029

The Bonett confidence interval now contains the specified value for the standard deviation, 0.3 miles. The director of statistics concludes that overall team performance is acceptable. An uncomfortable meeting is still planned but for a smaller group. ◀

Stored results

`ci means` and `cii means` store the following in `r()`:

Scalars

<code>r(N)</code>	number of observations or, if <code>poisson</code> is specified, exposure
<code>r(mean)</code>	mean
<code>r(se)</code>	estimate of standard error
<code>r(lb)</code>	lower bound of confidence interval
<code>r(ub)</code>	upper bound of confidence interval
<code>r(level)</code>	confidence level of confidence interval

Macros

<code>r(citype)</code>	normal or poisson; type of confidence interval
<code>r(exposure)</code>	name of exposure variable with poisson

`ci proportions` and `cii proportions` store the following in `r()`:

Scalars

<code>r(N)</code>	number of observations
<code>r(proportion)</code>	proportion
<code>r(se)</code>	estimate of standard error
<code>r(lb)</code>	lower bound of confidence interval
<code>r(ub)</code>	upper bound of confidence interval
<code>r(level)</code>	confidence level of confidence interval

Macros

<code>r(citype)</code>	exact, wald, wilson, agresti, or jeffreys; type of confidence interval
------------------------	--

`ci variances` and `cii variances` store the following in `r()`:

Scalars

<code>r(N)</code>	number of observations
<code>r(Var)</code>	variance

r(sd)	standard deviation, if sd is specified
r(kurtosis)	kurtosis, only if bonett is specified
r(lb)	lower bound of confidence interval
r(ub)	upper bound of confidence interval
r(level)	confidence level of confidence interval
Macros	
r(citype)	normal or bonett, type of confidence interval

Methods and formulas

Methods and formulas are presented under the following headings:

Normal mean
Poisson mean
Binomial proportion
Variance and standard deviation

Normal mean

Define n , \bar{x} , and s^2 as, respectively, the number of observations, (weighted) average, and (unbiased) estimated variance of the variable in question; see [R] [summarize](#).

The standard error of the mean, s_{μ} , is defined as $\sqrt{s^2/n}$.

Let α be $1-l/100$, where l is the confidence level specified by the user in the `level()` option. Define $t_{\alpha/2}$ as the two-sided t statistic corresponding to a significance level of α with $n-1$ degrees of freedom; $t_{\alpha/2}$ is obtained from Stata as `invttail(n-1,0.5*\alpha)`. The lower and upper confidence bounds are, respectively, $\bar{x} - s_{\mu}t_{\alpha/2}$ and $\bar{x} + s_{\mu}t_{\alpha/2}$.

Poisson mean

Given the total cases, k , the estimate of the expected count λ is k , and its standard error is \sqrt{k} . `ci` means with the `poisson` option calculates the exact confidence interval $[\lambda_1, \lambda_2]$ such that

$$\Pr(K \geq k | \lambda = \lambda_1) = \alpha/2$$

and

$$\Pr(K \leq k | \lambda = \lambda_2) = \alpha/2$$

where K is Poisson with mean λ . Solution is obtained by Newton's method. If $k = 0$, the calculation of λ_1 is skipped. All values are then reported as rates, which are the above numbers divided by the total exposure.

Binomial proportion

Given k successes of n trials, the estimated probability of a success is $\hat{p} = k/n$ with standard error $\sqrt{\hat{p}(1-\hat{p})/n}$. `ci` calculates the exact (Clopper–Pearson) confidence interval $[p_1, p_2]$ such that

$$\Pr(K \geq k | p = p_1) = \alpha/2$$

and

$$\Pr(K \leq k | p = p_2) = \alpha/2$$

where K is distributed as $\text{binomial}(n, p)$. The endpoints may be obtained directly by using Stata's `invbinomial()` function. If $k = 0$ or $k = n$, the calculation of the appropriate tail is skipped.

The Wald interval is $\hat{p} \pm z_{\alpha/2} \sqrt{\hat{p}(1-\hat{p})/n}$, where $z_{\alpha/2}$ is the $1-\alpha/2$ quantile of the standard normal. The interval is obtained by inverting the acceptance region of the large-sample Wald test of $H_0: p = p_0$ versus the two-sided alternative. That is, the confidence interval is the set of all p_0 such that

$$\left| \frac{\hat{p} - p_0}{\sqrt{n^{-1}\hat{p}(1-\hat{p})}} \right| \leq z_{\alpha/2}$$

The Wilson interval is a variation on the Wald interval, using the null standard error $\sqrt{n^{-1}p_0(1-p_0)}$ in place of the estimated standard error $\sqrt{n^{-1}\hat{p}(1-\hat{p})}$ in the above expression. Inverting this acceptance region is more complicated yet results in the closed form

$$\frac{k + z_{\alpha/2}^2/2}{n + z_{\alpha/2}^2} \pm \frac{z_{\alpha/2}n^{1/2}}{n + z_{\alpha/2}^2} \left\{ \hat{p}(1-\hat{p}) + \frac{z_{\alpha/2}^2}{4n} \right\}^{1/2}$$

The Agresti–Coull interval is basically a Wald interval that borrows its center from the Wilson interval. Defining $\tilde{k} = k + z_{\alpha/2}^2/2$, $\tilde{n} = n + z_{\alpha/2}^2$, and (hence) $\tilde{p} = \tilde{k}/\tilde{n}$, the Agresti–Coull interval is

$$\tilde{p} \pm z_{\alpha/2} \sqrt{\tilde{p}(1-\tilde{p})/\tilde{n}}$$

When $\alpha = 0.05$, $z_{\alpha/2}$ is near enough to 2 that \tilde{p} can be thought of as a typical estimate of proportion where two successes and two failures have been added to the sample (Agresti and Coull 1998). This typical estimate of proportion makes the Agresti–Coull interval an easy-to-present alternative for introductory statistics students.

The Jeffreys interval is a Bayesian credible interval and is based on the Jeffreys prior, which is the $\text{Beta}(1/2, 1/2)$ distribution. Assigning this prior to p results in a posterior distribution for p that is Beta with parameters $k+1/2$ and $n-k+1/2$. The Jeffreys interval is then taken to be the $1-\alpha$ central posterior probability interval, namely, the $\alpha/2$ and $1-\alpha/2$ quantiles of the $\text{Beta}(k+1/2, n-k+1/2)$ distribution. These quantiles may be obtained directly by using Stata's `invibeta()` function. See [BAYES] [bayesstats summary](#) for more details about credible intervals.

Variance and standard deviation

Let X_1, \dots, X_n be a random sample and assume that $X_i \sim N(\mu, \sigma^2)$. Because $(n-1)s^2/\sigma^2 \sim \chi_{n-1}^2$, we have $\Pr\{\chi_{n-1, \alpha/2}^2 \leq (n-1)s^2/\sigma^2 \leq \chi_{n-1, 1-\alpha/2}^2\} = 1 - \alpha$, where $\chi_{n-1, \alpha/2}^2$ and $\chi_{n-1, 1-\alpha/2}^2$ are the $\alpha/2$ and $1 - \alpha/2$ quantiles of the χ_{n-1}^2 distribution. Thus, the normal-based confidence interval for the population variance σ^2 with $100(1 - \alpha)\%$ confidence level is given by

$$I_{\text{normal}} = \left[\frac{(n-1)s^2}{\chi_{n-1, 1-\alpha/2}^2}, \frac{(n-1)s^2}{\chi_{n-1, \alpha/2}^2} \right]$$

$\chi_{n-1, 1-\alpha/2}^2$ and $\chi_{n-1, \alpha/2}^2$ are obtained from Stata as `invchi2tail(n-1, 0.5*\alpha)` and `invchi2(n-1, 0.5*\alpha)`, respectively.

The normal-based confidence interval is very sensitive to minor departures from the normality assumption, and its performance does not improve with increasing sample size. For scenarios in which the population distribution is not normal, the actual coverage probability of the normal-based confidence interval can be drastically lower than the nominal confidence level α .

Bonett (2006) proposed an alternative to the normal-based confidence interval that is nearly exact under normality and has coverage probability close to $1 - \alpha$ under moderate nonnormality. It also has $1 - \alpha$ asymptotic coverage probability for nonnormal distributions with finite fourth moment. Instead of assuming that $X_i \sim N(\mu, \sigma^2)$, Bonett's approach requires continuous i.i.d. random variables with finite fourth moment. The variance of s^2 may be expressed as $\sigma^4 \{\gamma_4 - (n-3)/(n-1)\} / n$ (see Casella and Berger [2002, ex. 5.8, 257]), where $\gamma_4 = \mu^4/\sigma^4$ is the kurtosis and $\mu^4 = E(X_i - \mu)^4$ is the population fourth central moment. The variance-stabilizing transformation $\ln(s^2)$ and the delta method can be used to construct an asymptotic $100(1 - \alpha)\%$ confidence interval for σ^2 ,

$$\left[\exp\{\ln(s^2) - z_{\alpha/2} se\}, \exp\{\ln(s^2) + z_{\alpha/2} se\} \right]$$

where $se = \{\hat{\gamma}_4 - (n-3)/(n-1)\} / n \approx \text{Var}\{\ln(s^2)\}$ and $\hat{\gamma}_4$ is an estimate of the kurtosis. Bonett introduced three adjustments to improve the small-sample properties of the above confidence interval. First, he swapped the inner and outer denominator in the expression for se and changed it to $\{\hat{\gamma}_4 - (n-3)/n\} / (n-1)$. This was suggested by Shoemaker (2003) who used it to improve the small-sample performance of his variance test. Second, with regard to the estimation of kurtosis, Bonett proposed $\hat{\gamma}_4 = n \sum (X_i - m)^4 / \left\{ \sum (X_i - \bar{X})^2 \right\}^2$, where m is a trimmed mean with a trim-proportion equal to $1/\{2(n-4)^{1/2}\}$. This kurtosis estimator reduces the negative bias in symmetric and skewed heavy-tailed distributions. Last, he empirically derived a small-sample correction factor $c = n/(n - z_{\alpha/2})$ that helps equalize the tail probabilities. These modifications yield

$$I_{\text{Bonett}} = \left[\exp\{\ln(cs^2) - z_{\alpha/2} se\}, \exp\{\ln(cs^2) + z_{\alpha/2} se\} \right]$$

where $z_{\alpha/2}$ is the $1 - \alpha/2$ quantile of the standard normal and $se = c\{\{\hat{\gamma}_4 - (n-3)/n\} / (n-1)\}$.

Taking the square root of the endpoints of both intervals gives confidence intervals for the standard deviation σ .

Edwin Bidwell (E. B.) Wilson (1879–1964) majored in mathematics at Harvard and studied and taught at Yale and MIT before returning to Harvard in 1922. He worked in mathematics, physics, and statistics. His method for binomial intervals can be considered a precursor, for a particular problem, of Neyman’s concept of confidence intervals.

Jerzy Neyman (1894–1981) was born in Bendery, Russia, now Moldavia. He studied and then taught at Kharkov University, moving from physics to mathematics. In 1921, Neyman moved to Poland, where he worked in statistics at Bydgoszcz and then Warsaw. Neyman received a Rockefeller Fellowship to work with Karl Pearson at University College London. There he collaborated with Egon Pearson, Karl’s son, on the theory of hypothesis testing. Life in Poland became progressively more difficult, and Neyman returned to UCL to work there from 1934 to 1938. At this time, he published on the theory of confidence intervals. He then was offered a post in California at Berkeley, where he settled. Neyman established an outstanding statistics department and remained highly active in research, including applications in astronomy, meteorology, and medicine. He was one of the great statisticians of the 20th century.

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Also see

- [R] **ameans** — Arithmetic, geometric, and harmonic means
- [R] **bitest** — Binomial probability test
- [R] **centile** — Report centile and confidence interval
- [R] **prtest** — Tests of proportions
- [R] **sctest** — Variance-comparison tests
- [R] **summarize** — Summary statistics
- [R] **ttest** — t tests (mean-comparison tests)
- [D] **pctile** — Create variable containing percentiles

clogit — Conditional (fixed-effects) logistic regression

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Description

`clogit` fits a conditional logistic regression model for matched case–control data, also known as a fixed-effects logit model for panel data. `clogit` can compute robust and cluster–robust standard errors and adjust results for complex survey designs.

See [\[CM\]](#) [**cmlogit**](#) if you want to fit McFadden’s choice model ([McFadden 1974](#)).

Quick start

Conditional logistic regression model of y on x with matched case–control pairs data identified by `idvar`

```
clogit y x, group(idvar)
```

Fixed-effects logistic regression model with panels identified by `idvar`

```
clogit y x, group(idvar)
```

Add categorical variable `a` and report results as odds ratios

```
clogit y x i.a, group(idvar) or
```

Same as above, but using sampling probability weight `wvar`

```
clogit y x i.a [pweight = wvar], group(idvar) or
```

Menu

Statistics > Binary outcomes > Conditional logistic regression

Syntax

```
clogit depvar [indepvars] [if] [in] [weight] , group(varname) [options]
```

depvar is treated as binary regardless of values; *depvar* equal to nonzero and nonmissing (typically equal to 1) indicates a positive outcome, whereas *depvar* equal to 0 indicates a negative outcome.

<i>options</i>	Description
Model	
* <u>group</u> (<i>varname</i>)	matched group variable
<u>offset</u> (<i>varname</i>)	include <i>varname</i> in model with coefficient constrained to 1
<u>constraints</u> (<i>constraints</i>)	apply specified linear constraints
SE/Robust	
<u>vce</u> (<i>vcetype</i>)	<i>vcetype</i> may be <code>oim</code> , <code>robust</code> , <code>cluster</code> <i>clustvar</i> , <code>opg</code> , <code>bootstrap</code> , or <code>jackknife</code>
<code>nonest</code>	do not check that panels are nested within clusters
Reporting	
<u>level</u> (#)	set confidence level; default is <code>level(95)</code>
<code>or</code>	report odds ratios
<u>nocnsreport</u>	do not display constraints
<u>display_options</u>	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Maximization	
<u>maximize_options</u>	control the maximization process; seldom used
<u>collinear</u>	keep collinear variables
<u>coeflegend</u>	display legend instead of statistics

*group(*varname*) is required.

indepvars may contain factor variables; see [U] 11.4.3 **Factor variables**.

`bayes`, `bootstrap`, `by`, `collect`, `fp`, `jackknife`, `mfp`, `mi estimate`, `nestreg`, `rolling`, `statsby`, `stepwise`, and `svy` are allowed; see [U] 11.1.10 **Prefix commands**. For more details, see [BAYES] **bayes: clogit**.

`vce(bootstrap)` and `vce(jackknife)` are not allowed with the `mi estimate` prefix; see [MI] **mi estimate**.

Weights are not allowed with the `bootstrap` prefix; see [R] **bootstrap**.

`vce()`, `nonest`, and weights are not allowed with the `svy` prefix; see [SVY] **svy**.

`fwweights`, `iweights`, and `pweights` are allowed (see [U] 11.1.6 **weight**), but they are interpreted to apply to groups as a whole, not to individual observations. See *Use of weights* below.

`collinear` and `coeflegend` do not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

Options

Model

group(*varname*) is required; it specifies an identifier variable (numeric or string) for the matched groups. strata(*varname*) is a synonym for `group()`.

offset(*varname*), constraints(*constraints*); see [R] **Estimation options**.

SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce_option](#).

`nonest`, available only with `vce(cluster clustvar)`, prevents checking that matched groups are nested within clusters. It is the user's responsibility to verify that the standard errors are theoretically correct.

Reporting

`level(#)`; see [R] [Estimation options](#).

or reports the estimated coefficients transformed to odds ratios, that is, e^b rather than b . Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated. or may be specified at estimation or when replaying previously estimated results.

`nocnsreport`; see [R] [Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no!stretch`; see [R] [Estimation options](#).

Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

Setting the optimization type to `technique(bhhh)` resets the default `vcetype` to `vce(opg)`.

The following options are available with `clogit` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

Remarks and examples

Remarks are presented under the following headings:

[Introduction](#)
[Matched case-control data](#)
[Use of weights](#)
[Fixed-effects logit](#)

Introduction

`clogit` fits maximum likelihood models with a dichotomous dependent variable coded as 0/1 (more precisely, `clogit` interprets 0 and not 0 to indicate the dichotomy). Conditional logistic analysis differs from regular logistic regression in that the data are grouped and the likelihood is calculated relative to each group; that is, a conditional likelihood is used. See [Methods and formulas](#) at the end of this entry.

Biostatisticians and epidemiologists call these models conditional logistic regression for matched case–control groups (see, for example, Hosmer, Lemeshow, and Sturdivant [2013, chap. 7]) and fit them when analyzing matched case–control studies with 1 : 1 matching, 1 : k_{2i} matching, or $k_{1i} : k_{2i}$ matching, where i denotes the i th matched group for $i = 1, 2, \dots, n$, where n is the total number of groups. `clogit` fits a model appropriate for all of these matching schemes or for any mix of the schemes because the matching $k_{1i} : k_{2i}$ can vary from group to group. `clogit` always uses the true conditional likelihood, not an approximation. Biostatisticians and epidemiologists sometimes refer to the matched groups as “strata”, but we will stick to the more generic term “group”.

Economists and other social scientists typically call the model fit by `clogit` a fixed-effects logit model for panel data (see, for example, Chamberlain [1980]). The data used to fit a fixed-effects logit model look exactly like the data biostatisticians and epidemiologists call $k_{1i} : k_{2i}$ matched case–control data. In terms of how the data are arranged, $k_{1i} : k_{2i}$ matching means that in the i th group, the dependent variable is 1 a total of k_{1i} times and 0 a total of k_{2i} times. There are a total of $T_i = k_{1i} + k_{2i}$ observations for the i th group. This data arrangement is what economists and other social scientists call “panel data”, “longitudinal data”, or “cross-sectional time-series data”.

So no matter what terminology you use, the computation and the use of the `clogit` command is the same. The following example shows how your data should be arranged to use `clogit`.

► Example 1

Suppose that we have grouped data with the variable `id` containing a unique identifier for each group. Our outcome variable, `y`, contains 0s and 1s. If we were biostatisticians, $y = 1$ would indicate a case, $y = 0$ would be a control, and `id` would be an identifier variable that indicates the groups of matched case–control subjects.

If we were economists, $y = 1$ might indicate that a person was unemployed at any time during a year and $y = 0$, that a person was employed all year, and `id` would be an identifier variable for persons.

If we list the first few observations of this dataset, it looks like

```
. use https://www.stata-press.com/data/r19/clogitid
. list y x1 x2 id in 1/11
```

	y	x1	x2	id
1.	0	0	4	1014
2.	0	1	4	1014
3.	0	1	6	1014
4.	1	1	8	1014
5.	0	0	1	1017
6.	0	0	7	1017
7.	1	1	10	1017
8.	0	0	1	1019
9.	0	1	7	1019
10.	1	1	7	1019
11.	1	1	9	1019

Pretending that we are biostatisticians, we describe our data as follows. The first group ($id = 1014$) consists of four matched persons: 1 case ($y = 1$) and three controls ($y = 0$), that is, 1 : 3 matching. The second group has 1 : 2 matching, and the third 2 : 2.

Pretending that we are economists, we describe our data as follows. The first group consists of 4 observations (one per year) for person 1014. This person had a period of unemployment during 1 year of 4. The second person had a period of unemployment during 1 year of 3, and the third had a period of 2 years of 4.

Our independent variables are x_1 and x_2 . To fit the conditional (fixed-effects) logistic model, we type

```
. clogit y x1 x2, group(id)
note: multiple positive outcomes within groups encountered.
Iteration 0:  Log likelihood = -123.42828
Iteration 1:  Log likelihood = -123.41386
Iteration 2:  Log likelihood = -123.41386
Conditional (fixed-effects) logistic regression      Number of obs =   369
LR chi2(2)      =    9.07
Prob > chi2     =  0.0107
Pseudo R2      =  0.0355
Log likelihood = -123.41386
```

	y	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
	x1	.653363	.2875215	2.27	0.023	.0898312	1.216895
	x2	.0659169	.0449555	1.47	0.143	-.0221943	.1540281



□ Technical note

The message “note: multiple positive outcomes within groups encountered” at the top of the `clogit` output for the previous example merely informs us that we have $k_{1i} : k_{2i}$ matching with $k_{1i} > 1$ for at least one group. If your data should be 1 : k_{2i} matched, this message tells you that there is an error in the data somewhere.

We can see the distribution of k_{1i} and $T_i = k_{1i} + k_{2i}$ for the data of the [example 1](#) by using the following steps:

```
. by id, sort: generate k1 = sum(y)
. by id: replace k1 = . if _n < _N
(303 real changes made, 303 to missing)
. by id: generate T = sum(y<.)
. by id: replace T = . if _n < _N
(303 real changes made, 303 to missing)
. tabulate k1
```

k1	Freq.	Percent	Cum.
1	48	72.73	72.73
2	12	18.18	90.91
3	4	6.06	96.97
4	2	3.03	100.00
Total	66	100.00	

```
. tabulate T
```

T	Freq.	Percent	Cum.
2	5	7.58	7.58
3	5	7.58	15.15
4	12	18.18	33.33
5	11	16.67	50.00
6	13	19.70	69.70
7	8	12.12	81.82
8	3	4.55	86.36
9	7	10.61	96.97
10	2	3.03	100.00
Total	66	100.00	

We see that k_{1i} ranges from 1 to 4 and T_i ranges from 2 to 10 for these data. □

□ Technical note

For $k_{1i} : k_{2i}$ matching (and hence in the general case of fixed-effects logit), `clogit` uses a recursive algorithm to compute the likelihood, which means that there are no limits on the size of T_i . However, computation time is proportional to $\sum T_i \min(k_{1i}, k_{2i})$, so `clogit` will take roughly 10 times longer to fit a model with 10 : 10 matching than one with 1 : 10 matching. But `clogit` is fast, so computation time becomes an issue only when $\min(k_{1i}, k_{2i})$ is around 100 or more. See [Methods and formulas](#) for details. □

Matched case–control data

Here we give a more detailed example of matched case–control data.

▷ Example 2

[Hosmer, Lemeshow, and Sturdivant \(2013, 24\)](#) present data on matched pairs of infants, each pair having one with low birthweight and another with regular birthweight. The data are matched on age of the mother. Several possible maternal exposures are considered: race (three categories), smoking status, presence of hypertension, presence of uterine irritability, previous preterm delivery, and weight at the last menstrual period.

```
. use https://www.stata-press.com/data/r19/lowbirth2, clear
(Applied Logistic Regression, Hosmer & Lemeshow)
. describe
Contains data from https://www.stata-press.com/data/r19/lowbirth2.dta
Observations:      112                Applied Logistic Regression,
                                Hosmer & Lemeshow
Variables:          9                 30 Jan 2024 08:46
```

Variable name	Storage type	Display format	Value label	Variable label
pairid	byte	%8.0g		Case-control pair ID
low	byte	%8.0g		Baby has low birthweight
age	byte	%8.0g		Age of mother
lwt	int	%8.0g		Mother's last menstrual weight
smoke	byte	%8.0g		Mother smoked during pregnancy
ptd	byte	%8.0g		Mother had previous preterm baby
ht	byte	%8.0g		Mother has hypertension
ui	byte	%8.0g		Uterine irritability
race	byte	%9.0g	race	Race of mother

Sorted by:

We list the case–control indicator variable, low; the match identifier variable, pairid; and two of the covariates, lwt and smoke, for the first 10 observations.

```
. list low lwt smoke pairid in 1/10
```

	low	lwt	smoke	pairid
1.	0	135	0	1
2.	1	101	1	1
3.	0	98	0	2
4.	1	115	0	2
5.	0	95	0	3
6.	1	130	0	3
7.	0	103	0	4
8.	1	130	1	4
9.	0	122	1	5
10.	1	110	1	5

We fit a conditional logistic model of low birthweight on mother’s weight, race, smoking behavior, and history.

```
. clogit low lwt smoke ptd ht ui i.race, group(pairid) nolog
Conditional (fixed-effects) logistic regression      Number of obs =   112
                                                    LR chi2(7)      =  26.04
                                                    Prob > chi2     =  0.0005
Log likelihood = -25.794271                        Pseudo R2      =  0.3355
```

low	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
lwt	-.0183757	.0100806	-1.82	0.068	-.0381333	.0013819
smoke	1.400656	.6278396	2.23	0.026	.1701131	2.631199
ptd	1.808009	.7886502	2.29	0.022	.2622828	3.353735
ht	2.361152	1.086128	2.17	0.030	.2323796	4.489924
ui	1.401929	.6961585	2.01	0.044	.0374836	2.766375
race						
Black	.5713643	.689645	0.83	0.407	-.7803149	1.923044
Other	-.0253148	.6992044	-0.04	0.971	-1.39573	1.345101

We might prefer to see results presented as odds ratios. We could have specified the or option when we first fit the model, or we can now redisplay results and specify or:

```
. clogit, or
Conditional (fixed-effects) logistic regression      Number of obs =   112
                                                    LR chi2(7)      =  26.04
                                                    Prob > chi2     =  0.0005
Log likelihood = -25.794271                        Pseudo R2      =  0.3355
```

low	Odds ratio	Std. err.	z	P> z	[95% conf. interval]	
lwt	.9817921	.009897	-1.82	0.068	.9625847	1.001383
smoke	4.057862	2.547686	2.23	0.026	1.185439	13.89042
ptd	6.098293	4.80942	2.29	0.022	1.299894	28.60938
ht	10.60316	11.51639	2.17	0.030	1.261599	89.11467
ui	4.06303	2.828513	2.01	0.044	1.038195	15.90088
race						
Black	1.770681	1.221141	0.83	0.407	.4582617	6.84175
Other	.975003	.6817263	-0.04	0.971	.2476522	3.838573

Smoking, previous preterm delivery, hypertension, uterine irritability, and possibly the mother’s weight all contribute to low birthweight. Race of black and race of other are statistically insignificant when compared with the race of white omitted group, although the race of black effect is large. We can test the joint statistical significance of race being black (2.race) and race being other (3.race) by using test:

```
. test 2.race 3.race
( 1) [low]2.race = 0
( 2) [low]3.race = 0
      chi2( 2) =    0.88
      Prob > chi2 =  0.6436
```

For a more complete description of `test`, see [R] [test](#). `test` presents results in coefficients rather than odds ratios. Jointly testing that the coefficients on `2.race` and `3.race` are 0 is equivalent to jointly testing that the odds ratios are 1.

Here one case was matched to one control, that is, 1 : 1 matching. From `clogit`'s point of view, that was not important— k_1 cases could have been matched to k_2 controls ($k_1 : k_2$ matching), and we would have fit the model in the same way. Furthermore, the matching can change from group to group, which we have denoted as $k_{1i} : k_{2i}$ matching, where i denotes the group. `clogit` does not care. To fit the conditional logistic regression model, we specified the `group(varname)` option, `group(pairid)`. The case and control are stored in separate observations. `clogit` knew that they were linked (in the same group) because the related observations share the same value of `pairid`.



□ Technical note

`clogit` provides a way to extend McNemar's test to multiple controls per case (1 : k_{2i} matching) and to multiple controls matched with multiple cases ($k_{1i} : k_{2i}$ matching).

In Stata, McNemar's test is calculated by the `mcc` command; see [R] [EpiTab](#). The `mcc` command, however, requires that the matched case and control appear in one observation, so the data will need to be manipulated from 1 to 2 observations per stratum before using `clogit`. Alternatively, if you begin with `clogit`'s 2-observations-per-group organization, you will have to change it to 1 observation per group if you wish to use `mcc`. In either case, `reshape` provides an easy way to change the organization of the data. We will demonstrate its use below, but we direct you to [D] [reshape](#) for a more thorough discussion.

In [example 2](#), we used `clogit` to analyze the relationship between low birthweight and various characteristics of the mother. Assume that we now want to assess the relationship between low birthweight and smoking, ignoring the mother's other characteristics. Using `clogit`, we obtain the following results:

```
. clogit low smoke, group(pairid) or
Iteration 0:  Log likelihood = -35.425931
Iteration 1:  Log likelihood = -35.419283
Iteration 2:  Log likelihood = -35.419282
Conditional (fixed-effects) logistic regression
Number of obs =   112
LR chi2(1)    =    6.79
Prob > chi2   =  0.0091
Pseudo R2    =  0.0875
Log likelihood = -35.419282
```

low	Odds ratio	Std. err.	z	P> z	[95% conf. interval]	
smoke	2.75	1.135369	2.45	0.014	1.224347	6.176763

Let's compare our estimated odds ratio and 95% confidence interval with that produced by `mcc`. We begin by reshaping the data:

```
. keep low smoke pairid
. reshape wide smoke, i(pairid) j(low 0 1)
```

Data	Long	->	Wide
Number of observations	112	->	56
Number of variables	3	->	3
j variable (2 values)	low	->	(dropped)
xij variables:			
	smoke	->	smoke0 smoke1

We now have the variables `smoke0` (formed from `smoke` and `low = 0`), recording 1 if the control mother smoked and 0 otherwise; and `smoke1` (formed from `smoke` and `low = 1`), recording 1 if the case mother smoked and 0 otherwise. We can now use `mcc`:

```
. mcc smoke1 smoke0
```

Cases	Controls		Total
	Exposed	Unexposed	
Exposed	8	22	30
Unexposed	8	18	26
Total	16	40	56

```
McNemar's chi2(1) = 6.53 Prob > chi2 = 0.0106
Exact McNemar significance probability = 0.0161
```

Proportion with factor

Cases	.5357143	[95% conf. interval]	
Controls	.2857143		
difference	.25	.0519726	.4480274
ratio	1.875	1.148685	3.060565
rel. diff.	.35	.1336258	.5663742
odds ratio	2.75	1.179154	7.143667 (exact)

Both methods estimated the same odds ratio, and the 95% confidence intervals are similar. `clogit` produced a confidence interval of [1.22, 6.18], whereas `mcc` produced a confidence interval of [1.18, 7.14].



Use of weights

With `clogit`, weights apply to groups as a whole, not to individual observations. For example, if there is a group in your dataset with a frequency weight of 3, there are a total of three groups in your sample with the same values of the dependent and independent variables as this one group. Weights must have the same value for all observations belonging to the same group; otherwise, an error message will be displayed.

▷ Example 3

We use the example from the above discussion of the `mcc` command. Here we have a total of 56 matched case–control groups, each with one case matched to one control. We had 8 matched pairs in which both the case and the control are exposed, 22 pairs in which the case is exposed and the control is unexposed, 8 pairs in which the case is unexposed and the control is exposed, and 18 pairs in which they are both unexposed.

With weights, it is easy to enter these data into Stata and run `clogit`.

```
. clear
. input id case exposed weight
      id      case  exposed  weight
1.  1  1  1  8
2.  1  0  1  8
3.  2  1  1  22
4.  2  0  0  22
5.  3  1  0  8
6.  3  0  1  8
7.  4  1  0  18
8.  4  0  0  18
9.  end

. clogit case exposed [w=weight], group(id) or
(frequency weights assumed)

Iteration 0:  Log likelihood = -35.425931
Iteration 1:  Log likelihood = -35.419283
Iteration 2:  Log likelihood = -35.419282

Conditional (fixed-effects) logistic regression      Number of obs =   112
LR chi2(1)      =    6.79
Prob > chi2     =  0.0091
Pseudo R2      =  0.0875

Log likelihood = -35.419282
```

case	Odds ratio	Std. err.	z	P> z	[95% conf. interval]	
exposed	2.75	1.135369	2.45	0.014	1.224347	6.176763



Fixed-effects logit

The fixed-effects logit model can be written as

$$\Pr(y_{it} = 1 \mid \mathbf{x}_{it}) = F(\alpha_i + \mathbf{x}_{it}\beta)$$

where F is the cumulative logistic distribution

$$F(z) = \frac{\exp(z)}{1 + \exp(z)}$$

$i = 1, 2, \dots, n$ denotes the independent units (called “groups” by `clogit`), and $t = 1, 2, \dots, T_i$ denotes the observations for the i th unit (group).

Fitting this model by using a full maximum-likelihood approach leads to difficulties, however. When T_i is fixed, the maximum likelihood estimates for α_i and β are inconsistent (Andersen 1970; Chamberlain 1980). This difficulty can be circumvented by looking at the probability of $\mathbf{y}_i = (y_{i1}, \dots, y_{iT_i})$ conditional on $\sum_{t=1}^{T_i} y_{it}$. This conditional probability does not involve the α_i , so they are never estimated when the resulting conditional likelihood is used. See Hamerle and Ronning (1995) for a succinct and lucid development. See *Methods and formulas* for the estimation equation.

▷ Example 4

We are studying unionization of women in the United States by using the union dataset; see [XT] xt. We fit the fixed-effects logit model:

```
. use https://www.stata-press.com/data/r19/union, clear
(NLS Women 14-24 in 1968)

. clogit union age grade not_smsa south black, group(idcode)
note: multiple positive outcomes within groups encountered.
note: 2,744 groups (14,165 obs) omitted because of all positive or
      all negative outcomes.
note: black omitted because of no within-group variance.

Iteration 0:  Log likelihood = -4521.3385
Iteration 1:  Log likelihood = -4516.1404
Iteration 2:  Log likelihood = -4516.1385
Iteration 3:  Log likelihood = -4516.1385

Conditional (fixed-effects) logistic regression          Number of obs = 12,035
LR chi2(4)      = 68.09
Prob > chi2    = 0.0000
Pseudo R2      = 0.0075

Log likelihood = -4516.1385
```

union	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
age	.0170301	.004146	4.11	0.000	.0089042	.0251561
grade	.0853572	.0418781	2.04	0.042	.0032777	.1674368
not_smsa	.0083678	.1127963	0.07	0.941	-.2127088	.2294445
south	-.748023	.1251752	-5.98	0.000	-.9933619	-.5026842
black	0	(omitted)				

We received three messages at the top of the output. The first one, “multiple positive outcomes within groups encountered”, we expected. Our data do indeed have multiple positive outcomes (union = 1) in many groups. (Here a group consists of all the observations for a particular individual.)

The second message tells us that 2,744 groups were “omitted” by clogit. When either union = 0 or union = 1 for all observations for an individual, this individual’s contribution to the log likelihood is zero. Although these are perfectly valid observations in every sense, they have no effect on the estimation, so they are not included in the total “Number of obs”. Hence, the reported “Number of obs” gives the effective sample size of the estimation. Here it is 12,035 observations—only 46% of the total 26,200.

We can easily check that there are indeed 2,744 groups with union either all 0 or all 1. We will generate a variable that contains the fraction of observations for each individual who has union = 1.

```
. by idcode, sort: generate fraction = sum(union)/sum(union < .)
. by idcode: replace fraction = . if _n < _N
(21,766 real changes made, 21,766 to missing)
. tabulate fraction
```

fraction	Freq.	Percent	Cum.
0	2,481	55.95	55.95
.0833333	30	0.68	56.63
.0909091	33	0.74	57.37
.1	53	1.20	58.57
<i>(output omitted)</i>			
.9	10	0.23	93.59
.9090909	11	0.25	93.84
.9166667	10	0.23	94.07
1	263	5.93	100.00
Total	4,434	100.00	

Because $2481 + 263 = 2744$, we confirm what `clogit` did.

The third warning message from `clogit` said “black omitted because of no within-group variance”. Obviously, race stays constant for an individual across time. Any such variables are collinear with the α_i (that is, the fixed effects), and just as the α_i drop out of the conditional likelihood, so do all variables that are unchanging within groups. Thus, they cannot be estimated with the conditional fixed-effects model.

There are other estimators implemented in Stata that we could use with these data, such as

```
cloglog ... , vce(cluster idcode)
logit ... , vce(cluster idcode)
probit ... , vce(cluster idcode)
scobit ... , vce(cluster idcode)
xtcloglog ...
xtgee ... , family(binomial) link(logit) corr(exchangeable)
xtlogit ...
xtprobit ...
```

See [R] [cloglog](#), [R] [logit](#), [R] [probit](#), [R] [scobit](#), [XT] [xtcloglog](#), [XT] [xtgee](#), [XT] [xtlogit](#), and [XT] [xtprobit](#) for details.

Stored results

clogit stores the following in e():

Scalars

e(N)	number of observations
e(N_drop)	number of observations dropped because of all positive or all negative outcomes
e(N_group_drop)	number of groups dropped because of all positive or all negative outcomes
e(k)	number of parameters
e(k_eq)	number of equations in e(b)
e(k_eq_model)	number of equations in overall model test
e(k_dv)	number of dependent variables
e(df_m)	model degrees of freedom
e(r2_p)	pseudo- R^2
e(ll)	log likelihood
e(ll_0)	log likelihood, constant-only model
e(N_clust)	number of clusters
e(chi2)	χ^2
e(p)	p -value for model test
e(rank)	rank of e(V)
e(ic)	number of iterations
e(rc)	return code
e(converged)	1 if converged, 0 otherwise

Macros

e(cmd)	clogit
e(cmdline)	command as typed
e(depvar)	name of dependent variable
e(group)	name of group() variable
e(multiple)	multiple if multiple positive outcomes within group
e(wtype)	weight type
e(wexp)	weight expression
e(title)	title in estimation output
e(clustvar)	name of cluster variable
e(offset)	linear offset variable
e(chi2type)	Wald or LR; type of model χ^2 test
e(vce)	<i>vcetype</i> specified in vce()
e(vcetype)	title used to label Std. err.
e(opt)	type of optimization
e(which)	max or min; whether optimizer is to perform maximization or minimization
e(ml_method)	type of ml method
e(user)	name of likelihood-evaluator program
e(technique)	maximization technique
e(properties)	b V
e(predict)	program used to implement predict
e(marginsok)	predictions allowed by margins
e(marginsnotok)	predictions disallowed by margins
e(marginsdefault)	default predict() specification for margins
e(asbalanced)	factor variables fvset as asbalanced
e(asobserved)	factor variables fvset as asobserved

Matrices

e(b)	coefficient vector
e(Cns)	constraints matrix
e(ilog)	iteration log (up to 20 iterations)
e(gradient)	gradient vector
e(V)	variance-covariance matrix of the estimators
e(V_modelbased)	model-based variance

Functions

e(sample)	marks estimation sample
-----------	-------------------------

In addition to the above, the following is stored in `r()`:

Matrices

`r(table)`

matrix containing the coefficients with their standard errors, test statistics, p -values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

Methods and formulas

Breslow and Day (1980, 247–279), Collett (2003, 251–267), and Hosmer, Lemeshow, and Sturdivant (2013, 243–268) provide a biostatistical point of view on conditional logistic regression. Hamerle and Ronning (1995) give a succinct and lucid review of fixed-effects logit; Chamberlain (1980) is a standard reference for this model. Greene (2018, chap. 18) provides a straightforward textbook description of conditional logistic regression from an economist’s point of view, as well as a brief description of choice models.

Let $i = 1, 2, \dots, n$ denote the groups and let $t = 1, 2, \dots, T_i$ denote the observations for the i th group. Let y_{it} be the dependent variable taking on values 0 or 1. Let $\mathbf{y}_i = (y_{i1}, \dots, y_{iT_i})$ be the outcomes for the i th group as a whole. Let \mathbf{x}_{it} be a row vector of covariates. Let

$$k_{1i} = \sum_{t=1}^{T_i} y_{it}$$

be the observed number of ones for the dependent variable in the i th group. Biostatisticians would say that there are k_{1i} cases matched to $k_{2i} = T_i - k_{1i}$ controls in the i th group.

We consider the probability of a possible value of \mathbf{y}_i conditional on $\sum_{t=1}^{T_i} y_{it} = k_{1i}$ (Hamerle and Ronning 1995, eq. 8.33; Hosmer, Lemeshow, and Sturdivant 2013, eq. 7.4),

$$\Pr(\mathbf{y}_i \mid \sum_{t=1}^{T_i} y_{it} = k_{1i}) = \frac{\exp(\sum_{t=1}^{T_i} y_{it} \mathbf{x}_{it} \boldsymbol{\beta})}{\sum_{\mathbf{d}_i \in S_i} \exp(\sum_{t=1}^{T_i} d_{it} \mathbf{x}_{it} \boldsymbol{\beta})}$$

where d_{it} is equal to 0 or 1 with $\sum_{t=1}^{T_i} d_{it} = k_{1i}$, and S_i is the set of all possible combinations of k_{1i} ones and k_{2i} zeros. Clearly, there are $\binom{T_i}{k_{1i}}$ such combinations, but we need not count all of these combinations to compute the denominator of the above equation. It can be computed recursively.

Denote the denominator by

$$f_i(T_i, k_{1i}) = \sum_{\mathbf{d}_i \in S_i} \exp\left(\sum_{t=1}^{T_i} d_{it} \mathbf{x}_{it} \boldsymbol{\beta}\right)$$

Consider, computationally, how f_i changes as we go from a total of 1 observation in the group to 2 observations to 3, etc. Doing this, we derive the recursive formula

$$f_i(T, k) = f_i(T - 1, k) + f_i(T - 1, k - 1) \exp(\mathbf{x}_{iT} \boldsymbol{\beta})$$

where we define $f_i(T, k) = 0$ if $T < k$ and $f_i(T, 0) = 1$.

The conditional log-likelihood is

$$\ln L = \sum_{i=1}^n \left\{ \sum_{t=1}^{T_i} y_{it} \mathbf{x}_{it} \boldsymbol{\beta} - \log f_i(T_i, k_{1i}) \right\}$$

The derivatives of the conditional log-likelihood can also be computed recursively by taking derivatives of the recursive formula for f_i .

Computation time is roughly proportional to

$$p^2 \sum_{i=1}^n T_i \min(k_{1i}, k_{2i})$$

where p is the number of independent variables in the model. If $\min(k_{1i}, k_{2i})$ is small, computation time is not an issue. But if it is large—say, 100 or more—patience may be required.

If T_i is large for all groups, the bias of the unconditional fixed-effects estimator is not a concern, and we can confidently use `logit` with an indicator variable for each group (provided, of course, that the number of groups is held within a Stata matrix; see [R] [Limits](#)).

This command supports the clustered version of the Huber/White/sandwich estimator of the variance using `vce(robust)` and `vce(cluster clustvar)`. See [P] [_robust](#), particularly *Maximum likelihood estimators* and *Methods and formulas*. Specifying `vce(robust)` is equivalent to specifying `vce(cluster groupvar)`, where `groupvar` is the variable for the matched groups.

`clogit` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] [Variance estimation](#).

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Also see

- [R] **clogit postestimation** — Postestimation tools for clogit
- [R] **logistic** — Logistic regression, reporting odds ratios
- [R] **mlogit** — Multinomial (polytomous) logistic regression
- [R] **ologit** — Ordered logistic regression
- [R] **scobit** — Skewed logistic regression
- [BAYES] **bayes: clogit** — Bayesian conditional logistic regression
- [CM] **cmlogit** — Conditional logit (McFadden's) choice model
- [CM] **nlogit** — Nested logit regression
- [MI] **Estimation** — Estimation commands for use with mi estimate
- [SVY] **svy estimation** — Estimation commands for survey data
- [XT] **xtgee** — GEE population-averaged panel-data models
- [XT] **xtlogit** — Fixed-effects, random-effects, and population-averaged logit models
- [XT] **xtmlogit** — Fixed-effects and random-effects multinomial logit models
- [U] **20 Estimation and postestimation commands**

Postestimation commands Methods and formulas	predict Reference	margins Also see	Remarks and examples
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Postestimation commands

The following standard postestimation commands are available after `clogit`:

Command	Description
<code>contrast</code>	contrasts and ANOVA-style joint tests of parameters
<code>estat ic</code>	Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively)
<code>estat summarize</code>	summary statistics for the estimation sample
<code>estat vce</code>	variance–covariance matrix of the estimators (VCE)
<code>estat (svy)</code>	postestimation statistics for survey data
<code>estimates</code>	cataloging estimation results
<code>etable</code>	table of estimation results
* <code>hausman</code>	Hausman's specification test
<code>lincom</code>	point estimates, standard errors, testing, and inference for linear combinations of parameters
<code>linktest</code>	link test for model specification
* <code>lrtest</code>	likelihood-ratio test
<code>margins</code>	marginal means, predictive margins, marginal effects, and average marginal effects
<code>marginsplot</code>	graph the results from margins (profile plots, interaction plots, etc.)
<code>nlcom</code>	point estimates, standard errors, testing, and inference for nonlinear combinations of parameters
<code>predict</code>	probabilities, influence statistics, residuals, etc.
<code>predictnl</code>	point estimates, standard errors, testing, and inference for generalized predictions
<code>pwcompare</code>	pairwise comparisons of parameters
<code>suest</code>	seemingly unrelated estimation
<code>test</code>	Wald tests of simple and composite linear hypotheses
<code>testnl</code>	Wald tests of nonlinear hypotheses

*`hausman` and `lrtest` are not appropriate with `svy` estimation results.

predict

Description for predict

`predict` creates a new variable containing predictions such as probabilities, linear predictions, standard errors, influence statistics, lack-of-fit statistics, Hosmer and Lemeshow leverages, Pearson residuals, and the equation-level score.

Menu for predict

Statistics > Postestimation

Syntax for predict

```
predict [type] newvar [if] [in] [ , statistic nooffset ]
```

<i>statistic</i>	Description
------------------	-------------

Main	
<code>pc1</code>	probability of a positive outcome; the default
<code>pu0</code>	probability of a positive outcome, assuming fixed effect is zero
<code>xb</code>	linear prediction
<code>stdp</code>	standard error of the linear prediction
* <code>dbeta</code>	Delta- β influence statistic
* <code>dx2</code>	Delta- χ^2 lack-of-fit statistic
* <code>gdbeta</code>	Delta- β influence statistic for each group
* <code>gdx2</code>	Delta- χ^2 lack-of-fit statistic for each group
* <code>hat</code>	Hosmer and Lemeshow leverage
* <code>residuals</code>	Pearson residuals
* <code>rstandard</code>	standardized Pearson residuals
<code>score</code>	first derivative of the log likelihood with respect to $\mathbf{x}_j\beta$

Unstarred statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample. Starred statistics are calculated only for the estimation sample, even when `if e(sample)` is not specified.

Starred statistics are available for multiple controls per case-matching design only. They are not available if `vce(robust)`, `vce(cluster clustvar)`, or `pweights` were specified with `clogit`.

`dbeta`, `dx2`, `gdbeta`, `gdx2`, `hat`, and `rstandard` are not available if `constraints()` was specified with `clogit`.

Options for predict

Main

`pc1`, the default, calculates the probability of a positive outcome conditional on one positive outcome within group.

`pu0` calculates the probability of a positive outcome, assuming that the fixed effect is zero.

`xb` calculates the linear prediction.

`stdp` calculates the standard error of the linear prediction.

- `dbeta` calculates the Delta- β influence statistic, a standardized measure of the difference in the coefficient vector that is due to deletion of the observation.
- `dx2` calculates the Delta- χ^2 influence statistic, reflecting the decrease in the Pearson χ^2 that is due to deletion of the observation.
- `gdbeta` calculates the approximation to the Pregibon stratum-specific Delta- β influence statistic, a standardized measure of the difference in the coefficient vector that is due to deletion of the entire stratum.
- `gdx2` calculates the approximation to the Pregibon stratum-specific Delta- χ^2 influence statistic, reflecting the decrease in the Pearson χ^2 that is due to deletion of the entire stratum.
- `hat` calculates the Hosmer and Lemeshow leverage or the diagonal element of the hat matrix.
- `residuals` calculates the Pearson residuals.
- `rstandard` calculates the standardized Pearson residuals.
- `score` calculates the equation-level score, $\partial \ln L / \partial (\mathbf{x}_{it} \boldsymbol{\beta})$.
- `nooffset` is relevant only if you specified `offset(varname)` for `clogit`. It modifies the calculations made by `predict` so that they ignore the offset variable; the linear prediction is treated as $\mathbf{x}_j \mathbf{b}$ rather than as $\mathbf{x}_j \mathbf{b} + \text{offset}_j$. This option cannot be specified with `dbeta`, `dx2`, `gdbeta`, `gdx2`, `hat`, and `rstandard`.

margins

Description for margins

`margins` estimates margins of response for probabilities and linear predictions.

Menu for margins

Statistics > Postestimation

Syntax for margins

```
margins [marginlist] [, options]  
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

<i>statistic</i>	Description
<code>pu0</code>	probability of a positive outcome, assuming fixed effect is zero; the default
<code>xb</code>	linear prediction
<code>pc1</code>	not allowed with margins
<code>stdp</code>	not allowed with margins
<code>dbeta</code>	not allowed with margins
<code>dx2</code>	not allowed with margins
<code>gdbeta</code>	not allowed with margins
<code>gdx2</code>	not allowed with margins
<code>hat</code>	not allowed with margins
<code>residuals</code>	not allowed with margins
<code>rstandard</code>	not allowed with margins
<code>score</code>	not allowed with margins

Statistics not allowed with margins are functions of stochastic quantities other than $e(b)$.

For the full syntax, see [R] [margins](#).

Remarks and examples

`predict` may be used after `clogit` to obtain predicted values of the index $\mathbf{x}_{it}\beta$. Predicted probabilities for conditional logistic regression must be interpreted carefully. Probabilities are estimated for each group as a whole, not for individual observations. Furthermore, the probabilities are conditional on the number of positive outcomes in the group (that is, the number of cases and the number of controls), or it is assumed that the fixed effect is zero. `predict` may also be used to obtain influence and lack-of-fit statistics for an individual observation and for the whole group, to compute Pearson, standardized Pearson residuals, and leverage values.

`predict` may be used for both within-sample and out-of-sample predictions.

▷ Example 1

Suppose that we have $1 : k_{2i}$ matched data and that we have previously fit the following model:

```
. use https://www.stata-press.com/data/r19/clogitid
. clogit y x1 x2, group(id)
(output omitted)
```

To obtain the predicted values of the index, we could type `predict idx, xb` to create a new variable called `idx`. From `idx`, we could then calculate the predicted probabilities. Easier, however, would be to type

```
. predict phat
(option pc1 assumed; probability of success given one success within group)
```

`phat` would then contain the predicted probabilities.

As noted previously, the predicted probabilities are really predicted probabilities for the group as a whole (that is, they are the predicted probability of observing $y_{it} = 1$ and $y_{it'} = 0$ for all $t' \neq t$). Thus, if we want to obtain the predicted probabilities for the estimation sample, it is important that, when we make the calculation, predictions be restricted to the same sample on which we estimated the data. We cannot predict the probabilities and then just keep the relevant ones because the entire sample determines each probability. Thus, assuming that we are not attempting to make out-of-sample predictions, we type

```
. predict phat2 if e(sample)
(option pc1 assumed; probability of success given one success within group)
```



Methods and formulas

Recall that $i = 1, \dots, n$ denote the groups and $t = 1, \dots, T_i$ denote the observations for the i th group.

`predict` produces probabilities of a positive outcome within group conditional on there being one positive outcome (`pc1`),

$$\Pr \left(y_{it} = 1 \mid \sum_{t=1}^{T_i} y_{it} = 1 \right) = \frac{\exp(\mathbf{x}_{it}\boldsymbol{\beta})}{\sum_{t=1}^{T_i} \exp(\mathbf{x}_{it}\boldsymbol{\beta})}$$

or `predict` calculates the unconditional `pu0`:

$$\Pr(y_{it} = 1) = \frac{\exp(\mathbf{x}_{it}\boldsymbol{\beta})}{1 + \exp(\mathbf{x}_{it}\boldsymbol{\beta})}$$

Let $N = \sum_{j=1}^n T_j$ denote the total number of observations, p denote the number of covariates, and $\hat{\theta}_{it}$ denote the conditional predicted probabilities of a positive outcome (`pc1`).

For the multiple control per case ($1 : k_{2i}$) matching, Hosmer, Lemeshow, and Sturdivant (2013, 248–251) propose the following diagnostics:

The Pearson residual is

$$r_{it} = \frac{(y_{it} - \hat{\theta}_{it})}{\sqrt{\hat{\theta}_{it}}}$$

The leverage (hat) value is defined as

$$h_{it} = \hat{\theta}_{it} \tilde{\mathbf{x}}_{it}^T (\tilde{\mathbf{X}}^T \mathbf{U} \tilde{\mathbf{X}})^{-1} \tilde{\mathbf{x}}_{it}$$

where $\tilde{\mathbf{x}}_{it} = \mathbf{x}_{it} - \sum_{j=1}^{T_i} \mathbf{x}_{ij} \hat{\theta}_{ij}$ is the $1 \times p$ row vector of centered by a weighted stratum-specific mean covariate values, $\mathbf{U}_N = \text{diag}\{\hat{\theta}_{it}\}$, and the rows of $\tilde{\mathbf{X}}_{N \times p}$ are composed of $\tilde{\mathbf{x}}_{it}$ values.

The standardized Pearson residual is

$$r_{sit} = \frac{r_{it}}{\sqrt{1 - h_{it}}}$$

The lack-of-fit and influence diagnostics for an individual observation are (respectively) computed as

$$\Delta \chi_{it}^2 = r_{sit}^2$$

and

$$\Delta \hat{\beta}_{it} = \Delta \chi_{it}^2 \frac{h_{it}}{1 - h_{it}}$$

The lack-of-fit and influence diagnostics for the groups are the group-specific totals of the respective individual diagnostics shown above.

Reference

Hosmer, D. W., Jr., S. A. Lemeshow, and R. X. Sturdivant. 2013. *Applied Logistic Regression*. 3rd ed. Hoboken, NJ: Wiley.

Also see

[R] [clomit](#) — Conditional (fixed-effects) logistic regression

[U] [20 Estimation and postestimation commands](#)

[Description](#)[Options](#)[Acknowledgment](#)[Quick start](#)[Remarks and examples](#)[References](#)[Menu](#)[Stored results](#)[Also see](#)[Syntax](#)[Methods and formulas](#)

Description

`cloglog` fits a complementary log–log model for a binary dependent variable, typically with one of the outcomes rare relative to the other. It can also be used to fit a gompit model. `cloglog` can compute robust and cluster–robust standard errors and adjust results for complex survey designs.

Quick start

Complementary log–log model of y on x_1 and x_2

```
cloglog y x1 x2
```

With robust standard errors

```
cloglog y x1 x2, vce(robust)
```

Adjust for complex survey design using `svyset` data

```
svy: cloglog y x1 x2
```

Menu

Statistics > Binary outcomes > Complementary log–log regression

Syntax

```
cloglog deprvar [indepvars] [if] [in] [weight] [, options]
```

<i>options</i>	Description
Model	
<code>noconstant</code>	suppress constant term
<code>offset(<i>varname</i>)</code>	include <i>varname</i> in model with coefficient constrained to 1
<code>asis</code>	retain perfect predictor variables
<code>constraints(<i>constraints</i>)</code>	apply specified linear constraints
SE/Robust	
<code>vce(<i>vcetype</i>)</code>	<i>vcetype</i> may be <code>oim</code> , <code>robust</code> , <code>cluster <i>clustvar</i></code> , <code>opg</code> , <code>bootstrap</code> , or <code>jackknife</code>
Reporting	
<code>level(#)</code>	set confidence level; default is level(95)
<code>eform</code>	report exponentiated coefficients
<code>nocnsreport</code>	do not display constraints
<code>display_options</code>	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Maximization	
<code>maximize_options</code>	control the maximization process; seldom used
<code>collinear</code>	keep collinear variables
<code>coeflegend</code>	display legend instead of statistics

indepvars may contain factor variables; see [U] 11.4.3 **Factor variables**.

deprvar and *indepvars* may contain time-series operators; see [U] 11.4.4 **Time-series varlists**.

`bayes`, `bayesboot`, `bootstrap`, `by`, `collect`, `fmm`, `fp`, `jackknife`, `mi estimate`, `nestreg`, `rolling`, `statsby`, `stepwise`, and `svy` are allowed; see [U] 11.1.10 **Prefix commands**. For more details, see [BAYES] **bayes: cloglog** and [FMM] **fmm: cloglog**.

`vce(bootstrap)` and `vce(jackknife)` are not allowed with the `mi estimate` prefix; see [MI] **mi estimate**.

Weights are not allowed with the `bootstrap` prefix; see [R] **bootstrap**.

`vce()` and weights are not allowed with the `svy` prefix; see [SVY] **svy**.

`fweights`, `iweights`, and `pweights` are allowed; see [U] 11.1.6 **weight**.

`collinear` and `coeflegend` do not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

Options

Model

`noconstant`, `offset(varname)`; see [R] **Estimation options**.

`asis` forces retention of perfect predictor variables and their associated perfectly predicted observations and may produce instabilities in maximization; see [R] **probit**.

`constraints(constraints)`; see [R] **Estimation options**.

SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce_option](#).

Reporting

`level(#)`; see [R] [Estimation options](#).

`eform` displays the exponentiated coefficients and corresponding standard errors and confidence intervals.

`nocnsreport`; see [R] [Estimation options](#).

`display_options`: `noc1`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

Setting the optimization type to `technique(bhhh)` resets the default `vcetype` to `vce(opg)`.

The following options are available with `cloglog` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

Remarks and examples

Remarks are presented under the following headings:

[Introduction to complementary log–log regression](#)
[Robust standard errors](#)

Introduction to complementary log–log regression

`cloglog` fits maximum likelihood models with dichotomous dependent variables coded as 0/1 (or, more precisely, coded as 0 and not 0).

▷ Example 1

We have data on the make, weight, and mileage rating of 22 foreign and 52 domestic automobiles. We wish to fit a model explaining whether a car is foreign based on its weight and mileage. Here is an overview of our data:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. keep make mpg weight foreign
. describe
Contains data from https://www.stata-press.com/data/r19/auto.dta
Observations:      74      1978 automobile data
Variables:         4      13 Apr 2024 17:45
                    (_dta has notes)
```

Variable name	Storage type	Display format	Value label	Variable label
make	str18	%-18s		Make and model
mpg	int	%8.0g		Mileage (mpg)
weight	int	%8.0gc		Weight (lbs.)
foreign	byte	%8.0g	origin	Car origin

Sorted by: foreign
 Note: Dataset has changed since last saved.

```
. inspect foreign
```

```
foreign: Car origin
```

		Number of observations		
		Total	Integers	Nonintegers
#	Negative	-	-	-
#	Zero	52	52	-
#	Positive	22	22	-
#				
# #	Total	74	74	-
# #	Missing	-		
0	1	74		

(2 unique values)

foreign is labeled and all values are documented in the label.

The variable `foreign` takes on two unique values, 0 and 1. The value 0 denotes a domestic car, and 1 denotes a foreign car.

The model that we wish to fit is

$$\Pr(\text{foreign} = 1) = F(\beta_0 + \beta_1 \text{weight} + \beta_2 \text{mpg})$$

where $F(z) = 1 - \exp\{-\exp(z)\}$.

To fit this model, we type

```
. cloglog foreign weight mpg
Iteration 0: Log likelihood = -34.054593
Iteration 1: Log likelihood = -27.869915
Iteration 2: Log likelihood = -27.742997
Iteration 3: Log likelihood = -27.742769
Iteration 4: Log likelihood = -27.742769
Complementary log-log regression      Number of obs   =       74
                                      Zero outcomes   =       52
                                      Nonzero outcomes =       22
                                      LR chi2(2)      =      34.58
                                      Prob > chi2     =      0.0000
Log likelihood = -27.742769
```

foreign	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
weight	-.0029153	.0006974	-4.18	0.000	-.0042823	-.0015483
mpg	-.1422911	.076337	-1.86	0.062	-.2920069	.0074247
_cons	10.09694	3.351841	3.01	0.003	3.527448	16.66642

We find that heavier cars are less likely to be foreign and that cars yielding better gas mileage are also less likely to be foreign, at least when holding the weight of the car constant.

See [R] [Maximize](#) for an explanation of the output.



□ Technical note

Stata interprets a value of 0 as a negative outcome (failure) and treats all other values (except missing) as positive outcomes (successes). Thus, if your dependent variable takes on the values 0 and 1, 0 is interpreted as failure and 1 as success. If your dependent variable takes on the values 0, 1, and 2, 0 is still interpreted as failure, but both 1 and 2 are treated as successes.

If you prefer a more formal mathematical statement, when you type `cloglog y x`, Stata fits the model

$$\Pr(y_j \neq 0 \mid \mathbf{x}_j) = 1 - \exp\left\{-\exp(\mathbf{x}_j\boldsymbol{\beta})\right\}$$



Robust standard errors

If you specify the `vce(robust)` option, `cloglog` reports robust standard errors, as described in [U] [20.22 Obtaining robust variance estimates](#). For the model of `foreign` on `weight` and `mpg`, the robust calculation increases the standard error of the coefficient on `mpg` by 44%:

```
. cloglog foreign weight mpg, vce(robust)
Iteration 0: Log pseudolikelihood = -34.054593
Iteration 1: Log pseudolikelihood = -27.869915
Iteration 2: Log pseudolikelihood = -27.742997
Iteration 3: Log pseudolikelihood = -27.742769
Iteration 4: Log pseudolikelihood = -27.742769
Complementary log-log regression          Number of obs   =          74
                                          Zero outcomes   =          52
                                          Nonzero outcomes =          22
                                          Wald chi2(2)    =          29.74
                                          Prob > chi2     =          0.0000
Log pseudolikelihood = -27.742769
```

foreign	Robust		z	P> z	[95% conf. interval]	
	Coefficient	std. err.				
weight	-.0029153	.0007484	-3.90	0.000	-.0043822	-.0014484
mpg	-.1422911	.1102466	-1.29	0.197	-.3583704	.0737882
_cons	10.09694	4.317305	2.34	0.019	1.635174	18.5587

Without `vce(robust)`, the standard error for the coefficient on `mpg` was reported to be 0.076, with a resulting confidence interval of $[-0.29, 0.01]$.

The `vce(cluster clustvar)` option can relax the independence assumption required by the complementary log–log estimator to being just independence between clusters. To demonstrate this ability, we will switch to a different dataset.

We are studying unionization of women in the United States by using the `union` dataset; see [XT] `xt`. We fit the following model, ignoring that women are observed an average of 5.9 times each in this dataset:

```
. use https://www.stata-press.com/data/r19/union, clear
(NLS Women 14–24 in 1968)
. cloglog union age grade not_smsa south#c.year
Iteration 0: Log likelihood = -13606.373
Iteration 1: Log likelihood = -13540.726
Iteration 2: Log likelihood = -13540.607
Iteration 3: Log likelihood = -13540.607
Complementary log-log regression          Number of obs   =        26,200
                                          Zero outcomes   =        20,389
                                          Nonzero outcomes =         5,811
                                          LR chi2(6)      =         647.24
                                          Prob > chi2     =         0.0000
Log likelihood = -13540.607
```

union	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
age	.0185346	.0043616	4.25	0.000	.009986	.0270833
grade	.0452772	.0057125	7.93	0.000	.0340809	.0564736
not_smsa	-.1886592	.0317801	-5.94	0.000	-.2509471	-.1263712
1.south	-1.422292	.3949381	-3.60	0.000	-2.196356	-.648227
year	-.0133007	.0049576	-2.68	0.007	-.0230174	-.0035839
south#c.year						
1	.0105659	.0049234	2.15	0.032	.0009161	.0202157
_cons	-1.219801	.2952374	-4.13	0.000	-1.798455	-.6411462

The reported standard errors in this model are probably meaningless. Women are observed repeatedly, and so the observations are not independent. Looking at the coefficients, we find a large southern effect against unionization and a different time trend for the south. The `vce(cluster clustvar)` option provides a way to fit this model and obtains correct standard errors:

```
. cloglog union age grade not_smsa south##c.year, vce(cluster id) nolog
Complementary log-log regression      Number of obs   =   26,200
                                      Zero outcomes    =   20,389
                                      Nonzero outcomes =    5,811
                                      Wald chi2(6)     =   160.76
                                      Prob > chi2     =    0.0000
Log pseudolikelihood = -13540.607
                                      (Std. err. adjusted for 4,434 clusters in idcode)
```

union	Coefficient	Robust std. err.	z	P> z	[95% conf. interval]	
age	.0185346	.0084873	2.18	0.029	.0018999	.0351694
grade	.0452772	.0125776	3.60	0.000	.0206255	.069929
not_smsa	-.1886592	.0642068	-2.94	0.003	-.3145021	-.0628162
1.south	-1.422292	.506517	-2.81	0.005	-2.415047	-.4295365
year	-.0133007	.0090628	-1.47	0.142	-.0310633	.004462
south#c.year						
1	.0105659	.0063175	1.67	0.094	-.0018162	.022948
_cons	-1.219801	.5175129	-2.36	0.018	-2.234107	-.2054942

These standard errors are larger than those reported by the inappropriate conventional calculation. By comparison, another way we could fit this model is with an equal-correlation population-averaged complementary log-log model:

```
. xtcloglog union age grade not_smsa south##c.year, pa nolog
GEE population-averaged model      Number of obs   =   26,200
Group variable: idcode             Number of groups =    4,434
Family: Binomial                   Obs per group:
Link: Complementary log-log        min =           1
Correlation: exchangeable         avg =           5.9
                                      max =           12
                                      Wald chi2(6)    =   234.66
Scale parameter = 1                Prob > chi2     =    0.0000
```

union	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
age	.0153737	.0081156	1.89	0.058	-.0005326	.03128
grade	.0549518	.0095093	5.78	0.000	.0363139	.0735897
not_smsa	-.1045232	.0431082	-2.42	0.015	-.1890138	-.0200326
1.south	-1.714868	.3384558	-5.07	0.000	-2.378229	-1.051507
year	-.0115881	.0084125	-1.38	0.168	-.0280763	.0049001
south#c.year						
1	.0149796	.0041687	3.59	0.000	.0068091	.0231501
_cons	-1.488278	.4468005	-3.33	0.001	-2.363991	-.6125652

The coefficient estimates are similar, but these standard errors are smaller than those produced by `cloglog, vce(cluster clustvar)`. This finding is as we would expect. If the within-panel correlation assumptions are valid, the population-averaged estimator should be more efficient.

In addition to this estimator, we may use the `xtgee` command to fit a panel estimator (with complementary log–log link) and any number of assumptions on the within-idcode correlation.

`cloglog, vce(cluster clustvar)` is robust to assumptions about within-cluster correlation. That is, it inefficiently sums within cluster for the standard error calculation rather than attempting to exploit what might be assumed about the within-cluster correlation (as do the `xtgee` population-averaged models).

Stored results

`cloglog` stores the following in `e()`:

Scalars

<code>e(N)</code>	number of observations
<code>e(k)</code>	number of parameters
<code>e(k_eq)</code>	number of equations in <code>e(b)</code>
<code>e(k_eq_model)</code>	number of equations in overall model test
<code>e(k_dv)</code>	number of dependent variables
<code>e(N_f)</code>	number of zero outcomes
<code>e(N_s)</code>	number of nonzero outcomes
<code>e(df_m)</code>	model degrees of freedom
<code>e(ll)</code>	log likelihood
<code>e(ll_0)</code>	log likelihood, constant-only model
<code>e(N_clust)</code>	number of clusters
<code>e(chi2)</code>	χ^2
<code>e(p)</code>	p -value for model test
<code>e(rank)</code>	rank of <code>e(V)</code>
<code>e(ic)</code>	number of iterations
<code>e(rc)</code>	return code
<code>e(converged)</code>	1 if converged, 0 otherwise

Macros

<code>e(cmd)</code>	<code>cloglog</code>
<code>e(cmdline)</code>	command as typed
<code>e(depvar)</code>	name of dependent variable
<code>e(wtype)</code>	weight type
<code>e(wexp)</code>	weight expression
<code>e(title)</code>	title in estimation output
<code>e(clustvar)</code>	name of cluster variable
<code>e(offset)</code>	linear offset variable
<code>e(chi2type)</code>	Wald or LR; type of model χ^2 test
<code>e(vce)</code>	<i>vcetype</i> specified in <code>vce()</code>
<code>e(vcetype)</code>	title used to label Std. err.
<code>e(opt)</code>	type of optimization
<code>e(which)</code>	max or min; whether optimizer is to perform maximization or minimization
<code>e(ml_method)</code>	type of ml method
<code>e(user)</code>	name of likelihood-evaluator program
<code>e(technique)</code>	maximization technique
<code>e(properties)</code>	<code>b V</code>
<code>e(predict)</code>	program used to implement <code>predict</code>
<code>e(marginsok)</code>	predictions allowed by <code>margins</code>
<code>e(marginsnotok)</code>	predictions disallowed by <code>margins</code>
<code>e(asbalanced)</code>	factor variables <code>fvset</code> as <code>asbalanced</code>
<code>e(asobserved)</code>	factor variables <code>fvset</code> as <code>asobserved</code>

Matrices	
e(b)	coefficient vector
e(Cns)	constraints matrix
e(iolog)	iteration log (up to 20 iterations)
e(gradient)	gradient vector
e(V)	variance–covariance matrix of the estimators
e(V_modelbased)	model-based variance
Functions	
e(sample)	marks estimation sample

In addition to the above, the following is stored in `r()`:

Matrices	
r(table)	matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

Methods and formulas

Complementary log–log analysis (related to the gompit model, so named because of its relationship to the Gompertz distribution) is an alternative to logit and probit analysis, but it is unlike these other estimators in that the transformation is not symmetric. Typically, this model is used when the positive (or negative) outcome is rare.

The log-likelihood function for complementary log–log is

$$\ln L = \sum_{j \in S} w_j \ln F(\mathbf{x}_j \mathbf{b}) + \sum_{j \notin S} w_j \ln \{1 - F(\mathbf{x}_j \mathbf{b})\}$$

where *S* is the set of all observations *j* such that $y_j \neq 0$, $F(z) = 1 - \exp\{-\exp(z)\}$, and w_j denotes the optional weights. $\ln L$ is maximized as described in [R] **Maximize**.

We can fit a gompit model by reversing the success–failure sense of the dependent variable and using `cloglog`.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] **_robust**, particularly *Maximum likelihood estimators* and *Methods and formulas*. The scores are calculated as $\mathbf{u}_j = [\exp(\mathbf{x}_j \mathbf{b}) \exp\{-\exp(\mathbf{x}_j \mathbf{b})\} / F(\mathbf{x}_j \mathbf{b})] \mathbf{x}_j$ for the positive outcomes and $\{-\exp(\mathbf{x}_j \mathbf{b})\} \mathbf{x}_j$ for the negative outcomes.

`cloglog` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] **Variance estimation**.

Acknowledgment

We thank Joseph Hilbe (1944–2017), coauthor of the Stata Press book *Generalized Linear Models and Extensions*, for providing the inspiration for the `cloglog` command.

References

- Long, J. S. 1997. *Regression Models for Categorical and Limited Dependent Variables*. Thousand Oaks, CA: Sage.
- Long, J. S., and J. Freese. 2014. *Regression Models for Categorical Dependent Variables Using Stata*. 3rd ed. College Station, TX: Stata Press.
- Xu, J., and J. S. Long. 2005. Confidence intervals for predicted outcomes in regression models for categorical outcomes. *Stata Journal* 5: 537–559.

Also see

- [R] **cloglog postestimation** — Postestimation tools for cloglog
- [R] **clogit** — Conditional (fixed-effects) logistic regression
- [R] **glm** — Generalized linear models
- [R] **logistic** — Logistic regression, reporting odds ratios
- [R] **scobit** — Skewed logistic regression
- [BAYES] **bayes: cloglog** — Bayesian complementary log–log regression
- [FMM] **fmm: cloglog** — Finite mixtures of complementary log–log regression models
- [ME] **mecloglog** — Multilevel mixed-effects complementary log–log regression
- [MI] **Estimation** — Estimation commands for use with mi estimate
- [SVY] **svy estimation** — Estimation commands for survey data
- [XT] **xtcloglog** — Random-effects and population-averaged cloglog models
- [U] **20 Estimation and postestimation commands**

Postestimation commands

The following postestimation commands are available after `cloglog`:

Command	Description
<code>contrast</code>	contrasts and ANOVA-style joint tests of parameters
<code>estat ic</code>	Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively)
<code>estat summarize</code>	summary statistics for the estimation sample
<code>estat vce</code>	variance-covariance matrix of the estimators (VCE)
<code>estat (svy)</code>	postestimation statistics for survey data
<code>estimates</code>	cataloging estimation results
<code>etable</code>	table of estimation results
* <code>forecast</code>	dynamic forecasts and simulations
* <code>hausman</code>	Hausman's specification test
<code>lincom</code>	point estimates, standard errors, testing, and inference for linear combinations of parameters
<code>linktest</code>	link test for model specification
* <code>lrtest</code>	likelihood-ratio test
<code>margins</code>	marginal means, predictive margins, marginal effects, and average marginal effects
<code>marginsplot</code>	graph the results from margins (profile plots, interaction plots, etc.)
<code>nlcom</code>	point estimates, standard errors, testing, and inference for nonlinear combinations of parameters
<code>predict</code>	probabilities, linear predictions and their SEs, etc.
<code>predictnl</code>	point estimates, standard errors, testing, and inference for generalized predictions
<code>pwcompare</code>	pairwise comparisons of parameters
<code>suest</code>	seemingly unrelated estimation
<code>test</code>	Wald tests of simple and composite linear hypotheses
<code>testnl</code>	Wald tests of nonlinear hypotheses

*`forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results. `forecast` is also not appropriate with `mi` estimation results.

predict

Description for predict

`predict` creates a new variable containing predictions such as probabilities, linear predictions, standard errors, and the equation-level score.

Menu for predict

Statistics > Postestimation

Syntax for predict

```
predict [type] newvar [if] [in] [ , statistic nooffset ]
```

<i>statistic</i>	Description
------------------	-------------

Main

<code>pr</code>	probability of a positive outcome; the default
<code>xb</code>	linear prediction
<code>stdp</code>	standard error of the linear prediction
<code>score</code>	first derivative of the log likelihood with respect to $\mathbf{x}_j\boldsymbol{\beta}$

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

Options for predict

Main

`pr`, the default, calculates the probability of a positive outcome.

`xb` calculates the linear prediction.

`stdp` calculates the standard error of the linear prediction.

`score` calculates the equation-level score, $\partial \ln L / \partial (\mathbf{x}_j\boldsymbol{\beta})$.

`nooffset` is relevant only if you specified `offset` (*varname*) for `cloglog`. It modifies the calculations made by `predict` so that they ignore the offset variable; the linear prediction is treated as $\mathbf{x}_j\mathbf{b}$ rather than as $\mathbf{x}_j\mathbf{b} + \text{offset}_j$.

margins

Description for margins

`margins` estimates margins of response for probabilities and linear predictions.

Menu for margins

Statistics > Postestimation

Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

<i>statistic</i>	Description
<code>pr</code>	probability of a positive outcome; the default
<code>xb</code>	linear prediction
<code>stdp</code>	not allowed with <code>margins</code>
<code>score</code>	not allowed with <code>margins</code>

Statistics not allowed with `margins` are functions of stochastic quantities other than $e(b)$.

For the full syntax, see [R] [margins](#).

Remarks and examples

Once you have fit a model, you can obtain the predicted probabilities by using the `predict` command for both the estimation sample and other samples; see [U] [20 Estimation and postestimation commands](#) and [R] [predict](#). Here we will make only a few comments.

`predict` without arguments calculates the predicted probability of a positive outcome. With the `xb` option, it calculates the linear combination $\mathbf{x}_j\mathbf{b}$, where \mathbf{x}_j are the independent variables in the j th observation and \mathbf{b} is the estimated parameter vector.

With the `stdp` option, `predict` calculates the standard error of the linear prediction, which is not adjusted for replicated covariate patterns in the data.

▷ Example 1

In [example 1](#) in [\[R\] cloglog](#), we fit the complementary log–log model `cloglog foreign weight mpg`. To obtain predicted probabilities,

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. cloglog foreign weight mpg
(output omitted)
. predict p
(option pr assumed; Pr(foreign))
. summarize foreign p
```

Variable	Obs	Mean	Std. dev.	Min	Max
foreign	74	.2972973	.4601885	0	1
p	74	.2928348	.29732	.0032726	.9446067

◀

Also see

[\[R\] cloglog](#) — Complementary log–log regression

[\[U\] 20 Estimation and postestimation commands](#)

Description

`cls` clears the Results window, causing all text to be removed. This operation cannot be undone.

Syntax

`cls`

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

Description

`cnsreg` fits constrained linear regression models.

Quick start

Linear regression with coefficients for x_1 and x_2 constrained to equality

```
constraint 1 x1 = x2
```

```
cnsreg y x1 x2 x3, constraints(1)
```

Add constraint $x_2 = x_3$ to impose $x_1 = x_2 = x_3$

```
constraint 2 x2 = x3
```

```
cnsreg y x1 x2 x3, constraints(1 2)
```

Constrain the coefficient for x_4 to be -1

```
constraint 3 x4 = -1
```

```
cnsreg y x1 x2 x3 x4, constraints(1-3)
```

Menu

Statistics > Linear models and related > Constrained linear regression

Syntax

cnsmreg *depvar indepvars* [*if*] [*in*] [*weight*], constraints(*constraints*) [*options*]

<i>options</i>	Description
Model	
* <u>constraints</u> (<i>constraints</i>)	apply specified linear constraints
<u>noconstant</u>	suppress constant term
SE/Robust	
<u>vce</u> (<i>vcetype</i>)	<i>vcetype</i> may be <u>ols</u> , <u>robust</u> , <u>cluster</u> <i>clustvar</i> , <u>bootstrap</u> , or <u>jackknife</u>
Reporting	
<u>level</u> (#)	set confidence level; default is level(95)
<u>nocnsreport</u>	do not display constraints
<u>display_options</u>	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
<u>mse1</u>	force MSE to be 1
<u>collinear</u>	keep collinear variables
<u>coeflegend</u>	display legend instead of statistics

* constraints(*constraints*) is required.

indepvars may contain factor variables; see [U] 11.4.3 **Factor variables**.

depvar and *indepvars* may contain time-series operators; see [U] 11.4.4 **Time-series varlists**.

bayesboot, bootstrap, by, collect, fp, jackknife, mi estimate, rolling, statsby, and svy are allowed; see [U] 11.1.10 **Prefix commands**.

vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] **mi estimate**.

With the fp prefix (see [R] **fp**), constraints cannot be specified for the variable containing fractional polynomial terms.

Weights are not allowed with the bootstrap prefix; see [R] **bootstrap**.

aweights are not allowed with the jackknife prefix; see [R] **jackknife**.

vce(), mse1, and weights are not allowed with the svy prefix; see [SVY] **svy**.

aweights, fweights, iweights, and pweights are allowed; see [U] 11.1.6 **weight**.

mse1, collinear, and coeflegend do not appear in the dialog.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

Options

Model

constraints(*constraints*), noconstant; see [R] **Estimation options**.

SE/Robust

vce(*vcetype*) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (ols), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster *clustvar*), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] **vce_option**.

vce(ols), the default, uses the standard variance estimator for ordinary least-squares regression.

Reporting

level(#), nocnsreport; see [R] [Estimation options](#).

display_options: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no stretch`; see [R] [Estimation options](#).

The following options are available with `cnsreg` but are not shown in the dialog box:

`mse1` is used only in programs and ado-files that use `cnsreg` to fit models other than constrained linear regression. `mse1` sets the mean squared error to 1, thus forcing the variance–covariance matrix of the estimators to be $(\mathbf{X}'\mathbf{DX})^{-1}$ (see [Methods and formulas](#) in [R] [regress](#)) and affecting calculated standard errors. Degrees of freedom for t statistics are calculated as n rather than $n - p + c$, where p is the total number of parameters (prior to restrictions and including the constant) and c is the number of constraints.

`mse1` is not allowed with the `svy` prefix.

`collinear`, `coeflegend`; see [R] [Estimation options](#).

Remarks and examples

For a discussion of constrained linear regression, see [Greene \(2018, 126–127\)](#); [Hill, Griffiths, and Lim \(2018, 271–273\)](#); or [Davidson and MacKinnon \(1993, 17\)](#).

▷ Example 1: One constraint

In principle, we can obtain constrained linear regression estimates by modifying the list of independent variables. For instance, if we wanted to fit the model

$$\text{mpg} = \beta_0 + \beta_1 \text{price} + \beta_2 \text{weight} + u$$

and constrain $\beta_1 = \beta_2$, we could write

$$\text{mpg} = \beta_0 + \beta_1(\text{price} + \text{weight}) + u$$

and run a regression of `mpg` on `price + weight`. The estimated coefficient on the sum would be the constrained estimate of β_1 and β_2 . Using `cnsreg`, however, is easier:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
```

```
. constraint 1 price = weight
. cnsreg mpg price weight, constraint(1)
```

```
Constrained linear regression
```

```
Number of obs =    74
F(1, 72)       =   37.59
Prob > F       =   0.0000
Root MSE      =   4.7220
```

```
( 1) price - weight = 0
```

	mpg	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
	price	-.0009875	.0001611	-6.13	0.000	-.0013086	-.0006664
	weight	-.0009875	.0001611	-6.13	0.000	-.0013086	-.0006664
	_cons	30.36718	1.577958	19.24	0.000	27.22158	33.51278

We define constraints by using the `constraint` command; see [R] [constraint](#). We fit the model with `cnsreg` and specify the constraint number or numbers in the `constraints()` option.

Just to show that the results above are correct, here is the result of applying the constraint by hand:

```
. generate x = price + weight
. regress mpg x
```

Source	SS	df	MS	Number of obs	=	74
Model	838.065767	1	838.065767	F(1, 72)	=	37.59
Residual	1605.39369	72	22.2971346	Prob > F	=	0.0000
Total	2443.45946	73	33.4720474	R-squared	=	0.3430
				Adj R-squared	=	0.3339
				Root MSE	=	4.722

mpg	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
x	-.0009875	.0001611	-6.13	0.000	-.0013086	-.0006664
_cons	30.36718	1.577958	19.24	0.000	27.22158	33.51278



▷ Example 2: Multiple constraints

Models can be fit subject to multiple simultaneous constraints. We simply define the constraints and then include the constraint numbers in the `constraints()` option. For instance, say that we wish to fit the model

$$\text{mpg} = \beta_0 + \beta_1 \text{price} + \beta_2 \text{weight} + \beta_3 \text{displ} + \beta_4 \text{gear_ratio} + \beta_5 \text{foreign} + \beta_6 \text{length} + u$$

subject to the constraints

$$\begin{aligned} \beta_1 &= \beta_2 = \beta_3 = \beta_6 \\ \beta_4 &= -\beta_5 = \beta_0/20 \end{aligned}$$

(This model, like the one in example 1, is admittedly senseless.) We fit the model by typing

```
. constraint 1 price=weight
. constraint 2 displ=weight
. constraint 3 length=weight
. constraint 5 gear_ratio = -foreign
. constraint 6 gear_ratio = _cons/20
```

```
. cnsreg mpg price weight displ gear_ratio foreign length, c(1-3,5-6)
Constrained linear regression                                Number of obs =    74
                                                           F(2, 72)          = 785.20
                                                           Prob > F          = 0.0000
                                                           Root MSE         = 4.6823

( 1) price - weight = 0
( 2) - weight + displacement = 0
( 3) - weight + length = 0
( 4) gear_ratio + foreign = 0
( 5) gear_ratio - .05*_cons = 0
```

mpg	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
price	-.000923	.0001534	-6.02	0.000	-.0012288	-.0006172
weight	-.000923	.0001534	-6.02	0.000	-.0012288	-.0006172
displacement	-.000923	.0001534	-6.02	0.000	-.0012288	-.0006172
gear_ratio	1.326114	.0687589	19.29	0.000	1.189046	1.463183
foreign	-1.326114	.0687589	-19.29	0.000	-1.463183	-1.189046
length	-.000923	.0001534	-6.02	0.000	-.0012288	-.0006172
_cons	26.52229	1.375178	19.29	0.000	23.78092	29.26365

There are many ways we could have specified the `constraints()` option (which we abbreviated `c()` above). We typed `c(1-3,5-6)`, meaning that we want constraints 1 through 3 and 5 and 6; those numbers correspond to the constraints we defined. The only reason we did not use the number 4 was to emphasize that constraints do not have to be consecutively numbered. We typed `c(1-3,5-6)`, but we could have typed `c(1,2,3,5,6)` or `c(1-3,5,6)` or `c(1-2,3,5,6)` or even `c(1-6)`, which would have worked as long as constraint 4 was not defined. If we had previously defined a constraint 4, then `c(1-6)` would have included it.

◀

Stored results

`cnsreg` stores the following in `e()`:

Scalars

<code>e(N)</code>	number of observations
<code>e(df_m)</code>	model degrees of freedom
<code>e(df_r)</code>	residual degrees of freedom
<code>e(F)</code>	<i>F</i> statistic
<code>e(p)</code>	<i>p</i> -value for model test
<code>e(rmse)</code>	root mean squared error
<code>e(ll)</code>	log likelihood
<code>e(N_clust)</code>	number of clusters
<code>e(rank)</code>	rank of $e(V)$

Macros

<code>e(cmd)</code>	<code>cnsreg</code>
<code>e(cmdline)</code>	command as typed
<code>e(depvar)</code>	name of dependent variable
<code>e(wtype)</code>	weight type
<code>e(wexp)</code>	weight expression
<code>e(title)</code>	title in estimation output
<code>e(clustvar)</code>	name of cluster variable
<code>e(vce)</code>	<i>vce</i> type specified in <code>vce()</code>
<code>e(vctype)</code>	title used to label Std. err.
<code>e(properties)</code>	<code>b V</code>

e(predict)	program used to implement predict
e(marginsok)	predictions allowed by margins
e(asbalanced)	factor variables fvset as asbalanced
e(asobserved)	factor variables fvset as asobserved
Matrices	
e(b)	coefficient vector
e(Cns)	constraints matrix
e(V)	variance–covariance matrix of the estimators
e(V_modelbased)	model-based variance
Functions	
e(sample)	marks estimation sample

In addition to the above, the following is stored in `r()`:

Matrices	
r(table)	matrix containing the coefficients with their standard errors, test statistics, p -values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

Methods and formulas

Let n be the number of observations, p be the total number of parameters (prior to restrictions and including the constant), and c be the number of constraints. The coefficients are calculated as $\mathbf{b}' = \mathbf{T}\{(\mathbf{T}'\mathbf{X}'\mathbf{W}\mathbf{X}\mathbf{T})^{-1}(\mathbf{T}'\mathbf{X}'\mathbf{W}\mathbf{y} - \mathbf{T}'\mathbf{X}'\mathbf{W}\mathbf{X}\mathbf{a}')\} + \mathbf{a}'$, where \mathbf{T} and \mathbf{a} are as defined in [P] [makecns](#). $\mathbf{W} = \mathbf{I}$ if no weights are specified. If weights are specified, let $\mathbf{v}: 1 \times n$ be the specified weights. If `weight` frequency weights are specified, $\mathbf{W} = \text{diag}(\mathbf{v})$. If `weight` analytic weights are specified, then $\mathbf{W} = \text{diag}[\mathbf{v}/(\mathbf{1}'\mathbf{v})(\mathbf{1}'\mathbf{1})]$, meaning that the weights are normalized to sum to the number of observations.

The mean squared error is $s^2 = (\mathbf{y}'\mathbf{W}\mathbf{y} - 2\mathbf{b}'\mathbf{X}'\mathbf{W}\mathbf{y} + \mathbf{b}'\mathbf{X}'\mathbf{W}\mathbf{X}\mathbf{b})/(n - p + c)$. The variance–covariance matrix is $s^2\mathbf{T}(\mathbf{T}'\mathbf{X}'\mathbf{W}\mathbf{X}\mathbf{T})^{-1}\mathbf{T}'$.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] [_robust](#), particularly [Introduction](#) and [Methods and formulas](#).

`cnsreg` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] [Variance estimation](#).

References

- Christodoulou, D. 2020. [Stata tip 137: Interpreting constraints on slopes of rank-deficient design matrices](#). *Stata Journal* 20: 493–498.
- Davidson, R., and J. G. MacKinnon. 1993. *Estimation and Inference in Econometrics*. New York: Oxford University Press.
- Greene, W. H. 2018. *Econometric Analysis*. 8th ed. New York: Pearson.
- Hill, R. C., W. E. Griffiths, and G. C. Lim. 2018. *Principles of Econometrics*. 5th ed. Hoboken, NJ: Wiley.

Also see

- [R] [cnsreg postestimation](#) — Postestimation tools for cnsreg
- [R] [regress](#) — Linear regression
- [MI] [Estimation](#) — Estimation commands for use with mi estimate
- [SVY] [svy estimation](#) — Estimation commands for survey data
- [U] [20 Estimation and postestimation commands](#)

Postestimation commands

The following postestimation commands are available after `cnsreg`:

Command	Description
<code>contrast</code>	contrasts and ANOVA-style joint tests of parameters
<code>estat ic</code>	Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively)
<code>estat summarize</code>	summary statistics for the estimation sample
<code>estat vce</code>	variance–covariance matrix of the estimators (VCE)
<code>estat (svy)</code>	postestimation statistics for survey data
<code>estimates</code>	cataloging estimation results
<code>etable</code>	table of estimation results
* <code>forecast</code>	dynamic forecasts and simulations
* <code>hausman</code>	Hausman's specification test
<code>lincom</code>	point estimates, standard errors, testing, and inference for linear combinations of parameters
<code>linktest</code>	link test for model specification
* <code>lrtest</code>	likelihood-ratio test
<code>margins</code>	marginal means, predictive margins, marginal effects, and average marginal effects
<code>marginsplot</code>	graph the results from margins (profile plots, interaction plots, etc.)
<code>nlcom</code>	point estimates, standard errors, testing, and inference for nonlinear combinations of parameters
<code>predict</code>	predictions and their SEs, residuals, etc.
<code>predictnl</code>	point estimates, standard errors, testing, and inference for generalized predictions
<code>pwcompare</code>	pairwise comparisons of parameters
<code>suest</code>	seemingly unrelated estimation
<code>test</code>	Wald tests of simple and composite linear hypotheses
<code>testnl</code>	Wald tests of nonlinear hypotheses

*`forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results. `forecast` is also not appropriate with `mi` estimation results.

predict

Description for predict

`predict` creates a new variable containing predictions such as linear predictions, residuals, standard errors, probabilities, and expected values.

Menu for predict

Statistics > Postestimation

Syntax for predict

```
predict [type] newvar [if] [in] [ , statistic]
```

<i>statistic</i>	Description
Main	
<code>xb</code>	linear prediction; the default
<code>residuals</code>	residuals
<code>stdp</code>	standard error of the prediction
<code>stdf</code>	standard error of the forecast
<code>pr(a,b)</code>	$\Pr(a < y_j < b)$
<code>e(a,b)</code>	$E(y_j a < y_j < b)$
<code>ystar(a,b)</code>	$E(y_j^*), y_j^* = \max\{a, \min(y_j, b)\}$
<code>score</code>	equivalent to residuals

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

`stdf` is not allowed with `svy` estimation results.

where a and b may be numbers or variables; a missing ($a \geq .$) means $-\infty$, and b missing ($b \geq .$) means $+\infty$; see [U] 12.2.1 Missing values.

Options for predict

Main

`xb`, the default, calculates the linear prediction.

`residuals` calculates the residuals, that is, $y_j - \mathbf{x}_j\mathbf{b}$.

`stdp` calculates the standard error of the prediction, which can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. The standard error of the prediction is also referred to as the standard error of the fitted value.

`stdf` calculates the standard error of the forecast, which is the standard error of the point prediction for 1 observation. It is commonly referred to as the standard error of the future or forecast value. By construction, the standard errors produced by `stdf` are always larger than those produced by `stdp`; see *Methods and formulas* in [R] regress postestimation.

`pr(a,b)` calculates $\Pr(a < \mathbf{x}_j\mathbf{b} + u_j < b)$, the probability that $y_j|\mathbf{x}_j$ would be observed in the interval (a, b) .

a and b may be specified as numbers or variable names; lb and ub are variable names;

`pr(20,30)` calculates $\Pr(20 < \mathbf{x}_j\mathbf{b} + u_j < 30)$;

`pr(lb,ub)` calculates $\Pr(lb < \mathbf{x}_j\mathbf{b} + u_j < ub)$; and

`pr(20,ub)` calculates $\Pr(20 < \mathbf{x}_j\mathbf{b} + u_j < ub)$.

a missing ($a \geq .$) means $-\infty$; `pr(.,30)` calculates $\Pr(-\infty < \mathbf{x}_j\mathbf{b} + u_j < 30)$;

`pr(lb,30)` calculates $\Pr(-\infty < \mathbf{x}_j\mathbf{b} + u_j < 30)$ in observations for which $lb \geq .$

and calculates $\Pr(lb < \mathbf{x}_j\mathbf{b} + u_j < 30)$ elsewhere.

b missing ($b \geq .$) means $+\infty$; `pr(20,.)` calculates $\Pr(+\infty > \mathbf{x}_j\mathbf{b} + u_j > 20)$;

`pr(20,ub)` calculates $\Pr(+\infty > \mathbf{x}_j\mathbf{b} + u_j > 20)$ in observations for which $ub \geq .$

and calculates $\Pr(20 < \mathbf{x}_j\mathbf{b} + u_j < ub)$ elsewhere.

`e(a,b)` calculates $E(\mathbf{x}_j\mathbf{b} + u_j \mid a < \mathbf{x}_j\mathbf{b} + u_j < b)$, the expected value of $y_j \mid \mathbf{x}_j$ conditional on $y_j \mid \mathbf{x}_j$ being in the interval (a, b) , meaning that $y_j \mid \mathbf{x}_j$ is truncated. a and b are specified as they are for `pr()`.

`ystar(a,b)` calculates $E(y_j^*)$, where $y_j^* = a$ if $\mathbf{x}_j\mathbf{b} + u_j \leq a$, $y_j^* = b$ if $\mathbf{x}_j\mathbf{b} + u_j \geq b$, and $y_j^* = \mathbf{x}_j\mathbf{b} + u_j$ otherwise, meaning that y_j^* is censored. a and b are specified as they are for `pr()`.

`score` is equivalent to `residuals` for linear regression models.

margins

Description for margins

`margins` estimates margins of response for linear predictions, probabilities, and expected values.

Menu for margins

Statistics > Postestimation

Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

<i>statistic</i>	Description
<code>xb</code>	linear prediction; the default
<code>pr(<i>a</i>,<i>b</i>)</code>	$\Pr(a < y_j < b)$
<code>e(<i>a</i>,<i>b</i>)</code>	$E(y_j a < y_j < b)$
<code>ystar(<i>a</i>,<i>b</i>)</code>	$E(y_j^*, y_j^* = \max\{a, \min(y_j, b)\})$
<code>stdp</code>	not allowed with <code>margins</code>
<code>stdf</code>	not allowed with <code>margins</code>
<code>residuals</code>	not allowed with <code>margins</code>
<code>score</code>	not allowed with <code>margins</code>

Statistics not allowed with `margins` are functions of stochastic quantities other than $e(b)$.

For the full syntax, see [R] [margins](#).

Also see

[R] [cnsreg](#) — Constrained linear regression

[U] [20 Estimation and postestimation commands](#)

Description

`constraint` defines, lists, and drops linear constraints. Constraints are for use by models that allow constrained estimation.

Constraints are defined by the `constraint` command. The currently defined constraints can be listed by either `constraint list` or `constraint dir`; both do the same thing. Existing constraints can be eliminated by `constraint drop`.

`constraint get` and `constraint free` are programmer's commands. `constraint get` returns the contents of the specified constraint in macro `r(contents)` and returns in scalar `r(defined)` 0 or 1—1 being returned if the constraint was defined. `constraint free` returns the number of a free (unused) constraint in macro `r(free)`.

Quick start

For single-equation models

Define constraint 1 to constrain the coefficient for `x1` to 0

```
constraint define 1 x1=0
```

Same as above

```
constraint 1 x1
```

Constrain coefficients for `x1` and `x2` to 0

```
constraint 2 x1 x2
```

Overwrite constraint 2 to constrain `x2` and `x3` to equality

```
constraint 2 x2 = x3
```

Constrain the coefficients for [factor indicators](#) `2.a` and `3.a` to equality

```
constraint 3 2.a = 3.a
```

Constrain the coefficient on `x1` to equal 1

```
constraint 4 x1 = 1
```

For multiple-equation models

Constrain coefficient for `x4` to 0 in all equations

```
constraint 11 x4
```

Constrain coefficients for `x4` and `x5` to equality in the equation for `y2`

```
constraint 12 [y2]x4 = [y2]x5
```

Constrain the coefficient for x5 to equality in equations for y1 and y2

```
constraint 13 [y1=y2] x5
```

Constrain coefficient for x1 to 0 in equation where the dependent variable equals cat2

```
constraint 14 [cat2] x1
```

Constrain the coefficients for [factor indicators](#) 1.a and 1.b to equality in the equation for category cat3

```
constraint 15 [cat3] : 1.a = 1.b
```

Constrain coefficients for x1 to equality in the equations for categories cat2 and cat3

```
constraint 16 [cat2=cat3] : x1
```

Listing constraints

List existing constraints

```
constraint dir
```

Same as above

```
constraint list
```

Menu

Statistics > Other > Manage constraints

Syntax

Define constraints

```
constraint [define] # [exp=exp | coeflist]
```

List constraints

```
constraint dir [numlist | _all]
```

```
constraint list [numlist | _all]
```

Drop constraints

```
constraint drop { numlist | _all }
```

Programmer's commands

```
constraint get #
```

```
constraint free
```

where *coeflist* is as defined in [R] [test](#) and # is restricted to the range 1 to 1,999, inclusive.

Remarks and examples

Using constraints is discussed in [R] [cnsreg](#), [R] [mlogit](#), and [R] [reg3](#); this entry is concerned only with practical aspects of defining and manipulating constraints.

▷ Example 1

Constraints are numbered from 1 to 1,999, and we assign the number when we define the constraint:

```
. use https://www.stata-press.com/data/r19/sysdsn1
(Health insurance data)
. constraint 2 [indemnity]2.site = 0
```

The currently defined constraints can be listed by `constraint list`:

```
. constraint list
      2: [indemnity]2.site = 0
```

`constraint drop` drops constraints:

```
. constraint drop 2
. constraint list
```

The empty list after `constraint list` indicates that no constraints are defined. Below, we demonstrate the various syntaxes allowed by `constraint`:

```
. constraint 1 [Indemnity]
. constraint 10 [Indemnity]: 1.site 2.site
. constraint 11 [Indemnity]: 3.site
. constraint 21 [Prepaid=Uninsure]: nonwhite
. constraint 30 [Prepaid]
. constraint 31 [Insure]
. constraint list
  1: [Indemnity]
 10: [Indemnity]: 1.site 2.site
 11: [Indemnity]: 3.site
 21: [Prepaid=Uninsure]: nonwhite
 30: [Prepaid]
 31: [Insure]
. constraint drop 21-25, 31
. constraint list
  1: [Indemnity]
 10: [Indemnity]: 1.site 2.site
 11: [Indemnity]: 3.site
 30: [Prepaid]
. constraint drop _all
. constraint list
```

◀

□ Technical note

The `constraint` command does not check the syntax of the constraint itself because a constraint can be interpreted only in the context of a model. Thus, `constraint` is willing to define constraints that later will not make sense. Any errors in the constraints will be detected and mentioned at the time of estimation.

□

Reference

Buis, M. L. 2012. [Stata tip 108: On adding and constraining](#). *Stata Journal* 12: 342–344.

Also see

[R] [cnsreg](#) — Constrained linear regression

[P] [makecns](#) — Constrained estimation

Description

Quick start

Menu

Syntax

Options

Remarks and examples

Stored results

Methods and formulas

References

Also see

Description

`contrast` tests linear hypotheses and forms contrasts involving factor variables and their interactions from the most recently fit model. The tests include ANOVA-style tests of main effects, simple effects, interactions, and nested effects. `contrast` can use named contrasts to decompose these effects into comparisons against reference categories, comparisons of adjacent levels, comparisons against the grand mean, orthogonal polynomials, and such. Custom contrasts may also be specified.

`contrast` can be used with `svy` estimation results; see [\[SVY\] svy postestimation](#).

Contrasts can also be computed for margins of linear and nonlinear responses; see [\[R\] margins, contrast](#).

Quick start

Contrasts for one-way models

Test the main effect of categorical variable `a` after `regress y i.a` or `anova y a`

```
contrast a
```

Reference category contrasts of cell means of `y` with the smallest value of `a` as the base category

```
contrast r.a
```

Same as above, but specify `a = 3` as the base category for comparisons

```
contrast rb3.a
```

Report tests instead of confidence intervals for each contrast

```
contrast r.a, pffects
```

Report tests and confidence intervals for each contrast

```
contrast r.a, effects
```

Contrasts of the cell mean of `y` for each level of `a` with the grand mean of `y`

```
contrast g.a
```

Same as above, but compute grand mean as a weighted average of cell means with weights based on the number of observations for each level of `a`

```
contrast gw.a
```

User-defined contrast comparing the cell mean of `y` for `a = 1` with the average of the cell means for `a = 3` and `a = 4`

```
contrast {a -1 0 .5 .5}
```

Contrasts for two-way models

Test of the interaction effect after `regress y a##b` or `anova y a##b`
`contrast a#b`

Test of the main and interaction effects
`contrast a b a#b`

Same as above
`contrast a##b`

Individual reference category contrasts for the interaction of a and b
`contrast r.a#r.b`

Joint tests of the simple effects of a within each level of b
`contrast a@b`

Individual reference category contrasts for the simple effects of a within each level of b
`contrast r.a@b`

Orthogonal polynomial contrasts for a within each level of b
`contrast p.a@b`

Reference category contrasts of the marginal means of y for levels of a
`contrast r.a`

Same as above, but with marginal means for a computed as a weighted average of cell means, using the marginal frequencies of b rather than equal weights for each level
`contrast r.a, asobserved`

Contrasts of the marginal mean of y for each level of a with the previous level—reverse-adjacent contrasts
`contrast ar.a`

Contrasts for models with continuous covariates

Test of the interaction effect after `regress y a##c.x` or `anova y a##c.x`
`contrast a#c.x`

Reference category effects of a on the slope of x
`contrast r.a#c.x`

Reference category effects of a on the intercept
`contrast r.a`

Contrasts for nonlinear models

Orthogonal polynomial contrasts of log odds across levels of a after `logit y i.a`
`contrast p.a`

Test the main and interaction effects after `logit y a##b`
`contrast a##b`

Simple reference category effects for a within each level of b
`contrast r.a@b`

Contrasts for multiple-equation models

Test the main and interaction effects in the equation for y2 after `mvreg y1 y2 y3 = a##b`
`contrast a##b, equation(y2)`

Reference category contrasts of estimated marginal means of y3 for levels of a
`contrast r.a, equation(y3)`

Test for a difference in the overall estimated marginal means of y1, y2, and y3
`contrast _eqns`

Contrasts of estimated marginal means of y2 and y3 with y1
`contrast r._eqns`

Test whether interaction effects differ across equations
`contrast a#b#_eqns`

Menu

Statistics > Postestimation

Syntax

contrast *termlist* [, *options*]

where *termlist* is a list of factor variables or interactions that appear in the current estimation results. The variables may be typed with or without [contrast operators](#), and you may use any factor-variable syntax:

See the *operators (op.)* table below for the list of contrast operators.

<i>options</i>	Description
Main	
<u>overall</u>	add a joint hypothesis test for all specified contrasts
<u>asobserved</u>	treat all factor variables as observed
<u>lincom</u>	treat user-defined contrasts as linear combinations
Equations	
<u>equation</u> (<i>eqspec</i>)	perform contrasts in <i>termlist</i> for equation <i>eqspec</i>
<u>atequations</u>	perform contrasts in <i>termlist</i> within each equation
Advanced	
<u>emptycells</u> (<i>empspec</i>)	treatment of empty cells for balanced factors
<u>noestimcheck</u>	suppress estimability checks
Reporting	
<u>level</u> (#)	confidence level; default is level(95)
<u>mcompare</u> (<i>method</i>)	adjust for multiple comparisons; default is mcompare(noadjust)
<u>noeffects</u>	suppress table of individual contrasts
<u>cieffects</u>	show effects table with confidence intervals
<u>pveffects</u>	show effects table with <i>p</i> -values
<u>effects</u>	show effects table with confidence intervals and <i>p</i> -values
<u>nowald</u>	suppress table of Wald tests
<u>noatlevels</u>	report only the overall Wald test for terms that use the within @ or nested operator
<u>nosvyadjust</u>	compute unadjusted Wald tests for survey results
<u>sort</u>	sort the individual contrast values in each term
<u>post</u>	post contrasts and their VCEs as estimation results
<u>display_options</u>	control column formats, row spacing, line width, and factor-variable labeling
<u>eform_option</u>	report exponentiated contrasts
<u>df</u> (#)	use <i>t</i> distribution with # degrees of freedom for computing <i>p</i> -values and confidence intervals

collect is allowed; see [U] [11.1.10 Prefix commands](#).

df(#) does not appear in the dialog box.

Term	Description
Main effects	
A	joint test of the main effects of A
$r . A$	individual contrasts that decompose A using r .
Interaction effects	
A#B	joint test of the two-way interaction effects of A and B
A#B#C	joint test of the three-way interaction effects of A, B, and C
$r . A \# g . B$	individual contrasts for each interaction of A and B defined by r . and g .
Partial interaction effects	
$r . A \# B$	joint tests of interactions of A and B within each contrast defined by r . A
A# r . B	joint tests of interactions of A and B within each contrast defined by r . B
Simple effects	
A@B	joint tests of the effects of A within each level of B
A@B#C	joint tests of the effects of A within each combination of the levels of B and C
$r . A @ B$	individual contrasts of A that decompose A@B using r .
$r . A @ B \# C$	individual contrasts of A that decompose A@B#C using r .
Other conditional effects	
A#B@C	joint tests of the interaction effects of A and B within each level of C
A#B@C#D	joint tests of the interaction effects of A and B within each combination of the levels of C and D
$r . A \# g . B @ C$	individual contrasts for each interaction of A and B that decompose A#B@C using r . and g .
Nested effects	
A B	joint tests of the effects of A nested in each level of B
A B#C	joint tests of the effects of A nested in each combination of the levels of B and C
A#B C	joint tests of the interaction effects of A and B nested in each level of C
A#B C#D	joint tests of the interaction effects of A and B nested in each combination of the levels of C and D
$r . A B$	individual contrasts of A that decompose A B using r .
$r . A B \# C$	individual contrasts of A that decompose A B#C using r .
$r . A \# g . B C$	individual contrasts for each interaction of A and B defined by r . and g . nested in each level of C
Slope effects	
A#c . x	joint test of the effects of A on the slopes of x
A#c . x#c . y	joint test of the effects of A on the slopes of the product (interaction) of x and y
A#B#c . x	joint test of the interaction effects of A and B on the slopes of x
A#B#c . x#c . y	joint test of the interaction effects of A and B on the slopes of the product (interaction) of x and y
$r . A \# c . x$	individual contrasts of A's effects on the slopes of x using r .
Denominators	
... / term2	use <i>term2</i> as the denominator in the F tests of the preceding terms
... /	use the residual as the denominator in the F tests of the preceding terms (the default if no other /s are specified)

A, B, C, and D represent any factor variable in the current estimation results.

x and y represent any continuous variable in the current estimation results.

r. and g. represent any contrast operator. See the table below.

c. specifies that a variable be treated as continuous; see [U] 11.4.3 Factor variables.

Operators are allowed on any factor variable that does not appear to the right of @ or |. Operators decompose the effects of the associated factor variable into one-degree-of-freedom effects (contrasts).

Higher-level interactions are allowed anywhere an interaction operator (#) appears in the table.

Time-series operators are allowed if they were used in the estimation.

_eqns designates the equations in `manova`, `mlogit`, `mprobit`, and `mvreg` and can be specified anywhere a factor variable appears.

/ is allowed only after `anova`, `cnsreg`, `manova`, `mvreg`, or `regress`.

<i>operators (op.)</i>	Description
r.	differences from the reference (base) level; the default
a.	differences from the next level (adjacent contrasts)
ar.	differences from the previous level (reverse adjacent contrasts)
As-balanced operators	
g.	differences from the balanced grand mean
h.	differences from the balanced mean of subsequent levels (Helmert contrasts)
j.	differences from the balanced mean of previous levels (reverse Helmert contrasts)
p.	orthogonal polynomial in the level values
q.	orthogonal polynomial in the level sequence
As-observed operators	
gw.	differences from the observation-weighted grand mean
hw.	differences from the observation-weighted mean of subsequent levels
jw.	differences from the observation-weighted mean of previous levels
pw.	observation-weighted orthogonal polynomial in the level values
qw.	observation-weighted orthogonal polynomial in the level sequence

One or more individual contrasts may be selected by using the *op#*. or *op(numlist)* . syntax. For example, `a3.A` selects the adjacent contrast for level 3 of A, and `p(1/2).B` selects the linear and quadratic effects of B. Also see *Orthogonal polynomial contrasts* and *Beyond linear models*.

Custom contrasts	Description
{A <i>numlist</i> }	user-defined contrast on the levels of factor A
{A#B <i>numlist</i> }	user-defined contrast on the levels of interaction between A and B

Custom contrasts may be part of a term, such as {A *numlist*}#B, {A *numlist*}@B, {A *numlist*}|B, {A#B *numlist*}, and {A *numlist*}#{B *numlist*}. The same is true of higher-order custom contrasts, such as {A#B *numlist*}@C, {A#B *numlist*}#r.C, and {A#B *numlist*}#c.x.

Higher-order interactions with at most eight factor variables are allowed with custom contrasts.

<i>method</i>	Description
<code>noadjust</code>	do not adjust for multiple comparisons; the default
<code>bonferroni [adjustall]</code>	Bonferroni's method; adjust across all terms
<code>sidak [adjustall]</code>	Šidák's method; adjust across all terms
<code>scheffe</code>	Scheffé's method

Options

Main

`overall` specifies that a joint hypothesis test over all terms be performed.

`asobserved` specifies that factor covariates be evaluated using the cell frequencies observed in the estimation sample. The default is to treat all factor covariates as though there were an equal number of observations in each level.

`lincom` specifies that user-defined contrasts be treated as linear combinations. The default is to require that all user-defined contrasts sum to zero. (Summing to zero is part of the definition of a contrast.)

Equations

`equation(eqspec)` specifies the equation from which contrasts are to be computed. The default is to compute contrasts from the first equation.

`atequations` specifies that the contrasts be computed within each equation.

Advanced

`emptycells(empspec)` specifies how empty cells are handled in interactions involving factor variables that are being treated as balanced.

`emptycells(strict)` is the default; it specifies that contrasts involving empty cells be treated as not estimable.

`emptycells(reweight)` specifies that the effects of the observed cells be increased to accommodate any missing cells. This makes the contrast estimable but changes its interpretation.

`noestimcheck` specifies that contrast not check for estimability. By default, the requested contrasts are checked and those found not estimable are reported as such. Nonestimability is usually caused by empty cells. If `noestimcheck` is specified, estimates are computed in the usual way and reported even though the resulting estimates are manipulable, which is to say they can differ across equivalent models having different parameterizations.

Reporting

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [\[U\] 20.8 Specifying the width of confidence intervals](#).

`mcompare(method)` specifies the method for computing *p*-values and confidence intervals that account for multiple comparisons within a factor-variable term.

Most methods adjust the comparisonwise error rate, α_c , to achieve a prespecified experimentwise error rate, α_e .

`mcompare(noadjust)` is the default; it specifies no adjustment.

$$\alpha_c = \alpha_e$$

`mcompare(bonferroni)` adjusts the comparisonwise error rate based on the upper limit of the Bonferroni inequality

$$\alpha_e \leq m\alpha_c$$

where m is the number of comparisons within the term.

The adjusted comparisonwise error rate is

$$\alpha_c = \alpha_e/m$$

`mcompare(sidak)` adjusts the comparisonwise error rate based on the upper limit of the probability inequality

$$\alpha_e \leq 1 - (1 - \alpha_c)^m$$

where m is the number of comparisons within the term.

The adjusted comparisonwise error rate is

$$\alpha_c = 1 - (1 - \alpha_e)^{1/m}$$

This adjustment is exact when the m comparisons are independent.

`mcompare(scheffe)` controls the experimentwise error rate using the F or χ^2 distribution with degrees of freedom equal to the rank of the term.

`mcompare(method adjustall)` specifies that the multiple-comparison adjustments count all comparisons across all terms rather than performing multiple comparisons term by term. This leads to more conservative adjustments when multiple variables or terms are specified in *marginslist*. This option is compatible only with the `bonferroni` and `sidak` methods.

`noeffects` suppresses the table of individual contrasts with confidence intervals. This table is produced by default when the `mcompare()` option is specified or when a term in *termlist* implies all individual contrasts.

`cieffects` specifies that a table containing a confidence interval for each individual contrast be reported.

`pveffects` specifies that a table containing a p -value for each individual contrast be reported.

`effects` specifies that a single table containing a confidence interval and p -value for each individual contrast be reported.

`nowald` suppresses the table of Wald tests.

`noatlevels` indicates that only the overall Wald test be reported for each term containing within or nested (@ or |) operators.

`nosvadjust` is for use with `svy` estimation commands. It specifies that the Wald test be carried out without the default adjustment for the design degrees of freedom. That is to say the test is carried out as $W/k \sim F(k, d)$ rather than as $(d - k + 1)W/(kd) \sim F(k, d - k + 1)$, where k is the dimension of the test and d is the total number of sampled PSUs minus the total number of strata.

`sort` specifies that the table of individual contrasts be sorted by the contrast values within each term.

post causes contrast to behave like a Stata estimation (`e-class`) command. `contrast` posts the vector of estimated contrasts along with the estimated variance–covariance matrix to `e()`, so you can treat the estimated contrasts just as you would results from any other estimation command. For example, you could use `test` to perform simultaneous tests of hypotheses on the contrasts, or you could use `lincom` to create linear combinations.

display_options: `vsquish`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`.

`vsquish` specifies that the blank space separating factor-variable terms or time-series–operated variables from other variables in the model be suppressed.

`nofvlabel` displays factor-variable level values rather than attached value labels. This option overrides the `fvlabel` setting; see [R] [set showbaselevels](#).

`fvwrap(#)` specifies how many lines to allow when long value labels must be wrapped. Labels requiring more than `#` lines are truncated. This option overrides the `fvwrap` setting; see [R] [set showbaselevels](#).

`fvwrapon(style)` specifies whether value labels that wrap will break at word boundaries or break based on available space.

`fvwrapon(word)`, the default, specifies that value labels break at word boundaries.

`fvwrapon(width)` specifies that value labels break based on available space.

This option overrides the `fvwrapon` setting; see [R] [set showbaselevels](#).

`cformat(%fmt)` specifies how to format contrasts, standard errors, and confidence limits in the table of estimated contrasts.

`pformat(%fmt)` specifies how to format p -values in the table of estimated contrasts.

`sformat(%fmt)` specifies how to format test statistics in the table of estimated contrasts.

`nolstretch` specifies that the width of the table of estimated contrasts not be automatically widened to accommodate longer variable names. The default, `lstretch`, is to automatically widen the table of estimated contrasts up to the width of the Results window. Specifying `lstretch` or `nolstretch` overrides the setting given by `set lstretch`. If `set lstretch` has not been set, the default is `lstretch`. `nolstretch` is not shown in the dialog box.

eform_option specifies that the contrasts table be displayed in exponentiated form. e^{contrast} is displayed rather than contrast. Standard errors and confidence intervals are also transformed. See [R] [eform_option](#) for the list of available options.

The following option is available with `contrast` but is not shown in the dialog box:

`df(#)` specifies that the t distribution with `#` degrees of freedom be used for computing p -values and confidence intervals. The default is to use `e(df_r)` degrees of freedom or the standard normal distribution if `e(df_r)` is missing.

Remarks and examples

Remarks are presented under the following headings:

- Introduction*
- One-way models*
 - Estimated cell means*
 - Testing equality of cell means*
 - Reference category contrasts*
 - Reverse adjacent contrasts*
 - Orthogonal polynomial contrasts*
- Two-way models*
 - Estimated interaction cell means*
 - Simple effects*
 - Interaction effects*
 - Main effects*
 - Partial interaction effects*
- Three-way and higher-order models*
- Contrast operators*
 - Differences from a reference level (r.)*
 - Differences from the next level (a.)*
 - Differences from the previous level (ar.)*
 - Differences from the grand mean (g.)*
 - Differences from the mean of subsequent levels (h.)*
 - Differences from the mean of previous levels (j.)*
 - Orthogonal polynomials (p. and q.)*
- User-defined contrasts*
- Empty cells*
- Empty cells, ANOVA style*
- Nested effects*
- Multiple comparisons*
- Unbalanced data*
 - Using observed cell frequencies*
 - Weighted contrast operators*
- Testing factor effects on slopes*
- Chow tests*
- Beyond linear models*
- Multiple equations*
- Video example*

Introduction

`contrast` performs ANOVA-style tests of main effects, interactions, simple effects, and nested effects. It can easily decompose these tests into constituent contrasts using either named contrasts (codings) or user-specified contrasts. Comparing levels of factor variables—whether as main effects, interactions, or simple effects—is as easy as adding a contrast operator to the variable. The operators can compare each level with the previous level, each level with a reference level, each level with the mean of previous levels, and more.

`contrast` tests and estimates contrasts. A contrast of the parameters $\mu_1, \mu_2, \dots, \mu_p$ is a linear combination $\sum_i c_i \mu_i$ whose c_i sum to zero. A difference of population means such as $\mu_1 - \mu_2$ is a contrast, as are most other comparisons of population or model quantities (Coster 2005). Some contrasts may be estimated with `lincom`, but `contrast` is much more powerful. `contrast` can handle multiple contrasts simultaneously, and the command's contrast operators make it easy to specify complicated linear combinations.

Both the contrast operation and the creation of the margins for comparison can be performed as though the data were balanced (typical for experimental designs) or using the observed frequencies in the estimation sample (typical for observational studies). `contrast` can perform these analyses on the results of almost all of Stata's estimators, not just the linear-models estimators.

Most of `contrast`'s computations can be considered comparisons of estimated cell means from a model fit. Tests of interactions are tests of whether the cell means for the interaction are all equal. Tests of main effects are tests of whether the marginal cell means for the factor are all equal. More focused comparisons of cell means (for example, is level 2 equal to level 1) are specified using contrast operators. More formally, all of `contrast`'s computations are comparisons of conditional expectations; cell means are one type of conditional expectation.

All contrasts can also easily be graphed; see [R] [marginsplot](#).

For a discussion of contrasts and testing for linear models, see [Searle and Gruber \(2017\)](#) and [Searle \(1997\)](#). For discussions specifically related to experimental design, see [Winer, Brown, and Michels \(1991\)](#) and [Milliken and Johnson \(2009\)](#). [Rosenthal, Rosnow, and Rubin \(2000\)](#) focus on contrasts with applications in behavioral sciences. [Mitchell \(2021, 2015\)](#) and [Baldwin \(2019\)](#) focus on contrasts in Stata.

`contrast` is a flexible tool for understanding the effects of categorical covariates. If your model contains categorical covariates, and especially if it contains interactions, you will want to use `contrast`.

One-way models

Suppose we have collected data on cholesterol levels for individuals from five age groups. To study the effect of age group on cholesterol, we can begin by fitting a one-way model using `regress`:

```
. use https://www.stata-press.com/data/r19/cholesterol
(Artificial cholesterol data)
. label list ages
ages:
      1 10-19
      2 20-29
      3 30-39
      4 40-59
      5 60-79
. regress chol i.agegrp
```

Source	SS	df	MS	Number of obs	=	75
Model	14943.3997	4	3735.84993	F(4, 70)	=	35.02
Residual	7468.21971	70	106.688853	Prob > F	=	0.0000
				R-squared	=	0.6668
				Adj R-squared	=	0.6477
Total	22411.6194	74	302.859722	Root MSE	=	10.329

chol	Coefficient	Std. err.	t	P> t	[95% conf. interval]
agegrp					
20-29	8.203575	3.771628	2.18	0.033	.6812991 15.72585
30-39	21.54105	3.771628	5.71	0.000	14.01878 29.06333
40-59	30.15067	3.771628	7.99	0.000	22.6284 37.67295
60-79	38.76221	3.771628	10.28	0.000	31.23993 46.28448
_cons	180.5198	2.666944	67.69	0.000	175.2007 185.8388

Estimated cell means

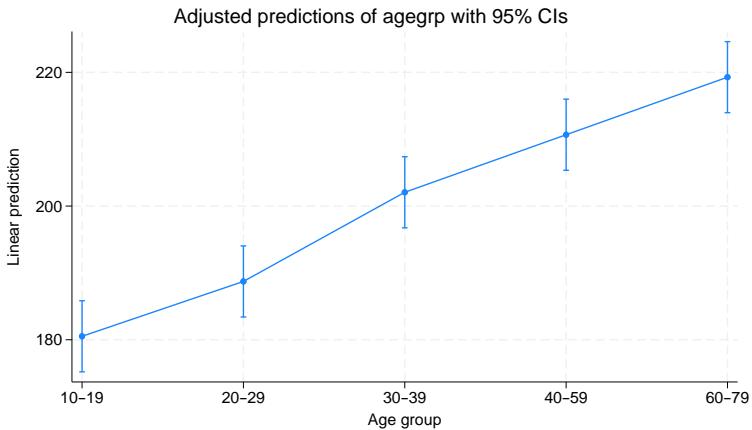
margins will show us the estimated cell means for each age group based on our fitted model:

```
. margins agegrp
Adjusted predictions          Number of obs = 75
Model VCE: OLS
Expression: Linear prediction, predict()
```

	Delta-method		t	P> t	[95% conf. interval]	
	Margin	std. err.				
agegrp						
10-19	180.5198	2.666944	67.69	0.000	175.2007	185.8388
20-29	188.7233	2.666944	70.76	0.000	183.4043	194.0424
30-39	202.0608	2.666944	75.76	0.000	196.7418	207.3799
40-59	210.6704	2.666944	78.99	0.000	205.3514	215.9895
60-79	219.282	2.666944	82.22	0.000	213.9629	224.601

We can graph those means with marginsplot:

```
. marginsplot
Variables that uniquely identify margins: agegrp
```



Testing equality of cell means

Are all the means equal? That is to say is there an effect of age group on cholesterol level? We can answer that by asking contrast to test whether the means of the age groups are identical.

```
. contrast agegrp
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
agegrp	4	35.02	0.0000
Denominator	70		

The means are clearly different. We could have obtained this same test directly had we fit our model using `anova` rather than `regress`.

```
. anova chol agegrp
```

	Number of obs =	75	R-squared =	0.6668	
	Root MSE =	10.329	Adj R-squared =	0.6477	
Source	Partial SS	df	MS	F	Prob>F
Model	14943.4	4	3735.8499	35.02	0.0000
agegrp	14943.4	4	3735.8499	35.02	0.0000
Residual	7468.2197	70	106.68885		
Total	22411.619	74	302.85972		

Achieving a more direct test result is why we recommend using `anova` instead of `regress` for models where our focus is on the categorical covariates. The models fit by `anova` and `regress` are identical; they merely parameterize the effects differently. The results of `contrast` will be identical regardless of which command is used to fit the model. If, however, we were fitting models whose responses are nonlinear functions of the covariates, such as logistic regression, then there would be no analogue to `anova`, and we would appreciate `contrast`'s ability to quickly test main effects and interactions.

Reference category contrasts

Now that we know that the overall effect of age group is statistically significant, we can explore the effects of each age group. One way to do that is to use the reference category operator, `r` :

```
. contrast r.agegrp
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
agegrp			
(20-29 vs 10-19)	1	4.73	0.0330
(30-39 vs 10-19)	1	32.62	0.0000
(40-59 vs 10-19)	1	63.91	0.0000
(60-79 vs 10-19)	1	105.62	0.0000
Joint	4	35.02	0.0000
Denominator	70		

	Contrast	Std. err.	[95% conf. interval]	
agegrp				
(20-29 vs 10-19)	8.203575	3.771628	.6812991	15.72585
(30-39 vs 10-19)	21.54105	3.771628	14.01878	29.06333
(40-59 vs 10-19)	30.15067	3.771628	22.6284	37.67295
(60-79 vs 10-19)	38.76221	3.771628	31.23993	46.28448

The cell mean of each age group is compared against the base age group (ages 10–19). The first table shows that each difference is significant. The second table gives an estimate and confidence interval for each contrast. These are the comparisons that linear regression gives with a factor covariate and no interactions. The contrasts are identical to the coefficients from our linear regression.

Reverse adjacent contrasts

We have far more flexibility with `contrast`. Age group is ordinal, so it is interesting to compare each age group with the preceding age group (rather than against one reference group). We specify that analysis by using the reverse adjacent operator, `ar`:

```
. contrast ar.agegrp
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
agegrp			
(20–29 vs 10–19)	1	4.73	0.0330
(30–39 vs 20–29)	1	12.51	0.0007
(40–59 vs 30–39)	1	5.21	0.0255
(60–79 vs 40–59)	1	5.21	0.0255
Joint	4	35.02	0.0000
Denominator	70		

	Contrast	Std. err.	[95% conf. interval]	
agegrp				
(20–29 vs 10–19)	8.203575	3.771628	.6812991	15.72585
(30–39 vs 20–29)	13.33748	3.771628	5.815204	20.85976
(40–59 vs 30–39)	8.60962	3.771628	1.087345	16.1319
(60–79 vs 40–59)	8.611533	3.771628	1.089257	16.13381

The 20–29 age group’s cholesterol level is 8.2 points higher than the 10–19 age group’s cholesterol level; the 30–39 age group’s level is 13.3 points higher than the 20–29 age group’s level; and so on. Each age group is statistically different from the preceding age group at the 5% level.

Orthogonal polynomial contrasts

The relationship between age group and cholesterol level looked almost linear in our graph. We can examine that relationship further by using the orthogonal polynomial operator, `p` :

```
. contrast p.agegrp, noeffects
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
agegrp			
(linear)	1	139.11	0.0000
(quadratic)	1	0.15	0.6962
(cubic)	1	0.37	0.5448
(quartic)	1	0.43	0.5153
Joint	4	35.02	0.0000
Denominator	70		

Only the linear effect is statistically significant.

We can even perform the joint test that all effects beyond linear are zero. We do that by selecting all polynomial contrasts above linear—that is, polynomial contrasts 2, 3, and 4.

```
. contrast p(2 3 4).agegrp, noeffects
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
agegrp			
(quadratic)	1	0.15	0.6962
(cubic)	1	0.37	0.5448
(quartic)	1	0.43	0.5153
Joint	3	0.32	0.8129
Denominator	70		

The joint test has three degrees of freedom and is clearly insignificant. A linear effect of age group seems adequate for this model.

Two-way models

Suppose we are investigating the effects of different dosages of a blood pressure medication and believe that the effects may be different for men and women. We can fit the following ANOVA model for `bpchange`, the change in diastolic blood pressure. Change is defined as the after measurement minus the before measurement, so that negative values of `bpchange` correspond to decreases in blood pressure.

```
. use https://www.stata-press.com/data/r19/bpchange
(Artificial blood pressure data)
. label list gender
gender:
      1 Male
      2 Female
. anova bpchange dose##gender
```

Source	Partial SS	df	MS	F	Prob>F
Model	1411.9087	5	282.38174	131.09	0.0000
dose	963.48179	2	481.7409	223.64	0.0000
gender	355.11882	1	355.11882	164.85	0.0000
dose#gender	93.308093	2	46.654046	21.66	0.0000
Residual	51.699253	24	2.1541355		
Total	1463.608	29	50.46924		

Estimated interaction cell means

Everything is significant, including the interaction. So increasing dosage is effective and differs by gender. Let's explore the effects. First, let's look at the estimated cell mean of blood pressure change for each combination of gender and dosage.

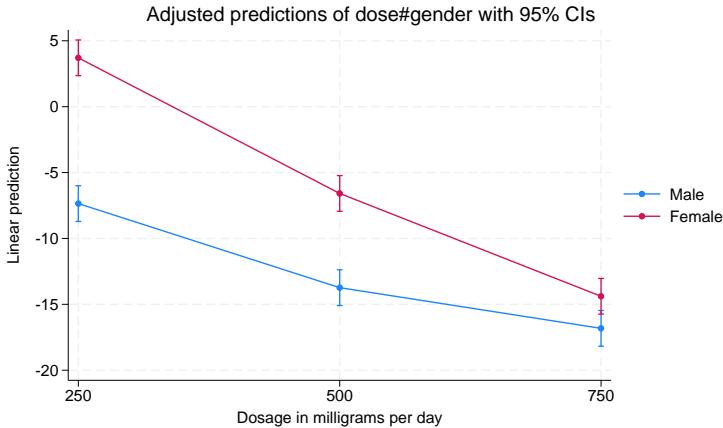
```
. margins dose#gender
Adjusted predictions                                Number of obs = 30
Expression: Linear prediction, predict()
```

	Delta-method				
	Margin	std. err.	t	P> t	[95% conf. interval]
dose#gender					
250#Male	-7.35384	.6563742	-11.20	0.000	-8.708529 -5.99915
250#Female	3.706567	.6563742	5.65	0.000	2.351877 5.061257
500#Male	-13.73386	.6563742	-20.92	0.000	-15.08855 -12.37917
500#Female	-6.584167	.6563742	-10.03	0.000	-7.938857 -5.229477
750#Male	-16.82108	.6563742	-25.63	0.000	-18.17576 -15.46639
750#Female	-14.38795	.6563742	-21.92	0.000	-15.74264 -13.03326

Our data are balanced, so these results will not be affected by the many different ways that margins can compute cell means. Moreover, because our model consists of only dose and gender, these are also the point estimates for each combination.

We can graph the results:

```
. marginsplot
Variables that uniquely identify margins: dose gender
```



The lines are not parallel, which we expected because the interaction term is significant. Males experience a greater decline in blood pressure at every dosage level, but the effect of increasing dosage is greater for females. In fact, it is not clear if we can tell the difference between male and female response at the maximum dosage.

Simple effects

We can contrast the male and female responses within dosage to see the simple effects of gender. Because there are only two levels in gender, the choice of contrast operator is largely irrelevant. Aside from orthogonal polynomials, all operators produce the same estimates, although the effects can change signs.

```
. contrast r.gender@dose
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
gender@dose			
(Female vs Male) 250	1	141.97	0.0000
(Female vs Male) 500	1	59.33	0.0000
(Female vs Male) 750	1	6.87	0.0150
Joint	3	69.39	0.0000
Denominator	24		

	Contrast	Std. err.	[95% conf. interval]	
gender@dose				
(Female vs Male) 250	11.06041	.9282533	9.144586	12.97623
(Female vs Male) 500	7.149691	.9282533	5.23387	9.065512
(Female vs Male) 750	2.433124	.9282533	.5173031	4.348944

The effect for females is about 11 points higher than for males at a dosage of 250, and that shrinks to 2.4 points higher at the maximum dosage of 750.

We can form the simple effects the other way by contrasting the effect of dose at each level of gender:

```
. contrast ar.dose@gender
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
dose@gender			
(500 vs 250) Male	1	47.24	0.0000
(500 vs 250) Female	1	122.90	0.0000
(750 vs 500) Male	1	11.06	0.0028
(750 vs 500) Female	1	70.68	0.0000
Joint	4	122.65	0.0000
Denominator	24		

	Contrast	Std. err.	[95% conf. interval]	
dose@gender				
(500 vs 250) Male	-6.380018	.9282533	-8.295839	-4.464198
(500 vs 250) Female	-10.29073	.9282533	-12.20655	-8.374914
(750 vs 500) Male	-3.087217	.9282533	-5.003038	-1.171396
(750 vs 500) Female	-7.803784	.9282533	-9.719605	-5.887963

Here we use the `ar.` reverse adjacent contrast operator so that first we are comparing a dosage of 500 with a dosage of 250, and then we are comparing 750 with 500. We see that increasing the dosage has a larger effect on females—10.3 points when going from 250 to 500 compared with 6.4 points for males, and 7.8 points when going from 500 to 750 versus 3.1 points for males.

Interaction effects

By specifying contrast operators on both factors, we can decompose the interaction effect into separate interaction contrasts.

```
. contrast ar.dose#r.gender
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
dose#gender			
(500 vs 250) (Female vs Male)	1	8.87	0.0065
(750 vs 500) (Female vs Male)	1	12.91	0.0015
Joint	2	21.66	0.0000
Denominator	24		

	Contrast	Std. err.	[95% conf. interval]	
dose#gender				
(500 vs 250)	-3.910716	1.312748	-6.620095	-1.201336
(750 vs 500)	-4.716567	1.312748	-7.425947	-2.007187

Look for departures from zero to indicate an interaction effect between dose and gender. Both contrasts are significantly different from zero. Of course, we already knew the overall interaction was significant from our ANOVA results. The effect of increasing dose from 250 to 500 is 3.9 points greater in females than in males, and the effect of increasing dose from 500 to 750 is 4.7 points greater in females than in males. The confidence intervals for both estimates easily exclude zero, meaning that there is an interaction effect.

The joint test of these two interaction effects reproduces the test of interaction effects in the anova output. We can see that the *F* statistic of 21.66 matches the statistic from our original ANOVA results.

Main effects

We can perform tests of the main effects by listing each variable individually in contrast.

```
. contrast dose gender
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
dose	2	223.64	0.0000
gender	1	164.85	0.0000
Denominator	24		

The F tests are equivalent to the tests of main effects in the anova output. This is true only for linear models. `contrast` provides an easy way to obtain main effects and other ANOVA-style tests for models whose responses are not linear in the parameters—logistic, probit, `glm`, etc.

If we include contrast operators on the variables, we can also decompose the main effects into individual contrasts:

```
. contrast ar.dose r.gender
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
dose			
(500 vs 250)	1	161.27	0.0000
(750 vs 500)	1	68.83	0.0000
Joint	2	223.64	0.0000
gender	1	164.85	0.0000
Denominator	24		

	Contrast	Std. err.	[95% conf. interval]	
dose				
(500 vs 250)	-8.335376	.6563742	-9.690066	-6.980687
(750 vs 500)	-5.4455	.6563742	-6.80019	-4.090811
gender				
(Female vs Male)	6.881074	.5359273	5.774974	7.987173

By specifying the `ar.` operator on `dose`, we decompose the main effect for `dose` into two one-degree-of-freedom contrasts, comparing the marginal mean of blood pressure change for each dosage level with that of the previous level. Because `gender` has only two levels, we cannot decompose this main effect any further. However, specifying a contrast operator on `gender` allowed us to calculate the difference in the marginal means for women and men.

Partial interaction effects

At this point, we have looked at the total interaction effects and at the main effects of each variable. The partial interaction effects are a midpoint between these two types of effects where we collect the individual interaction effects along the levels of one of the variables and perform a joint test of those interactions. If we think of the interaction effects as forming a table, with the levels of one factor variable forming the rows and the levels of the other forming the columns, partial interaction effects are joint tests of the interactions in a row or a column. To perform these tests, we specify a contrast operator on only one of the variables in our interaction. For this particular model, these are not very interesting because our variables have only two and three levels. Therefore, the tests of the partial interaction effects reproduce the tests that we obtained for the total interaction effects. We specify a contrast operator only on `dose` to decompose the overall test for interaction effects into joint tests for each `ar.dose` contrast:

```
. contrast ar.dose#gender
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
dose#gender			
(500 vs 250) (joint)	1	8.87	0.0065
(750 vs 500) (joint)	1	12.91	0.0015
Joint	2	21.66	0.0000
Denominator	24		

The first row is a joint test of all the interaction effects involving the (500 vs 250) comparison of dosages. The second row is a joint test of all the interaction effects involving the (750 vs 500) comparison. If we look back at our output in *Interaction effects*, we can see that there was only one of each of these interaction effects. Therefore, each test labeled (joint) has only one degree-of-freedom.

We could have instead included a contrast operator on gender to compute the partial interaction effects along the other dimension:

```
. contrast dose#r.gender
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
dose#gender	2	21.66	0.0000
Denominator	24		

Here we obtain a joint test of all the interaction effects involving the (Female vs Male) comparison for gender. Because gender has only two levels, the (Female vs Male) contrast is the only reference category contrast possible. Therefore, we obtain a single joint test of all the interaction effects.

Clearly, the partial interaction effects are not interesting for this particular model. However, if our factors had more levels, the partial interaction effects would produce tests that are not available in the total interaction effects. For example, if our model included factors for four dosage levels and three races, then typing

```
. contrast ar.dose#race
```

would produce three joint tests, one for each of the reverse adjacent contrasts for dosage. Each of these tests would be a two-degree-of-freedom test because race has three levels.

Three-way and higher-order models

All the contrasts and tests that we reviewed above for two-way models can be used with models that have more terms. For instance, we could fit a three-way full factorial model by using the `anova` command:

```
. use https://www.stata-press.com/data/r19/cont3way
. anova y race##sex##group
```

We could then test the simple effects of race within each level of the interaction between sex and group:

```
. contrast race@sex#group
```

To see the reference category contrasts that decompose these simple effects, type

```
. contrast r.race@sex#group
```

We could test the three-way interaction effects by typing

```
. contrast race#sex#group
```

or the interaction effects for the interaction of race and sex by typing

```
. contrast race#sex
```

To see the individual reference category contrasts that decompose this interaction effect, type

```
. contrast r.race#r.sex
```

We could even obtain joint tests for the interaction of race and sex within each level of group by typing

```
. contrast race#sex@group
```

For tests of the main effects of each factor, we can type

```
. contrast race sex group
```

We can calculate the individual reference category contrasts that decompose these main effects:

```
. contrast r.race r.sex r.group
```

For the partial interaction effects, we could type

```
. contrast r.race#group
```

to obtain a joint test of the two-way interaction effects of race and group for each of the individual `r.race` contrasts.

We could type

```
. contrast r.race#sex#group
```

to obtain a joint test of all the three-way interaction terms for each of the individual `r.race` contrasts.

Contrast operators

`contrast` recognizes a set of contrast operators that are used to specify commonly used contrasts. When these operators are used, `contrast` will report a test for each individual contrast in addition to the joint test for the term. We have already seen a few of these, like `r.` and `ar.`, in the previous examples. Here we will take a closer look at each of the unweighted operators.

Here we use the cholesterol dataset and the one-way ANOVA model from the example in [One-way models](#):

```
. use https://www.stata-press.com/data/r19/cholesterol
(Artificial cholesterol data)
. anova chol agegrp
(output omitted)
```

The `margins` command reports the estimated cell means, $\hat{\mu}_1, \dots, \hat{\mu}_5$, for each of the five age groups.

```
. margins agegrp
Adjusted predictions                                Number of obs = 75
Expression: Linear prediction, predict()
```

	Delta-method		t	P> t	[95% conf. interval]	
	Margin	std. err.				
agegrp						
10-19	180.5198	2.666944	67.69	0.000	175.2007	185.8388
20-29	188.7233	2.666944	70.76	0.000	183.4043	194.0424
30-39	202.0608	2.666944	75.76	0.000	196.7418	207.3799
40-59	210.6704	2.666944	78.99	0.000	205.3514	215.9895
60-79	219.282	2.666944	82.22	0.000	213.9629	224.601

Contrast operators provide an easy way to make certain types of comparisons of these cell means. We use the ordinal factor `agegrp` to demonstrate these operators because some types of contrasts are meaningful only when the levels of the factor have a natural ordering. We demonstrate these contrast operators using a one-way model; however, they are equally applicable to main effects, simple effects, and interactions for more complicated models.

Differences from a reference level (r.)

The `r.` operator specifies that each level of the attached factor variable be compared with a reference level. These are referred to as reference-level or reference-category contrasts (or effects), and `r.` is the reference-level operator.

In the following, we use the `r.` operator to test the effect of each category of age group when that category is compared with a reference category.

```
. contrast r.agegrp
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
agegrp			
(20-29 vs 10-19)	1	4.73	0.0330
(30-39 vs 10-19)	1	32.62	0.0000
(40-59 vs 10-19)	1	63.91	0.0000
(60-79 vs 10-19)	1	105.62	0.0000
Joint	4	35.02	0.0000
Denominator	70		

	Contrast	Std. err.	[95% conf. interval]	
agegrp				
(20-29 vs 10-19)	8.203575	3.771628	.6812991	15.72585
(30-39 vs 10-19)	21.54105	3.771628	14.01878	29.06333
(40-59 vs 10-19)	30.15067	3.771628	22.6284	37.67295
(60-79 vs 10-19)	38.76221	3.771628	31.23993	46.28448

In the first table, the row labeled (20-29 vs 10-19) is a test of $\mu_2 = \mu_1$, a test that the mean cholesterol levels for the 10-19 age group and the 20-29 age group are equal. The tests in the next three rows are defined similarly. The row labeled Joint provides the joint test for these four hypotheses, which is just the test of the main effects of age group.

The second table provides the contrasts of each category with the reference category along with confidence intervals. The contrast in the row labeled (20-29 vs 10-19) is the difference in the cell means of the second age group and the first age group, $\hat{\mu}_2 - \hat{\mu}_1$.

The first level of a factor is the default reference level, but we can specify a different reference level by using the `b.` operator; see [U] 11.4.3.2 Base levels. Here we use the last age group, (60-79), instead of the first as the reference category. We also include the `nowald` option so that only the table of contrasts and their confidence intervals is produced.

```
. contrast rb5.agegrp, nowald
Contrasts of marginal linear predictions
Margins: asbalanced
```

	Contrast	Std. err.	[95% conf. interval]	
agegrp				
(10-19 vs 60-79)	-38.76221	3.771628	-46.28448	-31.23993
(20-29 vs 60-79)	-30.55863	3.771628	-38.08091	-23.03636
(30-39 vs 60-79)	-17.22115	3.771628	-24.74343	-9.698877
(40-59 vs 60-79)	-8.611533	3.771628	-16.13381	-1.089257

Now, the first row is labeled (10-19 vs 60-79) and is the difference in the cell means of the first and fifth age groups.

Differences from the next level (a.)

The `a.` operator specifies that each level of the attached factor variable be compared with the next level. These are referred to as adjacent contrasts (or effects), and `a.` is the adjacent operator. This operator is meaningful only with factor variables that have a natural ordering in the levels.

We can use the `a.` operator to perform tests that each level of age group differs from the next adjacent level.

```
. contrast a.agegrp
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
agegrp			
(10-19 vs 20-29)	1	4.73	0.0330
(20-29 vs 30-39)	1	12.51	0.0007
(30-39 vs 40-59)	1	5.21	0.0255
(40-59 vs 60-79)	1	5.21	0.0255
Joint	4	35.02	0.0000
Denominator	70		

	Contrast	Std. err.	[95% conf. interval]	
agegrp				
(10-19 vs 20-29)	-8.203575	3.771628	-15.72585	-.6812991
(20-29 vs 30-39)	-13.33748	3.771628	-20.85976	-5.815204
(30-39 vs 40-59)	-8.60962	3.771628	-16.1319	-1.087345
(40-59 vs 60-79)	-8.611533	3.771628	-16.13381	-1.089257

In the first table, the row labeled (10-19 vs 20-29) tests the effect of belonging to the 10-19 age group instead of the 20-29 age group. Likewise, the rows labeled (20-29 vs 30-39), (30-39 vs 40-59), and (40-59 vs 60-79) are tests for the effects of being in the younger of the two age groups instead of the older one.

In the second table, the contrast in the row labeled (10-19 vs 20-29) is the difference in the cell means of the first and second age groups, $\hat{\mu}_1 - \hat{\mu}_2$. The contrasts in the other rows are defined similarly.

Differences from the previous level (ar.)

The `ar.` operator specifies that each level of the attached factor variable be compared with the previous level. These are referred to as reverse adjacent contrasts (or effects), and `ar.` is the reverse adjacent operator. As with the `a.` operator, this operator is meaningful only with factor variables that have a natural ordering in the levels.

In the following, we use the `ar.` operator to report tests for the individual reverse adjacent effects of `agegrp`.

```
. contrast ar.agegrp
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
agegrp			
(20-29 vs 10-19)	1	4.73	0.0330
(30-39 vs 20-29)	1	12.51	0.0007
(40-59 vs 30-39)	1	5.21	0.0255
(60-79 vs 40-59)	1	5.21	0.0255
Joint	4	35.02	0.0000
Denominator	70		

	Contrast	Std. err.	[95% conf. interval]	
agegrp				
(20-29 vs 10-19)	8.203575	3.771628	.6812991	15.72585
(30-39 vs 20-29)	13.33748	3.771628	5.815204	20.85976
(40-59 vs 30-39)	8.60962	3.771628	1.087345	16.1319
(60-79 vs 40-59)	8.611533	3.771628	1.089257	16.13381

Here the Wald tests in the first table for the individual reverse adjacent effects are equivalent to the tests for the adjacent effects in the [previous example](#). However, if we compare values of the contrasts in the bottom tables, we see the difference between the `r.` and the `ar.` operators. This time, the contrast in the first row is labeled (20-29 vs 10-19) and is the difference in the cell means of the second and first age groups, $\hat{\mu}_2 - \hat{\mu}_1$. This is the estimated effect of belonging to the 20-29 age group instead of the 10-19 age group. The remaining rows make similar comparisons with the previous level.

Differences from the grand mean (g.)

The g. operator specifies that each level of a factor variable be compared with the grand mean of all levels. For this operator, the grand mean is computed using a simple average of the cell means.

Here are the grand mean effects of agegrp:

```
. contrast g.agegrp
```

```
Contrasts of marginal linear predictions
```

```
Margins: asbalanced
```

	df	F	P>F
agegrp			
(10-19 vs mean)	1	68.42	0.0000
(20-29 vs mean)	1	23.36	0.0000
(30-39 vs mean)	1	0.58	0.4506
(40-59 vs mean)	1	19.08	0.0000
(60-79 vs mean)	1	63.65	0.0000
Joint	4	35.02	0.0000
Denominator	70		

	Contrast	Std. err.	[95% conf. interval]	
agegrp				
(10-19 vs mean)	-19.7315	2.385387	-24.48901	-14.974
(20-29 vs mean)	-11.52793	2.385387	-16.28543	-6.770423
(30-39 vs mean)	1.809552	2.385387	-2.947953	6.567057
(40-59 vs mean)	10.41917	2.385387	5.661668	15.17668
(60-79 vs mean)	19.0307	2.385387	14.2732	23.78821

There are five age groups in our estimation sample. Thus, the row labeled (10-19 vs mean) tests $\mu_1 = (\mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5)/5$. The row labeled (20-29 vs mean) tests $\mu_2 = (\mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5)/5$. The remaining rows perform similar tests for the third, fourth, and fifth age groups. In our example, the means for all age groups except the 30-39 age group are statistically different from the grand mean.

Differences from the mean of subsequent levels (h.)

The `h.` operator specifies that each level of the attached factor variable be compared with the mean of subsequent levels. These are referred to as Helmert contrasts (or effects), and `h.` is the Helmert operator. For this operator, the mean is computed using a simple average of the cell means. This operator is meaningful only with factor variables that have a natural ordering in the levels.

Here are the Helmert contrasts for `agegrp`:

```
. contrast h.agegrp
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
agegrp			
(10-19 vs >10-19)	1	68.42	0.0000
(20-29 vs >20-29)	1	50.79	0.0000
(30-39 vs >30-39)	1	15.63	0.0002
(40-59 vs 60-79)	1	5.21	0.0255
Joint	4	35.02	0.0000
Denominator	70		

	Contrast	Std. err.	[95% conf. interval]	
agegrp				
(10-19 vs >10-19)	-24.66438	2.981734	-30.61126	-18.7175
(20-29 vs >20-29)	-21.94774	3.079522	-28.08965	-15.80583
(30-39 vs >30-39)	-12.91539	3.266326	-19.42987	-6.400905
(40-59 vs 60-79)	-8.611533	3.771628	-16.13381	-1.089257

The row labeled (10-19 vs >10-19) tests $\mu_1 = (\mu_2 + \mu_3 + \mu_4 + \mu_5)/4$, that is, that the cell mean for the youngest age group is equal to the average of the cell means for the older age groups. The row labeled (20-29 vs >20-29) tests $\mu_2 = (\mu_3 + \mu_4 + \mu_5)/3$. The tests in the other rows are defined similarly.

Differences from the mean of previous levels (j.)

The `j.` operator specifies that each level of the attached factor variable be compared with the mean of the previous levels. These are referred to as reverse Helmert contrasts (or effects), and `j.` is the reverse Helmert operator. For this operator, the mean is computed using a simple average of the cell means. This operator is meaningful only with factor variables that have a natural ordering in the levels.

Here are the reverse Helmert contrasts of agegrp:

```
. contrast j.agegrp
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
agegrp			
(20-29 vs 10-19)	1	4.73	0.0330
(30-39 vs <30-39)	1	28.51	0.0000
(40-59 vs <40-59)	1	43.18	0.0000
(60-79 vs <60-79)	1	63.65	0.0000
Joint	4	35.02	0.0000
Denominator	70		

	Contrast	Std. err.	[95% conf. interval]	
agegrp				
(20-29 vs 10-19)	8.203575	3.771628	.6812991	15.72585
(30-39 vs <30-39)	17.43927	3.266326	10.92479	23.95375
(40-59 vs <40-59)	20.2358	3.079522	14.09389	26.37771
(60-79 vs <60-79)	23.78838	2.981734	17.8415	29.73526

The row labeled (20-29 vs 10-19) tests $\mu_2 = \mu_1$, that is, that the cell means for the 20-29 and the 10-19 age groups are equal. The row labeled (30-39 vs <30-39) tests $\mu_3 = (\mu_1 + \mu_2)/2$, that is, that the cell mean for the 30-39 age group is equal to the average of the cell means for the 10-19 and 20-29 age groups. The tests in the remaining rows are defined similarly.

Orthogonal polynomials (p. and q.)

The p. and q. operators specify that orthogonal polynomials be applied to the attached factor variable. Orthogonal polynomial contrasts allow us to partition the effects of a factor variable into linear, quadratic, cubic, and higher-order polynomial components. The p. operator applies orthogonal polynomials using the values of the factor variable. The q. operator applies orthogonal polynomials using the level indices. If the level values of the factor variable are equally spaced, as with our agegrp variable, then the p. and q. operators yield the same result. These operators are meaningful only with factor variables that have a natural ordering in the levels.

Because agegrp has five levels, contrast can test the linear, quadratic, cubic, and quartic effects of agegrp.

```
. contrast p.agegrp, noeffects
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
agegrp			
(linear)	1	139.11	0.0000
(quadratic)	1	0.15	0.6962
(cubic)	1	0.37	0.5448
(quartic)	1	0.43	0.5153
Joint	4	35.02	0.0000
Denominator	70		

The row labeled (linear) tests the linear effect of agegrp, the only effect that appears to be significant in this case.

The labels for our agegrp variable show the age ranges that correspond to each level.

```
. label list ages
ages:
    1 10-19
    2 20-29
    3 30-39
    4 40-59
    5 60-79
```

Notice that these groups do not have equal widths. Now, let’s refit our model using the agemidpt variable. The values of agemidpt indicate the midpoint of each age group that was defined by the agegrp variable and are, therefore, not equally spaced.

```
. anova chol agemidpt
```

	Number of obs =	75	R-squared =	0.6668	
	Root MSE =	10.329	Adj R-squared =	0.6477	
Source	Partial SS	df	MS	F	Prob>F
Model	14943.4	4	3735.8499	35.02	0.0000
agemidpt	14943.4	4	3735.8499	35.02	0.0000
Residual	7468.2197	70	106.68885		
Total	22411.619	74	302.85972		

Now if we use the `q.` operator, we will obtain the same results as above because the level indices of `agemidpt` are equivalent to the values of `agegrp`.

```
. contrast q.agemidpt, noeffects
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
agemidpt			
(linear)	1	139.11	0.0000
(quadratic)	1	0.15	0.6962
(cubic)	1	0.37	0.5448
(quartic)	1	0.43	0.5153
Joint	4	35.02	0.0000
Denominator	70		

However, if we use the `p.` operator, we will instead fit an orthogonal polynomial to the midpoint values.

```
. contrast p.agemidpt, noeffects
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
agemidpt			
(linear)	1	133.45	0.0000
(quadratic)	1	5.40	0.0230
(cubic)	1	0.05	0.8198
(quartic)	1	1.16	0.2850
Joint	4	35.02	0.0000
Denominator	70		

Using the values of the midpoints, the quadratic effect is also significant at the 5% level.

□ Technical note

We used the `noeffects` option when working with orthogonal polynomial contrasts. Apart from perhaps the sign of the contrast, the values of the individual contrasts are not meaningful for orthogonal polynomial contrasts. In addition, many textbooks provide tables with contrast coefficients that can be used to compute orthogonal polynomial contrasts where the levels of a factor are equally spaced. If we use these coefficients and calculate the contrasts manually with user-defined contrasts, as described below, the Wald tests for the polynomial terms will be equivalent, but the values of the individual contrasts will not necessarily match those that we obtain when using the polynomial contrast operator. When we use one of these contrast operators, an algorithm is used to calculate the coefficients of the polynomial contrast that will allow for unequal spacing in the levels of the factor as well as in the weights for the cell frequencies (when using `pw.` or `qw.`), as described in [Methods and formulas](#).

□

User-defined contrasts

In the previous examples, we performed tests using contrast operators. When there is not a contrast operator available to calculate the contrast in which we are interested, we can specify custom contrasts.

Here we fit a one-way model for cholesterol on the factor race, which has three levels:

```
. label list race
race:
    1 Black
    2 White
    3 Other
. anova chol race
```

	Number of obs =	75	R-squared =	0.0299	
	Root MSE =	17.3775	Adj R-squared =	0.0029	
Source	Partial SS	df	MS	F	Prob>F
Model	669.27823	2	334.63912	1.11	0.3357
race	669.27823	2	334.63912	1.11	0.3357
Residual	21742.341	72	301.97696		
Total	22411.619	74	302.85972		

margins calculates the estimated cell mean cholesterol level for each race:

```
. margins race
Adjusted predictions                                Number of obs = 75
Expression: Linear prediction, predict()
```

race	Delta-method				
	Margin	std. err.	t	P> t	[95% conf. interval]
Black	204.4279	3.475497	58.82	0.000	197.4996 211.3562
White	197.6132	3.475497	56.86	0.000	190.6849 204.5415
Other	198.7127	3.475497	57.18	0.000	191.7844 205.6409

Suppose we want to test the following linear combination:

$$\sum_{i=1}^3 c_i \mu_i$$

where μ_i is the cell mean of chol when race is equal to its i th level (the means estimated using margins above). Assuming the c_i elements sum to zero, this linear combination is a contrast. We can specify this type of custom contrast by using the following syntax:

```
{race c1 c2 c3}
```

The null hypothesis for the test of the main effects of race is

$$H_{0_{\text{race}}} : \mu_1 = \mu_2 = \mu_3$$

Although $H_{0_{\text{race}}}$ can be tested using any of several different contrasts on the cell means, we will test it by comparing the second and third cell means with the first. To test that the cell means for blacks and whites are equal, $\mu_1 = \mu_2$, we can specify the contrast

$$\{\text{race } -1 \ 1 \ 0\}$$

To test that the cell means for blacks and other races are equal, $\mu_1 = \mu_3$, we can specify the contrast

$$\{\text{race } -1 \ 0 \ 1\}$$

We can use both in a single call to contrast.

```
. contrast {race -1 1 0} {race -1 0 1}
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
race			
(1)	1	1.92	0.1699
(2)	1	1.35	0.2488
Joint	2	1.11	0.3357
Denominator	72		

	Contrast	Std. err.	[95% conf. interval]	
race				
(1)	-6.814717	4.915095	-16.61278	2.983345
(2)	-5.715261	4.915095	-15.51332	4.082801

The row labeled (1) is the test for $\mu_1 = \mu_2$, the first specified contrast. The row labeled (2) is the test for $\mu_1 = \mu_3$, the second specified contrast. The row labeled Joint is the overall test for the main effects of race.

Now, let's fit a model with two factors, race and age group:

```
. anova chol race##agegrp
Number of obs = 75 R-squared = 0.7524
Root MSE = 9.61785 Adj R-squared = 0.6946
```

Source	Partial SS	df	MS	F	Prob>F
Model	16861.438	14	1204.3884	13.02	0.0000
race	669.27823	2	334.63912	3.62	0.0329
agegrp	14943.4	4	3735.8499	40.39	0.0000
race#agegrp	1248.7601	8	156.09501	1.69	0.1201
Residual	5550.1814	60	92.503024		
Total	22411.619	74	302.85972		

The null hypothesis for the test of the main effects of race is now

$$H_{0_{\text{race}}} : \mu_1. = \mu_2. = \mu_3.$$

where $\mu_i.$ is the marginal mean of chol when race is equal to its i th level.

We can use the same syntax as above to perform this test by specifying contrasts on the marginal means of race:

```
. contrast {race -1 1 0} {race -1 0 1}
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
race			
(1)	1	6.28	0.0150
(2)	1	4.41	0.0399
Joint	2	3.62	0.0329
Denominator	60		

	Contrast	Std. err.	[95% conf. interval]	
race				
(1)	-6.814717	2.720339	-12.2562	-1.37323
(2)	-5.715261	2.720339	-11.15675	-.2737739

Custom contrasts may be specified on the cell means of interactions, too. Here we use margins to calculate the mean of chol for each cell in the interaction of race and agegrp:

```
. margins race#agegrp
Adjusted predictions                               Number of obs = 75
Expression: Linear prediction, predict()
```

	Delta-method				
	Margin	std. err.	t	P> t	[95% conf. interval]
race#agegrp					
Black#10-19	179.2309	4.301233	41.67	0.000	170.6271 187.8346
Black#20-29	196.4777	4.301233	45.68	0.000	187.874 205.0814
Black#30-39	210.6694	4.301233	48.98	0.000	202.0656 219.2731
Black#40-59	214.097	4.301233	49.78	0.000	205.4933 222.7008
Black#60-79	221.6646	4.301233	51.54	0.000	213.0609 230.2684
White#10-19	186.0727	4.301233	43.26	0.000	177.469 194.6765
White#20-29	184.6714	4.301233	42.93	0.000	176.0676 193.2751
White#30-39	196.2633	4.301233	45.63	0.000	187.6595 204.867
White#40-59	209.9953	4.301233	48.82	0.000	201.3916 218.5991
White#60-79	211.0633	4.301233	49.07	0.000	202.4595 219.667
Other#10-19	176.2556	4.301233	40.98	0.000	167.6519 184.8594
Other#20-29	185.0209	4.301233	43.02	0.000	176.4172 193.6247
Other#30-39	199.2498	4.301233	46.32	0.000	190.646 207.8535
Other#40-59	207.9189	4.301233	48.34	0.000	199.3152 216.5227
Other#60-79	225.118	4.301233	52.34	0.000	216.5143 233.7218

Now, we are interested in testing the following linear combination of these cell means:

$$\sum_{i=1}^3 \sum_{j=1}^5 c_{ij} \mu_{ij}$$

We can specify this type of custom contrast using the following syntax:

```
{race#agegrp c11 c12 ... c15 c21 c22 ... c25 c31 c32 ... c35}
```

Because the marginal means of `chol` for each level of `race` are linear combinations of the cell means, we can compose the test for the main effects of `race` in terms of the cell means directly. The constraint that the marginal means for blacks and whites are equal, $\mu_{1.} = \mu_{2.}$, translates to the following constraint on the cell means:

$$\frac{1}{5}(\mu_{11} + \mu_{12} + \mu_{13} + \mu_{14} + \mu_{15}) = \frac{1}{5}(\mu_{21} + \mu_{22} + \mu_{23} + \mu_{24} + \mu_{25})$$

Ignoring the common factor, we can specify this contrast as

```
{race#agegrp -1 -1 -1 -1 -1 1 1 1 1 1 0 0 0 0 0}
```

`contrast` will fill in the trailing zeros for us if we neglect to specify them, so

```
{race#agegrp -1 -1 -1 -1 -1 1 1 1 1 1}
```

is also allowed. The other constraint, $\mu_{1.} = \mu_{3.}$, translates to

$$\frac{1}{5}(\mu_{11} + \mu_{12} + \mu_{13} + \mu_{14} + \mu_{15}) = \frac{1}{5}(\mu_{31} + \mu_{32} + \mu_{33} + \mu_{34} + \mu_{35})$$

This can be specified to `contrast` as

```
{race#agegrp -1 -1 -1 -1 -1 0 0 0 0 0 1 1 1 1 1}
```

The following call to `contrast` yields the same test results as above.

```
. contrast {race#agegrp -1 -1 -1 -1 -1 1 1 1 1 1}
>          {race#agegrp -1 -1 -1 -1 -1 0 0 0 0 0 1 1 1 1 1}, noeffects
```

Contrasts of marginal linear predictions

Margins: asbalanced

	df	F	P>F
race#agegrp			
(1) (1)	1	6.28	0.0150
(2) (2)	1	4.41	0.0399
Joint	2	3.62	0.0329
Denominator	60		

The row labeled (1) (1) is the test for

$$\mu_{11} + \mu_{12} + \mu_{13} + \mu_{14} + \mu_{15} = \mu_{21} + \mu_{22} + \mu_{23} + \mu_{24} + \mu_{25}$$

It was the first specified contrast. The row labeled (2) (2) is the test for

$$\mu_{11} + \mu_{12} + \mu_{13} + \mu_{14} + \mu_{15} = \mu_{31} + \mu_{32} + \mu_{33} + \mu_{34} + \mu_{35}$$

It was the second specified contrast. The row labeled Joint tests (1) (1) and (2) (2) simultaneously.

We used the `noeffects` option above to suppress the table of contrasts. We can omit the $1/5$ from the equations for $\mu_{1.} = \mu_{2.}$ and $\mu_{1.} = \mu_{3.}$ and still obtain the appropriate tests. However, if we want to calculate the differences in the marginal means, we must include the $1/5 = 0.2$ on each of the contrast coefficients as follows:

```
. contrast {race#agegrp -0.2 -0.2 -0.2 -0.2 -0.2 ///
           0.2 0.2 0.2 0.2 0.2} ///
           {race#agegrp -0.2 -0.2 -0.2 -0.2 -0.2 ///
           0 0 0 0 0 ///
           0.2 0.2 0.2 0.2 0.2}
```

So far, we have reproduced the reference category contrasts by specifying user-defined contrasts on the marginal means and then on the cell means. For this test, it would have been easier to use the `r. contrast` operator:

```
. contrast r.race, noeffects
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
race			
(White vs Black)	1	6.28	0.0150
(Other vs Black)	1	4.41	0.0399
Joint	2	3.62	0.0329
Denominator	60		

In most cases, we can use contrast operators to perform tests. However, if we want to compare, for instance, the second and third age groups with the fourth and fifth age groups with the test

$$\frac{1}{2}(\mu_{.2} + \mu_{.3}) = \frac{1}{2}(\mu_{.4} + \mu_{.5})$$

there is not a contrast operator that corresponds to this particular contrast. A custom contrast is necessary.

```
. contrast {agegrp 0 -0.5 -0.5 0.5 0.5}
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
agegrp	1	62.19	0.0000
Denominator	60		

	Contrast	Std. err.	[95% conf. interval]	
agegrp (1)	19.58413	2.483318	14.61675	24.5515

Empty cells

An empty cell is a combination of the levels of factor variables that is not observed in the estimation sample. In the previous examples, we have seen data with three levels of `race`, five levels of `agegrp`, and all level combinations of `race` and `agegrp` present. Suppose there are no observations for white individuals in the second age group (ages 20–29).

```
. use https://www.stata-press.com/data/r19/cholesterol2
(Artificial cholesterol data, empty cells)
. label list
race:
      1 Black
      2 White
      3 Other
ages:
      1 10-19
      2 20-29
      3 30-39
      4 40-59
      5 60-79
```

```
. regress chol race##agegrp
note: 2.race#2.agegrp identifies no observations in the sample.
```

Source	SS	df	MS	Number of obs	=	70
Model	15751.6113	13	1211.66241	F(13, 56)	=	13.51
Residual	5022.71559	56	89.6913498	Prob > F	=	0.0000
				R-squared	=	0.7582
				Adj R-squared	=	0.7021
Total	20774.3269	69	301.077201	Root MSE	=	9.4706

chol	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
race						
White	12.84185	5.989703	2.14	0.036	.8430383	24.84067
Other	-.167627	5.989703	-0.03	0.978	-12.16644	11.83119
agegrp						
20-29	17.24681	5.989703	2.88	0.006	5.247991	29.24562
30-39	31.43847	5.989703	5.25	0.000	19.43966	43.43729
40-59	34.86613	5.989703	5.82	0.000	22.86732	46.86495
60-79	44.43374	5.989703	7.42	0.000	32.43492	56.43256
race#agegrp						
White#20-29	0 (empty)					
White#30-39	-22.83983	8.470719	-2.70	0.009	-39.80872	-5.870939
White#40-59	-14.67558	8.470719	-1.73	0.089	-31.64447	2.293306
White#60-79	-10.51115	8.470719	-1.24	0.220	-27.48004	6.457735
Other#20-29	-6.054425	8.470719	-0.71	0.478	-23.02331	10.91446
Other#30-39	-11.48083	8.470719	-1.36	0.181	-28.44971	5.488063
Other#40-59	-.6796112	8.470719	-0.08	0.936	-17.6485	16.28928
Other#60-79	-1.578052	8.470719	-0.19	0.853	-18.54694	15.39084
_cons	175.2309	4.235359	41.37	0.000	166.7464	183.7153

Now, let's use contrast to test the main effects of race:

```
. contrast race
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
race	(not testable)		
Denominator	56		

By “not testable”, contrast means that it cannot form a test for the main effects of race based on estimable functions of the model coefficients. agegrp has five levels, so contrast constructs an estimate of the i th margin for race as

$$\hat{\mu}_i = \frac{1}{5} \sum_{j=1}^5 \hat{\mu}_{ij} = \hat{\mu}_0 + \hat{\alpha}_i + \frac{1}{5} \sum_{j=1}^5 \{ \hat{\beta}_j + (\widehat{\alpha\beta})_{ij} \}$$

but $(\widehat{\alpha\beta})_{22}$ was constrained to zero because of the empty cell, so $\hat{\mu}_2$ is not an estimable function of the model coefficients.

See *Estimable functions* in *Methods and formulas of [R] margins* for a technical description of estimable functions. The `emptycells(reweight)` option causes contrast to estimate μ_2 by

$$\hat{\mu}_2 = \frac{\hat{\mu}_{21} + \hat{\mu}_{23} + \hat{\mu}_{24} + \hat{\mu}_{25}}{4}$$

which is an estimable function of the model coefficients.

```
. contrast race, emptycells(reweight)
Contrasts of marginal linear predictions
Margins:      asbalanced
Empty cells: reweight
```

	df	F	P>F
race	2	3.17	0.0498
Denominator	56		

We can reconstruct the effect of the `emptycells(reweight)` option by using custom contrasts.

```
. contrast {race#agegrp -4 -4 -4 -4 -4 5 0 5 5 5}
> {race#agegrp -1 -1 -1 -1 -1 0 0 0 0 1 1 1 1}, noeffects
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
race#agegrp			
(1) (1)	1	1.06	0.3080
(2) (2)	1	2.37	0.1291
Joint	2	3.17	0.0498
Denominator	56		

The row labeled (1) (1) is the test for

$$\frac{1}{5}(\mu_{11} + \mu_{12} + \mu_{13} + \mu_{14} + \mu_{15}) = \frac{1}{4}(\mu_{21} + \mu_{23} + \mu_{24} + \mu_{25})$$

It was the first specified contrast. The row labeled (2) (2) is the test for

$$\mu_{11} + \mu_{12} + \mu_{13} + \mu_{14} + \mu_{15} = \mu_{31} + \mu_{32} + \mu_{33} + \mu_{34} + \mu_{35}$$

It was the second specified contrast. The row labeled Joint is the overall test of the main effects of race.

Empty cells, ANOVA style

Let's refit the linear model from the [previous example](#) with anova to compare with contrast's test for the main effects of race.

```
. anova chol race#agegrp
```

	Number of obs =	70	R-squared =	0.7582	
	Root MSE =	9.47055	Adj R-squared =	0.7021	
Source	Partial SS	df	MS	F	Prob>F
Model	15751.611	13	1211.6624	13.51	0.0000
race	305.49046	2	152.74523	1.70	0.1914
agegrp	14387.856	4	3596.964	40.10	0.0000
race#agegrp	795.80757	7	113.6868	1.27	0.2831
Residual	5022.7156	56	89.69135		
Total	20774.327	69	301.0772		

contrast and anova handled the empty cell differently; the F statistic reported by contrast was 3.17, but anova reported 1.70. To see how they differ, consider the following table of the cell means and margins for our situation.

		agegrp					
		1	2	3	4	5	
race	1	μ_{11}	μ_{12}	μ_{13}	μ_{14}	μ_{15}	$\mu_{1\cdot}$
	2	μ_{21}		μ_{23}	μ_{24}	μ_{25}	
	3	μ_{31}	μ_{32}	μ_{33}	μ_{34}	μ_{35}	$\mu_{3\cdot}$
		$\mu_{\cdot 1}$		$\mu_{\cdot 3}$	$\mu_{\cdot 4}$	$\mu_{\cdot 5}$	

For testing the main effects of race, we know that we will be testing the equality of the marginal means for rows 1 and 3, that is, $\mu_{1\cdot} = \mu_{3\cdot}$. This translates into the following constraint:

$$\mu_{11} + \mu_{12} + \mu_{13} + \mu_{14} + \mu_{15} = \mu_{31} + \mu_{32} + \mu_{33} + \mu_{34} + \mu_{35}$$

Because row 2 contains an empty cell in column 2, anova dropped column 2 and tested the equality of the marginal mean for row 2 with the average of the marginal means from rows 1 and 3, using only the remaining cell means. This translates into the following constraint:

$$2(\mu_{21} + \mu_{23} + \mu_{24} + \mu_{25}) = \mu_{11} + \mu_{13} + \mu_{14} + \mu_{15} + \mu_{31} + \mu_{33} + \mu_{34} + \mu_{35} \tag{1}$$

Now that we know the constraints that anova used to test for the main effects of race, we can use custom contrasts to reproduce the anova test result.

```
. contrast {race#agegrp -1 -1 -1 -1 -1 0 0 0 0 0 1 1 1 1 1}
> {race#agegrp 1 0 1 1 1 -2 0 -2 -2 -2 1 0 1 1 1}, noeffects
```

Contrasts of marginal linear predictions

Margins: asbalanced

	df	F	P>F
race#agegrp			
(1) (1)	1	2.37	0.1291
(2) (2)	1	1.03	0.3138
Joint	2	1.70	0.1914
Denominator	56		

The row labeled (1) (1) is the test for $\mu_{1.} = \mu_{3.}$; it was the first specified contrast. The row labeled (2) (2) is the test for the constraint in (1); it was the second specified contrast. The row labeled Joint is an overall test for the main effects of race.

Nested effects

contrast has the | operator for computing simple effects when the levels of one factor are nested within the levels of another. Here is a fictional example where we are interested in the effect of five methods of teaching algebra on students' scores for the math portion of the SAT. Suppose three algebra classes are randomly sampled from classes using each of the five methods so that class is nested in method as demonstrated in the following tabulation.

```
. use https://www.stata-press.com/data/r19/sat
(Fictional SAT data)
. tabulate class method
```

Class ID	Five methods of teaching algebra					Total
	1	2	3	4	5	
1	5	0	0	0	0	5
2	5	0	0	0	0	5
3	5	0	0	0	0	5
4	0	5	0	0	0	5
5	0	5	0	0	0	5
6	0	5	0	0	0	5
7	0	0	5	0	0	5
8	0	0	5	0	0	5
9	0	0	5	0	0	5
10	0	0	0	5	0	5
11	0	0	0	5	0	5
12	0	0	0	5	0	5
13	0	0	0	0	5	5
14	0	0	0	0	5	5
15	0	0	0	0	5	5
Total	15	15	15	15	15	75

We will consider method as fixed and class nested in method as random. To use class nested in method as the error term for method, we can specify the following anova model:

```
. anova score method / class|method /
```

	Number of obs =	75	R-squared =	0.7599	
	Root MSE =	71.8517	Adj R-squared =	0.7039	
Source	Partial SS	df	MS	F	Prob>F
Model	980312	14	70022.286	13.56	0.0000
method	905872	4	226468	30.42	0.0000
class method	74440	10	7444		
class method	74440	10	7444	1.44	0.1845
Residual	309760	60	5162.6667		
Total	1290072	74	17433.405		

Like anova, contrast allows the | operator, which specifies that one variable is nested in the levels of another. We can use contrast to test the main effects of method and the simple effects of class within method.

```
. contrast method class|method
```

Contrasts of marginal linear predictions

Margins: asbalanced

	df	F	P>F
method	(not testable)		
class method			
1	2	2.80	0.0687
2	2	0.91	0.4089
3	2	1.10	0.3390
4	2	0.22	0.8025
5	2	2.18	0.1221
Joint	10	1.44	0.1845
Denominator	60		

Although `contrast` was able to perform the individual tests for the simple effects of `class` within `method`, empty cells in the interaction between `method` and `class` prevented `contrast` from testing for a main effect of `method`. Here we add the `emptycells(reweight)` option so that `contrast` can take the empty cells into account when computing the marginal means for `method`.

```
. contrast method class|method, emptycells(reweight)
Contrasts of marginal linear predictions
Margins:    asbalanced
Empty cells: reweight
```

	df	F	P>F
method	4	43.87	0.0000
class method			
1	2	2.80	0.0687
2	2	0.91	0.4089
3	2	1.10	0.3390
4	2	0.22	0.8025
5	2	2.18	0.1221
Joint	10	1.44	0.1845
Denominator	60		

Now, `contrast` does report a test for the main effects of `method`. However, if we compare this with the `anova` results, we will see that the results are different. They are different because `contrast` uses the residual error term to compute the F test by default. Using notation similar to `anova`, we can use the `/` operator to specify a different error term for the test. Therefore, we can reproduce the test of main effects from our `anova` command by typing

```
. contrast method / class|method /, emptycells(reweight)
Contrasts of marginal linear predictions
Margins:    asbalanced
Empty cells: reweight
```

	df	F	P>F
method	4	30.42	0.0000
class method	10 (denominator)		
class method			
1	2	2.80	0.0687
2	2	0.91	0.4089
3	2	1.10	0.3390
4	2	0.22	0.8025
5	2	2.18	0.1221
Joint	10	1.44	0.1845
Denominator	60		

Multiple comparisons

We have seen that `contrast` can report the individual linear combinations that make up the requested effects. Depending upon the specified option, `contrast` will report confidence intervals, p -values, or both in the effects table. By default, the reported confidence intervals and p -values are not adjusted for multiple comparisons. Use the `mcompare()` option to adjust the confidence intervals and p -values for multiple comparisons of the individual effects.

Let's compute the grand mean effects of `race` using the `g.` operator. We also specify the `mcompare(bonferroni)` option to compute p -values and confidence intervals using Bonferroni's adjustment.

```
. use https://www.stata-press.com/data/r19/cholesterol
(Artificial cholesterol data)
. anova chol race##agegrp
(output omitted)
. contrast g.race, mcompare(bonferroni)
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	Bonferroni	
			P>F	P>F
race				
(Black vs mean)	1	7.07	0.0100	0.0301
(White vs mean)	1	2.82	0.0982	0.2947
(Other vs mean)	1	0.96	0.3312	0.9936
Joint	2	3.62	0.0329	
Denominator	60			

Note: Bonferroni-adjusted p -values are reported for tests on individual contrasts only.

	Number of comparisons
race	3

	Contrast	Std. err.	Bonferroni	
			[95% conf. interval]	
race				
(Black vs mean)	4.17666	1.570588	.3083743	8.044945
(White vs mean)	-2.638058	1.570588	-6.506343	1.230227
(Other vs mean)	-1.538602	1.570588	-5.406887	2.329684

The last table reports a Bonferroni-adjusted confidence interval for each individual contrast. (Use the `effects` option to add p -values to the last table.) The first table includes a Bonferroni-adjusted p -value for each test that is not a joint test.

Joint tests are never adjusted for multiple comparisons. For example,

```
. contrast race@agegrp, mcompare(bonferroni)
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
race@agegrp			
10-19	2	1.37	0.2620
20-29	2	2.44	0.0958
30-39	2	3.12	0.0512
40-59	2	0.53	0.5889
60-79	2	2.90	0.0628
Joint	10	2.07	0.0409
Denominator	60		

Note: Bonferroni-adjusted p-values are reported for tests on individual contrasts only.

	Number of comparisons
race@agegrp	10

	Contrast	Std. err.	Bonferroni [95% conf. interval]	
race@agegrp				
(White vs base) 10-19	6.841855	6.082862	-10.88697	24.57068
(White vs base) 20-29	-11.80631	6.082862	-29.53513	5.922513
(White vs base) 30-39	-14.40607	6.082862	-32.13489	3.322751
(White vs base) 40-59	-4.101691	6.082862	-21.83051	13.62713
(White vs base) 60-79	-10.60137	6.082862	-28.33019	7.127448
(Other vs base) 10-19	-2.975244	6.082862	-20.70407	14.75358
(Other vs base) 20-29	-11.45679	6.082862	-29.18561	6.272031
(Other vs base) 30-39	-11.41958	6.082862	-29.1484	6.309244
(Other vs base) 40-59	-6.17807	6.082862	-23.90689	11.55075
(Other vs base) 60-79	3.453375	6.082862	-14.27545	21.1822

Here we have five tests of simple effects with two degrees of freedom each. No Bonferroni-adjusted *p*-values are available for these tests, but the confidence intervals for the individual contrasts are adjusted.

Unbalanced data

By default, `contrast` treats all factors as balanced when computing marginal means. By balanced, we mean that `contrast` assumes an equal number of observations in each level of each factor and an equal number of observations in each cell of each interaction. If our data are balanced, there is no issue. If, however, our data are not balanced, we might prefer that `contrast` use the actual cell frequencies from our data in computing marginal means. We instruct `contrast` to use observed frequencies by adding the `asobserved` option.

Even if our data are unbalanced, we might still want `contrast` to compute balanced marginal means. It depends on what we want to test and what our data represent. If we have data from a designed experiment that started with an equal number of males and females but the data became unbalanced because the data from a few males were unusable, we might still want our margins computed as though the data were balanced. If, however, we have a representative sample of individuals from Los Angeles with 40% of European descent, 34% African-American, 25% Hispanic, and 1% Australian, we probably want our margins computed using these representative frequencies. We do not want Australians receiving the same weight as Europeans.

The following examples will use an unbalanced version of our dataset.

```
. use https://www.stata-press.com/data/r19/cholesterol3
(Artificial cholesterol data, unbalanced)
. tab race agegrp
```

Race	Age group					Total
	10-19	20-29	30-39	40-59	60-79	
Black	1	5	5	4	3	18
White	4	5	7	4	4	24
Other	3	7	6	5	4	25
Total	8	17	18	13	11	67

The row labeled `Total` gives observed cell frequencies for age group. These can be obtained by summing frequencies from the cells in the corresponding column. In this respect, we can also refer to them as marginal frequencies. We use the terms marginal frequencies and cell frequencies interchangeably below.

We begin by fitting the two-factor model with an interaction.

```
. anova chol race##agegrp
```

Source	Partial SS	df	MS	F	Prob>F
Model	16379.993	14	1169.9995	16.68	0.0000
race	230.7544	2	115.3772	1.64	0.2029
agegrp	13857.988	4	3464.4969	49.39	0.0000
race#agegrp	857.81521	8	107.2269	1.53	0.1701
Residual	3647.2774	52	70.13995		
Total	20027.27	66	303.44349		

Using observed cell frequencies

Recall that the marginal means are computed from the cell means. Treating the factors as balanced yields the following marginal means for race:

$$\eta_{1.} = \frac{1}{5}(\mu_{11} + \mu_{12} + \mu_{13} + \mu_{14} + \mu_{15})$$

$$\eta_{2.} = \frac{1}{5}(\mu_{21} + \mu_{22} + \mu_{23} + \mu_{24} + \mu_{25})$$

$$\eta_{3.} = \frac{1}{5}(\mu_{31} + \mu_{32} + \mu_{33} + \mu_{34} + \mu_{35})$$

If we have a fixed population and unbalanced cells, then the $\eta_{i.}$ do not represent population means. If, however, our data are representative of the population, we can use the frequencies from our estimation sample to estimate the population marginal means, denoted $\mu_{i.}$.

Here are the results of testing for a main effect of race, treating all the factors as balanced.

```
. contrast r.race
```

```
Contrasts of marginal linear predictions
```

```
Margins: asbalanced
```

	df	F	P>F
race			
(White vs Black)	1	3.28	0.0757
(Other vs Black)	1	1.50	0.2263
Joint	2	1.64	0.2029
Denominator	52		

	Contrast	Std. err.	[95% conf. interval]	
race				
(White vs Black)	-5.324254	2.93778	-11.21934	.5708338
(Other vs Black)	-3.596867	2.93778	-9.491955	2.298221

The row labeled (White vs Black) is the test for $\eta_{2.} = \eta_{1.}$. The row labeled (Other vs Black) is the test for $\eta_{3.} = \eta_{1.}$.

If the observed marginal frequencies are representative of the distribution of the levels of agegrp, we can use them to form the marginal means of cho1 for each of the levels of race from the cell means.

$$\mu_{1.} = \frac{1}{67}(8\mu_{11} + 17\mu_{12} + 18\mu_{13} + 13\mu_{14} + 11\mu_{15})$$

$$\mu_{2.} = \frac{1}{67}(8\mu_{21} + 17\mu_{22} + 18\mu_{23} + 13\mu_{24} + 11\mu_{25})$$

$$\mu_{3.} = \frac{1}{67}(8\mu_{31} + 17\mu_{32} + 18\mu_{33} + 13\mu_{34} + 11\mu_{35})$$

Here are the results of testing for the main effects of race, using the observed marginal frequencies:

```
. contrast r.race, asobserved
Contrasts of marginal linear predictions
Margins: asobserved
```

	df	F	P>F
race			
(White vs Black)	1	7.25	0.0095
(Other vs Black)	1	3.89	0.0538
Joint	2	3.74	0.0304
Denominator	52		

	Contrast	Std. err.	[95% conf. interval]	
race				
(White vs Black)	-7.232433	2.686089	-12.62246	-1.842402
(Other vs Black)	-5.231198	2.651203	-10.55123	.0888295

The row labeled (White vs Black) is the test for $\mu_{2.} = \mu_{1.}$. The row labeled (Other vs Black) is the test for $\mu_{3.} = \mu_{1.}$. Both tests were insignificant when we tested the cell means resulting from balanced frequencies; however, when we tested the cell means from observed frequencies, the first test is significant beyond the 5% level (and the second test is nearly so).

Here we reproduce the results of the `asobserved` option with custom contrasts. Because we are modifying the way that the marginal means are constructed from the cell means, we will specify the contrasts on the predicted cell means. We use macro expansion, `=exp`, to evaluate the fractions instead of approximating them with decimals. Macro expansion guarantees that the contrast coefficients sum to zero. For more information, see [Macro expansion operators and function in \[P\] macro](#).

```
. contrast {race#agegrp -'=8/67' -'=17/67' -'=18/67' -'=13/67' -'=11/67'
>                '=8/67'  '=17/67'  '=18/67'  '=13/67'  '=11/67'}
>                {race#agegrp -'=8/67' -'=17/67' -'=18/67' -'=13/67' -'=11/67'
>                0          0          0          0          0
>                '=8/67'  '=17/67'  '=18/67'  '=13/67'  '=11/67'}
```

Contrasts of marginal linear predictions

Margins: asbalanced

	df	F	P>F
race#agegrp			
(1) (1)	1	7.25	0.0095
(2) (2)	1	3.89	0.0538
Joint	2	3.74	0.0304
Denominator	52		

	Contrast	Std. err.	[95% conf. interval]	
race#agegrp				
(1) (1)	-7.232433	2.686089	-12.62246	-1.842402
(2) (2)	-5.231198	2.651203	-10.55123	.0888295

Weighted contrast operators

`contrast` provides observation-weighted versions of five of the contrast operators—`gw.`, `hw.`, `jw.`, `pw.`, and `qw.`. The first three of these operators perform comparisons of means across cells, and like the marginal means just discussed, these means can be computed in two ways: 1) as though the cell frequencies were equal or 2) using the observed cell frequencies from the estimation sample. The weighted operators provide versions of the standard (as balanced) operators that weight these means by their cell frequencies. The two orthogonal polynomial operators involve similar adjustments for weighting.

Let's examine what this means by using the `gw.` operator. The `gw.` operator is a weighted version of the `g.` operator. The `gw.` operator computes the grand mean using the cell frequencies for race obtained from the model fit.

Here we test the effects of race, comparing each level with the weighted grand mean but otherwise treating the factors as balanced in the marginal mean calculations.

```
. contrast gw.race
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
race			
(Black vs mean)	1	2.78	0.1014
(White vs mean)	1	2.06	0.1573
(Other vs mean)	1	0.06	0.8068
Joint	2	1.64	0.2029
Denominator	52		

	Contrast	Std. err.	[95% conf. interval]	
race				
(Black vs mean)	3.24931	1.948468	-.6605779	7.159198
(White vs mean)	-2.074944	1.44618	-4.976915	.8270276
(Other vs mean)	-.347557	1.414182	-3.18532	2.490206

The observed marginal frequencies of race are 18, 24, and 25. Thus, the row labeled (Black vs Mean) tests $\eta_1 = (18\eta_1 + 24\eta_2 + 25\eta_3)/67$; the row labeled (White vs Mean) tests $\eta_2 = (18\eta_1 + 24\eta_2 + 25\eta_3)/67$; and the row labeled (Other vs Mean) tests $\eta_3 = (18\eta_1 + 24\eta_2 + 25\eta_3)/67$.

Now, we reproduce the above results using custom contrasts. We are weighting the calculation of the grand mean from the marginal means for each of the races, but we are not weighting the calculation of the marginal means themselves. Therefore, we can specify the custom contrast on the marginal means for race instead of on the cell means.

```
. contrast {race '49/67' '-24/67' '-25/67'}
> {race '-18/67' '43/67' '-25/67'}
> {race '-18/67' '-24/67' '42/67'}
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
race			
(1)	1	2.78	0.1014
(2)	1	2.06	0.1573
(3)	1	0.06	0.8068
Joint	2	1.64	0.2029
Denominator	52		

	Contrast	Std. err.	[95% conf. interval]	
race				
(1)	3.24931	1.948468	-.6605779	7.159198
(2)	-2.074944	1.44618	-4.976915	.8270276
(3)	-.347557	1.414182	-3.18532	2.490206

Now, we will test for each race the difference between the marginal mean and the weighted grand mean, treating the factors as observed in the marginal mean calculations.

```
. contrast gw.race, asobserved wald ci
Contrasts of marginal linear predictions
Margins: asobserved
```

	df	F	P>F
race			
(Black vs mean)	1	6.81	0.0118
(White vs mean)	1	3.74	0.0587
(Other vs mean)	1	0.26	0.6099
Joint	2	3.74	0.0304
Denominator	52		

	Contrast	Std. err.	[95% conf. interval]	
race				
(Black vs mean)	4.542662	1.740331	1.050432	8.034891
(White vs mean)	-2.689771	1.39142	-5.481859	.1023172
(Other vs mean)	-.6885363	1.341261	-3.379973	2.002901

The row labeled (Black vs Mean) tests $\mu_1 = (18\mu_1 + 24\mu_2 + 25\mu_3)/67$; the row labeled (White vs Mean) tests $\mu_2 = (18\mu_1 + 24\mu_2 + 25\mu_3)/67$; and the row labeled (Other vs Mean) tests $\mu_3 = (18\mu_1 + 24\mu_2 + 25\mu_3)/67$.

Here we use a custom contrast to reproduce the above result testing $\mu_1 = (18\mu_1 + 24\mu_2 + 25\mu_3)/67$. Because both the calculation of the marginal means and the calculation of the grand mean are adjusted, we specify the custom contrast on the cell means.

```
. contrast {race#agegrp ' =49/67*8/67' ' =49/67*17/67' ' =49/67*18/67'
> ' =49/67*13/67' ' =49/67*11/67'
> ' =24/67*8/67' -' =24/67*17/67' -' =24/67*18/67'
> -' =24/67*13/67' -' =24/67*11/67'
> -' =25/67*8/67' -' =25/67*17/67' -' =25/67*18/67'
> -' =25/67*13/67' -' =25/67*11/67'}, nowald
Contrasts of marginal linear predictions
Margins: asbalanced
```

	Contrast	Std. err.	[95% conf. interval]	
race#agegrp (1) (1)	4.542662	1.740331	1.050432	8.034891

The Helmert and reverse Helmert contrasts also involve calculating averages of the marginal means; therefore, weighted versions of these parameters are available as well. The `hw.` operator is a weighted version of the `h.` operator that computes the mean of the subsequent levels using the cell frequencies obtained from the model fit. The `jw.` operator is a weighted version of the `j.` operator that computes the mean of the previous levels using the cell frequencies obtained from the model fit.

For orthogonal polynomials, we can use the `pw.` and `qw.` operators, which are the weighted versions of the `p.` and `q.` operators. In this case, the cell frequencies from the model fit are used in the calculation of the orthogonal polynomial contrast coefficients.

Testing factor effects on slopes

For linear models where the independent variables are all factor variables, the linear prediction at fixed levels of the factor variables turns out to be a cell mean. With these models, `contrast` computes and tests the effects of the factor variables on the expected mean of the dependent variable. When factor variables are interacted with continuous variables, `contrast` distinguishes factor effects on the intercept from factor effects on the slope.

Here we have 1980 census data including information on the birthrate (`brate`), the median age (`medage`), and the region of the country (`region`) for each of the 50 states. We can fit an ANCOVA model for `brate` using main effects of the factor variable `region` and the continuous variable `medage`.

```
. use https://www.stata-press.com/data/r19/census3
(1980 Census data by state)
. label list cenreg
cenreg:
    1 NE
    2 NCentral
    3 South
    4 West
. anova brate i.region c.medage
```

	Number of obs =	50	R-squared =	0.8264	
	Root MSE =	12.7575	Adj R-squared =	0.8110	
Source	Partial SS	df	MS	F	Prob>F
Model	34872.859	4	8718.2147	53.57	0.0000
region	2197.7545	3	732.58484	4.50	0.0076
medage	15327.423	1	15327.423	94.18	0.0000
Residual	7323.9611	45	162.75469		
Total	42196.82	49	861.15959		

For those more comfortable with linear regression, this is equivalent to the regression model

```
. regress brate i.region c.medage
```

You may use either.

We can use contrast to compute reference category effects for region. These contrasts compare the adjusted means of NCentral, South, and West regions with the adjusted mean of the NE region.

```
. contrast r.region
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F	
region				
(NCentral vs NE)	1	2.24	0.1417	
(South vs NE)	1	0.78	0.3805	
(West vs NE)	1	10.33	0.0024	
Joint	3	4.50	0.0076	
Denominator	45			

	Contrast	Std. err.	[95% conf. interval]	
region				
(NCentral vs NE)	9.061063	6.057484	-3.139337	21.26146
(South vs NE)	5.06991	5.72396	-6.458738	16.59856
(West vs NE)	21.71328	6.755616	8.106774	35.31979

Let's add the interaction between region and medage to the model.

```
. anova brate region##c.medage
Number of obs = 50 R-squared = 0.9000
Root MSE = 10.0244 Adj R-squared = 0.8833
```

Source	Partial SS	df	MS	F	Prob>F
Model	37976.315	7	5425.1878	53.99	0.0000
region	3405.0704	3	1135.0235	11.30	0.0000
medage	5279.7145	1	5279.7145	52.54	0.0000
region#medage	3103.456	3	1034.4853	10.29	0.0000
Residual	4220.5051	42	100.48822		
Total	42196.82	49	861.15959		

The parameterization for the expected value of brate as a function of region and medage is given by

$$E(\text{brate} | \text{region} = i, \text{medage}) = \alpha_0 + \alpha_i + \beta_0 \text{medage} + \beta_i \text{medage}$$

where α_0 is the intercept and β_0 is the slope of medage. We are modeling the effects of region in two different ways. The α_i parameters measure the effect of region on the intercept, and the β_i parameters measure the effect of region on the slope of medage.

contrast computes and tests effects on slopes separately from effects on intercepts. First, we will compute the reference category effects of region on the intercept:

```
. contrast r.region
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F	
region				
(NCentral vs NE)	1	0.09	0.7691	
(South vs NE)	1	0.01	0.9389	
(West vs NE)	1	8.50	0.0057	
Joint	3	11.30	0.0000	
Denominator	42			

	Contrast	Std. err.	[95% conf. interval]	
region				
(NCentral vs NE)	-49.38396	167.1281	-386.6622	287.8942
(South vs NE)	-9.058983	117.424	-246.0302	227.9123
(West vs NE)	343.0024	117.6547	105.5656	580.4393

Now, we will compute the reference category effects of region on the slope of medage:

```
. contrast r.region#c.medage
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F	
region#c.medage				
(NCentral vs NE)	1	0.16	0.6917	
(South vs NE)	1	0.03	0.8558	
(West vs NE)	1	8.18	0.0066	
Joint	3	10.29	0.0000	
Denominator	42			

	Contrast	Std. err.	[95% conf. interval]	
region#c.medage				
(NCentral vs NE)	2.208539	5.530981	-8.953432	13.37051
(South vs NE)	.6928008	3.788735	-6.953175	8.338777
(West vs NE)	-10.94649	3.827357	-18.67041	-3.22257

At the 5% level, the slope of medage for the West region differs from that of the NE region, but at that level of significance, we cannot say that the slope for the NCentral or the South region differs from that of the NE region.

This model is simple enough that the reference category contrasts reproduce the coefficients for region and for the interactions in an equivalent model fit by regress.

```
. regress brate region#c.medage
```

Source	SS	df	MS	Number of obs	=	50
Model	37976.3149	7	5425.18784	F(7, 42)	=	53.99
Residual	4220.5051	42	100.488217	Prob > F	=	0.0000
				R-squared	=	0.9000
				Adj R-squared	=	0.8833
Total	42196.82	49	861.159592	Root MSE	=	10.024

brate	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
region						
NCentral	-49.38396	167.1281	-0.30	0.769	-386.6622	287.8942
South	-9.058983	117.424	-0.08	0.939	-246.0302	227.9123
West	343.0024	117.6547	2.92	0.006	105.5656	580.4393
medage	-8.802707	3.462865	-2.54	0.015	-15.79105	-1.814362
region#c.medage						
NCentral	2.208539	5.530981	0.40	0.692	-8.953432	13.37051
South	.6928008	3.788735	0.18	0.856	-6.953175	8.338777
West	-10.94649	3.827357	-2.86	0.007	-18.67041	-3.22257
_cons	411.8268	108.2084	3.81	0.000	193.4533	630.2002

This will not be the case for models that are more complicated.

Chow tests

Now, let's suppose we are fitting a model for birthrates on median age and marriage rate. We are also interested in whether the regression coefficients differ for states in the east versus states in the west. We use census divisions to create a new variable, west, that indicates which states are in the western half of the United States.

```
. generate west = inlist(division, 4, 7, 8, 9)
```

We fit a model that includes a separate intercept for west as well as an interaction between west and each of the other variables in our model.

```
. regress brate i.west#c.medage i.west#c.mrgrate
```

Source	SS	df	MS	Number of obs	=	50
Model	38516.2172	5	7703.24344	F(5, 44)	=	92.09
Residual	3680.60281	44	83.6500639	Prob > F	=	0.0000
				R-squared	=	0.9128
				Adj R-squared	=	0.9029
Total	42196.82	49	861.159592	Root MSE	=	9.146

brate	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
1.west	327.8733	58.71793	5.58	0.000	209.5351	446.2115
medage	-7.532304	1.387624	-5.43	0.000	-10.32888	-4.735731
west# c.medage 1	-10.11443	1.849103	-5.47	0.000	-13.84105	-6.387808
mrgrate	828.6813	643.3443	1.29	0.204	-467.8939	2125.257
west# c.mrgrate 1	-800.8036	645.488	-1.24	0.221	-2101.699	500.092
_cons	366.5325	47.08904	7.78	0.000	271.6308	461.4343

We can test the effects of west on the intercept and on the slopes of medage and mrgrate. We will specify all of these effects in a single contrast command and include the overall option to obtain a joint test of effects, that is, a test that the coefficients for eastern states and for western states are equal.

```
. contrast west west#c.medage west#c.mrgrate, overall
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
west	1	31.18	0.0000
west#c.medage	1	29.92	0.0000
west#c.mrgrate	1	1.54	0.2213
Overall	3	22.82	0.0000
Denominator	44		

This overall test is referred to as a Chow test in econometrics (Chow 1960).

Beyond linear models

`contrast` may be used after almost any estimation command, with the added benefit that `contrast` provides direct support for testing main and interaction effects that is not available in most estimation commands. To illustrate, we will use `contrast` with results from a logistic regression. Stata's `logit` command fits logistic regression models, reporting the fitted regression coefficients. The `logistic` command fits the same models but reports odds ratios. Although `contrast` can report odds ratios for the computed effects, the tests are all computed from linear combinations of the model coefficients regardless of which estimation command we used.

Suppose we have data on patient satisfaction for three hospitals in a city. Let's begin by fitting a model for `satisfied`, whether the patient was satisfied with his or her treatment, using the main effects of `hospital`:

```
. use https://www.stata-press.com/data/r19/hospital, clear
(Artificial hospital satisfaction data)
. logit satisfied i.hospital
Iteration 0:  Log likelihood = -393.72216
Iteration 1:  Log likelihood = -387.55736
Iteration 2:  Log likelihood = -387.4768
Iteration 3:  Log likelihood = -387.47679

Logistic regression
Log likelihood = -387.47679
Number of obs = 802
LR chi2(2) = 12.49
Prob > chi2 = 0.0019
Pseudo R2 = 0.0159
```

satisfied	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
hospital						
2	.5348129	.2136021	2.50	0.012	.1161604	.9534654
3	.7354519	.2221929	3.31	0.001	.2999618	1.170942
_cons	1.034708	.1391469	7.44	0.000	.7619855	1.307431

Because there are no other independent variables in this model, the reference category effects of `hospital` computed by `contrast` will match the fitted model coefficients, assuming a common reference level.

```
. contrast r.hospital
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	chi2	P>chi2
hospital			
(2 vs 1)	1	6.27	0.0123
(3 vs 1)	1	10.96	0.0009
Joint	2	12.55	0.0019

	Contrast	Std. err.	[95% conf. interval]	
hospital				
(2 vs 1)	.5348129	.2136021	.1161604	.9534654
(3 vs 1)	.7354519	.2221929	.2999618	1.170942

We see that the reference category effects are equal to the fitted coefficients. They also have the same interpretation, the difference in log odds from the reference category. The top table also provides a joint test of these effects, a test of the main effects of hospital.

We also have information on the condition for which each patient is being treated in the variable illness. Here we fit a logistic regression using a two-way crossed model of hospital and illness.

```
. label list illness
illness:
    1 Heart attack
    2 Stroke
    3 Pneumonia
    4 Lung disease
    5 Kidney failure

. logistic satisfied hospital##illness
Logistic regression                                Number of obs =   802
                                                    LR chi2(14)    =  38.51
                                                    Prob > chi2   = 0.0004
                                                    Pseudo R2     = 0.0489
Log likelihood = -374.46865
```

satisfied	Odds ratio	Std. err.	z	P> z	[95% conf. interval]	
hospital						
2	1.226496	.5492177	0.46	0.648	.509921	2.950049
3	1.711111	.8061016	1.14	0.254	.6796395	4.308021
illness						
Stroke	1.328704	.6044214	0.62	0.532	.544779	3.240678
Pneumonia	.7993827	.3408305	-0.53	0.599	.3466015	1.843653
Lung dise..	1.231481	.5627958	0.46	0.649	.5028318	3.016012
Kidney fa..	1.25	.5489438	0.51	0.611	.5285676	2.956102
hospital#illness						
2#Stroke	2.434061	1.768427	1.22	0.221	.5860099	10.11016
2#Pneumonia	4.045805	2.868559	1.97	0.049	1.008058	16.23769
2 #						
Lung dise..	.54713	.3469342	-0.95	0.342	.1578866	1.89599
2 #						
Kidney fa..	1.594425	1.081104	0.69	0.491	.4221288	6.022312
3#Stroke	.5416535	.3590089	-0.93	0.355	.1477555	1.985635
3#Pneumonia	1.579502	1.042504	0.69	0.489	.4332209	5.758783
3 #						
Lung dise..	3.137388	2.595748	1.38	0.167	.6198955	15.87881
3 #						
Kidney fa..	1.672727	1.226149	0.70	0.483	.3976256	7.036812
_cons	2.571429	.8099239	3.00	0.003	1.386983	4.767358

Note: **_cons** estimates baseline odds.

Using contrast, we can obtain an ANOVA-style table of tests for the main effects and interaction effects of hospital and illness.

```
. contrast hospital##illness
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	chi2	P>chi2
hospital	2	14.92	0.0006
illness	4	4.09	0.3937
hospital#illness	8	20.45	0.0088

Our interaction effect is significant, so we decide to evaluate the simple reference category effects of hospital within illness. We are particularly interested in patient satisfaction when being treated for a heart attack or stroke, so we will use the `i.` operator to limit our output to simple effects within the first two illnesses.

```
. contrast r.hospital@i(1 2).illness, nowald
Contrasts of marginal linear predictions
Margins: asbalanced
```

	Contrast	Std. err.	[95% conf. interval]	
hospital@illness				
(2 vs 1) Heart attack	.2041611	.4477942	-.6734995	1.081822
(2 vs 1) Stroke	1.093722	.5721288	-.0276296	2.215074
(3 vs 1) Heart attack	.5371429	.4710983	-.3861928	1.460479
(3 vs 1) Stroke	-.0759859	.4662325	-.9897847	.8378129

The row labeled (2 vs 1) heart attack estimates simple effects on the log odds when comparing hospital 2 with hospital 1 for patients having heart attacks. These effects are differences in the cell means of the linear predictions.

We can add the `or` option to report an odds ratio for each of these simple effects:

```
. contrast r.hospital@i(1 2).illness, nowald or
Contrasts of marginal linear predictions
Margins: asbalanced
```

	Odds ratio	Std. err.	[95% conf. interval]	
hospital@illness				
(2 vs 1) Heart attack	1.226496	.5492177	.509921	2.950049
(2 vs 1) Stroke	2.985366	1.708014	.9727486	9.162089
(3 vs 1) Heart attack	1.711111	.8061016	.6796395	4.308021
(3 vs 1) Stroke	.9268293	.4321179	.3716567	2.311306

These odds ratios are just the exponentiated version of the contrasts in the previous table.

For contrasts of the margins of nonlinear predictions, such as predicted probabilities, see [\[R\] margins, contrast](#).

Multiple equations

contrast works with models containing multiple equations. Commands such as intreg and gnbreg allow their ancillary parameters to be modeled as functions of independent variables, and contrast can compute and test effects within these equations. In addition, contrast allows a special pseudofactor for equation—called `_eqns`—when working with results from `manova`, `mvreg`, `mlogit`, and `mprobit`.

In example 4 of [MV] `manova`, we fit a two-way MANOVA model using data from Woodard (1931). Here we will fit this model using `mvreg`. The data represent patients with jaw fractures. `y1` is the patient's age, `y2` is blood lymphocytes, and `y3` is blood polymorphonuclears. Two factor variables, `gender` and `fracture`, are used as independent variables.

```
. use https://www.stata-press.com/data/r19/jaw
(Table 4.6. Two-way unbalanced data for fractures of the jaw, Rencher (1998))
```

```
. mvreg y1 y2 y3 = gender##fracture, vsquish nofvlabel
```

Equation	Obs	Parms	RMSE	"R-sq"	F	P>F
y1	27	6	10.21777	0.4086	2.902124	0.0382
y2	27	6	5.268768	0.4743	3.78967	0.0133
y3	27	6	4.993647	0.4518	3.460938	0.0195

	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
y1						
2.gender	-17.5	11.03645	-1.59	0.128	-40.45156	5.451555
fracture						
2	-12.625	5.518225	-2.29	0.033	-24.10078	-1.149222
3	5.666667	5.899231	0.96	0.348	-6.601456	17.93479
gender#						
fracture						
2 2	21.375	12.68678	1.68	0.107	-5.008595	47.75859
2 3	8.833333	13.83492	0.64	0.530	-19.93796	37.60463
_cons	39.5	4.171386	9.47	0.000	30.82513	48.17487
y2						
2.gender	20.5	5.69092	3.60	0.002	8.665083	32.33492
fracture						
2	-3.125	2.84546	-1.10	0.285	-9.042458	2.792458
3	.6666667	3.041925	0.22	0.829	-5.659362	6.992696
gender#						
fracture						
2 2	-19.625	6.541907	-3.00	0.007	-33.22964	-6.02036
2 3	-23.66667	7.133946	-3.32	0.003	-38.50252	-8.830813
_cons	35.5	2.150966	16.50	0.000	31.02682	39.97318
y3						
2.gender	-18.16667	5.393755	-3.37	0.003	-29.38359	-6.949739
fracture						
2	1.083333	2.696877	0.40	0.692	-4.52513	6.691797
3	-3	2.883083	-1.04	0.310	-8.9957	2.9957
gender#						
fracture						
2 2	19.91667	6.200305	3.21	0.004	7.022426	32.81091
2 3	23.5	6.76143	3.48	0.002	9.438837	37.56116
_cons	61.16667	2.038648	30.00	0.000	56.92707	65.40627

contrast computes Wald tests using the coefficients from the first equation by default.

```
. contrast gender##fracture
Contrasts of marginal linear predictions
Margins: asbalanced
```

		df	F	P>F
y1	gender	1	2.16	0.1569
	fracture	2	2.74	0.0880
	gender#fracture	2	1.69	0.2085
	Denominator	21		

Here we use the equation() option to compute the Wald tests in the y2 equation:

```
. contrast gender##fracture, equation(y2)
Contrasts of marginal linear predictions
Margins: asbalanced
```

		df	F	P>F
y2	gender	1	5.41	0.0301
	fracture	2	7.97	0.0027
	gender#fracture	2	5.97	0.0088
	Denominator	21		

Here we use the equation index to compute the Wald tests in the third equation:

```
. contrast gender##fracture, equation(#3)
Contrasts of marginal linear predictions
Margins: asbalanced
```

		df	F	P>F
y3	gender	1	2.23	0.1502
	fracture	2	6.36	0.0069
	gender#fracture	2	6.66	0.0058
	Denominator	21		

Here we use the `atequations` option to compute Wald tests for each equation in the model. We also use the `vsquish` option to suppress the extra blank lines between terms.

```
. contrast gender##fracture, atequations vsquish
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
y1			
gender	1	2.16	0.1569
fracture	2	2.74	0.0880
gender#fracture	2	1.69	0.2085
y2			
gender	1	5.41	0.0301
fracture	2	7.97	0.0027
gender#fracture	2	5.97	0.0088
y3			
gender	1	2.23	0.1502
fracture	2	6.36	0.0069
gender#fracture	2	6.66	0.0058
Denominator	21		

Because we are investigating the results from `mvreg`, we can use the special `_eqns` factor to test for a marginal effect on the means among the dependent variables:

```
. contrast _eqns
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
_eqns	2	49.19	0.0000
Denominator	21		

Here we test whether the main effects of `gender` differ among the dependent variables:

```
. contrast gender#_eqns
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
gender#_eqns	2	3.61	0.0448
Denominator	21		

Although it is not terribly interesting in this case, we can even calculate contrasts across equations:

```
. contrast gender#r._eqns
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
gender#_eqns			
(joint) (2 vs 1)	1	5.82	0.0251
(joint) (3 vs 1)	1	0.40	0.5352
Joint	2	3.61	0.0448
Denominator	21		

Video example

[Introduction to contrasts in Stata: One-way ANOVA](#)

Stored results

`contrast` stores the following in `r()`:

Scalars

`r(df_r)` variance degrees of freedom
`r(k_terms)` number of terms in *termlist*
`r(level)` confidence level of confidence intervals

Macros

`r(cmd)` `contrast`
`r(cmdline)` command as typed
`r(est_cmd)` `e(cmd)` from original estimation results
`r(est_cmdline)` `e(cmdline)` from original estimation results
`r(title)` title in output
`r(overall)` overall or empty
`r(emptycells)` *empspec* from `emptycells()`
`r(mcmethod)` *method* from `mcompare()`
`r(mctitle)` title for *method* from `mcompare()`
`r(mcadjustall)` `adjustall` or empty
`r(margin_method)` `asbalanced` or `asobserved`

Matrices

`r(b)` contrast estimates
`r(V)` variance-covariance matrix of the contrast estimates
`r(error)` contrast estimability codes;
0 means estimable,
8 means not estimable
`r(L)` matrix of contrasts applied to the model coefficients
`r(table)` matrix containing the contrasts with their standard errors,
test statistics, *p*-values, and confidence intervals
`r(F)` vector of *F* statistics; `r(df_r)` present
`r(chi2)` vector of χ^2 statistics; `r(df_r)` not present
`r(p)` vector of *p*-values corresponding to `r(F)` or `r(chi2)`
`r(df)` vector of degrees of freedom corresponding to `r(p)`
`r(df2)` vector of denominator degrees of freedom corresponding to `r(F)`

contrast with the post option stores the following in `e()`:

Scalars

`e(df_r)` variance degrees of freedom
`e(k_terms)` number of terms in *term*list

Macros

`e(cmd)` contrast
`e(cmdline)` command as typed
`e(properties)` b v
`e(est_cmd)` `e(cmd)` from original estimation results
`e(est_cmdline)` `e(cmdline)` from original estimation results
`e(title)` title in output
`e(overall)` overall or empty
`e(emptycells)` *empspec* from `emptycells()`
`e(margin_method)` asbalanced or asobserved
`e(asbalanced)` factor variables fvset as asbalanced
`e(asobserved)` factor variables fvset as asobserved

Matrices

`e(b)` contrast estimates
`e(V)` variance-covariance matrix of the contrast estimates
`e(error)` contrast estimability codes;
 0 means estimable,
 8 means not estimable
`e(L)` matrix of contrasts applied to the model coefficients
`e(F)` vector of unadjusted *F* statistics; `e(df_r)` present
`e(chi2)` vector of χ^2 statistics; `e(df_r)` not present
`e(p)` vector of unadjusted *p*-values corresponding to `e(F)` or `e(chi2)`
`e(df)` vector of degrees of freedom corresponding to `e(p)`
`e(df2)` vector of denominator degrees of freedom corresponding to `e(F)`

Methods and formulas

Methods and formulas are presented under the following headings:

Marginal linear predictions
Contrast operators
Reference level contrasts
Adjacent contrasts
Grand mean contrasts
Helmert contrasts
Reverse Helmert contrasts
Orthogonal polynomial contrasts
Contrasts within interactions
Multiple comparisons

Marginal linear predictions

contrast treats intercept effects separately from slope effects. To illustrate, consider the following parameterization for a quadratic regression of y on x that also models the effects of two factor variables A and B , where the levels of A are indexed by $i = 1, \dots, k_a$ and the levels of B are indexed by $j = 1, \dots, k_b$.

$$\begin{aligned} E(y|A = i, B = j, x) &= \eta_{0ij} + \eta_{1ij}x + \eta_{2ij}x^2 \\ \eta_{0ij} &= \eta_0 + \alpha_{0i} + \beta_{0j} + (\alpha\beta)_{0ij} \\ \eta_{1ij} &= \eta_1 + \alpha_{1i} + \beta_{1j} + (\alpha\beta)_{1ij} \\ \eta_{2ij} &= \eta_2 + \alpha_{2i} + \beta_{2j} + (\alpha\beta)_{2ij} \end{aligned}$$

We have partitioned the coefficients into three groups of parameters: η_{0ij} is a cell prediction for the intercept, η_{1ij} is a cell prediction for the slope on x , and η_{2ij} is a cell prediction for the slope on x^2 . For the intercept parameters, η_0 is the intercept, α_{0i} represents a main effect for factor A at its i th level, β_{0j} represents a main effect for factor B at its j th level, and $(\alpha\beta)_{0ij}$ represents an effect for the interaction of A and B at the ij th level. The individual coefficients in η_{1ij} and η_{2ij} have similar interpretations, but the effects are on the slopes of x and x^2 , respectively.

The marginal intercepts for A are given by

$$\eta_{0i.} = \sum_{j=1}^{k_b} f_{ij} \eta_{0ij}$$

where f_{ij} is a marginal relative frequency of the j th level of B and is controlled by the `asobserved` and `emptycells(reweight)` options according to

$$f_{ij} = \begin{cases} 1/k_b, & \text{default} \\ w_{.j}/w_{..}, & \text{asobserved} \\ 1/(k_b - e_{i.}), & \text{emptycells(reweight)} \\ w_{ij}/w_{i.}, & \text{emptycells(reweight) and asobserved} \end{cases}$$

Above, w_{ij} is the number of individuals with A at its i th level and B at its j th,

$$\begin{aligned} w_{i.} &= \sum_{j=1}^{k_b} w_{ij} \\ w_{.j} &= \sum_{i=1}^{k_a} w_{ij} \\ w_{..} &= \sum_{i=1}^{k_a} \sum_{j=1}^{k_b} w_{ij} \end{aligned}$$

and $e_{i.}$ is the number of empty cells where A is at its i th level. The marginal intercepts for B and marginal slopes on x and x^2 are similarly defined.

Estimates for the cell intercepts and slopes are computed using the corresponding linear combination of the coefficients from the fitted model. For example, the estimated cell intercepts are computed using

$$\hat{\eta}_{0ij} = \hat{\eta}_0 + \hat{\alpha}_{0i} + \hat{\beta}_{0j} + (\widehat{\alpha\beta})_{0ij}$$

and the estimated marginal intercepts for A are computed as

$$\hat{\eta}_{0i.} = \sum_{j=1}^{k_b} f_{ij} \hat{\eta}_{0ij}$$

Contrast operators

`contrast` performs Wald tests using linear combinations of marginal linear predictions. For example, the following linear combination can be used to test for a specific effect of factor A on the marginal intercepts.

$$\sum_{i=1}^{k_a} c_i \eta_{0i.}$$

If the c_i elements sum to zero, the linear combination is called a contrast. If the factor A is represented by a variable named A , then we specify this contrast using the following syntax:

$$\{A \ c_1 \ c_2 \ \dots \ c_{k_a}\}$$

Similarly, the following linear combination can be used to test for a specific interaction effect of factors A and B on the marginal slope of x .

$$\sum_{i=1}^{k_a} \sum_{j=1}^{k_b} c_{ij} \eta_{1ij}$$

If the factor B is represented by a variable named B , then we specify this contrast using the following syntax:

$$\{A\#B \ c_{11} \ c_{12} \ \dots \ c_{1k_b} \ c_{21} \ \dots \ c_{k_a k_b}\}$$

`contrast` has variable operators for several commonly used contrasts. Each contrast operator specifies a matrix of linear combinations that yield the requested set of contrasts to be applied to the marginal linear predictions associated with the attached factor variable.

Reference level contrasts

The *r.* operator compares each level with a reference level. Let \mathbf{R} be the corresponding contrast matrix for factor A , and then \mathbf{R} is a $(k_a - 1) \times k_a$ matrix with elements

$$\mathbf{R}_{ij} = \begin{cases} -1, & \text{if } j \text{ is the reference level} \\ 1, & \text{if } i = j \text{ and } j \text{ is less than the reference level} \\ 1, & \text{if } i + 1 = j \text{ and } j \text{ is greater than the reference level} \\ 0, & \text{otherwise} \end{cases}$$

If $k_a = 5$ and the reference level is the third level of A (specified as `rb(#3) . A`), then

$$\mathbf{R} = \begin{pmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{pmatrix}$$

Adjacent contrasts

The *a.* operator compares each level with the next level. Let \mathbf{A} be the corresponding contrast matrix for factor A , and then \mathbf{A} is a $(k_a - 1) \times k_a$ matrix with elements

$$\mathbf{A}_{ij} = \begin{cases} 1, & \text{if } i = j \\ -1, & \text{if } i + 1 = j \\ 0, & \text{otherwise} \end{cases}$$

If $k_a = 5$, then

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

The *ar.* operator compares each level with the previous level. If \mathbf{A} is the contrast matrix for the *a.* operator, then $-\mathbf{A}$ is the corresponding contrast matrix for the *ar.* operator.

Grand mean contrasts

The *g.* operator compares each level with the mean of all the levels. Let \mathbf{G} be the corresponding contrast matrix for factor A , and then \mathbf{G} is a $k_a \times k_a$ matrix with elements

$$\mathbf{G}_{ij} = \begin{cases} 1 - 1/k_a, & \text{if } i = j \\ -1/k_a, & \text{if } i \neq j \end{cases}$$

If $k_a = 5$, then

$$\mathbf{G} = \begin{pmatrix} 4/5 & -1/5 & -1/5 & -1/5 & -1/5 \\ -1/5 & 4/5 & -1/5 & -1/5 & -1/5 \\ -1/5 & -1/5 & 4/5 & -1/5 & -1/5 \\ -1/5 & -1/5 & -1/5 & 4/5 & -1/5 \\ -1/5 & -1/5 & -1/5 & -1/5 & 4/5 \end{pmatrix}$$

The *gw.* operator compares each level with the weighted mean of all the levels. The weights are taken from the observed weighted cell frequencies in the estimation sample of the fitted model. Let \mathbf{G}_w be the corresponding contrast matrix for factor A , and then \mathbf{G}_w is a $k_a \times k_a$ matrix with elements

$$\mathbf{G}_{ij} = \begin{cases} 1 - w_i/w., & \text{if } i = j \\ -w_j/w., & \text{if } i \neq j \end{cases}$$

where w_i is a marginal weight representing the number of individuals with A at its i th level and $w. = \sum_i w_i$.

Helmert contrasts

The *h.* operator compares each level with the mean of the subsequent levels. Let \mathbf{H} be the corresponding contrast matrix for factor A , and then \mathbf{H} is a $(k_a - 1) \times k_a$ matrix with elements

$$\mathbf{H}_{ij} = \begin{cases} 1, & \text{if } i = j \\ -1/(k_a - i), & \text{if } i < j \\ 0, & \text{otherwise} \end{cases}$$

If $k_a = 5$, then

$$\mathbf{H} = \begin{pmatrix} 1 & -1/4 & -1/4 & -1/4 & -1/4 \\ 0 & 1 & -1/3 & -1/3 & -1/3 \\ 0 & 0 & 1 & -1/2 & -1/2 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

The hw . operator compares each level with the weighted mean of the subsequent levels. Let \mathbf{H}_w be the corresponding contrast matrix for factor A , and then \mathbf{H}_w is a $(k_a - 1) \times k_a$ matrix with elements

$$\mathbf{H}_{wij} = \begin{cases} 1, & \text{if } i = j \\ -w_j / \sum_{l=j}^{k_a} w_l, & \text{if } i < j \\ 0, & \text{otherwise} \end{cases}$$

Reverse Helmert contrasts

The j . operator compares each level with the mean of the previous levels. Let \mathbf{J} be the corresponding contrast matrix for factor A , and then \mathbf{J} is a $(k_a - 1) \times k_a$ matrix with elements

$$\mathbf{J}_{ij} = \begin{cases} 1, & \text{if } i + 1 = j \\ -1/i, & \text{if } j \leq i \\ 0, & \text{otherwise} \end{cases}$$

If $k_a = 5$, then

$$\mathbf{H} = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 \\ -1/2 & -1/2 & 1 & 0 & 0 \\ -1/3 & -1/3 & -1/3 & 1 & 0 \\ -1/4 & -1/4 & -1/4 & -1/4 & 1 \end{pmatrix}$$

The jw . operator compares each level with the weighted mean of the previous levels. Let \mathbf{J}_w be the corresponding contrast matrix for factor A , and then \mathbf{J}_w is a $(k_a - 1) \times k_a$ matrix with elements

$$\mathbf{J}_{wij} = \begin{cases} 1, & \text{if } i + 1 = j \\ -w_j / \sum_{l=1}^i w_l, & \text{if } i \leq j \\ 0, & \text{otherwise} \end{cases}$$

Orthogonal polynomial contrasts

The $p.$ operator applies orthogonal polynomial contrasts using the level values of the attached factor variable. The $q.$ operator applies orthogonal polynomial contrasts using the level indices of the attached factor variable. These two operators are equivalent when the level values of the attached factor are equally spaced. The $pw.$ and $qw.$ operators are weighted versions of $p.$ and $q.$, where the weights are taken from the observed weighted cell frequencies in the estimation sample of the fitted model. `contrast` uses the Christoffel–Darboux recurrence formula for computing orthogonal polynomial contrasts (Abramowitz and Stegun 1964). The elements of the contrasts are normalized such that

$$Q'WQ = \frac{1}{w}I$$

where W is a diagonal matrix of the marginal cell weights w_1, w_2, \dots, w_k of the attached factor variable (all 1 for $p.$ and $q.$), and $w.$ is the sum of the weights (the number of levels k for $p.$ and $q.$).

Contrasts within interactions

Contrast operators are allowed to be specified on factor variables participating in interactions. In such cases, `contrast` applies the proper matrix product of the contrast matrices to the cell margins of the interacted factor variables.

For example, consider the contrasts implied by specifying `r . A#h . B`. Let M be the matrix of estimated cell margins for the levels of A and B , where the rows of M are indexed by the levels of A and the columns are indexed by the levels of B . `contrast` puts the estimated cell margins in the following vector form:

$$v = \text{vec}(M') = \begin{pmatrix} M_{11} \\ M_{12} \\ \vdots \\ M_{1k_b} \\ M_{21} \\ M_{22} \\ \vdots \\ M_{2k_b} \\ \vdots \\ M_{k_a k_b} \end{pmatrix}$$

The individual contrasts are then given by the elements of

$$(R \otimes H)v$$

where \otimes denotes the Kronecker direct product.

Multiple comparisons

See [R] **pwcompare** for details on the methods and formulas used to adjust p -values and confidence intervals for multiple comparisons. The formulas for Bonferroni's method and Šidák's method are presented with $m = k(k - 1)/2$, the number of pairwise comparisons for a factor term with k levels. For contrasts, m is instead the number of contrasts being performed on the factor term; often, $m = k - 1$ for a term with k levels.

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Also see

- [R] **contrast postestimation** — Postestimation tools for contrast
- [R] **lincom** — Linear combinations of parameters
- [R] **margins** — Marginal means, predictive margins, and marginal effects
- [R] **margins, contrast** — Contrasts of margins
- [R] **pwcompare** — Pairwise comparisons
- [R] **test** — Test linear hypotheses after estimation
- [U] **20 Estimation and postestimation commands**

Postestimation commands

The following postestimation commands are available after `contrast`, `post`:

Command	Description
<code>estat vce</code>	variance–covariance matrix of the estimators (VCE)
<code>estat (svy)</code>	postestimation statistics for survey data
<code>estimates</code>	cataloging estimation results
<code>etable</code>	table of estimation results
<code>lincom</code>	point estimates, standard errors, testing, and inference for linear combinations of parameters
<code>nlcom</code>	point estimates, standard errors, testing, and inference for nonlinear combinations of parameters
<code>test</code>	Wald tests of simple and composite linear hypotheses
<code>testnl</code>	Wald tests of nonlinear hypotheses

Remarks and examples

In *Orthogonal polynomial contrasts* in [R] `contrast`, we used the `p.` operator to test the orthogonal polynomial effects of age group.

```
. contrast p.agegrp, noeffects
```

We then used a second `contrast` command,

```
. contrast p(2 3 4).agegrp, noeffects
```

selecting levels to test whether the quadratic, cubic, and quartic contrasts were jointly significant.

We can perform the same joint test by using the `test` command after specifying the `post` option with our first `contrast` command.

```
. use https://www.stata-press.com/data/r19/cholesterol
(Artificial cholesterol data)
. anova chol agegrp
  (output omitted)
. contrast p.agegrp, noeffects post
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	F	P>F
agegrp			
(linear)	1	139.11	0.0000
(quadratic)	1	0.15	0.6962
(cubic)	1	0.37	0.5448
(quartic)	1	0.43	0.5153
Joint	4	35.02	0.0000
Denominator	70		

```
. test p2.agegrp p3.agegrp p4.agegrp
( 1) p2.agegrp = 0
( 2) p3.agegrp = 0
( 3) p4.agegrp = 0
      F( 3, 70) = 0.32
      Prob > F = 0.8129
```

Also see

[R] [contrast](#) — Contrasts and linear hypothesis tests after estimation

[U] [20 Estimation and postestimation commands](#)

Description

`copyright` presents copyright notifications concerning tools, libraries, etc., used in the construction of Stata.

Syntax

`copyright`

Remarks and examples

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Also see

Copyright page of this book

Description Also see

Description

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[Description](#)[Options for correlate](#)[Methods and formulas](#)[Quick start](#)[Options for pwcorr](#)[References](#)[Menu](#)[Remarks and examples](#)[Also see](#)[Syntax](#)[Stored results](#)

Description

The `correlate` command displays the correlation matrix or covariance matrix for a group of variables. If *varlist* is not specified, the matrix is displayed for all variables in the dataset.

`pwcorr` displays all the pairwise correlation coefficients between the variables in *varlist* or, if *varlist* is not specified, all the variables in the dataset.

Quick start

Correlation matrix for variables *v1*, *v2*, and *v3*

```
correlate v1 v2 v3
```

Same as above, but display covariances instead of correlations

```
correlate v1 v2 v3, covariance
```

Pairwise correlation coefficients between *v1*, *v2*, and *v3*

```
pwcorr v1 v2 v3
```

Also print significance level of each correlation coefficient

```
pwcorr v1 v2 v3, sig
```

Same as above, but star correlation coefficients significant at the 5% level

```
pwcorr v1 v2 v3, sig star(.05)
```

Same as above, but use Bonferroni-adjusted significance levels

```
pwcorr v1 v2 v3, sig star(.05) bonferroni
```

Menu

correlate

Statistics > Summaries, tables, and tests > Summary and descriptive statistics > Correlations and covariances

pwcorr

Statistics > Summaries, tables, and tests > Summary and descriptive statistics > Pairwise correlations

Syntax

Display correlation matrix or covariance matrix

```
correlate [varlist] [if] [in] [weight] [, correlate_options]
```

Display all pairwise correlation coefficients

```
pwcorr [varlist] [if] [in] [weight] [, pwcorr_options]
```

<i>correlate_options</i>	Description
--------------------------	-------------

Options

<code>means</code>	display means, standard deviations, minimums, and maximums with matrix
<code>noformat</code>	ignore display format associated with variables
<code>covariance</code>	display covariances
<code>wrap</code>	allow wide matrices to wrap

<i>pwcorr_options</i>	Description
-----------------------	-------------

Main

<code>obs</code>	print number of observations for each entry
<code>sig</code>	print significance level for each entry
<code>listwise</code>	use listwise deletion to handle missing values
<code>casewise</code>	synonym for listwise
<code>print(#)</code>	significance level for displaying coefficients
<code>star(#)</code>	significance level for displaying with a star
<code>bonferroni</code>	use Bonferroni-adjusted significance level
<code>sidak</code>	use Šidák-adjusted significance level

varlist may contain time-series operators; see [U] 11.4.4 Time-series varlists.

`by` and `collect` are allowed with `correlate` and `pwcorr`; see [U] 11.1.10 Prefix commands.

`aweight`s and `fweight`s are allowed; see [U] 11.1.6 weight.

Options for correlate

Options

`means` displays summary statistics (means, standard deviations, minimums, and maximums) with the matrix.

`noformat` displays the summary statistics requested by the `means` option in `g` format, regardless of the display formats associated with the variables.

`covariance` displays the covariances rather than the correlation coefficients.

`wrap` requests that no action be taken on wide correlation matrices to make them readable. It prevents Stata from breaking wide matrices into pieces to enhance readability. You might want to specify this option if you are displaying results in a window wider than 80 characters. Then you may need to set `linesize` to however many characters you can display across a line; see [R] log.

Options for pwcorr

Main

`obs` adds a line to each row of the matrix reporting the number of observations used to calculate the correlation coefficient.

`sig` adds a line to each row of the matrix reporting the significance level of each correlation coefficient.

`listwise` handles missing values through listwise deletion, meaning that the entire observation is omitted from the estimation sample if any of the variables in *varlist* is missing for that observation. By default, `pwcorr` handles missing values by pairwise deletion; all available observations are used to calculate each pairwise correlation without regard to whether variables outside that pair are missing.

`correlate` uses listwise deletion. Thus, `listwise` allows users of `pwcorr` to mimic `correlate`'s treatment of missing values while retaining access to `pwcorr`'s features.

`casewise` is a synonym for `listwise`.

`print(#)` specifies the significance level of correlation coefficients to be printed. Correlation coefficients with larger significance levels are left blank in the matrix. Typing `pwcorr, print(.10)` would list only correlation coefficients significant at the 10% level or better.

`star(#)` specifies the significance level of correlation coefficients to be starred. Typing `pwcorr, star(.05)` would star all correlation coefficients significant at the 5% level or better.

`bonferroni` makes the Bonferroni adjustment to calculated significance levels. This option affects printed significance levels and the `print()` and `star()` options. Thus, `pwcorr, print(.05) bonferroni` prints coefficients with Bonferroni-adjusted significance levels of 0.05 or less.

`sidak` makes the Šidák adjustment to calculated significance levels. This option affects printed significance levels and the `print()` and `star()` options. Thus, `pwcorr, print(.05) sidak` prints coefficients with Šidák-adjusted significance levels of 0.05 or less.

Remarks and examples

Remarks are presented under the following headings:

[correlate](#)

[pwcorr](#)

[Video example](#)

correlate

Typing `correlate` by itself produces a correlation matrix for all variables in the dataset. If you specify the *varlist*, a correlation matrix for just those variables is displayed.

▷ Example 1

We have state data on demographic characteristics of the population. To obtain a correlation matrix, we type

```
. use https://www.stata-press.com/data/r19/census13
(1980 Census data by state)
. correlate
(obs=50)
```

	state	brate	pop	medage	division	region	mrgrate
state	1.0000						
brate	0.0208	1.0000					
pop	-0.0540	-0.2830	1.0000				
medage	-0.0624	-0.8800	0.3294	1.0000			
division	-0.1345	0.6356	-0.1081	-0.5207	1.0000		
region	-0.1339	0.6086	-0.1515	-0.5292	0.9688	1.0000	
mrgrate	0.0509	0.0677	-0.1502	-0.0177	0.2280	0.2490	1.0000
dvcrate	-0.0655	0.3508	-0.2064	-0.2229	0.5522	0.5682	0.7700
medagesq	-0.0621	-0.8609	0.3324	0.9984	-0.5162	-0.5239	-0.0202
		dvcrate	medagesq				
dvcrate		1.0000					
medagesq		-0.2192	1.0000				

Because we did not specify the wrap option, Stata did its best to make the result readable by breaking the table into two parts.

To obtain the correlations between mrgrate, dvcrate, and medage, we type

```
. correlate mrgrate dvcrate medage
(obs=50)
```

	mrgrate	dvcrate	medage
mrgrate	1.0000		
dvcrate	0.7700	1.0000	
medage	-0.0177	-0.2229	1.0000

◀

▷ Example 2

The pop variable in [example 1](#) represents the total population of the state. Thus, to obtain population-weighted correlations among mrgrate, dvcrate, and medage, we type

```
. correlate mrgrate dvcrate medage [w=pop]
(analytic weights assumed)
(sum of wgt is 225,907,472)
(obs=50)
```

	mrgrate	dvcrate	medage
mrgrate	1.0000		
dvcrate	0.5854	1.0000	
medage	-0.1316	-0.2833	1.0000

◀

With the `covariance` option, `correlate` can be used to obtain covariance matrices, as well as correlation matrices, for both weighted and unweighted data.

▷ Example 3

To obtain the matrix of covariances between `mrgrate`, `dvcrate`, and `medage`, we type `correlate mrgrate dvcrate medage, covariance`:

```
. correlate mrgrate dvcrate medage, covariance
(obs=50)
```

	mrgrate	dvcrate	medage
mrgrate	.000662		
dvcrate	.000063	1.0e-05	
medage	-.000769	-.001191	2.86775

We could have obtained the pop-weighted covariance matrix by typing `correlate mrgrate dvcrate medage [w=pop], covariance`.

◀

pwcorr

`correlate` calculates correlation coefficients by using casewise deletion; when you request correlations of variables x_1, x_2, \dots, x_k , any observation for which any of x_1, x_2, \dots, x_k is missing is not used. Thus if x_3 and x_4 have no missing values, but x_2 is missing for half the data, the correlation between x_3 and x_4 is calculated using only the half of the data for which x_2 is not missing. Of course, you can obtain the correlation between x_3 and x_4 by using all the data by typing `correlate x3 x4`.

`pwcorr` makes obtaining such pairwise correlation coefficients easier.

▷ Example 4

Using `auto.dta`, we investigate the correlation between several of the variables.

```
. use https://www.stata-press.com/data/r19/auto1
(Automobile models)
```

```
. pwcorr mpg price rep78 foreign, obs sig
```

	mpg	price	rep78	foreign
mpg	1.0000			
	74			
price	-0.4594	1.0000		
	0.0000	74		
rep78	0.3739	0.0066	1.0000	
	0.0016	0.9574	69	
foreign	0.3613	0.0487	0.5922	1.0000
	0.0016	0.6802	0.0000	74
	74	74	69	74

```
. pwcorr mpg price headroom rear_seat trunk rep78 foreign, print(.05) star(.01)
```

	mpg	price	headroom	rear_s~t	trunk	rep78	foreign
mpg	1.0000						
price	-0.4594*	1.0000					
headroom	-0.4220*		1.0000				
rear_seat	-0.5213*	0.4194*	0.5238*	1.0000			
trunk	-0.5703*	0.3143*	0.6620*	0.6480*	1.0000		
rep78	0.3739*					1.0000	
foreign	0.3613*		-0.2939	-0.2409	-0.3594*	0.5922*	1.0000

```
. pwcorr mpg price headroom rear_seat trunk rep78 foreign, print(.05) bon
```

	mpg	price	headroom	rear_s~t	trunk	rep78	foreign
mpg	1.0000						
price	-0.4594	1.0000					
headroom	-0.4220		1.0000				
rear_seat	-0.5213	0.4194	0.5238	1.0000			
trunk	-0.5703		0.6620	0.6480	1.0000		
rep78	0.3739					1.0000	
foreign	0.3613				-0.3594	0.5922	1.0000

◀

□ Technical note

The `correlate` command will report the correlation matrix of the data, but there are occasions when you need the matrix stored as a Stata matrix so that you can further manipulate it. You can obtain the matrix by typing

```
. matrix accum R = varlist, noconstant deviations
. matrix R = corr(R)
```

The first line places the cross-product matrix of the data in matrix `R`. The second line converts that to a correlation matrix. Also see [P] [matrix define](#) and [P] [matrix accum](#).

□

Video example

[Pearson's correlation coefficient in Stata](#)

Stored results

`correlate` stores the following in `r()`:

Scalars

<code>r(N)</code>	number of observations
<code>r(rho)</code>	ρ (first and second variables)
<code>r(cov_12)</code>	covariance (covariance only)
<code>r(Var_1)</code>	variance of first variable (covariance only)
<code>r(Var_2)</code>	variance of second variable (covariance only)
<code>r(sum_w)</code>	sum of weights

Matrices

<code>r(C)</code>	correlation or covariance matrix
-------------------	----------------------------------

`pwcorr` stores the following in `r()`:

Scalars

<code>r(N)</code>	number of observations (first and second variables)
<code>r(rho)</code>	ρ (first and second variables)

Matrices

<code>r(Nobs)</code>	number of observations for each correlation coefficient
<code>r(C)</code>	pairwise correlation matrix
<code>r(sig)</code>	significance level of each correlation coefficient

Methods and formulas

For a discussion of correlation, see, for instance, [Snedecor and Cochran \(1989, 177–195\)](#); for an introductory explanation using Stata examples, see [Acock \(2023, 205–210\)](#).

According to [Snedecor and Cochran \(1989, 180\)](#), the term “co-relation” was first proposed by [Galton \(1888\)](#). The product-moment correlation coefficient is often called the Pearson product-moment correlation coefficient because [Pearson \(1896\)](#) and [Pearson and Filon \(1898\)](#) were partially responsible for popularizing its use. See [Stigler \(1986\)](#) for information on the history of correlation.

The estimate of the product-moment correlation coefficient, ρ , is

$$\hat{\rho} = \frac{\sum_{i=1}^n w_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n w_i (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n w_i (y_i - \bar{y})^2}}$$

where w_i are the weights, if specified, or $w_i = 1$ if weights are not specified. $\bar{x} = (\sum w_i x_i) / (\sum w_i)$ is the mean of x , and \bar{y} is similarly defined.

The unadjusted significance level is calculated by `pwcorr` as

$$p = 2 * \text{ttail}(n - 2, |\hat{\rho}| \sqrt{n - 2} / \sqrt{1 - \hat{\rho}^2})$$

Let v be the number of variables specified so that $k = v(v - 1)/2$ correlation coefficients are to be estimated. If `bonferroni` is specified, the adjusted significance level is $p' = \min(1, kp)$. If `sidak` is specified, $p' = \min\{1, 1 - (1 - p)^k\}$. In both cases, see [Methods and formulas](#) in [\[R\] oneway](#) for a more complete description of the logic behind these adjustments.

Carlo Emilio Bonferroni (1892–1960) was born in Bergamo, Italy, in 1892. Before enrolling in the mathematics department at the University of Turin, he studied conducting and the piano at the Music Conservatory of Turin. Bonferroni, like many men of his generation, fought in World War I, during which he was part of The Engineer Corps of the Italian Army.

After the war, Bonferroni was appointed as an assistant professor at the Polytechnic University of Turin. He taught geometry, mechanics, and analysis. In 1923, he moved to Bari and began teaching at the Economics Institute, where he got to teach financial mathematics, a topic that was of great interest to him. In 1933, Bonferroni moved to Florence, where he was chair of the mathematics department until his death in 1960.

Bonferroni's interests had a large breadth. He published on actuarial mathematics, probability, statistics, analysis, geometry, and mechanics. His work on probability inequalities has been applied to simultaneous statistical inference. However, the application of Bonferroni's theory to the construction of confidence intervals is the work of Olive Jean Dunn.

Olive Jean Dunn (1915–2008) was born in the United States in 1915. She obtained her bachelor's degree in 1936, her master's degree in 1951, and her PhD in 1956, all from the University of California in Los Angeles (UCLA). After spending one year as an assistant professor at Iowa State College, she returned to UCLA to serve in the biostatistics and preventive medicine and health departments. She remained at UCLA for the rest of her career. Dunn died in 2008.

Dunn is best known for her application of Bonferroni's inequalities to construct corrections to confidence intervals for multiple comparisons. Although the literature refers to it as the Bonferroni correction, it is Dunn who developed the application we use today.

Dunn is also well known for her textbooks *Basic Statistics: A Primer for the Biomedical Sciences*, written in 1977 with later editions coauthored with Virginia A. Clark, and *Applied Statistics: An Analysis of Variance and Regression*, which was also coauthored with Clark.

In 1968, Dunn became a Fellow of the American Statistical Association. She also was a fellow of the American Public Health Association and the American Association for the Advancement of Science. In 1974, she was awarded the honor of UCLA Woman of Science.

Florence Nightingale David (1909–1993) was born in Ivington, England, to parents who were friends with Florence Nightingale, David’s namesake. She began her studies in statistics under the direction of Karl Pearson at University College London and continued her studies under the direction of Jerzy Neyman. After receiving her doctorate in statistics in 1938, David became a senior statistician for various departments within the British military. She developed statistical models to forecast the toll on life and infrastructure that would occur if a large city were bombed. In 1938, she also published her book *Tables of the Correlation Coefficient*, dealing with the distributions of correlation coefficients. After the war, she returned to University College London, serving as a lecturer until her promotion to professor in 1962. In 1967, David joined the University of California–Riverside, eventually becoming chair of the Department of Statistics. One of her most well-known works is the book *Games, Gods and Gambling: The Origins and History of Probability and Statistical Ideas from the Earliest Times to the Newtonian Era*, a history of statistics. David published over 100 papers on topics including combinatorics, symmetric functions, the history of statistics, and applications of statistics, including ecological diversity. She published under the name F. N. David to avoid revealing her gender in a male-dominated profession.

Karl Pearson (1857–1936) studied mathematics at Cambridge. He was professor of applied mathematics (1884–1911) and eugenics (1911–1933) at University College London. His publications include literary, historical, philosophical, and religious topics. Statistics became his main interest in the early 1890s after he learned about its application to biological problems. His work centered on distribution theory, the method of moments, correlation, and regression. Pearson introduced the χ^2 test and the terms coefficient of variation, contingency table, heteroskedastic, histogram, homoskedastic, kurtosis, mode, random sampling, random walk, skewness, standard deviation, and truncation. Despite many strong qualities, he also fell into prolonged disagreements with others, most notably, William Bateson and R. A. Fisher.

Zbyněk Šidák (1933–1999) was a notable Czech statistician and probabilist. He worked on Markov chains, rank tests, multivariate distribution theory and multiple-comparison methods, and he served as the chief editor of *Applications of Mathematics*.

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Also see

- [R] **esize** — Effect size based on mean comparison
- [R] **estat vce** — Display covariance matrix estimates
- [R] **icc** — Intraclass correlation coefficients
- [R] **pcorr** — Partial and semipartial correlation coefficients
- [R] **spearman** — Spearman’s and Kendall’s correlations
- [R] **summarize** — Summary statistics
- [R] **tetrachoric** — Tetrachoric correlations for binary variables

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

Description

`cpoisson` fits a Poisson model of a count dependent variable with some censored values. The command can be used when the dependent variable is left-censored (has a lower limit), is right-censored (has an upper limit), or is interval-censored (has a lower and an upper limit).

Quick start

Censored Poisson regression of `y` on `x` without options `ll()` and `ul()`, equivalent to Poisson regression

```
cpoisson y x
```

Add categorical variable `a` using [factor-variable](#) syntax, and specify censoring at an upper limit of 4

```
cpoisson y x i.a, ul(4)
```

Also specify a lower-censoring limit that varies across observations by using the variable `lower`

```
cpoisson y x i.a, ul(4) ll(lower)
```

Add offset variable `v`, and report results as incidence-rate ratios

```
cpoisson y x i.a, ul(4) ll(lower) offset(v) irr
```

Constrain the coefficient for `x` to 2

```
constraint define 1 x=2  
cpoisson y x i.a, ul(4) constraints(1)
```

Menu

Statistics > Count outcomes > Censored Poisson regression

Syntax

`cpoisson devar [indepvars] [if] [in] [weight] [, options]`

<i>options</i>	Description
Model	
<code>noconstant</code>	suppress constant term
<code>ll[(<i>varname</i> #)]</code>	left-censoring variable or limit
<code>ul[(<i>varname</i> #)]</code>	right-censoring variable or limit
<code>exposure(<i>varname</i>_e)</code>	include $\ln(\text{varname}_e)$ in model with coefficient constrained to 1
<code>offset(<i>varname</i>_o)</code>	include <i>varname</i> _o in model with coefficient constrained to 1
<code>constraints(<i>constraints</i>)</code>	apply specified linear constraints
SE/Robust	
<code>vce(<i>vcetype</i>)</code>	<i>vcetype</i> may be <code>oim</code> , <code>robust</code> , <code>cluster <i>clustvar</i></code> , <code>bootstrap</code> , or <code>jackknife</code>
Reporting	
<code>level(#)</code>	set confidence level; default is <code>level(95)</code>
<code>irr</code>	report incidence-rate ratios
<code>nocnsreport</code>	do not display constraints
<code>display-options</code>	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Maximization	
<code>maximize-options</code>	control the maximization process; seldom used
<code>collinear</code>	keep collinear variables
<code>coeflegend</code>	display legend instead of statistics

indepvars may contain factor variables; see [U] 11.4.3 **Factor variables**.

devar and *indepvars* may contain time-series operators; see [U] 11.4.4 **Time-series varlists**.

`bayesboot`, `bootstrap`, `by`, `collect`, `fp`, `jackknife`, `rolling`, `statsby`, and `svy` are allowed; see [U] 11.1.10 **Prefix commands**.

Weights are not allowed with the `bootstrap` prefix; see [R] **bootstrap**.

`vce()` and weights are not allowed with the `svy` prefix; see [SVY] **svy**.

`fweights`, `iweights`, and `pweights` are allowed; see [U] 11.1.6 **weight**.

`collinear` and `coeflegend` do not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

Options

Model

`noconstant`; see [R] [Estimation options](#).

`ll[(varname | #)]` and `ul[(varname | #)]` indicate the lower and upper limits for censoring, respectively. Observations with `depvar ≤ ll()` are left-censored; observations with `depvar ≥ ul()` are right-censored; and remaining observations are not censored. You do not have to specify the censoring values. If you specify `ll`, the lower limit is the minimum of `depvar`. If you specify `ul`, the upper limit is the maximum of `depvar`.

`exposure(varnamee)`, `offset(varnameo)`, `constraints(constraints)`; see [R] [Estimation options](#).

SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce_option](#).

Reporting

`level(#)`; see [R] [Estimation options](#).

`irr` reports estimated coefficients transformed to incidence-rate ratios, that is, e^{β_i} rather than β_i . Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated. `irr` may be specified at estimation or when replaying previously estimated results.

`nocnsreport`; see [R] [Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

The following options are available with `cpoisson` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

Remarks and examples

Censored Poisson regression is a method for analyzing censored count data. One of the most common sources of censored count data is top coding, data that record only the value x when x or greater is observed. Not observing subjects for a sufficient period of time is another common cause.

Censored count data models have been studied by [Terza \(1985\)](#) and [Brännäs \(1992\)](#), among others. For an introduction to censored Poisson regression, see [Cameron and Trivedi \(2005, 2013\)](#) and [Winkelmann \(2008\)](#). [Raciborski \(2011\)](#) discusses a command for right-censored Poisson regression and presented Monte Carlo results indicating that the estimator performs well in finite samples. See [Creel and Loomis \(1990\)](#) and [Gurmu and Trivedi \(1996\)](#) for some examples of survey applications with top coding.

Censored data can be right-censored, left-censored, or interval-censored. Right-censoring occurs when we observe the covariates but only observe that the dependent variable is greater than or equal to an upper limit. Left-censoring occurs when we observe the covariates but only observe that the dependent variable is less than or equal to a lower limit.

When the dependent variable is censored, we must use estimation methods that account for this limitation. If we do not account for censoring when our data are censored, our estimates will not converge to the true values. More formally, failure to control for censoring when it exists leads to inconsistent parameter estimation.

Censored Poisson regression provides an alternative to standard Poisson regression that produces consistent estimates when the dependent variable is censored. If the dependent variable is not censored, standard Poisson regression may be more appropriate; see [\[R\] cpoisson](#).

Censoring differs from truncation. For censored observations, we observe complete covariate information but only a censored value of the dependent variable. When the data are truncated, we do not observe either the dependent variable or the covariates. Different research designs can give rise to censored data or truncated data.

For example, consider a study about the use of national parks. We could ask a random sample of people in the population how many national parks each has visited in the past year. Suppose we decide to record three for those individuals who visited three or more parks. In this case, individuals who visit four or more parks will have observations that are right-censored at three visits. Now suppose that instead of sending out surveys to a random sample from the population, we ask questions only of individuals who come to parks. We will have no information about individuals who do not visit at least one park, and the data will be truncated at zero visits.

Censoring and truncation are different statistical phenomenon and require different analytic methods. See [\[R\] cpoisson](#) for information on truncated Poisson regression.

▷ Example 1: Poisson model with top-coded data

Imagine that we have collected survey data about how many times a household has visited the ABC amusement park from a random sample of households in the state in which ABC is located. Respondents were asked about the number of visits to the park in the last year (`trips`), their income (`income`), and the number of children in the household (`children`). The number of trips recorded in `trips` was top coded at “three or more” visits.

We model right-censored trips as a function of income and children.

```
. use https://www.stata-press.com/data/r19/trips
(Visits to the ABC amusement park)
. cpoisson trips income children, ul(3)
Initial:      Log likelihood = -620.68749
Rescale:     Log likelihood = -620.68749
Iteration 0: Log likelihood = -620.68749
Iteration 1: Log likelihood = -600.96763
Iteration 2: Log likelihood = -600.78416
Iteration 3: Log likelihood = -600.78415
Censored Poisson regression
Limits: Lower = 0
        Upper = 3
Log likelihood = -600.78415
Number of obs   = 500
Uncensored     = 278
Left-censored  = 0
Right-censored = 222
LR chi2(2)     = 49.29
Prob > chi2    = 0.0000
```

trips	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
income	.0740477	.0137653	5.38	0.000	.0470683	.1010272
children	.1346922	.028617	4.71	0.000	.078604	.1907805
_cons	.0033918	.1455473	0.02	0.981	-.2818756	.2886592

Both income and the number of children have positive effects on the expected number of trips to the amusement park. The estimated parameters provide the sign, but not the magnitude of the effect, because the model is nonlinear; see [R] [cpoisson postestimation](#).



Stored results

cpoisson stores the following in e():

Scalars

- e(N) number of observations
- e(N_unc) number of uncensored observations
- e(N_lc) number of left-censored observations
- e(N_rc) number of right-censored observations
- e(k) number of parameters
- e(k_eq) number of equations in e(b)
- e(k_eq_model) number of equations in overall model test
- e(k_dv) number of dependent variables
- e(df_m) model degrees of freedom
- e(ll) log likelihood
- e(ll_0) log likelihood, constant-only model
- e(N_clust) number of clusters
- e(chi2) χ^2
- e(p) *p*-value for model test
- e(rank) rank of e(V)
- e(ic) number of iterations
- e(rc) return code
- e(converged) 1 if converged, 0 otherwise

Macros

- e(cmd) cpoisson
- e(cmdline) command as typed
- e(depvar) name of dependent variable

e(llopt)	contents of l1(), if specified
e(ulopt)	contents of u1(), if specified
e(wtype)	weight type
e(wexp)	weight expression
e(title)	title in estimation output
e(clustvar)	name of cluster variable
e(offset)	linear offset variable
e(chi2type)	Wald or LR; type of model χ^2 test
e(vce)	<i>vcetype</i> specified in vce()
e(vcetype)	title used to label Std. err.
e(opt)	type of optimization
e(which)	max or min; whether optimizer is to perform maximization or minimization
e(ml_method)	type of ml method
e(user)	name of likelihood-evaluator program
e(technique)	maximization technique
e(properties)	b V
e(predict)	program used to implement predict
e(marginsok)	predictions allowed by margins
e(marginsnotok)	predictions disallowed by margins
e(asbalanced)	factor variables fvset as asbalanced
e(asobserved)	factor variables fvset as asobserved

Matrices

e(b)	coefficient vector
e(Cns)	constraints matrix
e(ilog)	iteration log (up to 20 iterations)
e(gradient)	gradient vector
e(V)	variance–covariance matrix of the estimators
e(V_modelbased)	model-based variance

Functions

e(sample)	marks estimation sample
-----------	-------------------------

In addition to the above, the following is stored in `r()`:

Matrices

r(table)	matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals
----------	--

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

Methods and formulas

We let y_j be the observed, interval-censored dependent variable for observation j and let y_j^* be the uncensored, latent dependent variable. When y_j is not censored, it is the same as y_j^* . When y_j is censored, only the censoring point is observed. Letting L_j denote the left-censoring point (lower limit) and U_j denote the right-censoring point (upper limit), we see that

$$y_j = \begin{cases} L_j & \text{if } y_j^* \leq L_j \\ y_j^* & \text{if } L_j < y_j^* < U_j \\ U_j & \text{if } y_j^* \geq U_j \end{cases}$$

Note that L_j and U_j may vary over the observations so that individuals may have different left- and right-censoring points.

Although `cpoisson` may be used with data that are left-censored, right-censored, or censored from both sides (which is known as interval-censored), we present the formulas for the interval-censored case because it applies to all three cases.

Let $f(y_j|\mathbf{x}_j)$ denote the probability mass function of the Poisson distribution. Defining $\xi_j = \mathbf{x}_j\boldsymbol{\beta} + \text{offset}_j$ implies that the conditional mean of the uncensored variable is given by $E(y_j^*|\mathbf{x}_j) = \exp(\xi_j)$. The log likelihood for observation j is given by

$$l_j = w_j \left[d_j \{-\exp(\xi_j) + y_j \xi_j - \ln(y_j!)\} + (1 - d_j) \ln \left\{ 1 - \sum_{k=0}^{U_j-1} f(k|\mathbf{x}_j) + \sum_{k=0}^{L_j} f(k|\mathbf{x}_j) \right\} \right]$$

where d_j equals 1 when $L_j < y_j^* < U_j$ and equals 0 when $y_j^* \leq L_j$ or $y_j^* \geq U_j$. The log likelihood is thus

$$\ln L = \sum_{j=1}^N l_j$$

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] **_robust**, particularly *Maximum likelihood estimators* and *Methods and formulas*.

`cpoisson` also supports estimation with survey data. For details on variance–covariance estimates with survey data, see [SVY] **Variance estimation**.

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Also see

[R] **cpoisson postestimation** — Postestimation tools for cpoisson

[R] **nbreg** — Negative binomial regression

[R] **poisson** — Poisson regression

[R] **tnbreg** — Truncated negative binomial regression

[R] **tpoisson** — Truncated Poisson regression

[R] **zinb** — Zero-inflated negative binomial regression

[R] **zip** — Zero-inflated Poisson regression

[SVY] **svy estimation** — Estimation commands for survey data

[XT] **xtpoisson** — Fixed-effects, random-effects, and population-averaged Poisson models

[U] **20 Estimation and postestimation commands**

Postestimation commands
Methods and formulas

predict
Also see

margins

Remarks and examples

Postestimation commands

The following postestimation commands are available after `cpoisson`:

Command	Description
<code>contrast</code>	contrasts and ANOVA-style joint tests of parameters
<code>estat ic</code>	Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively)
<code>estat summarize</code>	summary statistics for the estimation sample
<code>estat vce</code>	variance–covariance matrix of the estimators (VCE)
<code>estat (svy)</code>	postestimation statistics for survey data
<code>estimates</code>	cataloging estimation results
<code>etable</code>	table of estimation results
* <code>forecast</code>	dynamic forecasts and simulations
* <code>hausman</code>	Hausman's specification test
<code>lincom</code>	point estimates, standard errors, testing, and inference for linear combinations of parameters
* <code>lrtest</code>	likelihood-ratio test
<code>margins</code>	marginal means, predictive margins, marginal effects, and average marginal effects
<code>marginsplot</code>	graph the results from margins (profile plots, interaction plots, etc.)
<code>nlcom</code>	point estimates, standard errors, testing, and inference for nonlinear combinations of parameters
<code>predict</code>	number of events, incidence rates, probabilities, etc.
<code>predictnl</code>	point estimates, standard errors, testing, and inference for generalized predictions
<code>pwcompare</code>	pairwise comparisons of parameters
<code>suest</code>	seemingly unrelated estimation
<code>test</code>	Wald tests of simple and composite linear hypotheses
<code>testnl</code>	Wald tests of nonlinear hypotheses

*`forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results.

predict

Description for predict

predict creates a new variable containing predictions such as numbers of events, incidence rates, conditional means, probabilities, conditional probabilities, linear predictions, standard errors, and the equation-level score.

Menu for predict

Statistics > Postestimation

Syntax for predict

```
predict [type] newvar [if] [in] [, statistic noffset]
```

statistic	Description
Main	
n	number of events; the default
ir	incidence rate
cm	conditional mean, $E(y_j y_j > L_j)$, $E(y_j y_j < U_j)$, or $E(y_j L_j < y_j < U_j)$
pr(n)	probability $\Pr(y_j = n)$
pr(a,b)	probability $\Pr(a \leq y_j \leq b)$
cpr(n)	conditional probability $\Pr(y_j = n y_j > L_j)$, $\Pr(y_j = n y_j < U_j)$, or $\Pr(y_j = n L_j < y_j < U_j)$
cpr(a,b)	conditional probability $\Pr(a \leq y_j \leq b y_j > L_j)$, $\Pr(a \leq y_j \leq b y_j < U_j)$, or $\Pr(a \leq y_j \leq b L_j < y_j < U_j)$
xb	linear prediction
stdp	standard error of the linear prediction
score	first derivative of the log likelihood with respect to $\mathbf{x}_j\beta$

These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

Options for predict

Main

n, the default, calculates the predicted number of events, which is $\exp(\mathbf{x}_j\beta)$ if neither offset() nor exposure() was specified when the model was fit; $\exp(\mathbf{x}_j\beta + \text{offset}_j)$ if offset() was specified; or $\exp(\mathbf{x}_j\beta) \times \text{exposure}_j$ if exposure() was specified.

ir calculates the incidence rate $\exp(\mathbf{x}_j\beta)$, which is the predicted number of events when exposure is 1. This is equivalent to specifying both the n and the noffset options.

cm calculates the conditional mean,

$$E(y_j | \Omega_j) = \frac{E(y_j)}{\Pr(\Omega_j)}$$

where Ω_j represents $y_j > L_j$ for a left-censored model, $y_j < U_j$ for a right-censored model, and $L_j < y_j < U_j$ for an interval-censored model. L_j is the left-censoring point found in `e(1lopt)`, and U_j is the right-censoring point found in `e(1uopt)`.

`pr(n)` calculates the probability $\Pr(y_j = n)$, where n is a nonnegative integer that may be specified as a number or a variable.

`pr(a,b)` calculates the probability $\Pr(a \leq y_j \leq b)$, where a and b are nonnegative integers that may be specified as numbers or variables;

b missing ($b \geq .$) means $+\infty$;

`pr(20,.)` calculates $\Pr(y_j \geq 20)$;

`pr(20,b)` calculates $\Pr(y_j \geq 20)$ in observations for which $b \geq .$ and calculates $\Pr(20 \leq y_j \leq b)$ elsewhere.

`pr(.,b)` produces a syntax error. A missing value in an observation of the variable a causes a missing value in that observation for `pr(a,b)`.

`cpr(n)` calculates the conditional probability $\Pr(y_j = n | \Omega_j)$, where Ω_j represents $y_j > L_j$ for a left-censored model, $y_j < U_j$ for a right-censored model, and $L_j < y_j < U_j$ for an interval-censored model. L_j is the left-censoring point found in `e(1lopt)`, and U_j is the right-censoring point found in `e(1uopt)`. n is an integer in the noncensored range.

`cpr(a,b)` calculates the conditional probability $\Pr(a \leq y_j \leq b | \Omega_j)$, where Ω_j represents $y_j > L_j$ for a left-censored model, $y_j < U_j$ for a right-censored model, and $L_j < y_j < U_j$ for an interval-censored model. L_j is the left-censoring point found in `e(1lopt)`, and U_j is the right-censoring point found in `e(1uopt)`. a and b must fall in the noncensored range if they are not missing. A missing value in an observation of the variable a causes a missing value in that observation for `cpr(a,b)`.

`xb` calculates the linear prediction, which is $\mathbf{x}_j\beta$ if neither `offset()` nor `exposure()` was specified when the model was fit; $\mathbf{x}_j\beta + \text{offset}_j$ if `offset()` was specified; or $\mathbf{x}_j\beta + \ln(\text{exposure}_j)$ if `exposure()` was specified; see `nooffset` below.

`stdp` calculates the standard error of the linear prediction.

`score` calculates the equation-level score, $\partial \ln L / \partial (\mathbf{x}_j\beta)$.

`nooffset` is relevant only if you specified `offset()` or `exposure()` when you fit the model. It modifies the calculations made by `predict` so that they ignore the offset or exposure variable; the linear prediction is treated as $\mathbf{x}_j\beta$ rather than as $\mathbf{x}_j\beta + \text{offset}_j$ or $\mathbf{x}_j\beta + \ln(\text{exposure}_j)$. Specifying `predict . . . , nooffset` is equivalent to specifying `predict . . . , ir`.

margins

Description for margins

`margins` estimates margins of response for numbers of events, incidence rates, conditional means, probabilities, and linear predictions.

Menu for margins

Statistics > Postestimation

Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

<i>statistic</i>	Description
<code>n</code>	number of events; the default
<code>ir</code>	incidence rate
<code>cm</code>	conditional mean, $E(y_j y_j > L_j)$, $E(y_j y_j < U_j)$, or $E(y_j L_j < y_j < U_j)$
<code>pr(<i>n</i>)</code>	probability $\Pr(y_j = n)$
<code>pr(<i>a</i>,<i>b</i>)</code>	probability $\Pr(a \leq y_j \leq b)$
<code>cpr(<i>n</i>)</code>	conditional probability $\Pr(y_j = n y_j > L_j)$, $\Pr(y_j = n y_j < U_j)$, or $\Pr(y_j = n L_j < y_j < U_j)$
<code>cpr(<i>a</i>,<i>b</i>)</code>	conditional probability $\Pr(a \leq y_j \leq b y_j > L_j)$, $\Pr(a \leq y_j \leq b y_j < U_j)$, or $\Pr(a \leq y_j \leq b L_j < y_j < U_j)$
<code>xb</code>	linear prediction
<code>stdp</code>	not allowed with <code>margins</code>
<code>score</code>	not allowed with <code>margins</code>

Statistics not allowed with `margins` are functions of stochastic quantities other than `e(b)`.

For the full syntax, see [R] [margins](#).

Remarks and examples

▷ Example 1: Obtaining marginal effects

Continuing with [example 1](#) of [R] [cpoisson](#), we estimate the effect of having another child on the uncensored number of trips to amusement parks. We use `margins` to estimate the average number of trips when each household has its actual number of children and when each household has one additional child. We include the `post` option so that we can use the results in subsequent commands.

```
. use https://www.stata-press.com/data/r19/trips
(Visits to the ABC amusement park)
. cpoisson trips income children, ul(3)
(output omitted)
```

```
. margins, at(children = generate(children))
>       at(children = generate(children+1)) post
Predictive margins                                     Number of obs = 500
Model VCE: OIM
Expression: Predicted number of events, predict()
1._at: children = children
2._at: children = children+1
```

	Delta-method		z	P> z	[95% conf. interval]	
_at	Margin	std. err.				
1	2.525517	.0836237	30.20	0.000	2.361618	2.689417
2	2.889658	.1337997	21.60	0.000	2.627416	3.151901

An average of 2.53 trips are taken when each household has its observed number of children. If each household has one additional child, then the average number of trips increases to 2.89.

We now use `contrast` to compute the effect of having an additional child. The Wald test, in this case, is superfluous, so we suppress it with the `nowald` option.

```
. contrast r._at, nowald
Contrasts of predictive margins                       Number of obs = 500
Model VCE: OIM
Expression: Predicted number of events, predict()
1._at: children = children
2._at: children = children+1
```

	Delta-method		[95% conf. interval]	
(2 vs 1)	Contrast	std. err.		
	.3641407	.0849951	.1975535	.530728

Adding one child to each household increases the average by 0.36 trips.



Methods and formulas

Using the notation under *Methods and formulas* of [R] `cpoisson`, we see that the equation-level score is given by

$$\text{score}(\mathbf{x}\beta)_j = d_j \{-\exp(\xi_j) + y_j\} + (1 - d_j) \frac{\Psi_1(L_j | \mathbf{x}_j) - \Psi_1(U_j - 1 | \mathbf{x}_j)}{1 - F(U_j - 1 | \mathbf{x}_j) + F(L_j | \mathbf{x}_j)}$$

where $\Psi_1(C) = \sum_{k=0}^C f(k | \mathbf{x}_j) \{k - \exp(\xi_j)\}$; $f(y_j | \mathbf{x}_j)$ and $F(y_j | \mathbf{x}_j)$ denote the probability mass function and the cumulative distribution function of the Poisson, respectively. L_j is the left-censoring point found in `e(llopt)`, and U_j is the right-censoring point found in `e(u1opt)`.

Also see

[R] [cpoisson](#) — Censored Poisson regression

[U] [20 Estimation and postestimation commands](#)

[Description](#)
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Description

`cumul` creates *newvar*, defined as the empirical cumulative distribution function of *varname*.

Quick start

Create new variable `ecd` containing the empirical cumulative distribution of `v`

```
cumul v, gen(ecd)
```

Use frequency as the unit for `v` to generate `ecdf`

```
cumul v, gen(ecdf) freq
```

Give equal values of `v` the same value in generated `ecde`

```
cumul v, gen(ecde) equal
```

Graph the empirical cumulative distribution of `v`

```
line ecd v, sort
```

Graph the distributions of variables `v1` and `v2`

```
cumul v1, gen(ecd1) equal
```

```
cumul v2, gen(ecd2) equal
```

```
stack ecd1 v1 ecd2 v2, into(ecd v) wide clear
```

```
line ecd1 ecd2 v, sort
```

Menu

Statistics > Summaries, tables, and tests > Distributional plots and tests > Generate cumulative distribution

Syntax

```
cumul varname [if] [in] [weight], generate(newvar) [options]
```

<i>options</i>	Description
Main	
* <u>generate</u> (<i>newvar</i>)	create variable <i>newvar</i>
<u>freq</u>	use frequency units for cumulative
<u>equal</u>	generate equal cumulatives for tied values

*generate(*newvar*) is required.

by is allowed; see [D] [by](#).

fweights and awweights are allowed; see [U] [11.1.6 weight](#).

Options

Main

generate(*newvar*) is required. It specifies the name of the new variable to be created.

freq specifies that the cumulative be in frequency units; otherwise, it is normalized so that *newvar* is 1 for the largest value of *varname*.

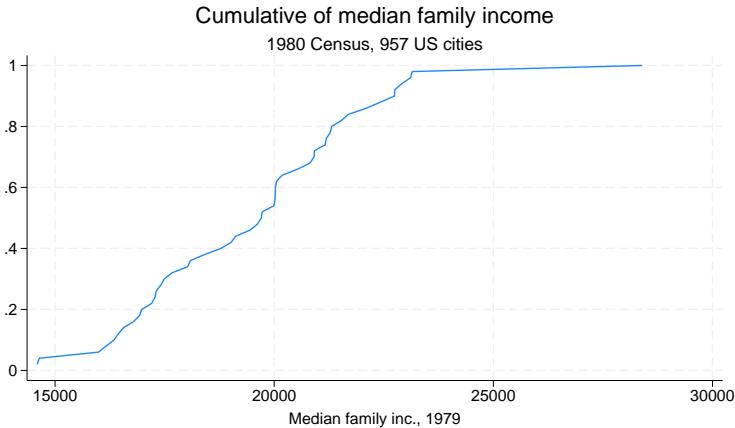
equal requests that observations with equal values in *varname* get the same cumulative value in *newvar*.

Remarks and examples

▷ Example 1

`cumul` is most often used with `graph` to graph the empirical cumulative distribution. For instance, we have data on the median family income of 957 US cities:

```
. use https://www.stata-press.com/data/r19/hsng
(1980 Census housing data)
. cumul faminc, gen(cum)
. sort cum
. line cum faminc, ytitle("") xlabel(, format(%6.0f))
> title("Cumulative of median family income")
> subtitle("1980 Census, 957 US cities")
```



It would have been enough to type `line cum faminc`, but we wanted to make the graph look better; see [\[G-2\] graph twoway line](#).

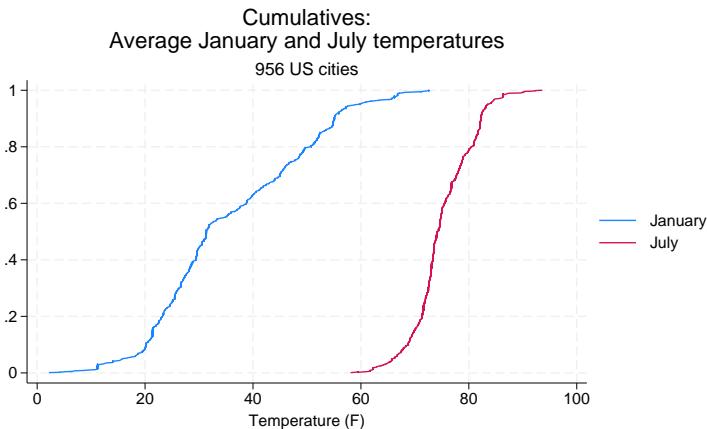
If we had wanted a weighted cumulative, we would have typed `cumul faminc [w=pop]` at the first step.



▷ Example 2

To graph two (or more) cumulatives on the same graph, use `cumul` and `stack`; see [\[D\] stack](#). For instance, we have data on the average January and July temperatures of 956 US cities:

```
. use https://www.stata-press.com/data/r19/citytemp, clear
(City temperature data)
. cumul tempjan, gen(cjan)
. cumul tempjuly, gen(cjuly)
. stack cjan tempjan cjuly tempjuly, into(c temp) wide clear
. line cjan cjuly temp, sort ytitle("") xtitle("Temperature (F)")
> title("Cumulatives:" "Average January and July temperatures")
> subtitle("956 US cities") legend(label(1 January) label(2 July))
```



As before, it would have been enough to type `line cjan cjuly temp, sort`. See [D] [stack](#) for an explanation of how the `stack` command works.



□ Technical note

According to [Beniger and Robyn \(1978\)](#), [Fourier \(1821\)](#) published the first graph of a cumulative frequency distribution, which was later given the name “ogive” by [Galton \(1875\)](#).



[Jean Baptiste Joseph Fourier \(1768–1830\)](#) was born in Auxerre in France. As a young man, Fourier became entangled in the complications of the French Revolution. As a result, he was arrested and put into prison, where he feared he might meet his end at the guillotine. When he was not in prison, he was studying, researching, and teaching mathematics. Later, he served Napoleon’s army in Egypt as a scientific adviser. Upon his return to France in 1801, he was appointed Prefect of the Department of Isère. While prefect, Fourier worked on the mathematical basis of the theory of heat, which is based on what are now called Fourier series. This work was published in 1822, despite the skepticism of Lagrange, Laplace, Legendre, and others—who found the work lacking in generality and even rigor—and disagreements of both priority and substance with Biot and Poisson.

Acknowledgment

The `equal` option was added by Nicholas J. Cox of the Department of Geography at Durham University, UK, who is coeditor of the *Stata Journal* and author of *Speaking Stata Graphics*.

References

- Beniger, J. R., and D. L. Robyn. 1978. Quantitative graphics in statistics: A brief history. *American Statistician* 32: 1–11. <https://doi.org/10.2307/2683467>.
- Fourier, J. B. J. 1821. Notions générales, sur la population. *Recherches Statistiques sur la Ville de Paris et le Département de la Seine* 1: 1–70.
- Galton, F. 1875. Statistics by intercomparison, with remarks on the law of frequency of error. *Philosophical Magazine* 49: 33–46. <https://doi.org/10.1080/14786447508641172>.
- Wilk, M. B., and R. Gnanadesikan. 1968. Probability plotting methods for the analysis of data. *Biometrika* 55: 1–17. <https://doi.org/10.2307/2334448>.

Also see

- [R] [Diagnostic plots](#) — Distributional diagnostic plots
- [R] [kdensity](#) — Univariate kernel density estimation
- [D] [stack](#) — Stack data

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Acknowledgment](#)[Syntax](#)[Reference](#)[Options](#)[Also see](#)

Description

`cusum` graphs the cumulative sum (cusum) of a binary (0/1) variable, *yvar*, against a (usually) continuous variable, *xvar*.

Quick start

Cusum statistics for binary variable *y* and graph of cumulative sum against values of *x*

```
cusum y x
```

Also generate *cs* to store the cumulative sum

```
cusum y x, generate(cs)
```

Set the seed first for reproducible results

```
set seed 87534690
```

```
cusum y x, generate(cs)
```

Cumulative sum of *y* against a variable containing fitted values *yhat*

```
cusum y x, yfit(yhat)
```

Menu

Statistics > Other > Quality control > Cusum plots and tests for binary variables

Syntax

```
cusum yvar xvar [if] [in] [, options]
```

<i>options</i>	Description
Main	
<code>generate(<i>newvar</i>)</code>	save cumulative sum in <i>newvar</i>
<code>yfit(<i>fitvar</i>)</code>	calculate cumulative sum against <i>fitvar</i>
<code>nograph</code>	suppress the plot
<code>nocalc</code>	suppress cusum test statistics
Cusum plot	
<code>connect_<i>options</i></code>	affect the rendition of the plotted line
Add plots	
<code>addplot(<i>plot</i>)</code>	add plots to the generated graph
Y axis, X axis, Titles, Legend, Overall	
<code>twoway_<i>options</i></code>	any options other than by() documented in [G-3] twoway_<i>options</i>

collect is allowed; see [U] [11.1.10 Prefix commands](#).

Options

Main

`generate(newvar)` saves the cusum in *newvar*.

`yfit(fitvar)` calculates a cusum against *fitvar*, that is, the running sums of the “residuals” *fitvar* minus *yvar*. Typically, *fitvar* is the predicted probability of a positive outcome obtained from a logistic regression analysis.

`nograph` suppresses the plot.

`nocalc` suppresses calculation of the cusum test statistics.

Cusum plot

`connect_options` affect the rendition of the plotted line; see [G-3] [connect_*options*](#).

Add plots

`addplot(plot)` provides a way to add other plots to the generated graph. See [G-3] [addplot_*option*](#).

Y axis, X axis, Titles, Legend, Overall

`twoway_options` are any of the options documented in [G-3] [twoway_*options*](#), excluding by(). These include options for titling the graph (see [G-3] [title_*options*](#)) and for saving the graph to disk (see [G-3] [saving_*option*](#)).

Remarks and examples

The cusum is the running sum of the proportion of ones in the sample, a constant number, minus $yvar$,

$$c_j = \sum_{k=1}^j f - yvar_{(k)}, \quad 1 \leq j \leq N$$

where $f = (\sum yvar)/N$ and $yvar_{(k)}$ refers to the corresponding value of $yvar$ when $xvar$ is placed in ascending order: $xvar_{(k+1)} \geq xvar_{(k)}$. Tied values of $xvar$ are broken at random. If you want them broken the same way in two runs, you must set the random-number seed to the same value before giving the cusum command; see [R] [set seed](#).

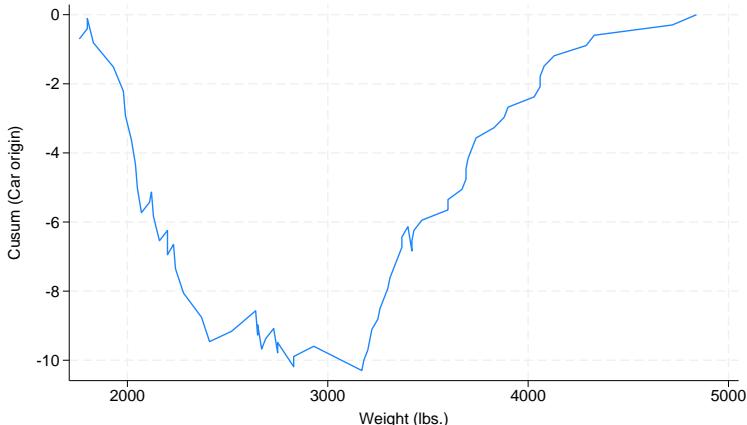
A U-shaped or inverted U-shaped cusum indicates, respectively, a negative or a positive trend of $yvar$ with $xvar$. A sinusoidal shape is evidence of a nonmonotonic (for example, quadratic) trend. `cusum` displays the maximum absolute cusum for monotonic and nonmonotonic trends of $yvar$ on $xvar$. These are nonparametric tests of departure from randomness of $yvar$ with respect to $xvar$. Approximate values for the tests are given.

► Example 1

For the automobile dataset, `auto.dta`, we wish to investigate the relationship between `foreign` (0 = domestic, 1 = foreign) and car weight as follows:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. cusum foreign weight
```

Variable	Obs	Pr(1)	CusumL	zL	Pr>zL	CusumQ	zQ	Pr>zQ
foreign	74	0.2973	10.30	3.963	0.000	2.92	0.064	0.475



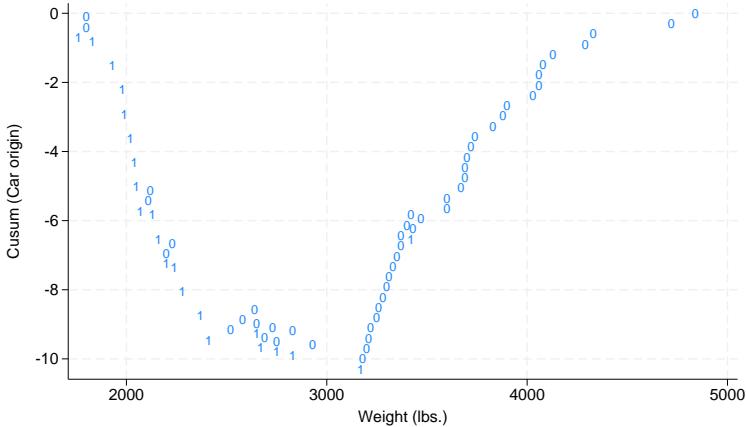
The resulting plot, which is U-shaped, suggests a negative monotonic relationship. The trend is confirmed by a highly significant linear cusum statistic, labeled `CusumL` in the output above.

Some 29.73% of the cars are foreign (coded 1). The proportion of foreign cars diminishes with increasing weight. The domestic cars are crudely heavier than the foreign ones. We could have discovered that by typing `table foreign, statistics(mean weight)`, but such an approach does not give the

full picture of the relationship. The quadratic cusum (CusumQ) is not significant, so we do not suspect any tendency for the very heavy cars to be foreign rather than domestic. A slightly enhanced version of the plot shows the preponderance of domestic (coded 0) cars at the heavy end of the weight axis:

```
. label values foreign
. cusum foreign weight, s(none) recast(scatter) mlabel(foreign) mlabp(0)
```

Variable	Obs	Pr(1)	CusumL	zL	Pr>zL	CusumQ	zQ	Pr>zQ
foreign	74	0.2973	10.30	3.963	0.000	3.32	0.469	0.320



The example is, of course, artificial, because we would not really try to model the probability of a car being foreign given its weight.



Stored results

cusum stores the following in `r()`:

Scalars

<code>r(N)</code>	number of observations	<code>r(P_z1)</code>	<i>p</i> -value for test (linear)
<code>r(prop1)</code>	proportion of positive outcomes	<code>r(cusumq)</code>	quadratic cusum
<code>r(cusuml)</code>	cusum	<code>r(zq)</code>	test (quadratic)
<code>r(zL)</code>	test (linear)	<code>r(P_zq)</code>	<i>p</i> -value for test (quadratic)

Acknowledgment

cusum was written by Patrick Royston of the MRC Clinical Trials Unit, London, and coauthor of the Stata Press book *Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model*.

Reference

Royston, P. 1992. The use of cusums and other techniques in modelling continuous covariates in logistic regression. *Statistics in Medicine* 11: 1115–1129. <https://doi.org/10.1002/sim.4780110813>.

Also see

[R] **logistic** — Logistic regression, reporting odds ratios

[R] **logit** — Logistic regression, reporting coefficients

[R] **probit** — Probit regression

Description

db opens the dialog box for the specified command. Programmers who wish to allow the launching of dialogs from a help file, see [P] [smcl](#) for information on the `dialog` SMCL directive.

`set maxdb` sets the maximum number of dialog boxes whose contents are remembered from one invocation to the next during a session. The default value of `maxdb` is 50.

Syntax

Syntax for db

```
db commandname
```

For programmers

```
db commandname [ , message(string) debug dryrun ]
```

Set system parameter

```
set maxdb # [ , permanently ]
```

where # must be between 5 and 1,000.

Options

`message(string)` specifies that *string* be passed to the dialog box, where it can be referred to from the `__MESSAGE STRING` property.

`debug` specifies that the underlying dialog box be loaded with debug messaging turned on.

`dryrun` specifies that, rather than launching the dialog, db show the commands it would issue to launch the dialog.

`permanently` specifies that, in addition to making the change right now, the `maxdb` setting be remembered and become the default setting when you invoke Stata.

Remarks and examples

The usual way to launch a dialog is to open the **Data, Graphics, or Statistics** menu and to make your selection from there. When you know the name of the command that you want to run, however, db provides a way to invoke the dialog from the command line.

db follows the same abbreviation rules that Stata's command-line interface follows. So, to launch the dialog for `regress`, you can type

```
. db regress
```

or

```
. db reg
```

Say that you use the dialog box for `regress`, either by selecting **Statistics > Linear models and related > Linear regression** or by typing

```
. db regress
```

You fit a regression.

Much later during the session, you return to the `regress` dialog box. It will have the contents as you left them if 1) you have not typed `clear all` between the first and second invocations; 2) you have not typed `discard` between the two invocations; and 3) you have not used more than 50 different dialog boxes—regardless of how many times you have used each—between the first and second invocations of `regress`. If you use 51 or more, the contents of the `regress` dialog box will be forgotten.

`set maxdb` determines how many different dialog boxes are remembered. A dialog box takes, on average, about 20 KB of memory, so the 50 default corresponds to allowing dialog boxes to consume about 1 MB of memory.

Also see

[\[R\] query](#) — Display system parameters

[Description](#)[Options](#)[Acknowledgment](#)[Quick start](#)[Remarks and examples](#)[References](#)[Menu](#)[Stored results](#)[Also see](#)[Syntax](#)[Methods and formulas](#)

Description

`demandsys` fits demand systems, sets of equations derived from economic theory that describe consumers' purchases of various goods or services. `demandsys` allows you to fit eight different demand systems, including the Cobb–Douglas system, the almost ideal demand system (AIDS) of [Deaton and Muellbauer \(1980b\)](#), the translog indirect utility demand system of [Christensen, Jorgenson, and Lau \(1975\)](#), and variants of the latter two. You can also include demographic variables that affect a consumer's or household's demands.

Quick start

Fit an AIDS demand system with four goods with expenditure shares w_1 , w_2 , w_3 , and w_4 ; prices, p_1 , p_2 , p_3 , and p_4 ; and total expenditure, `totexp`

```
demandsys aids w1 w2 w3 w4, prices(p1 p2 p3 p4)    ///
expenditure(totexp)
```

Same as above, reporting Marshallian (uncompensated) elasticities rather than coefficients

```
demandsys aids w1 w2 w3 w4, prices(p1 p2 p3 p4)    ///
expenditure(totexp) elasticities(uncompensated)    ///
nocoefftable
```

Same as above, labeling goods in output for easier interpretation

```
demandsys aids w1 w2 w3 w4, prices(p1 p2 p3 p4)    ///
expenditure(totexp) elasticities(uncompensated)    ///
nocoefftable labels("apples bananas carrots dates")
```

Fit a quadratic AIDS model with four goods, controlling for x_1 and x_2 using demographic translation

```
demandsys quaid w1 w2 w3 w4, prices(p1 p2 p3 p4)    ///
expenditure(totexp) demographics(x1 x2)
```

Same as above, but use demographic scaling rather than translating

```
demandsys quaid w1 w2 w3 w4, prices(p1 p2 p3 p4)    ///
expenditure(totexp) demographics(x1 x2, scaling)
```

Menu

[Statistics](#) > [Linear models and related](#) > [Multiple-equation models](#) > [Demand system](#)

Syntax

```
demandsys model varlists [if] [in] [weight], prices(varlistp)
    expenditure(varname) [options]
```

<i>model</i>	Description
<u>cdouglas</u>	Cobb–Douglas demand system
<u>les</u>	linear expenditure system
<u>translog</u>	basic translog demand system
<u>gtranslog</u>	generalized translog demand system
<u>aids</u>	almost ideal demand system (AIDS)
<u>gaids</u>	generalized AIDS
<u>quaid</u> s	quadratic AIDS
<u>gquaid</u> s	generalized quadratic AIDS

varlist_s indicates the list of G variables containing the expenditure shares of the G goods in the model.

<i>options</i>	Description
Main	
* <u>prices</u> (<i>varlist_p</i>)	specify the variables containing prices
* <u>expenditure</u> (<i>varname</i>)	specify the variable containing total expenditure
<u>demographics</u> (<i>varlist_d</i> [<i>scaling</i>])	specify other variables affecting a consumer's demand for the goods; use demographic scaling instead of translating
<u>pic</u> constant(#)	specify constant term in transcendental logarithmic price index
SE/Robust	
<u>vce</u> (<i>vcetype</i>)	<i>vcetype</i> may be <u>gnr</u> , <u>robust</u> , <u>cluster</u> <i>clustvar</i> , <u>bootstrap</u> , or <u>jackknife</u>
Reporting	
<u>e</u> lasticities(<i>e_type</i>)	report elasticities; <i>e_type</i> may be <u>expenditure</u> , <u>compensated</u> , or <u>uncompensated</u>
<u>l</u> abels(<i>string</i>)	specify labels for goods
<u>l</u> evel(#)	set confidence level; default is level(95)
<u>n</u> oheader	suppress the summary header at the top of the output
<u>n</u> ocoefftable	do not display the table of estimated coefficients
<u>d</u> isplay_ <u>o</u> ptions	control columns and column formats and line width
Optimization	
<u>o</u> ptimization_ <u>o</u> ptions	control the optimization process; seldom used
<u>c</u> oeff <u>l</u> egend	display legend instead of statistics

*prices() and expenditure() are required.

bayesboot and collect are allowed; see [U] 11.1.10 Prefix commands.

aweight, fweight, iweight, and pweight are allowed; see [U] 11.1.6 weight.

coeflegend does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

Options

Main

prices(*varlist_p*) specifies a list of G variables corresponding to the prices of the G goods faced by each consumer. The number of variables specified here must match the number specified in *varlist_s*, and the price variables must be specified in the same order as the share variables. All the price variables must be strictly positive for all the demand systems implemented. prices() is required.

expenditure(*varname*) specifies the variable corresponding to the total expenditure on all goods within the system by each consumer. This variable must be strictly positive. expenditure() is required.

demographics(*varlist_d*[, *scaling*]) specifies one or more demographic variables that affect each consumer's demand for the goods in the system. Suboption *scaling*, available only with models *aids* and *quads*, requests that demographics be incorporated via demographic scaling rather than demographic translation, the default.

piconstant(*#*) specifies the value of the constant term in the transcendental logarithmic price index; by default, this is set to the logarithm of the minimum expenditure observed in the sample. This option is relevant only for models *aids* and *quads*.

SE/Robust

vce(*vcetype*) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (*gnr*), that are robust to some kinds of misspecification (*robust*), that allow for intragroup correlation (*cluster clustvar*), and that use bootstrap or jackknife methods (*bootstrap*, *jackknife*); see [R] *vce_option*.

vce(*gnr*), the default, uses the conventionally derived variance estimator for nonlinear models fit using Gauss–Newton regression.

Reporting

elasticities(*e_type*) requests that elasticities be reported instead of, or in addition to, the parameter estimates. *e_type* may be *expenditure*, *compensated*, or *uncompensated*. *expenditure* computes demand elasticity to changes in expenditure. *compensated* computes demand elasticity to changes in prices, ignoring income effects. These elasticities are also known as Hicksian price elasticities. *uncompensated* computes demand elasticity to changes in prices. These elasticities are also known as Marshallian price elasticities. The elasticities are computed at the estimation sample means of the prices, expenditures, and any demographic variables specified. For more flexibility in obtaining elasticities, use the postestimation command *estat elasticities*.

`labels(string)` specifies a set of names with which the goods are to be labeled. By default, if you specify a four-good demand system, then the goods will be labeled generically: “Good 1”, “Good 2”, “Good 3”, and “Good 4”.

If you specify `labels("shelter fuel food other")`, then the four goods will be labeled “shelter”, “fuel”, “food”, and “other” in the output. If you specify a demand system with G goods, then you must supply G labels separated by spaces.

`level(#)`; see [R] [Estimation options](#).

`noheader` requests that the header summarizing the model, estimation sample, and other statistics not be shown in the output.

`nocoefftable` requests that the table containing the parameter estimates, their standard errors, and so on not be displayed. Typically, you would use this option if you specify the `elasticities()` option.

`display_options`: `nocl`, `nopvalues`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

Optimization

`optimization_options`: `iterate(#)`, `[no]log`, `trace`, `eps(#)`, `ifgnliterate(#)`, `ifgnlseps(#)`, and `delta(#)`.

`iterate(#)` specifies the maximum number of iterations to use for nonlinear least squares at each round of feasible generalized nonlinear least-squares (FGNLS) estimation. The default is the number set using `set maxiter`, which is 300 by default.

`log` and `nolog` specify whether to display the iteration log. The iteration log is displayed by default unless you used `set iterlog off` to suppress it; see `set iterlog` in [R] [set iter](#).

`trace` specifies that the iteration log should include the current parameter vector.

`eps(#)` specifies the convergence criterion for successive parameter estimates and for the residual sum of squares. The default is `eps(1e-5)` (0.00001). `eps()` also specifies the convergence criterion for successive parameter estimates between rounds of iterative FGNLS.

`ifgnliterate(#)` specifies the maximum number of FGNLS iterations to perform. The default is the number set using `set maxiter`, which is 300 by default.

`ifgnlseps(#)` specifies the convergence criterion for successive estimates of the error covariance matrix during iterative FGNLS estimation. The default is `ifgnlseps(1e-10)`.

`delta(#)` specifies the relative change in a parameter, δ , to be used in computing the numeric derivatives. The derivative for parameter β_{i+1} is computed as

$$\{f_i(\mathbf{x}_i, \beta_1, \beta_2, \dots, \beta_i + d, \beta_{i+1}, \dots) - f_i(\mathbf{x}_i, \beta_1, \beta_2, \dots, \beta_i, \beta_{i+1}, \dots)\} / d$$

where $d = \delta(|\beta_i| + \delta)$. The default is `delta(4e-7)`.

The following option is available with `demandsys` but is not shown in the dialog box:

`coeflegend`; see [R] [Estimation options](#).

Remarks and examples

Remarks are presented under the following headings:

- Introduction*
- Some notation*
- Cobb–Douglas*
- Linear expenditure system (LES)*
- Translog*
 - Basic translog*
 - Generalized translog*
- AIDS*
- QUAIDS*
- Controlling for demographic factors*
- Demographic translation*
- Demographic scaling*
- Epilogue*

Introduction

demandsys fits demand systems, sets of equations derived from economic theory that describe consumers’ purchases of various goods or services. Typically, you will have a large cross-sectional survey containing consumers’ data on their expenditures on various goods and services along with the prices paid for them. We often refer to the items as “goods” for brevity, but of course some of the items may be services. Whether they are physical goods or intangible services, in the context of utility-maximizing consumers, they are both goods in the sense that “more is better”. We also use the term “purchase” somewhat loosely; in some cases, what we are interested in is not the actual purchase of goods but rather their consumption over a fixed time period.

To fit a demand system, you must first decide on what is in the consumption basket or bundle and the set of goods whose demands you wish to model. Using the parameters from the model, you may then obtain the elasticities—the effects of changes in prices or changes in expenditure on demand of goods. You may also perform welfare analysis by contrasting changes in demand or utility that occur at different price or expenditure levels. We assume that you have decided which consumption bundle to model already or else that you have several alternative baskets to model and compare.

The left-hand-side variables you specify with demandsys are expenditure shares, the shares of total expenditure spent on goods or services. Given p_g (the price of good g), q_g (the quantity of good g purchased), and m (the consumer’s total expenditure on all goods within the demand system), the expenditure share for good g is defined as

$$w_g = \frac{p_g q_g}{m}$$

demandsys requires that you have the prices of all the goods and that you have the total expenditure across all the goods. demandsys does not need the quantities of the goods purchased, though you may need them to calculate the expenditure shares.

Consumer theory in microeconomics presents demand models in terms of quantities. In demand system analysis, we model expenditure shares, not quantities. By our definition of total expenditure, $0 \leq w_g \leq 1$ for all g , and $\sum_g w_g = 1$. For all but the simplest utility or cost functions, the algebra to obtain expenditure shares is arguably easier than that to obtain quantities. demandsys checks your expenditure shares for all goods and will exit with an error message if an expenditure share is found outside that range or if the sum is not equal to one (allowing for small rounding errors).

You may also specify additional variables that may affect a consumer's purchase decisions. These are often demographic variables, such as the number of children and adults in a household, or a set of indicator variables to denote the region of a country in which a consumer is located. If you have data that were collected over the course of different months or years, you may wish to include indicators to control for the time at which the consumer's purchases were observed. We call all of these variables demographics, even if some of these variables are not really demographic characteristics.

Based on the theoretical considerations in [Lewbel \(2001\)](#), we strongly encourage you to include demographic variables in your model. He shows that econometrically estimated demand functions will not satisfy rationality conditions unless other variables that affect demand and are correlated with expenditures are included. Conversely, if we include all such variables in our model, then the estimated demands will satisfy rationality even if preferences vary among households. On the other hand, you should also bear in mind that including too many demographic variables can greatly increase the number of parameters in your model, especially if your demand system includes many goods.

demandsys offers just a small sampling of demand systems that have been proposed in the literature. As a practical matter, which one should you use? We have included the Cobb–Douglas model mainly for pedagogical purposes because most students are familiar with Cobb–Douglas utility, though soon thereafter they learn about its severe restrictions on consumer behavior. Stone's (1954) linear expenditure system (LES) is historically important as an early demand system but also places somewhat strong restrictions on consumer behavior. Christensen, Jorgenson, and Lau's (1975) translog model relaxes some of the LES's restrictions. It is not as widely used as some of the newer models, though a very similar translog production function those authors proposed continues to see extensive use. Shortly after the translog model arrived, [Deaton and Muellbauer \(1980b\)](#) proposed their more flexible AIDS, which has been a workhorse model of demand system analysis ever since its arrival. [Banks, Blundell, and Lewbel \(1997\)](#) provided a quadratic extension to AIDS that is also popular.

[Banks, Blundell, and Lewbel \(1997\)](#) suggest using QUAIDS because of its flexibility. Moreover, QUAIDS satisfies certain theoretical properties developed in [Gorman \(1981\)](#); and it allows goods to be either luxuries or necessities depending on a consumer's income, which [Banks, Blundell, and Lewbel \(1997\)](#) show to be important in the data they consider.

With the AIDS and QUAIDS models, you can include demographics via two different methods: demographic translation and demographic scaling; see [Demographic translation](#) and [Demographic scaling](#). If you believe that subsistence or committed quantities are appropriate for the goods you are modeling, you can use what we call the generalized QUAIDS model. If you do that, you should check to see that the subsistence quantity interpretation holds; we do that in [example 2](#) of [\[R\] demandsys postestimation](#). You can also do a simple likelihood-ratio test to see whether the quadratic terms of these QUAIDS models increase their explanatory power over Deaton and Muellbauer's AIDS and its generalized variant; see [example 5](#) below.

Throughout the rest of the discussion, we assume that you are familiar with a few basic microeconomic concepts, including utility maximization, the expenditure function, and the indirect utility function. The presentations in standard texts like [Varian \(1992, chap. 7–10\)](#) and [Mas-Colell, Whinston, and Green \(1995, chap. 3\)](#) provide good introductions to these concepts. The classic monograph by [Deaton and Muellbauer \(1980a\)](#) and the book by [Pollak and Wales \(1992\)](#) describe demand system analysis in much greater detail than space permits us here. More recent survey papers include [Holt and Goodwin \(2009\)](#) and [Barnett and Serletis \(2008\)](#). [Fisher, Fleissig, and Serletis \(2001\)](#) compare many flexible functional forms for demand system analysis, including several that are implemented by demandsys.

Some notation

It will be useful to establish at the outset some standard notation we will use throughout this discussion. We use subscript $i = 1, \dots, N$ to index observations, $g = 1, \dots, G$ to index goods, and $d = 1, \dots, D$ to index demographic variables. In formulas where we must use double summations across goods or else have used subscript g elsewhere, we also use $h = 1, \dots, G$ and $j = 1, \dots, G$ to index goods. When no confusion could arise, we omit the observation subscript to reduce the number of subscripts.

Let w_g be the expenditure share for good g defined as $w_g \equiv (p_g q_g)/m$, where p_g is the price of good g , q_g is the quantity of good g consumed, and m denotes total expenditure on all the goods in the system being modeled. We use the notation \mathbf{p} to refer to the G -length vector of all G prices. When we refer to w_g , we are referring to the observed expenditure share for good g for consumer i . When we refer to, say, $w_g(\mathbf{p}, m; \alpha, \beta)$, we are referring to an expenditure-share equation or function for good g that depends on the prices of all the goods and total expenditure as well as parameter vectors α and β . We include the parameter vectors and matrices in the arguments for expenditure shares as well as some other functions to emphasize that they are estimated and will appear in the output from `demandsys` and that statistics available via `predict` or `estat elasticities` after estimation depend on those parameters.

The observed w_g is assumed to be related to the expenditure-share function $w_g(\mathbf{p}, m; \alpha, \beta)$ as

$$w_g = w_g(\mathbf{p}, m; \alpha, \beta) + \epsilon_g$$

where ϵ_g is a zero-mean error term that we discuss in more detail in [Methods and formulas](#).

Cobb–Douglas

Although widely used in many economic models, the Cobb–Douglas utility function is arguably too simple for serious demand system analysis because of its severe restrictions on the expenditure-share equations and elasticities. For instance, it restricts expenditure elasticities to be identically equal to 1 for all goods. Nevertheless, we include it because it serves as a good starting point for our discussion and allows us to present various aspects of `demandsys`. For three goods, consumers maximize their Cobb–Douglas utility function subject to the constraint that total expenditure does not exceed the allocated budget,

$$\begin{aligned} \max_{q_1, q_2, q_3} & q_1^{\alpha_1} q_2^{\alpha_2} q_3^{(1-\alpha_1-\alpha_2)} \\ \text{subject to} & p_1 q_1 + p_2 q_2 + p_3 q_3 \leq m \end{aligned}$$

where, without loss of generality, we have made the sum of the exponents of the Cobb–Douglas utility function sum to 1. It is easy to show that the optimal quantities q_1^* , q_2^* , and q_3^* are

$$q_1^* = \alpha_1 \frac{m}{p_1} \quad q_2^* = \alpha_2 \frac{m}{p_2} \quad q_3^* = (1 - \alpha_1 - \alpha_2) \frac{m}{p_3}$$

To obtain the expenditure-share function of, say, good 1, we have

$$w_1(\mathbf{p}, m; \boldsymbol{\alpha}) = \frac{p_1 q_1}{m} = \alpha_1 \frac{m}{p_1} \frac{p_1}{m} = \alpha_1$$

and likewise for goods 2 and 3. Notice that the expenditure-share function for good g is simply its coefficient α_g in the Cobb–Douglas utility function, or, equivalently, the coefficients in the Cobb–Douglas utility function are simply the shares of total expenditure allocated to each good. Most importantly, the expenditure shares for the Cobb–Douglas model are not functions of prices, income, or any other variables.

We made the sum of the exponents in the Cobb–Douglas utility function sum to 1, and it is clear that $w_1 + w_2 + w_3 = 1$. Without the constraint on the sum, we would have to carry around the divisor $\alpha_1 + \alpha_2 + \alpha_3$ in our expenditure-share equations, but we would gain absolutely no additional insight. Moreover, with the constraint we need estimate only two parameters rather than three.

More generally, for a Cobb–Douglas utility function of the form

$$u(\mathbf{q}; \boldsymbol{\alpha}) = \prod_{g=1}^G q_g^{\alpha_g} \quad \text{with} \quad \sum_{g=1}^G \alpha_g = 1$$

the expenditure-share function for the g th good is simply $w_g(\mathbf{p}, m; \boldsymbol{\alpha}) = \alpha_g$.

▷ Example 1

We first describe the dataset we will use in this example and all others in this manual entry:

```
. use https://www.stata-press.com/data/r19/food_consumption
(Food consumption)
. describe
Contains data from https://www.stata-press.com/data/r19/food_consumption.dta
Observations:      4,160      Food consumption
Variables:         13        17 Jul 2024 16:03
                        (_dta has notes)
```

Variable name	Storage type	Display format	Value label	Variable label
w_dairy	float	%10.6f		Expenditure share on dairy
w_proteins	float	%10.6f		Expenditure share on meats and proteins
w_fruitveg	float	%10.6f		Expenditure share on fruits and vegetables
w_flours	float	%10.6f		Expenditure share on flours, breads, pasta, and cereals
w_misc	float	%10.6f		Expenditure share on misc. food items
p_dairy	float	%10.6f		Price of dairy
p_proteins	float	%10.6f		Price of meats and proteins
p_fruitveg	float	%10.6f		Price of fruits and vegetables
p_flours	float	%10.6f		Price of flours, breads, pasta, and cereals
p_misc	float	%10.6f		Price of misc. food items
expfd	float	%10.6f		Total expenditure on all food categories
n_adults	byte	%8.0g		# adults in household
n_kids	byte	%8.0g		# kids in household

Sorted by:

The observation level of this dataset is the household, and we have data on 4,160 households. The data include five categories of food: dairy products; proteins, including meats and fish; fruits and vegetables; flour-based products, including breads, pastas, and cereals; and a catchall category. For each of the five categories, we have the expenditure shares and prices, and we have the household’s total expenditure on all five categories in the week that the household was surveyed. We also have demographic data for the household, including the numbers of children and adults.

Although we constructed the expenditure shares ourselves and know they sum to one, it is a good idea to verify that you have created your expenditure-share variables properly:

```
. egen wsum = rowtotal(w_dairy w_proteins w_fruitveg w_flours w_misc)
. summarize wsum
```

Variable	Obs	Mean	Std. dev.	Min	Max
wsum	4,160	1	4.40e-08	.9999999	1

demandsys will check that for you and exit with an error message if it is not the case, but doing so during your data management tasks may make debugging easier. demandsys will also check that all your price variables and expenditure variable are strictly positive, though again you might want to check those facts earlier in your data pipeline.

With a dataset in hand, we are ready to fit our first Cobb–Douglas demand system.

```
. demandsys cdouglas w_dairy w_proteins w_fruitveg w_flours w_misc,
> prices(p_dairy p_proteins p_fruitveg p_flours p_misc) expenditure(expdf)
Calculating NLS estimates:
Iteration 0: Residual SS = 180.2506
Iteration 1: Residual SS = 180.2506
Calculating FG-NLS estimates:
Iteration 0: Scaled RSS = 16640
FG-NLS iteration 2:
Iteration 0: Scaled RSS = 16640
Parameter change = 0.00e+00
Covariance matrix change = 0.00e+00
Cobb-Douglas model                                Number of obs = 4,160
Expenditure variable: expfd                       Number of goods = 5
Uncentered R2 of model for
Good 1 = 0.7519
Good 2 = 0.8851
Good 3 = 0.8329
Good 4 = 0.7467
Good 5 = 0.6836
```

		Estimate	Std. err.	z	P> z	[95% conf. interval]	
alpha							
Good							
	1	.1505844	.0013411	112.29	0.000	.147956	.1532129
	2	.3986526	.0022265	179.05	0.000	.3942887	.4030166
	3	.2406533	.0016711	144.01	0.000	.237378	.2439286
	4	.1031129	.0009312	110.73	0.000	.1012877	.104938

Note: **alpha** estimates are expenditure shares.

Normalized parameter

		Estimate	Std. err.	z	P> z	[95% conf. interval]	
alpha	Good						
	5	.1069968	.0011286	94.80	0.000	.1047847	.1092088

Note: **alpha** estimates sum to 1.

Shares: w_dairy w_proteins w_fruitveg w_flours w_misc

Prices: p_dairy p_proteins p_fruitveg p_flours p_misc

Expenditure: expfd

We specified the command name, `demandsys`, followed by the model we wish to fit, `cdouglas`. We then specified the five expenditure-share variables, `w_dairy` through `w_misc`. The price variables, `p_dairy` through `p_misc`, go into the `prices()` option, and the expenditure variable, `expfd`, goes into the `expenditure()` option. `demandsys` uses a nonlinear multiple-equation estimator, so the top of the output includes an iteration log showing the model’s convergence. In fact, the Cobb–Douglas model is linear, but that is the only model fit by `demandsys` that is. Stata uses the same nonlinear estimator for `cdouglas` as we use for the other demand systems. The header of the output includes R^2 values to get some idea of how well the model fits the data.

The table of coefficients is a bit different from most other estimators in Stata, which report coefficients organized by equations. The Cobb–Douglas model is unique in that each equation has its own parameter. However, that is not the case for any other demand system that `demandsys` fits. For most demand systems, many of the parameters appear in multiple equations, and there is no way to link parameters and equations. Hence, we cannot provide a coefficient table organized by equations as other multiple-equation estimators such as `sureg` provide. Instead, `demandsys` organizes estimates by parameter type.

We mentioned having a five-good demand system, so why do we see only four estimated parameters in the main output table? Recall that we used the normalization that the sum of the parameters in the Cobb–Douglas utility function is one. The upshot is that we really have only four free parameters because the fifth parameter must equal one minus the sum of the other four parameters. We provide the estimate of the fifth parameter in a separate table. It is computed separately using the parameter estimates of our fitted demand system and their standard errors. Given how the fifth parameter was computed, it does not share a covariance matrix with the other parameters and cannot be used for testing. We provide it here for reference.

`demandsys` shows only unconstrained parameters in the main estimation output because, with many goods and more complicated models, the output becomes long. Moreover, the estimated parameters themselves are of less interest than elasticities and other statistics derived from the fitted model.

□ Technical note

In the header of the output in the previous example, we see that the equation-level R^2 values are labeled as uncentered. Recall that the standard R^2 in regression analysis measures the ability of the model to predict the regressand compared with a model that contains just a constant term. For the Cobb–Douglas demand system, the expenditure shares are simply constant terms to be estimated. Hence, the traditional R^2 is by definition zero. We therefore report the uncentered R^2 , which is an alternative that measures the model’s explanatory power as a fraction of the sum of squares of an equation’s left-hand-side variable. A

higher uncentered R^2 is presumably better than a lower uncentered R^2 , but you cannot compare the uncentered R^2 of a Cobb–Douglas share equation with the centered R^2 of a share equation from a different demand system. □

The command `estat elasticities`, which is available after `demandsys`, allows you to calculate expenditure and price elasticities after fitting your demand system, and `estat elasticities` provides options to specify how those elasticities are calculated and for which observations. The full syntax for that command is listed in [R] [demandsys postestimation](#), but the basic command is easy to pick up as we work through examples below.

Here we use `estat elasticities` to obtain the expenditure elasticities for the estimation sample.

```
. estat elasticities, expenditure
Expenditure elasticities                                Number of obs = 4,160
```

Expenditure	Elasticity
Good	
1	1
2	1
3	1
4	1
5	1

Note: No standard errors are displayed because all elasticities are identically equal to one.

The expenditure elasticities for all five goods are identically equal to one, and so the standard errors, test statistics, and confidence interval are not displayed because there is no sampling variance. The Cobb–Douglas function is an example of a utility function with “homothetic preferences”, meaning that the ratio of two goods demanded by a consumer with such preferences depends only on the goods’ relative prices and not on income. Homothetic preferences also imply that, regardless of changes in income, the shares consumed of each good remain the same; the expansion path of consumption is linear in income. The Cobb–Douglas case is a particularly extreme example, where the expenditure elasticities are one, the uncompensated own-price elasticities are minus one, and uncompensated cross-price elasticities are zero, which you can verify by typing

```
estat elasticities, uncompensated
```

Particularly with more complex demand systems, the coefficients themselves can be more difficult to interpret. Therefore, `demandsys` allows you to request that expenditure, uncompensated price, or compensated price elasticities be displayed in addition to, or in place of, the table of estimated coefficients. The elasticities reported directly by `demandsys` are calculated at the estimation sample means of prices, expenditures, and any demographics that you specify. To obtain, say, the uncompensated price elasticities instead of the coefficient table from a Cobb–Douglas demand system, you would type

```
demandsys cdouglas ..., prices(...) expenditure(...) ///  
elasticities(uncompensated) nocoefstable
```

`estat elasticities` gives you much more flexibility in terms of the sample used, and you can even specify particular prices or expenditures at which you want the elasticities to be calculated.



Linear expenditure system (LES)

Nobel laureate Sir Richard Stone’s estimation of the LES, developed based on theory by Nobel laureates Paul Samuelson and Lawrence Klein, together with Herman Rubin, represents the genesis of flexible demand system estimation (Stone 1954). LES provides for more flexible consumption patterns than the Cobb–Douglas model. The LES begins with the utility function

$$u(\mathbf{q}; \boldsymbol{\beta}, \boldsymbol{\mu}) = \prod_g (q_g - \mu_g)^{\beta_g}$$

where μ_g is the g th element of $G \times 1$ vector $\boldsymbol{\mu}$, which is to be estimated. This utility function results from translating or shifting the origin of the Cobb–Douglas utility function; utility does not accrue from good g unless its consumption exceeds an amount μ_g . That parameter is known as a subsistence or committed quantity that a consumer must purchase for survival. β_g is the g th element of $G \times 1$ vector $\boldsymbol{\beta}$. β_g measures the share of supernumerary expenditure, namely, the share of remaining expenditure once all subsistence quantities have been purchased, on good g . As in the Cobb–Douglas case, we normalize the $G \times 1$ parameter vector $\boldsymbol{\beta}$ so that $\sum_g \beta_g = 1$. Thus, the model has $2G - 1$ parameters we must estimate.

Solving

$$\max_{\mathbf{q}} \prod_g (q_g - \mu_g)^{\beta_g} \quad \text{s. t.} \quad \mathbf{p}'\mathbf{q} \leq m$$

yields the expenditure-share functions

$$w_g(\mathbf{p}, m; \boldsymbol{\beta}, \boldsymbol{\mu}) = \frac{p_g \mu_g}{m} + \beta_g \left(1 - \sum_h \frac{p_h \mu_h}{m} \right)$$

The term to the left of the plus sign is the fraction of expenditure that must be spent on good g to maintain subsistence. To the right, the term bound in parentheses is the fraction of expenditure left after the subsistence level for all other goods has been spent. Of this amount, a share β_g is spent on good g . The normalization that $\sum_g \beta_g = 1$ ensures that $\sum_g w_g(\mathbf{p}, m) = 1$, as must be true of a demand system.

While we have interpreted the μ_g as minimum required amounts of each good, there is no requirement that $\mu_g > 0$ for all g , nor do we impose any such constraints during estimation. Moreover, for this interpretation to hold we must have that for each household i , $\sum_g p_{gi} \mu_{gi} \leq m_i$.

► Example 2

Here we fit an LES to the same data as in [example 1](#).

```
. use https://www.stata-press.com/data/r19/food_consumption, clear
(Food consumption)

. demandsys les w_dairy w_proteins w_fruitveg w_flours w_misc,
> prices(p_dairy p_proteins p_fruitveg p_flours p_misc)
> expenditure(expfd) nolog

Calculating NLS estimates ...
Calculating FG-NLS estimates ...
FG-NLS iteration 2 ...
FG-NLS iteration 3 ...
FG-NLS iteration 4 ...
FG-NLS iteration 5 ...

Linear expenditure system                                Number of obs = 4,160
Expenditure variable: expfd                            Number of goods = 5

Centered R2 of model for
  Good 1 = -0.0508
  Good 2 = 0.0494
  Good 3 = 0.0366
  Good 4 = 0.0829
  Good 5 = 0.0603
```

	Estimate	Std. err.	z	P> z	[95% conf. interval]	
beta						
Good						
1	.1650894	.0018783	87.89	0.000	.161408	.1687708
2	.4108231	.0033135	123.98	0.000	.4043288	.4173175
3	.2262407	.0024899	90.86	0.000	.2213607	.2311208
4	.0949082	.0013152	72.16	0.000	.0923304	.097486
mu						
Good						
1	.1794068	.0383858	4.67	0.000	.1041721	.2546416
2	.6975436	.0800327	8.72	0.000	.5406824	.8544048
3	2.368491	.1351367	17.53	0.000	2.103628	2.633354
4	.4215745	.0202499	20.82	0.000	.3818854	.4612636
5	.3742182	.0243646	15.36	0.000	.3264644	.421972

Note: **beta** estimates measure how expenditure shares respond to increases in supernumerary income.

Note: **mu** estimates are subsistence levels of consumption for each good.

Normalized parameter

	Estimate	Std. err.	z	P> z	[95% conf. interval]	
beta						
Good						
5	.1029385	.0015833	65.02	0.000	.0998354	.1060416

Note: **beta** estimates sum to 1.

Shares: w_dairy w_proteins w_fruitveg w_flours w_misc

Prices: p_dairy p_proteins p_fruitveg p_flours p_misc

Expenditure: expfd

We specified the `noLog` option to suppress the detailed iteration log. We first noticed that the R^2 for the model for Good 1 is actually negative. Because the equations here are nonlinear, the value of R^2 is not constrained to be between 0 and 1; that is only true for linear regression. The footer of the table with normalized parameters reemphasizes the fact that we have made $\sum_g \beta_g = 1$.

All the μ parameters are greater than zero. The μ parameter for Good 3, fruits and vegetables, is largest, implying the physical quantity of them required is much higher than the other four goods. However, we should also look at the average prices of the goods:

```
. summarize p_*
```

Variable	Obs	Mean	Std. dev.	Min	Max
p_dairy	4,160	.4387958	.4238236	.1041262	18
p_proteins	4,160	1.729284	.69554	.3333333	9.258823
p_fruitveg	4,160	.5472199	.1945325	.1073369	2.672269
p_flours	4,160	1.49421	.6816424	.0805687	7.202127
p_misc	4,160	1.47115	.8588024	.1428571	15.74629

Based on the average price of each good, the dollar amount of protein required for survival is only 9 cents less than the dollar amount of fruits and vegetables required ($2.368 \times 0.547 = \$1.30$ versus $0.698 \times 1.729 = \$1.21$).

◁

As we mentioned in the [introductory remarks](#), if you have demographic variables, you should incorporate them into your model. `demandsys` incorporates demographics into models by applying Pollak and Wales's (1978) "demographic translation" to the expenditure-share equations. Just as the linear expenditure system is a translated version of the Cobb–Douglas system, Pollak and Wales incorporate demographics by translating the consumer's available level of expenditures.

Suppose for each consumer we have a $D \times 1$ vector of demographic characteristics \mathbf{d} . We have G expenditure-share equations, so associated with \mathbf{d} is a $G \times D$ matrix of parameters \mathbf{N} with typical row ν_g . Let $\mathbf{c} = \mathbf{N}\mathbf{d}$ with typical element c_g . In demographic translation, we write each translated demand function as

$$\bar{x}_g(\mathbf{p}, m, \mathbf{c}) = c_g + x_g(\mathbf{p}, m - \sum_h p_h c_h)$$

For the LES, we have

$$\bar{x}_g(\mathbf{p}, m, \mathbf{d}; \boldsymbol{\beta}, \boldsymbol{\mu}, \mathbf{N}) = (\mu_g + c_g) + \frac{\beta_g}{p_g} \left(m - \sum_h p_h \mu_h - \sum_h p_h c_h \right)$$

so that

$$\bar{w}_g(\mathbf{p}, m, \mathbf{d}; \boldsymbol{\beta}, \boldsymbol{\mu}, \mathbf{N}) = \frac{p_g(\mu_g + \nu_g \mathbf{d})}{m} + \beta_g \left\{ 1 - \frac{\sum_h p_h (\mu_h + \nu_h \mathbf{d})}{m} \right\}$$

For the LES, the effect of the demographic variables is to adjust the μ parameters, though we must be cautious in interpreting the term $\mu_g + \nu_g \mathbf{d}$ as a minimum required quantity because there is nothing preventing it from being negative.

▷ Example 3

The dataset described in [example 1](#) also includes two demographic characteristics, the numbers of children and adults in each household. Here we refit the LES to these data, controlling for these two demographics.

```
. use https://www.stata-press.com/data/r19/food_consumption
(Food consumption)
. demandsys les w_dairy w_proteins w_fruitveg w_flours w_misc,
> prices(p_dairy p_proteins p_fruitveg p_flours p_misc)
> expenditure(expfd) demographics(n_kids n_adults) nolog
Calculating NLS estimates ...
Calculating FGNLS estimates ...
FGNLS iteration 2 ...
FGNLS iteration 3 ...
FGNLS iteration 4 ...
FGNLS iteration 5 ...
```

```
Linear expenditure system           Number of obs           = 4,160
Expenditure variable: expfd         Number of goods         = 5
Demographic method: Translating     Number of demographics = 2
```

```
Centered R2 of model for
Good 1 = -0.0444
Good 2 = 0.0665
Good 3 = 0.0584
Good 4 = 0.1576
Good 5 = 0.0922
```

		Estimate	Std. err.	z	P> z	[95% conf. interval]	
beta							
	Good						
	1	.1656925	.0023639	70.09	0.000	.1610594	.1703256
	2	.4170346	.0042631	97.82	0.000	.408679	.4253901
	3	.2434237	.0032072	75.90	0.000	.2371377	.2497098
	4	.0800666	.0015889	50.39	0.000	.0769525	.0831808
mu							
	Good						
	1	-.3932844	.164016	-2.40	0.016	-.71475	-.0718189
	2	-.53048	.1798223	-2.95	0.003	-.8829251	-.1780349
	3	1.010915	.3046076	3.32	0.001	.4138951	1.607935
	4	-.0269827	.0426005	-0.63	0.526	-.1104782	.0565128
	5	.0097685	.0526521	0.19	0.853	-.0934277	.1129647
Nu							
	Good#						
	c.n_kids						
	1	.4332196	.160145	2.71	0.007	.1193413	.747098
	2	-.0680289	.1197109	-0.57	0.570	-.302658	.1666002
	3	-.7945652	.2003774	-3.97	0.000	-1.187298	-.4018328
	4	.3923541	.0273698	14.34	0.000	.3387103	.4459979
	5	.3563529	.0368047	9.68	0.000	.284217	.4284888
	Good#						
	c.n_adults						
	1	.4856673	.1525998	3.18	0.001	.1865771	.7847574
	2	.8669319	.1316001	6.59	0.000	.6090005	1.124863
	3	.843552	.227083	3.71	0.000	.3984775	1.288626
	4	.3576213	.0318802	11.22	0.000	.2951373	.4201054
	5	.2639693	.0374888	7.04	0.000	.1904925	.3374461

Note: **beta** estimates measure how expenditure shares respond to increases in supernumerary income.
 Note: **mu** estimates are subsistence levels of consumption for each good.
 Note: **Nu** estimates measure the effect of demographic variables on subsistence levels of consumption.

Normalized parameter

		Estimate	Std. err.	z	P> z	[95% conf. interval]	
beta	Good						
	5	.0937826	.0019264	48.68	0.000	.090007	.0975582

Note: **beta** estimates sum to 1.

Shares: w_dairy w_proteins w_fruitveg w_flours w_misc

Prices: p_dairy p_proteins p_fruitveg p_flours p_misc

Expenditure: expfd

Demographics: n_kids n_adults

The estimated parameter matrix **N** is shown in the last block of the coefficient table, and `demandsys` labels the coefficients using factor-variable notation. Yet you should not think of the terms as you would of interactions in a regression. Here the notation means something different. The first five rows of the output for **N** correspond to the demographic variable `n_kids` and how it affects the consumption of each good. The remaining five correspond to `n_adults`. For instance, all the coefficients for `n_adults` are positive, as we would expect, meaning that having more adults in the family increases consumption of all goods. The coefficient for `n_kids` on the consumption of Good 3, fruits and vegetables, is negative, meaning consumption decreases as the number of children increases.

Directly interpreting the coefficients on demographic variables is not difficult for the LES, but for more complicated models, it can be. One easy way to see the practical effect of that negative coefficient is to use the `margins` command to see how our predicted quantity for Good 3 changes as the number of children changes.

```
. margins, predict(quantities equation(#3)) at(n_kids=1 n_kids=2 n_kids=3)
Predictive margins                                Number of obs = 4,160
Model VCE: OIM
Expression: Predicted quantity of good 3, predict(quantities equation(#3))
1._at: n_kids = 1
2._at: n_kids = 2
3._at: n_kids = 3
```

	Delta-method		z	P> z	[95% conf. interval]	
	Margin	std. err.				
_at						
1	22.57831	.1734791	130.15	0.000	22.2383	22.91833
2	21.407	.2254155	94.97	0.000	20.96519	21.84881
3	20.23569	.323032	62.64	0.000	19.60256	20.86882

We asked `margins` to produce predictions of quantities of Good 3, fruits and vegetables, by specifying the `equation(#3)` option, and we asked it to do so when the number of children is equal to one, again when the number of children is equal to two, and finally when the number of children is equal to three. `margins` first sets `n_kids` equal to one for all 4,160 observations in the estimation sample and obtains the predicted quantities; the mean of those predictions is 22.6. When `n_kids` is equal to two for all observations, the mean is 21.4, and when `n_kids` is equal to three, the mean is 20.2.

Kids do not always eat their fruits and vegetables, but finding that the quantity of fruits and vegetables actually declines as the household grows is rather surprising. Perhaps we have omitted other relevant demographic variables from our model, biasing our estimates, or perhaps the model itself is simply too rigid.

Finally, we use estat elasticities to obtain uncompensated price elasticities. We will obtain two sets of results, one for households with fewer than three kids and one for households with three or more kids.

```
. estat elasticities if n_kids <= 2, uncompensated
```

Uncompensated (Marshallian) price elasticities		Number of obs = 3,742				
Price	Elasticity	Std. err.	z	P> z	[95% conf. interval]	
Good 1						
Good 1						
1	-.9537446	.0083361	-114.41	0.000	-.970083	-.9374061
2	-.00852	.0015032	-5.67	0.000	-.0114662	-.0055738
3	-.0083598	.0014808	-5.65	0.000	-.0112621	-.0054574
4	-.0062373	.0011228	-5.56	0.000	-.0084379	-.0040367
5	-.0071346	.0012983	-5.50	0.000	-.0096793	-.00459
Good 2						
Good 2						
1	-.0621117	.012096	-5.13	0.000	-.0858194	-.038404
2	-.9289446	.0085912	-108.13	0.000	-.945783	-.9121063
3	-.0499231	.0062304	-8.01	0.000	-.0621346	-.0377117
4	-.0390871	.0048967	-7.98	0.000	-.0486844	-.0294897
5	-.0445883	.0056323	-7.92	0.000	-.0556274	-.0335492
Good 3						
Good 3						
1	-.0503597	.0118715	-4.24	0.000	-.0736274	-.0270921
2	-.0410463	.0036302	-11.31	0.000	-.0481614	-.0339312
3	-.886701	.0100534	-88.20	0.000	-.9064052	-.8669968
4	-.030438	.0027264	-11.16	0.000	-.0357816	-.0250944
5	-.0348829	.0032109	-10.86	0.000	-.0411761	-.0285897
Good 4						
Good 4						
1	-.0428307	.0038014	-11.27	0.000	-.0502814	-.0353801
2	-.0373442	.0014421	-25.90	0.000	-.0401707	-.0345178
3	-.0357288	.0013809	-25.87	0.000	-.0384354	-.0330223
4	-.7107582	.0101357	-70.12	0.000	-.7306238	-.6908925
5	-.0306015	.0012718	-24.06	0.000	-.0330941	-.0281089
Good 5						
Good 5						
1	-.0347514	.0047892	-7.26	0.000	-.0441381	-.0253647
2	-.0296409	.0016063	-18.45	0.000	-.0327893	-.0264926
3	-.0285217	.0015907	-17.93	0.000	-.0316394	-.025404
4	-.0212131	.0011859	-17.89	0.000	-.0235375	-.0188887
5	-.7824229	.0109448	-71.49	0.000	-.8038744	-.7609714

. estat elasticities if n_kids >= 3, uncompensated

Uncompensated (Marshallian) price elasticities Number of obs = 418

Price	Elasticity	Std. err.	z	P> z	[95% conf. interval]	
Good 1						
Good						
1	-.9395485	.0146154	-64.28	0.000	-.9681941	-.9109029
2	-.0126359	.0032625	-3.87	0.000	-.0190303	-.0062416
3	-.0146746	.0042999	-3.41	0.001	-.0231005	-.0062486
4	-.0082076	.002108	-3.89	0.000	-.0123393	-.0040759
5	-.0093949	.0024408	-3.85	0.000	-.0141788	-.0046109
Good 2						
Good						
1	-.023357	.0100783	-2.32	0.020	-.04311	-.0036039
2	-.9674982	.0135281	-71.52	0.000	-.9940128	-.9409836
3	-.028745	.0152303	-1.89	0.059	-.0585959	.0011058
4	-.0152942	.0064352	-2.38	0.017	-.0279071	-.0026813
5	-.0178107	.007578	-2.35	0.019	-.0326633	-.0029581
Good 3						
Good						
1	-.0000379	.0055043	-0.01	0.995	-.0108261	.0107504
2	.0001398	.0056976	0.02	0.980	-.0110273	.0113068
3	-1.000682	.0201954	-49.55	0.000	-1.040264	-.9610998
4	-.0000842	.0035771	-0.02	0.981	-.0070951	.0069268
5	-.0000569	.0041395	-0.01	0.989	-.0081702	.0080565
Good 4						
Good						
1	-.0451601	.0025582	-17.65	0.000	-.0501741	-.0401461
2	-.0463532	.0024489	-18.93	0.000	-.051153	-.0415534
3	-.052944	.0045994	-11.51	0.000	-.0619587	-.0439293
4	-.6843304	.0131677	-51.97	0.000	-.7101385	-.6585222
5	-.0334225	.0018152	-18.41	0.000	-.0369803	-.0298648
Good 5						
Good						
1	-.0356486	.002797	-12.75	0.000	-.0411305	-.0301666
2	-.0365794	.0027339	-13.38	0.000	-.0419377	-.0312212
3	-.0401475	.0035616	-11.27	0.000	-.0471281	-.033167
4	-.0234098	.0017471	-13.40	0.000	-.0268341	-.0199856
5	-.7542306	.0156676	-48.14	0.000	-.7849385	-.7235226

For example, in the table for households with 3 or more children, the price elasticity for Good 5 with respect to Good 1 is -0.0356486 . Among these households, if the price of Good 5 (miscellaneous items) increases by 1%, then the quantity of Good 1 (dairy products) purchased will decrease by an average of 0.036%.

We summarize the own-price elasticities in table 1. We again focus on the demand for fruits and vegetables. Among households with two children or fewer, it is inelastic, while it is (barely) elastic for households with three or more children. Again, we have no explanation for why that would be the case, though we suspect the model is simply too rigid to accurately model the consumption patterns seen in our data.

Table 1: Own-price elasticities based on number of children

Good	# of Children	
	≤ 2	≥ 3
Dairy	-0.954	-0.940
Proteins	-0.939	-0.967
Fruits & vegetables	-0.887	-1.001
Flours	-0.711	-0.684
Miscellany	-0.782	-0.754

◀

Translog

Pollak and Wales (1992, sec. 3.1) discuss an entire class of demand systems that they describe as the transcendental logarithmic or “translog” family. We implement the version they call the “basic translog” function, which is probably the most well known, because it corresponds to the translog indirect utility function proposed by Christensen, Jorgenson, and Lau (1975). We also implement the version Pollak and Wales call the “generalized translog” function, which is a more flexible version of the basic translog that has been translated in a way analogous to how the LES is a shifted version of the Cobb–Douglas utility function.

Basic translog

Christensen, Jorgenson, and Lau (1975) proposed both direct and indirect utility functions with the “translog” functional form. For their direct utility function, they obtain expenditure shares that are a function of expenditure and quantities, which is not the focus of demandsys. We instead implement the demand system based on their translog indirect utility function. The starting point is the indirect utility function

$$\ln v(\mathbf{p}, m; \boldsymbol{\alpha}, \boldsymbol{\Gamma}) = - \sum_g \alpha_g \ln \frac{p_g}{m} - \frac{1}{2} \sum_g \sum_h \gamma_{gh} \ln \frac{p_g}{m} \ln \frac{p_h}{m} \tag{1}$$

Equation (1) is twice differentiable in prices, and based on Young’s (1909) theorem, we must have $\gamma_{gh} = \gamma_{hg}$ for all g and h . (Young’s theorem states that cross-partial derivatives are equal.)

Roy (1943) introduced what would be known as Roy’s identity, which equates a consumer’s Marshallian demand function for a good with a function of their indirect utility function. Applying Roy’s identity, we have the expenditure-share functions

$$w_g(\mathbf{p}, m; \boldsymbol{\alpha}, \boldsymbol{\Gamma}) = \frac{\alpha_g + \sum_h \gamma_{gh} \ln \frac{p_h}{m}}{1 + \sum_j \sum_h \gamma_{jh} \ln \frac{p_h}{m}}$$

where we have imposed the normalization $\sum_g \alpha_g = 1$. This ensures that the expenditure shares sum to 1, a property known as “additivity”.

As we remarked in the introduction, if one has relevant demographic variables, one should include them in the demand system, so we will not present an example of the translog model without demographic variables. Moreover, it is in fact easier to describe how we incorporate demographic variables into the translog system if we first describe the generalized translog model.

Generalized translog

Another way to perform translation and introduce subsistence or committed quantities into a demand system is to translate the indirect utility function, so that consumers' effective expenditures are their expenditures after the subsistence quantities of all goods are purchased. Pollak and Wales (1992, 56) do this to Christensen, Jorgenson, and Lau's (1975) indirect utility function to obtain

$$\begin{aligned} \ln v(\mathbf{p}, m; \boldsymbol{\alpha}, \boldsymbol{\Gamma}, \boldsymbol{\mu}) &= - \sum_g \alpha_g \ln \frac{p_g}{\bar{m}} - \frac{1}{2} \sum_g \sum_h \gamma_{gh} \ln \frac{p_g}{\bar{m}} \ln \frac{p_h}{\bar{m}} \\ \bar{m} &= m - \sum_h p_h \mu_h \quad \sum_h \alpha_h = 1 \quad \gamma_{gh} = \gamma_{hg} \end{aligned}$$

where again $\boldsymbol{\mu}$ is a $G \times 1$ vector of subsistence quantities with typical element μ_g . A tedious but straightforward application of Roy's identity gives the expenditure-share functions as

$$\begin{aligned} w_g(\mathbf{p}, m; \boldsymbol{\alpha}, \boldsymbol{\Gamma}, \boldsymbol{\mu}) &= \frac{p_g \mu_g}{m} + \frac{\bar{m}}{m} \left(\frac{\alpha_g + \sum_h \gamma_{gh} \ln \frac{p_h}{\bar{m}}}{1 + \sum_j \sum_h \gamma_{jh} \ln \frac{p_h}{\bar{m}}} \right) \\ \bar{m} &= m - \sum_h p_h \mu_h \quad \sum_h \alpha_h = 1 \quad \gamma_{hj} = \gamma_{jh} \end{aligned}$$

It is apparent that the sum of $w_g(\mathbf{p}, m; \boldsymbol{\alpha}, \boldsymbol{\Gamma}, \boldsymbol{\mu})$ across all g is equal to one, as must be true of a demand system. Again, suppose for each consumer that we have a $D \times 1$ vector of demographics \mathbf{d} and an associated $G \times D$ matrix \mathbf{N} , and again, let c_g denote the g th element of the vector $\mathbf{c} = \mathbf{N}\mathbf{d}$.

To apply both demographic characteristics and committed quantities, we can translate the translog indirect utility function, this time with the value $(\mu_g + c_g)$ replacing μ_g in the developments above. Thus, the generalized translog system with demographic variables has expenditure-share functions of the form

$$\begin{aligned} w_g(\mathbf{p}, m, \mathbf{d}; \boldsymbol{\alpha}, \boldsymbol{\Gamma}, \boldsymbol{\mu}, \mathbf{N}) &= \frac{p_g(\mu_g + \boldsymbol{\nu}_g \mathbf{d})}{m} + \frac{\bar{m}}{m} \left(\frac{\alpha_g + \sum_h \gamma_{gh} \ln \frac{p_h}{\bar{m}}}{1 + \sum_j \sum_h \gamma_{jh} \ln \frac{p_h}{\bar{m}}} \right) \\ \bar{m} &= m - \sum_h p_h(\mu_h + \boldsymbol{\nu}_h \mathbf{d}) \quad \sum_h \alpha_h = 1 \quad \gamma_{hj} = \gamma_{jh} \end{aligned}$$

The basic translog model results if we set $\boldsymbol{\mu} = \mathbf{0}$ and $\mathbf{N} = \mathbf{0}$, and the basic translog model with demographics results if we set just $\boldsymbol{\mu} = \mathbf{0}$. The generalized translog model results if we set just $\mathbf{N} = \mathbf{0}$. The generalized translog model is essentially a translog model with a constant term included among the demographic variables.

▷ Example 4

We first fit a basic translog model to our food consumption data, controlling for the number of children and adults in each household. We will also store these estimation results so we can use them later.

```
. use https://www.stata-press.com/data/r19/food_consumption
(Food consumption)

. demandsys translog w_dairy w_proteins w_fruitveg w_flours w_misc,
> prices(p_dairy p_proteins p_fruitveg p_flours p_misc)
> demographics(n_kids n_adults)
> labels("dairy proteins fruitveg flours misc")
> expenditure(expfd) elasticities(expenditure) nolog nocoefstable

Calculating NLS estimates ...
Calculating FGMLS estimates ...
FGMLS iteration 2 ...
FGMLS iteration 3 ...
FGMLS iteration 4 ...

Basic translog model                Number of obs      = 4,160
Expenditure variable: expfd         Number of goods    = 5
Demographic method: Translating     Number of demographics = 2

Centered R2 of model for
  dairy      = 0.0448
  proteins   = 0.1593
  fruitveg   = 0.0976
  flours     = 0.1881
  misc       = 0.1678

Calculating expenditure elasticities ...
```

Expenditure elasticities

Expenditure	Elasticity	Std. err.	z	P> z	[95% conf. interval]	
dairy	.90727	.0157948	57.44	0.000	.8763129	.9382272
proteins	1.0912	.0102476	106.48	0.000	1.071115	1.111285
fruitveg	.9629558	.0128444	74.97	0.000	.9377813	.9881303
flours	.8728076	.0152504	57.23	0.000	.8429174	.9026979
misc	1.000333	.0177894	56.23	0.000	.9654664	1.0352

Note: Elasticities are calculated at prices', demographic variables', and expenditure means.

```
. estimates store translog
```

We specified the `nocoefstable` option to suppress the coefficient table and the `elasticities(expenditure)` option to obtain expenditure elasticities. Finally, we used the `labels()` option to name our five goods. Had we not done that, then the output would simply have numbered the goods from one to five.

For comparison's sake, we also fit the equivalent generalized translog model. We note that despite the long commands, the following one differs from the previous by just one character:

```
. demandsys gtranslog w_dairy w_proteins w_fruitveg w_flours w_misc,
> prices(p_dairy p_proteins p_fruitveg p_flours p_misc)
> demographics(n_kids n_adults)
> labels("dairy proteins fruitveg flours misc")
> expenditure(expdf) elasticities(expenditure) nolog nocoeftable

Calculating NLS estimates ...
Calculating FGNLS estimates ...
FGNLS iteration 2 ...
FGNLS iteration 3 ...
FGNLS iteration 4 ...

Generalized translog model                Number of obs      = 4,160
Expenditure variable: expfd              Number of goods    = 5
Demographic method: Translating          Number of demographics = 2

Centered R2 of model for
  dairy      = 0.0478
  proteins   = 0.1615
  fruitveg   = 0.0988
  flours     = 0.1903
  misc       = 0.1732

Calculating expenditure elasticities ...
Expenditure elasticities
```

Expenditure	Elasticity	Std. err.	z	P> z	[95% conf. interval]	
dairy	.906851	.0174393	52.00	0.000	.8726706	.9410313
proteins	1.074059	.01144	93.89	0.000	1.051637	1.096481
fruitveg	.9716098	.0144251	67.36	0.000	.943337	.9998826
flours	.892461	.0170431	52.36	0.000	.8590571	.9258649
misc	1.023297	.0194942	52.49	0.000	.9850886	1.061504

Note: Elasticities are calculated at prices', demographic variables', and expenditure means.

```
. estimates store gtranslog
```

The R^2 statistics from the generalized translog model are trivially higher than for the basic translog model. Because `demandsys` is based on maximum likelihood estimation and because the translog model is nested within the generalized translog model, we can use a likelihood-ratio test to see whether the inclusion of committed quantities is warranted here.

```
. lrtest gtranslog translog

Likelihood-ratio test
Assumption: translog nested within gtranslog
LR chi2(5) = 59.00
Prob > chi2 = 0.0000
```

The generalized translog model adds additional G parameters over the basic translog model, so the χ^2 statistic has five degrees of freedom in this example. The result indicates that the use of the generalized version of the translog model is warranted. Of course, if you do this test for your own models, you must use the same demographic specification in both.

As we cautioned with the LES, interpreting the μ parameters of the generalized translog model as subsistence quantities requires care. We do not force the estimates to be positive, nor does `demandsys` check that $\sum_g p_{gi}(\mu_g + \nu'_g \mathbf{d}_i) \leq m_i$ for each observation i in the dataset or that $\mu_g + \nu'_g \mathbf{d}_i \geq 0$ for each observation and each good, both of which must be true if the subsistence argument is to be credible. We have noted that the generalized translog model can produce negative estimates of μ 's quite frequently and changing the demographic specification is often enough to produce negative estimates.

For example, if we refit the previous example controlling for just the number of children but not the number of adults, we obtain negative estimates for some of the μ parameters:

```
. demandsys gtranslog w_dairy w_proteins w_fruitveg w_flours w_misc,
> prices(p_dairy p_proteins p_fruitveg p_flours p_misc)
> demographics(n_kids) expenditure(expfd) nolog

Calculating NLS estimates ...
Calculating FG-NLS estimates ...
FG-NLS iteration 2 ...
FG-NLS iteration 3 ...
FG-NLS iteration 4 ...

Generalized translog model                Number of obs      = 4,160
Expenditure variable: expfd              Number of goods    = 5
Demographic method: Translating          Number of demographics = 1

Centered R2 of model for
  Good 1 = 0.0421
  Good 2 = 0.1600
  Good 3 = 0.0929
  Good 4 = 0.1825
  Good 5 = 0.1706
```

	Estimate	Std. err.	z	P> z	[95% conf. interval]	
alpha						
Good						
1	.1745891	.0097353	17.93	0.000	.1555082	.1936699
2	.3416641	.0188744	18.10	0.000	.304671	.3786572
3	.3634044	.0192262	18.90	0.000	.3257218	.401087
4	.0598082	.0066154	9.04	0.000	.0468423	.0727741
Gamma						
Good_g#						
Good_h						
1#1	.0232919	.002496	9.33	0.000	.0183998	.0281841
1#2	-.0146955	.0032932	-4.46	0.000	-.02115	-.0082411
1#3	.0005704	.0018905	0.30	0.763	-.0031349	.0042758
1#4	.001351	.0009723	1.39	0.165	-.0005546	.0032566
1#5	.0026193	.0010776	2.43	0.015	.0005073	.0047312
2#2	.120652	.0085591	14.10	0.000	.1038764	.1374275
2#3	-.0301533	.0055644	-5.42	0.000	-.0410593	-.0192474
2#4	-.0184411	.0027052	-6.82	0.000	-.0237432	-.013139
2#5	-.0228452	.0032034	-7.13	0.000	-.0291238	-.0165667
3#3	.0698867	.0067428	10.36	0.000	.056671	.0831023
3#4	.0006331	.0013394	0.47	0.636	-.0019921	.0032583
3#5	-.0065477	.0017672	-3.71	0.000	-.0100113	-.003084
4#4	.0233853	.0023855	9.80	0.000	.0187097	.0280608
4#5	-.0074747	.0011361	-6.58	0.000	-.0097014	-.005248
5#5	.0323539	.0030878	10.48	0.000	.0263019	.0384059

mu							
	Good						
	1	-.1560689	.0474664	-3.29	0.001	-.2491014	-.0630364
	2	-.811357	.2053694	-3.95	0.000	-1.213874	-.4088403
	3	-.7661874	.3576452	-2.14	0.032	-1.467159	-.0652156
	4	.2121302	.0367754	5.77	0.000	.1400517	.2842087
	5	.0887401	.0446207	1.99	0.047	.0012853	.176195
Nu							
	Good#						
	c.n_kids						
	1	-.5508244	.1957429	-2.81	0.005	-.9344734	-.1671754
	2	-1.350384	.1934558	-6.98	0.000	-1.72955	-.9712175
	3	-2.749218	.303765	-9.05	0.000	-3.344586	-2.153849
	4	.0446036	.0442943	1.01	0.314	-.0422117	.1314189
	5	-.103696	.0559887	-1.85	0.064	-.2134318	.0060399

Note: **alpha** estimates are constant expenditures for each good.

Note: **Gamma** estimates measure the effect of price on expenditures shares across goods.

Note: **mu** estimates are subsistence levels of consumption for each good.

Note: **Nu** estimates measure the effect of demographic variables on shares and subsistence levels of consumption.

Normalized parameter

		Estimate	Std. err.	z	P> z	[95% conf. interval]	
alpha	Good						
	5	.0605342	.007785	7.78	0.000	.045276	.0757925

Note: **alpha** estimates sum to 1.

Shares: w_dairy w_proteins w_fruitveg w_flours w_misc

Prices: p_dairy p_proteins p_fruitveg p_flours p_misc

Expenditure: expfd

Demographics: n_kids

Here, again, we see factor-variable notation for the parameter matrix **N** but also for Γ . As we mentioned before, you should not interpret the factor-variable notation as you would in a regression output. We see the estimate labeled 1#2 under Good_g#Good_h. This is the estimate of γ_{12} , the coefficient associated with $\ln p_2 / \bar{m}$ in the equation for Good 1.

AIDS

One of the most commonly used demand systems is the celebrated AIDS model of [Deaton and Muellbauer \(1980b\)](#). Among other benefits, the AIDS model can be viewed as a first-order approximation to any demand system. Historically, the AIDS model was also favored because it is “almost linear” in the sense that replacing the price index with an approximation results in share equations that are linear. Given the speed with which computers can fit nonlinear system of equations, we do not implement that approximation.

An additional nice property of the AIDS model is that the functional form for its budget shares is a member of the “price-independent generalized log-linear” and the broader “price-independent generalized linear” families. An advantage to price-independent generalized linear expenditure shares is that they satisfy certain conditions required for the existence of a representative consumer and the related

theory of aggregation. In short, expenditure shares of this form can be fit to data aggregated across consumers. The implied market demand curves are consistent with a single utility-maximizing representative agent and satisfy the same conditions like Slutsky symmetry that hold for demand curves implied by a single consumer's utility-maximizing behavior. For more on price-independent generalized linear demands, see, for example, [Muellbauer \(1975\)](#), [Lewbel \(1989\)](#), and [Pollak and Wales \(1992, chap. 2\)](#).

The starting point for the AIDS model is the expenditure function

$$\ln e(\mathbf{p}, u; \alpha_0, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\Gamma}) = (1 - \ln u) \ln a(\mathbf{p}) + \ln u \{ \ln a(\mathbf{p}) + b(\mathbf{p}) \} \quad (2)$$

where u is utility, the price index $\ln a(\mathbf{p})$ is defined as

$$\ln a(\mathbf{p}) \equiv \alpha_0 + \sum_g \alpha_g \ln p_g + \frac{1}{2} \sum_g \sum_h \gamma_{gh} \ln p_g \ln p_h$$

where p_g is the price of good g , and the price aggregator $b(\mathbf{p})$ is defined as

$$b(\mathbf{p}) \equiv \beta_0 \prod_g p_g^{\beta_g}$$

Parameter β_0 cannot be distinguished from u , so we take $\beta_0 = 1$ in the following. In their original formulation, [Deaton and Muellbauer \(1980b\)](#) wrote their expenditure function slightly differently, with u in place of $\ln u$ in (2). We have chosen to use $\ln u$ to draw some comparisons with the QUAIDS model later, and because utility is an ordinal concept, it will not affect the expenditure-share equations.

Suppose that all prices are equal to one. Then (2) with $\beta_0 = 1$ implies that $\ln e(p, u) = \alpha_0$. Thus, $\exp(\alpha_0)$ can be interpreted as the level of expenditure needed for minimal subsistence when all prices are equal to one. In practice, estimating α_0 is difficult, particularly when prices are correlated, as they often are. Therefore, by default, we set α_0 equal to the natural logarithm of the minimum level of expenditure in the estimation sample. You can override the default by specifying the `piconstant()` option. Of course, prices are generally not all equal to one, so you will want to try various values for α_0 to see how sensitive elasticities and other calculations you may perform are to its value.

Expenditure functions consistent with rational consumer choice must be homogeneous of degree one in prices and expenditure, which imply that we must have $\sum_g \alpha_g = 1$, $\sum_g \beta_g = 0$, and $\sum_g \gamma_{gh} = \sum_h \gamma_{gh} = 0$. Slutsky symmetry further requires that $\gamma_{gh} = \gamma_{hg}$ for all g and h . Note that `demandsys` imposes these restrictions on the model being fit. An alternative is to fit the model with and without one or more of those restrictions and then test whether they hold. However, such unrestricted models can often fail to converge because of the increased number of parameters or because the models are “too flexible” and contain unidentifiable parameters.

Shephard's (1970) lemma equates the Hicksian demand functions with the partial derivatives of the expenditure function with respect to the price of the goods. Using Shephard's lemma, we can write the expenditure-share functions as

$$w_g(\mathbf{p}, m; \alpha_0, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\Gamma}) = \alpha_g + \sum_h \gamma_{gh} \ln p_h + \beta_g \ln \left\{ \frac{m}{a(\mathbf{p})} \right\} \quad (3)$$

Notice the simplicity of these share equations. Good g 's expenditure share depends linearly on the log prices of all the goods as well as the log of an expenditure term. The expenditure term $m/a(\mathbf{p})$ essentially adjusts each household's expenditure to control the cost of living faced by that household. This term is sometimes referred to as deflated expenditure. Equation (3) also makes clear that luxury goods have $\beta_g > 1$ while necessities have $\beta_g < 1$; inferior goods have $\beta_g < 0$.

Setting $e(\mathbf{p}, u) = m$ and manipulating (2) yields the indirect utility function

$$\ln v(\mathbf{p}, m; \alpha_0, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\Gamma}) = \frac{\ln m - \ln a(\mathbf{p})}{b(\mathbf{p})} \quad (4)$$

We will come back to this equation below. We show how the indirect utility function plays a role in computing welfare measures in [R] **demandsys postestimation**.

QUAIDS

The Engel curves for the AIDS model have the so-called “Working–Leser” form after [Working \(1943\)](#) and [Leser \(1963\)](#),

$$w_g = \omega_g + \psi_g \ln m$$

so that the share of expenditures spent on good g is a linear function of the logarithm of total expenditures. [Banks, Blundell, and Lewbel \(1997\)](#) provided evidence that in fact linear Engel curves provide a poor fit for many goods. They therefore consider Engel curves with an additional expenditure term,

$$w_g = A_g(\mathbf{p}) + B_g(\mathbf{p}) \ln \left\{ \frac{m}{a(\mathbf{p})} \right\} + C_g(\mathbf{p}) f \left\{ \frac{m}{a(\mathbf{p})} \right\} \quad (5)$$

where $A_g(\mathbf{p})$, $B_g(\mathbf{p})$, and $C_g(\mathbf{p})$ are differentiable functions of prices and $f\{m/a(\mathbf{p})\}$ is a differentiable function of real expenditures. $a(\mathbf{p})$ is the same translog price index as in the AIDS model.

[Lewbel \(1991\)](#), building on the work of [Gorman \(1981\)](#), showed that any exactly aggregable demand system must have the form of (5) and that additional terms containing functions of $m/a(\mathbf{p})$ are not theoretically possible if exact aggregability is to hold. [Lewbel \(1991\)](#) termed demand systems of the form (5) as having rank three because there are three linearly independent terms; [Gorman \(1981\)](#) showed that the maximum rank of demand systems is three if exact aggregability is to hold. In contrast, the AIDS model has rank two.

[Banks, Blundell, and Lewbel \(1997\)](#) further showed that the only indirect utility functions consistent with rank-three expenditure-share equations like (5) have the form

$$\ln v(\mathbf{p}, m) = \left[\left\{ \frac{\ln m - \ln a(\mathbf{p})}{b(\mathbf{p})} \right\}^{-1} + \lambda(\mathbf{p}) \right]^{-1}$$

where in their application they use the same definitions for $a(\mathbf{p})$ and $b(\mathbf{p})$ as in the AIDS model and they take $\lambda(\mathbf{p}) = \sum_g \lambda_g \ln p_g$ with $\sum_g \lambda_g = 0$. As with the restrictions implied by economic theory for the AIDS model, **demandsys** imposes the restriction that the λ 's sum to zero. Also notice that if we restrict $\lambda_g = 0$ for all g , we are left with the indirect utility function for the AIDS model shown in (4).

Solving for $\ln m$ yields the cost function

$$\ln e(\mathbf{p}, u) = \frac{b(\mathbf{p})}{\frac{1}{\ln u} - \lambda(\mathbf{p})} + \ln a(\mathbf{p})$$

Applying Shephard’s lemma, we have the expenditure-share functions

$$w_g(\mathbf{p}, m; \boldsymbol{\alpha}, \boldsymbol{\Gamma}, \boldsymbol{\Lambda}) = \alpha_g + \sum_h \gamma_{gh} \ln p_h + \beta_g \ln \left\{ \frac{m}{a(\mathbf{p})} \right\} + \frac{\lambda_g}{b(\mathbf{p})} \left[\ln \left\{ \frac{m}{a(\mathbf{p})} \right\} \right]^2$$

This equation also makes clear our earlier claim that if $\lambda_g = 0$ for all g then the QUAIDS model reduces to the AIDS model; see (3).

Controlling for demographic factors

demandsys allows you to choose between two different methods of incorporating demographic characteristics into AIDS and QUAIDS. The first method is the same as we have used with demand systems we have already covered: Pollak and Wales’s (1978) demographic translation. An added benefit of translation is that it allows us to introduce constant terms that might be interpreted as subsistence or committed quantities. A possible downside to demographic translation is that the intuition behind how demographic variables affect expenditure shares may not strike all users as entirely lucid. Essentially, demographic variables alter the level of subsistence for each household or observation. However, as we cautioned above in our discussion of the generalized translog model, there are no restrictions to force $\sum_g p_{gi}(\mu_g + \nu'_g \mathbf{d}_i) \leq m_i$, nor are there any restrictions to force $(\mu_g + \nu'_g \mathbf{d}_i) \geq 0$.

The second method of incorporating demographics that demandsys implements is based on Ray (1983) and Poi (2002) and is known as demographic scaling. This method explicitly allows for demographic variables to have “scale” and “composition” effects on expenditures as we explain below. There are two possible downsides to demographic scaling. First, it does not allow one to incorporate subsistence quantities into the demand system. While one may be tempted to include a constant term in the list of demographic variables because that is essentially what demographic translation does to incorporate subsistence quantities, the resulting parameter estimates do not have such a clearcut interpretation. Second, in our experience, models that incorporate demographic scaling sometimes take many iterations to converge. Specifying fewer demographic characteristics may help achieve convergence. We think this method is more intuitive than demographic translation, but either method should prove adequate for most applications.

Demographic translation

We have already discussed demographic translation in the context of the LES and the translog and generalized translog demand systems, so we will not show the details here. In *Methods and formulas*, we show the formula for the expenditure shares for the generalized QUAIDS model, and we discuss the parameter restrictions that give rise to the generalized AIDS model and their nongeneralized variants.

demandsys provides for four models in the AIDS family that can incorporate demographic variables via demographic translation: `aids`, `quaid`s, `gaids`, and `gquaid`s. The first two models do not allow for committed quantities, while the latter two represent generalized variants that do. The arguments in Pollak and Wales (1992, 75) suggest that the generalized variants are to be preferred because doing so ensures that a constant term is included in each demand equation (compare with share equation). Whether the constants allow for a subsistence interpretation is another matter, and in [R] **demandsys postestimation**, we provide an example of how to see whether that interpretation holds.

Demographic scaling

demandsys also implements the demographic scaling method used by Ray (1983) for the AIDS model and extended by Poi (2002) to the QUAIDS model. For each consumer, we again have a D -length vector of demographic characteristics \mathbf{d} . Suppose \mathbf{d} is the number of children in a household and we are modeling expenditures on household goods, rent, utilities, and food. Then the household's expenditures will presumably increase by virtue of there being more members in the household; call this the scale effect. Second, the consumption pattern of the household may shift as more money is spent on items consumed by children versus adults; think of that as a composition effect. Ray (1983) does this by writing the expenditure function as

$$e(\mathbf{p}, \mathbf{d}, u) = e^R(\mathbf{p}, u) \bar{m}_0(\mathbf{d}) \phi(\mathbf{p}, \mathbf{d}, u)$$

Here $e^R(\mathbf{p}, u)$ is the expenditure function for a reference household, where $\mathbf{d} = \mathbf{0}$. $\bar{m}_0(\mathbf{d})$ increases total expenditure of the household and accounts for the scale effect. $\phi(\mathbf{p}, \mathbf{d}, u)$ controls for composition effects. For AIDS, Ray (1983) controlled for both of these effects by taking

$$\bar{m}_0(\mathbf{d}) = 1 + \boldsymbol{\rho}' \mathbf{d}$$

for $G \times 1$ parameter vector $\boldsymbol{\rho}$ and

$$\phi(\mathbf{p}, \mathbf{d}, u) = \exp \left\{ u \prod_i p_i^{\beta_g} \left(\prod_g p_g^{\boldsymbol{\eta}_g \mathbf{d}} - 1 \right) \right\}$$

where $\boldsymbol{\eta}_g$ is the g th row of $G \times D$ parameter matrix \mathbf{H} . In *Methods and formulas*, we show that the expenditure-share functions for the AIDS model with demographic scaling are

$$w_g(\mathbf{p}, m, \mathbf{d}; \alpha_0, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\Gamma}, \boldsymbol{\lambda}, \boldsymbol{\rho}, \mathbf{H}) = \alpha_g + \sum_h \gamma_{gh} \ln p_h + (\beta_g + \boldsymbol{\eta}_g \mathbf{d}) \ln \left\{ \frac{m}{\bar{m}_0(\mathbf{d}; \boldsymbol{\rho}) a(\mathbf{p})} \right\}$$

Basically, the function $\bar{m}_0(\mathbf{d}; \boldsymbol{\rho})$ acts to reduce the effective amount of money available for spending. If \mathbf{d} represents a set of indicator variables for regions of a country, then $\bar{m}_0(\mathbf{d}; \boldsymbol{\rho})$ is controlling for the cost of living in different regions of the country. The function $\phi(\mathbf{p}, \mathbf{d}, u)$ is to make each good's expenditure expansion path a function of the household's demographics. Rather than all households increasing their expenditure share of good g by the same β_g in response to a change in m , now their responses can vary by household composition.

For the QUAIDS model, we show in *Methods and formulas* that an appropriate choice for $\phi(\mathbf{p}, \mathbf{d}, u)$ leads to expenditure share equations

$$\begin{aligned} w_g(\mathbf{p}, m, \mathbf{d}; \alpha_0, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\Gamma}, \boldsymbol{\lambda}, \boldsymbol{\rho}, \mathbf{H}) = & \alpha_g + \sum_h \gamma_{gh} \ln p_h + (\beta_g + \boldsymbol{\eta}_g \mathbf{d}) \ln \left\{ \frac{m}{\bar{m}_0(\mathbf{d}; \boldsymbol{\rho}) a(\mathbf{p})} \right\} \\ & + \frac{\lambda_g}{b(\mathbf{p}) c(\mathbf{p}, \mathbf{d})} \left[\ln \left\{ \frac{m}{\bar{m}_0(\mathbf{d}; \boldsymbol{\rho}) a(\mathbf{p})} \right\} \right]^2 \end{aligned}$$

The models `aids` and `quaid`s allow for demographic scaling. To request demographic scaling, rather than demographic translation, we specify the `scaling` suboption of the `demographics()` option, as the next example shows.

► Example 5

Here we fit a QUAIDS model to our food consumption data, controlling for the numbers of children and adults in each household. We request that demographic scaling be used rather than the default demographic translation.

```
. use https://www.stata-press.com/data/r19/food_consumption
(Food consumption)

. demandsys quaid5 w_dairy w_proteins w_fruitveg w_flours w_misc,
> prices(p_dairy p_proteins p_fruitveg p_flours p_misc)
> demographics(n_kids n_adults, scaling)
> labels("dairy proteins fruitveg flours misc")
> expenditure(expfd) elasticities(expenditure) nolog nocoeftable

Calculating NLS estimates ...
Calculating FG-NLS estimates ...
FG-NLS iteration 2 ...
FG-NLS iteration 3 ...
FG-NLS iteration 4 ...

Quadratic AIDS model          Number of obs      = 4,160
Expenditure variable: expfd   Number of goods    = 5
                               Price index constant = 1.615
Demographic method: Scaling   Number of demographics = 2

Centered R2 of model for
  dairy      = 0.0453
  proteins   = 0.1530
  fruitveg   = 0.0949
  flours     = 0.1813
  misc       = 0.1726

Calculating expenditure elasticities ...
Expenditure elasticities
```

Expenditure	Elasticity	Std. err.	z	P> z	[95% conf. interval]	
dairy	.8918188	.0176252	50.60	0.000	.857274	.9263636
proteins	1.077283	.0117431	91.74	0.000	1.054267	1.100299
fruitveg	1.043209	.014428	72.30	0.000	1.01493	1.071487
flours	.8409016	.0162384	51.78	0.000	.8090749	.8727284
misc	.9460643	.0189172	50.01	0.000	.9089872	.9831413

```
Note: Elasticities are calculated at prices', demographic variables', and
      expenditure means.

. estimates store quaid5_s
```

Because the QUAIDS model with demographic scaling for a 5-good system with 2 demographic variables has 32 estimated parameters, we instructed `demandsys` to report expenditure elasticities rather than the estimated parameters. We draw your attention to this part of the command:

```
demographics(n_kids n_adults, scaling)
```

Notice that we specified the `scaling` suboption. Had we not included this suboption, then `demandsys` would have used demographic translation instead. Finally, we asked Stata to store these estimation results as `quaid5_s`.

The header of the output shows that `demandsys` fit a QUAIDS model as requested and that our demographic variables were incorporated using demographic scaling. Take note of the line that reads

```
Price index constant = 1.615
```

In our discussion of the basic AIDS model, we discussed the price index $a(\mathbf{p})$ that includes a constant term α_0 . We discussed that when all prices are equal to one, then $\exp(\alpha_0)$ can be interpreted as the level of expenditure needed for minimal subsistence. `demandsys` looked at our expenditure variable `expfd` and found that the minimum value within the estimation sample is \$5.03. It therefore set $\alpha_0 = \ln 5.03 = 1.615$. You can specify your own value for α_0 by using the `piconstant()` option, which stands for price index constant.

The QUAIDS model nests the AIDS model, so we can easily use a likelihood-ratio test to see whether the quadratic terms in the expenditure-share equations contribute to the explanatory power of the model. We include `quietly` to fit the corresponding `aids` model without displaying the results and store the estimation results:

```
. quietly demandsys aids w_dairy w_proteins w_fruitveg w_flours w_misc,
> prices(p_dairy p_proteins p_fruitveg p_flours p_misc)
> demographics(n_kids n_adults, scaling)
> labels("dairy proteins fruitveg flours misc") expenditure(expfd)
. estimates store aids_s
```

Because we specified `quietly`, we did not request expenditure elasticities. Doing so would have required Stata to do more computations that we would not have seen.

We now perform the likelihood-ratio test:

```
. lrtest quaid_s aids_s
Likelihood-ratio test
Assumption: aids_s nested within quaid_s
LR chi2(4) = 36.58
Prob > chi2 = 0.0000
```

In this example, we reject the null hypothesis that the four free λ parameters are jointly equal to zero. Hence, the use of the QUAIDS model instead of the linear AIDS model is justified here. (Recall that to satisfy the adding-up constraint, we impose $\sum_g \lambda_g = 0$ so that in our example the fifth parameter is determined by the first four and is not estimated.)

◀

When one conducts tests of one model versus another using the likelihood-ratio principle, the more restrictive model must be nested within the more general model. For example, each of the AIDS models implemented here is nested within the corresponding QUAIDS models, and the AIDS model with demographic translation is nested within the generalized AIDS model. But it would make no sense to fit a generalized AIDS model and compare it with a generalized translog model because one is not a restricted variant of the other. In *Methods and formulas*, we describe the most flexible variant within each class of demand system, and we describe the parameter restrictions that give rise to less flexible variants; using `lrtest` to choose among members within the same class is valid.

Also, for a likelihood-ratio test to be valid, the model specifications—apart from the component we are testing—must be identical. That is, the two models must include the same estimation sample, the same goods, and the same demographic specification, including the method by which demographic variables are introduced for AIDS and QUAIDS models. Moreover, for AIDS and QUAIDS models, you must use the same value for the price index constant α_0 . If you do not specify the `piconstant()` option, `demandsys` will use the same value, assuming you use the same estimation sample.

► Example 6

In this example, we will fit a generalized QUAIDS model, again controlling for the number of children and adults. Because we are fitting a generalized model, the demographics will be incorporated via demographic translation. To demonstrate the use of the `piconstant()` option, we will specify $\alpha_0 = 3$.

```
. use https://www.stata-press.com/data/r19/food_consumption
(Food consumption)

. demandsys gquaid5 w_dairy w_proteins w_fruitveg w_flours w_misc,
> prices(p_dairy p_proteins p_fruitveg p_flours p_misc)
> demographics(n_kids n_adults) piconstant(3) expenditure(expfd) nolog

Calculating NLS estimates ...
Calculating FGNLS estimates ...
FGNLS iteration 2 ...
FGNLS iteration 3 ...
FGNLS iteration 4 ...

Generalized quadratic AIDS model                    Number of obs      = 4,160
Expenditure variable: expfd                        Number of goods    = 5
                                                    Price index constant = 3
Demographic method: Translating                    Number of demographics = 2

Centered R2 of model for
  Good 1 = 0.0486
  Good 2 = 0.1596
  Good 3 = 0.0996
  Good 4 = 0.1893
  Good 5 = 0.1715
```

		Estimate	Std. err.	z	P> z	[95% conf. interval]	
alpha							
	Good						
	1	.1911178	.0084914	22.51	0.000	.1744749	.2077607
	2	.2916652	.0180614	16.15	0.000	.2562654	.3270649
	3	.3606389	.0168958	21.34	0.000	.3275237	.3937542
	4	.0716379	.0045291	15.82	0.000	.062761	.0805149
beta							
	Good						
	1	-.0123296	.0075589	-1.63	0.103	-.0271448	.0024856
	2	.0007156	.0191226	0.04	0.970	-.036764	.0381953
	3	-.0083998	.0137386	-0.61	0.541	-.0353269	.0185273
	4	.0118892	.0045299	2.62	0.009	.0030106	.0207677
Gamma							
	Good_g#						
	Good_h						
	1#1	.0381355	.0030825	12.37	0.000	.032094	.044177
	1#2	-.0326577	.0029964	-10.90	0.000	-.0385305	-.026785
	1#3	-.0106876	.0026841	-3.98	0.000	-.0159483	-.0054269
	1#4	.0021791	.0012889	1.69	0.091	-.000347	.0047052
	2#2	.1579944	.0060979	25.91	0.000	.1460428	.169946
	2#3	-.0694334	.0044839	-15.49	0.000	-.0782216	-.0606452
	2#4	-.024068	.0020371	-11.82	0.000	-.0280606	-.0200755
	3#3	.0923088	.005649	16.34	0.000	.0812371	.1033806
	3#4	-.0001376	.0017618	-0.08	0.938	-.0035908	.0033156
	4#4	.0313475	.0023698	13.23	0.000	.0267027	.0359922

lambda							
	Good						
	1	.0043566	.0028362	1.54	0.125	-.0012021	.0099154
	2	.0005706	.0059707	0.10	0.924	-.0111316	.0122729
	3	-.0068449	.004352	-1.57	0.116	-.0153747	.0016848
	4	-.0001594	.0017033	-0.09	0.925	-.0034979	.003179
mu							
	Good						
	1	1.06405	.2097966	5.07	0.000	.6528563	1.475244
	2	.6058156	.4529227	1.34	0.181	-.2818965	1.493528
	3	2.146931	.6621706	3.24	0.001	.8490999	3.444761
	4	.3204353	.0685528	4.67	0.000	.1860743	.4547964
	5	.3588502	.0854053	4.20	0.000	.191459	.5262415
Nu							
	Good#						
	c.n_kids						
	1	-.2399129	.1953618	-1.23	0.219	-.6228149	.1429892
	2	-1.176653	.1938486	-6.07	0.000	-1.556589	-.7967167
	3	-2.604984	.3121795	-8.34	0.000	-3.216845	-1.993123
	4	.1120979	.0416122	2.69	0.007	.0305396	.1936563
	5	-.0296108	.0541832	-0.55	0.585	-.1358079	.0765863
	Good#						
	c.n_adults						
	1	-1.271932	.2071674	-6.14	0.000	-1.677972	-.865891
	2	-1.278973	.2249348	-5.69	0.000	-1.719837	-.8381089
	3	-2.698807	.3809636	-7.08	0.000	-3.445482	-1.952132
	4	-.0562651	.0483113	-1.16	0.244	-.1509536	.0384234
	5	-.2693881	.0569314	-4.73	0.000	-.3809716	-.1578045

Note: **alpha** estimates are constant terms in expenditure-share equations and also appear in the price index.

Note: **beta** estimates measure sensitivity of expenditure shares to changes in deflated expenditure and also appear in the price aggregator function.

Note: **Gamma** estimates measure the effect of price on expenditures shares across goods.

Note: **lambda** estimates measure the sensitivity of expenditure shares to changes in deflated expenditure.

Note: **Nu** estimates measure the effect of demographic variables on expenditure shares of each good.

Normalized parameters

	Estimate	Std. err.	z	P> z	[95% conf. interval]	
alpha						
Good						
5	.0849402	.0049286	17.23	0.000	.0752803	.0946001
beta						
Good						
5	.0081246	.0050562	1.61	0.108	-.0017854	.0180345
Gamma						
Good_g#						
Good_h						
1#5	.0030308	.0014661	2.07	0.039	.0001572	.0059044
2#5	-.0318352	.0023155	-13.75	0.000	-.0363734	-.0272969
3#5	-.0120502	.0019986	-6.03	0.000	-.0159674	-.008133
4#5	-.0093209	.0011512	-8.10	0.000	-.0115772	-.0070646
lambda						
Good						
5	.0020771	.0019631	1.06	0.290	-.0017705	.0059247

Note: **alpha** estimates sum to 1.
 Note: **beta** estimates sum to 0.
 Note: **Gamma** estimates sum to 0 over goods.
 Note: **lambda** estimates sum to 0 over goods.

Shares: w_dairy w_proteins w_fruitveg w_flours w_misc
 Prices: p_dairy p_proteins p_fruitveg p_flours p_misc
 Expenditure: expfd
 Demographics: n_kids n_adults

We see in the header of the output that `demandsys` did set the price index constant $\alpha_0 = 3$ as per our request.

The μ parameter estimates are all greater than zero, but it is difficult to interpret them in isolation. Because we control for the number of children and the number of adults in each household, the μ parameters themselves would represent subsistence expenditure shares for a hypothetical household with neither any children nor any adults! In a somewhat technical [example](#) in [\[R\] demandsys postestimation](#), we continue this example and show how to recover the estimated μ and **N** parameters and then use those estimates to calculate the actual subsistence shares for each household. We then compare the calculated subsistence expenditures with actual expenditures to see whether the subsistence interpretation even holds.



Epilogue

`demandsys` provides easy access to some of the most commonly used demand systems and their extensions. You can control for demographic characteristics. Options and postestimation commands make obtaining elasticities trivial.

Despite what may appear to be a thorough implementation, we have only scratched the surface. `demandsys` is designed for large cross-sectional datasets. The Rotterdam model of [Theil \(1965\)](#) and [Barten \(1966\)](#) and its extensions are widely used in the analysis of time-series data.

A frequent concern is that datasets often have expenditure shares that are zero for some households. [Deaton and Irish \(1984\)](#) is an early contribution to the literature on how to deal with zero shares. [Heien and Wessells \(1990\)](#) proposed a two-step Heckman estimation procedure, but it has been shown to lead to inconsistent estimates ([Vermeulen 2001](#)). [Tauchmann \(2010\)](#) provides an alternative Heckman-type estimator that is consistent. [Shonkwiler and Yen \(1999\)](#) proposed an alternative estimator for zero shares. See also [Yen and Lin \(2006\)](#) for yet another approach and [Meyerhoefer, Ranney, and Sahn \(2005\)](#), who develop an estimator that controls for censoring with panel data. More recently, [Caro et al. \(2021\)](#) have developed the community-contributed Stata command `quaidisce`, which provides an implementation of Schonkwiler and Yen’s estimator.

All of our estimators treat price as an exogenous variable. There are multiple reasons why one may want to allow for endogenous prices. In the classical supply and demand setting, which may occur for example in small villages, there are a small number of buyers and sellers, so the decisions that buyers make will influence prices, causing them to be endogenous in expenditure-share equations. A more pertinent concern is that prices may be measured with error. Consumption is often collected by survey data, and consumers are often not able to recall the exact price paid for an item. Unobserved quality differences also cause prices to be measured with error, especially when the goods being modeled are aggregates of individual items. For example, you may purchase filet mignon every week, while I purchase economy-grade ground beef. Both items are lumped together as “meat” even though, arguably, the two products are not even close substitutes; [Nelson \(1991\)](#) considers this type of problem. Spatial patterns may also make prices endogenous; see [Case \(1991\)](#) as an example.

[Lecocq and Robin \(2015\)](#) provide a community-contributed command called `aidsills` that allows one to fit an AIDS model with endogenous variables. Their command, however, provides the AIDS and QUAIDS models with only one method of including demographic variables that is similar to the demographic translation approach used here.

We have presented the QUAIDS model as somewhat of an endpoint because it has rank 3, which [Gorman \(1981\)](#) and [Lewbel \(1991\)](#) showed is the maximum rank of an exactly aggregable demand system. [Lewbel and Pendakur \(2009\)](#) develop what they call a theory of “implicit Marshallian demands” that are not within the class of demand equations considered by Gorman and Lewbel and hence can have any rank. Lewbel and Pendakur propose a demand system they call the exact affine Stone index that allows for interactions between prices and expenditures and allows for much more flexible Engel curves. Their empirical application shows that observed demands often deviate from those implied by demands that are linear or quadratic in income. Moreover, their model can be fit using the generalized method of moments, so the possibility of controlling for price endogeneity exists.

Stored results

`demandsys` stores the following in `e()`:

Scalars

<code>e(N)</code>	number of observations
<code>e(k)</code>	number of parameters
<code>e(k_eq)</code>	number of equations in <code>e(b)</code>
<code>e(n_demos)</code>	number of demographic factors
<code>e(n_goods)</code>	number of goods
<code>e(mss_#)</code>	model sum of squares for the <code>#</code> th equation
<code>e(rss_#)</code>	residual sum of squares for the <code>#</code> th equation
<code>e(tss_#)</code>	total sum of squares for the <code>#</code> th equation
<code>e(r2_#)</code>	R^2 for the <code>#</code> th equation
<code>e(l1)</code>	Gaussian log likelihood
<code>e(N_clust)</code>	number of clusters

e(piconstant)	number of constant term
e(rank)	rank of $e(V)$
e(ic)	number of iterations
e(converged)	1 if converged, 0 otherwise

Macros

e(cmd)	demandsys
e(cmdline)	command as typed
e(wtype)	weight type
e(wexp)	weight expression
e(model)	demand model
e(model_eval)	demand evaluator
e(title)	title in estimation output
e(clustvar)	name of cluster variable
e(vce)	<i>vce</i> type specified in <code>vce()</code>
e(vctype)	title used to label Std. err.
e(demo_type)	demographic variable specification
e(demos)	demographic variables
e(has_demos)	1 if demographic variables are used, 0 otherwise
e(expenditures)	expenditures variable
e(prices)	price variables
e(shares)	expenditure-share variables
e(r2_type)	R^2 type computed
e(p_index)	whether model has price index
e(properties)	b V
e(estat_cmd)	program used to implement <code>estat</code>
e(predict)	program used to implement <code>predict</code>
e(marginsok)	predictions allowed by <code>margins</code>

Matrices

e(b)	coefficient vector
e(Sigma)	error covariance matrix ($\widehat{\Sigma}$)
e(V)	variance-covariance matrix of the estimators
e(b_normalized)	coefficient vector of the normalized parameters
e(V_normalized)	variance-covariance matrix of the normalized parameters

Functions

e(sample)	marks estimation sample
-----------	-------------------------

In addition to the above, the following is stored in `r()`:

Matrices

r(table)	matrix containing the coefficients with their standard errors, test statistics, p -values, and confidence intervals
----------	---

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

Methods and formulas

Methods and formulas are presented under the following headings:

- [Introduction](#)
- [LES](#)
- [Generalized translog](#)
- [QUAIDS with demographic translation](#)
- [QUAIDS with demographic scaling](#)
- [Estimation](#)

Introduction

As in the text in *Remarks and examples*, we continue to use subscript $i = 1, \dots, N$ to index observations, $g = 1, \dots, G$ to index goods, and $d = 1, \dots, D$ to index demographic variables. In formulas where we must use double summations across goods, we use $h = 1, \dots, G$ and $j = 1, \dots, G$ to index goods. We omit the observation subscript for most of this discussion; all variables and expressions without i subscripts implicitly refer to the i th observation. Only when we sum across observations do we make the i subscript explicit.

Let w_g be the expenditure share for good g defined as $w_g \equiv (p_g q_g)/m$, where p_g is the price of good g , q_g is the quantity of good g consumed, and m denotes total expenditure on all the goods in the system being modeled. We use \mathbf{w} , \mathbf{p} , and \mathbf{q} to represent the $G \times 1$ vectors of expenditure shares, prices, and quantities for observation i . Let \mathbf{d} represent the $D \times 1$ vector of demographic characteristics.

We use the notation w_g to denote the observed expenditure share for good g for observation i . We use the notation $w_g(\mathbf{p}, m; \boldsymbol{\theta})$ to represent an expenditure-share functions where we will replace generic parameter vector $\boldsymbol{\theta}$ with each model's parameter vectors and matrices. We adorn direct and indirect utility functions and expenditure functions analogously.

We use the notation E_h^g to denote the uncompensated (Marshallian) elasticity of the quantity of good g with respect to the price of good h . We use the notation E^g to note the expenditure elasticity of good g . Then, given the definition of w_g , one can easily verify the following facts that are useful in obtaining the formulas for the elasticities:

$$\begin{aligned}
 E^g &= \frac{1}{w_g} \frac{\partial w_g}{\partial \ln m} + 1 = \frac{m}{w_g} \frac{\partial w_g}{\partial m} + 1 \\
 E_g^g &= \frac{1}{w_g} \frac{\partial w_g}{\partial \ln p_g} - 1 = \frac{p_g}{w_g} \frac{\partial w_g}{\partial p_g} - 1 \\
 E_h^g &= \frac{1}{w_g} \frac{\partial w_g}{\partial \ln p_h} = \frac{p_h}{w_g} \frac{\partial w_g}{\partial p_h}
 \end{aligned}$$

The Slutsky equation can be written in elasticity form to obtain the compensated (Hicksian) elasticity of the quantity of good g with respect to the price of good h : $\tilde{E}_h^g = E^g \hat{w}_h + E_h^g$, where \hat{w}_h is the predicted expenditure share for good h . We discuss elasticities in more detail, including why we use \hat{w}_h rather than w_h , in [R] **demandsys postestimation**.

For each demand system, we provide the equations for the g th expenditure share, the direct utility function (if available), the indirect utility function, the expenditure function (if available), and the elasticities of the quantity of good g with respect to expenditure and the price of good h . Predicted values of the first four items are available via `predict`; the elasticities are available via `estat elasticities`. When one demand system is nested within a more general demand system, we provide the equations for only the more general demand system and note what parameter restrictions would result in the less general demand system. Moreover, we include demographic variables in our exposition; versions of models without demographic variables result when the corresponding parameter vectors and matrices are set to zero. We also note what parameter restrictions are imposed at estimation to ensure adding up, homogeneity, and Slutsky symmetry.

LES

Let $c_g = \mu_g + \nu_g \mathbf{d}$, where ν_g is the g th row of $G \times D$ parameter matrix \mathbf{N} . The LES begins with the utility function

$$u(\mathbf{q}, \mathbf{d}; \boldsymbol{\beta}, \boldsymbol{\mu}, \mathbf{N}) = \prod_g (q_g - c_g)^{\beta_g}$$

Utility maximization yields the expenditure-share equations

$$w_g(\mathbf{p}, m, \mathbf{d}; \boldsymbol{\beta}, \boldsymbol{\mu}, \mathbf{N}) = \frac{p_g c_g}{m} + \beta_g \left(1 - \sum_h \frac{p_h c_h}{m} \right)$$

To enforce adding up, we impose the constraint $\sum_g \beta_g = 1$. Slutsky symmetry and homogeneity are implied by the functional form of the LES model. Straightforward algebra shows the indirect utility function is

$$v(\mathbf{p}, m, \mathbf{d}; \boldsymbol{\beta}, \boldsymbol{\mu}, \mathbf{N}) = \frac{\prod_g \beta_g^{\beta_g}}{\prod_g p_g^{\beta_g}} (m - \sum_g p_g c_g)$$

and inverting provides the expenditure function

$$e(\mathbf{p}, u, \mathbf{d}; \boldsymbol{\beta}, \boldsymbol{\mu}, \mathbf{N}) = u \frac{\prod_g p_g^{\beta_g}}{\prod_g \beta_g^{\beta_g}} + \sum_g p_g c_g$$

The expenditure and uncompensated price elasticities are given by

$$E^g = \frac{\beta_g m}{p_g c_g + \beta_g (m - \sum_h p_h c_h)}$$

$$E_h^g = \begin{cases} \frac{p_g c_g (1 - \beta_g)}{p_g c_g + \beta_g (m - \sum_h p_h c_h)} - 1 & g = h \\ \frac{-\beta_g p_h c_h}{p_g c_g + \beta_g (m - \sum_h p_h c_h)} & g \neq h \end{cases}$$

The Cobb–Douglas demand system results if we set $\boldsymbol{\mu} = \mathbf{0}$. Models without demographics result if we set $\mathbf{N} = \mathbf{0}$.

Generalized translog

Continuing to use $c_g = \mu_g + \nu_g \mathbf{d}$, we can obtain the generalized translog expenditure-share equations with demographics by first obtaining the regular translog expenditure-share equations and then applying translation with both a constant term and demographic characteristics to those expenditure-share equations to obtain

$$w_g(\mathbf{p}, m, \mathbf{d}; \alpha, \Gamma, \mu, \mathbf{N}) = \frac{p_g c_g}{m} + \frac{\bar{m}}{m} \frac{\alpha_g + \sum_h \gamma_{gh} \ln\left(\frac{p_h}{\bar{m}}\right)}{1 + \sum_h \sum_j \gamma_{hj} \ln\left(\frac{p_j}{\bar{m}}\right)}$$

$$\bar{m} = m - \sum_j p_j c_j$$

The normalization $\sum_g \alpha_g = 1$ ensures the expenditure shares sum to 1, and Slutsky symmetry requires that $\gamma_{jh} = \gamma_{hj}$. Homogeneity is implied by the functional form of the expenditure-share equation.

Noting that the translated indirect utility function has the same form as the untranslated version except with \bar{m} replacing m , we have the indirect utility function

$$\ln v(\mathbf{p}, m, \mathbf{d}; \alpha, \Gamma, \mu, \mathbf{N}) = - \sum_g \alpha_g \ln\left(\frac{p_g}{\bar{m}}\right) - \frac{1}{2} \sum_g \sum_h \gamma_{gh} \ln\left(\frac{p_g}{\bar{m}}\right) \ln\left(\frac{p_h}{\bar{m}}\right)$$

A limitation of the generalized translog model is that we cannot solve the indirect utility function for a closed-form expression for the cost function, nor does a closed-form expression for the direct utility function exist. (The direct utility function described in [Christensen, Jorgenson, and Lau \[1975\]](#) is not the utility function that is implied by their indirect utility function.)

Demand systems with translated demographics and committed quantities tend to produce elasticity formulas that are rather long. Omitting function arguments for clarity, we begin by writing the expenditure-share equation for good g as

$$w_g = A_g + B_g \times \frac{N_g}{D_g}$$

where

$$A_g = \frac{p_g c_g}{m}$$

$$B_g = \frac{\bar{m}}{m} = 1 - \frac{\sum_h p_h c_h}{m}$$

$$N_g = \alpha_g + \sum_h \gamma_{gh} \ln p_h - \ln(m - \sum_h p_h c_h) \sum_h \gamma_{gh}$$

$$D_g = 1 + \sum_j \sum_h \gamma_{jh} \ln p_h - \ln(m - \sum_h p_h c_h) \sum_j \sum_h \gamma_{jh}$$

Applying the chain and quotient rules of elementary calculus, we have

$$\frac{\partial w_g}{\partial m} = \frac{\partial A_g}{\partial m} + B_g \left(\frac{D_g \frac{\partial N_g}{\partial m} - N_g \frac{\partial D_g}{\partial m}}{D_g^2} \right) + \frac{N_g}{D_g} \frac{\partial B_g}{\partial m}$$

where

$$\begin{aligned} \frac{\partial A_g}{\partial m} &= -\frac{p_g c_g}{m^2} & \frac{\partial B_g}{\partial m} &= \frac{\sum_h p_h c_h}{m^2} \\ \frac{\partial N_g}{\partial m} &= -\frac{1}{m - \sum_h p_h c_h} \sum_h \gamma_{gh} & \frac{\partial D_g}{\partial m} &= -\frac{1}{m - \sum_h p_h c_h} \sum_j \sum_h \gamma_{jh} \end{aligned}$$

Given $\partial w_g / \partial m$, the expenditure elasticity of good g is calculated as $E^g = (m / \widehat{w}_g) (\partial w_g / \partial m) + 1$. We also have

$$\frac{\partial w_g}{\partial p_h} = \frac{\partial A_g}{\partial p_h} + B_g \left(\frac{D_g \frac{\partial N_g}{\partial p_h} - N_g \frac{\partial D_g}{\partial p_h}}{D_g^2} \right) + \frac{N_g}{D_g} \frac{\partial B_g}{\partial p_h}$$

where

$$\begin{aligned} \frac{\partial A_g}{\partial p_h} &= \begin{cases} c_g / m & h = g \\ 0 & h \neq g \end{cases} \\ \frac{\partial B_g}{\partial p_h} &= -\frac{c_g}{m} \\ \frac{\partial N_g}{\partial p_h} &= \frac{\gamma_{gh}}{c_h} + \frac{c_h}{m - \sum_j p_j c_j} \sum_j \gamma_{gj} \\ \frac{\partial D_g}{\partial p_h} &= \frac{1}{p_h} \sum_j \gamma_{jh} + \frac{c_h}{m - \sum_j p_j c_j} \sum_{jk} \gamma_{jk} \end{aligned}$$

Given $\partial w_g / \partial p_h$, the uncompensated price elasticity of good g with respect to price h is calculated as $E_h^g = (p_h / w_g) (\partial w_g / \partial p_h) + \mathbf{I}_{h=g}$, where \mathbf{I}_x is the indicator function that takes on the value one if x is true and zero otherwise.

The basic translog results if we set $\boldsymbol{\mu} = \mathbf{0}$, and models without demographics have $\mathbf{N} = \mathbf{0}$.

QUAIDS with demographic translation

Translating Banks, Blundell, and Lewbel's (1997) indirect utility function, we have

$$\begin{aligned} \ln V(\mathbf{p}, m, \mathbf{d}; \alpha_0, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\Gamma}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \mathbf{N}) &= \left[\left\{ \frac{\ln \bar{m} - \ln a(\mathbf{p})}{b(\mathbf{p})} \right\}^{-1} + \lambda(\mathbf{p}) \right]^{-1} \\ \ln a(\mathbf{p}) &= \alpha_0 + \sum_g \alpha_g \ln p_g + \frac{1}{2} \sum_g \sum_h \gamma_{gh} \ln p_g \ln p_h \\ b(\mathbf{p}) &= \prod_g p_g^{\beta_g} \\ \lambda(\mathbf{p}) &= \sum_g \lambda_g \ln p_g \end{aligned}$$

where α_0 is the value specified in the `piconstant()` option. Solving for \bar{m} and recalling that $m = \bar{m} + \sum_g p_g c_g$ for $c_g = \mu_g + \nu_g d$, we have the expenditure function

$$e(\mathbf{p}, u, \mathbf{d}; \alpha_0, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\Gamma}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \mathbf{N}) = \exp \left\{ \frac{b(\mathbf{p}) \ln u}{1 - \lambda(\mathbf{p}) \ln u} + \ln a(\mathbf{p}) \right\} + \sum_g p_g c_g \quad (6)$$

Applying Shepherd's lemma yields the expenditure-share equations

$$\begin{aligned} w_g(\mathbf{p}, m, \mathbf{d}; \alpha_0, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\Gamma}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \mathbf{N}) &= \\ \frac{p_g c_g}{m} + \frac{\bar{m}}{m} \left(\alpha_g + \sum_h \gamma_{gh} \ln p_h + \beta_g \ln \left\{ \frac{\bar{m}}{a(\mathbf{p})} \right\} + \frac{\lambda_g}{b(\mathbf{p})} \left[\ln \left\{ \frac{\bar{m}}{a(\mathbf{p})} \right\} \right]^2 \right) \end{aligned} \quad (7)$$

To obtain the expenditure elasticities, we first write (7) as

$$w_g = R_g + S_g \times T_g \quad (8)$$

where

$$\begin{aligned} R_g &= \frac{p_g c_g}{m} & S_g &= \frac{\bar{m}}{m} \\ T_g &= \alpha_g + \sum_h \gamma_{gh} \ln p_h + \beta_g \ln \left\{ \frac{\bar{m}}{a(\mathbf{p})} \right\} + \frac{\lambda_g}{b(\mathbf{p})} \left[\ln \left\{ \frac{\bar{m}}{a(\mathbf{p})} \right\} \right]^2 \end{aligned}$$

Taking the derivative with respect to $\ln m$, we have

$$\frac{\partial w_g}{\partial \ln m} = \frac{\partial R_g}{\partial \ln m} + S_g \frac{\partial T_g}{\partial \ln m} + T_g \frac{\partial S_g}{\partial \ln m}$$

where

$$\begin{aligned} \frac{\partial R_g}{\partial \ln m} &= -\frac{p_g c_g}{m} & \frac{\partial S_g}{\partial \ln m} &= 1 - S_g \\ \frac{\partial T_g}{\partial \ln m} &= \frac{1}{S_g} \left[\beta_g + 2 \frac{\lambda_g}{b(\mathbf{p})} \ln \left\{ \frac{\bar{m}}{a(\mathbf{p})} \right\} \right] \end{aligned}$$

Then the expenditure elasticity for good g is given by $E^g = (1/w_g)(\partial w_g/\partial \ln m) + 1$.

To obtain the uncompensated price elasticity of good g with respect to price h , we again use (8). Taking the partial derivative with respect to $\ln p_h$, we have

$$\frac{\partial w_g}{\partial \ln p_h} = \frac{\partial R_g}{\partial \ln p_h} + S_g \frac{\partial T_g}{\partial \ln p_h} + T_g \frac{\partial S_g}{\partial \ln p_h}$$

The required partials are

$$\begin{aligned} \frac{\partial R_g}{\partial \ln p_h} &= \begin{cases} \frac{p_h c_h}{m} & h = g \\ 0 & h \neq g \end{cases} \\ \frac{\partial S_g}{\partial \ln p_h} &= -\frac{p_h c_h}{m} \\ \frac{\partial T_g}{\partial \ln p_h} &= \gamma_{gh} + \beta_g \frac{\partial}{\partial \ln p_h} \left[\ln \left\{ \frac{\bar{m}}{a(\mathbf{p})} \right\} \right] + \frac{\partial}{\partial \ln p_h} \left(\frac{\lambda_g}{b(\mathbf{p})} \left[\ln \left\{ \frac{\bar{m}}{a(\mathbf{p})} \right\} \right]^2 \right) \end{aligned}$$

where

$$\begin{aligned} \frac{\partial}{\partial \ln p_h} \left[\ln \left\{ \frac{\bar{m}}{a(\mathbf{p})} \right\} \right] &= - \left(\frac{p_h c_h}{m - \sum_j p_j c_j} + \alpha_h + \sum_j \gamma_{hj} \ln p_j \right) \\ \frac{\partial}{\partial \ln p_h} \left(\frac{\lambda_g}{b(\mathbf{p})} \left[\ln \left\{ \frac{\bar{m}}{a(\mathbf{p})} \right\} \right]^2 \right) &= 2 \frac{\lambda_g}{b(\mathbf{p})} \ln \left\{ \frac{\bar{m}}{a(\mathbf{p})} \right\} \frac{\partial}{\partial \ln p_h} \left[\ln \left\{ \frac{\bar{m}}{a(\mathbf{p})} \right\} \right] \\ &\quad - \frac{\lambda_g \beta_h}{b(\mathbf{p})} \left[\ln \left\{ \frac{\bar{m}}{a(\mathbf{p})} \right\} \right]^2 \end{aligned}$$

The uncompensated price elasticity of good g with respect to price h is $E_h^g = (p_h/w_g)(\partial w_g/\partial p_h) + \mathbf{I}_{h=g}$.

The AIDS model results if $\lambda = \mathbf{0}$. The nongeneralized variants result if $\mu = \mathbf{0}$. The variants without demographics result if $\mathbf{N} = \mathbf{0}$.

QUAIDS with demographic scaling

As can be gleaned from (6), the expenditure function for a QUAIDS model without any form of translation or scaling is

$$\begin{aligned}
 e(\mathbf{p}, u; \alpha_0, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\Gamma}, \boldsymbol{\lambda}) &= \exp \left\{ \frac{b(\mathbf{p}) \ln u}{1 - \lambda(\mathbf{p}) \ln u} + \ln a(\mathbf{p}) \right\} = \exp \left\{ \frac{b(\mathbf{p})}{\frac{1}{\ln u} - \lambda(\mathbf{p})} + \ln a(\mathbf{p}) \right\} \\
 a(\mathbf{p}) &= \alpha_0 + \sum_j \alpha_j \ln p_j + \frac{1}{2} \sum_j \sum_k \gamma_{jk} \ln p_j \ln p_k \\
 b(\mathbf{p}) &= \prod_j p_j^{\beta_j} \\
 \lambda(\mathbf{p}) &= \sum_j \lambda_j \ln p_j
 \end{aligned}$$

The AIDS case results when $\boldsymbol{\lambda} = \mathbf{0}$, and we obtain the expenditure function given by (2).

To implement Ray's (1983) demographic scaling as extended to the QUAIDS case in Poi (2002), we require a function that can be split into two parts, one part that depends on prices and utility and one part that does not:

$$m_0(\mathbf{p}, u, \mathbf{d}) = \bar{m}_0(\mathbf{d}) \phi(\mathbf{p}, \mathbf{d}, u)$$

For $\bar{m}_0(\mathbf{d})$, we use the same function as in Ray,

$$\bar{m}_0(\mathbf{d}; \boldsymbol{\rho}) = 1 + \boldsymbol{\rho}' \mathbf{d}$$

where $\boldsymbol{\rho}$ is a $D \times 1$ parameter vector. For $\phi(\mathbf{p}, \boldsymbol{\delta}, u)$, we follow Poi (2002) and use

$$\phi(\mathbf{p}, \mathbf{d}, u; \boldsymbol{\beta}, \boldsymbol{\lambda}, \mathbf{H}) = \exp \left\{ \frac{b(\mathbf{p}) \left(\prod_j p_j^{\boldsymbol{\eta}_j \mathbf{d}} - 1 \right)}{\frac{1}{\ln u} - \lambda(\mathbf{p})} \right\}$$

where $\boldsymbol{\eta}_j$ is the j th row of $G \times D$ parameter matrix \mathbf{H} . For AIDS when $\boldsymbol{\lambda} = \mathbf{0}$, we are left with the function used by Ray (1983). The expenditure function for the QUAIDS model with demographic scaling is then

$$e(\mathbf{p}, u, \mathbf{d}; \alpha_0, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\Gamma}, \boldsymbol{\lambda}, \boldsymbol{\rho}, \mathbf{H}) = e(\mathbf{p}, u; \alpha_0, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\Gamma}, \boldsymbol{\lambda}) \times \bar{m}_0(\mathbf{d}; \boldsymbol{\rho}) \times \phi(\mathbf{p}, \mathbf{d}, u; \boldsymbol{\beta}, \mathbf{H})$$

Applying Shepherd’s lemma, we have the share equations

$$w_g(\mathbf{p}, m, \mathbf{d}; \alpha_0, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\Gamma}, \boldsymbol{\lambda}, \boldsymbol{\rho}, \mathbf{H}) = \alpha_g + \sum_h \gamma_{gh} \ln p_h + (\beta_g + \boldsymbol{\eta}_g \mathbf{d}) \ln \left[\frac{m}{\bar{m}_0(\mathbf{d}; \boldsymbol{\rho}) a(\mathbf{p})} \right] + \frac{\lambda_g}{b(\mathbf{p}) c(\mathbf{p}, \mathbf{d})} \left[\ln \left\{ \frac{m}{\bar{m}_0(\mathbf{d}; \boldsymbol{\rho}) a(\mathbf{p})} \right\} \right]^2$$

where the demographic-adjusted price aggregator is defined as $c(\mathbf{p}, \mathbf{d}) \equiv \prod_j p_j^{\boldsymbol{\eta}_j \mathbf{d}}$. The indirect utility function is

$$\ln V(\mathbf{p}, m, \mathbf{d}; \alpha_0, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \boldsymbol{\rho}, \mathbf{H}) = \left[\frac{b(\mathbf{p}) c(\mathbf{p}, \mathbf{d})}{\ln \{m - \bar{m}_0(\mathbf{d}; \boldsymbol{\rho}) - a(\mathbf{p})\}} + \lambda(\mathbf{p}) \right]^{-1}$$

Because we are not dealing with demographic translation here, the partial derivatives required for elasticities are not as involved as they are for the QUAIDS model with demographic translation or the generalized translog model. We have

$$\frac{\partial w_g}{\partial \ln m} = \beta_g + \boldsymbol{\eta}_g \mathbf{d} + \frac{2\lambda_g}{b(\mathbf{p}) c(\mathbf{p}, \mathbf{d})} \ln \left\{ \frac{m}{(1 + \boldsymbol{\rho}' \mathbf{d}) a(\mathbf{p})} \right\}$$

and

$$\begin{aligned} \frac{\partial w_g}{\partial \ln p_h} &= \gamma_{gh} - \left[\beta_g + \boldsymbol{\eta}_g \mathbf{d} + \frac{2\lambda_g}{b(\mathbf{p}) c(\mathbf{p}, \mathbf{d})} \ln \left\{ \frac{m}{(1 + \boldsymbol{\rho}' \mathbf{d}) a(\mathbf{p})} \right\} \right] \times \left(\alpha_h + \sum_k \gamma_{hk} \ln p_k \right) \\ &+ \frac{(\beta_h + \boldsymbol{\eta}_h \mathbf{d}) \lambda_g}{b(\mathbf{p}) c(\mathbf{p}, \mathbf{d})} \left[\ln \left\{ \frac{m}{(1 + \boldsymbol{\rho}' \mathbf{d}) a(\mathbf{p})} \right\} \right]^2 \end{aligned}$$

The AIDS model with demographic scaling results if $\boldsymbol{\lambda} = \mathbf{0}$, and models without demographic scaling result if $\boldsymbol{\rho} = \mathbf{0}$ and $\mathbf{H} = \mathbf{0}$.

Estimation

The expenditure shares of a demand system represent a set of nonlinear seemingly unrelated regression (SUR) equations. Hence, estimation is performed using `nlsur` with just one complication. Our system of equations for observation i and generic parameter vector $\boldsymbol{\theta}$ can be written as

$$\begin{aligned} w_{1i} &= w_1(\mathbf{p}_i, m_i, \mathbf{d}_i; \boldsymbol{\theta}) + \epsilon_{1i} \\ w_{2i} &= w_2(\mathbf{p}_i, m_i, \mathbf{d}_i; \boldsymbol{\theta}) + \epsilon_{2i} \\ &\vdots \\ w_{Gi} &= w_G(\mathbf{p}_i, m_i, \mathbf{d}_i; \boldsymbol{\theta}) + \epsilon_{Gi} \end{aligned}$$

where $\boldsymbol{\epsilon}_i = (\epsilon_{1i}, \epsilon_{2i}, \dots, \epsilon_{Gi})$ is a vector of zero-mean disturbances. Because $\sum_g w_{gi} = 1$ and by the construction of our demand systems we also have $\sum_g w_g(\mathbf{p}_i, m_i, \mathbf{d}_i; \boldsymbol{\theta}) = 1$, it must be the case that $\sum_g \epsilon_{gi} = 0$. Therefore, $\det(\boldsymbol{\Sigma}) = \det\{E(\boldsymbol{\epsilon}\boldsymbol{\epsilon}')\} = \mathbf{0}$, and neither the (quasi)maximum likelihood estimator nor the feasible generalized nonlinear SUR estimator is defined.

Barten (1969) showed that to obtain a well-defined likelihood function, we can drop any one of the expenditure-share equations and fit the system containing the remaining $G - 1$ equations. Parameter restrictions that force the demand system’s equations to sum to 1 can then be used to recover the parameters of the dropped equation. `demandsys` drops the final equation from the demand system, but which equation is dropped should not concern you: Barten (1969) showed that you obtain the same likelihood function regardless of which one is dropped.

Moreover, the nonlinear SUR model satisfies the so-called Oberhofer–Kmenta (1974) conditions that allow us to cycle between estimating the parameters θ and the error covariance matrix Σ as `nlsur` does when we specify the `ifgnls` option. The upshot is that we fit our expenditure-share equations by calling `nlsur` with the `ifgnls` option. The resulting parameter estimate $\hat{\theta}$ is the (quasi)maximum likelihood estimate of the parameters of our demand system. The $(G - 1) \times (G - 1)$ estimated error covariance matrix is returned in matrix `e(Sigma)`.

Estimation weights, variance–covariance matrix options, and options to control the optimization process available with `demandsys` are simply passed to `nlsur`. See *Methods and formulas* in [R] `nlsur`.

When you type `predict ... , shares eq(#)`, you obtain the predicted shares for the `#`th equation, based on the expenditure-share equation for the demand system estimated. By construction, the sum of predicted shares for each observation is equal to one.

When you type `predict ... , residuals eq(#)`, you obtain the residuals for the `#`th equation defined as $r_{\#i} = w_{\#i} - \widehat{w}_{\#i}$, where $\widehat{w}_{\#i}$ is the predicted share for equation `#` for observation `i`.

When you type `predict ... , quantities eq(#)`, `predict` first computes the predicted shares $\widehat{w}_{\#i}$ and then computes $\widehat{q}_{\#i} = m_i \widehat{w}_{\#i} / p_{\#i}$.

When you type `predict ... , iuf`, `predict` evaluates the indirect utility function based on the formulas given above.

When you type `predict ... , ef utilities(u)`, `predict` evaluates the expenditure function based on the formulas given above at the levels of utility specified in variable `u`.

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Also see

- [R] **demandsys postestimation** — Postestimation tools for demandsys
- [R] **nlsur** — Estimation of nonlinear system of equations
- [U] **20 Estimation and postestimation commands**

Postestimation commands [predict](#) [margins](#) [estat](#) [Remarks and examples](#)
 References [Also see](#)

Postestimation commands

The following postestimation commands are of special interest after `demandsys`:

Command	Description
<code>estat elasticities</code>	price and expenditure elasticities
<code>estat parameters</code>	estimated parameter vectors and matrices

The following standard postestimation commands are also available:

Command	Description
<code>estat ic</code>	Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively)
<code>estat summarize</code>	summary statistics for the estimation sample
<code>estat vce</code>	variance–covariance matrix of the estimators (VCE)
<code>estimates</code>	cataloging estimation results
<code>lincom</code>	point estimates, standard errors, testing, and inference for linear combinations of parameters
<code>lrtest</code>	likelihood-ratio test
<code>margins</code>	marginal means, predictive margins, marginal effects, and average marginal effects
<code>marginsplot</code>	graph the results from <code>margins</code> (profile plots, interaction plots, etc.)
<code>nlcom</code>	point estimates, standard errors, testing, and inference for nonlinear combinations of parameters
<code>predict</code>	predicted shares, residuals, quantities, and functions
<code>predictnl</code>	point estimates, standard errors, testing, and inference for generalized predictions
<code>test</code>	Wald tests of simple and composite linear hypotheses
<code>testnl</code>	Wald tests of nonlinear hypotheses

predict

Description for predict

predict creates a new variable containing statistics such as predicted shares and residuals based on the demandsys estimation results currently active.

Menu for predict

Statistics > Postestimation

Syntax for predict

```
predict [type] newvar [if] [in] [ , statistic options ]
```

<i>statistic</i>	Description
Main	
<u>shares</u>	predicted shares; the default
<u>quantities</u>	predicted quantities
<u>residuals</u>	residuals
<u>iuf</u>	indirect utility function
<u>ef</u>	expenditure function

<i>options</i>	Description
Main	
<u>equation</u> (<i>eqno</i>)	display only specified goods; available only with shares, quantities, and residuals
<u>prices</u> (<i>varlist_p</i>)	specify set of counterfactual price variables
<u>expenditures</u> (<i>varname</i>)	specify a counterfactual total expenditure variable
<u>utilities</u> (<i>varname</i>)	specify the variable containing the utilities at which to compute the expenditure function; available only with and required with ef
<u>demographics</u> (<i>varlist</i>)	specify a set of counterfactual demographic variables

Options for predict

Main

shares, the default, calculates the predicted shares.

quantities calculates the predicted quantities given actual prices, actual expenditures, and predicted shares.

residuals calculates the residuals.

iuf calculates the indirect utility function $v(\mathbf{p}, m)$.

ef calculates the expenditure function $e(\mathbf{p}, u)$. You must also specify option utilities() with this option. This option is not available after translog and generalized translog models.

`equation(eqno)` specifies to which good you are referring. `equation(#1)` would mean the calculation is to be made for the first good, `equation(#2)` would mean the second, and so on. If you do not specify `equation()`, results are the same as if you specified `equation(#1)`.

`prices(varlistp)` specifies a set of counterfactual price variables at which to compute the requested statistic. You must specify G variables, where G is the number of goods in the demand system. By default, `predict` uses the same price variables that were used to fit the demand system. This option aids in the computation of welfare and other policy measures by allowing you to contrast predicted values at different price levels.

`expenditures(varname)` specifies a counterfactual total expenditure variable at which to compute the requested statistic. By default, `predict` uses the same expenditure variable that was used to fit the demand system. This option aids in the computation of welfare and other policy measures by allowing you to contrast predicted values at different expenditure levels.

`utilities(varname)` specifies the variable containing the utilities at which the expenditure function is to be computed. Almost surely, *varname* is a variable that was created by calling `predict ...`, `iuf ...`. This option aids in the computation of welfare and other policy measures by allowing you to contrast predicted values at different levels of utility.

`demographics(varlist)` specifies a set of counterfactual demographic variables at which to compute the requested statistic. If you specified D demographic variables at estimation, then you must specify precisely D variables here. By default, `predict` uses the same demographic variables (if any) that were used to fit the demand system.

margins

Description for margins

`margins` estimates margins for the predicted share of the first good.

Menu for margins

Statistics > Postestimation

Syntax for margins

```
margins [ , options ]
margins, predict(statistic ...) [ options ]
```

<i>statistic</i>	Description
<u>s</u> hares	predicted shares; the default
<u>q</u> uantities	predicted quantities
<u>r</u> esiduals	residuals
<u>i</u> uf	indirect utility function
* <u>e</u> f	expenditure function

*You must specify the `predict` option `utilities()` with statistic `ef`.

Statistics not allowed with `margins` are functions of stochastic quantities other than `e(b)`.

For the full syntax, see [\[R\] margins](#).

estat

Description for estat

`estat elasticities` computes expenditure and price elasticities after fitting a demand system using `demandsys`. Elasticities are calculated at the price and expenditure levels in the data. However, you may compute elasticities at the means of prices and expenditures or use other counterfactual prices or expenditures.

`estat parameters` returns the estimated parameter vectors and matrices based on the currently active `demandsys` estimation results. Results are stored in `r()` and vary depending on the demand model fitted. See [\[U\] 18.8 Accessing results calculated by other programs](#).

Menu for estat

Statistics > Postestimation

Syntax for estat

Compute price and expenditure elasticities

```
estat elasticities [if] [in], { expenditure | compensated | uncompensated }
    [[ atmeans | at(atspec) ] generate(stub) ]
```

Report estimated parameter vectors and matrices

```
estat parameters
```

Options for estat elasticities

Main

`expenditure`, `compensated`, and `uncompensated` indicate the type of elasticities to be reported. You must specify one of these.

`expenditure` requests that the G expenditure elasticities be calculated, where G is the number of goods in the system.

`compensated` requests that compensated price elasticities be calculated. There is a total of G^2 such elasticities because there are G goods, each of whose quantity consumed can be affected by any one of the G prices changing.

`uncompensated` requests that the G^2 uncompensated price elasticities be calculated.

`atmeans` requests that elasticities be calculated at the means of the price, expenditure, and demographic variables. By default, elasticities are computed for each observation and then averaged.

`at(atspec)` specifies the prices, expenditures, and demographic variables' values at which elasticities are to be calculated. If you do not specify this option, elasticities are calculated at the observed values unless you specify option `atmeans`, in which case they are calculated at the means of the variables. See [example 1](#) below. You may not specify option `at()` if you request compensated price elasticities.

`generate(stub)` requests that new variables be generated holding the observation-level elasticities. For expenditure elasticities, G new variables `stub_1`, `stub_2`, ..., `stub_G` will be created, where G is the number of goods. For price elasticities, G^2 new variables will be created of the form `stub_g_h` for $g = 1, \dots, G$ and $h = 1, \dots, G$. Variable `stub_g_h` will contain the percentage change in the quantity of good h with respect to the price of good g .

Remarks and examples

Remarks are presented under the following headings:

- [Introduction](#)
- [Elasticities](#)
- [Evaluating elasticities](#)
- [Compensating and equivalent variation](#)

Introduction

`estat elasticities` allows you to compute expenditure and price elasticities after you fit a demand model and is much more flexible than the options available with `demandsys` to display elasticities in lieu of coefficient estimates. With `estat elasticities`, you can calculate elasticities at the same prices, expenditures, and demographic variables' levels as was used at estimation time, or you can specify your own values. For example, you could calculate price elasticities for every observation in your dataset assuming each household had two children, even if the number of children varies by household. [Example 1](#) below shows you how that can be done.

`predict` allows you to obtain values of the indirect utility function and of the expenditure function underlying the demand system you fit. [Example 3](#) leverages that ability and shows you how to compute compensated and equivalent variation in response to a price change.

`estat parameters` allows you to obtain the estimated parameter vectors and matrices for the demand system based on the coefficient vector stored in `e(b)`. This may be useful if you wish to conduct further policy experiments or calculate your own welfare measures. For instance, in [example 2](#) below, we show how `estat parameters` with just a bit of coding can be used to check whether a fitted generalized translog model can be interpreted as having subsistence quantities for each good.

Elasticities

Although one can have `demandsys` report expenditure or price elasticities for the estimation sample, the `estat elasticities` command gives you much more flexibility. We illustrate features of `estat elasticities` in the next example. We often request expenditure elasticities to keep the output shorter, but all our commands would work regardless of the type of elasticity requested.

► Example 1

Before we can obtain elasticities, we first need to fit a demand system to our data. Here we include the `quietly` prefix command to omit the results of fitting a quadratic almost ideal demand system (QUAIDS) model with demographic translation, and we use the `labels()` option to provide short names for each of our goods. We type

```
. use https://www.stata-press.com/data/r19/food_consumption
(Food consumption)
. quietly demandsys quaid5 w_dairy w_proteins w_fruitveg w_flours w_misc,
> prices(p_dairy p_proteins p_fruitveg p_flours p_misc)
> expenditures(expdf)
> demographics(n_kids n_adults)
> labels(Dairy Meats FruitVeg Flours Misc)
```

Suppose we want the average expenditure elasticity among households with two children. We type

```
. estat elasticities if n_kids == 2, expenditure
```

Expenditure elasticities					Number of obs = 751	
Expenditure	Elasticity	Std. err.	z	P> z	[95% conf. interval]	
Dairy	.9028921	.0240869	37.48	0.000	.8556826	.9501016
Meats	1.087763	.0156852	69.35	0.000	1.05702	1.118505
FruitVeg	.9978958	.0215996	46.20	0.000	.9555614	1.04023
Flours	.8523383	.0220511	38.65	0.000	.8091189	.8955576
Misc	.9971274	.0265993	37.49	0.000	.9449938	1.049261

The header of the output reports that there are 751 households in the dataset with two children. Had we not specified the `labels()` option when we fit our model, `estat elasticities` would have simply numbered the goods from one through five.

What is displayed in the table of output is an average elasticity. Behind the scenes, `estat elasticities` calculated the five expenditure elasticities for each of the 751 households in this sample. What is reported as the expenditure elasticity for dairy products is the average over the 751 individual expenditure elasticities calculated. The column labeled `Std. err.` contains the standard error of the estimated mean. Looking to the right, we see the 95% confidence interval for the mean dairy elasticity is [0.856, 0.950]. More importantly, the column labeled `Std. err.` is not the sample standard deviation of the 751 individual elasticities, nor does the 95% confidence interval report something resembling the central 95% of the individual elasticities.

To save the individual elasticities for the 751 observations, you can use the `generate()` option like so:

```
. quietly estat elasticities if n_kids == 2, expenditure generate(ee)
. summarize ee*
```

Variable	Obs	Mean	Std. dev.	Min	Max
ee_1	751	.9028921	.0753773	.5095888	1.44884
ee_2	751	1.087763	.0664746	.8448506	1.525114
ee_3	751	.9978958	.1755427	.7566877	5.07766
ee_4	751	.8523383	.1053903	.3864381	1.110392
ee_5	751	.9971274	.082875	.3257591	1.296199

The means of these variables match the point estimates in the previous `estat elasticities` output. The standard deviations reported here measure the spread of the individual observations' elasticities rather than the standard errors of the means as in the previous output.

A second way to compute elasticities is to first compute the means of the price variables, the expenditure variable, and any demographics and then compute the elasticity for this hypothetical household. To have `estat elasticities` do that, you use the `atmeans` option. Here we obtain the expenditure elasticities at the means for the subset of households with two children:

```
. estat elasticities if n_kids == 2, expenditure atmeans
Expenditure elasticities                                Number of obs = 751
```

Expenditure	Elasticity	Std. err.	z	P> z	[95% conf. interval]	
Dairy	.9155493	.0232877	39.31	0.000	.8699063	.9611923
Meats	1.078329	.0149179	72.28	0.000	1.04909	1.107567
FruitVeg	.9804481	.0201581	48.64	0.000	.9409389	1.019957
Flours	.8695964	.0212882	40.85	0.000	.8278722	.9113206
Misc	1.004098	.0246939	40.66	0.000	.9556994	1.052498

Note: Elasticities are calculated at prices', demographic variables', and expenditure means.

With the `atmeans` option, we calculate the elasticities for a hypothetical household that may have, for example, 0.87 children and 1.93 adults. Without the `atmeans` option, we are calculating elasticities for households that do exist in the data, though taking the average of household elasticities poses its own conceptual issues.

Consider the following table:

Table 1. Average expenditure elasticities

Consumer	m	E^m	Q	m'	Q'
<i>A</i>	100	2	20	110	24
<i>B</i>	200	1	30	220	33
<i>C</i>	400	0.5	30	440	31.5
Sum	700		80	770	88.5
Mean		1.17			

Column m represents each consumer's initial total expenditure, E^m is the consumer's expenditure elasticity for the good in question, and Q is the quantity consumed. Column m' represents a 10% increase in each consumer's expenditure, and Q' represents the level of consumption based on the increases in total expenditure and the expenditure elasticities. The mean elasticity, analogous to what `estat elasticities` reports without the `atmeans` option, is 1.17. However, based on the aggregate data, the expenditure elasticity works out to 1.06.

So far in this example, we computed the expenditure elasticities for the subset of households with two children. We can also calculate the average elasticity for all households in our dataset under the counterfactual that all of them have two children. One way to do that is to type

```
. generate n_kids_backup = n_kids
. replace n_kids = 2
(3,409 real changes made)
```

```
. estat elasticities, expenditure
Expenditure elasticities                                     Number of obs = 4,160
```

Expenditure	Elasticity	Std. err.	z	P> z	[95% conf. interval]	
Dairy	.8464408	.0266003	31.82	0.000	.7943051	.8985764
Meats	1.20188	.0252998	47.51	0.000	1.152294	1.251467
FruitVeg	1.130297	.0323216	34.97	0.000	1.066948	1.193646
Flours	.7604338	.0229378	33.15	0.000	.7154766	.8053911
Misc	.9082414	.0287833	31.55	0.000	.8518271	.9646556

```
. replace n_kids = n_kids_backup
(3,409 real changes made)
```

Here we first created a backup variable holding the true values of `n_kids`, and then we set `n_kids` equal to two for all observations. After calling `estat elasticities`, we restored the true values.



□ Technical note

In principle, a second way to obtain the expenditure elasticities under the counterfactual that all households have two children would be to type

```
estat elasticities, expenditure at(n_kids=2)
```

The key is the `at()` option, which allows us to request that variables be set to the values we specify before calculating the elasticities. For more information about the `at()` option, see [R] [margins](#), especially *Syntax of at()*.

However, in this example, if you issue the previous command, you would obtain the error message

```
. estat elasticities, expenditure at(n_kids=2)
Option at() specified; using margins.
could not calculate numerical derivatives -- discontinuous region with missing
values encountered
r(459);
```

The issue is that for most demand systems the predicted quantities are nonlinear functions of prices. As the output indicates, when you specify the `at()` option, `estat elasticities` calls `margins` to obtain the desired results. Depending on the exact demand system and demographic specification, `margins` may not be able to obtain all the required derivatives numerically.

When you do not use the `at()` option, `estat elasticities` uses analytic formulas for the expenditure and price elasticities; *Methods and formulas* of [R] [demandsys](#) includes the elasticity formulas for all the demand systems implemented. Evaluating analytic formulas is quick, and most of the time is spent computing the variance–covariance matrix of the estimated mean elasticities, because that does use numerical derivatives. Because those derivatives are for a sample mean, rather than individual observations, you are unlikely to obtain the same message as `margins` issued in the previous command.

In contrast, `margins` uses numerical derivatives for all its calculations, and, moreover, it must incur the added expense of having to call `predict` to evaluate predicted quantities. One case where you would need to use `margins` is to obtain, say, a quantity elasticity with respect to a change in a demographic variable, because those elasticities are not programmed into `demandsys`.



In the text after [example 4](#) and in [example 6](#) of [R] [demandsys](#), we cautioned that after fitting a generalized model that allows for subsistence or committed quantities, you should check whether that interpretation actually holds. If for some household $\sum_g p_g(\mu_g + \nu'_g \mathbf{d}) > m$, then either the household

is not consuming enough to meet what we think is a minimum subsistence or our assumption that this quantity represents a minimum subsistence is wrong. Moreover, if $\mu_g + \nu'_g \mathbf{d} < 0$, then we are saying that the minimum required level of g is less than 0, which implies that good g is in fact not needed for subsistence.

▶ Example 2

This example is a continuation of [example 6](#) in [R] [demandsys](#). In that example, we fit a generalized QUAIDS model, which we refit here:

```
. use https://www.stata-press.com/data/r19/food_consumption, clear
(Food consumption)
. demandsys gquaid5 w_dairy w_proteins w_fruitveg w_flours w_misc,
> prices(p_dairy p_proteins p_fruitveg p_flours p_misc)
> demographics(n_kids n_adults) piconstant(3) expenditure(expfid) nolog
(output omitted)
```

Next, we compute variables containing each household's implied minimum quantity of each good conditional on that household's demographics. Given those variables, we can then calculate the expenditure needed to acquire the minimum quantities given the prices each household faces. We type

```
. estat parameters
. return list
macros:
      r(cmdline) : "estat parameters"
      r(names)   : "alpha beta Gamma lambda mu Nu"
matrices:
      r(Nu)      : 5 x 2
      r(mu)      : 5 x 1
      r(lambda)  : 5 x 1
      r(Gamma)   : 5 x 5
      r(beta)    : 5 x 1
      r(alpha)   : 5 x 1
. matrix mu = r(mu)
. matrix Nu = r(Nu)
. forvalues g = 1/5 {
2.     generate minq_`g' = mu[ `g', 1] + Nu[ `g', 1]*n_kids + Nu[ `g', 2]*n_adults
3. }
. generate sub_exp = p_dairy*minq_1 + p_proteins*minq_2
> + p_fruitveg*minq_3 + p_flour*minq_4 + p_misc*minq_5
```

We first issued the command `estat parameters` to obtain the estimated parameter matrices from our demand system. The following command shows that six vectors and matrices were stored, following the layout of the coefficient table from `demandsys`. To compute minimum quantities, we need matrices μ and \mathbf{N} , so we transferred those `r()` results to Stata matrices.

Because we specified `n_kids` first in the `demographics()` option of `demandsys`, that variable's coefficients are stored in the first column of our matrix `r(Nu)`, which we converted to `Nu`. The second column corresponds to coefficients associated with `n_adults`. We looped over the five goods in our system and created variables named `minq_g` containing the minimum quantities. The final `generate` statement calculates the minimum expenditure for each household given the prices it faced and our calculated minimum quantities.

Now we can look at the results.

```
. summarize minq_*
```

Variable	Obs	Mean	Std. dev.	Min	Max
minq_1	4,160	-1.604524	.9955552	-7.287278	-.2078816
minq_2	4,160	-2.893994	1.743902	-13.71866	-.6731574
minq_3	4,160	-5.344705	3.802853	-29.30052	-.5518766
minq_4	4,160	.3094948	.1354879	-.0171551	1.328884
minq_5	4,160	-.1878337	.2030023	-1.346311	.0894622

For the first three goods in our system, the calculated minimum quantity is negative for every single household in our sample. That is hardly encouraging if we wish to take the subsistence interpretation seriously, because it implies three of the five categories of food we have are not required for subsistence. Moreover, on average, good 5 is not required either. Even for good 4, for some households, the minimum quantity is negative. Finally, we look at the expenditure needed to purchase the minimum quantities of the five goods:

```
. summarize sub_exp
```

Variable	Obs	Mean	Std. dev.	Min	Max
sub_exp	4,160	-8.101177	5.480523	-43.8327	.2678587

Not surprisingly given the previous summary statistics, the minimum required expenditure for most households is less than zero.

◀

What are we to make of the results in the previous example? We think there are at least four explanations:

1. There are minimum quantities of each good consumers must purchase for subsistence, but our demand system is misspecified. If you believe this to be true, you might try changing the demographic variables for which you control. Alternatively, try a different model; for example, use a generalized QUAIDS model rather than a generalized translog or AIDS model.
2. The data are simply not compatible with a minimum subsistence interpretation. That does not imply the results of the fitted demand system must be ignored. As [Pollak and Wales \(1992, 75\)](#) write, any demand system that includes demographic translation should include a constant term in the demand equations—which is the key characteristic of the models we refer to as “generalized”. For example, rather than using the translog model with demographic translation, you should instead use the generalized translog model with demographic translation. In other words, the μ parameters do belong in the model, but they should not be construed as minimum quantities required for subsistence.
3. Items for which the calculated minimum quantities are positive for all observations must be purchased to achieve subsistence, but items for which the minimum quantities are negative need not be consumed for subsistence. Under this interpretation, in our example, households must consume breads, cereals, and pastas but need not consume any other foods for minimal subsistence. That is not particularly palatable to us.

- The procedure we used is useful as a diagnostic tool, but it is not a formal statistical test. Our estimated parameters are subject to sampling variation, and therefore our calculated minimum quantities are also subject to sampling variation. We have not accounted for this. If we were to conduct a test of the hypothesis that all minimum quantities are zero or more versus the alternative that some are negative, we may not be able to reject the null hypothesis.

Evaluating elasticities

The elasticities that are reported with `estat elasticities` refer to the slope of the fitted demand curve. Let us explain what we mean. Look at [figure 1](#). We have plotted a fitted demand curve D^F as a straight line for simplicity, and the large dots represent observed (price, quantity) pairs for two observations. The dotted lines represent each observation’s residual, and the plus signs represent the predicted quantities given the two observations’ prices. While quantity is a function of price here, following the influential English economist Alfred Marshall, we plot price on the ordinate; hence, the lines connecting observations to the regression line are horizontal, not vertical.

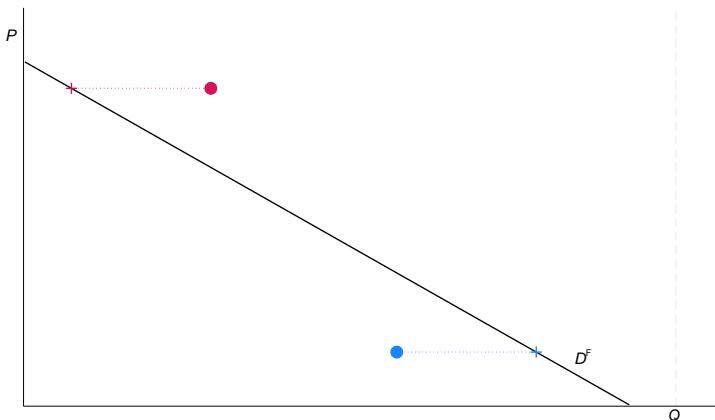


Figure 1. Actual and predicted quantities

For the AIDS model without demographic variables included, we can show that the expenditure elasticity for the g th good is given by

$$E^g = 1 + \frac{\beta_g}{w_g}$$

We do not observe the true value β_g , so to compute the elasticity for household i , we have no choice but to use the estimated $\hat{\beta}_g$. However, we observe both household i ’s true expenditure share w_{gi} , and we can compute its predicted expenditure share \widehat{w}_{gi} .

In terms of [figure 1](#), the issue is whether we evaluate the expenditure or any other elasticity formula at the red and blue dots or at their corresponding fitted values shown by the pluses on the demand curve. We believe the correct procedure is to evaluate elasticity formulas at their fitted values. That is, `estat elasticities` evaluates the expenditure share elasticity for household i as

$$\widehat{E}_i^g = 1 + \frac{\widehat{\beta}_g}{\widehat{w}_{gi}}$$

After all, for policy or other analyses, what we want to know is essentially the slope of the fitted demand curve. We need to be able to predict the effect of a policy on a household, and in general to do that, we predict the household's response with the policy imposed and compare it with the predicted response in the absence of the policy. In other words, our elasticity computations measure the slope of the fitted demand curve, not the slope of an imaginary line that passes through one of our dots that may or may not be parallel to the fitted demand curve.

In contrast, the postestimation tools accompanying the community-contributed `quaid` command of [Poi \(2012\)](#) use the actual values w_g to calculate elasticities. Thus, users of that command will see minor differences when refitting models using `demandsys` and `estat elasticities`. We believe, however, that evaluating the formulas at the predicted expenditure shares, or equivalently, evaluating the elasticity formulas along the demand curve rather than at points in space that need not lie on the demand curve, is conceptually more sound. Moreover, the elasticity results reported after `quaid` cannot be replicated using the `margins` command, but the results reported by `estat elasticities` are identical to those one would obtain using `margins`.

Compensating and equivalent variation

Although students in introductory economics courses learn of consumer's surplus as a way to measure whether consumers are better or worse off in response to a policy such as a tax, it has limitations. Perhaps most importantly, consumer's surplus is not well defined in most cases; see [Auerbach \(1985\)](#). Also see [Harberger \(1964\)](#), who develops a method for calculating consumer welfare when multiple prices change but assumes the timing of the price changes is scattered rather than occurring simultaneously. [Hines \(1999\)](#) provides a historical overview of consumer's welfare measurement.

The workhorse metrics used to evaluate a policy are compensating and equivalent variation (CV and EV, respectively). These metrics assign a monetary amount to a household's change in welfare and can accommodate policies that cause multiple prices to change simultaneously. Because `predict` after `demandsys` allows us to obtain the predicted values of the expenditure and indirect utility functions, these metrics are easy to obtain.

CV measures how much a social planner would have to compensate a consumer to offset the effect of, say, a tax that causes prices to rise. Consider figure 2, which illustrates CV for two goods and would be familiar to those who have studied intermediate microeconomics.

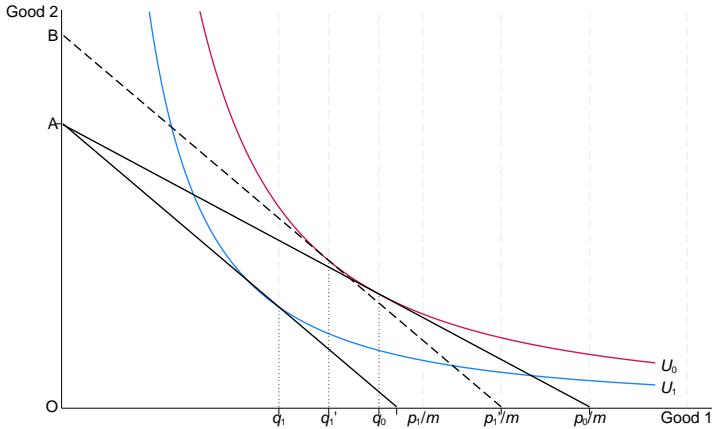


Figure 2. CV if the price of good 1 rises

Initially, the price of good 1 is p_0 , and the line segment $\overline{A \cdot p_0/m}$ represents the budget constraint. The consumer purchases q_0 units of good 1 and enjoys utility level U_0 . Now suppose the price of good 1 rises to p_1 : consumption of good 1 falls to q_1 and utility falls to U_1 ; line segment $\overline{A \cdot p_1/m}$ represents the new budget constraint.

The dotted line segment $\overline{B \cdot p_1'/m}$ represents a budget constraint based on price p_1 but shifted outward so that the consumer can still achieve utility level U_0 when facing price p_1 , by consuming q_1' . The line segment $\overline{B \cdot A}$ represents the CV. It tells us, in the face of a higher price for good 1, how much the social planner would have to compensate this consumer so that he or she can enjoy the same level of utility as before the price increase. In this case, CV is negative because the social planner must pay the consumer.

Notice the close similarity to the concept of the income and substitution effects that cause demand curves to slope downward. CV, and EV as we will see shortly, is a measure of the income effect.

Mathematically, CV is defined as

$$\begin{aligned} CV &= e(\mathbf{p}^1, u^1) - e(\mathbf{p}^1, u^0) \\ &= e(\mathbf{p}^0, u^0) - e(\mathbf{p}^1, u^0) \\ &= m - e(\mathbf{p}^1, u^0) \end{aligned}$$

where \mathbf{p}^0 and \mathbf{p}^1 are the price vectors before and after the policy is implemented and u^0 is the household's initial level of utility. Where do we get that level of utility, considering that utility is unobserved? The answer is the indirect utility function, which expresses utility as a function of observed prices and expenditure; we can evaluate the indirect utility function by calling `predict` with the `iuf` option. With the indirect utility variable in hand, we then call `predict` with the `ef` option and set the price variables to \mathbf{p}^1 .

▷ Example 3

We compute household CV in response to a 10% increase in the price of dairy products, holding all other prices constant. See Poi (2002) for similar computations for an expanded look at the welfare implications of lower dairy and other prices. First, we fit a generalized QUAIDS model controlling for the number of children and adults in each household. We type

```
. use https://www.stata-press.com/data/r19/food_consumption, clear
(Food consumption)
. quietly demandsys gquaid5 w_dairy w_proteins w_fruitveg w_flours w_misc,
> prices(p_dairy p_proteins p_fruitveg p_flours p_misc)
> expenditures(expfd) demographics(n_kids n_adults) nolog
. estimates store gquaid5
```

We stored our estimation results for later use by specifying the estimates store command. Next, we obtain each household’s initial utility u^0 by evaluating the indirect utility function:

```
. predict u0, iuf
```

Now, we create a new dairy price variable containing the price each household will face after the imposition of the tax. Given that price variable and initial utility u^0 , we can calculate each household’s new level of expenditure. Finally, we compute the CV. We type

```
. generate ptax_dairy = p_dairy * 1.10
. predict eplu0, ef utilities(u0)
> prices(ptax_dairy p_proteins p_fruitveg p_flours p_misc)
. generate cv = expfd - eplu0
```

Here we used the prices() option of predict to specify the new prices \mathbf{p}^1 at which we want the expenditure function. Had we not specified that option, predict would have used the prices used at estimation, \mathbf{p}^0 . Because we wanted to evaluate the function at the level of expenditures m , the same as used at estimation, we did not need to specify the expenditures() option of predict.

We summarize our results by looking at the average CV by household size:

```
. mean cv, over(n_kids)
```

Mean estimation Number of obs = 4,160

	Mean	Std. err.	[95% conf. interval]	
c.cv@n_kids				
0	-.5142129	.0066562	-.5272627	-.5011632
1	-.7762653	.013486	-.8027049	-.7498256
2	-.906099	.014986	-.9354796	-.8767183
3	-1.067159	.0268302	-1.11976	-1.014557
4	-1.222379	.0506473	-1.321675	-1.123084
5	-1.550713	.1581971	-1.860864	-1.240562
6	-1.304842	.2223386	-1.740745	-.8689394
7	-1.902751	.1828133	-2.261162	-1.544339
8	-2.923615	.	.	.
10	-1.743889	.	.	.

The standard errors of the means are missing for households with 8 and 10 children because only one household of each size exists in our data. For the most part, CV becomes more negative with the number of children, as we expected, because larger households are likely to consume more dairy products by virtue of there being more people within the household. To reiterate, CV is negative here, indicating that the social planner must transfer money to the consumer. For a price decrease, CV would be positive. ◀

EV is similar to CV, except that we use the postpolicy level of utility and the initial price vector rather than the initial level of utility and postpolicy price vector. We have

$$\begin{aligned} \text{EV} &= e(\mathbf{p}^0, u^1) - e(\mathbf{p}^0, u^0) \\ &= e(\mathbf{p}^0, u^1) - e(\mathbf{p}^1, u^1) \\ &= e(\mathbf{p}^0, u^1) - m \end{aligned}$$

To aid in understanding EV, you may wish to sketch a diagram similar to [figure 2](#). EV tells us how much consumers would be willing to pay to trade at the original price vector rather than the postpolicy one. In other words, EV tells us the change in expenditure that would yield u^1 if total expenditure were to change rather than the price vector. A negative value indicates that consumers are worse off.

▷ Example 4

Continuing the previous example, to calculate EV, we first need to calculate the new level of utility u^1 each household will enjoy based on the postpolicy dairy price. Because the preferences underlying all the demand systems implemented by `demandsys` are locally nonsatiated, each household will exhaust its full budget. Thus, the level of expenditure we supply as an argument to the indirect utility function is simply m , the value we used during estimation. To get u^1 , we can therefore call `predict, iuf` at postpolicy prices.

```
. estimates restore gquaid5
(results gquaid5 are active now)
. predict u1, iuf prices(ptax_dairy p_proteins p_fruitveg p_flours p_misc)
```

We had to restore our estimation results because the `means` command we used previously overwrites results in `e()`.

Now we evaluate the expenditure function at original prices and our computed level of utility u^1 . Because the original prices were used at estimation, we do not need to supply alternative prices via the `prices()` option. We type

```
. predict ep0u1, ef utilities(u1)
```

We are now ready to compute EV and summarize the results:

```
. generate ev = ep0u1 - expfd
. mean ev, over(n_kids)
Mean estimation                               Number of obs = 4,160
```

	Mean	Std. err.	[95% conf. interval]	
c.ev@n_kids				
0	-.5073723	.0065622	-.5202378	-.4945068
1	-.7660212	.013292	-.7920806	-.7399619
2	-.8940227	.014762	-.9229642	-.8650813
3	-1.052523	.0264206	-1.104322	-1.000725
4	-1.205508	.0498827	-1.303304	-1.107711
5	-1.528723	.1554491	-1.833486	-1.22396
6	-1.285957	.2186523	-1.714633	-.8572819
7	-1.871658	.1780595	-2.22075	-1.522567
8	-2.877365	.	.	.
10	-1.716629	.	.	.



For small price changes, CV and EV will be similar; however, they will not match because CV is based on the original level of utility and postpolicy prices and EV is based on the postpolicy level of utility and original prices. Which one is more appropriate in any situation depends on the policy and goals. If the government is implementing, say, a carbon tax to reduce consumption of goods and services that emit carbon dioxide, CV may be a better choice because it tells how much the government must compensate consumers so that their overall utilities do not change. If one is evaluating the introduction of a new product competing with an existing product into the market, EV may be a better metric because it could be interpreted as a measure of how much better off consumers would be thanks to lower prices.

As with our analysis of estimated minimum quantities in [example 2](#), here we have not attempted to obtain measures of the sampling variation associated with the CV or EV. One alternative to obtain, say, the sampling variation of the mean CV across households would be to use the bootstrap, being sure to bootstrap the entire procedure, including refitting the model at each bootstrap replication as well as obtaining the desired statistics.

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Also see

[R] [demandsys](#) — Estimation of flexible demand systems

[U] [20 Estimation and postestimation commands](#)

Description	Quick start	Menu
Syntax	Options for symplot, quantile, and qqplot	Options for qnorm and pnorm
Options for qchi and pchi	Remarks and examples	Methods and formulas
Acknowledgments	References	Also see

Description

`symplot` graphs a symmetry plot of *varname*.

`quantile` plots the ordered values of *varname* against the quantiles of a uniform distribution.

`qqplot` plots the quantiles of *varname*₁ against the quantiles of *varname*₂ (Q–Q plot).

`qnorm` plots the quantiles of *varname* against the quantiles of the normal distribution (Q–Q plot).

`pnorm` graphs a standardized normal probability plot (P–P plot).

`qchi` plots the quantiles of *varname* against the quantiles of a χ^2 distribution (Q–Q plot).

`pchi` graphs a χ^2 probability plot (P–P plot).

See [R] [regress postestimation diagnostic plots](#) for regression diagnostic plots and [R] [logistic postestimation](#) for logistic regression diagnostic plots.

Quick start

Symmetry plot for *v1*

```
symplot v1
```

Change marker color and size

```
symplot v1, mcolor(red) msize(large)
```

Plot ordered values of *v1* against quantiles of the uniform distribution

```
quantile v1
```

Same as above, but only for observations with *v2* greater than 5

```
quantile v1 if v2 > 5
```

Plot quantiles of *v1* against quantiles of *v2*

```
qqplot v1 v2
```

Change thickness of the reference line

```
qqplot v1 v2, rlopts(lwidth(thick))
```

Plot quantiles of *v1* against quantiles of the normal distribution

```
qnorm v1
```

Add grid lines

```
qnorm v1, grid
```

Standardized normal probability plot for v1

```
pnorm v1
```

Change labels on the x and y axes

```
pnorm v1, xlabel(0(0.1)1) ylabel(0(0.1)1)
```

Plot quantiles of v1 against quantiles of the χ_1^2 distribution

```
qchi v1
```

Same as above, but comparing with quantiles of the χ_2^2 distribution

```
qchi v1, df(2)
```

χ^2 probability plot for v1

```
pchi v1
```

Add “ $\chi^2(1)$ P-P plot” to graph

```
pchi v1, title("&chi^{sup:2}(1) P-P plot")
```

Menu

symplot

Statistics > Summaries, tables, and tests > Distributional plots and tests > Symmetry plot

quantile

Statistics > Summaries, tables, and tests > Distributional plots and tests > Quantiles plot

qqplot

Statistics > Summaries, tables, and tests > Distributional plots and tests > Quantile–quantile plot

qnorm

Statistics > Summaries, tables, and tests > Distributional plots and tests > Normal quantile plot

pnorm

Statistics > Summaries, tables, and tests > Distributional plots and tests > Normal probability plot, standardized

qchi

Statistics > Summaries, tables, and tests > Distributional plots and tests > Chi-squared quantile plot

pchi

Statistics > Summaries, tables, and tests > Distributional plots and tests > Chi-squared probability plot

Syntax

Symmetry plot

```
sympplot varname [if] [in] [ , options1 ]
```

Ordered values of *varname* against quantiles of uniform distribution

```
quantile varname [if] [in] [ , options1 ]
```

Quantiles of *varname*₁ against quantiles of *varname*₂

```
qqplot varname1 varname2 [if] [in] [ , options1 ]
```

Quantiles of *varname* against quantiles of normal distribution

```
qnorm varname [if] [in] [ , options2 ]
```

Standardized normal probability plot

```
pnorm varname [if] [in] [ , options2 ]
```

Quantiles of *varname* against quantiles of χ^2 distribution

```
qchi varname [if] [in] [ , options3 ]
```

χ^2 probability plot

```
pchi varname [if] [in] [ , options3 ]
```

<i>options</i> ₁	Description
Plot	
<i>marker_options</i>	change look of markers (color, size, etc.)
<i>marker_label_options</i>	add marker labels; change look or position
Reference line	
<i>rlopts</i> (<i>cline_options</i>)	affect rendition of the reference line
Add plots	
<i>addplot</i> (<i>plot</i>)	add other plots to the generated graph
Y axis, X axis, Titles, Legend, Overall	
<i>twoway_options</i>	any options other than by() documented in [G-3] <i>twoway_options</i>

<i>options</i> ₂	Description
Main	
<code>grid</code>	add grid lines
Plot	
<code>marker_options</code>	change look of markers (color, size, etc.)
<code>marker_label_options</code>	add marker labels; change look or position
Reference line	
<code>rlopts</code> (<i>cline_options</i>)	affect rendition of the reference line
Add plots	
<code>addplot</code> (<i>plot</i>)	add other plots to the generated graph
Y axis, X axis, Titles, Legend, Overall	
<i>twoway_options</i>	any options other than <code>by()</code> documented in [G-3] <i>twoway_options</i>

<i>options</i> ₃	Description
Main	
<code>grid</code>	add grid lines
<code>df</code> (#)	degrees of freedom of χ^2 distribution; default is <code>df(1)</code>
Plot	
<code>marker_options</code>	change look of markers (color, size, etc.)
<code>marker_label_options</code>	add marker labels; change look or position
Reference line	
<code>rlopts</code> (<i>cline_options</i>)	affect rendition of the reference line
Add plots	
<code>addplot</code> (<i>plot</i>)	add other plots to the generated graph
Y axis, X axis, Titles, Legend, Overall	
<i>twoway_options</i>	any options other than <code>by()</code> documented in [G-3] <i>twoway_options</i>

Options for `symplot`, `quantile`, and `qqplot`

Plot

marker_options affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] *marker_options*.

marker_label_options specify if and how the markers are to be labeled; see [G-3] *marker_label_options*.

Reference line

`rlopts` (*cline_options*) affect the rendition of the reference line; see [G-3] *cline_options*.

Add plots

`addplot(plot)` provides a way to add other plots to the generated graph; see [G-3] [addplot_option](#).

Y axis, X axis, Titles, Legend, Overall

`twoway_options` are any of the options documented in [G-3] [twoway_options](#), excluding `by()`. These include options for titling the graph (see [G-3] [title_options](#)) and for saving the graph to disk (see [G-3] [saving_option](#)).

Options for qnorm and pnorm

Main

`grid` adds grid lines at the 0.05, 0.10, 0.25, 0.50, 0.75, 0.90, and 0.95 quantiles when specified with `qnorm`. With `pnorm`, `grid` is equivalent to `yline(.25, .5, .75) xline(.25, .5, .75)`.

Plot

`marker_options` affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] [marker_options](#).

`marker_label_options` specify if and how the markers are to be labeled; see [G-3] [marker_label_options](#).

Reference line

`rlopts(cline_options)` affect the rendition of the reference line; see [G-3] [cline_options](#).

Add plots

`addplot(plot)` provides a way to add other plots to the generated graph; see [G-3] [addplot_option](#).

Y axis, X axis, Titles, Legend, Overall

`twoway_options` are any of the options documented in [G-3] [twoway_options](#), excluding `by()`. These include options for titling the graph (see [G-3] [title_options](#)) and for saving the graph to disk (see [G-3] [saving_option](#)).

Options for qchi and pchi

Main

`grid` adds grid lines at the 0.05, 0.10, 0.25, 0.50, 0.75, 0.90, and .95 quantiles when specified with `qchi`.

With `pchi`, `grid` is equivalent to `yline(.25, .5, .75) xline(.25, .5, .75)`.

`df(#)` specifies the degrees of freedom of the χ^2 distribution. The default is `df(1)`.

Plot

`marker_options` affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] [marker_options](#).

`marker_label_options` specify if and how the markers are to be labeled; see [G-3] [marker_label_options](#).

Reference line

`rlopts` (*cline_options*) affect the rendition of the reference line; see [G-3] *cline_options*.

Add plots

`addplot` (*plot*) provides a way to add other plots to the generated graph; see [G-3] *addplot_option*.

Y axis, X axis, Titles, Legend, Overall

twoway_options are any of the options documented in [G-3] *twoway_options*, excluding `by()`. These include options for titling the graph (see [G-3] *title_options*) and for saving the graph to disk (see [G-3] *saving_option*).

Remarks and examples

Remarks are presented under the following headings:

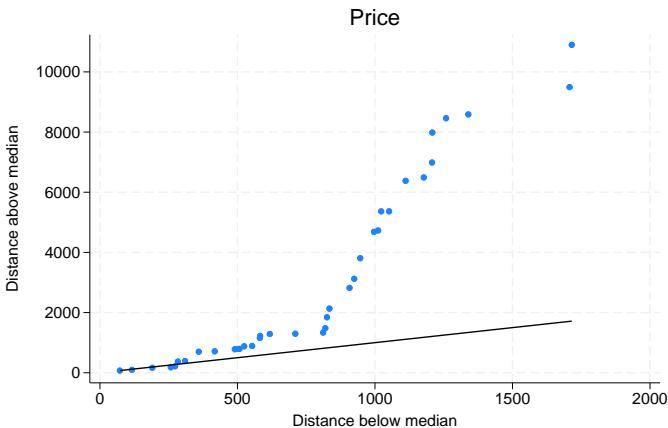
symplot
quantile
qqplot
qnorm
pnorm
qchi
pchi

symplot

► Example 1

We have data on 74 automobiles. To make a symmetry plot of the variable price, we type

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. symplot price
```



All points would lie along the reference line (defined as $y = x$) if car prices were symmetrically distributed. The points in this plot lie above the reference line, indicating that the distribution of car prices is skewed to the right—the most expensive cars are far more expensive than the least expensive cars are inexpensive.

The logic works as follows: a variable, z , is distributed symmetrically if

$$\text{median} - z_{(i)} = z_{(N+1-i)} - \text{median}$$

where $z_{(i)}$ indicates the i th-order statistic of z . `symplot` graphs $y_i = \text{median} - z_{(i)}$ versus $x_i = z_{(N+1-i)} - \text{median}$.

For instance, consider the largest and smallest values of `price` in the example above. The most expensive car costs \$15,906 and the least expensive, \$3,291. Let's compare these two cars with the typical car in the data and see how much more it costs to buy the most expensive car, and compare that with how much less it costs to buy the least expensive car. If the automobile price distribution is symmetric, the price differences would be the same.

Before we can make this comparison, we must agree on a definition for the word “typical”. Let's agree that “typical” means median. The price of the median car is \$5,006.50, so the most expensive car costs \$10,899.50 more than the median car, and the least expensive car costs \$1,715.50 less than the median car. We now have one piece of evidence that the car price distribution is not symmetric. We can repeat the experiment for the second-most-expensive car and the second-least-expensive car. We find that the second-most-expensive car costs \$9,494.50 more than the median car, and the second-least-expensive car costs \$1,707.50 less than the median car. We now have more evidence. We can continue doing this with the third most expensive and the third least expensive, and so on.

Once we have all of these numbers, we want to compare each pair and ask how similar, on average, they are. The easiest way to do that is to plot all the pairs.

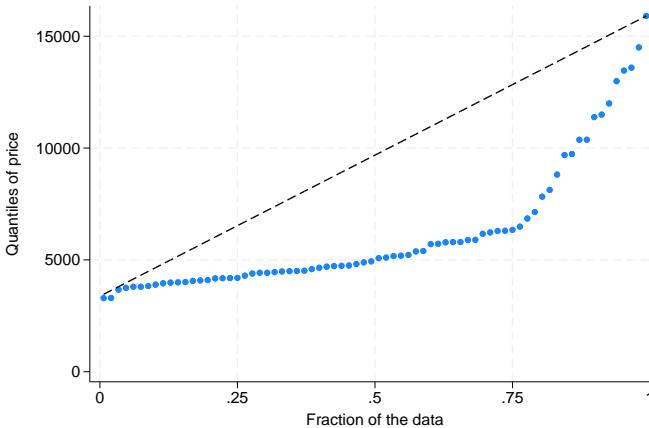


quantile

▷ Example 2

We have data on the prices of 74 automobiles. To make a quantile plot of price, we type

```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
. quantile price, rlopts(clpattern(dash)) ytitle(Quantiles of price)
```



We changed the pattern of the reference line by specifying `rlopts(clpattern(dash))`.

In a quantile plot, each value of the variable is plotted against the fraction of the data that have values less than that fraction. The diagonal line is a reference line. If automobile prices were rectangularly distributed, all the data would be plotted along the line. Because all the points are below the reference line, we know that the price distribution is skewed right.

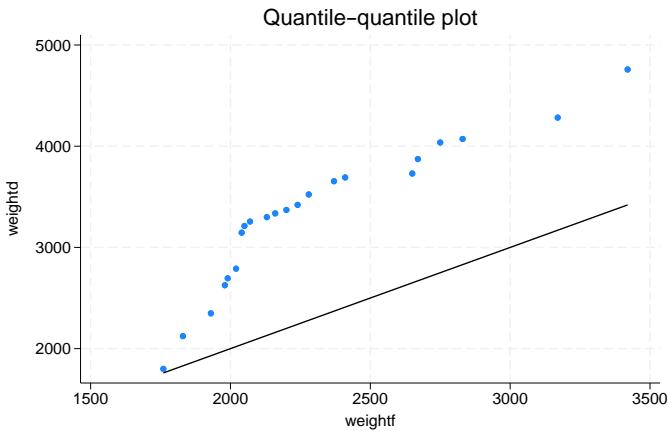


qqplot

▷ Example 3

We have data on the weight and country of manufacture of 74 automobiles. We wish to compare the distributions of weights for domestic and foreign automobiles:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. generate weightd=weight if !foreign
(22 missing values generated)
. generate weightf=weight if foreign
(52 missing values generated)
. qqplot weightd weightf
```

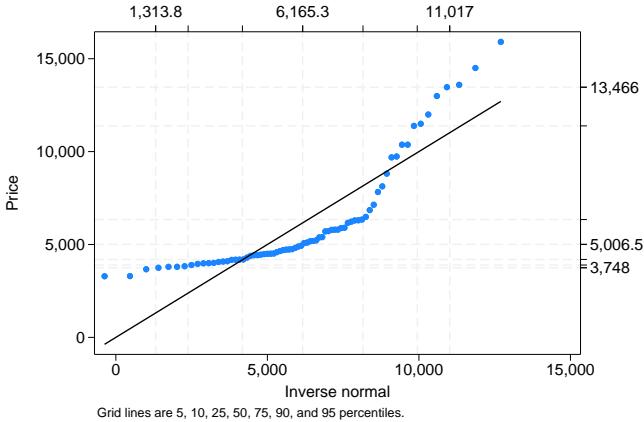


qnorm

▷ Example 4

Continuing with our price data on 74 automobiles, we now wish to compare the distribution of price with the normal distribution:

```
. qnorm price, grid
```



The result shows that the distributions are different.



□ Technical note

The idea behind `qnorm` is recommended strongly by [Miller \(1997\)](#): he calls it probit plotting. His recommendations from much practical experience should interest many users. “My recommendation for detecting nonnormality is *probit plotting*” ([Miller 1997](#), 10). “If a deviation from normality cannot be spotted by eye on probit paper, it is not worth worrying about. I never use the Kolmogorov–Smirnov test (or one of its cousins) or the χ^2 test as a preliminary test of normality. They do not tell you how the sample is differing from normality, and I have a feeling they are more likely to detect irregularities in the middle of the distribution than in the tails” ([Miller 1997](#), 13–14).

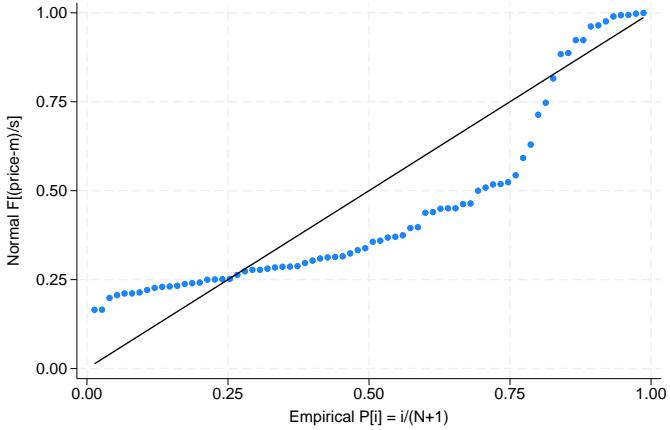


pnorm

▷ Example 5

Quantile–normal plots emphasize the tails of the distribution. Normal probability plots put the focus on the center of the distribution:

```
. pnorm price, grid
```

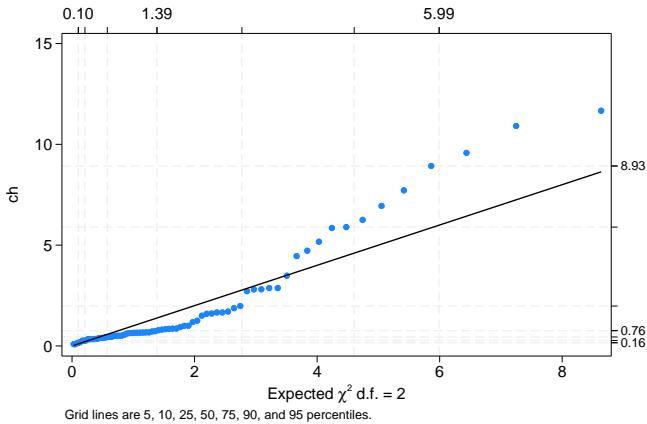


qchi

▷ Example 6

Suppose that we want to examine the distribution of the sum of squares of `price` and `mpg`, standardized for their variances.

```
. egen c1 = std(price)
. egen c2 = std(mpg)
. generate ch = c1^2 + c2^2
. qchi ch, df(2) grid ylabel(, axis(2) labsize(*.8) format(%4.2f))
> xlabel(, axis(2) format(%4.2f))
```



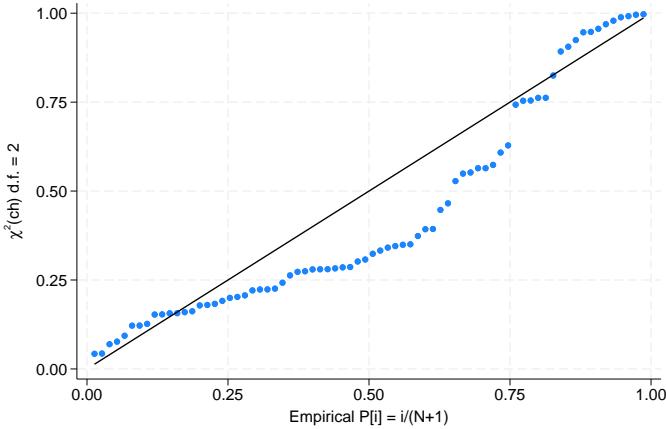
The quadratic form is clearly not χ^2 with 2 degrees of freedom.

pchi

▷ Example 7

We can focus on the center of the distribution by doing a probability plot:

```
. pchi ch, df(2) grid
```



◀

Methods and formulas

Let $x_{(1)}, x_{(2)}, \dots, x_{(N)}$ be the data sorted in ascending order.

If a continuous variable, x , has a cumulative distribution function $F(x) = P(X \leq x) = p$, the quantiles x_{p_i} are such that $F(x_{p_i}) = p_i$. For example, if $p_i = 0.5$, then $x_{0.5}$ is the median. When we plot data, the probabilities, p_i , are often referred to as plotting positions. There are many different conventions for choice of plotting positions, given $x_{(1)} \leq \dots \leq x_{(N)}$. Most belong to the family $(i - a)/(N - 2a + 1)$. $a = 0.5$ (suggested by Hazen) and $a = 0$ (suggested by Weibull) are popular choices.

For a wider discussion of the calculation of plotting positions, see [Cox \(2002\)](#).

symplot plots median $- x_{(i)}$ versus $x_{(N+1-i)} - \text{median}$.

quantile plots $x_{(i)}$ versus $(i - 0.5)/N$ (the Hazen position).

qnorm plots $x_{(i)}$ against $q_i \times \hat{\sigma} + \hat{\mu}$, where $q_i = \Phi^{-1}(p_i)$, Φ is the cumulative normal distribution, $p_i = i/(N + 1)$ (the Weibull position), $\hat{\sigma}$ is the standard deviation, and $\hat{\mu}$ is the mean of the data.

pnorm plots $\Phi\{(x_i - \hat{\mu})/\hat{\sigma}\}$ versus $p_i = i/(N + 1)$, where $\hat{\mu}$ is the mean of the data and $\hat{\sigma}$ is the standard deviation.

qchi and pchi are similar to qnorm and pnorm; the cumulative χ^2 distribution is used in place of the cumulative normal distribution.

qqplot is just a two-way scatterplot of one variable against the other after both variables have been sorted into ascending order, and both variables have the same number of nonmissing observations. If the variables have unequal numbers of nonmissing observations, interpolated values of the variable with more data are plotted against the variable with fewer data.

Ramanathan Gnanadesikan (1932–2015) was born in Madras. He obtained degrees from the Universities of Madras and North Carolina. He worked in industry at Procter & Gamble, Bell Labs, and Bellcore, as well as in universities, retiring from Rutgers in 1998. Among many contributions to statistics, he is especially well known for work on probability plotting, robustness, outlier detection, clustering, classification, and pattern recognition.

Martin Bradbury Wilk (1922–2013) was born in Montreal. He obtained degrees in chemical engineering and statistics from McGill and Iowa State Universities. After holding several statistics-related posts in industry and at universities (including periods at Princeton, Bell Labs, and Rutgers), Wilk was appointed Chief Statistician at Statistics Canada (1980–1986). He is especially well known for his work with Gnanadesikan on probability plotting and with Shapiro on tests for normality.

Acknowledgments

We thank Peter A. Lachenbruch (1937–2021), Emeritus Appointment, Biostatistics, College of Public Health and Human Sciences, Oregon State University for writing the original versions of `qchi` and `pchi`. Patrick Royston of the MRC Clinical Trials Unit, London, and coauthor of the Stata Press book *Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model* also published a similar command in Royston (1996).

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Also see

- [R] **cumul** — Cumulative distribution
- [R] **kdensity** — Univariate kernel density estimation
- [R] **logistic postestimation** — Postestimation tools for logistic
- [R] **lv** — Letter-value displays
- [R] **regress postestimation diagnostic plots** — Postestimation plots for regress

Description

`display` displays strings and values of scalar expressions.

`display` really has many more features and a more complex syntax diagram, but the diagram shown above is adequate for interactive use. For a full discussion of `display`'s capabilities, see [P] [display](#).

Quick start

Perform calculations interactively

```
display 100*100
```

Same as above, but include comma in the result

```
display %6.0fc 100*100
```

Verify choice of datetime function

```
display %tm monthly("January 1983", "MY")
```

View formatted mean after summarize

```
display %5.2f r(mean)
```

Add the variance with a different format on its own line

```
display "mean = " %5.2f r(mean) _newline "variance = " %10.4f r(Var)
```

Syntax

```
display exp
```

Remarks and examples

`display` can be used as a substitute for a hand calculator.

▷ Example 1

`display 2+2` produces the output 4. Stata variables may also appear in the expression, such as in `display myvar/2`. Because `display` works only with scalars, the resulting calculation is performed only for the first observation. You could type `display myvar[10]/2` to display the calculation for the 10th observation. Here are more examples:

```
. display sqrt(2)/2
.70710678
. display normal(-1.1)
.13566606
. di (57.2-3)/(12-2)
5.42
. display myvar/10
7
. display myvar[10]/2
3.5
```

◀

Also see

[P] [display](#) — Display strings and values of scalar expressions

[U] [13 Functions and expressions](#)

do — Execute commands from a file

Description	Quick start	Menu	Syntax
Option	Reference	Also see	

Description

`do` and `run` cause Stata to execute the commands stored in *filename* just as if they were entered from the keyboard. `do` echoes the commands as it executes them, whereas `run` is silent.

Quick start

Execute commands stored in `myfile.do`

```
do myfile
```

Same as above, passing `first arg` in local macro 1 and `arg2` in local macro 2 for use by `myfile.do`

```
do myfile "first arg" arg2
```

Execute commands stored in `myfile.do`, continuing execution even if an error occurs in one or more commands

```
do myfile, nostop
```

Execute commands stored in `myfile.do` silently

```
run myfile
```

Menu

File > Do...

Syntax

```
{do | run} filename [arguments] [ , nostop ]
```

filename (called a *do-file*) can be created using Stata's Do-file Editor; see [R] [doedit](#). This file will be a standard text file. *filename* can also be created by using a non-Stata text editor; see [D] [shell](#) for a way to invoke your favorite editor from inside Stata. Ensure that you save the file in ASCII or UTF-8 format.

If *filename* is specified without an extension, `.do` is assumed.

If the path or *filename* contains spaces, it should be enclosed in double quotes.

A complete discussion of do-files, including information on *arguments*, can be found in [U] [16 Do-files](#).

Option

`nostop` allows the do-file to continue executing even if an error occurs. Normally, Stata stops executing the do-file when it detects an error (nonzero return code).

Reference

Jenkins, S. P. 2006. *Stata tip 32: Do not stop*. *Stata Journal* 6: 281.

Also see

[R] **doedit** — Edit do-files and other text files

[P] **include** — Include commands from file

[GSM] **13 Using the Do-file Editor—automating Stata**

[GSU] **13 Using the Do-file Editor—automating Stata**

[GSW] **13 Using the Do-file Editor—automating Stata**

[U] **15 Saving and printing output—log files**

[U] **16 Do-files**

Description

doedit opens the Do-file Editor. This text editor lets you create and edit do-files, which typically contain a series of Stata commands. If you specify *filename*, doedit will open a text file, such as a do-file or an ado-file, saved to disk.

Quick start

Open a new untitled do-file in the Do-file Editor

```
doedit
```

Open new or existing do-file `myfile.do` in the Do-file Editor

```
doedit myfile
```

Menu

Window > Do-file Editor

Syntax

```
doedit [filename]
```

Remarks and examples

Clicking on the **Do-file Editor** button is equivalent to typing doedit.

doedit, typed by itself, invokes the Editor with an empty document. If you specify *filename*, that file is displayed in the Editor.

You may have more than one Do-file Editor open at once. Each time you submit the doedit command, a new window will be opened.

A tutorial discussion of doedit can be found in the *Getting Started with Stata* manual. Read [\[U\] 16 Do-files](#) for an explanation of do-files, and then read [\[GSW\] 13 Using the Do-file Editor—automating Stata](#) to learn how to use the Do-file Editor to create and execute do-files.

Also see

[\[R\] do](#) — Execute commands from a file

[\[GSM\] 13 Using the Do-file Editor—automating Stata](#)

[\[GSU\] 13 Using the Do-file Editor—automating Stata](#)

[\[GSW\] 13 Using the Do-file Editor—automating Stata](#)

[\[U\] 16 Do-files](#)

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Acknowledgments](#)[Syntax](#)[Reference](#)[Options](#)

Description

A dot plot is a scatterplot with values grouped together vertically (“binning”, as in a histogram) and with plotted points separated horizontally. The aim is to display all the data for several variables or groups in one compact graphic.

Quick start

Dotplot of $v1$

```
dotplot v1
```

Columns with separate dot plots of $v1$ for each level of categorical variable a

```
dotplot v1, over(a)
```

Same as above, but with the dots centered in each column

```
dotplot v1, over(a) center
```

Dotplots for $v1$, $v2$, and $v3$ in separate columns

```
dotplot v1 v2 v3
```

Add a horizontal line of pluses at the mean of each variable

```
dotplot v1 v2 v3, mean
```

Add pluses for the medians and dashed lines for the upper and lower quartiles

```
dotplot v1 v2 v3, median bar
```

Menu

Graphics > Distributional graphs > Distribution dot plot

Syntax

Dotplot of *varname*, with one column per value of *groupvar*

```
dotplot varname [if] [in] [, options]
```

Dotplot for each variable in *varlist*, with one column per variable

```
dotplot varlist [if] [in] [, options]
```

<i>options</i>	Description
Options	
<i>over</i> (<i>groupvar</i>)	display one columnar dot plot for each value of <i>groupvar</i>
<i>nx</i> (#)	horizontal dot density; default is <i>nx</i> (0)
<i>ny</i> (#)	vertical dot density; default is <i>ny</i> (35)
<i>incr</i> (#)	label every # group; default is <i>incr</i> (1)
<i>mean</i> <i>median</i>	plot a horizontal line of pluses at the mean or median
<i>bounded</i>	use minimum and maximum as boundaries
<i>bar</i>	plot horizontal dashed lines at shoulders of each group
<i>nogroup</i>	use the actual values of <i>yvar</i>
<i>center</i>	center the dot for each column
Plot	
<i>marker_options</i>	change look of markers (color, size, etc.)
<i>marker_label_options</i>	add marker labels; change look or position
Y axis, X axis, Titles, Legend, Overall	
<i>twoway_options</i>	any options other than <i>by</i> () documented in [G-3] <i>twoway_options</i>
collect is allowed; see [U] 11.1.10 Prefix commands.	

Options

Options

over(*groupvar*) identifies the variable for which *dotplot* will display one columnar dot plot for each value of *groupvar*. *over*() may not be specified in the second syntax.

nx(#) sets the horizontal dot density. A larger value of # will increase the dot density, reducing the horizontal separation between dots. This option will increase the separation between columns if two or more groups or variables are used.

ny(#) sets the vertical dot density (number of “bins” on the *y* axis). A larger value of # will result in more bins and a plot that is less spread out horizontally. # should be determined in conjunction with *nx*() to give the most pleasing appearance.

incr(#) specifies how the *x* axis is to be labeled. *incr*(1), the default, labels all groups. *incr*(2) labels every second group.

[*mean* | *median*] plots a horizontal line of pluses at the mean or median of each group.

`bounded` forces the minimum and maximum of the variable to be used as boundaries of the smallest and largest bins. It should be used with one variable whose support is not the whole of the real line and whose density does not tend to zero at the ends of its support, for example, a uniform random variable or an exponential random variable.

`bar` plots horizontal dashed lines at the “shoulders” of each group. The shoulders are taken to be the upper and lower quartiles unless mean has been specified; here they will be the mean plus or minus the standard deviation.

`nogroup` uses the actual values of *yvar* rather than grouping them (the default). This option may be useful if *yvar* takes on only a few values.

`center` centers the dots for each column on a hidden vertical line.

Plot

marker_options affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] *marker_options*.

marker_label_options specify if and how the markers are to be labeled; see [G-3] *marker_label_options*. *marker_label_options* are not allowed if *varlist* is specified.

Y axis, X axis, Titles, Legend, Overall

twoway_options are any of the options documented in [G-3] *twoway_options*, excluding `by()`. These include options for titling the graph (see [G-3] *title_options*) and for saving the graph to disk (see [G-3] *saving_option*).

Remarks and examples

`dotplot` produces a figure that has elements of a boxplot, a histogram, and a scatterplot. Like a boxplot, it is most useful for comparing the distributions of several variables or the distribution of 1 variable in several groups. Like a histogram, the figure provides a crude estimate of the density, and, as with a scatterplot, each symbol (dot) represents 1 observation.

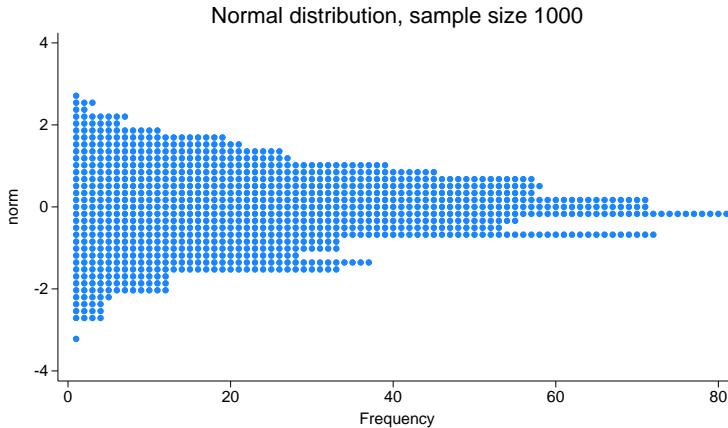
► Example 1

`dotplot` may be used as an alternative to Stata’s histogram graph for displaying the distribution of one variable.

```

. set seed 123456789
. set obs 1000
Number of observations (_N) was 0, now 1,000.
. generate norm = rnormal()
. dotplot norm, title("Normal distribution, sample size 1000")

```



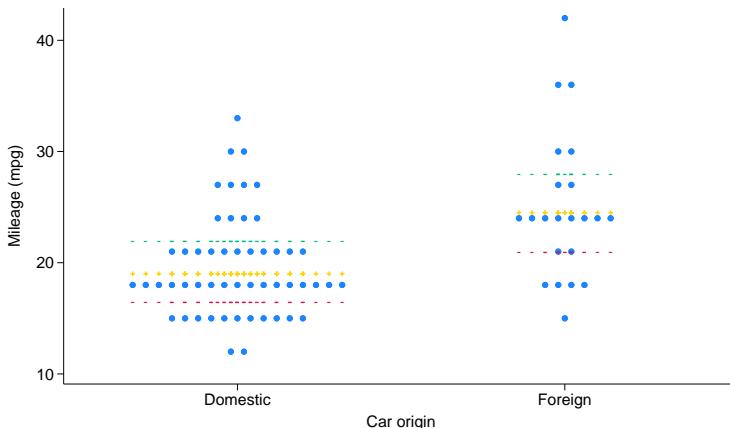
▶ Example 2

The `over()` option lets us use `dotplot` to compare the distribution of one variable within different levels of a grouping variable. The `center`, `median`, and `bar` options create a graph that may be compared with Stata's `boxplot`; see [\[G-2\] graph box](#). The next graph illustrates this option with Stata's automobile dataset.

```

. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
. dotplot mpg, over(foreign) nx(25) ny(10) center median bar

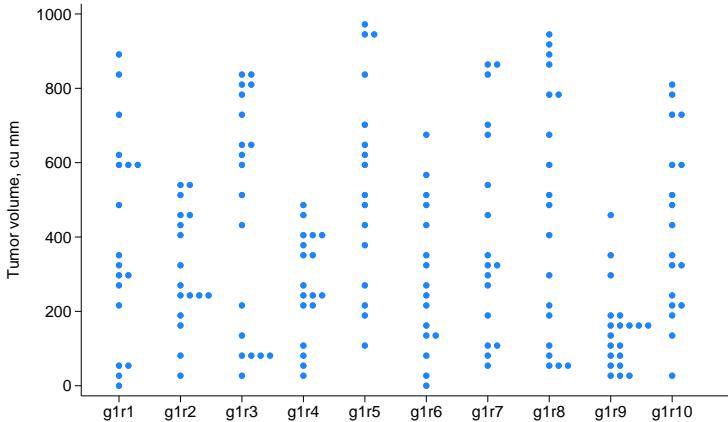
```



▶ Example 3

The second version of `dotplot` lets us compare the distribution of several variables. In the next graph, all 10 variables contain measurements on tumor volume.

```
. use https://www.stata-press.com/data/r19/dotgr
. dotplot g1r1-g1r10, ytitle("Tumor volume, cu mm")
```

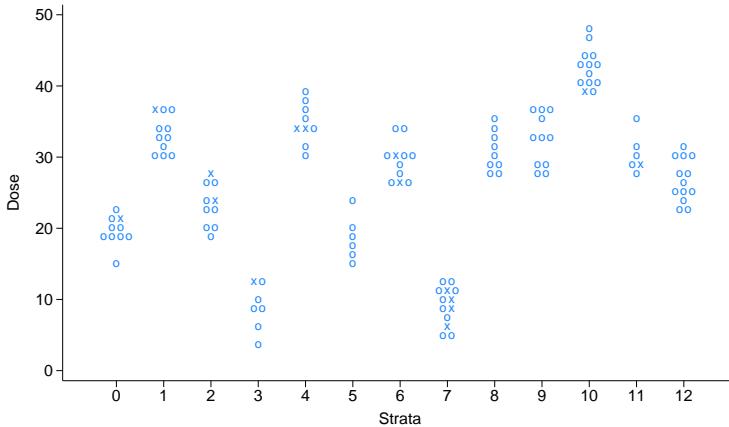


▶ Example 4

When using the first form with the `over()` option, we can encode a third dimension in a dot plot by using a different plotting symbol for different groups. The third dimension cannot be encoded with a varlist. The example is of a hypothetical matched case–control study. The next graph shows the exposure of each individual in each matched stratum. Cases are marked by the letter 'x', and controls are marked by the letter 'o'.

```
. use https://www.stata-press.com/data/r19/dotdose
. label define symbol 0 "o" 1 "x"
. label values case symbol
```

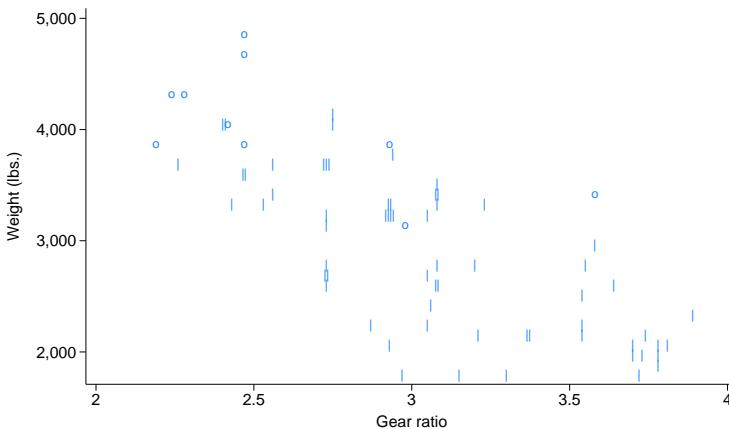
```
. dotplot dose, over(strata) m(none) mlabels(case) mlabp(0) center
```



▷ Example 5

dotplot can also be used with two virtually continuous variables as an alternative to jittering the data to distinguish ties. We must use the `xlabel()` option, because otherwise dotplot will attempt to label too many points on the x axis. It is often useful in such instances to use a value of `nx` that is smaller than the default. That was not necessary in this example, partly because of our choice of symbols.

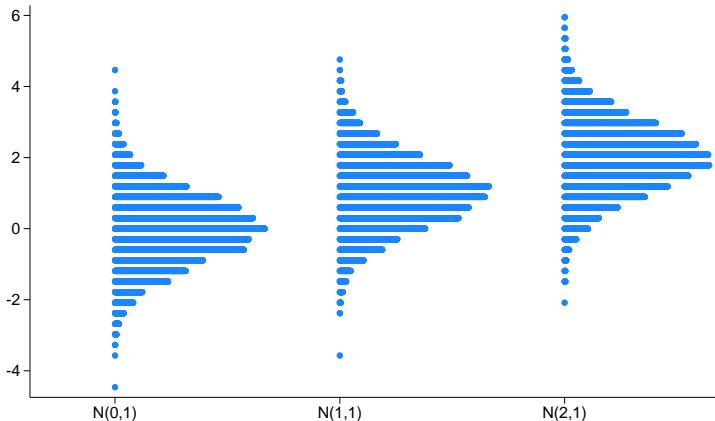
```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
. generate byte hi_price = (price>10000) if price < .
. label define symbol 0 "|" 1 "o"
. label values hi_price symbol
. dotplot weight, over(gear_ratio) m(none) mlabels(hi_price) mlabp(0) center
> xlabel(#5)
```



▶ Example 6

The following figure is included mostly for aesthetic reasons. It also demonstrates dotplot's ability to cope with even very large datasets. The sample size for each variable is 10,000, so it may take a long time to print.

```
. set seed 123456789
. set obs 10000
Number of observations (_N) was 0, now 10,000.
. generate norm0 = rnormal()
. generate norm1 = rnormal() + 1
. generate norm2 = rnormal() + 2
. label variable norm0 "N(0,1)"
. label variable norm1 "N(1,1)"
. label variable norm2 "N(2,1)"
. dotplot norm0 norm1 norm2
```



Stored results

dotplot stores the following in `r()`:

Scalars

<code>r(nx)</code>	horizontal dot density
<code>r(ny)</code>	vertical dot density

Acknowledgments

dotplot was written by Peter Sasieni of the King's Clinical Trials Unit at King's College London, and Patrick Royston of the MRC Clinical Trials Unit, London, and coauthor of the Stata Press book *Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model*.

Reference

Sasieni, P. D., and P. Royston. 1996. Dotplots. *Journal of the Royal Statistical Society, C ser.*, 45: 219–234. <https://doi.org/10.2307/2986156>.

[Description](#)[Options for dstdize](#)[Methods and formulas](#)[Quick start](#)[Options for istdize](#)[Acknowledgments](#)[Menu](#)[Remarks and examples](#)[References](#)[Syntax](#)[Stored results](#)[Also see](#)

Description

`dstdize` produces standardized rates, a weighted average of the stratum-specific rates.

`istdize` produces indirectly standardized rates that are appropriate when the stratum-specific rates for the population being studied are either unavailable or unreliable.

`istdize` also calculates a point estimate and exact confidence interval for the study population's standardized mortality ratio (SMR) or the standardized incidence ratio (SIR).

Quick start

Direct standardization

Use reference population saved in `mypop.dta` to standardize `v1` with stratum identifier `svar` and stratum size `v2` for `catvar`

```
dstdize v1 v2 svar, by(catvar) using(mypop)
```

Same as above, but with reference population in memory where `catvar = 1`

```
dstdize v1 v2 svar, by(catvar) base(1)
```

Same as above, but with reference population in memory where `catvar = "nation"`

```
dstdize v1 v2 svar, by(catvar) base("nation")
```

Indirect standardization

Use population cases and size saved in `cases` and `pop` to standardize study cases `v3` and stratum size `v4` at each level of `svar`

```
istdize v3 v4 svar using mypop.dta, popvars(cases pop)
```

Same as above, but standardize subpopulations identified by levels of `catvar`

```
istdize v3 v4 svar using mypop.dta, popvars(cases pop) by(catvar)
```

Same as above, but standardize by population stratum-specific and crude rates saved in `srate` and `crate` and display summary of standard population

```
istdize v3 v4 svar using mypop.dta, rate(srate crate) by(catvar) print
```

Same as above, but indicate that the crude population rate is 0.01

```
istdize v3 v4 svar using mypop.dta, rate(srate .01) by(catvar) print
```

Menu

dstdize

Statistics > Epidemiology and related > Other > Direct standardization

istdize

Statistics > Epidemiology and related > Other > Indirect standardization

Syntax

Direct standardization

```
dstdize charvar popvar stratavars [if] [in], by(groupvars) [dstdize_options]
```

Indirect standardization

```
istdize casevars popvars stratavars [if] [in] using filename,
    { popvars(casevarp popvarp) | rate(ratevarp {#| crudevarp}) }
    [istdize_options]
```

charvar is the characteristic to be standardized across different subpopulations identified by *groupvars*.

popvar defines the weights used in standardization.

stratavars defines the strata across which the weights are to be averaged in *dstdize*. For *istdize*, *stratavars* defines the strata for which *casevar*_s is measured.

*casevar*_s is the variable name for the study population's number of cases. If *by*(*groupvars*) is specified, *casevar*_s must be constant or missing within each group defined by combinations of *groupvars*.

*popvar*_s identifies the number of subjects in each strata in the study population.

filename must be a Stata dataset and contain *popvar* and *stratavars*.

<i>dstdize_options</i>	Description
Main	
* <i>by</i> (<i>groupvars</i>)	study populations
<u>using</u> (<i>filename</i>)	use standard population from Stata dataset
<u>base</u> (# <i>string</i>)	use standard population from a value of grouping variable
<u>level</u> (#)	set confidence level; default is <code>level(95)</code>
Options	
<u>saving</u> (<i>filename</i>)	save computed standard population distribution as a Stata dataset
<u>format</u> (% <i>fmt</i>)	final summary table display format; default is %10.0g
<u>print</u>	include table summary of standard population in output
<u>nores</u>	suppress storing results in <code>r()</code>

**by*(*groupvars*) is required.

<i>istdize_options</i>	Description
Main	
* <code>popvars(<i>casevar_p</i> <i>popvar_p</i>)</code>	for standard population, <i>casevar_p</i> is number of cases and <i>popvar_p</i> is number of individuals
* <code>rate(<i>ratevar_p</i> {# <i>crudevar_p</i>})</code>	<i>ratevar_p</i> is stratum-specific rates and # or <i>crudevar_p</i> is the crude case rate value or variable
<code>level(#)</code>	set confidence level; default is level(95)
Options	
<code>by(<i>groupvars</i>)</code>	variables identifying study populations
<code>format(%<i>fmt</i>)</code>	final summary table display format; default is %10.0g
<code>print</code>	include table summary of standard population in output
*Either <code>popvars(<i>casevar_p</i> <i>popvar_p</i>)</code> or <code>rate(<i>ratevar_p</i> {# <i>crudevar_p</i>})</code> must be specified.	

`collect` is allowed with `dstdize` and `istdize`; see [\[U\] 11.1.10 Prefix commands](#).

Options for dstdize

Main

`by(groupvars)` is required for the `dstdize` command; it specifies the variables identifying the study populations. If `base()` is also specified, there must be only one variable in the `by()` group. If you do not have a variable for this option, you can generate one by using something like `generate newvar=1` and then use `newvar` as the argument to this option.

`using(filename)` or `base(# | string)` may be used to specify the standard population. You may not specify both options. `using(filename)` supplies the name of a `.dta` file containing the standard population. The standard population must contain the *popvar* and the *stratavars*. If `using()` is not specified, the standard population distribution will be obtained from the data. `base(# | string)` lets you specify one of the values of *groupvar*—either a numeric value or a string—to be used as the standard population. If neither `base()` nor `using()` is specified, the entire dataset is used to determine an estimate of the standard population.

`level(#)` specifies the confidence level, as a percentage, for a confidence interval of the adjusted rate. The default is `level(95)` or as set by `set level`; see [\[U\] 20.8 Specifying the width of confidence intervals](#).

Options

`saving(filename)` saves the computed standard population distribution as a Stata dataset that can be used in further analyses.

`format(%fmt)` specifies the format in which to display the final summary table. The default is %10.0g.

`print` includes a table summary of the standard population before displaying the study population results.

`nores` suppresses storing results in `r()`. This option is seldom specified. Some results are stored in matrices. If there are more groups than can fit in a matrix, `dstdize` will report the “unable to allocate matrix” error message. In this case, you must specify `nores`. The `nores` option does not change how results are calculated but specifies that results need not be left behind for use by other programs.

Options for istdize

Main

`popvars` ($casevar_p$ $popvar_p$) or `rate` ($ratevar_p$ $\{ \# | crudevar_p \}$) must be specified with `istdize`. Only one of these two options is allowed. These options are used to describe the standard population's data.

With `popvars` ($casevar_p$ $popvar_p$), $casevar_p$ records the number of cases (deaths) for each stratum in the standard population, and $popvar_p$ records the total number of individuals in each stratum (individuals at risk).

With `rate` ($ratevar_p$ $\{ \# | crudevar_p \}$), $ratevar_p$ contains the stratum-specific rates. $\# | crudevar_p$ specifies the crude case rate either by a variable name or by the crude case rate value. If a crude rate variable is used, it must be the same for all observations, although it could be missing for some.

`level` ($\#$) specifies the confidence level, as a percentage, for a confidence interval of the adjusted rate.

The default is `level(95)` or as set by `set level`; see [\[U\] 20.8 Specifying the width of confidence intervals](#).

Options

`by` ($groupvars$) specifies variables identifying study populations when more than one exists in the data.

If this option is not specified, the entire study population is treated as one group.

`format` ($\%fmt$) specifies the format in which to display the final summary table. The default is `%10.0g`.

`print` outputs a table summary of the standard population before displaying the study population results.

Remarks and examples

Remarks are presented under the following headings:

[Direct standardization](#)
[Indirect standardization](#)

In epidemiology and other fields, you will often need to compare rates for some characteristic across different populations. These populations often differ on factors associated with the characteristic under study; thus, directly comparing overall rates may be misleading.

See van Belle et al. (2004, 642–684), [Fleiss, Levin, and Paik \(2003, chap. 19\)](#), or [Kirkwood and Sterne \(2003, chap. 25\)](#) for a discussion of direct and indirect standardization.

Direct standardization

The direct method of adjusting for differences among populations involves computing the overall rates that would result if, instead of having different distributions, all populations had the same standard distribution. The standardized rate is defined as a weighted average of the stratum-specific rates, with the weights taken from the standard distribution. Direct standardization may be applied only when the specific rates for a given population are available.

dstdize generates adjusted summary measures of occurrence, which can be used to compare prevalence, incidence, or mortality rates between populations that may differ on certain characteristics (for example, age, gender, race). These underlying differences may affect the crude prevalence, mortality, or incidence rates.

► Example 1

We have data (Rothman 1986, 42) on mortality rates for Sweden and Panama for 1962, and we wish to compare mortality in these two countries:

```
. use https://www.stata-press.com/data/r19/mortality
(1962 Mortality, Sweden & Panama)

. describe
Contains data from https://www.stata-press.com/data/r19/mortality.dta
Observations:      6                1962 Mortality, Sweden & Panama
Variables:         4                14 Apr 2024 16:18
```

Variable name	Storage type	Display format	Value label	Variable label
nation	str6	%9s		Nation
age_category	byte	%9.0g	age_lbl	Age category
population	float	%10.0gc		Population in age category
deaths	float	%9.0gc		Deaths in age category

Sorted by:

```
. list, sepby(nation) abbrev(12) divider
```

	nation	age_category	population	deaths
1.	Sweden	0-29	3145000	3,523
2.	Sweden	30-59	3057000	10,928
3.	Sweden	60+	1294000	59,104
4.	Panama	0-29	741,000	3,904
5.	Panama	30-59	275,000	1,421
6.	Panama	60+	59,000	2,456

We divide the total number of cases in the population by the population to obtain the *crude rate*:

```
. collapse (sum) pop deaths, by(nation)
. list, abbrev(10) divider
```

	nation	population	deaths
1.	Panama	1075000	7,781
2.	Sweden	7496000	73,555

```
. generate crude = deaths/pop
. list, abbrev(10) divider
```

	nation	population	deaths	crude
1.	Panama	1075000	7,781	.0072381
2.	Sweden	7496000	73,555	.0098126

If we examine the total number of deaths in the two nations, the total crude mortality rate in Sweden is higher than that in Panama. From the original data, we see one possible explanation: Swedes are older than Panamanians, making direct comparison of the mortality rates difficult.

Direct standardization lets us remove the distortion caused by the different age distributions. The adjusted rate is defined as the weighted sum of the crude rates, where the weights are given by the standard distribution. Suppose that we wish to standardize these mortality rates to the following age distribution:

```
. use https://www.stata-press.com/data/r19/1962, clear
(Standard population distribution)
. list, abbrev(12) divider
```

	age_category	population
1.	0-29	.35
2.	30-59	.35
3.	60+	.3

```
. save 1962
file 1962.dta saved
```

If we multiply the above weights for the age strata by the crude rate for the corresponding age category, the sum gives us the standardized rate.

```
. use https://www.stata-press.com/data/r19/mortality
(1962 Mortality, Sweden & Panama)
. generate crude=deaths/pop
. drop pop
. merge m:1 age_cat using 1962
```

Result	Number of obs
Not matched	0
Matched	6 (_merge==3)

```
. list, sepby(age_category) abbrev(12)
```

	nation	age_category	deaths	crude	population	_merge
1.	Sweden	0-29	3,523	.0011202	.35	Matched (3)
2.	Panama	0-29	3,904	.0052686	.35	Matched (3)
3.	Sweden	30-59	10,928	.0035747	.35	Matched (3)
4.	Panama	30-59	1,421	.0051673	.35	Matched (3)
5.	Sweden	60+	59,104	.0456754	.3	Matched (3)
6.	Panama	60+	2,456	.0416271	.3	Matched (3)

```
. generate product = crude*pop
. by nation, sort: egen adj_rate = sum(product)
. drop _merge
```

```
. list, sepby(nation)
```

	nation	age_cat	deaths	crude	popula-tion	product	adj_rate
1.	Panama	60+	2,456	.0416271	.3	.0124881	.0161407
2.	Panama	30-59	1,421	.0051673	.35	.0018085	.0161407
3.	Panama	0-29	3,904	.0052686	.35	.001844	.0161407
4.	Sweden	0-29	3,523	.0011202	.35	.0003921	.0153459
5.	Sweden	60+	59,104	.0456754	.3	.0137026	.0153459
6.	Sweden	30-59	10,928	.0035747	.35	.0012512	.0153459

Comparing the standardized rates indicates that the Swedes have a slightly lower mortality rate.

To perform the above analysis with dstdize, type

```
. use https://www.stata-press.com/data/r19/mortality, clear
(1962 Mortality, Sweden & Panama)
```

```
. dstdize deaths pop age_cat, by(nation) using(1962)
```

Direct standardization

```
-> nation = Panama
```

Stratum	Pop.	Unadjusted			Std. pop. dist.	s*P
		Cases	Pop. dist.	Stratum rate		
0-29	741,000	3,904	0.689	0.0053	0.350	0.0018
30-59	275,000	1,421	0.256	0.0052	0.350	0.0018
60+	59,000	2,456	0.055	0.0416	0.300	0.0125

```
Total: 1,075,000 7,781
```

```
Note: s*P is Stratum rate multiplied by Std. pop. dist.
```

```
Adjusted cases = 17,351.2
```

```
Crude rate = 0.0072
```

```
Adjusted rate = 0.0161
```

```
95% conf. interval: [0.0156, 0.0166]
```

```
-> nation = Sweden
```

Stratum	Pop.	Unadjusted			Std. pop. dist.	s*P
		Cases	Pop. dist.	Stratum rate		
0-29	3,145,000	3,523	0.420	0.0011	0.350	0.0004
30-59	3,057,000	10,928	0.408	0.0036	0.350	0.0013
60+	1,294,000	59,104	0.173	0.0457	0.300	0.0137

```
Total: 7,496,000 73,555
```

```
Note: s*P is Stratum rate multiplied by Std. pop. dist.
```

```
Adjusted cases = 115,032.5
```

```
Crude rate = 0.0098
```

```
Adjusted rate = 0.0153
```

```
95% conf. interval: [0.0152, 0.0155]
```

Summary of study populations

nation	N	Crude rate	Adjusted rate	[95% conf. interval]	
Panama	1,075,000	0.007238	0.016141	0.015645	0.016637
Sweden	7,496,000	0.009813	0.015346	0.015235	0.015457

The summary table above lets us make a quick inspection of the results within the study populations, and the detail tables give the behavior among the strata within the study populations.



▷ Example 2

We have individual-level data on persons in four cities over several years. Included in the data is a variable indicating whether the person has high blood pressure, together with information on the person's age, sex, and race. We wish to obtain standardized high blood pressure rates for each city for 1990 and 1992, using, as the standard, the age, sex, and race distribution of the four cities and two years combined.

Our dataset contains

```
. use https://www.stata-press.com/data/r19/hbp
. describe
```

```
Contains data from https://www.stata-press.com/data/r19/hbp.dta
Observations:      1,130
Variables:         7                21 Feb 2024 06:42
```

Variable name	Storage type	Display format	Value label	Variable label
id	str10	%10s		Record identification number
city	byte	%8.0g		City
year	int	%8.0g		Year
sex	byte	%8.0g	sexfmt	Sex
age_group	byte	%8.0g	agefmt	Age group
race	byte	%8.0g	racefmt	Race
hbp	byte	%8.0g	yn	High blood pressure

Sorted by:

The `dstdize` command is designed to work with aggregate data but will work with individual-level data only if we create a variable recording the population represented by each observation. For individual-level data, this is one:

```
. generate pop = 1
```

On the next page, we specify `print` to obtain a listing of the standard population and `level(90)` to request 90% rather than 95% confidence intervals. Typing `if year==1990 | year==1992` restricts the data to the two years for both summary tables and the standard population.

```
. dstdize hbp pop age race sex if year==1990 | year==1992, by(city year) print
> level(90)
```

Standard population

Stratum			Pop.	Dist.
15-19	Black	Female	35	0.077
15-19	Black	Male	44	0.097
15-19	Hispanic	Female	5	0.011
15-19	Hispanic	Male	10	0.022
15-19	White	Female	7	0.015
15-19	White	Male	5	0.011
20-24	Black	Female	43	0.095
20-24	Black	Male	67	0.147
20-24	Hispanic	Female	14	0.031
20-24	Hispanic	Male	13	0.029
20-24	White	Female	4	0.009
20-24	White	Male	21	0.046
25-29	Black	Female	17	0.037
25-29	Black	Male	44	0.097
25-29	Hispanic	Female	7	0.015
25-29	Hispanic	Male	13	0.029
25-29	White	Female	9	0.020
25-29	White	Male	16	0.035
30-34	Black	Female	16	0.035
30-34	Black	Male	32	0.070
30-34	Hispanic	Female	2	0.004
30-34	Hispanic	Male	3	0.007
30-34	White	Female	5	0.011
30-34	White	Male	23	0.051

Total: 455
 (6 observations excluded because of missing values)

Direct standardization

```
-> city year = 1 1990
```

Stratum			Pop.	Unadjusted Pop. Stratum Cases dist. rate	Std. pop. dist.	s*P
15-19	Black	Female	6	2 0.128 0.3333	0.077	0.0256
15-19	Black	Male	6	0 0.128 0.0000	0.097	0.0000
15-19	Hispanic	Male	1	0 0.021 0.0000	0.022	0.0000
20-24	Black	Female	3	0 0.064 0.0000	0.095	0.0000
20-24	Black	Male	11	0 0.234 0.0000	0.147	0.0000
25-29	Black	Female	4	0 0.085 0.0000	0.037	0.0000
25-29	Black	Male	6	1 0.128 0.1667	0.097	0.0161
25-29	Hispanic	Female	2	0 0.043 0.0000	0.015	0.0000
25-29	White	Female	1	0 0.021 0.0000	0.020	0.0000
30-34	Black	Female	1	0 0.021 0.0000	0.035	0.0000
30-34	Black	Male	6	0 0.128 0.0000	0.070	0.0000

Total: 47 3
 Note: s*P is Stratum rate multiplied by Std. pop. dist.

Adjusted cases = 2.0
 Crude rate = 0.0638
 Adjusted rate = 0.0418
 90% conf. interval: [0.0074, 0.0761]

-> city year = 1 1992

Stratum			Pop.	Cases	Unadjusted		Std. pop. dist.	s*P
					Pop. dist.	Stratum rate		
15-19	Black	Female	3	0	0.054	0.0000	0.077	0.0000
15-19	Black	Male	9	0	0.161	0.0000	0.097	0.0000
15-19	Hispanic	Male	1	0	0.018	0.0000	0.022	0.0000
20-24	Black	Female	7	0	0.125	0.0000	0.095	0.0000
20-24	Black	Male	9	0	0.161	0.0000	0.147	0.0000
20-24	Hispanic	Female	1	0	0.018	0.0000	0.031	0.0000
25-29	Black	Female	2	0	0.036	0.0000	0.037	0.0000
25-29	Black	Male	11	1	0.196	0.0909	0.097	0.0088
25-29	Hispanic	Male	1	0	0.018	0.0000	0.029	0.0000
30-34	Black	Female	7	0	0.125	0.0000	0.035	0.0000
30-34	Black	Male	4	0	0.071	0.0000	0.070	0.0000
30-34	White	Female	1	0	0.018	0.0000	0.011	0.0000

Total: 56 1

Note: s*P is Stratum rate multiplied by Std. pop. dist.

Adjusted cases = 0.5

Crude rate = 0.0179

Adjusted rate = 0.0088

90% conf. interval: [0.0000, 0.0226]

-> city year = 2 1990

Stratum			Pop.	Cases	Unadjusted		Std. pop. dist.	s*P
					Pop. dist.	Stratum rate		
15-19	Black	Female	5	0	0.078	0.0000	0.077	0.0000
15-19	Black	Male	7	1	0.109	0.1429	0.097	0.0138
15-19	Hispanic	Male	1	0	0.016	0.0000	0.022	0.0000
20-24	Black	Female	7	1	0.109	0.1429	0.095	0.0135
20-24	Black	Male	8	0	0.125	0.0000	0.147	0.0000
20-24	Hispanic	Female	5	0	0.078	0.0000	0.031	0.0000
20-24	Hispanic	Male	2	0	0.031	0.0000	0.029	0.0000
20-24	White	Male	2	0	0.031	0.0000	0.046	0.0000
25-29	Black	Female	3	0	0.047	0.0000	0.037	0.0000
25-29	Black	Male	9	0	0.141	0.0000	0.097	0.0000
25-29	Hispanic	Female	2	0	0.031	0.0000	0.015	0.0000
25-29	White	Female	1	0	0.016	0.0000	0.020	0.0000
25-29	White	Male	2	1	0.031	0.5000	0.035	0.0176
30-34	Black	Female	1	0	0.016	0.0000	0.035	0.0000
30-34	Black	Male	5	0	0.078	0.0000	0.070	0.0000
30-34	Hispanic	Female	2	0	0.031	0.0000	0.004	0.0000
30-34	White	Female	1	0	0.016	0.0000	0.011	0.0000
30-34	White	Male	1	0	0.016	0.0000	0.051	0.0000

Total: 64 3

Note: s*P is Stratum rate multiplied by Std. pop. dist.

Adjusted cases = 2.9

Crude rate = 0.0469

Adjusted rate = 0.0449

90% conf. interval: [0.0091, 0.0807]

-> city year = 2 1992

Stratum			Pop.	Cases	Unadjusted Pop. Stratum dist. rate		Std. pop. dist.	s*P
15-19	Black	Female	1	0	0.015	0.0000	0.077	0.0000
15-19	Black	Male	5	0	0.075	0.0000	0.097	0.0000
15-19	Hispanic	Female	3	0	0.045	0.0000	0.011	0.0000
15-19	Hispanic	Male	1	0	0.015	0.0000	0.022	0.0000
15-19	White	Male	1	0	0.015	0.0000	0.011	0.0000
20-24	Black	Female	8	0	0.119	0.0000	0.095	0.0000
20-24	Black	Male	11	0	0.164	0.0000	0.147	0.0000
20-24	Hispanic	Female	6	0	0.090	0.0000	0.031	0.0000
20-24	Hispanic	Male	4	2	0.060	0.5000	0.029	0.0143
20-24	White	Female	1	0	0.015	0.0000	0.009	0.0000
20-24	White	Male	2	0	0.030	0.0000	0.046	0.0000
25-29	Black	Female	2	0	0.030	0.0000	0.037	0.0000
25-29	Black	Male	3	0	0.045	0.0000	0.097	0.0000
25-29	Hispanic	Female	2	0	0.030	0.0000	0.015	0.0000
25-29	Hispanic	Male	4	0	0.060	0.0000	0.029	0.0000
25-29	White	Female	4	0	0.060	0.0000	0.020	0.0000
25-29	White	Male	2	0	0.030	0.0000	0.035	0.0000
30-34	Black	Female	1	0	0.015	0.0000	0.035	0.0000
30-34	Black	Male	2	0	0.030	0.0000	0.070	0.0000
30-34	Hispanic	Male	1	0	0.015	0.0000	0.007	0.0000
30-34	White	Female	2	0	0.030	0.0000	0.011	0.0000
30-34	White	Male	1	0	0.015	0.0000	0.051	0.0000

Total: 67 2
 Note: s*P is Stratum rate multiplied by Std. pop. dist.

Adjusted cases = 1.0
 Crude rate = 0.0299
 Adjusted rate = 0.0143
 90% conf. interval: [0.0025, 0.0260]

-> city year = 3 1990

Stratum			Pop.	Cases	Unadjusted Pop. Stratum dist. rate		Std. pop. dist.	s*P
15-19	Black	Female	3	0	0.043	0.0000	0.077	0.0000
15-19	Black	Male	1	0	0.014	0.0000	0.097	0.0000
15-19	Hispanic	Female	1	0	0.014	0.0000	0.011	0.0000
15-19	White	Female	3	0	0.043	0.0000	0.015	0.0000
15-19	White	Male	1	0	0.014	0.0000	0.011	0.0000
20-24	Black	Female	1	0	0.014	0.0000	0.095	0.0000
20-24	Black	Male	9	0	0.130	0.0000	0.147	0.0000
20-24	Hispanic	Male	3	0	0.043	0.0000	0.029	0.0000
20-24	White	Female	2	0	0.029	0.0000	0.009	0.0000
20-24	White	Male	8	1	0.116	0.1250	0.046	0.0058
25-29	Black	Female	1	0	0.014	0.0000	0.037	0.0000
25-29	Black	Male	8	3	0.116	0.3750	0.097	0.0363
25-29	Hispanic	Male	4	0	0.058	0.0000	0.029	0.0000
25-29	White	Female	1	0	0.014	0.0000	0.020	0.0000
25-29	White	Male	6	0	0.087	0.0000	0.035	0.0000
30-34	Black	Male	6	2	0.087	0.3333	0.070	0.0234
30-34	White	Male	11	5	0.159	0.4545	0.051	0.0230

Total: 69 11

Note: s*P is Stratum rate multiplied by Std. pop. dist.

Adjusted cases = 6.1
 Crude rate = 0.1594
 Adjusted rate = 0.0885
 90% conf. interval: [0.0501, 0.1268]

-> city year = 3 1992

Stratum			Pop.	Cases	Unadjusted Pop. dist.	Stratum rate	Std. pop. dist.	s*P
15-19	Black	Female	2	0	0.054	0.0000	0.077	0.0000
15-19	Hispanic	Male	3	0	0.081	0.0000	0.022	0.0000
15-19	White	Female	2	0	0.054	0.0000	0.015	0.0000
15-19	White	Male	1	0	0.027	0.0000	0.011	0.0000
20-24	Black	Male	3	0	0.081	0.0000	0.147	0.0000
20-24	Hispanic	Female	1	0	0.027	0.0000	0.031	0.0000
20-24	Hispanic	Male	3	0	0.081	0.0000	0.029	0.0000
20-24	White	Female	1	0	0.027	0.0000	0.009	0.0000
20-24	White	Male	6	1	0.162	0.1667	0.046	0.0077
25-29	Hispanic	Male	1	0	0.027	0.0000	0.029	0.0000
25-29	White	Male	5	1	0.135	0.2000	0.035	0.0070
30-34	Black	Male	1	0	0.027	0.0000	0.070	0.0000
30-34	White	Male	8	5	0.216	0.6250	0.051	0.0316

Total: 37 7

Note: s*P is Stratum rate multiplied by Std. pop. dist.

Adjusted cases = 1.7
 Crude rate = 0.1892
 Adjusted rate = 0.0463
 90% conf. interval: [0.0253, 0.0674]

-> city year = 5 1990

Stratum			Pop.	Cases	Unadjusted Pop. dist.	Stratum rate	Std. pop. dist.	s*P
15-19	Black	Female	9	0	0.196	0.0000	0.077	0.0000
15-19	Black	Male	7	0	0.152	0.0000	0.097	0.0000
15-19	Hispanic	Male	1	0	0.022	0.0000	0.022	0.0000
15-19	White	Male	1	0	0.022	0.0000	0.011	0.0000
20-24	Black	Female	4	0	0.087	0.0000	0.095	0.0000
20-24	Black	Male	6	0	0.130	0.0000	0.147	0.0000
20-24	Hispanic	Female	1	0	0.022	0.0000	0.031	0.0000
25-29	Black	Female	3	1	0.065	0.3333	0.037	0.0125
25-29	Black	Male	5	0	0.109	0.0000	0.097	0.0000
25-29	Hispanic	Female	1	0	0.022	0.0000	0.015	0.0000
25-29	White	Female	2	1	0.043	0.5000	0.020	0.0099
30-34	Black	Female	2	0	0.043	0.0000	0.035	0.0000
30-34	Black	Male	3	0	0.065	0.0000	0.070	0.0000
30-34	White	Male	1	0	0.022	0.0000	0.051	0.0000

Total: 46 2

Note: s*P is Stratum rate multiplied by Std. pop. dist.

Adjusted cases = 1.0
 Crude rate = 0.0435
 Adjusted rate = 0.0223
 90% conf. interval: [0.0020, 0.0426]

-> city year = 5 1992

Stratum			Pop.	Cases	Unadjusted		Std. pop. dist.	s*P
					Pop. dist.	Stratum rate		
15-19	Black	Female	6	0	0.087	0.0000	0.077	0.0000
15-19	Black	Male	9	0	0.130	0.0000	0.097	0.0000
15-19	Hispanic	Female	1	0	0.014	0.0000	0.011	0.0000
15-19	Hispanic	Male	2	0	0.029	0.0000	0.022	0.0000
15-19	White	Female	2	0	0.029	0.0000	0.015	0.0000
15-19	White	Male	1	0	0.014	0.0000	0.011	0.0000
20-24	Black	Female	13	0	0.188	0.0000	0.095	0.0000
20-24	Black	Male	10	0	0.145	0.0000	0.147	0.0000
20-24	Hispanic	Male	1	0	0.014	0.0000	0.029	0.0000
20-24	White	Male	3	0	0.043	0.0000	0.046	0.0000
25-29	Black	Female	2	0	0.029	0.0000	0.037	0.0000
25-29	Black	Male	2	0	0.029	0.0000	0.097	0.0000
25-29	Hispanic	Male	3	0	0.043	0.0000	0.029	0.0000
25-29	White	Male	1	0	0.014	0.0000	0.035	0.0000
30-34	Black	Female	4	0	0.058	0.0000	0.035	0.0000
30-34	Black	Male	5	0	0.072	0.0000	0.070	0.0000
30-34	Hispanic	Male	2	0	0.029	0.0000	0.007	0.0000
30-34	White	Female	1	0	0.014	0.0000	0.011	0.0000
30-34	White	Male	1	1	0.014	1.0000	0.051	0.0505

Total: 69 1

Note: s*P is Stratum rate multiplied by Std. pop. dist.

Adjusted cases = 3.5

Crude rate = 0.0145

Adjusted rate = 0.0505

90% conf. interval: [0.0505, 0.0505]

Summary of study populations

city year	N	Crude rate	Adjusted rate	[90% conf. interval]	
1					
1990	47	0.063830	0.041758	0.007427	0.076089
1					
1992	56	0.017857	0.008791	0.000000	0.022579
2					
1990	64	0.046875	0.044898	0.009072	0.080724
2					
1992	67	0.029851	0.014286	0.002537	0.026035
3					
1990	69	0.159420	0.088453	0.050093	0.126813
3					
1992	37	0.189189	0.046319	0.025271	0.067366
5					
1990	46	0.043478	0.022344	0.002044	0.042644
5					
1992	69	0.014493	0.050549	0.050549	0.050549

Indirect standardization

Standardization of rates can be performed via the indirect method whenever the stratum-specific rates are either unknown or unreliable. If the stratum-specific rates are known, the direct standardization method is preferred.

To apply the indirect method, you must have the following information:

- The observed number of cases in each population to be standardized, O . For example, if deathrates in two states are being standardized using the US deathrate for the same period, you must know the total number of deaths in each state.
- The distribution across the various strata for the population being studied, n_1, \dots, n_k . If you are standardizing the deathrate in the two states, adjusting for age, you must know the number of individuals in each of the k age groups.
- The stratum-specific rates for the standard population, p_1, \dots, p_k . For example, you must have the US deathrate for each stratum (age group).
- The crude rate of the standard population, C . For example, you must have the US mortality rate for the year.

The indirect adjusted rate is then

$$R_{\text{indirect}} = C \frac{O}{E}$$

where E is the expected number of cases (deaths) in each population. See [Methods and formulas](#) for a more detailed description of calculations.

► Example 3

This example is borrowed from [Kahn and Sempos \(1989, 95–105\)](#). We want to compare 1970 mortality rates in California and Maine, adjusting for age. Although we have age-specific population counts for the two states, we lack age-specific deathrates. Direct standardization is not feasible here. We can use the US population census data for the same year to produce indirectly standardized rates for these two states.

From the US census, the standard population for this example was entered into Stata and saved in `popkahn.dta`.

- ```
. use https://www.stata-press.com/data/r19/popkahn, clear
. list age pop deaths rate, sep(4)
```

|    | age   | population | deaths  | rate   |
|----|-------|------------|---------|--------|
| 1. | <15   | 57,900,000 | 103,062 | .00178 |
| 2. | 15–24 | 35,441,000 | 45,261  | .00128 |
| 3. | 25–34 | 24,907,000 | 39,193  | .00157 |
| 4. | 35–44 | 23,088,000 | 72,617  | .00315 |
| 5. | 45–54 | 23,220,000 | 169,517 | .0073  |
| 6. | 55–64 | 18,590,000 | 308,373 | .01659 |
| 7. | 65–74 | 12,436,000 | 445,531 | .03583 |
| 8. | 75+   | 7,630,000  | 736,758 | .09656 |

The standard population contains for each age stratum the total number of individuals (pop) and both the age-specific mortality rate (rate) and the number of deaths. The standard population need not contain all three. If we have only the age-specific mortality rate, we can use the `rate(ratevarp crudevarp)` or `rate(ratevarp #)` option, where `crudevarp` refers to the variable containing the total population's crude deathrate or `#` is the total population's crude deathrate.

Now, let's look at the states' data (study population):

```
. use https://www.stata-press.com/data/r19/kahn
. list, sep(4)
```

|     | state      | age   | populat-n | death   | st | death_~e |
|-----|------------|-------|-----------|---------|----|----------|
| 1.  | California | <15   | 5,524,000 | 166,285 | 1  | .0016    |
| 2.  | California | 15-24 | 3,558,000 | 166,285 | 1  | .0013    |
| 3.  | California | 25-34 | 2,677,000 | 166,285 | 1  | .0015    |
| 4.  | California | 35-44 | 2,359,000 | 166,285 | 1  | .0028    |
| 5.  | California | 45-54 | 2,330,000 | 166,285 | 1  | .0067    |
| 6.  | California | 55-64 | 1,704,000 | 166,285 | 1  | .0154    |
| 7.  | California | 65-74 | 1,105,000 | 166,285 | 1  | .0328    |
| 8.  | California | 75+   | 696,000   | 166,285 | 1  | .0917    |
| 9.  | Maine      | <15   | 286,000   | 11,051  | 2  | .0019    |
| 10. | Maine      | 15-24 | 168,000   | .       | 2  | .0011    |
| 11. | Maine      | 25-34 | 110,000   | .       | 2  | .0014    |
| 12. | Maine      | 35-44 | 109,000   | .       | 2  | .0029    |
| 13. | Maine      | 45-54 | 110,000   | .       | 2  | .0069    |
| 14. | Maine      | 55-64 | 94,000    | .       | 2  | .0173    |
| 15. | Maine      | 65-74 | 69,000    | .       | 2  | .039     |
| 16. | Maine      | 75+   | 46,000    | .       | 2  | .1041    |

For each state, the number of individuals in each stratum (age group) is contained in the pop variable. The death variable is the total number of deaths observed in the state during the year. It must have the same value for all observations in the group, as for California, or it could be missing in all but one observation per group, as for Maine.

To match these two datasets, the strata variables must have the same name in both datasets and ideally the same levels. If a level is missing from either dataset, that level will not be included in the standardization.

With `kahn.dta` in memory, we now execute the command. We will use the `print` option to obtain the standard population's summary table, and because we have both the standard population's age-specific count and deaths, we will specify the `popvars(casevarp popvarp)` option. Or, we could specify the `rate(rate 0.00945)` option because we know that 0.00945 is the US crude deathrate for 1970.

```
. istdize death pop age using https://www.stata-press.com/data/r19/popkahn,
> by(state) pop(deaths pop) print
```

Standard population

| Stratum | Rate    |
|---------|---------|
| <15     | 0.00178 |
| 15-24   | 0.00128 |
| 25-34   | 0.00157 |
| 35-44   | 0.00315 |
| 45-54   | 0.00730 |
| 55-64   | 0.01659 |
| 65-74   | 0.03583 |
| 75+     | 0.09656 |

Crude rate = 0.00945

Indirect standardization

-> state = California

| Stratum | Standard population rate | Observed population | Expected cases |
|---------|--------------------------|---------------------|----------------|
| <15     | 0.0018                   | 5,524,000           | 9,832.72       |
| 15-24   | 0.0013                   | 3,558,000           | 4,543.85       |
| 25-34   | 0.0016                   | 2,677,000           | 4,212.46       |
| 35-44   | 0.0031                   | 2,359,000           | 7,419.59       |
| 45-54   | 0.0073                   | 2,330,000           | 17,010.10      |
| 55-64   | 0.0166                   | 1,704,000           | 28,266.14      |
| 65-74   | 0.0358                   | 1,105,000           | 39,587.63      |
| 75+     | 0.0966                   | 696,000             | 67,206.23      |

Total: 19,953,000 178,078.73  
 Observed cases = 166,285  
 SMR (Obs/Exp) = 0.93  
 SMR exact 95% conf. interval: [0.9293, 0.9383]  
 Crude rate = 0.0083  
 Adjusted rate = 0.0088  
 95% conf. interval: [0.0088, 0.0089]

-> state = Maine

| Stratum | Standard population rate | Observed population | Expected cases |
|---------|--------------------------|---------------------|----------------|
| <15     | 0.0018                   | 286,000             | 509.08         |
| 15-24   | 0.0013                   | 168,000             | 214.55         |
| 25-34   | 0.0016                   | 110,000             | 173.09         |
| 35-44   | 0.0031                   | 109,000             | 342.83         |
| 45-54   | 0.0073                   | 110,000             | 803.05         |
| 55-64   | 0.0166                   | 94,000              | 1,559.28       |
| 65-74   | 0.0358                   | 69,000              | 2,471.99       |
| 75+     | 0.0966                   | 46,000              | 4,441.79       |

Total: 992,000 10,515.67

```

Observed cases = 11,051
SMR (Obs/Exp) = 1.05
SMR exact 95% conf. interval: [1.0314, 1.0707]
Crude rate = 0.0111
Adjusted rate = 0.0099
95% conf. interval: [0.0097, 0.0101]

```

## Summary of study populations, reporting rates

| state      | Observed cases | Crude rate | Adjusted rate | [95% conf. interval] |          |
|------------|----------------|------------|---------------|----------------------|----------|
| California | 166,285        | 0.008334   | 0.008824      | 0.008782             | 0.008866 |
| Maine      | 11,051         | 0.011140   | 0.009931      | 0.009747             | 0.010118 |

## Summary of study populations, reporting SMRs

| state      | Observed cases | Expected cases | SMR   | Exact [95% conf. interval] |          |
|------------|----------------|----------------|-------|----------------------------|----------|
| California | 166,285        | 178,078.73     | 0.934 | 0.929290                   | 0.938271 |
| Maine      | 11,051         | 10,515.67      | 1.051 | 1.031405                   | 1.070688 |



## Stored results

`dstdize` stores the following in `r()`:

## Scalars

`r(k)` number of populations

## Macros

`r(by)` variable names specified in `by()`

`r(c#)` values of `r(by)` for `#th` group

## Matrices

`r(se)`  $1 \times k$  vector of standard errors of adjusted rates

`r(ub_adj)`  $1 \times k$  vector of upper bounds of confidence intervals for adjusted rates

`r(lb_adj)`  $1 \times k$  vector of lower bounds of confidence intervals for adjusted rates

`r(Nobs)`  $1 \times k$  vector of number of observations

`r(crude)`  $1 \times k$  vector of crude rates (\*)

`r(adj)`  $1 \times k$  vector of adjusted rates (\*)

(\*) If, in a group, the number of observations is 0, then 9 is stored for the corresponding crude and adjusted rates.

`istdize` stores the following in `r()`:

## Scalars

`r(k)` number of populations

## Macros

`r(by)` variable names specified in `by()`

`r(c#)` values of `r(by)` for `#th` group

## Matrices

`r(cases_obs)`  $1 \times k$  vector of number of observed cases

`r(cases_exp)`  $1 \times k$  vector of number of expected cases

`r(ub_adj)`  $1 \times k$  vector of upper bounds of confidence intervals for adjusted rates

`r(lb_adj)`  $1 \times k$  vector of lower bounds of confidence intervals for adjusted rates

`r(crude)`  $1 \times k$  vector of crude rates

|                     |                                                                      |
|---------------------|----------------------------------------------------------------------|
| $r(\text{adj})$     | $1 \times k$ vector of adjusted rates                                |
| $r(\text{smr})$     | $1 \times k$ vector of SMRs                                          |
| $r(\text{ub\_smr})$ | $1 \times k$ vector of upper bounds of confidence intervals for SMRs |
| $r(\text{lb\_smr})$ | $1 \times k$ vector of lower bounds of confidence intervals for SMRs |

## Methods and formulas

The directly standardized rate,  $S_R$ , is defined by

$$S_R = \frac{\sum_{i=1}^k w_i R_i}{\sum_{i=1}^k w_i}$$

(Rothman 1986, 44), where  $R_i$  is the stratum-specific rate in stratum  $i$  and  $w_i$  is the weight for stratum  $i$  derived from the standard population.

If  $n_i$  is the population of stratum  $i$ , the standard error,  $\text{se}(S_R)$ , in stratified sampling for proportions (ignoring the finite population correction) is

$$\text{se}(S_R) = \frac{1}{\sum w_i} \sqrt{\sum_{i=1}^k \frac{w_i^2 R_i (1 - R_i)}{n_i}}$$

(Cochran 1977, 108), from which the confidence intervals are calculated.

For indirect standardization, define  $O$  as the observed number of cases in each population to be standardized;  $n_1, \dots, n_k$  as the distribution across the various strata for the population being studied;  $R_1, \dots, R_k$  as the stratum-specific rates for the standard population; and  $C$  as the crude rate of the standard population. The expected number of cases (deaths),  $E$ , in each population is obtained by applying the standard population stratum-specific rates,  $R_1, \dots, R_k$ , to the study populations:

$$E = \sum_{i=1}^k n_i R_i$$

The indirectly adjusted rate is then

$$R_{\text{indirect}} = C \frac{O}{E}$$

and  $O/E$  is the study population's SMR if death is the event of interest or the SIR for studies of disease (or other) incidence.

The exact confidence interval is calculated for each estimated SMR by assuming a Poisson process as described in Breslow and Day (1987, 69–71). These intervals are obtained by first calculating the upper and lower bounds for the confidence interval of the Poisson-distributed observed events,  $O$ —say,  $L$  and  $U$ , respectively—and then computing  $\text{SMR}_L = L/E$  and  $\text{SMR}_U = U/E$ .

## Acknowledgments

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## Also see

[R] [EpiTab](#) — Tables for epidemiologists

[SVY] [Direct standardization](#) — Direct standardization of means, proportions, and ratios

## **dtable** — Create a table of descriptive statistics

[Description](#)

[Remarks and examples](#)

[Also see](#)

[Quick start](#)

[Methods and formulas](#)

[Menu](#)

[Appendix](#)

[Syntax](#)

[Acknowledgments](#)

[Options](#)

[References](#)

## Description

The `dtable` command allows you to easily create a table of descriptive (summary) statistics, commonly known as “Table 1”. Optionally, you can add  $p$ -values from test statistics comparing groups on summary statistics. You can also create a table complete with a title, notes, and more, and then export it to a variety of file types.

## Quick start

Create a table describing continuous variables `x1` and `x2` and factor variables `f1` and `f2`

```
dtable x1 x2 i.f1 i.f2
```

Same as above, but report statistics for each level of variable group, with and without the total sample

```
dtable x1 x2 i.f1 i.f2, by(group)
dtable x1 x2 i.f1 i.f2, by(group, nototals)
```

Same as above, but test each variable for equality between groups using linear regressions for continuous variables and Pearson  $\chi^2$  tests for factor variables

```
dtable x1 x2 i.f1 i.f2, by(group, tests)
```

Same as above, but rather than putting sample frequencies in the first row, put them in the column header under each group label

```
dtable x1 x2 i.f1 i.f2, by(group, tests) sample(, place(seplabels))
```

Create a table of the median and interquartile interval for variables `x1-x4`

```
dtable, continuous(x1-x4, statistics(q2 iqi)) ///
define(iqi = q1 q3) sformat("%s" iqi)
```

Same as above, but add the Kruskal–Wallis test for equality between levels of variable group

```
dtable, continuous(x1-x4, statistics(q2 iqi) test(kwallis)) ///
define(iqi = q1 q3) sformat("%s" iqi) by(group, tests)
```

Create a table to describe variables using survey weights

```
dtable x* i.f*, svy
```

Same as above, but restrict the sample to the subpopulation where group equals 4

```
dtable x* i.f*, svy subpop(if group==4)
```

## Menu

Statistics > Summaries, tables, and tests > Table of descriptive statistics

## Syntax

```
dtable [varlist] [if] [in] [weight] [, options]
```

| <i>options</i>                                                          | Description                                                 |
|-------------------------------------------------------------------------|-------------------------------------------------------------|
| <b>Main</b>                                                             |                                                             |
| <code>by(<i>varname</i> [, <i>by_opts</i>])</code>                      | group results by categories of <i>varname</i>               |
| <code>column(<i>col_opts</i>)</code>                                    | change column header styles and titles                      |
| <b>Survey</b>                                                           |                                                             |
| <code>svy</code>                                                        | use survey settings                                         |
| <code>subpop([<i>varname</i>] [<i>if</i>])</code>                       | identify a survey subpopulation                             |
| <b>Sample</b>                                                           |                                                             |
| <code>nosample</code>                                                   | suppress sample frequency statistics                        |
| <code>sample([<i>title</i>] [, <i>samp_opts</i>])</code>                | select sample frequency statistics                          |
| <b>Continuous</b>                                                       |                                                             |
| <code>continuous([<i>varlist_c</i>] [, <i>cont_opts</i>])</code>        | select continuous variable statistics                       |
| <b>Factors</b>                                                          |                                                             |
| <code>factor([<i>varlist_f</i>] [, <i>fact_opts</i>])</code>            | select factor-variable statistics                           |
| <b>Composite</b>                                                        |                                                             |
| <code>define(<i>name</i> = <i>elements</i> [, <i>def_opts</i>])</code>  | define a composite result                                   |
| <b>Formats</b>                                                          |                                                             |
| <code>nformat(<i>%fmt</i> [<i>results</i>] [, <i>basestyle</i>])</code> | specify numeric format                                      |
| <code>sformat(<i>sfmt</i> [<i>results</i>])</code>                      | specify string format                                       |
| <b>Title</b>                                                            |                                                             |
| <code>title(<i>string</i>)</code>                                       | add table title                                             |
| <code>titlestyles(<i>text_styles</i>)</code>                            | change table title styles                                   |
| <b>Notes</b>                                                            |                                                             |
| <code>note(<i>string</i>)</code>                                        | add table note                                              |
| <code>notestyles(<i>text_styles</i>)</code>                             | change table note styles                                    |
| <b>Export</b>                                                           |                                                             |
| <code>export(<i>filename.suffix</i> [, <i>export_opts</i>])</code>      | export table                                                |
| <b>Options</b>                                                          |                                                             |
| <code>[no] varlabel</code>                                              | display or suppress variable names or labels                |
| <code>[no] fvlabel</code>                                               | display or suppress factor values or labels                 |
| <code>[no] listwise</code>                                              | specify how to handle missing values                        |
| <code>halign(<i>hvalue</i>)</code>                                      | specify horizontal alignment of item cells                  |
| <code>name(<i>cname</i>)</code>                                         | work with collection <i>cname</i> ; default is name(DTable) |
| <code>replace</code>                                                    | replace the collection                                      |
| <code>label(<i>filename</i>)</code>                                     | specify the collection labels                               |
| <code>style(<i>filename</i> [, <i>override</i>])</code>                 | specify the collection style                                |
| <code>warn</code>                                                       | show collect warnings                                       |

warn does not appear in the dialog box.

| <i>by_opts</i> | Description                                               |
|----------------|-----------------------------------------------------------|
| [no]tests      | show or suppress tests across groups                      |
| [no]testnotes  | show or suppress notes about tests across groups          |
| [no]totals     | show or suppress statistics for the total sample          |
| [no]missing    | show or suppress numeric missing values in <i>varname</i> |

| <i>col_opts</i>                                 | Description                                                                                        |
|-------------------------------------------------|----------------------------------------------------------------------------------------------------|
| summary([ <i>title</i> ][, <i>hcol_opts</i> ])  | change header styles for the summary column                                                        |
| by([ <i>by_style</i> ][, <i>by_hcol_opts</i> ]) | change header styles for the by() variable columns;<br><i>by_style</i> may be label, name, or hide |
| test([ <i>title</i> ][, <i>hcol_opts</i> ])     | change header styles for the test column                                                           |
| total([ <i>title</i> ][, <i>hcol_opts</i> ])    | change header styles for the total sample column                                                   |

| <i>hcol_opts</i>        | Description                                         |
|-------------------------|-----------------------------------------------------|
| [no]hide                | show or hide the column header title                |
| halign( <i>hvalue</i> ) | specify horizontal alignment of column header cells |
| <i>text_styles</i>      | change column header text styles                    |

nohide and hide do not appear in the dialog box for summary().

| <i>by_hcol_opts</i>     | Description                                         |
|-------------------------|-----------------------------------------------------|
| [no]fvlabel             | display by() variable values or value labels        |
| halign( <i>hvalue</i> ) | specify horizontal alignment of column header cells |
| <i>text_styles</i>      | change column header text styles                    |

| <i>text_styles</i>                               | Description                                              |
|--------------------------------------------------|----------------------------------------------------------|
| font([ <i>fontfamily</i> ][, <i>font_opts</i> ]) | specify font style                                       |
| smcl( <i>smcl</i> )                              | specify formatting for SMCL files                        |
| latex( <i>latex</i> )                            | specify L <sup>A</sup> T <sub>E</sub> X macro            |
| shading( <i>sspec</i> )                          | set background color, foreground color, and fill pattern |

| <i>font_opts</i>             | Description                              |
|------------------------------|------------------------------------------|
| size(# [ <i>unit</i> ])      | specify font size                        |
| color( <i>color</i> )        | specify font color                       |
| variant( <i>variant</i> )    | specify font variant and capitalization  |
| [no]bold                     | specify whether to format text as bold   |
| [no]italic                   | specify whether to format text as italic |
| [no]strikeout                | specify whether to strike out text       |
| [no]underline                | specify whether to underline text        |
| underline( <i>upattern</i> ) | specify underline pattern for text       |

| <i>samp_opts</i>                       | Description                                                                                                                                 |
|----------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------|
| <code>statistics(<i>nstats</i>)</code> | select sample frequency statistics                                                                                                          |
| <code>place(<i>place</i>)</code>       | select location for sample frequency statistics; <i>place</i> may be <code>items</code> , <code>inlabels</code> , or <code>seplabels</code> |
| <code>text_styles</code>               | change row header text styles                                                                                                               |

| <i>cont_opts</i>                       | Description                           |
|----------------------------------------|---------------------------------------|
| <code>statistics(<i>cstats</i>)</code> | select continuous variable statistics |
| <code>test(<i>ctest</i>)</code>        | select continuous variable test       |
| <code>text_styles</code>               | change row header text styles         |

| <i>fact_opts</i>                       | Description                       |
|----------------------------------------|-----------------------------------|
| <code>statistics(<i>fstats</i>)</code> | select factor-variable statistics |
| <code>test(<i>fctest</i>)</code>       | select factor-variable test       |
| <code>text_styles</code>               | change row header text styles     |

| <i>def_opts</i>                     | Description                                        |
|-------------------------------------|----------------------------------------------------|
| <code>delimiter(<i>char</i>)</code> | use character as delimiter between <i>elements</i> |
| <code>[no]trim</code>               | preserve or trim extra spaces from numeric formats |
| <code>[no]override</code>           | preserve or override trim property of elements     |
| <code>replace</code>                | redefine <i>name</i> if it already exists          |

| <i>suffix</i>         | <i>fileformat</i>         | Output format                      |
|-----------------------|---------------------------|------------------------------------|
| <code>docx</code>     | <code>as(docx)</code>     | Microsoft Word                     |
| <code>html</code>     | <code>as(html)</code>     | HTML 5 with CSS                    |
| <code>pdf</code>      | <code>as(pdf)</code>      | PDF                                |
| <code>xlsx</code>     | <code>as(xlsx)</code>     | Microsoft Excel 2007/2010 or newer |
| <code>xls</code>      | <code>as(xls)</code>      | Microsoft Excel 1997/2003          |
| <code>tex</code>      | <code>as(tex)</code>      | L <sup>A</sup> T <sub>E</sub> X    |
| <code>smcl</code>     | <code>as(smcl)</code>     | SMCL                               |
| <code>txt</code>      | <code>as(txt)</code>      | plain text                         |
| <code>markdown</code> | <code>as(markdown)</code> | Markdown                           |
| <code>md</code>       | <code>as(md)</code>       | Markdown                           |

| <i>export_opts</i>              | Description                                         |
|---------------------------------|-----------------------------------------------------|
| <i>as</i> ( <i>fileformat</i> ) | specify document type                               |
| <i>replace</i>                  | overwrite existing file                             |
| <i>docx_options</i>             | available when exporting to .docx files             |
| <i>html_options</i>             | available when exporting to .html files             |
| <i>pdf_options</i>              | available when exporting to .pdf files              |
| <i>excel_options</i>            | available when exporting to .xls and .xlsx files    |
| <i>tex_options</i>              | available when exporting to .tex files              |
| <i>smcl_option</i>              | available when exporting to .smcl files             |
| <i>txt_option</i>               | available when exporting to .txt files              |
| <i>md_option</i>                | available when exporting to .markdown and .md files |

| <i>docx_options</i>                                   | Description                                                         |
|-------------------------------------------------------|---------------------------------------------------------------------|
| <i>noisily</i>                                        | show the putdocx commands used to export to the Microsoft Word file |
| <i>dofile</i> ( <i>filename</i> [ , <i>replace</i> ]) | save the putdocx commands used for exporting to the named do-file   |

| <i>html_options</i>               | Description                                                    |
|-----------------------------------|----------------------------------------------------------------|
| <i>append</i>                     | append to an existing file                                     |
| <i>tableonly</i>                  | export only the table to the specified file                    |
| <i>cssfile</i> ( <i>cssfile</i> ) | define the styles in <i>cssfile</i> instead of <i>filename</i> |
| <i>prefix</i> ( <i>prefix</i> )   | use <i>prefix</i> to identify style classes                    |

| <i>pdf_options</i>                                    | Description                                                      |
|-------------------------------------------------------|------------------------------------------------------------------|
| <i>noisily</i>                                        | show the putpdf commands used to export to the PDF file          |
| <i>dofile</i> ( <i>filename</i> [ , <i>replace</i> ]) | save the putpdf commands used for exporting to the named do-file |

| <i>excel_options</i>                                  | Description                                                                                             |
|-------------------------------------------------------|---------------------------------------------------------------------------------------------------------|
| <i>noisily</i>                                        | show the putexcel commands used to export to the Excel file                                             |
| <i>dofile</i> ( <i>filename</i> [ , <i>replace</i> ]) | save the putexcel commands used for exporting to the named do-file                                      |
| <i>sheet</i> ( <i>sheetname</i> [ , <i>replace</i> ]) | specify the worksheet to use; the default sheet name is Sheet1                                          |
| <i>cell</i> ( <i>cell</i> )                           | specify the Excel upper-left cell as the starting position to export the table; the default is cell(A1) |
| <i>modify</i>                                         | modify Excel file                                                                                       |
| <i>noopen</i>                                         | do not open Excel file in memory                                                                        |

*noopen* does not appear in the dialog box.

| <i>tex_options</i> | Description                                 |
|--------------------|---------------------------------------------|
| append             | append to an existing file                  |
| tableonly          | export only the table to the specified file |

| <i>smcl_option</i> | Description                |
|--------------------|----------------------------|
| append             | append to an existing file |

| <i>txt_option</i> | Description                |
|-------------------|----------------------------|
| append            | append to an existing file |

| <i>md_option</i> | Description                |
|------------------|----------------------------|
| append           | append to an existing file |

*varlist* may contain `i.` notation, including level selection. Time-series operators and interactions are not allowed.

*fweights*, *aweight*s, *iweight*s, and *pweight*s are allowed; see [U] 11.1.6 **weight**.

*results* is a list of names taken from *nstats*, *cstats*, *ctest*, *fstats*, *frest*, and composite results.

*hvalue* may be `left`, `center`, or `right`.

*fontfamily* specifies a valid font family.

*unit* may be `in` (inch), `pt` (point), or `cm` (centimeter). An inch is equivalent to 72 points and 2.54 centimeters. The default is `pt`.

*variant* may be `allcaps`, `smallcaps`, or `normal`.

`variant(allcaps)` changes the text to all uppercase letters; applicable when publishing items from a collection to Microsoft Word, PDF, L<sup>A</sup>T<sub>E</sub>X, and HTML files.

`variant(smallcaps)` changes the text to use large capitals for uppercase letters and smaller capitals for lowercase letters; applicable when publishing items from a collection to Microsoft Word, L<sup>A</sup>T<sub>E</sub>X, and HTML files.

`variant(normal)` changes the font variant back to normal and leaves the capitalization unchanged from the original text; applicable when publishing items from a collection to Microsoft Word, PDF, L<sup>A</sup>T<sub>E</sub>X, and HTML files.

*upattern* may be any of the patterns listed in the [Appendix](#). For example, `underline(none)` removes the underline from the text, and `underline(single)` underlines the text. All other *upattern*s are available only when publishing items from a collection to Microsoft Word.

*smcl* specifies the name of the SMCL directive to render text for SMCL output. The supported SMCL directives are `input`, `error`, `result`, and `text`.

*latex* specifies the name of a L<sup>A</sup>T<sub>E</sub>X macro to render text for L<sup>A</sup>T<sub>E</sub>X output. Example L<sup>A</sup>T<sub>E</sub>X macro names are `textbf`, `textsf`, `textrm`, and `texttt`. Custom L<sup>A</sup>T<sub>E</sub>X macros are also allowed. If *text* is to be rendered in a cell, title, or note, then *latex* is translated to the following when you export to L<sup>A</sup>T<sub>E</sub>X:

```
\latex {text}
```

*sspec* is

```
[background(bgcolor) foreground(fgcolor) pattern(fpattern)]
```

*bgcolor* specifies the background color.

*fgcolor* specifies the foreground color.

*fpattern* specifies the fill pattern. A complete list of fill patterns is shown in the [Appendix](#).

*bgcolor*, *fgcolor*, and *color* may be one of the colors listed in the [Appendix](#); a valid RGB value in the form ### # ## # ##, for example, 171 248 103; or a valid RRGGBB hex value in the form #####, for example, ABF867.

*fmt* is the specification for a string format in option `sformat()` and may contain a mix of text and `%s`. Here `%s` refers to the statistic value that is formatted as specified using `nformat()`. The text will be placed around the statistic as it is placed around `%s` in this option. For instance, to place parentheses around the statistic, you can specify `sformat("(%s)")`.

Two text characters must be specified using a special character sequence if you want them to be displayed in your collection. To include `%`, type `%%`. To include `\`, type `\\`. For instance, to place a percent sign after a statistic, you can specify `sformat("%s%")`.

## Options

Main

`by(varname [ , by_opts ])` specifies that the statistics be displayed separately for each unique value of *varname*; *varname* may be numeric or string.

*by\_opts* are the following:

`tests` and `notests` specify whether to show the tests comparing groups. `tests` displays the tests comparing groups; `notests` suppresses them.

When `tests` is combined with the `missing` suboption, the test will be performed with the missing values composing an additional group. By default, missing values are excluded from tests.

`testnotes` and `notestnotes` specify whether to show a note detailing each test comparing groups. `testnotes` displays the notes; `notestnotes` suppresses them.

`totals` and `nototals` specify whether to show the statistics for the total sample. `totals` displays the statistics for the total sample; `nototals` suppresses them.

`missing` and `nomissing` specify whether to show numeric missing values in *varname*. `missing` displays the numeric missing values; `nomissing` suppresses them.

When `missing` is combined with the `tests` suboption, missing values will also be included in the test performed. By default, missing values are excluded from tests.

`column(col_opts)` specifies the column header styles and titles. *col\_opts* are one or more of the following.

`summary([title][, [no]hide halign(hvalue) text_styles])` specifies the column header and styles when global option `by()` is not specified. The default *title* is `Summary`.

`hide` and `nohide` specify whether to show the title. `hide` suppresses the title; `nohide` displays the title.

`halign(hvalue)` specifies how the column header cell is horizontally aligned. *hvalue* may be `left`, `center`, or `right`.

*text\_styles* affect the text styles for the column header.

`by([by_style][, [no]fvlabel halign(hvalue) text_styles])` specifies the `by()` variable's column header and styles.

*by\_style* specifies the column header style for the `by()` variable. *by\_style* may be `label`, `name`, or `hide`.

`label` specifies that the `by()` variable's label be used. If the `by()` variable does not have a label, then the name is used.

`name` specifies that the `by()` variable's name be used.

`hide` specifies that the `by()` variable not be shown in the column header.

`fvlabel` and `nofvlabel` specify whether value labels for the levels of the `by()` variable should be displayed. `fvlabel` displays the value labels; `nofvlabel` displays the values.

`halign(hvalue)` specifies how the column header cells are horizontally aligned. *hvalue* may be `left`, `center`, or `right`.

*text\_styles* affect the text styles for the column header.

`test([title][, [no]hide halign(hvalue) text_styles])` specifies the column header and text styles for the test column. The default *title* is `Test`.

`hide` and `nohide` specify whether to show the title for the test column. `hide` suppresses the title; `nohide` displays the title.

`halign(hvalue)` specifies how the test column header cell is horizontally aligned. *hvalue* may be `left`, `center`, or `right`.

*text\_styles* affect the text styles for the test column header.

`total([title][, [no]hide halign(hvalue) text_styles])` specifies the column header and text styles for the total sample. The default *title* is `Total`.

`hide` and `nohide` specify whether to show the title for the total sample. `hide` suppresses the title; `nohide` displays the title.

`halign(hvalue)` specifies how the total sample column header cell is horizontally aligned. *hvalue* may be `left`, `center`, or `right`.

*text\_styles* affect the text styles for the total sample column header.

*text\_styles* are `font([fontfamily][, font_opts])`, `smcl(smcl)`, `latex(latex)`, and `shading(sspec)`.

`font([fontfamily][, size(#[unit]) color(color) variant(variant) [no]bold [no]italic [no]strikeout [no]underline underline(upattern)])` specifies the font style. These font style properties are applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files, unless otherwise specified.

*fontfamily* specifies a valid font family. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`size(#[unit])` specifies the font size as a number optionally followed by units. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`color(color)` specifies the text color.

`variant(variant)` specifies the font variant and capitalization.

`bold` and `nobold` specify the font weight. `bold` changes the font weight to bold; `nobold` changes the font weight back to normal.

`italic` and `noitalic` specify the font style. `italic` changes the font style to italic; `noitalic` changes the font style back to normal.

`strikeout` and `nostrikeout` specify whether to add a strikeout mark to the text. `strikeout` adds a strikeout mark to the text; `nostrikeout` changes the text back to normal.

`underline(upattern)`, `underline`, and `nounderline` specify how to underline the text.

Only one of `strikeout` or `underline` is allowed when publishing to HTML files.

`smcl(smcl)` specifies how to render the text for SMCL output. This style property is applicable only when publishing items from a collection to a SMCL file.

`latex(latex)` specifies how to render the text for  $\LaTeX$  output. This style property is applicable only when publishing items from a collection to a  $\LaTeX$  file.

`shading(sspec)` sets the background color, foreground color, and fill pattern. The background color is applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files. The foreground color and fill pattern are applicable when exporting the table to Microsoft Word and Microsoft Excel.

#### Survey

`svy` specifies that `dtable` use the survey design information from `svyset`. This means all statistics will be computed using the specified survey weights, where applicable. This option changes the list of supported tests in `continuous()` and `factor()`.

`subpop([varname] [if])` specifies that estimates be computed for the single subpopulation identified by the observations for which  $varname \neq 0$  and that also meet the `if` conditions. Typically,  $varname = 1$  defines the subpopulation, and  $varname = 0$  indicates observations not belonging to the subpopulation. For observations whose subpopulation status is uncertain, *varname* should be set to a missing value; such observations are dropped from the estimation sample. This option implies the `svy` option.

See [\[SVY\] Subpopulation estimation](#) for more information.

## Sample

`nosample` and `sample()` control the display of sample frequency statistics.

`nosample` prevents reporting any sample frequency statistic.

`sample([title][, statistics(nstats) place(place) text_styles])` specifies sample frequency statistics, their row header title, and how they are composed in the table.

*title* specifies the row header for the sample frequency statistics. The default *title* is `N`.

*statistics*(*nstats*) specifies the sample frequency statistics to report.

*nstats* may include the following sample frequency statistics and any composite result defined from them.

| <i>nstats</i>        | Definition            |
|----------------------|-----------------------|
| <u>frequency</u>     | frequency             |
| <u>sumw</u>          | sum of weights        |
| <u>percent</u>       | percentage            |
| <u>proportion</u>    | proportion            |
| <u>rawpercent</u>    | unweighted percentage |
| <u>rawproportion</u> | unweighted proportion |

Without weights, the default sample frequency statistic is `frequency`.

With weights, the default sample frequency statistic is `sumw`.

With option `by()`, `percent` is added to the default sample frequency statistic.

`place`(*place*) controls where the sample frequency statistics are reported in the table.

`place`(`items`), the default, specifies that the sample frequency statistics be reported in the first row of items in the table.

`place`(`inlabels`) specifies that the sample frequency statistics be added to the column header labels.

`place`(`seplabels`) specifies that the sample frequency statistics be stacked in the column header as separate labels.

*text\_styles* specifies the text styles for *title*.

## Continuous

`continuous([varlistc][, statistics(cstats) test(ctest) text_styles])` specifies continuous variables, their row header text styles, their statistics, and their test. If `varlistc` is not specified, then the specified text styles, statistics, and test become the default for all continuous variables in `varlist`.

This option is repeatable, and when multiple text styles, statistics, or tests apply to a variable, the rightmost specification is applied.

`statistics(cstats)` specifies the statistics to report for `varlistc`.

`cstats` may contain the following continuous variable statistics and any composite result defined from them.

| <i>cstats</i>           | Description                          |
|-------------------------|--------------------------------------|
| <code>mean</code>       | mean                                 |
| <code>semean</code>     | standard error of the mean           |
| <code>sebinomial</code> | standard error of the mean, binomial |
| <code>sepoisson</code>  | standard error of the mean, Poisson  |
| <code>variance</code>   | variance                             |
| <code>sd</code>         | standard deviation                   |
| <code>skewness</code>   | skewness                             |
| <code>kurtosis</code>   | kurtosis                             |
| <code>cv</code>         | coefficient of variation             |
| <code>svy cv</code>     | coefficient of variation (svy)       |
| <code>geomean</code>    | geometric mean                       |
| <code>geosd</code>      | geometric standard deviation         |
| <code>count</code>      | number of nonmissing values          |
| <code>median</code>     | median                               |
| <code>p#</code>         | #th percentile                       |
| <code>q1</code>         | first quartile                       |
| <code>q2</code>         | second quartile                      |
| <code>q3</code>         | third quartile                       |
| <code>iqr</code>        | interquartile range                  |
| <code>min</code>        | minimum value                        |
| <code>max</code>        | maximum value                        |
| <code>range</code>      | range                                |
| <code>first</code>      | first value                          |
| <code>last</code>       | last value                           |
| <code>firstnm</code>    | first nonmissing value               |
| <code>lastnm</code>     | last nonmissing value                |
| <code>total</code>      | total                                |
| <code>rawtotal</code>   | unweighted total                     |

The default continuous variable statistics are `mean` and `sd`.

`test(ctest)` specifies that `ctest` be computed for the continuous variables in `varlistc`. This option has no effect if the `by()` option is not specified.

`ctest` is one of the following:

| <i>ctest</i>              | Description                                    |
|---------------------------|------------------------------------------------|
| <code>regress</code>      | main effects test from a linear regression     |
| <code>poisson</code>      | main effects test from a Poisson regression    |
| <code>lnormal</code>      | main effects test from a log-normal regression |
| <code>kwallis</code>      | Kruskal–Wallis rank test                       |
| <code>kwallis_ties</code> | Kruskal–Wallis rank test, adjusted for ties    |
| <code>none</code>         | suppress the test                              |

`kwallis` and `kwallis_ties` are not allowed with `weights` or the `svy` option.

The default test for continuous variables is `regress`.

For nonsurvey data, `regress` is equivalent to a one-way ANOVA, and when the `by()` variable has two levels, `regress` is equivalent to a pooled *t* test.

`text_styles` specifies the row header text styles for `varlistc`.

#### Factors

`factor([varlistf][, statistics(fstats) test(ftest) text_styles])` specifies factor variables, their row header text styles, their statistics, and their test. If `varlistf` is not specified, then the specified text styles, statistics, and test become the default for all factor variables in `varlist`.

This option is repeatable, and when multiple text styles, statistics, or tests apply to a variable, the rightmost specification is applied.

`statistics(fstats)` specifies the statistics to report for `varlistf`.

`fstats` may contain the following factor-variable statistics and any composite result defined from them.

| <i>fstats</i>                | Definition                                              |
|------------------------------|---------------------------------------------------------|
| <code>fvfrequency</code>     | frequency of each factor-variable level                 |
| <code>fvpercent</code>       | percentage within each factor-variable level            |
| <code>fvproportion</code>    | proportion within each factor-variable level            |
| <code>fvrawfrequency</code>  | unweighted frequency of each factor-variable level      |
| <code>fvrawpercent</code>    | unweighted percentage within each factor-variable level |
| <code>fvrawproportion</code> | unweighted proportion within each factor-variable level |

The default factor-variable statistics are `fvfrequency` and `fvpercent`.

`test(ftest)` specifies that *ftest* be computed for the factor variables in *varlist<sub>f</sub>*. This option has no effect if the `by()` option is not specified.

*ftest* is one of the following:

| <i>ftest</i> | Description                           |
|--------------|---------------------------------------|
| pearson      | Pearson's $\chi^2$ test               |
| fisher       | Fisher's exact test                   |
| lrchi2       | likelihood-ratio $\chi^2$ test        |
| gamma        | Goodman and Kruskal's gamma           |
| kendall      | Kendall's $\tau_b$                    |
| cramer       | Cramér's $V$                          |
| svylr        | survey-adjusted likelihood-ratio test |
| svywald      | survey-adjusted Wald test             |
| svyllwald    | survey-adjusted log-linear Wald test  |
| none         | suppress the test                     |

`fisher`, `lrchi2`, `gamma`, `kendall`, and `cramer` are not allowed with `aweight`s, `iweight`s, `pweight`s, or the `svy` option. `pearson` is ignored when `aweight`s, `iweight`s, or `pweight`s are specified.

`svylr`, `svywald`, and `svyllwald` are allowed only with the `svy` option.

The default test for factor variables is `pearson`. No test is computed when `aweight`s, `iweight`s, or `pweight`s are specified with the default `pearson`.

`text_styles` specifies the row header text styles for *varlist<sub>f</sub>*.

#### Composite

`define(name=elements[ , def_opts])` defines a composite result. Composite results give more control over how statistics are composed and formatted in a table cell.

*name* is the name you choose for the new composite result.

*elements* is a list of statistics and other composite results.

*def\_opts* are `delimiter(char)`, `[no]trim`, `[no]override`, and `replace`.

`delimiter(char)` changes the delimiter between elements. The default is `delimiter(" ")`.

`notrim` and `trim` control the handling of extra spaces caused by numeric formats applied to the elements. `notrim` preserves the extra spaces; `trim`, the default, removes the extra spaces.

`nooverride` and `override` control handling of the `trim` property when an element is a composite result. `nooverride`, the default, does not change the `trim` property of elements; `override` applies the specified `trim` property to all elements.

`replace` permits you to redefine *name* if it already exists.

## Formats

`nformat(%fmt [results] [, basestyle])` changes the numeric format, such as the number of decimal places, for specified results. If *results* are not specified, the numeric format is changed for all results. *results* is a list of names taken from *nstats*, *cstats*, *ctest*, *fstats*, *ftest*, and composite results.

This option is repeatable, and when multiple formats apply to one result, the rightmost specification is applied.

This option does not affect the format of factor variables specified in *varlist* or the `factor()` option. The default format of these variables is taken from the dataset.

*basestyle* indicates that the format be applied to results that do not already have their own format instead of overriding the format for all results.

`sformat(%fmt [results])` changes the string format for specified results. You can, for instance, add symbols or text to the values reported in the table by modifying the string format.

*%fmt* may contain a mix of text and *%s*. Here *%s* refers to the numeric value that is formatted as specified using `nformat()`. The text will be placed around the numeric values in your table as it is placed around *%s* in this option. For instance, to place parentheses around the percent statistics, you can specify `sformat("(%s) fvpercent percent)`.

*results* is a list of names taken from *nstats*, *cstats*, *ctest*, *fstats*, *ftest*, and composite results.

Two text characters must be specified using a special character sequence if you want them to be displayed in your table. To include %, type `%%`. To include \, type `\\`. For instance, to place a percent sign following percent statistics, you can specify `sformat("%s%" fvpercent percent)`.

This option is repeatable, and when multiple formats apply to one result, the rightmost specification is applied.

## Title

`title(string)` adds the text *string* as a title to the table.

`titlestyles(text_styles)` changes the style for the table title. *text\_styles* are the following:

`font([fontfamily] [, size(# [unit]) color(color) variant(variant) [no]bold [no]italic [no]strikeout [no]underline)` specifies the font style. These font style properties are applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and L<sup>A</sup>T<sub>E</sub>X files, unless otherwise specified.

*fontfamily* specifies a valid font family. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`size(# [unit])` specifies the font size as a number optionally followed by units. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`color(color)` specifies the text color.

`variant(variant)` specifies the font variant and capitalization.

`bold` and `nobold` specify the font weight. `bold` changes the font weight to bold; `nobold` changes the font weight back to normal.

`italic` and `noitalic` specify the font style. `italic` changes the font style to italic; `noitalic` changes the font style back to normal.

`strikeout` and `nostrikeout` specify whether to add a strikeout mark to the title. `strikeout` adds a strikeout mark to the title; `nostrikeout` changes the title back to normal.

`underline` and `nounderline` specify whether to underline the table title. `underline` adds a single line under the title; `nounderline` removes the underline.

Only one of `strikeout` or `underline` is allowed when publishing to HTML files.

`smcl` (`smcl`) specifies how to render the table title for SMCL output. This style property is applicable only when publishing items from a collection to a SMCL file.

`latex` (`latex`) specifies how to render the table title for L<sup>A</sup>T<sub>E</sub>X output. This style property is applicable only when publishing items from a collection to a L<sup>A</sup>T<sub>E</sub>X file.

`shading` (`spec`) sets the background color, foreground color, and fill pattern. The background color is applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and L<sup>A</sup>T<sub>E</sub>X files. The foreground color and fill pattern are applicable when exporting the table to Microsoft Word and Microsoft Excel.

#### Notes

`note` (`string`) adds the text `string` as a note to the table. `note()` may be specified multiple times to add multiple notes. Each note is placed on a new line.

`notestyles` (`text_styles`) changes the style for the table notes. `text_styles` are the following:

`font` (`[fontfamily]` [`,` `size`(# `[unit]`) `color`(`color`) `variant`(`variant`) [`no`]`bold` [`no`]`italic` [`no`]`strikeout` [`no`]`underline`) specifies the font style. These font style properties are applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and L<sup>A</sup>T<sub>E</sub>X files, unless otherwise specified.

`fontfamily` specifies a valid font family. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`size`(# `[unit]`) specifies the font size as a number optionally followed by units. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`color`(`color`) specifies the text color.

`variant`(`variant`) specifies the font variant and capitalization.

`bold` and `nobold` specify the font weight. `bold` changes the font weight to bold; `nobold` changes the font weight back to normal.

`italic` and `noitalic` specify the font style. `italic` changes the font style to italic; `noitalic` changes the font style back to normal.

`strikeout` and `nostrikeout` specify whether to add a strikeout mark to the notes. `strikeout` adds a strikeout mark to the note; `nostrikeout` changes the note back to normal.

`underline` and `nounderline` specify whether to underline the table notes. `underline` adds a single line under the notes; `nounderline` removes the underline.

Only one of `strikeout` or `underline` is allowed when publishing to HTML files.

`smcl` (`smcl`) specifies how to render the table notes for SMCL output. This style property is applicable only when publishing items from a collection to a SMCL file.

`latex` (*latex*) specifies how to render the table notes for L<sup>A</sup>T<sub>E</sub>X output. This style property is applicable only when publishing items from a collection to a L<sup>A</sup>T<sub>E</sub>X file.

`shading` (*sspec*) sets the background color, foreground color, and fill pattern. The background color is applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and L<sup>A</sup>T<sub>E</sub>X files. The foreground color and fill pattern are applicable when exporting the table to Microsoft Word and Microsoft Excel.

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Export

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`export` (*filename*.*suffix* [ , *export\_opts* ]) exports the table to the specified file. *export\_opts* are the following:

`as` (*fileformat*) specifies the file format to which the table is to be exported. This option is rarely specified because, by default, `dtable` determines the format from the suffix of the file being created.

`replace` permits `dtable` to overwrite an existing file.

`noisily` specifies that `dtable` show the commands used to export the table to Microsoft Word, Microsoft Excel, and PDF files. The `putdocx`, `putexcel`, or `putpdf` command used to export the table will be displayed.

`dofile` (*filename* [ , `replace` ]) specifies that `dtable` save to *filename* the commands used to export the table to Microsoft Word, Microsoft Excel, and PDF files.

If *filename* already exists, it can be overwritten by specifying `replace`. If *filename* is specified without an extension, `.do` is assumed.

`append` specifies that `dtable` append the table to an existing file.

This option is applicable when you export the table to an HTML, a L<sup>A</sup>T<sub>E</sub>X, a SMCL, a `txt`, or a Markdown file. When you export to HTML and L<sup>A</sup>T<sub>E</sub>X files, the `append` option implies the `tableonly` option. Furthermore, when you export to HTML files, if the target CSS file already exists, `dtable` will also append to it.

`tableonly` specifies that only the table be exported to the specified HTML or L<sup>A</sup>T<sub>E</sub>X document. With this option, the produced file may be included in other HTML or L<sup>A</sup>T<sub>E</sub>X documents. By default, `dtable` produces complete HTML and L<sup>A</sup>T<sub>E</sub>X documents.

When you export to an HTML file, if the `cssfile()` option is not specified, a CSS filename is constructed from *filename*, with the extension replaced with `.css`.

`cssfile` (*cssfile*) specifies that `dtable` define the styles in *cssfile* instead of *filename* when you export to HTML.

`prefix` (*prefix*) specifies that `dtable` use *prefix* to identify style classes when you export to HTML.

`sheet` (*sheetname* [ , `replace` ]) saves to the worksheet named *sheetname*. For more information about this option, see [RPT] `putexcel`.

`cell` (*cell*) specifies an Excel upper-left cell as the starting position to publish the table. The default is `cell(A1)`.

`modify` permits `putexcel` set to modify an Excel file. For more information about this option, see [RPT] [putexcel](#).

`noopen` prevents `putexcel` from opening the Excel file in memory for modification. It does not appear in the dialog box. For more information about this option, see [RPT] [putexcel](#).

#### Options

`varlabel` and `novarlabel` specify whether variable labels for the continuous and factor variables should be displayed. `varlabel` displays the variable label; `novarlabel` displays variable names.

`fvlabel` and `nofvlabel` specify whether value labels for the factor variables should be displayed. `fvlabel` displays value labels; `nofvlabel` displays the values.

`listwise` and `nolistwise` specify how to handle missing values across the continuous and factor variables. `listwise` handles missing values through listwise deletion, meaning that the entire observation is omitted from the sample if any continuous or factor variable is missing for that observation; `nolistwise` specifies that `dtable` omit an observation only if all the continuous and factor variables are missing for that observation.

`halign` (*hvalue*) specifies how the item cells are horizontally aligned. *hvalue* may be `left`, `center`, or `right`.

`name` (*cname*) specifies the collection for `dtable` to work with. The default is `name(DTable)`.

`replace` permits `dtable` to overwrite the existing collection. This option is implied for `name(DTable)`.

`label` (*filename*) specifies the *filename* containing the collection labels to use for your table. Labels in *filename* will be loaded for the table, and default labels will be used for any labels not specified in *filename*.

`style` (*filename* [, *override* ]) specifies the *filename* containing the collection styles to use for your table. This might be a style you saved with [collect style save](#) or a [predefined style](#) shipped with Stata. The `dtable` collection styles will be discarded, and only the collection styles in *filename* will be applied. Note that the layout specification saved in *filename* will not be applied; `dtable` will always use its predefined layout.

If you prefer the `dtable` collection styles but also want to apply any styles in *filename*, specify *override*. If there are conflicts between the default collection styles and those in *filename*, the ones in *filename* will take precedence.

The default is to use only the collection styles set in `c(dtable_style)`; see [TABLES] [set dtable\\_style](#).

The following option is available with `dtable` but is not shown in the dialog box:

`warn` specifies that `dtable` display warnings from `collect`. By default, these warnings are suppressed.

## Remarks and examples

Remarks are presented under the following headings:

*Introduction*  
*A first example*  
*Describe variables across groups*  
*Survey data*  
*Save your style choices for next time*  
*Composite results*  
*The default style*

### Introduction

dtable allows you to easily create a table of summary statistics and export it to a variety of file types, without any knowledge of the collection system. You can specify the continuous and factor variables that you want described in your table. You can also customize the table by formatting the results, changing the variable-specific statistics, adding a variable to identify sample groups, testing each variable for equality between groups, adding a title and notes, and more.

In most cases, you will use dtable to easily create a table and export it to another format. However, you can customize the table beyond the options that are available with dtable. When you issue a dtable command, the results are stored in a collection called DTable. This collection is replaced with each new dtable command, unless you specify a different name for the collection in the name() option. You can make additional changes to the collection with the collect suite of commands. To learn more about the collect commands, see [TABLES] [Intro](#) and the entries discussed therein.

See [Mitchell \(2025, chap. 3\)](#) for further examples.

### A first example

In its simplest specification, you type dtable and a list of variables you want described in your table. We will use data from the Second National Health and Nutrition Examination Survey (NHANES II) ([McDowell et al. 1981](#)). Our table describes continuous variables for systolic blood pressure, age, weight, and factor variables for race and health status.

```
. use https://www.stata-press.com/data/r19/nhanes2l
(Second National Health and Nutrition Examination Survey)
. dtable bpsystol age weight i.race i.hlthstat
```

| Summary                 |                  |
|-------------------------|------------------|
| N                       | 10,351           |
| Systolic blood pressure | 130.882 (23.333) |
| Age (years)             | 47.580 (17.215)  |
| Weight (kg)             | 71.898 (15.356)  |
| Race                    |                  |
| White                   | 9,065 (87.6%)    |
| Black                   | 1,086 (10.5%)    |
| Other                   | 200 (1.9%)       |
| Health status           |                  |
| Excellent               | 2,407 (23.3%)    |
| Very good               | 2,591 (25.1%)    |
| Good                    | 2,938 (28.4%)    |
| Fair                    | 1,670 (16.2%)    |
| Poor                    | 729 (7.1%)       |

The default continuous variable statistics are the mean and standard deviation. The default factor-variable statistics are the factor-level frequencies and percentages. You can change the statistics reported for each continuous and factor variable; see the `continuous()` and `factor()` options for the list of supported statistics.

Many options are available to change the formatting and style of your table. Here we set the format of means and standard deviations to show two decimal places, remove the first row of the table with the overall sample size, and change the column header to be more descriptive of the table results.

```
. dtable bpsystol age weight i.race i.hlthstat, nformat(%16.2fc mean sd)
> nosample column(summary(M(SD) / n(%)))
```

|                         | M(SD) / n(%)   |
|-------------------------|----------------|
| Systolic blood pressure | 130.88 (23.33) |
| Age (years)             | 47.58 (17.21)  |
| Weight (kg)             | 71.90 (15.36)  |
| Race                    |                |
| White                   | 9,065 (87.6%)  |
| Black                   | 1,086 (10.5%)  |
| Other                   | 200 (1.9%)     |
| Health status           |                |
| Excellent               | 2,407 (23.3%)  |
| Very good               | 2,591 (25.1%)  |
| Good                    | 2,938 (28.4%)  |
| Fair                    | 1,670 (16.2%)  |
| Poor                    | 729 (7.1%)     |

Additionally, you can complete your table with a title and notes and even export it to another format. We now add a title and export our table to the file `mydtable.html`.

```
. dtable bpsystol age weight i.race i.hlthstat, nformat(%16.2fc mean sd)
> nosample column(summary(M(SD) / n(%))) title(Table 1)
> export(mydtable.html)
```

Table 1

|                         | M(SD) / n(%)   |
|-------------------------|----------------|
| Systolic blood pressure | 130.88 (23.33) |
| Age (years)             | 47.58 (17.21)  |
| Weight (kg)             | 71.90 (15.36)  |
| Race                    |                |
| White                   | 9,065 (87.6%)  |
| Black                   | 1,086 (10.5%)  |
| Other                   | 200 (1.9%)     |
| Health status           |                |
| Excellent               | 2,407 (23.3%)  |
| Very good               | 2,591 (25.1%)  |
| Good                    | 2,938 (28.4%)  |
| Fair                    | 1,670 (16.2%)  |
| Poor                    | 729 (7.1%)     |

(collection DTable exported to file `mydtable.html`)

We could also export this table to a Microsoft Word, Microsoft Excel, L<sup>A</sup>T<sub>E</sub>X, Markdown, SMCL, PDF, or plain text file by specifying the appropriate [file extension](#).

## Describe variables across groups

Describing variables across samples in your dataset is easy with the `by()` option. In the following, we revisit our original table, but we add the `by()` option to get summary statistics from the urban, rural, and total samples.

```
. dtable bpsystol age weight i.race i.hlthstat, by(rural) sample(N (%))
```

|                         | Urban            | Rural<br>Rural   | Total            |
|-------------------------|------------------|------------------|------------------|
| N (%)                   | 6,548 (63.3%)    | 3,803 (36.7%)    | 10,351 (100.0%)  |
| Systolic blood pressure | 130.460 (23.526) | 131.607 (22.980) | 130.882 (23.333) |
| Age (years)             | 46.835 (17.484)  | 48.862 (16.666)  | 47.580 (17.215)  |
| Weight (kg)             | 71.427 (15.363)  | 72.708 (15.314)  | 71.898 (15.356)  |
| Race                    |                  |                  |                  |
| White                   | 5,419 (82.8%)    | 3,646 (95.9%)    | 9,065 (87.6%)    |
| Black                   | 968 (14.8%)      | 118 (3.1%)       | 1,086 (10.5%)    |
| Other                   | 161 (2.5%)       | 39 (1.0%)        | 200 (1.9%)       |
| Health status           |                  |                  |                  |
| Excellent               | 1,609 (24.6%)    | 798 (21.0%)      | 2,407 (23.3%)    |
| Very good               | 1,713 (26.2%)    | 878 (23.1%)      | 2,591 (25.1%)    |
| Good                    | 1,878 (28.7%)    | 1,060 (27.9%)    | 2,938 (28.4%)    |
| Fair                    | 950 (14.5%)      | 720 (19.0%)      | 1,670 (16.2%)    |
| Poor                    | 389 (5.9%)       | 340 (9.0%)       | 729 (7.1%)       |

Next, we add the `tests` suboption in the `by()` option to report tests of equality between samples and the `nototals` option to suppress the column of statistics from the total sample. We also specify that we would like sample statistics to be in the column headers rather than the first row of the table using the `sample()` option. Here we also add the `column(by(hide))` option to suppress the redundant label from variable `rural` and add notes that detail what is being reported.

```
. dtable bpsystol age weight i.race i.hlthstat,
> by(rural, nototals tests)
> column(by(hide))
> sample(, place(seplabels))
> note(Mean (Standard deviation): p-value from a pooled t-test.)
> note(Frequency (Percent%): p-value from Pearson test.)
note: using test regress across levels of rural for bpsystol, age, and
weight.
note: using test pearson across levels of rural for race and hlthstat.
```

|                         | Urban<br>6,548 (63.3%) | Rural<br>3,803 (36.7%) | Test   |
|-------------------------|------------------------|------------------------|--------|
| Systolic blood pressure | 130.460 (23.526)       | 131.607 (22.980)       | 0.016  |
| Age (years)             | 46.835 (17.484)        | 48.862 (16.666)        | <0.001 |
| Weight (kg)             | 71.427 (15.363)        | 72.708 (15.314)        | <0.001 |
| Race                    |                        |                        |        |
| White                   | 5,419 (82.8%)          | 3,646 (95.9%)          | <0.001 |
| Black                   | 968 (14.8%)            | 118 (3.1%)             |        |
| Other                   | 161 (2.5%)             | 39 (1.0%)              |        |
| Health status           |                        |                        |        |
| Excellent               | 1,609 (24.6%)          | 798 (21.0%)            | <0.001 |
| Very good               | 1,713 (26.2%)          | 878 (23.1%)            |        |
| Good                    | 1,878 (28.7%)          | 1,060 (27.9%)          |        |
| Fair                    | 950 (14.5%)            | 720 (19.0%)            |        |
| Poor                    | 389 (5.9%)             | 340 (9.0%)             |        |

Mean (Standard deviation): p-value from a pooled t-test.  
Frequency (Percent%): p-value from Pearson test.

For continuous variables, the Test column reports the  $p$ -value from linear regressions, which are equivalent to pooled  $t$  tests because rural has two categories. For factor variables, the Test column reports the  $p$ -values from Pearson  $\chi^2$  tests.

## Survey data

dtable supports survey data by using the survey data characteristics in your dataset when you specify the svy option. Your data must be svyset prior to calling dtable with the svy option. Our data already have survey data characteristics.

```
. svyset
Sampling weights: finalwgt
 VCE: linearized
Single unit: missing
Strata 1: strata
Sampling unit 1: psu
FPC 1: <zero>
```

The following table reports the survey weighted frequency, mean, standard deviation, factor-level frequency, and factor-level percent of the specified variables.

```
. dtable bpsystol age weight i.race i.hlthstat, svy
```

| Summary                 |                     |
|-------------------------|---------------------|
| N                       | 117,157,513         |
| Systolic blood pressure | 126.946 (21.401)    |
| Age (years)             | 42.253 (15.502)     |
| Weight (kg)             | 71.901 (15.433)     |
| Race                    |                     |
| White                   | 102,999,549 (87.9%) |
| Black                   | 11,189,236 (9.6%)   |
| Other                   | 2,968,728 (2.5%)    |
| Health status           |                     |
| Excellent               | 32,187,335 (27.5%)  |
| Very good               | 32,176,310 (27.5%)  |
| Good                    | 32,715,395 (28.0%)  |
| Fair                    | 14,380,261 (12.3%)  |
| Poor                    | 5,537,956 (4.7%)    |

Option `by()` is allowed with `svy`. `dtable` also has options for tests of equality between groups that are allowed with `svy`. These tests account for the survey data characteristics in your data. In the following, we revisit the urban and rural summary table, but we add the `svy` option, a title, and some notes. We also add the `column(by(hide))` option to suppress the redundant label from variable `rural` and add the `test()` suboption to rename the Test column.

```
. dtable bpsystol age weight i.race i.hlthstat, svy
> by(rural, nototals tests)
> column(by(hide) test(p-value))
> title(Survey data summary)
> note(Mean (Standard deviation): p-value from linear regression.)
> note(Frequency (Percent%): p-value from Pearson test.)
> note(Statistics computed using the survey weights.)
> note(Tests adjusted for the survey design.)
note: using test regress across levels of rural for bpsystol, age, and
weight.
note: using test pearson across levels of rural for race and hlthstat.
Survey data summary
```

|                         | Urban              | Rural              | p-value |
|-------------------------|--------------------|--------------------|---------|
| N                       | 79,965,794 (68.3%) | 37,191,719 (31.7%) |         |
| Systolic blood pressure | 126.607 (21.438)   | 127.675 (21.305)   | 0.406   |
| Age (years)             | 41.805 (15.662)    | 43.215 (15.112)    | 0.024   |
| Weight (kg)             | 71.322 (15.371)    | 73.144 (15.493)    | <0.001  |
| Race                    |                    |                    |         |
| White                   | 67,579,394 (84.5%) | 35,420,155 (95.2%) | <0.001  |
| Black                   | 9,936,159 (12.4%)  | 1,253,077 (3.4%)   |         |
| Other                   | 2,450,241 (3.1%)   | 518,487 (1.4%)     |         |
| Health status           |                    |                    |         |
| Excellent               | 22,781,784 (28.5%) | 9,405,551 (25.3%)  | <0.001  |
| Very good               | 22,867,496 (28.6%) | 9,308,814 (25.1%)  |         |
| Good                    | 22,089,942 (27.7%) | 10,625,453 (28.6%) |         |
| Fair                    | 8,892,926 (11.1%)  | 5,487,335 (14.8%)  |         |
| Poor                    | 3,229,798 (4.0%)   | 2,308,158 (6.2%)   |         |

Mean (Standard deviation): p-value from linear regression.  
Frequency (Percent%): p-value from Pearson test.  
Statistics computed using the survey weights.  
Tests adjusted for the survey design.

## Save your style choices for next time

If you find yourself repeating certain options each time you use `dtable`, you can save most of those option choices for future tables. Use `collect style save` to store your `dtable` option choices to a collection style file and then specify this file in the `style()` option the next time you use `dtable`. If you store the collection style file in your `PERSONAL` folder, you can set that file as the default style for `dtable`.

In the following, we use `dtable` to create a table with some option choices, save our style to a file in the `PERSONAL` directory, and set that style file as the default for `dtable`.

```
. sysuse auto
(1978 automobile data)

. dtable mpg turn i.rep78,
> sample(Sample, statistic(frequency) place(seplabels))
> sformat("n=%s" frequency)
> by(foreign, tests nototals missing)
> continuous(, test(kwallis))
> factor(, test(pearson))
> title(Table 1. Summary by group with a test.)
> note(Kruskal-Wallis test for continuous variables.)
> note(Pearson's test for factor variables.)
note: using test kwallis across levels of foreign for mpg and turn.
note: using test pearson across levels of foreign for rep78.
```

Table 1. Summary by group with a test.

|                    | Car origin       |                 | Test   |
|--------------------|------------------|-----------------|--------|
|                    | Domestic<br>n=52 | Foreign<br>n=22 |        |
| Mileage (mpg)      | 19.827 (4.743)   | 24.773 (6.611)  | 0.002  |
| Turn circle (ft.)  | 41.442 (3.968)   | 35.409 (1.501)  | <0.001 |
| Repair record 1978 |                  |                 |        |
| 1                  | 2 (4.2%)         | 0 (0.0%)        | <0.001 |
| 2                  | 8 (16.7%)        | 0 (0.0%)        |        |
| 3                  | 27 (56.2%)       | 3 (14.3%)       |        |
| 4                  | 9 (18.8%)        | 9 (42.9%)       |        |
| 5                  | 2 (4.2%)         | 9 (42.9%)       |        |

Kruskal-Wallis test for continuous variables.

Pearson's test for factor variables.

```
. personal dir
your personal ado-directory is C:\ado\personal\
(output omitted)

. collect style save "C:\ado\personal\mydtable"
(style from DTable saved to file C:\ado\personal\mydtable.stjson)

. set dtable_style mydtable

. display c(dtable_style)
mydtable
```

Now these style choices are the defaults for tables created in the future. Here we create a similarly styled table but from different data.

```
. use https://www.stata-press.com/data/r19/lbw
(Hosmer & Lemeshow data)
. dtable age i.smoke i.ht, by(low)
note: using test kwallis across levels of low for age.
note: using test pearson across levels of low for smoke and ht.
Table 1. Summary by group with a test.
```

|                             | Birthweight<2500g |                | Test  |
|-----------------------------|-------------------|----------------|-------|
|                             | 0<br>n=130        | 1<br>n=59      |       |
| Age of mother               | 23.662 (5.585)    | 22.305 (4.511) | 0.248 |
| Smoked during pregnancy     |                   |                |       |
| Nonsmoker                   | 86 (66.2%)        | 29 (49.2%)     | 0.026 |
| Smoker                      | 44 (33.8%)        | 30 (50.8%)     |       |
| Has history of hypertension |                   |                |       |
| 0                           | 125 (96.2%)       | 52 (88.1%)     | 0.036 |
| 1                           | 5 (3.8%)          | 7 (11.9%)      |       |

Kruskal-Wallis test for continuous variables.  
Pearson's test for factor variables.

To reset the dtable style back to its original default, type

```
. set dtable_style dtable
```

You can add the permanently option to set the default dtable style for future Stata sessions.

## Composite results

dtable's default style does not define any composite results, so you will need to define your own with the `define()` option. Stata has a collection style named `dtable_composites` that defines the following composite results for you.

| <i>composite</i> | <i>elements</i> | Description            |
|------------------|-----------------|------------------------|
| idi              | p10 p90         | Interdecile interval   |
| iqi              | q1 q3           | Interquartile interval |
| rangei           | min max         | Range interval         |

All of these results are bound in square brackets without a delimiter between their elements. To get access to these composite results, use the `style(dtable_composites, override)` option. You can then specify `idi`, `iqi`, and `rangei` in the `statistic()` suboption of `continuous()`, just as you would any other statistic.

You may want different formatting for these composite results, or you may want to create composite results based on other statistics. The `define()` option allows you to customize your own composite results. In the following, we use `dtable` to create a table that defines our own interval results with some other option choices and then save the collection's style to a file.

```
. sysuse auto
(1978 automobile data)

. dtable, define(idi = p10 p90, delimiter("; "))
> define(iqi = q1 q3, delimiter("; "))
> define(rangei = min max, delimiter("; "))
> sample(Sample, statistic(frequency) place(seplabels))
> sformat("n=%s" frequency)
> sformat("{%s}" idi)
> sformat("[%s]" iq_i)
> sformat("<%s>" rangei)
> continuous(mpg, statistic(p50 idi))
> continuous(turn, statistic(q2 iq_i))
> continuous(trunk, statistic(mean rangei))
> title(Table 1. Location and interval.)
> note(Median {Interdecile interval})
> note(Median [Interquartile interval])
> note(Mean <Range interval>)
```

Table 1. Location and interval.

| Summary                         |                         |
|---------------------------------|-------------------------|
| n=74                            |                         |
| Mileage (mpg)                   | 20.000 {14.000; 29.000} |
| Turn circle (ft.)               | 40.000 [36.000; 43.000] |
| Trunk space (cu. ft.)           | 13.757 <5.000; 23.000>  |
| Median {Interdecile interval}   |                         |
| Median [Interquartile interval] |                         |
| Mean <Range interval>           |                         |

```
. collect style save dtable-intervals
(style from DTable saved to file dtable-intervals.stjson)
```

Now we can use this style and simply reference our composite results without having to redefine them.

```
. dtable, continuous(mpg, statistic(p50 idi))
> continuous(turn, statistic(q2 iq_i))
> continuous(trunk, statistic(mean rangei))
> style(dtable-intervals)
```

Table 1. Location and interval.

| Summary                         |                         |
|---------------------------------|-------------------------|
| n=74                            |                         |
| Mileage (mpg)                   | 20.000 {14.000; 29.000} |
| Turn circle (ft.)               | 40.000 [36.000; 43.000] |
| Trunk space (cu. ft.)           | 13.757 <5.000; 23.000>  |
| Median {Interdecile interval}   |                         |
| Median [Interquartile interval] |                         |
| Mean <Range interval>           |                         |

## The default style

Most of dtable's option defaults are controllable in its collection style. The following options are not part of dtable's collection style:

```
svy
subpop()
export()
name()
replace
label()
style()
warn
```

When the `by()` option is not specified, the default `column()` header options are

```
summary("Summary", nohide
 halign(center)
 font(, nobold
 noitalic
 nostrikeout
 nounderline)
 smcl(text))
```

When a variable is specified in the `by()` option, its default options are

```
notest
testnotes
totals
nomissing
```

and its default `column()` header options are

```
by(label, nohide
 fvlabel
 halign(center)
 font(, nobold
 noitalic
 nostrikeout
 nounderline)
 smcl(text))
total("Total", nohide
 halign(center)
 font(, nobold
 noitalic
 nostrikeout
 nounderline)
 smcl(text))
```

```
test("Test", nohide
 halign(center)
 font(, nobold
 noitalic
 nostrikeout
 nounderline)
 smcl(text))
```

The default sample frequency title is “N”. When weights are not specified, the default sample frequency statistic is frequency; otherwise, sumw is reported. When the by() option is specified, percent is added to the default sample frequency statistic. The other default sample() options are

```
font(, nobold
 noitalic
 nostrikeout
 nounderline)
smcl(text)
place(items)
```

The defaults for the continuous() option are

```
statistic(mean sd)
test(regress)
font(, nobold
 noitalic
 nostrikeout
 nounderline)
smcl(text)
```

The defaults for the factor() option are

```
statistic(fvfrequency fvpercent)
test(pearson)
font(, nobold
 noitalic
 nostrikeout
 nounderline)
smcl(text)
```

There are no predefined composite results in dtable’s default style.

The default formats are

```
nformat("%21.3fc", basestyle)
nformat("%21.0fc" N count sumw frequency fvfrequency fvrawfrequency)
nformat("%9.1fc" percent rawpercent fvpercent fvrawpercent)
sformat("%s%%" percent rawpercent fvpercent fvrawpercent)
sformat("%s)" sd)
nformat("%6.3f" regress poisson lnnormal kwallis kwallis_ties pearson
 fisher lrchi2 svylr svywald svyllwald)
```

There is no default table title. The defaults for the `titlestyles()` option are

```
font(, nobold
 noitalic
 nostrikeout
 nounderline)
smcl(text)
```

There are no default table notes. The defaults for the `notestyles()` option are

```
font(, nobold
 noitalic
 nostrikeout
 nounderline)
smcl(text)
```

The remaining default options for `dtable` are

```
varlabel
fvlabel
nolistwise
halign(right)
```

## Methods and formulas

See *Methods and formulas* in [R] [table](#) for details on the statistics supported by `dtable`. When the `svy` or `subpop()` option is specified, the weights from `svyset` are used to compute statistics that support weights. In addition, the subpopulation variance  $\hat{\sigma}^2$  is computed as

$$\hat{\sigma}^2 = s^2 \frac{M}{n-1} \cdot \frac{n_s - 1}{M_s}$$

where  $s^2$  is the sample variance as documented in [R] [table](#),  $M$  is the estimated population size,  $n$  is the full sample size,  $M_s$  is the estimated subpopulation size, and  $n_s$  is the subpopulation sample size. The subpopulation standard deviation is computed as  $\hat{\sigma} = \sqrt{\hat{\sigma}^2}$  and coefficient of variation (`cv`) is computed as  $\hat{\sigma}/\bar{x}$ .

Let `y` be a continuous variable and `by(group, tests)` be in effect. For `test(regress)`, `dtable` reports the  $p$ -value from `test [#1]` after

```
regress y i.group [if] [in] [weight]
```

For `test(poisson)`, `dtable` reports the  $p$ -value from `test [#1]` after

```
poisson y i.group [if] [in] [weight]
```

For `test(lnormal)`, `dtable` reports the  $p$ -value from `test [#1]` after

```
gsem y <- i.group [if] [in] [weight], family(lognormal)
```

For `test(kwallis)` and `test(kwallis_ties)`, `dtable` reports the  $p$ -values corresponding with the  $\chi^2$  statistics reported by

```
kwallis y [if] [in], by(group)
```

When the `svy` or `subpop()` option is specified, the prefix

```
svy, subpop([varname] [if]) :
```

is used with `regress`, `poisson`, and `gsem`. For more details, see [R] [test](#), [R] [regress](#), [R] [poisson](#), [SEM] [gsem](#), [R] [kwallis](#), and [SVY] [svy](#).

Let `f` be a factor variable and `by(group, tests)` be in effect. For nonsurvey data, the factor-variable test across groups is performed using

```
tabulate f group [if] [in] [weight], opt
```

where `opt` is an option of `tabulate` corresponding with the specified test (*ftest*) using the following table:

| <i>ftest</i> | <i>opt</i> |
|--------------|------------|
| pearson      | chi2       |
| fisher       | exact      |
| lrchi2       | lrchi2     |
| gamma        | gamma      |
| kendall      | taub       |
| cramer       | V          |

For survey data, `dtable` reports the  $p$ -value from

```
svy, subpop([varname] [if]): tabulate f group [if] [in], opt
```

where `opt` is an option of `svy: tabulate` corresponding with the specified test (*ftest*) using the following table:

| <i>ftest</i> | <i>opt</i> |
|--------------|------------|
| pearson      | pearson    |
| svylr        | lr         |
| svywald      | wald       |
| svyllwald    | llwald     |

`pearson`, `fisher`, `lrchi2`, `svylr`, `svywald`, and `svyllwald` report a  $p$ -value. `gamma`, `kendall`, and `cramer` report the computed measure of association. For more details, see [R] [tabulate twoway](#) and [SVY] [svy: tabulate twoway](#).

## Appendix

### Colors

*bgcolor*, *fgcolor*, and *color*

---

|                |                      |                   |               |
|----------------|----------------------|-------------------|---------------|
| aliceblue      | darkslategray        | lightsalmon       | palevioletred |
| antiquewhite   | darkturquoise        | lightseagreen     | papayawhip    |
| aqua           | darkviolet           | lightskyblue      | peachpuff     |
| aquamarine     | deeppink             | lightslategray    | peru          |
| azure          | deepskyblue          | lightsteelblue    | pink          |
| beige          | dimgray              | lightyellow       | plum          |
| bisque         | dodgerblue           | lime              | powderblue    |
| black          | firebrick            | limegreen         | purple        |
| blanchedalmond | floralwhite          | linen             | red           |
| blue           | forestgreen          | magenta           | rosybrown     |
| blueviolet     | fuchsia              | maroon            | royalblue     |
| brown          | gainsboro            | mediumaquamarine  | saddlebrown   |
| burlywood      | ghostwhite           | mediumblue        | salmon        |
| cadetblue      | gold                 | mediumorchid      | sandybrown    |
| chartreuse     | goldenrod            | mediumpurple      | seagreen      |
| chocolate      | gray                 | mediumseagreen    | seashell      |
| coral          | green                | mediumslateblue   | sienna        |
| cornflowerblue | greenyellow          | mediumspringgreen | silver        |
| cornsilk       | honeydew             | mediumturquoise   | skyblue       |
| crimson        | hotpink              | mediumvioletred   | slateblue     |
| cyan           | indianred            | midnightblue      | slategray     |
| darkblue       | indigo               | mintcream         | snow          |
| darkcyan       | ivory                | mistyrose         | springgreen   |
| darkgoldenrod  | khaki                | moccasin          | steelblue     |
| darkgray       | lavender             | navajowhite       | tan           |
| darkgreen      | lavenderblush        | navy              | teal          |
| darkkhaki      | lawngreen            | oldlace           | thistle       |
| darkmagenta    | lemonchiffon         | olive             | tomato        |
| darkolivegreen | lightblue            | olivedrab         | turquoise     |
| darkorange     | lightcoral           | orange            | violet        |
| darkorchid     | lightcyan            | orangered         | wheat         |
| darkred        | lightgoldenrodyellow | orchid            | white         |
| darksalmon     | lightgray            | palegoldenrod     | whitesmoke    |
| darkseagreen   | lightgreen           | palegreen         | yellow        |
| darkslateblue  | lightpink            | paleturquoise     | yellowgreen   |

---

## Underline patterns

*upattern*


---

|             |                 |
|-------------|-----------------|
| none        | dashLong        |
| single      | dashLongHeavy   |
| words       | dotDash         |
| double      | dashDotHeavy    |
| thick       | dotDotDash      |
| dotted      | dashDotDotHeavy |
| dottedHeavy | wave            |
| dash        | wavyHeavy       |
| dashedHeavy | wavyDouble      |

---

## Shading patterns

*fpattern*


---

|                       |       |
|-----------------------|-------|
| nil                   | pct20 |
| clear                 | pct25 |
| solid                 | pct30 |
| horzStripe            | pct35 |
| vertStripe            | pct37 |
| reverseDiagStripe     | pct40 |
| diagStripe            | pct45 |
| horzCross             | pct50 |
| diagCross             | pct55 |
| thinHorzStripe        | pct60 |
| thinVertStripe        | pct62 |
| thinReverseDiagStripe | pct65 |
| thinDiagStripe        | pct70 |
| thinHorzCross         | pct75 |
| thinDiagCross         | pct80 |
| pct5                  | pct85 |
| pct10                 | pct87 |
| pct12                 | pct90 |
| pct15                 | pct95 |

---

## Acknowledgments

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## References

- McDowell, A., A. Engel, J. T. Massey, and K. Maurer. 1981. “Plan and operation of the Second National Health and Nutrition Examination Survey, 1976–1980”. In *Vital and Health Statistics*, ser. 1, no. 15. Hyattsville, MD: National Center for Health Statistics.
- Mitchell, M. N. 2025. *Create and Export Tables Using Stata*. College Station, TX: Stata Press.

## Also see

- [R] [table intro](#) — Introduction to tables of frequencies, summaries, and command results
- [R] [table summary](#) — Table of summary statistics
- [TABLES] [Intro](#) — Introduction

[Description](#)[Options for dydx](#)[Methods and formulas](#)[Quick start](#)[Options for integ](#)[Acknowledgment](#)[Menu](#)[Remarks and examples](#)[Reference](#)[Syntax](#)[Stored results](#)[Also see](#)

## Description

`dydx` and `integ` calculate derivatives and integrals of numeric “functions”.

## Quick start

For variables `y` and `x` corresponding to function  $y = f(x)$ , compute  $dy/dx$  using cubic splines and store result in `dy`

```
dydx y x, generate(dy)
```

Evaluate the integral of  $f(x)$  using cubic splines

```
integ y x
```

Evaluate the integral using the trapezoidal rule

```
integ y x, trapezoid
```

Same as above, and generate variable `iy` containing integral evaluated for each value of `x`

```
integ y x, trapezoid generate(iy)
```

## Menu

### **dydx**

Data > Create or change data > Other variable-creation commands > Calculate numerical derivatives

### **integ**

Data > Create or change data > Other variable-creation commands > Calculate numeric integrals

## Syntax

*Derivatives of numeric functions*

```
dydx yvar xvar [if] [in] , generate(newvar) [dydx_options]
```

*Integrals of numeric functions*

```
integ yvar xvar [if] [in] [, integ_options]
```

*dydx\_options*

Description

Main

|                            |                                               |
|----------------------------|-----------------------------------------------|
| *generate( <i>newvar</i> ) | store results in variable named <i>newvar</i> |
| replace                    | overwrite the existing variable               |
| double                     | store new variable as double                  |

\*generate(*newvar*) is required.

*integ\_options*

Description

Main

|                           |                                                                     |
|---------------------------|---------------------------------------------------------------------|
| trapezoid                 | use trapezoidal rule to compute integrals; default is cubic splines |
| generate( <i>newvar</i> ) | store results in variable named <i>newvar</i>                       |
| replace                   | overwrite the existing variable                                     |
| double                    | store new variable as double                                        |
| initial(#)                | initial value of integral; default is initial(0)                    |

by and collect are allowed with dydx and integ; see [\[U\] 11.1.10 Prefix commands](#).

## Options for dydx

Main

generate(*newvar*) specifies that results be stored in a new variable. generate() is required.

replace specifies that if an existing variable is specified for generate(), it should be overwritten.

double specifies that the new variable in generate() be stored as double. If the double option is not specified, the variable is stored using the current type as set by [set type](#), which is float by default.

## Options for integ

Main

trapezoid requests that the trapezoidal rule [the sum of  $(x_i - x_{i-1})(y_i + y_{i-1})/2$ ] be used to compute integrals. The default is cubic splines, which give superior results for most smooth functions; for irregular functions, trapezoid may give better results.

generate(*newvar*) specifies that results be stored in a new variable.

replace specifies that if an existing variable is specified for generate(), it should be overwritten.

`double` specifies that the new variable in `generate()` be stored as double. If the `double` option is not specified, the variable is stored using the current type as set by `set type`.

`initial(#)` specifies the initial condition for calculating definite integrals; see [Methods and formulas](#) below. The default is `initial(0)`.

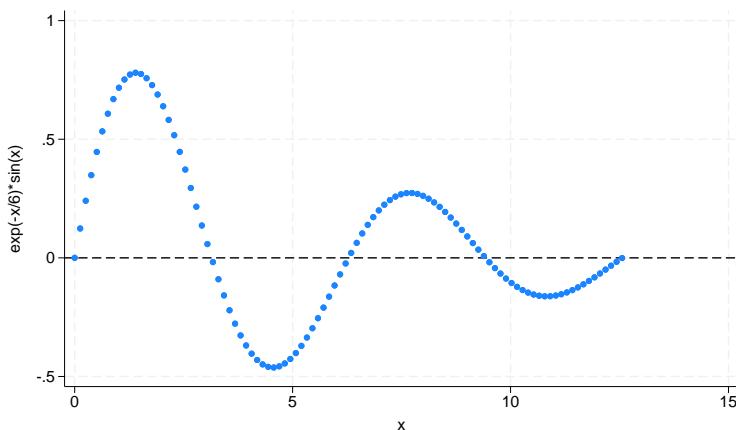
## Remarks and examples

`dydx` and `integ` lets you extend Stata's graphics capabilities beyond data analysis and into mathematics.

### ► Example 1

We graph  $y = e^{-x/6}\sin(x)$  over the interval  $[0, 12.56]$ :

```
. range x 0 12.56 100
Number of observations (_N) was 0, now 100.
. generate y = exp(-x/6)*sin(x)
. label variable y "exp(-x/6)*sin(x)"
. twoway connected y x, connect(i) yline(0)
```

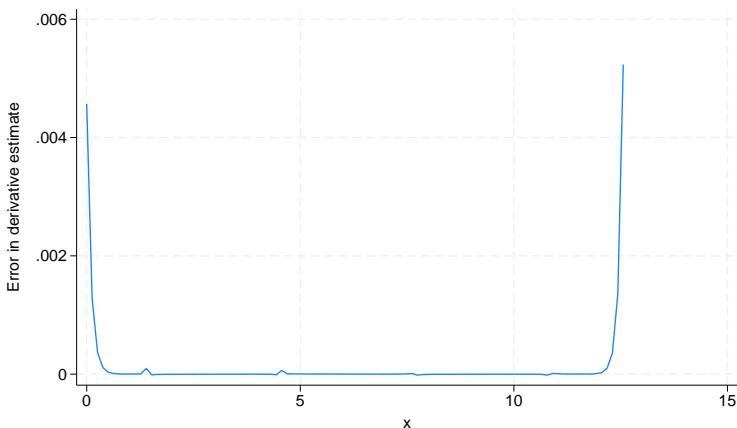


We estimate the derivative by using `dydx` and compute the relative difference between this estimate and the true derivative.

```
. dydx y x, gen(dy)
. generate dytrue = exp(-x/6)*(cos(x) - sin(x)/6)
. generate error = abs(dy - dytrue)/dytrue
```

The error is greatest at the endpoints, as we would expect. The error is approximately 0.5% at each endpoint, but the error quickly falls to less than 0.01%.

```
. label variable error "Error in derivative estimate"
. twoway line error x, ylabel(0(.002).006)
```

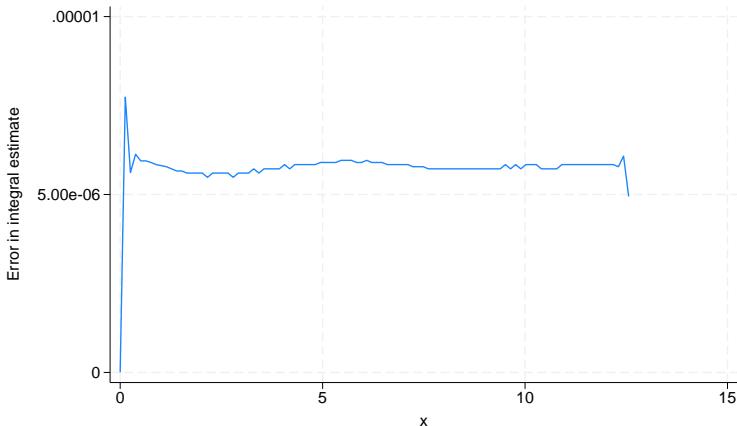


We now estimate the integral by using `integ`:

```
. integ y x, gen(iy)
number of points = 100
integral = .85316397
. generate iytrue = (36/37)*(1 - exp(-x/6))*(cos(x) + sin(x)/6)
. display iytrue[_N]
.85315901
. display abs(r(integral) - iytrue[_N])/iytrue[_N]
5.811e-06
. generate diff = iy - iytrue
```

The relative difference between the estimate [stored in `r(integral)`] and the true value of the integral is about  $6 \times 10^{-6}$ . A graph of the absolute difference (`diff`) is shown below. Here error is cumulative. Again, most of the error is due to a relatively poorer fit near the endpoints.

```
. label variable diff "Error in integral estimate"
. twoway line diff x, ylabel(0(5.00e-06).00001)
```



## Stored results

dydx stores the following in  $r()$ :

Macros

$r(y)$  name of *yvar*

integ stores the following in  $r()$ :

Scalars

$r(N\_points)$  number of unique  $x$  points

$r(integral)$  estimate of the integral

## Methods and formulas

Consider a set of data points,  $(x_1, y_1), \dots, (x_n, y_n)$ , generated by a function  $y = f(x)$ . dydx and integ first fit these points with a cubic spline, which is then analytically differentiated (integrated) to give an approximation for the derivative (integral) of  $f$ .

The cubic spline (see, for example, Press et al. [2007]) consists of  $n - 1$  cubic polynomials  $P_i(x)$ , with the  $i$ th one defined on the interval  $[x_i, x_{i+1}]$ ,

$$P_i(x) = y_i a_i(x) + y_{i+1} b_i(x) + y_i'' c_i(x) + y_{i+1}'' d_i(x)$$

where

$$a_i(x) = \frac{x_{i+1} - x}{x_{i+1} - x_i}$$

$$b_i(x) = \frac{x - x_i}{x_{i+1} - x_i}$$

$$c_i(x) = \frac{1}{6}(x_{i+1} - x_i)^2 a_i(x) [\{a_i(x)\}^2 - 1]$$

$$d_i(x) = \frac{1}{6}(x_{i+1} - x_i)^2 b_i(x) [\{b_i(x)\}^2 - 1]$$

and  $y_i''$  and  $y_{i+1}''$  are constants whose values will be determined as described below. The notation for these constants is justified because  $P_i''(x_i) = y_i''$  and  $P_i''(x_{i+1}) = y_{i+1}''$ .

Because  $a_i(x_i) = 1$ ,  $a_i(x_{i+1}) = 0$ ,  $b_i(x_i) = 0$ , and  $b_i(x_{i+1}) = 1$ . Therefore,  $P_i(x_i) = y_i$ , and  $P_i(x_{i+1}) = y_{i+1}$ . Thus, the  $P_i$  jointly define a function that is continuous at the interval boundaries. The first derivative should be continuous at the interval boundaries; that is,

$$P_i'(x_{i+1}) = P_{i+1}'(x_{i+1})$$

The above  $n - 2$  equations (one equation for each point except the two endpoints) and the values of the first derivative at the endpoints,  $P_1'(x_1)$  and  $P_{n-1}'(x_n)$ , determine the  $n$  constants  $y_i''$ .

The value of the first derivative at an endpoint is set to the value of the derivative obtained by fitting a quadratic to the endpoint and the two adjacent points; namely, we use

$$P_1'(x_1) = \frac{y_1 - y_2}{x_1 - x_2} + \frac{y_1 - y_3}{x_1 - x_3} - \frac{y_2 - y_3}{x_2 - x_3}$$

and a similar formula for the upper endpoint.

dydx approximates  $f'(x_i)$  by using  $P_i'(x_i)$ .

integ approximates  $F(x_i) = F(x_1) + \int_{x_1}^{x_i} f(x) dx$  by using

$$I_0 + \sum_{k=1}^{i-1} \int_{x_k}^{x_{k+1}} P_k(x) dx$$

where  $I_0$  (an estimate of  $F(x_1)$ ) is the value specified by the `initial(#)` option. If the trapezoid option is specified, `integ` approximates the integral by using the trapezoidal rule:

$$I_0 + \sum_{k=1}^{i-1} \frac{1}{2} (x_{k+1} - x_k)(y_{k+1} + y_k)$$

If there are ties among the  $x_i$ , the mean of  $y_i$  is computed at each set of ties and the cubic spline is fit to these values.

## Acknowledgment

The present versions of `dydx` and `integ` were inspired by the `dydx2` command written by Patrick Royston of the MRC Clinical Trials Unit, London, and coauthor of the Stata Press book *Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model*.

**Maria Gaetana Agnesi** (1718–1799) was an Italian mathematician and philosopher.

Born in Milan into a wealthy family, she was recognized as a child prodigy. At age nine, she published a detailed argument in Latin on the importance of education for women. At age 15, her father, a mathematics professor at the University of Bologna, presented her talents in language and philosophical reasoning to Bologna’s intellectual elite. Uncomfortable with public life, she educated her twenty siblings and published on mathematics. After her father’s death in 1752, she studied theology and devoted the rest of her life to helping the poor, homeless, and sick.

Agnesi’s best known work, *Instituzioni analitiche ad uso della gioventù italiana* (*Analytical Institutions for the Use of Italian Youth*), written in 1748, helped develop the analysis of finite quantities and infinitesimals. At the time, it was hailed as the best introduction to calculus. The work also discussed an asymptotic curve that, because of mistranslation, would come to be known as the “Witch of Agnesi”. In 1750, Pope Benedict XIV appointed her to the chair of mathematics and natural philosophy at Bologna, though she never served.

In addition to being recognized as an important mathematician, Agnesi is revered in the Basilica of San Nazaro in Milan. At her death, she was mourned by radical authors as a proponent for women’s rights and by the Catholic faithful as a symbol of personal piety.

## Reference

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery. 2007. *Numerical Recipes: The Art of Scientific Computing*. 3rd ed. New York: Cambridge University Press.

## Also see

[D] **obs** — Increase the number of observations in a dataset

[D] **range** — Generate numerical range

## Description

An *eform\_option* causes the coefficient table to be displayed in exponentiated form: for each coefficient,  $e^b$  rather than  $b$  is displayed. Standard errors and confidence intervals (CIs) are also transformed.

An *eform\_option* is one of the following:

| <i>eform_option</i>               | Description                                                     |
|-----------------------------------|-----------------------------------------------------------------|
| <code>eform(<i>string</i>)</code> | use <i>string</i> for the column title                          |
| <code>eform</code>                | exponentiated coefficient, <i>string</i> is <code>exp(b)</code> |
| <code>hr</code>                   | hazard ratio, <i>string</i> is <code>Haz. ratio</code>          |
| <code>shr</code>                  | subhazard ratio, <i>string</i> is <code>SHR</code>              |
| <code>irr</code>                  | incidence-rate ratio, <i>string</i> is <code>IRR</code>         |
| <code>or</code>                   | odds ratio, <i>string</i> is <code>Odds ratio</code>            |
| <code>rrr</code>                  | relative-risk ratio, <i>string</i> is <code>RRR</code>          |

## Remarks and examples

### ▶ Example 1

Here is a simple example of the `or` option with `svy: logit`. The CI for the odds ratio is computed by transforming (by exponentiating) the endpoints of the CI for the corresponding coefficient.

```
. use https://www.stata-press.com/data/r19/nhanes2d
. svy, or: logit highbp female black
(running logit on estimation sample)
(output omitted)
```

| highbp | Odds ratio | Linearized<br>std. err. | t     | P> t  | [95% conf. interval] |          |
|--------|------------|-------------------------|-------|-------|----------------------|----------|
| female | .6107011   | .0326159                | -9.23 | 0.000 | .5476753             | .6809798 |
| black  | 1.384865   | .1336054                | 3.37  | 0.002 | 1.137507             | 1.686011 |
| _cons  | .7249332   | .0551062                | -4.23 | 0.000 | .6208222             | .8465035 |

Note: `_cons` estimates baseline odds.

We also could have specified the following command and received the same results as above:

```
. svy: logit highbp female black, or
```



## Reference

Buis, M. L. 2012. *Stata tip 107: The baseline is now reported*. *Stata Journal* 12: 165–166.

## Also see

[R] [ml](#) — Maximum likelihood estimation

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

## Description

`eivreg` fits errors-in-variables regression models when one or more of the independent variables are measured with error. To use `eivreg`, you must have an estimate of each independent variable's reliability or assume it is measured without error.

## Quick start

Regression of `y` on `x1`, `x2`, and `x3` adjusted for `x1` measured with 90% reliability

```
eivreg y x1 x2 x3, reliab(x1 .9)
```

Same as above, but also specify 80% reliability for `x2`

```
eivreg y x1 x2 x3, reliab(x1 .9 x2 .8)
```

## Menu

Statistics > Linear models and related > Errors-in-variables regression

## Syntax

```
eivreg depvar [indepvars] [if] [in] [weight] [, options]
```

| <i>options</i>                                                    | Description                                                                                                                                      |
|-------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                                      |                                                                                                                                                  |
| <code>reliab(<i>indepvar</i> # [<i>indepvar</i> # [...] ])</code> | specify measurement reliability for each <i>indepvar</i> measured with error                                                                     |
| <b>Reporting</b>                                                  |                                                                                                                                                  |
| <code>level(#)</code>                                             | set confidence level; default is level(95)                                                                                                       |
| <code>display_options</code>                                      | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| <code>coeflegend</code>                                           | display legend instead of statistics                                                                                                             |

*indepvars* may contain factor variables; see [U] 11.4.3 **Factor variables**.

`bootstrap`, `by`, `collect`, `jackknife`, `rolling`, and `statsby` are allowed; see [U] 11.1.10 **Prefix commands**.

Weights are not allowed with the `bootstrap` prefix; see [R] **bootstrap**.

`aweight`s are not allowed with the `jackknife` prefix; see [R] **jackknife**.

`aweight`s and `fweight`s are allowed; see [U] 11.1.6 **weight**.

`coeflegend` does not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options

### Model

`reliab(indepvar # [indepvar # [...] ])` specifies the measurement reliability for each independent variable measured with error. Reliabilities are specified as pairs consisting of an independent variable name (a name that appears in *indepvars*) and the corresponding reliability  $r$ ,  $0 < r \leq 1$ . Independent variables for which no reliability is specified are assumed to have reliability 1. If the option is not specified, all variables are assumed to have reliability 1, and the result is thus the same as that produced by `regress` (the ordinary least-squares results).

### Reporting

`level(#)`; see [R] **Estimation options**.

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] **Estimation options**.

The following option is available with `eivreg` but is not shown in the dialog box:

`coeflegend`; see [R] **Estimation options**.

## Remarks and examples

For an introduction to errors-in-variables regression, see [Draper and Smith \(1998, 89–91\)](#) or [Kmenta \(1997, 352–357\)](#). [Treiman \(2009, 258–261\)](#) compares the results of errors-in-variables regression with conventional regression. Also see [Lockwood and McCaffrey \(2020\)](#) for how to use `sem` (see [\[SEM\] sem](#)) to fit errors-in-variables regression.

Errors-in-variables regression models are useful when one or more of the independent variables are measured with additive noise. Standard regression (as performed by `regress`) would underestimate the effect of the variable, and the other coefficients in the model can be biased to the extent that they are correlated with the poorly measured variable. You can adjust for the biases if you know the reliability:

$$r = 1 - \frac{\text{noise variance}}{\text{total variance}}$$

That is, given the model  $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$ , for some variable  $\mathbf{x}_i$  in  $\mathbf{X}$ , the  $\mathbf{x}_i$  is observed with error,  $\mathbf{x}_i = \mathbf{x}_i^* + \mathbf{e}$ , and the noise variance is the variance of  $\mathbf{e}$ . The total variance is the variance of  $\mathbf{x}_i$ .

### ▷ Example 1

Say that in our automobile data, the weight of cars was measured with error, and the reliability of our measured weight is 0.85. The result of this would be to underestimate the effect of `weight` in a regression of, say, `price` on `weight` and `foreign`, and it would also bias the estimate of the coefficient on `foreign` (because being of foreign manufacture is correlated with the weight of cars). We would ignore all of this if we fit the model with `regress`:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. regress price weight foreign
```

| Source   | SS        | df | MS         | Number of obs | = | 74     |
|----------|-----------|----|------------|---------------|---|--------|
| Model    | 316859273 | 2  | 158429637  | F(2, 71)      | = | 35.35  |
| Residual | 318206123 | 71 | 4481776.38 | Prob > F      | = | 0.0000 |
| Total    | 635065396 | 73 | 8699525.97 | R-squared     | = | 0.4989 |
|          |           |    |            | Adj R-squared | = | 0.4848 |
|          |           |    |            | Root MSE      | = | 2117   |

| price   | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|---------|-------------|-----------|-------|-------|----------------------|-----------|
| weight  | 3.320737    | .3958784  | 8.39  | 0.000 | 2.531378             | 4.110096  |
| foreign | 3637.001    | 668.583   | 5.44  | 0.000 | 2303.885             | 4970.118  |
| _cons   | -4942.844   | 1345.591  | -3.67 | 0.000 | -7625.876            | -2259.812 |

With `eivreg`, we can account for our measurement error:

```
. eivreg price weight foreign, reliab(weight .85)
Errors-in-variables regression
```

| Variable | Assumed<br>reliability |                    |  |  |  |  |
|----------|------------------------|--------------------|--|--|--|--|
| weight   | 0.8500                 | Number of obs = 74 |  |  |  |  |
| *        | 1.0000                 | F( 2, 71) = 18.46  |  |  |  |  |
|          |                        | Prob > F = 0.0000  |  |  |  |  |
|          |                        | R-squared = 0.6483 |  |  |  |  |
|          |                        | Root MSE = 1773.54 |  |  |  |  |

| price   | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|---------|-------------|-----------|-------|-------|----------------------|-----------|
| weight  | 4.31985     | .7134251  | 6.06  | 0.000 | 2.89732              | 5.742379  |
| foreign | 4637.32     | 849.0221  | 5.46  | 0.000 | 2944.418             | 6330.222  |
| _cons   | -8257.017   | 2390.337  | -3.45 | 0.001 | -13023.21            | -3490.821 |

The effect of `weight` is increased—as we knew it would be—and here the effect of `foreign` manufacture is also increased. A priori, we knew only that the estimate of `foreign` might be biased; we did not know the direction.



## □ Technical note

Swept under the rug in our example is how we would determine the reliability,  $r$ . We can easily see that a variable is measured with error, but we may not know the reliability because the ingredients for calculating  $r$  depend on the unobserved noise.

For our example, we made up a value for  $r$ , and in fact we do not believe that `weight` is measured with error at all, so the reported `eivreg` results have no validity. The `regress` results were the statistically correct results here.

But let's say that we do suspect that `weight` is measured with error and that we do not know  $r$ . We could then experiment with various values of  $r$  to describe the sensitivity of our estimates to possible error levels. We may not know  $r$ , but  $r$  does have a simple interpretation, and we could probably produce a sensible range for  $r$  by thinking about how the data were collected.

If the reliability,  $r$ , is less than the  $R^2$  from a regression of the poorly measured variable on all the other variables, including the dependent variable, the information might as well not have been collected; no adjustment to the final results is possible. For our automobile data, running a regression of `weight` on `foreign` and `price` would result in an  $R^2$  of 0.6743. Thus, the reliability must be at least 0.6743 here. If we specify a reliability that is too small, `eivreg` will inform us and refuse to fit the model:

```
. eivreg price weight foreign, reliab(weight .6742)
reliability r() too small
r(399);
```

Returning to our problem of how to estimate  $r$ , too small or not, if the measurements are summaries of scaled items, the reliability may be estimated using the `alpha` command; see [MV] [alpha](#). If the score is computed from factor analysis and the data are scored using `predict`'s default options (see [MV] [factor postestimation](#)), the square of the standard deviation of the score is an estimate of the reliability.



## ► Example 2

Consider a model with more than one variable measured with error. For instance, say that our model is that price is a function of weight, foreign, and mpg and that both weight and mpg are measured with error.

```
. eivreg price weight foreign mpg, reliab(weight .85 mpg .95)
```

```
Errors-in-variables regression
```

| Variable | Assumed<br>reliability |               |   |         |
|----------|------------------------|---------------|---|---------|
| weight   | 0.8500                 | Number of obs | = | 74      |
| mpg      | 0.9500                 | F( 3, 70)     | = | 9.58    |
| *        | 1.0000                 | Prob > F      | = | 0.0000  |
|          |                        | R-squared     | = | 0.8522  |
|          |                        | Root MSE      | = | 1158.04 |

| price   | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|---------|-------------|-----------|-------|-------|----------------------|----------|
| weight  | 9.69903     | 3.768985  | 2.57  | 0.012 | 2.182027             | 17.21603 |
| foreign | 6918.624    | 2259.531  | 3.06  | 0.003 | 2412.132             | 11425.12 |
| mpg     | 627.6764    | 431.0284  | 1.46  | 0.150 | -231.9826            | 1487.335 |
| _cons   | -38545.27   | 20960.72  | -1.84 | 0.070 | -80350.11            | 3259.564 |

◀

## Stored results

eivreg stores the following in `e()`:

### Scalars

|                      |                             |
|----------------------|-----------------------------|
| <code>e(N)</code>    | number of observations      |
| <code>e(df_m)</code> | model degrees of freedom    |
| <code>e(df_r)</code> | residual degrees of freedom |
| <code>e(r2)</code>   | $R^2$                       |
| <code>e(F)</code>    | $F$ statistic               |
| <code>e(rmse)</code> | root mean squared error     |
| <code>e(rank)</code> | rank of $e(V)$              |

### Macros

|                            |                                               |
|----------------------------|-----------------------------------------------|
| <code>e(cmd)</code>        | eivreg                                        |
| <code>e(cmdline)</code>    | command as typed                              |
| <code>e(depvar)</code>     | name of dependent variable                    |
| <code>e(rellist)</code>    | <i>indepvars</i> and associated reliabilities |
| <code>e(wtype)</code>      | weight type                                   |
| <code>e(wexp)</code>       | weight expression                             |
| <code>e(properties)</code> | b V                                           |
| <code>e(predict)</code>    | program used to implement predict             |
| <code>e(marginsok)</code>  | predictions allowed by margins                |
| <code>e(asbalanced)</code> | factor variables fvset as asbalanced          |
| <code>e(asobserved)</code> | factor variables fvset as asobserved          |

### Matrices

|                   |                                              |
|-------------------|----------------------------------------------|
| <code>e(b)</code> | coefficient vector                           |
| <code>e(V)</code> | variance-covariance matrix of the estimators |

### Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

Matrices

`r(table)`

matrix containing the coefficients with their standard errors, test statistics,  $p$ -values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

Let the model to be fit be

$$\begin{aligned} \mathbf{y} &= \mathbf{X}^* \boldsymbol{\beta} + \boldsymbol{\epsilon} \\ \mathbf{X} &= \mathbf{X}^* + \mathbf{U} \end{aligned}$$

where  $\mathbf{X}^*$  are the true values and  $\mathbf{X}$  are the observed values.  $\boldsymbol{\epsilon}$  and  $\mathbf{U}$  are assumed to be independent and have zero means and finite fourth moments.  $\text{Var}(\mathbf{U})$  is assumed to be diagonal.

Let  $\mathbf{W}$  be the user-specified weights. If no weights are specified,  $\mathbf{W} = \mathbf{I}$ . If weights are specified, let  $\mathbf{v}$  be the specified weights. If `weight` frequency weights are specified, then  $\mathbf{W} = \text{diag}(\mathbf{v})$ . If `aweight` analytic weights are specified, then  $\mathbf{W} = \text{diag}\{\mathbf{v}/(\mathbf{1}'\mathbf{v})(\mathbf{1}'\mathbf{1})\}$ , meaning that the weights are normalized to sum to the number of observations.

The estimates  $\mathbf{b}$  of  $\boldsymbol{\beta}$  are obtained as  $\mathbf{A}^{-1}\mathbf{X}'\mathbf{W}\mathbf{y}$ , where  $\mathbf{A} = \mathbf{X}'\mathbf{W}\mathbf{X} - \mathbf{S}$ .  $\mathbf{S}$  is a diagonal matrix with elements  $(1 - r_j)s_j^2$ . Here  $r_j$  is the user-specified reliability coefficient for the  $j$ th explanatory variable (or 1 if not specified), and  $s_j^2$  is the (appropriately weighted) sample variance of the variable.

The root mean squared error is  $(\mathbf{y}'\mathbf{W}\mathbf{y} - \mathbf{b}\mathbf{A}\mathbf{b}')/(n - p)$ , where  $n$  is the number of observations and  $p$  is the number of estimated parameters. The variance–covariance matrix of the estimators is obtained based on the formulas provided in [Stefanski and Boos \(2002\)](#), [Buonaccorsi \(2010\)](#), and [Fuller \(1987\)](#). For each  $i = 1, 2, \dots, n$ , let residual  $e_i = y_i - \mathbf{x}_i\mathbf{b}$ , where  $\mathbf{x}_i$  is the  $i$ th row of  $\mathbf{X}$ . Consider matrix  $\mathbf{H}$ , where the  $i$ th row of  $\mathbf{H}$ ,  $\mathbf{h}_i$ , is

$$\mathbf{h}_i' = \begin{pmatrix} e_i x_{i1} + (x_{i1} - \bar{x}_1)^2 (1 - r_1) b_1 \\ e_i x_{i2} + (x_{i2} - \bar{x}_2)^2 (1 - r_2) b_2 \\ \vdots \\ e_i x_{ip} + (x_{ip} - \bar{x}_p)^2 (1 - r_p) b_p \end{pmatrix}$$

where  $\bar{x}_j$  is the weighted mean of the  $j$ th variable.

If analytic weights, `aweight`s, are specified, the variance–covariance matrix is  $\mathbf{A}^{-1}\mathbf{H}'\mathbf{W}\mathbf{H}\mathbf{A}^{-1}$ ; otherwise, it is  $\mathbf{A}^{-1}\mathbf{H}'\mathbf{W}\mathbf{H}\mathbf{A}^{-1}$ .

## References

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## Also see

- [R] [eivreg postestimation](#) — Postestimation tools for eivreg
- [R] [regress](#) — Linear regression
- [SEM] [Example 24](#) — Reliability
- [U] [20 Estimation and postestimation commands](#)

## Postestimation commands

The following postestimation commands are available after `eivreg`:

| Command                      | Description                                                                                       |
|------------------------------|---------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                               |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                      |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                |
| <code>estimates</code>       | cataloging estimation results                                                                     |
| <code>etable</code>          | table of estimation results                                                                       |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters    |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                           |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters |
| <code>predict</code>         | linear predictions                                                                                |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                              |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                |

## predict

### Description for predict

`predict` creates a new variable containing the linear prediction assuming that values of the covariates used for the prediction were measured without error.

### Menu for predict

Statistics > Postestimation

### Syntax for predict

```
predict [type] newvar [if] [in]
```

Available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

## margins

### Description for margins

margins estimates margins of response for linear predictions.

### Menu for margins

Statistics > Postestimation

### Syntax for margins

```
margins [marginlist] [, options]
```

### Remarks and examples

#### ▷ Example 1

We return to [example 1](#) from [R] [eivreg](#):

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. eivreg price weight foreign, reliab(weight .85)
Errors-in-variables regression
```

| Variable | Assumed<br>reliability |                    |  |  |  |  |
|----------|------------------------|--------------------|--|--|--|--|
| weight   | 0.8500                 | Number of obs = 74 |  |  |  |  |
| *        | 1.0000                 | F( 2, 71) = 18.46  |  |  |  |  |
|          |                        | Prob > F = 0.0000  |  |  |  |  |
|          |                        | R-squared = 0.6483 |  |  |  |  |
|          |                        | Root MSE = 1773.54 |  |  |  |  |

| price   | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|---------|-------------|-----------|-------|-------|----------------------|-----------|
| weight  | 4.31985     | .7134251  | 6.06  | 0.000 | 2.89732              | 5.742379  |
| foreign | 4637.32     | 849.0221  | 5.46  | 0.000 | 2944.418             | 6330.222  |
| _cons   | -8257.017   | 2390.337  | -3.45 | 0.001 | -13023.21            | -3490.821 |

We wish to predict the price of a foreign car that weighs 2,300 pounds. We can use `predict` because 2,300 pounds is the true weight, not the result of an error-prone measurement.

To make this prediction, first we add the new observation to the dataset.

```
. set obs 75
Number of observations (_N) was 74, now 75.
. replace foreign = 1 in 75
(1 real change made)
. replace weight = 2300 in 75
(1 real change made)
```

Now, we use `predict` to predict the price of the car.

```
. predict newprice in 75
(option xb assumed; fitted values)
(74 missing values generated)
(predictions assume covariates measured without error)
. list weight foreign newprice in 75
```

|     | weight | foreign | newprice |
|-----|--------|---------|----------|
| 75. | 2,300  | Foreign | 6315.957 |

`predict` issued a note reminding us that the computed predictions assume that the covariates used for prediction are measured without error. In general, you should avoid using `predict` to obtain in-sample predictions unless you first replace the measurement-error covariates with values that are error free.



## Also see

[R] [eivreg](#) — Errors-in-variables regression

[U] [20 Estimation and postestimation commands](#)

|                 |                      |                |                      |
|-----------------|----------------------|----------------|----------------------|
| Description     | Quick start          | Menu           | Syntax               |
| Options         | Remarks and examples | Stored results | Methods and formulas |
| Acknowledgments | References           | Also see       |                      |

## Description

`ir` is used with incidence-rate (incidence-density or person-time) data. It calculates point estimates and confidence intervals for the incidence-rate ratio (IRR) and incidence-rate difference (IRD), along with attributable or prevented fractions for the exposed and total population. `iri` is the immediate form of `ir`; see [U] 19 **Immediate commands**. Also see [R] **poisson** and [ST] **stcox** for related commands.

`cs` is used with cohort study data with equal follow-up time per subject and sometimes with cross-sectional data. Risk is then the proportion of subjects who become cases. It calculates point estimates and confidence intervals for the risk difference, risk ratio, and (optionally) the odds ratio, along with attributable or prevented fractions for the exposed and total population. `csi` is the immediate form of `cs`; see [U] 19 **Immediate commands**. Also see [R] **logistic** for related commands.

`cc` is used with case–control and cross-sectional data. It calculates point estimates and confidence intervals for the odds ratio, along with attributable or prevented fractions for the exposed and total population. `cci` is the immediate form of `cc`; see [U] 19 **Immediate commands**. Also see [R] **logistic** for related commands.

`tabodds` is used with case–control and cross-sectional data. It tabulates the odds of failure against a categorical explanatory variable `expvar`. If `expvar` is specified, `tabodds` performs an approximate  $\chi^2$  test of homogeneity of odds and a test for linear trend of the log odds against the numerical code used for the categories of `expvar`. Both tests are based on the score statistic and its variance; see *Methods and formulas*. When `expvar` is absent, the overall odds are reported. The variable `varcase` is coded 0/1 for individual and simple frequency records and equals the number of cases for binomial frequency records.

Optionally, `tabodds` tabulates adjusted or unadjusted odds ratios, using either the lowest levels of `expvar` or a user-defined level as the reference group. If `adjust (varlist)` is specified, it produces odds ratios adjusted for the variables in `varlist` along with a (score) test for trend.

`mhodds` is used with case–control and cross-sectional data. It estimates the ratio of the odds of failure for two categories of `expvar`, controlled for specified confounding variables, `varsadjust`, and tests whether this odds ratio is equal to one. When `expvar` has more than two categories but none are specified with the `compare()` option, `mhodds` assumes that `expvar` is a quantitative variable and calculates a 1-degree-of-freedom test for trend. It also calculates an approximate estimate of the log odds-ratio for a one-unit increase in `expvar`. This is a one-step Newton–Raphson approximation to the maximum likelihood estimate calculated as the ratio of the score statistic,  $U$ , to its variance,  $V$  (Clayton and Hills 1993, 103).

`mcc` is used with matched case–control data. It calculates McNemar’s  $\chi^2$ ; point estimates and confidence intervals for the difference, ratio, and relative difference of the proportion with the factor; and the odds ratio and its confidence interval. `mcci` is the immediate form of `mcc`; see [U] 19 **Immediate commands**. Also see [R] **clogit** and [R] **symmetry** for related commands.

## Quick start

### *Cohort studies*

IRR and IRD for the number of cases stored in cases for exposure indicator exposed given time exposed time

```
ir cases exposed time
```

Crude and Mantel–Haenszel combined IRRs with test of homogeneity for strata defined by svar

```
ir cases exposed time, by(svar)
```

Same as above, and standardize the IRR by weighting variable wvar1

```
ir cases exposed time, by(svar) standard(wvar1)
```

Same as above, but use person-time of the unexposed group as weights

```
ir cases exposed time, by(svar) estandard
```

IRR and IRD for 10 cases over 50 person-years in the exposed group and 15 cases over 100 person-years in the unexposed group

```
iri 10 15 50 100
```

Risk difference and ratio with binary indicators case and exposed using cumulative incidence data

```
cs case exposed [fweight=wvar2]
```

Add odds ratios and calculate Fisher's exact  $p$

```
cs case exposed [fweight=wvar2], or exact
```

Internally standardized risk ratio for strata defined by svar

```
cs case exposed [fweight=wvar2], by(svar) istandard
```

Risk difference and ratio for 12 cases and 55 noncases among exposed subjects and 16 cases and 125 noncases among unexposed subjects

```
csi 12 16 55 125
```

### *Case-control studies*

Odds ratios from summary data with binary indicators case and exposed and frequency weight wvar3

```
cc case exposed [fweight=wvar3]
```

Same as above, but stratify analysis by svar and perform Breslow–Day and Tarone's homogeneity tests

```
cc case exposed [fweight=wvar3], by(svar) bd tarone
```

Odds ratios for 37 exposed cases, 148 unexposed cases, 7 exposed controls, and 137 unexposed controls

```
cci 37 148 7 137
```

Odds of binary event against catvar using summary data with frequency weight wvar4

```
tabodds event catvar [fweight=wvar4]
```

Same as above, but report odds ratios with the fourth level of catvar as the reference

```
tabodds event catvar [fweight=wvar4] , or base(4)
```

Same as above, but tabulate Mantel–Haenszel adjusted odds ratios adjusting for values of categorical variable a

```
tabodds event catvar [fweight=wvar4] , base(4) adjust(a)
```

Graph odds and confidence intervals against categories of catvar

```
tabodds event catvar [fweight=wvar4] , ciplot
```

Odds ratios for the effect of catvar on event controlling for categorical variable a using summary data with frequency weight wvar5

```
mhodds event catvar a [fweight=wvar5]
```

Same as above, but calculate odds ratios for each level of svar

```
mhodds event catvar a [fweight=wvar5] , by(svar)
```

Maximum likelihood estimate of odds ratio for a equal to 4 compared with a equal to 1

```
mhodds event a [fweight=wvar5] , compare(4,1)
```

Statistics on the difference in the proportion with the factor for exposed cases indicated in expcase and exposed controls indicated in expcontrol using summary data with frequency weight wvar6

```
mcc expcase expcontrol [fweight=wvar6]
```

Same as above, but indicate that there are 4 pairs where both cases and controls were exposed, 9 pairs where the case was exposed but the control was not, 3 pairs where the control was exposed but the case was not, and 14 pairs where neither subject was exposed

```
mcci 4 9 3 14
```

## Menu

### **ir**

Statistics > Epidemiology and related > Tables for epidemiologists > Incidence-rate ratio

### **iri**

Statistics > Epidemiology and related > Tables for epidemiologists > Incidence-rate-ratio calculator

### **cs**

Statistics > Epidemiology and related > Tables for epidemiologists > Cohort study risk-ratio etc.

### **csi**

Statistics > Epidemiology and related > Tables for epidemiologists > Cohort study risk-ratio etc. calculator

### **cc**

Statistics > Epidemiology and related > Tables for epidemiologists > Case-control odds ratio

### **cci**

Statistics > Epidemiology and related > Tables for epidemiologists > Case-control odds-ratio calculator

### **tabodds**

Statistics > Epidemiology and related > Tables for epidemiologists > Tabulate odds of failure by category

### **mhodds**

Statistics > Epidemiology and related > Tables for epidemiologists > Ratio of odds of failure for two categories

### **mcc**

Statistics > Epidemiology and related > Tables for epidemiologists > Matched case-control studies

### **mcci**

Statistics > Epidemiology and related > Tables for epidemiologists > Matched case-control calculator

## Syntax

### Cohort studies

```
ir varcase varexposed vartime [if] [in] [weight] [, ir_options]
```

```
iri #a #b #N1 #N2 [, iri_options]
```

```
cs varcase varexposed [if] [in] [weight] [, cs_options]
```

```
csi #a #b #c #d [, csi_options]
```

### Case–control studies

```
cc varcase varexposed [if] [in] [weight] [, cc_options]
```

```
cci #a #b #c #d [, cci_options]
```

```
tabodds varcase [expvar] [if] [in] [weight] [, tabodds_options]
```

```
mhodds varcase expvar [varsadjust] [if] [in] [weight] [, mhodds_options]
```

### Matched case–control studies

```
mcc varexposedcase varexposedcontrol [if] [in] [weight] [, level(#)]
```

```
mcci #a #b #c #d [, level(#)]
```

| <i>ir_options</i>                        | Description                                                                                         |
|------------------------------------------|-----------------------------------------------------------------------------------------------------|
| Options                                  |                                                                                                     |
| by( <i>varname</i> [ , <u>missing</u> ]) | stratify on <i>varname</i>                                                                          |
| <u>e</u> standard                        | combine external weights with within-stratum statistics                                             |
| <u>i</u> standard                        | combine internal weights with within-stratum statistics                                             |
| <u>s</u> tandard( <i>varname</i> )       | combine user-specified weights with within-stratum statistics                                       |
| <u>p</u> ool                             | display pooled estimate                                                                             |
| <u>n</u> ocrude                          | do not display crude estimate                                                                       |
| <u>n</u> ohom                            | do not display homogeneity test                                                                     |
| ird                                      | calculate standardized IRD                                                                          |
| midp                                     | display <i>p</i> -values calculated using mid- <i>p</i> adjustment (unstratified only); the default |
| exact                                    | display exact <i>p</i> -values without mid- <i>p</i> adjustment (unstratified only)                 |
| <u>l</u> evel(#)                         | set confidence level; default is level(95)                                                          |

| <i>iri_options</i> | Description                                                                     |
|--------------------|---------------------------------------------------------------------------------|
| midp               | display <i>p</i> -values calculated using mid- <i>p</i> adjustment; the default |
| exact              | display exact <i>p</i> -values without mid- <i>p</i> adjustment                 |
| <u>l</u> evel(#)   | set confidence level; default is level(95)                                      |

| <i>cs_options</i>                                 | Description                                                      |
|---------------------------------------------------|------------------------------------------------------------------|
| Options                                           |                                                                  |
| <code>by(<i>varlist</i>[, <i>missing</i>])</code> | stratify on <i>varlist</i>                                       |
| <code>estandard</code>                            | combine external weights with within-stratum statistics          |
| <code>istandard</code>                            | combine internal weights with within-stratum statistics          |
| <code>standard(<i>varname</i>)</code>             | combine user-specified weights with within-stratum statistics    |
| <code>pool</code>                                 | display pooled estimate                                          |
| <code>nocrude</code>                              | do not display crude estimate                                    |
| <code>nohom</code>                                | do not display homogeneity test                                  |
| <code>rd</code>                                   | calculate standardized risk difference                           |
| <code>binomial(<i>varname</i>)</code>             | number of subjects variable                                      |
| <code>or</code>                                   | report odds ratio                                                |
| <code>woolf</code>                                | use Woolf approximation to calculate SE and CI of the odds ratio |
| <code>exact</code>                                | calculate Fisher's exact <i>p</i>                                |
| <code>level(#)</code>                             | set confidence level; default is level(95)                       |

| <i>csi_options</i>    | Description                                                      |
|-----------------------|------------------------------------------------------------------|
| <code>or</code>       | report odds ratio                                                |
| <code>woolf</code>    | use Woolf approximation to calculate SE and CI of the odds ratio |
| <code>exact</code>    | calculate Fisher's exact <i>p</i>                                |
| <code>level(#)</code> | set confidence level; default is level(95)                       |

| <i>cc_options</i>                                 | Description                                                      |
|---------------------------------------------------|------------------------------------------------------------------|
| Options                                           |                                                                  |
| <code>by(<i>varname</i>[, <i>missing</i>])</code> | stratify on <i>varname</i>                                       |
| <code>estandard</code>                            | combine external weights with within-stratum statistics          |
| <code>istandard</code>                            | combine internal weights with within-stratum statistics          |
| <code>standard(<i>varname</i>)</code>             | combine user-specified weights with within-stratum statistics    |
| <code>pool</code>                                 | display pooled estimate                                          |
| <code>nocrude</code>                              | do not display crude estimate                                    |
| <code>nohom</code>                                | do not display homogeneity test                                  |
| <code>bd</code>                                   | perform Breslow–Day homogeneity test                             |
| <code>tarone</code>                               | perform Tarone's homogeneity test                                |
| <code>binomial(<i>varname</i>)</code>             | number of subjects variable                                      |
| <code>cornfield</code>                            | use Cornfield approximation to calculate CI of the odds ratio    |
| <code>woolf</code>                                | use Woolf approximation to calculate SE and CI of the odds ratio |
| <code>exact</code>                                | calculate Fisher's exact <i>p</i>                                |
| <code>level(#)</code>                             | set confidence level; default is level(95)                       |

| <i>cci_options</i>     | Description                                                      |
|------------------------|------------------------------------------------------------------|
| <code>cornfield</code> | use Cornfield approximation to calculate CI of the odds ratio    |
| <code>woolf</code>     | use Woolf approximation to calculate SE and CI of the odds ratio |
| <code>exact</code>     | calculate Fisher's exact $p$                                     |
| <code>level(#)</code>  | set confidence level; default is level(95)                       |

| <i>tabodds_options</i>                  | Description                                                            |
|-----------------------------------------|------------------------------------------------------------------------|
| Main                                    |                                                                        |
| <code>binomial(varname)</code>          | number of subjects variable                                            |
| <code>level(#)</code>                   | set confidence level; default is level(95)                             |
| <code>or</code>                         | report odds ratio                                                      |
| <code>adjust(varlist)</code>            | report odds ratios adjusted for the variables in <i>varlist</i>        |
| <code>base(#)</code>                    | reference group of control variable for odds ratio                     |
| <code>cornfield</code>                  | use Cornfield approximation to calculate CI of the odds ratio          |
| <code>woolf</code>                      | use Woolf approximation to calculate SE and CI of the odds ratio       |
| <code>graph</code>                      | graph odds against categories                                          |
| <code>ciplot</code>                     | same as <code>graph</code> option, except include confidence intervals |
| CI plot                                 |                                                                        |
| <code>ciopts(rcap_options)</code>       | affect rendition of the confidence bands                               |
| Plot                                    |                                                                        |
| <code>marker_options</code>             | change look of markers (color, size, etc.)                             |
| <code>marker_label_options</code>       | add marker labels; change look or position                             |
| <code>cline_options</code>              | affect rendition of the plotted points                                 |
| Add plots                               |                                                                        |
| <code>addplot(plot)</code>              | add other plots to the generated graph                                 |
| Y axis, X axis, Titles, Legend, Overall |                                                                        |
| <code>twoway_options</code>             | any options other than by() documented in [G-3] <i>twoway_options</i>  |

| <i>mhodds_options</i>                              | Description                                 |
|----------------------------------------------------|---------------------------------------------|
| Options                                            |                                             |
| <code>by(varlist[, missing])</code>                | stratify on <i>varlist</i>                  |
| <code>binomial(varname)</code>                     | number of subjects variable                 |
| <code>compare(v<sub>1</sub>, v<sub>2</sub>)</code> | override categories of the control variable |
| <code>level(#)</code>                              | set confidence level; default is level(95)  |

`collect` is allowed with `ir`, `iri`, `cs`, `csi`, `cc`, `cci`, `tabodds`, `mhodds`, `mcc`, and `mcci`; see [U] 11.1.10 Prefix commands.  
`fweights` are allowed; see [U] 11.1.6 `weight`.

## Options

Options are listed in the order that they appear in the syntax tables above. The commands for which the option is valid are indicated in parentheses immediately after the option name.

Options (ir, cs, cc, and mhodds) / Main (tabodds)

`by(varname[, missing])` (ir, cs, cc, and mhodds) specifies that the tables be stratified on *varname*.

Missing categories in *varname* are omitted from the stratified analysis, unless option `missing` is specified within `by()`. Within-stratum statistics are shown and then combined with Mantel–Haenszel weights. If `estandard`, `istandard`, or `standard()` is also specified (see below), the weights specified are used in place of Mantel–Haenszel weights.

`estandard`, `istandard`, and `standard(varname)` (ir, cs, and cc) request that within-stratum statistics be combined with external, internal, or user-specified weights to produce a standardized estimate. These options are mutually exclusive and can be used only when `by()` is also specified. (When `by()` is specified without one of these options, Mantel–Haenszel weights are used.)

`estandard` external weights are the person-time for the unexposed (ir), the total number of unexposed (cs), or the number of unexposed controls (cc).

`istandard` internal weights are the person-time for the exposed (ir), the total number of exposed (cs), or the number of exposed controls (cc). `istandard` can be used to produce, among other things, standardized mortality ratios (SMRs).

`standard(varname)` allows user-specified weights. *varname* must contain a constant within stratum and be nonnegative. The scale of *varname* is irrelevant.

`pool` (ir, cs, and cc) specifies that, in a stratified analysis, the directly pooled estimate also be displayed. The pooled estimate is a weighted average of the stratum-specific estimates using inverse-variance weights, which are the inverse of the variance of the stratum-specific estimate. `pool` is relevant only if `by()` is also specified.

`nocrude` (ir, cs, and cc) specifies that in a stratified analysis the crude estimate—an estimate obtained without regard to strata—not be displayed. `nocrude` is relevant only if `by()` is also specified.

`nohom` (ir, cs, and cc) specifies that a  $\chi^2$  test of homogeneity not be included in the output of a stratified analysis. This tests whether the exposure effect is the same across strata and can be performed for any pooled estimate—directly pooled or Mantel–Haenszel. `nohom` is relevant only if `by()` is also specified.

`ird` (ir) may be used only with `estandard`, `istandard`, or `standard()`. It requests that ir calculate the standardized IRD rather than the default IRR.

`midp` (ir without `by()` and `iri`), the default, displays mid-*p*-adjusted *p*-values for one-sided and two-sided tests of IRD. The tests of IRD are not available with `ir` for stratified analysis, so `midp` is not allowed in combination with `by()`. Only one of `exact` or `midp` may be specified.

`exact` (ir without `by()` and `iri`) displays exact *p*-values for one-sided and two-sided tests of IRD instead of the default mid-*p*-adjusted *p*-values. This option produces *p*-values that are more conservative than the mid-*p*-adjusted *p*-values. When counts of exposed and unexposed cases are both large, `exact` and `midp` give similar results. The tests of IRD are not available with `ir` for stratified analysis, so `exact` is not allowed in combination with `by()`. Only one of `exact` or `midp` may be specified.

`rd` (cs) may be used only with `estandard`, `istandard`, or `standard()`. It requests that cs calculate the standardized risk difference rather than the default risk ratio.

- `bd(cc)` specifies that Breslow and Day's  $\chi^2$  test of homogeneity be included in the output of a stratified analysis. This tests whether the exposure effect is the same across strata. `bd` is relevant only if `by()` is also specified.
- `tarone(cc)` specifies that Tarone's  $\chi^2$  test of homogeneity, which is a correction to the Breslow–Day test, be included in the output of a stratified analysis. This tests whether the exposure effect is the same across strata. `tarone` is relevant only if `by()` is also specified.
- `binomial(varname)` (`cs`, `cc`, `tabodds`, and `mhodds`) supplies the number of subjects (cases plus controls) for binomial frequency records. For individual and simple frequency records, this option is not used.
- `or(cs, csi, and tabodds)`, for `cs` and `csi`, reports the calculation of the odds ratio in addition to the risk ratio if `by()` is not specified. With `by()`, `or` specifies that a Mantel–Haenszel estimate of the combined odds ratio be made rather than the Mantel–Haenszel estimate of the risk ratio. In either case, this is the same calculation that would be made by `cc` and `cci`. Typically, `cc`, `cci`, or `tabodds` is preferred for calculating odds ratios. For `tabodds`, `or` specifies that odds ratios be produced; see `base()` for details about selecting a reference category. By default, `tabodds` will calculate odds.
- `adjust(varlist)` (`tabodds`) specifies that odds ratios adjusted for the variables in `varlist` be calculated.
- `base(#)` (`tabodds`) specifies that the `#`th category of `expvar` be used as the reference group for calculating odds ratios. If `base()` is not specified, the first category, corresponding to the minimum value of `expvar`, is used as the reference group.
- `cornfield(cc, cci, and tabodds)` requests that the [Cornfield \(1956\)](#) approximation be used to calculate the confidence interval of the odds ratio. By default, `cc` and `cci` report an exact interval and `tabodds` reports a standard-error–based interval, with the standard error coming from the square root of the variance of the score statistic.
- `woolf(cs, csi, cc, cci, and tabodds)` requests that the [Woolf \(1955\)](#) approximation, also known as the Taylor expansion, be used for calculating the standard error and confidence interval for the odds ratio. By default, `cs` and `csi` with the `or` option report the [Cornfield \(1956\)](#) interval; `cc` and `cci` report an exact interval; and `tabodds` reports a standard-error–based interval, with the standard error coming from the square root of the variance of the score statistic.
- `exact(cs, csi, cc, and cci)` requests that Fisher's exact  $p$  be calculated rather than the  $\chi^2$  and its significance level. We recommend specifying `exact` whenever samples are small. When the least-frequent cell contains 1,000 cases or more, there will be no appreciable difference between the exact significance level and the significance level based on the  $\chi^2$ , but the exact significance level will take considerably longer to calculate. `exact` does not affect whether exact confidence intervals are calculated. Commands always calculate exact confidence intervals where they can, unless `cornfield` or `woolf` is specified.
- `compare(v1, v2)` (`mhodds`) indicates the categories of `expvar` to be compared;  $v_1$  defines the numerator and  $v_2$ , the denominator. When `compare()` is not specified and there are only two categories, the second is compared with the first; when there are more than two categories, an approximate estimate of the odds ratio for a unit increase in `expvar`, controlled for specified confounding variables, is given.
- `level(#)` (`ir`, `iri`, `cs`, `csi`, `cc`, `cci`, `tabodds`, `mhodds`, `mcc`, and `mcci`) specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [\[R\] level](#).

The following options are for use only with `tabodds`.

#### Main

`graph` (`tabodds`) produces a graph of the odds against the numerical code used for the categories of *expvar*. All graph options except `connect()` are allowed. This option is not allowed with the `or` option or the `adjust()` option.

`ciplot` (`tabodds`) produces the same plot as the `graph` option, except that it also includes the confidence intervals. This option may not be used with either the `or` option or the `adjust()` option.

#### CI plot

`ciopts` (*rcap\_options*) (`tabodds`) is allowed only with the `ciplot` option. It affects the rendition of the confidence bands; see [G-3] *rcap\_options*.

#### Plot

*marker\_options* (`tabodds`) affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] *marker\_options*.

*marker\_label\_options* (`tabodds`) specify if and how the markers are to be labeled; see [G-3] *marker\_label\_options*.

*cline\_options* (`tabodds`) affect whether lines connect the plotted points and the rendition of those lines; see [G-3] *cline\_options*.

#### Add plots

`addplot` (*plot*) (`tabodds`) provides a way to add other plots to the generated graph; see [G-3] *addplot\_option*.

#### Y axis, X axis, Titles, Legend, Overall

*twoway\_options* (`tabodds`) are any of the options documented in [G-3] *twoway\_options*, excluding `by()`. These include options for titling the graph (see [G-3] *title\_options*) and options for saving the graph to disk (see [G-3] *saving\_option*).

## Remarks and examples

Remarks are presented under the following headings:

- Incidence-rate data*
- Stratified incidence-rate data*
- Standardized estimates with stratified incidence-rate data*
- Cumulative incidence data*
- Stratified cumulative incidence data*
- Standardized estimates with stratified cumulative incidence data*
- Case-control data*
- Stratified case-control data*
- Case-control data with multiple levels of exposure*
- Case-control data with confounders and possibly multiple levels of exposure*
- Standardized estimates with stratified case-control data*
- Matched case-control data*
- Video examples*
- Glossary*

To calculate appropriate statistics and suppress inappropriate statistics, the `ir`, `cs`, `cc`, `tabodds`, `mhodds`, and `mcc` commands, along with their immediate counterparts, are organized in the way epidemiologists conceptualize data. `ir` processes incidence-rate data from prospective studies; `cs`, cohort study data with equal follow-up time (cumulative incidence); `cc`, `tabodds`, and `mhodds`, case–control or cross-sectional (prevalence) data; and `mcc`, matched case–control data. With the exception of `mcc`, these commands work with both simple and stratified tables.

Epidemiological data are often summarized in a contingency table from which various statistics are calculated. The rows of the table reflect cases and noncases or cases and person-time, and the columns reflect exposure to a *risk factor*. To an epidemiologist, *cases* and *noncases* refer to the outcomes of the process being studied. For instance, a case might be a person with cancer and a noncase might be a person without cancer.

A *factor* is something that might affect the chances of being ultimately designated a case or a noncase. Thus, a case might be a cancer patient, and the factor might be smoking behavior. A person is said to be *exposed* or *unexposed* to the factor. Exposure can be classified as a dichotomy, smokes or does not smoke, or as multiple levels, such as number of cigarettes smoked per week.

For an introduction to epidemiological methods, see Walker (1991). For an intermediate treatment, see Clayton and Hills (1993) and Schneider and Lilienfeld (2015). For other advanced discussions, see Kleinbaum, Kupper, and Morgenstern (1982) and Lash et al. (2021). For an analysis of incidence rates, see, for instance, Cummings (2019). For an anthology of writings on epidemiology since World War II, see Greenland (1987). See Jewell (2004) for a text aimed at graduate students in the medical professions that uses Stata for much of the analysis. See Dohoo, Martin, and Stryhn (2010) for a graduate-level text on the principles and methods of veterinary epidemiologic research; Stata datasets and do-files are available. Also see Dohoo, Martin, and Stryhn (2012) for a text that is a revision of their veterinary epidemiology text, but examples from human epidemiology are used.

## Incidence-rate data

In incidence-rate data from a prospective study, you observe the transformation of noncases into cases. Starting with a group of noncase subjects, you monitor them to determine whether they become cases (for example, stricken with cancer). You monitor two populations: those exposed and those unexposed to the factor (for example, multiple X-rays). A summary of the data is

|             | Exposed | Unexposed | Total       |
|-------------|---------|-----------|-------------|
| Cases       | $a$     | $b$       | $a + b$     |
| Person-time | $N_1$   | $N_0$     | $N_1 + N_0$ |

► Example 1: iri

It will be easiest to understand these commands if we start with the immediate forms. Remember, in the immediate form, we specify the data on the command line rather than specifying names of variables containing the data; see [U] 19 Immediate commands. We have data (Boice and Monson [1977]; reported in Lash et al. [2021, 408]) on breast cancer cases and person-years of observation for women with tuberculosis repeatedly exposed to multiple X-ray fluoroscopies, and those not so exposed:

|                     | X-ray fluoroscopy |           |
|---------------------|-------------------|-----------|
|                     | Exposed           | Unexposed |
| Breast cancer cases | 41                | 15        |
| Person-years        | 28,010            | 19,017    |

Using iri, the immediate form of ir, we specify the values in the table following the command:

```
. iri 41 15 28010 19017
```

Incidence-rate comparison

|                 | Exposed        | Unexposed | Total                |
|-----------------|----------------|-----------|----------------------|
| Cases           | 41             | 15        | 56                   |
| Person-time     | 28010          | 19017     | 47027                |
| Incidence rate  | .0014638       | .0007888  | .0011908             |
|                 | Point estimate |           | [95% conf. interval] |
| Inc. rate diff. | .000675        |           | .0012751             |
| Inc. rate ratio | 1.855759       |           | 3.6093 (exact)       |
| Attr. frac. ex. | .4611368       |           | .722938 (exact)      |
| Attr. frac. pop | .337618        |           |                      |

Mid-p-values for tests of incidence-rate difference:

Adj Pr(Exposed cases <= 41) = 0.9823 (lower one-sided)

Adj Pr(Exposed cases >= 41) = 0.0177 (upper one-sided)

Two-sided p-value = 0.0355

iri shows the table, reports the incidence rates for the exposed and unexposed populations, and then shows the point estimates of the difference and ratio of the two incidence rates along with their confidence intervals. The incidence rate is simply the frequency with which noncases are transformed into cases.

Next, iri reports the attributable fraction among the exposed (AFE), an estimate of the proportion of exposed cases attributable to exposure. We estimate that 46.1% of the 41 breast cancer cases among the exposed were due to exposure. (Had the IRR been less than 1, iri would have reported the prevented fraction among the exposed (PFE), an estimate of the net proportion of all potential cases in the exposed population that was prevented by exposure; see the following technical note.)

After that, the table shows the attributable fraction for the population (AFP), which is the net proportion of all cases attributable to exposure. This number, of course, depends on the proportion of cases that are exposed in the base population, which here is taken to be 41/56 and may not be relevant in all situations. We estimate that 33.8% of the 56 cases were due to exposure. We estimate that 18.9 cases were caused by exposure; that is,  $0.338 \times 56 = 0.461 \times 41 = 18.9$ .

At the bottom of the table, iri reports one- and two-sided tests of the IRD. For the one-sided test of the number of exposed cases being 41 or greater, the p-value is 0.0177. The two-sided test is twice the smallest one-sided p-value and is 0.0355. These p-values are calculated using the mid-p adjustment to exact p-values.

Exact *p*-values can be seen by specifying the exact option.

```
. iri 41 15 28010 19017, exact
Incidence-rate comparison
```

|                 | Exposed        | Unexposed | Total                |
|-----------------|----------------|-----------|----------------------|
| Cases           | 41             | 15        | 56                   |
| Person-time     | 28010          | 19017     | 47027                |
| Incidence rate  | .0014638       | .0007888  | .0011908             |
|                 | Point estimate |           | [95% conf. interval] |
| Inc. rate diff. | .000675        | .0000749  | .0012751             |
| Inc. rate ratio | 1.855759       | 1.005684  | 3.6093 (exact)       |
| Attr. frac. ex. | .4611368       | .0056519  | .722938 (exact)      |
| Attr. frac. pop | .337618        |           |                      |

Exact *p*-values for tests of incidence-rate difference:

```
Pr(Exposed cases <= 41) = 0.9884 (lower one-sided)
Pr(Exposed cases >= 41) = 0.0238 (upper one-sided)
Two-sided p-value = 0.0477
```

The exact *p*-values are slightly larger than those calculated using the mid-*p* adjustment. This is always the case. However, when counts of exposed and unexposed cases are both large, they will be nearly identical. See [Methods and formulas](#) below.



□ Technical note

When the IRR is less than 1, *iri* (and *ir*, *cs*, *csi*, *cc*, and *cci*) substitutes the prevented fraction for the attributable fraction. Let's reverse the roles of exposure in the above data, treating as exposed a person who did not receive the X-ray fluoroscopy. You can think of this as a new treatment for preventing breast cancer—the suggested treatment being not to use fluoroscopy.

```
. iri 15 41 19017 28010
Incidence-rate comparison
```

|                 | Exposed        | Unexposed | Total                |
|-----------------|----------------|-----------|----------------------|
| Cases           | 15             | 41        | 56                   |
| Person-time     | 19017          | 28010     | 47027                |
| Incidence rate  | .0007888       | .0014638  | .0011908             |
|                 | Point estimate |           | [95% conf. interval] |
| Inc. rate diff. | -.000675       | -.0012751 | -.0000749            |
| Inc. rate ratio | .5388632       | .277062   | .9943481 (exact)     |
| Prev. frac. ex. | .4611368       | .0056519  | .722938 (exact)      |
| Prev. frac. pop | .1864767       |           |                      |

Mid-*p*-values for tests of incidence-rate difference:

```
Adj Pr(Exposed cases <= 15) = 0.0177 (lower one-sided)
Adj Pr(Exposed cases >= 15) = 0.9823 (upper one-sided)
Two-sided p-value = 0.0355
```

The PFE is the net proportion of all potential cases in the exposed population that were prevented by exposure. We estimate that 46.1% of potential cases among the women receiving the new “treatment” were prevented by the treatment. (Previously, we estimated that the same percentage of actual cases among women receiving the X-rays was caused by the X-rays.)

The prevented fraction for the population (PFP), which is the net proportion of all potential cases in the total population that was prevented by exposure, as with the attributable fraction, depends on the proportion of cases that are exposed in the base population—here taken as 15/56—so it may not be relevant in all situations. We estimate that 18.6% of the potential cases were prevented by exposure.

See [Greenland and Robins \(1988\)](#) for a discussion of how to interpret attributable and prevented fractions.

□

### ▷ Example 2: ir

`ir` works like `iri`, except that it obtains the entries in the tables by summing data. You specify three variables: the first represents the number of cases represented by this observation, the second indicates whether the observation is for subjects exposed to the factor, and the third records the total time the subjects in this observation were observed. An observation may reflect one subject or a group of subjects.

For instance, here is a 2-observation dataset for the table in the [previous example](#):

```
. use https://www.stata-press.com/data/r19/irxmpl
. list
```

|    | cases | exposed | time  |
|----|-------|---------|-------|
| 1. | 41    | 0       | 28010 |
| 2. | 15    | 1       | 19017 |

If we typed `ir cases exposed time`, we would obtain the same output that we obtained above. Another way the data might be recorded is

```
. use https://www.stata-press.com/data/r19/irxmpl2
. list
```

|    | cases | exposed | time  |
|----|-------|---------|-------|
| 1. | 20    | 0       | 14000 |
| 2. | 21    | 0       | 14010 |
| 3. | 15    | 1       | 19017 |

Here the first 2 observations will be automatically summed by `ir` because both are exposed. Finally, the data might be individual-level data:

```
. use https://www.stata-press.com/data/r19/irxmpl3
. list in 1/5
```

|    | cases | exposed | time |
|----|-------|---------|------|
| 1. | 1     | 1       | 10   |
| 2. | 0     | 1       | 8    |
| 3. | 0     | 0       | 9    |
| 4. | 1     | 0       | 2    |
| 5. | 0     | 1       | 1    |

The first observation represents a woman who got cancer, was exposed, and was observed for 10 years. The second is a woman who did not get cancer, was exposed, and was observed for 8 years, and so on.



### □ Technical note

`ir` (and all the other commands) assumes that a subject was exposed if the exposed variable is nonzero and not missing, assumes the subject was not exposed if the variable is zero, and ignores the observation if the variable is missing. For `ir`, the case variable and the time variable are restricted to nonnegative integers and are summed within the exposed and unexposed groups to obtain the entries in the table.



## Stratified incidence-rate data

### ▷ Example 3: `ir` with stratified data

`ir` can work with stratified tables, as well as with single tables. For instance, [Rothman \(1986, 185\)](#) discusses data from [Rothman and Monson \(1973\)](#) on mortality by sex and age for patients with trigeminal neuralgia:

|              | Age through 64 |         | Age 65+ |         |
|--------------|----------------|---------|---------|---------|
|              | Males          | Females | Males   | Females |
| Deaths       | 14             | 10      | 76      | 121     |
| Person-years | 1516           | 1701    | 949     | 2245    |

Entering the data into Stata, we have the following dataset:

```
. use https://www.stata-press.com/data/r19/rm
(Rothman and Monson 1973 data)
. list
```

|    | age | male   | deaths | pyears |
|----|-----|--------|--------|--------|
| 1. | <65 | Male   | 14     | 1516   |
| 2. | <65 | Female | 10     | 1701   |
| 3. | 65+ | Male   | 76     | 949    |
| 4. | 65+ | Female | 121    | 2245   |

The stratified analysis of the IRR is

```
. ir deaths male pyears, by(age)
Stratified incidence-rate analysis
```

| Age category | IRR      | [95% conf. interval] |          | M-H weight       |
|--------------|----------|----------------------|----------|------------------|
| <65          | 1.570844 | .6489373             | 3.952809 | 4.712465 (exact) |
| 65+          | 1.485862 | 1.100305             | 1.99584  | 35.95147 (exact) |
| Crude        | 1.099794 | .831437              | 1.449306 | (exact)          |
| M-H combined | 1.49571  | 1.141183             | 1.960377 |                  |

Test of homogeneity (M-H):  $\chi^2(1) = 0.02$        $\text{Pr} > \chi^2 = 0.8992$

The row labeled MH combined reflects the combined Mantel–Haenszel estimates.

As with the [previous example](#), it is not important that each entry in the table correspond to 1 observation in the data—`ir` sums the time (`pyears`) and case (`deaths`) variables within the exposure (`male`) category.

The difference between the unadjusted crude estimate and the Mantel–Haenszel estimate suggests confounding by age: women in the study are older, and older patients are more likely to die. But we should not use the Mantel–Haenszel estimate without checking its homogeneity assumption. The  $\chi^2$  test of homogeneity gives a  $p$ -value of 0.8992, so we have no evidence that the exposure effect (the effect of being male) differs across age categories. We are justified in using the Mantel–Haenszel estimate. ◀

## □ Technical note

Stratification is one way to deal with confounding; that is, perhaps sex affects the incidence of trigeminal neuralgia and so does age, so the table was stratified by age in an attempt to uncover the sex effect. (We are concerned that age may confound the true association between sex and the incidence of trigeminal neuralgia because the age distributions are so different for males and females. If age affects incidence, the difference in the age distributions would induce different incidences for males and females and thus confound the true effect of sex.)

We do not, however, have to use tables to uncover effects; the estimation alternative when we have aggregate data is Poisson regression, and we can use the same data on which we ran `ir` with `poisson`. Poisson regression also works with individual-level data.

(Although age in the [previous example](#) appears to be a string, it is actually a numeric variable taking on values 1 and 2. We attached a value label to produce the labels <65 and 65+ to make `ir`'s output look better; see [\[U\] 12.6.3 Value labels](#). Stata's estimation commands will ignore this labeling.)

```
. poisson deaths male age, exposure(pyears) irr
Iteration 0: Log likelihood = -10.836732
Iteration 1: Log likelihood = -10.734087
Iteration 2: Log likelihood = -10.733944
Iteration 3: Log likelihood = -10.733944
Poisson regression
Log likelihood = -10.733944
Number of obs = 4
LR chi2(2) = 164.01
Prob > chi2 = 0.0000
Pseudo R2 = 0.8843
```

| deaths     | IRR      | Std. err.  | z      | P> z  | [95% conf. interval] |          |
|------------|----------|------------|--------|-------|----------------------|----------|
| male       | 1.495096 | .2060997   | 2.92   | 0.004 | 1.141118             | 1.95888  |
| age        | 8.888775 | 1.934943   | 10.04  | 0.000 | 5.801616             | 13.61867 |
| _cons      | .0006805 | .0002908   | -17.07 | 0.000 | .0002945             | .0015724 |
| ln(pyears) | 1        | (exposure) |        |       |                      |          |

Note: **\_cons** estimates baseline incidence rate.

Compare these results with the Mantel–Haenszel estimates produced by `irr`:

| Source                               | IRR  | 95% conf. interval |      |
|--------------------------------------|------|--------------------|------|
| Mantel–Haenszel ( <code>irr</code> ) | 1.50 | 1.14               | 1.96 |
| poisson                              | 1.50 | 1.14               | 1.96 |

The results from `poisson` agree with the Mantel–Haenszel estimates to two decimal places. But `poisson` also estimates an IRR for `age`. Here the estimate is not of much interest, because the outcome variable is total mortality and we already knew that older people have a higher mortality rate. In other contexts, however, the estimate might be of greater interest.

See [\[R\] poisson](#) for an explanation of the `poisson` command.



## □ Technical note

Both the model fit above and the preceding table asserted that exposure effects are the same across age categories and, if they are not, then both of the previous results are equally inappropriate. The table presented a test of homogeneity, reassuring us that the exposure effects do indeed appear to be constant. The Poisson-regression alternative can be used to reproduce that test by including interactions between the age groups and exposure:

```
. poisson deaths male age male#c.age, exposure(pyyears) irr
Iteration 0: Log likelihood = -10.898799
Iteration 1: Log likelihood = -10.726225
Iteration 2: Log likelihood = -10.725904
Iteration 3: Log likelihood = -10.725904

Poisson regression Number of obs = 4
LR chi2(3) = 164.03
Prob > chi2 = 0.0000
Pseudo R2 = 0.8843

Log likelihood = -10.725904
```

| deaths      | IRR      | Std. err.  | z      | P> z  | [95% conf. interval] |          |
|-------------|----------|------------|--------|-------|----------------------|----------|
| male        | 1.660688 | 1.396496   | 0.60   | 0.546 | .3195218             | 8.631283 |
| age         | 9.167973 | 3.01659    | 6.73   | 0.000 | 4.810583             | 17.47226 |
| male#c.age  |          |            |        |       |                      |          |
| Male        | .9459    | .41539     | -0.13  | 0.899 | .3999832             | 2.236911 |
| _cons       | .0006412 | .0004097   | -11.51 | 0.000 | .0001833             | .0022434 |
| ln(pyyears) | 1        | (exposure) |        |       |                      |          |

Note: `_cons` estimates baseline incidence rate.

The significance level of the `male#c.age` effect is 0.899, the same as previously reported by `irr`.

Here forming the male-times-age interaction was easy because there were only two age groups. Had there been more groups, the test would have been slightly more difficult—see the following technical note.

□

## □ Technical note

A word of caution is in order when applying `poisson` (or any estimation technique) to more than two age categories. Say that in our data, we had three age categories, which we will call categories 0, 1, and 2, and that they are stored in the variable `agecat`. We might think of the categories as corresponding to age less than 35, 35–64, and 65 and above.

With such data, we might type `irr deaths male pyyears, by(agecat)`, but we would *not* type `poisson deaths male agecat, exposure(pyyears)` to obtain the equivalent Poisson-regression estimated results. Such a model might be reasonable, but it is not equivalent because we would be constraining the age effect in category 2 to be (multiplicatively) twice the effect in category 1.

To `poisson` (and all of Stata's estimation commands other than `anova`), `agecat` is simply one variable, and only one estimated coefficient is associated with it. Thus, the model is

$$\text{Poisson index} = P = \beta_0 + \beta_1 \text{male} + \beta_2 \text{agecat}$$

The expected number of deaths is then  $e^P$ , and the IRR associated with a variable is  $e^\beta$ ; see [R] poisson. Thus, the value of the Poisson index when male==0 and agecat==1 is  $\beta_0 + \beta_2$ , and the possibilities are

|           | male==0              | male==1                        |
|-----------|----------------------|--------------------------------|
| agecat==0 | $\beta_0$            | $\beta_0 + \beta_1$            |
| agecat==1 | $\beta_0 + \beta_2$  | $\beta_0 + \beta_2 + \beta_1$  |
| agecat==2 | $\beta_0 + 2\beta_2$ | $\beta_0 + 2\beta_2 + \beta_1$ |

The age effect for agecat==2 is constrained to be twice the age effect for agecat==1—the only difference between lines 3 and 2 of the table is that  $\beta_2$  is replaced with  $2\beta_2$ . Under certain circumstances, such a constraint might be reasonable, but it does not correspond to the assumptions made in generating the Mantel–Haenszel combined results.

To obtain results equivalent to the Mantel–Haenszel result, we must estimate a separate effect for each age group, meaning that we must replace  $2\beta_2$ , the constrained effect, with  $\beta_3$ , a new coefficient that is free to take on any value. We can achieve this by creating two new variables and using them in place of agecat. agecat1 will take on the value 1 when agecat is 1 and 0 otherwise; agecat2 will take on the value 1 when agecat is 2 and 0 otherwise:

```
. generate agecat1 = (agecat==1)
. generate agecat2 = (agecat==2)
. poisson deaths male agecat1 agecat2 [fweight=pop], exposure(pyyears) irr
```

In Stata, we do not have to generate these variables for ourselves. We could use factor variables:

```
. poisson deaths male i.agecat [fweight=pop], exposure(pyyears) irr
```

See [U] 11.4.3 Factor variables.

To reproduce the homogeneity test with multiple age categories, we could type

```
. poisson deaths agecat##male [fweight=pop], exp(pyyears) irr
. testparm agecat#male
```

Poisson regression combined with factor variables generalizes to multiway tables. Suppose that there are three exposure categories. Assume exposure variable burn takes on the values 1, 2, and 3 for first-, second-, and third-degree burns. The table itself is estimated by typing

```
. poisson deaths i.burn i.agecat [fweight=pop], exp(pyyears) irr
```

and the test of homogeneity is estimated by typing

```
. poisson deaths burn##agecat [fweight=pop], exp(pyyears) irr
. testparm burn#agecat
```



## Standardized estimates with stratified incidence-rate data

The `by()` option specifies that the data are stratified and, by default, will produce a Mantel–Haenszel combined estimate of the IRR. With the `estandard`, `istandard`, or `standard` (*varname*) options, you can specify your own weights and obtain standardized estimates of the IRR or IRD.

### ▷ Example 4: `ir` with stratified data, using standardized estimates

Lash et al. (2021, 417) report results from Doll and Hill (1966) on age-specific coronary disease deaths among British male doctors from cigarette smoking:

| Age   | Smokers |              | Nonsmokers |              |
|-------|---------|--------------|------------|--------------|
|       | Deaths  | Person-years | Deaths     | Person-years |
| 35–44 | 32      | 52,407       | 2          | 18,790       |
| 45–54 | 104     | 43,248       | 12         | 10,673       |
| 55–64 | 206     | 28,612       | 28         | 5,710        |
| 65–74 | 186     | 12,663       | 28         | 2,585        |
| 75–84 | 102     | 5,317        | 31         | 1,462        |

We have entered these data into Stata:

```
. use https://www.stata-press.com/data/r19/dollhill3
(Doll and Hill (1966))
. list
```

|     | agecat | smokes | deaths | pyears |
|-----|--------|--------|--------|--------|
| 1.  | 35-44  | 1      | 32     | 52,407 |
| 2.  | 45-54  | 1      | 104    | 43,248 |
| 3.  | 55-64  | 1      | 206    | 28,612 |
| 4.  | 65-74  | 1      | 186    | 12,663 |
| 5.  | 75-84  | 1      | 102    | 5,317  |
| 6.  | 35-44  | 0      | 2      | 18,790 |
| 7.  | 45-54  | 0      | 12     | 10,673 |
| 8.  | 55-64  | 0      | 28     | 5,710  |
| 9.  | 65-74  | 0      | 28     | 2,585  |
| 10. | 75-84  | 0      | 31     | 1,462  |

We can obtain the Mantel–Haenszel combined estimate along with the crude estimate for ignoring stratification of the IRR and 90% confidence intervals by typing

```
. ir deaths smokes pyears, by(age) level(90)
Stratified incidence-rate analysis
```

| Age category | IRR      | [90% conf. interval] |          | M-H weight       |
|--------------|----------|----------------------|----------|------------------|
| 35-44        | 5.736638 | 1.704271             | 33.61646 | 1.472169 (exact) |
| 45-54        | 2.138812 | 1.274552             | 3.813282 | 9.624747 (exact) |
| 55-64        | 1.46824  | 1.044915             | 2.110422 | 23.34176 (exact) |
| 65-74        | 1.35606  | .9626026             | 1.953505 | 23.25315 (exact) |
| 75-84        | .9047304 | .6375194             | 1.305412 | 24.31435 (exact) |
| Crude        | 1.719823 | 1.437544             | 2.0688   | (exact)          |
| M-H combined | 1.424682 | 1.194375             | 1.699399 |                  |

Test of homogeneity (M-H):  $\chi^2(4) = 10.41$

$\text{Pr} > \chi^2 = 0.0340$

Note the presence of heterogeneity revealed by the test; the effect of smoking is not the same across age categories. Moreover, the listed stratum-specific estimates show an effect that appears to be declining with age. (Even if the test of homogeneity is not significant, you should always examine estimates carefully when stratum-specific effects occur on both sides of 1 for ratios and 0 for differences.)

Lash et al. (2021, 422) obtain the standardized IRR and 90% confidence intervals, weighting each age category by the population of the exposed group, thus producing the standardized mortality ratio (SMR). This calculation can be reproduced by specifying by(age) to indicate that the table is stratified and istandard to specify that we want the internally standardized rate. We may also specify that we would like to see the pooled estimate (weighted average where the weights are based on the variance of the strata calculations):

```
. ir deaths smokes pyears, by(age) level(90) istandard pool
Stratified incidence-rate analysis
```

| Age category    | IRR      | [90% conf. interval] |          | Weight        |
|-----------------|----------|----------------------|----------|---------------|
| 35-44           | 5.736638 | 1.704271             | 33.61646 | 52407 (exact) |
| 45-54           | 2.138812 | 1.274552             | 3.813282 | 43248 (exact) |
| 55-64           | 1.46824  | 1.044915             | 2.110422 | 28612 (exact) |
| 65-74           | 1.35606  | .9626026             | 1.953505 | 12663 (exact) |
| 75-84           | .9047304 | .6375194             | 1.305412 | 5317 (exact)  |
| Crude           | 1.719823 | 1.437544             | 2.0688   | (exact)       |
| Pooled (direct) | 1.355343 | 1.134356             | 1.619382 |               |
| I. standardized | 1.417609 | 1.186541             | 1.693676 |               |

```
Test of homogeneity (direct): chi2(4) = 10.20 Pr>chi2 = 0.0372
```

We obtained the simple pooled results because we specified the pool option. Note the significance of the homogeneity test; it provides the motivation for standardizing the rate ratios.

If we wanted the externally standardized ratio (weights proportional to the population of the unexposed group), we would substitute estandard for istandard in the above command.

We are not limited to IRRs; ir can also estimate IRDs. Differences may be standardized internally or externally. We will obtain the internally weighted difference (Lash et al. 2021 , 418-419):

```
. ir deaths smokes pyears, by(age) level(90) istandard ird
Stratified incidence-rate analysis
```

| Age category    | IRD       | [90% conf. interval] |          | Weight |
|-----------------|-----------|----------------------|----------|--------|
| 35-44           | .0005042  | .0002877             | .0007206 | 52407  |
| 45-54           | .0012804  | .0006205             | .0019403 | 43248  |
| 55-64           | .0022961  | .0005628             | .0040294 | 28612  |
| 65-74           | .0038567  | .0000521             | .0076614 | 12663  |
| 75-84           | -.0020201 | -.0090201            | .00498   | 5317   |
| Crude           | .0018537  | .001342              | .0023654 |        |
| I. standardized | .0013047  | .000712              | .0018974 |        |

► Example 5: ir with user-specified weights

In addition to calculating results by using internal or external weights, `ir` (and `cs` and `cc`) can calculate results for arbitrary weights. If we wanted to obtain the IRR weighting each age category equally, we would type

```
. generate conswgt=1
. ir deaths smokes pyears, by(age) standard(conswgt)
Stratified incidence-rate analysis
```

| Age category | IRR      | [95% conf. interval] |          | Weight    |
|--------------|----------|----------------------|----------|-----------|
| 35-44        | 5.736638 | 1.463557             | 49.40468 | 1 (exact) |
| 45-54        | 2.138812 | 1.173714             | 4.272545 | 1 (exact) |
| 55-64        | 1.46824  | .9863624             | 2.264107 | 1 (exact) |
| 65-74        | 1.35606  | .9081925             | 2.096412 | 1 (exact) |
| 75-84        | .9047304 | .6000757             | 1.399687 | 1 (exact) |
| Crude        | 1.719823 | 1.391992             | 2.14353  | (exact)   |
| Standardized | 1.155026 | .9006199             | 1.481295 |           |



□ Technical note

`estandard` and `istandard` are convenience features; they do nothing different from what you could accomplish by creating the appropriate weights and using the `standard()` option. For instance, we could duplicate the previously shown results of `istandard` (example before last) by typing

```
. sort age smokes
. by age: generate wgt=pyears[_N]
. list in 1/4
```

|    | agecat | smokes | deaths | pyears | conswgt | wgt   |
|----|--------|--------|--------|--------|---------|-------|
| 1. | 35-44  | 0      | 2      | 18,790 | 1       | 52407 |
| 2. | 35-44  | 1      | 32     | 52,407 | 1       | 52407 |
| 3. | 45-54  | 0      | 12     | 10,673 | 1       | 43248 |
| 4. | 45-54  | 1      | 104    | 43,248 | 1       | 43248 |

```
. ir deaths smokes pyears, by(age) level(90) standard(wgt) ird
(output omitted)
```

`sort age smokes` made the exposed group (`smokes = 1`) the last observation within each age category. `by age: gen wgt=pyears[_N]` created `wgt` equal to the last observation in each age category.



## Cumulative incidence data

Cumulative incidence data are “follow-up data with denominators consisting of persons rather than person-time” (Rothman 1986, 172). A group of noncases is monitored for some time, during which some become cases. Each subject is also known to be exposed or unexposed. A summary of the data is

|          | Exposed | Unexposed | Total           |
|----------|---------|-----------|-----------------|
| Cases    | $a$     | $b$       | $a + b$         |
| Noncases | $c$     | $d$       | $c + d$         |
| Total    | $a + c$ | $b + d$   | $a + b + c + d$ |

Data of this type are generally summarized using the risk ratio,  $\{a/(a+c)\}/\{b/(b+d)\}$ . A ratio of 2 means that an exposed subject is twice as likely to become a case than is an unexposed subject, a ratio of one-half means half as likely, and so on. The “null” value—the number corresponding to no effect—is a ratio of 1. If cross-sectional data are analyzed in this format, the risk ratio becomes a prevalence ratio.

### ▷ Example 6: csi

We have data on diarrhea during a 10-day follow-up period among 30 breast-fed infants colonized with *Vibrio cholerae* 01 according to antilipoplysaccharide antibody titers in the mother’s breast milk (Glass et al. [1983]; reported in Lash et al. [2021, 403]):

|             | Antibody level |     |
|-------------|----------------|-----|
|             | High           | Low |
| Diarrhea    | 7              | 12  |
| No diarrhea | 9              | 2   |

The `csi` command works much like the `iri` command. Our sample is small, so we will specify the `exact` option.

```
. csi 7 12 9 2, exact
```

|                 | Exposed        | Unexposed | Total                |           |
|-----------------|----------------|-----------|----------------------|-----------|
| Cases           | 7              | 12        | 19                   |           |
| Noncases        | 9              | 2         | 11                   |           |
| Total           | 16             | 14        | 30                   |           |
| Risk            | .4375          | .8571429  | .6333333             |           |
|                 | Point estimate |           | [95% conf. interval] |           |
| Risk difference | -.4196429      |           | -.7240828            | -.1152029 |
| Risk ratio      | .5104167       |           | .2814332             | .9257086  |
| Prev. frac. ex. | .4895833       |           | .0742914             | .7185668  |
| Prev. frac. pop | .2611111       |           |                      |           |

1-sided Fisher’s exact P = 0.0212

2-sided Fisher’s exact P = 0.0259

We find that high antibody levels reduce the risk of diarrhea (the risk falls from 0.86 to 0.44). The difference is just significant at the 2.59% two-sided level. (Had we not specified the `exact` option, a  $\chi^2$  value and its significance level would have been reported in place of Fisher’s exact  $p$ . The calculated  $\chi^2$  two-sided significance level would have been 0.0173, but this calculation is inferior for small samples.)

□ Technical note

By default, `cs` and `csi` do not report the odds ratio, but they will if you specify the `or` option. If you want odds ratios, however, use the `cc` or `cci` commands—the commands appropriate for case–control data—because `cs` and `csi` calculate the attributable (prevented) fraction with the risk ratio, even if you specify `or`:

```
. csi 7 12 9 2, or exact
```

|                 | Exposed        | Unexposed | Total                |                      |
|-----------------|----------------|-----------|----------------------|----------------------|
| Cases           | 7              | 12        | 19                   |                      |
| Noncases        | 9              | 2         | 11                   |                      |
| Total           | 16             | 14        | 30                   |                      |
| Risk            | .4375          | .8571429  | .6333333             |                      |
|                 | Point estimate |           | [95% conf. interval] |                      |
| Risk difference | -.4196429      |           | -.7240828            | -.1152029            |
| Risk ratio      | .5104167       |           | .2814332             | .9257086             |
| Prev. frac. ex. | .4895833       |           | .0742914             | .7185668             |
| Prev. frac. pop | .2611111       |           |                      |                      |
| Odds ratio      | .1296296       |           | .0246233             | .7180882 (Cornfield) |

1-sided Fisher's exact P = 0.0212  
 2-sided Fisher's exact P = 0.0259



□ Technical note

As with `iri` and `ir`, `csi` and `cs` report the AFE, AFP, PFE, or PFP; see the discussion under *Incidence-rate data* above. In [example 6](#), we estimated that 49% of potential cases in the exposed population were prevented by exposure. We also estimated that exposure accounted for a 26% reduction in cases over the entire population, but that is based on the exposure distribution of the (small) population (16/30) and probably is of little interest.

[Fleiss, Levin, and Paik \(2003, 128\)](#) report infant mortality by birthweight for 72,730 live white births in 1974 in New York City:

```
. csi 618 422 4597 67093
```

|                 | Exposed        | Unexposed | Total                |          |
|-----------------|----------------|-----------|----------------------|----------|
| Cases           | 618            | 422       | 1040                 |          |
| Noncases        | 4597           | 67093     | 71690                |          |
| Total           | 5215           | 67515     | 72730                |          |
| Risk            | .1185043       | .0062505  | .0142995             |          |
|                 | Point estimate |           | [95% conf. interval] |          |
| Risk difference | .1122539       |           | .1034617             | .121046  |
| Risk ratio      | 18.95929       |           | 16.80661             | 21.38769 |
| Attr. frac. ex. | .9472554       |           | .9404996             | .9532441 |
| Attr. frac. pop | .5628883       |           |                      |          |

chi2(1) = 4327.92 Pr>chi2 = 0.0000

In these data, exposed means a premature baby (birthweight  $\leq 2,500$  g), and a case is a baby who is dead at the end of one year. We find that being premature accounts for 94.7% of deaths among the premature population. We also estimate, paraphrasing from [Fleiss, Levin, and Paik \(2003, 128\)](#), that 56.3% of all white infant deaths in New York City in 1974 could have been prevented if prematurity had been eliminated. (Moreover, Fleiss, Levin, and Paik put a standard error on the AFP. The formula is given in [Methods and formulas](#) but is appropriate only for the population on which the estimates are based because other populations may have different probabilities of exposure.)

□

### ▷ Example 7: cs

cs works like csi, except that it obtains its information from the data. The data equivalent to typing csi 7 12 9 2 are

```
. use https://www.stata-press.com/data/r19/csxmpl, clear
. list
```

|    | case | exp | pop |
|----|------|-----|-----|
| 1. | 1    | 1   | 7   |
| 2. | 1    | 0   | 12  |
| 3. | 0    | 1   | 9   |
| 4. | 0    | 0   | 2   |

We could then type `cs case exp [fweight=pop]`. If we had individual-level data, so that each observation reflected a patient and we had 30 observations, we would type `cs case exp`.

◀

## Stratified cumulative incidence data

### ▷ Example 8: cs with stratified data

[Lash et al. \(2021, 419\)](#) reprint the following age-specific information for deaths from all causes for tolbutamide and placebo treatment groups ([University Group Diabetes Program 1970](#)):

|           | Age through 54 |         | Age 55 and above |         |
|-----------|----------------|---------|------------------|---------|
|           | Tolbutamide    | Placebo | Tolbutamide      | Placebo |
| Dead      | 8              | 5       | 22               | 16      |
| Surviving | 98             | 115     | 76               | 69      |

The data corresponding to these results are

```
. use https://www.stata-press.com/data/r19/ugdp
(University Group Diabetes Program 1970)
. list
```

|    | age | case      | exposed     | pop |
|----|-----|-----------|-------------|-----|
| 1. | <55 | Surviving | Placebo     | 115 |
| 2. | <55 | Surviving | Tolbutamide | 98  |
| 3. | <55 | Dead      | Placebo     | 5   |
| 4. | <55 | Dead      | Tolbutamide | 8   |
| 5. | 55+ | Surviving | Placebo     | 69  |
| 6. | 55+ | Surviving | Tolbutamide | 76  |
| 7. | 55+ | Dead      | Placebo     | 16  |
| 8. | 55+ | Dead      | Tolbutamide | 22  |

The order of the observations is unimportant. If we were now to type `cs case exposed [fweight=pop]`, we would obtain a summary for all the data, ignoring the stratification by age. To incorporate the stratification, we type

```
. cs case exposed [fweight=pop], by(age)
```

| Age category | Risk ratio | [95% conf. interval] | M-H weight |
|--------------|------------|----------------------|------------|
| <55          | 1.811321   | .6112044 5.367898    | 2.345133   |
| 55+          | 1.192602   | .6712664 2.11883     | 8.568306   |
| Crude        | 1.435574   | .8510221 2.421645    |            |
| M-H combined | 1.325555   | .797907 2.202132     |            |

```
Test of homogeneity (M-H) chi2(1) = 0.447 Pr>chi2 = 0.5037
```

Mantel–Haenszel weights are appropriate when the risks may differ according to the strata but the risk ratio is believed to be the same (homogeneous across strata). Under these assumptions, Mantel–Haenszel weights are designed to use the information efficiently. They are not intended to measure a composite risk ratio when the within-stratum risk ratios differ. Then, we want a standardized ratio (see below).

The risk ratios above appear to differ markedly, but the confidence intervals are also broad because of the small sample sizes. The test of homogeneity shows that the differences can be attributed to chance; the use of the Mantel–Haenszel combined test is sensible.

□ Technical note

Stratified cumulative incidence tables are not the only way to control for confounding. Another way is logistic regression. However, logistic regression measures effects with odds ratios, not with risk ratios. So before we fit a logistic model, let's use `cs` to estimate the Mantel–Haenszel odds ratio:

```
. cs case exposed [fweight=pop], by(age) or
```

| Age category | Odds ratio | [95% conf. interval] |          | M-H weight           |
|--------------|------------|----------------------|----------|----------------------|
| <55          | 1.877551   | .6238165             | 5.637046 | 2.168142 (Cornfield) |
| 55+          | 1.248355   | .6112772             | 2.547411 | 6.644809 (Cornfield) |
| Crude        | 1.510673   | .8381198             | 2.722012 |                      |
| M-H combined | 1.403149   | .7625152             | 2.582015 |                      |

```
Test of homogeneity (M-H) chi2(1) = 0.347 Pr>chi2 = 0.5556
Test that combined odds ratio = 1:
Mantel-Haenszel chi2(1) = 1.19
Pr>chi2 = 0.2750
```

The Mantel–Haenszel odds ratio is 1.40. It measures the association between death and treatment while adjusting for age. A more general way to adjust for age is logistic regression; the outcome variable is case, and it is explained by age and exposed. (As in the incidence-rate example, age may appear to be a string variable in our data—we listed the data in the [previous example](#)—but it is actually a numeric variable taking on values 0 and 1 with value labels disguising that fact; see [\[U\] 12.6.3 Value labels.](#))

```
. logistic case exposed age [fweight=pop]
```

Logistic regression

Log likelihood = -142.6212

| case    | Odds ratio | Std. err. | z     | P> z  | [95% conf. interval] |          |
|---------|------------|-----------|-------|-------|----------------------|----------|
| exposed | 1.404674   | .4374454  | 1.09  | 0.275 | .7629451             | 2.586175 |
| age     | 4.216299   | 1.431519  | 4.24  | 0.000 | 2.167361             | 8.202223 |
| _cons   | .0513818   | .0170762  | -8.93 | 0.000 | .0267868             | .0985593 |

Note: `_cons` estimates baseline odds.

Compare these results with the Mantel–Haenszel estimates obtained with `cs`:

| Source               | Odds ratio | 95% conf. interval |      |
|----------------------|------------|--------------------|------|
| Mantel–Haenszel (cs) | 1.40       | 0.76               | 2.58 |
| logistic             | 1.40       | 0.76               | 2.59 |

They are virtually identical.

Logistic regression has advantages over the stratified-table approach. First, we obtained an estimate of the age effect: being 55 years or over significantly increases the odds of death. In addition to the point estimate, 4.22, we have a confidence interval for the effect: 2.17 to 8.20.

A discrete effect at age 55 is not a plausible model of aging. It would be more reasonable to assume that a 54-year-old patient has a higher probability of death, due merely to age, than does a 53-year-old patient; a 53-year-old, a higher probability than a 52-year-old patient; and so on. If we had the underlying

data, where each patient's age is presumably known, we could include the actual age in the model and so better control for the age effect. This would improve our estimate of the effect of being exposed to tolbutamide.

See [R] [logistic](#) for an explanation of the `logistic` command. Also see the [technical note](#) in *Stratified incidence-rate data* concerning categorical variables, which applies to logistic regression as well as Poisson regression.

□

## Standardized estimates with stratified cumulative incidence data

As with `ir`, `cs` can produce standardized estimates, and the method is basically the same, although the options for which estimates are to be combined or standardized make it confusing. We showed above that `cs` can produce Mantel–Haenszel weighted estimates of the risk ratio (the default) or the odds ratio (obtained by specifying `or`). `cs` can also produce standardized estimates of the risk ratio (the default) or the risk difference (obtained by specifying `rd`).

### ▷ Example 9: `cs` with stratified data, using standardized estimates

To produce an estimate of the internally standardized risk ratio by using our age-specific data on deaths from all causes for tolbutamide and placebo treatment groups ([example above](#)), we type

```
. cs case exposed [fweight=pop], by(age) istandard
```

| Age category    | Risk ratio | [95% conf. interval] |          | Weight |
|-----------------|------------|----------------------|----------|--------|
| <55             | 1.811321   | .6112044             | 5.367898 | 106    |
| 55+             | 1.192602   | .6712664             | 2.11883  | 98     |
| Crude           | 1.435574   | .8510221             | 2.421645 |        |
| I. Standardized | 1.312122   | .7889772             | 2.182147 |        |

We could obtain externally standardized estimates by substituting `estandard` for `istandard`.

To produce an estimate of the risk ratio weighting each age category equally, we could type

```
. generate wgt=1
. cs case exposed [fweight=pop], by(age) standard(wgt)
```

| Age category | Risk ratio | [95% conf. interval] |          | Weight |
|--------------|------------|----------------------|----------|--------|
| <55          | 1.811321   | .6112044             | 5.367898 | 1      |
| 55+          | 1.192602   | .6712664             | 2.11883  | 1      |
| Crude        | 1.435574   | .8510221             | 2.421645 |        |
| Standardized | 1.304737   | .7844994             | 2.169967 |        |

If we instead wanted the risk difference, we would type

```
. cs case exposed [fweight=pop], by(age) standard(wgt) rd
```

| Age category | Risk diff. | [95% conf. interval] |          | Weight |
|--------------|------------|----------------------|----------|--------|
| <55          | .033805    | -.0278954            | .0955055 | 1      |
| 55+          | .0362545   | -.0809204            | .1534294 | 1      |
| Crude        | .0446198   | -.0192936            | .1085332 |        |
| Standardized | .0350298   | -.0311837            | .1012432 |        |

If we wanted to weight the less-than-55 age group five times as heavily as the 55-and-over group, we would create `wgt` to contain 5 for the first age group and 1 for the second (or 10 for the first group and 2 for the second—the scale of the weights does not matter).

◀

### Case-control data

In case-control data, you select a sample on the basis of the outcome under study; that is, cases and noncases are sampled at different rates. If you were examining the link between coffee consumption and heart attacks, for instance, you could select a sample of subjects with and without the heart problem and then examine their coffee-drinking behavior. A subject who has suffered a heart attack is called a *case* just as with cohort study data. A subject who has never suffered a heart attack, however, is called a *control* rather than merely a noncase, emphasizing that the sampling was performed with respect to the outcome.

In case-control data, all hope of identifying the risk (that is, incidence) of the outcome (heart attacks) associated with the factor (coffee drinking) vanishes, at least without information on the underlying sampling fractions, but you can examine the proportion of coffee drinkers among the two populations and reason that, if there is a difference, coffee drinking may be associated with the risk of heart attacks. Remarkably, even without the underlying sampling fractions, you can also measure the ratio of the odds of heart attacks if a subject drinks coffee to the odds if a subject does not—the so-called odds ratio.

What is lost is the ability to compare absolute rates, which is not always the same as comparing relative rates; see [Fleiss, Levin, and Paik \(2003, 123\)](#).

#### ▷ Example 10: `cci`

`cci` calculates the odds ratio and the attributable risk associated with a  $2 \times 2$  table. [Rothman et al. \(1979; reprinted in Rothman \[1986, 161\], and Lash et al. \[2021, 411\]\)](#) present case-control data on the history of chlordiazopoxide use in early pregnancy for mothers of children born with and without congenital heart defects:

|                 | Chlordiazopoxide use |      |
|-----------------|----------------------|------|
|                 | Yes                  | No   |
| Case mothers    | 4                    | 386  |
| Control mothers | 4                    | 1250 |

```
. cci 4 386 4 1250, level(90)
```

|                 | Exposed        | Unexposed | Total                | Proportion exposed |
|-----------------|----------------|-----------|----------------------|--------------------|
| Cases           | 4              | 386       | 390                  | 0.0103             |
| Controls        | 4              | 1250      | 1254                 | 0.0032             |
| Total           | 8              | 1636      | 1644                 | 0.0049             |
|                 | Point estimate |           | [90% conf. interval] |                    |
| Odds ratio      | 3.238342       |           | .7698467             | 13.59664 (exact)   |
| Attr. frac. ex. | .6912          |           | -.2989599            | .9264524 (exact)   |
| Attr. frac. pop | .0070892       |           |                      |                    |

$\chi^2(1) = 3.07$   $\text{Pr} > \chi^2 = 0.0799$

We obtain a point estimate of the odds ratio as 3.24 and a  $\chi^2$  value, which is a test that the odds ratio is 1, significant at the 10% level.

◀

## □ Technical note

The epitab commands can calculate three different confidence intervals for the odds ratio: the exact, Woolf, and Cornfield intervals. The exact interval, illustrated in [example 10](#), is the default. The interval is “exact” because it uses an exact sampling distribution—a distribution with no unknown parameters under the null hypothesis. An exact interval does not use a normal or  $\chi^2$  approximation. “Exact” does not describe the coverage probability; the coverage probability of a 90% exact interval is not exactly 90%. The coverage probability is actually bounded below by 90% ([Agresti 2013](#), 606), so a 90% exact interval will always cover the odds ratio with probability at least 90% (if the model is correct).

The Woolf and Cornfield intervals, on the other hand, are approximate. They approximate the exact sampling distribution with a normal model and are not guaranteed to maintain their nominal coverage: the coverage probability of a 90% approximate interval fluctuates above and below 90%. The coverage approaches 90% only in the limit as the sample size increases. Exact intervals are conservative; approximate intervals can be conservative or anticonservative ([Agresti 2013](#), 607).

If you wish to maintain nominal coverage, then you should use the exact interval. But you will pay a price for the coverage: the exact interval will usually be wider than the approximate intervals. [Example 10](#) is no exception:

| Method    | 90% conf. interval |       | Command        |
|-----------|--------------------|-------|----------------|
| exact     | 0.77               | 13.60 | cci            |
| Woolf     | 1.01               | 10.40 | cci, woolf     |
| Cornfield | 1.07               | 9.83  | cci, cornfield |

The exact interval is the widest of the three—so wide that it includes the null value of one—even though the  $\chi^2$  test  $p$ -value of 0.0799 was significant at the 10% level. The exact interval and  $\chi^2$  test come from different models, so we should not expect them to always agree on sharp conclusions such as statistical significance.

The odds-ratio intervals are all frequentist methods, so we cannot compare them rigorously with one example. See [Brown \(1981\)](#), [Gart and Thomas \(1982\)](#), and [Agresti \(1999\)](#) for more rigorous comparisons. [Agresti \(1999\)](#) found that the Woolf interval performed well, even for small samples. □

Jerome Cornfield (1912–1979) was born in New York City. He majored in history at New York University and took courses in statistics at the US Department of Agriculture Graduate School but otherwise had little formal training. Cornfield held positions at the Bureau of Labor Statistics, the National Cancer Institute, the National Institutes of Health, Johns Hopkins University, the University of Pittsburgh, and George Washington University. He worked on many problems in biomedical statistics, including the analysis of clinical trials, epidemiology (especially case–control studies), and Bayesian approaches.

Barnet Woolf (1902–1983) was born in London. His parents were immigrants from Lithuania. Woolf was educated at Cambridge, where he studied physiology and biochemistry, and proposed methods for linearizing plots in enzyme chemistry that were later rediscovered by others (see [Haldane \[1957\]](#)). His later career in London, Birmingham, Rothamsted, and Edinburgh included lasting contributions to nutrition, epidemiology, public health, genetics, and statistics. He was also active in left-wing causes and penned humorous poems, songs, and revues.

□ Technical note

By default, `cc` and `cci` report exact confidence intervals but an approximate significance test. You can replace the approximate test with Fisher’s exact test by specifying the `exact` option. We recommend specifying `exact` whenever any cell count is less than 1,000.

```
. cci 4 386 4 1250, exact level(90)
```

|                 | Exposed        | Unexposed | Total                | Proportion exposed |
|-----------------|----------------|-----------|----------------------|--------------------|
| Cases           | 4              | 386       | 390                  | 0.0103             |
| Controls        | 4              | 1250      | 1254                 | 0.0032             |
| Total           | 8              | 1636      | 1644                 | 0.0049             |
|                 | Point estimate |           | [90% conf. interval] |                    |
| Odds ratio      | 3.238342       |           | .7698467             | 13.59664 (exact)   |
| Attr. frac. ex. | .6912          |           | -.2989599            | .9264524 (exact)   |
| Attr. frac. pop | .0070892       |           |                      |                    |

1-sided Fisher’s exact P = 0.0964

2-sided Fisher’s exact P = 0.0964

In this table, the one- and two-sided significance values are equal. This is not a mistake, but it does not happen often. Exact significance values are calculated by summing the probabilities for tables that have the same marginals (row and column sums) but that are less likely (given an odds ratio of 1) than the observed table. When considering each possible table, we might ask if the table is in the same or opposite tail as the observed table. If it is in the same tail, we would count the table under consideration in the one-sided test and, either way, we would count it in the two-sided test. Here all the tables more extreme than this table are in the same tail, so the one- and two-sided tests are the same.

The *p*-value of 0.0964 is significant at the 10% level, but the exact confidence interval is not (it includes the null odds ratio of one). It was not surprising that the exact interval disagreed with the  $\chi^2$  test; after all, they come from different models. Now, the exact interval and Fisher’s exact test also disagree, even though they come from the same model!

The test and interval disagree because the exact sampling distribution is asymmetric, and the test and interval handle the asymmetry differently. The two-sided test, as we have seen, sums the probabilities of all tables at least as unlikely as the observed table, and in example 10, all the unlikely tables fall in the same tail of the distribution. The other tail does not contribute to the *p*-value. The exact interval, on the other hand, must always use both tails of the distribution, because the interval inverts two one-sided tests, not one two-sided test (Breslow and Day 1980, 128–129).

□

□ Technical note

The reported value of the AFE or PFE is calculated using the odds ratio as a proxy for the risk ratio. This can be justified only if the outcome is rare in the population. The extrapolation to the AFP or PFP assumes that the control group is a random sample of the corresponding group in the underlying population.

□

### ▷ Example 11: cc equivalent to cci

Equivalent to typing `cci 4 386 4 1250` would be typing `cc case exposed [fweight=pop]` with the following data:

```
. use https://www.stata-press.com/data/r19/ccxmpl, clear
. list
```

|    | case | exposed | pop  |
|----|------|---------|------|
| 1. | 1    | 1       | 4    |
| 2. | 1    | 0       | 386  |
| 3. | 0    | 1       | 4    |
| 4. | 0    | 0       | 1250 |

◀

## Stratified case–control data

### ▷ Example 12: cc with stratified data

`cc` can work with stratified tables. [Lash et al. \(2021, 429\)](#) reprint and discuss data from a case–control study on infants with congenital heart disease and Down syndrome and healthy controls, according to maternal spermicide use before conception and maternal age at delivery ([Rothman 1982](#)):

|               | Maternal age to 34 |          | Maternal age 35+ |          |
|---------------|--------------------|----------|------------------|----------|
|               | Spermicide used    | not used | Spermicide used  | not used |
| Down syndrome | 3                  | 9        | 1                | 3        |
| Controls      | 104                | 1059     | 5                | 86       |

The data corresponding to these tables are

```
. use https://www.stata-press.com/data/r19/downs
(Congenital heart disease and Down syndrome)
. list
```

|    | case | exposed | pop  | age |
|----|------|---------|------|-----|
| 1. | 1    | 1       | 3    | <35 |
| 2. | 1    | 0       | 9    | <35 |
| 3. | 0    | 1       | 104  | <35 |
| 4. | 0    | 0       | 1059 | <35 |
| 5. | 1    | 1       | 1    | 35+ |
| 6. | 1    | 0       | 3    | 35+ |
| 7. | 0    | 1       | 5    | 35+ |
| 8. | 0    | 0       | 86   | 35+ |

The stratified results for the odds ratio are

```
. cc case exposed [fweight=pop], by(age) woolf
```

| Maternal age | Odds ratio | [95% conf. interval] |          | M-H weight       |
|--------------|------------|----------------------|----------|------------------|
| <35          | 3.394231   | .9048403             | 12.73242 | .7965957 (Woolf) |
| 35+          | 5.733333   | .5016418             | 65.52706 | .1578947 (Woolf) |
| Crude        | 3.501529   | 1.110362             | 11.04208 | (Woolf)          |
| M-H combined | 3.781172   | 1.18734              | 12.04142 |                  |

```
Test of homogeneity (M-H) chi2(1) = 0.14 Pr>chi2 = 0.7105
Test that combined odds ratio = 1:
Mantel-Haenszel chi2(1) = 5.81
Pr>chi2 = 0.0159
```

For no particular reason, we also specified the `woolf` option to obtain Woolf approximations to the within-stratum confidence intervals rather than the default. Had we wanted Tarone's test of homogeneity, we would have used

```
. cc case exposed [fweight=pop], by(age) tarone
```

| Maternal age | Odds ratio | [95% conf. interval] |          | M-H weight       |
|--------------|------------|----------------------|----------|------------------|
| <35          | 3.394231   | .5812415             | 13.87412 | .7965957 (exact) |
| 35+          | 5.733333   | .0911619             | 85.89602 | .1578947 (exact) |
| Crude        | 3.501529   | .8080857             | 11.78958 | (exact)          |
| M-H combined | 3.781172   | 1.18734              | 12.04142 |                  |

```
Test of homogeneity (M-H) chi2(1) = 0.14 Pr>chi2 = 0.7105
Test of homogeneity (Tarone) chi2(1) = 0.14 Pr>chi2 = 0.7092
Test that combined odds ratio = 1:
Mantel-Haenszel chi2(1) = 5.81
Pr>chi2 = 0.0159
```

Whatever method you choose for calculating confidence intervals, Stata will report a test of homogeneity, which here is  $\chi^2(1) = 0.14$  and not significant. That is, the odds of Down syndrome might vary with maternal age, but we cannot reject the hypothesis that the association between Down syndrome and spermicide is the same in the two maternal age strata. This is thus a test to reject the appropriateness of the single, Mantel–Haenszel combined odds ratio—a rejection not justified by these data.

◀

## □ Technical note

The `cc` command includes four tests of homogeneity: Mantel–Haenszel (the default); directly pooled, also known as the Woolf test (available with the `pool` option); Tarone (available with the `tarone` option); and Breslow–Day (available with the `bd` option). The preferred test is Tarone's (Tarone 1985, 94), which corrected an error in the Breslow–Day test; see Breslow (1996, 17–18) for details of the error and Tarone's correction.

The other two homogeneity tests, the Mantel–Haenszel and directly pooled, are less useful: they use the logs of the stratum-specific odds ratios, so they are undefined when any stratum has a zero cell. The `epitab` commands deal with the problem differently: `cs` omits the offending strata, while `cc` substitutes the Tarone test. The Tarone test does not use the stratum-specific odds ratios, so it can still be calculated when there are zero cells.

None of the tests is appropriate for finely stratified (many strata with only a few observations each) studies (Lash et al. 2021 , 429). If you have fine stratification, one alternative is multilevel logistic regression; see [ME] [melogit](#).



□ Technical note

As with cohort study data, an alternative to stratified tables for uncovering effects is logistic regression. From the logistic point of view, case–control data are no different from cohort study data—you must merely ignore the estimated intercept. The intercept is meaningless in case–control data because it reflects the baseline prevalence of the outcome, which you controlled by sampling.

The data we used with `cc` can be used directly by `logistic`. (The `age` variable, which appears to be a string, is really numeric with an associated value label; see [U] [12.6.3 Value labels](#). `age` takes on the value 0 for the age-less-than-35 group and 1 for the 35+ group.)

```
. logistic case exposed age [fweight=pop]
Logistic regression Number of obs = 1,270
 LR chi2(2) = 8.74
 Prob > chi2 = 0.0127
Log likelihood = -81.517532 Pseudo R2 = 0.0509
```

| case    | Odds ratio | Std. err. | z      | P> z  | [95% conf. interval] |          |
|---------|------------|-----------|--------|-------|----------------------|----------|
| exposed | 3.787779   | 2.241922  | 2.25   | 0.024 | 1.187334             | 12.0836  |
| age     | 4.582857   | 2.717352  | 2.57   | 0.010 | 1.433594             | 14.65029 |
| _cons   | .0082631   | .0027325  | -14.50 | 0.000 | .0043218             | .0157988 |

Note: `_cons` estimates baseline odds.

We compare the results with those presented by `cc` in the [previous example](#):

| Source               | Odds ratio | 95% CI |       |
|----------------------|------------|--------|-------|
| Mantel–Haenszel (cc) | 3.78       | 1.19   | 12.04 |
| logistic             | 3.79       | 1.19   | 12.08 |

As with the cohort study data in [example 8](#), results are virtually identical, and all the same comments we made previously apply once again.

To demonstrate an advantage of logistic regression, let’s now ask a question that would be difficult to answer on the basis of a stratified table analysis. We now know that spermicide use appears to increase the risk of having a baby with Down syndrome, and we know that the mother’s age also increases the risk. Is the effect of spermicide use statistically different for mothers in the two age groups?

```
. logistic case exposed age c.age#exposed [fweight=pop]
Logistic regression
Log likelihood = -81.451332
Number of obs = 1,270
LR chi2(3) = 8.87
Prob > chi2 = 0.0311
Pseudo R2 = 0.0516
```

| case              | Odds ratio | Std. err. | z      | P> z  | [95% conf. interval] |          |
|-------------------|------------|-----------|--------|-------|----------------------|----------|
| exposed           | 3.394231   | 2.289544  | 1.81   | 0.070 | .9048403             | 12.73242 |
| age               | 4.104651   | 2.774868  | 2.09   | 0.037 | 1.091034             | 15.44237 |
| exposed#<br>c.age |            |           |        |       |                      |          |
| 1                 | 1.689141   | 2.388785  | 0.37   | 0.711 | .1056563             | 27.0045  |
| _cons             | .0084986   | .0028449  | -14.24 | 0.000 | .0044097             | .0163789 |

Note: `_cons` estimates baseline odds.

The answer is no. The odds ratio and confidence interval reported for `exposed` now measure the spermicide effect for an `age==0` (age < 35) mother. The odds ratio and confidence interval reported for `c.age#exposed` are the (multiplicative) difference in the spermicide odds ratio for an `age==1` (age 35+) mother relative to a young mother. The point estimate is that the effect is larger for older mothers, suggesting grounds for future research, but the difference is not significant.

See [R] [logistic](#) for an explanation of the `logistic` command. Also see the [technical note](#) under *Incidence-rate data* above concerning Poisson regression, which applies equally to logistic regression. □

### Case–control data with multiple levels of exposure

In a case–control study, subjects with the disease of interest (cases) are compared with disease-free individuals (controls) to assess the relationship between exposure to one or more risk factors and disease incidence. Often exposure is measured qualitatively at several discrete levels or measured on a continuous scale and then grouped into three or more levels. The data can be summarized as

|          | Exposure level        |                       |     |                              | Total                 |
|----------|-----------------------|-----------------------|-----|------------------------------|-----------------------|
|          | 1                     | 2                     | ... | <i>k</i>                     |                       |
| Cases    | <i>a</i> <sub>1</sub> | <i>a</i> <sub>2</sub> | ... | <i>a</i> <sub><i>k</i></sub> | <i>M</i> <sub>1</sub> |
| Controls | <i>c</i> <sub>1</sub> | <i>c</i> <sub>2</sub> | ... | <i>c</i> <sub><i>k</i></sub> | <i>M</i> <sub>0</sub> |
| Total    | <i>N</i> <sub>1</sub> | <i>N</i> <sub>2</sub> | ... | <i>N</i> <sub><i>k</i></sub> | <i>T</i>              |

An advantage afforded by having multiple levels of exposure is the ability to examine dose–response relationships. If the association between a risk factor and a disease or outcome is real, we expect the strength of that association to increase with the level and duration of exposure. A dose–response relationship provides strong support for a direct or even causal relationship between the risk factor and the outcome. On the other hand, the lack of a dose–response is usually seen as an argument against causality.

We can use the `tabodds` command to tabulate the odds of failure or odds ratios against a categorical exposure variable. The test for trend calculated by `tabodds` can serve as a test for dose–response if the exposure variable is at least ordinal. If the exposure variable has no natural ordering, the trend test is meaningless and should be ignored. See the technical note at the end of this section for more information regarding the test for trend.

Before looking at an example, consider three possible data arrangements for case–control and prevalence studies. The most common data arrangement is individual records, where each subject in the study has his or her own record. Closely related are frequency records where identical individual records are included only once, but with a variable giving the frequency with which the record occurs. The `fweight` `weight` option is used for these data to specify the frequency variable. Data can also be arranged as binomial frequency records where each record contains a variable, `D`, the number of cases; another variable, `N`, the number of total subjects (cases plus controls); and other variables. An advantage of binomial frequency records is that large datasets can be entered succinctly into a Stata database.

### ▷ Example 13: `tabodds`

Consider the following data from the Ille-et-Vilaine study of esophageal cancer, discussed in [Breslow and Day \(1980, chap. 4 and app. I\)](#), corresponding to subjects age 55–64 who use from 0 to 9 g of tobacco per day:

|          | Alcohol consumption (g/day) |       |        |      | Total |
|----------|-----------------------------|-------|--------|------|-------|
|          | 0–39                        | 40–79 | 80–119 | 120+ |       |
| Cases    | 2                           | 9     | 9      | 5    | 25    |
| Controls | 47                          | 31    | 9      | 5    | 92    |
| Total    | 49                          | 40    | 18     | 10   | 117   |

The study included 24 such tables, each representing one of four levels of tobacco use and one of six age categories. We can create a binomial frequency-record dataset by typing

```
. input alcohol D N agegrp tobacco
 alcohol D N agegrp tobacco
1. 1 2 49 4 1
2. 2 9 40 4 1
3. 3 9 18 4 1
4. 4 5 10 4 1
5. end
```

where `D` is the number of esophageal cancer cases and `N` is the number of total subjects (cases plus controls) for each combination of six age groups (`agegrp`), four levels of alcohol consumption in g/day (`alcohol`), and four levels of tobacco use in g/day (`tobacco`).

Both the `tabodds` and `mhodds` commands can correctly handle all three data arrangements. Binomial frequency records require that the number of total subjects (cases plus controls) represented by each record `N` be specified with the `binomial()` option.

We could also enter the data as frequency-weighted data:

```
. input alcohol case freq agegrp tobacco
 alcohol case freq agegrp tobacco
1. 1 1 2 4 1
2. 1 0 47 4 1
3. 2 1 9 4 1
4. 2 0 31 4 1
5. 3 1 9 4 1
6. 3 0 9 4 1
7. 4 1 5 4 1
8. 4 0 5 4 1
9. end
```

If you are planning on using any of the other estimation commands, such as `poisson` or `logistic`, we recommend that you enter your data either as individual records or as frequency-weighted records and not as binomial frequency records, because the estimation commands currently do not recognize the `binomial()` option.

We have entered all the esophageal cancer data into Stata as a frequency-weighted record dataset as previously described. In our data, `case` indicates the esophageal cancer cases and controls, and `freq` is the number of subjects represented by each record (the weight).

We added value labels to the `agegrp`, `alcohol`, and `tobacco` variables in our dataset to ease interpretation in outputs, but these variables are numeric.

We are interested in the association between alcohol consumption and esophageal cancer. We first use `tabodds` to tabulate the odds of esophageal cancer against alcohol consumption:

```
. use https://www.stata-press.com/data/r19/bdesop, clear
(ILLE-et-Vilaine study of esophageal cancer)
. tabodds case alcohol [fweight=freq]
```

| alcohol | Cases | Controls | Odds    | [95% conf. interval] |         |
|---------|-------|----------|---------|----------------------|---------|
| 0-39    | 29    | 386      | 0.07513 | 0.05151              | 0.10957 |
| 40-79   | 75    | 280      | 0.26786 | 0.20760              | 0.34560 |
| 80-119  | 51    | 87       | 0.58621 | 0.41489              | 0.82826 |
| 120+    | 45    | 22       | 2.04545 | 1.22843              | 3.40587 |

```
Test of homogeneity (equal odds): chi2(3) = 158.79
Pr>chi2 = 0.0000
Score test for trend of odds: chi2(1) = 152.97
Pr>chi2 = 0.0000
```

The test of homogeneity clearly indicates that the odds of esophageal cancer differ by level of alcohol consumption, and the test for trend indicates a significant increase in odds with increasing alcohol use. This suggests a strong dose–response relation. The `graph` option can be used to study the shape of the relationship of the odds with alcohol consumption. Most of the heterogeneity in these data can be “explained” by the linear increase in risk of esophageal cancer with increased dosage (alcohol consumption).

We also could have requested that the odds ratios at each level of alcohol consumption be calculated by specifying the `or` option. For example, `tabodds case alcohol [fweight=freq]`, or would produce odds ratios using the minimum value of `alcohol`—that is, `alcohol = 1` (0–39)—as the reference group, and the command `tabodds case alcohol [fweight=freq]`, or `base(2)` would use `alcohol = 2` (40–79) as the reference group.

Although our results appear to provide strong evidence supporting an association between alcohol consumption and esophageal cancer, we need to be concerned with the possible existence of confounders, specifically age and tobacco use, in our data. We can again use `tabodds` to tabulate the odds of esophageal cancer against age and against tobacco use, independently:

```
. tabodds case agegrp [fweight=freq]
```

| agegrp | Cases | Controls | Odds    | [95% conf. interval] |         |
|--------|-------|----------|---------|----------------------|---------|
| 25-34  | 1     | 115      | 0.00870 | 0.00121              | 0.06226 |
| 35-44  | 9     | 190      | 0.04737 | 0.02427              | 0.09244 |
| 45-54  | 46    | 167      | 0.27545 | 0.19875              | 0.38175 |
| 55-64  | 76    | 166      | 0.45783 | 0.34899              | 0.60061 |
| 65-74  | 55    | 106      | 0.51887 | 0.37463              | 0.71864 |
| 75+    | 13    | 31       | 0.41935 | 0.21944              | 0.80138 |

```
Test of homogeneity (equal odds): chi2(5) = 96.94
Pr>chi2 = 0.0000
```

```
Score test for trend of odds: chi2(1) = 83.37
Pr>chi2 = 0.0000
```

```
. tabodds case tobacco [fweight=freq]
```

| tobacco | Cases | Controls | Odds    | [95% conf. interval] |         |
|---------|-------|----------|---------|----------------------|---------|
| 0-9     | 78    | 447      | 0.17450 | 0.13719              | 0.22194 |
| 10-19   | 58    | 178      | 0.32584 | 0.24228              | 0.43823 |
| 20-29   | 33    | 99       | 0.33333 | 0.22479              | 0.49428 |
| 30+     | 31    | 51       | 0.60784 | 0.38899              | 0.94983 |

```
Test of homogeneity (equal odds): chi2(3) = 29.33
Pr>chi2 = 0.0000
```

```
Score test for trend of odds: chi2(1) = 26.93
Pr>chi2 = 0.0000
```

We can see that there is evidence to support our concern that both age and tobacco use are potentially important confounders. Clearly, before we can make any statements regarding the association between esophageal cancer and alcohol use, we must examine and, if necessary, adjust for the effect of any confounder. We will return to this example in the following section.

◀

## □ Technical note

The score test for trend performs a test for linear trend of the log odds against the numerical code used for the exposure variable. The test depends not only on the relationship between dose level and the outcome but also on the numeric values assigned to each level or, to be more accurate, to the distance between the numeric values assigned. For example, the trend test on a dataset with four exposure levels coded 1, 2, 3, and 4 gives the same results as coding the levels 10, 20, 30, and 40 because the distance between the levels in each case is constant. In the first case, the distance is 1 unit, and in the second case, it is 10 units. However, if we code the exposure levels as 1, 10, 100, and 1,000, we would obtain different results because the distance between exposure levels is not constant. Thus, be careful when assigning values to exposure levels. You must determine whether equally spaced numbers make sense for your data or if other more meaningful values should be used.

Remember that we are testing whether a log-linear relationship exists between the odds and the exposure variable. For your particular problem, this relationship may not be correct or even make sense, so you must be careful in interpreting the output of this trend test. □

## Case-control data with confounders and possibly multiple levels of exposure

In the esophageal cancer data example introduced earlier, we determined that the apparent association between alcohol consumption and esophageal cancer could be confounded by age and tobacco use. You can adjust for the effect of possible confounding factors by stratifying on these factors. This is the method used by both `tabodds` and `mhodds` to adjust for other variables in the dataset. We will compare and contrast these two commands in the following example.

### ▷ Example 14: `tabodds`, adjusting for confounding factors

We begin by using `tabodds` to tabulate unadjusted odds ratios.

```
. tabodds case alcohol [fweight=freq], or
```

| alcohol | Odds ratio | chi2   | P>chi2 | [95% conf. interval] |           |
|---------|------------|--------|--------|----------------------|-----------|
| 0-39    | 1.000000   | .      | .      | .                    | .         |
| 40-79   | 3.565271   | 32.70  | 0.0000 | 2.237981             | 5.679744  |
| 80-119  | 7.802616   | 75.03  | 0.0000 | 4.497054             | 13.537932 |
| 120+    | 27.225705  | 160.41 | 0.0000 | 12.507808            | 59.262107 |

```
Test of homogeneity (equal odds): chi2(3) = 158.79
Pr>chi2 = 0.0000
Score test for trend of odds: chi2(1) = 152.97
Pr>chi2 = 0.0000
```

The `alcohol = 1` group (0-39) was used by `tabodds` as the reference category for calculating the odds ratios. We could have selected a different group by specifying the `base()` option; however, because the lowest dosage level is most often the appropriate reference group, as it is in these data, the `base()` option is seldom used.

We use `tabodds` with the `adjust()` option to tabulate Mantel-Haenszel age-adjusted odds ratios:

```
. tabodds case alcohol [fweight=freq], adjust(age)
Mantel-Haenszel odds ratios adjusted for agegrp
```

| alcohol | Odds ratio | chi2   | P>chi2 | [95% conf. interval] |           |
|---------|------------|--------|--------|----------------------|-----------|
| 0-39    | 1.000000   | .      | .      | .                    | .         |
| 40-79   | 4.268155   | 37.36  | 0.0000 | 2.570025             | 7.088314  |
| 80-119  | 8.018305   | 59.30  | 0.0000 | 4.266893             | 15.067922 |
| 120+    | 28.570426  | 139.70 | 0.0000 | 12.146409            | 67.202514 |

```
Score test for trend of odds: chi2(1) = 135.09
Pr>chi2 = 0.0000
```

We observe that the age-adjusted odds ratios are just slightly higher than the unadjusted ones, so it appears that age is not as strong a confounder as it first appeared. Even after adjusting for age, the dose-response relationship, as measured by the trend test, remains strong.

We now perform the same analysis but this time adjust for tobacco use instead of age.

```
. tabodds case alcohol [fweight=freq], adjust(tobacco)
Mantel-Haenszel odds ratios adjusted for tobacco
```

| alcohol | Odds ratio | chi2   | P>chi2 | [95% conf. interval] |           |
|---------|------------|--------|--------|----------------------|-----------|
| 0-39    | 1.000000   | .      | .      | .                    | .         |
| 40-79   | 3.261178   | 28.53  | 0.0000 | 2.059764             | 5.163349  |
| 80-119  | 6.771638   | 62.54  | 0.0000 | 3.908113             | 11.733306 |
| 120+    | 19.919526  | 123.93 | 0.0000 | 9.443830             | 42.015528 |

```
Score test for trend of odds: chi2(1) = 135.04
Pr>chi2 = 0.0000
```

Again we observe a significant dose–response relationship and not much difference between the adjusted and unadjusted odds ratios. We could also adjust for the joint effect of both age and tobacco use by specifying `adjust(tobacco age)`, but we will not bother here.

◀

A different approach to analyzing these data is to use the `mhodds` command. This command estimates the ratio of the odds of failure for two categories of an exposure variable, controlling for any specified confounding variables, and it tests whether this odds ratio is equal to one. For multiple exposures, if two exposure levels are not specified with `compare()`, then `mhodds` assumes that exposure is quantitative and calculates a 1-degree-of-freedom test for trend. This test for trend is the same one that `tabodds` reports.

### ▷ Example 15: `mhodds`, controlling for confounding factors

We first use `mhodds` to estimate the effect of alcohol controlled for age:

```
. mhodds case alcohol agegrp [fweight=freq]
Score test for trend of odds with alcohol
controlling for agegrp
```

| Odds ratio | chi2(1) | P>chi2 | [95% conf. interval] |          |
|------------|---------|--------|----------------------|----------|
| 2.845895   | 135.09  | 0.0000 | 2.385749             | 3.394792 |

Note: The Odds ratio estimate is an approximation to the odds ratio for a one-unit increase in **alcohol**.

Because `alcohol` has more than two levels, `mhodds` estimates and reports an approximate age-adjusted odds ratio for a one-unit increase in alcohol consumption. The  $\chi^2$  value reported is identical to that reported by `tabodds` for the score test for trend on the previous page.

We now use `mhodds` to estimate the effect of alcohol controlled for age, and while we are at it, we do this by levels of tobacco consumption:

```
. mhodds case alcohol agegrp [fweight=freq], by(tobacco)
Score test for trend of odds with alcohol
controlling for agegrp
by tobacco
```

| tobacco | Odds ratio | chi2(1) | P>chi2 | [95% conf. interval] |         |
|---------|------------|---------|--------|----------------------|---------|
| 0-9     | 3.579667   | 75.95   | 0.0000 | 2.68710              | 4.76871 |
| 10-19   | 2.303580   | 25.77   | 0.0000 | 1.66913              | 3.17920 |
| 20-29   | 2.364135   | 13.27   | 0.0003 | 1.48810              | 3.75589 |
| 30+     | 2.217946   | 8.84    | 0.0029 | 1.31184              | 3.74992 |

Notes: Only 19 of the 24 strata formed in this analysis contribute information about the effect of the explanatory variable.  
The Odds ratio estimate is an approximation to the odds ratio for a one-unit increase in **alcohol**.

Mantel-Haenszel estimate controlling for **agegrp** and **tobacco**

| Odds ratio | chi2(1) | P>chi2 | [95% conf. interval] |          |
|------------|---------|--------|----------------------|----------|
| 2.751236   | 118.37  | 0.0000 | 2.292705             | 3.301471 |

Approximate test of homogeneity of odds ratios:  $\chi^2(3) = 5.46$   
 $Pr>\chi^2 = 0.1409$

The first table reports estimates of the effect of alcohol for each level of tobacco use, controlling for age.

From the second table, we find that the effect of alcohol is about  $\times 2.8$  when we control for both age and tobacco use. Again, because `alcohol` has more than two levels, `mhodds` estimates and reports an approximate Mantel-Haenszel age and tobacco-use adjusted odds ratio for a one-unit increase in alcohol consumption.

The  $\chi^2$  test for trend reported with the Mantel-Haenszel estimate is again the same one that `tabodds` produces if `adjust(agegrp tobacco)` is specified.

To instead estimate the effect of tobacco use for each level of alcohol consumption, controlling for age, we type

```
. mhodds case tobacco agegrp [fweight=freq], by(alcohol)
Score test for trend of odds with tobacco
controlling for agegrp
by alcohol
```

| alcohol | Odds ratio | chi2(1) | P>chi2 | [95% conf. interval] |         |
|---------|------------|---------|--------|----------------------|---------|
| 0-39    | 2.420650   | 15.61   | 0.0001 | 1.56121              | 3.75320 |
| 40-79   | 1.427713   | 5.75    | 0.0165 | 1.06717              | 1.91007 |
| 80-119  | 1.472218   | 3.38    | 0.0659 | 0.97483              | 2.22339 |
| 120+    | 1.214815   | 0.59    | 0.4432 | 0.73876              | 1.99763 |

Notes: Only 18 of the 24 strata formed in this analysis contribute information about the effect of the explanatory variable.  
The Odds ratio estimate is an approximation to the odds ratio for a one-unit increase in **tobacco**.

Mantel-Haenszel estimate controlling for **agegrp** and **alcohol**

| Odds ratio | chi2(1) | P>chi2 | [95% conf. interval] |          |
|------------|---------|--------|----------------------|----------|
| 1.553437   | 20.07   | 0.0000 | 1.281160             | 1.883580 |

Approximate test of homogeneity of odds ratios: chi2(3) = 5.26  
Pr>chi2 = 0.1540

From the second table, we find that the effect of tobacco, controlled for both age and alcohol consumption, is about  $\times 1.6$ .

Comparisons between particular levels of alcohol and tobacco consumption can be made by generating a new variable with levels corresponding to all combinations of alcohol and tobacco, as in

```
. egen alctob = group(alcohol tobacco)
. mhdods case alctob [fweight=freq], compare(16,1)
Maximum likelihood estimate of the odds ratio comparing alctob==16
vs. alctob==1
```

| Odds ratio | chi2(1) | P>chi2 | [95% conf. interval] |            |
|------------|---------|--------|----------------------|------------|
| 93.333333  | 103.21  | 0.0000 | 14.766136            | 589.938431 |

which yields an odds ratio of 93 between subjects with the highest levels of alcohol and tobacco and those with the lowest levels. Similar results can be obtained simultaneously for all levels of alctob using alctob = 1 as the comparison group by specifying tabodds D alctob, binomial(N) or.

◀

## Standardized estimates with stratified case–control data

### ▶ Example 16: cc with stratified data, using standardized estimates

You obtain standardized estimates (here for the odds ratio) by using `cc` just as you obtain standardized estimates by using `ir` or `cs`. Along with the `by()` option, you specify one of `estandard`, `istandard`, or `standard(varname)`.

Case–control studies can provide standardized rate-ratio estimates when density sampling is used, or when the disease is rare (Lash et al. 2021, 422). Lash et al. (2021, 429) report the SMR for the case–control study on infants with congenital heart disease and Down syndrome. We can reproduce their estimates along with the pooled estimates by typing

```
. use https://www.stata-press.com/data/r19/downs, clear
(Congenital heart disease and Down syndrome)
. cc case exposed [fweight=pop], by(age) istandard pool
```

| Maternal age    | Odds ratio | [95% conf. interval] |          | Weight      |
|-----------------|------------|----------------------|----------|-------------|
| <35             | 3.394231   | .5812415             | 13.87412 | 104 (exact) |
| 35+             | 5.733333   | .0911619             | 85.89602 | 5 (exact)   |
| Crude           | 3.501529   | .8080857             | 11.78958 | (exact)     |
| Pooled (direct) | 3.824166   | 1.196437             | 12.22316 |             |
| I. Standardized | 3.779749   | 1.180566             | 12.10141 |             |

Test of homogeneity (direct) chi2(1) = 0.14 Pr>chi2 = 0.7109

Using the distribution of the nonexposed subjects in the source population as the standard, we can obtain an estimate of the standardized rate ratio (SRR):

```
. cc case exposed [fweight=pop], by(age) estandard
```

| Maternal age    | Odds ratio | [95% conf. interval] |          | Weight       |
|-----------------|------------|----------------------|----------|--------------|
| <35             | 3.394231   | .5812415             | 13.87412 | 1059 (exact) |
| 35+             | 5.733333   | .0911619             | 85.89602 | 86 (exact)   |
| Crude           | 3.501529   | .8080857             | 11.78958 | (exact)      |
| E. Standardized | 3.979006   | 1.176096             | 13.46191 |              |

Finally, if we wanted to weight the two age groups equally, we could type

```
. generate wgt=1
. cc case exposed [fweight=pop], by(age) standard(wgt)
```

| Maternal age | Odds ratio | [95% conf. interval] |          | Weight    |
|--------------|------------|----------------------|----------|-----------|
| <35          | 3.394231   | .5812415             | 13.87412 | 1 (exact) |
| 35+          | 5.733333   | .0911619             | 85.89602 | 1 (exact) |
| Crude        | 3.501529   | .8080857             | 11.78958 | (exact)   |
| Standardized | 5.275104   | .6233794             | 44.6385  |           |

◀

### Matched case–control data

Matched case–control studies are performed to gain sample-size efficiency and to control for important confounding factors. In a matched case–control design, each case is matched with a control on the basis of demographic characteristics, clinical characteristics, etc. Thus, their difference with respect to the outcome must be due to something other than the matching variables. If the only difference between them was exposure to the factor, we could attribute any difference in outcome to the factor.

A summary of the data is

| Cases     | Controls |           | Total               |
|-----------|----------|-----------|---------------------|
|           | Exposed  | Unexposed |                     |
| Exposed   | <i>a</i> | <i>b</i>  | $M_1$               |
| Unexposed | <i>c</i> | <i>d</i>  | $M_0$               |
| Total     | $N_1$    | $N_0$     | $T = a + b + c + d$ |

Each entry in the table represents the number of case–control pairs. For instance, in *a* of the pairs, both members were exposed; in *b* of the pairs, the case was exposed but the control was not; and so on. In total, *T* pairs were observed.

## ▷ Example 17: mcci

Rothman (1986, 257) discusses data from Jick et al. (1973) on a matched case–control study of myocardial infarction and drinking six or more cups of coffee per day (persons drinking from one to five cups per day were excluded):

| Cases   | Controls |        |
|---------|----------|--------|
|         | 6+ cups  | 0 cups |
| 6+ cups | 8        | 8      |
| 0 cups  | 3        | 8      |

mcci analyzes matched case–control data:

```
. mcci 8 8 3 8
```

| Cases     | Controls |           | Total |
|-----------|----------|-----------|-------|
|           | Exposed  | Unexposed |       |
| Exposed   | 8        | 8         | 16    |
| Unexposed | 3        | 8         | 11    |
| Total     | 11       | 16        | 27    |

```
McNemar's chi2(1) = 2.27 Prob > chi2 = 0.1317
Exact McNemar significance probability = 0.2266
```

```
Proportion with factor
```

|            |          |                      |                 |
|------------|----------|----------------------|-----------------|
| Cases      | .5925926 |                      |                 |
| Controls   | .4074074 | [95% conf. interval] |                 |
| difference | .1851852 | -.0822542            | .4526246        |
| ratio      | 1.454545 | .891101              | 2.374257        |
| rel. diff. | .3125    | -.0243688            | .6493688        |
| odds ratio | 2.666667 | .6400364             | 15.6064 (exact) |

The point estimate states that the odds of drinking 6 or more cups of coffee per day is 2.67 times greater among the myocardial infarction patients. The confidence interval is wide, however, and the  $p$ -value of 0.1317 from McNemar's test is not statistically significant.

◀

mcc works like the other nonimmediate commands but does not handle stratified data. If you have stratified matched case–control data, you can use conditional logistic regression to estimate odds ratios; see [R] [clogit](#).

Matched case–control studies can also be analyzed using mhodds by controlling on the variable used to identify the matched sets. For example, if the variable set is used to identify the matched set for each subject,

```
. mhodds fail xvar set
```

will do the job. Any attempt to control for further variables will restrict the analysis to the comparison of cases and matched controls that share the same values of these variables. In general, this would lead to the omission of many records from the analysis. Similar considerations usually apply when investigating effect modification by using the by() option. An important exception to this rule is that a variable used in matching cases to controls may appear in the by() option without loss of data.

### ► Example 18: mhodds with matched case–control data

Let's use `mhodds` to analyze matched case–control studies using the study of endometrial cancer and exposure to estrogen described in [Breslow and Day \(1980, chap. 5\)](#). In this study, there are four controls matched to each case. Cases and controls are matched on age, marital status, and time living in the community. The data collected include information on the daily dose of conjugated estrogen therapy. Breslow and Day created four levels of the dose variable and began by analyzing the 1:1 study formed by using the first control in each set. We examine the effect of exposure to estrogen:

```
. use https://www.stata-press.com/data/r19/bdendo11, clear
(Endometrial cancer and estrogen exposure)
. describe
Contains data from https://www.stata-press.com/data/r19/bdendo11.dta
Observations: 126 Endometrial cancer and estrogen
 exposure
Variables: 13 3 Mar 2024 23:29
 (_dta has notes)
```

| Variable name         | Storage type | Display format | Value label | Variable label      |
|-----------------------|--------------|----------------|-------------|---------------------|
| <code>set</code>      | byte         | %8.0g          |             | Set number          |
| <code>fail</code>     | byte         | %8.0g          | fail        | Case or control     |
| <code>gall</code>     | byte         | %8.0g          |             | Gallbladder dis     |
| <code>hyp</code>      | byte         | %8.0g          |             | Hypertension        |
| <code>ob</code>       | byte         | %8.0g          |             | Obesity             |
| <code>est</code>      | byte         | %8.0g          |             | Estrogen            |
| <code>dos</code>      | byte         | %8.0g          |             | Ordinal dose        |
| <code>dur</code>      | byte         | %8.0g          |             | Ordinal duration    |
| <code>non</code>      | byte         | %8.0g          |             | Nonestrogen drug    |
| <code>duration</code> | byte         | %8.0g          |             | Months              |
| <code>age</code>      | byte         | %8.0g          |             | Years               |
| <code>cest</code>     | byte         | %8.0g          |             | Conjugated est dose |
| <code>agegrp</code>   | byte         | %9.0g          |             | Age group of set    |

Sorted by: `set`

```
. mhodds fail est set
```

```
Mantel-Haenszel estimate of the odds ratio comparing est==1 vs.
est==0
controlling for set
```

| Odds ratio | chi2(1) | P>chi2 | [95% conf. interval] |
|------------|---------|--------|----------------------|
| 9.666667   | 21.12   | 0.0000 | 2.944702 31.733072   |

Note: Only 32 of the 63 strata formed in this analysis contribute information about the effect of the explanatory variable.

For the 1:1 matched study, the Mantel–Haenszel methods are equivalent to conditional likelihood methods. The maximum conditional likelihood estimate of the odds ratio is given by the ratio of the off-diagonal frequencies in the two-way (case–control) table below. The data must be in the 1-observation-per-group format; that is, the matched case and control must appear in 1 observation (the same format as required by the `mcc` command; see also [\[R\] `clogit`](#)).

```

. keep fail est set
. reshape wide est, i(set) j(fail)
(j = 0 1)
Data Long -> Wide

Number of observations 126 -> 63
Number of variables 3 -> 3
j variable (2 values) fail -> (dropped)
xij variables:
 est -> est0 est1

```

```

. rename est1 case
. rename est0 control
. label variable case case
. label variable control control
. tabulate case control

```

| case  | control |    | Total |
|-------|---------|----|-------|
|       | 0       | 1  |       |
| 0     | 4       | 3  | 7     |
| 1     | 29      | 27 | 56    |
| Total | 33      | 30 | 63    |

The odds ratio is  $29/3 = 9.67$ , which agrees with the value obtained from `mhods`. In the more general  $1:m$  matched study, however, the Mantel–Haenszel methods are no longer equivalent to maximum conditional likelihood, although they are usually close.

To illustrate the use of the `by()` option in matched case–control studies, we look at the effect of exposure to estrogen, stratified by `age3`, which codes the sets into three age groups (55–64, 65–74, and 75+) as follows:

```

. use https://www.stata-press.com/data/r19/bdendo11, clear
(Endometrial cancer and estrogen exposure)
. generate age3 = agegrp
. recode age3 1/2=1 3/4=2 5/6=3
(124 changes made to age3)
. mhods fail est set, by(age3)
Mantel-Haenszel estimate of the odds ratio comparing est==1 vs. est==0
controlling for set
by age3

```

| age3 | Odds ratio | chi2(1) | P>chi2 | [95% conf. interval] |           |
|------|------------|---------|--------|----------------------|-----------|
| 1    | 6.000000   | 3.57    | 0.0588 | 0.72235              | 49.83724  |
| 2    | 15.000000  | 12.25   | 0.0005 | 1.98141              | 113.55557 |
| 3    | 8.000000   | 5.44    | 0.0196 | 1.00059              | 63.96252  |

Note: Only 32 of the 63 strata formed in this analysis contribute information about the effect of the explanatory variable.

Mantel-Haenszel estimate controlling for **set** and **age3**

| Odds ratio | chi2(1) | P>chi2 | [95% conf. interval] |           |
|------------|---------|--------|----------------------|-----------|
| 9.666667   | 21.12   | 0.0000 | 2.944702             | 31.733072 |

Approximate test of homogeneity of odds ratios: chi2(2) = 0.41  
 Pr>chi2 = 0.8128

There is no further loss of information when we stratify by age3 because age was one of the matching variables.

The full set of matched controls can be used in the same way. For example, the effect of exposure to estrogen is obtained (using the full dataset) with

```
. use https://www.stata-press.com/data/r19/bdendo, clear
(Endometrial cancer and estrogen exposure)
. mhodds fail est set
Mantel-Haenszel estimate of the odds ratio comparing est==1 vs.
est==0
controlling for set
```

| Odds ratio | chi2(1) | P>chi2 | [95% conf. interval] |           |
|------------|---------|--------|----------------------|-----------|
| 8.461538   | 31.16   | 0.0000 | 3.437773             | 20.826746 |

Note: Only 58 of the 63 strata formed in this analysis contribute information about the effect of the explanatory variable.

The effect of exposure to estrogen, stratified by age3, is obtained with

```
. generate age3 =agegrp
. recode age3 1/2=1 3/4=2 5/6=3
(310 changes made to age3)
. mhodds fail est set, by(age3)
Mantel-Haenszel estimate of the odds ratio comparing est==1 vs. est==0
controlling for set
by age3
```

| age3 | Odds ratio | chi2(1) | P>chi2 | [95% conf. interval] |           |
|------|------------|---------|--------|----------------------|-----------|
| 1    | 3.800000   | 3.38    | 0.0660 | 0.82165              | 17.57438  |
| 2    | 10.666667  | 18.69   | 0.0000 | 2.78773              | 40.81376  |
| 3    | 13.500000  | 9.77    | 0.0018 | 1.59832              | 114.02620 |

Note: Only 58 of the 63 strata formed in this analysis contribute information about the effect of the explanatory variable.

Mantel-Haenszel estimate controlling for **set** and **age3**

| Odds ratio | chi2(1) | P>chi2 | [95% conf. interval] |           |
|------------|---------|--------|----------------------|-----------|
| 8.461538   | 31.16   | 0.0000 | 3.437773             | 20.826746 |

Approximate test of homogeneity of odds ratios: chi2(2) = 1.41  
 Pr>chi2 = 0.4943

## Video examples

[Incidence-rate ratios calculator](#)

[Risk ratios calculator](#)

[Odds ratios for case–control data](#)

[Stratified analysis of case–control data](#)

[Odds ratios calculator](#)

## Glossary

**attributable fraction.** An attributable fraction is the reduction in the risk of a disease or other condition of interest when a particular risk factor is removed.

**case–control studies.** In case–control studies, cases meeting a fixed criterion are matched to noncases ex post to study differences in possible covariates. Relative sample sizes are usually fixed at 1:1 or 1:2 but sometimes vary once the survey is complete. In any case, sample sizes do not reflect the distribution in the underlying population.

**cohort studies.** In cohort studies, a group that is well defined is monitored over time to track the transition of noncases to cases. Cohort studies differ from incidence studies in that they can be retrospective as well as prospective.

**confounding.** In the analysis of contingency tables, factor or interaction effects are said to be confounded when the effect of one factor is combined with that of another. For example, the effect of alcohol consumption on esophageal cancer may be confounded with the effects of age, smoking, or both. In the presence of confounding, it is often useful to stratify on the confounded factors that are not of primary interest, in the above example, age and smoking.

**cross-sectional or prevalence studies.** Cross-sectional studies sample distributions of healthy and diseased subjects in the population at one point in time.

**crude estimate.** A crude estimate has not been adjusted for the effects of other variables. Disregarding a stratification variable, for example, yields a crude estimate.

**incidence and incidence rate.** Incidence is the number of new failures (for example, number of new cases of a disease) that occur during a specified period in a population at risk (for example, of the disease).

Incidence rate is incidence divided by the sum of the length of time each individual was exposed to the risk.

Do not confuse incidence with prevalence. Prevalence is the fraction of a population that has the disease. Incidence refers to the rate at which people contract a disease, whereas prevalence is the total number actually sick at a given time.

**incidence studies, longitudinal studies, and follow-up studies.** Whichever word is used, these studies monitor a population for a time to track the transition of noncases into cases. Incidence studies are prospective. Also see *cohort studies*.

**matched case–control study.** Also known as a retrospective study, a matched case–control study is a study in which persons with positive outcomes are each matched with one or more persons with negative outcomes but with similar characteristics.

**odds and odds ratio.** The odds in favor of an event are  $o = p/(1 - p)$ , where  $p$  is the probability of the event. Thus if  $p = 0.2$ , the odds are 0.25, and if  $p = 0.8$ , the odds are 4.

The log of the odds is  $\ln(o) = \text{logit}(p) = \ln\{p/(1 - p)\}$ , and logistic-regression models, for instance, fit  $\ln(o)$  as a linear function of the covariates.

The odds ratio is a ratio of two odds:  $o_1/o_0$ . The individual odds that appear in the ratio are usually for an experimental group and a control group, or two different demographic groups.

**prevented fraction.** A prevented fraction is the reduction in the risk of a disease or other condition of interest caused by including a protective risk factor or public-health intervention.

**prospective study.** Also known as a prospective longitudinal study, a prospective study is a study based on observations over the same subjects for a given period.

**risk factor.** This is a variable associated with an increased or decreased risk of failure.

**risk ratio.** In a log-linear model, this is the ratio of probability of survival associated with a one-unit increase in a risk factor relative to that calculated without such an increase, that is,  $R(x + 1)/R(x)$ . Given the exponential form of the model,  $R(x + 1)/R(x)$  is constant and is given by the exponentiated coefficient.

**SMR.** See *standardized mortality (morbidity) ratio*.

**standardized mortality (morbidity) ratio.** Standardized mortality (morbidity) ratio (SMR) is the observed number of deaths divided by the expected number of deaths. It is calculated using indirect standardization: you take the population of the group of interest—say, by age, sex, and other factors—and calculate the expected number of deaths in each cell (expected being defined as the number of deaths that would have been observed if those in the cell had the same mortality as some other population). You then take the ratio to compare the observed with the expected number of deaths. For instance,

|       | (1)<br>Population<br>of group | (2)<br>Deaths per 100,000<br>in general pop. | (1)×(2)<br>Expected #<br>of deaths | (4)<br>Observed<br>deaths |
|-------|-------------------------------|----------------------------------------------|------------------------------------|---------------------------|
| Age   |                               |                                              |                                    |                           |
| 25–34 | 95,965                        | 105.2                                        | 100.9                              | 92                        |
| 34–44 | 78,280                        | 203.6                                        | 159.4                              | 180                       |
| 44–54 | 52,393                        | 428.9                                        | 224.7                              | 242                       |
| 55–64 | 28,914                        | 964.6                                        | 278.9                              | 312                       |
| Total |                               |                                              | 763.9                              | 826                       |

$$\text{SMR} = 826/763.9 = 1.08$$

**stratified test.** A stratified test is performed separately for each stratum. The stratum-specific results are then combined into an overall test statistic.

## Stored results

`ir` (without `by()`) and `iri` store the following in `r()`:

Scalars

|                        |                        |
|------------------------|------------------------|
| <code>r(ird)</code>    | IRD                    |
| <code>r(lb_ird)</code> | lower CI bound for IRD |
| <code>r(ub_ird)</code> | upper CI bound for IRD |
| <code>r(irr)</code>    | IRR                    |

|                                  |                                                     |
|----------------------------------|-----------------------------------------------------|
| <code>r(lb_irr)</code>           | lower CI bound for IRR                              |
| <code>r(ub_irr)</code>           | upper CI bound for IRR                              |
| <code>r(afe)</code>              | AFE                                                 |
| <code>r(lb_afe)</code>           | lower CI bound for AFE                              |
| <code>r(ub_afe)</code>           | upper CI bound for AFE                              |
| <code>r(afp)</code>              | AFP                                                 |
| <code>r(p_lower_midp)</code>     | lower one-sided $p$ -value with mid- $p$ adjustment |
| <code>r(p_upper_midp)</code>     | upper one-sided $p$ -value with mid- $p$ adjustment |
| <code>r(p_twosided_midp)</code>  | two-sided $p$ -value with mid- $p$ adjustment       |
| <code>r(p_lower_exact)</code>    | lower one-sided exact $p$ -value                    |
| <code>r(p_upper_exact)</code>    | upper one-sided exact $p$ -value                    |
| <code>r(p_twosided_exact)</code> | two-sided exact $p$ -value                          |

`ir`, `by()` stores the following in `r()`:

#### Scalars

|                           |                                                                        |
|---------------------------|------------------------------------------------------------------------|
| <code>r(irr)</code>       | Mantel–Haenszel IRR, if option <code>ird</code> is not specified       |
| <code>r(lb_irr)</code>    | lower CI bound for Mantel–Haenszel IRR                                 |
| <code>r(ub_irr)</code>    | upper CI bound for Mantel–Haenszel IRR                                 |
| <code>r(ird)</code>       | Mantel–Haenszel IRD, if option <code>ird</code> is specified           |
| <code>r(lb_ird)</code>    | lower CI bound for Mantel–Haenszel IRD                                 |
| <code>r(ub_ird)</code>    | upper CI bound for Mantel–Haenszel IRD                                 |
| <code>r(crude)</code>     | crude IRR or, if option <code>ird</code> is specified, crude IRD       |
| <code>r(lb_crude)</code>  | lower CI bound for the crude IRR or IRD                                |
| <code>r(ub_crude)</code>  | upper CI bound for the crude IRR or IRD                                |
| <code>r(pooled)</code>    | pooled IRR or, if option <code>ird</code> is specified, pooled IRD     |
| <code>r(lb_pooled)</code> | lower CI bound for pooled IRR or IRD                                   |
| <code>r(ub_pooled)</code> | upper CI bound for pooled IRR or IRD                                   |
| <code>r(df)</code>        | degrees of freedom for homogeneity $\chi^2$ test                       |
| <code>r(chi2_mh)</code>   | Mantel–Haenszel homogeneity $\chi^2$                                   |
| <code>r(chi2_p)</code>    | pooled homogeneity $\chi^2$ , if option <code>pool</code> is specified |

`cs` and `csi` store the following in `r()`:

#### Scalars

|                           |                                                                |
|---------------------------|----------------------------------------------------------------|
| <code>r(p)</code>         | two-sided $p$ -value                                           |
| <code>r(rd)</code>        | risk difference                                                |
| <code>r(lb_rd)</code>     | lower CI bound for risk difference                             |
| <code>r(ub_rd)</code>     | upper CI bound for risk difference                             |
| <code>r(rr)</code>        | risk ratio                                                     |
| <code>r(lb_rr)</code>     | lower CI bound for risk ratio                                  |
| <code>r(ub_rr)</code>     | upper CI bound for risk ratio                                  |
| <code>r(or)</code>        | odds ratio                                                     |
| <code>r(lb_or)</code>     | lower CI bound for odds ratio                                  |
| <code>r(ub_or)</code>     | upper CI bound for odds ratio                                  |
| <code>r(afe)</code>       | AFE                                                            |
| <code>r(lb_afe)</code>    | lower CI bound for AFE                                         |
| <code>r(ub_afe)</code>    | upper CI bound for AFE                                         |
| <code>r(afp)</code>       | AFP                                                            |
| <code>r(crude)</code>     | crude estimate ( <code>cs</code> only)                         |
| <code>r(lb_crude)</code>  | lower CI bound for crude estimate                              |
| <code>r(ub_crude)</code>  | upper CI bound for crude estimate                              |
| <code>r(pooled)</code>    | pooled estimate ( <code>cs</code> only)                        |
| <code>r(lb_pooled)</code> | lower CI bound for pooled estimate                             |
| <code>r(ub_pooled)</code> | upper CI bound for pooled estimate                             |
| <code>r(chi2_mh)</code>   | Mantel–Haenszel heterogeneity $\chi^2$ ( <code>cs</code> only) |
| <code>r(chi2_p)</code>    | pooled heterogeneity $\chi^2$                                  |
| <code>r(df)</code>        | degrees of freedom ( <code>cs</code> only)                     |

|                          |                                         |
|--------------------------|-----------------------------------------|
| <code>r(chi2)</code>     | $\chi^2$                                |
| <code>r(p_exact)</code>  | 2-sided Fisher's exact $p$ (exact only) |
| <code>r(p1_exact)</code> | 1-sided Fisher's exact $p$ (exact only) |

`cc` and `cci` store the following in `r()`:

Scalars

|                            |                                                  |
|----------------------------|--------------------------------------------------|
| <code>r(p)</code>          | two-sided $p$ -value                             |
| <code>r(p1_exact)</code>   | one-sided $p$ -value for Fisher's exact test     |
| <code>r(p_exact)</code>    | two-sided $p$ -value for Fisher's exact test     |
| <code>r(or)</code>         | odds ratio                                       |
| <code>r(lb_or)</code>      | lower CI bound for odds ratio                    |
| <code>r(ub_or)</code>      | upper CI bound for odds ratio                    |
| <code>r(afe)</code>        | AFE                                              |
| <code>r(lb_afe)</code>     | lower CI bound for AFE                           |
| <code>r(ub_afe)</code>     | upper CI bound for AFE                           |
| <code>r(afp)</code>        | AFP                                              |
| <code>r(crude)</code>      | crude estimate ( <code>cc</code> only)           |
| <code>r(lb_crude)</code>   | lower CI bound for crude estimate                |
| <code>r(ub_crude)</code>   | upper CI bound for crude estimate                |
| <code>r(poolled)</code>    | pooled estimate ( <code>cc</code> only)          |
| <code>r(lb_poolled)</code> | lower CI bound for pooled estimate               |
| <code>r(ub_poolled)</code> | upper CI bound for pooled estimate               |
| <code>r(chi2_p)</code>     | pooled heterogeneity $\chi^2$                    |
| <code>r(chi2_bd)</code>    | Breslow–Day $\chi^2$                             |
| <code>r(df_bd)</code>      | degrees of freedom for Breslow–Day $\chi^2$ test |
| <code>r(chi2_t)</code>     | Tarone $\chi^2$                                  |
| <code>r(df_t)</code>       | degrees of freedom for Tarone $\chi^2$ test      |
| <code>r(df)</code>         | degrees of freedom                               |
| <code>r(chi2)</code>       | $\chi^2$                                         |

`tabodds` stores the following in `r()`:

Scalars

|                          |                                            |
|--------------------------|--------------------------------------------|
| <code>r(odds)</code>     | odds                                       |
| <code>r(lb_odds)</code>  | lower CI bound for odds                    |
| <code>r(ub_odds)</code>  | upper CI bound for odds                    |
| <code>r(chi2_hom)</code> | $\chi^2$ for test of homogeneity           |
| <code>r(p_hom)</code>    | $p$ -value for test of homogeneity         |
| <code>r(df_hom)</code>   | degrees of freedom for test of homogeneity |
| <code>r(chi2_tr)</code>  | $\chi^2$ for score test for trend          |
| <code>r(p_trend)</code>  | $p$ -value for score test for trend        |

`mhodds` stores the following in `r()`:

Scalars

|                          |                                            |
|--------------------------|--------------------------------------------|
| <code>r(p)</code>        | two-sided $p$ -value                       |
| <code>r(or)</code>       | odds ratio                                 |
| <code>r(lb_or)</code>    | lower CI bound for odds ratio              |
| <code>r(ub_or)</code>    | upper CI bound for odds ratio              |
| <code>r(chi2_hom)</code> | $\chi^2$ for test of homogeneity           |
| <code>r(df_hom)</code>   | degrees of freedom for test of homogeneity |
| <code>r(chi2)</code>     | $\chi^2$                                   |

Matrices

|                              |                                                        |
|------------------------------|--------------------------------------------------------|
| <code>r(strata_table)</code> | odds ratios for strata, if <code>by()</code> specified |
|------------------------------|--------------------------------------------------------|

`mcc` and `mcci` store the following in `r()`:

Scalars

|                         |                                                      |
|-------------------------|------------------------------------------------------|
| <code>r(p_exact)</code> | two-sided $p$ -value for McNemar's test              |
| <code>r(or)</code>      | odds ratio                                           |
| <code>r(lb_or)</code>   | lower CI bound for odds ratio                        |
| <code>r(ub_or)</code>   | upper CI bound for odds ratio                        |
| <code>r(D_f)</code>     | difference in proportion with factor                 |
| <code>r(lb_D_f)</code>  | lower CI bound for difference in proportion          |
| <code>r(ub_D_f)</code>  | upper CI bound for difference in proportion          |
| <code>r(R_f)</code>     | ratio of proportion with factor                      |
| <code>r(lb_R_f)</code>  | lower CI bound for ratio of proportion               |
| <code>r(ub_R_f)</code>  | upper CI bound for ratio of proportion               |
| <code>r(RD_f)</code>    | relative difference in proportion with factor        |
| <code>r(lb_RD_f)</code> | lower CI bound for relative difference in proportion |
| <code>r(ub_RD_f)</code> | upper CI bound for relative difference in proportion |
| <code>r(chi2)</code>    | $\chi^2$                                             |

## Methods and formulas

The notation for incidence-rate data is

|             | Exposed | Unexposed | Total |
|-------------|---------|-----------|-------|
| Cases       | $a$     | $b$       | $M_1$ |
| Person-time | $N_1$   | $N_0$     | $T$   |

The notation for  $2 \times 2$  tables is

|          | Exposed | Unexposed | Total |
|----------|---------|-----------|-------|
| Cases    | $a$     | $b$       | $M_1$ |
| Controls | $c$     | $d$       | $M_0$ |
| Total    | $N_1$   | $N_0$     | $T$   |

The notation for  $2 \times k$  tables is

|          | Exposure level |       |     |       | Total |
|----------|----------------|-------|-----|-------|-------|
|          | 1              | 2     | ... | k     |       |
| Cases    | $a_1$          | $a_2$ | ... | $a_k$ | $M_1$ |
| Controls | $c_1$          | $c_2$ | ... | $c_k$ | $M_0$ |
| Total    | $N_1$          | $N_2$ | ... | $N_k$ | $T$   |

If the tables are stratified, all quantities are indexed by  $i$ , the stratum number.

We will refer to [Fleiss, Levin, and Paik \(2003\)](#); [Kleinbaum, Kupper, and Morgenstern \(1982\)](#); and [Rothman \(1986\)](#) so often that we will adopt the notation F-23 to mean [Fleiss, Levin, and Paik \(2003\)](#) page 23; KKM-52 to mean [Kleinbaum, Kupper, and Morgenstern \(1982\)](#) page 52; and R-164 to mean [Rothman \(1986\)](#) page 164.

We usually avoid making the continuity corrections to  $\chi^2$  statistics, following the advice of KKM-292: "... the use of a continuity correction has been the subject of considerable debate in the statistical literature ... On the basis of our evaluation of this debate and other evidence, we do *not* recommend the use of the continuity correction." [Breslow and Day \(1980, 133\)](#), on the other hand, argue for inclusion of the correction, but not strongly. Their summary is that for small datasets, one should use exact statistics. In practice, we believe that the adjustment makes little difference for reasonably sized datasets.

Methods and formulas are presented under the following headings:

- Unstratified incidence-rate data (ir and iri)*
- Unstratified cumulative incidence data (cs and csi)*
- Unstratified case-control data (cc and cci)*
- Unstratified matched case-control data (mcc and mcci)*
- Stratified incidence-rate data (ir with the by() option)*
- Stratified cumulative incidence data (cs with the by() option)*
- Stratified case-control data (cc with by() option, mhodds, tabodds)*

### Unstratified incidence-rate data (ir and iri)

The IRD is defined as  $I_d = a/N_1 - b/N_0$  (R-164). The standard error of the difference is  $s_{I_d} \approx \sqrt{a/N_1^2 + b/N_0^2}$  (R-170), from which confidence intervals are calculated.

The IRR is defined as  $I_r = (a/N_1)/(b/N_0)$  (R-164). Let  $p_l$  and  $p_u$  be the exact confidence interval of the binomial probability for observing  $a$  successes in  $M_1$  trials (obtained from `cii` proportions; see [R] `ci`). The exact confidence interval for the incidence ratio is then  $(p_l N_0)/\{(1 - p_l)N_1\}$  to  $(p_u N_0)/\{(1 - p_u)N_1\}$  (R-166).

The AFE is defined as  $AFE = (I_r - 1)/I_r$  for  $I_r \geq 1$  (KKM-164; R-38); the confidence interval is obtained by similarly transforming the interval values of  $I_r$ . The AFP is  $AFP = AFE \cdot a/M_1$  (KKM-161); no confidence interval is reported. For  $I_r < 1$ , the PFE is defined as  $PFE = 1 - I_r$  (KKM-166; R-39); the confidence interval is obtained by similarly transforming the interval values of  $I_r$ . The PFP is  $PFP = PFE \cdot N_1/T$  (KKM-165); no confidence interval is reported.

Exact one-sided  $p$ -values are calculated as the binomial probabilities (with  $n = M_1$  and  $p = N_1/T$ )  $\Pr(k \leq a)$  and  $\Pr(k \geq a)$ . Exact  $p$ -values tend to be overly conservative, so the mid- $p$  adjustment (R-155) reduces the exact  $p$ -values by subtracting half the probability of the observed result from each one-sided  $p$ -value. That is, one-sided  $p$ -values with the mid- $p$  adjustment are the binomial probabilities  $\Pr(k \leq a) - \Pr(k = a)/2$  and  $\Pr(k \geq a) - \Pr(k = a)/2$ . The two-sided  $p$ -value is twice the smallest one-sided  $p$ -value for both the exact and mid- $p$ -adjustment calculations. Rather than using twice the smallest one-sided  $p$ -value for the two-sided  $p$ -value, there is another formula for the two-sided  $p$ -value that is sometimes used. The command `bitest` uses this alternative; see [R] `bitest` for details.

### Unstratified cumulative incidence data (cs and csi)

The risk difference is defined as  $R_d = a/N_1 - b/N_0$  (R-164). Its standard error is

$$s_{R_d} \approx \left\{ \frac{ac}{N_1^3} + \frac{bd}{N_0^3} \right\}^{1/2}$$

(R-172), from which confidence intervals are calculated.

The risk ratio is defined as  $R_r = (a/N_1)/(b/N_0)$  (R-165). The standard error of  $\ln R_r$  is

$$s_{\ln R_r} \approx \left( \frac{c}{aN_1} + \frac{d}{bN_0} \right)^{1/2}$$

(R-173), from which confidence intervals are calculated.

For  $R_r \geq 1$ , the AFE is calculated as  $AFE = (R_r - 1)/R_r$  (KKM-164; R-38); the confidence interval is obtained by similarly transforming the interval values for  $R_r$ . The AFP is calculated as  $AFP = AFE \cdot a/M_1$  (KKM-161); no confidence interval is reported, but F-128 provides

$$\left\{ \frac{c + (a + d)AFP}{bT} \right\}^{1/2}$$

as the approximate standard error of  $\ln(1 - AFP)$ .

For  $R_r < 1$ , the PFE is calculated as  $PFE = 1 - R_r$  (KKM-166; R-39); the confidence interval is obtained by similarly transforming the interval values for  $R_r$ . The PFP is calculated as  $PFP = PFE \cdot N_1/T$ ; no confidence interval is reported.

The odds ratio, available with the `or` option, is defined as  $\psi = (ad)/(bc)$  (R-165). Several confidence intervals are available. The default interval for `cs` and `csi` is the Cornfield (1956) approximate interval. If we let  $z_\alpha$  be the index from a normal distribution for an  $\alpha$  significance level, the Cornfield interval  $(\psi_l, \psi_u)$  is calculated from

$$\begin{aligned} \psi_l &= a_l(M_0 - N_1 + a_l) / \left\{ (N_1 - a_l)(M_1 - a_l) \right\} \\ \psi_u &= a_u(M_0 - N_1 + a_u) / \left\{ (N_1 - a_u)(M_1 - a_u) \right\} \end{aligned}$$

where  $a_u$  and  $a_l$  are determined iteratively from

$$a_{i+1} = a \pm z_\alpha \left( \frac{1}{a_i} + \frac{1}{N_1 - a_i} + \frac{1}{M_1 - a_i} + \frac{1}{M_0 - N_1 + a_i} \right)^{-1/2}$$

(Newman 2001, sec. 4.4).  $a_{i+1}$  converges to  $a_u$  using the plus sign and  $a_l$  using the minus sign.  $a_0$  is taken as  $a$ . With small numbers, the iterative technique may fail. It is then restarted by decrementing ( $a_l$ ) or incrementing ( $a_u$ )  $a_0$ . If that fails,  $a_0$  is again decremented or incremented and iterations restarted, and so on, until a terminal condition is met ( $a_0 < 0$  or  $a_0 > M_1$ ), at which point the value is not calculated.

The Woolf odds-ratio confidence intervals are available with `cs` and `csi`. The Woolf method (Woolf 1955; R-173; Schlesselman 1982, 176), available with the `woolf` option, estimates the standard error of  $\ln\psi$  by

$$s \ln\psi = \left( \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} \right)^{1/2}$$

from which confidence intervals are calculated. The Woolf interval cannot be calculated when there exists a zero cell. Sometimes the Woolf interval is called the “logit interval” (Breslow and Day 1980, 134).

The  $\chi^2$  statistic, reported by default, can be calculated as

$$\chi^2 = \frac{(ad - bc)^2 T}{M_1 M_0 N_1 N_0}$$

(Schlesselman 1982, 179).

Fisher’s exact test, available with the `exact` option, is calculated as described in [R] `tabulate twoway`.

### Unstratified case–control data (cc and cci)

cc and cci report by default the same odds ratio,  $\psi$ , that is available with the `or` option in `cs` and `csi`. But `cc` and `cci` calculate the confidence interval differently: they default to the exact odds-ratio interval, not the Cornfield interval, but you can request the Cornfield interval with the `cornfield` option. The  $1 - \alpha$  exact interval ( $\underline{R}, \overline{R}$ ) is calculated from

$$\alpha/2 = \frac{\sum_{k=a}^{\min(N_1, M_1)} \binom{N_1}{k} \binom{N_0}{M_1-k} \underline{R}^k}{\sum_{k=\max(0, M_1-N_0)}^{\min(N_1, M_1)} \binom{N_1}{k} \binom{N_0}{M_1-k} \underline{R}^k}$$

and

$$1 - \alpha/2 = \frac{\sum_{k=a+1}^{\min(N_1, M_1)} \binom{N_1}{k} \binom{N_0}{M_1-k} \overline{R}^k}{\sum_{k=\max(0, M_1-N_0)}^{\min(N_1, M_1)} \binom{N_1}{k} \binom{N_0}{M_1-k} \overline{R}^k}$$

(R-169). The equations invert two one-sided Fisher exact tests.

cc and cci also report the same tests of significance as `cs` and `csi`: the  $\chi^2$  statistic is the default, and Fisher’s exact test is obtained with the `exact` option. The odds ratio,  $\psi$ , is used as an estimate of the risk ratio in calculating attributable or prevented fractions. For  $\psi \geq 1$ , the AFE is calculated as  $AFE = (\psi - 1)/\psi$  (KKM-164); the confidence interval is obtained by similarly transforming the interval values for  $\psi$ . The AFP is calculated as  $AFP = AFE \cdot a/M_1$  (KKM-161). No confidence interval is reported; however, F-152 provides

$$\left( \frac{a}{M_1 b} + \frac{c}{M_0 d} \right)^{1/2}$$

as the standard error of  $\ln(1 - AFP)$ .

For  $\psi < 1$ , the PFE is calculated as  $PFE = 1 - \psi$  (KKM-166); the confidence interval is obtained by similarly transforming the interval values for  $\psi$ . The PFP is calculated as  $PFP = \{(a/M_1)PFE\}/\{(a/M_1)PFE + \psi\}$  (KKM-165); no confidence interval is reported.

### Unstratified matched case–control data (mcc and mcci)

Referring to the table at the beginning of *Matched case–control data* under *Remarks and examples* above, the columns of the table indicate controls; the rows are cases. Each entry in the table reflects a pair of a matched case and control.

McNemar’s (1947)  $\chi^2$  is defined as

$$\chi^2 = \frac{(b - c)^2}{b + c}$$

(KKM-389).

The proportion of controls with the factor is  $p_1 = N_1/T$ , and the proportion of cases with the factor is  $p_2 = M_1/T$ .

The difference in the proportions is  $P_d = p_2 - p_1$ . An estimate of its standard error when the two underlying proportions are *not* hypothesized to be equal is

$$s_{P_d} \approx \frac{\{(a + d)(b + c) + 4bc\}^{1/2}}{T^{3/2}}$$

(F-378), from which confidence intervals are calculated. The confidence interval uses a continuity correction (F-378, eq. 13.15).

The ratio of the proportions is  $P_r = p_2/p_1$  (R-276, R-278). The standard error of  $\ln P_r$  is

$$s_{\ln P_r} \approx \left( \frac{b+c}{M_1 N_1} \right)^{1/2}$$

(R-276), from which confidence intervals are calculated.

The relative difference in the proportions is  $P_e = (b-c)/(b+d)$  (F-379). Its standard error is

$$s_{P_e} \approx (b+d)^{-2} \{ (b+c+d)(bc+bd+cd) - bcd \}^{1/2}$$

(F-379), from which confidence intervals are calculated.

The odds ratio is  $\psi = b/c$  (F-376), and the exact Fisher confidence interval is obtained by transforming into odds ratios the exact binomial confidence interval for the binomial parameter from observing  $b$  successes in  $b+c$  trials (R-264). Binomial confidence limits are obtained from [cii](#) proportions (see [\[R\] ci](#)) and are transformed by  $p/(1-p)$ .

The exact McNemar significance probability is a two-tailed exact test of  $H_0 : \psi = 1$ . The  $p$ -value, calculated from the binomial distribution, is

$$\min \left\{ 1, 2 \sum_{k=0}^{\min(b,c)} \binom{b+c}{k} \left( \frac{1}{2} \right)^{b+c} \right\}$$

([Agresti 2013](#), 416).

Quinn McNemar (1900–1986) was born in West Virginia and attended college there and in Pennsylvania. After a brief spell of high school teaching, he began graduate study of psychology at Stanford and then joined the faculty. McNemar’s text *Psychological Statistics*, first published in 1949, was widely influential, and he made many substantive and methodological contributions to the application of statistics in psychology.

### Stratified incidence-rate data (ir with the by() option)

Statistics presented for each stratum are calculated independently according to the formulas in [Unstratified incidence-rate data \(ir and iri\)](#) above. Within strata, the Mantel–Haenszel style weight is  $W_i = b_i N_{1i}/T_i$ , and the Mantel–Haenszel combined incidence-rate ratio ([Rothman and Boice 1982](#)) is

$$I_{mh} = \frac{\sum_i a_i N_{0i}/T_i}{\sum_i W_i}$$

(R-196). The standard error for the log of the incidence-rate ratio was derived by [Greenland and Robins \(1985, 63\)](#) and appears in R-213:

$$s_{\ln I_{mh}} \approx \left\{ \frac{\sum_i M_{1i} N_{1i} N_{0i}/T_i^2}{(\sum_i a_i N_{0i}/T_i)(\sum_i b_i N_{1i}/T_i)} \right\}^{1/2}$$

The confidence interval is calculated first on the log scale and then is transformed.

For standardized rates, let  $w_i$  be the user-specified weight within stratum  $i$ . The standardized rate difference (the `ird` option) and rate ratio are defined as

$$\text{SRD} = \frac{\sum_i w_i (R_{1i} - R_{0i})}{\sum_i w_i}$$

$$\text{SRR} = \frac{\sum_i w_i R_{1i}}{\sum_i w_i R_{0i}}$$

(R-229). The standard error of SRD is

$$s_{\text{SRD}} \approx \left\{ \frac{1}{(\sum_i w_i)^2} \sum_i w_i^2 \left( \frac{a_i}{N_{1i}^2} + \frac{b_i}{N_{0i}^2} \right) \right\}^{1/2}$$

(R-231), from which confidence intervals are calculated. The standard error of  $\ln(\text{SRR})$  is

$$s_{\ln(\text{SRR})} \approx \left\{ \frac{\sum_i w_i^2 a_i / N_{1i}^2}{(\sum_i w_i R_{1i})^2} + \frac{\sum_i w_i^2 b_i / N_{0i}^2}{(\sum_i w_i R_{0i})^2} \right\}^{1/2}$$

(R-231), from which confidence intervals are calculated.

Internally and externally standardized measures are calculated using  $w_i = N_{1i}$  and  $w_i = N_{0i}$ , respectively, and are obtained with the `istandard` and `estandard` options, respectively.

Directly pooled estimates are available with the `pool` option. The directly pooled estimate is a weighted average of stratum-specific estimates; each weight,  $w_i$ , is inversely proportional to the variance of the estimate for stratum  $i$ . The variances for rate differences come from the formulas in *Unstratified incidence-rate data (ir and iri)*, while the variances of log rate-ratios are estimated by  $(1/a_i + 1/b_i)$  (R-184). Ratios are averaged in the log scale before being exponentiated. The standard error of the directly pooled estimate is calculated as  $1/\sqrt{\sum w_i}$ , from which confidence intervals are calculated (R-183–185); the calculation for ratios again uses the log scale.

For rate differences, the  $\chi^2$  test of homogeneity is calculated as  $\sum (R_{di} - \widehat{R}_d)^2 / \text{var}(R_{di})$ , where  $R_{di}$  are the stratum-specific rate differences and  $\widehat{R}_d$  is the directly pooled estimate. The number of degrees of freedom is one less than the number of strata (R-222).

For rate ratios, the same calculation is made, except that it is made on a logarithmic scale using  $\ln(R_{ri})$  (R-222), and  $\ln(\widehat{R}_d)$  may be the log of either the directly pooled estimate or the Mantel–Haenszel estimate.

### Stratified cumulative incidence data (cs with the `by()` option)

Statistics presented for each stratum are calculated independently according to the formulas in *Unstratified cumulative incidence data (cs and csi)* above. The Mantel–Haenszel  $\chi^2$  test (Mantel and Haenszel 1959) is

$$\chi_{\text{mh}}^2 = \frac{\left\{ \sum_i (a_i - N_{1i} M_{1i} / T_i) \right\}^2}{\sum_i (N_{1i} N_{0i} M_{1i} M_{0i}) / \{ T_i^2 (T_i - 1) \}}$$

(R-206).

For the odds ratio (available with the `or` option), the Mantel–Haenszel weight is  $W_i = b_i c_i / T_i$ , and the combined odds ratio (Mantel and Haenszel 1959) is

$$\psi_{\text{mh}} = \frac{\sum_i a_i d_i / T_i}{\sum_i W_i}$$

(R-195). The standard error (Robins, Breslow, and Greenland 1986) is

$$s \ln \psi_{\text{mh}} \approx \left\{ \frac{\sum_i P_i R_i}{2(\sum_i R_i)^2} + \frac{\sum_i P_i S_i + Q_i R_i}{2 \sum_i R_i \sum_i S_i} + \frac{\sum_i Q_i S_i}{2(\sum_i S_i)^2} \right\}^{1/2}$$

where

$$\begin{aligned} P_i &= (a_i + d_i) / T_i \\ Q_i &= (b_i + c_i) / T_i \\ R_i &= a_i d_i / T_i \\ S_i &= b_i c_i / T_i \end{aligned}$$

(R-220).

For the risk ratio (the default), the Mantel–Haenszel-style weight is  $W_i = b_i N_{1i} / T_i$ , and the combined risk ratio (Rothman and Boice 1982) is

$$R_{\text{mh}} = \frac{\sum_i a_i N_{0i} / T_i}{\sum_i W_i}$$

(R-196). The standard error (Greenland and Robins 1985) is

$$s \ln R_{\text{mh}} \approx \left\{ \frac{\sum_i (M_{1i} N_{1i} N_{0i} - a_i b_i T_i) / T_i^2}{(\sum_i a_i N_{0i} / T_i)(\sum_i b_i N_{1i} / T_i)} \right\}^{1/2}$$

(R-216), from which confidence intervals are calculated.

For standardized rates, let  $w_i$  be the user-specified weight within stratum  $i$ . The standardized rate difference (SRD, the `rd` option) and rate ratios (SRR, the default) are defined as in *Stratified incidence-rate data (ir with the by() option)*, where the individual risks are defined  $R_{1i} = a_i / N_{1i}$  and  $R_{0i} = b_i / N_{0i}$ . The standard error of SRD is

$$s_{\text{SRD}} \approx \left[ \frac{1}{(\sum_i w_i)^2} \sum_i w_i^2 \left\{ \frac{a_i (N_{1i} - a_i)}{N_{1i}^3} + \frac{b_i (N_{0i} - b_i)}{N_{0i}^3} \right\} \right]^{1/2}$$

(R-231), from which confidence intervals are calculated. The standard error of  $\ln(\text{SRR})$  is

$$s \ln(\text{SRR}) \approx \left\{ \frac{\sum_i w_i^2 a_i (N_{1i} - a_i) / N_{1i}^3}{(\sum_i w_i R_{1i})^2} + \frac{\sum_i w_i^2 b_i (N_{0i} - b_i) / N_{0i}^3}{(\sum_i w_i R_{0i})^2} \right\}^{1/2}$$

(R-231), from which confidence intervals are calculated.

Internally and externally standardized measures are calculated using  $w_i = N_{1i}$  and  $w_i = N_{0i}$ , respectively, and are obtained with the `istandard` and `estandard` options, respectively.

Directly pooled estimates of the odds ratio are available when you specify both the `pool` and `or` options. The directly pooled estimate is a weighted average of stratum-specific log odds-ratios; each weight,  $w_i$ , is inversely proportional to the variance of the log odds-ratio for stratum  $i$ . The variances of the log odds-ratios are estimated by Woolf's method, described under *Unstratified cumulative incidence data (cs and csi)*. The standard error of the directly pooled log odds-ratio is calculated as  $1/\sqrt{\sum w_i}$ , from which confidence intervals are calculated and then exponentiated (Kahn and Sempos 1989, 113–115).

Direct pooling is also available for risk ratios and risk differences; the variance formulas may be found in *Unstratified cumulative incidence data (cs and csi)*. The directly pooled risk ratio is provided when the `pool` option is specified. The directly pooled risk difference is provided only when you specify the `pool` and `rd` options, and one of the `estandard`, `istandard`, and `standard()` options.

For risk differences, the  $\chi^2$  test of homogeneity is calculated as  $\sum (R_{di} - \widehat{R}_d)^2 / \text{var}(R_{di})$ , where  $R_{di}$  are the stratum-specific risk differences and  $\widehat{R}_d$  is the directly pooled estimate. The number of degrees of freedom is one less than the number of strata (R-222).

For risk and odds ratios, the same calculation is made, except that it is made in the log scale using  $\ln(R_{ri})$  or  $\ln(\psi_i)$  (R-222), and  $\ln(\widehat{R}_d)$  may be the log of either the directly pooled estimate or the Mantel–Haenszel estimate.

### Stratified case–control data (cc with by() option, mhodds, tabodds)

Statistics presented for each stratum are calculated independently according to the formulas in *Unstratified cumulative incidence data (cs and csi)* above. The combined odds ratio,  $\psi_{mh}$ , and the test that  $\psi_{mh} = 1$  ( $\chi^2_{mh}$ ) are calculated as described in *Stratified cumulative incidence data (cs with the by() option)* above.

For standardized weights, let  $w_i$  be the user-specified weight within stratum  $i$ . The standardized odds ratio (the `standard()` option) is calculated as

$$\text{SOR} = \frac{\sum_i w_i a_i / c_i}{\sum_i w_i b_i / d_i}$$

(Greenland 1986, 473). The standard error of  $\ln(\text{SOR})$  is

$$s \ln(\text{SOR}) = \left\{ \frac{\sum_i (w_i a_i / c_i)^2 \left( \frac{1}{a_i} + \frac{1}{b_i} + \frac{1}{c_i} + \frac{1}{d_i} \right)}{\left( \sum_i w_i a_i / c_i \right)^2} \right\}^{1/2}$$

from which confidence intervals are calculated. The internally and externally standardized odds ratios are calculated using  $w_i = c_i$  and  $w_i = d_i$ , respectively.

The directly pooled estimate of the odds ratio (the `pool` option) is calculated as described in *Stratified cumulative incidence data (cs with the by() option)* above.

The directly pooled and Mantel–Haenszel  $\chi^2$  tests of homogeneity are calculated as  $\sum \{ \ln(R_{ri}) - \ln(\widehat{R}_r) \}^2 / \text{var}\{ \ln(R_{ri}) \}$ , where  $R_{ri}$  are the stratum-specific odds ratios and  $\widehat{R}_r$  is the pooled estimate (Mantel–Haenszel or directly pooled). The number of degrees of freedom is one less than the number of strata (R-222).

The Breslow–Day  $\chi^2$  test of homogeneity is available with the `bd` option. Let  $\widehat{\psi}$  be the Mantel–Haenszel estimate of the common odds ratio, and let  $A_i(\widehat{\psi})$  be the fitted count for cell  $a$ ;  $A_i(\widehat{\psi})$  is found by solving the quadratic equation

$$A(M_0 - N_1 + A) = (\widehat{\psi})(M_1 - A)(N_1 - A)$$

and choosing the root that makes all cells in stratum  $i$  positive. Let  $\text{Var}(a_i; \widehat{\psi})$  be the estimated variance of  $a_i$  conditioned on the margins and on an odds ratio of  $\widehat{\psi}$ :

$$\text{Var}(a_i; \widehat{\psi}) = \left\{ \frac{1}{A_i(\widehat{\psi})} + \frac{1}{M_{1i} - A_i(\widehat{\psi})} + \frac{1}{N_{1i} - A_i(\widehat{\psi})} + \frac{1}{M_{0i} - N_{1i} + A_i(\widehat{\psi})} \right\}^{-1}$$

The Breslow–Day  $\chi^2$  statistic is then

$$\sum_i \frac{\{a_i - A_i(\widehat{\psi})\}^2}{\text{Var}(a_i; \widehat{\psi})}$$

The Tarone  $\chi^2$  test of homogeneity (the tarone option) is calculated as

$$\sum_i \frac{\{a_i - A_i(\widehat{\psi})\}^2}{\text{Var}(a_i; \widehat{\psi})} - \frac{\{\sum_i a_i - \sum_i A_i(\widehat{\psi})\}^2}{\sum_i \text{Var}(a_i; \widehat{\psi})}$$

Tarone (1985) provides this correction to the Breslow–Day statistic to ensure that its distribution is asymptotically  $\chi^2$ . Without the correction, the Breslow–Day statistic does not necessarily follow a  $\chi^2$  distribution because it is based on the Mantel–Haenszel estimate,  $\widehat{\psi}$ , which is an inefficient estimator of the common odds ratio.

When the exposure variable has multiple levels, mhodds calculates an approximate estimate of the log odds-ratio for a one-unit increase in exposure as the ratio of the score statistic,  $U$ , to its variance,  $V$  (Clayton and Hills 1993, 103), which are defined below. This is a one-step Newton-Raphson approximation to the maximum likelihood estimate. Within-stratum estimates are combined with Mantel–Haenszel weights.

By default, both tabodds and mhodds produce test statistics and confidence intervals based on score statistics (Clayton and Hills 1993). tabodds reports confidence intervals for the odds of the  $i$ th exposure level, unless the adjust() or or option is specified. The confidence interval for odds $_i$ ,  $i = 1, \dots, k$ , is given by

$$\text{odds}_i \cdot \exp\left(\pm z\sqrt{1/a_i + 1/c_i}\right)$$

The score  $\chi^2$  test of homogeneity of odds is calculated as

$$\chi_{k-1}^2 = \frac{T(T-1)}{M_1 M_0} \sum_{i=1}^k \frac{(a_i - E_i)^2}{N_i}$$

where  $E_i = (M_1 N_i)/T$ .

Let  $l_i$  denote the value of the exposure at the  $i$ th level. The score  $\chi^2$  test for trend of odds is calculated as

$$\chi_1^2 = \frac{U^2}{V}$$

where

$$U = \frac{M_1 M_0}{T} \left( \sum_{i=1}^k \frac{a_i l_i}{M_1} - \sum_{i=1}^k \frac{c_i l_i}{M_0} \right)$$

and

$$V = \frac{M_1 M_0}{T} \left\{ \frac{\sum_{i=1}^k N_i l_i^2 - (\sum_{i=1}^k N_i l_i)^2 / T}{T - 1} \right\}$$

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**John Snow** (1813–1858) was born in York, England. From age 14, he worked as an apprentice and assistant to surgeons in northeast England and Yorkshire. In 1836, Snow moved to London; he was admitted to the Royal College of Surgeons in 1838 and the Royal College of Physicians in 1850. He made outstanding contributions to the adoption of anesthesia and is considered one of the originators of modern epidemiology. Snow died following a stroke in 1858.

Snow calculated dosages for ether and chloroform. He personally administered chloroform to Queen Victoria for the births of her last two children, which helped obstetric anesthesia gain wider acceptance.

Snow was skeptical of the miasma theory that cholera was caused by foul air. His essay *On the Mode of Communication of Cholera* was first published in 1849 and then greatly enlarged in 1855 with the results of his very detailed investigation of the role of water supply in the epidemic of 1854 in the Soho district of London. Snow identified the source of the outbreak as the public water pump on Broad Street (now Broadwick Street), leading the local council to remove the pump handle. It was later discovered that the well had been dug very close to an old cesspit. He also mapped the clustering of cholera cases around the pump and related mortality to water sources, clearly showing higher death rates in areas supplied by the Southwark and Vauxhall Waterworks Company, which was taking water from sewage-polluted sections of the River Thames. Snow is widely regarded as a pioneer in public health, epidemiology, and medical geography.

Janet Elizabeth Lane-Claypon (1877–1967) was a pioneer in the use of cohort and case–control studies. She was born in Lincolnshire county, England, and began her studies at the London School of Medicine for Women in 1898. From 1907 to 1912, she was at the Lister Institute of Preventive Medicine, where she was a colleague of Major Greenwood. By the end of her studies, she had obtained a doctorate in both physiology and medicine.

In 1912, Lane-Claypon published one of the first retrospective cohort studies, examining the weight gain of babies fed cow’s milk versus babies fed breast milk. Using statistical techniques, she determined that babies fed breast milk gained weight faster; she later employed that knowledge to become a public health advocate for breast feeding.

She also conducted one of the first case–control studies, examining risk factors associated with breast cancer. Her study included 500 women without breast cancer and 500 women with breast cancer. To obtain what was at the time a remarkably large sample, she coordinated data collection from nine different hospitals. Carefully controlling for variables including occupation and infant mortality, she determined that factors like age at first pregnancy, age at menopause, and number of children all influence the incidence of breast cancer; these factors are still considered to be among the prime determinants.

In conjunction with the Ministry of Health, in 1926 Lane-Claypon published one of the first studies to contain long-term follow-up results. In that study, she followed patients who had undergone surgery for breast cancer for up to 10 years after the operation. As is still the case today, her study showed that the sooner the cancer was treated, the better the woman’s chance for long-term survival. Notably, her study was also among the first to consider survivorship bias.

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## Also see

- [ST] **stcox** — Cox proportional hazards model
- [R] **bitest** — Binomial probability test
- [R] **ci** — Confidence intervals for means, proportions, and variances
- [R] **clomit** — Conditional (fixed-effects) logistic regression
- [R] **stdize** — Direct and indirect standardization
- [R] **logistic** — Logistic regression, reporting odds ratios
- [R] **poisson** — Poisson regression
- [R] **rerri** — Relative excess risk due to interaction
- [R] **symmetry** — Symmetry and marginal homogeneity tests
- [R] **tabulate twoway** — Two-way table of frequencies
- [META] **meta** — Introduction to meta
- [U] **19 Immediate commands**

## Description

Whenever Stata detects that something is wrong—that what you typed is uninterpretable, that you are trying to do something you should not be trying to do, or that you requested the impossible—Stata responds by typing a message describing the problem, together with a *return code*. For instance,

```
. lsit
command lsit is unrecognized
r(199);
. list myvar
variable myvar not found
r(111);
. test a=b
last estimates not found
r(301);
```

In each case, the message is probably sufficient to guide you to a solution. When we typed `lsit`, Stata responded with “unrecognized command”. We meant to type `list`. When we typed `list myvar`, Stata responded with “variable `myvar` not found”. There is no variable named `myvar` in our data. When we typed `test a=b`, Stata responded with “last estimates not found”. `test` tests hypotheses about previously fit models, and we have not yet fit a model.

The numbers in parentheses in the `r(199)`, `r(111)`, and `r(301)` messages are called the *return codes*. To find out more about these messages, type `search rc #`, where `#` is the number returned in the parentheses.

### ▶ Example 1

```
. search rc 301
[P] error messages Return code 301
last estimates not found;
You typed an estimation command, such as regress, without arguments
or attempted to perform a test or typed predict, but there were no
previous estimation results.
```

◀

Programmers should see [\[P\] error](#) for details on programming error messages.

## Also see

[\[R\] search](#) — Search Stata documentation and other resources

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

## Description

`esize` calculates effect sizes for comparing the difference between the means of a continuous variable for two groups. In the first form, `esize` calculates effect sizes for the difference between the mean of *varname* for two groups defined by *groupvar*. In the second form, `esize` calculates effect sizes for the difference between *varname*<sub>1</sub> and *varname*<sub>2</sub>, assuming unpaired data.

`esizei` is the immediate form of `esize`; see [U] 19 **Immediate commands**. In the first form, `esizei` calculates the effect size for comparing the difference between the means of two groups. In the second form, `esizei` calculates the effect size for an *F* test after an ANOVA.

## Quick start

Cohen's *d* and Hedges's *g* comparing the difference in means of *v* for two independent groups in *catvar*  
`esize twosample v, by(catvar)`

Same as above, but with group data stored in *v1* and *v2*  
`esize unpaired v1==v2`

Same as above, but use 90% confidence level  
`esize unpaired v1==v2, level(90)`

Cohen's *d* and Hedges's *g* for means of *v* for groups in *catvar1* calculated over each level of *catvar2*  
by *catvar2*: `esize twosample v, by(catvar1)`

## Menu

### **esize**

Statistics > Summaries, tables, and tests > Classical tests of hypotheses > Effect size based on mean comparison

### **esizei**

Statistics > Summaries, tables, and tests > Classical tests of hypotheses > Effect-size calculator

## Syntax

Effect sizes for two independent samples using groups

```
esize twosample varname [if] [in], by(groupvar) [options]
```

Effect sizes for two independent samples using variables

```
esize unpaired varname_1 == varname_2 [if] [in], [options]
```

Immediate form of effect sizes for two independent samples

```
esizei #_obs1 #_mean1 #_sd1 #_obs2 #_mean2 #_sd2 [, options]
```

Immediate form of effect sizes for F tests after an ANOVA

```
esizei #_df1 #_df2 #_F [, level(#)]
```

| options           | Description                                                                          |
|-------------------|--------------------------------------------------------------------------------------|
| Main              |                                                                                      |
| <u>cohensd</u>    | report Cohen's <i>d</i> (1988)                                                       |
| <u>hedgesg</u>    | report Hedges's <i>g</i> (1981)                                                      |
| <u>glassdelta</u> | report Glass's $\Delta$ (Smith and Glass 1977) using each group's standard deviation |
| <u>pbcrr</u>      | report the point-biserial correlation coefficient (Pearson 1909)                     |
| <u>all</u>        | report all estimates of effect size                                                  |
| <u>unequal</u>    | use unequal variances                                                                |
| <u>welch</u>      | use Welch's (1947) approximation                                                     |
| <u>level(#)</u>   | set confidence level; default is level(95)                                           |

by is allowed with esize, and collect is allowed with esize and esizei; see [U] 11.1.10 Prefix commands.

## Options

Main

by(*groupvar*) specifies the *groupvar* that defines the two groups that esize will use to estimate the effect sizes. Do not confuse the by() option with the by prefix; you can specify both.

cohensd specifies that Cohen's *d* (1988) be reported.

hedgesg specifies that Hedges's *g* (1981) be reported.

glassdelta specifies that Glass's  $\Delta$  (Smith and Glass 1977) be reported.

pbcrr specifies that the point-biserial correlation coefficient (Pearson 1909) be reported.

all specifies that all estimates of effect size be reported. The default is Cohen's *d* and Hedges's *g*.

unequal specifies that the data not be assumed to have equal variances.

welch specifies that the approximate degrees of freedom for the test be obtained from Welch's formula (1947) rather than from Satterthwaite's approximation formula (1946), which is the default when unequal is specified. Specifying welch implies unequal.

level(#) specifies the confidence level, as a percentage, for confidence intervals. The default is level(95) or as set by set level; see [U] 20.8 Specifying the width of confidence intervals.

## Remarks and examples

Remarks are presented under the following headings:

*Introduction*  
*Estimating effect sizes*  
*Immediate form*  
*Video example*

### Introduction

Whereas  $p$ -values are used to assess the statistical significance of a result, measures of effect size are used to assess the practical significance of a result. Effect sizes can be broadly categorized as “measures of group differences” (the  $d$  family) and “measures of association” (the  $r$  family); see Ellis (2010, table 1.1). The  $d$  family includes estimators such as Cohen’s  $d$ , Hedges’s  $g$ , and Glass’s  $\Delta$ . The  $r$  family includes estimators such as the point-biserial correlation coefficient,  $\eta^2$ ,  $\varepsilon^2$ , and  $\omega^2$  (also see `estat esize` in [R] `regress postestimation`). For an introduction to the concepts and calculation of effect sizes, see Kline (2013) and Thompson (2006). For a more detailed discussion, see Kirk (1996), Ellis (2010), Cumming (2012), Grissom and Kim (2012), and Kelley and Preacher (2012).

Note that there is much variation in the definitions of measures of effect size (Kline 2013). As Ellis (2010, 27) cautions, “However, beware the inconsistent terminology. What is labeled here as  $g$  was labeled by Hedges and Olkin as  $d$  and vice versa. For these authors writing in the early 1980s,  $g$  was the mainstream effect-size index developed by Cohen and refined by Glass (hence  $g$  for Glass). However, since then  $g$  has become synonymous with Hedges’s equation (not Glass’s) and the reason it is called Hedges’s  $g$  and not Hedges’s  $h$  is because it was originally named after Glass—even though it was developed by Larry Hedges. Confused?”

To avoid confusion, `esize` and `esizei` closely follow the notation of Hedges (1981), Smithson (2001), Kline (2013), and Ellis (2010).

## Estimating effect sizes

### ▷ Example 1: Effect size for two independent samples using by()

Suppose we are interested in question 1 from the fictitious depression .dta: “My statistical software makes me feel sad”. We might have conducted a *t* test to test the null hypothesis that there is no difference in response by sex. We could then compute various measures of effect size to describe the magnitude of the effect of sex.

```
. use https://www.stata-press.com/data/r19/depression
(Fictitious depression inventory data based on the Beck Depression Inventory)
. esize twosample qu1, by(sex) all
Effect size based on mean comparison
```

```
Obs per group:
Female = 712
Male = 288
```

| Effect size             | Estimate  | [95% conf. interval] |          |
|-------------------------|-----------|----------------------|----------|
| Cohen's <i>d</i>        | -.0512417 | -.1881184            | .0856607 |
| Hedges's <i>g</i>       | -.0512032 | -.187977             | .0855963 |
| Glass's Delta 1         | -.0517793 | -.1886587            | .0851364 |
| Glass's Delta 2         | -.0499786 | -.1868673            | .086997  |
| Point-biserial <i>r</i> | -.0232208 | -.0849629            | .0387995 |

Cohen's *d*, Hedges's *g*, and both estimates of Glass's  $\Delta$  indicate that the score for females is 0.05 standard deviations lower than the score for males. The point-biserial correlation coefficient indicates that there is a small, negative correlation between the scores for females and males.



### □ Technical note

Glass's  $\Delta$  has traditionally been estimated for experimental studies using the control group standard deviation rather than the pooled standard deviation. Kline (2013) notes that the choice of group becomes arbitrary for data arising from observational studies and recommends the reporting of Glass's  $\Delta$  using each group standard deviation.



▷ Example 2: Effect size for two independent samples by a third variable

If we are interested in the same effect sizes from [example 1](#) stratified by race, we could use the by prefix with the sort option to accomplish this task.

```
. by race, sort: esize twosample q1, by(sex)
```

```
-> race = Hispanic
```

Effect size based on mean comparison

```
Obs per group:
 Female = 88
 Male = 45
```

| Effect size       | Estimate  | [95% conf. interval] |          |
|-------------------|-----------|----------------------|----------|
| Cohen's <i>d</i>  | -.1042883 | -.463503             | .2553235 |
| Hedges's <i>g</i> | -.1036899 | -.4608434            | .2538584 |

```
-> race = Black
```

Effect size based on mean comparison

```
Obs per group:
 Female = 259
 Male = 95
```

| Effect size       | Estimate  | [95% conf. interval] |          |
|-------------------|-----------|----------------------|----------|
| Cohen's <i>d</i>  | -.1720681 | -.4073814            | .063489  |
| Hedges's <i>g</i> | -.1717011 | -.4065127            | .0633536 |

```
-> race = White
```

Effect size based on mean comparison

```
Obs per group:
 Female = 365
 Male = 148
```

| Effect size       | Estimate | [95% conf. interval] |          |
|-------------------|----------|----------------------|----------|
| Cohen's <i>d</i>  | .0479511 | -.1430932            | .2389486 |
| Hedges's <i>g</i> | .0478807 | -.1428831            | .2385977 |



▷ Example 3: Bootstrap confidence intervals for effect sizes

Simulation studies have shown that bootstrap confidence intervals may be preferable to confidence intervals based on the noncentral *t* distribution when the variable of interest does not have a normal distribution (Kelley 2005; Algina, Keselman, and Penfield 2006). Bootstrap confidence intervals can be easily estimated for effect sizes using the bootstrap prefix.

```
. use https://www.stata-press.com/data/r19/depression
(Fictitious depression inventory data based on the Beck Depression Inventory)
. set seed 12345
. bootstrap r(d) r(g), reps(1000) nodots nowarn: esize twosample qu1, by(sex)
Bootstrap results Number of obs = 1,000
 Replications = 1,000

Command: esize twosample qu1, by(sex)
 _bs_1: r(d)
 _bs_2: r(g)
```

|       | Observed<br>coefficient | Bootstrap<br>std. err. | z     | P> z  | Normal-based<br>[95% conf. interval] |          |
|-------|-------------------------|------------------------|-------|-------|--------------------------------------|----------|
| _bs_1 | -.0512417               | .0742692               | -0.69 | 0.490 | -.1968066                            | .0943233 |
| _bs_2 | -.0512032               | .0742134               | -0.69 | 0.490 | -.1966587                            | .0942523 |



▷ Example 4: Effect sizes for two independent samples using variables

Sometimes, the data of interest are stored in two separate variables. We can calculate effect sizes for the two groups by using the unpaired version of esize.

```
. use https://www.stata-press.com/data/r19/fuel
. esize unpaired mpg1==mpg2
Effect size based on mean comparison

Number of obs = 24
```

| Effect size       | Estimate  | [95% conf. interval] |          |
|-------------------|-----------|----------------------|----------|
| Cohen's <i>d</i>  | -.5829654 | -1.394934            | .2416105 |
| Hedges's <i>g</i> | -.5628243 | -1.34674             | .2332631 |



## Immediate form

### ▷ Example 5: Immediate form for effect sizes for two means

Often we do not have access to raw data, but we are given summary statistics in a report or manuscript. To calculate the effect sizes from summary statistics, we can use the immediate command `esizei`. For example, [Kline \(2013\)](#) in table 4.2 shows summary statistics for a hypothetical sample where  $\text{mean}_1 = 13$ ,  $\text{sd}_1 = 2.74$ ,  $\text{mean}_2 = 11$ , and  $\text{sd}_2 = 2.24$ ; there are 30 people in each group. We can estimate the effect sizes from these summary data using `esizei`:

```
. esizei 30 13 2.74 30 11 2.24
Effect size based on mean comparison

 Obs per group:
 Group 1 = 30
 Group 2 = 30
```

| Effect size       | Estimate | [95% conf. interval] |          |
|-------------------|----------|----------------------|----------|
| Cohen's <i>d</i>  | .7991948 | .2695509             | 1.322465 |
| Hedges's <i>g</i> | .7888081 | .2660477             | 1.305277 |



### ▷ Example 6: Immediate form for effect sizes for F tests after an ANOVA

`esizei` can also be used to compute  $\eta^2$ ,  $\varepsilon^2$ , and  $\omega^2$  for *F* tests after an ANOVA. The following example from [Smithson \(2001, 623\)](#) illustrates the use of `esizei` for  $\text{df}_{\text{num}} = 4$ ,  $\text{df}_{\text{den}} = 50$ , and  $F = 4.2317$ :

```
. esizei 4 50 4.2317, level(90)
Effect sizes for linear models
```

| Effect size     | Estimate | [90% conf. interval] |          |
|-----------------|----------|----------------------|----------|
| Eta-squared     | .2529151 | .0521585             | .3603621 |
| Epsilon-squared | .1931483 |                      |          |
| Omega-squared   | .1903049 |                      |          |



## Video example

[Tour of effect sizes](#)

## Stored results

`esize` and `esizei` for comparing two means store the following in `r()`:

Scalars

|                           |                                                                       |
|---------------------------|-----------------------------------------------------------------------|
| <code>r(d)</code>         | Cohen's $d$                                                           |
| <code>r(lb_d)</code>      | lower confidence bound for Cohen's $d$                                |
| <code>r(ub_d)</code>      | upper confidence bound for Cohen's $d$                                |
| <code>r(g)</code>         | Hedges's $g$                                                          |
| <code>r(lb_g)</code>      | lower confidence bound for Hedges's $g$                               |
| <code>r(ub_g)</code>      | upper confidence bound for Hedges's $g$                               |
| <code>r(delta1)</code>    | Glass's $\Delta$ for group 1                                          |
| <code>r(lb_delta1)</code> | lower confidence bound for Glass's $\Delta$ for group 1               |
| <code>r(ub_delta1)</code> | upper confidence bound for Glass's $\Delta$ for group 1               |
| <code>r(delta2)</code>    | Glass's $\Delta$ for group 2                                          |
| <code>r(lb_delta2)</code> | lower confidence bound for Glass's $\Delta$ for group 2               |
| <code>r(ub_delta2)</code> | upper confidence bound for Glass's $\Delta$ for group 2               |
| <code>r(r_pb)</code>      | point-biserial correlation coefficient                                |
| <code>r(lb_r_pb)</code>   | lower confidence bound for the point-biserial correlation coefficient |
| <code>r(ub_r_pb)</code>   | upper confidence bound for the point-biserial correlation coefficient |
| <code>r(N_1)</code>       | sample size $n_1$                                                     |
| <code>r(N_2)</code>       | sample size $n_2$                                                     |
| <code>r(df_t)</code>      | degrees of freedom                                                    |
| <code>r(level)</code>     | confidence level                                                      |

`esizei` for  $F$  tests after ANOVA stores the following in `r()`:

Scalars

|                          |                                     |
|--------------------------|-------------------------------------|
| <code>r(eta2)</code>     | $\eta^2$                            |
| <code>r(lb_eta2)</code>  | lower confidence bound for $\eta^2$ |
| <code>r(ub_eta2)</code>  | upper confidence bound for $\eta^2$ |
| <code>r(epsilon2)</code> | $\epsilon^2$                        |
| <code>r(omega2)</code>   | $\omega^2$                          |
| <code>r(level)</code>    | confidence level                    |

## Methods and formulas

For the  $d$  family, the effect-size parameter of interest is the scaled difference between the means given by

$$\delta = \frac{(\mu_1 - \mu_2)}{\sigma}$$

One of the most popular estimators of effect size is Cohen's  $d$ , given by

$$\text{Cohen's } d = \frac{(\bar{x}_1 - \bar{x}_2)}{s^*}$$

where

$$s^* = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

Hedges (1981) showed that Cohen's  $d$  is biased and proposed the unbiased estimator

$$\text{Hedges's } g = \text{Cohen's } d \times c(m)$$

where  $m = n_1 + n_2 - 2$  and

$$c(m) = \frac{\Gamma(\frac{m}{2})}{\sqrt{\frac{m}{2}} \Gamma(\frac{m-1}{2})}$$

Glass (Smith and Glass 1977) proposed an estimator for  $\delta$  in the context of designed experiments,

$$\text{Glass's } \Delta = \frac{(\bar{x}_{\text{treated}} - \bar{x}_{\text{control}})}{s_{\text{control}}}$$

where  $s_{\text{control}}$  is the standard deviation for the control group.

As noted above, `esize` and `esizei` report two estimates of Glass's  $\Delta$ : one using the standard deviation for group 1 and the other using the standard deviation for group 2:

$$\text{Glass's } \Delta_1 = \frac{(\bar{x}_1 - \bar{x}_2)}{s_1}$$

and

$$\text{Glass's } \Delta_2 = \frac{(\bar{x}_1 - \bar{x}_2)}{s_2}$$

For the  $r$  family, the effect-size parameter of interest is the ratio of the variance attributable to an effect and the total variance:

$$\eta^2 = \frac{\sigma_{\text{effect}}^2}{\sigma_{\text{total}}^2}$$

A popular estimator of  $\eta$  when there are two groups is the point-biserial correlation coefficient,

$$r_{\text{PB}} = \frac{t}{\sqrt{t^2 + \text{df}}}$$

where  $t$  is the  $t$  statistic for the difference between the means of the two groups, and  $df$  is the corresponding degrees of freedom. Satterthwaite's or Welch's adjustment (see [R] `ttest` for details) to the degrees of freedom can be used to calculate  $r_{\text{PB}}$  by specifying the `unequal` or `welch` option, respectively.

When more than two means are being compared, as in the case of an ANOVA with  $p$  groups, a popular estimator of effect size is the correlation ratio denoted  $\eta^2$  (Fisher 1925; Kerlinger and Lee 2000).  $\eta^2$  can be computed directly as the ratio of the  $SS_{\text{effect}}$  and the  $SS_{\text{total}}$  or as a function of the  $F$  statistic with numerator degrees of freedom equal to  $df_{\text{num}}$  and denominator degrees of freedom equal to  $df_{\text{den}}$ .

$$\hat{\eta}^2 = \frac{F}{F + df_{\text{den}}/df_{\text{num}}}$$

Like its equivalent estimator  $R^2$ ,  $\eta^2$  has an upward bias. Less biased estimators of effect size are  $\varepsilon^2$  and  $\omega^2$  (Grissom and Kim 2012).

$$\hat{\varepsilon}^2 = \frac{F - 1}{F + \text{df}_{\text{den}}/\text{df}_{\text{num}}} = \hat{\eta}^2 - \frac{\text{df}_{\text{num}}}{\text{df}_{\text{den}}}(1 - \hat{\eta}^2)$$

$$\hat{\omega}^2 = \frac{F - 1}{F + (\text{df}_{\text{den}} + 1)/\text{df}_{\text{num}}}$$

To calculate  $\hat{\eta}^2$ ,  $\hat{\varepsilon}^2$ , and  $\hat{\omega}^2$  directly after anova or regress, see `estat esize` in [R] [regress postestimation](#).

Cohen's  $d$ , Hedges's  $g$ , and Glass's  $\Delta$  have been shown to have a noncentral  $t$  distribution (Hedges 1981) with noncentrality parameter equal to

$$\lambda = \delta \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$$

Confidence intervals are calculated by finding the noncentrality parameters  $\lambda_{\text{lower}}$  and  $\lambda_{\text{upper}}$  that correspond to

$$\Pr(\text{df}, \delta, \lambda_{\text{lower}}) = 1 - \frac{\alpha}{2}$$

and

$$\Pr(\text{df}, \delta, \lambda_{\text{upper}}) = \frac{\alpha}{2}$$

using the function `npnt(df, t, p)`. The noncentrality parameters are then transformed back to the effect-size scale:

$$\delta_{\text{lower}} = \lambda_{\text{lower}} \sqrt{\frac{n_1 + n_2}{n_1 n_2}}$$

and

$$\delta_{\text{upper}} = \lambda_{\text{upper}} \sqrt{\frac{n_1 + n_2}{n_1 n_2}}$$

(see Venables [1975]; Steiger and Fouladi [1997]; Cumming and Finch [2001]; Smithson [2001]).

Confidence intervals for the point-biserial correlation coefficient are calculated similarly and transformed back to the effect-size scale as

$$r_{\text{lower}} = \frac{\lambda_{\text{lower}}}{\sqrt{\lambda_{\text{lower}}^2 + \text{df}}}$$

and

$$r_{\text{upper}} = \frac{\lambda_{\text{upper}}}{\sqrt{\lambda_{\text{upper}}^2 + \text{df}}}$$

Following Smithson's (2001) notation, the  $F$  statistic is written as

$$F_{df_{\text{num}}, df_{\text{den}}} = f^2(df_{\text{num}}/df_{\text{den}})$$

This equation has a noncentral  $F$  distribution with noncentrality parameter:

$$\lambda = f^2(df_{\text{num}} + df_{\text{den}} + 1)$$

where  $f^2 = \eta^2 / (1 - \eta^2)$ .

Confidence intervals for  $\hat{\eta}^2$  are calculated by finding the noncentrality parameters  $\lambda_{\text{lower}}$  and  $\lambda_{\text{upper}}$  for a noncentral  $F$  distribution that correspond to

$$\Pr(df_{\text{num}}, df_{\text{den}}, F, \lambda_{\text{lower}}) = 1 - \frac{\alpha}{2}$$

and

$$\Pr(df_{\text{num}}, df_{\text{den}}, F, \lambda_{\text{upper}}) = \frac{\alpha}{2}$$

using the function `npnF(df1, df2, f, p)`. The noncentrality parameters are transformed back to the  $\hat{\eta}^2$  scale as

$$\hat{\eta}_{\text{lower}}^2 = \frac{\lambda_{\text{lower}}}{\lambda_{\text{lower}} + df_{\text{num}} + df_{\text{den}} + 1}$$

and

$$\hat{\eta}_{\text{upper}}^2 = \frac{\lambda_{\text{upper}}}{\lambda_{\text{upper}} + df_{\text{num}} + df_{\text{den}} + 1}$$

While confidence intervals for  $\varepsilon^2$  can be constructed using the same transformation that links it with  $\hat{\eta}^2$ , there are several arguments for not using them in practice. See Smithson (2003, 54) for further details.

Fred Nichols Kerlinger (1910–1991) was born in New York City. He studied music at New York University and graduated magna cum laude with a degree in education and philosophy. After graduation, he joined the US Army and served as a counterintelligence officer in Japan in 1946. Kerlinger earned an MA and a PhD in educational psychology from the University of Michigan and held faculty appointments at several universities, including New York University. He was president of the American Educational Research Association and is best known for his popular and influential book *Foundations of Behavioral Research* (1964), which introduced Fisher's (1925)  $\eta^2$  statistic to behavioral researchers.

William Lee Hays (1926–1995) was born in Clarksville, Texas. He studied mathematics and psychology at Paris Junior College in Paris, Texas, and at East Texas State College. He earned BS and MS degrees from North Texas State University. Upon completion of his PhD in psychology at the University of Michigan, he joined the faculty, where he eventually became associate vice president for academic affairs. In 1977, Hays accepted an appointment as vice president for academic affairs at the University of Texas at Austin, where he remained until his death in 1995. Hays is best known for his book *Statistics for Psychologists* (1963), which introduced the  $\omega^2$  statistic (and is actually denoted here by  $\varepsilon^2$ ).

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## Also see

- [R] **bitest** — Binomial probability test
- [R] **ci** — Confidence intervals for means, proportions, and variances
- [R] **mean** — Estimate means
- [R] **oneway** — One-way analysis of variance
- [R] **prtest** — Tests of proportions
- [R] **sdtest** — Variance-comparison tests
- [R] **ttest** —  $t$  tests (mean-comparison tests)

## Description

`estat` displays scalar- and matrix-valued statistics after estimation; it complements `predict`, which calculates variables after estimation. Exactly what statistics `estat` can calculate depends on the previous estimation command.

Three sets of statistics are so commonly used that they are available after all estimation commands that store the model log likelihood. `estat ic` displays Akaike's and Schwarz's Bayesian information criteria. `estat summarize` summarizes the variables used by the command and automatically restricts the sample to `e(sample)`; it also summarizes the weight variable and cluster structure, if specified. `estat vce` displays the covariance or correlation matrix of the parameter estimates of the previous model.

## Syntax

| Command                                                             | Reference                           |
|---------------------------------------------------------------------|-------------------------------------|
| <i>Display information criteria</i>                                 |                                     |
| <code>estat ic [ , n(#) ]</code>                                    | [R] <a href="#">estat ic</a>        |
| <i>Summarize estimation sample</i>                                  |                                     |
| <code>estat summarize [eqlist] [ , estat_summ_options ]</code>      | [R] <a href="#">estat summarize</a> |
| <i>Display covariance matrix estimates</i>                          |                                     |
| <code>estat vce [ , estat_vce_options ]</code>                      | [R] <a href="#">estat vce</a>       |
| <i>Command-specific</i>                                             |                                     |
| <code>estat subcommand<sub>1</sub> [ , options<sub>1</sub> ]</code> |                                     |

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu for estat](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

## Description

`estat classification` reports various summary statistics, including the classification table.

`estat classification` requires that the current estimation results be from `logistic`, `logit`, `probit`, or `ivprobit`; see [\[R\] logistic](#), [\[R\] logit](#), [\[R\] probit](#), or [\[R\] ivprobit](#).

## Quick start

Display classification table and related statistics for current estimation results

```
estat classification
```

Change probability threshold for assignment to positive outcome to 75%

```
estat classification, cutoff(.75)
```

Classification for observations with `catvar = 2`

```
estat classification if catvar==2
```

## Menu for estat

Statistics > Postestimation

## Syntax

```
estat classification [if] [in] [weight] [, options]
```

*options*

Description

---

|                        |                                                                 |
|------------------------|-----------------------------------------------------------------|
| Main                   |                                                                 |
| <code>all</code>       | display summary statistics for all observations in the data     |
| <code>cutoff(#)</code> | positive outcome threshold; default is <code>cutoff(0.5)</code> |

---

`estat classification` is not appropriate with `svy` estimation results.

`collect` is allowed; see [U] 11.1.10 Prefix commands.

`fweights` are allowed; see [U] 11.1.6 weight.

## Options

Main

`all` requests that the statistic be computed for all observations in the data, ignoring any `if` or `in` restrictions specified by the estimation command.

`cutoff(#)` specifies the value for determining whether an observation has a predicted positive outcome. An observation is classified as positive if its predicted probability is  $\geq \#$ . The default is 0.5.

## Remarks and examples

`estat classification` presents the classification statistics and classification table after `logistic`, `logit`, `probit`, or `ivprobit`.

Statistics are produced either for the estimation sample (the default) or for any set of observations. When weights, `if`, or `in` is used with the estimation command, it is not necessary to repeat the qualifier when you want statistics computed for the estimation sample. Specify `if`, `in`, or the `all` option only when you want statistics computed for a set of observations other than the estimation sample. Specify weights only when you want to use a different set of weights.

► Example 1

We illustrate `estat classification` after `logistic`; see [R] [logistic](#).

```
. use https://www.stata-press.com/data/r19/lbw
(Hosmer & Lemeshow data)
. logistic low age lwt i.race smoke ptl ht ui
(output omitted)
. estat classification
Logistic model for low
```

| Classified | True |     | Total |
|------------|------|-----|-------|
|            | D    | ~D  |       |
| +          | 21   | 12  | 33    |
| -          | 38   | 118 | 156   |
| Total      | 59   | 130 | 189   |

Classified + if predicted Pr(D) >= .5  
 True D defined as low != 0

|                               |            |        |
|-------------------------------|------------|--------|
| Sensitivity                   | Pr( +  D)  | 35.59% |
| Specificity                   | Pr( -  ~D) | 90.77% |
| Positive predictive value     | Pr( D  +)  | 63.64% |
| Negative predictive value     | Pr( ~D  -) | 75.64% |
| False + rate for true ~D      | Pr( +  ~D) | 9.23%  |
| False - rate for true D       | Pr( -  D)  | 64.41% |
| False + rate for classified + | Pr( ~D  +) | 36.36% |
| False - rate for classified - | Pr( D  -)  | 24.36% |
| Correctly classified          |            | 73.54% |

The overall rate of correct classification is estimated to be 73.54, with 90.77% of the normal weight group correctly classified (specificity) and only 35.59% of the low weight group correctly classified (sensitivity). Classification is sensitive to the relative sizes of each component group, and always favors classification into the larger group. This phenomenon is evident here.

By default, `estat classification` uses a cutoff of 0.5, although you can vary this with the `cutoff()` option. You can use the `lsens` command to review the potential cutoffs; see [R] [lsens](#).

## Stored results

`estat classification` stores the following in `r()`:

### Scalars

|                        |                                               |
|------------------------|-----------------------------------------------|
| <code>r(P_corr)</code> | percent correctly classified                  |
| <code>r(P_p1)</code>   | sensitivity                                   |
| <code>r(P_n0)</code>   | specificity                                   |
| <code>r(P_p0)</code>   | false-positive rate given true negative       |
| <code>r(P_n1)</code>   | false-negative rate given true positive       |
| <code>r(P_1p)</code>   | positive predictive value                     |
| <code>r(P_0n)</code>   | negative predictive value                     |
| <code>r(P_0p)</code>   | false-positive rate given classified positive |
| <code>r(P_1n)</code>   | false-negative rate given classified negative |

### Matrices

|                        |                      |
|------------------------|----------------------|
| <code>r(ctable)</code> | classification table |
|------------------------|----------------------|

## Methods and formulas

Let  $j$  index observations. Define  $c$  as the `cutoff()` specified by the user or, if not specified, as 0.5. Let  $p_j$  be the predicted probability of a positive outcome and  $y_j$  be the actual outcome, which we will treat as 0 or 1, although Stata treats it as 0 and non-0, excluding missing observations.

A prediction is classified as *positive* if  $p_j \geq c$  and otherwise is classified as *negative*. The classification is *correct* if it is *positive* and  $y_j = 1$  or if it is *negative* and  $y_j = 0$ .

*Sensitivity* is the fraction of  $y_j = 1$  observations that are correctly classified. *Specificity* is the percentage of  $y_j = 0$  observations that are correctly classified.

## References

- Hosmer, D. W., Jr., S. A. Lemeshow, and R. X. Sturdivant. 2013. *Applied Logistic Regression*. 3rd ed. Hoboken, NJ: Wiley.
- Kohler, U., and F. Kreuter. 2012. *Data Analysis Using Stata*. 3rd ed. College Station, TX: Stata Press.

## Also see

- [R] **logistic** — Logistic regression, reporting odds ratios
- [R] **logit** — Logistic regression, reporting coefficients
- [R] **probit** — Probit regression
- [R] **ivprobit** — Probit model with continuous endogenous covariates
- [R] **lroc** — Compute area under ROC curve and graph the curve
- [R] **lsens** — Graph sensitivity and specificity versus probability cutoff
- [R] **estat gof** — Pearson or Hosmer–Lemeshow goodness-of-fit test
- [R] **roc** — Receiver operating characteristic (ROC) analysis
- [U] **20 Estimation and postestimation commands**

|                      |                |                      |            |          |
|----------------------|----------------|----------------------|------------|----------|
| Description          | Quick start    | Menu for estat       | Syntax     | Options  |
| Remarks and examples | Stored results | Methods and formulas | References | Also see |

## Description

`estat gof` reports the Pearson goodness-of-fit test or the Hosmer–Lemeshow goodness-of-fit test.

`estat gof` requires that the current estimation results be from `logistic`, `logit`, or `probit`; see [\[R\] logistic](#), [\[R\] logit](#), or [\[R\] probit](#). For `estat gof` after `poisson`, see [\[R\] poisson postestimation](#). For `estat gof` after `sem`, see [\[SEM\] estat gof](#).

## Quick start

Pearson goodness-of-fit test for current estimation results

```
estat gof
```

Same as above, but apply to all observations in dataset instead of just those in `e(sample)`

```
estat gof, all
```

Hosmer–Lemeshow goodness-of-fit test

```
estat gof, group(10)
```

Same as above, and display table of groups used for the test

```
estat gof, group(10) table
```

## Menu for estat

Statistics > Postestimation

## Syntax

```
estat gof [if] [in] [weight] [, options]
```

| <i>options</i>         | Description                                                     |
|------------------------|-----------------------------------------------------------------|
| Main                   |                                                                 |
| <code>group(#)</code>  | perform Hosmer–Lemeshow goodness-of-fit test using # quantiles  |
| <code>all</code>       | execute test for all observations in the data                   |
| <code>outsample</code> | adjust degrees of freedom for samples outside estimation sample |
| <code>table</code>     | display table of groups used for test                           |

`estat gof` is not appropriate with `svy` estimation results.

`collect` is allowed; see [U] 11.1.10 Prefix commands.

`fweights` are allowed; see [U] 11.1.6 weight.

## Options

Main

`group(#)` specifies the number of quantiles to be used to group the data for the Hosmer–Lemeshow goodness-of-fit test. `group(10)` is typically specified. If this option is not given, the Pearson goodness-of-fit test is computed using the covariate patterns in the data as groups.

`all` requests that the statistic be computed for all observations in the data, ignoring any `if` or `in` restrictions specified by the estimation command.

`outsample` adjusts the degrees of freedom for the Pearson and Hosmer–Lemeshow goodness-of-fit tests for samples outside the estimation sample. See *Samples other than the estimation sample* later in this entry.

`table` displays a table of the groups used for the Hosmer–Lemeshow or Pearson goodness-of-fit test with predicted probabilities, observed and expected counts for both outcomes, and totals for each group.

## Remarks and examples

Remarks are presented under the following headings:

[Introduction](#)

[Samples other than the estimation sample](#)

### Introduction

`estat gof` computes goodness-of-fit tests: either the Pearson  $\chi^2$  test or the Hosmer–Lemeshow test.

By default, `estat gof` computes statistics for the estimation sample by using the last model fit by `logistic`, `logit`, or `probit`. However, samples other than the estimation sample can be specified; see *Samples other than the estimation sample* later in this entry.

## ▷ Example 1

estat gof, typed without options, presents the Pearson  $\chi^2$  goodness-of-fit test for the fitted model. The Pearson  $\chi^2$  goodness-of-fit test is a test of the observed against expected number of responses using cells defined by the covariate patterns; see *predict with the number option* in [R] [logistic postestimation](#) for the definition of covariate patterns.

```
. use https://www.stata-press.com/data/r19/lbw
(Hosmer & Lemeshow data)
. logistic low age lwt i.race smoke ptl ht ui
(output omitted)
. estat gof

Goodness-of-fit test after logistic model
Variable: low

 Number of observations = 189
 Number of covariate patterns = 182
 Pearson chi2(173) = 179.24
 Prob > chi2 = 0.3567
```

Our model fits reasonably well. However, the number of covariate patterns is close to the number of observations, making the applicability of the Pearson  $\chi^2$  test questionable but not necessarily inappropriate. [Hosmer, Lemeshow, and Sturdivant \(2013, 157–160\)](#) suggest regrouping the data by ordering on the predicted probabilities and then forming, say, 10 nearly equal-sized groups. estat gof with the group() option does this:

```
. estat gof, group(10)
note: obs collapsed on 10 quantiles of estimated probabilities.

Goodness-of-fit test after logistic model
Variable: low

 Number of observations = 189
 Number of groups = 10
Hosmer–Lemeshow chi2(8) = 9.65
 Prob > chi2 = 0.2904
```

Again we cannot reject our model. If we specify the table option, estat gof displays the groups along with the expected and observed number of positive responses (low-birthweight babies):

```
. estat gof, group(10) table
note: obs collapsed on 10 quantiles of estimated probabilities.
```

```
Goodness-of-fit test after logistic model
Variable: low
```

Table collapsed on quantiles of estimated probabilities

| Group | Prob   | Obs_1 | Exp_1 | Obs_0 | Exp_0 | Total |
|-------|--------|-------|-------|-------|-------|-------|
| 1     | 0.0827 | 0     | 1.2   | 19    | 17.8  | 19    |
| 2     | 0.1276 | 2     | 2.0   | 17    | 17.0  | 19    |
| 3     | 0.2015 | 6     | 3.2   | 13    | 15.8  | 19    |
| 4     | 0.2432 | 1     | 4.3   | 18    | 14.7  | 19    |
| 5     | 0.2792 | 7     | 4.9   | 12    | 14.1  | 19    |
| 6     | 0.3138 | 7     | 5.6   | 12    | 13.4  | 19    |
| 7     | 0.3872 | 6     | 6.5   | 13    | 12.5  | 19    |
| 8     | 0.4828 | 7     | 8.2   | 12    | 10.8  | 19    |
| 9     | 0.5941 | 10    | 10.3  | 9     | 8.7   | 19    |
| 10    | 0.8391 | 13    | 12.8  | 5     | 5.2   | 18    |

```
Number of observations = 189
Number of groups = 10
Hosmer-Lemeshow chi2(8) = 9.65
Prob > chi2 = 0.2904
```

In this table, the column Prob shows the upper boundaries of predicted probabilities for these 10 groups, which are the 10th, 20th, . . . , and 100th percentiles in this case.

◀

## □ Technical note

estat gof with the group() option puts all observations with the same predicted probabilities into the same group. If, as in the previous example, we request 10 groups, the groups that estat gof makes are  $[p_0, p_{10}]$ ,  $(p_{10}, p_{20}]$ ,  $(p_{20}, p_{30}]$ , . . . ,  $(p_{90}, p_{100}]$ , where  $p_k$  is the  $k$ th percentile of the predicted probabilities, with  $p_0$  the minimum and  $p_{100}$  the maximum.

If there are many ties at the quantile boundaries, as will often happen if all independent variables are categorical and there are only a few of them, the sizes of the groups will be uneven. If the totals in some of the groups are small, the  $\chi^2$  statistic for the Hosmer–Lemeshow test may be unreliable. In this case, fewer groups should be specified, or the Pearson goodness-of-fit test may be a better choice.

□

## ▷ Example 2

The table option can be used without the group() option. We would not want to specify this for our current model because there were 182 covariate patterns in the data, caused by including the two continuous variables, age and lwt, in the model. As an aside, we fit a simpler model and specify table with estat gof:

. logistic low i.race smoke ui

Logistic regression

Number of obs = 189  
 LR chi2(4) = 18.80  
 Prob > chi2 = 0.0009  
 Pseudo R2 = 0.0801

Log likelihood = -107.93404

|       | low | Odds ratio | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-------|-----|------------|-----------|-------|-------|----------------------|----------|
| race  |     |            |           |       |       |                      |          |
| Black |     | 3.052746   | 1.498087  | 2.27  | 0.023 | 1.166747             | 7.987382 |
| Other |     | 2.922593   | 1.189229  | 2.64  | 0.008 | 1.316457             | 6.488285 |
| smoke |     | 2.945742   | 1.101838  | 2.89  | 0.004 | 1.415167             | 6.131715 |
| ui    |     | 2.419131   | 1.047359  | 2.04  | 0.041 | 1.035459             | 5.651788 |
| _cons |     | .1402209   | .0512295  | -5.38 | 0.000 | .0685216             | .2869447 |

Note: **\_cons** estimates baseline odds.

. estat gof, table

Goodness-of-fit test after logistic model

Variable: low

| Group | Prob   | Obs_1 | Exp_1 | Obs_0 | Exp_0 | Total |
|-------|--------|-------|-------|-------|-------|-------|
| 1     | 0.1230 | 3     | 4.9   | 37    | 35.1  | 40    |
| 2     | 0.2533 | 1     | 1.0   | 3     | 3.0   | 4     |
| 3     | 0.2907 | 16    | 13.7  | 31    | 33.3  | 47    |
| 4     | 0.2923 | 15    | 12.6  | 28    | 30.4  | 43    |
| 5     | 0.2997 | 3     | 3.9   | 10    | 9.1   | 13    |
| 6     | 0.4978 | 4     | 4.0   | 4     | 4.0   | 8     |
| 7     | 0.4998 | 4     | 4.5   | 5     | 4.5   | 9     |
| 8     | 0.5087 | 2     | 1.5   | 1     | 1.5   | 3     |
| 9     | 0.5469 | 2     | 4.4   | 6     | 3.6   | 8     |
| 10    | 0.5577 | 6     | 5.6   | 4     | 4.4   | 10    |
| 11    | 0.7449 | 3     | 3.0   | 1     | 1.0   | 4     |

| Group | Prob   | race  | smoke     | ui |
|-------|--------|-------|-----------|----|
| 1     | 0.1230 | White | Nonsmoker | 0  |
| 2     | 0.2533 | White | Nonsmoker | 1  |
| 3     | 0.2907 | Other | Nonsmoker | 0  |
| 4     | 0.2923 | White | Smoker    | 0  |
| 5     | 0.2997 | Black | Nonsmoker | 0  |
| 6     | 0.4978 | Other | Nonsmoker | 1  |
| 7     | 0.4998 | White | Smoker    | 1  |
| 8     | 0.5087 | Black | Nonsmoker | 1  |
| 9     | 0.5469 | Other | Smoker    | 0  |
| 10    | 0.5577 | Black | Smoker    | 0  |
| 11    | 0.7449 | Other | Smoker    | 1  |

Number of observations = 189  
 Number of covariate patterns = 11  
 Pearson chi2(6) = 5.71  
 Prob > chi2 = 0.4569

## □ Technical note

logistic, logit, or probit and estat gof keep track of the estimation sample. If you type, for instance, `logistic ... if x==1`, then when you type `estat gof`, the statistics will be calculated on the `x==1` subsample of the data automatically.

You should specify `if` or `in` with `estat gof` only when you wish to calculate statistics for a set of observations other than the estimation sample. See *Samples other than the estimation sample* later in this entry.

If the logistic model was fit with `fweights`, `estat gof` properly accounts for the weights in its calculations. (`estat gof` allows only `fweights`.) You do not have to specify the weights when you run `estat gof`. Weights should be specified with `estat gof` only when you wish to use a different set of weights.

□

## Samples other than the estimation sample

`estat gof` can be used with samples other than the estimation sample. By default, `estat gof` remembers the estimation sample used with the last `logistic`, `logit`, or `probit` command. To override this, simply use an `if` or `in` restriction to select another set of observations, or specify the `all` option to force the command to use all the observations in the dataset.

If you use `estat gof` with a sample that is completely different from the estimation sample (that is, no overlap), you should also specify the `outsample` option so that the  $\chi^2$  statistic properly adjusts the degrees of freedom upward. For an overlapping sample, the conservative thing to do is to leave the degrees of freedom the same as they are for the estimation sample.

## ▷ Example 3

We want to develop a model for predicting low-birthweight babies. One approach would be to divide our data into two groups, a developmental sample and a validation sample. See *Lemeshow and Gall (1994)* and *Tilford, Roberson, and Fiser (1995)* for more information on developing prediction models and severity-scoring systems.

We will do this with the low-birthweight data that we considered previously. First, we randomly divide the data into two samples.

```
. use https://www.stata-press.com/data/r19/lbw, clear
(Hosmer & Lemeshow data)
. set seed 101
. generate r = runiform()
. sort r
. generate group = 1 if _n <= _N/2
(95 missing values generated)
. replace group = 2 if group==.
(95 real changes made)
```

Then, we fit a model using the first sample (`group = 1`), which is our developmental sample.

```
. logistic low age lwt i.race smoke ptl ht ui if group==1
Logistic regression Number of obs = 94
 LR chi2(8) = 28.03
 Prob > chi2 = 0.0005
Log likelihood = -42.351112 Pseudo R2 = 0.2487
```

| low   | Odds ratio | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-------|------------|-----------|-------|-------|----------------------|----------|
| age   | .922865    | .0555349  | -1.33 | 0.182 | .8201924             | 1.03839  |
| lwt   | .9825782   | .0114438  | -1.51 | 0.131 | .9604029             | 1.005265 |
| race  |            |           |       |       |                      |          |
| Black | 5.975476   | 4.936135  | 2.16  | 0.030 | 1.183652             | 30.16621 |
| Other | 3.364479   | 2.760784  | 1.48  | 0.139 | .6736724             | 16.803   |
| smoke | 3.442716   | 2.53779   | 1.68  | 0.094 | .8117831             | 14.60032 |
| ptl   | 3.467274   | 2.337648  | 1.84  | 0.065 | .9249222             | 12.99784 |
| ht    | 5.928512   | 6.047106  | 1.74  | 0.081 | .8030021             | 43.76982 |
| ui    | 4.045883   | 2.947396  | 1.92  | 0.055 | .9703295             | 16.8697  |
| _cons | 3.120871   | 5.977489  | 0.59  | 0.552 | .0731049             | 133.231  |

Note: `_cons` estimates baseline odds.

To test calibration in the developmental sample, we calculate the Hosmer–Lemeshow goodness-of-fit test by using `estat gof`.

```
. estat gof, group(10)
note: obs collapsed on 10 quantiles of estimated probabilities.
Goodness-of-fit test after logistic model
Variable: low
Number of observations = 94
Number of groups = 10
Hosmer-Lemeshow chi2(8) = 5.64
Prob > chi2 = 0.6871
```

We did not specify an `if` statement with `estat gof` because we wanted to use the estimation sample. Because the test is not significant, we are satisfied with the fit of our model.

Running `lroc` (see [\[R\] lroc](#)) gives a measure of the discrimination:

```
. lroc, nograph
Logistic model for low
Number of observations = 94
Area under ROC curve = 0.8145
```

Now, we test the calibration of our model by performing a goodness-of-fit test on the validation sample. We specify the `outsample` option so that the number of degrees of freedom is 10 rather than 8.

```
. estat gof if group==2, group(10) table outsample
note: obs collapsed on 10 quantiles of estimated probabilities.
```

Goodness-of-fit test after logistic model  
Variable: low

Table collapsed on quantiles of estimated probabilities

| Group | Prob   | Obs_1 | Exp_1 | Obs_0 | Exp_0 | Total |
|-------|--------|-------|-------|-------|-------|-------|
| 1     | 0.0276 | 0     | 0.2   | 10    | 9.8   | 10    |
| 2     | 0.0496 | 2     | 0.4   | 7     | 8.6   | 9     |
| 3     | 0.0875 | 1     | 0.7   | 9     | 9.3   | 10    |
| 4     | 0.1536 | 4     | 1.1   | 5     | 7.9   | 9     |
| 5     | 0.2283 | 4     | 2.0   | 6     | 8.0   | 10    |
| 6     | 0.2842 | 4     | 2.2   | 5     | 6.8   | 9     |
| 7     | 0.4190 | 3     | 3.6   | 7     | 6.4   | 10    |
| 8     | 0.5248 | 5     | 4.3   | 4     | 4.7   | 9     |
| 9     | 0.6413 | 5     | 5.8   | 5     | 4.2   | 10    |
| 10    | 0.9787 | 4     | 7.3   | 5     | 1.7   | 9     |

```
Number of observations = 95
Number of groups = 10
Hosmer-Lemeshow chi2(10) = 29.30
Prob > chi2 = 0.0011
```

We must acknowledge that our model does not fit well on the validation sample. The model’s discrimination in the validation sample is appreciably lower, as well.

```
. lroc if group==2, nograph
Logistic model for low
Number of observations = 95
Area under ROC curve = 0.6835
```



## Stored results

`estat gof` stores the following in `r()`:

Scalars

- `r(N)`            number of observations
- `r(m)`            number of covariate patterns or groups
- `r(df)`            degrees of freedom
- `r(chi2)`          $\chi^2$
- `r(p)`             $p$ -value for  $\chi^2$  test

## Methods and formulas

Let  $M$  be the total number of covariate patterns among the  $N$  observations. View the data as collapsed on covariate patterns  $j = 1, 2, \dots, M$ , and define  $m_j$  as the total number of observations having covariate pattern  $j$  and  $y_j$  as the total number of positive responses among observations with covariate pattern  $j$ . Define  $p_j$  as the predicted probability of a positive outcome in covariate pattern  $j$ .

The Pearson  $\chi^2$  goodness-of-fit statistic is

$$\chi^2 = \sum_{j=1}^M \frac{(y_j - m_j p_j)^2}{m_j p_j (1 - p_j)}$$

This  $\chi^2$  statistic has approximately  $M - k$  degrees of freedom for the estimation sample, where  $k$  is the number of independent variables, including the constant. For a sample outside the estimation sample, the statistic has  $M$  degrees of freedom.

The Hosmer–Lemeshow goodness-of-fit  $\chi^2$  (Hosmer and Lemeshow 1980; Lemeshow and Hosmer 1982; Hosmer, Lemeshow, and Klar 1988) is calculated similarly, except that rather than using the  $M$  covariate patterns as the group definition, the quantiles of the predicted probabilities are used to form groups. Let  $G = \#$  be the number of quantiles requested with group ( $\#$ ). The smallest index  $1 \leq q(i) \leq M$ , such that

$$W_{q(i)} = \sum_{j=1}^{q(i)} m_j \geq \frac{N}{G}$$

gives  $p_{q(i)}$  as the upper boundary of the  $i$ th quantile for  $i = 1, 2, \dots, G$ . Let  $q(0) = 1$  denote the first index.

The groups are then

$$[p_{q(0)}, p_{q(1)}], (p_{q(1)}, p_{q(2)}], \dots, (p_{q(G-1)}, p_{q(G)}]$$

If the `table` option is given, the upper boundaries  $p_{q(1)}, \dots, p_{q(G)}$  of the groups appear next to the group number on the output.

The resulting  $\chi^2$  statistic has approximately  $G - 2$  degrees of freedom for the estimation sample. For a sample outside the estimation sample, the statistic has  $G$  degrees of freedom.

## References

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## Also see

- [R] **logistic** — Logistic regression, reporting odds ratios
- [R] **logit** — Logistic regression, reporting coefficients
- [R] **probit** — Probit regression
- [R] **estat classification** — Classification statistics and table
- [R] **iroc** — Compute area under ROC curve and graph the curve
- [R] **lsens** — Graph sensitivity and specificity versus probability cutoff
- [U] **20 Estimation and postestimation commands**

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu for estat](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

## Description

`estat ic` computes Akaike's (AIC), consistent Akaike's (CAIC), corrected Akaike's (AICc), and Schwarz's Bayesian (BIC) information criteria.

## Quick start

Display AIC and BIC

```
estat ic
```

Display CAIC and BIC

```
estat ic, aicconsistent
```

Display AICc and BIC

```
estat ic, aiccorrected
```

Display AIC, BIC, AICc, and CAIC

```
estat ic, all
```

Specify  $N$  to be used in calculating BIC as 500

```
estat ic, n(500)
```

Specify  $N$  and degrees of freedom to be used in calculating all information criteria as 500 and 10, respectively

```
estat ic, n(500) df(10) all
```

## Menu for estat

Statistics > Postestimation

## Syntax

```
estat ic [, options]
```

| <i>options</i>             | Description                                                                          |
|----------------------------|--------------------------------------------------------------------------------------|
| <code>aiccorrected</code>  | report AICc instead of AIC                                                           |
| <code>aicconsistent</code> | report CAIC instead of AIC                                                           |
| <code>all</code>           | report all four information criteria: AIC, BIC, AICc, and CAIC                       |
| <code>n(#)</code>          | specify $N$ to be used in calculating BIC, AICc, and CAIC;<br>see [R] <b>IC note</b> |
| <code>df(#)</code>         | specify degrees of freedom $k$ to be used in calculating AIC, BIC, AICc, and CAIC    |

`collect` is allowed; see [U] **11.1.10 Prefix commands**.

## Options

`aiccorrected` specifies that AICc be computed instead of AIC. This information criterion is a second-order approximation and is recommended for small sample sizes.

Only one of `aiccorrected`, `aicconsistent`, or `all` is allowed.

`aicconsistent` specifies that CAIC be computed instead of AIC. This information criterion is a consistent version of AIC; that is, the probability of selecting the “true model” approaches 1 as sample size increases.

Only one of `aicconsistent`, `aiccorrected`, or `all` is allowed.

`all` produces a table showing all four information criteria: AIC, BIC, AICc, and CAIC.

Only one of `all`, `aiccorrected`, or `aicconsistent` is allowed.

`n(#)` specifies  $N$  to be used in calculating BIC, AICc, and CAIC; see [R] **IC note**.

`df(#)` specifies degrees of freedom  $k$  to be used in calculating AIC, BIC, AICc, and CAIC. By default,  $k$  is the number of estimated parameters.

## Remarks and examples

`estat ic` calculates four information criteria used to compare models fit to the same dataset. Unlike likelihood-ratio, Wald, and similar testing procedures, the models need not be nested to compare the information criteria. The information criteria are constructed as a function of the log likelihood  $\ln L$ , the number of estimated parameters (degrees of freedom)  $k$ , and, in some cases, the number of observations  $N$ . Because they are based on the log-likelihood function, information criteria are available only after commands that report the log likelihood.

The use of information criteria is subjective, and no formal inference can be drawn from the reported values. In a typical approach, a set of potential models is selected, and a superior model is selected from the values of information criteria. A superior model is the model with the lowest value of information criterion. For example, given two models, the model with the lowest AIC fits the data better than the model with the larger AIC. For details, see *Methods and formulas*.

## ▷ Example 1

In [R] **mlogit**, we fit a model explaining the type of insurance a person has on the basis of age, gender, race, and site of study. Here we refit the model with and without the site dummies and compare the models.

```
. use https://www.stata-press.com/data/r19/sysdsn1
(Health insurance data)
. mlogit insure age male nonwhite
(output omitted)
. estat ic
Akaike's information criterion and Bayesian information criterion
```

| Model | N   | ll(null)  | ll(model) | df | AIC      | BIC     |
|-------|-----|-----------|-----------|----|----------|---------|
| .     | 615 | -555.8545 | -545.5833 | 8  | 1107.167 | 1142.54 |

Note: BIC uses N = number of observations. See [R] **IC note**.

```
. mlogit insure age male nonwhite i.site
(output omitted)
. estat ic
Akaike's information criterion and Bayesian information criterion
```

| Model | N   | ll(null)  | ll(model) | df | AIC      | BIC      |
|-------|-----|-----------|-----------|----|----------|----------|
| .     | 615 | -555.8545 | -534.3616 | 12 | 1092.723 | 1145.783 |

Note: BIC uses N = number of observations. See [R] **IC note**.

The AIC indicates that the model including the site dummies fits the data better, whereas BIC indicates the opposite. As is often the case, different model-selection criteria have led to conflicting conclusions.



## ▷ Example 2

In [example 1](#), we compared AIC and BIC. Here we focus on comparing AIC and AICc for small sample size. For simplicity, we are using the same health insurance dataset but running `mlogit` with the `age < 30` condition to reduce the sample size.

```
. mlogit insure age male nonwhite if age < 30
 (output omitted)
. estat ic, all
Information criteria
```

| Model | N  | ll(null)  | ll(model) | df |
|-------|----|-----------|-----------|----|
| .     | 87 | -76.93025 | -70.36684 | 8  |

Note: BIC, AICc, and CAIC use N = number of observations.  
See [\[R\] IC note](#).

| Model | AIC      | BIC      | AICc     | CAIC     |
|-------|----------|----------|----------|----------|
| .     | 156.7337 | 176.4609 | 158.5798 | 184.4609 |

Legend: AIC is Akaike's information criterion.  
BIC is Bayesian information criterion.  
AICc is corrected Akaike's information criterion.  
CAIC is consistent Akaike's information criterion.

```
. mlogit insure age male nonwhite i.site if age < 30
 (output omitted)
. estat ic, all
Information criteria
```

| Model | N  | ll(null)  | ll(model) | df |
|-------|----|-----------|-----------|----|
| .     | 87 | -76.93025 | -66.03298 | 12 |

Note: BIC, AICc, and CAIC use N = number of observations.  
See [\[R\] IC note](#).

| Model | AIC     | BIC      | AICc     | CAIC     |
|-------|---------|----------|----------|----------|
| .     | 156.066 | 185.6569 | 160.2822 | 197.6569 |

Legend: AIC is Akaike's information criterion.  
BIC is Bayesian information criterion.  
AICc is corrected Akaike's information criterion.  
CAIC is consistent Akaike's information criterion.

[Burnham and Anderson \(2002\)](#) recommend using AICc when the ratio  $N/k < 40$ . The AIC suggests that the model with the site dummies is preferred, whereas AICc reports the opposite result.

### ▷ Example 3

As we discuss in the [technical note](#) below, for the linear mixed models fit using restricted maximum likelihood (REML), one needs to be careful when comparing models using the standard information criteria, especially when the fixed-effects specifications differ across models. In this example, we show how to use `n()` and `df()` to modify the the standard  $N$  and  $k$  used in the information criteria when we compare such models. As in [\[ME\] mixed](#), we consider the dataset from [Munnell \(1990\)](#) and estimate a Cobb–Douglas production function, which examines the productivity of public capital in each state’s private output ([Baltagi, Song, and Jung 2001](#)).

Suppose we want to compare two models:

```
. use https://www.stata-press.com/data/r19/productivity
(Public capital productivity)
. mixed gsp private emp hwy water other unemp || region: || state:, reml
(output omitted)
. estimates store model1
. mixed gsp private emp hwy unemp || region: hwy || state: unemp, reml
(output omitted)
. estimates store model2
```

The two models differ in both their fixed-effects and random-effects specifications. By default, the number of degrees of freedom in `estat ic` is calculated as  $k = k_f + k_r$ , where  $k_f$  and  $k_r$  are the number of estimated fixed-effects and random-effects parameters, respectively. For REML, [Gurka \(2006\)](#) evaluates the performance of various information criteria. He discusses using  $k = k_r$  and different possible values for  $N$ . Here, we follow the [Vonesh and Chinchilli \(1997\)](#) approach and choose  $N - k_f$ . Finally, we run `estat ic` to compare the models:

```
. estimates restore model1
(results model1 are active now)
. estat ic, n(809) df(3)
Akaike’s information criterion and Bayesian information criterion
```

| Model  | N   | ll(null) | ll(model) | df | AIC      | BIC       |
|--------|-----|----------|-----------|----|----------|-----------|
| model1 | 809 | .        | 1404.71   | 3  | -2803.42 | -2789.333 |

```
. estimates restore model2
(results model2 are active now)
. estat ic, n(811) df(5)
Akaike’s information criterion and Bayesian information criterion
```

| Model  | N   | ll(null) | ll(model) | df | AIC       | BIC       |
|--------|-----|----------|-----------|----|-----------|-----------|
| model2 | 811 | .        | 1413.557  | 5  | -2817.114 | -2793.623 |

Both AIC and BIC indicate that the second model is preferable.

## □ Technical note

`glm` and `binreg`, `m1` report a slightly different version of AIC and BIC; see [R] `glm` for the formulas used. That version is commonly used within the generalized linear models literature; see, for example, [Hardin and Hilbe \(2018\)](#). The literature on information criteria is vast; see, among others, [Akaike \(1973\)](#), [Sawa \(1978\)](#), and [Raftery \(1995\)](#). [Judge et al. \(1985\)](#) discuss the use of information criteria in econometrics. [Royston and Sauerbrei \(2008, chap. 2\)](#) examine the use of information criteria as an alternative to stepwise procedures for selecting model variables.

After the `regress` command, the number of parameters  $k$  does not include the error variance that would be estimated through the maximum likelihood approach. While excluding the error variance from the parameter count results in slightly lower AIC and BIC values, the relative difference between models remains unchanged because the penalty term  $2k$  is consistent across models. As long as you are comparing models fit with `regress`, excluding the error variance from the parameter count will not affect comparisons, because the relative ranking of models will not be affected. However, when comparing models fit with `regress` with models fit with other commands, we recommend using the `df()` option to explicitly account for the estimation of the error variance.

For linear mixed models, when restricted maximum likelihood is used, the information criteria with default degrees of freedom and the number of observations cannot be used to compare models with varying sets of fixed effects, because the likelihood of restricted maximum likelihood is dependent on the fixed-effects design matrix ([Harville 1974](#); [Gurka 2006](#)). By default, the degrees of freedom in `estat ic` is the sum of the dimension of fixed-effect parameters and the number of covariance parameters. Therefore, only models with the same sets of fixed effects can be compared. However, for each model, the `df()` option can be specified manually to allow comparison with different sets of fixed effects. There are also different views on which number should be used as  $N$  to calculate BIC, AICc, and CAIC. For example, see [Vonesh and Chinchilli \(1997\)](#) and [Kass and Raftery \(1995\)](#). Use the `n()` option to pass a desired number of observations to the `estat ic` command. For details, see [R] **IC note**.

□

## Stored results

`estat ic` stores the following in `r()`:

|                   |                                                                                                                                         |
|-------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| Matrices          |                                                                                                                                         |
| <code>r(S)</code> | row vector with columns ( <code>N</code> , <code>ll(null)</code> , <code>ll(mode1)</code> , <code>df</code> , and information criteria) |

## Methods and formulas

There are two main large-sample notions of information criteria: efficiency and consistency ([Burnham and Anderson 2002](#)). Efficient criteria target the best finite dimension model under the assumption that the unknown “true model” has infinite dimension. In contrast, assuming that the true data-generating model is finite and fixed, the consistent criterion selects the correct model with probability approaching 1 as  $N \rightarrow \infty$ . The AIC and AICc belong to the efficient class, while the BIC and CAIC to the consistent class.

Akaike’s (1974) information criterion is defined as

$$\text{AIC} = -2 \ln L + 2k$$

where  $\ln L$  is the maximized log-likelihood of the model and  $k$  is the number of parameters estimated. Some authors define AIC as the expression above divided by the sample size.

AIC performs poorly when there are too many parameters in relation to the sample size. Hurvich and Tsai (1989) derived a second-order variant of AIC called AICc,

$$\text{AICc} = \text{AIC} + \frac{2k(k+1)}{N-k-1}$$

where  $N$  is the sample size. See [R] **IC note** for additional information on calculating and interpreting  $N$ . Compared with AIC, AICc has an additional bias-correction term, and for large  $N$  and small  $k$ , this term is negligible. Burnham and Anderson (2002) recommend using AICc when the ratio  $N/k < 40$ .

Schwarz's (1978) Bayesian information criterion is another measure of fit defined as

$$\text{BIC} = -2 \ln L + k \ln N$$

Bozdogan (1987) proposed a consistent version of AIC called CAIC,

$$\text{CAIC} = -2 \ln L + k(\ln N + 1)$$

Burnham and Anderson (2002, chap. 6) argue that employing and comparing consistent and efficient information criteria in the same situation contrasts with the fact that they were designed to answer different questions. Thus, one needs to be careful when interpreting the results.

**Hirotsugu Akaike** (1927–2009) was born in Fujinomiya City, Shizuoka Prefecture, Japan. He was the son of a silkworm farmer. He gained BA and DSc degrees from the University of Tokyo. Akaike's career from 1952 at the Institute of Statistical Mathematics in Japan culminated in service as Director General; after 1994, he was Professor Emeritus. His best-known work in a prolific career is on what is now known as the Akaike information criterion (AIC), which was formulated to help selection of the most appropriate model from a number of candidates.

Gideon E. Schwarz (1933–2007) was a professor of statistics at the Hebrew University, Jerusalem. He was born in Salzburg, Austria, and obtained an MSc in 1956 from the Hebrew University and a PhD in 1961 from Columbia University. His interests included stochastic processes, sequential analysis, probability, and geometry. He is best known for the Bayesian information criterion (BIC).

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## Also see

- [R] **estat** — Postestimation statistics
- [R] **estat summarize** — Summarize estimation sample
- [R] **estat vce** — Display covariance matrix estimates
- [R] **estimates stats** — Model-selection statistics

[Description](#)[Quick start](#)[Menu for estat](#)[Syntax](#)[Options](#)[Remarks and examples](#)[Stored results](#)[Also see](#)

## Description

`estat summarize` summarizes the variables used by the command and automatically restricts the sample to the estimation sample; it also summarizes the weight variable and cluster structure, if specified.

## Quick start

Summary statistics for all variables in the model using estimation sample

```
estat summarize
```

Add variable labels to output

```
estat summarize, labels
```

Obtain summary of estimation sample for each equation

```
estat summarize, equation
```

Ignore weights when calculating summary statistics after weighted estimation

```
estat summarize, noweights
```

## Menu for estat

Statistics > Postestimation

## Syntax

```
estat summarize [eqlist] [, estat_summ_options]
```

| <i>estat_summ_options</i> | Description                                                                                                          |
|---------------------------|----------------------------------------------------------------------------------------------------------------------|
| <u>equation</u>           | display summary by equation                                                                                          |
| <u>group</u>              | display summary by group; only after <code>sem</code> and <code>gsem</code>                                          |
| <u>labels</u>             | display variable labels                                                                                              |
| <u>noheader</u>           | suppress the header                                                                                                  |
| <u>noweights</u>          | ignore weights                                                                                                       |
| <i>display_options</i>    | control row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |

*eqlist* is rarely used and specifies the variables, with optional equation name, to be summarized. *eqlist* may be *varlist* or (*eqname*<sub>1</sub>: *varlist*) (*eqname*<sub>2</sub>: *varlist*) ... *varlist* may contain time-series operators; see [U] 11.4.4 Time-series varlists.

`collect` is allowed; see [U] 11.1.10 Prefix commands.

## Options

`equation` requests that the dependent variables and the independent variables in the equations be displayed in the equation-style format of estimation commands, repeating the summary information about variables entered in more than one equation.

`group` displays summary information separately for each group. `group` is only allowed after `sem` or `gsem` with a `group()` variable specified.

`labels` displays variable labels.

`noheader` suppresses the header.

`noweights` ignores the weights, if any, from the previous estimation command. The default when weights are present is to perform a weighted summarize on all variables except the weight variable itself. An unweighted summarize is performed on the weight variable.

*display\_options*: `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, and `fvwrapon(style)`; see [R] Estimation options.

## Remarks and examples

Often when fitting a model, you will also be interested in obtaining summary statistics, such as the sample means and standard deviations of the variables in the model. `estat summarize` makes this process simple. The output displayed is similar to that obtained by typing

```
. summarize varlist if e(sample)
```

without the need to type the *varlist* containing the dependent and independent variables.

▷ Example 1

Continuing with the [example](#) in [R] `estat ic`, here we summarize the variables by using `estat summarize`.

```
. use https://www.stata-press.com/data/r19/sysdsn1
(Health insurance data)
. mlogit insure age male nonwhite i.site
(output omitted)
. estat summarize, noomitted
Estimation sample mlogit Number of obs = 615
```

| Variable | Mean     | Std. dev. | Min      | Max      |
|----------|----------|-----------|----------|----------|
| insure   | 1.596748 | .6225846  | 1        | 3        |
| age      | 44.46832 | 14.18523  | 18.11087 | 86.07254 |
| male     | .2504065 | .4335998  | 0        | 1        |
| nonwhite | .196748  | .3978638  | 0        | 1        |
| site     |          |           |          |          |
| 2        | .3707317 | .4833939  | 0        | 1        |
| 3        | .3138211 | .4644224  | 0        | 1        |



The output in the previous example contains all the variables in one table, though `mlogit` presents its results in a multiple-equation format. For models in which the same variables appear in all equations, that is fine; but for other multiple-equation models, we may prefer to have the variables separated by the equation in which they appear. The `equation` option makes this possible.



## Description

`estat vce` displays the covariance or correlation matrix of the parameter estimates of the previous model.

## Quick start

Display variance–covariance matrix of the estimates (VCE) from the previous model

```
estat vce
```

Matrix of correlations rather than covariances

```
estat vce, correlation
```

Same as above, but report correlations using three decimal places

```
estat vce, correlation format(%6.3f)
```

After fitting a multiple-equation model, display VCE for each equation in separate blocks

```
estat vce, block
```

Show VCE for equation `y1` only

```
estat vce, equation(y1)
```

## Menu for estat

Statistics > Postestimation

## Syntax

```
estat vce [, estat_vce_options]
```

| <i>estat_vce_options</i>           | Description                                                   |
|------------------------------------|---------------------------------------------------------------|
| <code>covariance</code>            | display as covariance matrix; the default                     |
| <code>correlation</code>           | display as correlation matrix                                 |
| <code>equation(<i>spec</i>)</code> | display only specified equations                              |
| <code>block</code>                 | display submatrices by equation                               |
| <code>diag</code>                  | display submatrices by equation; diagonal blocks only         |
| <code>format(<i>%fmt</i>)</code>   | display format for covariances and correlations               |
| <code>nolines</code>               | suppress lines between equations                              |
| <code>display_options</code>       | control display of omitted variables and base and empty cells |

collect is allowed; see [U] 11.1.10 Prefix commands.

## Options

`covariance` displays the matrix as a variance–covariance matrix; this is the default.

`correlation` displays the matrix as a correlation matrix rather than a variance–covariance matrix. `rho` is a synonym.

`equation(spec)` selects part of the VCE to be displayed. If *spec* is *eqlist*, the VCE for the listed equations is displayed. If *spec* is *eqlist1* \ *eqlist2*, the part of the VCE associated with the equations in *eqlist1* (rowwise) and *eqlist2* (columnwise) is displayed. If *spec* is \*, all equations are displayed. `equation()` implies `block` if `diag` is not specified.

`block` displays the submatrices pertaining to distinct equations separately.

`diag` displays the diagonal submatrices pertaining to distinct equations separately.

`format(%fmt)` specifies the number format for displaying the elements of the matrix. The default is `format(%10.0g)` for covariances and `format(%8.4f)` for correlations. See [U] 12.5 Formats: Controlling how data are displayed for more information.

`nolines` suppresses lines between equations.

*display\_options*: `noomitted`, `noemptycells`, `baselevels`, `allbaselevels`; see [R] Estimation options.

## Remarks and examples

`estat vce` allows you to display the VCE of the parameters of the previously fit model, as either a covariance matrix or a correlation matrix.

► Example 1

Returning to the [example in \[R\] estat ic](#), here we display the covariance matrix of the parameters of the mlogit model by using estat vce.

```
. use https://www.stata-press.com/data/r19/sysdsn1
(Health insurance data)
. mlogit insure age male nonwhite
(output omitted)
. estat vce, block
```

Covariance matrix of coefficients of mlogit model

Covariances of equation Indemnity

|            | o.<br>age | o.<br>male | o.<br>nonwhite | o.<br>_cons |
|------------|-----------|------------|----------------|-------------|
| o.age      | 0         |            |                |             |
| o.male     | 0         | 0          |                |             |
| o.nonwhite | 0         | 0          | 0              |             |
| o._cons    | 0         | 0          | 0              | 0           |

Covariances of equation Prepaid (row) by equation Indemnity (column)

|          | o.<br>age | o.<br>male | o.<br>nonwhite | o.<br>_cons |
|----------|-----------|------------|----------------|-------------|
| age      | 0         |            |                |             |
| male     | 0         | 0          |                |             |
| nonwhite | 0         | 0          | 0              |             |
| _cons    | 0         | 0          | 0              | 0           |

Covariances of equation Prepaid

|          | age        | male       | nonwhite   | _cons     |
|----------|------------|------------|------------|-----------|
| age      | .00003711  |            |            |           |
| male     | -.00015303 | .0402091   |            |           |
| nonwhite | -.00008948 | .00470608  | .04795135  |           |
| _cons    | -.00159095 | -.00398961 | -.00628886 | .08000462 |

Covariances of equation Uninsure (row) by equation Indemnity (column)

|          | o.<br>age | o.<br>male | o.<br>nonwhite | o.<br>_cons |
|----------|-----------|------------|----------------|-------------|
| age      | 0         |            |                |             |
| male     | 0         | 0          |                |             |
| nonwhite | 0         | 0          | 0              |             |
| _cons    | 0         | 0          | 0              | 0           |

Covariances of equation Uninsure (row) by equation Prepaid (column)

|          | age        | male       | nonwhite   | _cons      |
|----------|------------|------------|------------|------------|
| age      | .00001753  | -.00007926 | -.00004564 | -.00076886 |
| male     | -.00007544 | .02188398  | .0023186   | -.00145923 |
| nonwhite | -.00004577 | .00250588  | .02813553  | -.00263872 |
| _cons    | -.00077045 | -.00130535 | -.00257593 | .03888032  |

Covariances of equation Uninsure

|          | age        | male       | nonwhite   | _cons     |
|----------|------------|------------|------------|-----------|
| age      | .00013022  |            |            |           |
| male     | -.00050406 | .13248095  |            |           |
| nonwhite | -.00026145 | .01505449  | .16861327  |           |
| _cons    | -.00562159 | -.01686629 | -.02474852 | .28607591 |

The `block` option is particularly useful for multiple-equation estimators. The first block of output here corresponds to the VCE of the estimated parameters for the first equation—the square roots of the diagonal elements of this matrix are equal to the standard errors of the first equation’s parameters. Similarly, the final block corresponds to the VCE of the parameters for the second equation. The middle block shows the covariances between the estimated parameters of the first and second equations.



## Stored results

`estat vce` stores the following in `r()`:

|                   |                           |
|-------------------|---------------------------|
| Matrices          |                           |
| <code>r(V)</code> | VCE or correlation matrix |

## Also see

[R] [estat](#) — Postestimation statistics

[R] [estat ic](#) — Display information criteria

[R] [estat summarize](#) — Summarize estimation sample

[Description](#)[Syntax](#)[Remarks and examples](#)[Also see](#)

## Description

`estimates` allows you to store and manipulate estimation results:

- You can save estimation results in a file for use in later sessions.
- You can store estimation results in memory so that you can
  - a. switch among separate estimation results and
  - b. form tables combining separate estimation results.

## Syntax

| Command                                                             | Reference                              |
|---------------------------------------------------------------------|----------------------------------------|
| <i>Save and use results from disk</i>                               |                                        |
| <code><u>estimates</u> save <i>filename</i></code>                  | [R] <a href="#">estimates save</a>     |
| <code><u>estimates</u> use <i>filename</i></code>                   | [R] <a href="#">estimates save</a>     |
| <code><u>estimates</u> <u>describe</u> using <i>filename</i></code> | [R] <a href="#">estimates describe</a> |
| <code><u>estimates</u> <u>esample</u>: ...</code>                   | [R] <a href="#">estimates save</a>     |
| <i>Store and restore estimates in memory</i>                        |                                        |
| <code><u>estimates</u> <u>store</u> <i>name</i></code>              | [R] <a href="#">estimates store</a>    |
| <code><u>estimates</u> <u>restore</u> <i>name</i></code>            | [R] <a href="#">estimates store</a>    |
| <code><u>estimates</u> <u>query</u></code>                          | [R] <a href="#">estimates store</a>    |
| <code><u>estimates</u> <u>dir</u></code>                            | [R] <a href="#">estimates store</a>    |
| <code><u>estimates</u> <u>drop</u> <i>namelist</i></code>           | [R] <a href="#">estimates store</a>    |
| <code><u>estimates</u> <u>clear</u></code>                          | [R] <a href="#">estimates store</a>    |
| <i>Set titles and notes</i>                                         |                                        |
| <code><u>estimates</u> <u>title</u>: <i>text</i></code>             | [R] <a href="#">estimates title</a>    |
| <code><u>estimates</u> <u>title</u></code>                          | [R] <a href="#">estimates title</a>    |
| <code><u>estimates</u> <u>notes</u>: <i>text</i></code>             | [R] <a href="#">estimates notes</a>    |
| <code><u>estimates</u> <u>notes</u></code>                          | [R] <a href="#">estimates notes</a>    |
| <code><u>estimates</u> <u>notes</u> <u>list</u> ...</code>          | [R] <a href="#">estimates notes</a>    |
| <code><u>estimates</u> <u>notes</u> <u>drop</u> ...</code>          | [R] <a href="#">estimates notes</a>    |
| <i>Report</i>                                                       |                                        |
| <code><u>estimates</u> <u>describe</u> [<i>name</i>]</code>         | [R] <a href="#">estimates describe</a> |
| <code><u>estimates</u> <u>replay</u> [<i>namelist</i>]</code>       | [R] <a href="#">estimates replay</a>   |

| Command, continued                         | Reference, continued                   |
|--------------------------------------------|----------------------------------------|
| <i>Tables and statistics</i>               |                                        |
| <code>estimates table [namelist]</code>    | [R] <a href="#">estimates table</a>    |
| <code>etable</code>                        | [R] <a href="#">etable</a>             |
| <code>estimates selected [namelist]</code> | [R] <a href="#">estimates selected</a> |
| <code>estimates stats [namelist]</code>    | [R] <a href="#">estimates stats</a>    |
| <code>estimates for namelist: ...</code>   | [R] <a href="#">estimates for</a>      |

## Remarks and examples

`estimates` is for use after you have fit a model, be it with `regress`, `logistic`, etc. You can use `estimates` after any estimation command, whether it be an official estimation command of Stata or a community-contributed one.

`estimates` has three separate but related capabilities:

1. You can save estimation results in a file on disk so that you can use them later, even in a different Stata session.
2. You can store up to 300 estimation results in memory so that they are at your fingertips.
3. You can make tables comparing any results you have stored in memory.

Remarks are presented under the following headings:

[Saving and using estimation results](#)  
[Storing and restoring estimation results](#)  
[Comparing estimation results](#)  
[Jargon](#)

## Saving and using estimation results

After you have fit a model, say, with `regress`, type

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. regress mpg weight displ foreign
(output omitted)
```

You can save the results in a file:

```
. estimates save basemodel
file basemodel.ster saved
```

Later, say, in a different session, you can reload those results:

```
. estimates use basemodel
```

The situation is now nearly identical to what it was immediately after you fit the model. You can replay estimation results:

```
. regress
(output omitted)
```

You can perform tests:

```
. test foreign==0
 (output omitted)
```

And you can use any postestimation command or postestimation capability of Stata. The only difference is that Stata no longer knows what the estimation sample, `e(sample)` in Stata jargon, was. When you reload the estimation results, you might not even have the original data in memory. That is okay. Stata will know to refuse to calculate anything that can be calculated only on the original estimation sample.

If it is important that you use a postestimation command that can be used only on the original estimation sample, there is a way you can do that. You use the original data and then use `estimates esample:` to tell Stata what the original sample was.

See [R] [estimates save](#) for details.

## Storing and restoring estimation results

Storing and restoring estimation results in memory is much like saving them to disk. You type

```
. estimates store base
```

to save the current estimation results under the name `base`, and you type

```
. estimates restore base
```

to get them back later. You can find out what you have stored by typing

```
. estimates dir
```

Saving estimation results to disk is more permanent than storing them in memory, so why would you want merely to store them? The answer is that, once they are stored, you can use other `estimates` commands to produce tables and reports from them.

See [R] [estimates store](#) for details about the `estimates store` and `restore` commands.

## Comparing estimation results

Let's say that you have done the following:

```
. use https://www.stata-press.com/data/r19/auto
 (1978 automobile data)
. regress mpg weight displ
 (output omitted)
. estimates store base
. regress mpg weight displ foreign
 (output omitted)
. estimates store alt
```

You can now get a table comparing the coefficients:

```
. estimates table base alt
```

| Variable     | base       | alt        |
|--------------|------------|------------|
| weight       | -.00656711 | -.00677449 |
| displacement | .00528078  | .00192865  |
| foreign      |            | -1.6006312 |
| _cons        | 40.084522  | 41.847949  |

`estimates table` can do much more; see [R] [estimates table](#). `etable` also produces tables from stored estimates, customizes the tables, and exports to a variety of formats, including Word, PDF, L<sup>A</sup>T<sub>E</sub>X, Excel, and HTML; see [R] [etable](#). Also see [R] [estimates stats](#). `estimates stats` works like `estimates table` but produces model comparisons in terms of BIC and AIC.

## Jargon

You know that if you fit a model, say, by typing

```
. regress mpg weight displacement
```

then you can later replay the results by typing

```
. regress
```

and you can do tests and calculate other postestimation statistics by typing

```
. test displacement==0
. estat vif
. predict mpghat
```

As a result, we often refer to the *estimation results* or the *current estimation results* or the *most recent estimation results* or the *last estimation results* or the *estimation results in memory*.

With `estimates store` and `estimates restore`, you can have many estimation results in memory. One set of those—the set most recently estimated or the set most recently restored—is the *current* or *active* estimation results, which you can replay, which you can test, or from which you can calculate postestimation statistics.

*Current* and *active* are the two words we will use interchangeably from now on.

## Also see

[P] [\\_estimates](#) — Manage estimation results

[Description](#)  
[Stored results](#)[Quick start](#)  
[Also see](#)[Menu](#)[Syntax](#)[Option](#)[Remarks and examples](#)

## Description

`estimates describe` describes the current (active) estimates. Reported are the command line that produced the estimates, any title that was set by `estimates title` (see [R] [estimates title](#)), and any notes that were added by `estimates notes` (see [R] [estimates notes](#)).

`estimates describe name` does the same but reports results for estimates stored by `estimates store` (see [R] [estimates store](#)).

`estimates describe using filename` does the same but reports results for estimates saved by `estimates save` (see [R] [estimates save](#)). If *filename* contains multiple sets of estimates (saved in it by `estimates save, append`), the number of sets of estimates is also reported. If *filename* is specified without an extension, `.ster` is assumed.

## Quick start

Describe current estimation results

```
estimates describe
```

Describe estimation results stored in `m1`

```
estimates describe m1
```

Describe estimates from first model saved in `mymodels.ster`

```
estimates describe using mymodels
```

Same as above

```
estimates describe using mymodels, number(1)
```

Describe estimates from third model saved in `mymodels.ster`

```
estimates describe using mymodels, number(3)
```

## Menu

Statistics > Postestimation

## Syntax

```
estimates describe
```

```
estimates describe name
```

```
estimates describe using filename [, number(#)]
```

collect is allowed; see [U] 11.1.10 Prefix commands.

## Option

number(#) specifies that the #th set of estimation results from *filename* be described. This assumes that multiple sets of estimation results have been saved in *filename* by estimates save, append. The default is number(1).

## Remarks and examples

estimates describe can be used to describe the estimation results currently in memory,

```
. estimates describe
Estimation results produced by
. regress mpg weight displ if foreign
```

or to describe results saved by estimates save in a .ster file:

```
. estimates describe using final
Estimation results "Final results" saved on 12apr2024 14:20, produced by
. logistic myopic age sex drug1 drug2 if complete==1
```

Notes:

1. Used file patient.dta
2. "datasignature myopic age sex drug1 drug2 if complete==1" reports 148:5(58763):2252897466:3722318443
3. must be reviewed by rgg

## ▷ Example 1

```

. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. regress mpg weight displ if foreign
(output omitted)
. estimates notes: file 'c(filename)'
. datasignature
74:12(71728):3831085005:1395876116
. estimates notes: datasignature report 'r(datasignature)'
. estimates save foreign
file foreign.ster saved
. regress mpg weight displ if !foreign
(output omitted)
. estimates describe using foreign
Estimation results saved on 27mar2025 19:02, produced by
. regress mpg weight displ if foreign
Notes:
1. file https://www.stata-press.com/data/r19/auto.dta
2. datasignature report 74:12(71728):3831085005:1395876116

```

**Stored results**

`estimates describe` and `estimates describe name` store the following in `r()`:

Macros

|                         |                       |
|-------------------------|-----------------------|
| <code>r(title)</code>   | title                 |
| <code>r(cmdline)</code> | original command line |

`estimates describe using filename` stores the above and the following in `r()`:

Scalars

|                             |                                              |
|-----------------------------|----------------------------------------------|
| <code>r(datetime)</code>    | %tc value of date/time file saved            |
| <code>r(nestresults)</code> | number of sets of estimation results in file |

**Also see**

[R] [estimates](#) — Save and manipulate estimation results

## Description

`estimates for` performs *postestimation\_command* on each estimation result specified.

## Quick start

Test for no effect of continuous covariate `x1` in stored estimates `m1` and `m2`

```
estimates for m1 m2: test x1==0
```

Same as above, but test interaction of binary covariate `a` and `x1`

```
estimates for m1 m2: test 0.a#c.x1==1.a#c.x1
```

Linear combination of coefficients of `x1` and `x2` in all stored estimates

```
estimates for _all: lincom x1 + x2
```

Tables of margins for each level of `a` and confidence intervals using estimates `m1` and `m2`

```
estimates for m1 m2: pwcompare i.a, cimargins
```

## Syntax

```
estimates for namelist [, options] : postestimation_command
```

where *namelist* is a name, a list of names, `_all`, or `*`. A name may be `.`, meaning the current (active) estimates. `_all` and `*` mean the same thing.

| <i>options</i>   | Description                  |
|------------------|------------------------------|
| <u>n</u> oheader | do not display title         |
| <u>n</u> ostop   | do not stop if command fails |

## Options

`noheader` suppresses the display of the header as *postestimation\_command* is executed each time.

`nostop` specifies that execution of *postestimation\_command* is to be performed on the remaining models even if it fails on some.

## Remarks and examples

In the example that follows, we fit a model two different ways, store the results, and then use `estimates for` to perform the same test on both of them:

### ▷ Example 1

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. generate gpm = 1/mpg
. regress gpm i.foreign i.foreign#c.weight displ
(output omitted)
. estimates store reg
. qreg gpm i.foreign i.foreign#c.weight displ
(output omitted)
. estimates store qreg
. estimates for reg qreg: test 0.foreign#c.weight==1.foreign#c.weight
```

---

#### Model **reg**

---

```
(1) 0b.foreign#c.weight - 1.foreign#c.weight = 0
 F(1, 69) = 4.87
 Prob > F = 0.0307
```

---

#### Model **qreg**

---

```
(1) 0b.foreign#c.weight - 1.foreign#c.weight = 0
 F(1, 69) = 0.03
 Prob > F = 0.8554
```



## Also see

[R] [estimates](#) — Save and manipulate estimation results

## Description

`estimates notes: text` adds a note to the current (active) estimation results.

`estimates notes` and `estimates notes list` list the current notes.

`estimates notes drop in noterange` eliminates the specified notes.

## Quick start

Add “My note” to current estimation results

```
estimates notes: My note
```

List all notes for current estimation results

```
estimates notes list
```

Same as above

```
estimates notes
```

Drop notes 1 to 3 from current estimation results

```
estimates notes drop in 1/3
```

Drop last note applied to current estimation results

```
estimates notes drop in 1
```

## Syntax

```
estimates notes: text
```

```
estimates notes
```

```
estimates notes list [in noterange]
```

```
estimates notes drop in noterange
```

where *noterange* is # or #/# and where # may be a number, the letter f (meaning first), or the letter l (meaning last).

## Remarks and examples

After adding or removing notes, if estimates have been stored, do not forget to store them again. If estimates have been saved, do not forget to save them again.

Notes are most useful when you intend to save estimation results in a file; see [R] [estimates save](#). For instance, after fitting a model, you might type

```
. estimates note: I think these are final
. estimates save lock2
```

and later when going through your files, you could type

```
. estimates use lock2
. estimates notes
1. I think these are final
```

Up to 9,999 notes can be attached to estimation results. If estimation results are important, we recommend that you add a note identifying the `.dta` dataset you used. The best way to do that is to type

```
. estimates notes: file `c(filename)'
```

because ``c(filename)'` will expand to include not just the name of the file but also its full path; see [P] [creturn](#).

If estimation results took a long time to estimate—say, they were produced by `cmmprobit` or `stmgintcox`—it is also a good idea to add a data signature. A data signature takes less time to compute than reestimation when you need proof that you really have the right dataset. The easy way to do that is to type

```
. datasignature
74:12(71728):3831085005:1395876116
. estimates notes: datasignature reports `r(datasignature)'
```

Now when you ask to see the notes, you will see

```
. estimates notes
1. I think these are final
2. file C:\project\one\pat4.dta
3. datasignature reports 74:12(71728):3831085005:1395876116
```

See [D] [datasignature](#).

Notes need not be positive. You might set a note to be, “I need to check that age is defined correctly.”

## ▷ Example 1

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. regress mpg weight displ if foreign
(output omitted)
. estimates notes: file 'c(filename)'
. datasignature
74:12(71728):3831085005:1395876116
. estimates notes: datasignature report 'r(datasignature)'
. estimates save foreign
file foreign.ster saved
. estimates notes list in 1/2
1. file https://www.stata-press.com/data/r19/auto.dta
2. datasignature report 74:12(71728):3831085005:1395876116
. estimates notes drop in 2
(1 note dropped)
. estimates notes
1. file https://www.stata-press.com/data/r19/auto.dta
```

**Also see**

[R] [estimates](#) — Save and manipulate estimation results

## Description

`estimates replay` redisplays the current (active) estimation results, just as typing the name of the estimation command would do.

`estimates replay namelist` redisplays each specified estimation result. The active estimation results are left unchanged.

## Quick start

Redisplay current estimation results

```
estimates replay
```

Redisplay estimation results stored as `m1`

```
estimates replay m1
```

Redisplay all stored estimation results

```
estimates replay *
```

Same as above

```
estimates replay _all
```

## Menu

Statistics > Postestimation

## Syntax

```
estimates replay
```

```
estimates replay namelist
```

where *namelist* is a name, a list of names, `_all`, or `*`. A name may be `.`, meaning the current (active) estimates. `_all` and `*` mean the same thing.

## Remarks and examples

In the example that follows, we fit a model two different ways, store the results, use `estimates for` to perform the same test on both of them, and then replay the results:

### ► Example 1

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. generate gpm = 1/mpg
. regress gpm i.foreign i.foreign#c.weight displ
(output omitted)
. estimates store reg
. qreg gpm i.foreign i.foreign#c.weight displ
(output omitted)
. estimates store qreg
. estimates for reg qreg: test 0.foreign#c.weight==1.foreign#c.weight
```

---

Model **reg**

---

```
(1) 0b.foreign#c.weight - 1.foreign#c.weight = 0
 F(1, 69) = 4.87
 Prob > F = 0.0307
```

---

Model **qreg**

---

```
(1) 0b.foreign#c.weight - 1.foreign#c.weight = 0
 F(1, 69) = 0.03
 Prob > F = 0.8554
```



Description  
Stored results

Quick start  
Also see

Menu

Syntax

Options

Remarks and examples

## Description

`estimates save filename` saves the current (active) estimation results in *filename*.

`estimates use filename` loads the results saved in *filename* into the current (active) estimation results.

In both cases, if *filename* is specified without an extension, `.ster` is assumed.

`estimates esample:` (note the colon) resets `e(sample)`. After `estimates use filename`, `e(sample)` is set to contain 0, meaning that none of the observations currently in memory was used in obtaining the estimates.

`estimates esample` (without a colon) displays how `e(sample)` is currently set.

## Quick start

Save current estimation results to `mymodels.ster`

```
estimates save mymodels
```

Add current estimation results to existing file `mymodels.ster`

```
estimates save mymodels, append
```

Make active third estimation results saved in `mymodels.ster`

```
estimates use mymodels, number(3)
```

Reset `e(sample)` to original estimation sample assuming command `regress y x1 x2`

```
estimates esample: y x1 x2
```

## Menu

Statistics > Postestimation

## Syntax

```
estimates save filename [, append replace]
```

```
estimates use filename [, number(#)]
```

```
estimates esample: [varlist] [if] [in] [weight]
[, replace stringvars(varlist) zeroweight]
```

```
estimates esample
```

collect is allowed with estimates esample (without a colon); see [U] 11.1.10 Prefix commands.

## Options

append, used with estimates save, specifies that results be appended to an existing file. If the file does not already exist, a new file is created.

replace, used with estimates save, specifies that *filename* can be replaced if it already exists.

number(#), used with estimates use, specifies that the #th set of estimation results from *filename* be loaded. This assumes that multiple sets of estimation results have been saved in *filename* by estimates save, append. The default is number(1).

replace, used with estimates esample:, specifies that e(sample) can be replaced even if it is already set.

stringvars(*varlist*), used with estimates esample:, specifies string variables. Observations containing variables that contain "" will be omitted from e(sample).

zeroweight, used with estimates esample:, specifies that observations with zero weights are to be included in e(sample).

## Remarks and examples

See [R] estimates for an overview of the estimates commands.

For a description of estimates save and estimates use, see *Saving and using estimation results* in [R] estimates.

The rest of this entry concerns e(sample).

Remarks are presented under the following headings:

- Setting e(sample)*
- Resetting e(sample)*
- Determining who set e(sample)*

## Setting e(sample)

After `estimates use filename`, the situation is nearly identical to what it was immediately after you fit the model. The one difference is that `e(sample)` is set to 0.

`e(sample)` is Stata's function to mark which observations among those currently in memory were used in producing the estimates. For instance, you might type

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. regress mpg weight displ if foreign
(output omitted)
. summarize mpg if e(sample)
(output omitted)
```

and `summarize` would report the summary statistics for the observations `regress` in fact used, which would exclude not only observations for which `foreign = 0` but also any observations for which `mpg`, `weight`, or `displ` was missing.

If you saved the above estimation results and then reloaded them, however, `summarize mpg if e(sample)` would produce

```
. summarize mpg if e(sample)
```

| Variable | Obs | Mean | Std. dev. | Min | Max |
|----------|-----|------|-----------|-----|-----|
| mpg      | 0   |      |           |     |     |

Stata thinks that none of these observations was used in producing the estimates currently loaded.

What else could Stata think? When you `estimates use filename`, you do not have to have the original data in memory. Even if you do have data in memory that look like the original data, they might not be. Setting `e(sample)` to 0 is the safe thing to do. There are some postestimation statistics, for instance, that are appropriate only when calculated on the estimation sample. Setting `e(sample)` to 0 ensures that if you ask for one of them, you will get back a null result.

We recommend that you leave `e(sample)` set to 0. But what if you really need to calculate that postestimation statistic? Well, you can get it, but you are going to be responsible for setting `e(sample)` correctly. Here we just happen to know that all the observations with `foreign = 1` were used, so we can type

```
. estimates esample: if foreign
```

If all the observations had been used, we could simply type

```
. estimates esample:
```

The safe thing to do, however, is to look at the estimation command—`estimates describe` will show it to you—and then type

```
. estimates esample: mpg weight displ if foreign
```

We include all observations with `foreign = 1`, excluding any with missing values in the `mpg`, `weight`, or `displ` variable, that are to be treated as the estimation sample.

## Resetting e(sample)

`estimates esample:` will allow you to not only set but also reset `e(sample)`. If `e(sample)` has already been set (say that you just fit the model) and you try to set it, you will see

```
. estimates esample: mpg weight displ if foreign
no; e(sample) already set
r(322);
```

Here you can specify the `replace` option:

```
. estimates esample: mpg weight displ if foreign, replace
```

We do not recommend resetting `e(sample)`, but the situation can arise where you need to. Imagine that you `estimates use filename`, you set `e(sample)`, and then you realize that you set it wrong. Here you would want to reset it.

## Determining who set e(sample)

`estimates esample` without a colon will report whether and how `e(sample)` was set. You might see

```
. estimates esample
e(sample) set by estimation command
```

or

```
. estimates esample
e(sample) set by user
```

or

```
. estimates esample
e(sample) not set (0 assumed)
```

## Stored results

`estimates esample` without the colon saves macro `r(who)`, which will contain `cmd`, `user`, or `zero'd`.

## Also see

[\[R\] estimates](#) — Save and manipulate estimation results

Description      Quick start      Menu      Syntax      Options      Remarks and examples  
 Stored results      Also see

## Description

`estimates selected` reports on coefficients from one or more estimation results. It creates a table that indicates which coefficients were estimated in each model and, if requested, reports the value of those coefficients. The results may be sorted based on the values of the estimated coefficients or based on variable names.

## Quick start

Compare coefficients for stored estimates `m1` and `m2`

```
estimates selected m1 m2
```

Same as above, but display a `u` for covariates that are not specified in the model

```
estimates selected m1 m2, display(u)
```

Compare stored estimates `l1` and `l2`, and order the rows by absolute values of the coefficients

```
estimates selected l1 l2, sort(coef)
```

## Menu

Statistics > Postestimation

## Syntax

```
estimates selected [namelist] [, options]
```

*namelist* is the name given to previously stored estimation results, a list of names, `_all`, or `*`. A name may be `.`, meaning the current (active) estimates. `_all` and `*` mean the same thing.

| <i>options</i>                      | Description                                                                                |
|-------------------------------------|--------------------------------------------------------------------------------------------|
| Main                                |                                                                                            |
| <code>display(<i>info</i>)</code>   | display <i>info</i> ; default is <code>display(x)</code>                                   |
| <code>sort(<i>on</i>)</code>        | sort rows in order of <i>on</i>                                                            |
| Reporting                           |                                                                                            |
| <code>noabbrev</code>               | do not abbreviate variable names                                                           |
| <code>display_<i>options</i></code> | control row spacing, line width, and display of omitted variables and base and empty cells |

`collect` is allowed; see [U] 11.1.10 Prefix commands.

## Options

### Main

`display(info)` specifies what to display in the table. The default is `display(x)`.

Blank cells in the table indicate that the corresponding covariate does not have a fitted value. For some covariates without fitted values, a code that indicates the reason for omission is reported in the table. Base levels of factors and interactions are coded with the letter b. Empty levels of factors and interactions are coded with the letter e. Covariates omitted because of collinearity are coded with the letter o.

`display(x)` displays an x in the cell of the table where a covariate has a fitted value. This is the default.

`display(u)` is the same as `display(x)`, except that when a covariate was not specified in the model, u (for unavailable) is displayed instead of a blank cell.

`display(coef [ , eform fformat(%fmt) ])` specifies that coefficient values be displayed in the table.

`eform` displays coefficients in exponentiated form. For each coefficient,  $e^b$  rather than  $b$  is displayed. This option can be used to display odds ratios, incidence-rate ratios, relative-risk ratios, hazard ratios, and subhazard ratios after the appropriate estimation command.

`format(%fmt)` specifies the display format for the coefficients in the table. The default is `format(%9.0g)`.

`sort(on)` specifies how to sort the rows of the table. By default, coefficients are displayed in the order in which they appear in the estimation results.

`sort(none)` specifies that the rows are not sorted. This is the default. The order of the coefficients is taken from their order in `e(b)`.

`sort(names)` orders rows alphabetically by the variable names of the covariates. In the case of factor variables, main effects and nonfactor variables are displayed first in alphabetical order; then all two-way interactions are displayed in alphabetical order, then all three-way interactions, and so on.

`sort(coef)` orders rows in descending order by the absolute values of the coefficients. When results from two or more estimation results are displayed, results are sorted first by the ordering for the first estimation result with rows representing coefficients not in the first estimation result last. Within the rows representing coefficients not in the first estimation result, the rows are sorted by the ordering for the second estimation result with rows representing coefficients not in the first or second estimation results last. And so on.

### Reporting

`noabbrev` prevents variable names from being abbreviated in the row titles of the table. Long variable names are split onto multiple lines if they do not fit.

`display_options`: `vsquish`, `fvwrap(#)`, `fvwrapon(style)`, and `no1stretch`; see [R] [Estimation options](#).

## Remarks and examples

`estimates selected` produces a table based on estimated coefficients from one or more models. Results can be sorted by the values of the estimated coefficients or by variable names. Multiple models are displayed side-by-side, making it easy to compare which covariates were included in each model or to compare the estimated values of those coefficients.

### ► Example 1: Compare coefficients across models

To compare coefficients from two or more models, we first need to store the results of each model using `estimates store`; see [R] [estimates store](#). Then we use `estimates selected` to obtain a table comparing which coefficients were estimated in each model. By default, the table reports an `x` for each coefficient that was estimated.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. regress mpg gear turn
(output omitted)
. estimates store small
. regress mpg gear turn length
(output omitted)
. estimates store large
. estimates selected small large
```

|            | small | large |
|------------|-------|-------|
| gear_ratio | x     | x     |
| turn       | x     | x     |
| length     |       | x     |
| _cons      | x     | x     |

#### Legend:

```
b - base level
e - empty cell
o - omitted
x - estimated
```

There is only one difference in the two `regress` commands; the `large` model includes `length` as a covariate, but the `small` model does not. Therefore, the above table displays `x`'s for all but the `length` coefficient in the `small` model. When working with larger models or with more models, this type of table makes it easy to spot the differences in the model specification.

By default, the rows of the table in the order that covariates appear in the models. To sort the results on the covariate names, we add the `sort(names)` option.

```
. estimates selected small large, sort(names)
```

|            | small | large |
|------------|-------|-------|
| gear_ratio | x     | x     |
| length     |       | x     |
| turn       | x     | x     |
| _cons      | x     | x     |

Legend:

b - base level  
 e - empty cell  
 o - omitted  
 x - estimated

We could have instead sorted on the values of the estimated coefficients by including the `sort(coef)` option.

To display the coefficients values instead of the x's, we add the `display(coef)` option.

```
. estimates selected small large, sort(names) display(coef)
```

|            | small     | large     |
|------------|-----------|-----------|
| gear_ratio | 3.032884  | 1.35666   |
| length     |           | -.1665899 |
| turn       | -.7330502 | -.1219185 |
| _cons      | 41.21801  | 53.3487   |

Legend:

b - base level  
 e - empty cell  
 o - omitted

4

## Stored results

`estimates selected` stores the following in `r()`:

Macros

`r(names)` names of results used

Matrices

`r(coef)` matrix  $M$ :  $n \times m$   
 $M[i, j]$  =  $i$ th coefficient estimate for model  $j$ ;  $i = 1, \dots, n$ ;  $j = 1, \dots, m$

## Also see

[R] [estimates](#) — Save and manipulate estimation results

[LASSO] [lassocoeff](#) — Display coefficients after lasso estimation results

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Methods and formulas](#)[Syntax](#)[Also see](#)[Options](#)

## Description

`estimates stats` reports model-selection statistics, including the Akaike (AIC), consistent Akaike's (CAIC), corrected Akaike's (AICc), and Schwarz's Bayesian (BIC) information criteria. These measures are appropriate for maximum likelihood models.

If `estimates stats` is used for a non-likelihood-based model, such as `qreg`, missing values are reported.

## Quick start

Display AIC and BIC for last estimation command

```
estimates stats
```

Display AIC and BIC for stored estimates `m1` and `m2`

```
estimates stats m1 m2
```

Same as above, but display CAIC instead of AIC

```
estimates stats m1 m2, aicconsistent
```

Same as above, but display AICc instead of AIC

```
estimates stats m1 m2, aiccorrected
```

Same as above, but display all AIC, BIC, AICc, and CAIC

```
estimates stats m1 m2, all
```

Specify  $N = 1,000$  for calculation of BIC

```
estimates stats, n(1000)
```

Same as above, but use 10 degrees of freedom

```
estimates stats, n(1000) df(10)
```

## Menu

Statistics > Postestimation

## Syntax

```
estimates stats [namelist] [, options]
```

where *namelist* is a name, a list of names, `_all`, or `*`. A name may be `.`, meaning the current (active) estimates. `_all` and `*` mean the same thing.

| <i>options</i>             | Description                                                                               |
|----------------------------|-------------------------------------------------------------------------------------------|
| <code>aiccorrected</code>  | report AICc instead of AIC                                                                |
| <code>aicconsistent</code> | report CAIC instead of AIC                                                                |
| <code>all</code>           | report all four information criteria: AIC, BIC, AICc, and CAIC                            |
| <code>n(#)</code>          | specify <i>N</i> to be used in calculating BIC, AICc, and CAIC;<br>see [R] <b>IC note</b> |
| <code>df(#)</code>         | specify degrees of freedom <i>k</i> to be used in calculating AIC, BIC,<br>AICc, and CAIC |
| <code>icdetail</code>      | produce a table showing the type of <i>N</i> used in BIC, AICc, and<br>CAIC calculation   |

`collect` is allowed; see [U] **11.1.10 Prefix commands**.

## Options

`aiccorrected` specifies that AICc be computed instead of AIC. This information criterion is a second-order approximation and is recommended for small sample sizes.

Only one of `aiccorrected`, `aicconsistent`, or `all` is allowed.

`aicconsistent` specifies that CAIC be computed instead of AIC. This information criterion is a consistent version of AIC; that is, the probability of selecting the “true model” approaches 1 as sample size increases.

Only one of `aicconsistent`, `aiccorrected`, or `all` is allowed.

`all` produces a table showing all four information criteria: AIC, BIC, AICc, and CAIC.

Only one of `all`, `aiccorrected`, or `aicconsistent` is allowed.

`n(#)` specifies *N* to be used in calculating BIC, AICc, and CAIC; see [R] **IC note**.

`df(#)` specifies degrees of freedom *k* to be used in calculating AIC, BIC, AICc, and CAIC. By default, *k* is the number of estimated parameters.

`icdetail` produces a table showing the type of *N* used in BIC, AICc, and CAIC calculations. Most estimation commands use the number of observations in the estimation sample for the information criteria. For some models, however, other types of *N*, such as the number of cases in choice models, should be used. When the default table of `estimates stats` contains more than one type of *N*, specifying `icdetail` allows you to see the different types of *N* used for BIC, AICc, and CAIC.

## Remarks and examples

If you type `estimates stats` without arguments, a table for the most recent estimation results will be shown:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. logistic foreign mpg weight displ
(output omitted)
. estimates stats
Akaike's information criterion and Bayesian information criterion
```

| Model | N  | ll(null)  | ll(model) | df | AIC      | BIC      |
|-------|----|-----------|-----------|----|----------|----------|
| .     | 74 | -45.03321 | -20.59083 | 4  | 49.18167 | 58.39793 |

Note: BIC uses  $N$  = number of observations. See [\[R\] IC note](#).

Regarding the note at the bottom of the table,  $N$  is an ingredient in the calculation of BIC, AICc, and CAIC; see [\[R\] IC note](#). The note changes if you specify the `n()` option, which tells `estimates stats` what  $N$  to use. By default,  $N$  is the number of observations used in fitting the model.

Regarding the table itself, `ll(null)` is the log likelihood for the constant-only model, `ll(model)` is the log likelihood for the model, `df` is the number of degrees of freedom, and AIC and BIC are the Akaike and Bayesian information criteria, respectively.

Models with smaller values of an information criterion are considered preferable.

`estimates stats` can compare estimation results:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. logistic foreign mpg weight displ
(output omitted)
. estimates store full
. logistic foreign mpg weight
(output omitted)
. estimates store sub
. estimates stats full sub
Akaike's information criterion and Bayesian information criterion
```

| Model | N  | ll(null)  | ll(model) | df | AIC      | BIC      |
|-------|----|-----------|-----------|----|----------|----------|
| full  | 74 | -45.03321 | -20.59083 | 4  | 49.18167 | 58.39793 |
| sub   | 74 | -45.03321 | -27.17516 | 3  | 60.35031 | 67.26251 |

Note: BIC uses  $N$  = number of observations. See [\[R\] IC note](#).

You can use the `all` option to compare the models based on all four information criteria

```
. estimates stats full sub, all
```

Information criteria

| Model | N  | ll(null)  | ll(model) | df |
|-------|----|-----------|-----------|----|
| full  | 74 | -45.03321 | -20.59083 | 4  |
| sub   | 74 | -45.03321 | -27.17516 | 3  |

Note: BIC, AICc, and CAIC use  $N$  = number of observations.  
See [\[R\] IC note](#).

| Model | AIC      | BIC      | AICc     | CAIC     |
|-------|----------|----------|----------|----------|
| full  | 49.18167 | 58.39793 | 49.76138 | 62.39793 |
| sub   | 60.35031 | 67.26251 | 60.69317 | 70.26251 |

Legend: AIC is Akaike's information criterion.  
BIC is Bayesian information criterion.  
AICc is corrected Akaike's information criterion.  
CAIC is consistent Akaike's information criterion.

All four information criteria suggest that the full model is preferable.

## Stored results

`estimates stats` stores the following in `r()`:

Matrices

`r(S)` matrix with columns ( $N$ ,  $ll(\text{null})$ ,  $ll(\text{model})$ ,  $df$ , and information criteria) and rows corresponding to models in the table

## Methods and formulas

See [\[R\] IC note](#).

## Also see

[\[R\] estimates](#) — Save and manipulate estimation results

|                                |                             |                          |                        |                        |                                      |
|--------------------------------|-----------------------------|--------------------------|------------------------|------------------------|--------------------------------------|
| <a href="#">Description</a>    | <a href="#">Quick start</a> | <a href="#">Menu</a>     | <a href="#">Syntax</a> | <a href="#">Option</a> | <a href="#">Remarks and examples</a> |
| <a href="#">Stored results</a> | <a href="#">References</a>  | <a href="#">Also see</a> |                        |                        |                                      |

## Description

`estimates store name` stores the current (active) estimation results under the name *name*.

`estimates restore name` loads the results stored under *name* into the current (active) estimation results.

`estimates query` tells you whether the current (active) estimates have been stored and, if so, the name.

`estimates dir` displays a list of the stored estimates.

`estimates drop namelist` drops the specified stored estimation results.

`estimates clear` drops all stored estimation results.

`estimates clear`, `estimates drop _all`, and `estimates drop *` do the same thing. `estimates drop` and `estimates clear` do not eliminate the current (active) estimation results.

## Quick start

Store estimation results as `m1` for use later in the same session

```
estimates store m1
```

Restore estimation results from `m2`

```
estimates restore m2
```

Find out whether the current estimation results have been stored

```
estimates query
```

Display table of information about all stored results

```
estimates dir
```

Drop stored estimation results `m3`

```
estimates drop m3
```

Drop all stored results

```
estimates clear
```

## Menu

Statistics > Postestimation

## Syntax

```
estimates store name [, nocopy]
```

```
estimates restore name
```

```
estimates query
```

```
estimates dir [namelist]
```

```
estimates drop namelist
```

```
estimates clear
```

where *namelist* is a name, a list of names, `_all`, or `*`. `_all` and `*` mean the same thing.

`collect` is allowed with `estimates dir`; see [U] 11.1.10 Prefix commands.

## Option

`nocopy`, used with `estimates store`, specifies that the current (active) estimation results are to be moved into *name* rather than copied. Typing

```
. estimates store hold, nocopy
```

is the same as typing

```
. estimates store hold
. ereturn clear
```

except that the former is faster. The `nocopy` option is sometimes used by programmers.

## Remarks and examples

`estimates store` stores estimation results in memory so that you can access them later.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. regress mpg weight displ
(output omitted)
. estimates store myreg

. ... you do other things, including fitting other models ...

. estimates restore myreg
. regress
(same output shown again)
```

After `estimates restore myreg`, things are once again just as they were, estimationwise, just after you typed `regress mpg weight displ`.

`estimates store` stores results in memory. When you exit Stata, those stored results vanish. If you wish to make a permanent copy of your estimation results, see [R] [estimates save](#).

The purpose of making copies in memory is 1) so that you can quickly switch between them and 2) so that you can make tables comparing estimation results. Concerning the latter, see [R] [estimates table](#), [R] [etable](#), and [R] [estimates stats](#).

## Stored results

`estimates dir` stores the following in `r()`:

```
Macros
 r(names) names of stored results
```

## References

- Jann, B. 2005. [Making regression tables from stored estimates](#). *Stata Journal* 5: 288–308.  
 ———. 2007. [Making regression tables simplified](#). *Stata Journal* 7: 227–244.

## Also see

- [R] [estimates](#) — Save and manipulate estimation results  
 [LASSO] [estimates store](#) — Saving and restoring estimates in memory and on disk

|                |             |          |        |         |                      |
|----------------|-------------|----------|--------|---------|----------------------|
| Description    | Quick start | Menu     | Syntax | Options | Remarks and examples |
| Stored results | References  | Also see |        |         |                      |

## Description

`estimates table` organizes estimation results from one or more models in a single formatted table.

## Quick start

Display a table of coefficients for stored estimates `m1` and `m2`

```
estimates table m1 m2
```

Same as above, but include standard errors

```
estimates table m1 m2, se
```

Same as above, but display coefficients and standard errors to 2 decimal places

```
estimates table m1 m2, b(%7.2f) se(%7.2f)
```

Same as above, but include  $p$ -values displayed to 3 decimal places

```
estimates table m1 m2, b(%7.2f) se(%7.2f) p(%4.3f)
```

Table of coefficients for `m1` and `m2` with sample size and adjusted  $R^2$

```
estimates table m1 m2, stats(N r2_a)
```

Same as above, but replace variable names with labels

```
estimates table m1 m2, stats(N r2_a) varlabel
```

Table of coefficients with stars to denote significance

```
estimates table m1 m2, star
```

Display coefficients in exponentiated form

```
estimates table m3 m4, eform
```

Display only a subset of variables and reorder variables in table

```
estimates table m1 m2, keep(v2 v1 v3 _cons)
```

## Menu

Statistics > Postestimation

## Syntax

```
estimates table [namelist] [, options]
```

*namelist* is the name given to previously stored estimation results, a list of names, `_all`, or `*`. A name may be `.`, meaning the current (active) estimates. `_all` and `*` mean the same thing.

| <i>options</i>                           | Description                                                                                |
|------------------------------------------|--------------------------------------------------------------------------------------------|
| <b>Main</b>                              |                                                                                            |
| <code>stats(<i>scalarlist</i>)</code>    | report <i>scalarlist</i> in table                                                          |
| <code>star[ (#1 #2 #3) ]</code>          | use stars to denote significance                                                           |
| <b>Options</b>                           |                                                                                            |
| <code>keep(<i>coeflist</i>)</code>       | report coefficients in order specified                                                     |
| <code>drop(<i>coeflist</i>)</code>       | omit specified coefficients from table                                                     |
| <code>equations(<i>matchlist</i>)</code> | match equations of models as specified                                                     |
| <b>Numerical formats</b>                 |                                                                                            |
| <code>b[ (%<i>fmt</i>) ]</code>          | how to format coefficients, which are always reported                                      |
| <code>se[ (%<i>fmt</i>) ]</code>         | report standard errors and use optional format                                             |
| <code>t[ (%<i>fmt</i>) ]</code>          | report <i>t</i> or <i>z</i> and use optional format                                        |
| <code>p[ (%<i>fmt</i>) ]</code>          | report <i>p</i> -values and use optional format                                            |
| <code>stfmt(%<i>fmt</i>)</code>          | how to format scalar statistics                                                            |
| <b>General format</b>                    |                                                                                            |
| <code>varwidth(#)</code>                 | use # characters to display variable names and statistics                                  |
| <code>modelwidth(#)</code>               | use # characters to display model names                                                    |
| <code>eform</code>                       | display coefficients in exponentiated form                                                 |
| <code>varlabel</code>                    | display variable labels rather than variable names                                         |
| <code>newpanel</code>                    | display statistics in separate table from coefficients                                     |
| <code>style(online)</code>               | put vertical line after variable names; the default                                        |
| <code>style(columns)</code>              | put vertical line separating every column                                                  |
| <code>style(noline)</code>               | suppress all vertical lines                                                                |
| <code>coded</code>                       | display compact table                                                                      |
| <b>Reporting</b>                         |                                                                                            |
| <code>display_options</code>             | control row spacing, line width, and display of omitted variables and base and empty cells |
| <code>title(<i>string</i>)</code>        | title for table                                                                            |

`collect` is allowed; see [U] 11.1.10 Prefix commands.

`title()` does not appear in the dialog box.

## Options

### Main

`stats(scalarlist)` specifies a list of any of or all the names of scalars stored in `e()` to be displayed in the table. *scalarlist* may also contain the following:

|                   |                                                           |
|-------------------|-----------------------------------------------------------|
| <code>aic</code>  | Akaike's information criterion                            |
| <code>bic</code>  | Schwarz's Bayesian information criterion                  |
| <code>rank</code> | rank of <code>e(V)</code> (# of free parameters in model) |

The specified statistics do not have to be available for all estimation results being displayed.

For example, `stats(N l1 chi2 aic)` specifies that `e(N)`, `e(l1)`, `e(chi2)`, and AIC be included. In Stata, `e(N)` records the number of observations; `e(l1)`, the log likelihood; and `e(chi2)`, the  $\chi^2$  test that all coefficients in the first equation of the model are equal to zero.

`star` and `star(#1 #2 #3)` specify that stars (asterisks) are to be used to mark significance. The second syntax specifies the significance for one, two, and three stars. If you specify simply `star`, that is equivalent to specifying `star(.05 .01 .001)`, which means one star (\*) if  $p < 0.05$ , two stars (\*\*) if  $p < 0.01$ , and three stars (\*\*\*) if  $p < 0.001$ .

The `star` and `star()` options may not be combined with the `se`, `t`, or `p` option.

### Options

`keep(coeflist)` and `drop(coeflist)` are alternatives; they specify coefficients to be included or omitted from the table. The default is to display all coefficients.

If `keep()` is specified, it specifies not only the coefficients to be included but also the order in which they appear.

A *coeflist* is a list of coefficient names, each name of which may be simple (for example, `price`), an equation name followed by a colon (for example, `mean:`), or a full name (for example, `mean:price`). Names are separated from each other by blanks.

When full names are not specified, all coefficients that match the partial specification are included. For instance, `drop(_cons)` would omit `_cons` for all equations.

`equations(matchlist)` specifies how the equations of the models in *namelist* are to be matched. The default is to match equations by name. Matching by name usually works well when all results were fit by the same estimation command. When you are comparing results from different estimation commands, however, specifying `equations()` may be necessary.

The most common usage is `equations(1)`, which indicates that all first equations are to be matched into one equation named #1.

*matchlist* has the syntax

```
term [, term . . .]
```

*term* is

```
[eqname =] #:#. . .:# (syntax 1)
```

```
[eqname =] # (syntax 2)
```

In syntax 1, each # is a number or a period (.). If a number, it specifies the position of the equation in the corresponding model; 1:3:1 would indicate that equation 1 in the first model matches equation 3 in the second, which matches equation 1 in the third. A period indicates that there is no corresponding equation in the model; 1:.:1 indicates that equation 1 in the first matches equation 1 in the third.

In syntax 2, you specify just one number, say, 1 or 2, and that is shorthand for 1:1...:1 or 2:2...:2, meaning that equation 1 matches across all models specified or that equation 2 matches across all models specified.

Now that you can specify a *term*, you can put that together into a *matchlist* by separating one term from the other by commas. In what follows, we will assume that three names were specified,

```
. estimates table alpha beta gamma, ...
```

`equations(1)` is equivalent to `equations(1:1:1)`; we would be saying that the first equations match across the board.

`equations(1:.:1)` would specify that equation 1 matches in models alpha and gamma but that there is nothing corresponding in model beta.

`equations(1,2)` is equivalent to `equations(1:1:1, 2:2:2)`. We would be saying that the first equations match across the board and so do the second equations.

`equations(1, 2:.:2)` would specify that the first equations match across the board, that the second equations match for models alpha and gamma, and that there is nothing equivalent to equation 2 in model beta.

If `equations()` is specified, equations not matched by position are matched by name.

#### Numerical formats

`b(%fmt)` specifies how the coefficients are to be displayed. You might specify `b(%9.2f)` to make decimal points line up. There is also a `b` option, which specifies that coefficients are to be displayed, but that is just included for consistency with the `se`, `t`, and `p` options. Coefficients are always displayed.

`se`, `t`, and `p` specify that standard errors, *t* or *z* statistics, and *p*-values are to be displayed. The default is not to display them. `se(%fmt)`, `t(%fmt)`, and `p(%fmt)` specify that each is to be displayed and specifies the display format to be used to format them.

`stfmt(%fmt)` specifies the format for displaying the scalar statistics included by the `stats()` option.

#### General format

`varwidth(#)` specifies the number of character positions used to display the names of the variables and statistics. The default is 12.

`modelwidth(#)` specifies the number of character positions used to display the names of the models. The default is 12.

`eform` displays coefficients in exponentiated form. For each coefficient,  $\exp(\beta)$  rather than  $\beta$  is displayed, and standard errors are transformed appropriately. Display of the intercept, if any, is suppressed.

`varlabel` specifies that variable labels be displayed instead of variable names.

`newpanel` specifies that the statistics be displayed in a table separated by a blank line from the table with coefficients rather than in the style of another equation in the table of coefficients.

`style(stylespec)` specifies the style of the coefficient table.

`style(online)` specifies that a vertical line be displayed after the variables but not between the models. This is the default.

`style(columns)` specifies that vertical lines be displayed after each column.

`style(noline)` specifies that no vertical lines be displayed.

`coded` specifies that a compact table be displayed. This format is especially useful for comparing variables that are included in a large collection of models.

---

#### Reporting

---

`display_options:` `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, and `fvwrapon(style)`; see [R] [Estimation options](#).

The following option is available with `estimates table` but is not shown in the dialog box:

`title(string)` specifies the title to appear above the table.

## Remarks and examples

`estimates table` lets you format estimation results and organize results from multiple models in a single table. You achieve this by combining `estimates table` with `estimates store`; see [R] [estimates store](#). When combined with `putdocx` or `putpdf`, `estimates table` lets you create customized tables of results in `.docx` or `.pdf` format.

Alternatively, you can use `etable` to create a table with the results stored with `estimates store`. The advantage of using `etable` is that you can add notes to your table and export it to a variety of file types, such as HTML and  $\text{\LaTeX}$ .

### ► Example 1: Creating the default table

If you type `estimates table` without arguments, a table of the most recent estimation results will be shown:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. regress mpg weight
(output omitted)
. regress mpg weight displ
(output omitted)
. estimates table
```

| Variable     | Active     |
|--------------|------------|
| weight       | -.00656711 |
| displacement | .00528078  |
| _cons        | 40.084522  |

In the above output table, only the results from the second `regress` command are displayed because they are the current (active) estimates.

By default, `estimates` table displays only coefficient estimates. You can request additional model statistics by specifying the `stats()` option. Estimation commands store e-class results. To see a list of available results from the last estimation command that can be specified in the `stats()` option, you can use the `ereturn list` command.

### ► Example 2: Creating a table with multiple models

If we want to display more than one set of estimation results in a single table, we can use `estimates store` to store each set of estimation results in memory so that they can be accessed later by `estimates table`. This is one of the primary uses of `estimates table`—comparing estimation results:

```
. regress mpg weight displ
 (output omitted)
. estimates store base
. regress mpg weight displ i.foreign
 (output omitted)
. estimates store alt
. qreg mpg weight displ i.foreign
 (output omitted)
. estimates store qreg
. estimates table base alt qreg, stats(r2)
```

| Variable     | base       | alt        | qreg       |
|--------------|------------|------------|------------|
| weight       | -.00656711 | -.00677449 | -.00595056 |
| displacement | .00528078  | .00192865  | .00018552  |
| foreign      |            |            |            |
| Foreign      |            | -1.6006312 | -2.1326005 |
| _cons        | 40.084522  | 41.847949  | 39.213348  |
| r2           | .6529307   | .66287957  |            |

`estimates table` automatically lines up the point estimates of coefficients on covariates that are common across each model. The `stats(r2)` option specifies that the coefficient of determination ( $R^2$ ) be placed under the models for which it is computed.

The `foreign` variable was included in the models `alt` and `qreg` as a factor variable. Because the values of `foreign` are labeled, the value labels are displayed by default. See [D] [label](#) for information about managing value labels.

### ▶ Example 3: Creating and exporting a formatted table

We can add estimates of the standard errors to the table and format them along with the original coefficient estimates by using the `b()` and `se()` options.

As shown in [example 2](#), the labeled values of any factor variables are displayed by default. To see the labels corresponding to the variables that are included in the model rather than the names of the variables, we can specify the `varlabel` option.

```
. estimates table alt, b(%5.4f) se(%5.4f) varlabel
```

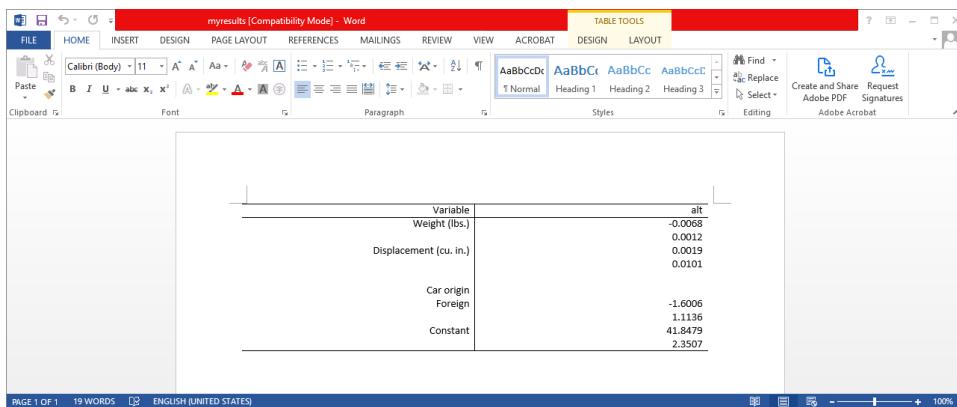
| Variable               | alt               |
|------------------------|-------------------|
| Weight (lbs.)          | -0.0068<br>0.0012 |
| Displacement (cu. in.) | 0.0019<br>0.0101  |
| Car origin             |                   |
| Foreign                | -1.6006<br>1.1136 |
| Constant               | 41.8479<br>2.3507 |

Legend: b/se

If we wanted this table to appear in a Word document, we could use the `putdocx` command with the `etable` output type to write it to a new document. To create the new document `myresults.docx`, we would type the following `putdocx` commands:

```
. putdocx begin
. putdocx table results = etable
. putdocx save myresults.docx
successfully created "myresults.docx"
```

This creates a table in Word that looks like



| Variable               | alt               |
|------------------------|-------------------|
| Weight (lbs.)          | -0.0068<br>0.0012 |
| Displacement (cu. in.) | 0.0019<br>0.0101  |
| Car origin             |                   |
| Foreign                | -1.6006<br>1.1136 |
| Constant               | 41.8479<br>2.3507 |

## Stored results

`estimates` table stores the following in `r()`:

### Macros

`r(names)` names of results used

### Matrices

`r(coef)` matrix  $M: n \times 2 * m$

$M[i, 2j - 1] = i$ th parameter estimate for model  $j$ ;

$M[i, 2j] =$  variance of  $M[i, 2j - 1]$ ;  $i = 1, \dots, n$ ;  $j = 1, \dots, m$

`r(stats)` matrix  $S: k \times m$  (if option `stats()` specified)

$S[i, j] = i$ th statistic for model  $j$ ;  $i = 1, \dots, k$ ;  $j = 1, \dots, m$

## References

Gallup, J. L. 2012. [A new system for formatting estimation tables](#). *Stata Journal* 12: 3–28.

Weiss, M. 2010. [Stata tip 90: Displaying partial results](#). *Stata Journal* 10: 500–502.

## Also see

[R] [estimates](#) — Save and manipulate estimation results

[R] [etable](#) — Create a table of estimation results

[R] [table regression](#) — Table of regression results

## Description

`estimates title:` (note the colon) sets or clears the title for the current estimation results. The title is used by `estimates table`, `estimates stats`, and `estimates dir`.

`estimates title` without the colon displays the current title.

## Quick start

Set title for the current estimation results

```
estimates title: Base model
```

Display the title of the current estimation results

```
estimates title
```

## Menu

Statistics > Postestimation

## Syntax

```
estimates title: [text]
```

```
estimates title
```

## Remarks and examples

After setting the title, if estimates have been stored, do not forget to store them again:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. regress mpg gear turn
(output omitted)
. estimates store reg
```

Now let's add a title:

```
. estimates title: "My regression"
. estimates store reg
```

## Also see

[\[R\] estimates](#) — Save and manipulate estimation results

## Description

This entry describes the options common to many estimation commands. Not all the options documented here work with all estimation commands. See the documentation for the particular estimation command; if an option is listed there, it is applicable.

## Syntax

*estimation\_cmd* ... [ , *options* ]

| <i>options</i>                                    | Description                                                                             |
|---------------------------------------------------|-----------------------------------------------------------------------------------------|
| <b>Model</b>                                      |                                                                                         |
| <code>noconstant</code>                           | suppress constant term                                                                  |
| <code>offset(<i>varname</i><sub>o</sub>)</code>   | include <i>varname</i> <sub>o</sub> in model with coefficient constrained to 1          |
| <code>exposure(<i>varname</i><sub>e</sub>)</code> | include ln( <i>varname</i> <sub>e</sub> ) in model with coefficient constrained to 1    |
| <code>constraints(<i>constraints</i>)</code>      | apply specified linear constraints                                                      |
| <b>Reporting</b>                                  |                                                                                         |
| <code>level(#)</code>                             | set confidence level; default is level(95)                                              |
| <code>lrmodel</code>                              | perform likelihood-ratio model test instead of the default Wald test                    |
| <code>nocnsreport</code>                          | do not display constraints                                                              |
| <code>noci</code>                                 | suppress confidence intervals                                                           |
| <code>nopvalues</code>                            | suppress <i>p</i> -values and their test statistics                                     |
| <code>noomitted</code>                            | do not display omitted collinear variables                                              |
| <code>vsquish</code>                              | suppress blank space separating factor variables or time-series variables               |
| <code>noemptycells</code>                         | do not display empty interaction cells of factor variables                              |
| <code>baselevels</code>                           | report base levels for factor variables and interactions                                |
| <code>allbaselevels</code>                        | display all base levels for factor variables and interactions                           |
| <code>nofvlabel</code>                            | display factor-variable level values rather than value labels                           |
| <code>fvwrap(#)</code>                            | allow # lines when wrapping long value labels                                           |
| <code>fvwrapon(<i>style</i>)</code>               | apply <i>style</i> for wrapping long value labels;<br><i>style</i> may be word or width |
| <code>cformat(<i>%fmt</i>)</code>                 | format for coefficients, standard errors, and confidence limits                         |
| <code>pformat(<i>%fmt</i>)</code>                 | format for <i>p</i> -values                                                             |
| <code>sformat(<i>%fmt</i>)</code>                 | format for test statistics                                                              |
| <b>Integration</b>                                |                                                                                         |
| <code>intmethod(<i>intmethod</i>)</code>          | integration method for random-effects models                                            |
| <code>intpoints(#)</code>                         | use # integration (quadrature) points                                                   |
| <code>nostretch</code>                            | do not automatically widen coefficient table for long variable names                    |
| <code>collinear</code>                            | keep collinear variables                                                                |
| <code>coeflegend</code>                           | display legend instead of statistics                                                    |

## Options

### Model

`noconstant` suppresses the constant term (intercept) in the model.

`offset(varnameo)` specifies that *varname<sub>o</sub>* be included in the model with the coefficient constrained to be 1.

`exposure(varnamee)` specifies a variable that reflects the amount of exposure over which the *depvar* events were observed for each observation;  $\ln(\text{varname}_e)$  with the coefficient constrained to be 1 is entered into the log-link function.

`constraints(numlist | matname)` specifies the linear constraints to be applied during estimation. The default is to perform unconstrained estimation. See [R] [reg3](#) for the use of constraints in multiple-equation contexts.

`constraints(numlist)` specifies the constraints by number after they have been defined by using the `constraint` command; see [R] [constraint](#). Some commands (for example, `slogit`) allow only `constraints(numlist)`.

`constraints(matname)` specifies a matrix containing the constraints; see [P] [makecns](#).

### Reporting

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [U] [20.8 Specifying the width of confidence intervals](#).

`lrmmodel` specifies to conduct a likelihood-ratio test of the full maximum-likelihood model versus the restricted model that includes only a constant term in the regression equation instead of conducting the default Wald test that all coefficients are zero. This option can substantially increase estimation time.

`lrmmodel` may not be specified with the `vce(robust)`, `vce(cluster clustvar)`, `vce(jackknife)`, `vce(bootstrap)`, or `noconstant` option.

`lrmmodel` may not be combined with `constraints`; see [constraints\(\*constraints\*\)](#). In some cases, a likelihood-ratio test is valid for models with constraints. To compute a likelihood-ratio test when constraints have been applied during estimation, use `lrtest`; see [R] [lrtest](#).

`lrmmodel` may not be specified with the `jackknife`, `bootstrap`, `svy`, or `mi` estimate prefix. In addition, `lrmmodel` may not be specified on `replay`.

`nocnsreport` specifies that no constraints be reported. The default is to display user-specified constraints above the coefficient table.

`noci` suppresses confidence intervals from being reported in the coefficient table.

`nopvalues` suppresses *p*-values and their test statistics from being reported in the coefficient table.

`noomitted` specifies that variables that were omitted because of collinearity not be displayed. The default is to include in the table any variables omitted because of collinearity and to label them as “(omitted)”.

`vsquish` specifies that the blank space separating factor-variable terms or time-series-operated variables from other variables in the model be suppressed.

`noemptycells` specifies that empty cells for interactions of factor variables not be displayed. The default is to include in the table interaction cells that do not occur in the estimation sample and to label them as “(empty)”.

`baselevels` and `allbaselevels` control whether the base levels of factor variables and interactions are displayed. The default is to exclude from the table all base categories.

`baselevels` specifies that base levels be reported for factor variables and for interactions whose bases cannot be inferred from their component factor variables.

`allbaselevels` specifies that all base levels of factor variables and interactions be reported.

`nofvlabel` displays factor-variable level values rather than attached value labels. This option overrides the `fvlabel` setting; see [R] [set showbaselevels](#).

`fvwrap(#)` specifies how many lines to allow when long value labels must be wrapped. Labels requiring more than # lines are truncated. This option overrides the `fvwrap` setting; see [R] [set showbaselevels](#).

`fvwrapon(style)` specifies whether value labels that wrap will break at word boundaries or break based on available space.

`fvwrapon(word)`, the default, specifies that value labels break at word boundaries.

`fvwrapon(width)` specifies that value labels break based on available space.

This option overrides the `fvwrapon` setting; see [R] [set showbaselevels](#).

`cformat(%fmt)` specifies how to format coefficients, standard errors, and confidence limits in the coefficient table. The maximum format width is 9. See [R] [set cformat](#).

`pformat(%fmt)` specifies how to format *p*-values in the coefficient table. The maximum format width is 5. See [R] [set cformat](#).

`sformat(%fmt)` specifies how to format test statistics in the coefficient table. The maximum format width is 8. See [R] [set cformat](#).

### Integration

`intmethod(intmethod)` specifies the integration method to be used for the random-effects model. It accepts one of four arguments: `mvaghermite`, the default for all but a crossed random-effects model, performs mean and variance adaptive Gauss–Hermite quadrature; `mcaghermite` performs mode and curvature adaptive Gauss–Hermite quadrature; `ghermite` performs nonadaptive Gauss–Hermite quadrature; and `laplace`, the default for crossed random-effects models, performs the Laplacian approximation.

`intpoints(#)` specifies the number of integration points to use for integration by quadrature. The default is `intpoints(12)`; the maximum is `intpoints(195)`. Increasing this value improves the accuracy but also increases computation time. Computation time is roughly proportional to its value.

The following options are not shown in the dialog box:

`no1stretch` specifies that the width of the coefficient table not be automatically widened to accommodate longer variable names. The default, `1stretch`, is to automatically widen the coefficient table up to the width of the Results window. Specifying `1stretch` or `no1stretch` overrides the setting given by `set 1stretch`. If `set 1stretch` has not been set, the default is `1stretch`.

`collinear` specifies that the estimation command not omit collinear variables. This option is seldom used because collinear variables make a model unidentified. However, you can add constraints to a model that will identify it even with collinear variables. For example, if variables `x1` and `x2` are

`collinear`, but you constrain the coefficient on `x2` to be a multiple of the coefficient on `x1`, then your model is identified even with collinear variables. In such cases, you specify `collinear` so that both `x1` and `x2` are retained in the model.

`coeflegend` specifies that the legend of the coefficients and how to specify them in an expression be displayed rather than displaying the statistics for the coefficients.

## Also see

[U] [20 Estimation and postestimation commands](#)

## **etable** — Create a table of estimation results

|             |                 |            |          |         |                      |
|-------------|-----------------|------------|----------|---------|----------------------|
| Description | Quick start     | Menu       | Syntax   | Options | Remarks and examples |
| Appendix    | Acknowledgments | References | Also see |         |                      |

### **Description**

The `etable` command allows you to easily create a table of estimation results and export it to a variety of file types. You can create a table complete with a title, notes, stars for indicating significant results, and more.

### **Quick start**

Create a table from the active estimation results, reporting the coefficients, standard errors, and number of observations

```
etable
```

Same as above, and display stars for significant results and a note indicating what the stars represent

```
etable, showstars showstarsnote
```

Create a table with title “My title” and note “My note”

```
etable, title("My title") note("My note")
```

Create a table with stored estimates `model1` and `model2`, along with the number of observations and adjusted  $R^2$  value for each model

```
etable, estimates(model1 model2) mstat(N) mstat(r2_a)
```

Same as above, and export the table to `myfile.tex`

```
etable, estimates(model1 model2) mstat(N) mstat(r2_a) export(myfile.tex)
```

### **Menu**

Statistics > Summaries, tables, and tests > Table of estimation results

## Syntax

```
etable [, options]
```

| <i>options</i>                                                       | Description                                                 |
|----------------------------------------------------------------------|-------------------------------------------------------------|
| <b>Main</b>                                                          |                                                             |
| <code>estimates(<i>namelist</i>)</code>                              | work with previously stored estimation results              |
| <code>margins</code>                                                 | consume results from <code>margins</code>                   |
| <code>replay</code>                                                  | report table without consuming results                      |
| <code>column(<i>column_header</i>)</code>                            | select column header                                        |
| <code>name(<i>cname</i>)</code>                                      | work with collection <i>cname</i> ; default is name(ETable) |
| <code>append</code>                                                  | append to the collection                                    |
| <code>replace</code>                                                 | replace the collection                                      |
| <b>Coefficients</b>                                                  |                                                             |
| <code>keep(<i>coeflist</i>)</code>                                   | report coefficients in order specified                      |
| <code>cstat(<i>cstat</i> [ , <i>cstat_opts</i> ])</code>             | report coefficient statistic                                |
| <b>Model</b>                                                         |                                                             |
| <code>mstat(<i>mstat</i> [ , <i>mstat_opts</i> ])</code>             | report model statistic                                      |
| <b>Equations</b>                                                     |                                                             |
| <code>equations(<i>eqlist</i>)</code>                                | report equations in order specified                         |
| <code>eqrcode(<i>oldeq</i> = <i>neweq</i>)</code>                    | recode equation                                             |
| <code>[no]showeq</code>                                              | display or suppress equations                               |
| <b>Stars</b>                                                         |                                                             |
| <code>stars([<i>starspec</i>] [ , <i>stars_opts</i> ])</code>        | customize rules for star labels                             |
| <code>[no]showstars</code>                                           | display or suppress star labels                             |
| <code>[no]showstarsnote</code>                                       | display or suppress note explaining star labels             |
| <b>Title</b>                                                         |                                                             |
| <code>title(<i>string</i>)</code>                                    | add table title                                             |
| <code>titlestyles(<i>text_styles</i>)</code>                         | change table title styles                                   |
| <b>Notes</b>                                                         |                                                             |
| <code>note(<i>string</i>)</code>                                     | add table note                                              |
| <code>notestyles(<i>text_styles</i>)</code>                          | change table note styles                                    |
| <b>Export</b>                                                        |                                                             |
| <code>export(<i>filename.suffix</i> [ , <i>export_opts</i> ])</code> | export table                                                |
| <b>Options</b>                                                       |                                                             |
| <code>[no]varlabel</code>                                            | display or suppress variable names or labels                |
| <code>[no]fvlabel</code>                                             | display or suppress factor values or labels                 |
| <code>[no]center</code>                                              | center or right-align item cells                            |
| <code>label(<i>filename</i> [ , <i>replace</i> ])</code>             | specify the collection labels                               |
| <code>style(<i>filename</i> [ , <i>override</i> ])</code>            | specify the collection style                                |
| <code>warn</code>                                                    | show collect warnings                                       |

warn does not appear in the dialog box.

| <i>column_header</i>                                           | Description                                               |
|----------------------------------------------------------------|-----------------------------------------------------------|
| <code>depvar</code>                                            | show dependent variable name; the default                 |
| <code>dvlabel</code>                                           | show variable label for dependent variable                |
| <code>command</code>                                           | show command name                                         |
| <code>title</code>                                             | show command title                                        |
| <code>estimates</code>                                         | show estimates name                                       |
| <code>index</code>                                             | show result set index                                     |
| <hr/>                                                          |                                                           |
| <i>cstat_opts</i>                                              | Description                                               |
| <code>label(<i>string</i>)</code>                              | specify coefficient statistic label                       |
| <code>font([<i>fontfamily</i>] [, <i>font_opts</i>])</code>    | specify font style                                        |
| <code>smcl(<i>smcl</i>)</code>                                 | specify formatting for SMCL files                         |
| <code>latex(<i>latex</i>)</code>                               | specify L <sup>A</sup> T <sub>E</sub> X macro             |
| <code>shading(<i>sspec</i>)</code>                             | set background color, foreground color, and fill pattern  |
| <code>nformat(<i>%fmt</i>)</code>                              | specify numeric format                                    |
| <code>sformat(<i>sfmt</i>)</code>                              | specify string format                                     |
| <code>cidelimiter(<i>char</i>)</code>                          | use character as delimiter for confidence interval limits |
| <code>cridelimiter(<i>char</i>)</code>                         | use character as delimiter for credible interval limits   |
| <hr/>                                                          |                                                           |
| <i>mstat_opts</i>                                              | Description                                               |
| <code>label(<i>string</i>)</code>                              | specify model statistic label                             |
| <code>font([<i>fontfamily</i>][ [, <i>font_opts</i> ]])</code> | specify font style                                        |
| <code>smcl(<i>smcl</i>)</code>                                 | specify formatting for SMCL files                         |
| <code>latex(<i>latex</i>)</code>                               | specify L <sup>A</sup> T <sub>E</sub> X macro             |
| <code>shading(<i>sspec</i>)</code>                             | set background color, foreground color, and fill pattern  |
| <code>nformat(<i>%fmt</i>)</code>                              | specify numeric format                                    |
| <code>sformat(<i>sfmt</i>)</code>                              | specify string format                                     |
| <hr/>                                                          |                                                           |
| <i>font_opts</i>                                               | Description                                               |
| <code>size(# [<i>unit</i>])</code>                             | specify font size                                         |
| <code>color(<i>color</i>)</code>                               | specify font color                                        |
| <code>variant(<i>variant</i>)</code>                           | specify font variant and capitalization                   |
| <code>[no]bold</code>                                          | specify whether to format statistic as bold               |
| <code>[no]italic</code>                                        | specify whether to format statistic as italic             |
| <code>[no]strikeout</code>                                     | specify whether to strike out statistic                   |
| <code>underline(<i>upattern</i>)</code>                        | specify whether to underline statistic                    |

| <i>stars_opts</i>                   | Description                                                      |
|-------------------------------------|------------------------------------------------------------------|
| <code>attach(<i>cstat</i>)</code>   | attach star labels to coefficient statistic <i>cstat</i>         |
| <code>increasing</code>             | compose stars note with increasing <i>p</i> -values; the default |
| <code>decreasing</code>             | compose stars note with decreasing <i>p</i> -values              |
| <code>pvalue(<i>string</i>)</code>  | <i>p</i> -value name for stars note                              |
| <code>delimiter(<i>char</i>)</code> | use character as delimiter for labels in stars note              |
| <code>nformat(<i>%fmt</i>)</code>   | numeric format for stars note                                    |
| <code>prefix(<i>string</i>)</code>  | prefix for stars note                                            |
| <code>suffix(<i>string</i>)</code>  | suffix for stars note                                            |
| <code>clear</code>                  | remove previous star properties                                  |

| <i>text_styles</i>                                          | Description                                              |
|-------------------------------------------------------------|----------------------------------------------------------|
| <code>font([<i>fontfamily</i>] [, <i>text_opts</i>])</code> | font style                                               |
| <code>smcl(<i>smcl</i>)</code>                              | specify formatting for SMCL files                        |
| <code>latex(<i>latex</i>)</code>                            | specify L <sup>A</sup> T <sub>E</sub> X macro            |
| <code>shading(<i>sspec</i>)</code>                          | set background color, foreground color, and fill pattern |

| <i>text_opts</i>                     | Description                                   |
|--------------------------------------|-----------------------------------------------|
| <code>size(# [<i>unit</i>])</code>   | specify font size                             |
| <code>color(<i>color</i>)</code>     | specify font color                            |
| <code>variant(<i>variant</i>)</code> | specify font variant and capitalization       |
| <code>[no]bold</code>                | specify whether to format statistic as bold   |
| <code>[no]italic</code>              | specify whether to format statistic as italic |
| <code>[no]strikeout</code>           | specify whether to strike out statistic       |
| <code>[no]underline</code>           | specify whether to underline statistic        |

| <i>suffix</i>         | <i>fileformat</i>         | Output format                      |
|-----------------------|---------------------------|------------------------------------|
| <code>docx</code>     | <code>as(docx)</code>     | Microsoft Word                     |
| <code>html</code>     | <code>as(html)</code>     | HTML 5 with CSS                    |
| <code>pdf</code>      | <code>as(pdf)</code>      | PDF                                |
| <code>xlsx</code>     | <code>as(xlsx)</code>     | Microsoft Excel 2007/2010 or newer |
| <code>xls</code>      | <code>as(xls)</code>      | Microsoft Excel 1997/2003          |
| <code>tex</code>      | <code>as(tex)</code>      | L <sup>A</sup> T <sub>E</sub> X    |
| <code>smcl</code>     | <code>as(smcl)</code>     | SMCL                               |
| <code>txt</code>      | <code>as(txt)</code>      | plain text                         |
| <code>markdown</code> | <code>as(markdown)</code> | Markdown                           |
| <code>md</code>       | <code>as(md)</code>       | Markdown                           |

| <i>export_opts</i>              | Description                                         |
|---------------------------------|-----------------------------------------------------|
| <i>as</i> ( <i>fileformat</i> ) | specify document type                               |
| <i>replace</i>                  | overwrite existing file                             |
| <i>docx_options</i>             | available when exporting to .docx files             |
| <i>html_options</i>             | available when exporting to .html files             |
| <i>pdf_options</i>              | available when exporting to .pdf files              |
| <i>excel_options</i>            | available when exporting to .xls and .xlsx files    |
| <i>tex_options</i>              | available when exporting to .tex files              |
| <i>smcl_option</i>              | available when exporting to .smcl files             |
| <i>txt_option</i>               | available when exporting to .txt files              |
| <i>md_option</i>                | available when exporting to .markdown and .md files |

| <i>docx_options</i>                                   | Description                                                         |
|-------------------------------------------------------|---------------------------------------------------------------------|
| <i>noisily</i>                                        | show the putdocx commands used to export to the Microsoft Word file |
| <i>dofile</i> ( <i>filename</i> [ , <i>replace</i> ]) | save the putdocx commands used for exporting to the named do-file   |

| <i>html_options</i>               | Description                                                    |
|-----------------------------------|----------------------------------------------------------------|
| <i>append</i>                     | append to an existing file                                     |
| <i>tableonly</i>                  | export only the table to the specified file                    |
| <i>cssfile</i> ( <i>cssfile</i> ) | define the styles in <i>cssfile</i> instead of <i>filename</i> |
| <i>prefix</i> ( <i>prefix</i> )   | use <i>prefix</i> to identify style classes                    |

| <i>pdf_options</i>                                    | Description                                                      |
|-------------------------------------------------------|------------------------------------------------------------------|
| <i>noisily</i>                                        | show the putpdf commands used to export to the PDF file          |
| <i>dofile</i> ( <i>filename</i> [ , <i>replace</i> ]) | save the putpdf commands used for exporting to the named do-file |

| <i>excel_options</i>                                  | Description                                                                                              |
|-------------------------------------------------------|----------------------------------------------------------------------------------------------------------|
| <i>noisily</i>                                        | show the putexcel commands used to export to the Excel file                                              |
| <i>dofile</i> ( <i>filename</i> [ , <i>replace</i> ]) | save the putexcel commands used for exporting to the named do-file                                       |
| <i>sheet</i> ( <i>sheetname</i> [ , <i>replace</i> ]) | specify the worksheet to use; the default sheet name is Sheet1                                           |
| <i>cell</i> ( <i>cell</i> )                           | specify the Excel upper-left cell as the starting position to export the table; the default is cell (A1) |
| <i>modify</i>                                         | modify Excel file                                                                                        |
| <i>noopen</i>                                         | do not open Excel file in memory                                                                         |

*noopen* does not appear in the dialog box.

| <i>tex_options</i> | Description                                 |
|--------------------|---------------------------------------------|
| <i>append</i>      | append to an existing file                  |
| <i>tableonly</i>   | export only the table to the specified file |

| <i>smcl_option</i> | Description                |
|--------------------|----------------------------|
| append             | append to an existing file |

| <i>txt_option</i> | Description                |
|-------------------|----------------------------|
| append            | append to an existing file |

| <i>md_option</i> | Description                |
|------------------|----------------------------|
| append           | append to an existing file |

*fontfamily* specifies a valid font family.

*unit* may be in (inch), pt (point), or cm (centimeter). An inch is equivalent to 72 points and 2.54 centimeters. The default is pt.

*variant* may be allcaps, smallcaps, or normal.

*variant*(allcaps) changes the text to all uppercase letters; applicable when publishing items from a collection to Microsoft Word, PDF,  $\LaTeX$ , and HTML files.

*variant*(smallcaps) changes the text to use large capitals for uppercase letters and smaller capitals for lowercase letters; applicable when publishing items from a collection to Microsoft Word,  $\LaTeX$ , and HTML files.

*variant*(normal) changes the font variant back to normal and leaves the capitalization unchanged from the original text; applicable when publishing items from a collection to Microsoft Word, PDF,  $\LaTeX$ , and HTML files.

*upattern* may be any of the patterns listed in the [Appendix](#). For example, *underline*(none) removes the underline from the statistic, and *underline*(single) underlines the statistic. All other *upatterns* are available only when publishing items from a collection to Microsoft Word.

*smcl* specifies the name of the SMCL directive to render text for SMCL output. The supported SMCL directives are *input*, *error*, *result*, and *text*.

*latex* specifies the name of a  $\LaTeX$  macro to render text for  $\LaTeX$  output. Example  $\LaTeX$  macro names are *textbf*, *textsf*, *textrm*, and *texttt*. Custom  $\LaTeX$  macros are also allowed. If *text* is to be rendered in a cell, title, or note, then *latex* is translated to the following when you export to  $\LaTeX$ :

$$\backslash latex \{text\}$$

*sspec* is

[ *background*(*bcolor*) *foreground*(*fgcolor*) *pattern*(*fpattern*) ]

*bcolor* specifies the background color.

*fgcolor* specifies the foreground color.

*fpattern* specifies the fill pattern. A complete list of fill patterns is shown in the [Appendix](#).

*bgcolor*, *fgcolor*, and *color* may be one of the colors listed in the [Appendix](#); a valid RGB value in the form `### ## #`, for example, 171 248 103; or a valid RRGGBB hex value in the form `#####`, for example, ABF867.

*sfmt* is the specification for a string format in option `sformat()` and may contain a mix of text and `%s`. Here `%s` refers to the statistic value that is formatted as specified using `nformat()`. The text will be placed around the statistic as it is placed around `%s` in this option. For instance, to place parentheses around the statistic, you can specify `sformat("(%s")`.

Two text characters must be specified using a special character sequence if you want them to be displayed in your collection. To include `%`, type `%`. To include `\`, type `\\`. For instance, to place a percent sign after a statistic, you can specify `sformat("%s%")`.

## Options

Main

`estimates(namelist)` specifies the estimation results to be included in the table. These are the names specified with `estimates store`. By default, `etable` creates a table with the active estimation results.

`margins` creates a table with the results of the immediately preceding `margins` command.

`replay` specifies that `etable` redisplays the table without consuming results.

`column(column_header)` specifies the content to be used in the column headers. *column\_header* may be `depvar`, `dvlabeled`, `command`, `title`, `estimates`, or `index`.

`depvar` specifies that `etable` use the dependent variable name for the column headers. This name is obtained from the `eclass` macro `e(depvar)`. Note that this macro may contain multiple names after fitting a multivariate model.

`dvlabeled` specifies that `etable` use the variable label for the dependent variable name for the column headers. `etable` uses the variable label associated with the variable name in the `eclass` macro `e(depvar)`. If the variable does not have a variable label, the variable name will be used. `column(dvlabeled)` will not be helpful when the estimation command stores multiple names in `e(depvar)`.

`command` specifies that `etable` use the command name for the column headers. This name is obtained from the `eclass` macro `e(cmd)`.

`title` specifies that `etable` use the command title for the column headers. This title is obtained from the `eclass` macro `e(title)`.

`estimates` specifies that `etable` use the name given to previously stored estimation results for the column headers; these are the names specified with `estimates store`.

`index` specifies that `etable` use the result set index for the column headers.

`name(cname)` specifies the collection from which estimation results will be obtained, instead of the current collection. The default is `name(ETable)`.

`append` specifies that `etable` append the results into the collection named in `name()`.

`replace` permits `etable` to overwrite the existing collection. This option is implied for `name(ETable)` when `append` and `replay` are not specified.

## Coefficients

`keep(coeflist)` specifies the coefficients to be included in the table and the order in which to display them. Names are separated from each other by blanks. The default is to display all coefficients.

`cstat(cstat [, cstat_opts])` specifies the coefficient statistic to be included in the table. Optionally, you may specify the label and style for this statistic. `cstat()` may be repeated to request multiple coefficient statistics.

The default is to display the coefficients (`_r_b`) and their standard errors (`_r_se`), both formatted to three decimals. Standard errors are enclosed in parentheses.

Available coefficient statistics are

| <i>cstat</i>          | Description                                                      |
|-----------------------|------------------------------------------------------------------|
| <code>_r_b</code>     | coefficients reported by estimation                              |
| <code>_r_se</code>    | standard errors of <code>_r_b</code>                             |
| <code>_r_z</code>     | test statistics for <code>_r_b</code>                            |
| <code>_r_z_abs</code> | absolute values of <code>_r_z</code>                             |
| <code>_r_p</code>     | <i>p</i> -values for <code>_r_b</code>                           |
| <code>_r_lb</code>    | lower bounds of confidence intervals (CIs) for <code>_r_b</code> |
| <code>_r_ub</code>    | upper bounds of CIs for <code>_r_b</code>                        |
| <code>_r_ci</code>    | CIs for <code>_r_b</code>                                        |
| <code>_r_crlb</code>  | lower bounds of credible intervals for <code>_r_b</code>         |
| <code>_r_crub</code>  | upper bounds of credible intervals for <code>_r_b</code>         |
| <code>_r_cri</code>   | credible intervals of Bayesian estimates                         |
| <code>_hide</code>    | hide coefficient statistics                                      |

*cstat\_opts* are `label(string)`, `font([fontfamily] [, font_opts])`, `smcl(smcl)`, `latex(latex)`, `shading(sspec)`, `nformat(%fmt)`, `sformat(sfmt)`, `cidelimiter(char)`, and `cridelimiter(char)`.

`label(string)` is used to modify the label for the specified coefficient statistic.

`font([fontfamily] [, size(# [unit]) color(color) variant(variant) [no]bold [no]italic [no]strikeout [no]underline underline(upattern))` specifies the font style for the coefficient statistic. These font style properties are applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and L<sup>A</sup>T<sub>E</sub>X files, unless otherwise specified.

*fontfamily* specifies a valid font family. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

*size*(# [*unit*]) specifies the font size as a number optionally followed by units. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

*color*(*color*) specifies the text color.

*variant*(*variant*) specifies the font variant and capitalization.

`bold` and `nobold` specify the font weight. `bold` changes the font weight to bold; `nobold` changes the font weight back to normal.

`italic` and `noitalic` specify the font style. `italic` changes the font style to italic; `noitalic` changes the font style back to normal.

`strikeout` and `nostrikeout` specify whether to add a strikeout mark to the coefficient statistic. `strikeout` adds a strikeout mark to the statistic; `nostrikeout` changes the statistic back to normal.

`underline` (*upattern*), `underline`, and `nounderline` specify how to underline the coefficient statistic.

Only one of `strikeout` or `underline` is allowed when publishing to HTML files.

`smcl` (*smcl*) specifies how to render the statistic value for SMCL output. This style property is applicable only when publishing items from a collection to a SMCL file.

`latex` (*latex*) specifies how to render the statistic value for L<sup>A</sup>T<sub>E</sub>X output. This style property is applicable only when publishing items from a collection to a L<sup>A</sup>T<sub>E</sub>X file.

`shading` (*sspec*) sets the background color, foreground color, and fill pattern. The background color is applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and L<sup>A</sup>T<sub>E</sub>X files. The foreground color and fill pattern are applicable when exporting the table to Microsoft Word and Microsoft Excel.

`nformat` (*%fmt*) applies the Stata numeric format *%fmt* to the coefficient statistic.

`sformat` (*sfmt*) applies a string format to the coefficient statistic.

`cidelimiter` (*char*) changes the delimiter between confidence interval limits for coefficient statistic `_r_ci`. The default is `cidelimiter(" ")`.

`cridelimiter` (*char*) changes the delimiter between credible interval limits for coefficient statistic `_r_cri`. The default is `cridelimiter(" ")`.

---

#### Model

`mstat` (*mstat* [, *mstat\_opts*]) specifies the model statistics to be included in the table.

*mstat* may be a *result identifier* or a *named expression*. `mstat()` may be repeated to request multiple model statistics. The default is to display the number of observations, with zero decimal digits.

*result identifier* is one of the following:

| Identifier         | Result                                          |
|--------------------|-------------------------------------------------|
| N                  | number of observations                          |
| aic                | Akaike's information criteria                   |
| bic                | Schwarz's Bayesian information criteria         |
| F                  | <i>F</i> statistic                              |
| chi2               | $\chi^2$                                        |
| ll                 | log likelihood of fitted model                  |
| r2                 | $R^2$                                           |
| r2_a               | adjusted $R^2$                                  |
| rank               | rank of fitted model                            |
| <i>escalar</i>     | any <code>e()</code> scalar, by default         |
| <i>rscalar</i>     | any <code>r()</code> scalar with option margins |
| <code>_hide</code> | hide model statistics                           |

*named expression* is specified as *name* = *exp*, where *name* may be any valid Stata name and *exp* is a scalar expression, typically an expression that involves one or more scalars in `e()`. For example, `aic=(-2*e(ll) + 2*e(rank))`.

With option `margin`, or if you ran a postestimation command like `estat, exp` may also involve one or more scalars in `r()`. For example, suppose you fit a logistic model and also called `estat gof`, then you can show its goodness-of-fit statistic with `chi2_gof=(r(chi2))`.

`mstat_opts` are `label(string)`, `font([fontfamily] [, font_opts])`, `smcl(smcl)`, `latex(latex)`, `shading(sspec)`, `nformat(%fmt)`, and `sformat(sfmt)`.

`label(string)` is used to modify the label for the specified model statistic.

`font([fontfamily] [, size(# [unit]) color(color) variant(variant) [no]bold [no]italic [no]strikeout [no]underline underline(upattern)])` specifies the font style for the model statistic. These font style properties are applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files, unless otherwise specified.

`fontfamily` specifies a valid font family. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`size(# [unit])` specifies the font size as a number optionally followed by units. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`color(color)` specifies the text color.

`variant(variant)` specifies the font variant and capitalization.

`bold` and `nobold` specify the font weight. `bold` changes the font weight to bold; `nobold` changes the font weight back to normal.

`italic` and `noitalic` specify the font style. `italic` changes the font style to italic; `noitalic` changes the font style back to normal.

`strikeout` and `nostrikeout` specify whether to add a strikeout mark to the model statistic. `strikeout` adds a strikeout mark to the statistic; `nostrikeout` changes the text back to normal.

`underline(upattern)`, `underline`, and `nunderline` specify how to underline the model statistic.

Only one of `strikeout` or `underline` is allowed when publishing to HTML files.

`smcl(smcl)` specifies how to render the statistic value for SMCL output. This style property is applicable only when publishing items from a collection to a SMCL file.

`latex(latex)` specifies how to render the statistic value for  $\LaTeX$  output. This style property is applicable only when publishing items from a collection to a  $\LaTeX$  file.

`shading(sspec)` sets the background color, foreground color, and fill pattern.

`nformat(%fmt)` applies the Stata numeric format `%fmt` to the model statistic.

`sformat(sfmt)` applies a string format to the model statistic.

#### Equations

`equations(eqlist)` specifies the equations to be included in the table and the order in which they are reported.

`eqrcode(oldeq = neweq)` changes the equation name from `oldeq` to `neweq`. `eqrcode()` may be repeated to recode multiple equations.

`showeq` and `nshoweq` specify whether equations should be displayed. `showeq` displays the equations; `nshoweq` suppresses the equations.

---

#### Stars

`stars([#1 "label1" [#2 "label2" [#3 "label3" [#4 "label4" [#5 "label5" ]]]])`  
`[, stars_opts]` manages the display of stars for indicating the significance of results.

The default is `stars(.05 "*" .01 "**", attach(_r_b))`, which will display one star (\*) for  $p$ -values less than 0.05 and two stars (\*\*) for  $p$ -values less than 0.01; the stars will be attached to the coefficients (`_r_b`).

`stars_opts` are `attach(cstat)`, `increasing`, `decreasing`, `pvname(string)`, `delimiter(char)`, `nformat(%fmt)`, `prefix(string)`, `suffix(string)`, and `clear`.

`attach(cstat)` specifies that the star labels be attached to coefficient statistic `cstat`. The default is `attach(_r_b)`.

`increasing` and `decreasing` control the order of  $p$ -values in the stars note.

`increasing` specifies that the stars note be composed with increasing  $p$ -values. This is the default.

`decreasing` specifies that the stars note be composed with decreasing  $p$ -values.

`pvname(string)` specifies the name for the  $p$ -value in the stars note that is displayed with `showstarsnote`. The default is `pvname(p)`.

`delimiter(char)` changes the delimiter between labels in the stars note. The default is `cridelimiter(",")`.

`nformat(%fmt)` specifies the numeric format for the numbers displayed in the stars note. The default is `nformat(%9.0g)`.

`prefix(string)` adds a prefix to the stars note.

`suffix(string)` adds a suffix to the stars note.

`clear` removes existing star properties.

`showstars` and `nshowstars` specify whether star labels should be displayed. `showstars` displays star labels; `nshowstars` suppresses the star labels.

`showstarsnote` and `nshowstarsnote` specify whether to display the note explaining what the star labels represent. `showstarsnote` displays the note; `nshowstarsnote` suppresses the note.

`showstarsnote` is ignored if `nshowstars` is in effect.

---

#### Title

`title(string)` adds the text `string` as a title to the table.

`titlestyles(text_styles)` changes the style for the table title. `text_styles` are the following:

`font([fontfamily] [, size(# [unit]) color(color) variant(variant) [no]bold`  
`[no]italic [no]strikeout [no]underline])` specifies the font style. These font style properties are applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and L<sup>A</sup>T<sub>E</sub>X files, unless otherwise specified.

`fontfamily` specifies a valid font family. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`size(# [unit])` specifies the font size as a number optionally followed by units. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`color(color)` specifies the text color.

`variant(variant)` specifies the font variant and capitalization.

`bold` and `nobold` specify the font weight. `bold` changes the font weight to bold; `nobold` changes the font weight back to normal.

`italic` and `noitalic` specify the font style. `italic` changes the font style to italic; `noitalic` changes the font style back to normal.

`strikeout` and `nostrikeout` specify whether to add a strikeout mark to the title. `strikeout` adds a strikeout mark to the title; `nostrikeout` changes the title back to normal.

`underline` and `nounderline` specify whether to underline the table title. `underline` adds a single line under the title; `nounderline` removes the underline.

Only one of `strikeout` or `underline` is allowed when publishing to HTML files.

`smcl(smcl)` specifies how to render the table title for SMCL output. This style property is applicable only when publishing items from a collection to a SMCL file.

`latex(latex)` specifies how to render the table title for L<sup>A</sup>T<sub>E</sub>X output. This style property is applicable only when publishing items from a collection to a L<sup>A</sup>T<sub>E</sub>X file.

`shading(sspec)` sets the background color, foreground color, and fill pattern. The background color is applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and L<sup>A</sup>T<sub>E</sub>X files. The foreground color and fill pattern are applicable when exporting the table to Microsoft Word and Microsoft Excel.

#### Notes

`note(string)` adds the text *string* as a note to the table. `note()` may be specified multiple times to add multiple notes. Each note is placed on a new line.

`notestyles(text_styles)` changes the style for the table notes. *text\_styles* are the following:

`font([fontfamily] [, size(# [unit]) color(color) variant(variant) [no]bold [no]italic [no]strikeout [no]underline])` specifies the font style. These font style properties are applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and L<sup>A</sup>T<sub>E</sub>X files, unless otherwise specified.

*fontfamily* specifies a valid font family. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`size(# [unit])` specifies the font size as a number optionally followed by units. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`color(color)` specifies the text color.

`variant(variant)` specifies the font variant and capitalization.

`bold` and `nobold` specify the font weight. `bold` changes the font weight to bold; `nobold` changes the font weight back to normal.

*italic* and *noitalic* specify the font style. *italic* changes the font style to italic; *noitalic* changes the font style back to normal.

*strikeout* and *nostrikeout* specify whether to add a strikeout mark to the notes. *strikeout* adds a strikeout mark to the note; *nostrikeout* changes the note back to normal.

*underline* and *nounderline* specify whether to underline the table notes. *underline* adds a single line under the notes; *nounderline* removes the underline.

Only one of *strikeout* or *underline* is allowed when publishing to HTML files.

*smcl* (*smcl*) specifies how to render the table notes for SMCL output. This style property is applicable only when publishing items from a collection to a SMCL file.

*latex* (*latex*) specifies how to render the table notes for L<sup>A</sup>T<sub>E</sub>X output. This style property is applicable only when publishing items from a collection to a L<sup>A</sup>T<sub>E</sub>X file.

*shading* (*spec*) sets the background color, foreground color, and fill pattern. The background color is applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and L<sup>A</sup>T<sub>E</sub>X files. The foreground color and fill pattern are applicable when exporting the table to Microsoft Word and Microsoft Excel.

#### Export

`export(filename.suffix[ , export_opts ])` exports the table to the specified file. *export\_opts* are the following:

*as* (*fileformat*) specifies the file format to which the table is to be exported. This option is rarely specified because, by default, *etable* determines the format from the suffix of the file being created.

*replace* permits *etable* to overwrite an existing file.

*noisily* specifies that *etable* show the commands used to export the table to Microsoft Word, Microsoft Excel, and PDF files. The `putdocx`, `putexcel`, or `putpdf` command used to export the table will be displayed.

*dofile* (*filename*[ , *replace* ]) specifies that *etable* save to *filename* the commands used to export the table to Microsoft Word, Microsoft Excel, and PDF files.

If *filename* already exists, it can be overwritten by specifying *replace*. If *filename* is specified without an extension, `.do` is assumed.

*append* specifies that *etable* append the table to an existing file.

This option is applicable when you export the table to an HTML, a L<sup>A</sup>T<sub>E</sub>X, a SMCL, a `txt`, or a Markdown file. When you export to HTML and L<sup>A</sup>T<sub>E</sub>X files, the *append* option implies the *tableonly* option. Furthermore, when you export to HTML files, if the target CSS file already exists, *etable* will also append to it.

*tableonly* specifies that only the table be exported to the specified HTML or L<sup>A</sup>T<sub>E</sub>X document. With this option, the produced file may be included in other HTML or L<sup>A</sup>T<sub>E</sub>X documents. By default, *etable* produces complete HTML and L<sup>A</sup>T<sub>E</sub>X documents.

When you export to an HTML file, if the `cssfile()` option is not specified, a CSS filename is constructed from *filename*, with the extension replaced with `.css`.

`cssfile(cssfile)` specifies that `etable` define the styles in *cssfile* instead of *filename* when you export to HTML.

`prefix(prefix)` specifies that `etable` use *prefix* to identify style classes when you export to HTML.

`sheet(sheetname [, replace])` saves to the worksheet named *sheetname*. For more information about this option, see [RPT] [putexcel](#).

`cell(cell)` specifies an Excel upper-left cell as the starting position to publish the table. The default is `cell(A1)`.

`modify` permits `putexcel` set to modify an Excel file. For more information about this option, see [RPT] [putexcel](#).

`noopen` prevents `putexcel` from opening the Excel file in memory for modification. It does not appear in the dialog box. For more information about this option, see [RPT] [putexcel](#). This option is necessary only when you need to force `etable` to produce do-files as it did when `etable` was first introduced in Stata 17.

---

#### Options

`varlabel` and `novarlabel` specify whether variable labels should be displayed. `varlabel` displays variable labels; `novarlabel` displays variable names.

`fvlabel` and `nofvlabel` specify whether value labels should be displayed. `fvlabel` displays value labels; `nofvlabel` displays the values of the factor variable.

`center` and `nocenter` specify how item cells are horizontally aligned. `center` specifies that item cells are centered; `nocenter` specifies that item cells are right-aligned.

`label(filename[, replace])` specifies the *filename* containing the collection labels to use for your table. Labels in *filename* will be loaded for the table, and default labels will be used for any labels not specified in *filename*.

If you prefer to replace the labels used by `etable` with those specified in *filename*, specify `replace`. The `etable` labels will be discarded, and only the labels in *filename* will be applied.

`style(filename[, override])` specifies the *filename* containing the collection styles to use for your table. This might be a style you saved with [collect style save](#) or a [predefined style](#) shipped with Stata. The `etable` collection styles will be discarded, and only the collection styles in *filename* will be applied. Note that the layout specification saved in *filename* will not be applied; `etable` will always use its predefined layout.

If you prefer the `etable` collection styles but also want to apply any styles in *filename*, specify `override`. If there are conflicts between the default collection styles and those in *filename*, the ones in *filename* will take precedence.

The default is to use only the collection styles set in `c(etable_style)`; see [TABLES] [set etable\\_style](#).

The following option is available with `etable` but is not shown in the dialog box:

`warn` specifies that `etable` display warnings from `collect`. By default, these warnings are suppressed.

## Remarks and examples

Remarks are presented under the following headings:

*Introduction*  
*A first example*  
*Table comparing regression results*  
*Multiple-equation models*

### Introduction

`etable` allows you to easily create a table of estimation results and export it to a variety of file types, without any knowledge of the collection system. You can make a standard estimation table with the active estimation results, results from a `margins` command, or with stored estimates. You can also customize the table by formatting the results, adding model statistics and coefficient-specific statistics, labeling statistically significant results, adding a title and notes, and more.

In most cases, you will use `etable` to easily create a table and export it to another format. However, you can customize the table beyond the options that are available with `etable`. When you issue an `etable` command, the results are stored in a collection called `ETable`. This collection is replaced with each new `etable` command, unless you specify the `append` or `replay` option. You can make additional changes to the collection with the `collect` suite of commands. To learn more about the `collect` commands, see [TABLES] [Intro](#) and the entries discussed therein.

### A first example

In its simplest specification, you type `etable` after fitting a model, and you get a table with coefficients, standard errors, and the number of observations. For example, below, we use data from the Second National Health and Nutrition Examination Survey (NHANES II) (McDowell et al. 1981). We fit a simple model for systolic blood pressure and then create our table of estimation results:

```
. use https://www.stata-press.com/data/r19/nhanes21
(Second National Health and Nutrition Examination Survey)
. quietly: regress bpsystol age weight i.region
. etable
```

|                        | bpsystol          |
|------------------------|-------------------|
| Age (years)            | 0.638<br>(0.011)  |
| Weight (kg)            | 0.407<br>(0.012)  |
| Region                 |                   |
| MW                     | -0.240<br>(0.564) |
| S                      | -0.619<br>(0.560) |
| W                      | -0.862<br>(0.570) |
| Intercept              | 71.708<br>(1.108) |
| Number of observations | 10351             |

You can also include statistics that pertain to the coefficients, such as test statistics and confidence intervals, and model statistics, such as the  $R^2$  value and the  $F$  statistic. You can look at the `cstat()` and `mstat()` options for additional statistics.

Additionally, you can complete your table with a title, notes, and labels for significant results. For example, below, we add a title to our table, and we display stars for statistically significant results:

```
. etable, title(Model for systolic blood pressure) showstars showstarsnote
Model for systolic blood pressure
```

|                        | bpsystol             |
|------------------------|----------------------|
| Age (years)            | 0.638 **<br>(0.011)  |
| Weight (kg)            | 0.407 **<br>(0.012)  |
| Region                 |                      |
| MW                     | -0.240<br>(0.564)    |
| S                      | -0.619<br>(0.560)    |
| W                      | -0.862<br>(0.570)    |
| Intercept              | 71.708 **<br>(1.108) |
| Number of observations | 10351                |

\*\* p<.01, \* p<.05

The `showstars` option displays stars next to the coefficients that are significant either at the 1% or 5% levels, and `showstarsnote` adds the note we see at the bottom, explaining what the stars represent. You can look at the `stars()` option to create your own rules for displaying stars or to specify your own labels for significance.

Suppose we have finalized our table and we are ready to export it to another format. Below, we export our table to the file `mytable.html`:

```
. etable, title(Model for systolic blood pressure)
> showstars showstarsnote export(mytable.html)
```

```
Model for systolic blood pressure
```

|                        | bpsystol             |
|------------------------|----------------------|
| Age (years)            | 0.638 **<br>(0.011)  |
| Weight (kg)            | 0.407 **<br>(0.012)  |
| Region                 |                      |
| MW                     | -0.240<br>(0.564)    |
| S                      | -0.619<br>(0.560)    |
| W                      | -0.862<br>(0.570)    |
| Intercept              | 71.708 **<br>(1.108) |
| Number of observations | 10351                |

\*\* p<.01, \* p<.05  
(collection ETable exported to file mytable.html)

We could also export this table to a Microsoft Word, Microsoft Excel,  $\LaTeX$ , Markdown, SMCL, PDF, or plain text file by specifying the appropriate file extension.

## Table comparing regression results

If your goal is to create a table comparing regression results, you can store the results from each model with `estimates store` and then specify which of those models you want to include in your table with the `estimates()` option.

For example, below, we fit two different models for systolic blood pressure and store them under the names `model1` and `model2`.

```
. quietly: regress bpsystol i.sex weight
. estimates store model1
. quietly: regress bpsystol i.sex i.agegrp weight
. estimates store model2
```

To include results from both of these models in our table, we specify `estimates(model1 model2)`; the models are presented in the order we list them. Additionally, we report the number of observations and the  $R^2$  adjusted for degrees of freedom.

```
. etable, estimates(model1 model2) mstat(N) mstat(r2_a)
```

|                        | bpsystol          | bpsystol          |
|------------------------|-------------------|-------------------|
| Sex                    |                   |                   |
| Female                 | 1.420<br>(0.475)  | 1.041<br>(0.415)  |
| Weight (kg)            | 0.452<br>(0.015)  | 0.436<br>(0.014)  |
| Age group              |                   |                   |
| 30–39                  |                   | 1.195<br>(0.633)  |
| 40–49                  |                   | 7.252<br>(0.684)  |
| 50–59                  |                   | 15.942<br>(0.681) |
| 60–69                  |                   | 22.839<br>(0.546) |
| 70+                    |                   | 30.466<br>(0.741) |
| Intercept              | 97.634<br>(1.246) | 86.710<br>(1.116) |
| Number of observations | 10351             | 10351             |
| Adjusted R-squared     | 0.08              | 0.30              |

We would like to make a few changes to finalize this table. First, because both models have the same dependent variable, we want to display the index of result sets instead of the variable name. Second, we add stars for significance and a note explaining what the stars represent. Third, instead of reporting standard errors, we want to report confidence intervals (`_r_ci`). We format the intervals with one decimal place and use a comma as the delimiter. `etable` will automatically report coefficients, unless you specify `cstat()`, in which case it will report only the coefficient statistics you specify; therefore, we add `cstat(_r_b)`. Finally, we add a title to our table:

```
. etable, estimates(model1 model2) mstat(N) mstat(r2_a) column(index)
> showstars showstarsnote cstat(_r_b)
> title("Models for systolic blood pressure")
> cstat(_r_ci, nformat(%6.1f) cidelimiter(", "))
Models for systolic blood pressure
```

|                        | 1             | 2            |
|------------------------|---------------|--------------|
| Sex                    |               |              |
| Female                 | 1.420 **      | 1.041 *      |
|                        | [0.5, 2.4]    | [0.2, 1.9]   |
| Weight (kg)            | 0.452 **      | 0.436 **     |
|                        | [0.4, 0.5]    | [0.4, 0.5]   |
| Age group              |               |              |
| 30-39                  |               | 1.195        |
|                        |               | [-0.0, 2.4]  |
| 40-49                  |               | 7.252 **     |
|                        |               | [5.9, 8.6]   |
| 50-59                  |               | 15.942 **    |
|                        |               | [14.6, 17.3] |
| 60-69                  |               | 22.839 **    |
|                        |               | [21.8, 23.9] |
| 70+                    |               | 30.466 **    |
|                        |               | [29.0, 31.9] |
| Intercept              | 97.634 **     | 86.710 **    |
|                        | [95.2, 100.1] | [84.5, 88.9] |
| Number of observations | 10351         | 10351        |
| Adjusted R-squared     | 0.08          | 0.30         |

\*\* p<.01, \* p<.05

Now our table is complete.

An alternative way to create this table is to build up the estimation table with the `append` option. For example, we can fit the first model and create the table. Then, after fitting the second model, we append the results as follows:

```
. quietly: regress bpsystol i.sex weight
. etable
. quietly: regress bpsystol i.sex i.agegrp weight
. etable, append mstat(N) mstat(r2_a) column(index)
> showstars showstarsnote cstat(_r_b)
> title("Models for systolic blood pressure")
> cstat(_r_ci, nformat(%6.1f) cidelimiter(", "))
(output omitted)
```

## Multiple-equation models

When you work with multiple-equation models, there is an additional option that will prove useful when creating the table of estimation results. For example, below, we fit a multivariate regression with `mvreg`:

```
. mvreg bpsystol bpdiaast = age weight
```

| Equation | Obs    | Parms | RMSE     | "R-sq" | F        | P>F    |
|----------|--------|-------|----------|--------|----------|--------|
| bpsystol | 10,351 | 3     | 19.48051 | 0.3031 | 2250     | 0.0000 |
| bpdiaast | 10,351 | 3     | 11.51474 | 0.2067 | 1348.469 | 0.0000 |

|                 | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |
|-----------------|-------------|-----------|-------|-------|----------------------|
| <b>bpsystol</b> |             |           |       |       |                      |
| age             | .6379892    | .0111315  | 57.31 | 0.000 | .6161692 .6598091    |
| weight          | .4069041    | .0124786  | 32.61 | 0.000 | .3824435 .4313646    |
| _cons           | 71.27096    | 1.041742  | 68.42 | 0.000 | 69.22894 73.31297    |
| <b>bpdiaast</b> |             |           |       |       |                      |
| age             | .187733     | .0065797  | 28.53 | 0.000 | .1748355 .2006306    |
| weight          | .3116502    | .007376   | 42.25 | 0.000 | .2971918 .3261086    |
| _cons           | 50.37585    | .615764   | 81.81 | 0.000 | 49.16884 51.58287    |

Next, we create our table of estimation results:

```
. etable
```

|                        | bpsystol | bpdiaast |
|------------------------|----------|----------|
| Age (years)            | 0.638    |          |
|                        | (0.011)  |          |
| Weight (kg)            | 0.407    |          |
|                        | (0.012)  |          |
| Intercept              | 71.271   |          |
|                        | (1.042)  |          |
| Age (years)            | 0.188    |          |
|                        | (0.007)  |          |
| Weight (kg)            | 0.312    |          |
|                        | (0.007)  |          |
| Intercept              | 50.376   |          |
|                        | (0.616)  |          |
| Number of observations | 10351    |          |

The results for both models are placed in a single column, so below we add the `showeq` option to display the equation names (`bpsystol` and `bpdiast`). This will help us identify which results correspond to each model. Note that there are two dependent variables in this model, and both variable names are displayed in the column header. These names are collected from the returned result `e(depvar)`. Instead of displaying both names, we will display the index of result sets we have collected for our table by typing `command(index)`.

```
. etable, showeq column(index)
```

|                          | 1                 |
|--------------------------|-------------------|
| Systolic blood pressure  |                   |
| Age (years)              | 0.638<br>(0.011)  |
| Weight (kg)              | 0.407<br>(0.012)  |
| Intercept                | 71.271<br>(1.042) |
| Diastolic blood pressure |                   |
| Age (years)              | 0.188<br>(0.007)  |
| Weight (kg)              | 0.312<br>(0.007)  |
| Intercept                | 50.376<br>(0.616) |
| Number of observations   | 10351             |

## Appendix

### Colors

*bgcolor*, *fgcolor*, and *color*

---

|                |                      |                   |               |
|----------------|----------------------|-------------------|---------------|
| aliceblue      | darkslategray        | lightsalmon       | palevioletred |
| antiquewhite   | darkturquoise        | lightseagreen     | papayawhip    |
| aqua           | darkviolet           | lightskyblue      | peachpuff     |
| aquamarine     | deeppink             | lightslategray    | peru          |
| azure          | deepskyblue          | lightsteelblue    | pink          |
| beige          | dimgray              | lightyellow       | plum          |
| bisque         | dodgerblue           | lime              | powderblue    |
| black          | firebrick            | limegreen         | purple        |
| blanchedalmond | floralwhite          | linen             | red           |
| blue           | forestgreen          | magenta           | rosybrown     |
| blueviolet     | fuchsia              | maroon            | royalblue     |
| brown          | gainsboro            | mediumaquamarine  | saddlebrown   |
| burlywood      | ghostwhite           | mediumblue        | salmon        |
| cadetblue      | gold                 | mediumorchid      | sandybrown    |
| chartreuse     | goldenrod            | mediumpurple      | seagreen      |
| chocolate      | gray                 | mediumseagreen    | seashell      |
| coral          | green                | mediumslateblue   | sienna        |
| cornflowerblue | greenyellow          | mediumspringgreen | silver        |
| cornsilk       | honeydew             | mediumturquoise   | skyblue       |
| crimson        | hotpink              | mediumvioletred   | slateblue     |
| cyan           | indianred            | midnightblue      | slategray     |
| darkblue       | indigo               | mintcream         | snow          |
| darkcyan       | ivory                | mistyrose         | springgreen   |
| darkgoldenrod  | khaki                | moccasin          | steelblue     |
| darkgray       | lavender             | navajowhite       | tan           |
| darkgreen      | lavenderblush        | navy              | teal          |
| darkkhaki      | lawngreen            | oldlace           | thistle       |
| darkmagenta    | lemonchiffon         | olive             | tomato        |
| darkolivegreen | lightblue            | olivedrab         | turquoise     |
| darkorange     | lightcoral           | orange            | violet        |
| darkorchid     | lightcyan            | orangered         | wheat         |
| darkred        | lightgoldenrodyellow | orchid            | white         |
| darksalmon     | lightgray            | palegoldenrod     | whitesmoke    |
| darkseagreen   | lightgreen           | palegreen         | yellow        |
| darkslateblue  | lightpink            | paleturquoise     | yellowgreen   |

---

## Underline patterns

*upattern*


---

|             |                 |
|-------------|-----------------|
| none        | dashLong        |
| single      | dashLongHeavy   |
| words       | dotDash         |
| double      | dashDotHeavy    |
| thick       | dotDotDash      |
| dotted      | dashDotDotHeavy |
| dottedHeavy | wave            |
| dash        | wavyHeavy       |
| dashedHeavy | wavyDouble      |

---

## Shading patterns

*fpattern*


---

|                       |       |
|-----------------------|-------|
| nil                   | pct20 |
| clear                 | pct25 |
| solid                 | pct30 |
| horzStripe            | pct35 |
| vertStripe            | pct37 |
| reverseDiagStripe     | pct40 |
| diagStripe            | pct45 |
| horzCross             | pct50 |
| diagCross             | pct55 |
| thinHorzStripe        | pct60 |
| thinVertStripe        | pct62 |
| thinReverseDiagStripe | pct65 |
| thinDiagStripe        | pct70 |
| thinHorzCross         | pct75 |
| thinDiagCross         | pct80 |
| pct5                  | pct85 |
| pct10                 | pct87 |
| pct12                 | pct90 |
| pct15                 | pct95 |

---

## Acknowledgments

We thank Roger Newson of the King's College London, UK, Ben Jann of the Institute of Sociology at the University of Bern, Switzerland, and John Luke Gallup of Portland State University for writing similar commands for exporting tables of estimation results.

## References

- McDowell, A., A. Engel, J. T. Massey, and K. Maurer. 1981. “Plan and operation of the Second National Health and Nutrition Examination Survey, 1976–1980”. In *Vital and Health Statistics*, ser. 1, no. 15. Hyattsville, MD: National Center for Health Statistics.
- Mitchell, M. N. 2025. *Create and Export Tables Using Stata*. College Station, TX: Stata Press.

## Also see

- [\[R\] table intro](#) — Introduction to tables of frequencies, summaries, and command results
- [\[TABLES\] Intro](#) — Introduction

## Description

Typing `exit` causes Stata to stop processing and return control to the operating system. If the dataset in memory has changed since the last `save` command, you must specify the `clear` option before Stata will let you exit.

If you wish to use `exit` in do-files or programs to set return codes or terminate programs, see [P] [exit](#).

Stata for Windows users may also exit Stata by clicking on the **Close** button or by pressing `Alt+F4`.

Stata for Mac users may also exit Stata by pressing `Command+Q`.

Stata for Unix(GUI) users may also exit Stata by clicking on the **Close** button.

## Syntax

```
exit [, clear]
```

## Option

`clear` permits you to `exit`, even if the current dataset has not been saved.

## Remarks and examples

Type `exit` to leave Stata and return to the operating system. If the dataset in memory has changed since the last time it was saved, however, Stata will refuse. At that point, you can either save the dataset and then type `exit`, or type `exit, clear`:

```
. exit
no; dataset in memory has changed since last saved
r(4);
. exit, clear
```

## Also see

[P] [exit](#) — Exit from a program or do-file

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

## Description

`exlogistic` fits an exact logistic regression model, which produces more accurate inference in small samples than the standard maximum-likelihood-based logistic regression estimator. It can also better deal with completely determined outcomes. `exlogistic` with the `group()` option conditions on the number of positive outcomes within stratum and is an alternative to the conditional (fixed-effects) logistic regression estimator.

Unlike Stata's other estimation commands, `exlogistic` must perform hypothesis tests during estimation rather than after estimation with standard postestimation commands.

## Quick start

Exact logistic regression of `y` on `x1`, `x2`, and `x3`

```
exlogistic y x1 x2 x3
```

Same as above, but condition on values of `x3` to save time and memory

```
exlogistic y x1 x2, condvars(x3)
```

Same as above, and allow more memory for computing the conditional distribution of sufficient statistics

```
exlogistic y x1 x2, condvars(x3) memory(100m)
```

Using data stored in binomial form with `ys` successes out of `n` trials

```
exlogistic ys x1 x2 x3, binomial(n)
```

Report coefficients rather than odds ratios

```
exlogistic y x1 x2 x3, coef
```

Report conditional scores tests

```
exlogistic y x1 x2 x3, test(score)
```

Fit a model with strata identified by `svar`

```
exlogistic y x1 x2 x3, group(svar)
```

## Menu

Statistics > Exact statistics > Exact logistic regression

## Syntax

```
exlogistic depvar indepvars [if] [in] [weight] [, options]
```

*depvar* can be specified as a zero or nonzero variable or the number of positive outcomes within each trial. For a zero or nonzero variable, zero indicates failure and nonzero indicates success. To specify *depvar* as the number of positive outcomes, you must also specify `binomial(varname | #)`.

| <i>options</i>                                    | Description                                                                                                                                      |
|---------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| Model                                             |                                                                                                                                                  |
| <code>condvars(<i>varlist<sub>c</sub></i>)</code> | condition on variables in <i>varlist<sub>c</sub></i>                                                                                             |
| <code>group(<i>varname</i>)</code>                | groups or strata are stratified by unique values of <i>varname</i>                                                                               |
| <code>binomial(<i>varname</i>   #)</code>         | data are in binomial form and the number of trials is contained in <i>varname</i> or in #                                                        |
| <code>estconstant</code>                          | estimate constant term; do not condition on the number of successes                                                                              |
| <code>noconstant</code>                           | suppress constant term                                                                                                                           |
| Options                                           |                                                                                                                                                  |
| <code>memory(#[b k m g])</code>                   | set limit on memory usage; default is memory(10m)                                                                                                |
| <code>saving(<i>filename</i>[, replace])</code>   | save the joint conditional distribution to <i>filename</i>                                                                                       |
| Reporting                                         |                                                                                                                                                  |
| <code>level(#)</code>                             | set confidence level; default is level(95)                                                                                                       |
| <code>coef</code>                                 | report estimated coefficients                                                                                                                    |
| <code>test(<i>testopt</i>)</code>                 | report <i>p</i> -value for observed sufficient statistic, conditional scores test, or conditional probabilities test                             |
| <code>mue(<i>varlist<sub>m</sub></i>)</code>      | compute the median unbiased estimates for <i>varlist<sub>m</sub></i>                                                                             |
| <code>midp</code>                                 | use the mid- <i>p</i> -value rule                                                                                                                |
| <code>[no]log</code>                              | display or suppress the enumeration log; default is to display                                                                                   |
| <code>display_options</code>                      | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| <code>coeflegend</code>                           | display legend instead of statistics                                                                                                             |

*indepvars*, *varlist<sub>c</sub>*, and *varlist<sub>m</sub>* may contain factor variables; see [U] 11.4.3 Factor variables.

`by`, `collect`, and `statsby` are allowed; see [U] 11.1.10 Prefix commands.

`fweights` are allowed; see [U] 11.1.6 weight.

`coeflegend` does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Options

### Model

`condvars(varlistc)` specifies variables whose parameter estimates are not of interest to you. You can save substantial computer time and memory by moving such variables from *indepvars* to `condvars()`. Understand that you will get the same results for `x1` and `x3` whether you type

```
. exlogistic y x1 x2 x3 x4
```

or

```
. exlogistic y x1 x3, condvars(x2 x4)
```

`group(varname)` specifies the variable defining the strata, if any. A constant term is assumed for each stratum identified in *varname*, and the sufficient statistics for *indepvars* are conditioned on the observed number of successes within each group. This makes the fitted model equivalent to that fit by `clogit`, Stata's conditional logistic regression command (see [R] [clogit](#)). `group()` may not be specified with `estconstant` or `noconstant`.

`binomial(varname | #)` indicates that the data are in binomial form and *depvar* contains the number of successes. *varname* contains the number of trials for each observation. If all observations have the same number of trials, you can instead specify the number as an integer. The number of trials must be a positive integer at least as great as the number of successes. If `binomial()` is not specified, the data are assumed to be Bernoulli, meaning that *depvar* equaling zero or nonzero records one failure or success.

`estconstant` estimates the constant term. By default, the models are assumed to have an intercept (constant), but the value of the intercept is not calculated. That is, the conditional distribution of the sufficient statistics for the *indepvars* is computed given the number of successes in *depvar*, thus conditioning out the constant term of the model. Use `estconstant` if you want the estimate of the intercept reported. `estconstant` may not be specified with `group()`.

`noconstant`; see [R] [Estimation options](#). `noconstant` may not be specified with `group()`.

### Options

`memory(#[b|k|m|g])` sets a limit on the amount of memory `exlogistic` can use when computing the conditional distribution of the parameter sufficient statistics. The default is `memory(10m)`, where *m* stands for megabyte, or 1,048,576 bytes. The following are also available: *b* stands for byte; *k* stands for kilobyte, which is equal to 1,024 bytes; and *g* stands for gigabyte, which is equal to 1,024 megabytes. The minimum setting allowed is `1m`, and the maximum is `2048m` or `2g`, but do not attempt to use more memory than is available on your computer. Also see the first [technical note](#) under example 4 on counting the conditional distribution.

`saving(filename [, replace])` saves the joint conditional distribution to *filename*. This distribution is conditioned on those variables specified in `condvars()`. Use `replace` to replace an existing file with *filename*. A Stata data file is created containing all the feasible values of the parameter sufficient statistics. The variable names are the same as those in *indepvars*, in addition to a variable named `_f_` containing the feasible value frequencies (sometimes referred to as the condition numbers).

## Reporting

`level(#)`; see [R] [Estimation options](#). The `level()` option will not work on replay because confidence intervals are based on estimator-specific enumerations. To change the confidence level, you must refit the model.

`coef` reports the estimated coefficients rather than odds ratios (exponentiated coefficients). `coef` may be specified when the model is fit or upon replay. `coef` affects only how results are displayed and not how they are estimated.

`test(sufficient | score | probability)` reports the  $p$ -value associated with the observed sufficient statistics, the conditional scores tests, or the conditional probabilities tests, respectively. The default is `test(sufficient)`. If factor variables are included in the specification, the conditional scores test and the conditional probabilities test are applied to each term providing conditional inference for several parameters simultaneously. All the statistics are computed at estimation time regardless of which is specified. Each statistic may thus also be displayed when replaying results after estimation without having to refit the model; see [R] [exlogistic postestimation](#).

`mue(varlistm)` specifies that median unbiased estimates (MUEs) be reported for the specified variables. By default, the conditional maximum likelihood estimates (CMLEs) are reported, except for those parameters for which the CMLEs are infinite. Specify `mue(_all)` if you want MUEs for all the *indepvars*.

`midp` instructs `exlogistic` to use the mid- $p$ -value rule when computing the MUEs,  $p$ -values, and confidence intervals. This adjustment is for the discreteness of the distribution and halves the value of the discrete probability of the observed statistic before adding it to the  $p$ -value. The mid- $p$ -value rule cannot be applied to MUEs whose corresponding parameter CMLE is infinite.

`log` and `nolog` specify whether to display the enumeration log, which shows the progress of computing the conditional distribution of the sufficient statistics. The enumeration log is displayed by default unless you used `set iterlog off` to suppress it; see `set iterlog` in [R] [set iter](#).

*display\_options*: `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, and `sformat(%fmt)`; see [R] [Estimation options](#).

Note that the maximum widths for `cformat()`, `pformat()`, and `sformat()` differ from those widths listed in [R] [Estimation options](#). The maximum width for each format is 9 for `exlogistic`.

The following option is available with `exlogistic` but is not shown in the dialog box:

`coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Exact logistic regression is the estimation of the logistic model parameters by using the conditional distribution of the parameter sufficient statistics. The estimates are referred to as the conditional maximum likelihood estimates (CMLEs). This technique was first introduced by [Cox and Snell \(1989\)](#) as an alternative to using maximum likelihood estimation, which can perform poorly for small sample sizes. For stratified data, exact logistic regression is a small-sample alternative to conditional logistic regression. See [R] [logit](#), [R] [logistic](#), and [R] [clogit](#) to obtain maximum likelihood estimates (MLEs) for the logistic model and the conditional logistic model. For a comprehensive overview of exact logistic regression, see [Mehta and Patel \(1995\)](#).

Let  $Y_i$  denote a Bernoulli random variable where we observe the outcome  $Y_i = y_i, i = 1, \dots, n$ . Associated with each independent observation is a  $1 \times p$  vector of covariates,  $\mathbf{x}_i$ . We will denote  $\pi_i = \Pr(Y_i | \mathbf{x}_i)$  and let the logit function model the relationship between  $Y_i$  and  $\mathbf{x}_i$ ,

$$\log\left(\frac{\pi_i}{1 - \pi_i}\right) = \theta + \mathbf{x}_i\boldsymbol{\beta}$$

where the constant term  $\theta$  and the  $p \times 1$  vector of regression parameters  $\boldsymbol{\beta}$  are unknown. The probability of observing  $Y_i = y_i, i = 1, \dots, n$ , is

$$\Pr(\mathbf{Y} = \mathbf{y}) = \prod_{i=1}^n \pi_i^{y_i} (1 - \pi_i)^{1-y_i}$$

where  $\mathbf{Y} = (Y_1, \dots, Y_n)$  and  $\mathbf{y} = (y_1, \dots, y_n)$ . The MLEs for  $\theta$  and  $\boldsymbol{\beta}$  maximize the log of this function.

The sufficient statistics for  $\theta$  and  $\beta_j, j = 1, \dots, p$ , are  $M = \sum_{i=1}^n Y_i$  and  $T_j = \sum_{i=1}^n Y_i x_{ij}$ , respectively, and we observe  $M = m$  and  $T_j = t_j$ . By default, `exlogistic` tallies the conditional distribution

of  $\mathbf{T} = (T_1, \dots, T_p)$  given  $M = m$ . This distribution will have a size of  $\binom{n}{m}$ . (It would have

a size of  $2^n$  without conditioning on  $M = m$ .) Denote one of these vectors  $\mathbf{T}^{(k)} = (t_1^{(k)}, \dots, t_p^{(k)})$ ,  $k = 1, \dots, N$ , with combinatorial coefficient (frequency)  $c_k$ ,  $\sum_{k=1}^N c_k = \binom{n}{m}$ . For each independent variable  $x_j, j = 1, \dots, p$ , we reduce the conditional distribution further by conditioning on all other observed sufficient statistics  $T_l = t_l, l \neq j$ . The conditional probability of observing  $T_j = t_j$  has the form

$$\Pr(T_j = t_j | T_l = t_l, l \neq j, M = m) = \frac{c e^{t_j \beta_j}}{\sum_k c_k e^{t_j^{(k)} \beta_j}}$$

where the sum is over the subset of  $\mathbf{T}$  vectors such that  $(T_1^{(k)} = t_1, \dots, T_j^{(k)} = t_j^{(k)}, \dots, T_p^{(k)} = t_p)$  and  $c$  is the combinatorial coefficient associated with the observed  $\mathbf{t}$ . The CMLE for  $\beta_j$  maximizes the log of this function.

Specifying nuisance variables in `condvars()` will reduce the size of the conditional distribution by conditioning on their observed sufficient statistics as well as conditioning on  $M = m$ . This reduces the amount of memory consumed at the cost of not obtaining regression estimates for those variables specified in `condvars()`.

Inferences from MLEs rely on asymptotics, and if your sample size is small, these inferences may not be valid. On the other hand, inferences from the CMLEs are exact in that they use the conditional distribution of the sufficient statistics outlined above.

For small datasets, the dependent variable can be completely determined by the data. Here the MLEs and the CMLEs are unbounded. When this occurs, `exlogistic` will compute the MUE, the regression estimate that places the observed sufficient statistic at the median of the conditional distribution.

► Example 1

One example presented by [Mehta and Patel \(1995\)](#) is data from a prospective study of perinatal infection and human immunodeficiency virus type 1 (HIV-1). We use a variation of this dataset. There was an investigation ([Hutto et al. 1991](#)) into whether the blood serum levels of glycoproteins CD4 and CD8 measured in infants at 6 months of age might predict their development of HIV infection. The blood serum levels are coded as ordinal values 0, 1, and 2.

```
. use https://www.stata-press.com/data/r19/hiv1
(Prospective study of perinatal infection of HIV-1)
. list in 1/5
```

|    | hiv      | cd4 | cd8 |
|----|----------|-----|-----|
| 1. | Positive | 0   | 0   |
| 2. | Negative | 0   | 0   |
| 3. | Positive | 0   | 2   |
| 4. | Positive | 1   | 0   |
| 5. | Negative | 1   | 0   |

We first obtain the MLEs from `logistic` so that we can compare the estimates and associated statistics with the CMLEs from `exlogistic`.

```
. logistic hiv cd4 cd8, coef
Logistic regression
Log likelihood = -20.751687
Number of obs = 47
LR chi2(2) = 15.75
Prob > chi2 = 0.0004
Pseudo R2 = 0.2751
```

| hiv   | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|-------|-------------|-----------|-------|-------|----------------------|-----------|
| cd4   | -2.541669   | .8392231  | -3.03 | 0.002 | -4.186517            | -.8968223 |
| cd8   | 1.658586    | .821113   | 2.02  | 0.043 | .0492344             | 3.267938  |
| _cons | .5132389    | .6809007  | 0.75  | 0.451 | -.8213019            | 1.84778   |

```
. exlogistic hiv cd4 cd8, coef
Enumerating sample-space combinations:
Observation 1: Enumerations = 2
Observation 2: Enumerations = 3
(output omitted)
Observation 46: Enumerations = 601
Observation 47: Enumerations = 326
Exact logistic regression
Number of obs = 47
Model score = 13.34655
Pr >= score = 0.0006
```

| hiv | Coefficient | Suff. | 2*Pr(Suff.) | [95% conf. interval] |           |
|-----|-------------|-------|-------------|----------------------|-----------|
| cd4 | -2.387632   | 10    | 0.0004      | -4.699633            | -.8221807 |
| cd8 | 1.592366    | 12    | 0.0528      | -.0137905            | 3.907876  |

exlogistic produced a log showing how many records are generated as it processes each observation. The primary purpose of the log is to provide feedback because generating the distribution can be time consuming, but we also see from the last entry that the joint distribution for the sufficient statistics for cd4 and cd8 conditioned on the total number of successes has 326 unique values (but a size of  $\binom{47}{14} = 341,643,774,795$ ).

The statistics for logistic are based on asymptotics: for a large sample size, each  $Z$  statistic will be approximately normally distributed (with a mean of zero and a standard deviation of one) if the associated regression parameter is zero. The question is whether a sample size of 47 is large enough.

On the other hand, the  $p$ -values computed by exlogistic are from the conditional distributions of the sufficient statistics for each parameter given the sufficient statistics for all other parameters. In this sense, these  $p$ -values are exact. By default, exlogistic reports the sufficient statistics for the regression parameters and the probability of observing a more extreme value. These are single-parameter tests for  $H_0: \beta_{cd4} = 0$  and  $H_0: \beta_{cd8} = 0$  versus the two-sided alternatives. The conditional scores test, located in the coefficient table header, is testing that both  $H_0: \beta_{cd4} = 0$  and  $H_0: \beta_{cd8} = 0$ . We find these  $p$ -values to be in fair agreement with the Wald and likelihood-ratio tests from logistic.

The confidence intervals for exlogistic are computed from the exact conditional distributions. The exact confidence intervals are asymmetrical about the estimate and are wider than the normal-based confidence intervals from logistic.

Both estimation techniques indicate that the incidence of HIV infection decreases with increasing CD4 blood serum levels and increases with increasing CD8 blood serum levels. The constant term is missing from the exact logistic coefficient table because we conditioned out its observed sufficient statistic when tallying the joint distribution of the sufficient statistics for the cd4 and cd8 parameters.

The test() option provides two other test statistics used in exact logistic regression: the conditional scores test, test(score), and the conditional probabilities test, test(probability). For comparison, we display the individual parameter conditional scores tests.

```
. exlogistic, test(score) coef
Exact logistic regression Number of obs = 47
 Model score = 13.34655
 Pr >= score = 0.0006
```

| hiv | Coefficient | Score    | Pr>=Score | [95% conf. interval] |           |
|-----|-------------|----------|-----------|----------------------|-----------|
| cd4 | -2.387632   | 12.88022 | 0.0003    | -4.699633            | -.8221807 |
| cd8 | 1.592366    | 4.604816 | 0.0410    | -.0137905            | 3.907876  |

For the probabilities test, the probability statistic is computed from (1) in *Methods and formulas* with  $\beta = 0$ . For this example, the  $p$ -value for the conditional probabilities tests matches the conditional scores tests, so they are not displayed here.

## □ Technical note

Typically, the value of  $\theta$ , the constant term, is of little interest, as well as perhaps some of the parameters in  $\beta$ , but we need to include all parameters in the model to correctly specify it. By conditioning out the nuisance parameters, we can reduce the size of the joint conditional distribution that is used to estimate the regression parameters of interest. The `condvars()` option allows you to specify *varlist* of nuisance variables. By default, `exlogistic` conditions on the sufficient statistic of  $\theta$ , which is the number of successes. You can save computation time and computer memory by using the `condvars()` option because infeasible values of the sufficient statistics associated with the variables in `condvars()` can be omitted from consideration before all  $n$  observations are processed.

Specifying some of your independent variables in `condvars()` will not change the estimated regression coefficients of the remaining independent variables. For instance, in [example 1](#), if we instead type

```
. exlogistic hiv cd4, condvars(cd8) coef
```

the regression coefficient for `cd4` (as well as all associated inference) will be identical. □

## □ Technical note

If you fit a `clogit` (see [\[R\] clogit](#)) model to the HIV data from [example 1](#), you will find that the estimates differ from those with `exlogistic`. (To fit the `clogit` model, you will have to create a group variable that includes all observations.) The regression estimates will be different because `clogit` conditions on the constant term only, whereas the estimates from `exlogistic` condition on the sufficient statistic of the other regression parameter as well as the constant term. □

## ▷ Example 2

The HIV data presented in table IV of [Mehta and Patel \(1995\)](#) are in a binomial form, where the variable `hiv` contains the HIV cases that tested positive and the variable `n` contains the number of individuals with the same CD4 and CD8 levels, the binomial number-of-trials parameter. Here *depvar* is `hiv`, and we use the `binomial(n)` option to identify the number-of-trials variable.

```
. use https://www.stata-press.com/data/r19/hiv_n
(Prospective study of perinatal infection of HIV-1; binomial form)
. list
```

|    | cd4 | cd8 | hiv | n  |
|----|-----|-----|-----|----|
| 1. | 0   | 2   | 1   | 1  |
| 2. | 1   | 2   | 2   | 2  |
| 3. | 0   | 0   | 4   | 7  |
| 4. | 1   | 1   | 4   | 12 |
| 5. | 2   | 2   | 1   | 3  |
| 6. | 1   | 0   | 2   | 7  |
| 7. | 2   | 0   | 0   | 2  |
| 8. | 2   | 1   | 0   | 13 |

Further, the `cd4` and `cd8` variables are coded with ordinal values (0, 1, 2). Another approach is to specify these variables as factor variables in the model so that indicators for all but the base level of `cd4` and of `cd8` will be included in the model. Similar to [Mehta and Patel \(1995\)](#), we will estimate the odds ratio of level 0 versus 2 and level 1 versus 2 by typing `ib2.cd4` and `ib2.cd8`, indicating that 2 is the base level.

```
. exlogistic hiv ib2.cd4 ib2.cd8, binomial(n) test(probability)
> saving(dist, replace) nolog
note: saving distribution to file dist.dta.
note: CMLE estimate for 0.cd4 is +inf; computing MUE.
note: CMLE estimate for 1.cd4 is +inf; computing MUE.
note: CMLE estimate for 0.cd8 is -inf; computing MUE.
note: CMLE estimate for 1.cd8 is -inf; computing MUE.

Exact logistic regression Number of obs = 47
Binomial variable: n Model prob. = 0.0000
 Pr <= prob. = 0.0011
```

|  | hiv | Odds ratio | Prob.  | Pr<=Prob. | [95% conf. interval] |          |
|--|-----|------------|--------|-----------|----------------------|----------|
|  | cd4 |            | 0.0007 | 0.0055    |                      |          |
|  | 0   | 18.82831*  | 0.0072 | 0.0072    | 1.714079             | +inf     |
|  | 1   | 11.53732*  | 0.0064 | 0.0105    | 1.575285             | +inf     |
|  | cd8 |            | 0.0053 | 0.0323    |                      |          |
|  | 0   | .1056887*  | 0.0290 | 0.0290    | 0                    | 1.072531 |
|  | 1   | .0983388*  | 0.0242 | 0.0242    | 0                    | .9837203 |

(\*) median unbiased estimates (MUE)

```
. matrix list e(sufficient)
e(sufficient)[1,6]
 0. 1. 2b. 0. 1. 2b.
 cd4 cd4 cd4 cd8 cd8 cd8
r1 5 8 0 6 4 0

. display e(n_possible)
1091475
```

Because we included `cd4` and `cd8` as factor variables, we obtained a conditional probabilities test for `cd4`, simultaneously testing both `0.cd4` and `1.cd4`, and for `cd8`, simultaneously testing both `0.cd8` and `1.cd8`. The *p*-values for the two terms are 0.0055 and 0.0323, respectively.

This example also illustrates instances where the dependent variable is completely determined by the independent variables and CMLs are infinite. If we try to obtain MLEs, `logistic` will omit each variable and then terminate with a no-data error, error number 2000.

```
. use https://www.stata-press.com/data/r19/hiv_n, clear
(Prospective study of perinatal infection of HIV-1; binomial form)

. expand n
(39 observations created)

. capture logistic hiv ib2.cd4 ib2.cd8
```

The previous `exlogistic` command generated the joint conditional distribution of  $T_{0.cd4}$ ,  $T_{1.cd4}$ ,  $T_{0.cd8}$ , and  $T_{1.cd8}$  given  $M = 14$  (the number of individuals that tested positive), and for reference, we listed the observed sufficient statistics that are stored in the matrix `e(sufficient)`. Below, we take that distribution and further condition on  $T_{1.cd4} = 8$ ,  $T_{0.cd8} = 6$ , and  $T_{1.cd8} = 4$ , giving the conditional distribution of  $T_{0.cd4}$ . Here we see that the observed sufficient statistic  $T_{0.cd4} = 5$  is last in the sorted listing or, equivalently,  $T_{0.cd4}$  is at the domain boundary of the conditional probability distribution. When this occurs, the conditional probability distribution is monotonically increasing in  $\beta_{0.cd4}$  and a maximum does not exist.

```
. use dist, clear
. keep if _1_cd4==8 & _0_cd8==6 & _1_cd8==4
(4,139 observations deleted)
. list, sep(0)
```

|    | _f_      | _0_cd4 | _1_cd4 | _0_cd8 | _1_cd8 | _2b_cd4 | _2b_cd8 |
|----|----------|--------|--------|--------|--------|---------|---------|
| 1. | 1668667  | 0      | 8      | 6      | 4      | 0       | 0       |
| 2. | 18945542 | 1      | 8      | 6      | 4      | 0       | 0       |
| 3. | 55801053 | 2      | 8      | 6      | 4      | 0       | 0       |
| 4. | 55867350 | 3      | 8      | 6      | 4      | 0       | 0       |
| 5. | 17423175 | 4      | 8      | 6      | 4      | 0       | 0       |
| 6. | 1091475  | 5      | 8      | 6      | 4      | 0       | 0       |

When the CMLEs are infinite, the MUEs are computed (Hirji, Tsiatis, and Mehta 1989). For the `0.cd4` estimate, we compute the value  $\bar{\beta}_{0.cd4}$  such that

$$\Pr(T_{0.cd4} \geq 5 \mid \beta_{0.cd4} = \bar{\beta}_{0.cd4}, T_{1.cd4} = 8, T_{0.cd8} = 6, T_{1.cd8} = 4, M = 14) = 1/2$$

using (1) in *Methods and formulas*.

The output is in agreement with [example 1](#): there is an increase in risk of HIV infection for a CD4 blood serum level of 0 relative to a level of 2 and for a level of 1 relative to a level of 2; there is a decrease in risk of HIV infection for a CD8 blood serum level of 0 relative to a level of 2 and for a level of 1 relative to a level of 2.

We also displayed `e(n_possible)`. This is the combinatorial coefficient associated with the observed sufficient statistics. The same value is found in the `_f_` variable of the conditional distribution dataset listed above. The size of the distribution is  $\binom{47}{14} = 341,643,774,795$ . This can be verified by summing the `_f_` variable of the generated conditional distribution dataset.

```
. use dist, clear
. summarize _f_, meanonly
. di %15.1f r(sum)
341643774795.0
```

▷ Example 3

One can think of exact logistic regression as a covariate-adjusted exact binomial. To demonstrate this point, we will use `exlogistic` to compute a binomial confidence interval for  $m$  successes of  $n$  trials by fitting the constant-only model, and we will compare it with the confidence interval computed by `ci proportions` (see [R] `ci`). We will use the `saving()` option to retain the dataset containing the feasible values for the constant term sufficient statistic, namely, the number of successes,  $m$ , given  $n$  trials and their associated combinatorial coefficients  $\binom{n}{m}$ ,  $m = 0, 1, \dots, n$ .

```
. input y
 y
1. 1
2. 0
3. 1
4. 0
5. 1
6. 1
7. end
. ci proportions y
```

| Variable | Obs | Proportion | Std. err. | Binomial exact<br>[95% conf. interval] |          |
|----------|-----|------------|-----------|----------------------------------------|----------|
| y        | 6   | .6666667   | .1924501  | .2227781                               | .9567281 |

```
. exlogistic y, estconstant nolog coef saving(binom)
note: saving distribution to file binom.dta.
Exact logistic regression
Number of obs = 6
```

| y     | Coefficient | Suff. | 2*Pr(Suff.) | [95% conf. interval] |          |
|-------|-------------|-------|-------------|----------------------|----------|
| _cons | .6931472    | 4     | 0.6875      | -1.24955             | 3.096017 |

We use the postestimation command `estat predict` to transform the estimated constant term and its confidence bounds by using the inverse logit function, `invlogit()` (see [FN] **Mathematical functions**). The standard error for the estimated probability is computed using the delta method.

```
. estat predict
```

| y           | Predicted | Std. err. | [95% conf. interval] |        |
|-------------|-----------|-----------|----------------------|--------|
| Probability | 0.6667    | 0.1925    | 0.2228               | 0.9567 |

```
. use binom, replace
. list, sep(0)
```

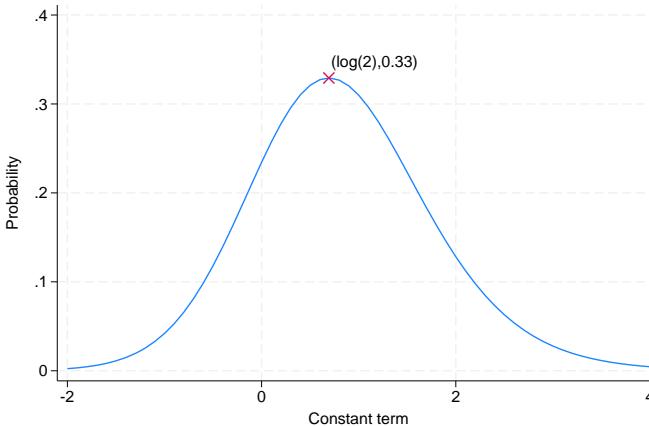
|    | _f_ | _cons_ |
|----|-----|--------|
| 1. | 1   | 0      |
| 2. | 6   | 1      |
| 3. | 15  | 2      |
| 4. | 20  | 3      |
| 5. | 15  | 4      |
| 6. | 6   | 5      |
| 7. | 1   | 6      |

Examining the listing of the generated data, the values contained in the variable `_cons_` are the feasible values of  $M$ , and the values contained in the variable `_f_` are the binomial coefficients  $\binom{6}{m}$  with total  $\sum_{m=0}^6 \binom{6}{m} = 2^6 = 64$ . In the coefficient table, the sufficient statistic for the constant term, labeled `Suff.`, is  $m = 4$ . This value is located at record 5 of the dataset. Therefore, the two-tailed probability of the sufficient statistic is computed as  $0.6875 = 2(15 + 6 + 1)/64$ .

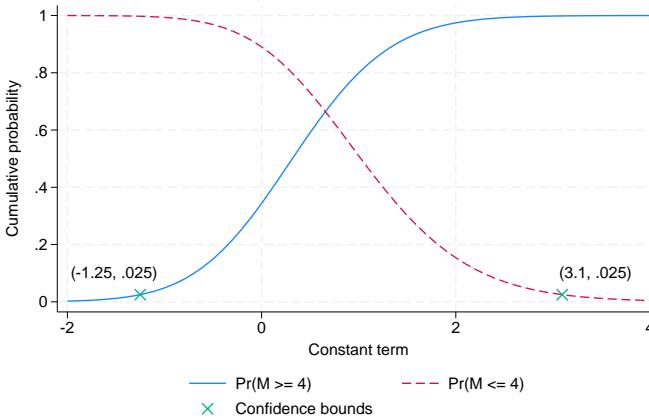
The constant term is the value of  $\theta$  that maximizes the probability of observing  $M = 4$ ; see (1) of *Methods and formulas*:

$$\Pr(M = 4|\theta) = \frac{15e^{4\alpha}}{1 + 6e^\alpha + 15e^{2\alpha} + 20e^{3\alpha} + 15e^{4\alpha} + 6e^{5\alpha} + e^{6\alpha}}$$

The maximum is at the value  $\theta = \log 2$ , which is demonstrated in the figure below.



The lower and upper confidence bounds are the values of  $\theta$  such that  $\Pr(M \geq 4|\theta) = 0.025$  and  $\Pr(M \leq 4|\theta) = 0.025$ , respectively. These probabilities are plotted in the figure below for  $\theta \in [-2, 4]$ .



#### ► Example 4

This example demonstrates the `group()` option, which allows the analysis of stratified data. Here the logistic model is

$$\log\left(\frac{\pi_{ik}}{1 - \pi_{ik}}\right) = \theta_k + \mathbf{x}_{ki}\beta$$

where  $k$  indexes the  $s$  strata,  $k = 1, \dots, s$ , and  $\theta_k$  is the strata-specific constant term whose sufficient statistic is  $M_k = \sum_{i=1}^{n_k} Y_{ki}$ .

Mehta and Patel (1995) use a case-control study to demonstrate this model, which is useful in comparing the estimates from `exlogistic` and `clogit`. This study was intended to determine the role of birth complications in people with schizophrenia (Garsd 1988). Siblings from seven families took part in the study, and each individual was classified as normal or schizophrenic. A birth complication index is recorded for each individual that ranges from 0, an uncomplicated birth, to 15, a very complicated birth. Some of the frequencies contained in variable `f` are greater than 1, and these count different births at different times where the individual has the same birth complications index, found in variable `BCindex`.

```
. use https://www.stata-press.com/data/r19/schizophrenia, clear
(Case-control study on birth complications for people with schizophrenia)
. list, sepby(family)
```

|     | family | BCindex | schizo | f |
|-----|--------|---------|--------|---|
| 1.  | 1      | 6       | 0      | 1 |
| 2.  | 1      | 7       | 0      | 1 |
| 3.  | 1      | 3       | 0      | 2 |
| 4.  | 1      | 2       | 0      | 3 |
| 5.  | 1      | 5       | 0      | 1 |
| 6.  | 1      | 0       | 0      | 1 |
| 7.  | 1      | 15      | 1      | 1 |
|     |        |         |        |   |
| 8.  | 2      | 2       | 1      | 1 |
| 9.  | 2      | 0       | 0      | 1 |
|     |        |         |        |   |
| 10. | 3      | 2       | 0      | 1 |
| 11. | 3      | 9       | 1      | 1 |
| 12. | 3      | 1       | 0      | 1 |
|     |        |         |        |   |
| 13. | 4      | 2       | 1      | 1 |
| 14. | 4      | 0       | 0      | 4 |
|     |        |         |        |   |
| 15. | 5      | 3       | 1      | 1 |
| 16. | 5      | 6       | 0      | 1 |
| 17. | 5      | 0       | 1      | 1 |
|     |        |         |        |   |
| 18. | 6      | 3       | 0      | 1 |
| 19. | 6      | 0       | 1      | 1 |
| 20. | 6      | 0       | 0      | 2 |
|     |        |         |        |   |
| 21. | 7      | 2       | 0      | 1 |
| 22. | 7      | 6       | 1      | 1 |

```
. exlogistic schizo BCindex [fw=f], group(family) test(score) coef
```

Enumerating sample-space combinations:

```
Observation 1: Enumerations = 2
Observation 2: Enumerations = 3
Observation 3: Enumerations = 4
Observation 4: Enumerations = 5
Observation 5: Enumerations = 6
Observation 6: Enumerations = 7
(output omitted)
Observation 21: Enumerations = 72
Observation 22: Enumerations = 40
```

Exact logistic regression  
Group variable: family

```
Number of obs = 29
Number of groups = 7
Obs per group:
 min = 2
 avg = 4.1
 max = 10
Model score = 6.328033
Pr >= score = 0.0167
```

| schizo  | Coefficient | Score    | Pr>=Score | [95% conf. interval] |          |
|---------|-------------|----------|-----------|----------------------|----------|
| BCindex | .3251178    | 6.328033 | 0.0167    | .0223423             | .7408832 |

The asymptotic alternative for this model can be estimated using `clogit` (equivalently, `xtlogit, fe`) and is listed below for comparison. We must expand the data because `clogit` will not accept frequency weights if they are not constant within the groups.

```
. expand f
(7 observations created)
. clogit schizo BCindex, group(family) nolog
note: multiple positive outcomes within groups encountered.
Conditional (fixed-effects) logistic regression Number of obs = 29
 LR chi2(1) = 5.20
 Prob > chi2 = 0.0226
 Pseudo R2 = 0.2927

Log likelihood = -6.2819819
```

|         | schizo | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |
|---------|--------|-------------|-----------|------|-------|----------------------|
| BCindex |        | .3251178    | .1678981  | 1.94 | 0.053 | -.0039565 .654192    |

Both techniques compute the same regression estimate for the `BCindex`, which might not be too surprising because both estimation techniques condition on the total number of successes in each group. The difference lies in the  $p$ -values and confidence intervals. The  $p$ -value testing  $H_0: \beta_{\text{BCindex}} = 0$  is approximately 0.0167 for the exact conditional scores test and 0.053 for the asymptotic Wald test. Moreover, the exact confidence interval is asymmetric about the estimate and does not contain zero.

◀

### □ Technical note

The `memory()` option limits the amount of memory that `exlogistic` will consume when computing the conditional distribution of the parameter sufficient statistics. `memory()` is independent of the data maximum memory setting (see `set max_memory` in [D] [memory](#)), and it is possible for `exlogistic` to exceed the memory limit specified in `set max_memory` without terminating. By default, a log is provided that displays the number of enumerations (the size of the conditional distribution) after processing each observation. Typically, you will see the number of enumerations increase, and then at some point they will decrease as the multivariate shift algorithm (Hirji, Mehta, and Patel 1987) determines that some of the enumerations cannot achieve the observed sufficient statistics of the conditioning variables. When the algorithm is complete, however, it is necessary to store the conditional distribution of the parameter sufficient statistics as a dataset. It is possible, therefore, to get a memory error when the algorithm has completed if there is not enough memory to store the conditional distribution.

□

### □ Technical note

Computing the conditional distributions and reported statistics requires data sorting and numerical comparisons. If there is at least one single-precision variable specified in the model, `exlogistic` will make comparisons with a relative precision of  $2^{-5}$ . Otherwise, a relative precision of  $2^{-11}$  is used. Be careful if you use `recast` to promote a single-precision variable to double precision (see [D] [recast](#)). You might try listing the data in full precision (maybe `%20.15g`; see [D] [format](#)) to make sure that this is really what you want. See [D] [Data types](#) for information on precision of numeric storage types.

□

## Stored results

exlogistic stores the following in `e()`:

### Scalars

|                            |                                                                                                                    |
|----------------------------|--------------------------------------------------------------------------------------------------------------------|
| <code>e(N)</code>          | number of observations                                                                                             |
| <code>e(k_groups)</code>   | number of groups                                                                                                   |
| <code>e(n_possible)</code> | number of distinct possible outcomes where <code>sum(sufficient)</code> equals observed <code>e(sufficient)</code> |
| <code>e(n_trials)</code>   | binomial number-of-trials parameter                                                                                |
| <code>e(sum_y)</code>      | sum of <i>depvar</i>                                                                                               |
| <code>e(k_indvars)</code>  | number of independent variables                                                                                    |
| <code>e(k_condvars)</code> | number of conditioning variables                                                                                   |
| <code>e(condcons)</code>   | conditioned on the constant(s) indicator                                                                           |
| <code>e(midp)</code>       | mid- <i>p</i> -value rule indicator                                                                                |
| <code>e(eps)</code>        | relative difference tolerance                                                                                      |

### Macros

|                                   |                                                |
|-----------------------------------|------------------------------------------------|
| <code>e(cmd)</code>               | exlogistic                                     |
| <code>e(cmdline)</code>           | command as typed                               |
| <code>e(title)</code>             | title in estimation output                     |
| <code>e(depvar)</code>            | name of dependent variable                     |
| <code>e(indvars)</code>           | independent variables                          |
| <code>e(condvars)</code>          | conditional variables                          |
| <code>e(groupvar)</code>          | group variable                                 |
| <code>e(binomial)</code>          | binomial number-of-trials variable             |
| <code>e(fvvarlist)</code>         | independent factor variables                   |
| <code>e(level)</code>             | confidence level                               |
| <code>e(wtype)</code>             | weight type                                    |
| <code>e(wexp)</code>              | weight expression                              |
| <code>e(datasignature)</code>     | the checksum                                   |
| <code>e(datasignaturevars)</code> | variables used in calculation of checksum      |
| <code>e(properties)</code>        | b                                              |
| <code>e(estat_cmd)</code>         | program used to implement <code>estat</code>   |
| <code>e(marginsnotok)</code>      | predictions disallowed by <code>margins</code> |

### Matrices

|                                |                                                                                 |
|--------------------------------|---------------------------------------------------------------------------------|
| <code>e(b)</code>              | coefficient vector                                                              |
| <code>e(mue_indicators)</code> | indicator for elements of <code>e(b)</code> estimated using MUE instead of CMLE |
| <code>e(se)</code>             | <code>e(b)</code> standard errors (CMLEs only)                                  |
| <code>e(ci)</code>             | matrix of <code>e(level)</code> confidence intervals for <code>e(b)</code>      |
| <code>e(sum_y_groups)</code>   | sum of <code>e(depvar)</code> for each group                                    |
| <code>e(N_g)</code>            | number of observations in each group                                            |
| <code>e(sufficient)</code>     | sufficient statistics for <code>e(b)</code>                                     |
| <code>e(p_sufficient)</code>   | <i>p</i> -value for <code>e(sufficient)</code>                                  |
| <code>e(scoretest)</code>      | conditional scores tests for <i>indepvars</i>                                   |
| <code>e(p_scoretest)</code>    | <i>p</i> -values for <code>e(scoretest)</code>                                  |
| <code>e(probtest)</code>       | conditional probabilities tests for <i>indepvars</i>                            |
| <code>e(p_probtest)</code>     | <i>p</i> -value for <code>e(probtest)</code>                                    |
| <code>e(scoretest_m)</code>    | conditional scores tests for model terms                                        |
| <code>e(p_scoretest_m)</code>  | <i>p</i> -value for <code>e(scoretest_m)</code>                                 |
| <code>e(probtest_m)</code>     | conditional probabilities tests for model terms                                 |
| <code>e(p_probtest_m)</code>   | <i>p</i> -value for <code>e(probtest_m)</code>                                  |

### Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

## Methods and formulas

Methods and formulas are presented under the following headings:

*Sufficient statistics*  
*Conditional distribution and CMLE*  
*MUEs and exact confidence intervals*  
*Conditional hypothesis tests*  
*Sufficient-statistic p-value*

### Sufficient statistics

Let  $\{Y_1, Y_2, \dots, Y_n\}$  be a set of  $n$  independent Bernoulli random variables, each of which can realize two outcomes,  $\{0, 1\}$ . For each  $i = 1, \dots, n$ , we observe  $Y_i = y_i$ , and associated with each observation is the covariate row vector of length  $p$ ,  $\mathbf{x}_i = (x_{i1}, \dots, x_{ip})$ . Denote  $\boldsymbol{\beta} = (\beta_1, \dots, \beta_p)^T$  to be the column vector of regression parameters and  $\theta$  to be the constant. The sufficient statistic for  $\beta_j$  is  $T_j = \sum_{i=1}^n Y_i x_{ij}$ ,  $j = 1, \dots, p$ , and for  $\theta$  is  $M = \sum_{i=1}^n Y_i$ . We observe  $T_j = t_j$ ,  $t_j = \sum_{i=1}^n y_i x_{ij}$ , and  $M = m$ ,  $m = \sum_{i=1}^n y_i$ . The probability of observing  $(Y_1 = y_1, Y_2 = y_2, \dots, Y_n = y_n)$  is

$$\Pr(Y_1 = y_1, \dots, Y_n = y_n \mid \boldsymbol{\beta}, \mathbf{X}) = \frac{\exp(m\theta + \mathbf{t}\boldsymbol{\beta})}{\prod_{i=1}^n \{1 + \exp(\theta + \mathbf{x}_i\boldsymbol{\beta})\}}$$

where  $\mathbf{t} = (t_1, \dots, t_p)$  and  $\mathbf{X} = (\mathbf{x}_1^T, \dots, \mathbf{x}_n^T)^T$ .

The joint distribution of the sufficient statistics  $\mathbf{T}$  is obtained by summing over all possible binary sequences  $Y_1, \dots, Y_n$  such that  $\mathbf{T} = \mathbf{t}$  and  $M = m$ . This probability function is

$$\Pr(T_1 = t_1, \dots, T_p = t_p, M = m \mid \boldsymbol{\beta}, \mathbf{X}) = \frac{c(\mathbf{t}, m) \exp(m\theta + \mathbf{t}\boldsymbol{\beta})}{\prod_{i=1}^n \{1 + \exp(\theta + \mathbf{x}_i\boldsymbol{\beta})\}}$$

where  $c(\mathbf{t}, m)$  is the combinatorial coefficient of  $(\mathbf{t}, m)$  or the number of distinct binary sequences  $Y_1, \dots, Y_n$  such that  $\mathbf{T} = \mathbf{t}$  and  $M = m$  (Cox and Snell 1989).

### Conditional distribution and CMLE

Without loss of generality, we will restrict our discussion to computing the CMLE of  $\beta_1$ . If we condition on observing  $M = m$  and  $T_2 = t_2, \dots, T_p = t_p$ , the probability function of  $(T_1 \mid \beta_1, T_2 = t_2, \dots, T_p = t_p, M = m)$  is

$$\Pr(T_1 = t_1 \mid \beta_1, T_2 = t_2, \dots, T_p = t_p, M = m) = \frac{c(\mathbf{t}, m) e^{t_1 \beta_1}}{\sum_u c(u, t_2, \dots, t_p, m) e^{u \beta_1}} \quad (1)$$

where the sum in the denominator is over all possible values of  $T_1$  such that  $M = m$  and  $T_2 = t_2, \dots, T_p = t_p$  and  $c(u, t_2, \dots, t_p, m)$  is the combinatorial coefficient of  $(u, t_2, \dots, t_p, m)$  (Cox and Snell 1989). The CMLE for  $\beta_1$  is the value  $\hat{\beta}_1$  that maximizes the log of (1). This optimization task is carried out by `m1`, using the conditional frequency distribution of  $(T_1 \mid T_2 = t_2, \dots, T_p = t_p, M = m)$  as a dataset. Generating the joint conditional distribution is efficiently computed using the multivariate shift algorithm described by Hirji, Mehta, and Patel (1987).

Difficulties in computing  $\hat{\beta}_1$  arise if the observed  $(T_1 = t_1, \dots, T_p = t_p, M = m)$  lies on the boundaries of the distribution of  $(T_1 | T_2 = t_2, \dots, T_p = t_p, M = m)$ , where the conditional probability function is monotonically increasing (or decreasing) in  $\beta_1$ . Here the CMLE is plus infinity if it is on the upper boundary,  $\Pr(T_1 \leq t_1 | T_2 = t_2, \dots, T_p = t_p, M = m) = 1$ , and is minus infinity if it is on the lower boundary of the distribution,  $\Pr(T_1 \geq t_1 | T_2 = t_2, \dots, T_p = t_p, M = m) = 1$ . This concept is demonstrated in [example 2](#). When infinite CMLEs occur, the MUE is computed.

### MUEs and exact confidence intervals

The MUE is computed using the technique outlined by [Hirji, Tsiatis, and Mehta \(1989\)](#). First, we find the values of  $\beta_1^{(u)}$  and  $\beta_1^{(l)}$  such that

$$\begin{aligned} \Pr(T_1 \leq t_1 | \beta_1 = \beta_1^{(u)}, T_2 = t_2, \dots, T_p = t_p, M = m) &= \\ \Pr(T_1 \geq t_1 | \beta_1 = \beta_1^{(l)}, T_2 = t_2, \dots, T_p = t_p, M = m) &= 1/2 \end{aligned} \tag{2}$$

The MUE is then  $\bar{\beta}_1 = (\beta_1^{(l)} + \beta_1^{(u)})/2$ . However, if  $T_1$  is equal to the minimum of the domain of the conditional distribution,  $\beta_1^{(l)}$  does not exist and  $\bar{\beta}_1 = \beta_1^{(u)}$ . If  $T_1$  is equal to the maximum of the domain of the conditional distribution,  $\beta_1^{(u)}$  does not exist and  $\bar{\beta}_1 = \beta_1^{(l)}$ .

Confidence bounds for  $\beta$  are computed similarly, except that we substitute  $\alpha/2$  for  $1/2$  in (2), where  $1 - \alpha$  is the confidence level. Here  $\beta_1^{(l)}$  would then be the lower confidence bound and  $\beta_1^{(u)}$  would be the upper confidence bound (see [example 3](#)).

### Conditional hypothesis tests

To test  $H_0: \beta_1 = 0$  versus  $H_1: \beta_1 \neq 0$ , we obtain the exact  $p$ -value from  $\sum_{u \in E} f_1(u) - f_1(t_1)/2$  if the mid- $p$ -value rule is used and  $\sum_{u \in E} f_1(u)$  otherwise. Here  $E$  is a critical region, and we define  $f_1(u) = \Pr(T_1 = u | \beta_1 = 0, T_2 = t_2, \dots, T_p = t_p, M = m)$  for ease of notation. There are two popular ways to define the critical region: the conditional probabilities test and the conditional scores test ([Mehta and Patel 1995](#)). The critical region when using the conditional probabilities test is all values of the sufficient statistic for  $\beta_1$  that have a probability less than or equal to that of the observed  $t_1$ ,  $E_p = \{u : f_1(u) \leq f_1(t_1)\}$ . The critical region of the conditional scores test is defined as all values of the sufficient statistic for  $\beta_1$  such that its score is greater than or equal to that of  $t_1$ ,

$$E_s = \{u : (u - \mu_1)^2/\sigma_1^2 \geq (t_1 - \mu_1)^2/\sigma_1^2\}$$

Here  $\mu_1$  and  $\sigma_1^2$  are the mean and variance of  $(T_1 | \beta_1 = 0, T_2 = t_2, \dots, T_p = t_p, M = m)$ .

The score statistic is defined as

$$\left\{ \frac{\partial \ell(\beta)}{\partial \beta} \right\}^2 \left[ -E \left\{ \frac{\partial^2 \ell(\beta)}{\partial \beta^2} \right\} \right]^{-1}$$

evaluated at  $H_0: \beta = 0$ , where  $\ell$  is the log of (1). The score test simplifies to  $(t - E[T|\beta])^2/\text{var}(T|\beta)$  ([Hirji 2006](#)), where the mean and variance are computed from the conditional distribution of the sufficient statistic with  $\beta = 0$  and  $t$  is the observed sufficient statistic.

## Sufficient-statistic p-value

The  $p$ -value for testing  $H_0 : \beta_1 = 0$  versus the two-sided alternative when  $(T_1 = t_1 | T_2 = t_2, \dots, T_p = t_p)$  is computed as  $2 \times \min(p_l, p_u)$ , where

$$p_l = \frac{\sum_{u \leq t_1} c(u, t_2, \dots, t_p, m)}{\sum_u c(u, t_2, \dots, t_p, m)}$$

$$p_u = \frac{\sum_{u \geq t_1} c(u, t_2, \dots, t_p, m)}{\sum_u c(u, t_2, \dots, t_p, m)}$$

It is the probability of observing a more extreme  $T_1$ .

## References

- Cox, D. R., and E. J. Snell. 1989. *Analysis of Binary Data*. 2nd ed. London: Chapman and Hall. <https://doi.org/10.1201/9781315137391>.
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- Hirji, K. F., A. A. Tsiatis, and C. R. Mehta. 1989. Median unbiased estimation for binary data. *American Statistician* 43: 7–11. <https://doi.org/10.2307/2685158>.
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- Mehta, C. R., and N. R. Patel. 1995. Exact logistic regression: Theory and examples. *Statistics in Medicine* 14: 2143–2160. <https://doi.org/10.1002/sim.4780141908>.

## Also see

- [R] **exlogistic postestimation** — Postestimation tools for exlogistic
- [R] **binreg** — Generalized linear models: Extensions to the binomial family
- [R] **clgit** — Conditional (fixed-effects) logistic regression
- [R] **expoisson** — Exact Poisson regression
- [R] **logistic** — Logistic regression, reporting odds ratios
- [R] **logit** — Logistic regression, reporting coefficients
- [U] **20 Estimation and postestimation commands**

[Postestimation commands](#)   [estat](#)   [Remarks and examples](#)   [Stored results](#)   [Reference](#)  
Also see

## Postestimation commands

The following postestimation commands are of special interest after `exlogistic`:

| Command                    | Description                                                     |
|----------------------------|-----------------------------------------------------------------|
| <code>estat predict</code> | single-observation prediction                                   |
| <code>estat se</code>      | report ORs or coefficients and their asymptotic standard errors |

The following standard postestimation commands are also available:

| Command                      | Description                                  |
|------------------------------|----------------------------------------------|
| <code>estat summarize</code> | summary statistics for the estimation sample |
| <code>estimates</code>       | cataloging estimation results                |
| <code>etable</code>          | table of estimation results                  |

`estat summarize` is not allowed if the `binomial()` option was specified in `exlogistic`.

## estat

### Description for estat

`estat predict` computes a predicted probability (or linear predictor), its asymptotic standard error, and its exact confidence interval for 1 observation. Predictions are carried out by estimating the constant coefficient after shifting the independent variables and conditioned variables by the values specified in the `at()` option or by their medians. Therefore, predictions must be done with the estimation sample in memory. If a different dataset is used or if the dataset is modified, then an error will result.

`estat se` reports odds ratio or coefficients and their asymptotic standard errors. The estimates are stored in the matrix `r(estimate)`.

### Menu for estat

Statistics > Postestimation

### Syntax for estat

*Single-observation prediction*

```
estat predict [, pred_options]
```

*Report ORs or coefficients and their asymptotic standard errors*

```
estat se [, coef]
```

| <i>pred_options</i>                                               | Description                                                                     |
|-------------------------------------------------------------------|---------------------------------------------------------------------------------|
| <code>pr</code>                                                   | probability; the default                                                        |
| <code>xb</code>                                                   | linear predictor                                                                |
| <code>at(<i>atspec</i>)</code>                                    | use the specified values for the <i>indepvars</i> and <code>condvars()</code>   |
| <code>level(#)</code>                                             | set confidence level for the predicted value; default is <code>level(95)</code> |
| <code>memory(#[<i>b</i>   <i>k</i>   <i>m</i>   <i>g</i>])</code> | set limit on memory usage; default is <code>memory(10m)</code>                  |
| <code>[no]log</code>                                              | display or suppress the enumeration log; default is to display                  |

`collect` is allowed with `estat predict`; see [\[U\] 11.1.10 Prefix commands](#).

These statistics are available only for the estimation sample.

### Options for estat predict

`pr`, the default, calculates the probability.

`xb` calculates the linear predictor.

`at(varname = # [[varname = #] [...]])` specifies values to use in computing the predicted value. Here *varname* is one of the independent variables, *indepvars*, or the conditioned variables, `condvars()`.

The default is to use the median of each independent and conditioned variable.

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [\[U\] 20.8 Specifying the width of confidence intervals](#).

`memory(#[b|k|m|g])` sets a limit on the amount of memory `estat predict` can use when generating the conditional distribution of the constant parameter sufficient statistic. The default is `memory(10m)`, where `m` stands for megabyte, or 1,048,576 bytes. The following are also available: `b` stands for byte; `k` stands for kilobyte, which is equal to 1,024 bytes; and `g` stands for gigabyte, which is equal to 1,024 megabytes. The minimum setting allowed is `1m` and the maximum is `512m` or `0.5g`, but do not attempt to use more memory than is available on your computer. Also see [Remarks and examples](#) in [\[R\] exlogistic](#) for details on enumerating the conditional distribution.

`log` and `no log` specify whether to display the enumeration log, which shows the progress of enumerating the distribution of the observed successes conditioned on the independent variables shifted by the values specified in `at()` (or by their medians). See [Methods and formulas](#) in [\[R\] exlogistic](#) for details of the computations. The enumeration log is displayed by default unless you used `set iterlog off` to suppress it; see `set iterlog` in [\[R\] set iter](#).

## Option for `estat se`

`coef` requests that the estimated coefficients and their asymptotic standard errors be reported. The default is to report the odds ratios and their asymptotic standard errors.

## Remarks and examples

Predictions must be done using the estimation sample. This is because the prediction is really an estimated constant coefficient (the intercept) after shifting the independent variables and conditioned variables by the values specified in `at()` or by their medians. The justification for this approach can be seen by rewriting the model as

$$\log\left(\frac{\pi_i}{1-\pi_i}\right) = (\alpha + \mathbf{x}_0\beta) + (\mathbf{x}_i - \mathbf{x}_0)\beta$$

where  $\mathbf{x}_0$  are the specified values for the *indepvars* (Mehta and Patel 1995). Because the estimation of the constant term is required, this technique is not appropriate for stratified models that used the `group()` option.

### ▷ Example 1

To demonstrate, we return to the [example 2](#) in [\[R\] exlogistic](#) using data from a prospective study of perinatal infection and HIV-1. Here there was an investigation into whether the blood serum levels of CD4 and CD8 measured in infants at 6 months of age might predict their development of HIV infection. The blood serum levels are coded as ordinal values 0, 1, and 2. These data are used by [Mehta and Patel \(1995\)](#) as an exposition of exact logistic.

```
. use https://www.stata-press.com/data/r19/hiv_n
(Prospective study of perinatal infection of HIV-1; binomial form)
. exlogistic hiv ib2.cd4 ib2.cd8, binomial(n) test(probability)
(output omitted)
```

```
. estat predict
Enumerating sample-space combinations:
Observation 1: Enumerations = 3
Observation 2: Enumerations = 12
Observation 3: Enumerations = 5
Observation 4: Enumerations = 5
Observation 5: Enumerations = 5
Observation 6: Enumerations = 35
Observation 7: Enumerations = 15
Observation 8: Enumerations = 15
Observation 9: Enumerations = 9
Observation 10: Enumerations = 9
Observation 11: Enumerations = 5
Observation 12: Enumerations = 18
note: CMLE estimate for _cons is -inf; computing MUE.
Predicted value at 0.cd4 = 0, 1.cd4 = 0, 0.cd8 = 0, 1.cd8 = 1
```

| hiv         | Predicted | Std. err. | [95% conf. interval] |        |
|-------------|-----------|-----------|----------------------|--------|
| Probability | 0.0390*   | N/A       | 0.0000               | 0.1962 |

(\*) identifies median unbiased estimates (MUE); because an MUE is computed, there is no SE estimate

Because we did not specify values by using the `at()` option, the median values of the *indepvars* are used for the prediction. By default, medians are used instead of means because we want to use values that are observed in the dataset. If the means of the indicators for levels of factor variables (0.cd4–1.cd8) were used, we would have created floating point variables in (0, 1) that not only do not properly represent the indicator variables but also would be a source of computational inefficiency in generating the conditional distribution. Because the MUE is computed for the predicted value, there is no standard error estimate.

From the example discussions in [R] **exlogistic**, the infants at highest risk are those with a CD4 level of 0 and a CD8 level of 2. Below, we use the `at()` option to make a prediction at these blood serum levels.

```
. estat predict, at(cd4=0 cd8=2) nolog
note: 2b.cd8 is a base level; setting all other levels of cd8 to zero implies
2b.cd8 = 1.
note: CMLE estimate for _cons is +inf; computing MUE.
Predicted value at 0.cd4 = 1, 1.cd4 = 0, 0.cd8 = 0, 1.cd8 = 0
```

| hiv         | Predicted | Std. err. | [95% conf. interval] |        |
|-------------|-----------|-----------|----------------------|--------|
| Probability | 0.9063*   | N/A       | 0.4637               | 1.0000 |

(\*) identifies median unbiased estimates (MUE); because an MUE is computed, there is no SE estimate

## Stored results

`estat predict` stores the following in `r()`:

### Scalars

|                      |                                                     |
|----------------------|-----------------------------------------------------|
| <code>r(imue)</code> | 1 if <code>r(pred)</code> is an MUE and 0 if a CMLE |
| <code>r(pred)</code> | estimated probability or the linear effect          |
| <code>r(se)</code>   | asymptotic standard error of <code>r(pred)</code>   |

### Macros

|                          |                                                     |
|--------------------------|-----------------------------------------------------|
| <code>r(estimate)</code> | prediction type: <code>pr</code> or <code>xb</code> |
| <code>r(level)</code>    | confidence level                                    |

### Matrices

|                    |                                               |
|--------------------|-----------------------------------------------|
| <code>r(ci)</code> | confidence interval                           |
| <code>r(x)</code>  | <i>indepvars</i> and <i>condvars()</i> values |

## Reference

Mehta, C. R., and N. R. Patel. 1995. Exact logistic regression: Theory and examples. *Statistics in Medicine* 14: 2143–2160.  
<https://doi.org/10.1002/sim.4780141908>.

## Also see

[R] **exlogistic** — Exact logistic regression

[U] **20 Estimation and postestimation commands**

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

## Description

`expoisson` fits an exact Poisson regression model, which produces more accurate inference in small samples than standard maximum-likelihood-based Poisson regression. For stratified data, `expoisson` conditions on the number of events in each stratum and is an alternative to fixed-effects Poisson regression.

## Quick start

Exact Poisson regression of  $y$  on  $x_1$ ,  $x_2$ , and  $x_3$

```
expoisson y x1 x2 x3
```

Add exposure variable `evar`

```
expoisson y x1 x2 x3, exposure(evar)
```

Same as above, but condition on values of  $x_3$  to save time and memory

```
expoisson y x1 x2, exposure(evar) condvars(x3)
```

Same as above, and allow more memory for computing the conditional distribution of sufficient statistics

```
expoisson y x1 x2, exposure(evar) condvars(x3) memory(100m)
```

Report incidence-rate ratios rather than coefficients

```
expoisson y x1 x2 x3, irr
```

Report conditional scores tests

```
expoisson y x1 x2 x3, test(score)
```

Fit a model with strata identified by `svar`

```
expoisson y x1 x2 x3, group(svar)
```

## Menu

Statistics > Exact statistics > Exact Poisson regression

## Syntax

expoisson *depvar indepvars* [*if*] [*in*] [*weight*] [, *options*]

| <i>options</i>                                    | Description                                                                                                                                      |
|---------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                      |                                                                                                                                                  |
| <code>condvars(<i>varlist<sub>c</sub></i>)</code> | condition on variables in <i>varlist<sub>c</sub></i>                                                                                             |
| <code>group(<i>varname</i>)</code>                | groups or strata are stratified by unique values of <i>varname</i>                                                                               |
| <code>exposure(<i>varname<sub>e</sub></i>)</code> | include $\ln(\text{varname}_e)$ in model with coefficient constrained to 1                                                                       |
| <code>offset(<i>varname<sub>o</sub></i>)</code>   | include <i>varname<sub>o</sub></i> in model with coefficient constrained to 1                                                                    |
| <b>Options</b>                                    |                                                                                                                                                  |
| <code>memory(#[b k m g])</code>                   | set limit on memory usage; default is memory(25m)                                                                                                |
| <code>saving(<i>filename</i> [, replace])</code>  | save the joint conditional distribution to <i>filename</i>                                                                                       |
| <b>Reporting</b>                                  |                                                                                                                                                  |
| <code>level(#)</code>                             | set confidence level; default is level(95)                                                                                                       |
| <code>irr</code>                                  | report incidence-rate ratios                                                                                                                     |
| <code>test(<i>testopt</i>)</code>                 | report <i>p</i> -value for observed sufficient statistic, conditional scores test, or conditional probabilities test                             |
| <code>mue(<i>varlist<sub>m</sub></i>)</code>      | compute the median unbiased estimates for <i>varlist<sub>m</sub></i>                                                                             |
| <code>midp</code>                                 | use the mid- <i>p</i> -value rule                                                                                                                |
| <code>[no]log</code>                              | display or suppress the enumeration log; default is to display                                                                                   |
| <code>display_options</code>                      | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| <code>coeflegend</code>                           | display legend instead of statistics                                                                                                             |

*indepvars*, *varlist<sub>c</sub>*, and *varlist<sub>m</sub>* may contain factor variables; see [U] 11.4.3 Factor variables.

`by`, `collect`, and `statsby` are allowed; see [U] 11.1.10 Prefix commands.

`fweights` are allowed; see [U] 11.1.6 weight.

`coeflegend` does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Options

### Model

`condvars(varlistc)` specifies variables whose parameter estimates are not of interest to you. You can save substantial computer time and memory by moving such variables from *indepvars* to `condvars()`. Understand that you will get the same results for `x1` and `x3` whether you type

```
. expoission y x1 x2 x3 x4
```

or

```
. expoission y x1 x3, condvars(x2 x4)
```

`group(varname)` specifies the variable defining the strata, if any. A constant term is assumed for each stratum identified in *varname*, and the sufficient statistics for *indepvars* are conditioned on the observed number of successes within each group (as well as other variables in the model). The group variable must be integer valued.

`exposure(varnamee)`, `offset(varnameo)`; see [R] Estimation options.

Options

`memory(#[ b | k | m | g ])` sets a limit on the amount of memory `expoisson` can use when computing the conditional distribution of the parameter sufficient statistics. The default is `memory(25m)`, where *m* stands for megabyte, or 1,048,576 bytes. The following are also available: *b* stands for byte; *k* stands for kilobyte, which is equal to 1,024 bytes; and *g* stands for gigabyte, which is equal to 1,024 megabytes. The minimum setting allowed is `1m` and the maximum is `2048m` or `2g`, but do not attempt to use more memory than is available on your computer. Also see the first [technical note](#) under example 3 on counting the conditional distribution.

`saving(filename [ , replace ])` saves the joint conditional distribution for each independent variable specified in *indepvars*. There is one file for each variable, and it is named using the prefix *filename* with the variable name appended. For example, `saving(mydata)` with an independent variable named *X* would generate a data file named `mydata_X.dta`. Use `replace` to replace an existing file. Each file contains the conditional distribution for one of the independent variables specified in *indepvars* conditioned on all other *indepvars* and those variables specified in `condvars()`. There are two variables in each data file: the feasible sufficient statistics for the variable's parameter and their associated weights. The weights variable is named `_w_`.

Reporting

`level(#)`; see [R] Estimation options. The `level()` option will not work on replay because confidence intervals are based on estimator-specific enumerations. To change the confidence level, you must refit the model.

`irr` reports estimated coefficients transformed to incidence-rate ratios, that is,  $\exp(\beta)$  rather than  $\beta$ . Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated or stored. `irr` may be specified at estimation or when replaying previously estimated results.

`test(sufficient | score | probability)` reports the *p*-value associated with the observed sufficient statistics, the conditional scores tests, or the conditional probabilities tests, respectively. The default is `test(sufficient)`. All the statistics are computed at estimation time regardless of which is specified. Each statistic may thus also be displayed when replaying results after estimation without having to refit the model; see [R] [expoisson postestimation](#).

`mue(varlistm)` specifies that median unbiased estimates (MUEs) be reported for the specified variables. By default, the conditional maximum likelihood estimates (CMLEs) are reported, except for those parameters for which the CMLEs are infinite. Specify `mue(_all)` if you want MUEs for all the *indepvars*.

`midp` instructs `expoisson` to use the mid-*p*-value rule when computing the MUEs, *p*-values, and confidence intervals. This adjustment is for the discreteness of the distribution and halves the value of the discrete probability of the observed statistic before adding it to the *p*-value. The mid-*p*-value rule cannot be applied to MUEs whose corresponding parameter CMLE is infinite.

`log` and `nolog` specify whether to display the enumeration log, which shows the progress of computing the conditional distribution of the sufficient statistics. The enumeration log is displayed by default unless you used `set iterlog off` to suppress it; see `set iterlog` in [R] [set iter](#).

`display_options`: `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, and `sformat(%fmt)`; see [R] [Estimation options](#).

Note that the maximum widths for `cformat()`, `pformat()`, and `sformat()` differ from those widths listed in [R] [Estimation options](#). The maximum width for each format is 9 for `expoisson`.

The following option is available with `expoisson` but is not shown in the dialog box:

`coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Exact Poisson regression estimates the model parameters by using the conditional distributions of the parameters' sufficient statistics, and the resulting parameter estimates are known as CMLEs. Exact Poisson regression is a small-sample alternative to the maximum-likelihood Poisson model. See [R] [poisson](#) and [XT] [xtpoisson](#) to obtain maximum likelihood estimates (MLEs) for the Poisson model and the fixed-effects Poisson model.

Let  $Y_i$  denote a Poisson random variable where we observe the outcome  $Y_i = y_i$ ,  $i = 1, \dots, n$ . Associated with each independent observation is a  $1 \times p$  vector of covariates,  $\mathbf{x}_i$ . We will denote  $\mu_i = E[Y_i | \mathbf{x}_i]$  and use the log-linear model to model the relationship between  $Y_i$  and  $\mathbf{x}_i$ ,

$$\log(\mu_i) = \theta + \mathbf{x}_i\boldsymbol{\beta}$$

where the constant term,  $\theta$ , and the  $p \times 1$  vector of regression parameters,  $\boldsymbol{\beta}$ , are unknown. The probability of observing  $Y_i = y_i$ ,  $i = 1, \dots, n$ , is

$$\Pr(\mathbf{Y} = \mathbf{y}) = \prod_{i=1}^n \frac{\mu_i^{y_i} e^{-\mu_i}}{y_i!}$$

where  $\mathbf{Y} = (Y_1, \dots, Y_n)$  and  $\mathbf{y} = (y_1, \dots, y_n)$ . The MLEs for  $\theta$  and  $\boldsymbol{\beta}$  maximize the log of this function.

The sufficient statistics for  $\theta$  and  $\beta_j$ ,  $j = 1, \dots, p$ , are  $M = \sum_{i=1}^n Y_i$  and  $T_j = \sum_{i=1}^n Y_i x_{ij}$ , respectively, and we observe  $M = m$  and  $T_j = t_j$ . `expoisson` tallies the conditional distribution for each  $T_j$ , given the other sufficient statistics  $T_l = t_l$ ,  $l \neq j$  and  $M = m$ . Denote one of these values to be  $t_j^{(k)}$ ,  $k = 1, \dots, N$ , with weight  $w_k$  that accounts for all the generated  $\mathbf{Y}$  vectors that give rise to  $t_j^{(k)}$ . The conditional probability of observing  $T_j = t_j$  has the form

$$\Pr(T_j = t_j | T_l = t_l, l \neq j, M = m) = \frac{w e^{t_j \beta_j}}{\sum_k w_k e^{t_j^{(k)} \beta_j}} \tag{1}$$

where the sum is over the subset of  $\mathbf{T}$  vectors such that  $(T_1^{(k)} = t_1, \dots, T_j^{(k)} = t_j^{(k)}, \dots, T_p^{(k)} = t_p)$  and  $w$  is the weight associated with the observed  $\mathbf{t}$ . The CMLE for  $\beta_j$  maximizes the log of this function.

Specifying nuisance variables in `condvars()` prevents `exppoisson` from estimating their associated regression coefficients. These variables are still conditional variables when tallying the conditional distribution for the variables in `indepvars`.

Inferences from MLEs rely on asymptotics, and if your sample size is small, these inferences may not be valid. On the other hand, inferences from the CMLEs are exact in that they use the conditional distribution of the sufficient statistics outlined above.

For small datasets, the dependent variable can be completely determined by the data. Here the MLEs and the CMLEs are unbounded. When this occurs, `exppoisson` will compute the MUE, the regression estimate that places the observed sufficient statistic at the median of the conditional distribution.

See [R] [exlogistic](#) for a more thorough discussion of exact estimation and related statistics.

▷ Example 1

Armitage, Berry, and Matthews (2002, 499–501) fit a log-linear model to data containing the number of cerebrovascular accidents experienced by 41 men during a fixed period, each of whom had recovered from a previous cerebrovascular accident and was hypertensive. Sixteen men received treatment, and in the original data, there are three age groups (40–49, 50–59,  $\geq 60$ ), but we pool the first two age groups to simplify the example. Armitage, Berry, and Matthews point out that this was not a controlled trial, but the data are useful to inquire whether there is evidence of fewer accidents for the treatment group and if age may be an important factor. The dependent variable `count` contains the number of accidents, variable `treat` is an indicator for the treatment group (1 = treatment, 0 = control), and variable `age` is an indicator for the age group (0 = 40–59; 1 =  $\geq 60$ ).

First, we load the dataset, list it, and tabulate the cerebrovascular accident counts by treatment and age group.

```
. use https://www.stata-press.com/data/r19/cerebacc
(Cerebrovascular accidents in hypotensive-treated and control groups)
. list
```

|     | treat                   | count | age   |
|-----|-------------------------|-------|-------|
| 1.  | Control                 | 0     | 40/59 |
| 2.  | Control                 | 0     | >=60  |
| 3.  | Control                 | 1     | 40/59 |
| 4.  | Control                 | 1     | >=60  |
| 5.  | Control                 | 2     | 40/59 |
| 6.  | Control                 | 2     | >=60  |
| 7.  | Control                 | 3     | 40/59 |
|     | <i>(output omitted)</i> |       |       |
| 35. | Treatment               | 0     | 40/59 |
| 36. | Treatment               | 0     | 40/59 |
| 37. | Treatment               | 0     | 40/59 |
| 38. | Treatment               | 0     | 40/59 |
| 39. | Treatment               | 1     | 40/59 |
| 40. | Treatment               | 1     | 40/59 |
| 41. | Treatment               | 1     | 40/59 |

```
. tabulate treat age [fw=count]
```

| Hypotensive drug treatment | Age group |      | Total |
|----------------------------|-----------|------|-------|
|                            | 40/59     | >=60 |       |
| Control                    | 15        | 10   | 25    |
| Treatment                  | 4         | 0    | 4     |
| Total                      | 19        | 10   | 29    |

Next, we estimate the CMLE with `expoisson` and, for comparison, the MLE with `poisson`.

```
. expoisson count i.treat i.age
Estimating: 1.treat
Enumerating sample-space combinations:
Observation 1: Enumerations = 11
Observation 2: Enumerations = 11
Observation 3: Enumerations = 11
(output omitted)
Observation 39: Enumerations = 410
Observation 40: Enumerations = 410
Observation 41: Enumerations = 30
Estimating: 1.age
Enumerating sample-space combinations:
Observation 1: Enumerations = 5
Observation 2: Enumerations = 15
Observation 3: Enumerations = 15
(output omitted)
Observation 39: Enumerations = 455
Observation 40: Enumerations = 455
Observation 41: Enumerations = 30
```

Exact Poisson regression

Number of obs = 41

| count              | Coefficient | Suff. | 2*Pr(Suff.) | [95% conf. interval] |           |
|--------------------|-------------|-------|-------------|----------------------|-----------|
| treat<br>Treatment | -1.594306   | 4     | 0.0026      | -3.005089            | -.4701708 |
| age<br>>=60        | -.5112067   | 10    | 0.2794      | -1.416179            | .3429232  |

```
. poisson count i.treat i.age, nolog
```

Poisson regression

Number of obs = 41  
 LR chi2(2) = 10.64  
 Prob > chi2 = 0.0049  
 Pseudo R2 = 0.1201

Log likelihood = -38.97981

| count              | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|--------------------|-------------|-----------|-------|-------|----------------------|-----------|
| treat<br>Treatment | -1.594306   | .5573614  | -2.86 | 0.004 | -2.686714            | -.5018975 |
| age<br>>=60        | -.5112067   | .4043525  | -1.26 | 0.206 | -1.303723            | .2813096  |
| _cons              | .233344     | .2556594  | 0.91  | 0.361 | -.2677391            | .7344271  |

expoisson generates an enumeration log for each independent variable in *indepvars*. The conditional distribution of the parameter sufficient statistic is tallied for each independent variable. The conditional distribution for `treat`, for example, has 30 records containing the weights,  $w_k$ , and feasible sufficient statistics,  $t_{\text{treat}}^{(k)}$ . In essence, the set of points  $(w_k, t_{\text{treat}}^{(k)})$ ,  $k = 1, \dots, 30$ , tallied by `expoisson` now become the data to estimate the regression coefficient for `treat`, using (1) as the likelihood. Remember that 1 of the 30  $(w_k, t_{\text{treat}}^{(k)})$  must contain the observed sufficient statistic,  $t_{\text{treat}} = \sum_{i=1}^{41} \text{treat}_i \times \text{count}_i = 4$ , and its relative position in the sorted set of points (sorted by  $t_{\text{treat}}^{(k)}$ ) is how the sufficient-statistic  $p$ -value is computed. This algorithm is repeated for the `age` variable.

The regression coefficients for `treat` and `age` are numerically identical for both Poisson models. Both models provide evidence that the treatment reduces the rate of cerebrovascular accidents,  $\approx e^{-1.59} \approx 0.204$ , or a reduction of about 80%. There is no evidence that `age` plays a role in the rate of accidents.

The results based on the sufficient statistic provide stronger evidence that treatment reduces the rate of cerebrovascular accidents than the corresponding asymptotic statistics. However, the exact confidence intervals are wider than their asymptotic counterparts.

◀

## ▷ Example 2

Agresti (2013, 129) used the data from Laird and Olivier (1981) to demonstrate the Poisson model for modeling rates. The data consist of patient survival after heart valve replacement operations. The sample consists of 109 patients that are classified by type of heart valve (aortic, mitral) and by age ( $<55$ ,  $\geq 55$ ). Follow-up observations cover lengths from 3 to 97 months, and the time at risk, or exposure, is stored in the variable `TAR`. The response is whether the subject died. First, we take a look at the data and then estimate the incidence rates (IRs) with `expoisson` and `poisson`.

```
. use https://www.stata-press.com/data/r19/heartvalve
(Heart valve replacement data)
. list
```

|    | age  | valve  | deaths | TAR  |
|----|------|--------|--------|------|
| 1. | <55  | Aortic | 4      | 1259 |
| 2. | <55  | Mitral | 1      | 2082 |
| 3. | >=55 | Aortic | 7      | 1417 |
| 4. | >=55 | Mitral | 9      | 1647 |

The `age` variable is coded 0 for `age < 55` and 1 for `age ≥ 55`, and the `valve` variable is coded 0 for the aortic valve and 1 for the mitral valve. The total number of deaths,  $M = 21$ , is small enough that enumerating the conditional distributions for `age` and `valve` type is feasible and asymptotic inferences associated with standard maximum-likelihood Poisson regression may be questionable.

```
. expoisson deaths i.age i.valve, exposure(TAR) irr
```

```
Estimating: 1.age
```

```
Enumerating sample-space combinations:
```

```
Observation 1: Enumerations = 11
Observation 2: Enumerations = 11
Observation 3: Enumerations = 132
Observation 4: Enumerations = 22
```

```
Estimating: 1.valve
```

```
Enumerating sample-space combinations:
```

```
Observation 1: Enumerations = 17
Observation 2: Enumerations = 17
Observation 3: Enumerations = 102
Observation 4: Enumerations = 22
```

```
Exact Poisson regression
```

Number of obs = 4

| deaths          | IRR          | Suff. | 2*Pr(Suff.) | [95% conf. interval] |          |
|-----------------|--------------|-------|-------------|----------------------|----------|
| age<br>>=55     | 3.390401     | 16    | 0.0194      | 1.182297             | 11.86935 |
| valve<br>Mitral | .7190197     | 10    | 0.5889      | .2729881             | 1.870068 |
| ln(TAR)         | 1 (exposure) |       |             |                      |          |

```
. poisson deaths i.age i.valve, exposure(TAR) irr nolog
```

```
Poisson regression
```

```
Number of obs = 4
LR chi2(2) = 7.62
Prob > chi2 = 0.0222
Pseudo R2 = 0.3178
```

```
Log likelihood = -8.1747285
```

| deaths          | IRR          | Std. err. | z      | P> z  | [95% conf. interval] |          |
|-----------------|--------------|-----------|--------|-------|----------------------|----------|
| age<br>>=55     | 3.390401     | 1.741967  | 2.38   | 0.017 | 1.238537             | 9.280965 |
| valve<br>Mitral | .7190197     | .3150492  | -0.75  | 0.452 | .3046311             | 1.6971   |
| _cons           | .0018142     | .0009191  | -12.46 | 0.000 | .0006722             | .0048968 |
| ln(TAR)         | 1 (exposure) |           |        |       |                      |          |

Note: **\_cons** estimates baseline incidence rate.

The CMLE and the MLE are numerically identical. We have strong evidence that the death rate for the older age group is higher than the younger age group, specifically 3.4 times higher ( $p = 0.017$ ). This means that for every death in the younger group each month, we would expect about three deaths in the older group. The IR estimate for valve type is approximately 0.72, but we do not have enough evidence to claim that it is different from one. The exact Poisson confidence intervals are a bit wider than the asymptotic confidence intervals.

You can use `ir` (see [R] [Epitab](#)) to estimate IRs and exact confidence intervals for one covariate, and we compare these confidence intervals with those from `expoisson`, where we estimate the IR by using age only.

```
. ir deaths age TAR
```

```
Incidence-rate comparison
```

|                  | Age of patient |           | Total                     |
|------------------|----------------|-----------|---------------------------|
|                  | Exposed        | Unexposed |                           |
| Number of deaths | 16             | 5         | 21                        |
| Time at risk     | 3064           | 3341      | 6405                      |
| Incidence rate   | .0052219       | .0014966  | .0032787                  |
|                  | Point estimate |           | [95% conf. interval]      |
| Inc. rate diff.  | .0037254       |           | .00085 .0066007           |
| Inc. rate ratio  | 3.489295       |           | 1.221441 12.17875 (exact) |
| Attr. frac. ex.  | .7134092       |           | .1812948 .9178898 (exact) |
| Attr. frac. pop  | .5435498       |           |                           |

```
Mid-p-values for tests of incidence-rate difference:
```

```
Adj Pr(Exposed Number of deaths <= 16) = 0.9951 (lower one-sided)
```

```
Adj Pr(Exposed Number of deaths >= 16) = 0.0049 (upper one-sided)
```

```
Two-sided p-value = 0.0099
```

```
. expoisson deaths age, exposure(TAR) irr midp nolog
```

```
Exact Poisson regression
```

|         | Number of obs = 4 |       |             |                      |          |
|---------|-------------------|-------|-------------|----------------------|----------|
| deaths  | IRR               | Suff. | 2*Pr(Suff.) | [95% conf. interval] |          |
| age     | 3.489295          | 16    | 0.0099      | 1.324926             | 10.64922 |
| ln(TAR) | 1 (exposure)      |       |             |                      |          |

```
Note: Mid-p-value computed for the probabilities and CIs.
```

Both `ir` and `expoisson` give identical IRs and  $p$ -values. Both report the two-sided exact  $p$ -value by using the mid- $p$ -value rule that accounts for the discreteness in the distribution by subtracting  $p_{1/2} = \Pr(T = t)/2$  from  $p_l = \Pr(T \leq t)$  and  $p_g = \Pr(T \geq t)$ , computing  $2 \times \min(p_l - p_{1/2}, p_g - p_{1/2})$ . By default, `expoisson` will not use the mid- $p$ -value rule (when you exclude the `midp` option), and here the two-sided exact  $p$ -value would be  $2 \times \min(p_l, p_g) = 0.0158$ . The confidence intervals differ because `expoisson` uses the mid- $p$ -value rule when computing the confidence intervals, yet `ir` does not. You can verify this by executing `expoisson` without the `midp` option for this example; you will get the same confidence intervals as `ir`.

You can replay `expoisson` to view the conditional scores test or the conditional probabilities test by using the `test()` option.

```
. expoisson, test(score) irr
Exact Poisson regression
Number of obs = 4
```

| deaths         | IRR           | Score                 | Pr>=Score | [95% conf. interval] |          |
|----------------|---------------|-----------------------|-----------|----------------------|----------|
| age<br>ln(TAR) | 3.489295<br>1 | 6.76528<br>(exposure) | 0.0113    | 1.324926             | 10.64922 |

Note: Mid-p-value computed for the probabilities and CIs.

All the statistics for `expoisson` are defined in *Methods and formulas* of [R] **exlogistic**. Apart from enumerating the conditional distributions for the logistic and Poisson sufficient statistics, computationally, the primary difference between `exlogistic` and `expoisson` is the weighting values in the likelihood for the parameter sufficient statistics.



### ▷ Example 3

In this example, we fabricate data that will demonstrate the difference between the CMLE and the MUE when the CMLE is not infinite. A difference in these estimates will be more pronounced when the probability of the coefficient sufficient statistic is skewed when plotted as a function of the regression coefficient.

```
. clear
. input y x
 y x
1. 0 2
2. 1 1
3. 1 0
4. 0 0
5. 0 .5
6. 1 .5
7. 2 .01
8. 3 .001
9. 4 .0001
10. end
```

```
. expoiss y x, test(score)
Enumerating sample-space combinations:
Observation 1: Enumerations = 13
Observation 2: Enumerations = 91
Observation 3: Enumerations = 169
Observation 4: Enumerations = 169
Observation 5: Enumerations = 313
Observation 6: Enumerations = 313
Observation 7: Enumerations = 1469
Observation 8: Enumerations = 5525
Observation 9: Enumerations = 5479
```

Exact Poisson regression

Number of obs = 9

| y | Coefficient | Score    | Pr>=Score | [95% conf. interval] |          |
|---|-------------|----------|-----------|----------------------|----------|
| x | -1.534468   | 2.955316 | 0.0810    | -3.761718            | .0485548 |

```
. expoiss y x, test(score) mue(x) nolog
```

Exact Poisson regression

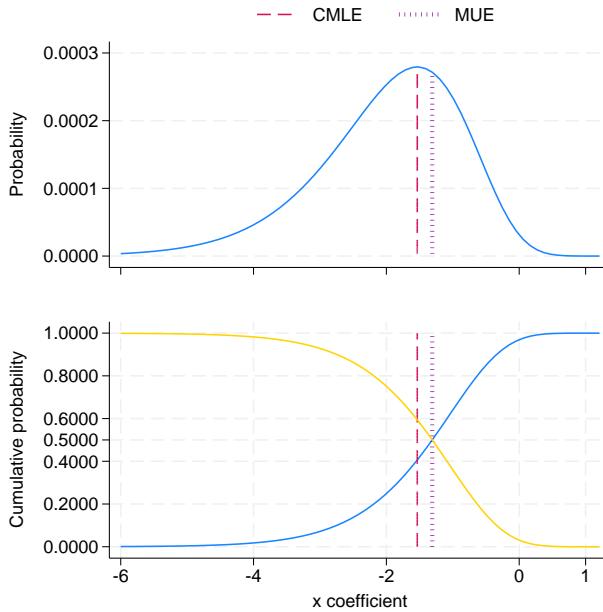
Number of obs = 9

| y | Coefficient | Score    | Pr>=Score | [95% conf. interval] |          |
|---|-------------|----------|-----------|----------------------|----------|
| x | -1.309268*  | 2.955316 | 0.0810    | -3.761718            | .0485548 |

(\*) median unbiased estimates (MUE)

We observe  $(x_i, y_i), i = 1, \dots, 9$ . If we condition on  $m = \sum_{i=1}^9 y_i = 12$ , the conditional distribution of  $T_x = \sum_i Y_i x_i$  has a size of 5,479 elements. For each entry in this enumeration, a realization of  $Y_i = y_i^{(k)}, k = 1, \dots, 5,479$ , is generated such that  $\sum_i y_i^{(k)} = 12$ . One of these realizations produces the observed  $t_x = \sum_i y_i x_i \approx 1.5234$ .

Below is a graphical display comparing the CMLE with the MUE. We plot  $\Pr(T_x = t_x \mid M = 12, \beta_x)$  versus  $\beta_x$ ,  $-6 \leq \beta_x \leq 1$ , in the upper panel and the cumulative probabilities,  $\Pr(T_x \leq t_x \mid M = 12, \beta_x)$  and  $\Pr(T_x \geq t_x \mid M = 12, \beta_x)$ , in the lower panel.



The location of the CMLE, indicated by the dashed line, is at the mode of the probability profile, and the MUE, indicated by the dotted line, is to the right of the mode. If we solve for the  $\beta_x^{(u)}$  and  $\beta_x^{(l)}$  such that  $\Pr(T_x \leq t_x \mid M = 12, \beta_x^{(u)}) = 1/2$  and  $\Pr(T_x \geq t_x \mid M = 12, \beta_x^{(l)}) = 1/2$ , the MUE is  $(\beta_x^{(u)} + \beta_x^{(l)})/2$ . As you can see in the lower panel, the MUE cuts through the intersection of these cumulative probability profiles.



## □ Technical note

The `memory()` option limits the amount of memory that `expoissou` will consume when computing the conditional distribution of the parameter sufficient statistics. `memory()` is independent of the data maximum memory setting (see `set max_memory` in [D] [memory](#)), and it is possible for `expoissou` to exceed the memory limit specified in `set max_memory` without terminating. By default, a log is provided that displays the number of enumerations (the size of the conditional distribution) after processing each observation. Typically, you will see the number of enumerations increase, and then at some point they will decrease as the multivariate shift algorithm (Hirji, Mehta, and Patel 1987) determines that some of the enumerations cannot achieve the observed sufficient statistics of the conditioning variables. When the algorithm is complete, however, it is necessary to store the conditional distribution of the parameter sufficient statistics as a dataset. It is possible, therefore, to get a memory error when the algorithm has completed if there is not enough memory to store the conditional distribution.



## □ Technical note

Computing the conditional distributions and reported statistics requires data sorting and numerical comparisons. If there is at least one single-precision variable specified in the model, `exppoisson` will make comparisons with a relative precision of  $2^{-5}$ . Otherwise, a relative precision of  $2^{-11}$  is used. Be careful if you use `recast` to promote a single-precision variable to double precision (see [D] [recast](#)). You might try listing the data in full precision (maybe `%20.15g`; see [D] [format](#)) to make sure that this is really what you want. See [D] [Data types](#) for information on precision of numeric storage types. □

## Stored results

`exppoisson` stores the following in `e()`:

### Scalars

|                                 |                                                                                          |
|---------------------------------|------------------------------------------------------------------------------------------|
| <code>e(N)</code>               | number of observations                                                                   |
| <code>e(k_groups)</code>        | number of groups                                                                         |
| <code>e(relative_weight)</code> | relative weight for the observed <code>e(sufficient)</code> and <code>e(condvars)</code> |
| <code>e(sum_y)</code>           | sum of <i>depvar</i>                                                                     |
| <code>e(k_indvars)</code>       | number of independent variables                                                          |
| <code>e(k_condvars)</code>      | number of conditioning variables                                                         |
| <code>e(midp)</code>            | mid- <i>p</i> -value rule indicator                                                      |
| <code>e(eps)</code>             | relative difference tolerance                                                            |

### Macros

|                                   |                                                |
|-----------------------------------|------------------------------------------------|
| <code>e(cmd)</code>               | <code>exppoisson</code>                        |
| <code>e(cmdline)</code>           | command as typed                               |
| <code>e(title)</code>             | title in estimation output                     |
| <code>e(depvar)</code>            | name of dependent variable                     |
| <code>e(indvars)</code>           | independent variables                          |
| <code>e(condvars)</code>          | conditional variables                          |
| <code>e(groupvar)</code>          | group variable                                 |
| <code>e(exposure)</code>          | exposure variable                              |
| <code>e(offset)</code>            | linear offset variable                         |
| <code>e(level)</code>             | confidence level                               |
| <code>e(wtype)</code>             | weight type                                    |
| <code>e(wexp)</code>              | weight expression                              |
| <code>e(datasignature)</code>     | the checksum                                   |
| <code>e(datasignaturevars)</code> | variables used in calculation of checksum      |
| <code>e(properties)</code>        | <code>b</code>                                 |
| <code>e(estat_cmd)</code>         | program used to implement <code>estat</code>   |
| <code>e(marginsnotok)</code>      | predictions disallowed by <code>margins</code> |

### Matrices

|                                |                                                                                 |
|--------------------------------|---------------------------------------------------------------------------------|
| <code>e(b)</code>              | coefficient vector                                                              |
| <code>e(mue_indicators)</code> | indicator for elements of <code>e(b)</code> estimated using MUE instead of CMLE |
| <code>e(se)</code>             | <code>e(b)</code> standard errors (CMLEs only)                                  |
| <code>e(ci)</code>             | matrix of <code>e(level)</code> confidence intervals for <code>e(b)</code>      |
| <code>e(sum_y_groups)</code>   | sum of <code>e(depvar)</code> for each group                                    |
| <code>e(N_g)</code>            | number of observations in each group                                            |
| <code>e(sufficient)</code>     | sufficient statistics for <code>e(b)</code>                                     |
| <code>e(p_sufficient)</code>   | <i>p</i> -value for <code>e(sufficient)</code>                                  |
| <code>e(scoretest)</code>      | conditional scores tests for <i>indepvars</i>                                   |
| <code>e(p_scoretest)</code>    | <i>p</i> -values for <code>e(scoretest)</code>                                  |
| <code>e(probtest)</code>       | conditional probabilities tests for <i>indepvars</i>                            |
| <code>e(p_probtest)</code>     | <i>p</i> -value for <code>e(probtest)</code>                                    |

### Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

## Methods and formulas

Let  $\{Y_1, Y_2, \dots, Y_n\}$  be a set of  $n$  independent Poisson random variables. For each  $i = 1, \dots, n$ , we observe  $Y_i = y_i \geq 0$ , and associated with each observation is the covariate row vector of length  $p$ ,  $\mathbf{x}_i = (x_{i1}, \dots, x_{ip})$ . Denote  $\boldsymbol{\beta} = (\beta_1, \dots, \beta_p)^T$  to be the column vector of regression parameters and  $\theta$  to be the constant. The sufficient statistic for  $\beta_j$  is  $T_j = \sum_{i=1}^n Y_i x_{ij}$ ,  $j = 1, \dots, p$ , and for  $\theta$  is  $M = \sum_{i=1}^n Y_i$ . We observe  $T_j = t_j$ ,  $t_j = \sum_{i=1}^n y_i x_{ij}$ , and  $M = m$ ,  $m = \sum_{i=1}^n y_i$ . Let  $\kappa_i$  be the exposure for the  $i$ th observation. Then the probability of observing  $(Y_1 = y_1, Y_2 = y_2, \dots, Y_n = y_n)$  is

$$\Pr(Y_1 = y_1, \dots, Y_n = y_n \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\kappa}) = \frac{\exp(m\theta + \mathbf{t}\boldsymbol{\beta})}{\exp\{\sum_{i=1}^n \kappa_i \exp(\theta + \mathbf{x}_i\boldsymbol{\beta})\}} \prod_{i=1}^n \frac{\kappa_i^{y_i}}{y_i!}$$

where  $\mathbf{t} = (t_1, \dots, t_p)$ ,  $\mathbf{X} = (\mathbf{x}_1^T, \dots, \mathbf{x}_n^T)^T$ , and  $\boldsymbol{\kappa} = (\kappa_1, \dots, \kappa_n)^T$ .

The joint distribution of the sufficient statistics  $(\mathbf{T}, M)$  is obtained by summing over all possible sequences  $Y_1 \geq 0, \dots, Y_n \geq 0$  such that  $\mathbf{T} = \mathbf{t}$  and  $M = m$ . This probability function is

$$\Pr(T_1 = t_1, \dots, T_p = t_p, M = m \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\kappa}) = \frac{\exp(m\theta + \mathbf{t}\boldsymbol{\beta})}{\exp\{\sum_{i=1}^n \kappa_i \exp(\theta + \mathbf{x}_i\boldsymbol{\beta})\}} \left( \sum_{\mathbf{u}} \prod_{i=1}^n \frac{\kappa_i^{u_i}}{u_i!} \right)$$

where the sum  $\sum_{\mathbf{u}}$  is over all nonnegative vectors  $\mathbf{u}$  of length  $n$  such that  $\sum_{i=1}^n u_i = m$  and  $\sum_{i=1}^n u_i \mathbf{x}_i = \mathbf{t}$ .

## Conditional distribution

Without loss of generality, we will restrict our discussion to the conditional distribution of the sufficient statistic for  $\beta_1$ ,  $T_1$ . If we condition on observing  $M = m$  and  $T_2 = t_2, \dots, T_p = t_p$ , the probability function of  $(T_1 \mid \beta_1, T_2 = t_2, \dots, T_p = t_p, M = m)$  is

$$\Pr(T_1 = t_1 \mid \beta_1, T_2 = t_2, \dots, T_p = t_p, M = m) = \frac{\left( \sum_{\mathbf{u}} \prod_{i=1}^n \frac{\kappa_i^{u_i}}{u_i!} \right) e^{t_1 \beta_1}}{\sum_{\mathbf{v}} \left( \prod_{i=1}^n \frac{\kappa_i^{v_i}}{v_i!} \right) e^{\beta_1 \sum_i v_i x_{i1}}} \quad (2)$$

where the sum  $\sum_{\mathbf{u}}$  is over all nonnegative vectors  $\mathbf{u}$  of length  $n$  such that  $\sum_{i=1}^n u_i = m$  and  $\sum_{i=1}^n u_i \mathbf{x}_i = \mathbf{t}$ , and the sum  $\sum_{\mathbf{v}}$  is over all nonnegative vectors  $\mathbf{v}$  of length  $n$  such that  $\sum_{i=1}^n v_i = m$ ,  $\sum_{i=1}^n v_i x_{i2} = t_2, \dots, \sum_{i=1}^n v_i x_{ip} = t_p$ . The CMLE for  $\beta_1$  is the value that maximizes the log of (2). This optimization task is carried out by `m1` (see [R] `m1`), using the conditional distribution of  $(T_1 \mid T_2 = t_2, \dots, T_p = t_p, M = m)$  as a dataset. This dataset consists of the feasible values and weights for  $T_1$ ,

$$\left\{ \left( s_1, \prod_{i=1}^n \frac{\kappa_i^{v_i}}{v_i!} \right) : \sum_{i=1}^n v_i = m, \sum_{i=1}^n v_i x_{i1} = s_1, \sum_{i=1}^n v_i x_{i2} = t_2, \dots, \sum_{i=1}^n v_i x_{ip} = t_p \right\}$$

Computing the CMLE, MUE, confidence intervals, conditional hypothesis tests, and sufficient statistic  $p$ -values is discussed in *Methods and formulas* of [R] **exlogistic**. The only difference between the two techniques is the use of the weights; that is, the weights for exact logistic are the combinatorial coefficients,  $c(\mathbf{t}, m)$ , in the *Methods and formulas* in [R] **exlogistic**. `expoisson` and `exlogistic` use the same `m1` likelihood evaluator to compute the CMLEs as well as the same ado-programs and Mata functions to compute the MUEs and estimate statistics.

## References

- Agresti, A. 2013. *Categorical Data Analysis*. 3rd ed. Hoboken, NJ: Wiley.
- Armitage, P., G. Berry, and J. N. S. Matthews. 2002. *Statistical Methods in Medical Research*. 4th ed. Oxford: Blackwell.
- Cox, D. R., and E. J. Snell. 1989. *Analysis of Binary Data*. 2nd ed. London: Chapman and Hall. <https://doi.org/10.1201/9781315137391>.
- Hirji, K. F., C. R. Mehta, and N. R. Patel. 1987. Computing distributions for exact logistic regression. *Journal of the American Statistical Association* 82: 1110–1117. <https://doi.org/10.2307/2289388>.
- Laird, N. M., and D. Olivier. 1981. Covariance analysis of censored survival data using log-linear analysis techniques. *Journal of the American Statistical Association* 76: 231–240. <https://doi.org/10.2307/2287816>.

## Also see

[R] **expoisson postestimation** — Postestimation tools for `expoisson`

[R] **poisson** — Poisson regression

[XT] **xtpoisson** — Fixed-effects, random-effects, and population-averaged Poisson models

[U] **20 Estimation and postestimation commands**

## Postestimation commands

The following postestimation command is of special interest after `expoisson`:

| Command               | Description                                                      |
|-----------------------|------------------------------------------------------------------|
| <code>estat se</code> | report coefficients or IRRs and their asymptotic standard errors |

The following standard postestimation commands are also available:

| Command                      | Description                                  |
|------------------------------|----------------------------------------------|
| <code>estat summarize</code> | summary statistics for the estimation sample |
| <code>estimates</code>       | cataloging estimation results                |
| <code>etable</code>          | table of estimation results                  |

## estat

### Description for `estat`

`estat se` reports regression coefficients or incidence-rate asymptotic standard errors. The estimates are stored in the matrix `r(estimates)`.

### Menu for `estat`

Statistics > Postestimation

### Syntax for `estat`

```
estat se [, irr]
```

### Option for `estat`

`irr` requests that the incidence-rate ratios and their asymptotic standard errors be reported. The default is to report the coefficients and their asymptotic standard errors.

## Remarks and examples

### ▷ Example 1

To demonstrate `estat se` after `expoission`, we use the British physicians smoking data.

```
. use https://www.stata-press.com/data/r19/smokes
(Cigarette smoking and lung cancer among British physicians (45–49 years))
. expoission cases smokes, exposure(peryrs) irr nolog
```

Exact Poisson regression

Number of obs = 7

| cases                | IRR                      | Suff. | 2*Pr(Suff.) | [95% conf. interval] |          |
|----------------------|--------------------------|-------|-------------|----------------------|----------|
| smokes<br>ln(peryrs) | 1.077718<br>1 (exposure) | 797.4 | 0.0000      | 1.04552              | 1.111866 |

```
. estat se, irr
```

| cases  | IRR      | Std. err. |
|--------|----------|-----------|
| smokes | 1.077718 | .0168547  |



### Also see

[R] [expoission](#) — Exact Poisson regression

[U] [20 Estimation and postestimation commands](#)

[Description](#)[Options](#)[Acknowledgment](#)[Quick start](#)[Remarks and examples](#)[References](#)[Menu](#)[Stored results](#)[Also see](#)[Syntax](#)[Methods and formulas](#)

## Description

`fp <term>`: `est_cmd` fits models with the “best”-fitting fractional polynomial substituted for `<term>` wherever it appears in `est_cmd`. `fp <weight>`: `regress mpg <weight> foreign` would fit a regression model of `mpg` on a fractional polynomial in `weight` and (linear) `foreign`.

By specifying option `fp()`, you may set the exact powers to be used. Otherwise, a search through all possible fractional polynomials up to the degree set by `dimension()` with the powers set by `powers()` is performed.

`fp` without arguments redisplay the previous estimation results, just as typing `est_cmd` would. You can type either one. `fp` will include a fractional polynomial comparison table.

`fp generate` creates fractional polynomial power variables for a given set of powers. For instance, `fp <weight>`: `regress mpg <weight> foreign` might produce the fractional polynomial  $\text{weight}^{(-2,-1)}$  and store  $\text{weight}^{-2}$  in `weight_1` and  $\text{weight}^{-1}$  in `weight_2`. Typing `fp generate weight^(-2 -1)` would allow you to create the same variables in another dataset.

See [\[R\] mfp](#) for multivariable fractional polynomial models.

## Quick start

*Fit models with fractional polynomials*

Find optimal second-degree fractional polynomial of `x1` in regression of `y` on `x2` and `x3`

```
fp <x1>: regress y <x1> x2 x3
```

Same as above, but search only powers of  $-1$ ,  $-0.5$ ,  $1$ , and  $2$ .

```
fp <x1>, power(-1 -.5 1 2): regress y <x1> x2 x3
```

Same as above, but allow search to include third-degree fractional polynomials

```
fp <x1>, power(-1 -.5 1 2) dimension(3): regress y <x1> x2 x3
```

Fit model including  $x1^{-2}$  and  $x1^2$  without performing search

```
fp <x1>, fp(-2 2): regress y <x1> x2 x3
```

Rescale `x1` to nonextreme positive values when computing fractional polynomials

```
fp <x1>, scale: regress y <x1> x2 x3
```

Same as above, and center fractional polynomial of `x1` at its scaled mean

```
fp <x1>, center scale: regress y <x1> x2 x3
```

Set fractional polynomial to zero for nonpositive values of  $x_1$

```
fp <x1>, zero: regress y <x1> x2 x3
```

Same as above, and include an indicator variable in the model for nonpositive values of  $x_1$

```
fp <x1>, catzero: regress y <x1> x2 x3
```

Create variables corresponding to fractional polynomial powers

Generate  $x1\_1$  and  $x1\_2$  corresponding to  $x_1^{-2}$  and  $x_1^2$

```
fp generate x1^(-2 2)
```

Same as above, but generate fractional polynomial variables with automatic scaling and centering

```
fp generate x1^(-2 2), center scale
```

Note: In the above examples, `regress` could be replaced with any estimation command allowing the `fp` prefix.

## Menu

### fp

Statistics > Linear models and related > Fractional polynomials > Fractional polynomial regression

### fp generate

Statistics > Linear models and related > Fractional polynomials > Create fractional polynomial variables

## Syntax

*Estimation*

```
fp <term> [, est_options] : est_cmd
```

*Specify that fractional powers of varname be calculated during estimation*

```
fp <term>(varname) [, est_options] : est_cmd
```

*Replay estimation results*

```
fp [, replay_options]
```

*Create specified fractional polynomial power variables*

```
fp generate [type] [newvar =] varnam@(numlist) [if] [in] [, gen_options]
```

*est\_cmd* may be almost any estimation command that stores the `e(11)` result. To confirm whether `fp` works with a specific *est\_cmd*, see the documentation for that *est\_cmd*. *est\_cmd* may not contain other prefix commands; see [\[U\] 11.1.10 Prefix commands](#).

Instances of  $\langle term \rangle$  (with the angle brackets) that occur within *est\_cmd* are replaced in *est\_cmd* by a varlist containing the fractional powers of the variable *term*. These variables will be named *term\_1*, *term\_2*, ...

fp performs *est\_cmd* with this substitution, fitting a fractional polynomial regression in *term*.

| <i>est_options</i>            | Description                                                                                                            |
|-------------------------------|------------------------------------------------------------------------------------------------------------------------|
| Main                          |                                                                                                                        |
| <code>powers(## ... #)</code> | powers to be searched; default is <code>powers(-2 -1 - .5 0 .5 1 2 3)</code>                                           |
| <code>dimension(#)</code>     | maximum degree of fractional polynomial; default is <code>dimension(2)</code>                                          |
| <code>fp(## ... #)</code>     | use specified fractional polynomial                                                                                    |
| Options                       |                                                                                                                        |
| <code>classic</code>          | perform automatic scaling and centering and omit comparison table                                                      |
| <code>replace</code>          | replace existing fractional polynomial power variables named <i>term_1</i> , <i>term_2</i> , ...                       |
| <code>all</code>              | generate <i>term_1</i> , <i>term_2</i> , ... in all observations; default is in observations if <code>esample()</code> |
| <code>scale(#_a #_b)</code>   | use $(term+a)/b$ ; default is to use variable <i>term</i> as is                                                        |
| <code>scale</code>            | specify <i>a</i> and <i>b</i> automatically                                                                            |
| <code>center(#_c)</code>      | report centered-on- <i>c</i> results; default is uncentered results                                                    |
| <code>center</code>           | specify <i>c</i> to be the mean of (scaled) <i>term</i>                                                                |
| <code>zero</code>             | set <i>term_1</i> , <i>term_2</i> , ... to zero if scaled $term \leq 0$ ; default is to issue an error message         |
| <code>catzero</code>          | same as <code>zero</code> and include $term_0 = (term \leq 0)$ among fractional polynomial power variables             |
| Reporting                     |                                                                                                                        |
| <code>replay_options</code>   | specify how results are displayed                                                                                      |

| <i>replay_options</i>          | Description                                                            |
|--------------------------------|------------------------------------------------------------------------|
| Reporting                      |                                                                        |
| <code>nocompare</code>         | do not display model-comparison test results                           |
| <code>reporting_options</code> | any options allowed by <i>est_cmd</i> for replaying estimation results |

| <i>gen_options</i>          | Description                                                                                                    |
|-----------------------------|----------------------------------------------------------------------------------------------------------------|
| Main                        |                                                                                                                |
| <code>replace</code>        | replace existing fractional polynomial power variables named <i>term_1</i> , <i>term_2</i> , ...               |
| <code>scale(#_a #_b)</code> | use $(term+a)/b$ ; default is to use variable <i>term</i> as is                                                |
| <code>scale</code>          | specify <i>a</i> and <i>b</i> automatically                                                                    |
| <code>center(#_c)</code>    | report centered-on- <i>c</i> results; default is uncentered results                                            |
| <code>center</code>         | specify <i>c</i> to be the mean of (scaled) <i>term</i>                                                        |
| <code>zero</code>           | set <i>term_1</i> , <i>term_2</i> , ... to zero if scaled $term \leq 0$ ; default is to issue an error message |
| <code>catzero</code>        | same as <code>zero</code> and include $term_0 = (term \leq 0)$ among fractional polynomial power variables     |

collect is allowed with fp and fp generate; see [\[U\] 11.1.10 Prefix commands](#).

## Options

Options are presented under the following headings:

*Options for fp*  
*Options for fp generate*

## Options for fp

### Main

`powers(# # ... #)` specifies that a search be performed and details about the search provided. `powers()` works with the `dimension()` option; see below. The default is `powers(-2 -1 -.5 0 .5 1 2 3)`.

`dimension(#)` specifies the maximum degree of the fractional polynomial to be searched. The default is `dimension(2)`.

If the defaults for both `powers()` and `dimension()` are used, then the fractional polynomial could be any of the following 44 possibilities:

$$\begin{array}{c} term^{(-2)} \\ term^{(-1)} \\ \vdots \\ term^{(3)} \\ term^{(-2)}, term^{(-2)} \\ term^{(-2)}, term^{(-1)} \\ \vdots \\ term^{(-2)}, term^{(3)} \\ term^{(-1)}, term^{(-2)} \\ \vdots \\ term^{(3)}, term^{(3)} \end{array}$$

`fp(# # ... #)` specifies that no search be performed and that the fractional polynomial specified be used. `fp()` is an alternative to `powers()` and `dimension()`.

### Options

`classic` performs automatic scaling and centering and omits the comparison table. Specifying `classic` is equivalent to specifying `scale`, `center`, and `nocompare`.

`replace` replaces existing fractional polynomial power variables named `term_1`, `term_2`, ...

`all` specifies that `term_1`, `term_2`, ... be filled in for all observations in the dataset rather than just for those in `e(sample)`.

`scale(#_a #_b)` specifies that `term` be scaled in the way specified, namely, that  $(term+a)/b$  be calculated. All values of the scaled term are required to be greater than zero unless you specify options `zero` or `catzero`. Values should not be too large or too close to zero, because by default, cubic powers and squared reciprocal powers will be considered. When `scale(a b)` is specified, values in the variable `term` are not modified; `fp` merely remembers to scale the values whenever powers are calculated.

You will probably not use `scale(a b)` for values of  $a$  and  $b$  that you create yourself, although you could. It is usually easier just to generate a scaled variable. For instance, if *term* is age, and age in your data is required to be greater than or equal to 20, you might generate an `age5` variable, for use as *term*:

```
. generate age5 = (age-19)/5
```

`scale(a b)` is useful when you previously fit a model using automatic scaling (option `scale`) in one dataset and now want to create the fractional polynomials in another. In the first dataset, `fp` with `scale` added notes to the dataset concerning the values of  $a$  and  $b$ . You can see them by typing

```
. notes
```

You can then use `fp generate, scale(a b)` in the second dataset.

The default is to use *term* as it is used in calculating fractional powers; thus, *term*'s values are required to be greater than zero unless you specify options `zero` or `catzero`. Values should not be too large, because by default, cubic powers will be considered.

`scale` specifies that *term* be scaled to be greater than zero and not too large in calculating fractional powers. See [Scaling](#) for more details. When `scale` is specified, values in the variable *term* are not modified; `fp` merely remembers to scale the values whenever powers are calculated.

`center(#_c)` reports results for the fractional polynomial in (scaled) *term*, centered on  $c$ . The default is to perform no centering.

$term^{(p_1, p_2, \dots, p_m)} - c^{(p_1, p_2, \dots, p_m)}$  is reported. This makes the constant coefficient (intercept) easier to interpret. See [Centering](#) for more details.

`center` performs `center(c)`, where  $c$  is the mean of (scaled) *term*.

`zero` and `catzero` specify how nonpositive values of *term* are to be handled. By default, nonpositive values of *term* are not allowed, because we will be calculating natural logarithms and fractional powers of *term*. Thus, an error message is issued.

`zero` sets the fractional polynomial value to zero for nonpositive values of (scaled) *term*.

`catzero` sets the fractional polynomial value to zero for nonpositive values of (scaled) *term* and includes a dummy variable indicating where nonpositive values of (scaled) *term* appear in the model.

---

#### Reporting

`nocompare` suppresses display of the comparison tests.

`reporting_options` are any options allowed by `est_cmd` for replaying estimation results.

## Options for fp generate

Main

`replace` replaces existing fractional polynomial power variables named `term_1`, `term_2`, ...

`scale(#_a #_b)` specifies that `term` be scaled in the way specified, namely, that  $(term+a)/b$  be calculated.

All values of the scaled term are required to be greater than zero unless you specify options `zero` or `catzero`. Values should not be too large or too close to zero, because by default, cubic powers and squared reciprocal powers will be considered. When `scale(a b)` is specified, values in the variable `term` are not modified; `fp` merely remembers to scale the values whenever powers are calculated.

You will probably not use `scale(a b)` for values of `a` and `b` that you create yourself, although you could. It is usually easier just to generate a scaled variable. For instance, if `term` is `age`, and `age` in your data is required to be greater than or equal to 20, you might generate an `age5` variable, for use as `term`:

```
. generate age5 = (age-19)/5
```

`scale(a b)` is useful when you previously fit a model using automatic scaling (option `scale`) in one dataset and now want to create the fractional polynomials in another. In the first dataset, `fp` with `scale` added notes to the dataset concerning the values of `a` and `b`. You can see them by typing

```
. notes
```

You can then use `fp generate`, `scale(a b)` in the second dataset.

The default is to use `term` as it is used in calculating fractional powers; thus, `term`'s values are required to be greater than zero unless you specify options `zero` or `catzero`. Values should not be too large, because by default, cubic powers will be considered.

`scale` specifies that `term` be scaled to be greater than zero and not too large in calculating fractional powers. See [Scaling](#) for more details. When `scale` is specified, values in the variable `term` are not modified; `fp` merely remembers to scale the values whenever powers are calculated.

`center(#_c)` reports results for the fractional polynomial in (scaled) `term`, centered on `c`. The default is to perform no centering.

$term^{(p_1:p_2:\dots:p_m)} - c^{(p_1:p_2:\dots:p_m)}$  is reported. This makes the constant coefficient (intercept) easier to interpret. See [Centering](#) for more details.

`center` performs `center(c)`, where `c` is the mean of (scaled) `term`.

`zero` and `catzero` specify how nonpositive values of `term` are to be handled. By default, nonpositive values of `term` are not allowed, because we will be calculating natural logarithms and fractional powers of `term`. Thus, an error message is issued.

`zero` sets the fractional polynomial value to zero for nonpositive values of (scaled) `term`.

`catzero` sets the fractional polynomial value to zero for nonpositive values of (scaled) `term` and includes a dummy variable indicating where nonpositive values of (scaled) `term` appear in the model.

## Remarks and examples

Remarks are presented under the following headings:

*Fractional polynomial regression*

*Scaling*

*Centering*

*Examples*

## Fractional polynomial regression

Regression models based on fractional polynomial functions of a continuous covariate are described by [Royston and Altman \(1994\)](#).

Fractional polynomials increase the flexibility afforded by the family of conventional polynomial models. Although polynomials are popular in data analysis, linear and quadratic functions are limited in their range of curve shapes, whereas cubic and higher-order curves often produce undesirable artifacts such as edge effects and waves.

Fractional polynomials differ from regular polynomials in that 1) they allow logarithms, 2) they allow noninteger powers, and 3) they allow powers to be repeated.

We will write a fractional polynomial in  $x$  as

$$x^{(p_1, p_2, \dots, p_m)'} \boldsymbol{\beta}$$

We will write  $x^{(p)}$  to mean a regular power except that  $x^{(0)}$  is to be interpreted as meaning  $\ln(x)$  rather than  $x^{(0)} = 1$ .

Then if there are no repeated powers in  $(p_1, p_2, \dots, p_m)$ ,

$$x^{(p_1, p_2, \dots, p_m)'} \boldsymbol{\beta} = \beta_0 + \beta_1 x^{(p_1)} + \beta_2 x^{(p_2)} + \dots + \beta_m x^{(p_m)}$$

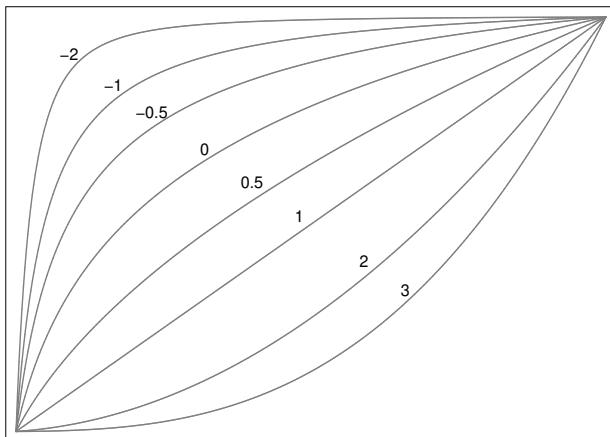
Powers are allowed to repeat in fractional polynomials. Each time a power repeats, it is multiplied by another  $\ln(x)$ . As an extreme case, consider the fractional polynomial with all-repeated powers, say,  $m$  of them,

$$x^{(p, p, \dots, p)'} \boldsymbol{\beta} = \beta_0 + \beta_1 x^{(p)} + \beta_2 x^{(p)} \ln(x) + \dots + \beta_m x^{(p)} \{\ln(x)\}^{m-1}$$

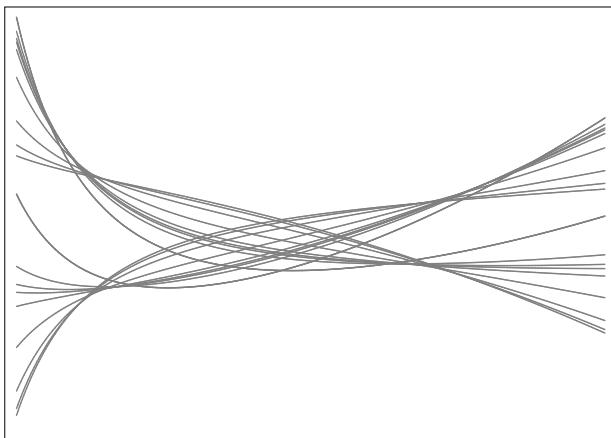
Thus, the fractional polynomial  $x^{(0, 0, 2)'} \boldsymbol{\beta}$  would be

$$\begin{aligned} x^{(0, 0, 2)'} \boldsymbol{\beta} &= \beta_0 + \beta_1 x^{(0)} + \beta_2 x^{(0)} \ln(x) + \beta_3 x^{(2)} \\ &= \beta_0 + \beta_1 \ln(x) + \beta_2 \{\ln(x)\}^2 + \beta_3 x^2 \end{aligned}$$

With this definition, we can obtain a much wider range of shapes than can be obtained with regular polynomials. The following graphs appeared in [Royston and Sauerbrei \(2008, sec. 4.5\)](#). The first graph shows the shapes of differing fractional polynomials.



The second graph shows some of the curve shapes available with different  $\beta$ s for the degree-2 fractional polynomial,  $x^{(-2,2)}$ .



In modeling a fractional polynomial, [Royston and Sauerbrei \(2008\)](#) recommend choosing powers from among  $\{-2, -1, -0.5, 0, 0.5, 1, 2, 3\}$ . By default, `fp` chooses powers from this set, but other powers can be explicitly specified in the `powers()` option.

`fp <term>`: `est_cmd` fits models with the terms of the best-fitting fractional polynomial substituted for `<term>` wherever it appears in `est_cmd`. We will demonstrate with `auto.dta`, which contains repair records and other information about a variety of vehicles in 1978.

We use `fp` to find the best fractional polynomial in automobile weight (lbs.) (`weight`) for the linear regression of miles per gallon (`mpg`) on `weight` and an indicator of whether the vehicle is foreign (`foreign`).

By default, `fp` will fit degree-2 fractional polynomial (FP2) models and choose the fractional powers from the set  $\{-2, -1, -0.5, 0, 0.5, 1, 2, 3\}$ . Because car weight is measured in pounds and will have a cubic transformation applied to it, we shrink it to a smaller scale before estimation by dividing by 1,000.

We modify the existing `weight` variable for conciseness and to facilitate the comparison of tables. When applying a data transformation in practice, rather than modifying the existing variables, you should create new variables that hold the transformed values.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)

. replace weight = weight/1000
variable weight was int now float
(74 real changes made)

. fp <weight>: regress mpg <weight> foreign
(fitting 44 models)
(...10%...20%...30%...40%...50%...60%...70%...80%...90%...100%)
```

Fractional polynomial comparisons:

| weight  | Test<br>df | Deviance | Residual<br>std. dev. | Deviance<br>diff. | P     | Powers |
|---------|------------|----------|-----------------------|-------------------|-------|--------|
| omitted | 4          | 456.347  | 5.356                 | 75.216            | 0.000 |        |
| linear  | 3          | 388.366  | 3.407                 | 7.236             | 0.082 | 1      |
| m = 1   | 2          | 381.806  | 3.259                 | 0.675             | 0.733 | -.5    |
| m = 2   | 0          | 381.131  | 3.268                 | 0.000             | --    | -2 -2  |

Note: **Test df** is degrees of freedom, and **P** =  $P > F$  is sig. level for tests comparing models vs. model with  $m = 2$  based on deviance difference,  $F(df, 68)$ .

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
|          |            |    |            | F(3, 70)      | = | 52.95  |
| Model    | 1696.05949 | 3  | 565.353163 | Prob > F      | = | 0.0000 |
| Residual | 747.399969 | 70 | 10.6771424 | R-squared     | = | 0.6941 |
|          |            |    |            | Adj R-squared | = | 0.6810 |
| Total    | 2443.45946 | 73 | 33.4720474 | Root MSE      | = | 3.2676 |

| mpg      | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|----------|-------------|-----------|-------|-------|----------------------|-----------|
| weight_1 | 15.88527    | 20.60329  | 0.77  | 0.443 | -25.20669            | 56.97724  |
| weight_2 | 127.9349    | 47.53106  | 2.69  | 0.009 | 33.13723             | 222.7326  |
| foreign  | -2.222515   | 1.053782  | -2.11 | 0.039 | -4.324218            | -.1208131 |
| _cons    | 3.705981    | 3.367949  | 1.10  | 0.275 | -3.011182            | 10.42314  |

`fp` begins by showing the model-comparison table. This table shows the best fractional polynomial model of `weight` for each examined degree,  $m$ , which is obtained by searching through all possible power combinations. The row labeled `omitted` describes the null model, which entirely omits `weight` from the model. A separate row is provided for the model with a linear function of `weight` because it is often the default when including a predictor in the model.

The fractional powers of the models are shown in the `Powers` column. An estimate of the residual standard error is given in the `Residual std. dev.` column. The model deviance, which we define as twice the negative log likelihood, is given in the `Deviance` column. The `Deviance diff.` column reports the difference in deviance compared with the model with the lowest deviance, which is always the model with the highest-degree fractional polynomial.

The `Test df` column displays the degrees of freedom used when testing a model's fit against the fit of the model with the lowest deviance. For normal error models such as linear regression, a partial  $F$  test is performed, and `Test df` is the numerator degrees of freedom of the  $F$  test. In other settings, a likelihood-ratio test is performed, and `Test df` is the degrees of freedom of the  $\chi^2$  statistic. In both cases, the  $p$ -value for the test is reported in column `P`.

Under robust variance estimation and some other cases (see [R] `lrtest`), the likelihood-ratio test cannot be performed. When the likelihood-ratio test cannot be performed on the model specified in `est_cmd`, `fp` still reports the model-comparison table, but the comparison tests are not performed.

`fp` reports the “best” model as the model with the lowest deviance; however, users may choose a more efficient model based on the comparison table. They may choose the lowest degree model that the partial  $F$  test (or likelihood-ratio test) fails to reject in favor of the lowest deviance model.

After the comparison table, the results of the estimation command for the lowest deviance model are shown. Here the best model has terms `weight(-2,-2)`. However, based on the model-comparison table, we can reject the model without `weight` and the linear model at the 0.1 significance level. We fail to reject the  $m = 1$  model at any reasonable level. We will choose the FP1 model, which includes `weight(-5)`.

We use `fp` again to estimate the parameters for this model. We use the `fp()` option to specify what powers we want to use; this option specifies that we do not want to perform a search for the best powers. We also specify the `replace` option to overwrite the previously created fractional polynomial power variables.

```
. fp <weight>, fp(-.5) replace: regress mpg <weight> foreign
-> regress mpg weight_1 foreign
```

| Source   | SS         | df | MS         |               |   |        |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 1689.20865 | 2  | 844.604325 | Number of obs | = | 74     |
| Residual | 754.25081  | 71 | 10.6232508 | F(2, 71)      | = | 79.51  |
|          |            |    |            | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.6913 |
|          |            |    |            | Adj R-squared | = | 0.6826 |
| Total    | 2443.45946 | 73 | 33.4720474 | Root MSE      | = | 3.2593 |

| mpg      | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|----------|-------------|-----------|-------|-------|----------------------|-----------|
| weight_1 | 66.89665    | 6.021749  | 11.11 | 0.000 | 54.88963             | 78.90368  |
| foreign  | -2.095622   | 1.043513  | -2.01 | 0.048 | -4.176329            | -.0149157 |
| _cons    | -17.58651   | 3.397992  | -5.18 | 0.000 | -24.36192            | -10.81111 |

Alternatively, we can use `fp generate` to create the fractional polynomial variable corresponding to `weight(-5)` and then use `regress`. We store `weight(-5)` in the new variable `wgt_nsqr`.

```
. fp generate wgt_nsqr=weight(-5)
. regress mpg wgt_nsqr foreign
```

| Source   | SS         | df | MS         |               |   |        |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 1689.20874 | 2  | 844.604371 | Number of obs | = | 74     |
| Residual | 754.250718 | 71 | 10.6232495 | F(2, 71)      | = | 79.51  |
|          |            |    |            | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.6913 |
|          |            |    |            | Adj R-squared | = | 0.6826 |
| Total    | 2443.45946 | 73 | 33.4720474 | Root MSE      | = | 3.2593 |

| mpg        | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|------------|-------------|-----------|-------|-------|----------------------|-----------|
| wgt_nsqr_1 | 66.89665    | 6.021748  | 11.11 | 0.000 | 54.88963             | 78.90368  |
| foreign    | -2.095622   | 1.043513  | -2.01 | 0.048 | -4.176328            | -.0149155 |
| _cons      | -17.58651   | 3.397991  | -5.18 | 0.000 | -24.36191            | -10.81111 |

## Scaling

Fractional polynomials are defined only for positive *term* variables. By default, `fp` will assume that the variable  $x$  is positive and attempt to compute fractional powers of  $x$ . If the positive value assumption is incorrect, an error will be reported and estimation will not be performed.

If the values of the variable are too large or too small, the reported results of `fp` may be difficult to interpret. By default, cubic powers and squared reciprocal powers will be considered in the search for the best fractional polynomial in *term*.

We can scale the variable  $x$  to 1) make it positive and 2) ensure its magnitude is not too large or too small.

Suppose you have data on hospital patients with `age` as a fractional polynomial variable of interest. `age` is required to be greater than or equal to 20, so you might generate an `age5` variable by typing

```
. generate age5 = (age-19)/5
```

A unit change in `age5` is equivalent to a five-year change in `age`, and the minimum value of `age5` is  $1/5$  instead of 20.

In the automobile example of *Fractional polynomial regression*, our *term* variable was automobile weight (lbs.). Cars weigh in the thousands of pounds, so cubing their weight figures results in large numbers. We prevented this from being a problem by shrinking the weight by 1,000; that is, we typed

```
. replace weight = weight/1000
```

Calendar year is another type of variable that can have a problematically large magnitude. We can shrink this by dividing by 10, making a unit change correspond to a decade.

```
. generate decade = calendar_year/10
```

You may also have a variable that measures deviation from zero. Perhaps  $x$  has already been demeaned and is symmetrically about zero. The fractional polynomial in  $x$  will be undefined for half of its domain. We can shift the location of  $x$ , making it positive by subtracting its minimum and adding a small number to it. Suppose  $x$  ranges from  $-4$  to  $4$ ; we could use

```
. generate newx = x+5
```

Rescaling ourselves provides easily communicated results. We can tell exactly how the scaling was performed and how it should be performed in similar applications.

Alternatively, `fp` can scale the fractional polynomial variable so that its values are positive and the magnitude of the values are not too large. This can be done automatically or by directly specifying the scaling values.

Scaling can be automatically performed with `fp` by specifying the `scale` option. If *term* has non-positive values, the minimum value of *term* is subtracted from each observation of *term*. In this case, the counting interval, the minimum distance between the sorted values of *term*, is also added to each observation of *term*.

After adjusting the location of *term* so that its minimum value is positive, creating  $term^*$ , automatic scaling will divide each observation of *term* by a power of ten. The exponent of this scaling factor is given by

$$p = \log_{10} \{ \max(term^*) - \min(term^*) \}$$

$$p^* = \text{sign}(p) \text{floor}(|p|)$$

Rather than letting `fp` automatically choose the scaling of *term*, you may specify adjustment and scale factors *a* and *b* by using the `scale(a b)` option. Fractional powers are then calculated using the  $(term+a)/b$  values.

When `scale` or `scale(a b)` is specified, values in the variable *term* are not modified; `fp` merely remembers to scale the values whenever powers are calculated.

In addition to `fp`, both `scale` and `scale(a b)` may be used with `fp generate`.

You will probably not use `scale(a b)` with `fp` for values of *a* and *b* that you create yourself, although you could. As we demonstrated earlier, it is usually easier just to generate a scaled variable.

`scale(a b)` is useful when you previously fit a model using `scale` in one dataset and now want to create the fractional polynomials in another. In the first dataset, `fp` with `scale` added notes to the dataset concerning the values of *a* and *b*. You can see them by typing

```
. notes
```

You can then use `fp generate`, `scale(a b)` in the second dataset.

When you apply the scaling rules of a previously fit model to new data with the `scale(a b)` option, it is possible that the scaled term may have nonpositive values. `fp` will be unable to calculate the fractional powers of the term in this case and will issue an error.

The options `zero` and `catzero` cause `fp` and `fp generate` to output zero values for each fractional polynomial variable when the input (scaled) fractional polynomial variable is nonpositive. Specifying `catzero` causes a dummy variable indicating nonpositive values of the (scaled) fractional polynomial variable to be included in the model. A detailed example of the use of `catzero` and `zero` is shown in [example 3](#) below.

Using the scaling options, we can fit our [previous model](#) again using the `auto.dta`. We specify `scale(0 1000)` so that `fp` will shrink the magnitude of `weight` in estimating the regression. This is done for demonstration purposes because our scaling rule is simple. As mentioned before, in practice, you would probably only use `scale(a b)` when applying the scaling rules from a previous analysis. Allowing `fp` to scale does have the advantage of not altering the original variable, `weight`.

```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
. fp <weight>, fp(-.5) scale(0 1000): regress mpg <weight> foreign
-> regress mpg weight_1 foreign
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 1689.20861 | 2  | 844.604307 | F(2, 71)      | = | 79.51  |
| Residual | 754.250846 | 71 | 10.6232514 | Prob > F      | = | 0.0000 |
| Total    | 2443.45946 | 73 | 33.4720474 | R-squared     | = | 0.6913 |
|          |            |    |            | Adj R-squared | = | 0.6826 |
|          |            |    |            | Root MSE      | = | 3.2593 |

| mpg      | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |
|----------|-------------|-----------|-------|-------|----------------------|
| weight_1 | 66.89665    | 6.021749  | 11.11 | 0.000 | 54.88963 78.90368    |
| foreign  | -2.095622   | 1.043513  | -2.01 | 0.048 | -4.176329 -.0149159  |
| _cons    | -17.58651   | 3.397992  | -5.18 | 0.000 | -24.36192 -10.81111  |

The scaling is clearly indicated in the variable notes for the generated variable `weight_1`.

```
. notes weight_1
weight_1:
1. fp term 1 of $x^{(-.5)}$, where x is weight scaled.
2. Scaling was user specified: $x = (\text{weight}+a)/b$ where $a=0$ and $b=1000$
3. Fractional polynomial variables created by fp <weight>, fp(-.5)
scale(0 1000): regress mpg <weight> foreign
4. To re-create the fractional polynomial variables, for instance, in
another dataset, type fp gen double weight(-.5), scale(0 1000)
```

## Centering

The fractional polynomial of *term*, centered on *c* is

$$(term^{(p_1, \dots, p_m)} - c^{(p_1, \dots, p_m)})' \beta$$

The intercept of a centered fractional polynomial can be interpreted as the effect at zero for all the covariates. When we center the fractional polynomial terms using *c*, the intercept is now interpreted as the effect at *term* = *c* and zero values for the other covariates.

Suppose we wanted to center the fractional polynomial of *x* with powers (0, 0, 2) at  $x = c$ .

$$\begin{aligned} & (x^{(0,0,2)} - c^{(0,0,2)})' \beta \\ &= \beta_0 + \beta_1 (x^{(0)} - c^{(0)}) + \beta_2 \{x^{(0)} \ln(x) - c^{(0)} \ln(c)\} + \beta_3 (x^{(2)} - c^{(2)}) \\ &= \beta_0 + \beta_1 \{\ln(x) - \ln(c)\} + \beta_2 [\{\ln(x)\}^2 - \{\ln(c)\}^2] + \beta_3 (x^2 - c^2) \end{aligned}$$

When `center` is specified, `fp` centers based on the sample mean of (scaled) *term*. A previously chosen value for centering, *c*, may also be specified in `center(c)`. This would be done when applying the results of a previous model fitting to a new dataset.

The `center` and `center(c)` options may be used in `fp` or `fp generate`.

Returning to the model of mileage per gallon based on automobile weight and foreign origin, we refit the model with the fractional polynomial of weight centered at its scaled mean.

```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
. fp <weight>, fp(-.5) scale(0 1000) center: regress mpg <weight> foreign
-> regress mpg weight_1 foreign
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
|          |            |    |            | F(2, 71)      | = | 79.51  |
| Model    | 1689.20861 | 2  | 844.604307 | Prob > F      | = | 0.0000 |
| Residual | 754.250846 | 71 | 10.6232514 | R-squared     | = | 0.6913 |
|          |            |    |            | Adj R-squared | = | 0.6826 |
| Total    | 2443.45946 | 73 | 33.4720474 | Root MSE      | = | 3.2593 |

| mpg      | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |
|----------|-------------|-----------|-------|-------|----------------------|
| weight_1 | 66.89665    | 6.021749  | 11.11 | 0.000 | 54.88963 78.90368    |
| foreign  | -2.095622   | 1.043513  | -2.01 | 0.048 | -4.176329 -.0149159  |
| _cons    | 20.91163    | .4624143  | 45.22 | 0.000 | 19.9896 21.83366     |

Note that the coefficients for `weight_1` and `foreign` do not change. Only the intercept `_cons` changes. It can be interpreted as the estimated average miles per gallon of an American-made car of average weight.

## Examples

### ▷ Example 1: Linear regression

Consider the serum immunoglobulin G (IgG) dataset from Isaacs et al. (1983), which consists of 298 independent observations in young children. The dependent variable `sqrtigg` is the square root of the IgG concentration, and the independent variable `age` is the age of each child. (Preliminary Box–Cox analysis shows that a square root transformation removes the skewness in IgG.)

The aim is to find a model that accurately predicts the mean of `sqrtigg` given `age`. We use `fp` to find the best FP2 model (the default option). We specify `center` for automatic centering. The age of each child is small in magnitude and positive, so we do not use the scaling options of `fp` or scale ourselves.

```
. use https://www.stata-press.com/data/r19/igg, clear
(Immunoglobulin in children)
. fp <age>, scale center: regress sqrtigg <age>
(fitting 44 models)
(. . . .10%20%30%40%50%60%70%80%90%100%)
Fractional polynomial comparisons:
```

| age     | Test<br>df | Deviance | Residual<br>std. dev. | Deviance<br>diff. | P     | Powers |
|---------|------------|----------|-----------------------|-------------------|-------|--------|
| omitted | 4          | 427.539  | 0.497                 | 108.090           | 0.000 |        |
| linear  | 3          | 337.561  | 0.428                 | 18.113            | 0.000 | 1      |
| m = 1   | 2          | 327.436  | 0.421                 | 7.987             | 0.020 | 0      |
| m = 2   | 0          | 319.448  | 0.416                 | 0.000             | --    | -2 2   |

Note: **Test df** is degrees of freedom, and **P** =  $P > F$  is sig. level for tests comparing models vs. model with  $m = 2$  based on deviance difference,  $F(df, 293)$ .

| Source   | SS         | df  | MS         | Number of obs | = | 298    |
|----------|------------|-----|------------|---------------|---|--------|
| Model    | 22.2846976 | 2   | 11.1423488 | F(2, 295)     | = | 64.49  |
| Residual | 50.9676492 | 295 | .172771692 | Prob > F      | = | 0.0000 |
| Total    | 73.2523469 | 297 | .246640898 | R-squared     | = | 0.3042 |
|          |            |     |            | Adj R-squared | = | 0.2995 |
|          |            |     |            | Root MSE      | = | .41566 |

| sqrtigg | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|---------|-------------|-----------|-------|-------|----------------------|----------|
| age_1   | -.1562156   | .027416   | -5.70 | 0.000 | -.2101713            | -.10226  |
| age_2   | .0148405    | .0027767  | 5.34  | 0.000 | .0093757             | .0203052 |
| _cons   | 2.283145    | .0305739  | 74.68 | 0.000 | 2.222974             | 2.343315 |

The new variables created by `fp` contain the best-fitting fractional polynomial powers of `age`, as centered by `fp`. For example, `age_1` is centered by subtracting the mean of `age` raised to the power  $-2$ .

The variables created by `fp` and `fp generate` are centered or scaled as specified by the user, which is reflected in the estimated regression coefficients and intercept. Centering does have its advantages (see [Centering](#) earlier in this entry). By default, `fp` will not perform scaling or centering. For a more detailed discussion, see [Royston and Sauerbrei \(2008, sec. 4.11\)](#).

The fitted curve has an asymmetric S shape. The best model has powers  $(-2, 2)$  and deviance 319.448. We reject lesser degree models: the null, linear, and natural log power models at the 0.05 level. As many as 44 models have been fit in the search for the best powers. Now let's look at models of degree  $\leq 4$ . The highest allowed degree is specified in `dimension()`. We overwrite the previously generated fractional polynomial power variables by including `replace`.

```
. fp <age>, dimension(4) center replace: regress sqrtigg <age>
(fitting 494 models)
(...10%...20%...30%...40%...50%...60%...70%...80%...90%...100%)
```

Fractional polynomial comparisons:

| age     | Test<br>df | Deviance | Residual<br>std. dev. | Deviance<br>diff. | P     | Powers  |
|---------|------------|----------|-----------------------|-------------------|-------|---------|
| omitted | 8          | 427.539  | 0.497                 | 109.795           | 0.000 |         |
| linear  | 7          | 337.561  | 0.428                 | 19.818            | 0.007 | 1       |
| m = 1   | 6          | 327.436  | 0.421                 | 9.692             | 0.149 | 0       |
| m = 2   | 4          | 319.448  | 0.416                 | 1.705             | 0.798 | -2 2    |
| m = 3   | 2          | 319.275  | 0.416                 | 1.532             | 0.476 | -2 1 1  |
| m = 4   | 0          | 317.744  | 0.416                 | 0.000             | --    | 0 3 3 3 |

Note: **Test df** is degrees of freedom, and **P** =  $P > F$  is sig. level for tests comparing models vs. model with  $m = 4$  based on deviance difference,  $F(df, 289)$ .

| Source   | SS         | df  | MS         | Number of obs | = | 298    |
|----------|------------|-----|------------|---------------|---|--------|
| Model    | 22.5754541 | 4   | 5.64386353 | F(4, 293)     | = | 32.63  |
| Residual | 50.6768927 | 293 | .172958678 | Prob > F      | = | 0.0000 |
|          |            |     |            | R-squared     | = | 0.3082 |
|          |            |     |            | Adj R-squared | = | 0.2987 |
| Total    | 73.2523469 | 297 | .246640898 | Root MSE      | = | .41588 |

| sqrtigg | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|---------|-------------|-----------|-------|-------|----------------------|-----------|
| age_1   | .8761824    | .1898721  | 4.61  | 0.000 | .5024962             | 1.249869  |
| age_2   | -.1922029   | .0684934  | -2.81 | 0.005 | -.3270044            | -.0574015 |
| age_3   | .2043794    | .074947   | 2.73  | 0.007 | .0568767             | .3518821  |
| age_4   | -.0560067   | .0212969  | -2.63 | 0.009 | -.097921             | -.0140924 |
| _cons   | 2.238735    | .0482705  | 46.38 | 0.000 | 2.143734             | 2.333736  |

It appears that the FP4 model is not significantly different from the other fractional polynomial models (at the 0.05 level).

Let's compare the curve shape from the  $m = 2$  model with that from a conventional quartic polynomial whose fit turns out to be significantly better than a cubic (not shown). We use the ability of `fp` both to generate the required powers of `age`, namely,  $(1, 2, 3, 4)$  for the quartic and  $(-2, 2)$  for the second-degree fractional polynomial, and to fit the model. The `fp()` option is used to specify the powers. We use `predict` to obtain the fitted values of each regression. We fit both models with `fp` and graph the resulting curves with `twoway scatter`.

```
. fp <age>, center fp(1 2 3 4) replace: regress sqrttigg <age>
-> regress sqrttigg age_1 age_2 age_3 age_4
```

| Source   | SS         | df  | MS         | Number of obs | = | 298    |
|----------|------------|-----|------------|---------------|---|--------|
| Model    | 22.5835458 | 4   | 5.64588646 | F(4, 293)     | = | 32.65  |
| Residual | 50.668801  | 293 | .172931061 | Prob > F      | = | 0.0000 |
|          |            |     |            | R-squared     | = | 0.3083 |
|          |            |     |            | Adj R-squared | = | 0.2989 |
| Total    | 73.2523469 | 297 | .246640898 | Root MSE      | = | .41585 |

| sqrttigg | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|----------|-------------|-----------|-------|-------|----------------------|-----------|
| age_1    | 2.047831    | .4595962  | 4.46  | 0.000 | 1.143302             | 2.952359  |
| age_2    | -1.058902   | .2822803  | -3.75 | 0.000 | -1.614456            | -.5033479 |
| age_3    | .2284917    | .0667591  | 3.42  | 0.001 | .0971037             | .3598798  |
| age_4    | -.0168534   | .0053321  | -3.16 | 0.002 | -.0273475            | -.0063594 |
| _cons    | 2.240012    | .0480157  | 46.65 | 0.000 | 2.145512             | 2.334511  |

```
. predict fit1
(option xb assumed; fitted values)
. label variable fit1 "Quartic"
. fp <age>, center fp(-2 2) replace: regress sqrttigg <age>
-> regress sqrttigg age_1 age_2
```

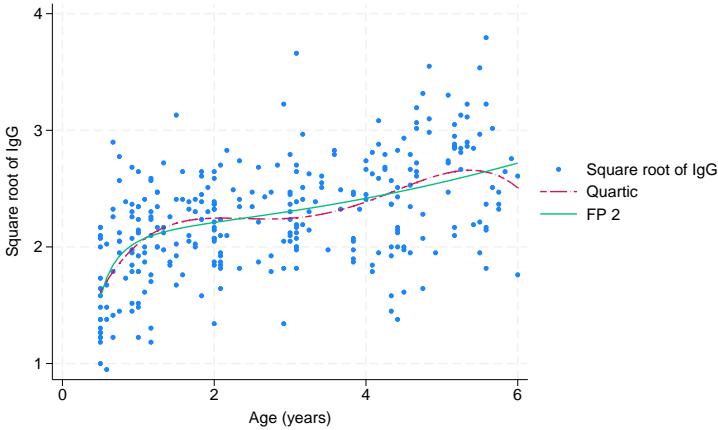
| Source   | SS         | df  | MS         | Number of obs | = | 298    |
|----------|------------|-----|------------|---------------|---|--------|
| Model    | 22.2846976 | 2   | 11.1423488 | F(2, 295)     | = | 64.49  |
| Residual | 50.9676492 | 295 | .172771692 | Prob > F      | = | 0.0000 |
|          |            |     |            | R-squared     | = | 0.3042 |
|          |            |     |            | Adj R-squared | = | 0.2995 |
| Total    | 73.2523469 | 297 | .246640898 | Root MSE      | = | .41566 |

| sqrttigg | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|----------|-------------|-----------|-------|-------|----------------------|----------|
| age_1    | -.1562156   | .027416   | -5.70 | 0.000 | -.2101713            | -.10226  |
| age_2    | .0148405    | .0027767  | 5.34  | 0.000 | .0093757             | .0203052 |
| _cons    | 2.283145    | .0305739  | 74.68 | 0.000 | 2.222974             | 2.343315 |

```
. predict fit2
(option xb assumed; fitted values)
. label variable fit2 "FP 2"
```

```
. scatter sqrtigg fit1 fit2 age, c(. 1 1) m(o i i) msize(small)
> lpattern(. -_) ytitle("Square root of IgG") xtitle("Age (years)")
```



The quartic curve has an unsatisfactory wavy appearance that is implausible for the known behavior of IgG, the serum level of which increases throughout early life. The fractional polynomial curve (FP2) increases monotonically and is therefore biologically the more plausible curve. The two models have approximately the same deviance.

◀

## ▷ Example 2: Cox regression

Data from Smith et al. (1992) contain times to complete healing of leg ulcers in a randomized, controlled clinical trial of two treatments in 192 elderly patients. Several covariates were available, of which an important one is `mothson`, the number of months since the recorded onset of the ulcer. This time is recorded in whole months, not fractions of a month; therefore, some zero values are recorded.

Because the response variable is time to an event of interest and some (in fact, about one-half) of the times are censored, using Cox regression to analyze the data is appropriate. We consider fractional polynomials in `mothson`, adjusting for four other covariates: `age`; `ulcarearea`, the area of tissue initially affected by the ulcer; `deepppg`, a binary variable indicating the presence or absence of deep vein involvement; and `treat`, a binary variable indicating treatment type.

We fit fractional polynomials of degrees 1 and 2 with `fp`. We specify `scale` to perform automatic scaling on `mothson`. This makes it positive and ensures that its magnitude is not too large. (See [Scaling](#) for more details.) The display option `nohr` is specified before the colon so that the coefficients and not the hazard ratios are displayed.

The `center` option is specified to obtain automatic centering. `age` and `ulcarearea` are also demeaned by using `summarize` and then subtracting the returned result `r(mean)`.

In Cox regression, there is no constant term, so we cannot see the effects of centering in the table of regression estimates. The effects would be present if we were to graph the baseline hazard or survival function because these functions are defined with all predictors set equal to 0.

In these graphs, we will see the estimated baseline hazard or survival function under no deep vein involvement or treatment and under mean age, ulcer area, and number of months since the recorded onset of the ulcer.

```
. use https://www.stata-press.com/data/r19/legulcer2, clear
(Leg ulcer clinical trial)
. stset ttevent, fail(healed)
Survival-time data settings
 Failure event: healed!=0 & healed<.
Observed time interval: (0, ttevent]
Exit on or before: failure
```

---

```
192 total observations
 0 exclusions
```

---

```
192 observations remaining, representing
 92 failures in single-record/single-failure data
13,825 total analysis time at risk and under observation
 At risk from t = 0
Earliest observed entry t = 0
Last observed exit t = 206
```

```
. quietly sum age
. replace age = age - r(mean)
variable age was byte now float
(192 real changes made)
. quietly sum ulcarea
. replace ulcarea = ulcarea - r(mean)
variable ulcarea was int now float
(192 real changes made)
. fp $stcox$, center scale nohr: $stcox$ age $ulcarea$ $depppg$ $treat$
(fitting 44 models)
(...10%...20%...30%...40%...50%...60%...70%...80%...90%...100%)
Fractional polynomial comparisons:
```

| mthson  | Test |          | Deviance |       | P     | Powers |
|---------|------|----------|----------|-------|-------|--------|
|         | df   | Deviance | diff.    |       |       |        |
| omitted | 4    | 754.345  | 17.636   | 0.001 |       |        |
| linear  | 3    | 751.680  | 14.971   | 0.002 | 1     |        |
| m = 1   | 2    | 738.969  | 2.260    | 0.323 | -.5   |        |
| m = 2   | 0    | 736.709  | 0.000    | --    | .5 .5 |        |

Note: **Test df** is degrees of freedom, and **P** =  $P > \chi^2$  is sig. level for tests comparing models vs. model with  $m = 2$  based on deviance difference,  $\chi^2$ .

Cox regression with Breslow method for ties

```
No. of subjects = 192 Number of obs = 192
No. of failures = 92
Time at risk = 13,825
Log likelihood = -368.35446 LR $\chi^2(6)$ = 108.59
 Prob > χ^2 = 0.0000
```

| _t       | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|----------|-------------|-----------|-------|-------|----------------------|-----------|
| mthson_1 | -2.81425    | .6996385  | -4.02 | 0.000 | -4.185516            | -1.442984 |
| mthson_2 | 1.541451    | .4703143  | 3.28  | 0.001 | .6196521             | 2.46325   |
| age      | -.0261111   | .0087983  | -2.97 | 0.003 | -.0433556            | -.0088667 |
| ulcarea  | -.0017491   | .000359   | -4.87 | 0.000 | -.0024527            | -.0010455 |
| depppg   | -.5850499   | .2163173  | -2.70 | 0.007 | -1.009024            | -.1610758 |
| treat    | -.1624663   | .2171048  | -0.75 | 0.454 | -.5879838            | .2630513  |

The best-fitting fractional polynomial of degree 2 has powers (0.5, 0.5) and deviance 736.709. However, this model does not fit significantly better than the fractional polynomial of degree 1 (at the 0.05 level), which has power  $-0.5$  and deviance 738.969. We prefer the model with  $m = 1$ .

```
. fp <mathson>, replace center scale nohr fp(-.5): stcox <mathson> age ulcarea
> deeppepg treat
-> stcox mathson_1 age ulcarea deeppepg treat
Cox regression with Breslow method for ties
No. of subjects = 192 Number of obs = 192
No. of failures = 92
Time at risk = 13,825
Log likelihood = -369.48426 LR chi2(5) = 106.33
 Prob > chi2 = 0.0000
```

| _t        | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|-----------|-------------|-----------|-------|-------|----------------------|-----------|
| mathson_1 | .1985592    | .0493922  | 4.02  | 0.000 | .1017523             | .2953662  |
| age       | -.02691     | .0087875  | -3.06 | 0.002 | -.0441331            | -.0096868 |
| ulcarea   | -.0017416   | .0003482  | -5.00 | 0.000 | -.0024241            | -.0010591 |
| deeppepg  | -.5740759   | .2185134  | -2.63 | 0.009 | -1.002354            | -.1457975 |
| treat     | -.1798575   | .2175726  | -0.83 | 0.408 | -.6062921            | .246577   |

The hazard for healing is much higher for patients whose ulcer is of recent onset than for those who have had an ulcer for many months.

A more appropriate analysis of this dataset, if one wanted to model all the predictors, possibly with fractional polynomial functions, would be to use `mfp`; see [R] `mfp`.

◀

### ▷ Example 3: Logistic regression

The `zero` option permits fitting a fractional polynomial model to the positive values of a covariate, taking nonpositive values as zero. An application is the assessment of the effect of cigarette smoking as a risk factor. Whitehall 1 is an epidemiological study, which was examined in [Royston and Sauerbrei \(2008\)](#), of 18,403 male British Civil Servants employed in London. We examine the data collected in Whitehall 1 and use logistic regression to model the odds of death based on a fractional polynomial in the number of cigarettes smoked.

Nonsmokers may be qualitatively different from smokers, so the effect of smoking (regarded as a continuous variable) may not be continuous between zero cigarettes and one cigarette. To allow for this possibility, we model the risk as a constant for the nonsmokers and as a fractional polynomial function of the number of cigarettes for the smokers, adjusted for age.

The dependent variable `all110` is an indicator of whether the individual passed away in the 10 years under study. `cigs` is the number of cigarettes consumed per day. After loading the data, we demean `age` and create a dummy variable, `nonsmoker`. We then use `fp` to fit the model.

```
. use https://www.stata-press.com/data/r19/smoking, clear
(Smoking and mortality data)
. quietly sum age
. replace age = age - r(mean)
variable age was byte now float
(17,260 real changes made)
. generate byte nonsmoker = cond(cigs==0, 1, 0) if cigs < .
```

```
. fp <cigs>, zero: logit all10 <cigs> nonsmoker age
(fitting 44 models)
(....10%...20%...30%...40%...50%...60%...70%...80%...90%...100%)
```

Fractional polynomial comparisons:

| cigs    | Test<br>df | Deviance | Deviance<br>diff. | P     | Powers |
|---------|------------|----------|-------------------|-------|--------|
| omitted | 4          | 9990.804 | 46.096            | 0.000 |        |
| linear  | 3          | 9958.801 | 14.093            | 0.003 | 1      |
| m = 1   | 2          | 9946.603 | 1.895             | 0.388 | 0      |
| m = 2   | 0          | 9944.708 | 0.000             | --    | -1 -1  |

Note: **Test df** is degrees of freedom, and **P** =  $P > \chi^2$  is sig. level for tests comparing models vs. model with  $m = 2$  based on deviance difference,  $\chi^2$ .

Logistic regression

Number of obs = 17,260

LR  $\chi^2(4)$  = 1029.03

Prob >  $\chi^2$  = 0.0000

Pseudo R2 = 0.0938

Log likelihood = -4972.3539

| all10     | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |           |
|-----------|-------------|-----------|--------|-------|----------------------|-----------|
| cigs_1    | -1.285867   | .3358483  | -3.83  | 0.000 | -1.944117            | -.6276162 |
| cigs_2    | -1.982424   | .572109   | -3.47  | 0.001 | -3.103736            | -.8611106 |
| nonsmoker | -1.223749   | .1119583  | -10.93 | 0.000 | -1.443183            | -1.004315 |
| age       | .1194541    | .0045818  | 26.07  | 0.000 | .1104739             | .1284343  |
| _cons     | -1.591489   | .1052078  | -15.13 | 0.000 | -1.797693            | -1.385286 |

Omission of the zero option would cause `fp` to halt with an error message because nonpositive covariate values (for example, values of `cigs`) are invalid unless the `scale` option is specified.

A closely related approach involves the `catzero` option. Here we no longer need to have `nonsmoker` in the model, because `fp` creates its own dummy variable `cigs_0` to indicate whether the individual does not smoke on that day.

```
. fp <cigs>, catzero replace: logit all10 <cigs> age
(fitting 44 models)
(...10%...20%...30%...40%...50%...60%...70%...80%...90%...100%)
```

Fractional polynomial comparisons:

| cigs    | Test<br>df | Deviance | Deviance<br>diff. | P     | Powers |
|---------|------------|----------|-------------------|-------|--------|
| omitted | 5          | 10175.75 | 231.047           | 0.000 |        |
| linear  | 3          | 9958.80  | 14.093            | 0.003 | 1      |
| m = 1   | 2          | 9946.60  | 1.895             | 0.388 | 0      |
| m = 2   | 0          | 9944.71  | 0.000             | --    | -1 -1  |

Note: **Test df** is degrees of freedom, and **P** =  $P > \chi^2$  is sig. level for tests comparing models vs. model with  $m = 2$  based on deviance difference,  $\chi^2$ .

Logistic regression

Number of obs = 17,260

LR  $\chi^2(4)$  = 1029.03

Prob >  $\chi^2$  = 0.0000

Pseudo R2 = 0.0938

Log likelihood = -4972.3539

| all10  | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |
|--------|-------------|-----------|--------|-------|----------------------|
| cigs_0 | -1.223749   | .1119583  | -10.93 | 0.000 | -1.443183 -1.004315  |
| cigs_1 | -1.285867   | .3358483  | -3.83  | 0.000 | -1.944117 -.6276162  |
| cigs_2 | -1.982424   | .572109   | -3.47  | 0.001 | -3.103736 -.8611106  |
| age    | .1194541    | .0045818  | 26.07  | 0.000 | .1104739 .1284343    |
| _cons  | -1.591489   | .1052078  | -15.13 | 0.000 | -1.797693 -1.385286  |

Under both approaches, the comparison table suggests that we can accept the FP1 model instead of the FP2 model. We estimate the parameters of the accepted model—that is, the one that uses the natural logarithm of *cigs*—with *fp*.

```
. fp <cigs>, catzero replace fp(0): logit all10 <cigs> age
-> logit all10 cigs_0 cigs_1 age
```

Logistic regression

Number of obs = 17,260

LR  $\chi^2(3)$  = 1027.13

Prob >  $\chi^2$  = 0.0000

Pseudo R2 = 0.0936

Log likelihood = -4973.3016

| all10  | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |
|--------|-------------|-----------|--------|-------|----------------------|
| cigs_0 | .1883732    | .1553093  | 1.21   | 0.225 | -.1160274 .4927738   |
| cigs_1 | .3469842    | .0543552  | 6.38   | 0.000 | .2404499 .4535185    |
| age    | .1194976    | .0045818  | 26.08  | 0.000 | .1105174 .1284778    |
| _cons  | -3.003767   | .1514909  | -19.83 | 0.000 | -3.300683 -2.70685   |

The high  $p$ -value for *cigs\_0* in the output indicates that we cannot reject that there is no extra effect at zero for nonsmokers.

## Stored results

In addition to the results that `est_cmd` stores, `fp` stores the following in `e()`:

### Scalars

|                                |                                           |
|--------------------------------|-------------------------------------------|
| <code>e(fp_dimension)</code>   | degree of fractional polynomial           |
| <code>e(fp_center_mean)</code> | value used for centering or .             |
| <code>e(fp_scale_a)</code>     | value used for scaling or .               |
| <code>e(fp_scale_b)</code>     | value used for scaling or .               |
| <code>e(fp_compare_df2)</code> | denominator degree of freedom in $F$ test |

### Macros

|                                 |                                                                                        |
|---------------------------------|----------------------------------------------------------------------------------------|
| <code>e(fp_cmd)</code>          | <code>fp</code> , <code>search()</code> : or <code>fp</code> , <code>powers()</code> : |
| <code>e(fp_cmdline)</code>      | full <code>fp</code> command as typed                                                  |
| <code>e(fp_variable)</code>     | fractional polynomial variable                                                         |
| <code>e(fp_terms)</code>        | generated <code>fp</code> variables                                                    |
| <code>e(fp_gen_cmdline)</code>  | <code>fp generate</code> command to re-create <code>e(fp_terms)</code> variables       |
| <code>e(fp_catzero)</code>      | <code>catzero</code> , if specified                                                    |
| <code>e(fp_zero)</code>         | <code>zero</code> , if specified                                                       |
| <code>e(fp_compare_type)</code> | <code>F</code> or <code>chi2</code>                                                    |

### Matrices

|                                   |                                                                                  |
|-----------------------------------|----------------------------------------------------------------------------------|
| <code>e(fp_fp)</code>             | powers used in fractional polynomial                                             |
| <code>e(fp_compare)</code>        | results of model comparisons                                                     |
| <code>e(fp_compare_stat)</code>   | $F$ test statistics                                                              |
| <code>e(fp_compare_df1)</code>    | <code>chi2</code> degrees of freedom or numerator degrees of freedom of $F$ test |
| <code>e(fp_compare_fp)</code>     | powers of comparison models                                                      |
| <code>e(fp_compare_length)</code> | encoded string for display of row titles                                         |
| <code>e(fp_powers)</code>         | powers that are searched                                                         |

`fp generate` stores the following in `r()`:

### Scalars

|                                |                               |
|--------------------------------|-------------------------------|
| <code>r(fp_center_mean)</code> | value used for centering or . |
| <code>r(fp_scale_a)</code>     | value used for scaling or .   |
| <code>r(fp_scale_b)</code>     | value used for scaling or .   |

### Macros

|                             |                                                |
|-----------------------------|------------------------------------------------|
| <code>r(fp_cmdline)</code>  | full <code>fp generate</code> command as typed |
| <code>r(fp_variable)</code> | fractional polynomial variable                 |
| <code>r(fp_terms)</code>    | generated <code>fp</code> variables            |
| <code>r(fp_catzero)</code>  | <code>catzero</code> , if specified            |
| <code>r(fp_zero)</code>     | <code>zero</code> , if specified               |

### Matrices

|                       |                                      |
|-----------------------|--------------------------------------|
| <code>r(fp_fp)</code> | powers used in fractional polynomial |
|-----------------------|--------------------------------------|

## Methods and formulas

The general definition of a fractional polynomial, accommodating possible repeated powers, may be written for functions  $H_1(x), \dots, H_m(x)$  of  $x > 0$  as

$$\beta_0 + \sum_{j=1}^m \beta_j H_j(x)$$

where  $H_1(x) = x^{(p_1)}$  and for  $j = 2, \dots, m$ ,

$$H_j(x) = \begin{cases} x^{(p_j)} & \text{if } p_j \neq p_{j-1} \\ H_{j-1}(x) \ln(x) & \text{if } p_j = p_{j-1} \end{cases}$$

For example, a fractional polynomial of degree 3 with powers (1, 3, 3) has  $H_1(x) = x$ ,  $H_2(x) = x^3$ , and  $H_3(x) = x^3 \ln(x)$  and equals  $\beta_0 + \beta_1 x + \beta_2 x^3 + \beta_3 x^3 \ln(x)$ .

We can express a fractional polynomial in vector notation by using  $\mathbf{H}(x) = [H_1(x), \dots, H_d(x)]'$ . We define  $x^{(p_1, p_2, \dots, p_m)} = [\mathbf{H}(x)', 1]'$ . Under this notation, we can write

$$x^{(1,3,3)'} \boldsymbol{\beta} = \beta_0 + \beta_1 x + \beta_2 x^3 + \beta_3 x^3 \ln(x)$$

The fractional polynomial may be centered so that the intercept can be more easily interpreted. When centering the fractional polynomial of  $x$  at  $c$ , we subtract  $c^{(p_1, p_2, \dots, p_m)}$  from  $x^{(p_1, p_2, \dots, p_m)}$ , where  $c^{(p_1, p_2, \dots, p_d)} = [\mathbf{H}(c)', 0]'$ . The centered fractional polynomial is

$$(x^{(p_1, \dots, p_d)} - c^{(p_1, \dots, p_d)})' \boldsymbol{\beta}$$

The definition may be extended to allow  $x \leq 0$  values. For these values, the fractional polynomial is equal to the intercept  $\beta_0$  or equal to a zero-offset term  $\alpha_0$  plus the intercept  $\beta_0$ .

The deviance  $D$  of a model is defined as  $-2$  times its maximized log likelihood. For normal error models, we use the formula

$$D = n \left( 1 - \bar{l} + \ln \frac{2\pi \text{RSS}}{n} \right)$$

where  $n$  is the sample size,  $\bar{l}$  is the mean of the lognormalized weights ( $\bar{l} = 0$  if the weights are all equal), and RSS is the residual sum of squares as fit by `regress`.

When `fp` is used to search for the best combination of powers, it reports a table comparing fractional polynomial models of degree  $k < m$  with the degree  $m$  fractional polynomial model, which will have the lowest deviance. The comparison table also includes the linear model, in which `<term>` is not raised to a power, and the null model, in which `<term>` is omitted.

The `Test df` column of the model comparison table does not correspond to the model degrees of freedom for the individual models but rather to the degrees of freedom of a test comparing that model with the model with the lowest deviance. For normal error models, this is the numerator degrees of freedom of a partial  $F$  test; for other models, it is the degrees of freedom of the likelihood-ratio  $\chi^2$  test. When calculating the test degrees of freedom, the command accounts for the two types of parameters that are being estimated by `fp`: coefficients ( $\beta_j$ ) and powers. Because the powers in a fractional polynomial are chosen from a finite set rather than from the entire real line, the degrees of freedom defined in this way are approximate and generally yield somewhat conservative tests (Royston and Altman 1994).

The  $p$ -values reported by `fp` are calculated differently for normal error models than for other models. Let  $D_k$  and  $D_m$  be the deviances of the models with degrees  $k$  and  $m$ , respectively. For normal error models, a variance ratio  $F$  is calculated as

$$F = \frac{d_2}{d_1} \left\{ \exp \left( \frac{D_k - D_m}{n} \right) - 1 \right\}$$

where  $d_1$  is the numerator df, the number of additional parameters estimated by the degree  $m$  model over the degree  $k$  model.  $d_2$  is the denominator degrees of freedom and equals the residual degrees of freedom of the degree  $m$  model minus the number of powers estimated,  $m$ . The  $p$ -value is obtained by referring  $F$  to an  $F$  distribution on  $(d_1, d_2)$  df.

For nonnormal models, the  $p$ -value is obtained by referring  $D_k - D_m$  to a  $\chi^2$  distribution on  $d_1$  degrees of freedom, with  $d_1$  defined as above.

## Acknowledgment

We thank Patrick Royston of the MRC Clinical Trials Unit, London, and coauthor of the Stata Press book *Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model* for writing `fracpoly` and `fracgen`, the commands on which `fp` and `fp generate` are based. We also thank Professor Royston for his advice on and review of the new `fp` commands.

## References

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## Also see

- [R] [fp postestimation](#) — Postestimation tools for `fp`
- [R] [mfp](#) — Multivariable fractional polynomial models
- [U] [20 Estimation and postestimation commands](#)

|                         |                                      |                                |                                                        |
|-------------------------|--------------------------------------|--------------------------------|--------------------------------------------------------|
| Postestimation commands | <a href="#">predict</a>              | <a href="#">margins</a>        | <a href="#">fp plot</a> and <a href="#">fp predict</a> |
| Remarks and examples    | <a href="#">Methods and formulas</a> | <a href="#">Acknowledgment</a> | <a href="#">Reference</a>                              |
| Also see                |                                      |                                |                                                        |

## Postestimation commands

The following postestimation commands are of special interest after `fp`:

| Command                 | Description                                                                     |
|-------------------------|---------------------------------------------------------------------------------|
| <code>fp plot</code>    | component-plus-residual plot from most recently fit fractional polynomial model |
| <code>fp predict</code> | create variable containing prediction or SEs of fractional polynomials          |

The following standard postestimation commands are also available if available after `est_cmd`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance-covariance matrix of the estimators (VCE)                                                                                      |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| <code>forecast</code>        | dynamic forecasts and simulations                                                                                                       |
| <code>hausman</code>         | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| <code>linktest</code>        | link test for model specification                                                                                                       |
| <code>lrtest</code>          | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | predictions, residuals, influence statistics, and other diagnostic measures                                                             |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

## predict

The behavior of `predict` following `fp` is determined by `est_cmd`. See the corresponding `est_cmd` postestimation entry for available `predict` options.

Also see information on `fp predict` below.

## margins

The behavior of `margins` following `fp` is determined by `est_cmd`. See the corresponding `est_cmd` postestimation entry for available `margins` options.

## fp plot and fp predict

### Description for fp plot and fp predict

`fp plot` produces a component-plus-residual plot. The fractional polynomial comprises the component, and the residual is specified by the user in `residuals()`. The component-plus-residuals are plotted against the fractional polynomial variable. If you only want to plot the component fit, without residuals, you would specify `residuals(none)`.

`fp predict` generates the fractional polynomial or the standard error of the fractional polynomial. The fractional polynomial prediction is equivalent to the fitted values prediction given by `predict, xb`, with the covariates other than the fractional polynomial variable set to zero. The standard error may be quite large if the range of the other covariates is far from zero. In this situation, the covariates would be centered and their range would include, or come close to including, zero.

These postestimation commands can be used only when the fractional polynomial variables do not interact with other variables in the specification of `est_cmd`. See [U] 11.4.3 [Factor variables](#) for more information about interactions.

### Menu for fp plot and fp predict

#### fp plot

Statistics > Linear models and related > Fractional polynomials > Component-plus-residual plot

#### fp predict

Statistics > Linear models and related > Fractional polynomials > Fractional polynomial prediction

### Syntax for fp plot and fp predict

*Component-plus-residual plot for most recently fit fractional polynomial model*

```
fp plot [if] [in], residuals(res_option) [graph_options]
```

*Create variable containing the prediction or SEs of fractional polynomials*

```
fp predict [type] newvar [if] [in] [, predict_options]
```

| <i>graph_options</i>                       | Description                                                                                                                                   |
|--------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|
| Main                                       |                                                                                                                                               |
| * <u>residuals</u> ( <i>res_option</i> )   | residual option name to use in <code>predict</code> after <i>est_cmd</i> , or <code>residuals(none)</code> if residuals are not to be graphed |
| <u>equation</u> ( <i>eqno</i> )            | specify equation                                                                                                                              |
| <u>level</u> (#)                           | set confidence level; default is <code>level(95)</code>                                                                                       |
| Plot                                       |                                                                                                                                               |
| <u>plotopts</u> ( <i>scatter_options</i> ) | affect rendition of the component-plus-residual scatter points                                                                                |
| Fitted line                                |                                                                                                                                               |
| <u>lineopts</u> ( <i>cline_options</i> )   | affect rendition of the fitted line                                                                                                           |
| CI plot                                    |                                                                                                                                               |
| <u>ciopts</u> ( <i>area_options</i> )      | affect rendition of the confidence bands                                                                                                      |
| Add plots                                  |                                                                                                                                               |
| <u>addplot</u> ( <i>plot</i> )             | add other plots to the generated graph                                                                                                        |
| Y axis, X axis, Titles, Legend, Overall    |                                                                                                                                               |
| <u>twoway_options</u>                      | any options other than <code>by()</code> documented in [G-3] <i>twoway_options</i>                                                            |

\*residuals(*res\_option*) is required.

| <i>predict_options</i>          | Description                                               |
|---------------------------------|-----------------------------------------------------------|
| Main                            |                                                           |
| <u>fp</u>                       | calculate the fractional polynomial; the default          |
| <u>stdp</u>                     | calculate the standard error of the fractional polynomial |
| <u>equation</u> ( <i>eqno</i> ) | specify equation                                          |

## Options for fp plot

### Main

residuals(*res\_option*) specifies what type of residuals to plot in the component-plus-residual plot. *res\_option* is the same option that would be specified to `predict` after *est\_cmd*. Residuals can be omitted from the plot by specifying `residuals(none)`. `residuals()` is required.

equation(*eqno*) is relevant only when you have previously fit a multiple-equation model in *est\_cmd*. It specifies the equation to which you are referring.

equation(#1) would mean that the calculation is to be made for the first equation, equation(#2) would mean the second, and so on. You could also refer to the equations by their names: equation(*income*) would refer to the equation named *income*, and equation(*hours*) would refer to the equation named *hours*.

If you do not specify equation(), the results are the same as if you specified equation(#1).

level(#); see [R] [Estimation options](#).

## Plot

`plotopts`(*scatter\_options*) affects the rendition of the component-plus-residual scatter points; see [G-2] [graph twoway scatter](#).

## Fitted line

`lineopts`(*cline\_options*) affects the rendition of the fitted line; see [G-3] [cline\\_options](#).

## CI plot

`ciopts`(*area\_options*) affects the rendition of the confidence bands; see [G-3] [area\\_options](#).

## Add plots

`addplot`(*plot*) provides a way to add other plots to the generated graph. See [G-3] [addplot\\_option](#).

## Y axis, X axis, Titles, Legend, Overall

*twoway\_options* are any of the options documented in [G-3] [twoway\\_options](#), excluding `by()`. These include options for titling the graph (see [G-3] [title\\_options](#)) and for saving the graph to disk (see [G-3] [saving\\_option](#)).

## Options for fp predict

## Main

`fp` calculates the fractional polynomial, the linear prediction with other variables set to zero. This is the default.

`stdp` calculates the standard error of the fractional polynomial.

`equation`(*eqno*) is relevant only when you have previously fit a multiple-equation model in *est\_cmd*. It specifies the equation to which you are referring.

`equation`(#1) would mean that the calculation is to be made for the first equation, `equation`(#2) would mean the second, and so on. You could also refer to the equations by their names: `equation`(*income*) would refer to the equation named *income*, and `equation`(*hours*) would refer to the equation named *hours*.

If you do not specify `equation()`, the results are the same as if you specified `equation`(#1).

## Remarks and examples

After a model is fit using `fp`, the estimated fractional polynomial may be of interest. This is the linear combination of the fractional polynomial terms and the constant intercept using the model coefficients estimated by `fp`. It is equivalent to the fitted values prediction given by `predict, xb`, with the covariates and the fractional polynomial variable set to zero. When these other covariates have been centered, the prediction is made at the centering values of the covariates.

A component-plus-residual plot is generated by `fp plot`. The fractional polynomial comprises the component, and the residual is specified by the user in `residuals()`. The `residuals()` option takes the same argument that would be supplied to `predict` after *est\_cmd* to obtain the desired type of residuals. If you only want to plot the component fit, without residuals, you would specify `residuals(none)`.

`fp predict` generates the fractional polynomial. If the `stdp` option is specified, the standard error of the fractional polynomial is generated instead. This standard error may be quite large if the range of the other covariates is far from zero. In this situation, the covariates would be centered and their range would include, or come close to including, zero.

These postestimation commands can be used only when the fractional polynomial terms do not interact with other variables in the specification of `est_cmd`. See [U] 11.4.3 **Factor variables** for more information about interactions.

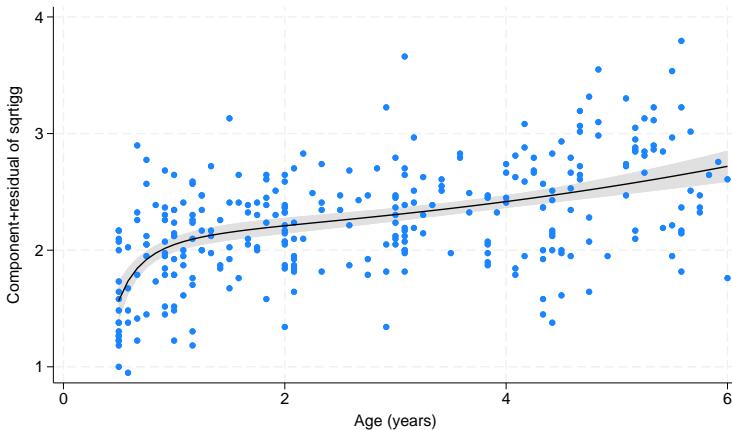
## Examples

### ▷ Example 1: fp plot after linear regression

In [example 1](#) of [R] `fp`, we modeled the mean of the square root of a child's serum immunoglobulin G (IgG) level as a fractional polynomial function of the child's age. An FP2 model with powers  $(-2, 2)$  is chosen.

We load the data and then fit the model with `fp`. Then, we use `fp plot` to draw the component-plus-residual plot. A 95% confidence interval is produced for the fractional polynomial in age (the component). The `residuals` prediction option for `regress` is specified in the `residuals()` option in `fp plot` so that the residuals are rendered.

```
. use https://www.stata-press.com/data/r19/igg
(Immunoglobulin in children)
. fp <age>, scale center: regress sqrtigg <age>
(output omitted)
. fp plot, residuals(residuals)
```

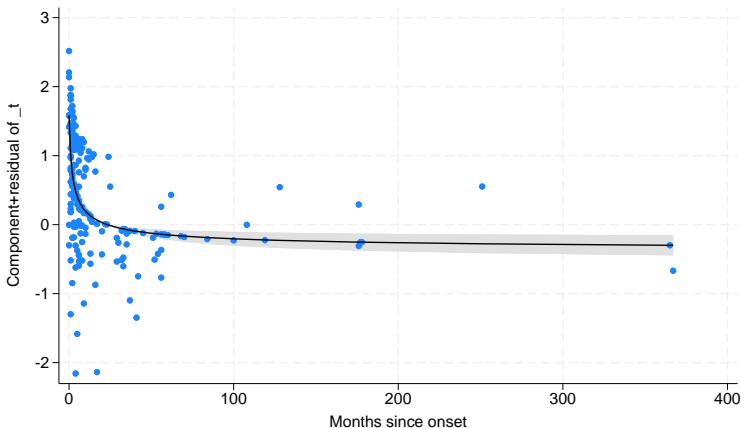


### ▷ Example 2: fp plot after Cox regression

In [example 2](#) of [\[R\] fp](#), we modeled the time to complete healing of leg ulcers for 192 elderly patients using a Cox regression. A one-degree fractional polynomial in `monthson`, the number of months since the onset of the ulcer, is used as a predictor in the regression. The power  $-0.5$  is used for `monthson`. Other covariates are `age` (`age`), ulcer area (`ulcarea`), treatment type, and a binary indicator of deep vein involvement (`deepppg`).

We load the data and then demean ulcer area and `age`. Then, we fit the model with `fp` and draw the component-plus-residual plot with `fp plot`. `mgale` is specified in the `residuals()` option to obtain martingale residuals. See [\[ST\] stcox postestimation](#) for more details.

```
. use https://www.stata-press.com/data/r19/legulcer2, clear
(Leg ulcer clinical trial)
. quietly stset ttevent, failure(healed)
. quietly summarize age
. replace age = age - r(mean)
variable age was byte now float
(192 real changes made)
. quietly summarize ulcarea
. replace ulcarea = ulcarea - r(mean)
variable ulcarea was int now float
(192 real changes made)
. fp <monthson>, replace center scale nohr fp(-.5): stcox <monthson> age ulcarea
> deepppg treat
(output omitted)
. fp plot, residuals(mgale)
```



### ▷ Example 3: fp plot and fp predict after logistic regression

In [example 3](#) of [\[R\] fp](#), we used logistic regression to model the odds of death for male civil servants in Britain conditional on cigarette consumption. The dependent variable `a1110` is an indicator of whether the individual passed away in the 10 years under study.

Nonsmokers may be qualitatively different from smokers, so the effect of smoking (regarded as a continuous variable) may not be continuous between zero cigarettes and one cigarette. To allow for this possibility, we model the risk as constant intercept for the nonsmokers and as a fractional polynomial function of the number of cigarettes for the smokers, `cigs`, adjusted for age. An FP1 model with power 0 is chosen.

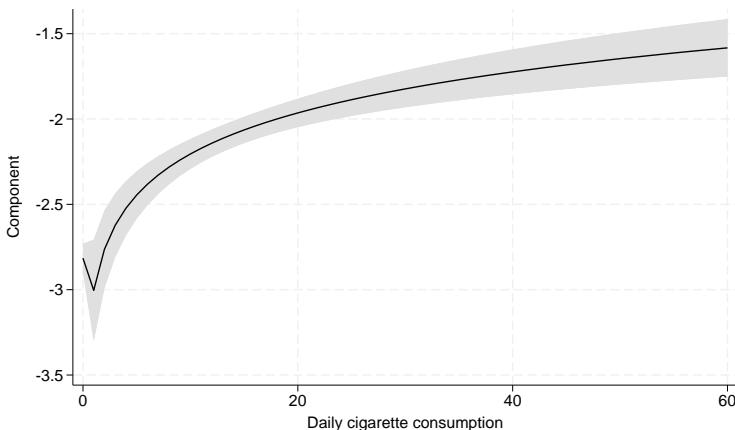
We load the data and demean age. Then, we fit the model using `fp` and graph the fit of the model and 95% confidence interval using `fp plot`. Only the component fit is graphed by specifying `residuals(none)`.

```
. use https://www.stata-press.com/data/r19/smoking, clear
(Smoking and mortality data)
. quietly summarize age
. replace age = age - r(mean)
variable age was byte now float
(17,260 real changes made)
. fp <cigs>, catzero replace fp(0): logit all10 <cigs> age
-> logit all10 cigs_0 cigs_1 age
```

|                             |                        |
|-----------------------------|------------------------|
| Logistic regression         | Number of obs = 17,260 |
|                             | LR chi2(3) = 1027.13   |
|                             | Prob > chi2 = 0.0000   |
|                             | Pseudo R2 = 0.0936     |
| Log likelihood = -4973.3016 |                        |

|        | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |          |
|--------|-------------|-----------|--------|-------|----------------------|----------|
| all10  |             |           |        |       |                      |          |
| cigs_0 | .1883732    | .1553093  | 1.21   | 0.225 | -.1160274            | .4927738 |
| cigs_1 | .3469842    | .0543552  | 6.38   | 0.000 | .2404499             | .4535185 |
| age    | .1194976    | .0045818  | 26.08  | 0.000 | .1105174             | .1284778 |
| _cons  | -3.003767   | .1514909  | -19.83 | 0.000 | -3.300683            | -2.70685 |

```
. fp plot, residuals(none)
```



We see a small spike at zero for `cigs` because of the effect of `cigs_0` on the fractional polynomial; however, the high  $p$ -value for `cigs_0` in the model output indicates that we cannot reject that there is no extra effect at zero for nonsmokers.

We can also use `fp predict` to predict the fractional polynomial for nonsmokers and the mean of age. This is the value at the spike. We store the result in `fp0`. We see it is equivalent to the sum of the constant intercept estimate and the estimate of the `cigs_0` coefficient.

```
. fp predict fp0 if cigs == 0
(7,157 missing values generated)
. summarize fp0
```

| Variable | Obs    | Mean      | Std. dev. | Min       | Max       |
|----------|--------|-----------|-----------|-----------|-----------|
| fp0      | 10,103 | -2.815393 | 0         | -2.815393 | -2.815393 |

```
. display _b[cigs_0]+_b[_cons]
-2.8153935
```

◀

## Methods and formulas

Let the data consist of triplets  $(y_i, x_i, \mathbf{z}_i)$ ,  $i = 1, \dots, n$ , where  $\mathbf{z}_i$  is the vector of covariates for the  $i$ th observation and  $x_i$  is the fractional polynomial variable.

`fp predict` calculates the fractional polynomial at the centering value  $x_0$ ,  $\hat{\eta}_i = (x_i^{(p_1, \dots, p_d)} - x_0^{(p_1, \dots, p_m)})' \hat{\beta}$ . This is equivalent to the linear predictor of the model at  $\mathbf{z}_i = \mathbf{0}$ . The standard error is calculated from the variance–covariance matrix of  $\hat{\beta}$ , ignoring estimation of the powers. When  $x_i \leq 0$ ,  $\mathbf{H}(x_i)$ , and thus  $x_i^{(p_1, \dots, p_m)}$ , is either undefined or zero. A zero offset term,  $\alpha_0$ , may be added to  $\hat{\eta}_i$  for these nonpositive  $x_i$  values.

The values  $\hat{\eta}_i$  represent the behavior of the fractional polynomial model for  $x$  at fixed values  $\mathbf{z} = \mathbf{0}$  of the (centered) covariates. The  $i$ th component-plus-residual is defined as  $\hat{\eta}_i + d_i$ , where  $d_i$  is the residual for the  $i$ th observation. The definition of  $d_i$  will change according to the type of model used and the preference of the user. `fp plot` plots  $\hat{\eta}_i + d_i$  versus  $x_i$ , overlaying  $\hat{\eta}_i$  and its confidence interval.

## Acknowledgment

We thank Patrick Royston of the MRC Clinical Trials Unit, London, and coauthor of the Stata Press book *Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model* for writing `fracplot` and `fracpred`, the commands on which `fp plot` and `fp predict` are based. We also thank Professor Royston for his advice on and review of `fp plot` and `fp predict`.

## Reference

Royston, P. 2017. [Model selection for univariable fractional polynomials](#). *Stata Journal* 17: 619–629.

## Also see

[R] [fp](#) — Fractional polynomial regression

[U] [20 Estimation and postestimation commands](#)

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

## Description

`fracreg` fits a fractional response model for a dependent variable that is greater than or equal to 0 and less than or equal to 1. It uses a probit, logit, or heteroskedastic probit model for the conditional mean. These models are often used for outcomes such as rates, proportions, and fractional data.

## Quick start

Fractional probit model for  $y$  with values between 0 and 1 on continuous variable  $x_1$

```
fracreg probit y x1
```

Same as above, but use logit distribution

```
fracreg logit y x1
```

Fractional probit model for  $y$  on  $x_1$  and use  $x_2$  to model the variance of  $y$

```
fracreg probit y x1, het(x2)
```

## Menu

Statistics > Fractional outcomes > Fractional regression

## Syntax

Syntax for fractional probit regression

```
fracreg probit depvar [indepvars] [if] [in] [weight] [, options]
```

Syntax for fractional logistic regression

```
fracreg logit depvar [indepvars] [if] [in] [weight] [, options]
```

Syntax for fractional heteroskedastic probit regression

```
fracreg probit depvar [indepvars] [if] [in] [weight],
het (varlist [, offset (varnameo)]) [options]
```

| <i>options</i>                                                                    | Description                                                                                                                                      |
|-----------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                                                      |                                                                                                                                                  |
| <u>noconstant</u>                                                                 | suppress constant term                                                                                                                           |
| <u>offset</u> ( <i>varname</i> )                                                  | include <i>varname</i> in model with coefficient constrained to 1                                                                                |
| <u>constraints</u> ( <i>constraints</i> )                                         | apply specified linear constraints                                                                                                               |
| * <u>het</u> ( <i>varlist</i> [, <u>offset</u> ( <i>varname</i> <sub>o</sub> )] ) | independent variables to model the variance and optional offset variable with fracreg probit                                                     |
| <b>SE/Robust</b>                                                                  |                                                                                                                                                  |
| <u>vce</u> ( <i>vcetype</i> )                                                     | <i>vcetype</i> may be <u>robust</u> , <u>cluster</u> <i>clustvar</i> , <u>bootstrap</u> , or <u>jackknife</u>                                    |
| <b>Reporting</b>                                                                  |                                                                                                                                                  |
| <u>level</u> (#)                                                                  | set confidence level; default is level(95)                                                                                                       |
| <u>or</u>                                                                         | report odds ratios; only valid with fracreg logit                                                                                                |
| <u>nocnsreport</u>                                                                | do not display constraints                                                                                                                       |
| <u>display_options</u>                                                            | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| <b>Maximization</b>                                                               |                                                                                                                                                  |
| <u>maximize_options</u>                                                           | control the maximization process; seldom used                                                                                                    |
| <u>nocoef</u>                                                                     | do not display the coefficient table; seldom used                                                                                                |
| <u>collinear</u>                                                                  | keep collinear variables                                                                                                                         |
| <u>coeflegend</u>                                                                 | display legend instead of statistics                                                                                                             |

\*het() may be used only with fracreg probit to compute fractional heteroskedastic probit regression.

*indepvars* may contain factor variables; see [U] 11.4.3 **Factor variables**.

*devar* and *indepvars* may contain time-series operators; see [U] 11.4.4 **Time-series varlists**.

bayes, bayesboot, bootstrap, by, collect, fp, jackknife, mi estimate, rolling, statsby, and svy are allowed; see [U] 11.1.10 **Prefix commands**. For more details, see [BAYES] **bayes: fracreg**.

vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] **mi estimate**.

Weights are not allowed with the bootstrap prefix; see [R] **bootstrap**.

vce(), nocoef, and weights are not allowed with the svy prefix; see [SVY] **svy**.

fweights, iweights, and pweights are allowed; see [U] 11.1.6 **weight**.

nocoef, collinear, and coeflegend do not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options

### Model

noconstant, offset(*varname*), constraints(*constraints*); see [R] **Estimation options**.

het(*varlist*[, offset(*varname*<sub>o</sub>)] ) specifies the independent variables and, optionally, the offset variable in the variance function. het() may only be used with fracreg probit to compute fractional heteroskedastic probit regression.

offset(*varname*<sub>o</sub>) specifies that selection offset *varname*<sub>o</sub> be included in the model with the coefficient constrained to be 1.

### SE/Robust

vce(*vcetype*) specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster *clustvar*), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] **vce\_option**.

### Reporting

level(#); see [R] **Estimation options**.

or reports the estimated coefficients transformed to odds ratios, that is,  $e^b$  rather than  $b$ . Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated. or may be specified at estimation or when replaying previously estimated results. This option may only be used with fracreg logit.

nocnsreport; see [R] **Estimation options**.

*display\_options*: nocl, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt), sformat(%fmt), and nolstretch; see [R] **Estimation options**.

### Maximization

*maximize\_options*: difficult, technique(algorithm\_spec), iterate(#), [no]log, trace, gradient, showstep, hessian, showtolerance, tolerance(#), ltolerance(#), nrtolerance(#), nonrtolerance, and from(init\_specs); see [R] **Maximize**. These options are seldom used.

The following options are available with `fracreg` but are not shown in the dialog box:

`nocoef` specifies that the coefficient table not be displayed. This option is sometimes used by programmers but is of no use interactively.

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Fractional response data may occur when the outcome of interest is measured as a fraction, for example, a patient's oxygen saturation or Gini coefficient values. These data are also often observed when proportions are generated from aggregated binary outcomes. For example, rather than having data on whether individual students passed an exam, we might simply have data on the proportion of students in each school that passed.

These models are appropriate when you have a dependent variable that takes values between 0 and 1 and may also be equal to 0 or 1, denoted for conciseness with the notation  $[0, 1]$ . If the dependent variable takes only values between 0 and 1, `betareg` might be a valid alternative. `betareg` provides more flexibility in the distribution of the mean of the dependent variable but is misspecified if the dependent variable is equal to 0 or 1. See [R] [betareg](#) for more information.

These models have been applied to various topics. For example, [Papke and Wooldridge \(1996\)](#) studied the participation rates of employees in firms' 401(k) retirement plans. [Papke and Wooldridge \(2008\)](#) also evaluated an education policy by studying the pass rates for an exam administered to fourth grade Michigan students over time.

The models fit by `fracreg` are quasilielihood estimators like the generalized linear models described in [R] [glm](#). Fractional regression is a model of the mean of the dependent variable  $y$  conditional on covariates  $\mathbf{x}$ , which we denote by  $\mu_{\mathbf{x}}$ . Because  $y$  is in  $[0, 1]$ , we must ensure that  $\mu_{\mathbf{x}}$  is also in  $[0, 1]$ . We do this by using a probit, logit, or heteroskedastic probit model for  $\mu_{\mathbf{x}}$ .

The key insight from quasilielihood estimation is that you do not need to know the true distribution of the entire model to obtain consistent parameter estimates. In fact, the only information that you need is the correct specification of the conditional mean.

This means that the true model does not need to be, for example, a probit. If the true model is a probit, then fitting a probit regression via maximum likelihood gives you consistent parameter estimates and asymptotically efficient standard errors.

By contrast, if the conditional mean of the model is the same as the conditional mean of a probit but the model is not a probit, the point estimates are consistent, but the standard errors are not asymptotically efficient. The standard errors are not efficient, because no assumptions about the distribution of the unobserved components in the model are made. Thus `fracreg` uses robust standard errors by default.

For further discussion on quasilielihood estimation in the context of fractional regression, please see [Papke and Wooldridge \(1996\)](#) and [Wooldridge \(2010\)](#).

### ► Example 1: Fractional probit model of rates

In this example, we look at the expected participation rate in 401(k) plans for a cross-section of firms. Participation rate (`prate`) is defined as the fraction of eligible employees in a firm that participate in a 401(k) plan. We use `summarize` to see the range of the participation rate.

```
. use https://www.stata-press.com/data/r19/401k
(Firm-level data on 401k participation)

. summarize prate
```

| Variable | Obs   | Mean    | Std. dev. | Min      | Max |
|----------|-------|---------|-----------|----------|-----|
| prate    | 4,075 | .840607 | .1874841  | .0036364 | 1   |

The variable has values between 0 and 1 but also has at least 1 firm for which the participation rate is exactly 1.

As in [Papke and Wooldridge \(1996\)](#), we surmise that the expected participation rate depends on the matching rate of employee 401(k) contributions (`mrate`), the natural log of the total number of employees (`ltotemp`), the age of the plan (`age`), and whether the 401(k) plan is the only retirement plan offered by the employer (`sole`). We include `ltotemp` and `age`, along with their squares, using factor-variable notation; see [\[U\] 11.4.3 Factor variables](#).

If we believe that the functional form of the expected participation rate is a cumulative normal density, we may use `fracreg probit`.

```
. fracreg probit prate mrate c.ltotemp##c.ltotemp c.age##c.age i.sole
Iteration 0: Log pseudolikelihood = -1769.6832
Iteration 1: Log pseudolikelihood = -1675.2763
Iteration 2: Log pseudolikelihood = -1674.6234
Iteration 3: Log pseudolikelihood = -1674.6232
Iteration 4: Log pseudolikelihood = -1674.6232

Fractional probit regression
Log pseudolikelihood = -1674.6232
Number of obs = 4,075
Wald chi2(6) = 815.88
Prob > chi2 = 0.0000
Pseudo R2 = 0.0632
```

| prate                   | Coefficient | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |           |
|-------------------------|-------------|---------------------|-------|-------|----------------------|-----------|
| mrate                   | .5859715    | .0387616            | 15.12 | 0.000 | .5100002             | .6619429  |
| ltotemp                 | -.6102767   | .0615052            | -9.92 | 0.000 | -.7308246            | -.4897288 |
| c.ltotemp#<br>c.ltotemp | .0313576    | .003975             | 7.89  | 0.000 | .0235667             | .0391484  |
| age                     | .0273266    | .0031926            | 8.56  | 0.000 | .0210691             | .033584   |
| c.age#c.age             | -.0003159   | .0000875            | -3.61 | 0.000 | -.0004874            | -.0001443 |
| sole                    |             |                     |       |       |                      |           |
| Only plan               | .0683196    | .0272091            | 2.51  | 0.012 | .0149908             | .1216484  |
| _cons                   | 3.25991     | .2323929            | 14.03 | 0.000 | 2.804429             | 3.715392  |

Like those obtained from `probit`, the parameters provide the sign of the marginal effect of the covariates on the outcome, but the magnitude is difficult to interpret. We can use `margins` to estimate conditional or population-averaged effects; see [example 2](#). The standard errors are robust by default because the true data-generating process need not be a probit, even though we use the probit likelihood to obtain our parameter estimates.

◀

### ▷ Example 2: Changing the distribution of the conditional mean

Continuing with [example 1](#), we may instead believe that the expected participation rate follows a fractional logistic response. In this case, we should use fractional logistic regression instead of fractional probit regression to obtain consistent estimates of the parameters of the conditional mean.

```
. fracreg logit prate mrate c.ltotemp#c.ltotemp c.age#c.age i.sole
Iteration 0: Log pseudolikelihood = -1983.8372
Iteration 1: Log pseudolikelihood = -1682.4496
Iteration 2: Log pseudolikelihood = -1673.6458
Iteration 3: Log pseudolikelihood = -1673.5566
Iteration 4: Log pseudolikelihood = -1673.5566

Fractional logistic regression
Log pseudolikelihood = -1673.5566
Number of obs = 4,075
Wald chi2(6) = 817.73
Prob > chi2 = 0.0000
Pseudo R2 = 0.0638
```

| prate       | Robust      |           | z     | P> z  | [95% conf. interval] |           |
|-------------|-------------|-----------|-------|-------|----------------------|-----------|
|             | Coefficient | std. err. |       |       |                      |           |
| mrate       | 1.143516    | .074748   | 15.30 | 0.000 | .9970125             | 1.290019  |
| ltotemp     | -1.103275   | .1130667  | -9.76 | 0.000 | -1.324882            | -.8816687 |
| c.ltotemp#  |             |           |       |       |                      |           |
| c.ltotemp   | .0565782    | .0072883  | 7.76  | 0.000 | .0422934             | .070863   |
| age         | .0512643    | .0059399  | 8.63  | 0.000 | .0396223             | .0629064  |
| c.age#c.age | -.0005891   | .0001645  | -3.58 | 0.000 | -.0009114            | -.0002667 |
| sole        |             |           |       |       |                      |           |
| Only plan   | .1137479    | .0507762  | 2.24  | 0.025 | .0142284             | .2132674  |
| _cons       | 5.747761    | .4294386  | 13.38 | 0.000 | 4.906077             | 6.589445  |

Like those obtained from `logit`, the parameters provide the sign of the marginal effect of the covariates on the outcome, but the magnitude is again difficult to interpret. As with `fracreg probit` in [example 1](#), we would use `margins` to obtain the marginal effects or other predictions of interest.

◀

### ► Example 3: Odds ratios from a fractional logit model

When the conditional mean of our outcome is interpretable as a probability, it is possible to adopt an odds-ratio interpretation of the results of a fractional logit model. In [example 2](#), this is plausible because expected participation rates can be viewed as estimates of the probability of participation. We obtain the odds ratios by specifying the option `or`.

```
. fracreg logit prate mrate c.ltotemp##c.ltotemp c.age##c.age i.sole, or
Iteration 0: Log pseudolikelihood = -1983.8372
Iteration 1: Log pseudolikelihood = -1682.4496
Iteration 2: Log pseudolikelihood = -1673.6458
Iteration 3: Log pseudolikelihood = -1673.5566
Iteration 4: Log pseudolikelihood = -1673.5566
Fractional logistic regression Number of obs = 4,075
 Wald chi2(6) = 817.73
 Prob > chi2 = 0.0000
Log pseudolikelihood = -1673.5566 Pseudo R2 = 0.0638
```

| prate                   | Odds ratio | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |          |
|-------------------------|------------|---------------------|-------|-------|----------------------|----------|
| mrate                   | 3.137781   | .2345429            | 15.30 | 0.000 | 2.710173             | 3.632857 |
| ltotemp                 | .3317826   | .0375136            | -9.76 | 0.000 | .2658343             | .4140913 |
| c.ltotemp#<br>c.ltotemp | 1.058209   | .0077125            | 7.76  | 0.000 | 1.043201             | 1.073434 |
| age                     | 1.052601   | .0062524            | 8.63  | 0.000 | 1.040418             | 1.064927 |
| c.age#c.age             | .9994111   | .0001644            | -3.58 | 0.000 | .999089              | .9997333 |
| sole                    |            |                     |       |       |                      |          |
| Only plan               | 1.12047    | .0568932            | 2.24  | 0.025 | 1.01433              | 1.237716 |
| _cons                   | 313.4879   | 134.6238            | 13.38 | 0.000 | 135.1083             | 727.3771 |

Note: `_cons` estimates baseline odds.

Among other things, we see that if the 401(k) is the only plan offered by the employer, then the odds of an employee participating increase by a factor of 1.12. We can also see that if the matching rate goes from 0 to 1:1 (exactly matching employee contributions) or from 1:1 to 2:1 (doubling employee contributions), then the odds of participating increase by 3.1.

The use of an odds-ratio interpretation is not appropriate if the conditional mean cannot be viewed as a probability. For example, if the fractional outcome were a Gini coefficient, we could not interpret the expected values of our outcomes as probabilities. The Gini coefficient is a measure of inequality between zero and one and cannot be interpreted as a probability. In this case, using the odds-ratio option would not be sensible.

## Stored results

fracreg stores the following in `e()`:

### Scalars

|                            |                                           |
|----------------------------|-------------------------------------------|
| <code>e(N)</code>          | number of observations                    |
| <code>e(k)</code>          | number of parameters                      |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>  |
| <code>e(k_eq_model)</code> | number of equations in overall model test |
| <code>e(k_dv)</code>       | number of dependent variables             |
| <code>e(df_m)</code>       | model degrees of freedom                  |
| <code>e(r2_p)</code>       | pseudo- $R^2$                             |
| <code>e(ll)</code>         | log likelihood                            |
| <code>e(ll_0)</code>       | log likelihood, constant-only model       |
| <code>e(N_clust)</code>    | number of clusters                        |
| <code>e(chi2)</code>       | $\chi^2$                                  |
| <code>e(p)</code>          | $p$ -value for model test                 |
| <code>e(rank)</code>       | rank of <code>e(V)</code>                 |
| <code>e(ic)</code>         | number of iterations                      |
| <code>e(rc)</code>         | return code                               |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise               |

### Macros

|                              |                                                                          |
|------------------------------|--------------------------------------------------------------------------|
| <code>e(cmd)</code>          | fracreg                                                                  |
| <code>e(cmdline)</code>      | command as typed                                                         |
| <code>e(estimator)</code>    | model for conditional mean; logit, probit, or hetprobit                  |
| <code>e(depvar)</code>       | name of dependent variable                                               |
| <code>e(wtype)</code>        | weight type                                                              |
| <code>e(wexp)</code>         | weight expression                                                        |
| <code>e(title)</code>        | title in estimation output                                               |
| <code>e(clustvar)</code>     | name of cluster variable                                                 |
| <code>e(offset)</code>       | offset                                                                   |
| <code>e(chi2type)</code>     | Wald; type of model $\chi^2$ test                                        |
| <code>e(vce)</code>          | <i>vcetype</i> specified in <code>vce()</code>                           |
| <code>e(vcetype)</code>      | title used to label Std. err.                                            |
| <code>e(opt)</code>          | type of optimization                                                     |
| <code>e(which)</code>        | max or min; whether optimizer is to perform maximization or minimization |
| <code>e(ml_method)</code>    | type of ml method                                                        |
| <code>e(user)</code>         | name of likelihood-evaluator program                                     |
| <code>e(technique)</code>    | maximization technique                                                   |
| <code>e(properties)</code>   | b V                                                                      |
| <code>e(estat_cmd)</code>    | program used to implement <code>estat</code>                             |
| <code>e(predict)</code>      | program used to implement <code>predict</code>                           |
| <code>e(marginsnotok)</code> | predictions disallowed by <code>margins</code>                           |
| <code>e(asbalanced)</code>   | factor variables <code>fvset</code> as <code>asbalanced</code>           |
| <code>e(asobserved)</code>   | factor variables <code>fvset</code> as <code>asobserved</code>           |

### Matrices

|                              |                                              |
|------------------------------|----------------------------------------------|
| <code>e(b)</code>            | coefficient vector                           |
| <code>e(mns)</code>          | vector of means of the independent variables |
| <code>e(Cns)</code>          | constraints matrix                           |
| <code>e(i_log)</code>        | iteration log (up to 20 iterations)          |
| <code>e(gradient)</code>     | gradient vector                              |
| <code>e(V)</code>            | variance-covariance matrix of the estimators |
| <code>e(V_modelbased)</code> | model-based variance                         |

### Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

Matrices

`r(table)`

matrix containing the coefficients with their standard errors, test statistics, *p*-values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

## Methods and formulas

The log-likelihood function for fractional models is of the form

$$\ln L = \sum_{j=1}^N w_j y_j \ln \{ G(\mathbf{x}'_j \boldsymbol{\beta}) \} + w_j (1 - y_j) \ln \{ 1 - G(\mathbf{x}'_j \boldsymbol{\beta}) \}$$

where  $N$  is the sample size,  $y_j$  is the dependent variable,  $w_j$  denotes the optional weights,  $\ln L$  is maximized, as described in [R] **Maximize**, and  $G(\cdot)$  can be

| Model     | Functional form for $G(\mathbf{x}'_j \boldsymbol{\beta})$                                 |
|-----------|-------------------------------------------------------------------------------------------|
| probit    | $\Phi(\mathbf{x}'_j \boldsymbol{\beta})$                                                  |
| logit     | $\exp(\mathbf{x}'_j \boldsymbol{\beta}) / \{1 + \exp(\mathbf{x}'_j \boldsymbol{\beta})\}$ |
| hetprobit | $\Phi \{ \mathbf{x}'_j \boldsymbol{\beta} / \exp(\mathbf{z}'_j \boldsymbol{\gamma}) \}$   |

where  $\mathbf{x}_j$  are the covariates for individual  $j$ ,  $\mathbf{z}_j$  are the covariates used to model the variance of the outcome for the heteroskedastic probit model, and  $\Phi$  is the standard normal cumulative density function.

## References

- Gray, L. A., and M. Hernández-Alava. 2018. [A command for fitting mixture regression models for bounded dependent variables using the beta distribution](#). *Stata Journal* 18: 51–75.
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- Xu, J., and J. S. Long. 2005. Confidence intervals for predicted outcomes in regression models for categorical outcomes. *Stata Journal* 5: 537–559.

## Also see

- [R] [fracreg postestimation](#) — Postestimation tools for fracreg
- [R] [betareg](#) — Beta regression
- [R] [glm](#) — Generalized linear models
- [R] [ivfprobit](#) — Fractional probit model with continuous endogenous covariates
- [BAYES] [bayes: fracreg](#) — Bayesian fractional response regression
- [MI] [Estimation](#) — Estimation commands for use with mi estimate
- [SVY] [svy estimation](#) — Estimation commands for survey data
- [U] [20 Estimation and postestimation commands](#)

## Postestimation commands

The following standard postestimation commands are available after `fracreg`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | conditional means, linear predictions, etc.                                                                                             |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`forecast` and `hausman` are not appropriate with `svy` estimation results. `forecast` is also not appropriate with `mi` estimation results.

## predict

### Description for predict

`predict` creates a new variable containing predictions such as conditional means, linear predictions, standard errors, and equation-level scores.

### Menu for predict

Statistics > Postestimation

### Syntax for predict

```
predict [type] newvar [if] [in] [, statistic nooffset]
```

```
predict [type] stub* [if] [in] , scores
```

| <i>statistic</i> | Description                                                    |
|------------------|----------------------------------------------------------------|
| cm               | conditional mean; the default                                  |
| xb               | linear prediction                                              |
| sigma            | standard deviation of the error term (for <code>het()</code> ) |
| stdp             | standard error of the linear prediction                        |

### Options for predict

#### Main

`cm`, the default, calculates the conditional mean of the outcome.

`xb` calculates the linear prediction.

`sigma` calculates the standard deviation of the error term. It is available only when `het()` is specified.

`stdp` calculates the standard error of the linear prediction.

`nooffset` is relevant only if you specified `offset(varname)`. It modifies the calculations made by `predict` so that they ignore the offset variable; the linear prediction is treated as  $\mathbf{x}_j\mathbf{b}$  rather than as  $\mathbf{x}_j\mathbf{b} + \text{offset}_j$ .

`scores` calculates the equation-level scores. In the case of `fracreg probit` and `fracreg logit`,  $\partial \ln L / \partial (\mathbf{x}_j\boldsymbol{\beta})$  is calculated, and if the option `het()` was specified with `fracreg probit`, then  $\partial \ln L / \partial (\mathbf{z}_j\boldsymbol{\gamma})$  is also calculated.

## margins

### Description for margins

`margins` estimates margins of response for conditional means and linear predictions.

### Menu for margins

Statistics > Postestimation

### Syntax for margins

```
margins [marginlist] [, options]
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i>    | Description                                                    |
|---------------------|----------------------------------------------------------------|
| <code>cm</code>     | conditional mean; the default                                  |
| <code>xb</code>     | linear prediction                                              |
| <code>sigma</code>  | standard deviation of the error term (for <code>het()</code> ) |
| <code>stdp</code>   | not allowed with <code>margins</code>                          |
| <code>scores</code> | not allowed with <code>margins</code>                          |

Statistics not allowed with `margins` are functions of stochastic quantities other than `e(b)`.

For the full syntax, see [R] [margins](#).

## Remarks and examples

Remarks are presented under the following headings:

*Obtaining predicted values*  
*Performing hypothesis tests*

### Obtaining predicted values

Once you have fit a model using `fracreg`, you can obtain the conditional mean of the fractional response by using the `predict` command for both the estimation sample and other samples; see [U] [20 Estimation and postestimation commands](#) and [R] [predict](#).

When you use the fractional probit estimator, `fracreg probit`, with the option `het()`, there is an additional statistic available, `sigma`. With the `sigma` option, `predict` calculates the predicted standard deviation,  $\sigma_j = \exp(\mathbf{z}_j\boldsymbol{\gamma})$ .

## Performing hypothesis tests

### ▷ Example 1: Conditional means

In [example 1](#) of [\[R\] fracreg](#), we fit a fractional probit model to see how participation rate (`prate`) in 401(k) plans is affected by the matching rate of employer contributions (`mrate`). To obtain the predicted conditional means, we use `predict` and do not specify the default `cm` option.

```
. use https://www.stata-press.com/data/r19/401k
(Firm-level data on 401k participation)
. fracreg probit prate mrate c.ltotemp##c.ltotemp c.age##c.age i.sole
Iteration 0: Log pseudolikelihood = -1769.6832
Iteration 1: Log pseudolikelihood = -1675.2763
Iteration 2: Log pseudolikelihood = -1674.6234
Iteration 3: Log pseudolikelihood = -1674.6232
Iteration 4: Log pseudolikelihood = -1674.6232
Fractional probit regression
Log pseudolikelihood = -1674.6232
Number of obs = 4,075
Wald chi2(6) = 815.88
Prob > chi2 = 0.0000
Pseudo R2 = 0.0632
```

| prate                   | Coefficient | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |           |
|-------------------------|-------------|---------------------|-------|-------|----------------------|-----------|
| mrage                   | .5859715    | .0387616            | 15.12 | 0.000 | .5100002             | .6619429  |
| ltotemp                 | -.6102767   | .0615052            | -9.92 | 0.000 | -.7308246            | -.4897288 |
| c.ltotemp#<br>c.ltotemp | .0313576    | .003975             | 7.89  | 0.000 | .0235667             | .0391484  |
| age                     | .0273266    | .0031926            | 8.56  | 0.000 | .0210691             | .033584   |
| c.age#c.age             | -.0003159   | .0000875            | -3.61 | 0.000 | -.0004874            | -.0001443 |
| sole                    |             |                     |       |       |                      |           |
| Only plan               | .0683196    | .0272091            | 2.51  | 0.012 | .0149908             | .1216484  |
| _cons                   | 3.25991     | .2323929            | 14.03 | 0.000 | 2.804429             | 3.715392  |

```
. predict mpart
(option cm assumed)
```

We can then summarize these conditional mean estimates (`cmean`) over the population to get the population average conditional mean participation rate in our sample.

```
. summarize mpart
```

| Variable | Obs   | Mean     | Std. dev. | Min      | Max      |
|----------|-------|----------|-----------|----------|----------|
| mpart    | 4,075 | .8405767 | .0828094  | .6251739 | .9964518 |

The average of the conditional mean of participation rate in our sample is 84% with a range between 62.5% and 99.6%.

► Example 2: Average marginal effects

In [example 2](#) of [R] `fracreg`, we used the outcome variable and covariates of [example 1](#) but instead of fitting a fractional probit regression, we fit a fractional logit. Using margins, we explore the average marginal effect of `mrater` on `prater` for both specifications.

Below, we use margins after `fracreg logit` with the option `post` to post the average marginal effects as estimates. We then store our results with the name `logit`. We do the same with our probit estimates.

```
. use https://www.stata-press.com/data/r19/401k, clear
(Firm-level data on 401k participation)
. fracreg logit prate mrater c.ltotemp#c.ltotemp c.age#c.age i.sole, or
Iteration 0: Log pseudolikelihood = -1983.8372
Iteration 1: Log pseudolikelihood = -1682.4496
Iteration 2: Log pseudolikelihood = -1673.6458
Iteration 3: Log pseudolikelihood = -1673.5566
Iteration 4: Log pseudolikelihood = -1673.5566
Fractional logistic regression
Log pseudolikelihood = -1673.5566
Number of obs = 4,075
Wald chi2(6) = 817.73
Prob > chi2 = 0.0000
Pseudo R2 = 0.0638
```

| prate                   | Odds ratio | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |          |
|-------------------------|------------|---------------------|-------|-------|----------------------|----------|
| mrater                  | 3.137781   | .2345429            | 15.30 | 0.000 | 2.710173             | 3.632857 |
| ltotemp                 | .3317826   | .0375136            | -9.76 | 0.000 | .2658343             | .4140913 |
| c.ltotemp#<br>c.ltotemp | 1.058209   | .0077125            | 7.76  | 0.000 | 1.043201             | 1.073434 |
| age                     | 1.052601   | .0062524            | 8.63  | 0.000 | 1.040418             | 1.064927 |
| c.age#c.age             | .9994111   | .0001644            | -3.58 | 0.000 | .999089              | .9997333 |
| sole                    |            |                     |       |       |                      |          |
| Only plan               | 1.12047    | .0568932            | 2.24  | 0.025 | 1.01433              | 1.237716 |
| _cons                   | 313.4879   | 134.6238            | 13.38 | 0.000 | 135.1083             | 727.3771 |

Note: `_cons` estimates baseline odds.

```
. margins, dydx(mrater) post
Average marginal effects
Model VCE: Robust
Expression: Conditional mean of prate, predict()
dy/dx wrt: mrater
Number of obs = 4,075
```

|        | dy/dx    | Delta-method<br>std. err. | z     | P> z  | [95% conf. interval] |          |
|--------|----------|---------------------------|-------|-------|----------------------|----------|
| mrater | .1450106 | .0094558                  | 15.34 | 0.000 | .1264776             | .1635436 |

```
. estimates store logit
```

The marginal effects from `fracreg logit` suggest that a small change in the matching rate of employers can increase participation by more than 14%.

```
. quietly fracreg probit prate mrate c.ltotemp##c.ltotemp c.age##c.age i.sole
. margins, dydx(mrate) post
Average marginal effects Number of obs = 4,075
Model VCE: Robust
Expression: Conditional mean of prate, predict()
dy/dx wrt: mrate
```

|       | Delta-method |           |       |       | [95% conf. interval] |          |
|-------|--------------|-----------|-------|-------|----------------------|----------|
|       | dy/dx        | std. err. | z     | P> z  |                      |          |
| mrate | .1335505     | .0087385  | 15.28 | 0.000 | .1164233             | .1506776 |

```
. estimates store probit
```

For the probit model, a change in the matching rate increases participation by more than 13%.

Because we stored our margins results as estimates, we can now produce a table showing both the logit and probit results.

```
. estimates table logit probit, se
```

| Variable | logit                  | probit                 |
|----------|------------------------|------------------------|
| mrate    | .14501059<br>.00945578 | .13355046<br>.00873852 |

Legend: b/se

As indicated by the standard errors in the table, both average marginal effects are significant. The difference between the two estimates is approximately one percentage point. This relatively small difference is consistent with the intuition that marginal effects obtained from probit and logit conditional means give us analogous results.



▷ Example 3: Average marginal effects for different levels of participation

We can also use margins to find the expected participation rate for various levels of employer matching. Using our probit model, we obtain the following by typing

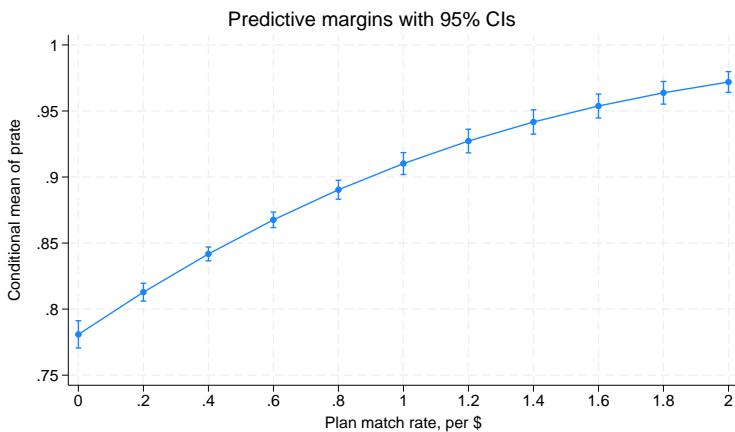
```
. quietly fracreg probit prate mrate c.ltotemp##c.ltotemp c.age##c.age i.sole
. margins, at(mrate=(0(.2)2))
Predictive margins Number of obs = 4,075
Model VCE: Robust
Expression: Conditional mean of prate, predict()
1._at: mrate = 0
2._at: mrate = .2
3._at: mrate = .4
4._at: mrate = .6
5._at: mrate = .8
6._at: mrate = 1
7._at: mrate = 1.2
8._at: mrate = 1.4
9._at: mrate = 1.6
10._at: mrate = 1.8
11._at: mrate = 2
```

|     | Delta-method |           |        |       |                      |
|-----|--------------|-----------|--------|-------|----------------------|
|     | Margin       | std. err. | z      | P> z  | [95% conf. interval] |
| _at |              |           |        |       |                      |
| 1   | .780858      | .0052738  | 148.06 | 0.000 | .7705216 .7911944    |
| 2   | .8128364     | .003441   | 236.22 | 0.000 | .8060923 .8195806    |
| 3   | .8417642     | .002672   | 315.03 | 0.000 | .8365271 .8470013    |
| 4   | .8675979     | .0029882  | 290.34 | 0.000 | .8617412 .8734547    |
| 5   | .8903734     | .0036591  | 243.33 | 0.000 | .8832018 .8975451    |
| 6   | .9101957     | .0042293  | 215.21 | 0.000 | .9019065 .9184849    |
| 7   | .9272265     | .0045767  | 202.60 | 0.000 | .9182563 .9361966    |
| 8   | .9416712     | .004694   | 200.61 | 0.000 | .9324711 .9508712    |
| 9   | .9537652     | .0046115  | 206.82 | 0.000 | .9447268 .9628035    |
| 10  | .9637608     | .004372   | 220.44 | 0.000 | .9551919 .9723298    |
| 11  | .9719159     | .0040207  | 241.73 | 0.000 | .9640355 .9797964    |

Going from no matching to equal matching changes the participation rate from 78% to 91%, and double matching moves participation all the way to 97.2%.

We can also see these results in a graph by using `marginplot`.

```
. marginplot
```



## Also see

[R] [fracreg](#) — Fractional response regression

[U] [20 Estimation and postestimation commands](#)

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

## Description

`frontier` fits stochastic production or cost frontier models; the default is a production frontier model. It provides estimators for the parameters of a linear model with a disturbance that is assumed to be a mixture of two components, which have a strictly nonnegative and symmetric distribution, respectively. `frontier` can fit models in which the nonnegative distribution component (a measurement of inefficiency) is assumed to be from a half-normal, exponential, or truncated-normal distribution. See [Kumbhakar and Lovell \(2000\)](#) for a detailed introduction to frontier analysis.

## Quick start

Cobb–Douglas production frontier model of `lny1` as a function of `lnx1` and `lnx2`

```
frontier lny1 lnx1 lnx2
```

Same as above, but use exponential instead of half-normal distribution for the inefficiency term

```
frontier lny1 lnx1 lnx2, distribution(exponential)
```

Include `x3` as an explanatory variable in the idiosyncratic error variance function

```
frontier lny1 lnx1 lnx2, vhet(x3)
```

Same as above, and include `x4` as an explanatory variable in the technical inefficiency variance function

```
frontier lny1 lnx1 lnx2, vhet(x3) uheth(x4)
```

Conditional mean model with the mean modeled as a linear function of `x3`

```
frontier lny1 lnx1 lnx2, distribution(tnormal) cm(x3)
```

Cost frontier model of `y2` as a function of `lnx1` and `lnx2`

```
frontier y2 lnx1 lnx2, distribution(tnormal) cost
```

## Menu

Statistics > Linear models and related > Frontier models

## Syntax

```
frontier depvar [indepvars] [if] [in] [weight] [, options]
```

| <i>options</i>                                                 | Description                                                                                                                                                                 |
|----------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                                   |                                                                                                                                                                             |
| <code>noconstant</code>                                        | suppress constant term                                                                                                                                                      |
| <code>distribution(hnormal)</code>                             | half-normal distribution for the inefficiency term                                                                                                                          |
| <code>distribution(exponential)</code>                         | exponential distribution for the inefficiency term                                                                                                                          |
| <code>distribution(tnormal)</code>                             | truncated-normal distribution for the inefficiency term                                                                                                                     |
| <code>ufrom(<i>matrix</i>)</code>                              | specify untransformed log likelihood; only with <code>d(tnormal)</code>                                                                                                     |
| <code>cm(<i>varlist</i> [, <code>noconstant</code>])</code>    | fit conditional mean model; only with <code>d(tnormal)</code> ; use <code>noconstant</code> to suppress constant term                                                       |
| <b>Model 2</b>                                                 |                                                                                                                                                                             |
| <code>constraints(<i>constraints</i>)</code>                   | apply specified linear constraints                                                                                                                                          |
| <code>uhet(<i>varlist</i> [, <code>noconstant</code>])</code>  | explanatory variables for technical inefficiency variance function; use <code>noconstant</code> to suppress constant term                                                   |
| <code>vhhet(<i>varlist</i> [, <code>noconstant</code>])</code> | explanatory variables for idiosyncratic error variance function; use <code>noconstant</code> to suppress constant term                                                      |
| <code>cost</code>                                              | fit cost frontier model; default is production frontier model                                                                                                               |
| <b>SE/Robust</b>                                               |                                                                                                                                                                             |
| * <code>vce(<i>vcetype</i>)</code>                             | <i>vcetype</i> may be <code>oim</code> , <code>robust</code> , <code>cluster <i>clustvar</i></code> , <code>opg</code> , <code>bootstrap</code> , or <code>jackknife</code> |
| <b>Reporting</b>                                               |                                                                                                                                                                             |
| <code>level(#)</code>                                          | set confidence level; default is <code>level(95)</code>                                                                                                                     |
| <code>nocnsreport</code>                                       | do not display constraints                                                                                                                                                  |
| <code>display_options</code>                                   | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling                            |
| <b>Maximization</b>                                            |                                                                                                                                                                             |
| <code>maximize_options</code>                                  | control the maximization process; seldom used                                                                                                                               |
| <code>collinear</code>                                         | keep collinear variables                                                                                                                                                    |
| <code>coeflegend</code>                                        | display legend instead of statistics                                                                                                                                        |

\*`vce(robust)` and `vce(cluster clustvar)` may not be specified with `distribution(tnormal)`.

*indepvars* and *varlist* may contain factor variables; see [U] 11.4.3 Factor variables.

`bayesboot`, `bootstrap`, `by`, `collect`, `fp`, `jackknife`, `rolling`, and `statsby` are allowed; see [U] 11.1.10 Prefix commands.

Weights are not allowed with the `bootstrap` prefix; see [R] `bootstrap`.

`fweights`, `iweights`, and `pweights` are allowed; see [U] 11.1.6 `weight`.

`collinear` and `coeflegend` do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Options

### Model

`noconstant`; see [R] [Estimation options](#).

`distribution(distname)` specifies the distribution for the inefficiency term as half-normal (`hnormal`), exponential, or truncated-normal (`tnormal`). The default is `hnormal`.

`ufrom(matrix)` specifies a  $1 \times K$  matrix of untransformed starting values when the distribution is truncated-normal (`tnormal`). `frontier` can estimate the parameters of the model by maximizing either the log likelihood or a transformed log likelihood (see [Methods and formulas](#)). `frontier` automatically transforms the starting values before passing them on to the transformed log likelihood. The matrix must have the same number of columns as there are parameters to estimate.

`cm(varlist [, noconstant])` may be used only with `distribution(tnormal)`. Here `frontier` will fit a conditional mean model in which the mean of the truncated-normal distribution is modeled as a linear function of the set of covariates specified in *varlist*. Specifying `noconstant` suppresses the constant in the mean function.

### Model 2

`constraints(constraints)`; see [R] [Estimation options](#).

By default, when fitting the truncated-normal model or the conditional mean model, `frontier` maximizes a transformed log likelihood. When constraints are applied, `frontier` will maximize the untransformed log likelihood with constraints defined in the untransformed metric.

`uhet(varlist [, noconstant])` specifies that the technical inefficiency component is heteroskedastic, with the variance function depending on a linear combination of *varlist*<sub>u</sub>. Specifying `noconstant` suppresses the constant term from the variance function. This option may not be specified with `distribution(tnormal)`.

`vhet(varlist [, noconstant])` specifies that the idiosyncratic error component is heteroskedastic, with the variance function depending on a linear combination of *varlist*<sub>v</sub>. Specifying `noconstant` suppresses the constant term from the variance function. This option may not be specified with `distribution(tnormal)`.

`cost` specifies that `frontier` fit a cost frontier model.

### SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

`vce(robust)` and `vce(cluster clustvar)` may not be specified with `distribution(tnormal)`.

### Reporting

`level(#)`, `nocnsreport`; see [R] [Estimation options](#).

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

## Maximization

`maximize_options`: `difficult`, `technique` (`algorithm_spec`), `iterate`(#), `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance`(#), `ltolerance`(#), `nrtolerance`(#), `nonrtolerance`, and `from` (`init_specs`); see [R] [Maximize](#). These options are seldom used.

Setting the optimization type to `technique` (`bhhh`) resets the default `vcetype` to `vce(opg)`.

The following options are available with `frontier` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Stochastic production frontier models were introduced by [Aigner, Lovell, and Schmidt \(1977\)](#) and [Meeusen and van den Broeck \(1977\)](#). Since then, stochastic frontier models have become a popular subfield in econometrics. [Kumbhakar and Lovell \(2000\)](#) provide a good introduction.

`frontier` fits three stochastic frontier models with distinct parameterizations of the inefficiency term and can fit stochastic production or cost frontier models.

Let's review the nature of the stochastic frontier problem. Suppose that a producer has a production function  $f(\mathbf{z}_i, \boldsymbol{\beta})$ . In a world without error or inefficiency, the  $i$ th firm would produce

$$q_i = f(\mathbf{z}_i, \boldsymbol{\beta})$$

Stochastic frontier analysis assumes that each firm potentially produces less than it might due to a degree of inefficiency. Specifically,

$$q_i = f(\mathbf{z}_i, \boldsymbol{\beta})\xi_i$$

where  $\xi_i$  is the level of efficiency for firm  $i$ ;  $\xi_i$  must be in the interval  $(0, 1]$ . If  $\xi_i = 1$ , the firm is achieving the optimal output with the technology embodied in the production function  $f(\mathbf{z}_i, \boldsymbol{\beta})$ . When  $\xi_i < 1$ , the firm is not making the most of the inputs  $\mathbf{z}_i$  given the technology embodied in the production function  $f(\mathbf{z}_i, \boldsymbol{\beta})$ . Because the output is assumed to be strictly positive (that is,  $q_i > 0$ ), the degree of technical efficiency is assumed to be strictly positive (that is,  $\xi_i > 0$ ).

Output is also assumed to be subject to random shocks, implying that

$$q_i = f(\mathbf{z}_i, \boldsymbol{\beta})\xi_i \exp(v_i)$$

Taking the natural log of both sides yields

$$\ln(q_i) = \ln\{f(\mathbf{z}_i, \boldsymbol{\beta})\} + \ln(\xi_i) + v_i$$

Assuming that there are  $k$  inputs and that the production function is linear in logs, defining  $u_i = -\ln(\xi_i)$  yields

$$\ln(q_i) = \beta_0 + \sum_{j=1}^k \beta_j \ln(z_{ji}) + v_i - u_i$$

Because  $u_i$  is subtracted from  $\ln(q_i)$ , restricting  $u_i \geq 0$  implies that  $0 < \xi_i \leq 1$ , as specified above.

Kumbhakar and Lovell (2000) provide a detailed version of the above derivation, and they show that performing an analogous derivation in the dual cost function problem allows us to specify the problem as

$$\ln(c_i) = \beta_0 + \beta_q \ln(q_i) + \sum_{j=1}^k \beta_j \ln(p_{ji}) + v_i + u_i$$

where  $q_i$  is output,  $z_{ji}$  are input quantities,  $c_i$  is cost, and the  $p_{ji}$  are input prices.

Intuitively, the inefficiency effect is required to lower output or raise expenditure, depending on the specification.

□ Technical note

The model that `frontier` actually fits is of the form

$$y_i = \beta_0 + \sum_{j=1}^k \beta_j x_{ji} + v_i - s u_i$$

where

$$s = \begin{cases} 1, & \text{for production functions} \\ -1, & \text{for cost functions} \end{cases}$$

so, in the context of the discussion above,  $y_i = \ln(q_i)$ , and  $x_{ji} = \ln(z_{ji})$  for a production function; and for a cost function,  $y_i = \ln(c_i)$ , and the  $x_{ji}$  are the  $\ln(p_{ji})$  and  $\ln(q_i)$ . You must take the natural logarithm of the data before fitting a stochastic frontier production or cost model. `frontier` performs no transformations on the data.

□

Different specifications of the  $u_i$  and the  $v_i$  terms give rise to distinct models. `frontier` provides estimators for the parameters of three basic models in which the idiosyncratic component,  $v_i$ , is assumed to be independently  $N(0, \sigma_v)$  distributed over the observations. The basic models differ in their specification of the inefficiency term,  $u_i$ , as follows:

`exponential`: the  $u_i$  are independently exponentially distributed with variance  $\sigma_u^2$

`hnormal`: the  $u_i$  are independently half-normally  $N^+(0, \sigma_u^2)$  distributed

`tnormal`: the  $u_i$  are independently  $N^+(\mu, \sigma_u^2)$  distributed with truncation point at 0

For half-normal or exponential distributions, `frontier` can fit models with heteroskedastic error components, conditional on a set of covariates. For a truncated-normal distribution, `frontier` can also fit a conditional mean model in which the mean is modeled as a linear function of a set of covariates.

## ▶ Example 1: The half-normal and the exponential models

For our first example, we demonstrate the half-normal and exponential models by reproducing a study found in [Greene \(2003, 505\)](#), which uses data originally published in [Zellner and Revankar \(1969\)](#). In this study of the transportation equipment manufacturing industry, observations on value added, capital, and labor are used to estimate a Cobb–Douglas production function. The variable `lnv` is the log-transformed value added, `lnk` is the log-transformed capital, and `lnl` is the log-transformed labor. OLS estimates are compared with those from stochastic frontier models using both the half-normal and exponential distribution for the inefficiency term.

```
. use https://www.stata-press.com/data/r19/greene9
(Transportation equipment manufacturing industry)
. regress lnv lnk ln1
```

| Source   | SS         | df | MS         | Number of obs | = | 25     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 44.1727741 | 2  | 22.086387  | F(2, 22)      | = | 397.54 |
| Residual | 1.22225984 | 22 | .055557265 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.9731 |
|          |            |    |            | Adj R-squared | = | 0.9706 |
| Total    | 45.3950339 | 24 | 1.89145975 | Root MSE      | = | .23571 |

```
. regress lnv lnk ln1
```

| lnv   | Coefficient | Std. err. | t    | P> t  | [95% conf. interval] |          |
|-------|-------------|-----------|------|-------|----------------------|----------|
| lnk   | .2454281    | .1068574  | 2.30 | 0.032 | .0238193             | .4670368 |
| ln1   | .805183     | .1263336  | 6.37 | 0.000 | .5431831             | 1.067183 |
| _cons | 1.844416    | .2335928  | 7.90 | 0.000 | 1.359974             | 2.328858 |

```
. frontier lnv lnk ln1
Iteration 0: Log likelihood = 2.3357572
Iteration 1: Log likelihood = 2.4673009
Iteration 2: Log likelihood = 2.4695125
Iteration 3: Log likelihood = 2.4695222
Iteration 4: Log likelihood = 2.4695222
Stoc. frontier normal/half-normal model
Log likelihood = 2.4695222
Number of obs = 25
Wald chi2(2) = 743.71
Prob > chi2 = 0.0000
```

| lnv      | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|----------|-------------|-----------|-------|-------|----------------------|-----------|
| lnk      | .2585478    | .098764   | 2.62  | 0.009 | .0649738             | .4521218  |
| ln1      | .7802451    | .1199399  | 6.51  | 0.000 | .5451672             | 1.015323  |
| _cons    | 2.081135    | .281641   | 7.39  | 0.000 | 1.529128             | 2.633141  |
| /lnsig2v | -3.48401    | .6195353  | -5.62 | 0.000 | -4.698277            | -2.269743 |
| /lnsig2u | -3.014599   | 1.11694   | -2.70 | 0.007 | -5.203761            | -.8254368 |
| sigma_v  | .1751688    | .0542616  |       |       | .0954514             | .3214633  |
| sigma_u  | .2215073    | .1237052  |       |       | .074134              | .6618486  |
| sigma2   | .0797496    | .0426989  |       |       | -.0039388            | .163438   |
| lambda   | 1.264536    | .1678684  |       |       | .9355204             | 1.593552  |

```
LR test of sigma_u=0: chibar2(01) = 0.43
Prob >= chibar2 = 0.256
. predict double u_h, u
```

. frontier lnv lnk ln1, distribution(exponential)

Iteration 0: Log likelihood = 2.7270659  
 Iteration 1: Log likelihood = 2.8551532  
 Iteration 2: Log likelihood = 2.8604815  
 Iteration 3: Log likelihood = 2.8604897  
 Iteration 4: Log likelihood = 2.8604897

Stoc. frontier normal/exponential model Number of obs = 25  
Wald chi2(2) = 845.68  
Prob > chi2 = 0.0000  
 Log likelihood = 2.8604897

| lnv      | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|----------|-------------|-----------|-------|-------|----------------------|-----------|
| lnk      | .2624859    | .0919988  | 2.85  | 0.004 | .0821717             | .4428002  |
| ln1      | .7703795    | .1109569  | 6.94  | 0.000 | .5529079             | .9878511  |
| _cons    | 2.069242    | .2356159  | 8.78  | 0.000 | 1.607444             | 2.531041  |
| /lnsig2v | -3.527598   | .4486176  | -7.86 | 0.000 | -4.406873            | -2.648324 |
| /lnsig2u | -4.002457   | .9274575  | -4.32 | 0.000 | -5.820241            | -2.184674 |
| sigma_v  | .1713925    | .0384448  |       |       | .1104231             | .2660258  |
| sigma_u  | .1351691    | .0626818  |       |       | .0544692             | .3354317  |
| sigma2   | .0476461    | .0157921  |       |       | .016694              | .0785981  |
| lambda   | .7886525    | .087684   |       |       | .616795              | .9605101  |

LR test of sigma\_u=0: chibar2(01) = 1.21 Prob >= chibar2 = 0.135

. predict double u\_e, u  
 . list state u\_h u\_e

|     | state         | u_h       | u_e       |
|-----|---------------|-----------|-----------|
| 1.  | Alabama       | .2011338  | .14592865 |
| 2.  | California    | .14480966 | .0972165  |
| 3.  | Connecticut   | .1903485  | .13478797 |
| 4.  | Florida       | .51753139 | .5903303  |
| 5.  | Georgia       | .10397912 | .07140994 |
| 6.  | Illinois      | .12126696 | .0830415  |
| 7.  | Indiana       | .21128212 | .15450664 |
| 8.  | Iowa          | .24933153 | .20073081 |
| 9.  | Kansas        | .10099517 | .06857629 |
| 10. | Kentucky      | .05626919 | .04152443 |
| 11. | Louisiana     | .20332731 | .15066405 |
| 12. | Maine         | .22263164 | .17245793 |
| 13. | Maryland      | .13534062 | .09245501 |
| 14. | Massachusetts | .15636999 | .10932923 |
| 15. | Michigan      | .15809566 | .10756915 |
| 16. | Missouri      | .10288047 | .0704146  |
| 17. | NewJersey     | .09584337 | .06587986 |
| 18. | NewYork       | .27787793 | .22249416 |
| 19. | Ohio          | .22914231 | .16981857 |
| 20. | Pennsylvania  | .1500667  | .10302905 |
| 21. | Texas         | .20297875 | .14552218 |
| 22. | Virginia      | .14000132 | .09676078 |
| 23. | Washington    | .11047581 | .07533251 |
| 24. | WestVirginia  | .15561392 | .11236153 |
| 25. | Wisconsin     | .14067066 | .0970861  |

The parameter estimates and the estimates of the inefficiency terms closely match those published in [Greene \(2003, 505\)](#), but the standard errors of the parameter estimates are estimated differently (see the technical note below).

The output from `frontier` includes estimates of the standard deviations of the two error components,  $\sigma_v$  and  $\sigma_u$ , which are labeled `sigma_v` and `sigma_u`, respectively. In the log likelihood, they are parameterized as  $\ln\sigma_v^2$  and  $\ln\sigma_u^2$ , and these estimates are labeled `/lnsig2v` and `/lnsig2u` in the output. `frontier` also reports two other useful parameterizations. The estimate of the total error variance,  $\sigma_S^2 = \sigma_v^2 + \sigma_u^2$ , is labeled `sigma2`, and the estimate of the ratio of the standard deviation of the inefficiency component to the standard deviation of the idiosyncratic component,  $\lambda = \sigma_u/\sigma_v$ , is labeled `lambda`.

At the bottom of the output, `frontier` reports the results of a test that there is no technical inefficiency component in the model. This is a test of the null hypothesis  $H_0 : \sigma_u^2 = 0$  against the alternative hypotheses  $H_1 : \sigma_u^2 > 0$ . If the null hypothesis is true, the stochastic frontier model reduces to an OLS model with normal errors. However, because the test lies on the boundary of the parameter space of  $\sigma_u^2$ , the standard likelihood-ratio test is not valid, and a one-sided generalized likelihood-ratio test must be constructed; see [Gutierrez, Carter, and Drukker \(2001\)](#). For this example, the output shows LR = 0.43 with a  $p$ -value of 0.256 for the half-normal model and LR = 1.21 with a  $p$ -value of 0.135 for the exponential model. There are several possible reasons for the failure to reject the null hypothesis, but the fact that the test is based on an asymptotic distribution and the sample size was 25 is certainly a leading candidate among those possibilities.

◀

## □ Technical note

`frontier` maximizes the log-likelihood function of a stochastic frontier model by using the Newton–Raphson method, and the estimated variance–covariance matrix is calculated as the inverse of the negative Hessian (matrix of second partial derivatives); see [\[R\] ml](#). When comparing the results with those published using other software, be aware of the difference in the optimization methods, which may result in different, yet asymptotically equivalent, variance estimates.

□

## ▷ Example 2: Models with heteroskedasticity

Often the error terms may not have constant variance. `frontier` allows you to model heteroskedasticity in either error term as a linear function of a set of covariates. The variance of either the technical inefficiency or the idiosyncratic component may be modeled as

$$\sigma_i^2 = \exp(\mathbf{w}_i\boldsymbol{\delta})$$

The default constant included in  $\mathbf{w}_i$  may be suppressed by appending a `noconstant` option to the list of covariates. Also, you can simultaneously specify covariates for both  $\sigma_{u_i}$  and  $\sigma_{v_i}$ .

In this example, we use a sample of 756 observations of fictional firms producing a manufactured good by using capital and labor. The firms are hypothesized to use a constant returns-to-scale technology, but the sizes of the firms differ. Believing that this size variation will introduce heteroskedasticity into the idiosyncratic error term, we estimate the parameters of a Cobb–Douglas production function. To do this, we use a conditional heteroskedastic half-normal model, with the size of the firm as an explanatory variable in the variance function for the idiosyncratic error. We also perform a test of the hypothesis that the firms use a constant returns-to-scale technology.

```

. use https://www.stata-press.com/data/r19/frontier1, clear
. frontier lnoutput lnlabor lncapital, vhet(size)
Iteration 0: Log likelihood = -1508.3692
Iteration 1: Log likelihood = -1501.583
Iteration 2: Log likelihood = -1500.3942
Iteration 3: Log likelihood = -1500.3794
Iteration 4: Log likelihood = -1500.3794

Stoc. frontier normal/half-normal model Number of obs = 756
 Wald chi2(2) = 9.68
Log likelihood = -1500.3794 Prob > chi2 = 0.0079

```

| lnoutput  | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|-----------|-------------|-----------|-------|-------|----------------------|-----------|
| lnoutput  |             |           |       |       |                      |           |
| lnlabor   | .7090933    | .2349374  | 3.02  | 0.003 | .2486244             | 1.169562  |
| lncapital | .3931345    | .5422173  | 0.73  | 0.468 | -.6695919            | 1.455861  |
| _cons     | 1.252199    | 3.14656   | 0.40  | 0.691 | -4.914946            | 7.419344  |
| lnsig2v   |             |           |       |       |                      |           |
| size      | -.0016951   | .0004748  | -3.57 | 0.000 | -.0026256            | -.0007645 |
| _cons     | 3.156091    | .9265826  | 3.41  | 0.001 | 1.340023             | 4.97216   |
| lnsig2u   |             |           |       |       |                      |           |
| _cons     | 1.947487    | .1017653  | 19.14 | 0.000 | 1.748031             | 2.146943  |
| sigma_u   | 2.647838    | .134729   |       |       | 2.396514             | 2.925518  |

```

. test _b[lnlabor] + _b[lncapital] = 1
(1) [lnoutput]lnlabor + [lnoutput]lncapital = 1
 chi2(1) = 0.03
 Prob > chi2 = 0.8622

```

The output above indicates that the variance of the idiosyncratic error term is a function of firm size. Also, we failed to reject the hypothesis that the firms use a constant returns-to-scale technology.



## □ Technical note

In small samples, the conditional heteroskedastic estimators will lack precision for the variance parameters and may fail to converge altogether.



## ▷ Example 3: The truncated-normal model

Let's turn our attention to the truncated-normal model. Once again, we will use fictional data. For this example, we have 1,231 observations on the quantity of output, the total cost of production for each firm, the prices that each firm paid for labor and capital services, and a categorical variable measuring the quality of each firm's management. After taking the natural logarithm of the costs ( $\ln\text{cost}$ ), prices ( $\ln p_k$  and  $\ln p_l$ ), and output ( $\ln\text{out}$ ), we fit a stochastic cost frontier model and specify the distribution for the inefficiency term to be truncated normal.

```

. use https://www.stata-press.com/data/r19/frontier2
. frontier lncost lnp_k lnp_l lnout, distribution(tnormal) cost
Iteration 0: Log likelihood = -2386.9523
Iteration 1: Log likelihood = -2386.5146
Iteration 2: Log likelihood = -2386.2704
Iteration 3: Log likelihood = -2386.2504
Iteration 4: Log likelihood = -2386.2493
Iteration 5: Log likelihood = -2386.2493
Stoc. frontier normal/truncated-normal model Number of obs = 1,231
 Wald chi2(3) = 8.82
Log likelihood = -2386.2493 Prob > chi2 = 0.0318

```

| lncost    | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |          |
|-----------|-------------|-----------|------|-------|----------------------|----------|
| lnp_k     | .3410717    | .2363861  | 1.44 | 0.149 | -.1222366            | .80438   |
| lnp_l     | .6608628    | .4951499  | 1.33 | 0.182 | -.3096131            | 1.631339 |
| lnout     | .7528653    | .3468968  | 2.17 | 0.030 | .0729601             | 1.432771 |
| _cons     | 2.602609    | 1.083004  | 2.40 | 0.016 | .4799595             | 4.725259 |
| /mu       | 1.095705    | .881517   | 1.24 | 0.214 | -.632037             | 2.823446 |
| /lnsigma2 | 1.5534      | .1873464  | 8.29 | 0.000 | 1.186208             | 1.920592 |
| /lgtgamma | 1.257862    | .2589522  | 4.86 | 0.000 | .7503255             | 1.765399 |
| sigma2    | 4.727518    | .8856833  |      |       | 3.274641             | 6.825001 |
| gamma     | .7786579    | .0446303  |      |       | .6792496             | .8538846 |
| sigma_u2  | 3.681119    | .7503408  |      |       | 2.210478             | 5.15176  |
| sigma_v2  | 1.046399    | .2660035  |      |       | .5250413             | 1.567756 |

HO: No inefficiency component: z = 5.595 Prob >= z = 0.0000

In addition to the coefficients, the output reports estimates for several parameters. `sigma_v2` is the estimate of  $\sigma_v^2$ . `sigma_u2` is the estimate of  $\sigma_u^2$ . `gamma` is the estimate of  $\gamma = \sigma_u^2 / \sigma_S^2$ . `sigma2` is the estimate of  $\sigma_S^2 = \sigma_v^2 + \sigma_u^2$ . Because  $\gamma$  must be between 0 and 1, the optimization is parameterized in terms of the logit of  $\gamma$ , and this estimate is reported as `lgtgamma`. Because  $\sigma_S^2$  must be positive, the optimization is parameterized in terms of  $\ln(\sigma_S^2)$ , whose estimate is reported as `lnsigma2`. Finally, `mu` is the estimate of  $\mu$ , the mean of the truncated-normal distribution.

In the output above, the generalized log-likelihood test for the presence of the inefficiency term has been replaced with a test based on the third moment of the OLS residuals. When  $\mu = 0$  and  $\sigma_u = 0$ , the truncated-normal model reduces to a linear regression model with normally distributed errors. However, the distribution of the test statistic under the null hypothesis is not well established, because it becomes impossible to evaluate the log likelihood as  $\sigma_u$  approaches zero, prohibiting the use of the likelihood-ratio test.

However, [Coelli \(1995\)](#) noted that the presence of an inefficiency term would negatively skew the residuals from an OLS regression. By identifying negative skewness in the residuals with the presence of an inefficiency term, Coelli derived a one-sided test for the presence of the inefficiency term. The results of this test are given at the bottom of the output. For this example, the null hypothesis of no inefficiency component is rejected.

In the example below, we fit a truncated model and detect a statistically significant inefficiency term in the model. We might question whether the inefficiency term is identically distributed over all firms or whether there might be heterogeneity across firms. `frontier` provides an extension to the truncated normal model by allowing the mean of the inefficiency term to be modeled as a linear function of a set of covariates. In our dataset, we have a categorical variable that measures the quality of a firm’s management. We refit the model, including the `cm()` option, specifying a set of binary indicator variables representing the different categories of the quality-measurement variable as covariates.

```
. frontier lncost lnp_k lnp_l lnout, distribution(tnormal) cm(i.quality) cost
Iteration 0: Log likelihood = -2386.9523
Iteration 1: Log likelihood = -2384.936
Iteration 2: Log likelihood = -2382.3942
Iteration 3: Log likelihood = -2382.324
Iteration 4: Log likelihood = -2382.3233
Iteration 5: Log likelihood = -2382.3233
Stoc. frontier normal/truncated-normal model Number of obs = 1,231
Log likelihood = -2382.3233 Wald chi2(3) = 9.31
 Prob > chi2 = 0.0254
```

|               | lncost    | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |          |
|---------------|-----------|-------------|-----------|------|-------|----------------------|----------|
| <b>lncost</b> |           |             |           |      |       |                      |          |
|               | lnp_k     | .3611204    | .2359749  | 1.53 | 0.126 | -.1013819            | .8236227 |
|               | lnp_l     | .680446     | .4934935  | 1.38 | 0.168 | -.2867835            | 1.647675 |
|               | lnout     | .7605533    | .3466102  | 2.19 | 0.028 | .0812098             | 1.439897 |
|               | _cons     | 2.550769    | 1.078911  | 2.36 | 0.018 | .4361417             | 4.665396 |
| <b>mu</b>     |           |             |           |      |       |                      |          |
|               | quality   |             |           |      |       |                      |          |
|               | 2         | .5056067    | .3382907  | 1.49 | 0.135 | -.1574309            | 1.168644 |
|               | 3         | .783223     | .376807   | 2.08 | 0.038 | .0446947             | 1.521751 |
|               | 4         | .5577511    | .3355061  | 1.66 | 0.096 | -.0998288            | 1.215331 |
|               | 5         | .6792882    | .3428073  | 1.98 | 0.048 | .0073981             | 1.351178 |
|               | _cons     | .6014025    | .990167   | 0.61 | 0.544 | -1.339289            | 2.542094 |
|               | /lnsigma2 | 1.541784    | .1790926  | 8.61 | 0.000 | 1.190769             | 1.892799 |
|               | /lgtgamma | 1.242302    | .2588968  | 4.80 | 0.000 | .734874              | 1.749731 |
|               | sigma2    | 4.67292     | .8368852  |      |       | 3.289611             | 6.637923 |
|               | gamma     | .7759645    | .0450075  |      |       | .6758739             | .8519189 |
|               | sigma_u2  | 3.62602     | .7139576  |      |       | 2.226689             | 5.025351 |
|               | sigma_v2  | 1.0469      | .2583469  |      |       | .5405491             | 1.553251 |

The conditional mean model was developed in the context of panel-data estimators, and we can apply `frontier`’s conditional mean model to panel data.



## Stored results

`frontier` stores the following in `e()`:

### Scalars

|                            |                                              |
|----------------------------|----------------------------------------------|
| <code>e(N)</code>          | number of observations                       |
| <code>e(df_m)</code>       | model degrees of freedom                     |
| <code>e(k)</code>          | number of parameters                         |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>     |
| <code>e(k_eq_model)</code> | number of equations in overall model test    |
| <code>e(k_dv)</code>       | number of dependent variables                |
| <code>e(chi2)</code>       | $\chi^2$                                     |
| <code>e(ll)</code>         | log likelihood                               |
| <code>e(ll_c)</code>       | log likelihood for $H_0 : \sigma_u = 0$      |
| <code>e(z)</code>          | test for negative skewness of OLS residuals  |
| <code>e(sigma_u)</code>    | standard deviation of technical inefficiency |
| <code>e(sigma_v)</code>    | standard deviation of $v_i$                  |
| <code>e(p)</code>          | $p$ -value for model test                    |
| <code>e(chi2_c)</code>     | LR test statistic                            |
| <code>e(p_z)</code>        | $p$ -value for $z$                           |
| <code>e(rank)</code>       | rank of <code>e(V)</code>                    |
| <code>e(ic)</code>         | number of iterations                         |
| <code>e(rc)</code>         | return code                                  |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise                  |

### Macros

|                            |                                                                          |
|----------------------------|--------------------------------------------------------------------------|
| <code>e(cmd)</code>        | <code>frontier</code>                                                    |
| <code>e(cmdline)</code>    | command as typed                                                         |
| <code>e(depvar)</code>     | name of dependent variable                                               |
| <code>e(function)</code>   | production or cost                                                       |
| <code>e(wtype)</code>      | weight type                                                              |
| <code>e(wexp)</code>       | weight expression                                                        |
| <code>e(title)</code>      | title in estimation output                                               |
| <code>e(chi2type)</code>   | Wald; type of model $\chi^2$ test                                        |
| <code>e(dist)</code>       | distribution assumption for $u_i$                                        |
| <code>e(het)</code>        | heteroskedastic components                                               |
| <code>e(u_hetvar)</code>   | <i>varlist</i> in <code>uhet()</code>                                    |
| <code>e(v_hetvar)</code>   | <i>varlist</i> in <code>vhet()</code>                                    |
| <code>e(vce)</code>        | <i>vcetype</i> specified in <code>vce()</code>                           |
| <code>e(vcetype)</code>    | title used to label Std. err.                                            |
| <code>e(opt)</code>        | type of optimization                                                     |
| <code>e(which)</code>      | max or min; whether optimizer is to perform maximization or minimization |
| <code>e(ml_method)</code>  | type of ml method                                                        |
| <code>e(user)</code>       | name of likelihood-evaluator program                                     |
| <code>e(technique)</code>  | maximization technique                                                   |
| <code>e(properties)</code> | <code>b V</code>                                                         |
| <code>e(predict)</code>    | program used to implement <code>predict</code>                           |
| <code>e(asbalanced)</code> | factor variables <code>fvset</code> as <code>asbalanced</code>           |
| <code>e(asobserved)</code> | factor variables <code>fvset</code> as <code>asobserved</code>           |

### Matrices

|                              |                                              |
|------------------------------|----------------------------------------------|
| <code>e(b)</code>            | coefficient vector                           |
| <code>e(Cns)</code>          | constraints matrix                           |
| <code>e(ilog)</code>         | iteration log (up to 20 iterations)          |
| <code>e(gradient)</code>     | gradient vector                              |
| <code>e(V)</code>            | variance–covariance matrix of the estimators |
| <code>e(V_modelbased)</code> | model-based variance                         |

### Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

Matrices

`r(table)`

matrix containing the coefficients with their standard errors, test statistics,  $p$ -values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

Consider an equation of the form

$$y_i = \mathbf{x}_i\boldsymbol{\beta} + v_i - s u_i$$

where  $y_i$  is the dependent variable,  $\mathbf{x}_i$  is a  $1 \times k$  vector of observations on the independent variables included as indent covariates,  $\boldsymbol{\beta}$  is a  $k \times 1$  vector of coefficients, and

$$s = \begin{cases} 1, & \text{for production functions} \\ -1, & \text{for cost functions} \end{cases}$$

The log-likelihood functions are as follows.

Normal/half-normal model:

$$\ln L = \sum_{i=1}^N \left\{ \frac{1}{2} \ln \left( \frac{2}{\pi} \right) - \ln \sigma_S + \ln \Phi \left( -\frac{s\epsilon_i \lambda}{\sigma_S} \right) - \frac{\epsilon_i^2}{2\sigma_S^2} \right\}$$

Normal/exponential model:

$$\ln L = \sum_{i=1}^N \left\{ -\ln \sigma_u + \frac{\sigma_v^2}{2\sigma_u^2} + \ln \Phi \left( \frac{-s\epsilon_i - \frac{\sigma_v^2}{\sigma_u}}{\sigma_v} \right) + \frac{s\epsilon_i}{\sigma_u} \right\}$$

Normal/truncated-normal model:

$$\ln L = \sum_{i=1}^N \left\{ -\frac{1}{2} \ln (2\pi) - \ln \sigma_S - \ln \Phi \left( \frac{\mu}{\sigma_S \sqrt{\gamma}} \right) + \ln \Phi \left[ \frac{(1-\gamma)\mu - s\gamma\epsilon_i}{\{\sigma_S^2\gamma(1-\gamma)\}^{1/2}} \right] - \frac{1}{2} \left( \frac{\epsilon_i + s\mu}{\sigma_S} \right)^2 \right\}$$

where  $\sigma_S = (\sigma_u^2 + \sigma_v^2)^{1/2}$ ,  $\lambda = \sigma_u/\sigma_v$ ,  $\gamma = \sigma_u^2/\sigma_S^2$ ,  $\epsilon_i = y_i - \mathbf{x}_i\boldsymbol{\beta}$ , and  $\Phi(\cdot)$  is the cumulative distribution function of the standard normal distribution.

To obtain estimation for  $u_i$ , you can use either the mean or the mode of the conditional distribution  $f(u|\epsilon)$ .

$$E(u_i | \epsilon_i) = \mu_{*i} + \sigma_* \left\{ \frac{\phi(-\mu_{*i}/\sigma_*)}{\Phi(\mu_{*i}/\sigma_*)} \right\}$$

$$M(u_i | \epsilon_i) = \begin{cases} \mu_{*i} & \text{if } \mu_{*i} \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Then, the technical efficiency ( $s = 1$ ) or cost efficiency ( $s = -1$ ) will be estimated by

$$E_i = E \{ \exp(-su_i) | \epsilon_i \}$$

$$= \left\{ \frac{1 - \Phi(s\sigma_* - \mu_{*i}/\sigma_*)}{1 - \Phi(-\mu_{*i}/\sigma_*)} \right\} \exp \left( -s\mu_{*i} + \frac{1}{2}\sigma_*^2 \right)$$

where  $\mu_{*i}$  and  $\sigma_*$  are defined for the normal/half-normal model as

$$\mu_{*i} = -s\epsilon_i\sigma_u^2/\sigma_S^2$$

$$\sigma_* = \sigma_u\sigma_v/\sigma_S$$

for the normal/exponential model as

$$\mu_{*i} = -s\epsilon_i - \sigma_v^2/\sigma_u$$

$$\sigma_* = \sigma_v$$

and for the normal/truncated-normal model as

$$\mu_{*i} = \frac{-s\epsilon_i\sigma_u^2 + \mu\sigma_v^2}{\sigma_S^2}$$

$$\sigma_* = \sigma_u\sigma_v/\sigma_S$$

In the half-normal and exponential models, when heteroskedasticity is assumed, the standard deviations,  $\sigma_u$  or  $\sigma_v$ , will be replaced in the above equations by

$$\sigma_i^2 = \exp(\mathbf{w}_i\boldsymbol{\delta})$$

where  $\mathbf{w}$  is the vector of explanatory variables in the variance function.

In the conditional mean model, the mean parameter of the truncated normal distribution,  $\mu$ , is modeled as a linear combination of the set of covariates,  $\mathbf{w}$ .

$$\mu = \mathbf{w}_i\boldsymbol{\delta}$$

Therefore, the log-likelihood function can be rewritten as

$$\ln L = \sum_{i=1}^N \left[ -\frac{1}{2} \ln(2\pi) - \ln \sigma_S - \ln \Phi \left( \frac{\mathbf{w}_i \boldsymbol{\delta}}{\sqrt{\sigma_S^2 \gamma}} \right) + \ln \Phi \left\{ \frac{(1-\gamma) \mathbf{w}_i \boldsymbol{\delta} - s\gamma \epsilon_i}{\sqrt{\sigma_S^2 \gamma (1-\gamma)}} \right\} - \frac{1}{2} \left( \frac{\epsilon_i + s\mathbf{w}_i \boldsymbol{\delta}}{\sigma_S} \right)^2 \right]$$

The  $z$  test reported in the output of the truncated-normal model is a third-moment test developed by Coelli (1995) as an extension of a test previously developed by Pagan and Hall (1983). Coelli shows that under the null of normally distributed errors, the statistic

$$z = \frac{m_3}{\left( \frac{6m_2^3}{N} \right)^{1/2}}$$

has a standard normal distribution, where  $m_3$  is the third moment from the OLS regression. Because the residuals are either negatively skewed (production function) or positively skewed (cost function), a one-sided  $p$ -value is used.

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## Also see

- [R] **frontier postestimation** — Postestimation tools for frontier
- [R] **regress** — Linear regression
- [XT] **xtfrontier** — Stochastic frontier models for panel data
- [U] **20 Estimation and postestimation commands**

Postestimation commands    [predict](#)    [margins](#)    [Remarks and examples](#)    [Reference](#)  
 Also see

## Postestimation commands

The following postestimation commands are available after `frontier`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| <code>hausman</code>         | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| <code>linktest</code>        | link test for model specification                                                                                                       |
| <code>lrtest</code>          | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | predictions and their SEs, technical efficiency, etc.                                                                                   |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

# predict

## Description for predict

`predict` creates a new variable containing predictions such as linear predictions, standard errors, and estimates of technical efficiency.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic]
```

```
predict [type] stub* [if] [in], scores
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

Main

|                   |                                                                                                         |
|-------------------|---------------------------------------------------------------------------------------------------------|
| <code>xb</code>   | linear prediction; the default                                                                          |
| <code>stdp</code> | standard error of the prediction                                                                        |
| <code>u</code>    | estimates of minus the natural log of the technical efficiency via $E(u_i   \epsilon_i)$                |
| <code>m</code>    | estimates of minus the natural log of the technical efficiency via $M(u_i   \epsilon_i)$                |
| <code>te</code>   | estimates of the technical efficiency via $E\{\exp(-su_i)   \epsilon_i\}$                               |
|                   | $s = \begin{cases} 1, & \text{for production functions} \\ -1, & \text{for cost functions} \end{cases}$ |

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

## Options for predict

Main

`xb`, the default, calculates the linear prediction.

`stdp` calculates the standard error of the linear prediction.

`u` produces estimates of minus the natural log of the technical efficiency via  $E(u_i | \epsilon_i)$ .

`m` produces estimates of minus the natural log of the technical efficiency via  $M(u_i | \epsilon_i)$ .

`te` produces estimates of the technical efficiency via  $E\{\exp(-su_i) | \epsilon_i\}$ .

`scores` calculates equation-level score variables.

The first new variable will contain  $\partial \ln L / \partial (\mathbf{x}_i \boldsymbol{\beta})$ .

The second new variable will contain  $\partial \ln L / \partial (\ln \text{sig}2v)$ .

The third new variable will contain  $\partial \ln L / \partial (\ln \text{sig}2u)$ .

`scores` may not be specified after estimation with option `distribution(tnormal)`.

# margins

## Description for margins

`margins` estimates margins of response for linear predictions.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [options]
```

| <i>statistic</i>  | Description                           |
|-------------------|---------------------------------------|
| <code>xb</code>   | linear prediction; the default        |
| <code>stdp</code> | not allowed with <code>margins</code> |
| <code>u</code>    | not allowed with <code>margins</code> |
| <code>m</code>    | not allowed with <code>margins</code> |
| <code>te</code>   | not allowed with <code>margins</code> |

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [R] [margins](#).

## Remarks and examples

### ▷ Example 1

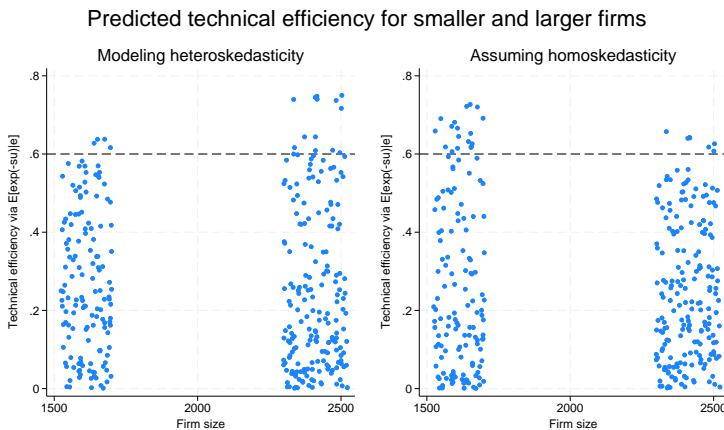
In [example 2](#) of [\[R\] frontier](#), we modeled heteroskedasticity by specifying the `vhet()` option. We would like to compare the predicted efficiency in that case with respect to a model specification without accounting for the presence of heteroskedasticity in the error term. [Kumbhakar and Lovell \(2000, 117\)](#) show that failing to account for heteroskedasticity associated with firm size may lead to bias in the estimation of the technical efficiency. By incorrectly assuming homoskedasticity, the estimates for relatively small firms would be biased upward, while the estimates for relatively large firms would be biased downward. Let's refit the model and use the `te` option of `predict`:

```
. use https://www.stata-press.com/data/r19/frontier1
. frontier lnoutput lnlabor lncapital, vhet(size)
 (output omitted)
. predict te_vhet, te
```

Next, we fit the model assuming homoskedasticity and then again predict the technical efficiency with the `te` option of `predict`:

```
. frontier lnoutput lnlabor lncapital
 (output omitted)
. predict te, te
```

The graph below shows the estimates for technical efficiency for the smaller and larger firms. The technical efficiency tends to be smaller for smaller firms when the model specification accounts for the presence of heteroskedasticity, whereas the predictions for the technical efficiency tends to be smaller for larger firms assuming homoskedasticity. These results agree with the theoretical statement in [Kumbhakar and Lovell \(2000\)](#) because the firm size was actually relevant to model heteroskedasticity in the idiosyncratic component of the error term.



## ▷ Example 2

We also test in [example 2](#) of [\[R\] frontier](#) whether the firms use constant returns to scale. We can use `lincom` as an alternative to perform an equivalent test based on the normal distribution.

```
. use https://www.stata-press.com/data/r19/frontier1, clear
. frontier lnoutput lnlabor lncapital, vhet(size)
 (output omitted)
. lincom _b[lnlabor] + _b[lncapital]-1
 (1) [lnoutput]lnlabor + [lnoutput]lncapital = 1
```

| lnoutput | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |          |
|----------|-------------|-----------|------|-------|----------------------|----------|
| (1)      | .1022278    | .5888511  | 0.17 | 0.862 | -1.051899            | 1.256355 |

The  $p$ -value is exactly the same as the one we obtained with the `test` command in [example 2](#) of [\[R\] frontier](#). However, notice that by using `lincom`, we obtained an estimate of the deviation from the constant returns-to-scale assumption, which is not significantly different from zero in this case.



## Reference

Kumbhakar, S. C., and C. A. K. Lovell. 2000. *Stochastic Frontier Analysis*. Cambridge: Cambridge University Press. <https://doi.org/10.1017/CBO9781139174411>.

## Also see

[\[R\] frontier](#) — Stochastic frontier models

[\[U\] 20 Estimation and postestimation commands](#)

## Description

`fvrevar` creates a variable list that includes equivalent, temporary variables in place of the factor variables, interactions, or time-series-operated variables in *varlist*. The resulting variable list can be used by commands that do not otherwise support factor variables or time-series-operated variables. The resulting list also could be used in a program to speed execution at the cost of using more memory.

## Quick start

Create temporary indicator variables for the levels of categorical variable `a` and store names in `r(varlist)`

```
fvrevar i.a
```

Create temporary variables corresponding to the levels of `a`, `b`, and their interaction

```
fvrevar i.a##i.b
```

Same as above, and create a temporary variable for the lag of `x` using `tsset` data

```
fvrevar i.a##i.b L.x
```

Return the list of unoperated variables (`a`, `b`, and `x`) in `r(varlist)`

```
fvrevar i.a##i.b L.x, list
```

Create new variables `a_1`, `a_2`, ..., corresponding to the levels of `a`

```
fvrevar i.a, stub(a_)
```

Create new variables `ab_1`, `ab_2`, ..., corresponding to the levels of the interaction between `a` and `b`

```
fvrevar i.a#i.b, stub(ab_)
```

## Syntax

```
fvrevar [varlist] [if] [in] [, substitute tsonly list stub(stub)]
```

You must `tsset` your data before using `fvrevar` if *varlist* contains time-series operators; see [TS] `tsset`. `collect` is allowed; see [U] 11.1.10 Prefix commands.

## Options

`substitute` specifies that equivalent, temporary variables be substituted for any factor variables, interactions, or time-series-operated variables in *varlist*. `substitute` is the default action taken by `fvrevar`; you do not need to specify the option.

`tsonly` specifies that equivalent, temporary variables be substituted for only the time-series-operated variables in *varlist*.

`list` specifies that all factor-variable operators and time-series operators be removed from *varlist* and the resulting list of base variables be returned in `r(varlist)`. No new variables are created with this option.

`stub(stub)` specifies that `fvrevar` generate named variables instead of temporary variables. The new variables will be named *stub#*.

## Remarks and examples

`fvrevar` might create no new variables, one new variable, or many new variables, depending on the number of factor variables, interactions, and time-series operators appearing in *varlist*. Any new variables created are temporary. The new, equivalent *varlist* is returned in `r(varlist)`. The new *varlist* corresponds one to one with the original *varlist*.

### ▷ Example 1

Typing

```
. use https://www.stata-press.com/data/r19/auto2
. fvrevar i.rep78 mpg turn
```

creates five temporary variables corresponding to the levels of `rep78`. No new variables are created for variables `mpg` and `turn` because they do not contain factor-variable or time-series operators.

The resulting variable list is

```
. display "r(varlist)"
__000000 __000001 __000002 __000003 __000004 mpg turn
```

(Your temporary variable names may be different, but that is of no consequence.)

Temporary variables automatically vanish when the program concludes.

## ▷ Example 2

Suppose we want to create temporary variables for specific levels of a factor variable. To do this, we can use the parenthesis notation of factor-variable syntax.

```
. fvrevar i(2,3)bn.rep78 mpg
```

creates two temporary variables corresponding to levels 2 and 3 of `rep78`. Notice that we specified that neither level 2 nor 3 be set as the base level by using the `bn` notation. If we did not specify `bn`, level 2 would have been treated as the base level.

The resulting variable list is

```
. display "r(varlist)"
__000005 __000002 mpg
```

We can see the results by listing the new variables alongside the original value of `rep78`.

```
. list rep78 'r(varlist)' in 1/5
```

|    | rep78   | __000005 | __000002 | mpg |
|----|---------|----------|----------|-----|
| 1. | Average | 0        | 1        | 22  |
| 2. | Average | 0        | 1        | 17  |
| 3. | .       | .        | .        | 22  |
| 4. | Average | 0        | 1        | 20  |
| 5. | Good    | 0        | 0        | 15  |

If we had needed only the base-variable names, we could have specified

```
. fvrevar i(2,3)bn.rep78 mpg, list
. display "r(varlist)"
mpg rep78
```

The order of the list will probably differ from that of the original list; base variables are listed only once.



## ▷ Example 3

Now let's assume we have `varlist` containing both an interaction and time-series-operated variables. If we want to create temporary variables for the entire equivalent `varlist`, we can specify `fvrevar` with no options.

```
. generate t = _n
. tsset t
 time variable: t, 1 to 74
 delta: 1 unit
. fvrevar c.turn#i(2,3).rep78 L.mpg
```

The resulting variable list is

```
. display "r(varlist)"
__000006 __000007 __000008
```

If we want to create temporary variables only for the time-series–operated variables, we can specify the `tsonly` option.

```
. fvrevar c.turn#i(2,3).rep78 L.mpg, tsonly
```

The resulting variable list is

```
. display "r(varlist)'"
2.rep78#c.turn 3.rep78#c.turn __000008
```

Notice that `fvrevar` returned the expanded factor-variable list with the `tsonly` option.



## □ Technical note

`fvrevar`, substitute avoids creating duplicate variables. Consider

```
. fvrevar i.rep78 turn mpg i.rep78
```

`i.rep78` appears twice in the `varlist`. `fvrevar` will create only one set of new variables for the five levels of `rep78` and will use these new variables once in the resulting `r(varlist)`. Moreover, `fvrevar` will do this even across multiple calls:

```
. fvrevar i.rep78 turn mpg
. fvrevar i.rep78
```

`i.rep78` appears in two separate calls. At the first call, `fvrevar` creates five temporary variables corresponding to the five levels of `rep78`. At the second call, `fvrevar` remembers what it has done and uses the same temporary variables for `i.rep78`.



## Stored results

`fvrevar` stores the following in `r()`:

```
Macros
 r(varlist) the modified variable list or list of base-variable names
```

## Also see

[TS] [tsrevar](#) — Time-series operator programming command

[P] [fvexpand](#) — Expand factor varlists

[P] [syntax](#) — Parse Stata syntax

[P] [unab](#) — Unabbreviate variable list

[U] [11 Language syntax](#)

[U] [11.4.4 Time-series varlists](#)

[U] [18 Programming Stata](#)

## Description

`fvset base`, `fvset design`, and `fvset clear` manage factor-variable settings, which identify the base level and specify how to accumulate statistics over levels. `fvset base` declares the base level for each specified variable; the default for factor variables without a declared base level is the lowest value. `fvset design` specifies how the `margins` command is to accumulate over the levels of a factor variable. `fvset clear` removes factor-variable settings for each variable in *varlist*. `fvset clear _all` removes all factor-variable settings from all variables.

`fvset report` reports the current factor-variable settings for each variable in *varlist*. `fvset` without arguments is a synonym for `fvset report`.

## Quick start

Set the base category of categorical variable `a1` to 3

```
fvset base 3 a1
```

Set the base category of `a2`, `a3`, and `a4` to each variable's largest observed value

```
fvset base last a2 a3 a4
```

Set the base category of `a5` to the most frequent category

```
fvset base frequent a5
```

Set `a6` to have no base category

```
fvset base none a6
```

Restore the default base category (first) for `a5`

```
fvset base default a5
```

Specify that `margins` should treat `a2` as though it is balanced

```
fvset design asbalanced a2
```

Clear factor-variable settings for `a2` to `a4`

```
fvset clear a2-a4
```

List factor-variable settings for all factor variables

```
fvset report
```

## Syntax

Declare base settings

```
fvset base base_spec varlist
```

Declare design settings

```
fvset design design_spec varlist
```

Clear the current settings

```
fvset clear varlist
```

Report the current settings

```
fvset report [varlist] [, base(base_spec) design(design_spec)]
```

| <i>base_spec</i>             | Description                     |
|------------------------------|---------------------------------|
| <code>default</code>         | default base                    |
| <code><u>f</u>irst</code>    | lowest level value; the default |
| <code><u>l</u>ast</code>     | highest level value             |
| <code><u>f</u>requent</code> | most frequent level value       |
| <code><u>n</u>one</code>     | no base                         |
| <code>#</code>               | nonnegative integer value       |

| <i>design_spec</i>                    | Description                                                 |
|---------------------------------------|-------------------------------------------------------------|
| <code>default</code>                  | default design                                              |
| <code><u>a</u>s<u>b</u>alanced</code> | accumulate using $1/k$ , $k$ = number of levels             |
| <code><u>a</u>s<u>o</u>bserved</code> | accumulate using observed relative frequencies; the default |

`collect` is allowed; see [U] 11.1.10 Prefix commands.

## Options

`base(base_spec)` restricts `fvset report` to report only the factor-variable settings for variables with the specified *base\_spec*.

`design(design_spec)` restricts `fvset report` to report only the factor-variable settings for variables with the specified *design\_spec*.

## Remarks and examples

### ▷ Example 1

Using `auto2.dta`, we include factor variable `i.rep78` in a regression:

```
. use https://www.stata-press.com/data/r19/auto2
(1978 automobile data)
. regress mpg i.rep78, baselevels
```

| Source   | SS         | df | MS         | Number of obs | = | 69     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 549.415777 | 4  | 137.353944 | F(4, 64)      | = | 4.91   |
| Residual | 1790.78712 | 64 | 27.9810488 | Prob > F      | = | 0.0016 |
|          |            |    |            | R-squared     | = | 0.2348 |
|          |            |    |            | Adj R-squared | = | 0.1869 |
| Total    | 2340.2029  | 68 | 34.4147485 | Root MSE      | = | 5.2897 |

| mpg       | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|-----------|-------------|-----------|-------|-------|----------------------|----------|
| rep78     |             |           |       |       |                      |          |
| Poor      | 0           | (base)    |       |       |                      |          |
| Fair      | -1.875      | 4.181884  | -0.45 | 0.655 | -10.22927            | 6.479274 |
| Average   | -1.566667   | 3.863059  | -0.41 | 0.686 | -9.284014            | 6.150681 |
| Good      | .6666667    | 3.942718  | 0.17  | 0.866 | -7.209818            | 8.543152 |
| Excellent | 6.363636    | 4.066234  | 1.56  | 0.123 | -1.759599            | 14.48687 |
| _cons     | 21          | 3.740391  | 5.61  | 0.000 | 13.52771             | 28.47229 |

We specified the `baselevels` option so that the base level would be included in the output. By default, the first level is the base level. We can change the base level to 2:

```
. fvset base 2 rep78
. regress mpg i.rep78, baselevels
```

| Source   | SS         | df | MS         | Number of obs | = | 69     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 549.415777 | 4  | 137.353944 | F(4, 64)      | = | 4.91   |
| Residual | 1790.78712 | 64 | 27.9810488 | Prob > F      | = | 0.0016 |
|          |            |    |            | R-squared     | = | 0.2348 |
|          |            |    |            | Adj R-squared | = | 0.1869 |
| Total    | 2340.2029  | 68 | 34.4147485 | Root MSE      | = | 5.2897 |

| mpg       | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|-----------|-------------|-----------|-------|-------|----------------------|----------|
| rep78     |             |           |       |       |                      |          |
| Poor      | 1.875       | 4.181884  | 0.45  | 0.655 | -6.479274            | 10.22927 |
| Fair      | 0           | (base)    |       |       |                      |          |
| Average   | .3083333    | 2.104836  | 0.15  | 0.884 | -3.896559            | 4.513226 |
| Good      | 2.541667    | 2.247695  | 1.13  | 0.262 | -1.948621            | 7.031954 |
| Excellent | 8.238636    | 2.457918  | 3.35  | 0.001 | 3.32838              | 13.14889 |
| _cons     | 19.125      | 1.870195  | 10.23 | 0.000 | 15.38886             | 22.86114 |

Let's set rep78 to have no base level and fit a cell-means regression:

```
. fvset base none rep78
. regress mpg i.rep78, noconstant
```

| Source   | SS         | df | MS         | Number of obs | = | 69     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 31824.2129 | 5  | 6364.84258 | F(5, 64)      | = | 227.47 |
| Residual | 1790.78712 | 64 | 27.9810488 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.9467 |
|          |            |    |            | Adj R-squared | = | 0.9426 |
| Total    | 33615      | 69 | 487.173913 | Root MSE      | = | 5.2897 |

| mpg       | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|-----------|-------------|-----------|-------|-------|----------------------|----------|
| rep78     |             |           |       |       |                      |          |
| Poor      | 21          | 3.740391  | 5.61  | 0.000 | 13.52771             | 28.47229 |
| Fair      | 19.125      | 1.870195  | 10.23 | 0.000 | 15.38886             | 22.86114 |
| Average   | 19.43333    | .9657648  | 20.12 | 0.000 | 17.504               | 21.36267 |
| Good      | 21.66667    | 1.246797  | 17.38 | 0.000 | 19.1759              | 24.15743 |
| Excellent | 27.36364    | 1.594908  | 17.16 | 0.000 | 24.17744             | 30.54983 |



### ▷ Example 2

By default, margins assumes that factor variables are to be treated as observed and accumulates a margin by using the observed relative frequencies of the factor levels or the sum of the weights if weights have been specified.

```
. regress mpg i.foreign
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 378.153515 | 1  | 378.153515 | F(1, 72)      | = | 13.18  |
| Residual | 2065.30594 | 72 | 28.6848048 | Prob > F      | = | 0.0005 |
|          |            |    |            | R-squared     | = | 0.1548 |
|          |            |    |            | Adj R-squared | = | 0.1430 |
| Total    | 2443.45946 | 73 | 33.4720474 | Root MSE      | = | 5.3558 |

| mpg     | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|---------|-------------|-----------|-------|-------|----------------------|----------|
| foreign |             |           |       |       |                      |          |
| Foreign | 4.945804    | 1.362162  | 3.63  | 0.001 | 2.230384             | 7.661225 |
| _cons   | 19.82692    | .7427186  | 26.70 | 0.000 | 18.34634             | 21.30751 |

```
. margins
Predictive margins Number of obs = 74
Model VCE: OLS
Expression: Linear prediction, predict()
```

|       | Delta-method |           | t     | P> t  | [95% conf. interval] |          |
|-------|--------------|-----------|-------|-------|----------------------|----------|
|       | Margin       | std. err. |       |       |                      |          |
| _cons | 21.2973      | .6226014  | 34.21 | 0.000 | 20.05616             | 22.53843 |

Let's set foreign to always accumulate using equal relative frequencies:

```
. fvset design asbalanced foreign
. regress mpg i.foreign
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 378.153515 | 1  | 378.153515 | F(1, 72)      | = | 13.18  |
| Residual | 2065.30594 | 72 | 28.6848048 | Prob > F      | = | 0.0005 |
|          |            |    |            | R-squared     | = | 0.1548 |
|          |            |    |            | Adj R-squared | = | 0.1430 |
| Total    | 2443.45946 | 73 | 33.4720474 | Root MSE      | = | 5.3558 |

| mpg     | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|---------|-------------|-----------|-------|-------|----------------------|----------|
| foreign |             |           |       |       |                      |          |
| Foreign | 4.945804    | 1.362162  | 3.63  | 0.001 | 2.230384             | 7.661225 |
| _cons   | 19.82692    | .7427186  | 26.70 | 0.000 | 18.34634             | 21.30751 |

```
. margins
Adjusted predictions Number of obs = 74
Model VCE: OLS
Expression: Linear prediction, predict()
At: foreign (asbalanced)
```

|       | Delta-method |           | t     | P> t  | [95% conf. interval] |          |
|-------|--------------|-----------|-------|-------|----------------------|----------|
|       | Margin       | std. err. |       |       |                      |          |
| _cons | 22.29983     | .6810811  | 32.74 | 0.000 | 20.94211             | 23.65754 |

Suppose that we issued the fvset design command earlier in our session and that we cannot remember which variables we set as asbalanced. We can retrieve this information by using the fvset report command:

```
. fvset report, design(asbalanced)
```

| Variable | Base | Design     |
|----------|------|------------|
| foreign  |      | asbalanced |

## □ Technical note

`margins` is aware of a factor variable's design setting only through the estimation results it is working with. The design setting is stored by the estimation command; thus changing the design setting between the estimation command and `margins` will have no effect. For example, the output from the following two calls to `margins` yields the same results:

```
. fvset clear foreign
. regress mpg i.foreign
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 378.153515 | 1  | 378.153515 | F(1, 72)      | = | 13.18  |
| Residual | 2065.30594 | 72 | 28.6848048 | Prob > F      | = | 0.0005 |
|          |            |    |            | R-squared     | = | 0.1548 |
|          |            |    |            | Adj R-squared | = | 0.1430 |
| Total    | 2443.45946 | 73 | 33.4720474 | Root MSE      | = | 5.3558 |

| mpg     | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|---------|-------------|-----------|-------|-------|----------------------|----------|
| foreign |             |           |       |       |                      |          |
| Foreign | 4.945804    | 1.362162  | 3.63  | 0.001 | 2.230384             | 7.661225 |
| _cons   | 19.82692    | .7427186  | 26.70 | 0.000 | 18.34634             | 21.30751 |

```
. margins
Predictive margins Number of obs = 74
Model VCE: OLS
Expression: Linear prediction, predict()
```

|       | Delta-method |           | t     | P> t  | [95% conf. interval] |          |
|-------|--------------|-----------|-------|-------|----------------------|----------|
|       | Margin       | std. err. |       |       |                      |          |
| _cons | 21.2973      | .6226014  | 34.21 | 0.000 | 20.05616             | 22.53843 |

```
. fvset design asbalanced foreign
. margins
Predictive margins Number of obs = 74
Model VCE: OLS
Expression: Linear prediction, predict()
```

|       | Delta-method |           | t     | P> t  | [95% conf. interval] |          |
|-------|--------------|-----------|-------|-------|----------------------|----------|
|       | Margin       | std. err. |       |       |                      |          |
| _cons | 21.2973      | .6226014  | 34.21 | 0.000 | 20.05616             | 22.53843 |

□

## Stored results

`fvset` stores the following in `r()`:

Macros

```
r(varlist) varlist
r(baselist) base setting for each variable in varlist
r(designlist) design setting for each variable in varlist
```

[Description](#)[Options](#)[Acknowledgments](#)[Quick start](#)[Remarks and examples](#)[References](#)[Menu](#)[Stored results](#)[Also see](#)[Syntax](#)[Methods and formulas](#)

## Description

`glm` fits generalized linear models. It can fit models by using either IRLS (maximum quasilielihood) or Newton–Raphson (maximum likelihood) optimization, which is the default.

See [\[U\] 27 Overview of Stata estimation commands](#) for a description of all of Stata’s estimation commands, several of which fit models that can also be fit using `glm`.

## Quick start

Model of  $y$  as a function of  $x$  when  $y$  is a proportion

```
glm y x, family(binomial)
```

Logit model of  $y$  events occurring in 15 trials as a function of  $x$

```
glm y x, family(binomial 15) link(logit)
```

Probit model of  $y$  events as a function of  $x$  using grouped data with group sizes  $n$

```
glm y x, family(binomial n) link(probit)
```

Model of discrete  $y$  with user-defined family `myfamily` and link `mylink`

```
glm y x, family(myfamily) link(mylink)
```

Bootstrap standard errors in a model of  $y$  as a function of  $x$  with a gamma family and log link

```
glm y x, family(gamma) link(log) vce(bootstrap)
```

## Menu

Statistics > Generalized linear models > Generalized linear models (GLM)

## Syntax

```
glm depvar [indepvars] [if] [in] [weight] [, options]
```

| <i>options</i>                            | Description                                                                                                                                                                                                            |
|-------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                              |                                                                                                                                                                                                                        |
| <u>family</u> ( <i>familyname</i> )       | distribution of <i>depvar</i> ; default is family( <i>gaussian</i> )                                                                                                                                                   |
| <u>link</u> ( <i>linkname</i> )           | link function; default is canonical link for family() specified                                                                                                                                                        |
| <b>Model 2</b>                            |                                                                                                                                                                                                                        |
| <u>noconstant</u>                         | suppress constant term                                                                                                                                                                                                 |
| <u>exposure</u> ( <i>varname</i> )        | include ln( <i>varname</i> ) in model with coefficient constrained to 1                                                                                                                                                |
| <u>offset</u> ( <i>varname</i> )          | include <i>varname</i> in model with coefficient constrained to 1                                                                                                                                                      |
| <u>constraints</u> ( <i>constraints</i> ) | apply specified linear constraints                                                                                                                                                                                     |
| <u>asis</u>                               | retain perfect predictor variables                                                                                                                                                                                     |
| <u>mu</u> ( <i>varname</i> )              | use <i>varname</i> as the initial estimate for the mean of <i>depvar</i>                                                                                                                                               |
| <u>init</u> ( <i>varname</i> )            | synonym for mu( <i>varname</i> )                                                                                                                                                                                       |
| <b>SE/Robust</b>                          |                                                                                                                                                                                                                        |
| <u>vce</u> ( <i>vcetype</i> )             | <i>vcetype</i> may be <u>oim</u> , <u>robust</u> , <u>cluster</u> <i>clustvar</i> , <u>eim</u> , <u>opg</u> , <u>bootstrap</u> , <u>jackknife</u> , <u>hac</u> <i>kernel</i> , <u>jackknifef1</u> , or <u>unbiased</u> |
| <u>vfactor</u> (#)                        | multiply variance matrix by scalar #                                                                                                                                                                                   |
| <u>disp</u> (#)                           | quasilikelihood multiplier                                                                                                                                                                                             |
| <u>scale</u> (x2   dev   #)               | set the scale parameter                                                                                                                                                                                                |
| <b>Reporting</b>                          |                                                                                                                                                                                                                        |
| <u>level</u> (#)                          | set confidence level; default is level(95)                                                                                                                                                                             |
| <u>eform</u>                              | report exponentiated coefficients                                                                                                                                                                                      |
| <u>nocnsreport</u>                        | do not display constraints                                                                                                                                                                                             |
| <u>display_options</u>                    | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling                                                                       |
| <b>Maximization</b>                       |                                                                                                                                                                                                                        |
| <u>ml</u>                                 | use maximum likelihood optimization; the default                                                                                                                                                                       |
| <u>irls</u>                               | use iterated, reweighted least-squares optimization of the deviance                                                                                                                                                    |
| <u>maximize_options</u>                   | control the maximization process; seldom used                                                                                                                                                                          |
| <u>fisher</u> (#)                         | use the Fisher scoring Hessian or expected information matrix (EIM)                                                                                                                                                    |
| <u>search</u>                             | search for good starting values                                                                                                                                                                                        |
| <u>noheader</u>                           | suppress header table from above coefficient table                                                                                                                                                                     |
| <u>notable</u>                            | suppress coefficient table                                                                                                                                                                                             |
| <u>nodisplay</u>                          | suppress the output; iteration log is still displayed                                                                                                                                                                  |
| <u>collinear</u>                          | keep collinear variables                                                                                                                                                                                               |
| <u>coeflegend</u>                         | display legend instead of statistics                                                                                                                                                                                   |

| <i>familyname</i>                                                                    | Description        |
|--------------------------------------------------------------------------------------|--------------------|
| <code>gaussian</code>                                                                | Gaussian (normal)  |
| <code>igaussian</code>                                                               | inverse Gaussian   |
| <code>binomial</code> [ <i>varname</i> <sub><i>N</i></sub>   # <sub><i>N</i></sub> ] | Bernoulli/binomial |
| <code>poisson</code>                                                                 | Poisson            |
| <code>nbinomial</code> [# <sub><i>k</i></sub>   ml]                                  | negative binomial  |
| <code>gamma</code>                                                                   | gamma              |

| <i>linkname</i>        | Description       |
|------------------------|-------------------|
| <code>identity</code>  | identity          |
| <code>log</code>       | log               |
| <code>logit</code>     | logit             |
| <code>probit</code>    | probit            |
| <code>cloglog</code>   | cloglog           |
| <code>power #</code>   | power             |
| <code>opower #</code>  | odds power        |
| <code>nbinomial</code> | negative binomial |
| <code>loglog</code>    | log–log           |
| <code>logc</code>      | log-complement    |

*indepvars* may contain factor variables; see [U] 11.4.3 Factor variables.

*depvar* and *indepvars* may contain time-series operators; see [U] 11.4.4 Time-series varlists.

`bayes`, `bayesboot`, `bootstrap`, `by`, `collect`, `fmm`, `fp`, `jackknife`, `mfp`, `mi estimate`, `nestreg`, `rolling`, `statsby`, `stepwise`, and `svy` are allowed; see [U] 11.1.10 Prefix commands. For more details, see [BAYES] `bayes: glm` and [FMM] `fmm: glm`.

`vce(bootstrap)`, `vce(jackknife)`, and `vce(jackknife1)` are not allowed with the `mi estimate` prefix; see [MI] `mi estimate`.

Weights are not allowed with the `bootstrap` prefix; see [R] `bootstrap`.

`aweight`s are not allowed with the `jackknife` prefix; see [R] `jackknife`.

`svy()`, `vfactor()`, `disp()`, `scale()`, `irls`, `fisher()`, `noheader`, `notable`, `nodisplay`, and `weights` are not allowed with the `svy` prefix; see [SVY] `svy`.

`fweights`, `aweight`s, `iweight`s, and `pweight`s are allowed; see [U] 11.1.6 `weight`.

`noheader`, `notable`, `nodisplay`, `collinear`, and `coeflegend` do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Options

Model

`family`(*familyname*) specifies the distribution of *depvar*; `family(gaussian)` is the default.

`link`(*linkname*) specifies the link function; the default is the canonical link for the `family()` specified (except for `family(nbinomial)`).

## Model 2

`noconstant`, `exposure(varname)`, `offset(varname)`, `constraints(constraints)`; see [R] [Estimation options](#). `constraints(constraints)` is not allowed with `irls`.

`asis` forces retention of perfect predictor variables and their associated, perfectly predicted observations and may produce instabilities in maximization; see [R] [probit](#). This option is allowed only with option `family(binomial)` with a denominator of 1.

`mu(varname)` specifies *varname* as the initial estimate for the mean of *depvar*. This option can be useful with models that experience convergence difficulties, such as `family(binomial)` models with power or odds-power links. `init(varname)` is a synonym.

## SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

In addition to the standard *vcetypes*, `glm` allows the following alternatives:

`vce(eim)` specifies that the EIM estimate of variance be used.

`vce(jackknife1)` specifies that the one-step jackknife estimate of variance be used.

`vce(hac kernel [#])` specifies that a heteroskedasticity- and autocorrelation-consistent (HAC) variance estimate be used. HAC refers to the general form for combining weighted matrices to form the variance estimate. There are three kernels built into `glm`. *kernel* is a user-written program or one of

`nwest` | `gallant` | `anderson`

`#` specifies the number of lags. If `#` is not specified,  $N - 2$  is assumed. If you wish to specify `vce(hac . . .)`, you must `tsset` your data before calling `glm`.

`vce(unbiased)` specifies that the unbiased sandwich estimate of variance be used.

`vfactor(#)` specifies a scalar by which to multiply the resulting variance matrix. This option allows you to match output with other packages, which may apply degrees of freedom or other small-sample corrections to estimates of variance.

`disp(#)` multiplies the variance of *depvar* by `#` and divides the deviance by `#`. The resulting distributions are members of the quasilielihood family. This option is allowed only with option `irls`.

`scale(x2 | dev | #)` overrides the default scale parameter. This option is allowed only with Hessian (information matrix) variance estimates.

By default, `scale(1)` is assumed for the discrete distributions (binomial, Poisson, and negative binomial), and `scale(x2)` is assumed for the continuous distributions (Gaussian, gamma, and inverse Gaussian).

`scale(x2)` specifies that the scale parameter be set to the Pearson  $\chi^2$  (or generalized  $\chi^2$ ) statistic divided by the residual degrees of freedom, which is recommended by [McCullagh and Nelder \(1989\)](#) as a good general choice for continuous distributions.

`scale(dev)` sets the scale parameter to the deviance divided by the residual degrees of freedom. This option provides an alternative to `scale(x2)` for continuous distributions and overdispersed or underdispersed discrete distributions. This option is allowed only with option `irls`.

`scale(#)` sets the scale parameter to `#`. For example, using `scale(1)` in `family(gamma)` models results in exponential-errors regression. Additional use of `link(log)` rather than the default `link(power -1)` for `family(gamma)` essentially reproduces Stata's `streg`, `dist(exp)` `nohr` command (see [ST] **streg**) if all the observations are uncensored.

---

#### Reporting

`level(#)`; see [R] **Estimation options**.

`eform` displays the exponentiated coefficients and corresponding standard errors and confidence intervals. For `family(binomial) link(logit)` (that is, logistic regression), exponentiation results are odds ratios; for `family(nbinomial) link(log)` (that is, negative binomial regression) and for `family(poisson) link(log)` (that is, Poisson regression), exponentiated coefficients are incidence-rate ratios.

`nocnsreport`; see [R] **Estimation options**.

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] **Estimation options**.

---

#### Maximization

`m1` requests that optimization be carried out using Stata's `m1` commands and is the default.

`irls` requests iterated, reweighted least-squares (IRLS) optimization of the deviance instead of Newton–Raphson optimization of the log likelihood. If the `irls` option is not specified, the optimization is carried out using Stata's `m1` commands, in which case all options of `m1 maximize` are also available.

*maximize\_options*: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] **Maximize**. These options are seldom used.

Setting the optimization method to `technique(bhhh)` resets the default `vcetype` to `vce(opg)`.

If option `irls` is specified, only *maximize\_options* `iterate()`, `nolog`, `trace`, and `ltolerance()` are allowed. With `irls` specified, the convergence criterion is satisfied when the absolute change in deviance from one iteration to the next is less than or equal to `ltolerance()`, where `ltolerance(1e-6)` is the default.

`fisher(#)` specifies the number of Newton–Raphson steps that should use the Fisher scoring Hessian or EIM before switching to the observed information matrix (OIM). This option is useful only for Newton–Raphson optimization (and not when using `irls`).

`search` specifies that the command search for good starting values. This option is useful only for Newton–Raphson optimization (and not when using `irls`).

The following options are available with `glm` but are not shown in the dialog box:

`noheader` suppresses the header information from the output. The coefficient table is still displayed.

`notable` suppresses the table of coefficients from the output. The header information is still displayed.

`nodisplay` suppresses the output. The iteration log is still displayed.

`collinear`, `coeflegend`; see [\[R\] Estimation options](#). `collinear` is not allowed with `irls`.

## Remarks and examples

Remarks are presented under the following headings:

*General use*

*Variance estimators*

*User-defined functions*

### General use

`glm` fits generalized linear models of  $y$  with covariates  $\mathbf{x}$ :

$$g\{E(y)\} = \mathbf{x}\boldsymbol{\beta}, \quad y \sim F$$

$g(\cdot)$  is called the link function, and  $F$  is the distributional family. Substituting various definitions for  $g(\cdot)$  and  $F$  results in a surprising array of models. For instance, if  $y$  is distributed as Gaussian (normal) and  $g(\cdot)$  is the identity function, we have

$$E(y) = \mathbf{x}\boldsymbol{\beta}, \quad y \sim \text{Normal}$$

or linear regression. If  $g(\cdot)$  is the logit function and  $y$  is distributed as Bernoulli, we have

$$\text{logit}\{E(y)\} = \mathbf{x}\boldsymbol{\beta}, \quad y \sim \text{Bernoulli}$$

or logistic regression. If  $g(\cdot)$  is the natural log function and  $y$  is distributed as Poisson, we have

$$\ln\{E(y)\} = \mathbf{x}\boldsymbol{\beta}, \quad y \sim \text{Poisson}$$

or Poisson regression, also known as the log-linear model. Other combinations are possible.

Although `glm` can be used to perform linear regression (and, in fact, does so by default), this regression should be viewed as an instructional feature; `regress` produces such estimates more quickly, and many postestimation commands are available to explore the adequacy of the fit; see [\[R\] regress](#) and [\[R\] regress postestimation](#).

In any case, you specify the link function by using the `link()` option and specify the distributional family by using `family()`. The available link functions are

| Link function         | glm option                   |
|-----------------------|------------------------------|
| identity              | <code>link(identity)</code>  |
| log                   | <code>link(log)</code>       |
| logit                 | <code>link(logit)</code>     |
| probit                | <code>link(probit)</code>    |
| complementary log-log | <code>link(cloglog)</code>   |
| odds power            | <code>link(opower #)</code>  |
| power                 | <code>link(power #)</code>   |
| negative binomial     | <code>link(nbinomial)</code> |
| log-log               | <code>link(loglog)</code>    |
| log-complement        | <code>link(logc)</code>      |

Define  $\mu = E(y)$  and  $\eta = g(\mu)$ , meaning that  $g(\cdot)$  maps  $E(y)$  to  $\eta = \mathbf{x}\beta + \text{offset}$ .

Link functions are defined as follows:

`identity` is defined as  $\eta = g(\mu) = \mu$ .

`log` is defined as  $\eta = \ln(\mu)$ .

`logit` is defined as  $\eta = \ln\{\mu/(1 - \mu)\}$ , the natural log of the odds.

`probit` is defined as  $\eta = \Phi^{-1}(\mu)$ , where  $\Phi^{-1}(\cdot)$  is the inverse Gaussian cumulative.

`cloglog` is defined as  $\eta = \ln\{-\ln(1 - \mu)\}$ .

`opower` is defined as  $\eta = [\{\mu/(1 - \mu)\}^n - 1]/n$ , the power of the odds. The function is generalized so that `link(opower 0)` is equivalent to `link(logit)`, the natural log of the odds.

`power` is defined as  $\eta = \mu^n$ . Specifying `link(power 1)` is equivalent to specifying `link(identity)`. The power function is generalized so that  $\mu^0 \equiv \ln(\mu)$ . Thus, `link(power 0)` is equivalent to `link(log)`. Negative powers are, of course, allowed.

`nbinomial` is defined as  $\eta = \ln\{\mu/(\mu + k)\}$ , where  $k = 1$  if `family(nbinomial)` is specified,  $k = \#_k$  if `family(nbinomial #_k)` is specified, and  $k$  is estimated via maximum likelihood if `family(nbinomial ml)` is specified.

`loglog` is defined as  $\eta = -\ln\{-\ln(\mu)\}$ .

`logc` is defined as  $\eta = \ln(1 - \mu)$ .

The available distributional families are

| Family             | glm option                     |
|--------------------|--------------------------------|
| Gaussian (normal)  | <code>family(gaussian)</code>  |
| inverse Gaussian   | <code>family(igaussian)</code> |
| Bernoulli/binomial | <code>family(binomial)</code>  |
| Poisson            | <code>family(poisson)</code>   |
| negative binomial  | <code>family(nbinomial)</code> |
| gamma              | <code>family(gamma)</code>     |

`family(normal)` is a synonym for `family(gaussian)`.

The binomial distribution can be specified as 1) `family(binomial)`, 2) `family(binomial #N)`, or 3) `family(binomial varnameN)`. In case 2, #<sub>N</sub> is the value of the binomial denominator *N*, the number of trials. Specifying `family(binomial 1)` is the same as specifying `family(binomial)`. In case 3, *varname<sub>N</sub>* is the variable containing the binomial denominator, allowing the number of trials to vary across observations.

The negative binomial distribution can be specified as 1) `family(nbinomial)`, 2) `family(nbinomial #k)`, or 3) `family(nbinomial ml)`. Omitting #<sub>k</sub> is equivalent to specifying `family(nbinomial 1)`. In case 3, the value of #<sub>k</sub> is estimated via maximum likelihood. The value #<sub>k</sub> enters the variance and deviance functions. Typical values range between 0.01 and 2; see the [technical note](#) below.

You do not have to specify both `family()` and `link()`; the default `link()` is the canonical link for the specified `family()` (except for `nbinomial`):

| Family                         | Default link                |
|--------------------------------|-----------------------------|
| <code>family(gaussian)</code>  | <code>link(identity)</code> |
| <code>family(igaussian)</code> | <code>link(power -2)</code> |
| <code>family(binomial)</code>  | <code>link(logit)</code>    |
| <code>family(poisson)</code>   | <code>link(log)</code>      |
| <code>family(nbinomial)</code> | <code>link(log)</code>      |
| <code>family(gamma)</code>     | <code>link(power -1)</code> |

If you specify both `family()` and `link()`, not all combinations make sense. You may choose from the following combinations:

|                   | identity | log | logit | probit | cloglog | power | opower | nbinomial | loglog | logc |
|-------------------|----------|-----|-------|--------|---------|-------|--------|-----------|--------|------|
| Gaussian          | x        | x   |       |        |         | x     |        |           |        |      |
| inverse Gaussian  | x        | x   |       |        |         | x     |        |           |        |      |
| binomial          | x        | x   | x     | x      | x       | x     | x      |           | x      | x    |
| Poisson           | x        | x   |       |        |         | x     |        |           |        |      |
| negative binomial | x        | x   |       |        |         | x     |        | x         |        |      |
| gamma             | x        | x   |       |        |         | x     |        |           |        |      |

### □ Technical note

Some `family()` and `link()` combinations result in models already fit by Stata. These are

| family()  | link()   | Options                                                      | Equivalent Stata command                                    |
|-----------|----------|--------------------------------------------------------------|-------------------------------------------------------------|
| gaussian  | identity | <i>nothing</i>   irls   irls vce(oim)                        | regress                                                     |
| gaussian  | identity | t( <i>var</i> ) vce(hac nwest #)<br>vfactor(# <sub>v</sub> ) | newey, t( <i>var</i> ) lag(#) (see <a href="#">note 1</a> ) |
| binomial  | cloglog  | <i>nothing</i>   irls vce(oim)                               | cloglog (see <a href="#">note 2</a> )                       |
| binomial  | probit   | <i>nothing</i>   irls vce(oim)                               | probit (see <a href="#">note 2</a> )                        |
| binomial  | logit    | <i>nothing</i>   irls   irls vce(oim)                        | logit or logistic (see <a href="#">note 3</a> )             |
| poisson   | log      | <i>nothing</i>   irls   irls vce(oim)                        | poisson (see <a href="#">note 3</a> )                       |
| nbinomial | log      | <i>nothing</i>   irls vce(oim)                               | nbreg (see <a href="#">note 4</a> )                         |
| gamma     | log      | scale(1)                                                     | streg, dist(exp) nohr (see <a href="#">note 5</a> )         |

Notes:

1. The variance factor  $\#_v$  should be set to  $n/(n - k)$ , where  $n$  is the number of observations and  $k$  the number of regressors. If the number of regressors is not specified, the estimated standard errors will, as a result, differ by this factor.
2. Because the link is not the canonical link for the binomial family, you must specify the `vce(oim)` option if using `irls` to get equivalent standard errors. If `irls` is used without `vce(oim)`, the regression coefficients will be the same but the standard errors will be only asymptotically equivalent. If no options are specified (*nothing*), `glm` will optimize using Newton–Raphson, making it equivalent to the other Stata command.  
See [R] [cloglog](#) and [R] [probit](#) for more details about these commands.
3. Because the canonical link is being used, the standard errors will be equivalent whether the EIM or the OIM estimator of variance is used.
4. Family negative binomial, log-link models—also known as negative binomial regression models—are used for data with an overdispersed Poisson distribution. Although `glm` can be used to fit such models, using Stata’s maximum likelihood `nbreg` command is probably better. In the GLM approach, you specify `family(nbinomial #k)` and then search for a  $\#_k$  that results in the deviance-based dispersion being 1. You can also specify `family(nbinomial ml)` to estimate  $\#_k$  via maximum likelihood, which will report the same value returned from `nbreg`. However, `nbreg` also reports a confidence interval for it; see [R] [nbreg](#) and Rogers (1993). Of course, `glm` allows links other than `log`, and for those links, including the canonical `nbinomial` link, you will need to use `glm`.
5. `glm` can be used to estimate parameters from exponential regressions, but this method requires specifying `scale(1)`. However, censoring is not available. Censored exponential regression may be modeled using `glm` with `family(poisson)`. The log of the original response is entered into a Poisson model as an offset, whereas the new response is the censor variable. The result of such modeling is identical to the log relative hazard parameterization of `streg`, `dist(exp) nohr`. See [ST] [streg](#) for details about the `streg` command.

In general, where there is overlap between a capability of `glm` and that of some other Stata command, we recommend using the other Stata command. Our recommendation is not because of some inferiority of the GLM approach. Rather, those other commands, by being specialized, provide options and ancillary commands that are missing in the broader `glm` framework. Nevertheless, `glm` does produce the same answers where it should.

*Special note.* When equivalence is expected, for some datasets, you may still see very slight differences in the results, most often only in the later digits of the standard errors. When you compare `glm` output to an equivalent Stata command, these tiny discrepancies arise for many reasons:

- a. `glm` uses a general methodology for starting values, whereas the equivalent Stata command may be more specialized in its treatment of starting values.
- b. When using a canonical link, `glm`, `irls` should be equivalent to the maximum likelihood method of the equivalent Stata command, yet the convergence criterion is different (one is for deviance, the other for log likelihood). These discrepancies are easily resolved by adjusting one convergence criterion to correspond to the other.

- c. When both `glm` and the equivalent Stata command use Newton–Raphson, small differences may still occur if the Stata command has a different default convergence criterion from that of `glm`. Adjusting the convergence criterion will resolve the difference. See [R] `ml` and [R] `Maximize` for more details. □

### ▷ Example 1

In [example 1](#) of [R] `logistic`, we fit a model based on data from a study of risk factors associated with low birthweight (Hosmer, Lemeshow, and Sturdivant 2013, 24). We can replicate the estimation by using `glm`:

```
. use https://www.stata-press.com/data/r19/lbw
(Hosmer & Lemeshow data)
. glm low age lwt i.race smoke ptl ht ui, family(binomial) link(logit)
Iteration 0: Log likelihood = -101.0213
Iteration 1: Log likelihood = -100.72519
Iteration 2: Log likelihood = -100.724
Iteration 3: Log likelihood = -100.724

Generalized linear models Number of obs = 189
Optimization : ML Residual df = 180
 Scale parameter = 1
Deviance = 201.4479911 (1/df) Deviance = 1.119156
Pearson = 182.0233425 (1/df) Pearson = 1.011241
Variance function: V(u) = u*(1-u) [Bernoulli]
Link function : g(u) = ln(u/(1-u)) [Logit]
Log likelihood = -100.7239956 AIC = 1.1611
 BIC = -742.0665
```

| low   | OIM         |           | z     | P> z  | [95% conf. interval] |           |
|-------|-------------|-----------|-------|-------|----------------------|-----------|
|       | Coefficient | std. err. |       |       |                      |           |
| age   | -.0271003   | .0364504  | -0.74 | 0.457 | -.0985418            | .0443412  |
| lwt   | -.0151508   | .0069259  | -2.19 | 0.029 | -.0287253            | -.0015763 |
| race  |             |           |       |       |                      |           |
| Black | 1.262647    | .5264101  | 2.40  | 0.016 | .2309024             | 2.294392  |
| Other | .8620792    | .4391532  | 1.96  | 0.050 | .0013548             | 1.722804  |
| smoke | .9233448    | .4008266  | 2.30  | 0.021 | .137739              | 1.708951  |
| ptl   | .5418366    | .346249   | 1.56  | 0.118 | -.136799             | 1.220472  |
| ht    | 1.832518    | .6916292  | 2.65  | 0.008 | .4769494             | 3.188086  |
| ui    | .7585135    | .4593768  | 1.65  | 0.099 | -.1418484            | 1.658875  |
| _cons | .4612239    | 1.20459   | 0.38  | 0.702 | -1.899729            | 2.822176  |

`glm`, by default, presents coefficient estimates, whereas `logistic` presents the exponentiated coefficients—the odds ratios. `glm`'s `eform` option reports exponentiated coefficients, and `glm`, like Stata's other estimation commands, replays results.

```
. glm, eform
Generalized linear models Number of obs = 189
Optimization : ML Residual df = 180
 Scale parameter = 1
Deviance = 201.4479911 (1/df) Deviance = 1.119156
Pearson = 182.0233425 (1/df) Pearson = 1.011241
Variance function: V(u) = u*(1-u) [Bernoulli]
Link function : g(u) = ln(u/(1-u)) [Logit]
 AIC = 1.1611
Log likelihood = -100.7239956 BIC = -742.0665
```

|       | OIM        |           |       |       | [95% conf. interval] |          |
|-------|------------|-----------|-------|-------|----------------------|----------|
| low   | Odds ratio | std. err. | z     | P> z  |                      |          |
| age   | .9732636   | .0354759  | -0.74 | 0.457 | .9061578             | 1.045339 |
| lwt   | .9849634   | .0068217  | -2.19 | 0.029 | .9716834             | .9984249 |
| race  |            |           |       |       |                      |          |
| Black | 3.534767   | 1.860737  | 2.40  | 0.016 | 1.259736             | 9.918406 |
| Other | 2.368079   | 1.039949  | 1.96  | 0.050 | 1.001356             | 5.600207 |
| smoke | 2.517698   | 1.00916   | 2.30  | 0.021 | 1.147676             | 5.523162 |
| ptl   | 1.719161   | .5952579  | 1.56  | 0.118 | .8721455             | 3.388787 |
| ht    | 6.249602   | 4.322408  | 2.65  | 0.008 | 1.611152             | 24.24199 |
| ui    | 2.1351     | .9808153  | 1.65  | 0.099 | .8677528             | 5.2534   |
| _cons | 1.586014   | 1.910496  | 0.38  | 0.702 | .1496092             | 16.8134  |

Note: `_cons` estimates baseline odds.

These results are the same as those reported in [example 1](#) of [\[R\] logistic](#).

Included in the output header are values for the [Akaike \(1973\)](#) information criterion (AIC) and the Bayesian information criterion (BIC) ([Raftery 1995](#)). Both are measures of model fit adjusted for the number of parameters that can be compared across models. In both cases, a smaller value generally indicates a better model fit. AIC is based on the log likelihood and thus is available only when Newton–Raphson optimization is used. BIC is based on the deviance and thus is always available.



### □ Technical note

The values for AIC and BIC reported in the output after `glm` are different from those reported by `estat ic`:

```
. estat ic
Akaike's information criterion and Bayesian information criterion
```

| Model | N   | ll(null) | ll(model) | df | AIC     | BIC      |
|-------|-----|----------|-----------|----|---------|----------|
| .     | 189 | .        | -100.724  | 9  | 219.448 | 248.6237 |

Note: BIC uses N = number of observations. See [\[R\] IC note](#).

There are various definitions of these information criteria (IC) in the literature; `glm` and `estat ic` use different definitions. `glm` bases its computation of the BIC on deviance, whereas `estat ic` uses the likelihood. Both `glm` and `estat ic` use the likelihood to compute the AIC; however, the AIC from `estat ic` is equal to  $N$ , the number of observations, times the AIC from `glm`. Refer to [Methods and formulas](#) in this entry and [\[R\] estat ic](#) for the references and formulas used by `glm` and `estat ic`, respectively, to compute AIC and BIC. Inferences based on comparison of IC values reported by `glm` for different GLM models will be equivalent to those based on comparison of IC values reported by `estat ic` after `glm`. □

## ▷ Example 2

We use data from an early insecticide experiment, given in [Pregibon \(1980\)](#). The variables are `ldose`, the log dose of insecticide; `n`, the number of flour beetles subjected to each dose; and `r`, the number killed.

```
. use https://www.stata-press.com/data/r19/ldose
. list, sep(4)
```

|    | ldose  | n  | r  |
|----|--------|----|----|
| 1. | 1.6907 | 59 | 6  |
| 2. | 1.7242 | 60 | 13 |
| 3. | 1.7552 | 62 | 18 |
| 4. | 1.7842 | 56 | 28 |
| 5. | 1.8113 | 63 | 52 |
| 6. | 1.8369 | 59 | 53 |
| 7. | 1.861  | 62 | 61 |
| 8. | 1.8839 | 60 | 60 |

The aim of the analysis is to estimate a dose–response relationship between  $p$ , the proportion killed, and  $X$ , the log dose.

As a first attempt, we will formulate the model as a linear logistic regression of  $p$  on `ldose`; that is, we will take the logit of  $p$  and represent the dose–response curve as a straight line in  $X$ :

$$\ln\{p/(1-p)\} = \beta_0 + \beta_1 X$$

Because the data are grouped, we cannot use Stata's logistic command to fit the model. Instead, we will fit the model by using glm:

```
. glm r ldose, family(binomial n) link(logit)
Iteration 0: Log likelihood = -18.824848
Iteration 1: Log likelihood = -18.715271
Iteration 2: Log likelihood = -18.715123
Iteration 3: Log likelihood = -18.715123

Generalized linear models Number of obs = 8
Optimization : ML Residual df = 6
 Scale parameter = 1
Deviance = 11.23220702 (1/df) Deviance = 1.872035
Pearson = 10.0267936 (1/df) Pearson = 1.671132

Variance function: V(u) = u*(1-u/n) [Binomial]
Link function : g(u) = ln(u/(n-u)) [Logit]

Log likelihood = -18.71512262 AIC = 5.178781
 BIC = -1.244442
```

| r     | OIM         |           |        |       |                      |
|-------|-------------|-----------|--------|-------|----------------------|
|       | Coefficient | std. err. | z      | P> z  | [95% conf. interval] |
| ldose | 34.27034    | 2.912141  | 11.77  | 0.000 | 28.56265 39.97803    |
| _cons | -60.71747   | 5.180713  | -11.72 | 0.000 | -70.87149 -50.56346  |

We specified family(binomial n), meaning that variable n contains the denominator.

An alternative model, which gives asymmetric sigmoid curves for p, involves the complementary log–log, or cloglog, function:

$$\ln\{-\ln(1-p)\} = \beta_0 + \beta_1 X$$

We fit this model by using glm:

```
. glm r ldose, family(binomial n) link(cloglog)
Iteration 0: Log likelihood = -14.883594
Iteration 1: Log likelihood = -14.822264
Iteration 2: Log likelihood = -14.822228
Iteration 3: Log likelihood = -14.822228

Generalized linear models Number of obs = 8
Optimization : ML Residual df = 6
 Scale parameter = 1
Deviance = 3.446418004 (1/df) Deviance = .574403
Pearson = 3.294675153 (1/df) Pearson = .5491125

Variance function: V(u) = u*(1-u/n) [Binomial]
Link function : g(u) = ln(-ln(1-u/n)) [Complementary log-log]

Log likelihood = -14.82222811 AIC = 4.205557
 BIC = -9.030231
```

| r     | OIM         |           |        |       |                      |
|-------|-------------|-----------|--------|-------|----------------------|
|       | Coefficient | std. err. | z      | P> z  | [95% conf. interval] |
| ldose | 22.04118    | 1.793089  | 12.29  | 0.000 | 18.52679 25.55557    |
| _cons | -39.57232   | 3.229047  | -12.26 | 0.000 | -45.90114 -33.24351  |

The cloglog model is preferred; the deviance for the logistic model, 11.23, is much higher than the deviance for the cloglog model, 3.45. This change also is evident by comparing log likelihoods, or equivalently, AIC values.

This example also shows the advantage of the `glm` command—we can vary assumptions easily. Note the minor difference in what we typed to obtain the logistic and cloglog models:

```
. glm r ldose, family(binomial n) link(logit)
. glm r ldose, family(binomial n) link(cloglog)
```

If we were performing this work for ourselves, we would have typed the commands in a more abbreviated form:

```
. glm r ldose, f(b n) l(1)
. glm r ldose, f(b n) l(c1)
```

◀

## □ Technical note

Factor variables may be used with `glm`. Say that, in the [example above](#), we had `ldose`, the log dose of insecticide; `n`, the number of flour beetles subjected to each dose; and `r`, the number killed—all as before—except that now we have results for three different kinds of beetles. Our hypothetical data include `beetle`, which contains the values 1 (“Destructive flour”), 2 (“Red flour”), and 3 (“Mealworm”).

```
. use https://www.stata-press.com/data/r19/beetle
. list, sep(0)
```

|     | beetle                  | ldose  | n  | r  |
|-----|-------------------------|--------|----|----|
| 1.  | Destructive flour       | 1.6907 | 59 | 6  |
| 2.  | Destructive flour       | 1.7242 | 60 | 13 |
| 3.  | Destructive flour       | 1.7552 | 62 | 18 |
| 4.  | Destructive flour       | 1.7842 | 56 | 28 |
| 5.  | Destructive flour       | 1.8113 | 63 | 52 |
|     | <i>(output omitted)</i> |        |    |    |
| 23. | Mealworm                | 1.861  | 64 | 23 |
| 24. | Mealworm                | 1.8839 | 58 | 22 |

Let's assume that, at first, we wish merely to add a shift factor for the type of beetle. We could type

```
. glm r i.beetle ldose, family(bin n) link(cloglog)
Iteration 0: Log likelihood = -79.012269
Iteration 1: Log likelihood = -76.94951
Iteration 2: Log likelihood = -76.945645
Iteration 3: Log likelihood = -76.945645

Generalized linear models Number of obs = 24
Optimization : ML Residual df = 20
 Scale parameter = 1
Deviance = 73.76505595 (1/df) Deviance = 3.688253
Pearson = 71.8901173 (1/df) Pearson = 3.594506

Variance function: V(u) = u*(1-u/n) [Binomial]
Link function : g(u) = ln(-ln(1-u/n)) [Complementary log-log]

Log likelihood = -76.94564525 AIC = 6.74547
 BIC = 10.20398
```

| r         | OIM         |           | z      | P> z  | [95% conf. interval] |           |
|-----------|-------------|-----------|--------|-------|----------------------|-----------|
|           | Coefficient | std. err. |        |       |                      |           |
| beetle    |             |           |        |       |                      |           |
| Red flour | -.0910396   | .1076132  | -0.85  | 0.398 | -.3019576            | .1198783  |
| Mealworm  | -1.836058   | .1307125  | -14.05 | 0.000 | -2.09225             | -1.579867 |
| ldose     | 19.41558    | .9954265  | 19.50  | 0.000 | 17.46458             | 21.36658  |
| _cons     | -34.84602   | 1.79333   | -19.43 | 0.000 | -38.36089            | -31.33116 |

We find strong evidence that the insecticide works differently on the mealworm. We now check whether the curve is merely shifted or also differently sloped:

```
. glm r beetle#c.ldose, family(bin n) link(cloglog)
Iteration 0: Log likelihood = -67.270188
Iteration 1: Log likelihood = -65.149316
Iteration 2: Log likelihood = -65.147978
Iteration 3: Log likelihood = -65.147978

Generalized linear models Number of obs = 24
Optimization : ML Residual df = 18
 Scale parameter = 1
Deviance = 50.16972096 (1/df) Deviance = 2.787207
Pearson = 49.28422567 (1/df) Pearson = 2.738013

Variance function: V(u) = u*(1-u/n) [Binomial]
Link function : g(u) = ln(-ln(1-u/n)) [Complementary log-log]

Log likelihood = -65.14797776 AIC = 5.928998
 BIC = -7.035248
```

| r              | OIM         |           |        |       |                      |           |
|----------------|-------------|-----------|--------|-------|----------------------|-----------|
|                | Coefficient | std. err. | z      | P> z  | [95% conf. interval] |           |
| beetle         |             |           |        |       |                      |           |
| Red flour      | -.79933     | 4.470882  | -0.18  | 0.858 | -9.562098            | 7.963438  |
| Mealworm       | 17.78741    | 4.586429  | 3.88   | 0.000 | 8.798172             | 26.77664  |
| ldose          | 22.04118    | 1.793089  | 12.29  | 0.000 | 18.52679             | 25.55557  |
| beetle#c.ldose |             |           |        |       |                      |           |
| Red flour      | .3838708    | 2.478477  | 0.15   | 0.877 | -4.473855            | 5.241596  |
| Mealworm       | -10.726     | 2.526412  | -4.25  | 0.000 | -15.67768            | -5.774321 |
| _cons          | -39.57232   | 3.229047  | -12.26 | 0.000 | -45.90114            | -33.24351 |

We find that the (complementary log–log) dose–response curve for the mealworm has roughly half the slope of that for the destructive flour beetle.

See [U] 26 [Working with categorical data and factor variables](#); what is said there concerning linear regression is applicable to any GLM model.

□

## Variance estimators

glm offers many variance options and gives different types of standard errors when used in various combinations. We highlight some of them here, but for a full explanation, see [Hardin and Hilbe \(2018\)](#).

### ▷ Example 3

Continuing with our flour beetle data, we rerun the most recently displayed model, this time requesting estimation via IRLS.

```
. use https://www.stata-press.com/data/r19/beetle
. glm r beetle#c.ldose, f(bin n) l(cloglog) ltol(1e-13) irls
Iteration 1: Deviance = 54.41414
Iteration 2: Deviance = 50.19424
Iteration 3: Deviance = 50.16973
(output omitted)
Generalized linear models Number of obs = 24
Optimization : MQL Fisher scoring Residual df = 18
 (IRLS EIM) Scale parameter = 1
Deviance = 50.16972096 (1/df) Deviance = 2.787207
Pearson = 49.28422528 (1/df) Pearson = 2.738013
Variance function: V(u) = u*(1-u/n) [Binomial]
Link function : g(u) = ln(-ln(1-u/n)) [Complementary log-log]
BIC = -7.035248
```

| r              | EIM         |           | z      | P> z  | [95% conf. interval] |           |
|----------------|-------------|-----------|--------|-------|----------------------|-----------|
|                | Coefficient | std. err. |        |       |                      |           |
| beetle         |             |           |        |       |                      |           |
| Red flour      | -.79933     | 4.586649  | -0.17  | 0.862 | -9.788997            | 8.190337  |
| Mealworm       | 17.78741    | 4.624834  | 3.85   | 0.000 | 8.7229               | 26.85192  |
| ldose          | 22.04118    | 1.799356  | 12.25  | 0.000 | 18.5145              | 25.56785  |
| beetle#c.ldose |             |           |        |       |                      |           |
| Red flour      | .3838708    | 2.544068  | 0.15   | 0.880 | -4.602411            | 5.370152  |
| Mealworm       | -10.726     | 2.548176  | -4.21  | 0.000 | -15.72033            | -5.731665 |
| _cons          | -39.57232   | 3.240274  | -12.21 | 0.000 | -45.92314            | -33.2215  |

Note our use of the `ltol()` option, which, although unrelated to our discussion on variance estimation, was used so that the regression coefficients would match those of the previous Newton–Raphson (NR) fit.

Because IRLS uses the EIM for optimization, the variance estimate is also based on EIM. If we want optimization via IRLS but the variance estimate based on OIM, we specify `glm, irls vce(oim)`:

```
. glm r beetle#c.ldose, f(b n) l(c1) ltol(1e-15) irls vce(oim) noheader nolog
```

| r              | OIM         |           |        |       |                      |           |
|----------------|-------------|-----------|--------|-------|----------------------|-----------|
|                | Coefficient | std. err. | z      | P> z  | [95% conf. interval] |           |
| beetle         |             |           |        |       |                      |           |
| Red flour      | -.79933     | 4.470882  | -0.18  | 0.858 | -9.562098            | 7.963438  |
| Mealworm       | 17.78741    | 4.586429  | 3.88   | 0.000 | 8.798172             | 26.77664  |
| ldose          | 22.04118    | 1.793089  | 12.29  | 0.000 | 18.52679             | 25.55557  |
| beetle#c.ldose |             |           |        |       |                      |           |
| Red flour      | .3838708    | 2.478477  | 0.15   | 0.877 | -4.473855            | 5.241596  |
| Mealworm       | -10.726     | 2.526412  | -4.25  | 0.000 | -15.67768            | -5.774321 |
| _cons          | -39.57232   | 3.229047  | -12.26 | 0.000 | -45.90114            | -33.24351 |

This approach is identical to NR except for the convergence path. Because the cloglog link is not the canonical link for the binomial family, EIM and OIM produce different results. Both estimators, however, are asymptotically equivalent.

Going back to NR, we can also specify `vce(robust)` to get the Huber/White/sandwich estimator of variance:

```
. glm r beetle#c.ldose, f(b n) l(c1) vce(robust) noheader nolog
```

| r              | Robust      |           |        |       |                      |           |
|----------------|-------------|-----------|--------|-------|----------------------|-----------|
|                | Coefficient | std. err. | z      | P> z  | [95% conf. interval] |           |
| beetle         |             |           |        |       |                      |           |
| Red flour      | -.79933     | 5.733049  | -0.14  | 0.889 | -12.0359             | 10.43724  |
| Mealworm       | 17.78741    | 5.158477  | 3.45   | 0.001 | 7.676977             | 27.89784  |
| ldose          | 22.04118    | .8998551  | 24.49  | 0.000 | 20.27749             | 23.80486  |
| beetle#c.ldose |             |           |        |       |                      |           |
| Red flour      | .3838708    | 3.174427  | 0.12   | 0.904 | -5.837892            | 6.605633  |
| Mealworm       | -10.726     | 2.800606  | -3.83  | 0.000 | -16.21508            | -5.236912 |
| _cons          | -39.57232   | 1.621306  | -24.41 | 0.000 | -42.75003            | -36.39462 |

The sandwich estimator gets its name from the form of the calculation—it is the multiplication of three matrices, with the outer two matrices (the “bread”) set to the OIM variance matrix. When `irls` is used along with `vce(robust)`, the EIM variance matrix is instead used as the bread. Using a result from [McCullagh and Nelder \(1989\)](#), [Newson \(1999\)](#) points out that the EIM and OIM variance matrices are equivalent under the canonical link. Thus if `irls` is specified with the canonical link, the resulting variance is labeled “Robust”. When the noncanonical link for the family is used, which is the case in the example below, the EIM and OIM variance matrices differ, so the resulting variance is labeled “Semirobust”.

```
. glm r beetle##c.ldose, f(b n) l(c1) irls ltol(1e-15) vce(robust) noheader
> nolog
```

| r              | Semirobust  |           | z      | P> z  | [95% conf. interval] |           |
|----------------|-------------|-----------|--------|-------|----------------------|-----------|
|                | Coefficient | std. err. |        |       |                      |           |
| beetle         |             |           |        |       |                      |           |
| Red flour      | -.79933     | 6.288963  | -0.13  | 0.899 | -13.12547            | 11.52681  |
| Mealworm       | 17.78741    | 5.255307  | 3.38   | 0.001 | 7.487194             | 28.08762  |
| ldose          | 22.04118    | .9061566  | 24.32  | 0.000 | 20.26514             | 23.81721  |
| beetle#c.ldose |             |           |        |       |                      |           |
| Red flour      | .3838708    | 3.489723  | 0.11   | 0.912 | -6.455861            | 7.223603  |
| Mealworm       | -10.726     | 2.855897  | -3.76  | 0.000 | -16.32345            | -5.128542 |
| _cons          | -39.57232   | 1.632544  | -24.24 | 0.000 | -42.77205            | -36.3726  |

The outer product of the gradient (OPG) estimate of variance is one that avoids the calculation of second derivatives. It is equivalent to the “middle” part of the sandwich estimate of variance and can be specified by using `glm, vce(opg)`, regardless of whether NR or IRLS optimization is used.

```
. glm r beetle##c.ldose, f(b n) l(c1) vce(opg) noheader nolog
```

| r              | OPG         |           | z     | P> z  | [95% conf. interval] |           |
|----------------|-------------|-----------|-------|-------|----------------------|-----------|
|                | Coefficient | std. err. |       |       |                      |           |
| beetle         |             |           |       |       |                      |           |
| Red flour      | -.79933     | 6.664045  | -0.12 | 0.905 | -13.86062            | 12.26196  |
| Mealworm       | 17.78741    | 6.838505  | 2.60  | 0.009 | 4.384183             | 31.19063  |
| ldose          | 22.04118    | 3.572983  | 6.17  | 0.000 | 15.03826             | 29.0441   |
| beetle#c.ldose |             |           |       |       |                      |           |
| Red flour      | .3838708    | 3.700192  | 0.10  | 0.917 | -6.868372            | 7.636114  |
| Mealworm       | -10.726     | 3.796448  | -2.83 | 0.005 | -18.1669             | -3.285097 |
| _cons          | -39.57232   | 6.433101  | -6.15 | 0.000 | -52.18097            | -26.96368 |

The OPG estimate of variance is a component of the BHHH (Berndt et al. 1974) optimization technique. This method of optimization is also available with `glm` with the `technique()` option; however, the `technique()` option is not allowed with the `irls` option.



#### ▷ Example 4

The Newey–West (1987) estimator of variance is a sandwich estimator with the “middle” of the sandwich modified to account for possible autocorrelation between the observations. These estimators are a generalization of those given by the Stata command `newey` for linear regression. See [TS] [newey](#) for more details.

For example, consider the dataset given in [TS] `newey`, which has time-series measurements on `usr` and `idle`. We want to perform a linear regression with Newey–West standard errors.

```
. use https://www.stata-press.com/data/r19/idle2
. list usr idle time
```

|                  | usr | idle | time |
|------------------|-----|------|------|
| 1.               | 0   | 100  | 1    |
| 2.               | 0   | 100  | 2    |
| 3.               | 0   | 97   | 3    |
| 4.               | 1   | 98   | 4    |
| 5.               | 2   | 94   | 5    |
| (output omitted) |     |      |      |
| 29.              | 1   | 98   | 29   |
| 30.              | 1   | 98   | 30   |

Examining *Methods and formulas* of [TS] `newey`, we see that the variance estimate is multiplied by a correction factor of  $n/(n - k)$ , where  $k$  is the number of regressors. `glm, vce(hac ...)` does not make this correction, so to get the same standard errors, we must use the `vfactor()` option within `glm` to make the correction manually.

```
. display 30/28
1.0714286
. tsset time
Time variable: time, 1 to 30
 Delta: 1 unit
. glm usr idle, vce(hac nwest 3) vfactor(1.0714286)
Iteration 0: Log likelihood = -71.743396
Generalized linear models Number of obs = 30
Optimization : ML Residual df = 28
 Scale parameter = 7.493297
Deviance = 209.8123165 (1/df) Deviance = 7.493297
Pearson = 209.8123165 (1/df) Pearson = 7.493297
Variance function: V(u) = 1 [Gaussian]
Link function : g(u) = u [Identity]
HAC kernel (lags): Newey-West (3)
Log likelihood = -71.74339627 AIC = 4.916226
 BIC = 114.5788
```

| usr   | HAC         |           |       | z     | P> z      | [95% conf. interval] |  |
|-------|-------------|-----------|-------|-------|-----------|----------------------|--|
|       | Coefficient | std. err. |       |       |           |                      |  |
| idle  | -.2281501   | .0690928  | -3.30 | 0.001 | -.3635694 | -.0927307            |  |
| _cons | 23.13483    | 6.327033  | 3.66  | 0.000 | 10.73407  | 35.53558             |  |

The `glm` command above reproduces the results given in [TS] `newey`. We may now generalize this output to models other than simple linear regression and to different kernel weights.

```
. glm usr idle, fam(gamma) link(log) vce(hac gallant 3)
Iteration 0: Log likelihood = -61.76593
Iteration 1: Log likelihood = -60.963233
Iteration 2: Log likelihood = -60.95097
Iteration 3: Log likelihood = -60.950965

Generalized linear models
Optimization : ML
Deviance = 9.908506707
Pearson = 12.07628677
Variance function: V(u) = u^2
Link function : g(u) = ln(u)
HAC kernel (lags): Gallant (3)

Number of obs = 30
Residual df = 28
Scale parameter = .431296
(1/df) Deviance = .3538752
(1/df) Pearson = .431296
[Gamma]
[Log]
AIC = 4.196731
BIC = -85.32502
Log likelihood = -60.95096484
```

| usr   | HAC         |           | z     | P> z  | [95% conf. interval] |           |
|-------|-------------|-----------|-------|-------|----------------------|-----------|
|       | Coefficient | std. err. |       |       |                      |           |
| idle  | -.0796609   | .0184647  | -4.31 | 0.000 | -.115851             | -.0434708 |
| _cons | 7.771011    | 1.510198  | 5.15  | 0.000 | 4.811078             | 10.73094  |

glm also offers variance estimators based on the bootstrap (resampling your data with replacement) and the jackknife (refitting the model with each observation left out in succession). Also included is the one-step jackknife estimate, which, instead of performing full reestimation when each observation is omitted, calculates a one-step NR estimate, with the full data regression coefficients as starting values.

```
. set seed 1
. glm usr idle, fam(gamma) link(log) vce(bootstrap, reps(100) nodots)
Generalized linear models
Optimization : ML
Deviance = 9.908506707
Pearson = 12.07628677
Variance function: V(u) = u^2
Link function : g(u) = ln(u)

Number of obs = 30
Residual df = 28
Scale parameter = .431296
(1/df) Deviance = .3538752
(1/df) Pearson = .431296
[Gamma]
[Log]
AIC = 4.196731
BIC = -85.32502
Log likelihood = -60.95096484
```

| usr   | Observed    | Bootstrap | z     | P> z  | Normal-based |                      |
|-------|-------------|-----------|-------|-------|--------------|----------------------|
|       | coefficient | std. err. |       |       |              | [95% conf. interval] |
| idle  | -.0796609   | .016657   | -4.78 | 0.000 | -.1123081    | -.0470137            |
| _cons | 7.771011    | 1.378037  | 5.64  | 0.000 | 5.070108     | 10.47192             |



See [Hardin and Hilbe \(2018\)](#) for a full discussion of the variance options that go with glm and, in particular, of how the different variance estimators are modified when vce(cluster *clustvar*) is specified. Finally, not all variance options are supported with all types of weights. See help glm for a current table of the variance options that are supported with the different weights.

## User-defined functions

glm may be called with a community-contributed link function, variance (family) function, Newey–West kernel-weight function, or any combination of the three.

### Syntax of link functions

```

program programe
version 19.5 // (or version 19 if you do not have StataNow)
args todo eta mu return

if `todo' == -1 {
 /* Set global macros for output */
 global SGLM_lt "title for link function"
 global SGLM_lf "subtitle showing link definition"
 exit
}
if `todo' == 0 {
 /* set $\eta = g(\mu)$ */
 /* Intermediate calculations go here */
 generate double `eta' = ...
 exit
}
if `todo' == 1 {
 /* set $\mu = g^{-1}(\eta)$ */
 /* Intermediate calculations go here */
 generate double `mu' = ...
 exit
}
if `todo' == 2 {
 /* set return= $\partial\mu/\partial\eta$ */
 /* Intermediate calculations go here */
 generate double `return' = ...
 exit
}
if `todo' == 3 {
 /* set return= $\partial^2\mu/\partial\eta^2$ */
 /* Intermediate calculations go here */
 generate double `return' = ...
 exit
}
display as error "Unknown call to glm link function"
exit 198
end

```

## Syntax of variance functions

```

program programe
 version 19.5 // (or version 19 if you do not have StataNow)
 args todo eta mu return

 if `todo' == -1 {
 /* Set global macros for output */
 /* Also check that devar is in proper range */
 /* Note: For this call, eta contains indicator for whether each obs. is in est. sample */
 global SGLM_vt "title for variance function"
 global SGLM_vf "subtitle showing function definition"
 global SGLM_mu "program to call to enforce boundary conditions on μ "
 exit
 }
 if `todo' == 0 {
 /* set η to initial value. */
 /* Intermediate calculations go here */
 generate double `eta' = ...
 exit
 }
 if `todo' == 1 {
 /* set return = $V(\mu)$ */
 /* Intermediate calculations go here */
 generate double `return' = ...
 exit
 }
 if `todo' == 2 {
 /* set return = $\partial V(\mu)/\partial \mu$ */
 /* Intermediate calculations go here */
 generate double `return' = ...
 exit
 }
 if `todo' == 3 {
 /* set return = squared deviance (per observation) */
 /* Intermediate calculations go here */
 generate double `return' = ...
 exit
 }
 if `todo' == 4 {
 /* set return = Anscombe residual */
 /* Intermediate calculations go here */
 generate double `return' = ...
 exit
 }
 if `todo' == 5 {
 /* set return = log likelihood */
 /* Intermediate calculations go here */
 generate double `return' = ...
 exit
 }
 if `todo' == 6 {
 /* set return = adjustment for deviance residuals */
 /* Intermediate calculations go here */
 generate double `return' = ...
 exit
 }
 display as error "Unknown call to glm variance function"
 exit 198
end

```

## Syntax of Newey–West kernel-weight functions

```

program progrname, rclass
 version 19.5 // (or version 19 if you do not have StataNow)
 args G j
 /* G is the maximum lag */
 /* j is the current lag */

 /* Intermediate calculations go here */

 return scalar wt = computed weight
 return local setype "Newey-West"
 return local sewtype "name of kernel"
end

```

## Global macros available for community-contributed programs

| Global macro | Description                                                                                                        |
|--------------|--------------------------------------------------------------------------------------------------------------------|
| SGLM_V       | program name of variance (family) evaluator                                                                        |
| SGLM_L       | program name of link evaluator                                                                                     |
| SGLM_y       | dependent variable name                                                                                            |
| SGLM_m       | binomial denominator                                                                                               |
| SGLM_a       | negative binomial $k$                                                                                              |
| SGLM_p       | power if <code>power()</code> or <code>opower()</code> is used, or an argument from a user-specified link function |
| SGLM_s1      | indicator; set to one if scale is equal to one                                                                     |
| SGLM_ph      | value of scale parameter                                                                                           |

### ▷ Example 5

Suppose that we wish to perform Poisson regression with a log-link function. Although this regression is already possible with standard `glm`, we will write our own version for illustrative purposes.

Because we want a log link,  $\eta = g(\mu) = \ln(\mu)$ , and for a Poisson family the variance function is  $V(\mu) = \mu$ .

The Poisson density is given by

$$f(y_i) = \frac{e^{-\exp(\mu_i)} e^{\mu_i y_i}}{y_i!}$$

resulting in a log likelihood of

$$L = \sum_{i=1}^n \{-e^{\mu_i} + \mu_i y_i - \ln(y_i!)\}$$

The squared deviance of the  $i$ th observation for the Poisson family is given by

$$d_i^2 = \begin{cases} 2\hat{\mu}_i & \text{if } y_i = 0 \\ 2\{y_i \ln(y_i/\hat{\mu}_i) - (y_i - \hat{\mu}_i)\} & \text{otherwise} \end{cases}$$

We now have enough information to write our own Poisson-log glm module. We create the file `mylog.ado`, which contains

```

program mylog
 version 19.5 // (or version 19 if you do not have StataNow)
 args todo eta mu return

 if `todo' == -1 {
 global SGLM_lt "My Log" // Titles for output
 global SGLM_lf "ln(u)"
 exit
 }
 if `todo' == 0 {
 gen double `eta' = ln(`mu') // $\eta = \ln(\mu)$
 exit
 }
 if `todo' == 1 {
 gen double `mu' = exp(`eta') // $\mu = \exp(\eta)$
 exit
 }
 if `todo' == 2 {
 gen double `return' = `mu' // $\partial\mu/\partial\eta = \exp(\eta) = \mu$
 exit
 }
 if `todo' == 3 {
 gen double `return' = `mu' // $\partial^2\mu/\partial\eta^2 = \exp(\eta) = \mu$
 exit
 }
 di as error "Unknown call to glm link function"
 exit 198
end

```

and we create the file `mypoiso.ado`, which contains

```

program mypoiso
 version 19.5 // (or version 19 if you do not have StataNow)
 args todo eta mu return

 if `todo' == -1 {
 local y "$SGLM_y"
 local touse "`eta'"
 capture assert `y'>=0 if `touse' // check range of y
 if _rc {
 di as error "'dependent variable `y' has negative values'"
 exit 499
 }
 global SGLM_vt "My Poisson" // Titles for output
 global SGLM_vf "u"
 global SGLM_mu "glim_mu 0 ." // see note 1
 exit
 }
 if `todo' == 0 {
 gen double `eta' = ln(`mu') // Initialization of η ; see note 2
 exit
 }

```

```

if `todo' == 1 {
 gen double `return' = `mu' // $V(\mu) = \mu$
 exit
}
if `todo' == 2 {
 gen byte `return' = 1 // $\partial V(\mu)/\partial \mu$
 exit
}
if `todo' == 3 {
 local y "$SGLM_y"
 if "`y'" == "" {
 local y "e(depvar)"
 }
 gen double `return' = cond(`y'==0, 2*`mu', /*
 / 2(`y'*ln(`y'/`mu')-(`y'-`mu'))
 exit
}
if `todo' == 4 {
 local y "$SGLM_y"
 if "`y'" == "" {
 local y "e(depvar)"
 }
 gen double `return' = 1.5*(`y'^(2/3)-`mu'^(2/3)) / `mu'^(1/6)
 exit
}
if `todo' == 5 {
 local y "$SGLM_y"
 if "`y'" == "" {
 local y "e(depvar)"
 }
 gen double `return' = -`mu'+`y'*ln(`mu')-lngamma(`y'+1)
 exit
}
if `todo' == 6 {
 gen double `return' = 1/(6*sqrt(`mu')) // adjustment to residual; see note 5
 exit
}
di as error "Unknown call to glm variance function"
error 198
end

```

## Notes:

1. `glm_mu` is a Stata program that will, at each iteration, bring  $\hat{\mu}$  back into its plausible range, should it stray out of it. Here `glm_mu` is called with the arguments zero and missing, meaning that zero is the lower bound of  $\hat{\mu}$  and there exists no upper bound—such is the case for Poisson models.
2. Here the initial value of  $\eta$  is easy because we intend to fit this model with our user-defined log link. In general, however, the initialization may need to vary according to the link to obtain convergence. If so, the global macro `SGLM_L` is used to determine which link is being utilized.
3. The Anscombe formula is given here because we know it. If we were not interested in Anscombe residuals, we could merely set ``return'` to missing. Also, the local macro `y` is set either to `SGLM_y` if it is in current estimation or to `e(depvar)` if this function is being accessed by `predict`.
4. If we were not interested in ML estimation, we could omit this code entirely and just leave an `exit` statement in its place. Similarly, if we were not interested in deviance or IRLS optimization, we could set ``return'` in the deviance portion of the code (``todo'==3`) to missing.

5. This code defines the term to be added to the predicted residuals if the adjusted option is specified. Again, if we were not interested, we could set 'return' to missing.

We can now test our Poisson-log module by running it on the airline data presented in [R] poisson.

```
. use https://www.stata-press.com/data/r19/airline
. list airline injuries n XYZowned
```

|    | airline | injuries | n      | XYZowned |
|----|---------|----------|--------|----------|
| 1. | 1       | 11       | 0.0950 | 1        |
| 2. | 2       | 7        | 0.1920 | 0        |
| 3. | 3       | 7        | 0.0750 | 0        |
| 4. | 4       | 19       | 0.2078 | 0        |
| 5. | 5       | 9        | 0.1382 | 0        |
| 6. | 6       | 4        | 0.0540 | 1        |
| 7. | 7       | 3        | 0.1292 | 0        |
| 8. | 8       | 1        | 0.0503 | 0        |
| 9. | 9       | 3        | 0.0629 | 1        |

```
. generate lnN=ln(n)
. glm injuries XYZowned lnN, f(mypois) l(mylog) scale(1)
```

```
Iteration 0: Log likelihood = -22.557572
Iteration 1: Log likelihood = -22.332861
Iteration 2: Log likelihood = -22.332276
Iteration 3: Log likelihood = -22.332276
```

```
Generalized linear models Number of obs = 9
Optimization : ML Residual df = 6
Scale parameter = 1
Deviance = 12.70432823 (1/df) Deviance = 2.117388
Pearson = 12.7695081 (1/df) Pearson = 2.128251
Variance function: V(u) = u [My Poisson]
Link function : g(u) = ln(u) [My Log]
Log likelihood = -22.33227605 AIC = 5.629395
BIC = -.4790192
```

| injuries | OIM         |           |      |       |                      |          |
|----------|-------------|-----------|------|-------|----------------------|----------|
|          | Coefficient | std. err. | z    | P> z  | [95% conf. interval] |          |
| XYZowned | .6840668    | .3895877  | 1.76 | 0.079 | -.0795111            | 1.447645 |
| lnN      | 1.424169    | .3725155  | 3.82 | 0.000 | .6940517             | 2.154286 |
| _cons    | 4.863891    | .7090501  | 6.86 | 0.000 | 3.474178             | 6.253603 |

(Standard errors scaled using dispersion equal to square root of 1.)

These are precisely the results given in [R] poisson and are those that would have been given had we run glm, family(poisson) link(log). The only minor adjustment we needed to make was to specify the scale(1) option. If scale() is left unspecified, glm assumes scale(1) for discrete distributions and scale(x2) for continuous ones. By default, glm assumes that any user-defined family is continuous because it has no way of checking. Thus, we needed to specify scale(1) because our model is discrete.

Because we were careful in defining the squared deviance, we could have fit this model with IRLS. Because log is the canonical link for the Poisson family, we would not only get the same regression coefficients but also the same standard errors.

## ▶ Example 6

Suppose now that we wish to use our log link (`mylog.ado`) with `glm`'s binomial family. This task requires some modification because our current function is not equipped to deal with the binomial denominator, which we are allowed to specify. This denominator is accessible to our link function through the global macro `SGLM_m`. We now make the modifications and store them in `mylog2.ado`.

```

program mylog2 // <-- changed
version 19.5 // (or version 19 if you do not have StataNow)
args todo eta mu return
if 'todo' == -1 {
 global SGLM_lt "My Log, Version 2" // <-- changed
 if "$SGLM_m" == "1" { // <-- changed
 global SGLM_lf "ln(u)" // <-- changed
 } // <-- changed
 else global SGLM_lf "ln(u/$SGLM_m)" // <-- changed
 exit
}
if 'todo' == 0 {
 gen double 'eta' = ln('mu'/$SGLM_m) // <-- changed
 exit
}
if 'todo' == 1 {
 gen double 'mu' = $SGLM_m*exp('eta') // <-- changed
 exit
}
if 'todo' == 2 {
 gen double 'return' = 'mu'
 exit
}
if 'todo' == 3 {
 gen double 'return' = 'mu'
 exit
}
di as error "Unknown call to glm link function"
exit 198
end

```

We can now run our new log link with `glm`'s binomial family. Using the flour beetle data from earlier, we have

```

. use https://www.stata-press.com/data/r19/beetle, clear
. glm r ldose, f(bin n) l(mylog2) irls

Iteration 1: Deviance = 2212.108
Iteration 2: Deviance = 452.9352
Iteration 3: Deviance = 429.95
Iteration 4: Deviance = 429.2745
Iteration 5: Deviance = 429.2192
Iteration 6: Deviance = 429.2082
Iteration 7: Deviance = 429.2061
Iteration 8: Deviance = 429.2057
Iteration 9: Deviance = 429.2056
Iteration 10: Deviance = 429.2056
Iteration 11: Deviance = 429.2056
Iteration 12: Deviance = 429.2056

Generalized linear models Number of obs = 24
Optimization : MQL Fisher scoring Residual df = 22
 (IRLS EIM) Scale parameter = 1
Deviance = 429.205599 (1/df) Deviance = 19.50935
Pearson = 413.088142 (1/df) Pearson = 18.77673

Variance function: V(u) = u*(1-u/n) [Binomial]
Link function : g(u) = ln(u/n) [My Log, Version 2]
 BIC = 359.2884

```

| r     | EIM         |           |        |       |                      |
|-------|-------------|-----------|--------|-------|----------------------|
|       | Coefficient | std. err. | z      | P> z  | [95% conf. interval] |
| ldose | 8.478908    | .4702808  | 18.03  | 0.000 | 7.557175 9.400642    |
| _cons | -16.11006   | .8723167  | -18.47 | 0.000 | -17.81977 -14.40035  |

◀

For a more detailed discussion on user-defined functions, and for an example of a user-defined Newey–West kernel weight, see [Hardin and Hilbe \(2018\)](#).

John Ashworth Nelder (1924–2010) was born in Somerset, England. He studied mathematics and statistics at Cambridge and worked as a statistician at the National Vegetable Research Station and then Rothamsted Experimental Station. In retirement, he was actively affiliated with Imperial College London. Nelder was especially well known for his contributions to the theory of linear models and to statistical computing. He was the principal architect of generalized and hierarchical generalized linear models and of the programs GenStat and GLIM.

Robert William Maclagan Wedderburn (1947–1975) was born in Edinburgh and studied mathematics and statistics at Cambridge. At Rothamsted Experimental Station, he developed the theory of generalized linear models with Nelder and originated the concept of quaslikelihood. He died of anaphylactic shock from an insect bite on a canal holiday.

## Stored results

`glm`, `ml` stores the following in `e()`:

### Scalars

|                             |                                                                        |
|-----------------------------|------------------------------------------------------------------------|
| <code>e(N)</code>           | number of observations                                                 |
| <code>e(k)</code>           | number of parameters                                                   |
| <code>e(k_eq)</code>        | number of equations in <code>e(b)</code>                               |
| <code>e(k_eq_model)</code>  | number of equations in overall model test                              |
| <code>e(k_dv)</code>        | number of dependent variables                                          |
| <code>e(df_m)</code>        | model degrees of freedom                                               |
| <code>e(df)</code>          | residual degrees of freedom                                            |
| <code>e(phi)</code>         | scale parameter                                                        |
| <code>e(aic)</code>         | model AIC                                                              |
| <code>e(bic)</code>         | model BIC                                                              |
| <code>e(ll)</code>          | log likelihood, if NR                                                  |
| <code>e(N_clust)</code>     | number of clusters                                                     |
| <code>e(chi2)</code>        | $\chi^2$                                                               |
| <code>e(p)</code>           | <i>p</i> -value for model test                                         |
| <code>e(deviance)</code>    | deviance                                                               |
| <code>e(deviance_s)</code>  | scaled deviance                                                        |
| <code>e(deviance_p)</code>  | Pearson deviance                                                       |
| <code>e(deviance_ps)</code> | scaled Pearson deviance                                                |
| <code>e(dispers)</code>     | dispersion                                                             |
| <code>e(dispers_s)</code>   | scaled dispersion                                                      |
| <code>e(dispers_p)</code>   | Pearson dispersion                                                     |
| <code>e(dispers_ps)</code>  | scaled Pearson dispersion                                              |
| <code>e(nbml)</code>        | 1 if negative binomial parameter estimated via ML, 0 otherwise         |
| <code>e(vf)</code>          | factor set by <code>vfactor()</code> , 1 if not set                    |
| <code>e(power)</code>       | power set by <code>link(power #)</code> or <code>link(opower #)</code> |
| <code>e(rank)</code>        | rank of <code>e(V)</code>                                              |
| <code>e(ic)</code>          | number of iterations                                                   |
| <code>e(rc)</code>          | return code                                                            |
| <code>e(converged)</code>   | 1 if converged, 0 otherwise                                            |

### Macros

|                            |                                               |
|----------------------------|-----------------------------------------------|
| <code>e(cmd)</code>        | <code>glm</code>                              |
| <code>e(cmdline)</code>    | command as typed                              |
| <code>e(depvar)</code>     | name of dependent variable                    |
| <code>e(varfunc)</code>    | program to calculate variance function        |
| <code>e(varfunct)</code>   | variance title                                |
| <code>e(varfuncf)</code>   | variance function                             |
| <code>e(link)</code>       | program to calculate link function            |
| <code>e(linkt)</code>      | link title                                    |
| <code>e(linkf)</code>      | link function                                 |
| <code>e(m)</code>          | number of binomial trials                     |
| <code>e(wtype)</code>      | weight type                                   |
| <code>e(wexp)</code>       | weight expression                             |
| <code>e(title)</code>      | title in estimation output                    |
| <code>e(clustvar)</code>   | name of cluster variable                      |
| <code>e(offset)</code>     | linear offset variable                        |
| <code>e(chi2type)</code>   | Wald; type of model $\chi^2$ test             |
| <code>e(cons)</code>       | noconstant, if specified                      |
| <code>e(hac_kernel)</code> | HAC kernel                                    |
| <code>e(hac_lag)</code>    | HAC lag                                       |
| <code>e(vce)</code>        | <i>vctype</i> specified in <code>vce()</code> |
| <code>e(vcetype)</code>    | title used to label Std. err.                 |
| <code>e(opt)</code>        | <code>ml</code> or <code>irls</code>          |
| <code>e(opt1)</code>       | optimization title, line 1                    |
| <code>e(opt2)</code>       | optimization title, line 2                    |

|                 |                                                                          |
|-----------------|--------------------------------------------------------------------------|
| e(which)        | max or min; whether optimizer is to perform maximization or minimization |
| e(ml_method)    | type of ml method                                                        |
| e(user)         | name of likelihood-evaluator program                                     |
| e(technique)    | maximization technique                                                   |
| e(properties)   | b V                                                                      |
| e(predict)      | program used to implement predict                                        |
| e(marginsok)    | predictions allowed by margins                                           |
| e(marginsnotok) | predictions disallowed by margins                                        |
| e(asbalanced)   | factor variables fvset as asbalanced                                     |
| e(asobserved)   | factor variables fvset as asobserved                                     |

## Matrices

|                 |                                              |
|-----------------|----------------------------------------------|
| e(b)            | coefficient vector                           |
| e(Cns)          | constraints matrix                           |
| e(ilog)         | iteration log (up to 20 iterations)          |
| e(gradient)     | gradient vector                              |
| e(V)            | variance-covariance matrix of the estimators |
| e(V_modelbased) | model-based variance                         |

## Functions

|           |                         |
|-----------|-------------------------|
| e(sample) | marks estimation sample |
|-----------|-------------------------|

In addition to the above, the following is stored in `r()`:

## Matrices

|          |                                                                                                                            |
|----------|----------------------------------------------------------------------------------------------------------------------------|
| r(table) | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |
|----------|----------------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

`glm`, `irls` stores the following in `e()`:

## Scalars

|                |                                                                        |
|----------------|------------------------------------------------------------------------|
| e(N)           | number of observations                                                 |
| e(k)           | number of parameters                                                   |
| e(k_eq_model)  | number of equations in overall model test                              |
| e(df_m)        | model degrees of freedom                                               |
| e(df)          | residual degrees of freedom                                            |
| e(phi)         | scale parameter                                                        |
| e(disp)        | dispersion parameter                                                   |
| e(bic)         | model BIC                                                              |
| e(N_clust)     | number of clusters                                                     |
| e(deviance)    | deviance                                                               |
| e(deviance_s)  | scaled deviance                                                        |
| e(deviance_p)  | Pearson deviance                                                       |
| e(deviance_ps) | scaled Pearson deviance                                                |
| e(dispers)     | dispersion                                                             |
| e(dispers_s)   | scaled dispersion                                                      |
| e(dispers_p)   | Pearson dispersion                                                     |
| e(dispers_ps)  | scaled Pearson dispersion                                              |
| e(nbml)        | 1 if negative binomial parameter estimated via ML, 0 otherwise         |
| e(vf)          | factor set by <code>vfactor()</code> , 1 if not set                    |
| e(power)       | power set by <code>link(power #)</code> or <code>link(opower #)</code> |
| e(rank)        | rank of <code>e(V)</code>                                              |
| e(rc)          | return code                                                            |

## Macros

|            |                            |
|------------|----------------------------|
| e(cmd)     | <code>glm</code>           |
| e(cmdline) | command as typed           |
| e(depvar)  | name of dependent variable |

|                              |                                                    |
|------------------------------|----------------------------------------------------|
| <code>e(varfunc)</code>      | program to calculate variance function             |
| <code>e(varfunct)</code>     | variance title                                     |
| <code>e(varfunctf)</code>    | variance function                                  |
| <code>e(link)</code>         | program to calculate link function                 |
| <code>e(linkt)</code>        | link title                                         |
| <code>e(linkf)</code>        | link function                                      |
| <code>e(m)</code>            | number of binomial trials                          |
| <code>e(wtype)</code>        | weight type                                        |
| <code>e(wexp)</code>         | weight expression                                  |
| <code>e(clustvar)</code>     | name of cluster variable                           |
| <code>e(offset)</code>       | linear offset variable                             |
| <code>e(cons)</code>         | noconstant, if specified                           |
| <code>e(hac_kernel)</code>   | HAC kernel                                         |
| <code>e(hac_lag)</code>      | HAC lag                                            |
| <code>e(vce)</code>          | <i>vce</i> type specified in <code>vce()</code>    |
| <code>e(vcetype)</code>      | title used to label Std. err.                      |
| <code>e(opt)</code>          | ml or irls                                         |
| <code>e(opt1)</code>         | optimization title, line 1                         |
| <code>e(opt2)</code>         | optimization title, line 2                         |
| <code>e(properties)</code>   | b V                                                |
| <code>e(predict)</code>      | program used to implement predict                  |
| <code>e(marginsok)</code>    | predictions allowed by margins                     |
| <code>e(marginsnotok)</code> | predictions disallowed by margins                  |
| <code>e(asbalanced)</code>   | factor variables <i>fvset</i> as <i>asbalanced</i> |
| <code>e(asobserved)</code>   | factor variables <i>fvset</i> as <i>asobserved</i> |
| Matrices                     |                                                    |
| <code>e(b)</code>            | coefficient vector                                 |
| <code>e(V)</code>            | variance–covariance matrix of the estimators       |
| <code>e(V_modelbased)</code> | model-based variance                               |
| Functions                    |                                                    |
| <code>e(sample)</code>       | marks estimation sample                            |

In addition to the above, the following is stored in `r()`:

|                       |                                                                                                                            |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|
| Matrices              |                                                                                                                            |
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

## Methods and formulas

The canonical reference on GLM is [McCullagh and Nelder \(1989\)](#). The term “generalized linear model” is from [Nelder and Wedderburn \(1972\)](#). Many people use the acronym GLIM for GLM models because of the classic GLM software tool GLIM, by [Baker and Nelder \(1985\)](#). See [Dobson and Barnett \(2018\)](#) for a concise introduction and overview. See [Rabe-Hesketh and Everitt \(2007\)](#) for more examples of GLM using Stata. [Hoffmann \(2004\)](#) focuses on applying generalized linear models, using real-world datasets, along with interpreting computer output, which for the most part is obtained using Stata.

This discussion highlights the details of parameter estimation and predicted statistics. For a more detailed treatment, and for information on variance estimation, see [Hardin and Hilbe \(2018\)](#). `glm` supports estimation with survey data. For details on VCEs with survey data, see [\[SVY\] Variance estimation](#).

`glm` obtains results by IRLS, as described in [McCullagh and Nelder \(1989\)](#), or by maximum likelihood using Newton–Raphson. The implementation here, however, allows user-specified weights, which we denote as  $v_j$  for the  $j$ th observation. Let  $M$  be the number of “observations” ignoring weights. Define

$$w_j = \begin{cases} 1 & \text{if no weights are specified} \\ v_j & \text{if fweights or iweights are specified} \\ Mv_j/(\sum_k v_k) & \text{if aweights or pweights are specified} \end{cases}$$

The number of observations is then  $N = \sum_j w_j$  if `fweights` are specified and  $N = M$  otherwise. Each IRLS step is performed by `regress` using  $w_j$  as the weights.

Let  $d_j^2$  denote the squared deviance residual for the  $j$ th observation:

For the Gaussian family,  $d_j^2 = (y_j - \hat{\mu}_j)^2$ .

For the Bernoulli family (binomial with denominator 1),

$$d_j^2 = \begin{cases} -2\ln(1 - \hat{\mu}_j) & \text{if } y_j = 0 \\ -2\ln(\hat{\mu}_j) & \text{otherwise} \end{cases}$$

For the binomial family with denominator  $m_j$ ,

$$d_j^2 = \begin{cases} 2y_j\ln(y_j/\hat{\mu}_j) + 2(m_j - y_j)\ln\{(m_j - y_j)/(m_j - \hat{\mu}_j)\} & \text{if } 0 < y_j < m_j \\ 2m_j\ln\{m_j/(m_j - \hat{\mu}_j)\} & \text{if } y_j = 0 \\ 2y_j\ln(y_j/\hat{\mu}_j) & \text{if } y_j = m_j \end{cases}$$

For the Poisson family,

$$d_j^2 = \begin{cases} 2\hat{\mu}_j & \text{if } y_j = 0 \\ 2\{y_j\ln(y_j/\hat{\mu}_j) - (y_j - \hat{\mu}_j)\} & \text{otherwise} \end{cases}$$

For the gamma family,  $d_j^2 = -2\{\ln(y_j/\hat{\mu}_j) - (y_j - \hat{\mu}_j)/\hat{\mu}_j\}$ .

For the inverse Gaussian,  $d_j^2 = (y_j - \hat{\mu}_j)^2/(\hat{\mu}_j^2 y_j)$ .

For the negative binomial,

$$d_j^2 = \begin{cases} 2\ln(1 + k\hat{\mu}_j)/k & \text{if } y_j = 0 \\ 2y_j\ln(y_j/\hat{\mu}_j) - 2\{(1 + ky_j)/k\}\ln\{(1 + ky_j)/(1 + k\hat{\mu}_j)\} & \text{otherwise} \end{cases}$$

Let  $\phi = 1$  if the scale parameter is set to one; otherwise, define  $\phi = \hat{\phi}_0(n - k)/n$ , where  $\hat{\phi}_0$  is the estimated scale parameter and  $k$  is the number of covariates in the model (including intercept).

Let  $\ln L_j$  denote the log likelihood for the  $j$ th observation:

For the Gaussian family,

$$\ln L_j = -\frac{1}{2} \left[ \left\{ \frac{(y_j - \hat{\mu}_j)^2}{\phi} \right\} + \ln(2\pi\phi) \right]$$

For the binomial family with denominator  $m_j$  (Bernoulli if all  $m_j = 1$ ),

$$\ln L_j = \phi \times \begin{cases} \ln\{\Gamma(m_j + 1)\} - \ln\{\Gamma(y_j + 1)\} - \ln\{\Gamma(m_j - y_j + 1)\} & \text{if } 0 < y_j < m_j \\ \quad + (m_j - y_j) \ln(1 - \hat{\mu}_j/m_j) + y_j \ln(\hat{\mu}_j/m_j) & \\ m_j \ln(1 - \hat{\mu}_j/m_j) & \text{if } y_j = 0 \\ m_j \ln(\hat{\mu}_j/m_j) & \text{if } y_j = m_j \end{cases}$$

For the Poisson family,

$$\ln L_j = \phi [y_j \ln(\hat{\mu}_j) - \hat{\mu}_j - \ln\{\Gamma(y_j + 1)\}]$$

For the gamma family,  $\ln L_j = -y_j/\hat{\mu}_j + \ln(1/\hat{\mu}_j)$ .

For the inverse Gaussian,

$$\ln L_j = -\frac{1}{2} \left\{ \frac{(y_j - \hat{\mu}_j)^2}{y_j \hat{\mu}_j^2} + 3 \ln(y_j) + \ln(2\pi) \right\}$$

For the negative binomial (let  $m = 1/k$ ),

$$\ln L_j = \phi \left[ \ln\{\Gamma(m + y_j)\} - \ln\{\Gamma(y_j + 1)\} - \ln\{\Gamma(m)\} \right. \\ \left. - m \ln(1 + \hat{\mu}_j/m) + y_j \ln\{\hat{\mu}_j/(\hat{\mu}_j + m)\} \right]$$

The overall deviance reported by `glm` is  $D^2 = \sum_j w_j d_j^2$ . The dispersion of the deviance is  $D^2$  divided by the residual degrees of freedom.

The Akaike information criterion (AIC) and Bayesian information criterion (BIC) are given by

$$\text{AIC} = \frac{-2 \ln L + 2k}{N}$$

$$\text{BIC} = D^2 - (N - k) \ln(N)$$

where  $\ln L = \sum_j w_j \ln L_j$  is the overall log likelihood.

The Pearson deviance reported by `glm` is  $\sum_j w_j r_j^2$ . The corresponding Pearson dispersion is the Pearson deviance divided by the residual degrees of freedom. `glm` also calculates the scaled versions of all of these quantities by dividing by the estimated scale parameter.

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## Also see

- [R] **glm postestimation** — Postestimation tools for glm
- [R] **cloglog** — Complementary log–log regression
- [R] **logistic** — Logistic regression, reporting odds ratios
- [R] **nbreg** — Negative binomial regression
- [R] **poisson** — Poisson regression
- [R] **regress** — Linear regression
- [BAYES] **bayes: glm** — Bayesian generalized linear models
- [FMM] **fmm: glm** — Finite mixtures of generalized linear regression models
- [ME] **meglm** — Multilevel mixed-effects generalized linear models
- [MI] **Estimation** — Estimation commands for use with mi estimate
- [SVY] **svy estimation** — Estimation commands for survey data
- [XT] **xtgee** — GEE population-averaged panel-data models
- [U] **20 Estimation and postestimation commands**

|                         |                            |                          |                      |
|-------------------------|----------------------------|--------------------------|----------------------|
| Postestimation commands | <a href="#">predict</a>    | <a href="#">margins</a>  | Remarks and examples |
| Methods and formulas    | <a href="#">References</a> | <a href="#">Also see</a> |                      |

## Postestimation commands

The following postestimation commands are available after `glm`:

| Command                         | Description                                                                                                                             |
|---------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <a href="#">contrast</a>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| * <a href="#">estat ic</a>      | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <a href="#">estat summarize</a> | summary statistics for the estimation sample                                                                                            |
| <a href="#">estat vce</a>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <a href="#">estat (svy)</a>     | postestimation statistics for survey data                                                                                               |
| <a href="#">estimates</a>       | cataloging estimation results                                                                                                           |
| <a href="#">etable</a>          | table of estimation results                                                                                                             |
| † <a href="#">forecast</a>      | dynamic forecasts and simulations                                                                                                       |
| † <a href="#">hausman</a>       | Hausman's specification test                                                                                                            |
| <a href="#">lincom</a>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| <a href="#">linktest</a>        | link test for model specification                                                                                                       |
| *† <a href="#">lrtest</a>       | likelihood-ratio test                                                                                                                   |
| <a href="#">margins</a>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <a href="#">marginsplot</a>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <a href="#">nlcom</a>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <a href="#">predict</a>         | predictions, residuals, influence statistics, and other diagnostic measures                                                             |
| <a href="#">predictnl</a>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <a href="#">pwcompare</a>       | pairwise comparisons of parameters                                                                                                      |
| <a href="#">suest</a>           | seemingly unrelated estimation                                                                                                          |
| <a href="#">test</a>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <a href="#">testnl</a>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*[estat ic](#) and [lrtest](#) are not appropriate after `glm`, `irls`.

†[forecast](#), [hausman](#), and [lrtest](#) are not appropriate with `svy` estimation results. [forecast](#) is also not appropriate with `mi` estimation results.

# predict

## Description for predict

predict creates a new variable containing predictions such as expected values, linear predictions, standard errors, residuals, Cook’s distance, diagonals of the “hat” matrix, weighted averages, differences between the observed and fitted outcomes, and equation-level scores.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic options]
```

| <i>statistic</i>  | Description                                                                    |
|-------------------|--------------------------------------------------------------------------------|
| Main              |                                                                                |
| <u>mu</u>         | expected value of $y$ ; the default                                            |
| <u>xb</u>         | linear prediction $\eta = \mathbf{x}_j\hat{\beta}$                             |
| <u>eta</u>        | synonym of <u>xb</u>                                                           |
| <u>stdp</u>       | standard error of the linear prediction                                        |
| <u>anscombe</u>   | <a href="#">Anscombe (1953)</a> residuals                                      |
| <u>cooksd</u>     | Cook’s distance                                                                |
| <u>deviance</u>   | deviance residuals                                                             |
| <u>hat</u>        | diagonals of the “hat” matrix                                                  |
| <u>likelihood</u> | a weighted average of standardized deviance and standardized Pearson residuals |
| <u>pearson</u>    | Pearson residuals                                                              |
| <u>response</u>   | differences between the observed and fitted outcomes                           |
| <u>score</u>      | first derivative of the log likelihood with respect to $\mathbf{x}_j\beta$     |
| <u>working</u>    | working residuals                                                              |

| <i>options</i>      | Description                                                                                 |
|---------------------|---------------------------------------------------------------------------------------------|
| Options             |                                                                                             |
| <u>nooffset</u>     | modify calculations to ignore offset variable                                               |
| <u>adjusted</u>     | adjust deviance residual to speed up convergence                                            |
| <u>standardized</u> | multiply residual by the factor $(1 - h)^{-1/2}$                                            |
| <u>studentized</u>  | multiply residual by one over the square root of the estimated scale parameter              |
| <u>modified</u>     | modify denominator of residual to be a reasonable estimate of the variance of <i>depvar</i> |

These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

mu, xb, stdp, and score are the only statistics allowed with svy estimation results.

## Options for predict

## Main

`mu`, the default, specifies that `predict` calculate the expected value of  $y$ , equal to  $g^{-1}(\mathbf{x}\hat{\boldsymbol{\beta}})$  [ $ng^{-1}(\mathbf{x}\hat{\boldsymbol{\beta}})$  for the binomial family].

`xb` calculates the linear prediction  $\eta = \mathbf{x}\hat{\boldsymbol{\beta}}$ .

`eta` is a synonym for `xb`.

`stdp` calculates the standard error of the linear prediction.

`anscombe` calculates the [Anscombe \(1953\)](#) residuals to produce residuals that closely follow a normal distribution.

`cooks` calculates Cook's distance, which measures the aggregate change in the estimated coefficients when each observation is left out of the estimation.

`deviance` calculates the deviance residuals. Deviance residuals are recommended by [McCullagh and Nelder \(1989\)](#) and by others as having the best properties for examining the goodness of fit of a GLM. They are approximately normally distributed if the model is correct. They may be plotted against the fitted values or against a covariate to inspect the model's fit. Also see the `pearson` option below.

`hat` calculates the diagonals of the "hat" matrix, analogous to linear regression.

`likelihood` calculates a weighted average of standardized deviance and standardized Pearson residuals.

`pearson` calculates the Pearson residuals. Pearson residuals often have markedly skewed distributions for nonnormal family distributions. Also see the `deviance` option above.

`response` calculates the differences between the observed and fitted outcomes.

`score` calculates the equation-level score,  $\partial \ln L / \partial (\mathbf{x}_j \boldsymbol{\beta})$ .

`working` calculates the working residuals, which are response residuals weighted according to the derivative of the link function.

## Options

`nooffset` is relevant only if you specified `offset` (*varname*) for `glm`. It modifies the calculations made by `predict` so that they ignore the offset variable; the linear prediction is treated as  $\mathbf{x}_j \mathbf{b}$  rather than as  $\mathbf{x}_j \mathbf{b} + \text{offset}_j$ .

`adjusted` adjusts the deviance residual to speed up the convergence to the limiting normal distribution. The adjustment deals with adding to the deviance residual a higher-order term that depends on the variance function family. This option is allowed only when `deviance` is specified.

`standardized` requests that the residual be multiplied by the factor  $(1-h)^{-1/2}$ , where  $h$  is the diagonal of the hat matrix. This operation is done to account for the correlation between *depvar* and its predicted value.

`studentized` requests that the residual be multiplied by one over the square root of the estimated scale parameter.

modified requests that the denominator of the residual be modified to be a reasonable estimate of the variance of *depyar*. The base residual is multiplied by the factor  $(k/w)^{-1/2}$ , where  $k$  is either one or the user-specified dispersion parameter and  $w$  is the specified weight (or one if left unspecified).

## margins

### Description for margins

`margins` estimates margins of response for expected values and linear predictions.

### Menu for margins

Statistics > Postestimation

### Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i>  | Description                                      |
|-------------------|--------------------------------------------------|
| <u>mu</u>         | expected value of $y$ ; the default              |
| <u>xb</u>         | linear prediction $\eta = \mathbf{x}\hat{\beta}$ |
| <u>eta</u>        | synonym for <code>xb</code>                      |
| <u>stdp</u>       | not allowed with <code>margins</code>            |
| <u>anscombe</u>   | not allowed with <code>margins</code>            |
| <u>cooksd</u>     | not allowed with <code>margins</code>            |
| <u>deviance</u>   | not allowed with <code>margins</code>            |
| <u>hat</u>        | not allowed with <code>margins</code>            |
| <u>likelihood</u> | not allowed with <code>margins</code>            |
| <u>pearson</u>    | not allowed with <code>margins</code>            |
| <u>response</u>   | not allowed with <code>margins</code>            |
| <u>score</u>      | not allowed with <code>margins</code>            |
| <u>working</u>    | not allowed with <code>margins</code>            |

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [\[R\] margins](#).

## Remarks and examples

Remarks are presented under the following headings:

*Predictions*

*Other postestimation commands*

## Predictions

### ▷ Example 1

After `glm` estimation, `predict` may be used to obtain various predictions based on the model. In [example 2](#) of [\[R\] glm](#), we mentioned that the complementary log–log link seemed to fit the data better than the logit link. Now, we go back and obtain the fitted values and deviance residuals:

```
. use https://www.stata-press.com/data/r19/ldose
. glm r ldose, family(binomial n) link(logit)
 (output omitted)
. predict mu_logit
(option mu assumed; predicted mean r)
. predict dr_logit, deviance
. quietly glm r ldose, f(binomial n) l(cloglog)
. predict mu_cl
(option mu assumed; predicted mean r)
. predict dr_cl, d
. format mu_logit dr_logit mu_cl dr_cl %9.5f
. list r mu_logit dr_logit mu_cl dr_cl, sep(4)
```

|    | r  | mu_logit | dr_logit | mu_cl    | dr_cl    |
|----|----|----------|----------|----------|----------|
| 1. | 6  | 3.45746  | 1.28368  | 5.58945  | 0.18057  |
| 2. | 13 | 9.84167  | 1.05969  | 11.28067 | 0.55773  |
| 3. | 18 | 22.45139 | -1.19611 | 20.95422 | -0.80330 |
| 4. | 28 | 33.89761 | -1.59412 | 30.36942 | -0.63439 |
| 5. | 52 | 50.09584 | 0.60614  | 47.77644 | 1.28883  |
| 6. | 53 | 53.29092 | -0.12716 | 54.14273 | -0.52366 |
| 7. | 61 | 59.22216 | 1.25107  | 61.11331 | -0.11878 |
| 8. | 60 | 58.74297 | 1.59398  | 59.94723 | 0.32495  |

In six of the eight cases,  $|\text{dr\_logit}| > |\text{dr\_cl}|$ . The above represents only one of the many available options for `predict`. See [Hardin and Hilbe \(2018\)](#) for a more in-depth examination.



## Other postestimation commands

### □ Technical note

After `glm` estimation, you may perform any of the postestimation commands that you would perform after any other kind of estimation in Stata; see [\[U\] 20 Estimation and postestimation commands](#). Below, we test the joint significance of all the interaction terms.

```
. use https://www.stata-press.com/data/r19/beetle, clear
. glm r beetle#c.ldose, family(binomial n) link(cloglog)
(output omitted)
. testparm i.beetle beetle#c.ldose
(1) [r]2.beetle = 0
(2) [r]3.beetle = 0
(3) [r]2.beetle#c.ldose = 0
(4) [r]3.beetle#c.ldose = 0
 chi2(4) = 249.69
 Prob > chi2 = 0.0000
```

If you wanted to print the variance–covariance matrix of the estimators, you would type `estat vce`.

If you use the `linktest` postestimation command, you must also specify the `family()` and `link()` options; see [R] [linktest](#).

□

## Methods and formulas

We follow the terminology used in *Methods and formulas* of [R] [glm](#).

The deviance residual calculated by `predict` following `glm` is  $r_j^D = \text{sign}(y_j - \hat{\mu}_j) \sqrt{d_j^2}$ .

The Pearson residual calculated by `predict` following `glm` is

$$r_j^P = \frac{y_j - \hat{\mu}_j}{\sqrt{V(\hat{\mu}_j)}}$$

where  $V(\hat{\mu}_j)$  is the family-specific variance function.

$$V(\hat{\mu}_j) = \begin{cases} \hat{\mu}_j(1 - \hat{\mu}_j/m_j) & \text{if binomial or Bernoulli } (m_j = 1) \\ \hat{\mu}_j^2 & \text{if gamma} \\ 1 & \text{if Gaussian} \\ \hat{\mu}_j^3 & \text{if inverse Gaussian} \\ \hat{\mu}_j + k\hat{\mu}_j^2 & \text{if negative binomial} \\ \hat{\mu}_j & \text{if Poisson} \end{cases}$$

The response residuals are given by  $r_j^R = y_j - \hat{\mu}_j$ . The working residuals are

$$r_j^W = (y_j - \hat{\mu}_j) \left( \frac{\partial \eta}{\partial \mu} \right)_j$$

and the score residuals are

$$r_j^S = \frac{y_j - \hat{\mu}_j}{V(\hat{\mu}_j)} \left( \frac{\partial \eta}{\partial \mu} \right)_j^{-1}$$

Define  $\widehat{W} = V(\hat{\mu})$  and  $X$  to be the covariate matrix.  $h_j$ , then, is the  $j$ th diagonal of the hat matrix given by

$$\widehat{H} = \widehat{W}^{1/2} X (X^T \widehat{W} X)^{-1} X^T \widehat{W}^{1/2}$$

As a result, the likelihood residuals are given by

$$r_j^L = \text{sign}(y_j - \hat{\mu}_j) \{h_j(r_j^{P'})^2 + (1 - h_j)(r_j^{D'})^2\}^{1/2}$$

where  $r_j^{P'}$  and  $r_j^{D'}$  are the standardized Pearson and standardized deviance residuals, respectively. By *standardized*, we mean that the residual is divided by  $\{1 - h_j\}^{1/2}$ .

Cook's distance is an overall measure of the change in the regression coefficients caused by omitting the  $i$ th observation from the analysis. Computationally, Cook's distance is obtained as

$$C_j = \frac{(r_j^{P'})^2 h_j}{k(1 - h_j)}$$

where  $k$  is the number of regressors, including the constant.

Anscombe residuals are given by

$$r_j^A = \frac{A(y_j) - A(\hat{\mu}_j)}{A'(\hat{\mu}_j)\{V(\hat{\mu}_j)\}^{1/2}}$$

where

$$A(\cdot) = \int \frac{d\mu}{V^{1/3}(\mu)}$$

Deviance residuals may be adjusted (`predict, adjusted`) to make the following correction:

$$r_j^{Da} = r_j^D + \frac{1}{6}\rho_3(\theta)$$

where  $\rho_3(\theta)$  is a family-specific correction. See [Hardin and Hilbe \(2018\)](#) for the exact forms of  $\rho_3(\theta)$  for each family.

## References

- Anscombe, F. J. 1953. Contribution of discussion paper by H. Hotelling "New light on the correlation coefficient and its transforms". *Journal of the Royal Statistical Society*, B ser., 15: 229–230. <https://doi.org/10.1111/j.2517-6161.1953.tb00136.x>.
- Hardin, J. W., and J. M. Hilbe. 2018. *Generalized Linear Models and Extensions*. 4th ed. College Station, TX: Stata Press.
- McCullagh, P., and J. A. Nelder. 1989. *Generalized Linear Models*. 2nd ed. London: Chapman and Hall/CRC.
- Newson, R. B. 2013. Attributable and unattributable risks and fractions and other scenario comparisons. *Stata Journal* 13: 672–698.

## Also see

[R] [glm](#) — Generalized linear models

[R] [regress postestimation](#) — Postestimation tools for regress

[U] [20 Estimation and postestimation commands](#)

|                                |                                      |                            |                          |                                      |
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## Description

`gmm` performs generalized method of moments (GMM) estimation. With the interactive version of the command, you enter the residual equation for each moment condition directly into the dialog box or on the command line by using substitutable expressions. The moment-evaluator program version gives you greater flexibility in exchange for increased complexity; with this version, you write a program in an ado-file that calculates the moments based on a vector of parameters passed to it.

`gmm` can fit both single- and multiple-equation models. It allows moment conditions of the form  $E\{\mathbf{z}_i u_i(\boldsymbol{\beta})\} = \mathbf{0}$ , where  $\mathbf{z}_i$  is a vector of instruments, and  $u_i(\boldsymbol{\beta})$  is an error term, as well as more general moment conditions of the form  $E\{\mathbf{h}_i(\mathbf{z}_i; \boldsymbol{\beta})\} = \mathbf{0}$ . `gmm` works with cross-sectional, time-series, and longitudinal (panel) data.

## Menu

Statistics > Endogenous covariates > Generalized method of moments estimation

## Syntax

### Interactive version

```
gmm ([reqname1:]rexp1) ([reqname2:]rexp2)...[if] [in] [weight] [, options]
```

### Moment-evaluator program version

```
gmm moment_prog [if] [in] [weight] , { equations(namelist) | nequations(#) }
```

```
{ parameters(namelist) | nparameters(#) } [options] [program_options]
```

*reqname<sub>j</sub>* is the *j*th residual equation name,

*rexp<sub>j</sub>* is the substitutable expression for the *j*th residual equation, and

*moment\_prog* is a moment-evaluator program.

| <i>options</i> | Description |
|----------------|-------------|
|----------------|-------------|

#### Model

```
derivative([reqname|#]/name = dexpjk)
```

specify derivative of *reqname* (or #) with respect to parameter *name*;  
can be specified more than once (interactive version only)

\* twostep

use two-step GMM estimator; the default

\* onestep

use one-step GMM estimator

\* igmm

use iterative GMM estimator

```
variables(varlist)
```

specify variables in model

```
nocommonesample
```

do not restrict estimation sample to be the same for all equations

#### Instruments

```
instruments([reqlist:]varlist [, noconstant])
```

specify instruments; can be specified more than once

```
xtinstruments([reqlist:]varlist , lags(#1/#2))
```

specify panel-style instruments; can be specified more than once

#### Weight matrix

```
wmatrix(wmtype [, independent])
```

specify weight matrix; *wmtype* may be robust, cluster *clustvar*,  
hac *hacspec*, or unadjusted

```
center
```

center moments in weight-matrix computation

```
winitial(iwtype [, independent])
```

specify initial weight matrix; *iwtype* may be unadjusted,  
identify, xt *xtspec*, or the name of a Stata matrix

#### SE/Robust

```
vce(vcetype [, independent])
```

*vcetype* may be robust, cluster *clustvar*, bootstrap,  
jackknife, hac *hacspec*, or unadjusted

```
quickderivatives
```

use alternative method of computing numerical derivatives  
for VCE

## Reporting

|                              |                                                                                                                                                  |
|------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>level(#)</code>        | set confidence level; default is <code>level(95)</code>                                                                                          |
| <code>title(string)</code>   | display <i>string</i> as title above the table of parameter estimates                                                                            |
| <code>title2(string)</code>  | display <i>string</i> as subtitle                                                                                                                |
| <code>display_options</code> | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |

## Optimization

|                                   |                                                                                                        |
|-----------------------------------|--------------------------------------------------------------------------------------------------------|
| <code>from(initial_values)</code> | specify initial values for parameters                                                                  |
| ‡ <code>igmmiterate(#)</code>     | specify maximum number of iterations for iterated GMM estimator                                        |
| ‡ <code>igmmeps(#)</code>         | specify # for iterated GMM parameter convergence criterion; default is <code>igmmeps(1e-6)</code>      |
| ‡ <code>igmmweps(#)</code>        | specify # for iterated GMM weight-matrix convergence criterion; default is <code>igmmweps(1e-6)</code> |
| <code>optimization_options</code> | control the optimization process; seldom used                                                          |
| <code>coeflegend</code>           | display legend instead of statistics                                                                   |

\*You can specify at most one of these options.

‡These options may be specified only when `igmm` is specified.

| <i>program_options</i> | Description |
|------------------------|-------------|
|------------------------|-------------|

## Model

|                                     |                                                                    |
|-------------------------------------|--------------------------------------------------------------------|
| <code>evaluator_options</code>      | additional options to be passed to the moment-evaluator program    |
| * <code>hasderivatives</code>       | moment-evaluator program can calculate parameter-level derivatives |
| * <code>haslfdderivatives</code>    | moment-evaluator program can calculate linear-form derivatives     |
| † <code>equations(namelist)</code>  | specify residual equation names                                    |
| † <code>nequations(#)</code>        | specify number of residual equations                               |
| ‡ <code>parameters(namelist)</code> | specify parameter names                                            |
| ‡ <code>nparameters(#)</code>       | specify number of parameters                                       |

\*You may not specify both `hasderivatives` and `haslfdderivatives`.

†You must specify `equations(namelist)` or `nequations(#)`; you may specify both.

‡You must specify `parameters(namelist)` or `nparameters(#)`; you may specify both.

$rexp_j$  and  $dexp_{jk}$  may contain factor variables and time-series operators; see [U] 11.4.3 Factor variables and [U] 11.4.4 Time-series varlists.

`bayesboot`, `bootstrap`, `by`, `collect`, `jackknife`, `rolling`, and `statsby` are allowed; see [U] 11.1.10 Prefix commands.

Weights are not allowed with the `bootstrap` prefix; see [R] `bootstrap`.

`aweights` are not allowed with the `jackknife` prefix; see [R] `jackknife`.

`aweights`, `fweights`, `iweights`, and `pweights` are allowed; see [U] 11.1.6 weight.

`coeflegend` does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

$rexp_j$  and  $dexp_{jk}$  are substitutable expressions, that is, Stata expressions that also contain parameters to be estimated. The parameters are enclosed in curly braces and must satisfy the naming requirements for variables; `{beta}` is an example of a parameter. The notation `{lcname:varlist}` is allowed for linear combinations of multiple covariates and their parameters. For example, `{xb: mpg price turn _cons}` defines a linear combination of the variables `mpg`, `price`, `turn`, and `_cons` (the constant term). See *Substitutable expressions* under *Remarks and examples* below.

## Options

Model

`derivative([reqname|#]/name = dexpjk)` specifies the derivative of residual equation *reqname* or # with respect to parameter *name*. If *reqname* or # is not specified, gmm assumes that the derivative applies to the first residual equation.

For a moment condition of the form  $E\{\mathbf{z}_{ji}u_{ji}(\boldsymbol{\beta})\} = \mathbf{0}$ , `derivative(j/ $\beta_k = dexp_{jk}$ )` is to contain a substitutable expression for  $\partial u_{ji}/\partial \beta_k$ . If you specified *m* as the *reqname*, then for a moment condition of the form  $E\{\mathbf{z}_{mi}u_{mi}(\boldsymbol{\beta})\} = \mathbf{0}$ , you can specify `derivative(m/ $\beta_k = dexp_{mk}$ )`, where *m* is the index of *m*.

*dexp<sub>jk</sub>* uses the same substitutable expression syntax as is used to specify residual equations. If you declare a linear combination in a residual equation, you provide the derivative for the linear combination; gmm then applies the chain rule for you. See [Specifying derivatives](#) under *Remarks and examples* below for examples.

If you do not specify the `derivative()` option, gmm calculates derivatives numerically. You must either specify no derivatives or specify a derivative for each of the *k* parameters that appears in each of the *j* residual equations unless the derivative is identically zero. You cannot specify some analytic derivatives and have gmm compute the rest numerically.

`twostep`, `onestep`, and `igmm` specify which estimator is to be used. You can specify at most one of these options. `twostep` is the default.

`twostep` requests the two-step GMM estimator. gmm obtains parameter estimates based on the initial weight matrix, computes a new weight matrix based on those estimates, and then reestimates the parameters based on that weight matrix.

`onestep` requests the one-step GMM estimator. The parameters are estimated based on an initial weight matrix, and no updating of the weight matrix is performed except when calculating the appropriate variance–covariance (VCE) matrix.

`igmm` requests the iterative GMM estimator. gmm obtains parameter estimates based on the initial weight matrix, computes a new weight matrix based on those estimates, reestimates the parameters based on that weight matrix, computes a new weight matrix, and so on, to convergence. Convergence is declared when the relative change in the parameter vector is less than `igmmeps()`, the relative change in the weight matrix is less than `igmmweps()`, or `igmmiterate()` iterations have been completed. [Hall \(2005, sec. 2.4 and 3.6\)](#) mentions that there may be gains to finite-sample efficiency from using the iterative estimator.

`variables(varlist)` specifies the variables in the model. gmm ignores observations for which any of these variables has a missing value. If you do not specify `variables()`, then gmm assumes all the observations are valid and issues an error message if any residual equations evaluate to missing for any observations at the initial value of the parameter vector.

`nocommonesample` requests that gmm not restrict the estimation sample to be the same for all equations. By default, gmm will restrict the estimation sample to observations that are available for all equations in the model, mirroring the behavior of other multiple-equation estimators such as `nlsur`, `sureg`, or `reg3`. For certain models, however, different equations can have different numbers of observations. For these models, you should specify `nocommonesample`. See [Dynamic panel-data models](#) below for one application of this option. You cannot specify weights if you specify `nocommonesample`.

## Instruments

`instruments([reqlist:]varlist[, noconstant])` specifies a list of instrumental variables to be used.

If you specify a single residual equation, then you do not need to specify the equations to which the instruments apply; you can omit the *reqlist* and simply specify `instruments(varlist)`. By default, a constant term is included in *varlist*; to omit the constant term, specify `instruments(varlist, noconstant)`.

If your model has multiple moment conditions of the form

$$E \begin{Bmatrix} \mathbf{z}_{1i} u_{1i}(\beta) \\ \dots \\ \mathbf{z}_{qi} u_{qi}(\beta) \end{Bmatrix} = \mathbf{0}$$

then you can specify multiple corresponding residual equations. Then, specify the *reqname* or an *reqlist* to indicate the residual equations for which the list of variables is to be used as instruments if you do not want that list applied to all the residual equations. For example, you might type

```
gmm (main:exp1) (exp2) (exp3), instruments(z1 z2) ///
 instruments(2: z3) instruments(main 3: z4)
```

Variables `z1` and `z2` will be used as instruments for all three equations, `z3` will be used as an instrument for the second equation, and `z4` will be used as an instrument for the first and third equations. Notice that we chose to supply a name for the first residual equation but not the second two, identifying each by its equation number.

*varlist* may contain factor variables and time-series operators; see [U] 11.4.3 Factor variables and [U] 11.4.4 Time-series varlists, respectively.

`xtinstruments([reqlist:]varlist, lags(#1/2))` is for use with panel-data models in which the set of available instruments depends on the time period. As with `instruments()`, you can prefix the list of variables with residual equation names or numbers to target instruments to specific equations. Unlike with `instruments()`, a constant term is not included in *varlist*. You must `xtset` your data before using this option; see [XT] `xtset`.

If you specify

```
gmm ..., xtinstruments(x, lags(1/.)) ...
```

then for panel *i* and period *t*, `gmm` uses  $x_{i,t-1}, x_{i,t-2}, \dots, x_{i1}$  as instruments. More generally, specifying `xtinstruments(x, lags(#1, #2))` uses  $x_{i,t-\#_1}, \dots, x_{i,t-\#_2}$  as instruments; setting `#2 = .` requests all available lags. `#1` and `#2` must be zero or positive integers.

`gmm` automatically excludes observations for which no valid instruments are available. It does, however, include observations for which only a subset of the lags is available. For example, if you request that lags one through three be used, then `gmm` will include the observations for the second and third time periods even though fewer than three lags are available as instruments.

## Weight matrix

`wmatrix(wmtype[, independent])` specifies the type of weight matrix to be used in conjunction with the two-step and iterated GMM estimators.

`wmatrix(robust)`, the default, requests a weight matrix that is appropriate when the errors are independent but not necessarily identically distributed.

`wmatrix(cluster clustvar)` requests a weight matrix that accounts for arbitrary correlation among observations within clusters identified by *clustvar*.

`wmatrix(hac hacspec)` requests a heteroskedasticity- and autocorrelation-consistent (HAC) weight matrix. The full syntax of *hacspec* is one of the following:

`wmatrix(hac kernel [#])` requests a HAC weight matrix using the specified kernel (see below) with optional # lags. The bandwidth of a kernel is equal to # + 1. If # is not specified, a kernel with  $N - 2$  lags is used, where  $N$  is the sample size.

`wmatrix(hac kernel opt [#])` requests a HAC weight matrix using the specified kernel (see below), and the lag order is selected using Newey and West's (1994) optimal lag-selection algorithm. # is an optional tuning parameter that affects the lag order selected; see the [discussion](#) in *Methods and formulas*.

*kernel* may be one of the following:

`bartlett` or `nwest` requests the Bartlett (Newey–West) kernel.

`parzen` or `gallant` requests the Parzen (Gallant 1987) kernel.

`quadraticspectral` or `andrews` requests the quadratic spectral (Andrews 1991) kernel.

`wmatrix(unadjusted)` requests a weight matrix that is suitable when the errors are homoskedastic. In some applications, the GMM estimator so constructed is known as the (nonlinear) two-stage least-squares (2SLS) estimator.

`independent` creates a weight matrix that assumes moment conditions are independent. It is often used to replicate other models that can be motivated outside the GMM framework, such as the estimation of a system of equations by system-wide 2SLS. `independent` has no effect if only one residual equation is specified.

`wmatrix()` has no effect if `onestep` is also specified.

`center` requests that the sample moments be centered (demeaned) when computing GMM weight matrices. By default, centering is not done.

`winitial(iwtype[ , independent ])` specifies the weight matrix to use to obtain the first-step parameter estimates.

`winitial(unadjusted)`, the default, requests a weight matrix that assumes the moment conditions are independent and identically distributed. This matrix is of the form  $(\mathbf{Z}'\mathbf{Z})^{-1}$ , where  $\mathbf{Z}$  represents all the instruments specified in the `instruments()` option. To avoid a singular weight matrix, you should specify at least  $q - 1$  moment conditions of the form  $E\{\mathbf{z}_{hi}u_{hi}(\beta)\} = \mathbf{0}$ , where  $q$  is the number of moment conditions, or you should specify `independent`.

`winitial(identity)` requests that the identity matrix be used.

`winitial(xt xtspec)` is for use with dynamic panel-data models in which one of the residual equations is specified in first-differences form. *xtspec* is a string consisting of the letters “L” and “D”, the length of which is equal to the number of residual equations in the model. You specify “L” for a residual equation if that residual equation is written in levels, and you specify “D” for a residual equation if it is written in first differences; *xtspec* is not case sensitive. When you specify this option, you can specify at most one residual equation in levels and one residual equation in first differences. See the examples listed in [Dynamic panel-data models](#) under *Remarks and examples* below.

`winitial(matname)` requests that Stata matrix *matname* be used. You cannot specify `independent` if you specify `winitial(matname)`.

`independent` creates a weight matrix that assumes moment conditions are independent. Elements of the weight matrix corresponding to covariances between two moment conditions are set equal to zero. `independent` has no effect if only one residual equation is specified.

---

#### SE/Robust

`vce(vcetype[, independent])` specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster` `clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

`vce(unadjusted)` specifies that an unadjusted (nonrobust) VCE matrix be used; this, along with the `twostep` option, results in the “optimal two-step GMM” estimates often discussed in textbooks.

The default `vcetype` is based on the `wmtype` specified in the `wmatrix()` option. If `wmatrix()` is specified but `vce()` is not, then `vcetype` is set equal to `wmtype`. To override this behavior and obtain an unadjusted (nonrobust) VCE matrix, specify `vce(unadjusted)`.

The syntax for `vcetypes` other than `bootstrap` and `jackknife` is identical to those for `wmatrix()`.

`vce(bootstrap)` or `vce(jackknife)` results in standard errors based on the bootstrap or jackknife, respectively. See [R] [vce\\_option](#), [R] [bootstrap](#), and [R] [jackknife](#) for more information on these VCEs.

`quickderivatives` requests that an alternative method be used to compute the numerical derivatives for the VCE. This option has no effect if you specify the `derivatives()`, `hasderivatives`, or `haslfdderivatives` option.

The VCE depends on a matrix of partial derivatives that `gmm` must compute numerically unless you supply analytic derivatives. This Jacobian matrix will be especially large if your model has many instruments, residual equations, or parameters.

By default, `gmm` computes each element of the Jacobian matrix individually, searching for an optimal step size each time. Although this procedure results in accurate derivatives, it is computationally taxing: `gmm` may have to evaluate the moments of your model five or more times for each element of the Jacobian matrix.

When you specify the `quickderivatives` option, `gmm` computes all derivatives corresponding to a parameter at once, using a fixed step size proportional to the parameter’s value. This method requires just two evaluations of the model’s moments to compute an entire column of the Jacobian matrix and therefore has the most impact when you specify many instruments or residual equations.

Most of the time, the two methods produce virtually identical results, but the `quickderivatives` method may fail if a residual equation is highly nonlinear or if instruments differ by orders of magnitude. In the rare case where you specify `quickderivatives` and obtain suspiciously large or small standard errors, try refitting your model without this option.

---

#### Reporting

`level(#)`; see [R] [Estimation options](#).

`title(string)` specifies an optional title that will be displayed just above the table of parameter estimates.

`title2(string)` specifies an optional subtitle that will be displayed between the title specified in `title()` and the table of parameter estimates. If `title2()` is specified but `title()` is not, `title2()` has the same effect as `title()`.

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

#### Optimization

`from(initial_values)` specifies the initial values to begin the estimation. You can specify a parameter name, its initial value, another parameter name, its initial value, and so on, or you can specify a  $1 \times k$  matrix, where  $k$  is the number of parameters in the model. For example, to initialize `alpha` to 1.23 and `delta` to 4.57, you would type

```
gmm ..., from(alpha 1.23 delta 4.57) ...
```

or equivalently

```
matrix define initval = (1.23, 4.57)
gmm ..., from(initval) ...
```

Initial values declared in the `from()` option override any that are declared within substitutable expressions. If you specify a parameter that does not appear in your model, `gmm` exits with an error message. If you specify a matrix, the values must be in the same order in which the parameters are declared in your model.

`igmmiterate(#)`, `igmmeps(#)`, and `igmmweps(#)` control the iterative process for the iterative GMM estimator. These options can be specified only if you also specify `igmm`.

`igmmiterate(#)` specifies the maximum number of iterations to perform with the iterative GMM estimator. The default is the number set using `set maxiter`, which is 300 by default.

`igmmeps(#)` specifies the convergence criterion used for successive parameter estimates when the iterative GMM estimator is used. The default is `igmmeps(1e-6)`. Convergence is declared when the relative difference between successive parameter estimates is less than `igmmeps()` and the relative difference between successive estimates of the weight matrix is less than `igmmweps()`.

`igmmweps(#)` specifies the convergence criterion used for successive estimates of the weight matrix when the iterative GMM estimator is used. The default is `igmmweps(1e-6)`. Convergence is declared when the relative difference between successive parameter estimates is less than `igmmeps()` and the relative difference between successive estimates of the weight matrix is less than `igmmweps()`.

*optimization\_options*: `technique()`, `conv_maxiter()`, `conv_ptol()`, `conv_vtol()`, `conv_nrtol()`, `tracelevel()`. `technique()` specifies the optimization technique to use; `gn` (the default), `nr`, `dfp`, and `bfgs` are allowed. `conv_maxiter()` specifies the maximum number of iterations; `conv_ptol()`, `conv_vtol()`, and `conv_nrtol()` specify the convergence criteria for the parameters, gradient, and scaled Hessian, respectively. `tracelevel()` allows you to obtain additional details during the iterative process. See [M-5] [optimize\(\)](#).

The following options pertain only to the moment-evaluator program version of `gmm`.

Model

`evaluator_options` refer to any options allowed by your `moment_prog`.

`hasderivatives` and `haslfdderivatives` indicate that you have written your moment-evaluator program to compute derivatives. You may specify one or the other but not both. If you do not specify either of these options, `gmm` computes the derivatives numerically.

`hasderivatives` indicates that your moment-evaluator program computes parameter-level derivatives.

`haslfdderivatives` indicates that your moment-evaluator program computes equation-level derivatives and is useful only when you specify the parameters of your model using the `{lcname:varlist}` syntax of the `parameters()` option.

See *Details of moment-evaluator programs* below for more information.

`equations(namelist)` specifies the names of the residual equations in the model. If you specify both `equations()` and `nequations()`, the number of names in the former must match the number specified in the latter.

`nequations(#)` specifies the number of residual equations in the model. If you do not specify names with the `equations()` option, `gmm` numbers the residual equations 1, 2, 3, .... If you specify both `equations()` and `nequations()`, the number of names in the former must match the number specified in the latter.

`parameters(namelist)` specifies the names of the parameters in the model. The names of the parameters must comply with the naming conventions of Stata's variables; see [U] 11.3 **Naming conventions**.

Alternatively, you can use parameter equation notation to specify linear combinations of parameters. Each linear combination is of the form `{lcname:varlist}`, where `varlist` is one or more variable names. Specify the **system variable** `_cons` in `varlist` to include a constant term. Distinguish between `{lcname:varlist}`, in which `lcname` identifies the linear combination, and `(reqname:rexp)`, in which `reqname` identifies the residual equation. When you use linear-combination syntax, `gmm` prepends each element of the parameter vector passed to your evaluator program with `lcname:` to generate unique names.

If you specify both `parameters()` and `nparameters()`, the number of names in the former must match the number specified in the latter.

`nparameters(#)` specifies the number of parameters in the model. If you do not specify names with the `parameters()` option, `gmm` names them `b1`, `b2`, ..., `b#`. If you specify both `parameters()` and `nparameters()`, the number of names in the former must match the number specified in the latter.

The following option is available with `gmm` but is not shown in the dialog box:

`coeflegend`; see [R] **Estimation options**.

## Remarks and examples

Remarks are presented under the following headings:

*Introduction*  
*Substitutable expressions*  
*The weight matrix and two-step estimation*  
*Obtaining standard errors*  
*Factor-variable coefficients in multiple residual functions*  
*Parameter interpretation using margins*  
*Exponential (Poisson) regression models*  
*Specifying derivatives*  
*Exponential regression models with panel data*  
*Rational-expectations models*  
*System estimators*  
*Dynamic panel-data models*  
*Details of moment-evaluator programs*

## Introduction

The GMM estimator is a workhorse of modern econometrics and is discussed in all the leading textbooks, including Cameron and Trivedi (2005, 2022), Davidson and MacKinnon (1993), Greene (2018, 500–534), Ruud (2000), Hayashi (2000), Wooldridge (2010), Hamilton (1994), and Baum (2006). An excellent treatise on GMM with a focus on time-series applications is Hall (2005). The collection of papers by Mátyás (1999) provides both theoretical and applied aspects of GMM. Here we give a brief introduction to the methodology and emphasize how the various options of `gmm` are used.

The starting point for the GMM estimator is the analogy principle, which says we can estimate a parameter by replacing a population moment condition with its sample analogue. For example, the mean of an independent and identically distributed (i.i.d.) population is defined as the value  $\mu$  such that the first (central) population moment is zero; that is,  $\mu$  solves  $E(y - \mu) = 0$ , where  $y$  is a random draw from the population. The analogy principle tells us that to obtain an estimate,  $\hat{\mu}$ , of  $\mu$ , we replace the population-expectations operator with its sample analogue (Manski 1988; Wooldridge 2010),

$$E(y - \mu) = 0 \longrightarrow \frac{1}{N} \sum_{i=1}^N (y_i - \hat{\mu}) = 0 \longrightarrow \hat{\mu} = \frac{1}{N} \sum_{i=1}^N y_i$$

where  $N$  denotes sample size, and  $y_i$  represents the  $i$ th observation of  $y$  in our dataset. The estimator  $\hat{\mu}$  is known as the method of moments (MM) estimator because we started with a population moment condition and then applied the analogy principle to obtain an estimator that depends on the observed data.

Ordinary least-squares (OLS) regression can also be viewed as an MM estimator. In the model

$$y = \mathbf{x}'\boldsymbol{\beta} + u$$

we assume that  $u$  has mean zero conditional on  $\mathbf{x}$ :  $E(u|\mathbf{x}) = 0$ . This conditional expectation implies the unconditional expectation  $E(\mathbf{x}u) = \mathbf{0}$  because, with the law of iterated expectations,

$$E(\mathbf{x}u) = E_{\mathbf{x}} \{E(\mathbf{x}u|\mathbf{x})\} = E_{\mathbf{x}} \{\mathbf{x} E(u|\mathbf{x})\} = \mathbf{0}$$

(Using the law of iterated expectations to derive unconditional expectations based on conditional expectations, perhaps motivated by subject theory, is extremely common in GMM estimation.) Continuing, we see that

$$E(\mathbf{x}u) = E\{\mathbf{x}(y - \mathbf{x}'\boldsymbol{\beta})\} = \mathbf{0}$$

Applying the analogy principle, we obtain

$$E\{\mathbf{x}(y - \mathbf{x}'\boldsymbol{\beta})\} \rightarrow \frac{1}{N} \sum_{i=1}^N \mathbf{x}_i(y_i - \mathbf{x}'_i\boldsymbol{\beta}) = \mathbf{0}$$

so that

$$\widehat{\boldsymbol{\beta}} = \left( \sum_i \mathbf{x}_i \mathbf{x}'_i \right)^{-1} \sum_i \mathbf{x}_i y_i$$

which is just the more familiar formula  $\widehat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'\mathbf{y}$  written with summation notation.

In both of the previous examples, the number of parameters we were estimating equaled the number of moment conditions. In the first example, we estimated one parameter,  $\mu$ , and had one moment condition  $E(y - \mu) = 0$ . In the second example, the parameter vector  $\boldsymbol{\beta}$  had  $k$  elements, as did the vector of regressors  $\mathbf{x}$ , yielding  $k$  moment conditions. Ignoring peculiar cases, we see that a model of  $m$  equations in  $m$  unknowns has a unique solution; and because the residual equations in these examples were linear, we could solve for the parameters analytically. If the moment conditions had been nonlinear, we would have had to use numerical techniques to solve for the parameters, but that is not a significant limitation with modern computers.

What if we have more moment conditions than parameters? Say we have  $q$  moment conditions and  $k$  parameters. A model of  $q > k$  equations in  $k$  unknowns does not have a unique solution. Any size- $k$  subset of the moment conditions would yield a consistent parameter estimate, though the parameter estimate would in general be different depending on which  $k$  moment conditions we used.

For concreteness, let's return to our regression model,

$$y = \mathbf{x}'\boldsymbol{\beta} + u$$

Now, however, we no longer wish to assume that  $E(\mathbf{x}u) = \mathbf{0}$ ; we suspect that the error term  $u$  affects one or more elements of  $\mathbf{x}$ . Thus, we can no longer use the OLS estimator. Suppose we have a vector  $\mathbf{z}$  with the properties that  $E(\mathbf{z}u) = \mathbf{0}$ , that the rank of  $E(\mathbf{z}'\mathbf{z})$  equals  $q$ , and that the rank of  $E(\mathbf{z}'\mathbf{x}) = k$ . The first assumption simply states that  $\mathbf{z}$  is not correlated with the error term. The second assumption rules out perfect collinearity among the elements of  $\mathbf{z}$ . The third assumption, known as the rank condition in econometrics, ensures that  $\mathbf{z}$  is sufficiently correlated with  $\mathbf{x}$  and that the estimator is feasible. If some elements of  $\mathbf{x}$  are not correlated with  $u$ , then they should also appear in  $\mathbf{z}$ .

If  $q < k$ , then the rank of  $E(\mathbf{z}'\mathbf{x}) < k$ , which violates the rank condition.

If  $q = k$ , then we can use the simpler MM estimator we already discussed; we would obtain what is sometimes called the simple instrumental-variables estimator  $\widehat{\boldsymbol{\beta}} = \left( \sum_i \mathbf{z}_i \mathbf{z}'_i \right)^{-1} \sum_i \mathbf{z}_i y_i$ . The rank condition ensures that  $\sum_i \mathbf{z}_i \mathbf{z}'_i$  is invertible, at least in the population.

If  $q > k$ , the GMM estimator chooses the value,  $\widehat{\boldsymbol{\beta}}$ , that minimizes a quadratic function of the moment conditions. We could define

$$\widehat{\boldsymbol{\beta}} \equiv \arg \min_{\boldsymbol{\beta}} \left\{ \frac{1}{N} \sum_i \mathbf{z}_i u_i(\boldsymbol{\beta}) \right\}' \left\{ \frac{1}{N} \sum_i \mathbf{z}_i u_i(\boldsymbol{\beta}) \right\}$$

where for our linear regression example  $u_i(\boldsymbol{\beta}) = y_i - \mathbf{x}'_i\boldsymbol{\beta}$ . This estimator tries to make the moment conditions as close to zero as possible. This simple estimator, however, applies equal weight to each of the moment conditions; and as we will see later, we can obtain more efficient estimators by choosing to weight some moment conditions more highly than others.

Consider the quadratic function

$$Q(\beta) = \left\{ \frac{1}{N} \sum_i \mathbf{z}_i u_i(\beta) \right\}' \mathbf{W} \left\{ \frac{1}{N} \sum_i \mathbf{z}_i u_i(\beta) \right\}$$

where  $\mathbf{W}$  is a symmetric positive-definite matrix known as a weight matrix. Then we define the GMM estimator as

$$\widehat{\beta} \equiv \arg \min_{\beta} Q(\beta) \quad (1)$$

Continuing with our regression model example, if we choose

$$\mathbf{W} = \left( \frac{1}{N} \sum_i \mathbf{z}_i \mathbf{z}_i' \right)^{-1}$$

then we obtain

$$\widehat{\beta} = \left\{ \left( \frac{1}{N} \sum_i \mathbf{x}_i \mathbf{z}_i' \right) \left( \frac{1}{N} \sum_i \mathbf{z}_i \mathbf{z}_i' \right)^{-1} \left( \frac{1}{N} \sum_i \mathbf{z}_i \mathbf{x}_i' \right) \right\}^{-1} \times \left( \frac{1}{N} \sum_i \mathbf{x}_i \mathbf{z}_i' \right) \left( \frac{1}{N} \sum_i \mathbf{z}_i \mathbf{z}_i' \right)^{-1} \left( \frac{1}{N} \sum_i \mathbf{z}_i y_i \right)$$

which is the well-known two-stage least-squares (2SLS) estimator. Our choice of weight matrix here was based on the assumption that  $u$  was homoskedastic. A feature of GMM estimation is that by selecting different weight matrices, we can obtain estimators that can tolerate heteroskedasticity, clustering, autocorrelation, and other features of  $u$ . See [R] [ivregress](#) for more information about the 2SLS and linear GMM estimators.

Returning to the case where the model is “just identified”, meaning that  $q = k$ , if we apply the GMM estimator, we will obtain the same estimate,  $\widehat{\beta}$ , regardless of our choice of  $\mathbf{W}$ . Because  $q = k$ , if a unique solution exists, it will set all the sample moment conditions jointly to zero, so  $\mathbf{W}$  has no impact on the value of  $\beta$  that minimizes the objective function.

We will highlight other features of the GMM estimator and the `gmm` command as we proceed through examples. First, though, we discuss how to specify moment conditions by using substitutable expressions.

## Substitutable expressions

To use the interactive version of `gmm`, you define the moment conditions by using substitutable expressions. Your moment conditions are of the form  $E\{z_i' u_i(\beta)\} = \mathbf{0}$ , where  $u_i(\beta)$  is a residual expression that depends on the parameter vector  $\beta$  as well as variables in your dataset, though we suppress expressing the variables for notational simplicity.

`gmm` requires you to write a substitutable expression for  $u_i(\beta)$ . This substitutable expression is the right-hand side of the model written in terms of  $u$ , or in the language of Stata syntax, a “residual equation”. For example, suppose you want to fit the function  $y = f(\mathbf{x}; \beta) + u$ . In this example,  $u_i(\beta) = u = y - f(\mathbf{x}; \beta)$ , so you would type

```
gmm (y - expression for f(x; beta)), ...
```

Note that we are not restricted to models with additive error terms.

In general, there are three rules to follow when defining substitutable expressions:

1. Parameters of the model are bound in curly braces: `{b0}`, `{param}`, etc. Parameter names must follow the same conventions as variable names. See [U] 11.3 [Naming conventions](#).
2. Initial values for parameters are given by including an equal sign and the initial value inside the curly braces: `{b0=1}`, `{param=3.571}`, etc.

You can also specify initial values by using the `from()` option. Initial values specified in `from()` override whatever initial values are given within the substitutable expression. If you do not specify an initial value for a parameter, it is initialized to 0.

3. Linear combinations of variables can be included using the notation `{lcname:varlist}`: `{xb: mpg price weight}`, `{score: w x z}`, etc. Parameters of linear combinations are initialized to 0.

Substitutable expressions may use any mathematical expression involving scalars and variables. See [U] 13.2 [Operators](#) and [U] 13.3 [Functions](#) for more information on expressions.

The notation `{xb: x1 x2 x3}` tells `gmm` that you want a linear combination of the variables `x1`, `x2`, and `x3`. We named this linear combination `xb`, so `gmm` will name the three parameters `xb:x1`, `xb:x2`, and `xb:x3`, which corresponds to the three variables `x1`, `x2`, and `x3` in the `xb` equation. Specify `_cons` to include a constant term in a linear combination. Factor variables and time-series operators are allowed; see [U] 11.4.3 [Factor variables](#) and [U] 11.4.4 [Time-series varlists](#).

Once we have declared the variables in the linear combination `xb`, we can refer to the linear combination in our substitutable expression by using the notation `xb:.` The colon is not optional; it tells `gmm` that you are referring to a previously declared linear combination, not an individual parameter. This shorthand notation is also handy when specifying derivatives, as we will show later.

## ▷ Example 1: OLS regression

In [Introduction](#), we stated that OLS is an MM estimator. Say that we want to fit the model

$$\text{mpg} = \beta_0 + \beta_1 \text{weight} + \beta_2 \text{length} + u$$

where  $u$  is an i.i.d. error term. Recall that the moment condition for OLS regression is  $E(\mathbf{x}u) = \mathbf{0}$ , where  $\mathbf{x}$ , the list of instruments, is the same as the list of regressors in the model. Writing this in the form required for a `gmm` substitutable expression, we have

$$u = \text{mpg} - \beta_0 - \beta_1 \text{weight} - \beta_2 \text{length}$$

The right-hand side of the equation is the substitutable expression that we will provide to `gmm`. We give  $\beta_0$ ,  $\beta_1$ , and  $\beta_2$  the parameter names `b0`, `b1`, and `b2` and enclose them in curly braces. Because linear combinations declared in substitutable expressions do not include a constant term by default, we include our own (`b0`). We specify the regressors, `weight` and `length`, with their respective parameters and also in the `instruments()` option. `gmm` includes a constant term in the instrument list by default, so we do not need to add an additional term there. Thus, our command is

```

. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. gmm (mpg - {b1}*weight - {b2}*length - {b0}), instruments(weight length)
Step 1:
Iteration 0: GMM criterion Q(b) = 475.4138
Iteration 1: GMM criterion Q(b) = 2.696e-20
Iteration 2: GMM criterion Q(b) = 3.329e-27
Step 2:
Iteration 0: GMM criterion Q(b) = 5.109e-28
Iteration 1: GMM criterion Q(b) = 7.237e-32
note: model is exactly identified.
GMM estimation
Number of parameters = 3
Number of moments = 3
Initial weight matrix: Unadjusted Number of obs = 74
GMM weight matrix: Robust

```

|     | Robust      |           |       |       |                      |           |
|-----|-------------|-----------|-------|-------|----------------------|-----------|
|     | Coefficient | std. err. | z     | P> z  | [95% conf. interval] |           |
| /b1 | -.0038515   | .0019472  | -1.98 | 0.048 | -.0076678            | -.0000351 |
| /b2 | -.0795935   | .0677528  | -1.17 | 0.240 | -.2123866            | .0531996  |
| /b0 | 47.88487    | 7.50599   | 6.38  | 0.000 | 33.1734              | 62.59634  |

```
Instruments for equation 1: weight length _cons
```

Because the number of moments equals the number of parameters we are estimating, the model is said to be “just identified” or “exactly identified”. Therefore, the choice of weight matrix has no impact on the solution to (1), and the criterion function  $Q(\beta)$  achieves its minimum value at 0.

The OLS estimator is a one-step GMM estimator, but we did not bother to specify the `onestep` option, because the model is just identified. Doing a second step of GMM estimation affects neither the point estimates nor the standard errors, so to keep the syntax as simple as possible, we did not include the `onestep` option. The first step of estimation resulted in  $Q(\beta) = 0$  as expected, and the second step of estimation did not change the minimized value of  $Q(\beta)$ . (The final iterations of the first and second steps result in  $Q(\beta)$  values of 0 for all practical purposes.)

When you do not specify either the `wmatrix()` or the `vce()` option, `gmm` reports heteroskedasticity-robust standard errors. The parameter estimates reported here match those that we would obtain from the command

```
. regress mpg weight length, vce(robust)
```

The standard errors reported by that `regress` command would be larger than those reported by `gmm` by a factor of  $\sqrt{74/71}$  because `regress` makes a small-sample adjustment to the estimated variance matrix while `gmm` does not. Likewise, if we had specified the `vce(unadjusted)` option with our `gmm` command, then our standard errors would differ by a factor of  $\sqrt{74/71}$  from those reported by `regress` without the `vce(robust)` option.

We could have submitted our substitutable expression using the notation for linear combinations of parameters. If we select `xb` as the name of our parameter equation, we could type

```
. gmm (mpg - {xb: weight length _cons}), instruments(weight length)
```



Note how we specified the `instruments()` option. In *Introduction*, we mentioned that the moment conditions for the 2SLS estimator are  $E(\mathbf{z}u) = \mathbf{0}$ , and we mentioned that if some elements of  $\mathbf{x}$  (the regressors) are not endogenous, then they should also appear in  $\mathbf{z}$ . In this model, we assume the regressor `pcurban` is exogenous, so we included it in the list of instrumental variables. Commands like `ivregress`, `ivprobit`, and `ivtobit` accept standard *varlists*, so they can deduce the exogenous regressors in the model. Because `gmm` accepts arbitrary functions in the form of substitutable expressions, it has no way of discerning the exogenous variables of the model on its own.

Also notice that we specified the `onestep` option. The 2SLS estimator is a one-step GMM estimator that is based on a weight matrix that assumes the error terms are i.i.d. Unlike the previous example, this example had more instruments than parameters, so the minimized value of  $Q(\beta)$  is nonzero. We discuss the weight matrix and its relationship to two-step estimation next.

◀

## The weight matrix and two-step estimation

Recall our definition of the GMM estimator given in (1). The estimator,  $\widehat{\beta}$ , depends on the choice of the weight matrix,  $\mathbf{W}$ . Under relatively mild assumptions, our estimator,  $\widehat{\beta}$ , is consistent regardless of the choice of  $\mathbf{W}$ , so how are we to decide what  $\mathbf{W}$  to use? The most common solution is to use the two-step estimator, which we now describe.

A key result in Hansen's (1982) seminal paper is that if we denote by  $\mathbf{S}$  the covariance matrix of the moment conditions, then the optimal (in a way we make precise later) GMM estimator is the one that uses a weight matrix equal to the inverse of the moment covariance matrix. That is, if we let  $\mathbf{S} = \text{Cov}(\mathbf{z}u)$ , then we want to use  $\mathbf{W} = \mathbf{S}^{-1}$ . But how do we obtain  $\mathbf{S}$  in the first place?

If we assume that the errors are i.i.d., then

$$\text{Cov}(\mathbf{z}u) = E(u^2 \mathbf{z}\mathbf{z}') = \sigma^2 E(\mathbf{z}\mathbf{z}')$$

where  $\sigma^2$  is the variance of  $u$ . Because  $\sigma^2$  is a positive scalar, we can ignore it when solving (1). Thus, we compute

$$\widehat{\mathbf{W}}_1 = \left( \frac{1}{N} \sum_i \mathbf{z}_i \mathbf{z}_i' \right)^{-1} \quad (2)$$

which does not depend on any unknown model parameters. (Notice that  $\widehat{\mathbf{W}}_1$  is the same weight matrix used in 2SLS.) Given  $\widehat{\mathbf{W}}_1$ , we can solve (1) to obtain an initial estimate, say,  $\widehat{\beta}_1$ .

Our estimate,  $\widehat{\beta}_1$ , is consistent, so by Slutsky's theorem, the sample residuals  $\widehat{u}$  computed at this value of  $\beta$  will also be consistent. Using virtually the same arguments used to justify the Huber/Eicker/White heteroskedasticity-robust VCE, if we assume that the residuals are independent though not identically distributed, we can estimate  $\mathbf{S}$  as

$$\widehat{\mathbf{S}} = \frac{1}{N} \sum_i \widehat{u}_i^2 \mathbf{z}_i \mathbf{z}_i'$$

Then, in the second step, we re-solve (1), using  $\widehat{\mathbf{W}}_2 = \widehat{\mathbf{S}}^{-1}$ , which yields the two-step GMM estimate  $\widehat{\beta}_2$ . If the residuals exhibit clustering, you can specify `wmatrix(cluster varname)` so that `gmm` computes a weight matrix that does not assume the  $u_i$ 's are independent within clusters identified by *varname*. You can specify `wmatrix(hac ...)` to obtain weight matrices that are suitable for when the  $u_i$ 's exhibit autocorrelation as well as heteroskedasticity.

We could take the point estimates from the second round of estimation and use them to compute yet another weight matrix,  $\widehat{\mathbf{W}}_3$ , say, re-solve (1) yet again, and so on, stopping when the parameters or weight matrix do not change much from one iteration to the next. This procedure is known as the iterative GMM estimator and is obtained with the `igmm` option. Asymptotically, the two-step and iterative GMM estimators have the same distribution. However, Hall (2005, 90) suggests that the iterative estimator may have better finite-sample properties.

Instead of computing  $\widehat{\mathbf{W}}_1$  as in (2), we could simply choose  $\widehat{\mathbf{W}}_1 = \mathbf{I}$ , the identity matrix. The initial estimate,  $\widehat{\beta}_1$ , would still be consistent. You can request this behavior by specifying the `winitial(identity)` option. However, if you specify all of your moment conditions of the form  $E(\mathbf{z}u) = \mathbf{0}$ , we recommend using the default `winitial(unadjusted)` instead; the rescaling of the moment conditions implied by using a homoskedastic initial weight matrix makes the numerical routines used to solve (1) more stable.

If you fit a model with more than one of the moment conditions of the form  $E\{h(\mathbf{z}; \beta)\} = \mathbf{0}$ , then you must use `winitial(identity)` or `winitial(unadjusted, independent)`. With moment conditions of that form, you do not specify a list of instruments, and `gmm` cannot evaluate (2)—the matrix expression in parentheses would necessarily be singular, so it cannot be inverted.

### ▷ Example 3: Two-step linear GMM estimator

From the previous discussion and the comments in [Introduction](#), we see that the linear 2SLS estimator is a one-step GMM estimator where we use the weight matrix defined in (2) that assumes the errors are i.i.d. If we use the 2SLS estimate of  $\beta$  to obtain the sample residuals, compute a new weight matrix based on those residuals, and then do a second step of GMM estimation, we obtain the linear two-step GMM estimator as implemented by `ivregress gmm`.

In [example 3](#) of [\[R\] ivregress](#), we fit the model of rental rates as discussed in [example 2](#) above. We now allow the residuals to be heteroskedastic, though we will maintain our assumption that they are independent. We type

```
. gmm (rent - {xb:hsngval pcturban _cons}), instruments(pcturban faminc reg2-reg4)
Step 1:
Iteration 0: GMM criterion Q(b) = 56115.03
Iteration 1: GMM criterion Q(b) = 110.91583
Iteration 2: GMM criterion Q(b) = 110.91583
Step 2:
Iteration 0: GMM criterion Q(b) = .2406087
Iteration 1: GMM criterion Q(b) = .13672801
Iteration 2: GMM criterion Q(b) = .13672801
GMM estimation
Number of parameters = 3
Number of moments = 6
Initial weight matrix: Unadjusted
GMM weight matrix: Robust
Number of obs = 50
```

|          | Robust      |           |       |       |                      |          |
|----------|-------------|-----------|-------|-------|----------------------|----------|
|          | Coefficient | std. err. | z     | P> z  | [95% conf. interval] |          |
| hsngval  | .0014643    | .0004473  | 3.27  | 0.001 | .0005877             | .002341  |
| pcturban | .7615482    | .2895105  | 2.63  | 0.009 | .1941181             | 1.328978 |
| _cons    | 112.1227    | 10.80234  | 10.38 | 0.000 | 90.95052             | 133.2949 |

Instruments for equation 1: pcturban faminc reg2 reg3 reg4 \_cons

By default, `gmm` computes a heteroskedasticity-robust weight matrix before the second step of estimation, though we could have specified `wmatrix(robust)` if we wanted to be explicit. Because we did not specify the `vce()` option, `gmm` used a heteroskedasticity-robust one. Our results match those in [example 3](#) of [R] `ivregress`. Moreover, the only substantive difference between this example and [example 2](#) is that here we did not specify the `onestep` option, so we obtain the two-step estimates. ◀

## Obtaining standard errors

This section is a bit more theoretical and can be skipped on first reading. However, the information is sufficiently important that you should return to this section at some point.

So far in our discussion, we have focused on point estimation without much mention of how we obtain the standard errors of the estimates. We also mentioned that if we choose  $\mathbf{W}$  to be the inverse of the covariance matrix of the moment conditions, then we obtain the “optimal” GMM estimator. We elaborate those points now.

Using mostly standard statistical arguments, we can show that for the GMM estimator defined in (1), the variance of  $\hat{\beta}$  is given by

$$\text{Var}(\hat{\beta}) = \frac{1}{N} \{ \overline{\mathbf{G}}(\hat{\beta})' \mathbf{W} \overline{\mathbf{G}}(\hat{\beta}) \}^{-1} \overline{\mathbf{G}}(\hat{\beta})' \mathbf{W} \mathbf{S} \overline{\mathbf{G}}(\hat{\beta}) \{ \overline{\mathbf{G}}(\hat{\beta})' \mathbf{W} \overline{\mathbf{G}}(\hat{\beta}) \}^{-1} \quad (3)$$

where

$$\overline{\mathbf{G}}(\hat{\beta}) = \frac{1}{N} \sum_i \mathbf{z}_i \left. \frac{\partial u_i}{\partial \beta} \right|_{\beta=\hat{\beta}} \quad \text{or} \quad \overline{\mathbf{G}}(\hat{\beta}) = \frac{1}{N} \sum_i \left. \frac{\partial \mathbf{h}_i}{\partial \beta} \right|_{\beta=\hat{\beta}}$$

as the case may be and  $\mathbf{S} = E(\mathbf{z}u u' \mathbf{z}')$ .

Assuming the `vce(unadjusted)` option is not specified, `gmm` reports standard errors based on the robust variance matrix defined in (3). For the two-step estimator,  $\mathbf{W}$  is the weight matrix requested with the `wmatrix()` option, and it is calculated based on the residuals obtained after the first estimation step. The second-step point estimates and residuals are obtained, and  $\mathbf{S}$  is calculated based on the specification of the `vce()` option. For the iterated estimator,  $\mathbf{W}$  is calculated based on the second-to-last round of estimation, while  $\mathbf{S}$  is based on the residuals obtained after the last round of estimation. Computation of the covariance matrix for the one-step estimator is, perhaps surprisingly, more involved; we discuss the covariance matrix with the one-step estimator in the technical note at the end of this section.

If the model is exactly identified, the matrix  $\overline{\mathbf{G}}(\hat{\beta})$  is square, and (3) simplifies to the following:

$$\text{Var}(\hat{\beta}) = \frac{1}{N} \overline{\mathbf{G}}(\hat{\beta})^{-1} \mathbf{S} (\overline{\mathbf{G}}(\hat{\beta}))^{-1}$$

If we choose the weight matrix to be the inverse of the covariance matrix of the moment conditions so that  $\mathbf{W} = \mathbf{S}^{-1}$ , then (3) simplifies substantially:

$$\text{Var}(\hat{\beta}) = \frac{1}{N} \{ \overline{\mathbf{G}}(\hat{\beta})' \mathbf{W} \overline{\mathbf{G}}(\hat{\beta}) \}^{-1} \quad (4)$$

The GMM estimator constructed using this choice of weight matrix along with the covariance matrix in (4) is known as the “optimal” GMM estimator. One can show that if in fact  $\mathbf{W} = \mathbf{S}^{-1}$ , then the variance in (4) is smaller than the variance in (3) of any other GMM estimator based on the same moment conditions but with a different choice of weight matrix. Thus, the optimal GMM estimator is also known as the efficient GMM estimator because it has the smallest variance of any estimator based on the given moment conditions.

To obtain standard errors from `gmm` based on the optimal GMM estimator, you specify the `vce(unadjusted)` option. We call that VCE unadjusted because we do not recompute the residuals after estimation to obtain the matrix  $\mathbf{S}$  required in (3) or allow for the fact that those residuals may not be i.i.d. Some statistical packages by default report standard errors based on (4) and offer standard errors based on (3) only as an option or not at all. While the optimal GMM estimator is theoretically appealing, Cameron and Trivedi (2005, 177) suggest that in finite samples, it need not perform better than the GMM estimator that uses (3) to obtain standard errors.

#### □ Technical note

Computing the covariance matrix of the parameters after using the one-step estimator is actually a bit more complex than after using the two-step or iterative estimator. We can illustrate most of the intricacies by using linear regression with moment conditions of the form  $E\{\mathbf{x}(y - \mathbf{x}'\boldsymbol{\beta})\} = \mathbf{0}$ .

If you specify `winitial(unadjusted)` and `vce(unadjusted)`, then the initial weight matrix will be computed as

$$\widehat{\mathbf{W}}_1 = \left( \frac{1}{N} \sum_i \mathbf{x}_i \mathbf{x}'_i \right)^{-1} \quad (5)$$

Moreover, for linear regression, we can show that

$$\overline{\mathbf{G}}(\widehat{\boldsymbol{\beta}}) = \frac{1}{N} \sum_i \mathbf{x}_i \mathbf{x}'_i$$

so that (4) becomes

$$\begin{aligned} \text{Var}(\widehat{\boldsymbol{\beta}}) &= \frac{1}{N} \left\{ \left( \frac{1}{N} \sum_i \mathbf{x}_i \mathbf{x}'_i \right) \left( \frac{1}{N} \sum_i \mathbf{x}_i \mathbf{x}'_i \right)^{-1} \left( \frac{1}{N} \sum_i \mathbf{x}_i \mathbf{x}'_i \right) \right\}^{-1} \\ &= \left( \sum_i \mathbf{x}_i \mathbf{x}'_i \right)^{-1} \\ &= (\mathbf{X}'\mathbf{X})^{-1} \end{aligned} \quad (6)$$

However, we know that the nonrobust covariance matrix for the OLS estimator is actually  $\widehat{\sigma}^2(\mathbf{X}'\mathbf{X})^{-1}$ . What is missing from (6) is the scalar  $\widehat{\sigma}^2$ , the estimated variance of the residuals. When you use the one-step estimator and specify `winitial(unadjusted)`, the weight matrix (5) does not include the  $\widehat{\sigma}^2$  term because `gmm` does not have a consistent estimate of  $\boldsymbol{\beta}$  from which it can then estimate  $\sigma^2$ . The point estimates are still correct because multiplying the weight matrix by a scalar factor does not affect the solution to the minimization problem.

To circumvent this issue, if you specify `winitial(unadjusted)` and `vce(unadjusted)`, `gmm` uses the estimated  $\widehat{\boldsymbol{\beta}}$  (which is consistent) to obtain a new unadjusted weight matrix that does include the term  $\widehat{\sigma}^2$  so that evaluating (4) will yield correct standard errors.

If you use the two-step or iterated GMM estimator, this extra effort is not needed to obtain standard errors because the first-step (and subsequent steps') estimate of  $\boldsymbol{\beta}$  is consistent and can be used to estimate  $\sigma^2$  or some other weight matrix based on the `wmatrix()` option. Straightforward algebra shows that this extra effort is also not needed if you request any type of adjusted (robust) covariance matrix with the one-step estimator.

A similar issue arises when you specify `winitial(identity)` and `vce(unadjusted)` with the one-step estimator. Again the solution is to compute an unadjusted weight matrix after obtaining  $\widehat{\beta}$  so that (4) provides the correct standard errors.

We have illustrated the problem and solution using a single-equation linear model. However, the problem arises whenever you use the one-step estimator with an unadjusted VCE, regardless of the number of equations, and `gmm` handles all the details automatically. Computation of Hansen's  $J$  statistic presents an identical issue, and `gmm` takes care of that as well.

If you supply your own initial weight matrix by using `winitial(matname)`, then the standard errors (as well as the  $J$  statistic reported by `estat overid`) are based on that weight matrix. You should verify that the weight matrix you provide will yield appropriate statistics. □

## Factor-variable coefficients in multiple residual functions

The long example in this section uses `gmm` to replicate the results produced by `regress` with factor variables and `margins`. It illustrates how to refer to the coefficients on factor variables in linear combinations in subsequent residual functions. The example also shows how to use `gmm` to address the two-step estimation problem, or the inconsistency of standard errors produced by two-step estimators that depend on previously estimated parameters.

### ► Example 4: Means of linear combinations of factor variables

The mean of a variable when everyone in a population receives a given treatment level is known as a potential-outcome mean. We use `regress` and `margins` to estimate the potential-outcome means of a mother's smoking behavior while pregnant on the birthweight of her baby after controlling for the mother's age and an indicator for whether the mother had a prenatal visit in the first trimester. We use an extract of data from Cattaneo (2010) in which `bweight` is the baby's birthweight in grams, `mbsmoke` is a binary variable indicating whether a mother smoked while pregnant, `mage` is the mother's age, and `prenatal1` is a binary variable indicating whether the mother had a prenatal visit in the first trimester.

We use regress to estimate the regression coefficients.

```
. use https://www.stata-press.com/data/r19/cattaneo2
(Excerpt from Cattaneo (2010) Journal of Econometrics 155: 138-154)
. regress bweight ibn.mbsmoke ibn.mbsmoke#(c.mage i.prenatal1),
> noconstant vce(robust)
```

Linear regression

|               |   |          |
|---------------|---|----------|
| Number of obs | = | 4,642    |
| F(6, 4636)    | = | 27751.75 |
| Prob > F      | = | 0.0000   |
| R-squared     | = | 0.9726   |
| Root MSE      | = | 565.08   |

| bweight               | Robust      |           |       |       |                      |          |
|-----------------------|-------------|-----------|-------|-------|----------------------|----------|
|                       | Coefficient | std. err. | t     | P> t  | [95% conf. interval] |          |
| mbsmoke               |             |           |       |       |                      |          |
| Nonsmoker             | 3073.201    | 48.68899  | 63.12 | 0.000 | 2977.748             | 3168.655 |
| Smoker                | 3217.973    | 93.637    | 34.37 | 0.000 | 3034.4               | 3401.546 |
| mbsmoke#<br>c.mage    |             |           |       |       |                      |          |
| Nonsmoker             | 9.737189    | 1.825552  | 5.33  | 0.000 | 6.158237             | 13.31614 |
| Smoker                | -4.962403   | 3.852613  | -1.29 | 0.198 | -12.51536            | 2.590552 |
| mbsmoke#<br>prenatal1 |             |           |       |       |                      |          |
| Nonsmoker #<br>Yes    | 95.11727    | 26.82039  | 3.55  | 0.000 | 42.53654             | 147.698  |
| Smoker#Yes            | 64.61752    | 39.72317  | 1.63  | 0.104 | -13.25879            | 142.4938 |

We used factor variables to interact mbsmoke with the other covariates to allow for separate coefficients for smoking and nonsmoking mothers.

The postestimation command margins uses the estimated regression coefficients to estimate the potential-outcome means of bweight, first assuming that no mother smoked and then assuming that all mothers smoked.

```
. margins i.mbsmoke, vce(unconditional)
Predictive margins
```

Number of obs = 4,642

Expression: Linear prediction, predict()

| mbsmoke   | Unconditional |           |        |       |                      |          |
|-----------|---------------|-----------|--------|-------|----------------------|----------|
|           | Margin        | std. err. | t      | P> t  | [95% conf. interval] |          |
| Nonsmoker | 3407.506      | 9.346894  | 364.56 | 0.000 | 3389.181             | 3425.83  |
| Smoker    | 3138.23       | 21.20463  | 148.00 | 0.000 | 3096.659             | 3179.801 |

Note that the standard errors for the estimated means account for the estimation error in the estimated coefficients used to compute them.

Before using `gmm` to simultaneously estimate the regression coefficients and the potential-outcome means, we demonstrate the equivalence of point estimates from `gmm` and `regress` and illustrate the two-step estimation problem. First, we use `gmm` to estimate just the regression coefficients. Note that we specify the factor variables in the linear combination `xb`: and in the instrument list.

```
. gmm (eq1: bweight - {xb:ibn.mbsmoke ibn.mbsmoke#(c.mage i.prenatal1)}),
> instruments(eq1: ibn.mbsmoke ibn.mbsmoke#(c.mage i.prenatal1), noconstant)
> coeflegend onestep

Step 1:
Iteration 0: GMM criterion Q(b) = 11316945
Iteration 1: GMM criterion Q(b) = 7.143e-19
Iteration 2: GMM criterion Q(b) = 2.051e-26
note: model is exactly identified.

GMM estimation
Number of parameters = 6
Number of moments = 6
Initial weight matrix: Unadjusted Number of obs = 4,642
```

|             | Coefficient | Legend                      |
|-------------|-------------|-----------------------------|
| mbsmoke     |             |                             |
| Nonsmoker   | 3073.201    | _b[0bn.mbsmoke]             |
| Smoker      | 3217.973    | _b[1.mbsmoke]               |
| mbsmoke#    |             |                             |
| c.mage      |             |                             |
| Nonsmoker   | 9.737189    | _b[0bn.mbsmoke#c.mage]      |
| Smoker      | -4.962403   | _b[1.mbsmoke#c.mage]        |
| mbsmoke#    |             |                             |
| prenatal1   |             |                             |
| Nonsmoker # |             |                             |
| Yes         | 95.11727    | _b[0bn.mbsmoke#1.prenatal1] |
| Smoker#Yes  | 64.61752    | _b[1.mbsmoke#1.prenatal1]   |

```
Instruments for equation eq1: 0.mbsmoke 1.mbsmoke 0.mbsmoke#c.mage
1.mbsmoke#c.mage 0o.mbsmoke#0b.prenatal1 0.mbsmoke#1.prenatal1
1o.mbsmoke#0b.prenatal1 1.mbsmoke#1.prenatal1
```

We specified the `coeflegend` option to learn the names of the coefficients on the factor variables in the linear combination. As expected, the point estimates are the same as those reported by `regress`.

Next, we illustrate the effect of the two-step estimation problem if we calculate the potential-outcome means by hand. We can calculate the mean when no mothers smoke by accessing these coefficients and then estimate the standard errors in the estimated means:

```
. generate mean0 = _b[xb:0.mbsmoke] + _b[xb:0.mbsmoke#c.mage]*mage
> + _b[xb:0.mbsmoke#1.prenatal1]*prenatal1
. mean mean0
Mean estimation Number of obs = 4,642
```

|       | Mean     | Std. err. | [95% conf. interval] |          |
|-------|----------|-----------|----------------------|----------|
| mean0 | 3407.506 | 1.085503  | 3405.378             | 3409.634 |

The estimated potential-outcome mean for no mothers smoking is the same as that reported by margins, but the standard error is much smaller because mean ignores the estimation error in the coefficients. This underscores the importance of accounting for the estimation error when estimating the standard errors.

Now, we use gmm to estimate the coefficients and the potential-outcome means simultaneously.

```
. gmm (eq1: bweight - {xb:ibn.mbsmoke ibn.mbsmoke#{c.mage i.prenatal1}})
> (eq2: {xb:0.mbsmoke} + {xb:0bn.mbsmoke#c.mage}*mage
> + {xb:0bn.mbsmoke#1.prenatal1}*1.prenatal1 - {m0})
> (eq3: {xb:1.mbsmoke} + {xb:1.mbsmoke#c.mage}*mage
> + {xb:1.mbsmoke#1.prenatal1}*1.prenatal1 - {m1}),
> instruments(eq1: ibn.mbsmoke ibn.mbsmoke#{c.mage i.prenatal1},
> noconstant)
> instruments(eq2 eq3:) winitial(identity) onestep
```

Step 1:

Iteration 0: GMM criterion Q(b) = 5.819e+09

Iteration 1: GMM criterion Q(b) = 3.108e-13

Iteration 2: GMM criterion Q(b) = 1.010e-22

note: model is exactly identified.

GMM estimation

Number of parameters = 8

Number of moments = 8

Initial weight matrix: Identity Number of obs = 4,642

|                       | Coefficient | Robust<br>std. err. | z      | P> z  | [95% conf. interval] |          |
|-----------------------|-------------|---------------------|--------|-------|----------------------|----------|
| mbsmoke               |             |                     |        |       |                      |          |
| Nonsmoker             | 3073.201    | 48.65751            | 63.16  | 0.000 | 2977.834             | 3168.568 |
| Smoker                | 3217.973    | 93.57647            | 34.39  | 0.000 | 3034.566             | 3401.379 |
| mbsmoke#<br>c.mage    |             |                     |        |       |                      |          |
| Nonsmoker             | 9.737189    | 1.824372            | 5.34   | 0.000 | 6.161485             | 13.31289 |
| Smoker                | -4.962403   | 3.850123            | -1.29  | 0.197 | -12.5085             | 2.583699 |
| mbsmoke#<br>prenatal1 |             |                     |        |       |                      |          |
| Nonsmoker #           |             |                     |        |       |                      |          |
| Yes                   | 95.11727    | 26.80305            | 3.55   | 0.000 | 42.58425             | 147.6503 |
| Smoker#Yes            | 64.61752    | 39.69749            | 1.63   | 0.104 | -13.18813            | 142.4232 |
| /m0                   |             |                     |        |       |                      |          |
| /m1                   | 3407.506    | 9.340852            | 364.80 | 0.000 | 3389.198             | 3425.813 |
|                       | 3138.23     | 21.19093            | 148.09 | 0.000 | 3096.696             | 3179.763 |

```
Instruments for equation eq1: 0.mbsmoke 1.mbsmoke 0.mbsmoke#c.mage
1.mbsmoke#c.mage 0o.mbsmoke#0b.prenatal1 0.mbsmoke#1.prenatal1
1o.mbsmoke#0b.prenatal1 1.mbsmoke#1.prenatal1
```

```
Instruments for equation eq2: _cons
```

```
Instruments for equation eq3: _cons
```

This output has five noteworthy features.

1. We specify three different residual equations. The first, eq1:, defines the moment conditions for the regression using the covariates as instruments; eq2: is the moment condition for the potential outcome when no mothers smoke; and eq3: is the moment condition for the potential outcome when all mothers smoke.

2. In `eq2:` and `eq3:`, we refer to the individual coefficients on the factor variables in the linear combination `xb:` by enclosing their names in curly braces.
3. The `instruments()` option is repeatable. We first specify that the covariates in the regression are the instruments for the residual equation `eq1:` and then specify that only the unit variable, also known as the constant, is an instrument for each of `eq2:` and `eq3:`.
4. The point estimates and the standard errors match those reported by `regress` and `margins`, after accounting for the small-sample adjustment performed by `regress`.
5. Although the point estimates match, the standard errors reported by `gmm` are much larger than those reported by `mean` because `gmm` takes into account that the regression coefficients are estimated.



## Parameter interpretation using margins

In the last section, we demonstrated how to use `gmm` to estimate potential-outcome means in a linear regression model jointly with the coefficients of the models. However, you can also estimate the potential-outcome mean, or any other predictive margins, by using the `margins` command after `gmm`. Using `margins` after `gmm` can allow more flexibility in the predictive margins that we estimate and is also more convenient. See [Obtaining margins of responses](#) in [R] [margins](#) for more information about predictive margins.

### ▷ Example 5: Predicting treatment effects after estimation

In [example 4](#), we used `gmm` to estimate potential-outcome means of a mother's smoking behavior on her baby's birthweight (in grams) after controlling for age and whether she had a prenatal visit in the first trimester. Here we demonstrate how to use `margins` after `gmm` to estimate the potential-outcome means.

First, we load the data and reestimate the regression coefficients.

```
. use https://www.stata-press.com/data/r19/cattaneo2
(Excerpt from Cattaneo (2010) Journal of Econometrics 155: 138-154)
. gmm (eq1: bweight - {xb:ibn.mbsmoke ibn.mbsmoke#(c.mage i.prenatal1)}),
> instruments(eq1: ibn.mbsmoke ibn.mbsmoke#(c.mage i.prenatal1), noconstant)
> onestep

Step 1:
Iteration 0: GMM criterion Q(b) = 11316945
Iteration 1: GMM criterion Q(b) = 7.143e-19
Iteration 2: GMM criterion Q(b) = 2.051e-26
note: model is exactly identified.

GMM estimation
Number of parameters = 6
Number of moments = 6
Initial weight matrix: Unadjusted Number of obs = 4,642
```

|                       | Coefficient | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |          |
|-----------------------|-------------|---------------------|-------|-------|----------------------|----------|
| mbsmoke               |             |                     |       |       |                      |          |
| Nonsmoker             | 3073.201    | 48.65752            | 63.16 | 0.000 | 2977.834             | 3168.568 |
| Smoker                | 3217.973    | 93.57647            | 34.39 | 0.000 | 3034.566             | 3401.379 |
| mbsmoke#<br>c.mage    |             |                     |       |       |                      |          |
| Nonsmoker             | 9.737189    | 1.824372            | 5.34  | 0.000 | 6.161484             | 13.31289 |
| Smoker                | -4.962403   | 3.850123            | -1.29 | 0.197 | -12.5085             | 2.583699 |
| mbsmoke#<br>prenatal1 |             |                     |       |       |                      |          |
| Nonsmoker #           |             |                     |       |       |                      |          |
| Yes                   | 95.11727    | 26.80305            | 3.55  | 0.000 | 42.58425             | 147.6503 |
| Smoker#Yes            | 64.61752    | 39.69749            | 1.63  | 0.104 | -13.18813            | 142.4232 |

```
Instruments for equation eq1: 0.mbsmoke 1.mbsmoke 0.mbsmoke#c.mage
1.mbsmoke#c.mage 0o.mbsmoke#0b.prenatal1 0.mbsmoke#1.prenatal1
1o.mbsmoke#0b.prenatal1 1.mbsmoke#1.prenatal1
```

Now, we use margins to estimate the potential-outcome means. We specify `vce(unconditional)` to obtain standard errors for the potential-outcome means of the population rather than the sample. When we specify this option, the standard errors for the estimated means will account for the estimation error in the estimated coefficients from `gmm`.

```
. margins i.mbsmoke, vce(unconditional)
Predictive margins Number of obs = 4,642
Expression: Linear prediction, predict()
```

|           | Margin   | Unconditional<br>std. err. | z      | P> z  | [95% conf. interval] |          |
|-----------|----------|----------------------------|--------|-------|----------------------|----------|
| mbsmoke   |          |                            |        |       |                      |          |
| Nonsmoker | 3407.506 | 9.341858                   | 364.76 | 0.000 | 3389.196             | 3425.815 |
| Smoker    | 3138.23  | 21.19321                   | 148.08 | 0.000 | 3096.692             | 3179.768 |

Our point estimates of the potential-outcome means exactly match those that appear as  $\mu_0$  and  $\mu_1$  in [example 4](#). However, the standard errors are slightly higher because `gmm` and `margins` use different values in the denominator for the formula for the robust covariance matrix. `gmm` uses  $N$  while `margins` uses  $N - 1$ , so the standard errors differ by a factor of  $\sqrt{\{N/(N - 1)\}} = \sqrt{4,642/4,641} \approx 1.0002$ . More details about the calculation of the standard errors are provided in [Marginal predictions with unconditional standard errors](#) in the *Methods and formulas*.

In addition to potential-outcome means, we can use `margins` to estimate the average treatment effect (ATE) of the mother’s smoking behavior on birthweight. We use the contrast operator `r.` to instruct `margins` to difference the potential-outcome means and estimate a treatment effect. We specify the `contrast(nowald)` option to suppress the Wald test that `margins` displays by default for contrasts.

```
. margins r.mbsmoke, vce(unconditional) contrast(nowald)
Contrasts of predictive margins Number of obs = 4,642
Expression: Linear prediction, predict()
```

|                                  | Unconditional |           |                      |
|----------------------------------|---------------|-----------|----------------------|
|                                  | Contrast      | std. err. | [95% conf. interval] |
| mbsmoke<br>(Smoker vs Nonsmoker) | -269.2759     | 23.16069  | -314.67 -223.8818    |

The ATE of  $-269.28$  is interpreted as the difference between the average birthweight if all mothers in the population smoked and the average birthweight if all mothers in the population did not smoke. The average birthweight if all mothers were to smoke would be 269.28 grams less than if they did not smoke. ◀

## Exponential (Poisson) regression models

Exponential regression models are frequently encountered in applied work. For example, they can be used as alternatives to linear regression models on log-transformed dependent variables, obviating the need for post-hoc transformations to obtain predicted values in the original metric of the dependent variable. When the dependent variable represents a discrete count variable, exponential regression models are also known as Poisson regression models; see [Cameron and Trivedi \(2013\)](#).

For now, we consider models of the form

$$y = \exp(\mathbf{x}'\boldsymbol{\beta}) + u \quad (7)$$

where  $u$  is a zero-mean additive error term so that  $E(y) = \exp(\mathbf{x}'\boldsymbol{\beta})$ . Because the error term is additive, if  $\mathbf{x}$  represents strictly exogenous regressors, then we have the population moment condition

$$E[\mathbf{x}\{y - \exp(\mathbf{x}'\boldsymbol{\beta})\}] = \mathbf{0} \quad (8)$$

Moreover, because the number of parameters in the model is equal to the number of instruments, there is no point to using the two-step GMM estimator.

## ▷ Example 6: Exponential regression

Cameron and Trivedi (2022, 584) fit a model of the number of doctor visits based on whether the patient has private insurance, whether the patient has a chronic disease, gender, and income. Here we fit that model by using `gmm`. To allow for potential excess dispersion, we will obtain a robust VCE matrix, which is the default for `gmm` anyway. We type

```
. use https://www.stata-press.com/data/r19/docvisits
. gmm (docvis - exp({xb:private chronic female income _cons})),
> instruments(private chronic female income) onestep

Step 1:
Iteration 0: GMM criterion Q(b) = 16.853973
Iteration 1: GMM criterion Q(b) = 2.2706472
Iteration 2: GMM criterion Q(b) = .19088097
Iteration 3: GMM criterion Q(b) = .00041101
Iteration 4: GMM criterion Q(b) = 3.939e-09
Iteration 5: GMM criterion Q(b) = 6.572e-19
note: model is exactly identified.

GMM estimation
Number of parameters = 5
Number of moments = 5
Initial weight matrix: Unadjusted Number of obs = 4,412
```

|         | Coefficient | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |           |
|---------|-------------|---------------------|-------|-------|----------------------|-----------|
| private | .7986654    | .1089891            | 7.33  | 0.000 | .5850507             | 1.01228   |
| chronic | 1.091865    | .0559888            | 19.50 | 0.000 | .9821291             | 1.201601  |
| female  | .4925481    | .0585298            | 8.42  | 0.000 | .3778317             | .6072644  |
| income  | .003557     | .0010824            | 3.29  | 0.001 | .0014356             | .0056784  |
| _cons   | -.2297263   | .1108607            | -2.07 | 0.038 | -.4470093            | -.0124434 |

Instruments for equation 1: private chronic female income \_cons

Our point estimates agree with those reported by Cameron and Trivedi (2022) to at least six significant digits; the small discrepancies are attributable to different optimization techniques and convergence criteria being used by `gmm` and `poisson`. The standard errors differ by a factor of  $\sqrt{4412/4411}$  because `gmm` uses  $N$  in the denominator of the formula for the robust covariance matrix, while the robust covariance matrix estimator used by `poisson` uses  $N - 1$ .

◀

## □ Technical note

That the GMM and maximum likelihood estimators of the exponential regression model coincide is not a general property of these two classes of estimators. The maximum likelihood estimator solves the score equations

$$\frac{1}{N} \sum_{i=1}^N \frac{\partial \ln \ell_i}{\partial \beta} = \mathbf{0}$$

where  $\ell_i$  is the likelihood for the  $i$ th observation. These score equations can be viewed as the sample analogues of the population moment conditions

$$E \left( \frac{\partial \ln \ell_i}{\partial \beta} \right) = \mathbf{0}$$

establishing that maximum likelihood estimators represent a subset of the class of GMM estimators.

For the Poisson model,

$$\ln \ell_i = -\exp(\mathbf{x}'_i \boldsymbol{\beta}) + y_i \mathbf{x}'_i \boldsymbol{\beta} - \ln y_i!$$

so the score equations are

$$\frac{1}{N} \sum_{i=1}^N \mathbf{x}_i \{y_i - \exp(\mathbf{x}'_i \boldsymbol{\beta})\} = \mathbf{0}$$

which are just the sample moment conditions implied by (8) that we used in the [previous example](#). That is why our results using `gmm` match Cameron and Trivedi's (2022) results using `poisson`.

On the other hand, an intuitive set of moment conditions to consider for GMM estimation of a probit model is

$$E[\mathbf{x}\{y - \Phi(\mathbf{x}'\boldsymbol{\beta})\}] = \mathbf{0}$$

where  $\Phi(\cdot)$  is the standard normal cumulative distribution function. Differentiating the likelihood function for the maximum-likelihood probit estimator, we can show that the corresponding score equations are

$$\frac{1}{N} \sum_{i=1}^N \left[ \mathbf{x}_i \left\{ y_i \frac{\phi(\mathbf{x}'_i \boldsymbol{\beta})}{\Phi(\mathbf{x}'_i \boldsymbol{\beta})} - (1 - y_i) \frac{\phi(\mathbf{x}'_i \boldsymbol{\beta})}{1 - \Phi(\mathbf{x}'_i \boldsymbol{\beta})} \right\} \right] = \mathbf{0}$$

where  $\phi(\cdot)$  is the standard normal density function. These two moment conditions are not equivalent, so the maximum likelihood and GMM probit estimators are distinct. □

### ▷ Example 7: Comparison of GMM and maximum likelihood

Using the automobile dataset, we fit a probit model of `foreign` on `gear_ratio`, `length`, and `headroom` using first the score equations and then the intuitive set of GMM equations. We type

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. gmm (foreign*normalden({xb:gear_ratio length headroom _cons})/
> normal({xb:}) - (1-foreign)*normalden({xb:})/(1-normal({xb:}))),
> instruments(gear_ratio length headroom) onestep
(output omitted)
. estimates store ml
. gmm (foreign - normal({xb:gear_ratio length headroom _cons})),
> instruments(gear_ratio length headroom) onestep
(output omitted)
. estimates store gmm
. estimates table ml gmm, b se
```

| Variable   | ml                      | gmm                     |
|------------|-------------------------|-------------------------|
| gear_ratio | 2.9586277<br>.64042341  | 2.8489213<br>.63570247  |
| length     | -.02148933<br>.01382043 | -.02056033<br>.01396954 |
| headroom   | .01136927<br>.27278528  | .02240761<br>.2849891   |
| _cons      | -6.0222289<br>3.5594588 | -5.8595615<br>3.5188029 |

Legend: b/se

The coefficients on `gear_ratio` and `length` are close for the two estimators. The GMM estimate of the coefficient on `headroom` is twice that of the maximum likelihood estimate, though the relatively large standard errors imply that this difference is not significant. You can verify that the coefficients in the column marked “m1” match those you would obtain with `probit`. We have not discussed the differences among standard errors based on the various GMM and maximum-likelihood covariance matrix estimators to avoid tedious algebra. However, you can verify that the robust covariance matrix after one-step GMM estimation differs by only a finite-sample adjustment factor of  $(N/N - 1)$  from the robust covariance matrix reported by `probit`. Both the maximum likelihood and GMM `probit` estimators require the normality assumption, and the maximum likelihood estimator is efficient if that normality assumption is correct; therefore, in this example, there is no reason to prefer the GMM estimator.

◀

We can modify (8) easily to allow for endogenous regressors. Suppose that  $x_j$  is endogenous in the sense that  $E(u|x_j) \neq 0$ . Then, (8) is no longer a valid moment condition. However, suppose we have some variables other than  $\mathbf{x}$  such that  $E(u|\mathbf{z}) = \mathbf{0}$ . We can instead use the moment conditions

$$E(\mathbf{z}u) = E[\mathbf{z}\{y - \exp(\mathbf{x}'\boldsymbol{\beta})\}] = \mathbf{0}$$

As usual, if some elements of  $\mathbf{x}$  are exogenous, then they should appear in  $\mathbf{z}$  as well.

### ► Example 8: Exponential regression with endogenous regressors

Returning to the model discussed in [example 6](#), we treat `income` as endogenous; unobservable factors that determine a person’s income may also affect the number of times a person visits a doctor. We use a person’s age and race as instruments. These are valid instruments if we believe that age and race influence a person’s income but do not have a direct impact on the number of doctor visits. (Whether this belief is justified is another matter; we test that belief in [\[R\] gmm postestimation](#).) Because we have more instruments (seven) than parameters (five), we have an overidentified model. Therefore, the choice of weight matrix does matter. We will use the default two-step GMM estimator. In the first step, we will use a weight matrix that assumes the errors are i.i.d. In the second step, we will use a weight matrix that assumes heteroskedasticity. When you specify `twostep`, these are the defaults for the first- and second-step weight matrices, so we do not have to use the `winitial()` or `wmatrix()` options. We will again obtain a robust VCE, which is also the default. We type

```
. use https://www.stata-press.com/data/r19/docvisits
. gmm (docvis - exp({xb:private chronic female income _cons})),
> instruments(private chronic female age black hispanic) twostep
```

Step 1:

```
Iteration 0: GMM criterion Q(b) = 16.910173
Iteration 1: GMM criterion Q(b) = .82276104
Iteration 2: GMM criterion Q(b) = .21832032
Iteration 3: GMM criterion Q(b) = .12685935
Iteration 4: GMM criterion Q(b) = .12672369
Iteration 5: GMM criterion Q(b) = .12672365
```

Step 2:

```
Iteration 0: GMM criterion Q(b) = .00234641
Iteration 1: GMM criterion Q(b) = .00215957
Iteration 2: GMM criterion Q(b) = .00215911
Iteration 3: GMM criterion Q(b) = .00215911
```

GMM estimation

Number of parameters = 5

Number of moments = 7

Initial weight matrix: Unadjusted

Number of obs = 4,412

GMM weight matrix: Robust

|         | Coefficient | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |          |
|---------|-------------|---------------------|-------|-------|----------------------|----------|
| private | .535335     | .1599039            | 3.35  | 0.001 | .2219291             | .8487409 |
| chronic | 1.090126    | .0617659            | 17.65 | 0.000 | .9690668             | 1.211185 |
| female  | .6636579    | .0959884            | 6.91  | 0.000 | .4755241             | .8517918 |
| income  | .0142855    | .0027162            | 5.26  | 0.000 | .0089618             | .0196092 |
| _cons   | -.5983477   | .138433             | -4.32 | 0.000 | -.8696713            | -.327024 |

Instruments for equation 1: private chronic female age black hispanic \_cons

Once we control for the endogeneity of income, we find that its coefficient has quadrupled in size. Additionally, access to private insurance has less of an impact on the number of doctor visits and gender has more of an impact.

◀

## □ Technical note

Although you may be tempted to try, you cannot, as you can in a Poisson model, replace  $\mathbf{x}$  in the moment conditions for the probit (or logit) model with a vector of instruments,  $\mathbf{z}$ , if you have endogenous regressors. See [Wilde \(2008\)](#).

□

[Mullahy \(1997\)](#) considers a slightly more complicated version of the exponential regression model that incorporates nonadditive unobserved heterogeneity. His model can be written as

$$y_i = \exp(\mathbf{x}'_i \boldsymbol{\beta}) \eta_i + \epsilon_i$$

where  $\eta_i > 0$  is an unobserved heterogeneity term that may be correlated with  $\mathbf{x}_i$ . One result from his article is that instead of using the additive moment condition (8), we can use the multiplicative moment condition

$$E \left\{ \mathbf{z} \frac{y - \exp(\mathbf{x}'\boldsymbol{\beta})}{\exp(\mathbf{x}'\boldsymbol{\beta})} \right\} = E[\mathbf{z}\{y\exp(-\mathbf{x}'\boldsymbol{\beta}) - 1\}] = \mathbf{0}$$

[Windmeijer and Santos Silva \(1997\)](#) discuss the use of additive versus multiplicative moment conditions with endogenous regressors and note that a set of instruments that satisfies the additive moment conditions will not also satisfy the multiplicative moment conditions. They remark that the decision about which to use is an empirical issue that can at least partially be settled by using the test of overidentifying restrictions that is implemented by `estat overid` after `gmm` to see whether the instruments for a given model are valid. See [\[R\] gmm postestimation](#) for information on the test of overidentifying restrictions.

## Specifying derivatives

By default, `gmm` calculates derivatives numerically, and the method used produces accurate results for the vast majority of applications. However, if you refit the same model repeatedly or else have the derivatives available, then `gmm` will run more quickly if you supply it with analytic derivatives.

When you use the interactive version of `gmm`, you specify derivatives using substitutable expressions in much the same way you specify the residual equations. There are three rules you must follow:

1. As with the substitutable expressions that define residual equations, you bind parameters of the model in curly braces: `{b0}`, `{param}`, etc.
2. You must specify a derivative for each parameter that appears in each residual equation. If a parameter does not appear in a residual equation, then you do not specify a derivative for that parameter in that residual equation.
3. If you declare a linear combination in an equation, then you specify a derivative with respect to that linear combination. `gmm` applies the chain rule to obtain the derivatives with respect to the individual parameters encompassed by that linear combination.

### ▷ Example 9: Derivatives for a single-equation model

Consider a simple exponential regression model with one exogenous regressor and a constant term. We have

$$u_i = y_i - \exp(\beta_0 + \beta_1 x_i)$$

Now,

$$\frac{\partial u_i}{\partial \beta_0} = -\exp(\beta_0 + \beta_1 x_i) \quad \text{and} \quad \frac{\partial u_i}{\partial \beta_1} = -x_i \exp(\beta_0 + \beta_1 x_i)$$

In Stata, we type

```
. gmm (docvis - exp({b0} + {b1}*income)), instruments(income)
> deriv(/b0 = -1*exp({b0} + {b1}*income))
> deriv(/b1 = -1*income*exp({b0}+{b1}*income)) onestep
```

Step 1:

```
Iteration 0: GMM criterion Q(b) = 9.1548611
Iteration 1: GMM criterion Q(b) = 3.5146131
Iteration 2: GMM criterion Q(b) = .01344695
Iteration 3: GMM criterion Q(b) = 3.690e-06
Iteration 4: GMM criterion Q(b) = 4.606e-13
Iteration 5: GMM criterion Q(b) = 1.502e-26
```

note: model is exactly identified.

GMM estimation

Number of parameters = 2

Number of moments = 2

Initial weight matrix: Unadjusted

Number of obs = 4,412

|     | Robust      |           | z     | P> z  | [95% conf. interval] |          |
|-----|-------------|-----------|-------|-------|----------------------|----------|
|     | Coefficient | std. err. |       |       |                      |          |
| /b0 | 1.204888    | .0462355  | 26.06 | 0.000 | 1.114268             | 1.295507 |
| /b1 | .0046702    | .0009715  | 4.81  | 0.000 | .0027662             | .0065743 |

Instruments for equation 1: income\_cons

Notice how we specified the `derivative()` option for each parameter. We simply specified a slash, the name of the parameter, an equal sign, then a substitutable expression that represents the derivative. Because our model has only one residual equation, we do not need to specify equation numbers in the `derivative()` options.

When you specify a linear combination of variables, your derivative should be with respect to the entire linear combination. For example, say we have the residual equation

$$u_i = y - \exp(\mathbf{x}'_i\boldsymbol{\beta} + \beta_0)$$

for which we would type

```
. gmm (y - exp({xb: x1 x2 x3} + {b0})) ...
```

Then, in addition to the derivative  $\partial u_i/\partial\beta_0$ , we are to compute and specify

$$\frac{\partial u_i}{\partial \mathbf{x}'_i\boldsymbol{\beta}} = -\exp(\mathbf{x}'_i\boldsymbol{\beta} + \beta_0)$$

Using the chain rule,  $\partial u_i/\partial\beta_j = \partial u_i/\partial(\mathbf{x}'_i\boldsymbol{\beta}) \times \partial(\mathbf{x}'_i\boldsymbol{\beta})/\partial\beta_j = -x_{ij}\exp(\mathbf{x}'_i\boldsymbol{\beta} + \beta_0)$ . Stata does this last calculation automatically. It knows the variables in the linear combination, so all it needs is the derivative of the residual equation with respect to the linear combination. This allows you to change the variables in your linear combination without having to change the derivatives.

### ▷ Example 10: Derivatives with a linear combination

We refit the model described in the [example](#) illustrating exponential regression with endogenous regressors, now providing analytic derivatives. We type

```
. gmm (docvis - exp({xb:private chronic female income _cons})),
> instruments(private chronic female age black hispanic)
> derivative(/xb = -1*exp({xb:}))
```

Step 1:

```
Iteration 0: GMM criterion Q(b) = 16.910173
Iteration 1: GMM criterion Q(b) = .82270871
Iteration 2: GMM criterion Q(b) = .21831995
Iteration 3: GMM criterion Q(b) = .12685934
Iteration 4: GMM criterion Q(b) = .12672369
Iteration 5: GMM criterion Q(b) = .12672365
```

Step 2:

```
Iteration 0: GMM criterion Q(b) = .00234641
Iteration 1: GMM criterion Q(b) = .00215957
Iteration 2: GMM criterion Q(b) = .00215911
Iteration 3: GMM criterion Q(b) = .00215911
```

GMM estimation

```
Number of parameters = 5
Number of moments = 7
Initial weight matrix: Unadjusted
GMM weight matrix: Robust
Number of obs = 4,412
```

|         | Robust      |           |       |       |                      |
|---------|-------------|-----------|-------|-------|----------------------|
|         | Coefficient | std. err. | z     | P> z  | [95% conf. interval] |
| private | .535335     | .159904   | 3.35  | 0.001 | .221929 .848741      |
| chronic | 1.090126    | .0617659  | 17.65 | 0.000 | .9690668 1.211185    |
| female  | .6636579    | .0959885  | 6.91  | 0.000 | .475524 .8517918     |
| income  | .0142855    | .0027162  | 5.26  | 0.000 | .0089618 .0196092    |
| _cons   | -.5983477   | .138433   | -4.32 | 0.000 | -.8696714 -.327024   |

Instruments for equation 1: private chronic female age black hispanic \_cons

In the first `derivative()` option, we specified the name of the linear combination, `xb`, instead of an individual parameter's name. We already declared the variables of our linear combination in the substitutable expression for the residual equation, so in our substitutable expressions for the derivatives, we can use the shorthand notation `{xb:}` to refer to it.

Our point estimates are identical to those we obtained earlier. The standard errors and confidence intervals differ by only trivial amounts.

◀

## Exponential regression models with panel data

In addition to supporting cross-sectional and time-series data, `gmm` also works with panel-data models. Here we illustrate `gmm`'s panel-data capabilities by expanding our discussion of exponential regression models to allow for panel data. This also provides us the opportunity to demonstrate the moment-evaluator program version of `gmm`. Our discussion is based on [Blundell, Griffith, and Windmeijer \(2002\)](#). Also see [Wooldridge \(1999\)](#) for further discussion of nonlinear panel-data models.

First, we expand (7) for panel data. With individual heterogeneity term  $\eta_i$ , we have

$$E(y_{it} | \mathbf{x}_{it}, \eta_i) = \exp(\mathbf{x}'_{it}\boldsymbol{\beta} + \eta_i) = \mu_{it}\nu_i$$

where  $\mu_{it} = \exp(\mathbf{x}'_{it}\boldsymbol{\beta})$  and  $\nu_i = \exp(\eta_i)$ . Note that there is no constant term in this model, because its effect cannot be disentangled from  $\nu_i$ . With an additive idiosyncratic error term, we have the regression model

$$y_{it} = \mu_{it}\nu_i + \epsilon_{it}$$

We do not impose the assumption  $E(\mathbf{x}_{it}\eta_i) = \mathbf{0}$ , so  $\eta_i$  can be considered a fixed effect in the sense that it may be correlated with the regressors.

As discussed by [Blundell, Griffith, and Windmeijer \(2002\)](#), if  $\mathbf{x}_{it}$  is strictly exogenous, meaning  $E(\mathbf{x}_{it}\epsilon_{is}) = \mathbf{0}$  for all  $t$  and  $s$ , then we can estimate the parameters of the model by using the sample moment conditions

$$\sum_i \sum_t \mathbf{x}_{it} \left( y_{it} - \mu_{it} \frac{\bar{y}_i}{\bar{\mu}_i} \right) = \mathbf{0} \quad (9)$$

where  $\bar{y}_i$  and  $\bar{\mu}_i$  are the means of  $y_{it}$  and  $\mu_{it}$  for panel  $i$ , respectively. Because  $\bar{\mu}_i$  depends on the parameters of the model, it must be recomputed each time `gmm` needs to evaluate the residual equation. Therefore, we cannot use the substitutable expression version of `gmm`. Instead, we must use the moment-evaluator program version.

The moment-evaluator program version of gmm functions much like the function-evaluator program versions of nl and nlsur. The program you write is passed one or more variables to be filled in with the residuals evaluated at the parameter values specified in an option passed to your program. For the fixed-effects Poisson model with strictly exogenous regressors, our first crack at a function-evaluator program is

```

program gmm_poi
 version 19.5 // (or version 19 if you do not have StataNow)
 syntax varlist if, at(name)
 quietly {
 tempvar mu mubar ybar
 generate double `mu' = exp(x1*`at'[1,1] + x2*`at'[1,2] ///
 + x3*`at'[1,3]) `if'
 egen double `mubar' = mean(`mu') `if', by(id)
 egen double `ybar' = mean(y) `if', by(id)
 replace `varlist' = y - `mu'*`ybar'/'mubar' `if'
 }
end

```

You can save your program in an ado-file named *name*.ado, where *name* is the name you use for your program; here we would save the program in the ado-file `gmm_poi.ado`. Alternatively, if you are working from within a do-file, you can simply define the program before calling `gmm`. The `syntax` statement declares we are expecting to receive *varlist* that will contain the names of variables whose values we are to replace with the values of the residual equations and an `if` expression that will mark the estimation sample; because our model has one residual equation, *varlist* will consist of one variable. `at()` is a required option to our program, and it will contain the name of a matrix containing the parameter values at which we are to evaluate the residual equation. All moment-evaluator programs must accept the *varlist*, `if` condition, and `at()` option.

The first part of our program computes  $\mu_{it}$ . In the model we will fit shortly, we have three regressors, named `x1`, `x2`, and `x3`. The `'at'` vector will have three elements, one for each of those variables. Notice that we included `'if'` at the end of each statement that affects variables to restrict the computations to the relevant estimation sample. The two `egen` statements compute  $\bar{\mu}_i$  and  $\bar{y}_i$ ; in the example dataset we will use shortly, the panel variable is named `id`, and for simplicity, we hardcoded that variable into our program as well. Finally, we compute the residual equation, which is the portion of (9) bound in parentheses.

### ► Example 11: Panel Poisson with strictly exogenous regressors

To fit our model, we type

```

. use https://www.stata-press.com/data/r19/poisson1
. gmm gmm_poi, nequations(1) parameters(b1 b2 b3)
> instruments(x1 x2 x3, noconstant) vce(cluster id) onestep

Step 1:
Iteration 0: GMM criterion Q(b) = 51.99142
Iteration 1: GMM criterion Q(b) = .04345191
Iteration 2: GMM criterion Q(b) = 8.720e-06
Iteration 3: GMM criterion Q(b) = 7.115e-13
Iteration 4: GMM criterion Q(b) = 5.130e-27

note: model is exactly identified.

```

## GMM estimation

Number of parameters = 3

Number of moments = 3

Initial weight matrix: Unadjusted

Number of obs = 409

(Std. err. adjusted for 45 clusters in id)

|     | Coefficient | Robust<br>std. err. | z      | P> z  | [95% conf. interval] |           |
|-----|-------------|---------------------|--------|-------|----------------------|-----------|
| /b1 | 1.94866     | .1000265            | 19.48  | 0.000 | 1.752612             | 2.144709  |
| /b2 | -2.966119   | .0923592            | -32.12 | 0.000 | -3.14714             | -2.785099 |
| /b3 | 1.008634    | .1156561            | 8.72   | 0.000 | .781952              | 1.235315  |

Instruments for equation 1: x1 x2 x3

All three of our regressors are strictly exogenous, so they can serve as their own regressors. There is no constant term in the model (it would be unidentified), so we exclude a constant term from our list of instruments. We have one residual equation as indicated by `nequations(1)`, and we have three parameters, named `b1`, `b2`, and `b3`. The order in which you declare parameters in the `parameters()` option determines the order in which they appear in the ‘at’ vector in the moment-evaluator program. We specified `vce(cluster id)` to obtain standard errors that allow for correlation among observations within each panel.

◀

The program we just wrote is sufficient to fit the model to the `poisson1` dataset, but if we want to fit that model to other datasets, we need to change the variable names and perhaps account for having a different number of parameters as well. Despite those limitations, if you just want to fit a single model, that program is adequate.

Next, we take advantage of the ability to specify full equation names in the `parameters()` option and rewrite our evaluator program so that we can more easily change the variables in our model. This approach is particularly useful if some of the residual equations are linear in the parameters because then we can use `matrix score` (see [P] [matrix score](#)) to evaluate those moments.

Our new evaluator program is

```

program gmm_poieq
 version 19.5 // (or version 19 if you do not have StataNow)
 syntax varlist if, at(name)
 quietly {
 tempvar mu mubar ybar
 matrix score double `mu' = `at' `if', eq(#1)
 replace `mu' = exp(`mu')
 egen double `mubar' = mean(`mu') `if', by(id)
 egen double `ybar' = mean(y) `if', by(id)
 replace `varlist' = y - `mu'*`ybar'/'mubar' `if'
 }
end

```

Rather than using `generate` to compute the temporary variable ‘mu’, we used `matrix score` to obtain the linear combination  $\mathbf{x}'_{it}\beta$  and then called `replace` to compute  $\exp(\mathbf{x}'_{it}\beta)$ .



Typing

```
. matrix score double eq1 = 'at', eq(#1)
```

is equivalent to typing

```
. generate double eq1 = x1*'at'[1,1] + x2*'at'[1,2] + 'at'[1,3]
```

with one important difference. If we change some of the variables in the `parameters()` option when we call `gmm`, `matrix score` will compute the correct linear combination. If we were to use the `generate` statement instead, then every time we wanted to change the variables in our model, we would have to modify that statement as well.

The command

```
. matrix score double alpha = 'at', eq(#2)
```

is equivalent to

```
. scalar alpha = 'at'[1,4]
```

Thus, even if you specify linear combination and variable names in the `parameters()` option, you can still have scalar parameters in your model. □

When past values of the idiosyncratic error term affect the value of a regressor, we say that regressor is predetermined. When one or more regressors are predetermined, sample moment condition (8) is no longer valid. However, [Chamberlain \(1992\)](#) shows that a simple alternative is to consider moment conditions of the form

$$\sum_i \sum_{t=2}^T \mathbf{x}_{i,t-1} \left( y_{i,t-1} - \mu_{i,t-1} \frac{y_{it}}{\mu_{it}} \right) = \mathbf{0} \quad (10)$$

Also see [Wooldridge \(1997\)](#) and [Windmeijer \(2000\)](#) for other moment conditions that can be used with predetermined regressors.

### ▷ Example 13: Panel Poisson with predetermined regressors

Here we refit the previous model, treating all the regressors as predetermined and using the moment conditions in (10). Our moment-evaluator program is

```
program gmm_poipre
 version 19.5 // (or version 19 if you do not have StataNow)
 syntax varlist if, at(name) mylhs(varlist)
 quietly {
 tempvar mu
 matrix score double 'mu' = 'at' 'if', eq(#1)
 replace 'mu' = exp('mu')
 replace 'varlist' = L.'mylhs' - L.'mu'*'mylhs'/'mu' 'if'
 }
end
```

To compute the residual equation, we used lag-operator notation so that Stata properly handles gaps in our panel dataset. We also made our program accept an additional option that we will use to pass in the dependent variable. When we specify this option in our `gmm` statement, it will get passed to our evaluator program because `gmm` will not recognize the option as one of its own. Equation (10) shows that we are to use the first lags of the regressors as instruments, so we type

```
. gmm gmm_poipre, mylhs(y) nequations(1) vce(cluster id) onestep
> parameters({y:x1 x2 x3}) instruments(L.(x1 x2 x3), noconstant)
note: 45 missing values returned for equation 1 at initial values.
```

Step 1:

```
Iteration 0: GMM criterion Q(b) = 76.652367
Iteration 1: GMM criterion Q(b) = 1.9118192
Iteration 2: GMM criterion Q(b) = .06634724
Iteration 3: GMM criterion Q(b) = .0000233
Iteration 4: GMM criterion Q(b) = 2.998e-12
Iteration 5: GMM criterion Q(b) = 4.888e-26
```

note: model is exactly identified.

GMM estimation

Number of parameters = 3

Number of moments = 3

Initial weight matrix: Unadjusted

Number of obs = 364

(Std. err. adjusted for 45 clusters in id)

|    | Robust      |           | z      | P> z  | [95% conf. interval] |           |
|----|-------------|-----------|--------|-------|----------------------|-----------|
|    | Coefficient | std. err. |        |       |                      |           |
| x1 | 2.088246    | .2513626  | 8.31   | 0.000 | 1.595584             | 2.580907  |
| x2 | -2.905504   | .2133908  | -13.62 | 0.000 | -3.323742            | -2.487266 |
| x3 | 1.121081    | .201654   | 5.56   | 0.000 | .7258459             | 1.516315  |

Instruments for equation 1: L.x1 L.x2 L.x3

Here, like earlier with strictly exogenous regressors, the number of instruments equals the number of parameters, so there is no gain to using the two-step or iterated estimator. However, if you do have more instruments than parameters, you will most likely want to use one of those other estimators instead.

The note at the top of the output is given because we have 45 panels in our dataset. Our residual equation includes lagged terms and therefore cannot be evaluated for the first time period within each panel. Notes like this can be ignored once you know why they occurred. If you receive a note that you were not expecting, you should first investigate the cause of the note before trusting the results.

Instead of making our program accept the `mylhs()` option, we could have used Stata's `coleq` macro function to determine the dependent variable based on the column names attached to the 'at' vector; see [\[P\] macro](#). Then, we could refit our model with a different dependent variable by changing the `lname` used in the `parameters()` option. In the [next example](#), we take this approach.

◀

In the [previous example](#), we used  $\mathbf{x}_{i,t-1}$  as instruments. A more efficient GMM estimator would also use  $\mathbf{x}_{i,t-2}, \mathbf{x}_{i,t-3}, \dots, \mathbf{x}_{i,1}$  as instruments in period  $t$  as well. `gmm`'s `xtinstruments()` option allows you to specify instrument lists that grow as  $t$  increases. Later, we discuss the `xtinstruments()` option in detail in the context of linear dynamic panel-data models.

When a regressor is contemporaneously correlated with the idiosyncratic error term, we say that regressor is endogenous. [Windmeijer \(2000\)](#) shows that we can use the moment condition

$$\sum_i \sum_{t=3}^T \mathbf{x}_{i,t-2} \left( \frac{y_{it}}{\mu_{it}} - \frac{y_{i,t-1}}{\mu_{i,t-1}} \right)$$

Here we use the second lag of the endogenous regressor as an instrument. If a variable is strictly exogenous, it can of course serve as its own instrument.

## ▶ Example 14: Panel Poisson with endogenous regressors

Here we refit the model, treating  $x_3$  as endogenous and  $x_1$  and  $x_2$  as strictly exogenous. Our moment-evaluator program is

```

program gmm_poiend
 version 19.5 // (or version 19 if you do not have StataNow)
 syntax varlist if, at(name)
 quietly {
 tempvar mu
 matrix score double `mu' = `at' `if', eq(#1)
 replace `mu' = exp(`mu')
 local mylhs : coleq `at'
 local mylhs : word 1 of `mylhs'
 replace `varlist' = `mylhs'/'mu' - L.`mylhs'/L.`mu' `if'
 }
end

```

Now, we call `gmm` using  $x_1$ ,  $x_2$ , and `L2.x3` as instruments:

```

. use https://www.stata-press.com/data/r19/poisson2
. gmm gmm_poiend, nequations(1) vce(cluster id) onestep
> parameters(y:x1 y:x2 y:x3) instruments(x1 x2 L2.x3, noconstant)
note: 500 missing values returned for equation 1 at initial values.

Step 1:
Iteration 0: GMM criterion Q(b) = 61.832288
Iteration 1: GMM criterion Q(b) = .03402584
Iteration 2: GMM criterion Q(b) = .01101288
Iteration 3: GMM criterion Q(b) = 6.339e-06
Iteration 4: GMM criterion Q(b) = 1.620e-12
Iteration 5: GMM criterion Q(b) = 1.312e-25

note: model is exactly identified.

GMM estimation
Number of parameters = 3
Number of moments = 3
Initial weight matrix: Unadjusted
 Number of obs = 3,766
 (Std. err. adjusted for 500 clusters in id)

```

|    | Coefficient | Robust<br>std. err. | z      | P> z  | [95% conf. interval] |           |
|----|-------------|---------------------|--------|-------|----------------------|-----------|
| x1 | 1.8141      | .2688318            | 6.75   | 0.000 | 1.2872               | 2.341001  |
| x2 | -2.982671   | .1086666            | -27.45 | 0.000 | -3.195653            | -2.769688 |
| x3 | 4.126518    | 6.369334            | 0.65   | 0.517 | -8.357147            | 16.61018  |

Instruments for equation 1: x1 x2 L2.x3

The note at the top of the output is given because that we have 500 panels in our dataset. As in the [previous example](#), our residual equation includes lagged terms and therefore cannot be evaluated for the first time period within each panel. Instead of using just  $x_{i,t-2}$  as an instrument, we could use all further lags of  $x_{it}$  as instruments as well.

## Rational-expectations models

Macroeconomic models typically assume that agents' expectations about the future are formed rationally. By rational expectations, we mean that agents use all information available when forming their forecasts, so the forecast error is uncorrelated with the information available when the forecast was made. Say that at time  $t$ , people make a forecast,  $\hat{y}_{t+1}$ , of variable  $y$  in the next period. If  $\Omega_t$  denotes all available information at time  $t$ , then rational expectations implies that  $E\{(\hat{y}_{t+1} - y_{t+1})|\Omega_t\} = 0$ . If  $\Omega_t$  denotes observable variables such as interest rates or prices, then this conditional expectation can serve as the basis of a moment condition for GMM estimation.

### ▷ Example 15: Fitting a Euler equation

In a well-known article, [Hansen and Singleton \(1982\)](#) consider a model of portfolio decision making and discuss parameter estimation using GMM. We will consider a simple example with one asset in which the agent can invest. A consumer wants to maximize the present value of his or her lifetime utility derived from buying a good. On the one hand, the consumer is impatient, so he or she would rather buy today than wait until tomorrow. On the other hand, by buying less today, the consumer can invest more money, earning more interest that can be used to buy more of the good tomorrow. Thus, there is a tradeoff between having cake today or sacrificing a bit today to have more cake tomorrow.

If we assume a specific form for the agent's utility function, known as the constant relative-risk aversion utility function, we can show that the Euler equation is

$$E[\mathbf{z}_t \{1 - \beta(1 + r_{t+1})(c_{t+1}/c_t)^{-\gamma}\}] = \mathbf{0}$$

where  $\beta$  and  $\gamma$  are the parameters to estimate,  $r_t$  is the return to the financial asset, and  $c_t$  is consumption in period  $t$ .  $\beta$  measures the agent's discount factor. If  $\beta$  is near 1, the agent is patient and is more willing to forgo consumption this period. If  $\beta$  is close to 0, the agent is less patient and prefers to consume more now. The parameter  $\gamma$  characterizes the agent's utility function. If  $\gamma = 0$ , the utility function is linear. As  $\gamma$  tends toward 1, the utility function tends toward  $u = \log(c)$ .

We have data on three-month Treasury bills ( $r_t$ ) and consumption expenditures ( $c_t$ ). As instruments, we will use lagged rates of return and past growth rates of consumption. We will use the two-step estimator and a weight matrix that allows for heteroskedasticity and autocorrelation up to four lags with the Bartlett kernel. In Stata, we type

```
. use https://www.stata-press.com/data/r19/cr
. generate cgrowth = c / L.c
(1 missing value generated)
. gmm (1 - {b=1}*(1+F.r)*(F.c/c)^(-1*{gamma=1})),
> inst(L.r L2.r cgrowth L.cgrowth) wmat(hac nw 4) twostep
note: 1 missing value returned for equation 1 at initial values.

Step 1:
Iteration 0: GMM criterion Q(b) = .00226482
Iteration 1: GMM criterion Q(b) = .00054369
Iteration 2: GMM criterion Q(b) = .00053904
Iteration 3: GMM criterion Q(b) = .00053904

Step 2:
Iteration 0: GMM criterion Q(b) = .0600729
Iteration 1: GMM criterion Q(b) = .0596369
Iteration 2: GMM criterion Q(b) = .0596369

GMM estimation
Number of parameters = 2
Number of moments = 5
Initial weight matrix: Unadjusted Number of obs = 239
GMM weight matrix: HAC Bartlett 4
```

|        | HAC         |           |       |       |                      |           |
|--------|-------------|-----------|-------|-------|----------------------|-----------|
|        | Coefficient | std. err. | z     | P> z  | [95% conf. interval] |           |
| /b     | .9204617    | .0134646  | 68.36 | 0.000 | .8940716             | .9468518  |
| /gamma | -4.222361   | 1.473895  | -2.86 | 0.004 | -7.111143            | -1.333579 |

HAC standard errors based on Bartlett kernel with 4 lags.  
 Instruments for equation 1: L.r L2.r cgrowth L.cgrowth\_cons

The note at the top of the output is given because the forward operator in our substitutable expression says that residuals cannot be computed for the last observation. In addition, two observations are omitted because the L2.r instrument has missing values in the first two time periods. Therefore, of the 242 observations in our dataset, 239 are used to fit the model. Our estimate of  $\beta$  is near 1, in line with expectations and published results. However, our estimate of  $\gamma$  implies risk-loving behavior and therefore a poorly specified model.

Also notice our use of the forward operator to refer to the values of  $r$  and  $c$  one period ahead; time-series operators are allowed in substitutable expressions as long as you have previously `tsset` (see [TS] `tsset`) your data. See [U] 13.10 Time-series operators for more information on time-series operators.

◀

## System estimators

In many economic models, two or more variables are determined jointly through a system of simultaneous equations. Indeed, some of the earliest work in econometrics, including that of the Cowles Commission, was centered around estimation of the parameters of simultaneous equations. The 2SLS and

instrumental-variables estimators we have already discussed are used in some circumstances to estimate such parameters. Here we focus on the joint estimation of all the parameters of systems of equations, and we begin with the well-known three-stage least-squares (3SLS) estimator.

Recall that the 2SLS estimator is based on the moment conditions  $E(\mathbf{z}\mathbf{u}) = \mathbf{0}$ . The 2SLS estimator can be used to estimate the parameters of one equation of a system of structural equations. Moreover, with the 2SLS estimator, we do not even need to specify the structural relationship among all the endogenous variables; we need to specify only the equation on which interest focuses and simply assume reduced-form relationships among the endogenous regressors of the equation of interest and the exogenous variables of the model. If we are willing to specify the complete system of structural equations, then assuming our model is correctly specified, by estimating all the equations jointly, we can obtain estimates that are more efficient than equation-by-equation 2SLS.

In [R] **reg3**, we fit a simple two-equation macroeconomic model,

$$\text{consump} = \beta_0 + \beta_1 \text{wagepriv} + \beta_2 \text{wagegovt} + \epsilon_1 \quad (11)$$

$$\text{wagepriv} = \beta_3 + \beta_4 \text{consump} + \beta_5 \text{govt} + \beta_6 \text{capital1} + \epsilon_2 \quad (12)$$

where `consump` represents aggregate consumption; `wagepriv` and `wagegovt` are total wages paid by the private and government sectors, respectively; `govt` is government spending; and `capital1` is the previous period's capital stock. We are not willing to assume that  $\epsilon_1$  and  $\epsilon_2$  are independent, so we must treat both `consump` and `wagepriv` as endogenous. Suppose that a random shock makes  $\epsilon_2$  positive. Then by (12), `wagepriv` will be higher than it otherwise would. Moreover,  $\epsilon_1$  will be either higher or lower, depending on the correlation between it and  $\epsilon_2$ . The shock to  $\epsilon_2$  has made both `wagepriv` and  $\epsilon_1$  move, which implies that in (11), `wagepriv` is an endogenous regressor. A similar argument shows that `consump` is an endogenous regressor in the second equation. In our model, `wagegovt`, `govt`, and `capital1` are all exogenous variables.

Let  $\mathbf{z}_1$  and  $\mathbf{z}_2$  denote the instruments for the first and second equations, respectively; we will discuss what comprises them shortly. We have two sets of moment conditions:

$$E \left\{ \begin{array}{l} \mathbf{z}_1 (\text{consump} - \beta_0 - \beta_1 \text{wagepriv} - \beta_2 \text{wagegovt}) \\ \mathbf{z}_2 (\text{wagepriv} - \beta_3 - \beta_4 \text{consump} - \beta_5 \text{govt} - \beta_6 \text{capital1}) \end{array} \right\} = \mathbf{0} \quad (13)$$

One of the defining characteristics of 3SLS is that the errors are homoskedastic conditional on the instrumental variables. Using this assumption, we have

$$E \left[ \begin{array}{l} \left\{ \begin{array}{l} \mathbf{z}_1 \epsilon_1 \\ \mathbf{z}_2 \epsilon_2 \end{array} \right\} \left\{ \begin{array}{ll} \mathbf{z}'_1 \epsilon_1 & \mathbf{z}'_2 \epsilon_2 \end{array} \right\} \end{array} \right] = \begin{array}{cc} \left\{ \begin{array}{ll} \sigma_{11} E(\mathbf{z}_1 \mathbf{z}'_1) & \sigma_{12} E(\mathbf{z}_1 \mathbf{z}'_2) \\ \sigma_{21} E(\mathbf{z}_2 \mathbf{z}'_1) & \sigma_{22} E(\mathbf{z}_2 \mathbf{z}'_2) \end{array} \right\} \end{array} \quad (14)$$

where  $\sigma_{ij} = \text{cov}(\epsilon_i, \epsilon_j)$ . Let  $\Sigma$  denote the  $2 \times 2$  matrix with typical element  $\sigma_{ij}$ .

The second defining characteristic of the 3SLS estimator is that it uses all the exogenous variables as instruments for all equations; here  $\mathbf{z}_1 = \mathbf{z}_2 = (\text{wagegovt}, \text{govt}, \text{capital1}, 1)$ , where the 1 indicates a constant term. From our discussion on the weight matrix and two-step estimation, we want to use the sample analogue of the matrix inverse of the right-hand side of (14) as our weight matrix.

To implement the 3SLS estimator, we apparently need to know  $\Sigma$  or at least have a consistent estimator of it. The solution is to fit (11) and (12) by 2SLS, use the sample residuals  $\widehat{\epsilon}_1$  and  $\widehat{\epsilon}_2$  to estimate  $\Sigma$ , then estimate the parameters of (13) via GMM by using the weight matrix just discussed.

### ► Example 16: 3SLS estimation

3SLS is easier to do using `gmm` than it sounds. The 3SLS estimator is a two-step GMM estimator. In the first step, we do the equivalent of 2SLS on each equation, and then we compute a weight matrix based on (14). Finally, we perform a second step of GMM with this weight matrix.

In Stata, we type

```
. use https://www.stata-press.com/data/r19/klein, clear
. gmm (eq1: consump - {xb: wagepriv wagegovt _cons})
> (eq2: wagepriv - {xc: consump govt capital1 _cons}),
> instruments(eq1: wagegovt govt capital1)
> instruments(eq2: wagegovt govt capital1)
> winitial(unadjusted, independent) wmatrix(unadjusted) twostep

Step 1:
Iteration 0: GMM criterion Q(b) = 4195.4487
Iteration 1: GMM criterion Q(b) = .22175631
Iteration 2: GMM criterion Q(b) = .22175631 (backed up)

Step 2:
Iteration 0: GMM criterion Q(b) = .09716589
Iteration 1: GMM criterion Q(b) = .07028208
Iteration 2: GMM criterion Q(b) = .07028208

GMM estimation
Number of parameters = 7
Number of moments = 8
Initial weight matrix: Unadjusted Number of obs = 22
GMM weight matrix: Unadjusted
```

|           | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-----------|-------------|-----------|-------|-------|----------------------|----------|
| <b>xb</b> |             |           |       |       |                      |          |
| wagepriv  | .8012754    | .1279329  | 6.26  | 0.000 | .5505314             | 1.052019 |
| wagegovt  | 1.029531    | .3048424  | 3.38  | 0.001 | .432051              | 1.627011 |
| _cons     | 19.3559     | 3.583772  | 5.40  | 0.000 | 12.33184             | 26.37996 |
| <b>xc</b> |             |           |       |       |                      |          |
| consump   | .4026076    | .2567312  | 1.57  | 0.117 | -.1005764            | .9057916 |
| govt      | 1.177792    | .5421253  | 2.17  | 0.030 | .1152461             | 2.240338 |
| capital1  | -.0281145   | .0572111  | -0.49 | 0.623 | -.1402462            | .0840173 |
| _cons     | 14.63026    | 10.26693  | 1.42  | 0.154 | -5.492552            | 34.75306 |

```
Instruments for equation eq1: wagegovt govt capital1 _cons
Instruments for equation eq2: wagegovt govt capital1 _cons
```

The independent suboption of the `winitial()` option tells `gmm` to assume that the residuals are independent across moment conditions; this suboption sets  $\sigma_{21} = \sigma_{12} = 0$  in (14). Assuming both homoskedasticity and cross-equation independence is equivalent to fitting the two equations of our model independently by 2SLS. The `wmatrix()` option controls how the weight matrix is computed on the basis of the first-step parameter estimates before the second step of estimation; here we request a weight matrix that assumes conditional homoskedasticity but that does not impose the cross-equation independence like the initial weight matrix we used. In this example, we also illustrated how to name residual equations and how equation names can be used in the `instruments()` option. Our results are identical to those in [R] `reg3`.

We could have specified our instruments with the syntax

```
instruments(wagegovt govt capital1)
```

because `gmm` uses the variables listed in the `instruments()` option for all equations unless you specify which equations the list of instruments is to be used with. However, we wanted to emphasize that the same instruments are being used for both equations; in a moment, we will discuss an estimator that does not use the same instruments in all equations. ◀

In the [previous example](#), if we omit the `twostep` option, the resulting coefficients will be equivalent to equation-by-equation 2SLS, which [Wooldridge \(2010, 216\)](#) calls the “system 2SLS estimator”. Eliminating the `twostep` option makes the `wmatrix()` option irrelevant, so that option can be eliminated as well.

So far, we have developed the traditional 3SLS estimator. [Wooldridge \(2010, chap. 8\)](#) discusses the “GMM 3SLS” estimator, which extends the traditional 3SLS estimator by allowing for heteroskedasticity and different instruments for different equations.

Generalizing (14) to an arbitrary number of equations, we have

$$E(\mathbf{Z}'\epsilon\epsilon'\mathbf{Z}) = E(\mathbf{Z}'\Sigma\mathbf{Z}) \quad (15)$$

where

$$\mathbf{Z} = \begin{bmatrix} \mathbf{z}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{z}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{z}_m \end{bmatrix}$$

and  $\Sigma$  is now  $m \times m$ . Equation (15) is the multivariate analogue of a homoskedasticity assumption; for each equation, the error variance is constant for all observations, as is the covariance between any two equations’ errors.

We can relax this homoskedasticity assumption by considering different weight matrices. For example, if we continue to assume that observations are independent but not necessarily identically distributed, then by specifying `wmatrix(robust)`, we would obtain a weight matrix that allows for heteroskedasticity:

$$\widehat{W} = \frac{1}{N} \sum_i \mathbf{z}'_i \widehat{\epsilon}_i \widehat{\epsilon}'_i \mathbf{z}_i$$

This is the weight matrix in [Wooldridge’s \(2010, 218\) Procedure 8.1](#), “GMM with Optimal Weighting Matrix”. By default, `gmm` would report standard errors based on his covariance matrix (8.27); specifying `vce(unadjusted)` would provide the optimal GMM standard errors. If you have multiple observations for each individual or firm in your dataset, you could specify `wmatrix(cluster id)`, where `id` identifies individuals or firms. This would allow arbitrary within-individual correlation, though it does not account for an individual-specific fixed or random effect. In both cases, we would continue to use `winitial(unadjusted, independent)` so that the first-step estimates are the system 2SLS estimates.

[Wooldridge \(2010, sec. 9.6\)](#) discusses instances where it is necessary to use different instruments in different equations. The GMM 3SLS estimator with different instruments in different equations but with conditional homoskedasticity is what [Hayashi \(2000, 275\)](#) calls the “full-information instrumental-variables efficient” (FIVE) estimator. Implementing the FIVE estimator is easy with `gmm`. For example, say we have a two-equation system where `kids`, `age`, `income`, and `education` are all valid instruments for the first equation but where `education` is not a valid instrument for the second equation. Then, our syntax would take the form

```
gmm (exp_1) (exp_2), instruments(1:kids age income education) ///
instruments(2:kids age income)
```

The following syntax is equivalent:

```
gmm (exp_1) (exp_2), instruments(kids age income)
instruments(1:education)
```

Because we did not specify a list of equations in the second example's first `instruments()` option, those variables are used as instruments in both equations. You can use whichever syntax you prefer. The first requires a bit more typing but is arguably more transparent.

If all the regressors in the model are exogenous, then the traditional 3SLS estimator is the seemingly unrelated regression (SUR) estimator. Here you would specify all the regressors as instruments.

## Dynamic panel-data models

Commands in Stata that work with panel data expect the data to be in the “long” format, meaning that each row of the dataset consists of one subobservation that is a member of a logical observation (represented by the panel identifier variable). See [D] [reshape](#) for a discussion of the long versus “wide” data forms. `gmm` is no exception in this respect when used with panel data. From a theoretical perspective, however, it is sometimes easier to view GMM estimators for panel data as system estimators in which we have  $N$  observations on a system of  $T$  equations, where  $N$  and  $T$  are the number of observations and panels, respectively, rather than a single-equation estimator with  $NT$  observations. Usually, each of the  $T$  equations will in fact be the same, though we will want to specify different instruments for each of these equations.

In a dynamic panel-data model, lagged values of the dependent variable are included as regressors. Here we consider a simple model with one lag of the dependent variable  $y$  as a regressor and a vector of strictly exogenous regressors,  $\mathbf{x}_{it}$ :

$$y_{it} = \rho y_{i,t-1} + \mathbf{x}'_{it}\boldsymbol{\beta} + u_i + \epsilon_{it} \quad (16)$$

$u_i$  can be either a fixed- or a random-effect term in the sense that we do not require  $\mathbf{x}_{it}$  to be independent of it. Even with the assumption that  $\epsilon_{it}$  is i.i.d., the presence of both  $y_{i,t-1}$  and  $u_i$  in (16) renders both the standard fixed- and random-effects estimators to be inconsistent because of the well-known [Nickell \(1981\)](#) bias. OLS regression of  $y_{it}$  on  $y_{i,t-1}$  and  $\mathbf{x}_{it}$  also produces inconsistent estimates because  $y_{i,t-1}$  will be correlated with the error term.

### □ Technical note

Stata has the `xtabond`, `xtdpd`, and `xtdpdsys` commands (see [XT] [xtabond](#), [XT] [xtdpd](#), and [XT] [xtdpdsys](#)) to fit equations like (16); for everyday use, those commands are preferred because they offer features such as [Windmeijer \(2005\)](#) bias-corrected standard errors to account for the bias of traditional two-step GMM standard errors seen in dynamic panel-data models and, being linear estimators, only require you to specify variable names instead of complete equations. However, using `gmm` has several pedagogical advantages, including the ability to tie those model-specific commands into a more general framework, a clear illustration of how certain types of instrument matrices for panel-data models are formed, and demonstrations of several advanced features of `gmm`. □

First-differencing (16) removes the panel-specific  $u_i$  term:

$$y_{it} - y_{i,t-1} = \rho(y_{i,t-1} - y_{i,t-2}) + (\mathbf{x}_{it} - \mathbf{x}_{i,t-1})'\boldsymbol{\beta} + (\epsilon_{it} - \epsilon_{i,t-1}) \quad (17)$$

However, now  $(y_{i,t-1} - y_{i,t-2})$  is correlated with  $(\epsilon_{it} - \epsilon_{i,t-1})$ . Thus, we need an instrument that is correlated with the former but not the latter. The lagged variables in (17) mean that the equation is not estimable for  $t < 3$ , so consider when  $t = 3$ . We have

$$y_{i3} - y_{i2} = \rho(y_{i2} - y_{i1}) + (\mathbf{x}_{i3} - \mathbf{x}_{i2})' \boldsymbol{\beta} + (\epsilon_{i3} - \epsilon_{i2}) \tag{18}$$

In the Arellano–Bond (1991) estimator, lagged levels of the dependent variable are used as instruments. With our assumption that the  $\epsilon_{it}$  are i.i.d., (16) intimates that  $y_{i1}$  can serve as an instrumental variable when we fit (18).

Next, consider (17) when  $t = 4$ . We have

$$y_{i4} - y_{i3} = \rho(y_{i3} - y_{i2}) + (\mathbf{x}_{i4} - \mathbf{x}_{i3})' \boldsymbol{\beta} + (\epsilon_{i4} - \epsilon_{i3})$$

Now, (16) shows that both  $y_{i1}$  and  $y_{i2}$  are uncorrelated with the error term  $(\epsilon_{i4} - \epsilon_{i3})$ , so we have two instruments available. For  $t = 5$ , you can show that  $y_{i1}$ ,  $y_{i2}$ , and  $y_{i3}$  can serve as instruments. As may now be apparent, one of the key features of these dynamic panel-data models is that the available instruments depend on the time period,  $t$ , as was the case for some of the panel Poisson models we considered earlier. Because the  $\mathbf{x}_{it}$  are strictly exogenous by assumption, they can serve as their own instruments.

The initial weight matrix that is appropriate for the GMM dynamic panel-data estimator is slightly more involved than the unadjusted matrix that we have used in most of our previous examples and that assumes the errors are i.i.d. First, rewrite (17) for panel  $i$  as

$$\mathbf{y}_i - \mathbf{y}_i^L = \rho(\mathbf{y}_i^L - \mathbf{y}_i^{LL}) + (\mathbf{X}_i - \mathbf{X}_i^L) \boldsymbol{\beta} + (\boldsymbol{\epsilon}_i - \boldsymbol{\epsilon}_i^L)$$

where  $\mathbf{y}_i = (y_{i3}, \dots, y_{iT})$  and  $\mathbf{y}_i^L = (y_{i2}, \dots, y_{i,T-1})$ ,  $\mathbf{y}_i^{LL} = (y_{i1}, \dots, y_{i,T-2})$ , and  $\mathbf{X}_i, \mathbf{X}_i^L, \boldsymbol{\epsilon}_i$ , and  $\boldsymbol{\epsilon}_i^L$  are defined analogously. Let  $\mathbf{Z}$  denote the full matrix of instruments for panel  $i$ , including the variables specified in both the `instruments()` and `xtinstruments()` options; the exact structure is detailed in [Methods and formulas](#).

By assumption,  $\epsilon_{it}$  is i.i.d., so the first difference  $(\epsilon_{it} - \epsilon_{i,t-1})$  is necessarily autocorrelated with correlation  $-0.5$ . Therefore, we should not use a weight matrix that assumes the errors are independent. For dynamic panel-data models, we can show that the appropriate initial weight matrix is

$$\widehat{\mathbf{W}} = \left( \frac{1}{N} \sum_i \mathbf{Z}_i' \mathbf{H}_D \mathbf{Z}_i \right)^{-1}$$

where

$$\mathbf{H}_D = \begin{bmatrix} 1 & -0.5 & 0 & \dots & 0 & 0 \\ -0.5 & 1 & -0.5 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & -0.5 \\ 0 & 0 & 0 & \dots & -0.5 & 1 \end{bmatrix}$$

We can obtain this initial weight matrix by specifying `winitial(xt D)`. The letter D indicates that the equation we are estimating is specified in first differences.

## ▶ Example 17: Arellano–Bond estimator

Say we want to fit the model

$$\mathbf{n}_{it} = \rho \mathbf{n}_{i,t-1} + \beta_1 \mathbf{w}_{it} + \beta_2 \mathbf{w}_{i,t-1} + \beta_3 \mathbf{k}_{it} + \beta_4 \mathbf{k}_{i,t-1} + u_i + \epsilon_{it} \quad (19)$$

where we assume that  $\mathbf{w}_{it}$  and  $\mathbf{k}_{it}$  are strictly exogenous. First-differencing, our residual equation is

$$\begin{aligned} \epsilon_{it}^* = (\epsilon_{it} - \epsilon_{i,t-1}) = & \mathbf{n}_{it} - \mathbf{n}_{i,t-1} - \rho (\mathbf{n}_{i,t-1} - \mathbf{n}_{i,t-2}) - \beta_1 (\mathbf{w}_{it} - \mathbf{w}_{i,t-1}) \\ & - \beta_2 (\mathbf{w}_{i,t-1} - \mathbf{w}_{i,t-2}) - \beta_3 (\mathbf{k}_{it} - \mathbf{k}_{i,t-1}) - \beta_4 (\mathbf{k}_{i,t-1} - \mathbf{k}_{i,t-2}) \end{aligned} \quad (20)$$

In Stata, we type

```
. use https://www.stata-press.com/data/r19/abdata
. gmm (D.n - {rho}*LD.n - {xb:D.w LD.w D.k LD.k}),
> xtinstruments(n, lags(2/.)) instruments(D.w LD.w D.k LD.k, noconstant)
> deriv(/rho = -1*LD.n) deriv(/xb = -1) winitial(xt D) onestep

Step 1:
Iteration 0: GMM criterion Q(b) = .0011455
Iteration 1: GMM criterion Q(b) = .00009103
Iteration 2: GMM criterion Q(b) = .00009103

GMM estimation
Number of parameters = 5
Number of moments = 32
Initial weight matrix: XT D
 Number of obs = 751
 (Std. err. adjusted for 140 clusters in id)
```

|       | Coefficient | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |           |
|-------|-------------|---------------------|-------|-------|----------------------|-----------|
| rho   |             |                     |       |       |                      |           |
| _cons | .8041712    | .1199819            | 6.70  | 0.000 | .5690111             | 1.039331  |
| xb    |             |                     |       |       |                      |           |
| w     |             |                     |       |       |                      |           |
| D1.   | -.5600476   | .1619472            | -3.46 | 0.001 | -.8774583            | -.242637  |
| LD.   | .3946699    | .1092229            | 3.61  | 0.000 | .1805969             | .6087429  |
| k     |             |                     |       |       |                      |           |
| D1.   | .3520286    | .0536546            | 6.56  | 0.000 | .2468676             | .4571897  |
| LD.   | -.2160435   | .0679689            | -3.18 | 0.001 | -.3492601            | -.0828269 |

```
Instruments for equation 1:
XT-style: L(2/.).n
Standard: D.w LD.w D.k LD.k
```

Because  $\mathbf{w}$  and  $\mathbf{k}$  are strictly exogenous, we specified their variants that appear in (20) in the `instruments()` option; because there is no constant term in the model, we specified `noconstant` to omit the constant from the instrument list.

We specified `xtinstruments(n, lags(2/.))` to tell `gmm` what instruments to use for the lagged dependent variable included as a regressor in (19). On the basis of our previous discussion, lags two and higher of  $\mathbf{n}_{it}$  can serve as instruments. The `lags(2/.)` suboption tells `gmm` that the first available instrument for  $\mathbf{n}_{it}$  is the lag-two value  $\mathbf{n}_{i,t-2}$ . The “.” tells `gmm` to use all further lags of  $\mathbf{n}_{it}$  as instruments as well. The instrument matrices in dynamic panel-data models can become large if the dataset has many time periods per panel. In those cases, you could specify, for example, `lags(2/4)` to use just lags two through four instead of using all available lags.

Our results are identical to those we would obtain using `xtabond` with the syntax

```
xtabond n L(0/1) .w L(0/1) .k, lags(1) noconstant vce(robust)
```

If we had left off the `vce(robust)` option in our call to `xtabond`, we would have had to specify `vce(unadjusted)` in our call to `gmm` to obtain the same standard errors.

◀

### □ Technical note

`gmm` automatically excludes observations for which there are no valid observations for the panel-style instruments. However, it keeps in the estimation sample those observations for which fewer than the maximum number of instruments you requested are available. For example, if you specify the `lags(2/4)` suboption, you have requested three instruments, but `gmm` will keep observations even if only one or two instruments are available.

□

### ▷ Example 18: Two-step Arellano–Bond estimator

Here we refit the model from [example 17](#), using the two-step GMM estimator.

```
. gmm (D.n - {rho}*LD.n - {xb:D.w LD.w D.k LD.k}),
> xt instruments(n, lags(2/.)) instruments(D.w LD.w D.k LD.k, noconstant)
> deriv(/rho = -1*LD.n) deriv(/xb = -1) winitial(xt D) wmatrix(robust)
> vce(unadjusted)
```

Step 1:

Iteration 0: GMM criterion Q(b) = .0011455

Iteration 1: GMM criterion Q(b) = .00009103

Iteration 2: GMM criterion Q(b) = .00009103

Step 2:

Iteration 0: GMM criterion Q(b) = .44107941

Iteration 1: GMM criterion Q(b) = .4236729

Iteration 2: GMM criterion Q(b) = .4236729 (backed up)

GMM estimation

Number of parameters = 5

Number of moments = 32

Initial weight matrix: XT D

Number of obs = 751

GMM weight matrix: Robust

|       | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |           |
|-------|-------------|-----------|--------|-------|----------------------|-----------|
| rho   |             |           |        |       |                      |           |
| _cons | .8044783    | .0534763  | 15.04  | 0.000 | .6996667             | .90929    |
| xb    |             |           |        |       |                      |           |
| w     |             |           |        |       |                      |           |
| D1.   | -.5154978   | .0335506  | -15.36 | 0.000 | -.5812557            | -.4497399 |
| LD.   | .4059309    | .0637294  | 6.37   | 0.000 | .2810235             | .5308384  |
| k     |             |           |        |       |                      |           |
| D1.   | .3556204    | .0390892  | 9.10   | 0.000 | .2790071             | .4322337  |
| LD.   | -.2204521   | .046439   | -4.75  | 0.000 | -.3114709            | -.1294332 |

Instruments for equation 1:

XT-style: L(2/.)n

Standard: D.w LD.w D.k LD.k

Our results match those you would obtain with the command

```
xtabond n L(0/1) . (w k) , lags(1) noconstant twostep
```



## □ Technical note

If we had specified `vce(robust)` in our call to `gmm`, we would have obtained the traditional sandwich-based robust covariance matrix, but our standard errors would not match those we would obtain by specifying `vce(robust)` with the `xtabond` command. The `xtabond`, `xtdpd`, and `xtdpdsys` commands implement a bias-corrected robust VCE for the two-step GMM dynamic panel-data estimator. Traditional VCEs computed after the two-step dynamic panel-data estimator have been shown to exhibit often-severe bias; see [Windmeijer \(2005\)](#).



Neither of the two dynamic panel-data examples (17 and 18) we have fit so far include a constant term. When a constant term is included, the dynamic panel-data estimator is in fact a two-equation system estimator. For notational simplicity, consider a simple model containing just a constant term and one lag of the dependent variable:

$$y_{it} = \alpha + \rho y_{i,t-1} + u_i + \epsilon_{it}$$

First-differencing to remove the  $u_i$  term, we have

$$y_{it} - y_{i,t-1} = \rho(y_{i,t-1} - y_{i,t-2}) + (\epsilon_{it} - \epsilon_{i,t-1}) \quad (21)$$

This has also eliminated the constant term. If we assume  $E(u_i) = 0$ , which is reasonable if a constant term is included in the model, then we can recover  $\alpha$  by including the moment condition

$$y_{it} = \alpha + \rho y_{i,t-1} + \epsilon'_{it} \quad (22)$$

where  $\epsilon'_{it} = u_i + \epsilon_{it}$ . The parameter  $\rho$  continues to be identified by (21), so the only instrument we use with (22) is a constant term. As before, the error term  $(\epsilon_{i,t} - \epsilon_{i,t-1})$  is necessarily autocorrelated with correlation coefficient  $-0.5$ , though the error term  $\epsilon'_{it}$  is white noise. Therefore, our initial weight matrix should be

$$\widehat{\mathbf{W}} = \left( \frac{1}{N} \sum_i \mathbf{z}'_i \mathbf{H} \mathbf{z}_i \right)^{-1}$$

where

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_D & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$$

and  $\mathbf{I}$  is a conformable identity matrix.

One complication arises concerning the relevant estimation sample. Looking at (21), we apparently lose the first two observations from each panel because of the presence of  $y_{i,t-2}$ , but in (22), we need to sacrifice only one observation for  $y_{i,t-1}$ . For most multiple-equation models, we need to use the same estimation sample for all equations. However, in dynamic panel-data models, we can use more observations to fit the equation in level form [(22) here] than the equation in first differences [equation (21)]. To request this behavior, we specify the `nocommonesample` option to `gmm`. That option tells `gmm` to use as many observations as possible for each equation, ignoring the loss of observations due to lagging or differencing.

## ▷ Example 19: Arellano–Bond estimator with constant term

Here we fit the model

$$\mathbf{n}_{it} = \alpha + \rho \mathbf{n}_{i,t-1} + u_i + \epsilon_{it}$$

Without specifying derivatives, our command would be

```
. gmm (D.n - {rho}*LD.n) (n - {alpha} - {rho}*L.n),
> xtinstruments(1: n, lags(2/.)) instruments(1:, noconstant) onestep
> winitial(xt DL) vce(unadj) nocommonesample
```

We would specify `winitial(xt DL)` to obtain the required initial weight matrix. The notation `DL` indicates that our first residual equation is in first differences and that the second residual equation is in levels (not first-differenced). We exclude a constant in the instrument list for the first equation because first-differencing removed the constant term. Because we do not specify the `instruments()` option for the second residual equation, a constant is used by default.

This example also provides us the opportunity to illustrate how to specify derivatives for multiple-equation GMM models. Within the `derivative()` option, instead of specifying just the parameter name, now you must specify the equation name or number, a slash, and the parameter name to which the derivative applies. In Stata, we type

```
. gmm (D.n - {rho}*LD.n) (n - {alpha} - {rho}*L.n),
> xtinstruments(1: n, lags(2/.)) instruments(1:, noconstant)
> derivative(1/rho = -1*LD.n) derivative(2/alpha = -1)
> derivative(2/rho = -1*L.n) winitial(xt DL) vce(unadj) nocommonesample onestep
```

Step 1:

```
Iteration 0: GMM criterion Q(b) = .09894466
Iteration 1: GMM criterion Q(b) = .00023508
Iteration 2: GMM criterion Q(b) = .00023508
```

GMM estimation

Number of parameters = 2

Number of moments = 29

Initial weight matrix: XT DL Number of obs = \*

|        | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|--------|-------------|-----------|-------|-------|----------------------|----------|
| /rho   | 1.023349    | .0608293  | 16.82 | 0.000 | .9041259             | 1.142572 |
| /alpha | -.0690864   | .0660343  | -1.05 | 0.295 | -.1985112            | .0603384 |

\* Number of observations for equation 1: 751

Number of observations for equation 2: 891

Instruments for equation 1:

XT-style: L(2/.).n

Instruments for equation 2:

Standard: \_cons

These results are identical to those we would obtain by typing

```
xtabond n, lags(1)
```

Because we specified `nocommonesample`, `gmm` did not report the number of observations used in the header of the output. In this dataset, there are in fact 1,031 observations on 140 panels. In the second equation, the presence of the lagged value of `n` reduces the sample size for that equation to  $1031 - 140 = 891$ . In the first equation, we lose the first two observations per panel because of lagging and differencing, which leads to 751 usable observations. These tallies are listed after the coefficient table in the output.

## □ Technical note

## Specifying

```
xtinstruments(x1 x2 x3, lags(1/3))
```

differs from

```
instruments(L(1/3).(x1 x2 x3))
```

in how observations are excluded from the estimation sample. When you use the latter syntax, `gmm` must exclude the first three observations from each panel when computing the residual equation: you requested that three lags of each regressor be used as instruments, so the first residual that could be interacted with those instruments is the one for  $t = 4$ . On the other hand, when you use `xtinstruments()`, you are telling `gmm` that you would like to use up to the first three lags of `x1`, `x2`, and `x3` as instruments but that using just one lag is acceptable. Because most panel datasets have a relatively modest number of observations per panel, dynamic instrument lists are typically used so that the number of usable observations is maximized. Dynamic instrument lists also accommodate the fact that there are more valid instruments for later time periods than earlier time periods.

Specifying panel-style instruments using the `xtinstruments()` option also affects how the standard instruments specified in the `instruments()` option are treated. To illustrate, we will suppose that we have a balanced panel dataset with  $T = 5$  observations per panel and that we specify

```
. gmm ..., xtinstruments(w, lags(1/2)) instruments(x)
```

We will lose the first observation because we need at least one lag of `w` to serve as an instrument. Our instrument matrix for panel  $i$  will therefore be

$$\mathbf{Z}_i = \begin{bmatrix} w_{i1} & 0 & 0 & 0 \\ 0 & w_{i1} & 0 & 0 \\ 0 & w_{i2} & 0 & 0 \\ 0 & 0 & w_{i2} & 0 \\ 0 & 0 & w_{i3} & 0 \\ 0 & 0 & 0 & w_{i3} \\ 0 & 0 & 0 & w_{i4} \\ x_{i2} & x_{i3} & x_{i4} & x_{i5} \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad (23)$$

The vector of ones in the final row represents the constant term implied by the `instruments()` option. Because we lost the first observation, the residual vector  $\mathbf{u}_i$  will be  $4 \times 1$ . Thus, our moment conditions for the  $i$ th panel can be written in matrix notation as

$$E\{\mathbf{Z}_i \mathbf{u}_i(\boldsymbol{\beta})\} = E\left\{\mathbf{Z}_i \begin{bmatrix} u_{i2}(\boldsymbol{\beta}) \\ u_{i3}(\boldsymbol{\beta}) \\ u_{i4}(\boldsymbol{\beta}) \\ u_{i5}(\boldsymbol{\beta}) \end{bmatrix}\right\} = \mathbf{0}$$

The moment conditions corresponding to the final two rows of (23) say that

$$E\left\{\sum_{t=2}^{T-4} x_{it} u_{it}(\boldsymbol{\beta})\right\} = 0 \quad \text{and} \quad E\left\{\sum_{t=2}^{T-4} u_{it}(\boldsymbol{\beta})\right\} = 0$$

Because we specified panel-style instruments with the `xtinstruments()` option, `gmm` no longer uses moment conditions for strictly exogenous variables of the form  $E\{x_{it}u_{it}(\beta)\} = 0$  for each  $t$ . Instead, the moment conditions now stipulate that the average (over  $t$ ) of  $x_{it}u_{it}(\beta)$  has expectation zero. This corresponds to the approach proposed by [Arellano and Bond \(1991, 280\)](#) and others.

When you request panel-style instruments with the `xtinstruments()` option, the number of instruments in the  $Z_i$  matrix increases quadratically in the number of periods. The dynamic panel-data estimators we have discussed in this section are designed for datasets that contain a large number of panels and a modest number of time periods. When the number of time periods is large, estimators that use standard (non-panel-style) instruments are more appropriate. □

We have focused on the Arellano–Bond dynamic panel-data estimator because of its relative simplicity. `gmm` can additionally fit any models that can be formulated using the `xtdpd` and `xtdpdsys` commands; see [\[XT\] xtdpd](#) and [\[XT\] xtdpdsys](#). The key is to determine the appropriate instruments to use for the level and difference equations. You may find it useful to fit a version of your model with those commands to determine what instruments and XT-style instruments to use. We conclude this section with an example using the Arellano–Bover/Blundell–Bond estimator.

### ▷ Example 20: Arellano–Bover/Blundell–Bond estimator

We fit a small model that includes one lag of the dependent variable `n` as a regressor as well as the contemporaneous and first lag of `w`, which we assume are strictly exogenous. When we apply virtually all the syntax issues we have discussed so far, the `gmm` command is

```
. gmm (n - {rho}*L.n - {w}*w - {lagw}*L.w - {c})
> (D.n - {rho}*LD.n - {w}*D.w - {lagw}*LD.w),
> xtinst(1: D.n, lags(1/1)) xtinst(2: n, lags(2/.))
> inst(2: D.w LD.w, noconstant)
> deriv(1/rho = -1*L.n) deriv(1/w = -1*w)
> deriv(1/lagw = -1*L.w) deriv(1/c = -1)
> deriv(2/rho = -1*LD.n) deriv(2/w = -1*D.w)
> deriv(2/lagw = -1*LD.w)
> winit(xt LD) wmatrix(robust) vce(unadjusted) nocommonesample
```

Step 1:

```
Iteration 0: GMM criterion Q(b) = .10170339
Iteration 1: GMM criterion Q(b) = .00022772
Iteration 2: GMM criterion Q(b) = .00022772
```

Step 2:

```
Iteration 0: GMM criterion Q(b) = .59965014
Iteration 1: GMM criterion Q(b) = .56578186
Iteration 2: GMM criterion Q(b) = .56578186
```

GMM estimation

Number of parameters = 4

Number of moments = 39

Initial weight matrix: XT LD

Number of obs = \*

GMM weight matrix: Robust

|       | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |           |
|-------|-------------|-----------|--------|-------|----------------------|-----------|
| /rho  | 1.122738    | .0206512  | 54.37  | 0.000 | 1.082263             | 1.163214  |
| /w    | -.6719909   | .0246148  | -27.30 | 0.000 | -.7202351            | -.6237468 |
| /lagw | .571274     | .0403243  | 14.17  | 0.000 | .4922398             | .6503083  |
| /c    | .154309     | .17241    | 0.90   | 0.371 | -.1836084            | .4922263  |

\* Number of observations for equation 1: 891

Number of observations for equation 2: 751

Instruments for equation 1:

XT-style: LD.n

Standard: \_cons

Instruments for equation 2:

XT-style: L(2/.)n

Standard: D.w LD.w

Alternatively, we could have fit this model with `xtdpdsys` by typing the following:

```
xtdpdsys n L(0/1).w, lags(1) twostep
```

◀

## Details of moment-evaluator programs

In examples 11, 12, 13, and 14, we used moment-evaluator programs to evaluate moment conditions that could not be specified using the interactive version of `gmm`. In example 13, we also showed how to pass additional information to an evaluator program. Here we discuss how to make moment-evaluator programs provide derivatives and accept weights.

The complete specification for a moment-evaluator program's syntax statement is

```
syntax varlist if [weight], at(name) options [derivatives(varlist)]
```

The macro 'varlist' contains the list of variables that we are to fill in with the values of our residual equations. The macro 'if' represents an if condition that restricts the estimation sample. The macro 'at' represents a vector containing the parameter values at which we are to evaluate our residual equations. options represent other options that you specify in your call to `gmm` and want to have passed to your moment-evaluator programs. In example 13, we included the `mylhs()` option so that we could pass the name of the dependent variable to our evaluator program.

Two new elements of the syntax statement allow for weights and derivatives. `weight` specifies the types of weights your program allows. The interactive version of `gmm` allows for `fweights`, `awweights`, and `pweights`. However, unless you explicitly allow your moment evaluator program to accept weights, you cannot specify weights in your call to `gmm` with the moment-evaluator program version.

The `derivatives()` option is used to pass to your program a set of variables that you are to fill in with the derivatives of your residual equations with respect to the parameters.

To indicate that your program can calculate derivatives, you specify either the `hasderivatives` or the `haslfdderivatives` option to `gmm`. The `hasderivatives` option indicates that your program calculates parameter-level derivatives; that method requires more work but can be applied to any GMM

problem. The `haslfdderivatives` option requires less work but can be used only when the model's residual equations satisfy certain restrictions and when you use the `{lname:varlist}` syntax with the `parameters()` option.

We first consider how to write the derivative computation logic to work with the `hasderivatives` option and provide an example; then, we do the same for the `haslfdderivatives` option.

Say that you specify  $k$  parameters in the `nparameters()` or `parameters()` option and  $q$  equations in the `nequations()` or `equations()` option and that you specify `hasderivatives`. Then, `'derivatives'` will contain  $k \times q$  variables. The first  $k$  variables are for the derivatives of the first residual equation with respect to the  $k$  parameters, the second  $k$  variables are for the derivatives of the second residual equation, and so on.

### ▷ Example 21: Specifying derivatives with simple parameter names

To focus on how to specify derivatives, we return to the simple moment-evaluator program we used in [example 11](#), in which we had three regressors, and extend it to supply derivatives. The residual equation corresponding to moment condition (9) is

$$u_{it}(\beta) = y_{it} - \mu_{it} \frac{\bar{y}_i}{\bar{\mu}_i}$$

where  $\mu_{it}$ ,  $\bar{\mu}_i$ , and  $\bar{y}_i$  were defined previously. Now,

$$\frac{\partial}{\partial \beta_j} u_{it}(\beta) = -\mu_{it} \frac{\bar{y}_i}{\bar{\mu}_i^2} \left( x_{it}^{(j)} \bar{\mu}_i - \frac{1}{T} \sum_{l=1}^T x_{il}^{(j)} \mu_{il} \right) \quad (24)$$

where  $x_{it}^{(j)}$  represents the  $j$ th element of  $\mathbf{x}_{it}$ .

Our moment-evaluator program is

```

program gmm_poideriv
 version 19.5 // (or version 19 if you do not have StataNow)
 syntax varlist if, at(name) [derivatives(varlist)]
 quietly {
 // Calculate residuals as before
 tempvar mu mubar ybar
 generate double 'mu' = exp(x1*'at'[1,1] + x2*'at'[1,2] //
 + x3*'at'[1,3]) 'if'
 egen double 'mubar' = mean('mu') 'if', by(id)
 egen double 'ybar' = mean(y) 'if', by(id)
 replace 'varlist' = y - 'mu'*'ybar'/'mubar' 'if'
 // Did -gmm- request derivatives?
 if "'derivatives'" == "" {
 exit // no, so we are done
 }
 // Calculate derivatives
 // We need the panel means of x1*mu, x2*mu, and x3*mu
 tempvar work x1mubar x2mubar x3mubar
 generate double 'work' = x1*'mu' 'if'
 egen double 'x1mubar' = mean('work') 'if', by(id)
 replace 'work' = x2*'mu' 'if'
 egen double 'x2mubar' = mean('work') 'if', by(id)
 }

```

```

replace 'work' = x3*'mu' 'if'
egen double 'x3mubar' = mean('work') 'if', by(id)

local d1: word 1 of 'derivatives'
local d2: word 2 of 'derivatives'
local d3: word 3 of 'derivatives'
replace 'd1' = -1*'mu'*'ybar'/'mubar'^2*(x1*'mubar' - 'x1mubar')
replace 'd2' = -1*'mu'*'ybar'/'mubar'^2*(x2*'mubar' - 'x2mubar')
replace 'd3' = -1*'mu'*'ybar'/'mubar'^2*(x3*'mubar' - 'x3mubar')
}

end

```

The `derivatives()` option is made optional in the `syntax` statement by placing it in square brackets. If `gmm` needs to evaluate your residual equations but does not need derivatives at that time, then the `derivatives()` option will be empty. In our program, we check to see whether that is the case and, if so, `exit` without calculating derivatives. As is often the case with [R] `ml` as well, the portion of our program devoted to derivatives is longer than the code to compute the objective function.

The first part of our derivative code computes the term

$$\frac{1}{T} \sum_{l=1}^{l=T} x_{il}^{(j)} \mu_{il} \quad (25)$$

for  $x_{it}^{(j)} = x_1, x_2,$  and  $x_3$ . The `'derivatives'` macro contains three variable names corresponding to the three parameters of the `'at'` matrix. We extract those names into local macros `'d1'`, `'d2'`, and `'d3'` and then fill in the variables those macros represent with the derivatives shown in (24).

With our program written, we fit our model by typing

```

. use https://www.stata-press.com/data/r19/poisson1, clear
. gmm gmm_poideriv, nequations(1) parameters(b1 b2 b3)
> instruments(x1 x2 x3, noconstant) vce(cluster id) onestep hasderivatives

Step 1:
Iteration 0: GMM criterion Q(b) = 51.99142
Iteration 1: GMM criterion Q(b) = .04345191
Iteration 2: GMM criterion Q(b) = 8.720e-06
Iteration 3: GMM criterion Q(b) = 7.115e-13
Iteration 4: GMM criterion Q(b) = 5.130e-27

note: model is exactly identified.

GMM estimation

Number of parameters = 3
Number of moments = 3
Initial weight matrix: Unadjusted Number of obs = 409
 (Std. err. adjusted for 45 clusters in id)

```

|     | Robust      |           |        |       |                      |
|-----|-------------|-----------|--------|-------|----------------------|
|     | Coefficient | std. err. | z      | P> z  | [95% conf. interval] |
| /b1 | 1.94866     | .1000265  | 19.48  | 0.000 | 1.752612 2.144709    |
| /b2 | -2.966119   | .0923592  | -32.12 | 0.000 | -3.14714 -2.785099   |
| /b3 | 1.008634    | .1156561  | 8.72   | 0.000 | .781952 1.235315     |

Instruments for equation 1: x1 x2 x3

Our results are identical to those in [example 11](#). Another way to verify that our program calculates derivatives correctly would be to type

```
. gmm gmm_poideriv, nequations(1) parameters(b1 b2 b3)
> instruments(x1 x2 x3, noconstant) vce(cluster id) onestep
```

Without the `hasderivatives` or `haslfdderivatives` option, `gmm` will not request derivatives from your program, even if it contains code to compute them. If you have trouble obtaining convergence with the `hasderivatives` or `haslfdderivatives` option but do not have trouble without specifying one of them, then you need to recheck your derivatives. ◀

After [example 11](#), we remarked that the evaluator program would have to be changed to accommodate different regressors. We then showed how you can specify parameters using the syntax `{lname:varlist}` and then use `matrix score` to compute linear combinations of variables. To specify derivatives when you specify parameters using this notation, ensure that your residual equations satisfy the “linear-form restriction” analogous to the restrictions of linear-form evaluators used by `ml`. See [\[R\] ml](#) and [Pitblado, Poi, and Gould \(2024\)](#) for more information about linear-form evaluators.

A GMM residual equation satisfies the linear-form restriction if the equation can be written in terms of a single observation in the dataset and if the equation for observation  $i$  does not depend on any observations  $j \neq i$ . Cross-sectional models satisfy the linear-form restriction. Time-series models satisfy the linear-form restriction only when no lags or leads are used.

Panel-data models often do not satisfy the linear-form restriction. For example, recall moment condition (9) for a panel Poisson model. That residual equation included panel-level mean terms  $\bar{y}_i$  and  $\bar{\mu}_i$ , so the residual equation for an individual observation depends on all the observations in the same panel.

When a residual equation does not satisfy the linear-form restriction, neither will its derivatives. To apply the chain rule, we need a way to multiply the `lname`-level derivative by each of the variables in the equation to obtain parameter-level derivatives. In (24), for example, there is no way to factor out each  $x_{it}^{(j)}$  variable and obtain an `lname`-level derivative that we then multiply by each of the  $x_{it}^{(j)}$ s.

Suppose we do have a model with  $q = 2$  moment conditions, both of which do satisfy the linear-form restriction, and we specify the `parameters()` option like this:

```
. gmm ..., parameters({eq1:x1 x2 _cons} {eq2:_cons} {eq3:x1 x2 _cons})
```

We have specified  $n = 3$  `lcnames` in the `parameters()` option: `eq1`, `eq2`, and `eq3`. When we specify the `haslfdderivatives` option, `gmm` will pass  $n \times q = 3 \times 2 = 6$  variables in the `derivatives()` option. The first three variables are to be filled with

$$\frac{\partial}{\partial \text{eq1}} u_{1i}(\beta) \quad \frac{\partial}{\partial \text{eq2}} u_{1i}(\beta) \quad \text{and} \quad \frac{\partial}{\partial \text{eq3}} u_{1i}(\beta)$$

where  $u_{1i}(\beta)$  is the  $i$ th observation for the first moment equation. Then, the second three variables are to be filled with

$$\frac{\partial}{\partial \text{eq1}} u_{2i}(\beta) \quad \frac{\partial}{\partial \text{eq2}} u_{2i}(\beta) \quad \text{and} \quad \frac{\partial}{\partial \text{eq3}} u_{2i}(\beta)$$

where  $u_{2i}(\beta)$  is the second moment equation. In this example, we filled in a total of six variables with derivatives. If we instead used the `hasderivatives` option, we would have filled  $k \times q = 7 \times 2 = 14$  variables; moreover, if we wanted to change the number of variables in our model, we would have modified our evaluator program.

► Example 22: Specifying derivatives with linear-form residual equations

In examples 9 and 10, we showed how to specify derivatives with an exponential regression model when using the interactive version of `gmm`. Here we show how to write a moment-evaluator program for the exponential regression model, including derivatives.

The residual equation for observation  $i$  is

$$u_i = y_i - \exp(\mathbf{x}'_i\boldsymbol{\beta})$$

where  $\mathbf{x}_i$  may include a constant term. The derivative with respect to the linear combination  $\mathbf{x}'_i\boldsymbol{\beta}$  is

$$\frac{\partial u_i}{\partial \mathbf{x}'_i\boldsymbol{\beta}} = -\exp(\mathbf{x}'_i\boldsymbol{\beta})$$

To verify that this residual equation satisfies the linear-form restriction, we see that for the  $j$ th element of  $\boldsymbol{\beta}$ , we have

$$\frac{\partial u_i}{\partial \beta_j} = -x_{ij} \exp(\mathbf{x}'_i\boldsymbol{\beta}) = \frac{\partial u_i}{\partial \mathbf{x}'_i\boldsymbol{\beta}} \times x_{ij}$$

so that given  $\partial u_i / \partial \mathbf{x}'_i\boldsymbol{\beta}$ , `gmm` can apply the chain rule to obtain the derivatives with respect to the individual parameters.

Our moment-evaluator program is

```

program gmm_poideriv2
 version 19.5 // (or version 19 if you do not have StataNow)
 syntax varlist if, at(name) [derivatives(varlist)]
 quietly {
 tempvar mu
 matrix score double 'mu' = 'at' 'if', eq(#1)
 replace 'mu' = exp('mu')
 local depvar : coleq 'at'
 local depvar : word 1 of 'depvar'
 replace 'varlist' = 'depvar' - 'mu' 'if'
 // Did -gmm- request derivatives?
 if "'derivatives'" == "" {
 exit // no, so we are done
 }
 // Calculate derivatives
 // The derivatives macro only has one variable
 // for this model.
 replace 'derivatives' = -1*'mu' 'if'
 }
end

```

To fit our model of doctor visits treating income as an endogenous regressor, we type

```
. use https://www.stata-press.com/data/r19/docvisits
. gmm gmm_poideriv2, nequations(1)
> instruments(private chronic female age black hispanic)
> parameters({docvis:private chronic female income _cons}) haslfdderivatives

Step 1:
Iteration 0: GMM criterion Q(b) = 16.910173
Iteration 1: GMM criterion Q(b) = .82270871
Iteration 2: GMM criterion Q(b) = .21831995
Iteration 3: GMM criterion Q(b) = .12685934
Iteration 4: GMM criterion Q(b) = .12672369
Iteration 5: GMM criterion Q(b) = .12672365

Step 2:
Iteration 0: GMM criterion Q(b) = .00234641
Iteration 1: GMM criterion Q(b) = .00215957
Iteration 2: GMM criterion Q(b) = .00215911
Iteration 3: GMM criterion Q(b) = .00215911

GMM estimation
Number of parameters = 5
Number of moments = 7
Initial weight matrix: Unadjusted Number of obs = 4,412
GMM weight matrix: Robust
```

|         | Robust      |           |       |       | [95% conf. interval] |          |
|---------|-------------|-----------|-------|-------|----------------------|----------|
|         | Coefficient | std. err. | z     | P> z  |                      |          |
| private | .535335     | .159904   | 3.35  | 0.001 | .221929              | .848741  |
| chronic | 1.090126    | .0617659  | 17.65 | 0.000 | .9690668             | 1.211185 |
| female  | .6636579    | .0959885  | 6.91  | 0.000 | .475524              | .8517918 |
| income  | .0142855    | .0027162  | 5.26  | 0.000 | .0089618             | .0196092 |
| _cons   | -.5983477   | .138433   | -4.32 | 0.000 | -.8696714            | -.327024 |

Instruments for equation 1: private chronic female age black hispanic \_cons

Our results match those shown in [example 10](#).

We can change the variables in our model just by changing the `parameters()` and `instruments()` options; we do not need to make any changes to the moment-evaluator program, because we used linear-form derivatives.

◀

Depending on your model, allowing your moment-evaluator program to accept weights may be as easy as modifying the syntax command to allow them, or it may require significantly more work. If your program uses only commands such as `generate` and `replace`, then just modifying the syntax command is all you need to do; `gmm` takes care of applying the weights to the observation-level residuals when computing the sample moments, derivatives, and weight matrices. On the other hand, if your moment-evaluator program computes residuals using statistics that depend on multiple observations, then you must apply the weights passed to your program when computing those statistics.

In our examples of panel Poisson with strictly exogenous regressors ([11](#) and [21](#)), we used the statistics  $\bar{\mu}_i$  and  $\bar{y}_i$  when computing the residuals. If we are to allow weights with our moment-evaluator program, then we must incorporate those weights when computing  $\bar{\mu}_i$  and  $\bar{y}_i$ . Moreover, looking at the derivative in [\(24\)](#), we see that the term highlighted in [\(25\)](#) is in fact a sample mean, so we must incorporate weights when computing it.

## ▶ Example 23: Panel Poisson with derivatives and weights

Here we modify the program in [example 21](#) to accept frequency weights. One complication arises: we had been using `egen` to compute  $\bar{\mu}_i$  and  $\bar{y}_i$ . `egen` does not accept weights, so we must compute  $\bar{\mu}_i$  and  $\bar{y}_i$  ourselves, incorporating any weights the user may specify. Our program is

```

program gmm_poiderivfw
 version 19.5 // (or version 19 if you do not have StataNow)
 syntax varlist if [fweight/], at(name) [derivatives(varlist)]
 quietly {
 if "'exp'" == "" { // no weights
 local exp 1 // weight each observation equally
 }

 // Calculate residuals as before
 tempvar mu mubar ybar sumwt
 generate double 'mu' = exp(x1*'at'[1,1] + x2*'at'[1,2] ///
 + x3*'at'[1,3]) 'if'

 bysort id: generate double 'sumwt' = sum('exp')
 by id: generate double 'mubar' = sum('mu'*'exp')
 by id: generate double 'ybar' = sum(y*'exp')
 by id: replace 'mubar' = 'mubar'[_N] / 'sumwt'[_N]
 by id: replace 'ybar' = 'ybar'[_N] / 'sumwt'[_N]
 replace 'varlist' = y - 'mu'*'ybar'/'mubar' 'if'

 // Did -gmm- request derivatives?
 if "'derivatives'" == "" {
 exit // no, so we are done
 }

 // Calculate derivatives
 // We need the panel means of x1*mu, x2*mu, and x3*mu
 tempvar work x1mubar x2mubar x3mubar
 generate double 'work' = x1*'mu' 'if'
 by id: generate double 'x1mubar' = sum('work'*'exp')
 by id: replace 'x1mubar' = 'x1mubar'[_N] / 'sumwt'[_N]
 replace 'work' = x2*'mu' 'if'
 by id: generate double 'x2mubar' = sum('work'*'exp')
 by id: replace 'x2mubar' = 'x2mubar'[_N] / 'sumwt'[_N]
 replace 'work' = x3*'mu' 'if'
 by id: generate double 'x3mubar' = sum('work'*'exp')
 by id: replace 'x3mubar' = 'x3mubar'[_N] / 'sumwt'[_N]

 local d1: word 1 of 'derivatives'
 local d2: word 2 of 'derivatives'
 local d3: word 3 of 'derivatives'
 replace 'd1' = -1*'mu'*'ybar'/'mubar'^2*(x1*'mubar' - 'x1mubar')
 replace 'd2' = -1*'mu'*'ybar'/'mubar'^2*(x2*'mubar' - 'x2mubar')
 replace 'd3' = -1*'mu'*'ybar'/'mubar'^2*(x3*'mubar' - 'x3mubar')
 }
end

```

Our syntax command now indicates that `fweights` are allowed. The first part of our code looks at the macro `'exp'`. If it is empty, then the user did not specify weights in their call to `gmm`; and we set the macro equal to 1 so that we weight each observation equally. After we compute  $\mu_{it}$ , we calculate  $\bar{\mu}_i$  and  $\bar{y}_i$ , accounting for weights. To compute frequency-weighted means for each panel, we just multiply each observation by its respective weight, sum over all observations in the panel, then divide by the sum of the weights for the panel. (See [\[U\] 20.24 Weighted estimation](#) for information on how to handle `aweights` and `pweights`.) We use the same procedure to compute the frequency-weighted variant of expression (25) in the derivative calculations. To use our program, we type

```
. use https://www.stata-press.com/data/r19/poissonwts
. gmm gmm_poiderivfw [fw=fwt], nequations(1) parameters(b1 b2 b3)
> instruments(x1 x2 x3, noconstant) vce(cluster id) onestep hasderivatives

Step 1:
Iteration 0: GMM criterion Q(b) = 49.8292
Iteration 1: GMM criterion Q(b) = .11136736
Iteration 2: GMM criterion Q(b) = .00008519
Iteration 3: GMM criterion Q(b) = 7.110e-11
Iteration 4: GMM criterion Q(b) = 5.596e-23

note: model is exactly identified.

GMM estimation

Number of parameters = 3
Number of moments = 3
Initial weight matrix: Unadjusted Number of obs = 819
 (Std. err. adjusted for 45 clusters in id)
```

|     | Robust      |           |        |       |                      |           |
|-----|-------------|-----------|--------|-------|----------------------|-----------|
|     | Coefficient | std. err. | z      | P> z  | [95% conf. interval] |           |
| /b1 | 1.967766    | .111795   | 17.60  | 0.000 | 1.748652             | 2.186881  |
| /b2 | -3.060838   | .0935561  | -32.72 | 0.000 | -3.244205            | -2.877472 |
| /b3 | 1.037594    | .1184227  | 8.76   | 0.000 | .80549               | 1.269698  |

```
Instruments for equation 1: x1 x2 x3
```

Testing whether our program works correctly with frequency weights is easy. A frequency-weighted dataset is just a compact form of a larger dataset in which identical observations are omitted and a frequency-weight variable is included to tell us how many times each observation in the smaller dataset appears in the larger dataset. Therefore, we can expand our smaller dataset by the frequency-weight variable and then refit our model without specifying frequency weights. If we obtain the same results, our program works correctly. When we type

```
. expand fw
. gmm gmm_poiderivfw, nequations(1) parameters(b1 b2 b3)
> instruments(x1 x2 x3, noconstant) vce(cluster id) onestep
```

we obtain the same results as before.

◀

## Stored results

`gmm` stores the following in `e()`:

### Scalars

|                            |                                                     |
|----------------------------|-----------------------------------------------------|
| <code>e(N)</code>          | number of observations                              |
| <code>e(k)</code>          | number of parameters                                |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>            |
| <code>e(k_eq_model)</code> | number of equations in overall model test           |
| <code>e(k_aux)</code>      | number of auxiliary parameters                      |
| <code>e(n_moments)</code>  | number of moments                                   |
| <code>e(n_eq)</code>       | number of equations in moment-evaluator program     |
| <code>e(Q)</code>          | criterion function                                  |
| <code>e(J)</code>          | Hansen $J$ $\chi^2$ statistic                       |
| <code>e(J_df)</code>       | $J$ statistic degrees of freedom                    |
| <code>e(k_i)</code>        | number of parameters in equation $i$                |
| <code>e(has_xtinst)</code> | 1 if panel-style instruments specified, 0 otherwise |
| <code>e(N_clust)</code>    | number of clusters                                  |

|                           |                                                                 |
|---------------------------|-----------------------------------------------------------------|
| <code>e(type)</code>      | 1 if interactive version, 2 if moment-evaluator program version |
| <code>e(rank)</code>      | rank of $e(V)$                                                  |
| <code>e(ic)</code>        | number of iterations used by iterative GMM estimator            |
| <code>e(converged)</code> | 1 if converged, 0 otherwise                                     |

## Macros

|                                 |                                                                    |
|---------------------------------|--------------------------------------------------------------------|
| <code>e(cmd)</code>             | <code>gmm</code>                                                   |
| <code>e(cmdline)</code>         | command as typed                                                   |
| <code>e(title)</code>           | title specified in <code>title()</code>                            |
| <code>e(title_2)</code>         | title specified in <code>title2()</code>                           |
| <code>e(clustvar)</code>        | name of cluster variable                                           |
| <code>e(inst_1)</code>          | equation $i$ instruments                                           |
| <code>e(eqnames)</code>         | equation names                                                     |
| <code>e(winit)</code>           | initial weight matrix used                                         |
| <code>e(winitname)</code>       | name of user-supplied initial weight matrix                        |
| <code>e(estimator)</code>       | <code>onestep</code> , <code>twostep</code> , or <code>igmm</code> |
| <code>e(rhs)</code>             | variables specified in <code>variables()</code>                    |
| <code>e(params_1)</code>        | equation $i$ parameters                                            |
| <code>e(wmatrix)</code>         | <code>wmtype</code> specified in <code>wmatrix()</code>            |
| <code>e(vce)</code>             | <code>vcetype</code> specified in <code>vce()</code>               |
| <code>e(vcetype)</code>         | title used to label Std. err.                                      |
| <code>e(params)</code>          | parameter names                                                    |
| <code>e(sexp_1)</code>          | substitutable expression for equation $i$                          |
| <code>e(evalprog)</code>        | moment-evaluator program                                           |
| <code>e(evalopts)</code>        | options passed to moment-evaluator program                         |
| <code>e(nocommonesample)</code> | <code>nocommonesample</code> , if specified                        |
| <code>e(technique)</code>       | optimization technique                                             |
| <code>e(properties)</code>      | <code>b V</code>                                                   |
| <code>e(estat_cmd)</code>       | program used to implement <code>estat</code>                       |
| <code>e(predict)</code>         | program used to implement <code>predict</code>                     |
| <code>e(marginsok)</code>       | predictions allowed by <code>margins</code>                        |
| <code>e(marginsnotok)</code>    | predictions disallowed by <code>margins</code>                     |
| <code>e(marginsprop)</code>     | signals to the <code>margins</code> command                        |
| <code>e(asbalanced)</code>      | factor variables <code>fvset</code> as <code>asbalanced</code>     |
| <code>e(asobserved)</code>      | factor variables <code>fvset</code> as <code>asobserved</code>     |

## Matrices

|                              |                                                                                |
|------------------------------|--------------------------------------------------------------------------------|
| <code>e(b)</code>            | coefficient vector                                                             |
| <code>e(init)</code>         | initial values of the estimators                                               |
| <code>e(Wuser)</code>        | user-supplied initial weight matrix                                            |
| <code>e(W)</code>            | weight matrix used for final round of estimation                               |
| <code>e(S)</code>            | moment covariance matrix used in robust VCE computations                       |
| <code>e(G)</code>            | averages of derivatives of moment conditions                                   |
| <code>e(N_byequation)</code> | number of observations per equation, if <code>nocommonesample</code> specified |
| <code>e(V)</code>            | variance–covariance matrix                                                     |
| <code>e(V_modelbased)</code> | model-based variance                                                           |

## Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

## Matrices

|                       |                                                                                                                       |
|-----------------------|-----------------------------------------------------------------------------------------------------------------------|
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, $p$ -values, and confidence intervals |
|-----------------------|-----------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

Let  $q$  denote the number of moment conditions. For observation  $i$ ,  $i = 1, \dots, N$ , write the  $j$ th moment equation as  $\mathbf{z}_{ij}u_{ij}(\beta_j)$  for  $j = 1, \dots, q$ .  $\mathbf{z}_{ij}$  is a  $1 \times m_j$  vector, where  $m_j$  is the number of instruments specified for equation  $j$ . Let  $m = m_1 + \dots + m_q$ .

Our notation can incorporate moment conditions of the form  $h_{ij}(\mathbf{w}_{ij}; \beta_j)$  with instruments  $\mathbf{w}_{ij}$  by defining  $\mathbf{z}_{ij} = 1$  and  $u_{ij}(\beta_j) = h_{ij}(\mathbf{w}_{ij}; \beta_j)$ , so except when necessary, we do not distinguish between the two types of moment conditions. We could instead use notation so that all our moment conditions are of the form  $h_{ij}(\mathbf{w}_{ij}; \beta_j)$ , or we could adopt notation that explicitly combines both forms of moment equations. However, because moment conditions of the form  $\mathbf{z}'_{ij}u_{ij}(\beta_j)$  are arguably more common, we use that notation.

Let  $\beta$  denote a  $k \times 1$  vector of parameters, consisting of all the unique parameters of  $\beta_1, \dots, \beta_q$ . Then, we can stack the moment conditions and write them more compactly as  $\mathbf{Z}'_i \mathbf{u}_i(\beta)$ , where

$$\mathbf{Z}'_i = \begin{bmatrix} \mathbf{z}_{i1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{z}_{i2} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{z}_{iq} \end{bmatrix} \quad \text{and} \quad \mathbf{u}_i(\beta) = \begin{bmatrix} u_{i1}(\beta_1) \\ u_{i2}(\beta_2) \\ \vdots \\ u_{iq}(\beta_q) \end{bmatrix}$$

The GMM estimator  $\hat{\beta}$  is the value of  $\beta$  that minimizes

$$Q(\beta) = \left\{ N^{-1} \sum_{i=1}^N \mathbf{Z}'_i \mathbf{u}_i(\beta) \right\}' \mathbf{W} \left\{ N^{-1} \sum_{i=1}^N \mathbf{Z}'_i \mathbf{u}_i(\beta) \right\} \quad (\text{A1})$$

for  $q \times q$  weight matrix  $\mathbf{W}$ .

By default, `gmm` minimizes (A1) using the Gauss–Newton method. See Hayashi (2000, 498) for a derivation. This technique is typically faster than quasi-Newton methods and does not require second-order derivatives.

Methods and formulas are presented under the following headings:

*Initial weight matrix*  
*Weight matrix*  
*Variance–covariance matrix*  
*Hansen’s J statistic*  
*Panel-style instruments*  
*Marginal predictions with unconditional standard errors*

### Initial weight matrix

If you specify `winitial(unadjusted)`, then we create matrix  $\Lambda$  with typical submatrix

$$\Lambda_{rs} = N^{-1} \sum_{i=1}^N \mathbf{z}'_{ir} \mathbf{z}_{is}$$

for  $r = 1, \dots, q$  and  $s = 1, \dots, q$ . If you include the `independent` suboption, then we set  $\Lambda_{rs} = \mathbf{0}$  for  $r \neq s$ . The weight matrix  $\mathbf{W}$  equals  $\Lambda^{-1}$ .

If you specify `winitial(identity)`, then we set  $\mathbf{W} = \mathbf{I}_q$ .

If you specify `winitial(xt xtspec)`, then you must specify one or two items in `xtspec`, one for each equation. `gmm` allows you to specify at most two moment equations when you specify `winitial(xt xtspec)`, one in first-differences, and one in levels. We create the block-diagonal matrix  $\mathbf{H}$  with typical block  $\mathbf{H}_j$ . If the  $j$ th element of `xtspec` is “L”, then  $\mathbf{H}_j$  is the identity matrix of suitable dimension. If the  $j$ th element of `xtspec` is “D”, then

$$\mathbf{H}_j = \begin{bmatrix} 1 & -0.5 & 0 & \dots & 0 & 0 \\ -0.5 & 1 & -0.5 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & -0.5 \\ 0 & 0 & 0 & \dots & -0.5 & 1 \end{bmatrix}$$

Then,

$$\mathbf{\Lambda}_H = N_G^{-1} \sum_{g=1}^{g=N_G} \mathbf{Z}'_g \mathbf{H} \mathbf{Z}_g$$

where  $g$  indexes panels in the dataset,  $N_G$  is the number of panels,  $\mathbf{Z}_g$  is the full instrument matrix for panel  $g$ , and  $\mathbf{W} = \mathbf{\Lambda}_H^{-1}$ . See [Panel-style instruments](#) below for a discussion of how  $\mathbf{Z}_g$  is formed.

If you specify `winitial(matname)`, then we set  $\mathbf{W}$  equal to Stata matrix `matname`.

## Weight matrix

Specification of the weight matrix applies only to the two-step and iterative estimators. When you use the `onestep` option, the `wmatrix()` option is ignored.

We first evaluate (A1) using the initial weight matrix described above and then compute  $\mathbf{u}_i(\hat{\beta})$ . In all cases,  $\mathbf{W} = \mathbf{\Lambda}^{-1}$ . If you specify `wmatrix(unadjusted)`, then we create  $\mathbf{\Lambda}$  to have typical submatrix

$$\mathbf{\Lambda}_{rs} = \sigma_{rs} N^{-1} \sum_{i=1}^N \mathbf{z}'_{ir} \mathbf{z}_{is}$$

where

$$\sigma_{rs} = N^{-1} \sum_{i=1}^N u_{ir}(\hat{\beta}) u_{is}(\hat{\beta})$$

and  $r$  and  $s$  index moment equations. For all types of weight matrices, if the independent suboption is specified, then  $\mathbf{\Lambda}_{rs} = \mathbf{0}$  for  $r \neq s$ , where  $\mathbf{\Lambda}_{rs}$  measures the covariance between moment conditions for equations  $r$  and  $s$ .

If you specify `wmatrix(robust)`, then

$$\mathbf{\Lambda} = N^{-1} \sum_{i=1}^N \mathbf{z}_i \mathbf{u}_i(\hat{\beta}) \mathbf{u}'_i(\hat{\beta}) \mathbf{z}'_i$$

If you specify `wmatrix(cluster clustvar)`, then

$$\mathbf{\Lambda} = N^{-1} \sum_{c=1}^{c=N_C} \mathbf{q}_c \mathbf{q}'_c$$

where  $c$  indexes clusters,  $N_C$  is the number of clusters, and

$$\mathbf{q}_c = \sum_{i \in c_j} \mathbf{z}_i \mathbf{u}_i(\hat{\beta})$$

If you specify `wmatrix(hac kernel [#])`, then

$$\begin{aligned} \Lambda = & N^{-1} \sum_{i=1}^N \mathbf{Z}_i \mathbf{u}_i(\hat{\beta}) \mathbf{u}_i(\hat{\beta})' \mathbf{Z}_i' + \\ & N^{-1} \sum_{l=1}^{l=N-1} \sum_{i=l+1}^N K(l, \lambda) \{ \mathbf{Z}_i \mathbf{u}_i(\hat{\beta}) \mathbf{u}_{i-l}'(\hat{\beta}) \mathbf{Z}_{i-l}' + \mathbf{Z}_{i-l} \mathbf{u}_{i-l}(\hat{\beta}) \mathbf{u}_i'(\hat{\beta}) \mathbf{Z}_i' \} \end{aligned}$$

where  $\lambda = \#$  if  $\#$  is specified and  $\lambda = N - 2$  otherwise. Define  $z = l/(\lambda + 1)$ . If `kernel` is `bartlett` or `nwest`, then

$$K(l, \lambda) = \begin{cases} 1 - z & 0 \leq z \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

If `kernel` is `parzen` or `gallant`, then

$$K(l, \lambda) = \begin{cases} 1 - 6z^2 + 6z^3 & 0 \leq z \leq 0.5 \\ 2(1 - z)^3 & 0.5 < z \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

If `kernel` is `quadraticspectral` or `andrews`, then

$$K(l, \lambda) = \begin{cases} 1 & z = 0 \\ 3\{\sin(\theta)/\theta - \cos(\theta)\}/\theta^2 & \text{otherwise} \end{cases}$$

where  $\theta = 6\pi z/5$ .

If `wmatrix(hac kernel opt)` is specified, then `gmm` uses Newey and West's (1994) automatic lag-selection algorithm, which proceeds as follows. Define  $\mathbf{h}$  to be an  $m \times 1$  vector of ones. Note that this definition of  $\mathbf{h}$  is slightly different from the one used by `ivregress`. There the element of  $\mathbf{h}$  corresponding to the constant term equals 0, effectively ignoring the effect of the constant in determining the optimal lag length. Here we include the effect of the constant term. Now, define

$$\begin{aligned} f_i &= \{ \mathbf{Z}_i' \mathbf{u}_i(\beta) \}' \mathbf{h} \\ \hat{\sigma}_j &= N^{-1} \sum_{i=j+1}^N f_i f_{i-j} \quad j = 0, \dots, \lambda^* \\ \hat{s}^{(q)} &= 2 \sum_{j=1}^{j=\lambda^*} \hat{\sigma}_j j^q \\ \hat{s}^{(0)} &= \hat{\sigma}_0 + 2 \sum_{j=1}^{j=\lambda^*} \hat{\sigma}_j \\ \hat{\gamma} &= c_\gamma \left\{ \left( \frac{\hat{s}^{(q)}}{\hat{s}^{(0)}} \right)^2 \right\}^{1/(2q+1)} \\ \lambda &= \hat{\gamma} N^{1/(2q+1)} \end{aligned}$$

where  $q$ ,  $\lambda^*$ , and  $c_\gamma$  depend on the kernel specified:

| Kernel                     | $q$ | $\lambda^*$                         | $c_\gamma$ |
|----------------------------|-----|-------------------------------------|------------|
| Bartlett/Newey–West        | 1   | $\text{int} \{ 20(T/100)^{2/9} \}$  | 1.1447     |
| Parzen/Gallant             | 2   | $\text{int} \{ 20(T/100)^{4/25} \}$ | 2.6614     |
| Quadratic spectral/Andrews | 2   | $\text{int} \{ 20(T/100)^{2/25} \}$ | 1.3221     |

Here  $\text{int}(x)$  denotes the integer obtained by truncating  $x$  toward zero. For the Bartlett and Parzen kernels, the optimal lag is  $\min\{\text{int}(\lambda), \lambda^*\}$ . For the quadratic spectral kernel, the optimal lag is  $\min\{\lambda, \lambda^*\}$ .

If `wmatrix(hac kernel opt #)` is specified, then `gmm` uses `#` instead of 20 in the definition of  $\lambda^*$  above to select the optimal lag.

## Variance–covariance matrix

If you specify `vce(unadjusted)`, then the VCE matrix is computed as

$$\text{Var}(\widehat{\beta}) = N^{-1} \{ \overline{\mathbf{G}}(\widehat{\beta})' \mathbf{W} \overline{\mathbf{G}}(\widehat{\beta}) \}^{-1} \quad (\text{A2})$$

where

$$\overline{\mathbf{G}}(\widehat{\beta}) = N^{-1} \sum_{i=1}^N \mathbf{z}'_i \left. \frac{\partial \mathbf{u}_i(\beta)}{\partial \beta'} \right|_{\beta=\widehat{\beta}} \quad (\text{A3})$$

For the two-step and iterated estimators, we use the weight matrix  $\mathbf{W}$  that was used to compute the final-round estimate  $\widehat{\beta}$ .

When you do not specify analytic derivatives, `gmm` must compute the Jacobian matrix (A3) numerically. By default, `gmm` computes each element of the matrix individually by using the Mata `deriv()` function; see [M-5] `deriv()`. This procedure results in accurate derivatives but can be slow if your model has many instruments or parameters.

When you specify the `quickderivatives` option, `gmm` computes all derivatives corresponding to parameter  $\beta_j$ ,  $j = 1, \dots, q$ , at once, using two-sided derivatives with a step size of  $|\beta_j| \epsilon^{1/3}$ , where  $\epsilon$  is the machine precision of a double precision number (approximately  $2.22045 \times 10^{-16}$ ). This method requires just two evaluations of the model's moments to compute an entire column of (A3) and therefore has the most impact when you specify many instruments or moment equations so that (A3) has many rows.

For the one-step estimator, how the unadjusted VCE is computed depends on the type of initial weight matrix requested and the form of the moment equations. If you specify two or more moment equations of the form  $h_{ij}(\mathbf{w}_{ij}; \beta_j)$ , then `gmm` issues a note and computes a heteroskedasticity-robust VCE because here the matrix  $\mathbf{Z}'\mathbf{Z}$  is necessarily singular; moreover, here you must use the identity matrix as the initial weight matrix. Otherwise, if you specify `winitial(unadjusted)` or `winitial(identity)`, then `gmm` first computes an unadjusted weight matrix based on  $\widehat{\beta}$  before evaluating (A2). If you specify `winitial(matname)`, then (A2) is evaluated on the basis of `matname`; the user is responsible for verifying that the VCE and other statistics so produced are appropriate.

All types of robust VCEs computed by `gmm` take the form

$$\text{Var}(\widehat{\beta}) = N^{-1} \{ \overline{\mathbf{G}}(\widehat{\beta})' \mathbf{W} \overline{\mathbf{G}}(\widehat{\beta}) \}^{-1} \overline{\mathbf{G}}(\widehat{\beta})' \mathbf{W} \mathbf{S} \overline{\mathbf{G}}(\widehat{\beta}) \{ \overline{\mathbf{G}}(\widehat{\beta})' \mathbf{W} \overline{\mathbf{G}}(\widehat{\beta}) \}^{-1}$$

For the one-step estimator,  $\mathbf{W}$  represents the initial weight matrix requested using the `winitial()` option, and  $\mathbf{S}$  is computed on the basis of the specification of the `vce()` option. The formulas for the  $\mathbf{S}$  matrix are identical to the ones that define the  $\mathbf{\Lambda}$  matrix in *Weight matrix* above, except that  $\mathbf{S}$  is computed after the moment equations are reevaluated using the final estimate of  $\widehat{\beta}$ . For the two-step and iterated GMM estimators, computation of  $\mathbf{W}$  is controlled by the `wmatrix()` option on the basis of the penultimate estimate of  $\widehat{\beta}$ .

For details on computation of the VCE matrix with dynamic panel-data models, see *Panel-style instruments* below.

## Hansen’s J statistic

Hansen’s (1982)  $J$  test of overidentifying restrictions is  $J = N \times Q(\widehat{\beta})$ .  $J \sim \chi^2(m - k)$ . If  $m < k$ , `gmm` issues an error message without estimating the parameters. If  $m = k$ , the model is just-identified and  $J$  is saved as missing (“.”). For the two-step and iterated GMM estimators, the  $J$  statistic is based on the last-computed weight matrix as determined by the `wmatrix()` option. For the one-step estimator, `gmm` recomputes a weight matrix as described in the second paragraph of [Variance–covariance matrix](#) above. To obtain Hansen’s  $J$  statistic, you use `estat overid`; see [\[R\] gmm postestimation](#).

## Panel-style instruments

Here we discuss several issues that arise only when you specify panel-style instruments by using the `xtinstruments()` option. When you specify the `xtinstruments()` option, we can no longer consider the instruments for one observation in isolation; instead, we must consider the instrument matrix for an entire panel at once. In the following discussion, we let  $T$  denote the number of time periods in a panel. To accommodate unbalanced datasets, conceptually we simply use zeros as instruments and residuals for time periods that are missing in a panel.

We consider the case where you specify both an equation in levels and an equation in differences, yielding two residual equations. Let  $u_{pt}^L(\beta)$  denote the residual for the level equation for panel  $p$  in period  $t$ , and let  $u_{pt}^D(\beta)$  denote the residual for the corresponding difference equation. Now, define the  $(2T - 1) \times 1$  vector  $\mathbf{u}_p(\beta)$  as

$$\mathbf{u}_p(\beta) = [u_{p1}^L(\beta), u_{p2}^L(\beta), \dots, u_{pT}^L(\beta), u_{p2}^D(\beta), u_{p3}^D(\beta), \dots, u_{pT}^D(\beta)]$$

The  $T + 1$  element of  $\mathbf{u}_p$  is  $u_{p2}^D(\beta)$  because we lose the first observation of the difference equation because of differencing.

We write the moment conditions for the  $p$ th panel as  $\mathbf{Z}_p \mathbf{u}_p(\beta)$ . To see how  $\mathbf{Z}_p$  is defined, we will let  $\mathbf{w}_{pt}^L$  and  $\mathbf{w}_{pt}^D$  denote the vectors of panel-style instruments for the level and difference equations, respectively, and let time be denoted by  $t$ ; we discuss their dimensions momentarily. Also let  $\mathbf{x}_{pt}^L$  and  $\mathbf{x}_{pt}^D$  denote the vectors of instruments specified in `instruments()` for the level and difference equations at time  $t$ . Without loss of generality, for our discussion, we assume that you specify the level equation first. Then,  $\mathbf{Z}_p$  has the form

$$\mathbf{Z}_p = \begin{bmatrix} \mathbf{w}_1^L & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{w}_2^L & \dots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{w}_T^L & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{x}_1^L & \mathbf{x}_2^L & \dots & \mathbf{x}_T^L & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{w}_1^D & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{w}_2^D & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{w}_T^D \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{x}_1^D & \mathbf{x}_2^D & \dots & \mathbf{x}_T^D \end{bmatrix} \tag{A4}$$

To see how the  $\mathbf{w}$  vectors are formed, we will suppose you specify

```
xtinstruments(1: d, lags(a/b))
```

Then,  $\mathbf{w}_t^L$  will be a  $(b - a + 1) \times 1$  vector consisting of  $d_{t-a}, \dots, d_{t-b}$ . If  $(t - a) \leq 0$ , then instead, we set  $\mathbf{w}_t^L = 0$ . If  $(t - a) > 0$  but  $(t - b) \leq 0$ , then we create  $\mathbf{w}_t^L$  to consist of  $d_{t-a}, \dots, d_1$ . With this definition,  $(b - a + 1)$  defines the maximum number of lags of  $d$  used, but `gmm` will proceed with fewer lags if all  $(b - a + 1)$  lags are not available. If you specify two panel-style instruments, `d` and `e`, say, then  $\mathbf{w}_t^L$  will consist of  $d_{t-a}, \dots, d_{t-b}, e_{t-a}, \dots, e_{t-b}$ .  $\mathbf{w}_t^D$  is handled analogously.

The  $\mathbf{x}_t^L$  vectors are simply  $j \times 1$  vectors, where  $j$  is the number of regular instruments specified with the `instruments()` option; these vectors include a “1” unless you specify the `noconstant` suboption.

Looking carefully at (A4), you will notice that for dynamic panel-data models, moment conditions corresponding to the instruments  $\mathbf{x}_{pt}^L$  take the form

$$E \left[ \sum_{t=1}^{t=T} \mathbf{x}_{pt}^L u_{pt}^L(\beta) \right] = \mathbf{0}$$

and likewise for  $\mathbf{x}_{pt}^D$ . Instead of having separate moment conditions for each time period, there is one moment condition equal to the average of individual periods’ moments. See [Arellano and Bond \(1991, 280\)](#). To include separate moment conditions for each time period, instead of specifying, say,

```
instruments(1: x)
```

you could instead first generate a variable called `one` equal to unity for all observations and specify

```
xtinstruments(1: x one)
```

(Creating the variable `one` is necessary because a constant is not automatically included in variable lists specified in `xtinstruments()`.)

Unbalanced panels are essentially handled by including zeros in rows and columns of  $\mathbf{Z}_p$  and  $\mathbf{u}_p(\beta)$  corresponding to missing time periods. However, the numbers of instruments and moment conditions reported by `gmm` do not reflect this trickery and instead reflect the numbers of instruments and moment conditions that are not manipulated in this way. Moreover, `gmm` includes code to work through these situations efficiently without actually having to fill in zeros.

When you specify `winitial(xt ...)`, the one-step unadjusted VCE is computed as

$$\text{Var}(\widehat{\beta}) = \widehat{\sigma}_1^2 \Lambda_H$$

where  $\Lambda_H$  is as defined previously,

$$\widehat{\sigma}_1^2 = (N - k)^{-1} \sum_{p=1}^{p=P} \mathbf{u}_p^D(\widehat{\beta})' \mathbf{u}_p^D(\widehat{\beta})$$

and  $\mathbf{u}_p^D(\widehat{\beta}) = [u_{p2}^D(\widehat{\beta}), \dots, u_{pT}^D(\widehat{\beta})]$ . Here we use  $(N - k)^{-1}$  instead of  $N^{-1}$  to match `xtdpd`.

## Marginal predictions with unconditional standard errors

Here we describe how `margins` computes unconditional standard errors when used with the `vce(unconditional)` option after `gmm`. These standard errors account for the estimation of parameters in `gmm` before `margins` is used to make marginal predictions. They also account for variation in the covariates over the population.

margins with the vce(unconditional) option uses linearization to estimate the unconditional variance of  $\widehat{\beta}$ . Linearization uses the variance estimator for the total of a score variable for the marginal prediction  $p(\widehat{\beta})$  as an approximate estimator for  $\text{Var}\{p(\widehat{\beta})\}$ . See [SVY] **Variance estimation** for more details. Our derivation of the standard errors here is similar to the derivation of the standard errors for two-step  $M$  estimators. See Wooldridge (2010, sec. 12.4) for the latter.

Let  $\mathbf{x}_i$  be a vector of covariate values, which includes all variables used in calculating the moment conditions, and let  $f(\mathbf{x}_i, \beta)$  be a scalar-valued function returning the value of the predictions of interest.

Let  $\delta_i(S_p)$  indicate whether observation  $i$  is in the subpopulation of interest,  $S_p$ ,

$$\delta_i(S_p) = \begin{cases} 1, & i \in S_p \\ 0, & i \notin S_p \end{cases}$$

margins computes  $\widehat{\theta} = p(\widehat{\beta})$  via

$$\widehat{\theta} = \frac{1}{w} \sum_{i=1}^N \delta_i(S_p) w_i f(\mathbf{x}_i, \widehat{\beta})$$

where

$$w = \sum_{i=1}^N \delta_i(S_p) w_i$$

and  $w_i$  is the weight for the  $i$ th observation.

In minimizing (A1), we see that the GMM estimator  $\widehat{\beta}$  is the value of  $\beta$  that solves the score equations

$$\mathbf{0} = \overline{\mathbf{G}}(\beta)' \mathbf{W} \left\{ N^{-1} \sum_{i=1}^N \mathbf{Z}'_i \mathbf{u}_i(\beta) \right\}$$

where  $\overline{\mathbf{G}}(\beta)$  was defined in (A3).

By the mean-value theorem, for some points  $\beta_1, \dots, \beta_q$  between  $\beta$  and  $\widehat{\beta}$ , we have

$$\mathbf{0} = \overline{\mathbf{G}}(\widehat{\beta})' \mathbf{W} \left[ \left\{ N^{-1} \sum_{i=1}^N \mathbf{Z}'_i \mathbf{u}_i(\beta) \right\} + \overline{\mathbf{G}}_m(\widehat{\beta} - \beta) \right]$$

where

$$\overline{\mathbf{G}}_{m(j,l)} = N^{-1} \sum_{i=1}^N \left[ \frac{\partial \{ \mathbf{Z}'_i \mathbf{u}_i(\beta_j) \}_l}{\partial \beta'_j} \right]_{j,l}$$

So we have

$$\sqrt{N}(\widehat{\beta} - \beta) = - \{ \overline{\mathbf{G}}(\widehat{\beta})' \mathbf{W} \overline{\mathbf{G}}_m \}^{-1} \overline{\mathbf{G}}(\widehat{\beta})' \mathbf{W} \left\{ N^{-0.5} \sum_{i=1}^N \mathbf{Z}'_i \mathbf{u}_i(\beta) \right\} \quad (\text{A5})$$

The margin  $\widehat{\theta}$  is the solution to the score equations

$$\frac{1}{N} \sum_{i=1}^N s_i(\theta, \widehat{\beta}) = \mathbf{0}$$

where

$$s_i(\theta, \beta) = w_i \delta_i(S_p) \{f(\mathbf{x}_i, \beta) - \theta\}$$

When we do a mean-value expansion about point  $\theta_1$  between  $\theta$  and  $\hat{\theta}$ , we get

$$\mathbf{0} = \frac{1}{N} \sum_{i=1}^N s_i(\theta, \hat{\beta}) - w.(\hat{\theta} - \theta)$$

So we have

$$\sqrt{N}(\hat{\theta} - \theta) = w.^{-1} N^{-0.5} \sum_{i=1}^N s_i(\theta, \hat{\beta})$$

Using the mean-value theorem again, for point  $\beta_m$  between  $\beta$  and  $\hat{\beta}$ , we have

$$w.^{-1} N^{-0.5} \sum_{i=1}^N s_i(\theta, \hat{\beta}) = w.^{-1} \left\{ N^{-0.5} \sum_{i=1}^N s_i(\theta, \beta) + \sqrt{N} \bar{\mathbf{J}}(\beta_m) (\hat{\beta} - \beta) \right\}$$

where  $\bar{\mathbf{J}}(\beta_m)$  is the Jacobian of the margin at  $\beta_m$ ,

$$\bar{\mathbf{J}}(\beta_m) = \left\{ N^{-1} \sum_{i=1}^N w_i \delta_i(S_p) \frac{\partial f(\mathbf{x}_i, \beta_m)}{\partial \beta'_m} \right\}$$

Using (A5), we get

$$\sqrt{N}(\hat{\theta} - \theta) = w.^{-1} N^{-0.5} \left[ \sum_{i=1}^N s_i(\theta, \beta) - \bar{\mathbf{J}}(\beta_m) \{ \bar{\mathbf{G}}(\hat{\beta})' \mathbf{W} \bar{\mathbf{G}}_m \}^{-1} \bar{\mathbf{G}}(\hat{\beta})' \mathbf{W} \left\{ \sum_{i=1}^N \mathbf{Z}'_i \mathbf{u}_i(\beta) \right\} \right]$$

$\hat{\theta}$  is asymptotically normal, and a consistent estimator of its variance is given by

$$\widehat{\text{Var}}\{\sqrt{N}(\hat{\theta} - \theta)\} = \sum_{i=1}^N w.^{-2} \left[ s_i(\hat{\theta}, \hat{\beta}) - \bar{\mathbf{J}}(\hat{\beta}) \{ \bar{\mathbf{G}}(\hat{\beta})' \mathbf{W} \bar{\mathbf{G}}(\hat{\beta}) \}^{-1} \bar{\mathbf{G}}(\hat{\beta})' \mathbf{W} \mathbf{Z}'_i \mathbf{u}_i(\hat{\beta}) \right]^2$$

See Wooldridge (2010, sec 12.4 and 12.5) for details.

`gmm` returns  $N^{-1} \{ \bar{\mathbf{G}}(\hat{\beta})' \mathbf{W} \bar{\mathbf{G}}(\hat{\beta}) \}^{-1}$  as the model-based variance. The scores are  $-\bar{\mathbf{G}}(\hat{\beta})' \mathbf{W} \mathbf{Z}'_i \mathbf{u}_i(\hat{\beta})$  and may be predicted in postestimation; see [R] [gmm postestimation](#). These scores correspond to derivatives of the criterion function  $Q(\beta)$ , scaled by  $-1/2$ . See Cameron and Trivedi (2005, sec. 6.3.2) for more details.

`margins` estimates the asymptotic standard error of  $\hat{\theta}$  from the model-based variance, the scores, and its own predictions of  $s_i$  and the Jacobian  $\bar{\mathbf{J}}(\hat{\beta})$ .

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## Also see

- [R] **gmm postestimation** — Postestimation tools for gmm
- [R] **ivregress** — Single-equation instrumental-variables regression
- [R] **ml** — Maximum likelihood estimation
- [R] **mlexp** — Maximum likelihood estimation of user-specified expressions
- [R] **nl** — Nonlinear least-squares estimation
- [R] **nlsur** — Estimation of nonlinear system of equations
- [XT] **xtabond** — Arellano–Bond linear dynamic panel-data estimation
- [XT] **xtdpd** — Linear dynamic panel-data estimation
- [XT] **xtdpdpsys** — Arellano–Bover/Blundell–Bond linear dynamic panel-data estimation
- [U] **20 Estimation and postestimation commands**

|                         |                            |                          |                       |                      |
|-------------------------|----------------------------|--------------------------|-----------------------|----------------------|
| Postestimation commands | <a href="#">predict</a>    | <a href="#">margins</a>  | <a href="#">estat</a> | Remarks and examples |
| Stored results          | <a href="#">References</a> | <a href="#">Also see</a> |                       |                      |

## Postestimation commands

The following postestimation command is of special interest after `gmm`:

| Command                   | Description                                  |
|---------------------------|----------------------------------------------|
| <code>estat overid</code> | perform test of overidentifying restrictions |

The following standard postestimation commands are also available:

| Command                  | Description                                                                                       |
|--------------------------|---------------------------------------------------------------------------------------------------|
| <code>contrast</code>    | contrasts and ANOVA-style joint tests of parameters                                               |
| <code>estat vce</code>   | variance–covariance matrix of the estimators (VCE)                                                |
| <code>estimates</code>   | cataloging estimation results                                                                     |
| <code>etable</code>      | table of estimation results                                                                       |
| <code>hausman</code>     | Hausman’s specification test                                                                      |
| <code>lincom</code>      | point estimates, standard errors, testing, and inference for linear combinations of parameters    |
| <code>margins</code>     | marginal means, predictive margins, marginal effects, and average marginal effects                |
| <code>marginsplot</code> | graph the results from margins (profile plots, interaction plots, etc.)                           |
| <code>nlcom</code>       | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters |
| <code>predict</code>     | residuals                                                                                         |
| <code>predictnl</code>   | point estimates, standard errors, testing, and inference for generalized predictions              |
| <code>pwcompare</code>   | pairwise comparisons of parameters                                                                |
| <code>test</code>        | Wald tests of simple and composite linear hypotheses                                              |
| <code>testnl</code>      | Wald tests of nonlinear hypotheses                                                                |

## predict

### Description for predict

`predict` creates a new variable containing predictions such as linear predictions, residuals, and scores.

### Menu for predict

Statistics > Postestimation

### Syntax for predict

```
predict [type] newvar [if] [in] [, xb equation(eqno | eqname)]
```

```
predict [type] newvar [if] [in], residuals [equation(eqno | eqname)]
```

```
predict [type] { stub* | newvar1 ... newvarq } [if] [in] [, residuals]
```

```
predict [type] stub* [if] [in], scores
```

Residuals are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

Scores are available only for observations within the estimation sample.

You specify one new variable and (optionally) `equation()`, or you specify `stub*` or  $q$  or  $p$  new variables, where  $q$  is the number of moment equations and  $p$  is the number of parameters in the model.

### Option for predict

Main

`xb`, the default, calculates the linear prediction.

`residuals` calculates the residuals, the predicted values of the moment equations. This option requires that the length of the new variable list be equal to the number of moment equations,  $q$ . Otherwise, use `stub*` to have `predict` generate enumerated variables with prefix `stub`. If `equation()` is not specified, the  $j$ th new variable will contain the residuals for the  $j$ th moment equation.

`equation(eqno | eqname)` with `xb` refers to a linear prediction in the model, whereas `equation(eqno | eqname)` with `residuals` refers to a moment equation.

For `xb`, specifying `equation(#1)` indicates that the calculation is to be made for the first linear prediction within curly braces. Specifying `equation(xb1)` would indicate that the calculation is to be made for the linear prediction `xb1`, assuming there is a linear prediction named `xb1` in the model, for instance, `{xb1:varlist}`.

For residuals, specifying `equation(#1)` indicates that the calculation is to be made for the first moment equation. Specifying `equation(demand)` would indicate that the calculation is to be made for the moment equation named `demand`, assuming there is a moment equation named `demand` in the model.

If you specify one new variable name and omit `equation()`, results are the same as if you had specified `equation(#1)`.

For more information on using `predict` after multiple-equation estimation commands, see [R] [predict](#).

`scores` calculates the parameter-level score equations, the first derivatives of the GMM criterion function with respect to the parameters scaled by  $-0.5$ . This option requires that the length of the new variable list be equal to the number of parameters,  $p$ . Otherwise, use `stub*` to have `predict` generate enumerated variables with prefix `stub`. The  $j$ th new variable will contain the  $j$ th score of the model.

## margins

### Description for margins

`margins` estimates margins of response for linear predictions.

### Menu for margins

Statistics > Postestimation

### Syntax for margins

```
margins [marginlist] [, options]
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i>       | Description                           |
|------------------------|---------------------------------------|
| <code>xb</code>        | linear prediction; the default        |
| <code>residuals</code> | not allowed with <code>margins</code> |

`xb` defaults to the first equation.

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [R] [margins](#).

## estat

### Description for estat

`estat overid` reports Hansen's  $J$  statistic, which is used to determine the validity of the overidentifying restrictions in a GMM model. If the model is correctly specified in the sense that  $E\{\mathbf{z}_i u_i(\boldsymbol{\beta})\} = \mathbf{0}$ , then the sample analog to that condition should hold at the estimated value of  $\boldsymbol{\beta}$ . Hansen's  $J$  statistic is valid only if the weight matrix is optimal, meaning that it equals the inverse of the covariance matrix of the moment conditions. Therefore, `estat overid` only reports Hansen's  $J$  statistic after two-step or iterated estimation, or if you specified `winitial(matname)` when calling `gmm`. In the latter case, it is your responsibility to determine the validity of the  $J$  statistic.

### Menu for estat

Statistics > Postestimation

### Syntax for estat

```
estat overid
```

`collect` is allowed with `estat overid`; see [\[U\] 11.1.10 Prefix commands](#).

### Remarks and examples

As we noted in [Introduction](#) of [\[R\] gmm](#), underlying generalized method of moments (GMM) estimators is a set of  $l$  moment conditions,  $E\{\mathbf{z}_i u_i(\boldsymbol{\beta})\} = \mathbf{0}$ . When  $l$  is greater than the number of parameters,  $k$ , any size- $k$  subset of the moment conditions would yield a consistent parameter estimate. We remarked that the parameter estimates we would obtain would in general depend on which  $k$  moment conditions we used. However, if all our moment conditions are indeed valid, then the parameter estimates should not differ too much regardless of which  $k$  moment conditions we used to estimate the parameters. The test of overidentifying restrictions is a model specification test based on this observation. The test of overidentifying restrictions requires that the number of moment conditions be greater than the number of parameters in the model.

Recall that the GMM criterion function is

$$Q = \left\{ \frac{1}{N} \sum_i \mathbf{z}_i u_i(\boldsymbol{\beta}) \right\}' \mathbf{W} \left\{ \frac{1}{N} \sum_i \mathbf{z}_i u_i(\boldsymbol{\beta}) \right\}$$

The test of overidentifying restrictions is remarkably simple. If  $\mathbf{W}$  is an optimal weight matrix, under the null hypothesis  $H_0: E\{\mathbf{z}_i u_i(\boldsymbol{\beta})\} = \mathbf{0}$ , the test statistic  $J = N \times Q \sim \chi^2(l - k)$ . A large test statistic casts doubt on the null hypothesis.

For the test to be valid,  $\mathbf{W}$  must be optimal, meaning that  $\mathbf{W}$  must be the inverse of the covariance matrix of the moment conditions:

$$\mathbf{W}^{-1} = E\{\mathbf{z}_i u_i(\boldsymbol{\beta}) u_i'(\boldsymbol{\beta}) \mathbf{z}_i'\}$$

Therefore, `estat overid` works only after the two-step and iterated estimators, or if you supplied your own initial weight matrix by using the `winitial(matname)` option to `gmm` and used the one-step estimator.

Often the overidentifying restrictions test is interpreted as a test of the validity of the instruments  $\mathbf{z}$ . However, other forms of model misspecification can sometimes lead to a significant test statistic. See Hall (2005, sec. 5.1) for a discussion of the overidentifying restrictions test and its behavior in correctly and misspecified models.

### ▷ Example 1

In [example 6](#) of [\[R\] gmm](#), we fit an exponential regression model of the number of doctor visits based on the person's gender, income, possession of private health insurance, and presence of a chronic disease. We argued that the variable `income` may be endogenous; we used the person's age and race as additional instrumental variables. Here we refit the model and test the specification of the model. We type

```
. use https://www.stata-press.com/data/r19/docvisits
. gmm (docvis - exp({xb:private chronic female income} + {b0})),
> instruments(private chronic female age black hispanic)
(output omitted)
. estat overid
Test of overidentifying restriction:
Hansen's J chi2(2) = 9.52598 (p = 0.0085)
```

The  $J$  statistic is significant even at the 1% significance level, so we conclude that our model is misspecified. One possibility is that age and race directly affect the number of doctor visits, so we are not justified in excluding them from the model.

A simple technique to explore whether any of the instruments is invalid is to examine the statistics

$$r_j = \mathbf{W}_{jj}^{1/2} \left\{ \frac{1}{N} \sum_{i=1}^N z_{ij} u_i(\widehat{\boldsymbol{\beta}}) \right\}$$

for  $j = 1, \dots, k$ , where  $\mathbf{W}_{jj}$  denotes the  $j$ th diagonal element of  $\mathbf{W}$ ,  $u_i(\widehat{\boldsymbol{\beta}})$  denotes the sample residuals, and  $k$  is the number of instruments. If all the instruments are valid, then the scaled sample moments should at least be on the same order of magnitude. If one (or more) instrument's  $r_j$  is large in absolute value relative to the others, then that could be an indication that instrument is not valid.

In Stata, we type

```
. predict double r if e(sample), residuals // obtain residual from the model
. matrix W = e(W) // retrieve weight matrix
. local i 1
. // loop over each instrument and compute r_j
. foreach var of varlist private chronic female age black hispanic {
2. generate double r'var' = r*'var'*sqrt(W['i', 'i'])
3. local '++i'
4. }
. summarize r*
```

| Variable  | Obs   | Mean      | Std. dev. | Min       | Max      |
|-----------|-------|-----------|-----------|-----------|----------|
| r         | 4,412 | .0344373  | 8.26176   | -151.1847 | 113.059  |
| rprivate  | 4,412 | .007988   | 3.824118  | -72.66254 | 54.33852 |
| rchronic  | 4,412 | .0026947  | 2.0707    | -43.7311  | 32.703   |
| rfemale   | 4,412 | .0028168  | 1.566397  | -12.7388  | 24.43621 |
| rage      | 4,412 | .0360978  | 4.752986  | -89.74112 | 55.58143 |
| rblack    | 4,412 | -.0379317 | 1.062027  | -24.39747 | 27.34512 |
| rhispanic | 4,412 | -.017435  | 1.08567   | -5.509386 | 31.53512 |

We notice that the  $r_j$  statistics for age, black, and hispanic are larger than those for the other instruments in our model, supporting our suspicion that age and race may have a direct impact on the number of doctor visits.



## Stored results

estat overid stores the following in `r()`:

Scalars

|         |                                  |
|---------|----------------------------------|
| r(J)    | Hansen's $J$ statistic           |
| r(J_df) | $J$ statistic degrees of freedom |
| r(J_p)  | $J$ statistic $p$ -value         |

## References

- Hall, A. R. 2005. *Generalized Method of Moments*. Oxford: Oxford University Press.
- Lindsey, C. 2016. Testing model specification and using the program version of gmm. *The Stata Blog: Not Elsewhere Classified*. <https://blog.stata.com/2016/02/11/testing-model-specification-and-using-the-program-version-of-gmm/>.

## Also see

[R] **gmm** — Generalized method of moments estimation

[U] **20 Estimation and postestimation commands**

## Description

`grmeanby` graphs the (optionally weighted) means or medians of *varname* according to the values of the variables in *varlist*. The variables in *varlist* may be string or numeric and, if numeric, may be labeled.

## Quick start

Graph means of *v1* for each level of categorical variables *cvar1*, *cvar2*, and *cvar3*

```
grmeanby cvar1 cvar2 cvar3, sum(v1)
```

Same as above, but graph medians

```
grmeanby cvar1 cvar2 cvar3, sum(v1) median
```

Same as above, but use + as the marker

```
grmeanby cvar1 cvar2 cvar3, sum(v1) median msymbol(+)
```

## Menu

Statistics > Summaries, tables, and tests > Summary and descriptive statistics > Graph means/medians by groups

## Syntax

```
grmeanby varlist [if] [in] [weight], summarize(varname) [options]
```

| <i>options</i>                          | Description                                                                        |
|-----------------------------------------|------------------------------------------------------------------------------------|
| Main                                    |                                                                                    |
| * <u>summarize</u> ( <i>varname</i> )   | graph mean (or median) of <i>varname</i>                                           |
| <u>median</u>                           | graph medians; default is to graph means                                           |
| Plot                                    |                                                                                    |
| <i>cline_options</i>                    | change look of the lines                                                           |
| <i>marker_options</i>                   | change look of markers (color, size, etc.)                                         |
| <i>marker_label_options</i>             | add marker labels; change look or position                                         |
| Y axis, X axis, Titles, Legend, Overall |                                                                                    |
| <i>twoway_options</i>                   | any options other than <code>by()</code> documented in [G-3] <i>twoway_options</i> |

\*`summarize`(*varname*) is required.

`aweights` and `fweights` are allowed; see [U] 11.1.6 **weight**.

## Options

### Main

`summarize`(*varname*) is required; it specifies the name of the variable whose mean or median is to be graphed.

`median` specifies that the graph is to be of medians, not means.

### Plot

*cline\_options* affect the rendition of the lines through the markers, including their color, pattern, and width; see [G-3] *cline\_options*.

*marker\_options* affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] *marker\_options*.

*marker\_label\_options* specify if and how the markers are to be labeled; see [G-3] *marker\_label\_options*.

### Y axis, X axis, Titles, Legend, Overall

*twoway\_options* are any of the options documented in [G-3] *twoway\_options*, excluding `by()`. These include options for titling the graph (see [G-3] *title\_options*) and for saving the graph to disk (see [G-3] *saving\_option*).

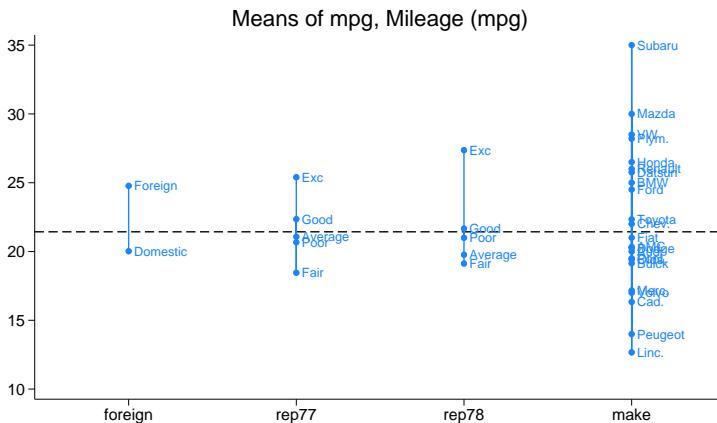
## Remarks and examples

The idea of graphing means of categorical variables was shown in [Chambers and Hastie \(1992, 3\)](#). Because this was shown in the context of an S function for making such graphs, it doubtless has roots going back further than that. `grmeanby` is, in any case, another implementation of what we will assume is their idea.

### ▶ Example 1

Using a variation of our auto dataset, we graph the mean of mpg by `foreign`, `rep77`, `rep78`, and `make`:

```
. use https://www.stata-press.com/data/r19/auto1
(Automobile models)
. grmeanby foreign rep77 rep78 make, sum(mpg)
```



If we had wanted a graph of medians rather than means, we could have typed

```
. grmeanby foreign rep77 rep78 make, sum(mpg) median
```



## References

- Chambers, J. M., and T. J. Hastie, eds. 1992. *Statistical Models in S*. Pacific Grove, CA: Wadsworth and Brooks/Cole. <https://doi.org/10.1201/9780203738535>.
- Cox, N. J. 2014. Speaking Stata: Design plots for graphical summary of a response given factors. *Stata Journal* 14: 975–990.

Description

Options

Acknowledgment

Quick start

Remarks and examples

References

Menu

Stored results

Also see

Syntax

Methods and formulas

## Description

`hausman` performs Hausman's (1978) specification test.

## Quick start

Hausman test for stored models `consistent` and `efficient`

```
hausman consistent efficient
```

Same as above, but compare fixed-effects and random-effects linear regression models

```
hausman fixed random, sigmamore
```

Endogeneity test after `ivprobit` and `probit` with estimates stored in `iv` and `noiv`

```
hausman iv noiv, equations(1:1)
```

Test of independence of irrelevant alternatives for model with all alternatives `all` and model with omitted alternative `omitted`

```
hausman omitted all, alleqs constant
```

## Menu

Statistics > Postestimation

## Syntax

```
hausman name-consistent [name-efficient] [, options]
```

*name-consistent* and *name-efficient* are names under which estimation results were stored via `estimates store`; see [R] [estimates store](#). A period (.) may be used to refer to the last estimation results, even if these were not already stored. Not specifying *name-efficient* is equivalent to specifying the last estimation results as “.”.

| <i>options</i>                              | Description                                                                                   |
|---------------------------------------------|-----------------------------------------------------------------------------------------------|
| Main                                        |                                                                                               |
| <code>constant</code>                       | include estimated intercepts in comparison; default is to exclude                             |
| <code>alleqs</code>                         | use all equations to perform test; default is first equation only                             |
| <code>skipeqs</code> ( <i>eqlist</i> )      | skip specified equations when performing test                                                 |
| <code>equations</code> ( <i>matchlist</i> ) | associate/compare the specified (by number) pairs of equations                                |
| <code>force</code>                          | force performance of test, even though assumptions are not met                                |
| <code>df</code> (#)                         | use # degrees of freedom                                                                      |
| <code>sigmamore</code>                      | base both (co)variance matrices on disturbance variance<br>estimate from efficient estimator  |
| <code>sigmaless</code>                      | base both (co)variance matrices on disturbance variance<br>estimate from consistent estimator |
| Advanced                                    |                                                                                               |
| <code>tconsistent</code> ( <i>string</i> )  | consistent estimator column header                                                            |
| <code>tefficient</code> ( <i>string</i> )   | efficient estimator column header                                                             |

`collect` is allowed; see [U] [11.1.10 Prefix commands](#).

## Options

### Main

`constant` specifies that the estimated intercept(s) be included in the model comparison; by default, they are excluded. The default behavior is appropriate for models in which the constant does not have a common interpretation across the two models.

`alleqs` specifies that all the equations in the models be used to perform the Hausman test; by default, only the first equation is used.

`skipeqs` (*eqlist*) specifies in *eqlist* the names of equations to be excluded from the test. Equation numbers are not allowed in this context, because the equation names, along with the variable names, are used to identify common coefficients.

`equations(matchlist)` specifies, by number, the pairs of equations that are to be compared.

The *matchlist* in `equations()` should follow the syntax

$$\#_c : \#_e [ , \#_c : \#_e [ , \dots ] ]$$

where  $\#_c$  ( $\#_e$ ) is an equation number of the always-consistent (efficient under  $H_0$ ) estimator. For instance, `equations(1:1)`, `equations(1:1, 2:2)`, or `equations(1:2)`.

If `equations()` is not specified, then equations are matched on equation names.

`equations()` handles the situation in which one estimator uses equation names and the other does not. For instance, `equations(1:2)` means that equation 1 of the always-consistent estimator is to be tested against equation 2 of the efficient estimator. `equations(1:1, 2:2)` means that equation 1 is to be tested against equation 1 and that equation 2 is to be tested against equation 2. If `equations()` is specified, the `alleqs` and `skipeqs` options are ignored.

`force` specifies that the Hausman test be performed, even though the assumptions of the Hausman test seem not to be met, for example, because the estimators were `pweighted` or the data were clustered.

`df(#)` specifies the degrees of freedom for the Hausman test. The default is the matrix rank of the variance of the difference between the coefficients of the two estimators.

`sigmamore` and `sigmaless` specify that the two covariance matrices used in the test be based on a common estimate of disturbance variance ( $\sigma^2$ ).

`sigmamore` specifies that the covariance matrices be based on the estimated disturbance variance from the efficient estimator. This option provides a proper estimate of the contrast variance for so-called tests of exogeneity and overidentification in instrumental-variables regression.

`sigmaless` specifies that the covariance matrices be based on the estimated disturbance variance from the consistent estimator.

These options can be specified only when both estimators store `e(sigma)` or `e(rmse)`, or with the `xtreg` command. `e(sigma_e)` is stored after the `xtreg` command with the `fe` or `mle` option. `e(rmse)` is stored after the `xtreg` command with the `re` option.

`sigmamore` or `sigmaless` are recommended when comparing fixed-effects and random-effects linear regression because they are much less likely to produce a non-positive-definite-differenced covariance matrix (although the tests are asymptotically equivalent whether or not one of the options is specified).

---

Advanced

`tconsistent(string)` and `tefficient(string)` are formatting options. They allow you to specify the headers of the columns of coefficients that default to the names of the models. These options will be of interest primarily to programmers.

## Remarks and examples

`hausman` is a general implementation of Hausman's (1978) specification test, which compares an estimator  $\hat{\theta}_1$  that is known to be consistent with an estimator  $\hat{\theta}_2$  that is efficient under the assumption being tested. The null hypothesis is that the estimator  $\hat{\theta}_2$  is indeed an efficient (and consistent) estimator of the true parameters. If this is the case, there should be no systematic difference between the two estimators. If there exists a systematic difference in the estimates, you have reason to doubt the assumptions on which the efficient estimator is based.

The assumption of efficiency is violated if the estimator is `pweighted` or the data are clustered, so `hausman` cannot be used. The test can be forced by specifying the `force` option with `hausman`. For an alternative to using `hausman` in these cases, see [R] [suest](#).

To use `hausman`, you

```
. (compute the always-consistent estimator)
. estimates store name-consistent
. (compute the estimator that is efficient under H_0)
. hausman name-consistent .
```

Alternatively, you can turn this around:

```
. (compute the estimator that is efficient under H_0)
. estimates store name-efficient
. (fit the less-efficient model)
. (compute the always-consistent estimator)
. hausman . name-efficient
```

You can, of course, also compute and store both the always-consistent and efficient-under- $H_0$  estimators and perform the Hausman test with

```
. hausman name-consistent name-efficient
```

### ▷ Example 1

We are studying the factors that affect the wages of young women in the United States between 1968 and 1988, and we have a panel-data sample of individual women over that time span.

```
. use https://www.stata-press.com/data/r19/nlswork4
(National Longitudinal Survey of Young Women, 14-24 years old in 1968)
. describe
Contains data from https://www.stata-press.com/data/r19/nlswork4.dta
Observations: 28,534 National Longitudinal Survey of
 Young Women, 14-24 years old in
 1968
Variables: 6 29 Jan 2024 16:35
 (_dta has notes)
```

| Variable name        | Storage type       | Display format     | Value label | Variable label               |
|----------------------|--------------------|--------------------|-------------|------------------------------|
| <code>idcode</code>  | <code>int</code>   | <code>%8.0g</code> |             | NLS ID                       |
| <code>year</code>    | <code>byte</code>  | <code>%8.0g</code> |             | Interview year               |
| <code>age</code>     | <code>byte</code>  | <code>%8.0g</code> |             | Age in current year          |
| <code>msp</code>     | <code>byte</code>  | <code>%8.0g</code> |             | 1 if married, spouse present |
| <code>ttl_exp</code> | <code>float</code> | <code>%9.0g</code> |             | Total work experience        |
| <code>ln_wage</code> | <code>float</code> | <code>%9.0g</code> |             | ln(wage/GNP deflator)        |

Sorted by: `idcode year`

We believe that a random-effects specification is appropriate for individual-level effects in our model. We fit a fixed-effects model that will capture all temporally constant individual-level effects.

```
. xtreg ln_wage age msp ttl_exp, fe
Fixed-effects (within) regression Number of obs = 28,494
Group variable: idcode Number of groups = 4,710
R-squared: Obs per group:
 Within = 0.1373 min = 1
 Between = 0.2571 avg = 6.0
 Overall = 0.1800 max = 15
 F(3, 23781) = 1262.01
corr(u_i, Xb) = 0.1476 Prob > F = 0.0000
```

| ln_wage | Coefficient                                 | Std. err. | t     | P> t  | [95% conf. interval] |           |
|---------|---------------------------------------------|-----------|-------|-------|----------------------|-----------|
| age     | -.005485                                    | .000837   | -6.55 | 0.000 | -.0071256            | -.0038443 |
| msp     | .0033427                                    | .0054868  | 0.61  | 0.542 | -.0074118            | .0140971  |
| ttl_exp | .0383604                                    | .0012416  | 30.90 | 0.000 | .0359268             | .0407941  |
| _cons   | 1.593953                                    | .0177538  | 89.78 | 0.000 | 1.559154             | 1.628752  |
| sigma_u | .37674223                                   |           |       |       |                      |           |
| sigma_e | .29751014                                   |           |       |       |                      |           |
| rho     | .61591044 (fraction of variance due to u_i) |           |       |       |                      |           |

F test that all u\_i=0: F(4709, 23781) = 7.76                      Prob > F = 0.0000

We assume that this model is consistent for the true parameters and store the results by using `estimates store` under a name, `fixed`:

```
. estimates store fixed
```

Now we fit a random-effects model as a fully efficient specification of the individual effects under the assumption that they are random and follow a normal distribution. We then compare these estimates with the previously stored results by using the `hausman` command.

```
. xtreg ln_wage age msp ttl_exp, re
Random-effects GLS regression Number of obs = 28,494
Group variable: idcode Number of groups = 4,710
R-squared: Obs per group:
 Within = 0.1373 min = 1
 Between = 0.2552 avg = 6.0
 Overall = 0.1797 max = 15
 Wald chi2(3) = 5100.33
corr(u_i, X) = 0 (assumed) Prob > chi2 = 0.0000
```

| ln_wage | Coefficient                                 | Std. err. | z      | P> z  | [95% conf. interval] |           |
|---------|---------------------------------------------|-----------|--------|-------|----------------------|-----------|
| age     | -.0069749                                   | .0006882  | -10.13 | 0.000 | -.0083238            | -.0056259 |
| msp     | .0046594                                    | .0051012  | 0.91   | 0.361 | -.0053387            | .0146575  |
| ttl_exp | .0429635                                    | .0010169  | 42.25  | 0.000 | .0409704             | .0449567  |
| _cons   | 1.609916                                    | .0159176  | 101.14 | 0.000 | 1.578718             | 1.641114  |
| sigma_u | .32648519                                   |           |        |       |                      |           |
| sigma_e | .29751014                                   |           |        |       |                      |           |
| rho     | .54633481 (fraction of variance due to u_i) |           |        |       |                      |           |

```
. hausman fixed ., sigmamore
```

|         | Coefficients |           | (b-B)<br>Difference | sqrt(diag(V_b-V_B))<br>Std. err. |
|---------|--------------|-----------|---------------------|----------------------------------|
|         | (b)<br>fixed | (B)<br>.  |                     |                                  |
| age     | -.005485     | -.0069749 | .0014899            | .0004803                         |
| msp     | .0033427     | .0046594  | -.0013167           | .0020596                         |
| t1l_exp | .0383604     | .0429635  | -.0046031           | .0007181                         |

b = Consistent under H0 and Ha; obtained from **xtreg**.

B = Inconsistent under Ha, efficient under H0; obtained from **xtreg**.

Test of H0: Difference in coefficients not systematic

$$\begin{aligned} \text{chi2}(3) &= (b-B)'[(V_b-V_B)^{-1}](b-B) \\ &= 260.40 \end{aligned}$$

Prob > chi2 = 0.0000

Under the current specification, our initial hypothesis that the individual-level effects are adequately modeled by a random-effects model is resoundingly rejected. This result is based on the rest of our model specification, and random effects might be appropriate for some alternate model of wages.

◀

Jerry Allen Hausman (1946–) is an American economist and econometrician. He was born in West Virginia and went on to study economics at Brown and Oxford. He joined the MIT faculty in 1972 and continues to teach there. He currently researches new goods and their effects on consumer welfare and its measurement in the Consumer Price Index along with regulation and competition in the telecommunications industry.

Hausman is best known for his many contributions to econometrics. In 1978, he published his now famous paper giving the Hausman specification test. The work remains one of the most widely cited econometrics papers. He has also done extensive work in applied microeconomics pertaining to government's role in the economy, including antitrust regulation, public finance, and taxation.

In 1980, Hausman received the Frisch Medal, a biennial award from the Econometric Society recognizing exceptional applied work, for his paper with David Wise on attrition bias. In 1985, he won the John Bates Clark Award from the American Economics Association, which is given for outstanding contributions to economics by an economist under 40 years of age. In 2012, the *Advances in Econometrics* book series devoted an entire volume to Hausman and his contributions to econometrics.

## ▷ Example 2

A stringent assumption of multinomial and conditional logit models is that outcome categories for the model have the property of independence of irrelevant alternatives (IIA). Stated simply, this assumption requires that the inclusion or exclusion of categories does not affect the relative risks associated with the regressors in the remaining categories.

One classic example of a situation in which this assumption would be violated involves the choice of transportation mode; see [McFadden \(1974\)](#). For simplicity, postulate a transportation model with the four possible outcomes: rides a train to work, takes a bus to work, drives the Ford to work, and drives the Chevrolet to work. Clearly, “drives the Ford” is a closer substitute to “drives the Chevrolet” than it is to “rides a train” (at least for most people). This means that excluding “drives the Ford” from the model could be expected to affect the relative risks of the remaining options and that the model would not obey the IIA assumption.

Using the data presented in [\[R\] mlogit](#), we will use a simplified model to test for IIA. The choice of insurance type among indemnity, prepaid, and uninsured is modeled as a function of age and gender. The indemnity category is allowed to be the base category, and the model including all three outcomes is fit. The results are then stored under the name `allcats`.

```
. use https://www.stata-press.com/data/r19/sysdsn3
(Health insurance data)
. mlogit insure age male
Iteration 0: Log likelihood = -555.85446
Iteration 1: Log likelihood = -551.32973
Iteration 2: Log likelihood = -551.32802
Iteration 3: Log likelihood = -551.32802
Multinomial logistic regression Number of obs = 615
LR chi2(4) = 9.05
Prob > chi2 = 0.0598
Pseudo R2 = 0.0081
Log likelihood = -551.32802
```

| insure    | Coefficient    | Std. err. | z     | P> z  | [95% conf. interval] |           |
|-----------|----------------|-----------|-------|-------|----------------------|-----------|
| Indemnity | (base outcome) |           |       |       |                      |           |
| Prepaid   |                |           |       |       |                      |           |
| age       | -.0100251      | .0060181  | -1.67 | 0.096 | -.0218204            | .0017702  |
| male      | .5095747       | .1977893  | 2.58  | 0.010 | .1219147             | .8972346  |
| _cons     | .2633838       | .2787575  | 0.94  | 0.345 | -.2829708            | .8097383  |
| Uninsure  |                |           |       |       |                      |           |
| age       | -.0051925      | .0113821  | -0.46 | 0.648 | -.0275011            | .0171161  |
| male      | .4748547       | .3618462  | 1.31  | 0.189 | -.2343508            | 1.18406   |
| _cons     | -1.756843      | .5309602  | -3.31 | 0.001 | -2.797506            | -.7161803 |

```
. estimates store allcats
```

Under the IIA assumption, we would expect no systematic change in the coefficients if we excluded one of the outcomes from the model. (For an extensive discussion, see [Hausman and McFadden \[1984\]](#).) We reestimate the parameters, excluding the uninsured outcome, and perform a Hausman test against the fully efficient full model.

```
. mlogit insure age male if insure != "Uninsure":insure
Iteration 0: Log likelihood = -394.8693
Iteration 1: Log likelihood = -390.4871
Iteration 2: Log likelihood = -390.48643
Iteration 3: Log likelihood = -390.48643
Multinomial logistic regression
Number of obs = 570
LR chi2(2) = 8.77
Prob > chi2 = 0.0125
Pseudo R2 = 0.0111
Log likelihood = -390.48643
```

| insure    | Coefficient    | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-----------|----------------|-----------|-------|-------|----------------------|----------|
| Indemnity | (base outcome) |           |       |       |                      |          |
| Prepaid   |                |           |       |       |                      |          |
| age       | -.0101521      | .0060049  | -1.69 | 0.091 | -.0219214            | .0016173 |
| male      | .5144003       | .1981735  | 2.60  | 0.009 | .1259874             | .9028133 |
| _cons     | .2678043       | .2775563  | 0.96  | 0.335 | -.276196             | .8118046 |

```
. hausman . allcats, alleqs constant
```

|       | Coefficients |                | (b-B)<br>Difference | sqrt(diag(V_b-V_B))<br>Std. err. |
|-------|--------------|----------------|---------------------|----------------------------------|
|       | (b)          | (B)<br>allcats |                     |                                  |
| age   | -.0101521    | -.0100251      | -.0001269           | .                                |
| male  | .5144003     | .5095747       | .0048256            | .0123338                         |
| _cons | .2678043     | .2633838       | .0044205            | .                                |

b = Consistent under H0 and Ha; obtained from **mlogit**.

B = Inconsistent under Ha, efficient under H0; obtained from **mlogit**.

Test of H0: Difference in coefficients not systematic

$$\begin{aligned} \text{chi2}(3) &= (b-B)' [(V_b-V_B)^{-1}] (b-B) \\ &= 0.08 \end{aligned}$$

Prob > chi2 = 0.9944

(V\_b-V\_B is not positive definite)

The syntax of the `if` condition on the `mlogit` command simply identified the "Uninsured" category with the `insure` value label; see [\[U\] 12.6.3 Value labels](#). On examining the output from `hausman`, we see that there is no evidence that the IIA assumption has been violated.

Because the Hausman test is a standardized comparison of model coefficients, using it with `mlogit` requires that the base outcome be the same in both competing models. In particular, if the most-frequent category (the default base outcome) is being removed to test for IIA, you must use the `baseoutcome()` option in `mlogit` to manually set the base outcome to something else. Or you can use the `equation()` option of the `hausman` command to align the equations of the two models.

Having the missing values for the square root of the diagonal of the covariance matrix of the differences is not comforting, but it is also not surprising. This covariance matrix is guaranteed to be positive definite only asymptotically (it is a consequence of the assumption that one of the estimators is efficient), and assurances are not made about the diagonal elements. Negative values along the diagonal are possible, and the fourth column of the table is provided mainly for descriptive use.

We can also perform the Hausman IIA test against the remaining alternative in the model:

```
. mlogit insure age male if insure != "Prepaid":insure
Iteration 0: Log likelihood = -132.59913
Iteration 1: Log likelihood = -131.78009
Iteration 2: Log likelihood = -131.76808
Iteration 3: Log likelihood = -131.76807

Multinomial logistic regression Number of obs = 338
LR chi2(2) = 1.66
Prob > chi2 = 0.4356
Pseudo R2 = 0.0063

Log likelihood = -131.76807
```

| insure         | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|----------------|-------------|-----------|-------|-------|----------------------|-----------|
| Indemnity      |             |           |       |       |                      |           |
| (base outcome) |             |           |       |       |                      |           |
| Uninsure       |             |           |       |       |                      |           |
| age            | -.0041055   | .0115807  | -0.35 | 0.723 | -.0268033            | .0185923  |
| male           | .4591074    | .3595663  | 1.28  | 0.202 | -.2456296            | 1.163844  |
| _cons          | -1.801774   | .5474476  | -3.29 | 0.001 | -2.874752            | -.7287968 |

```
. hausman . allcats, alleqs constant
----- Coefficients -----
 (b) (B) (b-B) sqrt(diag(V_b-V_B))
 . allcats Difference Std. err.

age -.0041055 -.0051925 .001087 .0021355
male .4591074 .4748547 -.0157473 .
_cons -1.801774 -1.756843 -.0449311 .1333421
```

b = Consistent under H0 and Ha; obtained from **mlogit**.  
 B = Inconsistent under Ha, efficient under H0; obtained from **mlogit**.

Test of H0: Difference in coefficients not systematic

```
chi2(3) = (b-B)' [(V_b-V_B)^(-1)](b-B)
 = -0.18
```

Warning: chi2 < 0 ==> model fitted on these data  
 fails to meet the asymptotic assumptions  
 of the Hausman test; see **suest** for a  
 generalized test.

Here the  $\chi^2$  statistic is actually negative. We might interpret this result as strong evidence that we cannot reject the null hypothesis. Such a result is not an unusual outcome for the Hausman test, particularly when the sample is relatively small—there are only 45 uninsured individuals in this dataset.

Are we surprised by the results of the Hausman test in this example? Not really. Judging from the  $z$  statistics on the original multinomial logit model, we were struggling to identify any structure in the data with the current specification. Even when we were willing to assume IIA and computed the efficient estimator under this assumption, few of the effects could be identified as statistically different from those on the base category. Trying to base a Hausman test on a contrast (difference) between two poor estimates is just asking too much of the existing data.

In [example 2](#), we encountered a case in which the Hausman was not well defined. Unfortunately, in our experience this happens fairly often. Stata provides an alternative to the Hausman test that overcomes this problem through an alternative estimator of the variance of the difference between the two estimators. This other estimator is guaranteed to be positive semidefinite. This alternative estimator also allows a widening of the scope of problems to which Hausman-type tests can be applied by relaxing the assumption that one of the estimators is efficient. For instance, you can perform Hausman-type tests to clustered observations and survey estimators. See [\[R\] suest](#) for details.

## Stored results

hausman stores the following in `r()`:

Scalars

|                      |                                      |
|----------------------|--------------------------------------|
| <code>r(chi2)</code> | $\chi^2$                             |
| <code>r(df)</code>   | degrees of freedom for the statistic |
| <code>r(p)</code>    | $p$ -value for the $\chi^2$          |
| <code>r(rank)</code> | rank of $(V_b - V_B)^{-1}$           |

## Methods and formulas

The Hausman statistic is distributed as  $\chi^2$  and is computed as

$$H = (\beta_c - \beta_e)'(V_c - V_e)^{-1}(\beta_c - \beta_e)$$

where

|           |                                                         |
|-----------|---------------------------------------------------------|
| $\beta_c$ | is the coefficient vector from the consistent estimator |
| $\beta_e$ | is the coefficient vector from the efficient estimator  |
| $V_c$     | is the covariance matrix of the consistent estimator    |
| $V_e$     | is the covariance matrix of the efficient estimator     |

When the difference in the variance matrices is not positive definite, a Moore–Penrose generalized inverse is used. As noted in [Gouriéroux and Monfort \(1995, 125–128\)](#), the choice of generalized inverse is not important asymptotically.

The number of degrees of freedom for the statistic is the rank of the difference in the variance matrices. When the difference is positive definite, this is the number of common coefficients in the models being compared.

## Acknowledgment

Portions of `hausman` are based on an early implementation by Jeroen Weesie of the Department of Sociology at Utrecht University, The Netherlands.

## References

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- Gouriéroux, C. S., and A. Monfort. 1995. *Statistics and Econometric Models*. Vol. 2, *Testing, Confidence Regions, Model Selection, and Asymptotic Theory*. Trans. Q. Vuong. Cambridge: Cambridge University Press. <https://doi.org/10.1017/CBO9780511751950>.
- Hausman, J. A. 1978. Specification tests in econometrics. *Econometrica* 46: 1251–1271. <https://doi.org/10.2307/1913827>.
- Hausman, J. A., and D. L. McFadden. 1984. Specification tests for the multinomial logit model. *Econometrica* 52: 1219–1240. <https://doi.org/10.2307/1910997>.
- McFadden, D. L. 1974. Measurement of urban travel demand. *Journal of Public Economics* 3: 303–328. [https://doi.org/10.1016/0047-2727\(74\)90003-6](https://doi.org/10.1016/0047-2727(74)90003-6).

## Also see

- [R] **lrtest** — Likelihood-ratio test after estimation
- [R] **suest** — Seemingly unrelated estimation
- [R] **test** — Test linear hypotheses after estimation
- [XT] **xtreg** — Linear models for panel data

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## Description

`heckman` fits regression models with selection by using either Heckman's two-step consistent estimator or full maximum likelihood.

## Quick start

Heckman model of  $y$  on  $x_1$  with  $v_1$  predicting selection when binary variable `selected` indicates selection status

```
heckman y x1, select(selected = v1 x1)
```

Same as above, and generate  $v_2$  containing the inverse of Mills's ratio

```
heckman y x1, select(selected = v1 x1) mills(v2)
```

Same as above

```
heckman y x1, select(selected = v1 x1) nshazard(v2)
```

Fit a Heckman model using the two-step estimation method

```
heckman y x1, select(selected = v1 x1) twostep
```

Same as above, and display first-stage probit estimates

```
heckman y x1, select(selected = v1 x1) twostep first
```

## Menu

Statistics > Sample-selection models > Heckman selection model

## Syntax

### Basic syntax

```
heckman depvar [indepvars], select(varlists) [twostep]
```

or

```
heckman depvar [indepvars], select(depvars = varlists) [twostep]
```

### Full syntax for maximum likelihood estimates only

```
heckman depvar [indepvars] [if] [in] [weight],
select([depvars =] varlists [, noconstant offset(varnameo)]) [heckman_ml_options]
```

### Full syntax for Heckman's two-step consistent estimates only

```
heckman depvar [indepvars] [if] [in], twostep
select([depvars =] varlists [, noconstant]) [heckman_ts_options]
```

| <i>heckman_ml_options</i>                 | Description                                                                                                                                      |
|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                              |                                                                                                                                                  |
| <u>mle</u>                                | use maximum likelihood estimator; the default                                                                                                    |
| * <u>select</u> ( )                       | specify selection equation: dependent and independent variables; whether to have constant term and offset variable                               |
| <u>noconstant</u>                         | suppress constant term                                                                                                                           |
| <u>offset</u> ( <i>varname</i> )          | include <i>varname</i> in model with coefficient constrained to 1                                                                                |
| <u>constraints</u> ( <i>constraints</i> ) | apply specified linear constraints                                                                                                               |
| <b>SE/Robust</b>                          |                                                                                                                                                  |
| <u>vce</u> ( <i>vcetype</i> )             | <i>vcetype</i> may be <u>oim</u> , <u>robust</u> , <u>cluster</u> <i>clustvar</i> , <u>opg</u> , <u>bootstrap</u> , or <u>jackknife</u>          |
| <b>Reporting</b>                          |                                                                                                                                                  |
| <u>level</u> (#)                          | set confidence level; default is <u>level</u> (95)                                                                                               |
| <u>first</u>                              | report first-step probit estimates                                                                                                               |
| <u>lrmodel</u>                            | perform the likelihood-ratio model test instead of the default Wald test                                                                         |
| <u>nshazard</u> ( <i>newvar</i> )         | generate nonselection hazard variable                                                                                                            |
| <u>mills</u> ( <i>newvar</i> )            | synonym for <u>nshazard</u> ( )                                                                                                                  |
| <u>nocnsreport</u>                        | do not display constraints                                                                                                                       |
| <u>display_options</u>                    | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| <b>Maximization</b>                       |                                                                                                                                                  |
| <u>maximize_options</u>                   | control the maximization process; seldom used                                                                                                    |
| <u>collinear</u>                          | keep collinear variables                                                                                                                         |
| <u>coeflegend</u>                         | display legend instead of statistics                                                                                                             |

\*select( ) is required.

The full specification is select( [*depvar*<sub>s</sub> = ] *varlist*<sub>s</sub> [, noconstant offset(*varname*<sub>o</sub>) ] ).

| <i>heckman_ts_options</i>     | Description                                                                                                                                      |
|-------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| Model                         |                                                                                                                                                  |
| * <code>twostep</code>        | produce two-step consistent estimate                                                                                                             |
| * <code>select()</code>       | specify selection equation: dependent and independent variables; whether to have constant term                                                   |
| <code>noconstant</code>       | suppress constant term                                                                                                                           |
| <code>rhosigma</code>         | truncate $\rho$ to $[-1, 1]$ with consistent $\sigma$                                                                                            |
| <code>hotrunc</code>          | truncate $\rho$ to $[-1, 1]$                                                                                                                     |
| <code>holimited</code>        | truncate $\rho$ in limited cases                                                                                                                 |
| <code>hoforce</code>          | do not truncate $\rho$                                                                                                                           |
| SE                            |                                                                                                                                                  |
| <code>vce(vctype)</code>      | <i>vctype</i> may be conventional, <code>bootstrap</code> , or <code>jackknife</code>                                                            |
| Reporting                     |                                                                                                                                                  |
| <code>level(#)</code>         | set confidence level; default is <code>level(95)</code>                                                                                          |
| <code>first</code>            | report first-step probit estimates                                                                                                               |
| <code>nshazard(newvar)</code> | generate nonselection hazard variable                                                                                                            |
| <code>mills(newvar)</code>    | synonym for <code>nshazard()</code>                                                                                                              |
| <code>display_options</code>  | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| <code>coeflegend</code>       | display legend instead of statistics                                                                                                             |

\*`twostep` and `select()` are required.

The full specification is `select([depvars =] varlists [, noconstant])`.

*indepvars* and *varlist<sub>s</sub>* may contain factor variables; see [U] 11.4.3 Factor variables.

*depvar*, *indepvars*, *varlist<sub>s</sub>*, and *depvar<sub>s</sub>* may contain time-series operators; see [U] 11.4.4 Time-series varlists.

`bayes`, `bayesboot`, `bootstrap`, `by`, `collect`, `fp`, `jackknife`, `rolling`, `statsby`, and `svy` are allowed; see [U] 11.1.10 Prefix commands. For more details, see [BAYES] bayes: heckman.

Weights are not allowed with the `bootstrap` prefix; see [R] bootstrap.

`twostep`, `vce()`, `first`, `lrmmodel`, and `weights` are not allowed with the `svy` prefix; see [SVY] svy.

`pweights`, `fweights`, and `iweights` are allowed with maximum likelihood estimation; see [U] 11.1.6 weight. No weights are allowed if `twostep` is specified.

`collinear` and `coeflegend` do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Options for Heckman selection model (ML)

### Model

`mle` requests that the maximum likelihood estimator be used. This is the default.

`select( [ depvars = ] varlists [ , noconstant offset(varnameo) ] )` specifies the variables and options for the selection equation. It is an integral part of specifying a Heckman model and is required. The selection equation should contain at least one variable that is not in the outcome equation.

If *depvar*<sub>s</sub> is specified, it should be coded as 0 or 1, with 0 indicating an observation not selected and 1 indicating a selected observation. If *depvar*<sub>s</sub> is not specified, observations for which *depvar* is not missing are assumed selected, and those for which *depvar* is missing are assumed not selected.

`noconstant` suppresses the selection constant term (intercept).

`offset(varnameo)` specifies that selection offset *varname*<sub>o</sub> be included in the model with the coefficient constrained to be 1.

`noconstant`, `offset(varname)`, `constraints(constraints)`; see [R] [Estimation options](#).

### SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

### Reporting

`level(#)`; see [R] [Estimation options](#).

`first` specifies that the first-step probit estimates of the selection equation be displayed before estimation.

`lrmodel`; see [R] [Estimation options](#).

`nshazard(newvar)` and `mills(newvar)` are synonyms; either will create a new variable containing the nonselection hazard—what Heckman (1979) referred to as the inverse of the Mills ratio—from the selection equation. The nonselection hazard is computed from the estimated parameters of the selection equation.

`nocnsreport`; see [R] [Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

### Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

Setting the optimization type to `technique(bhhh)` resets the default *vcetype* to `vce(opg)`.

The following options are available with `heckman` but are not shown in the dialog box: `collinear`, `coeflegend`; see [R] [Estimation options](#).

## Options for Heckman selection model (two-step)

### Model

`twostep` specifies that Heckman's (1979) two-step efficient estimates of the parameters, standard errors, and covariance matrix be produced.

`select( [ depvars = ] varlists [ , noconstant ] )` specifies the variables and options for the selection equation. It is an integral part of specifying a Heckman model and is required. The selection equation should contain at least one variable that is not in the outcome equation.

If *depvar*<sub>s</sub> is specified, it should be coded as 0 or 1, with 0 indicating an observation not selected and 1 indicating a selected observation. If *depvar*<sub>s</sub> is not specified, observations for which *depvar* is not missing are assumed selected, and those for which *depvar* is missing are assumed not selected.

`noconstant` suppresses the selection constant term (intercept).

`noconstant`; see [R] [Estimation options](#).

`rhosigma`, `rot trunc`, `rho limited`, and `rho force` are rarely used options to specify how the two-step estimator (option `twostep`) handles unusual cases in which the two-step estimate of  $\rho$  is outside the admissible range for a correlation,  $[-1, 1]$ . When  $\text{abs}(\rho) > 1$ , the two-step estimate of the coefficient variance–covariance matrix may not be positive definite and thus may be unusable for testing. The default is `rhosigma`.

`rhosigma` specifies that  $\rho$  be truncated, as with the `rot trunc` option, and that the estimate of  $\sigma$  be made consistent with  $\hat{\rho}$ , the truncated estimate of  $\rho$ . So,  $\hat{\sigma} = \beta_m \hat{\rho}$ ; see [Methods and formulas](#) for the definition of  $\beta_m$ . Both the truncated  $\rho$  and the new estimate of  $\hat{\sigma}$  are used in all computations to estimate the two-step covariance matrix.

`rot trunc` specifies that  $\rho$  be truncated to lie in the range  $[-1, 1]$ . If the two-step estimate is less than  $-1$ ,  $\rho$  is set to  $-1$ ; if the two-step estimate is greater than 1,  $\rho$  is set to 1. This truncated value of  $\rho$  is used in all computations to estimate the two-step covariance matrix.

`rho limited` specifies that  $\rho$  be truncated only in computing the diagonal matrix **D** as it enters  $\mathbf{V}_{\text{twostep}}$  and **Q**; see [Methods and formulas](#). In all other computations, the untruncated estimate of  $\rho$  is used.

`rho force` specifies that the two-step estimate of  $\rho$  be retained, even if it is outside the admissible range for a correlation. This option may, in rare cases, lead to a non–positive-definite covariance matrix.

These options have no effect when estimation is by maximum likelihood, the default. They also have no effect when the two-step estimate of  $\rho$  is in the range  $[-1, 1]$ .

### SE

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`conventional`) and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

`vce(conventional)`, the default, uses the two-step variance estimator derived by Heckman.

## Reporting

`level(#)`; see [R] [Estimation options](#).

`first` specifies that the first-step probit estimates of the selection equation be displayed before estimation.

`nshazard(newvar)` and `mills(newvar)` are synonyms; either will create a new variable containing the nonselection hazard—what Heckman (1979) referred to as the inverse of the Mills ratio—from the selection equation. The nonselection hazard is computed from the estimated parameters of the selection equation.

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no!stretch`; see [R] [Estimation options](#).

The following option is available with `heckman` but is not shown in the dialog box:

`coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

The Heckman selection model (Gronau 1974; Lewis 1974; Heckman 1976) assumes that there exists an underlying regression relationship,

$$y_j = \mathbf{x}_j\boldsymbol{\beta} + u_{1j} \qquad \text{regression equation}$$

The dependent variable, however, is not always observed. Rather, the dependent variable for observation  $j$  is observed if

$$\mathbf{z}_j\boldsymbol{\gamma} + u_{2j} > 0 \qquad \text{selection equation}$$

where

$$u_1 \sim N(0, \sigma)$$

$$u_2 \sim N(0, 1)$$

$$\text{corr}(u_1, u_2) = \rho$$

When  $\rho \neq 0$ , standard regression techniques applied to the first equation yield biased results. `heckman` provides consistent, asymptotically efficient estimates for all the parameters in such models.

In one classic example, the first equation describes the wages of women. Women choose whether to work, and thus, from our point of view as researchers, whether we observe their wages in our data. If women made this decision randomly, we could ignore that not all wages are observed and use ordinary regression to fit a wage model. Such an assumption of random participation, however, is unlikely to be true; women who would have low wages may be unlikely to choose to work, and thus the sample of observed wages is biased upward. In the jargon of economics, women choose not to work when their personal reservation wage is greater than the wage offered by employers. Thus, women who choose not to work might have even higher offer wages than those who do work—they may have high offer wages, but they have even higher reservation wages. We could tell a story that competency is related to wages, but competency is rewarded more at home than in the labor force.

In any case, in this problem—which is the paradigm for most such problems—a solution can be found if there are some variables that strongly affect the chances for observation (the reservation wage) but not the outcome under study (the offer wage). Such a variable might be the number of children in the home. (Theoretically, we do not need such identifying variables, but without them, we depend on functional form to identify the model. It would be difficult for anyone to take such results seriously because the functional form assumptions have no firm basis in theory.)

### ▷ Example 1

In the syntax for `heckman`, `depvar` and `indepvars` are the dependent variable and regressors for the underlying regression model to be fit ( $y = \mathbf{X}\beta$ ), and `varlists` are the variables ( $\mathbf{Z}$ ) thought to determine whether `depvar` is observed or unobserved (selected or not selected). In our female wage example, the number of children at home would be included in the second list. By default, `heckman` assumes that missing values (see [U] 12.2.1 Missing values) of `depvar` imply that the dependent variable is unobserved (not selected). With some datasets, it is more convenient to specify a binary variable (`depvars`) that identifies the observations for which the dependent is observed/selected (`depvars ≠ 0`) or not observed (`depvars = 0`); `heckman` will accommodate either type of data. Here we have a (fictional) dataset on 2,000 women, 1,343 of whom work:

```
. use https://www.stata-press.com/data/r19/womenwk
. summarize age educ married children wage
```

| Variable  | Obs   | Mean     | Std. dev. | Min     | Max      |
|-----------|-------|----------|-----------|---------|----------|
| age       | 2,000 | 36.208   | 8.28656   | 20      | 59       |
| education | 2,000 | 13.084   | 3.045912  | 10      | 20       |
| married   | 2,000 | .6705    | .4701492  | 0       | 1        |
| children  | 2,000 | 1.6445   | 1.398963  | 0       | 5        |
| wage      | 1,343 | 23.69217 | 6.305374  | 5.88497 | 45.80979 |

We will assume that the hourly wage is a function of education and age, whereas the likelihood of working (the likelihood of the wage being observed) is a function of marital status, the number of children at home, and (implicitly) the wage (via the inclusion of age and education, which we think determine the wage):

```

. heckman wage educ age, select(married children educ age)
Iteration 0: Log likelihood = -5178.7009
Iteration 1: Log likelihood = -5178.3049
Iteration 2: Log likelihood = -5178.3045
Heckman selection model Number of obs = 2,000
(regression model with sample selection) Selected = 1,343
 Nonselected = 657
 Wald chi2(2) = 508.44
Log likelihood = -5178.304 Prob > chi2 = 0.0000

```

| wage      | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |           |
|-----------|-------------|-----------|--------|-------|----------------------|-----------|
| wage      |             |           |        |       |                      |           |
| education | .9899537    | .0532565  | 18.59  | 0.000 | .8855729             | 1.094334  |
| age       | .2131294    | .0206031  | 10.34  | 0.000 | .1727481             | .2535108  |
| _cons     | .4857752    | 1.077037  | 0.45   | 0.652 | -1.625179            | 2.59673   |
| select    |             |           |        |       |                      |           |
| married   | .4451721    | .0673954  | 6.61   | 0.000 | .3130794             | .5772647  |
| children  | .4387068    | .0277828  | 15.79  | 0.000 | .3842534             | .4931601  |
| education | .0557318    | .0107349  | 5.19   | 0.000 | .0346917             | .0767718  |
| age       | .0365098    | .0041533  | 8.79   | 0.000 | .0283694             | .0446502  |
| _cons     | -2.491015   | .1893402  | -13.16 | 0.000 | -2.862115            | -2.119915 |
| /athrho   | .8742086    | .1014225  | 8.62   | 0.000 | .6754241             | 1.072993  |
| /lnsigma  | 1.792559    | .027598   | 64.95  | 0.000 | 1.738468             | 1.84665   |
| rho       | .7035061    | .0512264  |        |       | .5885365             | .7905862  |
| sigma     | 6.004797    | .1657202  |        |       | 5.68862              | 6.338548  |
| lambda    | 4.224412    | .3992265  |        |       | 3.441942             | 5.006881  |

```
LR test of indep. eqns. (rho = 0): chi2(1) = 61.20 Prob > chi2 = 0.0000
```

heckman assumes that wage is the dependent variable and that the first variable list (educ and age) are the determinants of wage. The variables specified in the select() option (married, children, educ, and age) are assumed to determine whether the dependent variable is observed (the selection equation). Thus, we fit the model

$$\text{wage} = \beta_0 + \beta_1 \text{educ} + \beta_2 \text{age} + u_1$$

and we assumed that wage is observed if

$$\gamma_0 + \gamma_1 \text{married} + \gamma_2 \text{children} + \gamma_3 \text{educ} + \gamma_4 \text{age} + u_2 > 0$$

where  $u_1$  and  $u_2$  have correlation  $\rho$ .

The reported results for the wage equation are interpreted exactly as though we observed wage data for all women in the sample; the coefficients on age and education level represent the estimated marginal effects of the regressors in the underlying regression equation. The results for the two ancillary parameters require some explanation. `heckman` does not directly estimate  $\rho$ ; to constrain  $\rho$  within its valid limits, and for numerical stability during optimization, it estimates the inverse hyperbolic tangent of  $\rho$ :

$$\operatorname{atanh} \rho = \frac{1}{2} \ln \left( \frac{1 + \rho}{1 - \rho} \right)$$

This estimate is reported as `/athrho`. In the bottom panel of the output, `heckman` undoes this transformation for you: the estimated value of  $\rho$  is 0.7035061. The standard error for  $\rho$  is computed using the delta method, and its confidence intervals are the transformed intervals of `/athrho`.

Similarly,  $\sigma$ , the standard error of the residual in the wage equation, is not directly estimated; for numerical stability, `heckman` instead estimates  $\ln \sigma$ . The untransformed `sigma` is reported at the end of the output: 6.004797.

Finally, some researchers—especially economists—are used to the selectivity effect summarized not by  $\rho$  but by  $\lambda = \rho\sigma$ . `heckman` reports this, too, along with an estimate of the standard error and confidence interval.

◀

## □ Technical note

If each of the equations in the model had contained many regressors, the `heckman` command could have become long. An alternate way of specifying our wage model would be to use Stata's global macros. The following lines are an equivalent way of specifying our model:

```
. global wageeq "wage educ age"
. global seleq "married children educ age"
. heckman $wageeq, select($seleq)
 (output omitted)
```

□

## □ Technical note

The reported model  $\chi^2$  test is a Wald test that all coefficients in the regression model (except the constant) are 0. `heckman` is an estimation command, so you can use `test`, `testnl`, or `lrtest` to perform tests against whatever nested alternate model you choose; see [\[R\] test](#), [\[R\] testnl](#), and [\[R\] lrtest](#).

The estimation of  $\rho$  and  $\sigma$  in the forms  $\operatorname{atanh} \rho$  and  $\ln \sigma$  extends the range of these parameters to infinity in both directions, thus avoiding boundary problems during the maximization. Tests of  $\rho$  must be made in the transformed units. However, because  $\operatorname{atanh}(0) = 0$ , the reported test for  $\operatorname{atanh} \rho = 0$  is equivalent to the test for  $\rho = 0$ .

The likelihood-ratio test reported at the bottom of the output is an equivalent test for  $\rho = 0$  and is computationally the comparison of the joint likelihood of an independent probit model for the selection equation and a regression model on the observed wage data against the Heckman model likelihood. Because  $\chi^2 = 61.20$ , this clearly justifies the Heckman selection equation with these data.

□

## ▶ Example 2

This command supports the Huber/White/sandwich estimator of variance under the `vce(robust)` and `vce(cluster clustvar)` options or when `pweights` are used for population-weighted data; see [U] 20.22 Obtaining robust variance estimates. We can obtain robust standard errors for our wage model by specifying clustering on county of residence (the `county` variable).

```
. heckman wage educ age, select(married children educ age) vce(cluster county)
Iteration 0: Log pseudolikelihood = -5178.7009
Iteration 1: Log pseudolikelihood = -5178.3049
Iteration 2: Log pseudolikelihood = -5178.3045
Heckman selection model Number of obs = 2,000
(regression model with sample selection) Selected = 1,343
 Nonselected = 657
 Wald chi2(1) = .
Log pseudolikelihood = -5178.304 Prob > chi2 = .
 (Std. err. adjusted for 10 clusters in county)
```

| wage      | Coefficient | Robust<br>std. err. | z      | P> z  | [95% conf. interval] |           |
|-----------|-------------|---------------------|--------|-------|----------------------|-----------|
| wage      |             |                     |        |       |                      |           |
| education | .9899537    | .0600061            | 16.50  | 0.000 | .8723438             | 1.107564  |
| age       | .2131294    | .020995             | 10.15  | 0.000 | .17198               | .2542789  |
| _cons     | .4857752    | 1.302103            | 0.37   | 0.709 | -2.066299            | 3.03785   |
| select    |             |                     |        |       |                      |           |
| married   | .4451721    | .0731472            | 6.09   | 0.000 | .3018062             | .5885379  |
| children  | .4387068    | .0312386            | 14.04  | 0.000 | .3774802             | .4999333  |
| education | .0557318    | .0110039            | 5.06   | 0.000 | .0341645             | .0772991  |
| age       | .0365098    | .004038             | 9.04   | 0.000 | .0285954             | .0444242  |
| _cons     | -2.491015   | .1153305            | -21.60 | 0.000 | -2.717059            | -2.264972 |
| /athrho   | .8742086    | .1403337            | 6.23   | 0.000 | .5991596             | 1.149258  |
| /lnsigma  | 1.792559    | .0258458            | 69.36  | 0.000 | 1.741902             | 1.843216  |
| rho       | .7035061    | .0708796            |        |       | .5364513             | .817508   |
| sigma     | 6.004797    | .155199             |        |       | 5.708189             | 6.316818  |
| lambda    | 4.224412    | .5186709            |        |       | 3.207835             | 5.240988  |

```
Wald test of indep. eqns. (rho = 0): chi2(1) = 38.81 Prob > chi2 = 0.0000
```

The robust standard errors tend to be a bit larger, but we notice no systematic differences. This finding is not surprising because the data were not constructed to have any county-specific correlations or any other characteristics that would deviate from the assumptions of the Heckman model.

### ► Example 3

Stata also produces Heckman's (1979) two-step efficient estimator of the model with the `twostep` option. Maximum likelihood estimation of the parameters can be time consuming with large datasets, and the two-step estimates may provide a good alternative in such cases. Continuing with the women's wage model, we can obtain the two-step estimates with Heckman's consistent covariance estimates by typing

```
. heckman wage educ age, select(married children educ age) twostep
Heckman selection model -- two-step estimates Number of obs = 2,000
(regression model with sample selection) Selected = 1,343
 Nonselected = 657
 Wald chi2(2) = 442.54
 Prob > chi2 = 0.0000
```

|               | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |           |
|---------------|-------------|-----------|--------|-------|----------------------|-----------|
| <b>wage</b>   |             |           |        |       |                      |           |
| education     | .9825259    | .0538821  | 18.23  | 0.000 | .8769189             | 1.088133  |
| age           | .2118695    | .0220511  | 9.61   | 0.000 | .1686502             | .2550888  |
| _cons         | .7340391    | 1.248331  | 0.59   | 0.557 | -1.712645            | 3.180723  |
| <b>select</b> |             |           |        |       |                      |           |
| married       | .4308575    | .074208   | 5.81   | 0.000 | .2854125             | .5763025  |
| children      | .4473249    | .0287417  | 15.56  | 0.000 | .3909922             | .5036576  |
| education     | .0583645    | .0109742  | 5.32   | 0.000 | .0368555             | .0798735  |
| age           | .0347211    | .0042293  | 8.21   | 0.000 | .0264318             | .0430105  |
| _cons         | -2.467365   | .1925635  | -12.81 | 0.000 | -2.844782            | -2.089948 |
| <b>/mills</b> |             |           |        |       |                      |           |
| lambda        | 4.001615    | .6065388  | 6.60   | 0.000 | 2.812821             | 5.19041   |
| rho           | 0.67284     |           |        |       |                      |           |
| sigma         | 5.9473529   |           |        |       |                      |           |



### □ Technical note

The Heckman selection model depends strongly on the model being correct, much more so than ordinary regression. Running a separate probit or logit for sample inclusion followed by a regression, referred to in the literature as the two-part model (Manning, Duan, and Rogers 1987)—not to be confused with Heckman's two-step procedure—is an especially attractive alternative if the regression part of the model arose because of taking a logarithm of zero values. When the goal is to analyze an underlying regression model or to predict the value of the dependent variable that would be observed in the absence of selection, however, the Heckman model is more appropriate. When the goal is to predict an actual response, the two-part model is usually the better choice.

The Heckman selection model can be unstable when the model is not properly specified or if a specific dataset simply does not support the model's assumptions. For example, let's examine the solution to another simulated problem.

```

. use https://www.stata-press.com/data/r19/twopart
. heckman yt x1 x2 x3, select(z1 z2) nonrtol
Iteration 0: Log likelihood = -111.94996
Iteration 1: Log likelihood = -110.82258
Iteration 2: Log likelihood = -110.17707
Iteration 3: Log likelihood = -107.70663 (not concave)
Iteration 4: Log likelihood = -107.07729 (not concave)
(output omitted)
Iteration 36: Log likelihood = -104.0825

Heckman selection model Number of obs = 150
(regression model with sample selection) Selected = 63
 Nonselected = 87

Log likelihood = -104.0825 Wald chi2(3) = 8.84e+08
 Prob > chi2 = 0.0000

```

|        | yt       | Coefficient | Std. err. | z        | P> z  | [95% conf. interval] |           |
|--------|----------|-------------|-----------|----------|-------|----------------------|-----------|
| yt     |          |             |           |          |       |                      |           |
|        | x1       | .8974192    | .0002164  | 4146.52  | 0.000 | .896995              | .8978434  |
|        | x2       | -2.525303   | .0001244  | -2.0e+04 | 0.000 | -2.525546            | -2.525059 |
|        | x3       | 2.855786    | .0002695  | 1.1e+04  | 0.000 | 2.855258             | 2.856314  |
|        | _cons    | .6975003    | .0907873  | 7.68     | 0.000 | .5195604             | .8754402  |
| select |          |             |           |          |       |                      |           |
|        | z1       | -.6826482   | .0889871  | -7.67    | 0.000 | -.8570598            | -.5082367 |
|        | z2       | 1.003678    | .1308344  | 7.67     | 0.000 | .7472471             | 1.260108  |
|        | _cons    | -.3605665   | .1219011  | -2.96    | 0.003 | -.5994883            | -.1216447 |
|        | /athrho  | 16.11489    | 260.7581  | 0.06     | 0.951 | -494.9617            | 527.1914  |
|        | /lnsigma | -.5396877   | .1303548  | -4.14    | 0.000 | -.7951785            | -.284197  |
|        | rho      | 1           | 1.05e-11  |          |       | -1                   | 1         |
|        | sigma    | .5829302    | .0759878  |          |       | .4515006             | .7526184  |
|        | lambda   | .5829302    | .0759878  |          |       | .433997              | .7318635  |

```
LR test of indep. eqns. (rho = 0): chi2(1) = 25.67 Prob > chi2 = 0.0000
```

The model has converged to a value of  $\rho$  that is 1.0—within machine-rounding tolerances. Given the form of the likelihood for the Heckman selection model, this implies a division by zero, and it is surprising that the model solution turns out as well as it does. Reparameterizing  $\rho$  has allowed the estimation to converge, but we clearly have problems with the estimates. Moreover, if this had occurred in a large dataset, waiting for convergence might take considerable time.

This dataset was not intentionally developed to cause problems. It is actually generated by a “Heckman process” and when generated starting from different random values can be easily estimated. The luck of the draw here merely led to data that, despite the source, did not support the assumptions of the Heckman model.

The two-step model is generally more stable when the data are problematic. It even tolerates estimates of  $\rho$  less than  $-1$  and greater than  $1$ . For these reasons, the two-step model may be preferred when exploring a large dataset. Still, if the maximum likelihood estimates cannot converge, or converge to a value of  $\rho$  that is at the boundary of acceptable values, there is scant support for fitting a Heckman selection model on the data. Heckman (1979) discusses the implications of  $\rho$  being exactly 1 or 0, together with the implications of other possible covariance relationships among the model’s determinants.

□

## Stored results

heckman (maximum likelihood) stores the following in `e()`:

### Scalars

|                               |                                                   |
|-------------------------------|---------------------------------------------------|
| <code>e(N)</code>             | number of observations                            |
| <code>e(N_selected)</code>    | number of selected observations                   |
| <code>e(N_nonselected)</code> | number of nonselected observations                |
| <code>e(k)</code>             | number of parameters                              |
| <code>e(k_eq)</code>          | number of equations in <code>e(b)</code>          |
| <code>e(k_eq_model)</code>    | number of equations in overall model test         |
| <code>e(k_aux)</code>         | number of auxiliary parameters                    |
| <code>e(k_dv)</code>          | number of dependent variables                     |
| <code>e(df_m)</code>          | model degrees of freedom                          |
| <code>e(ll)</code>            | log likelihood                                    |
| <code>e(ll_0)</code>          | log likelihood, constant-only model               |
| <code>e(N_clust)</code>       | number of clusters                                |
| <code>e(lambda)</code>        | $\lambda$                                         |
| <code>e(selambda)</code>      | standard error of $\lambda$                       |
| <code>e(sigma)</code>         | sigma                                             |
| <code>e(chi2)</code>          | $\chi^2$                                          |
| <code>e(chi2_c)</code>        | $\chi^2$ for comparison test                      |
| <code>e(p)</code>             | $p$ -value for model test                         |
| <code>e(p_c)</code>           | $p$ -value for comparison test                    |
| <code>e(rho)</code>           | $\rho$                                            |
| <code>e(rank)</code>          | rank of <code>e(V)</code>                         |
| <code>e(rank0)</code>         | rank of <code>e(V)</code> for constant-only model |
| <code>e(ic)</code>            | number of iterations                              |
| <code>e(rc)</code>            | return code                                       |
| <code>e(converged)</code>     | 1 if converged, 0 otherwise                       |

### Macros

|                              |                                                                                 |
|------------------------------|---------------------------------------------------------------------------------|
| <code>e(cmd)</code>          | heckman                                                                         |
| <code>e(cmdline)</code>      | command as typed                                                                |
| <code>e(depvar)</code>       | names of dependent variables                                                    |
| <code>e(wtype)</code>        | weight type                                                                     |
| <code>e(wexp)</code>         | weight expression                                                               |
| <code>e(title)</code>        | title in estimation output                                                      |
| <code>e(title2)</code>       | secondary title in estimation output                                            |
| <code>e(clustvar)</code>     | name of cluster variable                                                        |
| <code>e(offset1)</code>      | offset for regression equation                                                  |
| <code>e(offset2)</code>      | offset for selection equation                                                   |
| <code>e(mills)</code>        | variable containing nonselection hazard (inverse of Mills's ratio)              |
| <code>e(chi2type)</code>     | Wald or LR; type of model $\chi^2$ test                                         |
| <code>e(chi2_ct)</code>      | Wald or LR; type of model $\chi^2$ test corresponding to <code>e(chi2_c)</code> |
| <code>e(vce)</code>          | <i>vctype</i> specified in <code>vce()</code>                                   |
| <code>e(vctype)</code>       | title used to label Std. err.                                                   |
| <code>e(opt)</code>          | type of optimization                                                            |
| <code>e(which)</code>        | max or min; whether optimizer is to perform maximization or minimization        |
| <code>e(method)</code>       | m1                                                                              |
| <code>e(m1_method)</code>    | type of m1 method                                                               |
| <code>e(user)</code>         | name of likelihood-evaluator program                                            |
| <code>e(technique)</code>    | maximization technique                                                          |
| <code>e(properties)</code>   | b V                                                                             |
| <code>e(predict)</code>      | program used to implement predict                                               |
| <code>e(marginsok)</code>    | predictions allowed by margins                                                  |
| <code>e(marginsnotok)</code> | predictions disallowed by margins                                               |
| <code>e(asbalanced)</code>   | factor variables <code>fvset</code> as asbalanced                               |
| <code>e(asobserved)</code>   | factor variables <code>fvset</code> as asobserved                               |

|                 |                                              |
|-----------------|----------------------------------------------|
| Matrices        |                                              |
| e(b)            | coefficient vector                           |
| e(Cns)          | constraints matrix                           |
| e(i log)        | iteration log (up to 20 iterations)          |
| e(gradient)     | gradient vector                              |
| e(V)            | variance–covariance matrix of the estimators |
| e(V_modelbased) | model-based variance                         |

|           |                         |
|-----------|-------------------------|
| Functions |                         |
| e(sample) | marks estimation sample |

In addition to the above, the following is stored in `r()`:

|          |                                                                                                                       |
|----------|-----------------------------------------------------------------------------------------------------------------------|
| Matrices |                                                                                                                       |
| r(table) | matrix containing the coefficients with their standard errors, test statistics, $p$ -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r`-class command is run after the estimation command.

`heckman` (two-step) stores the following in `e()`:

|                  |                                               |
|------------------|-----------------------------------------------|
| Scalars          |                                               |
| e(N)             | number of observations                        |
| e(N_selected)    | number of selected observations               |
| e(N_nonselected) | number of nonselected observations            |
| e(df_m)          | model degrees of freedom                      |
| e(lambda)        | $\lambda$                                     |
| e(selambda)      | standard error of $\lambda$                   |
| e(sigma)         | $\sigma$                                      |
| e(chi2)          | $\chi^2$                                      |
| e(p)             | $p$ -value for comparison test                |
| e(rho)           | $\rho$                                        |
| e(rank)          | rank of $e(V)$                                |
| e(fconverged)    | 1 if first-stage model converged, 0 otherwise |

|                 |                                                                                                     |
|-----------------|-----------------------------------------------------------------------------------------------------|
| Macros          |                                                                                                     |
| e(cmd)          | <code>heckman</code>                                                                                |
| e(cmdline)      | command as typed                                                                                    |
| e(depvar)       | names of dependent variables                                                                        |
| e(title)        | title in estimation output                                                                          |
| e(title2)       | secondary title in estimation output                                                                |
| e(mills)        | variable containing nonselection hazard (inverse of Mills's ratio)                                  |
| e(chi2type)     | Wald or LR; type of model $\chi^2$ test                                                             |
| e(vce)          | <i>vce</i> type specified in <code>vce()</code>                                                     |
| e(rhometh)      | <code>rhosigma</code> , <code>rho</code> trunc, <code>rho</code> limited, or <code>rho</code> force |
| e(method)       | <code>twostep</code>                                                                                |
| e(properties)   | <code>b V</code>                                                                                    |
| e(predict)      | program used to implement <code>predict</code>                                                      |
| e(marginsok)    | predictions allowed by <code>margins</code>                                                         |
| e(marginsnotok) | predictions disallowed by <code>margins</code>                                                      |
| e(asbalanced)   | factor variables <code>fvset</code> as <code>asbalanced</code>                                      |
| e(asobserved)   | factor variables <code>fvset</code> as <code>asobserved</code>                                      |

|          |                                              |
|----------|----------------------------------------------|
| Matrices |                                              |
| e(b)     | coefficient vector                           |
| e(V)     | variance–covariance matrix of the estimators |

|           |                         |
|-----------|-------------------------|
| Functions |                         |
| e(sample) | marks estimation sample |

In addition to the above, the following is stored in `r()`:

Matrices

`r(table)`

matrix containing the coefficients with their standard errors, test statistics,  $p$ -values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

James Joseph Heckman (1944–) was born in Chicago in 1944 and studied mathematics at Colorado College and economics at Princeton. He has taught economics at Columbia and (since 1973) at the University of Chicago. He has worked on developing a scientific basis for economic policy evaluation, with emphasis on models of individuals or disaggregated groups and the problems and possibilities created by heterogeneity, diversity, and unobserved counterfactual states. In 2000, he shared the Nobel Prize in Economics with Daniel L. McFadden.

## Methods and formulas

Cameron and Trivedi (2022, 974–981) and Greene (2018, 950–957) provide good introductions to the Heckman selection model. Adkins and Hill (2011, sec. 16.8) describe the two-step estimator with an application using Stata. Jones (2007, 35–40) illustrates Heckman estimation with an application to health economics.

Regression estimates using the nonselection hazard (Heckman 1979) provide starting values for maximum likelihood estimation.

The regression equation is

$$y_j = \mathbf{x}_j\boldsymbol{\beta} + u_{1j}$$

The selection equation is

$$\mathbf{z}_j\boldsymbol{\gamma} + u_{2j} > 0$$

where

$$u_1 \sim N(0, \sigma)$$

$$u_2 \sim N(0, 1)$$

$$\text{corr}(u_1, u_2) = \rho$$

The log likelihood for observation  $j$ ,  $\ln L_j = l_j$ , is

$$l_j = \begin{cases} w_j \ln \Phi \left\{ \frac{\mathbf{z}_j\boldsymbol{\gamma} + (y_j - \mathbf{x}_j\boldsymbol{\beta})\rho/\sigma}{\sqrt{1 - \rho^2}} \right\} - \frac{w_j}{2} \left( \frac{y_j - \mathbf{x}_j\boldsymbol{\beta}}{\sigma} \right)^2 - w_j \ln(\sqrt{2\pi}\sigma) & y_j \text{ observed} \\ w_j \ln \Phi(-\mathbf{z}_j\boldsymbol{\gamma}) & y_j \text{ not observed} \end{cases}$$

where  $\Phi(\cdot)$  is the standard cumulative normal and  $w_j$  is an optional weight for observation  $j$ .

In the maximum likelihood estimation,  $\sigma$  and  $\rho$  are not directly estimated. Directly estimated are  $\ln \sigma$  and  $\text{atanh } \rho$ :

$$\text{atanh } \rho = \frac{1}{2} \ln \left( \frac{1 + \rho}{1 - \rho} \right)$$

The standard error of  $\lambda = \rho\sigma$  is approximated through the propagation of error (delta) method; that is,

$$\text{Var}(\lambda) \approx \mathbf{D} \text{Var}\{\text{atanh } \rho \text{ ln } \sigma\} \mathbf{D}'$$

where  $\mathbf{D}$  is the Jacobian of  $\lambda$  with respect to  $\text{atanh } \rho$  and  $\ln \sigma$ .

With maximum likelihood estimation, this command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] [\\_robust](#), particularly [Maximum likelihood estimators](#) and [Methods and formulas](#).

The maximum likelihood version of `heckman` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] [Variance estimation](#).

The two-step estimates are computed using Heckman's (1979) procedure.

Probit estimates of the selection equation

$$\Pr(y_j \text{ observed} \mid \mathbf{z}_j) = \Phi(\mathbf{z}_j \boldsymbol{\gamma})$$

are obtained. From these estimates, the nonselection hazard—what Heckman (1979) referred to as the inverse of the Mills ratio,  $m_j$ —for each observation  $j$  is computed as

$$m_j = \frac{\phi(\mathbf{z}_j \hat{\boldsymbol{\gamma}})}{\Phi(\mathbf{z}_j \hat{\boldsymbol{\gamma}})}$$

where  $\phi$  is the normal density. We also define

$$\delta_j = m_j(m_j + \hat{\boldsymbol{\gamma}} \mathbf{z}_j)$$

Following Heckman, the two-step parameter estimates of  $\boldsymbol{\beta}$  are obtained by augmenting the regression equation with the nonselection hazard  $\mathbf{m}$ . Thus, the regressors become  $[\mathbf{X} \ \mathbf{m}]$ , and we obtain the additional parameter estimate  $\boldsymbol{\beta}_m$  on the variable containing the nonselection hazard.

A consistent estimate of the regression disturbance variance is obtained using the residuals from the augmented regression and the parameter estimate on the nonselection hazard,

$$\hat{\sigma}^2 = \frac{\mathbf{e}'\mathbf{e} + \boldsymbol{\beta}_m^2 \sum_{j=1}^N \delta_j}{N}$$

The two-step estimate of  $\rho$  is then

$$\hat{\rho} = \frac{\boldsymbol{\beta}_m}{\hat{\sigma}}$$

Heckman derived consistent estimates of the coefficient covariance matrix on the basis of the augmented regression.

Let  $\mathbf{W} = [\mathbf{X} \mathbf{m}]$  and  $\mathbf{R}$  be a square, diagonal matrix of dimension  $N$ , with  $(1 - \hat{\rho}^2 \delta_j)$  as the diagonal elements. The conventional VCE is

$$\mathbf{V}_{\text{twostep}} = \hat{\sigma}^2 (\mathbf{W}'\mathbf{W})^{-1} (\mathbf{W}'\mathbf{R}\mathbf{W} + \mathbf{Q}) (\mathbf{W}'\mathbf{W})^{-1}$$

where

$$\mathbf{Q} = \hat{\rho}^2 (\mathbf{W}'\mathbf{D}\mathbf{Z}) \mathbf{V}_p (\mathbf{Z}'\mathbf{D}\mathbf{W})$$

where  $\mathbf{D}$  is the square, diagonal matrix of dimension  $N$  with  $\delta_j$  as the diagonal elements;  $\mathbf{Z}$  is the data matrix of selection equation covariates; and  $\mathbf{V}_p$  is the variance–covariance estimate from the probit estimation of the selection equation.

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## Also see

- [R] **heckman postestimation** — Postestimation tools for heckman
- [R] **heckoprobit** — Ordered probit model with sample selection
- [R] **heckpoisson** — Poisson regression with sample selection
- [R] **heckprobit** — Probit model with sample selection
- [R] **regress** — Linear regression
- [R] **tobit** — Tobit regression
- [BAYES] **bayes: heckman** — Bayesian Heckman selection model
- [CAUSAL] **etregress** — Linear regression with endogenous treatment effects
- [ERM] **eregress** — Extended linear regression
- [SVY] **svy estimation** — Estimation commands for survey data
- [XT] **xheckman** — Random-effects regression with sample selection
- [U] **20 Estimation and postestimation commands**

## Postestimation commands

The following postestimation commands are available after `heckman`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| * <code>estat ic</code>      | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| † <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| * † <code>lrtest</code>      | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from <code>margins</code> (profile plots, interaction plots, etc.)                                                    |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | linear predictions and their SEs, probabilities, etc.                                                                                   |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| * <code>suest</code>         | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\* `estat ic`, `lrtest`, and `suest` are not appropriate after `heckman`, `twostep`.

† `hausman` and `lrtest` are not appropriate with `svy` estimation results.

## predict

### Description for predict

`predict` creates a new variable containing predictions such as linear predictions, standard errors, probabilities, expected values, and nonselection hazards.

### Menu for predict

Statistics > Postestimation

### Syntax for predict

After ML or twostep

```
predict [type] newvar [if] [in] [, statistic nooffset]
```

After ML

```
predict [type] stub* [if] [in] , scores
```

| <i>statistic</i>                            | Description                                                    |
|---------------------------------------------|----------------------------------------------------------------|
| Main                                        |                                                                |
| <code>xb</code>                             | linear prediction; the default                                 |
| <code>stdp</code>                           | standard error of the prediction                               |
| <code>stdf</code>                           | standard error of the forecast                                 |
| <code>xbse1</code>                          | linear prediction for selection equation                       |
| <code>stdpse1</code>                        | standard error of the linear prediction for selection equation |
| <code>pr(a,b)</code>                        | $\Pr(y_j   a < y_j < b)$                                       |
| <code>e(a,b)</code>                         | $E(y_j   a < y_j < b)$                                         |
| <code>ystar(a,b)</code>                     | $E(y_j^*), y_j^* = \max\{a, \min(y_j, b)\}$                    |
| <code>ycond</code>                          | $E(y_j   y_j \text{ observed})$                                |
| <code>yexpected</code>                      | $E(y_j^*), y_j$ taken to be 0 where unobserved                 |
| <code>nshazard</code> or <code>mills</code> | nonselection hazard (also called the inverse of Mills's ratio) |
| <code>pse1</code>                           | $\Pr(y_j \text{ observed})$                                    |

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

`stdf` is not allowed with `svy` estimation results.

where  $a$  and  $b$  may be numbers or variables;  $a$  missing ( $a \geq .$ ) means  $-\infty$ , and  $b$  missing ( $b \geq .$ ) means  $+\infty$ ; see [U] 12.2.1 Missing values.

## Options for predict

Main

`xb`, the default, calculates the linear prediction  $\mathbf{x}_j\mathbf{b}$ .

`stdp` calculates the standard error of the prediction, which can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. The standard error of the prediction is also referred to as the standard error of the fitted value.

`stdf` calculates the standard error of the forecast, which is the standard error of the point prediction for 1 observation. It is commonly referred to as the standard error of the future or forecast value. By construction, the standard errors produced by `stdf` are always larger than those produced by `stdp`; see [Methods and formulas](#) in [R] [regress postestimation](#).

`xbse1` calculates the linear prediction for the selection equation.

`stdpse1` calculates the standard error of the linear prediction for the selection equation.

`pr(a,b)` calculates  $\Pr(a < \mathbf{x}_j\mathbf{b} + u_1 < b)$ , the probability that  $y_j|\mathbf{x}_j$  would be observed in the interval  $(a,b)$ .

$a$  and  $b$  may be specified as numbers or variable names;  $lb$  and  $ub$  are variable names;

`pr(20,30)` calculates  $\Pr(20 < \mathbf{x}_j\mathbf{b} + u_1 < 30)$ ; `pr(lb,ub)` calculates  $\Pr(lb < \mathbf{x}_j\mathbf{b} + u_1 < ub)$ ; and `pr(20,ub)` calculates  $\Pr(20 < \mathbf{x}_j\mathbf{b} + u_1 < ub)$ .

$a$  missing ( $a \geq .$ ) means  $-\infty$ ; `pr(.,30)` calculates  $\Pr(-\infty < \mathbf{x}_j\mathbf{b} + u_1 < 30)$ ;

`pr(lb,30)` calculates  $\Pr(-\infty < \mathbf{x}_j\mathbf{b} + u_j < 30)$  in observations for which  $lb \geq .$  and calculates  $\Pr(lb < \mathbf{x}_j\mathbf{b} + u_j < 30)$  elsewhere.

$b$  missing ( $b \geq .$ ) means  $+\infty$ ; `pr(20,.)` calculates  $\Pr(+\infty > \mathbf{x}_j\mathbf{b} + u_j > 20)$ ;

`pr(20,ub)` calculates  $\Pr(+\infty > \mathbf{x}_j\mathbf{b} + u_j > 20)$  in observations for which  $ub \geq .$  and calculates  $\Pr(20 < \mathbf{x}_j\mathbf{b} + u_j < ub)$  elsewhere.

`e(a,b)` calculates  $E(\mathbf{x}_j\mathbf{b} + u_1 | a < \mathbf{x}_j\mathbf{b} + u_1 < b)$ , the expected value of  $y_j|\mathbf{x}_j$  conditional on  $y_j|\mathbf{x}_j$  being in the interval  $(a,b)$ , meaning that  $y_j|\mathbf{x}_j$  is truncated.

$a$  and  $b$  are specified as they are for `pr()`.

`ystar(a,b)` calculates  $E(y_j^*)$ , where  $y_j^* = a$  if  $\mathbf{x}_j\mathbf{b} + u_j \leq a$ ,  $y_j^* = b$  if  $\mathbf{x}_j\mathbf{b} + u_j \geq b$ , and  $y_j^* = \mathbf{x}_j\mathbf{b} + u_j$  otherwise, meaning that  $y_j^*$  is censored.  $a$  and  $b$  are specified as they are for `pr()`.

`ycond` calculates the expected value of the dependent variable conditional on the dependent variable being observed, that is, selected;  $E(y_j | y_j \text{ observed})$ .

`yexpected` calculates the expected value of the dependent variable ( $y_j^*$ ), where that value is taken to be 0 when it is expected to be unobserved;  $y_j^* = \Pr(y_j \text{ observed})E(y_j | y_j \text{ observed})$ .

The assumption of 0 is valid for many cases where nonselection implies nonparticipation (for example, unobserved wage levels, insurance claims from those who are uninsured) but may be inappropriate for some problems (for example, unobserved disease incidence).

`nshazard` and `mills` are synonyms; both calculate the nonselection hazard—what Heckman (1979) referred to as the inverse of the Mills ratio—from the selection equation.

`pse1` calculates the probability of selection (or being observed):

$$\Pr(y_j \text{ observed}) = \Pr(\mathbf{z}_j\boldsymbol{\gamma} + u_{2j} > 0).$$

scores, not available with `twostep`, calculates equation-level score variables.

The first new variable will contain  $\partial \ln L / \partial (\mathbf{x}_j \boldsymbol{\beta})$ .

The second new variable will contain  $\partial \ln L / \partial (\mathbf{z}_j \boldsymbol{\gamma})$ .

The third new variable will contain  $\partial \ln L / \partial (\text{atanh } \rho)$ .

The fourth new variable will contain  $\partial \ln L / \partial (\ln \sigma)$ .

`nooffset` is relevant when you specify `offset(varname)` for `heckman`. It modifies the calculations made by `predict` so that they ignore the offset variable; the linear prediction is treated as  $\mathbf{x}_j \mathbf{b}$  rather than as  $\mathbf{x}_j \mathbf{b} + \text{offset}_j$ .

## margins

### Description for margins

`margins` estimates margins of response for linear predictions, probabilities, expected values, and nonselection hazards.

### Menu for margins

Statistics > Postestimation

### Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i>                                 | Description                                                    |
|--------------------------------------------------|----------------------------------------------------------------|
| <code>xb</code>                                  | linear prediction; the default                                 |
| <code>xbtsel</code>                              | linear prediction for selection equation                       |
| <code>pr(<i>a</i>,<i>b</i>)</code>               | $\Pr(y_j \mid a < y_j < b)$                                    |
| <code>e(<i>a</i>,<i>b</i>)</code>                | $E(y_j \mid a < y_j < b)$                                      |
| <code>y<sup>star</sup>(<i>a</i>,<i>b</i>)</code> | $E(y_j^*), y_j^* = \max\{a, \min(y_j, b)\}$                    |
| * <code>ycond</code>                             | $E(y_j \mid y_j \text{ observed})$                             |
| * <code>yexpected</code>                         | $E(y_j^*), y_j$ taken to be 0 where unobserved                 |
| <code>nshazard</code> or <code>mills</code>      | nonselection hazard (also called the inverse of Mills's ratio) |
| <code>p<sup>sel</sup></code>                     | $\Pr(y_j \text{ observed})$                                    |
| <code>stdp</code>                                | not allowed with margins                                       |
| <code>stdf</code>                                | not allowed with margins                                       |
| <code>stdp<sup>sel</sup></code>                  | not allowed with margins                                       |

\*`ycond` and `yexpected` are not allowed with margins after `heckman`, `twostep`.

Statistics not allowed with margins are functions of stochastic quantities other than `e(b)`.

For the full syntax, see [R] [margins](#).

## Remarks and examples

### ▷ Example 1

The default statistic produced by `predict` after `heckman` is the expected value of the dependent variable from the underlying distribution of the regression model. In the `wage model` of [R] `heckman`, this is the expected wage rate among all women, regardless of whether they were observed to participate in the labor force:

```
. use https://www.stata-press.com/data/r19/womenwk
. heckman wage educ age, select(married children educ age) vce(cluster county)
 (output omitted)
. predict heckwage
 (option xb assumed; fitted values)
```

It is instructive to compare these predicted wage values from the Heckman model with an ordinary regression model—a model without the selection adjustment:

```
. regress wage educ age
```

| Source   | SS         | df    | MS         | Number of obs | = | 1,343  |
|----------|------------|-------|------------|---------------|---|--------|
| Model    | 13524.0337 | 2     | 6762.01687 | F(2, 1340)    | = | 227.49 |
| Residual | 39830.8609 | 1,340 | 29.7245231 | Prob > F      | = | 0.0000 |
|          |            |       |            | R-squared     | = | 0.2535 |
|          |            |       |            | Adj R-squared | = | 0.2524 |
| Total    | 53354.8946 | 1,342 | 39.7577456 | Root MSE      | = | 5.452  |

| wage      | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|-----------|-------------|-----------|-------|-------|----------------------|----------|
| education | .8965829    | .0498061  | 18.00 | 0.000 | .7988765             | .9942893 |
| age       | .1465739    | .0187135  | 7.83  | 0.000 | .109863              | .1832848 |
| _cons     | 6.084875    | .8896182  | 6.84  | 0.000 | 4.339679             | 7.830071 |

```
. predict regwage
 (option xb assumed; fitted values)
. summarize heckwage regwage
```

| Variable | Obs   | Mean     | Std. dev. | Min      | Max      |
|----------|-------|----------|-----------|----------|----------|
| heckwage | 2,000 | 21.15532 | 3.83965   | 14.6479  | 32.85949 |
| regwage  | 2,000 | 23.12291 | 3.241911  | 17.98218 | 32.66439 |

Because this dataset was concocted, we know the true coefficients of the wage regression equation to be 1, 0.2, and 1, respectively. We can compute the true mean wage for our sample.

```
. generate truewage = 1 + .2*age + 1*educ
. summarize truewage
```

| Variable | Obs   | Mean    | Std. dev. | Min | Max  |
|----------|-------|---------|-----------|-----|------|
| truewage | 2,000 | 21.3256 | 3.797904  | 15  | 32.8 |

Whereas the mean of the predictions from `heckman` is within 18 cents of the true mean wage, ordinary regression yields predictions that are on average about \$1.80 per hour too high because of the selection effect. The regression predictions also show somewhat less variation than the true wages.

The coefficients from heckman are so close to the true values that they are not worth testing. Conversely, the regression equation is significantly off but seems to give the right sense. Would we be led far astray if we relied on the OLS coefficients? The effect of age is off by more than 5 cents per year of age, and the coefficient on education level is off by about 10%. We can test the OLS coefficient on education level against the true value by using `test`.

```
. test educ = 1
(1) education = 1
 F(1, 1340) = 4.31
 Prob > F = 0.0380
```

The OLS coefficient on education is substantially lower than the true parameter; moreover, the difference from the true parameter is also statistically significant beyond the 5% level. We can perform a similar test for the OLS age coefficient:

```
. test age = .2
(1) age = .2
 F(1, 1340) = 8.15
 Prob > F = 0.0044
```

We find even stronger evidence that the OLS regression results are biased away from the true parameters.



## ▷ Example 2

Several other interesting aspects of the Heckman model can be explored with `predict`. Continuing with our wage model, we can obtain the expected wages for women conditional on participating in the labor force with the `ycond` option. Let's get these predictions and compare them with actual wages for women participating in the labor force.

```
. use https://www.stata-press.com/data/r19/womenwk, clear
. heckman wage educ age, select(married children educ age)
 (output omitted)
. predict hcndwage, ycond
. summarize wage hcndwage if wage != .
```

| Variable | Obs   | Mean     | Std. dev. | Min      | Max      |
|----------|-------|----------|-----------|----------|----------|
| wage     | 1,343 | 23.69217 | 6.305374  | 5.88497  | 45.80979 |
| hcndwage | 1,343 | 23.68239 | 3.335087  | 16.18337 | 33.7567  |

We see that the average predictions from heckman are close to the observed levels but do not have the same mean. These conditional wage predictions are available for all observations in the dataset but can be directly compared only with observed wages, where individuals are participating in the labor force.

What if we were interested in making predictions about mean wages for all women? Here the expected wage is 0 for those who are not expected to participate in the labor force, with expected participation determined by the selection equation. These values can be obtained with the `yexpected` option of `predict`. For comparison, a variable can be generated where the wage is set to 0 for nonparticipants.

```
. predict hexpwage, yexpected
. generate wage0 = wage
(657 missing values generated)
. replace wage0 = 0 if wage == .
(657 real changes made)
. summarize hexpwage wage0
```

| Variable | Obs   | Mean     | Std. dev. | Min      | Max      |
|----------|-------|----------|-----------|----------|----------|
| hexpwage | 2,000 | 15.92511 | 5.979336  | 2.492469 | 32.45858 |
| wage0    | 2,000 | 15.90929 | 12.27081  | 0        | 45.80979 |

Again we note that the predictions from heckman are close to the observed mean hourly wage rate for all women. Why aren't the predictions using `ycond` and `yexpected` equal to their observed sample equivalents? For the Heckman model, unlike linear regression, the sample moments implied by the optimal solution to the model likelihood do not require that these predictions match observed data. Properly accounting for the additional variation from the selection equation requires that the model use more information than just the sample moments of the observed wages.

◀

## Reference

Heckman, J. J. 1979. Sample selection bias as a specification error. *Econometrica* 47: 153–161. <https://doi.org/10.2307/1912352>.

## Also see

[R] [heckman](#) — Heckman selection model

[U] [20 Estimation and postestimation commands](#)

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

## Description

`heckoprobit` fits maximum-likelihood ordered probit models with sample selection.

## Quick start

Ordered probit model of  $y$  on  $x$  with selection indicated by binary variable `selected` and predicted by  $v$

```
heckoprobit y x, select(selected = v x)
```

Add [indicator variables](#) for each level of categorical variable  $a$

```
heckoprobit y x i.a, select(selected = v x i.a)
```

Account for complex sampling design using [svyset](#) data

```
svy: heckoprobit y x, select(selected = v x)
```

## Menu

Statistics > Sample-selection models > Ordered probit model with selection

## Syntax

```
heckoprobit deprvar indepvars [if] [in] [weight] ,
 select([deprvars =] varlists [, noconstant offset(varnameo)]) [options]
```

| <i>options</i>                            | Description                                                                                                                                      |
|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                              |                                                                                                                                                  |
| * <u>select</u> ( )                       | specify selection equation: dependent and independent variables; whether to have constant term and offset variable                               |
| <u>offset</u> ( <i>varname</i> )          | include <i>varname</i> in model with coefficient constrained to 1                                                                                |
| <u>constraints</u> ( <i>constraints</i> ) | apply specified linear constraints                                                                                                               |
| <b>SE/Robust</b>                          |                                                                                                                                                  |
| <u>vce</u> ( <i>vcetype</i> )             | <i>vcetype</i> may be <u>oim</u> , <u>robust</u> , <u>cluster</u> <i>clustvar</i> , <u>opg</u> , <u>bootstrap</u> , or <u>jackknife</u>          |
| <b>Reporting</b>                          |                                                                                                                                                  |
| <u>level</u> (#)                          | set confidence level; default is <u>level</u> (95)                                                                                               |
| <u>first</u>                              | report first-step probit estimates                                                                                                               |
| <u>noheader</u>                           | do not display header above parameter table                                                                                                      |
| <u>nofootnote</u>                         | do not display footnotes below parameter table                                                                                                   |
| <u>nocnsreport</u>                        | do not display constraints                                                                                                                       |
| <u>display_options</u>                    | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| <b>Maximization</b>                       |                                                                                                                                                  |
| <u>maximize_options</u>                   | control the maximization process; seldom used                                                                                                    |
| <u>collinear</u>                          | keep collinear variables                                                                                                                         |
| <u>coeflegend</u>                         | display legend instead of statistics                                                                                                             |

\*select( ) is required.

The full specification is select( [*deprvar*<sub>s</sub> = ] *varlist*<sub>s</sub> [ , noconstant offset(*varname*<sub>o</sub>) ] ).

*indepvars* and *varlist*<sub>s</sub> may contain factor variables; see [U] 11.4.3 **Factor variables**.

*deprvar*, *indepvars*, *deprvar*<sub>s</sub>, and *varlist*<sub>s</sub> may contain time-series operators; see [U] 11.4.4 **Time-series varlists**.

bayes, bayesboot, bootstrap, by, collect, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 **Prefix commands**. For more details, see [BAYES] **bayes: heckoprobit**.

Weights are not allowed with the bootstrap prefix; see [R] **bootstrap**.

vce( ), first, and weights are not allowed with the svy prefix; see [SVY] **svy**.

pweights, fweights, and iweights are allowed; see [U] 11.1.6 **weight**.

collinear and coeflegend do not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options

### Model

`select( [ depvars = ] varlists [ , noconstant offset(varnameo) ] )` specifies the variables and options for the selection equation. It is an integral part of specifying a selection model and is required. The selection equation should contain at least one variable that is not in the outcome equation.

If *depvar*<sub>s</sub> is specified, it should be coded as 0 or 1, 0 indicating an observation not selected and 1 indicating a selected observation. If *depvar*<sub>s</sub> is not specified, observations for which *depvar* is not missing are assumed selected, and those for which *depvar* is missing are assumed not selected.

`noconstant` suppresses the selection constant term (intercept).

`offset(varnameo)` specifies that selection offset *varname*<sub>o</sub> be included in the model with the coefficient constrained to be 1.

`offset(varname)`, `constraints(constraints)`; see [R] [Estimation options](#).

### SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

### Reporting

`level(#)`; see [R] [Estimation options](#).

`first` specifies that the first-step probit estimates of the selection equation be displayed before estimation.

`noheader` suppresses the header above the parameter table.

`nofootnote` suppresses the footnotes displayed below the parameter table.

`nocnsreport`; see [R] [Estimation options](#).

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no!stretch`; see [R] [Estimation options](#).

### Maximization

*maximize\_options*: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrntolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

Setting the optimization type to `technique(bhhh)` resets the default *vcetype* to `vce(opg)`.

The following options are available with `heckoprobit` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

`heckoprobit` estimates the parameters of a regression model for an ordered categorical outcome from a nonrandom sample known as a selected sample. Selected samples suffer from “selection on unobservables” because the errors that determine whether a case is missing are correlated with the errors that determine the outcome.

For ordered categorical regression from samples that do not suffer from selection on unobservables, see [R] [oprobit](#) or [R] [ologit](#). For regression of a continuous outcome variable from a selected sample, see [R] [heckman](#).

Even though we are interested in modeling a single ordinal outcome, there are two dependent variables in the ordered probit sample-selection model because we must also model the sample-selection process. First, there is the ordinal outcome  $y_j$ . Second, there is a binary variable that indicates whether each case in the sample is observed or unobserved. To handle the sample-selection problem, we model both dependent variables jointly. Both variables are categorical. Their categorical values are determined by the values of linear combinations of covariates and normally distributed error terms relative to certain cutpoints that partition the real line. The error terms used in the determination of selection and the ordinal outcome value may be correlated.

The probability that the ordinal outcome  $y_j$  is equal to the value  $v_h$  is given by the probability that  $\mathbf{x}_j\boldsymbol{\beta} + u_{1j}$  falls within the cutpoints  $\kappa_{h-1}$  and  $\kappa_h$ ,

$$\Pr(y_j = v_h) = \Pr(\kappa_{h-1} < \mathbf{x}_j\boldsymbol{\beta} + u_{1j} \leq \kappa_h)$$

where  $\mathbf{x}_j$  is the outcome covariates,  $\boldsymbol{\beta}$  is the coefficients, and  $u_{1j}$  is a random-error term. The observed outcome values  $v_1, \dots, v_H$  are integers such that  $v_i < v_m$  for  $i < m$ .  $\kappa_0$  is taken as  $-\infty$  and  $\kappa_H$  is taken as  $+\infty$ .

We model the selection process for the outcome by

$$s_j = \mathbf{1}(\mathbf{z}_j\boldsymbol{\gamma} + u_{2j} > 0)$$

where  $s_j = 1$  if we observed  $y_j$  and 0 otherwise,  $\mathbf{z}_j$  is the covariates used to model the selection process,  $\boldsymbol{\gamma}$  is the coefficients for the selection process,  $\mathbf{1}(\cdot)$  denotes the indicator function, and  $u_{2j}$  is a random-error term.

$(u_{1j}, u_{2j})$  have bivariate normal distribution with mean zero and variance matrix

$$\begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

When  $\rho \neq 0$ , standard ordered probit techniques applied to the outcome equation yield inconsistent results. `heckoprobit` provides consistent, asymptotically efficient estimates for all the parameters in such models.

[De Luca and Perotti \(2011\)](#) describe the maximum likelihood estimator used in `heckoprobit`.

## ▷ Example 1

We have a simulated dataset containing a sample of 5,000 women, 3,480 of whom work. The outcome of interest is a woman's job satisfaction, and we suspect that unobservables that determine job satisfaction and the unobservables that increase the likelihood of employment are correlated. Women may make a decision to work based on how satisfying their job would be. We estimate the parameters of an ordered probit sample-selection model for the outcome of job satisfaction (`satisfaction`) with selection on employment (`work`). Age (`age`) and years of education (`education`) are used as outcome covariates, and we also expect that they affect selection. Additional covariates for selection are marital status (`married`) and the number of children at home (`children`).

Here we estimate the parameters of the model with `heckoprobit`. We use the factorial interaction of `married` and `children` in `select()`. This specifies that the number of children and marital status affect selection, and it allows the effect of the number of children to differ among married and nonmarried women. The factorial interaction is specified using factor-variable notation, which is described in [U] 11.4.3 **Factor variables**.

```
. use https://www.stata-press.com/data/r19/womensat
(Job satisfaction, female)
. heckoprobit satisfaction education age,
> select(work=education age i.married##c.children)
Fitting oprobit model:
Iteration 0: Log likelihood = -3934.1474
Iteration 1: Log likelihood = -3571.886
Iteration 2: Log likelihood = -3570.2616
Iteration 3: Log likelihood = -3570.2616
Fitting selection model:
Iteration 0: Log likelihood = -3071.0775
Iteration 1: Log likelihood = -2565.5092
Iteration 2: Log likelihood = -2556.8369
Iteration 3: Log likelihood = -2556.8237
Iteration 4: Log likelihood = -2556.8237
Comparison: Log likelihood = -6127.0853
Fitting full model:
Iteration 0: Log likelihood = -6127.0853
Iteration 1: Log likelihood = -6093.8868
Iteration 2: Log likelihood = -6083.215
Iteration 3: Log likelihood = -6083.0376
Iteration 4: Log likelihood = -6083.0372
```

```

Ordered probit model with sample selection Number of obs = 5,000
 Selected = 3,480
 Nonselected = 1,520
Log likelihood = -6083.037 Wald chi2(2) = 842.42
 Prob > chi2 = 0.0000

```

|              | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |           |
|--------------|-------------|-----------|--------|-------|----------------------|-----------|
| satisfaction |             |           |        |       |                      |           |
| education    | .1536381    | .0068266  | 22.51  | 0.000 | .1402583             | .1670179  |
| age          | .0334463    | .0024049  | 13.91  | 0.000 | .0287329             | .0381598  |
| work         |             |           |        |       |                      |           |
| education    | .0512494    | .0068095  | 7.53   | 0.000 | .037903              | .0645958  |
| age          | .0288084    | .0026528  | 10.86  | 0.000 | .023609              | .0340078  |
| 1.married    | .6120876    | .0700055  | 8.74   | 0.000 | .4748794             | .7492958  |
| children     | .5140995    | .0288529  | 17.82  | 0.000 | .4575489             | .5706501  |
| married#     |             |           |        |       |                      |           |
| c.children   |             |           |        |       |                      |           |
| 1            | -.1337573   | .035126   | -3.81  | 0.000 | -.202603             | -.0649117 |
| _cons        | -2.203036   | .125772   | -17.52 | 0.000 | -2.449545            | -1.956528 |
| /cut1        | 1.728757    | .1232063  | 14.03  | 0.000 | 1.487277             | 1.970237  |
| /cut2        | 2.64357     | .116586   | 22.67  | 0.000 | 2.415066             | 2.872075  |
| /cut3        | 3.642911    | .1178174  | 30.92  | 0.000 | 3.411993             | 3.873829  |
| /athrho      | .7430919    | .0780998  | 9.51   | 0.000 | .5900191             | .8961646  |
| rho          | .6310096    | .0470026  |        |       | .5299093             | .7144252  |

```

LR test of indep. eqns. (rho = 0): chi2(1) = 88.10 Prob > chi2 = 0.0000

```

The output shows several iteration logs. The first iteration log corresponds to running the ordered probit model for those observations in the sample where we have observed the outcome. The second iteration log corresponds to running the selection probit model, which models whether we observe our outcome of interest. If  $\rho = 0$ , the sum of the log likelihoods from these two models will equal the log likelihood of the ordered probit sample-selection model; this sum is printed in the iteration log as the comparison log likelihood. The final iteration log is for fitting the full ordered probit sample-selection model.

The Wald test in the header is highly significant, indicating a good model fit. All the covariates are statistically significant. The likelihood-ratio test in the footer indicates that we can reject the null hypothesis that the errors for outcome and selection are uncorrelated. This means that we should use the ordered probit sample-selection model instead of the simple ordered probit model.

The positive estimate of 0.63 for  $\rho$  indicates that unobservables that increase job satisfaction tend to occur with unobservables that increase the chance of having a job.

## Stored results

heckoprobit stores the following in `e()`:

### Scalars

|                               |                                              |
|-------------------------------|----------------------------------------------|
| <code>e(N)</code>             | number of observations                       |
| <code>e(N_selected)</code>    | number of selected observations              |
| <code>e(N_nonselected)</code> | number of nonselected observations           |
| <code>e(N_cd)</code>          | number of completely determined observations |
| <code>e(k_cat)</code>         | number of categories                         |
| <code>e(k)</code>             | number of parameters                         |
| <code>e(k_eq)</code>          | number of equations in <code>e(b)</code>     |
| <code>e(k_eq_model)</code>    | number of equations in overall model test    |
| <code>e(k_aux)</code>         | number of auxiliary parameters               |
| <code>e(k_dv)</code>          | number of dependent variables                |
| <code>e(df_m)</code>          | model degrees of freedom                     |
| <code>e(ll)</code>            | log likelihood                               |
| <code>e(ll_c)</code>          | log likelihood, comparison model             |
| <code>e(N_clust)</code>       | number of clusters                           |
| <code>e(chi2)</code>          | $\chi^2$                                     |
| <code>e(chi2_c)</code>        | $\chi^2$ for comparison test                 |
| <code>e(p)</code>             | <i>p</i> -value for model test               |
| <code>e(p_c)</code>           | <i>p</i> -value for comparison test          |
| <code>e(rho)</code>           | $\rho$                                       |
| <code>e(rank)</code>          | rank of <code>e(V)</code>                    |
| <code>e(ic)</code>            | number of iterations                         |
| <code>e(rc)</code>            | return code                                  |
| <code>e(converged)</code>     | 1 if converged, 0 otherwise                  |

### Macros

|                                |                                                                          |
|--------------------------------|--------------------------------------------------------------------------|
| <code>e(cmd)</code>            | heckoprobit                                                              |
| <code>e(cmdline)</code>        | command as typed                                                         |
| <code>e(depvar)</code>         | names of dependent variables                                             |
| <code>e(wtype)</code>          | weight type                                                              |
| <code>e(wexp)</code>           | weight expression                                                        |
| <code>e(title)</code>          | title in estimation output                                               |
| <code>e(clustvar)</code>       | name of cluster variable                                                 |
| <code>e(offset1)</code>        | offset for regression equation                                           |
| <code>e(offset2)</code>        | offset for selection equation                                            |
| <code>e(chi2type)</code>       | Wald or LR; type of model $\chi^2$ test                                  |
| <code>e(chi2_ct)</code>        | type of comparison $\chi^2$ test                                         |
| <code>e(vce)</code>            | <i>vcetype</i> specified in <code>vce()</code>                           |
| <code>e(vcetype)</code>        | title used to label Std. err.                                            |
| <code>e(opt)</code>            | type of optimization                                                     |
| <code>e(which)</code>          | max or min; whether optimizer is to perform maximization or minimization |
| <code>e(ml_method)</code>      | type of ml method                                                        |
| <code>e(user)</code>           | name of likelihood-evaluator program                                     |
| <code>e(technique)</code>      | maximization technique                                                   |
| <code>e(properties)</code>     | b V                                                                      |
| <code>e(predict)</code>        | program used to implement <code>predict</code>                           |
| <code>e(marginsok)</code>      | predictions allowed by margins                                           |
| <code>e(marginsnotok)</code>   | predictions disallowed by margins                                        |
| <code>e(marginsdefault)</code> | default <code>predict()</code> specification for margins                 |
| <code>e(asbalanced)</code>     | factor variables <code>fvset</code> as <code>asbalanced</code>           |
| <code>e(asobserved)</code>     | factor variables <code>fvset</code> as <code>asobserved</code>           |

### Matrices

|                          |                                     |
|--------------------------|-------------------------------------|
| <code>e(b)</code>        | coefficient vector                  |
| <code>e(Cns)</code>      | constraints matrix                  |
| <code>e(ilog)</code>     | iteration log (up to 20 iterations) |
| <code>e(gradient)</code> | gradient vector                     |

|                 |                                              |
|-----------------|----------------------------------------------|
| e(cat)          | category values                              |
| e(V)            | variance–covariance matrix of the estimators |
| e(V_modelbased) | model-based variance                         |
| Functions       |                                              |
| e(sample)       | marks estimation sample                      |

In addition to the above, the following is stored in `r()`:

|          |                                                                                                                       |
|----------|-----------------------------------------------------------------------------------------------------------------------|
| Matrices |                                                                                                                       |
| r(table) | matrix containing the coefficients with their standard errors, test statistics, $p$ -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any  $r$ -class command is run after the estimation command.

## Methods and formulas

De Luca and Perotti (2011) provide an introduction to this model.

The ordinal outcome equation is

$$y_j = \sum_{h=1}^H v_h 1(\kappa_{h-1} < \mathbf{x}_j \boldsymbol{\beta} + u_{1j} \leq \kappa_h)$$

where  $\mathbf{x}_j$  is the outcome covariates,  $\boldsymbol{\beta}$  is the coefficients, and  $u_{1j}$  is a random-error term. The observed outcome values  $v_1, \dots, v_H$  are integers such that  $v_i < v_m$  for  $i < m$ .  $\kappa_1, \dots, \kappa_{H-1}$  are real numbers such that  $\kappa_i < \kappa_m$  for  $i < m$ .  $\kappa_0$  is taken as  $-\infty$  and  $\kappa_H$  is taken as  $+\infty$ .

The selection equation is

$$s_j = \mathbf{1}(z_j \boldsymbol{\gamma} + u_{2j} > 0)$$

where  $s_j = 1$  if we observed  $y_j$  and 0 otherwise,  $z_j$  is the covariates used to model the selection process,  $\boldsymbol{\gamma}$  is the coefficients for the selection process, and  $u_{2j}$  is a random-error term.

$(u_{1j}, u_{2j})$  have bivariate normal distribution with mean zero and variance matrix

$$\begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

Let  $a_j = z_j \boldsymbol{\gamma} + \text{offset}_j^\gamma$  and  $b_j = x_j \boldsymbol{\beta} + \text{offset}_j^\beta$ . This yields the log likelihood

$$\ln L = \sum_{j \notin S} w_j \ln \{ \Phi(-a_j) \} + \sum_{h=1}^H \sum_{\substack{j \in S \\ y_j = v_h}} w_j \ln \{ \Phi_2(a_j, \kappa_h - b_j, -\rho) - \Phi_2(a_j, \kappa_{h-1} - b_j, -\rho) \}$$

where  $S$  is the set of observations for which  $y_j$  is observed,  $\Phi_2(\cdot)$  is the cumulative bivariate normal distribution function (with mean  $[0 \ 0]'$ ),  $\Phi(\cdot)$  is the standard cumulative normal, and  $w_j$  is an optional weight for observation  $j$ .

In the maximum likelihood estimation,  $\rho$  is not directly estimated. Directly estimated is  $\text{atanh } \rho$ :

$$\text{atanh } \rho = \frac{1}{2} \ln \left( \frac{1 + \rho}{1 - \rho} \right)$$

From the form of the likelihood, it is clear that if  $\rho = 0$ , the log likelihood for the ordered probit sample-selection model is equal to the sum of the ordered probit model for the outcome  $y$  and the selection model. We can perform a likelihood-ratio test by comparing the log likelihood of the full model with the sum of the log likelihoods for the ordered probit and selection models.

## References

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## Also see

- [R] **heckoprobit postestimation** — Postestimation tools for heckoprobit
- [R] **heckman** — Heckman selection model
- [R] **heckpoisson** — Poisson regression with sample selection
- [R] **heckprobit** — Probit model with sample selection
- [R] **oprobit** — Ordered probit regression
- [R] **probit** — Probit regression
- [R] **regress** — Linear regression
- [R] **tobit** — Tobit regression
- [BAYES] **bayes: heckoprobit** — Bayesian ordered probit model with sample selection
- [ERM] **eoprobit** — Extended ordered probit regression
- [SVY] **svy estimation** — Estimation commands for survey data
- [U] **20 Estimation and postestimation commands**

Postestimation commands  
Methods and formulas

predict  
Also see

margins

Remarks and examples

## Postestimation commands

The following postestimation commands are available after `heckoprobit`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | probabilities, linear predictions and their SEs, etc.                                                                                   |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`hausman` and `lrtest` are not appropriate with `svy` estimation results.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as probabilities, linear predictions, and standard errors.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] { stub* | newvar | newvarlist } [if] [in] [, statistic
outcome(outcome) nooffset]
```

```
predict [type] stub* [if] [in] , scores
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

---

|                         |                                                                |
|-------------------------|----------------------------------------------------------------|
| Main                    |                                                                |
| <code>pmargin</code>    | marginal probabilities; the default                            |
| <code>p1</code>         | bivariate probabilities of levels with selection               |
| <code>p0</code>         | bivariate probabilities of levels with no selection            |
| <code>pcond1</code>     | probabilities of levels conditional on selection               |
| <code>pcond0</code>     | probabilities of levels conditional on no selection            |
| <code>pselect</code>    | selection probability                                          |
| <code>xb</code>         | linear prediction                                              |
| <code>stdp</code>       | standard error of the linear prediction                        |
| <code>xbselect</code>   | linear prediction for selection equation                       |
| <code>stdpselect</code> | standard error of the linear prediction for selection equation |

---

If you do not specify `outcome()`, `pmargin` (with one new variable specified) assumes `outcome(#1)`.

You specify one or  $k$  new variables with `pmargin`, where  $k$  is the number of outcomes.

You specify one new variable with `pselect`, `xb`, `stdp`, `xbselect`, and `stdpselect`.

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

## Options for predict

Main

`pmargin`, the default, calculates the predicted marginal probabilities.

You specify one or  $k$  new variables, where  $k$  is the number of categories of the outcome variable  $y_j$ . If you specify the `outcome()` option, you must specify one new variable. If you specify one new variable and do not specify `outcome()`, `outcome(#1)` is assumed.

When `outcome()` is specified, the marginal probability that  $y_j$  is equal to the level `outcome()` is calculated. When `outcome()` is not specified, the marginal probabilities for each outcome level are calculated.

`p1` calculates the predicted bivariate probabilities of outcome levels with selection.

You specify one or  $k$  new variables, where  $k$  is the number of categories of the outcome variable  $y_j$ . If you specify the `outcome()` option, you must specify one new variable. If you specify one new variable and do not specify `outcome()`, `outcome(#1)` is assumed.

When `outcome()` is specified, the bivariate probability that  $y_j$  is equal to the level `outcome()` and that  $y_j$  is observed is calculated. When `outcome()` is not specified, the bivariate probabilities for each outcome level and selection are calculated.

`p0` calculates the predicted bivariate probabilities of outcome levels with no selection.

You specify one or  $k$  new variables, where  $k$  is the number of categories of the outcome variable  $y_j$ . If you specify the `outcome()` option, you must specify one new variable. If you specify one new variable and do not specify `outcome()`, `outcome(#1)` is assumed.

When `outcome()` is specified, the bivariate probability that  $y_j$  is equal to the level `outcome()` and that  $y_j$  is not observed is calculated. When `outcome()` is not specified, the bivariate probabilities for each outcome level and no selection are calculated.

`pcond1` calculates the predicted probabilities of outcome levels conditional on selection.

You specify one or  $k$  new variables, where  $k$  is the number of categories of the outcome variable  $y_j$ . If you specify the `outcome()` option, you must specify one new variable. If you specify one new variable and do not specify `outcome()`, `outcome(#1)` is assumed.

When `outcome()` is specified, the probability that  $y_j$  is equal to the level `outcome()` given that  $y_j$  is observed is calculated. When `outcome()` is not specified, the probabilities for each outcome level conditional on selection are calculated.

`pcond0` calculates the predicted probabilities of outcome levels conditional on no selection.

You specify one or  $k$  new variables, where  $k$  is the number of categories of the outcome variable  $y_j$ . If you specify the `outcome()` option, you must specify one new variable. If you specify one new variable and do not specify `outcome()`, `outcome(#1)` is assumed.

When `outcome()` is specified, the probability that  $y_j$  is equal to the level `outcome()` given that  $y_j$  is not observed is calculated. When `outcome()` is not specified, the probabilities for each outcome level conditional on no selection are calculated.

`pse1` calculates the predicted univariate (marginal) probability of selection.

`xb` calculates the linear prediction for the outcome variable, which is  $\mathbf{x}_j\beta$  if `offset()` was not specified and  $\mathbf{x}_j\beta + \text{offset}_j^\beta$  if `offset()` was specified.

`stdp` calculates the standard error of the prediction, which can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. The standard error of the prediction is also referred to as the standard error of the fitted value.

`xbse1` calculates the linear prediction for the selection equation, which is  $\mathbf{z}_j\boldsymbol{\gamma}$  if `offset()` was not specified in `select()` and  $\mathbf{z}_j\boldsymbol{\gamma} + \text{offset}_j$  if `offset()` was specified in `select()`.

`stdpse1` calculates the standard error of the linear prediction for the selection equation.

`outcome(outcome)` specifies for which outcome the predicted probabilities are to be calculated. `outcome()` should contain either one value of the dependent variable or one of #1, #2, ..., with #1 meaning the first category of the dependent variable, #2 meaning the second category, etc.

`nooffset` is relevant only if you specified `offset(varname)` for `heckoprobit`. It modifies the calculations made by `predict` so that they ignore the offset variable; the linear prediction is treated as  $\mathbf{x}_j\mathbf{b}$  rather than as  $\mathbf{x}_j\mathbf{b} + \text{offset}_j$ .

`scores` calculates equation-level score variables.

The first new variable will contain  $\partial \ln L / \partial (\mathbf{x}_j\boldsymbol{\beta})$ .

The second new variable will contain  $\partial \ln L / \partial (\mathbf{z}_j\boldsymbol{\gamma})$ .

When the dependent variable takes  $k$  different values, new variables three through  $k + 1$  will contain  $\partial \ln L / \partial (\kappa_{j-2})$ .

The last new variable will contain  $\partial \ln L / \partial (\text{atanh } \rho)$ .

## margins

### Description for margins

`margins` estimates margins of response for probabilities and linear predictions.

### Menu for margins

Statistics > Postestimation

### Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

*statistic*                      Description

---

|                                    |                                                     |
|------------------------------------|-----------------------------------------------------|
| <code>default</code>               | marginal probabilities for each outcome             |
| <code><u>p</u>margin</code>        | marginal probabilities                              |
| <code>p1</code>                    | bivariate probabilities of levels with selection    |
| <code>p0</code>                    | bivariate probabilities of levels with no selection |
| <code>pcond1</code>                | probabilities of levels conditional on selection    |
| <code>pcond0</code>                | probabilities of levels conditional on no selection |
| <code><u>p</u>sel</code>           | selection probability                               |
| <code>xb</code>                    | linear prediction                                   |
| <code><u>x</u>b<u>s</u>el</code>   | linear prediction for selection equation            |
| <code>stdp</code>                  | not allowed with margins                            |
| <code><u>s</u>td<u>p</u>sel</code> | not allowed with margins                            |

---

`pmargin`, `p1`, `p0`, `pcond1`, and `pcond0` default to the first outcome.

Statistics not allowed with margins are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [\[R\] margins](#).

## Remarks and examples

### ▷ Example 1

In [example 1](#) of [\[R\] heckoprobit](#), we examined a simulated dataset of 5,000 women, 3,480 of whom work and can thus report job satisfaction. Using job satisfaction (`satisfaction`) as the outcome variable and employment (`work`) as the selection variable, we estimated the parameters of an ordered probit sample-selection model. Covariates `age` (`age`), years of education (`education`), number of children (`children`), and marital status (`married`) are expected to affect selection. The outcome, job satisfaction, is affected by `age` (`age`) and education (`education`).

We first reestimate the parameters of the regression, but this time we request a robust variance estimator:

```
. use https://www.stata-press.com/data/r19/womensat
(Job satisfaction, female)
. heckoprobit satisfaction education age,
> select(work=education age i.married#c.children) vce(robust)
(output omitted)
```

We then use `margins` (see [\[R\] margins](#)) to estimate the average marginal effect of education on the probability of having low job satisfaction.

```
. margins, dydx(education) vce(unconditional)
Average marginal effects Number of obs = 5,000
dy/dx wrt: education
1._predict: Pr(satisfaction=1), predict(pmargi outcome(1))
2._predict: Pr(satisfaction=2), predict(pmargi outcome(2))
3._predict: Pr(satisfaction=3), predict(pmargi outcome(3))
4._predict: Pr(satisfaction=4), predict(pmargi outcome(4))
```

|           | Unconditional |           |        |       |                       |
|-----------|---------------|-----------|--------|-------|-----------------------|
|           | dy/dx         | std. err. | z      | P> z  | [95% conf. interval]  |
| education |               |           |        |       |                       |
| _predict  |               |           |        |       |                       |
| 1         | -.0234776     | .0019176  | -12.24 | 0.000 | -.027236    -.0197192 |
| 2         | -.0230858     | .0015143  | -15.24 | 0.000 | -.0260538   -.0201178 |
| 3         | -.0020603     | .001142   | -1.80  | 0.071 | -.0042987    .000178  |
| 4         | .0486238      | .0018593  | 26.15  | 0.000 | .0449795    .052268   |

The estimated average marginal effect of education on the probability of having low job satisfaction is approximately  $-0.023$ .

## Methods and formulas

The ordinal outcome equation is

$$y_j = \sum_{h=1}^H v_h 1(\kappa_{h-1} < \mathbf{x}_j \boldsymbol{\beta} + u_{1j} \leq \kappa_h)$$

where  $\mathbf{x}_j$  is the outcome covariates,  $\boldsymbol{\beta}$  is the coefficients, and  $u_{1j}$  is a random-error term. The observed outcome values  $v_1, \dots, v_H$  are integers such that  $v_i < v_m$  for  $i < m$ .  $\kappa_1, \dots, \kappa_{H-1}$  are real numbers such that  $\kappa_i < \kappa_m$  for  $i < m$ .  $\kappa_0$  is taken as  $-\infty$  and  $\kappa_H$  is taken as  $+\infty$ .

The selection equation is

$$s_j = 1(\mathbf{z}_j \boldsymbol{\gamma} + u_{2j} > 0)$$

where  $s_j = 1$  if we observed  $y_j$  and 0 otherwise,  $\mathbf{z}_j$  is the covariates used to model the selection process,  $\boldsymbol{\gamma}$  is the coefficients for the selection process, and  $u_{2j}$  is a random-error term.

$(u_{1j}, u_{2j})$  have bivariate normal distribution with mean zero and variance matrix

$$\begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

The probability of selection is

$$\Pr(s_j = 1) = \Phi(\mathbf{z}_j \boldsymbol{\gamma} + \text{offset}_j^\gamma)$$

$\Phi(\cdot)$  is the standard cumulative normal distribution function.

The probability of selection and the outcome  $y_j = v_h$  is

$$\begin{aligned} \Pr(y_j = v_h, s_j = 1) &= \Phi_2(\mathbf{z}_j \boldsymbol{\gamma} + \text{offset}_j^\gamma, \kappa_h - x_j \boldsymbol{\beta} - \text{offset}_j^\beta, -\rho) \\ &\quad - \Phi_2(\mathbf{z}_j \boldsymbol{\gamma} + \text{offset}_j^\gamma, \kappa_{h-1} - \mathbf{x}_j \boldsymbol{\beta} - \text{offset}_j^\beta, -\rho) \end{aligned}$$

$\Phi_2(\cdot)$  is the cumulative bivariate normal distribution function (with mean  $[0 \ 0]'$ ).

The probability of  $y_j$  not being selected and the outcome  $y_j = v_h$  is

$$\begin{aligned} \Pr(y_j = v_h, s_j = 0) &= \Phi_2(-\mathbf{z}_j \boldsymbol{\gamma} - \text{offset}_j^\gamma, \kappa_h - x_j \boldsymbol{\beta} - \text{offset}_j^\beta, \rho) \\ &\quad - \Phi_2(-\mathbf{z}_j \boldsymbol{\gamma} - \text{offset}_j^\gamma, \kappa_{h-1} - \mathbf{x}_j \boldsymbol{\beta} - \text{offset}_j^\beta, \rho) \end{aligned}$$

The probability of outcome  $y_j = v_h$  given selection is

$$\Pr(y_j = v_h | s_j = 1) = \frac{\Pr(y_j = v_h, s_j = 1)}{\Pr(s_j = 1)}$$

The probability of outcome  $y_j = v_h$  given  $y_j$  is not selected is

$$\Pr(y_j = v_h | s_j = 0) = \frac{\Pr(y_j = v_h, s_j = 0)}{\Pr(s_j = 0)}$$

The marginal probabilities of the outcome  $y_j$  are

$$\Pr(y_j = v_1) = \Phi(\kappa_1 - x_j\beta - \text{offset}_j^\beta)$$

$$\Pr(y_j = v_H) = 1 - \Phi(\kappa_{H-1} - x_j\beta - \text{offset}_j^\beta)$$

$$\Pr(y_j = v_h) = \Phi(\kappa_h - x_j\beta - \text{offset}_j^\beta) - \Phi(\kappa_{h-1} - x_j\beta - \text{offset}_j^\beta)$$

## Also see

[R] [heckoprobit](#) — Ordered probit model with sample selection

[U] [20 Estimation and postestimation commands](#)

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

## Description

`heckpoisson` fits a Poisson regression model with endogenous sample selection. This is sometimes called nonignorability of selection, missing not at random, or selection bias. Unlike the standard Poisson model, there is no assumption of equidispersion.

## Quick start

Poisson model of  $y$  on  $x_1$  with  $z_1$  predicting selection when binary variable `selected` indicates selection status

```
heckpoisson y x1, select(selected = z1)
```

Add categorical variable `a` using [factor-variables](#) syntax

```
heckpoisson y x1 i.a, select(selected = z1 i.a)
```

Report results as incidence-rate ratios

```
heckpoisson y x1 i.a, select(selected = z1 i.a) irr
```

Add robust standard errors

```
heckpoisson y x1 i.a, select(selected = z1 i.a) vce(robust)
```

Include exposure variable `expose` to account for different exposure levels

```
heckpoisson y x1 i.a, select(selected = z1 i.a) exposure(expose)
```

## Menu

Statistics > Sample-selection models > Poisson model with sample selection

## Syntax

```
heckpoisson depvar indepvars [if] [in] [weight] ,
 select([depvars =] indepvarss [, noconstant offset(varnameos)]) [options]
```

*options*

Description

### Model

\* select( )

specify selection equation: dependent and independent variables; whether to have constant term and offset variable

noconstant

suppress constant term

exposure(*varname*<sub>e</sub>)

include  $\ln(\text{varname}_e)$  in model with coefficient constrained to 1

offset(*varname*<sub>o</sub>)

include *varname*<sub>o</sub> in model with coefficient constrained to 1

constraints(*constraints*)

apply specified linear constraints

### SE/Robust

vce(*vcetype*)

*vcetype* may be oim, robust, cluster *clustvar*, opg, bootstrap, or jackknife

### Reporting

level(#)

set confidence level; default is level(95)

irr

report incidence-rate ratios

nocnsreport

do not display constraints

display\_options

control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling

### Integration

intpoints(#)

set the number of integration (quadrature) points; default is intpoints(25)

### Maximization

maximize\_options

control the maximization process; seldom used

collinear

keep collinear variables

coeflegend

display legend instead of statistics

\* select( ) is required.

The full specification is select( [*depvar*<sub>s</sub> = ] *indepvars*<sub>s</sub> [ , noconstant offset(*varname*<sub>os</sub>) ] ).

*indepvars* and *indepvars*<sub>s</sub> may contain factor variables; see [U] 11.4.3 Factor variables.

*indepvars* and *indepvars*<sub>s</sub> may contain time-series operators; see [U] 11.4.4 Time-series varlists.

bayesboot, bootstrap, by, collect, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands.

Weights are not allowed with the bootstrap prefix; see [R] bootstrap.

vce( ) and weights are not allowed with the svy prefix; see [SVY] svy.

fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.

collinear and coeflegend do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Options

### Model

`select([depvars = ] indepvarss [ , noconstant offset(varnameos) ])` specifies the variables and options for the selection equation. It is an integral part of specifying a sample-selection model and is required.

If *depvar*<sub>s</sub> is specified, it should be coded as 0 or 1, with 0 indicating an observation not selected and 1 indicating a selected observation. If *depvar*<sub>s</sub> is not specified, then observations for which *depvar* is not missing are assumed selected and those for which *depvar* is missing are assumed not selected.

`noconstant` suppresses the selection constant term (intercept).

`offset`(*varname*<sub>os</sub>) specifies that selection offset *varname*<sub>os</sub> be included in the model with the coefficient constrained to be 1.

`noconstant`, `exposure`(*varname*<sub>e</sub>), `offset`(*varname*<sub>o</sub>), `constraints`(*constraints*); see [R] [Estimation options](#).

### SE/Robust

`vce`(*vcetype*) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster` *clustvar*), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

### Reporting

`level`(#); see [R] [Estimation options](#).

`irr` reports estimated coefficients transformed to incidence-rate ratios, that is,  $e^{\beta_i}$  rather than  $\beta_i$ . Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated or stored. `irr` may be specified at estimation or when replaying previously estimated results.

`nocnsreport`; see [R] [Estimation options](#).

*display\_options*: `noci`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap`(#), `fvwrapon`(*style*), `cformat`(%*fmt*), `pformat`(%*fmt*), `sformat`(%*fmt*), and `nolstretch`; see [R] [Estimation options](#).

### Integration

`intpoints`(#) specifies the number of integration points to use for quadrature. The default is `intpoints`(25), which means that 25 quadrature points are used. The maximum number of allowed integration points is 128.

The more integration points, the more accurate the approximation to the log likelihood. However, computation time increases with the number of quadrature points and is roughly proportional to the number of points used.

Maximization

`maximize_options`: `difficult`, `technique` (`algorithm_spec`), `iterate`(#), `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance`(#), `ltolerance`(#), `nrtolerance`(#), `nonrtolerance`, and `from` (`init_specs`); see [R] [Maximize](#). These options are seldom used.

The following options are available with `heckpoisson` but are not shown in the dialog box: `collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

When analyzing observational data, we must consider the possibility that we cannot treat the observations for which we have data as if they were selected at random. Suppose we are interested in the number of after-school tutoring sessions a child attends. If unobservable variables that affect which students attend the sessions, for example, family stability, also affect the number of visits we observe, then a condition known as endogenous sample selection is present. This phenomenon is sometimes simply referred to as sample selection or called missing not at random, nonignorability of selection, or selection bias. When endogenous sample selection occurs, conventional estimation techniques are not appropriate. [Cameron and Trivedi \(2022, 974–981\)](#) and [Greene \(2018, 950–957\)](#) provide good introductions to the concept of endogenous sample selection.

The venerable Heckman estimator handles endogenous sample selection when the outcome of interest is modeled by linear regression; see [R] [heckman](#). However, the Heckman estimator is not appropriate for count outcomes because its linear model for the outcome could produce negative predicted values and does not restrict the predicted values to integers.

There are different methods for estimating the parameters of a count-data model with endogenous sample selection. `heckpoisson` implements the maximum likelihood estimator derived in [Terza \(1998\)](#); see also [Cameron and Trivedi \(2013, chap. 10\)](#) for a discussion of this estimator.

The model consists of one equation for the count outcome,  $y$ , and one equation for a binary selection indicator,  $s$ . The indicator  $s$  is always observed and takes values of 0 or 1. But the outcome  $y$  is observed only if  $s = 1$ , that is, we have complete information about the covariates of interest and selection status. However, the value of the primary outcome of interest,  $y$ , is sometimes unknown.

More formally, the count outcome  $y$  is assumed to have a Poisson distribution, conditional on the covariates, with conditional mean

$$E(y_j | \mathbf{x}_j, \epsilon_{1j}) = \exp(\mathbf{x}_j \boldsymbol{\beta} + \epsilon_{1j}) \quad \text{Poisson regression equation}$$

However, we only observe  $y$  for observation  $j$  if  $s_j = 1$ :

$$s_j = \begin{cases} 1, & \text{if } \mathbf{w}_j \boldsymbol{\gamma} + \epsilon_{2j} > 0 \\ 0, & \text{otherwise} \end{cases} \quad \text{selection equation}$$

where

$$\epsilon_1 \sim N(0, \sigma)$$

$$\epsilon_2 \sim N(0, 1)$$

$$\text{corr}(\epsilon_1, \epsilon_2) = \rho$$

When  $\rho \neq 0$ , standard Poisson regression based on the observed  $y$  yields biased estimates. `heckpoisson` provides consistent, asymptotically efficient estimates for the parameters in such models.

Unlike the standard Poisson regression, the Poisson model with sample selection allows underdispersion and overdispersion.

### ▷ Example 1: Poisson model with sample selection

Suppose we want to know the effect of research and development (R&D) expenditures on the number of patents obtained by a firm in the last two years. The patent dataset contains fictional data on the number of patents (`npatents`) of 10,000 firms in different sectors. After reading in the data, we tabulate the frequencies of `npatents` against an indicator for whether a firm applied for patents (`applied`).

```
. use https://www.stata-press.com/data/r19/patent
(Fictional data on patents and R&D)
. tabulate npatents applied, missing
```

| Number of<br>patents<br>(last 2<br>yrs) | Applied for patent |       | Total  |
|-----------------------------------------|--------------------|-------|--------|
|                                         | Not apply          | Apply |        |
| 0                                       | 0                  | 1,127 | 1,127  |
| 1                                       | 0                  | 1,455 | 1,455  |
| 2                                       | 0                  | 1,131 | 1,131  |
| 3                                       | 0                  | 710   | 710    |
| 4                                       | 0                  | 479   | 479    |
| 5                                       | 0                  | 266   | 266    |
| 6                                       | 0                  | 126   | 126    |
| 7                                       | 0                  | 98    | 98     |
| 8                                       | 0                  | 66    | 66     |
| 9                                       | 0                  | 42    | 42     |
| 10                                      | 0                  | 19    | 19     |
| 11                                      | 0                  | 24    | 24     |
| 12                                      | 0                  | 5     | 5      |
| 13                                      | 0                  | 7     | 7      |
| 14                                      | 0                  | 5     | 5      |
| 15                                      | 0                  | 10    | 10     |
| 17                                      | 0                  | 1     | 1      |
| 18                                      | 0                  | 1     | 1      |
| 19                                      | 0                  | 2     | 2      |
| 22                                      | 0                  | 1     | 1      |
| .                                       | 4,425              | 0     | 4,425  |
| Total                                   | 4,425              | 5,575 | 10,000 |

The output shows that `npatents` is missing for about half of the sample because some firms did not apply for any patents. Some firms prefer to keep their discoveries as trade secrets instead of applying for patents. The sample selection will be endogenous if the unobservable variables that affect which firms apply for patents also affect the number of patents obtained. Therefore, we do not want to use a standard Poisson model for these data.

We model `npatents` as a function of R&D expenditures (`expenditure`) and a categorical variable indicating whether the firm is in the information technology (IT) sector (`tech`). We model the selection indicator `applied` as a function of `expenditure`, `tech`, and firm size (`size`), which is excluded from the outcome model.

```
. heckpoisson npatents expenditure i.tech,
> select(applied = expenditure size i.tech)
Initial: Log likelihood = -17442.266
Rescale: Log likelihood = -17442.266
Rescale eq: Log likelihood = -17442.266
(setting technique to bhhh)
Iteration 0: Log likelihood = -17442.266
Iteration 1: Log likelihood = -17441.444
Iteration 2: Log likelihood = -17440.72
Iteration 3: Log likelihood = -17440.438
Iteration 4: Log likelihood = -17440.438

Poisson regression with endogenous selection Number of obs = 10,000
(25 quadrature points) Selected = 5,575
 Nonselected = 4,425

 Wald chi2(2) = 443.90
 Prob > chi2 = 0.0000

Log likelihood = -17440.44
```

| npatents    | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|-------------|-------------|-----------|-------|-------|----------------------|-----------|
| npatents    |             |           |       |       |                      |           |
| expenditure | .497821     | .0507866  | 9.80  | 0.000 | .398281              | .597361   |
| tech        |             |           |       |       |                      |           |
| IT sector   | .5833501    | .0300366  | 19.42 | 0.000 | .5244795             | .6422207  |
| _cons       | -1.855143   | .208204   | -8.91 | 0.000 | -2.263216            | -1.447071 |
| applied     |             |           |       |       |                      |           |
| expenditure | .1369954    | .0447339  | 3.06  | 0.002 | .0493185             | .2246723  |
| size        | .2774201    | .0469132  | 5.91  | 0.000 | .1854718             | .3693683  |
| tech        |             |           |       |       |                      |           |
| IT sector   | .2750208    | .0277032  | 9.93  | 0.000 | .2207236             | .329318   |
| _cons       | -1.660778   | .2631227  | -6.31 | 0.000 | -2.176489            | -1.145066 |
| /athrho     | 1.161677    | .2847896  | 4.08  | 0.000 | .6034999             | 1.719855  |
| /lnsigma    | -.3029685   | .0499674  | -6.06 | 0.000 | -.4009028            | -.2050342 |
| rho         | .8215857    | .0925557  |       |       | .5395353             | .9378455  |
| sigma       | .7386224    | .036907   |       |       | .6697151             | .8146195  |

Wald test of indep. eqns. (rho = 0): chi2(1) = 16.64 Prob > chi2 = 0.0000

The coefficient estimates reported by `heckpoisson` can be interpreted similarly to those reported by `poisson`. For example, the positive coefficient on `expenditure` tells us that increasing R&D expenditures is associated with an increasing number of patents. However, the magnitude of the effect cannot be directly determined by the coefficients. The best way to obtain interpretable effects is by using `margins`. See [example 1](#) in [R] `heckpoisson postestimation` for more information.

The estimated correlation between the selection errors and outcome errors is 0.8, and the Wald test in the footer indicates that we can reject the null hypothesis of zero correlation. This positive and significant correlation estimate implies that unobservable factors that increase the number of patents a firm is awarded tend to occur with unobservable factors that also increase the chance of a firm being willing to apply for patents.

□ Technical note

In practice, we rely on the strength of the relationship between size and applied and the fact that size does not appear in the model for npatents to pin down the parameter estimates. Technically, we do not need this exclusion restriction, but identification from the functional form alone tends to be weak. For a discussion of this point, see [Cameron and Trivedi \(2022, 977–981\)](#). □

▷ Example 2: Obtaining incidence-rate ratios

In some cases, we may wish to view the parameters as incidence-rate ratios (IRRs). That is, we want to hold all the  $x$ 's in the model constant except one, say, the  $i$ th. The IRR for a one-unit change in  $x_i$  is

$$\frac{e^{\ln(E)+\beta_1x_1+\dots+\beta_i(x_i+1)+\dots+\beta_kx_k+e_1}}{e^{\ln(E)+\beta_1x_1+\dots+\beta_ix_i+\dots+\beta_kx_k+e_1}} = e^{\beta_i}$$

For instance, we may want to know the relative incidence rate of patents as the expenditure changes or the relative incidence rate of patents as sectors change from non-IT to IT.

We can use option `irr` to display the coefficient estimates transformed to IRRs. This option may be specified when we originally fit our model or on replay. Because we have already fit the model, we specify `irr` below using the replay syntax.

```
. heckpoisson, irr
Poisson regression with endogenous selection Number of obs = 10,000
(25 quadrature points) Selected = 5,575
 Nonselected = 4,425
 Wald chi2(2) = 443.90
Log likelihood = -17440.44 Prob > chi2 = 0.0000
```

| npatents    | IRR       | Std. err. | z     | P> z  | [95% conf. interval] |           |
|-------------|-----------|-----------|-------|-------|----------------------|-----------|
| npatents    |           |           |       |       |                      |           |
| expenditure | 1.645133  | .0835508  | 9.80  | 0.000 | 1.489262             | 1.817316  |
| tech        |           |           |       |       |                      |           |
| IT sector   | 1.792032  | .0538265  | 19.42 | 0.000 | 1.689579             | 1.900697  |
| _cons       | .1564305  | .0325695  | -8.91 | 0.000 | .1040154             | .2352583  |
| applied     |           |           |       |       |                      |           |
| expenditure | .1369954  | .0447339  | 3.06  | 0.002 | .0493185             | .2246723  |
| size        | .2774201  | .0469132  | 5.91  | 0.000 | .1854718             | .3693683  |
| tech        |           |           |       |       |                      |           |
| IT sector   | .2750208  | .0277032  | 9.93  | 0.000 | .2207236             | .329318   |
| _cons       | -1.660778 | .2631227  | -6.31 | 0.000 | -2.176489            | -1.145066 |
| /athrho     | 1.161677  | .2847896  | 4.08  | 0.000 | .6034999             | 1.719855  |
| /lnsigma    | -.3029685 | .0499674  | -6.06 | 0.000 | -.4009028            | -.2050342 |
| rho         | .8215857  | .0925557  |       |       | .5395353             | .9378455  |
| sigma       | .7386224  | .036907   |       |       | .6697151             | .8146195  |

Note: Estimates are transformed only in the first equation to incidence-rate ratios.

Note: `_cons` estimates baseline incidence rate.

Wald test of indep. eqns. (rho = 0): chi2(1) = 16.64 Prob > chi2 = 0.0000

The IRR for IT is about 1.8, meaning that the expected number of patents in the IT sector is 1.8 times more than in the non-IT sector.



## Stored results

heckpoisson stores the following in `e()`:

### Scalars

|                               |                                           |
|-------------------------------|-------------------------------------------|
| <code>e(N)</code>             | number of observations                    |
| <code>e(N_selected)</code>    | number of selected observations           |
| <code>e(N_nonselected)</code> | number of nonselected observations        |
| <code>e(k)</code>             | number of parameters                      |
| <code>e(k_eq)</code>          | number of equations in <code>e(b)</code>  |
| <code>e(k_eq_model)</code>    | number of equations in overall model test |
| <code>e(k_aux)</code>         | number of auxiliary parameters            |
| <code>e(k_dv)</code>          | number of dependent variables             |
| <code>e(df_m)</code>          | model degrees of freedom                  |
| <code>e(ll)</code>            | log likelihood                            |
| <code>e(N_clust)</code>       | number of clusters                        |
| <code>e(chi2)</code>          | $\chi^2$                                  |
| <code>e(chi2_c)</code>        | $\chi^2$ for comparison, $\rho = 0$ test  |
| <code>e(n_quad)</code>        | number of quadrature points               |
| <code>e(p)</code>             | $p$ -value for model test                 |
| <code>e(p_c)</code>           | $p$ -value for comparison test            |
| <code>e(rank)</code>          | rank of <code>e(V)</code>                 |
| <code>e(ic)</code>            | number of iterations                      |
| <code>e(rc)</code>            | return code                               |
| <code>e(converged)</code>     | 1 if converged, 0 otherwise               |

### Macros

|                              |                                                                          |
|------------------------------|--------------------------------------------------------------------------|
| <code>e(cmd)</code>          | heckpoisson                                                              |
| <code>e(cmdline)</code>      | command as typed                                                         |
| <code>e(depvar)</code>       | name of dependent variable                                               |
| <code>e(wtype)</code>        | weight type                                                              |
| <code>e(wexp)</code>         | weight expression                                                        |
| <code>e(title)</code>        | title in estimation output                                               |
| <code>e(title2)</code>       | secondary title in estimation output                                     |
| <code>e(clustvar)</code>     | name of cluster variable                                                 |
| <code>e(offset1)</code>      | offset for regression equation                                           |
| <code>e(offset2)</code>      | offset for selection equation                                            |
| <code>e(chi2type)</code>     | Wald; type of model $\chi^2$ test                                        |
| <code>e(chi2_ct)</code>      | Wald; type of comparison $\chi^2$ test                                   |
| <code>e(vce)</code>          | <i>vcetype</i> specified in <code>vce()</code>                           |
| <code>e(vcetype)</code>      | title used to label Std. err.                                            |
| <code>e(opt)</code>          | type of optimization                                                     |
| <code>e(which)</code>        | max or min; whether optimizer is to perform maximization or minimization |
| <code>e(ml_method)</code>    | type of ml method                                                        |
| <code>e(user)</code>         | name of likelihood-evaluator program                                     |
| <code>e(technique)</code>    | maximization technique                                                   |
| <code>e(properties)</code>   | b V                                                                      |
| <code>e(predict)</code>      | program used to implement predict                                        |
| <code>e(marginsok)</code>    | predictions allowed by margins                                           |
| <code>e(marginsnotok)</code> | predictions disallowed by margins                                        |
| <code>e(asbalanced)</code>   | factor variables <code>fvset</code> as <code>asbalanced</code>           |
| <code>e(asobserved)</code>   | factor variables <code>fvset</code> as <code>asobserved</code>           |

## Matrices

|                 |                                              |
|-----------------|----------------------------------------------|
| e(b)            | coefficient vector                           |
| e(Cns)          | constraints matrix                           |
| e(iLog)         | iteration log (up to 20 iterations)          |
| e(gradient)     | gradient vector                              |
| e(V)            | variance–covariance matrix of the estimators |
| e(V_modelbased) | model-based variance                         |

## Functions

|           |                         |
|-----------|-------------------------|
| e(sample) | marks estimation sample |
|-----------|-------------------------|

In addition to the above, the following is stored in `r()`:

## Matrices

|          |                                                                                                                       |
|----------|-----------------------------------------------------------------------------------------------------------------------|
| r(table) | matrix containing the coefficients with their standard errors, test statistics, $p$ -values, and confidence intervals |
|----------|-----------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

`heckpoisson` implements [Terza's](#) maximum likelihood estimator for the parameters of a count-data model with endogenous sample selection ([Terza 1998](#)).

Suppose that the count outcome  $y_j$  has covariates  $\mathbf{x}_j$  and that  $y_j$  has a Poisson distribution, conditional on  $\mathbf{x}_j$ , with conditional mean

$$E(y_j | \mathbf{x}_j, \epsilon_{1j}) = \mu_j = \exp(\mathbf{x}_j \boldsymbol{\beta} + \epsilon_{1j})$$

and

$$\Pr(Y = y_j | \mathbf{x}_j, \epsilon_{1j}) = \frac{\mu_j^{y_j} e^{-\mu_j}}{y_j!}$$

We only observe  $y_j$  when  $s_j$ , the selection outcome, which is the binary outcome from a latent-variable model with covariates  $\mathbf{w}_j$ , is equal to 1.

$$s_j = \begin{cases} 1, & \text{if } \mathbf{w}_j \boldsymbol{\gamma} + \epsilon_{2j} > 0 \\ 0, & \text{otherwise} \end{cases}$$

The error terms  $\epsilon_1$  and  $\epsilon_2$  are assumed to have bivariate normal distribution with zero mean and covariance matrix

$$\begin{bmatrix} \sigma^2 & \sigma\rho \\ \sigma\rho & 1 \end{bmatrix}$$

where  $\sigma$  and  $\rho$  have their usual interpretation for the bivariate normal distribution. A nonzero  $\rho$  implies that the selected sample is not representative of the whole population and therefore that inference based on standard Poisson regression using the observed sample is incorrect.

In maximum likelihood estimation,  $\ln \sigma$  and  $\operatorname{atanh} \rho$  are estimated rather than directly estimating  $\sigma$  and  $\rho$ .

$$\operatorname{atanh} \rho = \frac{1}{2} \ln \left( \frac{1 + \rho}{1 - \rho} \right)$$

The joint log likelihood is given by

$$\ln L(\boldsymbol{\theta}) = \sum_{i=1}^N [s_j \times \ln\{\Pr(y_j, s_j = 1 | \mathbf{x}_j, \mathbf{w}_j, \boldsymbol{\theta})\} + (1 - s_j) \times \ln\{\Pr(s_j = 0 | \mathbf{w}_j, \boldsymbol{\theta})\}]$$

where  $\boldsymbol{\theta}$  denotes  $(\beta, \gamma, \rho, \sigma)$  for notational simplicity.

The joint probability  $\Pr(y_j, s_j = 1 | \mathbf{x}_j, \mathbf{w}_j, \boldsymbol{\theta})$  can be obtained by integrating the conditional probability  $\Pr(y_j, s_j = 1 | \mathbf{x}_j, \mathbf{w}_j, \boldsymbol{\theta}, \epsilon_1)$  over  $\epsilon_1$ . More precisely,

$$\Pr(y_j, s_j = 1 | \mathbf{x}_j, \mathbf{w}_j, \boldsymbol{\theta}) = \int_{-\infty}^{\infty} \Pr(y_j | \mathbf{x}_j, \epsilon_1) \Phi\left(\frac{\mathbf{w}_j \gamma + \rho / \sigma \epsilon_1}{\sqrt{1 - \rho^2}}\right) \phi(\epsilon_1 / \sigma) d\epsilon_1 \quad (1)$$

where  $\phi(\cdot)$  is the standard normal density function and  $\Phi(\cdot)$  is the standard normal cumulative density function.  $\Pr(s_j = 0 | \mathbf{w}_j, \boldsymbol{\theta})$  is similarly derived.

$$\Pr(s_j = 0 | \mathbf{w}_j, \boldsymbol{\theta}) = \int_{-\infty}^{\infty} \Phi\left(-\frac{\mathbf{w}_j \gamma + \rho / \sigma \epsilon_1}{\sqrt{1 - \rho^2}}\right) \phi(\epsilon_1 / \sigma) d\epsilon_1 \quad (2)$$

The integrations in (1) and (2) have no closed form and must be approximated using Gauss–Hermite quadrature.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] [\\_robust](#), particularly *Maximum likelihood estimators* and *Methods and formulas*.

`heckpoisson` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] [Variance estimation](#).

## References

- Cameron, A. C., and P. K. Trivedi. 2013. *Regression Analysis of Count Data*. 2nd ed. New York: Cambridge University Press.
- . 2022. *Microeconometrics Using Stata*. 2nd ed. College Station, TX: Stata Press.
- Greene, W. H. 2018. *Econometric Analysis*. 8th ed. New York: Pearson.
- Terza, J. V. 1998. Estimating count data models with endogenous switching: Sample selection and endogenous treatment effects. *Journal of Econometrics* 84: 129–154. [https://doi.org/10.1016/S0304-4076\(97\)00082-1](https://doi.org/10.1016/S0304-4076(97)00082-1).

## Also see

- [R] [heckpoisson postestimation](#) — Postestimation tools for `heckpoisson`
- [R] [heckman](#) — Heckman selection model
- [R] [heckprobit](#) — Ordered probit model with sample selection
- [R] [heckprobit](#) — Probit model with sample selection
- [R] [poisson](#) — Poisson regression
- [CAUSAL] [etpoisson](#) — Poisson regression with endogenous treatment effects
- [SVY] [svy estimation](#) — Estimation commands for survey data
- [U] [20 Estimation and postestimation commands](#)

Postestimation commands  
Methods and formulas

predict  
Also see

margins

Remarks and examples

## Postestimation commands

The following postestimation commands are available after `heckpoisson`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from <code>margins</code> (profile plots, interaction plots, etc.)                                                    |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | number of events, incidence rates, probabilities, etc.                                                                                  |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results.

## predict

### Description for predict

`predict` creates new variables containing predictions such as number of events, incidence rates, conditional predicted number of events, probabilities, linear predictions, and equation-level scores.

### Menu for predict

Statistics > Postestimation

### Syntax for predict

```
predict [type] newvar [if] [in] [, statistic nooffset]
```

```
predict [type] stub* [if] [in], scores
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

| <i>statistic</i>     | Description                                                    |
|----------------------|----------------------------------------------------------------|
| Main                 |                                                                |
| <code>n</code>       | number of events; the default                                  |
| <code>ir</code>      | incidence rate                                                 |
| <code>ncond</code>   | predicted number of events conditional on $y_j$ being observed |
| <code>pr(n)</code>   | $\Pr(y_j = n)$                                                 |
| <code>pr(a,b)</code> | $\Pr(a \leq y_j \leq b)$                                       |
| <code>pse1</code>    | $\Pr(y_j \text{ observed})$                                    |
| <code>xb</code>      | linear prediction                                              |
| <code>xbse1</code>   | linear prediction for selection equation                       |

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

### Options for predict

Main

`n`, the default, calculates the predicted number of events, which is  $\exp(\mathbf{x}_j\beta + \sigma^2/2)$  if neither `offset()` nor `exposure()` was specified when the model was fit; is  $\exp(\mathbf{x}_j\beta + \sigma^2/2 + \text{offset}_j)$  if `offset()` was specified; or is  $\exp(\mathbf{x}_j\beta + \sigma^2/2) \times \text{exposure}_i$  if `exposure()` was specified.

`ir` calculates the incidence rate  $\exp(\mathbf{x}_j\beta + \sigma^2/2)$ , which is the predicted number of events when exposure is 1. Specifying `ir` is equivalent to specifying `n` when neither `offset()` nor `exposure()` was specified when the model was fit.

`ncond` calculates the predicted number of events conditional on  $y_j$  being observed, which is  $\exp(\mathbf{x}_j\beta + \sigma^2/2)\Phi(\mathbf{w}_j\gamma + \rho\sigma)/\Phi(\mathbf{w}_j\gamma)$ .

`pr(n)` calculates the probability  $\Pr(y_j = n)$ , where  $n$  is a nonnegative integer that may be specified as a number or a variable.

`pr(a, b)` calculates the probability  $\Pr(a \leq y_j \leq b)$ , where  $a$  and  $b$  are nonnegative integers that may be specified as numbers or variables;

$b$  missing ( $b \geq .$ ) means  $+\infty$ ;

`pr(20, .)` calculates  $\Pr(y_j \geq 20)$ ;

`pr(20, b)` calculates  $\Pr(y_j \geq 20)$  in observations for which  $b \geq .$  and calculates  $\Pr(20 \leq y_j \leq b)$  elsewhere.

`pr(., b)` produces a syntax error. A missing value in an observation of the variable  $a$  causes a missing value in that observation for `pr(a, b)`.

`pselect` calculates the probability of selection (or being observed):

$$\Pr(y_j \text{ observed}) = \Pr(\mathbf{w}_j\gamma + \epsilon_{2j} > 0)$$

`xb` calculates the linear prediction for the dependent count variable, which is  $\mathbf{x}_j\beta$  if neither `offset()` nor `exposure()` was specified;  $\mathbf{x}_j\beta + \text{offset}_j^\beta$  if `offset()` was specified; or  $\mathbf{x}_j\beta + \ln(\text{exposure}_j)$  if `exposure()` was specified.

`xbselect` calculates the linear prediction for the selection equation, which is  $\mathbf{w}_j\gamma$  if `offset()` was not specified in `select()` and is  $\mathbf{w}_j\gamma + \text{offset}_j^\gamma$  if `offset()` was specified in `select()`.

`nooffset` is relevant only if you specified `offset()` or `exposure()` when you fit the model. It modifies the calculations made by `predict` so that they ignore the offset or exposure variable; the linear prediction is treated as  $\mathbf{x}_j\beta$  rather than as  $\mathbf{x}_j\beta + \text{offset}_j$  or  $\mathbf{x}_j\beta + \ln(\text{exposure}_j)$ .

`scores` calculates equation-level score variables.

The first new variable will contain  $\partial \ln L / \partial (\mathbf{x}_j\beta)$ .

The second new variable will contain  $\partial \ln L / \partial (\mathbf{w}_j\gamma)$ .

The third new variable will contain  $\partial \ln L / \partial \text{atanh } \rho$ .

The fourth new variable will contain  $\partial \ln L / \partial \ln \sigma$ .

# margins

## Description for margins

`margins` estimates margins of response for number of events, incidence rates, conditional predicted number of events, probabilities, and linear predictions.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i>                    | Description                                                    |
|-------------------------------------|----------------------------------------------------------------|
| <code>n</code>                      | number of events; the default                                  |
| <code>ir</code>                     | incidence rate                                                 |
| <code>ncond</code>                  | predicted number of events conditional on $y_j$ being observed |
| <code>pr(<i>n</i>)</code>           | $\Pr(y_j = n)$                                                 |
| <code>pr(<i>a</i>, <i>b</i>)</code> | $\Pr(a \leq y_j \leq b)$                                       |
| <code>pselect</code>                | $\Pr(y_j \text{ observed})$                                    |
| <code>xb</code>                     | linear prediction                                              |
| <code>xbselect</code>               | linear prediction for selection equation                       |

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [R] [margins](#).

## Remarks and examples

### ► Example 1: Obtaining margins for a count model with selection

In [example 1](#) of [R] [heckpoisson](#), we fit a model for the number of patents. In that example, we are interested in the effect of R&D expenditures on the number of patents received by a firm. We continue that example to determine the magnitude of the effect of R&D expenditures on the number of patents and compare this effect for IT and non-IT sectors.

After reading in the data and fitting the model, we use `margins` to estimate the effect of an increase of a million dollars in R&D expenditures (`expenditure`) on the number of patents (`npatents`) for firms in the IT and non-IT sectors (`tech`).

To do this, we use the `at()` option of `margins`. We use the observed values in our first scenario, so we tell `margins` to set `expenditure` equal to itself. For our second scenario, we tell `margins` to set `expenditure` equal to the observed value plus 1 because expenditures are measured in millions of dollars. We include the `post` option so that we can perform additional calculations later.

```
. use https://www.stata-press.com/data/r19/patent
(Fictional data on patents and R&D)
. quietly heckpoisson npatents expenditure i.tech,
> select(applied = expenditure size i.tech)
. margins i.tech, at(expenditure = generate(expenditure))
> at(expenditure = generate(expenditure+1)) post
Predictive margins Number of obs = 10,000
Model VCE: OIM
Expression: Predicted number of events, predict()
1._at: expenditure = expenditure
2._at: expenditure = expenditure+1
```

|             | Delta-method |           |       |       | [95% conf. interval] |          |
|-------------|--------------|-----------|-------|-------|----------------------|----------|
|             | Margin       | std. err. | z     | P> z  |                      |          |
| _at#tech    |              |           |       |       |                      |          |
| 1 #         |              |           |       |       |                      |          |
| Non-IT se.. | 1.276213     | .0556644  | 22.93 | 0.000 | 1.167112             | 1.385313 |
| 1#IT sector | 2.287013     | .080119   | 28.55 | 0.000 | 2.129983             | 2.444044 |
| 2 #         |              |           |       |       |                      |          |
| Non-IT se.. | 2.099539     | .131364   | 15.98 | 0.000 | 1.84207              | 2.357007 |
| 2#IT sector | 3.76244      | .2226221  | 16.90 | 0.000 | 3.326109             | 4.198771 |

The output indicates that the expected number of patents for non-IT firms is about 1.28 compared with 2.29 for firms in the IT sector.

The second scenario shows the expected number of patents after our hypothetical increase in R&D expenditures. In the non-IT sector, the expected number of patents received would be about 2.10 compared with 3.76 in the IT sector. It appears that increasing expenditures may have a larger effect for IT firms—the difference between the two scenarios is 1.47 for IT firms and only 0.82 for non-IT firms. We can test whether the effect of increasing expenditures is different for IT and non-IT firms. We use `lincom` to obtain an estimate of the difference in the differences between scenarios for the two sectors and a test of its significance. We ask for the differences by referring to the scenarios as 1.\_at and 2.\_at and by referring to the sector using the value that corresponds to the IT sector indicator, 1.tech for IT firms and 0.tech otherwise.

```
. lincom (_b[2._at#1.tech] - _b[1._at#1.tech]) -
> (_b[2._at#0.tech] - _b[1._at#0.tech])
(1) 1bn._at#0bn.tech - 1bn._at#1.tech - 2._at#0bn.tech + 2._at#1.tech = 0
```

|     | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |          |
|-----|-------------|-----------|------|-------|----------------------|----------|
| (1) | .6521006    | .0917299  | 7.11 | 0.000 | .4723134             | .8318878 |

We find that the expected effect of increasing R&D expenditures by one million dollars is 0.65 patents larger for IT firms than for non-IT firms, and this difference is significantly different from 0.



## Methods and formulas

Suppose that the count outcome  $y_j$  has covariates  $\mathbf{x}_j$  and the selection outcome  $s_j$  has covariates  $\mathbf{w}_j$ .  $y_j$  is assumed to have a Poisson distribution, conditional on  $\mathbf{x}_j$ , with conditional mean

$$E(y_j|\mathbf{x}_j, \epsilon_{1j}) = \mu_j = \exp(\mathbf{x}_j\boldsymbol{\beta} + \epsilon_{1j})$$

$s_j$  is a binary outcome from a latent-variable model:

$$s_j = \begin{cases} 1, & \text{if } \mathbf{w}_j\boldsymbol{\gamma} + \epsilon_{2j} > 0 \\ 0, & \text{otherwise} \end{cases}$$

The expectation of  $y_j$  conditional on covariates  $\mathbf{x}_j$  for the whole population is

$$E(y_j|\mathbf{x}_j) = \exp(\mathbf{x}_j\boldsymbol{\beta} + \sigma^2/2)$$

Furthermore, if we want the expectation of  $y_j$  only if it was observed, then the formula is

$$E(y_j|\mathbf{x}_j, \mathbf{w}_j, s_j = 1) = \exp(\mathbf{x}_j\boldsymbol{\beta} + \sigma^2/2) \frac{\Phi(\mathbf{w}_j\boldsymbol{\gamma} + \rho\sigma)}{\Phi(\mathbf{w}_j\boldsymbol{\gamma})}$$

We note that if  $\rho = 0$ , this expectation is the same as its population version.

We can also predict the probability of  $y_j$  conditional on  $\mathbf{x}_j$ . Note that although  $y_j$  is Poisson-distributed conditional on  $\epsilon_1$  and  $\mathbf{x}_j$ , the distribution of  $y_j$  is unknown unconditional on  $\epsilon_1$ .

$$\Pr(y_j = n|\mathbf{x}_j) = \int_{-\infty}^{\infty} \Pr(y_j = n|\mathbf{x}_j, \epsilon_1) \phi(\epsilon_1/\sigma) d\epsilon_1$$

As in the implementation of log likelihood, we approximate this integral by Gauss–Hermite quadrature.

## Also see

[R] [heckpoisson](#) — Poisson regression with sample selection

[U] [20 Estimation and postestimation commands](#)

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

## Description

`heckprobit` fits maximum-likelihood probit models with sample selection.

## Quick start

Probit model of  $y$  on  $x$  with sample selection indicated by binary variable `selected` and predicted by  $v$

```
heckprobit y x, select(selected = v x)
```

Suppress iteration log

```
heckprobit y x, select(selected = v x) nolog
```

With cluster-robust standard errors for clustering by levels of `cvar`

```
heckprobit y x, select(selected = v x) vce(cluster cvar)
```

## Menu

Statistics > Sample-selection models > Probit model with sample selection

## Syntax

```
heckprobit depvar indepvars [if] [in] [weight],
 select([depvars =] varlists [, noconstant offset(varnameo)]) [options]
```

| <i>options</i>                            | Description                                                                                                                                      |
|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                              |                                                                                                                                                  |
| * <u>select</u> ( )                       | specify selection equation: dependent and independent variables; whether to have constant term and offset variable                               |
| <u>noconstant</u>                         | suppress constant term                                                                                                                           |
| <u>offset</u> ( <i>varname</i> )          | include <i>varname</i> in model with coefficient constrained to 1                                                                                |
| <u>constraints</u> ( <i>constraints</i> ) | apply specified linear constraints                                                                                                               |
| <b>SE/Robust</b>                          |                                                                                                                                                  |
| vce( <i>vcetype</i> )                     | <i>vcetype</i> may be oim, <u>robust</u> , <u>cluster</u> <i>clustvar</i> , opg, <u>bootstrap</u> , or <u>jackknife</u>                          |
| <b>Reporting</b>                          |                                                                                                                                                  |
| <u>level</u> (#)                          | set confidence level; default is level(95)                                                                                                       |
| <u>first</u>                              | report first-step probit estimates                                                                                                               |
| <u>lrmodel</u>                            | perform the likelihood-ratio model test instead of the default Wald test                                                                         |
| <u>nocnsreport</u>                        | do not display constraints                                                                                                                       |
| <u>display_options</u>                    | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| <b>Maximization</b>                       |                                                                                                                                                  |
| <u>maximize_options</u>                   | control the maximization process; seldom used                                                                                                    |
| <u>collinear</u>                          | keep collinear variables                                                                                                                         |
| <u>coeflegend</u>                         | display legend instead of statistics                                                                                                             |

\*select() is required.

The full specification is `select([depvars = ] varlists [ , noconstant offset(varnameo) ])`.

*indepvars* and *varlist*<sub>s</sub> may contain factor variables; see [U] 11.4.3 Factor variables.

*depvar*, *indepvars*, *depvar*<sub>s</sub>, and *varlist*<sub>s</sub> may contain time-series operators; see [U] 11.4.4 Time-series varlists.

bayes, bayesboot, bootstrap, by, collect, fp, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands. For more details, see [BAYES] bayes: heckprobit.

Weights are not allowed with the bootstrap prefix; see [R] bootstrap.

vce(), first, lrmodel, and weights are not allowed with the svy prefix; see [SVY] svy.

pweights, fweights, and iweights are allowed; see [U] 11.1.6 weight.

collinear and coeflegend do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Options

### Model

`select( [ depvars = ] varlists [ , noconstant offset(varnameo) ] )` specifies the variables and options for the selection equation. It is an integral part of specifying a selection model and is required. The selection equation should contain at least one variable that is not in the outcome equation.

If *depvar*<sub>s</sub> is specified, it should be coded as 0 or 1, 0 indicating an observation not selected and 1 indicating a selected observation. If *depvar*<sub>s</sub> is not specified, observations for which *depvar* is not missing are assumed selected, and those for which *depvar* is missing are assumed not selected.

`noconstant` suppresses the selection constant term (intercept).

`offset(varnameo)` specifies that selection offset *varname*<sub>o</sub> be included in the model with the coefficient constrained to be 1.

`noconstant`, `offset(varname)`, `constraints(constraints)`; see [R] [Estimation options](#).

### SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

### Reporting

`level(#)`; see [R] [Estimation options](#).

`first` specifies that the first-step probit estimates of the selection equation be displayed before estimation.

`lrmodel`, `nocnsreport`; see [R] [Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

### Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

Setting the optimization type to `technique(bhhh)` resets the default *vcetype* to `vce(opg)`.

The following options are available with `heckprobit` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

The probit model with sample selection (Van de Ven and Van Pragg 1981) assumes that there exists an underlying relationship

$$y_j^* = \mathbf{x}_j\boldsymbol{\beta} + u_{1j} \quad \textit{latent equation}$$

such that we observe only the binary outcome

$$y_j^{\text{probit}} = (y_j^* > 0) \quad \textit{probit equation}$$

The dependent variable, however, is not always observed. Rather, the dependent variable for observation  $j$  is observed if

$$y_j^{\text{select}} = (\mathbf{z}_j\boldsymbol{\gamma} + u_{2j} > 0) \quad \textit{selection equation}$$

where

$$u_1 \sim N(0, 1)$$

$$u_2 \sim N(0, 1)$$

$$\text{corr}(u_1, u_2) = \rho$$

When  $\rho \neq 0$ , standard probit techniques applied to the first equation yield biased results. `heckprobit` provides consistent, asymptotically efficient estimates for all the parameters in such models.

For the model to be well identified, the selection equation should have at least one variable that is not in the probit equation. Otherwise, the model is identified only by functional form, and the coefficients have no structural interpretation.

### ▷ Example 1

We use the data from Pindyck and Rubinfeld (1998). In this dataset, the variables are whether children attend private school (`private`), number of years the family has been at the present residence (`years`), log of property tax (`logptax`), log of income (`loginc`), and whether one voted for an increase in property taxes (`vote`).

In this example, we alter the meaning of the data. Here we assume that we observe whether children attend private school only if the family votes for increasing the property taxes. This assumption is not true in the dataset, and we make it only to illustrate the use of this command.

We observe whether children attend private school only if the head of household voted for an increase in property taxes. We assume that the vote is affected by the number of years in residence, the current property taxes paid, and the household income. We wish to model whether children are sent to private school on the basis of the number of years spent in the current residence and the current property taxes paid.

```
. use https://www.stata-press.com/data/r19/school
. heckprobit private years logptax, select(vote=years loginc logptax)
```

Fitting probit model:

```
Iteration 0: Log likelihood = -17.122381
Iteration 1: Log likelihood = -16.243974
(output omitted)
Iteration 5: Log likelihood = -15.883655
```

Fitting selection model:

```
Iteration 0: Log likelihood = -63.036914
Iteration 1: Log likelihood = -58.534843
Iteration 2: Log likelihood = -58.497292
Iteration 3: Log likelihood = -58.497288
Comparison: Log likelihood = -74.380943
```

Fitting starting values:

```
Iteration 0: Log likelihood = -40.895684
Iteration 1: Log likelihood = -16.654497
(output omitted)
Iteration 6: Log likelihood = -15.753765
```

Fitting full model:

```
Iteration 0: Log likelihood = -75.010619 (not concave)
Iteration 1: Log likelihood = -74.287758
Iteration 2: Log likelihood = -74.250143
Iteration 3: Log likelihood = -74.245088
Iteration 4: Log likelihood = -74.244973
Iteration 5: Log likelihood = -74.244973
```

```
Probit model with sample selection Number of obs = 95
 Selected = 59
 Nonselected = 36
 Wald chi2(2) = 1.04
Log likelihood = -74.24497 Prob > chi2 = 0.5935
```

|                | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|----------------|-------------|-----------|-------|-------|----------------------|-----------|
| <b>private</b> |             |           |       |       |                      |           |
| years          | -.1142596   | .1461715  | -0.78 | 0.434 | -.4007505            | .1722313  |
| logptax        | .3516101    | 1.016483  | 0.35  | 0.729 | -1.64066             | 2.34388   |
| _cons          | -2.780667   | 6.905827  | -0.40 | 0.687 | -16.31584            | 10.75451  |
| <b>vote</b>    |             |           |       |       |                      |           |
| years          | -.0167511   | .0147735  | -1.13 | 0.257 | -.0457067            | .0122045  |
| loginc         | .9923023    | .4430008  | 2.24  | 0.025 | .1240368             | 1.860568  |
| logptax        | -1.278783   | .5717545  | -2.24 | 0.025 | -2.399401            | -.1581646 |
| _cons          | -.5458205   | 4.070417  | -0.13 | 0.893 | -8.523692            | 7.432051  |
| /athrho        | -.8663164   | 1.450017  | -0.60 | 0.550 | -3.708298            | 1.975665  |
| rho            | -.6994978   | .7405281  |       |       | -.9987983            | .9622674  |

LR test of indep. eqns. (rho = 0): chi2(1) = 0.27      Prob > chi2 = 0.6020

The output shows several iteration logs. The first iteration log corresponds to running the probit model for those observations in the sample where we have observed the outcome. The second iteration log corresponds to running the selection probit model, which models whether we observe our outcome of interest. If  $\rho = 0$ , the sum of the log likelihoods from these two models will equal the log likelihood of the probit model with sample selection; this sum is printed in the iteration log as the comparison log likelihood. The third iteration log shows starting values for the iterations.

The final iteration log is for fitting the full probit model with sample selection. A likelihood-ratio test of the log likelihood for this model and the comparison log likelihood is presented at the end of the output. If we had specified the `vce(robust)` option, this test would be presented as a Wald test instead of as a likelihood-ratio test.

◀

## ▷ Example 2

In [example 1](#), we could have obtained robust standard errors by specifying the `vce(robust)` option. We do this here and also eliminate the iteration logs by using the `noolog` option:

```
. heckprobit private years logptax, sel(vote=years loginc logptax) vce(robust)
> noolog
Probit model with sample selection Number of obs = 95
 Selected = 59
 Nonselected = 36
 Wald chi2(2) = 2.55
 Prob > chi2 = 0.2798
Log pseudolikelihood = -74.24497
```

|         | Robust      |           | z     | P> z  | [95% conf. interval] |           |
|---------|-------------|-----------|-------|-------|----------------------|-----------|
|         | Coefficient | std. err. |       |       |                      |           |
| private |             |           |       |       |                      |           |
| years   | -.1142596   | .1113968  | -1.03 | 0.305 | -.3325934            | .1040741  |
| logptax | .3516101    | .7358211  | 0.48  | 0.633 | -1.090573            | 1.793793  |
| _cons   | -2.780667   | 4.786652  | -0.58 | 0.561 | -12.16233            | 6.600998  |
| vote    |             |           |       |       |                      |           |
| years   | -.0167511   | .0173344  | -0.97 | 0.334 | -.0507259            | .0172237  |
| loginc  | .9923023    | .4228042  | 2.35  | 0.019 | .1636213             | 1.820983  |
| logptax | -1.278783   | .5095156  | -2.51 | 0.012 | -2.277415            | -.2801506 |
| _cons   | -.5458205   | 4.54389   | -0.12 | 0.904 | -9.451682            | 8.360041  |
| /athrho | -.8663164   | 1.63062   | -0.53 | 0.595 | -4.062272            | 2.329639  |
| rho     | -.6994978   | .8327621  |       |       | -.9994078            | .9812312  |

```
Wald test of indep. eqns. (rho = 0): chi2(1) = 0.28 Prob > chi2 = 0.5952
```

Regardless of whether we specify the `vce(robust)` option, the outcome is not significantly different from the outcome obtained by fitting the probit and selection models separately. This result is not surprising because the selection mechanism estimated was invented for the example rather than borne from any economic theory.

◀

## Stored results

heckprobit stores the following in `e()`:

### Scalars

|                               |                                           |
|-------------------------------|-------------------------------------------|
| <code>e(N)</code>             | number of observations                    |
| <code>e(N_selected)</code>    | number of selected observations           |
| <code>e(N_nonselected)</code> | number of nonselected observations        |
| <code>e(k)</code>             | number of parameters                      |
| <code>e(k_eq)</code>          | number of equations in <code>e(b)</code>  |
| <code>e(k_eq_model)</code>    | number of equations in overall model test |
| <code>e(k_aux)</code>         | number of auxiliary parameters            |
| <code>e(k_dv)</code>          | number of dependent variables             |
| <code>e(df_m)</code>          | model degrees of freedom                  |
| <code>e(ll)</code>            | log likelihood                            |
| <code>e(ll_0)</code>          | log likelihood, constant-only model       |
| <code>e(ll_c)</code>          | log likelihood, comparison model          |
| <code>e(N_clust)</code>       | number of clusters                        |
| <code>e(chi2)</code>          | $\chi^2$                                  |
| <code>e(chi2_c)</code>        | $\chi^2$ for comparison test              |
| <code>e(p)</code>             | $p$ -value for model test                 |
| <code>e(p_c)</code>           | $p$ -value for comparison test            |
| <code>e(rho)</code>           | $\rho$                                    |
| <code>e(rank)</code>          | rank of $e(V)$                            |
| <code>e(rank0)</code>         | rank of $e(V)$ for constant-only model    |
| <code>e(ic)</code>            | number of iterations                      |
| <code>e(rc)</code>            | return code                               |
| <code>e(converged)</code>     | 1 if converged, 0 otherwise               |

### Macros

|                            |                                                                          |
|----------------------------|--------------------------------------------------------------------------|
| <code>e(cmd)</code>        | heckprobit                                                               |
| <code>e(cmdline)</code>    | command as typed                                                         |
| <code>e(depvar)</code>     | names of dependent variables                                             |
| <code>e(wtype)</code>      | weight type                                                              |
| <code>e(wexp)</code>       | weight expression                                                        |
| <code>e(title)</code>      | title in estimation output                                               |
| <code>e(clustvar)</code>   | name of cluster variable                                                 |
| <code>e(offset1)</code>    | offset for regression equation                                           |
| <code>e(offset2)</code>    | offset for selection equation                                            |
| <code>e(chi2type)</code>   | Wald or LR; type of model $\chi^2$ test                                  |
| <code>e(chi2_ct)</code>    | type of comparison $\chi^2$ test                                         |
| <code>e(vce)</code>        | <i>vctype</i> specified in <code>vce()</code>                            |
| <code>e(vcetype)</code>    | title used to label Std. err.                                            |
| <code>e(opt)</code>        | type of optimization                                                     |
| <code>e(which)</code>      | max or min; whether optimizer is to perform maximization or minimization |
| <code>e(ml_method)</code>  | type of ml method                                                        |
| <code>e(user)</code>       | name of likelihood-evaluator program                                     |
| <code>e(technique)</code>  | maximization technique                                                   |
| <code>e(properties)</code> | <code>b V</code>                                                         |
| <code>e(predict)</code>    | program used to implement <code>predict</code>                           |
| <code>e(asbalanced)</code> | factor variables <code>fvset</code> as <code>asbalanced</code>           |
| <code>e(asobserved)</code> | factor variables <code>fvset</code> as <code>asobserved</code>           |

### Matrices

|                              |                                              |
|------------------------------|----------------------------------------------|
| <code>e(b)</code>            | coefficient vector                           |
| <code>e(Cns)</code>          | constraints matrix                           |
| <code>e(ilog)</code>         | iteration log (up to 20 iterations)          |
| <code>e(gradient)</code>     | gradient vector                              |
| <code>e(V)</code>            | variance-covariance matrix of the estimators |
| <code>e(V_modelbased)</code> | model-based variance                         |

Functions  
`e(sample)` marks estimation sample

In addition to the above, the following is stored in `r()`:

Matrices  
`r(table)` matrix containing the coefficients with their standard errors, test statistics,  $p$ -values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

Van de Ven and Van Pragg (1981) provide an introduction and an explanation of this model.

The probit equation is

$$y_j = (\mathbf{x}_j\boldsymbol{\beta} + u_{1j} > 0)$$

The selection equation is

$$\mathbf{z}_j\boldsymbol{\gamma} + u_{2j} > 0$$

where

$$u_1 \sim N(0, 1)$$

$$u_2 \sim N(0, 1)$$

$$\text{corr}(u_1, u_2) = \rho$$

The log likelihood is

$$\begin{aligned} \ln L = & \sum_{\substack{j \in S \\ y_j \neq 0}} w_j \ln \left\{ \Phi_2 \left( x_j\boldsymbol{\beta} + \text{offset}_j^\beta, z_j\boldsymbol{\gamma} + \text{offset}_j^\gamma, \rho \right) \right\} \\ & + \sum_{\substack{j \in S \\ y_j = 0}} w_j \ln \left\{ \Phi_2 \left( -x_j\boldsymbol{\beta} + \text{offset}_j^\beta, z_j\boldsymbol{\gamma} + \text{offset}_j^\gamma, -\rho \right) \right\} \\ & + \sum_{j \notin S} w_j \ln \left\{ 1 - \Phi \left( z_j\boldsymbol{\gamma} + \text{offset}_j^\gamma \right) \right\} \end{aligned}$$

where  $S$  is the set of observations for which  $y_j$  is observed,  $\Phi_2(\cdot)$  is the cumulative bivariate normal distribution function (with mean  $[0 \ 0]'$ ),  $\Phi(\cdot)$  is the standard cumulative normal, and  $w_j$  is an optional weight for observation  $j$ .

In the maximum likelihood estimation,  $\rho$  is not directly estimated. Directly estimated is  $\text{atanh } \rho$ :

$$\text{atanh } \rho = \frac{1}{2} \ln \left( \frac{1 + \rho}{1 - \rho} \right)$$

From the form of the likelihood, it is clear that if  $\rho = 0$ , the log likelihood for the probit model with sample selection is equal to the sum of the probit model for the outcome  $y$  and the selection model. We can perform a likelihood-ratio test by comparing the likelihood of the full model with the sum of the log likelihoods for the probit and selection models.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] [\\_robust](#), particularly *Maximum likelihood estimators* and *Methods and formulas*.

`heckprobit` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] [Variance estimation](#).

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## Also see

- [R] [heckprobit postestimation](#) — Postestimation tools for heckprobit
- [R] [heckman](#) — Heckman selection model
- [R] [heckoprobit](#) — Ordered probit model with sample selection
- [R] [heckpoisson](#) — Poisson regression with sample selection
- [R] [probit](#) — Probit regression
- [BAYES] [bayes: heckprobit](#) — Bayesian probit model with sample selection
- [CAUSAL] [etregress](#) — Linear regression with endogenous treatment effects
- [ERM] [eprobit](#) — Extended probit regression
- [SVY] [svy estimation](#) — Estimation commands for survey data
- [U] [20 Estimation and postestimation commands](#)

## Postestimation commands

The following postestimation commands are available after `heckprobit`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | probabilities, linear predictions and their SEs, etc.                                                                                   |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`hausman` and `lrtest` are not appropriate with `svy` estimation results.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as probabilities, linear predictions, and standard errors.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic nooffset]
```

```
predict [type] stub* [if] [in] , scores
```

| <i>statistic</i>         | Description                                                                                                                                      |
|--------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| Main                     |                                                                                                                                                  |
| <u>p</u> margin          | $\Phi(\mathbf{x}_j\mathbf{b})$ , success probability; the default                                                                                |
| <u>p</u> 11              | $\Phi_2(\mathbf{x}_j\mathbf{b}, \mathbf{z}_j\mathbf{g}, \rho)$ , predicted probability $\Pr(y_j^{\text{probit}} = 1, y_j^{\text{select}} = 1)$   |
| <u>p</u> 10              | $\Phi_2(\mathbf{x}_j\mathbf{b}, -\mathbf{z}_j\mathbf{g}, -\rho)$ , predicted probability $\Pr(y_j^{\text{probit}} = 1, y_j^{\text{select}} = 0)$ |
| <u>p</u> 01              | $\Phi_2(-\mathbf{x}_j\mathbf{b}, \mathbf{z}_j\mathbf{g}, -\rho)$ , predicted probability $\Pr(y_j^{\text{probit}} = 0, y_j^{\text{select}} = 1)$ |
| <u>p</u> 00              | $\Phi_2(-\mathbf{x}_j\mathbf{b}, -\mathbf{z}_j\mathbf{g}, \rho)$ , predicted probability $\Pr(y_j^{\text{probit}} = 0, y_j^{\text{select}} = 0)$ |
| <u>p</u> sel             | $\Phi(\mathbf{z}_j\mathbf{g})$ , selection probability                                                                                           |
| <u>p</u> cond            | $\Phi_2(\mathbf{x}_j\mathbf{b}, \mathbf{z}_j\mathbf{g}, \rho) / \Phi(\mathbf{z}_j\mathbf{g})$ , probability of success conditional on selection  |
| <u>x</u> b               | linear prediction                                                                                                                                |
| <u>s</u> tdp             | standard error of the linear prediction                                                                                                          |
| <u>x</u> b <u>s</u> el   | linear prediction for selection equation                                                                                                         |
| <u>s</u> td <u>p</u> sel | standard error of the linear prediction for selection equation                                                                                   |

$\Phi(\cdot)$  is the standard normal distribution function, and  $\Phi_2(\cdot)$  is the bivariate normal distribution function.

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

## Options for predict

Main

`pmargin`, the default, calculates the univariate (marginal) predicted probability of success

$$\Pr(y_j^{\text{probit}} = 1).$$

`p11` calculates the bivariate predicted probability  $\Pr(y_j^{\text{probit}} = 1, y_j^{\text{select}} = 1)$ .

`p10` calculates the bivariate predicted probability  $\Pr(y_j^{\text{probit}} = 1, y_j^{\text{select}} = 0)$ .

`p01` calculates the bivariate predicted probability  $\Pr(y_j^{\text{probit}} = 0, y_j^{\text{select}} = 1)$ .

`p00` calculates the bivariate predicted probability  $\Pr(y_j^{\text{probit}} = 0, y_j^{\text{select}} = 0)$ .

`pse1` calculates the univariate (marginal) predicted probability of selection  $\Pr(y_j^{\text{select}} = 1)$ .

`pcond` calculates the conditional (on selection) predicted probability of success

$$\Pr(y_j^{\text{probit}} = 1, y_j^{\text{select}} = 1) / \Pr(y_j^{\text{select}} = 1).$$

`xb` calculates the probit linear prediction  $\mathbf{x}_j\mathbf{b}$ .

`stdp` calculates the standard error of the prediction, which can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. The standard error of the prediction is also referred to as the standard error of the fitted value.

`xbse1` calculates the linear prediction for the selection equation.

`stdpse1` calculates the standard error of the linear prediction for the selection equation.

`scores` calculates equation-level score variables.

The first new variable will contain  $\partial \ln L / \partial (\mathbf{x}_j\boldsymbol{\beta})$ .

The second new variable will contain  $\partial \ln L / \partial (\mathbf{z}_j\boldsymbol{\gamma})$ .

The third new variable will contain  $\partial \ln L / \partial (\text{atanh } \rho)$ .

`nooffset` is relevant only if you specified `offset` (*varname*) for `heckprobit`. It modifies the calculations made by `predict` so that they ignore the offset variable; the linear prediction is treated as  $\mathbf{x}_j\mathbf{b}$  rather than as  $\mathbf{x}_j\mathbf{b} + \text{offset}_j$ .

# margins

## Description for margins

margins estimates margins of response for probabilities and linear predictions.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i>      | Description                                                                                                                                      |
|-----------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| <u>p</u> margin       | $\Phi(\mathbf{x}_j\mathbf{b})$ , success probability; the default                                                                                |
| p11                   | $\Phi_2(\mathbf{x}_j\mathbf{b}, \mathbf{z}_j\mathbf{g}, \rho)$ , predicted probability $\Pr(y_j^{\text{probit}} = 1, y_j^{\text{select}} = 1)$   |
| p10                   | $\Phi_2(\mathbf{x}_j\mathbf{b}, -\mathbf{z}_j\mathbf{g}, -\rho)$ , predicted probability $\Pr(y_j^{\text{probit}} = 1, y_j^{\text{select}} = 0)$ |
| p01                   | $\Phi_2(-\mathbf{x}_j\mathbf{b}, \mathbf{z}_j\mathbf{g}, -\rho)$ , predicted probability $\Pr(y_j^{\text{probit}} = 0, y_j^{\text{select}} = 1)$ |
| p00                   | $\Phi_2(-\mathbf{x}_j\mathbf{b}, -\mathbf{z}_j\mathbf{g}, \rho)$ , predicted probability $\Pr(y_j^{\text{probit}} = 0, y_j^{\text{select}} = 0)$ |
| <u>p</u> sel          | $\Phi(\mathbf{z}_j\mathbf{g})$ , selection probability                                                                                           |
| <u>p</u> cond         | $\Phi_2(\mathbf{x}_j\mathbf{b}, \mathbf{z}_j\mathbf{g}, \rho) / \Phi(\mathbf{z}_j\mathbf{g})$ , probability of success conditional on selection  |
| xb                    | linear prediction                                                                                                                                |
| <u>x</u> b <u>sel</u> | linear prediction for selection equation                                                                                                         |
| stdp                  | not allowed with margins                                                                                                                         |
| std <u>p</u> sel      | not allowed with margins                                                                                                                         |

Statistics not allowed with margins are functions of stochastic quantities other than  $\mathbf{e}(\mathbf{b})$ .

For the full syntax, see [\[R\] margins](#).

## Remarks and examples

### ▷ Example 1

It is instructive to compare the marginal predicted probabilities with the predicted probabilities that we would obtain by ignoring the selection mechanism. To compare the two approaches, we will synthesize data so that we know the “true” predicted probabilities.

First, we need to generate correlated error terms, which we can do using a standard Cholesky decomposition approach. For our example, we will clear any data from memory and then generate errors that have a correlation of 0.5 by using the following commands. We set the seed so that interested readers can type in these same commands and obtain the same results.

```
. set seed 12309
. set obs 5000
Number of observations (_N) was 0, now 5,000.
. generate c1 = rnormal()
. generate c2 = rnormal()
. matrix P = (1, .5 \ .5, 1)
. matrix A = cholesky(P)
. local fac1 = A[2,1]
. local fac2 = A[2,2]
. generate u1 = c1
. generate u2 = 'fac1'*c1 + 'fac2'*c2
```

We can check that the errors have the correct correlation by using the `correlate` command. We will also normalize the errors so that they have a standard deviation of one, so we can generate a bivariate probit model with known coefficients. We do that with the following commands:

```
. correlate u1 u2
(obs=5,000)
```

|    | u1     | u2     |
|----|--------|--------|
| u1 | 1.0000 |        |
| u2 | 0.5012 | 1.0000 |

```
. summarize u1
(output omitted)
. replace u1 = u1/r(sd)
(5,000 real changes made)
. summarize u2
(output omitted)
. replace u2 = u2/r(sd)
(5,000 real changes made)
. drop c1 c2
. generate x1 = runiform()-.5
. generate x2 = runiform()+1/3
. generate y1s = 0.5 + 4*x1 + u1
. generate y2s = 3 - 3*x2 + .5*x1 + u2
. generate y1 = (y1s>0)
. generate y2 = (y2s>0)
```

We have now created two dependent variables,  $y_1$  and  $y_2$ , which are defined by our specified coefficients. We also included error terms for each equation, and the error terms are correlated. We run `heckprobit` to verify that the data have been correctly generated according to the model

$$y_1 = 0.5 + 4x_1 + u_1$$

$$y_2 = 3 + 0.5x_1 - 3x_2 + u_2$$

where we assume that  $y_1$  is observed only if  $y_2 = 1$ .

```
. heckprobit y1 x1, sel(y2 = x1 x2) nolog
Probit model with sample selection Number of obs = 5,000
 Selected = 3,182
 Nonselected = 1,818
 Wald chi2(1) = 947.76
 Prob > chi2 = 0.0000
Log likelihood = -3612.401
```

|           | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |           |
|-----------|-------------|-----------|--------|-------|----------------------|-----------|
| <b>y1</b> |             |           |        |       |                      |           |
| x1        | 4.015564    | .130436   | 30.79  | 0.000 | 3.759914             | 4.271214  |
| _cons     | .4795158    | .0471276  | 10.17  | 0.000 | .3871473             | .5718842  |
| <b>y2</b> |             |           |        |       |                      |           |
| x1        | .5361114    | .0711951  | 7.53   | 0.000 | .3965715             | .6756513  |
| x2        | -3.017537   | .0817541  | -36.91 | 0.000 | -3.177772            | -2.857302 |
| _cons     | 2.990145    | .0765942  | 39.04  | 0.000 | 2.840024             | 3.140267  |
| /athrho   | .5339516    | .0854577  | 6.25   | 0.000 | .3664575             | .7014457  |
| rho       | .4883959    | .0650735  |        |       | .3508892             | .6052846  |

```
LR test of indep. eqns. (rho = 0): chi2(1) = 41.36 Prob > chi2 = 0.0000
```

Now that we have verified that we have generated data according to a known model, we can obtain and then compare predicted probabilities from the probit model with sample selection and a (usual) probit model.

```
. predict pmarg
(option pmargin assumed; Pr(y1=1))
. probit y1 x1 if y2==1
(output omitted)
. predict phat
(option pr assumed; Pr(y1))
```

Using the (marginal) predicted probabilities from the probit model with sample selection (`pmarg`) and the predicted probabilities from the (usual) probit model (`phat`), we can also generate the “true” predicted probabilities from the synthesized `y1s` variable and then compare the predicted probabilities:

```
. generate ptrue = normal(y1s)
. summarize pmarg ptrue phat
```

| Variable           | Obs   | Mean     | Std. dev. | Min      | Max     |
|--------------------|-------|----------|-----------|----------|---------|
| <code>pmarg</code> | 5,000 | .6089004 | .3249993  | .0632337 | .99354  |
| <code>ptrue</code> | 5,000 | .5967872 | .3534232  | 2.78e-07 | 1       |
| <code>phat</code>  | 5,000 | .6588519 | .3113716  | .0910951 | .997021 |

Here we see that ignoring the selection mechanism (comparing the `phat` variable with the true `ptrue` variable) results in predicted probabilities that are much higher than the true values. Looking at the marginal predicted probabilities from the model with sample selection, however, results in more accurate predictions.

◀

## Also see

[R] [heckprobit](#) — Probit model with sample selection

[U] [20 Estimation and postestimation commands](#)

## Description

The `help` command displays help information about the specified command or topic. `help` launches a new Viewer to display help for the specified command or topic or displays help on the console in Stata for Unix(console). If `help` is not followed by a command or a topic name, Stata displays advice for using the help system and documentation.

## Menu

Help > Stata command...

## Syntax

```
help [command_or_topic_name] [, nonew name(viewername) marker(markername)]
```

## Options

`nonew` specifies that a new Viewer window not be opened for the help topic if a Viewer window is already open. The default is for a new Viewer window to be opened each time `help` is typed so that multiple help files may be viewed at once. `nonew` causes the help file to be displayed in the topmost open Viewer.

`name(viewername)` specifies that help be displayed in a Viewer window named *viewername*. If the named window already exists, its contents will be replaced. If the named window does not exist, it will be created.

`marker(markername)` specifies that the help file be opened to the position of *markername* within the help file.

## Remarks and examples

To obtain help for any Stata command, type `help command` or select **Help > Stata command...** and fill in *command*.

help is best explained by examples.

| To obtain help for ...           | type                                                |
|----------------------------------|-----------------------------------------------------|
| regress                          | help regress                                        |
| postestimation tools for regress | help regress postestimation or<br>help regress post |
| graph option xlabel()            | help graph xlabel()                                 |
| Stata function strpos()          | help strpos()                                       |
| Mata function optimize()         | help mata optimize()                                |

Tips:

- help displays advice for using the help system and documentation.
- help guide displays a table of contents for basic Stata concepts.
- help estimation commands displays an alphabetical listing of all Stata estimation commands.
- help functions displays help on Stata functions by category.
- help mata functions displays a subject table of contents for Mata's functions.
- help ts glossary displays the glossary for the time-series manual, and similarly for the other Stata specialty manuals.

If you type help *topic* and help for *topic* is not found, Stata will automatically perform a search for *topic*.

For instance, try typing help forecasting. A forecasting help file is not found, so Stata executes search forecasting and displays the results in the Viewer.

See [U] 4 Stata's help and search facilities for a complete description of how to use help.

## □ Technical note

When you type help *topic*, Stata first looks along the adopath for *topic*.sthlp; see [U] 17.5 Where does Stata look for ado-files?

□

## Video examples

[Quick help in Stata](#)

## Reference

Chen, Y., and Y. Lian. 2024. Browse and cite Stata manuals easily: The wwwhelp command. *Stata Journal* 24: 161–168.

## Also see

[R] **net search** — Search the internet for installable packages

[R] **search** — Search Stata documentation and other resources

[GSM] **4 Getting help**

[GSW] **4 Getting help**

[GSU] **4 Getting help**

[U] **4 Stata's help and search facilities**

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

## Description

`hetoprobit` fits a heteroskedastic ordered probit model for an ordinal dependent variable. `hetoprobit` is a generalization of `oprobit` that allows the variance to be modeled as a function of independent variables and to differ between subjects or groups in the population.

## Quick start

Heteroskedastic ordinal probit model of  $y$  on  $x1$ , using  $x2$  to model the variance

```
hetoprobit y x1, het(x2)
```

With robust standard errors

```
hetoprobit y x1, het(x2) vce(robust)
```

Perform a Wald test on the variance instead of a likelihood-ratio (LR) test

```
hetoprobit y x1, het(x2) waldhet
```

## Menu

Statistics > Ordinal outcomes > Heteroskedastic ordered probit regression

## Syntax

```
hetoprobit depvar [indepvars] [if] [in] [weight],
 het(varlist[, offset(varnameo)] [options])
```

| <i>options</i>                            | Description                                                                                                                                      |
|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                              |                                                                                                                                                  |
| * <b>het</b> ( <i>varlist</i> [...])      | independent variables to model the variance and optional offset variable                                                                         |
| <b>offset</b> ( <i>varname</i> )          | include <i>varname</i> in model with coefficient constrained to 1                                                                                |
| <b>constraints</b> ( <i>constraints</i> ) | apply specified linear constraints                                                                                                               |
| <b>SE/Robust</b>                          |                                                                                                                                                  |
| <b>vce</b> ( <i>vcetype</i> )             | <i>vcetype</i> may be oim, <u>robust</u> , <u>cluster</u> <i>clustvar</i> , opg, <u>bootstrap</u> , or <u>jackknife</u>                          |
| <b>Reporting</b>                          |                                                                                                                                                  |
| <b>level</b> (#)                          | set confidence level; default is level(95)                                                                                                       |
| <b>waldhet</b>                            | perform Wald test on variance instead of LR test                                                                                                 |
| <b>nocnsreport</b>                        | do not display constraints                                                                                                                       |
| <b>display_options</b>                    | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| <b>Maximization</b>                       |                                                                                                                                                  |
| <b>maximize_options</b>                   | control the maximization process; seldom used                                                                                                    |
| <b>noheader</b>                           | do not display header above coefficient table                                                                                                    |
| <b>notable</b>                            | do not display coefficient table                                                                                                                 |
| <b>collinear</b>                          | keep collinear variables                                                                                                                         |
| <b>coeflegend</b>                         | display legend instead of statistics                                                                                                             |

\*het() is required. The full specification is het(*varlist* [, offset(*varname*<sub>o</sub>)]).

*indepvars* and *varlist* may contain factor variables; see [U] 11.4.3 **Factor variables**.

*depvar*, *indepvars*, and *varlist* may contain time-series operators; see [U] 11.4.4 **Time-series varlists**.

bayes, bayesboot, bootstrap, by, collect, fp, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 **Prefix commands**. For more details, see [BAYES] **bayes: hetoprobit**.

Weights are not allowed with the bootstrap prefix; see [R] **bootstrap**.

vce() and weights are not allowed with the svy prefix; see [SVY] **svy**.

fweights, iweights, and pweights are allowed; see [U] 11.1.6 **weight**.

noheader, notable, collinear, and coeflegend do not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options

### Model

`het(varlist [ , offset(varnameo) ])` specifies the independent variables and, optionally, the offset variable in the variance function. `het()` is required.

`offset(varnameo)` specifies that offset *varname<sub>o</sub>* be included in the variance model with the coefficient constrained to be 1.

`offset(varname)`, `constraints(constraints)`; see [R] [Estimation options](#).

### SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

### Reporting

`level(#)`; see [R] [Estimation options](#).

`waldhet` specifies that a Wald test of whether  $\ln\sigma = 0$  be performed instead of the LR test.

`nocnsreport`; see [R] [Estimation options](#).

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and  `nolstretch`; see [R] [Estimation options](#).

### Maximization

*maximize\_options*: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

The following options are available with `hetoprobit` but are not shown in the dialog box:

`noheader` suppresses the header above the coefficient table.

`notable` suppresses the display of the coefficient table.

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

`hetoprobit` fits a maximum-likelihood heteroskedastic ordered probit model, which is a generalization of the ordered probit model (see [R] [oprobit](#)).

In ordinal regression models, the outcome is an ordinal variable—a variable that is categorical and ordered, for instance, “poor”, “good”, and “excellent”. The specific values of the ordinal variable are irrelevant. It matters only that larger values are assumed to correspond to “higher” outcomes. To simplify the discussion in this entry, we assume without loss of generality that the dependent variable takes on the integer values  $0, 1, \dots, H$ , for some value  $H > 1$ .

In ordered probit models, an underlying score is estimated as a linear function of the independent variables and a set of cutpoints. The probability of observing outcome  $y_j = h$ , where  $h = 0, 1, \dots, H$ , corresponds to the probability that the value of the linear function, plus random error, is within the range of the cutpoints associated with the outcome

$$\begin{aligned} \Pr(y_j = h) &= \Pr(\kappa_h < \beta_1 x_{1j} + \beta_2 x_{2j} + \dots + \beta_k x_{kj} + u_j \leq \kappa_{h+1}) \\ &= \Phi(\kappa_{h+1} - \mathbf{x}_j \boldsymbol{\beta}) - \Phi(\kappa_h - \mathbf{x}_j \boldsymbol{\beta}) \end{aligned}$$

where  $\mathbf{x}_j = (x_{1j}, x_{2j}, \dots, x_{kj})$  are the  $k$  independent variables that model the mean function;  $\boldsymbol{\beta}$  is a column vector of unknown parameters in the mean function;  $u_j$ , where  $j = 1, \dots, N$ , are normally distributed error terms;  $\kappa_h$ , where  $h = 1, \dots, H$ , are the unknown cutpoints that separate the different possible values of  $h$ ; and  $\Phi(\cdot)$  is the cumulative distribution function of the standard normal distribution. Also, by convention, to complete the intervals for the lowest and highest values of the outcome,  $\kappa_0 = -\infty$  and  $\kappa_{H+1} = \infty$ .

In conventional ordinal probit models, the error term is assumed i.i.d. normal with unit variance for all observations. `hetoprobit` generalizes the ordered probit model by representing the variance of the error term  $u_j$  as a multiplicative function of explanatory variables  $\mathbf{z}_j = (z_{1j}, z_{2j}, \dots, z_{mj})$ . This approach was introduced by [Harvey \(1976\)](#), though we depart from Harvey slightly by modeling standard deviation rather than variance. More specifically, we model the natural logarithm of the standard deviation as a linear combination of the explanatory variables,

$$\ln \sigma_j = \mathbf{z}_j \boldsymbol{\gamma}$$

where  $\boldsymbol{\gamma}$  is a column vector of unknown parameters in the variance function.

With this generalization, the error variance may differ between subjects or between groups in the population, and

$$\Pr(y_j = h) = \Phi \left\{ \frac{\kappa_{h+1} - \mathbf{x}_j \boldsymbol{\beta}}{\exp(\mathbf{z}_j \boldsymbol{\gamma})} \right\} - \Phi \left\{ \frac{\kappa_h - \mathbf{x}_j \boldsymbol{\beta}}{\exp(\mathbf{z}_j \boldsymbol{\gamma})} \right\}$$

For the model to be identifiable, there can be no constant term in  $\mathbf{z}_j \boldsymbol{\gamma}$ . Also, as with [\[R\] oprobit](#), there is no constant term in  $\mathbf{x}_j \boldsymbol{\beta}$ . The role of the constant is subsumed by the cutpoints.

We estimate the coefficients  $\beta_1, \beta_2, \dots, \beta_k$  and  $\gamma_1, \gamma_2, \dots, \gamma_m$  together with the cutpoints  $\kappa_1, \kappa_2, \dots, \kappa_H$ . If the model has no independent variables in  $\mathbf{x}_j$ , only the cutpoints and the  $\boldsymbol{\gamma}$  parameters are estimated.

Modeling of heteroskedastic variance has both constructive and defensive uses. It is known that differences in variance between subjects or between groups in the population can cause biased coefficient estimates and can complicate comparison of distinct groups. Thus, incorporating a model for variance can be necessary for proper inference, even if the variance function itself is not a topic of interest to the researcher. For discussion, see [Williams \(2010\)](#) and the references cited therein. There are also cases where modeling the differences between variances of different subjects or different groups in the population is one of the principal purposes of the study. We will discuss such a scenario in the examples below. See [Reardon et al. \(2017\)](#) and [Alvarez and Brehm \(1995\)](#) for additional examples.

### ▷ Example 1: Modeling heteroskedasticity of reported health status

In this example, we will use a slightly modified subset of data from the 2015 Eating & Health Module of the American Time Use Survey (ATUS), conducted by the US Bureau of Labor Statistics. Our analysis will not account for the survey design. The ATUS measures the amount of time people spend doing various activities, such as working, caring for children, volunteering, and socializing. Of interest to us is an ordinal response variable, `health`, which contains individuals' self-assessments of their overall health status on a five-point scale: 1 for "poor", 2 for "fair", 3 for "good", 4 for "very good", and 5 for "excellent".

We want to examine the role that age and other factors play in an individual's self-assessment of health. Age is a natural variable to include when modeling mean or typical health status. But we also suspect that the variation in health status is greater in an older population, as compared with a youthful population, which consists mainly of healthy individuals. If our suspicion is true, quantifying the relationship between variation in health status and age may have value, for example, in planning a healthcare strategy that is appropriately tailored both for the older Medicare population and for a younger cohort.

Thus, we will use `hetoprobit` to model heteroskedasticity induced by age. In modeling the variance term, in addition to age, we will include a factor variable, `exercise`, which indicates whether or not an individual exercised during the previous week. For purposes of illustration, imagine that we are not interested in exercise as a topic in its own right, but we are concerned that health variability among those who exercise may differ from the variability among those who do not. Therefore, we include `exercise` in the variance term to help insulate our estimation results against a possible hidden bias.

Our model will include three explanatory variables for the mean function: `age`, `bmi` (body mass index), and `exercise`.

```
. use https://www.stata-press.com/data/r19/eathealth15
(2015 ATUS Eating & Health Module extract)
. hetoprobit health age bmi i.exercise, het(age i.exercise)
(output omitted)
```

Fitting ordered probit model:

```
Iteration 0: Log likelihood = -2905.7943
Iteration 1: Log likelihood = -2717.2752
Iteration 2: Log likelihood = -2716.9679
Iteration 3: Log likelihood = -2716.9679
```

Fitting full model:

```
Iteration 0: Log likelihood = -2716.9679
Iteration 1: Log likelihood = -2708.6752
Iteration 2: Log likelihood = -2708.5492
Iteration 3: Log likelihood = -2708.5491
```

```
Heteroskedastic ordered probit regression
Log likelihood = -2708.5491
Number of obs = 2,009
LR chi2(3) = 366.91
Prob > chi2 = 0.0000
```

| health   | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|----------|-------------|-----------|-------|-------|----------------------|-----------|
| health   |             |           |       |       |                      |           |
| age      | -.0083348   | .0015969  | -5.22 | 0.000 | -.0114646            | -.005205  |
| bmi      | -.0564072   | .0057392  | -9.83 | 0.000 | -.0676558            | -.0451586 |
| exercise |             |           |       |       |                      |           |
| Yes      | .6493794    | .0732137  | 8.87  | 0.000 | .5058833             | .7928755  |
| lnsigma  |             |           |       |       |                      |           |
| age      | .0041401    | .0011611  | 3.57  | 0.000 | .0018643             | .0064159  |
| exercise |             |           |       |       |                      |           |
| Yes      | -.0773166   | .0423038  | -1.83 | 0.068 | -.1602305            | .0055973  |
| /cut1    | -3.903773   | .2913163  |       |       | -4.474742            | -3.332803 |
| /cut2    | -2.776111   | .2262442  |       |       | -3.219541            | -2.33268  |
| /cut3    | -1.576396   | .174352   |       |       | -1.918119            | -1.234672 |
| /cut4    | -.4189882   | .1524084  |       |       | -.7177031            | -.1202733 |

```
LR test of lnsigma=0: chi2(2) = 16.84
Prob > chi2 = 0.0002
```

The LR test at the bottom of the output is a test of homogeneity of the variance function. The  $\chi^2(2)$  statistic of 16.84 is significant, indicating that heteroskedasticity is present. If you prefer the Wald test for heteroskedasticity, you can specify the `waldhet` option. The coefficients for the variance function are reported in the section of the table labeled `lnsigma`. Our results indicate that `age` is a significant contributor to the variance function but that `exercise` is not significant at a 0.05 level.

The LR test for the model that appears above the coefficient table is a joint test for inclusion of `age`, `bmi`, and `exercise` in the mean function. The null model for this test is the model consisting only of cutpoints and the heteroskedastic term. Coefficients for the mean function are reported in the section of the table labeled `health`. In this example, `age`, `bmi`, and `exercise` are significant components of the linear predictor of the mean.

The signs of the coefficients in the fitted model are directly interpretable. For example, the negative value for the coefficient of `bmi` implies that higher values of `bmi` predict lower values of health status. However, because of the probit link and the fact that we estimate variance with a log transformation, the numerical relationships between the coefficients of the model and the outcome variables are nonlinear. Postestimation commands recognize and account for these nonlinearities.



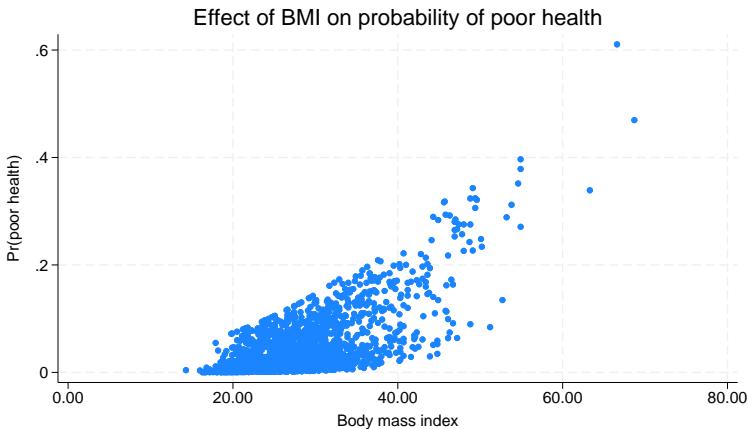
### ▷ Example 2: Predict the probability of a poor health rating

Ordered probit models allow us to look at the probabilities of different outcomes of interest. Suppose we are interested in predictions of a reported health status of "poor" (`health = 1`) and how it differs across levels of `bmi`. First, we obtain the predicted probability of poor health.

```
. predict pr1, pr outcome(1)
```

We can now visualize how the predicted probability of poor health status differs across the range of `bmi` values in our sample.

```
. twoway scatter pr1 bmi, ytitle("Pr(poor health)")
> title("Effect of BMI on probability of poor health")
```



We see that predicted probabilities of poor health increase as body mass index increases.



### ▷ Example 3: Predictive margins and average marginal effect

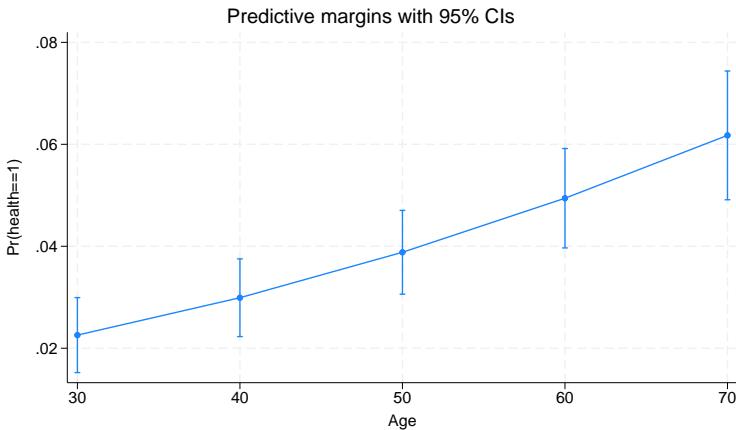
The graph [above](#) plots the predicted probability of poor health for each individual in our dataset. We may also want to evaluate how the average predicted probability changes across levels of the covariates in the model. For instance, we can use the `margins` command to obtain the expected probability of having poor health across a range of ages.

```
. margins, at(age = (30(10)70)) predict(outcome(1))
Predictive margins Number of obs = 2,009
Model VCE: OIM
Expression: Pr(health==1), predict(outcome(1))
1._at: age = 30
2._at: age = 40
3._at: age = 50
4._at: age = 60
5._at: age = 70
```

|     | Margin   | Delta-method<br>std. err. | z    | P> z  | [95% conf. interval] |          |
|-----|----------|---------------------------|------|-------|----------------------|----------|
| _at |          |                           |      |       |                      |          |
| 1   | .0225765 | .0037522                  | 6.02 | 0.000 | .0152224             | .0299306 |
| 2   | .0299079 | .0038887                  | 7.69 | 0.000 | .0222862             | .0375297 |
| 3   | .0388244 | .0041936                  | 9.26 | 0.000 | .0306051             | .0470438 |
| 4   | .0494246 | .0049748                  | 9.93 | 0.000 | .0396742             | .0591751 |
| 5   | .0617532 | .0064443                  | 9.58 | 0.000 | .0491226             | .0743839 |

Based on our model, what would we expect if everyone was 30 years old but had the same distributions of bmi and exercise that we observed in our data? The first line in this table reports that the average predicted probability of poor health is 0.0226 in this case. The second line shows the average predicted probability of poor health if we set age = 40, and so on. We find that for age = 70, the average probability of reporting a poor health status has increased to 0.0618. We can visualize this by typing marginsplot after margins.

```
. marginsplot
Variables that uniquely identify margins: age
```



We have focused on the prediction of poor health. We could instead simultaneously obtain average predicted probabilities of poor, fair, good, very good, and excellent health status and plot them across our requested age range. In that case, we would type

```
. margins, at(age = (30(10)70))
. marginsplot
```

We might also be interested in characterizing the relationship between bmi and the probability of reporting poor health. The coefficients and cutpoints reported in `hetoprobit` are not easily interpreted. We can, however, use `margins` to estimate the average marginal effect of `bmi` on the probability of reporting poor health. Because the average marginal effect depends on the value of `bmi`, we estimate it across a range of `bmi` values by typing

```
. margins, dydx(bmi) at(bmi = (20(5)35)) predict(outcome(1))
Average marginal effects Number of obs = 2,009
Model VCE: OIM
Expression: Pr(health==1), predict(outcome(1))
dy/dx wrt: bmi
1._at: bmi = 20
2._at: bmi = 25
3._at: bmi = 30
4._at: bmi = 35
```

|     |     | Delta-method |           | z    | P> z  | [95% conf. interval] |          |
|-----|-----|--------------|-----------|------|-------|----------------------|----------|
|     |     | dy/dx        | std. err. |      |       |                      |          |
| bmi |     |              |           |      |       |                      |          |
|     | _at |              |           |      |       |                      |          |
|     | 1   | .0015875     | .0001721  | 9.23 | 0.000 | .0012503             | .0019248 |
|     | 2   | .0024512     | .0002665  | 9.20 | 0.000 | .0019288             | .0029736 |
|     | 3   | .0036447     | .0004377  | 8.33 | 0.000 | .0027868             | .0045027 |
|     | 4   | .0052124     | .0007013  | 7.43 | 0.000 | .0038379             | .006587  |

The average marginal effect of `bmi` on the probability of reporting poor health increases as `bmi` itself increases.



### ► Example 4: Interpreting the variance function

From the output of our `hetoprobit` command, we determined that variance of health status is affected by age. Let's consider to what extent. In this example, we assess the effect of age on the variance by using the `margins` command. We use the `predict(sigma)` option to obtain the average predicted standard deviation of the errors. We will look at ages 15 and 85, which are the youngest and oldest ages, respectively, in our dataset.

```
. margins, predict(sigma) at(age = (15,85)) noatlegend
Predictive margins Number of obs = 2,009
Model VCE: OIM
Expression: Heteroskedastic standard deviation, predict(sigma)
```

|  |     | Delta-method |           | z     | P> z  | [95% conf. interval] |          |
|--|-----|--------------|-----------|-------|-------|----------------------|----------|
|  |     | Margin       | std. err. |       |       |                      |          |
|  | _at |              |           |       |       |                      |          |
|  | 1   | 1.014732     | .0320905  | 31.62 | 0.000 | .9518354             | 1.077628 |
|  | 2   | 1.355853     | .1398144  | 9.70  | 0.000 | 1.081822             | 1.629884 |

Variation increases with age. The expected standard deviation of the error term changes from 1.015 at age 15 to 1.356 at age 85.



## Stored results

hetoprobit stores the following in `e()`:

### Scalars

|                            |                                                       |
|----------------------------|-------------------------------------------------------|
| <code>e(N)</code>          | number of observations                                |
| <code>e(k_cat)</code>      | number of categories                                  |
| <code>e(k)</code>          | number of parameters                                  |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>              |
| <code>e(k_eq_model)</code> | number of equations in overall model test             |
| <code>e(k_aux)</code>      | number of auxiliary parameters                        |
| <code>e(k_dv)</code>       | number of dependent variables                         |
| <code>e(df_m)</code>       | model degrees of freedom ( $x\beta$ term)             |
| <code>e(ll)</code>         | log likelihood                                        |
| <code>e(ll_0)</code>       | log likelihood, cutpoint-only (heteroskedastic) model |
| <code>e(ll_c)</code>       | log likelihood, comparison (homoskedastic) model      |
| <code>e(N_clust)</code>    | number of clusters                                    |
| <code>e(chi2)</code>       | $\chi^2$                                              |
| <code>e(chi2_c)</code>     | $\chi^2$ for heteroskedasticity test                  |
| <code>e(p)</code>          | $p$ -value for model test                             |
| <code>e(p_c)</code>        | $p$ -value for heteroskedasticity test                |
| <code>e(df_m_c)</code>     | degrees of freedom for heteroskedasticity test        |
| <code>e(rank)</code>       | rank of $e(V)$                                        |
| <code>e(ic)</code>         | number of iterations                                  |
| <code>e(rc)</code>         | return code                                           |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise                           |

### Macros

|                                |                                                                                     |
|--------------------------------|-------------------------------------------------------------------------------------|
| <code>e(cmd)</code>            | hetoprobit                                                                          |
| <code>e(cmdline)</code>        | command as typed                                                                    |
| <code>e(depvar)</code>         | name of dependent variable                                                          |
| <code>e(wtype)</code>          | weight type                                                                         |
| <code>e(wexp)</code>           | weight expression                                                                   |
| <code>e(title)</code>          | title in estimation output                                                          |
| <code>e(clustvar)</code>       | name of cluster variable                                                            |
| <code>e(offset1)</code>        | offset for ordered probit equation                                                  |
| <code>e(offset2)</code>        | offset for variance equation                                                        |
| <code>e(chi2type)</code>       | LR; type of model $\chi^2$ test                                                     |
| <code>e(chi2_ct)</code>        | LR or Wald; type of heteroskedasticity test corresponding to <code>e(chi2_c)</code> |
| <code>e(vce)</code>            | <i>vcetype</i> specified in <code>vce()</code>                                      |
| <code>e(vcetype)</code>        | title used to label Std. err.                                                       |
| <code>e(opt)</code>            | type of optimization                                                                |
| <code>e(which)</code>          | max or min; whether optimizer is to perform maximization or minimization            |
| <code>e(ml_method)</code>      | type of ml method                                                                   |
| <code>e(user)</code>           | name of likelihood-evaluator program                                                |
| <code>e(technique)</code>      | maximization technique                                                              |
| <code>e(properties)</code>     | b V                                                                                 |
| <code>e(predict)</code>        | program used to implement <code>predict</code>                                      |
| <code>e(marginsok)</code>      | predictions allowed by <code>margins</code>                                         |
| <code>e(marginsnotok)</code>   | predictions disallowed by <code>margins</code>                                      |
| <code>e(marginsdefault)</code> | default <code>predict()</code> specification for <code>margins</code>               |
| <code>e(asbalanced)</code>     | factor variables <code>fvset</code> as <code>asbalanced</code>                      |
| <code>e(asobserved)</code>     | factor variables <code>fvset</code> as <code>asobserved</code>                      |

|                 |                                              |
|-----------------|----------------------------------------------|
| Matrices        |                                              |
| e(b)            | coefficient vector                           |
| e(Cns)          | constraints matrix                           |
| e(iolog)        | iteration log (up to 20 iterations)          |
| e(gradient)     | gradient vector                              |
| e(cat)          | category values                              |
| e(V)            | variance–covariance matrix of the estimators |
| e(V_modelbased) | model-based variance                         |
| Functions       |                                              |
| e(sample)       | marks estimation sample                      |

In addition to the above, the following is stored in `r()`:

|          |                                                                                                                            |
|----------|----------------------------------------------------------------------------------------------------------------------------|
| Matrices |                                                                                                                            |
| r(table) | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

## Methods and formulas

`hetoprobit` fits a cumulative probit model with heteroskedastic variance using maximum likelihood estimation. Namely, the model is that, for a subject with explanatory variables  $\mathbf{x}$  and  $\mathbf{z}$ ,

$$\Pr(Y \leq h) = \Phi \left\{ \frac{\kappa_{h+1} - \mathbf{x}\boldsymbol{\beta}}{\exp(\mathbf{z}\boldsymbol{\gamma})} \right\}$$

where  $Y$  is an ordinal outcome taking on values  $h = 0, 1, \dots, H$ , and  $\Phi(\cdot)$  is the cdf of the standard normal distribution. The value  $\kappa_{h+1}$  is a cutpoint that separates the region corresponding to  $Y = h$  from regions for higher-valued categories. The effects  $\boldsymbol{\beta}$  and the effects  $\boldsymbol{\gamma}$  are the same for each cumulative probability.

The log-likelihood function is

$$\ln L = \sum_{j=1}^N w_j \sum_{h=0}^H I_h(y_j) \ln \left[ \Phi \left\{ \frac{\kappa_{h+1} - \mathbf{x}_j \boldsymbol{\beta}}{\exp(\mathbf{z}_j \boldsymbol{\gamma})} \right\} - \Phi \left\{ \frac{\kappa_h - \mathbf{x}_j \boldsymbol{\beta}}{\exp(\mathbf{z}_j \boldsymbol{\gamma})} \right\} \right]$$

where

$$I_h(y_j) = \begin{cases} 1 & \text{if } y_j = h \\ 0 & \text{otherwise} \end{cases}$$

and  $y_j$ , where  $j = 1, \dots, N$ , is an observed value of  $Y$ ;  $w_j$  are optional weights;  $\kappa_0 = -\infty$  and  $\kappa_{H+1} = \infty$ ; and all other terminology is defined in [Remarks and examples](#) above.

The log-likelihood function is maximized as described in [\[R\] Maximize](#).

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [\[P\] \\_robust](#), particularly [Maximum likelihood estimators](#) and [Methods and formulas](#).

`hetoprobit` also supports estimation with survey data. For details on VCEs with survey data, see [\[SVY\] Variance estimation](#).

## References

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## Also see

- [R] **hetoprobit postestimation** — Postestimation tools for hetoprobit
- [R] **hetoprobit** — Heteroskedastic probit model
- [R] **oprobit** — Ordered probit regression
- [BAYES] **bayes: hetoprobit** — Bayesian heteroskedastic ordered probit regression
- [SVY] **svy estimation** — Estimation commands for survey data
- [U] **20 Estimation and postestimation commands**

## Postestimation commands

The following postestimation commands are available after `hetoprobit`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| <code>linktest</code>        | link test for model specification                                                                                                       |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | probabilities, linear predictions, etc.                                                                                                 |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results.

## predict

### Description for predict

`predict` creates a new variable containing predictions such as probabilities, linear predictions, and standard deviations.

### Menu for predict

Statistics > Postestimation

### Syntax for predict

```
predict [type] { stub* | newvar | newvarlist } [if] [in] [, statistic
outcome(outcome) nooffset]
```

```
predict [type] stub* [if] [in] , scores
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

Main

|                    |                                         |
|--------------------|-----------------------------------------|
| <code>pr</code>    | predicted probabilities; the default    |
| <code>xb</code>    | linear prediction                       |
| <code>stdp</code>  | standard error of the linear prediction |
| <code>sigma</code> | standard deviation of the error term    |

You specify one or  $k$  new variables with `pr`, where  $k$  is the number of outcomes. If you specify one new variable and you do not specify `outcome()`, then `outcome(#1)` is assumed.

You specify one new variable with `xb`, `stdp`, or `sigma`.

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

### Options for predict

Main

`pr`, the default, computes the predicted probabilities for all outcomes or for a specific outcome. To compute probabilities for all outcomes, you specify  $k$  new variables, where  $k$  is the number of categories of the dependent variable. Alternatively, you can specify `stub*`; in which case, `pr` will store predicted probabilities in variables `stub1`, `stub2`, ..., `stubk`. To compute the probability for a specific outcome, you specify one new variable and, optionally, the outcome value in option `outcome()`; if you omit `outcome()`, the first outcome value, `outcome(#1)`, is assumed.

Say that you fit a model by typing `estimation_cmd y x1 x2`, and `y` takes on four values. Then, you could type `predict p1 p2 p3 p4` to obtain all four predicted probabilities; alternatively, you could type `predict p*` to generate the four predicted probabilities. To compute specific probabilities one at a time, you can type `predict p1, outcome(#1)` (or simply `predict p1`), `predict p2, outcome(#2)`, and so on. See option `outcome()` for other ways to refer to outcome values.

`xb` calculates the linear prediction. The linear prediction is defined by ignoring the contribution of the estimated cutpoints.

`stdp` calculates the standard error of the linear prediction.

`sigma` calculates the standard deviation of the error term.

`outcome(outcome)` specifies for which outcome the predicted probabilities are to be calculated. `outcome()` should contain either one value of the dependent variable or one of #1, #2, ..., with #1 meaning the first category of the dependent variable, #2 meaning the second category, etc. `outcome()` is available only with the default `pr` option.

`nooffset` is relevant only if you specified `offset(varname)` for `hetoprobit` or within the `het()` option. `nooffset` modifies the calculations made by `predict` so that they ignore the offset variable: the linear prediction is treated as  $\mathbf{x}_j\mathbf{b}$  rather than as  $\mathbf{x}_j\mathbf{b} + \text{offset}_j^b$ , and the prediction of  $\ln(\sigma)$  is treated as  $\mathbf{z}_j\mathbf{g}$  rather than as  $\mathbf{z}_j\mathbf{g} + \text{offset}_j^g$ . `nooffset` is not allowed with `scores`.

`scores` calculates equation-level score variables.

The first new variable will contain  $\partial \ln L / \partial (\mathbf{x}_j\mathbf{b})$ .

The next new variable will contain  $\partial \ln L / \partial (\mathbf{z}_j\boldsymbol{\gamma})$ .

The next new variable will contain  $\partial \ln L / \partial \kappa_1$ .

The next new variable (if any) will contain  $\partial \ln L / \partial \kappa_2$ .

...

The last new variable will contain  $\partial \ln L / \partial \kappa_H$ , where  $\kappa_h$  for  $h = 1, 2, \dots, H$  refers to the  $h$ th cutpoint. If the linear predictor had no `indepvars`, the first new variable will contain  $\partial \ln L / \partial (\mathbf{z}_j\boldsymbol{\gamma})$ .

# margins

## Description for margins

`margins` estimates margins of response for probabilities, linear predictions, and standard deviations.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i> | Description                           |
|------------------|---------------------------------------|
| Main             |                                       |
| default          | probabilities for each outcome        |
| pr               | probability for a specified outcome   |
| xb               | linear prediction                     |
| stdp             | not allowed with <code>margins</code> |
| sigma            | standard deviation of the error term  |

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [R] [margins](#).

## Remarks and examples

See [U] [20 Estimation and postestimation commands](#) for an overview of postestimation commands, including information on obtaining the variance–covariance matrix of the estimators, predicted values, and hypothesis tests.

Once you have fit a model with `hetoprobit`, you may use the `predict` command to obtain the predicted probabilities for both the estimation sample and other samples. With the `pr` option, `predict` calculates the predicted probability of one or all ordinal value outcomes. With the `xb` option, `predict` calculates the linear prediction,  $\mathbf{x}_j\mathbf{b}$ , where  $\mathbf{x}_j$  are the independent variables in the  $j$ th observation and  $\mathbf{b}$  is the estimated parameter vector. The linear prediction is defined ignoring the contribution of the cutpoints. With the `sigma` option, `predict` calculates the predicted standard deviations of the error term,  $\sigma_j = \exp(\mathbf{z}_j\mathbf{g})$ , where  $\mathbf{g}$  is the estimated coefficient vector for the variance model.

See [example 2](#) in [R] [hetoprobit](#) for an example of `predict` after `hetoprobit`. In [example 3](#) and [example 4](#), we demonstrate how to use `margins` to obtain marginal effects, to compute expected probabilities of outcome levels across values of covariates, and to characterize the variance as a function of covariates.

## Methods and formulas

For definitions of the terminology used in this section, see *Remarks and examples* in [R] [hetoprobit](#).

For outcome  $h$ , the statistic `pr` is the estimated predicted probability

$$\widehat{\text{Pr}}(y_j = h) = \Phi \left\{ \frac{\widehat{\kappa}_{h+1} - \mathbf{x}_j \mathbf{b}}{\exp(\mathbf{z}_j \mathbf{g})} \right\} - \Phi \left\{ \frac{\widehat{\kappa}_h - \mathbf{x}_j \mathbf{b}}{\exp(\mathbf{z}_j \mathbf{g})} \right\}$$

where  $\mathbf{x}_j \mathbf{b}$  and  $\mathbf{z}_j \mathbf{g}$  are the linear predictions of the regression and variance models, respectively, for the  $j$ th subject.

The statistic `sigma` is the estimated standard deviation of the modeled heteroskedastic error, namely,

$$\widehat{\sigma}_j = \exp(\mathbf{z}_j \mathbf{g})$$

If you specified `offset(varname)` with `hetoprobit` or within the `het()` option (and if you do not specify option `nooffset` with `predict`), then the specified offsets are applied by `predict`. Namely, the linear prediction is computed as  $\mathbf{x}_j \mathbf{b} + \text{offset}_j^b$ ; the prediction of  $\ln(\sigma)$  is computed as  $\mathbf{z}_j \mathbf{g} + \text{offset}_j^g$ ; and all other statistics are based on the resulting predictions. If you specify `nooffset` with `predict`, then the linear prediction is  $\mathbf{x}_j \mathbf{b}$  and the prediction of  $\ln(\sigma)$  is  $\mathbf{z}_j \mathbf{g}$ , regardless of whether you specified the `offset()` option with `hetoprobit` or within `het()`.

## Also see

[R] [hetoprobit](#) — Heteroskedastic ordered probit regression

[U] [20 Estimation and postestimation commands](#)

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

## Description

`hetprobit` fits a maximum-likelihood heteroskedastic probit model.

## Quick start

Heteroskedastic probit model of  $y$  on  $x1$ , using  $x2$  to model the variance

```
hetprobit y x1, het(x2)
```

With robust standard errors

```
hetprobit y x1, het(x2) vce(robust)
```

After fitting a model, reprint the table as a coefficient legend

```
hetprobit, coeflegend
```

## Menu

Statistics > Binary outcomes > Heteroskedastic probit regression

## Syntax

```
hetprobit depvar [indepvars] [if] [in] [weight] ,
 het(varlist [, offset(varnameo)]) [options]
```

| <i>options</i>                            | Description                                                                                                                                      |
|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                              |                                                                                                                                                  |
| * <i>het</i> ( <i>varlist</i> [...])      | independent variables to model the variance and optional offset variable                                                                         |
| <i>noconstant</i>                         | suppress constant term                                                                                                                           |
| <i>offset</i> ( <i>varname</i> )          | include <i>varname</i> in model with coefficient constrained to 1                                                                                |
| <i>asis</i>                               | retain perfect predictor variables                                                                                                               |
| <i>constraints</i> ( <i>constraints</i> ) | apply specified linear constraints                                                                                                               |
| <b>SE/Robust</b>                          |                                                                                                                                                  |
| <i>vce</i> ( <i>vcetype</i> )             | <i>vcetype</i> may be <i>oim</i> , <i>robust</i> , <i>cluster</i> <i>clustvar</i> , <i>opg</i> , <i>bootstrap</i> , or <i>jackknife</i>          |
| <b>Reporting</b>                          |                                                                                                                                                  |
| <i>level</i> (#)                          | set confidence level; default is <i>level</i> (95)                                                                                               |
| <i>lrmodel</i>                            | perform the likelihood-ratio model test instead of the default Wald test                                                                         |
| <i>waldhet</i>                            | perform Wald test on variance instead of LR test                                                                                                 |
| <i>nocnsreport</i>                        | do not display constraints                                                                                                                       |
| <i>display_options</i>                    | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| <b>Maximization</b>                       |                                                                                                                                                  |
| <i>maximize_options</i>                   | control the maximization process; seldom used                                                                                                    |
| <i>collinear</i>                          | keep collinear variables                                                                                                                         |
| <i>coeflegend</i>                         | display legend instead of statistics                                                                                                             |

\**het*() is required. The full specification is *het*(*varlist* [ , *offset*(*varname*<sub>o</sub>) ]).

*indepvars* and *varlist* may contain factor variables; see [U] 11.4.3 **Factor variables**.

*depvar*, *indepvars*, and *varlist* may contain time-series operators; see [U] 11.4.4 **Time-series varlists**.

*bayes*, *bayesboot*, *bootstrap*, *by*, *collect*, *fp*, *jackknife*, *rolling*, *statsby*, and *svy* are allowed; see [U] 11.1.10 **Prefix commands**. For more details, see [BAYES] **bayes: hetprobit**.

Weights are not allowed with the *bootstrap* prefix; see [R] **bootstrap**.

*vce*() , *lrmodel*, and weights are not allowed with the *svy* prefix; see [SVY] **svy**.

*fweights*, *iweights*, and *pweights* are allowed; see [U] 11.1.6 **weight**.

*collinear* and *coeflegend* do not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options

### Model

`het` (*varlist* [ , `offset` (*varname<sub>o</sub>*) ]) specifies the independent variables and, optionally, the offset variable in the variance function. `het()` is required.

`offset` (*varname<sub>o</sub>*) specifies that selection offset *varname<sub>o</sub>* be included in the model with the coefficient constrained to be 1.

`noconstant`, `offset` (*varname*); see [R] [Estimation options](#).

`asis` forces the retention of perfect predictor variables and their associated perfectly predicted observations and may produce instabilities in maximization; see [R] [probit](#).

`constraints` (*constraints*); see [R] [Estimation options](#).

### SE/Robust

`vce` (*vcetype*) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster` *clustvar*), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

### Reporting

`level` (*#*), `lrmmodel`; see [R] [Estimation options](#).

`waldhet` specifies that a Wald test of whether  $\ln\sigma = 0$  be performed instead of the LR test.

`nocnsreport`; see [R] [Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `novlabel`, `fvwrap` (*#*), `fvwrapon` (*style*), `cformat` (*%fmt*), `pformat` (*%fmt*), `sformat` (*%fmt*), and `no1stretch`; see [R] [Estimation options](#).

### Maximization

`maximize_options`: `difficult`, `technique` (*algorithm\_spec*), `iterate` (*#*), `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance` (*#*), `ltolerance` (*#*), `nrtolerance` (*#*), `nonrtolerance`, and `from` (*init\_specs*); see [R] [Maximize](#). These options are seldom used.

Setting the optimization type to `technique` (`bhhh`) resets the default *vcetype* to `vce` (`opg`).

The following options are available with `hetprobit` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Remarks are presented under the following headings:

*Introduction*

*Robust standard errors*

### Introduction

`hetprobit` fits a maximum-likelihood heteroskedastic probit model, which is a generalization of the probit model. Let  $y_j, j = 1, \dots, N$ , be a binary outcome variable taking on the value 0 (failure) or 1 (success). In the probit model, the probability that  $y_j$  takes on the value 1 is modeled as a nonlinear function of a linear combination of the  $k$  independent variables  $\mathbf{x}_j = (x_{1j}, x_{2j}, \dots, x_{kj})$ ,

$$\Pr(y_j = 1) = \Phi(\mathbf{x}_j\mathbf{b})$$

in which  $\Phi(\cdot)$  is the cumulative distribution function (CDF) of a standard normal random variable, that is, a normally distributed (Gaussian) random variable with mean 0 and variance 1. The linear combination of the independent variables,  $\mathbf{x}_j\mathbf{b}$ , is commonly called the *index function*, or *index*. Heteroskedastic probit generalizes the probit model by generalizing  $\Phi(\cdot)$  to a normal CDF with a variance that is no longer fixed at 1 but can vary as a function of the independent variables. `hetprobit` models the variance as a multiplicative function of these  $m$  variables  $\mathbf{z}_j = (z_{1j}, z_{2j}, \dots, z_{mj})$ , following [Harvey \(1976\)](#):

$$\sigma_j^2 = \{ \exp(\mathbf{z}_j\boldsymbol{\gamma}) \}^2$$

Thus, the probability of success as a function of all the independent variables is

$$\Pr(y_j = 1) = \Phi\left\{ \mathbf{x}_j\mathbf{b} / \exp(\mathbf{z}_j\boldsymbol{\gamma}) \right\}$$

From this expression, it is clear that, unlike the index  $\mathbf{x}_j\mathbf{b}$ , no constant term can be present in  $\mathbf{z}_j\boldsymbol{\gamma}$  if the model is to be identifiable.

Suppose that the binary outcomes  $y_j$  are generated by thresholding an unobserved random variable,  $w$ , which is normally distributed with mean  $\mathbf{x}_j\mathbf{b}$  and variance 1 such that

$$y_j = \begin{cases} 1 & \text{if } w_j > 0 \\ 0 & \text{if } w_j \leq 0 \end{cases}$$

This process gives the probit model:

$$\Pr(y_j = 1) = \Pr(w_j > 0) = \Phi(\mathbf{x}_j\mathbf{b})$$

Now, suppose that the unobserved  $w_j$  are heteroskedastic with variance

$$\sigma_j^2 = \{ \exp(\mathbf{z}_j\boldsymbol{\gamma}) \}^2$$

Relaxing the homoskedastic assumption of the probit model in this manner yields our multiplicative heteroskedastic probit model:

$$\Pr(y_j = 1) = \Phi\left\{ \mathbf{x}_j\mathbf{b} / \exp(\mathbf{z}_j\boldsymbol{\gamma}) \right\}$$

► Example 1

For this example, we generate simulated data for a simple heteroskedastic probit model and then estimate the coefficients with `hetprobit`:

```
. set obs 1000
Number of observations (_N) was 0, now 1,000.
. set seed 1234567
. generate x = 1-2*runiform()
. generate xhet = runiform()
. generate sigma = exp(1.5*xhet)
. generate p = normal((0.3+2*x)/sigma)
. generate y = cond(runiform()<=p,1,0)
. hetprobit y x, het(xhet)
```

Fitting probit model:

```
Iteration 0: Log likelihood = -688.33746
Iteration 1: Log likelihood = -610.48362
Iteration 2: Log likelihood = -610.3626
Iteration 3: Log likelihood = -610.3626
```

Fitting full model:

```
Iteration 0: Log likelihood = -610.3626
Iteration 1: Log likelihood = -600.8767
Iteration 2: Log likelihood = -600.10154
Iteration 3: Log likelihood = -600.01544
Iteration 4: Log likelihood = -600.01521
Iteration 5: Log likelihood = -600.01521
```

Heteroskedastic probit model

```
Number of obs = 1,000
Zero outcomes = 451
Nonzero outcomes = 549
Wald chi2(1) = 54.20
Prob > chi2 = 0.0000
```

Log likelihood = -600.0152

|         | y     | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |          |
|---------|-------|-------------|-----------|------|-------|----------------------|----------|
| y       | x     | 1.782479    | .2421117  | 7.36 | 0.000 | 1.307949             | 2.257009 |
|         | _cons | .3140616    | .0871121  | 3.61 | 0.000 | .1433249             | .4847982 |
| lnsigma | xhet  | 1.31152     | .3011689  | 4.35 | 0.000 | .7212402             | 1.901801 |

LR test of lnsigma=0: chi2(1) = 20.69

Prob > chi2 = 0.0000

Above, we created two variables, `x` and `xhet`, and then simulated the model

$$\Pr(y = 1) = F\left\{(\beta_0 + \beta_1 x) / \exp(\gamma_1 xhet)\right\}$$

for  $\beta_0 = 0.3$ ,  $\beta_1 = 2$ , and  $\gamma_1 = 1.5$ . According to `hetprobit`'s output, all coefficients are significant, and, as we would expect, the Wald test of the full model versus the constant-only model—for example, the index consisting of  $\beta_0 + \beta_1 x$  versus that of just  $\beta_0$ —is significant with  $\chi^2(1) = 54$ . Likewise, the likelihood-ratio test of heteroskedasticity, which tests the full model with heteroskedasticity against the full model without, is significant with  $\chi^2(1) = 21$ . See [R] [Maximize](#) for more explanation of the output. For this simple model, `hetprobit` took five iterations to converge. As stated elsewhere ([Greene 2018, 764](#)), this is a difficult model to fit, and it is not uncommon for it to require many iterations or for the optimizer to print out warnings and informative messages during the optimization. Slow convergence is especially common for models in which one or more of the independent variables appear in both the index and variance functions.



□ Technical note

Stata interprets a value of 0 as a negative outcome (failure) and treats all other values (except missing) as positive outcomes (successes). Thus if your dependent variable takes on the values 0 and 1, then 0 is interpreted as failure and 1 as success. If your dependent variable takes on the values 0, 1, and 2, then 0 is still interpreted as failure, but both 1 and 2 are treated as successes.



### Robust standard errors

If you specify the `vce(robust)` option, `hetprobit` reports robust standard errors as described in [\[U\] 20.22 Obtaining robust variance estimates](#). To illustrate the effect of this option, we will reestimate our coefficients by using the same model and data in our example, this time adding `vce(robust)` to our `hetprobit` command.

▷ Example 2

```
. hetprobit y x, het(xhet) vce(robust) nolog
Heteroskedastic probit model Number of obs = 1,000
 Zero outcomes = 451
 Nonzero outcomes = 549
 Wald chi2(1) = 50.49
 Prob > chi2 = 0.0000
Log pseudolikelihood = -600.0152
```

| y       | Robust      |           | z    | P> z  | [95% conf. interval] |          |
|---------|-------------|-----------|------|-------|----------------------|----------|
|         | Coefficient | std. err. |      |       |                      |          |
| x       | 1.782479    | .2508447  | 7.11 | 0.000 | 1.290832             | 2.274126 |
| _cons   | .3140616    | .087195   | 3.60 | 0.000 | .1431625             | .4849607 |
| lnsigma |             |           |      |       |                      |          |
| xhet    | 1.31152     | .3059137  | 4.29 | 0.000 | .7119406             | 1.9111   |

```
Wald test of lnsigma=0: chi2(1) = 18.38 Prob > chi2 = 0.0000
```

The `vce(robust)` standard errors for two of the three parameters are larger than the previously reported conventional standard errors. This is to be expected, even though (by construction) we have perfect model specification because this option trades off efficient estimation of the coefficient variance–covariance matrix for robustness against misspecification.



Specifying the `vce(cluster clustvar)` option relaxes the usual assumption of independence between observations to the weaker assumption of independence just between clusters; that is, `hetprobit`, `vce(cluster clustvar)` is robust with respect to within-cluster correlation. This option is less efficient than the `xtgee` population-averaged models because `hetprobit` inefficiently sums within cluster for the standard error calculation rather than attempting to exploit what might be assumed about the within-cluster correlation.

## Stored results

`hetprobit` stores the following in `e()`:

### Scalars

|                            |                                                   |
|----------------------------|---------------------------------------------------|
| <code>e(N)</code>          | number of observations                            |
| <code>e(N_f)</code>        | number of zero outcomes                           |
| <code>e(N_s)</code>        | number of nonzero outcomes                        |
| <code>e(k)</code>          | number of parameters                              |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>          |
| <code>e(k_eq_model)</code> | number of equations in overall model test         |
| <code>e(k_dv)</code>       | number of dependent variables                     |
| <code>e(df_m)</code>       | model degrees of freedom                          |
| <code>e(ll)</code>         | log likelihood                                    |
| <code>e(ll_0)</code>       | log likelihood, constant-only model               |
| <code>e(ll_c)</code>       | log likelihood, comparison model                  |
| <code>e(N_clust)</code>    | number of clusters                                |
| <code>e(chi2)</code>       | $\chi^2$                                          |
| <code>e(chi2_c)</code>     | $\chi^2$ for heteroskedasticity test              |
| <code>e(p_c)</code>        | <i>p</i> -value for heteroskedasticity test       |
| <code>e(df_m_c)</code>     | degrees of freedom for heteroskedasticity test    |
| <code>e(p)</code>          | <i>p</i> -value for model test                    |
| <code>e(rank)</code>       | rank of <code>e(V)</code>                         |
| <code>e(rank0)</code>      | rank of <code>e(V)</code> for constant-only model |
| <code>e(ic)</code>         | number of iterations                              |
| <code>e(rc)</code>         | return code                                       |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise                       |

### Macros

|                          |                                                                                 |
|--------------------------|---------------------------------------------------------------------------------|
| <code>e(cmd)</code>      | <code>hetprobit</code>                                                          |
| <code>e(cmdline)</code>  | command as typed                                                                |
| <code>e(depvar)</code>   | name of dependent variable                                                      |
| <code>e(wtype)</code>    | weight type                                                                     |
| <code>e(wexp)</code>     | weight expression                                                               |
| <code>e(title)</code>    | title in estimation output                                                      |
| <code>e(clustvar)</code> | name of cluster variable                                                        |
| <code>e(offset1)</code>  | offset for probit equation                                                      |
| <code>e(offset2)</code>  | offset for variance equation                                                    |
| <code>e(chi2type)</code> | Wald or LR; type of model $\chi^2$ test                                         |
| <code>e(chi2_ct)</code>  | Wald or LR; type of model $\chi^2$ test corresponding to <code>e(chi2_c)</code> |
| <code>e(vce)</code>      | <i>vce</i> type specified in <code>vce()</code>                                 |
| <code>e(vcetype)</code>  | title used to label Std. err.                                                   |
| <code>e(opt)</code>      | type of optimization                                                            |
| <code>e(which)</code>    | max or min; whether optimizer is to perform maximization or minimization        |

|               |                                      |
|---------------|--------------------------------------|
| e(method)     | m1                                   |
| e(ml_method)  | type of m1 method                    |
| e(user)       | name of likelihood-evaluator program |
| e(technique)  | maximization technique               |
| e(properties) | b V                                  |
| e(predict)    | program used to implement predict    |
| e(asbalanced) | factor variables fvset as asbalanced |
| e(asobserved) | factor variables fvset as asobserved |

## Matrices

|                 |                                              |
|-----------------|----------------------------------------------|
| e(b)            | coefficient vector                           |
| e(Cns)          | constraints matrix                           |
| e(ilog)         | iteration log (up to 20 iterations)          |
| e(gradient)     | gradient vector                              |
| e(V)            | variance–covariance matrix of the estimators |
| e(V_modelbased) | model-based variance                         |

## Functions

|           |                         |
|-----------|-------------------------|
| e(sample) | marks estimation sample |
|-----------|-------------------------|

In addition to the above, the following is stored in `r()`:

## Matrices

|          |                                                                                                                            |
|----------|----------------------------------------------------------------------------------------------------------------------------|
| r(table) | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |
|----------|----------------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

## Methods and formulas

The heteroskedastic probit model is a generalization of the probit model because it allows the scale of the inverse link function to vary from observation to observation as a function of the independent variables.

The log-likelihood function for the heteroskedastic probit model is

$$\ln L = \sum_{j \in S} w_j \ln \Phi\{\mathbf{x}_j \boldsymbol{\beta} / \exp(\mathbf{z}_j \boldsymbol{\gamma})\} + \sum_{j \notin S} w_j \ln [1 - \Phi\{\mathbf{x}_j \boldsymbol{\beta} / \exp(\mathbf{z}_j \boldsymbol{\gamma})\}]$$

where  $S$  is the set of all observations  $j$  such that  $y_j \neq 0$  and  $w_j$  denotes the optional weights.  $\ln L$  is maximized as described in [R] [Maximize](#).

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] [\\_robust](#), particularly *Maximum likelihood estimators* and *Methods and formulas*.

`hetprobit` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] [Variance estimation](#).

## References

- Blevins, J. R., and S. Khan. 2013. [Distribution-free estimation of heteroskedastic binary response models in Stata](#). *Stata Journal* 13: 588–602.
- Greene, W. H. 2018. *Econometric Analysis*. 8th ed. New York: Pearson.
- Harvey, A. C. 1976. Estimating regression models with multiplicative heteroscedasticity. *Econometrica* 44: 461–465. <https://doi.org/10.2307/1913974>.

## Also see

[R] **hetprobit postestimation** — Postestimation tools for hetprobit

[R] **hetoprobit** — Heteroskedastic ordered probit regression

[R] **logistic** — Logistic regression, reporting odds ratios

[R] **probit** — Probit regression

[BAYES] **bayes: hetprobit** — Bayesian heteroskedastic probit regression

[SVY] **svy estimation** — Estimation commands for survey data

[XT] **xtprobit** — Random-effects and population-averaged probit models

[U] **20 Estimation and postestimation commands**

## Postestimation commands

The following postestimation commands are available after `hetprobit`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| <code>linktest</code>        | link test for model specification                                                                                                       |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | probabilities, linear predictions, etc.                                                                                                 |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as probabilities, linear predictions, and standard deviations.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic nooffset]
```

```
predict [type] stub* [if] [in] , scores
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

Main

|                    |                                                |
|--------------------|------------------------------------------------|
| <code>pr</code>    | probability of a positive outcome; the default |
| <code>xb</code>    | linear prediction                              |
| <code>sigma</code> | standard deviation of the error term           |

These statistics are available both in and out of sample; type `predict . . . if e(sample) . . .` if wanted only for the estimation sample.

## Options for predict

Main

`pr`, the default, calculates the probability of a positive outcome.

`xb` calculates the linear prediction.

`sigma` calculates the standard deviation of the error term.

`nooffset` is relevant only if you specified `offset(varname)` for `hetprobit`. It modifies the calculations made by `predict` so that they ignore the offset variable; the linear prediction is treated as  $\mathbf{x}_j\mathbf{b}$  rather than as  $\mathbf{x}_j\mathbf{b} + \text{offset}_j$ .

`scores` calculates equation-level score variables.

The first new variable will contain  $\partial \ln L / \partial (\mathbf{x}_j\boldsymbol{\beta})$ .

The second new variable will contain  $\partial \ln L / \partial (\mathbf{z}_j\boldsymbol{\gamma})$ .

# margins

## Description for margins

`margins` estimates margins of response for probabilities, linear predictions, and standard deviations.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```

margins [marginlist] [, options]
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]

```

| <i>statistic</i>   | Description                                    |
|--------------------|------------------------------------------------|
| <code>pr</code>    | probability of a positive outcome; the default |
| <code>xb</code>    | linear prediction                              |
| <code>sigma</code> | standard deviation of the error term           |

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [R] [margins](#).

## Remarks and examples

Once you have fit a model, you can use the `predict` command to obtain the predicted probabilities for both the estimation sample and other samples; see [U] [20 Estimation and postestimation commands](#) and [R] [predict](#). `predict` without arguments calculates the predicted probability of a positive outcome. With the `xb` option, `predict` calculates the index function combination,  $\mathbf{x}_j\mathbf{b}$ , where  $\mathbf{x}_j$  are the independent variables in the  $j$ th observation and  $\mathbf{b}$  is the estimated parameter vector. With the `sigma` option, `predict` calculates the predicted standard deviation,  $\sigma_j = \exp(\mathbf{z}_j\boldsymbol{\gamma})$ .

## ▷ Example 1

We use `predict` to compute the predicted probabilities and standard deviations based on the model in [example 2](#) in [\[R\] hetprobit](#) to compare these with the actual values:

```
. predict phat
(option pr assumed; Pr(y))
. generate diff_p = phat - p
. summarize diff_p
```

| Variable | Obs   | Mean     | Std. dev. | Min       | Max      |
|----------|-------|----------|-----------|-----------|----------|
| diff_p   | 1,000 | .0082805 | .0103027  | -.0169849 | .0396469 |

```
. predict sigmahat, sigma
. generate diff_s = sigmahat - sigma
. summarize diff_s
```

| Variable | Obs   | Mean      | Std. dev. | Min       | Max      |
|----------|-------|-----------|-----------|-----------|----------|
| diff_s   | 1,000 | -.2579493 | .2126614  | -.7661171 | -.000025 |

◀

**Also see**

[\[R\] hetprobit](#) — Heteroskedastic probit model

[\[U\] 20 Estimation and postestimation commands](#)

[Description](#)[Menu](#)[Options for maximum likelihood estimation](#)[Remarks and examples](#)[Methods and formulas](#)[Also see](#)[Quick start](#)[Syntax](#)[Options for two-step GLS estimation](#)[Stored results](#)[References](#)

## Description

`hetregress` fits a multiplicative heteroskedastic linear regression by modeling the variance as an exponential function of the specified variables using either maximum likelihood (ML; the default) or Harvey's two-step generalized least-squares (GLS) method.

## Quick start

Heteroskedastic regression model of  $y$  on  $x_1$ , using  $x_2$  to model the variance

```
hetregress y x1, het(x2)
```

Using Harvey's two-step GLS estimator instead of the default ML

```
hetregress y x1, het(x2) twostep
```

With robust standard errors

```
hetregress y x1, het(x2) vce(robust)
```

Perform a Wald test on the variance instead of a likelihood-ratio (LR) test

```
hetregress y x1, het(x2) waldhet
```

## Menu

Statistics > Linear models and related > Heteroskedastic linear regression

## Syntax

### Maximum likelihood estimation

```
hetregress depvar [indepvars] [if] [in] [weight] [, ml_options]
```

### Two-step GLS estimation

```
hetregress depvar [indepvars] [if] [in], twostep het(varlist) [ts_options]
```

### *ml\_options*

### Description

---

|                                            |                                                                                                                                                  |
|--------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| Model                                      |                                                                                                                                                  |
| <u>m</u> l                                 | use maximum likelihood estimator; the default                                                                                                    |
| het ( <i>varlist</i> )                     | independent variables to model the variance                                                                                                      |
| <u>n</u> oconstant                         | suppress constant term                                                                                                                           |
| <u>c</u> onstraints ( <i>constraints</i> ) | apply specified linear constraints                                                                                                               |
| SE/Robust                                  |                                                                                                                                                  |
| vce ( <i>vcetype</i> )                     | <i>vcetype</i> may be oim, r_robust, cluster <i>clustvar</i> , opg, bootstrap, or jackknife                                                      |
| Reporting                                  |                                                                                                                                                  |
| <u>l</u> evel (#)                          | set confidence level; default is level(95)                                                                                                       |
| lrmodel                                    | perform the LR model test instead of the default Wald model test                                                                                 |
| waldhet                                    | perform Wald test on variance instead of LR test                                                                                                 |
| <u>n</u> oconsreport                       | do not display constraints                                                                                                                       |
| <i>display_options</i>                     | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| Maximization                               |                                                                                                                                                  |
| <i>maximize_options</i>                    | control the maximization process; seldom used                                                                                                    |
| <u>c</u> ollinear                          | keep collinear variables                                                                                                                         |
| <u>c</u> oeflegend                         | display legend instead of statistics                                                                                                             |

---

| <i>ts_options</i>            | Description                                                                                                                                      |
|------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                 |                                                                                                                                                  |
| * <code>twostep</code>       | use two-step GLS estimator; default is maximum likelihood                                                                                        |
| * <code>het (varlist)</code> | independent variables to model the variance                                                                                                      |
| <code>noconstant</code>      | suppress constant term                                                                                                                           |
| <b>SE</b>                    |                                                                                                                                                  |
| <code>vce(vctype)</code>     | <i>vctype</i> may be <code>conventional</code> , <code>bootstrap</code> , or <code>jackknife</code>                                              |
| <b>Reporting</b>             |                                                                                                                                                  |
| <code>level(#)</code>        | set confidence level; default is <code>level(95)</code>                                                                                          |
| <code>display_options</code> | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| <code>coeflegend</code>      | display legend instead of statistics                                                                                                             |

\*`twostep` and `het()` are required.

*indepvars* and *varlist* may contain factor variables; see [U] 11.4.3 **Factor variables**.

*depvar*, *indepvars*, and *varlist* may contain time-series operators; see [U] 11.4.4 **Time-series varlists**.

`bayes`, `bayesboot`, `bootstrap`, `by`, `collect`, `fp`, `jackknife`, `rolling`, `statsby`, and `svy` are allowed; see [U] 11.1.10 **Prefix commands**. For more details, see [BAYES] **bayes: hetregress**.

Weights are not allowed with the `bootstrap` prefix; see [R] **bootstrap**.

`aweight`s are not allowed with the `jackknife` prefix; see [R] **jackknife**.

`vce()`, `lrmmodel`, `twostep`, and `weights` are not allowed with the `svy` prefix; see [SVY] **svy**.

`aweight`s, `fweight`s, `weight`s, and `pweight`s are allowed with maximum likelihood estimation; see [U] 11.1.6 **weight**.

`collinear` and `coeflegend` do not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options for maximum likelihood estimation

### Model

`mle` requests that the maximum likelihood estimator be used. This is the default.

`het (varlist)` specifies the independent variables in the variance function. When the `het()` option is not specified, homoskedasticity is assumed and the `waldhet` option is not allowed.

`noconstant`, `constraints(constraints)`; see [R] **Estimation options**.

### SE/Robust

`vce(vctype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] **vce\_option**.

## Reporting

`level(#)`, `lrmodel`; see [R] [Estimation options](#).

`waldhet` specifies that the Wald test of whether  $\ln\sigma^2 = 0$  be performed instead of the LR test.

`nocnsreport`; see [R] [Estimation options](#).

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

## Maximization

*maximize\_options*: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

Setting the optimization type to `technique(bhhh)` resets the default `vcetype` to `vce(opg)`.

The following options are available with `hetregress` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Options for two-step GLS estimation

## Model

`twostep` specifies that the model be fit using Harvey's two-step GLS estimator. This option requires that the independent variables be specified in the `het()` option to model the variance.

`het(varlist)` specifies the independent variables in the variance function.

`noconstant`; see [R] [Estimation options](#).

## SE

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`conventional`) and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

`vce(conventional)`, the default, uses the two-step variance estimator derived by Heckman.

## Reporting

`level(#)`; see [R] [Estimation options](#).

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

The following option is available with `hetregress` but is not shown in the dialog box:

`coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Remarks are presented under the following headings:

[Introduction](#)  
[Maximum likelihood estimation](#)  
[Two-step GLS estimation](#)

### Introduction

`hetregress` fits a multiplicative heteroskedastic linear regression model using either ML or Harvey's two-step GLS method. Multiplicative heteroskedasticity occurs when the variances of the error terms are assumed to be a multiplicative function of one or more variables. When variables are not specified in the `het()` option, `hetregress` fits a homoskedastic linear regression model.

Heteroskedasticity arises in a regression when the variances of the error terms are not constant across observations. For example, wages may be heteroskedastic when predicted by age group. While there is little variability in wages among workers in their teens and early 20s, wages among workers in their 50s may vary greatly because of a variety of factors. Heteroskedasticity is often found in time-series data and cross-sectional measurements and is a common issue in econometrics, social science, and many other fields. For more detailed information on how to detect the presence of heteroskedasticity, see [Tests for violation of assumptions](#) in [\[R\] regress postestimation](#).

We can use `hetregress` when the variance is assumed to have a form that is an exponential function of a linear combination of one or more variables. This is known as multiplicative heteroskedasticity and includes most of the useful formulations for variance as special cases. For example, in the special case of groupwise heteroskedasticity, the sample can be divided into groups where each group has a different variance.

A model with multiplicative heteroskedasticity can be written as

$$y_i = \mathbf{x}_i\boldsymbol{\beta} + \epsilon_i; \quad \sigma_i^2 = \exp(\mathbf{z}_i\boldsymbol{\alpha}) \quad (1)$$

where  $y_i$ ,  $i = 1, \dots, n$ , is the dependent variable;  $\mathbf{x}_i = (x_{1i}, x_{2i}, \dots, x_{ki})$  are the  $k$  independent variables that model the mean function; and  $\mathbf{z}_i = (z_{1i}, z_{2i}, \dots, z_{mi})$  are the  $m$  variables that model the variance function.  $\boldsymbol{\beta}$ 's are unknown parameters in the mean function, and  $\boldsymbol{\alpha}$ 's are unknown parameters in the variance function.  $\epsilon_i$ 's are errors that are independent and identically distributed with mean 0 and variance  $\sigma_i^2$ . Groupwise heteroskedasticity is modeled using (1) but where the  $\mathbf{z}_i$ 's are all indicator (dummy) variables for groups.

Harvey (1976) introduced two methods for dealing with multiplicative heteroskedasticity: ML estimation and two-step GLS estimation. By default, `hetregress` fits the multiplicative heteroskedastic regression model using ML. If the `twostep` option is specified, `hetregress` fits the model using the two-step GLS method. The ML estimates are more efficient than those obtained by the GLS estimator if the mean and variance function are correctly specified and the errors are normally distributed. By contrast, the two-step GLS estimates are more robust if the variance function is incorrect or the errors are nonnormal.

If the form of the variance is completely unknown, we may be better off using the OLS estimator instead of the ML and GLS estimators because it remains unbiased. However, we should then use the robust standard errors to correct for heteroskedasticity. Using robust standard errors for the OLS estimator allows us to make appropriate inferences without specifying any form for the variance. We discuss three modifications of the robust variance calculation in [Robust standard errors](#) of [\[R\] regress](#).

If the form of the variance is known and does not contain any unknown parameters, we can use the weighted least-squares estimator, also called the generalized least-squares estimator. For example, we can use weighted least squares to correct for heteroskedasticity if the variance is proportional to one of the regressors. See section 9.5.2 of [Greene \(2018\)](#) for details, and also see [Weighted regression](#) in [R] [regress](#).

[Greene \(2018\)](#) and [Hill, Griffiths, and Lim \(2018\)](#) compare the ML estimator and the GLS estimator with the robust OLS estimator. If the form of the heteroskedasticity is specified correctly, the ML and GLS estimators are more efficient than the robust OLS estimator. However, if the form of the heteroskedasticity is misspecified, the robust OLS estimator may be more efficient than the ML and GLS estimators.

## Maximum likelihood estimation

### ▷ Example 1: Multiplicative heteroskedasticity

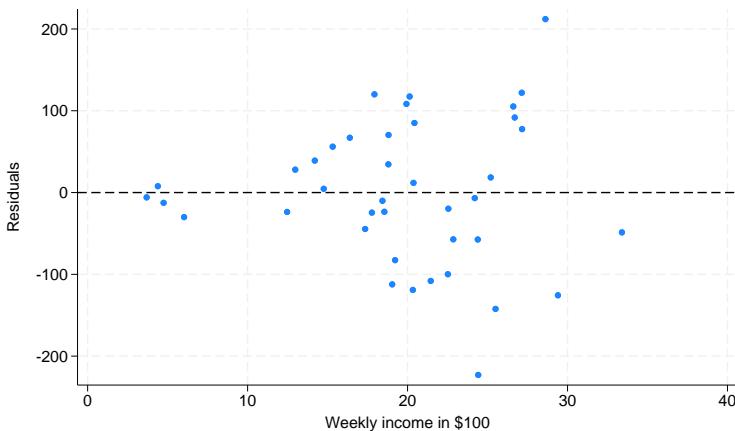
Consider the following dataset from a study of household expenditure on food described in [Hill, Griffiths, and Lim \(2018, chap. 8\)](#). We want to investigate the relationship between average household expenditure on food and household income by fitting a model of weekly food expenditure (`food_exp`) on weekly income (`income`) using OLS.

```
. use https://www.stata-press.com/data/r19/foodexp
(Household expenditure on food)

. regress food_exp income
(output omitted)
```

However, we suspect that the variance for low-income families may be lower than that for high-income families, because low-income families usually have less money to spend on food, while high-income families can choose to spend more or less on food. We plot the least-squares residuals against the value of income by using the `rvpplot` command after `regress`.

```
. rvpplot income, yline(0)
```



The graph confirms our suspicions about a relationship between income and the residuals. We believe that the variance is some power function of income. Therefore, we fit a multiplicative heteroskedastic regression model. The model for each observation is

$$\text{food\_exp}_i = \beta_0 + \beta_1 \times \text{income}_i + \epsilon_i$$

and the variance function can be written as

$$\sigma_i^2 = \sigma^2 \times \text{income}_i^\gamma$$

where  $\gamma$  is an unknown parameter of the variance function. To ensure that we will get positive values for the variance  $\sigma_i^2$  for all possible values of the unknown parameter  $\gamma$ , we rewrite this function so that  $\sigma_i^2$  is an exponential function of a linear combination of  $\ln(\text{income}_i)$  and a constant term:

$$\sigma_i^2 = \exp\{\alpha_0 + \alpha_1 \times \ln(\text{income}_i)\}$$

where  $\alpha_0 = \ln(\sigma^2)$  and  $\alpha_1 = \gamma$ .

To fit this model using `hetregress`, we first create a variable that contains the logarithm of income (`logincome`) and use it in the `het()` option to model the variance function. The constant term in the variance function is always assumed.

```
. generate double logincome = ln(income)
. hetregress food_exp income, het(logincome)
Fitting full model:
Iteration 0: Log likelihood = -227.889
Iteration 1: Log likelihood = -226.61039
Iteration 2: Log likelihood = -225.72188
Iteration 3: Log likelihood = -225.71519
Iteration 4: Log likelihood = -225.71519
Heteroskedastic linear regression Number of obs = 40
ML estimation Wald chi2(1) = 135.11
Log likelihood = -225.7152 Prob > chi2 = 0.0000
```

| food_exp  | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-----------|-------------|-----------|-------|-------|----------------------|----------|
| food_exp  |             |           |       |       |                      |          |
| income    | 10.63444    | .9148876  | 11.62 | 0.000 | 8.841295             | 12.42759 |
| _cons     | 76.07294    | 7.369143  | 10.32 | 0.000 | 61.62969             | 90.5162  |
| lnsigma2  |             |           |       |       |                      |          |
| logincome | 2.769762    | .4481606  | 6.18  | 0.000 | 1.891383             | 3.648141 |
| _cons     | .4684052    | 1.310337  | 0.36  | 0.721 | -2.099809            | 3.036619 |

```
LR test of lnsigma2=0: chi2(1) = 19.59 Prob > chi2 = 0.0000
```

The LR test at the bottom of the output is a test for the parameters of the variance function. The  $\chi^2(1)$  statistic of 19.59 is significant, indicating that heteroskedasticity is present. If we had preferred the Wald test for heteroskedasticity instead of the LR test, we would have specified the `waldhet` option.

In addition to the estimated parameters for the mean function (under `food_exp`), `hetregress` reports estimated parameters and test statistics for the variance function. The significant  $z$  statistic for `logincome` also suggests the presence of heteroskedasticity. Relating the output back to our model,  $\exp(0.47) \approx 1.60$  is our estimate of  $\sigma^2$ . The coefficient for `logincome` is 2.77. This is our estimate of  $\gamma$ , and it can be interpreted as the multiplicative factor of the variance associated with income.

We can obtain more formal results by using `nlcom`:

```
. nlcom (sigma2: exp(_b[lsigma2:_cons]))
 sigma2: exp(_b[lsigma2:_cons])
```

| food_exp | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |          |
|----------|-------------|-----------|------|-------|----------------------|----------|
| sigma2   | 1.597445    | 2.093191  | 0.76 | 0.445 | -2.505135            | 5.700024 |

Here `sigma2` refers to  $\sigma^2$  in the variance function given [above](#).



## Two-step GLS estimation

### ▷ Example 2: Groupwise heteroskedasticity

Here we will use a dataset of 725 faculty members' salaries described in [DeMaris \(2004\)](#) to determine whether there is evidence of a difference in salaries between male faculty and female faculty. In addition to `sex` (`female`), other variables that might affect the salaries are prior experience (`priorex`), years in rank (`yrrank`), years at the university (`yrbg`), and marketability of discipline (`salfac`). We will treat `female` as a factor variable and all other variables as continuous variables.

We could fit this model with regress by including main effects and the interaction terms between female and all other variables (by using `factor-variable notation`).

```
. use https://www.stata-press.com/data/r19/salary, clear
(DeMaris (2004) - Faculty salaries)
```

```
. regress salary i.female##(c.priorexp c.yrrank c.yrbg c.salfac)
```

| Source   | SS         | df  | MS         | Number of obs | = | 725    |
|----------|------------|-----|------------|---------------|---|--------|
| Model    | 8.9287e+10 | 9   | 9.9207e+09 | F(9, 715)     | = | 135.23 |
| Residual | 5.2453e+10 | 715 | 73360978.8 | Prob > F      | = | 0.0000 |
|          |            |     |            | R-squared     | = | 0.6299 |
|          |            |     |            | Adj R-squared | = | 0.6253 |
| Total    | 1.4174e+11 | 724 | 195773163  | Root MSE      | = | 8565.1 |

| salary                     | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|----------------------------|-------------|-----------|-------|-------|----------------------|-----------|
| 1.female                   | 5735.113    | 4987.433  | 1.15  | 0.251 | -4056.65             | 15526.88  |
| priorexp                   | 1042.845    | 82.62092  | 12.62 | 0.000 | 880.6366             | 1205.054  |
| yrrank                     | -47.80904   | 99.85936  | -0.48 | 0.632 | -243.8617            | 148.2436  |
| yrbg                       | 1009.12     | 75.5161   | 13.36 | 0.000 | 860.8607             | 1157.38   |
| salfac                     | 33601.3     | 2531.61   | 13.27 | 0.000 | 28631.02             | 38571.58  |
| female#<br>c.priorexp<br>1 | -662.7972   | 159.6171  | -4.15 | 0.000 | -976.1715            | -349.4228 |
| female#<br>c.yrrank<br>1   | -447.8621   | 218.8695  | -2.05 | 0.041 | -877.5659            | -18.15833 |
| female#c.yrbg<br>1         | 115.4784    | 157.9372  | 0.73  | 0.465 | -194.5976            | 425.5545  |
| female#<br>c.salfac<br>1   | -7618.157   | 5142.204  | -1.48 | 0.139 | -17713.78            | 2477.467  |
| _cons                      | 2137.718    | 2634.266  | 0.81  | 0.417 | -3034.103            | 7309.539  |

However, we believe that the variances differ between female faculty and male faculty. In this case, we will use `estat hettest` to perform the [Breusch and Pagan \(1979\)](#) test for heteroskedasticity. See [Tests for violation of assumptions in \[R\] regress postestimation](#) for more detailed information.

```
. estat hettest i.female
Breusch-Pagan/Cook-Weisberg test for heteroskedasticity
Assumption: Normal error terms
Variable: i.female
HO: Constant variance
chi2(1) = 11.80
Prob > chi2 = 0.0006
```

The results above suggest the presence of heteroskedasticity with respect to sex. This is a case of groupwise heteroskedasticity and can be modeled using `hetregress` by treating the sex variable (`female`) as a factor variable (`i.female`) in the `het()` option.



To obtain an estimate for female faculty of the effect of experience on salary, we can use `lincom`.

```
. lincom priorexp + 1.female#c.priorexp
(1) [salary]priorexp + [salary]1.female#c.priorexp = 0
```

| salary | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |          |
|--------|-------------|-----------|------|-------|----------------------|----------|
| (1)    | 380.0481    | 110.8702  | 3.43 | 0.001 | 162.7465             | 597.3497 |

We see that each additional year of experience increases salary by only \$380.05 and that this effect is significant.

We can estimate the effect of each of the other variables on the salaries of female faculty if we wish.

```
. lincom yrrank + 1.female#c.yrrank
(1) [salary]yrrank + [salary]1.female#c.yrrank = 0
```

| salary | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|--------|-------------|-----------|-------|-------|----------------------|-----------|
| (1)    | -495.6711   | 158.1108  | -3.13 | 0.002 | -805.5627            | -185.7796 |

The effect of `yrrank` is associated with a decrease in salary. The effect is significant for female faculty but not for male faculty.

◀

## Stored results

`hetregress (ML)` stores the following in `e()`:

### Scalars

|                            |                                                   |
|----------------------------|---------------------------------------------------|
| <code>e(N)</code>          | number of observations                            |
| <code>e(k)</code>          | number of parameters                              |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>          |
| <code>e(k_eq_model)</code> | number of equations in overall model test         |
| <code>e(k_dv)</code>       | number of dependent variables                     |
| <code>e(df_m)</code>       | model degrees of freedom                          |
| <code>e(ll)</code>         | log likelihood, full model                        |
| <code>e(ll_0)</code>       | log likelihood, constant-only model               |
| <code>e(ll_c)</code>       | log likelihood, comparison model                  |
| <code>e(N_clust)</code>    | number of clusters                                |
| <code>e(chi2)</code>       | $\chi^2$ for mean model test                      |
| <code>e(chi2_c)</code>     | $\chi^2$ for heteroskedasticity test              |
| <code>e(p_c)</code>        | $p$ -value for heteroskedasticity test            |
| <code>e(df_m_c)</code>     | degrees of freedom for heteroskedasticity test    |
| <code>e(p)</code>          | $p$ -value for the mean model test                |
| <code>e(rank)</code>       | rank of <code>e(V)</code>                         |
| <code>e(rank0)</code>      | rank of <code>e(V)</code> for constant-only model |
| <code>e(ic)</code>         | number of iterations                              |
| <code>e(rc)</code>         | return code                                       |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise                       |

### Macros

|                         |                            |
|-------------------------|----------------------------|
| <code>e(cmd)</code>     | <code>hetregress</code>    |
| <code>e(cmdline)</code> | command as typed           |
| <code>e(depvar)</code>  | name of dependent variable |
| <code>e(wtype)</code>   | weight type                |
| <code>e(wexp)</code>    | weight expression          |

|                              |                                                                                           |
|------------------------------|-------------------------------------------------------------------------------------------|
| <code>e(title)</code>        | title in estimation output                                                                |
| <code>e(title2)</code>       | secondary title in estimation output                                                      |
| <code>e(clustvar)</code>     | name of cluster variable                                                                  |
| <code>e(chi2type)</code>     | Wald or LR; type of model $\chi^2$ test                                                   |
| <code>e(chi2_ct)</code>      | Wald or LR; type of heteroskedastic $\chi^2$ test corresponding to <code>e(chi2_c)</code> |
| <code>e(vce)</code>          | <i>vce</i> type specified in <code>vce()</code>                                           |
| <code>e(vcetype)</code>      | title used to label Std. err.                                                             |
| <code>e(opt)</code>          | type of optimization                                                                      |
| <code>e(which)</code>        | max or min; whether optimizer is to perform maximization or minimization                  |
| <code>e(method)</code>       | m1                                                                                        |
| <code>e(ml_method)</code>    | type of ml method                                                                         |
| <code>e(user)</code>         | name of likelihood-evaluator program                                                      |
| <code>e(technique)</code>    | maximization technique                                                                    |
| <code>e(properties)</code>   | b V                                                                                       |
| <code>e(predict)</code>      | program used to implement predict                                                         |
| <code>e(marginsok)</code>    | predictions allowed by margins                                                            |
| <code>e(marginsnotok)</code> | predictions disallowed by margins                                                         |
| <code>e(asbalanced)</code>   | factor variables <code>fvset</code> as <code>asbalanced</code>                            |
| <code>e(asobserved)</code>   | factor variables <code>fvset</code> as <code>asobserved</code>                            |

## Matrices

|                              |                                              |
|------------------------------|----------------------------------------------|
| <code>e(b)</code>            | coefficient vector                           |
| <code>e(Cns)</code>          | constraints matrix                           |
| <code>e(ilog)</code>         | iteration log (up to 20 iterations)          |
| <code>e(gradient)</code>     | gradient vector                              |
| <code>e(V)</code>            | variance–covariance matrix of the estimators |
| <code>e(V_modelbased)</code> | model-based variance                         |

## Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

## Matrices

|                       |                                                                                                                            |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

`hetregress` (two-step GLS) stores the following in `e()`:

## Scalars

|                        |                                                |
|------------------------|------------------------------------------------|
| <code>e(N)</code>      | number of observations                         |
| <code>e(k)</code>      | number of parameters                           |
| <code>e(df_m)</code>   | model degrees of freedom                       |
| <code>e(chi2)</code>   | $\chi^2$ for mean model test                   |
| <code>e(chi2_c)</code> | $\chi^2$ for heteroskedasticity test           |
| <code>e(p_c)</code>    | <i>p</i> -value for heteroskedasticity test    |
| <code>e(df_m_c)</code> | degrees of freedom for heteroskedasticity test |
| <code>e(p)</code>      | <i>p</i> -value for the mean model test        |
| <code>e(rank)</code>   | rank of <code>e(V)</code>                      |

## Macros

|                          |                                                                                     |
|--------------------------|-------------------------------------------------------------------------------------|
| <code>e(cmd)</code>      | <code>hetregress</code>                                                             |
| <code>e(cmdline)</code>  | command as typed                                                                    |
| <code>e(depvar)</code>   | name of dependent variable                                                          |
| <code>e(title)</code>    | title in estimation output                                                          |
| <code>e(title2)</code>   | secondary title in estimation output                                                |
| <code>e(chi2type)</code> | Wald; type of model $\chi^2$ test                                                   |
| <code>e(chi2_ct)</code>  | Wald; type of heteroskedastic $\chi^2$ test corresponding to <code>e(chi2_c)</code> |

|                 |                                                                |
|-----------------|----------------------------------------------------------------|
| e(vce)          | <i>vcetype</i> specified in <code>vce()</code>                 |
| e(method)       | <code>twostep</code>                                           |
| e(properties)   | <code>b V</code>                                               |
| e(predict)      | program used to implement <code>predict</code>                 |
| e(marginsok)    | predictions allowed by <code>margins</code>                    |
| e(marginsnotok) | predictions disallowed by <code>margins</code>                 |
| e(asbalanced)   | factor variables <code>fvset</code> as <code>asbalanced</code> |
| e(asobserved)   | factor variables <code>fvset</code> as <code>asobserved</code> |
| Matrices        |                                                                |
| e(b)            | coefficient vector                                             |
| e(V)            | variance–covariance matrix of the estimators                   |
| Functions       |                                                                |
| e(sample)       | marks estimation sample                                        |

In addition to the above, the following is stored in `r()`:

|          |                                                                                                                            |
|----------|----------------------------------------------------------------------------------------------------------------------------|
| Matrices |                                                                                                                            |
| r(table) | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r`-class command is run after the estimation command.

## Methods and formulas

Methods and formulas are presented under the following headings:

*Maximum likelihood estimation*  
*Two-step GLS estimation*

### Maximum likelihood estimation

By default, `hetregress` fits a multiplicative heteroskedastic regression using ML estimation. The log-likelihood function is

$$\ln L = \sum_{i=1}^n \frac{w_i}{2} \left\{ \frac{(y_i - \mathbf{x}_i \boldsymbol{\beta})^2}{\exp(\mathbf{z}_i \boldsymbol{\alpha})} - \ln(2\pi) - \mathbf{z}_i \boldsymbol{\alpha} \right\}$$

where  $y_i$ ,  $i = 1, \dots, n$ , is the dependent variable;  $\mathbf{x}_i = (x_{1i}, x_{2i}, \dots, x_{ki})$  are the  $k$  independent variables that model the mean function;  $\mathbf{z}_i = (z_{1i}, z_{2i}, \dots, z_{mi})$  are the  $m$  variables that model the variance function; and  $w_i$  are the weights.  $\boldsymbol{\beta}$  is a column vector of unknown parameters in the mean function, and  $\boldsymbol{\alpha}$  is a column vector of unknown parameters in the variance function. The GLS estimates  $\widehat{\boldsymbol{\beta}}_{\text{GLS}}$  and  $\widehat{\boldsymbol{\alpha}}_{\text{GLS}}$  (described below) are used as the initial values in ML estimation. The  $\ln L$  function is maximized as described in [R] **Maximize**.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] **\_robust**, particularly *Maximum likelihood estimators* and *Methods and formulas*.

`hetregress` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] **Variance estimation**.

## Two-step GLS estimation

hetregress uses two-step GLS estimation when the `twostep` option is specified. Harvey (1976) describes the procedure in detail, but here are the main steps.

1. Use OLS to estimate regression coefficients  $\beta$  and compute residuals  $\mathbf{e}_i, i = 1, \dots, n$ .
2. Use OLS to regress the log-squared residuals,  $\ln(\mathbf{e}_i^2)$ , on  $\mathbf{z}$  and estimate  $\alpha$ .
3. Perform correction for the OLS estimates of  $\alpha$  to obtain  $\hat{\alpha}_c$  and their covariance matrix based on Harvey (1976).
4. Compute  $\hat{\sigma}_i^2 = \exp(\mathbf{z}_i \hat{\alpha}_c), i = 1, \dots, n$ .
5. Refit the original regression model using  $\hat{\sigma}_i^2$ 's as weights to obtain the GLS estimates  $\hat{\alpha}_{\text{GLS}}$  and  $\hat{\beta}_{\text{GLS}}$ .

## References

- Breusch, T. S., and A. R. Pagan. 1979. A simple test for heteroscedasticity and random coefficient variation. *Econometrica* 47: 1287–1294. <https://doi.org/10.2307/1911963>.
- DeMaris, A. 2004. *Regression with Social Data: Modeling Continuous and Limited Response Variables*. Hoboken, NJ: Wiley. <https://doi.org/10.1002/0471677566>.
- Greene, W. H. 2018. *Econometric Analysis*. 8th ed. New York: Pearson.
- Harvey, A. C. 1976. Estimating regression models with multiplicative heteroscedasticity. *Econometrica* 44: 461–465. <https://doi.org/10.2307/1913974>.
- Hill, R. C., W. E. Griffiths, and G. C. Lim. 2018. *Principles of Econometrics*. 5th ed. Hoboken, NJ: Wiley.

## Also see

- [R] **hetregress postestimation** — Postestimation tools for hetregress
- [R] **regress** — Linear regression
- [BAYES] **bayes: hetregress** — Bayesian heteroskedastic linear regression
- [SVY] **svy estimation** — Estimation commands for survey data
- [U] **20 Estimation and postestimation commands**

## Postestimation commands

The following postestimation commands are available after `hetregress`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| * <code>estat ic</code>      | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| * <code>estat (svy)</code>   | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| † <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| † <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| <code>linktest</code>        | link test for model specification                                                                                                       |
| *† <code>lrtest</code>       | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | linear predictions and their SEs, etc.                                                                                                  |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| * <code>suest</code>         | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`estat ic`, `estat (svy)`, `lrtest`, and `suest` are not appropriate after `hetregress`, `twostep`.

†`forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as linear predictions, standard errors, and standard deviations.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

*After ML or two-step*

```
predict [type] newvar [if] [in] [, statistic]
```

*After ML*

```
predict [type] stub* [if] [in] , scores
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

---

Main

|                    |                                         |
|--------------------|-----------------------------------------|
| <code>xb</code>    | linear prediction; the default          |
| <code>stdp</code>  | standard error of the linear prediction |
| <code>sigma</code> | standard deviation of the error term    |

---

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

## Options for predict

Main

`xb`, the default, calculates the linear prediction.

`stdp` calculates the standard errors of the linear prediction.

`sigma` calculates the standard deviations of the error term.

`scores` calculates equation-level score variables.

The first new variable will contain the derivative of the log likelihood with respect to the regression equation,  $\partial \ln L / \partial (\mathbf{x}_i \boldsymbol{\beta})$ .

The second new variable will contain the derivative of the log likelihood with respect to the scale equation (`lnsigma2`),  $\partial \ln L / \partial (\mathbf{z}_i \boldsymbol{\alpha})$ .

## margins

### Description for margins

`margins` estimates margins of response for linear predictions and of standard deviations.

### Menu for margins

Statistics > Postestimation

### Syntax for margins

```

margins [marginlist] [, options]
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]

```

| <i>statistic</i>   | Description                           |
|--------------------|---------------------------------------|
| <code>xb</code>    | linear prediction; the default        |
| <code>stdp</code>  | not allowed with <code>margins</code> |
| <code>sigma</code> | standard deviation of the error term  |

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [R] [margins](#).

### Remarks and examples

Once you have fit a heteroskedastic regression model, you can use the `predict` command to obtain the predicted values both for the estimation sample and for other samples; see [U] [20 Estimation and postestimation commands](#) and [R] [predict](#). `predict` without arguments calculates the linear prediction from the fitted model  $\mathbf{x}_i\mathbf{b}$ , where  $\mathbf{x}_i$  are the independent variables in the  $j$ th observation and  $\mathbf{b}$  is the estimated parameter vector for the mean model. With the `stdp` option, `predict` calculates the standard error of the linear prediction. With the `sigma` option, `predict` calculates the predicted standard deviations of the error term,  $\hat{\sigma}_j = \exp(0.5 \times \mathbf{z}_i\mathbf{a})$ , where  $\mathbf{g}$  is the estimated parameter vector for the variance model.

▷ Example 1: Predicting heteroskedastic standard deviation

We can use `predict` to compute the predicted values of the standard deviations for female and male faculty based on the model from [example 2](#) in [R] `hetregress`.

```
. use https://www.stata-press.com/data/r19/salary
(DeMaris (2004) - Faculty salaries)
. hetregress salary i.female##(c.priorexp c.yrrank c.yrbg c.salfac),
> het(i.female) twostep
(output omitted)
. predict sigma, sigma
. tabulate female, summarize(sigma)
```

| 1 = female;<br>0 = male | Summary of Heteroskedastic standard deviation |           |       |
|-------------------------|-----------------------------------------------|-----------|-------|
|                         | Mean                                          | Std. dev. | Freq. |
| 0                       | 7741.8481                                     | 0         | 511   |
| 1                       | 5828.6973                                     | 0         | 214   |
| Total                   | 7177.1388                                     | 873.22924 | 725   |

The predicted standard deviation for male faculty is  $7742/5829 \approx 1.3$  times the size for female faculty. We could have obtained the same results using `margins` with the `predict(sigma)` option.



▷ Example 2: Marginal means

We can use `margins` to compute the adjusted mean salary for male and female faculty when other factors are fixed at their means:

```
. margins female, atmeans
Adjusted predictions Number of obs = 725
Model VCE: Conventional
Expression: Linear prediction, predict()
At: priorexp = 2.89931 (mean)
 yrrank = 7.397241 (mean)
 yrbg = 12.52966 (mean)
 salfac = .9399862 (mean)
```

|        | Delta-method |           |        |       |                      |
|--------|--------------|-----------|--------|-------|----------------------|
|        | Margin       | std. err. | z      | P> z  | [95% conf. interval] |
| female |              |           |        |       |                      |
| 0      | 49036.29     | 420.3711  | 116.65 | 0.000 | 48212.37 49860.2     |
| 1      | 43822.74     | 569.9368  | 76.89  | 0.000 | 42705.69 44939.8     |

If everyone in the population were male faculty while holding all other factors at their mean values, the average salary would be \$49,036. If, instead, everyone were female faculty, the average salary would be \$43,823.



**Also see**

[R] `hetregress` — Heteroskedastic linear regression

[U] 20 Estimation and postestimation commands

[Description](#)  
[References](#)[Quick start](#)  
[Also see](#)[Menu](#)[Syntax](#)[Options](#)[Remarks and examples](#)

## Description

`histogram` draws histograms of *varname*, which is assumed to be the name of a continuous variable unless the `discrete` option is specified.

`hist` is a synonym for `histogram`.

## Quick start

Histogram of `v1`

```
histogram v1
```

Add a normal density curve to the graph

```
histogram v1, normal
```

Add a kernel density estimate to the graph

```
histogram v1, normal kdensity
```

Add “My Title” as the title of the graph

```
histogram v1, normal kdensity title("My Title")
```

Specify the number of bins as 10

```
histogram v1, bins(10)
```

Specify the width of the bins as 2

```
histogram v1, width(2)
```

Specify that `v2` should be treated as discrete

```
histogram v2, discrete
```

Same as above, but with narrower bars and space between the bars

```
histogram v2, discrete barwidth(.8)
```

Add labels to the bars on the *x* axis

```
histogram v2, discrete barwidth(.8) xlabel(1 "Category 1" ///
2 "Category 2" 3 "Category 3" 4 "Category 4")
```

Show frequencies on the *y* axis

```
histogram v1, frequency
```

Show percentages on the *y* axis

```
histogram v1, percent
```

Produce histograms for each value of categorical variable `catvar`

```
histogram v1, by(catvar)
```

Same as above, but with histograms arranged in a single column

```
histogram v1, by(cvar, cols(1))
```

## Menu

Graphics > Histogram

## Syntax

```
histogram varname [if] [in] [weight] [, [continuous_opts | discrete_opts] options]
```

### *continuous\_opts*

### Description

Main

|                  |                                   |
|------------------|-----------------------------------|
| <u>bins</u> (#)  | set number of bins to #           |
| <u>width</u> (#) | set width of bins to #            |
| <u>start</u> (#) | set lower limit of first bin to # |

### *discrete\_opts*

### Description

Main

|                  |                                    |
|------------------|------------------------------------|
| <u>discrete</u>  | specify that data are discrete     |
| <u>width</u> (#) | set width of bins to #             |
| <u>start</u> (#) | set theoretical minimum value to # |

### *options*

### Description

Main

|                                                   |                                                           |
|---------------------------------------------------|-----------------------------------------------------------|
| <u>density</u>                                    | draw as density; the default                              |
| <u>fraction</u>                                   | draw as fractions                                         |
| <u>frequency</u>                                  | draw as frequencies                                       |
| <u>percent</u>                                    | draw as percentages                                       |
| <i>bar_options</i>                                | rendition of bars                                         |
| <u>binrescale</u>                                 | recalculate bin sizes when <code>by()</code> is specified |
| <u>addlabels</u>                                  | add height labels to bars                                 |
| <u>addlabopts</u> ( <i>marker_label_options</i> ) | affect rendition of labels                                |

Density plots

|                                             |                                            |
|---------------------------------------------|--------------------------------------------|
| <u>normal</u>                               | add a normal density to the graph          |
| <u>normopts</u> ( <i>line_options</i> )     | affect rendition of normal density         |
| <u>kdensity</u>                             | add a kernel density estimate to the graph |
| <u>kdenopts</u> ( <i>kdensity_options</i> ) | affect rendition of kernel density         |

Add plots

|                                |                                  |
|--------------------------------|----------------------------------|
| <u>addplot</u> ( <i>plot</i> ) | add other plots to the histogram |
|--------------------------------|----------------------------------|

Y axis, X axis, Titles, Legend, Overall, By

|                       |                                                       |
|-----------------------|-------------------------------------------------------|
| <i>twoway_options</i> | any options documented in [G-3] <i>twoway_options</i> |
|-----------------------|-------------------------------------------------------|

`fweights` are allowed; see [U] 11.1.6 `weight`.

## Options

Options are presented under the following headings:

- Options for use in the continuous case*
- Options for use in the discrete case*
- Options for use in the continuous and discrete cases*

### Options for use in the continuous case

Main

`bins(#)` and `width(#)` are alternatives. They specify how the data are to be aggregated into bins: `bins()` by specifying the number of bins (from which the width can be derived) and `width()` by specifying the bin width (from which the number of bins can be derived).

If neither option is specified, results are the same as if `bins( $k$ )` had been specified, where

$$k = \min\{\text{sqrt}(N), 10 \ln(N)/\ln(10)\}$$

and where  $N$  is the (weighted) number of observations.

`start(#)` specifies the theoretical minimum of *varname*. The default is `start( $m$ )`, where  $m$  is the observed minimum value of *varname*.

Specify `start()` when you are concerned about sparse data, for instance, if you know that *varname* can have a value of 0, but you are concerned that 0 may not be observed.

`start(#)`, if specified, must be less than or equal to  $m$ , or else an error will be issued.

### Options for use in the discrete case

Main

`discrete` specifies that *varname* is discrete and that you want each unique value of *varname* to have its own bin (bar of histogram).

`width(#)` is rarely specified in the discrete case; it specifies the width of the bins. The default is `width( $d$ )`, where  $d$  is the observed minimum difference between the unique values of *varname*.

Specify `width()` if you are concerned that your data are sparse. For example, in theory *varname* could take on the values, say, 1, 2, 3, ..., 9, but because of the sparseness, perhaps only the values 2, 4, 7, and 8 are observed. Here the default width calculation would produce `width(2)`, and you would want to specify `width(1)`.

`start(#)` is also rarely specified in the discrete case; it specifies the theoretical minimum value of *varname*. The default is `start( $m$ )`, where  $m$  is the observed minimum value.

As with `width()`, specify `start(#)` if you are concerned that your data are sparse. In the previous example, you might also want to specify `start(1)`. `start()` does nothing more than add white space to the left side of the graph.

The value of  $\#$  in `start()` must be less than or equal to  $m$ , or an error will be issued.

## Options for use in the continuous and discrete cases

### Main

`density`, `fraction`, `frequency`, and `percent` specify whether you want the histogram scaled to density units, fractional units, frequencies, or percentages. `density` is the default.

`density` scales the height of the bars so that the sum of their areas equals 1.

`fraction` scales the height of the bars so that the sum of their heights equals 1.

`frequency` scales the height of the bars so that each bar's height is equal to the number of observations in the category. Thus the sum of the heights is equal to the total number of observations.

`percent` scales the height of the bars so that the sum of their heights equals 100.

`bar_options` are any of the options allowed by `graph twoway bar`; see [G-2] [graph twoway bar](#).

One of the most useful `bar_options` is `barwidth(#)`, which specifies the width of the bars in `varname` units. By default, `histogram` draws the bars so that adjacent bars just touch. If you want gaps between the bars, do not specify `histogram`'s `width()` option—which would change how the histogram is calculated—but specify the `bar_option` `barwidth()` or the `histogram` option `gap`, both of which affect only how the bar is rendered.

The `bar_option` `horizontal` cannot be used with the `addlabels` option.

`binrescale` specifies that bin size and plot range be recalculated for each group when `by()` is specified. If `normal` is specified, the mean and standard deviation of each overlaid normal density plot are recalculated in each group. Similarly, if `kdensity` is specified, the scaling of the overlaid kernel density plot is recalculated in each group.

`addlabels` specifies that the top of each bar be labeled with the density, fraction, or frequency, as determined by the `density`, `fraction`, and `frequency` options.

`addlabopts(marker_label_options)` specifies how to render the labels atop the bars. See [G-3] [marker\\_label\\_options](#). Do not specify the `marker_label_option` `mlabel(varname)`, which specifies the variable to be used; this is specified for you by `histogram`.

`addlabopts()` will accept more options than those documented in [G-3] [marker\\_label\\_options](#). All options allowed by `twoway scatter` are also allowed by `addlabopts()`; see [G-2] [graph twoway scatter](#). One particularly useful option is `yvarformat()`; see [G-3] [advanced\\_options](#).

### Density plots

`normal` specifies that the histogram be overlaid with an appropriately scaled normal density. The normal will have the same mean and standard deviation as the data.

`normopts(line_options)` specifies details about the rendition of the normal curve, such as the color and style of line used. See [G-2] [graph twoway line](#).

`kdensity` specifies that the histogram be overlaid with an appropriately scaled kernel density estimate of the density. By default, the estimate will be produced using the Epanechnikov kernel with an “optimal” half-width. This default corresponds to the default of `kdensity`; see [R] [kdensity](#). How the estimate is produced can be controlled using the `kdenopts()` option described below.

`kdenopts` (*kdensity\_options*) specifies details about how the kernel density estimate is to be produced along with details about the rendition of the resulting curve, such as the color and style of line used; see [G-2] [graph twoway kdensity](#). The kernel density estimate is described in [R] [kdensity](#). As an example, if you wanted to produce kernel density estimates by using the Gaussian kernel with optimal half-width, you would specify `kdenopts(gauss)` and if you also wanted a half-width of 5, you would specify `kdenopts(gauss width(5))`.

Add plots

`addplot` (*plot*) allows adding more graph twoway plots to the graph; see [G-3] [addplot\\_option](#).

Y axis, X axis, Titles, Legend, Overall, By

*twoway\_options* are any of the options documented in [G-3] [twoway\\_options](#). This includes, most importantly, options for titling the graph (see [G-3] [title\\_options](#)), options for saving the graph to disk (see [G-3] [saving\\_option](#)), and the `by()` option, which will allow you to simultaneously graph histograms for different subsets of the data (see [G-3] [by\\_option](#)).

## Remarks and examples

Remarks are presented under the following headings:

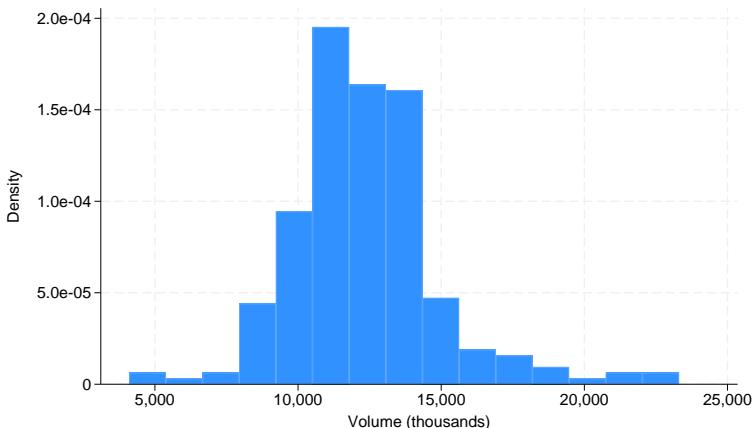
- [Histograms of continuous variables](#)
- [Overlaying normal and kernel density estimates](#)
- [Histograms of discrete variables](#)
- [Use with by\(\)](#)
- [Video example](#)

For an example of editing a histogram with the Graph Editor, see [Pollock \(2011, 29–31\)](#).

## Histograms of continuous variables

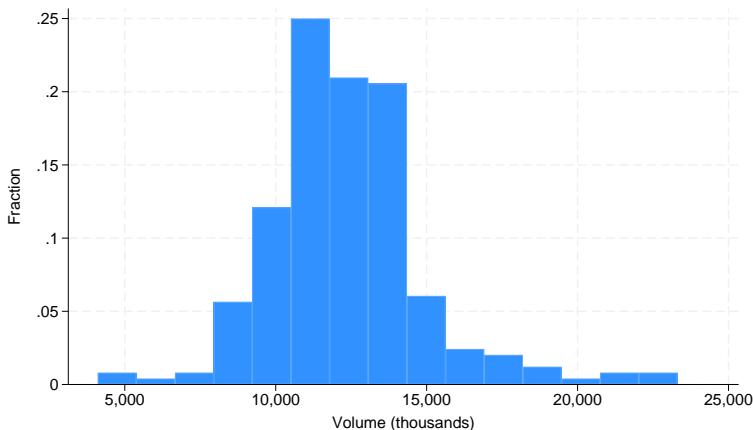
`histogram` assumes that the variable is continuous, so you need type only `histogram` followed by the variable name:

```
. use https://www.stata-press.com/data/r19/sp500
(S&P 500)
. histogram volume
(bin=15, start=4103, width=1280.3533)
```



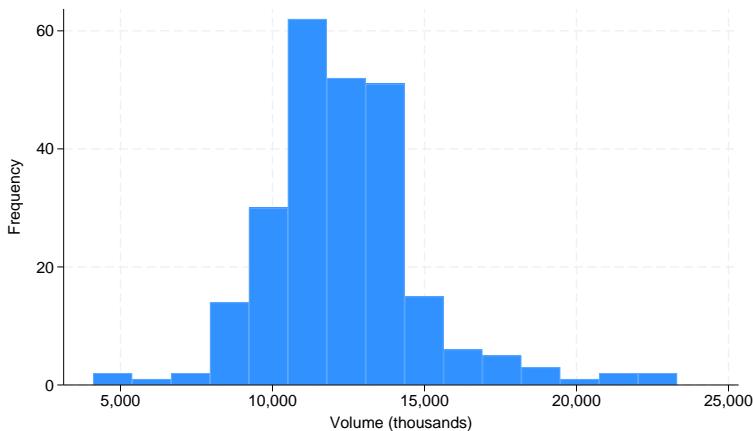
The small values reported for density on the  $y$  axis are correct; if you added up the area of the bars, you would get 1. Nevertheless, many people are used to seeing histograms scaled so that the bar heights sum to 1,

```
. histogram volume, fraction
(bin=15, start=4103, width=1280.3533)
```



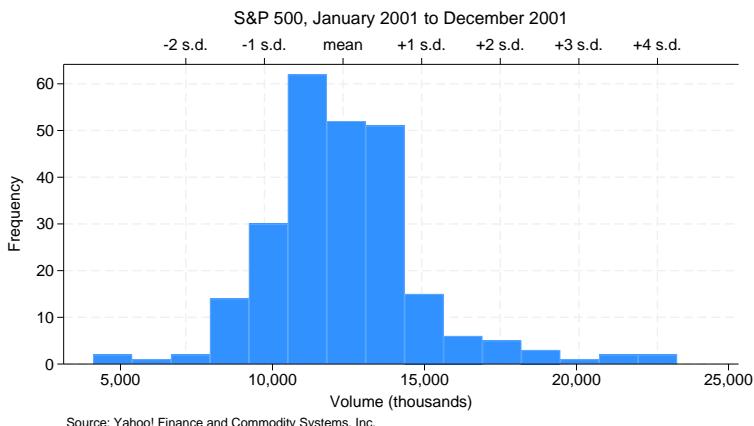
and others are used to seeing histograms so that the bar height reflects the number of observations,

```
. histogram volume, frequency
(bin=15, start=4103, width=1280.3533)
```



Regardless of the scale you prefer, you can specify other options to make the graph look more impressive:

```
. summarize volume
+-----+-----+-----+-----+-----+
| Variable | Obs | Mean | Std. dev. | Min | Max |
+-----+-----+-----+-----+-----+
| volume | 248 | 12320.68 | 2585.929 | 4103 | 23308.3 |
+-----+-----+-----+-----+-----+
. histogram volume, freq
> xaxis(1 2)
> ylabel(0(10)60)
> xlabel(12321 "mean"
> 9735 "-1 s.d."
> 14907 "+1 s.d."
> 7149 "-2 s.d."
> 17493 "+2 s.d."
> 20078 "+3 s.d."
> 22664 "+4 s.d."
>
> , axis(2) grid)
> xlabel(, axis(1) nogrid)
> xttitle("", axis(2))
> subtitle("S&P 500, January 2001 to December 2001")
> note("Source: Yahoo! Finance and Commodity Systems, Inc.")
(bin=15, start=4103, width=1280.3533)
```



For an explanation of the `xaxis()` option—it created the upper and lower  $x$  axis—see [G-3] [axis\\_choice\\_options](#). For an explanation of the `ylabel()` and `xlabel()` options, see [G-3] [axis\\_label\\_options](#). For an explanation of the `subtitle()` and `note()` options, see [G-3] [title\\_options](#).

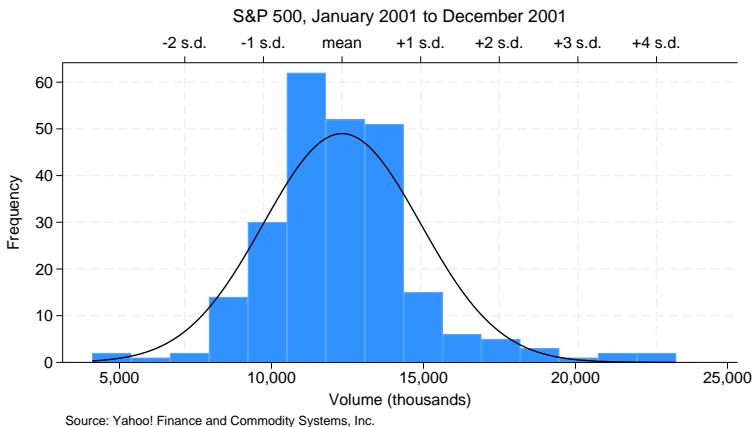
## Overlaying normal and kernel density estimates

Specifying `normal` will overlay a normal density over the histogram. It would be enough to type

```
. histogram volume, normal
```

but we will add the option to our more impressive rendition:

```
. summarize volume
+-----+-----+-----+-----+-----+
| Variable | Obs | Mean | Std. dev. | Min | Max |
+-----+-----+-----+-----+-----+
| volume | 248 | 12320.68 | 2585.929 | 4103 | 23308.3 |
+-----+-----+-----+-----+-----+
. histogram volume, freq normal
> xaxis(1 2)
> ylabel(0(10)60)
> xlabel(12321 "mean"
> 9735 "-1 s.d."
> 14907 "+1 s.d."
> 7149 "-2 s.d."
> 17493 "+2 s.d."
> 20078 "+3 s.d."
> 22664 "+4 s.d."
>
> , axis(2) grid
> xlabel(, axis(1) nogrid)
> xtitle("", axis(2))
> subtitle("S&P 500, January 2001 to December 2001")
> note("Source: Yahoo! Finance and Commodity Systems, Inc.")
(bin=15, start=4103, width=1280.3533)
```



If we instead wanted to overlay a kernel density estimate, we could specify `kdensity` in place of `normal`.

## Histograms of discrete variables

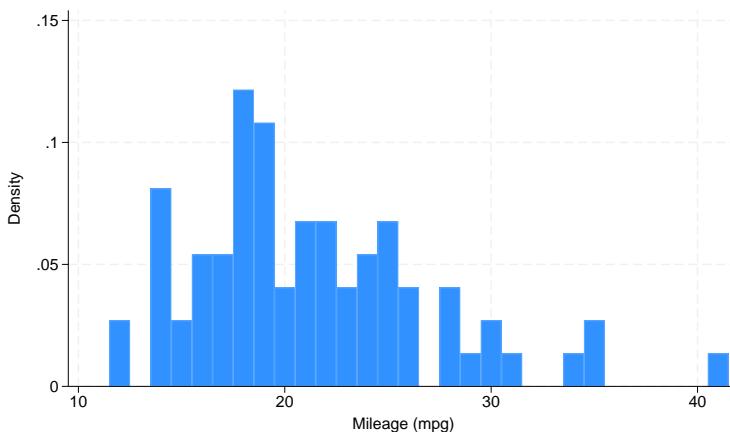
Specify histogram's discrete option when you wish to treat the data as discrete—when you wish each unique value of the variable to be assigned its own bin. For instance, in the automobile data, mpg is a continuous variable, but the mileage ratings have been measured to integer precision. If we were to type

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. histogram mpg
(bin=8, start=12, width=3.625)
```

mpg would be treated as continuous and categorized into eight bins by the default number-of-bins calculation, which is based on the number of observations, 74.

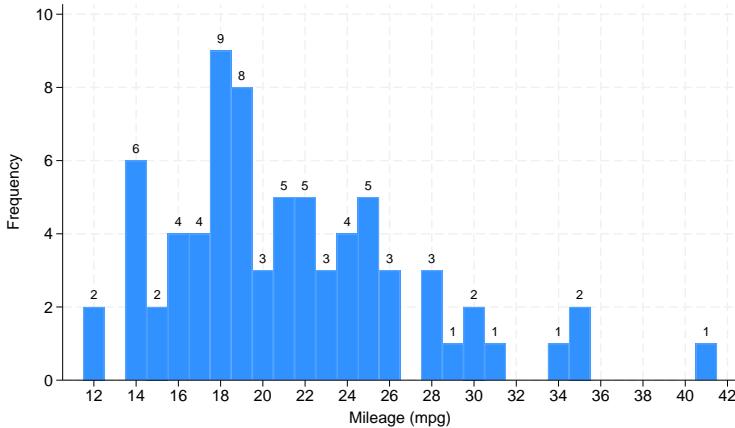
Adding the discrete option makes a histogram with a bin for each of the 21 unique values.

```
. histogram mpg, discrete
(start=12, width=1)
```



Just as in the continuous case, the  $y$  axis was reported in density, and we could specify the fraction or frequency options if we wanted it to be reported differently. Below, we specify frequency, we specify `addlabels` to add a report of frequencies printed above the bars, and we specify `xlabel(12(2)42)` to label the values 12, 14, ..., 42 on the  $x$  axis:

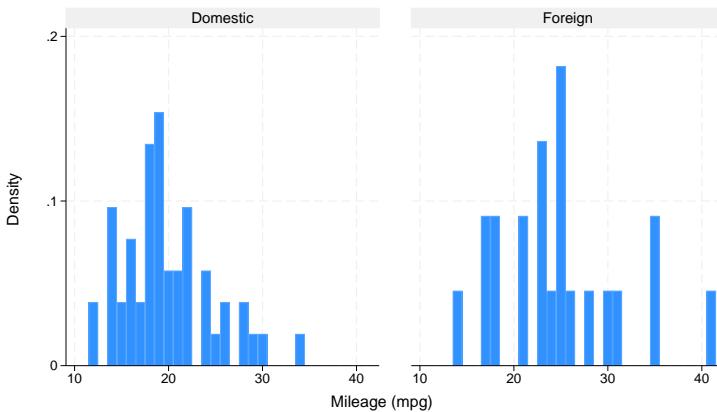
```
. histogram mpg, discrete freq addlabels xlabel(12(2)42)
(start=12, width=1)
```



### Use with by()

`histogram` may be used with `graph twoway's by()`; for example,

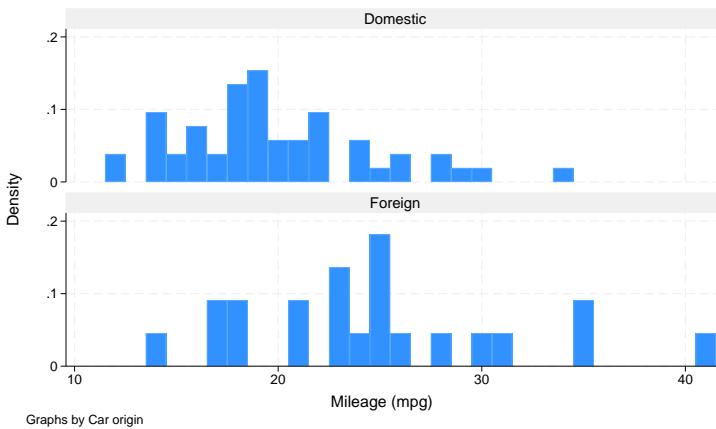
```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. histogram mpg, discrete by(foreign)
```



Graphs by Car origin

Here results would be easier to compare if the graphs were presented in one column:

```
. histogram mpg, discrete by(foreign, col(1))
```



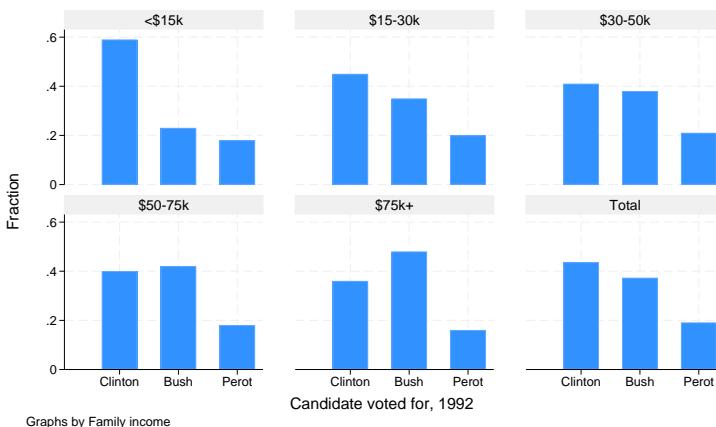
col(1) is a by() suboption—see [G-3] *by\_option*—and there are other useful suboptions, such as total, which will add an overall total histogram. total is a suboption of by(), not an option of histogram, so you would type

```
. histogram mpg, discrete by(foreign, total)
```

and not histogram mpg, discrete by(foreign) total.

As another example, [Lipset \(1993\)](#) reprinted data from the *New York Times* (November 5, 1992) collected by the Voter Research and Surveys based on questionnaires completed by 15,490 US presidential voters from 300 polling places on election day in 1992.

```
. sysuse voter, clear
(1992 US presidential voters)
. histogram candi [fweight=pop], discrete fraction by(inc, total)
> gap(40) xlabel(2 3 4, valuelabel)
```



We specified `gap(40)` to reduce the width of the bars by 40%. We also used `xlabel()`'s `valueLabel` suboption, which caused our bars to be labeled “Clinton”, “Bush”, and “Perot”, rather than 2, 3, and 4; see [G-3] *axis\_label\_options*.

## Video example

Histograms in Stata

## References

- Cox, N. J. 2004. *Speaking Stata: Graphing distributions*. *Stata Journal* 4: 66–88.
- . 2005. *Speaking Stata: Density probability plots*. *Stata Journal* 5: 259–273.
- Harrison, D. A. 2005. *Stata tip 20: Generating histogram bin variables*. *Stata Journal* 5: 280–281.
- Lipset, S. M. 1993. The significance of the 1992 election. *PS: Political Science and Politics* 26: 7–16. <https://doi.org/10.2307/419496>.
- Pinna, M. 2022. *Binned scatterplots with marginal histograms: binscatterhist*. *Stata Journal* 22: 430–445.
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## Also see

- [R] **kdensity** — Univariate kernel density estimation
- [R] **spikeplot** — Spike plots and rootograms
- [G-2] **graph twoway histogram** — Histogram plots

## Description

This entry discusses a statistical issue that arises when using the Bayesian (BIC), consistent Akaike's (CAIC), and corrected Akaike's (AICc) information criteria to compare models.

Stata calculates BIC, CAIC, and AICc using  $N = e(N)$ , unless `e(N_lic)` has been set; in that instance, it uses  $N = e(N\_lic)$ . For example, choice-model `cm` commands set `e(N_lic)` to the number of cases because these commands use a data arrangement in which multiple Stata observations represent a single statistical observation, which is called a case.

Sometimes, it would be better if a different  $N$  than  $e(N)$  were used. Commands that calculate BIC, CAIC, and AICc have an `n()` option, allowing you to specify the  $N$  to be used.

In summary,

1. if you are comparing results estimated by the same estimation command, using the default BIC, CAIC, or AICc calculation is probably fine. There is an issue, but most researchers would ignore it.
2. if you are comparing results estimated by different estimation commands, you need to be on your guard.
  - (a) If the different estimation commands share the same definitions of observations, independence, and the like, you are back to case 1.
  - (b) If they differ in these regards, you need to think about the value of  $N$  that should be used. For example, `logit` and `xtlogit` differ in that the former assumes independent observations and the latter, independent panels.
  - (c) If estimation commands differ in the events being used over which the likelihood function is calculated, the information criteria may not be comparable at all. We say information *criteria* because this would apply equally to the Akaike information criterion (AIC) and its possible extensions AICc and CAIC, as well as to the BIC. For instance, `streg` and `stcox` produce such incomparable results. The events used by `streg` are the actual survival times, whereas the events used by `stcox` are failures within risk pools, conditional on the times at which failures occurred.

## Remarks and examples

Remarks are presented under the following headings:

*Background*

*The problem of determining  $N$*

*The problem of conformable likelihoods*

*The first problem does not arise with AIC; the second problem does*

*Calculating BIC, AICc, and CAIC correctly*

## Background

The AIC and the BIC are two popular measures for comparing maximum likelihood models. AIC and BIC are defined as

$$\text{AIC} = -2 \ln L + 2k$$

$$\text{BIC} = -2 \ln L + k \ln N$$

where

$\ln L$  = maximized log-likelihood

$k$  = number of parameters estimated

$N$  = number of observations

However, when sample size is small, AIC is biased, and [Burnham and Anderson \(2002\)](#) suggest to use AICc,

$$\text{AICc} = \text{AIC} + \frac{2k(k+1)}{N-k-1}$$

CAIC is a consistent version of AIC and was proposed in [Bozdogan \(1987\)](#),

$$\text{CAIC} = -2 \ln L + k(\ln N + 1)$$

All four information criteria—AIC, BIC, CAIC, and AICc—can be viewed as measures that combine fit and complexity. Fit is measured negatively by  $-2 \ln L$ ; the larger the value, the worse the fit. Complexity is measured positively, for example, by  $2k$  (AIC) or  $k \ln N$  (BIC).

Given two models fit on the same data, the model with the smaller value of the information criterion is considered to be better.

There is substantial literature on these measures: see [Akaike \(1974\)](#); [Raftery \(1995\)](#); [Sakamoto, Ishiguro, and Kitagawa \(1986\)](#); [Schwarz \(1978\)](#); [Burnham and Anderson \(2002\)](#); and [Hurvich and Tsai \(1989\)](#).

When Stata calculates the above measures, it uses the rank of  $e(V)$  for  $k$ , and it uses  $e(N)$  for  $N$ .  $e(V)$  and  $e(N)$  are Stata notation for results stored by the estimation command.  $e(V)$  is the variance–covariance matrix of the estimated parameters, and  $e(N)$  is the number of observations in the dataset used in calculating the result.

## The problem of determining $N$

The difference between AIC and the other three information criteria is that AIC uses the constant 2 to weight  $k$ , whereas the complexity term for BIC, CAIC, and AICc depends on  $N$ .

Determining what value of  $N$  should be used is problematic. Despite appearances, the definition “ $N$  is the number of observations” is not easy to make operational.  $N$  does not appear in the likelihood function itself,  $N$  is not the output of a standard statistical formula, and what is an observation is often subjective.

## ▷ Example 1

Often what is meant by  $N$  is obvious. Consider a simple logit model. What is meant by  $N$  is the number of observations that is statistically independent and that corresponds to  $M$ , the number of observations in the dataset used in the calculation. We will write  $N = M$ .

But now assume that the same dataset has a grouping variable and the data are thought to be clustered within group. To keep the problem simple, let's pretend that there are  $G$  groups and  $m$  observations within group, so that  $M = G \times m$ . Because you are worried about intragroup correlation, you fit your model with `xtlogit`, grouping on the grouping variable. Now, you wish to calculate BIC. What is the  $N$  that should be used?  $N = M$  or  $N = G$ ?

That is a deep question. If the observations really are independent, then you should use  $N = M$ . If the observations within group are not just correlated but are duplicates of one another, and they had to be so, then you should use  $N = G$  (Kass and Raftery 1995). Between those two extremes, you should probably use a number between  $N$  and  $G$ , but determining what that number should be from measured correlations is difficult. Using  $N = M$  is conservative in that, if anything, it overweights complexity. Conservativeness, however, is subjective, too: using  $N = G$  could be considered more conservative in that fewer constraints are being placed on the data.

When the estimated correlation is high, our reaction would be that using  $N = G$  is probably more reasonable. Our first reaction, however, would be that using BIC to compare models is probably a misuse of the measure.

Stata uses  $N = M$ . An informal survey of web-based literature suggests that  $N = M$  is the popular choice.

There is another reason, not so good, to choose  $N = M$ . It makes across-model comparisons more likely to be valid when performed without thinking about the issue. Say that you wish to compare the `logit` and `xtlogit` results. Thus, you need to calculate

$$\text{BIC}_p = -2 \ln L_p + k \ln N_p$$

$$\text{BIC}_x = -2 \ln L_x + k \ln N_x$$

Whatever  $N$  you use, you must use the same  $N$  in both formulas. Stata's choice of  $N = M$  at least meets that test.

◀

## ▷ Example 2

In the above example, using  $N = M$  is reasonable. Now, let's look at when using  $N = M$  is wrong, even if popular.

Consider a model fit by `stcox`. Using  $N = M$  is certainly wrong if for no other reason than  $M$  is not even a well-defined number. The same data can be represented by different datasets with different numbers of observations. For example, in one dataset, there might be one observation per subject. In another, the same subjects could have two records each, the first recording the first half of the time at risk and the second recording the remaining part. All statistics calculated by Stata on either dataset would be the same, but  $M$  would be different.

Deciding on the right definition, however, is difficult. Viewed one way,  $N$  in the Cox regression case should be the number of risk pools,  $R$ , because the Cox regression calculation is made on the basis of the independent risk pools. Viewed another way,  $N$  should be the number of subjects,  $N_{\text{subj}}$ , because, even though the likelihood function is based on risk pools, the parameters estimated are at the subject level.

You can decide which argument you prefer.

For parametric survival models, in single-record data,  $N = M$  is unambiguously correct. For multi-record data, there is an argument for  $N = M$  and for  $N = N_{\text{subj}}$ .



## The problem of conformable likelihoods

The problem of conformable likelihoods does not concern  $N$ . Researchers sometimes use information criteria such as BIC and AIC to make comparisons across models. For that to be valid, the likelihoods must be conformable; that is, the likelihoods must all measure the same thing.

It is common to think of the likelihood function as the  $\Pr(\text{data} \mid \text{parameters})$ , but in fact, the likelihood is

$$\Pr(\text{particular events in the data} \mid \text{parameters})$$

You must ensure that the events are the same.

For instance, they are not the same in the semiparametric Cox regression and the various parametric survival models. In Cox regression, the events are, at each failure time, that the subjects observed to fail in fact failed, given that failures occurred at those times. In the parametric models, the events are that each subject failed exactly when the subject was observed to fail.

The formula for AIC, AICc, CAIC, and BIC can be written as

$$\text{measure} = -2 \ln L + \text{complexity}$$

When you are comparing models, if the likelihoods are measuring different events, even if the models obtain estimates of the same parameters, differences in the information measures are irrelevant.

## The first problem does not arise with AIC; the second problem does

Regardless of model, the problem of defining  $N$  never arises with AIC because  $N$  is not used in the AIC calculation. AIC uses a constant 2 to weight complexity as measured by  $k$ , rather than  $\ln N$ .

However, for all four information criteria—AIC, AICc, CAIC, and BIC—the likelihood functions must be conformable; that is, they must be measuring the same event.

## Calculating BIC, AICc, and CAIC correctly

When using BIC, AICc, or CAIC to compare results, and especially when using them to compare results from different models, you should think carefully about how  $N$  should be defined. Then, specify that number by using the `n()` option:

```
. estimates stats full sub, all n(74)
```

```
Information criteria
```

| Model | N  | ll(null)  | ll(model) | df |
|-------|----|-----------|-----------|----|
| full  | 74 | -45.03321 | -20.59083 | 4  |
| sub   | 74 | -45.03321 | -27.17516 | 3  |

| Model | AIC      | BIC      | AICc     | CAIC     |
|-------|----------|----------|----------|----------|
| full  | 49.18167 | 58.39793 | 49.76138 | 62.39793 |
| sub   | 60.35031 | 67.26251 | 60.69317 | 70.26251 |

Legend: AIC is Akaike's information criterion.

BIC is Bayesian information criterion.

AICc is corrected Akaike's information criterion.

CAIC is consistent Akaike's information criterion.

Both `estimates stats` and `estat ic` allow the `n()` option; see [\[R\] estimates stats](#) and [\[R\] estat ic](#).

## Methods and formulas

AIC, BIC, CAIC, and AICc are defined as

$$\text{AIC} = -2 \ln L + 2k$$

$$\text{BIC} = -2 \ln L + k \ln N$$

$$\text{CAIC} = -2 \ln L + k(\ln N + 1)$$

$$\text{AICc} = \text{AIC} + \frac{2k(k+1)}{N-k-1}$$

where  $\ln L$  is the maximized log-likelihood of the model;  $k$  is the model degrees of freedom calculated as the rank of variance-covariance matrix of the parameters  $e(V)$ , unless the `df()` option is specified; and  $N$  is the number of observations used in estimation or, more precisely, the number of independent terms in the likelihood. Operationally,  $N$  is defined as `e(N)`, unless the estimation command returns `e(N_ic)` or the `n()` option is specified with `estimates stats` or `estat ic`.

## References

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## Also see

[R] **estat ic** — Display information criteria

[R] **estimates stats** — Model-selection statistics

|                                      |                                              |                                                      |
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| <a href="#">Description</a>          | <a href="#">Quick start</a>                  | <a href="#">Menu</a>                                 |
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## Description

`icc` estimates intraclass correlations for one-way random-effects models, two-way random-effects models, or two-way mixed-effects models for both individual and average measurements. Intraclass correlations measuring consistency of agreement or absolute agreement of the measurements may be estimated.

## Quick start

Individual and average absolute-agreement intraclass correlation coefficients (ICCs) for ratings `y` of targets identified by `tid` in a one-way random-effects model

```
icc y tid
```

Same as above, but test that the individual and average ICCs are equal to 0.5

```
icc y tid, testvalue(.5)
```

Absolute-agreement ICCs for targets identified by `tid` and raters identified by `rid` in a two-way random-effects model

```
icc y tid rid
```

Same as above, but estimate consistency-of-agreement ICCs

```
icc y tid rid, consistency
```

Consistency-of-agreement ICCs when estimating random effects for targets and fixed effects for raters in a mixed-effects model

```
icc y tid rid, mixed
```

Same as above, but estimate absolute-agreement ICCs

```
icc y tid rid, mixed absolute
```

Same as above, but report 90% confidence intervals and test that ICCs are equal to 0.3

```
icc y tid rid, mixed absolute level(90) testvalue(.3)
```

## Menu

Statistics > Summaries, tables, and tests > Summary and descriptive statistics > Intraclass correlations

## Syntax

Calculate intraclass correlations for one-way random-effects model

```
icc depvar target [if] [in] [, oneway_options]
```

Calculate intraclass correlations for two-way random-effects model

```
icc depvar target rater [if] [in] [, twoway_re_options]
```

Calculate intraclass correlations for two-way mixed-effects model

```
icc depvar target rater [if] [in], mixed [twoway_me_options]
```

| <i>oneway_options</i>         | Description                                                                         |
|-------------------------------|-------------------------------------------------------------------------------------|
| Main                          |                                                                                     |
| <u>a</u> bsolute              | estimate absolute agreement; the default                                            |
| <u>t</u> estvalue(#)          | test whether intraclass correlations equal #;<br>default is testvalue(0)            |
| Reporting                     |                                                                                     |
| <u>l</u> evel(#)              | set confidence level; default is level(95)                                          |
| <u>f</u> ormat(% <i>fmt</i> ) | display format for statistics and confidence intervals;<br>default is format(%9.0g) |

| <i>twoway_re_options</i>      | Description                                                                         |
|-------------------------------|-------------------------------------------------------------------------------------|
| Main                          |                                                                                     |
| <u>a</u> bsolute              | estimate absolute agreement; the default                                            |
| <u>c</u> onsistency           | estimate consistency of agreement                                                   |
| <u>t</u> estvalue(#)          | test whether intraclass correlations equal #;<br>default is testvalue(0)            |
| Reporting                     |                                                                                     |
| <u>l</u> evel(#)              | set confidence level; default is level(95)                                          |
| <u>f</u> ormat(% <i>fmt</i> ) | display format for statistics and confidence intervals;<br>default is format(%9.0g) |

| <i>twoway_me_options</i> | Description                                                                                      |
|--------------------------|--------------------------------------------------------------------------------------------------|
| Main                     |                                                                                                  |
| * <b>mixed</b>           | estimate intraclass correlations for a mixed-effects model                                       |
| <b>consistency</b>       | estimate consistency of agreement; the default                                                   |
| <b>absolute</b>          | estimate absolute agreement                                                                      |
| <b>testvalue(#)</b>      | test whether intraclass correlations equal #;<br>default is <code>testvalue(0)</code>            |
| Reporting                |                                                                                                  |
| <b>level(#)</b>          | set confidence level; default is <code>level(95)</code>                                          |
| <b>format(%fmt)</b>      | display format for statistics and confidence intervals;<br>default is <code>format(%9.0g)</code> |

\* `mixed` is required.

`bootstrap`, `by`, `collect`, `jackknife`, and `statsby` are allowed; see [\[U\] 11.1.10 Prefix commands](#).

## Options for one-way RE model

### Main

`absolute` specifies that intraclass correlations measuring absolute agreement of the measurements be estimated. This is the default for random-effects models.

`testvalue(#)` tests whether intraclass correlations equal #. The default is `testvalue(0)`.

### Reporting

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [\[R\] level](#).

`format(%fmt)` specifies how the intraclass correlation estimates and confidence intervals are to be formatted. The default is `format(%9.0g)`.

## Options for two-way RE and ME models

### Main

`mixed` is required to calculate two-way mixed-effects models. `mixed` specifies that intraclass correlations for a mixed-effects model be estimated.

`absolute` specifies that intraclass correlations measuring absolute agreement of the measurements be estimated. This is the default for random-effects models. Only one of `absolute` or `consistency` may be specified.

`consistency` specifies that intraclass correlations measuring consistency of agreement of the measurements be estimated. This is the default for mixed-effects models. Only one of `absolute` or `consistency` may be specified.

`testvalue(#)` tests whether intraclass correlations equal #. The default is `testvalue(0)`.

## Reporting

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [\[R\] level](#).

`format(%fmt)` specifies how the intraclass correlation estimates and confidence intervals are to be formatted. The default is `format(%9.0g)`.

## Remarks and examples

Remarks are presented under the following headings:

*Introduction*  
*One-way random effects*  
*Two-way random effects*  
*Two-way mixed effects*  
*Adoption study*  
*Relationship between ICCs*  
*Tests against nonzero values*

## Introduction

In some disciplines, such as psychology and sociology, data are often measured with error that can seriously affect statistical interpretation of the results. Thus, it is important to assess the amount of measurement error by evaluating the consistency or reliability of measurements. The intraclass correlation coefficient (ICC) is often used to measure the consistency or homogeneity of measurements.

Several versions of ICCs are introduced in the literature depending on the experimental design and goals of the study (see, for example, [Shrout and Fleiss \[1979\]](#) and [McGraw and Wong \[1996a\]](#)). Following [Shrout and Fleiss \(1979\)](#), we describe various forms of ICCs in the context of a reliability study of ratings of different targets (or objects of measurements) by several raters.

Consider  $n$  targets (for example, students, patients, athletes) that are randomly sampled from a population of interest. Each target is rated independently by a set of  $k$  raters (for example, teachers, doctors, judges). One rating per target and rater is obtained. It is of interest to determine the extent of the agreement of the ratings.

As noted by [Shrout and Fleiss \(1979\)](#) and [McGraw and Wong \(1996a\)](#), you need to answer several questions to decide what version of ICC is appropriate to measure the agreement in your study:

1. Is a one-way or two-way analysis-of-variance model appropriate for your study?
2. Are differences between raters' mean ratings relevant to the reliability of interest?
3. Is the unit of analysis an individual rating or the mean rating over several raters?
4. Is the consistency of agreement or the absolute agreement of ratings of interest?

Three types of analysis-of-variance models are considered for the reliability study: one-way random effects, two-way random effects, and two-way mixed effects. Mixed models contain both fixed effects and random effects. In the one-way random-effects model, each target is rated by a different set of  $k$  independent raters, who are randomly drawn from the population of raters. The target is the only random effect in this model; the effects due to raters and possibly due to rater-and-target interaction cannot be separated from random error. In the two-way random-effects model, each target is rated by the same set of  $k$  independent raters, who are randomly drawn from the population of raters. The random effects in this model are target and rater and possibly their interaction, although in the absence of repeated measurements for each rater on each target, the effect of an interaction cannot be separated from random error. In the two-way mixed-effects model, each target is rated by the same set of  $k$  independent raters. Because they are the only raters of interest, rater is a fixed effect. The random effects are target and possibly target-and-rater interaction, but again the interaction effect cannot be separated from random error without repeated measurements for each rater and target. The definition of ICC depends on the chosen random-effects model; see *Methods and formulas* for details.

In summary, use a one-way model if there are no systematic differences in measurements due to raters and use a two-way model otherwise. If you want to generalize your results to a population of raters from which the observed raters are sampled, use a two-way random-effects model, treating raters as random. If you are interested only in the effects of the observed  $k$  raters, use a two-way mixed-effects model, treating raters as fixed. For example, suppose you compare judges' ratings of targets from different groups. If you use the combined data from  $k$  judges to compare the groups, the random-effects model is appropriate. If you compare groups separately for each judge and then pool the differences, the mixed-effects model is appropriate.

The definition of ICC also depends on the unit of analysis in a study—whether the agreement is measured between individual ratings (individual ICC) or between the averages of ratings over several raters (average ICC). The data on individual ratings are more common. The data on average ratings are typically used when individual ratings are deemed unreliable. The average ICC can also be used when teams of raters are used to rate a target. For example, the ratings of teams of physicians may be evaluated in this manner. When the unit of analysis is an average rating, you should remember that the interpretation of ICC pertains to average ratings and not individual ratings.

Finally, depending on whether consistency of agreement or absolute agreement is of interest, two types of ICC are used: consistency-of-agreement ICC (CA-ICC) and absolute-agreement ICC (AA-ICC). Under consistency of agreement, the scores are considered consistent if the scores from any two raters differ by the same constant value for all targets. This implies that raters give the same ranking to all targets. Under absolute agreement, the scores are considered in absolute agreement if the scores from all raters match exactly.

For example, suppose we observe three targets and two raters. The ratings are (2,4), (4,6), and (6,8), with rater 1 giving the scores (2,4,6) and rater 2 giving the scores (4,6,8), two points higher than rater 1. The CA-ICC between individual ratings is 1 because the scores from rater 1 and rater 2 differ by a constant value (two points) for all targets. That rater 1 gives lower scores than rater 2 is deemed irrelevant under the consistency measure of agreement. The raters have the same difference of opinion on every target, and the variation between raters that is caused by this difference is not relevant. On the other hand, the AA-ICC between individual ratings is  $8/12 = 0.67$ , where 8 is the estimated between-target variance and 12 is the estimated total variance of ratings.

Either CA-ICC or AA-ICC can serve as a useful measure of agreement depending on whether rater variability is relevant for determining the degree of agreement. As [McGraw and Wong \(1996a\)](#) point out, CA-ICC is useful when comparative judgments are made about objects of measurement. The CA-ICC represents correlation when the rater is fixed; the AA-ICC represents correlation when the rater is random.

See [Shrout and Fleiss \(1979\)](#) and [McGraw and Wong \(1996a\)](#) for more detailed guidelines about the choice of appropriate ICC.

[Shrout and Fleiss \(1979\)](#) and [McGraw and Wong \(1996a\)](#) describe 10 versions of ICCs based on the concepts above: individual and average AA-ICCs for a one-way model (consistency of agreement is not defined for this model); individual and average AA-ICCs and CA-ICCs for a two-way random-effects model; and individual and average AA-ICCs and CA-ICCs for a two-way mixed-effects model. Although each of these ICCs has its own definition and interpretation, the estimators for some are identical, leading to the same estimates of those ICCs; see [Relationship between ICCs](#) and [Methods and formulas](#) for details.

The `icc` command calculates ICCs for each of the three analysis-of-variance models. You can use option `absolute` to compute AA-ICCs or option `consistency` to compute CA-ICCs. By default, `icc` computes ICCs corresponding to the correlation between ratings and between average ratings made on the same target: AA-ICC for a random-effects model and CA-ICC for a mixed-effects model. As pointed out by [Shrout and Fleiss \(1979\)](#), although the data on average ratings might be needed for reliability, the generalization of interest might be individuals. For this reason, `icc` reports ICCs for both units, individual and average, for each model.

In addition to estimates of ICCs, `icc` provides confidence intervals and one-sided  $F$  tests. The  $F$  test of  $H_0: \rho = 0$  versus  $H_a: \rho > 0$  is the same for the individual and average ICCs, so `icc` reports one test. This is not true, however, for nonzero null hypotheses (see [Tests against nonzero values](#) for details), so `icc` reports a separate test in this case.

The `icc` command requires data in long form; see [\[D\] reshape](#) for how to convert data in wide form to long form. The data must also be balanced and contain one observation per target and rater. For unbalanced data, `icc` omits all targets with fewer than  $k$  ratings from computation. Under one-way models,  $k$  is determined as the largest number of observed ratings for a target. Under two-way models,  $k$  is the number of unique raters. If multiple observations per target and rater are detected, `icc` issues an error.

We demonstrate the use of `icc` using datasets from [Shrout and Fleiss \(1979\)](#) and [McGraw and Wong \(1996a\)](#). In the next three sections, we use an example from table 2 of [Shrout and Fleiss \(1979\)](#) with six targets and four judges. For instructional purposes, we analyze these data under each of the three different models: one-way random effects, two-way random effects, and two-way mixed effects.

## One-way random effects

In the one-way random-effects model, we assume that the  $n$  targets being rated are randomly selected from the population of potential targets. Each is rated by a different set of  $k$  raters randomly drawn from the population of potential raters. McGraw and Wong (1996a) describe an example of this setting, where behavioral genetics data are used to assess familial resemblance. Family units can be viewed as “targets”, and children can be viewed as “raters”. By taking a measurement on a child of the family unit, we obtain the “rating” of the family unit by the “child-rater”. In this case, we can use ICC to assess similarity between children within a family or, in other words, assess if there is a family effect in these data.

As we mentioned in the introduction, only AA-ICC is defined for a one-way model. The consistency of agreement is not defined in this case, as each target is evaluated by a different set of raters. Thus, there is no between-rater variability in this model.

In a one-way model, the AA-ICC corresponds to the correlation coefficient between ratings within a target. It is also a ratio of the between-target variance of ratings to the total variance of ratings, the sum of the between-target and error variances.

### ▷ Example 1: One-way random-effects ICCs

Consider data from table 2 of Shrout and Fleiss (1979) stored in `judges.dta`. The data contain 24 ratings of  $n = 6$  targets by  $k = 4$  judges. We list the first eight observations:

```
. use https://www.stata-press.com/data/r19/judges
(Ratings of targets by judges)
. list in 1/8, sepby(target)
```

|    | rating | target | judge |
|----|--------|--------|-------|
| 1. | 9      | 1      | 1     |
| 2. | 2      | 1      | 2     |
| 3. | 5      | 1      | 3     |
| 4. | 8      | 1      | 4     |
| 5. | 6      | 2      | 1     |
| 6. | 1      | 2      | 2     |
| 7. | 3      | 2      | 3     |
| 8. | 2      | 2      | 4     |

For a moment, let’s ignore that targets are rated by the same set of judges. Instead, we assume that a different set of four judges is used to rate each target. In this case, the only systematic variation in the data is due to targets, so the one-way random-effects model is appropriate.

We use `icc` to estimate the intraclass correlations for these data. To compute ICCs for a one-way model, we specify the dependent variable `rating` followed by the target variable `target`:

```
. icc rating target
Intraclass correlations
One-way random-effects model
Absolute agreement
Random effects: target Number of targets = 6
 Number of raters = 4
```

| rating     | ICC      | [95% conf. interval] |          |
|------------|----------|----------------------|----------|
| Individual | .1657418 | -.1329323            | .7225601 |
| Average    | .4427971 | -.8844422            | .9124154 |

```
F test that
ICC=0.00: F(5.0, 18.0) = 1.79 Prob > F = 0.165
```

Note: ICCs estimate correlations between individual measurements and between average measurements made on the same target.

`icc` reports the AA-ICCs for both individual and average ratings. The individual AA-ICC corresponds to  $ICC(1)$  in McGraw and Wong (1996a) or  $ICC(1,1)$  in Shrout and Fleiss (1979). The average AA-ICC corresponds to  $ICC(k)$  in McGraw and Wong (1996a) or  $ICC(1,k)$  in Shrout and Fleiss (1979).

The estimated correlation between individual ratings is 0.17, indicating little similarity between ratings within a target, low reliability of individual target ratings, or no target effect. The estimated intraclass correlation between ratings averaged over  $k = 4$  judges is higher, 0.44. (The average ICC will typically be higher than the individual ICC.) The estimated intraclass correlation measures the similarity or reliability of mean ratings from groups of four judges. We do not have statistical evidence that either ICC is different from zero based on reported confidence intervals and the one-sided  $F$  test.

Note that although the estimates of ICCs cannot be negative in this setting, the lower bound of the computed confidence interval may be negative. A common ad-hoc way of handling this is to truncate the lower bound at zero.

The estimates of both the individual and the average AA-ICC are also computed by the `lone` command (see [R] [lone](#)), which performs a one-way analysis of variance.

◀

## □ Technical note

Mean rating is commonly used when individual rating is unreliable because the reliability of a mean rating is always higher than the reliability of the individual rating when the individual reliability is positive.

In the [previous example](#), we estimated low reliability of the individual ratings of a target, 0.17. The reliability increased to 0.44 for the ratings averaged over four judges. What if we had more judges?

We can use the Spearman–Brown formula (Spearman 1910; Brown 1910) to compute the  $m$ -average ICC based on the individual ICC:

$$\text{ICC}(m) = \frac{m\text{ICC}(1)}{1 + (m - 1)\text{ICC}(1)}$$

Using this formula for the previous example, we find that the mean reliability over, say, 10 judges is  $10 \times 0.17 / (1 + 9 \times 0.17) = 0.67$ .

Alternatively, we can invert the Spearman–Brown formula to determine the number of judges (or the number of ratings of a target) we need to achieve the desired reliability. Suppose we would like an average reliability of 0.9, then

$$m = \frac{\text{ICC}(m)\{1 - \text{ICC}(1)\}}{\text{ICC}(1)\{1 - \text{ICC}(m)\}} = \frac{0.9(1 - 0.17)}{0.17(1 - 0.9)} = 44$$

See, for example, Bliese (2000) for other examples. □

## Two-way random effects

As before, we assume that the targets being rated are randomly selected from the population of potential targets. We now also assume that each target is evaluated by the same set of  $k$  raters, who have been randomly sampled from the population of raters. In this scenario, we want to generalize our findings to the population of raters from which the observed  $k$  raters were sampled. For example, suppose we want to estimate the reliability of doctors' evaluations of patients with a certain condition. Unless the reliability at a specific hospital is of interest, the doctors may be interchanged with others in the relevant population of doctors.

As for a one-way model, the AA-ICC corresponds to the correlation between measurements on the same target and is also a ratio of the between-target variance to the total variance of measurements in a two-way random-effects model. The total variance is now the sum of the between-target, between-rater, and error variances. Unlike a one-way model, the CA-ICC can be computed for a two-way random-effects model when the consistency of agreement is of interest rather than the absolute agreement. The CA-ICC is also the ratio of the between-target variance to the total variance, but the total variance does not include the between-rater variance because the between-rater variability is irrelevant for the consistency of agreement.

Again, the two versions, individual and average, are available for each ICC.

### ▷ Example 2: Two-way random-effects ICCs

Continuing with [example 1](#), recall that we previously ignored that each target is rated by the same set of four judges and instead assumed different sets of judges. We return to the original data setting. We want to evaluate the agreement between judges' ratings of targets in a population represented by the observed set of four judges.

In a two-way model, we must specify both the target and the rater variables. In `icc`, we now additionally specify the rater variable `judge` following the target variable `target`; the random-effects model is assumed by default.

```
. icc rating target judge
Intraclass correlations
Two-way random-effects model
Absolute agreement
Random effects: target Number of targets = 6
Random effects: judge Number of raters = 4
```

| rating     | ICC      | [95% conf. interval] |          |
|------------|----------|----------------------|----------|
| Individual | .2897638 | .0187865             | .7610844 |
| Average    | .6200505 | .0711368             | .927232  |

```
F test that
ICC=0.00: F(5.0, 15.0) = 11.03 Prob > F = 0.000
```

Note: ICCs estimate correlations between individual measurements and between average measurements made on the same target.

As for a one-way random-effects model, `icc` by default reports AA-ICCs that correspond to the correlation between ratings on a target. Notice that both individual and average ICCs are larger in the two-way random-effects model than in the previous one-way model—0.29 versus 0.17 and 0.62 versus 0.44, respectively. We also have statistical evidence to reject the null hypothesis that neither ICC is zero based on confidence intervals and the  $F$  test. If a one-way model is used when a two-way model is appropriate, the true ICC will generally be underestimated.

The individual AA-ICC corresponds to  $ICC(A,1)$  in [McGraw and Wong \(1996a\)](#) or  $ICC(2,1)$  in [Shrout and Fleiss \(1979\)](#). The average AA-ICC corresponds to  $ICC(A,k)$  in [McGraw and Wong \(1996a\)](#) or  $ICC(2,k)$  in [Shrout and Fleiss \(1979\)](#).

Instead of the absolute agreement, we can also assess the consistency of agreement. The individual and average CA-ICCs are considered in [McGraw and Wong \(1996a\)](#) and denoted as  $ICC(C,1)$  and  $ICC(C,k)$ , respectively. These ICCs are not considered in [Shrout and Fleiss \(1979\)](#) because they are not correlations in the strict sense. Although CA-ICCs do not estimate correlation, they can provide useful information about the reliability of the raters. [McGraw and Wong \(1996a\)](#) note that the practical value of the individual and average CA-ICCs in the two-way random-effects model setting is well documented in measurement theory, citing [Hartmann \(1982\)](#) and [Suen \(1988\)](#).

To estimate the individual and average CA-ICCs, we specify the consistency option:

```
. icc rating target judge, consistency
Intraclass correlations
Two-way random-effects model
Consistency of agreement
Random effects: target Number of targets = 6
Random effects: judge Number of raters = 4
```

| rating     | ICC      | [95% conf. interval] |          |
|------------|----------|----------------------|----------|
| Individual | .7148407 | .3424648             | .9458583 |
| Average    | .9093155 | .6756747             | .9858917 |

```
F test that
ICC=0.00: F(5.0, 15.0) = 11.03 Prob > F = 0.000
```

We estimate that the consistency of agreement of ratings in the considered population of raters is high, 0.71, based on the individual CA-ICC. On the other hand, the absolute agreement of ratings is low, 0.29, based on the individual AA-ICC from the previous output.

◀

The measure of consistency of agreement among means, the average CA-ICC, is equivalent to Cronbach's alpha (Cronbach 1951); see [MV] **alpha**. The individual CA-ICC can also be equivalent to the Pearson's correlation coefficient between raters when  $k = 2$ ; see McGraw and Wong (1996a) for details.

In the next example, we will see that the actual estimates of the individual and average AA-ICCs and CA-ICCs are the same whether we examine a random-effects model or a mixed-effects model. The differences between these ICCs are in their definitions and interpretations.

## Two-way mixed effects

As in a two-way random-effects model, we assume that the targets are randomly selected from the population of potential targets and that each is evaluated by the same set of  $k$  raters. In a mixed-effects model, however, we assume that these raters are the only raters of interest. So as before, the targets are random, but now the raters are fixed.

In the two-way mixed-effects model, the fixed effect of the rater does not contribute to the between-rater random variance component to the total variance. As such, the definitions and interpretations of ICCs are different in a mixed-effects model than in a random-effects model. However, the estimates of ICCs as well as test statistics and confidence intervals are the same. The only exceptions are average AA-ICCs and CA-ICCs. These are not estimable in a two-way mixed-effects model including an interaction term between target and rater; see *Relationship between ICCs* and *Methods and formulas* for details.

In a two-way mixed-effects model, the CA-ICC corresponds to the correlation between measurements on the same target. As pointed out by Shrout and Fleiss (1979), when the rater variance is ignored, the correlation coefficient is interpreted in terms of rater consistency rather than rater absolute agreement. Formally, the CA-ICC is the ratio of the covariance between measurements on the target to the total variance of the measurements. The AA-ICC corresponds to the same ratio, but includes a variance of the fixed factor, rater, in its denominator.

### ▶ Example 3: Two-way mixed-effects ICCs

Continuing with [example 2](#), suppose that we are now interested in assessing the agreement of ratings from only the observed four judges. The judges are now fixed effects, and the appropriate model is a two-way mixed-effects model.

To estimate ICCs for a two-way mixed-effects model, we specify the mixed option with `icc`:

```
. icc rating target judge, mixed
Intraclass correlations
Two-way mixed-effects model
Consistency of agreement
Random effects: target Number of targets = 6
Fixed effects: judge Number of raters = 4
```

| rating     | ICC      | [95% conf. interval] |          |
|------------|----------|----------------------|----------|
| Individual | .7148407 | .3424648             | .9458583 |
| Average    | .9093155 | .6756747             | .9858917 |

```
F test that
ICC=0.00: F(5.0, 15.0) = 11.03 Prob > F = 0.000
```

Note: ICCs estimate correlations between individual measurements and between average measurements made on the same target.

As we described in the introduction, `icc` by default reports ICCs corresponding to the correlations. So, for a mixed-effects model, `icc` reports CA-ICCs by default. The individual and average CA-ICCs are denoted as  $ICC(3,1)$  and  $ICC(3,k)$  in [Shrout and Fleiss \(1979\)](#) and  $ICC(C,1)$  and  $ICC(C,k)$  in [McGraw and Wong \(1996a\)](#).

Our estimates of the individual and average CA-ICCs are identical to the CA-ICC estimates obtained under the two-way random-effects model in [example 2](#), but our interpretation of the results is different. Under a mixed-effects model, 0.71 and 0.91 are the estimates, respectively, of the correlation between individual measurements and the correlation between average measurements made on the same target.

We can also estimate the AA-ICCs in this setting by specifying the `absolute` option:

```
. icc rating target judge, mixed absolute
Intraclass correlations
Two-way mixed-effects model
Absolute agreement
Random effects: target Number of targets = 6
Fixed effects: judge Number of raters = 4
```

| rating     | ICC      | [95% conf. interval] |          |
|------------|----------|----------------------|----------|
| Individual | .2897638 | .0187865             | .7610844 |
| Average    | .6200505 | .0711368             | .927232  |

```
F test that
ICC=0.00: F(5.0, 15.0) = 11.03 Prob > F = 0.000
```

The intraclass correlation estimates match the individual and average AA-ICCs obtained under the two-way random-effects model in [example 2](#); but in a mixed-effects model, they do not represent correlations. We demonstrate the use of an individual AA-ICC in a mixed-effects setting in the next example.

The AA-ICCs under a mixed-effects model are not considered by [Shrout and Fleiss \(1979\)](#). They are denoted as  $ICC(A,1)$  and  $ICC(A,k)$  in [McGraw and Wong \(1996a\)](#).

## Adoption study

In this section, we consider the adoption study described in [McGraw and Wong \(1996a\)](#). Adoption studies commonly include two effects of interest. One is the mean difference between the adopted child and its biological parents. It is used to determine if characteristics of adopted children differ on average from those of their biological parents. Another effect of interest is the correlation between genetically paired individuals and genetically unrelated individuals who live together. This effect is used to evaluate the impact of genetic differences on individual differences.

As discussed in [McGraw and Wong \(1996a\)](#), a consistent finding from adoption research using IQ as a trait characteristic is that while adopted children typically have higher IQs than their biological parents, their IQs correlate better with those of their biological parents than with those of their adoptive parents. Both effects are important, and there is additional need to reconcile the two findings. [McGraw and Wong \(1996a\)](#) propose to use the individual AA-ICC for this purpose.

### ► Example 4: Absolute-agreement ICC in a mixed-effects model

The `adoption.dta` dataset contains the data from table 6 of [McGraw and Wong \(1996a\)](#) on IQ scores:

```
. use https://www.stata-press.com/data/r19/adoption
(Biological mother and adopted child IQ scores)

. describe
Contains data from https://www.stata-press.com/data/r19/adoption.dta
Observations: 20 Biological mother and adopted
 child IQ scores
Variables: 5 15 May 2024 13:50
 (_dta has notes)
```

| Variable name | Storage type | Display format | Value label | Variable label                                  |
|---------------|--------------|----------------|-------------|-------------------------------------------------|
| family        | byte         | %9.0g          |             | Adoptive family ID                              |
| mc            | byte         | %9.0g          | mcvalues    | Whether mother or child                         |
| iq3           | int          | %9.0g          |             | IQ scores, mother-child<br>difference of 3 pts  |
| iq9           | int          | %9.0g          |             | IQ scores, mother-child<br>difference of 9 pts  |
| iq15          | int          | %9.0g          |             | IQ scores, mother-child<br>difference of 15 pts |

Sorted by:

The family variable contains adoptive family identifiers, the mc variable records a mother or a child, and the iq3, iq9, and iq15 variables record IQ scores with differences between mother and child mean IQ scores of 3, 9, and 15 points, respectively.

```
. by mc, sort: summarize iq*
```

```
-> mc = Mother
```

| Variable | Obs | Mean | Std. dev. | Min | Max |
|----------|-----|------|-----------|-----|-----|
| iq3      | 10  | 97   | 15.0037   | 62  | 116 |
| iq9      | 10  | 91   | 15.0037   | 56  | 110 |
| iq15     | 10  | 85   | 15.0037   | 50  | 104 |

```
-> mc = Child
```

| Variable | Obs | Mean | Std. dev. | Min | Max |
|----------|-----|------|-----------|-----|-----|
| iq3      | 10  | 100  | 15.0037   | 65  | 119 |
| iq9      | 10  | 100  | 15.0037   | 65  | 119 |
| iq15     | 10  | 100  | 15.0037   | 65  | 119 |

The variances of the mother and child IQ scores are the same.

Children are fixed effects, so the mixed-effects model is appropriate for these data. We want to compare individual CA-ICC with individual AA-ICC for each of the three IQ variables. We could issue a separate `icc` command for each of the three IQ variables to obtain the intraclass correlations. Instead, we use `reshape` to convert our data to long form with one `iq` variable and the new `diff` variable recording mean differences:

```
. reshape long iq, i(family mc) j(diff)
(j = 3 9 15)
```

| Data                   | Wide         | -> | Long |
|------------------------|--------------|----|------|
| Number of observations | 20           | -> | 60   |
| Number of variables    | 5            | -> | 4    |
| j variable (3 values)  |              | -> | diff |
| xij variables:         | iq3 iq9 iq15 | -> | iq   |

We can now use the `by` prefix with `icc` to estimate intraclass correlations for the three groups of interest:

```
. by diff, sort: icc iq family mc, mixed
```

```
-> diff = 3
```

```
Intraclass correlations
Two-way mixed-effects model
Consistency of agreement
```

```
Random effects: family Number of targets = 10
Fixed effects: mc Number of raters = 2
```

|            | iq | ICC      | [95% conf. interval] |          |
|------------|----|----------|----------------------|----------|
| Individual |    | .7142152 | .1967504             | .920474  |
| Average    |    | .8332853 | .3288078             | .9585904 |

```
F test that
```

```
ICC=0.00: F(9.0, 9.0) = 6.00 Prob > F = 0.007
```

```
Note: ICCs estimate correlations between individual measurements
and between average measurements made on the same target.
```

```
-> diff = 9
```

```
Intraclass correlations
Two-way mixed-effects model
Consistency of agreement
```

```
Random effects: family Number of targets = 10
Fixed effects: mc Number of raters = 2
```

|            | iq | ICC      | [95% conf. interval] |          |
|------------|----|----------|----------------------|----------|
| Individual |    | .7142152 | .1967504             | .920474  |
| Average    |    | .8332853 | .3288078             | .9585904 |

```
F test that
```

```
ICC=0.00: F(9.0, 9.0) = 6.00 Prob > F = 0.007
```

```
Note: ICCs estimate correlations between individual measurements
and between average measurements made on the same target.
```

```
-> diff = 15
```

```
(output omitted)
```

The estimated CA-ICCs are the same in all three groups and are equal to the corresponding estimates of the Pearson's correlation coefficients because mothers' and childrens' IQ scores have the same variability. The scores differ only in means, and mean differences are irrelevant when measuring the consistency of agreement.

The AA-ICCs, however, differ across the three groups:

```
. by diff, sort: icc iq family mc, mixed absolute
```

```
-> diff = 3
```

```
Intraclass correlations
Two-way mixed-effects model
Absolute agreement
```

```
Random effects: family Number of targets = 10
Fixed effects: mc Number of raters = 2
```

|            | iq | ICC      | [95% conf. interval] |          |
|------------|----|----------|----------------------|----------|
| Individual |    | .7204023 | .2275148             | .9217029 |
| Average    |    | .8374812 | .3706917             | .9592564 |

```
F test that
```

```
ICC=0.00: F(9.0, 9.0) = 6.00 Prob > F = 0.007
```

```
-> diff = 9
```

```
Intraclass correlations
Two-way mixed-effects model
Absolute agreement
```

```
Random effects: family Number of targets = 10
Fixed effects: mc Number of raters = 2
```

|            | iq | ICC      | [95% conf. interval] |          |
|------------|----|----------|----------------------|----------|
| Individual |    | .6203378 | .0293932             | .8905025 |
| Average    |    | .7656895 | .0571077             | .9420802 |

```
F test that
```

```
ICC=0.00: F(9.0, 9.0) = 6.00 Prob > F = 0.007
```

```
-> diff = 15
```

```
Intraclass correlations
Two-way mixed-effects model
Absolute agreement
```

```
Random effects: family Number of targets = 10
Fixed effects: mc Number of raters = 2
```

|            | iq | ICC      | [95% conf. interval] |          |
|------------|----|----------|----------------------|----------|
| Individual |    | .4854727 | -.1194157            | .8466905 |
| Average    |    | .6536272 | -.2712191            | .9169815 |

```
F test that
```

```
ICC=0.00: F(9.0, 9.0) = 6.00 Prob > F = 0.007
```

As the mean differences increase, the AA-ICCs decrease. Their attenuation reflects the difference in means between biological mother and child IQs while still measuring their agreement. Notice that for small mean differences, the estimates of AA-ICCs and CA-ICCs are very similar.

Note that our estimates match those given in [McGraw and Wong \(1996b\)](#), who correct the original table 6 of [McGraw and Wong \(1996a\)](#).

## Relationship between ICCs

In examples 2 and 3, we saw that the estimates of AA-ICCs and CA-ICCs are the same for two-way random-effects and two-way mixed-effects models. In this section, we consider the relationship between various forms of ICCs in more detail; also see *Methods and formulas*.

There are 10 different versions of ICCs, but only 6 different estimators are needed to compute them. These estimators include the two estimators for the individual and average AA-ICCs in a one-way model, the two estimators for the individual and average AA-ICCs in two-way models, and the two estimators for the individual and average CA-ICCs in two-way models.

Only individual and average AA-ICCs are defined for the one-way model. The estimates of AA-ICCs based on the one-way model will typically be smaller than individual and average estimates of AA-ICCs and CA-ICCs based on two-way models. The estimates of individual and average CA-ICCs will typically be larger than the estimates of individual and average AA-ICCs.

Although AA-ICCs and CA-ICCs have the same respective estimators in two-way random-effects and mixed-effects models, their definitions and interpretations are different. The AA-ICCs based on a random-effects model contain the between-rater variance component in the denominator of the variance ratio. The AA-ICCs based on a mixed-effects model contain the variance of the fixed-factor rater instead of the random between-rater variability. The AA-ICCs in a random-effects model represent correlations between any two measurements made on a target. The AA-ICCs in a mixed-effects model measure absolute agreement of measurements treating raters as fixed. The CA-ICCs for random-effects and mixed-effects models have the same definition but different interpretations. The CA-ICCs represent correlations between any two measurements made on a target in a mixed-effects model but estimate the degree of consistency among measurements treating raters as random in a random-effects model. The difference in the definitions of AA-ICCs and CA-ICCs is that CA-ICCs do not contain the between-rater variance in the denominator of the variance ratio.

For two-way models, the definitions and interpretations (but not the estimators) of ICCs also depend on whether the model contains an interaction between target and rater. For two-way models with interaction, ICCs include an additional variance component for the target-rater interaction in the denominator of the variance ratio. This component cannot be separated from random error because there is only one observation per target and rater.

Also, under a two-way mixed-effects model including interaction, the interaction components are not mutually independent, as they are in a two-way random-effects model. The considered version of the mixed-effects model places a constraint on the interaction effects—the sum of the interaction effects over levels of the fixed factor is zero; see, for example, chapter 7 in Kuehl (2000) for an introductory discussion of mixed models. In this version of the model, there is a correlation between the interaction effects. Specifically, the two interaction effects for the same target and two different raters are negatively correlated. As a result, the estimated intraclass correlation can be negative under a two-way mixed-effects model with interaction. Also, average AA-ICC and average CA-ICC cannot be estimated in a two-way mixed-effects model including interaction; see *Methods and formulas* and McGraw and Wong (1996a) for details.

## Tests against nonzero values

It may be of interest to test whether the intraclass correlation is equal to a value other than zero. `icc` supports testing against positive values through the use of the `testvalue()` option. Specifying `testvalue(#)` provides a one-sided hypothesis test of  $H_o: \rho = \#$  versus  $H_a: \rho > \#$ . The test is provided separately for both individual and average ICCs.

## ▷ Example 5: Testing ICC against a nonzero value

We return to the two-way random-effects model for the judge and target data from [Shrout and Fleiss \(1979\)](#). Suppose we want to test whether the individual and average AA-ICCs are each equal to 0.2. We specify the `testvalue(0.2)` option with `icc`:

```
. use https://www.stata-press.com/data/r19/judges, clear
(Ratings of targets by judges)
. icc rating target judge, testvalue(0.2)

Intraclass correlations
Two-way random-effects model
Absolute agreement

Random effects: target Number of targets = 6
Random effects: judge Number of raters = 4
```

| rating     | ICC      | [95% conf. interval] |          |
|------------|----------|----------------------|----------|
| Individual | .2897638 | .0187865             | .7610844 |
| Average    | .6200505 | .0711368             | .927232  |

```
F test that
ICC(1)=0.20: F(5.0, 5.3) = 1.54 Prob > F = 0.317
ICC(k)=0.20: F(5.0, 9.4) = 4.35 Prob > F = 0.026
```

Note: ICCs estimate correlations between individual measurements and between average measurements made on the same target.

We reject the null hypothesis that the average AA-ICC, labeled as  $ICC(k)$  in the output, is equal to 0.2, but we do not have statistical evidence to reject the null hypothesis that the individual AA-ICC, labeled as  $ICC(1)$ , is equal to 0.2.

◀

## Stored results

`icc` stores the following in `r()`:

### Scalars

|                             |                                                              |
|-----------------------------|--------------------------------------------------------------|
| <code>r(N_target)</code>    | number of targets                                            |
| <code>r(N_rater)</code>     | number of raters                                             |
| <code>r(icc_i)</code>       | intraclass correlation for individual measurements           |
| <code>r(icc_i_F)</code>     | <i>F</i> test statistic for individual ICC                   |
| <code>r(icc_i_df1)</code>   | numerator degrees of freedom for <code>r(icc_i_F)</code>     |
| <code>r(icc_i_df2)</code>   | denominator degrees of freedom for <code>r(icc_i_F)</code>   |
| <code>r(icc_i_p)</code>     | <i>p</i> -value for <i>F</i> test of individual ICC          |
| <code>r(icc_i_lb)</code>    | lower endpoint for confidence intervals of individual ICC    |
| <code>r(icc_i_ub)</code>    | upper endpoint for confidence intervals of individual ICC    |
| <code>r(icc_avg)</code>     | intraclass correlation for average measurements              |
| <code>r(icc_avg_F)</code>   | <i>F</i> test statistic for average ICC                      |
| <code>r(icc_avg_df1)</code> | numerator degrees of freedom for <code>r(icc_avg_F)</code>   |
| <code>r(icc_avg_df2)</code> | denominator degrees of freedom for <code>r(icc_avg_F)</code> |
| <code>r(icc_avg_p)</code>   | <i>p</i> -value for <i>F</i> test of average ICC             |
| <code>r(icc_avg_lb)</code>  | lower endpoint for confidence intervals of average ICC       |
| <code>r(icc_avg_ub)</code>  | upper endpoint for confidence intervals of average ICC       |
| <code>r(testvalue)</code>   | null hypothesis value                                        |
| <code>r(level)</code>       | confidence level                                             |

### Macros

|                        |                            |
|------------------------|----------------------------|
| <code>r(model)</code>  | analysis-of-variance model |
| <code>r(depvar)</code> | name of dependent variable |

|           |                                                 |
|-----------|-------------------------------------------------|
| r(target) | target variable                                 |
| r(rater)  | rater variable                                  |
| r(type)   | type of ICC estimated (absolute or consistency) |

## Methods and formulas

We observe  $y_{ij}$ , where  $i = 1, \dots, n$  and  $j = 1, \dots, k$ .  $y_{ij}$  is the  $j$ th rating on the  $i$ th target. Let  $\alpha = 1 - l/100$ , where  $l$  is the significance level specified by the user.

Methods and formulas are presented under the following headings:

*Mean squares*  
*One-way random effects*  
*Two-way random effects*  
*Two-way mixed effects*

## Mean squares

The mean squares within targets are

$$\text{WMS} = \sum_i \sum_j \frac{(y_{ij} - \bar{y}_{i.})^2}{n(k-1)}$$

where  $\bar{y}_{i.} = \sum_j y_{ij}/k$ .

The mean squares between targets are

$$\text{BMS} = \sum_i \frac{(\bar{y}_{i.} - \bar{y}_{..})^2}{n-1}$$

where  $\bar{y}_{..} = \sum_i \bar{y}_{i.}/n$ .

These are the only mean squares needed to estimate ICC in the one-way random-effects model. For the two-way models, we need two additional mean squares.

The mean squares between raters are

$$\text{JMS} = \sum_j \frac{(\bar{y}_{.j} - \bar{y}_{..})^2}{k-1}$$

where  $\bar{y}_{.j} = \sum_i y_{ij}/n$  and  $\bar{y}_{..} = \sum_j \bar{y}_{.j}/k$ .

The residual or error mean square is

$$\text{EMS} = \frac{\sum_i \sum_j (y_{ij} - \bar{y})^2 - (k-1)\text{JMS} - (n-1)\text{BMS}}{(n-1)(k-1)}$$

## One-way random effects

Under the one-way random-effects model, we observe

$$y_{ij} = \mu + r_i + \epsilon_{ij} \quad (\text{M1})$$

where  $\mu$  is the mean rating,  $r_i$  is the target random effect, and  $\epsilon_{ij}$  is random error. The  $r_i$ s are i.i.d.  $N(0, \sigma_r^2)$ ;  $\epsilon_{ij}$ s are i.i.d.  $N(0, \sigma_\epsilon^2)$  and are independent of  $r_i$ s. There is no rater effect separate from the residual error because each target is evaluated by a different set of raters.

The individual AA-ICC is the correlation between individual measurements on the same target:

$$\rho_1 = \text{ICC}(1) = \text{Corr}(y_{ij}, y_{ij'}) = \frac{\sigma_r^2}{\sigma_r^2 + \sigma_\epsilon^2}$$

The average AA-ICC is the correlation between average measurements of size  $k$  made on the same target:

$$\rho_k = \text{ICC}(k) = \text{Corr}(\bar{y}_i, \bar{y}'_i) = \frac{\sigma_r^2}{\sigma_r^2 + \sigma_\epsilon^2/k}$$

They are estimated by

$$\hat{\rho}_1 = \widehat{\text{ICC}}(1) = \frac{\text{BMS} - \text{WMS}}{\text{BMS} + (k-1)\text{WMS}}$$

$$\hat{\rho}_k = \widehat{\text{ICC}}(k) = \frac{\text{BMS} - \text{WMS}}{\text{BMS}}$$

**Confidence intervals.** Let  $F_{\text{obs}} = \text{BMS}/\text{WMS}$ , let  $F_l$  be the  $(1 - \alpha/2) \times 100$ th percentile of the  $F_{n-1, n(k-1)}$  distribution, and let  $F_u$  be the  $(1 - \alpha/2) \times 100$ th percentile of the  $F_{n(k-1), n-1}$  distribution. Let  $F_L = F_{\text{obs}}/F_l$  and  $F_U = F_{\text{obs}}F_u$ .

A  $(1 - \alpha) \times 100\%$  confidence interval for  $\rho_1$  is

$$\left( \frac{F_L - 1}{F_l + k - 1}, \frac{F_U - 1}{F_U + k - 1} \right) \quad (1)$$

A  $(1 - \alpha) \times 100\%$  confidence interval for  $\rho_k$  is

$$\left( 1 - \frac{1}{F_L}, 1 - \frac{1}{F_U} \right) \quad (2)$$

**Hypothesis tests.** Consider a one-sided hypothesis test of  $H_0: \text{ICC} = \rho_0$  versus  $H_a: \text{ICC} > \rho_0$ .

The test statistic for  $\rho_1$  is

$$F_{\rho_1} = \frac{\text{BMS}}{\text{WMS}} \frac{1 - \rho_0}{1 + (k-1)\rho_0} \quad (3)$$

The test statistic for  $\rho_k$  is

$$F_{\rho_k} = \frac{\text{BMS}}{\text{WMS}} (1 - \rho_0) \quad (4)$$

Under the null hypothesis, both  $F_{\rho_1}$  and  $F_{\rho_k}$  have the  $F_{n-1, n(k-1)}$  distribution. When  $\rho_0 = 0$ , the two test statistics coincide.

## Two-way random effects

In this setting, the target is evaluated by the same set of raters, who are randomly drawn from the population of raters. The underlying models with and without interaction are

$$y_{ij} = \mu + r_i + c_j + (rc)_{ij} + \epsilon_{ij} \quad (\text{M2})$$

$$y_{ij} = \mu + r_i + c_j + \epsilon_{ij} \quad (\text{M2A})$$

where  $y_{ij}$  is the rating of the  $i$ th target by the  $j$ th rater,  $\mu$  is the mean rating,  $r_i$  is the target random effect,  $c_j$  is the rater random effect,  $(rc)_{ij}$  is the target-rater random effect, and  $\epsilon_{ij}$  is random error. The  $r_i$ s are i.i.d.  $N(0, \sigma_r^2)$ ,  $c_j$ s are i.i.d.  $N(0, \sigma_c^2)$ ,  $(rc)_{ij}$ s are i.i.d.  $N(0, \sigma_{rc}^2)$ , and  $\epsilon_{ij}$ s are i.i.d.  $N(0, \sigma_\epsilon^2)$ . Each effect is mutually independent of the others.

Below, we provide formulas for ICCs for model (M2). The corresponding ICCs for model (M2A) can be obtained by setting  $\sigma_{rc}^2 = 0$ .

The individual AA-ICC is the correlation between individual measurements on the same target:

$$\rho_{A,1} = \text{ICC}(A,1) = \text{Corr}(y_{ij}, y_{ij'}) = \frac{\sigma_r^2}{\sigma_r^2 + \sigma_c^2 + (\sigma_{rc}^2 + \sigma_\epsilon^2)}$$

The average AA-ICC is the correlation between average measurements of size  $k$  made on the same target:

$$\rho_{A,k} = \text{ICC}(A,k) = \text{Corr}(\bar{y}_{i\cdot}, \bar{y}'_{i\cdot}) = \frac{\sigma_r^2}{\sigma_r^2 + (\sigma_c^2 + \sigma_{rc}^2 + \sigma_\epsilon^2)/k}$$

The consistency-of-agreement intraclass correlation for individual measurements, individual CA-ICC, is

$$\rho_{C,1} = \text{ICC}(C,1) = \frac{\sigma_r^2}{\sigma_r^2 + (\sigma_{rc}^2 + \sigma_\epsilon^2)}$$

The consistency-of-agreement intraclass correlation for average measurements of size  $k$ , average CA-ICC, is

$$\rho_{C,k} = \text{ICC}(C,k) = \frac{\sigma_r^2}{\sigma_r^2 + (\sigma_{rc}^2 + \sigma_\epsilon^2)/k}$$

With one observation per target and rater,  $\sigma_{rc}^2$  and  $\sigma_\epsilon^2$  cannot be estimated separately.

The estimators of intraclass correlations, confidence intervals, and test statistics are the same for models (M2) and (M2A). The estimators of ICCs are

$$\begin{aligned} \hat{\rho}_{A,1} &= \widehat{\text{ICC}}(A,1) = \frac{\text{BMS} - \text{EMS}}{\text{BMS} + (k-1)\text{EMS} + \frac{k}{n}(\text{JMS} - \text{EMS})} \\ \hat{\rho}_{A,k} &= \widehat{\text{ICC}}(A,k) = \frac{\text{BMS} - \text{EMS}}{\text{BMS} + \frac{1}{n}(\text{JMS} - \text{EMS})} \\ \hat{\rho}_{C,1} &= \widehat{\text{ICC}}(C,1) = \frac{\text{BMS} - \text{EMS}}{\text{BMS} + (k-1)\text{EMS}} \\ \hat{\rho}_{C,k} &= \widehat{\text{ICC}}(C,k) = \frac{\text{BMS} - \text{EMS}}{\text{BMS}} \end{aligned}$$

**Confidence intervals.** Let  $a = k\hat{\rho}_{A,1}/\{n(1 - \hat{\rho}_{A,1})\}$ ,  $b = 1 + k\hat{\rho}_{A,1}(n - 1)/\{n(1 - \hat{\rho}_{A,1})\}$ , and

$$v = \frac{(a\text{JMS} + b\text{EMS})^2}{\frac{a^2\text{JMS}^2}{k-1} + \frac{b^2\text{EMS}^2}{(n-1)(k-1)}} \tag{5}$$

Let  $F_l$  be the  $(1 - \alpha/2) \times 100$ th percentile of the  $F_{n-1,v}$  distribution and  $F_u$  be the  $(1 - \alpha/2) \times 100$ th percentile of the  $F_{v,n-1}$  distribution.

A  $(1 - \alpha) \times 100\%$  confidence interval for  $\rho_{A,1}$  is given by  $(L, U)$ , where

$$L = \frac{n(\text{BMS} - F_l\text{EMS})}{F_l\{k\text{JMS} + (kn - k - n)\text{EMS}\} + n\text{BMS}}$$

$$U = \frac{n(F_u\text{BMS} - \text{EMS})}{k\text{JMS} + (kn - k - n)\text{EMS} + nF_u\text{BMS}}$$
(6)

A  $(1 - \alpha) \times 100\%$  confidence intervals for  $\rho_{A,k}$  is a special case of (6) with  $k = 1$ , where  $a = \hat{\rho}_{A,k}/\{n(1 - \hat{\rho}_{A,k})\}$ ,  $b = 1 + \hat{\rho}_{A,k}(n - 1)/\{n(1 - \hat{\rho}_{A,k})\}$ , and  $v$  is defined in (5).

To define confidence intervals for  $\rho_{C,1}$  and  $\rho_{C,k}$ , let  $F_{\text{obs}} = \text{BMS}/\text{EMS}$ ,  $F_l$  be the  $(1 - \alpha/2) \times 100$ th percentile of the  $F_{n-1,(n-1)(k-1)}$  distribution, and  $F_u$  be the  $(1 - \alpha/2) \times 100$ th percentile of the  $F_{(n-1)(k-1),n-1}$  distribution. Let  $F_L = F_{\text{obs}}/F_l$  and  $F_U = F_{\text{obs}}F_u$ .

A  $(1 - \alpha) \times 100\%$  confidence intervals for  $\rho_{C,1}$  and  $\rho_{C,k}$  are then as given by (1) and (2) for model (M1).

**Hypothesis tests.** Consider a one-sided hypothesis test of  $H_0: \text{ICC} = \rho_0$  versus  $H_a: \text{ICC} > \rho_0$ . Let  $a = k\rho_0/\{n(1 - \rho_0)\}$  and  $b = 1 + k\rho_0(n - 1)/\{n(1 - \rho_0)\}$ .

The test statistic for  $\rho_{A,1}$  is

$$F_{\rho_{A,1}} = \frac{\text{BMS}}{a\text{JMS} + b\text{EMS}}$$

Under the null hypothesis,  $F_{\rho_{A,1}}$  has the  $F_{n-1,v}$  distribution, where  $v$  is defined in (5).

The test statistic for  $\rho_{A,k}$  is defined similarly, except  $a = \rho_0/\{n(1 - \rho_0)\}$  and  $b = 1 + \rho_0(n - 1)/\{n(1 - \rho_0)\}$ . Under the null hypothesis,  $F_{\rho_{A,k}}$  has the  $F_{n-1,v}$  distribution, where  $v$  is defined in (5). When  $\rho_0 = 0$ , then  $a = 0$ ,  $b = 1$ , and the two test statistics coincide.

The test statistics for  $\rho_{C,1}$  and  $\rho_{C,k}$  are defined by (3) and (4), respectively, with WMS replaced by EMS. Under the null hypothesis, both  $F_{\rho_{C,1}}$  and  $F_{\rho_{C,k}}$  have the  $F_{n-1,(n-1)(k-1)}$  distribution. They also both have the same value when  $\rho_0 = 0$ .

## Two-way mixed effects

In this setting, every target is evaluated by the same set of judges, who are the only judges of interest. The underlying models with and without interaction are

$$y_{ij} = \mu + r_i + c_j + (rc)_{ij} + \epsilon_{ij} \tag{M3}$$

$$y_{ij} = \mu + r_i + c_j + \epsilon_{ij} \tag{M3A}$$

where  $y_{ij}$  is the rating of the  $i$ th target by the  $j$ th rater,  $\mu$  is the mean rating,  $r_i$  is the target random effect,  $c_j$  is the rater random effect,  $(rc)_{ij}$  is an interaction effect between target and rater, and  $\epsilon_{ij}$  is random error. The  $r_i$ s are i.i.d.  $N(0, \sigma_r^2)$ ,  $(rc)_{ij}$ s are  $N(0, \sigma_{rc}^2)$ , and  $\epsilon_{ij}$ s are i.i.d.  $N(0, \sigma_\epsilon^2)$ . Each random effect is mutually independent of the others. The  $c_j$ s are fixed such that  $\sum_j c_j = 0$ . The variance of  $c_j$ s is  $\theta_c^2 = \sum c_j^2 / (k - 1)$ .

In the presence of an interaction, two versions of a mixed-effects model may be considered. One assumes that  $(rc)_{ij}$ s are i.i.d.  $N(0, \sigma_{rc}^2)$ . Another assumes that  $(rc)_{ij}$ s are  $N(0, \sigma_{rc}^2)$  with an additional constraint that  $\sum_j (rc)_{ij} = 0$  (for example, [Kuehl \[2000\]](#)), so only interaction terms involving different targets are independent. The latter model is considered here.

We now define the intraclass correlations for individual measurements for model (M3).

The individual CA-ICC, the correlation between individual measurements on the same target, is

$$\rho_{C,1} = \text{ICC}(C,1) = \text{Corr}(y_{ij}, y_{ij'}) = \frac{\sigma_r^2 - \sigma_{rc}^2 / (k - 1)}{\sigma_r^2 + (\sigma_{rc}^2 + \sigma_\epsilon^2)}$$

The absolute-agreement intraclass correlation for individual measurements, individual AA-ICC, is

$$\rho_{A,1} = \text{ICC}(A,1) = \frac{\sigma_r^2 - \sigma_{rc}^2 / (k - 1)}{\sigma_r^2 + \theta_c^2 + (\sigma_{rc}^2 + \sigma_\epsilon^2)}$$

[Shrout and Fleiss \(1979\)](#) show that the individual ICC could be negative in this case—a phenomenon first pointed out by [Sitgreaves \(1960\)](#). This can happen when the interaction term has a high variance relative to the targets and there are not many raters.

The individual intraclass correlations for model (M3A) have similar definitions with  $\sigma_{rc}^2 = 0$ . The individual CA-ICC is the correlation between individual measurements on the same target,  $\text{Corr}(y_{ij}, y_{ij'})$ .

We now discuss the intraclass correlations that correspond to average measurements. Neither average AA-ICC,  $\rho_{A,k}$ , nor average CA-ICC,  $\rho_{C,k}$ , can be estimated under model (M3) ([Shrout and Fleiss 1979](#); [McGraw and Wong 1996a](#)). The problem is that in this model,  $\sigma_r^2$ , which is the covariance between two means based on  $k$  raters, cannot be estimated.

Specifically, the parameter  $\sigma_r^2$  appears only in the expectation of the between-target mean squares BMS. Under the restriction  $\sum_j (rc)_{ij} = 0$ ,

$$E(\text{BMS}) = k\sigma_r^2 + \sigma_\epsilon^2$$

Note that  $\sigma_{rc}^2$  does not appear in the expectation of between-target mean squares. With one observation per target and rater,  $\sigma_r^2$  and  $\sigma_\epsilon^2$  cannot be estimated separately (only their sum  $\sigma_{rc}^2 + \sigma_\epsilon^2$  can be estimated), so BMS alone cannot be used to estimate  $\sigma_r^2$ .

Under model (M3A), however, there is no interaction (and thus no interaction variance component  $\sigma_{rc}^2$ ), so  $\rho_{A,k}$  or  $\rho_{C,k}$  can be estimated.

The average AA-ICC, the absolute-agreement intraclass correlation for average measurements of size  $k$ , is

$$\rho_{A,k} = \text{ICC}(A,k) = \frac{\sigma_r^2}{\sigma_r^2 + (\theta_c^2 + \sigma_\epsilon^2) / k}$$

The average CA-ICC, the correlation between average measurements of size  $k$  made on the same target, is

$$\rho_{C,k} = \text{ICC}(C,k) = \text{Corr}(\bar{y}_i, \bar{y}'_i) = \frac{\sigma_r^2}{\sigma_r^2 + \sigma_\epsilon^2 / k}$$

The estimators of ICCs, their confidence intervals, and hypothesis tests are as described for two-way random-effects models, except  $\rho_{A,k}$  and  $\rho_{C,k}$  are not defined under model (M3).

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## Also see

- [R] **anova** — Analysis of variance and covariance
- [R] **correlate** — Correlations of variables
- [R] **loneway** — Large one-way ANOVA, random effects, and reliability
- [MV] **alpha** — Compute interitem correlations (covariances) and Cronbach’s alpha

## Description

There are different approaches for measuring inequality and poverty, and different methods for summarizing the distribution of earnings, income, and wealth. Stata has some official features for these types of analyses. For example, the `roctab` command has an option to report Gini and Pietra indices, which are measures of income inequality; see [R] [roctab](#). With the `cumul` command, you can obtain the cumulative distribution function of, for example, household expenditures; see [R] [cumul](#). Additionally, you may want to look at the shape of income distributions; see [R] [kdensity](#) and [R] [histogram](#).

However, you might also be interested in the generalized Lorenz curve or a welfare index, which cannot be obtained from any official Stata commands. Fortunately, Stata users have developed an excellent suite of commands that create these graphs and report these types of indices, along with many other related measures. In this entry, we demonstrate how to find and install community-contributed commands related to inequality, poverty, and distributional summaries of earnings, income, and wealth.

## Remarks and examples

Suppose you are in search of a command to report inequality indices; you could type the following:

```
. search inequality
```

First, you will see official resources related to this keyword, including frequently asked questions (FAQs) and *Stata Journal* articles. Then, you will see community-contributed additions to Stata that are available from the internet, which typically include additions published in the *Stata Journal* and those posted on the Statistical Software Components (SSC) Archive, provided by <http://repec.org>. For example, in the output from `search`, we find `ineqdec0`, which calculates inequality indices with decomposition by subgroup.

### Search of web resources from Stata and other users

(output omitted)

```
ineqdec0 from http://fmwww.bc.edu/RePEc/bocode/i
'INEQDECO': module to calculate inequality indices with decomposition by
subgroup / ineqdec0 is a stripped-down version of ineqdeco (q.v.). /
Unlike the latter, it will provide estimates using samples /containing
zero or negative values for the variable of / interest. But as a result
ineqdecgini from http://fmwww.bc.edu/RePEc/bocode/i
'INEQDECGINI': module to estimate Gini coefficient with optional
decomposition by subgroups / ineqdecgini estimates the Gini coefficient
(a.k.a. Gini index) / of inequality plus, optionally, a decomposition by
population / subgroup into components representing inequality within /
```

(output omitted)

(end of search)

Let's take a closer look at this package:

1. Click on *ineqdec0* from <http://fmwww.bc.edu/RePEc/bocode/i>.
2. Click on *ineqdec0.hlp*.

This leads us to the help file, where we could learn more about which indices are computed with this package. This particular command supports *fweights* and *awweights*, but not all community-contributed commands do; it is important to look at the help file and confirm whether the command is a good fit for the analysis you want to perform. If it is, you can install it:

3. Click on the back button.
4. Click on *click here to install*.

Once you see a message indicating that the installation is complete, you may now use this command.

Another thing to keep in mind when installing packages is whether they are designed to work with complex survey data. This particular command is not. We can click on the back button once to return to the description of *ineqdec0* and then once more to return to the search results. Scrolling up, we come across

**Search of official help files, FAQs, Examples, and Stata Journals**

(output omitted)

```
SJ-16-2 st0427_1 Software update for conindex
. O. O'Donnell, S. O'Neill, T. Van Ourti, and B. Walsh
(help conindex if installed)
Q2/16 SJ 16(2):521--522
program can now be used on data from complex survey designs
```

(output omitted)

(end of search)

This is an update for a package called *conindex*; this updated version of the command can be used with data from complex survey designs. The original version of the package was introduced in *Stata Journal* Volume 16 Number 1 (March 2016), and this update was published the following quarter. If we scroll up a bit more in the search results, we will find an even more recent update:

**Search of official help files, FAQs, Examples, and Stata Journals**

(output omitted)

```
SJ-18-3 st0427_2 Software update for conindex
. O. O'Donnell, S. O'Neill, T. Van Ourti, and B. Walsh
(help conindex if installed)
Q3/18 SJ 18(3):758--759
uses the community-contributed lorenz command and fix a small bug
in the graph option
```

(output omitted)

(end of search)

It is best to install the most recent version, which we can do as follows:

1. Click on *st0427\_2*.
2. Click on *click here to install*.

The examples above demonstrate how to install community-contributed commands related to measures of inequality, but you would follow the same steps to find and install commands related to poverty measures, indices of social welfare, and the like.

For example, type the following:

```
. search poverty
```

In the list of results, you will find `g1curve`, which can be used to draw Lorenz curves, concentration curves, and other related curves. We can install this command as follows:

1. Click on `g1curve` from <http://fmwww.bc.edu/RePEc/bocode/g>.
2. Click on *click here to install*.

These are just a few examples of the many community-contributed commands that are available. You can use `search` to find more community-contributed commands used to obtain other measures of inequality and poverty, and distributional summaries of earnings, income, and wealth.

[Max Otto Lorenz](#) (1876–1959) was born in Burlington, Iowa. He did his undergraduate studies at the University of Iowa and received his PhD from the University of Wisconsin–Madison in 1906. In 1905, he published his only article, “Methods of measuring the concentration of wealth”, in a scientific journal. In the article, he introduces what we now call the Lorenz curve, a term first introduced in a statistics textbook in 1912.

Lorenz worked his whole life in governmental statistical institutions. He was the Deputy Commissioner of Labor and Industrial Statistics for Wisconsin, worked for the US Bureau of the Census and the Bureau of Railway Economics, and was the Director of the Bureau of Statistics and the Bureau of Transport and Economic Statistics.

His hobbies included calendar reform and Interlingua, a proposed international language.

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## Description

`intreg` fits a linear model with an outcome measured as point data, interval data, left-censored data, or right-censored data. As such, it is a generalization of the model fit by `tobit`.

## Quick start

Regression on `x1` and `x2` of an interval-measured dependent variable with lower endpoint `y_lower` and upper endpoint `y_upper`

```
intreg y_lower y_upper x1 x2
```

With robust standard errors

```
intreg y_lower y_upper x1 x2, vce(robust)
```

Model heteroskedasticity in the conditional variance as a function of `x3`

```
intreg y_lower y_upper x1 x2, het(x3)
```

Adjust for complex survey design using `svyset` data

```
svy: intreg y_lower y_upper x1 x2
```

## Menu

Statistics > Linear models and related > Censored regression > Interval regression

## Syntax

```
intreg depvar1 depvar2 [indepvars] [if] [in] [weight] [, options]
```

*depvar*<sub>1</sub> and *depvar*<sub>2</sub> should have the following form:

| Type of data        |                | <i>depvar</i> <sub>1</sub> | <i>depvar</i> <sub>2</sub> |
|---------------------|----------------|----------------------------|----------------------------|
| point data          | $a = [a, a]$   | <i>a</i>                   | <i>a</i>                   |
| interval data       | $[a, b]$       | <i>a</i>                   | <i>b</i>                   |
| left-censored data  | $(-\infty, b]$ | .                          | <i>b</i>                   |
| right-censored data | $[a, +\infty)$ | <i>a</i>                   | .                          |
| missing             |                | .                          | .                          |

| <i>options</i>                                               | Description                                                                                                                                                                 |
|--------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                                 |                                                                                                                                                                             |
| <code>noconstant</code>                                      | suppress constant term                                                                                                                                                      |
| <code>het(<i>varlist</i> [, <code>noconstant</code>])</code> | independent variables to model the variance; use <code>noconstant</code> to suppress constant term                                                                          |
| <code>offset(<i>varname</i>)</code>                          | include <i>varname</i> in model with coefficient constrained to 1                                                                                                           |
| <code>constraints(<i>constraints</i>)</code>                 | apply specified linear constraints                                                                                                                                          |
| <b>SE/Robust</b>                                             |                                                                                                                                                                             |
| <code>vce(<i>vcetype</i>)</code>                             | <i>vcetype</i> may be <code>oim</code> , <code>robust</code> , <code>cluster <i>clustvar</i></code> , <code>opg</code> , <code>bootstrap</code> , or <code>jackknife</code> |
| <b>Reporting</b>                                             |                                                                                                                                                                             |
| <code>level(#)</code>                                        | set confidence level; default is <code>level(95)</code>                                                                                                                     |
| <code>nocnsreport</code>                                     | do not display constraints                                                                                                                                                  |
| <code>display_options</code>                                 | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling                            |
| <b>Maximization</b>                                          |                                                                                                                                                                             |
| <code>maximize_options</code>                                | control the maximization process; seldom used                                                                                                                               |
| <code>collinear</code>                                       | keep collinear variables                                                                                                                                                    |
| <code>coeflegend</code>                                      | display legend instead of statistics                                                                                                                                        |

*indepvars* and *varlist* may contain factor variables; see [U] 11.4.3 Factor variables.

*depvar*<sub>1</sub>, *depvar*<sub>2</sub>, *indepvars*, and *varlist* may contain time-series operators; see [U] 11.4.4 Time-series varlists.

`bayes`, `bayesboot`, `bootstrap`, `by`, `collect`, `fmm`, `fp`, `jackknife`, `mfp`, `nestreg`, `rolling`, `statsby`, `stepwise`, and `svy` are allowed; see [U] 11.1.10 Prefix commands. For more details, see [BAYES] `bayes: intreg` and [FMM] `fmm: intreg`.

Weights are not allowed with the `bootstrap` prefix; see [R] `bootstrap`.

`aweight`s are not allowed with the `jackknife` prefix; see [R] `jackknife`.

`vce()` and weights are not allowed with the `svy` prefix; see [SVY] `svy`.

`aweight`s, `fweight`s, `ilweight`s, and `pweight`s are allowed; see [U] 11.1.6 `weight`.

`collinear` and `coeflegend` do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Options

### Model

`noconstant`; see [R] [Estimation options](#).

`het` (*varlist* [ , `noconstant`]) specifies that the logarithm of the standard deviation be modeled as a linear combination of *varlist*. The constant is included unless `noconstant` is specified.

`offset` (*varname*), `constraints` (*constraints*); see [R] [Estimation options](#).

### SE/Robust

`vce` (*vcetype*) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster` *clustvar*), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

### Reporting

`level` (#), `nocnsreport`; see [R] [Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap` (#), `fvwrapon` (*style*), `cformat` (*%fmt*), `pformat` (*%fmt*), `sformat` (*%fmt*), and `nolstretch`; see [R] [Estimation options](#).

### Maximization

`maximize_options`: `difficult`, `technique` (*algorithm\_spec*), `iterate` (#), `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance` (#), `ltolerance` (#), `nrtolerance` (#), `nonrtolerance`, and `from` (*init\_specs*); see [R] [Maximize](#). These options are seldom used.

Setting the optimization type to `technique` (`bhhh`) resets the default *vcetype* to `vce` (`opg`).

The following options are available with `intreg` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

`intreg` fits a linear model to an outcome that may be either observed exactly or unobserved but known to fall within some interval. The values of the outcome variable may be observed (point data), unobserved but known to fall within an interval with fixed endpoints (interval-censored data), unobserved but known to fall within an interval that has a fixed upper endpoint (left-censored data), or unobserved but known to fall within an interval that has a fixed lower endpoint (right-censored data). Such censored data arise naturally in many contexts, such as wage data. Often, you know only that, for example, a person's salary is between \$30,000 and \$40,000.

The interval regression model fit by `intreg` is a generalization of the models fit by `tobit` because it extends censoring beyond left-censored data or right-censored data; see [Cameron and Trivedi \(2022, 965–967\)](#) for additional discussion of these data types. See [Wooldridge \(2020, sec. 17.4\)](#) for an introduction to censored and truncated regression models.

Regardless of the type of censoring, `intreg` requires the outcome to be stored in the dataset as interval data. That is, two dependent variables, `depvar1` and `depvar2`, are used to hold the endpoints of the interval. If the data are left-censored, the lower endpoint is  $-\infty$  and is represented by a missing value in `depvar1`. If the data are right-censored, the upper endpoint is  $+\infty$  and is represented by a missing value in `depvar2`. Point data are represented by the two endpoints being equal. Truly missing values of the dependent variable must be represented by missing values in both `depvar1` and `depvar2`.

### ▷ Example 1: Interval regression

`womenwage2.dta` contains the yearly wages of working women in interval form. Women were asked to indicate a category for their yearly income from employment. The categories were \$5,000 or less, \$5,001–\$10,000, ..., \$25,001–\$30,000, \$30,001–\$40,000, \$40,001–\$50,000, and more than \$50,000. The lower and upper endpoints of the wage categories (in \$1,000s) are recorded in variables `wage1` and `wage2`. Below, we list the first 10 observations in `wage1` and `wage2`.

```
. use https://www.stata-press.com/data/r19/womenwage2
(Wages of women, fictional data)
. list wage1 wage2 in 1/10
```

|     | wage1 | wage2 |
|-----|-------|-------|
| 1.  | .     | 5     |
| 2.  | 5     | 10    |
| 3.  | 5     | 10    |
| 4.  | 10    | 15    |
| 5.  | 15    | 20    |
| 6.  | 20    | 25    |
| 7.  | 25    | 30    |
| 8.  | 30    | 40    |
| 9.  | 40    | 50    |
| 10. | 50    | .     |

We see, for example, that the first respondent made \$5,000 or less in a year, that the second respondent made between \$5,001 and \$10,000 in a year, and so on. The tenth respondent made at least \$50,000 a year.

We now fit an interval regression model of women’s wages using social and demographic characteristics as explanatory variables. The variables include the subject’s age (age), years of schooling (school), job tenure (tenure), a dummy for living in a rural area (rural), and a dummy for never being married (nev\_mar).

```
. intreg wage1 wage2 age c.age#c.age i.nev_mar i.rural school tenure
Fitting constant-only model:
Iteration 0: Log likelihood = -967.24956
Iteration 1: Log likelihood = -967.1368
Iteration 2: Log likelihood = -967.1368
Fitting full model:
Iteration 0: Log likelihood = -856.65324
Iteration 1: Log likelihood = -856.33294
Iteration 2: Log likelihood = -856.33293
Interval regression
Number of obs = 488
 Uncensored = 0
 Left-censored = 14
 Right-censored = 6
 Interval-cens. = 468
LR chi2(6) = 221.61
Prob > chi2 = 0.0000
Log likelihood = -856.33293
```

|             | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|-------------|-------------|-----------|-------|-------|----------------------|-----------|
| age         | .7914438    | .4433604  | 1.79  | 0.074 | -.0775265            | 1.660414  |
| c.age#c.age | -.0132624   | .0073028  | -1.82 | 0.069 | -.0275757            | .0010509  |
| 1.nev_mar   | -.2075022   | .8119581  | -0.26 | 0.798 | -1.798911            | 1.383906  |
| 1.rural     | -3.043044   | .7757324  | -3.92 | 0.000 | -4.563452            | -1.522637 |
| school      | 1.334721    | .1357873  | 9.83  | 0.000 | 1.068583             | 1.600859  |
| tenure      | .8000664    | .1045077  | 7.66  | 0.000 | .5952351             | 1.004898  |
| _cons       | -12.70238   | 6.367117  | -1.99 | 0.046 | -25.1817             | -.2230583 |
| /lnsigma    | 1.987823    | .0346543  | 57.36 | 0.000 | 1.919902             | 2.055744  |
| sigma       | 7.299626    | .2529634  |       |       | 6.82029              | 7.81265   |

Because the conditional mean modeled by interval regression is linear, the coefficients are interpreted the same way they are in ordinary least-squares regression; see [R] regress. For example, residing in a rural area lowers the expected income by \$3,043 and each additional year of schooling raises the expected income by \$1,335.



□ Technical note

Instead of using intervals to record wages, we could treat the outcome as categorical with a higher category corresponding to a higher wage. Here we fit an ordered probit model for wagecat, created based on groups defined by the intervals, by using `oprobit` (see [R] [oprobit](#)) with the same covariates:

```
. oprobit wagecat age c.age#c.age i.nev_mar i.rural school tenure
Iteration 0: Log likelihood = -881.1491
Iteration 1: Log likelihood = -764.31729
Iteration 2: Log likelihood = -763.31191
Iteration 3: Log likelihood = -763.31049
Iteration 4: Log likelihood = -763.31049
Ordered probit regression
Log likelihood = -763.31049
Number of obs = 488
LR chi2(6) = 235.68
Prob > chi2 = 0.0000
Pseudo R2 = 0.1337
```

| wagecat     | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|-------------|-------------|-----------|-------|-------|----------------------|-----------|
| age         | .1674519    | .0620333  | 2.70  | 0.007 | .0458689             | .289035   |
| c.age#c.age | -.0027983   | .0010214  | -2.74 | 0.006 | -.0048001            | -.0007964 |
| 1.nev_mar   | -.0046417   | .1126737  | -0.04 | 0.967 | -.225478             | .2161946  |
| 1.rural     | -.5270036   | .1100449  | -4.79 | 0.000 | -.7426875            | -.3113196 |
| school      | .2010587    | .0201189  | 9.99  | 0.000 | .1616263             | .2404911  |
| tenure      | .0989916    | .0147887  | 6.69  | 0.000 | .0700063             | .127977   |
| /cut1       | 2.650637    | .8957245  |       |       | .8950495             | 4.406225  |
| /cut2       | 3.941018    | .8979167  |       |       | 2.181134             | 5.700903  |
| /cut3       | 5.085205    | .9056582  |       |       | 3.310148             | 6.860263  |
| /cut4       | 5.875534    | .9120933  |       |       | 4.087864             | 7.663204  |
| /cut5       | 6.468723    | .918117   |       |       | 4.669247             | 8.268199  |
| /cut6       | 6.922726    | .9215455  |       |       | 5.11653              | 8.728922  |
| /cut7       | 7.34471     | .9237628  |       |       | 5.534168             | 9.155252  |
| /cut8       | 7.963441    | .9338881  |       |       | 6.133054             | 9.793828  |

We can directly compare the log likelihoods for the `intreg` and `oprobit` models because both likelihoods are discrete. If we had point data in our `intreg` estimation, the likelihood would be a mixture of discrete and continuous terms, and we could not compare it directly with the `oprobit` likelihood.

Here the oprobit log likelihood is significantly larger (that is, less negative), so it fits better than the intreg model. The intreg model assumes normality, but the distribution of wages is skewed and definitely nonnormal. Normality is more closely approximated if we model the log of wages.

```
. generate logwage1 = log(wage1)
(14 missing values generated)
. generate logwage2 = log(wage2)
(6 missing values generated)
. intreg logwage1 logwage2 age c.age#c.age i.nev_mar i.rural school tenure

Fitting constant-only model:
Iteration 0: Log likelihood = -889.23647
Iteration 1: Log likelihood = -889.06346
Iteration 2: Log likelihood = -889.06346

Fitting full model:
Iteration 0: Log likelihood = -773.81968
Iteration 1: Log likelihood = -773.36566
Iteration 2: Log likelihood = -773.36563

Interval regression
 Number of obs = 488
 Uncensored = 0
 Left-censored = 14
 Right-censored = 6
 Interval-cens. = 468
 LR chi2(6) = 231.40
 Prob > chi2 = 0.0000

Log likelihood = -773.36563
```

|             | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |           |
|-------------|-------------|-----------|--------|-------|----------------------|-----------|
| age         | .0645589    | .0249954  | 2.58   | 0.010 | .0155689             | .1135489  |
| c.age#c.age | -.0010812   | .0004115  | -2.63  | 0.009 | -.0018878            | -.0002746 |
| 1.nev_mar   | -.0058151   | .0454867  | -0.13  | 0.898 | -.0949674            | .0833371  |
| 1.rural     | -.2098361   | .0439454  | -4.77  | 0.000 | -.2959675            | -.1237047 |
| school      | .0804832    | .0076783  | 10.48  | 0.000 | .0654341             | .0955323  |
| tenure      | .0397144    | .0058001  | 6.85   | 0.000 | .0283464             | .0510825  |
| _cons       | .7084023    | .3593193  | 1.97   | 0.049 | .0041495             | 1.412655  |
| /lnsigma    | -.906989    | .0356265  | -25.46 | 0.000 | -.9768157            | -.8371623 |
| sigma       | .4037381    | .0143838  |        |       | .3765081             | .4329373  |

The log likelihood of this intreg model is close to the oprobit log likelihood, and the  $z$  statistics for both models are similar.

□

## Stored results

intreg stores the following in `e()`:

### Scalars

|                       |                                       |
|-----------------------|---------------------------------------|
| <code>e(N)</code>     | number of observations                |
| <code>e(N_unc)</code> | number of uncensored observations     |
| <code>e(N_lrc)</code> | number of left-censored observations  |
| <code>e(N_rc)</code>  | number of right-censored observations |
| <code>e(N_int)</code> | number of interval observations       |
| <code>e(k)</code>     | number of parameters                  |

|                              |                                                                          |
|------------------------------|--------------------------------------------------------------------------|
| <code>e(k_aux)</code>        | number of auxiliary parameters                                           |
| <code>e(k_eq)</code>         | number of equations in <code>e(b)</code>                                 |
| <code>e(k_eq_model)</code>   | number of equations in overall model test                                |
| <code>e(k_dv)</code>         | number of dependent variables                                            |
| <code>e(df_m)</code>         | model degrees of freedom                                                 |
| <code>e(ll)</code>           | log likelihood                                                           |
| <code>e(ll_0)</code>         | log likelihood, constant-only model                                      |
| <code>e(N_clust)</code>      | number of clusters                                                       |
| <code>e(chi2)</code>         | $\chi^2$                                                                 |
| <code>e(p)</code>            | $p$ -value for model $\chi^2$ test                                       |
| <code>e(sigma)</code>        | sigma                                                                    |
| <code>e(se_sigma)</code>     | standard error of sigma                                                  |
| <code>e(rank)</code>         | rank of <code>e(V)</code>                                                |
| <code>e(rank0)</code>        | rank of <code>e(V)</code> for constant-only model                        |
| <code>e(ic)</code>           | number of iterations                                                     |
| <code>e(rc)</code>           | return code                                                              |
| <code>e(converged)</code>    | 1 if converged, 0 otherwise                                              |
| Macros                       |                                                                          |
| <code>e(cmd)</code>          | <code>intreg</code>                                                      |
| <code>e(cmdline)</code>      | command as typed                                                         |
| <code>e(depvar)</code>       | names of dependent variables                                             |
| <code>e(wtype)</code>        | weight type                                                              |
| <code>e(wexp)</code>         | weight expression                                                        |
| <code>e(title)</code>        | title in estimation output                                               |
| <code>e(clustvar)</code>     | name of cluster variable                                                 |
| <code>e(offset)</code>       | linear offset variable                                                   |
| <code>e(chi2type)</code>     | Wald or LR; type of model $\chi^2$ test                                  |
| <code>e(vce)</code>          | <i>vctype</i> specified in <code>vce()</code>                            |
| <code>e(vcetype)</code>      | title used to label Std. err.                                            |
| <code>e(het)</code>          | heteroskedasticity, if <code>het()</code> specified                      |
| <code>e(ml_score)</code>     | program used to implement <code>scores</code>                            |
| <code>e(opt)</code>          | type of optimization                                                     |
| <code>e(which)</code>        | max or min; whether optimizer is to perform maximization or minimization |
| <code>e(ml_method)</code>    | type of ml method                                                        |
| <code>e(user)</code>         | name of likelihood-evaluator program                                     |
| <code>e(technique)</code>    | maximization technique                                                   |
| <code>e(properties)</code>   | <code>b V</code>                                                         |
| <code>e(predict)</code>      | program used to implement <code>predict</code>                           |
| <code>e(marginsok)</code>    | predictions allowed by <code>margins</code>                              |
| <code>e(asbalanced)</code>   | factor variables <code>fvset</code> as <code>asbalanced</code>           |
| <code>e(asobserved)</code>   | factor variables <code>fvset</code> as <code>asobserved</code>           |
| Matrices                     |                                                                          |
| <code>e(b)</code>            | coefficient vector                                                       |
| <code>e(Cns)</code>          | constraints matrix                                                       |
| <code>e(iolog)</code>        | iteration log (up to 20 iterations)                                      |
| <code>e(gradient)</code>     | gradient vector                                                          |
| <code>e(V)</code>            | variance–covariance matrix of the estimators                             |
| <code>e(V_modelbased)</code> | model-based variance                                                     |
| Functions                    |                                                                          |
| <code>e(sample)</code>       | marks estimation sample                                                  |

In addition to the above, the following is stored in `r()`:

|                       |                                                                                                                       |
|-----------------------|-----------------------------------------------------------------------------------------------------------------------|
| Matrices              |                                                                                                                       |
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, $p$ -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

The regression equation of interest is

$$y_j = \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j$$

where  $y_j$  is a continuous outcome for the  $j$ th observation—either observed or unobserved—with covariates  $\mathbf{x}_j$  and corresponding coefficients  $\boldsymbol{\beta}$ . The model assumes that the error term is normally distributed;  $\epsilon \sim N(0, \sigma^2)$ .

For observations  $j \in \mathcal{C}$ , we observe  $y_j$ , that is, point data. Observations  $j \in \mathcal{I}$  are intervals; we know only that the unobserved  $y_j$  is in the interval  $[y_{1j}, y_{2j}]$ . For these observations, the likelihood contribution is  $\Pr(y_{1j} \leq Y_j \leq y_{2j})$ , where  $Y_j$  denotes the random variable representing the dependent variable in the model. Observations  $j \in \mathcal{L}$  are left-censored; we know only that the unobserved  $y_j$  is less than or equal to  $y_{\mathcal{L}j}$ , a censoring value that we do know. Similarly, observations  $j \in \mathcal{R}$  are right-censored; we know only that the unobserved  $y_j$  is greater than or equal to  $y_{\mathcal{R}j}$ . The likelihoods for these censored observations contain terms of the form  $\Pr(Y_j \leq y_{\mathcal{L}j})$  for left-censored data and  $\Pr(Y_j \geq y_{\mathcal{R}j})$  for right-censored data.

The log likelihood is

$$\begin{aligned} \ln L = & -\frac{1}{2} \sum_{j \in \mathcal{C}} w_j \left\{ \left( \frac{y_j - \mathbf{x}_j\boldsymbol{\beta}}{\sigma} \right)^2 + \log 2\pi\sigma^2 \right\} \\ & + \sum_{j \in \mathcal{L}} w_j \log \Phi \left( \frac{y_{\mathcal{L}j} - \mathbf{x}_j\boldsymbol{\beta}}{\sigma} \right) \\ & + \sum_{j \in \mathcal{R}} w_j \log \left\{ 1 - \Phi \left( \frac{y_{\mathcal{R}j} - \mathbf{x}_j\boldsymbol{\beta}}{\sigma} \right) \right\} \\ & + \sum_{j \in \mathcal{I}} w_j \log \left\{ \Phi \left( \frac{y_{2j} - \mathbf{x}_j\boldsymbol{\beta}}{\sigma} \right) - \Phi \left( \frac{y_{1j} - \mathbf{x}_j\boldsymbol{\beta}}{\sigma} \right) \right\} \end{aligned}$$

where  $\Phi(\cdot)$  is the cumulative standard normal distribution and  $w_j$  is the weight for the  $j$ th observation. If no weights are specified,  $w_j = 1$ . If `awweights` are specified,  $w_j = 1$ , and  $\sigma$  is replaced by  $\sigma/\sqrt{a_j}$  in the above, where  $a_j$  are the `awweights` normalized to sum to  $N$ .

When the `het()` option is specified,  $\sigma$  is modeled as  $\ln(\sigma) = z'_j\boldsymbol{\gamma}$ , where  $z$  represents the variables in `het()` and  $\boldsymbol{\gamma}$  is a vector of the estimated parameters to model the variance.

Note that the likelihood for `intreg` subsumes that of the `tobit` models; see [\[R\] tobit](#).

Maximization is as described in [\[R\] Maximize](#). `intreg` stores the estimated  $\sigma$  in `e(b)` in the log metric; therefore, if you want to provide an initial value for  $\sigma$  or to specify a constraint on it, ensure you do so on the log scale.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [\[P\] \\_robust](#), particularly *Maximum likelihood estimators* and *Methods and formulas*.

`intreg` also supports estimation with survey data. For details on VCEs with survey data, see [\[SVY\] Variance estimation](#).

## References

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## Also see

- [R] [intreg postestimation](#) — Postestimation tools for intreg
- [R] [regress](#) — Linear regression
- [R] [tobit](#) — Tobit regression
- [BAYES] [bayes: intreg](#) — Bayesian interval regression
- [ERM] [eintreg](#) — Extended interval regression
- [FMM] [fmm: intreg](#) — Finite mixtures of interval regression models
- [ME] [meintreg](#) — Multilevel mixed-effects interval regression
- [ST] [stintreg](#) — Parametric models for interval-censored survival-time data
- [SVY] [svy estimation](#) — Estimation commands for survey data
- [XT] [xtintreg](#) — Random-effects interval-data regression model
- [XT] [xttobit](#) — Random-effects tobit model
- [U] [20 Estimation and postestimation commands](#)

## Postestimation commands

The following postestimation commands are available after `intreg`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | linear, censored, and truncated predictions                                                                                             |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`hausman` and `lrtest` are not appropriate with `svy` estimation results.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as linear predictions, standard errors, probabilities, and expected values.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic nooffset]
```

```
predict [type] stub* [if] [in] , scores
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

Main

|                                       |                                             |
|---------------------------------------|---------------------------------------------|
| <code>xb</code>                       | linear prediction; the default              |
| <code>stdp</code>                     | standard error of the prediction            |
| <code>stdf</code>                     | standard error of the forecast              |
| <code>pr(<i>a</i>,<i>b</i>)</code>    | $\Pr(a < y_j < b)$                          |
| <code>e(<i>a</i>,<i>b</i>)</code>     | $E(y_j   a < y_j < b)$                      |
| <code>ystar(<i>a</i>,<i>b</i>)</code> | $E(y_j^*), y_j^* = \max\{a, \min(y_j, b)\}$ |

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

`stdf` is not allowed with `svy` estimation results.

where *a* and *b* may be numbers or variables; *a* missing (*a* ≥ .) means  $-\infty$ , and *b* missing (*b* ≥ .) means  $+\infty$ ; see [U] 12.2.1 Missing values.

## Options for predict

Main

`xb`, the default, calculates the linear prediction.

`stdp` calculates the standard error of the prediction, which can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. The standard error of the prediction is also referred to as the standard error of the fitted value.

`stdf` calculates the standard error of the forecast, which is the standard error of the point prediction for 1 observation. It is commonly referred to as the standard error of the future or forecast value. By construction, the standard errors produced by `stdf` are always larger than those produced by `stdp`; see *Methods and formulas* in [R] [regress postestimation](#).

`pr(a,b)` calculates  $\Pr(a < \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j < b)$ , the probability that  $y_j|\mathbf{x}_j$  would be observed in the interval  $(a,b)$ .

$a$  and  $b$  may be specified as numbers or variable names;  $lb$  and  $ub$  are variable names;

`pr(20,30)` calculates  $\Pr(20 < \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j < 30)$ ;

`pr(lb,ub)` calculates  $\Pr(lb < \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j < ub)$ ; and

`pr(20,ub)` calculates  $\Pr(20 < \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j < ub)$ .

$a$  missing ( $a \geq .$ ) means  $-\infty$ ; `pr(.,30)` calculates  $\Pr(-\infty < \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j < 30)$ ;

`pr(lb,30)` calculates  $\Pr(-\infty < \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j < 30)$  in observations for which  $lb \geq .$

and calculates  $\Pr(lb < \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j < 30)$  elsewhere.

$b$  missing ( $b \geq .$ ) means  $+\infty$ ; `pr(20,.)` calculates  $\Pr(+\infty > \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j > 20)$ ;

`pr(20,ub)` calculates  $\Pr(+\infty > \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j > 20)$  in observations for which  $ub \geq .$

and calculates  $\Pr(20 < \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j < ub)$  elsewhere.

`e(a,b)` calculates  $E(\mathbf{x}_j\boldsymbol{\beta} + \epsilon_j | a < \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j < b)$ , the expected value of  $y_j|\mathbf{x}_j$  conditional on  $y_j|\mathbf{x}_j$  being in the interval  $(a,b)$ , meaning that  $y_j|\mathbf{x}_j$  is truncated.

$a$  and  $b$  are specified as they are for `pr()`.

`ystar(a,b)` calculates  $E(y_j^*)$ , where  $y_j^* = a$  if  $\mathbf{x}_j\boldsymbol{\beta} + \epsilon_j \leq a$ ,  $y_j^* = b$  if  $\mathbf{x}_j\boldsymbol{\beta} + \epsilon_j \geq b$ , and  $y_j^* = \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j$  otherwise, meaning that  $y_j^*$  is censored.  $a$  and  $b$  are specified as they are for `pr()`.

`nooffset` is relevant only if you specified `offset(varname)`. It modifies the calculations made by `predict` so that they ignore the offset variable; the linear prediction is treated as  $\mathbf{x}_j\boldsymbol{\beta}$  rather than as  $\mathbf{x}_j\boldsymbol{\beta} + \text{offset}_j$ .

`scores` calculates equation-level score variables.

The first new variable will contain  $\partial \ln L / \partial (\mathbf{x}_j\boldsymbol{\beta})$ .

The second new variable will contain  $\partial \ln L / \partial \ln \sigma$ .

# margins

## Description for margins

`margins` estimates margins of response for linear predictions, probabilities, and expected values.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i>                      | Description                                 |
|---------------------------------------|---------------------------------------------|
| <code>xb</code>                       | linear prediction; the default              |
| <code>pr(<i>a</i>,<i>b</i>)</code>    | $\Pr(a < y_j < b)$                          |
| <code>e(<i>a</i>,<i>b</i>)</code>     | $E(y_j   a < y_j < b)$                      |
| <code>ystar(<i>a</i>,<i>b</i>)</code> | $E(y_j^*), y_j^* = \max\{a, \min(y_j, b)\}$ |
| <code>stdp</code>                     | not allowed with <code>margins</code>       |
| <code>stdf</code>                     | not allowed with <code>margins</code>       |

Statistics not allowed with `margins` are functions of stochastic quantities other than `e(b)`.

For the full syntax, see [\[R\] margins](#).

## Remarks and examples

### ▷ Example 1: Marginal predictions

Continuing with [example 1](#) of [R] [intreg](#), we compute women's expected wages conditional on a woman's wage being higher than \$5,000. To do this, we can use the  $e(a, b)$  option.

```
. use https://www.stata-press.com/data/r19/womenwage2
(Wages of women, fictional data)
. intreg wage1 wage2 age c.age#c.age i.nev_mar i.rural school tenure
(output omitted)
. predict w1, e(5,.)
. summarize w1
```

| Variable | Obs | Mean     | Std. dev. | Min      | Max      |
|----------|-----|----------|-----------|----------|----------|
| w1       | 488 | 18.02362 | 4.583738  | 8.717687 | 35.31161 |

The predicted wages range from \$8,718 to \$35,312.

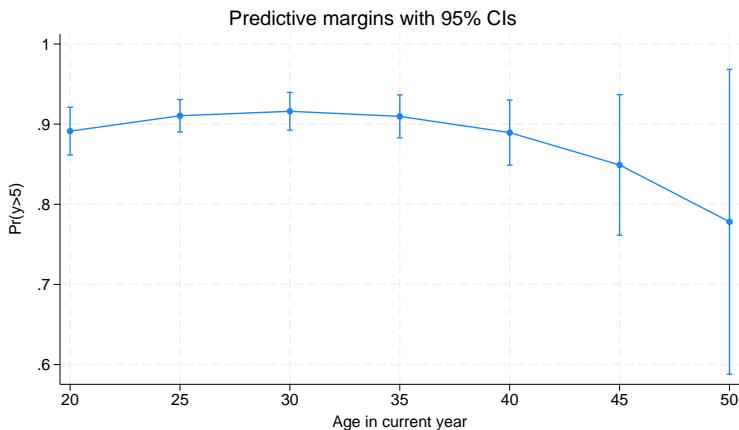
We can also examine whether the probability of earning more than \$5,000 varies with age. We can use `margins` to compute the marginal means of the predicted probabilities at different ages.

```
. margins, predict(pr(5,)) at(age=(20(5)50))
Predictive margins Number of obs = 488
Model VCE: OIM
Expression: Pr(y>5), predict(pr(5,))
1._at: age = 20
2._at: age = 25
3._at: age = 30
4._at: age = 35
5._at: age = 40
6._at: age = 45
7._at: age = 50
```

| _at | Delta-method |           |       |       |                      |
|-----|--------------|-----------|-------|-------|----------------------|
|     | Margin       | std. err. | z     | P> z  | [95% conf. interval] |
| 1   | .8912598     | .0151773  | 58.72 | 0.000 | .8615127 .9210068    |
| 2   | .9104568     | .0103467  | 87.99 | 0.000 | .8901775 .930736     |
| 3   | .9160005     | .0120025  | 76.32 | 0.000 | .892476 .9395251     |
| 4   | .9096667     | .0136693  | 66.55 | 0.000 | .8828753 .9364581    |
| 5   | .8894289     | .0206992  | 42.97 | 0.000 | .8488593 .9299985    |
| 6   | .8491103     | .0447429  | 18.98 | 0.000 | .7614159 .9368048    |
| 7   | .7781644     | .0970557  | 8.02  | 0.000 | .5879387 .9683902    |

We can visualize these results by using marginsplot:

```
. marginsplot
Variables that uniquely identify margins: age
```



The probability increases until age 30 and decreases thereafter.

◀

## Also see

[R] [intreg](#) — Interval regression

[U] [20 Estimation and postestimation commands](#)

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

## Description

`ivfprobit` fits a model for a fractional dependent variable, such as a rate or proportion, where one or more of the covariates are endogenous. The dependent variable must be greater than or equal to 0 and less than or equal to 1. `ivfprobit` assumes all endogenous covariates are continuous.

## Quick start

Fractional probit regression of  $y_1$  on  $x$  and endogenous regressor  $y_2$  that is instrumented using  $z$

```
ivfprobit y1 x (y2 = z)
```

Same as above, but with endogenous regressors  $y_2$  and  $y_3$ , using  $z_1$ ,  $z_2$ , and  $z_3$  as instruments

```
ivfprobit y1 x (y2 y3 = z1 z2 z3)
```

## Menu

Statistics > Endogenous covariates > Fractional probit with endogenous covariates

## Syntax

```
ivfprobit depvar [varlist1] (varlist2 = varlistiv) [if] [in] [weight] [, options]
```

*varlist*<sub>1</sub> is the list of exogenous variables.

*varlist*<sub>2</sub> is the list of endogenous variables.

*varlist*<sub>iv</sub> is the list of exogenous variables used with *varlist*<sub>1</sub> as instruments for *varlist*<sub>2</sub>.

| <i>options</i>                           | Description                                                                                                                                      |
|------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| Model                                    |                                                                                                                                                  |
| <code>noconstant</code>                  | suppress constant term                                                                                                                           |
| <code>constraints(<i>numlist</i>)</code> | apply specified linear constraints                                                                                                               |
| SE/Robust                                |                                                                                                                                                  |
| <code>vce(<i>vcetype</i>)</code>         | <i>vcetype</i> may be <code>robust</code> , <code>cluster <i>clustvar</i></code> , <code>bootstrap</code> , or <code>jackknife</code>            |
| Reporting                                |                                                                                                                                                  |
| <code>level(#)</code>                    | set confidence level; default is level(95)                                                                                                       |
| <code>first</code>                       | report first-stage regression                                                                                                                    |
| <code>nocnsreport</code>                 | do not display constraints                                                                                                                       |
| <code>display_options</code>             | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| Maximization                             |                                                                                                                                                  |
| <code>maximize_options</code>            | control the maximization process                                                                                                                 |
| <code>coeflegend</code>                  | display legend instead of statistics                                                                                                             |

*varlist*<sub>1</sub> and *varlist*<sub>iv</sub> may contain factor variables; see [U] 11.4.3 Factor variables.

*depvar*, *varlist*<sub>1</sub>, *varlist*<sub>2</sub>, and *varlist*<sub>iv</sub> may contain time-series operators; see [U] 11.4.4 Time-series varlists.

`bootstrap`, `by`, `collect`, `fp`, `jackknife`, `rolling`, `statsby`, and `svy` are allowed; see [U] 11.1.10 Prefix commands.

Weights are not allowed with the `bootstrap` prefix; see [R] `bootstrap`.

`vce()` and weights are not allowed with the `svy` prefix; see [SVY] `svy`.

`fweights`, `iweights`, and `pweights` are allowed. See [U] 11.1.6 `weight`.

`coeflegend` does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Options

### Model

`noconstant`, `constraints(numlist)`; see [R] [Estimation options](#).

### SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

### Reporting

`level(#)`; see [R] [Estimation options](#).

`first` requests that the parameters for the reduced-form equations showing the relationships between the endogenous variables and instruments be displayed. The default is not to show these parameter estimates.

`nocnsreport`; see [R] [Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

### Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#).

The following option is available with `ivfprobit` but is not shown in the dialog box:

`coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Remarks are presented under the following headings:

*Model setup*

*Model identification*

## Model setup

`ivfprobit` fits models for fractional dependent variables when one or more of the covariates is endogenous. Fractional variables can take any value in the interval  $[0, 1]$ ; thus, `ivfprobit` is useful for modeling outcomes such as rates and proportions.

Formally, we can write the model fit by `ivfprobit` as

$$\begin{aligned} E(y_{1i} | \mathbf{x}_{1i}, \mathbf{x}_{2i}, \mathbf{y}_{2i}, u_i) &= \Phi(\mathbf{y}_{2i}\boldsymbol{\beta} + \mathbf{x}_{1i}\boldsymbol{\gamma} + u_i) \\ \mathbf{y}_{2i} &= \mathbf{x}_{i1}\boldsymbol{\Pi}_1 + \mathbf{x}_{i2}\boldsymbol{\Pi}_2 + v_i \end{aligned} \quad (1)$$

where subscript  $i$  denotes the observation,  $\mathbf{y}_{2i}$  is a  $1 \times p$  vector of continuous endogenous variables,  $\mathbf{x}_{1i}$  is a  $1 \times k_1$  vector of exogenous covariates, and  $\mathbf{x}_{2i}$  is a  $1 \times k_2$  vector of excluded instruments. Endogeneity arises from the possible correlation between  $u_i$  and  $v_i$ . The coefficients in vectors  $\boldsymbol{\beta}$  and  $\boldsymbol{\gamma}$  are the parameters of interest. Matrices  $\boldsymbol{\Pi}_1$  and  $\boldsymbol{\Pi}_2$  contain the coefficients of the first stage for the reduced-form equation.

To obtain parameter estimates, `ivfprobit` maximizes the same likelihood as `ivprobit` but does not require a binary dependent variable and does not require the joint density of the errors in the model to be specified correctly; see [R] `ivprobit` for more information. `ivfprobit` fits the model via quasilielihood estimation rather than maximum likelihood estimation. The key insight behind quasilielihood estimation is that we do not need to know the true distribution of the entire model to obtain consistent parameter estimates. In fact, the only requirement is the correct specification of the conditional mean given in (1) after integrating over  $u_i$ . Specifying the full distribution of the model correctly is required only if we want to obtain asymptotically efficient standard errors from maximum likelihood estimation.

`ivfprobit` does not assume that the true model is a probit model that accounts for endogeneity, such as the one fit by `ivprobit`. Therefore, the standard errors provided by maximum likelihood estimation are not appropriate. Instead, `ivfprobit` takes the maximum quasilielihood approach and reports robust standard errors by default.

For further discussion on quasilielihood estimation in the context of fractional regression, see [Papke and Wooldridge \(1996\)](#) and [Wooldridge \(2010\)](#).

### ▷ Example 1

We use a corporate 401(k) participation dataset and fit a fractional probit regression of the 401(k) participation rate (`prate`), on corporate employment size (`ltotemp`) and its square, an indicator of whether the 401(k) is the sole pension plan (`sole`), and plan matching rate (`mrate`). The plan matching rate is endogenous and is instrumented using the age of the plan (`age`) and its square.

Our outcome variable `prate` has values between 0 and 1, including 1,351 firms with participation rates of 1. We assume that the functional form of the expected participation rate, after integrating over  $u_i$ , is a cumulative normal density as in (1).

We use `ivfprobit` to fit the fractional probit model, accounting for endogeneity of `mrate`.

```
. use https://www.stata-press.com/data/r19/401k
(Firm-level data on 401k participation)
. ivfprobit prate c.ltotemp#c.ltotemp 1.sole (mrate = c.age#c.age)

Fitting exogenous fractional probit model:
Iteration 0: Log pseudolikelihood = -1769.7046
Iteration 1: Log pseudolikelihood = -1675.4223
Iteration 2: Log pseudolikelihood = -1674.7663
Iteration 3: Log pseudolikelihood = -1674.7661
Iteration 4: Log pseudolikelihood = -1674.7661

Fitting full model:
Iteration 0: Log pseudolikelihood = -3712.498
Iteration 1: Log pseudolikelihood = -3712.4767
Iteration 2: Log pseudolikelihood = -3712.4767

Fractional probit model with endogenous regressors

Log pseudolikelihood = -3712.4767
Number of obs = 4,075
Wald chi2(4) = 907.06
Prob > chi2 = 0.0000
```

|                           | Coefficient | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |           |
|---------------------------|-------------|---------------------|-------|-------|----------------------|-----------|
| mrate                     | 1.907922    | .0946094            | 20.17 | 0.000 | 1.722491             | 2.093353  |
| ltotemp                   | -.4229273   | .0744177            | -5.68 | 0.000 | -.5687833            | -.2770713 |
| c.ltotemp#                |             |                     |       |       |                      |           |
| c.ltotemp                 | .0217492    | .0046476            | 4.68  | 0.000 | .01264               | .0308583  |
| sole                      |             |                     |       |       |                      |           |
| Only plan                 | -.1733119   | .0366136            | -4.73 | 0.000 | -.2450733            | -.1015504 |
| _cons                     | 1.904103    | .3199032            | 5.95  | 0.000 | 1.277104             | 2.531102  |
| corr(e.mrate,<br>e.prate) | -.5690386   | .0431738            |       |       | -.6476498            | -.4784406 |
| sd(e.mrate)               | .3989664    | .0061807            |       |       | .3870345             | .4112661  |

```
Wald test of exogeneity: chi2(1) = 102.40
Prob > chi2 = 0.0000
Endogenous: mrate
Exogenous: ltotemp c.ltotemp#c.ltotemp 1.sole age c.age#c.age
```

We find a positive effect of `mrate` on the participation rate. Additionally, we see that the correlation between the unobservables, `corr(e.mrate, e.prate)`, is different from zero. This means there is evidence to support our endogeneity conjecture.

◀

## Model identification

As in the basic linear instrumental-variables model, the order condition for identification requires that the number of excluded exogenous variables (that is, the additional instruments) be at least as great as the number of included endogenous variables ( $k_2 \geq p$ ). `ivfprobit` checks this for you and issues an error message if the order condition is not met.

`ivfprobit`, like `probit` and `ivprobit`, checks the exogenous and endogenous variables to see if any of them predict the outcome variable perfectly. It will drop any offending variables and observations and then fit the model on the remaining data. Instruments that are perfect predictors do not affect estimation, so they are not checked. See *Model identification* in [R] `probit` for more information.

## Stored results

`ivfprobit` stores the following in `e()`:

### Scalars

|                            |                                           |
|----------------------------|-------------------------------------------|
| <code>e(N)</code>          | number of observations                    |
| <code>e(k)</code>          | number of parameters                      |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>  |
| <code>e(k_eq_model)</code> | number of equations in overall model test |
| <code>e(k_dv)</code>       | number of dependent variables             |
| <code>e(df_m)</code>       | model degrees of freedom                  |
| <code>e(ll)</code>         | log likelihood                            |
| <code>e(N_clust)</code>    | number of clusters                        |
| <code>e(endog_ct)</code>   | number of endogenous covariates           |
| <code>e(p)</code>          | model Wald $p$ -value                     |
| <code>e(p_exog)</code>     | exogeneity test Wald $p$ -value           |
| <code>e(chi2)</code>       | model Wald $\chi^2$                       |
| <code>e(chi2_exog)</code>  | Wald $\chi^2$ test of exogeneity          |
| <code>e(rank)</code>       | rank of <code>e(V)</code>                 |
| <code>e(ic)</code>         | number of iterations                      |
| <code>e(rc)</code>         | return code                               |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise               |

### Macros

|                            |                                                                          |
|----------------------------|--------------------------------------------------------------------------|
| <code>e(cmd)</code>        | <code>ivfprobit</code>                                                   |
| <code>e(cmdline)</code>    | command as typed                                                         |
| <code>e(depvar)</code>     | name of dependent variable                                               |
| <code>e(endog)</code>      | names of endogenous variables                                            |
| <code>e(exog)</code>       | names of exogenous variables                                             |
| <code>e(wtype)</code>      | weight type                                                              |
| <code>e(wexp)</code>       | weight expression                                                        |
| <code>e(title)</code>      | title in estimation output                                               |
| <code>e(clustvar)</code>   | name of cluster variable                                                 |
| <code>e(chi2type)</code>   | Wald; type of model $\chi^2$ test                                        |
| <code>e(vce)</code>        | <i>vcetype</i> specified in <code>vce()</code>                           |
| <code>e(vcetype)</code>    | title used to label Std. err.                                            |
| <code>e(opt)</code>        | type of optimization                                                     |
| <code>e(which)</code>      | max or min; whether optimizer is to perform maximization or minimization |
| <code>e(ml_method)</code>  | type of ml method                                                        |
| <code>e(user)</code>       | name of likelihood-evaluator program                                     |
| <code>e(technique)</code>  | maximization technique                                                   |
| <code>e(properties)</code> | <code>b V</code>                                                         |
| <code>e(estat_cmd)</code>  | program used to implement <code>estat</code>                             |
| <code>e(predict)</code>    | program used to implement <code>predict</code>                           |
| <code>e(footnote)</code>   | program used to implement the footnote display                           |
| <code>e(marginsok)</code>  | predictions allowed by margins                                           |
| <code>e(asbalanced)</code> | factor variables <code>fvset</code> as <code>asbalanced</code>           |
| <code>e(asobserved)</code> | factor variables <code>fvset</code> as <code>asobserved</code>           |

### Matrices

|                              |                                              |
|------------------------------|----------------------------------------------|
| <code>e(b)</code>            | coefficient vector                           |
| <code>e(Cns)</code>          | constraints matrix                           |
| <code>e(ilog)</code>         | iteration log (up to 20 iterations)          |
| <code>e(gradient)</code>     | gradient vector                              |
| <code>e(Sigma)</code>        | $\Sigma$                                     |
| <code>e(V)</code>            | variance-covariance matrix of the estimators |
| <code>e(V_modelbased)</code> | model-based variance                         |

### Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

Matrices

`r(table)`

matrix containing the coefficients with their standard errors, test statistics,  $p$ -values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

See *Methods and formulas* in [R] [ivprobit](#).

## References

Papke, L. E., and J. M. Wooldridge. 1996. Econometric methods for fractional response variables with an application to 401(k) plan participation rates. *Journal of Applied Econometrics* 11: 619–632. [https://doi.org/10.1002/\(SICI\)1099-1255\(199611\)11:6<619::AID-JAE418>3.0.CO;2-1](https://doi.org/10.1002/(SICI)1099-1255(199611)11:6<619::AID-JAE418>3.0.CO;2-1).

Wooldridge, J. M. 2010. *Econometric Analysis of Cross Section and Panel Data*. 2nd ed. Cambridge, MA: MIT Press.

## Also see

[R] [ivfprobit postestimation](#) — Postestimation tools for `ivfprobit`

[R] [fracreg](#) — Fractional response regression

[R] [gmm](#) — Generalized method of moments estimation

[R] [ivprobit](#) — Probit model with continuous endogenous covariates

[R] [ivregress](#) — Single-equation instrumental-variables regression

[R] [ivtobit](#) — Tobit model with continuous endogenous covariates

[R] [probit](#) — Probit regression

[ERM] [eprobit](#) — Extended probit regression

[SVY] [svy estimation](#) — Estimation commands for survey data

[XT] [xtprobit](#) — Random-effects and population-averaged probit models

[U] [20 Estimation and postestimation commands](#)

|                         |                      |          |       |                      |
|-------------------------|----------------------|----------|-------|----------------------|
| Postestimation commands | predict              | margins  | estat | Remarks and examples |
| Stored results          | Methods and formulas | Also see |       |                      |

## Postestimation commands

The following postestimation commands are of special interest after `ivfprobit`:

| Command                        | Description                                                                                        |
|--------------------------------|----------------------------------------------------------------------------------------------------|
| <code>estat correlation</code> | report the correlation matrix of the errors of the dependent variable and the endogenous variables |
| <code>estat covariance</code>  | report the covariance matrix of the errors of the dependent variable and the endogenous variables  |

These commands are not appropriate after the `svy` prefix.

The following standard postestimation commands are also available:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | conditional means, linear predictions, etc.                                                                                             |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\* `forecast` is not appropriate with `svy` estimation results.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as conditional means, linear predictions, and standard errors.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic asfmethod]
```

```
predict [type] {stub* | newvarlist} [if] [in], scores
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

Main

|                   |                                         |
|-------------------|-----------------------------------------|
| <code>cm</code>   | conditional mean; the default           |
| <code>xb</code>   | linear prediction                       |
| <code>stdp</code> | standard error of the linear prediction |

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

| <i>asfmethod</i> | Description |
|------------------|-------------|
|------------------|-------------|

Main

|                       |                                          |
|-----------------------|------------------------------------------|
| <code>asf</code>      | average structural function; the default |
| <code>fixedasf</code> | fixed average structural function        |

## Options for predict

Main

`cm`, the default, calculates the conditional mean.

`xb` calculates the linear prediction.

`stdp` calculates the standard error of the linear prediction.

`asf` and `fixedasf` determine how the average structural function (ASF) of the conditional mean is computed. These options are not allowed with `xb` or `stdp`.

`asf` is the default estimator when the `cm` statistic is specified. `asf` computes the ASF of the conditional mean. It is the prediction conditional on the errors of the endogenous variable equations. Put another way, it is the conditional mean accounting for the correlation of the endogenous covariates with the errors of the main equation. Derivatives and contrasts based on `asf` have a structural interpretation. See [margins](#) below for computing derivatives and contrasts.

`fixedasf` calculates a fixed ASF. It is the prediction using only the coefficients and variables of the outcome equation. `fixedasf` does not condition on the errors of the endogenous variable equations. Contrasts between two fixed counterfactuals averaged over the whole sample have a potential-outcome interpretation. Intuitively, it is as if the values of the covariates were fixed at a value exogenously by fiat. See [margins](#) below for computing derivatives and contrasts.

To be clear, derivatives and contrasts between two fixed counterfactuals using the default `asf` option also have a potential-outcome interpretation. And, unlike `fixedasf`, they retain that interpretation when computed over subpopulations for both linear and nonlinear models.

`scores` calculates the equation-level score variables.

For models with one endogenous regressor, four new variables are created.

The first new variable will contain  $\partial \ln L / \partial (\mathbf{z}_i \boldsymbol{\delta})$ .

The second new variable will contain  $\partial \ln L / \partial (\mathbf{x}_i \boldsymbol{\Pi})$ .

The third new variable will contain  $\partial \ln L / \partial \operatorname{atanh} \rho$ .

The fourth new variable will contain  $\partial \ln L / \partial \ln \sigma$ .

For models with  $p$  endogenous regressors,  $p + \{(p + 1)(p + 2)\} / 2$  new variables are created.

The first new variable will contain  $\partial \ln L / \partial (\mathbf{z}_i \boldsymbol{\delta})$ .

The second through  $(p + 1)$ th new variables will contain  $\partial \ln L / \partial (\mathbf{x}_i \boldsymbol{\Pi}_k)$ ,  $k = 1, \dots, p$ , where  $\boldsymbol{\Pi}_k$  is the  $k$ th column of  $\boldsymbol{\Pi}$ .

The remaining score variables will contain the partial derivatives of  $\ln L$  with respect to  $s_{21}, s_{31}, \dots, s_{p+1,1}, s_{22}, \dots, s_{p+1,2}, \dots, s_{p+1,p+1}$ , where  $s_{m,n}$  denotes the  $(m, n)$  element of the Cholesky decomposition of the error covariance matrix.

# margins

## Description for margins

`margins` estimates margins of response for linear predictions and probabilities.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

---

Main

|                   |                                       |
|-------------------|---------------------------------------|
| <code>cm</code>   | conditional mean; the default         |
| <code>xb</code>   | linear prediction                     |
| <code>stdp</code> | not allowed with <code>margins</code> |

---

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [R] [margins](#).

## estat

### Description for estat

`estat correlation` displays the correlation matrix of the errors of the dependent variable and the endogenous variables.

`estat covariance` displays the covariance matrix of the errors of the dependent variable and the endogenous variables.

### Menu for estat

Statistics > Postestimation

### Syntax for estat

*Correlation matrix*

```
estat correlation [, border(bspec) left(#) format(%fmt)]
```

*Covariance matrix*

```
estat covariance [, border(bspec) left(#) format(%fmt)]
```

### Options for estat

#### Main

`border`(*bspec*) sets the border style of the matrix display. The default is `border(all)`.

`left`(#) sets the left indent of the matrix display. The default is `left(2)`.

`format`(%*fmt*) specifies the format for displaying the individual elements of the matrix. The default is `format(%9.0g)`.

## Remarks and examples

Remarks are presented under the following headings:

*Marginal effects*

*Obtaining predicted values*

### Marginal effects

#### ▷ Example 1

We can obtain marginal effects by using the `margins` command after `ivfprobit`. We will calculate average marginal effects by using the 401(k) participation model of [example 1](#) in [\[R\] ivfprobit](#).

```
. use https://www.stata-press.com/data/r19/401k
(Firm-level data on 401k participation)
. ivfprobit prate c.ltotemp##c.ltotemp i.sole (mrate = c.age##c.age)
(output omitted)
. margins, dydx(mrate) predict(cm)

Average marginal effects Number of obs = 4,075
Model VCE: Robust
Expression: Conditional mean of prate, predict(cm)
dy/dx wrt: mrate
```

|       | Delta-method |           | z     | P> z  | [95% conf. interval] |          |
|-------|--------------|-----------|-------|-------|----------------------|----------|
|       | dy/dx        | std. err. |       |       |                      |          |
| mrate | .5288314     | .0442438  | 11.95 | 0.000 | .4421152             | .6155477 |

The marginal effect from `ivfprobit` suggests that a \$1 increase in the matching rate (per dollar) given by employers can increase participation by approximately 50%.

◀

### Obtaining predicted values

After fitting your model with `ivfprobit`, you can obtain the conditional mean, or the linear prediction and its standard error for both the estimation sample and other samples, by using the `predict` command; see [\[U\] 20 Estimation and postestimation commands](#) and [\[R\] predict](#).

### Stored results

`estat correlation` stores the following results in `r()`:

Matrices

`r(corr)` correlation matrix of the errors

`estat covariance` stores the following results in `r()`:

Matrices

`r(cov)` covariance matrix of the errors

## Methods and formulas

Recall that the model fit by `ivfprobit` is given by two equations:

$$\begin{aligned} E[y_{1i} | \mathbf{x}_{1i}, \mathbf{x}_{2i}, y_{2i}, u_i] &= \Phi(\mathbf{y}_{2i}\boldsymbol{\beta} + \mathbf{x}_{1i}\boldsymbol{\gamma} + u_i) \\ \mathbf{y}_{2i} &= \mathbf{x}_{i1}\boldsymbol{\Pi}_1 + \mathbf{x}_{i2}\boldsymbol{\Pi}_2 + v_i \end{aligned}$$

The linear prediction for observation  $i$  is calculated as  $\mathbf{y}_{2i}\hat{\boldsymbol{\beta}} + \mathbf{x}_{1i}\hat{\boldsymbol{\gamma}}$ . The predicted conditional mean is given by  $\Phi(\hat{m}_i)$ , where  $\Phi(\cdot)$  is the standard normal distribution function and  $\hat{m}_i$  is the plugin estimator of

$$m_i = \frac{\mathbf{y}_{2i}\boldsymbol{\beta} + \mathbf{x}_{1i}\boldsymbol{\gamma} + \rho v_i / \sigma}{\sqrt{1 - \rho^2}}$$

where  $\sigma$  is the standard deviation of  $v_i$  and  $\rho$  is the correlation coefficient between  $u_i$  and  $v_i$ . The ASF uses  $\hat{m}_i$  instead of  $\mathbf{y}_{2i}\hat{\boldsymbol{\beta}} + \mathbf{x}_{1i}\hat{\boldsymbol{\gamma}}$  to evaluate  $\Phi(\cdot)$  and account for endogeneity in the model. The fixed ASF is evaluated at  $\mathbf{y}_{2i}\hat{\boldsymbol{\beta}} + \mathbf{x}_{1i}\hat{\boldsymbol{\gamma}}$ .

## Also see

[R] [ivprobit](#) — Fractional probit model with continuous endogenous covariates

[U] [20 Estimation and postestimation commands](#)

|                                      |                                |                                      |                            |                          |
|--------------------------------------|--------------------------------|--------------------------------------|----------------------------|--------------------------|
| <a href="#">Description</a>          | <a href="#">Quick start</a>    | <a href="#">Menu</a>                 | <a href="#">Syntax</a>     | <a href="#">Options</a>  |
| <a href="#">Remarks and examples</a> | <a href="#">Stored results</a> | <a href="#">Methods and formulas</a> | <a href="#">References</a> | <a href="#">Also see</a> |

## Description

`ivpoisson` estimates the parameters of a Poisson regression model in which some of the covariates are endogenous. The model is also known as an exponential conditional mean model in which some of the covariates are endogenous. The model may be specified using either additive or multiplicative error terms. The model is frequently used to model count outcomes and is also used to model nonnegative outcome variables.

## Quick start

Two-step GMM estimation of the Poisson regression of  $y_1$  on  $x$  and endogenous regressor  $y_2$  that is instrumented using  $z$

```
ivpoisson gmm y1 x (y2 = z)
```

Same as above, but specify multiplicative errors rather than additive

```
ivpoisson gmm y1 x (y2 = z), multiplicative
```

Use iterative GMM estimation

```
ivpoisson gmm y1 x (y2 = z), igmm
```

Specify a weight matrix that allows for correlation within clusters identified by `cvar`

```
ivpoisson gmm y1 x (y2 = z), wmatrix(cluster cvar)
```

Use the control-function estimator

```
ivpoisson cfunction y1 x (y2 = z)
```

## Menu

Statistics > Endogenous covariates > Poisson model with endogenous covariates

## Syntax

*Generalized method of moments estimator*

```
ivpoisson gmm depvar [varlist1] [(varlist2 = varlistiv)] [if] [in] [weight]
[, reg_err_opt options]
```

*Control-function estimator*

```
ivpoisson cfunction depvar [varlist1] (varlist2 = varlistiv) [if] [in] [weight]
[, options]
```

| <i>reg_err_opt</i>                                       | Description                                                                                                                                                                                 |
|----------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Model                                                    |                                                                                                                                                                                             |
| <u>additive</u>                                          | add regression errors to the conditional mean term; the default                                                                                                                             |
| <u>multiplicative</u>                                    | multiply regression errors by the conditional mean term                                                                                                                                     |
| <hr/>                                                    |                                                                                                                                                                                             |
| <i>options</i>                                           | Description                                                                                                                                                                                 |
| Model                                                    |                                                                                                                                                                                             |
| <u>noconstant</u>                                        | suppress constant term                                                                                                                                                                      |
| <u>exposure</u> ( <i>varname<sub>e</sub></i> )           | include $\ln(\text{varname}_e)$ in model with coefficient constrained to 1                                                                                                                  |
| <u>offset</u> ( <i>varname<sub>o</sub></i> )             | include <i>varname<sub>o</sub></i> in model with coefficient constrained to 1                                                                                                               |
| * <u>twostep</u>                                         | use two-step GMM estimator; the default for <code>ivpoisson gmm</code>                                                                                                                      |
| * <u>onestep</u>                                         | use one-step GMM estimator; the default for <code>ivpoisson cfunction</code>                                                                                                                |
| * <u>igmm</u>                                            | use iterative GMM estimator                                                                                                                                                                 |
| Weight matrix                                            |                                                                                                                                                                                             |
| <u>wmatrix</u> ( <i>wmtype</i> )                         | specify weight matrix; <i>wmtype</i> may be <u>robust</u> , <u>cluster</u> <i>clustvar</i> , or <u>unadjusted</u>                                                                           |
| <u>center</u>                                            | center moments in weight-matrix computation                                                                                                                                                 |
| <u>winitial</u> ( <i>iwtype</i> [, <u>independent</u> ]) | specify initial weight matrix; <i>iwtype</i> may be <u>unadjusted</u> , <u>identity</u> , or the name of a Stata matrix (independent may not be specified with <code>ivpoisson gmm</code> ) |
| SE/Robust                                                |                                                                                                                                                                                             |
| <u>vce</u> ( <i>vcetype</i> )                            | <i>vcetype</i> may be <u>robust</u> , <u>cluster</u> <i>clustvar</i> , <u>bootstrap</u> , <u>jackknife</u> , or <u>unadjusted</u>                                                           |
| Reporting                                                |                                                                                                                                                                                             |
| <u>level</u> (#)                                         | set confidence level; default is level(95)                                                                                                                                                  |
| <u>irr</u>                                               | report incidence-rate ratios                                                                                                                                                                |
| <i>display_options</i>                                   | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling                                            |
| Optimization                                             |                                                                                                                                                                                             |
| <u>from</u> ( <i>initial_values</i> )                    | specify initial values for parameters                                                                                                                                                       |
| † <u>igmmiterate</u> (#)                                 | specify maximum number of iterations for iterated GMM estimator                                                                                                                             |
| † <u>igmmeps</u> (#)                                     | specify # for iterated GMM parameter convergence criterion; default is <code>igmmeps(1e-6)</code>                                                                                           |
| † <u>igmmweps</u> (#)                                    | specify # for iterated GMM weight-matrix convergence criterion; default is <code>igmmweps(1e-6)</code>                                                                                      |
| <i>optimization_options</i>                              | control the optimization process; seldom used                                                                                                                                               |

\*You can specify at most one of these options.

†These options may be specified only when `igmm` is specified.

`varlist1` and `varlistv` may contain factor variables; see [U] 11.4.3 Factor variables.

`devar`, `varlist1`, `varlist2`, and `varlistv` may contain time-series operators; see [U] 11.4.4 Time-series varlists.

`bayesboot`, `bootstrap`, `by`, `collect`, `jackknife`, `rolling`, and `statsby` are allowed; see [U] 11.1.10 Prefix commands.

Weights are not allowed with the `bootstrap` prefix; see [R] `bootstrap`.

`aweights` are not allowed with the `jackknife` prefix; see [R] `jackknife`.

`aweights`, `fweights`, `iwweights`, and `pweights` are allowed; see [U] 11.1.6 weight.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Options

### Model

`noconstant`, `exposure(varnamee)`, `offset(varnameo)`; see [R] Estimation options.

`additive`, the default, specifies that the regression errors be added to the conditional mean term and have mean 0.

`multiplicative` specifies that the regression errors be multiplied by the conditional mean term and have mean 1.

`twostep`, `onestep`, and `igmm` specify which estimator is to be used.

`twostep` requests the two-step GMM estimator. `gmm` obtains parameter estimates based on the initial weight matrix, computes a new weight matrix based on those estimates, and then reestimates the parameters based on that weight matrix. `twostep` is the default for `ivpoisson gmm`.

`onestep` requests the one-step GMM estimator. The parameters are estimated based on an initial weight matrix, and no updating of the weight matrix is performed except when calculating the appropriate variance–covariance (VCE) matrix. `onestep` is the default for `ivpoisson cfunction`.

`igmm` requests the iterative GMM estimator. `gmm` obtains parameter estimates based on the initial weight matrix, computes a new weight matrix based on those estimates, reestimates the parameters based on that weight matrix, computes a new weight matrix, and so on, to convergence. Convergence is declared when the relative change in the parameter vector is less than `igmmeps()`, the relative change in the weight matrix is less than `igmmweps()`, or `igmmiterate()` iterations have been completed. Hall (2005, sec. 2.4 and 3.6) mentions that there may be gains to finite-sample efficiency from using the iterative estimator.

### Weight matrix

`wmatrix(wmtype)` specifies the type of weight matrix to be used in conjunction with the two-step and iterated GMM estimators.

Specifying `wmatrix(robust)` requests a weight matrix that is appropriate when the errors are independent but not necessarily identically distributed. `wmatrix(robust)` is the default.

Specifying `wmatrix(cluster clustvar)` requests a weight matrix that accounts for arbitrary correlation among observations within clusters identified by `clustvar`.

Specifying `wmatrix(unadjusted)` requests a weight matrix that is suitable when the errors are homoskedastic.

`wmatrix()` cannot be specified if `onestep` is also specified.

`center` requests that the sample moments be centered (demeaned) when computing GMM weight matrices. By default, centering is not done.

`winitial(wmtype[, independent])` specifies the weight matrix to use to obtain the first-step parameter estimates.

Specifying `winitial(unadjusted)` requests a weight matrix that assumes the error functions are independent and identically distributed. This matrix is of the form  $(\mathbf{Z}'\mathbf{Z})^{-1}$ , where  $\mathbf{Z}$  represents all the exogenous regressors and instrumental variables.

`winitial(identity)` requests that the identity matrix be used.

`winitial(matname)` requests that Stata matrix *matname* be used.

Including the `independent` suboption creates a weight matrix that assumes error functions are independent. Elements of the weight matrix corresponding to covariances between any two error functions are set equal to zero. This suboption only applies to `ivpoisson cfunction`.

`winitial(unadjusted)` is the default for `ivpoisson gmm`.

`winitial(unadjusted, independent)` is the default for `ivpoisson cfunction`.

---

#### SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

`vce(unadjusted)` specifies that an unadjusted (nonrobust) VCE matrix be used; this, along with the `twostep` option, results in the “optimal two-step GMM” estimates often discussed in textbooks. `vce(unadjusted)` may not be set in `ivpoisson cfunction`.

The default `vcetype` is based on the `wmtype` specified in the `wmatrix()` option. If `wmatrix()` is specified but `vce()` is not, then `vcetype` is set equal to `wmtype`. To override this behavior in `ivpoisson gmm` and obtain an unadjusted (nonrobust) VCE matrix, specify `vce(unadjusted)`. The default `vcetype` for `ivpoisson cfunction` is `robust`.

Specifying `vce(bootstrap)` or `vce(jackknife)` results in standard errors based on the bootstrap or jackknife, respectively. See [R] [vce\\_option](#), [R] [bootstrap](#), and [R] [jackknife](#) for more information on these VCEs.

The syntax for `vcetypes` is identical to those for `wmatrix()`.

---

#### Reporting

`level(#)`; see [R] [Estimation options](#).

`irr` reports estimated coefficients transformed to incidence-rate ratios, that is,  $e^{\beta_i}$  rather than  $\beta_i$ . Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated or stored. `irr` may be specified at estimation or when replaying previously estimated results. `irr` is not allowed with `additive`.

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

## Optimization

`from(initial_values)` specifies the initial values to begin the estimation. You can specify a  $1 \times k$  matrix, where  $k$  is the number of parameters in the model, or you can specify a parameter name, its initial value, another parameter name, its initial value, and so on. For example, to initialize the coefficient for `male` to 1.23 and the constant `_cons` to 4.57, you would type

```
ivpoisson ..., from(male 1.23 _cons 4.57) ...
```

Initial values declared using this option override any that are declared within substitutable expressions. If you specify a parameter that does not appear in your model, `ivpoisson` exits with error code 480. If you specify a matrix, the values must be in the same order in which the parameters are declared in your model. `ivpoisson` ignores the row and column names of the matrix.

`igmmiterate(#)`, `igmmeps(#)`, and `igmmweps(#)` control the iterative process for the iterative GMM estimator for `ivpoisson`. These options can be specified only if you also specify `igmm`.

`igmmiterate(#)` specifies the maximum number of iterations to perform with the iterative GMM estimator. The default is the number set using `set maxiter`, which is 300 by default.

`igmmeps(#)` specifies the convergence criterion used for successive parameter estimates when the iterative GMM estimator is used. The default is `igmmeps(1e-6)`. Convergence is declared when the relative difference between successive parameter estimates is less than `igmmeps()` and the relative difference between successive estimates of the weight matrix is less than `igmmweps()`.

`igmmweps(#)` specifies the convergence criterion used for successive estimates of the weight matrix when the iterative GMM estimator is used. The default is `igmmweps(1e-6)`. Convergence is declared when the relative difference between successive parameter estimates is less than `igmmeps()` and the relative difference between successive estimates of the weight matrix is less than `igmmweps()`.

*optimization\_options*: `technique()`, `conv_maxiter()`, `conv_ptol()`, `conv_vtol()`, `conv_nrtol()`, and `tracelevel()`. `technique()` specifies the optimization technique to use; `gn` (the default), `nr`, `dfp`, and `bfgs` are allowed. `conv_maxiter()` specifies the maximum number of iterations; `conv_ptol()`, `conv_vtol()`, and `conv_nrtol()` specify the convergence criteria for the parameters, gradient, and scaled Hessian, respectively. `tracelevel()` allows you to obtain additional details during the iterative process. See [M-5] [optimize\(\)](#).

## Remarks and examples

`ivpoisson` estimates the parameters of a Poisson regression model in which some of the covariates are endogenous. A regressor is endogenous if it is related to the unobserved error term. The model is also known as an exponential conditional mean model in which some of the covariates are endogenous. The model may be specified using either additive or multiplicative error terms.

The model is frequently used to model count outcomes and is also used to model nonnegative outcome variables. Poisson regression is a special exponential conditional mean model. See [R] [poisson](#) for more information on Poisson regression.

The exponential conditional mean model has an error form representation in which the dependent variable  $y$  is a function of the exogenous covariates  $\mathbf{x}$ , endogenous covariates  $\mathbf{y}_2$ , and an error  $\epsilon$ . The covariates  $\mathbf{x}$  are independent of  $\epsilon$ , while  $\mathbf{y}_2$  are not.

`ivpoisson` allows  $\epsilon$  to enter either additively,

$$y_i = \exp(\mathbf{x}'_i \boldsymbol{\beta}_1 + \mathbf{y}'_{2,i} \boldsymbol{\beta}_2) + \epsilon_i$$

or multiplicatively,

$$y_i = \exp(\mathbf{x}'_i\beta_1 + \mathbf{y}'_{2,i}\beta_2)\epsilon_i$$

Mullahy (1997), Cameron and Trivedi (2013), Windmeijer and Santos Silva (1997), and Wooldridge (2010) discuss the generalized method of moments (GMM) estimators implemented in `ivpoisson`. GMM is frequently used in modern econometrics. Many econometric and statistical models can be expressed as conditions on the population moments. The parameter estimates produced by GMM estimators make the sample-moment conditions as true as possible given the data. See [R] `gmm` for further information on GMM estimation and how Stata performs it.

The rest of the discussion is presented under the following headings:

*GMM estimator for additive model*  
*GMM estimator for multiplicative model*  
*CF estimator for multiplicative model*

## GMM estimator for additive model

The GMM estimator uses additional variables, known as instruments and denoted by  $\mathbf{z}_i$ , to specify moment conditions that hold in the population. The GMM parameter estimates make the sample versions of these population-moment conditions as close to true as possible. The instrumental variables are assumed to be correlated with the endogenous covariates  $\mathbf{y}_{2,i}$  but independent of the errors  $\epsilon_i$ .

Under additive errors, the dependent variable  $y_i$  is determined by exogenous covariates  $\mathbf{x}_i$ , endogenous covariates  $\mathbf{y}_{2,i}$ , and zero-mean error  $\epsilon_i$  as

$$y_i = \exp(\mathbf{x}'_i\beta_1 + \mathbf{y}'_{2,i}\beta_2) + \epsilon_i$$

This leads to the following error function

$$u(y_i, \mathbf{x}_i, \mathbf{y}_{2,i}, \beta_1, \beta_2) = y_i - \exp(\mathbf{x}'_i\beta_1 + \mathbf{y}'_{2,i}\beta_2)$$

The population-moment conditions for GMM estimation are  $E\{\tilde{\mathbf{z}}_i u(y_i, \mathbf{x}_i, \mathbf{y}_{2,i}, \beta_1, \beta_2)\} = \mathbf{0}$ , where the vector  $\tilde{\mathbf{z}}_i$  is partitioned as  $(\mathbf{x}'_i, \mathbf{z}'_i)$ . The sample-moment conditions are formed by replacing the expectation with the corresponding sample mean. The GMM estimator solves a minimization problem to make the sample-moment conditions as close to zero as possible. Details on how estimation is performed are given in *Methods and formulas*.

Now, we will demonstrate how `ivpoisson gmm` works in the additive error setting with an example.

### ► Example 1: `ivpoisson gmm` with additive errors

This example uses simulated data based on the following story. A news website randomly samples 500 young adults in a major city. The website wants to model the number of times the sampled individuals visit its website (`visits`) based on their overall time spent on the internet (`time`) and the number of times they receive an ad for the website through email or viewing another website (`ad`). The website also suspects the gender of the individual may matter, so an exogenous dummy variable, `female`, is included in the model.

We suspect time spent on the internet is correlated with unobserved factors that additively affect the number of times an individual visits the website. So we treat `time` as an endogenous regressor. Two instruments are used for this variable. The time spent on the phone (`phone`) is one instrument. The other instrument is the time spent interacting with friends and family that live out of town (`frfam`).

We model the number of visits the website receives using an exponential conditional mean model with additive errors and use `ivpoisson gmm` to estimate the parameters of the regression in the output below. To allow for heteroskedasticity of the errors, we use robust standard errors, which is the default; see *Obtaining standard errors* in [R] `gmm` for a discussion of why robust standard errors is the default.

```
. use https://www.stata-press.com/data/r19/website
(Visits to website)
. ivpoisson gmm visits ad female (time = phone frfam)

Step 1:
Iteration 0: GMM criterion Q(b) = .33829416
Iteration 1: GMM criterion Q(b) = .00362656
Iteration 2: GMM criterion Q(b) = .00131886
Iteration 3: GMM criterion Q(b) = .00131876

Step 2:
Iteration 0: GMM criterion Q(b) = .00027102
Iteration 1: GMM criterion Q(b) = .00025811
Iteration 2: GMM criterion Q(b) = .00025811

Exponential mean model with endogenous regressors
Number of parameters = 4 Number of obs = 500
Number of moments = 5
Initial weight matrix: Unadjusted
GMM weight matrix: Robust
```

| visits | Robust      |           | z     | P> z  | [95% conf. interval] |          |
|--------|-------------|-----------|-------|-------|----------------------|----------|
|        | Coefficient | std. err. |       |       |                      |          |
| time   | .0589294    | .0107942  | 5.46  | 0.000 | .0377732             | .0800857 |
| ad     | .137344     | .010157   | 13.52 | 0.000 | .1174366             | .1572515 |
| female | -.0247707   | .0376218  | -0.66 | 0.510 | -.098508             | .0489666 |
| _cons  | 1.041505    | .0385848  | 26.99 | 0.000 | .9658807             | 1.11713  |

Endogenous: time  
 Exogenous: ad female phone frfam

We find significant coefficients for all covariates but `female`. At fixed values of the other covariates, increased time spent on the internet will raise the expected number of website visits. Receiving additional advertisements will also cause an increase in the expected number of website visits.



## GMM estimator for multiplicative model

Under multiplicative errors, the dependent variable  $y_i$  is determined by exogenous covariates  $\mathbf{x}_i$ , endogenous covariates  $\mathbf{y}_{2,i}$ , and unit-mean errors  $\epsilon_i$  as

$$y_i = \exp(\mathbf{x}'_i\beta_1 + \mathbf{y}'_{2,i}\beta_2)\epsilon_i$$

This setting yields a different error function than the additive error case. This ratio formulation is

$$u(y_i, \mathbf{x}_i, \mathbf{y}_{2,i}, \beta_1, \beta_2) = y_i / \exp(\mathbf{x}'_i\beta_1 + \mathbf{y}'_{2,i}\beta_2) - 1$$

Given the instrumental variables  $\mathbf{z}$ , the population-moment conditions for GMM estimation are  $E\{\tilde{\mathbf{z}}_i u(y_i, \mathbf{x}_i, \mathbf{y}_{2,i}, \beta_1, \beta_2)\} = \mathbf{0}$ . The vector  $\tilde{\mathbf{z}}_i$  is partitioned as  $(\mathbf{x}'_i, \mathbf{z}'_i)$ . Same as above, the sample-moment conditions are the sample analogs of the population-moment conditions, and the GMM estimator solves a minimization problem to make the sample-moment conditions as close to zero as possible. Details on how estimation is performed are given in *Methods and formulas*.

## ► Example 2: ivpoisson gmm with multiplicative errors

In this example, we observe a simulated random sample of 5,000 households. We model the number of trips taken by members of the household in the 24-hour period immediately prior to the interview time by using an exponential conditional mean model with multiplicative errors. Exogenous covariates include the distance to the central business district from the household (*cbd*), the distance from the household to a public transit node (*ptn*), whether there is a full-time worker in the household (*worker*), and whether the examined period is on a weekend (*weekend*). We suspect that the endogenous regressor, the transportation cost of the household in the prior week (*tcost*), is correlated with unobserved factors that affect the number of trips taken. This transportation cost includes gasoline and bus, train tickets, etc.

The ratio of the cost of a public transit day pass in the sampled area to the national average cost of such a pass (*pt*) is also observed. This is used as an instrument for transportation cost.

In the output below, we estimate the parameters of the regression with `ivpoisson gmm`. To allow for heteroskedasticity of the errors, we use robust standard errors, which is the default.

```
. use https://www.stata-press.com/data/r19/trip
(Household trips)

. ivpoisson gmm trips cbd ptn worker weekend (tcost = pt), multiplicative

Step 1:
Iteration 0: GMM criterion Q(b) = .04949852
Iteration 1: GMM criterion Q(b) = .00011194
Iteration 2: GMM criterion Q(b) = 1.563e-08
Iteration 3: GMM criterion Q(b) = 3.685e-16

Step 2:
Iteration 0: GMM criterion Q(b) = 2.287e-16
Iteration 1: GMM criterion Q(b) = 1.342e-31

note: model is exactly identified.

Exponential mean model with endogenous regressors
Number of parameters = 6 Number of obs = 5,000
Number of moments = 6
Initial weight matrix: Unadjusted
GMM weight matrix: Robust
```

| trips          | Robust      |           | z     | P> z  | [95% conf. interval] |           |
|----------------|-------------|-----------|-------|-------|----------------------|-----------|
|                | Coefficient | std. err. |       |       |                      |           |
| <i>tcost</i>   | .0352185    | .0098182  | 3.59  | 0.000 | .0159752             | .0544617  |
| <i>cbd</i>     | -.008398    | .0020172  | -4.16 | 0.000 | -.0123517            | -.0044444 |
| <i>ptn</i>     | -.0113146   | .0021819  | -5.19 | 0.000 | -.015591             | -.0070383 |
| <i>worker</i>  | .6623018    | .0519909  | 12.74 | 0.000 | .5604015             | .764202   |
| <i>weekend</i> | .3009323    | .0362682  | 8.30  | 0.000 | .2298479             | .3720167  |
| <i>_cons</i>   | .2654423    | .1550127  | 1.71  | 0.087 | -.0383769            | .5692616  |

Endogenous: *tcost*

Exogenous: *cbd ptn worker weekend pt*

We find that all coefficients are significant. At fixed values of the other covariates, we see that additional mileage from the central business district and public transit nodes reduces the expected number of trips taken. Individuals who live farther away from the central business district may still be out of the house the same amount of time, but they will take fewer trips because the transit time has increased. The situation is similar for those who live farther from public transit.

To interpret the other parameters, we will look at the partial effects of their respective independent variables. The partial effects of a change in an independent variable on the modeled conditional expectation function vary over the data because the model is nonlinear. However, under the multiplicative error model, the ratio of the new value to the old value after a discrete change in an independent variable is constant over the data.

Let  $\mathbf{w} = (\mathbf{x}', \mathbf{y}'_2)'$ . If we add 1 to the  $j$ th independent variable in  $\mathbf{w}$ , the functional form of the model implies that

$$\frac{E(y|\mathbf{w}, w_j + 1, \epsilon)}{E(y|\mathbf{w}, w_j, \epsilon)} = \frac{E(y|w_1, \dots, w_j + 1, \dots, w_k, \epsilon)}{E(y|w_1, \dots, w_j, \dots, w_k, \epsilon)} = e^{\beta_j}$$

When  $y$  is a count variable, this normalized effect is called the incidence-rate ratio (IRR) for a one-unit change in  $w_j$ .

More generally, the IRR for a  $\Delta w_j$  change in  $w_j$  is  $e^{\beta_j \Delta w_j}$  under a multiplicative-error exponential conditional mean model. We can calculate incidence-rate ratios for different changes in the covariates by using `lincom`; see [R] [lincom](#).

Here we replay the `ivpoisson` results by typing the command name and we specify the `irr` option to get the incidence-rate ratios. Each significance test for a coefficient equaling zero becomes a test for the incidence-rate ratio equaling one.

```
. ivpoisson, irr
Exponential mean model with endogenous regressors
Number of parameters = 6 Number of obs = 5,000
Number of moments = 6
Initial weight matrix: Unadjusted
GMM weight matrix: Robust
```

| trips   | IRR      | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |          |
|---------|----------|---------------------|-------|-------|----------------------|----------|
| tcost   | 1.035846 | .0101701            | 3.59  | 0.000 | 1.016103             | 1.055972 |
| cbd     | .9916371 | .0020003            | -4.16 | 0.000 | .9877243             | .9955655 |
| ptn     | .9887491 | .0021573            | -5.19 | 0.000 | .9845299             | .9929864 |
| worker  | 1.939251 | .1008234            | 12.74 | 0.000 | 1.751376             | 2.14728  |
| weekend | 1.351118 | .0490026            | 8.30  | 0.000 | 1.258409             | 1.450657 |
| _cons   | 1.304008 | .2021377            | 1.71  | 0.087 | .9623501             | 1.766962 |

```
Note: _cons estimates baseline incidence rate.
Endogenous: tcost
Exogenous: cbd ptn worker weekend pt
```

Holding other covariates and the error constant, the expected number of trips made from houses with a full-time worker is nearly twice that of those houses without a full-time worker. Similarly, the expected number of trips made during a weekend day is close to 35% higher than the expected number of trips made on other days. For each additional dollar of weekly transportation cost, the expected number of household trips is increased by approximately 3.6%.

## CF estimator for multiplicative model

Control-function (CF) estimators can be used to account for endogenous covariates. As [Wooldridge \(2010, sec. 18.5\)](#) describes, CF estimators assume a certain structural relationship between the endogenous covariates and the exogenous covariates and use functions of first-stage parameter estimates to control for the endogeneity in the second stage.

[Wooldridge \(2010, sec. 18.5\)](#) notes that the VCE of the second-stage estimator must be adjusted to account for estimates from the first stage. `ivpoisson cfunction` solves this problem by stacking the moment conditions that define each stage and applying a single GMM estimator. See [Newey \(1984\)](#) and [Wooldridge \(2010, sec. 14.2\)](#) for a description of this technique. No adjustment to the VCE is necessary because there is only one stage.

The CF estimator augments the original multiplicative model with an estimated term that controls for the endogeneity of  $\mathbf{y}_{2,i}$ . When  $\mathbf{y}_{2,i}$  is exogenous, the coefficient on this control term is zero. Let  $\mathbf{z}$  be instrumental variables, and the vector  $\tilde{\mathbf{z}}_i$  be  $(\mathbf{x}'_i, \mathbf{z}'_i)$ .

The augmented model is

$$y_i = \exp(\mathbf{x}'_i\boldsymbol{\beta}_1 + \mathbf{y}'_{2,i}\boldsymbol{\beta}_2 + \mathbf{v}'_i\boldsymbol{\rho} + c_i)$$

where

$$\mathbf{y}_{2,i} = \mathbf{B}\tilde{\mathbf{z}}'_i + \mathbf{v}_i$$

The term  $\mathbf{v}'_i\boldsymbol{\rho}$  controls for the endogeneity of  $\mathbf{y}_{2,i}$ , and we normalize  $E\{\exp(c_i)\} = 1$ . The coefficient vector  $\boldsymbol{\rho}$  measures the strength of the endogeneity of  $\mathbf{y}_{2,i}$ ;  $\mathbf{y}_{2,i}$  is exogenous when  $\boldsymbol{\rho} = \mathbf{0}$ .

`ivpoisson cfunction` estimates  $\boldsymbol{\beta}_1$  and  $\boldsymbol{\beta}_2$  and the auxiliary parameters  $\boldsymbol{\rho}$  and  $\mathbf{B}$  by GMM; see [Methods and formulas](#) for details.

### ► Example 3: Control-function estimator

We return to the [previous example](#), where we estimated the parameters of an exponential conditional mean model for the number of trips taken by a household in a 24-hour period. We will estimate the parameters of the regression with the CF estimator method and compare our results with those obtained with the GMM estimator in [example 2](#).

In the output below, we estimate the parameters of the regression with the `ivpoisson` `cfunction` command.

```
. ivpoisson cfunction trips cbd ptn worker weekend (tcost = pt)
Step 1:
Iteration 0: GMM criterion Q(b) = .00056156
Iteration 1: GMM criterion Q(b) = 2.366e-07
Iteration 2: GMM criterion Q(b) = 5.552e-14
Iteration 3: GMM criterion Q(b) = 9.759e-27
note: model is exactly identified.
Exponential mean model with endogenous regressors
Number of parameters = 13 Number of obs = 5,000
Number of moments = 13
Initial weight matrix: Unadjusted
GMM weight matrix: Robust
```

|       |          | Robust      |           |        |       | [95% conf. interval] |           |
|-------|----------|-------------|-----------|--------|-------|----------------------|-----------|
|       | trips    | Coefficient | std. err. | z      | P> z  |                      |           |
| trips | cbd      | -.0082567   | .0020005  | -4.13  | 0.000 | -.0121777            | -.0043357 |
|       | ptn      | -.0113719   | .0021625  | -5.26  | 0.000 | -.0156102            | -.0071335 |
|       | worker   | .6903044    | .0521642  | 13.23  | 0.000 | .5880645             | .7925444  |
|       | weekend  | .2978149    | .0356474  | 8.35   | 0.000 | .2279472             | .3676825  |
|       | tcost    | .0320718    | .0092738  | 3.46   | 0.001 | .0138955             | .0502481  |
|       | _cons    | .2145986    | .1359327  | 1.58   | 0.114 | -.0518246            | .4810218  |
| tcost | cbd      | .0165466    | .0043693  | 3.79   | 0.000 | .0079829             | .0251102  |
|       | ptn      | -.040652    | .0045946  | -8.85  | 0.000 | -.0496573            | -.0316467 |
|       | worker   | 1.550985    | .0996496  | 15.56  | 0.000 | 1.355675             | 1.746294  |
|       | weekend  | .0423009    | .0779101  | 0.54   | 0.587 | -.1104002            | .1950019  |
|       | pt       | .7739176    | .0150072  | 51.57  | 0.000 | .7445041             | .8033312  |
|       | _cons    | 12.13934    | .1123471  | 108.05 | 0.000 | 11.91915             | 12.35954  |
|       | /c_tcost | .1599984    | .0111752  | 14.32  | 0.000 | .1380954             | .1819014  |

```
Endogenous: tcost
Exogenous: cbd ptn worker weekend pt
```

The output table presents results for the estimated coefficients in each of three equations. First, in the `trips` equation, we see the results for the estimated coefficients in the equation for the dependent variable `trips`. Second, in the `tcost` equation, we see the estimated coefficients in the regression of `tcost` on the instrumental and exogenous variables. Third, the `/c_tcost` ancillary parameter corresponds to the estimate of  $\rho$ , the coefficient on the residual variable included to control for the endogeneity of `tcost`.

We find that all coefficients are significant in the exponential conditional mean equation, `trips`. The coefficient estimates in the `trips` equation are similar to the estimates obtained by the GMM estimator in [example 2](#). That the estimated coefficient on the `tcost` control variable is significantly different from zero suggests that `tcost` is endogenous.

## Stored results

ivpoisson stores the following in `e()`:

### Scalars

|                           |                                                      |
|---------------------------|------------------------------------------------------|
| <code>e(N)</code>         | number of observations                               |
| <code>e(k)</code>         | number of parameters                                 |
| <code>e(k_eq)</code>      | number of equations                                  |
| <code>e(k_aux)</code>     | number of auxiliary parameters                       |
| <code>e(k_dv)</code>      | number of dependent variables                        |
| <code>e(Q)</code>         | criterion function                                   |
| <code>e(J)</code>         | Hansen $J$ $\chi^2$ statistic                        |
| <code>e(J_df)</code>      | $J$ statistic degrees of freedom                     |
| <code>e(N_clust)</code>   | number of clusters                                   |
| <code>e(rank)</code>      | rank of $e(V)$                                       |
| <code>e(ic)</code>        | number of iterations used by iterative GMM estimator |
| <code>e(converged)</code> | 1 if converged, 0 otherwise                          |

### Macros

|                                |                                                                |
|--------------------------------|----------------------------------------------------------------|
| <code>e(cmd)</code>            | ivpoisson                                                      |
| <code>e(cmdline)</code>        | command as typed                                               |
| <code>e(depvar)</code>         | dependent variable of regression                               |
| <code>e(endog)</code>          | names of endogenous variables                                  |
| <code>e(exog)</code>           | names of exogenous variables                                   |
| <code>e(wtype)</code>          | weight type                                                    |
| <code>e(wexp)</code>           | weight expression                                              |
| <code>e(title)</code>          | title in estimation output                                     |
| <code>e(clustvar)</code>       | name of cluster variable                                       |
| <code>e(offset1)</code>        | offset variable for first equation                             |
| <code>e(winit)</code>          | initial weight matrix used                                     |
| <code>e(winitname)</code>      | name of user-supplied initial weight matrix                    |
| <code>e(estimator)</code>      | gmm or cfunction                                               |
| <code>e(additive)</code>       | additive if additive errors specified                          |
| <code>e(multiplicative)</code> | multiplicative if multiplicative errors specified              |
| <code>e(gmmestimator)</code>   | onestep, twostep, or igmm                                      |
| <code>e(wmatrix)</code>        | <i>wmtype</i> specified in <code>wmatrix()</code>              |
| <code>e(vce)</code>            | <i>vcetype</i> specified in <code>vce()</code>                 |
| <code>e(vcetype)</code>        | title used to label Std. err.                                  |
| <code>e(technique)</code>      | optimization technique                                         |
| <code>e(properties)</code>     | b V                                                            |
| <code>e(estat_cmd)</code>      | program used to implement <code>estat</code>                   |
| <code>e(predict)</code>        | program used to implement <code>predict</code>                 |
| <code>e(footnote)</code>       | program used to implement footnote display                     |
| <code>e(marginsok)</code>      | predictions allowed by <code>margins</code>                    |
| <code>e(marginsnotok)</code>   | predictions disallowed by <code>margins</code>                 |
| <code>e(asbalanced)</code>     | factor variables <code>fvset</code> as <code>asbalanced</code> |
| <code>e(asobserved)</code>     | factor variables <code>fvset</code> as <code>asobserved</code> |

### Matrices

|                              |                                                          |
|------------------------------|----------------------------------------------------------|
| <code>e(b)</code>            | coefficient vector                                       |
| <code>e(V)</code>            | variance–covariance matrix                               |
| <code>e(init)</code>         | initial values of the estimators                         |
| <code>e(Wuser)</code>        | user-supplied initial weight matrix                      |
| <code>e(W)</code>            | weight matrix used for final round of estimation         |
| <code>e(S)</code>            | moment covariance matrix used in robust VCE computations |
| <code>e(V_modelbased)</code> | model-based variance                                     |

### Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in  $\mathbf{r}()$ :

Matrices

$\mathbf{r}(\text{table})$

matrix containing the coefficients with their standard errors, test statistics,  $p$ -values, and confidence intervals

Note that results stored in  $\mathbf{r}()$  are updated when the command is replayed and will be replaced when any  $\mathbf{r}$ -class command is run after the estimation command.

## Methods and formulas

The estimators in `ivpoisson` are GMM estimators that can be expressed in terms of error functions and the instruments that are used to form the moment conditions. When offsets  $o_j^\beta$  are used in the outcome variable equation, the following formulas apply with  $\mathbf{x}'_j\beta_1$  changed to  $\mathbf{x}'_j\beta_1 + o_j^\beta$ .

The error functions for the GMM estimators are given in the text.

Here we provide some details about the form of the error function used by the CF estimator.

Recall that the multiplicative model is

$$y_i = \exp(\mathbf{x}'_i\beta_1 + \mathbf{y}'_{2,i}\beta_2)\epsilon_i$$

We parameterize the endogenous variables in the form

$$\mathbf{y}_{2,i} = \mathbf{B}\tilde{\mathbf{z}}'_i + \mathbf{v}_i$$

This allows us to decompose  $\epsilon_i$  as

$$\epsilon_i = \exp(\mathbf{v}'_i\boldsymbol{\rho} + c_i)$$

Given this setup, we obtain the following conditional mean:

$$E(y|\mathbf{x}_i, \mathbf{z}_i, \mathbf{y}_{2,i}, \mathbf{v}_i) = \exp(\mathbf{x}'_i\beta_1 + \mathbf{y}'_{2,i}\beta_2 + \mathbf{v}'_i\boldsymbol{\rho})$$

We estimate  $\mathbf{v}_i$  as the residuals of the linear regression of  $\mathbf{y}_{2,i}$  on  $\tilde{\mathbf{z}}_i$ . The estimates of  $\mathbf{v}_i$  are used as additional covariates in the exponential conditional mean model for  $y$  to estimate  $\beta_1$ ,  $\beta_2$ , and  $\boldsymbol{\rho}$ . In essence, the estimates of  $\mathbf{v}_i$  control for the endogeneity.

The error functions for the endogenous covariates are defined as

$$\mathbf{u}_{en,i}(\mathbf{y}_{2,i}, \tilde{\mathbf{z}}_i, \mathbf{B}) = \mathbf{y}_{2,i} - \mathbf{B}\tilde{\mathbf{z}}'_i$$

Now, we define the error function for the dependent variable as

$$u_y(y_i, \mathbf{x}_i, \mathbf{y}_{2,i}, \mathbf{u}_{en,i}, \beta_1, \beta_2, \boldsymbol{\rho}) = y_i / \exp(\mathbf{x}'_i\beta_1 + \mathbf{y}'_{2,i}\beta_2 + \mathbf{u}'_{en,i}\boldsymbol{\rho}) - 1$$

$\mathbf{u}_{en,i}$  will be vector valued if we have multiple endogenous covariates  $\mathbf{y}_{2,i}$ . Call the dimension of  $\mathbf{y}_{2,i}$   $g$ .  $\mathbf{u}_{en,i}$  and  $u_{y,i}$  define  $g + 1$  separate error functions. We will use variables  $\tilde{\mathbf{z}}_i$  to instrument each error function in  $\mathbf{u}_{en,i}$ . So for error function  $j = 1, \dots, g$ , we have the error function  $u_{en,i,j}$  and the population-moment conditions  $E(\tilde{\mathbf{z}}_i u_{en,i,j}) = \mathbf{0}$ .

We calculate  $\hat{v}_{oi}$  previous to estimation as the residuals of the linear regression of  $y_{2,i}$  on  $\tilde{z}_i$ . We use variables  $x_i$ ,  $y_{2,i}$ , and  $\hat{v}_{oi}$  to instrument the error function  $u_y$ . This leads to the population-moment conditions  $E\{(\mathbf{x}'_i, y'_{2,i}, \hat{v}'_{oi})u_{y,i}\} = \mathbf{0}$

Details of GMM estimation can be found in *Methods and formulas* of [R] **gmm**. Determination of the weight matrix  $\mathbf{W}_N$  is discussed there.

Under the GMM estimation, the GMM estimators  $\hat{\beta}_1$  and  $\hat{\beta}_2$  are the values of  $\beta_1$  and  $\beta_2$  that minimize

$$Q(\beta_1, \beta_2) = \left\{ \frac{1}{N} \sum_i \tilde{z}_i u_i(y_i, \mathbf{x}_i, y_{2,i}, \beta_1, \beta_2) \right\}' \mathbf{W}_N \left\{ \frac{1}{N} \sum_i \tilde{z}_i u_i(y_i, \mathbf{x}_i, y_{2,i}, \beta_1, \beta_2) \right\} \quad (1)$$

for  $q \times q$  weight matrix  $\mathbf{W}_N$ , where  $q$  is the dimension of  $\tilde{z}_i$ . The error functions  $u_i$  were defined in the text.

In the CF method, we have multiple error functions as defined above. We can stack the moment conditions and write them more compactly as  $\mathbf{Z}'_i \mathbf{u}_i(\mathbf{B}, \beta_1, \beta_2, \rho)$ , where

$$\mathbf{Z}_i = \begin{bmatrix} \mathbf{x}'_i & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & y'_{2,i} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \tilde{z}_i & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \tilde{z}_i \end{bmatrix}$$

and

$$\mathbf{u}_i(\mathbf{B}, \beta_1, \beta_2, \rho) = \begin{bmatrix} u_y(y_i, \mathbf{x}_i, y_{2,i}, \mathbf{u}_{en,i}, \beta_1, \beta_2, \rho) \\ \mathbf{u}_{en}(y_{2,i}, \tilde{z}_i, \mathbf{B}) \end{bmatrix}$$

The matrix  $\mathbf{Z}_i$  has  $g + 1$  rows and  $k + gz$  columns, where  $k$  is the number of covariates for  $y_i$  and  $z$  is the number of exogenous covariates in  $\tilde{z}_i$ .

The GMM estimators  $\hat{\mathbf{B}}, \hat{\beta}_1, \hat{\beta}_2$ , and  $\hat{\rho}$  are the values of  $\mathbf{B}, \beta_1, \beta_2$ , and  $\rho$  that minimize

$$Q(\mathbf{B}, \beta_1, \beta_2, \rho) = \left\{ N^{-1} \sum_{i=1}^N \mathbf{Z}'_i \mathbf{u}_i(\mathbf{B}, \beta_1, \beta_2, \rho) \right\}' \mathbf{W}_N \left\{ N^{-1} \sum_{i=1}^N \mathbf{Z}'_i \mathbf{u}_i(\mathbf{B}, \beta_1, \beta_2, \rho) \right\} \quad (2)$$

for  $(k + gz) \times (k + gz)$  weight matrix  $\mathbf{W}_N$ .

By default, `ivpoisson` minimizes (1) and (2) using the Gauss–Newton method. See Hayashi (2000, 498) for a derivation. This technique is typically faster than quasi-Newton methods and does not require second-order derivatives.

## References

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## Also see

- [R] **ivpoisson postestimation** — Postestimation tools for ivpoisson
- [R] **gmm** — Generalized method of moments estimation
- [R] **ivprobit** — Probit model with continuous endogenous covariates
- [R] **ivregress** — Single-equation instrumental-variables regression
- [R] **ivtobit** — Tobit model with continuous endogenous covariates
- [R] **nl** — Nonlinear least-squares estimation
- [R] **nlsur** — Estimation of nonlinear system of equations
- [R] **poisson** — Poisson regression
- [R] **regress** — Linear regression
- [U] **20 Estimation and postestimation commands**

|                         |                                |                                      |                           |
|-------------------------|--------------------------------|--------------------------------------|---------------------------|
| Postestimation commands | <a href="#">predict</a>        | <a href="#">margins</a>              | <a href="#">estat</a>     |
| Remarks and examples    | <a href="#">Stored results</a> | <a href="#">Methods and formulas</a> | <a href="#">Reference</a> |
| Also see                |                                |                                      |                           |

## Postestimation commands

The following postestimation command is of special interest after `ivpoisson`:

| Command                   | Description                                  |
|---------------------------|----------------------------------------------|
| <code>estat overid</code> | perform test of overidentifying restrictions |

The following standard postestimation commands are also available:

| Command                      | Description                                                                                       |
|------------------------------|---------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                               |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                      |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                |
| <code>estimates</code>       | cataloging estimation results                                                                     |
| <code>etable</code>          | table of estimation results                                                                       |
| <code>forecast</code>        | dynamic forecasts and simulations                                                                 |
| <code>hausman</code>         | Hausman’s specification test                                                                      |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters    |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                           |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters |
| <code>predict</code>         | number of events, linear predictions, residuals, etc.                                             |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions              |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                              |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                |

# predict

## Description for predict

`predict` creates a new variable containing predictions such as numbers of events, linear predictions, and residuals.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic nooffset]
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

Main

|                        |                                                                                  |
|------------------------|----------------------------------------------------------------------------------|
| <code>n</code>         | number of events; the default                                                    |
| <code>xbtotal</code>   | linear prediction, using residual estimates for <code>ivpoisson cfunction</code> |
| <code>xb</code>        | linear prediction                                                                |
| <code>residuals</code> | residuals                                                                        |

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

## Options for predict

Main

`n`, the default, calculates the predicted number of events via the exponential-form estimate. This is  $\exp(\mathbf{x}'_j\beta_1 + \mathbf{y}'_{2,j}\beta_2)$  if neither `offset()` nor `exposure()` was specified,  $\exp(\mathbf{x}'_j\beta_1 + \mathbf{y}'_{2,j}\beta_2 + \text{offset}_j)$  if `offset()` was specified, or  $\exp(\mathbf{x}'_j\beta_1 + \mathbf{y}'_{2,j}\beta_2) \times \text{exposure}_j$  if `exposure()` was specified.

After generalized method of moments estimation, the exponential-form estimate is not a consistent estimate of the conditional mean of  $y_j$ , because it is not corrected for  $E(\epsilon_j | \mathbf{y}_{2,j})$ . More details are found in [Methods and formulas](#).

After control-function estimation, we correct the exponential-form estimate for  $E(\epsilon_j | \mathbf{y}_{2,j})$  by using the estimated residuals of  $\mathbf{y}_{2,j}$  and the `c_*` auxiliary parameters. This supplements the direct effect of  $\mathbf{y}_{2,j}$  and  $\mathbf{x}_j$  through  $\beta_1$  and  $\beta_2$  with the indirect effects of  $\mathbf{y}_{2,j}$ ,  $\mathbf{x}_j$ , and the instruments  $\mathbf{z}_j$  through the endogenous error  $\epsilon_j$ . Thus, the exponential-form estimate consistently estimates the conditional mean of  $y_j$ .

`xbtotal` calculates the linear prediction, which is  $\mathbf{x}'_j\beta_1 + \mathbf{y}'_{2,j}\beta_2$  if neither `offset()` nor `exposure()` was specified,  $\mathbf{x}'_j\beta_1 + \mathbf{y}'_{2,j}\beta_2 + \text{offset}_j$  if `offset()` was specified, or  $\mathbf{x}'_j\beta_1 + \mathbf{y}'_{2,j}\beta_2 + \ln(\text{exposure}_j)$  if `exposure()` was specified.

After control-function estimation, the estimate of the linear form  $\mathbf{x}'_j\beta_1$  includes the estimated residuals of the endogenous regressors with coefficients from the `c_*` auxiliary parameters.

`xb` calculates the linear prediction, which is  $\mathbf{x}'_j\beta_1 + \mathbf{y}'_{2,j}\beta_2$  if neither `offset()` nor `exposure()` was specified,  $\mathbf{x}'_j\beta_1 + \mathbf{y}'_{2,j}\beta_2 + \text{offset}_j$  if `offset()` was specified, or  $\mathbf{x}'_j\beta_1 + \mathbf{y}'_{2,j}\beta_2 + \ln(\text{exposure}_j)$  if `exposure()` was specified. See `nooffset` below.

`residuals` calculates the residuals. Under additive errors, these are calculated as  $y_j - \exp(\mathbf{x}'_j\beta_1 + \mathbf{y}'_{2,j}\beta_2)$ . Under multiplicative errors, they are calculated as  $y_j / \exp(\mathbf{x}'_j\beta_1 + \mathbf{y}'_{2,j}\beta_2) - 1$ .

When `offset()` or `exposure()` is specified,  $\mathbf{x}'_j\beta_1$  is not used directly in the residuals.  $\mathbf{x}'_j\beta_1 + \text{offset}_j$  is used if `offset()` was specified.  $\mathbf{x}'_j\beta_1 + \ln(\text{exposure}_j)$  is used if `exposure()` was specified. See `nooffset` below.

After control-function estimation, the estimate of the linear form  $\mathbf{x}'_j\beta_1$  includes the estimated residuals of the endogenous regressors with coefficients from the `c_*` auxiliary parameters.

`nooffset` is relevant only if you specified `offset()` or `exposure()` when you fit the model. It modifies the calculations made by `predict` so that they ignore the offset or exposure variable. `nooffset` removes the offset from calculations involving both the `treat()` equation and the dependent count variable.

## margins

### Description for margins

`margins` estimates margins of response for numbers of events and linear predictions.

### Menu for margins

Statistics > Postestimation

### Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i>       | Description                                                                      |
|------------------------|----------------------------------------------------------------------------------|
| <code>n</code>         | number of events; the default                                                    |
| <code>xbtotal</code>   | linear prediction, using residual estimates for <code>ivpoisson cfunction</code> |
| <code>xb</code>        | linear prediction                                                                |
| <code>residuals</code> | not allowed with <code>margins</code>                                            |

Statistics not allowed with `margins` are functions of stochastic quantities other than `e(b)`.

For the full syntax, see [R] [margins](#).

## estat

### Description for estat

`estat overid` reports Hansen's  $J$  statistic, which is used to determine the validity of the overidentifying restrictions in a GMM model. `ivpoisson gmm` uses GMM estimation to obtain parameter estimates. Under additive and multiplicative errors, Hansen's  $J$  statistic can be accurately reported when more instruments than endogenous regressors are specified. It is not appropriate to report the  $J$  statistic after `ivpoisson cfunction`, because a just-identified model is fit.

If the model is correctly specified in the sense that  $E\{\tilde{\mathbf{z}}_i u(y_i, \mathbf{x}_i, \mathbf{y}_{2,i}, \boldsymbol{\beta})\} = \mathbf{0}$ , then the sample analog to that condition should hold at the estimated value of  $\beta_1$  and  $\beta_2$ . The  $\tilde{\mathbf{z}}_i$  variables are the exogenous regressors  $\mathbf{x}_i$  and instrumental variables  $\mathbf{z}_i$  used in `ivpoisson gmm`. The  $\mathbf{y}_{2,i}$  are the endogenous regressors. The  $u$  function is the error function, which will have a different form for multiplicative and additive errors in the regression.

Hansen's  $J$  statistic is valid only if the weight matrix is optimal, meaning that it equals the inverse of the covariance matrix of the moment conditions. Therefore, `estat overid` only reports Hansen's  $J$  statistic after two-step or iterated estimation or if you specified `winitial(matname)` when calling `ivpoisson gmm`. In the latter case, it is your responsibility to determine the validity of the  $J$  statistic.

### Menu for estat

Statistics > Postestimation

### Syntax for estat

`estat overid`

### Remarks and examples

`estat overid` reports Hansen's  $J$  statistic, which is used to determine the validity of the overidentifying restrictions in a GMM model. It is not appropriate to use it after `ivpoisson cfunction`, because a just-identified model is fit.

Recall that the GMM criterion function is

$$Q(\boldsymbol{\beta}) = \left\{ \frac{1}{N} \sum_i \tilde{\mathbf{z}}_i u(y_i, \mathbf{x}_i, \mathbf{y}_{2,i}, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2) \right\}' \mathbf{W}_N \left\{ \frac{1}{N} \sum_i \tilde{\mathbf{z}}_i u(y_i, \mathbf{x}_i, \mathbf{y}_{2,i}, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2) \right\}$$

Our  $u$  function within this formula will change depending on whether we use additive or multiplicative errors. The  $\tilde{\mathbf{z}}$  vector contains the exogenous regressors and instrumental variables used. `ivpoisson gmm` estimates regression coefficients to minimize  $Q$ .

Let  $l$  be the dimension of  $\tilde{\mathbf{z}}$  and  $k$  the number of regressors. If  $\mathbf{W}_N$  is an optimal weight matrix, under the null hypothesis  $H_0: E\{\tilde{\mathbf{z}}_i u(y_i, \mathbf{x}_i, \mathbf{y}_{2,i}, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2)\} = \mathbf{0}$ , the test statistic  $J = N \times Q \sim \chi^2(l - k)$ . A large test statistic casts doubt on the null hypothesis.

Because the weight matrix  $\mathbf{W}_N$  must be optimal, `estat overid` works only after the two-step and iterated estimation or if you supplied your own initial weight matrix by using the `winitial(matname)` option of `ivpoisson gmm` and used the one-step estimator.

Often, the overidentifying restrictions test is interpreted as a test of the validity of the instruments  $\mathbf{z}$ . However, other forms of model misspecification can sometimes lead to a significant test statistic. See Hall (2005, sec. 5.1) for a discussion of the overidentifying restrictions test and its behavior in correctly specified and misspecified models.

Note that `ivpoisson gmm` defaults to the two-step estimator when other options are not specified to override the default. Thus, it is appropriate to perform the  $J$  test after the regression of [example 1](#) in [\[R\] ivpoisson](#).

### ▷ Example 1: Specification test

Recall [example 1](#) of [\[R\] ivpoisson](#). We estimated the parameters of an exponential conditional mean model for the number of visits to a website. Additive errors were used. Exogenous regressors included the gender of an individual and the number of ads received from the website.

An endogenous regressor, time spent on the internet, was also included in the model. Two instruments were used. One of the instruments measured the time spent interacting with friends and out-of-town family. The other measured the time spent on the phone.

We will reestimate the parameters of the regression here and then test the specification.

```
. use https://www.stata-press.com/data/r19/website
(Visits to website)
. ivpoisson gmm visits ad female (time = phone frfam)
(output omitted)
. estat overid
Test of overidentifying restriction:
Hansen's J chi2(1) = .129055 (p = 0.7194)
```

We have two instruments for one endogenous variable, so the  $J$  statistic has one degree of freedom. The  $J$  statistic is not significant. We fail to reject the null hypothesis that the model is correctly specified.



## Stored results

`estat overid` stores the following in `r()`:

Scalars

|                      |                                  |
|----------------------|----------------------------------|
| <code>r(J)</code>    | Hansen's $J$ statistic           |
| <code>r(J_df)</code> | $J$ statistic degrees of freedom |
| <code>r(J_p)</code>  | $J$ statistic $p$ -value         |

## Methods and formulas

The vector  $\mathbf{x}_i$  contains the exogenous regressors, and  $\mathbf{z}_i$  the instruments. The vector  $\tilde{\mathbf{z}}_i$  is partitioned as  $(\mathbf{x}_i, \mathbf{z}_i)$ . The vector  $\mathbf{y}_{2,i}$  contains the endogenous regressors.

Under multiplicative errors, the conditional mean of  $y_i$  is

$$\begin{aligned} E(y_i | \mathbf{y}_{2,i}, \tilde{\mathbf{z}}_i) &= E\{E(y_i | \mathbf{x}_i, \mathbf{y}_{2,i}, \epsilon_i) | \mathbf{y}_{2,i}, \tilde{\mathbf{z}}_i\} \\ &= E\{\exp(\mathbf{x}'_i \boldsymbol{\beta}_1 + \mathbf{y}'_{2,i} \boldsymbol{\beta}_2) \epsilon_i | \mathbf{y}_{2,i}, \tilde{\mathbf{z}}_i\} \\ &= \exp(\mathbf{x}'_i \boldsymbol{\beta}_1 + \mathbf{y}'_{2,i} \boldsymbol{\beta}_2) E(\epsilon_i | \mathbf{y}_{2,i}, \tilde{\mathbf{z}}_i) \end{aligned}$$

Under the CF estimator,

$$\begin{aligned} E(\epsilon_i | \mathbf{y}_{2,i}, \tilde{\mathbf{z}}_i) &= E\{E(\epsilon_i | \mathbf{v}_i, c_i) | \mathbf{y}_{2,i}, \tilde{\mathbf{z}}_i\} \\ &= E\{\exp(\mathbf{v}'_i \boldsymbol{\rho} + c_i) | \mathbf{y}_{2,i}, \tilde{\mathbf{z}}_i\} \\ &= \exp\{(\mathbf{y}_{2,i} - \mathbf{B}\tilde{\mathbf{z}}_i)' \boldsymbol{\rho}\} E(c_i | \mathbf{y}_{2,i}, \tilde{\mathbf{z}}_i) \\ &= \exp\{(\mathbf{y}_{2,i} - \mathbf{B}\tilde{\mathbf{z}}_i)' \boldsymbol{\rho}\} \end{aligned}$$

Thus under the CF estimator, we estimate the conditional mean of  $y_i$  as

$$E(y_i | \mathbf{y}_{2,i}, \tilde{\mathbf{z}}_i) = \exp(\mathbf{x}'_i \boldsymbol{\beta}_1 + \mathbf{y}'_{2,i} \boldsymbol{\beta}_2 + (\mathbf{y}_{2,i} - \mathbf{B}\tilde{\mathbf{z}}_i)' \boldsymbol{\rho})$$

The CF estimator explicitly models the functional form of the endogeneity of  $\mathbf{y}_{2,i}$  and  $\epsilon_i$  with the instruments and exogenous regressors  $\tilde{\mathbf{z}}_i$ . This allows it to correct the exponential-form estimator for the  $E(\epsilon_i | \mathbf{y}_{2,i}, \tilde{\mathbf{z}}_i)$  term.

In contrast, the GMM estimator does not model the functional form of the endogeneity of  $\mathbf{y}_{2,i}$  and  $\epsilon_i$ . Therefore,  $E(\epsilon_i | \mathbf{y}_{2,i}, \tilde{\mathbf{z}}_i)$  is not estimated, and the exponential-form estimator under GMM estimation simply ignores this term. Noting that because  $\tilde{\mathbf{z}}_i$  and  $\epsilon_i$  are independent,  $E(\epsilon_i | \mathbf{y}_{2,i}, \tilde{\mathbf{z}}_i) = E(\epsilon_i | \mathbf{y}_{2,i})$ , we can obviously see that ignoring the term will lead to inconsistent estimation of the conditional mean of  $y_i$ .  $y_{2,i}$  and  $\epsilon_i$  are not independent, so  $E(\epsilon_i | y_{2,i})$  may vary based on  $y_{2,i}$ .

In the additive errors setting, a similar derivation will show that the exponential-form estimator obtained from GMM estimation is inconsistent for the conditional mean of  $y_i$ .

## Reference

Hall, A. R. 2005. *Generalized Method of Moments*. Oxford: Oxford University Press.

## Also see

[R] [ivpoisson](#) — Poisson model with continuous endogenous covariates

[U] [20 Estimation and postestimation commands](#)

[Description](#)[Syntax](#)[Remarks and examples](#)[Acknowledgments](#)[Quick start](#)[Options for ML estimator](#)[Stored results](#)[References](#)[Menu](#)[Options for two-step estimator](#)[Methods and formulas](#)[Also see](#)

## Description

`ivprobit` fits models for binary dependent variables where one or more of the covariates are endogenous and errors are normally distributed. By default, `ivprobit` uses maximum likelihood, but Newey's (1987) minimum  $\chi^2$  (two-step) estimator can be requested. Both estimators assume that the endogenous covariates are continuous and so are not appropriate for use with discrete endogenous covariates.

## Quick start

Probit regression of  $y_1$  on  $x$  and endogenous regressor  $y_2$  that is instrumented using  $z$

```
ivprobit y1 x (y2 = z)
```

With robust standard errors

```
ivprobit y1 x (y2 = z), vce(robust)
```

Use Newey's two-step estimator

```
ivprobit y1 x (y2 = z), twostep
```

Same as above, and show first-stage regression results

```
ivprobit y1 x (y2 = z), twostep first
```

## Menu

Statistics > Endogenous covariates > Probit model with endogenous covariates

## Syntax

Maximum likelihood estimator

```
ivprobit depvar [varlist1] (varlist2 = varlistiv) [if] [in] [weight] [, mle_options]
```

Two-step estimator

```
ivprobit depvar [varlist1] (varlist2 = varlistiv) [if] [in] [weight] , twostep
[tse_options]
```

*varlist*<sub>1</sub> is the list of exogenous variables.

*varlist*<sub>2</sub> is the list of endogenous variables.

*varlist*<sub>iv</sub> is the list of exogenous variables used with *varlist*<sub>1</sub> as instruments for *varlist*<sub>2</sub>.

*mle\_options*

Description

---

|                                           |                                                                                                                                                        |
|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------|
| Model                                     |                                                                                                                                                        |
| <u>m</u> le                               | use conditional maximum-likelihood estimator; the default                                                                                              |
| <u>a</u> sis                              | retain perfect predictor variables                                                                                                                     |
| <u>c</u> onstraints( <i>constraints</i> ) | apply specified linear constraints                                                                                                                     |
| SE/Robust                                 |                                                                                                                                                        |
| vce( <i>vcetype</i> )                     | <i>vcetype</i> may be oim, <u>r</u> obust, <u>c</u> luster <i>clustvar</i> , opg, <u>b</u> ootstrap,<br>or <u>j</u> ackknife                           |
| Reporting                                 |                                                                                                                                                        |
| <u>l</u> evel(#)                          | set confidence level; default is level(95)                                                                                                             |
| <u>f</u> irst                             | report first-stage regression                                                                                                                          |
| <u>n</u> ocnsreport                       | do not display constraints                                                                                                                             |
| <i>display_options</i>                    | control columns and column formats, row spacing, line width,<br>display of omitted variables and base and empty cells, and<br>factor-variable labeling |
| Maximization                              |                                                                                                                                                        |
| <i>maximize_options</i>                   | control the maximization process                                                                                                                       |
| <u>c</u> oefflegend                       | display legend instead of statistics                                                                                                                   |

---

| <i>tse_options</i>           | Description                                                                                                                                      |
|------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                 |                                                                                                                                                  |
| * <code>twostep</code>       | use Newey's two-step estimator; the default is <code>mle</code>                                                                                  |
| <code>asis</code>            | retain perfect predictor variables                                                                                                               |
| <b>SE</b>                    |                                                                                                                                                  |
| <code>vce(vctype)</code>     | <i>vctype</i> may be <code>twostep</code> , <code>bootstrap</code> , or <code>jackknife</code>                                                   |
| <b>Reporting</b>             |                                                                                                                                                  |
| <code>level(#)</code>        | set confidence level; default is <code>level(95)</code>                                                                                          |
| <code>first</code>           | report first-stage regression                                                                                                                    |
| <code>display_options</code> | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| <code>coeflegend</code>      | display legend instead of statistics                                                                                                             |

\*`twostep` is required.

`varlist1` and `varlistiv` may contain factor variables; see [U] 11.4.3 **Factor variables**.

`devar`, `varlist1`, `varlist2`, and `varlistiv` may contain time-series operators; see [U] 11.4.4 **Time-series varlists**.

`bootstrap`, `by`, `collect`, `jackknife`, `rolling`, `statsby`, and `svy` are allowed; see [U] 11.1.10 **Prefix commands**. `fp` is allowed with the maximum likelihood estimator.

Weights are not allowed with the `bootstrap` prefix; see [R] **bootstrap**.

`vce()`, `first`, `twostep`, and `weights` are not allowed with the `svy` prefix; see [SVY] **svy**.

`fweights`, `iweights`, and `pweights` are allowed with the maximum likelihood estimator. `fweights` are allowed with Newey's two-step estimator. See [U] 11.1.6 **weight**.

`coeflegend` does not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options for ML estimator

### Model

`mle` requests that the conditional maximum-likelihood estimator be used. This is the default.

`asis` requests that all specified variables and observations be retained in the maximization process. This option is typically not used and may introduce numerical instability. Normally, `ivprobit` omits any endogenous or exogenous variables that perfectly predict success or failure in the dependent variable.

The associated observations are also excluded. For more information, see *Model identification* in [R] **probit**.

`constraints(constraints)`; see [R] **Estimation options**.

### SE/Robust

`vce(vctype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] *vce\_option*.

## Reporting

`level(#)`; see [R] [Estimation options](#).

`first` requests that the parameters for the reduced-form equations showing the relationships between the endogenous variables and instruments be displayed. For the two-step estimator, `first` shows the first-stage regressions. For the maximum likelihood estimator, these parameters are estimated jointly with the parameters of the probit equation. The default is not to show these parameter estimates.

`nocnsreport`; see [R] [Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

## Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#).

The following option is available with `ivprobit` but is not shown in the dialog box:

`coeflegend`; see [R] [Estimation options](#).

## Options for two-step estimator

## Model

`twostep` is required and requests that Newey's (1987) efficient two-step estimator be used to obtain the coefficient estimates.

`asis` requests that all specified variables and observations be retained in the maximization process. This option is typically not used and may introduce numerical instability. Normally, `ivprobit` omits any endogenous or exogenous variables that perfectly predict success or failure in the dependent variable. The associated observations are also excluded. For more information, see [Model identification](#) in [R] [probit](#).

## SE

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`twostep`) and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

## Reporting

`level(#)`; see [R] [Estimation options](#).

`first` requests that the parameters for the reduced-form equations showing the relationships between the endogenous variables and instruments be displayed. For the two-step estimator, `first` shows the first-stage regressions. For the maximum likelihood estimator, these parameters are estimated jointly with the parameters of the probit equation. The default is not to show these parameter estimates.

*display\_options*: `noc1`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

The following option is available with `ivprobit` but is not shown in the dialog box:

`coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Remarks are presented under the following headings:

[Model setup](#)

[Model identification](#)

### Model setup

`ivprobit` fits models with dichotomous dependent variables and endogenous covariates. You can use it to fit a probit model when you suspect that one or more of the covariates are correlated with the error term. `ivprobit` is to probit modeling what `ivregress` is to linear regression analysis; see [R] [ivregress](#) for more information.

Formally, the model is

$$\begin{aligned}y_{1i}^* &= \mathbf{y}_{2i}\boldsymbol{\beta} + \mathbf{x}_{1i}\boldsymbol{\gamma} + u_i \\ \mathbf{y}_{2i} &= \mathbf{x}_{1i}\boldsymbol{\Pi}_1 + \mathbf{x}_{2i}\boldsymbol{\Pi}_2 + \mathbf{v}_i\end{aligned}$$

where  $i = 1, \dots, N$ ,  $\mathbf{y}_{2i}$  is a  $1 \times p$  vector of endogenous variables,  $\mathbf{x}_{1i}$  is a  $1 \times k_1$  vector of exogenous variables,  $\mathbf{x}_{2i}$  is a  $1 \times k_2$  vector of additional instruments, and the equation for  $\mathbf{y}_{2i}$  is written in reduced form. By assumption,  $(u_i, \mathbf{v}_i) \sim N(\mathbf{0}, \boldsymbol{\Sigma})$ , where  $\sigma_{11}$  is normalized to one to identify the model.  $\boldsymbol{\beta}$  and  $\boldsymbol{\gamma}$  are vectors of structural parameters, and  $\boldsymbol{\Pi}_1$  and  $\boldsymbol{\Pi}_2$  are matrices of reduced-form parameters. This is a recursive model:  $\mathbf{y}_{2i}$  appears in the equation for  $y_{1i}^*$ , but  $y_{1i}^*$  does not appear in the equation for  $\mathbf{y}_{2i}$ . We do not observe  $y_{1i}^*$ ; instead, we observe

$$y_{1i} = \begin{cases} 0 & y_{1i}^* < 0 \\ 1 & y_{1i}^* \geq 0 \end{cases}$$

The order condition for identification of the structural parameters requires that  $k_2 \geq p$ . Presumably,  $\boldsymbol{\Sigma}$  is not block diagonal between  $u_i$  and  $\mathbf{v}_i$ ; otherwise,  $\mathbf{y}_{2i}$  would not be endogenous.

### □ Technical note

This model is derived under the assumption that  $(u_i, \mathbf{v}_i)$  is independent and identically distributed multivariate normal for all  $i$ . The `vce(cluster clustvar)` option can be used to control for a lack of independence. As with most probit models, if  $u_i$  is heteroskedastic, point estimates will be inconsistent. □

## ▶ Example 1

We have hypothetical data on 500 two-parent households, and we wish to model whether the woman is employed. We have a variable, `fem_work`, that is equal to 1 if she has a job and 0 otherwise. Her decision to work is a function of the number of children at home (`kids`), number of years of schooling completed (`fem_educ`), and other household income measured in thousands of dollars (`other_inc`). We suspect that unobservable shocks affecting the woman's decision to hold a job also affect the household's other income. Therefore, we treat `other_inc` as endogenous. As an instrument, we use the number of years of schooling completed by the man (`male_educ`).

The syntax for specifying the exogenous, endogenous, and instrumental variables is identical to that used in `ivregress`; see [R] [ivregress](#) for details.

```
. use https://www.stata-press.com/data/r19/laborsup
. ivprobit fem_work fem_educ kids (other_inc = male_educ)

Fitting exogenous probit model:
Iteration 0: Log likelihood = -344.63508
Iteration 1: Log likelihood = -252.10819
Iteration 2: Log likelihood = -252.04529
Iteration 3: Log likelihood = -252.04529

Fitting full model:
Iteration 0: Log likelihood = -2368.2142
Iteration 1: Log likelihood = -2368.2062
Iteration 2: Log likelihood = -2368.2062

Probit model with endogenous regressors Number of obs = 500
 Wald chi2(3) = 163.88
Log likelihood = -2368.2062 Prob > chi2 = 0.0000
```

|                                | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|--------------------------------|-------------|-----------|-------|-------|----------------------|-----------|
| other_inc                      | -.0542756   | .0060854  | -8.92 | 0.000 | -.0662028            | -.0423485 |
| fem_educ                       | .2111111    | .0268648  | 7.86  | 0.000 | .1584569             | .2637651  |
| kids                           | -.1820929   | .0478267  | -3.81 | 0.000 | -.2758315            | -.0883542 |
| _cons                          | .3672086    | .4480724  | 0.82  | 0.412 | -.5109971            | 1.245414  |
| corr(e.other~c,<br>e.fem_work) | .3720375    | .1300518  |       |       | .0946562             | .5958136  |
| sd(e.other~c)                  | 16.66621    | .5270318  |       |       | 15.66461             | 17.73186  |

```
Wald test of exogeneity (corr = 0): chi2(1) = 6.70 Prob > chi2 = 0.0096
Endogenous: other_inc
Exogenous: fem_educ kids male_educ
```

`ivprobit` used the default maximum likelihood estimator. The header of the output contains the sample size as well as a Wald statistic and  $p$ -value for the test of the hypothesis that all the slope coefficients are jointly zero. Below, the table of coefficients, Stata reminds us that the endogenous variable is `other_inc` and that `fem_educ`, `kids`, and `male_educ` were used as instruments.

At the bottom of the output is a Wald test of the exogeneity of the endogenous variables. We reject the null hypothesis of no endogeneity. If there is no endogeneity, a standard probit regression would be preferable (see [R] [probit](#)).

Below we fit our model with Newey's (1987) minimum  $\chi^2$  estimator using the twostep option.

## ▷ Example 2

Refitting our labor-supply model with the two-step estimator yields

```
. ivprobit fem_work fem_educ kids (other_inc = male_educ), twostep
Checking reduced-form model...
Two-step probit with endogenous regressors Number of obs = 500
 Wald chi2(3) = 93.97
 Prob > chi2 = 0.0000
```

|           | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|-----------|-------------|-----------|-------|-------|----------------------|-----------|
| other_inc | -.058473    | .0093364  | -6.26 | 0.000 | -.0767719            | -.040174  |
| fem_educ  | .227437     | .0281628  | 8.08  | 0.000 | .1722389             | .282635   |
| kids      | -.1961748   | .0496323  | -3.95 | 0.000 | -.2934522            | -.0988973 |
| _cons     | .3956061    | .4982649  | 0.79  | 0.427 | -.5809752            | 1.372187  |

```
Wald test of exogeneity: chi2(1) = 6.50 Prob > chi2 = 0.0108
Endogenous: other_inc
Exogenous: fem_educ kids male_educ
```

All the coefficients have the same signs as their counterparts in the maximum likelihood model. The Wald test at the bottom of the output confirms our earlier finding of endogeneity.

◀

## □ Technical note

In a standard probit model, the error is assumed to have a variance of 1. In the probit model with endogenous covariates, we assume that  $(u_i, \mathbf{v}_i)$  is multivariate normal with covariance matrix

$$\text{Var}(u_i, \mathbf{v}_i) = \Sigma = \begin{bmatrix} 1 & \Sigma'_{21} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}$$

From the properties of the multivariate normal distribution, it follows that  $\text{Var}(u_i | \mathbf{v}_i) = 1 - \Sigma'_{21} \Sigma_{22}^{-1} \Sigma_{21}$ . Newey's estimator and other two-step probit estimators yield estimates of  $\beta/\sigma$  and  $\gamma/\sigma$ , where  $\sigma$  is the square root of  $\text{Var}(u_i | \mathbf{v}_i)$ , instead of estimates of  $\beta$  and  $\gamma$ . Hence, we cannot directly compare the estimates obtained from Newey's estimator with those obtained from maximum likelihood, which estimate  $\beta$ ,  $\gamma$ , and  $\sigma$  separately. See Wooldridge (2010, 585–594) for a discussion about the interpretation of the estimates and the computation of marginal effects of two-step probit estimators under endogeneity. □

## Model identification

As in the linear simultaneous-equation model, the order condition for identification requires that the number of excluded exogenous variables (that is, the additional instruments) be at least as great as the number of included endogenous variables. `ivprobit` checks this for you and issues an error message if the order condition is not met.

Like `probit`, `logit`, and `logistic`, `ivprobit` checks the exogenous and endogenous variables to see if any of them predict the outcome variable perfectly. It will then omit offending variables and observations and fit the model on the remaining data. Instruments that are perfect predictors do not affect estimation, so they are not checked. See [Model identification](#) in [R] `probit` for more information.

ivprobit will also occasionally display messages such as

Note: 4 failures and 0 successes completely determined.

For an explanation of this message, see [R] [logit](#).

## Stored results

ivprobit, mle stores the following in `e()`:

### Scalars

|                            |                                           |
|----------------------------|-------------------------------------------|
| <code>e(N)</code>          | number of observations                    |
| <code>e(N_cds)</code>      | number of completely determined successes |
| <code>e(N_cdf)</code>      | number of completely determined failures  |
| <code>e(k)</code>          | number of parameters                      |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>  |
| <code>e(k_eq_model)</code> | number of equations in overall model test |
| <code>e(k_dv)</code>       | number of dependent variables             |
| <code>e(df_m)</code>       | model degrees of freedom                  |
| <code>e(ll)</code>         | log likelihood                            |
| <code>e(N_clust)</code>    | number of clusters                        |
| <code>e(endog_ct)</code>   | number of endogenous covariates           |
| <code>e(p)</code>          | model Wald $p$ -value                     |
| <code>e(p_exog)</code>     | exogeneity test Wald $p$ -value           |
| <code>e(chi2)</code>       | model Wald $\chi^2$                       |
| <code>e(chi2_exog)</code>  | Wald $\chi^2$ test of exogeneity          |
| <code>e(rank)</code>       | rank of <code>e(V)</code>                 |
| <code>e(ic)</code>         | number of iterations                      |
| <code>e(rc)</code>         | return code                               |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise               |

### Macros

|                             |                                                                          |
|-----------------------------|--------------------------------------------------------------------------|
| <code>e(cmd)</code>         | ivprobit                                                                 |
| <code>e(cmdline)</code>     | command as typed                                                         |
| <code>e(depvar)</code>      | name of dependent variable                                               |
| <code>e(endog)</code>       | names of endogenous variables                                            |
| <code>e(exog)</code>        | names of exogenous variables                                             |
| <code>e(wtype)</code>       | weight type                                                              |
| <code>e(wexp)</code>        | weight expression                                                        |
| <code>e(title)</code>       | title in estimation output                                               |
| <code>e(clustvar)</code>    | name of cluster variable                                                 |
| <code>e(chi2type)</code>    | Wald; type of model $\chi^2$ test                                        |
| <code>e(vce)</code>         | <i>vcetype</i> specified in <code>vce()</code>                           |
| <code>e(vcetype)</code>     | title used to label Std. err.                                            |
| <code>e(asis)</code>        | asis, if specified                                                       |
| <code>e(method)</code>      | m1                                                                       |
| <code>e(opt)</code>         | type of optimization                                                     |
| <code>e(which)</code>       | max or min; whether optimizer is to perform maximization or minimization |
| <code>e(ml_method)</code>   | type of ml method                                                        |
| <code>e(user)</code>        | name of likelihood-evaluator program                                     |
| <code>e(technique)</code>   | maximization technique                                                   |
| <code>e(properties)</code>  | b V                                                                      |
| <code>e(estat_cmd)</code>   | program used to implement <code>estat</code>                             |
| <code>e(predict)</code>     | program used to implement <code>predict</code>                           |
| <code>e(footnote)</code>    | program used to implement the footnote display                           |
| <code>e(marginsok)</code>   | predictions allowed by <code>margins</code>                              |
| <code>e(marginsprop)</code> | signals to the <code>margins</code> command                              |
| <code>e(asbalanced)</code>  | factor variables <code>fvset</code> as <code>asbalanced</code>           |
| <code>e(asobserved)</code>  | factor variables <code>fvset</code> as <code>asobserved</code>           |

|                 |                                              |
|-----------------|----------------------------------------------|
| Matrices        |                                              |
| e(b)            | coefficient vector                           |
| e(Cns)          | constraints matrix                           |
| e(rules)        | information about perfect predictors         |
| e(ilog)         | iteration log (up to 20 iterations)          |
| e(gradient)     | gradient vector                              |
| e(Sigma)        | $\Sigma$                                     |
| e(V)            | variance–covariance matrix of the estimators |
| e(V_modelbased) | model-based variance                         |
| Functions       |                                              |
| e(sample)       | marks estimation sample                      |

In addition to the above, the following is stored in `r()`:

|          |                                                                                                                       |
|----------|-----------------------------------------------------------------------------------------------------------------------|
| Matrices |                                                                                                                       |
| r(table) | matrix containing the coefficients with their standard errors, test statistics, $p$ -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

`ivprobit`, `twostep` stores the following in `e()`:

|                |                                                                |
|----------------|----------------------------------------------------------------|
| Scalars        |                                                                |
| e(N)           | number of observations                                         |
| e(N_cds)       | number of completely determined successes                      |
| e(N_cdf)       | number of completely determined failures                       |
| e(df_m)        | model degrees of freedom                                       |
| e(df_exog)     | degrees of freedom for $\chi^2$ test of exogeneity             |
| e(p)           | model Wald $p$ -value                                          |
| e(p_exog)      | exogeneity test Wald $p$ -value                                |
| e(chi2)        | model Wald $\chi^2$                                            |
| e(chi2_exog)   | Wald $\chi^2$ test of exogeneity                               |
| e(rank)        | rank of $e(V)$                                                 |
| Macros         |                                                                |
| e(cmd)         | <code>ivprobit</code>                                          |
| e(cmdline)     | command as typed                                               |
| e(depvar)      | name of dependent variable                                     |
| e(endog)       | names of endogenous variables                                  |
| e(exog)        | names of exogenous variables                                   |
| e(wtype)       | weight type                                                    |
| e(wexp)        | weight expression                                              |
| e(chi2type)    | Wald; type of model $\chi^2$ test                              |
| e(vce)         | <code>vce</code> type specified in <code>vce()</code>          |
| e(asis)        | <code>asis</code> , if specified                               |
| e(method)      | <code>twostep</code>                                           |
| e(properties)  | <code>b V</code>                                               |
| e(estat_cmd)   | program used to implement <code>estat</code>                   |
| e(predict)     | program used to implement <code>predict</code>                 |
| e(footnote)    | program used to implement the footnote display                 |
| e(marginsok)   | predictions allowed by <code>margins</code>                    |
| e(marginsprop) | signals to the <code>margins</code> command                    |
| e(asbalanced)  | factor variables <code>fvset</code> as <code>asbalanced</code> |
| e(asobserved)  | factor variables <code>fvset</code> as <code>asobserved</code> |

|          |                                              |
|----------|----------------------------------------------|
| Matrices |                                              |
| e(b)     | coefficient vector                           |
| e(rules) | information about perfect predictors         |
| e(V)     | variance–covariance matrix of the estimators |

Functions  
 e(sample) marks estimation sample

In addition to the above, the following is stored in `r()`:

Matrices  
 r(table) matrix containing the coefficients with their standard errors, test statistics,  $p$ -values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

Fitting limited-dependent variable models with endogenous covariates has received considerable attention in the econometrics literature. Building on the results of Amemiya (1978, 1979), Newey (1987) developed an efficient method of estimation that encompasses both Rivers and Vuong's (1988) simultaneous-equations probit model and Smith and Blundell's (1986) simultaneous-equations tobit model. An efficient alternative to two-step estimation, and `ivprobit`'s default, is to use maximum likelihood. For compactness, we write the model as

$$y_{1i}^* = \mathbf{z}_i \boldsymbol{\delta} + u_i \quad (1)$$

$$\mathbf{y}_{2i} = \mathbf{x}_i \boldsymbol{\Pi} + v_i \quad (2)$$

where  $\mathbf{z}_i = (\mathbf{y}_{2i}, \mathbf{x}_{1i})$ ,  $\mathbf{x}_i = (\mathbf{x}_{1i}, \mathbf{x}_{2i})$ ,  $\boldsymbol{\delta} = (\boldsymbol{\beta}', \boldsymbol{\gamma}')$ , and  $\boldsymbol{\Pi} = (\boldsymbol{\Pi}'_1, \boldsymbol{\Pi}'_2)'$ .

Deriving the likelihood function is straightforward because we can write the joint density  $f(y_{1i}, \mathbf{y}_{2i} | \mathbf{x}_i)$  as  $f(y_{1i} | \mathbf{y}_{2i}, \mathbf{x}_i) f(\mathbf{y}_{2i} | \mathbf{x}_i)$ . When there is an endogenous regressor, the log likelihood for observation  $i$  is

$$\ln L_i = w_i \left[ y_{1i} \ln \Phi(m_i) + (1 - y_{1i}) \ln \{1 - \Phi(m_i)\} + \ln \phi \left( \frac{y_{2i} - \mathbf{x}_i \boldsymbol{\Pi}}{\sigma} \right) - \ln \sigma \right]$$

where

$$m_i = \frac{\mathbf{z}_i \boldsymbol{\delta} + \rho (y_{2i} - \mathbf{x}_i \boldsymbol{\Pi}) / \sigma}{(1 - \rho^2)^{\frac{1}{2}}}$$

$\Phi(\cdot)$  and  $\phi(\cdot)$  are the standard normal distribution and density functions, respectively;  $\sigma$  is the standard deviation of  $v_i$ ;  $\rho$  is the correlation coefficient between  $u_i$  and  $v_i$ ; and  $w_i$  is the weight for observation  $i$  or one if no weights were specified. Instead of estimating  $\sigma$  and  $\rho$ , we estimate  $\ln \sigma$  and  $\operatorname{atanh} \rho$ , where

$$\operatorname{atanh} \rho = \frac{1}{2} \ln \left( \frac{1 + \rho}{1 - \rho} \right)$$

For multiple endogenous covariates, let

$$\operatorname{Var}(u_i, v_i) = \boldsymbol{\Sigma} = \begin{bmatrix} 1 & \boldsymbol{\Sigma}'_{21} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} \end{bmatrix}$$

As in any probit model, we have imposed the normalization  $\operatorname{Var}(u_i) = 1$  to identify the model. The log likelihood for observation  $i$  is

$$\ln L_i = w_i \left[ y_{1i} \ln \Phi(m_i) + (1 - y_{1i}) \ln \{1 - \Phi(m_i)\} + \ln f(\mathbf{y}_{2i} | \mathbf{x}_i) \right]$$

where

$$\ln f(\mathbf{y}_{2i} | \mathbf{x}_i) = -\frac{p}{2} \ln 2\pi - \frac{1}{2} \ln |\Sigma_{22}| - \frac{1}{2} (\mathbf{y}_{2i} - \mathbf{x}_i \mathbf{\Pi}) \Sigma_{22}^{-1} (\mathbf{y}_{2i} - \mathbf{x}_i \mathbf{\Pi})'$$

and

$$m_i = (1 - \Sigma'_{21} \Sigma_{22}^{-1} \Sigma_{21})^{-\frac{1}{2}} \{ \mathbf{z}_i \boldsymbol{\delta} + (\mathbf{y}_{2i} - \mathbf{x}_i \mathbf{\Pi}) \Sigma_{22}^{-1} \Sigma_{21} \}$$

With maximum likelihood estimation, this command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] **\_robust**, particularly *Maximum likelihood estimators* and *Methods and formulas*.

The maximum likelihood version of `ivprobit` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] **Variance estimation**.

The two-step estimates are obtained using Newey's (1987) minimum  $\chi^2$  estimator. The reduced-form equation for  $y_{1i}^*$  is

$$\begin{aligned} y_{1i}^* &= (\mathbf{x}_i \mathbf{\Pi} + \mathbf{v}_i) \boldsymbol{\beta} + \mathbf{x}_{1i} \boldsymbol{\gamma} + u_i \\ &= \mathbf{x}_i \boldsymbol{\alpha} + \mathbf{v}_i \boldsymbol{\beta} + u_i \\ &= \mathbf{x}_i \boldsymbol{\alpha} + \nu_i \end{aligned}$$

where  $\nu_i = \mathbf{v}_i \boldsymbol{\beta} + u_i$ . Because  $u_i$  and  $\mathbf{v}_i$  are jointly normal,  $\nu_i$  is also normal. Note that

$$\boldsymbol{\alpha} = \begin{bmatrix} \mathbf{\Pi}_1 \\ \mathbf{\Pi}_2 \end{bmatrix} \boldsymbol{\beta} + \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \boldsymbol{\gamma} = D(\mathbf{\Pi}) \boldsymbol{\delta}$$

where  $D(\mathbf{\Pi}) = (\mathbf{\Pi}, \mathbf{I}_1)$  and  $\mathbf{I}_1$  is defined such that  $\mathbf{x}_i \mathbf{I}_1 = \mathbf{x}_{1i}$ . Letting  $\widehat{\mathbf{z}}_i = (\mathbf{x}_i \widehat{\mathbf{\Pi}}, \mathbf{x}_{1i})$ ,  $\widehat{\mathbf{z}}_i \boldsymbol{\delta} = \mathbf{x}_i D(\widehat{\mathbf{\Pi}}) \boldsymbol{\delta}$ , where  $D(\widehat{\mathbf{\Pi}}) = (\widehat{\mathbf{\Pi}}, \mathbf{I}_1)$ . Thus, one estimator of  $\boldsymbol{\alpha}$  is  $D(\widehat{\mathbf{\Pi}}) \boldsymbol{\delta}$ ; denote this estimator by  $\widehat{D} \boldsymbol{\delta}$ .

$\boldsymbol{\alpha}$  could also be estimated directly as the solution to

$$\max_{\boldsymbol{\alpha}, \boldsymbol{\lambda}} \sum_{i=1}^N l(y_{1i}, \mathbf{x}_i \boldsymbol{\alpha} + \widehat{\mathbf{v}}_i \boldsymbol{\lambda}) \quad (3)$$

where  $l(\cdot)$  is the log likelihood for probit. Denote this estimator by  $\widetilde{\boldsymbol{\alpha}}$ . The inclusion of the  $\widehat{\mathbf{v}}_i \boldsymbol{\lambda}$  term follows because the multivariate normality of  $(u_i, \mathbf{v}_i)$  implies that, conditional on  $\mathbf{y}_{2i}$ , the expected value of  $u_i$  is nonzero. Because  $\mathbf{v}_i$  is unobservable, the least-squares residuals from fitting (2) are used.

Amemiya (1978) shows that the estimator of  $\boldsymbol{\delta}$  defined by

$$\max_{\boldsymbol{\delta}} (\widetilde{\boldsymbol{\alpha}} - \widehat{D} \boldsymbol{\delta})' \widehat{\boldsymbol{\Omega}}^{-1} (\widetilde{\boldsymbol{\alpha}} - \widehat{D} \boldsymbol{\delta})$$

where  $\widehat{\boldsymbol{\Omega}}$  is a consistent estimator of the covariance of  $\sqrt{N}(\widetilde{\boldsymbol{\alpha}} - \widehat{D} \boldsymbol{\delta})$ , is asymptotically efficient relative to all other estimators that minimize the distance between  $\widetilde{\boldsymbol{\alpha}}$  and  $D(\widehat{\mathbf{\Pi}}) \boldsymbol{\delta}$ . Thus, an efficient estimator of  $\boldsymbol{\delta}$  is

$$\widehat{\boldsymbol{\delta}} = (\widehat{D}' \widehat{\boldsymbol{\Omega}}^{-1} \widehat{D})^{-1} \widehat{D}' \widehat{\boldsymbol{\Omega}}^{-1} \widetilde{\boldsymbol{\alpha}} \quad (4)$$

and

$$\text{Var}(\widehat{\boldsymbol{\delta}}) = (\widehat{D}' \widehat{\boldsymbol{\Omega}}^{-1} \widehat{D})^{-1} \quad (5)$$

To implement this estimator, we need  $\widehat{\boldsymbol{\Omega}}^{-1}$ .

Consider the two-step maximum likelihood estimator that results from first fitting (2) by OLS and computing the residuals  $\widehat{v}_i = y_{2i} - x_i \widehat{\Pi}$ . The estimator is then obtained by solving

$$\max_{\delta, \lambda} \sum_{i=1}^N l(y_{1i}, z_i \delta + \widehat{v}_i \lambda)$$

This is the two-step instrumental-variables (2SIV) estimator proposed by Rivers and Vuong (1988), and its role will become apparent shortly.

From Proposition 5 of Newey (1987),  $\sqrt{N}(\widetilde{\alpha} - \widehat{D}\delta) \xrightarrow{d} N(\mathbf{0}, \Omega)$ , where

$$\Omega = J_{\alpha\alpha}^{-1} + (\lambda - \beta)' \Sigma_{22} (\lambda - \beta) Q^{-1}$$

and  $\Sigma_{22} = E\{v_i' v_i\}$ .  $J_{\alpha\alpha}^{-1}$  is simply the covariance matrix of  $\widetilde{\alpha}$ , ignoring that  $\widehat{\Pi}$  is an estimated parameter matrix. Moreover, Newey shows that the covariance matrix from an OLS regression of  $y_{2i}(\widehat{\lambda} - \widehat{\beta})$  on  $x_i$  is a consistent estimator of the second term.  $\widehat{\lambda}$  can be obtained from solving (3), and the 2SIV estimator yields a consistent estimate,  $\widehat{\beta}$ .

Mechanically, estimation proceeds in several steps.

1. Each of the endogenous right-hand-side variables is regressed on all the exogenous variables, and the fitted values and residuals are calculated. The matrix  $\widehat{D} = D(\widehat{\Pi})$  is assembled from the estimated coefficients.
2. probit is used to solve (3) and obtain  $\widetilde{\alpha}$  and  $\widehat{\lambda}$ . The portion of the covariance matrix corresponding to  $\alpha$ ,  $J_{\alpha\alpha}^{-1}$ , is also saved.
3. The 2SIV estimator is evaluated, and the parameters  $\widehat{\beta}$  corresponding to  $y_{2i}$  are collected.
4.  $y_{2i}(\widehat{\lambda} - \widehat{\beta})$  is regressed on  $x_i$ . The covariance matrix of the parameters from this regression is added to  $J_{\alpha\alpha}^{-1}$ , yielding  $\widehat{\Omega}$ .
5. Evaluating (4) and (5) yields the estimates  $\widehat{\delta}$  and  $\text{Var}(\widehat{\delta})$ .
6. A Wald test of the null hypothesis  $H_0 : \lambda = \mathbf{0}$ , using the 2SIV estimates, serves as our test of exogeneity.

The two-step estimates are not directly comparable with those obtained from the maximum likelihood estimator or from probit. The argument is the same for Newey's efficient estimator as for Rivers and Vuong's (1988) 2SIV estimator, so we consider the simpler 2SIV estimator. From the properties of the normal distribution,

$$E(u_i | v_i) = v_i \Sigma_{22}^{-1} \Sigma_{21} \quad \text{and} \quad \text{Var}(u_i | v_i) = 1 - \Sigma_{21}' \Sigma_{22}^{-1} \Sigma_{21}$$

We write  $u_i$  as  $u_i = v_i \Sigma_{22}^{-1} \Sigma_{21} + e_i = v_i \lambda + e_i$ , where  $e_i \sim N(0, 1 - \rho^2)$ ,  $\rho^2 = \Sigma_{21}' \Sigma_{22}^{-1} \Sigma_{21}$ , and  $e_i$  is independent of  $v_i$ . In the second stage of 2SIV, we use a probit regression to estimate the parameters of

$$y_{1i} = z_i \delta + v_i \lambda + e_i$$

Because  $v_i$  is unobservable, we use the sample residuals from the first-stage regressions.

$$\Pr(y_{1i} = 1 | z_i, v_i) = \Pr(z_i \delta + v_i \lambda + e_i > 0 | z_i, v_i) = \Phi \left\{ (1 - \rho^2)^{-\frac{1}{2}} (z_i \delta + v_i \lambda) \right\}$$

Hence, as mentioned previously, 2SIV and Newey's estimator do not estimate  $\delta$  and  $\lambda$  but rather

$$\delta_\rho = \frac{1}{(1 - \rho^2)^{\frac{1}{2}}} \delta \quad \text{and} \quad \lambda_\rho = \frac{1}{(1 - \rho^2)^{\frac{1}{2}}} \lambda$$

## Acknowledgments

The two-step estimator is based on the `probitiv` command written by Jonah Gelbach of the University of California at Berkeley Law School and the `ivprob` command written by Joe Harkness of the University of Connecticut.

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## Also see

- [R] [ivprobit postestimation](#) — Postestimation tools for `ivprobit`
- [R] [cfprobit](#) — Control-function probit regression
- [R] [gmm](#) — Generalized method of moments estimation
- [R] [ivfprobit](#) — Fractional probit model with continuous endogenous covariates
- [R] [ivregress](#) — Single-equation instrumental-variables regression
- [R] [ivtobit](#) — Tobit model with continuous endogenous covariates
- [R] [probit](#) — Probit regression
- [ERM] [eprobit](#) — Extended probit regression
- [SVY] [svy estimation](#) — Estimation commands for survey data
- [XT] [xtprobit](#) — Random-effects and population-averaged probit models
- [U] [20 Estimation and postestimation commands](#)

|                         |                                |                                      |                            |
|-------------------------|--------------------------------|--------------------------------------|----------------------------|
| Postestimation commands | <a href="#">predict</a>        | <a href="#">margins</a>              | <a href="#">estat</a>      |
| Remarks and examples    | <a href="#">Stored results</a> | <a href="#">Methods and formulas</a> | <a href="#">References</a> |
| Also see                |                                |                                      |                            |

## Postestimation commands

The following postestimation commands are of special interest after `ivprobit`:

| Command                           | Description                                                                                        |
|-----------------------------------|----------------------------------------------------------------------------------------------------|
| <code>estat classification</code> | report various summary statistics, including the classification table                              |
| <code>estat correlation</code>    | report the correlation matrix of the errors of the dependent variable and the endogenous variables |
| <code>estat covariance</code>     | report the covariance matrix of the errors of the dependent variable and the endogenous variables  |
| <code>lroc</code>                 | compute area under ROC curve and graph the curve                                                   |
| <code>lsens</code>                | graph sensitivity and specificity versus probability cutoff                                        |

These commands are not appropriate after the two-step estimator or the `svy` prefix.

The following standard postestimation commands are also available:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| * <code>estat ic</code>      | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| *† <code>forecast</code>     | dynamic forecasts and simulations                                                                                                       |
| † <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| *† <code>lrtest</code>       | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | linear predictions and their SEs, probabilities, etc.                                                                                   |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| * <code>suest</code>         | seemingly unrelated estimation                                                                                                          |

|                     |                                                      |
|---------------------|------------------------------------------------------|
| <code>test</code>   | Wald tests of simple and composite linear hypotheses |
| <code>testnl</code> | Wald tests of nonlinear hypotheses                   |

\*`estat ic`, `forecast`, `lrtest`, and `suest` are not appropriate after `ivprobit`, `twostep`.

†`forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results.

## predict

### Description for predict

`predict` creates a new variable containing predictions such as structural functions, linear predictions, standard errors, and probabilities.

### Menu for predict

Statistics > Postestimation

### Syntax for predict

After ML

```
predict [type] newvar [if] [in] [, statistic asfmethod rules asif]
```

```
predict [type] { stub* | newvarlist } [if] [in], scores
```

After twostep

```
predict [type] newvar [if] [in] [, twostep_statistic]
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

Main

|                   |                                                      |
|-------------------|------------------------------------------------------|
| <code>xb</code>   | linear prediction excluding endogeneity; the default |
| <code>pr</code>   | probability of a positive outcome                    |
| <code>stdp</code> | standard error of the linear prediction              |

| <i>asfmethod</i> | Description |
|------------------|-------------|
|------------------|-------------|

Main

|                       |                                          |
|-----------------------|------------------------------------------|
| <code>asf</code>      | average structural function; the default |
| <code>fixedasf</code> | fixed average structural function        |

| <i>twostep_statistic</i> | Description |
|--------------------------|-------------|
|--------------------------|-------------|

Main

|                   |                                         |
|-------------------|-----------------------------------------|
| <code>xb</code>   | linear prediction; the default          |
| <code>stdp</code> | standard error of the linear prediction |

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

## Options for predict

Main

`xb`, the default, calculates the linear prediction.

`pr` calculates the probability of a positive outcome. Results depend on how the endogeneity complication is handled, which is determined by the `asf` or `fixedasf` option. `pr` is not available with the two-step estimator.

`stdp` calculates the standard error of the linear prediction.

`asf` and `fixedasf` determine how the average structural function (ASF) of the specified statistic is computed. These options are not allowed with `xb` or `stdp`.

`asf` is the default for the ML estimator when the `pr` statistic is specified. `asf` computes the ASF of the specified statistic. It is the statistic conditional on the errors of the endogenous variable equations. Put another way, it is the statistic accounting for the correlation of the endogenous covariates with the errors of the main equation. Derivatives and contrasts based on `asf` have a structural interpretation. See [margins](#) for computing derivatives and contrasts.

`fixedasf` calculates a fixed ASF. It is the specified statistic using only the coefficients and variables of the outcome equation. `fixedasf` does not condition on the errors of the endogenous variable equations. Contrasts between two fixed counterfactuals averaged over the whole sample have a potential-outcome interpretation. Intuitively, it is as if the values of the covariates were fixed at a value exogenously by fiat. See [margins](#) for computing derivatives and contrasts.

To be clear, derivatives and contrasts between two fixed counterfactuals using the default `asf` option also have a potential-outcome interpretation. And, unlike `fixedasf`, they retain that interpretation when computed over subpopulations for both linear and nonlinear models.

`rules` requests that Stata use any rules that were used to identify the model when making the prediction. By default, Stata calculates missing for excluded observations. `rules` is not available with the two-step estimator.

`asif` requests that Stata ignore the rules and the exclusion criteria and calculate predictions for all observations possible using the estimated parameters from the model. `asif` is not available with the two-step estimator.

`scores`, not available with `twostep`, calculates equation-level score variables.

For models with one endogenous regressor, four new variables are created.

The first new variable will contain  $\partial \ln L / \partial (z_i \delta)$ .

The second new variable will contain  $\partial \ln L / \partial (x_i \Pi)$ .

The third new variable will contain  $\partial \ln L / \partial \operatorname{atanh} \rho$ .

The fourth new variable will contain  $\partial \ln L / \partial \ln \sigma$ .

For models with  $p$  endogenous regressors,  $p + \{(p + 1)(p + 2)\}/2$  new variables are created.

The first new variable will contain  $\partial \ln L / \partial (z_i \delta)$ .

The second through  $(p + 1)$ th new score variables will contain  $\partial \ln L / \partial (x_i \Pi_k)$ ,  $k = 1, \dots, p$ , where  $\Pi_k$  is the  $k$ th column of  $\Pi$ .

The remaining score variables will contain the partial derivatives of  $\ln L$  with respect to  $s_{21}, s_{31}, \dots, s_{p+1,1}, s_{22}, \dots, s_{p+1,2}, \dots, s_{p+1,p+1}$ , where  $s_{m,n}$  denotes the  $(m, n)$  element of the Cholesky decomposition of the error covariance matrix.

## margins

### Description for margins

`margins` estimates margins of response for linear predictions and probabilities.

### Menu for margins

Statistics > Postestimation

### Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

#### After ML

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

##### Main

|                   |                                                      |
|-------------------|------------------------------------------------------|
| <code>xb</code>   | linear prediction excluding endogeneity; the default |
| <code>pr</code>   | probability of a positive outcome                    |
| <code>stdp</code> | not allowed with margins                             |

#### After twostep

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

##### Main

|                   |                                |
|-------------------|--------------------------------|
| <code>xb</code>   | linear prediction; the default |
| <code>stdp</code> | not allowed with margins       |

Statistics not allowed with margins are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [\[R\] margins](#).

## estat

### Description for estat

`estat correlation` displays the correlation matrix of the errors of the dependent variable and the endogenous variables.

`estat covariance` displays the covariance matrix of the errors of the dependent variable and the endogenous variables.

`estat correlation` and `estat covariance` are not allowed after the `ivprobit` two-step estimator.

### Menu for estat

Statistics > Postestimation

### Syntax for estat

*Correlation matrix*

```
estat correlation [, border(bspec) left(#) format(%fmt)]
```

*Covariance matrix*

```
estat covariance [, border(bspec) left(#) format(%fmt)]
```

### Options for estat

Main

`border`(*bspec*) sets border style of the matrix display. The default is `border(a11)`.

`left`(#) sets the left indent of the matrix display. The default is `left(2)`.

`format`(%*fmt*) specifies the format for displaying the individual elements of the matrix. The default is `format(%9.0g)`.

## Remarks and examples

Remarks are presented under the following headings:

*Marginal effects*

*Obtaining predicted values*

### Marginal effects

Below, we discuss the interpretation of the predicted probability,  $\text{pr}$ , with the `asf` and `fixedasf` options for the ML estimator using `margins`.

The model is defined by two equations. The first is the equation for the outcome of interest, given by

$$y_{1i}^* = \mathbf{y}_{2i}\boldsymbol{\beta} + \mathbf{x}_{1i}\boldsymbol{\gamma} + u_i$$

where we do not observe  $y_{1i}^*$ ; instead, we observe

$$y_{1i} = \begin{cases} 0 & y_{1i}^* < 0 \\ 1 & y_{1i}^* \geq 0 \end{cases}$$

The second is the equation for the endogenous covariates,  $\mathbf{y}_{2i}$ ,

$$\mathbf{y}_{2i} = \mathbf{x}_{1i}\boldsymbol{\Pi}_1 + \mathbf{x}_{2i}\boldsymbol{\Pi}_2 + \mathbf{v}_i$$

This last equation is the difference between a standard probit model and the model fit by `ivprobit`.  $\mathbf{y}_{2i}$  is modeled as an exogenous component,  $\mathbf{x}_{1i}\boldsymbol{\Pi}_1 + \mathbf{x}_{2i}\boldsymbol{\Pi}_2$ , and a component that is correlated with  $u_i$  and causes the endogeneity problem,  $\mathbf{v}_i$ . The ASF predicted probability conditions on an estimate of  $\hat{\mathbf{v}}_i$ . It is given by

$$\begin{aligned} \Phi(\hat{m}_i) &= \hat{P}(y_{1i} | \mathbf{x}_{1i}, \mathbf{x}_{2i}, y_{2i}, \hat{\mathbf{v}}_i) \\ \hat{m}_i &= \mathbf{y}_{2i}\hat{\boldsymbol{\theta}}_1 + \mathbf{x}_{1i}\hat{\boldsymbol{\theta}}_2 + \hat{\mathbf{v}}_i\hat{\boldsymbol{\theta}}_3 \end{aligned}$$

Because the correlation between  $\mathbf{v}_i$  and  $u_i$  is the problem we intended to address, conditioning on  $\mathbf{v}_i$  purges the model of endogeneity. Using the ASF, we can get derivatives and contrast. See [Wooldridge \(2010\)](#) and [Blundell and Powell \(2003\)](#) for an in-depth discussion of ASFs and their interpretation.

The fixed ASF, estimated when the `fixedasf` option is specified, has a different interpretation. Suppose we wanted to analyze  $1(\mathbf{y}_{2i}\boldsymbol{\beta} + \mathbf{x}_{1i}\boldsymbol{\gamma} + u_i > 0)$  at two different values of  $\mathbf{y}_2$ , the original  $\mathbf{y}_2$  and  $\mathbf{y}_2 + 1$ .  $1(\cdot)$  is an indicator function that evaluates to 1 if the condition in parentheses is satisfied and 0 otherwise. We want the average difference at these two points for the given values of the other covariates. The values of the covariates are not arrived at via the model; they are fixed by fiat. You can think of them as potential outcomes. The difference of the two values of  $\mathbf{y}_2$  is given by

$$1\{(\mathbf{y}_{2i} + 1)\boldsymbol{\beta} + \mathbf{x}_{1i}\boldsymbol{\gamma} + u_i > 0\} - 1(\mathbf{y}_{2i}\boldsymbol{\beta} + \mathbf{x}_{1i}\boldsymbol{\gamma} + u_i > 0)$$

If we average over the distribution of  $u$ , we obtain

$$\Phi\{(\mathbf{y}_{2i} + 1)\boldsymbol{\beta} + \mathbf{x}_{1i}\boldsymbol{\gamma}\} - \Phi(\mathbf{y}_{2i}\boldsymbol{\beta} + \mathbf{x}_{1i}\boldsymbol{\gamma})$$

We do not account for endogeneity because the values of the covariates are given and fixed. If the research question you are pursuing after fitting the model has this interpretation, `fixedasf` gives you an adequate prediction. If, however, you do not want to treat the covariates as fixed, you should account for endogeneity and use `asf` predictions.

### ▷ Example 1

We can obtain marginal effects by using the `margins` command after `ivprobit`. Continuing with [example 1](#) in [\[R\] ivprobit](#), we calculate the difference in the probability of a woman working, `fem_work`, if `other_inc` increases by 10% versus the probability when `other_inc` is unchanged. The effect we get has an ASF interpretation. The probabilities are estimated conditional on the residual from the endogenous variable. In other words, the computed effects condition on the level of endogeneity in the model. See [Wooldridge \(2010\)](#) for a discussion about the interpretation of the estimates and the computation of marginal effects of probit estimators under endogeneity.

```
. use https://www.stata-press.com/data/r19/laborsup
. ivprobit fem_work fem_educ kids (other_inc = male_educ)
 (output omitted)
. margins, at(other_inc = generate(other_inc))
> at(other_inc = generate(other_inc*1.10))
> contrast(at(r) nowald) predict(pr)

Contrasts of predictive margins Number of obs = 500
Model VCE: OIM

Expression: Average structural function probabilities, predict(pr)
1. _at: other_inc = other_inc
2. _at: other_inc = other_inc*1.10
```

|                 | Delta-method |           |           | [95% conf. interval] |
|-----------------|--------------|-----------|-----------|----------------------|
|                 | Contrast     | std. err. |           |                      |
| _at<br>(2 vs 1) | -.0762151    | .0100472  | -.0959073 | -.0565229            |

Here we see that a 10% increase in `other_inc` leads to an average decrease of 0.076 in the probability of `fem_work`. The effect we get has an ASF interpretation. The probabilities are estimated conditional on the residual from the endogenous variable. In other words, the computed effects condition on the level of endogeneity in the model. See [Wooldridge \(2010\)](#) for a discussion about the interpretation of the estimates and the computation of marginal effects of probit estimators under endogeneity.

## Obtaining predicted values

After fitting your model with `ivprobit`, you can obtain the linear prediction and its standard error for both the estimation sample and other samples by using the `predict` command; see [U] 20 Estimation and postestimation commands and [R] `predict`. If you use the maximum likelihood estimator, you can also obtain the predicted probability or the linear prediction with an ASF or fixed ASF interpretation.

`predict`'s `pr` option calculates the probability of a positive outcome, remembering any rules used to identify the model, and calculates missing for excluded observations. `predict`'s `rules` option uses the rules in predicting probabilities, whereas `predict`'s `asif` option ignores both the rules and the exclusion criteria and calculates probabilities for all possible observations by using the estimated parameters from the model. See *Obtaining predicted values* in [R] `probit postestimation` for an example.

## Stored results

`estat correlation` stores the following results in `r()`:

Matrices

`r(corr)` correlation matrix of the errors

`estat covariance` stores the following results in `r()`:

Matrices

`r(cov)` covariance matrix of the errors

## Methods and formulas

The linear prediction is calculated as  $z_i \hat{\delta}$ , where  $\hat{\delta}$  is the estimated value of  $\delta$ , and  $z_i$  and  $\delta$  are defined in (1a) of [R] `ivprobit`. The probability of a positive outcome is evaluated at  $m_i$ ,  $\Phi(m_i)$ , where  $\Phi(\cdot)$  is the standard normal distribution function and  $m_i$  is defined in *Methods and formulas* of [R] `ivprobit`. The ASF uses  $\hat{m}_i$  instead of  $y_{2i} \hat{\beta} + x_{1i} \hat{\gamma}$  to evaluate  $\Phi(\cdot)$  and account for endogeneity in the model. The fixed ASF is evaluated at  $y_{2i} \hat{\beta} + x_{1i} \hat{\gamma}$ .

## References

Blundell, R. W., and J. L. Powell. 2003. "Endogeneity in nonparametric and semiparametric regression models". In *Theory and Applications: Eighth World Congress*. Advances in Economics and Econometrics, edited by M. Dewatripont, L. P. Hansen, and S. J. Turnovsky, vol. 2: 312–357. Cambridge: Cambridge University Press. <https://doi.org/10.1017/CBO9780511610257.011>.

Wooldridge, J. M. 2010. *Econometric Analysis of Cross Section and Panel Data*. 2nd ed. Cambridge, MA: MIT Press.

## Also see

[R] `ivprobit` — Probit model with continuous endogenous covariates

[R] `estat classification` — Classification statistics and table

[R] `lroc` — Compute area under ROC curve and graph the curve

[R] `lsens` — Graph sensitivity and specificity versus probability cutoff

[U] 20 Estimation and postestimation commands

[Description](#)[Options](#)[Acknowledgments](#)[Quick start](#)[Remarks and examples](#)[References](#)[Menu](#)[Stored results](#)[Also see](#)[Syntax](#)[Methods and formulas](#)

## Description

`ivqregress` fits a linear instrumental-variables quantile regression (IVQR) model that accounts for endogenous covariates using two estimators: the inverse quantile regression (IQR) estimator proposed in [Chernozhukov and Hansen \(2006\)](#) and the smoothed estimating equations (SEE) estimator outlined in [Kaplan and Sun \(2017\)](#).

## Quick start

Use the IQR estimator to fit the median IVQR model of  $y_1$  on exogenous  $x_1$  and endogenous  $y_2$  with instruments  $z_1$  and  $z_2$

```
ivqregress iqr y1 x1 (y2 = z1 z2)
```

Same as above, but estimate the 0.75 quantile

```
ivqregress iqr y1 x1 (y2 = z1 z2), quantile(0.75)
```

Same as above, but estimate the 0.1, 0.2, ..., 0.9 quantiles

```
ivqregress iqr y1 x1 (y2 = z1 z2), quantile(10(10)90)
```

Use the SEE estimator to estimate the 0.6 quantile regression of  $y_1$  on exogenous  $x_1$  and endogenous  $y_2$  and  $y_3$  with instruments  $z_1$  and  $z_2$

```
ivqregress smooth y1 x1 (y2 y3 = z1 z2), quantile(0.6)
```

Same as above, but estimate the 0.1, 0.2, ..., 0.9 quantiles

```
ivqregress smooth y1 x1 (y2 y3 = z1 z2), quantile(10(10)90)
```

### *IQR options to control optimization*

Use 50 grid points in the IQR estimator to fit the 0.5 and 0.75 IVQR model

```
ivqregress iqr y1 x1 (y2 = z1 z2), ngrid(50) quantile(50 75)
```

Same as above, but construct grid points between 1 and 5 for all the quantiles

```
ivqregress iqr y1 x1 (y2 = z1 z2), ngrid(50) quantile(50 75) bound(1 5)
```

Same as above, but construct grid points using different bounds for different quantiles

```
ivqregress iqr y1 x1 (y2 = z1 z2), ngrid(50) quantile(50 75) ///
bound(1 5, at(50)) bound(2 6, at(75))
```

*SEE options to control optimization*

Use 2 as the initial bandwidth in the SEE estimator to fit the 0.5 and 0.75 IVQR model

```
ivregress smooth y x1 (d1 d2 = z1 z2), quantile(50 75) ///
 initbwidth(2)
```

Same as above, but use different initial bandwidths for different quantiles

```
ivregress smooth y x1 (d1 d2 = z1 z2), quantile(50 75) ///
 initbwidth(2, at(50)) initbwidth(1, at(75))
```

## Menu

Statistics > Endogenous covariates > Quantile regression with endogenous covariates

## Syntax

*Inverse quantile regression (IQR) estimator*

```
ivregress iqr depvar [varlist1] (varname = varlistiv) [if] [in] [, options
 IQR_options]
```

*Smoothed estimating equations (SEE) estimator*

```
ivregress smooth depvar [varlist1] (varlist2 = varlistiv) [if] [in] [, options
 SEE_options]
```

*varlist*<sub>1</sub> is the list of exogenous variables.

*varname* is an endogenous variable.

*varlist*<sub>2</sub> is the list of endogenous variables.

*varlist*<sub>iv</sub> is the list of exogenous variables used with *varlist*<sub>1</sub> as instruments for *varlist*<sub>2</sub> and *varname*.

| <i>options</i>                                                                                                                                                                 | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Model<br><u>quantile</u> ( <i>numlist</i> )                                                                                                                                    | estimate quantiles specified in <i>numlist</i> ; default is <code>quantile(0.5)</code>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| SE/Robust<br><u>vce</u> ([ <i>vcetype</i> ][, <i>vceopts</i> ])                                                                                                                | technique used to estimate standard errors; <i>vcetype</i> may be <u>robust</u> (the default) or <u>bootstrap</u>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
| Reporting<br><u>level</u> (#)<br><i>display_options</i>                                                                                                                        | set confidence level; default is <code>level(95)</code><br>control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| Optimization<br>[ <u>no</u> ]log<br><u>verbose</u><br><u>coeflegend</u>                                                                                                        | suppress or display the iteration log<br>display a verbose iteration log<br>display legend instead of statistics                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| <i>IQR_options</i>                                                                                                                                                             | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| Options<br><u>bound</u> (# <sub>min</sub> # <sub>max</sub> [, <u>at</u> (# <sub>q</sub> )] )<br><u>ngrid</u> (# <sub>g</sub> )                                                 | specify the lower and upper bounds for the grid in the # <sub>q</sub> th quantile estimation; may be repeated<br>use # <sub>g</sub> grid points; default is <code>ngrid(30)</code>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| <i>SEE_options</i>                                                                                                                                                             | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| Options<br><u>initbwidth</u> (# <sub>b</sub> [, <u>at</u> (# <sub>q</sub> )] )<br><u>iterate</u> (#)<br><u>nosearchbwidth</u><br><u>tolerance</u> (#)<br><u>ztolerance</u> (#) | specify initial bandwidth # <sub>b</sub> to smooth the estimating equations for the # <sub>q</sub> th quantile estimation; default is the theoretical optimal bandwidth; may be repeated<br>perform maximum of # iterations when solving the estimating equation; default is <code>iterate(100)</code><br>do not search for feasible bandwidth if the initial bandwidth is not feasible; default is to search for feasible bandwidth<br>specify the tolerance for the coefficient vector; default is <code>tolerance(1e-9)</code><br>specify the tolerance to determine whether the proposed solution for a zero-finding problem is sufficiently close to 0; default is <code>ztolerance(1e-9)</code> |
| <i>vceopts</i>                                                                                                                                                                 | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| <u>kernel</u> ( <i>kernel</i> )<br><u>bwidth</u> (#  <i>bwrule</i> )                                                                                                           | use a nonparametric kernel density estimator;<br>default is <code>epanechnikov</code><br>specify the bandwidth to be used by the kernel density estimator;<br>default is <code>silverman</code> , which is Silverman's rule of thumb                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |

| <i>kernel</i>        | Description                               |
|----------------------|-------------------------------------------|
| <u>e</u> panechnikov | Epanechnikov kernel function; the default |
| <u>e</u> pan2        | alternative Epanechnikov kernel function  |
| <u>b</u> iweight     | biweight kernel function                  |
| <u>c</u> osine       | cosine trace kernel function              |
| <u>g</u> aussian     | Gaussian kernel function                  |
| <u>p</u> arzen       | Parzen kernel function                    |
| <u>r</u> ectangle    | rectangle kernel function                 |
| <u>t</u> riangle     | triangle kernel function                  |

| <i>bwrule</i>     | Description                            |
|-------------------|----------------------------------------|
| <u>s</u> ilverman | Silverman's rule of thumb; the default |
| <u>h</u> sheather | Hall–Sheather's bandwidth              |
| <u>b</u> ofinger  | Bofinger's bandwidth                   |

*varlist*<sub>1</sub>, *varname*, *varlist*<sub>2</sub>, and *varlist*<sub>iv</sub> may contain factor variables; see [U] 11.4.3 Factor variables.

*bootstrap*, *by*, *collect*, *rolling*, and *statsby* are allowed; see [U] 11.1.10 Prefix commands.

*coeflegend* does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Options

### Model

*quantile*(*numlist*) specifies the quantiles to be estimated and should contain numbers between 0 and 1, exclusive. Numbers larger than 1 are interpreted as percentages. The default is *quantile*(0.5), which corresponds to the median.

### Options

The following options apply only to the IQR estimator.

*bound*(*#*<sub>min</sub> *#*<sub>max</sub> [ , *at*(*#*<sub>q</sub>) ]) specifies the lower bound (*#*<sub>min</sub>) and the upper bound (*#*<sub>max</sub>) for the grid in the *#*<sub>q</sub>th quantile estimation. By default, the bounds are determined by the two-stage quantile regression, extending the two-stage median regression in Amemiya (1982). This option is repeatable as long as different quantiles *#*<sub>q</sub> are given in each specification.

The specified bound is required to be wider than the *#*<sub>level</sub> confidence interval (CI) that is robust to the weak instruments, which is also known as dual CI. The value of *#*<sub>level</sub> can be specified in the *level*() option; the default is 95% CI.

The grid points are *#*<sub>g</sub> equally spaced points between *#*<sub>min</sub> and *#*<sub>max</sub>, where *#*<sub>g</sub> is specified by the *ngrid*() option.

*ngrid*(*#*<sub>g</sub>) specifies the number of grid points in the IQR estimator. The default is *ngrid*(30); that is, 30 grid points are used.

The following options apply only to the SEE estimator.

`initbwidth(#b [ , at(#q )])` specifies initial bandwidth #<sub>b</sub> to smooth the estimating equations for the #<sub>q</sub>th quantile estimation. The default is the theoretical optimal bandwidth that minimizes the mean squared errors of the estimating equations; see [Kaplan and Sun \(2017\)](#). This option is repeatable as long as different quantiles #<sub>q</sub> are given in each specification.

`iterate(#)` specifies the maximum number of iterations to perform when solving the estimating equation; the default is `iterate(100)`.

`nosearchbwidth` specifies to not search for a feasible bandwidth if the initial bandwidth is not estimable; the default is to search for a feasible bandwidth.

`tolerance(#)` specifies the tolerance used to determine whether successive estimates of the solution have converged. The default is `tolerance(1e-9)`.

`ztolerance(#)` specifies the tolerance used to determine whether the proposed solution to a zero-finding problem is sufficiently close to 0; the default is `ztolerance(1e-9)`.

---

#### SE/Robust

`vce([vcetype] [ , vceopts])` specifies the type of VCE to compute and the density estimation method to use in computing the VCE.

`vcetype` specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (`robust`) and that use bootstrap methods (`bootstrap`); see [\[R\] vce\\_option](#).

`vceopts` available with `vcetype robust` are the following:

`kernel(kernel)` specifies the kernel method to be used by the nonparametric density estimator. The available kernel functions are `epanechnikov`, `epan2`, `biweight`, `cosine`, `gaussian`, `parzen`, `rectangle`, and `triangle`. The default is `epanechnikov`. See [\[R\] kdensity](#) for the kernel function forms.

`bwidth(#|bwrule)` specifies the bandwidth to be used by the nonparametric density estimator. If specified as a number, it is used as the bandwidth for the nonparametric density estimator. Otherwise, `bwrule` specifies the method used to compute the bandwidth. Available methods are `silverman` for Silverman's rule of thumb, `hsheather` for the Hall–Sheather bandwidth, and `bofinger` for the Bofinger bandwidth.

See [\[R\] kdensity](#) for Silverman's rule of thumb. See [Koenker \(2005, sec. 4.10\)](#) for a description of the Hall–Sheather and Bofinger bandwidth formulas.

---

#### Reporting

`level(#)`; see [\[R\] Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [\[R\] Estimation options](#).

## Optimization

`log` and `nolog` specify whether to display the log showing the progress of the estimation. By default, for the IQR estimator, one dot is shown for each grid point; for the SEE estimator, one line is shown for each bandwidth. The iteration log is displayed by default unless you used `set iterlog off` to suppress it; see `set iterlog` in [R] [set iter](#).

`verbose` displays a verbose log showing the iterations of each computation step. For the IQR estimator, each line is shown for each grid point. For the SEE estimator, iteration logs are shown when solving the estimating equations.

The following option is available with `ivqregress` but is not shown in the dialog box: `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Remarks are presented under the following headings:

[Overview](#)

[When quantile regression matters](#)

[Examples](#)

## Overview

`ivqregress` fits a linear IVQR model when some covariates are endogenous. The general IVQR model was first proposed by [Chernozhukov and Hansen \(2005\)](#). `ivqregress` is based on the linear IVQR model described in [Chernozhukov and Hansen \(2006, 2008\)](#). For an introduction to the IVQR model, see [Chernozhukov, Hansen, and Wüthrich \(2018\)](#). `ivqregress` implements two estimators: the IQR estimator proposed in [Chernozhukov and Hansen \(2006\)](#) and the SEE estimator outlined in [Kaplan and Sun \(2017\)](#).

In empirical applications, we are usually interested in the effects of some covariate on the outcome variable. The traditional linear regression model is an excellent way to model how the covariate affects the outcome's conditional mean. However, sometimes we would like to study features of the outcome distribution other than the mean to have a complete picture of the effects of covariates. For example, a policymaker may want to learn how participation in a 401(k) would affect the lower-level, median, and upper-level conditional quantiles of net wealth.

Quantile regression in [Koenker and Bassett \(1978\)](#) can help us grasp a better picture than regular linear regression by estimating the effects on different quantiles of the outcome's conditional distribution. For a general discussion, see [R] [qreg](#). For an illustration of when quantile regression matters, see [When quantile regression matters](#) below.

In practice, some covariates of interest are often endogenous for reasons such as self-selection, omission of some relevant variable, and measurement error. For example, participation in a voluntary savings plan for retirement, such as participation in a 401(k) program, may be endogenous because the people who do and do not participate may have different saving preferences, which will affect net wealth growth.

Endogenous covariates make quantile regression estimates inconsistent, as is the case for the linear regression model. Analogous to the instrumental-variable least-squares estimator, there are IVQR model estimators to consistently estimate the effects at different quantiles. For a discussion of instrumental-variables estimation, see [R] [ivregress](#).

`ivqregress` fits a quantile regression model that accounts for endogenous covariates using two estimators: the IQR estimator proposed in Chernozhukov and Hansen (2006) and the SEE estimator outlined in Kaplan and Sun (2017). Intuitively, `ivqregress` can be thought of as the `ivregress` version of `qreg`.

Here we outline the Stata commands to fit, visualize, infer, and diagnose the IVQR model. In particular, these Stata commands can be grouped into the following categories.

**Estimation:** `ivqregress iqr` fits the IVQR model by the IQR estimator proposed in Chernozhukov and Hansen (2006, 2008).

`ivqregress smooth` fits the IVQR model by the SEE estimator proposed in Kaplan and Sun (2017).

**Visualization:** `estat coefplot` allows us to visualize how one covariate's effects vary at different quantiles of the outcome.

**Inference:** `estat endogeffects` tests if

1. the endogenous variable does not affect the outcome variable,
2. the effects of the endogenous variable do not vary across estimated quantiles,
3. the effects of the endogenous variable are greater than zero across estimated quantiles, and
4. the variable is exogenous instead of endogenous.

`estat dualci` provides CIs that are robust to weak instruments for the effects of the endogenous variable. It is allowed only after `ivqregress iqr`.

**Diagnosis:** `estat waldplot` helps diagnose the convergence of the IQR estimator (`ivqregress iqr`). In particular, `estat waldplot` allows us to visualize the optimization process during the computation in `ivqregress iqr` and shows if the searching domain contains the true value of the parameter with a predefined probability level.

In addition, some other classical postestimation tools are also available; see [R] [ivqregress postestimation](#).

## When quantile regression matters

Here is an example illustrating the advantages of quantile regressions. Suppose we have a simple model  $E(y|x) = \beta_0 + x\beta_1$ , where  $y$  is the outcome variable and  $x$  is a covariate. For simplicity, we assume  $x$  can only take values in  $\{0, 1, 2, 3, 4, 5, 6\}$ . By definition,  $\beta_1$  fully characterizes the effects of increasing one unit of  $x$  on the conditional mean of outcome  $y$ ; that is,  $\beta_1 = E(y|x = a + 1) - E(y|x = a)$ . Now we consider two scenarios of the data-generating process.

1. The probability density function of the outcome conditional on  $x = a + 1$ ,  $f(y|x = a + 1)$ , is only location shifted relative to  $f(y|x = a)$ . In this case,  $\beta_1$  summarizes the effect of  $x$  not only on the conditional mean but also on each conditional quantile of  $y$ . This case is illustrated in the left panel of [figure 1](#).

2. The probability density function of the outcome conditional on  $x = a + 1$ ,  $f(y|x = a + 1)$ , is both location shifted and rescaled relative to  $f(y|x = a)$ . In this case,  $\beta_1$  summarizes the effect of  $x$  only on the conditional mean but not on conditional quantiles of  $y$ . This case is illustrated in the right panel of [figure 1](#).

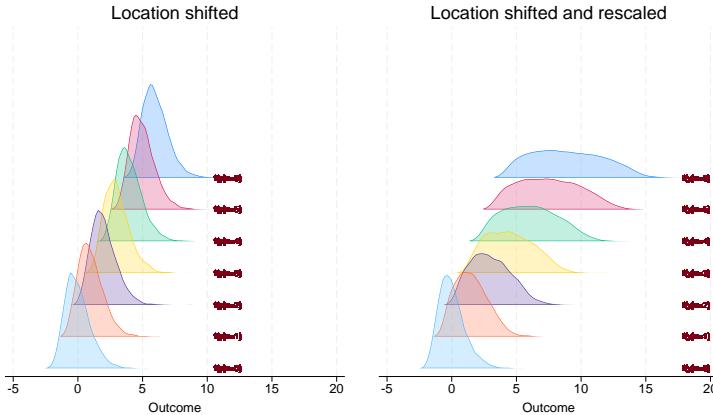


Figure 1.

In the left panel, we see that each conditional density is parallel relative to each other, and only the location has been shifted. In this case,  $\beta_1$  captures the shift in both conditional mean and any conditional quantiles of the outcome. As a result, running a linear regression provides as much information on  $\beta_1$  as quantile regression.

In contrast, in the right panel, conditional density for each level of  $x$  has different locations and different shapes. Thus,  $\beta_1$  can only summarize the shifts in conditional mean, which are generally different from the shifts in conditional quantiles. Quantile regression becomes necessary to learn about the effects of  $x$  on the conditional quantiles of the outcome.

## Examples

### ► Example 1: IVQR with the IQR estimator

Suppose that we want to estimate the effect of 401(k) participation (`p401k`) on different conditional quantiles of net financial assets (`assets`). We use data reported by [Chernozhukov and Hansen \(2004\)](#). These data are from a sample of households in the 1990 Survey of Income and Program Participation (SIPP). For the head of household, we have data on income (`income`), age (`age`), number of people in the family (`familysize`), years of education (`educ`), marital status (`married`), whether participated in an IRA (`ira`), whether received a pension benefit (`pension`), and whether owned a home (`ownhome`).

We suspect 401(k) participation is endogenous because it may depend on unobserved factors such as saving preference that also impact financial assets. Using 401(k) eligibility (`e401k`) as an instrument for 401(k) participation, we use `ivqregress` to estimate how `p401k` affects the entire range of `assets`' conditional distribution. One concern about using `e401k` as an instrument is that choosing to work for a company that offers a 401(k) program is not randomly assigned. [Poterba, Venti, and Wise \(1995\)](#) suggest that after conditioning on income, we can take working for a company that offers a 401(k) plan as exogenous.

The IVQR model we want to fit is

$$\text{assets}_i = \text{p401k}_i \alpha(U) + \text{covariates}'_i \beta(U)$$

where the distribution of  $U$  conditional on the instrument `e401k` and the covariates is assumed to be uniform between 0 and 1. The covariates `income`, `age`, `familysize`, and `educ` are included in the model as continuous variables. The covariates `i.married`, `i.ira`, `i.pension`, and `i.ownhome` are included as categorical (factor) variables. As discussed above, `e401k` is the instrument for `p401k`. The coefficients  $\alpha(U)$  and  $\beta(U)$  are random because they depend on the unobserved random variable  $U$ . In practice,  $U$  can be considered a ranking variable for the asset. When  $U$  is set to a fixed level  $\tau$ , we fit an IVQR model at a specific quantile index  $\tau$ . For example, when  $\tau = 0.5$ , we estimate how 401(k) participation affects the median of net financial assets conditional on other covariates.

The objective of the analysis is to estimate the quantile treatment effects of 401k participation on net financial assets. By definition, the  $\tau$ th conditional quantile of the asset when everyone participates in a 401(k) plan is

$$\text{assets}_{401(k)} = \alpha(\tau) + \text{covariates}'_i \beta(\tau)$$

In contrast, the  $\tau$ th conditional quantile of the asset when everyone does not participate in a 401(k) plan is

$$\text{assets}_{\text{no } 401(k)} = \text{covariates}'_i \beta(\tau)$$

Thus, the coefficient  $\alpha(\tau)$  can fully summarize the quantile treatment effect of `p401k` on `assets`. That is

$$\alpha(\tau) = \text{assets}_{401(k)} - \text{assets}_{\text{no } 401(k)}$$

In this example, we use the IQR estimator (`ivqregress iqr`) to estimate the effect of 401(k) participation on the conditional median of the net financial assets. The dependent variable is `assets`. The endogenous variable `i.p401k` and the instrument `i.e401k` are specified in parentheses; the other covariates follow as a regular variable list. `ivqregress` fits the IV median regression model by default. The estimation result is stored as `est_iqr` for later use.

```

. use https://www.stata-press.com/data/r19/assets2
(Excerpt from Chernozhukov and Hansen (2004))
. ivqregress iqr assets (i.p401k = i.e401k) income age familysize
> i.married i.ira i.pension i.ownhome educ
Initial grid:
Quantile = 0.50:10.....20.....30 done
Adaptive grid:
Quantile = 0.50:10.....20.....30 done
IV median regression Number of obs = 9,913
Estimator: Inverse quantile regression Wald chi2(9) = 1289.75
 Prob > chi2 = 0.0000

```

| assets      | Coefficient | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |           |
|-------------|-------------|---------------------|-------|-------|----------------------|-----------|
| p401k       |             |                     |       |       |                      |           |
| Yes         | 5313.397    | 573.2818            | 9.27  | 0.000 | 4189.786             | 6437.009  |
| income      | .1577512    | .0124889            | 12.63 | 0.000 | .1332735             | .1822289  |
| age         | 99.96526    | 8.561923            | 11.68 | 0.000 | 83.1842              | 116.7463  |
| familysize  | -197.8251   | 54.36773            | -3.64 | 0.000 | -304.3838            | -91.26627 |
| married     |             |                     |       |       |                      |           |
| Married     | -1359.124   | 227.3366            | -5.98 | 0.000 | -1804.696            | -913.5528 |
| ira         |             |                     |       |       |                      |           |
| Yes         | 22629.61    | 1022.706            | 22.13 | 0.000 | 20625.15             | 24634.08  |
| pension     |             |                     |       |       |                      |           |
| Receives .. | -693.8347   | 210.6176            | -3.29 | 0.001 | -1106.638            | -281.0317 |
| ownhome     |             |                     |       |       |                      |           |
| Yes         | -30.29657   | 154.7265            | -0.20 | 0.845 | -333.555             | 272.9618  |
| educ        | -96.43983   | 32.09465            | -3.00 | 0.003 | -159.3442            | -33.53547 |
| _cons       | -4998.673   | 570.1315            | -8.77 | 0.000 | -6116.11             | -3881.236 |

```

Endogenous: 1.p401k
Exogenous: income age familysize 1.married 1.ira 1.pension 1.ownhome educ
 1.e401k
. estimates store est_iqr

```

The coefficient for p401k is 5,313. It means participation in a 401(k) would increase the median net financial assets by \$5,313, conditional on other covariates, relative to a scenario where no one participates. We store the estimation result as `est_iqr` for later use.

After `ivqregress iqr`, we can use `estat dualci` to obtain the dual CI, which is robust to weak instruments, for the coefficient on the endogenous variable.

```

. estat dualci
Dual confidence interval Number of obs = 9,913

```

| assets | Coefficient | Robust<br>std. err. | z    | P> z  | Dual<br>[95% conf. interval] |          |
|--------|-------------|---------------------|------|-------|------------------------------|----------|
| p401k  |             |                     |      |       |                              |          |
| Yes    | 5313.397    | 573.2818            | 9.27 | 0.000 | 3683.916                     | 7304.986 |

The dual CI is usually wider than the regular CI, but it provides a more robust inference if the instruments are weak. In this example, we see that the dual 95% CI is [3684, 7305], which is wider than the regular 95% CI [4190, 6437].



### ▷ Example 2: IVQR with the smooth estimator

In this example, we use `ivqregress` to fit the IVQR model as in [example 1](#) but using the SEE estimator (`ivqregress smooth`). The model specification is the same as in [example 1](#). The estimation result is stored as `est_smooth` for later use.

```
. ivqregress smooth assets (i.p401k = i.e401k) income age familysize
> i.married i.ira i.pension i.ownhome educ
Fitting smoothed IV quantile regression:
Quantile = .5:
Step 1: Bandwidth = 1302.9736 GMM criterion Q(b) = 2.617e-08
Step 2: Bandwidth = 6079.6881 GMM criterion Q(b) = 2.391e-12
Step 3: Bandwidth = 1438.3068 GMM criterion Q(b) = 8.068e-13
IV median regression Number of obs = 9,913
Estimator: Smoothed estimating equations Wald chi2(9) = 1243.05
 Prob > chi2 = 0.0000
```

| assets      | Robust      |           |       |       | [95% conf. interval] |           |
|-------------|-------------|-----------|-------|-------|----------------------|-----------|
|             | Coefficient | std. err. | z     | P> z  |                      |           |
| p401k       |             |           |       |       |                      |           |
| Yes         | 5364.468    | 573.3728  | 9.36  | 0.000 | 4240.678             | 6488.258  |
| income      | .1679934    | .013419   | 12.52 | 0.000 | .1416925             | .1942942  |
| age         | 113.6318    | 9.352867  | 12.15 | 0.000 | 95.30052             | 131.9631  |
| familysize  | -228.7766   | 57.61072  | -3.97 | 0.000 | -341.6916            | -115.8617 |
| married     |             |           |       |       |                      |           |
| Married     | -1362.56    | 238.5988  | -5.71 | 0.000 | -1830.205            | -894.9153 |
| ira         |             |           |       |       |                      |           |
| Yes         | 22402.04    | 1043.504  | 21.47 | 0.000 | 20356.81             | 24447.27  |
| pension     |             |           |       |       |                      |           |
| Receives .. | -713.996    | 220.476   | -3.24 | 0.001 | -1146.121            | -281.8709 |
| ownhome     |             |           |       |       |                      |           |
| Yes         | -12.71396   | 161.3703  | -0.08 | 0.937 | -328.994             | 303.5661  |
| educ        | -102.2889   | 34.18527  | -2.99 | 0.003 | -169.2908            | -35.28701 |
| _cons       | -5672.645   | 619.7049  | -9.15 | 0.000 | -6887.244            | -4458.045 |

```
Endogenous: 1.p401k
Exogenous: income age familysize 1.married 1.ira 1.pension 1.ownhome educ
 1.e401k
. estimates store est_smooth
```

The interpretation of the coefficient estimates is the same as in [example 1](#). For example, the coefficient for `p401k` is 5,364. So participation in a 401(k) would increase the median of net financial assets by \$5,364, conditional on other covariates, relative to a scenario where no one participates.

Now we can compare the coefficient on p401k between the SEE estimator and the IQR estimator.

```
. estimates table est_iqr est_smooth, keep(i.p401k) se
```

| Variable | est_iqr   | est_smooth |
|----------|-----------|------------|
| p401k    | 5313.3974 | 5364.468   |
| Yes      | 573.28183 | 573.37279  |

Legend: b/se

We see that the point estimates from these two estimators are similar but not the same. It is normal to see different results from the IQR and SEE estimators because these two estimators approximate the original exact estimating equation differently. On one hand, the IQR estimator tries to find the solution by an exhaustive grid search. The estimation result critically depends on the range and finesse of grid points. On the other hand, the SEE estimator uses a kernel method to smooth the original estimating equation. Its result depends on how well the SEE approximates the original, mainly controlled by the bandwidth.

Both the IQR and SEE estimators have their advantages and weaknesses. The IQR estimator is numerically stable, and it allows computing the dual CI, which is robust to weak instruments (use `estat dualci`). However, the IQR becomes computationally intensive when there is more than one endogenous variable. Thus, `ivqregress iqr` allows only one endogenous variable. In contrast, the SEE estimator can handle multiple endogenous variables within a reasonable computation time. However, it does not allow `estat dualci` for inference that is robust to weak instruments. Suppose there is only one endogenous variable in the model. We recommend using both estimators, comparing the results, and using the IQR estimator as a benchmark because it can provide valid inference even if the instrument is weak. If there is more than one endogenous variable, only `ivqregress smooth` is available.

◀

### ► Example 3: IVQR at different quantiles

In the first two examples, we estimated the 401(k) participation (p401k) treatment effect on the conditional median of net financial assets (`assets`). From a policy designer's point of view, we may be more interested in estimating the treatment effect of p401k on other conditional quantiles of assets. For example, we can ask questions like 1) how 401(k) participation affects the lower quantile of assets and 2) whether 401(k) participation is unambiguously beneficial for the asset's lower and upper conditional quantiles. In addition, we might also want to know whether the 401(k) participation is endogenous in our model. In this example, we will show how to use `ivqregress` to fit the IVQR model at different quantiles and how to use the postestimation tools to answer the above questions.

First, we use the IQR estimator to fit the model at different quantiles. In particular, we specify the `quantile(10(10)90)` option to fit the IVQR model at the 10th, 20th, ..., 90th quantiles.

```
. ivregress iqr assets (i.p401k = i.e401k) income age familysize
> i.married i.ira i.pension i.ownhome educ, quantile(10(10)90)

Initial grid:
Quantile = 0.10:10.....20.....30 done
Quantile = 0.20:10.....20.....30 done
Quantile = 0.30:10.....20.....30 done
Quantile = 0.40:10.....20.....30 done
Quantile = 0.50:10.....20.....30 done
Quantile = 0.60:10.....20.....30 done
Quantile = 0.70:10.....20.....30 done
Quantile = 0.80:10.....20.....30 done
Quantile = 0.90:10.....20.....30 done

Adaptive grid:
Quantile = 0.10:10.....20.....30 done
Quantile = 0.20:10.....20.....30 done
Quantile = 0.30:10.....20.....30 done
Quantile = 0.40:10.....20.....30 done
Quantile = 0.50:10.....20.....30 done
Quantile = 0.60:10.....20.....30 done
Quantile = 0.70:10.....20.....30 done
Quantile = 0.80:10.....20.....30 done
Quantile = 0.90:10.....20.....30 done

IV quantile regression Number of obs = 9,913
Estimator: Inverse quantile regression Wald chi2(81) = 5121.46
 Prob > chi2 = 0.0000
```

|     | assets      | Coefficient | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |           |
|-----|-------------|-------------|---------------------|-------|-------|----------------------|-----------|
| q10 |             |             |                     |       |       |                      |           |
|     | p401k       |             |                     |       |       |                      |           |
|     | Yes         | 3240.08     | 475.6184            | 6.81  | 0.000 | 2307.885             | 4172.275  |
|     | income      | .0303072    | .0123138            | 2.46  | 0.014 | .0061725             | .0544419  |
|     | age         | 131.5908    | 15.13725            | 8.69  | 0.000 | 101.9223             | 161.2592  |
|     | familysize  | -329.2838   | 123.4665            | -2.67 | 0.008 | -571.2737            | -87.29385 |
|     | married     |             |                     |       |       |                      |           |
|     | Married     | -1504.648   | 380.0373            | -3.96 | 0.000 | -2249.508            | -759.7886 |
|     | ira         |             |                     |       |       |                      |           |
|     | Yes         | 7864.15     | 344.2198            | 22.85 | 0.000 | 7189.492             | 8538.809  |
|     | pension     |             |                     |       |       |                      |           |
|     | Receives .. | 63.88643    | 326.6017            | 0.20  | 0.845 | -576.2412            | 704.0141  |
|     | ownhome     |             |                     |       |       |                      |           |
|     | Yes         | 969.6861    | 300.4319            | 3.23  | 0.001 | 380.8503             | 1558.522  |
|     | educ        | -301.1635   | 52.02897            | -5.79 | 0.000 | -403.1384            | -199.1885 |
|     | _cons       | -7455.806   | 1192.112            | -6.25 | 0.000 | -9792.302            | -5119.311 |

(output omitted)

| q90 |             |           |          |       |       |           |           |
|-----|-------------|-----------|----------|-------|-------|-----------|-----------|
|     | p401k       |           |          |       |       |           |           |
|     | Yes         | 15983.42  | 3046.028 | 5.25  | 0.000 | 10013.32  | 21953.53  |
|     | income      | .8247356  | .0570029 | 14.47 | 0.000 | .713012   | .9364593  |
|     | age         | 485.8734  | 48.99224 | 9.92  | 0.000 | 389.8504  | 581.8965  |
|     | familysize  | -646.4962 | 185.913  | -3.48 | 0.001 | -1010.879 | -282.1134 |
|     | married     |           |          |       |       |           |           |
|     | Married     | -3265.007 | 753.4701 | -4.33 | 0.000 | -4741.782 | -1788.233 |
|     | ira         |           |          |       |       |           |           |
|     | Yes         | 68543.44  | 4952.261 | 13.84 | 0.000 | 58837.18  | 78249.69  |
|     | pension     |           |          |       |       |           |           |
|     | Receives .. | -4656.177 | 869.4887 | -5.36 | 0.000 | -6360.343 | -2952.01  |
|     | ownhome     |           |          |       |       |           |           |
|     | Yes         | 400.1957  | 680.2776 | 0.59  | 0.556 | -933.124  | 1733.515  |
|     | educ        | 48.4205   | 106.2844 | 0.46  | 0.649 | -159.8931 | 256.7341  |
|     | _cons       | -20594.85 | 2260.983 | -9.11 | 0.000 | -25026.3  | -16163.41 |

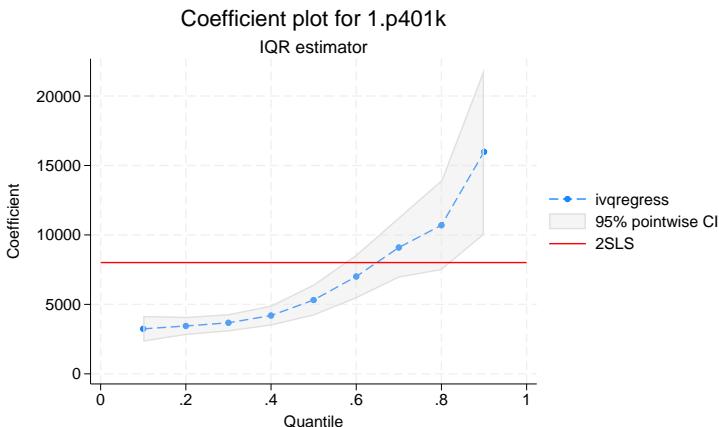
Endogenous: 1.p401k

Exogenous: income age familysize 1.married 1.ira 1.pension 1.ownhome educ  
1.e401k

The results show the estimates for the effect of 401(k) participation on each conditional quantile of the asset. The interpretation of the coefficient is similar to [example 1](#), except we are looking at different conditional quantiles. For example, for quantile q90, the estimate for the coefficient on p401k is 15,983. Thus, 401(k) participation would increase the 90% conditional quantile of net financial assets by \$15,983.

In addition to looking at the numerical estimates from the coefficient table, we can use `estat coefplot` to visualize the trend of p401k's treatment effect from the lower to the upper quantile. By default, `estat coefplot` shows the first endogenous variable, which is 1.p401k in our example. We specify the `name()` option for later reference of this graph and add a subtitle indicating which estimator we used.

```
. estat coefplot, name(cp_iqr) subtitle(IQR estimator)
```



The dots in the plot show the point estimates of p401k's treatment effect on different conditional quantiles of assets, and the gray bound shows the 95% pointwise CI. We see that there is an upward trend of p401k's treatment effect. At lower-level quantiles such as the 10th, 20th, ..., 40th quantiles, the treatment effect is relatively flat. However, we see the treatment effect increases significantly in the upper-level quantiles. The red line shows the two-stage least-squares estimates, which can be used as a benchmark.

`estat coefplot` is a good way to visualize the treatment effect's trend. If we want to test some hypotheses regarding the trend and the model statistically, we can use `estat endogeffects`. For example, we are interested in testing the following hypotheses:

- No effect:** 401(k) participation does not affect net financial assets for all the estimated quantiles.
- Constant effect:** 401(k) participation's treatment effect is constant for all the estimated quantiles.
- Dominance:** 401(k) participation is unambiguously positive for all the estimated quantiles; that is, the coefficient values are strictly positive.
- Exogeneity:** 401(k) participation is exogenous.

We will use `estat endogeffects` to show the Kolmogorov–Smirnov statistic and the 95% critical value for each hypothesis. We can reject the null hypothesis if the test statistic is greater than the critical value; otherwise, we cannot reject the null hypothesis. We specify the `rseed()` option to make the results reproducible because the critical values are generated from a bootstrap sample.

```
. estat endogeffects, rseed(12345671)

Tests for endogenous effects Replications = 100

Null hypothesis	KS statistic 95% critical value
No effect | 11.271 2.554
Constant effect | 5.395 2.446
Dominance | 0.000 2.467
Exogeneity | 4.145 2.478
-----|-----
```

Note: If the KS statistic < critical value, there is insufficient evidence to reject the null hypothesis. (KS = Kolmogorov-Smirnov)

In particular, we see that the test statistics are greater than the critical values in testing the hypotheses of no effect, constant effect, and exogeneity. Thus, with a 95% confidence level, we can reject these three hypotheses. In other words, we find that 401(k) participation has some effect, treatment is not constant across different quantiles, and 401(k) participation is endogenous. In contrast, we cannot reject the dominance hypothesis. Thus, we find that 401(k) participation is unambiguously beneficial for all the estimated quantiles of assets.

The test results are consistent with the result of the coefficient plot produced by `estat coefplot`, where we saw that the treatment effects are positive (dominance and no effect hypotheses) and upward trended (constant effect hypothesis).

For comparison, we can also use the SEE estimator to fit the model.

```
. ivregress smooth assets (i.p401k = i.e401k) income age familysize
> i.married i.ira i.pension i.ownhome educ, quantile(10(10)90)
Fitting smoothed IV quantile regression:
Quantile = .1:
Step 1: Bandwidth = 1327.0069 GMM criterion Q(b) = 9.224e-11
Step 2: Bandwidth = 1311.3131 GMM criterion Q(b) = 1.995e-10
Quantile = .2:
Step 1: Bandwidth = 1272.5204 GMM criterion Q(b) = 2.089e-10
Step 2: Bandwidth = 1237.7195 GMM criterion Q(b) = 3.075e-19
Quantile = .3:
Step 1: Bandwidth = 1504.4065 GMM criterion Q(b) = 5.407e-13
Step 2: Bandwidth = 1486.4224 GMM criterion Q(b) = 1.136e-10
Quantile = .4:
Step 1: Bandwidth = 1362.7753 GMM criterion Q(b) = 5.511e-17
Step 2: Bandwidth = 1362.6479 GMM criterion Q(b) = 8.561e-16
Quantile = .5:
Step 1: Bandwidth = 1302.9736 GMM criterion Q(b) = 2.617e-08
Step 2: Bandwidth = 6079.6881 GMM criterion Q(b) = 2.391e-12
Step 3: Bandwidth = 1438.3068 GMM criterion Q(b) = 8.068e-13
Quantile = .6:
Step 1: Bandwidth = 1533.5129 GMM criterion Q(b) = 2.679e-18
Step 2: Bandwidth = 1520.1182 GMM criterion Q(b) = 1.141e-19
Quantile = .7:
Step 1: Bandwidth = 2044.8617 GMM criterion Q(b) = 1.391e-10
Step 2: Bandwidth = 1977.2482 GMM criterion Q(b) = 1.827e-11
Quantile = .8:
Step 1: Bandwidth = 2503.7256 GMM criterion Q(b) = 3.623e-10
Step 2: Bandwidth = 2458.6714 GMM criterion Q(b) = 2.317e-10
Quantile = .9:
Step 1: Bandwidth = 3560.2178 GMM criterion Q(b) = 4.301e-12
Step 2: Bandwidth = 3529.3557 GMM criterion Q(b) = 2.932e-10
```

IV quantile regression  
 Estimator: Smoothed estimating equations

Number of obs = 9,913  
 Wald chi2(81) = 4932.84  
 Prob > chi2 = 0.0000

| assets           |             | Robust    |       | z     | P> z      | [95% conf. interval] |  |
|------------------|-------------|-----------|-------|-------|-----------|----------------------|--|
|                  | Coefficient | std. err. |       |       |           |                      |  |
| q10              |             |           |       |       |           |                      |  |
| p401k            |             |           |       |       |           |                      |  |
| Yes              | 3191.667    | 486.2193  | 6.56  | 0.000 | 2238.695  | 4144.639             |  |
| income           | .0318585    | .0123707  | 2.58  | 0.010 | .0076124  | .0561046             |  |
| age              | 128.9268    | 15.42632  | 8.36  | 0.000 | 98.69178  | 159.1618             |  |
| familysize       | -329.8374   | 125.4774  | -2.63 | 0.009 | -575.7687 | -83.90615            |  |
| married          |             |           |       |       |           |                      |  |
| Married          | -1480.013   | 386.4611  | -3.83 | 0.000 | -2237.463 | -722.5635            |  |
| ira              |             |           |       |       |           |                      |  |
| Yes              | 7914.049    | 342.9506  | 23.08 | 0.000 | 7241.878  | 8586.22              |  |
| pension          |             |           |       |       |           |                      |  |
| Receives ..      | -5.356704   | 334.9869  | -0.02 | 0.987 | -661.919  | 651.2056             |  |
| ownhome          |             |           |       |       |           |                      |  |
| Yes              | 1043.279    | 308.722   | 3.38  | 0.001 | 438.1945  | 1648.363             |  |
| educ             | -289.8807   | 53.06713  | -5.46 | 0.000 | -393.8904 | -185.8711            |  |
| _cons            | -7631.313   | 1214.725  | -6.28 | 0.000 | -10012.13 | -5250.496            |  |
| (output omitted) |             |           |       |       |           |                      |  |
| q90              |             |           |       |       |           |                      |  |
| p401k            |             |           |       |       |           |                      |  |
| Yes              | 15525.23    | 3035.965  | 5.11  | 0.000 | 9574.848  | 21475.61             |  |
| income           | .8311508    | .0574108  | 14.48 | 0.000 | .7186277  | .9436738             |  |
| age              | 486.9876    | 51.61654  | 9.43  | 0.000 | 385.821   | 588.1541             |  |
| familysize       | -586.2617   | 193.5936  | -3.03 | 0.002 | -965.6983 | -206.8252            |  |
| married          |             |           |       |       |           |                      |  |
| Married          | -3877.165   | 781.2296  | -4.96 | 0.000 | -5408.347 | -2345.983            |  |
| ira              |             |           |       |       |           |                      |  |
| Yes              | 67888.86    | 4902.106  | 13.85 | 0.000 | 58280.91  | 77496.81             |  |
| pension          |             |           |       |       |           |                      |  |
| Receives ..      | -4829.506   | 898.9147  | -5.37 | 0.000 | -6591.346 | -3067.665            |  |
| ownhome          |             |           |       |       |           |                      |  |
| Yes              | 715.6272    | 722.8727  | 0.99  | 0.322 | -701.1773 | 2132.432             |  |
| educ             | 14.5293     | 110.8781  | 0.13  | 0.896 | -202.7878 | 231.8464             |  |
| _cons            | -19953.21   | 2326.698  | -8.58 | 0.000 | -24513.45 | -15392.96            |  |

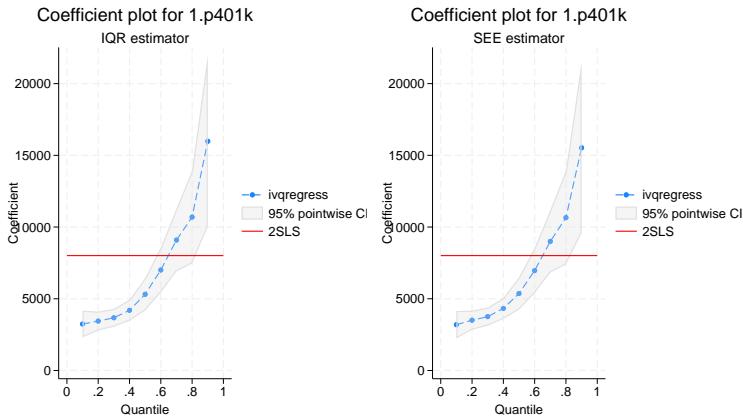
Endogenous: 1.p401k

Exogenous: income age familysize 1.married 1.ira 1.pension 1.ownhome educ  
 1.e401k

After `ivqregress smooth`, we can also use `estat coefplot` to visualize the treatment effect and `estat endoeffects` to test some hypotheses of particular interest in the context of the IVQR model.

First, we use `estat coefplot` to plot the coefficients and then use `graph combine` so that we can visually compare this plot with the coefficients plot for the IQR estimates.

```
. estat coefplot, name(cp_smooth) subtitle(SEE estimator)
. graph combine cp_iqr cp_smooth, xcommon ycommon altshrink
```



The left and right panels of the figure show the coefficient plots for the IQR and SEE estimates, respectively. We see that both estimators produce similar trends for the coefficients on 1.p401k at different quantiles.

Next, we can use `estat endogeffects` to see if we draw the same conclusion regarding the four hypotheses of interest as we did with the IQR estimator.

```
. estat endogeffects, rseed(12345671)
Tests for endogenous effects Replications = 100
```

| Null hypothesis | KS statistic | 95% critical value |
|-----------------|--------------|--------------------|
| No effect       | 11.507       | 2.593              |
| Constant effect | 5.351        | 2.391              |
| Dominance       | 0.000        | 2.556              |
| Exogeneity      | 4.195        | 2.526              |

Note: If the KS statistic < critical value, there is insufficient evidence to reject the null hypothesis. (KS = Kolmogorov-Smirnov)

The results align with those produced after `ivqregress iqr`. That is, the treatment effects are positive (dominance and no effect hypotheses), upward trended (constant effect hypothesis), and endogenous (exogeneity hypothesis).

#### ► Example 4: Robustness checks and diagnostics for the IQR estimator

In this example, we will take a closer look at the IQR estimator and show how to use `estat waldplot` to inspect the convergence visually. Nevertheless, let's first briefly discuss the intuition and algorithm behind the IQR estimator.

The IVQR model satisfies the following conditional probability:

$$\Pr(y \leq d\alpha(\tau) + \mathbf{x}'\beta(\tau) | \mathbf{x}, \mathbf{z}) = \tau$$

where  $y$  is the outcome variable,  $d$  is an endogenous variable,  $\mathbf{x}$  is a vector of exogenous covariates, and  $\mathbf{z}$  is a vector of instruments. The coefficients  $\alpha(\tau)$  and  $\beta(\tau)$  are indexed with the quantile level  $\tau$  to indicate that they are for the model of the  $\tau$  conditional quantile of the outcome  $y$ . We cannot fit the above model using the regular quantile regression because the conditional set is on  $\mathbf{x}$  and  $\mathbf{z}$  but the covariates contain  $\mathbf{x}$  and  $d$ . Now suppose we know the value of  $\alpha(\tau)$ . We can then rewrite this conditional probability as

$$\Pr(y - d\alpha(\tau) \leq \mathbf{x}'\beta(\tau) + \mathbf{z}'\mathbf{0} | \mathbf{x}, \mathbf{z}) = \tau$$

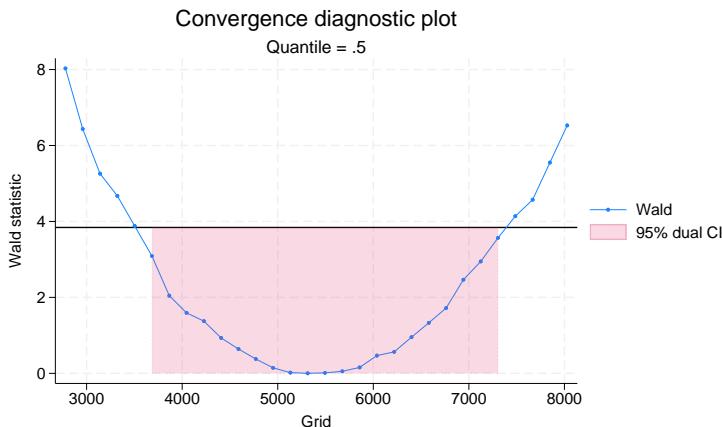
By the definition of quantile regression, we can fit this model by running a quantile regression of the transformed outcome variable,  $y - d\alpha(\tau)$ , on the covariates  $\mathbf{x}$  and instruments  $\mathbf{z}$ . Notice that if  $\alpha(\tau)$  is the true value, the coefficient on the instruments, which we denote as  $\gamma(\tau)$ , should be 0. In other words, to solve the original moment conditional for the IVQR model, we need to find a  $\alpha(\tau)$  such that the auxiliary quantile regression of  $y - d\alpha(\tau)$  on  $\mathbf{x}$  and  $\mathbf{z}$  produces 0s for the coefficients on the instrument  $\mathbf{z}$ . In practice, we want  $\gamma(\tau)$  as close to 0 as possible, where the closeness to 0 can be measured by the Wald statistic on  $\gamma(\tau)$ .

Based on the above intuition, here is an outline of the IQR estimator's algorithm.

1. Define a grid of  $A = \{\alpha_1, \dots, \alpha_J\}$  (see *IQR default grid algorithm* in *Methods and formulas*).
2. For each  $\alpha_j$  in  $A$ , run an auxiliary quantile regression of  $y - d\alpha_j$  on covariates  $\mathbf{x}$  and instruments  $\mathbf{z}$ .
3. IQR finds  $\alpha_k \in A$  as a solution such that the coefficient on  $\mathbf{z}$  is as close to 0 as possible in the corresponding auxiliary quantile regression, where the Wald statistic measures the closeness to 0.
4. The grid points boundary must be wider than the dual CI, which is robust to weak instruments; otherwise, `ivqregress iqr` will error out. Dual CI means it covers the true value of  $\alpha(\tau)$  with 95% probability (see [Chernozhukov and Hansen \[2008\]](#) and `estat dualci`).

We can use `estat waldplot` to visualize the above procedure. Using the estimation result in [example 1](#), we first restore the result `est_iqr` and then use `estat waldplot` to plot the Wald statistics corresponding to each grid point.

```
. estimates restore est_iqr
(results est_iqr are active now)
. estat waldplot
```



The horizontal axis shows the grid points for  $\alpha$ , and the vertical axis shows the values of the Wald statistics. The dots in the plot show the Wald statistics corresponding to each grid point. The red line is the 95% critical value of the Wald test. Thus, only the Wald statistics below the red line will not reject the hypothesis that  $\gamma_j$  equals 0. Respectively, the 95% dual CI corresponds to the  $\alpha$ 's for which the Wald statistics are below the critical value. See [example 1](#) for the use of `estat dualci` to show the numerical values of the dual CI.

By default, `ivqregress iqr` uses the dual CI to generate the lower and upper bounds for the grid points to make sure that the grid covers the true value of parameter  $\alpha$  with a large probability. Sometimes, we may want to customize the bounds. For example, suppose we want to search grid points between 3,000 and 6,000. We can use the `bound()` option for this purpose.

```
. ivqregress iqr assets (i.p401k = i.e401k) income age familysize
> i.married i.ira i.pension i.ownhome educ, bound(3000 6000)

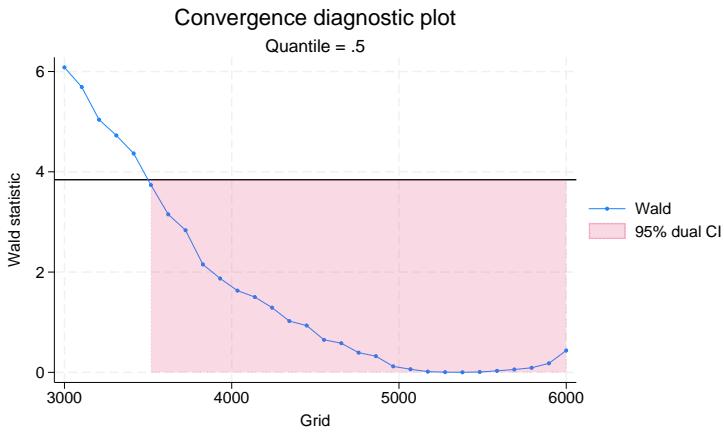
Initial grid:
Quantile = 0.50:10.....20.....30

convergence not achieved
The grid interval should be wider than the 95% dual confidence interval.
Try to set a wider bound using option bound(). Use estat waldplot for
diagnosis.
r(430);
```

We see that `ivqregress iqr` stops with a “convergence not achieved” error message. The reason is that the specified bound is too narrow to cover the true value of the parameter with a 95% probability.

We can now use `estat waldplot` to further visualize the issue.

```
. estat waldplot
```



The graph shows that the upper bound of 6,000 is too small because we need the Wald statistics to intersect with the 95% critical value at the lower and upper bounds.

We can increase the upper bound and see if the IQR estimator converges. For example, below we increase the upper bound to 8,000.

```
. ivqregress iqr assets (i.p401k = i.e401k) income age familysize
> i.married i.ira i.pension i.ownhome educ, bound(3000 8000)

Initial grid
quantile = 0.50:10.....20.....30

Adaptive grid
quantile = 0.50:10.....20.....30

IV median regression
Estimator: Inverse quantile regression

Number of obs = 9,913
Wald chi2(9) = 1290.41
Prob > chi2 = 0.0000
```

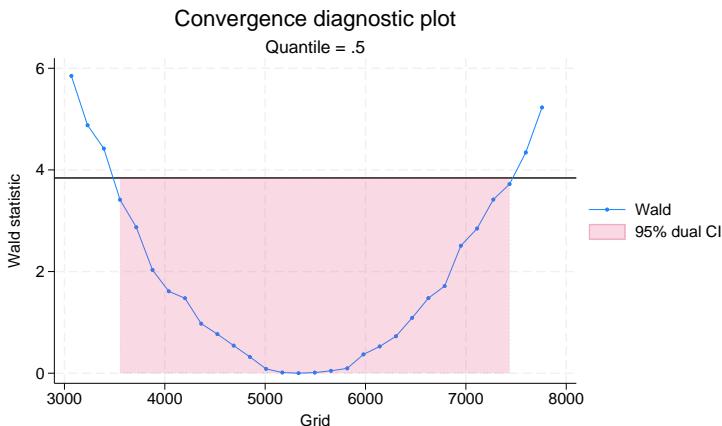
| assets      | Coefficient | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |           |
|-------------|-------------|---------------------|-------|-------|----------------------|-----------|
| p401k       |             |                     |       |       |                      |           |
| Yes         | 5332.937    | 574.5175            | 9.28  | 0.000 | 4206.903             | 6458.971  |
| income      | .157381     | .012478             | 12.61 | 0.000 | .1329246             | .1818374  |
| age         | 99.78981    | 8.553978            | 11.67 | 0.000 | 83.02432             | 116.5553  |
| familysize  | -199.6165   | 54.3519             | -3.67 | 0.000 | -306.1442            | -93.08872 |
| married     |             |                     |       |       |                      |           |
| Married     | -1351.309   | 227.0824            | -5.95 | 0.000 | -1796.382            | -906.2357 |
| ira         |             |                     |       |       |                      |           |
| Yes         | 22631.85    | 1022.023            | 22.14 | 0.000 | 20628.72             | 24634.98  |
| pension     |             |                     |       |       |                      |           |
| Receives .. | -694.1447   | 210.533             | -3.30 | 0.001 | -1106.782            | -281.5077 |
| ownhome     |             |                     |       |       |                      |           |
| Yes         | -30.67158   | 154.6947            | -0.20 | 0.843 | -333.8676            | 272.5244  |
| educ        | -96.30363   | 32.0715             | -3.00 | 0.003 | -159.1626            | -33.44465 |
| _cons       | -4983.758   | 569.4043            | -8.75 | 0.000 | -6099.77             | -3867.746 |

Endogenous: 1.p401k

Exogenous: income age familysize 1.married 1.ira 1.pension 1.ownhome educ  
1.e401k

Now that the IQR estimator converges, we can redraw the Wald plot to confirm that the proposed grid points interval is indeed wider than the dual CI.

```
. estat waldplot
```



◀

## Stored results

`ivqregress iqr` and `ivqregress smooth` store the following in `e()`:

### Scalars

|                                   |                                                                                                             |
|-----------------------------------|-------------------------------------------------------------------------------------------------------------|
| <code>e(N)</code>                 | number of observations                                                                                      |
| <code>e(q#)</code>                | the quantiles requested                                                                                     |
| <code>e(n_q)</code>               | number of quantiles requested                                                                               |
| <code>e(bwidth_q#)</code>         | bandwidth used in standard errors for $q_{\#}$ th quantile                                                  |
| <code>e(sm_init_bwidth_q#)</code> | initial bandwidth used in smoothing the indicator function in $q_{\#}$ th quantile estimation (smooth only) |
| <code>e(sm_bwidth_q#)</code>      | bandwidth used in smoothing the indicator function in $q_{\#}$ th quantile estimation (smooth only)         |
| <code>e(convcode)</code>          | 0 if converged; otherwise, return code for why nonconvergence                                               |
| <code>e(p)</code>                 | $p$ -value for model test                                                                                   |
| <code>e(df_m)</code>              | model degrees of freedom                                                                                    |
| <code>e(chi2)</code>              | $\chi^2$                                                                                                    |
| <code>e(rank)</code>              | rank of $e(V)$                                                                                              |

### Macros

|                            |                                                    |
|----------------------------|----------------------------------------------------|
| <code>e(cmd)</code>        | <code>ivqregress</code>                            |
| <code>e(cmdline)</code>    | command as typed                                   |
| <code>e(depvar)</code>     | name of dependent variable                         |
| <code>e(inst)</code>       | names of instrumental variables                    |
| <code>e(bwrule)</code>     | method to compute the bandwidth in standard errors |
| <code>e(kernel)</code>     | kernel function                                    |
| <code>e(title)</code>      | title in estimation output                         |
| <code>e(vce)</code>        | <i>vcetype</i> specified in <code>vce()</code>     |
| <code>e(vcetype)</code>    | title used to label Std. err.                      |
| <code>e(estimator)</code>  | <code>iqr</code> or <code>smooth</code>            |
| <code>e(exogr)</code>      | exogenous regressors                               |
| <code>e(endog)</code>      | endogenous regressors                              |
| <code>e(properties)</code> | <code>b V</code>                                   |
| <code>e(estat_cmd)</code>  | program used to implement <code>estat</code>       |
| <code>e(predict)</code>    | program used to implement <code>predict</code>     |

|                 |                                              |
|-----------------|----------------------------------------------|
| e(marginsok)    | predictions allowed by margins               |
| e(marginsnotok) | predictions disallowed by margins            |
| Matrices        |                                              |
| e(b)            | coefficient vector                           |
| e(V)            | variance–covariance matrix of the estimators |
| Functions       |                                              |
| e(sample)       | marks estimation sample                      |

In addition to the above, the following is stored in `r()`:

|          |                                                                                                                       |
|----------|-----------------------------------------------------------------------------------------------------------------------|
| Matrices |                                                                                                                       |
| r(table) | matrix containing the coefficients with their standard errors, test statistics, $p$ -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

Methods and formulas are presented under the following headings:

- The model*
- The IQR estimator*
- The IQR algorithm*
- The IQR default grid algorithm*
- The SEE estimator*
- The bandwidth selection algorithm*
- The robust standard errors*

## The model

The general IVQR model was first proposed by [Chernozhukov and Hansen \(2005\)](#). `ivqregress` fits a linear IVQR model described in [Chernozhukov and Hansen \(2006, 2008\)](#). For notational simplicity, we drop the observational subscript  $i$  to refer to a random variable and add the subscript  $i$  to refer to a realization of a random variable.

We can write the linear IVQR model in the form of a “random coefficients” model as

$$y = \mathbf{d}'\boldsymbol{\alpha}(u) + \mathbf{x}'\boldsymbol{\beta}(u)$$

where

- $y$  is a scalar outcome variable,  $\mathbf{d}$  is a vector of endogenous variables,  $\mathbf{x}$  is a vector of exogenous variables, and  $u$  is the unobserved error term;
- $\mathbf{d}$  depends on the exogenous covariates  $\mathbf{x}$ , and the instrumental variables  $\mathbf{z}$  and unobserved error term are correlated with  $u$ ;
- $u$  is a scalar random variable that characterizes the heterogeneity of the outcome and captures all the unobservables in the outcome from item 1 above. Conditional on  $\mathbf{z}$  and  $\mathbf{x}$ ,  $u$  is uniformly distributed between 0 and 1;
- $\boldsymbol{\alpha}(\cdot)$  and  $\boldsymbol{\beta}(\cdot)$  are random coefficient vectors that depend on  $u$ ;
- the function  $\tau \rightarrow \mathbf{d}'\boldsymbol{\alpha}(\tau) + \mathbf{x}'\boldsymbol{\beta}(\tau)$  is strictly increasing in  $\tau$ ; and
- the observable variables are  $\{y_i, \mathbf{x}_i, \mathbf{d}_i, \mathbf{z}_i\}_{i=1}^N$  with a sample of size  $N$ .

Under some regularity conditions (see Chernozhukov and Hansen [2005]), the IVQR model satisfies the conditional probability

$$\Pr\{y \leq \mathbf{d}'\boldsymbol{\alpha}(\tau) + \mathbf{x}'\boldsymbol{\beta}(\tau) | \mathbf{x}, \mathbf{z}\} = \tau \quad (1)$$

By the definition of probability and the law of iterated expectation, (1) implies the following unconditional moment condition:

$$E([\tau - I\{y \leq \mathbf{d}'\boldsymbol{\alpha}(\tau) + \mathbf{x}'\boldsymbol{\beta}(\tau)\}] \Psi) = 0 \quad (2)$$

where  $I(\cdot)$  is the indicator function,  $\Psi = (\hat{\mathbf{d}}', \mathbf{x}')$ , and  $\hat{\mathbf{d}}$  is some function of  $\mathbf{x}$  and  $\mathbf{z}$  and can be treated as instruments for  $\mathbf{d}$ . In practice,  $\hat{\mathbf{d}}$  is the linear prediction of  $\mathbf{d}$  using  $\mathbf{x}$  and  $\mathbf{z}$ .

Equation (2) can be used as the estimating equation for the IVQR model. However, the objective function based on (2) is nonconvex and nonsmooth because of the indicator function. Thus, it is computationally challenging to fit the IVQR model by directly using (2).

`ivqregress` implements two estimators that approximately solve the original moment condition in (2). In particular, `ivqregress iqr` implements the IQR estimator proposed in Chernozhukov and Hansen (2006), and `ivqregress smooth` implements the SEE estimator outlined in Kaplan and Sun (2017). Here are the main ideas behind these two estimators.

The IQR estimator reduces the original  $p$ -dimensional (where  $p$  is the dimension of  $\mathbf{x}$  and  $\mathbf{d}$ ) nonconvex problem into a low-dimensional nonconvex problem. Then, it solves the problem by doing an exhaustive grid search over a high-quality grid. The grid is high quality in the sense that it covers the true value of the parameter for  $\alpha(\tau)$  with a high probability (Chernozhukov and Hansen 2008). As a byproduct, the IQR estimator can also provide the CI that is robust to the weak instruments, which is also known as dual CI (see `estat dualci`). However, the IQR estimator becomes computationally intensive if there is more than one endogenous variable. As a result, `ivqregress iqr` allows only one endogenous variable.

The SEE estimator smooths the original moment condition in (2) using a kernel method to approximate the indicator function. Thus, the optimization problem reduces to solving a system of smooth nonlinear equations. One advantage of the SEE estimator compared with the IQR estimator is that it can handle more than one endogenous variable. However, it cannot provide the dual CI, which is robust to weak instruments like the IQR estimator.

While the IQR and SEE estimators are consistent for the IVQR model, their results are generally different. The reason is that the two estimators approximate the original moment condition in different ways. On one hand, the IQR estimator tries to find the solution by an exhaustive grid search. The estimation result critically depends on the range and finesse of grid points. On the other hand, the SEE estimator uses a kernel method to smooth the original estimating equation. Its result depends on how well the SEE estimator approximates the original, mainly controlled by the bandwidth.

In practice, suppose there is only one endogenous variable in the model. We recommend using both estimators, comparing the results, and using the IQR estimator as a benchmark because it can provide valid inference even if the instruments are weak.

## The IQR estimator

Before diving into the details, we discuss the intuition of the IQR estimator. The IVQR model satisfies the conditional probability

$$\Pr\{y \leq d\alpha(\tau) + \mathbf{x}'\boldsymbol{\beta}(\tau) | \mathbf{x}, \mathbf{z}\} = \tau$$

We cannot fit the above model using regular quantile regression because the conditional set is on  $\mathbf{x}$  and  $\mathbf{z}$ , but the covariates contain  $\mathbf{x}$  and  $d$ . Now suppose we know the value of  $\alpha(\tau)$ . We can then rewrite this conditional probability as

$$\Pr\{y - d\alpha(\tau) \leq \mathbf{x}'\beta(\tau) + \mathbf{z}' * 0 | \mathbf{x}, \mathbf{z}\} = \tau$$

By the definition of quantile regression, we can fit this model by running a quantile regression of the transformed outcome variable,  $y - d\alpha(\tau)$ , on the covariates  $\mathbf{x}$  and instruments  $\mathbf{z}$ . Notice that if  $\alpha(\tau)$  is the actual value, the coefficient on the instruments, which we denote as  $\gamma(\tau)$ , should be 0. In other words, to solve the original moment conditional for the IVQR model, we need to find a  $\alpha(\tau)$  such that the auxiliary quantile regression of  $y - d\alpha(\tau)$  on  $\mathbf{x}$  and  $\mathbf{z}$  produces zeros for the coefficients on the instruments  $\mathbf{z}$ . In practice, we want  $\gamma(\tau)$  as close to 0 as possible, where the closeness to 0 can be measured by the Wald statistic on  $\gamma(\tau)$ .

Based on the above intuition, here is an outline of the IQR estimator's algorithm.

### The IQR algorithm

1. Compute  $\hat{\mathbf{d}}$ , which is the linear projection of  $\mathbf{d}$  on  $\mathbf{x}$  and  $\mathbf{z}$ .  $\hat{\mathbf{d}}$  can be treated as instruments for  $\mathbf{d}$ .
2. Define a grid of  $A = \{\alpha_1, \dots, \alpha_J\}$ . For the algorithm of the default grid generation, see [The IQR default grid algorithm](#).
3. For each  $\alpha_j$  in  $A$ , run an auxiliary quantile regression of  $y - d\alpha_j$  on covariates  $\mathbf{x}$  and instruments  $\hat{\mathbf{d}}$ .
4. IQR finds  $\alpha_k \in A$  as a solution such that the coefficient on  $\hat{\mathbf{d}}$  is as close to 0 as possible in the corresponding auxiliary quantile regression, where the Wald statistic measures the closeness to 0.
5. The grid points boundary must be wider than the dual CI, which is robust to weak instruments; otherwise, `ivqregress iqr` will error out. Dual CI means it covers the true value of  $\alpha(\tau)$  with  $\#_{\text{level}}$  probability (see [Chernozhukov and Hansen \[2008\]](#) and `estat dualci`). The `level()` option specifies the confidence level  $\#_{\text{level}}$ ; the default is `level(95)`.

### The IQR default grid algorithm

The default grid algorithm can be divided into two stages: 1) the initial grid generation based on the two-stage quantile regression, which extends the two-stage median regression in [Amemiya \(1982\)](#); and 2) the adaptive grid that depends on the dual CI, which is robust to weak instruments ([Chernozhukov and Hansen 2008](#)).

1. Initial grid based on two-stage quantile regression
  - (a) Run a quantile regression of  $y$  on  $\mathbf{x}$  and  $\hat{\mathbf{d}}$ . Denote  $\tilde{\alpha}$  as the point estimate for the coefficient on  $\hat{\mathbf{d}}$  and  $\tilde{s}$  as its standard errors.  $\tilde{s}$  is computed by assuming the error term is normally distributed.
  - (b) Compute the lower and upper bounds of the grid. The lower bound is  $\text{lb} = \tilde{\alpha} - 4\tilde{s}$ , and the upper bound is  $\text{ub} = \tilde{\alpha} + 4\tilde{s}$ .
  - (c) By default, the grid points are  $\#_g$  equally spaced points between  $\text{lb}$  and  $\text{ub}$ , where the `ngrid()` option specifies the number of grid points  $\#_g$ .

## 2. Adaptive grid based on the dual CI

- (a) Given the initial grid, go through the steps 3–5 in *The IQR algorithm*.
- (b) Obtain the dual CI based on the initial grid.
- (c) Use the dual CI as the bound for the adaptive grid points and generate  $\#_g$  equally spaced points.

## The SEE estimator

The basic idea of the SEE estimator is to replace the indicator function in (2) with a smooth function. To be precise, we replace the moment condition in (2) with

$$E\left([\tau - \tilde{I}\{y - \mathbf{d}'\boldsymbol{\alpha}(\tau) - \mathbf{x}'\boldsymbol{\beta}(\tau) \leq 0\}] \Psi\right) = 0 \quad (3)$$

where  $\tilde{I}(v/h) = \max[0, \min\{1, (1 - v/h)/2\}]$  and  $h$  is the bandwidth. By default, the bandwidth is computed using the theoretical optimal bandwidth that minimizes the mean squared errors of the estimating equations. See proposition 2 in Kaplan and Sun (2017) for the optimal bandwidth.

Because  $\tilde{I}(\cdot)$  is a smooth function, the SEE estimator reduces to solve a system of smooth nonlinear equations. Let  $F(\theta)$  denote the left-hand side of (3), where  $\theta = \{\boldsymbol{\alpha}(\tau), \boldsymbol{\beta}(\tau)\}$ . Let  $\theta(i)$  denote the proposed solution at iteration  $i$ , and let  $\theta(i - 1)$  denote the proposed solution at the previous iteration. The convergence is declared if  $\text{mreldif}(\theta(i), \theta(i - 1)) < \text{itol}$  or  $F(\theta)'F(\theta) < \text{ztol}$ , where *itol* and *ztol* can be specified by using the `tolerance()` and `ztolerance()` options, respectively. The maximum number of iterations can be specified by using the `iterate()` option.

By default, the SEE estimator searches for the bandwidth as follows.

## The bandwidth selection algorithm

Denote  $\hat{\theta}_0$  as the initial values for the parameters  $\boldsymbol{\alpha}(\tau)$  and  $\boldsymbol{\beta}(\tau)$ . Denote  $\widehat{\mathbf{h}}_{\text{opt}}(\hat{\theta}_0)$  as the optimal bandwidths based on the initial values  $\hat{\theta}_0$ .  $\widehat{\mathbf{h}}_{\text{opt}}(\hat{\theta}_0)$  is a vector with elements  $(h_1, h_2, h_3)$ , where  $h_1$  is a nonparametrically estimated bandwidth,  $h_2$  assumes Gaussian distribution, and  $h_3$  uses the Silverman rule of thumb. Regardless of the assumption used, each element in  $\widehat{\mathbf{h}}_{\text{opt}}(\hat{\theta}_0)$  requires initial estimates of the error term  $\epsilon = y - \mathbf{d}'\boldsymbol{\alpha}(\tau) - \mathbf{x}'\boldsymbol{\beta}(\tau)$ . Thus, the optimal bandwidth is a function of the initial estimates for  $\boldsymbol{\alpha}(\tau)$  and  $\boldsymbol{\beta}(\tau)$ . For details, see section 5.4 in Kaplan (2022).

1. Let  $\hat{\theta}_0$  be the estimates of a quantile regression of  $y$  on  $\mathbf{d}$  and  $\mathbf{x}$ .
2. Based on  $\hat{\theta}_0$ , compute the optimal bandwidths  $\widehat{\mathbf{h}}_{\text{opt}}(\hat{\theta}_0)$ .
3. Define the initial bandwidth set as  $\mathbf{h}_0 = \{\widehat{\mathbf{h}}_{\text{opt}}(\hat{\theta}_0), h_{\text{init}}\}$ , where  $h_{\text{init}}$  is the bandwidth in the `initbwidth()` option if specified.  $\mathbf{h}_0 = \widehat{\mathbf{h}}_{\text{opt}}(\hat{\theta}_0)$  if `initbwidth()` is not specified.

4. Find the smallest element in  $\mathbf{h}_0$  such that it solves (3). The estimates for  $\alpha(\tau)$  are within the dual CI with  $\#_{\text{level}}$  probability (see Chernozhukov and Hansen [2008]).

- (a) If a valid bandwidth is found, go to step 5.
- (b) Otherwise, do a bisection search of the bandwidth with the upper bound as  $100 \times \min(\mathbf{h}_0)$  and the lower bound as  $\min(\mathbf{h}_0)/100$ .

If a valid bandwidth is found, denote it as  $h^*$ .

5. Update  $\hat{\theta}_0$  as the solution for the SEE estimator based on bandwidth  $h^*$ . Repeat steps 2–4 based on the updated  $\hat{\theta}_0$ .

By default, steps 1–5 are used to select the bandwidth. If the `nosearchbw` and `initbwidth()` options are both specified, steps 2, 4b, and 5 are omitted. Thus, in this case, `ivqregress smooth` will try to solve (3) with the specified initial bandwidth without searching for the optimal or feasible bandwidths. If only the `nosearchbw` option is specified, step 4b is omitted.

### The robust standard errors

The robust asymptotic variance–covariance estimator for the IQR and SEE estimators can be estimated as follows (see Chernozhukov and Hansen [2006] and de Castro et al. [2019]). Let  $\theta = \{\alpha(\tau), \beta(\tau)\}$  be the true values of parameters and  $\hat{\theta} = \{\widehat{\alpha(\tau)}, \widehat{\beta(\tau)}\}$  be the IQR or the SEE estimator. For any finite collection of quantile indices  $\tau_j, j \in T$

$$[\sqrt{n}\{\hat{\theta}(\tau) - \theta(\tau)\}]_{j \in T} \rightarrow N(0, [\mathbf{J}(\tau_k)^{-1} \mathbf{S}(\tau_j, \tau_k) \{\mathbf{J}(\tau_j)^{-1}\}']_{k, j \in T})$$

where

$$\begin{aligned} \mathbf{J}(\tau) &= E\{f_{\epsilon(\tau)}(0|\mathbf{x}, \mathbf{d}, \mathbf{z})\Psi[\mathbf{d}', \mathbf{x}']\} \\ \mathbf{S}(\tau_j, \tau_k) &= \{\min(\tau_j, \tau_k) - \tau_j\tau_k\}E(\Psi\Psi') \end{aligned}$$

and  $f_{\epsilon(\tau)}(0|\mathbf{x}, \mathbf{d}, \mathbf{z})$  is the conditional density of  $\epsilon(\tau)$  evaluated at 0, with  $\epsilon_i(\tau) = y_i - \mathbf{d}'_i\alpha(\tau) - \mathbf{x}'_i\beta(\tau)$ .

The components in the variance can be obtained by their sample counterparts. In particular,  $\mathbf{S}(\cdot)$  can be estimated as

$$\hat{\mathbf{S}}(\tau_k, \tau_j) = \{\min(\tau_k, \tau_j) - \tau_k\tau_j\} \frac{1}{N} \sum_{i=1}^N \Psi_i \Psi'_i$$

$\mathbf{J}(\cdot)$  can be estimated as

$$\hat{\mathbf{J}}(\tau) = \frac{1}{Nh_N} \sum_i^N K \left\{ \frac{-\widehat{\epsilon}_i(\tau)}{h_n} \right\} \Psi_i[\mathbf{d}'_i, \mathbf{x}'_i]$$

where  $\widehat{\epsilon}_i(\tau) = y_i - \mathbf{d}'_i\widehat{\alpha(\tau)} - \mathbf{x}'_i\widehat{\beta(\tau)}$ ,  $K(\cdot)$  is a kernel function, and  $h_n$  is the bandwidth.

`vce(vctype, kernel())` specifies the kernel function form  $K(\cdot)$ . See [R] `kdensity` for the function forms of the eight kernels.

`vce(vctype, bwidth())` specifies which bandwidth to use: `silverman` specifies to use  $h_s$ , `hshether` specifies to use  $h_k$  with  $h_1$  replaced by  $h_{hs}$ , and `bofinger` specifies to use  $h_k$  with  $h_1$  replaced by  $h_{bo}$ .

Silverman's rule of thumb bandwidth is

$$h_s = 0.9 \min \left\{ \widehat{\sigma(\hat{\epsilon})}, \frac{M}{1.349} \right\} N^{-\frac{1}{5}}$$

where  $\widehat{\sigma(\hat{\epsilon})}$  is the standard deviation of  $\hat{\epsilon}$  and  $M$  is the interquartile range of  $\hat{\epsilon}$ .

The bandwidth in Koenker (2005, 81) is

$$h_k = \min \left\{ \widehat{\sigma(\hat{\epsilon})}, \frac{M}{1.349} \right\} \{ \Phi^{-1}(\tau + h_1) - \Phi^{-1}(\tau - h_1) \}$$

where  $\Phi^{-1}(\cdot)$  is the inverse cumulative standard normal distribution and  $h_1$  can be one of the bandwidths in Hall and Sheather (1988) ( $h_{hs}$ ) or Bofinger (1975) ( $h_{bo}$ ). In particular,

$$h_{hs} = N^{-1/3} \Phi^{-1} \left( 1 - \frac{\alpha}{2} \right)^{2/3} \left[ \frac{3}{2} \times \frac{\phi \{ \Phi^{-1}(\tau) \}^2}{2\Phi^{-1}(\tau)^2 + 1} \right]^{1/3}$$

$$h_{bo} = N^{-1/5} \left[ \frac{9}{2} \times \frac{\phi \{ \Phi^{-1}(\tau) \}^4}{\{ 2\Phi^{-1}(\tau)^2 + 1 \}^2} \right]^{1/5}$$

where  $\phi(\cdot)$  is the standard normal probability density function.

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## Also see

- [R] **ivqregress postestimation** — Postestimation tools for ivqregress
- [R] **ivregress** — Single-equation instrumental-variables regression
- [R] **qreg** — Quantile regression
- [U] **20 Estimation and postestimation commands**

|                         |                                |                                      |                            |
|-------------------------|--------------------------------|--------------------------------------|----------------------------|
| Postestimation commands | <a href="#">predict</a>        | <a href="#">margins</a>              | <a href="#">estat</a>      |
| Remarks and examples    | <a href="#">Stored results</a> | <a href="#">Methods and formulas</a> | <a href="#">References</a> |
| Also see                |                                |                                      |                            |

## Postestimation commands

The following postestimation commands are of special interest after `ivqregress`:

| Command                        | Description                                                             |
|--------------------------------|-------------------------------------------------------------------------|
| <code>estat coefplot</code>    | plot coefficients and their confidence intervals at different quantiles |
| <code>estat endogeneity</code> | perform tests of endogeneity                                            |
| * <code>estat dualci</code>    | report dual confidence intervals for endogenous variable                |
| * <code>estat waldplot</code>  | plot Wald statistics corresponding to each grid point                   |

\*`estat dualci` and `estat waldplot` work only after `ivqregress iqr`.

The following postestimation commands are also available:

| Command                      | Description                                                                                       |
|------------------------------|---------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                               |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                      |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                |
| <code>estimates</code>       | cataloging estimation results                                                                     |
| <code>etable</code>          | table of estimation results                                                                       |
| <code>forecast</code>        | dynamic forecasts and simulations                                                                 |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters    |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                           |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters |
| <code>predict</code>         | linear predictions and residuals                                                                  |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions              |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                              |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                |

# predict

## Description for predict

`predict` creates a new variable containing predictions such as linear predictors and residuals.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic equation([eqno])]]
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

---

Main

|                        |                               |
|------------------------|-------------------------------|
| <code>xb</code>        | linear predictor; the default |
| <code>residuals</code> | residuals                     |

---

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

## Options for predict

Main

`xb`, the default, calculates the linear predictor.

`residuals` calculates the residuals, that is,  $y_j - \mathbf{x}_j\mathbf{b}$ .

`equation([eqno])` specifies the equation to which you are making the calculation.

`equation()` is filled in with one *eqno*. `equation(#1)` would mean that the calculation is to be made for the first equation, `equation(#2)` would mean the second, and so on. You could also refer to the equations by their names. `equation(p50)` would refer to the equation named p50 and `equation(p90)` to the equation named p90.

If you do not specify `equation()`, results are the same as if you had specified `equation(#1)`.

# margins

## Description for margins

`margins` estimates margins of response for linear predictors.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [options]
```

| <i>statistic</i>              | Description                   |
|-------------------------------|-------------------------------|
| <code>xb</code>               | linear predictor; the default |
| <code><u>r</u>esiduals</code> | not allowed with margins      |

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [\[R\] margins](#).

## estat

### Description for estat

`estat coefplot` plots the estimated coefficients and their confidence intervals (CIs) after `ivqregress`.

`estat endogeffects` tests four hypotheses for the coefficients on the endogenous variables; see [Chernozhukov and Hansen \(2006\)](#). In particular, `estat endogeffects` provides tests for the following null hypotheses:

1. No effect: the endogenous variables do not affect the outcome variable.
2. Constant effect: the effects of the endogenous variables do not vary across estimated quantiles.
3. Dominance: the effects of the endogenous variables are greater than 0 across estimated quantiles.
4. Exogeneity: the variables are exogenous instead of endogenous.

`estat dualci` computes the dual CIs for the coefficients on the endogenous variable (one for each quantile) after `ivqregress iqr`; see [Chernozhukov and Hansen \(2008\)](#). The dual CI is robust to the weak instruments, and it is usually wider than the traditional CI.

`estat waldplot` plots the Wald statistic corresponding to each grid point after `ivqregress iqr`.

### Menu for estat

Statistics > Postestimation

### Syntax for estat

*Plot coefficients and their CIs at different quantiles*

```
estat coefplot [varname] [, coefplot_options]
```

*Perform tests of endogeneity*

```
estat endogeffects [varlist] [, endogeffects_options]
```

*Report dual CIs for endogenous variable*

```
estat dualci [, level(#) display_options]
```

*Plot Wald statistics corresponding to each grid point*

```
estat waldplot [, waldplot_options]
```

*varname* is one of the endogenous regressors or exogenous variables specified when fitting `ivqregress`.

By default, *varname* is the endogenous variable specified with `ivqregress iqr` or the first endogenous variable specified with `ivqregress smooth`.

*varlist* contains one or more of the endogenous variables specified when fitting a model with `ivqregress`; the default is the first endogenous variable.

| <i>coefplot_options</i>                                        | Description                                                                                 |
|----------------------------------------------------------------|---------------------------------------------------------------------------------------------|
| <code>nocl</code>                                              | do not plot the CIs                                                                         |
| <code>no2sls</code>                                            | do not plot the 2SLS estimates                                                              |
| Plot                                                           |                                                                                             |
| <i><a href="#">connect_options</a></i>                         | change look of lines or connecting method                                                   |
| <i><a href="#">marker_options</a></i>                          | change look of markers (color, size, etc.)                                                  |
| CI plot                                                        |                                                                                             |
| <code>ciopts</code> ( <i><a href="#">area_options</a></i> )    | affect rendition of the pointwise CIs                                                       |
| Line options                                                   |                                                                                             |
| <code>lineopts</code> ( <i><a href="#">cline_options</a></i> ) | affect rendition of reference line identifying the 2SLS estimates                           |
| Y axis, X axis, Titles, Legend, Overall                        |                                                                                             |
| <i><a href="#">tway_options</a></i>                            | any options other than <code>by()</code> documented in <a href="#">[G-3] twoway_options</a> |

| <i>endogeffects_options</i> | Description                                                                      |
|-----------------------------|----------------------------------------------------------------------------------|
| <code>all</code>            | test four hypotheses; the default                                                |
| <code>noeffect</code>       | test of no effect                                                                |
| <code>constant</code>       | test of constant effect                                                          |
| <code>dominance</code>      | test of stochastic dominance                                                     |
| <code>exogeneity</code>     | test of exogeneity                                                               |
| <code>level(#)</code>       | confidence level of a test; default is <code>level(95)</code>                    |
| <code>rseed(#)</code>       | set random-number seed to <code>#</code>                                         |
| <code>reps(#)</code>        | perform <code>#</code> bootstrap replications; default is <code>reps(100)</code> |

| <i>waldplot_options</i>                                     | Description                                                                                 |
|-------------------------------------------------------------|---------------------------------------------------------------------------------------------|
| <code>quantile(#)</code>                                    | plot Wald statistics for the <code>#</code> th quantile estimation                          |
| <code>level(#)</code>                                       | set confidence level; default is <code>level(95)</code>                                     |
| Plot                                                        |                                                                                             |
| <i><a href="#">connect_options</a></i>                      | change look of lines or connecting method                                                   |
| <i><a href="#">marker_options</a></i>                       | change look of markers (color, size, etc.)                                                  |
| CI plot                                                     |                                                                                             |
| <code>ciopts</code> ( <i><a href="#">area_options</a></i> ) | affect rendition of the dual CI plot                                                        |
| Y axis, X axis, Titles, Legend, Overall                     |                                                                                             |
| <i><a href="#">tway_options</a></i>                         | any options other than <code>by()</code> documented in <a href="#">[G-3] twoway_options</a> |

## Options for estat

Options for `estat` are presented under the following headings:

*Options for estat coefplot*  
*Options for estat endogeffects*  
*Options for estat dualci*  
*Options for estat waldplot*

### Options for estat coefplot

`nocl` removes plots of the pointwise CIs. The default is to plot the CIs.

`no2s1s` removes the plot of the 2SLS estimates. The default is to plot the 2SLS reference line.

#### Plot

`connect_options` specify how points on a graph are to be connected; [G-3] *connect\_options*.

`marker_options` affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] *marker\_options*.

#### CI plot

`ciopts` (*area\_options*) affects rendition of the pointwise CIs; see [G-3] *area\_options*.

#### Line options

`lineopts` (*cline\_options*) affects rendition of reference line identifying the 2SLS estimates; see [G-3] *cline\_options*.

#### Y axis, X axis, Titles, Legend, Overall

`twoway_options` are any of the options documented in [G-3] *twoway\_options*, excluding `by()`. These include options for titling the graph (see [G-3] *title\_options*) and for saving the graph to disk (see [G-3] *saving\_option*).

### Options for estat endogeffects

`all` provides tests for all the four hypotheses, which is the default. If this option is specified with one of the options `noeffect`, `constant`, `dominance`, or `exogeneity`, then tests for all four hypotheses will be performed.

`noeffect` provides a test for the null hypothesis of no effect, that is, a test that the specified endogenous variables do not affect the outcome variable.

`constant` provides a test for the null hypothesis of constant effect, that is, a test that the effects of the specified endogenous variables do not vary across estimated quantiles.

`dominance` provides a test for the null hypothesis of dominance, that is, a test that the effects of the specified endogenous variables are greater than 0 across estimated quantiles.

`exogeneity` provides a test for the null hypothesis of exogeneity, that is, a test that the specified variables are exogenous instead of endogenous.

`level(#)` specifies the confidence level, as a percentage, for CIs. The default is `level(95)` or as set by `set level`; see [U] 20.8 *Specifying the width of confidence intervals*.

`rseed(#)` sets the random-number seed. Specifying this option makes the results reproducible because the critical values are drawn from a bootstrap sample.

`reps(#)` specifies the number of bootstrap replications to get the critical values of the test. The default is `reps(100)`.

### Options for `estat dualci`

`level(#)` specifies the confidence level, as a percentage, for the dual CIs. The default is `level(95)` or as set by `set level`; see [U] 20.8 Specifying the width of confidence intervals.

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] Estimation options.

### Options for `estat waldplot`

`quantile(#)` specifies to plot the Wald statistics for each grid points in the `#th` quantile estimation; the default is the first equation.

`level(#)` specifies the confidence level, as a percentage, for the dual CIs. The default is `level(95)` or as set by `set level`; see [U] 20.8 Specifying the width of confidence intervals.

Plot

`connect_options` specify how points on a graph are to be connected; [G-3] *connect\_options*.

`marker_options` affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] *marker\_options*.

CI plot

`ciopts(area_options)` affects rendition of the pointwise dual CI plot; see [G-3] *area\_options*.

Y axis, X axis, Titles, Legend, Overall

`twoway_options` are any of the options documented in [G-3] *twoway\_options*, excluding `by()`. These include options for titling the graph (see [G-3] *title\_options*) and for saving the graph to disk (see [G-3] *saving\_option*).

## Remarks and examples

For examples of using `estat coefplot` and `estat endogeffects`, see [example 3](#) in [R] `ivqregress`. For an example of using `estat dualci`, see [example 1](#) in [R] `ivqregress`. For an example of using `estat waldplot`, see [example 4](#) in [R] `ivqregress`.

### ▷ Example 1

In [example 1](#) in [R] `ivqregress`, we fit an instrumental-variables quantile regression (IVQR) model to estimate the effects of 401(k) participation on the conditional median of the net financial assets. Suppose that now we want to know the median of the net financial assets when everyone does or does not participate in a 401(k) conditional on other covariates; we can use `margins` to find the answer.

We specify `i.p401k` immediately after margins to obtain the median of the assets under 401(k) participation and under no 401(k) participation. The `at()` option specifies the values of other covariates when computing the median. In particular, the continuous variables such as `income`, `age`, `familysize`, and `educ` are fixed at the sample mean, and people are assumed to be married, participate in an IRA, receive pension benefits, and own a home.

```
. use https://www.stata-press.com/data/r19/assets2
(Excerpt from Chernozhukov and Hansen (2004))
. ivqregress iqr assets (i.p401k = i.e401k) income age familysize
> i.married i.ira i.pension i.ownhome educ
(output omitted)
. margins i.p401k, at((mean) income age familysize educ
> married = 1 ira = 1 pension = 1 ownhome = 1)
Adjusted predictions Number of obs = 9,913
Model VCE: Robust
Expression: Linear predictor, predict()
At: income = 37208.4 (mean)
 age = 41.05891 (mean)
 familysize = 2.865328 (mean)
 married = 1
 ira = 1
 pension = 1
 ownhome = 1
 educ = 13.20629 (mean)
```

|       | Delta-method |           |       |       |                      |
|-------|--------------|-----------|-------|-------|----------------------|
|       | Margin       | std. err. | z     | P> z  | [95% conf. interval] |
| p401k |              |           |       |       |                      |
| No    | 23681.37     | 1007.612  | 23.50 | 0.000 | 21706.49 25656.26    |
| Yes   | 28994.77     | 1123.076  | 25.82 | 0.000 | 26793.58 31195.96    |

The results show that the conditional median of assets when everyone participates in a 401(k) is \$28,995. In contrast, the conditional median of assets when no one participates in a 401(k) is only \$23,681. The difference between these two medians is \$5,313, which is the quantile treatment effect of `p401k` and is the same as the coefficient's value.

◀

## Stored results

`estat endogeffects` stores the following in `r()`:

### Scalars

`r(N_reps)`            number of replications  
`r(level)`             confidence level

### Macros

`r(endog)`            tested endogenous regressors

### Matrices

`r(table)`            matrix containing test statistics and critical values

estat dualci stores the following in `r()`:

Scalars

`r(N)` number of observations  
`r(level)` confidence level for the dual CI

Matrices

`r(table)` matrix containing test statistics,  $p$ -values, and confidence errors, test statistics,  $p$ -values, and confidence intervals

## Methods and formulas

Methods and formulas are presented under the following headings:

*Tests of effects of endogenous variables*  
*Dual CI*

## Tests of effects of endogenous variables

estat endogeffects implements the general inference procedure outlined in section 4 of Chernozhukov and Hansen (2006). It tests the following null hypotheses:

1. No effect: the endogenous variables do not affect the outcome variable.
2. Constant effect: the effects of the endogenous variables do not vary across estimated quantiles.
3. Dominance: the effects of the endogenous variables are greater than 0 across estimated quantiles.
4. Exogeneity: the variables are exogenous instead of endogenous.

It is convenient to write each null hypothesis in the following form:

$$\mathbf{R}(\tau)\{\boldsymbol{\theta}(\tau) - \mathbf{r}(\tau)\} = 0 \quad \text{for each } \tau \in T$$

where  $\mathbf{R}(\tau)$  is a  $q \times p$  matrix of rank  $q$  when  $q$  is smaller than the dimension of  $\boldsymbol{\theta}(\tau)$ .  $\boldsymbol{\theta}(\tau)$  is the  $p \times 1$  coefficient vector for the IVQR model in the  $\tau$ th quantile,  $\mathbf{r}(\tau) \in R^p$ , and  $T$  is a set of estimated quantile indexes. This form is different from the classical setting because  $\boldsymbol{\theta}(\cdot)$  and  $\mathbf{r}(\cdot)$  are functions, which need to be estimated in some cases.

Based on the IVQR model estimates  $\hat{\boldsymbol{\theta}}(\cdot)$ , we focus on the basic inference process

$$\mathbf{v}_n(\cdot) = \mathbf{R}(\cdot)\{\hat{\boldsymbol{\theta}}(\cdot) - \hat{\mathbf{r}}(\cdot)\}$$

where  $\hat{\mathbf{r}}(\cdot)$  is either a vector of constants or a vector of estimates from the classical quantile regression. We use the Kolmogorov–Smirnov statistic  $S_n = f\{\sqrt{n}\mathbf{v}_n(\cdot)\}$ , which is a function of  $\mathbf{v}_n(\cdot)$ .

$$S_n = \sqrt{n} \sup_{\tau \in T} \|\mathbf{v}_n(\tau)\|_{\widehat{\boldsymbol{\Lambda}}(\tau)}$$

where  $\|\mathbf{v}\|_{\boldsymbol{\Lambda}} = \sqrt{\mathbf{v}'\mathbf{A}\mathbf{v}}$ . For the choice of  $\widehat{\boldsymbol{\Lambda}}(\tau)$ , see section 4.4 in Chernozhukov and Hansen (2006). In essence,  $\widehat{\boldsymbol{\Lambda}}(\tau)$  is the empirical variance of the estimating scores implied by the IVQR model.

The null hypothesis is rejected if

$$S_n > c(1 - \alpha)$$

where the critical value  $c(1 - \alpha)$  with confidence level  $1 - \alpha$  can be obtained using the bootstrap resampling procedure described in section 4.3 of Chernozhukov and Hansen (2006).

Now we describe the formal definition of the four hypotheses. For notational simplicity, we assume that there is one endogenous variable but the case of multiple endogenous variables can be extended easily. Let  $\alpha(\tau)$  denote the endogenous coefficient for the  $\tau$ th quantile estimation. In this case,  $\mathbf{R} = (1, 0, \dots, 0)$  for the four hypotheses considered.

1. No effect: the null hypothesis is that the endogenous variable has no impact on the outcome:  $\alpha(\tau) = 0$ . In this case,

$$\begin{aligned} H_0 : \quad & \alpha(\tau) = 0 \quad \text{for all } \tau \in T \\ H_1 : \quad & \alpha(\tau) \neq 0 \quad \text{for some } \tau \in T \end{aligned}$$

and  $\hat{r}(\cdot) = 0$ .

2. Constant effect: the null hypothesis of a constant effect is that the endogenous variable only affects the location of the outcome but not other moments. That is,  $\alpha(\tau) = c$  for all  $\tau \in T$ , where  $c$  is a constant. In this case,

$$\begin{aligned} H_0 : \quad & \alpha(\tau) = c \quad \text{for all } \tau \in T \\ H_1 : \quad & \alpha(\tau) \neq c \quad \text{for some } \tau \in T \end{aligned}$$

and  $\hat{r}(\cdot)$  is the estimate of endogenous coefficient for one of the quantile indexes.

3. Dominance: the dominance test tests whether the effects of endogenous variable are unambiguously beneficial. That is,  $\alpha(\tau) > 0$  for all  $\tau \in T$ . For this hypothesis, we use the one-sided Kolmogorov–Smirnov statistic

$$S_n = \sqrt{n} \sup_{\tau \in T} \max(-\alpha(\tau), 0)$$

In this case,

$$\begin{aligned} H_0 : \quad & \alpha(\tau) > 0 \quad \text{for all } \tau \in T \\ H_1 : \quad & \alpha(\tau) \leq 0 \quad \text{for some } \tau \in T \end{aligned}$$

and  $\hat{r}(\cdot) = 0$ .

4. Exogeneity: if all the covariates are exogenous, we can fit the model by the regular quantile regression and denote  $\eta(\tau)$  as the quantile regression estimator. The difference between  $\theta(\tau)$  and  $\eta(\tau)$  can be used to formulate a Hausman test of exogeneity. In this case, the null and alternative are defined as

$$\begin{aligned} H_0 : \quad & \alpha(\tau) = \mathbf{R}\eta(\tau) \quad \text{for all } \tau \in T \\ H_1 : \quad & \alpha(\tau) \neq \mathbf{R}\eta(\tau) \quad \text{for some } \tau \in T \end{aligned}$$

and  $\hat{\mathbf{r}}(\tau) = \hat{\eta}(\tau)$ , where  $\hat{\eta}(\tau)$  is the regular quantile regression estimate.

## Dual CI

`estat dualci` computes the dual CI proposed in Chernozhukov and Hansen (2008) for the coefficient on the endogenous variable in the IVQR model. The dual CI is robust to the weak instrument, and it is usually wider than the classical CI. `estat dualci` is allowed only after `ivqregress iqr`. If you have not read about the methods for the inverse quantile regression (IQR) estimator, see *The IQR estimator in Methods and formulas* of [R] `ivqregress`.

Suppose we know the true value of the coefficient on the endogenous covariates, which we denote as  $\alpha(\tau)$ , and let  $W_n\{\alpha(\tau)\}$  be the Wald statistic for the coefficient on the instruments in the auxiliary quantile regression. Then by proposition 1 in Chernozhukov and Hansen (2008),

$$W_n\{\alpha(\tau)\} \rightarrow_d \chi^2(1)$$

and for the confidence region  $CR_p\{\alpha(\tau)\} = \{\alpha \in A : W_n(\alpha) < c_p\}$ , where  $P\{\chi^2(1) < c_p\} = p$ , and

$$P[\alpha(\tau) \in CR_p\{\alpha(\tau)\}] = P[W_n\{\alpha(\tau)\} < c_p] = p$$

Intuitively,  $W_n\{\alpha(\tau)\}$  is the Wald statistic for testing whether the coefficients for the instruments are 0. When  $\alpha$  equals the true value  $\alpha(\tau)$ ,  $W(\cdot)$  is  $\chi^2$  distributed with the degree of freedom of 1. Thus, a valid CI for  $\alpha$  can be constructed by the inversion of the Wald statistic. That is,

$$CR_p\{\alpha(\tau)\} = \{\alpha \in A : W_n(\alpha) < c_p\}$$

covers the true value of  $\alpha(\tau)$  with probability approaching  $p$ .

In practice, the dual CI is constructed by the lower and upper limits of the grid points such that the corresponding Wald statistic is smaller than the critical value  $c_p$ .

## References

- Chernozhukov, V., and C. B. Hansen. 2006. Instrumental quantile regression inference for structural and treatment effect models. *Journal of Econometrics* 132: 491–525. <https://doi.org/10.1016/j.jeconom.2005.02.009>.
- . 2008. Instrumental variable quantile regression: A robust inference approach. *Journal of Econometrics* 142: 379–398. <https://doi.org/10.1016/j.jeconom.2007.06.005>.

## Also see

- [R] `ivqregress` — Instrumental-variables quantile regression
- [U] 20 Estimation and postestimation commands

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

## Description

`ivregress` fits linear models where one or more of the regressors are endogenously determined. `ivregress` supports estimation via two-stage least squares (2SLS), limited-information maximum likelihood (LIML), and generalized method of moments (GMM).

## Quick start

2SLS estimation of a linear regression of `y1` on `x1` and endogenous regressor `y2` that is instrumented by `z1`

```
ivregress 2sls y1 x1 (y2 = z1)
```

Same as above, but with two endogenous regressors, `y2` and `y3` instrumented by `z1` and `z2`

```
ivregress 2sls y1 x1 (y2 y3 = z1 z2)
```

Same as above, but absorbing indicator variables for the levels of `cvar1` and `cvar2`

```
ivregress 2sls y1 x1 (y2 y3 = z1 z2), absorb(cvar1 cvar2)
```

With robust standard errors

```
ivregress 2sls y1 x1 (y2 y3 = z1 z2), vce(robust)
```

Report small-sample statistics

```
ivregress 2sls y1 x1 (y2 y3 = z1 z2), small
```

Use LIML estimation

```
ivregress liml y1 x1 (y2 y3 = z1 z2)
```

Use GMM estimation

```
ivregress gmm y1 x1 (y2 y3 = z1 z2)
```

Also specify a weight matrix that allows for correlation within clusters identified by `cvar`

```
ivregress gmm y1 x1 (y2 y3 = z1 z2), wmatrix(cluster cvar)
```

## Menu

Statistics > Endogenous covariates > Linear regression with endogenous covariates

## Syntax

```
ivregress estimator depvar [varlist1] (varlist2 = varlistiv) [if] [in] [weight]
[, options]
```

*varlist<sub>1</sub>* is the list of exogenous variables.

*varlist<sub>2</sub>* is the list of endogenous variables.

*varlist<sub>iv</sub>* is the list of exogenous variables used with *varlist<sub>1</sub>* as instruments for *varlist<sub>2</sub>*.

| <i>estimator</i> | Description                                   |
|------------------|-----------------------------------------------|
| 2sls             | two-stage least squares (2SLS)                |
| liml             | limited-information maximum likelihood (LIML) |
| gmm              | generalized method of moments (GMM)           |

| <i>options</i>                                      | Description                                                                                                                                                                         |
|-----------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Model                                               |                                                                                                                                                                                     |
| <code>absorb(varlist[ , method])<sup>1</sup></code> | specify categorical variables to be absorbed                                                                                                                                        |
| <code>† dfabsorb<sup>1</sup></code>                 | adjust degrees of freedom for collinearity among absorbed variables                                                                                                                 |
| <code>noconstant<sup>1</sup></code>                 | suppress constant term                                                                                                                                                              |
| <code>hascons</code>                                | has user-supplied constant                                                                                                                                                          |
| GMM <sup>2</sup>                                    |                                                                                                                                                                                     |
| <code>wmatrix(wmtype)</code>                        | <i>wmtype</i> may be <code>robust</code> , <code>cluster clustvar</code> , <code>hac hacspec</code> , or <code>unadjusted</code>                                                    |
| <code>center</code>                                 | center moments in weight matrix computation                                                                                                                                         |
| <code>igmm</code>                                   | use iterative instead of two-step GMM estimator                                                                                                                                     |
| <code>eps(#)<sup>3</sup></code>                     | specify parameter convergence criterion; default is <code>eps(1e-6)</code>                                                                                                          |
| <code>weps(#)<sup>3</sup></code>                    | specify weight-matrix convergence criterion; default is <code>weps(1e-6)</code>                                                                                                     |
| SE/Robust                                           |                                                                                                                                                                                     |
| <code>vce(vcetype)</code>                           | <i>vcetype</i> may be <code>unadjusted</code> , <code>robust</code> , <code>cluster clustvar</code> , <code>bootstrap</code> , <code>jackknife</code> , or <code>hac hacspec</code> |
| Reporting                                           |                                                                                                                                                                                     |
| <code>level(#)</code>                               | set confidence level; default is <code>level(95)</code>                                                                                                                             |
| <code>first</code>                                  | report first-stage regression                                                                                                                                                       |
| <code>small</code>                                  | make degrees-of-freedom adjustments and report small-sample statistics                                                                                                              |
| <code>noheader</code>                               | display only the coefficient table                                                                                                                                                  |
| <code>depname(depname)</code>                       | substitute dependent variable name                                                                                                                                                  |
| <code>eform(string)</code>                          | report exponentiated coefficients and use <i>string</i> to label them                                                                                                               |
| <code>display_options</code>                        | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling                                    |
| Optimization                                        |                                                                                                                                                                                     |
| <code>† optimization_options<sup>4</sup></code>     | control the optimization process; seldom used                                                                                                                                       |
| <code>perfect</code>                                | do not check for collinearity between endogenous regressors and excluded instruments                                                                                                |
| <code>coeflegend</code>                             | display legend instead of statistics                                                                                                                                                |

<sup>1</sup>These options may be specified only with `2s1s`.

<sup>2</sup>These options may be specified only with `gmm`.

<sup>3</sup>These options may be specified only with `igmm`.

<sup>4</sup>These options may be specified only with `igmm` or `2s1s` and `absorb()`.

<sup>†</sup>Ignored if only one absorbed variable is specified.

`varlist1`, `varlist2`, and `varlistiv` may contain factor variables; see [U] 11.4.3 Factor variables.

`depvar`, `varlist1`, `varlist2`, and `varlistiv` may contain time-series operators; see [U] 11.4.4 Time-series varlists.

`bayesboot`, `bootstrap`, `by`, `collect`, `fmm`, `jackknife`, `rolling`, `statsby`, and `svy` are allowed; see [U] 11.1.10 Prefix commands. For more details, see [FMM] `fmm`: `ivregress`.

Weights are not allowed with the `bootstrap` prefix; see [R] `bootstrap`.

`aweight`s are not allowed with the `jackknife` prefix; see [R] `jackknife`.

`absorb()`, `dfabsorb`, `hascons`, `vce()`, `noheader`, `depname()`, and `weights` are not allowed with the `svy` prefix; see [SVY] `svy`.

`fweights`, `ifweights`, and `pweights` are not allowed with `vce(hac)`.

`aweight`s, `fweights`, `ifweights`, and `pweights` are allowed; see [U] 11.1.6 weight.

`perfect` and `coeflegend` do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Options

### Model

`absorb(varlist [ , method ])` specifies the categorical variables to be absorbed. The results are adjusted as if indicator variables for each level of each variable in `varlist` were included in the regression.

The absorption of categorical variables involves projecting the `depvar` and all variables in `varlist1`, `varlist2`, and `varlistiv` via an alternating projection method (APM) iterative algorithm. `method` specifies the APM and is one of `halperin` or `cimmino`.

`halperin`, the default, uses the product of the projection matrices.

`cimmino` uses the mean of the projection matrices.

The two methods typically perform similarly. See [Stammann \(2018\)](#) for details.

`method` is ignored if only one absorbed variable is specified.

`absorb()` may not be combined with `vce(hac)`.

`dfabsorb` adjusts the degrees of freedom to account for collinearity among absorbed variables. The default degrees of freedom assumes that all absorbed variables are independent. This option is ignored if only one absorbed variable is specified in `absorb()`.

`noconstant`; see [R] Estimation options.

`hascons` indicates that a user-defined constant or its equivalent is specified among the independent variables.

### GMM

`wmatrix(wmtype)` specifies the type of weight matrix to be used in conjunction with the GMM estimator.

`wmatrix(robust)`, the default, requests a weight matrix that is optimal when the error term is heteroskedastic.

`wmatrix(cluster clustvar)` requests a weight matrix that accounts for arbitrary correlation among observations within clusters identified by *clustvar*.

`wmatrix(hac hacspec)` requests a heteroskedasticity- and autocorrelation-consistent (HAC) weight matrix. The full syntax of *hacspec* is one of the following:

`wmatrix(hac kernel [#])` requests a HAC weight matrix using the specified kernel (see below) with optional # lags. The bandwidth of a kernel is equal to # + 1. If # is not specified, a kernel with  $N - 2$  lags is used, where  $N$  is the sample size.

`wmatrix(hac kernel opt [#])` requests a HAC weight matrix using the specified kernel (see below), and the lag order is selected using Newey and West's (1994) optimal lag-selection algorithm. # is an optional tuning parameter that affects the lag order selected; see the [discussion](#) in *Methods and formulas*.

*kernel* may be one of the following:

`bartlett` or `nwest` requests the Bartlett (Newey–West) kernel.

`parzen` or `gallant` requests the Parzen (Gallant 1987) kernel.

`quadraticspectral` or `andrews` requests the quadratic spectral (Andrews 1991) kernel.

`wmatrix(unadjusted)` requests a weight matrix that is suitable when the errors are homoskedastic. The GMM estimator with this weight matrix is equivalent to the 2SLS estimator.

`center` requests that the sample moments be centered (demeaned) when computing GMM weight matrices. By default, centering is not done.

`igmm` requests that the iterative GMM estimator be used instead of the default two-step GMM estimator. Convergence is declared when the relative change in the parameter vector from one iteration to the next is less than `eps()` or the relative change in the weight matrix is less than `weps()`.

`eps(#)` specifies the convergence criterion for successive parameter estimates when the iterative GMM estimator is used. The default is `eps(1e-6)`. Convergence is declared when the relative difference between successive parameter estimates is less than `eps()` and the relative difference between successive estimates of the weight matrix is less than `weps()`.

`weps(#)` specifies the convergence criterion for successive estimates of the weight matrix when the iterative GMM estimator is used. The default is `weps(1e-6)`. Convergence is declared when the relative difference between successive parameter estimates is less than `eps()` and the relative difference between successive estimates of the weight matrix is less than `weps()`.

---

#### SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [\[R\] vce\\_option](#).

`vce(unadjusted)`, the default for `2s1s` and `liml`, specifies that an unadjusted (nonrobust) VCE matrix be used. The default for `gmm` is based on the *wmtype* specified in the `wmatrix()` option; see `wmatrix()` above. If `wmatrix()` is specified with `gmm` but `vce()` is not, then *vcetype* is set equal to *wmtype*. To override this behavior and obtain an unadjusted (nonrobust) VCE matrix, specify `vce(unadjusted)`.

`vce(hac hacspec)` specifies that a HAC covariance matrix be used. The syntax is identical to that for `wmatrix()`. `vce(hac)` may not be combined with `absorb()`.

## Reporting

`level(#)`; see [R] [Estimation options](#).

`first` requests that the first-stage regression results be displayed.

`small` requests that the degrees-of-freedom adjustment  $N/(N - k)$  be made to the variance–covariance matrix of parameters and that small-sample  $F$  and  $t$  statistics be reported, where  $N$  is the sample size and  $k$  is the number of parameters estimated. By default, no degrees-of-freedom adjustment is made, and Wald and  $z$  statistics are reported. Even with this option, no degrees-of-freedom adjustment is made to the weight matrix when the GMM estimator is used.

`noheader` suppresses the display of the summary statistics at the top of the output, displaying only the coefficient table.

`depname(depname)` is used only in programs and ado-files that use `ivregress` to fit models other than instrumental-variables regression. `depname()` may be specified only at estimation time. *depname* is recorded as the identity of the dependent variable, even though the estimates are calculated using *depvar*. This method affects the labeling of the output—not the results calculated—but could affect later calculations made by `predict`, where the residual would be calculated as deviations from *depname* rather than *depvar*. `depname()` is most typically used when *depvar* is a temporary variable (see [P] [macro](#)) used as a proxy for *depname*.

`eform(string)` is used only in programs and ado-files that use `ivregress` to fit models other than instrumental-variables regression. `eform()` specifies that the coefficient table be displayed in “exponentiated form”, as defined in [R] [Maximize](#), and that *string* be used to label the exponentiated coefficients in the table.

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

## Optimization

*optimization\_options*: `iterate(#)`, `[no]log`, and `tolerance(#)`. `iterate()` specifies the maximum number of iterations to perform in conjunction with the iterative GMM estimator. The default is the number set using `set maxiter`, which is 300 by default. `log/nolog` specifies whether to show the iteration log; see `set iterlog` in [R] [set iter](#). `tolerance()` is allowed only with `2sls` and the `absorb()` option and specifies the projection tolerance. These options are seldom used.

The following options are available with `ivregress` but are not shown in the dialog box:

`perfect` requests that `ivregress` not check for collinearity between the endogenous regressors and excluded instruments, allowing one to specify “perfect” instruments. This option cannot be used with the LIML estimator. This option may be required when using `ivregress` to implement other estimators.

`coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

`ivregress` performs instrumental-variables regression and weighted instrumental-variables regression. For a general discussion of instrumental variables, see [Baum \(2006\)](#), [Cameron and Trivedi \(2005; 2022, chap. 7\)](#) [Davidson and MacKinnon \(1993\)](#), [Greene \(2018, chap. 8\)](#), and [Wooldridge \(2010, 2020\)](#). See [Hall \(2005\)](#) for a lucid presentation of GMM estimation. [Angrist and Pischke \(2009, chap. 4\)](#) offer

a casual yet thorough introduction to instrumental-variables estimators, including their use in estimating treatment effects. Some of the earliest work on simultaneous systems can be found in Cowles Commission monographs—[Koopmans and Marschak \(1950\)](#) and [Koopmans and Hood \(1953\)](#)—with the first developments of 2SLS appearing in [Theil \(1953\)](#) and [Basmann \(1957\)](#). However, [Stock and Watson \(2019, 401–402\)](#) present an example of the method of instrumental variables that was first published in 1928 by Philip Wright.

The syntax for `ivregress` assumes that you want to fit one equation from a system of equations or an equation for which you do not want to specify the functional form for the remaining equations of the system. To fit a full system of equations, using either 2SLS equation-by-equation or three-stage least squares, see [\[R\] reg3](#). An advantage of `ivregress` is that you can fit one equation of a multiple-equation system without specifying the functional form of the remaining equations.

Formally, the model fit by `ivregress` is

$$y_i = \mathbf{y}_i\boldsymbol{\beta}_1 + \mathbf{x}_{1i}\boldsymbol{\beta}_2 + u_i \quad (1)$$

$$\mathbf{y}_i = \mathbf{x}_{1i}\boldsymbol{\Pi}_1 + \mathbf{x}_{2i}\boldsymbol{\Pi}_2 + \mathbf{v}_i \quad (2)$$

Here  $y_i$  is the dependent variable for the  $i$ th observation,  $\mathbf{y}_i$  represents the endogenous regressors (`varlist2` in the syntax diagram),  $\mathbf{x}_{1i}$  represents the included exogenous regressors (`varlist1` in the syntax diagram), and  $\mathbf{x}_{2i}$  represents the excluded exogenous regressors (`varlistiv` in the syntax diagram).  $\mathbf{x}_{1i}$  and  $\mathbf{x}_{2i}$  are collectively called the instruments.  $u_i$  and  $\mathbf{v}_i$  are zero-mean error terms, and the correlations between  $u_i$  and the elements of  $\mathbf{v}_i$  are presumably nonzero.

The rest of the discussion is presented under the following headings:

[2SLS and LIML estimators](#)

[GMM estimator](#)

[Video example](#)

## 2SLS and LIML estimators

The most common instrumental-variables estimator is 2SLS.

### ▷ Example 1: 2SLS estimator

We have state data from the 1980 census on the median dollar value of owner-occupied housing (`hshngval`) and the median monthly gross rent (`rent`). We want to model `rent` as a function of `hshngval` and the percentage of the population living in urban areas (`pcturban`):

$$\text{rent}_i = \beta_0 + \beta_1 \text{hshngval}_i + \beta_2 \text{pcturban}_i + u_i$$

where  $i$  indexes states and  $u_i$  is an error term.

Because random shocks that affect rental rates in a state probably also affect housing values, we treat `hshngval` as endogenous. We believe that the correlation between `hshngval` and  $u$  is not equal to zero. On the other hand, we have no reason to believe that the correlation between `pcturban` and  $u$  is nonzero, so we assume that `pcturban` is exogenous.

Because we are treating `hshngval` as an endogenous regressor, we must have one or more additional variables available that are correlated with `hshngval` but uncorrelated with  $u$ . Moreover, these excluded exogenous variables must not affect `rent` directly, because if they do then they should be included in the regression equation we specified above. In our dataset, we have a variable for family income (`faminc`) and for region of the country (`region`) that we believe are correlated with `hshngval` but not the error term. Together, `pcturban`, `faminc`, and factor variables `2.region`, `3.region`, and `4.region` constitute our set of instruments.

To fit the equation in Stata, we specify the dependent variable and the list of included exogenous variables. In parentheses, we specify the endogenous regressors, an equal sign, and the excluded exogenous variables. Only the additional exogenous variables must be specified to the right of the equal sign; the exogenous variables that appear in the regression equation are automatically included as instruments.

Here we fit our model with the 2SLS estimator:

```
. use https://www.stata-press.com/data/r19/hsng
(1980 Census housing data)
. ivregress 2sls rent pcturban (hsngval = faminc i.region)
Instrumental-variables 2SLS regression Number of obs = 50
 Wald chi2(2) = 90.76
 Prob > chi2 = 0.0000
 R-squared = 0.5989
 Root MSE = 22.166
```

| rent     | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |          |
|----------|-------------|-----------|------|-------|----------------------|----------|
| hsngval  | .0022398    | .0003284  | 6.82 | 0.000 | .0015961             | .0028836 |
| pcturban | .081516     | .2987652  | 0.27 | 0.785 | -.504053             | .667085  |
| _cons    | 120.7065    | 15.22839  | 7.93 | 0.000 | 90.85942             | 150.5536 |

Endogenous: hsngval

Exogenous: pcturban faminc 2.region 3.region 4.region

As we would expect, states with higher housing values have higher rental rates. The proportion of a state's population that is urban does not have a significant effect on rents.



## □ Technical note

In a simultaneous-equations framework, we could write the model we just fit as

$$\begin{aligned} \text{hsngval}_i &= \pi_0 + \pi_1 \text{faminc}_i + \pi_2 \text{2.region}_i + \pi_3 \text{3.region}_i + \pi_4 \text{4.region}_i + v_i \\ \text{rent}_i &= \beta_0 + \beta_1 \text{hsngval}_i + \beta_2 \text{pcturban}_i + u_i \end{aligned}$$

which here happens to be recursive (triangular), because `hsngval` appears in the equation for `rent` but `rent` does not appear in the equation for `hsngval`. In general, however, systems of simultaneous equations are not recursive. Because this system is recursive, we could fit the two equations individually via OLS if we were willing to assume that  $u$  and  $v$  were independent. For a more detailed discussion of triangular systems, see [Kmenta \(1997, 719–720\)](#).

Historically, instrumental-variables estimation and systems of simultaneous equations were taught concurrently, and older textbooks describe instrumental-variables estimation solely in the context of simultaneous equations. However, in recent decades, the treatment of endogeneity and instrumental-variables estimation has taken on a much broader scope, while interest in the specification of complete systems of simultaneous equations has waned. Most recent textbooks, such as [Cameron and Trivedi \(2005\)](#), [Davidson and MacKinnon \(1993\)](#), and [Wooldridge \(2010, 2020\)](#), treat instrumental-variables estimation as an integral part of the modern economists' toolkit and introduce it long before shorter discussions on simultaneous equations.



In addition to the 2SLS member of the  $\kappa$ -class estimators, `ivregress` implements the LIML estimator. Both theoretical and Monte Carlo exercises indicate that the LIML estimator may yield less bias and confidence intervals with better coverage rates than the 2SLS estimator. See [Poi \(2006\)](#) and [Stock, Wright, and Yogo \(2002\)](#) (and the papers cited therein) for Monte Carlo evidence.

## ▷ Example 2: LIML estimator

Here we refit our model with the LIML estimator:

```
. ivregress liml rent pcturban (hsngval = faminc i.region)
Instrumental-variables LIML regression Number of obs = 50
 Wald chi2(2) = 75.71
 Prob > chi2 = 0.0000
 R-squared = 0.4901
 Root MSE = 24.992
```

| rent     | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|----------|-------------|-----------|-------|-------|----------------------|----------|
| hsngval  | .0026686    | .0004173  | 6.39  | 0.000 | .0018507             | .0034865 |
| pcturban | -.1827391   | .3571132  | -0.51 | 0.609 | -.8826681            | .5171899 |
| _cons    | 117.6087    | 17.22625  | 6.83  | 0.000 | 83.84587             | 151.3715 |

Endogenous: hsngval

Exogenous: pcturban faminc 2.region 3.region 4.region

These results are qualitatively similar to the [2SLS results](#), although the coefficient on `hsngval` is about 19% higher.

◀

## GMM estimator

Since the celebrated paper of [Hansen \(1982\)](#), the GMM has been a popular method of estimation in economics and finance, and it lends itself well to instrumental-variables estimation. The basic principle is that we have some *moment* or *orthogonality* conditions of the form

$$E(\mathbf{z}_i u_i) = \mathbf{0} \quad (3)$$

From (1), we have  $u_i = y_i - \mathbf{y}_i \boldsymbol{\beta}_1 - \mathbf{x}_{1i} \boldsymbol{\beta}_2$ . What are the elements of the instrument vector  $\mathbf{z}_i$ ? By assumption,  $\mathbf{x}_{1i}$  is uncorrelated with  $u_i$ , as are the excluded exogenous variables  $\mathbf{x}_{2i}$ , and so we use  $\mathbf{z}_i = [\mathbf{x}_{1i} \ \mathbf{x}_{2i}]$ . The moment conditions are simply the mathematical representation of the assumption that the instruments are exogenous—that is, the instruments are orthogonal to (uncorrelated with)  $u_i$ .

If the number of elements in  $\mathbf{z}_i$  is just equal to the number of unknown parameters, then we can apply the analogy principle to (3) and solve

$$\frac{1}{N} \sum_i \mathbf{z}_i u_i = \frac{1}{N} \sum_i \mathbf{z}_i (y_i - \mathbf{y}_i \boldsymbol{\beta}_1 - \mathbf{x}_{1i} \boldsymbol{\beta}_2) = \mathbf{0} \quad (4)$$

This equation is known as the method of moments estimator. Here, where the number of instruments equals the number of parameters, the method of moments estimator coincides with the 2SLS estimator, which also coincides with what has historically been called the indirect least-squares estimator (Judge et al. 1985, 595).

The “generalized” in GMM addresses the case in which the number of instruments (columns of  $\mathbf{z}_i$ ) exceeds the number of parameters to be estimated. Here there is no unique solution to the population moment conditions defined in (3), so we cannot use (4). Instead, we define the objective function

$$Q(\beta_1, \beta_2) = \left( \frac{1}{N} \sum_i \mathbf{z}_i u_i \right)' \mathbf{W} \left( \frac{1}{N} \sum_i \mathbf{z}_i u_i \right) \quad (5)$$

where  $\mathbf{W}$  is a positive-definite matrix with the same number of rows and columns as the number of columns of  $\mathbf{z}_i$ .  $\mathbf{W}$  is known as the weight matrix, and we specify its structure with the `wmatrix()` option. The GMM estimator of  $(\beta_1, \beta_2)$  minimizes  $Q(\beta_1, \beta_2)$ ; that is, the GMM estimator chooses  $\beta_1$  and  $\beta_2$  to make the moment conditions as close to zero as possible for a given  $\mathbf{W}$ . For a more general GMM estimator, see [R] `gmm`. `gmm` does not restrict you to fitting a single linear equation, though the syntax is more complex.

A well-known result is that if we define the matrix  $\mathbf{S}_0$  to be the covariance of  $\mathbf{z}_i u_i$  and set  $\mathbf{W} = \mathbf{S}_0^{-1}$ , then we obtain the optimal two-step GMM estimator, where by optimal estimator we mean the one that results in the smallest variance given the moment conditions defined in (3).

Suppose that the errors  $u_i$  are heteroskedastic but independent among observations. Then

$$\mathbf{S}_0 = E(\mathbf{z}_i u_i u_i \mathbf{z}_i') = E(u_i^2 \mathbf{z}_i \mathbf{z}_i')$$

and the sample analogue is

$$\hat{\mathbf{S}} = \frac{1}{N} \sum_i \hat{u}_i^2 \mathbf{z}_i \mathbf{z}_i' \quad (6)$$

To implement this estimator, we need estimates of the sample residuals  $\hat{u}_i$ . `ivregress gmm` obtains the residuals by estimating  $\beta_1$  and  $\beta_2$  by 2SLS and then evaluates (6) and sets  $\mathbf{W} = \hat{\mathbf{S}}^{-1}$ . Equation (6) is the same as the center term of the “sandwich” robust covariance matrix available from most Stata estimation commands through the `vce(robust)` option.

### ▷ Example 3: GMM estimator

Here we refit our model of rents by using the GMM estimator, allowing for heteroskedasticity in  $u_i$ :

```
. ivregress gmm rent pcturban (hsngval = faminc i.region), wmatrix(robust)
Instrumental-variables GMM regression Number of obs = 50
 Wald chi2(2) = 112.09
 Prob > chi2 = 0.0000
 R-squared = 0.6616
GMM weight matrix: Robust Root MSE = 20.358
```

|          |             | Robust    |       |       |                      |          |
|----------|-------------|-----------|-------|-------|----------------------|----------|
| rent     | Coefficient | std. err. | z     | P> z  | [95% conf. interval] |          |
| hsngval  | .0014643    | .0004473  | 3.27  | 0.001 | .0005877             | .002341  |
| pcturban | .7615482    | .2895105  | 2.63  | 0.009 | .1941181             | 1.328978 |
| _cons    | 112.1227    | 10.80234  | 10.38 | 0.000 | 90.95052             | 133.2949 |

Endogenous: hsngval

Exogenous: pcturban faminc 2.region 3.region 4.region

Because we requested that a heteroskedasticity-consistent weight matrix be used during estimation but did not specify the `vce()` option, `ivregress` reported standard errors that are robust to heteroskedasticity. Had we specified `vce(unadjusted)`, we would have obtained standard errors that would be correct only if the weight matrix  $\mathbf{W}$  does in fact converge to  $\mathbf{S}_0^{-1}$ .

## □ Technical note

Many software packages that implement GMM estimation use the same heteroskedasticity-consistent weight matrix we used in the [previous example](#) to obtain the optimal two-step estimates but do not use a heteroskedasticity-consistent VCE, even though they may label the standard errors as being “robust”. To replicate results obtained from other packages, you may have to use the `vce(unadjusted)` option. See [Methods and formulas](#) below for a discussion of robust covariance matrix estimation in the GMM framework. □

By changing our definition of  $\mathbf{S}_0$ , we can obtain GMM estimators suitable for use with other types of data that violate the assumption that the errors are independent and identically distributed. For example, you may have a dataset that consists of multiple observations for each person in a sample. The observations that correspond to the same person are likely to be correlated, and the estimation technique should account for that lack of independence. Say that in your dataset, people are identified by the variable `personid` and you type

```
. ivregress gmm ..., wmatrix(cluster personid)
```

Here `ivregress` estimates  $\mathbf{S}_0$  as

$$\hat{\mathbf{S}} = \frac{1}{N} \sum_{c \in C} \mathbf{q}_c \mathbf{q}_c'$$

where  $C$  denotes the set of clusters and

$$\mathbf{q}_c = \sum_{i \in c_j} \hat{u}_i \mathbf{z}_i$$

where  $c_j$  denotes the  $j$ th cluster. This weight matrix accounts for the within-person correlation among observations, so the GMM estimator that uses this version of  $\mathbf{S}_0$  will be more efficient than the estimator that ignores this correlation.

## ▷ Example 4: GMM estimator with clustering

We have data from the National Longitudinal Survey on young women’s wages as reported in a series of interviews from 1968 through 1988, and we want to fit a model of wages as a function of each woman’s age and age squared, job tenure, birth year, and level of education. We believe that random shocks that affect a woman’s wage also affect her job tenure, so we treat tenure as endogenous. As additional instruments, we use her union status, number of weeks worked in the past year, and a dummy indicating whether she lives in a metropolitan area. Because we have several observations for each woman (corresponding to interviews done over several years), we want to control for clustering on each person.

```
. use https://www.stata-press.com/data/r19/nlswork
(National Longitudinal Survey of Young Women, 14-24 years old in 1968)
. ivregress gmm ln_wage age c.age#c.age birth_yr grade
> (tenure = union wks_work msp), wmatrix(cluster idcode)

Instrumental-variables GMM regression Number of obs = 18,625
 Wald chi2(5) = 1807.17
 Prob > chi2 = 0.0000
 Root MSE = .46951

GMM weight matrix: Cluster (idcode)
 (Std. err. adjusted for 4,110 clusters in idcode)
```

| ln_wage     | Coefficient | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |           |
|-------------|-------------|---------------------|-------|-------|----------------------|-----------|
| tenure      | .099221     | .0037764            | 26.27 | 0.000 | .0918194             | .1066227  |
| age         | .0171146    | .0066895            | 2.56  | 0.011 | .0040034             | .0302259  |
| c.age#c.age | -.0005191   | .0001111            | -4.68 | 0.000 | -.0007366            | -.0003016 |
| birth_yr    | -.0085994   | .0021932            | -3.92 | 0.000 | -.012898             | -.0043008 |
| grade       | .071574     | .0029938            | 23.91 | 0.000 | .0657062             | .0774417  |
| _cons       | .8575071    | .1616274            | 5.31  | 0.000 | .5407231             | 1.174291  |

Endogenous: tenure

Exogenous: age c.age#c.age birth\_yr grade union wks\_work msp

Both job tenure and years of schooling have significant positive effects on wages.

◀

Time-series data are often plagued by serial correlation. In these cases, we can construct a weight matrix to account for the fact that the error in period  $t$  is probably correlated with the errors in periods  $t - 1$ ,  $t - 2$ , etc. A HAC weight matrix can be used to account for both serial correlation and potential heteroskedasticity.

To request a HAC weight matrix, you specify the `wmatrix(hac kernel [#|opt])` option. *kernel* specifies which of three kernels to use: `bartlett`, `parzen`, or `quadraticspectral`. *kernel* determines the amount of weight given to lagged values when computing the HAC matrix, and `#` denotes the maximum number of lags to use. Many texts refer to the bandwidth of the kernel instead of the number of lags; the bandwidth is equal to the number of lags plus one. If neither `opt` nor `#` is specified, then  $N - 2$  lags are used, where  $N$  is the sample size.

If you specify `wmatrix(hac kernel opt)`, then `ivregress` uses Newey and West's (1994) algorithm for automatically selecting the number of lags to use. Although the authors' Monte Carlo simulations do show that the procedure may result in size distortions of hypothesis tests, the procedure is still useful when little other information is available to help choose the number of lags.

For more on GMM estimation, see [Baum \(2006\)](#); [Baum, Schaffer, and Stillman \(2003, 2007\)](#); [Cameron and Trivedi \(2005\)](#); [Davidson and MacKinnon \(1993\)](#); [Hayashi \(2000\)](#); or [Wooldridge \(2010\)](#). See [Newey and West \(1987\)](#) and [Wang and Wu \(2012\)](#) for an introduction to HAC covariance matrix estimation.

## Video example

[Instrumental variables regression using Stata](#)

## Stored results

`ivregress` stores the following in `e()`:

### Scalars

|                            |                                                                   |
|----------------------------|-------------------------------------------------------------------|
| <code>e(N)</code>          | number of observations                                            |
| <code>e(k_absorb)</code>   | total number of absorbed categories                               |
| <code>e(mss)</code>        | model sum of squares                                              |
| <code>e(df_m)</code>       | model degrees of freedom                                          |
| <code>e(rss)</code>        | residual sum of squares                                           |
| <code>e(df_r)</code>       | residual degrees of freedom                                       |
| <code>e(r2)</code>         | $R^2$                                                             |
| <code>e(r2_a)</code>       | adjusted $R^2$                                                    |
| <code>e(F)</code>          | $F$ statistic                                                     |
| <code>e(rmse)</code>       | root mean squared error                                           |
| <code>e(N_clust)</code>    | number of clusters                                                |
| <code>e(chi2)</code>       | $\chi^2$                                                          |
| <code>e(kappa)</code>      | $\kappa$ used in LIML estimator                                   |
| <code>e(J)</code>          | value of GMM objective function                                   |
| <code>e(wlagopt)</code>    | lags used in HAC weight matrix (if Newey–West algorithm used)     |
| <code>e(vcelagopt)</code>  | lags used in HAC VCE matrix (if Newey–West algorithm used)        |
| <code>e(hac_lag)</code>    | HAC lag                                                           |
| <code>e(rank)</code>       | rank of $e(V)$                                                    |
| <code>e(k_endog)</code>    | number of endogenous regressors (after factor-variable expansion) |
| <code>e(iterations)</code> | number of GMM iterations (0 if not applicable)                    |

### Macros

|                              |                                                                  |
|------------------------------|------------------------------------------------------------------|
| <code>e(cmd)</code>          | <code>ivregress</code>                                           |
| <code>e(cmdline)</code>      | command as typed                                                 |
| <code>e(depvar)</code>       | name of dependent variable                                       |
| <code>e(endog)</code>        | names of endogenous variables                                    |
| <code>e(exog)</code>         | names of exogenous variables                                     |
| <code>e(absvar)</code>       | names of absorbed variables                                      |
| <code>e(apm)</code>          | alternating projection method                                    |
| <code>e(constant)</code>     | <code>noconstant</code> or <code>hasconstant</code> if specified |
| <code>e(wtype)</code>        | weight type                                                      |
| <code>e(wexp)</code>         | weight expression                                                |
| <code>e(title)</code>        | title in estimation output                                       |
| <code>e(clustvar)</code>     | name of cluster variable                                         |
| <code>e(hac_kernel)</code>   | HAC kernel                                                       |
| <code>e(vce)</code>          | <i>vctype</i> specified in <code>vce()</code>                    |
| <code>e(vctype)</code>       | title used to label Std. err.                                    |
| <code>e(estimator)</code>    | <code>2sls</code> , <code>liml</code> , or <code>gmm</code>      |
| <code>e(exogr)</code>        | exogenous regressors                                             |
| <code>e(wmatrix)</code>      | <i>wmtype</i> specified in <code>wmatrix()</code>                |
| <code>e(moments)</code>      | centered if <code>center</code> specified                        |
| <code>e(small)</code>        | <code>small</code> if small-sample statistics                    |
| <code>e(properties)</code>   | <code>b V</code>                                                 |
| <code>e(estat_cmd)</code>    | program used to implement <code>estat</code>                     |
| <code>e(predict)</code>      | program used to implement <code>predict</code>                   |
| <code>e(footnote)</code>     | program used to implement footnote display                       |
| <code>e(marginsok)</code>    | predictions allowed by <code>margins</code>                      |
| <code>e(marginsnotok)</code> | predictions disallowed by <code>margins</code>                   |
| <code>e(asbalanced)</code>   | factor variables <code>fvset</code> as <code>asbalanced</code>   |
| <code>e(asobserved)</code>   | factor variables <code>fvset</code> as <code>asobserved</code>   |

### Matrices

|                   |                                                                         |
|-------------------|-------------------------------------------------------------------------|
| <code>e(b)</code> | coefficient vector                                                      |
| <code>e(W)</code> | weight matrix used to compute GMM estimates                             |
| <code>e(S)</code> | moment covariance matrix used to compute GMM variance–covariance matrix |
| <code>e(V)</code> | variance–covariance matrix of the estimators                            |

|                              |                                                        |
|------------------------------|--------------------------------------------------------|
| <code>e(V_modelbased)</code> | model-based variance                                   |
| <code>e(kabsorb)</code>      | number of levels for each absorbed variable            |
| <code>e(dfabsorb)</code>     | adjusted degrees of freedom for each absorbed variable |
| <code>e(ksingle)</code>      | number of singletons for each absorbed variable        |
| Functions                    |                                                        |
| <code>e(sample)</code>       | marks estimation sample                                |

In addition to the above, the following is stored in `r()`:

|                       |                                                                                                                       |
|-----------------------|-----------------------------------------------------------------------------------------------------------------------|
| Matrices              |                                                                                                                       |
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, $p$ -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

Methods and formulas are presented under the following headings:

*Notation*  
*2SLS and LIML estimators*  
*2SLS estimator with absorb() option GMM estimator*

## Notation

Items printed in lowercase and italicized (for example,  $x$ ) are scalars. Items printed in lowercase and boldfaced (for example,  $\mathbf{x}$ ) are vectors. Items printed in uppercase and boldfaced (for example,  $\mathbf{X}$ ) are matrices.

The model is

$$\mathbf{y} = \mathbf{Y}\beta_1 + \mathbf{X}_1\beta_2 + \mathbf{u} = \mathbf{X}\beta + \mathbf{u}$$

$$\mathbf{Y} = \mathbf{X}_1\Pi_1 + \mathbf{X}_2\Pi_2 + \mathbf{V} = \mathbf{Z}\Pi + \mathbf{V}$$

where  $\mathbf{y}$  is an  $N \times 1$  vector of the left-hand-side variable;  $N$  is the sample size;  $\mathbf{Y}$  is an  $N \times p$  matrix of  $p$  endogenous regressors;  $\mathbf{X}_1$  is an  $N \times k_1$  matrix of  $k_1$  included exogenous regressors;  $\mathbf{X}_2$  is an  $N \times k_2$  matrix of  $k_2$  excluded exogenous variables,  $\mathbf{X} = [\mathbf{Y} \ \mathbf{X}_1]$ ,  $\mathbf{Z} = [\mathbf{X}_1 \ \mathbf{X}_2]$ ;  $\mathbf{u}$  is an  $N \times 1$  vector of errors;  $\mathbf{V}$  is an  $N \times p$  matrix of errors;  $\beta = [\beta_1 \ \beta_2]$  is a  $k = (p + k_1) \times 1$  vector of parameters; and  $\Pi$  is a  $(k_1 + k_2) \times p$  vector of parameters. If a constant term is included in the model, then one column of  $\mathbf{X}_1$  contains all ones.

Let  $\mathbf{v}$  be a column vector of weights specified by the user. If no weights are specified,  $\mathbf{v} = \mathbf{1}$ . Let  $\mathbf{w}$  be a column vector of normalized weights. If no weights are specified or if the user specified `fweights` or `weights`,  $\mathbf{w} = \mathbf{v}$ ; otherwise,  $\mathbf{w} = \{\mathbf{v}/(\mathbf{1}'\mathbf{v})\}(\mathbf{1}'\mathbf{1})$ . Let  $\mathbf{D}$  denote the  $N \times N$  matrix with  $\mathbf{w}$  on the main diagonal and zeros elsewhere. If no weights are specified,  $\mathbf{D}$  is the identity matrix.

The weighted number of observations  $n$  is defined as  $\mathbf{1}'\mathbf{w}$ . For `weights`, this is truncated to an integer. The *sum of the weights* is  $\mathbf{1}'\mathbf{v}$ . Define  $c = 1$  if there is a constant in the regression and zero otherwise.

The order condition for identification requires that  $k_2 \geq p$ : the number of excluded exogenous variables must be at least as great as the number of endogenous regressors.

In the following formulas, if weights are specified,  $\mathbf{X}'_1\mathbf{X}_1$ ,  $\mathbf{X}'\mathbf{X}$ ,  $\mathbf{X}'\mathbf{y}$ ,  $\mathbf{y}'\mathbf{y}$ ,  $\mathbf{Z}'\mathbf{Z}$ ,  $\mathbf{Z}'\mathbf{X}$ , and  $\mathbf{Z}'\mathbf{y}$  are replaced with  $\mathbf{X}'_1\mathbf{D}\mathbf{X}_1$ ,  $\mathbf{X}'\mathbf{D}\mathbf{X}$ ,  $\mathbf{X}'\mathbf{D}\mathbf{y}$ ,  $\mathbf{y}'\mathbf{D}\mathbf{y}$ ,  $\mathbf{Z}'\mathbf{D}\mathbf{Z}$ ,  $\mathbf{Z}'\mathbf{D}\mathbf{X}$ , and  $\mathbf{Z}'\mathbf{D}\mathbf{y}$ , respectively. We suppress the  $\mathbf{D}$  below to simplify the notation.

## 2SLS and LIML estimators

Define the  $\kappa$ -class estimator of  $\beta$  as

$$\mathbf{b} = \{\mathbf{X}'(\mathbf{I} - \kappa\mathbf{M}_Z)\mathbf{X}\}^{-1}\mathbf{X}'(\mathbf{I} - \kappa\mathbf{M}_Z)\mathbf{y}$$

where  $\mathbf{M}_Z = \mathbf{I} - \mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'$ . The 2SLS estimator results from setting  $\kappa = 1$ . The LIML estimator results from selecting  $\kappa$  to be the minimum eigenvalue of  $(\mathbf{Y}'\mathbf{M}_Z\mathbf{Y})^{-1/2}\mathbf{Y}'\mathbf{M}_{\mathbf{X}_1}\mathbf{Y}(\mathbf{Y}'\mathbf{M}_Z\mathbf{Y})^{-1/2}$ , where  $\mathbf{M}_{\mathbf{X}_1} = \mathbf{I} - \mathbf{X}_1(\mathbf{X}_1'\mathbf{X}_1)^{-1}\mathbf{X}_1'$ .

The total sum of squares (TSS) equals  $\mathbf{y}'\mathbf{y}$  if there is no intercept and  $\mathbf{y}'\mathbf{y} - \{(\mathbf{1}'\mathbf{y})^2/n\}$  otherwise. The degrees of freedom is  $n - c$ . The error sum of squares (ESS) is defined as  $\mathbf{y}'\mathbf{y} - 2\mathbf{b}\mathbf{X}'\mathbf{y} + \mathbf{b}'\mathbf{X}'\mathbf{X}\mathbf{b}$ . The model sum of squares (MSS) equals TSS - ESS. The degrees of freedom is  $k - c$ .

The mean squared error,  $s^2$ , is defined as ESS/( $n - k$ ) if `small` is specified and ESS/ $n$  otherwise. The root mean squared error is  $s$ , its square root.

If  $c = 1$  and `small` is not specified, a Wald statistic,  $W$ , of the joint significance of the  $k - 1$  parameters of  $\beta$  except the constant term is calculated;  $W \sim \chi^2(k - 1)$ . If  $c = 1$  and `small` is specified, then an  $F$  statistic is calculated as  $F = W/(k - 1)$ ;  $F \sim F(k - 1, n - k)$ .

The  $R^2$  is defined as  $R^2 = 1 - \text{ESS}/\text{TSS}$ .

The adjusted  $R^2$  is  $R_a^2 = 1 - (1 - R^2)(n - c)/(n - k)$ .

The unadjusted (default) variance estimate is  $\text{Var}(\mathbf{b}) = s^2\{\mathbf{X}'(\mathbf{I} - \kappa\mathbf{M}_Z)\mathbf{X}\}^{-1}$ .

For a general discussion of robust variance estimates in regression, see [A general notation for the robust variance calculation](#) in [R] `regress`. `ivregress` uses the same definitions for terms discussed in [Robust calculation for regress](#) in its robust variance calculation, except for the following.

The vector of scores is given by

$$\mathbf{u}_j = (y_j - \mathbf{x}_j\mathbf{b})\hat{\mathbf{x}}_j$$

where  $\hat{\mathbf{x}}_j' = \mathbf{P}\mathbf{z}_j'$  and  $\mathbf{P} = (\mathbf{X}'\mathbf{Z})(\mathbf{Z}'\mathbf{Z})^{-1}$ . When the formulas in [R] `regress` are applied,  $q_c$  is given by its regressionlike definition. If `small` is not specified, then  $k = 0$  in the formulas given in [R] `regress`.

`ivregress 2sls` and `ivregress liml` also support estimation with survey data. For details on VCEs with survey data, see [SVY] [Variance estimation](#).

## 2SLS estimator with absorb() option

When absorbed variables are specified, we must project the dependent variable, instruments, and endogenous variables onto the orthogonal complement of the column space of the absorbed indicator matrices. Using the same notation found in the [Methods and formulas](#) of [R] `areg`, we have  $m_k$  categorical levels for the  $k$ th absorbed variable,  $C_k$ , and an  $N \times m_k$  indicator matrix  $\mathbf{D}_k$ . The orthonormal projection matrix for the  $k$ th variable is  $\mathbf{P}_k = \mathbf{D}_k(\mathbf{D}_k'\mathbf{D}_k)^{-1}\mathbf{D}_k'$ . Thus, the product  $\bar{\mathbf{y}}_k = \mathbf{P}_k\mathbf{y}$  is the projection of the dependent variable onto the column space of  $\mathbf{D}_k$ . That is,  $\bar{\mathbf{y}}_k$  is the  $N \times 1$  vector containing the (repeated) means of  $y_i$  for each level of  $C_k$  in the order that these levels appear in the sample. The product  $(\mathbf{I} - \mathbf{P}_k)\mathbf{y}$  is the vector of the demeaned dependent variable. The same projection (demeaning) is applied to the columns of matrices  $\mathbf{X}_1$ ,  $\mathbf{X}_2$ , and  $\mathbf{Y}$ . The Halperin or Cimmino iterative algorithm loops over the  $h$  absorbed variables computing projections as described in [Methods and formulas](#) of [R] `areg`.

See [Methods and formulas](#) in [R] `areg` for a description of the method used to adjust the degrees of freedom to account for collinearity among absorbed variables when the `dfabsorb` option is specified.

## GMM estimator

We obtain an initial consistent estimate of  $\beta$  by using the 2SLS estimator; see above. Using this estimate of  $\beta$ , we compute the weight matrix  $\mathbf{W}$  and calculate the GMM estimator

$$\mathbf{b}_{\text{GMM}} = \{\mathbf{X}'\mathbf{Z}\mathbf{W}\mathbf{Z}'\mathbf{X}\}^{-1}\mathbf{X}'\mathbf{Z}\mathbf{W}\mathbf{Z}'\mathbf{y}$$

The variance of  $\mathbf{b}_{\text{GMM}}$  is

$$\text{Var}(\mathbf{b}_{\text{GMM}}) = n\{\mathbf{X}'\mathbf{Z}\mathbf{W}\mathbf{Z}'\mathbf{X}\}^{-1}\mathbf{X}'\mathbf{Z}\mathbf{W}\widehat{\mathbf{S}}\mathbf{W}\mathbf{Z}'\mathbf{X}\{\mathbf{X}'\mathbf{Z}\mathbf{W}\mathbf{Z}'\mathbf{X}\}^{-1}$$

$\text{Var}(\mathbf{b}_{\text{GMM}})$  is of the sandwich form **DMD**; see [P] [\\_robust](#). If the user specifies the `small` option, `ivregress` implements a small-sample adjustment by multiplying the VCE by  $N/(N - k)$ .

If `vce(unadjusted)` is specified, then we set  $\widehat{\mathbf{S}} = \mathbf{W}^{-1}$  and the VCE reduces to the “optimal” GMM variance estimator

$$\text{Var}(\beta_{\text{GMM}}) = n\{\mathbf{X}'\mathbf{Z}\mathbf{W}\mathbf{Z}'\mathbf{X}\}^{-1}$$

However, if  $\mathbf{W}^{-1}$  is not a good estimator of  $E(\mathbf{z}_i u_i u_i \mathbf{z}_i')$ , then the optimal GMM estimator is inefficient, and inference based on the optimal variance estimator could be misleading.

$\mathbf{W}$  is calculated using the residuals from the initial 2SLS estimates, whereas  $\mathbf{S}$  is estimated using the residuals based on  $\mathbf{b}_{\text{GMM}}$ . The `wmatrix()` option affects the form of  $\mathbf{W}$ , whereas the `vce()` option affects the form of  $\mathbf{S}$ . Except for different residuals being used, the formulas for  $\mathbf{W}^{-1}$  and  $\mathbf{S}$  are identical, so we focus on estimating  $\mathbf{W}^{-1}$ .

If `wmatrix(unadjusted)` is specified, then

$$\mathbf{W}^{-1} = \frac{s^2}{n} \sum_i \mathbf{z}_i \mathbf{z}_i'$$

where  $s^2 = \sum_i u_i^2/n$ . This weight matrix is appropriate if the errors are homoskedastic.

If `wmatrix(robust)` is specified, then

$$\mathbf{W}^{-1} = \frac{1}{n} \sum_i u_i^2 \mathbf{z}_i \mathbf{z}_i'$$

which is appropriate if the errors are heteroskedastic.

If `wmatrix(cluster clustvar)` is specified, then

$$\mathbf{W}^{-1} = \frac{1}{n} \sum_c \mathbf{q}_c \mathbf{q}_c'$$

where  $c$  indexes clusters,

$$\mathbf{q}_c = \sum_{i \in c_j} u_i \mathbf{z}_i$$

and  $c_j$  denotes the  $j$ th cluster.

If `wmatrix(hac kernel [#])` is specified, then

$$\mathbf{W}^{-1} = \frac{1}{n} \sum_i u_i^2 \mathbf{z}_i \mathbf{z}_i' + \frac{1}{n} \sum_{l=1}^{l=n-1} \sum_{i=l+1}^{i=n} K(l, m) u_i u_{i-l} (\mathbf{z}_i \mathbf{z}_{i-l}' + \mathbf{z}_{i-l} \mathbf{z}_i')$$

where  $m = \#$  if  $\#$  is specified and  $m = n - 2$  otherwise. Define  $z = l/(m + 1)$ . If *kernel* is *nwest*, then

$$K(l, m) = \begin{cases} 1 - z & 0 \leq z \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

If *kernel* is *gallant*, then

$$K(l, m) = \begin{cases} 1 - 6z^2 + 6z^3 & 0 \leq z \leq 0.5 \\ 2(1 - z)^3 & 0.5 < z \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

If *kernel* is *quadraticspectral*, then

$$K(l, m) = \begin{cases} 1 & z = 0 \\ 3 \{ \sin(\theta)/\theta - \cos(\theta) \} / \theta^2 & \text{otherwise} \end{cases}$$

where  $\theta = 6\pi z/5$ .

If `wmatrix(hac kernel opt)` is specified, then `ivregress` uses Newey and West's (1994) automatic lag-selection algorithm, which proceeds as follows. Define  $\mathbf{h}$  to be a  $(k_1 + k_2) \times 1$  vector containing ones in all rows except for the row corresponding to the constant term (if present); that row contains a zero. Define

$$\begin{aligned} f_i &= (u_i \mathbf{z}_i) \mathbf{h} \\ \hat{\sigma}_j &= \frac{1}{n} \sum_{i=j+1}^n f_i f_{i-j} \quad j = 0, \dots, m^* \\ \hat{s}^{(q)} &= 2 \sum_{j=1}^{m^*} \hat{\sigma}_j j^q \\ \hat{s}^{(0)} &= \hat{\sigma}_0 + 2 \sum_{j=1}^{m^*} \hat{\sigma}_j \\ \hat{\gamma} &= c_\gamma \left\{ \left( \frac{\hat{s}^{(q)}}{\hat{s}^{(0)}} \right)^2 \right\}^{1/2q+1} \\ m &= \hat{\gamma} n^{1/(2q+1)} \end{aligned}$$

where  $q$ ,  $m^*$ , and  $c_\gamma$  depend on the kernel specified:

| Kernel             | $q$ | $m^*$                               | $c_\gamma$ |
|--------------------|-----|-------------------------------------|------------|
| Bartlett           | 1   | $\text{int} \{ 20(T/100)^{2/9} \}$  | 1.1447     |
| Parzen             | 2   | $\text{int} \{ 20(T/100)^{4/25} \}$ | 2.6614     |
| Quadratic spectral | 2   | $\text{int} \{ 20(T/100)^{2/25} \}$ | 1.3221     |

where  $\text{int}(x)$  denotes the integer obtained by truncating  $x$  toward zero. For the Bartlett and Parzen kernels, the optimal lag is  $\min\{\text{int}(m), m^*\}$ . For the quadratic spectral, the optimal lag is  $\min\{m, m^*\}$ .

If `wmatrix(hac kernel opt #)` is specified, then `ivregress` uses  $\#$  instead of 20 in the definition of  $m^*$  above to select the optimal lag.

If center is specified, when computing weight matrices `ivregress` replaces the term  $u_i z_i$  in the formulas above with  $u_i z_i - \bar{u}\bar{z}$ , where  $\bar{u}\bar{z} = \sum_i u_i z_i / N$ .

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## Also see

- [R] **ivregress postestimation** — Postestimation tools for ivregress
- [R] **cfregress** — Control-function linear regression
- [R] **gmm** — Generalized method of moments estimation
- [R] **ivprobit** — Probit model with continuous endogenous covariates
- [R] **ivqregress** — Instrumental-variables quantile regression
- [R] **ivtobit** — Tobit model with continuous endogenous covariates
- [R] **reg3** — Three-stage estimation for systems of simultaneous equations
- [R] **regress** — Linear regression
- [ERM] **eregress** — Extended linear regression
- [FMM] **fmm: ivregress** — Finite mixtures of linear regression models with endogenous covariates
- [SEM] **Intro 5** — Tour of models
- [SP] **spivregress** — Spatial autoregressive models with endogenous covariates
- [SVY] **svy estimation** — Estimation commands for survey data
- [TS] **forecast** — Econometric model forecasting
- [XT] **xtivreg** — Instrumental variables and two-stage least squares for panel-data models
- [U] **20 Estimation and postestimation commands**

|                         |                                |                                      |                                 |
|-------------------------|--------------------------------|--------------------------------------|---------------------------------|
| Postestimation commands | <a href="#">predict</a>        | <a href="#">margins</a>              | <a href="#">estat</a>           |
| Remarks and examples    | <a href="#">Stored results</a> | <a href="#">Methods and formulas</a> | <a href="#">Acknowledgments</a> |
| References              | <a href="#">Also see</a>       |                                      |                                 |

## Postestimation commands

The following postestimation commands are of special interest after `ivregress`:

| Command                       | Description                                                     |
|-------------------------------|-----------------------------------------------------------------|
| <code>estat endogenous</code> | perform tests of endogeneity                                    |
| <code>estat firststage</code> | report “first-stage” regression statistics                      |
| <code>estat overid</code>     | perform tests of overidentifying restrictions                   |
| <code>estat weakrobust</code> | perform inference robust to weak instruments                    |
| * <code>estat sbknown</code>  | perform tests for a structural break with a known break date    |
| * <code>estat single</code>   | perform tests for a structural break with an unknown break date |

These commands are not appropriate with `svy` estimation results.

\*`estat sbknown` and `estat sbsingle` work only after `ivregress 2s1s`.

The following postestimation commands are also available:

| Command                      | Description                                                                                       |
|------------------------------|---------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                               |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                      |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                         |
| <code>estimates</code>       | cataloging estimation results                                                                     |
| <code>etable</code>          | table of estimation results                                                                       |
| † <code>forecast</code>      | dynamic forecasts and simulations                                                                 |
| † <code>hausman</code>       | Hausman’s specification test                                                                      |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters    |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                           |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters |
| <code>predict</code>         | linear predictions and their SEs, probabilities, etc.                                             |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions              |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                              |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                |

†`forecast` and `hausman` are not appropriate with `svy` estimation results.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as linear predictions, residuals, standard errors, probabilities, and expected values.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic]
```

```
predict [type] stub* [if] [in], scores
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

Main

`xb`  $\mathbf{x}_j\mathbf{b}$ , fitted values; the default

`stdp` standard error of the prediction

`stdf` standard error of the forecast

`dresiduals`  $d_{\text{absorbvars}} + e_j = y_j - \mathbf{x}_j\mathbf{b}$

\* `xbd`  $\mathbf{x}_j\mathbf{b} + d_{\text{absorbvars}}$

\* `d`  $d_{\text{absorbvars}}$

`residuals` residuals

† `pr(a,b)`  $\Pr(a < y_j < b)$  under exogeneity and normal errors

† `e(a,b)`  $E(y_j | a < y_j < b)$  under exogeneity and normal errors

† `ystar(a,b)`  $E(y_j^*), y_j^* = \max\{a, \min(y_j, b)\}$  under exogeneity and normal errors

Unstarred statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

†Not allowed if `absorb()` is used at estimation.

`xb`, `stdp`, and `residuals` are the only options allowed with `svy` estimation results.

where *a* and *b* may be numbers or variables; *a* missing (*a* ≥ .) means  $-\infty$ , and *b* missing (*b* ≥ .) means  $+\infty$ ; see [U] 12.2.1 Missing values.

## Options for predict

Main

`xb`, the default, calculates the linear prediction,  $\mathbf{x}_j\mathbf{b}$ .

`stdp` calculates the standard error of the prediction, which can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. This is also referred to as the standard error of the fitted value.

`stdf` calculates the standard error of the forecast, which is the standard error of the point prediction for 1 observation. It is commonly referred to as the standard error of the future or forecast value. By construction, the standard errors produced by `stdf` are always larger than those produced by `stdp`; see *Methods and formulas* in [R] **regress postestimation**.

`dresiduals` calculates  $y_j - \mathbf{x}_j\mathbf{b}$ , which are the residuals plus the effects of the absorbed variables. These are based on the estimated equation when the observed values of the endogenous variables are used—not the projections of the instruments onto the endogenous variables.

`xbd` calculates  $\mathbf{x}_j\mathbf{b} + d_{\text{absorbvar}}$ , which are the fitted values including the individual effects of the absorbed variables.

`d` calculates  $d_{\text{absorbvar}}$ , the sum of individual coefficients for the absorbed variables.

`residuals` calculates the residuals, that is,  $y_j - (\mathbf{x}_j\mathbf{b} + d_{\text{absorbvar}})$ .

`pr(a,b)` calculates  $\Pr(a < \mathbf{x}_j\mathbf{b} + u_j < b)$ , the probability that  $y_j|\mathbf{x}_j$  would be observed in the interval  $(a,b)$  under exogeneity and assuming errors are normally distributed.

$a$  and  $b$  may be specified as numbers or variable names;  $lb$  and  $ub$  are variable names;

`pr(20,30)` calculates  $\Pr(20 < \mathbf{x}_j\mathbf{b} + u_j < 30)$ ;

`pr(lb,ub)` calculates  $\Pr(lb < \mathbf{x}_j\mathbf{b} + u_j < ub)$ ; and

`pr(20,ub)` calculates  $\Pr(20 < \mathbf{x}_j\mathbf{b} + u_j < ub)$ .

$a$  missing ( $a \geq .$ ) means  $-\infty$ ; `pr(. ,30)` calculates  $\Pr(-\infty < \mathbf{x}_j\mathbf{b} + u_j < 30)$ ;

`pr(lb,30)` calculates  $\Pr(-\infty < \mathbf{x}_j\mathbf{b} + u_j < 30)$  in observations for which  $lb \geq .$

and calculates  $\Pr(lb < \mathbf{x}_j\mathbf{b} + u_j < 30)$  elsewhere.

$b$  missing ( $b \geq .$ ) means  $+\infty$ ; `pr(20, .)` calculates  $\Pr(+\infty > \mathbf{x}_j\mathbf{b} + u_j > 20)$ ;

`pr(20,ub)` calculates  $\Pr(+\infty > \mathbf{x}_j\mathbf{b} + u_j > 20)$  in observations for which  $ub \geq .$

and calculates  $\Pr(20 < \mathbf{x}_j\mathbf{b} + u_j < ub)$  elsewhere.

`e(a,b)` calculates  $E(\mathbf{x}_j\mathbf{b} + u_j | a < \mathbf{x}_j\mathbf{b} + u_j < b)$ , the expected value of  $y_j|\mathbf{x}_j$  conditional on  $y_j|\mathbf{x}_j$  being in the interval  $(a,b)$ , meaning that  $y_j|\mathbf{x}_j$  is truncated.  $a$  and  $b$  are specified as they are for `pr()`. Exogeneity and normally distributed errors are assumed.

`ystar(a,b)` calculates  $E(y_j^*)$ , where  $y_j^* = a$  if  $\mathbf{x}_j\mathbf{b} + u_j \leq a$ ,  $y_j^* = b$  if  $\mathbf{x}_j\mathbf{b} + u_j \geq b$ , and  $y_j^* = \mathbf{x}_j\mathbf{b} + u_j$  otherwise, meaning that  $y_j^*$  is censored.  $a$  and  $b$  are specified as they are for `pr()`. Exogeneity and normally distributed errors are assumed.

`scores` calculates the scores for the model. A new score variable is created for each endogenous regressor, as well as an equation-level score that applies to all exogenous variables and constant term (if present). `scores` is not allowed if the `absorb()` option is used at estimation.

## margins

### Description for margins

`margins` estimates margins of response for linear predictions, probabilities, and expected values.

### Menu for margins

Statistics > Postestimation

### Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i>                      | Description                                                                    |
|---------------------------------------|--------------------------------------------------------------------------------|
| <code>xb</code>                       | linear prediction; the default                                                 |
| <code>stdp</code>                     | not allowed with <code>margins</code>                                          |
| <code>stdf</code>                     | not allowed with <code>margins</code>                                          |
| <code>dresiduals</code>               | not allowed with <code>margins</code>                                          |
| <code>xbd</code>                      | not allowed with <code>margins</code>                                          |
| <code>d</code>                        | not allowed with <code>margins</code>                                          |
| <code>residuals</code>                | not allowed with <code>margins</code>                                          |
| <code>pr(<i>a</i>,<i>b</i>)</code>    | $\Pr(a < y_j < b)$ under exogeneity and normal errors                          |
| <code>e(<i>a</i>,<i>b</i>)</code>     | $E(y_j   a < y_j < b)$ under exogeneity and normal errors                      |
| <code>ystar(<i>a</i>,<i>b</i>)</code> | $E(y_j^*, y_j^* = \max\{a, \min(y_j, b)\})$ under exogeneity and normal errors |

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [R] [margins](#).

## estat

### Description for estat

`estat endogenous` performs tests to determine whether endogenous regressors in the model are in fact exogenous. After GMM estimation, the  $C$  (difference-in-Sargan) statistic is reported. After 2SLS estimation with an unadjusted VCE, the Durbin (1954) and Wu–Hausman (Wu 1974; Hausman 1978) statistics are reported. After 2SLS estimation with a robust VCE, Wooldridge’s (1995) robust score test and a robust regression-based test are reported. In all cases, if the test statistic is significant, then the variables being tested must be treated as endogenous. `estat endogenous` is not available after LIML estimation.

`estat firststage` reports various statistics that measure the relevance of the excluded exogenous variables. By default, which statistics are reported depends on whether the equation has one or more than one endogenous regressor.

`estat overid` performs tests of overidentifying restrictions. If the 2SLS estimator was used, Sargan’s (1958) and Basman’s (1960)  $\chi^2$  tests are reported, as is Wooldridge’s (1995) robust score test; if the LIML estimator was used, Anderson and Rubin’s (1950)  $\chi^2$  test and Basman’s  $F$  test are reported; and if the GMM estimator was used, Hansen’s (1982)  $J$  statistic  $\chi^2$  test is reported. In all of these cases, a rejection of the test indicates that the instruments may not be valid.

`estat weakrobust` performs hypothesis tests on the coefficients of endogenous regressors that are robust to weak instruments. For just-identified models, the Anderson–Rubin (1949) test is reported. For overidentified models, if a homoskedastic VCE was used in estimation, the conditional likelihood-ratio (CLR) test of Moreira (2003) is reported. If a heteroskedastic or cluster–robust VCE was used instead, an appropriate generalization of the CLR test is used (see Finlay and Magnusson [2009]). When there is only one endogenous regressor, the confidence intervals associated with any of these tests can be requested.

### Menu for estat

Statistics > Postestimation

### Syntax for estat

*Perform tests of endogeneity*

```
estat endogenous [varlist] [, lags(#) forceweights forcenonrobust]
```

*Report “first-stage” regression statistics*

```
estat firststage [, all forcenonrobust]
```

*Perform tests of overidentifying restrictions*

```
estat overid [, lags(#) forceweights forcenonrobust]
```

*Perform inference robust to weak instruments*

```
estat weakrobust [, weak_options]
```

| <i>weak_options</i>                                 | Description                                                                       |
|-----------------------------------------------------|-----------------------------------------------------------------------------------|
| <code>betanull(<i>null_spec</i>)</code>             | specify the null hypothesis on the endogenous regressors                          |
| <code>ar</code>                                     | perform Anderson–Rubin test                                                       |
| <code>clr</code>                                    | perform conditional likelihood-ratio test                                         |
| <code>ci</code>                                     | report confidence interval instead of test                                        |
| <code>level(#)</code>                               | set confidence level; default is <code>level(95)</code>                           |
| <code>reps(#)</code>                                | perform # replications for simulations; default is <code>reps(25000)</code>       |
| <code>rseed(#)</code>                               | set random-number seed for simulations to #                                       |
| <code>bound(#<sub>min</sub> #<sub>max</sub>)</code> | specify bounds of confidence interval grid                                        |
| <code>ngrid(#)</code>                               | use # grid points for the confidence interval; default is <code>ngrid(500)</code> |
| <code>noadaptgrid</code>                            | suppress adaptive grid expansion                                                  |
| <code>[no]log</code>                                | suppress or display the iteration log                                             |
| <code>iterate(#)</code>                             | perform maximum of # adaptive iterations; default is <code>iterate(20)</code>     |

`collect` is allowed with `estat endogenous`, `estat firststage`, `estat overid`, and `estat weakrobust`; see [U] 11.1.10 Prefix commands.

## Options for estat

Options for `estat` are presented under the following headings:

[Options for estat endogenous](#)

[Options for estat firststage](#)

[Options for estat overid](#)

[Options for estat weakrobust](#)

## Options for estat endogenous

`lags(#)` specifies the number of lags to use for prewhitening when computing the heteroskedasticity- and autocorrelation-consistent (HAC) version of the score test of endogeneity. Specifying `lags(0)` requests no prewhitening. This option is valid only when the model was fit via 2SLS and a HAC covariance matrix was requested when the model was fit. The default is `lags(1)`.

`forceweights` requests that the tests of endogeneity be computed even though `aweight`s, `pweight`s, or `iweight`s were used in the previous estimation. By default, these tests are conducted only after unweighted or frequency-weighted estimation. The reported critical values may be inappropriate for weighted data, so the user must determine whether the critical values are appropriate for a given application.

`forcenonrobust` requests that the Durbin and Wu–Hausman tests be performed after 2SLS estimation even though a robust VCE was used at estimation time. This option is available only if the model was fit by 2SLS.

## Options for estat firststage

`all` requests that all first-stage goodness-of-fit statistics be reported regardless of whether the model contains one or more endogenous regressors. By default, if the model contains one endogenous regressor, then the first-stage  $R^2$ , adjusted  $R^2$ , partial  $R^2$ , and  $F$  statistics are reported, whereas if the model contains multiple endogenous regressors, then Shea's partial  $R^2$  and adjusted partial  $R^2$  are reported instead.

`forcenonrobust` requests that the minimum eigenvalue statistic and its critical values be reported even though a robust VCE was used at estimation time. The reported critical values assume that the errors are independent and identically distributed (i.i.d.) normal, so the user must determine whether the critical values are appropriate for a given application.

## Options for `estat overid`

`lags(#)` specifies the number of lags to use for prewhitening when computing the heteroskedasticity- and autocorrelation-consistent (HAC) version of the score test of overidentifying restrictions. Specifying `lags(0)` requests no prewhitening. This option is valid only when the model was fit via 2SLS and a HAC covariance matrix was requested when the model was fit. The default is `lags(1)`.

`forceweights` requests that the tests of overidentifying restrictions be computed even though `aweights`, `pweights`, or `iweights` were used in the previous estimation. By default, these tests are conducted only after unweighted or frequency-weighted estimation. The reported critical values may be inappropriate for weighted data, so the user must determine whether the critical values are appropriate for a given application.

`forcenonrobust` requests that the Sargan and Basman tests of overidentifying restrictions be performed after 2SLS or LIML estimation even though a robust VCE was used at estimation time. These tests assume that the errors are i.i.d. normal, so the user must determine whether the critical values are appropriate for a given application.

## Options for `estat weakrobust`

`betanull(null_spec)` specifies the null hypothesis of the test on the coefficients of the endogenous regressors. By default, the null hypothesis is that the coefficients of all endogenous regressors are zero. The null hypothesis can be specified using `numlist`, a list of equalities, or a matrix. When using `numlist`, you must specify the null values for all endogenous regressors in the order given in the estimation command and specify the `copy` suboption if there is more than one endogenous regressor. When you use a list of equalities or a matrix, null values are set to zero for endogenous regressors you did not specify. When you use a matrix, null values are assigned to coefficients by matrix column names.

`null_spec` is one of

`# [#... , copy]`

`varname = # [varname = # [...]] [, skip]`

`matname [ , skip copy]`

`skip` specifies that any variables that are not endogenous regressors in the model be ignored. The default action is to issue an error message.

`copy` specifies that the list of null values be assigned into the null hypothesis by position rather than by name.

`ar` requests that an Anderson–Rubin test be performed. The associated confidence interval of the test can be requested with the `ci` option. By default, the Anderson–Rubin test is performed when the model is just identified. If both `ar` and `clr` are specified, both tests are performed.

`clr` requests that a CLR test be performed. The associated confidence interval of the test can be requested with the `ci` option. By default, a CLR test is performed when the model is overidentified. If both `ar` and `clr` are specified, both tests are performed.

- `ci` requests that a confidence interval be returned instead of a test statistic and a  $p$ -value. This applies to all tests specified by the user. Option `ci` is available only when there is a single endogenous regressor.
- `level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [U] 20.8 Specifying the width of confidence intervals.
- `reps(#)` specifies the number of replications to use in simulating critical values when  $p$ -values cannot be computed analytically. The default is `reps(25000)`.
- `rseed(#)` sets the random-number seed. This option can be used to reproduce results for simulating critical values when  $p$ -values cannot be computed analytically. `rseed(#)` is equivalent to typing `set seed #` prior to calling `estat weakrobust`; see [R] set seed.
- `bound(#min #max)` specifies the lower (`#min`) and the upper bound (`#max`) for the grid used to approximate confidence intervals when they cannot be computed analytically. The specified bounds will be the starting bounds for the adaptive grid expansion procedure or, if `noadaptgrid` is specified, the fixed bounds of the grid. By default, the lower bound is set to five standard errors below the coefficient estimated by `ivregress`, and the upper bound is five standard errors above.
- `ngrid(#)` specifies the starting number of gridpoints for approximating confidence intervals. The default is `ngrid(500)`. Higher values will increase computation time but improve precision, which may be helpful when the confidence interval is wide. If `noadaptgrid` is specified, `ngrid()` specifies the fixed number of gridpoints.
- `noadaptgrid` requests that the default method for computing confidence intervals, where the grid is adaptively expanded, be turned off and the interval be estimated with a fixed grid. `noadaptgrid` is equivalent to specifying `iterate(0)`.
- `log` and `nolog` specify whether to display the iteration log showing the progress of the adaptive gridding procedure. The iteration log is displayed by default unless you used `set iterlog off` to suppress it; see `set iterlog` in [R] set iter.
- `iterate(#)` specifies the maximum number of iterations for the adaptive grid expansion procedure. Each iteration expands the grid by half the width of the starting bounds in each direction, and the procedure stops when the maximum number of iterations is reached or when the confidence interval has finite endpoints and was unchanged by the most recent expansion. The default is `iterate(20)`.

## Remarks and examples

Remarks are presented under the following headings:

*estat endogenous*  
*estat firststage*  
*estat overid*  
*estat weakrobust*

## estat endogenous

A natural question to ask is whether a variable presumed to be endogenous in the previously fit model could instead be treated as exogenous. If the endogenous regressors are in fact exogenous, then the OLS estimator is more efficient; and depending on the strength of the instruments and other factors, the sacrifice in efficiency by using an instrumental-variables estimator can be significant. Thus, unless an instrumental-variables estimator is really needed, OLS should be used instead. `estat endogenous` provides several tests of endogeneity after 2SLS and GMM estimation.

## ▷ Example 1

In [example 1](#) of [R] `ivregress`, we fit a model of the average rental rate for housing in a state as a function of the percentage of the population living in urban areas and the average value of houses. We treated `hshgval` as endogenous because unanticipated shocks that affect rental rates probably affect house prices as well. We used family income and region dummies as additional instruments for `hshgval`. Here we test whether we could treat `hshgval` as exogenous.

```
. use https://www.stata-press.com/data/r19/hshg
(1980 Census housing data)
. ivregress 2sls rent pcturban (hshgval = faminc i.region)
(output omitted)
. estat endogenous
Tests of endogeneity
H0: Variables are exogenous
Durbin (score) chi2(1) = 12.8473 (p = 0.0003)
Wu-Hausman F(1,46) = 15.9067 (p = 0.0002)
```

Because we did not specify any variable names after the `estat endogenous` command, Stata by default tested all the endogenous regressors (namely, `hshgval`) in our model. The null hypothesis of the Durbin and Wu–Hausman tests is that the variable under consideration can be treated as exogenous. Here both test statistics are highly significant, so we reject the null of exogeneity; we must continue to treat `hshgval` as endogenous.

◀

The difference between the Durbin and Wu–Hausman tests of endogeneity is that the former uses an estimate of the error term’s variance based on the model assuming the variables being tested are exogenous, while the latter uses an estimate of the error variance based on the model assuming the variables being tested are endogenous. Under the null hypothesis that the variables being tested are exogenous, both estimates of the error variance are consistent. What we label the Wu–Hausman statistic is Wu’s (1974) “ $T_2$ ” statistic, which Hausman (1978) showed can be calculated very easily via linear regression. Baum, Schaffer, and Stillman (2003, 2007) provide a lucid discussion of these tests.

When you fit a model with multiple endogenous regressors, you can test the exogeneity of a subset of the regressors while continuing to treat the others as endogenous. For example, say you have three endogenous regressors,  $y_1$ ,  $y_2$ , and  $y_3$ , and you fit your model by typing

```
. ivregress depvar ... (y1 y2 y3 = ...)
```

Suppose you are confident that  $y_1$  must be treated as endogenous, but you are undecided about  $y_2$  and  $y_3$ . To test whether  $y_2$  and  $y_3$  can be treated as exogenous, you would type

```
. estat endogenous y2 y3
```

The Durbin and Wu–Hausman tests assume that the error term is i.i.d. Therefore, if you requested a robust VCE at estimation time, `estat endogenous` will instead report Wooldridge’s (1995) score test and a regression-based test of exogeneity. Both these tests can tolerate heteroskedastic and autocorrelated errors, while only the regression-based test is amenable to clustering.

## ▷ Example 2

We refit our housing model, requesting robust standard errors, and then test the exogeneity of `hsngval`:

```
. use https://www.stata-press.com/data/r19/hsng
(1980 Census housing data)
. ivregress 2sls rent pcturban (hsngval = faminc i.region), vce(robust)
(output omitted)
. estat endogenous
Tests of endogeneity
H0: Variables are exogenous
Robust score chi2(1) = 2.10428 (p = 0.1469)
Robust regression F(1,46) = 4.31101 (p = 0.0435)
```

Wooldridge’s score test does not reject the null hypothesis that `hsngval` is exogenous at conventional significance levels ( $p = 0.1469$ ). However, the regression-based test does reject the null hypothesis at the 5% significance level ( $p = 0.0435$ ). Typically, these two tests yield the same conclusion; the fact that our dataset has only 50 observations could be contributing to the discrepancy. Here we would be inclined to continue to treat `hsngval` as endogenous. Even if `hsngval` is exogenous, the 2SLS estimates are still consistent. On the other hand, if `hsngval` is in fact endogenous, the OLS estimates would not be consistent. Moreover, as we will see in our discussion of the `estat overid` command, our additional instruments may be invalid. To test whether an endogenous variable can be treated as exogenous, we must have a valid set of instruments to use to fit the model in the first place!

◀

Unlike the Durbin and Wu–Hausman tests, Wooldridge’s score and the regression-based tests do not allow you to test a subset of the endogenous regressors in the model; you can test only whether all the endogenous regressors are in fact exogenous.

After GMM estimation, `estat endogenous` calculates what Hayashi (2000, 220) calls the  $C$  statistic, also known as the difference-in-Sargan statistic. The  $C$  statistic can be made robust to heteroskedasticity, autocorrelation, and clustering; and the version reported by `estat endogenous` is determined by the weight matrix requested via the `wmatrix()` option used when fitting the model with `ivregress`. Additionally, the test can be used to determine the exogeneity of a subset of the endogenous regressors, regardless of the type of weight matrix used.

If you fit your model using the LIML estimator, you can use the `hausman` command to carry out a traditional Hausman (1978) test between the OLS and LIML estimates.

## estat firststage

For an excluded exogenous variable to be a valid instrument, it must be sufficiently correlated with the included endogenous regressors but uncorrelated with the error term. In recent decades, researchers have paid considerable attention to the issue of instruments that are only weakly correlated with the endogenous regressors. In such cases, the usual 2SLS, GMM, and LIML estimators are biased toward the OLS estimator, and inference based on the standard errors reported by, for example, `ivregress` can be severely misleading. For more information on the theory behind instrumental-variables estimation with weak instruments, see Nelson and Startz (1990); Staiger and Stock (1997); Hahn and Hausman (2003); the survey article by Stock, Wright, and Yogo (2002); and Angrist and Pischke (2009, chap. 4).

When the instruments are only weakly correlated with the endogenous regressors, some Monte Carlo evidence suggests that the LIML estimator performs better than the 2SLS and GMM estimators; see, for example, [Poi \(2006\)](#) and [Stock, Wright, and Yogo \(2002\)](#) (and the papers cited therein). On the other hand, the LIML estimator often results in confidence intervals that are somewhat larger than those from the 2SLS estimator.

Moreover, using more instruments is not a solution, because the biases of instrumental-variables estimators increase with the number of instruments. See [Hahn and Hausman \(2003\)](#).

`estat firststage` produces several statistics for judging the explanatory power of the instruments and is most easily explained with examples.

### ▷ Example 3

Again building on the model fit in [example 1](#) of [\[R\] ivregress](#), we now explore the degree of correlation between the additional instruments `faminc`, `2.region`, `3.region`, and `4.region` and the endogenous regressor `hsngval`:

```
. use https://www.stata-press.com/data/r19/hsng
(1980 Census housing data)
. ivregress 2sls rent pcturban (hsngval = faminc i.region)
(output omitted)
. estat firststage
```

First-stage regression summary statistics

| Variable | R-sq.  | Adjusted R-sq. | Partial R-sq. | F(4,44) | Prob > F |
|----------|--------|----------------|---------------|---------|----------|
| hsngval  | 0.6908 | 0.6557         | 0.5473        | 13.2978 | 0.0000   |

Minimum eigenvalue statistic = 13.2978

|                          |                             |   |
|--------------------------|-----------------------------|---|
| Critical Values          | # of endogenous regressors: | 1 |
| H0: Instruments are weak | # of excluded instruments:  | 4 |

|                                   | 5%    | 10%   | 20%   | 30%  |
|-----------------------------------|-------|-------|-------|------|
| 2SLS relative bias                | 16.85 | 10.27 | 6.71  | 5.34 |
| 2SLS size of nominal 5% Wald test | 24.58 | 13.96 | 10.26 | 8.31 |
| LIML size of nominal 5% Wald test | 5.44  | 3.87  | 3.30  | 2.98 |

To understand these results, recall that the first-stage regression is

$$\text{hsngval}_i = \pi_0 + \pi_1 \text{pcturban}_i + \pi_2 \text{faminc} + \pi_3 \text{2.region} + \pi_4 \text{3.region} + \pi_5 \text{4.region} + v_i$$

where  $v_i$  is an error term. The column marked “R-sq.” is the simple  $R^2$  from fitting the first-stage regression by OLS, and the column marked “Adjusted R-sq.” is the adjusted  $R^2$  from that regression. Higher values purportedly indicate stronger instruments, and instrumental-variables estimators exhibit less bias when the instruments are strongly correlated with the endogenous variable.

Looking at just the  $R^2$  and adjusted  $R^2$  can be misleading, however. If `hsngval` were strongly correlated with the included exogenous variable `pcturban` but only weakly correlated with the additional instruments, then these statistics could be large even though a weak-instrument problem is present.

The partial  $R^2$  statistic measures the correlation between `hsgnval` and the additional instruments after *partialing out* the effect of `pcturban`. Unlike the  $R^2$  and adjusted  $R^2$  statistics, the partial  $R^2$  statistic will not be inflated because of strong correlation between `hsgnval` and `pcturban`. Bound, Jaeger, and Baker (1995) and others have promoted using this statistic.

The column marked “F(4, 44)” is an  $F$  statistic for the joint significance of  $\pi_2$ ,  $\pi_3$ ,  $\pi_4$ , and  $\pi_5$ , the coefficients on the additional instruments. Its  $p$ -value is listed in the column marked “Prob > F”. If the  $F$  statistic is not significant, then the additional instruments have no significant explanatory power for `hsgnval` after controlling for the effect of `pcturban`. However, Hall, Rudebusch, and Wilcox (1996) used Monte Carlo simulation to show that simply having an  $F$  statistic that is significant at the typical 5% or 10% level is not sufficient. Stock, Wright, and Yogo (2002) suggest that the  $F$  statistic should exceed 10 for inference based on the 2SLS estimator to be reliable when there is one endogenous regressor.

`estat firststage` also presents the Cragg and Donald (1993) minimum eigenvalue statistic as a further test of weak instruments. Stock and Yogo (2005) discuss two characterizations of weak instruments: first, weak instruments cause instrumental-variables estimators to be biased; second, hypothesis tests of parameters estimated by instrumental-variables estimators may suffer from severe size distortions. The test statistic in our example is 13.30, which is identical to the  $F$  statistic just discussed because our model contains one endogenous regressor.

The null hypothesis of each of Stock and Yogo’s tests is that the set of instruments is weak. To perform these tests, we must first choose either the largest relative bias of the 2SLS estimator we are willing to tolerate or the largest rejection rate of a nominal 5% Wald test we are willing to tolerate. If the test statistic exceeds the critical value, we can conclude that our instruments are not weak.

The row marked “2SLS relative bias” contains critical values for the test that the instruments are weak based on the bias of the 2SLS estimator *relative to* the bias of the OLS estimator. For example, from past experience we might know that the OLS estimate of a parameter  $\beta$  may be 50% too high. Saying that we are willing to tolerate a 10% relative bias means that we are willing to tolerate a bias of the 2SLS estimator no greater than 5% (that is, 10% of 50%). In our rental rate model, if we are willing to tolerate a 10% relative bias, then we can conclude that our instruments are not weak because the test statistic of 13.30 exceeds the critical value of 10.27. However, if we were willing to tolerate only a relative bias of 5%, we would conclude that our instruments are weak because  $13.30 < 16.85$ .

The rows marked “2SLS Size of nominal 5% Wald test” and “LIML Size of nominal 5% Wald test” contain critical values pertaining to Stock and Yogo’s (2005) second characterization of weak instruments. This characterization defines a set of instruments to be weak if a Wald test at the 5% level can have an actual rejection rate of no more than 10%, 15%, 20%, or 25%. Using the current example, suppose that we are willing to accept a rejection rate of at most 10%. Because  $13.30 < 24.58$ , we cannot reject the null hypothesis of weak instruments. On the other hand, if we use the LIML estimator instead, then we can reject the null hypothesis because  $13.30 > 5.44$ .

◀

## □ Technical note

Stock and Yogo (2005) tabulated critical values for 2SLS relative biases of 5%, 10%, 20%, and 30% for models with 1, 2, or 3 endogenous regressors and between 3 and 30 excluded exogenous variables (instruments). They also provide critical values for worst-case rejection rates of 5%, 10%, 20%, and 25% for nominal 5% Wald tests of the endogenous regressors with 1 or 2 endogenous regressors and between 1 and 30 instruments. If the model previously fit by `ivregress` has more instruments or endogenous regressors than these limits, the critical values are not shown. Stock and Yogo did not consider GMM estimators.

□

When the model being fit contains more than one endogenous regressor, the  $R^2$  and  $F$  statistics described above can overstate the relevance of the excluded instruments. Suppose that there are two endogenous regressors,  $Y_1$  and  $Y_2$ , and that there are two additional instruments,  $z_1$  and  $z_2$ . Say that  $z_1$  is highly correlated with both  $Y_1$  and  $Y_2$  but  $z_2$  is not correlated with either  $Y_1$  or  $Y_2$ . Then, the first-stage regression of  $Y_1$  on  $z_1$  and  $z_2$  (along with the included exogenous variables) will produce large  $R^2$  and  $F$  statistics, as will the regression of  $Y_2$  on  $z_1$ ,  $z_2$ , and the included exogenous variables. Nevertheless, the lack of correlation between  $z_2$  and  $Y_1$  and  $Y_2$  is problematic. Here, although the order condition indicates that the model is just identified (the number of excluded instruments equals the number of endogenous regressors), the irrelevance of  $z_2$  implies that the model is in fact not identified. Even if the model is overidentified, including irrelevant instruments can adversely affect the properties of instrumental-variables estimators, because their biases increase as the number of instruments increases.

#### ▷ Example 4

`estat firststage` presents different statistics when the model contains multiple endogenous regressors. For illustration, we refit our model of rental rates, assuming that both `hsngval` and `faminc` are endogenously determined. We use `i.region` along with `popden`, a measure of population density, as additional instruments.

```
. ivregress 2sls rent pcturban (hsngval faminc = i.region popden)
(output omitted)
```

```
. estat firststage
```

Shea's partial R-squared

| Variable | Shea's partial R-sq. | Shea's adj. partial R-sq. |
|----------|----------------------|---------------------------|
| hsngval  | 0.3477               | 0.2897                    |
| faminc   | 0.1893               | 0.1173                    |

Minimum eigenvalue statistic = 2.51666

|                          |                             |   |
|--------------------------|-----------------------------|---|
| Critical Values          | # of endogenous regressors: | 2 |
| H0: Instruments are weak | # of excluded instruments:  | 4 |

|                                   | 5%    | 10%  | 20%  | 30%  |
|-----------------------------------|-------|------|------|------|
| 2SLS relative bias                | 11.04 | 7.56 | 5.57 | 4.73 |
| 2SLS size of nominal 5% Wald test | 16.87 | 9.93 | 7.54 | 6.28 |
| LIML size of nominal 5% Wald test | 4.72  | 3.39 | 2.99 | 2.79 |

Consider the endogenous regressor `hsngval`. Part of its variation is attributable to its correlation with the other regressors `pcturban` and `faminc`. The other component of `hsngval`'s variation is peculiar to it and orthogonal to the variation in the other regressors. Similarly, we can think of the instruments as predicting the variation in `hsngval` in two ways, one stemming from the fact that the predicted values of `hsngval` are correlated with the predicted values of the other regressors and one from the variation in the predicted values of `hsngval` that is orthogonal to the variation in the predicted values of the other regressors.

What really matters for instrumental-variables estimation is whether the component of `hsngval` that is orthogonal to the other regressors can be explained by the component of the predicted value of `hsngval` that is orthogonal to the predicted values of the other regressors in the model. Shea's (1997) partial

$R^2$  statistic measures this correlation. Because the bias of instrumental-variables estimators increases as more instruments are used, Shea's adjusted partial  $R^2$  statistic is often used instead, as it makes a degrees-of-freedom adjustment for the number of instruments, analogous to the adjusted  $R^2$  measure used in OLS regression. Although what constitutes a "low" value for Shea's partial  $R^2$  depends on the specifics of the model being fit and the data used, these results, taken in isolation, do not strike us as being a particular cause for concern.

However, with this specification the minimum eigenvalue statistic is low. We cannot reject the null hypothesis of weak instruments for either of the characterizations we have discussed.

◀

By default, `estat firststage` determines which statistics to present based on the number of endogenous regressors in the model previously fit. However, you can specify the `all` option to obtain all the statistics.

### □ Technical note

If the previous estimation was conducted using `aweight`s, `pweight`s, or `iweight`s, then the first-stage regression summary statistics are computed using those weights. However, in these cases the minimum eigenvalue statistic and its critical values are not available.

If the previous estimation included a robust VCE, then the first-stage  $F$  statistic is based on a robust VCE as well; for example, if you fit your model with a HAC VCE using the Bartlett kernel and four lags, then the  $F$  statistic reported is based on regression results using a HAC VCE using the Bartlett kernel and four lags. By default, the minimum eigenvalue statistic and its critical values are not displayed. You can use the `forcenonrobust` option to obtain them in these cases; the minimum eigenvalue statistic is computed using the weights, though the critical values reported may not be appropriate.

□

### estat overid

In addition to the requirement that instrumental variables be correlated with the endogenous regressors, the instruments must also be uncorrelated with the structural error term. If the model is overidentified, meaning that the number of additional instruments exceeds the number of endogenous regressors, then we can test whether the instruments are uncorrelated with the error term. If the model is just identified, then we cannot perform a test of overidentifying restrictions.

The estimator you used to fit the model determines which tests of overidentifying restrictions `estat overid` reports. If you used the 2SLS estimator without a robust VCE, `estat overid` reports Sargan's (1958) and Basmann's (1960)  $\chi^2$  tests. If you used the 2SLS estimator and requested a robust VCE, Wooldridge's robust score test of overidentifying restrictions is performed instead; without a robust VCE, Wooldridge's test statistic is identical to Sargan's test statistic. If you used the LIML estimator, `estat overid` reports the Anderson–Rubin (1950) likelihood-ratio test and Basmann's (1960)  $F$  test. `estat overid` reports Hansen's (1982)  $J$  statistic if you used the GMM estimator. Davidson and MacKinnon (1993, 235–236) give a particularly clear explanation of the intuition behind tests of overidentifying restrictions. Also see Judge et al. (1985, 614–616) for a summary of tests of overidentifying restrictions for the 2SLS and LIML estimators.

Tests of overidentifying restrictions actually test two different things simultaneously. One, as we have discussed, is whether the instruments are uncorrelated with the error term. The other is that the equation is misspecified and that one or more of the excluded exogenous variables should in fact be included in the structural equation. Thus, a significant test statistic could represent either an invalid instrument or an incorrectly specified structural equation.

## ▶ Example 5

Here we refit the model that treated just `hsgnval` as endogenous using 2SLS, and then we perform tests of overidentifying restrictions:

```
. ivregress 2sls rent pcturban (hsgnval = faminc i.region)
(output omitted)
. estat overid
Tests of overidentifying restrictions:
Sargan (score) chi2(3) = 11.2877 (p = 0.0103)
Basman chi2(3) = 12.8294 (p = 0.0050)
```

Both test statistics are significant at the 5% test level, which means that either one or more of our instruments are invalid or that our structural model is specified incorrectly.

One possibility is that the error term in our structural model is heteroskedastic. Both Sargan's and Basman's tests assume that the errors are i.i.d.; if the errors are not i.i.d., then these tests are not valid. Here we refit the model by requesting heteroskedasticity-robust standard errors, and then we use `estat overid` to obtain Wooldridge's score test of overidentifying restrictions, which is robust to heteroskedasticity.

```
. ivregress 2sls rent pcturban (hsgnval = faminc i.region), vce(robust)
(output omitted)
. estat overid
Test of overidentifying restrictions:
Score chi2(3) = 6.8364 (p = 0.0773)
```

Here we no longer reject the null hypothesis that our instruments are valid at the 5% significance level, though we do reject the null at the 10% level. You can verify that the robust standard error on the coefficient for `hsgnval` is more than twice as large as its nonrobust counterpart and that the robust standard error for `pcturban` is nearly 50% larger.

◀

## □ Technical note

The test statistic for the test of overidentifying restrictions performed after GMM estimation is simply the sample size times the value of the objective function  $Q(\beta_1, \beta_2)$  defined in (5) of [R] `ivregress`, evaluated at the GMM parameter estimates. If the weight matrix  $\mathbf{W}$  is optimal, meaning that  $\mathbf{W} = \text{Var}(\mathbf{z}_i u_i)$ , then  $Q(\beta_1, \beta_2) \overset{A}{\sim} \chi^2(q)$ , where  $q$  is the number of overidentifying restrictions. However, if the estimated  $\mathbf{W}$  is not optimal, then the test statistic will not have an asymptotic  $\chi^2$  distribution.

Like the Sargan and Basman tests of overidentifying restrictions for the 2SLS estimator, the Anderson–Rubin and Basman tests after LIML estimation are predicated on the errors' being i.i.d. If the previous LIML results were reported with robust standard errors, then `estat overid` by default issues an error message and refuses to report the Anderson–Rubin and Basman test statistics. You can use the `forcenonrobust` option to override this behavior. You can also use `forcenonrobust` to obtain the Sargan and Basman test statistics after 2SLS estimation with robust standard errors.

□

By default, `estat overid` issues an error message if the previous estimation was conducted using `aweight`s, `pweight`s, or `iweight`s. You can use the `forceweights` option to override this behavior, though the test statistics may no longer have the expected  $\chi^2$  distributions.

## estat weakrobust

Instrumental-variables methods require that the instruments be correlated with the endogenous regressors. In principle, this is a low bar: even instruments only weakly correlated with the endogenous regressors allow for valid asymptotic inference using standard instrumental-variables methods. But weak instruments can lead to misleading inference in practice, even in relatively large samples. This is especially the case when models are overidentified or there is a high degree of endogeneity.

When instruments are weak, first-stage coefficients are small relative to the variance of their estimates. This causes the distribution of instrumental-variables estimators to be highly nonnormal and thus poorly approximated by standard inference methods. (See [Andrews, Stock, and Sun \[2019\]](#).)

The linear instrumental-variables model fit by `ivregress` is written in matrix form as

$$\begin{aligned} \mathbf{y} &= \mathbf{Y}\beta_1 + \mathbf{X}_1\beta_2 + \mathbf{u} \\ \mathbf{Y} &= \mathbf{X}_1\Pi_1 + \mathbf{X}_2\Pi_2 + \mathbf{V} \end{aligned}$$

where  $\mathbf{y}$  is a vector with the values of the dependent variable in the sample,  $\mathbf{Y}$  is a matrix of endogenous regressors,  $\mathbf{X}_1$  is a matrix of included exogenous regressors, and  $\mathbf{X}_2$  is a matrix of excluded exogenous regressors. The first equation is referred to as the structural equation, and the second equation is referred to as the first-stage equation. By substituting the endogenous regressors  $\mathbf{Y}$  into the structural equation and reparameterizing, we derive the reduced-form equation

$$\mathbf{y} = \mathbf{X}_1\delta_1 + \mathbf{X}_2\delta_2 + \epsilon$$

where  $\delta_1 \equiv \Pi_1\beta_1 + \beta_2$  and  $\delta_2 \equiv \Pi_2\beta_1$ .

The principle of the tests performed by `estat weakrobust` is that the reduced-form and first-stage equations are free of endogeneity. The estimates of  $\delta_1$ ,  $\delta_2$ ,  $\Pi_1$ , and  $\Pi_2$  can be tested for consistency with a null hypothesis about  $\beta_1$ , our parameter of interest. (In practice, the exogenous covariates  $\mathbf{X}_2$  are partialled out for simplicity.) `estat weakrobust` performs tests of the form  $\beta_1 = \mathbf{r}_1$ , where  $\mathbf{r}_1$  is a constant vector of the same dimension as  $\beta_1$ .

When the model is just identified, meaning there are no more instruments than there are endogenous regressors, the test of [Anderson and Rubin \(1949\)](#), or the associated confidence interval, is reported by default. When the model has been fit with a heteroskedastic, cluster-robust, or HAC VCE, an appropriate version of the Anderson–Rubin test or confidence interval is reported. In all cases, the Anderson–Rubin test statistic takes on a  $\chi^2$  distribution, and critical values and  $p$ -values can be exactly computed.

When the model is overidentified and has been fit with a homoskedastic VCE, the CLR test of [Moreira \(2003\)](#), or its associated confidence interval, is reported by default. The CLR test has better power properties than the Anderson–Rubin test in the overidentified case. When the model is fit with a nonhomoskedastic VCE, a generalization of the CLR test proposed by [Finlay and Magnusson \(2009\)](#) is used. This test is equivalent to the GMM quasi-LR statistic of [Kleibergen \(2007\)](#) in the linear instrumental-variables case considered here.

Regardless of the VCE specified, the CLR test statistic has a nonstandard distribution. However, when the model has been fit with a homoskedastic VCE and there is only a single endogenous regressor, the  $p$ -value for the CLR statistic can be computed exactly using the method of [Andrews, Moreira, and Stock \(2007\)](#). Otherwise, the  $p$ -value for the CLR test statistic is obtained by simulation.

Both the Anderson–Rubin and the CLR tests (and their CIs) can be requested for any model, even though only one test (or CI) is computed and reported by default. In the just-identified case, the CLR test is equivalent to the Anderson–Rubin test.

## ▷ Example 6

We revisit the model of average rental rate as a function of average housing values and the proportion of the population living in urban areas, this time supposing we do not have access to `faminc`, the variable for median family income. We may suspect that our region indicator variables are weak instruments for `hsgnval` and therefore perform inference robust to weak instruments using `estat weakrobust`.

```
. ivregress 2sls rent pcturban (hsgnval = i.region), vce(robust)
(output omitted)
. estat weakrobust, rseed(12345)
Test robust to weak instruments
Model VCE: Robust
(1) hsgnval = 0
Cond. likelihood-ratio (CLR) test = 5.48
 Prob > CLR = 0.0255
Notes: CLR test reported by default because
 model is overidentified.
 p-value computed by simulation
 (25,000 replications).
```

Here `estat weakrobust` reports a CLR test because there are more instruments than there are endogenous regressors. We find evidence to reject the null hypothesis that the coefficient on `hsgnval` is zero, but the  $p$ -value is substantially larger than the  $p$ -value of 0.003 that is reported by `ivregress`. Note that because the model is fit with a nonhomoskedastic VCE and a CLR test is being reported, the  $p$ -value is computed by simulation. We have included a random-number seed using the `rseed()` option for reproducibility, but this is optional.

We can request a confidence interval that is robust to weak instruments for `hsgnval` with the `ci` option:

```
. estat weakrobust, ci rseed(12345)
Searching for CI bounds:
Iteration 0: Grid points = 500
Iteration 1: Grid points = 1,000
(CI computed using 1,000 grid points on [-.003591, .006664])
Confidence interval robust to weak instruments
Model VCE: Robust
```

|         | Coefficient | CLR                  |         |
|---------|-------------|----------------------|---------|
|         |             | [95% conf. interval] |         |
| hsgnval | .0015365    | .0002263             | .002857 |

```
Notes: CLR CI reported by default because model is
 overidentified.
 Computed using simulation (25,000
 replications).
```

The confidence interval comes close to containing zero. Indeed, if we specified a confidence level of 99% using `level(99)`, zero would be included. Accounting for the possibility of weak instruments, we may conclude that we cannot rule out a zero coefficient on `hsgnval`.

◀

The `ci` option, which computes Anderson–Rubin and CLR confidence intervals, is available only in the single-endogenous regressor case. When the model has been fit with a homoskedastic VCE or is just identified, confidence intervals can be computed directly by inverting the relevant test. When the

model has been fit with a nonhomoskedastic VCE and is overidentified, there is no closed form for the endpoints of the conditional likelihood confidence interval, so a gridding procedure is used to estimate the confidence intervals.

The coefficient on the endogenous regressor, as fit by `ivregress`, is reported next to the confidence interval in the output of `estat weakrobust`, `ci` for reference. However, there is generally no guarantee that the estimated coefficient will be contained in the reported confidence interval. An exception is that when the model has been fit with homoskedastic errors, the CLR confidence set will contain the LIML estimator. In the just-identified case, the Anderson–Rubin confidence set will also contain the LIML estimator. See [Moreira \(2003\)](#) for a discussion.

## □ Technical note

Unlike conventional confidence intervals, the confidence “intervals” produced by inverting Anderson–Rubin and CLR tests are not always finite intervals and may not be intervals at all. Confidence intervals can take one of five forms:

1. Finite interval  $[a, b]$ . When instruments are strong and the model is well identified, both Anderson–Rubin and CLR confidence intervals typically take this form.
2. Union of finite intervals  $[a_1, b_1] \cup [a_2, b_2] \cup \dots \cup [a_m, b_m]$ . Confidence intervals can take this form when the model VCE is nonhomoskedastic.
3. Union of (possibly infinite) intervals  $(-\infty, b_1] \cup \dots \cup [a_m, +\infty)$ . Confidence sets may take this form when instruments are weak. When the model VCE is homoskedastic, this form may only be the union of two infinite intervals.
4. Real line  $(-\infty, +\infty)$ . Confidence sets may take this form when instruments are weak.
5. Empty set  $\emptyset$ . An empty confidence interval means the test rejects every possible null value for the endogenous regressor, thus rejecting the model. The only case in which the model can be rejected in this way is when the model is overidentified and an Anderson–Rubin confidence interval is used. In other cases, an empty confidence interval means the grid used to estimate the confidence interval was too narrow, and the bounds should be widened using the `bound()` option.

For detailed discussion of unconventional confidence intervals in inference that is robust to weak instruments, see [Mikusheva and Poi \(2006\)](#), [Mikusheva \(2010\)](#), and [Kleibergen \(2007\)](#).

□

## Stored results

After 2SLS estimation, `estat endogenous` stores the following in `r()`:

Scalars

|                             |                                                   |
|-----------------------------|---------------------------------------------------|
| <code>r(durbin)</code>      | Durbin $\chi^2$ statistic                         |
| <code>r(p_durbin)</code>    | $p$ -value for Durbin $\chi^2$ statistic          |
| <code>r(wu)</code>          | Wu–Hausman $F$ statistic                          |
| <code>r(p_wu)</code>        | $p$ -value for Wu–Hausman $F$ statistic           |
| <code>r(df)</code>          | degrees of freedom                                |
| <code>r(wudf_r)</code>      | denominator degrees of freedom for Wu–Hausman $F$ |
| <code>r(r_score)</code>     | robust score statistic                            |
| <code>r(p_r_score)</code>   | $p$ -value for robust score statistic             |
| <code>r(hac_score)</code>   | HAC score statistic                               |
| <code>r(p_hac_score)</code> | $p$ -value for HAC score statistic                |
| <code>r(lags)</code>        | lags used in prewhitening                         |
| <code>r(regF)</code>        | regression-based $F$ statistic                    |

|             |                                                          |
|-------------|----------------------------------------------------------|
| r(p_regF)   | <i>p</i> -value for regression-based <i>F</i> statistic  |
| r(regFdf_n) | regression-based <i>F</i> numerator degrees of freedom   |
| r(regFdf_r) | regression-based <i>F</i> denominator degrees of freedom |

After GMM estimation, `estat endogenous` stores the following in `r()`:

Scalars

|        |                                                 |
|--------|-------------------------------------------------|
| r(C)   | <i>C</i> $\chi^2$ statistic                     |
| r(p_C) | <i>p</i> -value for <i>C</i> $\chi^2$ statistic |
| r(df)  | degrees of freedom                              |

`estat firststage` stores the following in `r()`:

Scalars

|           |                              |
|-----------|------------------------------|
| r(mineig) | minimum eigenvalue statistic |
|-----------|------------------------------|

Matrices

|                  |                                                  |
|------------------|--------------------------------------------------|
| r(mineigcv)      | critical values for minimum eigenvalue statistic |
| r(multiresults)  | Shea's partial $R^2$ statistics                  |
| r(singleresults) | first-stage $R^2$ and <i>F</i> statistics        |

After 2SLS estimation, `estat overid` stores the following in `r()`:

Scalars

|              |                                                |
|--------------|------------------------------------------------|
| r(lags)      | lags used in prewhitening                      |
| r(df)        | $\chi^2$ degrees of freedom                    |
| r(score)     | score $\chi^2$ statistic                       |
| r(p_score)   | <i>p</i> -value for score $\chi^2$ statistic   |
| r(basmann)   | Basmann $\chi^2$ statistic                     |
| r(p_basmann) | <i>p</i> -value for Basmann $\chi^2$ statistic |
| r(sargan)    | Sargan $\chi^2$ statistic                      |
| r(p_sargan)  | <i>p</i> -value for Sargan $\chi^2$ statistic  |

After LIML estimation, `estat overid` stores the following in `r()`:

Scalars

|                 |                                                              |
|-----------------|--------------------------------------------------------------|
| r(ar)           | Anderson–Rubin (1950) $\chi^2$ statistic                     |
| r(p_ar)         | <i>p</i> -value for Anderson–Rubin (1950) $\chi^2$ statistic |
| r(ar_df)        | $\chi^2$ degrees of freedom                                  |
| r(basmann)      | Basmann <i>F</i> statistic                                   |
| r(p_basmann)    | <i>p</i> -value for Basmann <i>F</i> statistic               |
| r(basmann_df_n) | <i>F</i> numerator degrees of freedom                        |
| r(basmann_df_d) | <i>F</i> denominator degrees of freedom                      |

After GMM estimation, `estat overid` stores the following in `r()`:

Scalars

|              |                                                          |
|--------------|----------------------------------------------------------|
| r(HansenJ)   | Hansen's <i>J</i> $\chi^2$ statistic                     |
| r(p_HansenJ) | <i>p</i> -value for Hansen's <i>J</i> $\chi^2$ statistic |
| r(J_df)      | $\chi^2$ degrees of freedom                              |

After estimation of a just-identified model, or when `ar` is specified, `estat weakrobust` stores the following in `r()`:

Scalars

|              |                                                              |
|--------------|--------------------------------------------------------------|
| r(archi2)    | Anderson–Rubin (1949) $\chi^2$ statistic                     |
| r(p_archi2)  | <i>p</i> -value for Anderson–Rubin (1949) $\chi^2$ statistic |
| r(archi2_df) | $\chi^2$ degrees of freedom                                  |
| r(arF)       | Anderson–Rubin (1949) <i>F</i> statistic                     |
| r(p_arF)     | <i>p</i> -value for Anderson–Rubin (1949) <i>F</i> statistic |
| r(arF_df_n)  | <i>F</i> numerator degrees of freedom                        |

|                               |                                                                                          |
|-------------------------------|------------------------------------------------------------------------------------------|
| <code>r(arF_df_d)</code>      | $F$ denominator degrees of freedom                                                       |
| <code>r(ar_ngrid)</code>      | number of gridpoints used for Anderson–Rubin (1949) confidence interval                  |
| Macros                        |                                                                                          |
| <code>r(bound)</code>         | user-specified grid bounds                                                               |
| Matrices                      |                                                                                          |
| <code>r(betanull)</code>      | values of the null hypothesis for endogenous regressors                                  |
| <code>r(table)</code>         | matrix containing coefficient with reported confidence interval (omitted if CI is empty) |
| <code>r(ar_ci)</code>         | matrix containing the bounds of the Anderson–Rubin (1949) confidence interval            |
| <code>r(ar_finalbound)</code> | matrix containing the final upper and lower bounds used by the gridding procedure        |

After estimation of an overidentified model, or when `clr` is specified, `estat weakrobust` stores the following in `r()`:

|                                |                                                                                                   |
|--------------------------------|---------------------------------------------------------------------------------------------------|
| Scalars                        |                                                                                                   |
| <code>r(clr)</code>            | CLR statistic of <a href="#">Moreira (2003)</a>                                                   |
| <code>r(p_clr)</code>          | $p$ -value for CLR statistic                                                                      |
| <code>r(J)</code>              | $J$ statistic of <a href="#">Finlay and Magnusson (2009)</a> (when model VCE is nonhomoskedastic) |
| <code>r(rk)</code>             | rank statistic of <a href="#">Kleibergen and Paap (2006)</a> (when model VCE is nonhomoskedastic) |
| <code>r(reps)</code>           | number of simulation replications (when results are simulated)                                    |
| <code>r(rseed)</code>          | random seed used (when results are simulated)                                                     |
| <code>r(clr_ngrid)</code>      | number of gridpoints used for CLR confidence interval                                             |
| Macros                         |                                                                                                   |
| <code>r(bound)</code>          | user-specified grid bounds                                                                        |
| <code>r(rngstate)</code>       | random-number state used (when results are simulated)                                             |
| Matrices                       |                                                                                                   |
| <code>r(betanull)</code>       | values of the null hypothesis for endogenous regressors                                           |
| <code>r(Sbar)</code>           | $\bar{S}$ statistic of <a href="#">Moreira (2003)</a> (when model VCE is homoskedastic)           |
| <code>r(Tbar)</code>           | $\bar{T}$ statistic of <a href="#">Moreira (2003)</a> (when model VCE is homoskedastic)           |
| <code>r(table)</code>          | matrix containing coefficient with reported confidence interval (omitted if CI is empty)          |
| <code>r(clr_ci)</code>         | matrix containing the bounds of the CLR confidence interval                                       |
| <code>r(clr_finalbound)</code> | matrix containing the final upper and lower bounds used by the gridding procedure                 |

## Methods and formulas

Methods and formulas are presented under the following headings:

*Notation*  
*estat endogenous*  
*estat firststage*  
*estat overid*  
*estat weakrobust*  
*Homoskedastic errors*  
*Nonhomoskedastic errors*  
*Confidence intervals*

## Notation

Recall from [\[R\] ivregress](#) that the model is

$$\mathbf{y} = \mathbf{Y}\boldsymbol{\beta}_1 + \mathbf{X}_1\boldsymbol{\beta}_2 + \mathbf{u} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$$

$$\mathbf{Y} = \mathbf{X}_1\Pi_1 + \mathbf{X}_2\Pi_2 + \mathbf{V} = \mathbf{Z}\Pi + \mathbf{V}$$

where  $\mathbf{y}$  is an  $N \times 1$  vector of the left-hand-side variable,  $N$  is the sample size,  $\mathbf{Y}$  is an  $N \times p$  matrix of  $p$  endogenous regressors,  $\mathbf{X}_1$  is an  $N \times k_1$  matrix of  $k_1$  included exogenous regressors,  $\mathbf{X}_2$  is an  $N \times k_2$  matrix of  $k_2$  excluded exogenous variables,  $\mathbf{X} = [\mathbf{Y} \ \mathbf{X}_1]$ ,  $\mathbf{Z} = [\mathbf{X}_1 \ \mathbf{X}_2]$ ,  $\mathbf{u}$  is an  $N \times 1$  vector of errors,  $\mathbf{V}$  is an  $N \times p$  matrix of errors,  $\boldsymbol{\beta} = [\boldsymbol{\beta}_1 \ \boldsymbol{\beta}_2]$  is a  $k = (p + k_1) \times 1$  vector of parameters, and  $\boldsymbol{\Pi}$  is a  $(k_1 + k_2) \times p$  vector of parameters. If a constant term is included in the model, then one column of  $\mathbf{X}_1$  contains all ones.

### estat endogenous

Partition  $\mathbf{Y}$  as  $\mathbf{Y} = [\mathbf{Y}_1 \ \mathbf{Y}_2]$ , where  $\mathbf{Y}_1$  represents the  $p_1$  endogenous regressors whose endogeneity is being tested and  $\mathbf{Y}_2$  represents the  $p_2$  endogenous regressors whose endogeneity is not being tested. If the endogeneity of all endogenous regressors is being tested,  $\mathbf{Y} = \mathbf{Y}_1$  and  $p_2 = 0$ . After GMM estimation, `estat endogenous` refits the model treating  $\mathbf{Y}_1$  as exogenous using the same type of weight matrix as requested at estimation time with the `wmatrix()` option; denote the Sargan statistic from this model by  $J_e$  and the estimated weight matrix by  $\mathbf{W}_e$ . Let  $\mathbf{S}_e = \mathbf{W}_e^{-1}$ . `estat endogenous` removes from  $\mathbf{S}_e$  the rows and columns corresponding to the variables represented by  $\mathbf{Y}_1$ ; denote the inverse of the resulting matrix by  $\mathbf{W}'_e$ . Next, `estat endogenous` fits the model treating both  $\mathbf{Y}_1$  and  $\mathbf{Y}_2$  as endogenous, using the weight matrix  $\mathbf{W}'_e$ ; denote the Sargan statistic from this model by  $J_c$ . Then,  $C = (J_e - J_c) \sim \chi^2(p_1)$ . If one simply used the  $J$  statistic from the original model fit by `ivregress` in place of  $J_c$ , then in finite samples  $J_e - J$  might be negative. The procedure used by `estat endogenous` is guaranteed to yield  $C \geq 0$ ; see [Hayashi \(2000, 220\)](#).

Let  $\hat{\mathbf{u}}_c$  denote the residuals from the model treating both  $\mathbf{Y}_1$  and  $\mathbf{Y}_2$  as endogenous, and let  $\hat{\mathbf{u}}_e$  denote the residuals from the model treating only  $\mathbf{Y}_2$  as endogenous. Then, Durbin's (1954) statistic is

$$D = \frac{\hat{\mathbf{u}}'_e \mathbf{P}_{Z\mathbf{Y}_1} \hat{\mathbf{u}}_e - \hat{\mathbf{u}}'_c \mathbf{P}_Z \hat{\mathbf{u}}_c}{\hat{\mathbf{u}}'_e \hat{\mathbf{u}}_e / N}$$

where  $\mathbf{P}_Z = \mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'$  and  $\mathbf{P}_{Z\mathbf{Y}_1} = [\mathbf{Z} \ \mathbf{Y}_1][(\mathbf{Z} \ \mathbf{Y}_1)'(\mathbf{Z} \ \mathbf{Y}_1)]^{-1}[\mathbf{Z} \ \mathbf{Y}_1]'$ .  $D \sim \chi^2(p_1)$ . The Wu–Hausman ([Wu 1974](#); [Hausman 1978](#)) statistic is

$$WH = \frac{(\hat{\mathbf{u}}'_e \mathbf{P}_{Z\mathbf{Y}_1} \hat{\mathbf{u}}_e - \hat{\mathbf{u}}'_c \mathbf{P}_Z \hat{\mathbf{u}}_c) / p_1}{\{\hat{\mathbf{u}}'_e \hat{\mathbf{u}}_e - (\hat{\mathbf{u}}'_c \mathbf{P}_{Z\mathbf{Y}_1} \hat{\mathbf{u}}_e - \hat{\mathbf{u}}'_c \mathbf{P}_Z \hat{\mathbf{u}}_c)\} / (N - k_1 - p - p_1)}$$

$WH \sim F(p_1, N - k_1 - p - p_1)$ . [Baum, Schaffer, and Stillman \(2003, 2007\)](#) discuss these tests in more detail.

Next, we describe Wooldridge's (1995) score test. The nonrobust version of Wooldridge's test is identical to Durbin's test. Suppose a robust covariance matrix was used at estimation time. Let  $\hat{\mathbf{e}}$  denote the sample residuals obtained by fitting the model via OLS, treating  $\mathbf{Y}$  as exogenous. We then regress each variable represented in  $\mathbf{Y}$  on  $\mathbf{Z}$ ; call the residuals for the  $j$ th regression  $\hat{\mathbf{r}}_j$ ,  $j = 1, \dots, p$ . Define  $\hat{k}_{ij} = \hat{e}_i \hat{r}_{ij}$ ,  $i = 1, \dots, N$ . We then run the regression

$$\mathbf{1} = \theta_1 \hat{\mathbf{k}}_1 + \dots + \theta_p \hat{\mathbf{k}}_p + \boldsymbol{\epsilon}$$

where  $\mathbf{1}$  is an  $N \times 1$  vector of ones and  $\boldsymbol{\epsilon}$  is a regression error term.  $N - \text{RSS} \sim \chi^2(p)$ , where RSS is the residual sum of squares from the regression just described. If instead a HAC VCE was used at estimation time, then before running the final regression we prewhiten the  $\hat{\mathbf{k}}_j$  series by using a `VAR(q)` model, where  $q$  is the number of lags specified with the `lags()` option.

The regression-based test proceeds as follows. Following Hausman (1978, 1259), we regress  $\mathbf{Y}$  on  $\mathbf{Z}$  and obtain the residuals  $\widehat{\mathbf{V}}$ . Next, we fit the augmented regression

$$\mathbf{y} = \mathbf{Y}\beta_1 + \mathbf{X}_1\beta_2 + \widehat{\mathbf{V}}\gamma + \epsilon$$

by OLS regression, where  $\epsilon$  is a regression error term. A test of the exogeneity of  $\mathbf{Y}$  is equivalent to a test of  $\gamma = \mathbf{0}$ . As Cameron and Trivedi (2005, 276) suggest, this test can be made robust to heteroskedasticity, autocorrelation, or clustering by using the appropriate robust VCE when testing  $\gamma = \mathbf{0}$ . When a nonrobust VCE is used, this test is equivalent to the Wu–Hausman test described earlier. One cannot simply fit this augmented regression via 2SLS to test the endogeneity of a subset of the endogenous regressors; Davidson and MacKinnon (1993, 229–231) discuss a test of  $\gamma = \mathbf{0}$  for the homoskedastic version of the augmented regression fit by 2SLS, but an appropriate robust test is not apparent.

### estat firststage

When the structural equation includes one endogenous regressor, `estat firststage` fits the regression

$$\mathbf{Y} = \mathbf{X}_1\pi_1 + \mathbf{X}_2\pi_2 + \mathbf{v}$$

via OLS. The  $R^2$  and adjusted  $R^2$  from that regression are reported in the output, as well as the  $F$  statistic from the Wald test of  $H_0: \pi_2 = \mathbf{0}$ . To obtain the partial  $R^2$  statistic, `estat firststage` fits the regression

$$\mathbf{M}_{\mathbf{X}_1}\mathbf{y} = \mathbf{M}_{\mathbf{X}_1}\mathbf{X}_2\xi + \epsilon$$

by OLS, where  $\epsilon$  is a regression error term,  $\xi$  is a  $k_2 \times 1$  parameter vector, and  $\mathbf{M}_{\mathbf{X}_1} = \mathbf{I} - \mathbf{X}_1(\mathbf{X}'_1\mathbf{X}_1)^{-1}\mathbf{X}'_1$ ; that is, the partial  $R^2$  is the  $R^2$  between  $\mathbf{y}$  and  $\mathbf{X}_2$  after eliminating the effects of  $\mathbf{X}_1$ . If the model contains multiple endogenous regressors and the `all` option is specified, these statistics are calculated for each endogenous regressor in turn.

To calculate Shea’s partial  $R^2$ , let  $\mathbf{y}_1$  denote the endogenous regressor whose statistic is being calculated and  $\mathbf{Y}_0$  denote the other endogenous regressors. Define  $\tilde{\mathbf{y}}_1$  as the residuals obtained from regressing  $\mathbf{y}_1$  on  $\mathbf{Y}_0$  and  $\mathbf{X}_1$ . Let  $\hat{\mathbf{y}}_1$  denote the fitted values obtained from regressing  $\mathbf{y}_1$  on  $\mathbf{X}_1$  and  $\mathbf{X}_2$ ; that is,  $\hat{\mathbf{y}}_1$  are the fitted values from the first-stage regression for  $\mathbf{y}_1$ , and define the columns of  $\widehat{\mathbf{Y}}_0$  analogously. Finally, let  $\tilde{\tilde{\mathbf{y}}}_1$  denote the residuals from regressing  $\hat{\mathbf{y}}_1$  on  $\widehat{\mathbf{Y}}_0$  and  $\mathbf{X}_1$ . Shea’s partial  $R^2$  is the simple  $R^2$  from the regression of  $\tilde{\tilde{\mathbf{y}}}_1$  on  $\tilde{\tilde{\mathbf{y}}}_1$ ; denote this as  $R^2_S$ . Shea’s adjusted partial  $R^2$  is equal to  $1 - (1 - R^2_S)(N - 1)/(N - k_Z + 1)$  if a constant term is included and  $1 - (1 - R^2_S)(N - 1)/(N - k_Z)$  if there is no constant term included in the model, where  $k_Z = k_1 + k_2$ . For one endogenous regressor, one instrument, no exogenous regressors, and a constant term,  $R^2_S$  equals the adjusted  $R^2_S$ .

The Stock and Yogo minimum eigenvalue statistic, first proposed by Cragg and Donald (1993) as a test for underidentification, is the minimum eigenvalue of the matrix

$$\mathbf{G} = \frac{1}{k_Z} \widehat{\Sigma}_{\mathbf{V}\mathbf{V}}^{-1/2} \mathbf{Y}' \mathbf{M}'_{\mathbf{X}_1} \mathbf{X}_2 (\mathbf{X}'_2 \mathbf{M}_{\mathbf{X}_1} \mathbf{X}_2)^{-1} \mathbf{X}'_2 \mathbf{M}_{\mathbf{X}_1} \mathbf{Y} \widehat{\Sigma}_{\mathbf{V}\mathbf{V}}^{-1/2}$$

where

$$\widehat{\Sigma}_{\mathbf{V}\mathbf{V}} = \frac{1}{N - k_Z} \mathbf{Y}' \mathbf{M}_Z \mathbf{Y}$$

$\mathbf{M}_Z = \mathbf{I} - \mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'$ , and  $\mathbf{Z} = [\mathbf{X}_1 \ \mathbf{X}_2]$ . Critical values are obtained from the tables in Stock and Yogo (2005).

## estat overid

The Sargan (1958) and Basmann (1960)  $\chi^2$  statistics are calculated by running the auxiliary regression

$$\hat{\mathbf{u}} = \mathbf{Z}\boldsymbol{\delta} + \mathbf{e}$$

where  $\hat{\mathbf{u}}$  are the sample residuals from the model and  $\mathbf{e}$  is an error term. Then, Sargan's statistic is

$$S = N \left( 1 - \frac{\hat{\mathbf{e}}'\hat{\mathbf{e}}}{\hat{\mathbf{u}}'\hat{\mathbf{u}}} \right)$$

where  $\hat{\mathbf{e}}$  are the residuals from that auxiliary regression. Basmann's statistic is calculated as

$$B = S \frac{N - k_Z}{N - S}$$

Both  $S$  and  $B$  are distributed  $\chi^2(m)$ , where  $m$ , the number of overidentifying restrictions, is equal to  $k_Z - k$ , where  $k$  is the number of endogenous regressors.

Wooldridge's (1995) score test of overidentifying restrictions is identical to Sargan's (1958) statistic under the assumption of i.i.d. and therefore is not recomputed unless a robust VCE was used at estimation time. If a heteroskedasticity-robust VCE was used, Wooldridge's test proceeds as follows. Let  $\hat{\mathbf{Y}}$  denote the  $N \times k$  matrix of fitted values obtained by regressing the endogenous regressors on  $\mathbf{X}_1$  and  $\mathbf{X}_2$ . Let  $\mathbf{Q}$  denote an  $N \times m$  matrix of excluded exogenous variables; the test statistic to be calculated is invariant to whichever  $m$  of the  $k_2$  excluded exogenous variables is chosen. Define the  $i$ th element of  $\hat{\mathbf{k}}_j$ ,  $i = 1, \dots, N$ ,  $j = 1, \dots, m$ , as

$$k_{ij} = \hat{q}_{ij} \hat{u}_i$$

where  $\hat{q}_{ij}$  is the  $i$ th element of  $\hat{\mathbf{q}}_j$ , the residuals from regressing the  $j$ th column of  $\mathbf{Q}$  on  $\hat{\mathbf{Y}}$  and  $\mathbf{X}_1$ . Finally, fit the regression

$$\mathbf{1} = \theta_1 \hat{\mathbf{k}}_1 + \dots + \theta_m \hat{\mathbf{k}}_m + \boldsymbol{\epsilon}$$

where  $\mathbf{1}$  is an  $N \times 1$  vector of ones and  $\boldsymbol{\epsilon}$  is a regression error term, and calculate the residual sum of squares, RSS. Then, the test statistic is  $W = N - \text{RSS}$ .  $W \sim \chi^2(m)$ . If a HAC VCE was used at estimation, then the  $\hat{\mathbf{k}}_j$  are prewhitened using a VAR( $p$ ) model, where  $p$  is specified using the `lags()` option.

The Anderson–Rubin (AR; 1950) test of overidentifying restrictions for use after the LIML estimator is calculated as  $\text{AR} = N(\kappa - 1)$ , where  $\kappa$  is the minimal eigenvalue of a certain matrix defined in *Methods and formulas* of [R] **ivregress**.  $\text{AR} \sim \chi^2(m)$ . (Some texts define this statistic as  $N \ln(\kappa)$  because  $\ln(x) \approx (x - 1)$  for  $x$  near 1.) Basmann's  $F$  statistic for use after the LIML estimator is calculated as  $B_F = (\kappa - 1)(N - k_Z)/m$ .  $B_F \sim F(m, N - k_Z)$ .

Hansen's  $J$  statistic is simply the sample size times the value of the GMM objective function defined in (5) of [R] **ivregress**, evaluated at the estimated parameter values. Under the null hypothesis that the overidentifying restrictions are valid,  $J \sim \chi^2(m)$ .

John Denis Sargan (1924–1996) was born in Yorkshire, UK. He pioneered the theory of instrumental-variables (IV) estimation in an article published in 1958. In the article, he also developed overidentification tests, developed significance tests, and discussed possible instruments for applied work. A year later, he wrote an article extending the theory to models containing autoregressive errors. This extension was one of his many contributions to time-series econometric analysis. For example, in 1964 he published a paper in which he developed misspecification tests for dynamic equations, along with an IV estimator for models with nonlinear parameters, and a model with a long-run equilibrium. His paper laid the foundation for other econometric methods, such as cointegration analysis, and established what would be known as the London School of Economics (LSE) approach to econometric modeling. He spent twenty years at this institution, supervising the doctoral work of many econometricians who themselves made important contributions to econometrics. In addition to Sargan’s many lasting contributions to econometrics, he also left a lasting impression on his students and colleagues through his generosity.

## estat weakrobust

In *estat weakrobust* of *Remarks and examples*, we reintroduced the model fit by `ivregress`, written in matrix form as

$$\begin{aligned} \mathbf{y} &= \mathbf{Y}\boldsymbol{\beta}_1 + \mathbf{X}_1\boldsymbol{\beta}_2 + \mathbf{u} \\ \mathbf{Y} &= \mathbf{X}_1\boldsymbol{\Pi}_1 + \mathbf{X}_2\boldsymbol{\Pi}_2 + \mathbf{V} \end{aligned}$$

The first equation is referred to as the structural equation, and the second equation is referred to as the first-stage equation.

We then derived the reduced-form equation. Below, we write the model in the form of the reduced-form equation and the first-stage equations

$$\begin{aligned} \mathbf{y} &= \mathbf{X}_1\boldsymbol{\delta}_1 + \mathbf{X}_2\boldsymbol{\delta}_2 + \boldsymbol{\epsilon} \\ \mathbf{Y} &= \mathbf{X}_1\boldsymbol{\Pi}_1 + \mathbf{X}_2\boldsymbol{\Pi}_2 + \mathbf{V} \end{aligned}$$

where  $\boldsymbol{\delta}_1 \equiv \boldsymbol{\Pi}_1\boldsymbol{\beta}_1 + \boldsymbol{\beta}_2$  and  $\boldsymbol{\delta}_2 \equiv \boldsymbol{\Pi}_2\boldsymbol{\beta}_1$ . The reduced-form coefficients  $\boldsymbol{\delta}_1$  and  $\boldsymbol{\delta}_2$  are  $k_1 \times 1$  and  $k_2 \times 1$ , respectively. Without loss of generality, we can rewrite the model to omit included exogenous regressors  $\mathbf{X}_1$  by considering  $\mathbf{y}$ ,  $\mathbf{Y}$ ,  $\mathbf{X}_2$ ,  $\boldsymbol{\epsilon}$ , and  $\mathbf{V}$  to have been replaced by their partialled-out equivalents:

$$\begin{aligned} \mathbf{y} &= \mathbf{X}_2\boldsymbol{\delta}_2 + \boldsymbol{\epsilon} \\ \mathbf{Y} &= \mathbf{X}_2\boldsymbol{\Pi}_2 + \mathbf{V} \end{aligned}$$

Our null hypothesis  $\boldsymbol{\beta}_1 = \mathbf{r}_1$  implies that  $\boldsymbol{\delta}_2 - \boldsymbol{\Pi}_2\mathbf{r}_1 = 0$ .

## Homoskedastic errors

Let  $\widehat{\boldsymbol{\Omega}} = 1/N \begin{bmatrix} \tilde{\boldsymbol{\epsilon}}'\tilde{\boldsymbol{\epsilon}} & \tilde{\boldsymbol{\epsilon}}'\widehat{\mathbf{V}} \\ \widehat{\mathbf{V}}'\tilde{\boldsymbol{\epsilon}} & \widehat{\mathbf{V}}'\widehat{\mathbf{V}} \end{bmatrix}$  be the estimated  $(p+1) \times (p+1)$  covariance matrix of  $(\boldsymbol{\epsilon}', \mathbf{V}')'$ . When the model is assumed to have homoskedastic errors, we can proceed by computing the statistics

$$\bar{S} = (\mathbf{X}_2'\mathbf{X}_2)^{-1/2}\mathbf{X}_2'(\mathbf{y} - \mathbf{Y}\mathbf{r}_1)(\mathbf{b}_0'\widehat{\boldsymbol{\Omega}}\mathbf{b}_0)^{-1/2}$$

and

$$\bar{T} = (\mathbf{X}'_2 \mathbf{X}_2)^{-1/2} \mathbf{X}'_2 [\mathbf{y}, \mathbf{Y}']' \widehat{\boldsymbol{\Omega}}^{-1} \mathbf{A}_0 (\mathbf{A}'_0 \widehat{\boldsymbol{\Omega}}^{-1} \mathbf{A}_0)^{-1/2}$$

where  $\mathbf{b}_0 = [1, -\mathbf{r}_1]$  and  $\mathbf{A}_0 = [\mathbf{r}_1, \mathbf{I}_p]'$ .

The Anderson–Rubin (1949) statistic is computed as  $\text{AR} = \bar{S}' \bar{S}$  and follows a  $\chi^2(k_2)$  distribution. When the model has been fit with the `small` option in `ivregress`, an  $F$  version of the statistic is returned, which is distributed  $F(k_2, N - k_1 - k_2)$ . This  $F$  statistic is computed using  $N/(N - k_1 - k_2) \widehat{\boldsymbol{\Omega}}$  in place of  $\widehat{\boldsymbol{\Omega}}$ .

The CLR statistic is computed as  $\text{CLR} = \bar{S}' \bar{S} - \bar{\lambda}^{\min}$ , where  $\bar{\lambda}^{\min}$  is the minimum eigenvalue of the matrix  $(\bar{S}, \bar{T})' (\bar{S}, \bar{T})$ . When the model is fit with the `small` option in `ivregress`, the statistic is computed using  $N/(N - k_1 - k_2) \widehat{\boldsymbol{\Omega}}$ . When the model is just identified,  $\text{CLR} \sim \chi^2(k_2)$ . When the model is overidentified, CLR has a nonstandard distribution. The numerical method of Andrews, Moreira, and Stock (2007) is used to obtain  $p$ -values in the single-endogenous regressor case. In the multiple-endogenous regressor case,  $p$ -values are obtained by simulation conditional on  $\bar{T}$ , as outlined in Moreira (2003).

## Nonhomoskedastic errors

When the model is fit with a nonhomoskedastic VCE, let

$$\widehat{\boldsymbol{\Sigma}} = \begin{bmatrix} \widehat{\boldsymbol{\Sigma}}_{\delta} & \widehat{\boldsymbol{\Sigma}}_{\delta \Pi} \\ \widehat{\boldsymbol{\Sigma}}_{\Pi \delta} & \widehat{\boldsymbol{\Sigma}}_{\Pi} \end{bmatrix}$$

be the  $k_2(p + 1) \times k_2(p + 1)$  estimated covariance matrix of  $\{\delta'_2, \text{Vec}(\boldsymbol{\Pi}_2)'\}'$ , where  $\text{Vec}(\cdot)$  is the column-major vectorization operator.

We compute the Anderson–Rubin statistic as

$$\text{AR}(\mathbf{r}_1) = (\mathbf{y} - \mathbf{Y}\mathbf{r}_1)' \mathbf{X}_2 (\mathbf{X}'_2 \mathbf{X}_2)^{-1} \widehat{\boldsymbol{\Psi}}(\mathbf{r}_1)^{-1} (\mathbf{X}'_2 \mathbf{X}_2)^{-1} \mathbf{X}'_2 (\mathbf{y} - \mathbf{Y}\mathbf{r}_1)$$

where  $\widehat{\boldsymbol{\Psi}}(\mathbf{r}_1) = (\mathbf{b}_0 \otimes \mathbf{I}_k)' \widehat{\boldsymbol{\Sigma}} (\mathbf{b}_0 \otimes \mathbf{I}_k)$ .

As in the homoskedastic case, AR takes on a  $\chi^2(k_2)$  distribution under the null. When the model is fit with the `small` option, the statistic is computed using a small-sample adjustment, and an  $F$  version of the statistic is returned.

To compute the CLR test statistic, let  $J(\mathbf{r}_1)$  be defined as in Finlay and Magnusson (2009) (or, equivalently, as in Kleibergen [2007]), and let  $\text{rk}(\mathbf{r}_1)$  be the rank statistic of Kleibergen and Paap (2006) (the forms of these statistics are omitted here for brevity). The test statistic is given by

$$\text{CLR}(\mathbf{r}_1) = \frac{1}{2} \left[ \text{AR}(\mathbf{r}_1) - \text{rk}(\mathbf{r}_1) + \sqrt{\{\text{AR}(\mathbf{r}_1) + \text{rk}(\mathbf{r}_1)\}^2 - 4J(\mathbf{r}_1)\text{rk}(\mathbf{r}_1)} \right]$$

In the just-identified case, CLR is equal to AR and has a  $\chi^2(k_2)$  distribution. Otherwise, CLR has a nonstandard distribution, and  $p$ -values are obtained by simulation, as outlined in Kleibergen (2007). When the model is fit with the `small` option, the statistic is computed using a small-sample adjustment.

## Confidence intervals

Confidence intervals are obtained by inverting the relevant tests. When the model is fit with a homoskedastic VCE, or when it is just identified, the Anderson–Rubin test can be inverted analytically, so the bounds of the resulting interval (or union of intervals) are computed as a closed form (see Mikushcheva [2010] for details). When the model VCE is nonhomoskedastic and the model is overidentified, the Anderson–Rubin confidence interval is obtained by a gridding procedure.

The CLR test is equivalent to the Anderson–Rubin test in the just-identified case, so its confidence intervals can be computed in the same way. In the overidentified case when errors are homoskedastic, the CLR test is inverted numerically using the method of Mikusheva (2010). When errors are nonhomoskedastic, a gridding procedure is used to obtain a confidence interval.

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## Also see

[R] **ivregress** — Single-equation instrumental-variables regression

[U] **20 Estimation and postestimation commands**

|                                      |                                          |                                                |
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## Description

`ivtobit` fits tobit models where one or more of the covariates are endogenously determined. By default, `ivtobit` uses maximum likelihood estimation, but Newey's (1987) minimum  $\chi^2$  (two-step) estimator can be requested. Both estimators assume that the endogenous covariates are continuous and so are not appropriate for use with discrete endogenous covariates.

## Quick start

Tobit regression of  $y_1$  on  $x$  and endogenous regressor  $y_2$  that is instrumented by  $z$  where  $y_1$  is left-censored at its observed minimum

```
ivtobit y1 x (y2 = z), ll
```

Same as above, but specify that  $y_1$  is left-censored at 0 and right-censored at 20

```
ivtobit y1 x (y2 = z), ll(0) ul(20)
```

Use Newey's two-step estimator

```
ivtobit y1 x (y2 = z), ll(0) ul(20) twostep
```

Same as above, and show first-stage regression results

```
ivtobit y1 x (y2 = z), ll(0) ul(20) twostep first
```

## Menu

Statistics > Endogenous covariates > Tobit model with endogenous covariates

## Syntax

### Maximum likelihood estimator

```
ivtobit depvar [varlist1] (varlist2 = varlistiv) [if] [in] [weight] ,
 ll[(#)] ul[(#)] [mle_options]
```

### Two-step estimator

```
ivtobit depvar [varlist1] (varlist2 = varlistiv) [if] [in] [weight] , twostep
 ll[(#)] ul[(#)] [tse_options]
```

*varlist*<sub>1</sub> is the list of exogenous variables.

*varlist*<sub>2</sub> is the list of endogenous variables.

*varlist*<sub>iv</sub> is the list of exogenous variables used with *varlist*<sub>1</sub> as instruments for *varlist*<sub>2</sub>.

| <i>mle_options</i>                        | Description                                                                                                                                            |
|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------|
| Model                                     |                                                                                                                                                        |
| * ll[(#)]                                 | left-censoring limit                                                                                                                                   |
| * ul[(#)]                                 | right-censoring limit                                                                                                                                  |
| <u>m</u> le                               | use conditional maximum-likelihood estimator; the default                                                                                              |
| <u>c</u> onstraints( <i>constraints</i> ) | apply specified linear constraints                                                                                                                     |
| SE/Robust                                 |                                                                                                                                                        |
| vce( <i>vcetype</i> )                     | <i>vcetype</i> may be oim, <u>r</u> obust, <u>c</u> luster <i>clustvar</i> , opg, <u>b</u> ootstrap,<br>or <u>j</u> ackknife                           |
| Reporting                                 |                                                                                                                                                        |
| <u>l</u> evel(#)                          | set confidence level; default is level(95)                                                                                                             |
| <u>f</u> irst                             | report first-stage regression                                                                                                                          |
| <u>n</u> ocnsreport                       | do not display constraints                                                                                                                             |
| <i>display_options</i>                    | control columns and column formats, row spacing, line width,<br>display of omitted variables and base and empty cells, and<br>factor-variable labeling |
| Maximization                              |                                                                                                                                                        |
| <i>maximize_options</i>                   | control the maximization process                                                                                                                       |
| <u>c</u> oefflegend                       | display legend instead of statistics                                                                                                                   |

\*You must specify at least one of ll[(#)] and ul[(#)].

| <i>tse_options</i>            | Description                                                                                                                                      |
|-------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                  |                                                                                                                                                  |
| * <b>twostep</b>              | use Newey's two-step estimator; the default is <code>mle</code>                                                                                  |
| * <b>ll</b> [(#)]             | left-censoring limit                                                                                                                             |
| * <b>ul</b> [(#)]             | right-censoring limit                                                                                                                            |
| <b>SE</b>                     |                                                                                                                                                  |
| <b>vce</b> ( <i>vcetype</i> ) | <i>vcetype</i> may be <code>twostep</code> , <code>bootstrap</code> , or <code>jackknife</code>                                                  |
| <b>Reporting</b>              |                                                                                                                                                  |
| <b>level</b> (#)              | set confidence level; default is <code>level(95)</code>                                                                                          |
| <b>first</b>                  | report first-stage regression                                                                                                                    |
| <b>display_options</b>        | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| <b>coeflegend</b>             | display legend instead of statistics                                                                                                             |

\*`twostep` is required. You must specify at least one of `ll`[(#)] and `ul`[(#)].

*varlist*<sub>1</sub> and *varlist*<sub>iv</sub> may contain factor variables; see [U] 11.4.3 Factor variables.

*devar*, *varlist*<sub>1</sub>, *varlist*<sub>2</sub>, and *varlist*<sub>iv</sub> may contain time-series operators; see [U] 11.4.4 Time-series varlists.

`bayesboot`, `bootstrap`, `by`, `collect`, `jackknife`, `rolling`, `statsby`, and `svy` are allowed; see [U] 11.1.10 Prefix commands. `fp` is allowed with the maximum likelihood estimator.

Weights are not allowed with the `bootstrap` prefix; see [R] `bootstrap`.

`vce()`, `first`, `twostep`, and `weights` are not allowed with the `svy` prefix; see [SVY] `svy`.

`fweights`, `iwweights`, and `pweights` are allowed with the maximum likelihood estimator. `fweights` are allowed with Newey's two-step estimator. See [U] 11.1.6 `weight`.

`coeflegend` does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Options for ML estimator

### Model

`ll`[(#)] and `ul`[(#)] indicate the lower and upper limits for censoring, respectively. You may specify one or both. Observations with `devar` ≤ `ll`() are left-censored; observations with `devar` ≥ `ul`() are right-censored; and remaining observations are not censored. You do not have to specify the censoring values at all. It is enough to type `ll`, `ul`, or both. When you do not specify a censoring value, `ivtobit` assumes that the lower limit is the minimum observed in the data (if `ll` is specified) and that the upper limit is the maximum (if `ul` is specified).

`mle` requests that the conditional maximum-likelihood estimator be used. This is the default.

`constraints`(*constraints*); see [R] Estimation options.

### SE/Robust

`vce`(*vcetype*) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster` *clustvar*), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] `vce_option`.

## Reporting

`level(#)`; see [R] [Estimation options](#).

`first` requests that the parameters for the reduced-form equations showing the relationships between the endogenous variables and instruments be displayed. For the two-step estimator, `first` shows the first-stage regressions. For the maximum likelihood estimator, these parameters are estimated jointly with the parameters of the tobit equation. The default is not to show these parameter estimates.

`nocnsreport`; see [R] [Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

## Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#).

Setting the optimization type to `technique(bhhh)` resets the default `vcetype` to `vce(opg)`.

The following option is available with `ivtobit` but is not shown in the dialog box:

`coeflegend`; see [R] [Estimation options](#).

## Options for two-step estimator

## Model

`twostep` is required and requests that Newey's (1987) efficient two-step estimator be used to obtain the coefficient estimates.

`ll[ (#) ]` and `ul[ (#) ]` indicate the lower and upper limits for censoring, respectively. You may specify one or both. Observations with `depvar ≤ ll()` are left-censored; observations with `depvar ≥ ul()` are right-censored; and remaining observations are not censored. You do not have to specify the censoring values at all. It is enough to type `ll`, `ul`, or both. When you do not specify a censoring value, `ivtobit` assumes that the lower limit is the minimum observed in the data (if `ll` is specified) and that the upper limit is the maximum (if `ul` is specified).

## SE

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`twostep`) and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

## Reporting

`level(#)`; see [R] [Estimation options](#).

`first` requests that the parameters for the reduced-form equations showing the relationships between the endogenous variables and instruments be displayed. For the two-step estimator, `first` shows the first-stage regressions. For the maximum likelihood estimator, these parameters are estimated jointly with the parameters of the tobit equation. The default is not to show these parameter estimates.

*display\_options*: `noc1`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

The following option is available with `ivtobit` but is not shown in the dialog box:

`coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

`ivtobit` fits models with censored dependent variables and endogenous covariates. You can use it to fit a tobit model when you suspect that one or more of the covariates is correlated with the error term. `ivtobit` is to tobit what `ivregress` is to linear regression analysis; see [R] [ivregress](#) for more information.

Formally, the model is

$$\begin{aligned} y_{1i}^* &= \mathbf{y}_{2i} \boldsymbol{\beta} + \mathbf{x}_{1i} \boldsymbol{\gamma} + u_i \\ \mathbf{y}_{2i} &= \mathbf{x}_{1i} \boldsymbol{\Pi}_1 + \mathbf{x}_{2i} \boldsymbol{\Pi}_2 + \mathbf{v}_i \end{aligned}$$

where  $i = 1, \dots, N$ ;  $\mathbf{y}_{2i}$  is a  $1 \times p$  vector of endogenous variables;  $\mathbf{x}_{1i}$  is a  $1 \times k_1$  vector of exogenous variables;  $\mathbf{x}_{2i}$  is a  $1 \times k_2$  vector of additional instruments; and the equation for  $\mathbf{y}_{2i}$  is written in reduced form. By assumption,  $(u_i, \mathbf{v}_i) \sim N(\mathbf{0}, \boldsymbol{\Sigma})$ .  $\boldsymbol{\beta}$  and  $\boldsymbol{\gamma}$  are vectors of structural parameters, and  $\boldsymbol{\Pi}_1$  and  $\boldsymbol{\Pi}_2$  are matrices of reduced-form parameters. We do not observe  $y_{1i}^*$ ; instead, we observe

$$y_{1i} = \begin{cases} a & y_{1i}^* < a \\ y_{1i}^* & a \leq y_{1i}^* \leq b \\ b & y_{1i}^* > b \end{cases}$$

The order condition for identification of the structural parameters is that  $k_2 \geq p$ . Presumably,  $\boldsymbol{\Sigma}$  is not block diagonal between  $u_i$  and  $\mathbf{v}_i$ ; otherwise,  $\mathbf{y}_{2i}$  would not be endogenous.

### □ Technical note

This model is derived under the assumption that  $(u_i, \mathbf{v}_i)$  is independent and identically distributed multivariate normal for all  $i$ . The `vce(cluster clustvar)` option can be used to control for a lack of independence. As with the standard tobit model without endogeneity, if  $u_i$  is heteroskedastic, point estimates will be inconsistent.

□

## ► Example 1: Estimation and parameter interpretation

We model the number of hours per week that high school boys spend using social media (`hsocial`). The data collection process caused the observations on the number of hours spent to be censored at 12 hours. A tobit-type model is therefore reasonable for our data.

We model each boy's number of hours spent using social media as a function of whether he has a smartphone (`sphone`), whether he has a computer at home (`computer`), the year in high school in which he is enrolled (`year`), and the hours per week he spends studying (`hstudy`).

We believe that there are unobservable variables that simultaneously affect `hstudy` and `hsocial`, which is to say that `hstudy` is endogenous. Because `hstudy` is endogenous, we must model it as well. Our model for the endogenous `hstudy` always includes the exogenous covariates used to model the outcome `hsocial`. We must also include at least one covariate in the model for the endogenous `hstudy` that was not included in the model for the outcome `hsocial`.

We use `ivtobit` with the default maximum-likelihood estimator to model the endogenous variable `hstudy` as a function of the highest educational degree attained by their parents (`pedu`), the time spent watching television (`tvhours`), and the exogenous covariates used to model `hsocial`.

```
. use https://www.stata-press.com/data/r19/smedia
(Fictional data on hours spent on social media)
. ivtobit hsocial i.sphone i.computer i.year (hstudy = tvhours i.pedu), ul(12)
Fitting exogenous tobit model
Fitting full model
Iteration 0: Log likelihood = -3240.5279
Iteration 1: Log likelihood = -3186.8824
Iteration 2: Log likelihood = -3173.1147
Iteration 3: Log likelihood = -3172.8561
Iteration 4: Log likelihood = -3172.856
Tobit model with endogenous regressors
Limits: Lower = -inf
 Upper = 12
Log likelihood = -3172.856
Number of obs = 1,324
Uncensored = 928
Left-censored = 0
Right-censored = 396
Wald chi2(6) = 11610.73
Prob > chi2 = 0.0000
```

|                              | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |           |
|------------------------------|-------------|-----------|--------|-------|----------------------|-----------|
| hstudy                       | -.9610518   | .0327204  | -29.37 | 0.000 | -1.025183            | -.8969211 |
| 1.sphone                     | 6.041781    | .0625236  | 96.63  | 0.000 | 5.919237             | 6.164325  |
| 1.computer                   | 2.51903     | .0629128  | 40.04  | 0.000 | 2.395723             | 2.642337  |
| year                         |             |           |        |       |                      |           |
| 2                            | .4439009    | .0802309  | 5.53   | 0.000 | .2866513             | .6011505  |
| 3                            | .8574705    | .080476   | 10.65  | 0.000 | .6997404             | 1.015201  |
| 4                            | 1.478215    | .0816582  | 18.10  | 0.000 | 1.318168             | 1.638262  |
| _cons                        | 8.955813    | .2069144  | 43.28  | 0.000 | 8.550268             | 9.361357  |
| corr(e.hstudy,<br>e.hsocial) | .4667975    | .0340342  |        |       | .3975006             | .5308     |
| sd(e.hsocial)                | .9709836    | .0266364  |        |       | .9201559             | 1.024619  |
| sd(e.hstudy)                 | .9701792    | .0188547  |        |       | .9339197             | 1.007847  |

```
Wald test of exogeneity (corr = 0): chi2(1) = 135.19 Prob > chi2 = 0.0000
Endogenous: hstudy
Exogenous: 1.sphone 1.computer 2.year 3.year 4.year tvhours 2.pedu 3.pedu
```

The coefficients in the table tell us how much the linear prediction for the outcome changes when there is a change in a covariate.

Below the table, we see a Wald test for whether the correlation between the residuals from the main equation (predicting `hstudy`) and the residuals from the auxiliary equation (predicting `hsocial`) is 0. The correlation itself is 0.47 and shown in the table as `corr(e.hstudy, e.hsocia1)`. If the test statistic is not significant, there is not sufficient information in the sample to reject the null hypothesis of no endogeneity. In our example, we reject the null hypothesis that supports our choice of a tobit model that accounts for endogeneity.

◀

## □ Technical note

In the tobit model with endogenous covariates, we assume that  $(u_i, v_i)$  is multivariate normal with covariance matrix

$$\text{Var}(u_i, v_i) = \Sigma = \begin{bmatrix} \sigma_u^2 & \Sigma'_{21} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}$$

Using the properties of the multivariate normal distribution,  $\text{Var}(u_i | v_i) \equiv \sigma_{u|v}^2 = \sigma_u^2 - \Sigma'_{21} \Sigma_{22}^{-1} \Sigma_{21}$ . Calculating the marginal effects on the conditional expected values of the observed and latent dependent variables and on the probability of censoring requires an estimate of  $\sigma_{u|v}^2$ . Unlike the default maximum-likelihood estimator, the two-step estimator identifies only  $\sigma_{u|v}^2$ , not  $\sigma_u^2$ , so only the linear prediction and its standard error are available after you have used the `twostep` option.

□

## Stored results

`ivtobit, mle` stores the following in `e()`:

### Scalars

|                            |                                                           |
|----------------------------|-----------------------------------------------------------|
| <code>e(N)</code>          | number of observations                                    |
| <code>e(N_unc)</code>      | number of uncensored observations                         |
| <code>e(N_lc)</code>       | number of left-censored observations                      |
| <code>e(N_rc)</code>       | number of right-censored observations                     |
| <code>e(llopt)</code>      | minimum of <i>depvar</i> or contents of <code>ll()</code> |
| <code>e(ulopt)</code>      | maximum of <i>depvar</i> or contents of <code>ul()</code> |
| <code>e(k)</code>          | number of parameters                                      |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>                  |
| <code>e(k_eq_model)</code> | number of equations in overall model test                 |
| <code>e(k_dv)</code>       | number of dependent variables                             |
| <code>e(df_m)</code>       | model degrees of freedom                                  |
| <code>e(ll)</code>         | log likelihood                                            |
| <code>e(N_clust)</code>    | number of clusters                                        |
| <code>e(endog_ct)</code>   | number of endogenous covariates                           |
| <code>e(p)</code>          | model Wald <i>p</i> -value                                |
| <code>e(p_exog)</code>     | exogeneity test Wald <i>p</i> -value                      |
| <code>e(chi2)</code>       | model Wald $\chi^2$                                       |
| <code>e(chi2_exog)</code>  | Wald $\chi^2$ test of exogeneity                          |
| <code>e(rank)</code>       | rank of <code>e(V)</code>                                 |
| <code>e(ic)</code>         | number of iterations                                      |
| <code>e(rc)</code>         | return code                                               |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise                               |

### Macros

|                         |                      |
|-------------------------|----------------------|
| <code>e(cmd)</code>     | <code>ivtobit</code> |
| <code>e(cmdline)</code> | command as typed     |

|                |                                                                          |
|----------------|--------------------------------------------------------------------------|
| e(depvar)      | name of dependent variable                                               |
| e(endog)       | names of endogenous variables                                            |
| e(exog)        | names of exogenous variables                                             |
| e(wtype)       | weight type                                                              |
| e(wexp)        | weight expression                                                        |
| e(title)       | title in estimation output                                               |
| e(clustvar)    | name of cluster variable                                                 |
| e(chi2type)    | Wald; type of model $\chi^2$ test                                        |
| e(vce)         | <i>vcetype</i> specified in <code>vce()</code>                           |
| e(vcetype)     | title used to label Std. err.                                            |
| e(method)      | m1                                                                       |
| e(opt)         | type of optimization                                                     |
| e(which)       | max or min; whether optimizer is to perform maximization or minimization |
| e(m1_method)   | type of m1 method                                                        |
| e(user)        | name of likelihood-evaluator program                                     |
| e(technique)   | maximization technique                                                   |
| e(properties)  | b V                                                                      |
| e(estat_cmd)   | program used to implement <code>estat</code>                             |
| e(predict)     | program used to implement <code>predict</code>                           |
| e(footnote)    | program used to implement the footnote display                           |
| e(marginsok)   | predictions allowed by <code>margins</code>                              |
| e(marginsprop) | signals to the <code>margins</code> command                              |
| e(asbalanced)  | factor variables <code>fvset</code> as <code>asbalanced</code>           |
| e(asobserved)  | factor variables <code>fvset</code> as <code>asobserved</code>           |

## Matrices

|                 |                                              |
|-----------------|----------------------------------------------|
| e(b)            | coefficient vector                           |
| e(Cns)          | constraints matrix                           |
| e(ilog)         | iteration log (up to 20 iterations)          |
| e(gradient)     | gradient vector                              |
| e(Sigma)        | $\Sigma$                                     |
| e(V)            | variance-covariance matrix of the estimators |
| e(V_modelbased) | model-based variance                         |

## Functions

|           |                         |
|-----------|-------------------------|
| e(sample) | marks estimation sample |
|-----------|-------------------------|

In addition to the above, the following is stored in `r()`:

## Matrices

|          |                                                                                                                            |
|----------|----------------------------------------------------------------------------------------------------------------------------|
| r(table) | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |
|----------|----------------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

`ivtobit`, `twostep` stores the following in `e()`:

## Scalars

|            |                                                    |
|------------|----------------------------------------------------|
| e(N)       | number of observations                             |
| e(N_unc)   | number of uncensored observations                  |
| e(N_lc)    | number of left-censored observations               |
| e(N_rc)    | number of right-censored observations              |
| e(llopt)   | contents of <code>ll()</code>                      |
| e(ulopt)   | contents of <code>ul()</code>                      |
| e(df_m)    | model degrees of freedom                           |
| e(df_exog) | degrees of freedom for $\chi^2$ test of exogeneity |
| e(p)       | model Wald <i>p</i> -value                         |
| e(p_exog)  | exogeneity test Wald <i>p</i> -value               |
| e(chi2)    | model Wald $\chi^2$                                |

|                 |                                                                |
|-----------------|----------------------------------------------------------------|
| e(chi2_exog)    | Wald $\chi^2$ test of exogeneity                               |
| e(rank)         | rank of $e(V)$                                                 |
| Macros          |                                                                |
| e(cmd)          | ivtobit                                                        |
| e(cmdline)      | command as typed                                               |
| e(depvar)       | name of dependent variable                                     |
| e(endog)        | names of endogenous variables                                  |
| e(exog)         | names of exogenous variables                                   |
| e(wtype)        | weight type                                                    |
| e(wexp)         | weight expression                                              |
| e(chi2type)     | Wald; type of model $\chi^2$ test                              |
| e(vce)          | <i>vce</i> type specified in <code>vce()</code>                |
| e(method)       | twostep                                                        |
| e(properties)   | b V                                                            |
| e(estat_cmd)    | program used to implement <code>estat</code>                   |
| e(predict)      | program used to implement <code>predict</code>                 |
| e(footnote)     | program used to implement the footnote display                 |
| e(marginsok)    | predictions allowed by <code>margins</code>                    |
| e(marginsprop)  | signals to the <code>margins</code> command                    |
| e(asbalanced)   | factor variables <code>fvset</code> as <code>asbalanced</code> |
| e(asobserved)   | factor variables <code>fvset</code> as <code>asobserved</code> |
| Matrices        |                                                                |
| e(b)            | coefficient vector                                             |
| e(Cns)          | constraints matrix                                             |
| e(V)            | variance–covariance matrix of the estimators                   |
| e(V_modelbased) | model-based variance                                           |
| Functions       |                                                                |
| e(sample)       | marks estimation sample                                        |

In addition to the above, the following is stored in `r()`:

|                       |                                                                                                                            |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|
| Matrices              |                                                                                                                            |
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

## Methods and formulas

The estimation procedure used by `ivtobit` is similar to that used by `ivprobit`. For compactness, we write the model as

$$y_{1i}^* = z_i \delta + u_i$$

$$y_{2i} = x_i \Pi + v_i$$

where  $z_i = (y_{2i}, x_{1i})$ ,  $x_i = (x_{1i}, x_{2i})$ ,  $\delta = (\beta', \gamma')'$ , and  $\Pi = (\Pi_1', \Pi_2')'$ . We do not observe  $y_{1i}^*$ ; instead, we observe

$$y_{1i} = \begin{cases} a & y_{1i}^* < a \\ y_{1i}^* & a \leq y_{1i}^* \leq b \\ b & y_{1i}^* > b \end{cases}$$

$(u_i, v_i)$  is distributed multivariate normal with mean zero and covariance matrix

$$\Sigma = \begin{bmatrix} \sigma_u^2 & \Sigma'_{21} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}$$

Using the properties of the multivariate normal distribution, we can write  $u_i = \mathbf{v}'_i \boldsymbol{\alpha} + \epsilon_i$ , where  $\boldsymbol{\alpha} = \boldsymbol{\Sigma}_{22}^{-1} \boldsymbol{\Sigma}_{21}$ ;  $\epsilon_i \sim N(0; \sigma_{u|v}^2)$ , where  $\sigma_{u|v}^2 = \sigma_u^2 - \boldsymbol{\Sigma}'_{21} \boldsymbol{\Sigma}_{22}^{-1} \boldsymbol{\Sigma}_{21}$ ; and  $\epsilon_i$  is independent of  $\mathbf{v}_i$ ,  $\mathbf{z}_i$ , and  $\mathbf{x}_i$ .

The likelihood function is straightforward to derive because we can write the joint density  $f(y_{1i}, \mathbf{y}_{2i} | \mathbf{x}_i)$  as  $f(y_{1i} | \mathbf{y}_{2i}, \mathbf{x}_i) f(\mathbf{y}_{2i} | \mathbf{x}_i)$ . We have that

$$\ln f(\mathbf{y}_{2i} | \mathbf{x}_i) = -\frac{1}{2} (p \ln 2\pi + \ln |\boldsymbol{\Sigma}_{22}| + \mathbf{v}'_i \boldsymbol{\Sigma}_{22}^{-1} \mathbf{v}_i)$$

and

$$\ln f(y_{1i} | \mathbf{y}_{2i}, \mathbf{x}_i) = \begin{cases} \ln \left\{ 1 - \Phi \left( \frac{m_i - a}{\sigma_{u|v}} \right) \right\} & y_{1i} = a \\ -\frac{1}{2} \left\{ \ln 2\pi + \ln \sigma_{u|v}^2 + \frac{(y_{1i} - m_i)^2}{\sigma_{u|v}^2} \right\} & a < y_{1i} < b \\ \ln \Phi \left( \frac{m_i - b}{\sigma_{u|v}} \right) & y_{1i} = b \end{cases}$$

where

$$m_i = \mathbf{z}_i \boldsymbol{\delta} + (\mathbf{y}_{2i} - \mathbf{x}_i \boldsymbol{\Pi}) \boldsymbol{\Sigma}_{22}^{-1} \boldsymbol{\Sigma}_{21}$$

and  $\Phi(\cdot)$  is the normal distribution function so that the log likelihood for observation  $i$  is

$$\ln L_i = w_i \{ \ln f(y_{1i} | \mathbf{y}_{2i}, \mathbf{x}_i) + \ln f(\mathbf{y}_{2i} | \mathbf{x}_i) \}$$

where  $w_i$  is the weight for observation  $i$  or one if no weights were specified. Instead of estimating  $\sigma_{u|v}$  and  $\sigma_v$  directly, we estimate  $\ln \sigma_{u|v}$  and  $\ln \sigma_v$ .

With maximum likelihood estimation, this command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] [\\_robust](#), particularly [Maximum likelihood estimators](#) and [Methods and formulas](#).

The maximum likelihood version of `ivtobit` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] [Variance estimation](#).

The two-step estimates are obtained using Newey's (1987) minimum  $\chi^2$  estimator. For more details on the minimum  $\chi^2$  estimator, see [R] [ivprobit](#).

## Acknowledgments

The two-step estimator is based on the `tobitiv` command written by Jonah Gelbach of the University of California at Berkeley Law School and the `ivtobit` command written by Joe Harkness of the University of Connecticut.

## References

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- Miranda, A., and S. Rabe-Hesketh. 2006. [Maximum likelihood estimation of endogenous switching and sample selection models for binary, ordinal, and count variables](#). *Stata Journal* 6: 285–308.
- Newey, W. K. 1987. [Efficient estimation of limited dependent variable models with endogenous explanatory variables](#). *Journal of Econometrics* 36: 231–250. [https://doi.org/10.1016/0304-4076\(87\)90001-7](https://doi.org/10.1016/0304-4076(87)90001-7).

## Also see

- [R] **ivtobit postestimation** — Postestimation tools for ivtobit
  - [R] **gmm** — Generalized method of moments estimation
  - [R] **ivprobit** — Probit model with continuous endogenous covariates
  - [R] **ivregress** — Single-equation instrumental-variables regression
  - [R] **regress** — Linear regression
  - [R] **tobit** — Tobit regression
  - [ERM] **eintreg** — Extended interval regression
  - [SVY] **svy estimation** — Estimation commands for survey data
  - [XT] **xtintreg** — Random-effects interval-data regression model
  - [XT] **xttobit** — Random-effects tobit model
- [U] **20 Estimation and postestimation commands**

|                         |                      |                      |                    |
|-------------------------|----------------------|----------------------|--------------------|
| Postestimation commands | <code>predict</code> | <code>margins</code> | <code>estat</code> |
| Remarks and examples    | Stored results       | Methods and formulas | References         |
| Also see                |                      |                      |                    |

## Postestimation commands

The following postestimation commands are of special interest after `ivtobit`:

| Command                        | Description                                                                                        |
|--------------------------------|----------------------------------------------------------------------------------------------------|
| <code>estat correlation</code> | report the correlation matrix of the errors of the dependent variable and the endogenous variables |
| <code>estat covariance</code>  | report the covariance matrix of the errors of the dependent variable and the endogenous variables  |

These commands are not appropriate after the two-step estimator or with `svy` estimation results.

The following standard postestimation commands are also available:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| * <code>estat ic</code>      | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * † <code>forecast</code>    | dynamic forecasts and simulations                                                                                                       |
| † <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| * † <code>lrtest</code>      | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | linear predictions and their SEs, probabilities, etc.                                                                                   |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| * <code>suest</code>         | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`estat ic`, `forecast`, `lrtest`, and `suest` are not appropriate after `ivtobit`, `twostep`.

† `forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as structural functions, linear predictions, standard errors, probabilities, and expected values.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

After ML

```
predict [type] newvar [if] [in] [, statistic asfmethod]
```

```
predict [type] { stub* | newvarlist } [if] [in] , scores
```

After twostep

```
predict [type] newvar [if] [in] [, twostep_statistic]
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

|                         |                                                                        |
|-------------------------|------------------------------------------------------------------------|
| Main                    |                                                                        |
| <code>xb</code>         | linear prediction excluding endogeneity; the default                   |
| <code>mean</code>       | linear prediction accounting for endogeneity                           |
| <code>stdp</code>       | standard error of the linear prediction                                |
| <code>stdf</code>       | standard error of the forecast                                         |
| <code>pr(a,b)</code>    | $\Pr(a < y_j < b)$ accounting for endogeneity                          |
| <code>e(a,b)</code>     | $E(y_j   a < y_j < b)$ accounting for endogeneity                      |
| <code>ystar(a,b)</code> | $E(y_j^*, y_j^* = \max\{a, \min(y_j, b)\})$ accounting for endogeneity |

`stdf` is not allowed with `svy` estimation results.

where  $a$  and  $b$  may be numbers or variables;  $a$  missing ( $a \geq .$ ) means  $-\infty$ , and  $b$  missing ( $b \geq .$ ) means  $+\infty$ ; see [U] 12.2.1 Missing values.

| <i>asfmethod</i> | Description |
|------------------|-------------|
|------------------|-------------|

|                       |                                          |
|-----------------------|------------------------------------------|
| Main                  |                                          |
| <code>asf</code>      | average structural function; the default |
| <code>fixedasf</code> | fixed average structural function        |

| <i>twostep_statistic</i> | Description |
|--------------------------|-------------|
|--------------------------|-------------|

|                   |                                         |
|-------------------|-----------------------------------------|
| Main              |                                         |
| <code>xb</code>   | linear prediction; the default          |
| <code>stdp</code> | standard error of the linear prediction |

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

## Options for predict

Main

`xb`, the default, calculates the linear prediction.

`mean` calculates the linear prediction. Results depend on how the endogeneity complication is handled, which is determined by the `asf` or `fixedasf` option. `mean` is not available with the two-step estimator.

`stdp` calculates the standard error of the linear prediction. It can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. The standard error of the prediction is also referred to as the standard error of the fitted value.

`stdf` calculates the standard error of the forecast, which is the standard error of the point prediction for 1 observation. It is commonly referred to as the standard error of the future or forecast value. By construction, the standard errors produced by `stdf` are always larger than those produced by `stdp`; see *Methods and formulas* in [R] **regress postestimation**.

`pr(a,b)` calculates  $\Pr(a < y_j < b | \mathbf{z}_j)$ , the probability that  $y_j | \mathbf{z}_j$  would be observed in the interval  $(a, b)$ .

$a$  and  $b$  may be specified as numbers or variable names;  $lb$  and  $ub$  are variable names;

`pr(20,30)` calculates  $\Pr(20 < y_j < 30 | \mathbf{z}_j)$ ;

`pr(lb,ub)` calculates  $\Pr(lb < y_j < ub | \mathbf{z}_j)$ ; and

`pr(20,ub)` calculates  $\Pr(20 < y_j < ub | \mathbf{z}_j)$ .

$a$  missing ( $a \geq .$ ) means  $-\infty$ ; `pr(. ,30)` calculates  $\Pr(-\infty < y_j < 30 | \mathbf{z}_j)$ ;

`pr(lb,30)` calculates  $\Pr(-\infty < y_j < 30 | \mathbf{z}_j)$  in observations for which  $lb \geq .$

and calculates  $\Pr(lb < y_j < 30 | \mathbf{z}_j)$  elsewhere.

$b$  missing ( $b \geq .$ ) means  $+\infty$ ; `pr(20, .)` calculates  $\Pr(+\infty > y_j > 20 | \mathbf{z}_j)$ ;

`pr(20,ub)` calculates  $\Pr(+\infty > y_j > 20 | \mathbf{z}_j)$  in observations for which  $ub \geq .$

and calculates  $\Pr(20 < y_j < ub | \mathbf{z}_j)$  elsewhere.

Results depend on how the endogeneity complication is handled, which is determined by the `asf` or `fixedasf` option.

`pr(a,b)` is not available with the two-step estimator.

`e(a,b)` calculates  $E(y_j | a < y_j < b)$ , the expected value of  $y_j | \mathbf{z}_j$  conditional on  $y_j | \mathbf{z}_j$  being in the interval  $(a, b)$ , meaning that  $y_j | \mathbf{z}_j$  is truncated.  $a$  and  $b$  are specified as they are for `pr()`. Results depend on how the endogeneity complication is handled, which is determined by the `asf` or `fixedasf` option. `e(a,b)` is not available with the two-step estimator.

`ystar(a,b)` calculates  $E(y_j^*)$ , where  $y_j^* = a$  if  $\mathbf{z}_i \boldsymbol{\delta} + u_j \leq a$ ,  $y_j^* = b$  if  $\mathbf{z}_i \boldsymbol{\delta} + u_j \geq b$ , and  $y_j^* = \mathbf{z}_i \boldsymbol{\delta} + u_j$  otherwise, meaning that  $y_j^*$  is censored.  $a$  and  $b$  are specified as they are for `pr()`. Results depend on how the endogeneity complication is handled, which is determined by the `asf` or `fixedasf` option.

`ystar(a,b)` is not available with the two-step estimator.

`asf` and `fixedasf` determine how the average structural function (ASF) of the specified statistic is computed. These options are not allowed with `xb`, `stdp`, or `stdf`.

`asf` is the default for the ML estimator when the mean,  $\text{pr}(a, b)$ ,  $e(a, b)$ , or  $\text{ystar}(a, b)$  statistic is specified. `asf` computes the ASF of the specified statistic. It is the statistic conditional on the errors of the endogenous variable equations. Put another way, it is the statistic accounting for the correlation of the endogenous covariates with the errors of the main equation. Derivatives and contrasts based on `asf` have a structural interpretation. See `margins` for computing derivatives and contrasts.

`fixedasf` calculates a fixed ASF. It is the specified statistic using only the coefficients and variables of the outcome equation. `fixedasf` does not condition on the errors of the endogenous variable equations. Contrasts between two fixed counterfactuals averaged over the whole sample have a potential-outcome interpretation. Intuitively, it is as if the values of the covariates were fixed at a value exogenously by fiat. See `margins` for computing derivatives and contrasts.

To be clear, derivatives and contrasts between two fixed counterfactuals using the default `asf` option also have a potential-outcome interpretation. And, unlike `fixedasf`, they retain that interpretation when computed over subpopulations for both linear and nonlinear models.

`scores`, not available with `twostep`, calculates equation-level score variables.

For models with one endogenous regressor, five new variables are created.

The first new variable will contain  $\partial \ln L / \partial (z_i \delta)$ .

The second new variable will contain  $\partial \ln L / \partial (x_i \mathbf{\Pi})$ .

The third new variable will contain  $\partial \ln L / \partial \alpha$ .

The fourth new variable will contain  $\partial \ln L / \partial \ln \sigma_{u|v}$ .

The fifth new variable will contain  $\partial \ln L / \partial \ln \sigma_v$ .

For models with  $p$  endogenous regressors,  $p + \{(p + 1)(p + 2)\} / 2 + 1$  new variables are created.

The first new variable will contain  $\partial \ln L / \partial (z_i \delta)$ .

The second through  $(p + 1)$ th new score variables will contain  $\partial \ln L / \partial (x_i \mathbf{\Pi}_k)$ ,  $k = 1, \dots, p$ , where  $\mathbf{\Pi}_k$  is the  $k$ th column of  $\mathbf{\Pi}$ .

The remaining score variables will contain the partial derivatives of  $\ln L$  with respect to the  $(p + 1)(p + 2) / 2$  ancillary parameters.

# margins

## Description for margins

`margins` estimates margins of response for linear predictions, probabilities, and expected values.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

### After ML

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

|                                                  |                                                                        |
|--------------------------------------------------|------------------------------------------------------------------------|
| Main                                             |                                                                        |
| <code>xb</code>                                  | linear prediction excluding endogeneity; the default                   |
| <code>mean</code>                                | linear prediction accounting for endogeneity                           |
| <code>stdp</code>                                | not allowed with margins                                               |
| <code>stdf</code>                                | not allowed with margins                                               |
| <code>pr(<i>a</i>,<i>b</i>)</code>               | $\Pr(a < y_j < b)$ accounting for endogeneity                          |
| <code>e(<i>a</i>,<i>b</i>)</code>                | $E(y_j   a < y_j < b)$ accounting for endogeneity                      |
| <code>y<sub>star</sub>(<i>a</i>,<i>b</i>)</code> | $E(y_j^*, y_j^* = \max\{a, \min(y_j, b)\})$ accounting for endogeneity |

### After twostep

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

|                   |                                |
|-------------------|--------------------------------|
| Main              |                                |
| <code>xb</code>   | linear prediction; the default |
| <code>stdp</code> | not allowed with margins       |

Statistics not allowed with margins are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [\[R\] margins](#).

## estat

### Description for estat

`estat correlation` displays the correlation matrix of the errors of the dependent variable and the endogenous variables.

`estat covariance` displays the covariance matrix of the errors of the dependent variable and the endogenous variables.

`estat correlation` and `estat covariance` are not allowed after the `ivprobit` two-step estimator.

### Menu for estat

Statistics > Postestimation

### Syntax for estat

*Correlation matrix*

```
estat correlation [, border(bspec) left(#) format(%fmt)]
```

*Covariance matrix*

```
estat covariance [, border(bspec) left(#) format(%fmt)]
```

### Options for estat

Main

`border`(*bspec*) sets border style of the matrix display. The default is `border(a11)`.

`left`(#) sets the left indent of the matrix display. The default is `left(2)`.

`format`(%*fmt*) specifies the format for displaying the individual elements of the matrix. The default is `format(%9.0g)`.

## Remarks and examples

Remarks are presented under the following headings:

*Marginal effects*

*Obtaining predicted values*

### Marginal effects

Below, we discuss the interpretation of predictions with the `asf` and `fixedasf` options for the ML estimator using `margins`.

The model is defined by two equations. The first is the equation for the outcome of interest, given by

$$y_{1i}^* = \mathbf{y}_{2i}\boldsymbol{\beta} + \mathbf{x}_{1i}\boldsymbol{\gamma} + u_i$$

where we do not observe  $y_{1i}^*$ ; instead, we observe

$$y_{1i} = \begin{cases} a & y_{1i}^* < a \\ y_{1i}^* & a \leq y_{1i}^* \leq b \\ b & y_{1i}^* > b \end{cases}$$

The second is the equation for the endogenous covariates,  $\mathbf{y}_{2i}$ ,

$$\mathbf{y}_{2i} = \mathbf{x}_{1i}\boldsymbol{\Pi}_1 + \mathbf{x}_{2i}\boldsymbol{\Pi}_2 + \mathbf{v}_i$$

This last equation is the difference between a standard tobit model and the model fit by `ivtobit`.  $\mathbf{y}_{2i}$  is modeled by an exogenous component,  $\mathbf{x}_{1i}\boldsymbol{\Pi}_1 + \mathbf{x}_{2i}\boldsymbol{\Pi}_2$ , and a component that is correlated with  $u_i$  and causes the endogeneity problem,  $\mathbf{v}_i$ . The ASF linear prediction conditions on an estimate of  $\hat{\mathbf{v}}_i$ . It is given by

$$\begin{aligned} \widehat{m}_i &= \widehat{E}(y_{1i} | \mathbf{x}_{1i}, \mathbf{x}_{2i}, y_{2i}, \hat{\mathbf{v}}_i) \\ \widehat{m}_i &= \mathbf{y}_{2i}\widehat{\boldsymbol{\theta}}_1 + \mathbf{x}_{1i}\widehat{\boldsymbol{\theta}}_2 + \hat{\mathbf{v}}_i\widehat{\boldsymbol{\theta}}_3 \end{aligned}$$

Because the correlation between  $\mathbf{v}_i$  and  $u_i$  is the problem we intended to address, conditioning on  $\mathbf{v}_i$  purges the model of endogeneity. Using the ASF, we can get derivatives and contrast. See [Wooldridge \(2010\)](#) and [Blundell and Powell \(2003\)](#) for an in-depth discussion of ASFs and their interpretation.

The fixed ASF, estimated when the `fixedasf` option is specified, has a different interpretation. Suppose we wanted to analyze  $\mathbf{y}_{2i}\beta + \mathbf{x}_{1i}\gamma + u_i$  at two different values of  $\mathbf{y}_2$ , the original  $\mathbf{y}_2$  and  $\mathbf{y}_2 + 1$ . We want the average difference at these two points for the given values of the other covariates. The values of the covariates are not arrived at via the model; they are fixed by fiat. You can think of them as potential outcomes. The difference of the two values of  $\mathbf{y}_2$  is given by

$$\{(\mathbf{y}_{2i} + 1)\beta + \mathbf{x}_{1i}\gamma + u_i\} - (\mathbf{y}_{2i}\beta + \mathbf{x}_{1i}\gamma + u_i)$$

If we average over the distribution of  $u$ , we obtain

$$E\{(\mathbf{y}_{2i} + 1)\beta + \mathbf{x}_{1i}\gamma\} - E(\mathbf{y}_{2i}\beta + \mathbf{x}_{1i}\gamma)$$

We do not account for endogeneity because the values of the covariates are given and fixed. If the research question you are pursuing after fitting the model has this interpretation, `fixedasf` gives you an adequate prediction. If, however, you do not want to treat the covariates as fixed, you should account for endogeneity and use `asf` predictions.

### ▷ Example 1: Obtaining marginal effects

We can obtain average marginal effects by using the `margins` command after `ivtobit`. For the social-media model of [example 1](#) in [\[R\] ivtobit](#), suppose that we wanted to know the average marginal effects on the probability of spending more than 12 hours using social media. Average marginal effects are equivalent to estimating how a change in a covariate affects every individual in our sample and taking the average of these effects. The effect of each covariate is estimated with all other covariates kept at their observed values.

```
. use https://www.stata-press.com/data/r19/smedia
(Fictional data on hours spent on social media)
. ivtobit hsocial i.sphone i.computer i.year (hstudy = tvhours i.pedu), ul(12)
(output omitted)
. margins, dydx(*) predict(p(12, .))
Average marginal effects Number of obs = 1,324
Model VCE: OIM
Expression: Pr(hsocial>12), predict(p(12, .))
dy/dx wrt: hstudy 1.sphone 1.computer 2.year 3.year 4.year
```

|            | Delta-method |           |        | z     | P> z      | [95% conf. interval] |  |
|------------|--------------|-----------|--------|-------|-----------|----------------------|--|
|            | dy/dx        | std. err. |        |       |           |                      |  |
| hstudy     | -.1508219    | .0044544  | -33.86 | 0.000 | -.1595523 | -.1420914            |  |
| 1.sphone   | .3943071     | .0085181  | 46.29  | 0.000 | .3776119  | .4110022             |  |
| 1.computer | .3993489     | .0082484  | 48.42  | 0.000 | .3831824  | .4155154             |  |
| year       |              |           |        |       |           |                      |  |
| 2          | .0663908     | .0120746  | 5.50   | 0.000 | .042725   | .0900567             |  |
| 3          | .1334667     | .012488   | 10.69  | 0.000 | .1089908  | .1579426             |  |
| 4          | .2343393     | .0123729  | 18.94  | 0.000 | .2100888  | .2585897             |  |

Note: dy/dx for factor levels is the discrete change from the base level.

Having a smartphone increases the probability of spending more than 12 hours on social media by 0.39, on average. Any additional study time decreases the probability of spending more than 12 hours using social media by 0.15, on average. The other average marginal effects are interpreted similarly. All effects above have a structural interpretation because we are conditioning on the level of endogeneity. See Wooldridge (2010) and Blundell and Powell (2003) for an in-depth discussion of ASFs and their interpretation.



## Obtaining predicted values

After fitting your model with `ivtobit`, you can obtain the linear prediction and its standard error for both the estimation sample and other samples by using the `predict` command. If you used the ML estimator, you can also obtain the linear prediction, the conditional expected values of the observed and latent dependent variables, and the probability of observing the dependent variable in a specified interval—each of these can be computed with an ASF or a fixed ASF interpretation. In addition, with the ML estimator, you can obtain the standard error of the forecast. See [U] 20 Estimation and postestimation commands and [R] `predict`.

## Stored results

`estat correlation` stores the following results in `r()`:

Matrices

`r(corr)` correlation matrix of the errors

`estat covariance` stores the following results in `r()`:

Matrices

`r(cov)` covariance matrix of the errors

## Methods and formulas

The linear prediction is calculated as  $z_i \hat{\delta}$ , where  $\hat{\delta}$  is the estimated value of  $\delta$ , and  $z_i$  and  $\delta$  are defined in (1a) of [R] `ivtobit`. Expected values and probabilities are calculated using the same formulas as those used by the standard tobit model. However, when we use the default `asf` option with `mean`, `pr(a, b)`, `e(a, b)`, or `ystar(a, b)`, instead of evaluating the standard normal cumulative density and probability density at the linear prediction, we evaluate expected values and probabilities at  $\hat{m}_i$ , where  $\hat{m}_i$  is defined in *Methods and formulas* of [R] `ivtobit`. Using  $\hat{m}_i$  instead of  $z_i \hat{\delta}$  in the formulas produces the ASF, which accounts for endogeneity. The fixed ASF, obtained with the `fixedasf` option, evaluates the statistic at  $z_i \hat{\delta}$ .

## References

- Blundell, R. W., and J. L. Powell. 2003. "Endogeneity in nonparametric and semiparametric regression models". In *Theory and Applications: Eighth World Congress*. Advances in Economics and Econometrics, edited by M. Dewatripont, L. P. Hansen, and S. J. Turnovsky, vol. 2: 312–357. Cambridge: Cambridge University Press. <https://doi.org/10.1017/CBO9780511610257.011>.
- Wooldridge, J. M. 2010. *Econometric Analysis of Cross Section and Panel Data*. 2nd ed. Cambridge, MA: MIT Press.

## Also see

[R] [ivtobit](#) — Tobit model with continuous endogenous covariates

[U] [20 Estimation and postestimation commands](#)

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

## Description

`jackknife` performs jackknife estimation of the specified statistics (or expressions) for a Stata command or a user-written program. Statistics are jackknifed by estimating the command once for each observation or cluster in the dataset, leaving the associated observation or cluster out of the calculations. `jackknife` is designed for use with nonestimation commands, functions of coefficients, or user-written programs. To jackknife coefficients, we recommend using the `vce(jackknife)` option when allowed by the estimation command.

## Quick start

Jackknife estimate of the standard deviation of `v1` returned by `summarize` in `r(sd)`

```
jackknife sd=r(sd), rclass: summarize v1
```

Jackknife estimate of the statistic `r(mystat)` returned by `rclass` program `myprog1` that returns the sample size in `r(N)`

```
jackknife stat=r(mystat), rclass: myprog1 v1
```

Same as above, and save the results from each replication in `mydata.dta`

```
jackknife stat=r(mystat), rclass saving(mydata): myprog1 v1
```

Jackknife estimate of a difference in coefficients estimated by `regress`

```
jackknife diff=(_b[x2]-_b[x1]): regress y x1 x2 x3
```

Jackknife estimate of the statistic `e(mystat)` returned by `eclass` program `myprog2` that returns the sample size in `e(N)`

```
jackknife stat=e(mystat), eclass: myprog2 y x1 x2 x3
```

Jackknife estimates of coefficients stored in `e(b)` by `myprog2`

```
jackknife _b, eclass: myprog2 y x1 x2 x3
```

Add variables containing the pseudovalues of the coefficients to the dataset

```
jackknife _b, eclass keep: myprog2 y x1 x2 x3
```

## Menu

Statistics > Resampling > Jackknife estimation

## Syntax

jackknife *exp\_list* [ , *options eform\_option* ] : *command*

| <i>options</i>                            | Description                                                                                                                                            |
|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Main</b>                               |                                                                                                                                                        |
| <code>eclass</code>                       | number of observations used is stored in <code>e(N)</code>                                                                                             |
| <code>rclass</code>                       | number of observations used is stored in <code>r(N)</code>                                                                                             |
| <code>n(<i>exp</i>)</code>                | specify <i>exp</i> that evaluates to the number of observations used                                                                                   |
| <b>Options</b>                            |                                                                                                                                                        |
| <code>cluster(<i>varlist</i>)</code>      | variables identifying sample clusters                                                                                                                  |
| <code>idcluster(<i>newvar</i>)</code>     | create new cluster ID variable                                                                                                                         |
| <code>saving(<i>filename</i>, ...)</code> | save results to <i>filename</i> ; save statistics in double precision;<br>save results to <i>filename</i> every # replications                         |
| <code>keep</code>                         | keep pseudovalues                                                                                                                                      |
| <code>mse</code>                          | use MSE formula for variance estimation                                                                                                                |
| <b>Reporting</b>                          |                                                                                                                                                        |
| <code>level(#)</code>                     | set confidence level; default is <code>level(95)</code>                                                                                                |
| <code>notable</code>                      | suppress table of results                                                                                                                              |
| <code>noheader</code>                     | suppress table header                                                                                                                                  |
| <code>nolegend</code>                     | suppress table legend                                                                                                                                  |
| <code>verbose</code>                      | display the full table legend                                                                                                                          |
| <code>nodots</code>                       | suppress replication dots                                                                                                                              |
| <code>dots(#)</code>                      | display dots every # replications                                                                                                                      |
| <code>noisily</code>                      | display any output from <i>command</i>                                                                                                                 |
| <code>trace</code>                        | trace <i>command</i>                                                                                                                                   |
| <code>title(<i>text</i>)</code>           | use <i>text</i> as title for jackknife results                                                                                                         |
| <code>display_options</code>              | control columns and column formats, row spacing, line width,<br>display of omitted variables and base and empty cells, and<br>factor-variable labeling |
| <code>eform_option</code>                 | display coefficient table in exponentiated form                                                                                                        |
| <b>Advanced</b>                           |                                                                                                                                                        |
| <code>nodrop</code>                       | do not drop observations                                                                                                                               |
| <code>reject(<i>exp</i>)</code>           | identify invalid results                                                                                                                               |
| <code>coeflegend</code>                   | display legend instead of statistics                                                                                                                   |

*command* is any command that follows standard Stata syntax. All weight types supported by *command* are allowed except `aweight`s; see [\[U\] 11.1.6 weight](#).

`collect` and `svy` are allowed; see [\[U\] 11.1.10 Prefix commands](#).

`coeflegend` does not appear in the dialog box.

See [\[U\] 20 Estimation and postestimation commands](#) for more capabilities of estimation commands.

*exp\_list* contains     (*name*: *elist*)  
                           *elist*  
                           *eexp*

*elist* contains        *newvar* = (*exp*)  
                           (*exp*)

*eexp* is                *specname*  
                           [*eqno*]*specname*

*specname* is         \_b  
                           \_b []  
                           \_se  
                           \_se []

*eqno* is              ##  
                           *name*

*exp* is a standard Stata expression; see [U] 13 Functions and expressions.

Distinguish between [], which are to be typed, and [], which indicate optional arguments.

## Options

Main

*e*class, *r*class, and *n*(*exp*) specify where *command* stores the number of observations on which it based the calculated results. We strongly advise you to specify one of these options.

*e*class specifies that *command* store the number of observations in *e*(*N*).

*r*class specifies that *command* store the number of observations in *r*(*N*).

*n*(*exp*) specifies an expression that evaluates to the number of observations used. Specifying *n*(*r*(*N*)) is equivalent to specifying the *r*class option. Specifying *n*(*e*(*N*)) is equivalent to specifying the *e*class option. If *command* stores the number of observations in *r*(*N*1), specify *n*(*r*(*N*1)).

If you specify no options, *jackknife* will assume *e*class or *r*class, depending on which of *e*(*N*) and *r*(*N*) is not missing (in that order). If both *e*(*N*) and *r*(*N*) are missing, *jackknife* assumes that all observations in the dataset contribute to the calculated result. If that assumption is incorrect, the reported standard errors will be incorrect. For instance, say that you specify

```
. jackknife coef=_b[x2]: myreg y x1 x2 x3
```

where *myreg* uses *e*(*n*) instead of *e*(*N*) to identify the number of observations used in calculations. Further assume that observation 42 in the dataset has *x*3 equal to missing. The 42nd observation plays no role in obtaining the estimates, but *jackknife* has no way of knowing that and will use the wrong *N*. If, on the other hand, you specify

```
. jackknife coef=_b[x2], n(e(n)): myreg y x1 x2 x3
```

*jackknife* will notice that observation 42 plays no role. The *n*(*e*(*n*)) option is specified because *myreg* is an estimation command but it stores the number of observations used in *e*(*n*) (instead of the standard *e*(*N*)). When *jackknife* runs the regression omitting the 42nd observation, *jackknife* will observe that *e*(*n*) has the same value as when *jackknife* previously ran the regression using all the observations. Thus *jackknife* will know that *myreg* did not use the observation.

## Options

`cluster(varlist)` specifies the variables identifying sample clusters. If `cluster()` is specified, one cluster is left out of each call to *command*, instead of 1 observation.

`idcluster(newvar)` creates a new variable containing a unique integer identifier for each resampled cluster, starting at 1 and leading up to the number of clusters. This option may be specified only when the `cluster()` option is specified. `idcluster()` helps identify the cluster to which a pseudo-value belongs.

`saving(filename[ , suboptions ])` creates a Stata data file (.dta file) consisting of (for each statistic in *exp-list*) a variable containing the replicates.

`double` specifies that the results for each replication be saved as doubles, meaning 8-byte reals. By default, they are saved as floats, meaning 4-byte reals. This option may be used without the `saving()` option to compute the variance estimates by using double precision.

`every(#)` specifies that results be written to disk every #th replication. `every()` should be specified only in conjunction with `saving()` when *command* takes a long time for each replication. This option will allow recovery of partial results should some other software crash your computer. See [P] [postfile](#).

`replace` specifies that *filename* be overwritten if it exists. This option does not appear in the dialog box.

`keep` specifies that new variables be added to the dataset containing the pseudo-values of the requested statistics. For instance, if you typed

```
. jackknife coef=_b[x2], eclass keep: regress y x1 x2 x3
```

new variable `coef` would be added to the dataset containing the pseudo-values for `_b[x2]`. Let  $b$  be the value of `_b[x2]` when all observations are used to fit the model, and let  $b(j)$  be the value when the  $j$ th observation is omitted. The pseudo-values are defined as

$$\text{pseudo-value}_j = N\{b - b(j)\} + b(j)$$

where  $N$  is the number of observations used to produce  $b$ .

When the `cluster()` option is specified, each cluster is given at most one nonmissing pseudo-value. The `keep` option implies the `nodrop` option.

`mse` specifies that `jackknife` compute the variance by using deviations of the replicates from the observed value of the statistics based on the entire dataset. By default, `jackknife` computes the variance by using deviations of the pseudo-values from their mean.

## Reporting

`level(#)`; see [R] [Estimation options](#).

`notable` suppresses the display of the table of results.

`noheader` suppresses the display of the table header. This option implies `nolegend`.

`nolegend` suppresses the display of the table legend. The table legend identifies the rows of the table with the expressions they represent.

`verbose` specifies that the full table legend be displayed. By default, coefficients and standard errors are not displayed.

`nodots` and `dots(#)` specify whether to display replication dots. By default, one dot character is displayed for each successful replication. An “x” is displayed if *command* returns an error or if any value in *exp\_list* is missing. You can also control whether dots are displayed using `set dots`; see [R] [set](#).

`nodots` suppresses display of the replication dots.

`dots(#)` displays dots every # replications. `dots(0)` is a synonym for `nodots`.

`noisily` specifies that any output from *command* be displayed. This option implies the `nodots` option.

`trace` causes a trace of the execution of *command* to be displayed. This option implies the `noisily` option.

`title(text)` specifies a title to be displayed above the table of jackknife results; the default title is `Jackknife results` or what is produced in `e(title)` by an estimation command.

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

*eform\_option* causes the coefficient table to be displayed in exponentiated form; see [R] [eform\\_option](#). *command* determines which *eform\_option* is allowed (`eform(string)` and `eform` are always allowed).

*command* determines which of the following are allowed (`eform(string)` and `eform` are always allowed):

| <i>eform_option</i>        | Description                                                     |
|----------------------------|-----------------------------------------------------------------|
| <code>eform(string)</code> | use <i>string</i> for the column title                          |
| <code>eform</code>         | exponentiated coefficient, <i>string</i> is <code>exp(b)</code> |
| <code>hr</code>            | hazard ratio, <i>string</i> is <code>Haz. ratio</code>          |
| <code>shr</code>           | subhazard ratio, <i>string</i> is <code>SHR</code>              |
| <code>irr</code>           | incidence-rate ratio, <i>string</i> is <code>IRR</code>         |
| <code>or</code>            | odds ratio, <i>string</i> is <code>Odds ratio</code>            |
| <code>rrr</code>           | relative-risk ratio, <i>string</i> is <code>RRR</code>          |

#### Advanced

`nodrop` prevents observations outside `e(sample)` and the `if` and `in` qualifiers from being dropped before the data are resampled.

`reject(exp)` identifies an expression that indicates when results should be rejected. When *exp* is true, the resulting values are reset to missing values.

The following option is available with `jackknife` but is not shown in the dialog box:

`coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Remarks are presented under the following headings:

*Introduction*  
*Using jackknife*  
*Jackknifed standard deviation*  
*Collecting multiple statistics*  
*Collecting coefficients*

### Introduction

Although the jackknife—developed in the late 1940s and early 1950s—is of largely historical interest today, it is still useful in searching for overly influential observations. This feature is often forgotten. In any case, the jackknife is

- an alternative, first-order unbiased estimator for a statistic;
- a data-dependent way to calculate the standard error of the statistic and to obtain significance levels and confidence intervals; and
- a way of producing measures called pseudovalues for each observation, reflecting the observation’s influence on the overall statistic.

The idea behind the simplest form of the jackknife—the one implemented here—is to repeatedly calculate the statistic in question, each time omitting just one of the dataset’s observations. Assume that our statistic of interest is the sample mean. Let  $y_j$  be the  $j$ th observation of our data on some measurement  $y$ , where  $j = 1, \dots, N$  and  $N$  is the sample size. If  $\bar{y}$  is the sample mean of  $y$  using the entire dataset and  $\bar{y}_{(j)}$  is the mean when the  $j$ th observation is omitted, then

$$\bar{y} = \frac{(N-1)\bar{y}_{(j)} + y_j}{N}$$

Solving for  $y_j$ , we obtain

$$y_j = N\bar{y} - (N-1)\bar{y}_{(j)}$$

These are the pseudovalues that `jackknife` calculates. To move this discussion beyond the sample mean, let  $\hat{\theta}$  be the value of our statistic (not necessarily the sample mean) using the entire dataset, and let  $\hat{\theta}_{(j)}$  be the computed value of our statistic with the  $j$ th observation omitted. The pseudovalue for the  $j$ th observation is

$$\hat{\theta}_j^* = N\hat{\theta} - (N-1)\hat{\theta}_{(j)}$$

The mean of the pseudovalues is the alternative, first-order unbiased estimator mentioned above, and the standard error of the mean of the pseudovalues is an estimator for the standard error of  $\hat{\theta}$  (Tukey 1958).

The jackknife estimate of variance has been largely replaced by the bootstrap estimate (see [R] **bootstrap**), which is widely viewed as more efficient and robust. The use of jackknife pseudovalues to detect outliers is too often forgotten and is something the bootstrap does not provide. See Mosteller and Tukey (1977, 133–163) and Mooney and Duval (1993, 22–27) for more information.

## Using jackknife

### Typing

```
. jackknife exp_list: command
```

executes *command* once for each observation in the dataset, leaving the associated observation out of the calculations that make up *exp\_list*.

*command* defines the statistical command to be executed. Most Stata commands and user-written programs can be used with *jackknife*, as long as they follow standard Stata syntax and allow the *if* qualifier; see [U] 11 Language syntax. The *by* prefix may not be part of *command*.

*exp\_list* specifies the statistics to be collected from the execution of *command*. If *command* changes the contents in *e(b)*, *exp\_list* is optional and defaults to *\_b*.

When the *cluster()* option is given, clusters are omitted instead of observations, and *N* is the number of clusters instead of the sample size.

### ► Example 1

As our first example, we will show that the jackknife standard error of the sample mean is equivalent to the standard error of the sample mean computed using the classical formula in the *ci means* command. We use the *double* option to compute the standard errors with the same precision as the *ci means* command.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. jackknife r(mean), double: summarize mpg
(running summarize on estimation sample)
Jackknife replications (74):10.....20.....30.....40.....
> ...50.....60.....70.... done
Jackknife results Number of obs = 74
 Replications = 74

Command: summarize mpg
 _jk_1: r(mean)
 n(): r(N)
```

|       | Coefficient | Jackknife<br>std. err. | t     | P> t  | [95% conf. interval] |          |
|-------|-------------|------------------------|-------|-------|----------------------|----------|
| _jk_1 | 21.2973     | .6725511               | 31.67 | 0.000 | 19.9569              | 22.63769 |

```
. ci means mpg
```

| Variable | Obs | Mean    | Std. err. | [95% conf. interval] |          |
|----------|-----|---------|-----------|----------------------|----------|
| mpg      | 74  | 21.2973 | .6725511  | 19.9569              | 22.63769 |

## Jackknifed standard deviation

### ▷ Example 2

Mosteller and Tukey (1977, 139–140) request a 95% confidence interval for the standard deviation of the 11 values:

0.1, 0.1, 0.1, 0.4, 0.5, 1.0, 1.1, 1.3, 1.9, 1.9, 4.7

Stata's `summarize` command calculates the mean and standard deviation and stores them as `r(mean)` and `r(sd)`. To obtain the jackknifed standard deviation of the 11 values and save the pseudovalues as a new variable, `sd`, we would type

```
. clear
. input x
 x
 1. 0.1
 2. 0.1
 3. 0.1
 4. 0.4
 5. 0.5
 6. 1.0
 7. 1.1
 8. 1.3
 9. 1.9
10. 1.9
11. 4.7
12. end

. jackknife sd=r(sd), rclass keep: summarize x
(running summarize on estimation sample)
Jackknife replications (11):10. done

Jackknife results Number of obs = 11
 Replications = 11

Command: summarize x
 sd: r(sd)
 n(): r(N)
```

|    | Coefficient | Jackknife<br>std. err. | t    | P> t  | [95% conf. interval] |         |
|----|-------------|------------------------|------|-------|----------------------|---------|
| sd | 1.343469    | .624405                | 2.15 | 0.057 | -.047792             | 2.73473 |

Interpreting the output, the standard deviation reported by `summarize mpg` is 1.34. The jackknife standard error is 0.62. The 95% confidence interval for the standard deviation is  $-0.048$  to  $2.73$ .

By specifying `keep`, `jackknife` creates in our dataset a new variable, `sd`, for the pseudovalues.

```
. list, sep(4)
```

|     | x   | sd       |
|-----|-----|----------|
| 1.  | .1  | 1.139977 |
| 2.  | .1  | 1.139977 |
| 3.  | .1  | 1.139977 |
| 4.  | .4  | .8893147 |
| 5.  | .5  | .824267  |
| 6.  | 1   | .632489  |
| 7.  | 1.1 | .6203189 |
| 8.  | 1.3 | .6218889 |
| 9.  | 1.9 | .835419  |
| 10. | 1.9 | .835419  |
| 11. | 4.7 | 7.703949 |

The jackknife estimate is the average of the `sd` variable, so `sd` contains the individual values of our statistic. We can see that the last observation is substantially larger than the others. The last observation is certainly an outlier, but whether that reflects the considerable information it contains or indicates that it should be excluded from analysis depends on the context of the problem. Mosteller and Tukey created the data by sampling from an exponential distribution, so the observation is informative.

◀

### ▷ Example 3

Let's repeat the example above using the automobile dataset, obtaining the standard error of the standard deviation of `mpg`.

```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
. jackknife sd=r(sd), rclass keep: summarize mpg
(running summarize on estimation sample)
Jackknife replications (74):10.....20.....30.....40.....
> ...50.....60.....70.... done

Jackknife results Number of obs = 74
 Replications = 74

Command: summarize mpg
 sd: r(sd)
 n(): r(N)
```

|    | Coefficient | Jackknife<br>std. err. | t    | P> t  | [95% conf. interval] |          |
|----|-------------|------------------------|------|-------|----------------------|----------|
| sd | 5.785503    | .6072509               | 9.53 | 0.000 | 4.575254             | 6.995753 |

Let's look at sd more carefully:

```
. summarize sd, detail
 pseudovalues: r(sd)

 Percentiles Smallest
 1% 2.870471 2.870471
 5% 2.870471 2.870471
 10% 2.906255 2.870471 Obs 74
 25% 3.328489 2.870471 Sum of wgt. 74
 50% 3.948335 Mean 5.817374
 Largest 5.22377
 75% 6.844418 17.34316 Std. dev. 5.22377
 90% 9.597018 19.7617 Variance 27.28777
 95% 17.34316 19.7617 Skewness 4.07202
 99% 38.60905 38.60905 Kurtosis 23.37823

. list make mpg sd if sd > 30
```

|     | make      | mpg | sd       |
|-----|-----------|-----|----------|
| 71. | VW Diesel | 41  | 38.60905 |

Here the VW Diesel is the only diesel car in our dataset.



## Collecting multiple statistics

### ▷ Example 4

jackknife is not limited to collecting just one statistic. For instance, we can use `summarize`, `detail` and then obtain the jackknife estimate of the standard deviation and skewness. `summarize, detail` stores the standard deviation in `r(sd)` and the skewness in `r(skewness)`, so we might type

```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
. jackknife sd=r(sd) skew=r(skewness), rclass: summarize mpg, detail
(running summarize on estimation sample)
Jackknife replications (74):10.....20.....30.....40.....
> ...50.....60.....70.... done

Jackknife results Number of obs = 74
 Replications = 74

Command: summarize mpg, detail
 sd: r(sd)
 skew: r(skewness)
 n(): r(N)
```

|      | Coefficient | Jackknife<br>std. err. | t    | P> t  | [95% conf. interval] |          |
|------|-------------|------------------------|------|-------|----------------------|----------|
| sd   | 5.785503    | .6072509               | 9.53 | 0.000 | 4.575254             | 6.995753 |
| skew | .9487176    | .3367242               | 2.82 | 0.006 | .2776272             | 1.619808 |



## Collecting coefficients

### ▷ Example 5

jackknife can also collect coefficients from estimation commands. For instance, using `auto.dta`, we might wish to obtain the jackknife standard errors of the coefficients from a regression in which we model the mileage of a car by its weight and trunk space. To do this, we could refer to the coefficients as `_b[weight]`, `_b[trunk]`, `_se[weight]`, and `_se[trunk]` in the `exp_list`, or we could simply use the extended expressions `_b`. In fact, jackknife assumes `_b` by default when used with estimation commands.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)

. jackknife: regress mpg weight trunk
(running regress on estimation sample)

Jackknife replications (74):10.....20.....30.....40.....
> ...50.....60.....70.... done

Linear regression Number of obs = 74
 Replications = 74
 F(2, 73) = 78.10
 Prob > F = 0.0000
 R-squared = 0.6543
 Adj R-squared = 0.6446
 Root MSE = 3.4492
```

| mpg    | Coefficient | Jackknife<br>std. err. | t     | P> t  | [95% conf. interval] |           |
|--------|-------------|------------------------|-------|-------|----------------------|-----------|
| weight | -.0056527   | .0010216               | -5.53 | 0.000 | -.0076887            | -.0036167 |
| trunk  | -.096229    | .1486236               | -0.65 | 0.519 | -.3924354            | .1999773  |
| _cons  | 39.68913    | 1.873324               | 21.19 | 0.000 | 35.9556              | 43.42266  |

If you are going to use jackknife to estimate standard errors of model coefficients, we recommend using the `vce(jackknife)` option when it is allowed with the estimation command; see [R] [vce\\_option](#).

```
. regress mpg weight trunk, vce(jackknife, nodots)

Linear regression Number of obs = 74
 Replications = 74
 F(2, 73) = 78.10
 Prob > F = 0.0000
 R-squared = 0.6543
 Adj R-squared = 0.6446
 Root MSE = 3.4492
```

| mpg    | Coefficient | Jackknife<br>std. err. | t     | P> t  | [95% conf. interval] |           |
|--------|-------------|------------------------|-------|-------|----------------------|-----------|
| weight | -.0056527   | .0010216               | -5.53 | 0.000 | -.0076887            | -.0036167 |
| trunk  | -.096229    | .1486236               | -0.65 | 0.519 | -.3924354            | .1999773  |
| _cons  | 39.68913    | 1.873324               | 21.19 | 0.000 | 35.9556              | 43.42266  |

## □ Technical note

When the `jackknife` prefix is used with a user-defined program and when the expression list is `_b`, `jackknife` calls

```
set coeftabresults off
```

before entering the replication loop to prevent Stata from performing unnecessary calculations. This means that, provided option `noisily` is not specified, estimation commands will not build or post the coefficient table matrix `r(table)`.

If your program calls an estimation command and needs `r(table)` to exist to perform properly, then your program will need to call

```
set coeftabresults on
```

before calling other estimation commands.

□

**John Wilder Tukey** (1915–2000) was born in Massachusetts. He studied chemistry at Brown and mathematics at Princeton and afterward worked at both Princeton and Bell Labs, as well as being involved in a great many government projects, consultancies, and committees. He made outstanding contributions to several areas of statistics, including time series, multiple comparisons, robust statistics, and exploratory data analysis. Tukey was extraordinarily energetic and inventive, not least in his use of terminology: he is credited with inventing the terms `bit` and `software`, in addition to `ANOVA`, `boxplot`, `data analysis`, `hat matrix`, `jackknife`, `stem-and-leaf plot`, `trimming`, and `winsorizing`, among many others. Tukey's direct and indirect impacts mark him as one of the greatest statisticians of all time.

## Stored results

`jackknife` stores the following in `e()`:

### Scalars

|                           |                                                                        |
|---------------------------|------------------------------------------------------------------------|
| <code>e(N)</code>         | sample size                                                            |
| <code>e(N_reps)</code>    | number of complete replications                                        |
| <code>e(N_misreps)</code> | number of incomplete replications                                      |
| <code>e(N_clust)</code>   | number of clusters                                                     |
| <code>e(k_eq)</code>      | number of equations in <code>e(b)</code>                               |
| <code>e(k_extra)</code>   | number of extra equations                                              |
| <code>e(k_exp)</code>     | number of expressions                                                  |
| <code>e(k_eeexp)</code>   | number of extended expressions ( <code>_b</code> or <code>_se</code> ) |
| <code>e(df_r)</code>      | degrees of freedom                                                     |

### Macros

|                           |                                                                           |
|---------------------------|---------------------------------------------------------------------------|
| <code>e(cmdname)</code>   | command name from <i>command</i>                                          |
| <code>e(cmd)</code>       | same as <code>e(cmdname)</code> or <code>jackknife</code>                 |
| <code>e(command)</code>   | <i>command</i>                                                            |
| <code>e(cmdline)</code>   | command as typed                                                          |
| <code>e(prefix)</code>    | <code>jackknife</code>                                                    |
| <code>e(wtype)</code>     | weight type                                                               |
| <code>e(wexp)</code>      | weight expression                                                         |
| <code>e(title)</code>     | title in estimation output                                                |
| <code>e(cluster)</code>   | cluster variables                                                         |
| <code>e(pseudo)</code>    | new variables containing pseudovalues                                     |
| <code>e(nfunction)</code> | <code>e(N)</code> , <code>r(N)</code> , <code>n()</code> option, or empty |

|                            |                                  |
|----------------------------|----------------------------------|
| <code>e(exp#)</code>       | expression for the #th statistic |
| <code>e(mse)</code>        | from mse option                  |
| <code>e(vce)</code>        | jackknife                        |
| <code>e(vcetype)</code>    | title used to label Std. err.    |
| <code>e(properties)</code> | b V                              |

#### Matrices

|                              |                                      |
|------------------------------|--------------------------------------|
| <code>e(b)</code>            | observed statistics                  |
| <code>e(b_jk)</code>         | jackknife estimates                  |
| <code>e(V)</code>            | jackknife variance–covariance matrix |
| <code>e(V_modelbased)</code> | model-based variance                 |

In addition to the above, the following is stored in `r()`:

#### Matrices

|                       |                                                                                                                            |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

When `exp_list` is `_b`, `jackknife` will also carry forward most of the results already in `e()` from *command*.

## Methods and formulas

Let  $\hat{\theta}$  be the observed value of the statistic, that is, the value of the statistic calculated using the original dataset. Let  $\hat{\theta}_{(j)}$  be the value of the statistic computed by leaving out the *j*th observation (or cluster); thus  $j = 1, 2, \dots, N$  identifies an individual observation (or cluster), and *N* is the total number of observations (or clusters). The *j*th pseudo-value is given by

$$\hat{\theta}_j^* = \hat{\theta}_{(j)} + N\{\hat{\theta} - \hat{\theta}_{(j)}\}$$

When the `mse` option is specified, the standard error is estimated as

$$\widehat{\text{se}} = \left\{ \frac{N-1}{N} \sum_{j=1}^N (\hat{\theta}_{(j)} - \hat{\theta})^2 \right\}^{1/2}$$

and the jackknife estimate is

$$\bar{\theta}_{(\cdot)} = \frac{1}{N} \sum_{j=1}^N \hat{\theta}_{(j)}$$

Otherwise, the standard error is estimated as

$$\widehat{\text{se}} = \left\{ \frac{1}{N(N-1)} \sum_{j=1}^N (\hat{\theta}_j^* - \bar{\theta}^*)^2 \right\}^{1/2} \quad \bar{\theta}^* = \frac{1}{N} \sum_{j=1}^N \hat{\theta}_j^*$$

where  $\bar{\theta}^*$  is the jackknife estimate. The variance–covariance matrix is similarly computed.

## References

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## Also see

- [R] [jackknife postestimation](#) — Postestimation tools for jackknife
- [R] [bootstrap](#) — Bootstrap sampling and estimation
- [R] [permute](#) — Permutation tests
- [R] [simulate](#) — Monte Carlo simulations
- [SVY] [svy jackknife](#) — Jackknife estimation for survey data
- [U] [13.5 Accessing coefficients and standard errors](#)
- [U] [13.6 Accessing results from Stata commands](#)
- [U] [20 Estimation and postestimation commands](#)

## Postestimation commands

The following postestimation commands are available after `jackknife`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| <code>hausman</code>         | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | predictions, residuals, influence statistics, and other diagnostic measures                                                             |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

The postestimation command is allowed only if it may be used after *command*.

## predict

The syntax of `predict` (and whether `predict` is even allowed) following `jackknife` depends on the *command* used with `jackknife`.

## margins

The syntax of `margins` (and whether `margins` is even allowed) following `jackknife` depends on the *command* used with `jackknife`.

## Also see

[R] [jackknife](#) — Jackknife estimation

[U] [20 Estimation and postestimation commands](#)

## Description

`kap` and `kappa` calculate the kappa-statistic measure of interrater agreement. `kap` calculates the statistic for two unique raters or at least two nonunique raters. `kappa` calculates only the statistic for nonunique raters, but it handles the case where data have been recorded as rating frequencies. `kapwgt` defines weights used by `kap` in measuring the importance of disagreements.

## Quick start

Calculate interrater agreement for values `rater1` and `rater2`

```
kap rater1 rater2
```

Add table of assessments

```
kap rater1 rater2, tab
```

Same as above, and apply frequency weights defined by `wvar`

```
kap rater1 rater2 [fweight=wvar], tab
```

Agreement for values from three nonunique raters stored in `rater1`, `rater2`, and `rater3`

```
kap rater1 rater2 rater3
```

Add values from an additional three raters stored in `rater4`, `rater5`, and `rater6`

```
kap rater1-rater6
```

Use weights  $1 - |i - j| / (k - 1)$  to weight disagreements between rater 1 and rater 2

```
kap rater1 rater2, wgt(w)
```

Number of times each subject classified in categories stored in `poor`, `fair`, and `good`

```
kappa poor fair good
```

## Menu

### kap: two unique raters

Statistics > Epidemiology and related > Other > Interrater agreement, two unique raters

### kapwgt

Statistics > Epidemiology and related > Other > Define weights for the above (kap)

### kap: nonunique raters

Statistics > Epidemiology and related > Other > Interrater agreement, nonunique raters

### kappa

Statistics > Epidemiology and related > Other > Interrater agreement, nonunique raters with frequencies

## Syntax

*Interrater agreement, two unique raters*

```
kap varname1 varname2 [if] [in] [weight] [, options]
```

*Weights for weighting disagreements*

```
kapwgt wgtid [1 \ # 1 [\ # # 1 ...]]
```

*Interrater agreement, nonunique raters, variables record ratings for each rater*

```
kap varname1 varname2 varname3 [...] [if] [in] [weight]
```

*Interrater agreement, nonunique raters, variables record frequency of ratings*

```
kappa varlist [if] [in]
```

*options*

Description

---

|                          |                                                                                   |
|--------------------------|-----------------------------------------------------------------------------------|
| Main                     |                                                                                   |
| <code>tab</code>         | display table of assessments                                                      |
| <code>wgt (wgtid)</code> | specify how to weight disagreements; see <a href="#">Options</a> for alternatives |
| <code>absolute</code>    | treat rating categories as absolute                                               |

---

`collect` is allowed with `kap` and `kappa`; see [\[U\] 11.1.10 Prefix commands](#).

`fweights` are allowed; see [\[U\] 11.1.6 weight](#).

## Options

Main

`tab` displays a tabulation of the assessments by the two raters.

`wgt(wgtid)` specifies that `wgtid` be used to weight disagreements. You can define your own weights by using `kapwgt`; `wgt()` then specifies the name of the user-defined matrix. For instance, you might define

```
. kapwgt mine 1 \ .8 1 \ 0 .8 1 \ 0 0 .8 1
```

and then

```
. kap rata ratb, wgt(mine)
```

Also, two prerecorded weights are available.

`wgt(w)` specifies weights  $1 - |i - j| / (k - 1)$ , where  $i$  and  $j$  index the rows and columns of the ratings by the two raters and  $k$  is the maximum number of possible ratings.

`wgt(w2)` specifies weights  $1 - \{(i - j) / (k - 1)\}^2$ .

`absolute` is relevant only if `wgt()` is also specified. The `absolute` option modifies how  $i$ ,  $j$ , and  $k$  are defined and how corresponding entries are found in a user-defined weighting matrix. When `absolute` is not specified,  $i$  and  $j$  refer to the row and column index, not to the ratings themselves. Say that the ratings are recorded as  $\{0, 1, 1.5, 2\}$ . There are four ratings;  $k = 4$ , and  $i$  and  $j$  are still 1, 2, 3, and 4 in the formulas above. Index 3, for instance, corresponds to rating = 1.5. This system is convenient but can, with some data, lead to difficulties.

When `absolute` is specified, all ratings must be integers, and they must be coded from the set  $\{1, 2, 3, \dots\}$ . Not all values need be used; integer values that do not occur are simply assumed to be unobserved.

## Remarks and examples

Remarks are presented under the following headings:

*Two raters*

*More than two raters*

The kappa-statistic measure of agreement is scaled to be 0 when the amount of agreement is what would be expected to be observed by chance and 1 when there is perfect agreement. For intermediate values, Landis and Koch (1977a, 165) suggest the following interpretations:

|           |                |
|-----------|----------------|
| below 0.0 | Poor           |
| 0.00–0.20 | Slight         |
| 0.21–0.40 | Fair           |
| 0.41–0.60 | Moderate       |
| 0.61–0.80 | Substantial    |
| 0.81–1.00 | Almost perfect |

## Two raters

### ▷ Example 1

Consider the classification by two radiologists of 85 xeromammograms as normal, benign disease, suspicion of cancer, or cancer (a subset of the data from Boyd et al. [1982] and discussed in the context of kappa in Altman [1991, 403–405]).

```
. use https://www.stata-press.com/data/r19/rate2
(Altman p. 403)
. tabulate rada radb
```

| Radiologist<br>A's<br>assessment | Radiologist B's assessment |        |         |        | Total |
|----------------------------------|----------------------------|--------|---------|--------|-------|
|                                  | Normal                     | Benign | Suspect | Cancer |       |
| Normal                           | 21                         | 12     | 0       | 0      | 33    |
| Benign                           | 4                          | 17     | 1       | 0      | 22    |
| Suspect                          | 3                          | 9      | 15      | 2      | 29    |
| Cancer                           | 0                          | 0      | 0       | 1      | 1     |
| Total                            | 28                         | 38     | 16      | 3      | 85    |

Our dataset contains two variables: rada, radiologist A's assessment, and radb, radiologist B's assessment. Each observation is a patient.

We can obtain the kappa measure of interrater agreement by typing

```
. kap rada radb
```

| Agreement | Expected<br>agreement | Kappa  | Std. err. | Z    | Prob>Z |
|-----------|-----------------------|--------|-----------|------|--------|
| 63.53%    | 30.82%                | 0.4728 | 0.0694    | 6.81 | 0.0000 |

If each radiologist had made his determination randomly (but with probabilities equal to the overall proportions), we would expect the two radiologists to agree on 30.8% of the patients. In fact, they agreed on 63.5% of the patients, or 47.3% of the way between random agreement and perfect agreement. The amount of agreement indicates that we can reject the hypothesis that they are making their determinations randomly.

### ▷ Example 2: Weighted kappa, prerecorded weight $w$

There is a difference between two radiologists disagreeing about whether a xeromammogram indicates cancer or the suspicion of cancer and disagreeing about whether it indicates cancer or is normal. The weighted kappa attempts to deal with this. `kap` provides two “prerecorded” weights,  $w$  and  $w2$ :

```
. kap rada radb, wgt(w)
Ratings weighted by:
 1.0000 0.6667 0.3333 0.0000
 0.6667 1.0000 0.6667 0.3333
 0.3333 0.6667 1.0000 0.6667
 0.0000 0.3333 0.6667 1.0000
```

| Agreement | Expected agreement | Kappa  | Std. err. | Z    | Prob>Z |
|-----------|--------------------|--------|-----------|------|--------|
| 86.67%    | 69.11%             | 0.5684 | 0.0788    | 7.22 | 0.0000 |

The  $w$  weights are given by  $1 - |i - j| / (k - 1)$ , where  $i$  and  $j$  index the rows of columns of the ratings by the two raters and  $k$  is the maximum number of possible ratings. The weighting matrix is printed above the table. Here the rows and columns of the  $4 \times 4$  matrix correspond to the ratings normal, benign, suspicious, and cancerous.

A weight of 1 indicates that an observation should count as perfect agreement. The matrix has 1s down the diagonals—when both radiologists make the same assessment, they are in agreement. A weight of, say, 0.6667 means that they are in two-thirds agreement. In our matrix, they get that score if they are “one apart”—one radiologist assesses cancer and the other is merely suspicious, or one is suspicious and the other says benign, and so on. An entry of 0.3333 means that they are in one-third agreement, or, if you prefer, two-thirds disagreement. That is the score attached when they are “two apart”. Finally, they are in complete disagreement when the weight is zero, which happens only when they are three apart—one says cancer and the other says normal.

◀

### ▷ Example 3: Weighted kappa, prerecorded weight $w2$

The other prerecorded weight is  $w2$ , where the weights are given by  $1 - \{(i - j) / (k - 1)\}^2$ :

```
. kap rada radb, wgt(w2)
Ratings weighted by:
 1.0000 0.8889 0.5556 0.0000
 0.8889 1.0000 0.8889 0.5556
 0.5556 0.8889 1.0000 0.8889
 0.0000 0.5556 0.8889 1.0000
```

| Agreement | Expected agreement | Kappa  | Std. err. | Z    | Prob>Z |
|-----------|--------------------|--------|-----------|------|--------|
| 94.77%    | 84.09%             | 0.6714 | 0.1079    | 6.22 | 0.0000 |

The  $w2$  weight makes the categories even more alike and is probably inappropriate here.

◀

### ► Example 4: Weighted kappa, user-defined weights

In addition to using prerecorded weights, we can define our own weights with the `kapwgt` command. For instance, we might feel that suspicious and cancerous are reasonably similar, that benign and normal are reasonably similar, but that the suspicious/cancerous group is nothing like the benign/normal group:

```
. kapwgt xm 1 \ .8 1 \ 0 0 1 \ 0 0 .8 1
. kapwgt xm
1.0000
0.8000 1.0000
0.0000 0.0000 1.0000
0.0000 0.0000 0.8000 1.0000
```

We name the weights `xm`, and after the weight name, we enter the lower triangle of the weighting matrix, using `\` to separate rows. We have four outcomes, so we continued entering numbers until we had defined the fourth row of the weighting matrix. If we type `kapwgt` followed by a name and nothing else, it shows us the weights recorded under that name. Satisfied that we have entered them correctly, we now use the weights to recalculate kappa:

```
. kap rada radb, wgt(xm)
Ratings weighted by:
 1.0000 0.8000 0.0000 0.0000
 0.8000 1.0000 0.0000 0.0000
 0.0000 0.0000 1.0000 0.8000
 0.0000 0.0000 0.8000 1.0000
```

| Agreement | Expected agreement | Kappa  | Std. err. | Z    | Prob>Z |
|-----------|--------------------|--------|-----------|------|--------|
| 80.47%    | 52.67%             | 0.5874 | 0.0865    | 6.79 | 0.0000 |

◀

### □ Technical note

In addition to using weights for weighting the differences in categories, you can specify Stata's traditional weights for weighting the data. In the examples above, we have 85 observations in our dataset—one for each patient. If we only knew the table of outcomes—that there were 21 patients rated normal by both radiologists, etc.—it would be easier to enter the table into Stata and work from it. The easiest way to enter the data is with `tabi`; see [\[R\] tabulate twoway](#).

```
. tabi 21 12 0 0 \ 4 17 1 0 \ 3 9 15 2 \ 0 0 0 1, replace
```

| row   | col |    |    |   | Total |
|-------|-----|----|----|---|-------|
|       | 1   | 2  | 3  | 4 |       |
| 1     | 21  | 12 | 0  | 0 | 33    |
| 2     | 4   | 17 | 1  | 0 | 22    |
| 3     | 3   | 9  | 15 | 2 | 29    |
| 4     | 0   | 0  | 0  | 1 | 1     |
| Total | 28  | 38 | 16 | 3 | 85    |

Pearson chi2(9) = 77.8111 Pr = 0.000

tabi reported the Pearson  $\chi^2$  for this table, but we do not care about it. The important thing is that, with the `replace` option, tabi left the table in memory:

```
. list in 1/5
```

|    | row | col | pop |
|----|-----|-----|-----|
| 1. | 1   | 1   | 21  |
| 2. | 1   | 2   | 12  |
| 3. | 1   | 3   | 0   |
| 4. | 1   | 4   | 0   |
| 5. | 2   | 1   | 4   |

The variable `row` is radiologist A's assessment, `col` is radiologist B's assessment, and `pop` is the number so assessed by both. Thus,

```
. kap row col [fweight=pop]
```

| Agreement | Expected agreement | Kappa  | Std. err. | Z    | Prob>Z |
|-----------|--------------------|--------|-----------|------|--------|
| 63.53%    | 30.82%             | 0.4728 | 0.0694    | 6.81 | 0.0000 |

If we are going to keep these data, the names `row` and `col` are not indicative of what the data reflect. We could type (see [U] 12.6 **Dataset, variable, and value labels**)

```
. rename row rada
. rename col radb
. label var rada "Radiologist A's assessment"
. label var radb "Radiologist B's assessment"
. label define assess 1 normal 2 benign 3 suspect 4 cancer
. label values rada assess
. label values radb assess
. label data "Altman, page 403"
```

`kap`'s `tab` option, which can be used with or without weighted data, shows the table of assessments:

```
. kap rada radb [fweight=pop], tab
```

| Radiologist A's assessment | Radiologist B's assessment |        |         |        | Total |
|----------------------------|----------------------------|--------|---------|--------|-------|
|                            | normal                     | benign | suspect | cancer |       |
| normal                     | 21                         | 12     | 0       | 0      | 33    |
| benign                     | 4                          | 17     | 1       | 0      | 22    |
| suspect                    | 3                          | 9      | 15      | 2      | 29    |
| cancer                     | 0                          | 0      | 0       | 1      | 1     |
| Total                      | 28                         | 38     | 16      | 3      | 85    |

| Agreement | Expected agreement | Kappa  | Std. err. | Z    | Prob>Z |
|-----------|--------------------|--------|-----------|------|--------|
| 63.53%    | 30.82%             | 0.4728 | 0.0694    | 6.81 | 0.0000 |

## □ Technical note

You have data on individual patients. There are two raters, and the possible ratings are 1, 2, 3, and 4, but neither rater ever used rating 3:

```
. use https://www.stata-press.com/data/r19/rate2no3, clear
. tabulate ratera raterb
```

| Rater A | Rater B |   |    | Total |
|---------|---------|---|----|-------|
|         | 1       | 2 | 4  |       |
| 1       | 6       | 4 | 3  | 13    |
| 2       | 5       | 3 | 3  | 11    |
| 4       | 1       | 1 | 26 | 28    |
| Total   | 12      | 8 | 32 | 52    |

Here `kap` would determine that the ratings are from the set  $\{1, 2, 4\}$  because those were the only values observed. `kap` would expect a user-defined weighting matrix to be  $3 \times 3$ , and if it were not, `kap` would issue an error message. In the formula-based weights, the calculation would be based on  $i, j = 1, 2, 3$  corresponding to the three observed ratings  $\{1, 2, 4\}$ .

Specifying the `absolute` option would clarify that the ratings are 1, 2, 3, and 4; it just so happens that rating 3 was never assigned. If a user-defined weighting matrix were also specified, `kap` would expect it to be  $4 \times 4$  or larger (larger because we can think of the ratings being 1, 2, 3, 4, 5, ... and it just so happens that ratings 5, 6, ... were never observed, just as rating 3 was not observed). In the formula-based weights, the calculation would be based on  $i, j = 1, 2, 4$ .

```
. kap ratera raterb, wgt(w)
```

Ratings weighted by:

```
1.0000 0.5000 0.0000
0.5000 1.0000 0.5000
0.0000 0.5000 1.0000
```

| Agreement | Expected agreement | Kappa  | Std. err. | Z    | Prob>Z |
|-----------|--------------------|--------|-----------|------|--------|
| 79.81%    | 57.17%             | 0.5285 | 0.1169    | 4.52 | 0.0000 |

```
. kap ratera raterb, wgt(w) absolute
```

Ratings weighted by:

```
1.0000 0.6667 0.0000
0.6667 1.0000 0.3333
0.0000 0.3333 1.0000
```

| Agreement | Expected agreement | Kappa  | Std. err. | Z    | Prob>Z |
|-----------|--------------------|--------|-----------|------|--------|
| 81.41%    | 55.08%             | 0.5862 | 0.1209    | 4.85 | 0.0000 |

If all conceivable ratings are observed in the data, specifying `absolute` makes no difference. For instance, if rater A assigns ratings  $\{1, 2, 4\}$  and rater B assigns  $\{1, 2, 3, 4\}$ , the complete set of assigned ratings is  $\{1, 2, 3, 4\}$ , the same that `absolute` would specify. Without `absolute`, it makes no difference whether the ratings are coded  $\{1, 2, 3, 4\}$ ,  $\{0, 1, 2, 3\}$ ,  $\{1, 7, 9, 100\}$ ,  $\{0, 1, 1.5, 2.0\}$ , or otherwise.

□

## More than two raters

For more than two raters, the mathematics are such that the two raters are not considered unique. For instance, if there are three raters, there is no assumption that the three raters who rate the first subject are the same as the three raters who rate the second. Although we call this the “more than two raters” case, it can be used with two raters when the raters’ identities vary.

The nonunique rater case can be usefully broken down into three subcases: 1) there are two possible ratings, which we will call positive and negative; 2) there are more than two possible ratings, but the number of raters per subject is the same for all subjects; and 3) there are more than two possible ratings, and the number of raters per subject varies. `kappa` handles all these cases. To emphasize that there is no assumption of constant identity of raters across subjects, the variables specified contain counts of the number of raters rating the subject into a particular category.

Jacob Cohen (1923–1998) was born in New York City. After studying psychology at City College of New York and New York University, he worked as a medical psychologist until 1959 when he became a full professor in the Department of Psychology at New York University. He made many contributions to research methods, including the kappa measure. He persistently emphasized the value of multiple regression and the importance of power and of measuring effects rather than testing significance.

### ► Example 5: Two ratings

Fleiss, Levin, and Paik (2003, 612) offer the following hypothetical ratings by different sets of raters on 25 subjects:

| Subject | No. of raters | No. of pos. ratings | Subject | No. of raters | No. of pos. ratings |
|---------|---------------|---------------------|---------|---------------|---------------------|
| 1       | 2             | 2                   | 14      | 4             | 3                   |
| 2       | 2             | 0                   | 15      | 2             | 0                   |
| 3       | 3             | 2                   | 16      | 2             | 2                   |
| 4       | 4             | 3                   | 17      | 3             | 1                   |
| 5       | 3             | 3                   | 18      | 2             | 1                   |
| 6       | 4             | 1                   | 19      | 4             | 1                   |
| 7       | 3             | 0                   | 20      | 5             | 4                   |
| 8       | 5             | 0                   | 21      | 3             | 2                   |
| 9       | 2             | 0                   | 22      | 4             | 0                   |
| 10      | 4             | 4                   | 23      | 3             | 0                   |
| 11      | 5             | 5                   | 24      | 3             | 3                   |
| 12      | 3             | 3                   | 25      | 2             | 2                   |
| 13      | 4             | 4                   |         |               |                     |

We have entered these data into Stata, and the variables are called `subject`, `raters`, and `pos`. `kappa`, however, requires that we specify variables containing the number of positive ratings and negative ratings, that is, `pos` and `raters-pos`:

```
. use https://www.stata-press.com/data/r19/p612
. generate neg = raters-pos
. kappa pos neg
```

Two-outcomes, multiple raters:

| Kappa  | Z    | Prob>Z |
|--------|------|--------|
| 0.5415 | 5.28 | 0.0000 |

We would have obtained the same results if we had typed `kappa neg pos`.



▷ **Example 6: More than two ratings, constant number of raters, kappa**

Each of 10 subjects is rated into one of three categories by five raters (Fleiss, Levin, and Paik 2003, 615):

```
. use https://www.stata-press.com/data/r19/p615, clear
. list
```

|     | subject | cat1 | cat2 | cat3 |
|-----|---------|------|------|------|
| 1.  | 1       | 1    | 4    | 0    |
| 2.  | 2       | 2    | 0    | 3    |
| 3.  | 3       | 0    | 0    | 5    |
| 4.  | 4       | 4    | 0    | 1    |
| 5.  | 5       | 3    | 0    | 2    |
| 6.  | 6       | 1    | 4    | 0    |
| 7.  | 7       | 5    | 0    | 0    |
| 8.  | 8       | 0    | 4    | 1    |
| 9.  | 9       | 1    | 0    | 4    |
| 10. | 10      | 3    | 0    | 2    |

We obtain the kappa statistic:

```
. kappa cat1-cat3
```

| Outcome    | Kappa  | Z    | Prob>Z |
|------------|--------|------|--------|
| Category 1 | 0.2917 | 2.92 | 0.0018 |
| Category 2 | 0.6711 | 6.71 | 0.0000 |
| Category 3 | 0.3490 | 3.49 | 0.0002 |
| combined   | 0.4179 | 5.83 | 0.0000 |

The first part of the output shows the results of calculating kappa for each of the categories separately against an amalgam of the remaining categories. For instance, the `cat1` line is the two-rating kappa, where positive is `cat1` and negative is `cat2` or `cat3`. The test statistic, however, is calculated differently (see *Methods and formulas*). The combined kappa is the appropriately weighted average of the individual kappas. There is considerably less agreement about the rating of subjects into the first category than there is for the second.



▷ Example 7: More than two ratings, constant number of raters, kap

Now, suppose that we have the same data as in the previous example but that the data are organized differently:

```
. use https://www.stata-press.com/data/r19/p615b
. list
```

|     | subject | rater1 | rater2 | rater3 | rater4 | rater5 |
|-----|---------|--------|--------|--------|--------|--------|
| 1.  | 1       | 1      | 2      | 2      | 2      | 2      |
| 2.  | 2       | 1      | 1      | 3      | 3      | 3      |
| 3.  | 3       | 3      | 3      | 3      | 3      | 3      |
| 4.  | 4       | 1      | 1      | 1      | 1      | 3      |
| 5.  | 5       | 1      | 1      | 1      | 3      | 3      |
| 6.  | 6       | 1      | 2      | 2      | 2      | 2      |
| 7.  | 7       | 1      | 1      | 1      | 1      | 1      |
| 8.  | 8       | 2      | 2      | 2      | 2      | 3      |
| 9.  | 9       | 1      | 3      | 3      | 3      | 3      |
| 10. | 10      | 1      | 1      | 1      | 3      | 3      |

Here we would use `kap` rather than `kappa` because the variables record ratings for each rater.

```
. kap rater1 rater2 rater3 rater4 rater5
```

There are 5 raters per subject:

| Outcome  | Kappa  | Z    | Prob>Z |
|----------|--------|------|--------|
| 1        | 0.2917 | 2.92 | 0.0018 |
| 2        | 0.6711 | 6.71 | 0.0000 |
| 3        | 0.3490 | 3.49 | 0.0002 |
| combined | 0.4179 | 5.83 | 0.0000 |

It does not matter which rater is which when there are more than two raters.

▷ Example 8: More than two ratings, varying number of raters, kappa

In this unfortunate case, kappa can be calculated, but there is no test statistic for testing against  $\kappa > 0$ . We do nothing differently—kappa calculates the total number of raters for each subject, and, if it is not a constant, kappa suppresses the calculation of test statistics.

```
. use https://www.stata-press.com/data/r19/rvary
. list
```

|     | subject | cat1 | cat2 | cat3 |
|-----|---------|------|------|------|
| 1.  | 1       | 1    | 3    | 0    |
| 2.  | 2       | 2    | 0    | 3    |
| 3.  | 3       | 0    | 0    | 5    |
| 4.  | 4       | 4    | 0    | 1    |
| 5.  | 5       | 3    | 0    | 2    |
| 6.  | 6       | 1    | 4    | 0    |
| 7.  | 7       | 5    | 0    | 0    |
| 8.  | 8       | 0    | 4    | 1    |
| 9.  | 9       | 1    | 0    | 2    |
| 10. | 10      | 3    | 0    | 2    |

```
. kappa cat1-cat3
```

| Outcome    | Kappa  | Z | Prob>Z |
|------------|--------|---|--------|
| Category 1 | 0.2685 | . | .      |
| Category 2 | 0.6457 | . | .      |
| Category 3 | 0.2938 | . | .      |
| combined   | 0.3816 | . | .      |

Note: Number of ratings per subject vary; cannot calculate test statistics.

◀

▷ Example 9: More than two ratings, varying number of raters, kap

This case is similar to the [previous example](#), but the data are organized differently:

```
. use https://www.stata-press.com/data/r19/rvary2
. list
```

|     | subject | rater1 | rater2 | rater3 | rater4 | rater5 |
|-----|---------|--------|--------|--------|--------|--------|
| 1.  | 1       | 1      | 2      | 2      | .      | 2      |
| 2.  | 2       | 1      | 1      | 3      | 3      | 3      |
| 3.  | 3       | 3      | 3      | 3      | 3      | 3      |
| 4.  | 4       | 1      | 1      | 1      | 1      | 3      |
| 5.  | 5       | 1      | 1      | 1      | 3      | 3      |
| 6.  | 6       | 1      | 2      | 2      | 2      | 2      |
| 7.  | 7       | 1      | 1      | 1      | 1      | 1      |
| 8.  | 8       | 2      | 2      | 2      | 2      | 3      |
| 9.  | 9       | 1      | 3      | .      | .      | 3      |
| 10. | 10      | 1      | 1      | 1      | 3      | 3      |

Here we specify kap instead of kappa because the variables record ratings for each rater.

```
. kap rater1-rater5
```

There are between 3 and 5 (median = 5.00) raters per subject:

| Outcome  | Kappa  | Z | Prob>Z |
|----------|--------|---|--------|
| 1        | 0.2685 | . | .      |
| 2        | 0.6457 | . | .      |
| 3        | 0.2938 | . | .      |
| combined | 0.3816 | . | .      |

Note: Number of ratings per subject vary; cannot calculate test statistics.

◀

## Stored results

kap and kappa store the following in `r()`:

Scalars

|                        |                                             |                       |                                    |
|------------------------|---------------------------------------------|-----------------------|------------------------------------|
| <code>r(N)</code>      | number of subjects (kap only)               | <code>r(kappa)</code> | kappa                              |
| <code>r(prop_o)</code> | observed proportion of agreement (kap only) | <code>r(z)</code>     | z statistic                        |
| <code>r(prop_e)</code> | expected proportion of agreement (kap only) | <code>r(se)</code>    | standard error for kappa statistic |

## Methods and formulas

The kappa statistic was first proposed by [Cohen \(1960\)](#). The generalization for weights reflecting the relative seriousness of each possible disagreement is due to [Cohen \(1968\)](#). The analysis-of-variance approach for  $k = 2$  and  $m \geq 2$  is due to [Landis and Koch \(1977b\)](#). See [Altman \(1991, 403–409\)](#) or [Dunn \(2000, chap. 2\)](#) for an introductory treatment and [Fleiss, Levin, and Paik \(2003, chap. 18\)](#) for a more detailed treatment. All formulas below are as presented in [Fleiss, Levin, and Paik \(2003\)](#). Let  $m$  be the number of raters, and let  $k$  be the number of rating outcomes.

Methods and formulas are presented under the following headings:

*kap*:  $m = 2$

*kappa*:  $m > 2, k = 2$

*kappa*:  $m > 2, k > 2$

**kap: m = 2**

Define  $w_{ij}$  ( $i = 1, \dots, k$  and  $j = 1, \dots, k$ ) as the weights for agreement and disagreement ( $\text{wgt}()$ ), or, if the data are not weighted, define  $w_{ii} = 1$  and  $w_{ij} = 0$  for  $i \neq j$ . If  $\text{wgt}(w)$  is specified,  $w_{ij} = 1 - |i - j|/(k - 1)$ . If  $\text{wgt}(w2)$  is specified,  $w_{ij} = 1 - \{(i - j)/(k - 1)\}^2$ .

The observed proportion of agreement is

$$p_o = \sum_{i=1}^k \sum_{j=1}^k w_{ij} p_{ij}$$

where  $p_{ij}$  is the fraction of ratings  $i$  by the first rater and  $j$  by the second. The expected proportion of agreement is

$$p_e = \sum_{i=1}^k \sum_{j=1}^k w_{ij} p_{i \cdot} p_{\cdot j}$$

where  $p_{i \cdot} = \sum_j p_{ij}$  and  $p_{\cdot j} = \sum_i p_{ij}$ .

Kappa is given by  $\hat{\kappa} = (p_o - p_e)/(1 - p_e)$ .

The standard error of  $\hat{\kappa}$  for testing against 0 is

$$\hat{\sigma}_0 = \frac{1}{(1 - p_e)\sqrt{n}} \left( \left[ \sum_i \sum_j p_{i \cdot} p_{\cdot j} \{w_{ij} - (\bar{w}_i + \bar{w}_j)\}^2 \right] - p_e^2 \right)^{1/2}$$

where  $n$  is the number of subjects being rated,  $\bar{w}_i = \sum_j p_{\cdot j} w_{ij}$ , and  $\bar{w}_j = \sum_i p_i w_{ij}$ . The test statistic  $Z = \hat{\kappa}/\hat{\sigma}_0$  is assumed to be distributed  $N(0, 1)$ .

**kappa: m > 2, k = 2**

Each subject  $i$ ,  $i = 1, \dots, n$ , is found by  $x_i$  of  $m_i$  raters to be positive (the choice as to what is labeled positive is arbitrary).

The overall proportion of positive ratings is  $\bar{p} = \sum_i x_i/(n\bar{m})$ , where  $\bar{m} = \sum_i m_i/n$ . The between-subjects mean square is (approximately)

$$B = \frac{1}{n} \sum_i \frac{(x_i - m_i \bar{p})^2}{m_i}$$

and the within-subject mean square is

$$W = \frac{1}{n(\bar{m} - 1)} \sum_i \frac{x_i(m_i - x_i)}{m_i}$$

Kappa is then defined as

$$\hat{\kappa} = \frac{B - W}{B + (\bar{m} - 1)W}$$

The standard error for testing against 0 (Fleiss and Cuzick 1979) is approximately equal to and is calculated as

$$\hat{s}_0 = \frac{1}{(\bar{m} - 1)\sqrt{n\bar{m}_H}} \left\{ 2(\bar{m}_H - 1) + \frac{(\bar{m} - \bar{m}_H)(1 - 4\bar{p}\bar{q})}{\bar{m}\bar{p}\bar{q}} \right\}^{1/2}$$

where  $\bar{m}_H$  is the harmonic mean of  $m_i$  and  $\bar{q} = 1 - \bar{p}$ .

The test statistic  $Z = \hat{\kappa}/\hat{s}_0$  is assumed to be distributed  $N(0, 1)$ .

### kappa: $m > 2, k > 2$

Let  $x_{ij}$  be the number of ratings on subject  $i, i = 1, \dots, n$ , into category  $j, j = 1, \dots, k$ . Define  $\bar{p}_j$  as the overall proportion of ratings in category  $j, \bar{q}_j = 1 - \bar{p}_j$ , and let  $\hat{\kappa}_j$  be the kappa statistic given above for  $k = 2$  when category  $j$  is compared with the amalgam of all other categories. Kappa is

$$\bar{\kappa} = \frac{\sum_j \bar{p}_j \bar{q}_j \hat{\kappa}_j}{\sum_j \bar{p}_j \bar{q}_j}$$

(Landis and Koch 1977b). In the case where the number of raters per subject,  $\sum_j x_{ij}$ , is a constant  $m$  for all  $i$ , Fleiss, Nee, and Landis (1979) derived the following formulas for the approximate standard errors. The standard error for testing  $\hat{\kappa}_j$  against 0 is

$$\hat{s}_j = \left\{ \frac{2}{nm(m-1)} \right\}^{1/2}$$

and the standard error for testing  $\bar{\kappa}$  is

$$\bar{s} = \frac{\sqrt{2}}{\sum_j \bar{p}_j \bar{q}_j \sqrt{nm(m-1)}} \left\{ \left( \sum_j \bar{p}_j \bar{q}_j \right)^2 - \sum_j \bar{p}_j \bar{q}_j (\bar{q}_j - \bar{p}_j) \right\}^{1/2}$$

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## Description

`kdensity` produces kernel density estimates and graphs the result.

## Quick start

Graph of the kernel density estimate for `v1`

```
kdensity v1
```

Add a normal curve

```
kdensity v1, normal
```

With a kernel bandwidth of 2

```
kdensity v1, bwidth(2)
```

Gaussian kernel function for `v1`

```
kdensity v1, kernel(gaussian)
```

Kernel density estimate for `v1` and `v2` in the same graph area

```
twoway kdensity v1 || kdensity v2
```

Separate graphs of kernel density estimate of `v1` for each level of `catvar`

```
twoway kdensity v1, by(catvar)
```

Kernel density estimates of `v1` for `catvar = 0` and `1` in the same graph area

```
twoway kdensity v1 if catvar==0 || kdensity v1 if catvar==1
```

## Menu

Statistics > Nonparametric analysis > Kernel density estimation

## Syntax

kdensity *varname* [*if*] [*in*] [*weight*] [, *options*]

| <i>options</i>                                                           | Description                                                                                                    |
|--------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------|
| <b>Main</b>                                                              |                                                                                                                |
| <u>k</u> kernel ( <i>kernel</i> )                                        | specify kernel function; default is kernel (epanechnikov)                                                      |
| <u>b</u> width (#)                                                       | half-width of kernel                                                                                           |
| <u>g</u> enerate ( <i>newvar<sub>x</sub></i> <i>newvar<sub>d</sub></i> ) | store the estimation points in <i>newvar<sub>x</sub></i> and the density estimate in <i>newvar<sub>d</sub></i> |
| <u>n</u> (#)                                                             | estimate density using # points; default is min( <i>N</i> , 50)                                                |
| <u>a</u> t ( <i>var<sub>x</sub></i> )                                    | estimate density using the values specified by <i>var<sub>x</sub></i>                                          |
| <u>n</u> ograph                                                          | suppress graph                                                                                                 |
| <b>Kernel plot</b>                                                       |                                                                                                                |
| <u>c</u> line_ <i>options</i>                                            | affect rendition of the plotted kernel density estimate                                                        |
| <b>Density plots</b>                                                     |                                                                                                                |
| <u>n</u> ormal                                                           | add normal density to the graph                                                                                |
| <u>n</u> ormopts ( <i>cline_options</i> )                                | affect rendition of normal density                                                                             |
| <u>s</u> tudent (#)                                                      | add Student's <i>t</i> density with # degrees of freedom to the graph                                          |
| <u>s</u> topts ( <i>cline_options</i> )                                  | affect rendition of the Student's <i>t</i> density                                                             |
| <b>Add plots</b>                                                         |                                                                                                                |
| <u>a</u> ddplot ( <i>plot</i> )                                          | add other plots to the generated graph                                                                         |
| <b>Y axis, X axis, Titles, Legend, Overall</b>                           |                                                                                                                |
| <u>t</u> woway_ <i>options</i>                                           | any options other than by () documented in [G-3] <i>twoway_options</i>                                         |
| <hr/>                                                                    |                                                                                                                |
| <i>kernel</i>                                                            | Description                                                                                                    |
| <u>e</u> panechnikov                                                     | Epanechnikov kernel function; the default                                                                      |
| <u>e</u> pan2                                                            | alternative Epanechnikov kernel function                                                                       |
| <u>b</u> iweight                                                         | biweight kernel function                                                                                       |
| <u>c</u> osine                                                           | cosine trace kernel function                                                                                   |
| <u>g</u> aussian                                                         | Gaussian kernel function                                                                                       |
| <u>p</u> arzen                                                           | Parzen kernel function                                                                                         |
| <u>r</u> ectangle                                                        | rectangle kernel function                                                                                      |
| <u>t</u> riangle                                                         | triangle kernel function                                                                                       |

collect is allowed; see [U] 11.1.10 Prefix commands.

fweights, aweights, and iweights are allowed; see [U] 11.1.6 weight.

## Options

### Main

`kernel` (*kernel*) specifies the kernel function for use in calculating the kernel density estimate. The default kernel is the Epanechnikov kernel (`epanechnikov`).

`bw`width(*#*) specifies the half-width of the kernel, the width of the density window around each point. If `bw`width() is not specified, the “optimal” width is calculated and used. The optimal width is the width that would minimize the mean integrated squared error if the data were Gaussian and a Gaussian kernel were used, so it is not optimal in any global sense. In fact, for multimodal and highly skewed densities, this width is usually too wide and oversmooths the density (Silverman 1986).

`generate` (*newvar<sub>x</sub>* *newvar<sub>d</sub>*) stores the results of the estimation. *newvar<sub>x</sub>* will contain the points at which the density is estimated. *newvar<sub>d</sub>* will contain the density estimate.

`n`(*#*) specifies the number of points at which the density estimate is to be evaluated. The default is  $\min(N, 50)$ , where  $N$  is the number of observations in memory.

`at` (*var<sub>x</sub>*) specifies a variable that contains the values at which the density should be estimated. This option allows you to more easily obtain density estimates for different variables or different subsamples of a variable and then overlay the estimated densities for comparison.

`nograph` suppresses the graph. This option is often used with the `generate` () option.

### Kernel plot

`cline_options` affect the rendition of the plotted kernel density estimate. See [G-3] *cline\_options*.

### Density plots

`normal` requests that a normal density be overlaid on the density estimate for comparison.

`normopts` (*cline\_options*) specifies details about the rendition of the normal curve, such as the color and style of line used. See [G-3] *cline\_options*.

`student` (*#*) specifies that a Student’s  $t$  density with  $\#$  degrees of freedom be overlaid on the density estimate for comparison.

`stopts` (*cline\_options*) affects the rendition of the Student’s  $t$  density. See [G-3] *cline\_options*.

### Add plots

`addplot` (*plot*) provides a way to add other plots to the generated graph. See [G-3] *addplot\_option*.

### Y axis, X axis, Titles, Legend, Overall

`twoway_options` are any of the options documented in [G-3] *twoway\_options*, excluding `by` (). These include options for titling the graph (see [G-3] *title\_options*) and for saving the graph to disk (see [G-3] *saving\_option*).

## Remarks and examples

Kernel density estimators approximate the density  $f(x)$  from observations on  $x$ . Histograms do this, too, and the histogram itself is a kind of kernel density estimate. The data are divided into nonoverlapping intervals, and counts are made of the number of data points within each interval. Histograms are bar graphs that depict these frequency counts—the bar is centered at the midpoint of each interval—and its height reflects the average number of data points in the interval.

In more general kernel density estimates, the range is still divided into intervals, and estimates of the density at the center of intervals are produced. One difference is that the intervals are allowed to overlap. We can think of sliding the interval—called a window—along the range of the data and collecting the center-point density estimates. The second difference is that, rather than merely counting the number of observations in a window, a kernel density estimator assigns a weight between 0 and 1—based on the distance from the center of the window—and sums the weighted values. The function that determines these weights is called the kernel.

Kernel density estimates have the advantages of being smooth and of being independent of the choice of origin (corresponding to the location of the bins in a histogram).

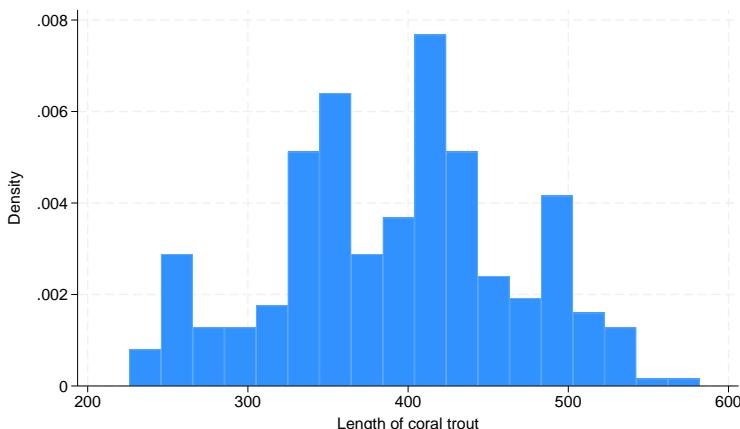
See [Salgado-Ugarte, Shimizu, and Taniuchi \(1993\)](#) and [Fox \(1990\)](#) for discussions of kernel density estimators that stress their use as exploratory data-analysis tools.

[Cox \(2007\)](#) gives a lucid introductory tutorial on kernel density estimation with several Stata produced examples. He provides tips and tricks for working with skewed or bounded distributions and applying the same techniques to estimate the intensity function of a point process.

### ▷ Example 1: Histogram and kernel density estimate

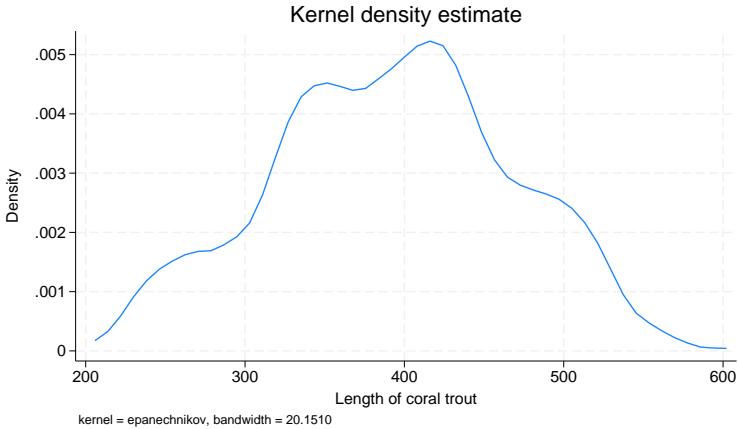
[Goeden \(1978\)](#) reports data consisting of 316 length observations of coral trout. We wish to investigate the underlying density of the lengths. To begin on familiar ground, we might draw a histogram. In [\[R\] histogram](#), we suggest setting the bins to  $\min(\sqrt{n}, 10 \cdot \log_{10} n)$ , which for  $n = 316$  is roughly 18:

```
. use https://www.stata-press.com/data/r19/trocolen
. histogram length, bins(18)
(bin=18, start=226, width=19.777778)
```



The kernel density estimate, on the other hand, is smooth.

```
. kdensity length
```



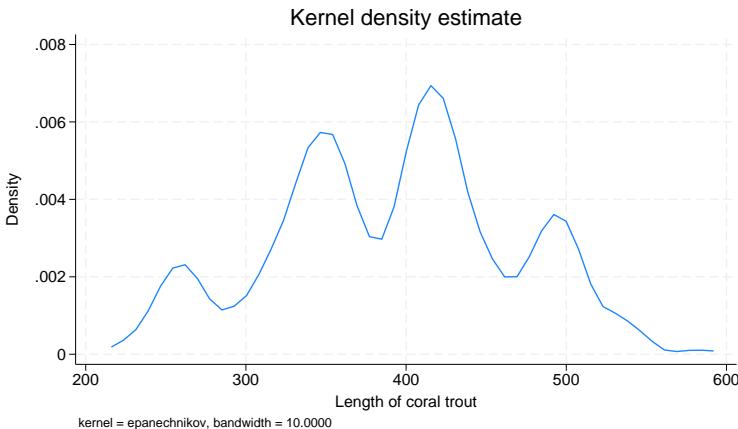
Kernel density estimators are, however, sensitive to an assumption, just as are histograms. In histograms, we specify a number of bins. For kernel density estimators, we specify a width. In the graph above, we used the default width. `kdensity` is smarter than `twoway histogram` in that its default width is not a fixed constant. Even so, the default width is not necessarily best.

`kdensity` stores the width in the returned scalar `bwidth`, so typing `display r(bwidth)` reveals it. Doing this, we discover that the width is approximately 20.

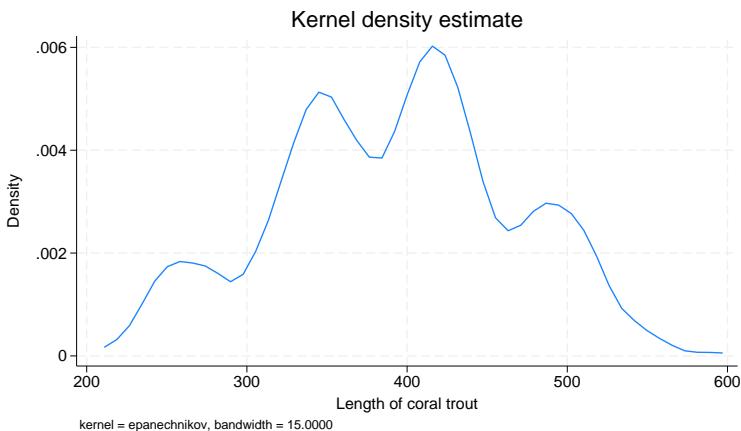
Widths are similar to the inverse of the number of bins in a histogram in that smaller widths provide more detail. The units of the width are the units of  $x$ , the variable being analyzed. The width is specified as a half-width, meaning that the kernel density estimator with half-width 20 corresponds to sliding a window of size 40 across the data.

We can specify half-widths for ourselves by using the `bwidth()` option. Smaller widths do not smooth the density as much:

```
. kdensity length, bwidth(10)
```



```
. kdensity length, bwidth(15)
```

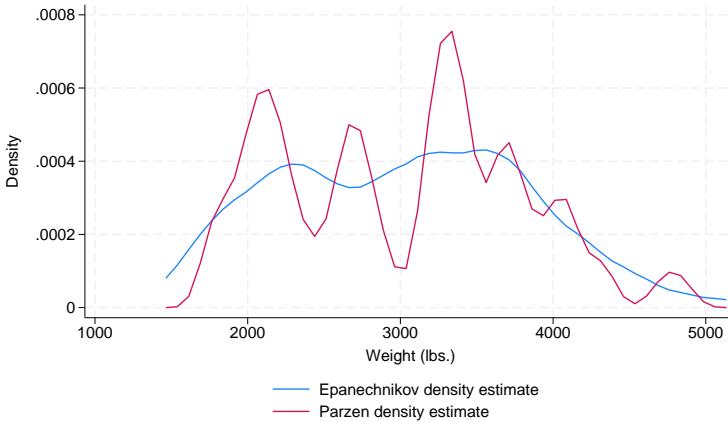


### ► Example 2: Different kernels can produce different results

When widths are held constant, different kernels can produce surprisingly different results. This is really an attribute of the kernel and width combination; for a given width, some kernels are more sensitive than others at identifying peaks in the density estimate.

We can see this when using a dataset with lots of peaks. In the automobile dataset, we characterize the density of `weight`, the weight of the vehicles. Below, we compare the Epanechnikov and Parzen kernels.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. kdensity weight, kernel(epanechnikov) nograph generate(x epan)
. kdensity weight, kernel(parzen) nograph generate(x2 parzen)
. label var epan "Epanechnikov density estimate"
. label var parzen "Parzen density estimate"
. line epan parzen x, sort ytitle("Density") legend(pos(6))
```



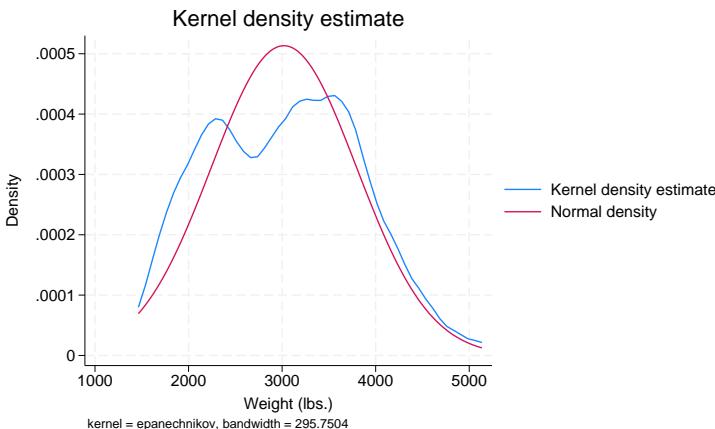
We did not specify a width, so we obtained the default width. That width is not a function of the selected kernel, but of the data. See [Methods and formulas](#) for the calculation of the optimal width.



▷ Example 3: Density with overlaid normal density

In examining the density estimates, we may wish to overlay a normal density or a Student's *t* density for comparison. Using automobile weights, we can get an idea of the distance from normality by using the normal option.

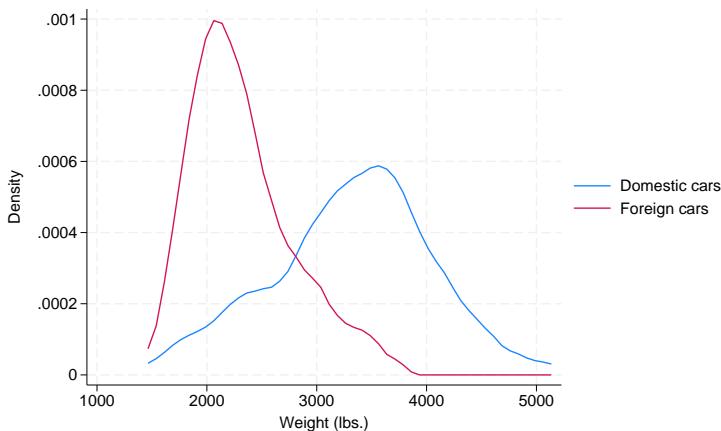
```
. kdensity weight, kernel(epanechnikov) normal
```



### ▷ Example 4: Compare two densities

We also may want to compare two or more densities. In this example, we will compare the density estimates of the weights for the foreign and domestic cars.

```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
. kdensity weight, nograph generate(x fx)
. kdensity weight if foreign==0, nograph generate(fx0) at(x)
. kdensity weight if foreign==1, nograph generate(fx1) at(x)
. label var fx0 "Domestic cars"
. label var fx1 "Foreign cars"
. line fx0 fx1 x, sort ytitle(Density)
```



◀

### □ Technical note

Although all the examples we included had densities of less than 1, the density may exceed 1.

The probability density  $f(x)$  of a continuous variable,  $x$ , has the units and dimensions of the reciprocal of  $x$ . If  $x$  is measured in meters,  $f(x)$  has units 1/meter. Thus, the density is not measured on a probability scale, so it is possible for  $f(x)$  to exceed 1.

To see this, think of a uniform density on the interval 0 to 1. The area under the density curve is 1: this is the product of the density, which is constant at 1, and the range, which is 1. If the variable is then transformed by doubling, the area under the curve remains 1 and is the product of the density, constant at 0.5, and the range, which is 2. Conversely, if the variable is transformed by halving, the area under the curve also remains at 1 and is the product of the density, constant at 2, and the range, which is 0.5. (Strictly, the range is measured in certain units, and the density is measured in the reciprocal of those units, so the units cancel on multiplication.)

□

## Stored results

kdensity stores the following in `r()`:

Scalars

`r(bwidth)` kernel bandwidth  
`r(n)` number of points at which the estimate was evaluated  
`r(scale)` density bin width

Macros

`r(kernel)` name of kernel

## Methods and formulas

A kernel density estimate is formed by summing the weighted values calculated with the kernel function  $K$ , as in

$$\hat{f}_K = \frac{1}{qh} \sum_{i=1}^n w_i K\left(\frac{x - X_i}{h}\right)$$

where  $q = \sum_i w_i$  if weights are frequency weights (`fweight`) or analytic weights (`aweight`), and  $q = 1$  if weights are importance weights (`iweights`). Analytic weights are rescaled so that  $\sum_i w_i = n$  (see [\[U\] 11 Language syntax](#)). If weights are not used, then  $w_i = 1$ , for  $i = 1, \dots, n$ . `kdensity` includes seven different kernel functions. The Epanechnikov is the default function if no other kernel is specified and is the most efficient in minimizing the mean integrated squared error.

| Kernel       | Formula                                                                                                                                                          |  |
|--------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| Biweight     | $K[z] = \begin{cases} \frac{15}{16}(1 - z^2)^2 & \text{if }  z  < 1 \\ 0 & \text{otherwise} \end{cases}$                                                         |  |
| Cosine       | $K[z] = \begin{cases} 1 + \cos(2\pi z) & \text{if }  z  < 1/2 \\ 0 & \text{otherwise} \end{cases}$                                                               |  |
| Epanechnikov | $K[z] = \begin{cases} \frac{3}{4}(1 - \frac{1}{5}z^2)/\sqrt{5} & \text{if }  z  < \sqrt{5} \\ 0 & \text{otherwise} \end{cases}$                                  |  |
| Epan2        | $K[z] = \begin{cases} \frac{3}{4}(1 - z^2) & \text{if }  z  < 1 \\ 0 & \text{otherwise} \end{cases}$                                                             |  |
| Gaussian     | $K[z] = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$                                                                                                                        |  |
| Parzen       | $K[z] = \begin{cases} \frac{4}{3} - 8z^2 + 8 z ^3 & \text{if }  z  \leq 1/2 \\ 8(1 -  z )^3/3 & \text{if } 1/2 <  z  \leq 1 \\ 0 & \text{otherwise} \end{cases}$ |  |
| Rectangular  | $K[z] = \begin{cases} 1/2 & \text{if }  z  < 1 \\ 0 & \text{otherwise} \end{cases}$                                                                              |  |
| Triangular   | $K[z] = \begin{cases} 1 -  z  & \text{if }  z  < 1 \\ 0 & \text{otherwise} \end{cases}$                                                                          |  |

From the definitions given in the table, we can see that the choice of  $h$  will drive how many values are included in estimating the density at each point. This value is called the *window width* or *bandwidth*. If the window width is not specified, it is determined as

$$m = \min \left( \sqrt{\text{variance}_x}, \frac{\text{interquartile range}_x}{1.349} \right)$$

$$h = \frac{0.9m}{n^{1/5}}$$

where  $x$  is the variable for which we wish to estimate the kernel and  $n$  is the number of observations.

Most researchers agree that the choice of kernel is not as important as the choice of bandwidth. There is a great deal of literature on choosing bandwidths under various conditions; see, for example, [Parzen \(1962\)](#) or [Tapia and Thompson \(1978\)](#). Also see [Newton \(1988\)](#) for a comparison with sample spectral density estimation in time-series applications.

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We gratefully acknowledge the previous work by Isaías H. Salgado-Ugarte of the Universidad Nacional Autónoma de México, Makoto Shimizu of the University of Tokyo, and Toru Taniuchi formerly of the University of Tokyo; see [Salgado-Ugarte, Shimizu, and Taniuchi \(1993\)](#). Their article provides a good overview of the subject of univariate kernel density estimation and presents arguments for its use in exploratory data analysis.

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## Also see

- [R] **histogram** — Histograms for continuous and categorical variables
- [R] **makespline** — Spline generation
- [R] **npregress kernel** — Nonparametric kernel regression
- [R] **npregress series** — Nonparametric series regression

[Description](#)[Options for two-sample test](#)[References](#)[Quick start](#)[Remarks and examples](#)[Also see](#)[Menu](#)[Stored results](#)[Syntax](#)[Methods and formulas](#)

## Description

`ksmirnov` performs one- and two-sample Kolmogorov–Smirnov tests of the equality of distributions. A one-sample test compares the distribution of the tested variable with the specified distribution. A two-sample test tests the equality of the distributions of two samples.

When testing for normality, please see [\[R\] sktest](#) and [\[R\] swilk](#).

## Quick start

One-sample test comparing the distribution of `v1` with a Student's  $t$  distribution with 5 degrees of freedom

```
ksmirnov v1 = t(5, v1)
```

Two-sample test comparing distributions of `v2` in two groups defined by `catvar`

```
ksmirnov v2, by(catvar)
```

Same as above, but calculate an exact  $p$ -value

```
ksmirnov v2, by(catvar) exact
```

## Menu

Statistics > Nonparametric analysis > Tests of hypotheses > Kolmogorov–Smirnov test

## Syntax

One-sample Kolmogorov–Smirnov test

```
ksmirnov varname = exp [if] [in]
```

Two-sample Kolmogorov–Smirnov test

```
ksmirnov varname [if] [in], by(groupvar) [exact]
```

In the first syntax, *varname* is the variable whose distribution is being tested, and *exp* must evaluate to the corresponding (theoretical) cumulative. In the second syntax, *groupvar* must take on two distinct values. The distribution of *varname* for the first value of *groupvar* is compared with that of the second value.

`collect` is allowed; see [U] 11.1.10 Prefix commands.

## Options for two-sample test

Main

`by(groupvar)` is required. It specifies a binary variable that identifies the two groups.

`exact` specifies that the exact *p*-value be computed.

## Remarks and examples

### ► Example 1: Two-sample test

Say that we have data on *x* that resulted from two different experiments, labeled as `group==1` and `group==2`. Our data contain

```
. use https://www.stata-press.com/data/r19/ksxmpl
. list
```

|    | group | x  |
|----|-------|----|
| 1. | 2     | 2  |
| 2. | 1     | 0  |
| 3. | 2     | 3  |
| 4. | 1     | 4  |
| 5. | 1     | 5  |
| 6. | 2     | 8  |
| 7. | 2     | 10 |

We wish to use the two-sample Kolmogorov–Smirnov test to determine if there are any differences in the distribution of  $x$  for these two groups:

```
. ksmirnov x, by(group)
Two-sample Kolmogorov-Smirnov test for equality of distribution functions
```

| Smaller group | D       | p-value |
|---------------|---------|---------|
| 1             | 0.5000  | 0.424   |
| 2             | -0.1667 | 0.909   |
| Combined K-S  | 0.5000  | 0.785   |

The first line tests the hypothesis that  $x$  for group 1 contains *smaller* values than for group 2. The largest difference between the distribution functions is 0.5. The approximate asymptotic  $p$ -value for this is 0.424, which is not significant.

The second line tests the hypothesis that  $x$  for group 1 contains *larger* values than for group 2. The largest difference between the distribution functions in this direction is 0.1667. The approximate asymptotic  $p$ -value for this small difference is 0.909.

Finally, the approximate asymptotic  $p$ -value for the combined test is 0.785. The approximate  $p$ -values `ksmirnov` calculates are based on the five-term approximation of the asymptotic distributions derived by [Smirnov \(1933\)](#). These approximations are not good for small samples ( $n < 50$ ). They are too conservative.

An exact  $p$ -value can be calculated using the exact option:

```
. ksmirnov x, by(group) exact
Two-sample Kolmogorov-Smirnov test for equality of distribution functions
```

| Smaller group | D       | p-value | Exact |
|---------------|---------|---------|-------|
| 1             | 0.5000  | 0.424   |       |
| 2             | -0.1667 | 0.909   |       |
| Combined K-S  | 0.5000  | 0.785   | 0.657 |

◀

## ► Example 2: One-sample test

Let's now test whether  $x$  in the example above is distributed normally. Kolmogorov–Smirnov is not a particularly powerful test in testing for normality, and we do not endorse such use of it; see [\[R\] sktest](#) and [\[R\] swilk](#) for better tests.

In any case, we will test against a normal distribution with the same mean and standard deviation:

```
. summarize x
```

| Variable | Obs | Mean     | Std. dev. | Min | Max |
|----------|-----|----------|-----------|-----|-----|
| x        | 7   | 4.571429 | 3.457222  | 0   | 10  |

```
. ksmirnov x = normal((x-4.571429)/3.457222)
One-sample Kolmogorov-Smirnov test against theoretical distribution
normal((x-4.571429)/3.457222)
```

| Smaller group | D       | p-value |
|---------------|---------|---------|
| x             | 0.1650  | 0.683   |
| Cumulative    | -0.1250 | 0.803   |
| Combined K-S  | 0.1650  | 0.991   |

Because Stata has no way of knowing that we based this calculation on the calculated mean and standard deviation of  $x$ , the  $p$ -values will be slightly conservative in addition to being approximations. Nevertheless, they clearly indicate that the data cannot be distinguished from normally distributed data. ◀

## Stored results

`ksmirnov` stores the following in `r()`:

### Scalars

|                         |                           |
|-------------------------|---------------------------|
| <code>r(D_1)</code>     | $D$ from line 1           |
| <code>r(p_1)</code>     | $p$ -value from line 1    |
| <code>r(D_2)</code>     | $D$ from line 2           |
| <code>r(p_2)</code>     | $p$ -value from line 2    |
| <code>r(D)</code>       | combined $D$              |
| <code>r(p)</code>       | combined $p$ -value       |
| <code>r(p_exact)</code> | exact combined $p$ -value |

### Macros

|                        |                           |
|------------------------|---------------------------|
| <code>r(group1)</code> | name of group from line 1 |
| <code>r(group2)</code> | name of group from line 2 |

## Methods and formulas

In general, the Kolmogorov–Smirnov test (Kolmogorov 1933; Smirnov 1933; also see Conover [1999], 428–465) is not very powerful against differences in the tails of distributions. In return for this, it is fairly powerful for alternative hypotheses that involve lumpiness or clustering in the data.

The directional hypotheses are evaluated with the statistics

$$D^+ = \max_x \{F(x) - G(x)\}$$

$$D^- = \min_x \{F(x) - G(x)\}$$

where  $F(x)$  and  $G(x)$  are the empirical distribution functions for the sample being compared. The combined statistic is

$$D = \max(|D^+|, |D^-|)$$

The  $p$ -value for this statistic may be obtained by evaluating the asymptotic limiting distribution. Let  $m$  be the sample size for the first sample, and let  $n$  be the sample size for the second sample. Smirnov (1933) shows that

$$\lim_{m,n \rightarrow \infty} \Pr \left\{ \sqrt{mn/(m+n)} D_{m,n} \leq z \right\} = 1 - 2 \sum_{i=1}^{\infty} (-1)^{i-1} \exp(-2i^2 z^2)$$

The first five terms form the approximation  $P_a$  used by Stata. The exact  $p$ -value is calculated by a counting algorithm; see Gibbons and Chakraborti (2011, 236–238).

Andrei Nikolayevich Kolmogorov (1903–1987), of Russia, was one of the great mathematicians of the twentieth century, making outstanding contributions in many different branches, including set theory, measure theory, probability and statistics, approximation theory, functional analysis, classical dynamics, and theory of turbulence. He was a faculty member at Moscow State University for more than 60 years.

Nikolai Vasilyevich Smirnov (1900–1966) was a Russian statistician whose work included contributions in nonparametric statistics, order statistics, and goodness of fit. After army service and the study of philosophy and philology, he turned to mathematics and eventually rose to be head of mathematical statistics at the Steklov Mathematical Institute in Moscow.

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## Also see

- [R] `runtest` — Test for random order
- [R] `sktest` — Skewness and kurtosis tests for normality
- [R] `swilk` — Shapiro–Wilk and Shapiro–Francia tests for normality

Description  
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Quick start  
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## Description

`kwallis` performs a Kruskal–Wallis test of the hypothesis that several samples are from the same population. This test is a multisample generalization of the two-sample Wilcoxon (Mann–Whitney) rank-sum test.

## Quick start

Test the equality of distribution of `v1` across all levels of categorical variable `cvar`

```
kwallis v1, by(cvar)
```

Same as above, but perform the test only for observations with `v2 = 1`

```
kwallis v1 if v2==1, by(cvar)
```

## Menu

Statistics > Nonparametric analysis > Tests of hypotheses > Kruskal–Wallis rank test

## Syntax

```
kwallis varname [if] [in], by(groupvar)
```

`collect` is allowed; see [U] [11.1.10 Prefix commands](#).

## Option

`by(groupvar)` is required. It specifies a variable that identifies the groups.

## Remarks and examples

### ▷ Example 1

We have data on the 50 states. The data contain the median age of the population, `medage`, and the region of the country, `region`, for each state. We wish to test for the equality of the median age distribution across all four regions simultaneously:

```
. use https://www.stata-press.com/data/r19/census
(1980 Census data by state)
. kwallis medage, by(region)
Kruskal-Wallis equality-of-populations rank test
```

| region  | Obs | Rank sum |
|---------|-----|----------|
| NE      | 9   | 376.50   |
| N Cntrl | 12  | 294.00   |
| South   | 16  | 398.00   |
| West    | 13  | 206.50   |

```
chi2(3) = 17.041
Prob = 0.0007
chi2(3) with ties = 17.062
Prob = 0.0007
```

From the output, we see that we can reject the hypothesis that the populations are the same at any level below 0.07%.

◀

## Stored results

kwallis stores the following in `r()`:

Scalars

```
r(df) degrees of freedom
r(chi2) χ^2
r(chi2_adj) χ^2 adjusted for ties
```

## Methods and formulas

The Kruskal–Wallis test (Kruskal and Wallis 1952, 1953; also see Altman [1991, 213–215]; Conover [1999, 288–297]; and Riffenburgh [2012, sec. 11.6]) is a multiple-sample generalization of the two-sample Wilcoxon (also called Mann–Whitney) rank-sum test (Wilcoxon 1945; Mann and Whitney 1947). Samples of sizes  $n_j$ ,  $j = 1, \dots, m$ , are combined and ranked in ascending order of magnitude. Tied values are assigned the average ranks. Let  $n$  denote the overall sample size, and let  $R_j = \sum_{i=1}^{n_j} R(X_{ji})$  denote the sum of the ranks for the  $j$ th sample. The Kruskal–Wallis one-way analysis-of-variance test,  $H$ , is defined as

$$H = \frac{1}{S^2} \left\{ \sum_{j=1}^m \frac{R_j^2}{n_j} - \frac{n(n+1)^2}{4} \right\}$$

where

$$S^2 = \frac{1}{n-1} \left\{ \sum_{\text{all ranks}} R(X_{ji})^2 - \frac{n(n+1)^2}{4} \right\}$$

If there are no ties, this equation simplifies to

$$H = \frac{12}{n(n+1)} \sum_{j=1}^m \frac{R_j^2}{n_j} - 3(n+1)$$

The sampling distribution of  $H$  is approximately  $\chi^2$  with  $m-1$  degrees of freedom.

William Henry Kruskal (1919–2005) was born in New York City. He studied mathematics and statistics at Antioch College, Harvard, and Columbia, and joined the University of Chicago in 1951. He made many outstanding contributions to linear models, nonparametric statistics, government statistics, and the history and methodology of statistics.

Wilson Allen Wallis (1912–1998) was born in Philadelphia. He studied psychology and economics at the Universities of Minnesota and Chicago and at Columbia. He taught at Yale, Stanford, and Chicago, before moving as president (later chancellor) to the University of Rochester in 1962. He also served in several Republican administrations. Wallis served as editor of the *Journal of the American Statistical Association*, coauthored a popular introduction to statistics, and contributed to nonparametric statistics.

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## Also see

- [R] **nptrend** — Tests for trend across ordered groups
- [R] **oneway** — One-way analysis of variance
- [R] **ranksum** — Equality tests on unmatched data
- [R] **sdtest** — Variance-comparison tests
- [R] **signrank** — Equality tests on matched data

## ladder — Ladder of powers

Description

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Acknowledgment

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References

## Description

`ladder` searches a subset of the ladder of powers (Tukey 1977) for a transform that converts *varname* into a normally distributed variable.

`gladder` and `qladder` each display a graph matrix. `gladder` displays nine histograms of transforms of *varname* according to the ladder of powers. `qladder` displays the quantiles of transforms of *varname* according to the ladder of powers against the quantiles of a normal distribution.

## Quick start

Table showing Tukey's ladder of powers transformations for *v*

```
ladder v
```

Same as above, but with separate tables for each level of the categorical variable *catvar*

```
by catvar: ladder v
```

Display transformations graphically using histograms

```
gladder v
```

Same as above, but using quantile plots

```
qladder v
```

## Menu

### ladder

Statistics > Summaries, tables, and tests > Distributional plots and tests > Ladder of powers

### gladder

Statistics > Summaries, tables, and tests > Distributional plots and tests > Ladder-of-powers histograms

### qladder

Statistics > Summaries, tables, and tests > Distributional plots and tests > Ladder-of-powers quantile-normal plots

## Syntax

*Ladder of powers*

```
ladder varname [if] [in] [, generate(newvar) noadjust]
```

*Ladder-of-powers histograms*

```
gladder varname [if] [in] [, histogram_options combine_options]
```

*Ladder-of-powers quantile–normal plots*

```
qladder varname [if] [in] [, qnorm_options combine_options]
```

by and collect are allowed with ladder; see [U] 11.1.10 Prefix commands.

## Options for ladder

Main

`generate(newvar)` saves the transformed values corresponding to the minimum  $\chi^2$  value from the table. We do not recommend using `generate()` because it is literal in interpreting the minimum, thus ignoring nearly equal but perhaps more interpretable transforms.

`noadjust` is the `noadjust` option to `sktest`; see [R] [sktest](#).

## Options for gladder

*histogram\_options* affect the rendition of the histograms across all relevant transformations; see [R] [histogram](#). Here the `normal` option is assumed, so you must supply the `nonnormal` option to suppress the overlaid normal density. Also, `gladder` does not allow the `width(#)` option of `histogram`.

*combine\_options* are any of the options documented in [G-2] [graph combine](#). These include options for titling the graph (see [G-3] [title\\_options](#)) and for saving the graph to disk (see [G-3] [saving\\_option](#)).

## Options for qladder

*qnorm\_options* affect the rendition of the quantile–normal plots across all relevant transformations. See [R] [Diagnostic plots](#).

*combine\_options* are any of the options documented in [G-2] [graph combine](#). These include options for titling the graph (see [G-3] [title\\_options](#)) and for saving the graph to disk (see [G-3] [saving\\_option](#)).

## Remarks and examples

### ▷ Example 1: ladder

We have data on the mileage rating of 74 automobiles and wish to find a transform that makes the variable normally distributed:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
```

```
. ladder mpg
```

| Transformation  | Formula               | chi2(2) | Prob > chi2 |
|-----------------|-----------------------|---------|-------------|
| Cubic           | $\text{mpg}^3$        | 43.59   | 0.000       |
| Square          | $\text{mpg}^2$        | 27.03   | 0.000       |
| Identity        | $\text{mpg}$          | 10.95   | 0.004       |
| Square root     | $\sqrt{\text{mpg}}$   | 4.94    | 0.084       |
| Log             | $\log(\text{mpg})$    | 0.87    | 0.647       |
| 1/(Square root) | $1/\sqrt{\text{mpg}}$ | 0.20    | 0.905       |
| Inverse         | $1/\text{mpg}$        | 2.36    | 0.307       |
| 1/Square        | $1/(\text{mpg}^2)$    | 11.99   | 0.002       |
| 1/Cubic         | $1/(\text{mpg}^3)$    | 24.30   | 0.000       |

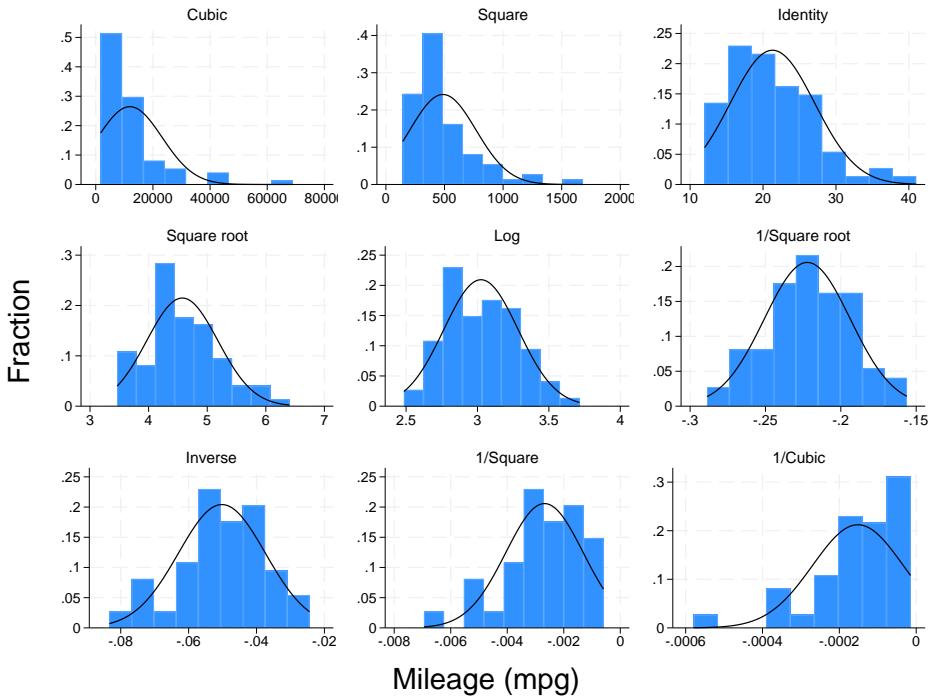
If we had typed `ladder mpg, gen(mpgx)`, the variable `mpgx` containing  $1/\sqrt{\text{mpg}}$  would have been automatically generated for us. This is the perfect example of why you should not, in general, specify the `generate()` option. We also cannot reject the hypothesis that the inverse of `mpg` is normally distributed and that  $1/\text{mpg}$ —gallons per mile—has a better interpretation. It is a measure of energy consumption.

◀

## ► Example 2: gladder

gladder explores the same transforms as ladder but presents results graphically:

```
. gladder mpg, fraction
```



Histograms by transformation



## □ Technical note

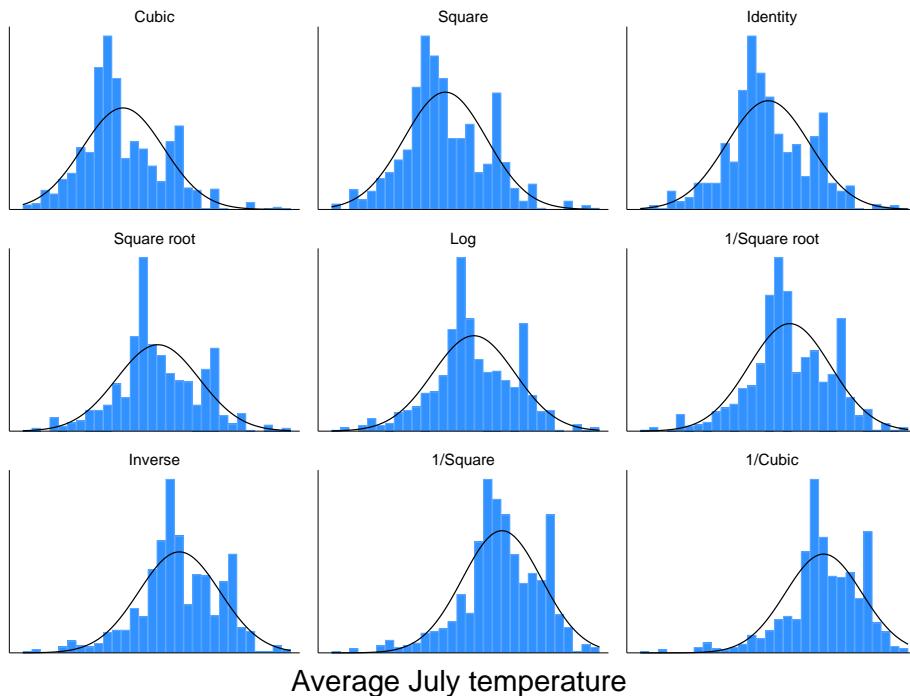
gladder is useful pedagogically, but be careful when using it for research work, especially with many observations. For instance, consider the following data on the average July temperature in degrees Fahrenheit for 954 US cities:

```
. use https://www.stata-press.com/data/r19/citytemp
(City temperature data)
. ladder tempjuly
```

| Transformation  | Formula                    | chi2(2) | Prob > chi2 |
|-----------------|----------------------------|---------|-------------|
| Cubic           | tempjuly <sup>3</sup>      | 47.49   | 0.000       |
| Square          | tempjuly <sup>2</sup>      | 19.70   | 0.000       |
| Identity        | tempjuly                   | 3.83    | 0.147       |
| Square root     | sqrt(tempjuly)             | 1.83    | 0.400       |
| Log             | log(tempjuly)              | 5.40    | 0.067       |
| 1/(Square root) | 1/sqrt(tempjuly)           | 13.72   | 0.001       |
| Inverse         | 1/tempjuly                 | 26.36   | 0.000       |
| 1/Square        | 1/(tempjuly <sup>2</sup> ) | 64.44   | 0.000       |
| 1/Cubic         | 1/(tempjuly <sup>3</sup> ) | 116.16  | 0.000       |

From the table, we see that there is certainly a difference in normality between the square and square-root transform. If, however, you can see the difference between the transforms in the diagram below, you have better eyes than we do:

```
. gladder tempjuly, l1title("") ylabel(none) xlabel(none)
```



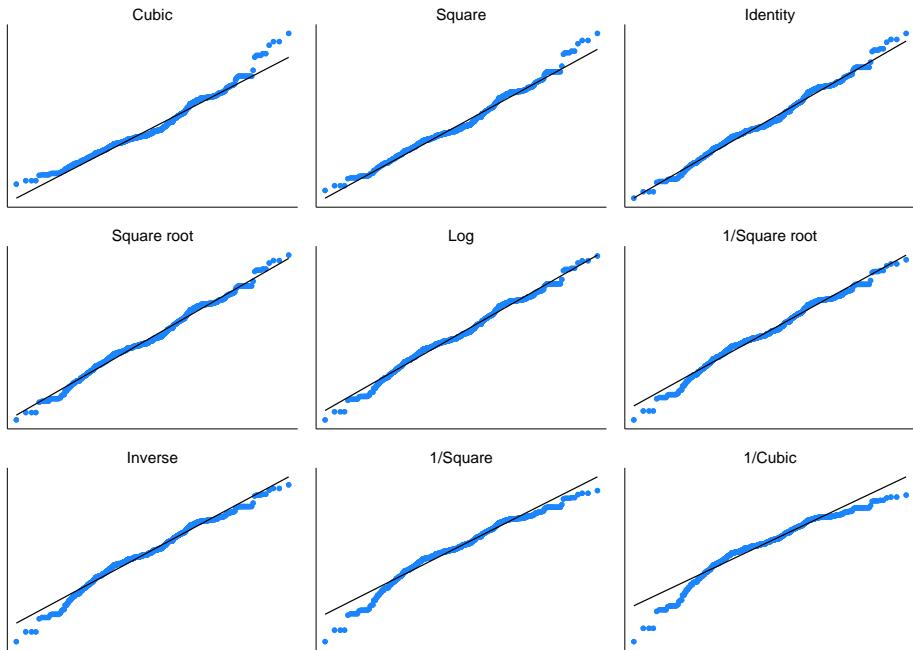
Histograms by transformation



### ▷ Example 3: qladder

A better graph for seeing normality is the quantile–normal graph, which can be produced by `qladder`.

```
. qladder tempjuly, ylabel(none) xlabel(none)
```



Average July temperature

Quantile–normal plots by transformation

This graph shows that for the square transform, the upper tail—and only the upper tail—diverges from what would be expected. This divergence is detected by `sktest` (see [R] [sktest](#)) as a problem with skewness, as we would learn from using `sktest` to examine `tempjuly squared` and `tempjuly sqrt`.

## Stored results

`ladder` stores the following in `r()`:

Scalars

|                           |                                                                   |
|---------------------------|-------------------------------------------------------------------|
| <code>r(N)</code>         | number of observations                                            |
| <code>r(invcube)</code>   | $\chi^2$ for inverse-cubic transformation                         |
| <code>r(P_invcube)</code> | $p$ -value for normality test after inverse-cubic transformation  |
| <code>r(invsq)</code>     | $\chi^2$ for inverse-square transformation                        |
| <code>r(P_invsq)</code>   | $p$ -value for normality test after inverse-square transformation |
| <code>r(inv)</code>       | $\chi^2$ for inverse transformation                               |
| <code>r(P_inv)</code>     | $p$ -value for normality test after inverse transformation        |
| <code>r(invsqrt)</code>   | $\chi^2$ for inverse-root transformation                          |
| <code>r(P_invsqrt)</code> | $p$ -value for normality test after inverse-root transformation   |
| <code>r(log)</code>       | $\chi^2$ for log transformation                                   |
| <code>r(P_log)</code>     | $p$ -value for normality test after log transformation            |
| <code>r(sqrt)</code>      | $\chi^2$ for square-root transformation                           |
| <code>r(P_sqrt)</code>    | $p$ -value for normality test after square-root transformation    |
| <code>r(ident)</code>     | $\chi^2$ for untransformed data                                   |
| <code>r(P_ident)</code>   | $p$ -value for normality test of untransformed data               |
| <code>r(square)</code>    | $\chi^2$ for square transformation                                |
| <code>r(P_square)</code>  | $p$ -value for normality test after square transformation         |
| <code>r(cube)</code>      | $\chi^2$ for cubic transformation                                 |
| <code>r(P_cube)</code>    | $p$ -value for normality test after cubic transformation          |

## Methods and formulas

For `ladder`, results are as reported by `sktest`; see [\[R\] sktest](#). If `generate()` is specified, the transform with the minimum  $\chi^2$  value is chosen.

`gladder` sets the number of bins to  $\min(\sqrt{n}, 10 \log_{10} n)$ , rounded to the closest integer, where  $n$  is the number of unique values of `varname`. See [\[R\] histogram](#) for a discussion of the optimal number of bins.

Also see [Findley \(1990\)](#) for a ladder-of-powers variable transformation program that produces one-way graphs with overlaid box plots, in addition to histograms with overlaid normals. [Buchner and Findley \(1990\)](#) discuss ladder-of-powers transformations as one aspect of preliminary data analysis. Also see [Hamilton \(1992, 18–23\)](#) and [Hamilton \(2013, 129–132\)](#).

## Acknowledgment

`qladder` was written by Jeroen Weesie of the Department of Sociology at Utrecht University, The Netherlands.

## References

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- Hamilton, L. C. 1992. *Regression with Graphics: A Second Course in Applied Statistics*. Belmont, CA: Duxbury.
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## Also see

- [R] **boxcox** — Box–Cox regression models
- [R] **Diagnostic plots** — Distributional diagnostic plots
- [R] **lnskew0** — Find zero-skewness log or Box–Cox transform
- [R] **lv** — Letter-value displays
- [R] **sktest** — Skewness and kurtosis tests for normality

## Description

`set level` specifies the default confidence level for confidence intervals for all commands that report confidence intervals. The initial value is 95, meaning 95% confidence intervals.

## Syntax

```
set level # [, permanently]
```

# is any number between 10.00 and 99.99 and may be specified with at most two digits after the decimal point.

## Option

`permanently` specifies that, in addition to making the change right now, the `level` setting be remembered and become the default setting when you invoke Stata.

## Remarks and examples

To change the level of confidence intervals reported by a particular command, you need not reset the default confidence level. All commands that report confidence intervals have a `level(#)` option. When you do not specify the option, the confidence intervals are calculated for the default level set by `set level`, or for 95% if you have not reset `set level`.

### ▷ Example 1

We use the `ci means` command to obtain the confidence interval for the mean of `mpg`:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. ci means mpg
```

| Variable | Obs | Mean    | Std. err. | [95% conf. interval] |          |
|----------|-----|---------|-----------|----------------------|----------|
| mpg      | 74  | 21.2973 | .6725511  | 19.9569              | 22.63769 |

To obtain 90% confidence intervals, we would type

```
. ci means mpg, level(90)
```

| Variable | Obs | Mean    | Std. err. | [90% conf. interval] |          |
|----------|-----|---------|-----------|----------------------|----------|
| mpg      | 74  | 21.2973 | .6725511  | 20.17683             | 22.41776 |

or

```
. set level 90
. ci means mpg
```

| Variable | Obs | Mean    | Std. err. | [90% conf. interval] |          |
|----------|-----|---------|-----------|----------------------|----------|
| mpg      | 74  | 21.2973 | .6725511  | 20.17683             | 22.41776 |

If we opt for the second alternative, the next time that we fit a model (say, with `regress`), 90% confidence intervals will be reported. If we wanted 95% confidence intervals, we could specify `level(95)` on the estimation command, or we could reset the default by typing `set level 95`.

The current setting of `level()` is stored as the c-class value `c(level)`; see [P] [creturn](#).



## Also see

[R] [query](#) — Display system parameters

[P] [creturn](#) — Return c-class values

[U] [20 Estimation and postestimation commands](#)

[U] [20.8 Specifying the width of confidence intervals](#)

## Description

This entry provides a quick reference for the size limits in Stata. Note that most of these limits are so high that you will never encounter them.

## Remarks and examples

Remarks are presented under the following headings:

*Maximum size limits*  
*Determining which edition of Stata you are running*

### Maximum size limits

|                                                  | Stata/BE      | Stata/MP and<br>Stata/SE                             |
|--------------------------------------------------|---------------|------------------------------------------------------|
| # of observations                                | 2,147,483,619 | 1,099,511,627,775 (MP) (1)<br>2,147,483,619 (SE) (1) |
| # of variables                                   | 2,048         | 120,000 (MP)<br>32,767 (SE)                          |
| # of RHS variables                               | 798           | 65,532 (MP)<br>10,998 (SE)                           |
| # characters in a command                        | 264,408       | 4,227,159                                            |
| # options for a command                          | 256           | 256                                                  |
| # of elements in a <i>numlist</i>                | 2,500         | 2,500                                                |
| # of interacted continuous variables             | 64            | 64                                                   |
| # of interacted factor variables                 | 8             | 8                                                    |
| # of unique time-series operators in a command   | 100           | 100                                                  |
| # seasonal suboperators per time-series operator | 8             | 8                                                    |
| # of dyadic operators in an expression           | 800           | 800                                                  |
| # of numeric literals in an expression           | 300           | 300                                                  |
| # of string literals in an expression            | 512           | 512                                                  |
| length of string in string expression (bytes)    | 2,000,000,000 | 2,000,000,000                                        |
| # of sum functions in an expression              | 5             | 5                                                    |
| # of pairs of nested parentheses                 | 249           | 249                                                  |
| # of characters in a macro (2)                   | 264,392       | 15,480,200 (MP)<br>4,227,143 (SE)                    |

| <i>continued</i>                                       | Stata/BE      | Stata/MP and<br>Stata/SE |
|--------------------------------------------------------|---------------|--------------------------|
| # of nested do-files                                   | 64            | 64                       |
| # of lines in a program                                | 3,500         | 3,500                    |
| # of bytes in a program                                | 135,600       | 135,600                  |
| length of a variable name (characters)                 | 32            | 32                       |
| length of ado-command name (characters)                | 32            | 32                       |
| length of a global macro name (characters)             | 32            | 32                       |
| length of a local macro name (characters)              | 31            | 31                       |
| length of a <code>str#</code> variable (bytes)         | 2,045         | 2,045                    |
| length of a <code>strL</code> variable (bytes)         | 2,000,000,000 | 2,000,000,000            |
| <i>anova</i>                                           |               |                          |
| # of variables in one <code>anova</code> term          | 8             | 8                        |
| # of terms in the <code>repeated()</code> option       | 4             | 4                        |
| <i>char</i>                                            |               |                          |
| length of one characteristic (bytes)                   | 67,784        | 67,784                   |
| <i>constraint</i>                                      |               |                          |
| # of constraints                                       | 1,999         | 1,999                    |
| <i>encode</i> and <i>decode</i>                        |               |                          |
| # of unique values                                     | 65,536        | 65,536                   |
| <i>_estimates hold</i>                                 |               |                          |
| # of stored estimation results                         | 300           | 300                      |
| <i>estimates store</i>                                 |               |                          |
| # of stored estimation results                         | 300           | 300                      |
| <i>exlogistic</i> and <i>expoisson</i>                 |               |                          |
| maximum memory specification in <code>memory(#)</code> | 2gb           | 2gb                      |
| <i>frames</i>                                          |               |                          |
| # of frames                                            | 100           | 100                      |
| <i>grmeanby</i>                                        |               |                          |
| # of unique values in <i>varlist</i>                   | $\_N/2$       | $\_N/2$                  |
| <i>graph</i>                                           |               |                          |
| minimum graph size (inches)                            | 1             | 1                        |
| maximum graph size (inches)                            | 100           | 100                      |
| <i>graph twoway</i>                                    |               |                          |
| # of variables in a plot                               | 100           | 100                      |
| # of styles in an option's <code>stylelist</code>      | 20            | 20                       |
| <i>import sas</i>                                      |               |                          |
| # of variables                                         | 2,048         | 32,766                   |
| <i>import spss</i>                                     |               |                          |
| # of variables                                         | 2,048         | 32,766                   |

| <i>continued</i>                           | Stata/BE  | Stata/MP and<br>Stata/SE                     |
|--------------------------------------------|-----------|----------------------------------------------|
| <i>infile</i> (free format)                |           |                                              |
| record length without dictionary           | none      | none                                         |
| <i>infile</i> (fixed format)               |           |                                              |
| record length with a dictionary            | 524,275   | 524,275                                      |
| <i>infix</i> (fixed format)                |           |                                              |
| record length with a dictionary            | 524,275   | 524,275                                      |
| <i>label</i>                               |           |                                              |
| length of dataset label (characters)       | 80        | 80                                           |
| length of variable label (characters)      | 80        | 80                                           |
| length of value label string (bytes)       | 32,000    | 32,000                                       |
| length of name of value label (characters) | 32        | 32                                           |
| # of codings within one value label        | 65,536    | 65,536                                       |
| <i>label language</i>                      |           |                                              |
| # of different languages                   | 100       | 100                                          |
| <i>macro</i>                               |           |                                              |
| # of nested macros                         | 20        | 20                                           |
| <i>manova</i>                              |           |                                              |
| # of variables in single manova term       | 8         | 8                                            |
| <i>matrix</i> (3)                          |           |                                              |
| dimension of single matrix                 | 800 × 800 | 65,534 × 65,534 (MP)<br>11,000 × 11,000 (SE) |
| <i>maximize</i> options                    |           |                                              |
| <i>iterate</i> () maximum                  | 16,000    | 16,000                                       |
| <i>mprobit</i>                             |           |                                              |
| # of categories in a <i>depvar</i>         | 30        | 30                                           |
| <i>net</i>                                 |           |                                              |
| # of description lines in .pkg file        | 100       | 100                                          |
| <i>nlogit</i> and <i>nlogittree</i>        |           |                                              |
| # of levels in model                       | 8         | 8                                            |
| <i>notes</i>                               |           |                                              |
| length of one note (bytes)                 | 67,784    | 67,784                                       |
| # of notes attached to <i>_dta</i>         | 9,999     | 9,999                                        |
| # of notes attached to each variable       | 9,999     | 9,999                                        |
| <i>numlist</i>                             |           |                                              |
| # of elements in the numeric list          | 2,500     | 2,500                                        |
| <i>_pctile</i>                             |           |                                              |
| # of percentiles                           | 4,096     | 4,096                                        |

| <i>continued</i>                                                                                                                                                                                                                                                                                                                                        | Stata/BE    | Stata/MP and<br>Stata/SE   |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------|----------------------------|
| <code>putdocx</code> and <code>putpdf</code><br># of tables added to a document                                                                                                                                                                                                                                                                         | 10,000      | 10,000                     |
| <code>reg3</code> , <code>sureg</code> , and other system estimators<br># of equations                                                                                                                                                                                                                                                                  | 800         | 65,534 (MP)<br>11,000 (SE) |
| <code>set adosize</code><br>memory ado-files may consume                                                                                                                                                                                                                                                                                                | 1000K       | 1000K                      |
| <code>set scrollbufsize</code><br>memory for Results window buffer                                                                                                                                                                                                                                                                                      | 2000K       | 2000k                      |
| <code>slogit</code><br># of categories in a <i>depvar</i>                                                                                                                                                                                                                                                                                               | 30          | 30                         |
| <code>snapshot</code><br>length of label (characters)<br># of saved snapshots                                                                                                                                                                                                                                                                           | 80<br>1,000 | 80<br>1,000                |
| <code>stcox</code><br># of variables in <code>strata()</code> option                                                                                                                                                                                                                                                                                    | 5           | 5                          |
| <code>stcurve</code><br># of curves plotted on the same graph                                                                                                                                                                                                                                                                                           | 10          | 10                         |
| <code>tabdisp</code><br># of by variables<br># of margins, i.e., sum of rows,<br>columns, supercolumns, and by groups                                                                                                                                                                                                                                   | 4<br>3,000  | 4<br>3,000                 |
| <code>tabulate oneway</code><br># of rows in one-way table                                                                                                                                                                                                                                                                                              | 3,000       | 12,000                     |
| <code>tabulate twoway</code><br># of rows & cols in two-way table                                                                                                                                                                                                                                                                                       | 300 × 20    | 1,200 × 80                 |
| <code>tabulate, summarize()</code><br># of cells (rows X cols)                                                                                                                                                                                                                                                                                          | 375         | 375                        |
| <code>teffects</code><br># of treatments                                                                                                                                                                                                                                                                                                                | 20          | 20                         |
| <code>xt</code> estimation commands (e.g., <code>xtgee</code> ,<br><code>xtgls</code> , <code>xtpoisson</code> , <code>xtprobit</code> , <code>xtreg</code><br>with <code>mle</code> option, and <code>xtpcse</code> when<br>neither option <code>hetonly</code> nor option<br><code>independent</code> is specified)<br># of time periods within panel | 800         | 65,534 (MP)<br>11,000 (SE) |
| # of integration points accepted<br>by <code>intpoints(#)</code>                                                                                                                                                                                                                                                                                        | 195         | 195                        |

- (1) For Stata/MP, the maximum number of observations is 1,099,511,627,775, and for Stata/SE, the maximum number is 2,147,483,619. In practice, both editions are limited by memory.
- (2) The maximum length of the contents of a macro are fixed in Stata/BE and settable in Stata/SE and Stata/MP. The currently set maximum length is recorded in `c(macrolen)`; type `display c(macrolen)`. The maximum length can be changed with `set maxvar`. If you set `maxvar` to a larger value, the maximum length increases; if you set `maxvar` to a smaller value, the maximum length decreases. The relationship between them is  $maximum\_length = 129 \times maxvar + 200$ .
- (3) In Mata, matrices are limited only by the amount of memory on your computer.

## Determining which edition of Stata you are running

Type

```
. about
```

The response will be Stata/MP, Stata/SE, or Stata/BE. Other information is also shown, including your serial number. See [R] [about](#).

## Also see

[R] [about](#) — Display information about your Stata

[D] [compress](#) — Compress data in memory

[D] [Data types](#) — Quick reference for data types

[D] [import](#) — Overview of importing data into Stata

[D] [infile \(fixed format\)](#) — Import text data in fixed format with a dictionary

[D] [infile \(free format\)](#) — Import unformatted text data

[D] [memory](#) — Memory management

[D] [obs](#) — Increase the number of observations in a dataset

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[References](#)[Syntax](#)[Also see](#)[Options](#)

## Description

`lincom` computes point estimates, standard errors,  $t$  or  $z$  statistics,  $p$ -values, and confidence intervals for linear combinations of parameters after any estimation command, including survey estimation. Results can optionally be displayed as odds ratios, hazard ratios, incidence-rate ratios, or relative-risk ratios.

## Quick start

Point estimate and confidence interval for sum of coefficients of `x1` and `x2`

```
lincom x1 + x2
```

Same as above, but report results as a relative-risk ratio

```
lincom x1 + x2, rrr
```

Same as above, but use coefficients from second equation of a multiequation model

```
lincom [2]x1 + [2]x2, rrr
```

Difference between coefficients of first and third level of categorical variable `a`

```
lincom 1.a - 3.a
```

Sum of coefficients of `x1` and `x2` after a model adjusted for complex survey design

```
lincom x1 + x2
```

## Menu

Statistics > Postestimation

## Syntax

```
lincom exp [, options]
```

| <i>options</i>               | Description                                                                                       |
|------------------------------|---------------------------------------------------------------------------------------------------|
| <code>eform</code>           | generic label; $\exp(b)$                                                                          |
| <code>or</code>              | odds ratio                                                                                        |
| <code>hr</code>              | hazard ratio                                                                                      |
| <code>shr</code>             | subhazard ratio                                                                                   |
| <code>irr</code>             | incidence-rate ratio                                                                              |
| <code>rrr</code>             | relative-risk ratio                                                                               |
| <code>level(#)</code>        | set confidence level; default is <code>level(95)</code>                                           |
| <code>display_options</code> | control column formats                                                                            |
| <code>df(#)</code>           | use $t$ distribution with # degrees of freedom for computing $p$ -values and confidence intervals |

*exp* is any linear combination of parameters that is a valid syntax for `test`; see [R] `test`. *exp* must not contain an equal sign.

`collect` is allowed; see [U] 11.1.10 **Prefix commands**.

`df(#)` does not appear in the dialog box.

## Options

`eform`, `or`, `hr`, `shr`, `irr`, and `rrr` all report coefficient estimates as  $\exp(\hat{\beta})$  rather than  $\hat{\beta}$ . Standard errors and confidence intervals are similarly transformed. `or` is the default after `logistic`. The only difference in these options is how the output is labeled.

| Option             | Label                   | Explanation          | Example commands             |
|--------------------|-------------------------|----------------------|------------------------------|
| <code>eform</code> | <code>exp(b)</code>     | Generic label        | <code>cloglog</code>         |
| <code>or</code>    | <code>Odds ratio</code> | Odds ratio           | <code>logistic, logit</code> |
| <code>hr</code>    | <code>Haz. ratio</code> | Hazard ratio         | <code>stcox, streg</code>    |
| <code>shr</code>   | <code>SHR</code>        | Subhazard ratio      | <code>stcrreg</code>         |
| <code>irr</code>   | <code>IRR</code>        | Incidence-rate ratio | <code>poisson</code>         |
| <code>rrr</code>   | <code>RRR</code>        | Relative-risk ratio  | <code>mlogit</code>          |

*exp* may not contain any additive constants when you use the `eform`, `or`, `hr`, `irr`, or `rrr` option.

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [U] 20.8 **Specifying the width of confidence intervals**.

`display_options`: `cformat(%fmt)`, `pformat(%fmt)`, and `sformat(%fmt)`; see [R] **Estimation options**.

The following option is available with `lincom` but is not shown in the dialog box:

`df(#)` specifies that the  $t$  distribution with # degrees of freedom be used for computing  $p$ -values and confidence intervals. The default is to use `e(df_r)` degrees of freedom or the standard normal distribution if `e(df_r)` is missing.

## Remarks and examples

Remarks are presented under the following headings:

*Using lincom*  
*Odds ratios and incidence-rate ratios*  
*Multiple-equation models*

### Using lincom

After fitting a model and obtaining estimates for coefficients  $\beta_1, \beta_2, \dots, \beta_k$ , you may want to view estimates for linear combinations of the  $\beta_i$ , such as  $\beta_1 - \beta_2$ . `lincom` can display estimates for any linear combination of the form  $c_0 + c_1\beta_1 + c_2\beta_2 + \dots + c_k\beta_k$ .

`lincom` works after any estimation command for which `test` works. Any valid expression for `test` syntax 1 (see [R] `test`) is a valid expression for `lincom`.

`lincom` is useful for viewing odds ratios, hazard ratios, etc., for one group (that is, one set of covariates) relative to another group (that is, another set of covariates). See the examples below.

#### ► Example 1

We perform a linear regression:

```
. use https://www.stata-press.com/data/r19/regress
. regress y x1 x2 x3
```

| Source   | SS         | df  | MS         | Number of obs | = | 148    |
|----------|------------|-----|------------|---------------|---|--------|
| Model    | 3259.3561  | 3   | 1086.45203 | F(3, 144)     | = | 96.12  |
| Residual | 1627.56282 | 144 | 11.3025196 | Prob > F      | = | 0.0000 |
|          |            |     |            | R-squared     | = | 0.6670 |
|          |            |     |            | Adj R-squared | = | 0.6600 |
| Total    | 4886.91892 | 147 | 33.2443464 | Root MSE      | = | 3.3619 |

| y     | Coefficient | Std. err. | t      | P> t  | [95% conf. interval] |           |
|-------|-------------|-----------|--------|-------|----------------------|-----------|
| x1    | 1.457113    | 1.07461   | 1.36   | 0.177 | -.666934             | 3.581161  |
| x2    | 2.221682    | .8610358  | 2.58   | 0.011 | .5197797             | 3.923583  |
| x3    | -.006139    | .0005543  | -11.08 | 0.000 | -.0072345            | -.0050435 |
| _cons | 36.10135    | 4.382693  | 8.24   | 0.000 | 27.43863             | 44.76407  |

To see the difference of the coefficients of `x2` and `x1`, we type

```
. lincom x2 - x1
(1) - x1 + x2 = 0
```

| y   | Coefficient | Std. err. | t    | P> t  | [95% conf. interval] |          |
|-----|-------------|-----------|------|-------|----------------------|----------|
| (1) | .7645682    | .9950282  | 0.77 | 0.444 | -1.20218             | 2.731316 |

The expression can be any linear combination.

```
. lincom 3*x1 + 500*x3
(1) 3*x1 + 500*x3 = 0
```

| y   | Coefficient | Std. err. | t    | P> t  | [95% conf. interval] |          |
|-----|-------------|-----------|------|-------|----------------------|----------|
| (1) | 1.301825    | 3.396624  | 0.38 | 0.702 | -5.411858            | 8.015507 |

Nonlinear expressions are not allowed.

```
. lincom x2/x1
not possible with test
r(131);
```

For information about estimating nonlinear expressions, see [R] [nlcom](#).



### □ Technical note

`lincom` uses the same shorthands for coefficients as does `test` (see [R] [test](#)). When you type `x1`, for instance, `lincom` knows that you mean the coefficient of `x1`. The formal syntax for referencing this coefficient is actually `_b[x1]`, or alternatively, `_coef[x1]`. So, more formally, in the last example we could have typed

```
. lincom 3*_b[x1] + 500*_b[x3]
```



## Odds ratios and incidence-rate ratios

After logistic regression, the `or` option can be specified with `lincom` to display odds ratios for any effect. Incidence-rate ratios after commands such as `poisson` can be similarly obtained by specifying the `irr` option.

### ▷ Example 2

Consider the low birthweight dataset from [Hosmer, Lemeshow, and Sturdivant \(2013, 24\)](#). We fit a logistic regression model of low birthweight (variable `low`) on the following variables:

| Variable                | Description                                                              | Coding                             |
|-------------------------|--------------------------------------------------------------------------|------------------------------------|
| <code>age</code>        | age in years                                                             |                                    |
| <code>race</code>       | race                                                                     | 1 if white, 2 if black, 3 if other |
| <code>smoke</code>      | smoking status                                                           | 1 if smoker, 0 if nonsmoker        |
| <code>ht</code>         | history of hypertension                                                  | 1 if yes, 0 if no                  |
| <code>ui</code>         | uterine irritability                                                     | 1 if yes, 0 if no                  |
| <code>lwd</code>        | maternal weight before pregnancy                                         | 1 if weight < 110 lb., 0 otherwise |
| <code>ptd</code>        | history of premature labor                                               | 1 if yes, 0 if no                  |
| <code>c.age##lwd</code> | age main effects, <code>lwd</code> main effects, and their interaction   |                                    |
| <code>smoke##lwd</code> | smoke main effects, <code>lwd</code> main effects, and their interaction |                                    |

We first fit a model without the interaction terms by using logit.

```
. use https://www.stata-press.com/data/r19/lbw3
(Hosmer & Lemeshow data)
. logit low age lwd i.race smoke ptd ht ui
Iteration 0: Log likelihood = -117.336
Iteration 1: Log likelihood = -99.3982
Iteration 2: Log likelihood = -98.780418
Iteration 3: Log likelihood = -98.777998
Iteration 4: Log likelihood = -98.777998
Logistic regression
Number of obs = 189
LR chi2(8) = 37.12
Prob > chi2 = 0.0000
Pseudo R2 = 0.1582
Log likelihood = -98.777998
```

| low   | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-------|-------------|-----------|-------|-------|----------------------|----------|
| age   | -.0464796   | .0373888  | -1.24 | 0.214 | -.1197603            | .0268011 |
| lwd   | .8420615    | .4055338  | 2.08  | 0.038 | .0472299             | 1.636893 |
| race  |             |           |       |       |                      |          |
| Black | 1.073456    | .5150753  | 2.08  | 0.037 | .0639273             | 2.082985 |
| Other | .815367     | .4452979  | 1.83  | 0.067 | -.0574008            | 1.688135 |
| smoke | .8071996    | .404446   | 2.00  | 0.046 | .0145001             | 1.599899 |
| ptd   | 1.281678    | .4621157  | 2.77  | 0.006 | .3759478             | 2.187408 |
| ht    | 1.435227    | .6482699  | 2.21  | 0.027 | .1646414             | 2.705813 |
| ui    | .6576256    | .4666192  | 1.41  | 0.159 | -.2569313            | 1.572182 |
| _cons | -1.216781   | .9556797  | -1.27 | 0.203 | -3.089878            | .656317  |

To get the odds ratio for black smokers relative to white nonsmokers (the reference group), we type

```
. lincom 2.race + smoke, or
(1) [low]2.race + [low]smoke = 0
```

| low | Odds ratio | Std. err. | z    | P> z  | [95% conf. interval] |          |
|-----|------------|-----------|------|-------|----------------------|----------|
| (1) | 6.557805   | 4.744692  | 2.60 | 0.009 | 1.588176             | 27.07811 |

lincom computed  $\exp(\beta_{2.race} + \beta_{smoke}) = 6.56$ . To see the odds ratio for white smokers relative to black nonsmokers, we type

```
. lincom smoke - 2.race, or
(1) - [low]2.race + [low]smoke = 0
```

| low | Odds ratio | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-----|------------|-----------|-------|-------|----------------------|----------|
| (1) | .7662425   | .4430176  | -0.46 | 0.645 | .2467334             | 2.379603 |

Now let's add the interaction terms to the model (Hosmer and Lemeshow 1989, table 4.10). This time, we will use logistic rather than logit. By default, logistic displays odds ratios.

```
. logistic low i.race ht ui ptd c.age##lwd smoke##lwd
Logistic regression Number of obs = 189
 LR chi2(10) = 42.66
 Prob > chi2 = 0.0000
Log likelihood = -96.00616 Pseudo R2 = 0.1818
```

| low       | Odds ratio | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-----------|------------|-----------|-------|-------|----------------------|----------|
| race      |            |           |       |       |                      |          |
| Black     | 2.95383    | 1.532789  | 2.09  | 0.037 | 1.068277             | 8.167465 |
| Other     | 2.137589   | .9919138  | 1.64  | 0.102 | .8608708             | 5.307752 |
| ht        |            |           |       |       |                      |          |
| ui        | 2.071284   | .9931388  | 1.52  | 0.129 | .8092926             | 5.301192 |
| ptd       | 3.426633   | 1.615282  | 2.61  | 0.009 | 1.360252             | 8.632089 |
| age       | .9194513   | .041896   | -1.84 | 0.065 | .8408967             | 1.005344 |
| 1.lwd     | .1772934   | .3312384  | -0.93 | 0.354 | .0045539             | 6.902367 |
| lwd#c.age |            |           |       |       |                      |          |
| 1         | 1.15883    | .09602    | 1.78  | 0.075 | .9851215             | 1.36317  |
| smoke     |            |           |       |       |                      |          |
| Smoker    | 3.168096   | 1.452378  | 2.52  | 0.012 | 1.289956             | 7.78076  |
| smoke#lwd |            |           |       |       |                      |          |
| Smoker#1  | .2447849   | .2003996  | -1.72 | 0.086 | .0491956             | 1.217988 |
| _cons     |            |           |       |       |                      |          |
|           | .599443    | .6519163  | -0.47 | 0.638 | .0711271             | 5.051971 |

Note: **\_cons** estimates baseline odds.

Hosmer and Lemeshow (1989, table 4.13) consider the effects of smoking (smoke = 1) and low maternal weight before pregnancy (lwd = 1). The effect of smoking among non-low-weight mothers (lwd = 0) is given by the odds ratio 3.17 for smoke in the logistic output. The effect of smoking among low-weight mothers is given by

```
. lincom 1.smoke + 1.smoke#1.lwd
(1) [low]1.smoke + [low]1.smoke#1.lwd = 0
```

| low | Odds ratio | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-----|------------|-----------|-------|-------|----------------------|----------|
| (1) | .7755022   | .574951   | -0.34 | 0.732 | .1813465             | 3.316323 |

We did not have to specify the or option. After logistic, lincom assumes or by default.

The effect of low weight (lwd = 1) is more complicated because we fit an age × lwd interaction. We must specify the age of mothers for the effect. The effect among 30-year-old nonsmokers is given by

```
. lincom 1.lwd + 30*1.lwd#c.age
(1) [low]1.lwd + 30*[low]1.lwd#c.age = 0
```

| low | Odds ratio | Std. err. | z    | P> z  | [95% conf. interval] |          |
|-----|------------|-----------|------|-------|----------------------|----------|
| (1) | 14.7669    | 13.5669   | 2.93 | 0.003 | 2.439264             | 89.39633 |

lincom computed  $\exp(\beta_{1wd} + 30\beta_{age1wd}) = 14.8$ . It may seem odd that we entered it as `1.1wd + 30*1.1wd#c.age`, but remember that these terms are just lincom's (and test's) shorthands for `_b[1.1wd]` and `_b[1.1wd#c.age]`. We could have typed

```
. lincom _b[1.1wd] + 30*_b[1.1wd#c.age]
(1) [low]1.1wd + 30*[low]1.1wd#c.age = 0
```

|     | low | Odds ratio | Std. err. | z    | P> z  | [95% conf. interval] |          |
|-----|-----|------------|-----------|------|-------|----------------------|----------|
| (1) |     | 14.7669    | 13.5669   | 2.93 | 0.003 | 2.439264             | 89.39633 |



## Multiple-equation models

lincom also works with multiple-equation models. The only difference is how you refer to the coefficients. Recall that for multiple-equation models, coefficients are referenced using the syntax

`[eqno]varname`

where *eqno* is the equation number or equation name and *varname* is the corresponding variable name for the coefficient; see [\[U\] 13.5 Accessing coefficients and standard errors](#) and [\[R\] test](#) for details.

### ▷ Example 3

Let's consider [example 4](#) from [\[R\] mlogit](#) (Tarlov et al. 1989; Wells et al. 1989).

```
. use https://www.stata-press.com/data/r19/sysdsn1
(Health insurance data)
. mlogit insure age male nonwhite i.site, nolog
(output omitted)
```

To see the estimate of the sum of the coefficient of `male` and the coefficient of `nonwhite` for the `Prepaid` outcome, we type

```
. lincom [Prepaid]male + [Prepaid]nonwhite
(1) [Prepaid]male + [Prepaid]nonwhite = 0
```

|     | insure | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |          |
|-----|--------|-------------|-----------|------|-------|----------------------|----------|
| (1) |        | 1.53647     | .3272489  | 4.70 | 0.000 | .8950741             | 2.177866 |

To view the estimate as a ratio of relative risks (see [\[R\] mlogit](#) for the definition and interpretation), we specify the `rrr` option.

```
. lincom [Prepaid]male + [Prepaid]nonwhite, rrr
(1) [Prepaid]male + [Prepaid]nonwhite = 0
```

|     | insure | RRR      | Std. err. | z    | P> z  | [95% conf. interval] |          |
|-----|--------|----------|-----------|------|-------|----------------------|----------|
| (1) |        | 4.648154 | 1.521103  | 4.70 | 0.000 | 2.447517             | 8.827451 |



## Stored results

lincom stores the following in `r()`:

Scalars

|                                        |                                    |
|----------------------------------------|------------------------------------|
| <code>r(estimate)</code>               | point estimate                     |
| <code>r(se)</code>                     | estimate of standard error         |
| <code>r(df)</code>                     | degrees of freedom                 |
| <code>r(t)</code> or <code>r(z)</code> | $t$ or $z$ statistic               |
| <code>r(p)</code>                      | $p$ -value                         |
| <code>r(lb)</code>                     | lower bound of confidence interval |
| <code>r(ub)</code>                     | upper bound of confidence interval |
| <code>r(level)</code>                  | confidence level                   |

## References

- Hosmer, D. W., Jr., and S. A. Lemeshow. 1989. *Applied Logistic Regression*. New York: Wiley.
- Hosmer, D. W., Jr., S. A. Lemeshow, and R. X. Sturdivant. 2013. *Applied Logistic Regression*. 3rd ed. Hoboken, NJ: Wiley.
- Tarlov, A. R., J. E. Ware, Jr., S. Greenfield, E. C. Nelson, E. Perrin, and M. Zubkoff. 1989. The medical outcomes study. An application of methods for monitoring the results of medical care. *Journal of the American Medical Association* 262: 925–930. <https://doi.org/10.1001/jama.1989.03430070073033>.
- Wells, K. B., R. D. Hays, M. A. Burnam, W. H. Rogers, S. Greenfield, and J. E. Ware, Jr. 1989. Detection of depressive disorder for patients receiving prepaid or fee-for-service care. Results from the Medical Outcomes Survey. *Journal of the American Medical Association* 262: 3298–3302. <https://doi.org/10.1001/jama.1989.03430230083030>.

## Also see

- [R] **nlcom** — Nonlinear combinations of parameters
- [R] **test** — Test linear hypotheses after estimation
- [R] **testnl** — Test nonlinear hypotheses after estimation
- [SVY] **svy postestimation** — Postestimation tools for svy
- [U] **13.5 Accessing coefficients and standard errors**
- [U] **20 Estimation and postestimation commands**

Description  
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References

Quick start  
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Also see

Menu  
Stored results

Syntax  
Methods and formulas

## Description

`linktest` performs a link test for model specification.

## Quick start

Specification link test after a single-equation estimation command without options

```
linktest
```

After `tobit` estimated with right-censoring limit at 24

```
linktest, ul(24)
```

After `stcox` estimated with Efron method for tied failures

```
linktest, efron
```

Perform test on full dataset when estimation used a subset of observations

```
linktest if e(sample) < .
```

## Menu

Statistics > Postestimation

## Syntax

```
linktest [if] [in] [, cmd_options]
```

When `if` and `in` are not specified, the link test is performed on the same sample as the previous estimation.

`collect` is allowed; see [U] [11.1.10 Prefix commands](#).

## Option

Main

*cmd\_options* must be the same options specified with the underlying estimation command, except the *display\_options* may differ.

## Remarks and examples

The form of the link test implemented here is based on an idea of [Tukey \(1949\)](#), which was further described by [Pregibon \(1980\)](#), elaborating on work in his unpublished thesis ([Pregibon 1979](#)). See *Methods and formulas* below for more details.

### ▷ Example 1

We want to explain the mileage ratings of cars in our automobile dataset by using the weight, engine displacement, and whether the car is manufactured outside the United States:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. regress mpg weight displ foreign
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 1619.71935 | 3  | 539.906448 | F(3, 70)      | = | 45.88  |
| Residual | 823.740114 | 70 | 11.7677159 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.6629 |
|          |            |    |            | Adj R-squared | = | 0.6484 |
| Total    | 2443.45946 | 73 | 33.4720474 | Root MSE      | = | 3.4304 |

| mpg          | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|--------------|-------------|-----------|-------|-------|----------------------|-----------|
| weight       | -.0067745   | .0011665  | -5.81 | 0.000 | -.0091011            | -.0044479 |
| displacement | .0019286    | .0100701  | 0.19  | 0.849 | -.0181556            | .0220129  |
| foreign      | -1.600631   | 1.113648  | -1.44 | 0.155 | -3.821732            | .6204699  |
| _cons        | 41.84795    | 2.350704  | 17.80 | 0.000 | 37.15962             | 46.53628  |

On the basis of the  $R^2$ , we are reasonably pleased with this model.

If our model really is specified correctly, then if we were to regress mpg on the prediction and the prediction squared, the prediction squared would have no explanatory power. This is what `linktest` does:

```
. linktest
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 1670.71514 | 2  | 835.357572 | F(2, 71)      | = | 76.75  |
| Residual | 772.744316 | 71 | 10.8837228 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.6837 |
|          |            |    |            | Adj R-squared | = | 0.6748 |
| Total    | 2443.45946 | 73 | 33.4720474 | Root MSE      | = | 3.299  |

| mpg    | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|--------|-------------|-----------|-------|-------|----------------------|----------|
| _hat   | -.4127198   | .6577736  | -0.63 | 0.532 | -1.724283            | .8988434 |
| _hatsq | .0338198    | .015624   | 2.16  | 0.034 | .0026664             | .0649732 |
| _cons  | 14.00705    | 6.713276  | 2.09  | 0.041 | .6211539             | 27.39294 |

We find that the prediction squared does have explanatory power, so our specification is not as good as we thought.

Although linktest is formally a test of the specification of the dependent variable, it is often interpreted as a test that, conditional on the specification, the independent variables are specified incorrectly. We will follow that interpretation and now include weight squared in our model:

```
. regress mpg weight c.weight#c.weight displ foreign
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 1699.02634 | 4  | 424.756584 | F(4, 69)      | = | 39.37  |
| Residual | 744.433124 | 69 | 10.7888859 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.6953 |
|          |            |    |            | Adj R-squared | = | 0.6777 |
| Total    | 2443.45946 | 73 | 33.4720474 | Root MSE      | = | 3.2846 |

| mpg                   | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|-----------------------|-------------|-----------|-------|-------|----------------------|-----------|
| weight                | -.0173257   | .0040488  | -4.28 | 0.000 | -.0254028            | -.0092486 |
| c.weight#<br>c.weight | 1.87e-06    | 6.89e-07  | 2.71  | 0.008 | 4.93e-07             | 3.24e-06  |
| displacement          | -.0101625   | .0106236  | -0.96 | 0.342 | -.031356             | .011031   |
| foreign               | -2.560016   | 1.123506  | -2.28 | 0.026 | -4.801349            | -.3186832 |
| _cons                 | 58.23575    | 6.449882  | 9.03  | 0.000 | 45.36859             | 71.10291  |

Now, we perform the link test on our new model:

```
. linktest
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 1699.39489 | 2  | 849.697445 | F(2, 71)      | = | 81.08  |
| Residual | 744.06457  | 71 | 10.4797827 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.6955 |
|          |            |    |            | Adj R-squared | = | 0.6869 |
| Total    | 2443.45946 | 73 | 33.4720474 | Root MSE      | = | 3.2372 |

| mpg    | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|--------|-------------|-----------|-------|-------|----------------------|----------|
| _hat   | 1.141987    | .7612218  | 1.50  | 0.138 | -.3758456            | 2.659821 |
| _hatsq | -.0031916   | .0170194  | -0.19 | 0.852 | -.0371272            | .0307441 |
| _cons  | -1.50305    | 8.196444  | -0.18 | 0.855 | -17.84629            | 14.84019 |

We now pass the link test.

## ▷ Example 2

Above, we followed a standard misinterpretation of the link test—when we discovered a problem, we focused on the explanatory variables of our model. We might consider varying exactly what the link test tests. The link test told us that our dependent variable was misspecified. For those with an engineering background, mpg is indeed a strange measure. It would make more sense to model energy consumption—gallons per mile—in terms of weight and displacement:

```
. gen gpm = 1/mpg
. regress gpm weight displ foreign
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | .009157962 | 3  | .003052654 | F(3, 70)      | = | 76.33  |
| Residual | .002799666 | 70 | .000039995 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.7659 |
|          |            |    |            | Adj R-squared | = | 0.7558 |
| Total    | .011957628 | 73 | .000163803 | Root MSE      | = | .00632 |

| gpm          | Coefficient | Std. err. | t    | P> t  | [95% conf. interval] |          |
|--------------|-------------|-----------|------|-------|----------------------|----------|
| weight       | .0000144    | 2.15e-06  | 6.72 | 0.000 | .0000102             | .0000187 |
| displacement | .0000186    | .0000186  | 1.00 | 0.319 | -.0000184            | .0000557 |
| foreign      | .0066981    | .0020531  | 3.26 | 0.002 | .0026034             | .0107928 |
| _cons        | .0008917    | .0043337  | 0.21 | 0.838 | -.0077515            | .009535  |

This model looks every bit as reasonable as our original model:

```
. linktest
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | .009175219 | 2  | .004587609 | F(2, 71)      | = | 117.06 |
| Residual | .002782409 | 71 | .000039189 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.7673 |
|          |            |    |            | Adj R-squared | = | 0.7608 |
| Total    | .011957628 | 73 | .000163803 | Root MSE      | = | .00626 |

| gpm    | Coefficient | Std. err. | t    | P> t  | [95% conf. interval] |          |
|--------|-------------|-----------|------|-------|----------------------|----------|
| _hat   | .6608413    | .515275   | 1.28 | 0.204 | -.3665877            | 1.68827  |
| _hatsq | 3.275857    | 4.936655  | 0.66 | 0.509 | -6.567553            | 13.11927 |
| _cons  | .008365     | .0130468  | 0.64 | 0.523 | -.0176496            | .0343795 |

Specifying the model in terms of gallons per mile also solves the specification problem and results in a more parsimonious specification.

## ▷ Example 3

The link test can be used with any single-equation estimation procedure, not solely regression. Let's turn our problem around and attempt to explain whether a car is manufactured outside the United States by its mileage rating and weight. To save paper, we will specify `logit`'s `nolog` option, which suppresses the iteration log:

```
. logit foreign mpg weight, nolog
Logistic regression Number of obs = 74
 LR chi2(2) = 35.72
 Prob > chi2 = 0.0000
Log likelihood = -27.175156 Pseudo R2 = 0.3966
```

| foreign | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|---------|-------------|-----------|-------|-------|----------------------|----------|
| mpg     | -.1685869   | .0919175  | -1.83 | 0.067 | -.3487418            | .011568  |
| weight  | -.0039067   | .0010116  | -3.86 | 0.000 | -.0058894            | -.001924 |
| _cons   | 13.70837    | 4.518709  | 3.03  | 0.002 | 4.851859             | 22.56487 |

When we run `linktest` after `logit`, the result is another logit specification:

```
. linktest, nolog
Logistic regression Number of obs = 74
 LR chi2(2) = 36.83
 Prob > chi2 = 0.0000
Log likelihood = -26.615714 Pseudo R2 = 0.4090
```

| foreign | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|---------|-------------|-----------|-------|-------|----------------------|----------|
| _hat    | .8438531    | .2738759  | 3.08  | 0.002 | .3070661             | 1.38064  |
| _hatsq  | -.1559115   | .1568642  | -0.99 | 0.320 | -.4633596            | .1515366 |
| _cons   | .2630557    | .4299598  | 0.61  | 0.541 | -.57965              | 1.105761 |

The link test reveals no problems with our specification.

If there had been a problem, we would have been virtually forced to accept the misinterpretation of the link test—we would have reconsidered our specification of the independent variables. When using `logit`, we have no control over the specification of the dependent variable other than to change likelihood functions.

We admit to having seen a dataset once for which the link test rejected the logit specification. We did change the likelihood function, refitting the model using `probit`, and satisfied the link test. `Probit` has thinner tails than `logit`. In general, however, you will not be so lucky.



## □ Technical note

You should specify the same options with `linktest` that you do with the estimation command, although you do not have to follow this advice as literally as we did in the preceding example. `logit`'s `nolog` option merely suppresses a part of the output, not what is estimated. We specified `nolog` both times to save space.

If you are testing a tobit model, you must specify the censoring points just as you do with the `tobit` command.

If you are not sure which options are important, duplicate exactly what you specified on the estimation command.

If you do not specify `if exp` or `in range` with `linktest`, Stata will by default perform the link test on the same sample as the previous estimation. Suppose that you omitted some data when performing your estimation but want to calculate the link test on all the data, which you might do if you believe the model is appropriate for all the data. You would type `linktest if e(sample) < .` to do this.

□

## Stored results

`linktest` stores the following in `r()`:

Scalars

|                    |                                           |
|--------------------|-------------------------------------------|
| <code>r(t)</code>  | <i>t</i> statistic on <code>_hatsq</code> |
| <code>r(df)</code> | degrees of freedom                        |

`linktest` is *not* an estimation command in the sense that it leaves previous estimation results unchanged. For instance, after running a regression and performing the link test, typing `regress` without arguments after the link test still replays the original regression.

For integrating an estimation command with `linktest`, `linktest` assumes that the name of the estimation command is stored in `e(cmd)` and that the name of the dependent variable is stored in `e(depvar)`. After estimation, it assumes that the number of degrees of freedom for the *t* test is given by `e(df_m)` if the macro is defined.

If the estimation command reports *z* statistics instead of *t* statistics, `linktest` will also report *z* statistics. The *z* statistic, however, is still returned in `r(t)`, and `r(df)` is set to a missing value.

## Methods and formulas

The link test is based on the idea that if a regression or regression-like equation is properly specified, you should be able to find no additional independent variables that are significant except by chance. One kind of specification error is called a link error. In regression, this means that the dependent variable needs a transformation or “link” function to properly relate to the independent variables. The idea of a link test is to add an independent variable to the equation that is especially likely to be significant if there is a link error.

Let

$$\mathbf{y} = f(\mathbf{X}\boldsymbol{\beta})$$

be the model and  $\widehat{\boldsymbol{\beta}}$  be the parameter estimates. `linktest` calculates

$$\_hat = \mathbf{X}\widehat{\boldsymbol{\beta}}$$

and

$$\_hatsq = \_hat^2$$

The model is then refit with these two variables, and the test is based on the significance of `_hatsq`. This is the form suggested by Pregibon (1979) based on an idea of Tukey (1949). Pregibon (1980) suggests a slightly different method that has come to be known as “Pregibon’s goodness-of-link test”. We prefer the older version because it is universally applicable, straightforward, and a good second-order approximation. It can be applied to any single-equation estimation technique, whereas Pregibon’s more recent tests are estimation-technique specific.

## References

- Pregibon, D. 1979. Data analytic methods for generalized linear models. PhD diss., University of Toronto.
- . 1980. Goodness of link tests for generalized linear models. *Journal of the Royal Statistical Society, C ser.*, 29: 15–24. <https://doi.org/10.2307/2346405>.
- Tukey, J. W. 1949. One degree of freedom for non-additivity. *Biometrics* 5: 232–242. <https://doi.org/10.2307/3001938>.

## Also see

[R] [regress postestimation](#) — Postestimation tools for regress

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## Description

`lnskew0` creates  $newvar = \ln(\pm exp - k)$ , choosing  $k$  and the sign of  $exp$  so that the skewness of  $newvar$  is zero.

`bcskew0` creates  $newvar = (exp^\lambda - 1)/\lambda$ , the Box–Cox power transformation (Box and Cox 1964), choosing  $\lambda$  so that the skewness of  $newvar$  is zero.  $exp$  must be strictly positive.

## Quick start

Generate `newv1`, the zero-skewness log transform of continuous variable `v1`

```
lnskew0 newv1 = v1
```

Same as above, but transform ratio of `v1` to `v2`

```
lnskew0 newv1 = v1/v2
```

Zero-skewness Box–Cox transform, `newv2`, of `v2`

```
bcskew0 newv2 = v2
```

Same as above, and change the value for convergence to 0.0001 from the default 0.001

```
bcskew0 newv2 = v2, zero(.0001)
```

## Menu

### **Inskew0**

Data > Create or change data > Other variable-creation commands > Zero-skewness log transform

### **bcskew0**

Data > Create or change data > Other variable-creation commands > Box–Cox transform

## Syntax

Zero-skewness log transform

```
lnskew0 newvar = exp [if] [in] [, options]
```

Zero-skewness Box–Cox transform

```
bcskew0 newvar = exp [if] [in] [, options]
```

*options*

Description

Main

| <i>options</i>   | Description                                                                                                   |
|------------------|---------------------------------------------------------------------------------------------------------------|
| <u>d</u> elta(#) | increment for derivative of skewness function; default is delta(0.02) for lnskew0 and delta(0.01) for bcskew0 |
| <u>z</u> ero(#)  | value for determining convergence; default is zero(0.001)                                                     |
| <u>l</u> evel(#) | compute the confidence interval at confidence level #; by default, no confidence interval is calculated       |

collect is allowed with lnskew0 and bcskew0; see [\[U\] 11.1.10 Prefix commands](#).

## Options

Main

`delta(#)` specifies the increment used for calculating the derivative of the skewness function with respect to  $k$  (lnskew0) or  $\lambda$  (bcskew0). The default values are 0.02 for lnskew0 and 0.01 for bcskew0.

`zero(#)` specifies a value for skewness to determine convergence that is small enough to be considered zero and is, by default, 0.001.

`level(#)` specifies the confidence level for the confidence interval for  $k$  (lnskew0) or  $\lambda$  (bcskew0). The confidence interval is calculated only if `level()` is specified. # is specified as an integer; 95 means 95% confidence intervals. The `level()` option is honored only if the number of observations exceeds 7.

## Remarks and examples

### ▷ Example 1: Inskew0

Using our automobile dataset (see [U] 1.2.2 Example datasets), we want to generate a new variable equal to  $\ln(\text{mpg} - k)$  to be approximately normally distributed. `mpg` records the miles per gallon for each of our cars. One feature of the normal distribution is that it has skewness 0.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. lnskew0 lnmpg = mpg
```

| Transform           | k        | [95% conf. interval] | Skewness  |
|---------------------|----------|----------------------|-----------|
| $\ln(\text{mpg}-k)$ | 5.383659 | (not calculated)     | -7.05e-06 |

This created the new variable `lnmpg = ln(mpg - 5.384)`:

```
. describe lnmpg
```

| Variable name      | Storage type | Display format | Value label | Variable label                |
|--------------------|--------------|----------------|-------------|-------------------------------|
| <code>lnmpg</code> | float        | %9.0g          |             | <code>ln(mpg-5.383659)</code> |

Because we did not specify the `level()` option, no confidence interval was calculated. At the outset, we could have typed

```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
. lnskew0 lnmpg = mpg, level(95)
```

| Transform           | k        | [95% conf. interval] | Skewness  |
|---------------------|----------|----------------------|-----------|
| $\ln(\text{mpg}-k)$ | 5.383659 | -17.12339 9.892416   | -7.05e-06 |

The confidence interval is calculated under the assumption that  $\ln(\text{mpg} - k)$  really does have a normal distribution. It would be perfectly reasonable to use `lnskew0`, even if we did not believe that the transformed variable would have a normal distribution—if we literally wanted the zero-skewness transform—although, then the confidence interval would be an approximation of unknown quality to the true confidence interval. If we now wanted to test the believability of the confidence interval, we could also test our new variable `lnmpg` by using `swilk` (see [R] [swilk](#)) with the `lnnormal` option.



□ Technical note

lnskew0 and bcskew0 report the resulting skewness of the variable merely to reassure you of the accuracy of its results. In our example above, lnskew0 found  $k$  such that the resulting skewness was  $-7 \times 10^{-6}$ , a number close enough to zero for all practical purposes. If we wanted to make it even smaller, we could specify the zero() option. Typing lnskew0 new=mpg, zero(1e-8) changes the estimated  $k$  to 5.383552 from 5.383659 and reduces the calculated skewness to  $-2 \times 10^{-11}$ .

When you request a confidence interval, lnskew0 may report the lower confidence interval as ‘.’, which should be taken as indicating the lower confidence limit  $k_L = -\infty$ . (This cannot happen with bcskew0.)

As an example, consider a sample of size  $n$  on  $x$  and assume that the skewness of  $x$  is positive, but not significantly so, at the desired significance level—say, 5%. Then, no matter how large and negative you make  $k_L$ , there is no value extreme enough to make the skewness of  $\ln(x - k_L)$  equal the corresponding percentile (97.5 for a 95% confidence interval) of the distribution of skewness in a normal distribution of the same sample size. You cannot do this because the distribution of  $\ln(x - k_L)$  tends to that of  $x$ —apart from location and scale shift—as  $x \rightarrow \infty$ . This “problem” never applies to the upper confidence limit,  $k_U$ , because the skewness of  $\ln(x - k_U)$  tends to  $-\infty$  as  $k$  tends upward to the minimum value of  $x$ . □

▷ Example 2: bcskew0

In example 1, using lnskew0 with a variable such as mpg is probably undesirable. mpg has a natural zero, and we are shifting that zero arbitrarily. On the other hand, use of lnskew0 with a variable such as temperature measured in Fahrenheit or Celsius would be more appropriate because the zero is indeed arbitrary.

For a variable like mpg, it makes more sense to use the Box–Cox power transform (Box and Cox 1964):

$$y^{(\lambda)} = \frac{y^\lambda - 1}{\lambda}$$

$\lambda$  is free to take on any value, but  $y^{(1)} = y - 1$ ,  $y^{(0)} = \ln(y)$ , and  $y^{(-1)} = 1 - 1/y$ .

bcskew0 works like lnskew0:

```
. bcskew0 bcmpg = mpg, level(95)
```

| Transform   | L         | [95% conf. interval] |          |  | Skewness |
|-------------|-----------|----------------------|----------|--|----------|
| (mpg^L-1)/L | -.3673283 | -1.212752            | .4339645 |  | .0001898 |

The 95% confidence interval includes  $\lambda = -1$  ( $\lambda$  is labeled L in the output), which has a rather more pleasing interpretation—gallons per mile—than  $(\text{mpg}^{-0.3673} - 1)/(-0.3673)$ . The confidence interval, however, is calculated assuming that the power transformed variable is normally distributed. It makes perfect sense to use bcskew0, even when you do not believe that the transformed variable will be normally distributed, but then the confidence interval is an approximation of unknown quality. If you believe that the transformed data are normally distributed, you can alternatively use boxcox to estimate  $\lambda$ ; see [R] boxcox.



## Stored results

lnskew0 and bcskew0 store the following in `r()`:

Scalars

|                          |                                            |
|--------------------------|--------------------------------------------|
| <code>r(gamma)</code>    | $k$ (lnskew0)                              |
| <code>r(lambda)</code>   | $\lambda$ (bcskew0)                        |
| <code>r(lb)</code>       | lower bound of confidence interval         |
| <code>r(ub)</code>       | upper bound of confidence interval         |
| <code>r(skewness)</code> | resulting skewness of transformed variable |

## Methods and formulas

Skewness is as calculated by `summarize`; see [R] [summarize](#). Newton's method with numeric, uncentered derivatives is used to estimate  $k$  (lnskew0) and  $\lambda$  (bcskew0). For lnskew0, the initial value is chosen so that the minimum of  $x - k$  is 1, and thus  $\ln(x - k)$  is 0. bcskew0 starts with  $\lambda = 1$ .

## Acknowledgment

lnskew0 and bcskew0 were written by Patrick Royston of the MRC Clinical Trials Unit, London, and coauthor of the Stata Press book *Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model*.

## Reference

Box, G. E. P., and D. R. Cox. 1964. An analysis of transformations. *Journal of the Royal Statistical Society, B ser.*, 26: 211–252. <https://doi.org/10.1111/j.2517-6161.1964.tb00553.x>.

## Also see

[R] [boxcox](#) — Box–Cox regression models

[R] [ladder](#) — Ladder of powers

[R] [swilk](#) — Shapiro–Wilk and Shapiro–Francia tests for normality

|                                                 |                                                          |                                          |
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## Description

`log` and its subcommands tell Stata to open a log file and create a record of what you type and any output that appears in the Results window, to suspend or resume logging, to check logging status, and to close the log file. The default format is Stata Markup and Control Language (SMCL) but can be plain text. You can have up to five SMCL and five text logs open at a time. `cmdlog` and its subcommands are similar to `log` but create a command log recording only what you type and can be only plain text. That is, `cmdlog` is designed only for interactive use. You can have only one command log open at a time.

`set logtype`, `set logmsg`, and `set linesize` are commands to control system parameters that relate to logs.

## Quick start

Begin recording your Stata session in `mylog.smcl`

```
log using mylog
```

Same as above, but use a text format that can be read by a word processor

```
log using mylog, text
```

Save a subset of output to `mylog2.smcl` while `mylog.smcl` is still open

```
log using mylog2, name(mylog2)
```

Close `mylog2.smcl` and keep `mylog.smcl` open

```
log close mylog2
```

Create a do-file from commands typed interactively

```
cmdlog using mydo.do
```

## Menu

File > Log

## Syntax

*Report status of log file*

```
log
```

```
log query [logname | _all]
```

*Open log file*

```
log using filename [, append replace [text | smc1] name(logname) [nomsg | msg]]
```

*Close log*

```
log close [logname | _all]
```

*Temporarily suspend logging or resume logging*

```
log { off | on } [logname]
```

*Report status of command log file*

```
cmdlog
```

*Open command log file*

```
cmdlog using filename [, append replace]
```

*Close command log, temporarily suspend logging, or resume logging*

```
cmdlog { close | on | off }
```

*Set default format for logs*

```
set logtype { text | smc1 } [, permanently]
```

*Set default for display of log messages*

```
set logmsg { on | off }
```

*Specify screen width*

```
set linesize #
```

In addition to using the log command, you may access the capabilities of log by selecting **File > Log** from the menu and choosing one of the options in the list.

collect is allowed with log query, log, and cmdlog; see [U] [11.1.10 Prefix commands](#).

## Options for use with both log and cmdlog

`append` specifies that results be appended to an existing file. If the file does not already exist, a new file is created.

`replace` specifies that *filename*, if it already exists, be overwritten. When you do not specify either `replace` or `append`, the file is assumed to be new. If the specified file already exists, an error message is issued and logging is not started.

## Options for use with log

`text` and `smcl` specify the format in which the log is to be recorded. The default is complicated to describe but is what you would expect:

If you specify the file as *filename*.`smcl`, the default is to write the log in SMCL format (regardless of the value of `set logtype`).

If you specify the file as *filename*.`log`, the default is to write the log in text format (regardless of the value of `set logtype`).

If you type *filename* without an extension and specify neither the `smcl` option nor the `text` option, the default is to write the file according to the value of `set logtype`. If you have not `set logtype`, then the default is SMCL. Also, the *filename* you specified will be fixed to read *filename*.`smcl` if a SMCL log is being created or *filename*.`log` if a text log is being created.

If you specify either the `text` or `smcl` option, then what you specify determines how the log is written. If *filename* was specified without an extension, the appropriate extension is added for you.

If you open multiple log files, you may choose a different format for each file.

`name(logname)` specifies an optional name you may use to refer to the log while it is open. You can start multiple log files, give each a different *logname*, and then close, temporarily suspend, or resume them each individually. The default *logname* is `<unnamed>`.

`nomsg` and `msg` specify whether to display or suppress the default message at the top and bottom of the log file.

`nomsg` suppresses the default message displayed at the top and bottom of the log file. This message consists of the log name (if specified in `name()`, otherwise `unnamed`), log path, log type, and date opened or closed.

`msg` displays the default message at the top and bottom of the log file. This option is for use when `set logmsg` is turned off.

## Option for use with set logtype

`permanently` specifies that, in addition to making the change right now, the `logtype` setting be remembered and become the default setting when you invoke Stata.

## Remarks and examples

A full log is a file containing what you type and Stata's output that is shown in the Results window. To begin logging your session, you type `log` using *filename*. If *filename* contains embedded spaces, remember to enclose it in double quotation marks.

When the default log format is SMCL, `log` will add the extension `.smcl` if *filename* is specified without one. If *text* is specified or the default log type is changed to `text`, `log` adds the extension `.log`.

We recommend using SMCL because it preserves fonts and colors. SMCL logs can be viewed and printed from the Viewer window, as can any text file; see [R] [view](#). Users of console Stata can use `translate` to produce printable versions of log files. `translate` also converts SMCL logs to text or other formats, such as PostScript or PDF; see [R] [translate](#).

When you open a full log, the default is to show the name of the file and a time and date stamp:

```
. log using myfile

 name: <unnamed>
 log: C:\data\proj1\myfile.smcl
 log type: smcl
opened on: 12 Jan 2025, 12:28:23
.
```

The above information will appear in the log. If you do not want this information to appear, precede the command by `quietly`:

```
. quietly log using myfile
```

`quietly` will not suppress any error messages or anything else you need to know.

Similarly, when you close a full log, the default is to show the full information,

```
. log close
 name: <unnamed>
 log: C:\data\proj1\myfile.smcl
 log type: smcl
closed on: 12 Jan 2025, 12:32:41

```

and that information will also appear in the log. If you want to suppress that, type `quietly log close`. Alternatively, specifying `nomsg` with `log using` will suppress these messages.

If you do not specify name (*logname*), Stata will use the name `<unnamed>`, as shown above. However, you can start multiple log files by specifying the `name()` option with each new `log using` command. To control a specific log, type, for example, `log close logname`; to close all log files at one time, type `log close _all`.

Stata also lets you log only your commands using `cmdlog`. Command logs are always text files, which makes them easy to convert to do-files. The default extension is `.txt` instead of `.do` to keep you from accidentally overwriting your important do-files. However, `cmdlog` will allow you to specify `.do` as the extension of *filename*.

You can have only one command log open at a time. However, you can have full logs open while logging your commands. Moreover, the text file you create for your command log does not count against the limit of five text logs.

`set logtype` specifies the default format in which full logs are to be recorded. Initially, full logs are recorded in SMCL format.

`set logmsg` specifies whether the default message is displayed at the top and bottom of the log file. The default is `on`. Specifying `set logmsg off` suppresses the default message from both `log using` and `log close` just as if the `nomsg` option were used with `log using`.

`set linesize` specifies the maximum width, in characters, of Stata output. Most commands in Stata do not respect `linesize`, because it is not important for most commands. Most users never need to set `linesize`, because it will automatically be reset if you resize your Results window. This is also why there is no permanently option allowed with `set linesize`. `set linesize` is for use with commands such as `list` and `display` and is typically used by programmers who wish the output of those commands to be wider or narrower than the current width of the Results window.

## Stored results

`log` and `cmdlog` store the following in `r()`:

Macros

|                          |                |
|--------------------------|----------------|
| <code>r(name)</code>     | <i>logname</i> |
| <code>r(filename)</code> | name of file   |
| <code>r(status)</code>   | on or off      |
| <code>r(type)</code>     | smcl or text   |

`log query _all` stores the following in `r()`:

Scalars

|                         |                          |
|-------------------------|--------------------------|
| <code>r(numlogs)</code> | number of open log files |
|-------------------------|--------------------------|

For each open log file, `log query _all` also stores

|                           |                |
|---------------------------|----------------|
| <code>r(name#)</code>     | <i>logname</i> |
| <code>r(filename#)</code> | name of file   |
| <code>r(status#)</code>   | on or off      |
| <code>r(type#)</code>     | smcl or text   |

where `#` varies between 1 and the value of `r(numlogs)`. Be aware that `#` will not necessarily represent the order in which the log files were first opened, nor will it necessarily remain constant for a given log file upon multiple calls to `log query`.

## Reference

Hansen, M. R. 2015. [graphlog: Creating log files with embedded graphics](#). *Stata Journal* 15: 594–596.

## Also see

[R] [query](#) — Display system parameters

[R] [translate](#) — Print and translate logs

[GSM] [16 Saving and printing results by using logs](#)

[GSW] [16 Saving and printing results by using logs](#)

[GSU] [16 Saving and printing results by using logs](#)

[U] [15 Saving and printing output—log files](#)

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## Description

`logistic` fits a logistic regression model of *depvar* on *indepvars*, where *depvar* is a 0/1 variable (or, more precisely, a 0/non-0 variable). Without arguments, `logistic` redisplay the last `logistic` estimates. `logistic` displays estimates as odds ratios; to view coefficients, type `logit` after running `logistic`. To obtain odds ratios for any covariate pattern relative to another, see [R] [lincom](#).

## Quick start

Report odds ratios from logistic regression of *y* on *x1* and *x2*

```
logistic y x1 x2
```

Add indicators for values of categorical variable *a*

```
logistic y x1 x2 i.a
```

Same as above, and apply frequency weights defined by *wvar*

```
logistic y x1 x2 i.a [fweight=wvar]
```

Show base level of *a*

```
logistic y x1 x2 i.a, baselevels
```

## Menu

Statistics > Binary outcomes > Logistic regression

## Syntax

```
logistic devar indepvars [if] [in] [weight] [, options]
```

| <i>options</i>                               | Description                                                                                                                                                                 |
|----------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Model                                        |                                                                                                                                                                             |
| <code>noconstant</code>                      | suppress constant term                                                                                                                                                      |
| <code>offset(<i>varname</i>)</code>          | include <i>varname</i> in model with coefficient constrained to 1                                                                                                           |
| <code>asis</code>                            | retain perfect predictor variables                                                                                                                                          |
| <code>constraints(<i>constraints</i>)</code> | apply specified linear constraints                                                                                                                                          |
| SE/Robust                                    |                                                                                                                                                                             |
| <code>vce(<i>vcetype</i>)</code>             | <i>vcetype</i> may be <code>oim</code> , <code>opg</code> , <code>robust</code> , <code>cluster <i>clustvar</i></code> , <code>bootstrap</code> , or <code>jackknife</code> |
| Reporting                                    |                                                                                                                                                                             |
| <code>level(#)</code>                        | set confidence level; default is <code>level(95)</code>                                                                                                                     |
| <code>coef</code>                            | report estimated coefficients                                                                                                                                               |
| <code>nocnsreport</code>                     | do not display constraints                                                                                                                                                  |
| <code>display_options</code>                 | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling                            |
| Maximization                                 |                                                                                                                                                                             |
| <code>maximize_options</code>                | control the maximization process; seldom used                                                                                                                               |
| <code>collinear</code>                       | keep collinear variables                                                                                                                                                    |
| <code>coeflegend</code>                      | display legend instead of statistics                                                                                                                                        |

*indepvars* may contain factor variables; see [U] 11.4.3 **Factor variables**.

*devar* and *indepvars* may contain time-series operators; see [U] 11.4.4 **Time-series varlists**.

`bayes`, `bayesboot`, `bootstrap`, `by`, `collect`, `fp`, `jackknife`, `mfp`, `mi estimate`, `nestreg`, `rolling`, `statsby`, `stepwise`, and `svy` are allowed; see [U] 11.1.10 **Prefix commands**. For more details, see [BAYES] **bayes: logistic**.

`vce(bootstrap)` and `vce(jackknife)` are not allowed with the `mi estimate` prefix; see [MI] **mi estimate**.

Weights are not allowed with the `bootstrap` prefix; see [R] **bootstrap**.

`vce()` and weights are not allowed with the `svy` prefix; see [SVY] **svy**.

`fweights`, `iweights`, and `pweights` are allowed; see [U] 11.1.6 **weight**.

`collinear` and `coeflegend` do not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options

Model

`noconstant`, `offset(varname)`, `constraints(constraints)`; see [R] **Estimation options**.

`asis` forces retention of perfect predictor variables and their associated perfectly predicted observations and may produce instabilities in maximization; see [R] **probit**.

## SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

## Reporting

`level(#)`; see [R] [Estimation options](#).

`coef` causes `logistic` to report the estimated coefficients rather than the odds ratios (exponentiated coefficients). `coef` may be specified when the model is fit or may be used later to redisplay results. `coef` affects only how results are displayed and not how they are estimated.

`nocnsreport`; see [R] [Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

## Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

The following options are available with `logistic` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Remarks are presented under the following headings:

[logistic and logit](#)  
[Robust estimate of variance](#)  
[Video examples](#)

## logistic and logit

`logistic` provides an alternative and preferred way to fit maximum-likelihood logit models, the other choice being `logit` ([R] [logit](#)).

First, let's dispose of some confusing terminology. We use the words `logit` and `logistic` to mean the same thing: maximum likelihood estimation. To some, one or the other of these words connotes transforming the dependent variable and using weighted least squares to fit the model, but that is not how we use either word here. Thus, the `logit` and `logistic` commands produce the same results.

The `logistic` command is generally preferred to the `logit` command because `logistic` presents the estimates in terms of odds ratios rather than coefficients. To some people, this may seem disadvantageous, but you can type `logit` without arguments after `logistic` to see the underlying coefficients. You should be cautious when interpreting the odds ratio of the constant term. Usually, this odds ratio represents the baseline odds of the model when all predictor variables are set to zero. However, you must verify that a zero value for all predictor variables in the model actually makes sense before continuing with this interpretation.

Nevertheless, `[R] logit` is still worth reading because `logistic` shares the same features as `logit`, including omitting variables due to collinearity or one-way causation.

For an introduction to logistic regression, see [Lemeshow and Hosmer \(2005\)](#), [Pagano and Gauvreau \(2022, 455–478\)](#), or [Pampel \(2021\)](#); for a complete but nonmathematical treatment, see [Kleinbaum and Klein \(2010\)](#); and for a thorough discussion, see [Hosmer, Lemeshow, and Sturdivant \(2013\)](#). See [Gould \(2000\)](#) for a discussion of the interpretation of logistic regression. See [Dupont \(2009\)](#) or [Hilbe \(2009\)](#) for a discussion of logistic regression with examples using Stata. For a discussion using Stata with an emphasis on model specification, see [Vittinghoff et al. \(2012\)](#).

Stata has a variety of commands for performing estimation when the dependent variable is dichotomous or polytomous. See [Long and Freese \(2014\)](#) for a book devoted to fitting these models with Stata. See `help estimation commands` for a complete list of all of Stata’s estimation commands.

## ▷ Example 1

Consider the following dataset from a study of risk factors associated with low birthweight described in [Hosmer, Lemeshow, and Sturdivant \(2013, 24\)](#).

```
. use https://www.stata-press.com/data/r19/lbw
(Hosmer & Lemeshow data)

. describe

Contains data from https://www.stata-press.com/data/r19/lbw.dta
Observations: 189 Hosmer & Lemeshow data
Variables: 11 15 Jan 2024 05:01
```

| Variable name | Storage type | Display format | Value label | Variable label                                     |
|---------------|--------------|----------------|-------------|----------------------------------------------------|
| id            | int          | %8.0g          |             | Identification code                                |
| low           | byte         | %8.0g          |             | Birthweight<2500g                                  |
| age           | byte         | %8.0g          |             | Age of mother                                      |
| lwt           | int          | %8.0g          |             | Weight at last menstrual period                    |
| race          | byte         | %8.0g          | race        | Race                                               |
| smoke         | byte         | %9.0g          | smoke       | Smoked during pregnancy                            |
| ptl           | byte         | %8.0g          |             | Premature labor history (count)                    |
| ht            | byte         | %8.0g          |             | Has history of hypertension                        |
| ui            | byte         | %8.0g          |             | Presence, uterine irritability                     |
| ftv           | byte         | %8.0g          |             | Number of visits to physician during 1st trimester |
| bwt           | int          | %8.0g          |             | Birthweight (grams)                                |

Sorted by:

We want to investigate the causes of low birthweight. Here `race` is a categorical variable indicating whether a person is white (`race = 1`), black (`race = 2`), or some other race (`race = 3`). We want indicator (dummy) variables for `race` included in the regression, so we will use factor variables.

```
. logistic low age lwt i.race smoke ptl ht ui
Logistic regression
Log likelihood = -100.724
Number of obs = 189
LR chi2(8) = 33.22
Prob > chi2 = 0.0001
Pseudo R2 = 0.1416
```

| low   | Odds ratio | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-------|------------|-----------|-------|-------|----------------------|----------|
| age   | .9732636   | .0354759  | -0.74 | 0.457 | .9061578             | 1.045339 |
| lwt   | .9849634   | .0068217  | -2.19 | 0.029 | .9716834             | .9984249 |
| race  |            |           |       |       |                      |          |
| Black | 3.534767   | 1.860737  | 2.40  | 0.016 | 1.259736             | 9.918406 |
| Other | 2.368079   | 1.039949  | 1.96  | 0.050 | 1.001356             | 5.600207 |
| smoke | 2.517698   | 1.00916   | 2.30  | 0.021 | 1.147676             | 5.523162 |
| ptl   | 1.719161   | .5952579  | 1.56  | 0.118 | .8721455             | 3.388787 |
| ht    | 6.249602   | 4.322408  | 2.65  | 0.008 | 1.611152             | 24.24199 |
| ui    | 2.1351     | .9808153  | 1.65  | 0.099 | .8677528             | 5.2534   |
| _cons | 1.586014   | 1.910496  | 0.38  | 0.702 | .1496092             | 16.8134  |

Note: `_cons` estimates baseline odds.

The odds ratios are for a one-unit change in the variable. If we wanted the odds ratio for age to be in terms of 4-year intervals, we would type

```
. generate age4 = age/4
. logistic low age4 lwt i.race smoke ptl ht ui
(output omitted)
```

After `logistic`, we can type `logit` to see the model in terms of coefficients and standard errors:

```
. logit
Logistic regression
Log likelihood = -100.724
Number of obs = 189
LR chi2(8) = 33.22
Prob > chi2 = 0.0001
Pseudo R2 = 0.1416
```

| low   | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|-------|-------------|-----------|-------|-------|----------------------|-----------|
| age4  | -.1084012   | .1458017  | -0.74 | 0.457 | -.3941673            | .1773649  |
| lwt   | -.0151508   | .0069259  | -2.19 | 0.029 | -.0287253            | -.0015763 |
| race  |             |           |       |       |                      |           |
| Black | 1.262647    | .5264101  | 2.40  | 0.016 | .2309024             | 2.294392  |
| Other | .8620792    | .4391532  | 1.96  | 0.050 | .0013548             | 1.722804  |
| smoke | .9233448    | .4008266  | 2.30  | 0.021 | .137739              | 1.708951  |
| ptl   | .5418366    | .346249   | 1.56  | 0.118 | -.136799             | 1.220472  |
| ht    | 1.832518    | .6916292  | 2.65  | 0.008 | .4769494             | 3.188086  |
| ui    | .7585135    | .4593768  | 1.65  | 0.099 | -.1418484            | 1.658875  |
| _cons | .4612239    | 1.20459   | 0.38  | 0.702 | -1.899729            | 2.822176  |

If we wanted to see the logistic output again, we would type `logistic` without arguments.

## ▶ Example 2

We can specify the confidence interval for the odds ratios with the `level()` option, and we can do this either at estimation time or when replaying the model. For instance, to see our first model in [example 1](#) with narrower, 90% confidence intervals, we might type

```
. logistic, level(90)
Logistic regression Number of obs = 189
 LR chi2(8) = 33.22
 Prob > chi2 = 0.0001
Log likelihood = -100.724 Pseudo R2 = 0.1416
```

| low   | Odds ratio | Std. err. | z     | P> z  | [90% conf. interval] |          |
|-------|------------|-----------|-------|-------|----------------------|----------|
| age4  | .8972675   | .1308231  | -0.74 | 0.457 | .7059409             | 1.140448 |
| lwt   | .9849634   | .0068217  | -2.19 | 0.029 | .9738063             | .9962483 |
| race  |            |           |       |       |                      |          |
| Black | 3.534767   | 1.860737  | 2.40  | 0.016 | 1.487028             | 8.402379 |
| Other | 2.368079   | 1.039949  | 1.96  | 0.050 | 1.149971             | 4.876471 |
| smoke |            |           |       |       |                      |          |
| ptl   | 2.517698   | 1.00916   | 2.30  | 0.021 | 1.302185             | 4.867819 |
| ht    | 1.719161   | .5952579  | 1.56  | 0.118 | .9726876             | 3.038505 |
| ui    | 6.249602   | 4.322408  | 2.65  | 0.008 | 2.003487             | 19.49478 |
| _cons | 2.1351     | .9808153  | 1.65  | 0.099 | 1.00291              | 4.545424 |
|       | 1.586014   | 1.910496  | 0.38  | 0.702 | .2186791             | 11.50288 |

Note: `_cons` estimates baseline odds.

◀

**Robust estimate of variance**

If you specify `vce(robust)`, Stata reports the robust estimate of variance described in [\[U\] 20.22 Obtaining robust variance estimates](#). Here is the model previously fit with the robust estimate of variance:

```
. logistic low age lwt i.race smoke ptl ht ui, vce(robust)
Logistic regression Number of obs = 189
 Wald chi2(8) = 29.02
 Prob > chi2 = 0.0003
Log pseudolikelihood = -100.724 Pseudo R2 = 0.1416
```

| low   | Odds ratio | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |          |
|-------|------------|---------------------|-------|-------|----------------------|----------|
| age   | .9732636   | .0329376            | -0.80 | 0.423 | .9108015             | 1.040009 |
| lwt   | .9849634   | .0070209            | -2.13 | 0.034 | .9712984             | .9988206 |
| race  |            |                     |       |       |                      |          |
| Black | 3.534767   | 1.793616            | 2.49  | 0.013 | 1.307504             | 9.556051 |
| Other | 2.368079   | 1.026563            | 1.99  | 0.047 | 1.012512             | 5.538501 |
| smoke |            |                     |       |       |                      |          |
| ptl   | 2.517698   | .9736417            | 2.39  | 0.017 | 1.179852             | 5.372537 |
| ht    | 1.719161   | .7072902            | 1.32  | 0.188 | .7675715             | 3.850476 |
| ui    | 6.249602   | 4.102026            | 2.79  | 0.005 | 1.726445             | 22.6231  |
| _cons | 2.1351     | 1.042775            | 1.55  | 0.120 | .8197749             | 5.560858 |
|       | 1.586014   | 1.939482            | 0.38  | 0.706 | .144345              | 17.42658 |

Note: `_cons` estimates baseline odds.

Also, you can specify `vce(cluster clustvar)` and then, within cluster, relax the assumption of independence. To illustrate this, we have made some fictional additions to the low-birthweight data.

Say that these data are not a random sample of mothers but instead are a random sample of mothers from a random sample of hospitals. In fact, that may be true—we do not know the history of these data.

Hospitals specialize, and it would not be too incorrect to say that some hospitals specialize in more difficult cases. We are going to show two extremes. In one, all hospitals are alike, but we are going to estimate under the possibility that they might differ. In the other, hospitals are strikingly different. In both cases, we assume that patients are drawn from 20 hospitals.

In both examples, we will fit the same model, and we will type the same command to fit it. Below are the same data we have been using but with a new variable, `hospid`, that identifies from which of the 20 hospitals each patient was drawn (and which we have made up):

```
. use https://www.stata-press.com/data/r19/hospid1, clear
. logistic low age lwt i.race smoke ptl ht ui, vce(cluster hospid)
Logistic regression Number of obs = 189
 Wald chi2(8) = 49.67
 Prob > chi2 = 0.0000
Log pseudolikelihood = -100.724 Pseudo R2 = 0.1416
 (Std. err. adjusted for 20 clusters in hospid)
```

| low   | Odds ratio | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |          |
|-------|------------|---------------------|-------|-------|----------------------|----------|
| age   | .9732636   | .0397476            | -0.66 | 0.507 | .898396              | 1.05437  |
| lwt   | .9849634   | .0057101            | -2.61 | 0.009 | .9738352             | .9962187 |
| race  |            |                     |       |       |                      |          |
| Black | 3.534767   | 2.013285            | 2.22  | 0.027 | 1.157563             | 10.79386 |
| Other | 2.368079   | .8451325            | 2.42  | 0.016 | 1.176562             | 4.766257 |
| smoke | 2.517698   | .8284259            | 2.81  | 0.005 | 1.321062             | 4.79826  |
| ptl   | 1.719161   | .6676221            | 1.40  | 0.163 | .8030814             | 3.680219 |
| ht    | 6.249602   | 4.066275            | 2.82  | 0.005 | 1.74591              | 22.37086 |
| ui    | 2.1351     | 1.093144            | 1.48  | 0.138 | .7827337             | 5.824014 |
| _cons | 1.586014   | 1.661913            | 0.44  | 0.660 | .2034094             | 12.36639 |

Note: `_cons` estimates baseline odds.

The standard errors are similar to the standard errors we have previously obtained, whether we used the robust or conventional estimators. In this example, we invented the hospital IDs randomly.

Here are the results of the estimation with the same data but with a different set of hospital IDs:

```
. use https://www.stata-press.com/data/r19/hospid2
. logistic low age lwt i.race smoke ptl ht ui, vce(cluster hospid)
Logistic regression Number of obs = 189
 Wald chi2(8) = 7.19
 Prob > chi2 = 0.5167
Log pseudolikelihood = -100.724 Pseudo R2 = 0.1416
 (Std. err. adjusted for 20 clusters in hospid)
```

| low   | Odds ratio | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |          |
|-------|------------|---------------------|-------|-------|----------------------|----------|
| age   | .9732636   | .0293064            | -0.90 | 0.368 | .9174862             | 1.032432 |
| lwt   | .9849634   | .0106123            | -1.41 | 0.160 | .9643817             | 1.005984 |
| race  |            |                     |       |       |                      |          |
| Black | 3.534767   | 3.120338            | 1.43  | 0.153 | .6265521             | 19.9418  |
| Other | 2.368079   | 1.297738            | 1.57  | 0.116 | .8089594             | 6.932114 |
| smoke | 2.517698   | 1.570287            | 1.48  | 0.139 | .7414969             | 8.548655 |
| ptl   | 1.719161   | .6799153            | 1.37  | 0.171 | .7919045             | 3.732161 |
| ht    | 6.249602   | 7.165454            | 1.60  | 0.110 | .660558              | 59.12808 |
| ui    | 2.1351     | 1.411977            | 1.15  | 0.251 | .5841231             | 7.804266 |
| _cons | 1.586014   | 1.946253            | 0.38  | 0.707 | .1431423             | 17.573   |

Note: **\_cons** estimates baseline odds.

Note the strikingly larger standard errors. What happened? In these data, women most likely to have low-birthweight babies are sent to certain hospitals, and the decision on likeliness is based not just on age, smoking history, etc., but on other things that doctors can see but that are not recorded in our data. Thus, merely because a woman is at one of the centers identifies her to be more likely to have a low-birthweight baby.

## Video examples

[Logistic regression, part 1: Binary predictors](#)

[Logistic regression, part 2: Continuous predictors](#)

[Logistic regression, part 3: Factor variables](#)

## Stored results

logistic stores the following in `e()`:

### Scalars

|                            |                                           |
|----------------------------|-------------------------------------------|
| <code>e(N)</code>          | number of observations                    |
| <code>e(N_cds)</code>      | number of completely determined successes |
| <code>e(N_cdf)</code>      | number of completely determined failures  |
| <code>e(k)</code>          | number of parameters                      |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>  |
| <code>e(k_eq_model)</code> | number of equations in overall model test |
| <code>e(k_dv)</code>       | number of dependent variables             |
| <code>e(df_m)</code>       | model degrees of freedom                  |
| <code>e(r2_p)</code>       | pseudo- $R^2$                             |
| <code>e(ll)</code>         | log likelihood                            |
| <code>e(ll_0)</code>       | log likelihood, constant-only model       |
| <code>e(N_clust)</code>    | number of clusters                        |
| <code>e(chi2)</code>       | $\chi^2$                                  |
| <code>e(p)</code>          | $p$ -value for model test                 |
| <code>e(rank)</code>       | rank of <code>e(V)</code>                 |
| <code>e(ic)</code>         | number of iterations                      |
| <code>e(rc)</code>         | return code                               |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise               |

### Macros

|                              |                                                                          |
|------------------------------|--------------------------------------------------------------------------|
| <code>e(cmd)</code>          | logistic                                                                 |
| <code>e(cmdline)</code>      | command as typed                                                         |
| <code>e(depvar)</code>       | name of dependent variable                                               |
| <code>e(wtype)</code>        | weight type                                                              |
| <code>e(wexp)</code>         | weight expression                                                        |
| <code>e(title)</code>        | title in estimation output                                               |
| <code>e(clustvar)</code>     | name of cluster variable                                                 |
| <code>e(offset)</code>       | linear offset variable                                                   |
| <code>e(chi2type)</code>     | Wald or LR; type of model $\chi^2$ test                                  |
| <code>e(vce)</code>          | <i>vcetype</i> specified in <code>vce()</code>                           |
| <code>e(vcetype)</code>      | title used to label Std. err.                                            |
| <code>e(opt)</code>          | type of optimization                                                     |
| <code>e(which)</code>        | max or min; whether optimizer is to perform maximization or minimization |
| <code>e(ml_method)</code>    | type of ml method                                                        |
| <code>e(user)</code>         | name of likelihood-evaluator program                                     |
| <code>e(technique)</code>    | maximization technique                                                   |
| <code>e(properties)</code>   | b V                                                                      |
| <code>e(estat_cmd)</code>    | program used to implement <code>estat</code>                             |
| <code>e(predict)</code>      | program used to implement <code>predict</code>                           |
| <code>e(marginsok)</code>    | predictions allowed by <code>margins</code>                              |
| <code>e(marginsnotok)</code> | predictions disallowed by <code>margins</code>                           |
| <code>e(asbalanced)</code>   | factor variables <code>fvset</code> as <code>asbalanced</code>           |
| <code>e(asobserved)</code>   | factor variables <code>fvset</code> as <code>asobserved</code>           |

### Matrices

|                              |                                              |
|------------------------------|----------------------------------------------|
| <code>e(b)</code>            | coefficient vector                           |
| <code>e(Cns)</code>          | constraints matrix                           |
| <code>e(ilog)</code>         | iteration log (up to 20 iterations)          |
| <code>e(gradient)</code>     | gradient vector                              |
| <code>e(mns)</code>          | vector of means of the independent variables |
| <code>e(rules)</code>        | information about perfect predictors         |
| <code>e(V)</code>            | variance-covariance matrix of the estimators |
| <code>e(V_modelbased)</code> | model-based variance                         |

### Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

Matrices

`r(table)`

matrix containing the coefficients with their standard errors, test statistics, *p*-values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

## Methods and formulas

Define  $\mathbf{x}_j$  as the (row) vector of independent variables, augmented by 1, and  $\mathbf{b}$  as the corresponding estimated parameter (column) vector. The logistic regression model is fit by `logit`; see [R] `logit` for details of estimation.

The odds ratio corresponding to the *i*th coefficient is  $\psi_i = \exp(b_i)$ . The standard error of the odds ratio is  $s_i^\psi = \psi_i s_i$ , where  $s_i$  is the standard error of  $b_i$  estimated by `logit`.

Define  $I_j = \mathbf{x}_j \mathbf{b}$  as the predicted index of the *j*th observation. The predicted probability of a positive outcome is

$$p_j = \frac{\exp(I_j)}{1 + \exp(I_j)}$$

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] `_robust`, particularly *Maximum likelihood estimators* and *Methods and formulas*.

`logistic` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] `Variance estimation`.

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## Also see

- [R] **logistic postestimation** — Postestimation tools for logistic
- [R] **brier** — Brier score decomposition
- [R] **cloglog** — Complementary log–log regression
- [R] **xlogistic** — Exact logistic regression
- [R] **logit** — Logistic regression, reporting coefficients
- [R] **npregress kernel** — Nonparametric kernel regression
- [R] **npregress series** — Nonparametric series regression
- [R] **roc** — Receiver operating characteristic (ROC) analysis
- [BAYES] **bayes: logistic** — Bayesian logistic regression, reporting odds ratios
- [FMM] **fmm: logit** — Finite mixtures of logistic regression models
- [LASSO] **Lasso intro** — Introduction to lasso
- [MI] **Estimation** — Estimation commands for use with mi estimate
- [SVY] **svy estimation** — Estimation commands for survey data
- [XT] **xtlogit** — Fixed-effects, random-effects, and population-averaged logit models
- [U] **20 Estimation and postestimation commands**

|                         |                            |                          |                                      |
|-------------------------|----------------------------|--------------------------|--------------------------------------|
| Postestimation commands | <a href="#">predict</a>    | <a href="#">margins</a>  | <a href="#">Remarks and examples</a> |
| Methods and formulas    | <a href="#">References</a> | <a href="#">Also see</a> |                                      |

## Postestimation commands

The following postestimation commands are of special interest after `logistic`:

| Command                           | Description                                                           |
|-----------------------------------|-----------------------------------------------------------------------|
| <code>estat classification</code> | report various summary statistics, including the classification table |
| <code>estat gof</code>            | Pearson or Hosmer–Lemeshow goodness-of-fit test                       |
| <code>lroc</code>                 | compute area under ROC curve and graph the curve                      |
| <code>lsens</code>                | graph sensitivity and specificity versus probability cutoff           |

These commands are not appropriate with `svy` estimation results.

The following standard postestimation commands are also available:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| <code>linktest</code>        | link test for model specification                                                                                                       |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from <code>margins</code> (profile plots, interaction plots, etc.)                                                    |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | probabilities, influence statistics, residuals, etc.                                                                                    |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\* `forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results. `forecast` is also not appropriate with `mi` estimation results.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as probabilities, linear predictions, standard errors, influence statistics, deviance residuals, leverages, sequential numbers, Pearson residuals, and the equation-level score.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic nooffset rules asif]
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

Main

`pr` probability of a positive outcome; the default

`xb` linear prediction

`stdp` standard error of the prediction

\* `dbeta` Pregibon (1981)  $\Delta \hat{\beta}$  influence statistic

\* `deviance` deviance residual

\* `dx2` Hosmer, Lemeshow, and Sturdivant (2013)  $\Delta \chi^2$  influence statistic

\* `ddeviance` Hosmer, Lemeshow, and Sturdivant (2013)  $\Delta D$  influence statistic

\* `hat` Pregibon (1981) leverage

\* `number` sequential number of the covariate pattern

\* `residuals` Pearson residuals; adjusted for number sharing covariate pattern

\* `rstandard` standardized Pearson residuals; adjusted for number sharing covariate pattern

`score` first derivative of the log likelihood with respect to  $\mathbf{x}_j\beta$

Unstarred statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample. Starred statistics are calculated only for the estimation sample, even when `if e(sample)` is not specified.

`pr`, `xb`, `stdp`, and `score` are the only options allowed with `svy` estimation results.

## Options for predict

## Main

`pr`, the default, calculates the probability of a positive outcome.

`xb` calculates the linear prediction.

`stdp` calculates the standard error of the linear prediction.

`dbeta` calculates the Pregibon (1981)  $\Delta\hat{\beta}$  influence statistic, a standardized measure of the difference in the coefficient vector that is due to deletion of the observation along with all others that share the same covariate pattern. In Hosmer, Lemeshow, and Sturdivant (2013, 154–155) jargon, this statistic is *M*-asymptotic; that is, it is adjusted for the number of observations that share the same covariate pattern.

`deviance` calculates the deviance residual.

`dx2` calculates the Hosmer, Lemeshow, and Sturdivant (2013, 191)  $\Delta\chi^2$  influence statistic, reflecting the decrease in the Pearson  $\chi^2$  that is due to deletion of the observation and all others that share the same covariate pattern.

`ddeviance` calculates the Hosmer, Lemeshow, and Sturdivant (2013, 191)  $\Delta D$  influence statistic, which is the change in the deviance residual that is due to deletion of the observation and all others that share the same covariate pattern.

`hat` calculates the Pregibon (1981) leverage or the diagonal elements of the hat matrix adjusted for the number of observations that share the same covariate pattern.

`number` numbers the covariate patterns—observations with the same covariate pattern have the same number. Observations not used in estimation have `number` set to missing. The first covariate pattern is numbered 1, the second 2, and so on.

`residuals` calculates the Pearson residual as given by Hosmer, Lemeshow, and Sturdivant (2013, 155) and adjusted for the number of observations that share the same covariate pattern.

`rstandard` calculates the standardized Pearson residual as given by Hosmer, Lemeshow, and Sturdivant (2013, 191) and adjusted for the number of observations that share the same covariate pattern.

`score` calculates the equation-level score,  $\partial \ln L / \partial (\mathbf{x}_j \boldsymbol{\beta})$ .

## Options

`nooffset` is relevant only if you specified `offset(varname)` for `logistic`. It modifies the calculations made by `predict` so that they ignore the offset variable; the linear prediction is treated as  $\mathbf{x}_j \mathbf{b}$  rather than as  $\mathbf{x}_j \mathbf{b} + \text{offset}_j$ .

`rules` requests that Stata use any rules that were used to identify the model when making the prediction. By default, Stata calculates missing for excluded observations. See [example 1](#) in [R] [logit postestimation](#).

`asif` requests that Stata ignore the rules and the exclusion criteria and calculate predictions for all observations possible by using the estimated parameter from the model. See [example 1](#) in [R] [logit postestimation](#).

# margins

## Description for margins

`margins` estimates margins of response for probabilities and linear predictions.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i>       | Description                                    |
|------------------------|------------------------------------------------|
| <code>pr</code>        | probability of a positive outcome; the default |
| <code>xb</code>        | linear prediction                              |
| <code>stdp</code>      | not allowed with margins                       |
| <code>dbeta</code>     | not allowed with margins                       |
| <code>deviance</code>  | not allowed with margins                       |
| <code>dx2</code>       | not allowed with margins                       |
| <code>ddeviance</code> | not allowed with margins                       |
| <code>hat</code>       | not allowed with margins                       |
| <code>number</code>    | not allowed with margins                       |
| <code>residuals</code> | not allowed with margins                       |
| <code>rstandard</code> | not allowed with margins                       |
| <code>score</code>     | not allowed with margins                       |

Statistics not allowed with margins are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [\[R\] margins](#).

## Remarks and examples

`predict` is used after `logistic` to obtain predicted probabilities, residuals, and influence statistics for the estimation sample. The suggested diagnostic graphs below are from [Hosmer, Lemeshow, and Sturdivant \(2013\)](#), where they are more elaborately explained. Also see [Collett \(2003, 129–168\)](#) for a thorough discussion of model checking.

Remarks are presented under the following headings:

*predict without options*  
*predict with the xb and stdp options*  
*predict with the residuals option*  
*predict with the number option*  
*predict with the deviance option*  
*predict with the rstandard option*  
*predict with the hat option*  
*predict with the dx2 option*  
*predict with the ddeviance option*  
*predict with the dbeta option*

### predict without options

Typing `predict newvar` after estimation calculates the predicted probability of a positive outcome.

In [example 1](#) of [\[R\] logistic](#), we ran the model `logistic low age lwt i.race smoke ptl ht ui`. We obtain the predicted probabilities of a positive outcome by typing

```
. use https://www.stata-press.com/data/r19/lbw
(Hosmer & Lemeshow data)
. logistic low age lwt i.race smoke ptl ht ui
(output omitted)
. predict p
(option pr assumed; Pr(low))
. summarize p low
```

| Variable | Obs | Mean     | Std. dev. | Min      | Max      |
|----------|-----|----------|-----------|----------|----------|
| p        | 189 | .3121693 | .1913915  | .0272559 | .8391283 |
| low      | 189 | .3121693 | .4646093  | 0        | 1        |

### predict with the xb and stdp options

`predict` with the `xb` option calculates the linear combination  $x_j\mathbf{b}$ , where  $x_j$  are the independent variables in the  $j$ th observation and  $\mathbf{b}$  is the estimated parameter vector. This is sometimes known as the index function because the cumulative distribution function indexed at this value is the probability of a positive outcome.

With the `stdp` option, `predict` calculates the standard error of the prediction, which is *not* adjusted for replicated covariate patterns in the data. The influence statistics described below are adjusted for replicated covariate patterns in the data.

## predict with the residuals option

`predict` can calculate more than predicted probabilities. The Pearson residual is defined as the square root of the contribution of the covariate pattern to the Pearson  $\chi^2$  goodness-of-fit statistic, signed according to whether the observed number of positive responses within the covariate pattern is less than or greater than expected. For instance,

```
. predict r, residuals
. summarize r, detail
```

| Pearson residual |             |           |             |  |           |
|------------------|-------------|-----------|-------------|--|-----------|
|                  | Percentiles | Smallest  |             |  |           |
| 1%               | -1.750923   | -2.283885 |             |  |           |
| 5%               | -1.129907   | -1.750923 |             |  |           |
| 10%              | -.9581174   | -1.636279 | Obs         |  | 189       |
| 25%              | -.6545911   | -1.636279 | Sum of wgt. |  | 189       |
| 50%              | -.3806923   |           | Mean        |  | -.0242299 |
|                  |             | Largest   | Std. dev.   |  | .9970949  |
| 75%              | .8162894    | 2.23879   |             |  |           |
| 90%              | 1.510355    | 2.317558  | Variance    |  | .9941981  |
| 95%              | 1.747948    | 3.002206  | Skewness    |  | .8618271  |
| 99%              | 3.002206    | 3.126763  | Kurtosis    |  | 3.038448  |

We notice the prevalence of a few large positive residuals:

```
. sort r
. list id r low p age race in -5/1
```

|      | id | r        | low | p        | age | race  |
|------|----|----------|-----|----------|-----|-------|
| 185. | 33 | 2.224501 | 1   | .1681123 | 19  | White |
| 186. | 57 | 2.23879  | 1   | .166329  | 15  | White |
| 187. | 16 | 2.317558 | 1   | .1569594 | 27  | Other |
| 188. | 77 | 3.002206 | 1   | .0998678 | 26  | White |
| 189. | 36 | 3.126763 | 1   | .0927932 | 24  | White |

## predict with the number option

Covariate patterns play an important role in logistic regression. Two observations are said to share the same covariate pattern if the independent variables for the two observations are identical. Although we might think of having individual observations, the statistical information in the sample can be summarized by the covariate patterns, the number of observations with that covariate pattern, and the number of positive outcomes within the pattern. Depending on the model, the number of covariate patterns can approach or be equal to the number of observations, or it can be considerably less.

Stata calculates all the residual and diagnostic statistics in terms of covariate patterns, not observations. That is, all observations with the same covariate pattern are given the same residual and diagnostic statistics. Hosmer, Lemeshow, and Sturdivant (2013, 154–155) argue that such “*M*-asymptotic” statistics are more useful than “*N*-asymptotic” statistics.

To understand the difference, think of an observed positive outcome with predicted probability of 0.8. Taking the observation in isolation, the residual must be positive—we expected 0.8 positive responses and observed 1. This may indeed be the correct residual, but not necessarily. Under the *M*-asymptotic definition, we ask how many successes we observed across all observations with this covariate pattern.

If that number were, say, six, and there were a total of 10 observations with this covariate pattern, then the residual is negative for the covariate pattern—we expected eight positive outcomes but observed six. `predict` makes this kind of calculation and then attaches the same residual to all observations in the covariate pattern.

Occasionally, you might want to find all observations sharing a covariate pattern. `number` allows you to do this:

```
. predict pattern, number
. summarize pattern
```

| Variable | Obs | Mean    | Std. dev. | Min | Max |
|----------|-----|---------|-----------|-----|-----|
| pattern  | 189 | 89.2328 | 53.16573  | 1   | 182 |

We previously fit the model `logistic low age lwt i.race smoke ptl ht ui` over 189 observations. There are 182 covariate patterns in our data.

### predict with the deviance option

The deviance residual is defined as the square root of the contribution to the likelihood-ratio test statistic of a saturated model versus the fitted model. It has slightly different properties from the Pearson residual (see [Hosmer, Lemeshow, and Sturdivant \[2013, 155–157\]](#)):

```
. predict d, deviance
. summarize d, detail
```

| deviance residual |             |           |             |  |           |
|-------------------|-------------|-----------|-------------|--|-----------|
|                   | Percentiles | Smallest  |             |  |           |
| 1%                | -1.843472   | -1.911621 |             |  |           |
| 5%                | -1.33477    | -1.843472 |             |  |           |
| 10%               | -1.148316   | -1.843472 | Obs         |  | 189       |
| 25%               | -.8445325   | -1.674869 | Sum of wgt. |  | 189       |
| 50%               | -.5202702   |           | Mean        |  | -.1228811 |
|                   |             | Largest   | Std. dev.   |  | 1.049237  |
| 75%               | .9129041    | 1.894089  |             |  |           |
| 90%               | 1.541558    | 1.924457  | Variance    |  | 1.100898  |
| 95%               | 1.673338    | 2.146583  | Skewness    |  | .6598857  |
| 99%               | 2.146583    | 2.180542  | Kurtosis    |  | 2.036938  |

## predict with the rstandard option

Pearson residuals do not have a standard deviation equal to 1. `rstandard` generates Pearson residuals normalized to have an *expected* standard deviation equal to 1.

```
. predict rs, rstandard
. summarize r rs
```

| Variable | Obs | Mean      | Std. dev. | Min       | Max      |
|----------|-----|-----------|-----------|-----------|----------|
| r        | 189 | -.0242299 | .9970949  | -2.283885 | 3.126763 |
| rs       | 189 | -.0279135 | 1.026406  | -2.4478   | 3.149081 |

```
. correlate r rs
(obs=189)
```

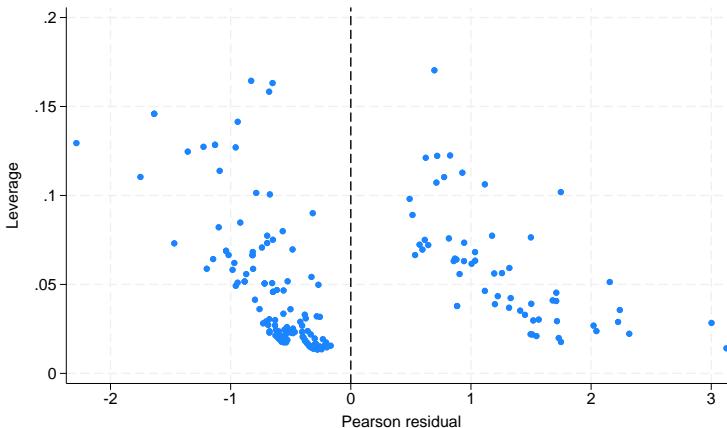
|    | r      | rs     |
|----|--------|--------|
| r  | 1.0000 |        |
| rs | 0.9998 | 1.0000 |

Remember that we previously created `r` containing the (unstandardized) Pearson residuals. In these data, whether we use standardized or unstandardized residuals does not matter much.

## predict with the hat option

`hat` calculates the leverage of a covariate pattern—a scaled measure of distance in terms of the independent variables. Large values indicate covariate patterns far from the average covariate pattern that can have a large effect on the fitted model even if the corresponding residual is small. Consider the following graph:

```
. predict h, hat
. scatter h r, xline(0) ytitle("Leverage")
```



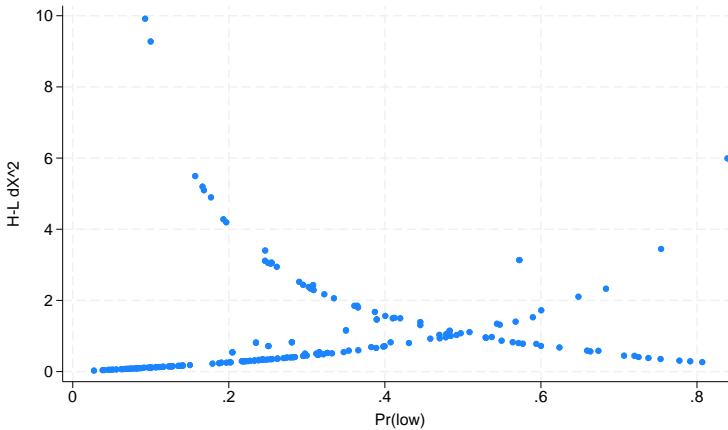
The points to the left of the vertical line are observed negative outcomes; here our data contain almost as many covariate patterns as observations, so most covariate patterns are unique. In such unique patterns, we observe either 0 or 1 success and expect  $p$ , thus forcing the sign of the residual. If we had fewer covariate patterns—if we did not have continuous variables in our model—there would be no such interpretation, and we would not have drawn the vertical line at 0.

Points on the left and right edges of the graph represent large residuals—covariate patterns that are not fit well by our model. Points at the top of our graph represent high leverage patterns. When analyzing the influence of observations on the model, we are most interested in patterns with high leverage and small residuals—patterns that might otherwise escape our attention.

## predict with the dx2 option

There are many ways to measure influence, and `hat` is one example. `dx2` measures the decrease in the Pearson  $\chi^2$  goodness-of-fit statistic that would be caused by deleting an observation (and all others sharing the covariate pattern):

```
. predict dx2, dx2
. scatter dx2 p
```



Paraphrasing [Hosmer, Lemeshow, and Sturdivant \(2013, 195–197\)](#), the points going from the top left to the bottom right correspond to covariate patterns with the number of positive outcomes equal to the number in the group; the points on the other curve correspond to 0 positive outcomes. In our data, most of the covariate patterns are unique, so the points tend to lie along one or the other curves; the points that are off the curves correspond to the few repeated covariate patterns in our data in which all the outcomes are not the same.

We examine this graph for large values of `dx2`—there are two at the top left.

## predict with the ddeviance option

Another measure of influence is the change in the deviance residuals due to deletion of a covariate pattern:

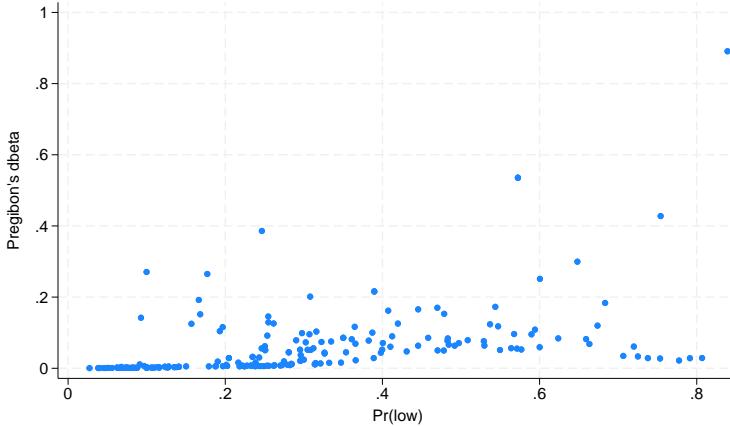
```
. predict dd, ddeviance
```

As with `dx2`, we typically graph `ddeviance` against the probability of a positive outcome. We direct you to [Hosmer, Lemeshow, and Sturdivant \(2013, 195\)](#) for an example and for the interpretation of this graph.

## predict with the dbeta option

One of the more direct measures of influence of interest to model fitters is the [Pregibon \(1981\)](#) dbeta measure, a measure of the change in the coefficient vector that would be caused by deleting an observation (and all others sharing the covariate pattern):

```
. predict db, dbeta
. scatter db p
```



One observation has a large effect on the estimated coefficients. We can easily find this point:

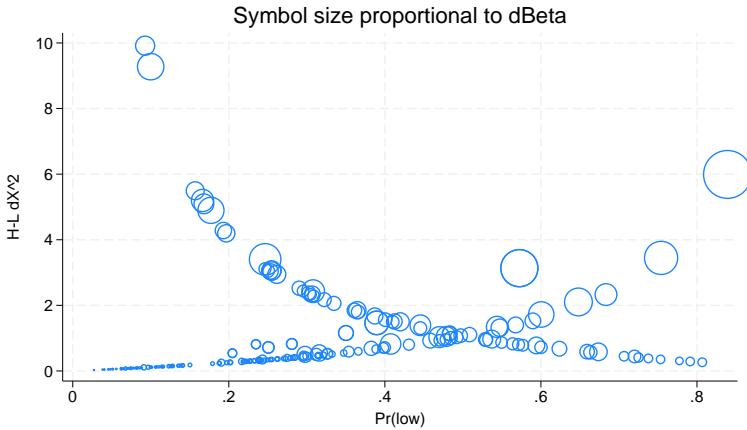
```
. sort db
. list in 1
```

189.

|          |     |           |     |          |           |     |          |    |          |      |
|----------|-----|-----------|-----|----------|-----------|-----|----------|----|----------|------|
| id       | low | age       | lwt | race     | smoke     | ptl | ht       | ui | ftv      | bwt  |
| 188      | 0   | 25        | 95  | White    | Smoker    | 3   | 0        | 1  | 0        | 3637 |
| p        |     | r         |     | pattern  | d         |     | rs       |    | h        |      |
| .8391283 |     | -2.283885 |     | 117      | -1.911621 |     | -2.4478  |    | .1294439 |      |
| dx2      |     |           |     | dd       |           |     | db       |    |          |      |
| 5.991726 |     |           |     | 4.197658 |           |     | .8909163 |    |          |      |

Hosmer, Lemeshow, and Sturdivant (2013, 196) suggest a graph that combines two of the influence measures:

```
. scatter dx2 p [aw=db], title("Symbol size proportional to dBeta") mfcolor(none)
```



We can easily spot the most influential points by the dbeta and dx2 measures.

## Methods and formulas

Let  $j$  index observations. Define  $M_j$  for each observation as the total number of observations sharing  $j$ 's covariate pattern. Define  $Y_j$  as the total number of positive responses among observations sharing  $j$ 's covariate pattern.

The Pearson residual for the  $j$ th observation is defined as

$$r_j = \frac{Y_j - M_j p_j}{\sqrt{M_j p_j (1 - p_j)}}$$

For  $M_j > 1$ , the deviance residual  $d_j$  is defined as

$$d_j = \pm \left( 2 \left[ Y_j \ln \left( \frac{Y_j}{M_j p_j} \right) + (M_j - Y_j) \ln \left\{ \frac{M_j - Y_j}{M_j (1 - p_j)} \right\} \right] \right)^{1/2}$$

where the sign is the same as the sign of  $(Y_j - M_j p_j)$ . In the limiting cases, the deviance residual is given by

$$d_j = \begin{cases} -\sqrt{2M_j |\ln(1 - p_j)|} & \text{if } Y_j = 0 \\ \sqrt{2M_j |\ln p_j|} & \text{if } Y_j = M_j \end{cases}$$

The *unadjusted* diagonal elements of the hat matrix  $h_{Uj}$  are given by  $h_{Uj} = (\mathbf{XVX}')_{jj}$ , where  $V$  is the estimated covariance matrix of parameters. The adjusted diagonal elements  $h_j$  created by hat are then  $h_j = M_j p_j (1 - p_j) h_{Uj}$ .

The standardized Pearson residual  $r_{Sj}$  is  $r_j / \sqrt{1 - h_j}$ .

The Pregibon (1981)  $\Delta\hat{\beta}_j$  influence statistic is

$$\Delta\hat{\beta}_j = \frac{r_j^2 h_j}{(1 - h_j)^2}$$

The corresponding change in the Pearson  $\chi^2$  is  $r_{Sj}^2$ . The corresponding change in the deviance residual is  $\Delta D_j = d_j^2 / (1 - h_j)$ .

## References

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## Also see

- [R] **logistic** — Logistic regression, reporting odds ratios
- [R] **estat classification** — Classification statistics and table
- [R] **estat gof** — Pearson or Hosmer–Lemeshow goodness-of-fit test
- [R] **lroc** — Compute area under ROC curve and graph the curve
- [R] **lsens** — Graph sensitivity and specificity versus probability cutoff
- [U] **20 Estimation and postestimation commands**

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## Description

`logit` fits a logit model for a binary response by maximum likelihood; it models the probability of a positive outcome given a set of regressors. `devar` equal to nonzero and nonmissing (typically `devar` equal to one) indicates a positive outcome, whereas `devar` equal to zero indicates a negative outcome.

Also see [\[R\] `logistic`](#); `logistic` displays estimates as odds ratios. Many users prefer the `logistic` command to `logit`. Results are the same regardless of which you use—both are the maximum-likelihood estimator. Several auxiliary commands that can be run after `logit`, `probit`, or `logistic` estimation are described in [\[R\] `logistic postestimation`](#).

## Quick start

Logit model of `y` on `x1` and `x2`

```
logit y x1 x2
```

Add indicators for categorical variable `a`

```
logit y x1 x2 i.a
```

With cluster-robust standard errors for clustering by levels of `cvar`

```
logit y x1 x2 i.a, vce(cluster cvar)
```

Save separate coefficient estimates for each level of `cvar` to `myresults.dta`

```
statsby _b, by(cvar) saving(myresults): logit y x1 x2 i.a
```

Adjust for complex survey design using `svyset` data

```
svy: logit y x1 x2 i.a
```

## Menu

Statistics > Binary outcomes > Logistic regression

## Syntax

```
logit deprvar [indepvars] [if] [in] [weight] [, options]
```

| <i>options</i>                               | Description                                                                                                                                                                 |
|----------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                 |                                                                                                                                                                             |
| <code>noconstant</code>                      | suppress constant term                                                                                                                                                      |
| <code>offset(<i>varname</i>)</code>          | include <i>varname</i> in model with coefficient constrained to 1                                                                                                           |
| <code>asis</code>                            | retain perfect predictor variables                                                                                                                                          |
| <code>constraints(<i>constraints</i>)</code> | apply specified linear constraints                                                                                                                                          |
| <b>SE/Robust</b>                             |                                                                                                                                                                             |
| <code>vce(<i>vcetype</i>)</code>             | <i>vcetype</i> may be <code>oim</code> , <code>opg</code> , <code>robust</code> , <code>cluster <i>clustvar</i></code> , <code>bootstrap</code> , or <code>jackknife</code> |
| <b>Reporting</b>                             |                                                                                                                                                                             |
| <code>level(#)</code>                        | set confidence level; default is level(95)                                                                                                                                  |
| <code>or</code>                              | report odds ratios                                                                                                                                                          |
| <code>nocnsreport</code>                     | do not display constraints                                                                                                                                                  |
| <code>display_options</code>                 | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling                            |
| <b>Maximization</b>                          |                                                                                                                                                                             |
| <code>maximize_options</code>                | control the maximization process; seldom used                                                                                                                               |
| <code>nocoeff</code>                         | do not display coefficient table; seldom used                                                                                                                               |
| <code>collinear</code>                       | keep collinear variables                                                                                                                                                    |
| <code>coeflegend</code>                      | display legend instead of statistics                                                                                                                                        |

*indepvars* may contain factor variables; see [U] 11.4.3 **Factor variables**.

*deprvar* and *indepvars* may contain time-series operators; see [U] 11.4.4 **Time-series varlists**.

`bayes`, `bayesboot`, `bootstrap`, `by`, `collect`, `fmm`, `fp`, `jackknife`, `mfp`, `mi estimate`, `nestreg`, `rolling`, `statsby`, `stepwise`, and `svy` are allowed; see [U] 11.1.10 **Prefix commands**. For more details, see [BAYES] **bayes: logit** and [FMM] **fmm: logit**.

`vce(bootstrap)` and `vce(jackknife)` are not allowed with the `mi estimate` prefix; see [MI] **mi estimate**.

Weights are not allowed with the `bootstrap` prefix; see [R] **bootstrap**.

`vce()`, `nocoeff`, and `weights` are not allowed with the `svy` prefix; see [SVY] **svy**.

`fweights`, `iwweights`, and `pweights` are allowed; see [U] 11.1.6 **weight**.

`nocoeff`, `collinear`, and `coeflegend` do not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options

### Model

`noconstant`, `offset(varname)`, `constraints(constraints)`; see [R] [Estimation options](#).

`asis` forces retention of perfect predictor variables and their associated perfectly predicted observations and may produce instabilities in maximization; see [R] [probit](#).

### SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

### Reporting

`level(#)`; see [R] [Estimation options](#).

`or` reports the estimated coefficients transformed to odds ratios, that is,  $e^b$  rather than  $b$ . Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated. `or` may be specified at estimation or when replaying previously estimated results.

`nocnsreport`; see [R] [Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

### Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

The following options are available with `logit` but are not shown in the dialog box:

`noccoef` specifies that the coefficient table not be displayed. This option is sometimes used by program writers but is of no use interactively.

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Remarks are presented under the following headings:

*Basic usage*

*Model identification*

## Basic usage

`logit` fits maximum likelihood models with dichotomous dependent (left-hand-side) variables coded as 0/1 (or, more precisely, coded as 0 and not-0).

For grouped data or data in binomial form, a probit model can be fit using `glm` with the family(binomial) and link(logit) options.

### ► Example 1

We have data on the make, weight, and mileage rating of 22 foreign and 52 domestic automobiles. We wish to fit a logit model explaining whether a car is foreign on the basis of its weight and mileage. Here is an overview of our data:

```
. use https://www.stata-press.com/data/r19/aut
(1978 automobile data)
. keep make mpg weight foreign
. describe
Contains data from https://www.stata-press.com/data/r19/aut.dta
Observations: 74 1978 automobile data
Variables: 4 13 Apr 2024 17:45
 (_dta has notes)
```

| Variable name | Storage type | Display format | Value label | Variable label |
|---------------|--------------|----------------|-------------|----------------|
| make          | str18        | %-18s          |             | Make and model |
| mpg           | int          | %8.0g          |             | Mileage (mpg)  |
| weight        | int          | %8.0gc         |             | Weight (lbs.)  |
| foreign       | byte         | %8.0g          | origin      | Car origin     |

Sorted by: foreign

Note: Dataset has changed since last saved.

```
. inspect foreign
```

| foreign: Car origin |          | Number of observations |          |             |
|---------------------|----------|------------------------|----------|-------------|
|                     |          | Total                  | Integers | Nonintegers |
| #                   | Negative | -                      | -        | -           |
| #                   | Zero     | 52                     | 52       | -           |
| #                   | Positive | 22                     | 22       | -           |
| #                   |          |                        |          |             |
| # #                 | Total    | 74                     | 74       | -           |
| # #                 | Missing  | -                      |          |             |
| 0                   |          | 74                     |          |             |
| 1                   |          |                        |          |             |

(2 unique values)

foreign is labeled and all values are documented in the label.

The variable `foreign` takes on two unique values, 0 and 1. The value 0 denotes a domestic car, and 1 denotes a foreign car.

The model that we wish to fit is

$$\Pr(\text{foreign} = 1) = F(\beta_0 + \beta_1 \text{weight} + \beta_2 \text{mpg})$$

where  $F(z) = e^z / (1 + e^z)$  is the cumulative logistic distribution.

To fit this model, we type

```
. logit foreign weight mpg
Iteration 0: Log likelihood = -45.03321
Iteration 1: Log likelihood = -29.238536
Iteration 2: Log likelihood = -27.244139
Iteration 3: Log likelihood = -27.175277
Iteration 4: Log likelihood = -27.175156
Iteration 5: Log likelihood = -27.175156

Logistic regression Number of obs = 74
 LR chi2(2) = 35.72
 Prob > chi2 = 0.0000
Log likelihood = -27.175156 Pseudo R2 = 0.3966
```

| foreign | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|---------|-------------|-----------|-------|-------|----------------------|----------|
| weight  | -.0039067   | .0010116  | -3.86 | 0.000 | -.0058894            | -.001924 |
| mpg     | -.1685869   | .0919175  | -1.83 | 0.067 | -.3487418            | .011568  |
| _cons   | 13.70837    | 4.518709  | 3.03  | 0.002 | 4.851859             | 22.56487 |

We find that heavier cars are less likely to be foreign and that cars yielding better gas mileage are also less likely to be foreign, at least holding the weight of the car constant.

◀

## □ Technical note

Stata interprets a value of 0 as a negative outcome (failure) and treats all other values (except missing) as positive outcomes (successes). Thus if your dependent variable takes on the values 0 and 1, then 0 is interpreted as failure and 1 as success. If your dependent variable takes on the values 0, 1, and 2, then 0 is still interpreted as failure, but both 1 and 2 are treated as successes.

If you prefer a more formal mathematical statement, when you type `logit y x`, Stata fits the model

$$\Pr(y_j \neq 0 \mid \mathbf{x}_j) = \frac{\exp(\mathbf{x}_j\boldsymbol{\beta})}{1 + \exp(\mathbf{x}_j\boldsymbol{\beta})}$$

□

## Model identification

The `logit` command has one more feature, and it is probably the most useful. `logit` automatically checks the model for identification and, if it is underidentified, drops whatever variables and observations are necessary for estimation to proceed. (`logistic`, `probit`, and `ivprobit` do this as well.)

## ▶ Example 2

Have you ever fit a logit model where one or more of your independent variables perfectly predicted one or the other outcome?

For instance, consider the following data:

| Outcome $y$ | Independent variable $x$ |
|-------------|--------------------------|
| 0           | 1                        |
| 0           | 1                        |
| 0           | 0                        |
| 1           | 0                        |

Say that we wish to predict the outcome on the basis of the independent variable. The outcome is always zero whenever the independent variable is one. In our data,  $\Pr(y = 0 \mid x = 1) = 1$ , which means that the logit coefficient on  $x$  must be minus infinity with a corresponding infinite standard error. At this point, you may suspect that we have a problem.

Unfortunately, not all such problems are so easily detected, especially if you have a lot of independent variables in your model. If you have ever had such difficulties, you have experienced one of the more unpleasant aspects of computer optimization. The computer has no idea that it is trying to solve for an infinite coefficient as it begins its iterative process. All it knows is that at each step, making the coefficient a little bigger, or a little smaller, works wonders. It continues on its merry way until either 1) the whole thing comes crashing to the ground when a numerical overflow error occurs or 2) it reaches some predetermined cutoff that stops the process. In the meantime, you have been waiting. The estimates that you finally receive, if you receive any at all, may be nothing more than numerical roundoff.

Stata watches for these sorts of problems, alerts us, fixes them, and properly fits the model.

Let's return to our automobile data. Among the variables we have in the data is one called `repair`, which takes on three values. A value of 1 indicates that the car has a poor repair record, 2 indicates an average record, and 3 indicates a better-than-average record. Here is a tabulation of our data:

```
. use https://www.stata-press.com/data/r19/repair, clear
(1978 automobile data)
. tabulate foreign repair
```

| Car origin | Repair |    |    | Total |
|------------|--------|----|----|-------|
|            | 1      | 2  | 3  |       |
| Domestic   | 10     | 27 | 9  | 46    |
| Foreign    | 0      | 3  | 9  | 12    |
| Total      | 10     | 30 | 18 | 58    |

All the cars with poor repair records (`repair = 1`) are domestic. If we were to attempt to predict `foreign` on the basis of the repair records, the predicted probability for the `repair = 1` category would have to be zero. This in turn means that the logit coefficient must be minus infinity, and that would set most computer programs buzzing.

Let's try Stata on this problem.

```
. logit foreign b3.repair
note: 1.repair != 0 predicts failure perfectly;
 1.repair omitted and 10 obs not used.
Iteration 0: Log likelihood = -26.992087
Iteration 1: Log likelihood = -22.483187
Iteration 2: Log likelihood = -22.230498
Iteration 3: Log likelihood = -22.229139
Iteration 4: Log likelihood = -22.229138

Logistic regression Number of obs = 48
 LR chi2(1) = 9.53
 Prob > chi2 = 0.0020
 Pseudo R2 = 0.1765

Log likelihood = -22.229138
```

|  | foreign | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|--|---------|-------------|-----------|-------|-------|----------------------|-----------|
|  | repair  |             |           |       |       |                      |           |
|  | 1       | 0           | (empty)   |       |       |                      |           |
|  | 2       | -2.197225   | .7698003  | -2.85 | 0.004 | -3.706005            | -.6884436 |
|  | _cons   | -1.85e-17   | .4714045  | -0.00 | 1.000 | -.9239359            | .9239359  |

Remember that all the cars with poor repair records (`repair = 1`) are domestic, so the model cannot be fit, or at least it cannot be fit if we restrict ourselves to finite coefficients. Stata noted that fact “note: 1.repair !=0 predicts failure perfectly”. This is Stata’s mathematically precise way of saying what we said in English. When `repair` is 1, the car is domestic.

Stata then went on to say “1.repair omitted and 10 obs not used”. This is Stata eliminating the problem. First `1.repair` had to be removed from the model because it would have an infinite coefficient. Then, the 10 observations that led to the problem had to be eliminated, as well, so as not to bias the remaining coefficients in the model. The 10 observations that are not used are the 10 domestic cars that have poor repair records.

Stata then fit what was left of the model, using the remaining observations. Because no observations remained for cars with poor repair records, Stata reports “(empty)” in the row for `repair = 1`.



## □ Technical note

Stata is pretty smart about catching problems like this. It will catch “one-way causation by a dummy variable”, as we demonstrated above.

Stata also watches for “two-way causation”, that is, a variable that perfectly determines the outcome, both successes and failures. Here Stata says, “so-and-so predicts outcome perfectly” and stops. Statistics dictates that no model can be fit.

Stata also checks your data for collinear variables; it will say, “so-and-so omitted because of collinearity”. No observations need to be eliminated in this case, and model fitting will proceed without the offending variable.

It will also catch a subtle problem that can arise with continuous data. For instance, if we were estimating the chances of surviving the first year after an operation, and if we included in our model age, and if all the persons over 65 died within the year, Stata would say, “age > 65 predicts failure perfectly”. It would then inform us about the fix-up it takes and fit what can be fit of our model.

logit (and logistic, probit, and ivprobit) will also occasionally display messages such as

Note: 4 failures and 0 successes completely determined.

There are two causes for a message like this. The first—and most unlikely—case occurs when a continuous variable (or a combination of a continuous variable with other continuous or dummy variables) is simply a great predictor of the dependent variable. Consider Stata’s auto.dta dataset with 6 observations removed.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. drop if foreign==0 & gear_ratio > 3.1
(6 observations deleted)
. logit foreign mpg weight gear_ratio, nolog
Logistic regression Number of obs = 68
 LR chi2(3) = 72.64
 Prob > chi2 = 0.0000
Log likelihood = -6.4874814 Pseudo R2 = 0.8484
```

| foreign    | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|------------|-------------|-----------|-------|-------|----------------------|----------|
| mpg        | -.4944907   | .2655508  | -1.86 | 0.063 | -1.014961            | .0259792 |
| weight     | -.0060919   | .003101   | -1.96 | 0.049 | -.0121698            | -.000014 |
| gear_ratio | 15.70509    | 8.166234  | 1.92  | 0.054 | -.300436             | 31.71061 |
| _cons      | -21.39527   | 25.41486  | -0.84 | 0.400 | -71.20747            | 28.41694 |

Note: 4 failures and 0 successes completely determined.

There are no missing standard errors in the output. If you receive the “completely determined” message and have one or more missing standard errors in your output, see the second case discussed below.

Note gear\_ratio’s large coefficient. logit thought that the 4 observations with the smallest predicted probabilities were essentially predicted perfectly.

```
. predict p
(option pr assumed; Pr(foreign))
. sort p
. list p in 1/4
```

|    | P        |
|----|----------|
| 1. | 1.34e-10 |
| 2. | 6.26e-09 |
| 3. | 7.84e-09 |
| 4. | 1.49e-08 |

If this happens to you, you do not have to do anything. Computationally, the model is sound. The second case discussed below requires careful examination.

The second case occurs when the independent terms are all dummy variables or continuous ones with repeated values (for example, age). Here one or more of the estimated coefficients will have missing standard errors. For example, consider this dataset consisting of 6 observations.

```
. use https://www.stata-press.com/data/r19/logitxmpl, clear
. list, separator(0)
```

|    | y | x1 | x2 |
|----|---|----|----|
| 1. | 0 | 0  | 0  |
| 2. | 0 | 0  | 0  |
| 3. | 0 | 1  | 0  |
| 4. | 1 | 1  | 0  |
| 5. | 0 | 0  | 1  |
| 6. | 1 | 0  | 1  |

```
. logit y x1 x2
Iteration 0: Log likelihood = -3.819085
Iteration 1: Log likelihood = -2.9527336
Iteration 2: Log likelihood = -2.8110282
Iteration 3: Log likelihood = -2.7811973
Iteration 4: Log likelihood = -2.7746107
Iteration 5: Log likelihood = -2.7730128
(output omitted)
Iteration 296: Log likelihood = -2.7725887 (not concave)
Iteration 297: Log likelihood = -2.7725887 (not concave)
Iteration 298: Log likelihood = -2.7725887 (not concave)
Iteration 299: Log likelihood = -2.7725887 (not concave)
Iteration 300: Log likelihood = -2.7725887 (not concave)
convergence not achieved

Logistic regression Number of obs = 6
LR chi2(1) = 2.09
Prob > chi2 = 0.1480
Pseudo R2 = 0.2740

Log likelihood = -2.7725887
```

|  | y     | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |          |
|--|-------|-------------|-----------|--------|-------|----------------------|----------|
|  | x1    | 18.3704     | 2         | 9.19   | 0.000 | 14.45047             | 22.29033 |
|  | x2    | 18.3704     | .         | .      | .     | .                    | .        |
|  | _cons | -18.3704    | 1.414214  | -12.99 | 0.000 | -21.14221            | -15.5986 |

```
Note: 2 failures and 0 successes completely determined.
convergence not achieved
r(430);
```

Three things are happening here. First, `logit` iterates almost forever and then declares nonconvergence. Second, `logit` can fit the outcome ( $y = 0$ ) for the covariate pattern  $x_1 = 0$  and  $x_2 = 0$  (that is, the first two observations) perfectly. This observation is the “2 failures and 0 successes completely determined”. Third, if this observation is excluded, then  $x_1$ ,  $x_2$ , and the constant are collinear.

This is the cause of the nonconvergence, the message “completely determined”, and the missing standard errors. It happens when you have a covariate pattern (or patterns) with only one outcome and there is collinearity when the observations corresponding to this covariate pattern are excluded.

If this happens to you, confirm the causes. First, identify the covariate pattern with only one outcome. (For your data, replace `x1` and `x2` with the independent variables of your model.)

```
. egen pattern = group(x1 x2)
. quietly logit y x1 x2, iterate(100)
convergence not achieved
. predict p
(option pr assumed; Pr(y))
. summarize p
```

| Variable | Obs | Mean     | Std. dev. | Min      | Max |
|----------|-----|----------|-----------|----------|-----|
| p        | 6   | .3333333 | .2581989  | 1.05e-08 | .5  |

If successes were completely determined, that means that there are predicted probabilities that are almost 1. If failures were completely determined, that means that there are predicted probabilities that are almost 0. The latter is the case here, so we locate the corresponding value of `pattern`:

```
. tabulate pattern if p < 1e-7
```

| group(x1<br>x2) | Freq. | Percent | Cum.   |
|-----------------|-------|---------|--------|
| 1               | 2     | 100.00  | 100.00 |
| Total           | 2     | 100.00  |        |

Once we omit this covariate pattern from the estimation sample, `logit` can deal with the collinearity:

```
. logit y x1 x2 if pattern != 1, nolog
note: x2 omitted because of collinearity.
```

```
Logistic regression
```

```
Number of obs = 4
LR chi2(1) = 0.00
Prob > chi2 = 1.0000
Pseudo R2 = 0.0000
```

```
Log likelihood = -2.7725887
```

| y     | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |          |
|-------|-------------|-----------|------|-------|----------------------|----------|
| x1    | 0           | 2         | 0.00 | 1.000 | -3.919928            | 3.919928 |
| x2    | 0 (omitted) |           |      |       |                      |          |
| _cons | 0           | 1.414214  | 0.00 | 1.000 | -2.771808            | 2.771808 |

We omit the collinear variable. Then we must decide whether to include or omit the observations with `pattern = 1`. We could include them,

```
. logit y x1, nolog
Logistic regression Number of obs = 6
 LR chi2(1) = 0.37
 Prob > chi2 = 0.5447
Log likelihood = -3.6356349 Pseudo R2 = 0.0480
```

| y     | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-------|-------------|-----------|-------|-------|----------------------|----------|
| x1    | 1.098612    | 1.825742  | 0.60  | 0.547 | -2.479776            | 4.677001 |
| _cons | -1.098612   | 1.154701  | -0.95 | 0.341 | -3.361784            | 1.164559 |

or exclude them,

```
. logit y x1 if pattern != 1, nolog
Logistic regression Number of obs = 4
 LR chi2(1) = 0.00
 Prob > chi2 = 1.0000
Log likelihood = -2.7725887 Pseudo R2 = 0.0000
```

| y     | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |          |
|-------|-------------|-----------|------|-------|----------------------|----------|
| x1    | 0           | 2         | 0.00 | 1.000 | -3.919928            | 3.919928 |
| _cons | 0           | 1.414214  | 0.00 | 1.000 | -2.771808            | 2.771808 |

If the covariate pattern that predicts outcome perfectly is meaningful, you may want to exclude these observations from the model. Here you would report that covariate pattern such and such predicted outcome perfectly and that the best model for the rest of the data is .... But, more likely, the perfect prediction was simply the result of having too many predictors in the model. Then you would omit the extraneous variables from further consideration and report the best model for all the data.

□

## Stored results

`logit` stores the following in `e()`:

Scalars

|                            |                                           |
|----------------------------|-------------------------------------------|
| <code>e(N)</code>          | number of observations                    |
| <code>e(N_cds)</code>      | number of completely determined successes |
| <code>e(N_cdf)</code>      | number of completely determined failures  |
| <code>e(k)</code>          | number of parameters                      |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>  |
| <code>e(k_eq_model)</code> | number of equations in overall model test |
| <code>e(k_dv)</code>       | number of dependent variables             |
| <code>e(df_m)</code>       | model degrees of freedom                  |
| <code>e(r2_p)</code>       | pseudo- $R^2$                             |
| <code>e(ll)</code>         | log likelihood                            |
| <code>e(ll_0)</code>       | log likelihood, constant-only model       |
| <code>e(N_clust)</code>    | number of clusters                        |
| <code>e(chi2)</code>       | $\chi^2$                                  |
| <code>e(p)</code>          | $p$ -value for model test                 |
| <code>e(rank)</code>       | rank of <code>e(V)</code>                 |
| <code>e(ic)</code>         | number of iterations                      |
| <code>e(rc)</code>         | return code                               |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise               |

## Macros

|                              |                                                                          |
|------------------------------|--------------------------------------------------------------------------|
| <code>e(cmd)</code>          | logit                                                                    |
| <code>e(cmdline)</code>      | command as typed                                                         |
| <code>e(depvar)</code>       | name of dependent variable                                               |
| <code>e(wtype)</code>        | weight type                                                              |
| <code>e(wexp)</code>         | weight expression                                                        |
| <code>e(title)</code>        | title in estimation output                                               |
| <code>e(clustvar)</code>     | name of cluster variable                                                 |
| <code>e(offset)</code>       | linear offset variable                                                   |
| <code>e(chi2type)</code>     | Wald or LR; type of model $\chi^2$ test                                  |
| <code>e(vce)</code>          | <i>vce</i> type specified in <code>vce()</code>                          |
| <code>e(vcetype)</code>      | title used to label Std. err.                                            |
| <code>e(opt)</code>          | type of optimization                                                     |
| <code>e(which)</code>        | max or min; whether optimizer is to perform maximization or minimization |
| <code>e(ml_method)</code>    | type of ml method                                                        |
| <code>e(user)</code>         | name of likelihood-evaluator program                                     |
| <code>e(technique)</code>    | maximization technique                                                   |
| <code>e(properties)</code>   | b V                                                                      |
| <code>e(estat_cmd)</code>    | program used to implement <code>estat</code>                             |
| <code>e(predict)</code>      | program used to implement <code>predict</code>                           |
| <code>e(marginsok)</code>    | predictions allowed by <code>margins</code>                              |
| <code>e(marginsnotok)</code> | predictions disallowed by <code>margins</code>                           |
| <code>e(asbalanced)</code>   | factor variables <code>fvset</code> as <code>asbalanced</code>           |
| <code>e(asobserved)</code>   | factor variables <code>fvset</code> as <code>asobserved</code>           |

## Matrices

|                              |                                              |
|------------------------------|----------------------------------------------|
| <code>e(b)</code>            | coefficient vector                           |
| <code>e(Cns)</code>          | constraints matrix                           |
| <code>e(ilog)</code>         | iteration log (up to 20 iterations)          |
| <code>e(gradient)</code>     | gradient vector                              |
| <code>e(mns)</code>          | vector of means of the independent variables |
| <code>e(rules)</code>        | information about perfect predictors         |
| <code>e(V)</code>            | variance–covariance matrix of the estimators |
| <code>e(V_modelbased)</code> | model-based variance                         |

## Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

## Matrices

|                       |                                                                                                                            |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

## Methods and formulas

Cramer (2003, chap. 9) surveys the prehistory and history of the logit model. The word “logit” was coined by Berkson (1944) and is analogous to the word “probit”. For an introduction to probit and logit, see, for example, Aldrich and Nelson (1984), Cameron and Trivedi (2022), Jones (2007), Long (1997), Long and Freese (2014), Pampel (2021), or Powers and Xie (2008).

The likelihood function for logit is

$$\ln L = \sum_{j \in S} w_j \ln F(\mathbf{x}_j \mathbf{b}) + \sum_{j \notin S} w_j \ln \{1 - F(\mathbf{x}_j \mathbf{b})\}$$

where  $S$  is the set of all observations  $j$ , such that  $y_j \neq 0$ ,  $F(z) = e^z/(1 + e^z)$ , and  $w_j$  denotes the optional weights. `lnL` is maximized as described in [R] **Maximize**.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] **\_robust**, particularly *Maximum likelihood estimators* and *Methods and formulas*. The scores are calculated as  $\mathbf{u}_j = \{1 - F(\mathbf{x}_j\mathbf{b})\}\mathbf{x}_j$  for the positive outcomes and  $-F(\mathbf{x}_j\mathbf{b})\mathbf{x}_j$  for the negative outcomes.

`logit` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] **Variance estimation**.

Joseph Berkson (1899–1982) was born in New York City and studied at the College of the City of New York, Columbia, and Johns Hopkins, earning both an MD and a doctorate in statistics. He then worked at Johns Hopkins before moving to the Mayo Clinic in 1931 as a biostatistician. Among many other contributions, his most influential one drew upon a long-sustained interest in the logistic function, especially his 1944 paper on bioassay, in which he introduced the term “logit”. Berkson was a frequent participant in controversy—sometimes humorous, sometimes bitter—on subjects such as the evidence for links between smoking and various diseases and the relative merits of probit and logit methods and of different calculation methods.

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## Also see

- [R] **logit postestimation** — Postestimation tools for logit
- [R] **brier** — Brier score decomposition
- [R] **cloglog** — Complementary log–log regression
- [R] **exlogistic** — Exact logistic regression
- [R] **logistic** — Logistic regression, reporting odds ratios
- [R] **npregress kernel** — Nonparametric kernel regression
- [R] **npregress series** — Nonparametric series regression
- [R] **probit** — Probit regression
- [R] **roc** — Receiver operating characteristic (ROC) analysis
- [R] **ziologit** — Zero-inflated ordered logit regression
- [BAYES] **bayes: logit** — Bayesian logistic regression, reporting coefficients
- [FMM] **fmm: logit** — Finite mixtures of logistic regression models
- [LASSO] **Lasso intro** — Introduction to lasso
- [ME] **melogit** — Multilevel mixed-effects logistic regression
- [MI] **Estimation** — Estimation commands for use with mi estimate
- [SVY] **svy estimation** — Estimation commands for survey data
- [XT] **xtlogit** — Fixed-effects, random-effects, and population-averaged logit models
- [U] **20 Estimation and postestimation commands**

|                         |                            |                          |                      |
|-------------------------|----------------------------|--------------------------|----------------------|
| Postestimation commands | <a href="#">predict</a>    | <a href="#">margins</a>  | Remarks and examples |
| Methods and formulas    | <a href="#">References</a> | <a href="#">Also see</a> |                      |

## Postestimation commands

The following postestimation commands are of special interest after `logit`:

| Command                           | Description                                                           |
|-----------------------------------|-----------------------------------------------------------------------|
| <code>estat classification</code> | report various summary statistics, including the classification table |
| <code>estat gof</code>            | Pearson or Hosmer–Lemeshow goodness-of-fit test                       |
| <code>lroc</code>                 | compute area under ROC curve and graph the curve                      |
| <code>lsens</code>                | graph sensitivity and specificity versus probability cutoff           |
| <code>lassogof</code>             | calculate goodness-of-fit predictions                                 |

These commands are not appropriate with `svy` estimation results.

The following standard postestimation commands are also available:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike’s, consistent Akaike’s, corrected Akaike’s, and Schwarz’s Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| * <code>hausman</code>       | Hausman’s specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| <code>linktest</code>        | link test for model specification                                                                                                       |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from <code>margins</code> (profile plots, interaction plots, etc.)                                                    |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | probabilities, influence statistics, residuals, etc.                                                                                    |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*forecast, hausman, and lrtest are not appropriate with svy estimation results. forecast is also not appropriate with mi estimation results.

## predict

### Description for predict

predict creates a new variable containing predictions such as probabilities, linear predictions, standard errors, influence statistics, deviance residuals, leverages, sequential numbers, Pearson residuals, and the equation-level score.

### Menu for predict

Statistics > Postestimation

### Syntax for predict

```
predict [type] newvar [if] [in] [, statistic nooffset rules asif]
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

Main

|                          |                                                                               |
|--------------------------|-------------------------------------------------------------------------------|
| <code>pr</code>          | probability of a positive outcome; the default                                |
| <code>xb</code>          | linear prediction                                                             |
| <code>stdp</code>        | standard error of the prediction                                              |
| * <code>dbeta</code>     | Pregibon (1981) $\Delta \hat{\beta}$ influence statistic                      |
| * <code>deviance</code>  | deviance residual                                                             |
| * <code>dx2</code>       | Hosmer, Lemeshow, and Sturdivant (2013) $\Delta \chi^2$ influence statistic   |
| * <code>ddeviance</code> | Hosmer, Lemeshow, and Sturdivant (2013) $\Delta D$ influence statistic        |
| * <code>hat</code>       | Pregibon (1981) leverage                                                      |
| * <code>number</code>    | sequential number of the covariate pattern                                    |
| * <code>residuals</code> | Pearson residuals; adjusted for number sharing covariate pattern              |
| * <code>rstandard</code> | standardized Pearson residuals; adjusted for number sharing covariate pattern |
| <code>score</code>       | first derivative of the log likelihood with respect to $\mathbf{x}_j\beta$    |

Unstarred statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample. Starred statistics are calculated only for the estimation sample, even when `if e(sample)` is not specified.

`pr`, `xb`, `stdp`, and `score` are the only options allowed with svy estimation results.

### Options for predict

Main

`pr`, the default, calculates the probability of a positive outcome.

`xb` calculates the linear prediction.

`stdp` calculates the standard error of the linear prediction.

`dbeta` calculates the Pregibon (1981)  $\Delta\hat{\beta}$  influence statistic, a standardized measure of the difference in the coefficient vector that is due to deletion of the observation along with all others that share the same covariate pattern. In Hosmer, Lemeshow, and Sturdivant (2013, 154–155) jargon, this statistic is  $M$ -asymptotic; that is, it is adjusted for the number of observations that share the same covariate pattern.

`deviance` calculates the deviance residual.

`dx2` calculates the Hosmer, Lemeshow, and Sturdivant (2013, 191)  $\Delta\chi^2$  influence statistic, reflecting the decrease in the Pearson  $\chi^2$  that is due to deletion of the observation and all others that share the same covariate pattern.

`ddeviance` calculates the Hosmer, Lemeshow, and Sturdivant (2013, 191)  $\Delta D$  influence statistic, which is the change in the deviance residual that is due to deletion of the observation and all others that share the same covariate pattern.

`hat` calculates the Pregibon (1981) leverage or the diagonal elements of the hat matrix adjusted for the number of observations that share the same covariate pattern.

`number` numbers the covariate patterns—observations with the same covariate pattern have the same number. Observations not used in estimation have `number` set to missing. The first covariate pattern is numbered 1, the second 2, and so on.

`residuals` calculates the Pearson residual as given by Hosmer, Lemeshow, and Sturdivant (2013, 155) and adjusted for the number of observations that share the same covariate pattern.

`rstandard` calculates the standardized Pearson residual as given by Hosmer, Lemeshow, and Sturdivant (2013, 191) and adjusted for the number of observations that share the same covariate pattern.

`score` calculates the equation-level score,  $\partial \ln L / \partial (\mathbf{x}_j \boldsymbol{\beta})$ .

---

#### Options

`nooffset` is relevant only if you specified `offset(varname)` for `logit`. It modifies the calculations made by `predict` so that they ignore the offset variable; the linear prediction is treated as  $\mathbf{x}_j \mathbf{b}$  rather than as  $\mathbf{x}_j \mathbf{b} + \text{offset}_j$ .

`rules` requests that Stata use any rules that were used to identify the model when making the prediction. By default, Stata calculates missing for excluded observations.

`asif` requests that Stata ignore the rules and exclusion criteria and calculate predictions for all observations possible by using the estimated parameter from the model.

# margins

## Description for margins

margins estimates margins of response for probabilities and linear predictions.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i>       | Description                                    |
|------------------------|------------------------------------------------|
| <code>pr</code>        | probability of a positive outcome; the default |
| <code>xb</code>        | linear prediction                              |
| <code>stdp</code>      | not allowed with margins                       |
| <code>dbeta</code>     | not allowed with margins                       |
| <code>deviance</code>  | not allowed with margins                       |
| <code>dx2</code>       | not allowed with margins                       |
| <code>ddeviance</code> | not allowed with margins                       |
| <code>hat</code>       | not allowed with margins                       |
| <code>number</code>    | not allowed with margins                       |
| <code>residuals</code> | not allowed with margins                       |
| <code>rstandard</code> | not allowed with margins                       |
| <code>score</code>     | not allowed with margins                       |

Statistics not allowed with margins are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [R] [margins](#).

## Remarks and examples

Once you have fit a logit model, you can obtain the predicted probabilities by using the `predict` command for both the estimation sample and other samples; see [U] [20 Estimation and postestimation commands](#) and [R] [predict](#). Here we will make only a few more comments.

`predict` without arguments calculates the predicted probability of a positive outcome, that is,  $\Pr(y_j = 1) = F(\mathbf{x}_j\mathbf{b})$ . With the `xb` option, `predict` calculates the linear combination  $\mathbf{x}_j\mathbf{b}$ , where  $\mathbf{x}_j$  are the independent variables in the  $j$ th observation and  $\mathbf{b}$  is the estimated parameter vector. This is sometimes known as the index function because the cumulative distribution function indexed at this value is the probability of a positive outcome.

In both cases, Stata remembers any rules used to identify the model and calculates missing for excluded observations, unless `rules` or `asif` is specified. For information about the other statistics available after `predict`, see [R] [logistic postestimation](#).

## ► Example 1: Predicted probabilities

In [example 2](#) of [\[R\] logit](#), we fit the logit model `logit foreign b3.repair`. To obtain predicted probabilities, type

```
. use https://www.stata-press.com/data/r19/repair
(1978 automobile data)
. logit foreign b3.repair
note: 1.repair != 0 predicts failure perfectly
 1.repair omitted and 10 obs not used
(output omitted)
. predict p
(option pr assumed; Pr(foreign))
(10 missing values generated)
. summarize foreign p
```

| Variable | Obs | Mean     | Std. dev. | Min | Max |
|----------|-----|----------|-----------|-----|-----|
| foreign  | 58  | .2068966 | .4086186  | 0   | 1   |
| p        | 48  | .25      | .1956984  | .1  | .5  |

Stata remembers any rules used to identify the model and sets predictions to missing for any excluded observations. `logit` omitted the variable `1.repair` from our model and excluded 10 observations. Thus when we typed `predict p`, those same 10 observations were again excluded, and their predictions were set to missing.

`predict's` `rules` option uses the rules in the prediction. During estimation, we were told “`1.repair != 0` predicts failure perfectly”, so the rule is that when `1.repair` is not zero, we should predict 0 probability of success or a positive outcome:

```
. predict p2, rules
(option pr assumed; Pr(foreign))
. summarize foreign p p2
```

| Variable | Obs | Mean     | Std. dev. | Min | Max |
|----------|-----|----------|-----------|-----|-----|
| foreign  | 58  | .2068966 | .4086186  | 0   | 1   |
| p        | 48  | .25      | .1956984  | .1  | .5  |
| p2       | 58  | .2068966 | .2016268  | 0   | .5  |

`predict's` `asif` option ignores the rules and exclusion criteria and calculates predictions for all observations possible by using the estimated parameters from the model:

```
. predict p3, asif
(option pr assumed; Pr(foreign))
. summarize foreign p p2 p3
```

| Variable | Obs | Mean     | Std. dev. | Min | Max |
|----------|-----|----------|-----------|-----|-----|
| foreign  | 58  | .2068966 | .4086186  | 0   | 1   |
| p        | 48  | .25      | .1956984  | .1  | .5  |
| p2       | 58  | .2068966 | .2016268  | 0   | .5  |
| p3       | 58  | .2931035 | .2016268  | .1  | .5  |

Which is right? What predict does by default is the most conservative approach. If many observations had been excluded because of a simple rule, we could be reasonably certain that the rules prediction is correct. The `asif` prediction is correct only if the exclusion is a fluke, and we would be willing to exclude the variable from the analysis anyway. Then, however, we would refit the model to include the excluded observations.



## ▷ Example 2: Predictive margins

We can use the command `margins, contrast` after `logit` to make comparisons on the probability scale. Let's fit a model predicting low birthweight from characteristics of the mother:

```
. use https://www.stata-press.com/data/r19/lbw, clear
(Hosmer & Lemeshow data)
. logit low age i.race i.smoke ptl i.ht i.ui

Iteration 0: Log likelihood = -117.336
Iteration 1: Log likelihood = -103.81846
Iteration 2: Log likelihood = -103.40486
Iteration 3: Log likelihood = -103.40384
Iteration 4: Log likelihood = -103.40384

Logistic regression Number of obs = 189
 LR chi2(7) = 27.86
 Prob > chi2 = 0.0002
 Pseudo R2 = 0.1187

Log likelihood = -103.40384
```

| low    | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|--------|-------------|-----------|-------|-------|----------------------|----------|
| age    | -.0403293   | .0357127  | -1.13 | 0.259 | -.1103249            | .0296663 |
| race   |             |           |       |       |                      |          |
| Black  | 1.009436    | .5025122  | 2.01  | 0.045 | .0245302             | 1.994342 |
| Other  | 1.001908    | .4248342  | 2.36  | 0.018 | .1692485             | 1.834568 |
| smoke  |             |           |       |       |                      |          |
| Smoker | .9631876    | .3904357  | 2.47  | 0.014 | .1979477             | 1.728427 |
| ptl    | .6288678    | .3399067  | 1.85  | 0.064 | -.0373371            | 1.295073 |
| i.ht   | 1.358142    | .6289555  | 2.16  | 0.031 | .125412              | 2.590872 |
| i.ui   | .8001832    | .4572306  | 1.75  | 0.080 | -.0959724            | 1.696339 |
| _cons  | -1.184127   | .9187461  | -1.29 | 0.197 | -2.984837            | .6165818 |

The coefficients are log odds-ratios: conditional on the other predictors, smoking during pregnancy is associated with an increase of 0.96 in the log odds of low birthweight. The model is linear in the log odds-scale, so the estimate of 0.96 has the same interpretation, whatever the values of the other predictors might be. We could convert 0.96 to an odds ratio by replaying the results with `logit, or`.

But what if we want to talk about the probability of low birthweight, and not the odds? Then we will need the command `margins, contrast`. We will use the `r.` contrast operator to compare each level of smoke with a reference level. (smoke has only two levels, so there will be only one comparison: a comparison of smokers with nonsmokers.)

```
. margins r.smoke, contrast
Contrasts of predictive margins Number of obs = 189
Model VCE: OIM
Expression: Pr(low), predict()
```

|       | df | chi2 | P>chi2 |  |
|-------|----|------|--------|--|
| smoke | 1  | 6.32 | 0.0119 |  |

|                                | Delta-method |           |                      |          |
|--------------------------------|--------------|-----------|----------------------|----------|
|                                | Contrast     | std. err. | [95% conf. interval] |          |
| smoke<br>(Smoker vs Nonsmoker) | .1832779     | .0728814  | .0404329             | .3261229 |

We see that maternal smoking is associated with an 18.3% increase in the probability of low birthweight. (We received a contrast in the probability scale because predicted probabilities are the default when `margins` is used after `logit`.)

The contrast of 18.3% is a difference of margins that are computed by averaging over the predictions for observations in the estimation sample. If the values of the other predictors were different, the contrast for smoke would be different, too. Let's estimate the contrast for 25-year-old mothers:

```
. margins r.smoke, contrast at(age=25)
Contrasts of predictive margins Number of obs = 189
Model VCE: OIM
Expression: Pr(low), predict()
At: age = 25
```

|       | df | chi2 | P>chi2 |  |
|-------|----|------|--------|--|
| smoke | 1  | 6.19 | 0.0129 |  |

|                                | Delta-method |           |                      |          |
|--------------------------------|--------------|-----------|----------------------|----------|
|                                | Contrast     | std. err. | [95% conf. interval] |          |
| smoke<br>(Smoker vs Nonsmoker) | .1808089     | .0726777  | .0383632             | .3232547 |

Specifying a maternal age of 25 changed the contrast to 18.1%. Our contrast of probabilities changed because the `logit` model is nonlinear in the probability scale. A contrast of log odds would not have changed.

## Methods and formulas

See *Methods and formulas* in [R] **logistic postestimation** for details on predictions. For all other postestimation commands, see *Methods and formulas* in the entries for the corresponding commands.

## References

- Hosmer, D. W., Jr., S. A. Lemeshow, and R. X. Sturdivant. 2013. *Applied Logistic Regression*. 3rd ed. Hoboken, NJ: Wiley.
- Newson, R. B. 2013. *Attributable and unattributable risks and fractions and other scenario comparisons*. *Stata Journal* 13: 672–698.
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- Pregibon, D. 1981. Logistic regression diagnostics. *Annals of Statistics* 9: 705–724. <https://doi.org/10.1214/aos/1176345513>.
- Zlotnik, A., and V. Abraira. 2015. *A general-purpose nomogram generator for predictive logistic regression models*. *Stata Journal* 15: 537–546.

## Also see

- [R] **logit** — Logistic regression, reporting coefficients
- [R] **estat classification** — Classification statistics and table
- [R] **estat gof** — Pearson or Hosmer–Lemeshow goodness-of-fit test
- [R] **iroc** — Compute area under ROC curve and graph the curve
- [R] **lsens** — Graph sensitivity and specificity versus probability cutoff
- [LASSO] **lassogof** — Goodness of fit after lasso for prediction
- [U] **20 Estimation and postestimation commands**

## loneway — Large one-way ANOVA, random effects, and reliability

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### Description

loneway fits one-way analysis-of-variance (ANOVA) models. loneway relaxes the restriction imposed by [oneway](#) that factors must have fewer than 376 levels. The command additionally reports the intraclass correlation, its standard error, and confidence interval; the estimated reliability of the group-averaged mean; the standard deviation of the group effect; and the standard deviation of the within-group effect.

### Quick start

One-way ANOVA model of  $y$  for factor  $a$

```
loneway y a
```

Report an exact 95% confidence interval for the intraclass correlation

```
loneway y a, exact
```

Same as above, but report a 90% confidence interval

```
loneway y a, exact level(90)
```

### Menu

Statistics > Linear models and related > ANOVA/MANOVA > Large one-way ANOVA

## Syntax

```
loneway response_var group_var [if] [in] [weight] [, options]
```

| options               | Description                                                       |
|-----------------------|-------------------------------------------------------------------|
| Main                  |                                                                   |
| <code>mean</code>     | expected value of $F$ distribution; default is 1                  |
| <code>median</code>   | median of $F$ distribution; default is 1                          |
| <code>exact</code>    | exact confidence intervals (groups must be equal with no weights) |
| <code>level(#)</code> | set confidence level; default is level(95)                        |

`by` and `collect` are allowed; see [\[U\] 11.1.10 Prefix commands](#).

`aweight`s are allowed; see [\[U\] 11.1.6 weight](#).

## Options

Main

`mean` specifies that the expected value of the  $F_{k-1, N-k}$  distribution be used as the reference point  $F_m$  in the estimation of  $\rho$  instead of the default value of 1.

`median` specifies that the median of the  $F_{k-1, N-k}$  distribution be used as the reference point  $F_m$  in the estimation of  $\rho$  instead of the default value of 1.

`exact` requests that exact confidence intervals be computed, as opposed to the default asymptotic confidence intervals. This option is allowed only if the groups are equal in size and weights are not used.

`level(#)` specifies the confidence level, as a percentage, for confidence intervals of the coefficients. The default is level(95) or as set by `set level`; see [\[U\] 20.8 Specifying the width of confidence intervals](#).

## Remarks and examples

`loneway` and `oneway` both fit ANOVA models, but `loneway` presents different ancillary statistics from `oneway`:

| Feature                                    | oneway | loneway |
|--------------------------------------------|--------|---------|
| Fit one-way model                          | x      | x       |
| on 375 or fewer levels                     | x      | x       |
| on more than 375 levels                    |        | x       |
| Bartlett's test for equal variance         | x      |         |
| Multiple-comparison tests                  | x      |         |
| Intraclass correlation and SE              |        | x       |
| Intraclass correlation confidence interval |        | x       |
| Est. reliability of group-averaged mean    |        | x       |
| Est. SD of group effect                    |        | x       |
| Est. SD within group                       |        | x       |

Remarks are presented under the following headings:

*The one-way ANOVA model*  
 $R^2$   
*The random-effects ANOVA model*  
*Intraclass correlation*  
*Estimated reliability of the group-averaged score*

## The one-way ANOVA model

### ▷ Example 1

loneway's output looks like that of oneway, except that loneway presents more information at the end. Using our automobile dataset, we have created a (numeric) variable called `manufacturer_grp` identifying the manufacturer of each car, and within each manufacturer we have retained a maximum of four models, selecting those with the lowest mpg. We can compute the intraclass correlation of mpg for all manufacturers with at least four models as follows:

```
. use https://www.stata-press.com/data/r19/auto7
(1978 automobile data)
. loneway mpg manufacturer_grp if nummake == 4
```

One-way analysis of variance for mpg: Mileage (mpg)

| Source                 | SS        | df | MS        | F    | Prob > F |
|------------------------|-----------|----|-----------|------|----------|
| Between manufacturer~p | 621.88889 | 8  | 77.736111 | 3.70 | 0.0049   |
| Within manufacturer~p  | 567.75    | 27 | 21.027778 |      |          |
| Total                  | 1189.6389 | 35 | 33.989683 |      |          |

Number of obs = 36  
R-squared = 0.5228

| Intraclass correlation | Asy. S.E. | [95% conf. interval] |         |
|------------------------|-----------|----------------------|---------|
| 0.40270                | 0.18770   | 0.03481              | 0.77060 |

Estimated SD of manufacturer~p effect 3.765247  
Estimated SD within manufacturer~p 4.585605  
Est. reliability of a manufacturer~p mean 0.72950  
(evaluated at n=4.00)

◀

In addition to the standard one-way ANOVA output, loneway produces the  $R^2$ , the estimated standard deviation of the group effect, the estimated standard deviation within group, the intragroup correlation, the estimated reliability of the group-averaged mean, and, for unweighted data, the asymptotic standard error and confidence interval for the intragroup correlation.

$R^2$

The  $R^2$  is, of course, simply the underlying  $R^2$  for a regression of `response_var` on the levels of `group_var`, or mpg on the various manufacturers here.

## The random-effects ANOVA model

loneway assumes that we observe a variable,  $y_{ij}$ , measured for  $n_i$  elements within  $k$  groups or classes such that

$$y_{ij} = \mu + \alpha_i + \epsilon_{ij}, \quad i = 1, 2, \dots, k, \quad j = 1, 2, \dots, n_i$$

and  $\alpha_i$  and  $\epsilon_{ij}$  are independent zero-mean random variables with variance  $\sigma_\alpha^2$  and  $\sigma_\epsilon^2$ , respectively. This is the random-effects ANOVA model, also known as the components-of-variance model, in which it is typically assumed that the  $y_{ij}$  are normally distributed.

The interpretation with respect to our example is that the observed value of our response variable, mpg, is created in two steps. First, the  $i$ th manufacturer is chosen, and a value,  $\alpha_i$ , is determined—the typical mpg for that manufacturer less the overall mpg  $\mu$ . Then a deviation,  $\epsilon_{ij}$ , is chosen for the  $j$ th model within this manufacturer. This is how much that particular automobile differs from the typical mpg value for models from this manufacturer.

For our sample of 36 car models, the estimated standard deviations are  $\sigma_\alpha = 3.8$  and  $\sigma_\epsilon = 4.6$ . Thus, a little more than half of the variation in mpg between cars is attributable to the car model, with the rest attributable to differences between manufacturers. These standard deviations differ from those that would be produced by a (standard) fixed-effects regression in that the regression would require the sum within each manufacturer of the  $\epsilon_{ij}$ ,  $\epsilon_i$ , for the  $i$ th manufacturer, to be zero, whereas these estimates merely impose the constraint that the sum is *expected* to be zero.

## Intraclass correlation

There are various estimators of the intraclass correlation, such as the pairwise estimator, which is defined as the Pearson product-moment correlation computed over all possible pairs of observations that can be constructed within groups. For a discussion of various estimators, see [Donner \(1986\)](#). loneway computes what is termed the analysis of variance, or ANOVA, estimator. This intraclass correlation is the theoretical upper bound on the variation in *response\_var* that is explainable by *group\_var*, of which  $R^2$  is an overestimate because of the serendipity of fitting. This correlation is comparable to an  $R^2$ —you do not have to square it.

In our example, the intra-manu correlation, the correlation of mpg within manufacturer, is 0.40. Because *aweights* were not used and the default correlation was computed (that is, the mean and median options were not specified), loneway also provided the asymptotic confidence interval and standard error of the intraclass correlation estimate.

## Estimated reliability of the group-averaged score

The estimated reliability of the group-averaged score or mean has an interpretation similar to that of the intragroup correlation; it is a comparable number if we average *response\_var* by *group\_var*, or mpg by manu in our example. It is the theoretical upper bound of a regression of manufacturer-averaged mpg on characteristics of manufacturers. Why would we want to collapse our 36-observation dataset into a 9-observation dataset of manufacturer averages? Because the 36 observations might be a mirage. When General Motors builds cars, do they sometimes put a Pontiac label and sometimes a Chevrolet label on them, so that it appears in our data as if we have two cars when we really have only one, replicated? If that were the case, and if it were the case for many other manufacturers, then we would be forced to admit that we do not have data on 36 cars; we instead have data on nine manufacturer-averaged characteristics.

## Stored results

loneway stores the following in `r()`:

Scalars

|                     |                               |                       |                                     |
|---------------------|-------------------------------|-----------------------|-------------------------------------|
| <code>r(N)</code>   | number of observations        | <code>r(rho_t)</code> | estimated reliability               |
| <code>r(rho)</code> | intraclass correlation        | <code>r(se)</code>    | asypm. SE of intraclass correlation |
| <code>r(lb)</code>  | lower bound of 95% CI for rho | <code>r(sd_w)</code>  | estimated SD within group           |
| <code>r(ub)</code>  | upper bound of 95% CI for rho | <code>r(sd_b)</code>  | estimated SD of group effect        |

## Methods and formulas

The mean squares in the loneway's ANOVA table are computed as

$$MS_{\alpha} = \sum_i w_i (\bar{y}_i - \bar{y}_{..})^2 / (k - 1)$$

and

$$MS_{\epsilon} = \sum_i \sum_j w_{ij} (y_{ij} - \bar{y}_i)^2 / (N - k)$$

in which

$$w_i = \sum_j w_{ij} \quad w_{..} = \sum_i w_i \quad \bar{y}_i = \sum_j w_{ij} y_{ij} / w_i \quad \text{and} \quad \bar{y}_{..} = \sum_i w_i \bar{y}_i / w_{..}$$

The corresponding expected values of these mean squares are

$$E(MS_{\alpha}) = \sigma_{\epsilon}^2 + g\sigma_{\alpha}^2 \quad \text{and} \quad E(MS_{\epsilon}) = \sigma_{\epsilon}^2$$

in which

$$g = \frac{w_{..} - \sum_i w_i^2 / w_{..}}{k - 1}$$

In the unweighted case, we get

$$g = \frac{N - \sum_i n_i^2 / N}{k - 1}$$

As expected,  $g = m$  for the case of no weights and equal group sizes in the data, that is,  $n_i = m$  for all  $i$ . Replacing the expected values with the observed values and solving yields the ANOVA estimates of  $\sigma_{\alpha}^2$  and  $\sigma_{\epsilon}^2$ . Substituting these into the definition of the intraclass correlation

$$\rho = \frac{\sigma_{\alpha}^2}{\sigma_{\alpha}^2 + \sigma_{\epsilon}^2}$$

yields the ANOVA estimator of the intraclass correlation:

$$\rho_A = \frac{F_{\text{obs}} - 1}{F_{\text{obs}} - 1 + g}$$

$F_{\text{obs}}$  is the observed value of the  $F$  statistic from the ANOVA table. For no weights and equal  $n_i$ ,  $\rho_A = \text{roh}$ , which is the intragroup correlation defined by [Kish \(1965\)](#). Two slightly different estimators are available through the `mean` and `median` options ([Gleason 1997](#)). If either of these options is specified, the estimate of  $\rho$  becomes

$$\rho = \frac{F_{\text{obs}} - F_m}{F_{\text{obs}} + (g - 1)F_m}$$

For the mean option,  $F_m = E(F_{k-1, N-K}) = (N - k)/(N - k - 2)$ , that is, the expected value of the ANOVA table's  $F$  statistic. For the median option,  $F_m$  is simply the median of the  $F$  statistic. Setting  $F_m$  to 1 gives  $\rho_A$ , so for large samples, these different point estimators are essentially the same. Also, because the intraclass correlation of the random-effects model is by definition nonnegative, for any of the three possible point estimators,  $\rho$  is truncated to zero if  $F_{\text{obs}}$  is less than  $F_m$ .

For no weighting, interval estimators for  $\rho_A$  are computed. If the groups are equal sized (all  $n_i$  equal) and the exact option is specified, the following exact (assuming that the  $y_{ij}$  are normally distributed)  $100(1 - \alpha)\%$  confidence interval is computed:

$$\left\{ \frac{F_{\text{obs}} - F_m F_u}{F_{\text{obs}} + (g - 1) F_m F_u}, \frac{F_{\text{obs}} - F_m F_l}{F_{\text{obs}} + (g - 1) F_m F_l} \right\}$$

with  $F_m = 1$ ,  $F_l = F_{\alpha/2, k-1, N-k}$ , and  $F_u = F_{1-\alpha/2, k-1, N-k}$ ,  $F_{\cdot, k-1, N-k}$  being the cumulative distribution function for the  $F$  distribution with  $k - 1$  and  $N - k$  degrees of freedom. If mean or median is specified,  $F_m$  is defined as above. If the groups are equal sized and exact is not specified, the following asymptotic  $100(1 - \alpha)\%$  confidence interval for  $\rho_A$  is computed,

$$\left[ \rho_A - z_{\alpha/2} \sqrt{V(\rho_A)}, \rho_A + z_{\alpha/2} \sqrt{V(\rho_A)} \right]$$

where  $z_{\alpha/2}$  is the  $100(1 - \alpha/2)$  percentile of the standard normal distribution and  $\sqrt{V(\rho_A)}$  is the asymptotic standard error of  $\rho$  defined below. This confidence interval is also available for unequal groups. It is not applicable and, therefore, not computed for the estimates of  $\rho$  provided by the mean and median options. Again, because the intraclass coefficient is nonnegative, if the lower bound is negative for either confidence interval, it is truncated to zero. As might be expected, the coverage probability of a truncated interval is higher than its nominal value.

The asymptotic standard error of  $\rho_A$ , assuming that the  $y_{ij}$  are normally distributed, is also computed when appropriate, namely, for unweighted data and when  $\rho_A$  is computed (neither the mean option nor the median option is specified):

$$V(\rho_A) = \frac{2(1 - \rho)^2}{g^2} (A + B + C)$$

with

$$A = \frac{\{1 + \rho(g - 1)\}^2}{N - k}$$

$$B = \frac{(1 - \rho)\{1 + \rho(2g - 1)\}}{k - 1}$$

$$C = \frac{\rho^2 \{ \sum n_i^2 - 2N^{-1} \sum n_i^3 + N^{-2} (\sum n_i^2)^2 \}}{(k - 1)^2}$$

and  $\rho_A$  is substituted for  $\rho$  (Donner 1986).

The estimated reliability of the group-averaged score, known as the Spearman–Brown prediction formula in the psychometric literature (Winer, Brown, and Michels 1991, 1014), is

$$\rho_t = \frac{t\rho}{1 + (t - 1)\rho}$$

for group size  $t$ . loneway computes  $\rho_t$  for  $t = g$ .

The estimated standard deviation of the group effect is  $\sigma_\alpha = \sqrt{(\text{MS}_\alpha - \text{MS}_\epsilon)/g}$ . This deviation comes from the assumption that an observation is derived by adding a group effect to a within-group effect.

The estimated standard deviation within group is the square root of the mean square due to error, or  $\sqrt{\text{MS}_\epsilon}$ .

## Acknowledgment

We thank John Gleason (retired) of Syracuse University for his contributions to improving loneway.

## References

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- Gleason, J. R. 1997. sg65: Computing intraclass correlations and large ANOVAs. *Stata Technical Bulletin* 35: 25–31. Reprinted in *Stata Technical Bulletin Reprints*, vol. 6, pp. 167–176. College Station, TX: Stata Press.
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## Also see

- [R] **anova** — Analysis of variance and covariance
- [R] **icc** — Intraclass correlation coefficients
- [R] **oneway** — One-way analysis of variance

## Description

`lowess` carries out a locally weighted regression of *yvar* on *xvar*, displays the graph, and optionally saves the smoothed variable.

Warning: `lowess` is computationally intensive and may therefore take a long time to run on a slow computer. Lowess calculations on 1,000 observations, for instance, require performing 1,000 regressions.

## Quick start

Locally weighted regression of *y1* on *x*

```
lowess y1 x
```

Same as above, but with a bandwidth of 0.4

```
lowess y1 x, bwidth(.4)
```

With running-mean smoothing

```
lowess y1 x, mean
```

Without the tricube weighting function

```
lowess y1 x, noweight
```

Generate a new variable *v* containing the smoothed values of *y1*

```
lowess y1 x, generate(v)
```

Adjust the mean of the smoothed values to equal the mean of the unsmoothed values

```
lowess y1 x, adjust
```

Lowess smoothing of categorical variable *y2* on *x* in terms of the log of the odds ratio

```
lowess y2 x, logit
```

## Menu

Statistics > Nonparametric analysis > Lowess smoothing

## Syntax

```
lowess yvar xvar [if] [in] [, options]
```

| <i>options</i>                                                                                      | Description                                                          |
|-----------------------------------------------------------------------------------------------------|----------------------------------------------------------------------|
| <b>Main</b>                                                                                         |                                                                      |
| <code>mean</code>                                                                                   | running-mean smooth; default is running-line least squares           |
| <code>nweight</code>                                                                                | suppress weighted regressions; default is tricube weighting function |
| <code>bwidth(#)</code>                                                                              | use # for the bandwidth; default is <code>bwidth(0.8)</code>         |
| <code>logit</code>                                                                                  | transform dependent variable to logits                               |
| <code>adjust</code>                                                                                 | adjust smoothed mean to equal mean of dependent variable             |
| <code>nograph</code>                                                                                | suppress graph                                                       |
| <code>generate(<i>newvar</i>)</code>                                                                | create <i>newvar</i> containing smoothed values of <i>yvar</i>       |
| <b>Plot</b>                                                                                         |                                                                      |
| <code>marker_options</code>                                                                         | change look of markers (color, size, etc.)                           |
| <code>marker_label_options</code>                                                                   | add marker labels; change look or position                           |
| <b>Smoothed line</b>                                                                                |                                                                      |
| <code>lineopts(<i>cline_options</i>)</code>                                                         | affect rendition of the smoothed line                                |
| <b>Add plots</b>                                                                                    |                                                                      |
| <code>addplot(<i>plot</i>)</code>                                                                   | add other plots to generated graph                                   |
| <b>Y axis, X axis, Titles, Legend, Overall, By</b>                                                  |                                                                      |
| <code>tway_options</code>                                                                           | any of the options documented in [G-3] <code>tway_options</code>     |
| <i>yvar</i> and <i>xvar</i> may contain time-series operators; see [U] 11.4.4 Time-series varlists. |                                                                      |

## Options

### Main

`mean` specifies running-mean smoothing; the default is running-line least-squares smoothing.

`nweight` prevents the use of Cleveland's (1979) tricube weighting function; the default is to use the weighting function.

`bwidth(#)` specifies the bandwidth. Centered subsets of `bwidth() × N` observations are used for calculating smoothed values for each point in the data except for the end points, where smaller, uncentered subsets are used. The greater the `bwidth()`, the greater the smoothing. The default is 0.8.

`logit` transforms the smoothed *yvar* into logits. Predicted values less than 0.0001 or greater than 0.9999 are set to  $1/N$  and  $1 - 1/N$ , respectively, before taking logits.

`adjust` adjusts the mean of the smoothed *yvar* to equal the mean of *yvar* by multiplying by an appropriate factor. This option is useful when smoothing binary (0/1) data.

`nograph` suppresses displaying the graph.

`generate(newvar)` creates *newvar* containing the smoothed values of *yvar*.

## Plot

*marker\_options* affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] *marker\_options*.

*marker\_label\_options* specify if and how the markers are to be labeled; see [G-3] *marker\_label\_options*.

## Smoothed line

*lineopts* (*cline\_options*) affects the rendition of the lowess-smoothed line; see [G-3] *cline\_options*.

## Add plots

*addplot* (*plot*) provides a way to add other plots to the generated graph; see [G-3] *addplot\_option*.

## Y axis, X axis, Titles, Legend, Overall, By

*twoway\_options* are any of the options documented in [G-3] *twoway\_options*. These include options for titling the graph (see [G-3] *title\_options*), options for saving the graph to disk (see [G-3] *saving\_option*), and the *by*() option (see [G-3] *by\_option*).

## Remarks and examples

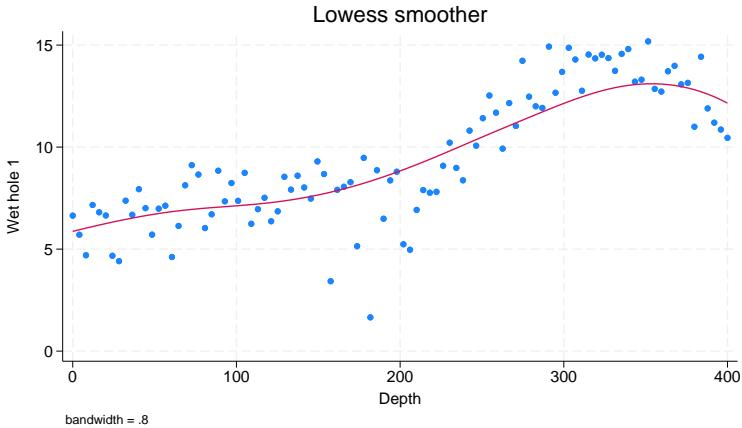
By default, *lowess* provides locally weighted scatterplot smoothing. The basic idea is to create a new variable (*newvar*) that, for each *yvar*  $y_i$ , contains the corresponding smoothed value. The smoothed values are obtained by running a regression of *yvar* on *xvar* by using only the data  $(x_i, y_i)$  and a few of the data near this point. In *lowess*, the regression is weighted so that the central point  $(x_i, y_i)$  gets the highest weight and points that are farther away (based on the distance  $|x_j - x_i|$ ) receive less weight. The estimated regression line is then used to predict the smoothed value  $\hat{y}_i$  for  $y_i$  only. The procedure is repeated to obtain the remaining smoothed values, which means that a separate weighted regression is performed for every point in the data.

Lowess is a desirable smoother because of its locality—it tends to follow the data. Polynomial smoothing methods, for instance, are global in that what happens on the extreme left of a scatterplot can affect the fitted values on the extreme right.

## ► Example 1

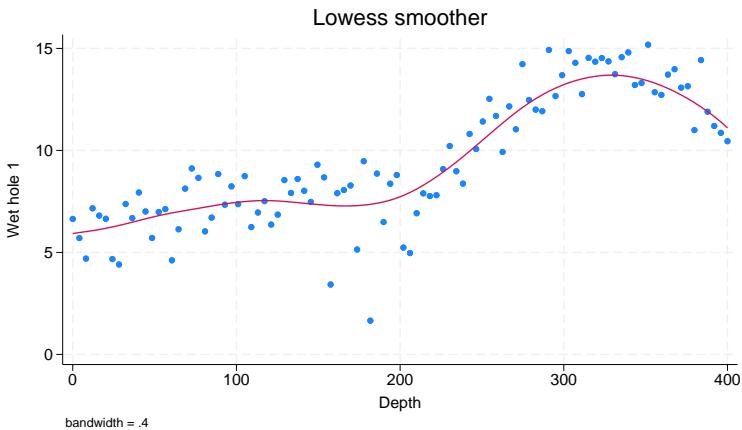
The amount of smoothing is affected by `bwidth(#)`. You are warned to experiment with different values. For instance,

```
. use https://www.stata-press.com/data/r19/lowess1
(Example data for lowess)
. lowess h1 depth
```



Now compare that with

```
. lowess h1 depth, bwidth(.4)
```

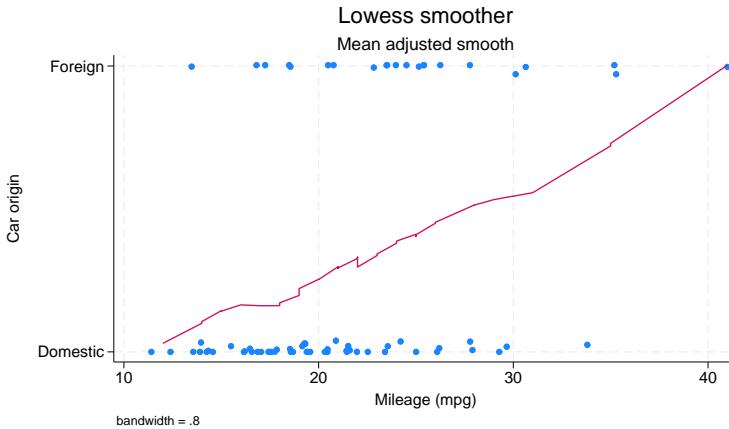


In the first case, the default bandwidth of 0.8 is used, meaning that 80% of the data are used in smoothing each point. In the second case, we explicitly specified a bandwidth of 0.4. Smaller bandwidths follow the original data more closely.

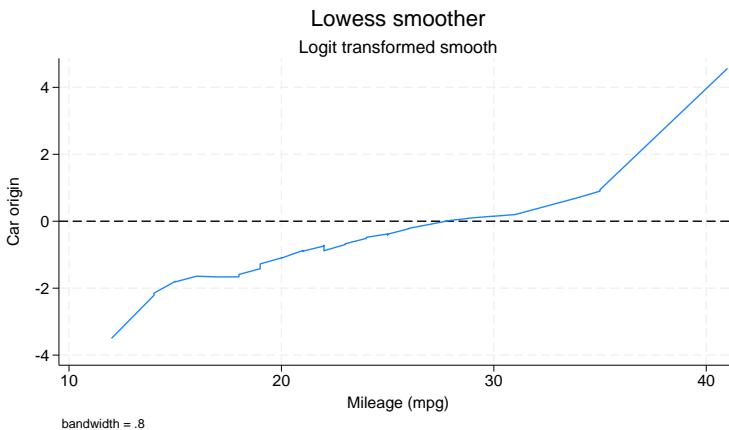
## ► Example 2

Two `lowess` options are especially useful with binary (0/1) data: `adjust` and `logit`. `adjust` adjusts the resulting curve (by multiplication) so that the mean of the smoothed values is equal to the mean of the unsmoothed values. `logit` specifies that the smoothed curve be in terms of the log of the odds ratio:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. lowess foreign mpg, ylabel(0 "Domestic" 1 "Foreign") jitter(5) adjust
```



```
. lowess foreign mpg, logit yline(0)
```



With binary data, if you do not use the `logit` option, it is a good idea to specify `graph's jitter()` option; see [G-2] [graph twoway scatter](#). Because the underlying data (whether the car was manufactured outside the United States here) take on only two values, raw data points are more likely to be on top of each other, thus making it impossible to tell how many points there are. `graph's jitter()` option adds some noise to the data to shift the points around. This noise affects only the location of points on the graph, not the lowess curve.

When you specify the `logit` option, the display of the raw data is suppressed.



### □ Technical note

`lowess` can be used for more than just lowess smoothing. Lowess can be usefully thought of as a combination of two smoothing concepts: the use of predicted values from regression (rather than means) for imputing a smoothed value and the use of the tricube weighting function (rather than a constant weighting function). `lowess` allows you to combine these concepts freely. You can use line smoothing without weighting (specify `noweight`), mean smoothing with tricube weighting (specify `mean`), or mean smoothing without weighting (specify `mean` and `noweight`).



## Methods and formulas

Let  $y_i$  and  $x_i$  be the two variables, and assume that the data are ordered so that  $x_i \leq x_{i+1}$  for  $i = 1, \dots, N - 1$ . For each  $y_i$ , a smoothed value  $y_i^s$  is calculated.

The subset used in calculating  $y_i^s$  is indices  $i_- = \max(1, i - k)$  through  $i_+ = \min(i + k, N)$ , where  $k = \lfloor (N \times \text{bwidth} - 0.5)/2 \rfloor$ . The weights for each of the observations between  $j = i_-, \dots, i_+$  are either 1 (`noweight`) or the tricube (default),

$$w_j = \left\{ 1 - \left( \frac{|x_j - x_i|}{\Delta} \right)^3 \right\}^3$$

where  $\Delta = 1.0001 \max(x_{i_+} - x_i, x_i - x_{i_-})$ . The smoothed value  $y_i^s$  is then the (weighted) mean or the (weighted) regression prediction at  $x_i$ .

William Swain Cleveland (1943–) studied mathematics and statistics at Princeton and Yale. He worked for several years at Bell Labs in New Jersey and now teaches statistics and computer science at Purdue. He has made key contributions in many areas of statistics, including graphics and data visualization, time series, environmental applications, and analysis of internet traffic data.

## Acknowledgment

`lowess` is a modified version of a command originally written by Patrick Royston of the MRC Clinical Trials Unit, London, and coauthor of the Stata Press book *Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model*.

## References

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## Also see

- [R] [lpoly](#) — Kernel-weighted local polynomial smoothing
- [R] [smooth](#) — Robust nonlinear smoother
- [D] [ipolate](#) — Linearly interpolate (extrapolate) values

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

## Description

`lpoly` performs a kernel-weighted local polynomial regression of  $yvar$  on  $xvar$  and displays a graph of the smoothed values with (optional) confidence bands.

## Quick start

Kernel-weighted local polynomial regression of  $y$  on  $x$

```
lpoly y x
```

Same as above, but specify a bandwidth of 2

```
lpoly y x, bwidth(2)
```

Same as above, but specify a degree of 1

```
lpoly y x, bwidth(2) degree(1)
```

Same as above, but use the alternative Epanechnikov kernel

```
lpoly y x, bwidth(2) degree(1) kernel(epan2)
```

Same as above, but create a new variable for the smoothing grid  $g$  and smoothed values  $s$

```
lpoly y x, bwidth(2) degree(1) kernel(epan2) generate(g s)
```

With 95% confidence bands

```
lpoly y x, ci
```

Use `twoway` to graph multiple local polynomial fits

```
twoway scatter y x || ///
 lpoly y x, degree(1) kernel(epan2) || ///
 lpoly y x, degree(1) kernel(epan2) bwidth(1) || ///
 lpoly y x, degree(1) kernel(epan2) bwidth(7) ||
```

## Menu

Statistics > Nonparametric analysis > Local polynomial smoothing

## Syntax

```
lpoly yvar xvar [if] [in] [weight] [, options]
```

| <i>options</i>                                                                             | Description                                                                                                        |
|--------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------|
| <b>Main</b>                                                                                |                                                                                                                    |
| <u>k</u> ernel( <i>kernel</i> )                                                            | specify kernel function; default is kernel(epanechnikov)                                                           |
| <u>b</u> width(#  <i>varname</i> )                                                         | specify kernel bandwidth                                                                                           |
| <u>d</u> egree(#)                                                                          | specify degree of the polynomial smooth; default is degree(0)                                                      |
| <u>g</u> enerate([ <i>newvar</i> <sub><i>x</i></sub> ] <i>newvar</i> <sub><i>s</i></sub> ) | store smoothing grid in <i>newvar</i> <sub><i>x</i></sub> and smoothed points in <i>newvar</i> <sub><i>s</i></sub> |
| <u>n</u> (#)                                                                               | obtain the smooth at # points; default is min( <i>N</i> , 50)                                                      |
| <u>a</u> t( <i>varname</i> )                                                               | obtain the smooth at the values specified by <i>varname</i>                                                        |
| <u>n</u> ograph                                                                            | suppress graph                                                                                                     |
| <u>n</u> osscatter                                                                         | suppress scatterplot only                                                                                          |
| <b>SE/CI</b>                                                                               |                                                                                                                    |
| <u>c</u> i                                                                                 | plot confidence bands                                                                                              |
| <u>l</u> evel(#)                                                                           | set confidence level; default is level(95)                                                                         |
| <u>s</u> e( <i>newvar</i> )                                                                | store standard errors in <i>newvar</i>                                                                             |
| <u>p</u> width(#)                                                                          | specify pilot bandwidth for standard error calculation                                                             |
| <u>v</u> ar(#  <i>varname</i> )                                                            | specify estimates of residual variance                                                                             |
| <b>Scatterplot</b>                                                                         |                                                                                                                    |
| <i>marker_options</i>                                                                      | change look of markers (color, size, etc.)                                                                         |
| <i>marker_label_options</i>                                                                | add marker labels; change look or position                                                                         |
| <b>Smoothed line</b>                                                                       |                                                                                                                    |
| <u>l</u> ineopts( <i>cline_options</i> )                                                   | affect rendition of the smoothed line                                                                              |
| <b>CI plot</b>                                                                             |                                                                                                                    |
| <u>c</u> iopts( <i>cline_options</i> )                                                     | affect rendition of the confidence bands                                                                           |
| <b>Add plots</b>                                                                           |                                                                                                                    |
| <u>a</u> ddplot( <i>plot</i> )                                                             | add other plots to the generated graph                                                                             |
| <b>Y axis, X axis, Titles, Legend, Overall</b>                                             |                                                                                                                    |
| <i>twoway_options</i>                                                                      | any options other than by() documented in [G-3] <i>twoway_options</i>                                              |

| <i>kernel</i>             | Description                               |
|---------------------------|-------------------------------------------|
| <code>epanechnikov</code> | Epanechnikov kernel function; the default |
| <code>epan2</code>        | alternative Epanechnikov kernel function  |
| <code>biweight</code>     | biweight kernel function                  |
| <code>cosine</code>       | cosine trace kernel function              |
| <code>gaussian</code>     | Gaussian kernel function                  |
| <code>parzen</code>       | Parzen kernel function                    |
| <code>rectangle</code>    | rectangle kernel function                 |
| <code>triangle</code>     | triangle kernel function                  |

`collect` is allowed; see [U] 11.1.10 Prefix commands.

`fweights` and `aweights` are allowed; see [U] 11.1.6 weight.

## Options

### Main

`kernel` (*kernel*) specifies the kernel function for use in calculating the weighted local polynomial estimate. The default is `kernel(epanechnikov)`.

`width` (`#` | *varname*) specifies the half-width of the kernel—the width of the smoothing window around each point. If `width()` is not specified, a rule-of-thumb (ROT) bandwidth estimator is calculated and used. A local variable bandwidth may be specified in *varname*, in conjunction with an explicit smoothing grid using the `at()` option.

`degree` (`#`) specifies the degree of the polynomial to be used in the smoothing. The default is `degree(0)`, meaning local-mean smoothing.

`generate`( [*newvar<sub>x</sub>* ] *newvar<sub>s</sub>*) stores the smoothing grid in *newvar<sub>x</sub>* and the smoothed values in *newvar<sub>s</sub>*. If `at()` is not specified, then both *newvar<sub>x</sub>* and *newvar<sub>s</sub>* must be specified. Otherwise, only *newvar<sub>s</sub>* is to be specified.

`n` (`#`) specifies the number of points at which the smooth is to be calculated. The default is  $\min(N, 50)$ , where  $N$  is the number of observations.

`at` (*varname*) specifies a variable that contains the values at which the smooth should be calculated. By default, the smoothing is done on an equally spaced grid, but you can use `at()` to instead perform the smoothing at the observed  $x$ 's, for example. This option also allows you to more easily obtain smooths for different variables or different subsamples of a variable and then overlay the estimates for comparison.

`nograph` suppresses drawing the graph of the estimated smooth. This option is often used with the `generate()` option.

`nosm` suppresses superimposing a scatterplot of the observed data over the smooth. This option is useful when the number of resulting points would be so large as to clutter the graph.

## SE/CI

`ci` plots confidence bands, using the confidence level specified in `level()`.

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [U] 20.8 [Specifying the width of confidence intervals](#).

`se(newvar)` stores the estimates of the standard errors in *newvar*. This option requires specifying `generate()` or `at()`.

`pwidth(#)` specifies the pilot bandwidth to be used for standard error computations. The default is chosen to be 1.5 times the value of the ROT bandwidth selector. If you specify `pwidth()` without specifying `se()` or `ci`, then the `ci` option is assumed.

`var(#|varname)` specifies an estimate of a constant residual variance or a variable containing estimates of the residual variances at each grid point required for standard error computation. By default, the residual variance at each smoothing point is estimated by the normalized weighted residual sum of squares obtained from locally fitting a polynomial of order  $p + 2$ , where  $p$  is the degree specified in `degree()`. `var(varname)` is allowed only if `at()` is specified. If you specify `var()` without specifying `se()` or `ci`, then the `ci` option is assumed.

## Scatterplot

`marker_options` affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] [marker\\_options](#).

`marker_label_options` specify if and how the markers are to be labeled; see [G-3] [marker\\_label\\_options](#).

## Smoothed line

`lineopts(cline_options)` affects the rendition of the smoothed line; see [G-3] [cline\\_options](#).

## CI plot

`ciopts(cline_options)` affects the rendition of the confidence bands; see [G-3] [cline\\_options](#).

## Add plots

`addplot(plot)` provides a way to add other plots to the generated graph; see [G-3] [addplot\\_option](#).

## Y axis, X axis, Titles, Legend, Overall

`twoway_options` are any of the options documented in [G-3] [twoway\\_options](#), excluding `by()`. These include options for titling the graph (see [G-3] [title\\_options](#)) and for saving the graph to disk (see [G-3] [saving\\_option](#)).

## Remarks and examples

Remarks are presented under the following headings:

[Introduction](#)  
[Local polynomial smoothing](#)  
[Choice of a bandwidth](#)  
[Confidence bands](#)

## Introduction

The last 25 years or so has seen a significant outgrowth in the literature on scatterplot smoothing, otherwise known as univariate nonparametric regression. Of most appeal is the idea of making no assumptions about the functional form for the expected value of a response given a regressor, but instead allowing the data to “speak for themselves”. Various methods and estimators fall into the category of nonparametric regression, including local mean smoothing as described independently by [Nadaraya \(1964\)](#) and [Watson \(1964\)](#), the [Gasser and Müller \(1979\)](#) estimator, locally weighted scatterplot smoothing (LOWESS) as described by [Cleveland \(1979\)](#), wavelets (for example, [Donoho \[1995\]](#)), and splines ([Eubank 1999](#)), to name a few. Much of the vast literature focuses on automating the amount of smoothing to be performed and dealing with the bias/variance tradeoff inherent to this type of estimation. For example, for Nadaraya–Watson the amount of smoothing is controlled by choosing a *bandwidth*.

Smoothing via local polynomials is by no means a new idea but instead one that has been rediscovered in recent years in articles such as [Fan \(1992\)](#). A natural extension of the local mean smoothing of Nadaraya–Watson, local polynomial regression involves fitting the response to a polynomial form of the regressor via locally weighted least squares. Higher-order polynomials have better bias properties than the zero-degree local polynomials of the Nadaraya–Watson estimator; in general, higher-order polynomials do not require bias adjustment at the boundary of the regression space. For a definitive reference on local polynomial smoothing, see [Fan and Gijbels \(1996\)](#).

## Local polynomial smoothing

Consider a set of scatterplot data  $\{(x_1, y_1), \dots, (x_n, y_n)\}$  from the model

$$y_i = m(x_i) + \sigma(x_i)\epsilon_i \quad (1)$$

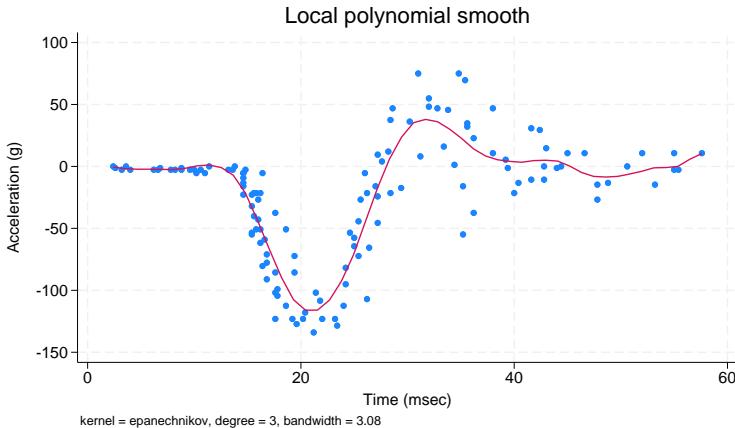
for some unknown mean and variance functions  $m(\cdot)$  and  $\sigma^2(\cdot)$ , and symmetric errors  $\epsilon_i$  with  $E(\epsilon_i) = 0$  and  $\text{Var}(\epsilon_i) = 1$ . The goal is to estimate  $m(x_0) = E[Y|X = x_0]$ , making no assumption about the functional form of  $m(\cdot)$ .

`lpoly` estimates  $m(x_0)$  as the constant term (intercept) of a regression, weighted by the kernel function specified in `kernel()`, of `yvar` on the polynomial terms  $(xvar - x_0), (xvar - x_0)^2, \dots, (xvar - x_0)^p$  for each smoothing point  $x_0$ . The degree of the polynomial,  $p$ , is specified in `degree()`, the amount of smoothing is controlled by the bandwidth specified in `width()`, and the chosen kernel function is specified in `kernel()`.

## ▷ Example 1

Consider the motorcycle data as examined (among other places) in [Fan and Gijbels \(1996\)](#). The data consist of 133 observations and measure the acceleration (accel measured in grams [g]) of a dummy's head during impact over time (time measured in milliseconds). For these data, we use `lpoly` to fit a local cubic polynomial with the default bandwidth (obtained using the ROT method) and the default Epanechnikov kernel.

```
. use https://www.stata-press.com/data/r19/motorcycle
(Motorcycle data from Fan & Gijbels (1996))
. lpoly accel time, degree(3)
```



## □ Technical note

`lpoly` allows specifying in `degree()` both odd and even orders of the polynomial to be used for the smoothing. However, the odd-order,  $2k + 1$ , polynomial approximations are preferable. They have an extra parameter compared with the even-order,  $2k$ , approximations, which leads to a significant bias reduction and there is no increase of variability associated with adding this extra parameter. Using an odd order when estimating the regression function is therefore usually sufficient. For a more thorough discussion, see [Fan and Gijbels \(1996\)](#).



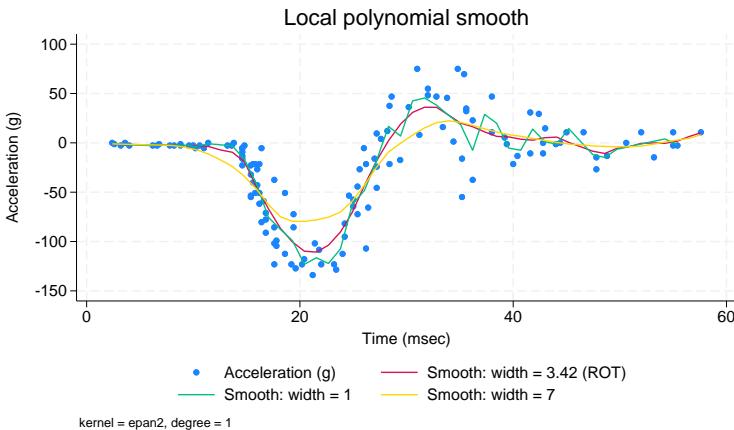
## Choice of a bandwidth

The choice of a bandwidth is crucial for many smoothing techniques, including local polynomial smoothing. In general, using a large bandwidth gives smooths with a large bias, whereas a small bandwidth may result in highly variable smoothed values. Various techniques exist for optimal bandwidth selection. By default, `lpoly` uses the ROT method to estimate the bandwidth used for the smoothing; see [Methods and formulas](#) for details.

### ► Example 2

Using the `motorcycle` data, we demonstrate how a local linear polynomial fit changes using different bandwidths.

```
. lpoly accel time, degree(1) kernel(epan2) bwidth(1)
> generate(at smooth1) nograph
. lpoly accel time, degree(1) kernel(epan2) bwidth(7) at(at)
> generate(smooth2) nograph
. label variable smooth1 "Smooth: width = 1"
. label variable smooth2 "Smooth: width = 7"
. lpoly accel time, degree(1) kernel(epan2) at(at) addplot(line smooth* at)
> legend(label(2 "Smooth: width = 3.42 (ROT)") cols(2) pos(6))
> note("kernel = epan2, degree = 1")
```

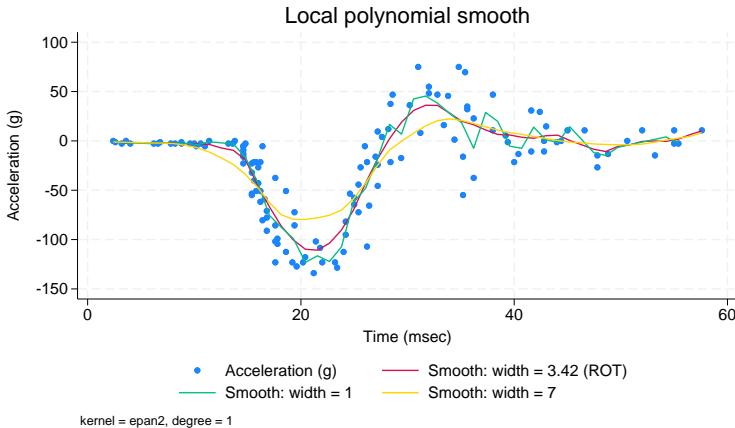


From this graph, we can see that the local linear polynomial fit with larger bandwidth (`width = 7`) corresponds to a smoother line but fails to fit the curvature of the scatterplot data. The smooth obtained using the width equal to one seems to fit most data points, but the corresponding line has several spikes indicating larger variability. The smooth obtained using the ROT bandwidth estimator seems to have a good tradeoff between the fit and variability in this example.

In the above, we also demonstrated how the `generate()` and `addplot()` options may be used to produce overlaid plots obtained from `lpoly` with different options. The `nograph` option saves time when you need to save only results with `generate()`.

However, to avoid generating variables manually, one can use `twoway lpoly` instead; see [G-2] [graph twoway lpoly](#) for more details.

```
. twoway scatter accel time ||
> lpoly accel time, degree(1) kernel(epan2) lpattern(solid) ||
> lpoly accel time, degree(1) kernel(epan2) bwidth(1) ||
> lpoly accel time, degree(1) kernel(epan2) bwidth(7) ||
> , legend(label(2 "Smooth: width = 3.42 (ROT)")
> label(3 "Smooth: width = 1")
> label(4 "Smooth: width = 7") cols(2) pos(6))
> title("Local polynomial smooth") note("kernel = epan2, degree = 1")
> xtitle("Time (msec)") ytitle("Acceleration (g)")
```



◀

The ROT estimate is commonly used as an initial guess for the amount of smoothing; this approach may be sufficient when the choice of a bandwidth is less important. In other cases, you can pick your own bandwidth.

When the shape of the regression function has a combination of peaked and flat regions, a variable bandwidth may be preferable over the constant bandwidth to allow for different degrees of smoothness in different regions. The `bwidth()` option allows you to specify the values of the local variable bandwidths as those stored in a variable in your data.

Similar issues with bias and variability arise when choosing a pilot bandwidth (the `pwidth()` option) used to compute standard errors of the local polynomial smoother. The default value is chosen to be  $1.5 \times \text{ROT}$ . For a review of methods for pilot bandwidth selection, see [Fan and Gijbels \(1996\)](#).

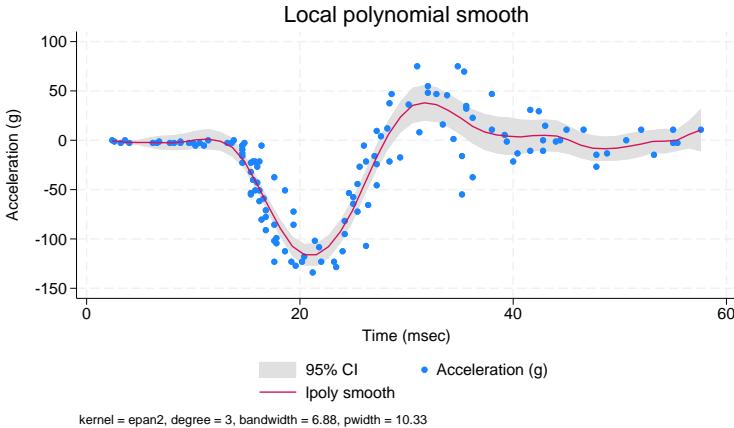
## Confidence bands

The established asymptotic normality of the local polynomial estimators under certain conditions allows the construction of approximate confidence bands. `lpoly` offers the `ci` option to plot these bands.

### ▷ Example 3

Let us plot the confidence bands for the local polynomial fit from [example 1](#).

```
. lpoly accel time, degree(3) kernel(epan2) ci legend(cols(2) pos(6))
```



You can obtain graphs with overlaid confidence bands by using `twoway lpolyci`; see [\[G-2\] graph twoway lpolyci](#) for examples.



Constructing the confidence intervals involves computing standard errors obtained by taking a square root of the estimate of the conditional variance of the local polynomial estimator at each grid point  $x_0$ . Estimating the conditional variance requires fitting a polynomial of a higher order locally by using a different bandwidth, the pilot bandwidth. The value of the pilot bandwidth may be supplied by using `pwidth()`. By default, the value of  $1.5 \times \text{ROT}$  is used. Also, estimates of the residual variance  $\sigma^2(x_0)$  at each grid point,  $x_0$ , are required to obtain the estimates of the conditional variances. These estimates may be supplied by using the `var()` option. By default, they are computed using the normalized weighted residual sum of squares from a local polynomial fit of a higher order. See [Methods and formulas](#) for details. The standard errors may be saved by using `se()`.

## Stored results

`lpoly` stores the following in `r()`:

### Scalars

|                        |                                  |                        |                         |
|------------------------|----------------------------------|------------------------|-------------------------|
| <code>r(degree)</code> | smoothing polynomial degree      | <code>r(bwidth)</code> | bandwidth of the smooth |
| <code>r(ngrid)</code>  | number of successful regressions | <code>r(pwidth)</code> | pilot bandwidth         |
| <code>r(N)</code>      | sample size                      |                        |                         |

### Macros

|                        |                |
|------------------------|----------------|
| <code>r(kernel)</code> | name of kernel |
|------------------------|----------------|

## Methods and formulas

Consider model (1), written in matrix notation,

$$\mathbf{y} = m(\mathbf{x}) + \epsilon$$

where  $\mathbf{y}$  and  $\mathbf{x}$  are the  $n \times 1$  vectors of scatterplot values,  $\epsilon$  is the  $n \times 1$  vector of errors with zero mean and covariance matrix  $\Sigma = \text{diag}\{\sigma(x_i)\}\mathbf{I}_n$ , and  $m(\cdot)$  and  $\sigma(\cdot)$  are some unknown functions. Define  $m(x_0) = E[Y|X = x_0]$  and  $\sigma^2(x_0) = \text{Var}[Y|X = x_0]$  to be the conditional mean and conditional variance of random variable  $Y$  (residual variance), respectively, for some realization  $x_0$  of random variable  $X$ .

The method of local polynomial smoothing is based on the approximation of  $m(x)$  locally by a  $p$ th order polynomial in  $(x - x_0)$  for some  $x$  in the neighborhood of  $x_0$ . For the scatterplot data  $\{(x_1, y_1), \dots, (x_n, y_n)\}$ , the  $p$ th-order local polynomial smooth  $\widehat{m}(x_0)$  is equal to  $\widehat{\beta}_0$ , an estimate of the intercept of the weighted linear regression,

$$\widehat{\beta} = (\mathbf{X}^T \mathbf{W} \mathbf{X})^{-1} \mathbf{X}^T \mathbf{W} \mathbf{y} \quad (2)$$

where  $\widehat{\beta} = (\widehat{\beta}_0, \widehat{\beta}_1, \dots, \widehat{\beta}_p)^T$  is the vector of estimated regression coefficients (with  $\{\widehat{\beta}_j = (j!)^{-1} \widehat{m}^{(j)}(x)|_{x=x_0}, j = 0, \dots, p\}$  also representing estimated coefficients from a corresponding Taylor expansion);  $\mathbf{X} = \{(x_i - x_0)^j\}_{i,j=1,0}^{n,p}$  is a design matrix; and  $\mathbf{W} = \text{diag}\{K_h(x_i - x_0)\}_{n \times n}$  is a weight matrix with weights  $K_h(\cdot)$  defined as  $K_h(x) = h^{-1}K(x/h)$ , with  $K(\cdot)$  being a kernel function and  $h$  defining a bandwidth. The kernels are defined in [Methods and formulas](#) of `[R] kdensity`.

The default bandwidth is obtained using the ROT method of bandwidth selection. The ROT bandwidth is the plugin estimator of the asymptotically optimal constant bandwidth. This is the bandwidth that minimizes the conditional weighted mean integrated squared error. The ROT plugin bandwidth selector for the smoothing bandwidth  $h$  is defined as follows; assuming constant residual variance  $\sigma^2(x_0) = \sigma^2$  and odd degree  $p$ :

$$\widehat{h} = C_{0,p}(K) \left[ \frac{\widehat{\sigma}^2 \int w_0(x) dx}{n \int \{\widehat{m}^{(p+1)}(x)\}^2 w_0(x) f(x) dx} \right]^{1/(2p+3)} \quad (3)$$

where  $C_{0,p}(K)$  is a constant, as defined in [Fan and Gijbels \(1996\)](#), that depends on the kernel function  $K(\cdot)$ , and the degree of a polynomial  $p$  and  $w_0$  is chosen to be an indicator function on the interval  $[\min_{\mathbf{x}} + 0.05 \times \text{range}_{\mathbf{x}}, \max_{\mathbf{x}} - 0.05 \times \text{range}_{\mathbf{x}}]$  with  $\min_{\mathbf{x}}$ ,  $\max_{\mathbf{x}}$ , and  $\text{range}_{\mathbf{x}}$  being, respectively, the minimum, maximum, and the range of  $\mathbf{x}$ . To obtain the estimates of a constant residual variance,  $\widehat{\sigma}^2$ , and  $(p + 1)$ th order derivative of  $m(x)$ , denoted as  $\widehat{m}^{(p+1)}(x)$ , a polynomial in  $\mathbf{x}$  of order  $(p + 3)$  is fit globally to  $\mathbf{y}$ .  $\widehat{\sigma}^2$  is estimated as a standardized residual sum of squares from this fit.

The expression for the asymptotically optimal constant bandwidth used in constructing the ROT bandwidth estimator is derived for the odd-order polynomial approximations. For even-order polynomial fits the expression would depend not only on  $m^{(p+1)}(x)$  but also on  $m^{(p+2)}(x)$  and the design density and its derivative,  $f(x)$  and  $f'(x)$ . Therefore, the ROT bandwidth selector would require estimation of these additional quantities. Instead, for an even-degree  $p$  of the local polynomial, `lpoly` uses the value of the ROT estimator (3) computed using degree  $p + 1$ . As such, for even degrees this is not a plugin estimator of the asymptotically optimal constant bandwidth.

The estimates of the conditional variance of local polynomial estimators are obtained using

$$\widehat{\text{Var}}\{\widehat{m}(x_0)|X = x_0\} = \widehat{\sigma}_m^2(x_0) = (\mathbf{X}^T\mathbf{W}\mathbf{X})^{-1}(\mathbf{X}^T\mathbf{W}^2\mathbf{X})(\mathbf{X}^T\mathbf{W}\mathbf{X})^{-1}\widehat{\sigma}^2(x_0) \quad (4)$$

where  $\widehat{\sigma}^2(x_0)$  is estimated by the normalized weighted residual sum of squares from the  $(p + 2)$ th order polynomial fit using pilot bandwidth  $h^*$ .

When the bias is negligible the normal-approximation method yields a  $(1 - \alpha) \times 100\%$  confidence interval for  $m(x_0)$ ,

$$\{\widehat{m}(x_0) - z_{(1-\alpha/2)}\widehat{\sigma}_m(x_0), \widehat{m}(x_0) + z_{(1-\alpha/2)}\widehat{\sigma}_m(x_0)\}$$

where  $z_{(1-\alpha/2)}$  is the  $(1 - \alpha/2)$ th quantile of the standard Gaussian distribution, and  $\widehat{m}(x_0)$  and  $\widehat{\sigma}_m(x_0)$  are as defined in (2) and (4), respectively.

## References

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## Also see

- [R] **kdensity** — Univariate kernel density estimation
- [R] **lowess** — Lowess smoothing
- [R] **makespline** — Spline generation
- [R] **npregress intro** — Introduction to nonparametric regression
- [R] **smooth** — Robust nonlinear smoother
- [G-2] **graph twoway lpoly** — Local polynomial smooth plots
- [G-2] **graph twoway lpolyci** — Local polynomial smooth plots with CIs

## **lroc** — Compute area under ROC curve and graph the curve

[Description](#)  
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### **Description**

`lroc` graphs the ROC curve and calculates the area under the curve.

`lroc` requires that the current estimation results be from `logistic`, `logit`, `probit`, or `ivprobit`; see [\[R\] logistic](#), [\[R\] logit](#), [\[R\] probit](#), or [\[R\] ivprobit](#).

### **Quick start**

Graph and compute area under ROC curve for current estimation results

```
lroc
```

Add “My Title” as title of graph

```
lroc, title(My Title)
```

Suppress graph

```
lroc, nograph
```

### **Menu**

Statistics > Binary outcomes > Postestimation > ROC curve after logistic/logit/probit/ivprobit

## Syntax

```
lroc [depvar] [if] [in] [weight] [, options]
```

| <i>options</i>                                                        | Description                                                                                 |
|-----------------------------------------------------------------------|---------------------------------------------------------------------------------------------|
| <b>Main</b>                                                           |                                                                                             |
| <code>all</code>                                                      | compute area under ROC curve and graph curve for all observations                           |
| <code>nograph</code>                                                  | suppress graph                                                                              |
| <b>Advanced</b>                                                       |                                                                                             |
| <code>beta(<i>matname</i>)</code>                                     | row vector containing model coefficients                                                    |
| <b>Plot</b>                                                           |                                                                                             |
| <code>cline_options</code>                                            | change look of the line                                                                     |
| <code>marker_options</code>                                           | change look of markers (color, size, etc.)                                                  |
| <code>marker_label_options</code>                                     | add marker labels; change look or position                                                  |
| <b>Reference line</b>                                                 |                                                                                             |
| <code>rlopts(<i>cline_options</i>)</code>                             | affect rendition of the reference line                                                      |
| <b>Add plots</b>                                                      |                                                                                             |
| <code>addplot(<i>plot</i>)</code>                                     | add other plots to the generated graph                                                      |
| <b>Y axis, X axis, Titles, Legend, Overall</b>                        |                                                                                             |
| <code>twoway_options</code>                                           | any options other than <code>by()</code> documented in [G-3] <a href="#">twoway_options</a> |
| collect is allowed; see [U] <a href="#">11.1.10 Prefix commands</a> . |                                                                                             |
| fweights are allowed; see [U] <a href="#">11.1.6 weight</a> .         |                                                                                             |
| lroc is not appropriate with svy estimation results.                  |                                                                                             |

## Options

### Main

`all` requests that the statistic be computed for all observations in the data, ignoring any `if` or `in` restrictions specified by the estimation command.

`nograph` suppresses graphical output.

### Advanced

`beta(matname)` specifies a row vector containing model coefficients. The columns of the row vector must be labeled with the corresponding names of the independent variables in the data. The dependent variable *depvar* must be specified immediately after the command name. See [Models other than the last fitted model](#) later in this entry.

### Plot

`cline_options`, `marker_options`, and `marker_label_options` affect the rendition of the ROC curve—the plotted points connected by lines. These options affect the size and color of markers, whether and how the markers are labeled, and whether and how the points are connected; see [G-3] [cline\\_options](#), [G-3] [marker\\_options](#), and [G-3] [marker\\_label\\_options](#).

## Reference line

`rlopts` (*cline\_options*) affects the rendition of the reference line; see [G-3] *cline\_options*.

## Add plots

`addplot` (*plot*) provides a way to add other plots to the generated graph; see [G-3] *addplot\_option*.

## Y axis, X axis, Titles, Legend, Overall

*twoway\_options* are any of the options documented in [G-3] *twoway\_options*, excluding `by()`. These include options for titling the graph (see [G-3] *title\_options*) and for saving the graph to disk (see [G-3] *saving\_option*).

## Remarks and examples

Remarks are presented under the following headings:

*Introduction*

*Samples other than the estimation sample*

*Models other than the last fitted model*

### Introduction

Stata also has a suite of commands for performing both parametric and nonparametric receiver operating characteristic (ROC) analysis. See [R] *roc* for an overview of these commands.

`lroc` graphs the ROC curve—a graph of sensitivity versus one minus specificity as the cutoff  $c$  is varied—and calculates the area under it. Sensitivity is the fraction of observed positive-outcome cases that are correctly classified; specificity is the fraction of observed negative-outcome cases that are correctly classified. When the purpose of the analysis is classification, you must choose a cutoff.

The curve starts at  $(0, 0)$ , corresponding to  $c = 1$ , and continues to  $(1, 1)$ , corresponding to  $c = 0$ . A model with no predictive power would be a 45° line. The greater the predictive power, the more bowed the curve, and hence the area beneath the curve is often used as a measure of the predictive power. A model with no predictive power has area 0.5; a perfect model has area 1.

The ROC curve was first discussed in signal detection theory (Peterson, Birdsall, and Fox 1954) and then was quickly introduced into psychology (Tanner and Swets 1954). It has since been applied in other fields, particularly medicine (for instance, Metz [1978]). For a classic text on ROC techniques, see Green and Swets (1966).

`lsens` also plots sensitivity and specificity; see [R] *lsens*.

### ▶ Example 1

Hardin and Hilbe (2018) examine data from the National Canadian Registry of Cardiovascular Disease (FASTRAK), sponsored by Hoffman-La Roche Canada. They model death within 48 hours based on whether a patient suffers an anterior infarct (heart attack) rather than an inferior infarct using a logistic regression and evaluate the model using an ROC curve. We replicate their analysis here.

Both anterior and inferior refer to sites on the heart where damage occurs. The model is also adjusted for `hcabg`, whether the subject has had a cardiac bypass surgery (CABG); `age`, a four-category age-group indicator; and `killip`, a four-level risk indicator.

We load the data and then estimate the parameters of the logistic regression with `logistic`. Factor-variable notation is used for each predictor, because they are categorical; see [U] 11.4.3 *Factor variables*.

```
. use https://www.stata-press.com/data/r19/heart
(Heart attacks)
. logistic death i.site i.hcabg i.killip i.age
Logistic regression
Log likelihood = -636.62553
Number of obs = 4,483
LR chi2(8) = 211.37
Prob > chi2 = 0.0000
Pseudo R2 = 0.1424
```

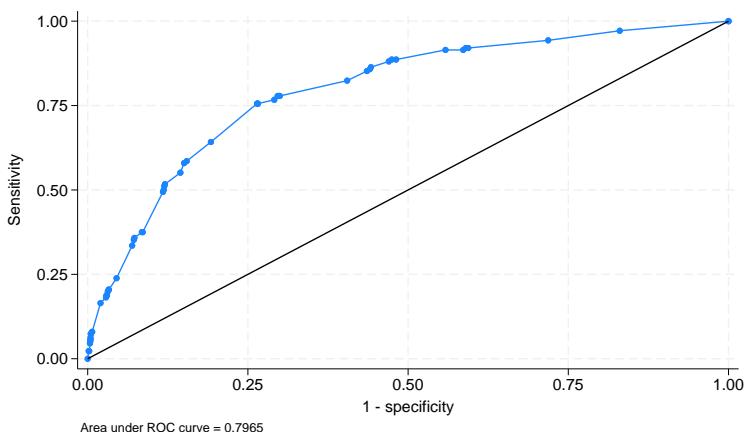
| death    | Odds ratio | Std. err. | z      | P> z  | [95% conf. interval] |          |
|----------|------------|-----------|--------|-------|----------------------|----------|
| site     |            |           |        |       |                      |          |
| Anterior | 1.901333   | .3185757  | 3.83   | 0.000 | 1.369103             | 2.640464 |
| 1.hcabg  | 2.105275   | .7430694  | 2.11   | 0.035 | 1.054076             | 4.204801 |
| killip   |            |           |        |       |                      |          |
| 2        | 2.251732   | .4064423  | 4.50   | 0.000 | 1.580786             | 3.207453 |
| 3        | 2.172105   | .584427   | 2.88   | 0.004 | 1.281907             | 3.680487 |
| 4        | 14.29137   | 5.087654  | 7.47   | 0.000 | 7.112964             | 28.71423 |
| age      |            |           |        |       |                      |          |
| 60-69    | 1.63726    | .5078582  | 1.59   | 0.112 | .8914261             | 3.007115 |
| 70-79    | 4.532029   | 1.206534  | 5.68   | 0.000 | 2.689568             | 7.636647 |
| >=80     | 8.893222   | 2.41752   | 8.04   | 0.000 | 5.219991             | 15.15125 |
| _cons    | .0063961   | .0016541  | -19.54 | 0.000 | .0038529             | .010618  |

Note: **\_cons** estimates baseline odds.

The odds ratios for a unit change in each covariate are reported by `logistic`. At fixed values of the other covariates, patients who enter Canadian hospitals with an anterior infarct have nearly twice the odds of death within 48 hours than those with an inferior infarct. Those who have had a previous CABG have approximately twice the risk of death of those who have not. Those with higher Killip risks and those who are older are also at greater risk of death.

We use `lroc` to draw the ROC curve for the model. The area under the curve of approximately 0.8 indicates acceptable discrimination for the model.

```
. lroc
Logistic model for death
Number of observations = 4483
Area under ROC curve = 0.7965
```



## Samples other than the estimation sample

`lroc` can be used with samples other than the estimation sample. By default, `lroc` remembers the estimation sample used with the last `logistic`, `logit`, `probit`, or `ivprobit` command. To override this, simply use an `if` or `in` restriction to select another set of observations, or specify the `all` option to force the command to use all the observations in the dataset.

See [example 3](#) in [\[R\] estat gof](#) for an example of using `lroc` with a sample other than the estimation sample.

## Models other than the last fitted model

By default, `lroc` uses the last model fit by `logistic`, `logit`, `probit`, or `ivprobit`. You may also directly specify the model to `lroc` by inputting a vector of coefficients with the `beta()` option and passing the name of the dependent variable `devar` to `lroc`.

### ▷ Example 2

Suppose that someone publishes the following logistic model of low birthweight:

$$\Pr(\text{low} = 1) = F(-0.02 \text{ age} - 0.01 \text{ lwt} + 1.3 \text{ black} + 1.1 \text{ smoke} + 0.5 \text{ pt1} + 1.8 \text{ ht} + 0.8 \text{ ui} + 0.5)$$

where  $F$  is the cumulative logistic distribution. These coefficients are not odds ratios; they are the equivalent of what `logit` produces.

We can see whether this model fits our data. First, we enter the coefficients as a row vector and label its columns with the names of the independent variables plus `_cons` for the constant (see [\[P\] matrix define](#) and [\[P\] matrix rownames](#)).

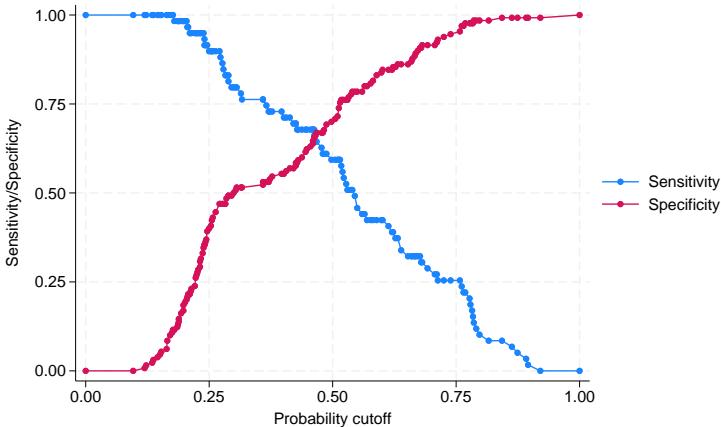
```
. use https://www.stata-press.com/data/r19/lbw3, clear
(Hosmer & Lemeshow data)
. matrix input b = (-.02, -.01, 1.3, 1.1, .5, 1.8, .8, .5)
. matrix colnames b = age lwt black smoke ptl ht ui _cons
```

Here we use `lroc` to examine the predictive ability of the model:

```
. lroc low, beta(b) nograph
Logistic model for low
Number of observations = 189
Area under ROC curve = 0.7275
```

The area under the curve indicates that this model does have some predictive power. We can obtain a graph of sensitivity and specificity as a function of the cutoff probability by typing

```
. lsens low, beta(b)
```



See [R] `lsens`.



## Stored results

`lroc` stores the following in `r()`:

Scalars

|                      |                          |
|----------------------|--------------------------|
| <code>r(N)</code>    | number of observations   |
| <code>r(area)</code> | area under the ROC curve |

## Methods and formulas

The ROC curve is a graph of *sensitivity* against  $(1 - \textit{specificity})$ . This is guaranteed to be a monotone nondecreasing function because the number of correctly predicted successes increases and the number of correctly predicted failures decreases as the classification cutoff  $c$  decreases.

The area under the ROC curve is the area on the bottom of this graph and is determined by integrating the curve. The vertices of the curve are determined by sorting the data according to the predicted index, and the integral is computed using the trapezoidal rule.

## References

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## Also see

- [R] **logistic** — Logistic regression, reporting odds ratios
- [R] **logit** — Logistic regression, reporting coefficients
- [R] **probit** — Probit regression
- [R] **ivprobit** — Probit model with continuous endogenous covariates
- [R] **lsens** — Graph sensitivity and specificity versus probability cutoff
- [R] **estat classification** — Classification statistics and table
- [R] **estat gof** — Pearson or Hosmer–Lemeshow goodness-of-fit test
- [R] **roc** — Receiver operating characteristic (ROC) analysis
- [U] **20 Estimation and postestimation commands**

Description  
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## Description

`lrtest` performs a likelihood-ratio test of the null hypothesis that the parameter vector of a statistical model satisfies some smooth constraint. To conduct the test, both the unrestricted and the restricted models must be fit using the maximum likelihood method (or some equivalent method), and the results of at least one must be stored using `estimates store`.

`lrtest` also supports composite models. In a composite model, we assume that the log likelihood and dimension (number of free parameters) of the full model are obtained as the sum of the log-likelihood values and dimensions of the constituting models.

## Quick start

Likelihood-ratio test that the coefficients for `x2` and `x3` are equal to 0

```
logit y x1 x2 x3
estimates store full
```

```
logit y x1 if e(sample)
estimates store restricted
```

```
lrtest full restricted
```

Display additional information, including AIC and BIC

```
lrtest full restricted, stats
```

Likelihood-ratio test that the coefficients for `x1` and `x3` are equal

```
constraint 1 x1=x3
logit y x1 x2 x3, constraints(1)
estimates store constrained
```

```
lrtest full constrained
```

Compare stored estimates `full` with the last model run

```
lrtest full
```

## Menu

Statistics > Postestimation

## Syntax

```
lrtest modelspec1 [modelspec2] [, options]
```

*modelspec*<sub>1</sub> and *modelspec*<sub>2</sub> specify the restricted and unrestricted model in any order. *modelspec*# is *name* | . | (*namelist*)

*name* is the name under which estimation results were stored using `estimates store` (see [R] [estimates store](#)), and “.” refers to the last estimation results, whether or not these were already stored. If *modelspec*<sub>2</sub> is not specified, the last estimation result is used; this is equivalent to specifying *modelspec*<sub>2</sub> as “.”.

If *namelist* is specified for a composite model, *modelspec*<sub>1</sub> and *modelspec*<sub>2</sub> cannot have names in common; for example, `lrtest (A B C) (C D E)` is not allowed because both model specifications include C.

| <i>options</i>     | Description                                                        |
|--------------------|--------------------------------------------------------------------|
| <code>stats</code> | display statistical information about the two models               |
| <code>dir</code>   | display descriptive information about the two models               |
| <code>df(#)</code> | override the automatic degrees-of-freedom calculation; seldom used |
| <code>force</code> | force testing even when apparently invalid                         |

`collect` is allowed; see [U] [11.1.10 Prefix commands](#).

## Options

`stats` displays statistical information about the unrestricted and restricted models, including the information indices of Akaike and Schwarz.

`dir` displays descriptive information about the unrestricted and restricted models; see `estimates dir` in [R] [estimates store](#).

`df(#)` is seldom specified; it overrides the automatic degrees-of-freedom calculation.

`force` forces the likelihood-ratio test calculations to take place in situations where `lrtest` would normally refuse to do so and issue an error. Such situations arise when one or more assumptions of the test are violated, for example, if the models were fit with `vce(robust)`, `vce(cluster clustvar)`, or `pweights`; when the dependent variables in the two models differ; when the null log likelihoods differ; when the samples differ; or when the estimation commands differ. If you use the `force` option, there is no guarantee as to the validity or interpretability of the resulting test.

## Remarks and examples

The standard way to use `lrtest` is to do the following:

1. Fit either the restricted model or the unrestricted model by using one of Stata’s estimation commands and then store the results using `estimates store name`.
2. Fit the alternative model (the unrestricted or restricted model) and then type `‘lrtest name .’`. `lrtest` determines for itself which of the two models is the restricted model by comparing the degrees of freedom.

Often, you may want to store the alternative model with estimates store `name2`, for instance, if you plan additional tests against models yet to be fit. The likelihood-ratio test is then obtained as `lrtest name name2`.

Remarks are presented under the following headings:

*Nested models*  
*Composite models*

## Nested models

`lrtest` may be used with any estimation command that reports a log likelihood, including `heckman`, `logit`, `poisson`, `stcox`, and `streg`. You must check that one of the model specifications implies a statistical model that is *nested within* the model implied by the other specification. Usually, this means that both models are fit with the same estimation command (for example, both are fit by `logit`, with the same dependent variables) and that the set of covariates of one model is a subset of the covariates of the other model. Second, `lrtest` is valid only for models that are fit by maximum likelihood or by some equivalent method, so it does not apply to models that were fit with probability weights or clusters. Specifying the `vce(robust)` option similarly would indicate that you are worried about the valid specification of the model, so you would not use `lrtest`. Third, `lrtest` assumes that under the null hypothesis, the test statistic is (approximately) distributed as  $\chi^2$ . This assumption is not true for likelihood-ratio tests of “boundary conditions”, such as tests for the presence of overdispersion or random effects (Gutierrez, Carter, and Drukker 2001).

### ► Example 1

We have data on infants born with low birthweights along with the characteristics of the mother (Hosmer, Lemeshow, and Sturdivant 2013; see also [R] [logistic](#)). We fit the following model:

```
. use https://www.stata-press.com/data/r19/lbw
(Hosmer & Lemeshow data)
. logistic low age lwt i.race smoke ptl ht ui
Logistic regression Number of obs = 189
 LR chi2(8) = 33.22
 Prob > chi2 = 0.0001
Log likelihood = -100.724 Pseudo R2 = 0.1416
```

|       | Odds ratio | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-------|------------|-----------|-------|-------|----------------------|----------|
| low   |            |           |       |       |                      |          |
| age   | .9732636   | .0354759  | -0.74 | 0.457 | .9061578             | 1.045339 |
| lwt   | .9849634   | .0068217  | -2.19 | 0.029 | .9716834             | .9984249 |
| race  |            |           |       |       |                      |          |
| Black | 3.534767   | 1.860737  | 2.40  | 0.016 | 1.259736             | 9.918406 |
| Other | 2.368079   | 1.039949  | 1.96  | 0.050 | 1.001356             | 5.600207 |
| smoke | 2.517698   | 1.00916   | 2.30  | 0.021 | 1.147676             | 5.523162 |
| ptl   | 1.719161   | .5952579  | 1.56  | 0.118 | .8721455             | 3.388787 |
| ht    | 6.249602   | 4.322408  | 2.65  | 0.008 | 1.611152             | 24.24199 |
| ui    | 2.1351     | .9808153  | 1.65  | 0.099 | .8677528             | 5.2534   |
| _cons | 1.586014   | 1.910496  | 0.38  | 0.702 | .1496092             | 16.8134  |

Note: `_cons` estimates baseline odds.

We now wish to test the constraint that the coefficients on age, lwt, pt1, and ht are all zero or, equivalently here, that the odds ratios are all 1. One solution is to type

```
. test age lwt pt1 ht
(1) [low]age = 0
(2) [low]lwt = 0
(3) [low]pt1 = 0
(4) [low]ht = 0

 chi2(4) = 12.38
 Prob > chi2 = 0.0147
```

This test is based on the inverse of the information matrix and is therefore based on a quadratic approximation to the likelihood function; see [R] [test](#). A more precise test would be to refit the model, applying the proposed constraints, and then calculate the likelihood-ratio test.

We first save the current model:

```
. estimates store full
```

We then fit the constrained model, which here is the model omitting age, lwt, pt1, and ht:

```
. logistic low i.race smoke ui
Logistic regression Number of obs = 189
 LR chi2(4) = 18.80
 Prob > chi2 = 0.0009
Log likelihood = -107.93404 Pseudo R2 = 0.0801
```

| low   | Odds ratio | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-------|------------|-----------|-------|-------|----------------------|----------|
| race  |            |           |       |       |                      |          |
| Black | 3.052746   | 1.498087  | 2.27  | 0.023 | 1.166747             | 7.987382 |
| Other | 2.922593   | 1.189229  | 2.64  | 0.008 | 1.316457             | 6.488285 |
| smoke |            |           |       |       |                      |          |
| ui    | 2.945742   | 1.101838  | 2.89  | 0.004 | 1.415167             | 6.131715 |
| _cons | 2.419131   | 1.047359  | 2.04  | 0.041 | 1.035459             | 5.651788 |
|       | .1402209   | .0512295  | -5.38 | 0.000 | .0685216             | .2869447 |

Note: **\_cons** estimates baseline odds.

That done, `lrtest` compares this model with the model we previously stored:

```
. lrtest full .
Likelihood-ratio test
Assumption: . nested within full

 LR chi2(4) = 14.42
 Prob > chi2 = 0.0061
```

Let's compare results. `test` reported that age, lwt, pt1, and ht were jointly significant at the 1.5% level; `lrtest` reports that they are significant at the 0.6% level. Given the quadratic approximation made by `test`, we could argue that `lrtest`'s results are more accurate.

`lrtest` explicates the assumption that, from a comparison of the degrees of freedom, it has assessed that the last fit model (`.`) is nested within the model stored as `full`. In other words, `full` is the unconstrained model and `.` is the constrained model.

The names in “(Assumption: . nested in full)” are actually links. Click on a name, and the results for that model are replayed.

Aside: The `nestreg` command provides a simple syntax for performing likelihood-ratio tests for nested model specifications; see [R] [nestreg](#). In the [previous example](#), we fit a full logistic model, used `estimates store` to store the full model, fit a constrained logistic model, and used `lrtest` to report a likelihood-ratio test between two models. To do this with one call to `nestreg`, use the `lrtable` option.

#### □ Technical note

`lrtest` determines the degrees of freedom of a model as the rank of the (co)variance matrix  $e(V)$ . There are two issues here. First, the *numerical* determination of the rank of a matrix is a subtle problem that can, for instance, be affected by the scaling of the variables in the model. The rank of a matrix depends on the number of (independent) linear combinations of coefficients that sum exactly to zero. In the world of numerical mathematics, it is hard to tell whether a very small number is really nonzero or is a real zero that happens to be slightly off because of roundoff error from the finite precision with which computers make floating-point calculations. Whether a small number is being classified as one or the other, typically on the basis of a threshold, affects the determined degrees of freedom. Although Stata generally makes sensible choices, it is bound to make mistakes occasionally. The moral of this story is to make sure that the calculated degrees of freedom is as you expect before interpreting the results. □

#### □ Technical note

A second issue involves `regress` and related commands such as `anova`. Mainly for historical reasons, `regress` does not treat the residual variance,  $\sigma^2$ , the same way that it treats the regression coefficients. Type `estat vce` after `regress`, and you will see the regression coefficients, not  $\hat{\sigma}^2$ . Most estimation commands for models with ancillary parameters (for example, `streg` and `heckman`) treat all parameters as equals. There is nothing technically wrong with `regress` here; we are usually focused on the regression coefficients, and their estimators are uncorrelated with  $\hat{\sigma}^2$ . But, formally,  $\sigma^2$  adds a degree of freedom to the model, which does not matter if you are comparing two regression models by a likelihood-ratio test. This test depends on the difference in the degrees of freedom, and hence being “off by 1” in each does not matter. But, if you are comparing a regression model with a larger model—for example, a heteroskedastic regression model fit by `arch`—the automatic determination of the degrees of freedom is incorrect, and you must specify the `df (#)` option. □

### ▷ Example 2

Returning to the low-birthweight data in [example 1](#), we now wish to test that the coefficient on `2.race` (black) is equal to that on `3.race` (other). The base model is still stored under the name `full`, so we need only fit the constrained model and perform the test. With  $z$  as the index of the logit model, the base model is

$$z = \beta_0 + \beta_1 \text{age} + \beta_2 \text{lw} + \beta_3 2.\text{race} + \beta_4 3.\text{race} + \dots$$

If  $\beta_3 = \beta_4$ , this can be written as

$$z = \beta_0 + \beta_1 \text{age} + \beta_2 \text{lw} + \beta_3 (2.\text{race} + 3.\text{race}) + \dots$$

We can fit the constrained model as follows:

```
. constraint 1 2.race = 3.race
. logistic low age lwt i.race smoke ptl ht ui, constraints(1)
Logistic regression Number of obs = 189
 Wald chi2(7) = 25.17
Log likelihood = -100.9997 Prob > chi2 = 0.0007
(1) [low]2.race - [low]3.race = 0
```

|       | low | Odds ratio | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-------|-----|------------|-----------|-------|-------|----------------------|----------|
| age   |     | .9716799   | .0352638  | -0.79 | 0.429 | .9049649             | 1.043313 |
| lwt   |     | .9864971   | .0064627  | -2.08 | 0.038 | .9739114             | .9992453 |
| race  |     |            |           |       |       |                      |          |
| Black |     | 2.728186   | 1.080207  | 2.53  | 0.011 | 1.255586             | 5.927907 |
| Other |     | 2.728186   | 1.080207  | 2.53  | 0.011 | 1.255586             | 5.927907 |
| smoke |     | 2.664498   | 1.052379  | 2.48  | 0.013 | 1.228633             | 5.778414 |
| ptl   |     | 1.709129   | .5924776  | 1.55  | 0.122 | .8663666             | 3.371691 |
| ht    |     | 6.116391   | 4.215585  | 2.63  | 0.009 | 1.58425              | 23.61385 |
| ui    |     | 2.09936    | .9699702  | 1.61  | 0.108 | .8487997             | 5.192407 |
| _cons |     | 1.309371   | 1.527398  | 0.23  | 0.817 | .1330839             | 12.8825  |

Note: **\_cons** estimates baseline odds.

Comparing this model with our original model, we obtain

```
. lrtest full .
Likelihood-ratio test
Assumption: . nested within full
LR chi2(1) = 0.55
Prob > chi2 = 0.4577
```

By comparison, typing `test 2.race=3.race` after fitting our base model results in a significance level of 0.4572. Alternatively, we can first store the restricted model, here using the name `equal`. Next, `lrtest` is invoked specifying the names of the restricted and unrestricted models (we do not care about the order). This time, we also add the option `stats` requesting a table of model statistics, including the model selection indices AIC and BIC.

```
. estimates store equal
. lrtest equal full, stats
Likelihood-ratio test
Assumption: equal nested within full
LR chi2(1) = 0.55
Prob > chi2 = 0.4577
Akaike's information criterion and Bayesian information criterion
```

| Model | N   | ll(null) | ll(model) | df | AIC      | BIC      |
|-------|-----|----------|-----------|----|----------|----------|
| equal | 189 | .        | -100.9997 | 8  | 217.9994 | 243.9334 |
| full  | 189 | -117.336 | -100.724  | 9  | 219.448  | 248.6237 |

Note: BIC uses N = number of observations. See [\[R\] IC note](#).

## Composite models

lrtest supports composite models; that is, models that can be fit by fitting a series of simpler models or by fitting models on subsets of the data. Theoretically, a composite model is one in which the likelihood function,  $L(\theta)$ , of the parameter vector,  $\theta$ , can be written as the product

$$L(\theta) = L_1(\theta_1) \times L_2(\theta_2) \times \cdots \times L_k(\theta_k)$$

of likelihood terms with  $\theta = (\theta_1, \dots, \theta_k)$  a partitioning of the full parameter vector. In such a case, the full-model likelihood  $L(\theta)$  is maximized by maximizing the likelihood terms  $L_j(\theta_j)$  in turn. Obviously,  $\log L(\hat{\theta}) = \sum_{j=1}^k \log L_j(\hat{\theta}_j)$ . The degrees of freedom for the composite model is obtained as the sum of the degrees of freedom of the constituting models.

### ▷ Example 3

As an example of the application of composite models, we consider a test of the hypothesis that the coefficients of a statistical model do not differ between different portions (“regimes”) of the covariate space. Economists call a test for such a hypothesis a *Chow test*.

We continue the analysis of the data on children of low birthweight by using logistic regression modeling and study whether the regression coefficients are the same among the three races: white, black, and other. A likelihood-ratio Chow test can be obtained by fitting the logistic regression model for each of the races and then comparing the combined results with those of the model previously stored as `full`. Because the full model included dummies for the three races, this version of the Chow test allows the intercept of the logistic regression model to vary between the regimes (races).

```
. logistic low age lwt smoke ptl ht ui if 1.race, nolog
Logistic regression Number of obs = 96
 LR chi2(6) = 13.86
 Prob > chi2 = 0.0312
Log likelihood = -45.927061 Pseudo R2 = 0.1311
```

|       | low | Odds ratio | Std. err. | z     | P> z  | [95% conf. interval] |
|-------|-----|------------|-----------|-------|-------|----------------------|
| age   |     | .9869674   | .0527757  | -0.25 | 0.806 | .8887649 1.096021    |
| lwt   |     | .9900874   | .0106101  | -0.93 | 0.353 | .9695089 1.011103    |
| smoke |     | 4.208697   | 2.680133  | 2.26  | 0.024 | 1.20808 14.66222     |
| ptl   |     | 1.592145   | .7474264  | 0.99  | 0.322 | .6344379 3.995544    |
| ht    |     | 2.900166   | 3.193537  | 0.97  | 0.334 | .3350554 25.1032     |
| ui    |     | 1.229523   | .9474768  | 0.27  | 0.789 | .2715165 5.567715    |
| _cons |     | .4891008   | .993785   | -0.35 | 0.725 | .0091175 26.23746    |

Note: `_cons` estimates baseline odds.

```
. estimates store white
```

```
. logistic low age lwt smoke ptl ht ui if 2.race, nolog
Logistic regression Number of obs = 26
 LR chi2(6) = 10.12
 Prob > chi2 = 0.1198
Log likelihood = -12.654157 Pseudo R2 = 0.2856
```

| low   | Odds ratio | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-------|------------|-----------|-------|-------|----------------------|----------|
| age   | .8735313   | .1377846  | -0.86 | 0.391 | .6412332             | 1.189983 |
| lwt   | .9747736   | .016689   | -1.49 | 0.136 | .9426065             | 1.008038 |
| smoke | 16.50373   | 24.37044  | 1.90  | 0.058 | .9133647             | 298.2083 |
| ptl   | 4.866916   | 9.33151   | 0.83  | 0.409 | .1135573             | 208.5895 |
| ht    | 85.05605   | 214.6382  | 1.76  | 0.078 | .6049308             | 11959.27 |
| ui    | 67.61338   | 133.3313  | 2.14  | 0.033 | 1.417399             | 3225.322 |
| _cons | 48.7249    | 169.9216  | 1.11  | 0.265 | .0523961             | 45310.94 |

Note: **\_cons** estimates baseline odds.

```
. estimates store black
. logistic low age lwt smoke ptl ht ui if 3.race, nolog
Logistic regression Number of obs = 67
 LR chi2(6) = 14.06
 Prob > chi2 = 0.0289
Log likelihood = -37.228444 Pseudo R2 = 0.1589
```

| low   | Odds ratio | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-------|------------|-----------|-------|-------|----------------------|----------|
| age   | .9263905   | .0665386  | -1.06 | 0.287 | .8047407             | 1.06643  |
| lwt   | .9724499   | .015762   | -1.72 | 0.085 | .9420424             | 1.003839 |
| smoke | .7979034   | .6340585  | -0.28 | 0.776 | .1680885             | 3.787586 |
| ptl   | 2.845675   | 1.777944  | 1.67  | 0.094 | .8363053             | 9.682908 |
| ht    | 7.767503   | 10.00537  | 1.59  | 0.112 | .6220764             | 96.98826 |
| ui    | 2.925006   | 2.046473  | 1.53  | 0.125 | .7423107             | 11.52571 |
| _cons | 49.09444   | 113.9165  | 1.68  | 0.093 | .5199275             | 4635.769 |

Note: **\_cons** estimates baseline odds.

. estimates store other

We are now ready to perform the likelihood-ratio Chow test:

```
. lrtest (full) (white black other), stats
Likelihood-ratio test
Assumption: full nested within (white, black, other)
LR chi2(12) = 9.83
Prob > chi2 = 0.6310
Akaike's information criterion and Bayesian information criterion
```

| Model | N   | ll(null)  | ll(model) | df | AIC      | BIC      |
|-------|-----|-----------|-----------|----|----------|----------|
| full  | 189 | -117.336  | -100.724  | 9  | 219.448  | 248.6237 |
| white | 96  | -52.85752 | -45.92706 | 7  | 105.8541 | 123.8046 |
| black | 26  | -17.71291 | -12.65416 | 7  | 39.30831 | 48.11499 |
| other | 67  | -44.26039 | -37.22844 | 7  | 88.45689 | 103.8897 |

Note: BIC uses N = number of observations. See [\[R\] IC note](#).

We cannot reject the hypothesis that the logistic regression model applies to each of the races at any reasonable significance level. By specifying the `stats` option, we can verify the degrees of freedom of the test:  $12 = 7 + 7 + 7 - 9$ . We can obtain the same test by fitting an expanded model with interactions between all covariates and race.

```
. logistic low race##c.(age lwt smoke ptl ht ui)
Logistic regression
Log likelihood = -95.809661
Number of obs = 189
LR chi2(20) = 43.05
Prob > chi2 = 0.0020
Pseudo R2 = 0.1835
```

| low          | Odds ratio | Std. err. | z     | P> z  | [95% conf. interval] |          |
|--------------|------------|-----------|-------|-------|----------------------|----------|
| race         |            |           |       |       |                      |          |
| Black        | 99.62137   | 402.0829  | 1.14  | 0.254 | .0365434             | 271578.9 |
| Other        | 100.3769   | 309.586   | 1.49  | 0.135 | .2378638             | 42358.38 |
| age          | .9869674   | .0527757  | -0.25 | 0.806 | .8887649             | 1.096021 |
| lwt          | .9900874   | .0106101  | -0.93 | 0.353 | .9695089             | 1.011103 |
| smoke        | 4.208697   | 2.680133  | 2.26  | 0.024 | 1.20808              | 14.66222 |
| ptl          | 1.592145   | .7474264  | 0.99  | 0.322 | .6344379             | 3.995544 |
| ht           | 2.900166   | 3.193537  | 0.97  | 0.334 | .3350554             | 25.1032  |
| ui           | 1.229523   | .9474768  | 0.27  | 0.789 | .2715165             | 5.567715 |
| race#c.age   |            |           |       |       |                      |          |
| Black        | .885066    | .1474079  | -0.73 | 0.464 | .638569              | 1.226714 |
| Other        | .9386232   | .0840486  | -0.71 | 0.479 | .7875366             | 1.118695 |
| race#c.lwt   |            |           |       |       |                      |          |
| Black        | .9845329   | .0198857  | -0.77 | 0.440 | .9463191             | 1.02429  |
| Other        | .9821859   | .0190847  | -0.93 | 0.355 | .9454839             | 1.020313 |
| race#c.smoke |            |           |       |       |                      |          |
| Black        | 3.921338   | 6.305992  | 0.85  | 0.395 | .167725              | 91.67917 |
| Other        | .1895844   | .1930601  | -1.63 | 0.102 | .025763              | 1.395113 |
| race#c.ptl   |            |           |       |       |                      |          |
| Black        | 3.05683    | 6.034089  | 0.57  | 0.571 | .0638301             | 146.3918 |
| Other        | 1.787322   | 1.396789  | 0.74  | 0.457 | .3863582             | 8.268285 |
| race#c.ht    |            |           |       |       |                      |          |
| Black        | 29.328     | 80.7482   | 1.23  | 0.220 | .1329492             | 6469.623 |
| Other        | 2.678295   | 4.538712  | 0.58  | 0.561 | .0966916             | 74.18702 |
| race#c.ui    |            |           |       |       |                      |          |
| Black        | 54.99155   | 116.4274  | 1.89  | 0.058 | .8672471             | 3486.977 |
| Other        | 2.378976   | 2.476124  | 0.83  | 0.405 | .309335              | 18.29579 |
| _cons        | .4891008   | .993785   | -0.35 | 0.725 | .0091175             | 26.23746 |

Note: `_cons` estimates baseline odds.

```
. lrtest full .
Likelihood-ratio test
Assumption: full nested within .
LR chi2(12) = 9.83
Prob > chi2 = 0.6310
```

Applying `lrtest` for the full model against the model with all interactions yields the same test statistic and  $p$ -value as for the full model against the composite model for the three regimes. Here the specification of the model with interactions was convenient, and `logistic` had no problem computing the estimates for the expanded model. In models with more complicated likelihoods, such as Heckman's selection model (see [R] [heckman](#)) or complicated survival-time models (see [ST] [streg](#)), fitting the models with all interactions may be numerically demanding and may be much more time consuming than fitting a series of models separately for each regime.

Given the model with all interactions, we could also test the hypothesis of no differences among the regions (races) by a Wald version of the Chow test by using the `testparm` command; see [R] [test](#).

```
. testparm race#c.(age lwt smoke pt1 ht ui)
(1) [low]2.race#c.age = 0
(2) [low]3.race#c.age = 0
(3) [low]2.race#c.lwt = 0
(4) [low]3.race#c.lwt = 0
(5) [low]2.race#c.smoke = 0
(6) [low]3.race#c.smoke = 0
(7) [low]2.race#c.pt1 = 0
(8) [low]3.race#c.pt1 = 0
(9) [low]2.race#c.ht = 0
(10) [low]3.race#c.ht = 0
(11) [low]2.race#c.ui = 0
(12) [low]3.race#c.ui = 0

 chi2(12) = 8.24
 Prob > chi2 = 0.7663
```

We conclude that, here, the Wald version of the Chow test is similar to the likelihood-ratio version of the Chow test.



## Stored results

`lrtest` stores the following in `r()`:

Scalars

|                      |                                      |
|----------------------|--------------------------------------|
| <code>r(p)</code>    | $p$ -value for likelihood-ratio test |
| <code>r(df)</code>   | degrees of freedom                   |
| <code>r(chi2)</code> | LR test statistic                    |

Programmers wishing their estimation commands to be compatible with `lrtest` should note that `lrtest` requires that the following results be returned:

|                     |                                              |
|---------------------|----------------------------------------------|
| <code>e(cmd)</code> | name of estimation command                   |
| <code>e(ll)</code>  | log likelihood                               |
| <code>e(V)</code>   | variance-covariance matrix of the estimators |
| <code>e(N)</code>   | number of observations                       |

`lrtest` also verifies that `e(N)`, `e(ll_0)`, and `e(depvar)` are consistent between two noncomposite models.

## Methods and formulas

Let  $L_0$  and  $L_1$  be the log-likelihood values associated with the full and constrained models, respectively. The test statistic of the likelihood-ratio test is  $LR = -2(L_1 - L_0)$ . If the constrained model is true, LR is approximately  $\chi^2$  distributed with  $d_0 - d_1$  degrees of freedom, where  $d_0$  and  $d_1$  are the model degrees of freedom associated with the full and constrained models, respectively (Greene 2018, 554–555).

`lrtest` determines the degrees of freedom of a model as the rank of  $e(V)$ , computed as the number of nonzero diagonal elements of `invsym(e(V))`.

## References

- Greene, W. H. 2018. *Econometric Analysis*. 8th ed. New York: Pearson.
- Gutierrez, R. G., S. L. Carter, and D. M. Drukker. 2001. `sg160`: On boundary-value likelihood-ratio tests. *Stata Technical Bulletin* 60: 15–18. Reprinted in *Stata Technical Bulletin Reprints*, vol. 10, pp. 269–273. College Station, TX: Stata Press.
- Hosmer, D. W., Jr., S. A. Lemeshow, and R. X. Sturdivant. 2013. *Applied Logistic Regression*. 3rd ed. Hoboken, NJ: Wiley.
- Raciborski, R. 2015. Spotlight on `irt`. *The Stata Blog: Not Elsewhere Classified*. <https://blog.stata.com/2015/07/31/spotlight-on-irt/>.
- Tauchmann, H. 2023. `lgrgtest`: Lagrange multiplier test after constrained maximum-likelihood estimation. *Stata Journal* 23: 386–401.

## Also see

- [R] `test` — Test linear hypotheses after estimation
- [R] `testnl` — Test nonlinear hypotheses after estimation
- [R] `nestreg` — Nested model statistics

[Description](#)  
[Options](#)  
[Reference](#)

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## Description

`lsens` graphs sensitivity and specificity versus probability cutoff and optionally creates new variables containing these data.

`lsens` requires that the current estimation results be from `logistic`, `logit`, `probit`, or `ivprobit`; see [\[R\] logistic](#), [\[R\] logit](#), [\[R\] probit](#), or [\[R\] ivprobit](#).

## Quick start

Graph sensitivity and specificity versus a probability cutoff using current estimation results

```
lsens
```

Generate variables `v1`, `v2`, and `v3` to contain probability cutoffs, sensitivity, and specificity

```
lsens, genprob(v1) gensens(v2) genspec(v3)
```

Add “My Title” to graph

```
lsens, genprob(v1) gensens(v2) genspec(v3) title(My Title)
```

## Menu

Statistics > Binary outcomes > Postestimation > Sensitivity/specificity plot

## Syntax

```
lsens [depvar] [if] [in] [weight] [, options]
```

| <i>options</i>                                                        | Description                                                                                 |
|-----------------------------------------------------------------------|---------------------------------------------------------------------------------------------|
| <b>Main</b>                                                           |                                                                                             |
| <code>all</code>                                                      | graph all observations in the data                                                          |
| <code>genprob(<i>varname</i>)</code>                                  | create variable containing probability cutoffs                                              |
| <code>gensens(<i>varname</i>)</code>                                  | create variable containing sensitivity                                                      |
| <code>genspec(<i>varname</i>)</code>                                  | create variable containing specificity                                                      |
| <code>replace</code>                                                  | overwrite existing variables                                                                |
| <code>nograph</code>                                                  | suppress the graph                                                                          |
| <b>Advanced</b>                                                       |                                                                                             |
| <code>beta(<i>matname</i>)</code>                                     | row vector containing model coefficients                                                    |
| <b>Plot</b>                                                           |                                                                                             |
| <code>connect_options</code>                                          | affect rendition of the plotted points connected by lines                                   |
| <b>Add plots</b>                                                      |                                                                                             |
| <code>addplot(<i>plot</i>)</code>                                     | add other plots to the generated graph                                                      |
| <b>Y axis, X axis, Titles, Legend, Overall</b>                        |                                                                                             |
| <code>twoway_options</code>                                           | any options other than <code>by()</code> documented in <a href="#">[G-3] twoway_options</a> |
| collect is allowed; see <a href="#">[U] 11.1.10 Prefix commands</a> . |                                                                                             |
| fweights are allowed; see <a href="#">[U] 11.1.6 weight</a> .         |                                                                                             |
| lsens is not appropriate with svy estimation results.                 |                                                                                             |

## Options

### Main

`all` requests that the statistic be computed for all observations in the data, ignoring any `if` or `in` restrictions specified by the estimation command.

`genprob(varname)`, `gensens(varname)`, and `genspec(varname)` specify the names of new variables created to contain, respectively, the probability cutoffs and the corresponding sensitivity and specificity.

`replace` requests that existing variables specified for `genprob()`, `gensens()`, or `genspec()` be overwritten.

`nograph` suppresses graphical output.

### Advanced

`beta(matname)` specifies a row vector containing model coefficients. The columns of the row vector must be labeled with the corresponding names of the independent variables in the data. The dependent variable *depvar* must be specified immediately after the command name. See [Models other than the last fitted model](#) later in this entry.

## Plot

*connect\_options* affect the rendition of the plotted points connected by lines; see *connect\_options* in [G-2] [graph twoway scatter](#).

## Add plots

`addplot(plot)` provides a way to add other plots to the generated graph. See [G-3] [addplot\\_option](#).

## Y axis, X axis, Titles, Legend, Overall

*twoway\_options* are any of the options documented in [G-3] [twoway\\_options](#), excluding `by()`. These include options for titling the graph (see [G-3] [title\\_options](#)) and for saving the graph to disk (see [G-3] [saving\\_option](#)).

## Remarks and examples

Remarks are presented under the following headings:

[Introduction](#)

[Models other than the last fitted model](#)

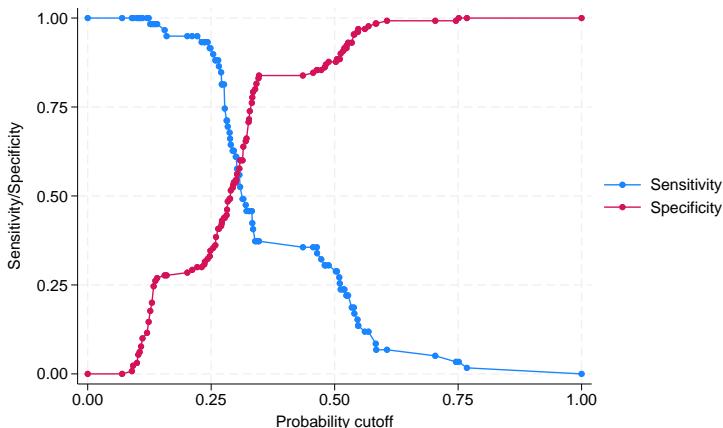
### Introduction

`lsens` plots sensitivity and specificity; it plots both sensitivity and specificity versus probability cutoff  $c$ . The graph is equivalent to what you would get from `estat classification` (see [R] [estat classification](#)) if you varied the cutoff probability  $c$  from 0 to 1.

#### ▷ Example 1

We illustrate `lsens` after `logistic`; see [R] [logistic](#).

```
. use https://www.stata-press.com/data/r19/lbw
(Hosmer & Lemeshow data)
. logistic low age i.race smoke ui
(output omitted)
. lsens
```



lsens optionally creates new variables containing the probability cutoff, sensitivity, and specificity.

```
. lsens, genprob(p) gensens(sens) genspec(spec) nograph
```

The variables created will have  $M + 2$  distinct nonmissing values: one for each of the  $M$  covariate patterns, one for  $c = 0$ , and another for  $c = 1$ . Values are recorded for  $p = 0$ , for each of the observed predicted probabilities, and for  $p = 1$ . The total number of observations required to do this can be fewer than  $\_N$ , the same as  $\_N$ , or  $\_N + 1$ , or  $\_N + 2$ . If more observations are added, they are added at the end of the dataset and the values of the original variables are set to missing in the added observations. How the values added align with existing observations is irrelevant.



## □ Technical note

logistic, logit, probit, or ivprobit and lsens keep track of the estimation sample. If you type, for instance, logistic ... if x==1, then when you type lsens, the statistics will be calculated on the x==1 subsample of the data automatically.

You should specify if or in with lsens only when you wish to produce graphs and calculate statistics for a set of observations other than the estimation sample.

If the logistic model was fit with fweights, lsens properly accounts for the weights in its calculations. You do not have to specify the weights when you run lsens. Weights should be specified with lsens only when you wish to use a different set of weights.



## Models other than the last fitted model

By default, lsens uses the last model fit. You may also directly specify the model to lsens by inputting a vector of coefficients with the beta() option and passing the name of the dependent variable *devar* to lsens.

## ▷ Example 2

Suppose that someone publishes the following logistic model of low birthweight:

$$\Pr(\text{low} = 1) = F(-0.02 \text{ age} - 0.01 \text{ lwt} + 1.3 \text{ black} + 1.1 \text{ smoke} + 0.5 \text{ pt1} + 1.8 \text{ ht} + 0.8 \text{ ui} + 0.5)$$

where  $F$  is the cumulative logistic distribution. These coefficients are not odds ratios; they are the equivalent of what logit produces.

We can see whether this model fits our data. First, we enter the coefficients as a row vector and label its columns with the names of the independent variables plus `_cons` for the constant (see [P] [matrix define](#) and [P] [matrix rownames](#)).

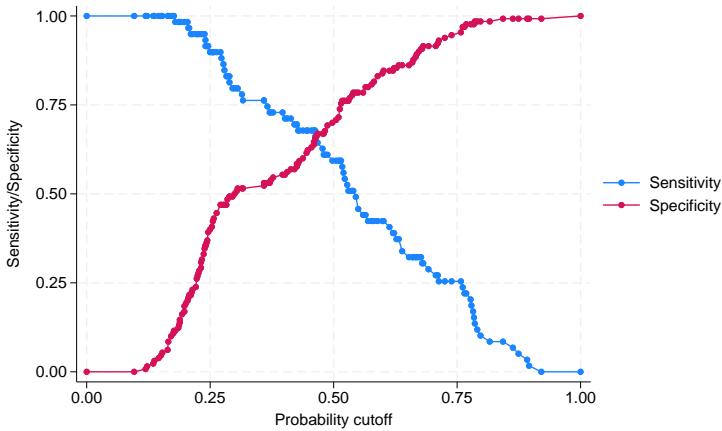
```
. use https://www.stata-press.com/data/r19/lbw3, clear
(Hosmer & Lemeshow data)
. matrix input b = (-0.02, -.01, 1.3, 1.1, .5, 1.8, .8, .5)
. matrix colnames b = age lwt black smoke pt1 ht ui _cons
```

We can use `lroc` (see [R] [lroc](#)) to examine the predictive ability of the model:

```
. lroc low, beta(b) nograph
Logistic model for low
Number of observations = 189
Area under ROC curve = 0.7275
```

The area under the curve indicates that this model does have some predictive power. We can obtain a graph of sensitivity and specificity as a function of the cutoff probability by typing

```
. lsens low, beta(b)
```



◀

## Stored results

`lsens` stores the following in `r()`:

```
Scalars
 r(N) number of observations
```

## Methods and formulas

Let  $j$  index observations and  $c$  be the cutoff probability. Let  $p_j$  be the predicted probability of a positive outcome and  $y_j$  be the actual outcome, which we will treat as 0 or 1, although Stata treats it as 0 and non-0, excluding missing observations.

A prediction is classified as *positive* if  $p_j \geq c$  and otherwise is classified as *negative*. The classification is *correct* if it is *positive* and  $y_j = 1$  or if it is *negative* and  $y_j = 0$ .

*Sensitivity* is the fraction of  $y_j = 1$  observations that are correctly classified. *Specificity* is the percentage of  $y_j = 0$  observations that are correctly classified.

## Reference

Hosmer, D. W., Jr., S. A. Lemeshow, and R. X. Sturdivant. 2013. *Applied Logistic Regression*. 3rd ed. Hoboken, NJ: Wiley.

## Also see

- [R] **logistic** — Logistic regression, reporting odds ratios
- [R] **logit** — Logistic regression, reporting coefficients
- [R] **probit** — Probit regression
- [R] **ivprobit** — Probit model with continuous endogenous covariates
- [R] **lroc** — Compute area under ROC curve and graph the curve
- [R] **estat classification** — Classification statistics and table
- [R] **estat gof** — Pearson or Hosmer–Lemeshow goodness-of-fit test
- [R] **roc** — Receiver operating characteristic (ROC) analysis
- [U] **20 Estimation and postestimation commands**

|             |                      |                |                      |
|-------------|----------------------|----------------|----------------------|
| Description | Quick start          | Menu           | Syntax               |
| Options     | Remarks and examples | Stored results | Methods and formulas |
| References  | Also see             |                |                      |

## Description

`lv` shows a letter-value display (Tukey 1977, 44–49; Hoaglin 1983) for each variable in *varlist*. If no variables are specified, letter-value displays are shown for each numeric variable in the data.

## Quick start

Letter-value display for all numeric variables in the dataset

```
lv
```

Letter-value display for `v1`

```
lv v1
```

Also generate new variables `_mid`, `_spread`, `_psigma`, and `_z2` containing midsummaries, spreads, pseudosigmias, and  $z^2$  values

```
lv v1, generate
```

Letter-value displays for `v1` separately for each value of `v2`

```
bysort v2: lv v1
```

## Menu

Statistics > Summaries, tables, and tests > Distributional plots and tests > Letter-value display

## Syntax

```
lv [varlist] [if] [in] [, generate tail(#)]
```

`by` and `collect` are allowed; see [U] 11.1.10 Prefix commands.

## Options

### Main

`generate` adds four new variables to the data: `_mid`, containing the midsummaries; `_spread`, containing the spreads; `_psigma`, containing the pseudosigmias; and `_z2`, containing the squared values from a standard normal distribution corresponding to the particular letter value. If the variables `_mid`, `_spread`, `_psigma`, and `_z2` already exist, their contents are replaced. At most, only the first 11 observations of each variable are used; the remaining observations contain missing. If *varlist* specifies more than one variable, the newly created variables contain results for the last variable specified. The `generate` option may not be used with the `by` prefix.

`tail(#)` indicates the inverse of the tail density through which letter values are to be displayed: 2 corresponds to the median (meaning half in each tail), 4 to the fourths (roughly the 25th and 75th percentiles), 8 to the eighths, and so on. # may be specified as 4, 8, 16, 32, 64, 128, 256, 512, or 1,024 and defaults to a value of # that has corresponding depth just greater than 1. The default is taken as 1,024 if the calculation results in a number larger than 1,024. Given the intelligent default, this option is rarely specified.

## Remarks and examples

Letter-value displays are a collection of observations drawn systematically from the data, focusing especially on the tails rather than the middle of the distribution. The displays are called letter-value displays because letters have been (almost arbitrarily) assigned to tail densities:

| Letter | Tail area | Letter | Tail area |
|--------|-----------|--------|-----------|
| M      | 1/2       | B      | 1/64      |
| F      | 1/4       | A      | 1/128     |
| E      | 1/8       | Z      | 1/256     |
| D      | 1/16      | Y      | 1/512     |
| C      | 1/32      | X      | 1/1024    |

### ▷ Example 1

We have data on the mileage ratings of 74 automobiles. To obtain a letter-value display, we type

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. lv mpg
```

| #           | 74   | Mileage (mpg) |       |      | spread  | pseudosigma |
|-------------|------|---------------|-------|------|---------|-------------|
| M           | 37.5 |               | 20    |      |         |             |
| F           | 19   | 18            | 21.5  | 25   | 7       | 5.216359    |
| E           | 10   | 15            | 21.5  | 28   | 13      | 5.771728    |
| D           | 5.5  | 14            | 22.25 | 30.5 | 16.5    | 5.576303    |
| C           | 3    | 14            | 24.5  | 35   | 21      | 5.831039    |
| B           | 2    | 12            | 23.5  | 35   | 23      | 5.732448    |
| A           | 1.5  | 12            | 25    | 38   | 26      | 6.040635    |
|             | 1    | 12            | 26.5  | 41   | 29      | 6.16562     |
|             |      |               |       |      | # below | # above     |
| inner fence |      | 7.5           |       | 35.5 | 0       | 1           |
| outer fence |      | -3            |       | 46   | 0       | 0           |

The decimal points can be made to line up and thus the output made more readable by specifying a display format for the variable; see [U] 12.5 Formats: Controlling how data are displayed.

```
. format mpg %9.2f
. lv mpg
```

| #           | 74   | Mileage (mpg) |       |       |         |             |
|-------------|------|---------------|-------|-------|---------|-------------|
| M           | 37.5 |               | 20.00 |       | spread  | pseudosigma |
| F           | 19   | 18.00         | 21.50 | 25.00 | 7.00    | 5.22        |
| E           | 10   | 15.00         | 21.50 | 28.00 | 13.00   | 5.77        |
| D           | 5.5  | 14.00         | 22.25 | 30.50 | 16.50   | 5.58        |
| C           | 3    | 14.00         | 24.50 | 35.00 | 21.00   | 5.83        |
| B           | 2    | 12.00         | 23.50 | 35.00 | 23.00   | 5.73        |
| A           | 1.5  | 12.00         | 25.00 | 38.00 | 26.00   | 6.04        |
|             | 1    | 12.00         | 26.50 | 41.00 | 29.00   | 6.17        |
|             |      |               |       |       | # below | # above     |
| inner fence |      | 7.50          |       | 35.50 | 0       | 1           |
| outer fence |      | -3.00         |       | 46.00 | 0       | 0           |

At the top, the number of observations is indicated as 74. The first line shows the statistics associated with M, the letter value that puts half the density in each tail, or the median. The median has *depth* 37.5 (that is, in the ordered data, M is 37.5 observations in from the extremes) and has value 20. The next line shows the statistics associated with F or the fourths. The fourths have depth 19 (that is, in the ordered data, the lower fourth is observation 19, and the upper fourth is observation  $74 - 19 + 1$ ), and the values of the lower and upper fourths are 18 and 25. The number in the middle is the point halfway between the fourths—called a midsummary. If the distribution were perfectly symmetric, the midsummary would equal the median. The spread is the difference between the lower and upper summaries ( $25 - 18 = 7$ ). For fourths, half the data lie within a 7-mpg band. The pseudosigma is a calculation of the standard deviation using only the lower and upper summaries and assuming that the variable is normally distributed. If the data really were normally distributed, all the pseudosigmas would be roughly equal.

After the letter values, the line labeled with depth 1 reports the minimum and maximum values. Here the halfway point between the extremes is 26.5, which is greater than the median, indicating that 41 is more extreme than 12, at least relative to the median. And with each letter value, the midsummaries are increasing—our data are skewed. The pseudosigmas are also increasing, indicating that the data are spreading out relative to a normal distribution, although, given the evident skewness, this elongation may be an artifact of the skewness.

At the end is an attempt to identify outliers, although the points so identified are merely outside some predetermined cutoff. Points outside the inner fence are called *outside values* or *mild outliers*. Points outside the outer fence are called *severe outliers*. The inner fence is defined as  $(3/2)$ IQR and the outer fence as 3IQR above and below the F summaries, where the interquartile range (IQR) is the spread of the fourths.

◀

## □ Technical note

The form of the letter-value display has varied slightly with different authors. `lv` displays appear as described by Hoaglin (1983) but as modified by Emerson and Stoto (1983), where they included the midpoint of each of the spreads. This format was later adopted by Hoaglin (1985). If the distribution is symmetric, the midpoints will all be roughly equal. On the other hand, if the midpoints vary systematically, the distribution is skewed.

The pseudosigmas are obtained from the lower and upper summaries for each letter value. For each letter value, they are the standard deviation a normal distribution would have if its spread for the given letter value were to equal the observed spread. If the pseudosigmas are all roughly equal, the data are said to have *neutral elongation*. If the pseudosigmas increase systematically, the data are said to be more elongated than a normal, that is, have thicker tails. If the pseudosigmas decrease systematically, the data are said to be less elongated than a normal, that is, have thinner tails.

Interpretation of the number of mild and severe outliers is more problematic. The following discussion is drawn from [Hamilton \(1991\)](#):

Obviously, the presence of any such outliers does not rule out that the data have been drawn from a normal distribution; in large datasets, there will most certainly be observations outside  $(3/2)IQR$  and  $3IQR$ . Severe outliers, however, make up about two per million (0.0002%) of a normal population. In samples, they lie far enough out to have substantial effects on means, standard deviations, and other classical statistics. The 0.0002%, however, should be interpreted carefully; outliers appear more often in small samples than one might expect from population proportions because of sampling variation in estimated quartiles. Monte Carlo simulation by [Hoaglin, Iglewicz, and Tukey \(1986\)](#) obtained these results on the percentages and numbers of outliers in random samples from a normal population:

| $n$      | Percentage   |        | Number       |          |
|----------|--------------|--------|--------------|----------|
|          | any outliers | severe | any outliers | severe   |
| 10       | 2.83         | 0.362  | 0.283        | 0.0362   |
| 20       | 1.66         | 0.074  | 0.332        | 0.0148   |
| 50       | 1.15         | 0.011  | 0.575        | 0.0055   |
| 100      | 0.95         | 0.002  | 0.95         | 0.002    |
| 200      | 0.79         | 0.001  | 1.58         | 0.002    |
| 300      | 0.75         | 0.001  | 2.25         | 0.003    |
| $\infty$ | 0.70         | 0.0002 | $\infty$     | $\infty$ |

Thus, the presence of any severe outliers in samples of less than 300 is sufficient to reject normality. [Hoaglin, Iglewicz, and Tukey \(1981\)](#) suggested the approximation  $0.00698 + 0.4/n$  for the fraction of mild outliers in a sample of size  $n$  or, equivalently,  $0.00698n + 0.4$  for the number of outliers.

□

## ► Example 2

The `generate` option adds the `_mid`, `_spread`, `_psigma`, and `_z2` variables to our data, making possible many of the diagnostic graphs suggested by [Hoaglin \(1985\)](#).

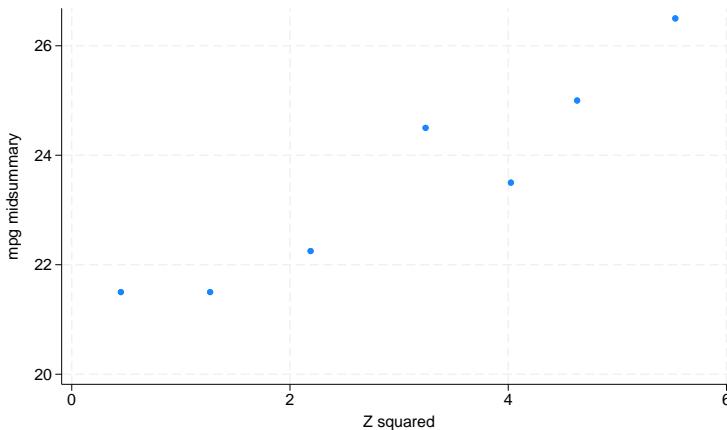
```
. lv mpg, generate
 (output omitted)
. list _mid _spread _psigma _z2 in 1/12
```

|     | _mid  | _spread | _psigma  | _z2      |
|-----|-------|---------|----------|----------|
| 1.  | 20    | .       | .        | .        |
| 2.  | 21.5  | 7       | 5.216359 | .4501955 |
| 3.  | 21.5  | 13      | 5.771728 | 1.26828  |
| 4.  | 22.25 | 16.5    | 5.576303 | 2.188846 |
| 5.  | 24.5  | 21      | 5.831039 | 3.24255  |
| 6.  | 23.5  | 23      | 5.732448 | 4.024532 |
| 7.  | 25    | 26      | 6.040635 | 4.631499 |
| 8.  | .     | .       | .        | .        |
| 9.  | .     | .       | .        | .        |
| 10. | .     | .       | .        | .        |
| 11. | 26.5  | 29      | 6.16562  | 5.53073  |
| 12. | .     | .       | .        | .        |

Observations 12 through the end are missing for these new variables. The definition of the observations is always the same. The first observation contains the M summary; the second, the F; the third, the E; and so on. Observation 11 always contains the summary for depth 1. Observations 8–10—corresponding to letter values Z, Y, and X—contain missing because these statistics were not calculated. We have only 74 observations, and their depth would be 1.

[Hoaglin \(1985\)](#) suggests graphing the midsummary against  $z^2$ . If the distribution is not skewed, the points in the resulting graph will be along a horizontal line:

```
. scatter _mid _z2
```



The graph clearly indicates the skewness of the distribution. We might also graph `_psigma` against `_z2` to examine elongation.

◀

## Stored results

`lv` stores the following in `r()`:

Scalars

|                        |                        |                     |              |
|------------------------|------------------------|---------------------|--------------|
| <code>r(N)</code>      | number of observations | <code>r(u_C)</code> | upper 32nd   |
| <code>r(min)</code>    | minimum                | <code>r(l_B)</code> | lower 64th   |
| <code>r(max)</code>    | maximum                | <code>r(u_B)</code> | upper 64th   |
| <code>r(median)</code> | median                 | <code>r(l_A)</code> | lower 128th  |
| <code>r(l_F)</code>    | lower 4th              | <code>r(u_A)</code> | upper 128th  |
| <code>r(u_F)</code>    | upper 4th              | <code>r(l_Z)</code> | lower 256th  |
| <code>r(l_E)</code>    | lower 8th              | <code>r(u_Z)</code> | upper 256th  |
| <code>r(u_E)</code>    | upper 8th              | <code>r(l_Y)</code> | lower 512th  |
| <code>r(l_D)</code>    | lower 16th             | <code>r(u_Y)</code> | upper 512th  |
| <code>r(u_D)</code>    | upper 16th             | <code>r(l_X)</code> | lower 1024th |
| <code>r(l_C)</code>    | lower 32nd             | <code>r(u_X)</code> | upper 1024th |

The lower/upper 8ths, 16ths, ..., 1024ths will be defined only if there are sufficient data.

## Methods and formulas

Let  $N$  be the number of (nonmissing) observations on  $x$ , and let  $x_{(i)}$  refer to the ordered data when  $i$  is an integer. Define  $x_{(i+0.5)} = (x_{(i)} + x_{(i+1)})/2$ ; the median is defined as  $x_{\{(N+1)/2\}}$ .

Define  $x_{[d]}$  as the pair of numbers  $x_{(d)}$  and  $x_{(N+1-d)}$ , where  $d$  is called the *depth*. Thus,  $x_{[1]}$  refers to the minimum and maximum of the data. Define  $m = (N + 1)/2$  as the depth of the median,  $f = (\lfloor m \rfloor + 1)/2$  as the depth of the fourths,  $e = (\lfloor f \rfloor + 1)/2$  as the depth of the eighths, and so on. Depths are reported on the far left of the letter-value display. The corresponding fourths of the data are  $x_{[f]}$ , the eighths are  $x_{[e]}$ , and so on. These values are reported inside the display. The middle value is defined as the corresponding midpoint of  $x_{[1]}$ . The spreads are defined as the difference in  $x_{[1]}$ .

The corresponding point  $z_i$  on a standard normal distribution is obtained as (Hoaglin 1985, 456–457)

$$z_i = \begin{cases} F^{-1}\{(d_i - 1/3)/(N + 1/3)\} & \text{if } d_i > 1 \\ F^{-1}\{0.695/(N + 0.390)\} & \text{otherwise} \end{cases}$$

where  $d_i$  is the depth of the letter value. The corresponding pseudosigma is obtained as the ratio of the spread to  $-2z_i$  (Hoaglin 1985, 431).

Define  $(F_l, F_u) = x_{[f]}$ . The inner fence has cutoffs  $F_l - \frac{3}{2}(F_u - F_l)$  and  $F_u + \frac{3}{2}(F_u - F_l)$ . The outer fence has cutoffs  $F_l - 3(F_u - F_l)$  and  $F_u + 3(F_u - F_l)$ .

The inner-fence values reported by `lv` are almost equal to those used by `graph`, `box` to identify outside points. The only difference is that `graph` uses a slightly different definition of fourths, namely, the 25th and 75th percentiles as defined by `summarize`; see [R] [summarize](#).

## References

- Cox, N. J. 2016. [Speaking Stata: Letter values as selected quantiles](#). *Stata Journal* 16: 1058–1071.
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- . 1985. “Using quantiles to study shape”. In *Exploring Data Tables, Trends, and Shapes*, edited by D. C. Hoaglin, C. F. Mosteller, and J. W. Tukey, 417–460. New York: Wiley.
- Hoaglin, D. C., B. Iglewicz, and J. W. Tukey. 1981. “Small-sample performance of a resistant rule for outlier detection”. In *1980 Proceedings of the Statistical Computing Section*. Washington, DC: American Statistical Association.
- . 1986. Performance of some resistant rules for outlier labeling. *Journal of the American Statistical Association* 81: 991–999. <https://doi.org/10.2307/2289073>.
- Tukey, J. W. 1977. *Exploratory Data Analysis*. Reading, MA: Addison–Wesley.

## Also see

- [R] [Diagnostic plots](#) — Distributional diagnostic plots
- [R] [stem](#) — Stem-and-leaf displays
- [R] [summarize](#) — Summary statistics

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

## Description

`makespline` generates a set of variables that form B-spline, piecewise polynomial spline, and restricted cubic spline basis functions from a list of existing variables. B-spline and piecewise polynomial spline bases may be first, second, or third order, with knots at percentiles of the data or uniformly spaced over the range of the variables. Restricted cubic splines, also known as natural splines, may only be of third order.

## Quick start

Generate variables from `x1` and `x2` that form third-order B-spline basis functions, with one knot at the median of each variable

```
makespline bspline x1 x2
```

Same as above, but with three knots at the 25th, 50th, and 75th percentiles

```
makespline bspline x1 x2, knots(3)
```

Same as above, but use second-order B-splines

```
makespline bspline x1 x2, knots(3) order(2)
```

Generate variables that form a linear spline for `x1` with knots at 10 and 20

```
makespline piecewise x1, knotslist(10 20) order(1)
```

Same as above, but do not rescale `x1` before creating spline variables

```
makespline piecewise x1, knotslist(10 20) order(1) norescalevars
```

Generate variables that form third-order piecewise polynomial splines for `x1` and `x2`, with knots at their 25th, 50th, and 75th percentiles

```
makespline piecewise x1 x2, knots(3)
```

Same as above, but with three knots at evenly spaced points over the range of `x1` and of `x2`

```
makespline piecewise x1 x2, knots(3) uniformknots
```

Specify values of knots in matrix `K` to generate variables forming restricted cubic splines

```
makespline rcs x1 x2, knotsmat(K)
```

Generate variables that form a linear spline for `x1` without rescaling the values of `x1`

```
makespline linear x1
```

## Menu

Data > Create or change data > Other variable-creation commands > Spline generation

## Syntax

```
makespline basis varlist [if] [in] [weight] [, options]
```

| <i>basis</i>           | Description                                                          |
|------------------------|----------------------------------------------------------------------|
| <code>bspline</code>   | B-spline                                                             |
| <code>piecewise</code> | piecewise polynomial spline                                          |
| <code>rbs</code>       | restricted cubic spline                                              |
| <code>linear</code>    | linear spline—piecewise basis of order 1 without rescaling variables |

| <i>options</i>                          | Description                                                                                                                       |
|-----------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------|
| <b>Main</b>                             |                                                                                                                                   |
| <code>bsepsilon(#)</code>               | specify the distance (#) from the variable's boundary for B-spline knot placement; default is <code>bsepsilon(0.01)</code>        |
| <code>local</code>                      | generate first-order polynomial spline variables centered around adjacent knots                                                   |
| <code>harrell</code>                    | place knots according to percentiles in <a href="#">Harrell (2001)</a> ; only for <code>rbs</code> basis                          |
| <code>order(#)</code>                   | use a spline basis of order #; default is <code>order(3)</code>                                                                   |
| <code>knots(#)</code>                   | use a spline basis function with # knots                                                                                          |
| <code>knotslist(knotvals)</code>        | use knots specified in <i>knotvals</i>                                                                                            |
| <code>knotsmat(matname)</code>          | use knots in matrix <i>matname</i>                                                                                                |
| <code>distinct(#)</code>                | set minimum number of distinct values required for variables used to construct splines to #; default is <code>distinct(10)</code> |
| <code>replace</code>                    | replace existing variables having the same names as the new basis and rescaled variables, if they exist                           |
| <code>norescalevars</code>              | do not rescale variables before generating spline basis                                                                           |
| <code>uniformknots</code>               | place knots at evenly spaced points over the range of each variable; default is placement at percentiles                          |
| <code>float</code>                      | set type for generated variables to float instead of double                                                                       |
| <code>basis(stub   newvarlist)</code>   | store elements of spline basis function using <i>stub</i> or <i>newvarlist</i>                                                    |
| <code>rescale(stub   newvarlist)</code> | store rescaled values of variables using <i>stub</i> or <i>newvarlist</i>                                                         |

`collect` is allowed; see [U] [11.1.10 Prefix commands](#).

`fweights`, `awweights`, and `iweights` are allowed; see [U] [11.1.6 weight](#).

## Options

### Main

`bsepsilon(#)` specifies the distance from the boundary of the variable where B-spline knots may be placed. The default is `bsepsilon(0.01)`.

`local` specifies that a basis function for a first-order piecewise polynomial be generated with variables centered around adjacent knots. When splines are generated for only one variable and used in estimation, the regression coefficients measure slopes for the intervals defined by knots.

`harrell` specifies that knots be placed according to the percentiles recommended in [Harrell \(2001, 23\)](#). This option may be used only with *basis* `rbs` and when specifying 3 to 7 knots.

`order(#)` specifies that a spline of order `#` be used as the basis. `order()` may be 1, 2, or 3 for *basis bspline* and *basis piecewise*. `order()` may only be 3 for *basis rcs*. For *basis linear* or when the `local` option is specified, `order()` may only be 1. The default is `order(3)`, cubic splines.

`knots(#)` specifies that a spline or B-spline basis function with `#` interior knots be used. The number of knots must be an integer greater than or equal to 1. The maximum number of knots is either 4,096 or two-thirds of the sample size, whichever is smaller. Also, the number of knots must be less than the number of distinct values in the variable used to generate the basis function. The default is `knots(1)` if exact knot values are not specified using `knotslist()` or `knotsmat()`. For *basis rcs*, the default is `knots(3)`, and the number of knots must be 3 or greater.

`knotslist(knotvals)` specifies in *knotvals* the values of knots to be used for each variable. The knot values must be specified in the order of *varlist*, and a backslash (`\`) must be used to separate knots for different variables. For example, if splines are generated for `x1` and `x2`, the knots may be specified as `knotslist(20 40 60 \ 5 10 15)`.

`knotsmat(matname)` specifies that, in *matname*, the knots for each variable be the values in each row. The number of knots should be the same for each variable, and there must be as many rows as there are variables. If rows of *matname* are not labeled with *varnames*, then rows are assumed to be in the order of *varlist*.

`distinct(#)` specifies the minimum number of distinct values required for the variables used to construct the basis functions. Intuitively, using discrete variables for continuous interpolation is difficult to justify. `#` specifies the number of distinct values necessary for a variable to be considered continuous. The default is `distinct(10)`.

`replace` specifies that the variables generated to form the basis function be replaced. If `basis(stub | newvarlist)` or `rescale(stub | newvarlist)` are specified, the variables named with *stub*, or those listed in *newvarlist*, are replaced. Otherwise, variables with the default names are replaced.

`norescalevars` specifies that the original values of the variables in *varlist* be used to generate the basis function. By default, variables are first rescaled to  $[0, 1]$ . `norescalevars` may not be used with *basis bspline* or *basis linear*.

`uniformknots` specifies that knots be placed at evenly spaced points over the range of each variable. The default is placement at percentiles of each of the specified variables.

`float` specifies that variables be generated as floats. Because of numerical precision and stability, the default is `double`.

`basis(stub | newvarlist)` specifies that the elements of the basis function be generated with the specified names.

When *stub* is specified, this prefix is used to generate enumerated variables for each element of the basis function.

When *newvarlist* is specified, variables with these names are generated for the elements of the basis function.

`rescale(stub | newvarlist)` specifies that the rescaled variables used to generate the basis function be stored with the specified names. This option applies only to *basis piecewise* and *basis rcs*.

When *stub* is specified, this prefix is used to generate enumerated variables for the rescaled variables.

When *newvarlist* is specified, variables with these names are generated for the rescaled variables.

## Remarks and examples

`makespline` generates new variables that form B-splines, piecewise polynomial splines, and restricted cubic splines from existing variables. Splines allow for different low-order polynomials in different regions of the original variables, and they approximate a smooth function by continuously connecting these low-order polynomials. Knots define the boundaries of the regions.

The standard piecewise polynomial variables created by `makespline` `piecewise` allow the functions to be linear, quadratic, or cubic in each region. `makespline` `linear` provides a convenient method for creating linear splines from the original variables, without rescaling. This is useful when you wish to directly interpret regression coefficients in the metric of the original variables. The terms in the standard piecewise polynomial spline function can be highly collinear and may be numerically unstable when used in estimation. B-splines, which can be created by `makespline` `bspline`, avoid this problem by creating orthogonal spline terms. For an introduction to piecewise polynomial splines and B-splines, see *Piecewise polynomial splines and B-splines* in [R] [npregress intro](#). `makespline` `rccs` creates restricted cubic splines, also known as natural splines, in which the function is linear before the first knot, cubic between adjacent knots, and linear again after the last knot. This can improve performance in the tails over the standard cubic spline.

In addition to selecting the type of spline, `makespline` allows you to specify the location of knots—the locations where the function changes. You can specify the number of knots you wish to allow, and `makespline` will place the knots based on percentiles of the data or uniformly spaced across the range of values in the data. Alternatively, you can specify the exact values at which you wish the knots to be placed.

Regardless of the type of spline, we can refer to our newly created variables as a spline basis function. A basis is a collection of terms that can approximate a smooth function arbitrarily well. A basis function, such as one of the spline functions created by `makespline`, is a subset of the basis terms that can be used to approximate the mean function.

The basis function variables generated by `makespline` are useful for nonparametric and semiparametric estimation. For instance, `makespline` can be used when we want to fit models such as

$$\mathbf{y} = \mathbf{x}_1\beta + g(\mathbf{x}_2, \mathbf{x}_3, \dots, \mathbf{x}_k) + \epsilon \quad (1)$$

In the expression above, the outcome  $\mathbf{y}$ , the covariates  $\mathbf{x}_1, \dots, \mathbf{x}_k$ , and the unobservable  $\epsilon$  are  $n \times 1$  vectors of covariates. The function  $g(\cdot)$  is unknown and  $\mathbf{x}_1$  enters the model linearly. These types of models are commonly used when we are interested in estimating the effect of  $\mathbf{x}_1$  on the mean of  $\mathbf{y}$ . We are agnostic about the functional form in which the controls,  $\mathbf{x}_2, \dots, \mathbf{x}_k$ , enter the model, but to get a precise estimate of the effect of  $\mathbf{x}_1$ , we need a reliable approximation of  $g(\cdot)$ . We may use `makespline` to generate the basis functions that best approximate  $g(\cdot)$  and then use the basis functions to fit the model in (1).

For instance, we can generate basis functions with `basis` as the stub name:

```
makespline bspline x2-x5, basis(basis)
```

This would generate a third-order B-spline basis function for each of the variables in `x2-x5`, with knots at the medians of `x2-x5`. Each of the basis functions would consist of five variables; see *Methods and formulas* in [R] [npregress series](#) for details.



The default naming convention is to give the elements of the basis function a name that starts with `_bsp` and two subscripts. The first subscript enumerates the basis functions, and the second subscript enumerates the elements within the basis function. For example, if we created basis functions for two variables, we would obtain the following:

```
. makespline bspline price mpg, replace
. describe _*
```

| Variable name         | Storage type | Display format | Value label | Variable label                  |
|-----------------------|--------------|----------------|-------------|---------------------------------|
| <code>_bsp_1_1</code> | double       | %10.0g         |             | B-spline basis term 1 for price |
| <code>_bsp_1_2</code> | double       | %10.0g         |             | B-spline basis term 2 for price |
| <code>_bsp_1_3</code> | double       | %10.0g         |             | B-spline basis term 3 for price |
| <code>_bsp_1_4</code> | double       | %10.0g         |             | B-spline basis term 4 for price |
| <code>_bsp_1_5</code> | double       | %10.0g         |             | B-spline basis term 5 for price |
| <code>_bsp_2_1</code> | double       | %10.0g         |             | B-spline basis term 1 for mpg   |
| <code>_bsp_2_2</code> | double       | %10.0g         |             | B-spline basis term 2 for mpg   |
| <code>_bsp_2_3</code> | double       | %10.0g         |             | B-spline basis term 3 for mpg   |
| <code>_bsp_2_4</code> | double       | %10.0g         |             | B-spline basis term 4 for mpg   |
| <code>_bsp_2_5</code> | double       | %10.0g         |             | B-spline basis term 5 for mpg   |

If we want to change the stub name `_bsp` to `autobasis`, we could use the `basis()` option as follows:

```
. makespline bspline price mpg, basis(autobasis)
. describe auto*
```

| Variable name              | Storage type | Display format | Value label | Variable label                  |
|----------------------------|--------------|----------------|-------------|---------------------------------|
| <code>autobasis_1_1</code> | double       | %10.0g         |             | B-spline basis term 1 for price |
| <code>autobasis_1_2</code> | double       | %10.0g         |             | B-spline basis term 2 for price |
| <code>autobasis_1_3</code> | double       | %10.0g         |             | B-spline basis term 3 for price |
| <code>autobasis_1_4</code> | double       | %10.0g         |             | B-spline basis term 4 for price |
| <code>autobasis_1_5</code> | double       | %10.0g         |             | B-spline basis term 5 for price |
| <code>autobasis_2_1</code> | double       | %10.0g         |             | B-spline basis term 1 for mpg   |
| <code>autobasis_2_2</code> | double       | %10.0g         |             | B-spline basis term 2 for mpg   |
| <code>autobasis_2_3</code> | double       | %10.0g         |             | B-spline basis term 3 for mpg   |
| <code>autobasis_2_4</code> | double       | %10.0g         |             | B-spline basis term 4 for mpg   |
| <code>autobasis_2_5</code> | double       | %10.0g         |             | B-spline basis term 5 for mpg   |

Alternatively, we could provide names for each of the variables that form a basis function. For example,

```
. makespline bspline mpg, basis(mpg1 mpg2 mpg3 mpg4 mpg5)
. describe mpg1-mpg5
```

| Variable name     | Storage type | Display format | Value label | Variable label                |
|-------------------|--------------|----------------|-------------|-------------------------------|
| <code>mpg1</code> | double       | %10.0g         |             | B-spline basis term 1 for mpg |
| <code>mpg2</code> | double       | %10.0g         |             | B-spline basis term 2 for mpg |
| <code>mpg3</code> | double       | %10.0g         |             | B-spline basis term 3 for mpg |
| <code>mpg4</code> | double       | %10.0g         |             | B-spline basis term 4 for mpg |
| <code>mpg5</code> | double       | %10.0g         |             | B-spline basis term 5 for mpg |

## ► Example 2: Generating and naming piecewise polynomial spline basis functions

Below, we generate a third-order piecewise polynomial spline with one knot at the median and show the variables we generated:

```
. makespline piecewise mpg
. describe *_sp*
```

| Variable name | Storage type | Display format | Value label | Variable label                            |
|---------------|--------------|----------------|-------------|-------------------------------------------|
| _rs_sp_1      | double       | %10.0g         |             | mpg rescaled to [0,1]                     |
| _sp_1_1       | double       | %10.0g         |             | Piecewise polynomial basis term 1 for mpg |

The only syntactical difference is that, after `makespline`, we specify `piecewise` instead of `bspline` to be the basis. `makespline` generates two variables in this case. They are the elements that are necessary to construct a basis function.

The default naming convention is to give the elements of the spline function a name that starts with `_sp` and has two subscripts. The first subscript enumerates the piecewise polynomial spline for a given variable, and the second subscript denotes the knot number. The rescaled variable starts with `_rs_sp` followed by a subscript denoting the element in the variable list.

Again, we may use a stub to modify the names that precede the subscripts, or we may specify a name for each new variable. Below, we also specified the names for the rescaled variables:

```
. makespline piecewise mpg price, basis(mpgsp pricesp) rescale(mpgrs pricers)
. describe mpgsp mpgrs pricesp pricers
```

| Variable name | Storage type | Display format | Value label | Variable label                              |
|---------------|--------------|----------------|-------------|---------------------------------------------|
| mpgsp         | double       | %10.0g         |             | Piecewise polynomial basis term 1 for mpg   |
| mpgrs         | double       | %10.0g         |             | mpg rescaled to [0,1]                       |
| pricesp       | double       | %10.0g         |             | Piecewise polynomial basis term 1 for price |
| pricers       | double       | %10.0g         |             | price rescaled to [0,1]                     |

The logic behind the variables generated is that they consist of all the elements needed to approximate the unknown function  $g(\cdot)$  of the specified variables nonparametrically. In this case, a third-order piecewise polynomial spline approximation of  $g(\cdot)$  consists of the levels, square, and cube of `mpgsp`, `mpgrs`, `pricesp`, and `pricers`. Specifically, to include the fully interacted basis functions in a model, we would need to include the term below in our specification:

```
c.(c.mpgrs##c.mpgrs##c.mpgrs mpgsp)##c.(c.pricers##c.pricers##c.pricers pricesp)
```

`makespline` simplifies this task by returning a local macro with the terms needed to fit  $g(\cdot)$ . The local macro has the name `r(regressors)`. In this case, it expands to the following:

```
. display "r(regressors)'"
c.(c.mpgrs##c.mpgrs##c.mpgrs mpgsp)##c.(c.pricers##c.pricers##c.pricers pricesp)
```

Note that when you generate basis functions for more than one variable, as we did above, `r(regressors)` fully interacts these basis functions. These fully interacted basis functions can be included when fitting a model by adding `'r(regressors)'` to your list of covariates.

### ▷ Example 3: Using makespline in semiparametric estimation

As we mentioned previously, basis functions are particularly useful for approximating unknown functions. For example, say we want to obtain the average marginal effect of  $x_1$  on the conditional mean of the continuous outcome  $y$ . We have two controls,  $x_2$  and  $x_3$ , but it is unclear whether they enter the model linearly or with another functional form.

To approximate the unknown function of  $x_2$  and  $x_3$ , we construct two B-spline basis functions with eight knots each. We use the simulated dataset and then the `makespline bspline` command:

```
. use https://www.stata-press.com/data/r19/splines, clear
(Simulated data)
. makespline bspline x2 x3, knots(8)
```

This yields basis functions with 12 elements. Once you fully interact the two basis functions, you get 168 regressors. Using all of them to approximate the unknown function would not be a sound idea. Thus, we will use `poregress` to perform partialing-out lasso linear regression. This estimator will select from the 168 covariates to provide a good approximation to the unknown function and at the same time provide a reliable estimate of the marginal effect of interest.

Rather than interact the basis terms manually, we can simply refer to the macro `r(regressors)`, which contains the full interaction of the basis functions:

```
. poregress y x1, controls('r(regressors)')
Estimating lasso for y using plugin
Estimating lasso for x1 using plugin
Partialing-out linear model
```

|  |                             |   |         |
|--|-----------------------------|---|---------|
|  | Number of obs               | = | 5,000   |
|  | Number of controls          | = | 168     |
|  | Number of selected controls | = | 19      |
|  | Wald chi2(1)                | = | 3535.78 |
|  | Prob > chi2                 | = | 0.0000  |

| y  | Robust      |           | z     | P> z  | [95% conf. interval] |          |
|----|-------------|-----------|-------|-------|----------------------|----------|
|    | Coefficient | std. err. |       |       |                      |          |
| x1 | 2.951242    | .049632   | 59.46 | 0.000 | 2.853965             | 3.048519 |

Note: Chi-squared test is a Wald test of the coefficients of the variables of interest jointly equal to zero. Lassos select controls for model estimation. Type `lassoinfo` to see number of selected variables in each lasso.

We obtain an average marginal effect of 2.95.

A researcher does not know the true value of the effect; however, we do. These are simulated data. The model is given by

$$\mathbf{y} = 3\mathbf{x}_1 + 3\sin\{3(\mathbf{x}_2 - \mathbf{x}_3)\} + \epsilon$$

The unknown function of  $\mathbf{x}_2$  and  $\mathbf{x}_3$  is complex, yet we obtained a precise estimate of the average marginal effect.

### ► Example 4: Using makespline for estimation and graphing

It is common to use linear splines to create a graph after estimation. The knots of a regressor define a piecewise polynomial that can be visualized conditional on the values of other covariates.

Below, we study the effect of mileage in miles per gallon (mpg) on car prices (price). We regress price on mpg, three linear polynomial basis terms defined by knots at the quartiles of mpg, and a dummy variable, foreign (1 if cars are foreign).

We first generate the variables that form the polynomial basis and then fit the regression.

```
. sysuse auto, clear
(1978 automobile data)
. makespline linear mpg, knots(3) basis(mpg)
. regress price mpg mpg_* i.foreign
```

| Source   | SS        | df | MS         | Number of obs | = | 74     |
|----------|-----------|----|------------|---------------|---|--------|
|          |           |    |            | F(5, 68)      | = | 13.49  |
| Model    | 316201619 | 5  | 63240323.8 | Prob > F      | = | 0.0000 |
| Residual | 318863777 | 68 | 4689173.19 | R-squared     | = | 0.4979 |
|          |           |    |            | Adj R-squared | = | 0.4610 |
| Total    | 635065396 | 73 | 8699525.97 | Root MSE      | = | 2165.4 |

| price   | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |
|---------|-------------|-----------|-------|-------|----------------------|
| mpg     | -1330.299   | 213.0425  | -6.24 | 0.000 | -1755.419 -905.1798  |
| mpg_1_1 | 1698.953    | 622.5153  | 2.73  | 0.008 | 456.7432 2941.163    |
| mpg_1_2 | -622.9298   | 651.5686  | -0.96 | 0.342 | -1923.115 677.2551   |
| mpg_1_3 | 139.4188    | 277.0762  | 0.50  | 0.616 | -413.4783 692.3158   |
| foreign |             |           |       |       |                      |
| Foreign | 1676.381    | 609.4723  | 2.75  | 0.008 | 460.1983 2892.564    |
| _cons   | 28796.47    | 3449.408  | 8.35  | 0.000 | 21913.28 35679.65    |

The regression line for our model is given by the following command:

```
generate double xb = _b[_cons] + _b[1.foreign]*foreign + ///
mpg*_b[mpg] + (mpg>18)*(mpg-18)*_b[mpg_1_1] + ///
(mpg>20)*(mpg-20)*_b[mpg_1_2] + ///
(mpg>25)*(mpg-25)*_b[mpg_1_3]
```

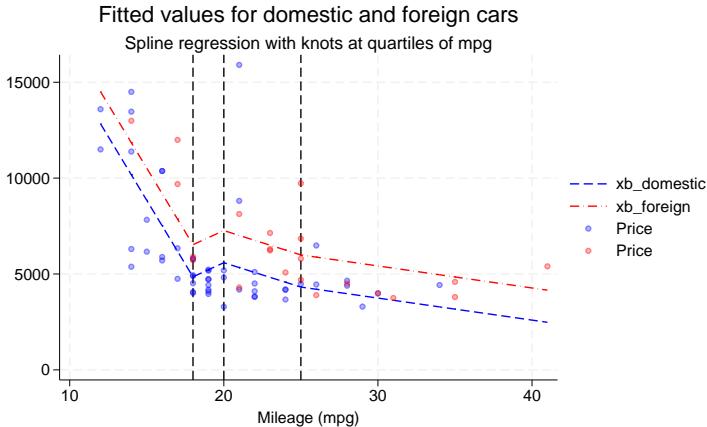
The effect of mpg changes at the knots. If mpg is less than or equal to 18, it is  $\_b[mpg]$ ; if it is greater than 18 but less than or equal to 20, it is  $(\_b[mpg] + \_b[mpg\_1\_1])$ ; if it is greater than 20 but less than or equal to 25, it is  $(\_b[mpg] + \_b[mpg\_1\_1] + \_b[mpg\_1\_2])$ ; and if it is greater than 25, it is  $(\_b[mpg] + \_b[mpg\_1\_1] + \_b[mpg\_1\_2] + \_b[mpg\_1\_3])$ .

We can plot regression lines for foreign and domestic cars. We first generate the predictions for foreign and domestic cars.

```
. generate xb_domestic = _b[_cons] + mpg*_b[mpg]
> + (mpg>18)*(mpg-18)*_b[mpg_1_1]
> + (mpg>20)*(mpg-20)*_b[mpg_1_2]
> + (mpg>25)*(mpg-25)*_b[mpg_1_3]
. generate xb_foreign = _b[_cons] + _b[1.foreign] + mpg*_b[mpg]
> + (mpg>18)*(mpg-18)*_b[mpg_1_1]
> + (mpg>20)*(mpg-20)*_b[mpg_1_2]
> + (mpg>25)*(mpg-25)*_b[mpg_1_3]
```

Then we plot both regression lines referencing the placement of the knots with vertical lines. In the graph, we also include the values of the dependent variable. We can inspect graphically how the effect of mpg differs across the regions defined by the knots.

```
. twoway line xb_domestic mpg,
> lcolor(blue) lpattern(dash) sort ||
> line xb_foreign mpg,
> lcolor(red) lpatter(dash_dot) sort ||
> scatter price mpg if foreign==0, mcolor(blue%30) ||
> scatter price mpg if foreign==1, mcolor(red%30)
> xline(18 20 25)
> title(Fitted values for domestic and foreign cars)
> subtitle(Spline regression with knots at quartiles of mpg)
```



4

## Stored results

`makespline` stores the following in `r()`:

### Scalars

|                           |                                                               |
|---------------------------|---------------------------------------------------------------|
| <code>r(N_knots)</code>   | number of knots                                               |
| <code>r(local)</code>     | 1 if <code>local</code> was specified, 0 otherwise            |
| <code>r(bsepsilon)</code> | distance from variable's boundary for B-spline knot placement |

### Macros

|                             |                                                 |
|-----------------------------|-------------------------------------------------|
| <code>r(basis)</code>       | spline type used to generate basis function     |
| <code>r(regressors)</code>  | regressors formed from basis functions          |
| <code>r(basisnames#)</code> | variable names of basis function for variable # |
| <code>r(wtype)</code>       | weight type                                     |
| <code>r(wexp)</code>        | weight expression                               |

### Matrices

|                        |                                      |
|------------------------|--------------------------------------|
| <code>r(minmax)</code> | minimum and maximum of all variables |
| <code>r(knots)</code>  | matrix of knots                      |

## Methods and formulas

See *Methods and formulas* in [R] [npregress series](#) for piecewise polynomial spline and B-spline computation.

When the `local` option is specified, let  $V_i$ ,  $i = 1, \dots, n$ , be the variables to be created;  $k_i$ ,  $i = 1, \dots, n - 1$ , be the corresponding knots; and  $\mathcal{V}$  be the original variable rescaled to be in  $[0, 1]$ . Then

$$\begin{aligned} V_1 &= \min(\mathcal{V}, k_1) \\ V_i &= \max\left\{\min(\mathcal{V}, k_i), k_{i-1}\right\} - k_{i-1} \quad i = 2, \dots, n - 1 \\ V_n &= \max(\mathcal{V}, k_{n-1}) - k_{n-1} \end{aligned}$$

When the `rcs` basis is specified, let  $k_i$ ,  $i = 1, \dots, n$ , be the knot values;  $V_i$ ,  $i = 1, \dots, n - 1$ , be the variables to be created; and  $\mathcal{V}$  be the original variable rescaled to be in  $[0, 1]$ . Then

$$\begin{aligned} V_1 &= \mathcal{V} \\ V_{i+1} &= \frac{(\mathcal{V} - k_i)_+^3 - (k_n - k_{n-1})^{-1}\{(\mathcal{V} - k_{n-1})_+^3(k_n - k_i) - (\mathcal{V} - k_n)_+^3(k_{n-1} - k_i)\}}{(k_n - k_1)^2} \\ & \quad i = 1, \dots, n - 2 \end{aligned}$$

where

$$(u)_+ = \begin{cases} u, & \text{if } u > 0 \\ 0, & \text{if } u \leq 0 \end{cases}$$

When the `harrell` option is specified, the knots are placed using the percentiles recommended in [Harrell \(2001, 23\)](#). These percentiles are based on the chosen number of knots as follows:

| No.<br>of knots | Percentiles |       |       |      |       |       |      |
|-----------------|-------------|-------|-------|------|-------|-------|------|
|                 | 10          | 50    | 90    |      |       |       |      |
| 3               | 10          | 50    | 90    |      |       |       |      |
| 4               | 5           | 35    | 65    | 95   |       |       |      |
| 5               | 5           | 27.5  | 50    | 72.5 | 95    |       |      |
| 6               | 5           | 23    | 41    | 59   | 77    | 95    |      |
| 7               | 2.5         | 18.33 | 34.17 | 50   | 65.83 | 81.67 | 97.5 |

## References

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## Also see

- [R] **npregress series** — Nonparametric series regression
- [R] **npregress series postestimation** — Postestimation tools for npregress series
- [R] **npregress intro** — Introduction to nonparametric regression
- [R] **kdensity** — Univariate kernel density estimation
- [R] **lpoly** — Kernel-weighted local polynomial smoothing

|                             |                                      |                                |                                      |
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| <a href="#">References</a>  | <a href="#">Also see</a>             |                                |                                      |

## Description

Margins are statistics calculated from predictions of a previously fit model at fixed values of some covariates and averaging or otherwise integrating over the remaining covariates.

The `margins` command estimates margins of responses for specified values of covariates and presents the results as a table.

Capabilities include estimated marginal means (also called least-squares means); adjusted predictions and adjusted treatment means; predictive margins (also called potential-outcome means); average and conditional adjusted predictions; and average and conditional marginal and partial effects, which may be reported as derivatives or as elasticities.

## Quick start

*Estimated marginal means (least-squares means)*

Estimated marginal mean of  $y$  for each level of  $a$  after `anova y a##b`  
`margins a, asbalanced`

Estimated marginal mean of  $y$  for each level of the interaction of  $a$  and  $b$  after `anova y a##b##c`  
`margins a#b, asbalanced`

Estimated marginal means of  $y_1$ ,  $y_2$ , and  $y_3$  for each level of  $a$  after `manova y1 y2 y3 = a##b`  
`margins a, asbalanced`

*Adjusted means and adjusted predictions*

Adjusted mean of  $y$  for each level of  $a$  when  $x$  is at its mean after `regress y i.a x`  
`margins a, atmeans`

Same as above, but set  $x$  to 10 rather than to its mean  
`margins a, at(x=10)`

Same as above, and also report adjusted means for  $x = 20$ ,  $x = 30$ , and  $x = 40$   
`margins a, at(x=(10(10)40))`

Adjusted predicted probability of  $y = 1$  for each level of  $a$  when  $x$  is at its mean after `logit y a##c.x`  
`margins a, atmeans`

Adjusted predicted probability for each level of the interaction of  $a$  and  $b$ , holding  $x$  at 25, after `logit y a##b##c.x`  
`margins a#b, at(x=25)`

Adjusted prediction for each level of  $a$  when  $x = 25$  and  $b = 1$   
`margins a, at(x=25 b=1)`

*Predictive margins and potential-outcome means*

Overall predictive margin, the average predicted probability of  $y = 1$ , after `logit y a##b x1 x2`  
`margins`

Predictive margins (potential-outcome means) for each level of `a`  
`margins a`

Predictive margins for `a`, when `x1` is set to 10, 20, 30, and 40  
`margins a, at(x1=(10(10)40))`

Predictive margins for levels of the interaction of `a` and `b`  
`margins a#b`

Predictive margins for `a` for all combinations of `x1 = 10, 20, 30` and `x2 = 50, 100, 150`  
`margins a, at(x1=(10(10)30) x2=(50(50)150))`

Predictive margins for `a`, first for `x1 = 10, 20, 30` with `x2` at its observed values, then for `x2 = 50, 100, 150` with `x1` at its observed values  
`margins a, at(x1=(10(10)30)) at(x2=(50(50)150))`

Predictive margins for `a` after `svy:logit y a##b x1 x2`  
`margins a, vce(unconditional)`

Average predicted probabilities of  $y = 1, y = 2, \dots$  after `mlogit y x1 x2 i.a`  
`margins`

Predictive margins for each level of `a` for each outcome of `y`  
`margins a`

*Average marginal effects and average partial effects*

Average marginal effect of `x1` on the predicted probability of  $y = 1$  after `probit y c.x1##c.x2##a` with continuous `x1` and `x2` and binary `a`  
`margins, dydx(x1)`

Average marginal effect (average partial effect) of binary `a`  
`margins, dydx(a)`

Average marginal effect of `x1` when `x2` is set to 10, 20, 30, and 40  
`margins, dydx(x1) at(x2=(10(10)40))`

Average marginal effect of `x1` when `a` is set to 0 and then to 1  
`margins a, dydx(x1)`

Average marginal effect of each variable in the model  
`margins, dydx(*)`

Average marginal effect of all variables on the truncated expected value of  $y, e(0, .)$ , after `tobit y x1 x2 x3, ll(0)`  
`margins, dydx(*) predict(e(0, .))`

Same as above, and report marginal effects for censored expected value of  $y$ ,  $y_{\text{star}}(0, .)$ , and for the linear prediction,  $xb$

```
margins, dydx(*) predict(e(0,.)) predict(ystar(0,.)) predict(xb)
```

### *Conditional marginal effects and conditional partial effects*

Marginal effect of  $x_1$  on the predicted probability of  $y = 1$ , setting all variables to their means, after `probit y c.x1#c.x2##a` with continuous  $x_1$  and  $x_2$  and binary  $a$

```
margins, dydx(x1) atmeans
```

Marginal effect (partial effect) of  $a$  when all variables are set to their means

```
margins, dydx(a) atmeans
```

Marginal effect of  $x_1$  when  $a = 0$  and  $x_1$  and  $x_2$  are set to their means

```
margins, dydx(x1) at(a=0 (mean) x1 x2)
```

Same as above

```
margins, dydx(x1) at(a=0) atmeans
```

Marginal effect of  $x_1$  when for all possible combinations of  $a = 0, 1$ ,  $x_1 = 50, 100$ , and  $x_2 = 10, 20, 30, 40$

```
margins a, dydx(x1) at(x1=(50 100) x2=(10(10)40))
```

Marginal effects of  $x_1$ ,  $x_2$ , and  $a$  with all variables set to their means

```
margins, dydx(*) atmeans
```

## Menu

Statistics > Postestimation

## Syntax

```
margins [marginlist] [if] [in] [weight] [, response_options options]
```

where *marginlist* is a list of factor variables or interactions that appear in the current estimation results. The variables may be typed with or without the *i.* prefix, and you may use any factor-variable syntax:

```
. margins i.sex i.group i.sex#i.group
. margins sex group sex#i.group
. margins sex##group
```

| <i>response_options</i>              | Description                                             |
|--------------------------------------|---------------------------------------------------------|
| Main                                 |                                                         |
| <u>predict</u> ( <i>pred_opt</i> )   | estimate margins for <i>predict</i> , <i>pred_opt</i>   |
| <u>expression</u> ( <i>pnl_exp</i> ) | estimate margins for <i>pnl_exp</i>                     |
| <u>dydx</u> ( <i>varlist</i> )       | estimate marginal effect of variables in <i>varlist</i> |
| <u>eyex</u> ( <i>varlist</i> )       | estimate elasticities of variables in <i>varlist</i>    |
| <u>dyex</u> ( <i>varlist</i> )       | estimate semielasticity— $d(\hat{y})/d(\ln x)$          |
| <u>eydx</u> ( <i>varlist</i> )       | estimate semielasticity— $d(\ln \hat{y})/d(x)$          |
| <u>continuous</u>                    | treat factor-level indicators as continuous             |

| <i>options</i>                          | Description                                                                                                        |
|-----------------------------------------|--------------------------------------------------------------------------------------------------------------------|
| <b>Main</b>                             |                                                                                                                    |
| <code>grand</code>                      | add the overall margin; default if no <i>marginlist</i>                                                            |
| <b>At</b>                               |                                                                                                                    |
| <code>at(<i>atspec</i>)</code>          | estimate margins at specified values of covariates                                                                 |
| <code>atmeans</code>                    | estimate margins at the means of covariates                                                                        |
| <code>asbalanced</code>                 | treat all factor variables as balanced                                                                             |
| <b>if/in/over</b>                       |                                                                                                                    |
| <code>over(<i>varlist</i>)</code>       | estimate margins at unique values of <i>varlist</i>                                                                |
| <code>subpop(<i>subspec</i>)</code>     | estimate margins for subpopulation                                                                                 |
| <b>Within</b>                           |                                                                                                                    |
| <code>within(<i>varlist</i>)</code>     | estimate margins at unique values of the nesting factors in <i>varlist</i>                                         |
| <b>Contrast</b>                         |                                                                                                                    |
| <code>contrast_options</code>           | any options documented in [R] <a href="#">margins, contrast</a>                                                    |
| <b>Pairwise comparisons</b>             |                                                                                                                    |
| <code>pwcompare_options</code>          | any options documented in [R] <a href="#">margins, pwcompare</a>                                                   |
| <b>SE</b>                               |                                                                                                                    |
| <code>vce(delta)</code>                 | estimate SEs using delta method; the default                                                                       |
| <code>vce(unconditional)</code>         | estimate SEs allowing for sampling of covariates                                                                   |
| <code>nose</code>                       | do not estimate SEs                                                                                                |
| <b>Advanced</b>                         |                                                                                                                    |
| <code>noweights</code>                  | ignore weights specified in estimation                                                                             |
| <code>noesample</code>                  | do not restrict margins to the estimation sample                                                                   |
| <code>emptycells(<i>empspec</i>)</code> | treatment of empty cells for balanced factors                                                                      |
| <code>estimtolerance(<i>tol</i>)</code> | specify numerical tolerance used to determine estimable functions;<br>default is <code>estimtolerance(1e-5)</code> |
| <code>noestimcheck</code>               | suppress estimability checks                                                                                       |
| <code>force</code>                      | estimate margins despite potential problems                                                                        |
| <code>chainrule</code>                  | use the chain rule when computing derivatives                                                                      |
| <code>nochainrule</code>                | do not use the chain rule                                                                                          |
| <b>Reporting</b>                        |                                                                                                                    |
| <code>level(#)</code>                   | set confidence level; default is <code>level(95)</code>                                                            |
| <code>mcompare(<i>method</i>)</code>    | adjust for multiple comparisons; default is <code>mcompare(noadjust)</code>                                        |
| <code>noatlegend</code>                 | suppress legend of fixed covariate values                                                                          |
| <code>post</code>                       | post margins and their VCE as estimation results                                                                   |
| <code>display_options</code>            | control columns and column formats, row spacing, line width and<br>factor-variable labeling                        |
| <code>df(#)</code>                      | use <i>t</i> distribution with # degrees of freedom for computing <i>p</i> -values<br>and confidence intervals     |

---

| <i>method</i>                       | Description                                         |
|-------------------------------------|-----------------------------------------------------|
| <code>noadjust</code>               | do not adjust for multiple comparisons; the default |
| <code>bonferroni [adjustall]</code> | Bonferroni's method; adjust across all terms        |
| <code>sidak [adjustall]</code>      | Šidák's method; adjust across all terms             |
| <code>scheffe</code>                | Scheffé's method                                    |

Time-series operators are allowed if they were used in the estimation.

See `at()` under *Options* for a description of *atspec*.

`collect` is allowed; see [U] 11.1.10 **Prefix commands**.

`fweights`, `awweights`, `iweights`, and `pweights` are allowed; see [U] 11.1.6 **weight**.

`df(#)` does not appear in the dialog box.

## Options

*Warning: The option descriptions are brief and use jargon. Skip to [Remarks and examples](#) if you are reading about margins for the first time.*

Main

`predict(pred_opt)` and `expression(pnl_exp)` are mutually exclusive; they specify the response. If neither is specified, the response will be the default prediction that would be produced by `predict` after the underlying estimation command. Some estimation commands, such as `mlogit`, document a different default prediction for margins than for `predict`.

`predict(pred_opt)` specifies the option(s) to be specified with the `predict` command to produce the variable that will be used as the response. After estimation by `logistic`, you could specify `predict(xb)` to obtain linear predictions rather than the `predict` command's default, the probabilities.

Multiple `predict()` options can be specified to compute margins of multiple predictions simultaneously.

`expression(pnl_exp)` specifies the response as an expression. See [R] [predictnl](#) for a full description of `pnl_exp`. After estimation by `logistic`, you might specify `expression(exp(predict(xb)))` to use relative odds rather than probabilities as the response. For examples, see [Example 12: Margins of a specified expression](#).

`dydx(varlist)`, `eyex(varlist)`, `dyex(varlist)`, and `eydx(varlist)` request that margins report derivatives of the response with respect to `varlist` rather than on the response itself. `eyex()`, `dyex()`, and `eydx()` report derivatives as elasticities; see [Expressing derivatives as elasticities](#).

`continuous` is relevant only when one of `dydx()` or `eydx()` is also specified. It specifies that the levels of factor variables be treated as continuous; see [Derivatives versus discrete differences](#). This option is implied if there is a single-level factor variable specified in `dydx()` or `eydx()`.

`grand` specifies that the overall margin be reported. `grand` is assumed when `marginlist` is empty.

At

`at(atspec)` specifies values for covariates to be treated as fixed. By default, margins are calculated using observed values of covariates.

`at(age=20)` fixes covariate `age` to the value specified. `at()` may be used to fix continuous or factor covariates.

`at(age=20 sex=1)` simultaneously fixes covariates `age` and `sex` at the values specified.

`at(age=(20 30 40 50))` fixes `age` first at 20, then at 30, .... `margins` produces separate results for each specified value.

`at(age=(20(10)50))` does the same as `at(age=(20 30 40 50))`; that is, you may specify a numlist.

`at((mean) age (median) distance)` fixes the covariates at the summary statistics specified.

`at((p25) _all)` fixes all covariates at their 25th percentile values. See [Syntax of at\(\)](#) for the full list of summary-statistic modifiers.

`at((mean) _all (median) x x2=1.2 z=(1 2 3))` is processed from general to specific, with settings for named covariates overriding general settings specified via `_all`. Thus, all covariates are fixed at their means except for `x` (fixed at its median), `x2` (fixed at 1.2), and `z` (fixed first at 1, then at 2, and finally at 3).

`at((means) _all (asobserved) x2)` is a convenient way to set all covariates except `x2` to the mean.

Multiple `at()` options can be specified, and each will produce a different set of margins.

See [Syntax of at\(\)](#) for more information.

`atmeans` specifies that covariates be fixed at their means and is shorthand for `at((mean) _all)`. `atmeans` differs from `at((mean) _all)` in that `atmeans` will affect subsequent `at()` options. For instance,

```
. margins ..., atmeans at((p25) x) at((p75) x)
```

produces two sets of margins with both sets evaluated at the means of all covariates except `x`.

`asbalanced` is shorthand for `at((asbalanced) _factor)` and specifies that factor covariates be evaluated as though there were an equal number of observations in each level; see [Obtaining margins as though the data were balanced](#). `asbalanced` differs from `at((asbalanced) _factor)` in that `asbalanced` will affect subsequent `at()` options in the same way as `atmeans` does.

When `asbalanced` is combined with the `atmeans` option, factor variables are treated as balanced, while continuous covariates are fixed at their means. The resulting margin is known as an [adjusted treatment mean](#).

if/in/over

`over(varlist)` specifies that separate sets of margins be estimated for the groups defined by `varlist`. The variables in `varlist` must contain nonnegative integer (or missing) values. The variables need not be covariates in your model. When `over()` is combined with the `vce(unconditional)` option, each group is treated as a subpopulation; see [\[SVY\] Subpopulation estimation](#).

`subpop([varname] [if])` is intended for use with the `vce(unconditional)` option. It specifies that margins be estimated for the single subpopulation identified by the indicator variable or by the `if` expression or by both. Zero or missing indicates that the observation be excluded; nonzero or non-

missing, that it be included. See [SVY] [Subpopulation estimation](#) for why `subpop()` is preferred to `if` expressions and `in` ranges when also using `vce(unconditional)`. If `subpop()` is used without `vce(unconditional)`, it is treated merely as an additional `if` qualifier.

#### Within

`within(varlist)` allows for nested designs. `varlist` contains the nesting variable(s) over which margins are to be estimated. See [Obtaining margins with nested designs](#). As with `over(varlist)`, when `within(varlist)` is combined with `vce(unconditional)`, each level of the variables in `varlist` is treated as a subpopulation.

#### Contrast

`contrast_options` are any of the options documented in [R] [margins, contrast](#).

#### Pairwise comparisons

`pwcompare_options` are any of the options documented in [R] [margins, pwcompare](#).

#### SE

`vce(delta)` and `vce(unconditional)` specify how the VCE and, correspondingly, standard errors are calculated.

`vce(delta)` is the default. The delta method is applied to the formula for the response and the VCE of the estimation command. This method assumes that values of the covariates used to calculate the response are given or, if all covariates are not fixed using `at()`, that the data are given.

`vce(unconditional)` specifies that the covariates that are not fixed be treated in a way that accounts for their having been sampled. The VCE is estimated using the linearization method. This method allows for heteroskedasticity or other violations of distributional assumptions and allows for correlation among the observations in the same manner as `vce(robust)` and `vce(cluster...)`, which may have been specified with the estimation command. This method also accounts for complex survey designs if the data are `svyset`. See [Obtaining margins with survey data and representative samples](#). When you use complex survey data, this method requires that the linearized variance estimation method be used for the model. See [SVY] [svy postestimation](#) for an example of margins with replication-based methods.

`nose` suppresses calculation of the VCE and standard errors. See [Requirements for model specification](#) for an example of the use of this option.

#### Advanced

`noweights` specifies that any weights specified on the previous estimation command be ignored by `margins`. By default, `margins` uses the weights specified on the estimator to average responses and to compute summary statistics. If weights are specified on the `margins` command, they override previously specified weights, making it unnecessary to specify `noweights`. The `noweights` option is not allowed after `svy:` estimation when the `vce(unconditional)` option is specified.

For multilevel models, such as `meglm`, the default behavior is to construct a single weight value for each observation by multiplying the corresponding multilevel weights within the given observation.

`noesample` specifies that `margins` not restrict its computations to the estimation sample used by the previous estimation command. See [Example 15: Margins evaluated out of sample](#).

With the default delta-method VCE, `noesample` margins may be estimated on samples other than the estimation sample; such results are valid under the assumption that the data used are treated as being given.

You can specify `noesample` and `vce(unconditional)` together, but if you do, you should be sure that the data in memory correspond to the original `e(sample)`. To show that you understand that, you must also specify the `force` option. Be aware that making the `vce(unconditional)` calculation on a sample different from the estimation sample would be equivalent to estimating the coefficients on one set of data and computing the scores used by the linearization on another set; see [P] [\\_robust](#).

`emptycells(strict)` and `emptycells(reweight)` are relevant only when the `asbalanced` option is also specified. `emptycells()` specifies how empty cells are handled in interactions involving factor variables that are being treated as balanced; see [Obtaining margins as though the data were balanced](#).

`emptycells(strict)` is the default; it specifies that margins involving empty cells be treated as not estimable.

`emptycells(reweight)` specifies that the effects of the observed cells be increased to accommodate any missing cells. This makes the margin estimable but changes its interpretation. `emptycells(reweight)` is implied when the `within()` option is specified.

`estimtolerance(tol)` specifies the numerical tolerance used to determine estimable functions. The default is `estimtolerance(1e-5)`.

A linear combination of the model coefficients  $z$  is found to be not estimable if

$$\text{mreldif}(z, z \times H) > \text{tol}$$

where  $H$  is defined in [Methods and formulas](#).

`noestimcheck` specifies that margins not check for estimability. By default, the requested margins are checked and those found not estimable are reported as such. Nonestimability is usually caused by empty cells. If `noestimcheck` is specified, estimates are computed in the usual way and reported even though the resulting estimates are manipulable, which is to say they can differ across equivalent models having different parameterizations. See [Estimability of margins](#).

`force` instructs margins to proceed in some situations where it would otherwise issue an error message because of apparent violations of assumptions. Do not be casual about specifying `force`. You need to understand and fully evaluate the statistical issues. For an example of the use of `force`, see [Using margins after the estimates use command](#).

`chainrule` and `nochainrule` specify whether margins uses the chain rule when numerically computing derivatives. You need not specify these options when using margins after any official Stata estimator; margins will choose the appropriate method automatically.

Specify `nochainrule` after estimation by a community-contributed command. We recommend using `nochainrule`, even though `chainrule` is usually safe and is always faster. `nochainrule` is safer because it makes no assumptions about how the parameters and covariates join to form the response.

`nochainrule` is implied when the `expression()` option is specified.

#### Reporting

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [U] [20.8 Specifying the width of confidence intervals](#).

`mcompare(method)` specifies the method for computing  $p$ -values and confidence intervals that account for multiple comparisons within a factor-variable term.

Most methods adjust the comparisonwise error rate,  $\alpha_c$ , to achieve a prespecified experimentwise error rate,  $\alpha_e$ .

`mcompare(noadjust)` is the default; it specifies no adjustment.

$$\alpha_c = \alpha_e$$

`mcompare(bonferroni)` adjusts the comparisonwise error rate based on the upper limit of the Bonferroni inequality

$$\alpha_e \leq m\alpha_c$$

where  $m$  is the number of comparisons within the term.

The adjusted comparisonwise error rate is

$$\alpha_c = \alpha_e/m$$

`mcompare(sidak)` adjusts the comparisonwise error rate based on the upper limit of the probability inequality

$$\alpha_e \leq 1 - (1 - \alpha_c)^m$$

where  $m$  is the number of comparisons within the term.

The adjusted comparisonwise error rate is

$$\alpha_c = 1 - (1 - \alpha_e)^{1/m}$$

This adjustment is exact when the  $m$  comparisons are independent.

`mcompare(scheffe)` controls the experimentwise error rate using the  $F$  or  $\chi^2$  distribution with degrees of freedom equal to the rank of the term.

`mcompare(method adjustall)` specifies that the multiple-comparison adjustments count all comparisons across all terms rather than performing multiple comparisons term by term. This leads to more conservative adjustments when multiple variables or terms are specified in *marginslist*. This option is compatible only with the `bonferroni` and `sidak` methods.

`noatlegend` specifies that the legend showing the fixed values of covariates be suppressed.

`post` causes `margins` to behave like a Stata estimation (e-class) command. `margins` posts the vector of estimated margins along with the estimated variance–covariance matrix to `e()`, so you can treat the estimated margins just as you would results from any other estimation command. For example, you could use `test` to perform simultaneous tests of hypotheses on the margins, or you could use `lincom` to create linear combinations. See [Example 10: Testing margins—contrasts of margins](#).

*display\_options*: `noci`, `nopvalues`, `vsquish`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `noIstretch`.

`noci` suppresses confidence intervals from being reported in the coefficient table.

`nopvalues` suppresses  $p$ -values and their test statistics from being reported in the coefficient table.

`vsquish` specifies that the blank space separating factor-variable terms or time-series–operated variables from other variables in the model be suppressed.

`nofvlabel` displays factor-variable level values rather than attached value labels. This option overrides the `fvlabel` setting; see [\[R\] set showbaselevels](#).

`fvwrap(#)` allows long value labels to wrap the first  $\#$  lines in the coefficient table. This option overrides the `fvwrap` setting; see [\[R\] set showbaselevels](#).

`fvwrapon(style)` specifies whether value labels that wrap will break at word boundaries or break based on available space.

`fvwrapon(word)`, the default, specifies that value labels break at word boundaries.

`fvwrapon(width)` specifies that value labels break based on available space.

This option overrides the `fvwrapon` setting; see [R] [set showbaselevels](#).

`cformat(%fmt)` specifies how to format margins, standard errors, and confidence limits in the table of estimated margins.

`pformat(%fmt)` specifies how to format  $p$ -values in the table of estimated margins.

`sformat(%fmt)` specifies how to format test statistics in the table of estimated margins.

`no1stretch` specifies that the width of the table of estimated margins not be automatically widened to accommodate longer variable names. The default, `1stretch`, is to automatically widen the table of estimated margins up to the width of the Results window. Specifying `1stretch` or `no1stretch` overrides the setting given by [set 1stretch](#). If `set 1stretch` has not been set, the default is `1stretch`. `no1stretch` is not shown in the dialog box.

The following option is available with `margins` but is not shown in the dialog box:

`df(#)` specifies that the  $t$  distribution with  $\#$  degrees of freedom be used for computing  $p$ -values and confidence intervals. The default typically is to use the standard normal distribution. However, if the estimation command computes the residual degrees of freedom (`e(df_r)`) and `predict(xb)` is specified with `margins`, the default is to use the  $t$  distribution with `e(df_r)` degrees of freedom.

## Remarks and examples

Remarks are presented under the following headings:

### Introduction

#### Obtaining margins of responses

Example 1: A simple case after regress

Example 2: A simple case after logistic

Example 3: Average response versus response at average

Example 4: Multiple margins from one command

Example 5: Margins with interaction terms

Example 6: Margins with continuous variables

Example 7: Margins of continuous variables

Example 8: Margins of interactions

Example 9: Decomposing margins

Example 10: Testing margins—contrasts of margins

Example 11: Margins of a specified prediction

Example 12: Margins of a specified expression

Example 13: Margins with multiple outcomes (responses)

Example 14: Margins with multiple equations

Example 15: Margins evaluated out of sample

#### Obtaining margins of derivatives of responses (a.k.a. marginal effects)

Use `at()` freely, especially with continuous variables

Expressing derivatives as elasticities

Derivatives versus discrete differences

Example 16: Average marginal effect (partial effects)

Example 17: Average marginal effect of all covariates

Example 18: Evaluating marginal effects over the response surface

#### Obtaining margins with survey data and representative samples

Example 19: Inferences for populations, margins of response

Example 20: Inferences for populations, marginal effects

*Example 21: Inferences for populations with svyset data*

- Standardizing margins*
- Obtaining margins as though the data were balanced*
  - Balancing using asbalanced*
  - Balancing by standardization*
  - Balancing nonlinear responses*
  - Treating a subset of covariates as balanced*
  - Balancing in the presence of continuous covariates*
  - Using fvset design*
  - Balancing in the presence of empty cells*
- Obtaining margins with nested designs*
  - Introduction to nested designs*
  - Margins with nested designs as though the data were balanced*
  - Coding of nested designs*
- Special topics*
  - Requirements for model specification*
  - Estimability of margins*
  - Manipulability of tests*
  - Using margins after the estimates use command*
  - Syntax of at()*
  - Estimation commands that may be used with margins*
- Video examples*
- Glossary*

## Introduction

`margins` is a postestimation command, a command for use after you have fit a model using an estimation command such as `regress` or `logistic`, or using almost any other estimation command.

`margins` estimates and reports margins of responses and margins of derivatives of responses, also known as marginal effects. A margin is a statistic based on a fitted model in which some of or all the covariates are fixed. Marginal effects are changes in the response for change in a covariate, which can be reported as a derivative, elasticity, or semielasticity.

For a brief overview of `margins`, see [Williams \(2012\)](#).

## Obtaining margins of responses

What we call margins of responses are also known as predictive margins, adjusted predictions, and recycled predictions. When applied to [balanced data](#), margins of responses are also called [estimated marginal means](#) and least-squares means.

A margin is a statistic based on a fitted model calculated over a dataset in which some of or all the covariates are fixed at values different from what they really are. For instance, after a linear regression fit on males and females, the marginal mean (margin of mean) for males is the predicted mean of the dependent variable, where every observation is treated as if it represents a male; thus, those observations that in fact do represent males are included, as well as those observations that represent females. The marginal mean for females would be similarly obtained by treating all observations as if they represented females.

In making the calculation, `sex` is treated as male or female everywhere it appears in the model. The model might be

```
. regress y age bp i.sex sex#c.age sex#c.bp
```

and then, in making the marginal calculation of the mean for males and females, `margins` not only accounts for the direct effect of `i.sex` but also for the indirect effects of `sex#c.age` and `sex#c.bp`.

The response being margined can be any statistic produced by `[R] predict`, or any expression of those statistics.

Standard errors are obtained by the delta method, at least by default. The delta method assumes that the values at which the covariates are evaluated to obtain the marginal responses are fixed. When your sample represents a population, whether you are using `svy` or not (see `[SVY] svy`), you can specify `margins' vce(unconditional)` option and `margins` will produce standard errors that account for the sampling variability of the covariates. Some researchers reserve the term predictive margins to describe this.

The best way to understand margins is to see some examples. You can run the following examples yourself if you type

```
. use https://www.stata-press.com/data/r19/margex
(Artificial data for margins)
```

### Example 1: A simple case after regress

```
. regress y i.sex i.group
(output omitted)
. margins sex
Predictive margins Number of obs = 3,000
Model VCE: OLS
Expression: Linear prediction, predict()
```

|        | Margin   | Delta-method<br>std. err. | t      | P> t  | [95% conf. interval] |          |
|--------|----------|---------------------------|--------|-------|----------------------|----------|
| sex    |          |                           |        |       |                      |          |
| Male   | 60.56034 | .5781782                  | 104.74 | 0.000 | 59.42668             | 61.69401 |
| Female | 78.88236 | .5772578                  | 136.65 | 0.000 | 77.7505              | 80.01422 |

The numbers reported in the Margin column are average values of the linear prediction of `y`, as noted above the output table after `Expression:.` Based on a linear regression of `y` on `sex` and `group`, 60.6 would be the average value of the linear prediction of `y` if everyone in the data were treated as if they were male, and 78.9 would be the average value if everyone were treated as if they were female.

### Example 2: A simple case after logistic

`margins` may be used after almost any estimation command.

```
. logistic outcome i.sex i.group
(output omitted)
. margins sex
Predictive margins Number of obs = 3,000
Model VCE: OIM
Expression: Pr(outcome), predict()
```

|        | Margin   | Delta-method<br>std. err. | z     | P> z  | [95% conf. interval] |          |
|--------|----------|---------------------------|-------|-------|----------------------|----------|
| sex    |          |                           |       |       |                      |          |
| Male   | .1286796 | .0111424                  | 11.55 | 0.000 | .106841              | .1505182 |
| Female | .1905087 | .0089719                  | 21.23 | 0.000 | .1729241             | .2080933 |

The numbers reported in the Margin column are average predicted probabilities. Based on a logistic regression of outcome on sex and group, 0.13 would be the average probability of outcome if everyone in the data were treated as if they were male, and 0.19 would be the average probability if everyone were treated as if they were female.

margins reports average values after regress and average probabilities after logistic. By default, margins makes tables of whatever it is that predict (see [R] predict) predicts by default. Alternatively, margins can make tables of anything that predict can produce if you use margins' predict() option; see *Example 11: Margins of a specified prediction*.

### Example 3: Average response versus response at average

In *example 2*, margins reported average probabilities of outcome for sex = 0 and sex = 1. If we instead wanted the predicted probabilities evaluated at the mean of the covariates, we would specify margins' atmeans option. We previously typed

```
. logistic outcome i.sex i.group
(output omitted)
. margins sex
(output omitted)
```

and now we type

```
. margins sex, atmeans
Adjusted predictions Number of obs = 3,000
Model VCE: OIM
Expression: Pr(outcome), predict()
At: 1.group = .3996667 (mean)
 2.group = .3726667 (mean)
 3.group = .2276667 (mean)
```

|        | Delta-method |           |       |       |                      |
|--------|--------------|-----------|-------|-------|----------------------|
|        | Margin       | std. err. | z     | P> z  | [95% conf. interval] |
| sex    |              |           |       |       |                      |
| Male   | .0966105     | .0089561  | 10.79 | 0.000 | .0790569 .1141641    |
| Female | .1508362     | .0118064  | 12.78 | 0.000 | .127696 .1739764     |

The average of the predictions, which we calculated in *example 2*, is different from the prediction at the average of the covariates that we calculate here. The first is the average of the probabilities among actual persons in the data. The second is the expected probability of a person with “average” characteristics; in this case, that person is 40% in group one, 37% in group two, and 23% in group three. Such a person cannot exist because membership in each group is exclusive, but margins uses regression coefficients to calculate the predicted response of the mythical “average” individual.

When you specify the atmeans or at() option, margins reports the values used for the covariates in the legend above the table. margins lists the values for all the covariates, including values it may not use, in the results that follow. In this example, margins reported means for sex even though those means were not used. They were not used because we asked for the margins of sex, so sex was fixed first at 0 and then at 1.

If you wish to suppress this legend, specify the nolegend option.

### Example 4: Multiple margins from one command

More than one margin can be reported by just one margins command. You can type

```
. margins sex group
```

and doing that is equivalent in terms of the output to typing

```
. margins sex
. margins group
```

When multiple margins are requested on the same command, each is estimated separately. There is, however, a difference when you also specify margins' post option. Then, the variance–covariance matrix for all margins requested is posted, and that is what allows you to test, for example, equality of margins. Testing equality of margins is covered in [Example 10: Testing margins—contrasts of margins](#).

In any case, below we request margins for sex and for group.

```
. margins sex group
Predictive margins Number of obs = 3,000
Model VCE: OIM
Expression: Pr(outcome), predict()
```

|              | Delta-method |           | z     | P> z  | [95% conf. interval] |          |
|--------------|--------------|-----------|-------|-------|----------------------|----------|
|              | Margin       | std. err. |       |       |                      |          |
| <b>sex</b>   |              |           |       |       |                      |          |
| Male         | .1286796     | .0111424  | 11.55 | 0.000 | .106841              | .1505182 |
| Female       | .1905087     | .0089719  | 21.23 | 0.000 | .1729241             | .2080933 |
| <b>group</b> |              |           |       |       |                      |          |
| 1            | .2826207     | .0146234  | 19.33 | 0.000 | .2539593             | .311282  |
| 2            | .1074814     | .0094901  | 11.33 | 0.000 | .0888812             | .1260817 |
| 3            | .0291065     | .0073417  | 3.96  | 0.000 | .0147169             | .043496  |

### Example 5: Margins with interaction terms

The estimation command on which margins bases its calculations may contain interaction terms, such as an interaction of `sex` and `group`:

```
. logistic outcome i.sex i.group sex#group
 (output omitted)
. margins sex group
Predictive margins Number of obs = 3,000
Model VCE: OIM
Expression: Pr(outcome), predict()
```

|              | Delta-method |           | z     | P> z  | [95% conf. interval] |          |
|--------------|--------------|-----------|-------|-------|----------------------|----------|
|              | Margin       | std. err. |       |       |                      |          |
| <b>sex</b>   |              |           |       |       |                      |          |
| Male         | .1561738     | .0132774  | 11.76 | 0.000 | .1301506             | .182197  |
| Female       | .1983749     | .0101546  | 19.54 | 0.000 | .1784723             | .2182776 |
| <b>group</b> |              |           |       |       |                      |          |
| 1            | .3211001     | .0176403  | 18.20 | 0.000 | .2865257             | .3556744 |
| 2            | .1152127     | .0099854  | 11.54 | 0.000 | .0956417             | .1347838 |
| 3            | .0265018     | .0109802  | 2.41  | 0.016 | .0049811             | .0480226 |

We fit the model by typing `logistic outcome i.sex i.group sex#group`, but the meaning would have been the same had we typed `logistic outcome sex##group`.

As mentioned in [example 4](#), the results for `sex` and the results for `group` are calculated independently, and we would have obtained the same results had we typed `margins sex` followed by `margins group`.

The margin for male (`sex = 0`) is 0.16. The probability 0.16 is the average probability if everyone in the data were treated as if `sex = 0`, including `sex = 0` in the main effect and `sex = 0` in the interaction of `sex` with `group`.

Had we specified `margins sex, atmeans`, we would have obtained not average probabilities but the probabilities evaluated at the average. Rather than obtaining 0.16, we would have obtained 0.10 for `sex = 0`. The 0.10 is calculated by taking the fitted model, plugging in `sex = 0` everywhere, and plugging in the average value of the group indicator variables everywhere they are used. That is, rather than treating the group indicators as being (1, 0, 0), (0, 1, 0), or (0, 0, 1) depending on observation, the group indicators are treated as being (0.40, 0.37, 0.23), which are the average values of `group = 1`, `group = 2`, and `group = 3`.

**Example 6: Margins with continuous variables**

To the [above example](#), we will add the continuous covariate age to the model and then rerun margins sex group.

```
. logistic outcome i.sex i.group sex#group age
 (output omitted)
. margins sex group
Predictive margins Number of obs = 3,000
Model VCE: OIM
Expression: Pr(outcome), predict()
```

|              | Delta-method |           | z     | P> z  | [95% conf. interval] |          |
|--------------|--------------|-----------|-------|-------|----------------------|----------|
|              | Margin       | std. err. |       |       |                      |          |
| <b>sex</b>   |              |           |       |       |                      |          |
| Male         | .1600644     | .0125653  | 12.74 | 0.000 | .1354368             | .184692  |
| Female       | .1966902     | .0100043  | 19.66 | 0.000 | .1770821             | .2162983 |
| <b>group</b> |              |           |       |       |                      |          |
| 1            | .2251302     | .0123233  | 18.27 | 0.000 | .200977              | .2492834 |
| 2            | .150603      | .0116505  | 12.93 | 0.000 | .1277685             | .1734376 |
| 3            | .0736157     | .0337256  | 2.18  | 0.029 | .0075147             | .1397167 |

Compared with the results presented in [example 5](#), results for sex change little, but results for groups 1 and 3 change markedly. The tables differ because now we are adjusting for the continuous covariate age, as well as for sex and group.

We will continue examining interactions in [example 8](#). Because we have added a continuous variable, let's take a detour to explain how to obtain margins for continuous variables and to explain their interpretation.

**Example 7: Margins of continuous variables**

Continuing with our example of

```
. logistic outcome i.sex i.group sex#group age
```

let's examine the continuous covariate age.

You are not allowed to type `margins age`; doing that will produce an error:

```
. margins age
factor age not found in list of covariates
r(322);
```

The message “age not found in list of covariates” is `margins`' way of saying, “Yes, age might be in the model, but if it is, it is not included as a factor variable; it is in as a continuous variable.” Sometimes, Stata is overly terse. `margins` might also say that because age is continuous there are an infinite number of values at which it could evaluate the margins. At what value(s) should age be fixed? `margins` requires more guidance with continuous covariates. We can provide that guidance by using the `at()` option and typing

```
. margins, at(age=40)
```

To understand why that yields the desired result, let us tell you that if you were to type

```
. margins
```

`margins` would report the overall margin—the margin that holds nothing constant. Because our model is logistic, the average value of the predicted probabilities would be reported. The `at()` option fixes one or more covariates to the value(s) specified and can be used with both factor and continuous variables. Thus, if you typed `margins, at(age=40)`, then `margins` would average over the data the responses for everybody, setting `age=40`. Here is what happens when you type that:

```
. margins, at(age=40)
Predictive margins Number of obs = 3,000
Model VCE: OIM
Expression: Pr(outcome), predict()
At: age = 40
```

|       | Delta-method |           |       |       |                      |
|-------|--------------|-----------|-------|-------|----------------------|
|       | Margin       | std. err. | z     | P> z  | [95% conf. interval] |
| _cons | .1133603     | .0070731  | 16.03 | 0.000 | .0994972 .1272234    |

Reported is the margin for `age = 40`, adjusted for the other covariates in our model.

If we wanted to obtain the margins for age 30, 35, 40, 45, and 50, we could type

```
. margins, at(age=(30 35 40 45 50))
```

or, equivalently,

```
. margins, at(age=(30(5)50))
```

### Example 8: Margins of interactions

Our model is

```
. logistic outcome i.sex i.group sex#group age
```

We can obtain the margins of all possible combinations of the levels of sex and the levels of group by typing

```
. margins sex#group
Predictive margins Number of obs = 3,000
Model VCE: OIM
Expression: Pr(outcome), predict()
```

|           | Delta-method |           |       |       | [95% conf. interval] |          |
|-----------|--------------|-----------|-------|-------|----------------------|----------|
|           | Margin       | std. err. | z     | P> z  |                      |          |
| sex#group |              |           |       |       |                      |          |
| Male#1    | .2379605     | .0237178  | 10.03 | 0.000 | .1914745             | .2844465 |
| Male#2    | .0658294     | .0105278  | 6.25  | 0.000 | .0451953             | .0864636 |
| Male#3    | .0538001     | .0136561  | 3.94  | 0.000 | .0270347             | .0805656 |
| Female#1  | .2158632     | .0112968  | 19.11 | 0.000 | .1937218             | .2380045 |
| Female#2  | .2054406     | .0183486  | 11.20 | 0.000 | .1694781             | .2414032 |
| Female#3  | .085448      | .0533914  | 1.60  | 0.110 | -.0191973            | .1900932 |

The first line in the table reports the marginal probability for sex = 0 (male) and group = 1. That is, it reports the estimated probability if everyone in the data were treated as if they were sex = 0 and group = 1.

Also reported are all the other combinations of sex and group.

By the way, we could have typed margins sex#group even if our fitted model did not include sex#group. Estimation is one thing, and asking questions about the nature of the estimates is another. margins does, however, require that i.sex and i.group appear somewhere in the model, because fixing a value outside the model would just produce the grand margin, and you can separately ask for that if you want it by typing margins without arguments.

### Example 9: Decomposing margins

We have the model

```
. logistic outcome i.sex i.group sex#group age
```

In [example 6](#), we typed `margins sex` and obtained 0.160 for males and 0.197 for females. We are going to decompose each of those numbers. Let us explain:

1. The margin for males, 0.160, treats everyone as if they were male, and that amounts to simultaneously
  - (a) treating males as males and
  - (b) treating females as males.
2. The margin for females, 0.197, treats everyone as if they were female, and that amounts to simultaneously
  - (a) treating males as females and
  - (b) treating females as females.

The margins 1a and 1b are the decomposition of 1, and the margins 2a and 2b are the decomposition of 2.

We could obtain 1a and 2a by typing

```
. margins if sex==0, at(sex=(0 1))
```

because the qualifier `if sex==0` would restrict margins to running on only the males. Similarly, we could obtain 1b and 2b by typing

```
. margins if sex==1, at(sex=(0 1))
```

We run these examples below:

```
. margins if sex==0, at(sex=(0 1))
```

```
Predictive margins Number of obs = 1,498
Model VCE: OIM
Expression: Pr(outcome), predict()
1._at: sex = 0
2._at: sex = 1
```

|     | Delta-method |           |       |       |                      |
|-----|--------------|-----------|-------|-------|----------------------|
|     | Margin       | std. err. | z     | P> z  | [95% conf. interval] |
| _at |              |           |       |       |                      |
| 1   | .0794393     | .0062147  | 12.78 | 0.000 | .0672586 .0916199    |
| 2   | .1335584     | .0127351  | 10.49 | 0.000 | .1085981 .1585187    |

```
. margins if sex==1, at(sex=(0 1))
Predictive margins Number of obs = 1,502
Model VCE: OIM
Expression: Pr(outcome), predict()
1. _at: sex = 0
2. _at: sex = 1
```

|     | Delta-method |           |       |       | [95% conf. interval] |          |
|-----|--------------|-----------|-------|-------|----------------------|----------|
|     | Margin       | std. err. | z     | P> z  |                      |          |
| _at |              |           |       |       |                      |          |
| 1   | .2404749     | .0199709  | 12.04 | 0.000 | .2013326             | .2796171 |
| 2   | .2596538     | .0104756  | 24.79 | 0.000 | .2391219             | .2801857 |

Putting together the results from [example 6](#) and the results above, we have

|                                           |       |
|-------------------------------------------|-------|
| Margin treating individuals as themselves | 0.170 |
| Margin treating individuals as male       | 0.160 |
| Margin treating male as male              | 0.079 |
| Margin treating female as male            | 0.240 |
| Margin treating individuals as female     | 0.197 |
| Margin treating male as female            | 0.134 |
| Margin treating female as female          | 0.260 |

### Example 10: Testing margins—contrasts of margins

Continuing with the previous example, it would be interesting to test the equality of [2b](#) and [1b](#), to test whether the average probability of a positive outcome for females treated as females is equal to that for females treated as males. That test would be different from testing the overall significance of `sex` in our model. The test performed on our model would be a test of whether the probability of a positive outcome differs between males and females when they have equal values of the other covariates. The test of equality of margins is a test of whether the *average* probabilities differ given the different pattern of values of the other covariates that the two sexes have in our data.

We can also perform such tests by treating the results from `margins` as estimation results. There are three steps required to perform tests on margins. First, you must arrange it so that all the margins of interest are reported by just one `margins` command. Second, you must specify `margins`' post option. Third, you perform the test with the `test` command.

Such tests and comparisons can be readily performed by contrasting margins; see [\[R\] margins, contrast](#). Also see *Contrasts of margins—effects (discrete marginal effects)* in [\[R\] marginsplot](#).

In the [previous example](#), we used two commands to obtain our results, namely,

```
. margins if sex==0, at(sex=(0 1))
. margins if sex==1, at(sex=(0 1))
```

We could, however, have obtained the same results by typing just one command:

```
. margins, over(sex) at(sex=(0 1))
```

Performing `margins, over(sex)` first restricts the sample to `sex==0` and then restricts it to `sex==1`, and that is equivalent to the two different `if` conditions that we specified before.

To test whether females treated as females is equal to females treated as males, we will need to type

```
. margins, over(sex) at(sex=(0 1)) post
. test _b[2._at#1.sex] = _b[1._at#1.sex]
```

We admit that the second command may seem to have come out of nowhere. When we specify `post` on the `margins` command, `margins` behaves as if it were an estimation command, which means that 1) it posts its estimates and full VCE to `e()`, 2) it gains the ability to replay results just as any estimation command can, and 3) it gains access to the standard postestimation commands. Item 3 explains why we could use `test`. We learned that we wanted to test `_b[2._at#1.sex]` and `_b[1._at#1.sex]` by replaying the estimation results, but this time with the standard estimation command `coeflegend` option. So, what we typed was

```
. margins, over(sex) at(sex=(0 1)) post
. margins, coeflegend
. test _b[2._at#1.sex] = _b[1._at#1.sex]
```

We will let you try `margins`, `coeflegend` for yourself. The results of running the other two commands are

```
. margins, over(sex) at(sex=(0 1)) post
Predictive margins Number of obs = 3,000
Model VCE: OIM
Expression: Pr(outcome), predict()
Over: sex
1._at: 0.sex
 sex = 0
 1.sex
 sex = 0
2._at: 0.sex
 sex = 1
 1.sex
 sex = 1
```

|          | Delta-method |           |       |       | [95% conf. interval] |          |
|----------|--------------|-----------|-------|-------|----------------------|----------|
|          | Margin       | std. err. | z     | P> z  |                      |          |
| _at#sex  |              |           |       |       |                      |          |
| 1#Male   | .0794393     | .0062147  | 12.78 | 0.000 | .0672586             | .0916199 |
| 1#Female | .2404749     | .0199709  | 12.04 | 0.000 | .2013326             | .2796171 |
| 2#Male   | .1335584     | .0127351  | 10.49 | 0.000 | .1085981             | .1585187 |
| 2#Female | .2596538     | .0104756  | 24.79 | 0.000 | .2391219             | .2801857 |

```
. test _b[2._at#1.sex] = _b[1._at#1.sex]
(1) - 1bn._at#1.sex + 2._at#1.sex = 0
 chi2(1) = 0.72
 Prob > chi2 = 0.3951
```

We can perform the same test in one command using contrasts of margins:

```
. logistic outcome i.sex i.group sex#group age
(output omitted)
. margins, over(sex) at(sex=(0 1)) contrast(atcontrast(r._at) wald)
Contrasts of predictive margins Number of obs = 3,000
Model VCE: OIM
Expression: Pr(outcome), predict()
Over: sex
1._at: 0.sex
 sex = 0
 1.sex
 sex = 0
2._at: 0.sex
 sex = 1
 1.sex
 sex = 1
```

|                 | df | chi2  | P>chi2 |
|-----------------|----|-------|--------|
| ._at@sex        |    |       |        |
| (2 vs 1) Male   | 1  | 14.59 | 0.0001 |
| (2 vs 1) Female | 1  | 0.72  | 0.3951 |
| Joint           | 2  | 16.13 | 0.0003 |

|                 | Delta-method |           |                      |          |
|-----------------|--------------|-----------|----------------------|----------|
|                 | Contrast     | std. err. | [95% conf. interval] |          |
| ._at@sex        |              |           |                      |          |
| (2 vs 1) Male   | .0541192     | .0141706  | .0263453             | .081893  |
| (2 vs 1) Female | .0191789     | .0225516  | -.0250215            | .0633793 |

We refit our logistic model because its estimation results were replaced when we posted our margins. The syntax to perform the contrast we want is admittedly not obvious. Contrasting (testing) across at() groups is more difficult than contrasting across the margins themselves or across over() groups, because we have no natural place for the contrast operators (r., in our case). We also explicitly requested Wald tests of the contrasts, which are not provided by default. Nevertheless, the  $\chi^2$  statistic and its *p*-value for (2 vs 1) for female matches the results of our test command. We also obtain the test of whether the response of males treated as males is equal to the response of males treated as females.

For a gentler introduction to contrasts of margins, see [\[R\] margins, contrast](#).

### Example 11: Margins of a specified prediction

We will fit the model

```
. use https://www.stata-press.com/data/r19/margex
. tobit ycn i.sex i.group sex#group age, ul(90)
```

and we will tell the following story about the variables: We run a peach orchard where we allow people to pick their own peaches. A person receives one empty basket in exchange for \$20, along with the right to enter the orchard. There is no official limit on how many peaches a person can pick, but only 90 peaches will fit into a basket. The dependent variable in the above tobit model, ycn, is the number of peaches picked. We use tobit, a special case of censored-normal regression, because ycn is censored at 90.

After fitting this model, if we typed

```
. margins sex
```

we would obtain the margins for males and for females of the uncensored number of peaches picked. We would obtain that because `predict` after `tobit` produces the uncensored number by default. To obtain the censored prediction, we would have to specify `predict's ystar(.,90)` option. If we want the margins based on that response, we type

```
. margins sex, predict(ystar(.,90))
```

The results of typing that are

```
. tobit ycn i.sex i.group sex#group age, ul(90)
(output omitted)
```

```
. margins sex, predict(ystar(.,90))
```

```
Predictive margins
```

```
Number of obs = 3,000
```

```
Model VCE: OIM
```

```
Expression: E(ycn|ycn<90), predict(ystar(.,90))
```

|        | Delta-method |           |        |       | [95% conf. interval] |          |
|--------|--------------|-----------|--------|-------|----------------------|----------|
|        | Margin       | std. err. | z      | P> z  |                      |          |
| sex    |              |           |        |       |                      |          |
| Male   | 62.21804     | .5996952  | 103.75 | 0.000 | 61.04266             | 63.39342 |
| Female | 78.34272     | .4555278  | 171.98 | 0.000 | 77.4499              | 79.23553 |

In our previous examples, `sex = 1` has designated females, so evidently the females visiting our orchard are better at filling baskets than the men.

### Example 12: Margins of a specified expression

Continuing with our peach orchard example and the previously fit model

```
. use https://www.stata-press.com/data/r19/margex
```

```
. tobit ycn i.sex i.group sex#group age, ul(90)
```

let's examine how well our baskets are working for us. What is the proportion of the number of peaches actually picked to the number that would have been picked were the baskets larger? As mentioned in [example 11](#), `predict, ystar(.,90)` produces the expected number picked given the limit of basket size. `predict, xb` would predict the expected number without a limit. We want the ratio of those two predictions. That ratio will measure as a proportion how well the baskets work. Thus, we could type

```
. margins sex, expression(predict(ystar(.,90))/predict(xb))
```

That would give us the proportion for everyone treated as male and everyone treated as female, but what we want to know is how well baskets work for true males and true females, so we will type

```
. margins, over(sex) expression(predict(ystar(0,90))/predict(xb))
Predictive margins Number of obs = 3,000
Model VCE: OIM
Expression: predict(ystar(0,90))/predict(xb)
Over: sex
```

|        | Delta-method |           |        |       |                      |
|--------|--------------|-----------|--------|-------|----------------------|
|        | Margin       | std. err. | z      | P> z  | [95% conf. interval] |
| sex    |              |           |        |       |                      |
| Male   | .9811785     | .0013037  | 752.60 | 0.000 | .9786233 .9837338    |
| Female | .9419962     | .0026175  | 359.88 | 0.000 | .9368659 .9471265    |

By the way, we could count the number of peaches saved by the limited basket size during the period of data collection by typing

```
. count
3,000
. margins, expression(3000*(predict(xb)-predict(ystar(.,90))))
(output omitted)
```

The number of peaches saved turns out to be 9,183.

### Example 13: Margins with multiple outcomes (responses)

Estimation commands such as `mlogit` and `mprobit` (see [R] [mlogit](#) and [R] [mprobit](#)) calculate multiple responses, and those multiple responses are reflected in the options available with `predict` after estimation. Obtaining margins for such estimators is thus the same as obtaining margins of a specified prediction, which was demonstrated in [example 11](#). The solution is to include the `predict_opt` that selects the desired response in margins' `predict(predict_opt)` option.

If we fit the multinomial logistic model

```
. mlogit group i.sex age
```

then to obtain the margins for the probability that `group = 1`, we would type

```
. margins sex, predict(outcome(1))
```

and to obtain the margins for the probability that `group = 3`, we would type

```
. margins sex, predict(outcome(3))
```

To obtain the margins for each of these outcomes simultaneously, type

```
. margins sex, predict(outcome(1)) predict(outcome(3))
```

We learned about the `outcome(1)` and `outcome(3)` options by looking in [R] [mlogit postestimation](#). For an example using margins with a multiple-outcome estimator, see [example 4](#) in [R] [mlogit postestimation](#).

**Example 14: Margins with multiple equations**

Estimation commands such as `mvreg`, `manova`, `sureg`, and `reg3` (see [\[MV\] mvreg](#), [\[MV\] manova](#), [\[R\] sureg](#), and [\[R\] reg3](#)) fit multiple equations. Obtaining margins for such estimators is the same as obtaining margins with multiple outcomes (see [example 13](#)), which in turn is the same as obtaining margins of a specified prediction (see [example 11](#)). You place the relevant option from the estimator's `predict` command into `margins'` `predict(predict_opt)` option.

If we fit the seemingly unrelated regression model

```
. sureg (y = i.sex age) (distance = i.sex i.group)
```

we can obtain the marginal means of `y` for males and females by typing

```
. margins sex, predict(equation(y))
```

and we can obtain the marginal means of `distance` by typing

```
. margins sex, predict(equation(distance))
```

We could obtain the difference between the margins of `y` and `distance` by typing

```
. margins sex, expression(predict(equation(y)) -
> predict(equation(distance)))
```

More examples can be found in [\[MV\] manova](#) and [\[MV\] manova postestimation](#).

**Example 15: Margins evaluated out of sample**

You can fit your model on one dataset and use `margins` on another if you specify `margins'` `noesample` option. Remember that `margins` reports estimated average responses, and, unless you lock all the covariates at fixed values by using the `at()` option, the remaining variables are allowed to vary as they are observed to vary in the data. That is indeed the point of using `margins`. The fitted model provides the basis for adjusting for the remaining variables, and the data provide their values. The predictions produced by `margins` are of interest assuming the data used by `margins` are in some sense interesting or representative. In some cases, you might need to fit your model on one set of data and perform `margins` on another.

In [example 11](#), we fit the model

```
. tobit ycn i.sex i.group sex#group age, ul(90)
```

and we told a story about our peach orchard in which we charged people \$20 to collect a basket of peaches, where baskets could hold at most 90 peaches. Let us now tell you that we believe the data on which we estimated those margins were unrepresentative, or at least, we have a more representative sample stored in another `.dta` file. That dataset includes the demographics of our customers but does not include counts of peaches picked. It is a lot of work counting those peaches.

Thus, we will fit our model just as we did previously using the detailed data, but we will bring the other, more representative dataset into memory before issuing the margins sex, predict(ystar(.,90)) command, and we will add noesample to it.

```
. use https://www.stata-press.com/data/r19/margex
(Artificial data for margins)
. tobit ycn i.sex i.group sex#group age, ul(90)
(output omitted)
. use https://www.stata-press.com/data/r19/peach
. margins sex, predict(ystar(.,90)) noesample
Predictive margins Number of obs = 2,727
Model VCE: OIM
Expression: E(ycn*|ycn<90), predict(ystar(.,90))
```

|     | Delta-method |           |        |       | [95% conf. interval] |          |
|-----|--------------|-----------|--------|-------|----------------------|----------|
|     | Margin       | std. err. | z      | P> z  |                      |          |
| sex |              |           |        |       |                      |          |
| 0   | 56.79774     | 1.003731  | 56.59  | 0.000 | 54.83046             | 58.76502 |
| 1   | 75.02146     | .6437446  | 116.54 | 0.000 | 73.75974             | 76.28317 |

In [example 12](#), we produced an estimate of the number of peaches saved by the limited-size baskets. We can update that estimate using the new demographic data by typing

```
. count
2,727
. margins, exp(2727*(predict(xb)-predict(ystar(.,90)))) noesample
(output omitted)
```

By running the above, we find that the updated number of peaches saved is 6,408.

### Obtaining margins of derivatives of responses (a.k.a. marginal effects)

Derivatives of responses are themselves responses, so everything said above in [Obtaining margins of responses](#) is equally true of derivatives of responses, and every example above could be repeated here substituting the derivative of the response for the response.

Derivatives are of interest because they are an informative way of summarizing fitted results. The change in a response for a change in the covariate is easy to understand and to explain. In simple models, one hardly needs margins to assist in obtaining such margins. Consider the simple linear regression

$$y = \beta_0 + \beta_1 \times \text{sex} + \beta_2 \times \text{age} + \epsilon$$

The derivatives of the responses are

$$dy/d(\text{sex}) = \beta_1$$

$$dy/d(\text{age}) = \beta_2$$

margins computes these derivatives using  $\hat{\beta}_1$  and  $\hat{\beta}_2$ , the coefficients from the linear predictor of  $y$ . How does the linear predictor  $\hat{y}$  change between males and females? It changes by  $\hat{\beta}_1$ . How does  $\hat{y}$  change with age? It changes by  $\hat{\beta}_2$  per year.

If you make the model a little more complicated, however, the need for margins arises. Consider the model

$$y = \beta_0 + \beta_1 \times \text{sex} + \beta_2 \times \text{age} + \beta_3 \times \text{age}^2 + \epsilon$$

Now, the derivative with respect to age is

$$dy/d(\text{age}) = \beta_2 + 2 \times \beta_3 \times \text{age}$$

The change in  $y$  for a change in age itself changes with age, and so to better understand the fitted results, you might want to make a table of the change in  $y$  for a change in age for age = 30, age = 40, and age = 50. `margins` can do that.

Consider an even more complicated model, such as

$$y = \beta_0 + \beta_1 \times \text{sex} + \beta_2 \times \text{age} + \beta_3 \times \text{age}^2 + \beta_4 \times \text{bp} + \beta_5 \times \text{sex} \times \text{bp} + \beta_6 \times \text{tmt} + \beta_7 \times \text{tmt} \times \text{age} + \beta_8 \times \text{tmt} \times \text{age}^2 + \epsilon \quad (1)$$

The derivatives are

$$dy/d(\text{sex}) = \beta_1 + \beta_5 \times \text{bp}$$

$$dy/d(\text{age}) = \beta_2 + 2 \times \beta_3 \times \text{age} + \beta_7 \times \text{tmt} + 2 \times \beta_8 \times \text{tmt} \times \text{age}$$

$$dy/d(\text{bp}) = \beta_4 + \beta_5 \times \text{sex}$$

$$dy/d(\text{tmt}) = \beta_6 + \beta_7 \times \text{age} + \beta_8 \times \text{age}^2$$

At this point, `margins` becomes indispensable.

### Use at() freely, especially with continuous variables

An option one tends to use frequently with derivatives of responses is `at()`. Such use is often to better understand or to communicate how the response varies, or, in technical jargon, to explore the nature of the response surface.

For instance, the effect  $dy/d(\text{tmt})$  in (1) is equal to  $\beta_6 + \beta_7 \times \text{age} + \beta_8 \times \text{age}^2$ , and so simply to understand how treatment varies with age, we may want to fix age at various values. We might type

```
. margins, dydx(tmt) at(age=(30 40 50))
```

### Expressing derivatives as elasticities

You specify the `dydx(varname)` option on the `margins` command to use  $dy/d(\text{varname})$  as the response variable. If you want that derivative expressed as an elasticity, you can specify `eyex(varname)`, `eydx(varname)`, or `dyex(varname)`. You substitute `e` for `d` where you want an elasticity. In the discussion below, when we talk about  $y$ , we are referring to the linear predictions from the model.

The formulas are

$$\text{dydx}() = dy/dx$$

$$\text{eyex}() = dy/dx \times (x/y)$$

$$\text{eydx}() = dy/dx \times (1/y) = d \ln(y)/dx$$

$$\text{dyex}() = dy/dx \times (x) = dy/d \ln(x)$$

and the interpretations are

|                       |              |                     |              |               |
|-----------------------|--------------|---------------------|--------------|---------------|
| <code>dydx()</code> : |              | change in $y$ for a |              | change in $x$ |
| <code>eyex()</code> : | proportional | change in $y$ for a | proportional | change in $x$ |
| <code>eydx()</code> : | proportional | change in $y$ for a |              | change in $x$ |
| <code>dyex()</code> : |              | change in $y$ for a | proportional | change in $x$ |

As `margins` always does with response functions, calculations are made at the observational level and are then averaged. Let's assume that in observation 5,  $dy/dx = 0.5$ ,  $y = 15$ , and  $x = 30$ ; then

$$\begin{aligned} \text{dydx}() &= 0.5 \\ \text{eyex}() &= 1.0 \\ \text{eydx}() &= 0.03 \\ \text{dyex}() &= 15.0 \end{aligned}$$

Many social scientists would informally explain the meaning of  $\text{eyex}() = 1$  as “ $y$  increases 100% when  $x$  increases 100%” or as “ $y$  doubles when  $x$  doubles”, although neither statement is literally true.  $\text{eyex}()$ ,  $\text{eydx}()$ , and  $\text{dyex}()$  are rates evaluated at a point, just as  $\text{dydx}()$  is a rate, and all such interpretations are valid only for small (infinitesimal) changes in  $x$ . It is true that  $\text{eyex}() = 1$  means  $y$  increases with  $x$  at a rate such that, if the rate were constant,  $y$  would double if  $x$  doubled. This issue of casual interpretation is no different from casually interpreting  $\text{dydx}()$  as if it represents the response to a unit change. It is not necessarily true that  $\text{dydx}() = 0.5$  means that “ $y$  increases by 0.5 if  $x$  increases by 1”. It is true that “ $y$  increases with  $x$  at a rate such that, if the rate were constant,  $y$  would increase by 0.5 if  $x$  increased by 1”.

$\text{dydx}()$ ,  $\text{eyex}()$ ,  $\text{eydx}()$ , and  $\text{dyex}()$  may be used with continuous  $x$  variables.  $\text{dydx}()$  and  $\text{eydx}()$  may also be used with factor variables. For  $\text{eydx}()$ , effects are based on  $d \ln(y)/dx$  for continuous covariates and on differences of  $\ln(y)$  for discrete covariates. Estimates based on logarithms have better computational properties and allow us to think of semielasticities as coefficients in a regression with log dependent variables. As we discuss next, when we are analyzing marginal effects of categorical variables, we think in terms of discrete differences instead of derivatives.

## Derivatives versus discrete differences

In (1),

$$y = \beta_0 + \beta_1 \times \text{sex} + \beta_2 \times \text{age} + \beta_3 \times \text{age}^2 + \beta_4 \times \text{bp} + \beta_5 \times \text{sex} \times \text{bp} + \beta_6 \times \text{tmt} + \beta_7 \times \text{tmt} \times \text{age} + \beta_8 \times \text{tmt} \times \text{age}^2 + \epsilon$$

Let us call your attention to the derivatives of  $y$  with respect to age and sex:

$$dy/d(\text{age}) = \beta_2 + 2 \times \beta_3 \times \text{age} + \beta_7 \times \text{tmt} + 2 \times \beta_8 \times \text{tmt} \times \text{age} \quad (2)$$

$$dy/d(\text{sex}) = \beta_1 + \beta_5 \times \text{bp} \quad (3)$$

`age` is presumably a continuous variable, and (2) is precisely how `margins` calculates its derivatives when you type `margins, dydx(age)`. `sex`, however, is presumably a factor variable, and `margins` does not necessarily make the calculation using (3) were you to type `margins, dydx(sex)`. We will explain, but let us first clarify what we mean by a continuous and a factor variable. Say that you fit (1) by typing

```
. regress y i.sex age c.age#c.age i.bp bp#sex
> i.tmt tmt#c.age tmt#c.age#c.age
```

It is important that `sex` entered the model as a factor variable. It would not do to type `regress y sex ...` because then `sex` would be a continuous variable, or at least it would be a continuous variable from Stata's point of view. The model estimates would be the same, but `margins`' understanding of those estimates would be a little different. With the model fit using `i.sex`, `margins` understands that either `sex` is 0 or `sex` is 1. With the model fit using `sex`, `margins` thinks `sex` is continuous and, for instance, `sex = 1.5` is a possibility.

`margins` calculates `dydx()` differently for continuous and for factor variables. For continuous variables, `margins` calculates  $dy/dx$  using the fitted values from `regress`. For factor variables, `margins` calculates the discrete first difference from the base category. To obtain that for `sex`, write down the model and then subtract from it the model evaluated at the base category for `sex`, which is `sex = 0`. If you do that, you will get the same formula as we obtained for the derivative, namely,

$$\text{discrete difference}\{(\text{sex} = 1) - (\text{sex} = 0)\} = \beta_1 + \beta_5 \times \text{bp}$$

We obtain the same formula because our model is linear regression. Outside of linear regression, and outside of linear response functions generally, the discrete difference is not equal to the derivative. The discrete difference is not equal to the derivative for logistic regression, probit, etc. The discrete difference calculation is generally viewed as better for factor variables than the derivative calculation because the discrete difference is what would actually be observed.

If you want the derivative calculation for your factor variables, specify the `continuous` option on the `margins` command.

### Example 16: Average marginal effect (partial effects)

Concerning the title of this example, the way we use the term marginal effect, the effects of factor variables are calculated using discrete first differences. If you wanted the continuous calculation, you would specify `margins`' `continuous` option in what follows.

```
. use https://www.stata-press.com/data/r19/margex
(Artificial data for margins)
. logistic outcome treatment##group age c.age#c.age treatment#c.age
(output omitted)
. margins, dydx(treatment)
Average marginal effects Number of obs = 3,000
Model VCE: OIM
Expression: Pr(outcome), predict()
dy/dx wrt: 1.treatment
```

|             | Delta-method |           |      |       |                      |          |
|-------------|--------------|-----------|------|-------|----------------------|----------|
|             | dy/dx        | std. err. | z    | P> z  | [95% conf. interval] |          |
| 1.treatment | .0385625     | .0162848  | 2.37 | 0.018 | .0066449             | .0704801 |

Note: `dy/dx` for factor levels is the discrete change from the base level.

The average marginal effect of treatment on the probability of a positive outcome is 0.039.

**Example 17: Average marginal effect of all covariates**

We will continue with the model

```
. logistic outcome treatment##group age c.age#c.age treatment#c.age
```

if we wanted the average marginal effects for all covariates, we would type margins, dydx(\*) or margins, dydx(\_all); they mean the same thing. This is probably the most common way margins, dydx() is used.

```
. margins, dydx(*)
```

Average marginal effects Number of obs = 3,000  
 Model VCE: OIM

Expression: Pr(outcome), predict()  
 dy/dx wrt: 1.treatment 2.group 3.group age

|             | Delta-method |           | z     | P> z  | [95% conf. interval] |           |
|-------------|--------------|-----------|-------|-------|----------------------|-----------|
|             | dy/dx        | std. err. |       |       |                      |           |
| 1.treatment | .0385625     | .0162848  | 2.37  | 0.018 | .0066449             | .0704801  |
| group       |              |           |       |       |                      |           |
| 2           | -.0776906    | .0181584  | -4.28 | 0.000 | -.1132805            | -.0421007 |
| 3           | -.1505652    | .0400882  | -3.76 | 0.000 | -.2291366            | -.0719937 |
| age         | .0095868     | .0007796  | 12.30 | 0.000 | .0080589             | .0111148  |

Note: dy/dx for factor levels is the discrete change from the base level.

**Example 18: Evaluating marginal effects over the response surface**

Continuing with the model

```
. logistic outcome treatment##group age c.age#c.age treatment#c.age
```

What follows maps out the entire response surface of our fitted model. We report the marginal effect of treatment evaluated at age = 20, 30, . . . , 60, by each level of group.

```
. margins group, dydx(treatment) at(age=(20(10)60))
Conditional marginal effects Number of obs = 3,000
Model VCE: OIM
Expression: Pr(outcome), predict()
dy/dx wrt: 1.treatment
1._at: age = 20
2._at: age = 30
3._at: age = 40
4._at: age = 50
5._at: age = 60
```

|             | Delta-method   |           |       |       |                      |
|-------------|----------------|-----------|-------|-------|----------------------|
|             | dy/dx          | std. err. | z     | P> z  | [95% conf. interval] |
| 0.treatment | (base outcome) |           |       |       |                      |
| 1.treatment |                |           |       |       |                      |
| _at#group   |                |           |       |       |                      |
| 1 1         | -.0208409      | .0152862  | -1.36 | 0.173 | -.0508013 .0091196   |
| 1 2         | .009324        | .0059896  | 1.56  | 0.120 | -.0024155 .0210635   |
| 1 3         | .0006558       | .0048682  | 0.13  | 0.893 | -.0088856 .0101972   |
| 2 1         | -.0436964      | .0279271  | -1.56 | 0.118 | -.0984325 .0110397   |
| 2 2         | .0382959       | .0120405  | 3.18  | 0.001 | .014697 .0618949     |
| 2 3         | .0064564       | .0166581  | 0.39  | 0.698 | -.0261929 .0391057   |
| 3 1         | -.055676       | .0363191  | -1.53 | 0.125 | -.1268601 .015508    |
| 3 2         | .1152235       | .0209858  | 5.49  | 0.000 | .074092 .156355      |
| 3 3         | .0284808       | .0471293  | 0.60  | 0.546 | -.0638908 .1208524   |
| 4 1         | -.027101       | .0395501  | -0.69 | 0.493 | -.1046177 .0504158   |
| 4 2         | .2447682       | .0362623  | 6.75  | 0.000 | .1736954 .315841     |
| 4 3         | .0824401       | .1025028  | 0.80  | 0.421 | -.1184616 .2833418   |
| 5 1         | .0292732       | .0587751  | 0.50  | 0.618 | -.0859239 .1444703   |
| 5 2         | .3757777       | .0578106  | 6.50  | 0.000 | .2624709 .4890844    |
| 5 3         | .1688268       | .1642191  | 1.03  | 0.304 | -.1530368 .4906904   |

Note: dy/dx for factor levels is the discrete change from the base level.

## Obtaining margins with survey data and representative samples

The standard errors and confidence intervals produced by `margins` are based by default on the delta method applied to the VCE of the current estimates. Delta-method standard errors treat the covariates at which the response is evaluated as given or fixed. Such standard errors are appropriate if you specify `at()` to fix the covariates, and they are appropriate when you are making inferences about groups exactly like your sample whether you specify `at()` or not.

On the other hand, if you have a representative sample of the population or if you have complex survey data and if you want to make inferences about the underlying population, you need to account for the variation in the covariates that would arise in repeated sampling. You do that using `vce(unconditional)`, which invokes a different standard error calculation based on [Korn and Graubard \(1999\)](#). Syntactically, there are three cases. They all involve specifying `vce(unconditional)` on the `margins` command:

1. *You have a representative random sample, and you have not svyset your data.*  
When you fit the model, you need to specify the `vce(robust)` or `vce(cluster clustvar)` option. When you issue the `margins` command, you need to specify the `vce(unconditional)` option.
2. *You have a weighted sample, and you have not svyset your data.*  
You need to specify `[pw=weight]` when you fit the model and, of course, specify the `vce(unconditional)` option on the `margins` command. You do not need to specify the weights on the `margins` command because `margins` will obtain them from the estimation results.
3. *You have svyset your data, whether it be a simple random sample or something more complex including weights, strata, sampling units, or poststratification, and you are using the linearized variance estimator.*  
You need to use the `svy` prefix when you fit the model. You need to specify `vce(unconditional)` when you issue the `margins` command. You do not need to respecify the weights.  
Even though the data are `svyset`, and even though the estimation was `svy` estimation, `margins` does not default to `vce(unconditional)`. It does not default to `vce(unconditional)` because there are valid reasons to want the data-specific, `vce(delta)` standard-error estimates. Whether you specify `vce(unconditional)` or not, `margins` uses the weights, so you do not need to respecify them even if you are using `vce(unconditional)`.

`vce(unconditional)` is allowed only after estimation with `vce(robust)`, `vce(cluster ...)`, or the `svy` prefix with the linearized variance estimator. If the VCE of the current estimates was specified as clustered, so will be the VCE estimates of `margins`. If the estimates were from a survey estimation, the survey settings in the dataset will be used by `margins`.

When you use `vce(unconditional)`, never specify `if exp` or `in range` on the `margins` command; instead, specify the `subpop(if exp)` option. You do that for the usual reasons; see [\[SVY\] Subpopulation estimation](#). If you specify `over(varlist)` to examine subgroups, the subgroups will automatically be treated as subpopulations.

If you are using a replication-based variance estimator, you may want to use this method to estimate the variance of your margins; see [\[SVY\] svy postestimation](#).

**Example 19: Inferences for populations, margins of response**

In [example 6](#), we fit the model

```
. logistic outcome i.sex i.group sex#group age
```

and we obtained margins by sex and margins by group,

```
. margins sex group
```

If our data were randomly drawn from the population of interest and we wanted to account for this, we would have typed

```
. logistic outcome i.sex i.group sex#group age, vce(robust)
. margins sex group, vce(unconditional)
```

We do that below:

```
. logistic outcome i.sex i.group sex#group age, vce(robust)
 (output omitted)
. margins sex group, vce(unconditional)
```

Predictive margins Number of obs = 3,000  
 Expression: Pr(outcome), predict()

|              | Unconditional |           | z     | P> z  | [95% conf. interval] |          |
|--------------|---------------|-----------|-------|-------|----------------------|----------|
|              | Margin        | std. err. |       |       |                      |          |
| <b>sex</b>   |               |           |       |       |                      |          |
| Male         | .1600644      | .0131685  | 12.16 | 0.000 | .1342546             | .1858743 |
| Female       | .1966902      | .0104563  | 18.81 | 0.000 | .1761963             | .2171841 |
| <b>group</b> |               |           |       |       |                      |          |
| 1            | .2251302      | .0127069  | 17.72 | 0.000 | .200225              | .2500354 |
| 2            | .150603       | .0118399  | 12.72 | 0.000 | .1273972             | .1738088 |
| 3            | .0736157      | .0343188  | 2.15  | 0.032 | .0063522             | .1408793 |

The estimated margins are the same as they were in [example 6](#), but the standard errors and confidence intervals differ, although not by much. Given that we have 3,000 observations in our randomly drawn sample, we should expect this.

### Example 20: Inferences for populations, marginal effects

In [example 17](#), we fit a logistic model and then obtained the average marginal effects for all covariates by typing

```
. logistic outcome treatment##group age c.age#c.age treatment#c.age
. margins, dydx(*)
```

To repeat that and also obtain standard errors for our population, we would type

```
. logistic outcome treatment##group age c.age#c.age treatment#c.age, vce(robust)
 (output omitted)
. margins, dydx(*) vce(unconditional)
```

```
Average marginal effects Number of obs = 3,000
Expression: Pr(outcome), predict()
dy/dx wrt: 1.treatment 2.group 3.group age
```

|             | dy/dx     | Unconditional<br>std. err. | z     | P> z  | [95% conf. interval] |           |
|-------------|-----------|----------------------------|-------|-------|----------------------|-----------|
| 1.treatment | .0385625  | .0163872                   | 2.35  | 0.019 | .0064442             | .0706808  |
| group       |           |                            |       |       |                      |           |
| 2           | -.0776906 | .0179573                   | -4.33 | 0.000 | -.1128863            | -.0424949 |
| 3           | -.1505652 | .0411842                   | -3.66 | 0.000 | -.2312848            | -.0698456 |
| age         | .0095868  | .0007814                   | 12.27 | 0.000 | .0080553             | .0111183  |

Note: dy/dx for factor levels is the discrete change from the base level.

### Example 21: Inferences for populations with svyset data

See [example 3](#) in [\[SVY\] svy postestimation](#).

## Standardizing margins

A standardized margin is the margin calculated on data different from the data used to fit the model. Typically, the word standardized is reserved for situations in which the alternate population is a reference population, which may be real or artificial, and which is treated as fixed.

Say that you work for a hospital and have fit a model of mortality on the demographic characteristics of the hospital's patients. At this stage, were you to type

```
. margins
```

you would obtain the mortality rate for your hospital. You have another dataset, `hstandard.dta`, that contains demographic characteristics of patients across all hospitals along with the population of each hospital recorded in the `pop` variable. You could obtain the expected mortality rate at your hospital if your patients matched the characteristics of the standard population by typing

```
. use https://www.stata-press.com/data/r19/hstandard, clear
. margins [fw=pop], noesample
```

You specified `noesample` because the margin is being calculated on data other than the data used to fit the model. You specified `[fw=pop]` because the reference dataset you are using included population counts, as many reference datasets do.

## Obtaining margins as though the data were balanced

Here we discuss what are commonly called estimated marginal means or least-squares means. These are margins that assume that the design is **balanced**, which means all levels of factor variables are equally likely. The seminal reference on these margins is [Searle, Speed, and Milliken \(1980\)](#).

In designed experiments, observations are often allocated in a balanced way so that the variances can be easily compared and decomposed. At the Acme Portable Widget Company, they are experimenting with a new machine. The machine has three temperature settings and two pressure settings; a combination of settings will be optimal on any particular day, determined by the weather. At start-up, one runs a quick test and chooses the optimal setting for the day. Across different days, each setting will be used about equally, says the manufacturer.

In experiments with the machine, 10 widgets were collected for stress testing at each of the settings over a six-week period. We wish to know the average stress-test value that can be expected from these machines over a long period.

## Balancing using asbalanced

The data were intended to be balanced, but unfortunately some samples were destroyed before the stress could be measured, rendering the data unbalanced. To estimate the margins as if the data were balanced, specify the `asbalanced` option.

```
. use https://www.stata-press.com/data/r19/acmemanuf
. regress y pressure##temp
 (output omitted)
. margins, asbalanced
Adjusted predictions Number of obs = 49
Model VCE: OLS
Expression: Linear prediction, predict()
At: pressure (asbalanced)
 temp (asbalanced)
```

|       | Delta-method |           |       |       |                      |
|-------|--------------|-----------|-------|-------|----------------------|
|       | Margin       | std. err. | t     | P> t  | [95% conf. interval] |
| _cons | 115.3758     | 1.530199  | 75.40 | 0.000 | 112.2899 118.4618    |

If we had omitted the `asbalanced` option, our calculations would have been different:

```
. margins
Predictive margins Number of obs = 49
Model VCE: OLS
Expression: Linear prediction, predict()
```

|       | Delta-method |           |       |       |                      |
|-------|--------------|-----------|-------|-------|----------------------|
|       | Margin       | std. err. | t     | P> t  | [95% conf. interval] |
| _cons | 109.9214     | 1.422629  | 77.27 | 0.000 | 107.0524 112.7904    |

To see what margins, asbalanced is doing, first recall that pressure and temp are categorical variables that take values {1, 2} and {1, 2, 3}, respectively. To re-create the asbalanced calculations using the at() option, we set each level of pressure equal to 1/2 and each level of temp equal to 1/3.

```
. margins, at(1.pressure=0.5 2.pressure=0.5
> 1.temp=0.333333 2.temp=0.333333 3.temp=0.333333)
Adjusted predictions Number of obs = 49
Model VCE: OLS
Expression: Linear prediction, predict()
At: 1.pressure = .5
 2.pressure = .5
 1.temp = .333333
 2.temp = .333333
 3.temp = .333333
```

|       | Margin   | Delta-method<br>std. err. | t     | P> t  | [95% conf. interval] |          |
|-------|----------|---------------------------|-------|-------|----------------------|----------|
| _cons | 115.3758 | 1.530199                  | 75.40 | 0.000 | 112.2899             | 118.4618 |

□ Technical note

Concerning how asbalanced calculations are performed, if a factor variable has  $l$  levels, then each level’s coefficient contributes to the response weighted by  $1/l$ . If two factors,  $a$  and  $b$ , interact, then each coefficient associated with their interaction is weighted by  $1/(l_a \times l_b)$ .

If a balanced factor interacts with a continuous variable, then each coefficient in the interaction is applied to the value of the continuous variable, and the results are weighted equally. So, if the factor being interacted has  $l_a$  levels, the effect of each coefficient on the value of the continuous covariate is weighted by  $1/l_a$ .



**Balancing by standardization**

To better understand the balanced results, we can perform the balancing ourselves by using the standardizing method shown in *Standardizing margins*. To do that, we will input a balanced dataset and then type margins, noesample.

```
. use https://www.stata-press.com/data/r19/acmemanuf
. regress y pressure##temp
 (output omitted)
. drop _all
```

```

. input pressure temp
 pressure temp
1. 1 1
2. 1 2
3. 1 3
4. 2 1
5. 2 2
6. 2 3
7. end

. margins, noesample
Predictive margins Number of obs = 6
Model VCE: OLS
Expression: Linear prediction, predict()

```

|       | Delta-method |           | t     | P> t  | [95% conf. interval] |          |
|-------|--------------|-----------|-------|-------|----------------------|----------|
|       | Margin       | std. err. |       |       |                      |          |
| _cons | 115.3758     | 1.530199  | 75.40 | 0.000 | 112.2899             | 118.4618 |

We obtain the same results as previously.

## Balancing nonlinear responses

If our testing had produced a binary outcome, say, acceptable/unacceptable, rather than a continuous variable, we would type

```

. use https://www.stata-press.com/data/r19/acmemanuf, clear
. logistic acceptable pressure##temp
. margins, asbalanced

```

The result of doing that would be 0.680. If we omitted the `asbalanced` option, the result would have been 0.667. The two results are so similar because `acmemanuf.dta` is nearly balanced.

Even though the `asbalanced` option can be used on both linear and nonlinear responses, such as probabilities, there is an issue of which you should be aware. The most widely used formulas for balancing responses apply the balancing to the linear prediction, average that as if it were balanced, and then apply the nonlinear transform. That is the calculation that produced 0.680.

An alternative would be to apply the standardization method. That amounts to making the linear predictions observation by observation, applying the nonlinear transform to each, and then averaging the nonlinear result as if it were balanced. You could do that by typing

```

. use https://www.stata-press.com/data/r19/acmemanuf, clear
. logistic acceptable pressure##temp
. clear
. input pressure temp
 (see above for entered data)
. margins, noesample

```

The result from the standardization procedure would be 0.672. These two ways of averaging nonlinear responses are discussed in detail in [Lane and Nelder \(1982\)](#) within the context of general linear models.

Concerning the method used by the `asbalanced` option, if your data start balanced and you have a nonlinear response, you will get different results with and without the `asbalanced` option!

## Treating a subset of covariates as balanced

So far, we have treated all the covariates as if they were balanced. `margins` will allow you to treat a subset of the covariates as balanced, too. For instance, you might be performing an experiment in which you are randomly allocating patients to a treatment arm and so want to balance on arm, but you do not want to balance the other characteristics.

In this example, we will imagine that the outcome of the experiment is continuous. We type

```
. use https://www.stata-press.com/data/r19/margex, clear
. regress y arm##sex sex##agegroup
. margins, at((asbalanced) arm)
```

If we wanted results balanced on `agegroup` as well, we could type

```
. margins, at((asbalanced) arm agegroup)
```

If we wanted results balanced on all three covariates, we could type

```
. margins, at((asbalanced) arm agegroup sex)
```

or we could type

```
. margins, at((asbalanced) _factor)
```

or we could type

```
. margins, asbalanced
```

## Balancing in the presence of continuous covariates

So far, we have considered only factor variables because the concept of balancing is applicable only to factor covariates. If your model contains both factor and continuous covariates, the `asbalanced` option will treat the factor covariates as if they were balanced, but it will not affect the continuous covariates.

Here we imagine that the outcome of the experiment is binary and that we have two factor covariates (`arm` and `sex`) and two continuous covariates (`distance` and `ycn`). We could type

```
. use https://www.stata-press.com/data/r19/margex, clear
. logistic outcome i.arm i.sex c.distance c.ycn
. margins, asbalanced
```

This calculates the marginal probability of `outcome = 1` while treating factor covariates `arm` and `sex` as balanced, but continuous covariates `distance` and `ycn` are left as observed, which is the default behavior.

If we set continuous covariates equal to their means while maintaining a balanced treatment of factor covariates, the resulting margin is called an **adjusted treatment mean**. To calculate this margin, we could type

```
. margins, asbalanced atmeans
```

or we could type

```
. margins, at((asbalanced) _factor (mean) _continuous)
```

## Using fvset design

As a convenience feature, equivalent to

```
. regress y arm##sex sex##agegroup
. margins, at((asbalanced) arm sex)
```

is

```
. fvset design asbalanced arm sex
. regress y arm##sex sex##agegroup
. margins
```

The advantage of the latter is that you have to set the variables as balanced only once. This is useful when balancing is a design characteristic of certain variables and you wish to avoid accidentally treating them as unbalanced.

If you save your data after `fvsetting`, the settings will be remembered in future sessions. If you want to clear the setting(s), type

```
. fvset clear varlist
```

See [R] [fvset](#).

## Balancing in the presence of empty cells

The issue of empty cells is not exclusively an issue of balancing, but there are special considerations when balancing. Empty cells are discussed generally in *Estimability of margins*.

An empty cell is an interaction of levels of two or more factor variables for which you have no data. Usually, margins involving empty cells cannot be estimated. When balancing, there is an alternate definition of the margin that allows the margin to be estimated. `margins` makes the alternate calculation when you specify the `emptycells(reweight)` option. By default, `margins` uses the `emptycells(strict)` option.

If you have empty cells in your data and you request margins involving the empty cells, those margins will be marked as not estimable even if you specify the `asbalanced` option.

```
. use https://www.stata-press.com/data/r19/estimability, clear
(margins estimability)
. regress y sex##group
(output omitted)
. margins sex, asbalanced
```

```
Adjusted predictions Number of obs = 69
Model VCE: OLS
Expression: Linear prediction, predict()
At: group (asbalanced)
```

|        | Delta-method |                 |       |       |                      |
|--------|--------------|-----------------|-------|-------|----------------------|
|        | Margin       | std. err.       | t     | P> t  | [95% conf. interval] |
| sex    |              |                 |       |       |                      |
| Male   | 21.91389     | 1.119295        | 19.58 | 0.000 | 19.67572 24.15206    |
| Female | .            | (not estimable) |       |       |                      |

This example is discussed in *Estimability of margins*, although without the `asbalanced` option. What is said there is equally relevant to the `asbalanced` case. For reasons explained there, the margin for `sex = 1` (female) cannot be estimated.

The margin for `sex = 1` can be estimated in the `asbalanced` case if you are willing to make an assumption. Remember that `margins` makes the balanced calculation by summing the responses associated with the levels and then dividing by the number of levels. If you specify `emptycells(reweight)`, `margins` sums what is available and divides by the number available. Thus, you are assuming that, whatever the responses in the empty cells, those responses are such that they would not change the overall mean of what is observed.

The results of specifying `emptycells(reweight)` are

```
. margins sex, asbalanced emptycells(reweight)
Adjusted predictions Number of obs = 69
Model VCE: OLS
Expression: Linear prediction, predict()
Empty cells: reweight
At: group (asbalanced)
```

|        | Delta-method |           |       |       |                      |
|--------|--------------|-----------|-------|-------|----------------------|
|        | Margin       | std. err. | t     | P> t  | [95% conf. interval] |
| sex    |              |           |       |       |                      |
| Male   | 21.91389     | 1.119295  | 19.58 | 0.000 | 19.67572 24.15206    |
| Female | 24.85185     | 1.232304  | 20.17 | 0.000 | 22.38771 27.316      |

## Obtaining margins with nested designs

### Introduction to nested designs

Factors whose meaning depends on other factors are called nested factors, and the factors on which their meaning depends are called the nesting factors. For instance, assume that we have a sample of patients and each patient is assigned to one doctor. Then, patient is nested within doctor. Let the identifiers of the first 5 observations of our data be

| Doctor | Patient | Name  |
|--------|---------|-------|
| 1      | 1       | Fred  |
| 1      | 2       | Mary  |
| 1      | 3       | Bob   |
| 2      | 1       | Karen |
| 2      | 2       | Hank  |

The first patient on one doctor’s list has nothing whatsoever to do with the first patient on another doctor’s list. The meaning of `patient = 1` is defined only when the value of `doctor` is supplied.

Nested factors enter into models as interactions of nesting and nested; the nested factor does not appear by itself. We might estimate a model such as

```
. regress y ... i.doctor doctor#patient ...
```

You do not include `i.patient` because the coding for patient has no meaning except within doctor. Patient 1 is Fred for doctor 1 and Karen for doctor 2, etc.

`margins` provides an option to help account for the structure of nested models. The `within(varlist)` option specifies that margins estimate and report a set of margins for the value combinations of `varlist`. We might type

```
. margins, within(doctor)
```

Margin calculations are performed first for `doctor = 1`, then for `doctor = 2`, and so on.

Sometimes you need to specify `within()`, and other times you do not. Let's consider the particular model

```
. regress y i.doctor doctor#patient i.sex sex#doctor#patient
```

The guidelines are the following:

1. You may compute overall margins by typing `margins`.
2. You may compute overall margins within levels of a nesting factor by typing `margins, within(doctor)`.
3. You may compute margins of a nested factor within levels of its nesting factor by typing `margins patient, within(doctor)`.
4. You may compute margins of factors in your model, as long as the factor does not nest other factors and is not nested within other factors, by typing `margins sex`.
5. You may not compute margins of a nesting factor, such as `margins doctor`, because they are not estimable.

For examples using `within()`, see [\[R\] anova](#).

### Margins with nested designs as though the data were balanced

To obtain margins with nested designs as though the data were balanced, the guidelines are the same as above except that 1) you add the `asbalanced` option and 2) whenever you do not specify `within()`, you specify `emptycells(reweight)`. The updated guidelines are

1. You may compute overall margins by typing `margins, asbalanced emptycells(reweight)`.
2. You may compute overall margins within levels of a nesting factor by typing `margins, asbalanced within(doctor)`.
3. You may compute margins of a nested factor within levels of its nesting factor by typing `margins patient, asbalanced within(doctor)`.
4. You may compute margins of factors in your model, as long as the factor does not nest other factors and is not nested within other factors, by typing `margins sex, asbalanced emptycells(reweight)`.
5. You may not compute margins of a nesting factor, such as `margins doctor`, because they are not estimable.

Just as explained in *Using fvset design*, rather than specifying the `asbalanced` option, you may set the balancing characteristic on the factor variables once and for all by using the command `fvset design asbalanced varlist`.

#### □ Technical note

Specifying either `emptycells(reweight)` or `within(varlist)` causes margins to rebalance over all empty cells in your model. If you have interactions in your model that are not involved in the nesting, margins will lose its ability to detect estimability. □

#### □ Technical note

Careful readers will note that the description of `within(varlist)` matches closely the description of `over(varlist)`. The concept of nesting is similar to the concept of subpopulations. `within()` differs from `over()` in that it gracefully handles the missing cells when margins are computed as balanced. □

### Coding of nested designs

In the *Introduction* to this section, we showed a coding of the nested variable `patient`, where the coding started over with each doctor:

| Doctor | Patient | Name  |
|--------|---------|-------|
| 1      | 1       | Fred  |
| 1      | 2       | Mary  |
| 1      | 3       | Bob   |
| 2      | 1       | Karen |
| 2      | 2       | Hank  |

That coding style is not required. The data could just as well have been coded

| Doctor | Patient | Name  |
|--------|---------|-------|
| 1      | 1       | Fred  |
| 1      | 2       | Mary  |
| 1      | 3       | Bob   |
| 2      | 4       | Karen |
| 2      | 5       | Hank  |

or even

| Doctor | Patient | Name  |
|--------|---------|-------|
| 1      | 1037239 | Fred  |
| 1      | 2223942 | Mary  |
| 1      | 0611393 | Bob   |
| 2      | 4433329 | Karen |
| 2      | 6110271 | Hank  |

Actually, either of the above two alternatives is better than the first one because margins will be better able to give you feedback about estimability should you make a mistake following the guidelines. On the other hand, both of these alternatives require more memory at the estimation step. If you run short of memory, you will need to recode your patient ID to the first coding style, which you could do by typing

```
. sort doctor patient
. by doctor: generate newpatient = _n
```

Alternatively, you can set `emptycells drop` and continue to use your patient ID variable just as it is coded. If you do this, we recommend that you remember to type `set emptycells keep` when you are finished; margins is better able to determine estimability that way. If you regularly work with large nested models, you can set `emptycells keep`, permanently so that the setting persists across sessions. See [R] [set emptycells](#).

## Special topics

### Requirements for model specification

The results that margins reports are based on the most recently fit model or, in Stata jargon, the most recently issued estimation command. Here we discuss 1) mechanical requirements for how you specify that estimation command, 2) workarounds to use when those restrictions prove impossible, and 3) requirements for margins' `predict(pred_opt)` option to work.

Concerning 1, when you specify the estimation command, covariates that are logically factor variables must be Stata factor variables, and that includes indicator variables, binary variables, and dummies. It will not do to type

```
. regress y ... sex ...
```

even if `sex` is a 0/1 variable. You must type

```
. regress y ... i.sex ...
```

If you violate this rule, you will not get incorrect results, but you will discover that you will be unable to obtain margins on `sex`:

```
. margins sex
factor sex not found in list of covariates
r(111);
```

It is also important that if the same continuous variable appears in your model more than once, differently transformed, those transforms be performed via Stata's factor-variable notation. It will not do to type

```
. generate age2 = age^2
. regress y ... age age2 ...
```

You must type

```
. regress y ... age c.age#c.age ...
```

You must do that because margins needs to know everywhere that variable appears in the model if it is to be able to set covariates to fixed values.

Concerning 2, sometimes the transformations you desire may not be achievable using the factor-variable notation; in those situations, there is a workaround. Let's assume you wish to estimate

```
. generate age1_5 = age^1.5
. regress y ... age age1_5 ...
```

There is no factor-variable notation for including `age` and `age1.5` in a model, so obviously you are going to obtain the estimates by typing just what we have shown. In what follows, it would be okay if there are interactions of `age` and `age1_5` with other variables specified by the factor-variable notation, so the model could just as well be

```
. regress y ... age age1_5 sex#c.age sex#c.age1_5 ...
```

Let's assume you have fit one of these two models. On any subsequent `margins` command where you leave `age` free to vary, there will be no issue. You can type

```
. margins sex
```

and results will be correct. Issues arise when you attempt to fix `age` at predetermined values. The following would produce incorrect results:

```
. margins sex, at(age=20)
```

The results would be incorrect because they leave `age1_5` free to vary, and, logically, fixing `age` implies that `age1_5` should also be fixed. Because we were unable to state the relationship between `age` and `age1_5` using the factor-variable notation, `margins` does not know to fix `age1_5` at `201.5` when it fixes `age` at 20. To get the correct results, you must fix the value of `age1_5` yourself:

```
. margins sex, at(age=20 age1_5=89.442719)
```

That command produces correct results. In the command, `89.442719` is `201.5`.

In summary, when there is a functional relationship between covariates of your model and that functional relationship is not communicated to `margins` via the factor-variable notation, then it becomes your responsibility to ensure that all variables that are functionally related are set to the appropriate fixed values when any one of them is set to a fixed value.

Concerning 3, we wish to amend our claim that you can calculate margins for anything that `predict` will produce. We need to add a qualifier. Let us show you an example where the statement is not true. After `regress`, `predict` will predict something it calls `pr(a, b)`, which is the probability  $a \leq y \leq b$ . Yet if we attempted to use `pr()` with `margins` after estimation by `regress`, we would obtain

```
. margins sex, predict(pr(10,20))
prediction is a function of possibly stochastic quantities other than e(b)
r(498);
```

What we should have stated was that you can calculate margins for anything that `predict` will produce for which all the estimated quantities used in its calculation appear in  $e(V)$ , the estimated VCE. `pr()` is a function of  $\beta$ , the estimated coefficients, and of  $s^2$ , the estimated variance of the residual. `regress` does not post the variance of the residual variance (`sic`) in  $e(V)$ , or even estimate it, and therefore, `predict(pr(10, 20))` cannot be specified with `margins` after estimation by `regress`.

It is unlikely that you will ever encounter these kinds of problems because there are so few predictions where the components are not posted to  $e(V)$ . If you do encounter the problem, the solution may be to specify `nose` to suppress the standard error calculation. If the problem is not with computing the margin, but with computing its standard error, `margins` will report the result:

```
. margins sex, predict(pr(10,20)) nose
(output appears with SEs, tests, and CIs left blank)
```

□ Technical note

Programmers: If you run into this after running an estimation command that you have written, be aware that as of Stata 11, you are supposed to set in `e(marginsok)` the list of options allowed with `predict` that are okay to use with `margins`. When that list is not set, `margins` looks for violations of its assumptions and, if it finds any, refuses to proceed. □

**Estimability of margins**

Sometimes `margins` will report that a margin cannot be estimated:

```
. use https://www.stata-press.com/data/r19/estimability, clear
(margins estimability)
. regress y sex##group
(output omitted)
. margins sex
Predictive margins Number of obs = 69
Model VCE: OLS
Expression: Linear prediction, predict()
```

|        | Delta-method |                 |       |       |                      |
|--------|--------------|-----------------|-------|-------|----------------------|
|        | Margin       | std. err.       | t     | P> t  | [95% conf. interval] |
| sex    |              |                 |       |       |                      |
| Male   | 21           | .8500245        | 24.71 | 0.000 | 19.30027 22.69973    |
| Female | .            | (not estimable) |       |       |                      |

In the above output, the margin for `sex = 0` (male) is estimated, but the margin for `sex = 1` (female) is not estimable. This occurs because of empty cells. An empty cell is an interaction of levels of two or more factor variables for which you have no data. In the example, the lack of estimability arises because we have two empty cells:

```
. table sex group, nototals
```

|        | Group |   |    |   |   |
|--------|-------|---|----|---|---|
|        | 1     | 2 | 3  | 4 | 5 |
| Sex    |       |   |    |   |   |
| Male   | 2     | 9 | 27 | 8 | 2 |
| Female | 9     | 9 | 3  |   |   |

To calculate the marginal mean response for `sex = 1`, we have no responses to average over for `group = 4` and `group = 5`. We obviously could calculate that mean for the observations that really are `sex = 1`, but remember, the marginal calculation for `sex = 1` treats everyone as if female, and we will thus have 8 and 2 observations for which we have no basis for estimating the response.

There is no solution for this problem unless you are willing to treat the data as if it were balanced and adjust your definition of a margin; see *Balancing in the presence of empty cells*.

## Manipulability of tests

Manipulability is a problem that arises with some tests, and in particular, arises with Wald tests. Tests of margins are based on Wald tests, hence our interest. This is a generic issue and not specific to the `margins` command.

Let's understand the problem. Consider performing a test of whether some statistic  $\phi$  is 0. Whatever the outcome of that test, it would be desirable if the outcome were the same were we to test whether the  $\sqrt{\phi}$  were 0, or whether  $\phi^2$  were 0, or whether any other monotonic transform of  $\phi$  were 0 (for  $\phi^2$ , we were considering only the positive half of the number line). If a test does not have that property, it is manipulable.

Wald tests are manipulable, and that means the tests produced by `margins` are manipulable. You can see this for yourself by typing

```
. use https://www.stata-press.com/data/r19/margex, clear
. replace y = y - 65
. regress y sex##group
. margins, df(.)
. margins, expression(predict(xb)^2)
```

To compare the results from the two `margins` commands, we added the `df(.)` option to the first one, forcing it to report a  $z$  statistic even though a  $t$  statistic would have been appropriate in this case. We would prefer if the test against zero produced by `margins, df(.)` was equal to the test produced by `margins, expression(predict(xb)^2)`. But alas, they produce different results. The first produces  $z = 12.93$ , and the second produces  $z = 12.57$ .

The difference is not much in our example, but behind the scenes, we worked to make it small. We subtracted 65 from `y` so that the experiment would be for a case where it might be reasonable that you would be testing against 0. One does not typically test whether the mean income in the United States is zero or whether the mean blood pressure of live patients is zero. Had we left `y` as it was originally, we would have obtained  $z = 190$  and  $z = 96$ . We did not want to show that comparison to you first because the mean of `y` is so far from 0 that you probably would never be testing it. The corresponding difference in  $\phi$  is tiny.

Regardless of the example, it is important that you base your tests in the metric where the likelihood surface is most quadratic. For further discussion on manipulability, see [Manipulability in \[R\] predictnl](#).

This manipulability is not limited to Wald tests after estimation; you can also see the manipulability of results produced by linear regression just by applying nonlinear transforms to a covariate (Phillips and Park 1988; Gould 1996).

## Using margins after the estimates use command

Assume you fit and used `estimates save` (see [\[R\] estimates save](#)) to save the estimation results:

```
. regress y sex##group age c.age*c.age if site==1
. ...
. estimates save mymodel
file mymodel.ster saved
```

Later, perhaps in a different Stata session, you reload the estimation results by typing

```
. estimates use mymodel
```

You plan to use margins with the reloaded results. You must remember that margins bases its results not only on the current estimation results but also on the current data in memory. Before you can use margins, you must reload the dataset on which you fit the model or, if you wish to produce standardized margins, some other dataset.

```
. use mydata, clear
(data for fitting models)
```

If the dataset you loaded contained the data for standardization, you can stop reading; you know that to produce standardized margins, you need to specify the noesample option.

We reloaded the original data and want to produce margins for the estimation sample. In addition to the data, margins requires that e(sample) be set, as margins will remind us:

```
. margins sex
e(sample) does not identify the estimation sample
r(322);
```

The best solution is to use estimates esample to rebuild e(sample):

```
. estimates esample: y sex group age if site==1
```

If we knew we had no missing values in y and the covariates, we could type

```
. estimates esample: if site==1
```

Either way, margins would now work:

```
. margins sex
(usual output appears)
```

There is an alternative. We do not recommend it, but we admit that we have used it. Rather than rebuilding e(sample), you can use margins' noesample option to tell margins to skip using e(sample). You could then specify the appropriate if statement (if necessary) to identify the estimation sample:

```
. estimates use mymodel
. use mydata, clear
. margins sex if !missing(y, sex, group age) & site==1, noesample
```

In the above, we are not really running on a sample different from the estimation sample; we are merely using noesample to fool margins, and then we are specifying on the margins command the conditions equivalent to re-create e(sample).

If we wish to obtain vce(unconditional) results, however, noesample will be insufficient. We must also specify the force option,

```
. margins sex if !missing(y, sex, group age) & site==1,
> vce(unconditional) noesample force
(usual output appears)
```

Regardless of the approach you choose—resetting e(sample) or specifying noesample and possibly force—make sure you are right. In the vce(delta) case, you want to be right to ensure that you obtain the results you want. In the vce(unconditional) case, you need to be right because otherwise results will be statistically invalid.

## Syntax of at()

In `at(atspec)`, `atspec` may contain one or more of the following specifications:

```
varlist
(stat) varlist
varname = #
varname = (numlist)
varname = generate(exp)
```

where

1. `varnames` must be covariates in the previously fit model (estimation command).
2. Variable names (whether in `varname` or `varlist`) may be continuous variables, factor variables, or specific level variables, such as `age`, `group`, or `3.group`.
3. `varlist` may also be one of three standard lists:
  - (a) `_all` (all covariates),
  - (b) `_factor` (all factor-variable covariates), or
  - (c) `_continuous` (all continuous covariates).
4. `stat` can be any of the following:

| <i>stat</i>        | Description                                | Variables allowed |
|--------------------|--------------------------------------------|-------------------|
| <u>as</u> observed | at observed values in the sample (default) | all               |
| mean               | means (default for <i>varlist</i> )        | all               |
| median             | medians                                    | continuous        |
| p1                 | 1st percentile                             | continuous        |
| p2                 | 2nd percentile                             | continuous        |
| ...                | 3rd–49th percentiles                       | continuous        |
| p50                | 50th percentile (same as median)           | continuous        |
| ...                | 51st–97th percentiles                      | continuous        |
| p98                | 98th percentile                            | continuous        |
| p99                | 99th percentile                            | continuous        |
| min                | minimums                                   | continuous        |
| max                | maximums                                   | continuous        |
| zero               | fixed at zero                              | continuous        |
| base               | base level                                 | factors           |
| <u>as</u> balanced | all levels equally probable and sum to 1   | factors           |

Any *stat* except `zero`, `base`, and `asbalanced` may be prefixed with an `o` to get the overall statistic—the sample over all `over()` groups. For example, `omean`, `omedian`, and `op25`. Overall statistics differ from their correspondingly named statistics only when the `over()` or `within()` option is specified. When no *stat* is specified, `mean` is assumed. If *stat* is not followed by a *varlist*, *stat* is ignored.

*atspec* can involve multiple settings for one covariate as well as settings for multiple covariates. The following rules are applied when more than one covariate or value is included:

1. When more than one covariate is referenced in *atspec* but each covariate is set to only one value, all settings are applied in combination. For example, `at(x1=5 x2=0)` results in *margins* being estimated under one scenario, with *x1* set to 5, *x2* set to 0, and all other covariates set to their observed values (the default).
2. When multiple values are specified for a covariate, the covariate will be set to each of the values in turn. For example, `at(x1=5 x1=10)` or, equivalently, `at(x1=(5 10))` specifies that *x1* be set first to 5 and then to 10.
3. When multiple values are specified for more than one covariate, all possible combinations of settings are applied in turn. For example, `at(x1=(5 10) x2=(0 1))` results in *margins* being estimated under four scenarios: (*x1* = 5 *x2* = 0), (*x1* = 5 *x2* = 1), (*x1* = 10 *x2* = 0), and (*x1* = 10 *x2* = 1).
4. Settings may be specified for groups of covariates using three general varlists—`_all`, `_factor`, and `_continuous`. When *atspec* includes both specifications with general varlists and specifications with named covariates, the specifications for named covariates take precedence over general ones. For example, `at(x1=10 (means) _all)` sets *x1* to 10 while setting all other covariates to their means.
5. Only one (*stat*) varlist specification can be applied to a covariate. If more than one is specified, the rightmost specification is respected. For example, `at((means) x1 x2 (medians) x1 x2)` sets both *x1* and *x2* to their medians.
6. When both a (*stat*) specification and another specification are included for a named covariate, the other specification takes precedence. For example, `at(x1=5 (means) x1)` sets *x1* to 5.

In addition, `at()` can be repeated. When multiple `at()` options are specified, *atspecs* are processed sequentially. For instance, `at(x1=5) at(x2=0)` results in *margins* being estimated under two scenarios. The first sets *x1* to 5 and all other covariates, including *x2*, to their observed values. The second sets *x2* to 0 and all other covariates to their observed values. Note that this is different from the single `at(x1=5 x2=0)` specification, which sets *x1* and *x2* to the specified values simultaneously.

### Estimation commands that may be used with margins

*margins* may be used after most estimation commands.

*margins* cannot be used after estimation commands that do not produce full variance matrices, such as `exlogistic` and `expoisson` (see [R] [exlogistic](#) and [R] [expoisson](#)).

*margins* is all about covariates and cannot be used after estimation commands that do not post the covariates, which eliminates `gmm` (see [R] [gmm](#)).

*margins* cannot be used after some estimation commands that have an odd data organization, such as `nlogit`. However, *margins* has been specially adapted to work with many choice model estimation commands; see [CM] [margins](#).

## Video examples

Introduction to margins, part 1: Categorical variables

Introduction to margins, part 2: Continuous variables

Introduction to margins, part 3: Interactions

## Glossary

**adjusted mean.** A *margin* when the response is the linear predictor from linear regression, ANOVA, etc. For some authors, adjusting also implies adjusting for unbalanced data. See *Obtaining margins of responses* and see *Obtaining margins as though the data were balanced*.

**adjusted treatment mean.** When calculating *estimated marginal means* (also known as least-squares means) in the presence of continuous covariates, some authors recommend setting the continuous covariates equal to their means. This yields an estimated marginal mean that *Searle, Speed, and Milliken (1980)* refer to as an adjusted treatment mean. Adjusted treatment means are obtained using option `at((asbalanced) _factor (mean) _continuous)` or, equivalently, options `asbalanced` and `atmeans` together.

**average marginal effect.** See *marginal effect and average marginal effect*.

**average partial effect.** See *partial effect and average partial effect*.

**balanced data.** When a dataset contains an equal number of observations at each level of a factor variable, it is said to be balanced with respect to that variable. The concept of balanced data is only applicable to factor variables, not continuous variables. Balancing is common in designed experiments, but even if the data are not balanced, the `asbalanced` option allows margins to be calculated as though the data were balanced. Also see *Obtaining margins as though the data were balanced*.

**conditional margin.** A *margin* when the response is evaluated at fixed values of all the covariates. If any covariates are left as observed, the margin is called a predictive margin.

**effect.** The effect of  $x$  is the derivative of the *response* with respect to covariate  $x$ , or it is the difference in responses caused by a discrete change in  $x$ . Also see *marginal effect*.

The effect of  $x$  measures the change in the response for a change in  $x$ . Derivatives or differences might be reported as elasticities. If  $x$  is continuous, the effect is measured continuously. If  $x$  is a factor, the effect is measured with respect to each level of the factor and may be calculated as a discrete difference or as a continuous change, as measured by the derivative. `margins` calculates the discrete difference by default and calculates the derivative if the `continuous` option is specified.

**elasticity and semielasticity.** The elasticity of  $y$  with respect to  $x$  is  $d(\ln y)/d(\ln x) = (x/y) \times (dy/dx)$ , which is approximately equal to the proportional change in  $y$  for a proportional change in  $x$ .

The semielasticity of  $y$  with respect to  $x$  is either 1)  $dy/d(\ln x) = x \times (dy/dx)$  or 2)  $d(\ln y)/dx = (1/y) \times (dy/dx)$ , which is approximately 1) the change in  $y$  for a proportional change in  $x$  or 2) the proportional change in  $y$  for a change in  $x$ .

`margins` uses model predictions or user-specified statistics in `expression()` in place of  $y$  above.

**empty cell.** An interaction of levels of two or more factor variables for which you have no data. For instance, you have `sex` interacted with `group` in your model, and in your data there are no females in group 1. Empty cells affect which margins can be estimated; see *Estimability of margins*.

**estimability.** Estimability concerns whether a margin can be uniquely estimated (identified); see *Estimability of margins*.

**estimated marginal mean.** An estimated marginal mean, as defined by Searle, Speed, and Milliken (1980), is a margin assuming the levels of each factor covariate are balanced (equally likely), including interaction terms. This is obtained using the `asbalanced` option.

If the model includes continuous covariates, setting them equal to their sample means yields a type of estimated marginal mean known as an *adjusted treatment mean*.

When some cells are empty, it is impossible to estimate their marginal means. To get around this limitation, Searle, Speed, and Milliken introduced the concept of modified estimated marginal means, in which margins involving empty cells are redefined so that they become estimable. This is invoked by the `emptycells(reweight)` option. See *Balancing in the presence of empty cells*.

**least-squares mean.** Synonym for estimated marginal mean.

**margin.** A statistic calculated from predictions or other statistics of a previously fit model at fixed values of some covariates and averaging or otherwise integrating over the remaining covariates. The prediction or other statistic on which the margin is based is called the response.

If all the covariates are fixed, then the margin is called a conditional margin. If any covariates are left to vary, the margin is called a predictive margin.

In this documentation, we divide margins on the basis of whether the statistic is a response or a derivative of a response; see *Obtaining margins of responses* and *Obtaining margins of derivatives of responses*.

**marginal effect and average marginal effect.** The marginal effect of  $x$  is the *margin* of the *effect* of  $x$ . The term is popular with social scientists, and because of that, you might think the word marginal in marginal effect means derivative because of terms like marginal cost and marginal revenue. Marginal used in that way, however, refers to the derivative of revenue and the derivative of cost; it refers to the numerator, whereas marginal effect refers to the denominator. Moreover, *effect* is already a derivative or difference.

Some researchers interpret marginal in marginal effect to mean instantaneous, and thus a marginal effect is the instantaneous derivative rather than the discrete first difference, corresponding to margins' continuous option. Researchers who use marginal in this way refer to the discrete difference calculation of an effect as a partial effect.

Other researchers define marginal effect to be the margin when all covariates are held fixed and the average marginal effect when some covariates are not fixed.

**out-of-sample prediction.** Predictions made in one dataset using the results from a model fit on another. Sample here refers to the sample on which the model was fit, and out-of-sample refers to the dataset on which the predictions are made.

**partial effect and average partial effect.** Some authors restrict the term *marginal effect* to mean derivatives and use the term partial effect to denote discrete differences; see *marginal effect and average marginal effect*.

**population marginal mean.** The theoretical (true) value that is estimated by *estimated marginal mean*. We avoid this term because it can be confused with the concept of a population in survey statistics, with which the population marginal mean has no connection.

**posting results, posting margins.** A Stata concept having to do with storing the results from the `margins` command in `e()` so that those results can be used as if they were estimation results, thus allowing the subsequent use of postestimation commands, such as `test`, `testnl`, `lincom`, and `nlcom` (see [R] [test](#), [R] [testnl](#), [R] [lincom](#), and [R] [nlcom](#)). This is achieved by specifying `margins' post` option. See [Example 10: Testing margins—contrasts of margins](#).

**predictive margin.** A *margin* in which all the covariates are not fixed. When all covariates are fixed, it is called a *conditional margin*.

**recycled prediction.** A synonym for *predictive margin*.

**response.** A prediction or other statistic derived from combining the parameter estimates of a fitted model with data or specified values on covariates. Derivatives of responses are themselves responses. Responses are what we take *margins* of.

**standardized margin.** The margin calculated on data different from the data used to fit the model. The term *standardized* is usually reserved for situations in which the alternate population is a reference population, which may be real or artificial, and which is treated as fixed.

**subpopulation.** A subset of your sample that represents a subset of the population, such as the males in a sample of people. In survey contexts when it is desired to account for sampling of the covariates, standard errors for marginal statistics and effects need to account for both the population and the subpopulation. This is accomplished by specifying the `vce(unconditional)` option and one of the `subpop()` or `over()` options. In fact, the above is allowed even when your data are not `svyset` because `vce(unconditional)` implies that the sample represents a population.

## Stored results

`margins` stores the following in `r()`:

### Scalars

|                              |                                               |
|------------------------------|-----------------------------------------------|
| <code>r(N)</code>            | number of observations                        |
| <code>r(N_sub)</code>        | subpopulation observations                    |
| <code>r(N_clust)</code>      | number of clusters                            |
| <code>r(N_psu)</code>        | number of sampled PSUs, survey data only      |
| <code>r(N_strata)</code>     | number of strata, survey data only            |
| <code>r(df_r)</code>         | variance degrees of freedom, survey data only |
| <code>r(N_poststrata)</code> | number of post strata, survey data only       |
| <code>r(k_predict)</code>    | number of <code>predict()</code> options      |
| <code>r(k_margins)</code>    | number of terms in <i>marginlist</i>          |
| <code>r(k_by)</code>         | number of subpopulations                      |
| <code>r(k_at)</code>         | number of <code>at()</code> options           |
| <code>r(level)</code>        | confidence level of confidence intervals      |

### Macros

|                                |                                                                                                              |
|--------------------------------|--------------------------------------------------------------------------------------------------------------|
| <code>r(cmd)</code>            | <code>margins</code>                                                                                         |
| <code>r(cmdline)</code>        | command as typed                                                                                             |
| <code>r(est_cmd)</code>        | <code>e(cmd)</code> from original estimation results                                                         |
| <code>r(est_cmdline)</code>    | <code>e(cmdline)</code> from original estimation results                                                     |
| <code>r(title)</code>          | title in output                                                                                              |
| <code>r(subpop)</code>         | <i>subspec</i> from <code>subpop()</code>                                                                    |
| <code>r(model_vce)</code>      | <i>vcetype</i> from estimation command                                                                       |
| <code>r(model_vcetype)</code>  | Std. err. title from estimation command                                                                      |
| <code>r(vce)</code>            | <i>vcetype</i> specified in <code>vce()</code>                                                               |
| <code>r(vcetype)</code>        | title used to label Std. err.                                                                                |
| <code>r(clustvar)</code>       | name of cluster variable                                                                                     |
| <code>r(margins)</code>        | <i>marginlist</i>                                                                                            |
| <code>r(predict#_opts)</code>  | the <i>#</i> th <code>predict()</code> option                                                                |
| <code>r(predict#_label)</code> | label from the <i>#</i> th <code>predict()</code> option                                                     |
| <code>r(expression)</code>     | response expression                                                                                          |
| <code>r(xvars)</code>          | <i>varlist</i> from <code>dydx()</code> , <code>dyex()</code> , <code>eydx()</code> , or <code>eyex()</code> |
| <code>r(derivatives)</code>    | “”, “dy/dx”, “dy/ex”, “ey/dx”, or “ey/ex”                                                                    |
| <code>r(over)</code>           | <i>varlist</i> from <code>over()</code>                                                                      |
| <code>r(within)</code>         | <i>varlist</i> from <code>within()</code>                                                                    |
| <code>r(by)</code>             | union of <code>r(over)</code> and <code>r(within)</code> lists                                               |
| <code>r(by#)</code>            | interaction notation identifying the <i>#</i> th subpopulation                                               |
| <code>r(atstats#)</code>       | the <i>#</i> th <code>at()</code> specification                                                              |
| <code>r(emptycells)</code>     | <i>empspec</i> from <code>emptycells()</code>                                                                |
| <code>r(mcmethod)</code>       | <i>method</i> from <code>mcompare()</code>                                                                   |
| <code>r(mcadjustall)</code>    | <code>adjustall</code> or <code>empty</code>                                                                 |

### Matrices

|                           |                                                                                                                       |
|---------------------------|-----------------------------------------------------------------------------------------------------------------------|
| <code>r(b)</code>         | estimates                                                                                                             |
| <code>r(V)</code>         | variance–covariance matrix of the estimates                                                                           |
| <code>r(Jacobian)</code>  | Jacobian matrix                                                                                                       |
| <code>r(_N)</code>        | sample size corresponding to each margin estimate                                                                     |
| <code>r(at)</code>        | matrix of values from the <code>at()</code> options                                                                   |
| <code>r(chainrule)</code> | chain rule information from the fitted model                                                                          |
| <code>r(error)</code>     | margin estimability codes;<br>0 means estimable,<br>8 means not estimable                                             |
| <code>r(table)</code>     | matrix containing the margins with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |

`margins` with the `post` option also stores the following in `e()`:

#### Scalars

|                              |                                               |
|------------------------------|-----------------------------------------------|
| <code>e(N)</code>            | number of observations                        |
| <code>e(N_sub)</code>        | subpopulation observations                    |
| <code>e(N_clust)</code>      | number of clusters                            |
| <code>e(N_psu)</code>        | number of sampled PSUs, survey data only      |
| <code>e(N_strata)</code>     | number of strata, survey data only            |
| <code>e(df_r)</code>         | variance degrees of freedom, survey data only |
| <code>e(N_poststrata)</code> | number of post strata, survey data only       |
| <code>e(k_predict)</code>    | number of <code>predict()</code> options      |
| <code>e(k_margins)</code>    | number of terms in <i>marginlist</i>          |
| <code>e(k_by)</code>         | number of subpopulations                      |
| <code>e(k_at)</code>         | number of <code>at()</code> options           |

#### Macros

|                                |                                                                                                              |
|--------------------------------|--------------------------------------------------------------------------------------------------------------|
| <code>e(cmd)</code>            | <code>margins</code>                                                                                         |
| <code>e(cmdline)</code>        | command as typed                                                                                             |
| <code>e(est_cmd)</code>        | <code>e(cmd)</code> from original estimation results                                                         |
| <code>e(est_cmdline)</code>    | <code>e(cmdline)</code> from original estimation results                                                     |
| <code>e(wtype)</code>          | weight type                                                                                                  |
| <code>e(wexp)</code>           | weight expression                                                                                            |
| <code>e(title)</code>          | title in estimation output                                                                                   |
| <code>e(subpop)</code>         | <i>subspec</i> from <code>subpop()</code>                                                                    |
| <code>e(model_vce)</code>      | <i>vcetype</i> from estimation command                                                                       |
| <code>e(model_vcetype)</code>  | Std. err. title from estimation command                                                                      |
| <code>e(vce)</code>            | <i>vcetype</i> specified in <code>vce()</code>                                                               |
| <code>e(vcetype)</code>        | title used to label Std. err.                                                                                |
| <code>e(clustvar)</code>       | name of cluster variable                                                                                     |
| <code>e(properties)</code>     | <code>b V</code> , or just <code>b</code> if <code>nose</code> is specified                                  |
| <code>e(margins)</code>        | <i>marginlist</i>                                                                                            |
| <code>e(asbalanced)</code>     | factor variables <code>fvset</code> as <code>asbalanced</code>                                               |
| <code>e(asobserved)</code>     | factor variables <code>fvset</code> as <code>asobserved</code>                                               |
| <code>e(predict#_opts)</code>  | the <code>#th predict()</code> option                                                                        |
| <code>e(predict#_label)</code> | label from the <code>#th predict()</code> option                                                             |
| <code>e(expression)</code>     | prediction expression                                                                                        |
| <code>e(xvars)</code>          | <i>varlist</i> from <code>dydx()</code> , <code>dyex()</code> , <code>eydx()</code> , or <code>eyex()</code> |
| <code>e(derivatives)</code>    | “”, “ <code>dy/dx</code> ”, “ <code>dy/ex</code> ”, “ <code>ey/dx</code> ”, or “ <code>ey/ex</code> ”        |
| <code>e(over)</code>           | <i>varlist</i> from <code>over()</code>                                                                      |
| <code>e(within)</code>         | <i>varlist</i> from <code>within()</code>                                                                    |
| <code>e(by)</code>             | union of <code>r(over)</code> and <code>r(within)</code> lists                                               |
| <code>e(by#)</code>            | interaction notation identifying the <code>#th</code> subpopulation                                          |
| <code>e(atstats#)</code>       | the <code>#th at()</code> specification                                                                      |
| <code>e(emptycells)</code>     | <i>empspec</i> from <code>emptycells()</code>                                                                |

#### Matrices

|                           |                                                     |
|---------------------------|-----------------------------------------------------|
| <code>e(b)</code>         | estimates                                           |
| <code>e(V)</code>         | variance–covariance matrix of the estimates         |
| <code>e(Jacobian)</code>  | Jacobian matrix                                     |
| <code>e(_N)</code>        | sample size corresponding to each margin estimate   |
| <code>e(error)</code>     | error code corresponding to <code>e(b)</code>       |
| <code>e(at)</code>        | matrix of values from the <code>at()</code> options |
| <code>e(chainrule)</code> | chain rule information from the fitted model        |

#### Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

## Methods and formulas

Margins are statistics calculated from predictions of a previously fit model at fixed values of some covariates and averaging or otherwise integrating over the remaining covariates. There are many names for the different statistics that margins can compute: estimates marginal means (see Searle, Speed, and Milliken [1980]), predictive margins (see Graubard and Korn [2004]), marginal effects (see Greene [2018]), and average marginal/partial effects (see Wooldridge [2010] and Bartus [2005]).

Methods and formulas are presented under the following headings:

- Notation*
- Marginal effects*
- Fixing covariates and balancing factors*
- Estimable functions*
- Standard errors conditional on the covariates*
- Unconditional standard errors*

## Notation

Let  $\theta$  be the vector of parameters in the current model fit, let  $\mathbf{z}$  be a vector of covariate values, and let  $f(\mathbf{z}, \theta)$  be a scalar-valued function returning the value of the predictions of interest. The following table illustrates the parameters and default prediction for several of Stata's estimation commands.

| Command  | $\theta$                           | $\mathbf{z}$                 | $f(\mathbf{z}, \theta)$                                  |
|----------|------------------------------------|------------------------------|----------------------------------------------------------|
| regress  | $\beta$                            | $\mathbf{x}$                 | $\mathbf{x}\beta$                                        |
| cloglog  | $\beta$                            | $\mathbf{x}$                 | $1 - e^{-e^{x\beta}}$                                    |
| logit    | $\beta$                            | $\mathbf{x}$                 | $1/(1 + e^{-x\beta})$                                    |
| poisson  | $\beta$                            | $\mathbf{x}$                 | $e^{x\beta}$                                             |
| probit   | $\beta$                            | $\mathbf{x}$                 | $\Phi(\mathbf{x}\beta)$                                  |
| biprobit | $\beta_1, \beta_2, \rho$           | $\mathbf{x}_1, \mathbf{x}_2$ | $\Phi_2(\mathbf{x}_1\beta_1, \mathbf{x}_2\beta_2, \rho)$ |
| mlogit   | $\beta_1, \beta_2, \dots, \beta_k$ | $\mathbf{x}$                 | $e^{-x\beta_1} / (\sum_i e^{-x\beta_i})$                 |
| nbreg    | $\beta, \ln\alpha$                 | $\mathbf{x}$                 | $e^{x\beta}$                                             |

$\Phi()$  and  $\Phi_2()$  are cumulative distribution functions:  $\Phi()$  for the standard normal distribution and  $\Phi_2()$  for the standard bivariate normal distribution.

margins computes estimates of

$$p(\theta) = \frac{1}{M_{S_p}} \sum_{j=1}^M \delta_j(S_p) f(\mathbf{z}_j, \theta)$$

where  $\delta_j(S_p)$  identifies elements within the subpopulation  $S_p$  (for the prediction of interest),

$$\delta_j(S_p) = \begin{cases} 1, & j \in S_p \\ 0, & j \notin S_p \end{cases}$$

$M_{S_p}$  is the subpopulation size,

$$M_{S_p} = \sum_{j=1}^M \delta_j(S_p)$$

and  $M$  is the population size.

Let  $\hat{\theta}$  be the vector of parameter estimates. Then, margins estimates  $p(\theta)$  via

$$\hat{p} = \frac{1}{w.} \sum_{j=1}^N \delta_j(S_p) w_j f(\mathbf{z}_j, \hat{\theta})$$

where

$$w. = \sum_{j=1}^N \delta_j(S_p) w_j$$

$\delta_j(S_p)$  indicates whether observation  $j$  is in subpopulation  $S_p$ ,  $w_j$  is the weight for the  $j$ th observation, and  $N$  is the sample size.

## Marginal effects

margins also computes marginal and partial effects. For the marginal effect of continuous covariate  $x$ , margins computes

$$\hat{p} = \frac{1}{w.} \sum_{j=1}^N \delta_j(S_p) w_j h(\mathbf{z}_j, \hat{\theta})$$

where

$$h(\mathbf{z}, \theta) = \frac{\partial f(\mathbf{z}, \theta)}{\partial x}$$

The marginal effect for level  $k$  of factor variable  $A$  is the simple contrast (also known as difference) comparing its margin with the margin at the base level.

$$h(\mathbf{z}, \theta) = f(\mathbf{z}, \theta|A = k) - f(\mathbf{z}, \theta|A = \text{base})$$

Elasticities and semielasticities given by `eyex()` and `dyex()` are only available for continuous covariates. Elasticities are computed using

$$h(\mathbf{z}, \theta) = \frac{\partial \ln\{f(\mathbf{z}, \theta)\}}{\partial \ln(x)}$$

and semielasticities, `dyex()`, using

$$h(\mathbf{z}, \theta) = \frac{\partial f(\mathbf{z}, \theta)}{\partial \ln(x)}$$

Semielasticities of the form `eydx()` for continuous covariates are computed using

$$h(\mathbf{z}, \theta) = \frac{\partial \ln\{f(\mathbf{z}, \theta)\}}{\partial x}$$

and for discrete covariates using

$$h(\mathbf{z}, \theta) = \ln\{f(\mathbf{z}, \theta|A = k)\} - \ln\{f(\mathbf{z}, \theta|A = \text{base})\}$$

## Fixing covariates and balancing factors

`margins` controls the values in each  $\mathbf{z}$  vector through the `marginlist`, the `at()` option, the `atmeans` option, and the `asbalanced` and `emptycells()` options. Suppose  $\mathbf{z}$  is composed of the elements from the equation specification

$$A\#B\ x$$

where  $A$  is a factor variable with  $a$  levels,  $B$  is a factor variable with  $b$  levels, and  $x$  is a continuous covariate. To simplify the notation for this discussion, assume the levels of  $A$  and  $B$  start with 1 and are contiguous. Then

$$\mathbf{z} = (A_1, \dots, A_a, B_1, \dots, B_b, A_1B_1, A_1B_2, \dots, A_aB_b, x, 1)$$

where  $A_i$ ,  $B_j$ , and  $A_iB_j$  represent the indicator values for the factor variables  $A$  and  $B$  and the interaction  $A\#B$ .

When factor  $A$  is in the `marginlist`, `margins` replaces  $A$  with  $i$  and then computes the mean of the subsequent prediction, for  $i = 1, \dots, a$ . When the interaction term  $A\#B$  is in the `marginlist`, `margins` replaces  $A$  with  $i$  and  $B$  with  $j$ , and then computes the mean of the subsequent prediction, for all combinations of  $i = 1, \dots, a$  and  $j = 1, \dots, b$ .

The `at()` option sets model covariates to fixed values. For example, `at(x=15)` causes `margins` to temporarily set  $x$  to 15 for each observation in the dataset before computing any predictions. Similarly, `at((median) x)` causes `margins` to temporarily set  $x$  to the median of  $x$  using the current dataset.

When factor variable  $A$  is specified as `asbalanced`, `margins` sets each  $A_i$  to  $1/a$ . Thus, each  $\mathbf{z}$  vector will look like

$$\mathbf{z} = (1/a, \dots, 1/a, B_1, \dots, B_b, B_1/a, B_2/a, \dots, B_b/a, x, 1)$$

If  $B$  is also specified as `asbalanced`, then each  $B_j$  is set to  $1/b$ , and each  $\mathbf{z}$  vector will look like

$$\mathbf{z} = (1/a, \dots, 1/a, 1/b, \dots, 1/b, 1/ab, 1/ab, \dots, 1/ab, x, 1)$$

If `emptycells(reweight)` is also specified, then `margins` uses a different balancing weight for each element of  $\mathbf{z}$ , depending on how many empty cells the element is associated with. Let  $\delta_{ij}$  indicate that the  $ij$ th cell of  $A\#B$  was observed in the estimation sample.

$$\delta_{ij} = \begin{cases} 0, & A = i \text{ and } B = j \text{ was an empty cell} \\ 1, & \text{otherwise} \end{cases}$$

For the grand margin, the affected elements of  $\mathbf{z}$  and their corresponding balancing weights are

$$A_i = \frac{\sum_j \delta_{ij}}{\sum_k \sum_j \delta_{kj}}$$

$$B_j = \frac{\sum_i \delta_{ij}}{\sum_i \sum_k \delta_{ik}}$$

$$A_iB_j = \frac{\delta_{ij}}{\sum_k \sum_l \delta_{kl}}$$

For the  $j$ th margin of  $B$ , the affected elements of  $\mathbf{z}$  and their corresponding balancing weights are

$$A_i = \frac{\delta_{ij}}{\sum_k \delta_{kj}}$$

$$B_l = \begin{cases} 1, & \text{if } l = j \text{ and not all } \delta_{ij} \text{ are zero} \\ 0, & \text{otherwise} \end{cases}$$

$$A_i B_l = \frac{\delta_{il}}{\sum_k \delta_{kl}} B_l$$

## Estimable functions

The fundamental idea behind estimable functions is clearly defined in the statistical literature for linear models; see [Searle and Gruber \(2017\)](#). Assume that we are working with the following linear model:

$$\mathbf{y} = \mathbf{X}\mathbf{b} + \mathbf{e}$$

where  $\mathbf{y}$  is an  $N \times 1$  vector of responses,  $\mathbf{X}$  is an  $N \times p$  matrix of covariate values,  $\mathbf{b}$  is a  $p \times 1$  vector of coefficients, and  $\mathbf{e}$  is a vector of random errors. Assuming a constant variance for the random errors, the normal equations for the least-squares estimator,  $\hat{\mathbf{b}}$ , are

$$\mathbf{X}'\mathbf{X}\hat{\mathbf{b}} = \mathbf{X}'\mathbf{y}$$

When  $\mathbf{X}$  is not of full column rank, we will need a generalized inverse (g-inverse) of  $\mathbf{X}'\mathbf{X}$  to solve for  $\hat{\mathbf{b}}$ . Let  $\mathbf{G}$  be a g-inverse of  $\mathbf{X}'\mathbf{X}$ .

[Searle and Gruber \(2017\)](#) defines a linear function of the parameters as *estimable* if it is identically equal to some linear function of the expected values of the  $\mathbf{y}$  vector. Let  $\mathbf{H} = \mathbf{G}\mathbf{X}'\mathbf{X}$ . Then this definition simplifies to the following rule:

$$\mathbf{z}\mathbf{b} \text{ is estimable if } \mathbf{z} = \mathbf{z}\mathbf{H}$$

`margins` generalizes this to nonlinear functions by assuming the prediction function  $f(\mathbf{z}, \boldsymbol{\theta})$  is a function of one or more of the linear predictions from the equations in the model that  $\boldsymbol{\theta}$  represents.

$$f(\mathbf{z}, \boldsymbol{\theta}) = h(\mathbf{z}_1\boldsymbol{\beta}_1, \mathbf{z}_2\boldsymbol{\beta}_2, \dots, \mathbf{z}_k\boldsymbol{\beta}_k)$$

$\mathbf{z}_i\boldsymbol{\beta}_i$  is considered estimable if  $\mathbf{z}_i = \mathbf{z}_i\mathbf{H}_i$ , where  $\mathbf{H}_i = \mathbf{G}_i\mathbf{X}'_i\mathbf{X}_i$ ,  $\mathbf{G}_i$  is a g-inverse for  $\mathbf{X}'_i\mathbf{X}_i$ , and  $\mathbf{X}_i$  is the matrix of covariates from the  $i$ th equation of the fitted model. `margins` considers  $p(\boldsymbol{\theta})$  to be estimable if every  $\mathbf{z}_i\boldsymbol{\beta}_i$  is estimable.

## Standard errors conditional on the covariates

By default, `margins` uses the delta method to estimate the variance of  $\hat{p}$ .

$$\widehat{\text{Var}}(\hat{p} | \mathbf{z}) = \mathbf{v}'\mathbf{V}\mathbf{v}$$

where  $\mathbf{V}$  is a variance estimate for  $\hat{\boldsymbol{\theta}}$  and

$$\mathbf{v} = \left. \frac{\partial \hat{p}}{\partial \boldsymbol{\theta}} \right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}}$$

This variance estimate is conditional on the  $\mathbf{z}$  vectors used to compute the marginalized predictions.

## Unconditional standard errors

margins with the vce(unconditional) option uses linearization to estimate the unconditional variance of  $\hat{\theta}$ . Linearization uses the variance estimator for the total of a score variable for  $\hat{p}$  as an approximate estimator for  $\text{Var}(\hat{p})$ ; see [SVY] **Variance estimation**. margins requires that the model was fit using some form of linearized variance estimator and that predict, scores computes the appropriate score values for the linearized variance estimator.

The score for  $\hat{p}$  from the  $j$ th observation is given by

$$s_j = \frac{\partial \hat{p}}{\partial w_j} = -\frac{\delta_j(S_p)}{w} \hat{p} + \frac{\delta_j(S_p)}{w} f(\mathbf{z}_j, \hat{\theta}) + \frac{1}{w} \sum_{i=1}^N \delta_i(S_p) w_i \frac{\partial f(\mathbf{z}_i, \hat{\theta})}{\partial w_j}$$

The remaining partial derivative can be decomposed using the chain rule.

$$\frac{\partial f(\mathbf{z}_i, \hat{\theta})}{\partial w_j} = \left( \frac{\partial f(\mathbf{z}_i, \theta)}{\partial \theta} \Big|_{\theta=\hat{\theta}} \right) \left( \frac{\partial \hat{\theta}}{\partial w_j} \right)'$$

This is the inner product of two vectors, the second of which is not a function of the  $i$  index. Thus, the score is

$$s_j = -\frac{\delta_j(S_p)}{w} \hat{p} + \frac{\delta_j(S_p)}{w} f(\mathbf{z}_j, \hat{\theta}) + \left( \frac{\partial \hat{p}}{\partial \theta} \Big|_{\theta=\hat{\theta}} \right) \left( \frac{\partial \hat{\theta}}{\partial w_j} \right)'$$

If  $\hat{\theta}$  was derived from a system of equations (such as in linear regression or maximum likelihood estimation), then  $\hat{\theta}$  is the solution to

$$\mathbf{G}(\theta) = \sum_{j=1}^N \delta_j(S_m) w_j \mathbf{g}(\theta, \mathbf{y}_j, \mathbf{x}_j) = \mathbf{0}$$

where  $S_m$  identifies the subpopulation used to fit the model,  $\mathbf{g}()$  is the model's gradient function, and  $\mathbf{y}_j$  and  $\mathbf{x}_j$  are the values of the dependent and independent variables for the  $j$ th observation. We can use linearization to derive a first-order approximation for  $\partial \hat{\theta} / \partial w_j$ .

$$\mathbf{G}(\hat{\theta}) \approx \mathbf{G}(\theta_0) + \frac{\partial \mathbf{G}(\theta)}{\partial \theta} \Big|_{\theta=\theta_0} (\hat{\theta} - \theta_0)$$

Let  $\mathbf{H}$  be the Hessian matrix

$$\mathbf{H} = \frac{\partial \mathbf{G}(\theta)}{\partial \theta} \Big|_{\theta=\theta_0}$$

Then

$$\hat{\theta} \approx \theta_0 + (-\mathbf{H})^{-1} \mathbf{G}(\theta_0)$$

and

$$\frac{\partial \hat{\theta}}{\partial w_j} \approx (-\mathbf{H})^{-1} \frac{\partial \mathbf{G}(\theta)}{\partial w_j} \Big|_{\theta=\hat{\theta}} = (-\mathbf{H})^{-1} \delta_j(S_m) \mathbf{g}(\hat{\theta}, \mathbf{y}_j, \mathbf{x}_j)$$

The computed value of the score for  $\hat{p}$  for the  $j$ th observation is

$$s_j = \mathbf{v}' \mathbf{u}_j$$

where

$$\mathbf{v} = \begin{bmatrix} -\frac{\hat{p}}{w_j} \\ \frac{1}{w_j} \\ \frac{\partial \hat{p}}{\partial \boldsymbol{\theta}} (-\mathbf{H})^{-1} \end{bmatrix}$$

and

$$\mathbf{u}_j = \begin{bmatrix} \delta_j(S_p) \\ \delta_j(S_p) f(\mathbf{z}_j, \hat{\boldsymbol{\theta}}) \\ \delta_j(S_m) \mathbf{g}(\hat{\boldsymbol{\theta}}, \mathbf{y}_j, \mathbf{x}_j) \end{bmatrix}$$

Thus, the variance estimate for  $\hat{p}$  is

$$\widehat{\text{Var}}(\hat{p}) = \mathbf{v}' \widehat{\text{Var}}(\widehat{\mathbf{U}}) \mathbf{v}$$

where

$$\widehat{\mathbf{U}} = \sum_{j=1}^N w_j \mathbf{u}_j$$

margins uses the model-based variance estimates for  $(-\mathbf{H})^{-1}$  and the scores from predict for  $\mathbf{g}(\hat{\boldsymbol{\theta}}, \mathbf{y}_j, \mathbf{x}_j)$ .

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## **Also see**

[R] **contrast** — Contrasts and linear hypothesis tests after estimation

[R] **margins, contrast** — Contrasts of margins

[R] **margins, pwcompare** — Pairwise comparisons of margins

[R] **margins postestimation** — Postestimation tools for margins

[R] **marginsplot** — Graph results from margins (profile plots, etc.)

[R] **lincom** — Linear combinations of parameters

[R] **nlcom** — Nonlinear combinations of parameters

[R] **predict** — Obtain predictions, residuals, etc., after estimation

[R] **predictnl** — Obtain nonlinear predictions, standard errors, etc., after estimation

[U] **20 Estimation and postestimation commands**

## Postestimation commands

The following standard postestimation command is available after `margins`:

| Command                  | Description                                                                         |
|--------------------------|-------------------------------------------------------------------------------------|
| <code>marginsplot</code> | graph the results from <code>margins</code> —profile plots, interaction plots, etc. |

For information on `marginsplot`, see [R] [marginsplot](#).

The following standard postestimation commands are available after `margins`, `post`:

| Command                      | Description                                                                                       |
|------------------------------|---------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                               |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                      |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                |
| <code>estimates</code>       | cataloging estimation results                                                                     |
| <code>etable</code>          | table of estimation results                                                                       |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters    |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                              |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                |

## Remarks and examples

Continuing with the example from *Example 8: Margins of interactions* in [R] [margins](#), we use the dataset and refit the logistic model of outcome:

```
. use https://www.stata-press.com/data/r19/margex
(Artificial data for margins)
. logistic outcome sex##group age
(output omitted)
```

We then estimate the margins for males and females and post the margins as estimation results with a full VCE.

```
. margins sex, post
Predictive margins Number of obs = 3,000
Model VCE: OIM
Expression: Pr(outcome), predict()
```

|        | Delta-method |           | z     | P> z  | [95% conf. interval] |          |
|--------|--------------|-----------|-------|-------|----------------------|----------|
|        | Margin       | std. err. |       |       |                      |          |
| sex    |              |           |       |       |                      |          |
| Male   | .1600644     | .0125653  | 12.74 | 0.000 | .1354368             | .184692  |
| Female | .1966902     | .0100043  | 19.66 | 0.000 | .1770821             | .2162983 |

We can now use nlcom (see [R] [nlcom](#)) to estimate a risk ratio of females to males using the average probabilities for females and males posted by margins:

```
. nlcom (risk_ratio: _b[1.sex] / _b[0.sex])
 risk_ratio: _b[1.sex] / _b[0.sex]
```

|            | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|------------|-------------|-----------|-------|-------|----------------------|----------|
| risk_ratio | 1.228819    | .1149538  | 10.69 | 0.000 | 1.003514             | 1.454124 |

We could similarly estimate the average risk difference between females and males:

```
. nlcom (risk_diff: _b[1.sex] - _b[0.sex])
 risk_diff: _b[1.sex] - _b[0.sex]
```

|           | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |         |
|-----------|-------------|-----------|------|-------|----------------------|---------|
| risk_diff | .0366258    | .0160632  | 2.28 | 0.023 | .0051425             | .068109 |

## Also see

[R] [margins](#) — Marginal means, predictive margins, and marginal effects

[R] [marginsplot](#) — Graph results from margins (profile plots, etc.)

[U] [20 Estimation and postestimation commands](#)

Description  
Suboptions  
Reference

Quick start  
Remarks and examples  
Also see

Menu  
Stored results

Syntax  
Methods and formulas

## Description

`margins` with the `contrast` option or with contrast operators performs contrasts of margins. This extends the capabilities of `contrast` to any of the nonlinear responses, predictive margins, or other margins that can be estimated by `margins`.

## Quick start

Reference category contrasts of the predictive margins for `a` after `logit y a##b x1`  
`margins r.a`

Contrasts of predictive margins for `a` with the previous level  
`margins ar.a`

Test the equality of predictive margins for `a`  
`margins a, contrast`

Reference category contrasts of predictive margins for `x1 = 10` and `x1 = 20` with `x1 = 0`  
`margins, at(x1=(0 10 20)) contrast(atcontrast(r))`

Average partial effect of increasing `x1` by 100 for each observation after `probit y x1 x2`  
`margins, at((asobserved) _all) at(x1=generate(x1+100)) ///  
contrast(atcontrast(r))`

## Menu

Statistics > Postestimation

## Syntax

```
margins [marginlist] [if] [in] [weight] [, contrast margins_options]
```

```
margins [marginlist] [if] [in] [weight] [, contrast(suboptions) margins_options]
```

where *marginlist* is a list of factor variables or interactions that appear in the current estimation results. The variables may be typed with or without [contrast operators](#), and you may use any factor-variable syntax:

```
. margins sex##group, contrast
. margins sex##g.group, contrast
. margins sex@group, contrast
```

See the [operators \(\*op.\*\)](#) table in [\[R\] contrast](#) for the list of contrast operators. Contrast operators may also be specified on the variables in `margins's` `over()` and `within()` options to perform contrasts across the levels of those variables.

| <i>suboptions</i>                                         | Description                                                                                                      |
|-----------------------------------------------------------|------------------------------------------------------------------------------------------------------------------|
| Contrast                                                  |                                                                                                                  |
| <code>overall</code>                                      | add a joint hypothesis test for all specified contrasts                                                          |
| <code>lincom</code>                                       | treat user-defined contrasts as linear combinations                                                              |
| <code>predict(<i>op</i>[ <code>._predict</code> ])</code> | apply the <i>op.</i> contrast operator to the groups defined by multiple <code>predict()</code> options          |
| <code>atcontrast(<i>op</i>[ <code>._at</code> ])</code>   | apply the <i>op.</i> contrast operator to the groups defined by <code>at()</code>                                |
| <code>predictjoint</code>                                 | test jointly across all groups defined by multiple <code>predict()</code> options                                |
| <code>atjoint</code>                                      | test jointly across all groups defined by <code>at()</code>                                                      |
| <code>overjoint</code>                                    | test jointly across all levels of the unoperated <code>over()</code> variables                                   |
| <code>withinjoint</code>                                  | test jointly across all levels of the unoperated <code>within()</code> variables                                 |
| <code>marginswithin</code>                                | perform contrasts within the levels of the unoperated terms in <i>marginlist</i>                                 |
| <code>cieffects</code>                                    | show effects table with confidence intervals                                                                     |
| <code>pveffects</code>                                    | show effects table with <i>p</i> -values                                                                         |
| <code>effects</code>                                      | show effects table with confidence intervals and <i>p</i> -values                                                |
| <code>nowald</code>                                       | suppress table of Wald tests                                                                                     |
| <code>noatlevels</code>                                   | report only the overall Wald test for terms that use the <code>within @</code> or nested <code> </code> operator |
| <code>nosvyadjust</code>                                  | compute unadjusted Wald tests for survey results                                                                 |

`collect` is allowed; see [\[U\] 11.1.10 Prefix commands](#).

`fweights`, `awweights`, `iweights`, and `pweights` are allowed; see [\[U\] 11.1.6 weight](#).

## Suboptions

### Contrast

`overall` specifies that a joint hypothesis test over all terms be performed.

`lincom` specifies that user-defined contrasts be treated as linear combinations. The default is to require that all user-defined contrasts sum to zero. (Summing to zero is part of the definition of a contrast.)

`predict` (`op` [`._predict`]) specifies that the `op` contrast operator be applied to the groups defined by multiple specifications of `margins`'s `predict()` option. The default behavior, by comparison, is to perform tests and contrasts within these groups.

`atcontrast` (`op` [`._at`]) specifies that the `op` contrast operator be applied to the groups defined by the `at()` option(s). The default behavior, by comparison, is to perform tests and contrasts within the groups defined by the `at()` option(s).

See [example 6](#) in *Remarks and examples*.

`predictjoint` specifies that joint tests be performed across all groups defined by multiple specifications of `margins`'s `predict()` option. The default behavior, by comparison, is to perform contrasts and tests within each group.

`atjoint` specifies that joint tests be performed across all groups defined by the `at()` option. The default behavior, by comparison, is to perform contrasts and tests within each group.

See [example 5](#) in *Remarks and examples*.

`overjoint` specifies how unoperated variables in the `over()` option are treated.

Each variable in the `over()` option may be specified either with or without a contrast operator. For contrast-operated variables, the specified contrast comparisons are always performed.

`overjoint` specifies that joint tests be performed across all levels of the unoperated variables. The default behavior, by comparison, is to perform contrasts and tests within each combination of levels of the unoperated variables.

See [example 3](#) in *Remarks and examples*.

`withinjoint` specifies how unoperated variables in the `within()` option are treated.

Each variable in the `within()` option may be specified either with or without a contrast operator. For contrast-operated variables, the specified contrast comparisons are always performed.

`withinjoint` specifies that joint tests be performed across all levels of the unoperated variables. The default behavior, by comparison, is to perform contrasts and tests within each combination of levels of the unoperated variables.

`marginswithin` specifies how unoperated variables in `marginlist` are treated.

Each variable in `marginlist` may be specified either with or without a contrast operator. For contrast-operated variables, the specified contrast comparisons are always performed.

`marginswithin` specifies that contrasts and tests be performed within each combination of levels of the unoperated variables. The default behavior, by comparison, is to perform joint tests across all levels of the unoperated variables.

See [example 4](#) in *Remarks and examples*.

`ci effects` specifies that a table containing a confidence interval for each individual contrast be reported.

`pveffects` specifies that a table containing a  $p$ -value for each individual contrast be reported.

`effects` specifies that a single table containing a confidence interval and  $p$ -value for each individual contrast be reported.

`nowald` suppresses the table of Wald tests.

`noatlevels` indicates that only the overall Wald test be reported for each term containing within or nested (@ or |) operators.

`nosvyadjust` is for use with `svy` estimation commands. It specifies that the Wald test be carried out without the default adjustment for the design degrees of freedom. That is to say the test is carried out as  $W/k \sim F(k, d)$  rather than as  $(d - k + 1)W/(kd) \sim F(k, d - k + 1)$ , where  $k$  is the dimension of the test and  $d$  is the total number of sampled PSUs minus the total number of strata.

## Remarks and examples

Remarks are presented under the following headings:

*Contrasts of margins*

*Contrasts and the `over()` option*

*The overjoint suboption*

*The marginswithin suboption*

*Contrasts and the `at()` option*

*Estimating treatment effects with margins*

*Conclusion*

## Contrasts of margins

### ▷ Example 1

Estimating contrasts of margins is as easy as adding a contrast operator to the variable name. Let's review [Example 2: A simple case after logistic](#) of [R] **margins**. Variable `sex` is coded 0 for males and 1 for females.

```
. use https://www.stata-press.com/data/r19/margex
(Artificial data for margins)

. logistic outcome i.sex i.group
(output omitted)

. margins sex

Predictive margins Number of obs = 3,000
Model VCE: OIM

Expression: Pr(outcome), predict()
```

|        | Delta-method |           |       |       |                      |
|--------|--------------|-----------|-------|-------|----------------------|
|        | Margin       | std. err. | z     | P> z  | [95% conf. interval] |
| sex    |              |           |       |       |                      |
| Male   | .1286796     | .0111424  | 11.55 | 0.000 | .106841 .1505182     |
| Female | .1905087     | .0089719  | 21.23 | 0.000 | .1729241 .2080933    |

The first margin, 0.13, is the average probability of a positive outcome, treating everyone as if they were male. The second margin, 0.19, is the average probability of a positive outcome, treating everyone as if they were female. We can compare females with males by rerunning `margins` and adding a contrast operator:

```
. margins r.sex
Contrasts of predictive margins Number of obs = 3,000
Model VCE: OIM
Expression: Pr(outcome), predict()
```

|     | df | chi2  | P>chi2 |
|-----|----|-------|--------|
| sex | 1  | 16.61 | 0.0000 |

|                         | Delta-method |           |                      |
|-------------------------|--------------|-----------|----------------------|
|                         | Contrast     | std. err. | [95% conf. interval] |
| sex<br>(Female vs Male) | .0618291     | .0151719  | .0320927 .0915656    |

The `r.` prefix for `sex` is the reference-category contrast operator—see [R] [contrast](#). (The default reference category is zero, the lowest value of `sex`.) Contrast operators in a *marginlist* work just as they do in the *termlist* of a `contrast` command.

The contrast estimate of 0.06 says that unconditional on group, females on average are about 6% more likely than males to have a positive outcome. The  $\chi^2$  statistic of 16.61 shows that the contrast is significantly different from zero.

You may be surprised that we did not need to include the `contrast` option to estimate our contrast. If we had included the option, our output would not have changed:

```
. margins r.sex, contrast
Contrasts of predictive margins Number of obs = 3,000
Model VCE: OIM
Expression: Pr(outcome), predict()
```

|     | df | chi2  | P>chi2 |
|-----|----|-------|--------|
| sex | 1  | 16.61 | 0.0000 |

|                         | Delta-method |           |                      |
|-------------------------|--------------|-----------|----------------------|
|                         | Contrast     | std. err. | [95% conf. interval] |
| sex<br>(Female vs Male) | .0618291     | .0151719  | .0320927 .0915656    |

The contrast option is useful mostly for its suboptions, which control the output and how contrasts are estimated in more complicated situations. But contrast may be specified on its own (without contrast operators or suboptions) if we do not need estimates or confidence intervals:

```
. margins sex group, contrast
Contrasts of predictive margins Number of obs = 3,000
Model VCE: OIM
Expression: Pr(outcome), predict()
```

|       | df | chi2   | P>chi2 |
|-------|----|--------|--------|
| sex   | 1  | 16.61  | 0.0000 |
| group | 2  | 225.76 | 0.0000 |

Each  $\chi^2$  statistic is a joint test of constituent contrasts. The test for group has two degrees of freedom because group has three levels.



## Contrasts and the over() option

### ▷ Example 2

It is common to estimate margins at combinations of factor levels, and margins, contrast includes several suboptions for contrasting such margins. Let's fit a model with two categorical predictors and their interaction:

```
. logistic outcome agegroup##group
Logistic regression Number of obs = 3,000
 LR chi2(8) = 520.64
 Prob > chi2 = 0.0000
Log likelihood = -1105.7504 Pseudo R2 = 0.1906
```

| outcome   | Odds ratio | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-----------|------------|-----------|-------|-------|----------------------|----------|
| agegroup  |            |           |       |       |                      |          |
| 30-39     | 3.54191    | 2.226951  | 2.01  | 0.044 | 1.032882             | 12.14576 |
| 40+       | 16.23351   | 9.611879  | 4.71  | 0.000 | 5.086452             | 51.80955 |
| group     |            |           |       |       |                      |          |
| 2         | .834507    | .5663738  | -0.27 | 0.790 | .2206611             | 3.15598  |
| 3         | .2146739   | .1772903  | -1.86 | 0.062 | .042541              | 1.083306 |
| agegroup# |            |           |       |       |                      |          |
| group     |            |           |       |       |                      |          |
| 30-39#2   | .4426927   | .3358505  | -1.07 | 0.283 | .1000772             | 1.958257 |
| 30-39#3   | 1.16088    | 1.103521  | 0.16  | 0.875 | .1801538             | 7.480508 |
| 40+#2     | .440672    | .3049393  | -1.18 | 0.236 | .1135259             | 1.711055 |
| 40+#3     | .4407892   | .4034666  | -0.89 | 0.371 | .0732998             | 2.650693 |
| _cons     | .0379747   | .0223371  | -5.56 | 0.000 | .0119897             | .1202762 |

Note: **\_cons** estimates baseline odds.

Each of agegroup and group has three levels. To compare each age group with the reference category on the probability scale, we can again use margins with the r. contrast operator.

```
. margins r.agegroup
Contrasts of predictive margins Number of obs = 3,000
Model VCE: OIM
Expression: Pr(outcome), predict()
```

|                  | df | chi2   | P>chi2 |  |
|------------------|----|--------|--------|--|
| agegroup         |    |        |        |  |
| (30-39 vs 20-29) | 1  | 10.04  | 0.0015 |  |
| (40+ vs 20-29)   | 1  | 224.44 | 0.0000 |  |
| Joint            | 2  | 238.21 | 0.0000 |  |

|                  | Contrast | Delta-method<br>std. err. | [95% conf. interval] |          |
|------------------|----------|---------------------------|----------------------|----------|
| agegroup         |          |                           |                      |          |
| (30-39 vs 20-29) | .044498  | .0140448                  | .0169706             | .0720253 |
| (40+ vs 20-29)   | .2059281 | .0137455                  | .1789874             | .2328688 |

Our model includes an interaction, though, so it would be nice to estimate the contrasts separately for each value of group. We need the `over()` option:

```
. margins r.agegroup, over(group)
Contrasts of predictive margins Number of obs = 3,000
Model VCE: OIM
Expression: Pr(outcome), predict()
Over: group
```

|                    | df | chi2   | P>chi2 |  |
|--------------------|----|--------|--------|--|
| agegroup@group     |    |        |        |  |
| (30-39 vs 20-29) 1 | 1  | 6.94   | 0.0084 |  |
| (30-39 vs 20-29) 2 | 1  | 1.18   | 0.2783 |  |
| (30-39 vs 20-29) 3 | 1  | 3.10   | 0.0783 |  |
| (40+ vs 20-29) 1   | 1  | 173.42 | 0.0000 |  |
| (40+ vs 20-29) 2   | 1  | 57.77  | 0.0000 |  |
| (40+ vs 20-29) 3   | 1  | 5.12   | 0.0236 |  |
| Joint              | 6  | 266.84 | 0.0000 |  |

|                    | Contrast | Delta-method<br>std. err. | [95% conf. interval] |          |
|--------------------|----------|---------------------------|----------------------|----------|
| agegroup@group     |          |                           |                      |          |
| (30-39 vs 20-29) 1 | .0819713 | .0311208                  | .0209757             | .142967  |
| (30-39 vs 20-29) 2 | .0166206 | .0153309                  | -.0134275            | .0466686 |
| (30-39 vs 20-29) 3 | .0243462 | .0138291                  | -.0027584            | .0514507 |
| (40+ vs 20-29) 1   | .3447797 | .0261811                  | .2934658             | .3960937 |
| (40+ vs 20-29) 2   | .1540882 | .0202722                  | .1143554             | .193821  |
| (40+ vs 20-29) 3   | .0470319 | .0207774                  | .006309              | .0877548 |

The effect of agegroup appears to be greatest for the first level of group.

Including a variable in the `over()` option is not equivalent to including the variable in the main *marginlist*. The variables in the *marginlist* are manipulated in the analysis, so that we can measure, for example, the effect of being in age group 3 and not age group 1. (The manipulation could be mimicked by running `replace` and then `predict`, but the manipulations actually performed by `margins` do not change the data in memory.) The variables in the `over()` option are not so manipulated—the values of the `over()` variables are left as they were observed, and the *marginlist* variables are manipulated separately for each observed `over()` group.



### The overjoint suboption

#### ▷ Example 3

Each variable in an `over()` option may be specified with or without contrast operators. Our `over(group)` option did not include a contrast operator, so `margins` estimated the contrasts separately for each level of group. If we had instead specified `over(r.group)`, we would have received differences of the contrasts:

```
. margins r.agegroup, over(r.group)
Contrasts of predictive margins Number of obs = 3,000
Model VCE: OIM
Expression: Pr(outcome), predict()
Over: group
```

|                           | df | chi2  | P>chi2 |
|---------------------------|----|-------|--------|
| group#agegroup            |    |       |        |
| (2 vs 1) (30-39 vs 20-29) | 1  | 3.55  | 0.0596 |
| (2 vs 1) (40+ vs 20-29)   | 1  | 33.17 | 0.0000 |
| (3 vs 1) (30-39 vs 20-29) | 1  | 2.86  | 0.0906 |
| (3 vs 1) (40+ vs 20-29)   | 1  | 79.36 | 0.0000 |
| Joint                     | 4  | 83.88 | 0.0000 |

|                           | Delta-method |           |                      |
|---------------------------|--------------|-----------|----------------------|
|                           | Contrast     | std. err. | [95% conf. interval] |
| group#agegroup            |              |           |                      |
| (2 vs 1) (30-39 vs 20-29) | -.0653508    | .0346921  | -.133346 .0026445    |
| (2 vs 1) (40+ vs 20-29)   | -.1906915    | .0331121  | -.25559 -.1257931    |
| (3 vs 1) (30-39 vs 20-29) | -.0576252    | .0340551  | -.1243719 .0091216   |
| (3 vs 1) (40+ vs 20-29)   | -.2977479    | .0334237  | -.3632572 -.2322386  |

The contrasts are double differences: the estimate of  $-0.19$ , for example, says that the difference in the probability of success between age group 3 and age group 1 is smaller in group 2 than in group 1. We can jointly test pairs of the double differences with the `overjoint` suboption:

```
. margins r.agegroup, over(group) contrast(overjoint)
Contrasts of predictive margins Number of obs = 3,000
Model VCE: OIM
Expression: Pr(outcome), predict()
Over: group
```

|                          | df | chi2  | P>chi2 |
|--------------------------|----|-------|--------|
| group#agegroup           |    |       |        |
| (joint) (30-39 vs 20-29) | 2  | 3.62  | 0.1641 |
| (joint) (40+ vs 20-29)   | 2  | 79.45 | 0.0000 |
| Joint                    | 4  | 83.88 | 0.0000 |

The `contrast(overjoint)` option overrides the default behavior of `over()` and requests joint tests over the levels of the unoperated variable `group`. The  $\chi^2$  statistic of 3.62 tests that the first and third contrasts from the previous table are jointly zero. The  $\chi^2$  statistic of 79.45 jointly tests the other pair of contrasts.



### The marginswithin suboption

#### ➤ Example 4

Another suboption that may usefully be combined with `over()` is `marginswithin`. `marginswithin` requests that contrasts be performed within the levels of unoperated variables in the main *marginlist*, instead of performing them jointly across the levels. `marginswithin` affects only unoperated variables because contrast operators take precedence over suboptions.

Let's first look at the default behavior, which occurs when marginswithin is not specified:

```
. margins agegroup, over(r.group) contrast(effects)
Contrasts of predictive margins Number of obs = 3,000
Model VCE: OIM
Expression: Pr(outcome), predict()
Over: group
```

|                  | df | chi2  | P>chi2 |
|------------------|----|-------|--------|
| group#agegroup   |    |       |        |
| (2 vs 1) (joint) | 2  | 33.94 | 0.0000 |
| (3 vs 1) (joint) | 2  | 83.38 | 0.0000 |
| Joint            | 4  | 83.88 | 0.0000 |

|                                                         | Delta-method |           |       |       | [95% conf. interval] |           |
|---------------------------------------------------------|--------------|-----------|-------|-------|----------------------|-----------|
|                                                         | Contrast     | std. err. | z     | P> z  |                      |           |
| group#<br>agegroup<br>(2 vs 1)<br>(30-39<br>vs<br>base) | -.0653508    | .0346921  | -1.88 | 0.060 | -.133346             | .0026445  |
| (2 vs 1)<br>(40+<br>vs<br>base)                         | -.1906915    | .0331121  | -5.76 | 0.000 | -.25559              | -.1257931 |
| (3 vs 1)<br>(30-39<br>vs<br>base)                       | -.0576252    | .0340551  | -1.69 | 0.091 | -.1243719            | .0091216  |
| (3 vs 1)<br>(40+<br>vs<br>base)                         | -.2977479    | .0334237  | -8.91 | 0.000 | -.3632572            | -.2322386 |

Here agegroup in the main *marginlist* is an unoperated variable, so margins by default performs joint tests across the levels of agegroup: the  $\chi^2$  statistic of 33.94, for example, jointly tests whether the first two contrast estimates in the lower table differ significantly from zero.

When we specify marginswithin, the contrasts will instead be performed within the levels of agegroup:

```
. margins agegroup, over(r.group) contrast(marginswithin effects)
Contrasts of predictive margins Number of obs = 3,000
Model VCE: OIM
Expression: Pr(outcome), predict()
Over: group
```

|                | df | chi2   | P>chi2 |
|----------------|----|--------|--------|
| group@agegroup |    |        |        |
| (2 vs 1) 20-29 | 1  | 0.06   | 0.7991 |
| (2 vs 1) 30-39 | 1  | 7.55   | 0.0060 |
| (2 vs 1) 40+   | 1  | 68.39  | 0.0000 |
| (3 vs 1) 20-29 | 1  | 1.80   | 0.1798 |
| (3 vs 1) 30-39 | 1  | 10.47  | 0.0012 |
| (3 vs 1) 40+   | 1  | 159.89 | 0.0000 |
| Joint          | 6  | 186.87 | 0.0000 |

|                | Delta-method |           |        |       |           | [95% conf. interval] |  |
|----------------|--------------|-----------|--------|-------|-----------|----------------------|--|
|                | Contrast     | std. err. | z      | P> z  |           |                      |  |
| group@agegroup |              |           |        |       |           |                      |  |
| (2 vs 1) 20-29 | -.0058686    | .0230533  | -0.25  | 0.799 | -.0510523 | .039315              |  |
| (2 vs 1) 30-39 | -.0712194    | .0259246  | -2.75  | 0.006 | -.1220308 | -.0204081            |  |
| (2 vs 1) 40+   | -.1965602    | .0237688  | -8.27  | 0.000 | -.2431461 | -.1499742            |  |
| (3 vs 1) 20-29 | -.0284991    | .0212476  | -1.34  | 0.180 | -.0701436 | .0131454             |  |
| (3 vs 1) 30-39 | -.0861243    | .0266137  | -3.24  | 0.001 | -.1382862 | -.0339624            |  |
| (3 vs 1) 40+   | -.326247     | .0258009  | -12.64 | 0.000 | -.3768159 | -.2756781            |  |

The joint tests in the top table have been replaced by one-degree-of-freedom tests, one for each combination of the two reference comparisons and three levels of agegroup. The reference-category contrasts for group have been performed within levels of agegroup.

## Contrasts and the at() option

### ► Example 5

The `at()` option of `margins` is used to set predictors to particular values. When `at()` is used, contrasts are by default performed within each `at()` level:

```
. margins r.agegroup, at(group=(1/3))
Contrasts of adjusted predictions Number of obs = 3,000
Expression: Pr(outcome), predict()
1._at: group = 1
2._at: group = 2
3._at: group = 3
```

|                    | df | chi2   | P>chi2 |
|--------------------|----|--------|--------|
| agegroup@_at       |    |        |        |
| (30-39 vs 20-29) 1 | 1  | 6.94   | 0.0084 |
| (30-39 vs 20-29) 2 | 1  | 1.18   | 0.2783 |
| (30-39 vs 20-29) 3 | 1  | 3.10   | 0.0783 |
| (40+ vs 20-29) 1   | 1  | 173.42 | 0.0000 |
| (40+ vs 20-29) 2   | 1  | 57.77  | 0.0000 |
| (40+ vs 20-29) 3   | 1  | 5.12   | 0.0236 |
| Joint              | 6  | 266.84 | 0.0000 |

|                    | Delta-method |           |                      |          |
|--------------------|--------------|-----------|----------------------|----------|
|                    | Contrast     | std. err. | [95% conf. interval] |          |
| agegroup@_at       |              |           |                      |          |
| (30-39 vs 20-29) 1 | .0819713     | .0311208  | .0209757             | .142967  |
| (30-39 vs 20-29) 2 | .0166206     | .0153309  | -.0134275            | .0466686 |
| (30-39 vs 20-29) 3 | .0243462     | .0138291  | -.0027584            | .0514507 |
| (40+ vs 20-29) 1   | .3447797     | .0261811  | .2934658             | .3960937 |
| (40+ vs 20-29) 2   | .1540882     | .0202722  | .1143554             | .193821  |
| (40+ vs 20-29) 3   | .0470319     | .0207774  | .006309              | .0877548 |

Our `at(group=(1/3))` option manipulates the values of `group` and is therefore not equivalent to `over(group)`. We see that the reference-category contrasts for `agegroup` have been performed within each `at()` level. For a similar example that uses the `._at` operator instead of the `at()` option, see *Contrasts of at() groups—discrete effects* in [R] [marginsplot](#).

The default within behavior of `at()` may be changed to joint behavior with the `atjoint` suboption:

```
. margins r.agegroup, at(group=(1/3)) contrast(atjoint)
Contrasts of adjusted predictions Number of obs = 3,000
Model VCE: OIM
Expression: Pr(outcome), predict()
1._at: group = 1
2._at: group = 2
3._at: group = 3
```

|                          | df | chi2  | P>chi2 |
|--------------------------|----|-------|--------|
| _at#agegroup             |    |       |        |
| (joint) (30-39 vs 20-29) | 2  | 3.62  | 0.1641 |
| (joint) (40+ vs 20-29)   | 2  | 79.45 | 0.0000 |
| Joint                    | 4  | 83.88 | 0.0000 |

Now, the tests are performed jointly over the levels of `group`, the `at()` variable. The `atjoint` suboption is the analogue for `at()` of the `overjoint` suboption from [example 3](#).



### ▷ Example 6

What if we would like to apply a contrast operator, like `r.`, to the `at()` levels? It is not possible to specify the operator inside the `at()` option. Instead, we need a new suboption, `atcontrast()`:

```
. margins r.agegroup, at(group=(1/3)) contrast(atcontrast(r))
Contrasts of adjusted predictions Number of obs = 3,000
Model VCE: OIM
Expression: Pr(outcome), predict()
1._at: group = 1
2._at: group = 2
3._at: group = 3
```

|                           | df | chi2  | P>chi2 |
|---------------------------|----|-------|--------|
| _at#agegroup              |    |       |        |
| (2 vs 1) (30-39 vs 20-29) | 1  | 3.55  | 0.0596 |
| (2 vs 1) (40+ vs 20-29)   | 1  | 33.17 | 0.0000 |
| (3 vs 1) (30-39 vs 20-29) | 1  | 2.86  | 0.0906 |
| (3 vs 1) (40+ vs 20-29)   | 1  | 79.36 | 0.0000 |
| Joint                     | 4  | 83.88 | 0.0000 |

|                           | Delta-method |           |                        |
|---------------------------|--------------|-----------|------------------------|
|                           | Contrast     | std. err. | [95% conf. interval]   |
| _at#agegroup              |              |           |                        |
| (2 vs 1) (30-39 vs 20-29) | -.0653508    | .0346921  | -.133346    .0026445   |
| (2 vs 1) (40+ vs 20-29)   | -.1906915    | .0331121  | -.25559    -.1257931   |
| (3 vs 1) (30-39 vs 20-29) | -.0576252    | .0340551  | -.1243719    .0091216  |
| (3 vs 1) (40+ vs 20-29)   | -.2977479    | .0334237  | -.3632572    -.2322386 |

When we specify `contrast(atcontrast(r))`, `margins` will apply the `r.` reference-category operator to the levels of `group`, the variable specified inside `at()`. The default reference category is 1, the lowest level of `group`.

## Estimating treatment effects with margins

margins with the contrast option can also be used to estimate treatment effects in certain cases. A treatment effect represents the change in an outcome variable that is attributable to a particular event, controlling for all other factors that could affect the outcome. For example, we might want to know how a person’s wage changes as a result of being in a union. Here the outcome variable is the person’s wage, and the “event” is membership in a union. The treatment effect measures the difference in a person’s wage as a result of being or not being in a union once we control for the person’s educational background, level of experience, industry, and other factors.

In fact, Stata has an entire manual dedicated to estimators designed specifically for estimating treatment effects; see the *Stata Causal Inference and Treatment-Effects Estimation Reference Manual*. Here we show how margins can be used to estimate treatment effects using the regression-adjustment estimator when the conditional independence assumption is met; see [CAUSAL] [teffects intro](#). Regression adjustment simply means that we are going to use a regression model to predict the outcome variable, controlling for treatment status and other characteristics. The conditional independence assumption implies that we have enough variables in our dataset so that once we control for them in our regression model, the outcomes one would obtain with and without treatment are independent of how treatment status is determined.

### ▷ Example 7: Regression adjustment with a binary treatment variable

nlsw88.dta contains women’s wages (wage) in dollars per hour, a binary variable indicating their union status (union), years of experience (ttl\_exp), and a variable, grade, indicating the number of years of schooling completed. We want to know how being in a union (the treatment) affects women’s wages. Traditionally, a wage equation of the form

$$\ln \text{wage}_i = \beta_0 + \beta_1 \text{union}_i + \beta_2 \text{grade}_i + \beta_3 \text{ttl\_exp} + \beta_4 \text{ttl\_exp}^2 + \epsilon_i$$

would be fit. However, there are two shortcomings that we will improve upon. First, to avoid the problem of predicting the level of a log-transformed dependent variable, we will use poisson with the vce(robust) option to fit an exponential regression model; see [Wooldridge \(2010, sec. 18.2\)](#) for background on this approach. Second, the previous equation implies that factors other than union status have the same impact on wages for both union and nonunion workers. Regression-adjustment estimators allow all the variables to have different impacts depending on the level of the treatment variable, and we can accomplish that here using factor-variable notation. In Stata, we fit our model by typing

```
. use https://www.stata-press.com/data/r19/nlsw88
(NLSW, 1988 extract)

. poisson wage i.union##(c.grade c.ttl_exp##c.ttl_exp), vce(robust)
note: noncount dependent variable encountered; results correspond to an
 exponential-mean model rather than a poisson model.

Iteration 0: Log pseudolikelihood = -4770.7957
Iteration 1: Log pseudolikelihood = -4770.7693
Iteration 2: Log pseudolikelihood = -4770.7693

Poisson regression
Log pseudolikelihood = -4770.7693
Number of obs = 1,876
Wald chi2(7) = 1047.11
Prob > chi2 = 0.0000
Pseudo R2 = 0.1195
```

| wage       | Coefficient | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |           |
|------------|-------------|---------------------|-------|-------|----------------------|-----------|
| union      |             |                     |       |       |                      |           |
| Union      | .8638376    | .168233             | 5.13  | 0.000 | .534107              | 1.193568  |
| grade      | .0895252    | .0056874            | 15.74 | 0.000 | .0783782             | .1006722  |
| ttl_exp    | .0805737    | .0114534            | 7.03  | 0.000 | .0581255             | .103022   |
| c.ttl_exp# |             |                     |       |       |                      |           |
| c.ttl_exp  | -.0015502   | .0004612            | -3.36 | 0.001 | -.0024541            | -.0006463 |
| union#     |             |                     |       |       |                      |           |
| c.grade    |             |                     |       |       |                      |           |
| Union      | -.0310298   | .0088259            | -3.52 | 0.000 | -.0483282            | -.0137314 |
| union#     |             |                     |       |       |                      |           |
| c.ttl_exp  |             |                     |       |       |                      |           |
| Union      | -.0404226   | .0230113            | -1.76 | 0.079 | -.085524             | .0046788  |
| union#     |             |                     |       |       |                      |           |
| c.ttl_exp# |             |                     |       |       |                      |           |
| c.ttl_exp  |             |                     |       |       |                      |           |
| Union      | .0011808    | .0008428            | 1.40  | 0.161 | -.0004711            | .0028327  |
| _cons      | .017488     | .0893602            | 0.20  | 0.845 | -.1576547            | .1926308  |

To see how union status affects wages, we can use margins:

```
. margins r.union, vce(unconditional)
Contrasts of predictive margins
Expression: Predicted number of events, predict()
Number of obs = 1,876
```

|       | df | chi2  | P>chi2 |
|-------|----|-------|--------|
| union | 1  | 26.22 | 0.0000 |

|                              | Contrast | Unconditional<br>std. err. | [95% conf. interval] |          |
|------------------------------|----------|----------------------------|----------------------|----------|
| union<br>(Union vs Nonunion) | 1.004119 | .1960944                   | .6197815             | 1.388457 |

The estimated contrast 1.004 indicates that on average, belonging to a union causes a woman’s wage to be slightly more than a dollar higher than if she were not in the union. This estimated contrast is called the average treatment effect (ATE). Conceptually, we predicted the wage of each woman in the estimation sample assuming she was in a union and obtained the sample mean. We then predicted each woman’s wage assuming she was not in a union and obtained that sample mean. The difference between these two sample means represents the ATE.

We obtain essentially the same results by using `teffects ra`:

```
. teffects ra (wage c.grade c.ttl_exp##c.ttl_exp, poisson) (union)
Iteration 0: EE criterion = 2.611e-13
Iteration 1: EE criterion = 1.112e-26
Treatment-effects estimation Number of obs = 1,876
Estimator : regression adjustment
Outcome model : Poisson
Treatment model: none
```

|        |                                    | Robust      |           | z     | P> z  | [95% conf. interval] |          |
|--------|------------------------------------|-------------|-----------|-------|-------|----------------------|----------|
| wage   |                                    | Coefficient | std. err. |       |       |                      |          |
| ATE    |                                    |             |           |       |       |                      |          |
|        | union<br>(Union<br>vs<br>Nonunion) | 1.004119    | .1960421  | 5.12  | 0.000 | .619884              | 1.388355 |
| POmean |                                    |             |           |       |       |                      |          |
|        | union<br>Nonunion                  | 7.346493    | .1096182  | 67.02 | 0.000 | 7.131645             | 7.561341 |

The point estimates of the ATE are identical to those we obtained using `margins`, though the standard errors differ slightly from those reported by `margins`. The standard errors from the two estimators are, however, asymptotically equivalent, meaning they would coincide with a sufficiently large dataset. The last statistic in this output table indicates the untreated potential-outcome mean (untreated POM), which is the mean predicted wage assuming each woman did not belong to a union.

If we specify the `pomeans` option with `teffects ra`, we can obtain both the treated and the untreated POMs, which represent the predicted mean wages assuming all women were or were not in the union:

```
. teffects ra (wage c.grade c.ttl_exp##c.ttl_exp, poisson) (union), pomeans
Iteration 0: EE criterion = 2.611e-13
Iteration 1: EE criterion = 1.112e-26
Treatment-effects estimation Number of obs = 1,876
Estimator : regression adjustment
Outcome model : Poisson
Treatment model: none
```

|         |                   | Robust      |           | z     | P> z  | [95% conf. interval] |          |
|---------|-------------------|-------------|-----------|-------|-------|----------------------|----------|
| wage    |                   | Coefficient | std. err. |       |       |                      |          |
| POmeans |                   |             |           |       |       |                      |          |
|         | union<br>Nonunion | 7.346493    | .1096182  | 67.02 | 0.000 | 7.131645             | 7.561341 |
|         | Union             | 8.350612    | .1757346  | 47.52 | 0.000 | 8.006179             | 8.695046 |

Notice that the difference between these two POMs equals 1.004119, which is the ATE we obtained earlier.

In some applications, the average treatment effect of the treated (ATET) is more germane than the ATE. For example, if the untreated subjects in the sample could not possibly receive treatment (perhaps because a medical condition precludes their taking an experimental drug), then considering the counterfactual outcome had those subjects taken the drug may not be relevant. In these cases, the ATET is a better statistic because it measures the effect of the treatment only for those subjects who actually did receive treatment. Like the ATE, the ATET involves computing predicted outcomes for each treatment level, obtaining the sample means, and computing the difference between those two means. Unlike the ATE, however, we use only observations corresponding to treated subjects.

▷ Example 8: Regression adjustment with a binary treatment variable (continued)

Here we calculate the ATET of union membership, first using margins. Because teffects ra overwrote our estimation results, we first quietly refit our poisson model. We then call margins to obtain the ATET:

```
. quietly poisson wage i.union##(c.grade c.ttl_exp##c.ttl_exp), vce(robust)
. margins r.union, subpop(union) vce(unconditional)
Contrasts of predictive margins Number of obs = 1,876
 Subpop. no. obs = 460
```

Expression: Predicted number of events, predict()

|       | df | chi2  | P>chi2 |
|-------|----|-------|--------|
| union | 1  | 18.86 | 0.0000 |

|                              | Unconditional |           |                      |
|------------------------------|---------------|-----------|----------------------|
|                              | Contrast      | std. err. | [95% conf. interval] |
| union<br>(Union vs Nonunion) | .901419       | .2075863  | .4945574 1.308281    |

The key here was specifying the subpop(union) option to restrict margin's computations to those women who are union members. The results indicate that being in the union causes the union members' wages to be about 90 cents higher than they would otherwise be.

To replicate these results using `teffects ra`, we include the `atet` option to obtain ATETs:

```
. teffects ra (wage c.grade c.ttl_exp#c.ttl_exp, poisson) (union), atet
Iteration 0: EE criterion = 2.611e-13
Iteration 1: EE criterion = 9.347e-27
Treatment-effects estimation Number of obs = 1,876
Estimator : regression adjustment
Outcome model : Poisson
Treatment model: none
```

|        |                                    | Robust      |           | z     | P> z  | [95% conf. interval] |          |
|--------|------------------------------------|-------------|-----------|-------|-------|----------------------|----------|
| wage   |                                    | Coefficient | std. err. |       |       |                      |          |
| ATET   |                                    |             |           |       |       |                      |          |
|        | union<br>(Union<br>vs<br>Nonunion) | .901419     | .2075309  | 4.34  | 0.000 | .4946658             | 1.308172 |
| P0mean |                                    |             |           |       |       |                      |          |
|        | union<br>Nonunion                  | 7.776417    | .162121   | 47.97 | 0.000 | 7.458665             | 8.094168 |

We obtain the same point estimate of the effect of union status as with `margins`. As before, the standard errors differ slightly between the two estimators, but they are asymptotically equivalent. The output also indicates that among the women who are in a union, their average wage would be \$7.78 if they were not in a union.



□ Technical note

One advantage of the ATET over the ATE is that the ATET can be consistently estimated with slightly weaker assumptions than are required to consistently estimate the ATE. See [Comparing the ATE and ATET in Remarks and examples of \[CAUSAL\] teffects intro advanced](#).



Both `margins` and `teffects` can estimate treatment effects using regression adjustment, so which should you use? In addition to regression adjustment, the `teffects` command implements other estimators of treatment effects; some of these estimators possess desirable robustness properties that we cannot replicate using `margins`. Moreover, all the `teffects` estimators use a common syntax and automatically present the estimated treatment effects, whereas we must first fit our own regression model and then call `margins` to obtain the treatment effects.

On the other hand, particularly with the `at()` option, `margins` gives us more flexibility in specifying our scenarios. The `teffects` commands allow us to measure the effect of a single binary or multinomial treatment, but we can have `margins` compute the effects of arbitrary interventions, as we illustrate in the next example.

## ► Example 9: Interventions involving multiple variables

Suppose we want to see how women's wages would be affected if we could increase each woman's education level by one year. That is, we want to measure the treatment effect of an additional year of schooling. We assume that if a woman attains another year of schooling, she cannot simultaneously work. Thus, an additional year of education implies her total work experience must decrease by a year. The flexible `at()` option of `margins` allows us to manipulate both variables at once:

```
. quietly poisson wage i.union##(c.grade c.ttl_exp##c.ttl_exp), vce(robust)
. margins, at((asobserved) _all)
> at(grade=generate(grade+1) ttl_exp=generate(ttl_exp-1))
> contrast(atcontrast(r._at))

Contrasts of predictive margins Number of obs = 1,876
Model VCE: Robust

Expression: Predicted number of events, predict()
1._at: (asobserved)
2._at: grade = grade+1
 ttl_exp = ttl_exp-1
```

|      | df | chi2  | P>chi2 |  |
|------|----|-------|--------|--|
| ._at | 1  | 58.53 | 0.0000 |  |

|                  | Delta-method |           |                      |          |
|------------------|--------------|-----------|----------------------|----------|
|                  | Contrast     | std. err. | [95% conf. interval] |          |
| ._at<br>(2 vs 1) | .3390392     | .0443161  | .2521813             | .4258971 |

The first `at()` option instructs `margins` to obtain predicted wages for all women in the sample using their existing values for `grade` and `ttl_exp` and to record the mean of those predictions. The second `at()` option instructs `margins` to obtain the mean predicted wage under the counterfactual scenario where each woman's education level is increased by one year and total work experience is simultaneously decreased by one year. The `contrast()` option instructs `margins` to compute the difference between the two means. The output indicates that increasing education by one year, which will necessarily decrease work experience by the same amount, will cause the average wage to increase by about 34 cents per hour, a statistically significant amount.

◀

## Conclusion

`margins, contrast` is a powerful command, and its abundance of suboptions may seem daunting. The suboptions are in the service of only three goals, however. There are three things that `margins, contrast` can do with a factor variable or a set of `at()` definitions:

1. Perform contrasts across the levels of the factor or set (as in [example 1](#)).
2. Perform a joint test across the levels of the factor or set (as in [example 5](#)).
3. Perform other tests and contrasts within each level of the factor or set (as in [example 4](#)).

The default behavior for variables specified inside `at()`, `over()`, and `within()` is to perform contrasts within groups; the default behavior for variables in the *marginalist* is to perform joint tests across groups.

◀

## Stored results

`margins`, `contrast` stores the following additional results in `r()`:

### Scalars

`r(k_terms)`                      number of terms participating in contrasts

### Macros

`r(cmd)`                              `contrast`  
`r(cmd2)`                             `margins`  
`r(overall)`                         `overall` or empty

### Matrices

`r(L)`                                 matrix of contrasts applied to the margins  
`r(chi2)`                              vector of  $\chi^2$  statistics  
`r(p)`                                 vector of  $p$ -values corresponding to `r(chi2)`  
`r(df)`                                vector of degrees of freedom corresponding to `r(p)`

`margins`, `contrast` with the `post` option also stores the following additional results in `e()`:

### Scalars

`e(k_terms)`                         number of terms participating in contrasts

### Macros

`e(cmd)`                              `contrast`  
`e(cmd2)`                             `margins`  
`e(overall)`                         `overall` or empty

### Matrices

`e(L)`                                 matrix of contrasts applied to the margins  
`e(chi2)`                              vector of  $\chi^2$  statistics  
`e(p)`                                 vector of  $p$ -values corresponding to `e(chi2)`  
`e(df)`                                vector of degrees of freedom corresponding to `e(p)`

## Methods and formulas

See *Methods and formulas* in [R] [margins](#) and *Methods and formulas* in [R] [contrast](#).

## Reference

Wooldridge, J. M. 2010. *Econometric Analysis of Cross Section and Panel Data*. 2nd ed. Cambridge, MA: MIT Press.

## Also see

- [R] [contrast](#) — Contrasts and linear hypothesis tests after estimation
- [R] [lincom](#) — Linear combinations of parameters
- [R] [margins](#) — Marginal means, predictive margins, and marginal effects
- [R] [margins postestimation](#) — Postestimation tools for margins
- [R] [margins, pwcompare](#) — Pairwise comparisons of margins
- [R] [pwcompare](#) — Pairwise comparisons

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## Description

`margins` with the `pwcompare` option performs pairwise comparisons of margins. `margins, pwcompare` extends the capabilities of `pwcompare` to any of the nonlinear responses, predictive margins, or other margins that can be estimated by `margins`.

## Quick start

All pairwise comparisons of the predictive margins for `a` after `logit y a##b x1`  
`margins a, pwcompare`

Same as above, but report both tests and confidence intervals for differences in predictive margins  
`margins a, pwcompare(effects)`

Same as above, but adjust  $p$ -values and confidence intervals for multiple comparisons using Bonferroni's method  
`margins a, pwcompare(effects) mcompare(bonferroni)`

Report predictive margins for the levels of `a`, and group those that are not significantly different  
`margins a, pwcompare(groups)`

All pairwise comparisons of the predictive margins for combinations of levels of `a` and `b`  
`margins a#b, pwcompare`

## Menu

Statistics > Postestimation

## Syntax

```
margins [marginlist] [if] [in] [weight] [, pwcompare margins_options]
```

```
margins [marginlist] [if] [in] [weight] [, pwcompare(suboptions) margins_options]
```

where *marginlist* is a list of factor variables or interactions that appear in the current estimation results. The variables may be typed with or without the `i.` prefix, and you may use any factor-variable syntax:

```
. margins i.sex i.group i.sex#i.group, pwcompare
. margins sex group sex#i.group, pwcompare
. margins sex##group, pwcompare
```

| <i>suboptions</i> | Description |
|-------------------|-------------|
|-------------------|-------------|

Pairwise comparisons

|                        |                                                                   |
|------------------------|-------------------------------------------------------------------|
| <code>cieffects</code> | show effects table with confidence intervals; the default         |
| <code>pveffects</code> | show effects table with <i>p</i> -values                          |
| <code>effects</code>   | show effects table with confidence intervals and <i>p</i> -values |
| <code>cimargins</code> | show table of margins and confidence intervals                    |
| <code>groups</code>    | show table of margins and group codes                             |
| <code>sort</code>      | sort the margins or contrasts in each term                        |

`collect` is allowed; see [\[U\] 11.1.10 Prefix commands](#).

`fweights`, `awweights`, `iweights`, and `pweights` are allowed; see [\[U\] 11.1.6 weight](#).

## Suboptions

Pairwise comparisons

`cieffects` specifies that a table of the pairwise comparisons with their standard errors and confidence intervals be reported. This is the default.

`pveffects` specifies that a table of the pairwise comparisons with their standard errors, test statistics, and *p*-values be reported.

`effects` specifies that a table of the pairwise comparisons with their standard errors, test statistics, *p*-values, and confidence intervals be reported.

`cimargins` specifies that a table of the margins with their standard errors and confidence intervals be reported.

`groups` specifies that a table of the margins with their standard errors and group codes be reported. Margins with the same letter in the group code are not significantly different at the specified significance level.

`sort` specifies that the reported tables be sorted on the margins or contrasts in each term.

## Remarks and examples

You should be familiar with the concepts and syntax of both `margins` and `pwcompare` before using the `pwcompare` option of `margins`. These remarks build on those in [R] [margins](#) and [R] [pwcompare](#).

`margins` can perform pairwise comparisons of any of the margins that it estimates.

We begin by fitting a logistic regression model using the NHANES II dataset, ignoring the complex survey nature of the data. Our dependent variable is `highbp`, an indicator for whether a person has high blood pressure. We fit an interacted model including two factor variables representing the region of the country as well as the continuous covariate `bmi`.

```
. use https://www.stata-press.com/data/r19/nhanes2
. logistic highbp region#c.bmi
 (output omitted)
```

By default, `margins` will compute the predictive margins of the probability of a positive outcome for each of the terms in `marginlist` after logistic regression. We will `margin` on `region` so that `margins` will estimate the average predicted probabilities of having high blood pressure conditional on being in each of the four regions and unconditional on BMI. We can specify the `pwcompare` option to obtain all possible pairwise comparisons of these predictive margins:

```
. margins region, pwcompare
Pairwise comparisons of predictive margins Number of obs = 10,351
Model VCE: OIM
Expression: Pr(highbp), predict()
```

|          | Delta-method |           | Unadjusted           |          |
|----------|--------------|-----------|----------------------|----------|
|          | Contrast     | std. err. | [95% conf. interval] |          |
| region   |              |           |                      |          |
| MW vs NE | -.0377194    | .0133571  | -.0638987            | -.01154  |
| S vs NE  | -.0156843    | .0133986  | -.041945             | .0105764 |
| W vs NE  | -.006873     | .0136595  | -.0336451            | .019899  |
| S vs MW  | .0220351     | .0124564  | -.0023789            | .0464492 |
| W vs MW  | .0308463     | .0127366  | .0058831             | .0558096 |
| W vs S   | .0088112     | .0127801  | -.0162373            | .0338598 |

This table gives each of the pairwise differences with confidence intervals. We can see that the confidence interval in the row labeled `MW vs NE` does not include 0. At the 5% level, the predictive margins for the first and second regions, the Northeast and the Midwest, are significantly different. The same is true of the second and fourth regions, the Midwest and the West. With many pairwise comparisons, output in this format can be difficult to sort through. We can organize it by adding the `group` suboption:

```
. margins region, pwcompare(group)
Pairwise comparisons of predictive margins Number of obs = 10,351
Model VCE: OIM
Expression: Pr(highbp), predict()
```

|        | Delta-method Unadjusted |           |        |
|--------|-------------------------|-----------|--------|
|        | Margin                  | std. err. | groups |
| region |                         |           |        |
| NE     | .4388358                | .010069   | B      |
| MW     | .4011164                | .0087764  | A      |
| S      | .4231516                | .0088395  | AB     |
| W      | .4319628                | .0092301  | B      |

Note: Margins sharing a letter in the group label are not significantly different at the 5% level.

The group output includes the predictive margins for each region and letters denoting margins that are not significantly different from one another. In this case, the Northeast (NE), South (S), and West (W) regions have the letter B in the “Unadjusted Groups” column. The letter B indicates that the average predicted probability for the Northeast region is not significantly different from the average predicted probabilities for the South and West regions at the 5% significance level. The Midwest (MW) region does not share a letter with the Northeast region nor the West region, which indicates that the average predicted probability for the Midwest region is significantly different for each of the other two regions at our 5% level.

We can also include the `mcompare(bonferroni)` option to perform tests using Bonferroni’s method to account for making multiple comparisons.

```
. margins region, pwcompare(group) mcompare(bonferroni)
Pairwise comparisons of predictive margins Number of obs = 10,351
Model VCE: OIM
Expression: Pr(highbp), predict()
```

|        | Number of comparisons |
|--------|-----------------------|
| region | 6                     |

|        | Delta-method Bonferroni |           |        |
|--------|-------------------------|-----------|--------|
|        | Margin                  | std. err. | groups |
| region |                         |           |        |
| NE     | .4388358                | .010069   | B      |
| MW     | .4011164                | .0087764  | A      |
| S      | .4231516                | .0088395  | AB     |
| W      | .4319628                | .0092301  | AB     |

Note: Margins sharing a letter in the group label are not significantly different at the 5% level.

We now see the letter A on the row corresponding to the West region. At the 5% level and with Bonferroni’s adjustment, the predictive margins for the probability in the Midwest and West regions are not significantly different.

## Stored results

`margins`, `pwcompare` stores the following additional results in `r()`:

### Scalars

`r(k_terms)` number of terms participating in pairwise comparisons

### Macros

`r(cmd)` `pwcompare`  
`r(cmd2)` `margins`  
`r(group#)` group code for the #th margin in `r(b)`  
`r(mcmethod_vs)` *method* from `mcompare()`  
`r(mctitle_vs)` title for *method* from `mcompare()`  
`r(mcadjustall_vs)` `adjustall` or empty

### Matrices

`r(b)` margin estimates  
`r(V)` variance-covariance matrix of the margin estimates  
`r(b_vs)` margin difference estimates  
`r(V_vs)` variance-covariance matrix of the margin difference estimates  
`r(error_vs)` margin difference estimability codes;  
 0 means estimable,  
 8 means not estimable  
`r(table_vs)` matrix containing the margin differences with their standard errors, test statistics, *p*-values, and confidence intervals  
`r(L)` matrix that produces the margin differences

`margins`, `pwcompare` with the `post` option also stores the following additional results in `e()`:

### Scalars

`e(k_terms)` number of terms participating in pairwise comparisons

### Macros

`e(cmd)` `pwcompare`  
`e(cmd2)` `margins`

### Matrices

`e(b)` margin estimates  
`e(V)` variance-covariance matrix of the margin estimates  
`e(b_vs)` margin difference estimates  
`e(V_vs)` variance-covariance matrix of the margin difference estimates  
`e(error_vs)` margin difference estimability codes;  
 0 means estimable,  
 8 means not estimable  
`e(L)` matrix that produces the margin differences

## Methods and formulas

See *Methods and formulas* in [\[R\] margins](#) and *Methods and formulas* in [\[R\] pwcompare](#).

## Also see

- [\[R\] contrast](#) — Contrasts and linear hypothesis tests after estimation
- [\[R\] margins](#) — Marginal means, predictive margins, and marginal effects
- [\[R\] margins, contrast](#) — Contrasts of margins
- [\[R\] margins postestimation](#) — Postestimation tools for margins
- [\[R\] pwcompare](#) — Pairwise comparisons

|                 |                      |                                    |
|-----------------|----------------------|------------------------------------|
| Description     | Menu                 | Syntax                             |
| Options         | Remarks and examples | Addendum: Advanced uses of dimlist |
| Acknowledgments | References           | Also see                           |

## Description

`marginsplot` graphs the results of the immediately preceding `margins` command; see [R] [margins](#). Common names for some of the graphs that `marginsplot` can produce are profile plots and interaction plots.

`marginsplot` is also available after `estat lcp` and `estat lcmean`; see [SEM] [estat lcp](#), [SEM] [estat lcmean](#), [FMM] [estat lcp](#), and [FMM] [estat lcmean](#).

## Menu

Statistics > Postestimation

## Syntax

`marginsplot` [ , *options* ]

*options*

Description

### Main

|                                                                           |                                                   |
|---------------------------------------------------------------------------|---------------------------------------------------|
| <code>x</code> dimension( <i>dimlist</i> [ , <i>dimopts</i> ])            | use <i>dimlist</i> to define <i>x</i> axis        |
| <code>plot</code> dimension( <i>dimlist</i> [ , <i>dimopts</i> ])         | create plots for groups in <i>dimlist</i>         |
| <code>by</code> dimension( <i>dimlist</i> [ , <i>dimopts</i> ])           | create subgraphs for groups in <i>dimlist</i>     |
| <code>graph</code> dimension( <i>dimlist</i> [ , <i>dimopts</i> ])        | create graphs for groups in <i>dimlist</i>        |
| <code>horizontal</code>                                                   | swap <i>x</i> and <i>y</i> axes                   |
| <code>noci</code>                                                         | do not plot confidence intervals                  |
| <code>deriv</code> labels                                                 | use labels attached to marginal-effects variables |
| <code>name</code> ( <i>name</i>   <i>stub</i> [ , <code>replace</code> ]) | name of graph, or <i>stub</i> if multiple graphs  |

### Labels

|                                          |                                                                           |
|------------------------------------------|---------------------------------------------------------------------------|
| <code>all</code> xlabels                 | place ticks and labels on the <i>x</i> axis for each value                |
| <code>no</code> labels                   | label groups with their values, not their labels                          |
| <code>all</code> simplelabels            | forgo variable name and equal signs in all labels                         |
| <code>no</code> simplelabels             | include variable name and equal signs in all labels                       |
| <code>separator</code> ( <i>string</i> ) | separator for labels when multiple variables are specified in a dimension |
| <code>no</code> separator                | do not use a separator                                                    |

|                                                |                                                       |
|------------------------------------------------|-------------------------------------------------------|
| Plot                                           |                                                       |
| <code>plotopts</code> ( <i>plot_options</i> )  | affect rendition of all margin plots                  |
| <code>plot#opts</code> ( <i>plot_options</i> ) | affect rendition of #th margin plot                   |
| <code>recast</code> ( <i>plotype</i> )         | plot margins using <i>plotype</i>                     |
| CI plot                                        |                                                       |
| <code>ciopts</code> ( <i>rcap_options</i> )    | affect rendition of all confidence interval plots     |
| <code>ci#opts</code> ( <i>rcap_options</i> )   | affect rendition of #th confidence interval plot      |
| <code>recastci</code> ( <i>plotype</i> )       | plot confidence intervals using <i>plotype</i>        |
| <code>mcompare</code> ( <i>method</i> )        | adjust for multiple comparisons                       |
| <code>level</code> (#)                         | set confidence level                                  |
| Pairwise                                       |                                                       |
| <code>unique</code>                            | plot only unique pairwise comparisons                 |
| <code>csort</code>                             | sort comparison categories first                      |
| Add plots                                      |                                                       |
| <code>addplot</code> ( <i>plot</i> )           | add other plots to the graph                          |
| Y axis, X axis, Titles, Legend, Overall, By    |                                                       |
| <code>twoway_options</code>                    | any options documented in [G-3] <i>twoway_options</i> |
| <code>byopts</code> ( <i>byopts</i> )          | how subgraphs are combined, labeled, etc.             |

where *dimlist* may be any of the dimensions across which margins were computed in the immediately preceding `margins` command; see [R] [margins](#). That is to say, *dimlist* may be any variable used in the `margins` command, including variables specified in the `at()`, `over()`, and `within()` options. More advanced specifications of *dimlist* are covered in [Addendum: Advanced uses of dimlist](#).

| <i>dimopts</i>                           | Description                                                                 |
|------------------------------------------|-----------------------------------------------------------------------------|
| <code>labels</code> ( <i>lablist</i> )   | list of quoted strings to label each level of the dimension                 |
| <code>elabels</code> ( <i>elablist</i> ) | list of enumerated labels                                                   |
| <code>no labels</code>                   | label groups with their values, not their labels                            |
| <code>all simple labels</code>           | forgo variable name and equal signs in all labels                           |
| <code>no simple labels</code>            | include variable name and equal signs in all labels                         |
| <code>separator</code> ( <i>string</i> ) | separator for labels when multiple variables are specified in the dimension |
| <code>no separator</code>                | do not use a separator                                                      |

where *lablist* is defined as

```
"label" ["label" [...]]
```

*elablist* is defined as

```
"label" [# "label" [...]]
```

and the #s are the indices of the levels of the dimension—1 is the first level, 2 is the second level, and so on.

| <i>plot_options</i>               | Description                                |
|-----------------------------------|--------------------------------------------|
| <code>marker_options</code>       | change look of markers (color, size, etc.) |
| <code>marker_label_options</code> | add marker labels; change look or position |
| <code>cline_options</code>        | change look of the line                    |

| <i>method</i>                       | Description                                  |
|-------------------------------------|----------------------------------------------|
| <code>noadjust</code>               | do not adjust for multiple comparisons       |
| <code>bonferroni [adjustall]</code> | Bonferroni's method; adjust across all terms |
| <code>sidak [adjustall]</code>      | Šidák's method; adjust across all terms      |
| <code>scheffe</code>                | Scheffé's method                             |

## Options

### Main

`xdimension()`, `plotdimension()`, `bydimension()`, and `graphdimension()` specify the variables from the preceding `margins` command whose group levels will be used for the graph's  $x$  axis, plots, `by()` subgraphs, and graphs.

`marginsplot` chooses default dimensions based on the `margins` command. In most cases, the first variable appearing in an `at()` option and evaluated over more than one value is used for the  $x$  axis. If no `at()` variable meets this condition, the first variable in the *marginlist* is usually used for the  $x$  axis and the remaining variables determine the plotted lines or markers. Pairwise comparisons and graphs of marginal effects (derivatives) have different defaults. In all cases, you may override the defaults and explicitly control which variables are used on each dimension of the graph by using these dimension options.

Each of these options supports [suboptions](#) that control the labeling of the dimension—axis labels for `xdimension()`, plot labels for `plotdimension()`, subgraph titles for `bydimension()`, and graph titles for `graphdimension()` titles.

For examples using the dimension options, see [Controlling the graph's dimensions](#).

`xdimension(dimlist [ , dimopts ])` specifies the variables for the  $x$  axis in *dimlist* and controls the content of those labels with *dimopts*.

`plotdimension(dimlist [ , dimopts ])` specifies in *dimlist* the variables whose group levels determine the plots and optionally specifies in *dimopts* the content of the plots' labels.

`bydimension(dimlist [ , dimopts ])` specifies in *dimlist* the variables whose group levels determine the `by()` subgraphs and optionally specifies in *dimopts* the content of the subgraphs' titles. For an example using `by()`, see [Three-way interactions](#).

`graphdimension(dimlist [ , dimopts ])` specifies in *dimlist* the variables whose group levels determine the graphs and optionally specifies in *dimopts* the content of the graphs' titles.

`horizontal` reverses the default  $x$  and  $y$  axes. By default, the  $y$  axis represents the estimates of the margins and the  $x$  axis represents one or more factors or continuous covariates. Specifying `horizontal` swaps the axes so that the  $x$  axis represents the estimates of the margins. This option can be useful if the labels on the factor or continuous covariates are long.

The `horizontal` option is discussed in [Horizontal is sometimes better](#).

`noci` removes plots of the pointwise confidence intervals. The default is to plot the confidence intervals.

`derivlabels` specifies that variable labels attached to marginal-effects variables be used in place of the variable names in titles and legends. Marginal-effects variables are the ones specified in `margins`'s option `dydx()`, `dyex()`, `eydx()`, or `eyex()`.

`name(name | stub [ , replace ])` specifies the name of the graph or graphs. If the `graphdimension()` option is specified, or if the default action is to produce multiple graphs, then the argument of `name()` is taken to be `stub` and graphs named `stub1`, `stub2`, ... are created.

The `replace` suboption causes existing graphs with the specified name or names to be replaced.

If `name()` is not specified, default names are used and the graphs may be replaced by subsequent `marginsplot` or other graphing commands.

### Labels

With the exception of `allxlabels`, all of these options may be specified either directly as options or as *dimopts* within options `xdimension()`, `plotdimension()`, `bydimension()`, and `graphdimension()`. When specified in one of the dimension options, only the labels for that dimension are affected. When specified outside the dimension options, all labels on all dimensions are affected. Specifications within the dimension options take precedence.

`allxlabels` specifies that tick marks and labels be placed on the  $x$  axis for each value of the  $x$ -dimension variables. By default, if there are more than 25 ticks, default graph axis labeling rules are applied. Labeling may also be specified using the standard graph `twoway`  $x$ -axis label rules and options—`xlabel()`; see [G-3] *axis\_label\_options*.

`nolabels` specifies that value labels not be used to construct graph labels and titles for the group levels in the dimension. By default, if a variable in a dimension has value labels, those labels are used to construct labels and titles for axis ticks, plots, subgraphs, and graphs.

Graphs of contrasts and pairwise comparisons are an exception to this rule and are always labeled with values rather than value labels.

`allsimplelabels` and `nosimplelabels` control whether graphs' labels and titles include just the values of the variables or include variable names and equal signs. The default is to use just the value label for variables that have value labels and to use variable names and equal signs for variables that do not have value labels. An example of the former is “Female” and the latter is “country=2”.

Sometimes, value labels are universally descriptive, and sometimes they have meaning only when considered in relation to their variable. For example, “Male” and “Female” are typically universal, regardless of the variable from which they are taken. “High” and “Low” may not have meaning unless you know they are in relation to a specific measure, say, blood-pressure level. The `allsimplelabels` and `nosimplelabels` options let you override the default labeling.

`allsimplelabels` specifies that all titles and labels use just the value or value label of the variable.

`nosimplelabels` specifies that all titles and labels include `varname=` before the value or value label of the variable.

`separator(string)` and `noseparator` control the separator between label sections when more than one variable is used to specify a dimension. The default separator is a comma followed by a space, but no separator may be requested with `noseparator` or the default may be changed to any string with `separator()`.

For example, if `plotdimension(a b)` is specified, the plot labels in our graph legend might be “a=1, b=1”, “a=1, b=2”, .... Specifying `separator(:)` would create labels “a=1:b=1”, “a=1:b=2”, ....

## Plot

`plotopts(plot_options)` affects the rendition of all margin plots. The `plot_options` can affect the size and color of markers, whether and how the markers are labeled, and whether and how the points are connected; see [G-3] [marker\\_options](#), [G-3] [marker\\_label\\_options](#), and [G-3] [cline\\_options](#).

These settings may be overridden for specific plots by using the `plot#opts()` option.

`plot#opts(plot_options)` affects the rendition of the #th margin plot. The `plot_options` can affect the size and color of markers, whether and how the markers are labeled, and whether and how the points are connected; see [G-3] [marker\\_options](#), [G-3] [marker\\_label\\_options](#), and [G-3] [cline\\_options](#).

`recast(plottype)` specifies that margins be plotted using `plottype`. `plottype` may be `scatter`, `line`, `connected`, `bar`, `area`, `spike`, `dropline`, or `dot`; see [G-2] [graph twoway](#). When `recast()` is specified, the plot-rendition options appropriate to the specified `plottype` may be used in lieu of `plot_options`. For details on those options, follow the appropriate link from [G-2] [graph twoway](#).

For an example using `recast()`, see [Continuous covariates](#).

You may specify `recast()` within a `plotopts()` or `plot#opts()` option. It is better, however, to specify it as documented here, outside those options. When specified outside those options, you have greater access to the plot-specific rendition options of your specified `plottype`.

## CI plot

`ciopts(rcap_options)` affects the rendition of all confidence interval plots; see [G-3] [rcap\\_options](#).

These settings may be overridden for specific confidence interval plots with the `ci#opts()` option.

`ci#opts(rcap_options)` affects the rendition of the #th confidence interval; see [G-3] [rcap\\_options](#).

`recastci(plottype)` specifies that confidence intervals be plotted using `plottype`. `plottype` may be `rarea`, `rbar`, `rspike`, `rcap`, `rcapsym`, `rline`, `rconnected`, or `rscatter`; see [G-2] [graph twoway](#). When `recastci()` is specified, the plot-rendition options appropriate to the specified `plottype` may be used in lieu of `rcap_options`. For details on those options, follow the appropriate link from [G-2] [graph twoway](#).

For an example using `recastci()`, see [Continuous covariates](#).

You may specify `recastci()` within a `ciopts()` or `ci#opts()` option. It is better, however, to specify it as documented here, outside those options. When specified outside those options, you have greater access to the plot-specific rendition options of your specified `plottype`.

`mcompare(method)` specifies the method for confidence intervals that account for multiple comparisons within a factor-variable term. The default is determined by the margins results stored in `r()`. If `marginsplot` is working from margins results stored in `e()`, the default is `mcompare(noadjust)`.

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is determined by the margins results stored in `r()`. If `marginsplot` is working from margins results stored in `e()`, the default is `level(95)` or as set by `set level`; see [U] [20.8 Specifying the width of confidence intervals](#).

## Pairwise

These options have an effect only when the `pwcompare` option was specified on the preceding `margins` command.

`unique` specifies that only unique pairwise comparisons be plotted. The default is to plot all pairwise comparisons, including those that are mirror images of each other—“male” versus “female” and “female” versus “male”. `margins` reports only the unique pairwise comparisons. `unique` also changes the default `xdimension()` for graphs of pairwise comparisons from the reference categories (`_pw0`) to the comparisons of each pairwise category (`_pw`).

Unique comparisons are often preferred with horizontal graphs that put all pairwise comparisons on the  $x$  axis, whereas including the full matrix of comparisons is preferred for charts showing the reference groups on an axis and the comparison groups as plots; see *Pairwise comparisons* and *Horizontal is sometimes better*.

`csort` specifies that comparison categories are sorted first, and then reference categories are sorted within comparison category. The default is to sort reference categories first, and then sort comparison categories within reference categories. This option has an observable effect only when `_pw` is also specified in one of the dimension options. It then determines the order of the labeling in the dimension where `_pw` is specified.

## Add plots

`addplot(plot)` provides a way to add other plots to the generated graph; see [G-3] *addplot\_option*.

For an example using `addplot()`, see *Adding scatterplots of the data*.

If multiple graphs are drawn by a single `marginsplot` command or if `plot` specifies plots with multiple  $y$  variables, for example, `scatter y1 y2 x`, then the graph’s legend will not clearly identify all the plots and will require customization using the `legend()` option; see [G-3] *legend\_options*.

## Y axis, X axis, Titles, Legend, Overall, By

*twoway\_options* are any of the options documented in [G-3] *twoway\_options*. These include options for titling the graph (see [G-3] *title\_options*); for saving the graph to disk (see [G-3] *saving\_option*); for controlling the labeling and look of the axes (see [G-3] *axis\_options*); for controlling the look, contents, position, and organization of the legend (see [G-3] *legend\_options*); for adding lines (see [G-3] *added\_line\_options*) and text (see [G-3] *added\_text\_options*); and for controlling other aspects of the graph’s appearance (see [G-3] *twoway\_options*).

The `label()` suboption of the `legend()` option has no effect on `marginsplot`. Use the `order()` or `lastlabel()` suboptions instead.

`byopts(byopts)` affects the appearance of the combined graph when `bydimension()` is specified or when the default graph has subgraphs, including the overall graph title, the position of the legend, and the organization of subgraphs. See [G-3] *by\_option*.

## Remarks and examples

Remarks are presented under the following headings:

- Introduction*
- Dataset*
- Profile plots*
- Interaction plots*
- Contrasts of margins—effects (discrete marginal effects)*
- Three-way interactions*
- Continuous covariates*
- Plots at every value of a continuous covariate*
- Contrasts of at() groups—discrete effects*
- Controlling the graph's dimensions*
- Pairwise comparisons*
- Horizontal is sometimes better*
- Marginal effects*
- Plotting a subset of the results from margins*
- Advanced usage*
  - Plots with multiple terms*
  - Plots with multiple at() options*
  - Adding scatterplots of the data*
- Video examples*

## Introduction

`marginsplot` is a post-`margins` command. It graphs the results of the `margins` command, whether those results are marginal means, predictive margins, marginal effects, contrasts, pairwise comparisons, or other statistics; see [R] [margins](#).

By default, the margins are plotted on the  $y$  axis, and all continuous and factor covariates specified in the `margins` command will usually be placed on the  $x$  axis or used to identify plots. Exceptions are discussed in the following sections and in *Addendum: Advanced uses of `dimlist`* below.

`marginsplot` produces classic plots, such as profile plots and interaction plots. Beyond that, anything that `margins` can compute, `marginsplot` can graph.

We will be using some relatively complicated `margins` commands with little explanation of the syntax. We will also avoid lengthy interpretations of the results of `margins`. See [R] [margins](#) for the complete syntax of `margins` and discussions of its results.

All graphs in this entry were drawn using the `s2gcolor` scheme; see [G-4] [Scheme s2](#).

[Mitchell \(2021\)](#) and [Baldwin \(2019\)](#) show in many examples how to use `marginsplot` to understand a fitted model.

## Dataset

For continuity, we will use one dataset for most examples—the Second National Health and Nutrition Examination Survey (NHANES II) ([McDowell et al. 1981](#)). NHANES II is part of a study to assess the health and nutritional status of adults and children in the United States. It is designed to be a nationally representative sample of the US population. This particular sample is from 1976 to 1980.

The survey nature of the dataset—weights, strata, and sampling units—will be ignored in our analyses. We are discussing graphing, not survey statistics. If you would like to see the results with the appropriate adjustments for the survey design, just add `svy:` before each estimation command, and if you wish, add `vce(unconditional)` as an option to each `margins` command. See [R] [margins](#), particularly the discussion and examples under *Obtaining margins with survey data and representative samples*, for reasons why you probably would want to add `vce(unconditional)` when analyzing survey data. For the most part, adjusting for survey design produces moderately larger confidence intervals and relatively small changes in point estimates.

## Profile plots

What does my estimation say about how my response varies as one (or more) of my covariates changes? That is the question that is answered by profile plots. Profile plots are also referred to as plots of estimated (or expected, or least-squares) means, though that is unnecessarily restrictive when considering models of binary, count, and ordered outcomes. In the latter cases, we might prefer to say they plot conditional expectations of responses, where a response might be a probability.

What we do with the other covariates depends on the questions we wish to answer. Sometimes, we wish to hold other covariates at fixed values, and sometimes we wish to average the response over their values. `margins` can do either, so you can graph either.

We can fit a fully factorial two-way ANOVA of systolic blood pressure on age group and sex using the NHANES II data.

```
. use https://www.stata-press.com/data/r19/nhanes2
. anova bpsystol agegrp##sex
```

|            | Number of obs = | 10,351  | R-squared =     | 0.2497 |        |
|------------|-----------------|---------|-----------------|--------|--------|
|            | Root MSE =      | 20.2209 | Adj R-squared = | 0.2489 |        |
| Source     | Partial SS      | df      | MS              | F      | Prob>F |
| Model      | 1407229.3       | 11      | 127929.93       | 312.88 | 0.0000 |
| agegrp     | 1243037.8       | 5       | 248607.56       | 608.02 | 0.0000 |
| sex        | 27728.379       | 1       | 27728.379       | 67.81  | 0.0000 |
| agegrp#sex | 88675.043       | 5       | 17735.009       | 43.37  | 0.0000 |
| Residual   | 4227440.7       | 10,339  | 408.88294       |        |        |
| Total      | 5634670         | 10,350  | 544.41256       |        |        |

If you are more comfortable with regression than ANOVA, then type

```
. regress bpsystol agegrp##sex
```

The `anova` and `regress` commands fit identical models. The output from `anova` displays all the terms in the model and thus tends to be more conducive to exploration with `margins` and `marginsplot`.

We estimate the predictive margins of systolic blood pressure for each age group using margins.

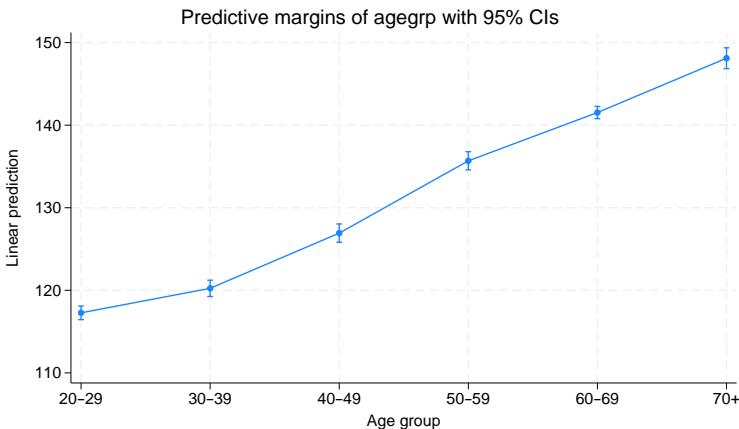
```
. margins agegrp
Predictive margins Number of obs = 10,351
Expression: Linear prediction, predict()
```

|        | Delta-method |           | t      | P> t  | [95% conf. interval] |          |
|--------|--------------|-----------|--------|-------|----------------------|----------|
|        | Margin       | std. err. |        |       |                      |          |
| agegrp |              |           |        |       |                      |          |
| 20-29  | 117.2684     | .419845   | 279.31 | 0.000 | 116.4454             | 118.0914 |
| 30-39  | 120.2383     | .5020813  | 239.48 | 0.000 | 119.2541             | 121.2225 |
| 40-49  | 126.9255     | .56699    | 223.86 | 0.000 | 125.8141             | 128.0369 |
| 50-59  | 135.682      | .5628593  | 241.06 | 0.000 | 134.5787             | 136.7853 |
| 60-69  | 141.5285     | .3781197  | 374.30 | 0.000 | 140.7873             | 142.2696 |
| 70+    | 148.1096     | .6445073  | 229.80 | 0.000 | 146.8463             | 149.373  |

The six predictive margins are just the averages of the predictions over the estimation sample, holding agegrp to each of its six levels. If this were a designed experiment rather than survey data, we might wish to assume the cells are balanced—that they have the same number of observations—and thus estimate what are often called expected means or least-squares means. To do that, we would simply add the `asbalanced` option to the `margins` command. The NHANES II data are decidedly unbalanced over `sex#agegrp` cells. So much so that it is unreasonable to assume the cells are balanced.

We graph the results:

```
. marginsplot
Variables that uniquely identify margins: agegrp
```



Profile plots are often drawn without confidence intervals (CIs). The CIs may be removed by adding the `noc1` option. We prefer to see the CIs.

Disciplines vary widely in their use of the term profile plot. Some disciplines consider any connected plot of a response over values of other variables to be a profile plot. By that definition, most graphs in this entry are profile plots.

## Interaction plots

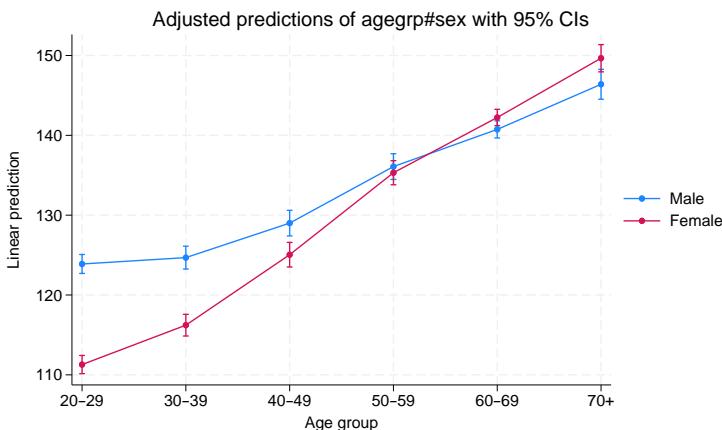
Interaction plots are often used to explore the form of an interaction. The interaction term in our ANOVA results is highly significant. Are the interaction effects also large enough to matter? What form do they take? We can answer these questions by fixing `agegrp` and `sex` to each possible combination of the two covariates and estimating the margins for those cells.

```
. margins agegrp#sex
```

Then, we can graph the results:

```
. marginsplot
```

```
Variables that uniquely identify margins: agegrp sex
```



It is clear that the effect of age differs by sex—there is an interaction. If there were no interaction, then the two lines would be parallel.

While males start out with higher systolic blood pressure, females catch up to the males as age increases and may even surpass males in the upper age groups. We say “may” because we cannot tell if the differences are statistically significant. The CIs overlap for the top three age groups. It is tempting to conclude from this overlap that the differences are not statistically significant. Do not fall into this trap. Likewise, do not fall into the trap that the first three age groups are different because their CIs do not overlap. The CIs are for the point estimates, not the differences. There is a covariance between the differences that we must consider if we are to make statements about those differences.

## Contrasts of margins—effects (discrete marginal effects)

To assess the differences, all we need to do is ask `margins` to contrast the sets of effects that we just estimated; see [\[R\] margins, contrast](#). With only two groups in sex, it does not matter much which contrast operator we choose. We will use the reference contrast. It will compare the difference between males and females, with males (the first category) as the reference category.

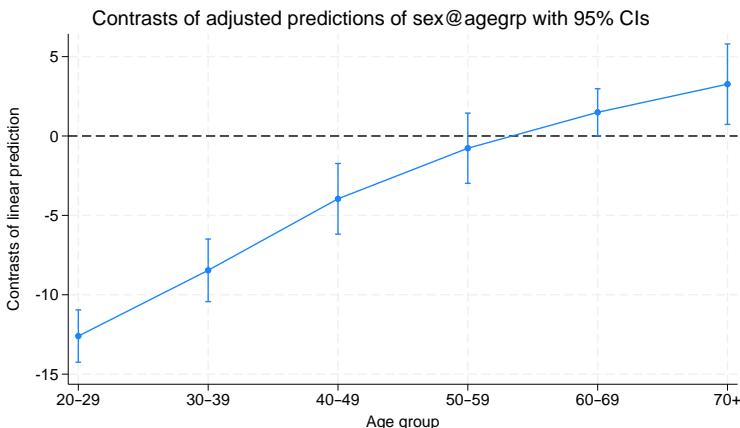
```
. margins r.sex@agegrp
Contrasts of adjusted predictions Number of obs = 10,351
Expression: Linear prediction, predict()
```

|                        | df    | F      | P>F    |
|------------------------|-------|--------|--------|
| sex@agegrp             |       |        |        |
| (Female vs Male) 20-29 | 1     | 224.92 | 0.0000 |
| (Female vs Male) 30-39 | 1     | 70.82  | 0.0000 |
| (Female vs Male) 40-49 | 1     | 12.15  | 0.0005 |
| (Female vs Male) 50-59 | 1     | 0.47   | 0.4949 |
| (Female vs Male) 60-69 | 1     | 3.88   | 0.0488 |
| (Female vs Male) 70+   | 1     | 6.37   | 0.0116 |
| Joint                  | 6     | 53.10  | 0.0000 |
| Denominator            | 10339 |        |        |

|                        | Delta-method |           |                      |
|------------------------|--------------|-----------|----------------------|
|                        | Contrast     | std. err. | [95% conf. interval] |
| sex@agegrp             |              |           |                      |
| (Female vs Male) 20-29 | -12.60132    | .8402299  | -14.24833 -10.9543   |
| (Female vs Male) 30-39 | -8.461161    | 1.005448  | -10.43203 -6.490288  |
| (Female vs Male) 40-49 | -3.956451    | 1.134878  | -6.181031 -1.731871  |
| (Female vs Male) 50-59 | -.7699782    | 1.128119  | -2.981309 1.441353   |
| (Female vs Male) 60-69 | 1.491684     | .756906   | .0080022 2.975367    |
| (Female vs Male) 70+   | 3.264762     | 1.293325  | .729594 5.79993      |

Because we are looking for effects that are different from 0, we will add a reference line at 0 to our graph.

```
. marginsplot, yline(0)
Variables that uniquely identify margins: agegrp
```



We can now say that females' systolic blood pressure is substantially and significantly lower than males' in the first three age groups but is significantly higher in the last two age groups. Despite the overlapping CIs for the last two age groups in the interaction graph, the effect of sex is significant in these age groups.

The terminology for what we just estimated and graphed varies widely across disciplines. Those versed in design of experiments refer to these values as contrasts or effects. Economists and some other social scientists call them marginal or partial effects. The latter groups might be more comfortable if we avoided the whole concept of contrasts and instead estimated the effects by typing

```
. margins agegrp, dydx(sex)
```

This will produce estimates that are identical to those shown above, and we can graph them by typing `marginsplot`.

The advantage of using the contrast notation and thinking in contrasts is most evident when we take marginal effects with respect to a categorical covariate with more than two levels. Marginal effects for each level of the covariate will be taken with respect to a specified base level. Contrasts are much more flexible. Using the `r.` operator, we can reproduce the marginal-effects results by taking derivatives with respect to a reference level (as we saw above.) We can also estimate the marginal effect of first moving from level 1 to level 2, then from level 2 to level 3, then from level 3 to level 4, ... using the `ar.` or “reverse adjacent” operator. Adjacent effects (marginal effects) can be valuable when evaluating an ordinal covariate, such as `agegrp` in our current model. For a discussion of contrasts, see [R] [contrast](#) and [R] [margins, contrast](#).

### Three-way interactions

`marginsplot` can handle any number of covariates in your `margins` command. Consider the three-way ANOVA model that results from adding an indicator for whether an individual has been diagnosed with diabetes. We will fully interact the new covariate with the others in the model.

```
. anova bpsystol agegrp##sex##diabetes
```

| Source              | Partial SS | df     | MS        | F      | Prob>F |
|---------------------|------------|--------|-----------|--------|--------|
| Model               | 1448983.2  | 23     | 62999.268 | 155.45 | 0.0000 |
| agegrp              | 107963.58  | 5      | 21592.716 | 53.28  | 0.0000 |
| sex                 | 1232.7927  | 1      | 1232.7927 | 3.04   | 0.0812 |
| agegrp#sex          | 11679.592  | 5      | 2335.9185 | 5.76   | 0.0000 |
| diabetes            | 7324.9892  | 1      | 7324.9892 | 18.07  | 0.0000 |
| agegrp#diabetes     | 5484.5462  | 5      | 1096.9092 | 2.71   | 0.0189 |
| sex#diabetes        | 102.98824  | 1      | 102.98824 | 0.25   | 0.6142 |
| agegrp#sex#diabetes | 4863.1497  | 5      | 972.62994 | 2.40   | 0.0349 |
| Residual            | 4184296.9  | 10,325 | 405.25878 |        |        |
| Total               | 5633280    | 10,348 | 544.38346 |        |        |

The three-way interaction is significant, as is the main effect of `diabetes` and its interaction with `agegrp`.

Again, if you are more comfortable with regression than ANOVA, you may type

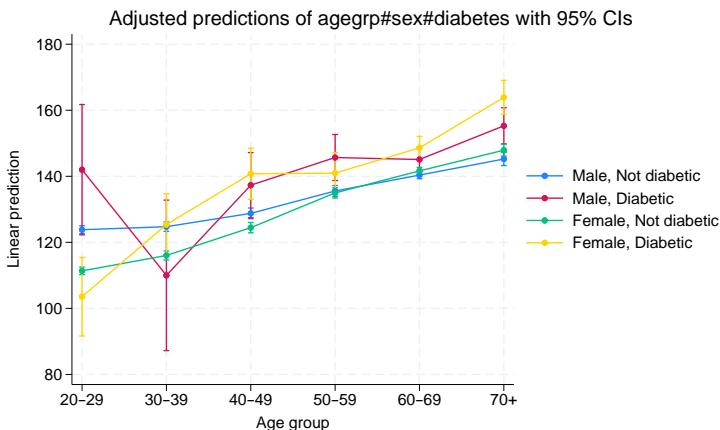
```
. regress bpsystol agegrp##sex##diabetes
```

The `margins` and `marginsplot` results will be the same.

We estimate the expected cell means for each combination of agegrp, sex, and diabetes, and then graph the results by typing

```
. margins agegrp#sex#diabetes
 (output omitted)
. marginsplot
```

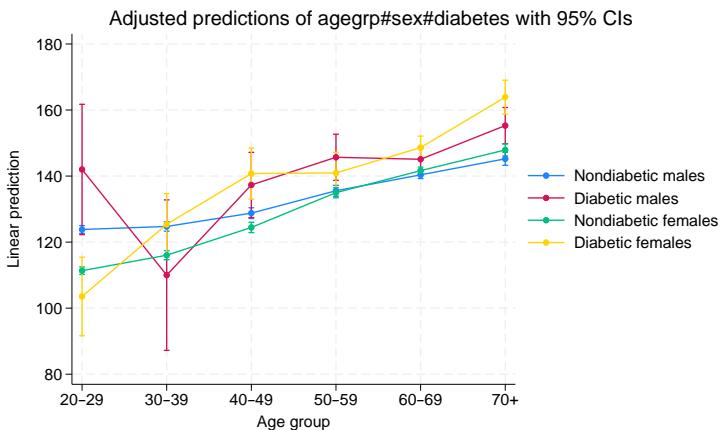
Variables that uniquely identify margins: **agegrp sex diabetes**



We wish to modify the text in the legend, and we can do so with the `lastlabel()` suboption of `legend()`. We label the first key as “Nondiabetic males”, the second as “Diabetic males”, and so on.

```
. marginsplot, legend(lastlabel(1 "Nondiabetic males")
> lastlabel(2 "Diabetic males") lastlabel(3 "Nondiabetic females")
> lastlabel(4 "Diabetic females"))
```

Variables that uniquely identify margins: **agegrp sex diabetes**

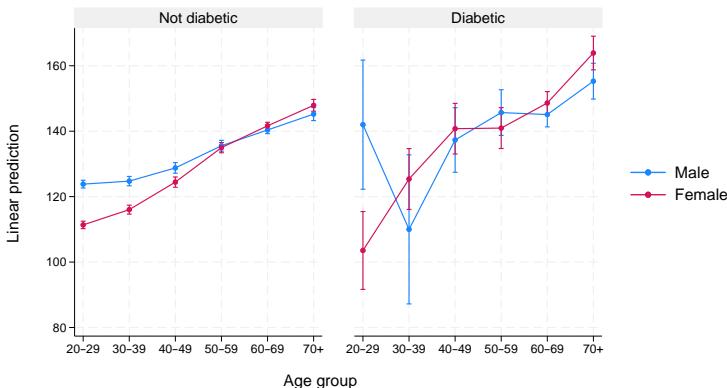


The last two graphs are busy and difficult to interpret.

We can make it better by putting those with diabetes on one subgraph and those without on another:

```
. marginsplot, by(diabetes)
Variables that uniquely identify margins: agegrp sex diabetes
```

Adjusted predictions of agegrp#sex#diabetes with 95% CIs

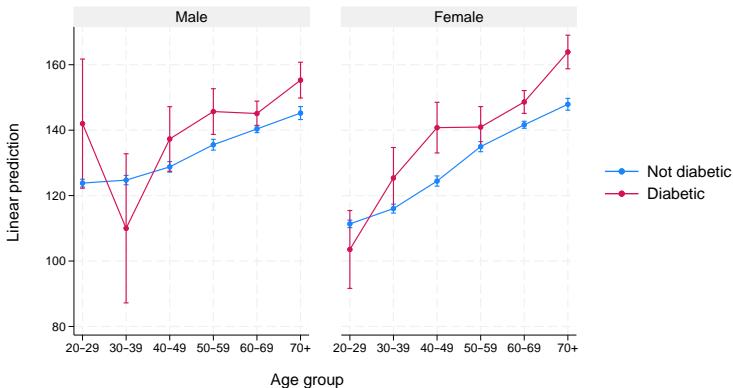


We notice much larger CIs for diabetics. That is not surprising because our sample contains only 499 diabetics compared with 9,850 nondiabetics.

A more interesting way to arrange the plots is by grouping the subgraphs on sex:

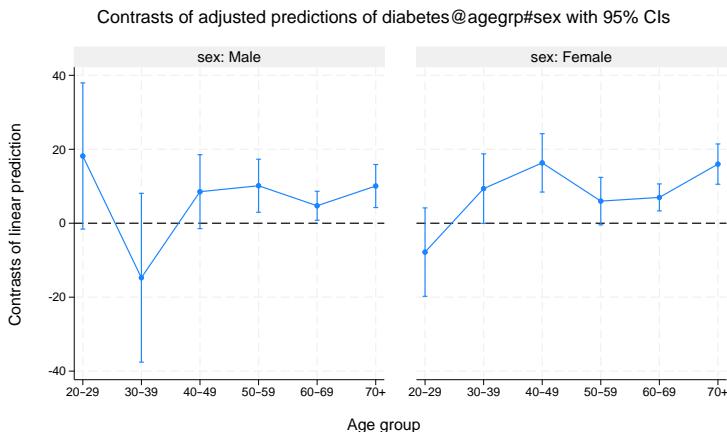
```
. marginsplot, by(sex)
Variables that uniquely identify margins: agegrp sex diabetes
```

Adjusted predictions of agegrp#sex#diabetes with 95% CIs



Aside from increased systolic blood pressure in the upper-age groups, which we saw earlier, it appears that those with diabetes are at greater risk of higher systolic blood pressure for many upper-age groups. We can check that by having `margins` estimate the differences between diabetics and nondiabetics, and graphing the results.

```
. margins r.diabetes@agegrp#sex
 (output omitted)
. marginsplot, by(sex) yline(0)
Variables that uniquely identify margins: agegrp sex
```



With CIs above 0 for six of eight age groups over 40, this graph provides evidence that diabetes is related to higher blood pressure in those over 40.

## Continuous covariates

`margins` and `marginsplot` are just as useful with continuous covariates as they are with factor variables. As a variation on our ANOVA/regression models, let's move to a logistic regression, using as our dependent variable an indicator for whether a person has high blood pressure. We introduce a continuous covariate—body mass index (BMI), a measure of weight relative to height. High BMI is often associated with high blood pressure. We will allow the effect of BMI to vary across sexes, age groups, and sex/age combinations by fully interacting the covariates.

```
. logistic highbp sex##agegrp##c.bmi
```

If we wished, we could perform all the analyses above on this model. Instead of estimating margins, contrasts, and marginal effects on the level of systolic blood pressure, we would be estimating margins, contrasts, and marginal effects on the probability of having high blood pressure. You can see those results by repeating any of the prior commands that involve `sex` and `agegrp`. In this section, we will focus on the continuous covariate `bmi`.

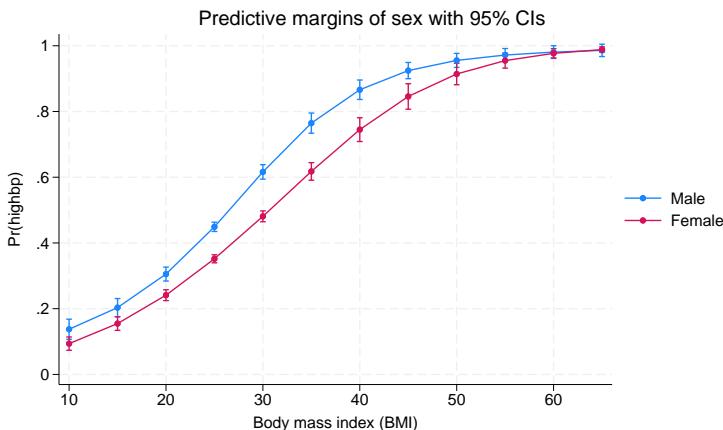
With continuous covariates, rather than specify them in the `marginlist` of `margins`, we specify the specific values at which we want the covariate evaluated in an `at()` option. `at()` options are very flexible, and there are many ways to specify values; see [Syntax of `at\(\)` in \[R\] margins](#).

BMI in our sample ranges from 12.4 to 61.1. Let's estimate the predictive margins for males and females at levels of BMI from 10 through 65 at intervals of 5 and graph the results:

```
. margins sex, at(bmi=(10(5)65))
(output omitted)
```

```
. marginsplot, xlabel(10(10)60)
```

Variables that uniquely identify margins: **bmi sex**

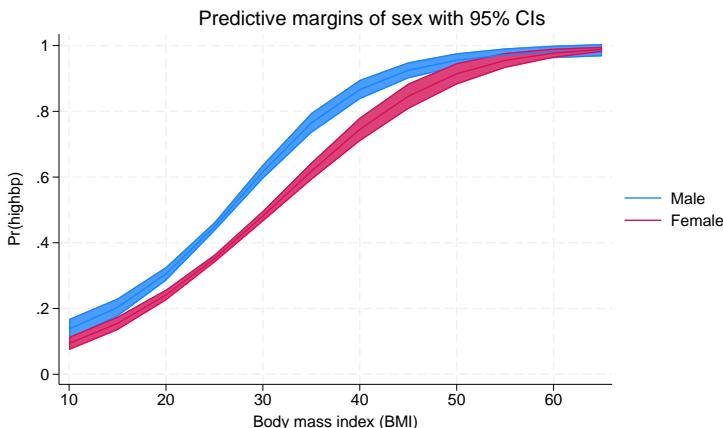


We added the `xlabel(10(10)60)` option to improve the labeling of the  $x$  axis. You may add any *twoway\_options* (see [G-3] *twoway\_options*) to the `marginsplot` command.

For a given BMI, males are generally more susceptible to high blood pressure, though the effect is attenuated by the logistic response when the probabilities approach 0 or 1.

Because `bmi` is continuous, we might prefer to see the response graphed using a line. We might also prefer that the CIs be plotted as areas. We change the plottype of the response by using the `recast()` option and the plottype of the CI by using the `recastci()` option:

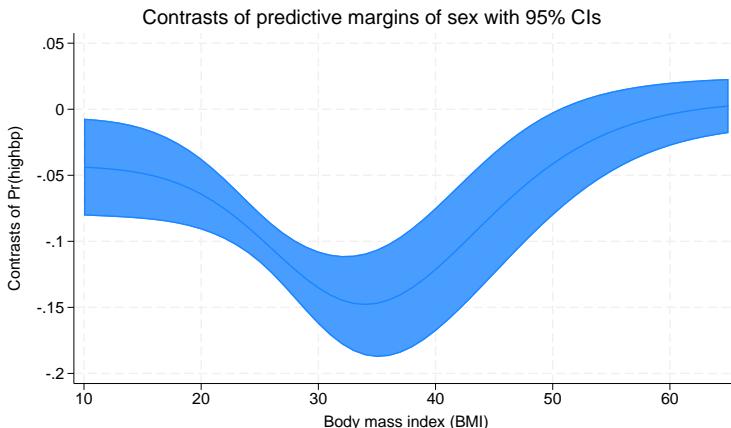
```
. marginsplot, xlabel(10(10)60) recast(line) recastci(rarea)
Variables that uniquely identify margins: bmi sex
```



The CIs are a little dark for our tastes. You can dim them a bit by reducing the intensity of their color. Adding `ciopts(color(*.8))` to our `marginsplot` command will do that. Any plot option accepted by `twoway rarea` (see [\[G-2\] graph twoway rarea](#)) may be specified in a `ciopts()` option.

Given their confidence regions, the male and female profiles appear to be statistically different over most of the range of BMI. As with the profiles of categorical covariates, we can check that assertion by contrasting the two profiles on `sex` and graphing the results. Let's improve the smoothness of the response by specifying intervals of 1 instead of 5.

```
. margins r.sex, at(bmi=(10(1)65))
(output omitted)
. marginsplot, xlabel(10(10)60) recast(line) recastci(rarea)
Variables that uniquely identify margins: bmi
```



We see that the difference between the sexes is largest at a BMI of about 35 and that the sexes respond more similarly with very high and very low BMI. This shape is largely determined by the response of the logistic function, which is attenuated near probabilities 0 and 1, combined with the fact that the lowest measured BMIs are associated with low probabilities of high blood pressure and the highest measured BMIs are associated with high probabilities of high blood pressure.

As when we contrasted profiles of categorical variables, different disciplines will think of this graph differently. Those familiar with designed experiments will be comfortable with the terms used above—this is a contrast of profiles, or a profile of effects, or a profile of a contrast. Many social scientists will prefer to think of this as a graph of marginal or partial effects. For them, this is a plot of the discrete marginal effect of being female for various levels of BMI. They can obtain an identical graph, with labeling more appropriate for the marginal effect's interpretation, by typing

```
. margins, at(bmi=(10(1)65)) dydx(sex)
. marginsplot, xlabel(10(10)60) recast(line) recastci(rarea)
```

We can also plot profiles of the response of BMI by levels of another continuous covariate (rather than by the categorical variable `sex`). To do so, we will need another continuous variable in our model. We have been using age groups as a covariate to emphasize the treatment of categorical variables and to allow the effect of age to be flexible. Our dataset also has age recorded in integer years. We replace `agegrp` with continuous `age` in our logistic regression.

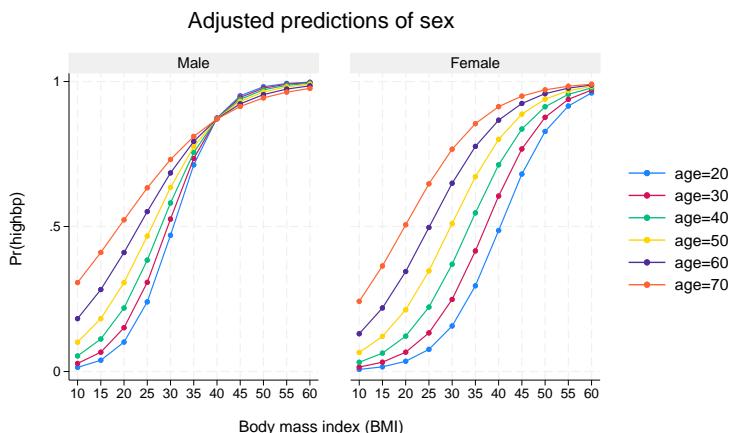
```
. logistic highbp sex##c.age##c.bmi
```

We can now obtain profiles of BMI for different ages by specifying ranges for both `bmi` and `age` in a single `at()` option on the `margins` command:

```
. margins sex, at(bmi=(10(5)60) age=(20(10)70))
```

With six ages specified, we have many profiles, so we will dispense with the CIs by adding the `nocl` option:

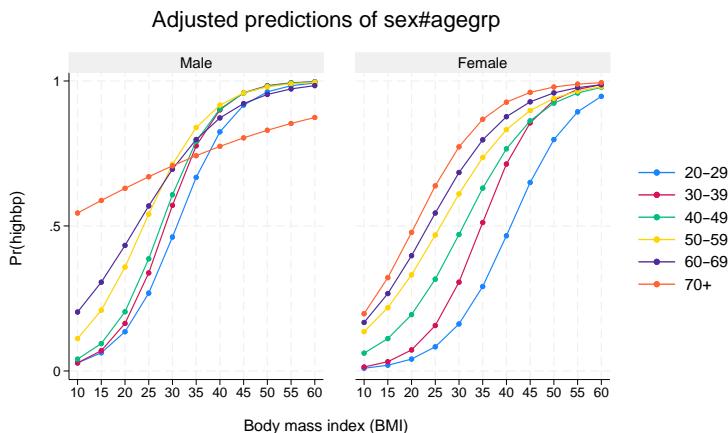
```
. marginsplot, noci by(sex)
Variables that uniquely identify margins: bmi age sex
```



Our model seems to indicate that males have a sharper reaction to body mass indices than do females. Likewise, younger subjects display a sharper response, while older subjects have a more gradual response with earlier onset. That interpretation might be a result of our parametric treatment of age. As it turns

out, the interpretation holds if we allow age to take more flexible forms or return to our use of age groups, which allows each of seven age groups to have unique BMI profiles. Here are the commands to perform that analysis:

```
. logistic highbp sex##agegrp##c.bmi
(output omitted)
. margins sex##agegrp, at(bmi=(10(5)60))
(output omitted)
. marginsplot, noci by(sex)
Variables that uniquely identify margins: bmi sex agegrp
```



## Plots at every value of a continuous covariate

In some cases, the specific values of a continuous covariate are important, and we want to plot the response at those specific values. Return to our logistic example with age treated as a continuous covariate.

```
. logistic highbp sex##c.age##c.bmi
```

We can use a programming trick to extract all the values of age and then supply them in an `at()` option, just as we would any list of values.

```
. levelsof age
. margins sex, at(age=('r(levels)'))
```

See [\[P\] levelsof](#) for a discussion of the `levelsof` command. `levelsof` returns in `r(levels)` the sorted list of unique values of the specified *varlist*, in our case, `age`.

We can then plot the results using `marginsplot`.

This is not a very interesting trick when using our `age` variable, which is recorded as integers from 20 to 74, but the approach will work with almost any continuous variable. In our model, `bmi` might seem more interesting, but there are 9,941 unique values of `bmi` in our dataset. A graph cannot resolve so many different values. For that reason, we usually recommend against plotting at every value of a covariate. Instead, graph at reasonable values over the range of the covariate by using the `at()` option, as we did earlier. This trick is best reserved for variables with a few, or at most a few dozen, unique values.

## Contrasts of at() groups—discrete effects

We have previously contrasted across the values of factor variables in our model. Put another way, we have estimated the discrete marginal effects of factor variables. We can do the same for the levels of variables in `at()` specifications and across separate `at()` specifications.

Returning to one of our logistic models and its margins, we earlier estimated the predictive margins of BMI at 5-unit intervals for both sexes. These are the commands we typed:

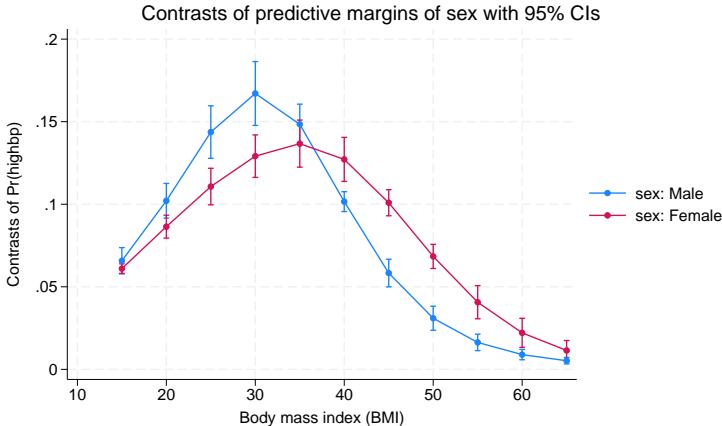
```
. logistic highbp sex##agegrp##c.bmi
. margins sex, at(bmi=(10(5)65))
. marginplot, xlabel(10(10)60)
```

We can estimate the discrete effects by sex of bmi moving from 10 to 15, then from 15 to 20, ..., and then from 60 to 65 by contrasting the levels of the `at()` groups using the reverse-adjacent contrast operator (`ar.`). We specify the operator within the `atcontrast()` suboption of the `contrast()` option. We need to specify one other option. By default, margins, contrast will apply a contrast to all variables in its *marginlist* when a contrast has been requested. In this case, we do not want to contrast across sexes but rather to contrast across the levels of BMI within each sex. To prevent margins from contrasting across the sexes, we specify the `marginswithin` option. Our margins command is

```
. margins sex, at(bmi=(10(5)65)) contrast(atcontrast(ar._at) marginswithin)
```

And we graph the results using `marginplot`:

```
. marginplot
Variables that uniquely identify margins: bmi sex
```

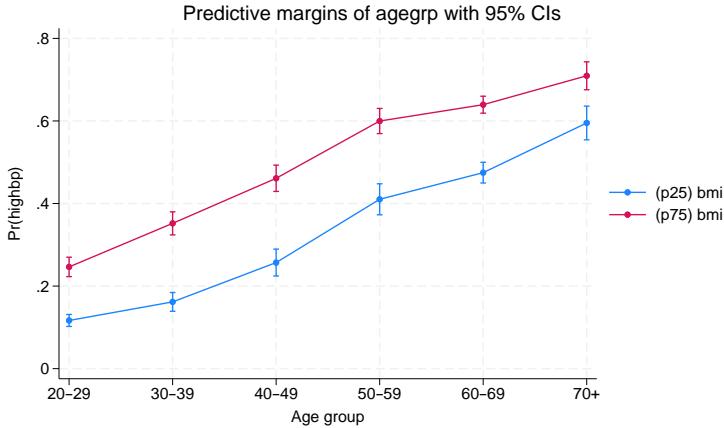


The graph shows the contrasts (or if you prefer, discrete changes) in the probability of high blood pressure by sex as one increases BMI in 5-unit increments.

We can even estimate contrasts (discrete effects) across `at()` options. To start, let's compare the age-group profiles of the probability of high blood pressure for those in the 25th and 75th percentile of BMI.

```
. margins agegrp, at((p25) bmi) at((p75) bmi)
 (output omitted)

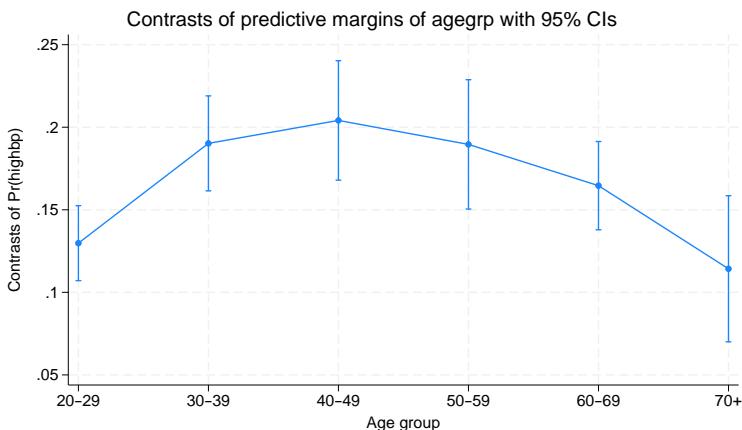
. marginsplot
Variables that uniquely identify margins: agegrp _atopt
Multiple at() options specified:
 _atoption=1: (p25) bmi
 _atoption=2: (p75) bmi
```



For each age group, people whose BMI is at the 75th percentile have a much higher probability of high blood pressure than those at the 25th percentile. What is that difference in probability and its CI? To contrast across the percentiles of BMI within age groups, we again specify a contrast operator on the `at()` groups using `atcontrast()`, and we also tell `margins` to perform that contrast within the levels of the *marginlist* by using the `marginswithin` option.

```
. margins agegrp, at((p25) bmi) at((p75) bmi)
> contrast(atcontrast(r._at) marginswithin)
(output omitted)
```

```
. marginsplot
Variables that uniquely identify margins: agegrp _atopt
Multiple at() options specified:
 _atoption=1: (p25) bmi
 _atoption=2: (p75) bmi
```



The differences in probability between 25th and 75th BMI percentiles are clearly significantly greater than 0. The differences appear to be smallest for those in the youngest and oldest age groups.

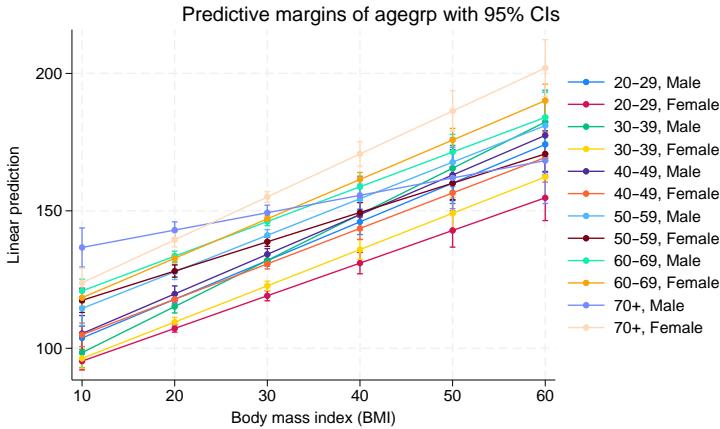
## Controlling the graph's dimensions

Thus far, `marginsplot` has miraculously done almost exactly what we want in most cases. The things we want on the  $x$  axis have been there, the choice of plots has made sense, etc. Some of that luck sprang from the relatively simple analyses we were performing, and some was from careful specification of our `margins` command. Sometimes, we will not be so lucky.

Consider the following `regress`, `margins`, and `marginsplot` commands:

```
. regress bpsystol agegrp##sex##c.bmi
(output omitted)
. margins agegrp, over(sex) at(bmi=(10(10)60))
(output omitted)
```

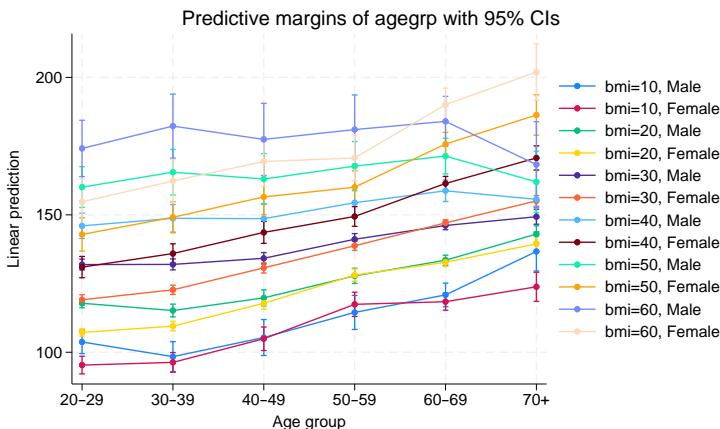
```
. marginsplot
Variables that uniquely identify margins: agegrp _atopt
Multiple at() options specified:
 _atoption=1: (p25) bmi
 _atoption=2: (p75) bmi
```



By default, `marginsplot` places the levels of the first multilevel `at()` specification on the  $x$  axis and then usually plots the levels of all remaining variables as connected lines. That is what we see in the graph above—`bmi`, the `at()` variable, is on the  $x$  axis, and each combination of `agegrp` and `sex` is plotted as a separate connected line. If there is no multilevel `at()` specification, then the first variable in `marginlist` becomes the  $x$  axis. There are many more rules, but it is usually best to simply type `marginsplot` and see what happens. If you do not like `marginsplot`'s choices, change them.

What if we wanted `agegrp` on the  $x$  axis instead of `BMI`? We tell `marginsplot` to make that change by specifying `agegrp` in the `xdimension()` option:

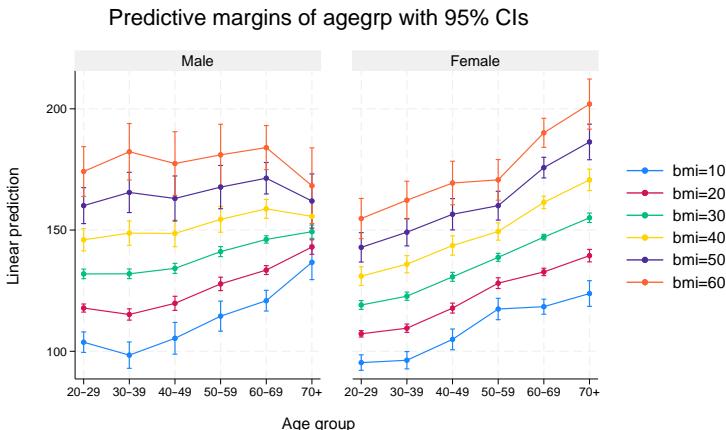
```
. marginsplot, xdimension(agegrp)
Variables that uniquely identify margins: bmi agegrp sex
```



We have been suppressing the Results window output for marginsplot, but that output is helpful if we want to change how things are plotted. You may specify any variable used in your margins command in any of the dimension options—`xdimension()`, `plotdimension()`, `bydimension()`, and `graphdimension()`. (In fact, there are some pseudovariables that you may also specify in some cases; see *Addendum: Advanced uses of dimlist* for details.) `marginsplot` tries to help you narrow your choices by listing a set of variables that uniquely identify all your margins. You are not restricted to this list.

We have a different *x* axis and a different set of plots, but our graph is still busy and difficult to read. We can make it better by creating separate graph panels for each sex. We do that by adding a `bydimension()` option with `sex` as the argument.

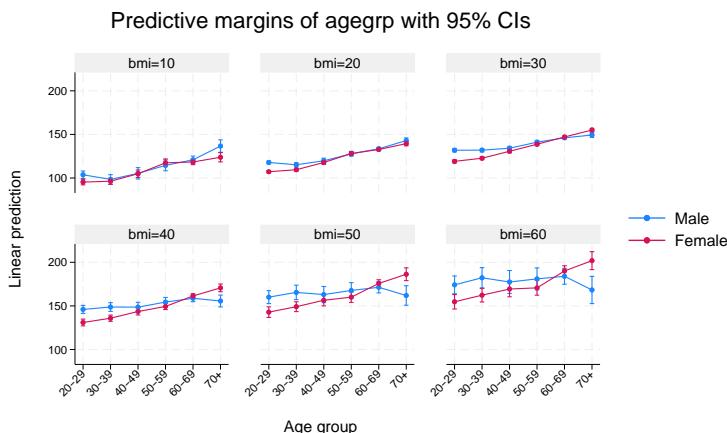
```
. marginsplot, xdimension(agegrp) bydimension(sex)
Variables that uniquely identify margins: bmi agegrp sex
```



The patterns and the differences between males and females are now easier to see.

If our interest is in comparing males and females, we might even choose to create a separate panel for each level of BMI:

```
. marginsplot, xdimension(agegrp) bydimension(bmi) xlabel(, angle(45))
Variables that uniquely identify margins: bmi agegrp sex
```



The  $x$ -axis labels did not fit, so we angled them.

We leave you to explore the use of the `graphdimension()` option. It is much like `bydimension()` but creates separate graphs rather than separate panels. Operationally, the `plotdimension()` option is rarely used. All variables not in the  $x$  dimension and not specified elsewhere become the plotted connected lines.

You will likely use the dimension options frequently. This is one of the rare cases where we recommend using the minimal abbreviations of the options—`x()` for `xdimension()`, `plot()` for `plotdimension()`, `by()` for `bydimension()`, and `graph()` for `graphdimension()`. The abbreviations are easy to read and just as meaningful as the full option names. The full names exist to reinforce the relationship between the dimension options.

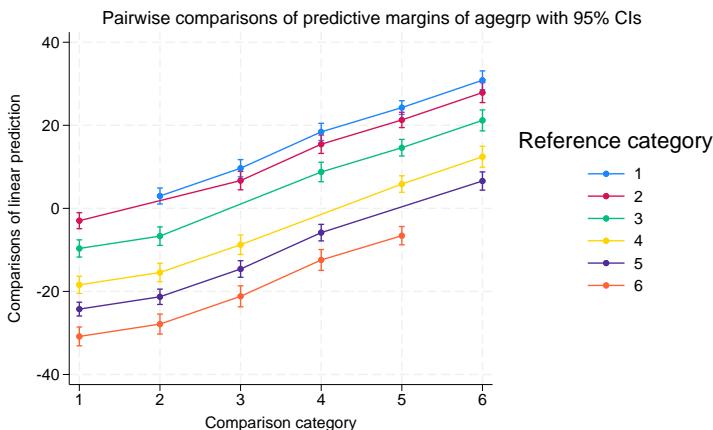
## Pairwise comparisons

`marginsplot` can graph the results of `margins`, `pwcompare`; see [R] [margins](#), [pwcompare](#). We return to one of our ANOVA examples. Here we request pairwise comparisons with the `pwcompare` option of `margins`, and we request Bonferroni-adjusted CIs with the `mcompare()` option:

```
. anova bpsystol agegrp##sex
(output omitted)
. margins agegrp, pwcompare mcompare(bonferroni)
(output omitted)
. marginsplot
```

Variables that uniquely identify margins: `_pw1` `_pw0`

`i_pw` enumerates all pairwise comparisons; `_pw0` enumerates the reference categories; `_pw1` enumerates the comparison categories.



Each connected line plot in the graph represents a reference age-group category for the pairwise comparison. The ticks on the  $x$  axis represent comparison age-group categories. So, each plot is a profile for a reference category showing its comparison to each other category.

## Horizontal is sometimes better

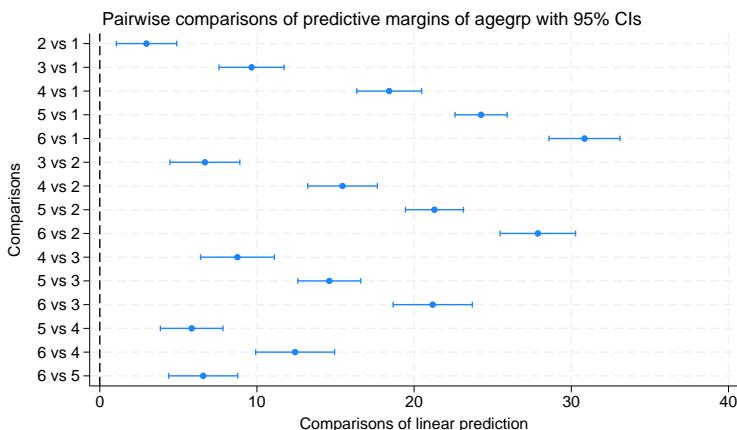
Another interesting way to graph pairwise comparisons is to simply plot each comparison and label the two categories being compared. This type of graph works better if it is oriented horizontally rather than vertically.

Continuing with the example above, we will switch the graph to horizontal. We will also make several changes to display the graph better. We specify that only unique comparisons be plotted. The graph above plotted both 1 versus 2 and 2 versus 1, which are the same comparison with opposite signs. We add a reference line at 0 because we are interested in comparisons that differ from 0. This graph looks better without the connecting lines, so we add the `recast(scatter)` option. We also reverse the  $y$  scale so that the smallest levels of age group appear at the top of the axis.

```
. marginsplot, horizontal unique xline(0) recast(scatter) yscale(reverse)
```

Variables that uniquely identify margins: `_pw1 _pw0`

`i_pw` enumerates all pairwise comparisons; `_pw0` enumerates the reference categories; `_pw1` enumerates the comparison categories.



All the comparisons differ from 0, so all our age groups are statistically different from each other.

The `horizontal` option can be useful outside of pairwise comparisons. Profile plots are usually oriented vertically. However, when your covariates have long labels or there are many levels at which the margins are being evaluated, the graph may be easier to read when rendered horizontally.

## Marginal effects

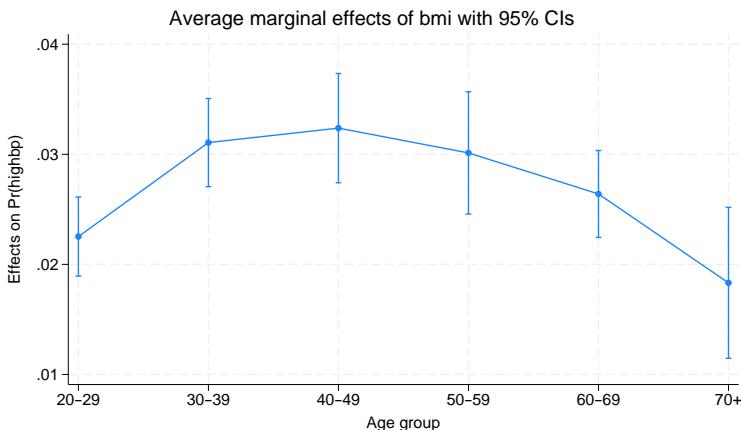
We have seen how to graph discrete effects for factor variables and continuous variables by using contrasts, and optionally by using the `dydx()` option of `margins`: [Contrasts of margins—effects \(discrete marginal effects\)](#) and [Continuous covariates](#). Let's now consider graphing instantaneous marginal effects for continuous covariates. Begin by refitting our logistic model of high blood pressure as a function of sex, age, and BMI:

```
. logistic highbp sex##agegrp##c.bmi
```

We estimate the average marginal effect of BMI on the probability of high blood pressure for each age group and then graph the results by typing

```
. margins agegrp, dydx(bmi)
 (output omitted)
. marginsplot
```

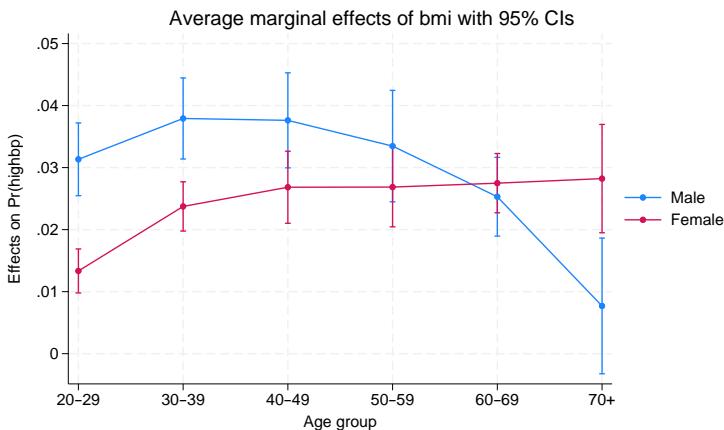
Variables that uniquely identify margins: **agegrp**



These are the conditional expectations of the marginal effects treating everyone in the sample as though they were in each age group. We can estimate fully conditional marginal effects that do not depend on averaging over the sample by also margining on our one remaining covariate—sex.

```
. margins agegrp#sex, dydx(bmi)
 (output omitted)
. marginsplot
```

Variables that uniquely identify margins: **agegrp sex**



The effect of BMI on the probability of high blood pressure looks to increase with age for females. The marginal effect is higher for males than females in the younger age groups but then decreases with age for males after the 40–49 age group.

You may want to test for differences in the marginal effect of BMI for males and females by contrasting across sexes within agegrp:

```
. margins r.sex@agegrp, dydx(bmi)
```

## Plotting a subset of the results from margins

`marginsplot` plots all the margins produced by the preceding `margins` command. If you want a graph that does not include all the margins, then enter a `margins` command that produces a reduced set of margins. Obvious ways to reduce the number of margins include not specifying some factors or interactions in the *marginlist* of `margins`, not specifying some `at()` or `over()` options, or reducing the values specified in an `at()` option. A less obvious technique uses selection lists in factor operators to select specific sets of levels from factor variables specified in the *marginlist*.

Instead of typing

```
. margins agegrp
```

which will give you margins for all six age groups in our sample, type

```
. margins i(2/4).agegrp
```

which will give you only three margins—those for groups 2, 3, and 4. See [\[U\] 11.4.3.4 Selecting levels](#).

## Advanced usage

`margins` is incredibly flexible in the statistics it can estimate and in the grouping of those estimates. Many of the estimates that `margins` can produce do not make convincing graphs. `marginsplot` plots the results of any `margins` command, regardless of whether the resulting graph is easily interpreted. Here we demonstrate some options that can make complicated `margins` into graphs that are somewhat more useful than those produced by `marginsplot`'s defaults. Others may find truly useful applications for these approaches.

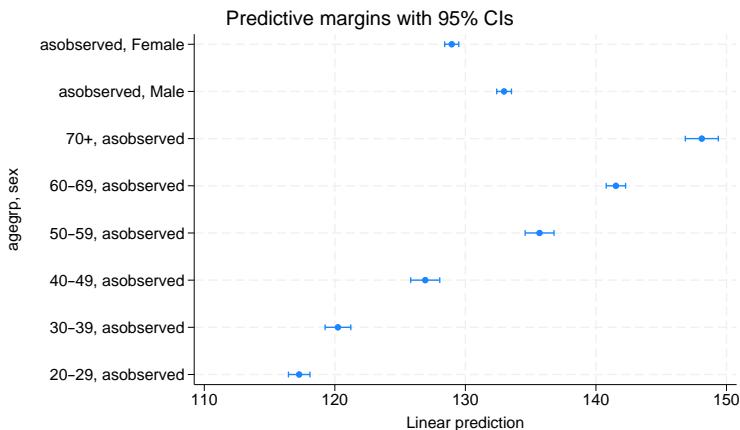
## Plots with multiple terms

Margins plots are rarely interesting when you specify multiple terms on your `margins` command, for example, `margins a b`. Such plots often compare things that are not comparable. The defaults for `marginsplot` rarely produce useful plots with multiple terms. Perhaps the most interesting graph in such cases puts all the levels of all the terms together on the vertical axis and plots their margins on the horizontal axis. We do that by including the *marginlist* from `margins` in an `xdimension()` option on `marginsplot`. The long labels on such graphs look better with a horizontal orientation, and there is no need to connect the margin estimates, so we specify the `recast(scatter)` option.

Using one of our ANOVA examples from earlier,

```
. anova bpsystol agegrp##sex
 (output omitted)
. margins agegrp sex
 (output omitted)
. marginsplot, xdimension(agegrp sex) horizontal recast(scatter)
```

Variables that uniquely identify margins: **agegrp sex**



The “asobserved” notations in the  $y$ -axis labels are informing us that, for example, when the margin for females is evaluated, the values of age group are taken as they are observed in the dataset. The margin is computed as an average over those values.

### Plots with multiple at() options

Some disciplines like to compute margins at the means of other covariates in their model and others like to compute the response for each observation and then take the means of the response. These correspond to the margins options `at((mean) _all)` and `at((asobserved) _all)`. For responses that are linear functions of the coefficients, such as `predict` after `regress`, the two computations yield identical results. For responses that are nonlinear functions of the coefficients, the two computations estimate different things.

Using one of our logistic models of high blood pressure,

```
. logistic highbp sex##agegrp##c.bmi
```

and computing both sets of margins for each age group,

```
. margins agegrp, at((mean) _all) at((asobserved) _all)
```

we can use marginsplot to compare the approaches:

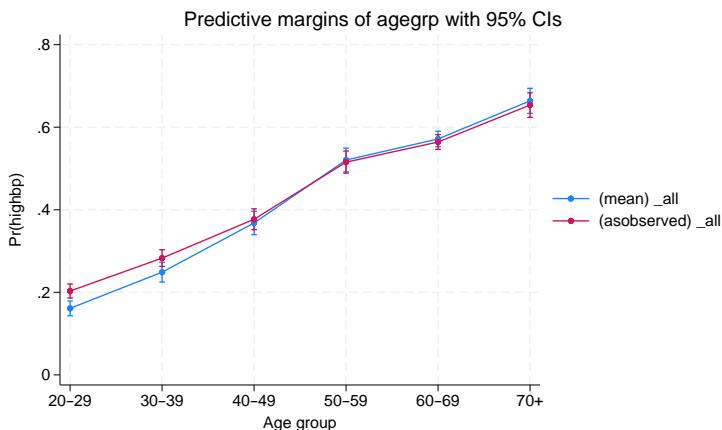
```
. marginsplot
```

Variables that uniquely identify margins: **agegrp \_atopt**

Multiple **at()** options specified:

\_atoption=1: (mean)\_all

\_atoption=2: (asobserved)\_all

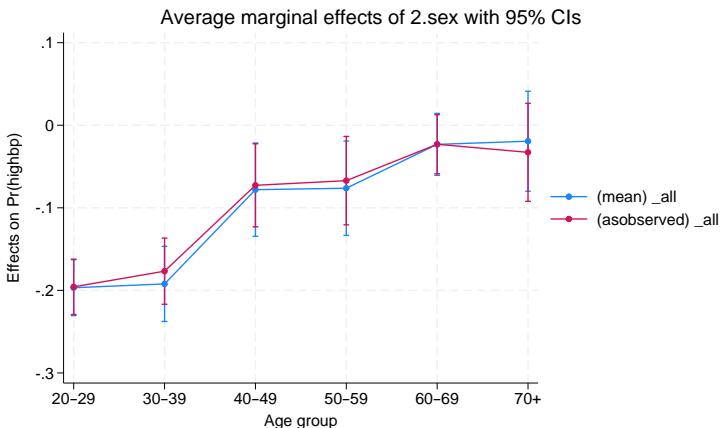


For the first three age groups, the probabilities of high blood pressure are lower at the means of sex and bpi than are the mean probabilities of high blood pressure averaged over the observed values of sex and bpi. The reverse is true for the last three age groups, although the values are very similar in these older age groups.

Such comparisons come up even more frequently when evaluating marginal effects. We can estimate the marginal effects of sex at each age group and graph the results by adding `dydx(sex)` to our `margins` command:

```
. margins agegrp, at((mean) _all) at((asobserved) _all) dydx(sex)
(output omitted)
```

```
. marginsplot
Variables that uniquely identify margins: agegrp _atopt
Multiple at() options specified:
 _atoption=1: (mean)_all
 _atoption=2: (asobserved) _all
```

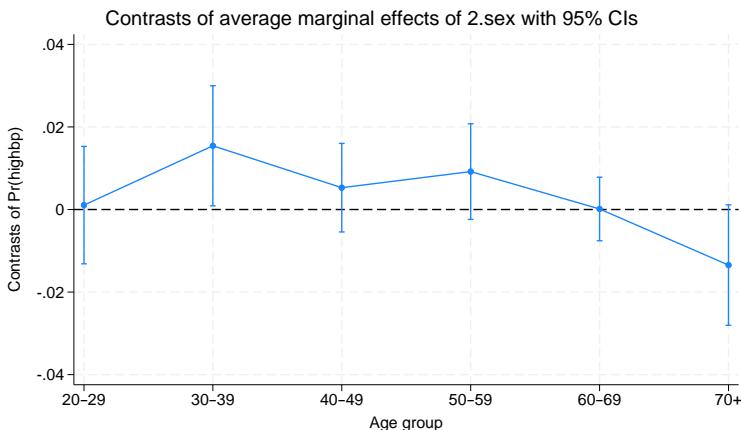


The average marginal effect is smaller for most age groups, but the CIs for both sets of estimates are wide. Can we tell the difference between the estimates? To answer that, we use the now-familiar tactic of taking the contrast of our estimated marginal-effects profiles. That means adding `contrast(atjoint marginswithin)` to our `margins` command. We will also add `mcompare(bonferroni)` to account for the fact that we will be comparing six contrasts.

```
. margins agegrp, at((mean) _all) at((asobserved) _all) dydx(sex)
> contrast(atjoint marginswithin) mcompare(bonferroni)
```

We will also add the familiar reference line at 0 to our graph of the contrasts.

```
. marginsplot, yline(0)
Variables that uniquely identify margins: agegrp _atopt
Multiple at() options specified:
 _atoption=1: (mean)_all
 _atoption=2: (asobserved)_all
```



While the difference in the estimates of marginal effects is not large, we can distinguish the estimates for the 30–39 and 70+ age groups.

The `at()` option of `margins` provides far more flexibility than demonstrated above. It can be used to evaluate a response or marginal effect at almost any point of interest or combinations of such points. See *Syntax of at()* in [R] `margins`.

## Adding scatterplots of the data

We can add scatterplots of the observed data to our plots of the margins. The NHANES II dataset is too large for this to be interesting, so for this example, we will use `auto.dta`. We fit mileage on whether the care is foreign and on a quadratic in the weight of the car. We convert the weight into tons (US definition) to improve the scaling, and we format the new tons variable to improve its labels on the graph. For our graph, we create separate variables for mileage of domestic and of foreign cars. We fit a fully interacted model so that the effect of weight on mileage can be different for foreign and for domestic cars.

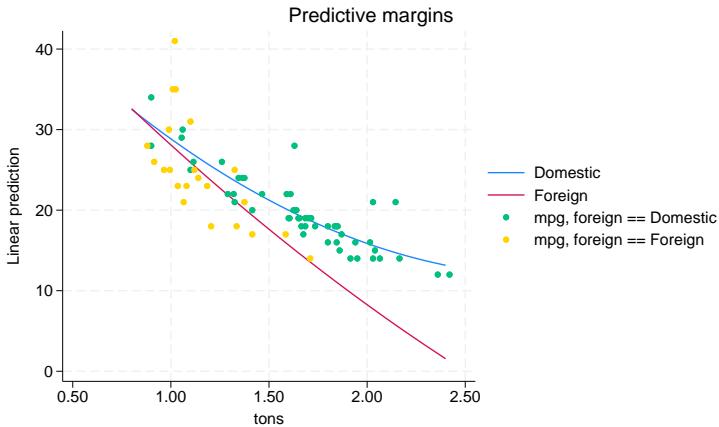
```
. use https://www.stata-press.com/data/r19/auto
. generate tons = weight/2000
. format tons %6.2f
. separate mpg, by(foreign)
. regress mpg foreign##c.tons##c.tons
```

We then estimate the margins over the range of tons, using the option `over(foreign)` to obtain separate estimates for foreign and domestic cars.

```
. margins, at(tons=(.8(.05)2.4)) over(foreign)
```

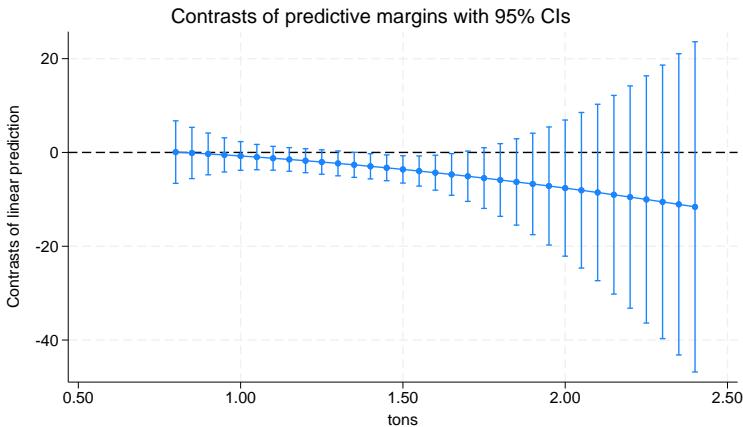
Adding scatterplots of mileage for domestic and foreign cars is easy. We insert into an `addplot()` option of `marginsplot` the same scatterplot syntax for `twoway` that we would type to produce a scatterplot of the data:

```
. marginsplot, addplot(scatter mpg0 tons || scatter mpg1 tons) recast(line) noci
Variables that uniquely identify margins: tons foreign
```



Many will be surprised that the mileage profile is higher in 1978 for domestic (US built) cars. Is the difference significant?

```
. margins, at(tons=(.8(.05)2.4)) over(r.for)
(output omitted)
. marginsplot, yline(0)
Variables that uniquely identify margins: tons
```



As we did earlier, we contrast the two profiles. We can discern some difference between the two profiles for midweight vehicles, but otherwise there is insufficient information to believe mileage differs across domestic and foreign cars.

## Video examples

[Profile plots and interaction plots, part 1: A single categorical variable](#)

[Profile plots and interaction plots, part 2: A single continuous variable](#)

[Profile plots and interaction plots, part 3: Interactions between categorical variables](#)

[Profile plots and interaction plots, part 4: Interactions of continuous and categorical variables](#)

[Profile plots and interaction plots, part 5: Interactions of two continuous variables](#)

## Addendum: Advanced uses of dimlist

*dimlist* specifies the dimensions from the immediately preceding *margins* command that are to be used for the *marginsplot*'s *x* axis, plots, subgraphs, and graphs. *dimlist* may contain:

| <i>dim</i>         | Description                                                                                                                                                                                                                                                                       |
|--------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>varname</i>     | Any variable referenced in the preceding <i>margins</i> command.                                                                                                                                                                                                                  |
| <i>_equation</i>   | If the estimation command being analyzed is multivariate and <i>margins</i> automatically produced estimates for more than one dependent-variable equation, then <i>dimlist</i> may contain <i>_equation</i> to enumerate those equations.                                        |
| <i>_outcome</i>    | If the estimation command being analyzed is ordinal and <i>margins</i> automatically produced estimates for more than one outcome level, then <i>dimlist</i> may contain <i>_outcome</i> to enumerate those outcomes.                                                             |
| <i>_predict</i>    | If the preceding <i>margins</i> command included multiple <i>predict()</i> options, then <i>dimlist</i> may contain <i>_predict</i> to enumerate those <i>predict()</i> options.                                                                                                  |
| <i>at(varname)</i> | If a variable is specified in both the <i>marginlist</i> or the <i>over()</i> option and in the <i>at()</i> option of <i>margins</i> , then the two uses can be distinguished in <i>marginsplot</i> by typing the <i>at()</i> variables as <i>at(varname)</i> in <i>dimlist</i> . |
| <i>_deriv</i>      | If the preceding <i>margins</i> command included a <i>dydx()</i> , <i>eyex()</i> , <i>dyex()</i> , or <i>eydx()</i> option, <i>dimlist</i> may also contain <i>_deriv</i> to specify all the variables over which derivatives were taken.                                         |
| <i>_term</i>       | If the preceding <i>margins</i> command included multiple terms (for example, <i>margins a b</i> ), then <i>dimlist</i> may contain <i>_term</i> to enumerate those terms.                                                                                                        |
| <i>_atopt</i>      | If the preceding <i>margins</i> command included multiple <i>at()</i> options, then <i>dimlist</i> may contain <i>_atopt</i> to enumerate those <i>at()</i> options.                                                                                                              |

When the *pairwise* option is specified on *margins*, you may specify dimensions that enumerate the pairwise comparisons.

|             |                                                         |
|-------------|---------------------------------------------------------|
| <i>_pw</i>  | enumerates all the pairwise comparisons                 |
| <i>_pw0</i> | enumerates the reference categories of the comparisons  |
| <i>_pw1</i> | enumerates the comparison categories of the comparisons |

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## Also see

- [R] **margins** — Marginal means, predictive margins, and marginal effects
- [R] **margins, contrast** — Contrasts of margins
- [R] **margins, pwcompare** — Pairwise comparisons of margins
- [R] **margins postestimation** — Postestimation tools for margins

|                |                      |                      |                      |
|----------------|----------------------|----------------------|----------------------|
| Description    | Syntax               | Maximization options | Remarks and examples |
| Stored results | Methods and formulas | References           | Also see             |

## Description

All Stata commands maximize likelihood functions using `moptimize()` and `optimize()`; see *Methods and formulas* below. Commands use the Newton–Raphson method with step halving and special fixes when they encounter nonconcave regions of the likelihood. For details, see [M-5] `moptimize()` and [M-5] `optimize()`. For more information about programming maximum likelihood estimators in ado-files and Mata, see [R] `ml` and Pitblado, Poi, and Gould (2024).

## Syntax

`mle_cmd ... [ , options ]`

| <i>options</i>                                | Description                                                                                                                                                                                                                                                  |
|-----------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>difficult</code>                        | use a different stepping algorithm in nonconcave regions                                                                                                                                                                                                     |
| <code>technique(<i>algorithm_spec</i>)</code> | maximization technique                                                                                                                                                                                                                                       |
| <code>iterate(#)</code>                       | perform maximum of # iterations; default is <code>iterate(300)</code>                                                                                                                                                                                        |
| <code>[no]log</code>                          | display an iteration log of the log likelihood; typically, the default                                                                                                                                                                                       |
| <code>trace</code>                            | display current parameter vector in iteration log                                                                                                                                                                                                            |
| <code>gradient</code>                         | display current gradient vector in iteration log                                                                                                                                                                                                             |
| <code>showstep</code>                         | report steps within an iteration in iteration log                                                                                                                                                                                                            |
| <code>hessian</code>                          | display current negative Hessian matrix in iteration log                                                                                                                                                                                                     |
| <code>showtolerance</code>                    | report the calculated result that is compared to the effective convergence criterion                                                                                                                                                                         |
| <code>tolerance(#)</code>                     | tolerance for the coefficient vector; see <i>Options</i> for the defaults                                                                                                                                                                                    |
| <code>ltolerance(#)</code>                    | tolerance for the log likelihood; see <i>Options</i> for the defaults                                                                                                                                                                                        |
| <code>nrtolerance(#)</code>                   | tolerance for the scaled gradient; see <i>Options</i> for the defaults                                                                                                                                                                                       |
| <code>qtolerance(#)</code>                    | when specified with algorithms <code>bhhh</code> , <code>dfp</code> , or <code>bfgs</code> , the $\mathbf{q} - \mathbf{H}$ matrix is used as the final check for convergence rather than <code>nrtolerance()</code> and the $\mathbf{H}$ matrix; seldom used |
| <code>nonrtolerance</code>                    | ignore the <code>nrtolerance()</code> option                                                                                                                                                                                                                 |
| <code>from(<i>init_specs</i>)</code>          | initial values for the coefficients                                                                                                                                                                                                                          |

*algorithm\_spec* is

`algorithm [ # [ algorithm [ # ] ] ... ]`

*algorithm* is { `nr` | `bhhh` | `dfp` | `bfgs` }

*init\_specs* is one of

`matname [ , skip copy ]`

{ [ *eqname*: ] *name* = # | / *eqname* = # } [ ... ]

# [ # ... ] , copy

## Maximization options

`difficult` specifies that the likelihood function is likely to be difficult to maximize because of nonconcave regions. When the message “not concave” appears repeatedly, `m1`’s standard stepping algorithm may not be working well. `difficult` specifies that a different stepping algorithm be used in nonconcave regions. There is no guarantee that `difficult` will work better than the default; sometimes it is better and sometimes it is worse. You should use the `difficult` option only when the default stepper declares convergence and the last iteration is “not concave” or when the default stepper is repeatedly issuing “not concave” messages and producing only tiny improvements in the log likelihood.

`technique(algorithm_spec)` specifies how the likelihood function is to be maximized. The following algorithms are allowed. For details, see [Pitblado, Poi, and Gould \(2024\)](#).

`technique(nr)` specifies Stata’s modified Newton–Raphson (NR) algorithm.

`technique(bhhh)` specifies the Berndt–Hall–Hall–Hausman (BHHH) algorithm.

`technique(dfp)` specifies the Davidon–Fletcher–Powell (DFP) algorithm.

`technique(bfgs)` specifies the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm.

The default is `technique(nr)`.

You can switch between algorithms by specifying more than one in the `technique()` option. By default, an algorithm is used for five iterations before switching to the next algorithm. To specify a different number of iterations, include the number after the `technique` in the option. For example, specifying `technique(bhhh 10 nr 1000)` requests that `m1` perform 10 iterations with the BHHH algorithm followed by 1000 iterations with the NR algorithm, and then switch back to BHHH for 10 iterations, and so on. The process continues until convergence or until the maximum number of iterations is reached.

`iterate(#)` specifies the maximum number of iterations. When the number of iterations equals `iterate()`, the optimizer stops and presents the current results. If convergence is declared before this threshold is reached, it will stop when convergence is declared. Specifying `iterate(0)` is useful for viewing results evaluated at the initial value of the coefficient vector. Specifying `iterate(0)` and `from()` together allows you to view results evaluated at a specified coefficient vector; however, not all commands allow the `from()` option. The default value of `iterate(#)` for both estimators programmed internally and estimators programmed with `m1` is the number set using `set maxiter`, which is 300 by default.

`log` and `nolog` specify whether an iteration log showing the progress of the log likelihood is to be displayed. For most commands, the log is displayed by default, and `nolog` suppresses it; see `set iterlog` in [\[R\] set iter](#). For a few commands (such as the `svy` maximum likelihood estimators), you must specify `log` to see the log.

`trace` adds to the iteration log a display of the current parameter vector.

`gradient` adds to the iteration log a display of the current gradient vector.

`showstep` adds to the iteration log a report on the steps within an iteration. This option was added so that developers at StataCorp could view the stepping when they were improving the `m1` optimizer code. At this point, it mainly provides entertainment.

`hessian` adds to the iteration log a display of the current negative Hessian matrix.

`showtolerance` adds to the iteration log the calculated value that is compared with the effective convergence criterion at the end of each iteration. Until convergence is achieved, the smallest calculated value is reported.

`shownrtolerance` is a synonym of `showtolerance`.

Below, we describe the three convergence tolerances. Convergence is declared when the `nrtolerance()` criterion is met and either the `tolerance()` or the `ltolerance()` criterion is also met.

`tolerance(#)` specifies the tolerance for the coefficient vector. When the relative change in the coefficient vector from one iteration to the next is less than or equal to `tolerance()`, the `tolerance()` convergence criterion is satisfied.

`tolerance(1e-4)` is the default for estimators programmed with `ml`.

`tolerance(1e-6)` is the default.

`ltolerance(#)` specifies the tolerance for the log likelihood. When the relative change in the log likelihood from one iteration to the next is less than or equal to `ltolerance()`, the `ltolerance()` convergence is satisfied.

`ltolerance(0)` is the default for estimators programmed with `ml`.

`ltolerance(1e-7)` is the default.

`nrtolerance(#)` specifies the tolerance for the scaled gradient. Convergence is declared when  $\mathbf{g}\mathbf{H}^{-1}\mathbf{g}' < \text{nrtolerance}()$ . The default is `nrtolerance(1e-5)`.

`qtolerance(#)` when specified with algorithms `bhhh`, `dfp`, or `bfgs` uses the  $\mathbf{q} - \mathbf{H}$  matrix as the final check for convergence rather than `nrtolerance()` and the  $\mathbf{H}$  matrix.

Beginning with Stata 12, by default, Stata now computes the  $\mathbf{H}$  matrix when the  $\mathbf{q} - \mathbf{H}$  matrix passes the convergence tolerance, and Stata requires that  $\mathbf{H}$  be concave and pass the `nrtolerance()` criterion before concluding convergence has occurred.

`qtolerance()` provides a way for the user to obtain Stata's earlier behavior.

`nonrtolerance` specifies that the default `nrtolerance()` criterion be turned off.

`from()` specifies initial values for the coefficients. Not all estimators in Stata support this option. You can specify the initial values in one of three ways: by specifying the name of a vector containing the initial values (for example, `from(b0)`, where `b0` is a properly labeled vector); by specifying coefficient names with the values (for example, `from(age=2.1 /sigma=7.4)`); or by specifying a list of values (for example, `from(2.1 7.4, copy)`). `from()` is intended for use when doing bootstraps (see [R] [bootstrap](#)) and in other special situations (for example, with `iterate(0)`). Even when the values specified in `from()` are close to the values that maximize the likelihood, only a few iterations may be saved. Poor values in `from()` may lead to convergence problems.

`skip` specifies that any parameters found in the specified initialization vector that are not also found in the model be ignored. The default action is to issue an error message.

`copy` specifies that the list of values or the initialization vector be copied into the initial-value vector by position rather than by name.

## Remarks and examples

Only in rare circumstances would you ever need to specify any of these options, except `nolog`. The `nolog` option is useful for reducing the amount of output appearing in log files; also see `set iterlog` in [R] *set iter*.

The following is an example of an iteration log:

```
Iteration 0: Log likelihood = -3791.0251
Iteration 1: Log likelihood = -3761.738
Iteration 2: Log likelihood = -3758.0632 (not concave)
Iteration 3: Log likelihood = -3758.0447
Iteration 4: Log likelihood = -3757.5861
Iteration 5: Log likelihood = -3757.474
Iteration 6: Log likelihood = -3757.4613
Iteration 7: Log likelihood = -3757.4606
Iteration 8: Log likelihood = -3757.4606
(table of results omitted$ $)
```

At iteration 8, the model converged. The message “not concave” at the second iteration is notable. This example was produced using the `heckman` command; its likelihood is not globally concave, so it is not surprising that this message sometimes appears. The other message that is occasionally seen is “backed up”. Neither of these messages should be of any concern unless they appear at the final iteration.

If a “not concave” message appears at the last step, there are two possibilities. One is that the result is valid, but there is collinearity in the model that the command did not otherwise catch. Stata checks for obvious collinearity among the independent variables before performing the maximization, but strange collinearities or near collinearities can sometimes arise between coefficients and ancillary parameters. The second, more likely cause for a “not concave” message at the final step is that the optimizer entered a flat region of the likelihood and prematurely declared convergence.

If a “backed up” message appears at the last step, there are also two possibilities. One is that Stata found a perfect maximum and could not step to a better point; if this is the case, all is fine, but this is a highly unlikely occurrence. The second is that the optimizer worked itself into a bad concave spot where the computed gradient and Hessian gave a bad direction for stepping.

If either of these messages appears at the last step, perform the maximization again with the `gradient` option. If the gradient goes to zero, the optimizer has found a maximum that may not be unique but is a maximum. From the standpoint of maximum likelihood estimation, this is a valid result. If the gradient is not zero, it is not a valid result, and you should try tightening up the convergence criterion, or try `lto1(0) to1(1e-7)` to see if the optimizer can work its way out of the bad region.

If you get repeated “not concave” steps with little progress being made at each step, try specifying the `difficult` option. Sometimes `difficult` works wonderfully, reducing the number of iterations and producing convergence at a good (that is, concave) point. Other times, `difficult` works poorly, taking much longer to converge than the default stepper.

## Stored results

Maximum likelihood estimators store the following in `e()`:

### Scalars

|                            |                                           |                                                                                                                                |
|----------------------------|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------|
| <code>e(N)</code>          | number of observations                    | always stored                                                                                                                  |
| <code>e(k)</code>          | number of parameters                      | always stored                                                                                                                  |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>  | usually stored                                                                                                                 |
| <code>e(k_eq_model)</code> | number of equations in overall model test | usually stored                                                                                                                 |
| <code>e(k_dv)</code>       | number of dependent variables             | usually stored                                                                                                                 |
| <code>e(df_m)</code>       | model degrees of freedom                  | always stored                                                                                                                  |
| <code>e(r2_p)</code>       | pseudo- $R^2$                             | sometimes stored                                                                                                               |
| <code>e(ll)</code>         | log likelihood                            | always stored                                                                                                                  |
| <code>e(ll_0)</code>       | log likelihood, constant-only model       | stored when constant-only model is fit                                                                                         |
| <code>e(N_clust)</code>    | number of clusters                        | stored when <code>vce(cluster clustvar)</code> is specified; see <a href="#">[U] 20.22 Obtaining robust variance estimates</a> |
| <code>e(chi2)</code>       | $\chi^2$                                  | usually stored                                                                                                                 |
| <code>e(p)</code>          | $p$ -value for model test                 | usually stored                                                                                                                 |
| <code>e(rank)</code>       | rank of $e(V)$                            | always stored                                                                                                                  |
| <code>e(rank0)</code>      | rank of $e(V)$ for constant-only model    | stored when constant-only model is fit                                                                                         |
| <code>e(ic)</code>         | number of iterations                      | usually stored                                                                                                                 |
| <code>e(rc)</code>         | return code                               | usually stored                                                                                                                 |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise               | usually stored                                                                                                                 |

### Macros

|                                 |                                                                          |                                                                                                                                |
|---------------------------------|--------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------|
| <code>e(cmd)</code>             | name of command                                                          | always stored                                                                                                                  |
| <code>e(cmdline)</code>         | command as typed                                                         | always stored                                                                                                                  |
| <code>e(depvar)</code>          | names of dependent variables                                             | always stored                                                                                                                  |
| <code>e(wtype)</code>           | weight type                                                              | stored when weights are specified or implied                                                                                   |
| <code>e(wexp)</code>            | weight expression                                                        | stored when weights are specified or implied                                                                                   |
| <code>e(title)</code>           | title in estimation output                                               | usually stored by commands using <code>ml</code>                                                                               |
| <code>e(clustvar)</code>        | name of cluster variable                                                 | stored when <code>vce(cluster clustvar)</code> is specified; see <a href="#">[U] 20.22 Obtaining robust variance estimates</a> |
| <code>e(chi2type)</code>        | Wald or LR; type of model $\chi^2$ test                                  | usually stored                                                                                                                 |
| <code>e(vce)</code>             | <i>vcetype</i> specified in <code>vce()</code>                           | stored when command allows <code>vce()</code>                                                                                  |
| <code>e(vcetype)</code>         | title used to label Std. err.                                            | sometimes stored                                                                                                               |
| <code>e(opt)</code>             | type of optimization                                                     | always stored                                                                                                                  |
| <code>e(which)</code>           | max or min; whether optimizer is to perform maximization or minimization | always stored                                                                                                                  |
| <code>e(ml_method)</code>       | type of <code>ml</code> method                                           | always stored by commands using <code>ml</code>                                                                                |
| <code>e(user)</code>            | name of likelihood-evaluator program                                     | always stored                                                                                                                  |
| <code>e(technique)</code>       | from <code>technique()</code> option                                     | sometimes stored                                                                                                               |
| <code>e(singularHmethod)</code> | m-marquardt or hybrid; method used when Hessian is singular              | sometimes stored <sup>1</sup>                                                                                                  |
| <code>e(crittype)</code>        | optimization criterion                                                   | always stored <sup>1</sup>                                                                                                     |
| <code>e(properties)</code>      | estimator properties                                                     | always stored                                                                                                                  |
| <code>e(predict)</code>         | program used to implement <code>predict</code>                           | usually stored                                                                                                                 |

Macros, *continued*

|                                   |                                                                                                |                                        |
|-----------------------------------|------------------------------------------------------------------------------------------------|----------------------------------------|
| <code>e(deriv_useminbound)</code> | off or on; whether minimum bounds are used for step sizes in numerical derivative computations | always stored by <code>ml</code> model |
|-----------------------------------|------------------------------------------------------------------------------------------------|----------------------------------------|

## Matrices

|                                |                                              |                                                                                                |
|--------------------------------|----------------------------------------------|------------------------------------------------------------------------------------------------|
| <code>e(b)</code>              | coefficient vector                           | always stored                                                                                  |
| <code>e(Cns)</code>            | constraints matrix                           | sometimes stored                                                                               |
| <code>e(ilog)</code>           | iteration log (up to 20 iterations)          | usually stored                                                                                 |
| <code>e(gradient)</code>       | gradient vector                              | usually stored                                                                                 |
| <code>e(V)</code>              | variance–covariance matrix of the estimators | always stored                                                                                  |
| <code>e(V_modelbased)</code>   | model-based variance                         | only stored when <code>e(V)</code> is robust, cluster–robust, bootstrap, or jackknife variance |
| <code>e(deriv_minbound)</code> | minimum values for step sizes                | only stored when <code>e(deriv_useminbound)</code> is on                                       |

## Functions

|                        |                         |               |
|------------------------|-------------------------|---------------|
| <code>e(sample)</code> | marks estimation sample | always stored |
|------------------------|-------------------------|---------------|

---

1. Type `ereturn list, all` to view these results; see [P] [return](#).

See *Stored results* in the manual entry for any maximum likelihood estimator for a list of returned results.

## Methods and formulas

Optimization is currently performed by `moptimize()` and `optimize()`, with the former implemented in terms of the latter; see [M-5] [moptimize\(\)](#) and [M-5] [optimize\(\)](#). Some estimators use `moptimize()` and `optimize()` directly, and others use the `ml` ado-file interface to `moptimize()`.

Prior to Stata 11, Stata had three separate optimization engines: an internal one used by estimation commands implemented in C code; `ml` implemented in ado-code separately from `moptimize()` and used by most estimators; and `moptimize()` and `optimize()` used by a few recently written estimators. These days, the internal optimizer and the old version of `ml` are used only under version control. In addition, `arch` and `arima` (see [TS] [arch](#) and [TS] [arima](#)) are currently implemented using the old `ml`.

Let  $L_1$  be the log likelihood of the full model (that is, the log-likelihood value shown on the output), and let  $L_0$  be the log likelihood of the “constant-only” model. The likelihood-ratio  $\chi^2$  model test is defined as  $2(L_1 - L_0)$ . The pseudo- $R^2$  (McFadden 1974) is defined as  $1 - L_1/L_0$ . This is simply the log likelihood on a scale where 0 corresponds to the “constant-only” model and 1 corresponds to perfect prediction for a discrete model (in which case the overall log likelihood is 0).

Some maximum likelihood routines can report coefficients in an exponentiated form, for example, odds ratios in `logistic`. Let  $b$  be the unexponentiated coefficient,  $s$  its standard error, and  $b_0$  and  $b_1$  the reported confidence interval for  $b$ . In exponentiated form, the point estimate is  $e^b$ , the standard error  $e^b s$ , and the confidence interval  $e^{b_0}$  and  $e^{b_1}$ . The displayed  $Z$  (or  $t$ ) statistics and  $p$ -values are the same as those for the unexponentiated results. This is justified because  $e^b = 1$  and  $b = 0$  are equivalent hypotheses, and normality is more likely to hold in the  $b$  metric.

## References

- McFadden, D. L. 1974. “Conditional logit analysis of qualitative choice behavior”. In *Frontiers in Econometrics*, edited by P. Zarembka, 105–142. New York: Academic Press.
- Pitblado, J. S., B. P. Poi, and W. W. Gould. 2024. *Maximum Likelihood Estimation with Stata*. 5th ed. College Station, TX: Stata Press.

## Also see

- [R] **ml** — Maximum likelihood estimation
- [R] **set iter** — Control iteration settings
- [SVY] **ml for svy** — Maximum pseudolikelihood estimation for survey data
- [M-5] **moptimize()** — Model optimization
- [M-5] **optimize()** — Function optimization

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## Description

`mean` produces estimates of means, along with standard errors.

## Quick start

Mean, standard error, and 95% confidence interval for `v1`

```
mean v1
```

Also compute statistics for `v2`

```
mean v1 v2
```

Same as above, but for each level of categorical variable `catvar1`

```
mean v1 v2, over(catvar1)
```

Weighting by probability weight `wvar`

```
mean v1 v2 [pweight=wvar]
```

Population mean using `svyset` data

```
svy: mean v3
```

Subpopulation means for each level of categorical variable `catvar2` using `svyset` data

```
svy: mean v3, over(catvar2)
```

Test equality of two subpopulation means

```
svy: mean v3, over(catvar2)
test v3@1.catvar2 = v3@2.catvar2
```

## Menu

Statistics > Summaries, tables, and tests > Summary and descriptive statistics > Means

## Syntax

```
mean varlist [if] [in] [weight] [, options]
```

| <i>options</i>                                | Description                                                                                                               |
|-----------------------------------------------|---------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                  |                                                                                                                           |
| <code>stdize(<i>varname</i>)</code>           | variable identifying strata for standardization                                                                           |
| <code>stdweight(<i>varname</i>)</code>        | weight variable for standardization                                                                                       |
| <code>nostdrescale</code>                     | do not rescale the standard weight variable                                                                               |
| <b>if/in/over</b>                             |                                                                                                                           |
| <code>over(<i>varlist<sub>o</sub></i>)</code> | group over subpopulations defined by <i>varlist<sub>o</sub></i>                                                           |
| <b>SE/Cluster</b>                             |                                                                                                                           |
| <code>vce(<i>vcetype</i>)</code>              | <i>vcetype</i> may be analytic, <code>cluster <i>clustvar</i></code> , <code>bootstrap</code> , or <code>jackknife</code> |
| <b>Reporting</b>                              |                                                                                                                           |
| <code>level(#)</code>                         | set confidence level; default is level(95)                                                                                |
| <code>noheader</code>                         | suppress table header                                                                                                     |
| <code>display_options</code>                  | control column formats, line width, display of omitted variables and base and empty cells, and factor-variable labeling   |
| <code>coeflegend</code>                       | display legend instead of statistics                                                                                      |

*varlist* may contain factor variables; see [U] 11.4.3 Factor variables.

bayesboot, bootstrap, collect, jackknife, mi estimate, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands.

vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.

Weights are not allowed with the bootstrap prefix; see [R] bootstrap.

aweight are not allowed with the jackknife prefix; see [R] jackknife.

vce() and weights are not allowed with the svy prefix; see [SVY] svy.

fweights, aweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.

coeflegend does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Options

### Model

`stdize(varname)` specifies that the point estimates be adjusted by direct standardization across the strata identified by *varname*. This option requires the `stdweight()` option.

`stdweight(varname)` specifies the weight variable associated with the standard strata identified in the `stdize()` option. The standardization weights must be constant within the standard strata.

`nostdrescale` prevents the standardization weights from being rescaled within the `over()` groups. This option requires `stdize()` but is ignored if the `over()` option is not specified.

### if/in/over

`over(varlisto)` specifies that estimates be computed for multiple subpopulations, which are identified by the different values of the variables in *varlist<sub>o</sub>*. Only numeric, nonnegative, integer-valued variables are allowed in `over(varlisto)`.

## SE/Cluster

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`analytic`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

`vce(analytic)`, the default, uses the analytically derived variance estimator associated with the sample mean.

## Reporting

`level(#)`; see [R] [Estimation options](#).

`noheader` prevents the table header from being displayed.

`display_options`: `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

The following option is available with `mean` but is not shown in the dialog box:

`coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

### ▷ Example 1

Using the fuel data from [example 3](#) of [R] [ttest](#), we estimate the average mileage of the cars without the fuel treatment (`mpg1`) and those with the fuel treatment (`mpg2`).

```
. use https://www.stata-press.com/data/r19/fuel
. mean mpg1 mpg2
```

| Mean estimation |       | Number of obs = 12 |                      |          |
|-----------------|-------|--------------------|----------------------|----------|
|                 | Mean  | Std. err.          | [95% conf. interval] |          |
| mpg1            | 21    | .7881701           | 19.26525             | 22.73475 |
| mpg2            | 22.75 | .9384465           | 20.68449             | 24.81551 |

Using these results, we can test the equality of the mileage between the two groups of cars.

```
. test mpg1 = mpg2
(1) mpg1 - mpg2 = 0
 F(1, 11) = 5.04
 Prob > F = 0.0463
```

## ▷ Example 2

In [example 1](#), the joint observations of `mpg1` and `mpg2` were used to estimate a covariance between their means.

```
. matrix list e(V)
symmetric e(V) [2,2]
 mpg1 mpg2
mpg1 .62121212
mpg2 .4469697 .88068182
```

If the data were organized this way out of convenience but the two variables represent independent samples of cars (coincidentally of the same sample size), we should reshape the data and use the `over()` option to ensure that the covariance between the means is zero.

```
. use https://www.stata-press.com/data/r19/fuel
. stack mpg1 mpg2, into(mpg) clear
. rename _stack trt
. label define trt_lab 1 "without" 2 "with"
. label values trt trt_lab
. label var trt "Fuel treatment"
. mean mpg, over(trt)
Mean estimation Number of obs = 24
```

|           | Mean  | Std. err. | [95% conf. interval] |          |
|-----------|-------|-----------|----------------------|----------|
| c.mpg@trt |       |           |                      |          |
| without   | 21    | .7881701  | 19.36955             | 22.63045 |
| with      | 22.75 | .9384465  | 20.80868             | 24.69132 |

```
. matrix list e(V)
symmetric e(V) [2,2]
 c.mpg@ c.mpg@
 1.trt 2.trt
c.mpg@1.trt .62121212
c.mpg@2.trt 0 .88068182
```

Now, we can test the equality of the mileage between the two independent groups of cars.

```
. test mpg@1.trt = mpg@2.trt
(1) c.mpg@1bn.trt - c.mpg@2.trt = 0
 F(1, 23) = 2.04
 Prob > F = 0.1667
```

### ► Example 3: standardized means

Suppose that we collected the blood pressure data from [example 2](#) of [R] [dstdize](#), and we wish to obtain standardized high blood pressure rates for each city in 1990 and 1992, using, as the standard, the age, sex, and race distribution of the four cities and two years combined. Our rate is really the mean of a variable that indicates whether a sampled individual has high blood pressure. First, we generate the strata and weight variables from our standard distribution, and then use `mean` to compute the rates.

```
. use https://www.stata-press.com/data/r19/hbp, clear
. egen strata = group(age race sex) if inlist(year, 1990, 1992)
(675 missing values generated)
. by strata, sort: gen stdw = _N
. mean hbp, over(city year) stdize(strata) stdweight(stdw)
Mean estimation
N. of std strata = 24 Number of obs = 455
```

|                 | Mean     | Std. err. | [95% conf. interval] |          |
|-----------------|----------|-----------|----------------------|----------|
| c.hbp@city#year |          |           |                      |          |
| 1 1990          | .058642  | .0296273  | .0004182             | .1168657 |
| 1 1992          | .0117647 | .0113187  | -.0104789            | .0340083 |
| 2 1990          | .0488722 | .0238958  | .0019121             | .0958322 |
| 2 1992          | .014574  | .007342   | .0001455             | .0290025 |
| 3 1990          | .1011211 | .0268566  | .0483425             | .1538998 |
| 3 1992          | .0810577 | .0227021  | .0364435             | .1256719 |
| 5 1990          | .0277778 | .0155121  | -.0027066            | .0582622 |
| 5 1992          | .0548926 | 0         | .                    | .        |

The standard error of the high blood pressure rate estimate is missing for city 5 in 1992 because there was only one individual with high blood pressure; that individual was the only person observed in the stratum of white males 30–35 years old.

By default, `mean` rescales the standard weights within the `over()` groups. In the following, we use the `nostdrescale` option to prevent this, thus reproducing the results in [R] [dstdize](#).

```
. mean hbp, over(city year) stdize(strata) stdweight(stdw) nostdrescale
Mean estimation
N. of std strata = 24 Number of obs = 455
```

|                 | Mean     | Std. err. | [95% conf. interval] |          |
|-----------------|----------|-----------|----------------------|----------|
| c.hbp@city#year |          |           |                      |          |
| 1 1990          | .0073302 | .0037034  | .0000523             | .0146082 |
| 1 1992          | .0015432 | .0014847  | -.0013745            | .004461  |
| 2 1990          | .0078814 | .0038536  | .0003084             | .0154544 |
| 2 1992          | .0025077 | .0012633  | .000025              | .0049904 |
| 3 1990          | .0155271 | .0041238  | .007423              | .0236312 |
| 3 1992          | .0081308 | .0022772  | .0036556             | .012606  |
| 5 1990          | .0039223 | .0021904  | -.0003822            | .0082268 |
| 5 1992          | .0088735 | 0         | .                    | .        |

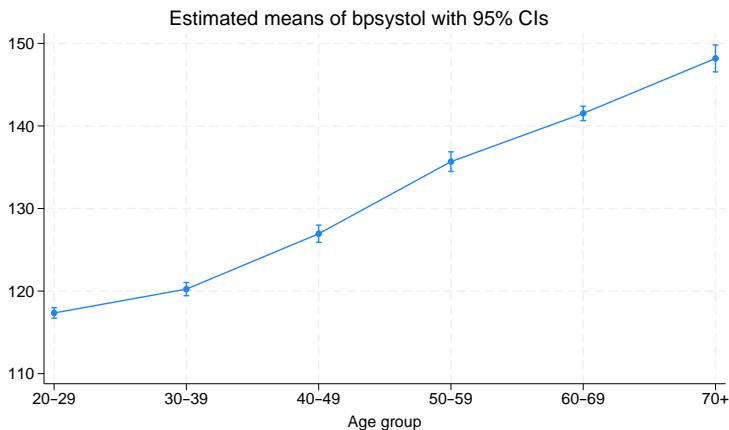
## ► Example 4: profile plots and contrasts

The [first example](#) in [\[R\] marginsplot](#) shows how to use margins and marginsplot to get profile plots from a linear regression. We can similarly explore the data using marginsplot after mean with the `over()` option. Here we use marginsplot to plot the means of systolic blood pressure for each age group.

```
. use https://www.stata-press.com/data/r19/nhanes2, clear
. mean bpsystol, over(agegrp)
Mean estimation Number of obs = 10,351
```

|                   | Mean     | Std. err. | [95% conf. interval] |          |
|-------------------|----------|-----------|----------------------|----------|
| c.bpsystol@agegrp |          |           |                      |          |
| 20-29             | 117.3466 | .3247329  | 116.71               | 117.9831 |
| 30-39             | 120.2374 | .4095845  | 119.4345             | 121.0402 |
| 40-49             | 126.9442 | .532033   | 125.9013             | 127.9871 |
| 50-59             | 135.6754 | .6061842  | 134.4872             | 136.8637 |
| 60-69             | 141.5227 | .4433527  | 140.6537             | 142.3918 |
| 70+               | 148.1765 | .8321116  | 146.5454             | 149.8076 |

```
. marginsplot
Variables that uniquely identify means: agegrp
```



We see that the mean systolic blood pressure increases with age. We can use `contrast` to formally test whether each mean is different from the mean in the previous age group using the `ar.` contrast operator; see [\[R\] contrast](#) for more information on this command.

```
. contrast ar.agegrp#c.bpsystol, effects nowald
```

Contrasts of means

|                                                                                                                                                       | Contrast | Std. err. | t     | P> t  | [95% conf. interval] |          |
|-------------------------------------------------------------------------------------------------------------------------------------------------------|----------|-----------|-------|-------|----------------------|----------|
| agegrp#<br>c.bpsystol<br>(30-39<br>vs<br>20-29)<br>(40-49<br>vs<br>30-39)<br>(50-59<br>vs<br>40-49)<br>(60-69<br>vs<br>50-59)<br>(70+<br>vs<br>60-69) | 2.89081  | .5226958  | 5.53  | 0.000 | 1.866225             | 3.915394 |
|                                                                                                                                                       | 6.706821 | .6714302  | 9.99  | 0.000 | 5.390688             | 8.022954 |
|                                                                                                                                                       | 8.731263 | .8065472  | 10.83 | 0.000 | 7.150275             | 10.31225 |
|                                                                                                                                                       | 5.847282 | .7510133  | 7.79  | 0.000 | 4.375151             | 7.319413 |
|                                                                                                                                                       | 6.653743 | .9428528  | 7.06  | 0.000 | 4.80557              | 8.501917 |

The first row of the output reports that the mean systolic blood pressure for the 30–39 age group is 2.89 higher than the mean for the 20–29 age group. The mean for the 40–49 age group is 6.71 higher than the mean for the 30–39 age group, and so on. Each of these differences is significantly different from zero.

We can include both agegrp and sex in the over() option to estimate means separately for men and women in each age group.

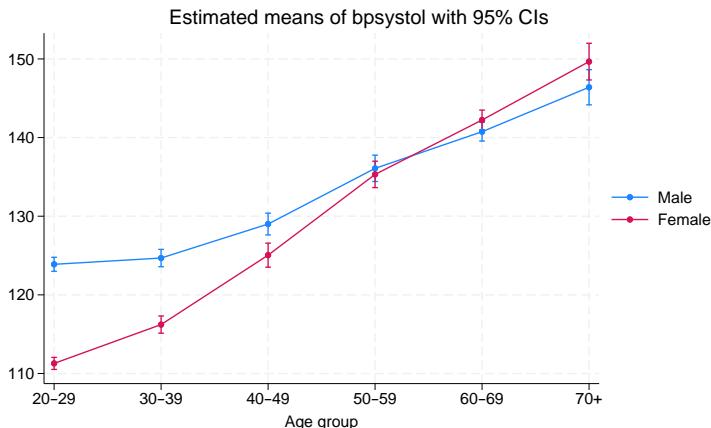
```
. mean bpsystol, over(agegrp sex)
```

Mean estimation

Number of obs = 10,351

|                       | Mean     | Std. err. | [95% conf. interval] |          |
|-----------------------|----------|-----------|----------------------|----------|
| c.bpsystol@agegrp#sex |          |           |                      |          |
| 20-29#Male            | 123.8862 | .4528516  | 122.9985             | 124.7739 |
| 20-29#Female          | 111.2849 | .3898972  | 110.5206             | 112.0492 |
| 30-39#Male            | 124.6818 | .5619855  | 123.5802             | 125.7834 |
| 30-39#Female          | 116.2207 | .5572103  | 115.1284             | 117.3129 |
| 40-49#Male            | 129.0033 | .7080788  | 127.6153             | 130.3912 |
| 40-49#Female          | 125.0468 | .7802558  | 123.5174             | 126.5763 |
| 50-59#Male            | 136.0864 | .855435   | 134.4096             | 137.7632 |
| 50-59#Female          | 135.3164 | .8556015  | 133.6393             | 136.9935 |
| 60-69#Male            | 140.7451 | .6059786  | 139.5572             | 141.9329 |
| 60-69#Female          | 142.2368 | .6427981  | 140.9767             | 143.4968 |
| 70+#Male              | 146.3951 | 1.141126  | 144.1583             | 148.6319 |
| 70+#Female            | 149.6599 | 1.189975  | 147.3273             | 151.9924 |

```
. marginsplot
Variables that uniquely identify means: agegrp sex
```



Are the means different for men and women within each age group? We can again perform the tests using contrast. This time, we will use `r.sex` to obtain contrasts comparing men and women and use `@agegrp` to request that the tests are performed for each age group.

```
. contrast r.sex#c.bpsystol@agegrp, effects nowald
Contrasts of means
```

|                                                              | Contrast  | Std. err. | t      | P> t  | [95% conf. interval] |           |
|--------------------------------------------------------------|-----------|-----------|--------|-------|----------------------|-----------|
| sex@agegrp#<br>c.bpsystol<br>(Female<br>vs<br>Male)<br>20-29 | -12.60132 | .5975738  | -21.09 | 0.000 | -13.77268            | -11.42996 |
| (Female<br>vs<br>Male)<br>30-39                              | -8.461161 | .7913981  | -10.69 | 0.000 | -10.01245            | -6.909868 |
| (Female<br>vs<br>Male)<br>40-49                              | -3.956451 | 1.053648  | -3.76  | 0.000 | -6.021805            | -1.891097 |
| (Female<br>vs<br>Male)<br>50-59                              | -.7699782 | 1.209886  | -0.64  | 0.525 | -3.141588            | 1.601631  |
| (Female<br>vs<br>Male)<br>60-69                              | 1.491684  | .8834022  | 1.69   | 0.091 | -.2399545            | 3.223323  |
| (Female<br>vs<br>Male)<br>70+                                | 3.264762  | 1.648699  | 1.98   | 0.048 | .0329927             | 6.496531  |

Using a 0.05 significance level, we find that the mean systolic blood pressure is different for men and women in all age groups except the fifties and sixties.

◀

## Video example

[Descriptive statistics in Stata](#)

## Stored results

mean stores the following in `e()`:

### Scalars

|                          |                                          |
|--------------------------|------------------------------------------|
| <code>e(N)</code>        | number of observations                   |
| <code>e(N_over)</code>   | number of subpopulations                 |
| <code>e(N_stdize)</code> | number of standard strata                |
| <code>e(N_clust)</code>  | number of clusters                       |
| <code>e(k_eq)</code>     | number of equations in <code>e(b)</code> |
| <code>e(df_r)</code>     | sample degrees of freedom                |
| <code>e(rank)</code>     | rank of <code>e(V)</code>                |

### Macros

|                              |                                                |
|------------------------------|------------------------------------------------|
| <code>e(cmd)</code>          | mean                                           |
| <code>e(cmdline)</code>      | command as typed                               |
| <code>e(varlist)</code>      | <i>varlist</i>                                 |
| <code>e(stdize)</code>       | <i>varname</i> from <code>stdize()</code>      |
| <code>e(stdweight)</code>    | <i>varname</i> from <code>stdweight()</code>   |
| <code>e(wtype)</code>        | weight type                                    |
| <code>e(wexp)</code>         | weight expression                              |
| <code>e(title)</code>        | title in estimation output                     |
| <code>e(clustvar)</code>     | name of cluster variable                       |
| <code>e(over)</code>         | <i>varlist</i> from <code>over()</code>        |
| <code>e(vce)</code>          | <i>vcetype</i> specified in <code>vce()</code> |
| <code>e(vcetype)</code>      | title used to label Std. err.                  |
| <code>e(properties)</code>   | b V                                            |
| <code>e(estat_cmd)</code>    | program used to implement <code>estat</code>   |
| <code>e(marginsnotok)</code> | predictions disallowed by <code>margins</code> |

### Matrices

|                           |                                                              |
|---------------------------|--------------------------------------------------------------|
| <code>e(b)</code>         | vector of mean estimates                                     |
| <code>e(V)</code>         | (co)variance estimates                                       |
| <code>e(sd)</code>        | vector of standard deviation estimates                       |
| <code>e(_N)</code>        | vector of numbers of nonmissing observations                 |
| <code>e(_N_stdsum)</code> | number of nonmissing observations within the standard strata |
| <code>e(_p_stdize)</code> | standardizing proportions                                    |
| <code>e(error)</code>     | error code corresponding to <code>e(b)</code>                |

### Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

### Matrices

|                       |                                                                                                                            |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

Methods and formulas are presented under the following headings:

*The mean estimator*  
*Survey data*  
*The survey mean estimator*  
*The standardized mean estimator*  
*The poststratified mean estimator*  
*The standardized poststratified mean estimator*  
*Subpopulation estimation*

### The mean estimator

Let  $y$  be the variable on which we want to calculate the mean and  $y_j$  an individual observation on  $y$ , where  $j = 1, \dots, n$  and  $n$  is the sample size. Let  $w_j$  be the weight, and if no weight is specified, define  $w_j = 1$  for all  $j$ . For  $n$  weights, the  $w_j$  are normalized to sum to  $n$ . See *The survey mean estimator* for **weighted data**.

Let  $W$  be the sum of the weights

$$W = \sum_{j=1}^n w_j$$

The mean is defined as

$$\bar{y} = \frac{1}{W} \sum_{j=1}^n w_j y_j$$

The default variance estimator for the mean is

$$\hat{V}(\bar{y}) = \frac{1}{W(W-1)} \sum_{j=1}^n w_j (y_j - \bar{y})^2$$

The standard error of the mean is the square root of the variance.

If  $x$ ,  $x_j$ , and  $\bar{x}$  are similarly defined for another variable (observed jointly with  $y$ ), the covariance estimator between  $\bar{x}$  and  $\bar{y}$  is

$$\widehat{\text{Cov}}(\bar{x}, \bar{y}) = \frac{1}{W(W-1)} \sum_{j=1}^n w_j (x_j - \bar{x})(y_j - \bar{y})$$

### Survey data

See **[SVY] Variance estimation**, **[SVY] Direct standardization**, and **[SVY] Poststratification** for discussions that provide background information for the following formulas. The following formulas are derived from the fact that the mean is a special case of the ratio estimator where the denominator variable is one,  $x_j = 1$ ; see **[R] ratio**.

## The survey mean estimator

Let  $Y_j$  be a survey item for the  $j$ th individual in the population, where  $j = 1, \dots, M$  and  $M$  is the size of the population. The associated population mean for the item of interest is  $\bar{Y} = Y/M$  where

$$Y = \sum_{j=1}^M Y_j$$

Let  $y_j$  be the survey item for the  $j$ th sampled individual from the population, where  $j = 1, \dots, m$  and  $m$  is the number of observations in the sample.

The estimator for the mean is  $\bar{y} = \hat{Y}/\widehat{M}$ , where

$$\hat{Y} = \sum_{j=1}^m w_j y_j \quad \text{and} \quad \widehat{M} = \sum_{j=1}^m w_j$$

and  $w_j$  is a sampling weight. The score variable for the mean estimator is

$$z_j(\bar{y}) = \frac{y_j - \bar{y}}{\widehat{M}} = \frac{\widehat{M}y_j - \hat{Y}}{\widehat{M}^2}$$

## The standardized mean estimator

Let  $D_g$  denote the set of sampled observations that belong to the  $g$ th standard stratum and define  $I_{D_g}(j)$  to indicate if the  $j$ th observation is a member of the  $g$ th standard stratum; where  $g = 1, \dots, L_D$  and  $L_D$  is the number of standard strata. Also, let  $\pi_g$  denote the fraction of the population that belongs to the  $g$ th standard stratum, thus  $\pi_1 + \dots + \pi_{L_D} = 1$ .  $\pi_g$  is derived from the `stdweight()` option.

The estimator for the standardized mean is

$$\bar{y}^D = \sum_{g=1}^{L_D} \pi_g \frac{\hat{Y}_g}{\widehat{M}_g}$$

where

$$\hat{Y}_g = \sum_{j=1}^m I_{D_g}(j) w_j y_j \quad \text{and} \quad \widehat{M}_g = \sum_{j=1}^m I_{D_g}(j) w_j$$

The score variable for the standardized mean is

$$z_j(\bar{y}^D) = \sum_{g=1}^{L_D} \pi_g I_{D_g}(j) \frac{\widehat{M}_g y_j - \hat{Y}_g}{\widehat{M}_g^2}$$

## The poststratified mean estimator

Let  $P_k$  denote the set of sampled observations that belong to poststratum  $k$  and define  $I_{P_k}(j)$  to indicate if the  $j$ th observation is a member of poststratum  $k$ ; where  $k = 1, \dots, L_P$  and  $L_P$  is the number of poststrata. Also let  $M_k$  denote the population size for poststratum  $k$ .  $P_k$  and  $M_k$  are identified by specifying the `poststrata()` and `postweight()` options on `svyset`; see [\[SVY\] svyset](#).

The estimator for the poststratified mean is

$$\bar{y}^P = \frac{\hat{Y}^P}{\widehat{M}^P} = \frac{\hat{Y}^P}{M}$$

where

$$\hat{Y}^P = \sum_{k=1}^{L_P} \frac{M_k}{\widehat{M}_k} \hat{Y}_k = \sum_{k=1}^{L_P} \frac{M_k}{\widehat{M}_k} \sum_{j=1}^m I_{P_k}(j) w_j y_j$$

and

$$\widehat{M}^P = \sum_{k=1}^{L_P} \frac{M_k}{\widehat{M}_k} \widehat{M}_k = \sum_{k=1}^{L_P} M_k = M$$

The score variable for the poststratified mean is

$$z_j(\bar{y}^P) = \frac{z_j(\hat{Y}^P)}{M} = \frac{1}{M} \sum_{k=1}^{L_P} I_{P_k}(j) \frac{M_k}{\widehat{M}_k} \left( y_j - \frac{\hat{Y}_k}{\widehat{M}_k} \right)$$

## The standardized poststratified mean estimator

The estimator for the standardized poststratified mean is

$$\bar{y}^{DP} = \sum_{g=1}^{L_D} \pi_g \frac{\hat{Y}_g^P}{\widehat{M}_g^P}$$

where

$$\hat{Y}_g^P = \sum_{k=1}^{L_p} \frac{M_k}{\widehat{M}_k} \hat{Y}_{g,k} = \sum_{k=1}^{L_p} \frac{M_k}{\widehat{M}_k} \sum_{j=1}^m I_{D_g}(j) I_{P_k}(j) w_j y_j$$

and

$$\widehat{M}_g^P = \sum_{k=1}^{L_p} \frac{M_k}{\widehat{M}_k} \widehat{M}_{g,k} = \sum_{k=1}^{L_p} \frac{M_k}{\widehat{M}_k} \sum_{j=1}^m I_{D_g}(j) I_{P_k}(j) w_j$$

The score variable for the standardized poststratified mean is

$$z_j(\bar{y}^{DP}) = \sum_{g=1}^{L_D} \pi_g \frac{\widehat{M}_g^P z_j(\hat{Y}_g^P) - \hat{Y}_g^P z_j(\widehat{M}_g^P)}{(\widehat{M}_g^P)^2}$$

where

$$z_j(\hat{Y}_g^P) = \sum_{k=1}^{L_P} I_{P_k}(j) \frac{M_k}{\widehat{M}_k} \left\{ I_{D_g}(j) y_j - \frac{\hat{Y}_{g,k}}{\widehat{M}_k} \right\}$$

and

$$z_j(\widehat{M}_g^P) = \sum_{k=1}^{L_P} I_{P_k}(j) \frac{M_k}{\widehat{M}_k} \left\{ I_{D_g}(j) - \frac{\widehat{M}_{g,k}}{\widehat{M}_k} \right\}$$

### Subpopulation estimation

Let  $S$  denote the set of sampled observations that belong to the subpopulation of interest, and define  $I_S(j)$  to indicate if the  $j$ th observation falls within the subpopulation.

The estimator for the subpopulation mean is  $\bar{y}^S = \hat{Y}^S / \widehat{M}^S$ , where

$$\hat{Y}^S = \sum_{j=1}^m I_S(j) w_j y_j \quad \text{and} \quad \widehat{M}^S = \sum_{j=1}^m I_S(j) w_j$$

Its score variable is

$$z_j(\bar{y}^S) = I_S(j) \frac{y_j - \bar{y}^S}{\widehat{M}^S} = I_S(j) \frac{\widehat{M}^S y_j - \hat{Y}^S}{(\widehat{M}^S)^2}$$

The estimator for the standardized subpopulation mean is

$$\bar{y}^{DS} = \sum_{g=1}^{L_D} \pi_g \frac{\hat{Y}_g^S}{\widehat{M}_g^S}$$

where

$$\hat{Y}_g^S = \sum_{j=1}^m I_{D_g}(j) I_S(j) w_j y_j \quad \text{and} \quad \widehat{M}_g^S = \sum_{j=1}^m I_{D_g}(j) I_S(j) w_j$$

Its score variable is

$$z_j(\bar{y}^{DS}) = \sum_{g=1}^{L_D} \pi_g I_{D_g}(j) I_S(j) \frac{\widehat{M}_g^S y_j - \hat{Y}_g^S}{(\widehat{M}_g^S)^2}$$

The estimator for the poststratified subpopulation mean is

$$\bar{y}^{PS} = \frac{\hat{Y}^{PS}}{\widehat{M}^{PS}}$$

where

$$\hat{Y}^{PS} = \sum_{k=1}^{L_P} \frac{M_k}{\widehat{M}_k} \hat{Y}_k^S = \sum_{k=1}^{L_P} \frac{M_k}{\widehat{M}_k} \sum_{j=1}^m I_{P_k}(j) I_S(j) w_j y_j$$

and

$$\widehat{M}^{PS} = \sum_{k=1}^{L_P} \frac{M_k}{\widehat{M}_k} \widehat{M}_k^S = \sum_{k=1}^{L_P} \frac{M_k}{\widehat{M}_k} \sum_{j=1}^m I_{P_k}(j) I_S(j) w_j$$

Its score variable is

$$z_j(\bar{y}^{PS}) = \frac{\widehat{M}^{PS} z_j(\hat{Y}^{PS}) - \hat{Y}^{PS} z_j(\widehat{M}^{PS})}{(\widehat{M}^{PS})^2}$$

where

$$z_j(\hat{Y}^{PS}) = \sum_{k=1}^{L_P} I_{P_k}(j) \frac{M_k}{\widehat{M}_k} \left\{ I_S(j) y_j - \frac{\hat{Y}_k^S}{\widehat{M}_k} \right\}$$

and

$$z_j(\widehat{M}^{PS}) = \sum_{k=1}^{L_P} I_{P_k}(j) \frac{M_k}{\widehat{M}_k} \left\{ I_S(j) - \frac{\widehat{M}_k^S}{\widehat{M}_k} \right\}$$

The estimator for the standardized poststratified subpopulation mean is

$$\bar{y}^{DPS} = \sum_{g=1}^{L_D} \pi_g \frac{\hat{Y}_g^{PS}}{\widehat{M}_g^{PS}}$$

where

$$\hat{Y}_g^{PS} = \sum_{k=1}^{L_p} \frac{M_k}{\widehat{M}_k} \hat{Y}_{g,k}^S = \sum_{k=1}^{L_p} \frac{M_k}{\widehat{M}_k} \sum_{j=1}^m I_{D_g}(j) I_{P_k}(j) I_S(j) w_j y_j$$

and

$$\widehat{M}_g^{PS} = \sum_{k=1}^{L_p} \frac{M_k}{\widehat{M}_k} \widehat{M}_{g,k}^S = \sum_{k=1}^{L_p} \frac{M_k}{\widehat{M}_k} \sum_{j=1}^m I_{D_g}(j) I_{P_k}(j) I_S(j) w_j$$

Its score variable is

$$z_j(\bar{y}^{DPS}) = \sum_{g=1}^{L_D} \pi_g \frac{\widehat{M}_g^{PS} z_j(\widehat{Y}_g^{PS}) - \widehat{Y}_g^{PS} z_j(\widehat{M}_g^{PS})}{(\widehat{M}_g^{PS})^2}$$

where

$$z_j(\widehat{Y}_g^{PS}) = \sum_{k=1}^{L_P} I_{P_k}(j) \frac{M_k}{\widehat{M}_k} \left\{ I_{D_g}(j) I_S(j) y_j - \frac{\widehat{Y}_{g,k}^S}{\widehat{M}_k} \right\}$$

and

$$z_j(\widehat{M}_g^{PS}) = \sum_{k=1}^{L_P} I_{P_k}(j) \frac{M_k}{\widehat{M}_k} \left\{ I_{D_g}(j) I_S(j) - \frac{\widehat{M}_{g,k}^S}{\widehat{M}_k} \right\}$$

## References

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- Stuart, A., and J. K. Ord. 1994. *Distribution Theory*. Vol. 1 of *Kendall's Advanced Theory of Statistics*, 6th ed. London: Arnold.

## Also see

- [R] [mean postestimation](#) — Postestimation tools for mean
- [R] [ameans](#) — Arithmetic, geometric, and harmonic means
- [R] [proportion](#) — Estimate proportions
- [R] [ratio](#) — Estimate ratios
- [R] [summarize](#) — Summary statistics
- [R] [total](#) — Estimate totals
- [MI] [Estimation](#) — Estimation commands for use with mi estimate
- [SVY] [Direct standardization](#) — Direct standardization of means, proportions, and ratios
- [SVY] [Poststratification](#) — Poststratification for survey data
- [SVY] [Subpopulation estimation](#) — Subpopulation estimation for survey data
- [SVY] [svy estimation](#) — Estimation commands for survey data
- [SVY] [Variance estimation](#) — Variance estimation for survey data
- [U] [20 Estimation and postestimation commands](#)

## Postestimation commands

The following postestimation commands are of special interest after `mean`:

| Command               | Description                  |
|-----------------------|------------------------------|
| <code>estat sd</code> | standard deviation estimates |

The following standard postestimation commands are also available:

| Command                  | Description                                                                                       |
|--------------------------|---------------------------------------------------------------------------------------------------|
| <code>contrast</code>    | contrasts and ANOVA-style joint tests of parameters                                               |
| <code>estat vce</code>   | variance–covariance matrix of the estimators (VCE)                                                |
| <code>estat (svy)</code> | postestimation statistics for survey data                                                         |
| <code>estimates</code>   | cataloging estimation results                                                                     |
| <code>etable</code>      | table of estimation results                                                                       |
| <code>hausman</code>     | Hausman’s specification test                                                                      |
| <code>lincom</code>      | point estimates, standard errors, testing, and inference for linear combinations of parameters    |
| <code>marginsplot</code> | graph the results from <code>mean</code>                                                          |
| <code>nlcom</code>       | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters |
| <code>pwcompare</code>   | pairwise comparisons of parameters                                                                |
| <code>test</code>        | Wald tests of simple and composite linear hypotheses                                              |
| <code>testnl</code>      | Wald tests of nonlinear hypotheses                                                                |

## estat sd

### Description for estat sd

`estat sd` reports standard deviations based on the estimation results from `mean`. `estat sd` is not appropriate with estimation results that used direct standardization.

`estat sd` can also report subpopulation standard deviations based on estimation results from `svy: mean`; see [\[SVY\] estat](#).

### Menu for estat sd

Statistics > Postestimation

### Syntax for estat sd

```
estat sd [, variance]
```

### Option for estat sd

`variance` requests that the variance be displayed instead of the standard deviation.

### Stored results for estat sd

`estat sd` stores the following in `r()`:

Matrices

|                          |                                        |
|--------------------------|----------------------------------------|
| <code>r(mean)</code>     | vector of mean estimates               |
| <code>r(sd)</code>       | vector of standard deviation estimates |
| <code>r(variance)</code> | vector of variance estimates           |

### Also see

[\[R\] mean](#) — Estimate means

[\[SVY\] estat](#) — Postestimation statistics for survey data

[\[U\] 20 Estimation and postestimation commands](#)

Description  
Options  
References

Quick start  
Remarks and examples  
Also see

Menu  
Stored results

Syntax  
Acknowledgments

## Description

`mfp` selects the multivariable fractional polynomial (MFP) model that best predicts the outcome variable from the right-hand-side variables in *xvarlist*.

For univariate fractional polynomials, `fp` can be used to fit a wider range of models than `mfp`. See [R] `fp` for more details.

## Quick start

Find optimal MFP model for regression of *y* on fractional polynomials of *x1*, *x2*, and *x3*

```
mfp: regress y x1 x2 x3
```

Same as above, but search only powers of  $-1$ ,  $-0.5$ ,  $1$ , and  $2$

```
mfp, xpowers(-1 -.5 1 2): regress y x1 x2 x3
```

Allow a maximum of 2 degrees of freedom for all covariates

```
mfp, dfdefault(2): regress y x1 x2 x3
```

Allow 4 degrees of freedom for *x1* and 2 degrees of freedom for *x2* and *x3*

```
mfp, dfdefault(2) df(x1:4): regress y x1 x2 x3
```

Same as above

```
mfp, df(x1:4, x2 x3:2): regress y x1 x2 x3
```

Use a 10% significance level when testing between fractional polynomials of different degrees

```
mfp, alpha(0.1): regress y x1 x2 x3
```

Perform backward selection using a nominal *p*-value of 0.05 for all variables

```
mfp, select(0.05): regress y x1 x2 x3
```

Same as above, but force *x3* into the model by setting its nominal *p*-value to 1

```
mfp, select(0.05, x3:1): regress y x1 x2 x3
```

Note: In the above examples, `regress` could be replaced with any estimation command allowing the `mfp` prefix.

## Menu

Statistics > Linear models and related > Fractional polynomials > Multivariable fractional polynomial models

## Syntax

```
mfp [, options] : regression_cmd [yvar_1 [yvar_2]] xvarlist [if] [in] [weight]
[, regression_cmd_options]
```

*regression\_cmd* may be `clogit`, `glm`, `intreg`, `logistic`, `logit`, `mlogit`, `nbreg`, `ologit`, `oprobit`, `poisson`, `probit`, `qreg`, `regress`, `rreg`, `stcox`, `stcrreg`, `streg`, or `xtgee`.

*yvar\_1* is not allowed for `streg`, `stcrreg`, and `stcox`. For these commands, you must first `stset` your data.

*yvar\_1* and *yvar\_2* must both be specified when *regression\_cmd* is `intreg`.

*xvarlist* has elements of type *varlist* or (*varlist*) or both, for example, `x1 x2 (x3 x4 x5)`. Elements enclosed in parentheses are tested jointly for inclusion in the model and are not eligible for fractional polynomial transformation.

| <i>options</i>                   | Description                                                                              |
|----------------------------------|------------------------------------------------------------------------------------------|
| Model 2                          |                                                                                          |
| <code>sequential</code>          | use the Royston and Altman model-selection algorithm; default uses closed-test procedure |
| <code>cycles(#)</code>           | maximum number of iteration cycles; default is <code>cycles(5)</code>                    |
| <code>dfdefault(#)</code>        | default maximum degrees of freedom; default is <code>dfdefault(4)</code>                 |
| <code>center(cent_list)</code>   | specification of centering for the independent variables                                 |
| <code>alpha(alpha_list)</code>   | <i>p</i> -values for testing between FP models; default is <code>alpha(0.05)</code>      |
| <code>df(df_list)</code>         | degrees of freedom for each predictor                                                    |
| <code>powers(numlist)</code>     | list of FP powers to use; default is <code>powers(-2 -1 (.5) 1 2 3)</code>               |
| Adv. model                       |                                                                                          |
| <code>xorder(+ - n)</code>       | order of entry into model-selection algorithm; default is <code>xorder(+)</code>         |
| <code>select(select_list)</code> | nominal <i>p</i> -values for selection on each predictor                                 |
| <code>xpowers(xp_list)</code>    | FP powers for each predictor                                                             |
| <code>zero(varlist)</code>       | treat nonpositive values of specified predictors as zero when FP is transformed          |
| <code>catzero(varlist)</code>    | add indicator variable for specified predictors                                          |
| <code>all</code>                 | include out-of-sample observations in generated variables                                |
| Reporting                        |                                                                                          |
| <code>level(#)</code>            | set confidence level; default is <code>level(95)</code>                                  |
| <code>display_options</code>     | control column formats and line width                                                    |

---

| <i>regression_cmd_options</i> | Description |
|-------------------------------|-------------|
|-------------------------------|-------------|

---

Adv. model

|                               |                                                      |
|-------------------------------|------------------------------------------------------|
| <i>regression_cmd_options</i> | options appropriate to the regression command in use |
|-------------------------------|------------------------------------------------------|

---

`collect` is allowed; see [U] 11.1.10 Prefix commands.

All weight types supported by *regression\_cmd* are allowed; see [U] 11.1.6 weight.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

`fp generate` may be used to create new variables containing fractional polynomial powers. See [R] fp.

## Options

Model 2

`sequential` chooses the sequential fractional polynomial (FP) selection algorithm (see *Methods of FP model selection*).

`cycles(#)` sets the maximum number of iteration cycles permitted. `cycles(5)` is the default.

`dfdefault(#)` determines the default maximum degrees of freedom (df) for a predictor. The default is `dfdefault(4)` (second-degree FP).

`center(cent_list)` defines the centering of the covariates  $xvar_1, xvar_2, \dots$  of *xvarlist*. The default is `center(mean)`, except for binary covariates, where it is `center(#)`, with # being the lower of the two distinct values of the covariate. A typical item in *cent\_list* is *varlist*:{mean|#|no}. Items are separated by commas. The first item is special in that *varlist* is optional, and if it is omitted, the default is reset to the specified value (mean, #, or no). For example, `center(no, age:mean)` sets the default to no (that is, no centering) and the centering of age to mean.

`alpha(alpha_list)` sets the significance levels for testing between FP models of different degrees. The rules for *alpha\_list* are the same as those for *df\_list* in the `df()` option (see below). The default nominal *p*-value (significance level, selection level) is 0.05 for all variables.

Example: `alpha(0.01)` specifies that all variables have an FP selection level of 1%.

Example: `alpha(0.05, weight:0.1)` specifies that all variables except `weight` have an FP selection level of 5%; `weight` has a level of 10%.

`df(df_list)` sets the df for each predictor. The df (not counting the regression constant, `_cons`) is twice the degree of the FP, so, for example, an *xvar* fit as a second-degree FP (FP2) has 4 df. The first item in *df\_list* may be either # or *varlist*:#. Subsequent items must be *varlist*:#. Items are separated by commas, and *varlist* is specified in the usual way for variables. With the first type of item, the df for all predictors is taken to be #. With the second type of item, all members of *varlist* (which must be a subset of *xvarlist*) have # df.

The default number of degrees of freedom for a predictor of type *varlist* specified in *xvarlist* but not in *df\_list* is assigned according to the number of distinct (unique) values of the predictor, as follows:

| # of distinct values | Default df                       |
|----------------------|----------------------------------|
| 1                    | (invalid predictor)              |
| 2–3                  | 1                                |
| 4–5                  | <code>min(2, dfdefault())</code> |
| ≥ 6                  | <code>dfdefault()</code>         |

Example: `df(4)`

All variables have 4 df.

Example: `df(2, weight displ:4)`

`weight` and `displ` have 4 df; all other variables have 2 df.

Example: `df(weight displ:4, mpg:2)`

`weight` and `displ` have 4 df, `mpg` has 2 df; all other variables have default df.

`powers(numlist)` is the set of FP powers to be used. The default set is `-2, -1, -0.5, 0, 0.5, 1, 2, 3` (0 means log).

## Adv. model

`xorder(+|-|n)` determines the order of entry of the covariates into the model-selection algorithm.

The default is `xorder(+)`, which enters them in decreasing order of significance in a multiple linear regression (most significant first). `xorder(-)` places them in reverse significance order, whereas `xorder(n)` respects the original order in *xvarlist*.

`select(select_list)` sets the nominal *p*-values (significance levels) for variable selection by backward elimination. A variable is dropped if its removal causes a nonsignificant increase in deviance. The rules for *select\_list* are the same as those for *df\_list* in the `df()` option (see above). Using the default selection level of 1 for all variables forces them all into the model. Setting the nominal *p*-value to be 1 for a given variable forces it into the model, leaving others to be selected or not. The nominal *p*-value for elements of *xvarlist* bound by parentheses is specified by including (*varlist*) in *select\_list*.

Example: `select(0.05)`

All variables have a nominal *p*-value of 5%.

Example: `select(0.05, weight:1)`

All variables except `weight` have a nominal *p*-value of 5%; `weight` is forced into the model.

Example: `select(a (b c):0.05)`

All variables except `a`, `b`, and `c` are forced into the model. `b` and `c` are tested jointly with 2 df at the 5% level, and `a` is tested singly at the 5% level.

`xpowers(xp_list)` sets the permitted FP powers for covariates individually. The rules for *xp\_list* are the same as for *df\_list* in the `df()` option. The default selection is the same as that for the `powers()` option.

Example: `xpowers(-1 0 1)`

All variables have powers  $-1, 0, 1$ .

Example: `xpowers(x5:-1 0 1)`

All variables except `x5` have default powers; `x5` has powers  $-1, 0, 1$ .

`zero(varlist)` treats negative and zero values of members of *varlist* as zero when FP transformations are applied. By default, such variables are subjected to a preliminary linear transformation to avoid negative and zero values, as described in the `scale` option of `[R] fp`. *varlist* must be part of *xvarlist*.

`catzero(varlist)` is a variation on `zero()`; see *Zeros and zero categories* below. *varlist* must be part of *xvarlist*.

*regression\_cmd\_options* may be any of the options appropriate to *regression\_cmd*.

`all` includes out-of-sample observations when generating the FP variables. By default, the generated FP variables contain missing values outside the estimation sample.

## Reporting

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [\[U\] 20.8 Specifying the width of confidence intervals](#).

*display\_options*: `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [\[R\] Estimation options](#).

## Remarks and examples

Remarks are presented under the following headings:

*Iteration report*  
*Estimation algorithm*  
*Methods of FP model selection*  
*Zeros and zero categories*

For elements in *xvarlist* not enclosed in parentheses, mfp leaves variables in the data named  $Ixvar\_1$ ,  $Ixvar\_2$ ,  $\dots$ , where *xvar* represents the first four letters of the name of  $xvar_1$ , and so on, for  $xvar_2$ ,  $xvar_3$ , etc. The new variables contain the best-fitting FP powers of  $xvar_1$ ,  $xvar_2$ ,  $\dots$ .

### Iteration report

By default, for each continuous predictor, *x*, mfp compares null, linear, and FP1 models for *x* with an FP2 model. The deviance for each of these nested submodels is given in the column labeled “Deviance”. The line labeled “Final” gives the deviance for the selected model and its powers. All the other predictors currently selected are included, with their transformations (if any). For models specified as having 1 df, the only choice is whether the variable enters the model.

### Estimation algorithm

The estimation algorithm in mfp processes the *xvars* in turn. Initially, mfp silently arranges *xvarlist* in order of increasing *p*-value (that is, of decreasing statistical significance) for omitting each predictor from the model comprising *xvarlist*, with each term linear. The aim is to model relatively important variables before unimportant ones. This approach may help to reduce potential model-fitting difficulties caused by collinearity or, more generally, “concurvity” among the predictors. See the `xorder()` option above for details on how to change the ordering.

At the initial cycle, the best-fitting FP function for  $xvar_1$  (the first of *xvarlist*) is determined, with all the other variables assumed to be linear. Either the default or the alternative procedure is used (see *Methods of FP model selection* below). The functional form (but not the estimated regression coefficients) for  $xvar_1$  is kept, and the process is repeated for  $xvar_2$ ,  $xvar_3$ , etc. The first iteration concludes when all the variables have been processed in this way. The next cycle is similar, except that the functional forms from the initial cycle are retained for all variables except the one currently being processed.

A variable whose functional form is prespecified to be linear (that is, to have 1 df) is tested for exclusion within the above procedure when its nominal *p*-value (selection level) according to `select()` is less than 1; otherwise, it is included.

Updating of FP functions and candidate variables continues until the functions and variables included in the overall model do not change (convergence). Convergence is usually achieved within 1–4 cycles.

## Methods of FP model selection

`mfp` includes two algorithms for FP model selection, both of which combine backward elimination with the selection of an FP function. For each continuous variable in turn, they start from a most-complex permitted FP model and attempt to simplify the model by reducing the degree. The default algorithm resembles a closed-test procedure, a sequence of tests maintaining the overall type I error rate at a pre-specified nominal level, such as 5%. All significance tests are approximate; therefore, the algorithm is not precisely a closed-test procedure (Royston and Sauerbrei 2008, chap. 6).

The closed-test algorithm for choosing an FP model with maximum permitted degree  $m = 2$  (that is, an FP2 model with 4 df) for one continuous predictor,  $x$ , is as follows:

1. Inclusion: Test FP2 against the null model for  $x$  on 4 df at the significance level determined by `select()`. If  $x$  is significant, continue; otherwise, drop  $x$  from the model.
2. Nonlinearity: Test FP2 against a straight line in  $x$  on 3 df at the significance level determined by `alpha()`. If significant, continue; otherwise, stop, with the chosen model for  $x$  being a straight line.
3. Simplification: Test FP2 against FP1 on 2 df at the significance level determined by `alpha()`. If significant, the final model is FP2; otherwise, it is FP1.

The first step is omitted if  $x$  is to be retained in the model, that is, if its nominal  $p$ -value, according to the `select()` option, is 1.

An alternative algorithm is available with the `sequential` option, as originally suggested by Royston and Altman (1994):

1. Test FP2 against FP1 on 2 df at the `alpha()` significance level. If significant, the final model is FP2; otherwise, continue.
2. Test FP1 against a straight line on 1 df at the `alpha()` level. If significant, the final model is FP1; otherwise, continue.
3. Test a straight line against omitting  $x$  on 1 df at the `select()` level. If significant, the final model is a straight line; otherwise, drop  $x$ .

The final step is omitted if  $x$  is to be retained in the model, that is, if its nominal  $p$ -value, according to the `select()` option, is 1.

If  $x$  is uninfluential, the overall type I error rate of this procedure is about double that of the closed-test procedure, for which the rate is close to the nominal value. This inflated type I error rate confers increased apparent power to detect nonlinear relationships.

## Zeros and zero categories

The `zero()` option permits fitting an FP model to the positive values of a covariate, taking nonpositive values as zero. An application is the assessment of the effect of cigarette smoking as a risk factor in an epidemiological study. Nonsmokers may be qualitatively different from smokers, so the effect of smoking (regarded as a continuous variable) may not be continuous between one and zero cigarettes. To allow for this, the risk may be modeled as constant for the nonsmokers and as an FP function of the number of cigarettes for the smokers:

```
. generate byte nonsmokr = cond(n_cigs==0, 1, 0) if n_cigs != .
. mfp, zero(n_cigs) df(4, nonsmokr:1): logit case n_cigs nonsmokr age
```

Omission of `zero(n_cigs)` would cause `n_cigs` to be transformed before analysis by the addition of a suitable constant, probably 1.

A closely related approach involves the `catzero()` option. The command

```
. mfp, catzero(n_cigs): logit case n_cigs age
```

would achieve a similar result to the previous command but with important differences. First, `mfp` would create the equivalent of the binary variable `nonsmokr` automatically and include it in the model. Second, the two smoking variables would be treated as one predictor in the model. With the `select()` option active, the two variables would be tested jointly for inclusion in the model. A modified version is described in [Royston and Sauerbrei \(2008, sec. 4.15\)](#).

### ▷ Example 1

We illustrate two of the analyses performed by [Sauerbrei and Royston \(1999\)](#). We use `brcancer.dta`, which contains prognostic factors data from the German Breast Cancer Study Group of patients with node-positive breast cancer. The response variable is recurrence-free survival time (`rectime`), and the censoring variable is `censrec`. There are 686 patients with 299 events. We use Cox regression to predict the log hazard of recurrence from prognostic factors of which five are continuous (`x1`, `x3`, `x5`, `x6`, `x7`) and three are binary (`x2`, `x4a`, `x4b`). Hormonal therapy (`hormon`) is known to reduce recurrence rates and is forced into the model. We use `mfp` to build a model from the initial set of eight predictors by using the backfitting model-selection algorithm. We set the nominal  $p$ -value for variable and FP selection to 0.05 for all variables except `hormon`, which it is set to 1:

```
. use https://www.stata-press.com/data/r19/brcancer
(German breast cancer data)
. stset rectime, fail(censrec)
(output omitted)
```

```
. mfp, alpha(.05) select(.05, hormon:1): stcox x1 x2 x3 x4a x4b x5 x6 x7 hormon,
> nohr
```

Deviance for model with all terms untransformed = 3471.637, 686 observations

| Variable                             | Model | (vs.) | Deviance | Dev diff. | P      | Powers | (vs.)  |
|--------------------------------------|-------|-------|----------|-----------|--------|--------|--------|
| x5                                   | null  | FP2   | 3503.610 | 61.366    | 0.000* | .      | .5 3   |
|                                      | Lin.  |       | 3471.637 | 29.393    | 0.000+ | 1      |        |
|                                      | FP1   |       | 3449.203 | 6.959     | 0.031+ | 0      |        |
|                                      | Final |       | 3442.244 |           |        | .5 3   |        |
| x6                                   | null  | FP2   | 3464.113 | 29.917    | 0.000* | .      | -2 .5  |
|                                      | Lin.  |       | 3442.244 | 8.048     | 0.045+ | 1      |        |
|                                      | FP1   |       | 3435.550 | 1.354     | 0.508  | .5     |        |
|                                      | Final |       | 3435.550 |           |        | .5     |        |
| [hormon included with 1 df in model] |       |       |          |           |        |        |        |
| x4a                                  | null  | Lin.  | 3440.749 | 5.199     | 0.023* | .      | 1      |
|                                      | Final |       | 3435.550 |           |        | 1      |        |
| x3                                   | null  | FP2   | 3436.832 | 3.560     | 0.469  | .      | -2 3   |
|                                      | Final |       | 3436.832 |           |        | .      |        |
| x2                                   | null  | Lin.  | 3437.589 | 0.756     | 0.384  | .      | 1      |
|                                      | Final |       | 3437.589 |           |        | .      |        |
| x4b                                  | null  | Lin.  | 3437.848 | 0.259     | 0.611  | .      | 1      |
|                                      | Final |       | 3437.848 |           |        | .      |        |
| x1                                   | null  | FP2   | 3437.893 | 18.085    | 0.001* | .      | -2 -.5 |
|                                      | Lin.  |       | 3437.848 | 18.040    | 0.000+ | 1      |        |
|                                      | FP1   |       | 3433.628 | 13.820    | 0.001+ | -2     |        |
|                                      | Final |       | 3419.808 |           |        | -2 -.5 |        |
| x7                                   | null  | FP2   | 3420.805 | 3.715     | 0.446  | .      | -.5 3  |
|                                      | Final |       | 3420.805 |           |        | .      |        |

End of Cycle 1: Deviance = 3420.805

|                                      |       |      |          |        |        |        |        |
|--------------------------------------|-------|------|----------|--------|--------|--------|--------|
| x5                                   | null  | FP2  | 3494.867 | 74.143 | 0.000* | .      | -2 -1  |
|                                      | Lin.  |      | 3451.795 | 31.071 | 0.000+ | 1      |        |
|                                      | FP1   |      | 3428.023 | 7.299  | 0.026+ | 0      |        |
|                                      | Final |      | 3420.724 |        |        | -2 -1  |        |
| x6                                   | null  | FP2  | 3452.093 | 32.704 | 0.000* | .      | 0 0    |
|                                      | Lin.  |      | 3427.703 | 8.313  | 0.040+ | 1      |        |
|                                      | FP1   |      | 3420.724 | 1.334  | 0.513  | .5     |        |
|                                      | Final |      | 3420.724 |        |        | .5     |        |
| [hormon included with 1 df in model] |       |      |          |        |        |        |        |
| x4a                                  | null  | Lin. | 3425.310 | 4.586  | 0.032* | .      | 1      |
|                                      | Final |      | 3420.724 |        |        | 1      |        |
| x3                                   | null  | FP2  | 3420.724 | 5.305  | 0.257  | .      | -.5 0  |
|                                      | Final |      | 3420.724 |        |        | .      |        |
| x2                                   | null  | Lin. | 3420.724 | 0.214  | 0.644  | .      | 1      |
|                                      | Final |      | 3420.724 |        |        | .      |        |
| x4b                                  | null  | Lin. | 3420.724 | 0.145  | 0.703  | .      | 1      |
|                                      | Final |      | 3420.724 |        |        | .      |        |
| x1                                   | null  | FP2  | 3440.057 | 19.333 | 0.001* | .      | -2 -.5 |
|                                      | Lin.  |      | 3440.038 | 19.314 | 0.000+ | 1      |        |
|                                      | FP1   |      | 3436.949 | 16.225 | 0.000+ | -2     |        |
|                                      | Final |      | 3420.724 |        |        | -2 -.5 |        |
| x7                                   | null  | FP2  | 3420.724 | 2.152  | 0.708  | .      | -1 3   |
|                                      | Final |      | 3420.724 |        |        | .      |        |

Fractional polynomial fitting algorithm converged after 2 cycles.

Transformations of covariates:

```
-> gen double Ix1__1 = X^-2-.0355294635 if e(sample)
-> gen double Ix1__2 = X^-1.5-.4341573547 if e(sample)
 (where: X = x1/10)
-> gen double Ix5__1 = X^-2-3.983723313 if e(sample)
-> gen double Ix5__2 = X^-1-1.99592668 if e(sample)
 (where: X = x5/10)
-> gen double Ix6__1 = X^.5-.3331600619 if e(sample)
 (where: X = (x6+1)/1000)
```

Final multivariable fractional polynomial model for \_t

| Variable | Initial |        |        | Final  |    |        |
|----------|---------|--------|--------|--------|----|--------|
|          | df      | Select | Alpha  | Status | df | Powers |
| x1       | 4       | 0.0500 | 0.0500 | in     | 4  | -2 -.5 |
| x2       | 1       | 0.0500 | 0.0500 | out    | 0  |        |
| x3       | 4       | 0.0500 | 0.0500 | out    | 0  |        |
| x4a      | 1       | 0.0500 | 0.0500 | in     | 1  | 1      |
| x4b      | 1       | 0.0500 | 0.0500 | out    | 0  |        |
| x5       | 4       | 0.0500 | 0.0500 | in     | 4  | -2 -1  |
| x6       | 4       | 0.0500 | 0.0500 | in     | 2  | .5     |
| x7       | 4       | 0.0500 | 0.0500 | out    | 0  |        |
| hormon   | 1       | 1.0000 | 0.0500 | in     | 1  | 1      |

Cox regression -- Breslow method for ties

```
Entry time _t0
Log likelihood = -1710.3619
Number of obs = 686
LR chi2(7) = 155.62
Prob > chi2 = 0.0000
Pseudo R2 = 0.0435
```

| _t     | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|--------|-------------|-----------|-------|-------|----------------------|-----------|
| Ix1__1 | 44.73377    | 8.256682  | 5.42  | 0.000 | 28.55097             | 60.91657  |
| Ix1__2 | -17.92302   | 3.909611  | -4.58 | 0.000 | -25.58571            | -10.26032 |
| x4a    | .5006982    | .2496324  | 2.01  | 0.045 | .0114276             | .9899687  |
| Ix5__1 | .0387904    | .0076972  | 5.04  | 0.000 | .0237041             | .0538767  |
| Ix5__2 | -.5490645   | .0864255  | -6.35 | 0.000 | -.7184554            | -.3796736 |
| Ix6__1 | -1.806966   | .3506314  | -5.15 | 0.000 | -2.494191            | -1.119741 |
| hormon | -.4024169   | .1280843  | -3.14 | 0.002 | -.6534575            | -.1513763 |

Deviance = 3420.724.

Some explanation of the output from the model-selection algorithm is desirable. Consider the first few lines of output in the iteration log:

```
1. Deviance for model with all terms untransformed = 3471.637, 686 observations
```

| Variable | Model (vs.) | Deviance | Dev diff. | P      | Powers | (vs.) |
|----------|-------------|----------|-----------|--------|--------|-------|
| 2. x5    | null FP2    | 3503.610 | 61.366    | 0.000* | .      | .5 3  |
| 3.       | lin.        | 3471.637 | 29.393    | 0.000+ | 1      |       |
| 4.       | FP1         | 3449.203 | 6.959     | 0.031+ | 0      |       |
| 5.       | Final       | 3442.244 |           |        | .5 3   |       |

Line 1 gives the deviance ( $-2 \times \log$  partial likelihood) for the Cox model with all terms linear, the place where the algorithm starts. The model is modified variable by variable in subsequent steps. The most significant linear term turns out to be x5, which is therefore processed first. Line 2 compares the best-fitting FP2 for x5 with a model omitting x5. The FP has powers (0.5, 3), and the test for inclusion of x5 is highly significant. The reported deviance of 3,503.610 is of the null model, not for the FP2 model.

The deviance for the FP2 model may be calculated by subtracting the deviance difference (`Dev diff.`) from the reported deviance, giving  $3,503.610 - 61.366 = 3,442.244$ . Line 3 shows that the FP2 model is also a significantly better fit than a straight line (`lin.`) and line 4 that FP2 is also somewhat better than FP1 ( $p = 0.031$ ). Thus at this stage in the model-selection procedure, the final model for `x5` (line 5) is FP2 with powers (0.5, 3). The overall model with an FP2 for `x5` and all other terms linear has a deviance of 3,442.244.

After all the variables have been processed (cycle 1) and reprocessed (cycle 2) in this way, convergence is achieved because the functional forms (FP powers and variables included) after cycle 2 are the same as they were after cycle 1. The model finally chosen is Model II as given in tables 3 and 4 of [Sauerbrei and Royston \(1999\)](#). Because of scaling of variables, the regression coefficients reported there are different, but the model and its deviance are identical. The model includes `x1` with powers  $(-2, -0.5)$ , `x4a`, `x5` with powers  $(-2, -1)$ , and `x6` with power 0.5. There is strong evidence of nonlinearity for `x1` and for `x5`, the deviance differences for comparison with a straight-line model (FP2 vs `lin.`) being, respectively, 19.3 and 31.1 at convergence (cycle 2). Predictors `x2`, `x3`, `x4b`, and `x7` are dropped, as may be seen from their status out in the table `Final multivariable fractional polynomial model for _t` (the assumed `depvar` when using `stcox`).

All predictors except `x4a` and `hormon`, which are binary, have been centered on the mean of the original variable. For example, the mean of `x1` (`age`) is 53.05 years. The first FP-transformed variable for `x1` is  $x1^{-2}$  and is created by the expression `generate double Ix1__1 = X^-2-.0355 if e(sample)`. The value 0.0355 is obtained from  $(53.05/10)^{-2}$ . The division by 10 is applied automatically to improve the scaling of the regression coefficient for `Ix1__1`.

According to [Sauerbrei and Royston \(1999\)](#), medical knowledge dictates that the estimated risk function for `x5` (number of positive nodes), which was based on the above FP with powers  $(-2, -1)$ , should be monotonic, but it was not. They improved Model II by estimating a preliminary exponential transformation,  $x5e = \exp(-0.12 \cdot x5)$ , for `x5` and fitting a degree 1 FP for `x5e`, thus obtaining a monotonic risk function. The value of  $-0.12$  was estimated univariately using nonlinear Cox regression with the `ado-file` `boxtid` ([Royston and Ambler 1999b, 1999d](#)). To ensure a negative exponent, [Sauerbrei and Royston \(1999\)](#) restricted the powers for `x5e` to be positive. Their Model III may be fit by using the following command:

```
. mfp, alpha(.05) select(.05, hormon:1) df(x5e:2) xpowers(x5e:0.5 1 2 3):
> stcox x1 x2 x3 x4a x4b x5e x6 x7 hormon
```



## Stored results

In addition to what *regression\_cmd* stores, *mfp* stores the following in `e()`:

### Scalars

|                        |                                                                             |
|------------------------|-----------------------------------------------------------------------------|
| <code>e(fp_nx)</code>  | number of predictors in <i>xvarlist</i>                                     |
| <code>e(fp_dev)</code> | deviance of final model fit                                                 |
| <code>e(Fp_id#)</code> | initial degrees of freedom for the #th element of <i>xvarlist</i>           |
| <code>e(Fp_fd#)</code> | final degrees of freedom for the #th element of <i>xvarlist</i>             |
| <code>e(Fp_al#)</code> | FP selection level for the #th element of <i>xvarlist</i>                   |
| <code>e(Fp_se#)</code> | backward elimination selection level for the #th element of <i>xvarlist</i> |

### Macros

|                          |                                                                    |
|--------------------------|--------------------------------------------------------------------|
| <code>e(fp_cmd)</code>   | <code>fracpoly</code>                                              |
| <code>e(fp_cmd2)</code>  | <code>mfp</code>                                                   |
| <code>e(cmdline)</code>  | command as typed                                                   |
| <code>e(fracpoly)</code> | command used to fit the selected model using <code>fracpoly</code> |
| <code>e(fp_fv1)</code>   | variables in final model                                           |
| <code>e(fp_depv)</code>  | <i>yvar</i> <sub>1</sub> ( <i>yvar</i> <sub>2</sub> )              |
| <code>e(fp_opts)</code>  | estimation command options                                         |
| <code>e(fp_x1)</code>    | first variable in <i>xvarlist</i>                                  |
| <code>e(fp_x2)</code>    | second variable in <i>xvarlist</i>                                 |
| ...                      |                                                                    |
| <code>e(fp_xN)</code>    | last variable in <i>xvarlist</i> , $N = e(fp\_nx)$                 |
| <code>e(fp_k1)</code>    | power for first variable in <i>xvarlist</i> (*)                    |
| <code>e(fp_k2)</code>    | power for second variable in <i>xvarlist</i> (*)                   |
| ...                      |                                                                    |
| <code>e(fp_kN)</code>    | power for last var. in <i>xvarlist</i> (*), $N = e(fp\_nx)$        |

Note: (\*) contains '.' if the variable is not selected in the final model.

In addition to the above, the following is stored in `r()`:

### Matrices

|                       |                                                                                                                            |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

## Acknowledgments

*mfp* is an update of *mfracpol* by Royston and Ambler (1998).

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## Also see

[R] [mfp postestimation](#) — Postestimation tools for mfp

[R] [fp](#) — Fractional polynomial regression

[U] [20 Estimation and postestimation commands](#)

Postestimation commands  
Methods and formulas

fracplot and fracpred  
Also see

Remarks and examples

## Postestimation commands

The following postestimation commands are of special interest after `mfp`:

| Command               | Description                                                                        |
|-----------------------|------------------------------------------------------------------------------------|
| <code>fracplot</code> | plot data and fit from most recently fit fractional polynomial model               |
| <code>fracpred</code> | create variable containing prediction, deviance residuals, or SEs of fitted values |

The following standard postestimation commands are also available if available after `regression_cmd`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| <code>linktest</code>        | link test for model specification                                                                                                       |
| <code>lrtest</code>          | likelihood-ratio test                                                                                                                   |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

## fracplot and fracpred

### Description for fracplot and fracpred

`fracplot` plots the data and fit, with 95% confidence limits, from the most recently fit fractional polynomial (FP) model. The data and fit are plotted against *varname*, which may be *xvar*<sub>1</sub> or another of the covariates (*xvar*<sub>2</sub>, ..., or a variable from *xvarlist*). If *varname* is not specified, *xvar*<sub>1</sub> is assumed.

`fracpred` creates *newvar* containing the fitted index or deviance residuals for the whole model, or the fitted index or its standard error for *varname*, which may be *xvar*<sub>1</sub> or another covariate.

### Menu for fracplot and fracpred

#### fracplot

Statistics > Linear models and related > Fractional polynomials > Multivariable fractional polynomial plot

#### fracpred

Statistics > Linear models and related > Fractional polynomials > Multivariable fractional polynomial prediction

### Syntax for fracplot and fracpred

*Plot data and fit from most recently fit fractional polynomial model*

```
fracplot [varname] [if] [in] [, fracplot_options]
```

*Create variable containing the prediction, deviance residuals, or SEs of fitted values*

```
fracpred newvar [, fracpred_options]
```

| <i>fracplot_options</i>                  | Description                                                           |
|------------------------------------------|-----------------------------------------------------------------------|
| Plot                                     |                                                                       |
| <i>marker_options</i>                    | change look of markers (color, size, etc.)                            |
| <i>marker_label_options</i>              | add marker labels; change look or position                            |
| Fitted line                              |                                                                       |
| <i>lineopts</i> ( <i>cline_options</i> ) | affect rendition of the fitted line                                   |
| CI plot                                  |                                                                       |
| <i>ciopts</i> ( <i>area_options</i> )    | affect rendition of the confidence bands                              |
| Add plots                                |                                                                       |
| <i>addplot</i> ( <i>plot</i> )           | add other plots to the generated graph                                |
| Y axis, X axis, Titles, Legend, Overall  |                                                                       |
| <i>twoway_options</i>                    | any options other than by() documented in [G-3] <i>twoway_options</i> |

| <i>fracpred_options</i>       | Description                                                 |
|-------------------------------|-------------------------------------------------------------|
| <i>for</i> ( <i>varname</i> ) | compute prediction for <i>varname</i>                       |
| <i>dresid</i>                 | compute deviance residuals                                  |
| <i>stdp</i>                   | compute standard errors of the fitted values <i>varname</i> |

`fracplot` is not allowed after `mfp` with `clogit`, `mlogit`, or `stcrreg`. `fracpred`, `dresid` is not allowed after `mfp` with `clogit`, `mlogit`, or `stcrreg`.

## Options for fracplot

### Plot

`marker_options` affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] [marker\\_options](#).

`marker_label_options` specify if and how the markers are to be labeled; see [G-3] [marker\\_label\\_options](#).

### Fitted line

`lineopts` (`cline_options`) affect the rendition of the fitted line; see [G-3] [cline\\_options](#).

### CI plot

`ciopts` (`area_options`) affect the rendition of the confidence bands; see [G-3] [area\\_options](#).

### Add plots

`addplot` (`plot`) provides a way to add other plots to the generated graph. See [G-3] [addplot\\_option](#).

### Y axis, X axis, Titles, Legend, Overall

`twoway_options` are any of the options documented in [G-3] [twoway\\_options](#), excluding `by()`. These include options for titling the graph (see [G-3] [title\\_options](#)) and for saving the graph to disk (see [G-3] [saving\\_option](#)).

## Options for fracpred

`for` (`varname`) specifies (partial) prediction for variable `varname`. The fitted values are adjusted to the value specified by the `center()` option in `mfp`.

`dresid` specifies that deviance residuals be calculated.

`stdp` specifies calculation of the standard errors of the fitted values `varname`, adjusted for all the other predictors at the values specified by `center()`.

## Remarks and examples

`fracplot` actually produces a component-plus-residual plot. For normal-error models with constant weights and one covariate, this amounts to a plot of the observations with the fitted line inscribed. For other normal-error models, weighted residuals are calculated and added to the fitted values.

For models with additional covariates, the line is the partial linear predictor for the variable in question (`xvar1` or a covariate) and includes the intercept  $\beta_0$ .

For generalized linear and Cox models, the fitted values are plotted on the scale of the “index” (linear predictor). Deviance residuals are added to the (partial) linear predictor to give component-plus-residual values. These values are plotted as small circles.

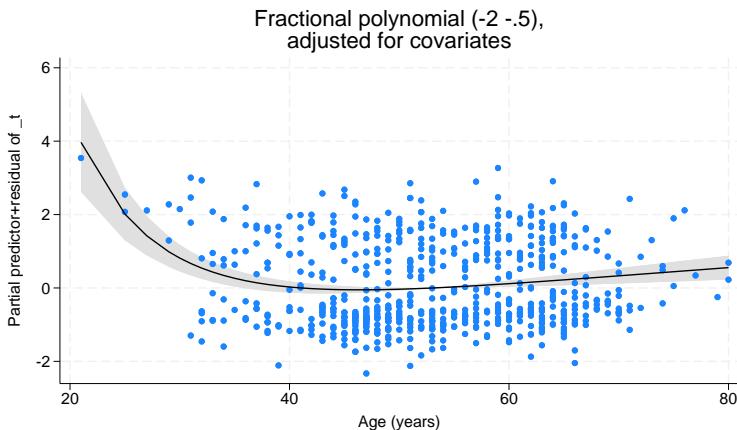
## ► Example 1

In [example 1](#) of [\[R\] mfp](#), we used Cox regression to predict the log hazard of breast cancer recurrence from prognostic factors of which five are continuous ( $x_1$ ,  $x_3$ ,  $x_5$ ,  $x_6$ ,  $x_7$ ) and three are binary ( $x_2$ ,  $x_{4a}$ ,  $x_{4b}$ ). We also controlled for hormonal therapy (`hormon`). We used `mfp` to build a model from the initial set of eight predictors by using the backfitting model-selection algorithm. The nominal  $p$ -value for variable and FP selection was set to 0.05 for all variables except `hormon`, which is set to 1.

```
. use https://www.stata-press.com/data/r19/brcancer
(German breast cancer data)
. stset rectime, fail(censrec)
(output omitted)
. mfp, alpha(.05) select(.05, hormon:1): stcox x1 x2 x3 x4a x4b x5 x6 x7 hormon,
> nohr
(output omitted)
```

We can use `fracplot` to produce component-plus-residual plots of the continuous variables. We produce the component-plus-residual plot for  $x_1$  with `fracplot` by specifying  $x_1$  after the command name.

```
. fracplot x1
```



We use `fracpred` with the `stdp` option to predict the standard error of the fractional polynomial prediction for  $x_1$ . The standard error prediction will be stored in variable `sepx1`. We specify that prediction is made for  $x_1$  with the `for()` option. After prediction, we use `summarize` to show how the standard error estimate varies over different values of  $x_1$ .

```
. fracpred sepx1, stdp for(x1)
```

```
. summarize sepx1
```

| Variable | Obs | Mean     | Std. dev. | Min      | Max      |
|----------|-----|----------|-----------|----------|----------|
| sepx1    | 686 | .0542654 | .0471993  | .0003304 | .6862065 |

◀

## Methods and formulas

The general definition of an FP, accommodating possible repeated powers, may be written for functions  $H_1(x), \dots, H_m(x)$  as

$$\beta_0 + \sum_{j=1}^m \beta_j H_j(x)$$

where  $H_1(x) = x^{(p_1)}$  and for  $j = 2, \dots, m$ ,

$$H_j(x) = \begin{cases} x^{(p_j)} & \text{if } p_j \neq p_{j-1} \\ H_{j-1}(x) \log x & \text{if } p_j = p_{j-1} \end{cases}$$

For example, an FP of degree 3 with powers (1, 3, 3) has  $H_1(x) = x$ ,  $H_2(x) = x^3$ , and  $H_3(x) = x^3 \log x$  and equals  $\beta_0 + \beta_1 x + \beta_2 x^3 + \beta_3 x^3 \log x$ .

The component-plus-residual values graphed by `fracplot` are calculated as follows: Let the data consist of triplets  $(y_i, x_i, \mathbf{z}_i)$ ,  $i = 1, \dots, n$ , where  $\mathbf{z}_i$  is the vector of covariates for the  $i$ th observation, after applying possible fractional polynomial transformation and adjustment as described earlier. Let  $\hat{\eta}_i = \hat{\beta}_0 + \{\mathbf{H}(x_i) - \mathbf{H}(x_0)\}' \hat{\beta} + \mathbf{z}_i' \hat{\gamma}$  be the linear predictor from the FP model, as given by the `fracpred` command or, equivalently, by the `predict` command with the `xb` option, following `mfp`. Here  $\mathbf{H}(x_i) = \{H_1(x_i), \dots, H_m(x_i)\}'$  is the vector of FP functions described above,  $\mathbf{H}(x_0) = \{H_1(x_0), \dots, H_m(x_0)\}'$  is the vector of adjustments to  $x_0$  (often,  $x_0$  is chosen to be the mean of the  $x_i$ ),  $\hat{\beta}$  is the estimated parameter vector, and  $\hat{\gamma}$  is the estimated parameter vector for the covariates. The values  $\hat{\eta}_i^* = \hat{\beta}_0 + \{\mathbf{H}(x_i) - \mathbf{H}(x_0)\}' \hat{\beta}$  represent the behavior of the FP model for  $x$  at fixed values  $\mathbf{z} = \mathbf{0}$  of the (adjusted) covariates. The  $i$ th component-plus-residual is defined as  $\hat{\eta}_i^* + d_i$ , where  $d_i$  is the deviance residual for the  $i$ th observation. For normal-errors models,  $d_i = \sqrt{w_i}(y_i - \hat{\eta}_i)$ , where  $w_i$  is the case weight (or 1, if *weight* is not specified). For logistic, Cox, and generalized linear regression models, see [R] [logistic](#), [R] [probit](#), [ST] [stcox](#), and [R] [glm](#) for the formula for  $d_i$ . The formula for poisson models is the same as that for `glm` with `family(poisson)`. For `stcox`,  $d_i$  is the partial martingale residual (see [ST] [stcox postestimation](#)).

`fracplot` plots the values of  $d_i$  and the curve represented by  $\hat{\eta}_i^*$  against  $x_i$ . The confidence interval for  $\hat{\eta}_i^*$  is obtained from the variance–covariance matrix of the entire model and takes into account the uncertainty in estimating  $\beta_0$ ,  $\beta$ , and  $\gamma$  (but not in estimating the FP powers for  $x$ ).

`fracpred` with the `for(varname)` option calculates the predicted index at  $x_i = x_0$  and  $\mathbf{z}_i = \mathbf{0}$ ; that is,  $\hat{\eta}_i = \hat{\beta}_0 + \{\mathbf{H}(x_i) - \mathbf{H}(x_0)\}' \hat{\beta}$ . The standard error is calculated from the variance–covariance matrix of  $(\hat{\beta}_0, \hat{\beta})$ , again ignoring estimation of the powers.

## Also see

[R] [mfp](#) — Multivariable fractional polynomial models

[U] 20 [Estimation and postestimation commands](#)

[Description](#)  
[Options](#)[Quick start](#)  
[Remarks and examples](#)[Menu](#)  
[Stored results](#)[Syntax](#)  
[Also see](#)

## Description

`misstable` makes tables that help you understand the pattern of missing values in your data.

## Quick start

*Tables with counts of missing values*

Missing observations in `v1`, `v2`, and `v3`

```
misstable summarize v1 v2 v3
```

Missing observations in `v1`–`v3` for cases where `v4 > 10`

```
misstable summarize v1 v2 v3 if v4>10
```

All variables with missing values

```
misstable summarize
```

Include variables with no missing values in the table

```
misstable summarize, all
```

Create 3 missing-data indicator variables with prefix `m` for `v1`, `v2`, and `v3`

```
misstable summarize v1 v2 v3, generate(m)
```

*Tables of missing-value patterns*

Missing-value patterns for `v1`, `v2`, and `v3`

```
misstable patterns v1 v2 v3
```

Same as above, but for all variables with missing values

```
misstable patterns
```

Show variables in the order listed in the command

```
misstable patterns v1 v3 v2, asis
```

Show frequencies, rather than percentages, in the output

```
misstable patterns v1 v2 v3, frequency
```

*Missing-value patterns displayed as trees*

A tree view of missing-value patterns for `v1`, `v2`, and `v3`

```
misstable tree v1 v2 v3
```

For all variables with missing values

```
misstable tree
```

*Nesting patterns of missing values*

Missing values for v1, v2, and v3

```
misstable nested v1 v2 v3
```

Same as above, but for all variables with missing values

```
misstable nested
```

Treat extended missing values (.a, .b, ..., .z) as nonmissing

```
misstable nested v1 v2 v3, exok
```

## Menu

Statistics > Summaries, tables, and tests > Other tables > Tabulate missing values

## Syntax

Report counts of missing values

```
misstable summarize [varlist] [if] [in] [, summarize_options]
```

Report pattern of missing values

```
misstable patterns [varlist] [if] [in] [, patterns_options]
```

Present a tree view of the pattern of missing values

```
misstable tree [varlist] [if] [in] [, tree_options]
```

List the nesting rules that describe the missing-value pattern

```
misstable nested [varlist] [if] [in] [, nested_options]
```

| <i>summarize_options</i>                 | Description                       |
|------------------------------------------|-----------------------------------|
| all                                      | show all variables                |
| showzeros                                | show zeros in table               |
| generate( <i>stub</i> [ , <i>exok</i> ]) | generate missing-value indicators |

| <i>patterns_options</i> | Description                                     |
|-------------------------|-------------------------------------------------|
| asis                    | use variables in order given                    |
| frequency               | report frequencies instead of percentages       |
| exok                    | treat .a, .b, ..., .z as nonmissing             |
| replace                 | replace data in memory with dataset of patterns |
| clear                   | okay to replace even if original unsaved        |
| bypatterns              | list by patterns rather than by frequency       |

| <i>tree_options</i> | Description                               |
|---------------------|-------------------------------------------|
| asis                | use variables in order given              |
| frequency           | report frequencies instead of percentages |
| exok                | treat .a, .b, ..., .z as nonmissing       |

| <i>nested_options</i> | Description                         |
|-----------------------|-------------------------------------|
| exok                  | treat .a, .b, ..., .z as nonmissing |

In addition, programmer's option `nopreserve` is allowed with all syntaxes; see [P] [nopreserve option](#).

`collect` is allowed; see [U] [11.1.10 Prefix commands](#).

## Options

Options are presented under the following headings:

*Options for misstable summarize*  
*Options for misstable patterns*  
*Options for misstable tree*  
*Option for misstable nested*  
*Common options*

### Options for misstable summarize

`all` specifies that the table should include all the variables specified or all the variables in the dataset. The default is to include only numeric variables that contain missing values.

`showzeros` specifies that zeros in the table should display as 0 rather than being omitted.

`generate(stub [ , exok ])` requests that a missing-value indicator *newvar*, a new binary variable containing 0 for complete observations and 1 for incomplete observations, be generated for every numeric variable in *varlist* containing missing values. If the `all` option is specified, missing-value indicators are created for all the numeric variables specified or for all the numeric variables in the dataset. If `exok` is specified within `generate()`, the extended missing values `.a`, `.b`, ..., `.z` are treated as if they do not designate missing.

For each variable in *varlist*, *newvar* is the corresponding variable name *varname* prefixed with *stub*. If the total length of *stub* and *varname* exceeds 32 characters, *newvar* is abbreviated so that its name does not exceed 32 characters.

### Options for misstable patterns

`asis`, `frequency`, and `exok` – see *Common options* below.

`replace` specifies that the data in memory be replaced with a dataset corresponding to the table just displayed; see *misstable patterns* under *Remarks and examples* below.

`clear` is for use with `replace`; it specifies that it is okay to change the data in memory even if they have not been saved to disk.

`bypatterns` specifies the table be ordered by pattern rather than by frequency. That is, `bypatterns` specifies that patterns containing one incomplete variable be listed first, followed by those for two incomplete variables, and so on. The default is to list the most frequent pattern first, followed by the next most frequent pattern, etc.

### Options for misstable tree

`asis`, `frequency`, and `exok` – see *Common options* below.

### Option for misstable nested

`exok` – see *Common options* below.

## Common options

`asis` specifies that the order of the variables in the table be the same as the order in which they are specified on the `misstable` command. The default is to order the variables by the number of missing values, and within that, by the amount of overlap of missing values.

`frequency` specifies that the table should report frequencies instead of percentages.

`exok` specifies that the extended missing values `.a`, `.b`, `. . .`, `.z` should be treated as if they do not designate missing. Some users use extended missing values to designate values that are missing for a known and valid reason.

`nopreserve` is a programmer's option allowed with all `misstable` commands; see [P] [nopreserve option](#).

## Remarks and examples

Remarks are presented under the following headings:

*misstable summarize*  
*misstable patterns*  
*misstable tree*  
*misstable nested*  
*Execution time of misstable nested*

In what follows, we will use data from a 125-observation, fictional, student-satisfaction survey:

```
. use https://www.stata-press.com/data/r19/studentsurvey
(Student survey)
. summarize
```

| Variable  | Obs | Mean     | Std. dev. | Min | Max |
|-----------|-----|----------|-----------|-----|-----|
| m1        | 125 | 2.456    | .8376619  | 1   | 4   |
| m2        | 125 | 2.472    | .8089818  | 1   | 4   |
| age       | 122 | 18.97541 | .8763477  | 17  | 21  |
| female    | 122 | .5245902 | .5014543  | 0   | 1   |
| dept      | 116 | 2.491379 | 1.226488  | 1   | 4   |
| offcampus | 125 | .36      | .4819316  | 0   | 1   |
| comment   | 0   |          |           |     |     |

The `m1` and `m2` variables record the student's satisfaction with teaching and with academics. `comment` is a string variable recording any comments the student might have had.

## misstable summarize

### ▷ Example 1

misstable summarize reports counts of missing values:

```
. misstable summarize
```

| Variable | Obs=. | Obs>. | Obs<. | Obs<.         |     |     |
|----------|-------|-------|-------|---------------|-----|-----|
|          |       |       |       | Unique values | Min | Max |
| age      | 3     |       | 122   | 5             | 17  | 21  |
| female   | 3     |       | 122   | 2             | 0   | 1   |
| dept     | 9     |       | 116   | 4             | 1   | 4   |

Stata provides 27 different missing values, namely, ., .a, .b, ..., .z. The first of those, ., is often called system missing. The remaining missing values are called extended missings. The nonmissing and missing values are ordered *nonmissing* < . < .a < .b < ... < .z. Thus, reported in the column “Obs=.” are counts of system missing values; in the column “Obs>.”, extended missing values; and in the column “Obs<.”, nonmissing values.

The rightmost portion of the table is included to remind you how the variables are encoded.

Our data contain seven variables and yet misstable reported only three of them. The omitted variables contain no missing values or are string variables. Even if we specified the varlist explicitly, those variables would not appear in the table unless we specified the all option.

We can also create missing-value indicators for each of the variables above using the generate() option:

```
. quietly misstable summarize, generate(miss_)
. describe miss_*
```

| Variable name | Storage type | Display format | Value label | Variable label |
|---------------|--------------|----------------|-------------|----------------|
| miss_age      | byte         | %8.0g          |             | (age>=.)       |
| miss_female   | byte         | %8.0g          |             | (female>=.)    |
| miss_dept     | byte         | %8.0g          |             | (dept>=.)      |

For each variable containing missing values, the generate() option creates a new binary variable containing 0 for complete observations and 1 for incomplete observations. In our example, three new missing-value indicators are generated, one for each of the incomplete variables age, female, and dept. The naming convention of generate() is to prefix the corresponding variable names with the specified stub, which is miss\_ in this example.

Missing-value indicators are useful, for example, for checking whether data are missing completely at random. They are also often used within the multiple-imputation context to identify the observed and imputed data; see [MI] [Intro substantive](#) for a general introduction to multiple imputation. Within Stata’s multiple-imputation commands, an incomplete value is identified by the system missing value, a dot. By default, misstable summarize, generate() marks the extended missing values as incomplete values, as well. You can use exok within generate() to treat extended missing values as complete when creating missing-value identifiers.

## misstable patterns

### ▷ Example 2

misstable patterns reports the pattern of missing values:

```
. misstable patterns
Missing-value patterns
(1 means complete)
```

| Percent | Pattern |   |   |
|---------|---------|---|---|
|         | 1       | 2 | 3 |
| 93%     | 1       | 1 | 1 |
| 5       | 1       | 1 | 0 |
| 2       | 0       | 0 | 0 |
| 100%    |         |   |   |

Variables are (1) age (2) female (3) dept

There are three patterns in these data: (1,1,1), (1,1,0), and (0,0,0). By default, the rows of the table are ordered by frequency. In larger tables that have more patterns, it is sometimes useful to order the rows by pattern. We could have obtained that by typing `mi misstable patterns, bypatterns`.

In a pattern, 1 indicates that all values of the variable are nonmissing and 0 indicates that all values are missing. Thus, pattern (1,1,1) means no missing values, and 93% of our data have that pattern. There are two patterns in which variables are missing, (1,1,0) and (0,0,0). Pattern (1,1,0) means that age is nonmissing, female is nonmissing, and dept is missing. The order of the variables in the patterns appears in the key at the bottom of the table. Five percent of the observations have pattern (1,1,0). The remaining 2% have pattern (0,0,0), meaning that all three variables contain missing.

As with `misstable summarize`, only numeric variables that contain missing are listed, so had we typed `misstable patterns comments age female offcampus dept`, we still would have obtained the same table. Variables that are automatically omitted contain no missing values or are string variables.

The variables in the table are ordered from lowest to highest frequency of missing values, although you cannot see that from the information presented in the table. The variables are ordered this way even if you explicitly specify the varlist with a different ordering. Typing `misstable patterns dept female age` would produce the same table as above. Specify the `asis` option if you want the variables in the order in which you specify them.

You can obtain a dataset of the patterns by specifying the `replace` option:

```
. misstable patterns, replace clear
Missing-value patterns
(1 means complete)
```

| Percent | Pattern |   |   |
|---------|---------|---|---|
|         | 1       | 2 | 3 |
| 93%     | 1       | 1 | 1 |
| 5       | 1       | 1 | 0 |
| 2       | 0       | 0 | 0 |
| 100%    |         |   |   |

Variables are (1) age (2) female (3) dept  
(summary data now in memory)

```
. list
```

|    | _freq | age | female | dept |
|----|-------|-----|--------|------|
| 1. | 3     | 0   | Male   | 0    |
| 2. | 6     | 1   | Female | 0    |
| 3. | 116   | 1   | Female | 1    |

The differences between the dataset and the printed table are that 1) the dataset always records frequency and 2) the rows are reversed.



## misstable tree

### ▷ Example 3

`misstable tree` presents a tree view of the pattern of missing values:

```
. use https://www.stata-press.com/data/r19/studentsurvey, clear
(Student survey)
```

```
. misstable tree, frequency
```

```
Nested pattern of missing values
```

| dept | age | female |
|------|-----|--------|
| 9    | 3   | 3      |
|      |     | 0      |
|      | 6   | 0      |
|      |     | 6      |
| 116  | 0   | 0      |
|      |     | 0      |
|      | 116 | 0      |
|      |     | 116    |

```
(number missing listed first)
```

In this example, we specified the `frequency` option to see the table in frequency rather than percentage terms. In the table, each column sums to the total number of observations in the data, 125. Variables are ordered from those with the most missing values to those with the least. Start with the first column. The `dept` variable is missing in 9 observations and, farther down, the table reports that it is not missing in 116 observations.

Go back to the first row and read across, but only to the second column. The `dept` variable is missing in 9 observations. Within those 9, `age` is missing in 3 of them and is not missing in the remaining 6. Reading down the second column, within the 116 observations that `dept` is not missing, `age` is missing in 0 and not missing in 116.

Reading straight across the first row again, `dept` is missing in 9 observations, and within the 9, `age` is missing in 3, and within the 3, `female` is also missing in 3. Skipping down just a little, within the 6 observations for which `dept` is missing and `age` is not missing, `female` is not missing, too.



## misstable nested

### ▷ Example 4

`misstable nested` lists the nesting rules that describe the missing-value pattern,

```
. misstable nested
 1. female(3) <-> age(3) -> dept(9)
```

This line says that in observations in which `female` is missing, so is `age` missing, and vice versa, and in observations in which `age` (or `female`) is missing, so is `dept`. The numbers in parentheses are counts of the missing values. The `female` variable happens to be missing in 3 observations, and the same is true for `age`; the `dept` variable is missing in 9 observations. Thus, `dept` is missing in the 3 observations for which `age` and `female` are missing, and in 6 more observations, too.

In these data, it turns out that the missing-value pattern can be summarized in one statement. In a larger dataset, you might see something like this:

```
. misstable nested
 1. female(50) <-> age(50) -> dept(120)
 2. female(50) -> m1(58)
 3. offcampus(11)
```

`misstable nested` accounts for every missing value. In the above, in addition to `female <-> age -> dept`, we have that `female -> m1`, and we have `offcampus`, the last all by itself. The last line says that the 11 missing values in `offcampus` are not themselves nested in the missing value of any other variable, nor do they imply the missing values in another variable. In some datasets, all the statements will be of this last form.

In our data, however, we have one statement:

```
. misstable nested
 1. female(3) <-> age(3) -> dept(9)
```

When the missing-value pattern can be summarized in one `misstable nested` statement, the pattern of missing values in the data is said to be monotone.

## Execution time of misstable nested

The execution time of `misstable nested` is affected little by the number of observations but can grow quickly with the number of variables, depending on the fraction of missing values within variable. The execution time of the example above, which has 3 variables containing missing, is instant. In worst-case scenarios, with 500 variables, the time might be 25 seconds; with 1,000 variables, the execution time might be closer to an hour.

In situations where `misstable nested` takes a long time to complete, it will produce thousands of rules that will defy interpretation. A 523-variable dataset we have seen ran in 20 seconds and produced 8,040 rules. Although we spotted a few rules in the output that did not surprise us, such as the year of the date being missing implied that the month and the day were also missing, mostly the output was not helpful.

If you have such a dataset, we recommend you run `misstable` on groups of variables that you have reason to believe the pattern of missing values might be related.

## Stored results

`misstable summarize` stores the following values of the last variable summarized in `r()`:

### Scalars

|                          |                                                   |
|--------------------------|---------------------------------------------------|
| <code>r(N_eq_dot)</code> | number of observations containing .               |
| <code>r(N_gt_dot)</code> | number of observations containing .a, .b, ..., .z |
| <code>r(N_lt_dot)</code> | number of observations containing nonmissing      |
| <code>r(K_uniq)</code>   | number of unique, nonmissing values               |
| <code>r(min)</code>      | variable's minimum value                          |
| <code>r(max)</code>      | variable's maximum value                          |

### Macros

|                         |                          |
|-------------------------|--------------------------|
| <code>r(vartype)</code> | numeric, string, or none |
|-------------------------|--------------------------|

`r(K_uniq)` contains . if the number of unique, nonmissing values is greater than 500. `r(vartype)` contains none if no variables are summarized, and in that case, the value of the scalars are all set to missing (.). Programmers intending to access results after `misstable summarize` should specify the all option.

`misstable patterns` stores the following in `r()`:

### Scalars

|                              |                                   |
|------------------------------|-----------------------------------|
| <code>r(N_complete)</code>   | number of complete observations   |
| <code>r(N_incomplete)</code> | number of incomplete observations |
| <code>r(K)</code>            | number of patterns                |

### Macros

|                      |                                   |
|----------------------|-----------------------------------|
| <code>r(vars)</code> | variables used in order presented |
|----------------------|-----------------------------------|

`r(N_complete)` and `r(N_incomplete)` are defined with respect to the variables specified if variables were specified and otherwise, defined with respect to all the numeric variables in the dataset. `r(N_complete)` is the number of observations that contain no missing values.

`misstable tree` stores the following in `r()`:

### Macros

|                      |                                   |
|----------------------|-----------------------------------|
| <code>r(vars)</code> | variables used in order presented |
|----------------------|-----------------------------------|

misstable nested stores the following in `r()`:

Scalars

`r(K)`                      number of statements

Macros

|                            |                                                 |
|----------------------------|-------------------------------------------------|
| <code>r(stmt1)</code>      | first statement                                 |
| <code>r(stmt2)</code>      | second statement                                |
| .                          | .                                               |
| .                          | .                                               |
| <code>r(stmt'r(K)')</code> | last statement                                  |
| <code>r(stmt1wc)</code>    | <code>r(stmt1)</code> with missing-value counts |
| <code>r(vars)</code>       | variables considered                            |

A statement is encoded “*varname*”, “*varname op varname*”, or “*varname op varname op varname*”, and so on; *op* is either “->” or “<->”.

## Also see

[MI] [mi misstable](#) — Tabulate pattern of missing values

[R] [summarize](#) — Summary statistics

[R] [tabulate oneway](#) — One-way table of frequencies

[R] [tabulate twoway](#) — Two-way table of frequencies

| Description    | Syntax               | Options    | Remarks and examples |
|----------------|----------------------|------------|----------------------|
| Stored results | Methods and formulas | References | Also see             |

## Description

`ml model` defines the current problem.

`ml clear` clears the current problem definition. This command is rarely used because when you type `ml model`, any previous problem is automatically cleared.

`ml query` displays a description of the current problem.

`ml check` verifies that the log-likelihood evaluator you have written works. We strongly recommend using this command.

`ml search` searches for (better) initial values. We recommend using this command.

`ml plot` provides a graphical way of searching for (better) initial values.

`ml init` provides a way to specify initial values.

`ml report` reports  $\ln L$ 's values, gradient, and Hessian at the initial values or current parameter estimates,  $\mathbf{b}_0$ .

`ml trace` traces the execution of the user-defined log-likelihood evaluation program.

`ml count` counts the number of times the user-defined log-likelihood evaluation program is called; this command is seldom used. `ml count clear` clears the counter. `ml count on` turns on the counter. `ml count` without arguments reports the current values of the counter. `ml count off` stops counting calls.

`ml maximize` maximizes the likelihood function and reports results. Once `ml maximize` has successfully completed, the previously mentioned `ml` commands may no longer be used unless `noclear` is specified. `ml graph` and `ml display` may be used whether or not `noclear` is specified.

`ml graph` graphs the log-likelihood values against the iteration number.

`ml display` redisplay results.

`ml footnote` displays a warning message when the model did not converge within the specified number of iterations.

`ml score` creates new variables containing the equation-level scores. The variables generated by `ml score` are equivalent to those generated by specifying the `score()` option of `ml maximize` (and `ml model ... , ... maximize`).

*progname* is the name of a Stata program you write to evaluate the log-likelihood function.

*funcname()* is the name of a Mata function you write to evaluate the log-likelihood function.

In this documentation, *progname* and *funcname()* are referred to as the user-written evaluator, the likelihood evaluator, or sometimes simply as the evaluator. The program you write is written in the style required by the method you choose. The methods are lf, d0, d1, d2, lf0, lf1, lf2, and gf0. Thus, if you choose to use method lf, your program is called a method-lf evaluator.

Method-lf evaluators are required to evaluate the observation-by-observation log likelihood  $\ln \ell_j, j = 1, \dots, N$ .

Method-d0 evaluators are required to evaluate the overall log likelihood  $\ln L$ . Method-d1 evaluators are required to evaluate the overall log likelihood and its gradient vector  $\mathbf{g} = \partial \ln L / \partial \mathbf{b}$ . Method-d2 evaluators are required to evaluate the overall log likelihood, its gradient, and its Hessian matrix  $H = \partial^2 \ln L / \partial \mathbf{b} \partial \mathbf{b}'$ .

Method-lf0 evaluators are required to evaluate the observation-by-observation log likelihood  $\ln \ell_j$ ,  $j = 1, \dots, N$ . Method-lf1 evaluators are required to evaluate the observation-by-observation log likelihood and its equation-level scores  $g_{ji} = \partial \ln \ell / \partial \mathbf{x}_{ji} \mathbf{b}_i$ . Method-lf2 evaluators are required to evaluate the observation-by-observation log likelihood, its equation-level scores, and its Hessian matrix  $H = \partial^2 \ln \ell / \partial \mathbf{b} \partial \mathbf{b}'$ .

Method-gf0 evaluators are required to evaluate the summable pieces of the log likelihood  $\ln \ell_k$ ,  $k = 1, \dots, K$ .

`mlevel` is a subroutine used by evaluators of methods d0, d1, d2, lf0, lf1, lf2, and gf0 to evaluate the coefficient vector,  $\mathbf{b}$ , that they are passed.

`mlsum` is a subroutine used by evaluators of methods d0, d1, and d2 to define the value,  $\ln L$ , that is to be returned.

`mlvecsum` is a subroutine used by evaluators of methods d1 and d2 to define the gradient vector,  $\mathbf{g}$ , that is to be returned. It is suitable for use only when the likelihood function meets the linear-form restrictions.

`mlmatsum` is a subroutine used by evaluators of methods d2 and lf2 to define the Hessian matrix,  $\mathbf{H}$ , that is to be returned. It is suitable for use only when the likelihood function meets the linear-form restrictions.

`mlmatbysum` is a subroutine used by evaluator of method d2 to help define the Hessian matrix,  $\mathbf{H}$ , that is to be returned. It is suitable for use when the likelihood function contains terms made up of grouped sums, such as in panel-data models. For such models, use `mlmatsum` to compute the observation-level outer products and `mlmatbysum` to compute the group-level outer products. `mlmatbysum` requires that the data be sorted by the variable identified in the `by()` option.

## Syntax

*ml model in interactive mode*

```
ml model method progname eq [eq ...] [if] [in] [weight]
 [, model_options svy diparm_options]
```

```
ml model method funcname() eq [eq ...] [if] [in] [weight]
 [, model_options svy diparm_options]
```

*ml model in noninteractive mode*

```
ml model method progname eq [eq ...] [if] [in] [weight], maximize
 [, model_options svy diparm_options noninteractive_options]
```

```
ml model method funcname() eq [eq ...] [if] [in] [weight], maximize
 [, model_options svy diparm_options noninteractive_options]
```

Noninteractive mode is invoked by specifying the `maximize` option. Use `maximize` when `ml` will be used as a subroutine of another ado-file or program and you want to carry forth the problem, from definition to posting of results, in one command.

```
ml clear

ml query

ml check

ml search [[/]eqname[:] #lb #ub] [...] [, search_options]

ml plot [eqname:]name [# [# [#]]] [, saving(filename[, replace])]

ml init { [eqname:]name=#|/eqname=# } [...]

ml init # [#...], copy

ml init matname [, copy skip]

ml report

ml trace { on|off }

ml count [clear|on|off]

ml maximize [, ml_maximize_options display_options eform_option]

ml graph [#] [, saving(filename[, replace])]

ml display [, display_options eform_option]

ml footnote

ml score newvar [if] [in] [, equation(eqname) missing]

ml score newvarlist [if] [in] [, missing]

ml score [type] stub* [if] [in] [, missing]
```

where *method* is one of

|    |         |          |     |
|----|---------|----------|-----|
| lf | d0      | lf0      | gf0 |
|    | d1      | lf1      |     |
|    | d1debug | lf1debug |     |
|    | d2      | lf2      |     |
|    | d2debug | lf2debug |     |

or *method* can be specified using one of the longer, more descriptive names

| <i>method</i> | Longer name      |
|---------------|------------------|
| lf            | linearform       |
| d0            | derivative0      |
| d1            | derivative1      |
| d1debug       | derivative1debug |
| d2            | derivative2      |
| d2debug       | derivative2debug |
| lf0           | linearform0      |
| lf1           | linearform1      |
| lf1debug      | linearform1debug |
| lf2           | linearform2      |
| lf2debug      | linearform2debug |
| gf0           | generalform0     |

*eq* is the equation to be estimated, enclosed in parentheses, and optionally with a name to be given to the equation, preceded by a colon,

( [*eqname*: ] [ *varlist*<sub>y</sub> = ] [ *varlist*<sub>x</sub> ] [ , *eq\_options* ] )

or *eq* is the name of a parameter, such as sigma, with a slash in front

*/eqname* which is equivalent to (*eqname*:, freeparm)

and *diparm\_options* is one or more `diparm(diparm_args)` options where *diparm\_args* is either `__sep__` or anything accepted by the “undocumented” `_diparm` command; see help `_diparm`.

| <i>eq_options</i>                                 | Description                                                                    |
|---------------------------------------------------|--------------------------------------------------------------------------------|
| <code>noconstant</code>                           | do not include an intercept in the equation                                    |
| <code>offset(<i>varname</i><sub>o</sub>)</code>   | include <i>varname</i> <sub>o</sub> in model with coefficient constrained to 1 |
| <code>exposure(<i>varname</i><sub>e</sub>)</code> | include $\ln(\text{varname}_e)$ in model with coefficient constrained to 1     |
| <code>freeparm</code>                             | <i>eqname</i> is a free parameter                                              |

| <i>model_options</i>                                   | Description                                                                                                                   |
|--------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------|
| <code>group(<i>varname</i>)</code>                     | use <i>varname</i> to identify groups                                                                                         |
| <code>vce(<i>vcetype</i>)</code>                       | <i>vcetype</i> may be <code>robust</code> , <code>cluster</code> <i>clustvar</i> , <code>oim</code> , or <code>opg</code>     |
| <code>constraints(<i>numlist</i>)</code>               | constraints by number to be applied                                                                                           |
| <code>constraints(<i>matname</i>)</code>               | matrix that contains the constraints to be applied                                                                            |
| <code>nocnsnotes</code>                                | do not display notes when constraints are dropped                                                                             |
| <code>title(<i>string</i>)</code>                      | place a title on the estimation output                                                                                        |
| <code>nopreserve</code>                                | do not preserve the estimation subsample in memory                                                                            |
| <code>collinear</code>                                 | keep collinear variables within equations                                                                                     |
| <code>missing</code>                                   | keep observations containing variables with missing values                                                                    |
| <code>lfl0(<i>#<sub>k</sub> #<sub>ll</sub></i>)</code> | number of parameters and log-likelihood value of the constant-only model                                                      |
| <code>continue</code>                                  | specifies that a model has been fit and sets the initial values $\mathbf{b}_0$ for the model to be fit based on those results |
| <code>waldtest(<i>#</i>)</code>                        | perform a Wald test; see <i>Options for use with ml model in interactive or noninteractive mode</i> below                     |
| <code>obs(<i>#</i>)</code>                             | number of observations                                                                                                        |
| <code>crittype(<i>string</i>)</code>                   | describe the criterion optimized by ml                                                                                        |
| <code>subpop(<i>varname</i>)</code>                    | compute estimates for the single subpopulation                                                                                |
| <code>nosvyadjust</code>                               | carry out Wald test as $W/k \sim F(k, d)$                                                                                     |
| <code>technique(nr)</code>                             | Stata's modified Newton–Raphson (NR) algorithm                                                                                |
| <code>technique(bhhh)</code>                           | Berndt–Hall–Hall–Hausman (BHHH) algorithm                                                                                     |
| <code>technique(dfpr)</code>                           | Davidon–Fletcher–Powell (DFP) algorithm                                                                                       |
| <code>technique(bfgs)</code>                           | Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm                                                                             |
| <code>deriv(<i>min_spec</i>)</code>                    | specify the minimum step size for computing numerical derivatives                                                             |

| <i>noninteractive_options</i>                | Description                                                            |
|----------------------------------------------|------------------------------------------------------------------------|
| <code>init(<i>ml_init_args</i>)</code>       | set the initial values $\mathbf{b}_0$                                  |
| <code>search(on)</code>                      | equivalent to <code>ml search, repeat(0)</code> ; the default          |
| <code>search(norescale)</code>               | equivalent to <code>ml search, repeat(0) norescale</code>              |
| <code>search(quietly)</code>                 | same as <code>search(on)</code> , except that output is suppressed     |
| <code>search(off)</code>                     | prevents calling <code>ml search</code>                                |
| <code>repeat(<i>#</i>)</code>                | <code>ml search</code> 's <code>repeat()</code> option; see below      |
| <code>bounds(<i>ml_search_bounds</i>)</code> | specify bounds for <code>ml search</code>                              |
| <code>nowarning</code>                       | suppress “convergence not achieved” message of <code>iterate(0)</code> |
| <code>novce</code>                           | substitute the zero matrix for the variance matrix                     |
| <code>negh</code>                            | indicates that the evaluator returns the negative Hessian matrix       |
| <code>score(<i>newvars</i>)</code>           | new variables containing the contribution to the score                 |
| <code>maximize_options</code>                | control the maximization process; seldom used                          |

| <i>search_options</i>               | Description                                                                                                                                                             |
|-------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>repeat(#)</code>              | number of random attempts to find better initial-value vector; default is <code>repeat(10)</code> in interactive mode and <code>repeat(0)</code> in noninteractive mode |
| <code>restart</code>                | use random actions to find starting values; not recommended                                                                                                             |
| <code>norescale</code>              | do not rescale to improve parameter vector; not recommended                                                                                                             |
| <i>maximize_options</i>             | control the maximization process; seldom used                                                                                                                           |
| <i>ml_maximize_options</i>          | Description                                                                                                                                                             |
| <code>nowarning</code>              | suppress “convergence not achieved” message of <code>iterate(0)</code>                                                                                                  |
| <code>novce</code>                  | substitute the zero matrix for the variance matrix                                                                                                                      |
| <code>negh</code>                   | indicates that the evaluator returns the negative Hessian matrix                                                                                                        |
| <code>score(newvars   stub*)</code> | new variables containing the contribution to the score                                                                                                                  |
| <code>nooutput</code>               | suppress display of final results                                                                                                                                       |
| <code>noskipline</code>             | suppress display of blank line before iteration log                                                                                                                     |
| <code>noclear</code>                | do not clear ml problem definition after model has converged                                                                                                            |
| <i>maximize_options</i>             | control the maximization process; seldom used                                                                                                                           |
| <i>display_options</i>              | Description                                                                                                                                                             |
| <code>noheader</code>               | suppress header display above the coefficient table                                                                                                                     |
| <code>nofootnote</code>             | suppress footnote display below the coefficient table                                                                                                                   |
| <code>level(#)</code>               | set confidence level; default is <code>level(95)</code>                                                                                                                 |
| <code>first</code>                  | display coefficient table reporting results for first equation only                                                                                                     |
| <code>neq(#)</code>                 | display coefficient table reporting first # equations                                                                                                                   |
| <code>showeqns</code>               | display equation names in the coefficient table                                                                                                                         |
| <code>plus</code>                   | display coefficient table ending in dashes–plus–sign–dashes                                                                                                             |
| <code>nocnsreport</code>            | suppress constraints display above the coefficient table                                                                                                                |
| <code>noomitted</code>              | suppress display of omitted variables                                                                                                                                   |
| <code>vsquish</code>                | suppress blank space separating factor-variable terms or time-series–operated variables from other variables                                                            |
| <code>noemptycells</code>           | suppress empty cells for interactions of factor variables                                                                                                               |
| <code>baselevels</code>             | report base levels of factor variables and interactions                                                                                                                 |
| <code>allbaselevels</code>          | display all base levels of factor variables and interactions                                                                                                            |
| <code>cformat(%fmt)</code>          | format the coefficients, standard errors, and confidence limits in the coefficient table                                                                                |
| <code>pformat(%fmt)</code>          | format the <i>p</i> -values in the coefficient table                                                                                                                    |
| <code>sformat(%fmt)</code>          | format the test statistics in the coefficient table                                                                                                                     |
| <code>no1stretch</code>             | do not automatically widen the coefficient table to accommodate longer variable names                                                                                   |
| <code>coeflegend</code>             | display legend instead of statistics                                                                                                                                    |

| <i>eform_option</i>        | Description                                                  |
|----------------------------|--------------------------------------------------------------|
| <code>eform(string)</code> | display exponentiated coefficients; column title is “string” |
| <code>eform</code>         | display exponentiated coefficients; column title is “exp(b)” |
| <code>hr</code>            | report hazard ratios                                         |
| <code>shr</code>           | report subhazard ratios                                      |
| <code>irr</code>           | report incidence-rate ratios                                 |
| <code>or</code>            | report odds ratios                                           |
| <code>rrr</code>           | report relative-risk ratios                                  |

`fweights`, `awweights`, `iweights`, and `pweights` are allowed; see [U] 11.1.6 **weight**. With all but method `lf`, you must write your likelihood-evaluation program carefully if `pweights` are to be specified, and `pweights` may not be specified with method `d0`, `d1`, `d1debug`, `d2`, or `d2debug`. See Pitblado, Poi, and Gould (2024, chap. 7) for details.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

To redisplay results, type `ml display`.

## Syntax of subroutines for use by evaluator programs

```

mleval newvar = vecname [, eq(#)]
mleval scalarname = vecname , scalar [eq(#)]

mlsum scalarnamelnf = exp [if] [, noweight]
mlvecsum scalarnamelnf rowvecname = exp [if] [, eq(#)]
mlmatsum scalarnamelnf matrixname = exp [if] [, eq(#[, #])]
mlmatbysum scalarnamelnf matrixname varnamea varnameb [varnamec] [if] ,
 by(varname) [eq(#[, #])]

```

## Syntax of user-written evaluator

### Summary of notation

The log-likelihood function is  $\ln L(\theta_{1j}, \theta_{2j}, \dots, \theta_{Ej})$ , where  $\theta_{ij} = \mathbf{x}_{ij}\mathbf{b}_i$ ,  $j = 1, \dots, N$  indexes observations, and  $i = 1, \dots, E$  indexes the linear equations defined by `ml model1`. If the likelihood satisfies the linear-form restrictions, it can be decomposed as  $\ln L = \sum_{j=1}^N \ln \ell(\theta_{1j}, \theta_{2j}, \dots, \theta_{Ej})$ .

### Method-`lf` evaluators

```

program progname
 version 19.5 // (or version 19 if you do not have StataNow)
 args lnfj theta1 theta2 ...
 // if you need to create any intermediate results:
 tempvar tmp1 tmp2 ...
 quietly generate double `tmp1' = ...
 ...
 quietly replace `lnfj' = ...
end

```

where

- 'lnfj' variable to be filled in with observation-by-observation values of  $\ln \ell_j$
- 'theta1' variable containing evaluation of first equation  $\theta_{1j} = \mathbf{x}_{1j} \mathbf{b}_1$
- 'theta2' variable containing evaluation of second equation  $\theta_{2j} = \mathbf{x}_{2j} \mathbf{b}_2$
- ...

## Method-d0 evaluators

```

program progname
 version 19.5 // (or version 19 if you do not have StataNow)
 args todo b lnf
 tempvar theta1 theta2 ...
 mlevel 'theta1' = 'b', eq(1)
 mlevel 'theta2' = 'b', eq(2) // if there is a θ_2
 ...
 // if you need to create any intermediate results:
 tempvar tmp1 tmp2 ...
 generate double 'tmp1' = ...
 ...
 mlsum 'lnf' = ...
end

```

where

- 'todo' always contains 0 (may be ignored)
- 'b' full parameter row vector  $\mathbf{b} = (\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_E)$
- 'lnf' scalar to be filled in with overall  $\ln L$

## Method-d1 evaluators

```

program progname
 version 19.5 // (or version 19 if you do not have StataNow)
 args todo b lnf g
 tempvar theta1 theta2 ...
 mlevel 'theta1' = 'b', eq(1)
 mlevel 'theta2' = 'b', eq(2) // if there is a θ_2
 ...
 // if you need to create any intermediate results:
 tempvar tmp1 tmp2 ...
 generate double 'tmp1' = ...
 ...
 mlsum 'lnf' = ...
 if ('todo'==0 | 'lnf'>=.) exit
 tempname d1 d2 ...
 mlvecsum 'lnf' 'd1' = formula for $\partial \ln \ell_{-j} / \partial \theta_{-1j}$, eq(1)
 mlvecsum 'lnf' 'd2' = formula for $\partial \ln \ell_{-j} / \partial \theta_{-2j}$, eq(2)
 ...
 matrix 'g' = ('d1', 'd2', ...)
end

```

where

- 'todo' contains 0 or 1
  - 0  $\Rightarrow$  'lnf' to be filled in;
  - 1  $\Rightarrow$  'lnf' and 'g' to be filled in
- 'b' full parameter row vector  $\mathbf{b} = (\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_E)$
- 'lnf' scalar to be filled in with overall  $\ln L$
- 'g' row vector to be filled in with overall  $\mathbf{g} = \partial \ln L / \partial \mathbf{b}$

## Method-d2 evaluators

```

program programe
 version 19.5 // (or version 19 if you do not have StataNow)
 args todo b lnf g H
 tempvar theta1 theta2 ...
 mlevel 'theta1' = 'b', eq(1)
 mlevel 'theta2' = 'b', eq(2) // if there is a θ_2
 ...
 // if you need to create any intermediate results:
 tempvar tmp1 tmp2 ...
 generate double 'tmp1' = ...
 ...
 mlsum 'lnf' = ...
 if ('todo'==0 | 'lnf'>=.) exit
 tempname d1 d2 ...
 mlvecsum 'lnf' 'd1' = formula for $\partial \ln \ell_{-j} / \partial \theta_{-1j}$, eq(1)
 mlvecsum 'lnf' 'd2' = formula for $\partial \ln \ell_{-j} / \partial \theta_{-2j}$, eq(2)
 ...
 matrix 'g' = ('d1', 'd2', ...)
 if ('todo'==1 | 'lnf'>=.) exit
 tempname d11 d12 d22 ...
 mlmatsum 'lnf' 'd11' = formula for $\partial^2 \ln \ell_{-j} / \partial \theta_{-1j}^2$, eq(1)
 mlmatsum 'lnf' 'd12' = formula for $\partial^2 \ln \ell_{-j} / \partial \theta_{-1j} \partial \theta_{-2j}$, eq(1,2)
 mlmatsum 'lnf' 'd22' = formula for $\partial^2 \ln \ell_{-j} / \partial \theta_{-2j}^2$, eq(2)
 ...
 matrix 'H' = ('d11', 'd12', ... \ 'd12', 'd22', ...)
end

```

where

```

'todo' contains 0, 1, or 2
 0 ⇒ 'lnf' to be filled in;
 1 ⇒ 'lnf' and 'g' to be filled in;
 2 ⇒ 'lnf', 'g', and 'H' to be filled in
'b' full parameter row vector $\mathbf{b} = (\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_E)$
'lnf' scalar to be filled in with overall $\ln L$
'g' row vector to be filled in with overall $\mathbf{g} = \partial \ln L / \partial \mathbf{b}$
'H' matrix to be filled in with overall Hessian $\mathbf{H} = \partial^2 \ln L / \partial \mathbf{b} \partial \mathbf{b}'$

```

## Method-lf0 evaluators

```

program programe
 version 19.5 // (or version 19 if you do not have StataNow)
 args todo b lnfj
 tempvar theta1 theta2 ...
 mlevel 'theta1' = 'b', eq(1)
 mlevel 'theta2' = 'b', eq(2) // if there is a θ_2
 ...
 // if you need to create any intermediate results:
 tempvar tmp1 tmp2 ...
 generate double 'tmp1' = ...
 ...
 quietly replace 'lnfj' = ...
end

```

where

```
'todo' always contains 0 (may be ignored)
'b' full parameter row vector $\mathbf{b} = (\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_E)$
'lnfj' variable to be filled in with observation-by-observation values of $\ln l_j$
```

## Method-lf1 evaluators

```
program programe
version 19.5 // (or version 19 if you do not have StataNow)
args todo b lnfj g1 g2 $...$
tempvar theta1 theta2 ...
mlevel 'theta1' = 'b', eq(1)
mlevel 'theta2' = 'b', eq(2) // if there is a θ_{-2}
...
// if you need to create any intermediate results:
tempvar tmp1 tmp2 ...
generate double 'tmp1' = ...
...
quietly replace 'lnfj' = ...
if ('todo'==0) exit
quietly replace 'g1' = formula for $\partial \ln l_{-j} / \partial \theta_{-1j}$
quietly replace 'g2' = formula for $\partial \ln l_{-j} / \partial \theta_{-2j}$
...
end
```

where

```
'todo' contains 0 or 1
 0 \Rightarrow 'lnfj' to be filled in;
 1 \Rightarrow 'lnfj', 'g1', 'g2', ..., to be filled in
'b' full parameter row vector $\mathbf{b} = (\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_E)$
'lnfj' variable to be filled in with observation-by-observation values of $\ln l_j$
'g1' variable to be filled in with $\partial \ln l_j / \partial \theta_{1j}$
'g2' variable to be filled in with $\partial \ln l_j / \partial \theta_{2j}$
...
```

## Method-lf2 evaluators

```
program programe
version 19.5 // (or version 19 if you do not have StataNow)
args todo b lnfj g1 g2 ... H
tempvar theta1 theta2 ...
mlevel 'theta1' = 'b', eq(1)
mlevel 'theta2' = 'b', eq(2) // if there is a θ_{-2}
...
// if you need to create any intermediate results:
tempvar tmp1 tmp2 ...
generate double 'tmp1' = ...
...
quietly replace 'lnfj' = ...
if ('todo'==0) exit
quietly replace 'g1' = formula for $\partial \ln l_{-j} / \partial \theta_{-1j}$
quietly replace 'g2' = formula for $\partial \ln l_{-j} / \partial \theta_{-2j}$
...
if ('todo'==1) exit
tempname d11 d12 d22 lnf ...
```

```

mlmatsum 'lnf' 'd11' = formula for $\partial^2 \ln \ell_{-j} / \partial \theta_{-1j}^2$, eq(1)
mlmatsum 'lnf' 'd12' = formula for $\partial^2 \ln \ell_{-j} / \partial \theta_{-1j} \partial \theta_{-2j}$, eq(1,2)
mlmatsum 'lnf' 'd22' = formula for $\partial^2 \ln \ell_{-j} / \partial \theta_{-2j}^2$, eq(2)
...
matrix 'H' = ('d11','d12', ... \ 'd12''','d22', ...)
end

```

where

```

'todo' contains 0 or 1
 0 \Rightarrow 'lnfj' to be filled in;
 1 \Rightarrow 'lnfj', 'g1', 'g2', ..., to be filled in
 2 \Rightarrow 'lnfj', 'g1', 'g2', ..., and 'H' to be filled in
'b' full parameter row vector $\mathbf{b} = (\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_E)$
'lnfj' scalar to be filled in with observation-by-observation $\ln L$
'g1' variable to be filled in with $\partial \ln \ell_j / \partial \theta_{1j}$
'g2' variable to be filled in with $\partial \ln \ell_j / \partial \theta_{2j}$
...
'H' matrix to be filled in with overall Hessian $\mathbf{H} = \partial^2 \ln L / \partial \mathbf{b} \partial \mathbf{b}'$

```

## Method-gf0 evaluators

```

program prognam
version 19.5 // (or version 19 if you do not have StataNow)
args todo b lnfj
tempvar theta1 theta2 ...
mlevel 'theta1' = 'b', eq(1)
mlevel 'theta2' = 'b', eq(2) // if there is a θ_{-2}
...
// if you need to create any intermediate results:
tempvar tmp1 tmp2 ...
generate double 'tmp1' = ...
...
quietly replace 'lnfj' = ...
end

```

where

```

'todo' always contains 0 (may be ignored)
'b' full parameter row vector $\mathbf{b} = (\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_E)$
'lnfj' variable to be filled in with the values of the log-likelihood $\ln \ell_j$

```

## Global macros for use by all evaluators

```

$ML_y1 name of first dependent variable
$ML_y2 name of second dependent variable, if any
...
$ML_samp variable containing 1 if observation to be used; 0 otherwise
$ML_w variable containing weight associated with observation or 1 if no weights specified

```

Method-lf evaluators can ignore \$ML\_samp, but restricting calculations to the \$ML\_samp==1 subsample will speed execution. Method-lf evaluators must ignore \$ML\_w; application of weights is handled by the method itself.

Methods `d0`, `d1`, `d2`, `lf0`, `lf1`, `lf2`, and `gf0` can ignore `$ML_samp` as long as `ml` model's `nopreserve` option is not specified. These methods will run more quickly if `nopreserve` is specified. These evaluators can ignore `$ML_w` only if they use `mlsum`, `mlvecsum`, `mlmatsum`, and `mlmatbysum` to produce all final results.

## Options

Options are presented under the following headings:

- Options for use with ml model in interactive or noninteractive mode*
- Options for use with ml model in noninteractive mode*
- Options for use when specifying equations*
- Options for use with ml search*
- Option for use with ml plot*
- Options for use with ml init*
- Options for use with ml maximize*
- Option for use with ml graph*
- Options for use with ml display*
- Options for use with mlevel*
- Option for use with mlsum*
- Option for use with mlvecsum*
- Option for use with mlmatsum*
- Options for use with mlmatbysum*
- Options for use with ml score*

### Options for use with ml model in interactive or noninteractive mode

`group(varname)` specifies the numeric variable that identifies groups. This option is typically used to identify panels for panel-data models.

`vce(vcetype)` specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that are derived from asymptotic theory (`oim`, `opg`); see [R] [vce\\_option](#).

`vce(robust)`, `vce(cluster clustvar)`, `pweight`, and `svy` will work with evaluators of methods `lf`, `lf0`, `lf1`, `lf2`, and `gf0`; all you need to do is specify them.

These options will not work with evaluators of methods `d0`, `d1`, or `d2`, and specifying these options will produce an error message.

`constraints(numlist | matname)` specifies the linear constraints to be applied during estimation. `constraints(numlist)` specifies the constraints by number. Constraints are defined by using the `constraint` command; see [R] [constraint](#). `constraint(matname)` specifies a matrix that contains the constraints.

`nocnsnotes` prevents notes from being displayed when constraints are dropped. A constraint will be dropped if it is inconsistent, contradicts other constraints, or causes some other error when the constraint matrix is being built. Constraints are checked in the order in which they are specified.

`title(string)` specifies the title for the estimation output when results are complete.

`nopreserve` specifies that `ml` need not ensure that only the estimation subsample is in memory when the user-written likelihood evaluator is called. `nopreserve` is irrelevant when you use method `lf`.

For the other methods, if `nopreserve` is not specified, `ml` saves the data in a file (preserves the original dataset) and drops the irrelevant observations before calling the user-written evaluator. This way, even if the evaluator does not restrict its attentions to the `$ML_samp==1` subsample, results will still be correct. Later, `ml` automatically restores the original dataset.

ml need not go through these machinations for method lf because the user-written evaluator calculates observation-by-observation values, and ml itself sums the components.

ml goes through these machinations if and only if the estimation sample is a subsample of the data in memory. If the estimation sample includes every observation in memory, ml does not preserve the original dataset. Thus, programmers must not alter the original dataset unless they preserve the data themselves.

We recommend that interactive users of ml not specify nopreserve; the speed gain is not worth the possibility of getting incorrect results.

We recommend that programmers specify nopreserve, but only after verifying that their evaluator really does restrict its attentions solely to the \$ML\_samp==1 subsample.

collinear specifies that ml not remove the collinear variables within equations. There is no reason to leave collinear variables in place, but this option is of interest to programmers who, in their code, have already removed collinear variables and do not want ml to waste computer time checking again.

missing specifies that observations containing variables with missing values not be eliminated from the estimation sample. There are two reasons you might want to specify missing:

Programmers may wish to specify missing because, in other parts of their code, they have already eliminated observations with missing values and do not want ml to waste computer time looking again.

You may wish to specify missing if your model explicitly deals with missing values. Stata's heckman command is a good example of this. In such cases, there will be observations where missing values are allowed and other observations where they are not—where their presence should cause the observation to be eliminated. If you specify missing, it is your responsibility to specify an if exp that eliminates the irrelevant observations.

lf0( $\#_k$   $\#_{ll}$ ) is typically used by programmers. It specifies the number of parameters and log-likelihood value of the constant-only model so that ml can report a likelihood-ratio test rather than a Wald test. These values may have been analytically determined, or they may have been determined by a previous fitting of the constant-only model on the estimation sample.

Also see the continue option directly below.

If you specify lf0(), it must be safe for you to specify the missing option, too, else how did you calculate the log likelihood for the constant-only model on the same sample? You must have identified the estimation sample, and done so correctly, so there is no reason for ml to waste time rechecking your results. All of which is to say, do not specify lf0() unless you are certain your code identifies the estimation sample correctly.

lf0(), even if specified, is ignored if vce(robust), vce(cluster *clustvar*), pweight, or svy is specified because, in that case, a likelihood-ratio test would be inappropriate.

continue is typically specified by programmers and does two things:

First, it specifies that a model has just been fit by either ml or some other estimation command, such as logit, and that the likelihood value stored in e(l1) and the number of parameters stored in e(b) as of that instant are the relevant values of the constant-only model. The current value of the log likelihood is used to present a likelihood-ratio test unless vce(robust), vce(cluster *clustvar*), pweight, svy, or constraints() is specified. A likelihood-ratio test is inappropriate when vce(robust), vce(cluster *clustvar*), pweight, or svy is specified. We suggest using lrtest when constraints() is specified; see [R] lrtest.

Second, `continue` sets the initial values,  $\mathbf{b}_0$ , for the model about to be fit according to the `e(b)` currently stored.

The comments made about specifying missing with `lfl0()` apply equally well here.

`waldtest(#)` is typically specified by programmers. By default, `ml` presents a Wald test, but that is overridden if the `lfl0()` or `continue` option is specified. A Wald test is performed if `vce(robust)`, `vce(cluster clustvar)`, or `pweight` is specified.

`waldtest(0)` prevents even the Wald test from being reported.

`waldtest(-1)` is the default. It specifies that a Wald test be performed by constraining all coefficients except the intercept to 0 in the first equation. Remaining equations are to be unconstrained. A Wald test is performed if neither `lfl0()` nor `continue` was specified, and a Wald test is forced if `vce(robust)`, `vce(cluster clustvar)`, or `pweight` was specified.

`waldtest(k)` for  $k \leq -1$  specifies that a Wald test be performed by constraining all coefficients except intercepts to 0 in the first  $|k|$  equations; remaining equations are to be unconstrained. A Wald test is performed if neither `lfl0()` nor `continue` was specified, and a Wald test is forced if `vce(robust)`, `vce(cluster clustvar)`, or `pweight` was specified.

`waldtest(k)` for  $k \geq 1$  works like the options above, except that it forces a Wald test to be reported even if the information to perform the likelihood-ratio test is available and even if none of `vce(robust)`, `vce(cluster clustvar)`, or `pweight` was specified. `waldtest(k)`,  $k \geq 1$ , may not be specified with `lfl0()`.

`obs(#)` is used mostly by programmers. It specifies that the number of observations reported and ultimately stored in `e(N)` be `#`. Ordinarily, `ml` works that out for itself. Programmers may want to specify this option when, for the likelihood evaluator to work for  $N$  observations, they first had to modify the dataset so that it contained a different number of observations.

`crittype(string)` is used mostly by programmers. It allows programmers to supply a string (up to 32 characters long) that describes the criterion that is being optimized by `ml`. The default is "Log likelihood" for nonrobust and "Log pseudolikelihood" for robust estimation.

`svy` indicates that `ml` is to pick up the `svy` settings set by `svyset` and use the robust variance estimator. This option requires the data to be `svyset`; see [\[SVY\] svyset](#). `svy` may not be specified with `vce()` or `weights`.

`subpop(varname)` specifies that estimates be computed for the single subpopulation defined by the observations for which `varname`  $\neq$  0. Typically, `varname` = 1 defines the subpopulation, and `varname` = 0 indicates observations not belonging to the subpopulation. For observations whose subpopulation status is uncertain, `varname` should be set to missing (`.'`). This option requires the `svy` option.

`nosvyadjust` specifies that the model Wald test be carried out as  $W/k \sim F(k, d)$ , where  $W$  is the Wald test statistic,  $k$  is the number of terms in the model excluding the constant term,  $d$  is the total number of sampled PSUs minus the total number of strata, and  $F(k, d)$  is an  $F$  distribution with  $k$  numerator degrees of freedom and  $d$  denominator degrees of freedom. By default, an adjusted Wald test is conducted:  $(d - k + 1)W/(kd) \sim F(k, d - k + 1)$ . See [Korn and Graubard \(1990\)](#) for a discussion of the Wald test and the adjustments thereof. This option requires the `svy` option.

`technique(algorithm_spec)` specifies how the likelihood function is to be maximized. The following algorithms are currently implemented in `ml`. For details, see [Pitblado, Poi, and Gould \(2024\)](#).

`technique(nr)` specifies Stata's modified Newton–Raphson (NR) algorithm.

`technique(bhhh)` specifies the Berndt–Hall–Hall–Hausman (BHHH) algorithm.

`technique(dfpr)` specifies the Davidon–Fletcher–Powell (DFP) algorithm.

`technique(bfgs)` specifies the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm.

The default is `technique(nr)`.

You can switch between algorithms by specifying more than one in the `technique()` option. By default, `ml` will use an algorithm for five iterations before switching to the next algorithm. To specify a different number of iterations, include the number after the technique in the option. For example, `technique(bhhh 10 nr 1000)` requests that `ml` perform 10 iterations using the BHHH algorithm, followed by 1,000 iterations using the NR algorithm, and then switch back to BHHH for 10 iterations, and so on. The process continues until convergence or until reaching the maximum number of iterations.

`deriv(min_spec)` specifies whether to use minimum values of step sizes for computing numerical derivatives and optionally specifies the minimum values.

`deriv(nomin)`, the default, suppresses the use of minimum values.

`deriv(min)` sets the minimum step size to  $1e-6$  for computing first-order derivatives and  $1e-4$  for computing second-order derivatives.

`deriv(min(#1, #2))` sets the minimum step size to #<sub>1</sub> for computing the first-order derivatives and #<sub>2</sub> for computing the second-order derivatives. Specifying `deriv(min(1e-6, 1e-4))` is equivalent to specifying `deriv(min)`.

`deriv(min(matname))` sets the minimum step size to the values in the  $1 \times 2$  matrix *matname*. The value in the first column of the row vector will set the minimum step size for the first-order derivatives, and the value in the second column will set the minimum step size for the second-order derivatives.

## Options for use with `ml model` in noninteractive mode

The following extra options are for use with `ml model` in noninteractive mode. Noninteractive mode is for programmers who use `ml` as a subroutine and want to issue one command that will carry forth the estimation from start to finish.

`maximize` is required. It specifies noninteractive mode.

`init(ml_init_args)` sets the initial values,  $\mathbf{b}_0$ . *ml\_init\_args* are whatever you would type after the `ml init` command.

`search(on|norescale|quietly|off)` specifies whether `ml search` is to be used to improve the initial values. `search(on)` is the default and is equivalent to separately running `ml search, repeat(0)`. `search(norescale)` is equivalent to separately running `ml search, repeat(0) norescale`. `search(quietly)` is equivalent to `search(on)`, except that it suppresses `ml search`'s output. `search(off)` prevents calling `ml search`.

`repeat(#)` is `ml search`'s `repeat()` option. `repeat(0)` is the default.

`bounds(ml_search_bounds)` specifies the search bounds. *ml\_search\_bounds* is specified as

`[eqn_name] lower_bound upper_bound ... [eqn_name] lower_bound upper_bound`

for instance, `bounds(100 100 lnsigma 0 10)`. The `ml model` command issues `ml search ml_search_bounds, repeat(#)`. Specifying search bounds is optional.

`nowarning`, `novce`, `negh`, and `score()` are `ml maximize`'s equivalent options.

*maximize\_options*: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] **Maximize**. These options are seldom used.

## Options for use when specifying equations

`noconstant` specifies that the equation not include an intercept.

`offset(varnameo)` specifies that the equation be  $\mathbf{xb} + \text{varname}_o$ —that it include `varnameo` with coefficient constrained to be 1.

`exposure(varnamee)` is an alternative to `offset(varnameo)`; it specifies that the equation be  $\mathbf{xb} + \ln(\text{varname}_e)$ . The equation is to include  $\ln(\text{varname}_e)$  with coefficient constrained to be 1.

`freeparm` specifies that the associated `eqname` is a free parameter. The corresponding full column name on `e(b)` will be `/eqname` instead of `eqname: _cons`. This option is not allowed with `varliste`.

## Options for use with ml search

`repeat(#)` specifies the number of random attempts that are to be made to find a better initial-value vector. The default is `repeat(10)`.

`repeat(0)` specifies that no random attempts be made. More precisely, `repeat(0)` specifies that no random attempts be made if the first initial-value vector is a feasible starting point. If it is not, `ml search` will make random attempts, even if you specify `repeat(0)`, because it has no alternative. The `repeat()` option refers to the number of random attempts to be made to improve the initial values. When the initial starting value vector is not feasible, `ml search` will make up to 1,000 random attempts to find starting values. It stops when it finds one set of values that works and then moves into its improve-initial-values logic.

`repeat(k)`,  $k > 0$ , specifies the number of random attempts to be made to improve the initial values.

`restart` specifies that random actions be taken to obtain starting values and that the resulting starting values not be a deterministic function of the current values. Generally, you should not specify this option because, with `restart`, `ml search` intentionally does not produce as good a set of starting values as it could. `restart` is included for use by the optimizer when it gets into serious trouble. The random actions ensure that the optimizer and `ml search`, working together, do not cause an endless loop.

`restart` implies `norescale`, which is why we recommend that you do not specify `restart`. In testing, sometimes `rescale` worked so well that, even after randomization, the rescaler would bring the starting values right back to where they had been the first time and thus defeat the intended randomization.

`norescale` specifies that `ml search` not engage in its rescaling actions to improve the parameter vector. We do not recommend specifying this option because rescaling tends to work so well.

*maximize\_options*: `[no]log` and `trace`; see [R] **Maximize**. These options are seldom used.

## Option for use with ml plot

`saving(filename[, replace])` specifies that the graph be saved in `filename.gph`.

See [G-3] *saving\_option*.

## Options for use with ml init

`copy` specifies that the list of numbers or the initialization vector be copied into the initial-value vector by position rather than by name.

`skip` specifies that any parameters found in the specified initialization vector that are not also found in the model be ignored. The default action is to issue an error message.

## Options for use with ml maximize

`nowarning` is allowed only with `iterate(0)`. `nowarning` suppresses the “convergence not achieved” message. Programmers might specify `iterate(0) nowarning` when they have a vector  $\mathbf{b}$  already containing the final estimates and want `ml` to calculate the variance matrix and postestimation results. Then, specify `init(b) search(off) iterate(0) nowarning nolog`.

`novce` is allowed only with `iterate(0)`. `novce` substitutes the zero matrix for the variance matrix, which in effect posts estimation results as fixed constants.

`negh` indicates that the evaluator returns the negative Hessian matrix. By default, `ml` assumes `d2` and `lf2` evaluators return the Hessian matrix.

`score(newvars | stub*)` creates new variables containing the contributions to the score for each equation and ancillary parameter in the model; see [U] 20.23 **Obtaining scores**.

If `score(newvars)` is specified, the `newvars` must contain  $k$  new variables. For evaluators of methods `lf`, `lf0`, `lf1`, and `lf2`,  $k$  is the number of equations. For evaluators of method `gf0`,  $k$  is the number of parameters. If `score(stub*)` is specified, variables named `stub1`, `stub2`, ..., `stubk` are created.

For evaluators of methods `lf`, `lf0`, `lf1`, and `lf2`, the first variable contains  $\partial \ln \ell_j / \partial (\mathbf{x}_{1j} \mathbf{b}_1)$ , the second variable contains  $\partial \ln \ell_j / \partial (\mathbf{x}_{2j} \mathbf{b}_2)$ , and so on.

For evaluators of method `gf0`, the first variable contains  $\partial \ln \ell_j / \partial \mathbf{b}_1$ , the second variable contains  $\partial \ln \ell_j / \partial \mathbf{b}_2$ , and so on.

`nooutput` suppresses display of results. This option is different from prefixing `ml maximize` with `quietly` in that the iteration log is still displayed (assuming that `nolog` is not specified).

`noskipline` suppresses display of a blank line before the iteration log. This is useful in programs when there is a header preceding the iteration log and a blank line is not wanted after the header.

`noclear` specifies that the `ml` problem definition not be cleared after the model has converged. Perhaps you are having convergence problems and intend to run the model to convergence. If so, use `ml search` to see if those values can be improved, and then restart the estimation.

`maximize_options`: `difficult`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, and `nonrntolerance`; see [R] **Maximize**. These options are seldom used.

`display_options`; see *Options for use with ml display* below.

`eform_option`; see *Options for use with ml display* below.

## Option for use with ml graph

`saving(filename[, replace])` specifies that the graph be saved in `filename.gph`.

See [G-3] *saving\_option*.

## Options for use with ml display

- `noheader` suppresses the header display above the coefficient table that displays the final log-likelihood value, the number of observations, and the model significance test.
- `nofootnote` suppresses the footnote display below the coefficient table, which displays a warning if the model fit did not converge within the specified number of iterations. Use `ml footnote` to display the warning if 1) you add to the coefficient table using the `plus` option or 2) you have your own footnotes and want the warning to be last.
- `level(#)` is the standard confidence-level option. It specifies the confidence level, as a percentage, for confidence intervals of the coefficients. The default is `level(95)` or as set by `set level`; see [\[U\] 20.8 Specifying the width of confidence intervals](#).
- `first` displays a coefficient table reporting results for the first equation only, and the report makes it appear that the first equation is the only equation. This option is used by programmers who estimate ancillary parameters in the second and subsequent equations and who wish to report the values of such parameters themselves.
- `neq(#)` is an alternative to `first`. `neq(#)` displays a coefficient table reporting results for the first # equations. This option is used by programmers who estimate ancillary parameters in the # + 1 and subsequent equations and who wish to report the values of such parameters themselves.
- `showeqns` is a seldom-used option that displays the equation names in the coefficient table. `ml display` uses the numbers stored in `e(k_eq)` and `e(k_aux)` to determine how to display the coefficient table. `e(k_eq)` identifies the number of equations, and `e(k_aux)` identifies how many of these are for ancillary parameters. The `first` option is implied when `showeqns` is not specified and all but the first equation are for ancillary parameters.
- `plus` displays the coefficient table, but rather than ending the table in a line of dashes, ends it in dashes–plus–sign–dashes. This is so that programmers can write additional display code to add more results to the table and make it appear as if the combined result is one table. Programmers typically specify `plus` with the `first` or `neq()` options. This option implies `nofootnote`.
- `nocnsreport` suppresses the display of constraints above the coefficient table. This option is ignored if constraints were not used to fit the model.
- `noomitted` specifies that variables that were omitted because of collinearity not be displayed. The default is to include in the table any variables omitted because of collinearity and to label them as “(omitted)”.
- `vsquish` specifies that the blank space separating factor-variable terms or time-series–operated variables from other variables in the model be suppressed.
- `noemptycells` specifies that empty cells for interactions of factor variables not be displayed. The default is to include in the table interaction cells that do not occur in the estimation sample and to label them as “(empty)”.
- `baselevels` and `allbaselevels` control whether the base levels of factor variables and interactions are displayed. The default is to exclude from the table all base categories.
- `baselevels` specifies that base levels be reported for factor variables and for interactions whose bases cannot be inferred from their component factor variables.
- `allbaselevels` specifies that all base levels of factor variables and interactions be reported.
- `cformat(%fmt)` specifies how to format coefficients, standard errors, and confidence limits in the coefficient table.

`pformat(%fmt)` specifies how to format  $p$ -values in the coefficient table.

`sformat(%fmt)` specifies how to format test statistics in the coefficient table.

`no1stretch` specifies that the width of the coefficient table not be automatically widened to accommodate longer variable names. The default, `1stretch`, is to automatically widen the coefficient table up to the width of the Results window. Specifying `1stretch` or `no1stretch` overrides the setting given by `set 1stretch`. If `set 1stretch` has not been set, the default is `1stretch`.

`coeflegend` specifies that the legend of the coefficients and how to specify them in an expression be displayed rather than displaying the statistics for the coefficients.

*eform\_option*: `eform(string)`, `eform`, `hr`, `shr`, `irr`, `or`, and `rrr` display the coefficient table in exponentiated form: for each coefficient,  $\exp(b)$  rather than  $b$  is displayed, and standard errors and confidence intervals are transformed. *string* is the table header that will be displayed above the transformed coefficients and must be 11 characters or shorter in length—for example, `eform("Odds ratio")`. The options `eform`, `hr`, `shr`, `irr`, `or`, and `rrr` provide a default *string* equivalent to “exp(b)”, “Haz. ratio”, “SHR”, “IRR”, “Odds ratio”, and “RRR”, respectively. These options may not be combined.

`ml display` looks at `e(k_iform)` to determine how many equations are affected by an *eform\_option*; by default, only the first equation is affected. Type `ereturn list, all` to view `e(k_iform)`; see [P] [ereturn](#).

## Options for use with `mlevel`

`eq(#)` specifies the equation number,  $i$ , for which  $\theta_{ij} = \mathbf{x}_{ij}\mathbf{b}_i$  is to be evaluated. `eq(1)` is assumed if `eq()` is not specified.

`scalar` asserts that the  $i$ th equation is known to evaluate to a constant, meaning that the equation was specified as `()`, `(name:)`, or `/name` on the `ml model` statement. If you specify this option, the new variable created is created as a scalar. If the  $i$ th equation does not evaluate to a scalar, an error message is issued.

## Option for use with `mlsum`

`noweight` specifies that weights (`$ML_w`) be ignored when summing the likelihood function.

## Option for use with `mlvecsum`

`eq(#)` specifies the equation for which a gradient vector  $\partial \ln L / \partial \mathbf{b}_i$  is to be constructed. The default is `eq(1)`.

## Option for use with `mlmatsum`

`eq([#, #])` specifies the equations for which the Hessian matrix is to be constructed. The default is `eq(1)`, which is the same as `eq(1,1)`, which means  $\partial^2 \ln L / \partial \mathbf{b}_1 \partial \mathbf{b}'_1$ . Specifying `eq(i,j)` results in  $\partial^2 \ln L / \partial \mathbf{b}_i \partial \mathbf{b}'_j$ .

## Options for use with `mlmatbysum`

`by(varname)` is required and specifies the group variable.

`eq(#[, #])` specifies the equations for which the Hessian matrix is to be constructed. The default is `eq(1)`, which is the same as `eq(1, 1)`, which means  $\partial^2 \ln L / \partial \mathbf{b}_1 \partial \mathbf{b}'_1$ . Specifying `eq(i, j)` results in  $\partial^2 \ln L / \partial \mathbf{b}_i \partial \mathbf{b}'_j$ .

## Options for use with ml score

`equation(eqname)` identifies from which equation the observation scores are to come. This option may be used only when generating one variable.

`missing` specifies that observations containing variables with missing values not be eliminated from the estimation sample.

## Remarks and examples

For a thorough discussion of `ml`, see the fifth edition of *Maximum Likelihood Estimation with Stata* (Pitblado, Poi, and Gould 2024). The book provides a tutorial introduction to `ml`, notes on advanced programming issues, and a discourse on maximum likelihood estimation from both theoretical and practical standpoints. See *Survey options and ml* at the end of *Remarks and examples* for examples of the new `svy` options. For more information about survey estimation, see `[SVY] Survey`, `[SVY] svy estimation`, and `[SVY] Variance estimation`.

`ml` requires that you write a program that evaluates the log-likelihood function and, possibly, its first and second derivatives. The style of the program you write depends upon the method you choose. Methods `lf`, `lf0`, `d0`, and `gf0` require that your program evaluate the log likelihood only. Methods `d1` and `lf1` require that your program evaluate the log likelihood and its first derivatives. Methods `d2` and `lf2` requires that your program evaluate the log likelihood and its first and second derivatives. Methods `lf`, `lf0`, `d0`, and `gf0` differ from each other in that, with methods `lf` and `lf0`, your program is required to produce observation-by-observation log-likelihood values  $\ln \ell_j$  and it is assumed that  $\ln L = \sum_j \ln \ell_j$ ; with method `d0`, your program is required to produce only the overall value  $\ln L$ ; and with method `gf0`, your program is required to produce the summable pieces of the log likelihood, such as those in panel-data models.

Once you have written the program—called an evaluator—you define a model to be fit using `ml model` and obtain estimates using `ml maximize`. You might type

```
. ml model ...
. ml maximize
```

but we recommend that you type

```
. ml model ...
. ml check
. ml search
. ml maximize
```

`ml check` verifies your evaluator has no obvious errors, and `ml search` finds better initial values.

You fill in the `ml model` statement with 1) the method you are using, 2) the name of your program, and 3) the “equations”. You write your evaluator in terms of  $\theta_1, \theta_2, \dots$ , each of which has a linear equation associated with it. That linear equation might be as simple as  $\theta_i = b_0$ , it might be  $\theta_i = b_1 \text{mpg} + b_2 \text{weight} + b_3$ , or it might omit the intercept  $b_3$ . The equations are specified in parentheses on the `ml model` line.

Suppose that you are using method lf and the name of your evaluator program is myprog. The statement

```
. ml model lf myprog (mpg weight)
```

would specify one equation with  $\theta_i = b_1\text{mpg} + b_2\text{weight} + b_3$ . If you wanted to omit  $b_3$ , you would type

```
. ml model lf myprog (mpg weight, nocons)
```

and if all you wanted was  $\theta_i = b_0$ , you would type

```
. ml model lf myprog ()
```

With multiple equations, you list the equations one after the other; so, if you typed

```
. ml model lf myprog (mpg weight) ()
```

you would be specifying  $\theta_1 = b_1\text{mpg} + b_2\text{weight} + b_3$  and  $\theta_2 = b_4$ . You would write your likelihood in terms of  $\theta_1$  and  $\theta_2$ . If the model was linear regression,  $\theta_1$  might be the  $\mathbf{x}\mathbf{b}$  part and  $\theta_2$  the variance of the residuals.

When you specify the equations, you also specify any dependent variables. If you typed

```
. ml model lf myprog (price = mpg weight) ()
```

price would be the one and only dependent variable, and that would be passed to your program in  $\$ML\_y1$ . If your model had two dependent variables, you could type

```
. ml model lf myprog (price displ = mpg weight) ()
```

Then,  $\$ML\_y1$  would be price and  $\$ML\_y2$  would be displ. You can specify however many dependent variables are necessary and specify them on any equation. It does not matter on which equation you specify them; the first one specified is placed in  $\$ML\_y1$ , the second in  $\$ML\_y2$ , and so on.

### ▷ Example 1: Method lf

Using method lf, we want to produce observation-by-observation values of the log likelihood. The probit log-likelihood function is

$$\ln \ell_j = \begin{cases} \ln \Phi(\theta_{1j}) & \text{if } y_j = 1 \\ \ln \Phi(-\theta_{1j}) & \text{if } y_j = 0 \end{cases}$$

$$\theta_{1j} = \mathbf{x}_j \mathbf{b}_1$$

The following is the method-lf evaluator for this likelihood function:

```
program myprobit
 version 19.5 // (or version 19 if you do not have StataNow)
 args lnf theta1
 quietly replace `lnf' = ln(normal(`theta1')) if $ML_y1==1
 quietly replace `lnf' = ln(normal(-`theta1')) if $ML_y1==0
end
```

If we wanted to fit a model of foreign on mpg and weight, we would type the following commands. The 'foreign =' part specifies that  $y$  is foreign. The 'mpg weight' part specifies that  $\theta_{1j} = b_1\text{mpg}_j + b_2\text{weight}_j + b_3$ .

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. ml model lf myprobit (foreign = mpg weight)
. ml maximize
```

```
Initial: Log likelihood = -51.292891
Alternative: Log likelihood = -45.055272
Rescale: Log likelihood = -45.055272
Iteration 0: Log likelihood = -45.055272
Iteration 1: Log likelihood = -27.904125
Iteration 2: Log likelihood = -26.858643
Iteration 3: Log likelihood = -26.844199
Iteration 4: Log likelihood = -26.844189
Iteration 5: Log likelihood = -26.844189
```

```
Number of obs = 74
Wald chi2(2) = 20.75
Prob > chi2 = 0.0000
```

```
Log likelihood = -26.844189
```

| foreign | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|---------|-------------|-----------|-------|-------|----------------------|-----------|
| mpg     | -.1039503   | .0515689  | -2.02 | 0.044 | -.2050235            | -.0028772 |
| weight  | -.0023355   | .0005661  | -4.13 | 0.000 | -.003445             | -.0012261 |
| _cons   | 8.275464    | 2.554142  | 3.24  | 0.001 | 3.269438             | 13.28149  |

◀

### ▷ Example 2: Method lf for two-equation, two-dependent-variable model

A two-equation, two-dependent-variable model is a little different. Rather than receiving one  $\theta$ , our program will receive two. Rather than there being one dependent variable in `$ML_y1`, there will be dependent variables in `$ML_y1` and `$ML_y2`. For instance, the Weibull regression log-likelihood function is

$$\ln \ell_j = -(t_j e^{-\theta_{1j}})^{\exp(\theta_{2j})} + d_j \{ \theta_{2j} - \theta_{1j} + (e^{\theta_{2j}} - 1)(\ln t_j - \theta_{1j}) \}$$

$$\theta_{1j} = \mathbf{x}_j \mathbf{b}_1$$

$$\theta_{2j} = s$$

where  $t_j$  is the time of failure or censoring and  $d_j = 1$  if failure and 0 if censored. We can make the log likelihood a little easier to program by introducing some extra variables:

$$p_j = \exp(\theta_{2j})$$

$$M_j = \{t_j \exp(-\theta_{1j})\}^{p_j}$$

$$R_j = \ln t_j - \theta_{1j}$$

$$\ln \ell_j = -M_j + d_j \{ \theta_{2j} - \theta_{1j} + (p_j - 1)R_j \}$$

The method-lf evaluator for this is

```

program myweib
 version 19.5 // (or version 19 if you do not have StataNow)
 args lnf theta1 theta2
 tempvar p M R
 quietly generate double `p' = exp(`theta2')
 quietly generate double `M' = ($ML_y1*exp(-`theta1'))^`p'
 quietly generate double `R' = ln($ML_y1)-`theta1'
 quietly replace `lnf' = -`M' + $ML_y2*(`theta2'-`theta1' + (`p'-1)*`R')
end

```

We can fit a model by typing

```

. ml model lf myweib (studytime died = i.drug age) ()
. ml maximize

```

Note that we specified `()` for the second equation. The second equation corresponds to the Weibull shape parameter  $s$ , and the linear combination we want for  $s$  contains just an intercept. Alternatively, we could type

```

. ml model lf myweib (studytime died = i.drug age) /s

```

Typing `/s` means the same thing as typing `(s : , freeparm)`. The `s`, either after a slash or in parentheses before a colon, labels the equation. The leading slash, `/`, is a shortcut for specifying that “ $s$ ” is a free parameter. Free parameters are labeled using this shortcut notation, `/s`, in the column names of `e(b)` and in the estimation results. If we instead specified `(s :)`, then `s` would be the equation label for a constant linear equation and would be labeled as `s : _cons` in the column names of `e(b)` and in the estimation results.

```

. use https://www.stata-press.com/data/r19/cancer, clear
(Patient survival in drug trial)
. ml model lf myweib (studytime died = i.drug age) /s
. ml maximize
Initial: Log likelihood = -744
Alternative: Log likelihood = -356.14276
Rescale: Log likelihood = -200.80201
Rescale eq: Log likelihood = -136.69232
Iteration 0: Log likelihood = -136.69232 (not concave)
Iteration 1: Log likelihood = -124.11726
Iteration 2: Log likelihood = -113.9591
Iteration 3: Log likelihood = -110.30683
Iteration 4: Log likelihood = -110.26748
Iteration 5: Log likelihood = -110.26736
Iteration 6: Log likelihood = -110.26736

 Number of obs = 48
 Wald chi2(3) = 35.25
 Prob > chi2 = 0.0000

Log likelihood = -110.26736

```

|       | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|-------|-------------|-----------|-------|-------|----------------------|-----------|
| drug  |             |           |       |       |                      |           |
| Other | 1.012966    | .2903917  | 3.49  | 0.000 | .4438086             | 1.582123  |
| NA    | 1.45917     | .2821195  | 5.17  | 0.000 | .9062261             | 2.012114  |
| age   | -.0671728   | .0205688  | -3.27 | 0.001 | -.1074868            | -.0268587 |
| _cons | 6.060723    | 1.152845  | 5.26  | 0.000 | 3.801188             | 8.320259  |
| /s    | .5573333    | .1402154  | 3.97  | 0.000 | .2825162             | .8321504  |

◀

### ▷ Example 3: Method d0

Method-d0 evaluators receive  $\mathbf{b} = (\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_E)$ , the coefficient vector, rather than the already evaluated  $\theta_1, \theta_2, \dots, \theta_E$ , and they are required to evaluate the overall log-likelihood  $\ln L$  rather than  $\ln \ell_j, j = 1, \dots, N$ .

Use `mlevel` to produce the thetas from the coefficient vector.

Use `mlsum` to sum the components that enter into  $\ln L$ .

In the case of Weibull,  $\ln L = \sum \ln \ell_j$ , and our method-d0 evaluator is

```

program weib0
 version 19.5 // (or version 19 if you do not have StataNow)
 args todo b lnf
 tempvar theta1 theta2
 mlevel 'theta1' = 'b', eq(1)
 mlevel 'theta2' = 'b', eq(2)
 local t "$ML_y1" // this is just for readability
 local d "$ML_y2"
 tempvar p M R
 quietly generate double 'p' = exp('theta2')
 quietly generate double 'M' = ('t'*exp(-'theta1'))^'p'
 quietly generate double 'R' = ln('t')-'theta1'
 mlsum 'lnf' = -'M' + 'd'*('theta2'-'theta1' + ('p'-1)*'R')
end

```

To fit our model using this evaluator, we would type

```

. ml model d0 weib0 (studytime died = i.drug age) /s
. ml maximize

```

◀

## □ Technical note

Method d0 does not require  $\ln L = \sum_j \ln \ell_j$ ,  $j = 1, \dots, N$ , as method lf does. Your likelihood function might have independent components only for groups of observations. Panel-data estimators have a log-likelihood value  $\ln L = \sum_i \ln L_i$ , where  $i$  indexes the panels, each of which contains multiple observations. Conditional logistic regression has  $\ln L = \sum_k \ln L_k$ , where  $k$  indexes the risk pools. Cox regression has  $\ln L = \sum_{(t)} \ln L_{(t)}$ , where  $(t)$  denotes the ordered failure times.

To evaluate such likelihood functions, first calculate the within-group log-likelihood contributions. This usually involves `generate` and `replace` statements prefixed with `by`, as in

```

tempvar sumd
by group: generate double 'sumd' = sum($ML_y1)

```

Structure your code so that the log-likelihood contributions are recorded in the last observation of each group. Say that a variable is named `'cont'`. To sum the contributions, code

```

tempvar last
quietly by group: generate byte 'last' = (_n==_N)
mlsum 'lnf' = 'cont' if 'last'

```

You must inform `mlsum` which observations contain log-likelihood values to be summed. First, you do not want to include intermediate results in the sum. Second, `mlsum` does not skip missing values. Rather, if `mlsum` sees a missing value among the contributions, it sets the overall result, `'lnf'`, to missing. That is how `ml maximize` is informed that the likelihood function could not be evaluated at the particular value of `b`. `ml maximize` will then take action to escape from what it thinks is an infeasible area of the likelihood function.

When the likelihood function violates the linear-form restriction  $\ln L = \sum_j \ln \ell_j$ ,  $j = 1, \dots, N$ , with  $\ln \ell_j$  being a function solely of values within the  $j$ th observation, use method d0. In the following examples, we will demonstrate methods d1 and d2 with likelihood functions that meet this linear-form

restriction. The d1 and d2 methods themselves do not require the linear-form restriction, but the utility routines `mlvecsum` and `mlmatsum` do. Using method d1 or d2 when the restriction is violated is difficult; however, `mlmatbysum` may be of some help for method-d2 evaluators. □

#### ▷ Example 4: Method d1

Method-d1 evaluators are required to produce the gradient vector  $\mathbf{g} = \partial \ln L / \partial \mathbf{b}$ , as well as the overall log-likelihood value. Using `mlvecsum`, we can obtain  $\partial \ln L / \partial \mathbf{b}$  from  $\partial \ln L / \partial \theta_i$ ,  $i = 1, \dots, E$ . The derivatives of the Weibull log-likelihood function are

$$\frac{\partial \ln \ell_j}{\partial \theta_{1j}} = p_j (M_j - d_j)$$

$$\frac{\partial \ln \ell_j}{\partial \theta_{2j}} = d_j - R_j p_j (M_j - d_j)$$

The method-d1 evaluator for this is

```

program weib1
 version 19.5 // (or version 19 if you do not have StataNow)
 args todo b lnf g // g is new
 tempvar t1 t2
 mlevel 't1' = 'b', eq(1)
 mlevel 't2' = 'b', eq(2)
 local t "$ML_y1"
 local d "$ML_y2"
 tempvar p M R
 quietly generate double 'p' = exp('t2')
 quietly generate double 'M' = ('t'*exp(-'t1'))^'p'
 quietly generate double 'R' = ln('t')-'t1'
 mlsum 'lnf' = -'M' + 'd'*('t2'-'t1') + ('p'-1)*'R'
 if ('todo'==0 | 'lnf'>=.) exit /* <-- new */
 tempname d1 d2 /* <-- new */
 mlvecsum 'lnf' 'd1' = 'p'*('M'-'d'), eq(1) /* <-- new */
 mlvecsum 'lnf' 'd2' = 'd' - 'R'*'p'*('M'-'d'), eq(2) /* <-- new */
 matrix 'g' = ('d1','d2') /* <-- new */
end

```

We obtained this code by starting with our method-d0 evaluator and then adding the extra lines that method d1 requires. To fit our model using this evaluator, we could type

```

. ml model d1 weib1 (studytime died = drug2 drug3 age) /s
. ml maximize

```

but we recommend substituting `method d1debug` for `method d1` and typing

```

. ml model d1debug weib1 (studytime died = drug2 drug3 age) /s
. ml maximize

```

Method `d1debug` will compare the derivatives we calculate with numerical derivatives and thus verify that our program is correct. Once we are certain the program is correct, then we would switch from `method d1debug` to `method d1`. ◀

## ▶ Example 5: Method d2

Method-d2 evaluators are required to produce  $\mathbf{H} = \partial^2 \ln L / \partial \mathbf{b} \partial \mathbf{b}'$ , the Hessian matrix, as well as the gradient and log-likelihood value. `mlmatsum` will help calculate  $\partial^2 \ln L / \partial \mathbf{b} \partial \mathbf{b}'$  from the second derivatives with respect to  $\theta$ . For the Weibull model, these second derivatives are

$$\begin{aligned}\frac{\partial^2 \ln \ell_j}{\partial \theta_{1j}^2} &= -p_j^2 M_j \\ \frac{\partial^2 \ln \ell_j}{\partial \theta_{1j} \partial \theta_{2j}} &= p_j (M_j - d_j + R_j p_j M_j) \\ \frac{\partial^2 \ln \ell_j}{\partial \theta_{2j}^2} &= -p_j R_j (R_j p_j M_j + M_j - d_j)\end{aligned}$$

The method-d2 evaluator is

```

program weib2
 version 19.5 // (or version 19 if you do not have StataNow)
 args todo b lnf g H // H added

 tempvar t1 t2
 mlevel 't1' = 'b', eq(1)
 mlevel 't2' = 'b', eq(2)

 local t "$ML_y1"
 local d "$ML_y2"

 tempvar p M R
 quietly generate double 'p' = exp('t2')
 quietly generate double 'M' = ('t'*exp(-'t1'))^-'p'
 quietly generate double 'R' = ln('t')-'t1'

 mlsum 'lnf' = -'M' + 'd'*('t2'-'t1' + ('p'-1)*'R')
 if ('todo'==0 | 'lnf'>=.) exit

 tempname d1 d2
 mlvecsum 'lnf' 'd1' = 'p'*('M'-'d'), eq(1)
 mlvecsum 'lnf' 'd2' = 'd' - 'R'*'p'*('M'-'d'), eq(2)
 matrix 'g' = ('d1','d2')
 if ('todo'==1 | 'lnf'>=.) exit // new from here down

 tempname d11 d12 d22
 mlmatsum 'lnf' 'd11' = -'p'^2 * 'M', eq(1)
 mlmatsum 'lnf' 'd12' = 'p'*('M'-'d' + 'R'*'p'*'M'), eq(1,2)
 mlmatsum 'lnf' 'd22' = -'p'*'R'*('R'*'p'*'M' + 'M' - 'd'), eq(2)
 matrix 'H' = ('d11','d12' \ 'd12''','d22')
end

```

We started with our previous method-d1 evaluator and added the lines that method d2 requires. We could now fit a model by typing

```
. ml model d2 weib2 (studytime died = drug2 drug3 age) /s
. ml maximize
```

but we would recommend substituting method `d2debug` for method `d2` and typing

```
. ml model d2debug weib2 (studytime died = drug2 drug3 age) /s
. ml maximize
```

Method `d2debug` will compare the first and second derivatives we calculate with numerical derivatives and thus verify that our program is correct. Once we are certain the program is correct, then we would switch from method `d2debug` to method `d2`.

As we stated earlier, to produce the robust variance estimator with method `lf`, there is nothing to do except specify `vce(robust)`, `vce(cluster clustvar)`, or `pweight`. For methods `d0`, `d1`, and `d2`, these options do not work. If your likelihood function meets the linear-form restrictions, you can use methods `lf0`, `lf1`, and `lf2`, then these options will work. The equation scores are defined as

$$\frac{\partial \ln \ell_j}{\partial \theta_{1j}}, \frac{\partial \ln \ell_j}{\partial \theta_{2j}}, \dots$$

Your evaluator will be passed variables, one for each equation, which you fill in with the equation scores. For *both* method `lf1` and `lf2`, these variables are passed in the fourth and subsequent positions of the argument list. That is, you must process the arguments as

```
args todo b lnf g1 g2 ... H
```

Note that for method `lf1`, the ‘H’ argument is not used and can be ignored.

### ▷ Example 6: Robust variance estimates

If you have used `mlvecsum` in your evaluator of method `d1` or `d2`, it is easy to turn it into an evaluator of method `lf1` or `lf2` that allows the computation of the robust variance estimator. The expression that you specified on the right-hand side of `mlvecsum` is the equation score.

Here we turn the program that we gave earlier in the `method-d1` example into a `method-lf1` evaluator that allows `vce(robust)`, `vce(cluster clustvar)`, or `pweight`.

```
program weib1
 version 19.5 // (or version 19 if you do not have StataNow)
 args todo b lnfj g1 g2 // g1 and g2 are new
 tempvar t1 t2
 mlevel 't1' = 'b', eq(1)
 mlevel 't2' = 'b', eq(2)
 local t "$ML_y1"
 local d "$ML_y2"
 tempvar p M R
 quietly generate double 'p' = exp('t2')
 quietly generate double 'M' = ('t'*exp(-'t1'))^-'p'
 quietly generate double 'R' = ln('t')-'t1'
 quietly replace 'lnfj' = -'M' + 'd'*('t2'-'t1' + ('p'-1)*'R')
 if ('todo'==0) exit
 quietly replace 'g1' = 'p'*('M'-'d') /* <-- new */
 quietly replace 'g2' = 'd' - 'R'*'p'*('M'-'d') /* <-- new */
end
```

To fit our model and get the robust variance estimates, we type

```
. ml model lf1 weib1 (studytime died = drug2 drug3 age) /s, vce(robust)
. ml maximize
```

◀

## Survey options and ml

`ml` can handle stratification, poststratification, multiple stages of clustering, and finite population corrections. Specifying the `svy` option implies that the data come from a survey design and also implies that the survey linearized variance estimator is to be used; see [\[SVY\] Variance estimation](#).

## ▶ Example 7

Suppose that we are interested in a probit analysis of data from a survey in which  $q_1$  is the answer to a yes/no question and  $x_1, x_2, x_3$  are demographic responses. The following is a `lf2` evaluator for the probit model that meets the requirements for `vce(robust)` (linear form and computes the scores).

```

program mylf2probit
 version 19.5 // (or version 19 if you do not have StataNow)
 args todo b lnfj g1 H

 tempvar z Fz lnf
 mlevel 'z' = 'b'

 quietly generate double 'Fz' = normal('z') if $ML_y1 == 1
 quietly replace 'Fz' = normal(-'z') if $ML_y1 == 0
 quietly replace 'lnfj' = log('Fz')
 if ('todo'==0) exit

 quietly replace 'g1' = normalden('z')/'Fz' if $ML_y1 == 1
 quietly replace 'g1' = -normalden('z')/'Fz' if $ML_y1 == 0
 if ('todo'==1) exit

 mlmatsum 'lnf' 'H' = -'g1'*('g1'+ 'z'), eq(1,1)
end

```

To fit a model, we `svyset` the data, then use `svy` with `ml`.

```

. svyset psuid [pw=w], strata(strid)
. ml model lf2 mylf2probit (q1 = x1 x2 x3), svy
. ml maximize

```

We could also use the `subpop()` option to make inferences about the subpopulation identified by the variable `sub`:

```

. svyset psuid [pw=w], strata(strid)
. ml model lf2 mylf2probit (q1 = x1 x2 x3), svy subpop(sub)
. ml maximize

```

◀

## Stored results

For results stored by `ml` without the `svy` option, see [R] [Maximize](#).

For results stored by `ml` with the `svy` option, see [SVY] [svy](#).

## Methods and formulas

`ml` is implemented using `moptimize()`; see [M-5] [moptimize\(\)](#).

## References

- Korn, E. L., and B. I. Graubard. 1990. Simultaneous testing of regression coefficients with complex survey data: Use of Bonferroni  $t$  statistics. *American Statistician* 44: 270–276. <https://doi.org/10.2307/2684345>.
- Pitblado, J. S., B. P. Poi, and W. W. Gould. 2024. *Maximum Likelihood Estimation with Stata*. 5th ed. College Station, TX: Stata Press.
- Royston, P. 2007. Profile likelihood for estimation and confidence intervals. *Stata Journal* 7: 376–387.

## Also see

- [R] **Maximize** — Details of iterative maximization
- [R] **mlexp** — Maximum likelihood estimation of user-specified expressions
- [R] **nl** — Nonlinear least-squares estimation
- [M-5] **moptimize()** — Model optimization
- [M-5] **optimize()** — Function optimization
- [U] **20 Estimation and postestimation commands**

|             |                      |                |                      |
|-------------|----------------------|----------------|----------------------|
| Description | Quick start          | Menu           | Syntax               |
| Options     | Remarks and examples | Stored results | Methods and formulas |
| References  | Also see             |                |                      |

## Description

`mlexp` performs maximum likelihood estimation of models that satisfy the linear-form restrictions, that is, models for which you can write the log likelihood for an individual observation and for which the overall log likelihood is the sum of the individual observations' log likelihoods.

You express the observation-level log-likelihood function by using a substitutable expression. Unlike models fit using `ml`, you do not need to do any programming. However, `ml` can fit classes of models that cannot be fit by `mlexp`.

## Quick start

Linear regression of `y` on `x1` and `x2`

```
mlexp (ln(normalden(y, {xb: x1 x2 _cons}, {sigma})))
```

Same as above

```
mlexp (ln(normalden(y, {b0} + {b1}*x1 + {b2}*x2, {sigma})))
```

Same as above, and set initial values for `b2` and `sigma`

```
mlexp (ln(normalden(y, {b0} + {b1}*x1 + {b2=0.5}*x2, {sigma=3})))
```

Constrain `sigma` and the coefficient on `x1` to be positive

```
mlexp (ln(normalden(y, {b0} + exp({lnb1})*x1+{b2}*x2, exp({lnsigma}))))
nlcom (b1: exp(_b[lnb1:_cons])) (sigma: exp(_b[lnsigma:_cons]))
```

Omit observations with missing values for `y`, `x1`, or `x2`

```
mlexp (ln(normalden(y, {xb: x1 x2 _cons}, {sigma}))), variables(y x1 x2)
```

## Menu

Statistics > Other > Maximum likelihood estimation of expression

## Syntax

```
mlexp (lexp) [if] [in] [weight] [, options]
```

where *lexp* is a substitutable expression representing the log-likelihood function.

| <i>options</i>                                      | Description                                                                                                                                                                    |
|-----------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                        |                                                                                                                                                                                |
| <code>variables(<i>varlist</i>)</code>              | specify variables in model                                                                                                                                                     |
| <code>from(<i>initial_values</i>)</code>            | specify initial values for parameters                                                                                                                                          |
| <code>constraints(<i>numlist</i>)</code>            | apply specified linear constraints                                                                                                                                             |
| <b>Derivatives</b>                                  |                                                                                                                                                                                |
| <code>derivative(/<i>name</i> = <i>dexp</i>)</code> | specify derivative of <i>lexp</i> with respect to parameter <i>name</i> ;<br>can be specified more than once                                                                   |
| <b>SE/Robust</b>                                    |                                                                                                                                                                                |
| <code>vce(<i>vcetype</i>)</code>                    | <i>vcetype</i> may be <code>oim</code> , <code>opg</code> , <code>robust</code> , <code>cluster <i>clustvar</i></code> ,<br><code>bootstrap</code> , or <code>jackknife</code> |
| <b>Reporting</b>                                    |                                                                                                                                                                                |
| <code>level(#)</code>                               | set confidence level; default is <code>level(95)</code>                                                                                                                        |
| <code>title(<i>string</i>)</code>                   | display <i>string</i> as title above the table of parameter estimates                                                                                                          |
| <code>title2(<i>string</i>)</code>                  | display <i>string</i> as subtitle                                                                                                                                              |
| <code>display_options</code>                        | control columns and column formats, row spacing, line width,<br>display of omitted variables and base and empty cells, and<br>factor-variable labeling                         |
| <b>Maximization</b>                                 |                                                                                                                                                                                |
| <code>maximize_options</code>                       | control the maximization process; seldom used                                                                                                                                  |
| <code>debug</code>                                  | display debug output                                                                                                                                                           |
| <code>collinear</code>                              | keep collinear variables                                                                                                                                                       |
| <code>coeflegend</code>                             | display legend instead of statistics                                                                                                                                           |

*lexp* and *dexp* may contain factor variables and time-series operators; see [U] 11.4.3 Factor variables and [U] 11.4.4 Time-series varlists.

`bayesboot`, `bootstrap`, `by`, `collect`, `jackknife`, `rolling`, `statsby`, and `svy` are allowed; see [U] 11.1.10 Prefix commands.

Weights are not allowed with the `bootstrap` prefix; see [R] `bootstrap`.

`aweights` are not allowed with the `jackknife` prefix; see [R] `jackknife`.

`vce()` and weights are not allowed with the `svy` prefix; see [SVY] `svy`.

`aweights`, `fweights`, `iweights`, and `pweights` are allowed; see [U] 11.1.6 `weight`.

`debug`, `collinear`, and `coeflegend` do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

*lexp* and *dexp* are substitutable expressions, Stata expressions that also contain parameters to be estimated. The parameters are enclosed in curly braces and must satisfy the naming requirements for variables; `{beta}` is an example of a parameter. The notation `{lc: varlist}` is allowed for linear combinations of multiple covariates and their parameters. For example, `{xb: mpg price turn _cons}` defines a linear combination of the variables `mpg`, `price`, `turn`, and `_cons` (the constant term). See *Substitutable expressions* under *Remarks and examples* below.

## Options

### Model

`variables(varlist)` specifies the variables in the model. `mlexp` excludes observations for which any of these variables has missing values. If you do not specify `variables()`, then `mlexp` assumes all observations are valid. `mlexp` will exit with an error message if the log likelihood cannot be calculated at the initial values for any observation.

`from(initial_values)` specifies the initial values to begin the estimation. You can specify parameter names and values, or you can specify a  $1 \times k$  matrix, where  $k$  is the number of parameters in the model. For example, to initialize `alpha` to 1.23 and `delta` to 4.57, you would type

```
mlexp ..., from(alpha=1.23 delta=4.57) ...
```

or equivalently

```
matrix define initval = (1.23, 4.57)
mlexp ..., from(initval) ...
```

Initial values declared in the `from()` option override any that are declared within substitutable expressions. If you specify a parameter that does not appear in your model, `mlexp` exits with an error. If you specify a matrix, the values must be in the same order in which the parameters are declared in your model.

`constraints(numlist)`; see [R] [Estimation options](#).

### Derivatives

`derivative(/name = dexp)` specifies the derivative of the observation-level log-likelihood function with respect to parameter *name*. If you wish to specify analytic derivatives, you must specify `derivative()` for each parameter in your model.

*dexp* uses the same substitutable expression syntax as is used to specify the log-likelihood function. If you declare a linear combination in the log-likelihood function, you provide the derivative for the linear combination; `mlexp` then applies the chain rule for you. See [Specifying derivatives](#) under *Remarks and examples* for examples.

If you do not specify the `derivative()` option, `mlexp` calculates derivatives numerically.

### SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

### Reporting

`level(#)`; see [R] [Estimation options](#).

`title(string)` specifies an optional title that will be displayed just above the table of parameter estimates.

`title2(string)` specifies an optional subtitle that will be displayed between the title specified in `title()` and the table of parameter estimates. If `title2()` is specified but `title()` is not, then `title2()` has the same effect as `title()`.

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

#### Maximization

*maximize\_options*: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, and `nonrtolerance`; see [R] [Maximize](#). These options are seldom used.

The following options are available with `mlexp` but are not shown in the dialog box:

`debug` specifies that differences between the numerically computed gradient and the gradient computed from your derivative expression are reported at each iteration. This option is only allowed with the `derivative()` option.

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Remarks are presented under the following headings:

- [Introduction](#)
- [Substitutable expressions](#)
- [Parameter interpretation using margins](#)
- [Parameter constraints](#)
- [Specifying derivatives](#)

## Introduction

`mlexp` performs maximum likelihood estimation of models that satisfy the linear-form restrictions, that is, models for which you can write the log likelihood for a single observation and for which the overall log likelihood is the sum of the individual observations' log likelihoods. Models designed for use with cross-sectional data usually meet the linear-form restrictions, including linear regression, many discrete choice models, limited-dependent-variable models, and selection models. Examples of models that do not satisfy the linear-form restrictions are random-effects panel-data models (because the likelihood function is defined at the panel level) and Cox proportional hazards models (because the likelihood function is defined for risk sets).

Because of its straightforward syntax and accessibility from the menu system, `mlexp` is particularly suited to users who are new to Stata and to those using Stata for pedagogical purposes. You specify the log-likelihood function that `mlexp` is to maximize by using [substitutable expressions](#) that are similar to those used by `nl`, `nlSUR`, and `gmm`. `mlexp` allows you to avoid the programming requirements of `m1`. However, `m1` can fit classes of models that cannot be fit by `mlexp`, including those that do not meet the linear-form restrictions. See [Pitblado, Poi, and Gould \(2024, chap. 2\)](#) for examples using `mlexp` and for further discussion of the advantages of both `mlexp` and `m1`.

## Substitutable expressions

Substitutable expressions allow you to distinguish between variables and parameters. There are three rules to follow when defining substitutable expressions:

1. Parameters of the model are bound in curly braces: `{b0}`, `{param}`, etc. Parameter names must follow the same conventions as variable names; see [\[U\] 11.3 Naming conventions](#).
2. Initial values for parameters are given by including an equal sign and the initial value inside the curly braces: `{b0=1}`, `{param=3.571}`, etc.
3. Linear combinations of variables can be included using the notation `{lc:varlist}`: `{xb: mpg price weight _cons}`, `{score: w x z}`, etc. Parameters of linear combinations are initialized to zero.

Substitutable expressions can include any mathematical expression involving scalars and variables. See [\[U\] 13.2 Operators](#) and [\[U\] 13.3 Functions](#) for more information on expressions.

### ► Example 1: The gamma density function

In an extract of the March 2014 Current Population Survey downloaded from Integrated Public Use Microdata Series (IPUMS), `wage` contains the wage and salary income earned in tens of thousands of dollars per year for working individuals; see [King et al. \(2010\)](#). We want to model `wage` using the two-parameter gamma distribution with shape parameter  $\alpha$  and rate parameter  $\beta$ . The density function for  $y > 0$  is

$$f(y) = \frac{\beta^\alpha}{\Gamma(\alpha)} y^{\alpha-1} \exp(-\beta y) \quad \beta > 0, \alpha > 0$$

so that the log likelihood for the  $i$ th observation is

$$\ln(\ell_i) = \alpha \ln(\beta) - \ln\{\Gamma(\alpha)\} + (\alpha - 1) \ln(y_i) - \beta y_i$$

To fit `wage` to the two-parameter gamma distribution, we let `a` be the parameter name that conforms to Stata naming conventions for  $\alpha$  and `b` be the parameter name for  $\beta$ . We enclose both in `{}` in our substitutable expression for the log-likelihood function:

```
. use https://www.stata-press.com/data/r19/cpswage
. mlexp ({a}*ln({b}) - lngamma({a}) + ({a}-1)*ln(wage) - {b}*wage)
Initial: Log likelihood = -<inf> (could not be evaluated)
Feasible: Log likelihood = -239367.36
Rescale: Log likelihood = -228037.02
Rescale eq: Log likelihood = -163560.64
Iteration 0: Log likelihood = -163560.64
Iteration 1: Log likelihood = -162820.41
Iteration 2: Log likelihood = -162808.55
Iteration 3: Log likelihood = -162808.55

Maximum likelihood estimation
Log likelihood = -162808.55 Number of obs = 64,748
```

|    | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |          |
|----|-------------|-----------|--------|-------|----------------------|----------|
| /a | 1.097287    | .0054134  | 202.70 | 0.000 | 1.086677             | 1.107897 |
| /b | .2406718    | .0014917  | 161.34 | 0.000 | .2377482             | .2435954 |

Because we did not specify initial values, mlexp initialized  $\alpha$  and  $\beta$  to be 0. When both parameters are 0, the log-likelihood function cannot be evaluated, because  $\ln(0)$  is undefined. Therefore, in the iteration log above the coefficient table, we see that mlexp reported the initial log likelihood to be  $-\infty$  (could not be evaluated). When this occurs, mlexp uses a search routine to find alternative initial values that do allow the log-likelihood function to be calculated.

We now initialize a to 1 and b to 0.1 the first time that we type them within the substitutable expression:

```
. mlexp ({a=1}*ln({b=.1}) - lngamma({a}) + ({a}-1)*ln(wage) - {b}*wage)
Initial: Log likelihood = -178608.12
Rescale: Log likelihood = -178608.12
Rescale eq: Log likelihood = -173389.94
Iteration 0: Log likelihood = -173389.94
Iteration 1: Log likelihood = -163081.69
Iteration 2: Log likelihood = -162813.54
Iteration 3: Log likelihood = -162808.55
Iteration 4: Log likelihood = -162808.55

Maximum likelihood estimation
Log likelihood = -162808.55 Number of obs = 64,748
```

|    | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |          |
|----|-------------|-----------|--------|-------|----------------------|----------|
| /a | 1.097287    | .0054134  | 202.70 | 0.000 | 1.086677             | 1.107897 |
| /b | .2406719    | .0014917  | 161.34 | 0.000 | .2377483             | .2435955 |

Even when mlexp can find alternative initial values, specifying your own values facilitates optimization.



By default, initial values of parameters are set to zero, and mlexp performs an iterative search for optimum starting values before beginning maximization. If mlexp cannot find initial parameter values for which it can calculate the log-likelihood function, it will exit with an error message. Restricting the sample or specifying starting values solves this problem.

Use the `variables()` option, an *if* qualifier, or an *in* qualifier to restrict the sample. You can also specify initial values by using the `from()` option or within the substitutable expression by including an equal sign and the initial value after the parameter. If you specify initial values by using `from()`, they

override whatever initial values are given within the substitutable expression. Regardless of whether you specify initial values, mlexp performs a search procedure for better starting values before commencing the first iteration of the maximization routine.

▷ Example 2: Linear combinations of covariates

We frequently want to model the parameters as linear combinations of variables. Continuing [example 1](#), we see that the mean of the two-parameter gamma distribution is  $E(y) = \alpha/\beta$ . By letting  $\alpha = a_1 \text{age} + a_0$ , we model the mean of wage conditional on age as  $E(\text{wage}|\text{age}) = (a_1 \text{age} + a_0)/\beta$ . Below, we specify {a: age \_cons} to model a as a linear combination of age and a constant term.

```
. mlexp ({a:age _cons}*ln({b=.1})-lngamma({a:})+({a:}-1)*ln(wage)-{b}*wage)
Initial: Log likelihood = -<inf> (could not be evaluated)
Feasible: Log likelihood = -239367.36
Rescale: Log likelihood = -228037.02
Rescale eq: Log likelihood = -163560.64
Iteration 0: Log likelihood = -163560.64
Iteration 1: Log likelihood = -160123.65
Iteration 2: Log likelihood = -159838.42
Iteration 3: Log likelihood = -159838.38
Iteration 4: Log likelihood = -159838.38

Maximum likelihood estimation
Log likelihood = -159838.38 Number of obs = 64,748
```

|          | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |          |
|----------|-------------|-----------|--------|-------|----------------------|----------|
| <b>a</b> |             |           |        |       |                      |          |
| age      | .0194396    | .0002689  | 72.31  | 0.000 | .0189126             | .0199665 |
| _cons    | .4108355    | .0096791  | 42.45  | 0.000 | .3918647             | .4298062 |
| /b       | .2667954    | .0016423  | 162.45 | 0.000 | .2635765             | .2700142 |

The notation {a: age \_cons} indicates to mlexp that you want a linear combination of the variable age and a constant term. We named the linear combination a, so mlexp names the parameters a:age and a:\_cons, respectively.

Once you have declared a linear combination, you can subsequently refer to the linear combination by specifying its name and a colon inside curly braces, as we did in this example. You cannot use the same name for both an individual parameter and a linear combination. However, after a linear combination has been declared, you can refer to the parameter of an individual variable within that linear combination by using the notation {lc: z}, where lc is the name of the linear combination, and z is the variable whose parameter you want to reference.



▷ Example 3: Linear combinations of factor variables

Factor variables and time-series operated variables can be included in a varlist defining a linear combination. Continuing [example 2](#), we want to allow different intercepts for males and females in  $\alpha$ , and we want different  $\beta$  parameters for males and females. We implement this model below by including `ibn.female` in each linear combination using [factor-variable](#) notation.

```
. mlexp ({a:age ibn.female}*ln({b:ibn.female}) - lngamma({a:}) +
> ({a:}-1)*ln(wage) - {b:}*wage)
Initial: Log likelihood = -<inf> (could not be evaluated)
Feasible: Log likelihood = -239367.36
Rescale: Log likelihood = -228037.02
Rescale eq: Log likelihood = -163560.64
Iteration 0: Log likelihood = -163560.64
Iteration 1: Log likelihood = -158999.51
Iteration 2: Log likelihood = -158135.16
Iteration 3: Log likelihood = -158130.94
Iteration 4: Log likelihood = -158130.93
Maximum likelihood estimation
Log likelihood = -158130.93 Number of obs = 64,748
```

|          | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |          |
|----------|-------------|-----------|--------|-------|----------------------|----------|
| <b>a</b> |             |           |        |       |                      |          |
| age      | .020543     | .0002781  | 73.88  | 0.000 | .019998              | .0210881 |
| female   |             |           |        |       |                      |          |
| Male     | .3917619    | .0111697  | 35.07  | 0.000 | .3698696             | .4136542 |
| Female   | .4475977    | .0116285  | 38.49  | 0.000 | .4248063             | .4703892 |
| <b>b</b> |             |           |        |       |                      |          |
| female   |             |           |        |       |                      |          |
| Male     | .2279862    | .0018677  | 122.07 | 0.000 | .2243256             | .2316468 |
| Female   | .3600628    | .0030156  | 119.40 | 0.000 | .3541523             | .3659733 |



## Parameter interpretation using margins

The `margins` command can be used after `mlexp` to estimate the effect of a covariate from a set of parameter estimates. The estimated covariate effects can be conditional on the other covariates or population-averaged effects that average out the other covariates.

### ▶ Example 4

Continuing [example 3](#), we use `margins` to estimate the average wage if everyone in the sample were male and if everyone in the sample were female. The `expression()` option is used to specify the formula for the mean. Wages, measured in tens of thousands of dollars, have a gamma distribution, so the mean is a ratio of the  $\alpha$  and  $\beta$  parameters. The linear prediction is specified in the expression with `xb()`.

```
. margins i.female, expression(xb(a)/xb(b))
Predictive margins Number of obs = 64,748
Model VCE: OIM
Expression: xb(a)/xb(b)
```

|        | Delta-method |           |        |       | [95% conf. interval] |          |
|--------|--------------|-----------|--------|-------|----------------------|----------|
|        | Margin       | std. err. | z      | P> z  |                      |          |
| female |              |           |        |       |                      |          |
| Male   | 5.452281     | .0267573  | 203.77 | 0.000 | 5.399838             | 5.504724 |
| Female | 3.607372     | .0178778  | 201.78 | 0.000 | 3.572332             | 3.642412 |

When everyone is male, the estimated average wage is 5.45; when everyone is female, the estimated average wage is 3.61.



## Parameter constraints

### ▷ Example 5

In examples 1, 2, and 3, we were lucky. The two-parameter gamma density function is defined only when both  $\alpha$  and  $\beta$  are positive. However, mlexp does not know this; when maximizing the log-likelihood function, it will consider all real values for the parameters. Also recall from above, mlexp will exit with an error message if it cannot find parameter values that produce nonmissing values for the likelihood for each sample observation.

We could reparameterize our model so that we avoid having to directly estimate parameters that are restricted. For example, consider the parameter  $\alpha > 0$ , and suppose we define the new parameter  $\theta = \ln(\alpha)$  so that  $\alpha = \exp(\theta)$ . With this parameterization, for any real value of  $\theta$  that mlexp might try to use when evaluating the log-likelihood function,  $\alpha$  is guaranteed to be positive. Below we apply this parameterization to the model in example 1.

```
. mlexp (exp({lna})*{lnb} - lngamma(exp({lna}))
> + (exp({lna})-1)*ln(wage) - exp({lnb})*wage)
Initial: Log likelihood = -295203.41
Alternative: Log likelihood = -249215.67
Rescale: Log likelihood = -230376.58
Rescale eq: Log likelihood = -166588.81
Iteration 0: Log likelihood = -166588.81
Iteration 1: Log likelihood = -162845.95
Iteration 2: Log likelihood = -162808.56
Iteration 3: Log likelihood = -162808.55
Iteration 4: Log likelihood = -162808.55
Maximum likelihood estimation
Log likelihood = -162808.55 Number of obs = 64,748
```

|      | Coefficient | Std. err. | z       | P> z  | [95% conf. interval] |           |
|------|-------------|-----------|---------|-------|----------------------|-----------|
| /lna | .0928409    | .0049334  | 18.82   | 0.000 | .0831716             | .1025103  |
| /lnb | -1.424321   | .0061979  | -229.81 | 0.000 | -1.436469            | -1.412173 |

We can use `nlcom` to obtain the back-transformed parameter estimates, but first we need to know how to refer to the parameters estimated by `mlexp`. We can replay the results and request the coefficient legend.

```
. mlexp, coeflegend
Maximum likelihood estimation
Log likelihood = -162808.55 Number of obs = 64,748
```

|      | Coefficient | Legend        |
|------|-------------|---------------|
| /lna | .0928409    | _b[lna:_cons] |
| /lnb | -1.424321   | _b[lnb:_cons] |

Now, we see that we can refer to `_b[lna:_cons]` and `_b[lnb:_cons]` in `nlcom`.

```
. nlcom (a: exp(_b[lna:_cons])) (b: exp(_b[lnb:_cons]))
 a: exp(_b[lna:_cons])
 b: exp(_b[lnb:_cons])
```

|   | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |          |
|---|-------------|-----------|--------|-------|----------------------|----------|
| a | 1.097287    | .0054134  | 202.70 | 0.000 | 1.086677             | 1.107897 |
| b | .2406718    | .0014917  | 161.34 | 0.000 | .2377482             | .2435954 |

The optimal value of the log-likelihood function and the back-transformed parameter estimates match those reported in [example 1](#).

However, when you impose nonlinear constraints on linear combinations of covariates, you change the model. These nonlinear constraints change the model because they are not invertible functions of the original parameters. For example, we can use the exponential function to ensure  $\alpha > 0$  and  $\beta > 0$  in the model from [example 3](#).

```
. mlexp (exp({a:age ibn.female})*{b:ibn.female} - lngamma(exp({a:}))
> + (exp({a:})-1)*ln(wage) - exp({b:})*wage)
Initial: Log likelihood = -295203.41
Alternative: Log likelihood = -249215.67
Rescale: Log likelihood = -230376.58
Rescale eq: Log likelihood = -166588.81
Iteration 0: Log likelihood = -166588.81
Iteration 1: Log likelihood = -160273.12
Iteration 2: Log likelihood = -158868.39
Iteration 3: Log likelihood = -158864.02
Iteration 4: Log likelihood = -158864.02
Maximum likelihood estimation
Log likelihood = -158864.02 Number of obs = 64,748
```

|          | Coefficient | Std. err. | z       | P> z  | [95% conf. interval] |           |
|----------|-------------|-----------|---------|-------|----------------------|-----------|
| <b>a</b> |             |           |         |       |                      |           |
| age      | .0126713    | .0001785  | 70.98   | 0.000 | .0123214             | .0130212  |
| female   |             |           |         |       |                      |           |
| Male     | -.3470203   | .009978   | -34.78  | 0.000 | -.3665769            | -.3274638 |
| Female   | -.3177398   | .0103376  | -30.74  | 0.000 | -.338001             | -.2974786 |
| <b>b</b> |             |           |         |       |                      |           |
| female   |             |           |         |       |                      |           |
| Male     | -1.502222   | .0085431  | -175.84 | 0.000 | -1.518966            | -1.485478 |
| Female   | -1.059358   | .0087602  | -120.93 | 0.000 | -1.076528            | -1.042189 |

The value of the log-likelihood function is now  $-158,864.02$ , which is less than the value  $-158,130.93$ , which was reported in [example 3](#). The constrained parameterization that keeps  $\alpha > 0$  and  $\beta > 0$  has changed the model, and the optimal value of the log-likelihood function in the constrained model is less than the optimal value in the unconstrained model.

Because neither the unconstrained nor the constrained parameterization generates missing predicted values for  $\alpha$  or  $\beta$ , we can choose between them. However, if any of the predicted values for  $\alpha$  or  $\beta$  produce missing values for the log-likelihood function using the unconstrained parameterization, we could not compute the log-likelihood function for the unconstrained parameterization, and we would have to use a constrained parameterization.



## Specifying derivatives

By default, `mlexp` calculates derivatives of the log-likelihood function numerically using an algorithm that produces accurate results. However, `mlexp` will fit your model more quickly (and even more accurately) if you specify analytic derivatives.

You specify derivatives by using substitutable expressions in much the same way as you specify the log-likelihood function. If you specify a linear combination in your log-likelihood function, then you supply a derivative with respect to that linear combination; `mlexp` then uses the chain rule to obtain the derivatives with respect to the individual parameters.

We will illustrate how to specify derivatives using the probit model for dichotomous outcomes. The log-likelihood function for the probit model is often written as

$$\ln \ell_i = \begin{cases} \ln \Phi(\mathbf{x}'_i \boldsymbol{\beta}) & y_i = 1 \\ \ln \Phi(-\mathbf{x}'_i \boldsymbol{\beta}) & y_i = 0 \end{cases}$$

using the fact that  $1 - \Phi(\mathbf{x}'_i \boldsymbol{\beta}) = \Phi(-\mathbf{x}'_i \boldsymbol{\beta})$ , where  $\Phi(\cdot)$  is the cumulative standard normal distribution function. If we use the trick suggested by [Greene \(2018, 742, fn. 16\)](#), we can simplify the log-likelihood function, making the derivative calculation easier. Let  $q_i = 2y_i - 1$ . Then, we can write the log-likelihood function as

$$\ln \ell_i = \ln \Phi(q_i \mathbf{x}'_i \boldsymbol{\beta})$$

and the first derivative as

$$\frac{\partial \ln \ell_i}{\partial \boldsymbol{\beta}} = \frac{q_i \phi(q_i \mathbf{x}'_i \boldsymbol{\beta})}{\Phi(q_i \mathbf{x}'_i \boldsymbol{\beta})} \mathbf{x}_i$$

### ► Example 6: Probit with a linear combination

Now, let's fit a probit model of the indicator for whether an individual is below the official poverty level of `fpov` on `age`, `female`, and a constant. We could specify the parameters and independent variables individually, but we will use a linear combination instead. First, note that

$$\frac{\partial \ln \ell_i}{\partial \mathbf{x}'_i \boldsymbol{\beta}} = \frac{q_i \phi(q_i \mathbf{x}'_i \boldsymbol{\beta})}{\Phi(q_i \mathbf{x}'_i \boldsymbol{\beta})}$$

When you specify a linear combination of variables, you specify the derivative with respect to the linear combination. That way, if you change the variables in the linear combination, you do not need to change the derivative. To see why this is the case, consider the function  $f(\mathbf{x}'_i \boldsymbol{\beta})$ , where  $\mathbf{x}'_i \boldsymbol{\beta}$  is a linear combination. Then, using the chain rule, we see that

$$\frac{\partial f(\mathbf{x}'_i \boldsymbol{\beta})}{\partial \beta_j} = \frac{\partial f(\mathbf{x}'_i \boldsymbol{\beta})}{\partial \mathbf{x}'_i \boldsymbol{\beta}} \times \frac{\partial \mathbf{x}'_i \boldsymbol{\beta}}{\partial \beta_j} = \frac{\partial f(\mathbf{x}'_i \boldsymbol{\beta})}{\partial \mathbf{x}'_i \boldsymbol{\beta}} \times x_{ij}$$

Once the derivative with respect to the linear combination is known, `mlexp` can then multiply it by each of the variables in the linear combination to get the full set of derivatives with respect to the parameters needed to maximize the likelihood function. Moreover, the derivative with respect to the linear combination does not depend on the variables within the linear combination, so even if you change the variables in it, you will not need to modify the specification of the corresponding `derivative()` option.

We type

```
. generate int q = 2*offpov - 1
. mlexp (ln(normal(q*({xb:age i.female _cons}))))
> deriv(/xb = q*normalden(q*{xb:})/normal(q*{xb:}))
Initial: Log likelihood = -44879.894
Alternative: Log likelihood = -27407.718
Rescale: Log likelihood = -17888.147
Iteration 0: Log likelihood = -17888.147
Iteration 1: Log likelihood = -15805.22
Iteration 2: Log likelihood = -15427.598
Iteration 3: Log likelihood = -15427.04
Iteration 4: Log likelihood = -15427.04

Maximum likelihood estimation
Log likelihood = -15427.04 Number of obs = 64,748
```

|        | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |           |
|--------|-------------|-----------|--------|-------|----------------------|-----------|
| xb     |             |           |        |       |                      |           |
| age    | -.0179139   | .0005852  | -30.61 | 0.000 | -.0190608            | -.016767  |
| female |             |           |        |       |                      |           |
| Female | .1514014    | .0154565  | 9.80   | 0.000 | .1211072             | .1816955  |
| _cons  | -.8773462   | .0244956  | -35.82 | 0.000 | -.9253567            | -.8293357 |

After defining *q*, we specified the log-likelihood function using *q* and the linear combination *xb*. We also use *xb* in specifying the derivative.



## Stored results

mlexp stores the following in *e()*:

### Scalars

- e(N)*                    number of observations
- e(k)*                    number of parameters
- e(k\_aux)*                number of ancillary parameters
- e(k\_eq)*                 number of equations in *e(b)*
- e(k\_eq\_model)*         number of equations in overall model test
- e(df\_m)*                 model degrees of freedom
- e(ll)*                    log likelihood
- e(N\_clust)*              number of clusters
- e(rank)*                 rank of *e(V)*
- e(ic)*                    number of iterations
- e(rc)*                    return code
- e(converged)*          1 if converged, 0 otherwise

### Macros

- e(cmd)*                    mlexp
- e(cmdline)*              command as typed
- e(lexp)*                  likelihood expression
- e(wtype)*                 weight type
- e(wexp)*                 weight expression
- e(usrtitle)*              user-specified title
- e(usrtitle2)*            user-specified secondary title
- e(clustvar)*              name of cluster variable
- e(vce)*                    *vcetype* specified in *vce()*
- e(vcetype)*              title used to label Std. err.

|                    |                                                                          |
|--------------------|--------------------------------------------------------------------------|
| e(params)          | names of parameters                                                      |
| e(hasderiv)        | yes, if <code>derivative()</code> is specified                           |
| e(d_j)             | derivative expression for parameter <i>j</i>                             |
| e(rhs)             | contents of <code>variables()</code>                                     |
| e(opt)             | type of optimization                                                     |
| e(ml_method)       | type of ml method                                                        |
| e(technique)       | maximization technique                                                   |
| e(singularHmethod) | n-marquardt or hybrid; method used when Hessian is singular <sup>1</sup> |
| e(crittype)        | optimization criterion <sup>1</sup>                                      |
| e(properties)      | b V                                                                      |
| e(estat_cmd)       | program used to implement <code>estat</code>                             |
| e(predict)         | program used to implement <code>predict</code>                           |
| e(marginsok)       | predictions allowed by <code>margins</code>                              |
| e(marginsnotok)    | predictions disallowed by <code>margins</code>                           |
| e(marginsprop)     | signals to the <code>margins</code> command                              |
| e(asbalanced)      | factor variables <code>fvset</code> as <code>asbalanced</code>           |
| e(asobserved)      | factor variables <code>fvset</code> as <code>asobserved</code>           |
| Matrices           |                                                                          |
| e(b)               | coefficient vector                                                       |
| e(Cns)             | constraints matrix                                                       |
| e(ilog)            | iteration log (up to 20 iterations)                                      |
| e(init)            | initial values                                                           |
| e(gradient)        | gradient vector                                                          |
| e(V)               | variance–covariance matrix of the estimators                             |
| e(V_modelbased)    | model-based variance                                                     |
| Functions          |                                                                          |
| e(sample)          | marks estimation sample                                                  |

1. Type `ereturn list, all` to view these results; see [P] [return](#).

In addition to the above, the following is stored in `r()`:

|          |                                                                                                                            |
|----------|----------------------------------------------------------------------------------------------------------------------------|
| Matrices |                                                                                                                            |
| r(table) | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

## Methods and formulas

Optimization is carried out using `moptimize()`; see [M-5] [moptimize\(\)](#).

## References

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- Rajbhandari, A. 2015. Estimating parameters by maximum likelihood and method of moments using mlexp and gmm. *The Stata Blog: Not Elsewhere Classified*. <https://blog.stata.com/2015/10/15/estimating-parameters-by-maximum-likelihood-and-method-of-moments-using-mlexp-and-gmm/>.

## Also see

- [R] **mlexp postestimation** — Postestimation tools for mlexp
- [R] **gmm** — Generalized method of moments estimation
- [R] **Maximize** — Details of iterative maximization
- [R] **ml** — Maximum likelihood estimation
- [R] **nl** — Nonlinear least-squares estimation
- [R] **nlsur** — Estimation of nonlinear system of equations
- [U] **20 Estimation and postestimation commands**

## Postestimation commands

The following postestimation commands are available after `mlexp`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | linear predictions and scores                                                                                                           |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`lrtest` is not appropriate with `svy` estimation results.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as linear predictions and equation-level scores.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, xb equation(eqno | eqname)]
```

```
predict [type] stub* [if] [in] , scores
```

Scores are only available for observations within the estimation sample.

## Options for predict

`xb` calculates the linear prediction.

`equation(eqno | eqname)` specifies the equation for which the linear prediction is desired. Specifying `equation(#1)` indicates that the calculation be made for the first equation. Specifying `equation(demand)` indicates that the calculation be made for the equation named `demand`, assuming there is an equation named `demand` in the model.

If you specify one new variable name and omit `equation()`, results are the same as if you had specified `equation(#1)`.

For more information on using `predict` after multiple-equation estimation commands, see [\[R\] predict](#).

`scores` calculates the equation-level score variables. The *j*th new variable will contain the scores for the *j*th equation of the model.

# margins

## Description for margins

`margins` estimates margins of response for linear predictions.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

---

|                 |                   |
|-----------------|-------------------|
| <code>xb</code> | linear prediction |
|-----------------|-------------------|

---

`xb` defaults to the first equation.

## Also see

[R] [mlexp](#) — Maximum likelihood estimation of user-specified expressions

[U] [20 Estimation and postestimation commands](#)

Description  
Options  
References

Quick start  
Remarks and examples  
Also see

Menu  
Stored results

Syntax  
Methods and formulas

## Description

`mlogit` fits a multinomial logit (MNL) model for a categorical dependent variable with outcomes that have no natural ordering. The actual values taken by the dependent variable are irrelevant. The MNL model is also known as the polytomous logistic regression model. Some people refer to conditional logistic regression as multinomial logistic regression. If you are one of them, see [\[R\] clogit](#).

## Quick start

MNL model of `y` on `x1`, `x2`, and categorical variable `a`

```
mlogit y x1 x2 i.a
```

Same as above, but use `y = 1` as the base outcome even if 1 is not the most frequent

```
mlogit y x1 x2 i.a, baseoutcome(1)
```

Report results as relative-risk ratios

```
mlogit y x1 x2 i.a, rrr
```

Constrain coefficient of `x1` to be equal for second and third outcomes

```
constraint 1 [#2=#3]:x1
mlogit y x1 x2 i.a, constraints(1)
```

## Menu

Statistics > Categorical outcomes > Multinomial logistic regression

## Syntax

```
mlogit depvar [indepvars] [if] [in] [weight] [, options]
```

| <i>options</i>                               | Description                                                                                                                                                                 |
|----------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                 |                                                                                                                                                                             |
| <code>noconstant</code>                      | suppress constant term                                                                                                                                                      |
| <code>baseoutcome(#)</code>                  | value of <i>depvar</i> that will be the base outcome                                                                                                                        |
| <code>constraints(<i>constraints</i>)</code> | apply specified linear constraints                                                                                                                                          |
| <b>SE/Robust</b>                             |                                                                                                                                                                             |
| <code>vce(<i>vcetype</i>)</code>             | <i>vcetype</i> may be <code>oim</code> , <code>opg</code> , <code>robust</code> , <code>cluster <i>clustvar</i></code> , <code>bootstrap</code> , or <code>jackknife</code> |
| <b>Reporting</b>                             |                                                                                                                                                                             |
| <code>level(#)</code>                        | set confidence level; default is <code>level(95)</code>                                                                                                                     |
| <code>rrr</code>                             | report relative-risk ratios                                                                                                                                                 |
| <code>nocnsreport</code>                     | do not display constraints                                                                                                                                                  |
| <code>display_options</code>                 | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling                            |
| <b>Maximization</b>                          |                                                                                                                                                                             |
| <code>maximize_options</code>                | control the maximization process; seldom used                                                                                                                               |
| <code>collinear</code>                       | keep collinear variables                                                                                                                                                    |
| <code>coeflegend</code>                      | display legend instead of statistics                                                                                                                                        |

*indepvars* may contain factor variables; see [U] 11.4.3 **Factor variables**.

*indepvars* may contain time-series operators; see [U] 11.4.4 **Time-series varlists**.

`bayes`, `bayesboot`, `bootstrap`, `by`, `collect`, `fmm`, `fp`, `jackknife`, `mfp`, `mi estimate`, `rolling`, `statsby`, and `svy` are allowed; see [U] 11.1.10 **Prefix commands**. For more details, see [BAYES] **bayes: mlogit** and [FMM] **fmm: mlogit**.

`vce(bootstrap)` and `vce(jackknife)` are not allowed with the `mi estimate` prefix; see [MI] **mi estimate**.

Weights are not allowed with the `bootstrap` prefix; see [R] **bootstrap**.

`vce()` and weights are not allowed with the `svy` prefix; see [SVY] **svy**.

`fweights`, `iweights`, and `pweights` are allowed; see [U] 11.1.6 **weight**.

`collinear` and `coeflegend` do not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options

Model

`noconstant`; see [R] **Estimation options**.

`baseoutcome(#)` specifies the value of *depvar* to be treated as the base outcome. The default is to choose the most frequent outcome.

`constraints(constraints)`; see [R] **Estimation options**.

## SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

If specifying `vce(bootstrap)` or `vce(jackknife)`, you must also specify `baseoutcome()`.

## Reporting

`level(#)`; see [R] [Estimation options](#).

`rrr` reports the estimated coefficients transformed to relative-risk ratios, that is,  $e^b$  rather than  $b$ ; see [Description of the model](#) below for an explanation of this concept. Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated. `rrr` may be specified at estimation or when replaying previously estimated results.

`nocnsreport`; see [R] [Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

## Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

The following options are available with `mlogit` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Remarks are presented under the following headings:

*Description of the model*  
*Fitting unconstrained models*  
*Fitting constrained models*

`mlogit` fits maximum likelihood models with discrete dependent (left-hand-side) variables when the dependent variable takes on more than two outcomes and the outcomes have no natural ordering. If the dependent variable takes on only two outcomes, estimates are identical to those produced by `logistic` or `logit`; see [R] [logistic](#) or [R] [logit](#). If the outcomes are ordered, see [R] [ologit](#). See [R] [logistic](#) for a list of related estimation commands.

## Description of the model

For an introduction to MNL models, see [Greene \(2018, 829–833\)](#), [Hosmer, Lemeshow, and Sturdivant \(2013, 269–289\)](#), [Long \(1997, chap. 6\)](#), [Long and Freese \(2014, chap. 8\)](#), and [Treiman \(2009, 336–341\)](#). For a description emphasizing the difference in assumptions and data requirements for conditional and multinomial logit, see [Davidson and MacKinnon \(1993\)](#).

Consider the outcomes  $1, 2, 3, \dots, m$  recorded in  $y$ , and the explanatory variables  $X$ . Assume that there are  $m = 3$  outcomes: “buy an American car”, “buy a Japanese car”, and “buy a European car”. The values of  $y$  are then said to be “unordered”. Even though the outcomes are coded 1, 2, and 3, the numerical values are arbitrary because  $1 < 2 < 3$  does not imply that outcome 1 (buy American) is less than outcome 2 (buy Japanese) is less than outcome 3 (buy European). This unordered categorical property of  $y$  distinguishes the use of `mlogit` from `regress` (which is appropriate for a continuous dependent variable), from `ologit` (which is appropriate for ordered categorical data), and from `logit` (which is appropriate for two outcomes, which can be thought of as ordered).

In the MNL model, you estimate a set of coefficients,  $\beta^{(1)}$ ,  $\beta^{(2)}$ , and  $\beta^{(3)}$ , corresponding to each outcome:

$$\Pr(y = 1) = \frac{e^{X\beta^{(1)}}}{e^{X\beta^{(1)}} + e^{X\beta^{(2)}} + e^{X\beta^{(3)}}}$$

$$\Pr(y = 2) = \frac{e^{X\beta^{(2)}}}{e^{X\beta^{(1)}} + e^{X\beta^{(2)}} + e^{X\beta^{(3)}}}$$

$$\Pr(y = 3) = \frac{e^{X\beta^{(3)}}}{e^{X\beta^{(1)}} + e^{X\beta^{(2)}} + e^{X\beta^{(3)}}}$$

The model, however, is unidentified in the sense that there is more than one solution to  $\beta^{(1)}$ ,  $\beta^{(2)}$ , and  $\beta^{(3)}$  that leads to the same probabilities for  $y = 1$ ,  $y = 2$ , and  $y = 3$ . To identify the model, you arbitrarily set one of  $\beta^{(1)}$ ,  $\beta^{(2)}$ , or  $\beta^{(3)}$  to 0—it does not matter which. That is, if you arbitrarily set  $\beta^{(1)} = 0$ , the remaining coefficients  $\beta^{(2)}$  and  $\beta^{(3)}$  will measure the change relative to the  $y = 1$  group. If you instead set  $\beta^{(2)} = 0$ , the remaining coefficients  $\beta^{(1)}$  and  $\beta^{(3)}$  will measure the change relative to the  $y = 2$  group. The coefficients will differ because they have different interpretations, but the predicted probabilities for  $y = 1, 2$ , and  $3$  will still be the same. Thus, either parameterization will be a solution to the same underlying model.

Setting  $\beta^{(1)} = 0$ , the equations become

$$\Pr(y = 1) = \frac{1}{1 + e^{X\beta^{(2)}} + e^{X\beta^{(3)}}}$$

$$\Pr(y = 2) = \frac{e^{X\beta^{(2)}}}{1 + e^{X\beta^{(2)}} + e^{X\beta^{(3)}}}$$

$$\Pr(y = 3) = \frac{e^{X\beta^{(3)}}}{1 + e^{X\beta^{(2)}} + e^{X\beta^{(3)}}}$$

The relative probability of  $y = 2$  to the base outcome is

$$\frac{\Pr(y = 2)}{\Pr(y = 1)} = e^{X\beta^{(2)}}$$

Let's call this ratio the relative risk, and let's further assume that  $X$  and  $\beta_k^{(2)}$  are vectors equal to  $(x_1, x_2, \dots, x_k)$  and  $(\beta_1^{(2)}, \beta_2^{(2)}, \dots, \beta_k^{(2)})'$ , respectively. The ratio of the relative risk for a one-unit change in  $x_i$  is then

$$\frac{e^{\beta_1^{(2)}x_1 + \dots + \beta_i^{(2)}(x_i+1) + \dots + \beta_k^{(2)}x_k}}{e^{\beta_1^{(2)}x_1 + \dots + \beta_i^{(2)}x_i + \dots + \beta_k^{(2)}x_k}} = e^{\beta_i^{(2)}}$$

Thus, the exponentiated value of a coefficient is the relative-risk ratio for a one-unit change in the corresponding variable (risk is measured as the risk of the outcome relative to the base outcome).

## Fitting unconstrained models

### ▷ Example 1: A first example

We have data on the type of health insurance available to 616 psychologically depressed subjects in the United States (Tarlov et al. 1989; Wells et al. 1989). The insurance is categorized as either an indemnity plan (that is, regular fee-for-service insurance, which may have a deductible or coinsurance rate) or a prepaid plan (a fixed up-front payment allowing subsequent unlimited use as provided, for instance, by an HMO). The third possibility is that the subject has no insurance whatsoever. We wish to explore the demographic factors associated with each subject's insurance choice. One of the demographic factors in our data is the race of the participant, coded as white or nonwhite:

```
. use https://www.stata-press.com/data/r19/sysdsn1
(Health insurance data)
. tabulate insure nonwhite, chi2 col
```

|                          |
|--------------------------|
| Key                      |
| <i>frequency</i>         |
| <i>column percentage</i> |

| Type of insurance | Nonwhite      |               | Total         |
|-------------------|---------------|---------------|---------------|
|                   | 0             | 1             |               |
| Indemnity         | 251<br>50.71  | 43<br>35.54   | 294<br>47.73  |
| Prepaid           | 208<br>42.02  | 69<br>57.02   | 277<br>44.97  |
| Uninsure          | 36<br>7.27    | 9<br>7.44     | 45<br>7.31    |
| Total             | 495<br>100.00 | 121<br>100.00 | 616<br>100.00 |

Pearson chi2(2) = 9.5599 Pr = 0.008

Although insure appears to take on the values Indemnity, Prepaid, and Uninsure, it actually takes on the values 1, 2, and 3. The words appear because we have associated a value label with the numeric variable insure; see [U] 12.6.3 Value labels.

When we fit an MNL model, we can tell mlogit which outcome to use as the base outcome, or we can let mlogit choose. To fit a model of insure on nonwhite, letting mlogit choose the base outcome, we type

```
. mlogit insure nonwhite
Iteration 0: Log likelihood = -556.59502
Iteration 1: Log likelihood = -551.78935
Iteration 2: Log likelihood = -551.78348
Iteration 3: Log likelihood = -551.78348

Multinomial logistic regression
Log likelihood = -551.78348
Number of obs = 616
LR chi2(2) = 9.62
Prob > chi2 = 0.0081
Pseudo R2 = 0.0086
```

| insure    | Coefficient    | Std. err. | z      | P> z  | [95% conf. interval] |           |
|-----------|----------------|-----------|--------|-------|----------------------|-----------|
| Indemnity | (base outcome) |           |        |       |                      |           |
| Prepaid   |                |           |        |       |                      |           |
| nonwhite  | .6608212       | .2157321  | 3.06   | 0.002 | .2379942             | 1.083648  |
| _cons     | -.1879149      | .0937644  | -2.00  | 0.045 | -.3716896            | -.0041401 |
| Uninsure  |                |           |        |       |                      |           |
| nonwhite  | .3779586       | .407589   | 0.93   | 0.354 | -.4209011            | 1.176818  |
| _cons     | -1.941934      | .1782185  | -10.90 | 0.000 | -2.291236            | -1.592632 |

mlogit chose the indemnity outcome as the base outcome and presented coefficients for the outcomes prepaid and uninsured. According to the model, the probability of prepaid for whites (nonwhite = 0) is

$$\Pr(\text{insure} = \text{Prepaid}) = \frac{e^{-.188}}{1 + e^{-.188} + e^{-1.942}} = 0.420$$

Similarly, for nonwhites, the probability of prepaid is

$$\Pr(\text{insure} = \text{Prepaid}) = \frac{e^{-.188+.661}}{1 + e^{-.188+.661} + e^{-1.942+.378}} = 0.570$$

These results agree with the column percentages presented by tabulate because the mlogit model is fully saturated. That is, there are enough terms in the model to fully explain the column percentage in each cell. The model  $\chi^2$  and the tabulate  $\chi^2$  are in almost perfect agreement; both test that the column percentages of insure are the same for both values of nonwhite.



▷ Example 2: Specifying the base outcome

By specifying the `baseoutcome()` option, we can control which outcome of the dependent variable is treated as the base. Left to its own, `mlogit` chose to make outcome 1, indemnity, the base outcome. To make outcome 2, prepaid, the base, we would type

```
. mlogit insure nonwhite, base(2)
Iteration 0: Log likelihood = -556.59502
Iteration 1: Log likelihood = -551.78935
Iteration 2: Log likelihood = -551.78348
Iteration 3: Log likelihood = -551.78348

Multinomial logistic regression Number of obs = 616
LR chi2(2) = 9.62
Prob > chi2 = 0.0081
Pseudo R2 = 0.0086

Log likelihood = -551.78348
```

| insure    | Coefficient    | Std. err. | z     | P> z  | [95% conf. interval] |           |
|-----------|----------------|-----------|-------|-------|----------------------|-----------|
| Indemnity |                |           |       |       |                      |           |
| nonwhite  | -.6608212      | .2157321  | -3.06 | 0.002 | -1.083648            | -.2379942 |
| _cons     | .1879149       | .0937644  | 2.00  | 0.045 | .0041401             | .3716896  |
| Prepaid   | (base outcome) |           |       |       |                      |           |
| Uninsure  |                |           |       |       |                      |           |
| nonwhite  | -.2828627      | .3977302  | -0.71 | 0.477 | -1.0624              | .4966742  |
| _cons     | -1.754019      | .1805145  | -9.72 | 0.000 | -2.107821            | -1.400217 |

The `baseoutcome()` option requires that we specify the numeric value of the outcome, so we could not type `base(Prepaid)`.

Although the coefficients now appear to be different, the summary statistics reported at the top are identical. With this parameterization, the probability of prepaid insurance for whites is

$$\Pr(\text{insure} = \text{Prepaid}) = \frac{1}{1 + e^{.188} + e^{-1.754}} = 0.420$$

This is the same answer we obtained previously.



▷ Example 3: Displaying relative-risk ratios

By specifying `rrr`, which we can do at estimation time or when we redisplay results, we see the model in terms of relative-risk ratios:

```
. mlogit, rrr
Multinomial logistic regression Number of obs = 616
 LR chi2(2) = 9.62
 Prob > chi2 = 0.0081
Log likelihood = -551.78348 Pseudo R2 = 0.0086
```

| insure    | RRR            | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-----------|----------------|-----------|-------|-------|----------------------|----------|
| Indemnity |                |           |       |       |                      |          |
| nonwhite  | .516427        | .1114099  | -3.06 | 0.002 | .3383588             | .7882073 |
| _cons     | 1.206731       | .1131483  | 2.00  | 0.045 | 1.004149             | 1.450183 |
| Prepaid   | (base outcome) |           |       |       |                      |          |
| Uninsure  |                |           |       |       |                      |          |
| nonwhite  | .7536233       | .2997387  | -0.71 | 0.477 | .3456255             | 1.643247 |
| _cons     | .1730769       | .0312429  | -9.72 | 0.000 | .1215024             | .2465434 |

Note: `_cons` estimates baseline relative risk for each outcome.

Looked at this way, the relative risk of choosing an indemnity over a prepaid plan is 0.516 for nonwhites relative to whites.

To illustrate, from the output and discussions of examples 1 and 2 we find that

$$\Pr(\text{insure} = \text{Indemnity} \mid \text{white}) = \frac{1}{1 + e^{-.188} + e^{-1.942}} = 0.507$$

and thus the relative risk of choosing indemnity over prepaid (for whites) is

$$\frac{\Pr(\text{insure} = \text{Indemnity} \mid \text{white})}{\Pr(\text{insure} = \text{Prepaid} \mid \text{white})} = \frac{0.507}{0.420} = 1.207$$

For nonwhites,

$$\Pr(\text{insure} = \text{Indemnity} \mid \text{not white}) = \frac{1}{1 + e^{-.188+.661} + e^{-1.942+.378}} = 0.355$$

and thus the relative risk of choosing indemnity over prepaid (for nonwhites) is

$$\frac{\Pr(\text{insure} = \text{Indemnity} \mid \text{not white})}{\Pr(\text{insure} = \text{Prepaid} \mid \text{not white})} = \frac{0.355}{0.570} = 0.623$$

The ratio of these two relative risks, hence the name “relative-risk ratio”, is  $0.623/1.207 = 0.516$ , as given in the output under the heading “RRR”.

□ Technical note

In models where only two categories are considered, the mlogit model reduces to standard logit. Consequently, the exponentiated regression coefficients, labeled as RRR within mlogit, are equal to the odds ratios as given when the or option is specified under logit; see [R] logit.

As such, always referring to mlogit’s exponentiated coefficients as odds ratios may be tempting. However, the discussion in example 3 demonstrates that doing so would be incorrect. In general mlogit models, the exponentiated coefficients are ratios of relative risks, not ratios of odds.



▷ Example 4: Model with continuous and multiple categorical variables

One of the advantages of mlogit over tabulate is that we can include continuous variables and multiple categorical variables in the model. In examining the data on insurance choice, we decide that we want to control for age, gender, and site of study (the study was conducted in three sites):

```
. mlogit insure age male nonwhite i.site
Iteration 0: Log likelihood = -555.85446
Iteration 1: Log likelihood = -534.67443
Iteration 2: Log likelihood = -534.36284
Iteration 3: Log likelihood = -534.36165
Iteration 4: Log likelihood = -534.36165

Multinomial logistic regression
Log likelihood = -534.36165
Number of obs = 615
LR chi2(10) = 42.99
Prob > chi2 = 0.000
Pseudo R2 = 0.0387
```

| insure                   | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|--------------------------|-------------|-----------|-------|-------|----------------------|-----------|
| Indemnity (base outcome) |             |           |       |       |                      |           |
| Prepaid                  |             |           |       |       |                      |           |
| age                      | -.011745    | .0061946  | -1.90 | 0.058 | -.0238862            | .0003962  |
| male                     | .5616934    | .2027465  | 2.77  | 0.006 | .1643175             | .9590693  |
| nonwhite                 | .9747768    | .2363213  | 4.12  | 0.000 | .5115955             | 1.437958  |
| site                     |             |           |       |       |                      |           |
| 2                        | .1130359    | .2101903  | 0.54  | 0.591 | -.2989296            | .5250013  |
| 3                        | -.5879879   | .2279351  | -2.58 | 0.010 | -1.034733            | -.1412433 |
| _cons                    | .2697127    | .3284422  | 0.82  | 0.412 | -.3740222            | .9134476  |
| Uninsure                 |             |           |       |       |                      |           |
| age                      | -.0077961   | .0114418  | -0.68 | 0.496 | -.0302217            | .0146294  |
| male                     | .4518496    | .3674867  | 1.23  | 0.219 | -.268411             | 1.17211   |
| nonwhite                 | .2170589    | .4256361  | 0.51  | 0.610 | -.6171725            | 1.05129   |
| site                     |             |           |       |       |                      |           |
| 2                        | -1.211563   | .4705127  | -2.57 | 0.010 | -2.133751            | -.2893747 |
| 3                        | -.2078123   | .3662926  | -0.57 | 0.570 | -.9257327            | .510108   |
| _cons                    | -1.286943   | .5923219  | -2.17 | 0.030 | -2.447872            | -.1260134 |

These results suggest that the inclination of nonwhites to choose prepaid care is even stronger than it was without controlling. We also see that subjects in site 2 are less likely to be uninsured.



## Fitting constrained models

mlogit can fit models with subsets of coefficients constrained to be zero, with subsets of coefficients constrained to be equal both within and across equations, and with subsets of coefficients arbitrarily constrained to equal linear combinations of other estimated coefficients.

Before fitting a constrained model, you define the constraints with the `constraint` command; see [R] [constraint](#). Once the constraints are defined, you estimate using `mlogit`, specifying the `constraint()` option. Typing `constraint(4)` would use the constraint you previously saved as 4. Typing `constraint(1,4,6)` would use the previously stored constraints 1, 4, and 6. Typing `constraint(1-4,6)` would use the previously stored constraints 1, 2, 3, 4, and 6.

Sometimes, you will not be able to specify the constraints without knowing the omitted outcome. In such cases, assume that the omitted outcome is whatever outcome is convenient for you, and include the `baseoutcome()` option when you specify the `mlogit` command.

### ▷ Example 5: Specifying constraints to test hypotheses

We can use constraints to test hypotheses, among other things. In our insurance-choice model, let's test the hypothesis that there is no distinction between having indemnity insurance and being uninsured. Indemnity-style insurance was the omitted outcome, so we type

```
. test [Uninsure]
(1) [Uninsure]age = 0
(2) [Uninsure]male = 0
(3) [Uninsure]nonwhite = 0
(4) [Uninsure]1b.site = 0
(5) [Uninsure]2.site = 0
(6) [Uninsure]3.site = 0
Constraint 4 dropped
 chi2(5) = 9.31
 Prob > chi2 = 0.0973
```

If indemnity had not been the omitted outcome, we would have typed `test [Uninsure=Indemnity]`.

The results produced by `test` are an approximation based on the estimated covariance matrix of the coefficients. Because the probability of being uninsured is low, the log likelihood may be nonlinear for the uninsured. Conventional statistical wisdom is not to trust the asymptotic answer under these circumstances but to perform a likelihood-ratio test instead.

To use Stata's `lrtest` (likelihood-ratio test) command, we must fit both the unconstrained and constrained models. The unconstrained model is the one we have previously fit. Following the instruction in [R] [lrtest](#), we first store the unconstrained model results:

```
. estimates store unconstrained
```

To fit the constrained model, we must refit our model with all the coefficients except the constant set to 0 in the Uninsure equation. We define the constraint and then refit:

```
. constraint 1 [Uninsure]
. mlogit insure age male nonwhite i.site, constraints(1)
Iteration 0: Log likelihood = -555.85446
Iteration 1: Log likelihood = -539.80523
Iteration 2: Log likelihood = -539.75644
Iteration 3: Log likelihood = -539.75643

Multinomial logistic regression
Log likelihood = -539.75643
Number of obs = 615
Wald chi2(5) = 29.70
Prob > chi2 = 0.0000

(1) [Uninsure]o.age = 0
(2) [Uninsure]o.male = 0
(3) [Uninsure]o.nonwhite = 0
(4) [Uninsure]2o.site = 0
(5) [Uninsure]3o.site = 0
```

| insure         | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |           |
|----------------|-------------|-----------|--------|-------|----------------------|-----------|
| Indemnity      |             |           |        |       |                      |           |
| (base outcome) |             |           |        |       |                      |           |
| Prepaid        |             |           |        |       |                      |           |
| age            | -.0107025   | .0060039  | -1.78  | 0.075 | -.0224699            | .0010649  |
| male           | .4963616    | .1939683  | 2.56   | 0.010 | .1161907             | .8765324  |
| nonwhite       | .9421369    | .2252094  | 4.18   | 0.000 | .5007346             | 1.383539  |
| site           |             |           |        |       |                      |           |
| 2              | .2530912    | .2029465  | 1.25   | 0.212 | -.1446767            | .6508591  |
| 3              | -.5521773   | .2187237  | -2.52  | 0.012 | -.9808678            | -.1234869 |
| _cons          | .1792752    | .3171372  | 0.57   | 0.572 | -.4423023            | .8008527  |
| Uninsure       |             |           |        |       |                      |           |
| age            | 0 (omitted) |           |        |       |                      |           |
| male           | 0 (omitted) |           |        |       |                      |           |
| nonwhite       | 0 (omitted) |           |        |       |                      |           |
| site           |             |           |        |       |                      |           |
| 2              | 0 (omitted) |           |        |       |                      |           |
| 3              | 0 (omitted) |           |        |       |                      |           |
| _cons          | -1.87351    | .1601099  | -11.70 | 0.000 | -2.18732             | -1.5597   |

We can now perform the likelihood-ratio test:

```
. lrtest unconstrained .
Likelihood-ratio test
Assumption: . nested within unconstrained
LR chi2(5) = 10.79
Prob > chi2 = 0.0557
```

The likelihood-ratio  $\chi^2$  is 10.79 with 5 degrees of freedom—just slightly greater than the magic  $p = 0.05$  level—so we should not call this difference significant.



## □ Technical note

In certain circumstances, you should fit an MNL model with conditional logit; see [R] [clogit](#). With substantial data manipulation, `clogit` can handle the same class of models with some interesting additions. For example, if we had available the price and deductible of the most competitive insurance plan of each type, `mlogit` could not use this information, but `clogit` could.

□

## Stored results

`mlogit` stores the following in `e()`:

### Scalars

|                            |                                                                    |
|----------------------------|--------------------------------------------------------------------|
| <code>e(N)</code>          | number of observations                                             |
| <code>e(N_cd)</code>       | number of completely determined observations                       |
| <code>e(k_out)</code>      | number of outcomes                                                 |
| <code>e(k)</code>          | number of parameters                                               |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>                           |
| <code>e(k_eq_model)</code> | number of equations in overall model test                          |
| <code>e(k_dv)</code>       | number of dependent variables                                      |
| <code>e(df_m)</code>       | model degrees of freedom                                           |
| <code>e(r2_p)</code>       | pseudo- $R^2$                                                      |
| <code>e(ll)</code>         | log likelihood                                                     |
| <code>e(ll_0)</code>       | log likelihood, constant-only model                                |
| <code>e(N_clust)</code>    | number of clusters                                                 |
| <code>e(chi2)</code>       | $\chi^2$                                                           |
| <code>e(p)</code>          | $p$ -value for model test                                          |
| <code>e(k_eq_base)</code>  | equation number of the base outcome                                |
| <code>e(baseout)</code>    | the value of <code>depvar</code> to be treated as the base outcome |
| <code>e(ibaseout)</code>   | index of the base outcome                                          |
| <code>e(rank)</code>       | rank of <code>e(V)</code>                                          |
| <code>e(ic)</code>         | number of iterations                                               |
| <code>e(rc)</code>         | return code                                                        |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise                                        |

### Macros

|                                |                                                                          |
|--------------------------------|--------------------------------------------------------------------------|
| <code>e(cmd)</code>            | <code>mlogit</code>                                                      |
| <code>e(cmdline)</code>        | command as typed                                                         |
| <code>e(depvar)</code>         | name of dependent variable                                               |
| <code>e(wtype)</code>          | weight type                                                              |
| <code>e(wexp)</code>           | weight expression                                                        |
| <code>e(title)</code>          | title in estimation output                                               |
| <code>e(clustvar)</code>       | name of cluster variable                                                 |
| <code>e(chi2type)</code>       | Wald or LR; type of model $\chi^2$ test                                  |
| <code>e(vce)</code>            | <code>vcetype</code> specified in <code>vce()</code>                     |
| <code>e(vcetype)</code>        | title used to label Std. err.                                            |
| <code>e(eqnames)</code>        | names of equations                                                       |
| <code>e(baselab)</code>        | value label corresponding to base outcome                                |
| <code>e(opt)</code>            | type of optimization                                                     |
| <code>e(which)</code>          | max or min; whether optimizer is to perform maximization or minimization |
| <code>e(ml_method)</code>      | type of ml method                                                        |
| <code>e(user)</code>           | name of likelihood-evaluator program                                     |
| <code>e(technique)</code>      | maximization technique                                                   |
| <code>e(properties)</code>     | <code>b V</code>                                                         |
| <code>e(predict)</code>        | program used to implement <code>predict</code>                           |
| <code>e(marginsnotok)</code>   | predictions disallowed by <code>margins</code>                           |
| <code>e(marginsdefault)</code> | default <code>predict()</code> specification for <code>margins</code>    |
| <code>e(asbalanced)</code>     | factor variables <code>fvset</code> as <code>asbalanced</code>           |
| <code>e(asobserved)</code>     | factor variables <code>fvset</code> as <code>asobserved</code>           |

|                 |                                              |
|-----------------|----------------------------------------------|
| Matrices        |                                              |
| e(b)            | coefficient vector                           |
| e(out)          | outcome values                               |
| e(Cns)          | constraints matrix                           |
| e(iilog)        | iteration log (up to 20 iterations)          |
| e(gradient)     | gradient vector                              |
| e(V)            | variance–covariance matrix of the estimators |
| e(V_modelbased) | model-based variance                         |
| Functions       |                                              |
| e(sample)       | marks estimation sample                      |

In addition to the above, the following is stored in `r()`:

|          |                                                                                                                            |
|----------|----------------------------------------------------------------------------------------------------------------------------|
| Matrices |                                                                                                                            |
| r(table) | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

## Methods and formulas

The MNL model is described in [Greene \(2018, 829–833\)](#).

Suppose that there are *k* categorical outcomes and—without loss of generality—let the base outcome be 1. The probability that the response for the *j*th observation is equal to the *i*th outcome is

$$p_{ij} = \Pr(y_j = i) = \begin{cases} \frac{1}{1 + \sum_{m=2}^k \exp(\mathbf{x}_j \boldsymbol{\beta}_m)}, & \text{if } i = 1 \\ \frac{\exp(\mathbf{x}_j \boldsymbol{\beta}_i)}{1 + \sum_{m=2}^k \exp(\mathbf{x}_j \boldsymbol{\beta}_m)}, & \text{if } i > 1 \end{cases}$$

where  $\mathbf{x}_j$  is the row vector of observed values of the independent variables for the *j*th observation and  $\boldsymbol{\beta}_m$  is the coefficient vector for outcome *m*. The log pseudolikelihood is

$$\ln L = \sum_j w_j \sum_{i=1}^k I_i(y_j) \ln p_{ik}$$

where  $w_j$  is an optional weight and

$$I_i(y_j) = \begin{cases} 1, & \text{if } y_j = i \\ 0, & \text{otherwise} \end{cases}$$

Newton–Raphson maximum likelihood is used; see [\[R\] Maximize](#).

For constrained equations, the set of constraints is orthogonalized, and a subset of maximizable parameters is selected. For example, a parameter that is constrained to zero is not a maximizable parameter. If two parameters are constrained to be equal to each other, only one is a maximizable parameter.

Let  $\mathbf{r}$  be the vector of maximizable parameters.  $\mathbf{r}$  is physically a subset of the solution parameters,  $\mathbf{b}$ . A matrix,  $\mathbf{T}$ , and a vector,  $\mathbf{m}$ , are defined as

$$\mathbf{b} = \mathbf{T}\mathbf{r} + \mathbf{m}$$

so that

$$\frac{\partial f}{\partial \mathbf{b}} = \frac{\partial f}{\partial \mathbf{r}} \mathbf{T}'$$

$$\frac{\partial^2 f}{\partial \mathbf{b}^2} = \mathbf{T} \frac{\partial^2 f}{\partial \mathbf{r}^2} \mathbf{T}'$$

$\mathbf{T}$  consists of a block form in which one part is a permutation of the identity matrix and the other part describes how to calculate the constrained parameters from the maximizable parameters.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] [\\_robust](#), particularly *Maximum likelihood estimators* and *Methods and formulas*.

`mlogit` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] [Variance estimation](#).

## References

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## Also see

- [R] **mlogit postestimation** — Postestimation tools for mlogit
- [R] **clgit** — Conditional (fixed-effects) logistic regression
- [R] **logistic** — Logistic regression, reporting odds ratios
- [R] **logit** — Logistic regression, reporting coefficients
- [R] **mprobit** — Multinomial probit regression
- [R] **ologit** — Ordered logistic regression
- [R] **slogit** — Stereotype logistic regression
- [BAYES] **bayes: mlogit** — Bayesian multinomial logistic regression
- [CM] **cmrologit** — Rank-ordered logit choice model
- [CM] **nlogit** — Nested logit regression
- [FMM] **fmm: mlogit** — Finite mixtures of multinomial (polytomous) logistic regression models
- [MI] **Estimation** — Estimation commands for use with mi estimate
- [SVY] **svy estimation** — Estimation commands for survey data
- [XT] **xtmlogit** — Fixed-effects and random-effects multinomial logit models
- [U] **20 Estimation and postestimation commands**

Postestimation commands    [predict](#)    [margins](#)    [Remarks and examples](#)  
 Reference    [Also see](#)

## Postestimation commands

The following postestimation commands are available after `mlogit`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | probabilities, linear predictions and their SEs, etc.                                                                                   |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\* `forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results. `forecast` is also not appropriate with `mi` estimation results.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as probabilities, linear predictions, and standard errors.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] { stub* | newvar | newvarlist } [if] [in] [, statistic outcome(outcome)]
```

```
predict [type] stub* [if] [in], scores
```

| <i>statistic</i>   | Description                                                |
|--------------------|------------------------------------------------------------|
| Main               |                                                            |
| <code>pr</code>    | predicted probabilities; the default                       |
| <code>xb</code>    | linear prediction                                          |
| <code>stdp</code>  | standard error of the linear prediction                    |
| <code>stddp</code> | standard error of the difference in two linear predictions |

You specify one or  $k$  new variables with `pr`, where  $k$  is the number of outcomes. If you specify one new variable and you do not specify `outcome()`, then `outcome(#1)` is assumed.

You specify one new variable with `xb`, `stdp`, and `stddp`. If you do not specify `outcome()`, then `outcome(#1)` is assumed. You must specify `outcome()` with the `stddp` option.

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

## Options for predict

Main

`pr`, the default, computes the predicted probabilities for all outcomes or for a specific outcome. To compute probabilities for all outcomes, you specify  $k$  new variables, where  $k$  is the number of categories of the dependent variable. Alternatively, you can specify `stub*`; in which case, `pr` will store predicted probabilities in variables `stub1`, `stub2`, ..., `stubk`. To compute the probability for a specific outcome, you specify one new variable and, optionally, the outcome value in option `outcome()`; if you omit `outcome()`, the first outcome value, `outcome(#1)`, is assumed.

Say that you fit a model by typing `estimation_cmd y x1 x2`, and `y` takes on four values. Then, you could type `predict p1 p2 p3 p4` to obtain all four predicted probabilities; alternatively, you could type `predict p*` to generate the four predicted probabilities. To compute specific probabilities one at a time, you can type `predict p1, outcome(#1)` (or simply `predict p1`), `predict p2, outcome(#2)`, and so on. See option `outcome()` for other ways to refer to outcome values.

`xb` calculates the linear prediction. You must also specify the `outcome(outcome)` option.

`stdp` calculates the standard error of the linear prediction. You must also specify the outcome (*outcome*) option.

`stddp` calculates the standard error of the difference in two linear predictions. You must specify the outcome (*outcome*) option, and here you specify the two particular outcomes of interest inside the parentheses, for example, `predict sed, stddp outcome(1,3)`.

`outcome(outcome)` specifies for which outcome the predicted probabilities are to be calculated. `outcome()` should contain either one value of the dependent variable or one of #1, #2, ..., with #1 meaning the first category of the dependent variable, #2 meaning the second category, etc. `outcome()` is not allowed with `scores`.

`scores` calculates equation-level score variables. The number of score variables created will be one less than the number of outcomes in the model. If the number of outcomes in the model were  $k$ , then

the first new variable will contain  $\partial \ln L / \partial (\mathbf{x}_j \beta_1)$ ;

the second new variable will contain  $\partial \ln L / \partial (\mathbf{x}_j \beta_2)$ ;

...

the  $(k - 1)$ th new variable will contain  $\partial \ln L / \partial (\mathbf{x}_j \beta_{k-1})$ .

## margins

### Description for margins

`margins` estimates margins of response for probabilities and linear predictions.

### Menu for margins

Statistics > Postestimation

### Syntax for margins

`margins` [*marginlist*] [, *options*]

`margins` [*marginlist*] , predict(*statistic* ...) [predict(*statistic* ...) ...] [*options*]

| <i>statistic</i> | Description                               |
|------------------|-------------------------------------------|
| default          | probabilities for each outcome            |
| pr               | probability for a specified outcome       |
| xb               | linear prediction for a specified outcome |
| stdp             | not allowed with margins                  |
| stddp            | not allowed with margins                  |

`pr` and `xb` default to the first outcome.

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [R] [margins](#).

## Remarks and examples

Remarks are presented under the following headings:

*Obtaining predicted values*  
*Calculating marginal effects*  
*Testing hypotheses about coefficients*

### Obtaining predicted values

#### ▷ Example 1: Obtaining predicted probabilities

After estimation, we can use `predict` to obtain predicted probabilities, index values, and standard errors of the index, or differences in the index. For instance, in [example 4 of \[R\] mlogit](#), we fit a model of insurance choice on various characteristics. We can obtain the predicted probabilities for outcome 1 by typing

```
. use https://www.stata-press.com/data/r19/sysdsn1
(Health insurance data)
. mlogit insure age i.male i.nonwhite i.site
(output omitted)
. predict p1 if e(sample), outcome(1)
(option pr assumed; predicted probability)
(29 missing values generated)
. summarize p1
```

| Variable | Obs | Mean     | Std. dev. | Min      | Max    |
|----------|-----|----------|-----------|----------|--------|
| p1       | 615 | .4764228 | .1032279  | .1698142 | .71939 |

We added the `i.` prefix to the `male`, `nonwhite`, and `site` variables to explicitly identify them as factor variables. That makes no difference in the estimated results, but we will take advantage of it in later examples. We also included `if e(sample)` to restrict the calculation to the estimation sample. In [example 4 of \[R\] mlogit](#), the multinomial logit model was fit on 615 observations, so there must be missing values in our dataset.

Although we typed `outcome(1)`, specifying 1 for the indemnity outcome, we could have typed `outcome(Indemnity)`. For instance, to obtain the probabilities for prepaid, we could type

```
. predict p2 if e(sample), outcome(Prepaid)
(option pr assumed; predicted probability)
(29 missing values generated)
. summarize p2
```

| Variable | Obs | Mean     | Std. dev. | Min      | Max      |
|----------|-----|----------|-----------|----------|----------|
| p2       | 615 | .4504065 | .1125962  | .1964103 | .7885724 |

We must specify the label exactly as it appears in the underlying value label (or how it appears in the `mlogit` output), including capitalization.

Here we have used `predict` to obtain probabilities for the same sample on which we estimated. That is not necessary. We could use another dataset that had the independent variables defined (in our example, `age`, `male`, `nonwhite`, and `site`) and use `predict` to obtain predicted probabilities; here, we would not specify `if e(sample)`.

► Example 2: Obtaining index values

predict can also be used to obtain the index values—the  $\sum x_i \hat{\beta}_i^{(k)}$ —as well as the probabilities:

```
. predict idx1, outcome(Indemnity) xb
(1 missing value generated)
. summarize idx1
```

| Variable | Obs | Mean | Std. dev. | Min | Max |
|----------|-----|------|-----------|-----|-----|
| idx1     | 643 | 0    | 0         | 0   | 0   |

The indemnity outcome was our base outcome—the outcome for which all the coefficients were set to 0—so the index is always 0. For the prepaid and uninsured outcomes, we type

```
. predict idx2, outcome(Prepaid) xb
(1 missing value generated)
. predict idx3, outcome(Uninsure) xb
(1 missing value generated)
. summarize idx2 idx3
```

| Variable | Obs | Mean      | Std. dev. | Min       | Max       |
|----------|-----|-----------|-----------|-----------|-----------|
| idx2     | 643 | -.0566113 | .4962973  | -1.298198 | 1.700719  |
| idx3     | 643 | -1.980747 | .6018139  | -3.112741 | -.8258458 |

We can obtain the standard error of the index by specifying the stdp option:

```
. predict se2, outcome(Prepaid) stdp
(1 missing value generated)
. list p2 idx2 se2 in 1/5
```

|    | p2       | idx2      | se2      |
|----|----------|-----------|----------|
| 1. | .3709022 | -.4831167 | .2437772 |
| 2. | .4977667 | .055111   | .1694686 |
| 3. | .4113073 | -.1712106 | .1793498 |
| 4. | .5424927 | .3788345  | .2513701 |
| 5. | .        | -.0925817 | .1452616 |

We obtained the probability, p2, in the [previous example](#).

Finally, `predict` can calculate the standard error of the difference in the index values between two outcomes with the `stdp` option:

```
. predict se_2_3, outcome(Prepaid,Uninsure) stdp
(1 missing value generated)
. list idx2 idx3 se_2_3 in 1/5
```

|    | idx2      | idx3      | se_2_3   |
|----|-----------|-----------|----------|
| 1. | -.4831167 | -3.073253 | .5469354 |
| 2. | .055111   | -2.715986 | .4331918 |
| 3. | -.1712106 | -1.579621 | .3053815 |
| 4. | .3788345  | -1.462007 | .4492552 |
| 5. | -.0925817 | -2.814022 | .4024784 |

In the first observation, the difference in the indexes is  $-0.483 - (-3.073) = 2.59$ . The standard error of that difference is 0.547.



### ▷ Example 3: Interpreting results using predictive margins

It is more difficult to interpret the results from `mlogit` than those from `clogit` or `logit` because there are multiple equations. For example, suppose that one of the independent variables in our model takes on the values 0 and 1, and we are attempting to understand the effect of this variable. Assume that the coefficient on this variable for the second outcome,  $\beta^{(2)}$ , is positive. We might then be tempted to reason that the probability of the second outcome is higher if the variable is 1 rather than 0. Most of the time, that will be true, but occasionally we will be surprised. The probability of some other outcome could increase even more (say,  $\beta^{(3)} > \beta^{(2)}$ ), and thus the probability of outcome 2 would actually fall relative to that outcome. We can use `predict` to help interpret such results.

Continuing with our previously fit insurance-choice model, we wish to describe the model's predictions by race. For this purpose, we can use the method of predictive margins (also known as recycled predictions), in which we vary characteristics of interest across the whole dataset and average the predictions. That is, we have data on both whites and nonwhites, and our individuals have other characteristics as well. We will first pretend that all the people in our data are white but hold their other characteristics constant. We then calculate the probabilities of each outcome. Next we will pretend that all the people in our data are nonwhite, still holding their other characteristics constant. Again, we calculate the probabilities of each outcome. The difference in those two sets of calculated probabilities, then, is the difference due to race, holding other characteristics constant.

```
. gen byte nonwhold=nonwhite // save real race
. replace nonwhite=0 // make everyone white
(126 real changes made)
. predict wpiid, outcome(Indemnity) // predict probabilities
(option pr assumed; predicted probability)
(1 missing value generated)
. predict wpp, outcome(Prepaid)
(option pr assumed; predicted probability)
(1 missing value generated)
. predict wpoi, outcome(Uninsure)
(option pr assumed; predicted probability)
(1 missing value generated)
. replace nonwhite=1 // make everyone nonwhite
(644 real changes made)
```

```
. predict nwpind, outcome(Indemnity)
(option pr assumed; predicted probability)
(1 missing value generated)
. predict nwpp, outcome(Prepaid)
(option pr assumed; predicted probability)
(1 missing value generated)
. predict nwpnoi, outcome(Uninsure)
(option pr assumed; predicted probability)
(1 missing value generated)
. replace nonwhite=nonwhold // restore real race
(518 real changes made)
. summarize wp* nwp*, sep(3)
```

| Variable | Obs | Mean     | Std. dev. | Min      | Max      |
|----------|-----|----------|-----------|----------|----------|
| wpind    | 643 | .5141673 | .0872679  | .3092903 | .71939   |
| wpp      | 643 | .4082052 | .0993286  | .1964103 | .6502247 |
| wpnoi    | 643 | .0776275 | .0360283  | .0273596 | .1302816 |
| nwpind   | 643 | .3112809 | .0817693  | .1511329 | .535021  |
| nwpp     | 643 | .630078  | .0979976  | .3871782 | .8278881 |
| nwpnoi   | 643 | .0586411 | .0287185  | .0209648 | .0933874 |

In example 1 of [R] [mlogit](#), we presented a cross-tabulation of insurance type and race. Those values were unadjusted. The means reported above are the values adjusted for age, sex, and site. Combining the results gives

|           | Unadjusted |          | Adjusted |          |
|-----------|------------|----------|----------|----------|
|           | white      | nonwhite | white    | nonwhite |
| Indemnity | 0.51       | 0.36     | 0.51     | 0.31     |
| Prepaid   | 0.42       | 0.57     | 0.41     | 0.63     |
| Uninsured | 0.07       | 0.07     | 0.08     | 0.06     |

We find, for instance, after adjusting for age, sex, and site, that although 57% of nonwhites in our data had prepaid plans, 63% of nonwhites chose prepaid plans.

Computing predictive margins by hand was instructive, but we can compute these values more easily using the margins command (see [R] [margins](#)). The two margins for the indemnity outcome can be estimated by typing

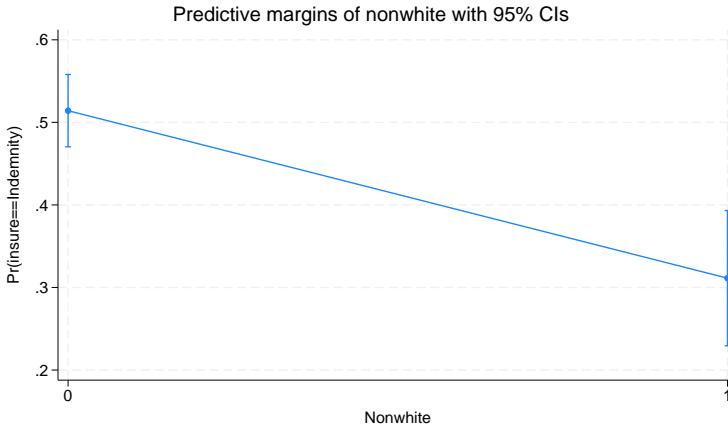
```
. margins nonwhite, predict(outcome(Indemnity)) noesample
Predictive margins Number of obs = 643
Model VCE: OIM
Expression: Pr(insure==Indemnity), predict(outcome(Indemnity))
```

|          | Delta-method |           |       |       | [95% conf. interval] |
|----------|--------------|-----------|-------|-------|----------------------|
|          | Margin       | std. err. | z     | P> z  |                      |
| nonwhite |              |           |       |       |                      |
| 0        | .5141673     | .0223485  | 23.01 | 0.000 | .470365 .5579695     |
| 1        | .3112809     | .0418049  | 7.45  | 0.000 | .2293448 .393217     |

margins also estimates the standard errors and confidence intervals of the margins. By default, margins uses only the estimation sample. We added the noesample option so that margins would use the entire sample and produce results comparable with our earlier analysis.

We can use `marginsplot` to graph the results from `margins`:

```
. marginsplot
Variables that uniquely identify margins: nonwhite
```



The margins for the other two outcomes can be computed by typing

```
. margins nonwhite, predict(outcome(Prepaid)) noesample
(output omitted)
. margins nonwhite, predict(outcome(Uninsure)) noesample
(output omitted)
```

The margins for each outcome is computed when no outcome is specified. For example,

```
. margins nonwhite, noesample
(output omitted)
```

◀

## □ Technical note

You can use `predict` to classify predicted values and compare them with the observed outcomes to interpret a multinomial logit model. This is a variation on the notions of sensitivity and specificity for logistic regression. Here we will classify indemnity and prepaid as definitely predicting indemnity, definitely predicting prepaid, and ambiguous.

```
. predict indem, outcome(Indemnity) index // obtain indexes
(1 missing value generated)
. predict prepaid, outcome(Prepaid) index
(1 missing value generated)
. gen diff = prepaid-indem // obtain difference
(1 missing value generated)
. predict sediff, outcome(Indemnity,Prepaid) stddp // & its standard error
(1 missing value generated)
. gen type = 1 if diff/sediff < -1.96 // definitely indemnity
(504 missing values generated)
. replace type = 3 if diff/sediff > 1.96 // definitely prepaid
(100 real changes made)
```

```
. replace type = 2 if type>=. & diff/sediff < . // ambiguous
(404 real changes made)
. label def type 1 "Def Ind" 2 "Ambiguous" 3 "Def Prep"
. label values type type // label results
. tabulate insure type
```

| Type of insurance | type    |           |          | Total |
|-------------------|---------|-----------|----------|-------|
|                   | Def Ind | Ambiguous | Def Prep |       |
| Indemnity         | 78      | 183       | 33       | 294   |
| Prepaid           | 44      | 177       | 56       | 277   |
| Uninsure          | 12      | 28        | 5        | 45    |
| Total             | 134     | 388       | 94       | 616   |

We can see that the predictive power of this model is modest. There are many misclassifications in both directions, though there are more correctly classified observations than misclassified observations.

Also, the uninsured look overwhelmingly as though they might have come from the indemnity system rather than from the prepaid system.



## Calculating marginal effects

### ▷ Example 4

We have already noted that the coefficients from multinomial logit can be difficult to interpret because they are relative to the base outcome. Another way to evaluate the effect of covariates is to examine the marginal effect of changing their values on the probability of observing an outcome.

The margins command can be used for this too. We can estimate the marginal effect of each covariate on the probability of observing the first outcome—indemnity insurance—by typing

```
. margins, dydx(*) predict(outcome(Indemnity))
Average marginal effects Number of obs = 615
Model VCE: OIM
Expression: Pr(insure==Indemnity), predict(outcome(Indemnity))
dy/dx wrt: age 1.male 1.nonwhite 2.site 3.site
```

|            | Delta-method |           |       |       | [95% conf. interval] |           |
|------------|--------------|-----------|-------|-------|----------------------|-----------|
|            | dy/dx        | std. err. | z     | P> z  |                      |           |
| age        | .0026655     | .001399   | 1.91  | 0.057 | -.0000765            | .0054074  |
| 1.male     | -.1295734    | .0450945  | -2.87 | 0.004 | -.2179571            | -.0411898 |
| 1.nonwhite | -.2032404    | .0482554  | -4.21 | 0.000 | -.2978192            | -.1086616 |
| site       |              |           |       |       |                      |           |
| 2          | .0070995     | .0479993  | 0.15  | 0.882 | -.0869775            | .1011765  |
| 3          | .1216165     | .0505833  | 2.40  | 0.016 | .022475              | .220758   |

Note: dy/dx for factor levels is the discrete change from the base level.

By default, `margins` estimates the average marginal effect over the estimation sample, and that is what we see above. Being male decreases the average probability of having indemnity insurance by 0.130. We also see, from the note at the bottom of the table, that the marginal effect was computed as a discrete change in the probability of being male rather than female. That is why we made `male` a factor variable when fitting the model.

The `dydx(*)` option requested that `margins` estimate the marginal effect for each regressor, `dydx(age)` would have produced estimates only for the effect of `age`. `margins` has many options for controlling how the marginal effect is computed, including the ability to average over subgroups or to compute estimates for specified values of the regressors; see [\[R\] margins](#).

`margins` will compute the marginal effects on each outcome when no outcome is specified.

```
. margins, dydx(*)
 (output omitted)
```

◀

## Testing hypotheses about coefficients

### ▷ Example 5

`test` tests hypotheses about the coefficients just as after any estimation command; see [\[R\] test](#). Note, however, `test`'s syntax for dealing with multiple-equation models. Because `test` bases its results on the estimated covariance matrix, we might prefer a likelihood-ratio test; see [example 5](#) in [\[R\] mlogit](#) for an example of `lrtest`.

If we simply list variables after the `test` command, we are testing that the corresponding coefficients are zero across all equations:

```
. test 2.site 3.site
(1) [Indemnity]2o.site = 0
(2) [Prepaid]2.site = 0
(3) [Uninsure]2.site = 0
(4) [Indemnity]3o.site = 0
(5) [Prepaid]3.site = 0
(6) [Uninsure]3.site = 0
 Constraint 1 dropped
 Constraint 4 dropped
 chi2(4) = 19.74
 Prob > chi2 = 0.0006
```

We can test that all the coefficients (except the constant) in an equation are zero by simply typing the outcome in square brackets:

```
. test [Uninsure]
(1) [Uninsure]age = 0
(2) [Uninsure]0b.male = 0
(3) [Uninsure]1.male = 0
(4) [Uninsure]0b.nonwhite = 0
(5) [Uninsure]1.nonwhite = 0
(6) [Uninsure]1b.site = 0
(7) [Uninsure]2.site = 0
(8) [Uninsure]3.site = 0
Constraint 2 dropped
Constraint 4 dropped
Constraint 6 dropped
 chi2(5) = 9.31
Prob > chi2 = 0.0973
```

We specify the outcome just as we do with `predict`; we can specify the label if the outcome variable is labeled, or we can specify the numeric value of the outcome. We would have obtained the same test as above if we had typed `test [3]` because 3 is the value of `insure` for the outcome `uninsured`.

We can combine the two syntaxes. To test that the coefficients on the site variables are 0 in the equation corresponding to the outcome `prepaid`, we can type

```
. test [Prepaid]: 2.site 3.site
(1) [Prepaid]2.site = 0
(2) [Prepaid]3.site = 0
 chi2(2) = 10.78
Prob > chi2 = 0.0046
```

We specified the outcome and then followed that with a colon and the variables we wanted to test.

We can also test that coefficients are equal across equations. To test that all coefficients except the constant are equal for the `prepaid` and `uninsured` outcomes, we can type

```
. test [Prepaid=Uninsure]
(1) [Prepaid]age - [Uninsure]age = 0
(2) [Prepaid]0b.male - [Uninsure]0b.male = 0
(3) [Prepaid]1.male - [Uninsure]1.male = 0
(4) [Prepaid]0b.nonwhite - [Uninsure]0b.nonwhite = 0
(5) [Prepaid]1.nonwhite - [Uninsure]1.nonwhite = 0
(6) [Prepaid]1b.site - [Uninsure]1b.site = 0
(7) [Prepaid]2.site - [Uninsure]2.site = 0
(8) [Prepaid]3.site - [Uninsure]3.site = 0
Constraint 2 dropped
Constraint 4 dropped
Constraint 6 dropped
 chi2(5) = 13.80
Prob > chi2 = 0.0169
```

To test that only the site variables are equal, we can type

```
. test [Prepaid=Uninsure]: 2.site 3.site
(1) [Prepaid]2.site - [Uninsure]2.site = 0
(2) [Prepaid]3.site - [Uninsure]3.site = 0
 chi2(2) = 12.68
Prob > chi2 = 0.0018
```

Finally, we can test any arbitrary constraint by simply entering the equation and specifying the coefficients as described in [U] [13.5 Accessing coefficients and standard errors](#). The following hypothesis is senseless but illustrates the point:

```
. test ([Prepaid]age+[Uninsure]2.site)/2 = 2-[Uninsure]1.nonwhite
(1) .5*[Prepaid]age + [Uninsure]1.nonwhite + .5*[Uninsure]2.site = 2
 chi2(1) = 22.45
 Prob > chi2 = 0.0000
```

See [R] [test](#) for more information about test. The [information](#) there about combining hypotheses across test commands (the `accumulate` option) also applies after `mlogit`.



## Reference

Fagerland, M. W., and D. W. Hosmer, Jr. 2012. [A generalized Hosmer–Lemeshow goodness-of-fit test for multinomial logistic regression models](#). *Stata Journal* 12: 447–453.

## Also see

[R] [mlogit](#) — Multinomial (polytomous) logistic regression

[U] [20 Estimation and postestimation commands](#)

## Description

`set more off`, which is the default, tells Stata not to pause or display a —more— message. `set more on` tells Stata to wait until you press a key before continuing when a —more— message is displayed.

`set pagesize #` sets the number of lines between —more— messages.

## Syntax

*Tell Stata to pause or not pause for —more— messages*

```
set more { on | off } [, permanently]
```

*Set number of lines between —more— messages*

```
set pagesize #
```

## Option

`permanently` specifies that, in addition to making the change right now, the `more` setting be remembered and become the default setting when you invoke Stata.

## Remarks and examples

—more— is Stata’s way of telling you that it has something more to show you but that showing it to you will cause the information on the screen to scroll off. If you type `set more off`, the —more— conditions will never arise, and Stata’s output will scroll by at full speed. If you type `set more on`, the —more— conditions will be restored at the appropriate places.

When you see —more— at the bottom of the screen, you can do the following:

| <u>Press ...</u>                | <u>and Stata ...</u>                |
|---------------------------------|-------------------------------------|
| letter <i>l</i> or <i>Enter</i> | displays the next line              |
| letter <i>q</i>                 | acts as if you pressed <i>Break</i> |
| Spacebar or any other key       | displays the next screen            |

You can also click on the **More** button or click on —more— to display the next screen.

If `set more` is used within a do-file or program, Stata automatically restores the previous `set more` setting when the do-file or program concludes.

Programmers should see [P] [more](#) for information on the `more` programming command.

## Also see

[R] **query** — Display system parameters

[P] **creturn** — Return c-class values

[P] **more** — Pause until key is pressed

[P] **sleep** — Pause for a specified time

[U] **7 -more- conditions**

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## Description

`mprobit` fits a multinomial probit (MNP) model for a categorical dependent variable with outcomes that have no natural ordering. The actual values taken by the dependent variable are irrelevant. The error terms are assumed to be independent, standard normal, random variables. `cmmprobit` relaxes the independence of irrelevant alternatives assumption by specifying correlated latent-variable errors. `cmmprobit` also allows heteroskedastic latent-variable errors and alternative-specific independent variables.

## Quick start

Multinomial probit model of  $y$  on  $x_1$ ,  $x_2$ , and categorical  $a$

```
mprobit y x1 x2 i.a
```

Same as above, but use  $a$  as the base outcome  $y = 3$

```
mprobit y x1 x2 i.a, baseoutcome(3)
```

Probit variance parameterization of differenced latent errors

```
mprobit y x1 x2 i.a, probitparam
```

Multiple-imputation estimates with Monte Carlo errors from `mi` set data

```
mi estimate, merror: mprobit y x1 x2 i.a
```

## Menu

Statistics > Categorical outcomes > Multinomial probit regression

## Syntax

```
mprobit depvar [indepvars] [if] [in] [weight] [, options]
```

| <i>options</i>                               | Description                                                                                                                                                                 |
|----------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                 |                                                                                                                                                                             |
| <code>noconstant</code>                      | suppress constant terms                                                                                                                                                     |
| <code>baseoutcome(#   <i>lbl</i>)</code>     | outcome used to normalize location                                                                                                                                          |
| <code>probitparam</code>                     | use the probit variance parameterization                                                                                                                                    |
| <code>constraints(<i>constraints</i>)</code> | apply specified linear constraints                                                                                                                                          |
| <b>SE/Robust</b>                             |                                                                                                                                                                             |
| <code>vce(<i>vcetype</i>)</code>             | <i>vcetype</i> may be <code>oim</code> , <code>robust</code> , <code>cluster <i>clustvar</i></code> , <code>opg</code> , <code>bootstrap</code> , or <code>jackknife</code> |
| <b>Reporting</b>                             |                                                                                                                                                                             |
| <code>level(#)</code>                        | set confidence level; default is <code>level(95)</code>                                                                                                                     |
| <code>nocnsreport</code>                     | do not display constraints                                                                                                                                                  |
| <code>display_options</code>                 | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling                            |
| <b>Integration</b>                           |                                                                                                                                                                             |
| <code>intpoints(#)</code>                    | number of quadrature points                                                                                                                                                 |
| <b>Maximization</b>                          |                                                                                                                                                                             |
| <code>maximize_options</code>                | control the maximization process; seldom used                                                                                                                               |
| <code>collinear</code>                       | keep collinear variables                                                                                                                                                    |
| <code>coeflegend</code>                      | display legend instead of statistics                                                                                                                                        |

*indepvars* may contain factor variables; see [U] 11.4.3 Factor variables.

`bayes`, `bayesboot`, `bootstrap`, `by`, `collect`, `fp`, `jackknife`, `mi estimate`, `rolling`, `statsby`, and `svy` are allowed; see [U] 11.1.10 Prefix commands. For more details, see [BAYES] bayes: mprobit.

`vce(bootstrap)` and `vce(jackknife)` are not allowed with the `mi estimate` prefix; see [MI] mi estimate.

Weights are not allowed with the `bootstrap` prefix; see [R] bootstrap.

`vce()` and weights are not allowed with the `svy` prefix; see [SVY] svy.

`fweights`, `iweights`, and `pweights` are allowed; see [U] 11.1.6 weight.

`collinear` and `coeflegend` do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Options

### Model

`noconstant` suppresses the  $J - 1$  constant terms.

`baseoutcome(# | lbl)` specifies the outcome used to normalize the location of the latent variable. The base outcome may be specified as a number or a label. The default is to use the most frequent outcome. The coefficients associated with the base outcome are zero.

`probitparam` specifies to use the probit variance parameterization by fixing the variance of the differenced latent errors between the scale and the base alternatives to be one. The default is to make the variance of the base and scale latent errors one, thereby making the variance of the difference to be two.

`constraints` (*constraints*); see [R] [Estimation options](#).

#### SE/Robust

`vce` (*vcetype*) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster` *clustvar*), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

If specifying `vce(bootstrap)` or `vce(jackknife)`, you must also specify `baseoutcome()`.

#### Reporting

`level(#)`, `nocnsreport`; see [R] [Estimation options](#).

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

#### Integration

`intpoints(#)` specifies the number of Gaussian quadrature points to use in approximating the likelihood. The default is `intpoints(15)`.

#### Maximization

*maximize\_options*: `difficult`, `technique` (*algorithm\_spec*), `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from` (*init\_specs*); see [R] [Maximize](#). These options are seldom used.

Setting the optimization type to `technique(bhhh)` resets the default *vcetype* to `vce(opg)`.

The following options are available with `mprobit` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

The MNP model is used with discrete dependent variables that take on more than two outcomes that do not have a natural ordering. The stochastic error terms for this implementation of the model are assumed to have independent, standard normal distributions. To use `mprobit`, you must have one observation for each decision maker in the sample. See [CM] [cmmprobit](#) for another implementation of the MNP model that permits correlated and heteroskedastic errors and is suitable when you have data for each alternative that a decision maker faced.

The MNP model is frequently motivated using a latent-variable framework. The latent variable for the  $j$ th alternative,  $j = 1, \dots, J$ , is

$$\eta_{ij} = \mathbf{z}_i \boldsymbol{\alpha}_j + \xi_{ij}$$

where the  $1 \times q$  row vector  $\mathbf{z}_i$  contains the observed independent variables for the  $i$ th decision maker. Associated with  $\mathbf{z}_i$  are the  $J$  vectors of regression coefficients  $\boldsymbol{\alpha}_j$ . The  $\xi_{i,1}, \dots, \xi_{i,J}$  are distributed independently and identically standard normal. The decision maker chooses the alternative  $k$  such that  $\eta_{ik} \geq \eta_{im}$  for  $m \neq k$ .

Suppose that case  $i$  chooses alternative  $k$ , and take the difference between latent variable  $\eta_{ik}$  and the  $J - 1$  others:

$$\begin{aligned} v_{ijk} &= \eta_{ij} - \eta_{ik} \\ &= \mathbf{z}_i (\boldsymbol{\alpha}_j - \boldsymbol{\alpha}_k) + \xi_{ij} - \xi_{ik} \\ &= \mathbf{z}_i \boldsymbol{\gamma}_{j'} + \epsilon_{ij'} \end{aligned} \tag{1}$$

where  $j' = j$  if  $j < k$  and  $j' = j - 1$  if  $j > k$  so that  $j' = 1, \dots, J - 1$ .  $\text{Var}(\epsilon_{ij'}) = \text{Var}(\xi_{ij} - \xi_{ik}) = 2$  and  $\text{Cov}(\epsilon_{ij'}, \epsilon_{il'}) = 1$  for  $j' \neq l'$ . The probability that alternative  $k$  is chosen is

$$\begin{aligned} \Pr(i \text{ chooses } k) &= \Pr(v_{i1k} \leq 0, \dots, v_{i,J-1,k} \leq 0) \\ &= \Pr(\epsilon_{i1} \leq -\mathbf{z}_i \boldsymbol{\gamma}_1, \dots, \epsilon_{i,J-1} \leq -\mathbf{z}_i \boldsymbol{\gamma}_{J-1}) \end{aligned}$$

Hence, evaluating the likelihood function involves computing probabilities from the multivariate normal distribution. That all the covariances are equal simplifies the problem somewhat; see [Methods and formulas](#) for details.

In (1), not all  $J$  of the  $\boldsymbol{\alpha}_j$  are identifiable. To remove the indeterminacy,  $\boldsymbol{\alpha}_l$  is set to the zero vector, where  $l$  is the base outcome as specified in the `baseoutcome()` option. That fixes the  $l$ th latent variable to zero so that the remaining variables measure the attractiveness of the other alternatives relative to the base.

### ► Example 1

As discussed in [example 1](#) of [\[R\] mlogit](#), we have data on the type of health insurance available to 616 psychologically depressed subjects in the United States (Tarlov et al. 1989; Wells et al. 1989). Patients may have either an indemnity (fee-for-service) plan or a prepaid plan such as an HMO, or the patient may be uninsured. Demographic variables include age, gender, race, and site. Indemnity insurance is the most popular alternative, so `mprobit` will choose it as the base outcome by default.

```
. use https://www.stata-press.com/data/r19/sysdsn1
(Health insurance data)
```

```
. mprobit insure age male nonwhite i.site
```

```
Iteration 0: Log likelihood = -535.89424
Iteration 1: Log likelihood = -534.56173
Iteration 2: Log likelihood = -534.52835
Iteration 3: Log likelihood = -534.52833
```

Multinomial probit regression

Number of obs = 615  
Wald chi2(10) = 40.18  
Prob > chi2 = 0.0000

Log likelihood = -534.52833

| insure         | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|----------------|-------------|-----------|-------|-------|----------------------|-----------|
| Indemnity      |             |           |       |       |                      |           |
| (base outcome) |             |           |       |       |                      |           |
| Prepaid        |             |           |       |       |                      |           |
| age            | -.0098536   | .0052688  | -1.87 | 0.061 | -.0201802            | .000473   |
| male           | .4774678    | .1718316  | 2.78  | 0.005 | .1406841             | .8142515  |
| nonwhite       | .8245003    | .1977582  | 4.17  | 0.000 | .4369013             | 1.212099  |
| site           |             |           |       |       |                      |           |
| 2              | .0973956    | .1794546  | 0.54  | 0.587 | -.2543289            | .4491201  |
| 3              | -.495892    | .1904984  | -2.60 | 0.009 | -.869262             | -.1225221 |
| _cons          | .22315      | .2792424  | 0.80  | 0.424 | -.324155             | .7704549  |
| Uninsure       |             |           |       |       |                      |           |
| age            | -.0050814   | .0075327  | -0.67 | 0.500 | -.0198452            | .0096823  |
| male           | .3332637    | .2432986  | 1.37  | 0.171 | -.1435929            | .8101203  |
| nonwhite       | .2485859    | .2767734  | 0.90  | 0.369 | -.29388              | .7910518  |
| site           |             |           |       |       |                      |           |
| 2              | -.6899485   | .2804497  | -2.46 | 0.014 | -1.23962             | -.1402771 |
| 3              | -.1788447   | .2479898  | -0.72 | 0.471 | -.6648957            | .3072063  |
| _cons          | -.9855917   | .3891873  | -2.53 | 0.011 | -1.748385            | -.2227986 |



The likelihood function for `mprobit` is derived under the assumption that all decision-making units face the same choice set, which is the union of all outcomes observed in the dataset. If that is not true for your model, then an alternative is to use the `cmmprobit` command, which does not require this assumption. To do that, you will need to expand the dataset so that each decision maker has  $k_i$  observations, where  $k_i$  is the number of alternatives in the choice set faced by decision maker  $i$ . You will also need to create a binary variable to indicate the choice made by each decision maker. Moreover, you will need to use the `correlation(independent)` and `stddev(homoskedastic)` options with `cmmprobit` unless you have alternative-specific variables.

## Stored results

mprobit stores the following in `e()`:

### Scalars

|                             |                                                              |
|-----------------------------|--------------------------------------------------------------|
| <code>e(N)</code>           | number of observations                                       |
| <code>e(k_out)</code>       | number of outcomes                                           |
| <code>e(k_points)</code>    | number of quadrature points                                  |
| <code>e(k)</code>           | number of parameters                                         |
| <code>e(k_eq)</code>        | number of equations in <code>e(b)</code>                     |
| <code>e(k_eq_model)</code>  | number of equations in overall model test                    |
| <code>e(k_indvars)</code>   | number of independent variables                              |
| <code>e(k_dv)</code>        | number of dependent variables                                |
| <code>e(df_m)</code>        | model degrees of freedom                                     |
| <code>e(ll)</code>          | log likelihood                                               |
| <code>e(N_clust)</code>     | number of clusters                                           |
| <code>e(chi2)</code>        | $\chi^2$                                                     |
| <code>e(p)</code>           | <i>p</i> -value for model test                               |
| <code>e(k_eq_base)</code>   | equation number of the base outcome                          |
| <code>e(baseout)</code>     | the value of <i>depvar</i> to be treated as the base outcome |
| <code>e(ibaseout)</code>    | index of the base outcome                                    |
| <code>e(const)</code>       | 0 if <code>noconstant</code> is specified, 1 otherwise       |
| <code>e(probitparam)</code> | 1 if <code>probitparam</code> is specified, 0 otherwise      |
| <code>e(rank)</code>        | rank of $e(V)$                                               |
| <code>e(ic)</code>          | number of iterations                                         |
| <code>e(rc)</code>          | return code                                                  |
| <code>e(converged)</code>   | 1 if converged, 0 otherwise                                  |

### Macros

|                                |                                                                          |
|--------------------------------|--------------------------------------------------------------------------|
| <code>e(cmd)</code>            | mprobit                                                                  |
| <code>e(cmdline)</code>        | command as typed                                                         |
| <code>e(depvar)</code>         | name of dependent variable                                               |
| <code>e(indvars)</code>        | independent variables                                                    |
| <code>e(wtype)</code>          | weight type                                                              |
| <code>e(wexp)</code>           | weight expression                                                        |
| <code>e(title)</code>          | title in estimation output                                               |
| <code>e(clustvar)</code>       | name of cluster variable                                                 |
| <code>e(chi2type)</code>       | Wald, type of model $\chi^2$ test                                        |
| <code>e(vce)</code>            | <i>vcetype</i> specified in <code>vce()</code>                           |
| <code>e(vcetype)</code>        | title used to label Std. err.                                            |
| <code>e(outeqs)</code>         | outcome equations                                                        |
| <code>e(out#)</code>           | outcome labels, $\# = 1, \dots, e(k\_out)$                               |
| <code>e(opt)</code>            | type of optimization                                                     |
| <code>e(which)</code>          | max or min; whether optimizer is to perform maximization or minimization |
| <code>e(ml_method)</code>      | type of ml method                                                        |
| <code>e(user)</code>           | name of likelihood-evaluator program                                     |
| <code>e(technique)</code>      | maximization technique                                                   |
| <code>e(properties)</code>     | b V                                                                      |
| <code>e(predict)</code>        | program used to implement <code>predict</code>                           |
| <code>e(marginsnotok)</code>   | predictions disallowed by <code>margins</code>                           |
| <code>e(marginsdefault)</code> | default <code>predict()</code> specification for <code>margins</code>    |
| <code>e(asbalanced)</code>     | factor variables <code>fvset</code> as <code>asbalanced</code>           |
| <code>e(asobserved)</code>     | factor variables <code>fvset</code> as <code>asobserved</code>           |

|                 |                                              |
|-----------------|----------------------------------------------|
| Matrices        |                                              |
| e(b)            | coefficient vector                           |
| e(outcomes)     | outcome values                               |
| e(Cns)          | constraints matrix                           |
| e(iilog)        | iteration log (up to 20 iterations)          |
| e(gradient)     | gradient vector                              |
| e(V)            | variance–covariance matrix of the estimators |
| e(V_modelbased) | model-based variance                         |
| Functions       |                                              |
| e(sample)       | marks estimation sample                      |

In addition to the above, the following is stored in `r()`:

|          |                                                                                                                       |
|----------|-----------------------------------------------------------------------------------------------------------------------|
| Matrices |                                                                                                                       |
| r(table) | matrix containing the coefficients with their standard errors, test statistics, $p$ -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any  $r$ -class command is run after the estimation command.

## Methods and formulas

See [Cameron and Trivedi \(2005, chap. 15\)](#) for a discussion of multinomial models, including multinomial probit. [Long and Freese \(2014, chap. 8\)](#) discuss the multinomial logistic, multinomial probit, and stereotype logistic regression models, with examples using Stata.

As discussed in [Remarks and examples](#), the latent variables for a  $J$ -alternative model are  $\eta_{ij} = \mathbf{z}_i \boldsymbol{\alpha}_j + \xi_{ij}$ , for  $j = 1, \dots, J$ ,  $i = 1, \dots, n$ , and  $\{\xi_{i,1}, \dots, \xi_{i,J}\} \sim i.i.d.N(0, 1)$ . The experimenter observes alternative  $k$  for the  $i$ th observation if  $\eta_{ik} > \eta_{il}$  for  $l \neq k$ . For  $j \neq k$ , let

$$\begin{aligned} v_{ij'} &= \eta_{ij} - \eta_{ik} \\ &= \mathbf{z}_i (\boldsymbol{\alpha}_j - \boldsymbol{\alpha}_k) + \xi_{ij} - \xi_{ik} \\ &= \mathbf{z}_i \boldsymbol{\gamma}_{j'} + \epsilon_{ij'} \end{aligned}$$

where  $j' = j$  if  $j < k$  and  $j' = j - 1$  if  $j > k$  so that  $j' = 1, \dots, J - 1$ .  $\boldsymbol{\epsilon}_i = (\epsilon_{i1}, \dots, \epsilon_{i,J-1}) \sim MVN(\mathbf{0}, \boldsymbol{\Sigma})$ , where

$$\boldsymbol{\Sigma} = \begin{pmatrix} 2 & 1 & 1 & \dots & 1 \\ 1 & 2 & 1 & \dots & 1 \\ 1 & 1 & 2 & \dots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \dots & 2 \end{pmatrix}$$

Denote the deterministic part of the model as  $\lambda_{ij'} = \mathbf{z}_i \boldsymbol{\gamma}_{j'}$ ; the probability that subject  $i$  chooses outcome  $k$  is

$$\begin{aligned} \Pr(y_i = k) &= \Pr(v_{i1} \leq 0, \dots, v_{i,J-1} \leq 0) \\ &= \Pr(\epsilon_{i1} \leq -\lambda_{i1}, \dots, \epsilon_{i,J-1} \leq -\lambda_{i,J-1}) \\ &= \frac{1}{(2\pi)^{(J-1)/2} |\boldsymbol{\Sigma}|^{1/2}} \int_{-\infty}^{-\lambda_{i1}} \dots \int_{-\infty}^{-\lambda_{i,J-1}} \exp\left(-\frac{1}{2} \mathbf{z}' \boldsymbol{\Sigma}^{-1} \mathbf{z}\right) d\mathbf{z} \end{aligned}$$

Because of the exchangeable correlation structure of  $\Sigma$  ( $\rho_{ij} = 1/2$  for all  $i \neq j$ ), we can use Dunnett's (1989) result to reduce the multidimensional integral to one dimension:

$$\Pr(y_i = k) = \frac{1}{\sqrt{\pi}} \int_0^\infty \left\{ \prod_{j=1}^{J-1} \Phi(-z\sqrt{2} - \lambda_{ij}) + \prod_{j=1}^{J-1} \Phi(z\sqrt{2} - \lambda_{ij}) \right\} e^{-z^2} dz$$

Gaussian quadrature is used to approximate this integral, resulting in the  $K$ -point quadrature formula

$$\Pr(y_i = k) \approx \frac{1}{2} \sum_{k=1}^K w_k \left\{ \prod_{j=1}^{J-1} \Phi(-\sqrt{2x_k} - \lambda_{ij}) + \prod_{j=1}^{J-1} \Phi(\sqrt{2x_k} - \lambda_{ij}) \right\}$$

where  $w_k$  and  $x_k$  are the weights and roots of the Laguerre polynomial of order  $K$ . In `mprobit`,  $K$  is specified by the `intpoints()` option.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] `_robust`, particularly *Maximum likelihood estimators* and *Methods and formulas*.

`mprobit` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] **Variance estimation**.

## References

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## Also see

- [R] **mprobit postestimation** — Postestimation tools for mprobit
- [R] **clgit** — Conditional (fixed-effects) logistic regression
- [R] **mlogit** — Multinomial (polytomous) logistic regression
- [R] **ologit** — Ordered logistic regression
- [R] **oprobit** — Ordered probit regression
- [BAYES] **bayes: mprobit** — Bayesian multinomial probit regression
- [CM] **cmmprobit** — Multinomial probit choice model
- [CM] **nlogit** — Nested logit regression
- [MI] **Estimation** — Estimation commands for use with mi estimate
- [SVY] **svy estimation** — Estimation commands for survey data
- [XT] **xtmlogit** — Fixed-effects and random-effects multinomial logit models
- [U] **20 Estimation and postestimation commands**

Postestimation commands  
References

predict  
Also see

margins

Remarks and examples

## Postestimation commands

The following postestimation commands are available after `mprobit`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from <code>margins</code> (profile plots, interaction plots, etc.)                                                    |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | probabilities, linear predictions and their SEs, etc.                                                                                   |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\* `forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results. `forecast` is also not appropriate with `mi` estimation results.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as probabilities, linear predictions, and standard errors.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] { stub* | newvar | newvarlist } [if] [in] [, statistic outcome(outcome)]
```

```
predict [type] stub* [if] [in] , scores
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

Main

|                   |                                         |
|-------------------|-----------------------------------------|
| <code>pr</code>   | predicted probabilities; the default    |
| <code>xb</code>   | linear prediction                       |
| <code>stdp</code> | standard error of the linear prediction |

You specify one or  $k$  new variables with `pr`, where  $k$  is the number of outcomes. If you specify one new variable and you do not specify `outcome()`, then `outcome(#1)` is assumed.

You specify one new variable with `xb` and `stdp`. If you do not specify `outcome()`, then `outcome(#1)` is assumed.

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

## Options for predict

Main

`pr`, the default, computes the predicted probabilities for all outcomes or for a specific outcome. To compute probabilities for all outcomes, you specify  $k$  new variables, where  $k$  is the number of categories of the dependent variable. Alternatively, you can specify `stub*`; in which case, `pr` will store predicted probabilities in variables `stub1`, `stub2`, ..., `stubk`. To compute the probability for a specific outcome, you specify one new variable and, optionally, the outcome value in option `outcome()`; if you omit `outcome()`, the first outcome value, `outcome(#1)`, is assumed.

Say that you fit a model by typing `estimation_cmd y x1 x2`, and `y` takes on four values. Then, you could type `predict p1 p2 p3 p4` to obtain all four predicted probabilities; alternatively, you could type `predict p*` to generate the four predicted probabilities. To compute specific probabilities one at a time, you can type `predict p1, outcome(#1)` (or simply `predict p1`), `predict p2, outcome(#2)`, and so on. See option `outcome()` for other ways to refer to outcome values.

`xb` calculates the linear prediction,  $\mathbf{x}_i \boldsymbol{\alpha}_j$ , for alternative  $j$  and individual  $i$ . The index,  $j$ , corresponds to the outcome specified in `outcome()`.

`stdp` calculates the standard error of the linear prediction.

`outcome(outcome)` specifies for which outcome the predicted probabilities are to be calculated. `outcome()` should contain either one value of the dependent variable or one of #1, #2, ..., with #1 meaning the first category of the dependent variable, #2 meaning the second category, etc. `outcome()` is not allowed with `scores`.

`scores` calculates equation-level score variables. The  $j$ th new variable will contain the scores for the  $j$ th fitted equation.

## margins

### Description for margins

`margins` estimates margins of response for probabilities and linear predictions.

### Menu for margins

Statistics > Postestimation

### Syntax for margins

```
margins [marginlist] [, options]
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i> | Description                               |
|------------------|-------------------------------------------|
| default          | probabilities for each outcome            |
| pr               | probability for a specified outcome       |
| xb               | linear prediction for a specified outcome |
| stdp             | not allowed with <code>margins</code>     |

`pr` and `xb` default to the first outcome.

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [\[R\] margins](#).

### Remarks and examples

Once you have fit a multinomial probit model, you can use `predict` to obtain probabilities that an individual will choose each of the alternatives for the estimation sample, as well as other samples; see [\[U\] 20 Estimation and postestimation commands](#) and [\[R\] predict](#).

## ▷ Example 1

In [example 1](#) of [\[R\] mprobit](#), we fit the multinomial probit model to a dataset containing the type of health insurance available to 616 psychologically depressed subjects in the United States (Tarlov et al. 1989; Wells et al. 1989). We can obtain the predicted probabilities by typing

```
. use https://www.stata-press.com/data/r19/sysdsn1
(Health insurance data)
. mprobit insure age male nonwhite i.site
(output omitted)
. predict p1-p3
(option pr assumed; predicted probabilities)
. list p1-p3 insure in 1/10
```

|     | p1       | p2       | p3       | insure    |
|-----|----------|----------|----------|-----------|
| 1.  | .5961306 | .3741824 | .029687  | Indemnity |
| 2.  | .4719296 | .4972289 | .0308415 | Prepaid   |
| 3.  | .4896086 | .4121961 | .0981953 | Indemnity |
| 4.  | .3730529 | .5416623 | .0852848 | Prepaid   |
| 5.  | .5063069 | .4629773 | .0307158 | .         |
| 6.  | .4768125 | .4923548 | .0308327 | Prepaid   |
| 7.  | .5035672 | .4657016 | .0307312 | Prepaid   |
| 8.  | .3326361 | .5580404 | .1093235 | .         |
| 9.  | .4758165 | .4384811 | .0857024 | Uninsure  |
| 10. | .5734057 | .3316601 | .0949342 | Prepaid   |

insure contains a missing value for observations 5 and 8. Because of that, those two observations were not used in the estimation. However, because none of the independent variables is missing, predict can still calculate the probabilities. Had we typed

```
. predict p1-p3 if e(sample)
```

predict would have filled in missing values for p1, p2, and p3 for those observations because they were not used in the estimation.



## References

- Tarlov, A. R., J. E. Ware, Jr., S. Greenfield, E. C. Nelson, E. Perrin, and M. Zubkoff. 1989. The medical outcomes study. An application of methods for monitoring the results of medical care. *Journal of the American Medical Association* 262: 925–930. <https://doi.org/10.1001/jama.1989.03430070073033>.
- Wells, K. B., R. D. Hays, M. A. Burnam, W. H. Rogers, S. Greenfield, and J. E. Ware, Jr. 1989. Detection of depressive disorder for patients receiving prepaid or fee-for-service care. Results from the Medical Outcomes Survey. *Journal of the American Medical Association* 262: 3298–3302. <https://doi.org/10.1001/jama.1989.03430230083030>.

## Also see

[\[R\] mprobit](#) — Multinomial probit regression

[\[U\] 20 Estimation and postestimation commands](#)

[Description](#)[Options for nbreg](#)[Methods and formulas](#)[Quick start](#)[Options for gnbreg](#)[References](#)[Menu](#)[Remarks and examples](#)[Also see](#)[Syntax](#)[Stored results](#)

## Description

`nbreg` fits a negative binomial regression model for a nonnegative count dependent variable. In this model, the count variable is believed to be generated by a Poisson-like process, except that the variation is allowed to be greater than that of a true Poisson. This extra variation is referred to as overdispersion.

`gnbreg` fits a generalization of the negative binomial mean-dispersion model; the shape parameter  $\alpha$  may also be parameterized.

## Quick start

Negative binomial model of  $y$  on  $x_1$  and categorical variable  $a$

```
nbreg y x1 i.a
```

Same as above, but report results as incidence-rate ratios

```
nbreg y x1 i.a, irr
```

Same as above, and specify exposure variable  $evar$

```
nbreg y x1 i.a, irr exposure(evar)
```

Generalized negative binomial model with shape parameter  $\alpha$  a function of  $x_2$  and  $x_3$

```
gnbreg y x1 i.a, lnalpha(x2 x3)
```

Add log of exposure,  $lnevar$ , as an offset

```
gnbreg y x1 i.a, lnalpha(x2 x3) offset(lnevar)
```

## Menu

### **nbreg**

Statistics > Count outcomes > Negative binomial regression

### **gnbreg**

Statistics > Count outcomes > Generalized negative binomial regression

## Syntax

Negative binomial regression model

```
nbreg depvar [indepvars] [if] [in] [weight] [, nbreg_options]
```

Generalized negative binomial model

```
gnbreg depvar [indepvars] [if] [in] [weight] [, gnbreg_options]
```

| <i>nbreg_options</i>                              | Description                                                                                                                                                                 |
|---------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Model                                             |                                                                                                                                                                             |
| <code>noconstant</code>                           | suppress constant term                                                                                                                                                      |
| <code>dispersion(<u>m</u>ean)</code>              | parameterization of dispersion; the default                                                                                                                                 |
| <code>dispersion(<u>c</u>onstant)</code>          | constant dispersion for all observations                                                                                                                                    |
| <code>exposure(<i>varname</i><sub>e</sub>)</code> | include $\ln(\textit{varname}_e)$ in model with coefficient constrained to 1                                                                                                |
| <code>offset(<i>varname</i><sub>o</sub>)</code>   | include $\textit{varname}_o$ in model with coefficient constrained to 1                                                                                                     |
| <code>constraints(<i>constraints</i>)</code>      | apply specified linear constraints                                                                                                                                          |
| SE/Robust                                         |                                                                                                                                                                             |
| <code>vce(<i>vcetype</i>)</code>                  | <i>vcetype</i> may be <code>oim</code> , <code>robust</code> , <code>cluster <i>clustvar</i></code> , <code>opg</code> , <code>bootstrap</code> , or <code>jackknife</code> |
| Reporting                                         |                                                                                                                                                                             |
| <code>level(#)</code>                             | set confidence level; default is level(95)                                                                                                                                  |
| <code>no<del>l</del>rtest</code>                  | suppress likelihood-ratio test                                                                                                                                              |
| <code>irr</code>                                  | report incidence-rate ratios                                                                                                                                                |
| <code>no<del>c</del>n<del>s</del>report</code>    | do not display constraints                                                                                                                                                  |
| <code>display_options</code>                      | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling                            |
| Maximization                                      |                                                                                                                                                                             |
| <code>maximize_options</code>                     | control the maximization process; seldom used                                                                                                                               |
| <code>collinear</code>                            | keep collinear variables                                                                                                                                                    |
| <code>coeflegend</code>                           | display legend instead of statistics                                                                                                                                        |

| <i>gnbreg_options</i>                             | Description                                                                                                                                                                 |
|---------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Model                                             |                                                                                                                                                                             |
| <code>noconstant</code>                           | suppress constant term                                                                                                                                                      |
| <code>lnalpha(<i>varlist</i>)</code>              | dispersion model variables                                                                                                                                                  |
| <code>exposure(<i>varname<sub>e</sub></i>)</code> | include $\ln(\text{varname}_e)$ in model with coefficient constrained to 1                                                                                                  |
| <code>offset(<i>varname<sub>o</sub></i>)</code>   | include <i>varname<sub>o</sub></i> in model with coefficient constrained to 1                                                                                               |
| <code>constraints(<i>constraints</i>)</code>      | apply specified linear constraints                                                                                                                                          |
| SE/Robust                                         |                                                                                                                                                                             |
| <code>vce(<i>vcetype</i>)</code>                  | <i>vcetype</i> may be <code>oim</code> , <code>robust</code> , <code>cluster <i>clustvar</i></code> , <code>opg</code> , <code>bootstrap</code> , or <code>jackknife</code> |
| Reporting                                         |                                                                                                                                                                             |
| <code>level(#)</code>                             | set confidence level; default is <code>level(95)</code>                                                                                                                     |
| <code>irr</code>                                  | report incidence-rate ratios                                                                                                                                                |
| <code>nocnsreport</code>                          | do not display constraints                                                                                                                                                  |
| <code>display_options</code>                      | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling                            |
| Maximization                                      |                                                                                                                                                                             |
| <code>maximize_options</code>                     | control the maximization process; seldom used                                                                                                                               |
| <code>collinear</code>                            | keep collinear variables                                                                                                                                                    |
| <code>coeflegend</code>                           | display legend instead of statistics                                                                                                                                        |

*indepvars* and *varlist* may contain factor variables; see [U] 11.4.3 Factor variables.

*devar*, *indepvars*, *varname<sub>e</sub>*, and *varname<sub>o</sub>* may contain time-series operators (nbreg only); see [U] 11.4.4 Time-series varlists.

`bayes`, `bayesboot` (nbreg only), `bootstrap`, `by` (nbreg only), `collect`, `fmm` (nbreg only), `fp` (nbreg only), `jackknife`, `mfp` (nbreg only), `mi` estimate, `nestreg` (nbreg only), `rolling`, `statsby`, `stepwise`, and `svy` are allowed; see [U] 11.1.10 Prefix commands. For more details, see [BAYES] `bayes: nbreg`, [BAYES] `bayes: nbreg`, and [FMM] `fmm: nbreg`.

`vce(bootstrap)` and `vce(jackknife)` are not allowed with the `mi` estimate prefix; see [MI] `mi estimate`.

Weights are not allowed with the `bootstrap` prefix; see [R] `bootstrap`.

`vce()` and weights are not allowed with the `svy` prefix; see [SVY] `svy`.

`fweights`, `iwweights`, and `pweights` are allowed; see [U] 11.1.6 `weight`.

`collinear` and `coeflegend` do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Options for nbreg

Model

`noconstant`; see [R] Estimation options.

`dispersion(mean | constant)` specifies the parameterization of the model. `dispersion(mean)`, the default, yields a model with dispersion equal to  $1 + \alpha \exp(\mathbf{x}_j\beta + \text{offset}_j)$ ; that is, the dispersion is a function of the expected mean:  $\exp(\mathbf{x}_j\beta + \text{offset}_j)$ . `dispersion(constant)` has dispersion equal to  $1 + \delta$ ; that is, it is a constant for all observations.

`exposure(varnamee)`, `offset(varnameo)`, `constraints(constraints)`; see [R] Estimation options.

## SE/Robust

`vce(vctype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

## Reporting

`level(#)`; see [R] [Estimation options](#).

`no1rtest` suppresses fitting the Poisson model. Without this option, a comparison Poisson model is fit, and the likelihood is used in a likelihood-ratio test of the null hypothesis that the dispersion parameter is zero.

`irr` reports estimated coefficients transformed to incidence-rate ratios, that is,  $e^{\beta_i}$  rather than  $\beta_i$ . Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated or stored. `irr` may be specified at estimation or when replaying previously estimated results.

`nocnsreport`; see [R] [Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

## Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `1tolerance(#)`, `nr1tolerance(#)`, `nonr1tolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

Setting the optimization type to `technique(bhhh)` resets the default `vctype` to `vce(opg)`.

The following options are available with `nbreg` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Options for `gnbreg`

## Model

`noconstant`; see [R] [Estimation options](#).

`lnalpha(varlist)` allows you to specify a linear equation for  $\ln\alpha$ . Specifying `lnalpha(male old)` means that  $\ln\alpha = \gamma_0 + \gamma_1\text{male} + \gamma_2\text{old}$ , where  $\gamma_0$ ,  $\gamma_1$ , and  $\gamma_2$  are parameters to be estimated along with the other model coefficients. If this option is not specified, `gnbreg` and `nbreg` will produce the same results because the shape parameter will be parameterized as a constant.

`exposure(varnamee)`, `offset(varnameo)`, `constraints(constraints)`; see [R] [Estimation options](#).

## SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

## Reporting

`level(#)`; see [R] [Estimation options](#).

`irr` reports estimated coefficients transformed to incidence-rate ratios, that is,  $e^{\beta_i}$  rather than  $\beta_i$ . Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated or stored. `irr` may be specified at estimation or when replaying previously estimated results.

`nocnsreport`; see [R] [Estimation options](#).

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

## Maximization

*maximize\_options*: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

Setting the optimization type to `technique(bhhh)` resets the default `vcetype` to `vce(opg)`.

The following options are available with `gnbreg` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Remarks are presented under the following headings:

[Introduction to negative binomial regression](#)  
[nbgreg](#)  
[gnbreg](#)

## Introduction to negative binomial regression

Negative binomial regression models the number of occurrences (counts) of an event when the event has extra-Poisson variation, that is, when it has overdispersion. The Poisson regression model is

$$y_j \sim \text{Poisson}(\mu_j)$$

where

$$\mu_j = \exp(\mathbf{x}_j\boldsymbol{\beta} + \text{offset}_j)$$

for observed counts  $y_j$  with covariates  $\mathbf{x}_j$  for the  $j$ th observation. One derivation of the negative binomial mean-dispersion model is that individual units follow a Poisson regression model, but there is an omitted variable  $\zeta_j$ , such that  $e^{\zeta_j}$  follows a gamma distribution with mean 1 and variance  $\alpha$ :

$$y_j \sim \text{Poisson}(\mu_j^*)$$

where

$$\mu_j^* = \exp(\mathbf{x}_j\boldsymbol{\beta} + \text{offset}_j + \zeta_j)$$

and

$$e^{\zeta_j} \sim \text{Gamma}(1/\alpha, \alpha)$$

With this parameterization, a  $\text{Gamma}(a, b)$  distribution will have expectation  $ab$  and variance  $ab^2$ .

We refer to  $\alpha$  as the overdispersion parameter. The larger  $\alpha$  is, the greater the overdispersion. The Poisson model corresponds to  $\alpha = 0$ . `nbreg` parameterizes  $\alpha$  as  $\ln\alpha$ . `gnbreg` allows  $\ln\alpha$  to be modeled as  $\ln\alpha_j = \mathbf{z}_j\boldsymbol{\gamma}$ , a linear combination of covariates  $\mathbf{z}_j$ .

`nbreg` will fit two different parameterizations of the negative binomial model. The default, described above and also given by the `dispersion(mean)` option, has dispersion for the  $j$ th observation equal to  $1 + \alpha \exp(\mathbf{x}_j\boldsymbol{\beta} + \text{offset}_j)$ . This is seen by noting that the above implies that

$$\mu_j^* \sim \text{Gamma}(1/\alpha, \alpha\mu_j)$$

and thus

$$\begin{aligned} \text{Var}(y_j) &= E\{\text{Var}(y_j|\mu_j^*)\} + \text{Var}\{E(y_j|\mu_j^*)\} \\ &= E(\mu_j^*) + \text{Var}(\mu_j^*) \\ &= \mu_j(1 + \alpha\mu_j) \end{aligned}$$

The alternative parameterization, given by the `dispersion(constant)` option, has dispersion equal to  $1 + \delta$ ; that is, it is constant for all observations. This is so because the constant-dispersion model assumes instead that

$$\mu_j^* \sim \text{Gamma}(\mu_j/\delta, \delta)$$

and thus  $\text{Var}(y_j) = \mu_j(1 + \delta)$ . The Poisson model corresponds to  $\delta = 0$ .

For detailed derivations of both models, see [Cameron and Trivedi \(2013, 80–89\)](#). In particular, note that the mean-dispersion model is known as the NB2 model in their terminology, whereas the constant-dispersion model is referred to as the NB1 model.

See [Long and Freese \(2014\)](#) and [Cameron and Trivedi \(2022, chap. 20\)](#) for a discussion of the negative binomial regression model with Stata examples and for a discussion of other regression models for count data.

[Hilbe \(2011\)](#) provides an extensive review of the negative binomial model and its variations, using Stata examples.

## nbreg

It is not uncommon to posit a Poisson regression model and observe a lack of model fit. The following data appeared in [Rodríguez \(1993\)](#):

```
. use https://www.stata-press.com/data/r19/rod93
. list, sepby(cohort)
```

|     | cohort    | age_mos | deaths | exposure |
|-----|-----------|---------|--------|----------|
| 1.  | 1941–1949 | 0.5     | 168    | 278.4    |
| 2.  | 1941–1949 | 2.0     | 48     | 538.8    |
| 3.  | 1941–1949 | 4.5     | 63     | 794.4    |
| 4.  | 1941–1949 | 9.0     | 89     | 1,550.8  |
| 5.  | 1941–1949 | 18.0    | 102    | 3,006.0  |
| 6.  | 1941–1949 | 42.0    | 81     | 8,743.5  |
| 7.  | 1941–1949 | 90.0    | 40     | 14,270.0 |
| 8.  | 1960–1967 | 0.5     | 197    | 403.2    |
| 9.  | 1960–1967 | 2.0     | 48     | 786.0    |
| 10. | 1960–1967 | 4.5     | 62     | 1,165.3  |
| 11. | 1960–1967 | 9.0     | 81     | 2,294.8  |
| 12. | 1960–1967 | 18.0    | 97     | 4,500.5  |
| 13. | 1960–1967 | 42.0    | 103    | 13,201.5 |
| 14. | 1960–1967 | 90.0    | 39     | 19,525.0 |
| 15. | 1968–1976 | 0.5     | 195    | 495.3    |
| 16. | 1968–1976 | 2.0     | 55     | 956.7    |
| 17. | 1968–1976 | 4.5     | 58     | 1,381.4  |
| 18. | 1968–1976 | 9.0     | 85     | 2,604.5  |
| 19. | 1968–1976 | 18.0    | 87     | 4,618.5  |
| 20. | 1968–1976 | 42.0    | 70     | 9,814.5  |
| 21. | 1968–1976 | 90.0    | 10     | 5,802.5  |

```
. generate logexp = ln(exposure)
```

```

. poisson deaths i.cohort, offset(logexp)
Iteration 0: Log likelihood = -2160.0544
Iteration 1: Log likelihood = -2159.5162
Iteration 2: Log likelihood = -2159.5159
Iteration 3: Log likelihood = -2159.5159

Poisson regression
Log likelihood = -2159.5159
Number of obs = 21
LR chi2(2) = 49.16
Prob > chi2 = 0.0000
Pseudo R2 = 0.0113

```

| deaths    | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |           |
|-----------|-------------|-----------|--------|-------|----------------------|-----------|
| cohort    |             |           |        |       |                      |           |
| 1960-1967 | -.3020405   | .0573319  | -5.27  | 0.000 | -.4144089            | -.1896721 |
| 1968-1976 | .0742143    | .0589726  | 1.26   | 0.208 | -.0413698            | .1897983  |
| _cons     | -3.899488   | .0411345  | -94.80 | 0.000 | -3.98011             | -3.818866 |
| logexp    | 1           | (offset)  |        |       |                      |           |

```

. estat gof
Deviance goodness-of-fit = 4190.689
Prob > chi2(18) = 0.0000
Pearson goodness-of-fit = 15387.67
Prob > chi2(18) = 0.0000

```

The extreme significance of the goodness-of-fit  $\chi^2$  indicates that the Poisson regression model is inappropriate, suggesting to us that we should try a negative binomial model:

```

. nbreg deaths i.cohort, offset(logexp) nolog
Negative binomial regression
Log likelihood = -131.3799
Number of obs = 21
LR chi2(2) = 0.40
Prob > chi2 = 0.8171
Pseudo R2 = 0.0015

```

| deaths    | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|-----------|-------------|-----------|-------|-------|----------------------|-----------|
| cohort    |             |           |       |       |                      |           |
| 1960-1967 | -.2676187   | .7237203  | -0.37 | 0.712 | -1.686084            | 1.150847  |
| 1968-1976 | -.4573957   | .7236651  | -0.63 | 0.527 | -1.875753            | .9609618  |
| _cons     | -2.086731   | .511856   | -4.08 | 0.000 | -3.08995             | -1.083511 |
| logexp    | 1           | (offset)  |       |       |                      |           |
| /lnalpha  | .5939963    | .2583615  |       |       | .0876171             | 1.100376  |
| alpha     | 1.811212    | .4679475  |       |       | 1.09157              | 3.005295  |

```
LR test of alpha=0: chibar2(01) = 4056.27 Prob >= chibar2 = 0.000
```

Our original Poisson model is a special case of the negative binomial—it corresponds to  $\alpha = 0$ . nbreg, however, estimates  $\alpha$  indirectly, estimating instead  $\ln\alpha$ . In our model,  $\ln\alpha = 0.594$ , meaning that  $\alpha = 1.81$  (nbreg undoes the transformation for us at the bottom of the output).

To test  $\alpha = 0$  (equivalent to  $\ln\alpha = -\infty$ ), nbreg performs a likelihood-ratio test. The staggering  $\chi^2$  value of 4,056 asserts that the probability that we would observe these data conditional on  $\alpha = 0$  is virtually zero, that is, conditional on the process being Poisson. The data are not Poisson. It is not accidental that this  $\chi^2$  value is close to the goodness-of-fit statistic from the Poisson regression itself.

### □ Technical note

The usual Gaussian test of  $\alpha = 0$  is omitted because this test occurs on the boundary, invalidating the usual theory associated with such tests. However, the likelihood-ratio test of  $\alpha = 0$  has been modified to be valid on the boundary. In particular, the null distribution of the likelihood-ratio test statistic is not the usual  $\chi_1^2$ , but rather a 50 : 50 mixture of a  $\chi_0^2$  (point mass at zero) and a  $\chi_1^2$ , denoted as  $\bar{\chi}_{01}^2$ . See [Gutierrez, Carter, and Drukker \(2001\)](#) for more details.

□

### □ Technical note

The negative binomial model deals with cases in which there is more variation than would be expected if the process were Poisson. The negative binomial model is not helpful if there is less than Poisson variation—if the variance of the count variable is less than its mean. However, underdispersion is uncommon. Poisson models arise because of independently generated events. Overdispersion comes about if some of the parameters (causes) of the Poisson processes are unknown. To obtain underdispersion, the sequence of events somehow would have to be regulated; that is, events would not be independent but controlled based on past occurrences.

□

## gnbreg

`gnbreg` is a generalization of `nbreg`, `dispersion(mean)`. Whereas in `nbreg`, one  $\ln\alpha$  is estimated, `gnbreg` allows  $\ln\alpha$  to vary, observation by observation, as a linear combination of another set of covariates:  $\ln\alpha_j = \mathbf{z}_j\boldsymbol{\gamma}$ .

We will assume that the number of deaths is a function of age, whereas the  $\ln\alpha$  parameter is a function of cohort. To fit the model, we type

```
. nbreg deaths age_mos, lnalpha(i.cohort) offset(logexp)
Fitting constant-only model:
Iteration 0: Log likelihood = -187.067 (not concave)
Iteration 1: Log likelihood = -138.13047
Iteration 2: Log likelihood = -133.83164
Iteration 3: Log likelihood = -131.59551
Iteration 4: Log likelihood = -131.5795
Iteration 5: Log likelihood = -131.57948
Iteration 6: Log likelihood = -131.57948
Fitting full model:
Iteration 0: Log likelihood = -124.34327
Iteration 1: Log likelihood = -117.76701
Iteration 2: Log likelihood = -117.56403
Iteration 3: Log likelihood = -117.56164
Iteration 4: Log likelihood = -117.56164
Generalized negative binomial regression
Log likelihood = -117.56164
Number of obs = 21
LR chi2(1) = 28.04
Prob > chi2 = 0.0000
Pseudo R2 = 0.1065
```

|                | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|----------------|-------------|-----------|-------|-------|----------------------|-----------|
| <b>deaths</b>  |             |           |       |       |                      |           |
| age_mos        | -.0516657   | .0051747  | -9.98 | 0.000 | -.061808             | -.0415233 |
| _cons          | -1.867225   | .2227944  | -8.38 | 0.000 | -2.303894            | -1.430556 |
| logexp         | 1           | (offset)  |       |       |                      |           |
| <b>lnalpha</b> |             |           |       |       |                      |           |
| cohort         |             |           |       |       |                      |           |
| 1960-1967      | .0939546    | .7187747  | 0.13  | 0.896 | -1.314818            | 1.502727  |
| 1968-1976      | .0815279    | .7365476  | 0.11  | 0.912 | -1.362079            | 1.525135  |
| _cons          | -.4759581   | .5156502  | -0.92 | 0.356 | -1.486614            | .5346978  |

We find that age is a significant determinant of the number of deaths. The standard errors for the variables in the  $\ln\alpha$  equation suggest that the overdispersion parameter does not vary across cohorts. We can test this assertion by typing

```
. test 2.cohort 3.cohort
(1) [lnalpha]2.cohort = 0
(2) [lnalpha]3.cohort = 0
 chi2(2) = 0.02
 Prob > chi2 = 0.9904
```

There is no evidence of variation by cohort in these data.

## □ Technical note

Note the intentional absence of a likelihood-ratio test for  $\alpha = 0$  in `gnbreg`. The test is affected by the same boundary condition that affects the comparison test in `nbreg`; however, when  $\alpha$  is parameterized by more than a constant term, the null distribution becomes intractable. For this reason, we recommend using `nbreg` to test for overdispersion and, if you have reason to believe that overdispersion exists, only then modeling the overdispersion using `gnbreg`.

□

## Stored results

`nbreg` stores the following in `e()`:

### Scalars

|                            |                                                   |
|----------------------------|---------------------------------------------------|
| <code>e(N)</code>          | number of observations                            |
| <code>e(k)</code>          | number of parameters                              |
| <code>e(k_aux)</code>      | number of auxiliary parameters                    |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>          |
| <code>e(k_eq_model)</code> | number of equations in overall model test         |
| <code>e(k_dv)</code>       | number of dependent variables                     |
| <code>e(df_m)</code>       | model degrees of freedom                          |
| <code>e(r2_p)</code>       | pseudo- $R^2$                                     |
| <code>e(ll)</code>         | log likelihood                                    |
| <code>e(ll_0)</code>       | log likelihood, constant-only model               |
| <code>e(ll_c)</code>       | log likelihood, comparison model                  |
| <code>e(alpha)</code>      | value of alpha                                    |
| <code>e(delta)</code>      | value of delta                                    |
| <code>e(N_clust)</code>    | number of clusters                                |
| <code>e(chi2)</code>       | $\chi^2$                                          |
| <code>e(chi2_c)</code>     | $\chi^2$ for comparison test                      |
| <code>e(p)</code>          | $p$ -value for model test                         |
| <code>e(rank)</code>       | rank of <code>e(V)</code>                         |
| <code>e(rank0)</code>      | rank of <code>e(V)</code> for constant-only model |
| <code>e(ic)</code>         | number of iterations                              |
| <code>e(rc)</code>         | return code                                       |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise                       |

### Macros

|                            |                                                                                 |
|----------------------------|---------------------------------------------------------------------------------|
| <code>e(cmd)</code>        | <code>nbreg</code>                                                              |
| <code>e(cmdline)</code>    | command as typed                                                                |
| <code>e(depvar)</code>     | name of dependent variable                                                      |
| <code>e(wtype)</code>      | weight type                                                                     |
| <code>e(wexp)</code>       | weight expression                                                               |
| <code>e(title)</code>      | title in estimation output                                                      |
| <code>e(clustvar)</code>   | name of cluster variable                                                        |
| <code>e(offset)</code>     | linear offset variable                                                          |
| <code>e(chi2type)</code>   | Wald or LR; type of model $\chi^2$ test                                         |
| <code>e(chi2_ct)</code>    | Wald or LR; type of model $\chi^2$ test corresponding to <code>e(chi2_c)</code> |
| <code>e(dispers)</code>    | mean or constant                                                                |
| <code>e(vce)</code>        | <code>vce</code> type specified in <code>vce()</code>                           |
| <code>e(vcetype)</code>    | title used to label Std. err.                                                   |
| <code>e(opt)</code>        | type of optimization                                                            |
| <code>e(which)</code>      | max or min; whether optimizer is to perform maximization or minimization        |
| <code>e(ml_method)</code>  | type of ml method                                                               |
| <code>e(user)</code>       | name of likelihood-evaluator program                                            |
| <code>e(technique)</code>  | maximization technique                                                          |
| <code>e(properties)</code> | <code>b V</code>                                                                |
| <code>e(predict)</code>    | program used to implement <code>predict</code>                                  |

|                 |                                              |
|-----------------|----------------------------------------------|
| e(asbalanced)   | factor variables fvset as asbalanced         |
| e(asobserved)   | factor variables fvset as asobserved         |
| Matrices        |                                              |
| e(b)            | coefficient vector                           |
| e(Cns)          | constraints matrix                           |
| e(iilog)        | iteration log (up to 20 iterations)          |
| e(gradient)     | gradient vector                              |
| e(V)            | variance–covariance matrix of the estimators |
| e(V_modelbased) | model-based variance                         |
| Functions       |                                              |
| e(sample)       | marks estimation sample                      |

In addition to the above, the following is stored in `r()`:

|          |                                                                                                                            |
|----------|----------------------------------------------------------------------------------------------------------------------------|
| Matrices |                                                                                                                            |
| r(table) | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r`-class command is run after the estimation command.

`gnbreg` stores the following in `e()`:

|               |                                                   |
|---------------|---------------------------------------------------|
| Scalars       |                                                   |
| e(N)          | number of observations                            |
| e(k)          | number of parameters                              |
| e(k_eq)       | number of equations in <code>e(b)</code>          |
| e(k_eq_model) | number of equations in overall model test         |
| e(k_dv)       | number of dependent variables                     |
| e(df_m)       | model degrees of freedom                          |
| e(r2_p)       | pseudo- $R^2$                                     |
| e(l1)         | log likelihood                                    |
| e(l1_0)       | log likelihood, constant-only model               |
| e(N_clust)    | number of clusters                                |
| e(chi2)       | $\chi^2$                                          |
| e(p)          | <i>p</i> -value for model test                    |
| e(rank)       | rank of <code>e(V)</code>                         |
| e(rank0)      | rank of <code>e(V)</code> for constant-only model |
| e(ic)         | number of iterations                              |
| e(rc)         | return code                                       |
| e(converged)  | 1 if converged, 0 otherwise                       |

|               |                                                                          |
|---------------|--------------------------------------------------------------------------|
| Macros        |                                                                          |
| e(cmd)        | <code>gnbreg</code>                                                      |
| e(cmdline)    | command as typed                                                         |
| e(depvar)     | name of dependent variable                                               |
| e(wtype)      | weight type                                                              |
| e(wexp)       | weight expression                                                        |
| e(title)      | title in estimation output                                               |
| e(clustvar)   | name of cluster variable                                                 |
| e(offset1)    | linear offset variable                                                   |
| e(chi2type)   | Wald or LR; type of model $\chi^2$ test                                  |
| e(vce)        | <i>vcetype</i> specified in <code>vce()</code>                           |
| e(vcetype)    | title used to label Std. err.                                            |
| e(opt)        | type of optimization                                                     |
| e(which)      | max or min; whether optimizer is to perform maximization or minimization |
| e(ml_method)  | type of ml method                                                        |
| e(user)       | name of likelihood-evaluator program                                     |
| e(technique)  | maximization technique                                                   |
| e(properties) | b V                                                                      |

e(predict) program used to implement predict  
 e(asbalanced) factor variables fvset as asbalanced  
 e(asobserved) factor variables fvset as asobserved

Matrices

e(b) coefficient vector  
 e(Cns) constraints matrix  
 e(iilog) iteration log (up to 20 iterations)  
 e(gradient) gradient vector  
 e(V) variance–covariance matrix of the estimators  
 e(V\_modelbased) model-based variance

Functions

e(sample) marks estimation sample

In addition to the above, the following is stored in `r()`:

Matrices

r(table) matrix containing the coefficients with their standard errors, test statistics, *p*-values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

## Methods and formulas

See [R] [poisson](#) and Johnson, Kemp, and Kotz (2005, chap. 4) for an introduction to the Poisson distribution.

Methods and formulas are presented under the following headings:

*Mean-dispersion model*  
*Constant-dispersion model*

### Mean-dispersion model

A negative binomial distribution can be regarded as a gamma mixture of Poisson random variables. The number of times something occurs,  $y_j$ , is distributed as  $\text{Poisson}(\nu_j \mu_j)$ . That is, its conditional likelihood is

$$f(y_j | \nu_j) = \frac{(\nu_j \mu_j)^{y_j} e^{-\nu_j \mu_j}}{\Gamma(y_j + 1)}$$

where  $\mu_j = \exp(\mathbf{x}_j \boldsymbol{\beta} + \text{offset}_j)$  and  $\nu_j$  is an unobserved parameter with a  $\text{Gamma}(1/\alpha, \alpha)$  density:

$$g(\nu) = \frac{\nu^{(1-\alpha)/\alpha} e^{-\nu/\alpha}}{\alpha^{1/\alpha} \Gamma(1/\alpha)}$$

This gamma distribution has mean 1 and variance  $\alpha$ , where  $\alpha$  is our ancillary parameter.

The unconditional likelihood for the  $j$ th observation is therefore

$$f(y_j) = \int_0^\infty f(y_j | \nu) g(\nu) d\nu = \frac{\Gamma(m + y_j)}{\Gamma(y_j + 1) \Gamma(m)} p_j^m (1 - p_j)^{y_j}$$

where  $p_j = 1/(1 + \alpha \mu_j)$  and  $m = 1/\alpha$ . Solutions for  $\alpha$  are handled by searching for  $\ln \alpha$  because  $\alpha$  must be greater than zero.

The log likelihood (with weights  $w_j$  and offsets) is given by

$$m = 1/\alpha \quad p_j = 1/(1 + \alpha\mu_j) \quad \mu_j = \exp(\mathbf{x}_j\boldsymbol{\beta} + \text{offset}_j)$$

$$\ln L = \sum_{j=1}^n w_j \left[ \ln\{\Gamma(m + y_j)\} - \ln\{\Gamma(y_j + 1)\} \right. \\ \left. - \ln\{\Gamma(m)\} + m \ln(p_j) + y_j \ln(1 - p_j) \right]$$

For `gnbreg`,  $\alpha$  can vary across the observations according to the parameterization  $\ln\alpha_j = \mathbf{z}_j\boldsymbol{\gamma}$ .

## Constant-dispersion model

The constant-dispersion model assumes that  $y_j$  is conditionally distributed as  $\text{Poisson}(\mu_j^*)$ , where  $\mu_j^* \sim \text{Gamma}(\mu_j/\delta, \delta)$  for some dispersion parameter  $\delta$  (by contrast, the mean-dispersion model assumes that  $\mu_j^* \sim \text{Gamma}(1/\alpha, \alpha\mu_j)$ ). The log likelihood is given by

$$m_j = \mu_j/\delta \quad p = 1/(1 + \delta)$$

$$\ln L = \sum_{j=1}^n w_j \left[ \ln\{\Gamma(m_j + y_j)\} - \ln\{\Gamma(y_j + 1)\} \right. \\ \left. - \ln\{\Gamma(m_j)\} + m_j \ln(p) + y_j \ln(1 - p) \right]$$

with everything else defined as before in the calculations for the mean-dispersion model.

`nbreg` and `gnbreg` support the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] [\\_robust](#), particularly [Maximum likelihood estimators](#) and [Methods and formulas](#).

These commands also support estimation with survey data. For details on VCEs with survey data, see [SVY] [Variance estimation](#).

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- Xu, X., and J. W. Hardin. 2016. Regression models for bivariate count outcomes. *Stata Journal* 16: 301–315.

## Also see

- [R] **nbreg postestimation** — Postestimation tools for nbreg and gnbreg
- [R] **glm** — Generalized linear models
- [R] **npregress kernel** — Nonparametric kernel regression
- [R] **npregress series** — Nonparametric series regression
- [R] **poisson** — Poisson regression
- [R] **tnbreg** — Truncated negative binomial regression
- [R] **zinb** — Zero-inflated negative binomial regression
- [BAYES] **bayes: gnbreg** — Bayesian generalized negative binomial regression
- [BAYES] **bayes: nbreg** — Bayesian negative binomial regression
- [FMM] **fmm: nbreg** — Finite mixtures of negative binomial regression models
- [ME] **menbreg** — Multilevel mixed-effects negative binomial regression
- [MI] **Estimation** — Estimation commands for use with mi estimate
- [SVY] **svy estimation** — Estimation commands for survey data
- [XT] **xtnbreg** — Fixed-effects, random-effects, & population-averaged negative binomial models
- [U] **20 Estimation and postestimation commands**

|                         |                                      |                           |
|-------------------------|--------------------------------------|---------------------------|
| Postestimation commands | <a href="#">predict</a>              | <a href="#">margins</a>   |
| Remarks and examples    | <a href="#">Methods and formulas</a> | <a href="#">Reference</a> |
| Also see                |                                      |                           |

## Postestimation commands

The following postestimation commands are available after nbreg and gnbreg:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| <code>linktest</code>        | link test for model specification                                                                                                       |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | number of events, incidence rates, probabilities, etc.                                                                                  |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results. `forecast` is also not appropriate with `mi` estimation results.

# predict

## Description for predict

predict creates a new variable containing predictions such as numbers of events, incidence rates, probabilities, linear predictions, standard errors, and predicted values.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic noffset]
```

```
predict [type] stub* [if] [in] , scores
```

| <i>statistic</i>  | Description                                           |
|-------------------|-------------------------------------------------------|
| Main              |                                                       |
| n                 | number of events; the default                         |
| ir                | incidence rate (equivalent to predict ..., n noffset) |
| pr( <i>n</i> )    | probability $\Pr(y_j = n)$                            |
| pr( <i>a, b</i> ) | probability $\Pr(a \leq y_j \leq b)$                  |
| xb                | linear prediction                                     |
| stdp              | standard error of the linear prediction               |

In addition, relevant only after gnbreg are the following:

| <i>statistic</i> | Description                               |
|------------------|-------------------------------------------|
| Main             |                                           |
| <u>a</u> lpha    | predicted values of $\alpha_j$            |
| <u>l</u> nalpha  | predicted values of $\ln\alpha_j$         |
| <u>s</u> tdplna  | standard error of predicted $\ln\alpha_j$ |

These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

## Options for predict

Main

n, the default, calculates the predicted number of events, which is  $\exp(\mathbf{x}_j\beta)$  if neither `offset(varnameo)` nor `exposure(varnamee)` was specified when the model was fit;  $\exp(\mathbf{x}_j\beta + \text{offset}_j)$  if `offset()` was specified; or  $\exp(\mathbf{x}_j\beta) \times \text{exposure}_j$  if `exposure()` was specified.

ir calculates the incidence rate  $\exp(\mathbf{x}_j\beta)$ , which is the predicted number of events when exposure is 1. This is equivalent to specifying both the n and the noffset options.

`pr(n)` calculates the probability  $\Pr(y_j = n)$ , where  $n$  is a nonnegative integer that may be specified as a number or a variable.

`pr(a,b)` calculates the probability  $\Pr(a \leq y_j \leq b)$ , where  $a$  and  $b$  are nonnegative integers that may be specified as numbers or variables;

$b$  missing ( $b \geq .$ ) means  $+\infty$ ;

`pr(20,.)` calculates  $\Pr(y_j \geq 20)$ ;

`pr(20,b)` calculates  $\Pr(y_j \geq 20)$  in observations for which  $b \geq .$  and calculates  $\Pr(20 \leq y_j \leq b)$  elsewhere.

`pr(.,b)` produces a syntax error. A missing value in an observation of the variable  $a$  causes a missing value in that observation for `pr(a,b)`.

`xb` calculates the linear prediction, which is  $\mathbf{x}_j\beta$  if neither `offset()` nor `exposure()` was specified;  $\mathbf{x}_j\beta + \text{offset}_j$  if `offset()` was specified; or  $\mathbf{x}_j\beta + \ln(\text{exposure}_j)$  if `exposure()` was specified; see `nooffset` below.

`stdp` calculates the standard error of the linear prediction.

`alpha`, `lnalpha`, and `stdplna` are relevant after `gnbreg` estimation only; they produce the predicted values of  $\alpha_j$ ,  $\ln\alpha_j$ , and the standard error of the predicted  $\ln\alpha_j$ , respectively.

`nooffset` is relevant only if you specified `offset()` or `exposure()` when you fit the model. It modifies the calculations made by `predict` so that they ignore the offset or exposure variable; the linear prediction is treated as  $\mathbf{x}_j\beta$  rather than as  $\mathbf{x}_j\beta + \text{offset}_j$  or  $\mathbf{x}_j\beta + \ln(\text{exposure}_j)$ . Specifying `predict ..., nooffset` is equivalent to specifying `predict ..., ir`.

`scores` calculates equation-level score variables.

The first new variable will contain  $\partial \ln L / \partial (\mathbf{x}_j\beta)$ .

The second new variable will contain  $\partial \ln L / \partial (\ln\alpha_j)$  for `dispersion(mean)` and `gnbreg`.

The second new variable will contain  $\partial \ln L / \partial (\ln\delta)$  for `dispersion(constant)`.

# margins

## Description for margins

`margins` estimates margins of response for numbers of events, incidence rates, probabilities, linear predictions, and predicted values.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i>             | Description                                                          |
|------------------------------|----------------------------------------------------------------------|
| <code>n</code>               | number of events; the default                                        |
| <code>ir</code>              | incidence rate (equivalent to <code>predict ..., n nooffset</code> ) |
| <code>pr(<i>n</i>)</code>    | probability $\Pr(y_j = n)$                                           |
| <code>pr(<i>a, b</i>)</code> | probability $\Pr(a \leq y_j \leq b)$                                 |
| <code>xb</code>              | linear prediction                                                    |
| <code>stdp</code>            | not allowed with <code>margins</code>                                |

In addition, relevant only after `gnbreg` are the following:

| <i>statistic</i>            | Description                           |
|-----------------------------|---------------------------------------|
| <code><u>alpha</u></code>   | predicted values of $\alpha_j$        |
| <code><u>lnalpha</u></code> | predicted values of $\ln\alpha_j$     |
| <code><u>stdplna</u></code> | not allowed with <code>margins</code> |

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [\[R\] margins](#).

## Remarks and examples

After `nbreg` and `gnbreg`, `predict` returns the expected number of deaths per cohort and the probability of observing the number of deaths recorded or fewer.

```
. use https://www.stata-press.com/data/r19/rod93
. nbreg deaths i.cohort, nolog
Negative binomial regression Number of obs = 21
 LR chi2(2) = 0.14
Dispersion: mean Prob > chi2 = 0.9307
Log likelihood = -108.48841 Pseudo R2 = 0.0007
```

| deaths    | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|-----------|-------------|-----------|-------|-------|----------------------|-----------|
| cohort    |             |           |       |       |                      |           |
| 1960–1967 | .0591305    | .2978419  | 0.20  | 0.843 | -.5246289            | .64289    |
| 1968–1976 | -.0538792   | .2981621  | -0.18 | 0.857 | -.6382662            | .5305077  |
| _cons     | 4.435906    | .2107213  | 21.05 | 0.000 | 4.0229               | 4.848912  |
| /lnalpha  | -1.207379   | .3108622  |       |       | -1.816657            | -.5980999 |
| alpha     | .29898      | .0929416  |       |       | .1625683             | .5498555  |

LR test of alpha=0:  $\text{chibar2}(01) = 434.62$       Prob  $\geq$   $\text{chibar2} = 0.000$

```
. predict count
(option n assumed; predicted number of events)
. predict p, pr(0, deaths)
. summarize deaths count p
```

| Variable | Obs | Mean     | Std. dev. | Min      | Max      |
|----------|-----|----------|-----------|----------|----------|
| deaths   | 21  | 84.66667 | 48.84192  | 10       | 197      |
| count    | 21  | 84.66667 | 4.00773   | 80       | 89.57143 |
| p        | 21  | .4991542 | .2743702  | .0070255 | .9801285 |

The expected number of deaths ranges from 80 to 90. The probability  $\Pr(y_i \leq \text{deaths})$  ranges from 0.007 to 0.98.

The estimated expected and observed mean number of deaths, 84.67, happen to be the same in our example because our model included only a categorical predictor. In general, in the presence of other continuous predictors, the two estimates may not always be the same.

## Methods and formulas

In the following, we use the same notation as in [R] `nbreg`.

Methods and formulas are presented under the following headings:

*Mean-dispersion model*  
*Constant-dispersion model*

## Mean-dispersion model

The equation-level scores are given by

$$\begin{aligned}\text{score}(\mathbf{x}\boldsymbol{\beta})_j &= p_j(y_j - \mu_j) \\ \text{score}(\tau)_j &= -m \left\{ \frac{\alpha_j(\mu_j - y_j)}{1 + \alpha_j\mu_j} - \ln(1 + \alpha_j\mu_j) + \psi(y_j + m) - \psi(m) \right\}\end{aligned}$$

where  $\tau_j = \ln\alpha_j$  and  $\psi(z)$  is the digamma function.

## Constant-dispersion model

The equation-level scores are given by

$$\begin{aligned}\text{score}(\mathbf{x}\boldsymbol{\beta})_j &= m_j \{ \psi(y_j + m_j) - \psi(m_j) + \ln(p) \} \\ \text{score}(\tau)_j &= y_j - (y_j + m_j)(1 - p) - \text{score}(\mathbf{x}\boldsymbol{\beta})_j\end{aligned}$$

where  $\tau_j = \ln\delta_j$ .

## Reference

Manjón, M., and O. Martínez. 2014. The chi-squared goodness-of-fit test for count-data models. *Stata Journal* 14: 798–816.

## Also see

[R] [nbreg](#) — Negative binomial regression

[U] [20 Estimation and postestimation commands](#)

[Description](#)  
[Options](#)  
[References](#)[Quick start](#)  
[Remarks and examples](#)  
[Also see](#)[Menu](#)  
[Stored results](#)[Syntax](#)  
[Acknowledgment](#)

## Description

`nestreg` fits nested models by sequentially adding blocks of variables and then reports comparison tests between the nested models.

## Quick start

Fit nested (hierarchical) models sequentially, including covariates `x1` and `x2` first and then adding `x3` and `x4`

```
nestreg: regress y (x1 x2) (x3 x4)
```

Also fit third model including indicators for categorical variable `a`

```
nestreg: regress y (x1 x2) (x3 x4) (i.a)
```

Report table of likelihood-ratio tests instead of Wald tests comparing models

```
nestreg, lrtable: regress y (x1 x2) (x3 x4) (i.a)
```

Fit nested models and adjust for complex survey design using `svyset` data

```
nestreg: svy: regress y (x1 x2) (x3 x4) (i.a)
```

Note: In the above examples, `regress` could be replaced with any estimation command allowing the `nestreg` prefix.

## Menu

Statistics > Other > Nested model statistics

## Syntax

Standard estimation command syntax

```
nestreg [, options]: command_name depvar (varlist) [(varlist) ...]
 [if] [in] [weight] [, command_options]
```

Survey estimation command syntax

```
nestreg [, options]: svy [vcetype] [, svy_options]: command_name depvar
 (varlist) [(varlist) ...] [if] [in] [, command_options]
```

| <i>options</i>                  | Description                                                       |
|---------------------------------|-------------------------------------------------------------------|
| Reporting                       |                                                                   |
| <code>waldtable</code>          | report Wald test results; the default                             |
| <code>lrtable</code>            | report likelihood-ratio test results                              |
| <code>quietly</code>            | suppress any output from <i>command_name</i>                      |
| <code>store(<i>stub</i>)</code> | store nested estimation results in <code>_est_<i>stub</i>#</code> |

by is allowed; see [U] 11.1.10 Prefix commands.

Weights are allowed if *command\_name* allows them; see [U] 11.1.6 weight.

A *varlist* in parentheses indicates that this list of variables is to be considered as a block. Each variable in *varlist* not bound in parentheses will be treated as its own block.

All postestimation commands behave as they would after *command\_name* without the `nestreg` prefix; see the postestimation manual entry for *command\_name*.

## Options

### Reporting

`waldtable` specifies that the table of Wald test results be reported. `waldtable` is the default.

`lrtable` specifies that the table of likelihood-ratio tests be reported. This option is not allowed if `pweights`, the `vce(robust)` option, or the `vce(cluster clustvar)` option is specified. `lrtable` is also not allowed with the `svy` prefix.

`quietly` suppresses the display of any output from *command\_name*.

`store(stub)` specifies that each model fit by `nestreg` be stored under the name `_est_stub#`, where `#` is the nesting order from first to last.

## Remarks and examples

Remarks are presented under the following headings:

- [Estimation commands](#)
- [Wald tests](#)
- [Likelihood-ratio tests](#)
- [Programming for nestreg](#)

## Estimation commands

nestreg removes collinear predictors and observations with missing values from the estimation sample before calling *command\_name*.

The following Stata commands are supported by nestreg:

betareg, clogit, cloglog, glm, intreg, logistic, logit, nbreg, ologit, oprobit, poisson, probit, qreg, regress, scobit, stcox, stcrreg, stintreg, streg, and tobit

You do not supply *depvar* for stcox, stintreg, stcrreg, or streg; otherwise, *depvar* is required. You must supply two *devars* for intreg.

## Wald tests

Use nestreg to test the significance of blocks of predictors, building the regression model one block at a time. Using the data from [example 1](#) of [\[R\] test](#), we wish to test the significance of the following predictors of birthrate: median age (medage), median age squared (c.medage#c.medage), and indicators of the census region (i.region).

```
. use https://www.stata-press.com/data/r19/census4
(Census data on birthrate, median age)
. nestreg: regress brate (medage) (c.medage#c.medage) (i.region)
note: 1.region omitted because of estimability.
```

Block 1: medage

| Source   | SS         | df | MS         | Number of obs | = |        |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 32675.1044 | 1  | 32675.1044 | F(1, 48)      | = | 164.72 |
| Residual | 9521.71561 | 48 | 198.369075 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.7743 |
|          |            |    |            | Adj R-squared | = | 0.7696 |
| Total    | 42196.82   | 49 | 861.159592 | Root MSE      | = | 14.084 |

| brate  | Coefficient | Std. err. | t      | P> t  | [95% conf. interval] |           |
|--------|-------------|-----------|--------|-------|----------------------|-----------|
| medage | -15.24893   | 1.188141  | -12.83 | 0.000 | -17.63785            | -12.86002 |
| _cons  | 618.3935    | 35.15416  | 17.59  | 0.000 | 547.7113             | 689.0756  |

Block 2: c.medage#c.medage

| Source   | SS         | df | MS         | Number of obs | = |        |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 36755.8566 | 2  | 18377.9283 | F(2, 47)      | = | 158.75 |
| Residual | 5440.96342 | 47 | 115.765179 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.8711 |
|          |            |    |            | Adj R-squared | = | 0.8656 |
| Total    | 42196.82   | 49 | 861.159592 | Root MSE      | = | 10.759 |

| brate             | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|-------------------|-------------|-----------|-------|-------|----------------------|-----------|
| medage            | -109.8926   | 15.96663  | -6.88 | 0.000 | -142.0133            | -77.77189 |
| c.medage#c.medage | 1.607334    | .2707229  | 5.94  | 0.000 | 1.06271              | 2.151958  |
| _cons             | 2007.073    | 235.4316  | 8.53  | 0.000 | 1533.445             | 2480.7    |

Block 3: 2.region 3.region 4.region

| Source   | SS         | df | MS         | Number of obs | = | 50     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 38803.4208 | 5  | 7760.68416 | F(5, 44)      | = | 100.63 |
| Residual | 3393.39921 | 44 | 77.1227094 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.9196 |
|          |            |    |            | Adj R-squared | = | 0.9104 |
| Total    | 42196.82   | 49 | 861.159592 | Root MSE      | = | 8.782  |

| brate                 | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|-----------------------|-------------|-----------|-------|-------|----------------------|-----------|
| medage                | -109.0958   | 13.52452  | -8.07 | 0.000 | -136.3527            | -81.83892 |
| c.medage#<br>c.medage | 1.635209    | .2290536  | 7.14  | 0.000 | 1.173582             | 2.096836  |
| region<br>N Cntrl     | 15.00283    | 4.252067  | 3.53  | 0.001 | 6.433353             | 23.57231  |
| South                 | 7.366445    | 3.953335  | 1.86  | 0.069 | -.6009775            | 15.33387  |
| West                  | 21.39679    | 4.650601  | 4.60  | 0.000 | 12.02412             | 30.76946  |
| _cons                 | 1947.611    | 199.8405  | 9.75  | 0.000 | 1544.859             | 2350.363  |

| Block | Block  |    | Residual |        | R2     | Change<br>in R2 |
|-------|--------|----|----------|--------|--------|-----------------|
|       | F      | df | df       | Pr > F |        |                 |
| 1     | 164.72 | 1  | 48       | 0.0000 | 0.7743 |                 |
| 2     | 35.25  | 1  | 47       | 0.0000 | 0.8711 | 0.0967          |
| 3     | 8.85   | 3  | 44       | 0.0001 | 0.9196 | 0.0485          |

This single call to nestreg ran regress three times, adding a block of predictors to the model for each run as in

. regress brate medage

| Source   | SS         | df | MS         | Number of obs | = | 50     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 32675.1044 | 1  | 32675.1044 | F(1, 48)      | = | 164.72 |
| Residual | 9521.71561 | 48 | 198.369075 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.7743 |
|          |            |    |            | Adj R-squared | = | 0.7696 |
| Total    | 42196.82   | 49 | 861.159592 | Root MSE      | = | 14.084 |

| brate  | Coefficient | Std. err. | t      | P> t  | [95% conf. interval] |           |
|--------|-------------|-----------|--------|-------|----------------------|-----------|
| medage | -15.24893   | 1.188141  | -12.83 | 0.000 | -17.63785            | -12.86002 |
| _cons  | 618.3935    | 35.15416  | 17.59  | 0.000 | 547.7113             | 689.0756  |

```
. regress brate medage c.medage#c.medage
```

| Source   | SS         | df | MS         | Number of obs | = | 50     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 36755.8566 | 2  | 18377.9283 | F(2, 47)      | = | 158.75 |
| Residual | 5440.96342 | 47 | 115.765179 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.8711 |
|          |            |    |            | Adj R-squared | = | 0.8656 |
| Total    | 42196.82   | 49 | 861.159592 | Root MSE      | = | 10.759 |

| brate             | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|-------------------|-------------|-----------|-------|-------|----------------------|-----------|
| medage            | -109.8926   | 15.96663  | -6.88 | 0.000 | -142.0133            | -77.77189 |
| c.medage#c.medage | 1.607334    | .2707229  | 5.94  | 0.000 | 1.06271              | 2.151958  |
| _cons             | 2007.073    | 235.4316  | 8.53  | 0.000 | 1533.445             | 2480.7    |

```
. regress brate medage c.medage#c.medage i.region
```

| Source   | SS         | df | MS         | Number of obs | = | 50     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 38803.4208 | 5  | 7760.68416 | F(5, 44)      | = | 100.63 |
| Residual | 3393.39921 | 44 | 77.1227094 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.9196 |
|          |            |    |            | Adj R-squared | = | 0.9104 |
| Total    | 42196.82   | 49 | 861.159592 | Root MSE      | = | 8.782  |

| brate             | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|-------------------|-------------|-----------|-------|-------|----------------------|-----------|
| medage            | -109.0958   | 13.52452  | -8.07 | 0.000 | -136.3527            | -81.83892 |
| c.medage#c.medage | 1.635209    | .2290536  | 7.14  | 0.000 | 1.173582             | 2.096836  |
| region            |             |           |       |       |                      |           |
| N Cntrl           | 15.00283    | 4.252067  | 3.53  | 0.001 | 6.433353             | 23.57231  |
| South             | 7.366445    | 3.953335  | 1.86  | 0.069 | -.6009775            | 15.33387  |
| West              | 21.39679    | 4.650601  | 4.60  | 0.000 | 12.02412             | 30.76946  |
| _cons             | 1947.611    | 199.8405  | 9.75  | 0.000 | 1544.859             | 2350.363  |

nestreg collected the  $F$  statistic for the corresponding block of predictors and the model  $R^2$  statistic from each model fit.

The  $F$  statistic for the first block, 164.72, is for a test of the joint significance of the first block of variables; it is simply the  $F$  statistic from the regression of `brate` on `medage`. The  $F$  statistic for the second block, 35.25, is for a test of the joint significance of the second block of variables in a regression of both the first and second blocks of variables. In our example, it is an  $F$  test of `c.medage#c.medage` in the regression of `brate` on `medage` and `c.medage#c.medage`. Similarly, the third block's  $F$  statistic of 8.85 corresponds to a joint test of the indicators for the `N Cntrl`, `South`, and `West` regions in the final regression.

## Likelihood-ratio tests

The nestreg command provides a simple syntax for performing likelihood-ratio tests for nested model specifications; also see lrtest. Using the data from [example 1](#) of [\[R\] lrtest](#), we wish to jointly test the significance of the following predictors of low birthweight: age, lwt, ptl, and ht.

```
. use https://www.stata-press.com/data/r19/lbw
(Hosmer & Lemeshow data)
. nestreg, lr: logistic low (i.race smoke ui) (age lwt ptl ht)
note: 1.race omitted because of estimability.
Block 1: 2.race 3.race smoke ui
Logistic regression
Log likelihood = -107.93404
Number of obs = 189
LR chi2(4) = 18.80
Prob > chi2 = 0.0009
Pseudo R2 = 0.0801
```

| low   | Odds ratio | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-------|------------|-----------|-------|-------|----------------------|----------|
| race  |            |           |       |       |                      |          |
| Black | 3.052746   | 1.498087  | 2.27  | 0.023 | 1.166747             | 7.987382 |
| Other | 2.922593   | 1.189229  | 2.64  | 0.008 | 1.316457             | 6.488285 |
| smoke | 2.945742   | 1.101838  | 2.89  | 0.004 | 1.415167             | 6.131715 |
| ui    | 2.419131   | 1.047359  | 2.04  | 0.041 | 1.035459             | 5.651788 |
| _cons | .1402209   | .0512295  | -5.38 | 0.000 | .0685216             | .2869447 |

Note: **\_cons** estimates baseline odds.

```
Block 2: age lwt ptl ht
Logistic regression
Log likelihood = -100.724
Number of obs = 189
LR chi2(8) = 33.22
Prob > chi2 = 0.0001
Pseudo R2 = 0.1416
```

| low   | Odds ratio | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-------|------------|-----------|-------|-------|----------------------|----------|
| race  |            |           |       |       |                      |          |
| Black | 3.534767   | 1.860737  | 2.40  | 0.016 | 1.259736             | 9.918406 |
| Other | 2.368079   | 1.039949  | 1.96  | 0.050 | 1.001356             | 5.600207 |
| smoke | 2.517698   | 1.00916   | 2.30  | 0.021 | 1.147676             | 5.523162 |
| ui    | 2.1351     | .9808153  | 1.65  | 0.099 | .8677528             | 5.2534   |
| age   | .9732636   | .0354759  | -0.74 | 0.457 | .9061578             | 1.045339 |
| lwt   | .9849634   | .0068217  | -2.19 | 0.029 | .9716834             | .9984249 |
| ptl   | 1.719161   | .5952579  | 1.56  | 0.118 | .8721455             | 3.388787 |
| ht    | 6.249602   | 4.322408  | 2.65  | 0.008 | 1.611152             | 24.24199 |
| _cons | 1.586014   | 1.910496  | 0.38  | 0.702 | .1496092             | 16.8134  |

Note: **\_cons** estimates baseline odds.

| Block | LL       | LR    | df | Pr > LR | AIC      | BIC      |
|-------|----------|-------|----|---------|----------|----------|
| 1     | -107.934 | 18.80 | 4  | 0.0009  | 225.8681 | 242.0768 |
| 2     | -100.724 | 14.42 | 4  | 0.0061  | 219.448  | 248.6237 |

The estimation results from the full model are left in `e()`, so we can later use `estat` and other post-estimation commands.

```
. estat gof
Goodness-of-fit test after logistic model
Variable: low
 Number of observations = 189
Number of covariate patterns = 182
 Pearson chi2(173) = 179.24
 Prob > chi2 = 0.3567
```

## Programming for nestreg

If you want your community-contributed command (*command\_name*) to work with `nestreg`, it must follow standard Stata syntax and allow the `if` qualifier. Furthermore, *command\_name* must have `sw` or `swml` as a program property; see [P] [program properties](#). If *command\_name* has `swml` as a property, *command\_name* must store the log-likelihood value in `e(l1)` and the model degrees of freedom in `e(df_m)`.

## Stored results

`nestreg` stores the following in `r()`:

```
Matrices
 r(wald) matrix corresponding to the Wald table
 r(lr) matrix corresponding to the likelihood-ratio table
```

## Acknowledgment

We thank Paul H. Bern of Syracuse University for developing the hierarchical regression command that inspired `nestreg`.

## References

- Acock, A. C. 2023. *A Gentle Introduction to Stata*. Rev. 6th ed. College Station, TX: Stata Press.
- Lindsey, C., and S. J. Sheather. 2015. [Best subsets variable selection in nonnormal regression models](#). *Stata Journal* 15: 1046–1059.

## Also see

[P] [program properties](#) — Properties of user-defined programs

[Description](#)

[Syntax](#)

[Options](#)

[Remarks and examples](#)

[Also see](#)

## Description

`net` downloads and installs additions to Stata. The additions can be obtained from the internet or from physical media. The additions can be ado-files (new commands), help files, or even datasets. Collections of files that may be installed as a group are bound together into a *package*.

`ado` manages the packages you have installed by using `net`. The `ado` command lets you list and uninstall previously installed packages.

You can also access the `net` and `ado` features by selecting **Help > SJ and community-contributed features**; this is the recommended method to find and install additions to Stata.

## Syntax

*Set current location for net*

```
net from directory_or_url
```

*Change to a different net directory*

```
net cd path_or_url
```

*Change to a different net site*

```
net link linkname
```

*Search for installed packages*

```
net search (see [R] net search)
```

*Report current net location*

```
net
```

*Describe a package*

```
net describe pkgname [, from(directory_or_url)]
```

*Set location where packages will be installed*

```
net set ado dirname
```

*Set location where ancillary files will be installed*

```
net set other dirname
```

Report net ‘from’, ‘ado’, and ‘other’ settings

```
net query
```

Install ado-files and help files from a package

```
net install pkgname [, all replace force from(directory_or_url)]
```

Install ancillary files from a package

```
net get pkgname [, all replace force from(directory_or_url)]
```

Shortcut to access Stata Journal (SJ) net site

```
net sj vol-issue [insert]
```

List installed packages

```
ado [, find(string) from(dirname)]
ado dir [pkgid] [, find(string) from(dirname)]
```

Describe installed packages

```
ado describe [pkgid] [, find(string) from(dirname)]
```

Update installed packages

```
ado update (see [R] ado update)
```

Uninstall an installed package

```
ado uninstall pkgid [, from(dirname)]
```

where

```
pkgname is name of a package
pkgid is name of a package
 or a number in square brackets: [#]
dirname is a directory name
 or PLUS (default)
 or PERSONAL
 or SITE
```

## Options

`all` is used with `net install` and `net get`. Typing it with either one makes the command equivalent to typing `net install` followed by `net get`.

`replace` is for use with `net install` and `net get`. It specifies that the downloaded files replace existing files if any of the files already exists.

`force` specifies that the downloaded files replace existing files if any of the files already exists, even if Stata thinks all the files are the same. `force` implies `replace`.

`find(string)` is for use with `ado`, `ado dir`, and `ado describe`. It specifies that the descriptions of the packages installed on your computer be searched, and that the package descriptions containing *string* be listed.

`from(dirname)`, when used with `ado`, specifies where the packages are installed. The default is `from(PLUS)`. PLUS is a code word that Stata understands to correspond to a particular directory on your computer that was set at installation time. On Windows computers, PLUS probably means the directory `c:\ado\plus`, but it might mean something else. You can find out what it means by typing `sysdir`, but doing so is irrelevant if you use the defaults.

`from(directory_or_url)`, when used with `net`, specifies the directory or URL where installable packages may be found. The directory or URL is the same as the one that would have been specified with `net from`.

## Remarks and examples

For an introduction to using `net` and `ado`, see [\[U\] 29 Using the internet to keep up to date](#). The purpose of this documentation is

- to briefly, but accurately, describe `net` and `ado` and all their features and
- to provide documentation to those who wish to set up their own sites to distribute additions to Stata.

Remarks are presented under the following headings:

*Definition of a package*

*The purpose of the net and ado commands*

*Content pages*

*Package-description pages*

*Where packages are installed*

*A summary of the net command*

*A summary of the ado command*

*Relationship of net and ado to the point-and-click interface*

*Creating your own site*

*Format of content and package-description files*

*Example 1*

*Example 2*

*Additional package directives*

*SMCL in content and package-description files*

*Error-free file delivery*

## Definition of a package

A *package* is a collection of files—typically, `.ado` and `.sthlp` files—that together provide a new feature in Stata. Packages contain additions that you wish had been part of Stata at the outset. For instance, the package named `zz49` might add the `xyz` command to Stata. At a minimum, such a package would contain `xyz.ado`, the code to implement the new command, and `xyz.sthlp`, the system help to describe it. We write such additions, and so do other users.

One source of these additions is the *Stata Journal*, a printed and electronic journal with corresponding software. If you want the journal, you must subscribe, but the software is available for free from our website.

## The purpose of the net and ado commands

The `net` command makes it easy to distribute and install packages. The goal is to get you quickly to a package-description page that summarizes the addition, for example,

---

```
. net describe rte_stat, from(http://www.wemakeitupaswego.edu/faculty/sgazer/)
```

---

```
package rte_stat from http://www.wemakeitupaswego.edu/faculty/sgazer/
```

---

### TITLE

```
rte_stat. The robust-to-everything statistic; update.
```

### DESCRIPTION/AUTHOR(S)

```
S. Gazer, Dept. of Applied Theoretical Mathematics, WMIUAWG Univ.
Aleph-0 100% confidence intervals proved too conservative for some
applications; Aleph-1 confidence intervals have been substituted.
The new robust-to-everything supplants the previous robust-to-
everything-conceivable statistic. See "Inference in the absence
of data" (forthcoming). After installation, see help rte.
```

### INSTALLATION FILES

```
rte.ado
rte.sthlp
nullset.ado
random.ado
```

---

```
(type net install rte_stat)
```

If you decide that the addition might prove useful, `net` makes the installation easy:

```
. net install rte_stat
checking rte_stat consistency and verifying not already installed...
installing into c:\ado\plus\ ...
installation complete.
```



A content page tells you about other content pages and package-description pages. The example above lists other content pages only. Below, we follow one of the links for the *Stata Journal*:

```
. net link sj
```

---

```
https://www.stata-journal.com/
The Stata Journal
```

---

The Stata Journal is a refereed, quarterly journal containing articles of interest to Stata users. For more details and subscription information, visit the Stata Journal website at <https://www.stata-journal.com>.

PLACES you could **-net link-** to:

|       |                   |
|-------|-------------------|
| stata | StataCorp website |
|-------|-------------------|

DIRECTORIES you could **-net cd-** to:

|            |                                                 |
|------------|-------------------------------------------------|
| production | Files for authors of the Stata Journal          |
| software   | Software associated with Stata Journal articles |

---

```
. net cd software
```

---

```
https://www.stata-journal.com/software/
The Stata Journal
```

---

PLACES you could **-net link-** to:

|       |                                                                |
|-------|----------------------------------------------------------------|
| stata | StataCorp website                                              |
| stb   | Historical materials published in the Stata Technical Bulletin |

DIRECTORIES you could **-net cd-** to:

|                  |                   |
|------------------|-------------------|
| (output omitted) |                   |
| sj7-1            | volume 7, issue 1 |
| (output omitted) |                   |
| sj1-1            | volume 1, issue 1 |

---

```
. net cd sj7-1
```

---

```
https://www.stata-journal.com/software/sj7-1/
Stata Journal volume 7, issue 1
```

---

DIRECTORIES you could **-net cd-** to:

|    |                      |
|----|----------------------|
| .. | Other Stata Journals |
|----|----------------------|

PACKAGES you could **-net describe-**:

|        |                                                                                             |
|--------|---------------------------------------------------------------------------------------------|
| dm0027 | File filtering in Stata: handling complex data formats and navigating log files efficiently |
| st0119 | Rasch analysis                                                                              |
| st0120 | Multivariable regression spline models                                                      |
| st0121 | mhbounds - Sensitivity Analysis for Average Treatment Effects                               |

---

dm0027, st0119, ..., st0121 are links to package-description pages.

1. When you type `net from`, you follow that with a location to display the location's content page.
  - a. The location could be a URL, such as <https://www.stata.com>. The content page at that location would then be listed.
  - b. The location could be `e:` on a Windows computer or a mounted volume on a Mac computer. The content page on that source would be listed. That would work if you had special media obtained from StataCorp or special media prepared by another user.

- c. The location could even be a directory on your computer, but that would work only if that directory contained the right kind of files.
2. Once you have specified a location, typing `net cd` will take you into subdirectories of that location, if there are any. Typing

```
. net from https://www.stata-journal.com
. net cd software
```

is equivalent to typing

```
. net from https://www.stata-journal.com/software
```

Typing `net cd` displays the content page from that location.

3. Typing `net` without arguments redisplay the current content page, which is the content page last displayed.
4. `net link` is similar to `net cd` in that the result is to change the location, but rather than changing to subdirectories of the current location, `net link` jumps to another location:

```
. net from https://www.stata-journal.com
```

---

```
https://www.stata-journal.com/
```

**The Stata Journal**

---

The Stata Journal is a refereed, quarterly journal containing articles of interest to Stata users. For more details and subscription information, visit the Stata Journal website at <https://www.stata-journal.com>.

PLACES you could **-net link-** to:

|       |                   |
|-------|-------------------|
| stata | StataCorp website |
|-------|-------------------|

DIRECTORIES you could **-net cd-** to:

|            |                                                 |
|------------|-------------------------------------------------|
| production | Files for authors of the Stata Journal          |
| software   | Software associated with Stata Journal articles |

---

Typing `net link stata` would jump to <https://www.stata.com>:

```
. net link stata
```

---

```
https://www.stata.com/
```

**StataCorp**

---

Welcome to StataCorp.

*(output omitted)*

---

## Package-description pages

Package-description pages describe what could be installed:

```
. net from https://www.stata-journal.com/software/sj7-1
```

---

```
https://www.stata-journal.com/software/sj7-1/
(output omitted)
```

```
. net describe st0119
```

---

```
package st0119 from https://www.stata-journal.com/software/sj7-1
```

---

### TITLE

SJ7-1 st0119. Rasch analysis

### DESCRIPTION/AUTHOR(S)

Rasch analysis

by Jean-Benoit Hardouin, University of Nantes, France

Support: jean-benoit.hardouin@univ-nantes.fr

After installation, type help **gammasy**, **gausshermite**,  
**geekel2d**, **raschtest**, and **raschtestv7**

### INSTALLATION FILES

(type **net install st0119**)

```
st0119/raschtest.ado
st0119/raschtest.hlp
st0119/raschtestv7.ado
st0119/raschtestv7.hlp
st0119/gammasy.ado
st0119/gammasy.hlp
st0119/gausshermite.ado
st0119/gausshermite.hlp
st0119/geekel2d.ado
st0119/geekel2d.hlp
```

### ANCILLARY FILES

(type **net get st0119**)

```
st0119/data.dta
st0119/outrasch.do
```

---

A package-description page describes the package and tells you how to install the component files. Package-description pages potentially describe two types of files:

1. Installation files: files that you type `net install` to install and that are required to make the addition work.
2. Ancillary files: additional files that you might want to install—you type `net get` to install them—but that you can ignore. Ancillary files are typically datasets that are useful for demonstration purposes. Ancillary files are not really installed in the sense of being copied to an official place for use by Stata itself. They are merely copied into the current directory so that you may use them if you wish.

You install the official files by typing `net install` followed by the package name. For example, to install `st0119`, you would type

```
. net install st0119
checking st0119 consistency and verifying not already installed...
installing into c:\ado\plus\ ...
installation complete.
```

You get the ancillary files—if there are any and if you want them—by typing `net get` followed by the package name:

```
. net get st0119
checking st0119 consistency and verifying not already installed...
copying into current directory...
 copying data.dta
 copying outrasch.do
ancillary files successfully copied.
```

Most users ignore the ancillary files.

Once you have installed a package—by typing `net install`—use `ado` to redisplay the package-description page whenever you wish:

```
. ado describe st0119
```

---

```
[1] package st0119 from https://www.stata-journal.com/software/sj7-1
```

---

**TITLE**

SJ7-1 st0119. Rasch analysis

**DESCRIPTION/AUTHOR(S)**

Rasch analysis  
 by Jean-Benoit Hardouin, University of Nantes, France  
 Support: [jean-benoit.hardouin@univ-nantes.fr](mailto:jean-benoit.hardouin@univ-nantes.fr)  
 After installation, type help **gammasym**, **gausshermite**,  
**geekel2d**, **raschtest**, and **raschtestv7**

**INSTALLATION FILES**

r/raschtest.ado  
 r/raschtest.hlp  
 r/raschtestv7.ado  
 r/raschtestv7.hlp  
 g/gammasym.ado  
 g/gammasym.hlp  
 g/gausshermite.ado  
 g/gausshermite.hlp  
 g/geekel2d.ado  
 g/geekel2d.hlp

**INSTALLED ON**

24 Feb 2025

---

The package-description page shown by `ado` includes the location from which we got the package and when we installed it. It does not mention the ancillary files that were originally part of this package because they are not tracked by `ado`.

## Where packages are installed

Packages should be installed in PLUS or SITE; these are code words that Stata understands and correspond to real directories on your computer. Typing `sysdir` will tell you where these are, if you care.

```
. sysdir
 STATA: C:\Program Files\Stata19\
 BASE: C:\Program Files\Stata19\ado\base\
 SITE: C:\Program Files\Stata19\ado\site\
 PLUS: c:\ado\plus\
 PERSONAL: c:\ado\personal\
 OLDPLACE: c:\ado\
```

If you type `sysdir`, you may obtain different results.

By default, `net` installs in the PLUS directory, and `ado` tells you about what is installed there. If you are on a multiple-user system, you may wish to install some packages in the SITE directory. This way, they will be available to other Stata users. To do that, before using `net install`, type

```
. net set ado SITE
```

and when reviewing what is installed or removing packages, redirect `ado` to that directory:

```
. ado ..., from(SITE)
```

In both cases, you type SITE because Stata will understand that SITE means the site `ado`-directory as defined by `sysdir`. To install into SITE, you must have write access to that directory.

If you reset where `net` installs and then, in the same session, wish to install into your private `ado`-directory, type

```
. net set ado PLUS
```

That is how things were originally. If you are confused as to where you are, type `net query`.

## A summary of the net command

The `net` command displays content pages and package-description pages. Such pages are provided over the internet, and most users get them there. We recommend that you start at <https://www.stata.com> and work out from there. We also recommend using `net search` to find packages of interest to you; see [R] [net search](#).

`net from` moves you to a location and displays the content page.

`net cd` and `net link` change from your current location to other locations. `net cd` enters subdirectories of the original location. `net link` jumps from one location to another, depending on the code on the content page.

`net describe` lists a package-description page. Packages are named, and you type `net describe pkgname`.

`net install` installs a package into your copy of Stata. `net get` copies any additional files (ancillary files) to your current directory.

net sj simplify loading files from the *Stata Journal*.

```
net sj vol-issue
```

is a synonym for typing

```
net from https://www.stata-journal.com/software/sjvol-issue
```

whereas

```
net sj vol-issue insert
```

is a synonym for typing

```
net from https://www.stata-journal.com/software/sjvol-issue
net describe insert
```

net set controls where net installs files. By default, net installs in the PLUS directory; see [P] **sysdir**. net set ado SITE would cause subsequent net commands to install in the SITE directory. net set other sets where ancillary files, such as .dta files, are installed. The default is the current directory.

net query displays the current net from, net set ado, and net set other settings.

## A summary of the ado command

The ado command lists the package descriptions of previously installed packages.

Typing ado without arguments is the same as typing ado dir. Both list the names and titles of the packages you have installed.

ado describe lists full package-description pages.

ado uninstall removes packages from your computer.

Because you can install packages from a variety of sources, the package names may not always be unique. Thus the packages installed on your computer are numbered sequentially, and you may refer to them by name or by number. For instance, say that you wanted to get rid of the robust-to-everything statistic command you installed. Type

```
. ado, find("robust-to-everything")
[15] package rte_stat from http://www.wemakeitupaswego.edu/faculty/sgazer
rte_stat. The robust-to-everything statistic; update.
```

You could then type

```
. ado uninstall rte_stat
```

or

```
. ado uninstall [15]
```

Typing ado uninstall rte\_stat would work only if the name rte\_stat were unique; otherwise, ado would refuse, and you would have to type the number.

The `find()` option is allowed with `ado dir` and `ado describe`. It searches the package description for the word or phrase you specify, ignoring case (alpha matches Alpha). The complete package description is searched, including the author's name and the name of the files. Thus if `rte` was the name of a command that you wanted to eliminate, but you could not remember the name of the package, you could type

```
. ado, find(rte)
[15] package rte_stat from http://www.wemakeitupaswego.edu/faculty/sgazer
rte_stat. The robust-to-everything statistic; update.
```

## Relationship of net and ado to the point-and-click interface

Users may instead select **Help > SJ and community-contributed features**. There are advantages and disadvantages:

1. Flipping through content and package-description pages is easier; it is much like a browser. See [\[GSM\] 19 Updating and extending Stata—internet functionality](#), [\[GSU\] 19 Updating and extending Stata—internet functionality](#), or [\[GSW\] 19 Updating and extending Stata—internet functionality](#).
2. When browsing a product-description page, note that the `.sthlp` files are highlighted. You may click on `.sthlp` files to review them before installing the package.
3. You may not redirect from where `ado` searches for files.

## Creating your own site

Users who wish to share content with the Stata community often do so via the Statistical Software Components (SSC) Archive. See [\[R\] ssc](#) and also see <http://repec.org/bocode/s/sscsubmit.html>.

The rest of this entry concerns how to create your own site to distribute additions to Stata. The idea is that you have written additions for use with Stata—say, `xyz.ado` and `xyz.sthlp`—and you wish to make them available so that others can easily install them. Or, perhaps you just have a dataset that you and others want to share.

In any case, all you need is a webpage. You place the files that you want to distribute on your webpage, and you add two more files—a content file and a package-description file—and you are done.

## Format of content and package-description files

The content file describes the content page. It must be named `stata.toc`:

---

```

OFF (to make site unavailable temporarily)
* lines starting with * are comments; they are ignored
* blank lines are ignored, too
* v indicates version---specify v 3, which is the current version of .toc files
v 3
* d lines display description text
* the first d line is the title, and the remaining ones are text
* blank d lines display a blank line
d title
d text
d text
d
...
* l lines display links
l word-to-show path-or-url [description]
l word-to-show path-or-url [description]
...
* t lines display other directories within the site
t path [description]
t path [description]
...
* p lines display packages
p pkgname [description]
p pkgname [description]
...

```

---

end stata.toc

Package files describe packages and are named `pkgname.pkg`:

---

```

* lines starting with * are comments; they are ignored
* blank lines are ignored, too
* v indicates version---specify v 3, which is the current version of .toc files
v 3
* d lines display package description text
* the first d line is the title, and the remaining ones are text
* blank d lines display a blank line
d title
d text
d Distribution-Date: date
d text
d
...
* f identifies the component files
f [path/]filename [description]
f [path/]filename [description]
...
* e line is optional; it means stop reading
e

```

---

end pkgname.pkg

Note the Distribution-Date description line. This line is optional but recommended. Stata can look for updates to community-contributed programs with the `ado update` command if the package files from which those programs were installed contain a Distribution-Date description line.

## Example 1

Say that we want the user to see the following:

```
. net from http://www.university.edu/~me

http://www.university.edu/~me
Chris Farrar, Uni University

PACKAGES you could -net describe-:
 xyz interval-truncated survival

. net describe xyz

package xyz from http://www.university.edu/~me

TITLE
 xyz. interval-truncated survival.
DESCRIPTION/AUTHOR(S)
 C. Farrar, Uni University.
INSTALLATION FILES (type net install xyz)
 xyz.ado
 xyz.sthlp
ANCILLARY FILES (type net get xyz)
 sample.dta

```

The files needed to do this would be

```
-----begin stata.toc-----
v 3
d Chris Farrar, Uni University
p xyz interval-truncated survival
-----end stata.toc-----
-----begin xyz.pkg-----
v 3
d xyz. interval-truncated survival.
d C. Farrar, Uni University.
f xyz.ado
f xyz.sthlp
f sample.dta
-----end xyz.pkg-----
```

On his homepage, Chris would place the following files:

|            |                                                             |
|------------|-------------------------------------------------------------|
| stata.toc  | (shown above)                                               |
| xyz.pkg    | (shown above)                                               |
| xyz.ado    | file to be delivered (for use by <code>net install</code> ) |
| xyz.sthlp  | file to be delivered (for use by <code>net install</code> ) |
| sample.dta | file to be delivered (for use by <code>net get</code> )     |

Chris does nothing to distinguish ancillary files from installation files.

## Example 2

S. Gazer wants to create a more complex site:

```
. net from http://www.wemakeitupaswego.edu/faculty/sgazer
```

---

```
http://www.wemakeitupaswego.edu/faculty/sgazer
```

```
Data-free inference materials
```

---

S. Gazer, Department of Applied Theoretical Mathematics

Also see my homepage for the preprint of "Irrefutable inference".

PLACES you could **-net link-** to:

```
stata StataCorp website
```

DIRECTORIES you could **-net cd-** to:

```
ir irrefutable inference programs (work in progress)
```

PACKAGES you could **-net describe-**:

```
rtec Robust-to-everything-conceivable statistic
```

```
rte Robust-to-everything statistic
```

```
. net describe rte
```

---

```
package rte from http://www.wemakeitupaswego.edu/faculty/sgazer/
```

---

### TITLE

```
rte. The robust-to-everything statistic; update.
```

### DESCRIPTION/AUTHOR(S)

```
S. Gazer, Dept. of Applied Theoretical Mathematics, WMIUWG Univ.
Aleph-0 100% confidence intervals proved too conservative for some
applications; Aleph-1 confidence intervals have been substituted.
The new robust-to-everything supplants the previous robust-to-
everything-conceivable statistic. See "Inference in the absence
of data" (forthcoming). After installation, see help rte.
```

```
Distribution-Date: 20190320
```

```
Support: email sgazer@wemakeitupaswego.edu
```

### INSTALLATION FILES

```
rte.ado
rte.sthlp
nullset.ado
random.ado
```

```
(type net install rte_stat)
```

### ANCILLARY FILES

```
empty.dta
```

```
(type net get rte_stat)
```

---

The files needed to do this would be

---

```

begin stata.toc
v 3
d Data-free inference materials
d S. Gazer, Department of Applied Theoretical Mathematics
d
d Also see my homepage for the preprint of "Irrefutable inference".
l stata https://www.stata.com
t ir irrefutable inference programs (work in progress)
p rtec Robust-to-everything-conceivable statistic
p rte Robust-to-everything statistic

```

---

end stata.toc

---

```

begin rte.pkg
v 3
d rte. The robust-to-everything statistic; update.
d {bf:S. Gazer, Dept. of Applied Theoretical Mathematics, WMIUAWG Univ.}
d Aleph-0 100% confidence intervals proved too conservative for some
d applications; Aleph-1 confidence intervals have been substituted.
d The new robust-to-everything supplants the previous robust-to-
d everything-conceivable statistic. See "Inference in the absence
d of data" (forthcoming). After installation, see help {bf:rte}.
d
d Distribution-Date: 20190320
d
d Support: email sgazer@wemakeitupaswego.edu
f rte.ado
f rte.sthlp
f nullset.ado
f random.ado
f empty.dta

```

---

end rte.pkg

On his homepage, Mr. Gazer would place the following files:

|              |                                                     |
|--------------|-----------------------------------------------------|
| stata.toc    | (shown above)                                       |
| rte.pkg      | (shown above)                                       |
| rte.ado      | (file to be delivered)                              |
| rte.sthlp    | (file to be delivered)                              |
| nullset.ado  | (file to be delivered)                              |
| random.ado   | (file to be delivered)                              |
| empty.dta    | (file to be delivered)                              |
| rtec.pkg     | the other package referred to in stata.toc          |
| rtec.ado     | the corresponding files to be delivered             |
| rtec.sthlp   |                                                     |
| ir/stata.toc | the contents file for when the user types net cd ir |
| ir/...       | whatever other .pkg files are referred to           |
| ir/...       | whatever other files are to be delivered            |

If Mr. Gazer later updated the `rte` package, he could change the Distribution-Date description line in his package. Then, if someone who had previously installed the `rte` packaged wanted to obtain the latest version, that person could use the `ado update` command; see [R] [ado update](#).

For complex sites, a different structure may prove more convenient:

|                              |                                                                  |
|------------------------------|------------------------------------------------------------------|
| <code>stata.toc</code>       | (shown above)                                                    |
| <code>rte.pkg</code>         | (shown above)                                                    |
| <code>rtec.pkg</code>        | the other package referred to in <code>stata.toc</code>          |
| <code>rte/</code>            | directory containing <code>rte</code> files to be delivered:     |
| <code>rte/rte.ado</code>     | (file to be delivered)                                           |
| <code>rte/rte.sthlp</code>   | (file to be delivered)                                           |
| <code>rte/nullset.ado</code> | (file to be delivered)                                           |
| <code>rte/random.ado</code>  | (file to be delivered)                                           |
| <code>rte/empty.dta</code>   | (file to be delivered)                                           |
| <code>rtec/</code>           | directory containing <code>rtec</code> files to be delivered:    |
| <code>rtec/...</code>        | (files to be delivered)                                          |
| <code>ir/stata.toc</code>    | the contents file for when the user types <code>net cd ir</code> |
| <code>ir/*.pkg</code>        | whatever other package files are referred to                     |
| <code>ir/*/...</code>        | whatever other files are to be delivered                         |

If you prefer this structure, it is simply a matter of changing the bottom of the `rte.pkg` from

```
f rte.ado
f rte.sthlp
f nullset.ado
f random.ado
f empty.dta
```

to

```
f rte/rte.ado
f rte/rte.sthlp
f rte/nullset.ado
f rte/random.ado
f rte/empty.dta
```

In writing paths and files, the directory separator forward slash (/) is used, regardless of operating system, because this is what the internet uses.

It does not matter whether the files you put out are in Windows, Mac, or Unix format (how lines end is recorded differently). When Stata reads the files over the internet, it will figure out the file format on its own and will automatically translate the files to what is appropriate for the receiver.

## Additional package directives

*F filename* is similar to *f filename*, except that, when the file is installed, it will always be copied to the system directories (and not the current directory).

With *f filename*, the file is installed into a directory according to the file's suffix. For instance, `xyz.ado` would be installed in the system directories, whereas `xyz.dta` would be installed in the current directory.

Coding `F xyz.ado` would have the same result as coding `f xyz.ado`.

Coding `F xyz.dta`, however, would state that `xyz.dta` is to be installed in the system directories.

*g platformname filename* is also a variation on *f filename*. It specifies that the file be installed only if the user's operating system is of type *platformname*; otherwise, the file is ignored. The platform names are WIN64 (64-bit x86-64) for Windows; MACARM64 (Apple Silicon, GUI) and OSX.ARM64 (Apple Silicon, console), MACINTEL64 (64-bit Intel, GUI) and OSX.X8664 (64-bit Intel, console) for Mac;

and LINUX64 (64-bit x86-64) and LINUX64P (64-bit x86-64, libpng v1.6) for Linux. For LINUX64P, the LINUX64-specific version of a file will be installed if no LINUX64P-specific one exists. LINUX64 will not install a LINUX64P-specific version of a file because of possible changes in system libraries.

*G platformname filename* is a variation on *F filename*. The file, if not ignored, is to be installed in the system directories.

*g platformname filename1 filename2* is a more detailed version of *g platformname filename*. In this case, *filename1* is the name of the file on the server (the file to be copied), and *filename2* is to be the name of the file on the user's system; for example, you might code

```
g WIN64 mydll.forwin mydll.plugin
g LINUX64 mydll.forlinux mydll.plugin
```

When you specify one *filename*, the result is the same as specifying two identical *filenames*.

*G platformname filename1 filename2* is the install-in-system-directories version of *g platformname filename1 filename2*.

*h filename* asserts that *filename* must be loaded, or this package is not to be installed; for example, you might code

```
g WIN64 mydll.forwin mydll.plugin
g LINUX64 mydll.forlinux mydll.plugin
h mydll.plugin
```

if you were offering the plugin `mydll.plugin` for Windows and Linux only.

## SMCL in content and package-description files

The text listed on the second and subsequent `d` lines in both `stata.toc` and `pkgname.pkg` may contain SMCL as long as you include `v 3`; see [P] [smcl](#).

Thus, in `rte.pkg`, S. Gazer coded the third line as

```
d {bf:S. Gazer, Dept. of Applied Theoretical Mathematics, WMIUAWG Univ.}
```

## Error-free file delivery

Most people transport files over the internet and never worry about the file being corrupted in the process because corruption rarely occurs. If, however, the files must be delivered perfectly or not at all, you can include checksum files in the directory.

For instance, say that `big.dta` is included in your package and that it must be sent perfectly. First, use Stata to make the checksum file for `big.dta`

```
. checksum big.dta, save
```

That command creates a small file called `big.sum`; see [D] [checksum](#). Then, copy both `big.dta` and `big.sum` to your homepage.

If you do this, be cautious. If you put `big.dta` and `big.sum` on your homepage and then later change `big.dta` without changing `big.sum`, people will think that there are transmission errors when they try to download `big.dta`.

## Also see

[R] **ado update** — Update community-contributed packages

[R] **net search** — Search the internet for installable packages

[R] **netio** — Control internet connections

[R] **search** — Search Stata documentation and other resources

[R] **sj** — Stata Journal installation instructions

[R] **ssc** — Install and uninstall packages from SSC

[R] **update** — Check for official updates

[D] **checksum** — Calculate checksum of file

[P] **smcl** — Stata Markup and Control Language

[GSM] **19 Updating and extending Stata—internet functionality**

[GSU] **19 Updating and extending Stata—internet functionality**

[GSW] **19 Updating and extending Stata—internet functionality**

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## Description

`net search` searches the internet for community-contributed additions to Stata, including, but not limited to, community-contributed additions published in the *Stata Journal* (SJ). `net search` lists the available additions that contain the specified keywords.

The community-contributed materials found are available for immediate download by using the `net` command or by clicking on the link.

In addition to typing `net search`, you may select **Help > Search...** and choose **Search net resources**. This is the recommended way to search for community-contributed additions to Stata.

## Quick start

Search community-contributed commands for all terms `word1`, `word2`, and `word3`

```
net search word1 word2 word3
```

Same as above, but search for any term

```
net search word1 word2 word3, or
```

Search internet sources other than the *Stata Journal*

```
net search word1 word2 word3, nosj
```

## Syntax

```
net search word [word ...] [, options]
```

| <i>options</i>          | Description                                                    |
|-------------------------|----------------------------------------------------------------|
| <code>or</code>         | list packages that contain any of the keywords; default is all |
| <code>nosj</code>       | search non-SJ sources                                          |
| <code>tocpkg</code>     | search both tables of contents and packages; the default       |
| <code>toc</code>        | search tables of contents only                                 |
| <code>pkg</code>        | search packages only                                           |
| <code>everywhere</code> | search packages for match                                      |
| <code>filenames</code>  | search filenames associated with package for match             |
| <code>errnone</code>    | make return code 111 instead of 0 when no matches found        |

## Options

`or` is relevant only when multiple keywords are specified. By default, `net search` lists only packages that include all the keywords. `or` changes the command to list packages that contain any of the keywords.

`nosj` specifies that `net search` not list matches that were published in the SJ.

`tocpkg`, `toc`, and `pkg` determine what is searched. `tocpkg` is the default, meaning that both tables of contents (tocs) and packages (pkgs) are searched. `toc` restricts the search to tables of contents. `pkg` restricts the search to packages.

`everywhere` and `filenames` determine where in packages `net search` looks for *keywords*. The default is `everywhere`. `filenames` restricts `net search` to search for matches only in the filenames associated with a package. Specifying `everywhere` implies `pkg`.

`errnone` is a programmer's option that causes the return code to be 111 instead of 0 when no matches are found.

## Remarks and examples

`net search` searches the internet for community-contributed additions to Stata. If you want to search the Stata documentation for a particular topic, command, or author, see [\[R\] search](#). `net search word [word ...]` (without options) is equivalent to typing `search word [word ...]`, `net`.

Remarks are presented under the following headings:

*Topic searches*  
*Author searches*  
*Command searches*  
*Where does net search look?*  
*How does net search work?*

## Topic searches

### Example: Find what is available about random effects

```
. net search random effect
```

Comments:

- It is best to search using the singular form of a word. `net search random effect` will find both “random effect” and “random effects”.
- `net search random effect` will also find “random-effect” because `net search` performs a string search and not a word search.
- `net search random effect` lists all packages containing the words “random” and “effect”, not necessarily used together.
- If you wanted all packages containing the word “random” or the word “effect”, you would type `net search random effect, or`.

## Author searches

### Example: Find what is available by author Jeroen Weesie

```
. net search weesie
```

Comments:

- You could type `net search jeroen weesie`, but that might list fewer results because sometimes the last name is used without the first.
- You could type `net search Weesie`, but it would not matter. Capitalization is ignored in the search.

### Example: Find what is available by Jeroen Weesie, excluding SJ materials

```
. net search weesie, nosj
```

- The SJ tends to dominate search results because so much has been published in it. If you know that what you are looking for is not in the SJ, specifying the `nosj` option will narrow the search.
- `net search weesie` lists everything that `net search weesie, nosj` lists, and more. If you just type `net search weesie`, look down the list. SJ materials are listed first, and non-SJ materials are listed last.

## Command searches

### Example: Find the community-contributed command kursus

```
. net search kursus, file
```

- You could just type `net search kursus`, and that will list everything `net search kursus, file` lists, and more. Because you know `kursus` is a command, however, there must be a `kursus.ado` file associated with the package. Typing `net search kursus, file` narrows the search.
- You could also type `net search kursus.ado, file` to narrow the search even more.

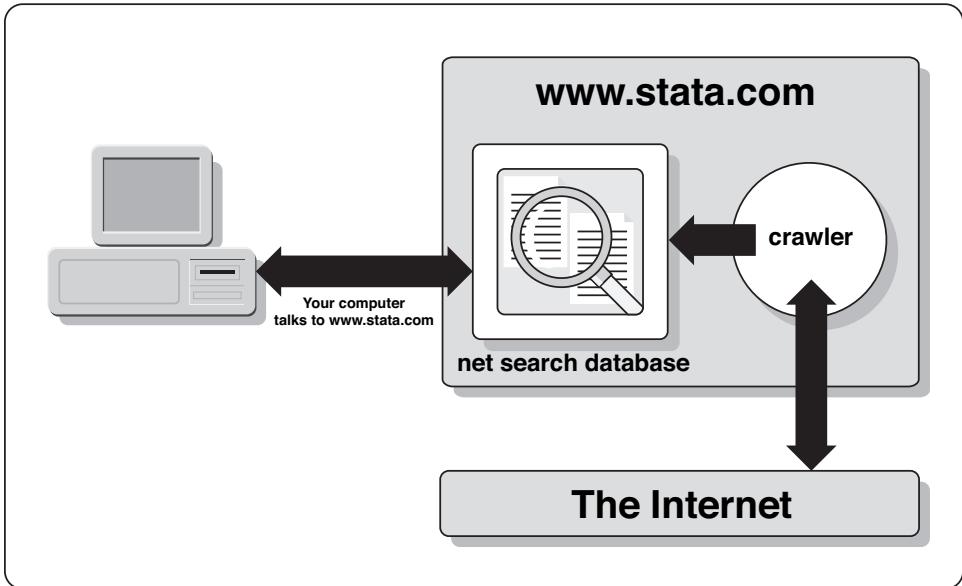
## Where does net search look?

`net search` looks everywhere, not just at <https://www.stata.com>.

`net search` begins by looking at <https://www.stata.com>, but then follows every link, which takes it to other places, and then follows every link again, which takes it to even more places, and so on.

Authors: Please let us know if you have a site that we should include in our search by sending an email to [webmaster@stata.com](mailto:webmaster@stata.com). We will then link to your site from ours to ensure that `net search` finds your materials. That is not strictly necessary, however, as long as your site is directly or indirectly linked from some site that is linked to ours.

## How does net search work?



Our website maintains a database of Stata resources. When you use `net search`, it contacts <https://www.stata.com> with your request, <https://www.stata.com> searches its database, and Stata returns the results to you.

Another part of the system is called the crawler, which searches the web for new Stata resources to add to the `net search` database and verifies that the resources already found are still available. When a new resource becomes available, the crawler takes about two days to add it to the database, and, similarly, if a resource disappears, the crawler takes roughly two days to remove it from the database.

## Also see

[R] `ado update` — Update community-contributed packages

[R] `net` — Install and manage community-contributed additions from the internet

[R] `search` — Search Stata documentation and other resources

[R] `sj` — Stata Journal installation instructions

[R] `ssc` — Install and uninstall packages from SSC

[R] `update` — Check for official updates

## Description

Some commands (for example, `net` and `update`) are designed specifically for use over the internet. Many other Stata commands that read a file (for example, `copy`, `type`, and `use`) can also read directly from a URL. All of these commands will usually work without your ever needing to concern yourself with the `set` commands discussed here. These `set` commands provide control over network system parameters.

If you experience problems when using Stata's network features, ask your system administrator if your site uses a proxy. A proxy is a server between your computer and the rest of the internet, and your computer may need to communicate with other computers on the internet through this proxy. If your site uses a proxy, your system administrator can provide you with its host name and the port your computer can use to communicate with it. If your site's proxy requires you to log in to it before it will respond, your system administrator will provide you with a user ID and password.

`set httpproxyhost` sets the name of the host to be used as a proxy server. `set httpproxyport` sets the port number. `set httpproxy` turns on or off the use of a proxy server, leaving the proxy host name and port intact, even when not in use.

Under the Mac and Windows operating systems, when you `set httpproxy on`, Stata will attempt to obtain the values of `httpproxyhost` and `httpproxyport` from the operating system if they have not been previously set. `set httpproxy on, init` attempts to obtain these values from the operating system, even if they have been previously set.

If the proxy requires authorization (user ID and password), `set authorization on` via `set httpproxyauth on`. The proxy user and proxy password must also be set to the appropriate user ID and password by using `set httpproxyuser` and `set httpproxypw`.

Stata remembers the various proxy settings between sessions and does not need a permanently option.

## Syntax

*Turn on or off the use of a proxy server*

```
set httpproxy {on | off} [, init]
```

*Set proxy host name*

```
set httpproxyhost ["name"]
```

*Set the proxy port number*

```
set httpproxyport #
```

*Turn on or off proxy authorization*

```
set httpproxyauth {on | off}
```

*Set proxy authorization user ID*

```
set httpproxyuser ["name"]
```

*Set proxy authorization password*

```
set httpproxypw ["password"]
```

## Options

`init` specifies that `set httpproxy` on attempts to initialize `httpproxyhost` and `httpproxyport` from the operating system (Mac and Windows only).

## Remarks and examples

If you receive an error message, see <https://www.stata.com/support/faqs/web/> for the latest information.

## Also see

[R] **query** — Display system parameters

[P] **creturn** — Return c-class values

[U] **29 Using the internet to keep up to date**

[Description](#)[Options](#)[Acknowledgments](#)[Quick start](#)[Remarks and examples](#)[References](#)[Menu](#)[Stored results](#)[Also see](#)[Syntax](#)[Methods and formulas](#)

## Description

`nl` fits an arbitrary nonlinear regression function by least squares. With the interactive version of the command, you enter the function directly on the command line or in the dialog box by using a *substitutable expression*. If you have a function that you use regularly, you can write a *substitutable expression program* and use the second syntax to avoid having to reenter the function every time. The function evaluator program version gives you the most flexibility in exchange for increased complexity; with this version, your program is given a vector of parameters and a variable list, and your program computes the regression function.

When you write a substitutable expression program or function evaluator program, the first two letters of the name must be `nl`. *sexp\_prog* and *func\_prog* refer to the name of the program without the first two letters. For example, if you wrote a function evaluator program named `nlregss`, you would type `nl regss @ . . .` to estimate the parameters.

## Quick start

Linear model of  $y$  as a function of  $x_1$  and a constant

```
nl (y = {b0} + {b1}*x1)
```

Same as above, but defining the linear combination `lc`

```
nl (y = {lc: x1, xb})
```

Same as above, but specify starting values

```
nl (y = {b0=.5} + {b1=2}*x1)
```

Add variables  $x_2$  and  $x_3$  to the linear combination

```
nl (y = {lc: x1 x2 x3})
```

Same as above, but explicitly specify a constant term `b0` with starting value

```
nl (y = {b0=.5} + {lc: x1 x2 x3, noconstant})
```

An exponential model

```
nl (y = {b0} + {b1}*{b2}^x1)
```

Same as above, but use `nl`'s built-in function `exp3` to specify the model

```
nl exp3: y x1
```

## Menu

Statistics > Linear models and related > Nonlinear least-squares estimation

## Syntax

### Interactive version

```
nl (devar = <sexp>) [if] [in] [weight] [, options]
```

### Programmed substitutable expression version

```
nl sexp_prog : devar [varlist] [if] [in] [weight] [, options]
```

### Function evaluator program version

```
nl func_prog @ devar [varlist] [if] [in] [weight] ,
 { parameters(namelist) | nparameters(#) } [options]
```

where

*devar* is the dependent variable;

<*sexp*> is a substitutable expression;

*sexp\_prog* is a substitutable expression program; and

*func\_prog* is a function evaluator program.

| <i>options</i>                                                                                                                                                                                                                                                                                                         | Description                                                                                                                                                                     |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Model                                                                                                                                                                                                                                                                                                                  |                                                                                                                                                                                 |
| <u>define</u> ( <i>name</i> :< <i>subexpr</i> >)                                                                                                                                                                                                                                                                       | define a function of model parameters; this option may be repeated (interactive version only)                                                                                   |
| * <u>parameters</u> ( <i>namelist</i> )                                                                                                                                                                                                                                                                                | specify parameters in model (function evaluator program version only)                                                                                                           |
| * <u>nparameters</u> (#)                                                                                                                                                                                                                                                                                               | specify number of parameters in model (function evaluator program version only)                                                                                                 |
| <u>variables</u> ( <i>varlist</i> )                                                                                                                                                                                                                                                                                    | specify variables in model                                                                                                                                                      |
| <i>sexp_options</i>                                                                                                                                                                                                                                                                                                    | options for substitutable expression program                                                                                                                                    |
| <i>func_options</i>                                                                                                                                                                                                                                                                                                    | options for function evaluator program                                                                                                                                          |
| Model 2                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                                 |
| <u>lnlsq</u> (#)                                                                                                                                                                                                                                                                                                       | use log least-squares where $\ln(\text{depvar} - \#)$ is assumed to be normally distributed                                                                                     |
| <u>noconstant</u>                                                                                                                                                                                                                                                                                                      | the model has no constant term; seldom used                                                                                                                                     |
| <u>hasconstant</u> ( <i>name</i> )                                                                                                                                                                                                                                                                                     | use <i>name</i> as constant term; seldom used                                                                                                                                   |
| <u>constraints</u> ( <i>constraints</i> )                                                                                                                                                                                                                                                                              | apply specified linear constraints                                                                                                                                              |
| SE/Robust                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                 |
| <u>vce</u> ( <i>vcetype</i> )                                                                                                                                                                                                                                                                                          | <i>vcetype</i> may be <u>gnr</u> , <u>robust</u> , <u>cluster</u> <i>clustvar</i> , <u>bootstrap</u> , <u>jackknife</u> , <u>hac</u> <i>kernel</i> , <u>hc2</u> , or <u>hc3</u> |
| Reporting                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                 |
| <u>level</u> (#)                                                                                                                                                                                                                                                                                                       | set confidence level; default is <u>level</u> (95)                                                                                                                              |
| <u>leave</u>                                                                                                                                                                                                                                                                                                           | create variables containing derivative of $E(y)$                                                                                                                                |
| <u>title</u> ( <i>string</i> )                                                                                                                                                                                                                                                                                         | display <i>string</i> as title above the table of parameter estimates                                                                                                           |
| <u>title2</u> ( <i>string</i> )                                                                                                                                                                                                                                                                                        | display <i>string</i> as subtitle                                                                                                                                               |
| <i>display_options</i>                                                                                                                                                                                                                                                                                                 | control column formats and line width                                                                                                                                           |
| Optimization                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                 |
| <i>optimization_options</i>                                                                                                                                                                                                                                                                                            | control the optimization process; seldom used                                                                                                                                   |
| <u>coeflegend</u>                                                                                                                                                                                                                                                                                                      | display legend instead of statistics                                                                                                                                            |
| *For the function evaluator program version, you must specify <u>parameters</u> ( <i>namelist</i> ) or <u>nparameters</u> (#).<br><u>bayesboot</u> , <u>bootstrap</u> , <u>by</u> , <u>collect</u> , <u>jackknife</u> , <u>rolling</u> , <u>statsby</u> , and <u>svy</u> are allowed; see [U] 11.1.10 Prefix commands. |                                                                                                                                                                                 |
| Weights are not allowed with the <u>bootstrap</u> prefix; see [R] <u>bootstrap</u> . <u>vce</u> ( <u>bootstrap</u> ) cannot be specified with weights.                                                                                                                                                                 |                                                                                                                                                                                 |
| <u>aweight</u> s are not allowed with the <u>jackknife</u> prefix; see [R] <u>jackknife</u> . <u>vce</u> ( <u>jackknife</u> ) cannot be specified with <u>aweight</u> s.                                                                                                                                               |                                                                                                                                                                                 |
| <u>vce</u> ( <u>hac</u> <i>kernel</i> ) cannot be specified with weights.                                                                                                                                                                                                                                              |                                                                                                                                                                                 |
| <u>vce</u> ( <u>hc3</u> ) cannot be specified with <u>pweight</u> s.                                                                                                                                                                                                                                                   |                                                                                                                                                                                 |
| <u>vce</u> ( <u> </u> ), <u>leave</u> , and weights are not allowed with the <u>svy</u> prefix; see [SVY] <u>svy</u> .                                                                                                                                                                                                 |                                                                                                                                                                                 |
| <u>aweight</u> s, <u>fweight</u> s, <u>weight</u> s, and <u>pweight</u> s are allowed; see [U] 11.1.6 <u>weight</u> .                                                                                                                                                                                                  |                                                                                                                                                                                 |
| <u>coeflegend</u> does not appear in the dialog box.                                                                                                                                                                                                                                                                   |                                                                                                                                                                                 |
| See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.                                                                                                                                                                                                                        |                                                                                                                                                                                 |

## Options

### Model

`define(name:<subexpr>)` defines a function of model parameters, <subexpr>, and labels it as *name*.

This option can be repeated to define multiple functions. The `define()` option is useful for expressions that appear multiple times in the main nonlinear specification of the command: you define the expression once and then simply refer to it by using `{name}`: in the nonlinear specification. This option can be used for notational convenience. See [Substitutable expressions](#) for how to specify <subexpr>.

`parameters(namelist)` specifies the names of the parameters in the model. This option applies only to the function evaluator program version. The names of the parameters must adhere to the naming conventions of Stata's variables; see [\[U\] 11.3 Naming conventions](#). If `parameters()` and `nparameters()` are both specified, the number in `nparameters()` must match the number of names specified in `parameters()`.

`nparameters(#)` specifies the number of parameters in the model. If you do not specify names with the `parameters()` option, `nl` names them `b1`, `b2`, ..., `b#`. This option applies only to the function evaluator program version. If `nparameters()` and `parameters()` are both specified, the number in `nparameters()` must match the number of names specified in `parameters()`.

`variables(varlist)` specifies computational variables used in a function evaluator program other than `depvar` and `varlist`. `nl` ignores observations for which any of these variables, in addition to variables in `depvar` and `varlist`, have missing values. This option is necessary only for the function evaluator program version and only when using variables other than `depvar` and `varlist`. In all other cases, the substitutable expression parser considers all the variables in your expressions. If you do not declare these extra computational variables in function evaluators programs, then `nl` exits with an error if the computed `depvar` contains missing values.

*sexp\_options* refer to any options allowed by your *sexp\_prog*.

*func\_options* refer to any options allowed by your *func\_prog*.

### Model 2

`lnlsq(#)` fits the model by using log least-squares, which we define as least squares with shifted log-normal errors. In other words,  $\ln(\text{depvar} - \#)$  is assumed to be normally distributed. Sums of squares and deviance are adjusted to the same scale as `depvar`.

`noconstant` indicates that the function does not include a constant term. By default, Stata will use a parameter as a constant if the coefficient of variation (over observations) of the partial derivative of our function with respect to the parameter is less than `ltolerance()`. The `noconstant` option overrides this default and is generally not needed.

`hasconstant(name)` indicates that parameter *name* be treated as the constant term in the model and that `nl` should not use its default algorithm to find a constant term. As with `noconstant`, this option is seldom used.

`constraints(constraints)`; see [\[R\] Estimation options](#).

## SE/Robust

`vce(vctype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`gnr`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

`vce(gnr)`, the default, uses the conventionally derived variance estimator for nonlinear models fit using Gauss–Newton regression.

`nl` also allows the following:

`vce(hac kernel [#])` specifies that a heteroskedasticity- and autocorrelation-consistent (HAC) variance estimate be used. HAC refers to the general form for combining weighted matrices to form the variance estimate. There are three kernels available for `nl`:

`nwest | gallant | anderson`

`#` specifies the number of lags. If `#` is not specified,  $N - 2$  is assumed.

`vce(hac kernel [#])` is not allowed if weights are specified.

`vce(hc2)` and `vce(hc3)` specify alternative bias corrections for the robust variance calculation. `vce(hc2)` and `vce(hc3)` may not be specified with the `svy` prefix. By default, `vce(robust)` uses  $\hat{\sigma}_j^2 = \{n/(n - k)\}u_j^2$  as an estimate of the variance of the  $j$ th observation, where  $u_j$  is the calculated residual and  $n/(n - k)$  is included to improve the overall estimate's small-sample properties.

`vce(hc2)` instead uses  $u_j^2/(1 - h_{jj})$  as the observation's variance estimate, where  $h_{jj}$  is the  $j$ th diagonal element of the hat (projection) matrix. This produces an unbiased estimate of the covariance matrix if the model is homoskedastic. `vce(hc2)` tends to produce slightly more conservative confidence intervals than `vce(robust)`.

`vce(hc3)` uses  $u_j^2/(1 - h_{jj})^2$  as suggested by Davidson and MacKinnon (1993 and 2004), who report that this often produces better results when the model is heteroskedastic. `vce(hc3)` produces confidence intervals that tend to be even more conservative.

See, in particular, Davidson and MacKinnon (2004, 239), who advocate the use of `vce(hc2)` or `vce(hc3)` instead of the plain robust estimator for nonlinear least squares.

## Reporting

`level(#)`; see [R] [Estimation options](#).

`leave` leaves behind after estimation a set of new variables with the same names as the estimated parameters containing the derivatives of  $E(y)$  with respect to the parameters. If the dataset contains an existing variable with the same name as a parameter, then using `leave` causes `nl` to issue an error message with return code 110.

`leave` may not be specified with `vce(cluster clustvar)` or the `svy` prefix.

`title(string)` specifies an optional title that will be displayed just above the table of parameter estimates.

`title2(string)` specifies an optional subtitle that will be displayed between the title specified in `title()` and the table of parameter estimates. If `title2()` is specified but `title()` is not, `title2()` has the same effect as `title()`.

*display\_options*: `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

### Optimization

*optimization\_options*: `iterate(#)`, `[no]log`, `trace`, `showstep`, `gradient`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrntolerance`, `difficult`, `search(on|off[, rseed(#)])`, `from(init_specs)`, `epsilon(#)`, `minderiv(#)`; see [R] [Maximize](#). Those that require special mention for `nl` are listed below.

`ltolerance(#)` specifies the tolerance for the residual sum of squares (RSS) of the Gauss–Newton algorithm. When the relative change in the RSS from one (alternating) iteration to the next is less than or equal to `ltolerance(#)`, the RSS convergence is satisfied. The default is `ltolerance(1e-7)`.

`difficult` specifies that a different stepping algorithm be used in nonconcave regions.

`search(on|off[, rseed(#)])` specifies whether an initial value search should be done. The default is `search(off)`. Use `rseed(#)` for reproducible results when searching for initial values. `search(on)` manipulates the values of the entire coefficient vector and will replace any initial values set in the expression.

`epsilon(#)` specifies the relative change in a parameter to be used in computing the numeric derivatives. The centered derivative for parameter  $\beta_i$  is computed as  $\{f(\mathbf{X}, \dots, \beta_i + \delta_i, \dots) - f(\mathbf{X}, \dots, \beta_i - \delta_i, \dots)\} / (2\delta_i)$ , where  $\delta_i = \epsilon(|\beta_i| + \epsilon)$ . The default is  $\epsilon = c(\text{epsdouble})^{1/3}$  (Press et al. 2007).

`minderiv(#)` specifies the minimum value for the derivative step  $\delta_i = \epsilon(|\beta_i| + \epsilon)$ . The default is  $\epsilon^2$ .

The following option is available with `nl` but is not shown in the dialog box:

`coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Remarks are presented under the following headings:

- [Substitutable expressions](#)
- [Substitutable expression programs](#)
- [Built-in functions](#)
- [Lognormal errors](#)
- [Other uses](#)
- [Weights](#)
- [Potential errors](#)
- [General comments on fitting nonlinear models](#)
- [Function evaluator programs](#)

`nl` fits an arbitrary nonlinear function by least squares. The interactive version allows you to enter the function directly on the command line or dialog box using *substitutable expressions*. You can write a *substitutable expression program* for functions that you fit frequently to save yourself time. Finally, *function evaluator programs* give you the most flexibility in defining your nonlinear function, though they are more complicated to use.

The next section explains the substitutable expressions that are used to define the regression function, and the section thereafter explains how to write substitutable expression program files so that you do not need to type in commonly used functions over and over. Later sections highlight other features of `nl`.

The final section discusses function evaluator programs. If you find substitutable expressions adequate to define your nonlinear function, then you can skip that section entirely. Function evaluator programs are generally needed only for complicated problems, such as multistep estimators. The program receives a vector of parameters at which it is to compute the function and a variable into which the results are to be placed.

## Substitutable expressions

You define the nonlinear function to be fit by `nl` by using a substitutable expression. Substitutable expressions are just like any other mathematical expressions involving scalars and variables, such as those you would use with Stata's `generate` command, except that the parameters to be estimated are bound in braces. See [U] 13.2 Operators and [U] 13.3 Functions for more information on expressions.

For example, suppose that you wish to fit the function

$$y_i = \beta_0(1 - e^{-\beta_1 x_i}) + \epsilon_i$$

where  $\beta_0$  and  $\beta_1$  are the parameters to be estimated and  $\epsilon_i$  is an error term. You would simply type

```
. nl (y = {b0}*(1 - exp(-{b1}*x)))
```

You must enclose the entire equation in parentheses. Because `b0` and `b1` are enclosed in braces, `nl` knows that they are parameters in the model. `nl` will initialize `b0` and `b1` to zero by default. To request that `nl` initialize `b0` to 1 and `b1` to 0.25, you would type

```
. nl (y = {b0=1}*(1 - exp(-{b1=0.25}*x)))
```

That is, inside the braces denoting a parameter, you put the parameter name followed by an equal sign and the initial value. If a parameter appears in your function multiple times, you need only specify an initial value only once (or never, if you wish to set the initial value to zero). If you do specify more than one initial value for the same parameter, `nl` will use the *last* value given. Parameter names must follow the same conventions as variable names. See [U] 11.3 Naming conventions.

Frequently, even nonlinear functions contain linear combinations of variables. As an example, suppose that you wish to fit the function

$$y_i = \beta_0 \{1 - e^{-(\beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i})}\} + \epsilon_i$$

`nl` allows you to declare the linear combination of variables by using the shorthand notation

```
. nl (y = {b0=1}*(1 - exp(-{lc: x1 x2 x3, noconstant})))
```

In the syntax `{lc: x1 x2 x3, noconstant}`, you are telling `nl` that you are declaring a linear combination named `lc` that is a function of three variables, `x1`, `x2`, and `x3`. `nl` will create three parameters, named `lc:x1`, `lc:x2`, and `lc:x3`, and initialize them to zero. Instead of typing the previous command, you could have typed

```
. nl (y = {b0=1}*(1 - exp(-({b1}*x1 + {b2}*x2 + {b3}*x3))))
```

or

```
. nl (y = {b0=1}*(1 - exp(-{lc:}))), define(lc: x1 x2 x3, noconstant)
```

and yielded the same result. Having defined this linear combination, you can refer to its individual parameters elsewhere in the function by using `{lc:x1}`, `{lc:x2}`, and `{lc:x3}` or, more generally, `{eqname:varname}`. When creating linear combinations, `nl` ensures that the parameter names it chooses are unique and have not yet been used in the function.

You may also refer to a “subset” of a previously defined linear combination. For example, suppose we wish to fit

$$y_i = \beta_0 \{1 - e^{-(\beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i})}\} + \beta_1 x_{1i} + \beta_3 x_{3i} + \epsilon_i$$

We can refer to it as

```
. nl (y = {b0=1}*(1 - exp(-{lc: x1 x2 x3, noconstant})) + {lc:x1}*x1 + {lc:x3}*x3)
```

To refer to an entire linear combination, you can simply use its name. For example, you can type

```
. nl (y = {b0=1}*(1 - exp(-{lc: x1 x2 x3, noconstant})) + {lc:})
```

The general syntax for defining a linear combination is

```
{eqname: varspec[, xb noconstant]}
```

The `xb` option is used to distinguish between, on the one hand, a linear combination of only one variable and, on the other, a free parameter that has the same name as its covariate and the same group name as the linear combination. For example, `{lc: x1, xb}` is equivalent to `{lc:_cons} + {lc:x1}*x1`, whereas `{lc:x1}` refers to either a free parameter `x1` with a group name `lc` or the coefficient of the `x1` variable, if `{lc:}` has been previously defined in the expression as a linear combination that involves variable `x1`. Thus, the `xb` option indicates that the specification is a linear combination rather than a single parameter to be estimated.

In general, there are three rules to follow when defining substitutable expressions:

1. Parameters of the model are bound in braces: `{b0}`, `{param}`, etc.
2. Initial values for parameters are given by including an equal sign and the initial value inside the braces: `{b0=1}`, `{param=3.571}`, etc.
3. Linear combinations of variables can be included using the notation `{eqname:varlist}`, for example, `{xb: mpg price weight}`, `{score: w x z}`, etc. Parameters of linear combinations are initialized to zero.

If you specify initial values by using the `from()` option, they override whatever initial values are given within the substitutable expression. Substitutable expressions are so named because, once values are assigned to the parameters, the resulting expression can be handled by `generate` and `replace`.

## ▷ Example 1

We wish to fit the CES production function

$$\ln Q_i = \beta_0 - \frac{1}{\rho} \ln \{ \delta K_i^{-\rho} + (1 - \delta) L_i^{-\rho} \} + \epsilon_i \quad (1)$$

where  $\ln Q_i$  is the log of output for firm  $i$ ;  $K_i$  and  $L_i$  are firm  $i$ 's capital and labor usage, respectively; and  $\epsilon_i$  is a regression error term. Because  $\rho$  appears in the denominator of a fraction, zero is not a feasible initial value; for a CES production function,  $\rho = 1$  is a reasonable choice. Setting  $\delta = 0.5$  implies that labor and capital have equal impacts on output, which is also a reasonable choice for an initial value. We type

```
. use https://www.stata-press.com/data/r19/production
. summarize lnoutput, meanonly
. local b0 = round(r(mean),.1)
. di "b0 = 'b0'"
b0 = 3
. nl (lnoutput = {b0='b0'} - ln({capital:} + {labor:})/{rho=1}),
> define(capital:{delta=0.5}*capital^{-(rho)})
> define(labor:(1 - {delta})*labor^{-(rho)})
Iteration 0: Residual SS = 80.334811
Iteration 1: Residual SS = 29.387341
Iteration 2: Residual SS = 29.36601
Iteration 3: Residual SS = 29.365808
Iteration 4: Residual SS = 29.365806
```

| Source   | SS        | df | MS         |                 |          |  |
|----------|-----------|----|------------|-----------------|----------|--|
| Model    | 91.144992 | 2  | 45.5724962 | Number of obs = | 100      |  |
| Residual | 29.365806 | 97 | .302740263 | R-squared =     | 0.7563   |  |
|          |           |    |            | Adj R-squared = | 0.7513   |  |
|          |           |    |            | Root MSE =      | .5502184 |  |
| Total    | 120.5108  | 99 | 1.21728079 | Res. dev. =     | 161.2538 |  |

| lnoutput | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|----------|-------------|-----------|-------|-------|----------------------|----------|
| /b0      | 3.792143    | .0996809  | 38.04 | 0.000 | 3.594303             | 3.989982 |
| /delta   | .4823479    | .0519785  | 9.28  | 0.000 | .379185              | .5855109 |
| /rho     | 1.386944    | .4725595  | 2.93  | 0.004 | .4490446             | 2.324844 |

Note: Parameter **b0** is used as a constant term during estimation.

`nl` will attempt to find a constant term in the model and, if one is found, mention it at the bottom of the output. `nl` found `b0` to be a constant because the partial derivative  $\partial \ln Q_i / \partial b_0$  has a coefficient of variation less than `tolerance()` in the estimation sample.

The elasticity of substitution for the CES production function is  $\sigma = 1/(1 + \rho)$ ; and, having fit the model, we can use `nlcom` to estimate it:

```
. nlcom (1/(1 + _b[/rho]))
 _nl_1: 1/(1 + _b[/rho])
```

| lnoutput | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |         |
|----------|-------------|-----------|------|-------|----------------------|---------|
| _nl_1    | .4189457    | .0829415  | 5.05 | 0.000 | .2563833             | .581508 |

See [R] `nlcom` and [U] 13.5 Accessing coefficients and standard errors for more information.

nl's output closely mimics that of regress; see [R] regress for more information. The  $R^2$ , sums of squares, and similar statistics are calculated in the same way that regress calculates them. If no "constant" term is specified, the usual caveats apply to the interpretation of the  $R^2$  statistic; see the comments and references in Goldstein (1992). Unlike regress, nl does not report a model  $F$  statistic, because a test of the joint significance of all the parameters except the constant term may not be relevant in a nonlinear model.

## Substitutable expression programs

If you fit the same model often or if you want to write an estimator that will operate on whatever variables you specify, then you will want to write a substitutable expression program. That program will return a macro containing a substitutable expression that nl can then evaluate, and it may optionally calculate initial values as well. The name of the program must begin with the letters nl.

To illustrate, suppose that you use the CES production function often in your work. Instead of typing in the formula each time, you can write a program like this:

```

program nlces, rclass
 version 19.5 // (or version 19 if you do not have StataNow)
 syntax varlist(min=3 max=3) [if]
 local logout : word 1 of `varlist'
 local capital : word 2 of `varlist'
 local labor : word 3 of `varlist'
 // Initial value for b0 given delta=0.5 and rho=1
 tempvar y
 generate double `y' = `logout' + ln(0.5*`capital'^-1 + 0.5*`labor'^-1)
 summarize `y' `if', meanonly
 local b0val = r(mean)
 // Terms for substitutable expression
 local capterm "{delta=0.5}*`capital'^(-1*{rho})"
 local labterm "(1-{delta})*`labor'^(-1*{rho})"
 local term2 "1/{rho=1}*ln(`capterm' + `labterm')
 // Return substitutable expression and title
 return local eq "logout' = {b0=`b0val}' - `term2'"
 return local title "CES ftn., ln Q=`logout', K=`capital', L=`labor'"
end

```

The program accepts three variables for log output, capital, and labor, and it accepts an *if exp* qualifier to restrict the estimation sample. All programs that you write to use with nl must accept an *if exp* qualifier because, when nl calls the program, it passes a binary variable that marks the estimation sample (the variable equals one if the observation is in the sample and zero otherwise). When calculating initial values, you will want to restrict your computations to the estimation sample, and you can do so by using *if* with any commands that accept *if exp* qualifiers. Even if your program does not calculate initial values or otherwise use the *if* qualifier, the *syntax* statement must still allow it. See [P] syntax for more information on the *syntax* command and the use of *if*.

As in the previous example, reasonable initial values for  $\delta$  and  $\rho$  are 0.5 and 1, respectively. Conditional on those values, (1) can be rewritten as

$$\beta_0 = \ln Q_i + \ln(0.5K_i^{-1} + 0.5L_i^{-1}) - \epsilon_i \quad (2)$$

so a good initial value for  $\beta_0$  is the mean of the right-hand side of (2) ignoring  $\epsilon_i$ . Lines 7–10 of the function evaluator program calculate that mean and store it in a local macro. Notice the use of *if* in the *summarize* statement so that the mean is calculated only for the estimation sample.

The final part of the program returns two macros. The macro `title` is optional and defines a short description of the model that will be displayed in the output immediately above the table of parameter estimates. The macro `eq` is required and defines the substitutable expression that `nl` will use. If the expression is short, you can define it all at once. However, because the expression used here is somewhat lengthy, defining local macros and then building up the final expression from them is easier.

To verify that there are no errors in your program, you can call it directly and then use `return list`:

```
. use https://www.stata-press.com/data/r19/production
. nlces lnoutput capital labor
 (output omitted)
. return list
macros:
 r(title) : "CES ftn., ln Q=lnoutput, K=capital, L=labor"
 r(eq) : "lnoutput = {b0=3.711606264663641} - 1/{rho=1}*
> ln({delta=0.5}*capital^{-1*{rho}} + (1-{delta})*labor^{-1*{rho}})"
```

The macro `r(eq)` contains the same substitutable expression that we specified at the command line in the [preceding example](#), except for the initial value for `b0`. In short, an `nl` substitutable expression program should return in `r(eq)` the same substitutable expression you would type at the command line. The only difference is that when writing a substitutable expression program, you do not bind the entire expression inside parentheses.

Having written the program, you can use it by typing

```
. nl ces: lnoutput capital labor
```

(There is a space between `nl` and `ces`.) The output is identical to that shown in [example 1](#), save for the title defined in the function evaluator program that appears immediately above the table of parameter estimates.

## □ Technical note

You will want to store `nlces` as an ado-file called `nlces.ado`. The alternative is to type the code into Stata interactively or to place the code in a do-file. While those alternatives are adequate for occasional use, if you save the program as an ado-file, you can use the function anytime you use Stata without having to redefine the program. When `nl` attempts to execute `nlces`, if the program is not in Stata's memory, Stata will search the disk(s) for an ado-file of the same name and, if found, automatically load it. All you have to do is name the file with the `.ado` suffix and then place it in a directory where Stata will find it. You should put the file in the directory Stata reserves for user-written ado-files, which, depending on your operating system, is `c:\ado\personal` (Windows), `~/ado/personal` (Unix), or `~:ado:personal` (Mac). See [\[U\] 17 Ado-files](#).

□

Sometimes you may want to pass additional options to the substitutable expression program. You can modify the `syntax` statement of your program to accept whatever options you wish. Then when you call `nl` with the `syntax`

```
. nl func_prog: varlist, options
```

any *options* that are not recognized by `nl` (see the table of options at the beginning of this entry) are passed on to your function evaluator program. The only other restriction is that your program cannot accept an option named `at` because `nl` uses that option with function evaluator programs.

## Built-in functions

Some functions are used so often that `nl` has them built in so that you do not need to write them yourself. `nl` automatically chooses initial values for the parameters, though you can use the `initial()` option to override them.

Three alternatives are provided for exponential regression with one asymptote:

$$\text{exp3} \quad y_i = \beta_0 + \beta_1 \beta_2^{x_i} + \epsilon_i$$

$$\text{exp2} \quad y_i = \beta_1 \beta_2^{x_i} + \epsilon_i$$

$$\text{exp2a} \quad y_i = \beta_1 (1 - \beta_2^{x_i}) + \epsilon_i$$

For instance, typing `nl exp3: ras dv1` fits the three-parameter exponential model (parameters  $\beta_0$ ,  $\beta_1$ , and  $\beta_2$ ) using  $y_i = \text{ras}$  and  $x_i = \text{dv1}$ .

Two alternatives are provided for the logistic function (symmetric sigmoid shape; not to be confused with logistic regression):

$$\text{log4} \quad y_i = \beta_0 + \beta_1 / [1 + \exp\{-\beta_2(x_i - \beta_3)\}] + \epsilon_i$$

$$\text{log3} \quad y_i = \beta_1 / [1 + \exp\{-\beta_2(x_i - \beta_3)\}] + \epsilon_i$$

Finally, two alternatives are provided for the Gompertz function (asymmetric sigmoid shape):

$$\text{gom4} \quad y_i = \beta_0 + \beta_1 \exp[-\exp\{-\beta_2(x_i - \beta_3)\}] + \epsilon_i$$

$$\text{gom3} \quad y_i = \beta_1 \exp[-\exp\{-\beta_2(x_i - \beta_3)\}] + \epsilon_i$$

## Lognormal errors

A nonlinear model with errors that are independent and identically distributed normal may be written as

$$y_i = f(\mathbf{x}_i, \boldsymbol{\beta}) + u_i, \quad u_i \sim N(0, \sigma^2) \quad (3)$$

for  $i = 1, \dots, n$ . If the  $y_i$  are thought to have a  $k$ -shifted lognormal instead of a normal distribution—that is,  $\ln(y_i - k) \sim N(\zeta_i, \tau^2)$ , and the systematic part  $f(\mathbf{x}_i, \boldsymbol{\beta})$  of the original model is still thought appropriate for  $y_i$ —the model becomes

$$\ln(y_i - k) = \zeta_i + v_i = \ln\{f(\mathbf{x}_i, \boldsymbol{\beta}) - k\} + v_i, \quad v_i \sim N(0, \tau^2) \quad (4)$$

This model is fit if `lnlsq(k)` is specified.

If model (4) is correct, the variance of  $(y_i - k)$  is proportional to  $\{f(\mathbf{x}_i, \boldsymbol{\beta}) - k\}^2$ . Probably the most common case is  $k = 0$ , sometimes called “proportional errors” because the standard error of  $y_i$  is proportional to its expectation,  $f(\mathbf{x}_i, \boldsymbol{\beta})$ . Assuming that the value of  $k$  is known, (4) is just another nonlinear model in  $\boldsymbol{\beta}$ , and it may be fit as usual. However, we may wish to compare the fit of (3) with that of (4) using the residual sum of squares (RSS) or the deviance  $D$ ,  $D = -2 \times \log\text{-likelihood}$ , from each model. To do so, we must allow for the change in scale introduced by the log transformation.

Assuming, then, the  $y_i$  to be normally distributed, [Atkinson \(1985, 85–87, 184\)](#), by considering the Jacobian  $\prod |\partial \ln(y_i - k) / \partial y_i|$ , showed that multiplying both sides of (4) by the geometric mean of  $y_i - k$ ,  $\dot{y}$ , gives residuals on the same scale as those of  $y_i$ . The geometric mean is given by

$$\dot{y} = e^{n^{-1} \sum \ln(y_i - k)}$$

which is a constant for a given dataset. The residual deviance for (3) and for (4) may be expressed as

$$D(\hat{\beta}) = \left\{ 1 + \ln(2\pi\hat{\sigma}^2) \right\} n \quad (5)$$

where  $\hat{\beta}$  is the maximum likelihood estimate (MLE) of  $\beta$  for each model and  $n\hat{\sigma}^2$  is the RSS from (3), or that from (4) multiplied by  $\dot{y}^2$ .

Because (3) and (4) are models with different error structures but the same functional form, the arithmetic difference in their RSS or deviances is not easily tested for statistical significance. However, if the deviance difference is large ( $>4$ , say), we would naturally prefer the model with the smaller deviance. Of course, the residuals for each model should be examined for departures from assumptions (nonconstant variance, nonnormality, serial correlations, etc.) in the usual way.

Alternatively, consider modeling

$$E(y_i) = 1/(C + Ae^{Bx_i}) \quad (6)$$

$$E(1/y_i) = E(\tilde{y}_i) = C + Ae^{Bx_i} \quad (7)$$

where  $C$ ,  $A$ , and  $B$  are parameters to be estimated. Using the data  $(y, x) = (0.04, 5)$ ,  $(0.06, 12)$ ,  $(0.08, 25)$ ,  $(0.1, 35)$ ,  $(0.15, 42)$ ,  $(0.2, 48)$ ,  $(0.25, 60)$ ,  $(0.3, 75)$ , and  $(0.5, 120)$  ([Danuso 1991](#)), fitting the models yields

| Model                          | $C$   | $A$   | $B$      | RSS       | Deviance |
|--------------------------------|-------|-------|----------|-----------|----------|
| (6)                            | 1.781 | 25.74 | -0.03926 | -0.001640 | -51.95   |
| (6) with <code>lnlsq(0)</code> | 1.799 | 25.45 | -0.04051 | -0.001431 | -53.18   |
| (7)                            | 1.781 | 25.74 | -0.03926 | 8.197     | 24.70    |
| (7) with <code>lnlsq(0)</code> | 1.799 | 27.45 | -0.04051 | 3.651     | 17.42    |

There is little to choose between the two versions of the logistic model (6), whereas for the exponential model (7), the fit using `lnlsq(0)` is much better (a deviance difference of 7.28). The reciprocal transformation has introduced heteroskedasticity into  $\tilde{y}_i$ , which is countered by the proportional errors property of the lognormal distribution implicit in `lnlsq(0)`. The deviances are not comparable between the logistic and exponential models because the change of scale has not been allowed for, although in principle it could be.

## Other uses

Even if you are fitting linear regression models, you may find that `nl` can save you some typing. Because you specify the parameters of your model explicitly, you can impose constraints on them directly.

## ▷ Example 2

In [example 2](#) of [\[R\] cnsreg](#), we showed how to fit the model

$$\text{mpg} = \beta_0 + \beta_1 \text{price} + \beta_2 \text{weight} + \beta_3 \text{displ} + \beta_4 \text{gear\_ratio} + \beta_5 \text{foreign} + \beta_6 \text{length} + u$$

subject to the constraints

$$\begin{aligned} \beta_1 &= \beta_2 = \beta_3 = \beta_6 \\ \beta_4 &= -\beta_5 = \beta_0/20 \end{aligned}$$

An alternative way is to use `nl`:

```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
. nl (mpg = {b0} + {b1}*price + {b1}*weight + {b1}*displ +
> {b0}/20*gear_ratio - {b0}/20*foreign + {b1}*length)
Iteration 0: Residual SS = 36008
Iteration 1: Residual SS = 1578.5223
Iteration 2: Residual SS = 1578.5223
```

| Source   | SS        | df | MS         |                 |          |
|----------|-----------|----|------------|-----------------|----------|
| Model    | 34429.478 | 2  | 17214.7389 | Number of obs = | 74       |
| Residual | 1578.5223 | 72 | 21.9239203 | R-squared =     | 0.9562   |
|          |           |    |            | Adj R-squared = | 0.9549   |
|          |           |    |            | Root MSE =      | 4.682299 |
| Total    | 36008     | 74 | 486.594595 | Res. dev. =     | 436.4562 |

| mpg | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|-----|-------------|-----------|-------|-------|----------------------|-----------|
| /b0 | 26.52229    | 1.375178  | 19.29 | 0.000 | 23.78092             | 29.26365  |
| /b1 | -.000923    | .0001534  | -6.02 | 0.000 | -.0012288            | -.0006172 |

The point estimates and standard errors for  $\beta_0$  and  $\beta_1$  are identical to those reported in [example 2](#) of [\[R\] cnsreg](#). To get the estimate for  $\beta_4$ , we can use `nlcom`:

```
. nlcom _b[/b0]/20
 _nl_1: _b[/b0]/20
```

| mpg   | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-------|-------------|-----------|-------|-------|----------------------|----------|
| _nl_1 | 1.326114    | .0687589  | 19.29 | 0.000 | 1.191349             | 1.460879 |

The advantage to using `nl` is that we do not need to use the `constraint` command six times. ◀

`nl` is also a useful tool when doing exploratory data analysis. For example, you may want to run a regression of  $y$  on a function of  $x$ , though you have not decided whether to use  $\sqrt{x}$  or  $\ln(x)$ . You can use `nl` to run both regressions without having first to generate two new variables:

```
. nl (y = {b0} + {b1}*ln(x))
. nl (y = {b0} + {b1}*sqrt(x))
```

[Poi \(2008\)](#) shows the advantages of using `nl` when marginal effects of transformed variables are desired as well.

## Weights

Weights are specified in the usual way—analytic and frequency weights as well as `iweights` are supported; see [U] 20.24 **Weighted estimation**. Use of analytic weights implies that the  $y_i$  have different variances. Therefore, model (3) may be rewritten as

$$y_i = f(\mathbf{x}_i, \boldsymbol{\beta}) + u_i, \quad u_i \sim N(0, \sigma^2/w_i) \quad (3a)$$

where  $w_i$  are (positive) weights, assumed to be known and normalized such that their sum equals the number of observations. The residual deviance for (3a) is

$$D(\widehat{\boldsymbol{\beta}}) = \{1 + \ln(2\pi\widehat{\sigma}^2)\}n - \sum \ln(w_i) \quad (5a)$$

[compare with (5)], where

$$n\widehat{\sigma}^2 = \text{RSS} = \sum w_i \{y_i - f(\mathbf{x}_i, \widehat{\boldsymbol{\beta}})\}^2$$

Defining and fitting a model equivalent to (4) when weights have been specified as in (3a) is not straightforward and has not been attempted. Thus, deviances using and not using the `lnlsq()` option may not be strictly comparable when analytic weights (other than 0 and 1) are used.

You do not need to modify your substitutable expression in any way to use weights. If, however, you write a substitutable expression program, then you should account for weights when obtaining initial values. When `nl` calls your program, it passes whatever weight expression (if any) was specified by the user. Here is an outline of a substitutable expression program that accepts weights:

```

program nl name, rclass
 version 19.5 // (or version 19 if you do not have StataNow)
 syntax varlist [aw fw iw pw] if
 ...
 // Obtain initial values allowing weights
 // Use the syntax ['weight' 'exp']. For example,
 summarize varname ['weight' 'exp'] 'if'
 regress depvar varlist ['weight' 'exp'] 'if'
 ...
 // Return substitutable expression
 return local eq "substitutable expression"
 return local title "description of estimator"
end

```

For details on how the `syntax` command processes weight expressions, see [P] **syntax**.

## Potential errors

`nl` is reasonably robust to the inability of your nonlinear function to be evaluated at some parameter values. `nl` does assume that your function can be evaluated at the initial values of the parameters. If your function cannot be evaluated at the initial values, an error message is issued with return code 480. Recall that if you do not specify an initial value for a parameter, then `nl` initializes it to zero. Many nonlinear functions cannot be evaluated when some parameters are zero, so in those cases specifying alternative initial values is crucial.

Thereafter, as `nl` changes the parameter values, it monitors your function for unexpected missing values. If these are detected, `nl` backs up. That is, `nl` finds a point between the previous, known-to-be-good parameter vector and the new, known-to-be-bad vector at which the function can be evaluated and continues its iterations from that point.

`nl` requires that once a parameter vector is found where the predictions can be calculated, small changes to the parameter vector be made to calculate numeric derivatives. If a boundary is encountered at this point, an error message is issued with return code 481.

When specifying `lnlsq()`, an attempt to take logarithms of  $y_i - k$  when  $y_i \leq k$  results in an error message with return code 482.

If `iterate()` iterations are performed and estimates still have not converged, results are presented with a warning, and the return code is set to 430.

If you use the programmed substitutable expression version of `nl` with a function evaluator program, or vice versa, Stata issues an error message. Verify that you are using the syntax appropriate for the program you have.

## General comments on fitting nonlinear models

Achieving convergence is often problematic. For example, a unique minimum of the sum-of-squares function may not exist. Much literature exists on different algorithms that have been used, on strategies for obtaining good initial parameter values, and on tricks for parameterizing the model to make its behavior as linear-like as possible. Selected references are [Kennedy and Gentle \(1980, chap. 10\)](#) for computational matters and [Ross \(1990\)](#) and [Ratkowsky \(1983\)](#) for all three aspects. Ratkowsky's book is particularly clear and approachable, with useful discussion on the meaning and practical implications of intrinsic and parameter-effects nonlinearity. An excellent text on nonlinear estimation is [Gallant \(1987\)](#). Also see [Davidson and MacKinnon \(1993 and 2004\)](#).

To enhance the success of `nl`, pay attention to the form of the model fit, along the lines of [Ratkowsky and Ross](#). For example, [Ratkowsky \(1983, 49–59\)](#) analyzes three possible three-parameter yield-density models for plant growth:

$$E(y_i) = \begin{cases} (\alpha + \beta x_i)^{-1/\theta} \\ (\alpha + \beta x_i + \gamma x_i^2)^{-1} \\ (\alpha + \beta x_i^\phi)^{-1} \end{cases}$$

All three models give similar fits. However, he shows that the second formulation is dramatically more linear-like than the other two and therefore has better convergence properties. In addition, the parameter estimates are virtually unbiased and normally distributed, and the asymptotic approximation to the standard errors, correlations, and confidence intervals is much more accurate than for the other models. Even within a given model, the way the parameters are expressed (for example,  $\phi^{x_i}$  or  $e^{\theta x_i}$ ) affects the degree of linearity and convergence behavior.

## Function evaluator programs

Occasionally, a nonlinear function may be so complex that writing a substitutable expression for it is impractical. For example, there could be many parameters in the model. Alternatively, if you are implementing a two-step estimator, writing a substitutable expression may be altogether impossible. Function evaluator programs can be used in these situations.

`nl` will pass to your function evaluator program a list of variables, a weight expression, a variable marking the estimation sample, and a vector of parameters. Your program is to replace the dependent variable, which is the first variable in the variables list, with the values of the nonlinear function evaluated at those parameters. As with substitutable expression programs, the first two letters of the name must be `nl`.

To focus on the mechanics of the function evaluator program, again let's compare the CES production function to the previous examples. The function evaluator program is

```

program nlces2
 version 19.5 // (or version 19 if you do not have StataNow)
 syntax varlist(min=3 max=3) if, at(name)
 local logout : word 1 of `varlist'
 local capital : word 2 of `varlist'
 local labor : word 3 of `varlist'

 // Retrieve parameters out of at matrix
 tempname b0 rho delta
 scalar `b0' = `at'[1, 1]
 scalar `rho' = `at'[1, 2]
 scalar `delta' = `at'[1, 3]

 tempvar kterm lterm
 generate double `kterm' = `delta'*`capital'^(-1*`rho') `if'
 generate double `lterm' = (1-`delta')*`labor'^(-1*`rho') `if'

 // Fill in dependent variable
 replace `logout' = `b0' - 1/`rho'*ln(`kterm' + `lterm') `if'

end

```

Unlike the previous `nlces` program, this one is not declared to be `r-class`. The `syntax` statement again accepts three variables: one for log output, one for capital, and one for labor. An `if exp` is again required because `nl` will pass a binary variable marking the estimation sample. All function evaluator programs must accept an option named `at()` that takes a name as an argument—that is how `nl` passes the parameter vector to your program.

The next part of the program retrieves the output, labor, and capital variables from the variables list. It then breaks up the temporary matrix `at` and retrieves the parameters `b0`, `rho`, and `delta`. Pay careful attention to the order in which the parameters refer to the columns of the `at` matrix because that will affect the syntax you use with `nl`. The temporary names you use inside this program are immaterial, however.

The rest of the program computes the nonlinear function, using some temporary variables to hold intermediate results. The final line of the program then replaces the dependent variable with the values of the function. Notice the use of `'if'` to restrict attention to the estimation sample. `nl` makes a copy of your dependent variable so that when the command is finished your data are left unchanged.

To use the program and fit your model, you type

```

. use https://www.stata-press.com/data/r19/production, clear
. nl ces2 @ lnoutput capital labor, parameters(b0 rho delta)
> initial(b0 0 rho 1 delta 0.5)

```

The output is again identical to that shown in [example 1](#). The order in which the parameters were specified in the `parameters()` option is the same in which they are retrieved from the `at` matrix in the program. To initialize them, you simply list the parameter name, a space, the initial value, and so on.

If you use the `nparameters()` option instead of the `parameters()` option, the parameters are named `b1`, `b2`, ..., `bk`, where `k` is the number of parameters. Thus, you could have typed

```
. nl ces2 @ lnoutput capital labor, nparameters(3) initial(b1 0 b2 1 b3 0.5)
```

With that syntax, the parameters called `b0`, `rho`, and `delta` in the program will be labeled `b1`, `b2`, and `b3`, respectively. In programming situations or if there are many parameters, instead of listing the parameter names and initial values in the `initial()` option, you may find it more convenient to pass a column vector. In those cases, you could type

```
. matrix myvals = (0, 1, 0.5)
. nl ces2 @ lnoutput capital labor, nparameters(3) initial(myvals, copy)
```

or by name

```
. matrix colnames myvals = b1 b2 b3
. nl ces2 @ lnoutput capital labor, nparameters(3) initial(myvals)
```

In summary, a function evaluator program receives a list of variables, the first of which is the dependent variable that you are to replace with the values of your nonlinear function. Additionally, it must accept an *if exp*, as well as an option named `at` that will contain the vector of parameters at which `nl` wants the function evaluated. You are then free to do whatever is necessary to evaluate your function and replace the dependent variable.

If you wish to use weights, your function evaluator program's `syntax` statement must accept them. If your program consists only of, for example, `generate` statements, you need not do anything with the weights passed to your program. However, if in calculating the nonlinear function you use commands such as `summarize` or `regress`, then you will want to use the weights with those commands.

As with substitutable expression programs, `nl` will pass to it any options specified that `nl` does not accept, providing you with a way to pass more information to your function.

## □ Technical note

Before version 9 of Stata, the `nl` command used a different syntax, which required you to write an *nlfcn* program, and it did not have a syntax for interactive use other than the seven functions that were built-in. The old syntax of `nl` still works, and you can still use those *nlfcn* programs. If `nl` does not see a colon, an `at` sign, or a set of parentheses surrounding the equation in your command, it assumes that the old syntax is being used.

The current version of `nl` uses scalars and matrices to store intermediate calculations instead of local and global macros as the old version did, so the current version produces more accurate results. In practice, however, any discrepancies are likely to be small.

□

## Stored results

nl stores the following in `e()`:

### Scalars

|                            |                                                                           |
|----------------------------|---------------------------------------------------------------------------|
| <code>e(N)</code>          | number of observations                                                    |
| <code>e(k)</code>          | number of parameters                                                      |
| <code>e(k_eq_model)</code> | number of equations in overall model test; always 0                       |
| <code>e(df_m)</code>       | model degrees of freedom                                                  |
| <code>e(df_r)</code>       | residual degrees of freedom                                               |
| <code>e(df_t)</code>       | total degrees of freedom                                                  |
| <code>e(mss)</code>        | model sum of squares                                                      |
| <code>e(rss)</code>        | residual sum of squares                                                   |
| <code>e(tss)</code>        | total sum of squares                                                      |
| <code>e(mms)</code>        | model mean square                                                         |
| <code>e(msr)</code>        | residual mean square                                                      |
| <code>e(ll)</code>         | log likelihood assuming i.i.d. normal errors                              |
| <code>e(r2)</code>         | $R^2$                                                                     |
| <code>e(r2_a)</code>       | adjusted $R^2$                                                            |
| <code>e(rmse)</code>       | root mean squared error                                                   |
| <code>e(dev)</code>        | residual deviance                                                         |
| <code>e(sum_w)</code>      | sum of weights                                                            |
| <code>e(N_clust)</code>    | number of clusters                                                        |
| <code>e(lnlsq)</code>      | value of <code>lnlsq</code> if specified                                  |
| <code>e(log_t)</code>      | 1 if <code>lnlsq</code> specified, 0 otherwise                            |
| <code>e(gm_2)</code>       | square of geometric mean of $(y - k)$ if <code>lnlsq</code> , 1 otherwise |
| <code>e(cons_j)</code>     | position of constant in <code>e(b)</code> or 0 if no constant             |
| <code>e(minderiv)</code>   | minimum value for derivative step                                         |
| <code>e(epsilon)</code>    | relative change used to compute derivatives                               |
| <code>e(rank)</code>       | rank of <code>e(V)</code>                                                 |
| <code>e(ic)</code>         | number of iterations                                                      |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise                                               |

### Macros

|                              |                                                                                                                                                                            |
|------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>e(cmd)</code>          | nl                                                                                                                                                                         |
| <code>e(cmdline)</code>      | command as typed                                                                                                                                                           |
| <code>e(depvar)</code>       | name of dependent variable                                                                                                                                                 |
| <code>e(wtype)</code>        | weight type                                                                                                                                                                |
| <code>e(wexp)</code>         | weight expression                                                                                                                                                          |
| <code>e(title)</code>        | title in estimation output                                                                                                                                                 |
| <code>e(title_2)</code>      | secondary title in estimation output                                                                                                                                       |
| <code>e(clustvar)</code>     | name of cluster variable                                                                                                                                                   |
| <code>e(hac_kernel)</code>   | HAC kernel                                                                                                                                                                 |
| <code>e(hac_lag)</code>      | HAC lag                                                                                                                                                                    |
| <code>e(vce)</code>          | <i>vctype</i> specified in <code>vce()</code>                                                                                                                              |
| <code>e(vcetype)</code>      | title used to label Std. err.                                                                                                                                              |
| <code>e(type)</code>         | <code>sexpress</code> : interactively entered expression<br><code>sexpprog</code> : substitutable expression program<br><code>funcprog</code> : function evaluator program |
| <code>e(sexp)</code>         | substitutable expression                                                                                                                                                   |
| <code>e(expressions)</code>  | expression names                                                                                                                                                           |
| <code>e(expr_name)</code>    | expression named <i>name</i>                                                                                                                                               |
| <code>e(params)</code>       | names of parameters                                                                                                                                                        |
| <code>e(funcprog)</code>     | function evaluator program                                                                                                                                                 |
| <code>e(varlist)</code>      | independent variables                                                                                                                                                      |
| <code>e(properties)</code>   | b V                                                                                                                                                                        |
| <code>e(predict)</code>      | program used to implement <code>predict</code>                                                                                                                             |
| <code>e(marginsok)</code>    | predictions allowed by <code>margins</code>                                                                                                                                |
| <code>e(marginsnotok)</code> | predictions disallowed by <code>margins</code>                                                                                                                             |

|                 |                                              |
|-----------------|----------------------------------------------|
| Matrices        |                                              |
| e(b)            | coefficient vector                           |
| e(Cns)          | constraints matrix                           |
| e(from)         | initial values vector                        |
| e(V)            | variance-covariance matrix of the estimators |
| e(V_modelbased) | model-based variance                         |
| Functions       |                                              |
| e(sample)       | marks estimation sample                      |

In addition to the above, the following is stored in `r()`:

|          |                                                                                                                       |
|----------|-----------------------------------------------------------------------------------------------------------------------|
| Matrices |                                                                                                                       |
| r(table) | matrix containing the coefficients with their standard errors, test statistics, $p$ -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

The derivation here is based on Davidson and MacKinnon (2004, chap. 6). Let  $\beta$  denote the  $k \times 1$  vector of parameters, and write the regression function using matrix notation as  $\mathbf{y} = \mathbf{f}(\mathbf{x}, \beta) + \mathbf{u}$  so that the objective function can be written as

$$\text{SSR}(\beta) = \{\mathbf{y} - \mathbf{f}(\mathbf{x}, \beta)\}' \mathbf{D} \{\mathbf{y} - \mathbf{f}(\mathbf{x}, \beta)\}$$

The  $\mathbf{D}$  matrix contains the weights and is defined in `[R] regress`; if no weights are specified, then  $\mathbf{D}$  is the  $N \times N$  identity matrix. Taking a second-order Taylor series expansion centered at  $\beta_0$  yields

$$\text{SSR}(\beta) \approx \text{SSR}(\beta_0) + \mathbf{g}'(\beta_0)(\beta - \beta_0) + \frac{1}{2}(\beta - \beta_0)'\mathbf{H}(\beta_0)(\beta - \beta_0) \quad (8)$$

where  $\mathbf{g}(\beta_0)$  denotes the  $k \times 1$  gradient of  $\text{SSR}(\beta)$  evaluated at  $\beta_0$  and  $\mathbf{H}(\beta_0)$  denotes the  $k \times k$  Hessian of  $\text{SSR}(\beta)$  evaluated at  $\beta_0$ . Letting  $\mathbf{X}$  denote the  $N \times k$  Jacobian matrix, the matrix of derivatives of  $\mathbf{f}(\mathbf{x}, \beta)$  with respect to  $\beta$ , the gradient  $\mathbf{g}(\beta)$  is

$$\mathbf{g}(\beta) = -2\mathbf{X}'\mathbf{D}\mathbf{u} \quad (9)$$

$\mathbf{X}$  and  $\mathbf{u}$  are obviously functions of  $\beta$ , though for notational simplicity that dependence is not shown explicitly. The  $(m, n)$  element of the Hessian can be written as

$$H_{mn}(\beta) = -2 \sum_{i=1}^{i=N} d_{ii} \left[ \frac{\partial^2 f_i}{\partial \beta_m \partial \beta_n} u_i - X_{im} X_{in} \right] \quad (10)$$

where  $d_{ii}$  is the  $i$ th diagonal element of  $\mathbf{D}$ . As discussed in Davidson and MacKinnon (2004, chap. 6), the first term inside the brackets of (10) has expectation zero, so the Hessian can be approximated as

$$\mathbf{H}(\beta) = 2\mathbf{X}'\mathbf{D}\mathbf{X} \quad (11)$$

Differentiating the Taylor series expansion of  $\text{SSR}(\beta)$  shown in (8) yields the first-order condition for a minimum

$$\mathbf{g}(\beta_0) + \mathbf{H}(\beta_0)(\beta - \beta_0) = \mathbf{0}$$

which suggests the iterative procedure

$$\beta_{j+1} = \beta_j - \alpha \mathbf{H}^{-1}(\beta_j) \mathbf{g}(\beta_j) \quad (12)$$

where  $\alpha$  is a “step size” parameter chosen at each iteration to improve convergence. Using (9) and (11), we can write (12) as

$$\beta_{j+1} = \beta_j + \alpha(\mathbf{X}'\mathbf{D}\mathbf{X})^{-1}\mathbf{X}'\mathbf{D}\mathbf{u} \quad (13)$$

where  $\mathbf{X}$  and  $\mathbf{u}$  are evaluated at  $\beta_j$ . Apart from the scalar  $\alpha$ , the second term on the right-hand side of (13) can be computed via a (weighted) regression of the columns of  $\mathbf{X}$  on the errors. This is implemented using Mata’s `moptimize()` function and the Gauss–Newton algorithm. Derivatives are computed numerically using Mata’s `deriv()` function. Convergence is declared when `mreldif( $\beta_{j+1}, \beta_j$ ) < tolerance()` or `reldif(SSR( $\beta_{j+1}$ ), SSR( $\beta_j$ )) < ltolerance()`, and  $\mathbf{g}'_{j+1}(\mathbf{H}_{j+1})^{-1}\mathbf{g}_{j+1} < \text{nrtolerance}()$ .

As derived, for example, in Davidson and MacKinnon (2004, chap. 6), an expedient way to obtain the covariance matrix is to compute  $\mathbf{u}$  and the columns of  $\mathbf{X}$  at the final estimate  $\hat{\beta}$  and then regress that  $\mathbf{u}$  on  $\mathbf{X}$ . The covariance matrix of the estimated parameters of that regression serves as an estimate of  $\text{Var}(\hat{\beta})$ . If that regression employs a robust covariance matrix estimator, then the covariance matrix for the parameters of the nonlinear regression will also be robust.

All other statistics are calculated analogously to those in linear regression, except that the nonlinear function  $f(\mathbf{x}_i, \beta)$  plays the role of the linear function  $\mathbf{x}'_i\beta$ . See [R] [regress](#).

This command supports estimation with survey data. For details on VCEs with survey data, see [SVY] [Variance estimation](#).

## Acknowledgments

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## Also see

[R] **nl postestimation** — Postestimation tools for nl

[R] **gmm** — Generalized method of moments estimation

[R] **ml** — Maximum likelihood estimation

[R] **mlexp** — Maximum likelihood estimation of user-specified expressions

[R] **nlcom** — Nonlinear combinations of parameters

[R] **nlsur** — Estimation of nonlinear system of equations

[R] **regress** — Linear regression

[ME] **menl** — Nonlinear mixed-effects regression

[SVY] **svy estimation** — Estimation commands for survey data

[U] **20 Estimation and postestimation commands**

## Postestimation commands

The following postestimation commands are available after `nl`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| † <code>margins</code>       | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | fitted values, residuals, etc.                                                                                                          |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results.

†You must specify the `variables()` option with `nl`.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as fitted values, residuals, probabilities, and expected values. It can also create multiple new variables containing predicted, named substitutable expressions.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

*Syntax for predicting without named substitutable expressions (parameters)*

```
predict [type] newvar [if] [in] [, statistic]
```

```
predict [type] stub* [if] [in] , scores
```

*Syntax for predicting with named substitutable expressions (parameters)*

*Predict all parameters*

```
predict [type] { stub* | newvarlist } [if] [in] , parameters
```

*Predict specific parameters*

```
predict [type] (newvar = {param:}) [(newvar = {param:}) [...]] [if] [in]
```

```
predict [type] { stub* | newvarlist } [if] [in] , parameters(paramnames)
```

| <i>statistic</i>                     | Description                                 |
|--------------------------------------|---------------------------------------------|
| <u>y</u> hat                         | fitted values; the default                  |
| <u>r</u> esiduals                    | residuals                                   |
| <u>pr</u> ( <i>a</i> , <i>b</i> )    | $\Pr(y_j   a < y_j < b)$                    |
| <u>e</u> ( <i>a</i> , <i>b</i> )     | $E(y_j   a < y_j < b)$                      |
| <u>ys</u> tar( <i>a</i> , <i>b</i> ) | $E(y_j^*), y_j^* = \max\{a, \min(y_j, b)\}$ |

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

## Options for predict

Main

`yhat`, the default, calculates the fitted values.

`residuals` calculates the residuals.

`pr(a,b)` calculates  $\Pr(a < f(\mathbf{x}_j, \mathbf{b}) + u_j < b)$ , the probability that  $y_j|\mathbf{x}_j$  would be observed in the interval  $(a, b)$ .

$a$  and  $b$  may be specified as numbers or variable names;  $lb$  and  $ub$  are variable names;

`pr(20,30)` calculates  $\Pr(20 < f(\mathbf{x}_j, \mathbf{b}) + u_j < 30)$ ;

`pr(lb,ub)` calculates  $\Pr(lb < f(\mathbf{x}_j, \mathbf{b}) + u_j < ub)$ ; and

`pr(20,ub)` calculates  $\Pr(20 < f(\mathbf{x}_j, \mathbf{b}) + u_j < ub)$ .

$a$  missing ( $a \geq .$ ) means  $-\infty$ ; `pr(. ,30)` calculates  $\Pr(-\infty < f(\mathbf{x}_j, \mathbf{b}) + u_j < 30)$ ;

`pr(lb,30)` calculates  $\Pr(-\infty < f(\mathbf{x}_j, \mathbf{b}) + u_j < 30)$  in observations for which  $lb \geq .$  and calculates  $\Pr(lb < f(\mathbf{x}_j, \mathbf{b}) + u_j < 30)$  elsewhere.

$b$  missing ( $b \geq .$ ) means  $+\infty$ ; `pr(20, .)` calculates  $\Pr(+\infty > f(\mathbf{x}_j, \mathbf{b}) + u_j > 20)$ ;

`pr(20,ub)` calculates  $\Pr(+\infty > f(\mathbf{x}_j, \mathbf{b}) + u_j > 20)$  in observations for which  $ub \geq .$  and calculates  $\Pr(20 < f(\mathbf{x}_j, \mathbf{b}) + u_j < ub)$  elsewhere.

`e(a,b)` calculates  $E(f(\mathbf{x}_j, \mathbf{b}) + u_j | a < f(\mathbf{x}_j, \mathbf{b}) + u_j < b)$ , the expected value of  $y_j|\mathbf{x}_j$  conditional on  $y_j|\mathbf{x}_j$  being in the interval  $(a, b)$ , meaning that  $y_j|\mathbf{x}_j$  is truncated.  $a$  and  $b$  are specified as they are for `pr()`.

`ystar(a,b)` calculates  $E(y_j^*)$ , where  $y_j^* = a$  if  $f(\mathbf{x}_j, \mathbf{b}) + u_j \leq a$ ,  $y_j^* = b$  if  $f(\mathbf{x}_j, \mathbf{b}) + u_j \geq b$ , and  $y_j^* = f(\mathbf{x}_j, \mathbf{b}) + u_j$  otherwise, meaning that  $y_j^*$  is censored.  $a$  and  $b$  are specified as they are for `pr()`.

`scores` calculates the scores. The  $j$ th new variable created will contain the score for the  $j$ th parameter in `e(b)`.

`parameters` and `parameters(paramnames)` calculate predictions for all or a subset of the named substitutable expressions in the model. `parameters()` does not appear in the dialog box.

`paramnames` is `param [param [ . . . ]]`, and `param` is a name of a substitutable expression as specified in one of `nl`'s `define()` options.

# margins

## Description for margins

`margins` estimates margins of response for fitted values.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
margins [marginlist] , predict(statistic ...) [options]
```

| <i>statistic</i>                      | Description                                            |
|---------------------------------------|--------------------------------------------------------|
| <code>yhat</code>                     | fitted values; the default                             |
| <code>pr(a,b)</code>                  | not allowed with margins                               |
| <code>e(a,b)</code>                   | not allowed with margins                               |
| <code>ystar(a,b)</code>               | not allowed with margins                               |
| <code>residuals</code>                | not allowed with margins                               |
| <code>parameters</code>               | predicted parameters                                   |
| <code>parameters(<i>param</i>)</code> | predicted, named substitutable expression <i>param</i> |

Statistics not allowed with margins are functions of stochastic quantities other than `e(b)`.

For the full syntax, see [R] [margins](#).

## Remarks and examples

### ▷ Example 1: Basic usage of predict and margins

Obtaining predictions after fitting a nonlinear regression model with `nl` is no more difficult than obtaining predictions after fitting a linear regression model with `regress`. Here we fit a model of `mpg` on `weight`, allowing for a nonlinear relationship:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. nl (mpg = {b0} + {b1}*weight^{gamma=-.5}), variables(weight) nolog
```

| Source   | SS        | df | MS         |                 |          |  |
|----------|-----------|----|------------|-----------------|----------|--|
| Model    | 1646.4376 | 2  | 823.218805 | Number of obs = | 74       |  |
| Residual | 797.02185 | 71 | 11.2256599 | R-squared =     | 0.6738   |  |
|          |           |    |            | Adj R-squared = | 0.6646   |  |
|          |           |    |            | Root MSE =      | 3.350472 |  |
| Total    | 2443.4595 | 73 | 33.4720474 | Res. dev. =     | 385.8874 |  |

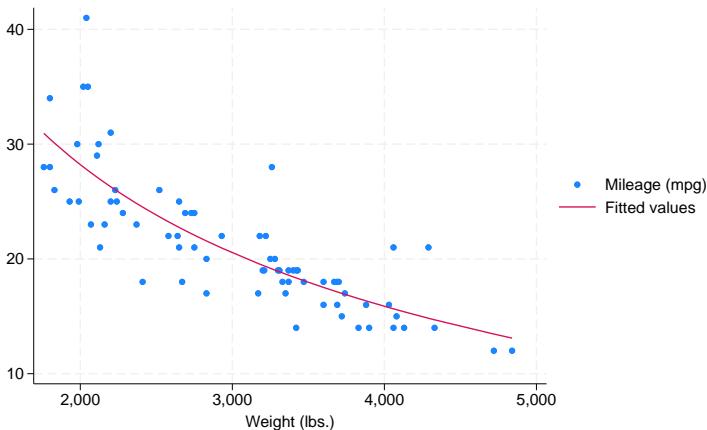
  

| mpg    | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|--------|-------------|-----------|-------|-------|----------------------|----------|
| /b0    | -18.17687   | 60.62086  | -0.30 | 0.765 | -139.0514            | 102.6977 |
| /b1    | 1377.163    | 5291.956  | 0.26  | 0.795 | -9174.697            | 11929.02 |
| /gamma | -.4460793   | .6763682  | -0.66 | 0.512 | -1.794719            | .9025606 |

Note: Parameter **b0** is used as a constant term during estimation.

Now, we obtain the predicted values of `mpg` and plot them in a graph along with the observed values:

```
. predict mpghat
(option yhat assumed; fitted values)
. scatter mpg weight || line mpghat weight, sort
```



Suppose we wanted to know how sensitive mpg is to changes in weight for cars that weigh 3,000 pounds. We can use margins to find out:

```
. margins, eyex(weight) at(weight = 3000)
Conditional marginal effects Number of obs = 74
Model VCE: GNR
Expression: Fitted values, predict()
ey/ex wrt: weight
At: weight = 3000
```

|        | ey/ex     | Delta-method<br>std. err. | z      | P> z  | [95% conf. interval] |          |
|--------|-----------|---------------------------|--------|-------|----------------------|----------|
| weight | -.8408143 | .0804327                  | -10.45 | 0.000 | -.9984596            | -.683169 |

With the `eyex()` option, margins reports elasticities. These results show that if we increase weight by 1%, then mpg decreases by about 0.84%.

◀

## □ Technical note

Observant readers will notice that margins issued a warning message stating that it could not perform its usual check for estimable functions. In the case of `nl`, as long as you do not specify the `predict()` option of margins or specify the default `predict(yhat)`, you can safely ignore that message. The predicted values that `nl` produces are suitable for use with margins. However, if you specify any `predict()` options other than `yhat`, then the output from margins after using `nl` will not be correct.

□

## ▷ Example 2: Predictions from named expressions

Continuing with [example 1](#), consider now a logistic regression that explains the origin of the cars as a function of their price, their fuel efficiency, and their length.

```
. logit foreign price mpg length
Iteration 0: Log likelihood = -45.03321
Iteration 1: Log likelihood = -27.130212
Iteration 2: Log likelihood = -23.885718
Iteration 3: Log likelihood = -23.468167
Iteration 4: Log likelihood = -23.460272
Iteration 5: Log likelihood = -23.460253
Iteration 6: Log likelihood = -23.460253

Logistic regression Number of obs = 74
LR chi2(3) = 43.15
Prob > chi2 = 0.0000
Pseudo R2 = 0.4790

Log likelihood = -23.460253
```

| foreign | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|---------|-------------|-----------|-------|-------|----------------------|-----------|
| price   | .0005602    | .0001889  | 2.97  | 0.003 | .0001899             | .0009306  |
| mpg     | -.0606663   | .0854276  | -0.71 | 0.478 | -.2281012            | .1067687  |
| length  | -.1718435   | .0538215  | -3.19 | 0.001 | -.2773316            | -.0663553 |
| _cons   | 27.7666     | 9.860407  | 2.82  | 0.005 | 8.440557             | 47.09264  |

After fitting this model, the `predict` command computes predicted probabilities by default or the linear prediction if we use the `xb` option:

```
. predict p_logit
(option pr assumed; Pr(foreign))
. predict xb_logit, xb
```

Next recall that logistic regression fits the logistic cumulative density function to our data via maximum likelihood. We could instead fit this (nonlinear) function via nonlinear least squares using the `nl` command:

```
. nl (foreign = 1/(1 + exp(-{lp: price mpg length})))
Iteration 0: Residual SS = 18.5
Iteration 1: Residual SS = 7.7385558
Iteration 2: Residual SS = 7.6199595
Iteration 3: Residual SS = 7.6149448
Iteration 4: Residual SS = 7.6143961
Iteration 5: Residual SS = 7.6143036
Iteration 6: Residual SS = 7.6142876
Iteration 7: Residual SS = 7.6142848
Iteration 8: Residual SS = 7.6142843
```

| Source   | SS        | df | MS         |                 |          |
|----------|-----------|----|------------|-----------------|----------|
| Model    | 7.8451752 | 3  | 2.61505839 | Number of obs = | 74       |
| Residual | 7.6142843 | 70 | .10877549  | R-squared =     | 0.5075   |
|          |           |    |            | Adj R-squared = | 0.4864   |
|          |           |    |            | Root MSE =      | .3298113 |
| Total    | 15.459459 | 73 | .211773417 | Res. dev. =     | 41.72401 |

| foreign | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|---------|-------------|-----------|-------|-------|----------------------|-----------|
| price   | .0006768    | .0002826  | 2.39  | 0.019 | .0001131             | .0012405  |
| mpg     | -.0703169   | .0704894  | -1.00 | 0.322 | -.2109036            | .0702698  |
| length  | -.2241772   | .0912304  | -2.46 | 0.016 | -.4061305            | -.0422224 |
| _cons   | 36.6067     | 15.28359  | 2.40  | 0.019 | 6.124531             | 67.08886  |

Note: Parameter `_cons` is used as a constant term during estimation.

Computing predicted probabilities from this model is still the default for `predict` because these are simply the values our model predicts for the dependent variable:

```
. predict p_nl
(option yhat assumed; fitted values)
```

To obtain the linear prediction, we need the predicted values of the named linear combination `{lp:}` we created. We can compute those using the `parameters()` option:

```
. predict xb_nl, parameters(lp)
```

◀

## Also see

[R] [nl](#) — Nonlinear least-squares estimation

[U] [20 Estimation and postestimation commands](#)

[Description](#)  
[Options](#)  
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## Description

`nlcom` computes point estimates, standard errors, test statistics, significance levels, and confidence intervals for (possibly) nonlinear combinations of parameter estimates after any Stata estimation command, including survey estimation. Results are displayed in the usual table format used for displaying estimation results. Standard error calculations are based on the “delta method”, an approximation appropriate in large samples.

## Quick start

Estimate the ratio of the coefficient of `x2` to the coefficient of `x1`

```
nlcom _b[x2]/_b[x1]
```

Also estimate the ratio of the coefficient of `x3` to coefficient of `x1`

```
nlcom (_b[x2]/_b[x1]) (_b[x3]/_b[x1])
```

Add labels to the ratios

```
nlcom (r21:_b[x2]/_b[x1]) (r31:_b[x3]/_b[x1])
```

Same as above, but post estimates and use the `test` command to test that both ratios are equal to 1

```
nlcom (r21:_b[x2]/_b[x1]) (r31:_b[x3]/_b[x1]), post
test (r21 = 1) (r31 = 1)
```

Estimate the ratio of the coefficients of factor indicators 2.a and 3.a

```
nlcom _b[2.a]/_b[3.a]
```

Estimate the ratio of the coefficients of `x1` in the equations for `y1` and `y2` in a multiequation model

```
nlcom _b[y1:x1]/_b[y2:x1]
```

## Menu

Statistics > Postestimation

## Syntax

Nonlinear combination of parameters—*one expression*

```
nlcom [name:]exp [, options]
```

Nonlinear combinations of parameters—*more than one expression*

```
nlcom ([name:]exp) ([[name:]exp) ... [, options]
```

| <i>options</i>               | Description                                                                                                 |
|------------------------------|-------------------------------------------------------------------------------------------------------------|
| <code>level(#)</code>        | set confidence level; default is <code>level(95)</code>                                                     |
| <code>iterate(#)</code>      | maximum number of iterations                                                                                |
| <code>display_options</code> | control column formats and line width                                                                       |
| <code>post</code>            | post estimation results                                                                                     |
| <code>eform</code>           | report exponentiated estimates; column title is “exp(b)”                                                    |
| <code>eform(string)</code>   | report exponentiated estimates; column title is “string”                                                    |
| <code>noheader</code>        | suppress output header                                                                                      |
| <code>df(#)</code>           | use <i>t</i> distribution with # degrees of freedom for computing <i>p</i> -values and confidence intervals |

`collect` is allowed; see [\[U\] 11.1.10 Prefix commands](#).

Only one of `eform` or `eform()` is allowed.

`noheader` and `df(#)` do not appear in the dialog box.

The second syntax means that if more than one expression is specified, each must be surrounded by parentheses. The optional *name* is any valid Stata name and labels the transformations.

*exp* is a possibly nonlinear expression containing

```
_b[coef]
_b[eqno:coef]
[eqno]coef
[eqno]_b[coef]
```

*eqno* is

```
##
name
```

*coef* identifies a coefficient in the model. *coef* is typically a variable name, a level indicator, an interaction indicator, or an interaction involving continuous variables. Level indicators identify one level of a factor variable and interaction indicators identify one combination of levels of an interaction; see [\[U\] 11.4.3 Factor variables](#). *coef* may contain time-series operators; see [\[U\] 11.4.4 Time-series varlists](#).

Distinguish between `[]`, which are to be typed, and `[_]`, which indicate optional arguments.

## Options

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [\[U\] 20.8 Specifying the width of confidence intervals](#).

`iterate(#)` specifies the maximum number of iterations used to find the optimal step size in calculating numerical derivatives of the transformation(s) with respect to the original parameters. By default, the maximum number of iterations is 100, but convergence is usually achieved after only a few iterations. You should rarely have to use this option.

`display_options:` `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

`post` causes `nlcom` to behave like a Stata estimation (`eclass`) command. When `post` is specified, `nlcom` will post the vector of transformed parameters and its estimated variance–covariance matrix to `e()`. This option, in essence, makes the transformation permanent. Thus you could, after posting, treat the transformed estimation results in the same way as you would treat results from other Stata estimation commands. For example, after posting, you could redisplay the results by typing `nlcom` without any arguments, or use `test` to perform simultaneous tests of hypotheses on linear combinations of the transformed parameters; see [R] [test](#).

Specifying `post` clears out the previous estimation results, which can be recovered only by refitting the original model or by storing the estimation results before running `nlcom` and then restoring them; see [R] [estimates store](#).

`eform` and `eform(string)` specify that the table of estimates be displayed in exponentiated form: for each estimate,  $e^b$  rather than  $b$  is displayed, where  $b$  is the nonlinear combination of parameters. `eform` labels the exponentiated results `exp(b)`, whereas `eform(string)` uses `string` to label them. Standard errors and confidence intervals are also transformed; see [R] [eform\\_option](#).

The following options are available with `nlcom` but are not shown in the dialog box:

`noheader` suppresses the output header.

`df(#)` specifies that the  $t$  distribution with  $\#$  degrees of freedom be used for computing  $p$ -values and confidence intervals.

## Remarks and examples

Remarks are presented under the following headings:

[Introduction](#)

[Basics](#)

[Using the post option](#)

[Reparameterizing ML estimates for univariate data](#)

[nlcom versus eform](#)

## Introduction

`nlcom` and `predictnl` both use the delta method. They take nonlinear transformations of the estimated parameter vector from some fitted model and apply the delta method to calculate the variance, standard error, Wald test statistic, etc., of the transformations. `nlcom` is designed for functions of the parameters, and `predictnl` is designed for functions of the parameters and of the data, that is, for predictions.

`nlcom` generalizes `lincom` (see [R] [lincom](#)) in two ways. First, `nlcom` allows the transformations to be nonlinear. Second, `nlcom` can be used to simultaneously estimate many transformations (whether linear or nonlinear) and to obtain the estimated variance–covariance matrix of these transformations.

## Basics

In [R] `lincom`, the following regression was performed:

```
. use https://www.stata-press.com/data/r19/regress
. regress y x1 x2 x3
```

| Source   | SS         | df  | MS         | Number of obs | = | 148    |
|----------|------------|-----|------------|---------------|---|--------|
| Model    | 3259.3561  | 3   | 1086.45203 | F(3, 144)     | = | 96.12  |
| Residual | 1627.56282 | 144 | 11.3025196 | Prob > F      | = | 0.0000 |
|          |            |     |            | R-squared     | = | 0.6670 |
|          |            |     |            | Adj R-squared | = | 0.6600 |
| Total    | 4886.91892 | 147 | 33.2443464 | Root MSE      | = | 3.3619 |

| y     | Coefficient | Std. err. | t      | P> t  | [95% conf. interval] |           |
|-------|-------------|-----------|--------|-------|----------------------|-----------|
| x1    | 1.457113    | 1.07461   | 1.36   | 0.177 | -.666934             | 3.581161  |
| x2    | 2.221682    | .8610358  | 2.58   | 0.011 | .5197797             | 3.923583  |
| x3    | -.006139    | .0005543  | -11.08 | 0.000 | -.0072345            | -.0050435 |
| _cons | 36.10135    | 4.382693  | 8.24   | 0.000 | 27.43863             | 44.76407  |

Then, `lincom` was used to estimate the difference between the coefficients of `x1` and `x2`:

```
. lincom _b[x2] - _b[x1]
(1) - x1 + x2 = 0
```

| y   | Coefficient | Std. err. | t    | P> t  | [95% conf. interval] |          |
|-----|-------------|-----------|------|-------|----------------------|----------|
| (1) | .7645682    | .9950282  | 0.77 | 0.444 | -1.20218             | 2.731316 |

It was noted, however, that nonlinear expressions are not allowed with `lincom`:

```
. lincom _b[x2]/_b[x1]
not possible with test
r(131);
```

Nonlinear transformations are instead estimated using `nlcom`:

```
. nlcom _b[x2]/_b[x1]
 _nl_1: _b[x2]/_b[x1]
```

| y     | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |          |
|-------|-------------|-----------|------|-------|----------------------|----------|
| _nl_1 | 1.524714    | .9812848  | 1.55 | 0.120 | -.3985686            | 3.447997 |

### □ Technical note

The notation `_b[name]` is the standard way in Stata to refer to regression coefficients; see [U] 13.5 [Accessing coefficients and standard errors](#). Some commands, such as `lincom` and `test`, allow you to drop the `_b[]` and just refer to the coefficients by *name*. `nlcom`, however, requires the full specification `_b[name]`.

□

Returning to our linear regression example, `nlcom` also allows simultaneous estimation of more than one combination:

```
. nlcom (_b[x2]/_b[x1]) (_b[x3]/_b[x1]) (_b[x3]/_b[x2])
 _nl_1: _b[x2]/_b[x1]
 _nl_2: _b[x3]/_b[x1]
 _nl_3: _b[x3]/_b[x2]
```

| y     | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-------|-------------|-----------|-------|-------|----------------------|----------|
| _nl_1 | 1.524714    | .9812848  | 1.55  | 0.120 | -.3985686            | 3.447997 |
| _nl_2 | -.0042131   | .0033483  | -1.26 | 0.208 | -.0107756            | .0023494 |
| _nl_3 | -.0027632   | .0010695  | -2.58 | 0.010 | -.0048594            | -.000667 |

We can also label the transformations to produce more informative names in the estimation table:

```
. nlcom (ratio21:_b[x2]/_b[x1]) (ratio31:_b[x3]/_b[x1]) (ratio32:_b[x3]/_b[x2])
 ratio21: _b[x2]/_b[x1]
 ratio31: _b[x3]/_b[x1]
 ratio32: _b[x3]/_b[x2]
```

| y       | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|---------|-------------|-----------|-------|-------|----------------------|----------|
| ratio21 | 1.524714    | .9812848  | 1.55  | 0.120 | -.3985686            | 3.447997 |
| ratio31 | -.0042131   | .0033483  | -1.26 | 0.208 | -.0107756            | .0023494 |
| ratio32 | -.0027632   | .0010695  | -2.58 | 0.010 | -.0048594            | -.000667 |

`nlcom` stores the vector of estimated combinations and its estimated variance–covariance matrix in `r()`.

```
. matrix list r(b)
r(b)[1,3]
 ratio21 ratio31 ratio32
c1 1.5247143 -.00421315 -.00276324
. matrix list r(V)
symmetric r(V)[3,3]
 ratio21 ratio31 ratio32
ratio21 .96291982
ratio31 -.00287781 .00001121
ratio32 -.00014234 2.137e-06 1.144e-06
```

## Using the post option

When used with the `post` option, `nlcom` stores the estimation vector and variance–covariance matrix in `e()`, making the transformation permanent:

```
. quietly nlcom (ratio21:_b[x2]/_b[x1]) (ratio31:_b[x3]/_b[x1])
> (ratio32:_b[x3]/_b[x2]), post
. matrix list e(b)
e(b) [1,3]
 ratio21 ratio31 ratio32
y1 1.5247143 -.00421315 -.00276324
. matrix list e(V)
symmetric e(V) [3,3]
 ratio21 ratio31 ratio32
ratio21 .96291982
ratio31 -.00287781 .00001121
ratio32 -.00014234 2.137e-06 1.144e-06
```

After posting, we can proceed as if we had just run a Stata estimation (`eclass`) command. For instance, we can replay the results,

```
. nlcom
```

| y       | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|---------|-------------|-----------|-------|-------|----------------------|----------|
| ratio21 | 1.524714    | .9812848  | 1.55  | 0.120 | -.3985686            | 3.447997 |
| ratio31 | -.0042131   | .0033483  | -1.26 | 0.208 | -.0107756            | .0023494 |
| ratio32 | -.0027632   | .0010695  | -2.58 | 0.010 | -.0048594            | -.000667 |

or perform other postestimation tasks in the transformed metric, this time making reference to the new “coefficients”:

```
. display _b[ratio31]
-.00421315
. estat vce, correlation
Correlation matrix of coefficients of nlcom model
 e(V) | ratio21 ratio31 ratio32
-----|-----
ratio21 | 1.0000
ratio31 | -0.8759 1.0000
ratio32 | -0.1356 0.5969 1.0000
. test _b[ratio21] = 1
(1) ratio21 = 1
 chi2(1) = 0.29
 Prob > chi2 = 0.5928
```

We see that testing `_b[ratio21]=1` in the transformed metric is equivalent to testing using `testnl _b[x2]/_b[x1]=1` in the original metric:

```
. quietly regress y x1 x2 x3
. testnl _b[x2]/_b[x1] = 1
(1) _b[x2]/_b[x1] = 1
 chi2(1) = 0.29
 Prob > chi2 = 0.5928
```

We needed to refit the regression model to recover the original parameter estimates.

## □ Technical note

In a previous technical note, we mentioned that commands such as `lincom` and `test` permit reference to *name* instead of `_b[name]`. This is not the case when `lincom` and `test` are used after `nlcom`, `post`. In the above, we used

```
. test _b[ratio21] = 1
```

rather than

```
. test ratio21 = 1
```

which would have returned an error. Consider this a limitation of Stata. For the shorthand notation to work, you need a variable named *name* in the data. In `nlcom`, however, *name* is just a coefficient label that does not necessarily correspond to any variable in the data. □

## Reparameterizing ML estimates for univariate data

When run using only a response and no covariates, Stata's maximum likelihood (ML) estimation commands will produce ML estimates of the parameters of some assumed univariate distribution for the response. The parameterization, however, is usually not one we are used to dealing with in a nonregression setting. In such cases, `nlcom` can be used to transform the estimation results from a regression model to those from a maximum likelihood estimation of the parameters of a univariate probability distribution in a more familiar metric.

### ▷ Example 1

Consider the following univariate data on  $Y = \#$  of traffic accidents at a certain intersection in a given year:

```
. use https://www.stata-press.com/data/r19/trafint
```

```
. summarize accidents
```

| Variable  | Obs | Mean     | Std. dev. | Min | Max |
|-----------|-----|----------|-----------|-----|-----|
| accidents | 12  | 13.83333 | 14.47778  | 0   | 41  |

A quick glance of the output from `summarize` leads us to quickly reject the assumption that  $Y$  is distributed as Poisson because the estimated variance of  $Y$  is much greater than the estimated mean of  $Y$ .

Instead, we choose to model the data as univariate negative binomial, of which a common parameterization is

$$\Pr(Y = y) = \frac{\Gamma(r + y)}{\Gamma(r)\Gamma(y + 1)} p^r (1 - p)^y \quad 0 \leq p \leq 1, \quad r > 0, \quad y = 0, 1, \dots$$

with

$$E(Y) = \frac{r(1 - p)}{p} \quad \text{Var}(Y) = \frac{r(1 - p)}{p^2}$$

There exist no closed-form solutions for the maximum likelihood estimates of  $p$  and  $r$ , yet they may be estimated by the iterative method of Newton–Raphson. One way to get these estimates would be to write our own Newton–Raphson program for the negative binomial. Another way would be to write our own ML evaluator; see [R] ml.

The easiest solution, however, would be to use Stata’s existing negative binomial ML regression command, nbreg. The only problem with this solution is that nbreg estimates a different parameterization of the negative binomial, but we can worry about that later.

```
. nbreg accidents
Fitting Poisson model:
Iteration 0: Log likelihood = -105.05361
Iteration 1: Log likelihood = -105.05361
Fitting constant-only model:
Iteration 0: Log likelihood = -43.948619
Iteration 1: Log likelihood = -43.891483
Iteration 2: Log likelihood = -43.89144
Iteration 3: Log likelihood = -43.89144
Fitting full model:
Iteration 0: Log likelihood = -43.89144
Iteration 1: Log likelihood = -43.89144
Negative binomial regression Number of obs = 12
 LR chi2(0) = 0.00
Dispersion: mean Prob > chi2 = .
Log likelihood = -43.89144 Pseudo R2 = 0.0000
```

| accidents | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |          |
|-----------|-------------|-----------|------|-------|----------------------|----------|
| _cons     | 2.627081    | .3192233  | 8.23 | 0.000 | 2.001415             | 3.252747 |
| /lnalpha  | .1402425    | .4187147  |      |       | -.6804233            | .9609083 |
| alpha     | 1.150553    | .4817534  |      |       | .5064026             | 2.61407  |

```
LR test of alpha=0: chibar2(01) = 122.32 Prob >= chibar2 = 0.000
```

```
. nbreg, coeflegend
Negative binomial regression Number of obs = 12
 LR chi2(0) = 0.00
Dispersion: mean Prob > chi2 = .
Log likelihood = -43.89144 Pseudo R2 = 0.0000
```

| accidents | Coefficient | Legend       |
|-----------|-------------|--------------|
| _cons     | 2.627081    | _b[_cons]    |
| /lnalpha  | .1402425    | _b[/lnalpha] |
| alpha     | 1.150553    |              |

```
LR test of alpha=0: chibar2(01) = 122.32 Prob >= chibar2 = 0.000
```

From this output, we see that, when used with univariate data, nbreg estimates a regression intercept,  $\beta_0$ , and the logarithm of some parameter  $\alpha$ . This parameterization is useful in regression models:  $\beta_0$  is the intercept meant to be augmented with other terms of the linear predictor, and  $\alpha$  is an overdispersion parameter used for comparison with the Poisson regression model.

However, we need to transform  $(\beta_0, \ln\alpha)$  to  $(p, r)$ . Examining *Methods and formulas* of [R] `nbreg` reveals the transformation as

$$p = \{1 + \alpha \exp(\beta_0)\}^{-1} \quad r = \alpha^{-1}$$

which we apply using `nlcom`:

```
. nlcom (p:1/(1 + exp(_b[/lnalpha] + _b[_cons])))
> (r:exp(-_b[/lnalpha]))
 p: 1/(1 + exp(_b[/lnalpha] + _b[_cons]))
 r: exp(-_b[/lnalpha])
```

| accidents | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |          |
|-----------|-------------|-----------|------|-------|----------------------|----------|
| p         | .0591157    | .0292857  | 2.02 | 0.044 | .0017168             | .1165146 |
| r         | .8691474    | .3639248  | 2.39 | 0.017 | .1558679             | 1.582427 |

Given the invariance of maximum likelihood estimators and the properties of the delta method, the above parameter estimates, standard errors, etc., are precisely those we would have obtained had we instead performed the Newton–Raphson optimization in the  $(p, r)$  metric.

◀

## □ Technical note

Note how we referred to the estimate of  $\ln\alpha$  above as `_b[/lnalpha]`. This is not entirely evident from the output of `nbreg`, which is why we redisplayed the results using the `coeflegend` option so that we would know how to refer to the coefficients; [U] [13.5 Accessing coefficients and standard errors](#). □

## nlcom versus eform

Many Stata estimation commands allow you to display exponentiated regression coefficients, some by default, some optionally. Known as “`eform`” in Stata terminology, this reparameterization serves many uses: it gives odds ratios for logistic models, hazard ratios in survival models, incidence-rate ratios in Poisson models, and relative-risk ratios in multinomial logit models, to name a few.

For example, consider the following estimation taken directly from the [technical note](#) in [R] `poisson`:

```
. use https://www.stata-press.com/data/r19/airline
. generate lnN = ln(n)
. poisson injuries XYZowned lnN
Iteration 0: Log likelihood = -22.333875
Iteration 1: Log likelihood = -22.332276
Iteration 2: Log likelihood = -22.332276

Poisson regression Number of obs = 9
LR chi2(2) = 19.15
Prob > chi2 = 0.0001
Pseudo R2 = 0.3001

Log likelihood = -22.332276
```

| injuries | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |          |
|----------|-------------|-----------|------|-------|----------------------|----------|
| XYZowned | .6840667    | .3895877  | 1.76 | 0.079 | -.0795111            | 1.447645 |
| lnN      | 1.424169    | .3725155  | 3.82 | 0.000 | .6940517             | 2.154285 |
| _cons    | 4.863891    | .7090501  | 6.86 | 0.000 | 3.474178             | 6.253603 |

When we replay results and specify the `irr` (incidence-rate ratios) option,

```
. poisson, irr
Poisson regression Number of obs = 9
 LR chi2(2) = 19.15
 Prob > chi2 = 0.0001
 Pseudo R2 = 0.3001

Log likelihood = -22.332276
```

| injuries | IRR      | Std. err. | z    | P> z  | [95% conf. interval] |          |
|----------|----------|-----------|------|-------|----------------------|----------|
| XYZowned | 1.981921 | .7721322  | 1.76 | 0.079 | .9235678             | 4.253085 |
| lnN      | 4.154402 | 1.547579  | 3.82 | 0.000 | 2.00181              | 8.621728 |
| _cons    | 129.5272 | 91.84126  | 6.86 | 0.000 | 32.2713              | 519.8828 |

Note: `_cons` estimates baseline incidence rate.

we obtain the exponentiated regression coefficients and their estimated standard errors.

Contrast this with what we obtain if we exponentiate the coefficients manually by using `nlcom`:

```
. nlcom (E_XYZowned:exp(_b[XYZowned])) (E_lnN:exp(_b[lnN]))
 E_XYZowned: exp(_b[XYZowned])
 E_lnN: exp(_b[lnN])
```

| injuries   | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |          |
|------------|-------------|-----------|------|-------|----------------------|----------|
| E_XYZowned | 1.981921    | .7721322  | 2.57 | 0.010 | .4685701             | 3.495273 |
| E_lnN      | 4.154402    | 1.547579  | 2.68 | 0.007 | 1.121203             | 7.187602 |

There are three things to note when comparing an estimation command specified with an *eform\_option* with the `nlcom` command specified with coefficients that are exponentiated manually:

1. The exponentiated coefficients and standard errors are identical. This is certainly good news.
2. The Wald test statistic ( $z$ ) and level of significance are different. When using *eform\_option* with an estimation command, the Wald test does not change from what you would have obtained without the *eform\_option*. You can see this by comparing both versions of the `poisson` output given [previously](#).

When you use *eform*, Stata knows that what is usually desired is a test of

$$H_0: \exp(\beta) = 1$$

and not the uninformative-by-comparison

$$H_0: \exp(\beta) = 0$$

The test of  $H_0: \exp(\beta) = 1$  is asymptotically equivalent to a test of  $H_0: \beta = 0$ , the Wald test in the original metric, but the latter has better small-sample properties. Thus if you specify *eform*, you get a test of  $H_0: \beta = 0$ .

`nlcom`, however, is general. It does not attempt to infer the test of greatest interest for a given transformation, and so a test of

$$H_0: \text{transformed coefficient} = 0$$

is always given, regardless of the transformation.

3. You may be surprised to see that, even though the coefficients and standard errors are identical, the confidence intervals (both 95%) are different.

iform confidence intervals reported by an estimation command are standard confidence intervals with the endpoints transformed. For example, the confidence interval for the coefficient on `lnN` is `[0.694, 2.154]`, whereas the confidence interval for the incidence-rate ratio due to `lnN` is `[exp(0.694), exp(2.154)] = [2.002, 8.619]`, which, except for some roundoff error, is what we see from the `output` of `poisson, irr`. For exponentiated coefficients, confidence intervals based on transform-the-endpoints methodology generally have better small-sample properties than their asymptotically equivalent counterparts.

The transform-the-endpoints method, however, gives valid coverage only when the transformation is monotonic. `nlcom` uses a more general and asymptotically equivalent method for calculating confidence intervals, as described in [Methods and formulas](#).

Notice that `nlcom` has an `iform` option that behaves the same way as the `iform` option of any estimation command. Thus, we could have obtained the exponentiated coefficients and confidence intervals after `poisson` by specifying the `iform` option with `nlcom`; that is, we would type

```
. nlcom (E_XYZowned: _b[XYZowned]) (E_lnN: _b[lnN]), iform
 E_XYZowned: _b[XYZowned]
 E_lnN: _b[lnN]
```

| injuries   | exp(b)   | Std. err. | z    | P> z  | [95% conf. interval] |          |
|------------|----------|-----------|------|-------|----------------------|----------|
| E_XYZowned | 1.981921 | .7721322  | 1.76 | 0.079 | .9235678             | 4.253085 |
| E_lnN      | 4.154402 | 1.547579  | 3.82 | 0.000 | 2.00181              | 8.621728 |

We obtain the same results as we did using `poisson` with the `irr` option. The interpretation of the coefficients and confidence intervals follows the discussion above.

## Stored results

`nlcom` stores the following in `r()`:

### Scalars

`r(N)` number of observations  
`r(df_r)` residual degrees of freedom

### Matrices

`r(b)` vector of transformed coefficients  
`r(V)` estimated variance–covariance matrix of the transformed coefficients  
`r(table)` matrix containing the estimates with their standard errors, test statistics, *p*-values, and confidence intervals

If `post` is specified, `nlcom` also stores the following in `e()`:

### Scalars

`e(N)` number of observations  
`e(df_r)` residual degrees of freedom  
`e(N_strata)` number of strata *L*, if used after `svy`  
`e(N_psu)` number of sampled PSUs *n*, if used after `svy`  
`e(rank)` rank of `e(V)`

### Macros

`e(cmd)` `nlcom`  
`e(predict)` program used to implement `predict`  
`e(properties)` `b V`

## Matrices

|            |                                                                                                      |
|------------|------------------------------------------------------------------------------------------------------|
| e(b)       | vector of transformed coefficients                                                                   |
| e(V)       | estimated variance–covariance matrix of the transformed coefficients                                 |
| e(V_srs)   | simple-random-sampling-without-replacement (co)variance $\widehat{V}_{\text{srswor}}$ , if svy       |
| e(V_srswr) | simple-random-sampling-with-replacement (co)variance $\widehat{V}_{\text{srswr}}$ , if svy and fpc() |
| e(V_msp)   | misspecification (co)variance $\widehat{V}_{\text{msp}}$ , if svy and available                      |

## Functions

|           |                         |
|-----------|-------------------------|
| e(sample) | marks estimation sample |
|-----------|-------------------------|

## Methods and formulas

Given a  $1 \times k$  vector of parameter estimates,  $\widehat{\boldsymbol{\theta}} = (\widehat{\theta}_1, \dots, \widehat{\theta}_k)$ , consider the estimated  $p$ -dimensional transformation

$$g(\widehat{\boldsymbol{\theta}}) = [g_1(\widehat{\boldsymbol{\theta}}), g_2(\widehat{\boldsymbol{\theta}}), \dots, g_p(\widehat{\boldsymbol{\theta}})]$$

The estimated variance–covariance of  $g(\widehat{\boldsymbol{\theta}})$  is given by

$$\widehat{\text{Var}}\{g(\widehat{\boldsymbol{\theta}})\} = \mathbf{G}\mathbf{V}\mathbf{G}'$$

where  $\mathbf{G}$  is the  $p \times k$  matrix of derivatives for which

$$\mathbf{G}_{ij} = \left. \frac{\partial g_i(\boldsymbol{\theta})}{\partial \theta_j} \right|_{\boldsymbol{\theta}=\widehat{\boldsymbol{\theta}}} \quad i = 1, \dots, p \quad j = 1, \dots, k$$

and  $\mathbf{V}$  is the estimated variance–covariance matrix of  $\widehat{\boldsymbol{\theta}}$ . Standard errors are obtained as the square roots of the variances.

The Wald test statistic for testing

$$H_0: g_i(\boldsymbol{\theta}) = 0$$

versus the two-sided alternative is given by

$$Z_i = \frac{g_i(\widehat{\boldsymbol{\theta}})}{[\widehat{\text{Var}}_{ii}\{g(\widehat{\boldsymbol{\theta}})\}]^{1/2}}$$

When the variance–covariance matrix of  $\widehat{\boldsymbol{\theta}}$  is an asymptotic covariance matrix,  $Z_i$  is approximately distributed as Gaussian. For linear regression,  $Z_i$  is taken to be approximately distributed as  $t_{1,r}$  where  $r$  is the residual degrees of freedom from the original fitted model.

A  $(1 - \alpha) \times 100\%$  confidence interval for  $g_i(\boldsymbol{\theta})$  is given by

$$g_i(\widehat{\boldsymbol{\theta}}) \pm z_{\alpha/2} [\widehat{\text{Var}}_{ii}\{g(\widehat{\boldsymbol{\theta}})\}]^{1/2}$$

for those cases where  $Z_i$  is Gaussian and

$$g_i(\widehat{\boldsymbol{\theta}}) \pm t_{\alpha/2,r} [\widehat{\text{Var}}_{ii}\{g(\widehat{\boldsymbol{\theta}})\}]^{1/2}$$

for those cases where  $Z_i$  is  $t$  distributed.  $z_p$  is the  $1 - p$  quantile of the standard normal distribution, and  $t_{p,r}$  is the  $1 - p$  quantile of the  $t$  distribution with  $r$  degrees of freedom.

## References

- Feiveson, A. H. 1999. FAQ: What is the delta method and how is it used to estimate the standard error of a transformed parameter? <https://www.stata.com/support/faqs/stat/deltam.html>.
- Oehlert, G. W. 1992. A note on the delta method. *American Statistician* 46: 27–29. <https://doi.org/10.2307/2684406>.
- Phillips, P. C. B., and J. Y. Park. 1988. On the formulation of Wald tests of nonlinear restrictions. *Econometrica* 56: 1065–1083. <https://doi.org/10.2307/1911359>.

## Also see

- [R] **lincom** — Linear combinations of parameters
- [R] **predictnl** — Obtain nonlinear predictions, standard errors, etc., after estimation
- [R] **test** — Test linear hypotheses after estimation
- [R] **testnl** — Test nonlinear hypotheses after estimation
- [SVY] **svy postestimation** — Postestimation tools for svy
- [U] **20 Estimation and postestimation commands**

|             |                      |                |                      |
|-------------|----------------------|----------------|----------------------|
| Description | Quick start          | Menu           | Syntax               |
| Options     | Remarks and examples | Stored results | Methods and formulas |
| References  | Also see             |                |                      |

## Description

`nlsur` fits a system of nonlinear equations by feasible generalized nonlinear least squares (FGNLS). With the interactive version of the command, you enter the system of equations on the command line or in the dialog box by using *substitutable expressions*. If you have a system that you use regularly, you can write a *substitutable expression program* and use the second syntax to avoid having to reenter the system every time. The function evaluator program version gives you the most flexibility in exchange for increased complexity; with this version, your program is given a vector of parameters and a variable list, and your program computes the system of equations.

When you write a substitutable expression program or a function evaluator program, the first five letters of the name must be `nlsur`. *sexp\_prog* and *func\_prog* refer to the name of the program without the first five letters. For example, if you wrote a function evaluator program named `nlsurregss`, you would type `nlsur regss @ ...` to estimate the parameters.

## Quick start

Two-parameter exponential model regressing  $y_1$  on  $x$  using the default FGNLS estimator

```
nlsur (y1 = {b1}*{b2}^x)
```

Add the `variables()` option to allow for missing values of  $y_1$  and  $x$

```
nlsur (y1 = {b1}*{b2}^x), variables(y1 x)
```

Two-equation, two-parameter exponential model

```
nlsur (y1 = {b1}*{b2}^x) (y2 = {g1}*{g2}^x), variables(y1 y2 x)
```

Same as above, but use the iterative FGNLS estimator

```
nlsur (y1 = {b1}*{b2}^x) (y2 = {g1}*{g2}^x), variables(y1 y2 x) ifgnls
```

Specify starting values for parameters  $b_1$  and  $g_2$

```
nlsur (y1 = {b1=.5}*{b2}^x) (y2 = {g1}*{g2=1}^x), variables(y1 y2 x)
```

Same as above

```
nlsur (y1 = {b1}*{b2}^x) (y2 = {g1}*{g2}^x), variables(y1 y2 x) ///
initial(b1 .5 g2 1)
```

Same as above, but specify starting values using the matrix `i`

```
matrix i = (.5,0,0,1)
nlsur (y1 = {b1}*{b2}^x) (y2 = {g1}*{g2}^x), variables(y1 y2 x) ///
initial(i)
```

## Menu

Statistics > Linear models and related > Multiple-equation models > Nonlinear seemingly unrelated regression

## Syntax

### Interactive version

```
nlsur (deivar_1 = <sexp_1>) (deivar_2 = <sexp_2>) ... [if] [in] [weight]
 [, options]
```

### Programmed substitutable expression version

```
nlsur sexp_prog : deivar_1 deivar_2 ... [varlist] [if] [in] [weight] [, options]
```

### Function evaluator program version

```
nlsur func_prog @ deivar_1 deivar_2 ... [varlist] [if] [in] [weight] ,
 nequations(#) { parameters(namelist) | nparameters(#) } [options]
```

where

*deivar*<sub>*j*</sub> is the dependent variable for equation *j*;  
 <*sexp*><sub>*j*</sub> is the substitutable expression for equation *j*;  
*sexp\_prog* is a substitutable expression program; and  
*func\_prog* is a function evaluator program.

| <i>options</i>                                | Description                                                                                                                                              |
|-----------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                  |                                                                                                                                                          |
| <code>fgnls</code>                            | use two-step FGNLS estimator; the default                                                                                                                |
| <code>ifgnls</code>                           | use iterative FGNLS estimator                                                                                                                            |
| <code>nls</code>                              | use NLS estimator                                                                                                                                        |
| <code>variables(<i>varlist</i>)</code>        | variables in model                                                                                                                                       |
| <code>initial(<i>initial_values</i>)</code>   | initial values for parameters                                                                                                                            |
| * <code>nequations</code> (#)                 | number of equations in model (function evaluator program version only)                                                                                   |
| * <code>parameters</code> ( <i>namelist</i> ) | parameters in model (function evaluator program version only)                                                                                            |
| * <code>nparameters</code> (#)                | number of parameters in model<br>(function evaluator program version only)                                                                               |
| <code>sexp_options</code>                     | options for substitutable expression program                                                                                                             |
| <code>func_options</code>                     | options for function evaluator program                                                                                                                   |
| <b>SE/Robust</b>                              |                                                                                                                                                          |
| <code>vce(<i>vcetype</i>)</code>              | <i>vcetype</i> may be <code>gnr</code> , <code>robust</code> , <code>cluster</code> <i>clustvar</i> , <code>bootstrap</code> , or <code>jackknife</code> |
| <b>Reporting</b>                              |                                                                                                                                                          |
| <code>level</code> (#)                        | set confidence level; default is <code>level(95)</code>                                                                                                  |
| <code>title(<i>string</i>)</code>             | display <i>string</i> as title above the table of parameter estimates                                                                                    |
| <code>title2(<i>string</i>)</code>            | display <i>string</i> as subtitle                                                                                                                        |
| <code>display_options</code>                  | control columns and column formats and line width                                                                                                        |
| <b>Optimization</b>                           |                                                                                                                                                          |
| <code>optimization_options</code>             | control the optimization process; seldom used                                                                                                            |
| <code>coeflegend</code>                       | display legend instead of statistics                                                                                                                     |

\*You must specify `nequations(#)` and one of `parameters(namelist)` or `nparameters(#)` or both.

`bayesboot`, `bootstrap`, `by`, `collect`, `jackknife`, `rolling`, and `statsby` are allowed; see [U] 11.1.10 [Prefix commands](#).

Weights are not allowed with the `bootstrap` prefix; see [R] [bootstrap](#).

`aweights` are not allowed with the `jackknife` prefix; see [R] [jackknife](#).

`aweights`, `fweights`, `iweights`, and `pweights` are allowed; see [U] 11.1.6 [weight](#).

`coeflegend` does not appear in the dialog box.

See [U] 20 [Estimation and postestimation commands](#) for more capabilities of estimation commands.

## Options

### Model

`fgnls` requests the two-step FGNLS estimator; this is the default.

`ifgnls` requests the iterative FGNLS estimator. For the nonlinear systems estimator, this is equivalent to maximum likelihood estimation.

`nls` requests the nonlinear least-squares (NLS) estimator.

`variables(varlist)` specifies the variables in the system. `nlsur` ignores observations for which any of these variables has missing values. If you do not specify `variables()`, `nlsur` issues an error message if the estimation sample contains any missing values.

`initial(initial_values)` specifies the initial values to begin the estimation. You can specify a  $1 \times k$  matrix, where  $k$  is the total number of parameters in the system, or you can specify a parameter name, its initial value, another parameter name, its initial value, and so on. For example, to initialize `alpha` to 1.23 and `delta` to 4.57, you would type

```
. nlsur ..., initial(alpha 1.23 delta 4.57) ...
```

Initial values declared using this option override any that are declared within substitutable expressions. If you specify a matrix, the values must be in the same order in which the parameters are declared in your model. `nlsur` ignores the row and column names of the matrix.

`nequations(#)` specifies the number of equations in the system.

`parameters(namelist)` specifies the names of the parameters in the system. The names of the parameters must adhere to the naming conventions of Stata's variables; see [U] 11.3 [Naming conventions](#). If you specify both `parameters()` and `nparameters()`, the number of names in the former must match the number specified in the latter.

`nparameters(#)` specifies the number of parameters in the system. If you do not specify names with the `parameters()` option, `nlsur` names them `b1`, `b2`, ..., `b#`. If you specify both `parameters()` and `nparameters()`, the number of names in the former must match the number specified in the latter.

`sexp_options` refer to any options allowed by your `sexp_prog`.

`func_options` refer to any options allowed by your `func_prog`.

### SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`gnr`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

`vce(gnr)`, the default, uses the conventionally derived variance estimator for nonlinear models fit using Gauss–Newton regression.

#### Reporting

`level(#)`; see [R] [Estimation options](#).

`title(string)` specifies an optional title that will be displayed just above the table of parameter estimates.

`title2(string)` specifies an optional subtitle that will be displayed between the title specified in `title()` and the table of parameter estimates. If `title2()` is specified but `title()` is not, `title2()` has the same effect as `title()`.

`display_options`: `noci`, `nopvalues`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

#### Optimization

`optimization_options`: `iterate(#)`, `[no]log`, `trace`, `eps(#)`, `ifgnlsiterate(#)`, `ifgnlseps(#)`, `delta(#)`, `noconstants`, and `hasconstants(namelist)`.

`iterate()` specifies the maximum number of iterations to use for NLS at each round of FG-NLS estimation. This option is different from `ifgnlsiterate()`, which controls the maximum rounds of FG-NLS estimation to use when the `ifgnls` option is specified.

`log` and `nolog` specify whether to show the iteration log; see `set iterlog` in [R] [set iter](#).

`trace` specifies that the iteration log should include the current parameter vector.

`eps(#)` specifies the convergence criterion for successive parameter estimates and for the residual sum of squares (RSS). The default is `eps(1e-5)` (0.00001). `eps()` also specifies the convergence criterion for successive parameter estimates between rounds of iterative FG-NLS estimation when `ifgnls` is specified.

`ifgnlsiterate(#)` specifies the maximum number of FG-NLS iterations to perform. The default is the number set using `set maxiter`, which is 300 by default. To use this option, you must also specify the `ifgnls` option.

`ifgnlseps(#)` specifies the convergence criterion for successive estimates of the error covariance matrix during iterative FG-NLS estimation. The default is `ifgnlseps(1e-10)`. To use this option, you must also specify the `ifgnls` option.

`delta(#)` specifies the relative change in a parameter,  $\delta$ , to be used in computing the numeric derivatives. The derivative for parameter  $\beta_i$  is computed as

$$\{f_i(\mathbf{x}_i, \beta_1, \beta_2, \dots, \beta_i + d, \beta_{i+1}, \dots) - f_i(\mathbf{x}_i, \beta_1, \beta_2, \dots, \beta_i, \beta_{i+1}, \dots)\} / d$$

where  $d = \delta(|\beta_i| + \delta)$ . The default is `delta(4e-7)`.

`noconstants` indicates that none of the equations in the system includes constant terms. This option is generally not needed, even if there are no constant terms in the system; though in rare cases without this option, `nlsur` may claim that there is one or more constant terms even if there are none.

`hasconstants(namelist)` indicates the parameters that are to be treated as constant terms in the system of equations. The number of elements of `namelist` must equal the number of equations in the system. The  $i$ th entry of `namelist` specifies the constant term in the  $i$ th equation. If an equation

does not include a constant term, specify a period (.) instead of a parameter name. This option is seldom needed with the interactive and programmed substitutable expression versions, because in those cases nlshr can almost always find the constant terms automatically.

The following options are available with nlshr but are not shown in the dialog box:

coeflegend; see [R] [Estimation options](#).

## Remarks and examples

Remarks are presented under the following headings:

*Introduction*  
*Substitutable expression programs*  
*Function evaluator programs*

### Introduction

nlshr fits a system of nonlinear equations by FGNLS. It can be viewed as a nonlinear variant of Zellner's seemingly unrelated regression model (Zellner 1962; Zellner and Huang 1962; Zellner 1963) and is therefore commonly called nonlinear SUR or nonlinear SURE. The model is also discussed in textbooks such as Davidson and MacKinnon (1993) and Greene (2012, 305–306). Formally, the model fit by nlshr is

$$\begin{aligned}y_{i1} &= f_1(\mathbf{x}_i, \boldsymbol{\beta}) + u_{i1} \\y_{i2} &= f_2(\mathbf{x}_i, \boldsymbol{\beta}) + u_{i2} \\&\vdots = \vdots \\y_{iM} &= f_M(\mathbf{x}_i, \boldsymbol{\beta}) + u_{iM}\end{aligned}$$

for  $i = 1, \dots, N$  observations and  $m = 1, \dots, M$  equations. The errors for the  $i$ th observation,  $u_{i1}, u_{i2}, \dots, u_{iM}$ , may be correlated, so fitting the  $m$  equations jointly may lead to more efficient estimates. Moreover, fitting the equations jointly allows us to impose cross-equation restrictions on the parameters. Not all elements of the parameter vector  $\boldsymbol{\beta}$  and data vector  $\mathbf{x}_i$  must appear in all the equations, though each element of  $\boldsymbol{\beta}$  must appear in at least one equation for  $\boldsymbol{\beta}$  to be identified. For this model, iterative FGNLS estimation is equivalent to maximum likelihood estimation with multivariate normal disturbances.

The syntax you use with nlshr closely mirrors that used with nl. In particular, you use substitutable expressions with the interactive and programmed substitutable expression versions to define the functions in your system. See [R] [nl](#) for more information on substitutable expressions. Here we reiterate the three rules that you must follow:

1. Parameters of the model are bound in braces: {b0}, {param}, etc.
2. Initial values for parameters are given by including an equal sign and the initial value inside the braces: {b0=1}, {param=3.571}, etc. If you do not specify an initial value, that parameter is initialized to zero. The `initial()` option overrides initial values in substitutable expressions.
3. Linear combinations of variables can be included using the notation {eqname:varlist}, for example, {xb: mpg price weight}, {score: w x z}, etc. Parameters of linear combinations are initialized to zero.

▷ Example 1: Interactive version using two-step FGNLS estimator

We have data from an experiment in which two closely related types of bacteria were placed in a Petri dish, and the number of each type of bacteria were recorded every hour. We suspect a two-parameter exponential growth model can be used to model each type of bacteria, but because they shared the same dish, we want to allow for correlation in the error terms. We want to fit the system of equations

$$p_1 = \beta_1 \beta_2^t + u_1$$

$$p_2 = \gamma_1 \gamma_2^t + u_2$$

where  $p_1$  and  $p_2$  are the two populations and  $t$  is time, and we want to allow for nonzero correlation between  $u_1$  and  $u_2$ . We type

```
. use https://www.stata-press.com/data/r19/petridish
. nlstur (p1 = {b1}*{b2}^t) (p2 = {g1}*{g2}^t)
(obs = 25)
```

Calculating NLS estimates:

```
Iteration 0: Residual SS = 335.5286
Iteration 1: Residual SS = 333.8583
Iteration 2: Residual SS = 219.9233
Iteration 3: Residual SS = 127.9355
Iteration 4: Residual SS = 14.86765
Iteration 5: Residual SS = 8.628459
Iteration 6: Residual SS = 8.281268
Iteration 7: Residual SS = 8.28098
Iteration 8: Residual SS = 8.280979
Iteration 9: Residual SS = 8.280979
```

Calculating FGNLS estimates:

```
Iteration 0: Scaled RSS = 49.99892
Iteration 1: Scaled RSS = 49.99892
Iteration 2: Scaled RSS = 49.99892
```

FGNLS regression

| Equation |    | Obs | Parms | RMSE     | R-sq    | Constant |
|----------|----|-----|-------|----------|---------|----------|
| 1        | p1 | 25  | 2     | .4337019 | 0.9734* | (none)   |
| 2        | p2 | 25  | 2     | .3783479 | 0.9776* | (none)   |

\* Uncentered R-sq

|     | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |          |
|-----|-------------|-----------|--------|-------|----------------------|----------|
| /b1 | .3926631    | .064203   | 6.12   | 0.000 | .2668275             | .5184987 |
| /b2 | 1.119593    | .0088999  | 125.80 | 0.000 | 1.102149             | 1.137036 |
| /g1 | .5090441    | .0669495  | 7.60   | 0.000 | .3778256             | .6402626 |
| /g2 | 1.102315    | .0072183  | 152.71 | 0.000 | 1.088167             | 1.116463 |

The header of the output contains a summary of each equation, including the number of observations and parameters and the root mean squared error of the residuals. `nlstur` checks to see whether each equation contains a constant term, and if an equation does contain a constant term, an  $R^2$  statistic is presented. If an equation does not have a constant term, an uncentered  $R^2$  is instead reported. The  $R^2$  statistic for each equation measures the percentage of variance explained by the nonlinear function and may be useful for descriptive purposes, though it does not have the same formal interpretation in the context of FGNLS as it does with NLS estimation. As we would expect,  $\beta_2$  and  $\gamma_2$  are both greater than one, indicating the two bacterial populations increased in size over time.

The model we fit in the next three examples is in fact linear in the parameters, so it could be fit using the `sureg` command. However, we will fit the model using `nlshr` so that we can focus on the mechanics of using the command. Moreover, using `nlshr` will obviate the need to generate several variables as well as the need to use the `constraint` command to impose parameter restrictions.

► **Example 2: Interactive version using iterative FGNLS estimator—the translog production function**

Greene (1997, sec. 15.6) discusses the transcendental logarithmic (translog) cost function and provides cost and input price data for capital, labor, energy, and materials for the US economy. One way to fit the translog production function to these data is to fit the system of three equations

$$s_k = \beta_k + \delta_{kk} \ln \left( \frac{p_k}{p_m} \right) + \delta_{kl} \ln \left( \frac{p_l}{p_m} \right) + \delta_{ke} \ln \left( \frac{p_e}{p_m} \right) + u_1$$

$$s_l = \beta_l + \delta_{kl} \ln \left( \frac{p_k}{p_m} \right) + \delta_{ll} \ln \left( \frac{p_l}{p_m} \right) + \delta_{le} \ln \left( \frac{p_e}{p_m} \right) + u_2$$

$$s_e = \beta_e + \delta_{ke} \ln \left( \frac{p_k}{p_m} \right) + \delta_{le} \ln \left( \frac{p_l}{p_m} \right) + \delta_{ee} \ln \left( \frac{p_e}{p_m} \right) + u_3$$

where  $s_k$  is capital's cost share,  $s_l$  is labor's cost share, and  $s_e$  is energy's cost share;  $p_k$ ,  $p_l$ ,  $p_e$ , and  $p_m$  are the prices of capital, labor, energy, and materials, respectively; the  $u$ 's are regression error terms; and the  $\beta$ 's and  $\delta$ 's are parameters to be estimated. There are three cross-equation restrictions on the parameters:  $\delta_{kl}$ ,  $\delta_{ke}$ , and  $\delta_{le}$  each appear in two equations. To fit this model by using the iterative FGNLS estimator, we type

```
. use https://www.stata-press.com/data/r19/mfgcost
(Manufacturing cost)
. nlshr (s_k = {bk} + {dkk}*ln(pk/pm) + {dkl}*ln(pl/pm) + {dke}*ln(pe/pm))
> (s_l = {bl} + {dkl}*ln(pk/pm) + {dll}*ln(pl/pm) + {dle}*ln(pe/pm))
> (s_e = {be} + {dke}*ln(pk/pm) + {dle}*ln(pl/pm) + {dee}*ln(pe/pm)),
> ifgnls
(obs = 25)

Calculating NLS estimates:
Iteration 0: Residual SS = .0009989
Iteration 1: Residual SS = .0009989

Calculating FGNLS estimates:
Iteration 0: Scaled RSS = 65.45197
Iteration 1: Scaled RSS = 65.45197
(output omitted)
FGNLS iteration 10:
Iteration 0: Scaled RSS = 75
Iteration 1: Scaled RSS = 75
Parameter change = 4.08e-06
Covariance matrix change = 6.26e-10
FGNLS regression
```

| Equation |     | Obs | Parms | RMSE     | R-sq   | Constant |
|----------|-----|-----|-------|----------|--------|----------|
| 1        | s_k | 25  | 4     | .0031722 | 0.4776 | bk       |
| 2        | s_l | 25  | 4     | .0053963 | 0.8171 | bl       |
| 3        | s_e | 25  | 4     | .00177   | 0.6615 | be       |

|      | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |           |
|------|-------------|-----------|--------|-------|----------------------|-----------|
| /bk  | .0568925    | .0013454  | 42.29  | 0.000 | .0542556             | .0595294  |
| /dkk | .0294833    | .0057956  | 5.09   | 0.000 | .0181241             | .0408425  |
| /dkl | -.0000471   | .0038478  | -0.01  | 0.990 | -.0075887            | .0074945  |
| /dke | -.0106749   | .0033882  | -3.15  | 0.002 | -.0173157            | -.0040341 |
| /bl  | .253438     | .0020945  | 121.00 | 0.000 | .2493329             | .2575432  |
| /dll | .0754327    | .0067572  | 11.16  | 0.000 | .0621889             | .0886766  |
| /dle | -.004756    | .002344   | -2.03  | 0.042 | -.0093501            | -.0001619 |
| /be  | .0444099    | .0008533  | 52.04  | 0.000 | .0427374             | .0460823  |
| /dee | .0183415    | .0049858  | 3.68   | 0.000 | .0085694             | .0281135  |

We draw your attention to the iteration log at the top of the output. When iterative FGNLS estimation is used, the final scaled RSS will equal the product of the number of observations in the estimation sample and the number of equations; see *Methods and formulas* for details. Because the RSS is scaled by the error covariance matrix during each round of FGNLS estimation, the scaled RSS is not comparable from one FGNLS iteration to the next.



### □ Technical note

You may have noticed that we mentioned having data for four factors of production, yet we fit only three share equations. Because the four shares sum to one, we must drop one of the equations to avoid having a singular error covariance matrix. The iterative FGNLS estimator is equivalent to maximum likelihood estimation, and thus it is invariant to which one of the four equations we choose to drop. The (linearly restricted) parameters of the fourth equation can be obtained using the `lincom` command. Nonlinear functions of the parameters, such as the elasticities of substitution, can be computed using `nlcom`.



## Substitutable expression programs

If you fit the same model repeatedly or you want to share code with colleagues, you can write a *substitutable expression program* to define your system of equations and avoid having to retype the system every time. The first five letters of the program's name must be `nlsur`, and the program must set the `r`-class macro `r(n_eq)` to the number of equations in your system. The first equation's substitutable expression must be returned in `r(eq_1)`, the second equation's in `r(eq_2)`, and so on. You may optionally set `r(title)` to label your output; that has the same effect as specifying the `title()` option.

### ► Example 3: Programmed substitutable expression version

We return to our translog cost function, for which a substitutable expression program is

```

program nlsurtranslog, rclass
 version 19.5 // (or version 19 if you do not have StataNow)
 syntax varlist(min=7 max=7) [if]
 tokenize `varlist'
 args sk sl se pk pl pe pm
 local pkpm ln(`pk'/'pm')
 local plpm ln(`pl'/'pm')
 local pepm ln(`pe'/'pm')
 return scalar n_eq = 3
 return local eq_1 "'sk'= {bk} + {dkk}*`pkpm' + {dkl}*`plpm' + {dke}*`pepm'"
 return local eq_2 "'s1'= {b1} + {dkl}*`pkpm' + {dll}*`plpm' + {dle}*`pepm'"
 return local eq_3 "'se'= {be} + {dke}*`pkpm' + {dle}*`plpm' + {dee}*`pepm'"
 return local title "4-factor translog cost function"
end

```

We made our program accept seven variables, for the three dependent variables  $s_k$ ,  $s_l$ , and  $s_e$ , and the four factor prices  $p_k$ ,  $p_l$ ,  $p_m$ , and  $p_e$ . The `tokenize` command assigns to macros '1', '2', ..., '7' the seven variables stored in 'varlist', and the `args` command transfers those numbered macros to macros 'sk', 's1', ..., 'pm'. Because we knew our substitutable expressions were going to be somewhat long, we created local macros to hold the log price ratios. These are simply macros that hold strings such as `ln(pk/pm)`, not variables, and they will save us some repetitious typing when we define our substitutable expressions. Our program returns the number of equations in `r(n_eq)`, and we defined our substitutable expressions in `eq_1`, `eq_2`, and `eq_3`. We do not bind the expressions in parentheses as we do with the interactive version of `nlsur`. Finally, we put a title in `r(title)` to label our output.

Our `syntax` command also accepts an `if` clause, and that is how `nlsur` indicates the estimation sample to our program. In this application, we can safely ignore it because our program does not compute initial values. However, had we used commands such as `summarize` or `regress` to obtain initial values, then we would need to restrict those commands to analyze only the estimation sample. In those cases, typically, you simply need to include 'if' with the commands you are using. For example, instead of the command

```
summarize `depvar', meanonly
```

you would use

```
summarize `depvar' `if', meanonly
```

We can check our program by typing

```

. nlsurtranslog s_k s_l s_e pk pl pe pm
. return list
scalars:
 r(n_eq) = 3
macros:
 r(title) : "4-factor translog cost function"
 r(eq_3) : "s_e= {be} + {dke}*ln(pk/pm) + {dle}*ln(pl/pm) + {..}"
 r(eq_2) : "s_l= {b1} + {dkl}*ln(pk/pm) + {dll}*ln(pl/pm) + {..}"
 r(eq_1) : "s_k= {bk} + {dkk}*ln(pk/pm) + {dkl}*ln(pl/pm) + {..}"

```

Now that we know that our program works, we fit our model by typing

```
. nlstur translog: s_k s_l s_e pk pl pe pm, ifgnls
(obs = 25)

Calculating NLS estimates:
Iteration 0: Residual SS = .0009989
Iteration 1: Residual SS = .0009989

Calculating FGMLS estimates:
Iteration 0: Scaled RSS = 65.45197
Iteration 1: Scaled RSS = 65.45197

FGMLS iteration 2:
Iteration 0: Scaled RSS = 73.28311
Iteration 1: Scaled RSS = 73.28311
Iteration 2: Scaled RSS = 73.28311
Parameter change = 6.54e-03
Covariance matrix change = 1.00e-06
(output omitted)

FGMLS iteration 10:
Iteration 0: Scaled RSS = 75
Iteration 1: Scaled RSS = 75
Parameter change = 4.08e-06
Covariance matrix change = 6.26e-10

FGMLS regression
```

| Equation |     | Obs | Parms | RMSE     | R-sq   | Constant |
|----------|-----|-----|-------|----------|--------|----------|
| 1        | s_k | 25  | 4     | .0031722 | 0.4776 | bk       |
| 2        | s_l | 25  | 4     | .0053963 | 0.8171 | bl       |
| 3        | s_e | 25  | 4     | .00177   | 0.6615 | be       |

4-factor translog cost function

|      | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |           |
|------|-------------|-----------|--------|-------|----------------------|-----------|
| /bk  | .0568925    | .0013454  | 42.29  | 0.000 | .0542556             | .0595294  |
| /dkk | .0294833    | .0057956  | 5.09   | 0.000 | .0181241             | .0408425  |
| /dkl | -.0000471   | .0038478  | -0.01  | 0.990 | -.0075887            | .0074945  |
| /dke | -.0106749   | .0033882  | -3.15  | 0.002 | -.0173157            | -.0040341 |
| /bl  | .253438     | .0020945  | 121.00 | 0.000 | .2493329             | .2575432  |
| /dll | .0754327    | .0067572  | 11.16  | 0.000 | .0621889             | .0886766  |
| /dle | -.004756    | .002344   | -2.03  | 0.042 | -.0093501            | -.0001619 |
| /be  | .0444099    | .0008533  | 52.04  | 0.000 | .0427374             | .0460823  |
| /dee | .0183415    | .0049858  | 3.68   | 0.000 | .0085694             | .0281135  |

Because we set `r(title)` in our substitutable expression program, the coefficient table has a title attached to it. The estimates are identical to those we obtained in [example 2](#).



## □ Technical note

`nlsur` accepts frequency and analytic weights as well as `pweights` (sampling weights) and `iweights` (importance weights). You do not need to modify your substitutable expressions in any way to perform weighted estimation, though you must make two changes to your substitutable expression program. The general outline of a *sexp-prog* program is

```

program nlsur name, rclass
 version 19.5 // (or version 19 if you do not have StataNow)
 syntax varlist [fw aw pw iw] [if]
 // Obtain initial values incorporating weights. For example,
 summarize varname ['weight' 'exp' 'if'
 ...
 // Return n_eqn and substitutable expressions
 return scalar n_eq = #
 return local eq_1 = ...
 ...
end

```

First, we wrote the `syntax` statement to accept a weight expression. Here we allow all four types of weights, but if you know that your estimator is valid, say, for only frequency weights, then you should modify the `syntax` line to accept only `fweights`. Second, if your program computes starting values, then any commands you use must incorporate the weights passed to the program; you do that by including `['weight' 'exp']` when calling those commands.

□

## Function evaluator programs

Although substitutable expressions are extremely flexible, there are some problems for which the nonlinear system cannot be defined using them. You can use the function evaluator program version of `nlsur` in these cases. We present two examples, a simple one to illustrate the mechanics of function evaluator programs and a more complicated one to illustrate the power of `nlsur`.

### ► Example 4: Function evaluator program version

Here we write a function evaluator program to fit the translog cost function used in examples 2 and 3. The function evaluator program is

```

program nlsurtranslog2
 version 19.5 // (or version 19 if you do not have StataNow)
 syntax varlist(min=7 max=7) [if], at(name)
 tokenize `varlist'
 args sk sl se pk pl pe pm
 tempname bk dkk dkl dke bl dll dle be dee
 scalar `bk' = `at'[1,1]
 scalar `dkk' = `at'[1,2]
 scalar `dkl' = `at'[1,3]
 scalar `dke' = `at'[1,4]
 scalar `bl' = `at'[1,5]
 scalar `dll' = `at'[1,6]
 scalar `dle' = `at'[1,7]
 scalar `be' = `at'[1,8]
 scalar `dee' = `at'[1,9]
 local pkpm ln(`pk'/'pm')
 local plpm ln(`pl'/'pm')
 local pepm ln(`pe'/'pm')
 quietly {
 replace `sk' = `bk' + `dkk'*`pkpm' + `dkl'*`plpm' + ///
 `dke'*`pepm' `if'
 replace `sl' = `bl' + `dkl'*`pkpm' + `dll'*`plpm' + ///
 `dle'*`pepm' `if'
 replace `se' = `be' + `dke'*`pkpm' + `dle'*`plpm' + ///
 `dee'*`pepm' `if'
 }
end

```

Unlike the substitutable expression program we wrote in [example 3](#), `nlsurtranslog2` is not declared as `r-class` because we will not be returning any stored results. We are again expecting seven variables: three shares and four factor prices, and `nlsur` will again mark the estimation sample with an `if` expression.

Our function evaluator program also accepts an option named `at()`, which will receive a parameter vector at which we are to evaluate the system of equations. All function evaluator programs must accept this option. Our model has nine parameters to estimate, and we created nine temporary scalars to hold the elements of the `'at'` matrix.

Because our model has three equations, the first three variables passed to our program are the dependent variables that we are to fill in with the function values. We replaced only the observations in our estimation sample by including the `'if'` qualifier in the `replace` statements. Here we could have ignored the `'if'` qualifier because `nlsur` will skip over observations not in the estimation sample and we did not perform any computations requiring knowledge of the estimation sample. However, including the `'if'` is good practice and may result in a slight speed improvement if the functions of your model are complicated and the estimation sample is much smaller than the dataset in memory.

We could have avoided creating temporary scalars to hold our individual parameters by writing the replace statements as, for example,

```
replace 'sk' = 'at'[1,1] + 'at'[1,2]*'pkpm' + 'at'[1,3]*'plpm' + 'at'[1,4]*'pepm' 'if'
```

You can use whichever method you find more appealing, though giving the parameters descriptive names reduces the chance for mistakes and makes debugging easier.

To fit our model by using the function evaluator program version of nlsur, we type

```
. nlsur translog2 @ s_k s_l s_e pk pl pe pm, ifgnls nequations(3)
> parameters(bk dkk dkl dke bl dll dle be dee)
> hasconstants(bk bl be)
(obs = 25)
```

```
Calculating NLS estimates:
Iteration 0: Residual SS = .0009989
Iteration 1: Residual SS = .0009989
```

```
Calculating FGNLS estimates:
Iteration 0: Scaled RSS = 65.45197
Iteration 1: Scaled RSS = 65.45197
```

```
FGNLS iteration 2:
Iteration 0: Scaled RSS = 73.28311
Iteration 1: Scaled RSS = 73.28311
Iteration 2: Scaled RSS = 73.28311
Parameter change = 6.54e-03
Covariance matrix change = 1.00e-06
```

```
FGNLS iteration 3:
Iteration 0: Scaled RSS = 74.7113
Iteration 1: Scaled RSS = 74.7113
Parameter change = 2.58e-03
Covariance matrix change = 3.96e-07
```

```
FGNLS iteration 4:
Iteration 0: Scaled RSS = 74.95356
Iteration 1: Scaled RSS = 74.95356
Parameter change = 1.02e-03
Covariance matrix change = 1.57e-07
```

```
FGNLS iteration 5:
Iteration 0: Scaled RSS = 74.99261
Iteration 1: Scaled RSS = 74.99261
Parameter change = 4.07e-04
Covariance matrix change = 6.25e-08
```

```
FGNLS iteration 6:
Iteration 0: Scaled RSS = 74.99883
Iteration 1: Scaled RSS = 74.99883
Parameter change = 1.62e-04
Covariance matrix change = 2.49e-08
```

```
FGNLS iteration 7:
Iteration 0: Scaled RSS = 74.99981
Iteration 1: Scaled RSS = 74.99981
Iteration 2: Scaled RSS = 74.99981
Parameter change = 6.45e-05
Covariance matrix change = 9.91e-09
```

```
FGNLS iteration 8:
Iteration 0: Scaled RSS = 74.99997
Iteration 1: Scaled RSS = 74.99997
Iteration 2: Scaled RSS = 74.99997
Iteration 3: Scaled RSS = 74.99997
Parameter change = 2.57e-05
Covariance matrix change = 3.95e-09
```

```

FGNLS iteration 9:
Iteration 0: Scaled RSS = 75
Iteration 1: Scaled RSS = 75
Iteration 2: Scaled RSS = 75
Parameter change = 1.02e-05
Covariance matrix change = 1.57e-09

FGNLS iteration 10:
Iteration 0: Scaled RSS = 75
Iteration 1: Scaled RSS = 75
Parameter change = 4.08e-06
Covariance matrix change = 6.26e-10

FGNLS regression

```

| Equation |     | Obs | Parms | RMSE     | R-sq   | Constant |
|----------|-----|-----|-------|----------|--------|----------|
| 1        | s_k | 25  | .     | .0031722 | 0.4776 | bk       |
| 2        | s_l | 25  | .     | .0053963 | 0.8171 | bl       |
| 3        | s_e | 25  | .     | .00177   | 0.6615 | be       |

|      | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |           |
|------|-------------|-----------|--------|-------|----------------------|-----------|
| /bk  | .0568925    | .0013454  | 42.29  | 0.000 | .0542556             | .0595294  |
| /dkk | .0294833    | .0057956  | 5.09   | 0.000 | .0181241             | .0408425  |
| /dkl | -.0000471   | .0038478  | -0.01  | 0.990 | -.0075887            | .0074945  |
| /dke | -.0106749   | .0033882  | -3.15  | 0.002 | -.0173157            | -.0040341 |
| /bl  | .253438     | .0020945  | 121.00 | 0.000 | .2493329             | .2575432  |
| /dll | .0754327    | .0067572  | 11.16  | 0.000 | .0621889             | .0886766  |
| /dle | -.004756    | .002344   | -2.03  | 0.042 | -.0093501            | -.0001619 |
| /be  | .0444099    | .0008533  | 52.04  | 0.000 | .0427374             | .0460823  |
| /dee | .0183415    | .0049858  | 3.68   | 0.000 | .0085694             | .0281135  |

When we use the function evaluator program version, `nlsur` requires us to specify the number of equations in `nequations()`, and it requires us to specify either the names for each of our parameters or the number of parameters in the model. Here we used the `parameters()` option to name our parameters; the order in which we specified them in this option is the same as the order in which we extracted them from the ‘at’ matrix in our program. Had we instead specified `nparameters(9)`, our parameters would have been labeled `/b1`, `/b2`, ..., `/b9` in the output.

`nlsur` has no way of telling how many parameters appear in each equation, so the `Parms` column in the header contains missing values. Moreover, the function evaluator program version of `nlsur` does not attempt to identify constant terms, so we used the `hasconstant` option to tell `nlsur` which parameter in each equation is a constant term.

The estimates are identical to those we obtained in examples 2 and 3.



### □ Technical note

As with substitutable expression programs, if you intend to do weighted estimation with a function evaluator program, you must modify your `func_prog` program’s `syntax` statement to accept weights. Moreover, if you use any statistical commands when computing your nonlinear functions, then you must include the weight expression with those commands.



► Example 5: Fitting the basic AIDS model using nlstur

Deaton and Muellbauer (1980) introduce the almost ideal demand system (AIDS), and Poi (2012) presents a set of commands and several extensions for fitting the AIDS automatically. Here we show how to fit the basic AIDS model, which is a common example of a nonlinear system of equations, by manually using `nlstur`. The dataset `food.dta` contains household expenditures, expenditure shares, and log prices for four broad food groups. For a four-good demand system, we need to fit the following system of three equations:

$$\begin{aligned}
 w_1 &= \alpha_1 + \gamma_{11} \ln p_1 + \gamma_{12} \ln p_2 + \gamma_{13} \ln p_3 + \beta_1 \ln \left\{ \frac{m}{P(\mathbf{p})} \right\} + u_1 \\
 w_2 &= \alpha_2 + \gamma_{12} \ln p_1 + \gamma_{22} \ln p_2 + \gamma_{23} \ln p_3 + \beta_2 \ln \left\{ \frac{m}{P(\mathbf{p})} \right\} + u_2 \\
 w_3 &= \alpha_3 + \gamma_{13} \ln p_1 + \gamma_{23} \ln p_2 + \gamma_{33} \ln p_3 + \beta_3 \ln \left\{ \frac{m}{P(\mathbf{p})} \right\} + u_3
 \end{aligned}$$

where  $w_k$  denotes a household's fraction of expenditures on good  $k$ ,  $\ln p_k$  denotes the logarithm of the price paid for good  $k$ ,  $m$  denotes a household's total expenditure on all four goods, the  $u$ 's are regression error terms, and

$$\ln P(\mathbf{p}) = \alpha_0 + \sum_{i=1}^4 \alpha_i \ln p_i + \frac{1}{2} \sum_{i=1}^4 \sum_{j=1}^4 \gamma_{ij} \ln p_i \ln p_j$$

The parameters for the fourth good's share equation can be recovered from the following constraints that are imposed by economic theory:

$$\sum_{i=1}^4 \alpha_i = 1 \quad \sum_{i=1}^4 \beta_i = 0 \quad \gamma_{ij} = \gamma_{ji} \quad \text{and} \quad \sum_{i=1}^4 \gamma_{ij} = 0 \quad \text{for all } j$$

Our model has a total of 12 unrestricted parameters. We will not estimate  $\alpha_0$  directly. Instead, we will set it equal to 5; see Deaton and Muellbauer (1980) for a discussion of why treating  $\alpha_0$  as fixed is acceptable.

Our function evaluator program is

```

program nlsuraids
 version 19.5 // (or version 19 if you do not have StataNow)
 syntax varlist(min=8 max=8) if, at(name)
 tokenize `varlist'
 args w1 w2 w3 lnp1 lnp2 lnp3 lnp4 lnm
 tempname a1 a2 a3 a4
 scalar `a1' = `at'[1,1]
 scalar `a2' = `at'[1,2]
 scalar `a3' = `at'[1,3]
 scalar `a4' = 1 - `a1' - `a2' - `a3'

 tempname b1 b2 b3
 scalar `b1' = `at'[1,4]
 scalar `b2' = `at'[1,5]
 scalar `b3' = `at'[1,6]

 tempname g11 g12 g13 g14
 tempname g21 g22 g23 g24
 tempname g31 g32 g33 g34
 tempname g41 g42 g43 g44
 scalar `g11' = `at'[1,7]
 scalar `g12' = `at'[1,8]
 scalar `g13' = `at'[1,9]
 scalar `g14' = -`g11' - `g12' - `g13'

 scalar `g21' = `g12'
 scalar `g22' = `at'[1,10]
 scalar `g23' = `at'[1,11]
 scalar `g24' = -`g21' - `g22' - `g23'

 scalar `g31' = `g13'
 scalar `g32' = `g23'
 scalar `g33' = `at'[1,12]
 scalar `g34' = -`g31' - `g32' - `g33'

 scalar `g41' = `g14'
 scalar `g42' = `g24'
 scalar `g43' = `g34'
 scalar `g44' = -`g41' - `g42' - `g43'

 quietly {
 tempvar lnpindex
 gen double `lnpindex' = 5 + `a1'*`lnp1' + `a2'*`lnp2' + ///
 `a3'*`lnp3' + `a4'*`lnp4'

 forvalues i = 1/4 {
 forvalues j = 1/4 {
 replace `lnpindex' = `lnpindex' + ///
 0.5*`g'`i'`j'*`lnp'`i'*`lnp'`j'
 }
 }

 replace `w1' = `a1' + `g11'*`lnp1' + `g12'*`lnp2' + ///
 `g13'*`lnp3' + `g14'*`lnp4' + ///
 `b1'*(`lnm' - `lnpindex')

 replace `w2' = `a2' + `g21'*`lnp1' + `g22'*`lnp2' + ///
 `g23'*`lnp3' + `g24'*`lnp4' + ///
 `b2'*(`lnm' - `lnpindex')

 replace `w3' = `a3' + `g31'*`lnp1' + `g32'*`lnp2' + ///
 `g33'*`lnp3' + `g34'*`lnp4' + ///
 `b3'*(`lnm' - `lnpindex')
 }
end

```

The syntax statement accepts eight variables: three expenditure share variables, all four log-price variables, and a variable for log expenditures ( $\ln m$ ). Most of the code simply extracts the parameters from the 'at' matrix. Although we are estimating only 12 parameters, to calculate the price index term and the expenditure share equations, we need the restricted parameters as well. Notice how we impose the constraints on the parameters. We then created a temporary variable to hold  $\ln P(\mathbf{p})$ , and we filled the three dependent variables with the predicted expenditure shares.

To fit our model, we type

```
. use https://www.stata-press.com/data/r19/food
(Four food groups)
. nlsur aids @ w1 w2 w3 lnp1 lnp2 lnp3 lnp4 lnexp,
> parameters(a1 a2 a3 b1 b2 b3
> g11 g12 g13 g22 g32 g33)
> neq(3) ifgnls
(obs = 4,048)
```

```
Calculating NLS estimates:
Iteration 0: Residual SS = 126.9713
Iteration 1: Residual SS = 125.669
Iteration 2: Residual SS = 125.669
Iteration 3: Residual SS = 125.669
Iteration 4: Residual SS = 125.669
```

```
Calculating FG-NLS estimates:
Iteration 0: Scaled RSS = 12080.14
Iteration 1: Scaled RSS = 12080.14
Iteration 2: Scaled RSS = 12080.14
Iteration 3: Scaled RSS = 12080.14
```

```
FG-NLS iteration 2:
Iteration 0: Scaled RSS = 12143.99
Iteration 1: Scaled RSS = 12143.99
Iteration 2: Scaled RSS = 12143.99
Parameter change = 1.97e-04
Covariance matrix change = 2.94e-06
```

```
FG-NLS iteration 3:
Iteration 0: Scaled RSS = 12144
Iteration 1: Scaled RSS = 12144
Parameter change = 2.18e-06
Covariance matrix change = 3.47e-08
```

FG-NLS regression

| Equation |    | Obs   | Parms | RMSE     | R-sq    | Constant |
|----------|----|-------|-------|----------|---------|----------|
| 1        | w1 | 4,048 | .     | .1333175 | 0.9017* | (none)   |
| 2        | w2 | 4,048 | .     | .1024166 | 0.8480* | (none)   |
| 3        | w3 | 4,048 | .     | .053777  | 0.7906* | (none)   |

\* Uncentered R-sq

|      | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |           |
|------|-------------|-----------|--------|-------|----------------------|-----------|
| /a1  | .3163958    | .0073871  | 42.83  | 0.000 | .3019175             | .3308742  |
| /a2  | .2712501    | .0056938  | 47.64  | 0.000 | .2600904             | .2824097  |
| /a3  | .1039898    | .0029004  | 35.85  | 0.000 | .0983051             | .1096746  |
| /b1  | .0161044    | .0034153  | 4.72   | 0.000 | .0094105             | .0227983  |
| /b2  | -.0260771   | .002623   | -9.94  | 0.000 | -.0312181            | -.0209361 |
| /b3  | .0014538    | .0013776  | 1.06   | 0.291 | -.0012463            | .004154   |
| /g11 | .1215838    | .0057186  | 21.26  | 0.000 | .1103756             | .1327921  |
| /g12 | -.0522943   | .0039305  | -13.30 | 0.000 | -.0599979            | -.0445908 |
| /g13 | -.0351292   | .0021788  | -16.12 | 0.000 | -.0393996            | -.0308588 |
| /g22 | .0644298    | .0044587  | 14.45  | 0.000 | .0556909             | .0731687  |
| /g32 | -.0011786   | .0019767  | -0.60  | 0.551 | -.0050528            | .0026957  |
| /g33 | .0424381    | .0017589  | 24.13  | 0.000 | .0389909             | .0458854  |

To get the restricted parameters for the fourth share equation, we can use `lincom`. For example, to obtain  $\alpha_4$ , we type

```
. lincom 1 - [a1]_cons - [a2]_cons - [a3]_cons
(1) - [a1]_cons - [a2]_cons - [a3]_cons = -1
```

|     | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-----|-------------|-----------|-------|-------|----------------------|----------|
| (1) | .3083643    | .0052611  | 58.61 | 0.000 | .2980528             | .3186758 |

For more information on `lincom`, see [\[R\] lincom](#).



## Stored results

`nlsur` stores the following in `e()`:

### Scalars

- `e(N)` number of observations
- `e(k)` number of parameters
- `e(k_#)` number of parameters for equation #
- `e(k_eq)` number of equation names in `e(b)`
- `e(k_eq_model)` number of equations in overall model test
- `e(n_eq)` number of equations
- `e(mss_#)` model sum of squares for equation #
- `e(rss_#)` RSS for equation #
- `e(rmse_#)` root mean squared error for equation #
- `e(r2_#)`  $R^2$  for equation #
- `e(ll)` Gaussian log likelihood (`iflgs` version only)
- `e(N_clust)` number of clusters
- `e(rank)` rank of `e(V)`
- `e(converged)` 1 if converged, 0 otherwise

### Macros

- `e(cmd)` `nlsur`
- `e(cmdline)` command as typed
- `e(method)` `fgnls`, `ifgnls`, or `nls`
- `e(depvar)` names of dependent variables
- `e(depvar_#)` dependent variable for equation #
- `e(wtype)` weight type
- `e(wexp)` weight expression

|               |                                                                                                                |
|---------------|----------------------------------------------------------------------------------------------------------------|
| e(title)      | title in estimation output                                                                                     |
| e(title_2)    | secondary title in estimation output                                                                           |
| e(clustvar)   | name of cluster variable                                                                                       |
| e(vce)        | <i>vce</i> type specified in <code>vce()</code>                                                                |
| e(vcetype)    | title used to label Std. err.                                                                                  |
| e(type)       | 1 = interactively entered expression<br>2 = substitutable expression program<br>3 = function evaluator program |
| e(sexpprog)   | substitutable expression program                                                                               |
| e(sexp_#)     | substitutable expression for equation #                                                                        |
| e(params)     | names of all parameters                                                                                        |
| e(params_#)   | parameters in equation #                                                                                       |
| e(funcprog)   | function evaluator program                                                                                     |
| e(rhs)        | contents of <code>variables()</code>                                                                           |
| e(constants)  | identifies constant terms                                                                                      |
| e(properties) | b V                                                                                                            |
| e(predict)    | program used to implement <code>predict</code>                                                                 |
| Matrices      |                                                                                                                |
| e(b)          | coefficient vector                                                                                             |
| e(init)       | initial values vector                                                                                          |
| e(Sigma)      | error covariance matrix ( $\widehat{\Sigma}$ )                                                                 |
| e(V)          | variance-covariance matrix of the estimators                                                                   |
| Functions     |                                                                                                                |
| e(sample)     | marks estimation sample                                                                                        |

In addition to the above, the following is stored in `r()`:

|          |                                                                                                                            |
|----------|----------------------------------------------------------------------------------------------------------------------------|
| Matrices |                                                                                                                            |
| r(table) | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

## Methods and formulas

Write the system of equations for the *i*th observation as

$$\mathbf{y}_i = \mathbf{f}(\mathbf{x}_i, \boldsymbol{\beta}) + \mathbf{u}_i \quad (1)$$

where  $\mathbf{y}_i$  and  $\mathbf{u}_i$  are  $1 \times M$  vectors, for  $i = 1, \dots, N$ ;  $\mathbf{f}$  is a function that returns a  $1 \times M$  vector;  $\mathbf{x}_i$  represents all the exogenous variables in the system; and  $\boldsymbol{\beta}$  is a  $1 \times k$  vector of parameters. The generalized nonlinear least-squares system estimator is defined as

$$\widehat{\boldsymbol{\beta}} \equiv \operatorname{argmin}_{\boldsymbol{\beta}} \sum_{i=1}^N \{\mathbf{y}_i - \mathbf{f}(\mathbf{x}_i, \boldsymbol{\beta})\} \boldsymbol{\Sigma}^{-1} \{\mathbf{y}_i - \mathbf{f}(\mathbf{x}_i, \boldsymbol{\beta})\}'$$

where  $\boldsymbol{\Sigma} = E(\mathbf{u}_i' \mathbf{u}_i)$  is an  $M \times M$  positive-definite weight matrix. Let  $\mathbf{T}$  be the Cholesky decomposition of  $\boldsymbol{\Sigma}^{-1}$ ; that is,  $\mathbf{T}\mathbf{T}' = \boldsymbol{\Sigma}^{-1}$ . Postmultiply (1) by  $\mathbf{T}$ :

$$\mathbf{y}_i \mathbf{T} = \mathbf{f}(\mathbf{x}_i, \boldsymbol{\beta}) \mathbf{T} + \mathbf{u}_i \mathbf{T} \quad (2)$$

Because  $E(\mathbf{T}'\mathbf{u}'_i\mathbf{u}_i\mathbf{T}) = \mathbf{I}$ , we can “stack” the columns of (2) and write

$$\begin{aligned}
 \mathbf{y}_1\mathbf{T}_1 &= \mathbf{f}(\mathbf{x}_1, \boldsymbol{\beta})\mathbf{T}_1 + \tilde{u}_{11} \\
 \mathbf{y}_1\mathbf{T}_2 &= \mathbf{f}(\mathbf{x}_1, \boldsymbol{\beta})\mathbf{T}_2 + \tilde{u}_{12} \\
 &\vdots \\
 \mathbf{y}_1\mathbf{T}_M &= \mathbf{f}(\mathbf{x}_1, \boldsymbol{\beta})\mathbf{T}_M + \tilde{u}_{1M} \\
 &\vdots \\
 \mathbf{y}_N\mathbf{T}_1 &= \mathbf{f}(\mathbf{x}_N, \boldsymbol{\beta})\mathbf{T}_1 + \tilde{u}_{N1} \\
 \mathbf{y}_N\mathbf{T}_2 &= \mathbf{f}(\mathbf{x}_N, \boldsymbol{\beta})\mathbf{T}_2 + \tilde{u}_{N2} \\
 &\vdots \\
 \mathbf{y}_N\mathbf{T}_M &= \mathbf{f}(\mathbf{x}_N, \boldsymbol{\beta})\mathbf{T}_M + \tilde{u}_{NM}
 \end{aligned} \tag{3}$$

where  $\mathbf{T}_j$  denotes the  $j$ th column of  $\mathbf{T}$ . By construction, all  $\tilde{u}_{ij}$  are independently distributed with unit variance. As a result, by transforming the model in (1) to that shown in (3), we have reduced the multivariate generalized nonlinear least-squares system estimator to a univariate nonlinear least-squares problem; and the same parameter estimation technique used by `nls` can be used here. See [R] `nls` for the details. Moreover, because the  $\tilde{u}_{ij}$  all have variance 1, the final scaled RSS reported by `nlsur` is equal to  $NM$ .

To make the estimator feasible, we require an estimate  $\widehat{\boldsymbol{\Sigma}}$  of  $\boldsymbol{\Sigma}$ . `nlsur` first sets  $\widehat{\boldsymbol{\Sigma}} = \mathbf{I}$ . Although not efficient, the resulting estimate,  $\widehat{\boldsymbol{\beta}}_{\text{NLS}}$ , is consistent. If the `nls` option is specified, estimation is complete. Otherwise, the residuals

$$\hat{\mathbf{u}}_i = \mathbf{y}_i - \mathbf{f}(\mathbf{x}_i, \widehat{\boldsymbol{\beta}}_{\text{NLS}})$$

are calculated and used to compute

$$\widehat{\boldsymbol{\Sigma}} = \frac{1}{N} \sum_{i=1}^N \hat{\mathbf{u}}_i' \hat{\mathbf{u}}_i$$

With  $\widehat{\boldsymbol{\Sigma}}$  in hand, a new estimate  $\widehat{\boldsymbol{\beta}}$  is then obtained.

If the `ifgnls` option is specified, the new  $\widehat{\boldsymbol{\beta}}$  is used to recompute the residuals and obtain a new estimate of  $\widehat{\boldsymbol{\Sigma}}$ , from which  $\widehat{\boldsymbol{\beta}}$  can then be reestimated. Iterations stop when the relative change in  $\widehat{\boldsymbol{\beta}}$  is less than `eps()`, the relative change in  $\widehat{\boldsymbol{\Sigma}}$  is less than `ifgnlseps()`, or if `ifgnlsiterate()` iterations have been performed.

If the `vce(robust)` and `vce(cluster clustvar)` options were not specified, then

$$V(\widehat{\boldsymbol{\beta}}) = \left( \sum_{i=1}^N \mathbf{X}'_i \widehat{\boldsymbol{\Sigma}}^{-1} \mathbf{X}_i \right)^{-1}$$

where the  $M \times k$  matrix  $\mathbf{X}_i$  has typical element  $X_{ist}$ , the derivative of the  $s$ th element of  $\mathbf{f}$  with respect to the  $t$ th element of  $\boldsymbol{\beta}$ , evaluated at  $\mathbf{x}_i$  and  $\widehat{\boldsymbol{\beta}}$ . As a practical matter, once the model is written in the form of (3), the variance–covariance matrix can be calculated via a Gauss–Newton regression; see Davidson and MacKinnon (1993, chap. 6).

If `robust` is specified, then

$$V_R(\widehat{\boldsymbol{\beta}}) = \left( \sum_{i=1}^N \mathbf{X}'_i \widehat{\boldsymbol{\Sigma}}^{-1} \mathbf{X}_i \right)^{-1} \sum_{i=1}^N \mathbf{X}'_i \widehat{\boldsymbol{\Sigma}}^{-1} \hat{\mathbf{u}}_i' \hat{\mathbf{u}}_i \widehat{\boldsymbol{\Sigma}}^{-1} \mathbf{X}_i \left( \sum_{i=1}^N \mathbf{X}'_i \widehat{\boldsymbol{\Sigma}}^{-1} \mathbf{X}_i \right)^{-1}$$

The cluster–robust variance matrix is

$$V_C(\widehat{\beta}) = \left( \sum_{i=1}^N \mathbf{X}'_i \widehat{\Sigma}^{-1} \mathbf{X}_i \right)^{-1} \sum_{c=1}^{N_C} \mathbf{w}_c \mathbf{w}'_c \left( \sum_{i=1}^N \mathbf{X}'_i \widehat{\Sigma}^{-1} \mathbf{X}_i \right)^{-1}$$

where  $N_C$  is the number of clusters and

$$\mathbf{w}_c = \sum_{j \in C_k} \mathbf{X}'_j \widehat{\Sigma}^{-1} \widehat{\mathbf{u}}'_j$$

with  $C_k$  denoting the set of observations in the  $k$ th cluster. In evaluating these formulas, we use the value of  $\widehat{\Sigma}$  used in calculating the final estimate of  $\widehat{\beta}$ . That is, we do not recalculate  $\widehat{\Sigma}$  after we obtain the final value of  $\widehat{\beta}$ .

The RSS for the  $j$ th equation,  $\text{RSS}_j$ , is

$$\text{RSS}_j = \sum_{i=1}^N (\widehat{y}_{ij} - y_{ij})^2$$

where  $\widehat{y}_{ij}$  is the predicted value of the  $i$ th observation on the  $j$ th dependent variable; the total sum of squares (TSS) for the  $j$ th equation,  $\text{TSS}_j$ , is

$$\text{TSS}_j = \sum_{i=1}^N (y_{ij} - \bar{y}_j)^2$$

if there is a constant term in the  $j$ th equation, where  $\bar{y}_j$  is the sample mean of the  $j$ th dependent variable, and

$$\text{TSS}_j = \sum_{i=1}^N y_{ij}^2$$

if there is no constant term in the  $j$ th equation; and the model sum of squares (MSS) for the  $j$ th equation,  $\text{MSS}_j$ , is  $\text{TSS}_j - \text{RSS}_j$ .

The  $R^2$  for the  $j$ th equation is  $\text{MSS}_j/\text{TSS}_j$ . If an equation does not have a constant term, then the reported  $R^2$  for that equation is “uncentered” and based on the latter definition of  $\text{TSS}_j$ .

Under the assumption that the  $\mathbf{u}_i$  are independent and identically distributed  $N(\mathbf{0}, \widehat{\Sigma})$ , the log likelihood for the model is

$$\ln L = -\frac{MN}{2} \{1 + \ln(2\pi)\} - \frac{N}{2} \ln |\widehat{\Sigma}|$$

The log likelihood is reported only when the `ifgnls` option is specified.

## References

- Canette, I. 2011. A tip to debug your nl/nlshr function evaluator program. *The Stata Blog: Not Elsewhere Classified*. <https://blog.stata.com/2011/12/05/a-tip-to-debug-your-nlshr-function-evaluator-program/>.
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- Greene, W. H. 1997. *Econometric Analysis*. 3rd ed. Upper Saddle River, NJ: Prentice Hall.
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## Also see

[R] **nlsur postestimation** — Postestimation tools for nlsur

[R] **nl** — Nonlinear least-squares estimation

[R] **demandsys** — Estimation of flexible demand systems

[R] **gmm** — Generalized method of moments estimation

[R] **ml** — Maximum likelihood estimation

[R] **mlexp** — Maximum likelihood estimation of user-specified expressions

[R] **reg3** — Three-stage estimation for systems of simultaneous equations

[R] **sureg** — Zellner’s seemingly unrelated regression

[U] **20 Estimation and postestimation commands**

## Postestimation commands

The following postestimation commands are available after `nlsur`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| <code>forecast</code>        | dynamic forecasts and simulations                                                                                                       |
| <code>hausman</code>         | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| <code>lrtest</code>          | likelihood-ratio test                                                                                                                   |
| * <code>margins</code>       | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | fitted values, residuals, etc.                                                                                                          |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*You must specify the `variables()` option with `nlsur`.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as fitted values and residuals.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, equation(eqno) yhat residuals]
```

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

## Options for predict

Main

`equation(eqno)` specifies to which equation you are referring. `equation(#1)` would mean that the calculation is to be made for the first equation, `equation(#2)` would mean the second, and so on. If you do not specify `equation()`, results are the same as if you had specified `equation(#1)`.

`yhat`, the default, calculates the fitted values for the specified equation.

`residuals` calculates the residuals for the specified equation.

# margins

## Description for margins

`margins` estimates margins of response for fitted values.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [options]
```

| <i>statistic</i>  | Description                           |
|-------------------|---------------------------------------|
| <u>y</u> hat      | fitted values; the default            |
| <u>r</u> esiduals | not allowed with <code>margins</code> |

Statistics not allowed with `margins` are functions of stochastic quantities other than `e(b)`.

For the full syntax, see [R] [margins](#).

## Remarks and examples

### ▷ Example 1

In [example 2](#) of [R] [nlsur](#), we fit a four-factor translog cost function to data for the US economy. The own-price elasticity for a factor measures the percentage change in its usage as a result of a 1% increase in the factor's price, assuming that output is held constant. For the translog production function, the own-price factor elasticities are

$$\eta_i = \frac{\delta_{ii} + s_i(s_i - 1)}{s_i}$$

Here we compute the elasticity for capital at the sample mean of capital's factor share. First, we use `summarize` to get the mean of `s_k` and store that value in a scalar:

```
. use https://www.stata-press.com/data/r19/mfgcost
(Manufacturing cost)
. nlsur (s_k = {bk} + {dkk}*ln(pk/pm) + {dkl}*ln(pl/pm) + {dke}*ln(pe/pm))
> (s_l = {bl} + {dkl}*ln(pk/pm) + {dll}*ln(pl/pm) + {dle}*ln(pe/pm))
> (s_e = {be} + {dke}*ln(pk/pm) + {dle}*ln(pl/pm) + {dee}*ln(pe/pm)),
> ifgnls
(output omitted)
. summarize s_k
Variable	Obs Mean Std. dev. Min Max
 s_k | 25 .053488 .0044795 .04602 .06185
. scalar kmean = r(mean)
```

Now, we can use `nlcom` to calculate the elasticity:

```
. nlcom (([dkk]_cons + kmean*(kmean-1)) / kmean)
 _nl_1: ([dkk]_cons + kmean*(kmean-1)) / kmean
```

|       | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|-------|-------------|-----------|-------|-------|----------------------|-----------|
| _nl_1 | -.3952986   | .1083535  | -3.65 | 0.000 | -.6076676            | -.1829295 |

If the price of capital increases by 1%, its usage will decrease by about 0.4%. To maintain its current level of output, a firm would increase its usage of other inputs to compensate for the lower capital usage. The standard error reported by `nlcom` reflects the sampling variance of the estimated parameter  $\widehat{\delta}_{kk}$ , but `nlcom` treats the sample mean of `s_k` as a fixed parameter that does not contribute to the sampling variance of the estimated elasticity.



## Also see

[R] [nlsur](#) — Estimation of nonlinear system of equations

[U] [20 Estimation and postestimation commands](#)

## Description

Nonparametric regression models the mean of an outcome given the covariates without making assumptions about its functional form. This makes nonparametric regression estimates robust to functional form misspecification. `npregress` implements the two most common nonparametric regression estimators: series regression and kernel regression.

Nonparametric series estimation regresses the outcome on a function of the covariates. The function of the covariates is known as a basis function. A basis is a collection of terms that approximates smooth functions arbitrarily well. A basis function includes a subset of these terms. The bases used by `npregress series` are polynomials, piecewise polynomial splines, and B-splines.

Nonparametric kernel estimation computes a weighted average of the outcome. The weights are functions called kernels, which give rise to the name of the method. `npregress kernel` performs local-linear and local-constant kernel regression.

Whether we choose to approximate the mean of our outcome using series regression or kernel regression, we obtain estimates that are robust to assumptions about functional form. This robustness comes at a cost; we need many observations and perhaps a long computation time to estimate the elements of the approximating function.

This entry introduces the intuition behind the nonparametric regression estimators implemented in `npregress`. If you are familiar with these methods, you may want to skip to [\[R\] npregress kernel](#) or [\[R\] npregress series](#).

## Remarks and examples

Remarks are presented under the following headings:

*[Overview](#)*

*[Nonparametric series regression](#)*

*[Runge's phenomenon](#)*

*[Piecewise polynomial splines and B-splines](#)*

*[Nonparametric kernel regression](#)*

*[Limitations of nonparametric methods](#)*

## Overview

Nonparametric regression is used when we are uncertain about the functional form of the mean of the outcome given the covariates. For example, when we estimate a linear regression, we assume that the functional form for the mean of the outcome is a linear combination of the specified covariates. Both parametric (linear) regression and nonparametric regression provide an estimate of the mean for the different values of the covariates. Consider the simulated data in figure 1. The mean of the outcome for all values of  $x$  is overlaid on these points.

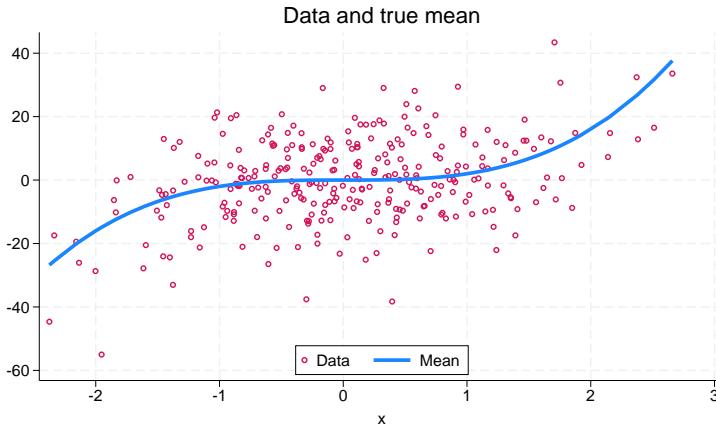


Figure 1.

Because the mean of the data in figure 1 is not linear in  $x$ , using a simple linear regression will not give us a correct picture about the effect of covariate  $x$  on the outcome. For example, if we perform a linear regression of the outcome on  $x$  for the data, we obtain the plot shown in figure 2.

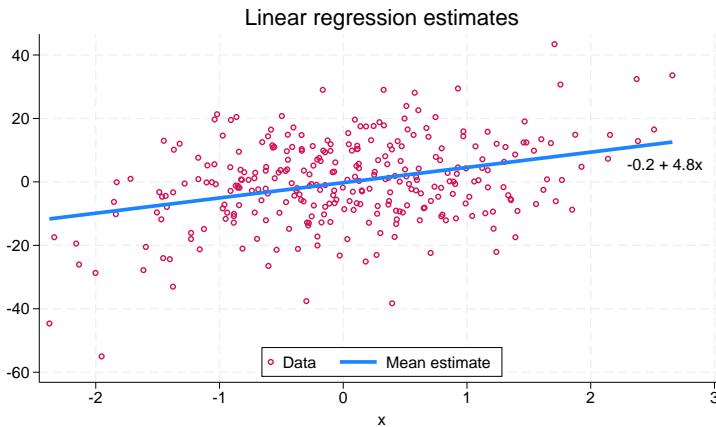


Figure 2.

The change in the predicted outcome when  $x$  changes is positive and constant, yet the true mean is nonlinear. If the assumption about the functional form of the mean is incorrect, the estimates we obtain are inconsistent. If we instead fit the model using `npregress` and graph the estimates, we obtain figure 3.

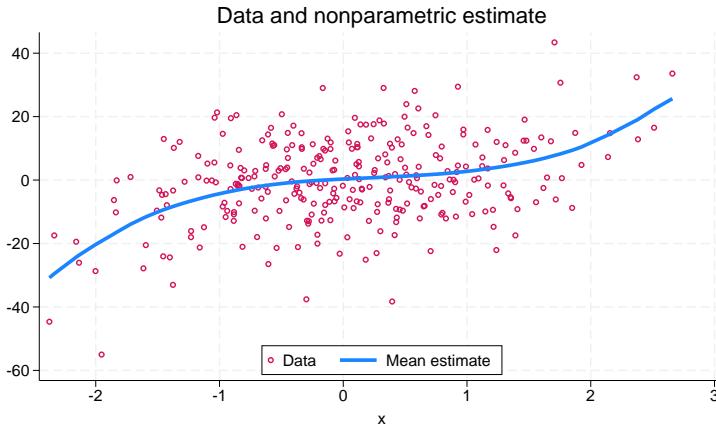


Figure 3.

`npregress` gives us the correct relationship between the outcome and the covariates. The nonparametric regression estimates are consistent as long as the true function is sufficiently smooth. If the linear regression assumptions are true, nonparametric regression is still consistent but less efficient.

Although nonparametric regression is a way to obtain estimates that are robust to functional form misspecification, this robustness comes at a cost. You need many observations and more time to compute the estimates. The cost increases with the number of covariates; this is referred to as the curse of dimensionality.

## Nonparametric series regression

The basis and the basis function are concepts essential to understanding series regression. A basis is a collection of terms that can approximate a smooth function arbitrarily well. A basis function uses a subset of these terms to approximate the mean function. `npregress series` allows you to use a polynomial basis, a piecewise polynomial spline basis, or a B-spline basis. For each basis, `npregress series` selects the basis function for you.

We use an example to illustrate the use of a basis and a basis function. Suppose a researcher has data on the outcome  $y$  and a covariate  $x$ . We plot their relationship in the figure 4 below.

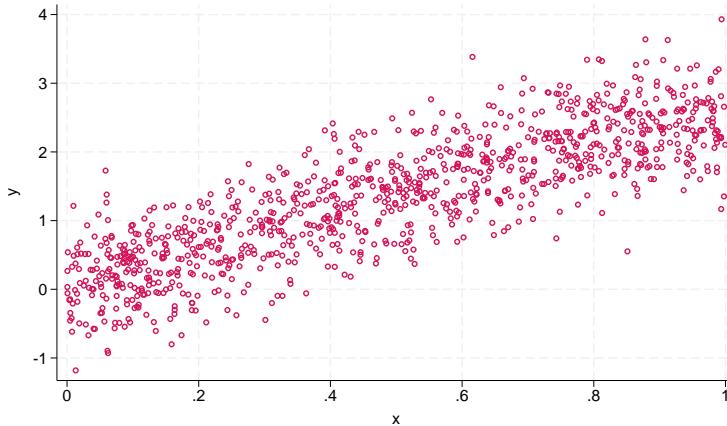


Figure 4.

In this case, a regression of  $y$  on  $x$  will do a good job of approximating the true function. If our data looked like the data in figure 5, however, a regression of  $y$  on  $x$  would be inadequate.

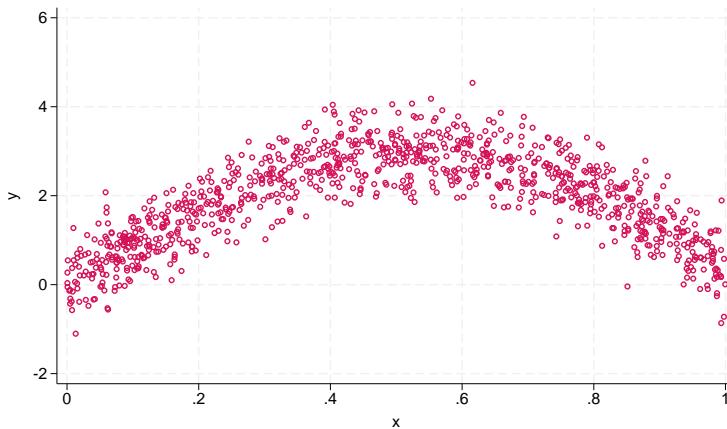


Figure 5.

In this case, a regression of  $y$  on  $x$  and  $x^2$  is more appropriate.

In each case, we include terms from a polynomial basis. In the first case, we need a constant and the linear term  $x$ . In the second case, we need a constant, the linear term  $x$ , and the quadratic term  $x^2$ . A more complex function would require a basis function that includes more terms from the polynomial basis.

If we want to use a polynomial basis function, `npregress` will select a degree of the polynomial for us. Additional terms reduce bias but increase the variance of the estimator. `npregress` will select the terms that optimally tradeoff bias and variance. In other words, `npregress` selects a basis function that includes the terms that minimize the mean squared error. Our example above used a polynomial basis function, but `npregress` can also select terms from a piecewise polynomial spline or B-spline basis.

## Runge's phenomenon

Polynomials are the most intuitive basis but not the preferred basis for nonparametric series estimation. The reason is that they are poor at interpolating. This problem shows up at the boundaries of the support of the covariates, where, as you increase the order of the polynomial, the polynomial approximation oscillates frequently, even when the true function does not behave this way.

Let us demonstrate. Below is an example for which we model a mean function using a third-order polynomial. We plot the data and the estimate of the mean function:

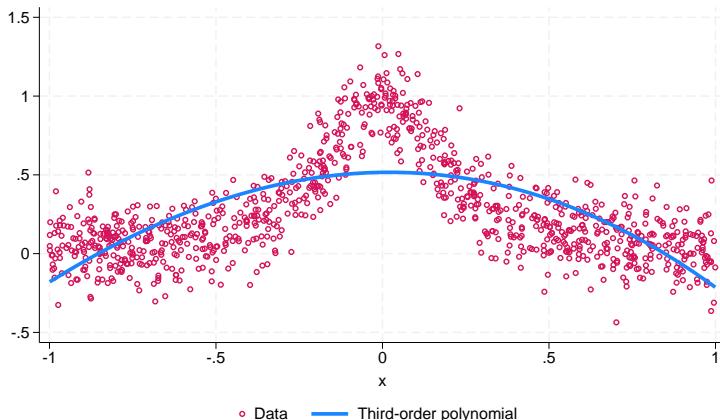


Figure 6.

Looking at the data, it appears that a higher-order polynomial would be a better fit for the data. Below is the mean function we get using a sixth-order and a tenth-order polynomial:

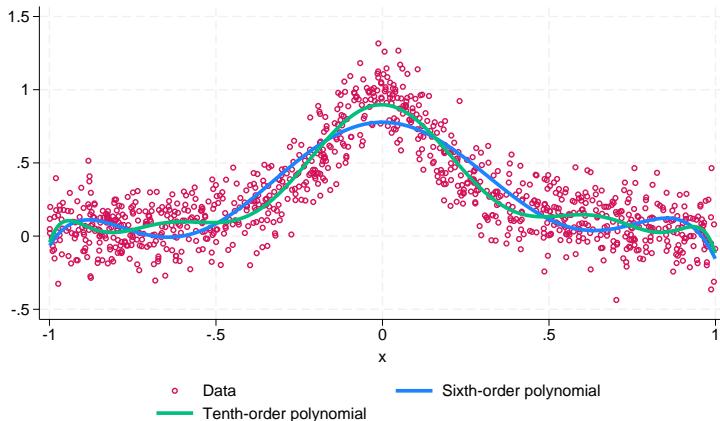


Figure 7.

The predictions improve at values near the middle of the range of  $x$  but become more variable at the edges of the parameter space.

What we illustrated above is referred to as Runge’s phenomenon. Increasing the complexity of the polynomial order did not improve our approximation. In fact, as we increased the polynomial order, the behavior at the edges of the parameter space became more variable. The way to address this is to use a basis that does a better job of interpolating: piecewise polynomial splines or B-splines.

## Piecewise polynomial splines and B-splines

Piecewise polynomial splines and B-splines are preferred to a polynomial basis because they are better at approximation. We discuss piecewise polynomial splines to provide intuition for both the piecewise polynomial spline basis and the B-spline basis.

Low-order polynomials do a great job of approximating functions in regions where the true function does not change too much. Splines continuously connect a set of low-order polynomials to create a basis to approximate a smooth function. The graph below illustrates what this definition means. We show in maroon a piecewise polynomial spline estimate of the mean function for the data in the example above.

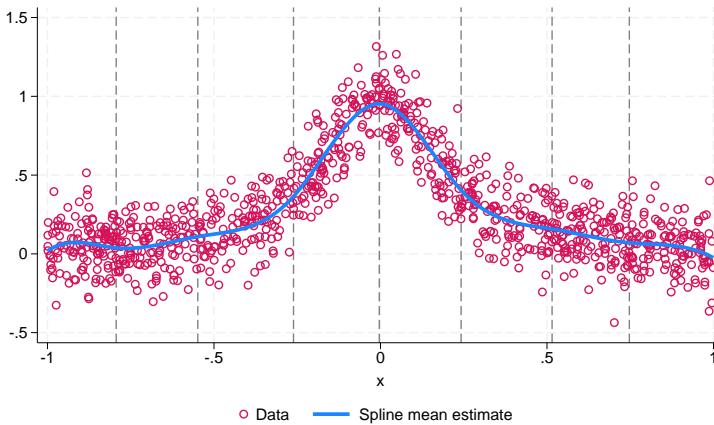


Figure 8.

To see that splines are better than polynomials, note that the spline approximation of the mean function fits the data well and that there are no regions where the approximation wiggles wildly.

Now, we delve into the definition above. The vertical lines in the graph partition the support of  $x$  into subregions. The piecewise polynomial spline basis allows for a different low-order polynomial in each subregion, and it forces the polynomials in neighboring regions to be continuously connected. In figure 8 above, the basis used is a third-order polynomial in each subregion. The graph illustrates that the polynomials are smoothly connected at the subregion boundaries. The subregion boundaries are known as the knot points, or just the knots, because they are where the different polynomials are tied together.

By default, `npregress` selects the number of knots for you. Alternatively, you may specify the number of knots yourself.

We now look at how the mean function at each region was computed. We show this mathematically and graphically.

Defining the seven knots as  $t_1, \dots, t_7$ , where  $t_1 < t_2 < \dots < t_6 < t_7$ , the third-order piecewise polynomial spline estimate is given by

$$\widehat{E}(y_i|x_i) = \widehat{\beta}_0 + \widehat{\beta}_1 x_i + \widehat{\beta}_2 x_i^2 + \widehat{\beta}_3 x_i^3 + \sum_{j=1}^7 \beta_{j+3} \max(x_i - t_j, 0)^3$$

Thus, for all  $x_i$  that are less than the smallest knot,  $t_1$ , the mean estimate is given by the third-order polynomial

$$\widehat{E}(y_i|x_i \leq t_1) = \widehat{\beta}_0 + \widehat{\beta}_1 x_i + \widehat{\beta}_2 x_i^2 + \widehat{\beta}_3 x_i^3$$

Here it is graphically:

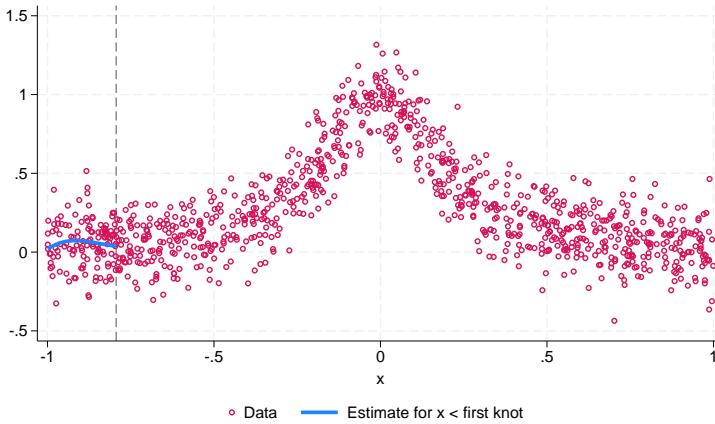


Figure 9.

Likewise, if  $x$  is less than the second knot,  $t_2$ , then the mean estimate for that region is different if  $x_i > t_1$  than if  $x_i \leq t_1$ , and is given by

$$\widehat{E}(y_i|x_i \leq t_2) = \widehat{\beta}_0 + \widehat{\beta}_1 x_i + \widehat{\beta}_2 x_i^2 + \widehat{\beta}_3 x_i^3 + \beta_4 (x_i - t_1)^3 (x_i > t_1)$$

Here it is graphically:

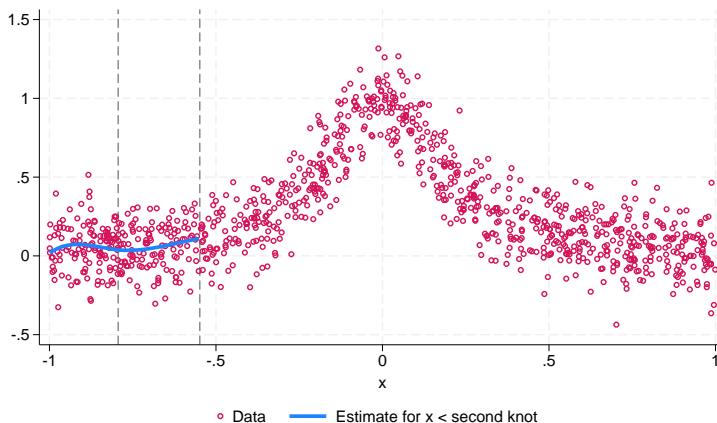


Figure 10.

As  $x$  increases, there are additional contributions from each subregion. If we continue plotting the resulting mean estimates, the following graphs would be what we would obtain:

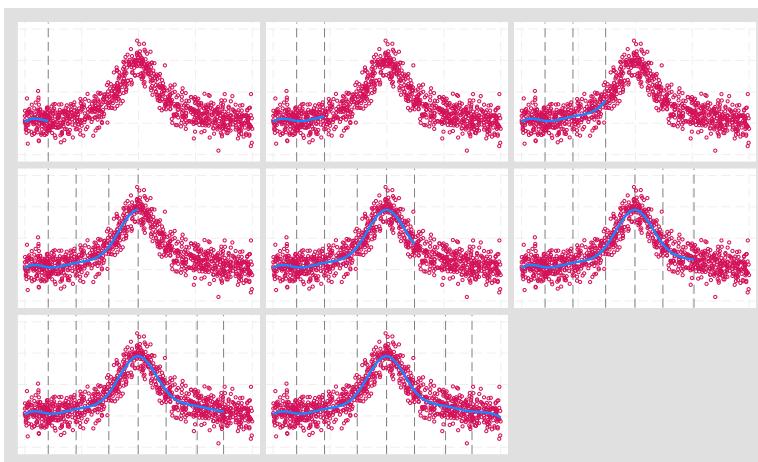


Figure 11.

This example illustrates how the terms in the spline basis approximate the mean function. Both the graph of the estimated function and the intuition in the example illustrate why the spline basis is better than the polynomial basis.

In the examples above, we used a piecewise polynomial spline basis. Specifically, we used a third-order piecewise polynomial spline basis function to obtain our estimates of the conditional mean. We could have also used second-order or first-order piecewise polynomial splines, where the order of the splines is defined by the order of the polynomial terms in the covariates used in each subregion.

As mentioned before, piecewise polynomial splines are preferred to a polynomial basis because they are better at approximation. However, piecewise polynomial splines also have some issues. In particular, they can be highly collinear and therefore numerically unstable. You can see this in the regions delineated in figure 11, which are defined by terms of the form  $\max(x_i - t_j, 0)$  that may overlap.

B-splines avoid this problem, so each term that goes into the conditional mean approximation is orthogonal. It is for this reason that B-splines are the default basis for `npregress` series. However, the intuition we obtain from piecewise polynomial splines and B-splines is equivalent. In fact, B-spline and piecewise polynomial spline bases can approximate the same functions. For a more detailed explanation of B-splines, see *Methods and formulas* in [R] `npregress` series.

In this section, we provided an intuitive and brief introduction to nonparametric series estimation. For detailed introductions to series estimators and the methods implemented by `npregress` series, see [de Boor \(2001\)](#), [Schumaker \(2007\)](#), [Eubank \(1999\)](#), [Schoenberg \(1969\)](#), [Newey \(1997\)](#), and [Chen \(2007\)](#).

## Nonparametric kernel regression

`npregress` `kernel` approximates the mean by using a kernel function. In [Overview](#), we plotted the following data and nonparametric estimate of the mean function:

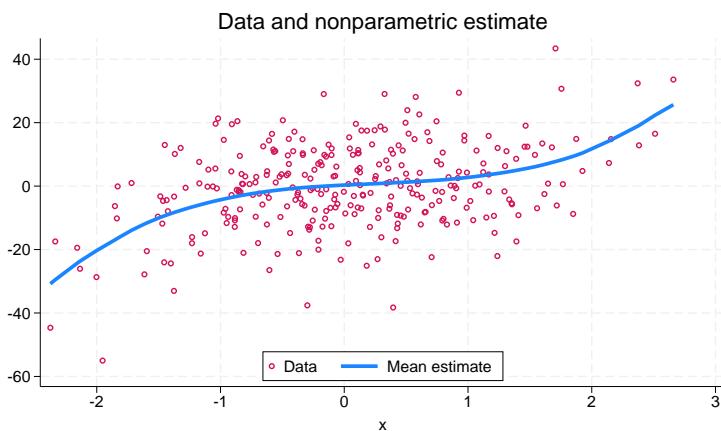


Figure 12.

We used kernel regression to estimate this mean function. With this method, we estimate the mean of the outcome at different values of the covariate  $x$ . In this section, we build our intuition for how kernel regression estimates these means, and we demonstrate this graphically.

Suppose covariate  $x$  is discrete. In this case, a consistent estimator of the mean of outcome  $y$  given that  $x = a$  is the average of the values of  $y$  for which  $x$  is equal to a given value  $a$ . For instance, the sample average of the yearly income for married individuals is a consistent estimator for the population mean yearly income for married individuals.

Now, consider estimating the mean of  $y$  given that  $x = a$  when  $x$  is continuous and  $a$  is a value observed for  $x$ . Because  $x$  is continuous, the probability of any observed value being exactly equal to  $a$  is 0. Therefore, we cannot compute an average for the values of  $y$  for which  $x$  is equal to a given value  $a$ . We use the average of  $y$  for the observations in which  $x$  is close to  $a$  to estimate the mean of  $y$  given that  $x = a$ . Specifically, we use the observations for which  $|x - a| < h$ , where  $h$  is small. The parameter  $h$  is called a bandwidth. In nonparametric kernel regression, a bandwidth determines the amount of information we use to estimate the conditional mean at each point  $a$ . We demonstrate how this works graphically.

For the simulated data in our example, we choose  $h = 0.25$  and  $a = -0.19$ . The vertical lines in figure 13 delimit the values of  $x$  around  $a$  for which we are computing the mean of  $y$ . The light blue square is our estimate of the conditional mean using the observations between the vertical lines.

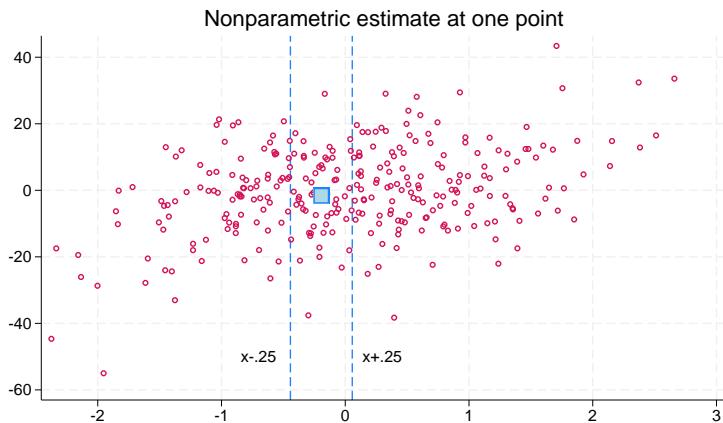


Figure 13.

Repeating this estimation when  $a = 2.66$  produces figure 14.

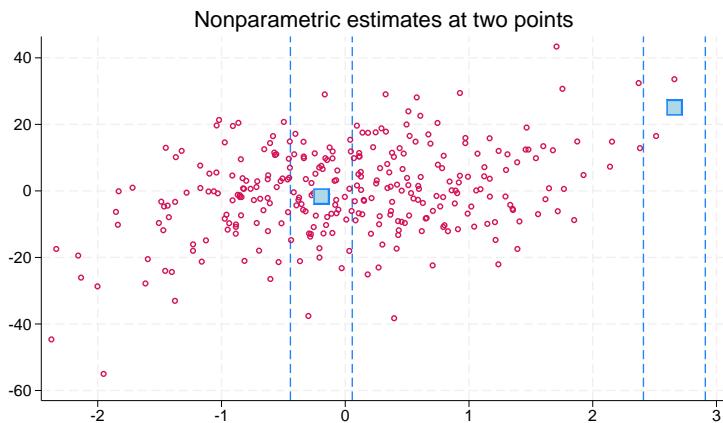


Figure 14.

Doing this estimation for each point in our data produces a nonparametric estimate of the mean for a given value of the covariates (see figure 15).

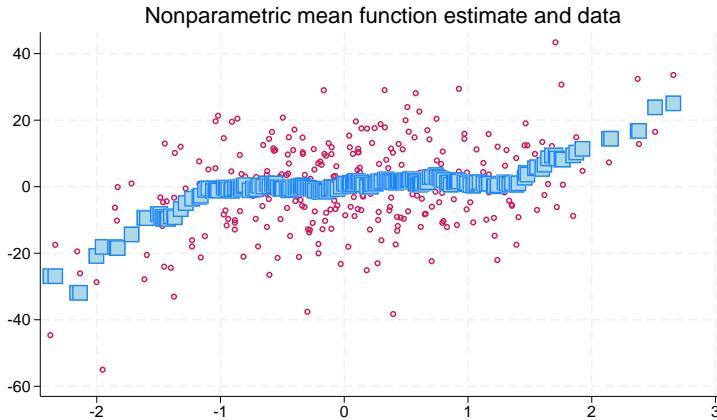


Figure 15.

The plotted blue squares in figure 15 form what is known as the conditional mean function. Because these are simulated data, we can compare our estimate with the true conditional mean function, a comparison we show in figure 16.

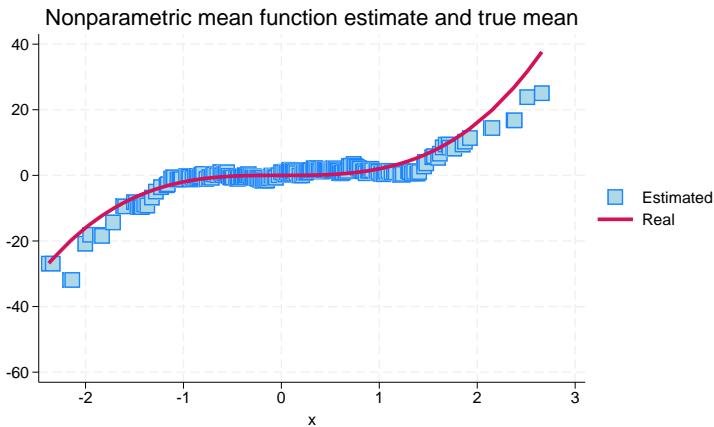


Figure 16.

We see that the estimate is a bit less smooth than the true function. The size of the bandwidth  $h$  determines the shape and smoothness of the estimated conditional mean function, because the bandwidth defines how many observations around each point are used. For example, if  $h$  is arbitrarily large—say,  $h = 300$ —then we get figure 17.

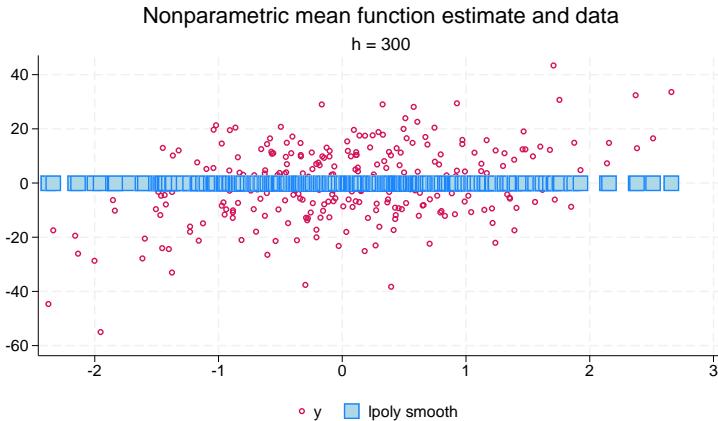


Figure 17.

In this case, all observations are used to estimate the conditional mean at each point, and the estimate is therefore a constant. On the other hand, a too-small bandwidth produces a jagged function with high variability, as illustrated in figure 18.

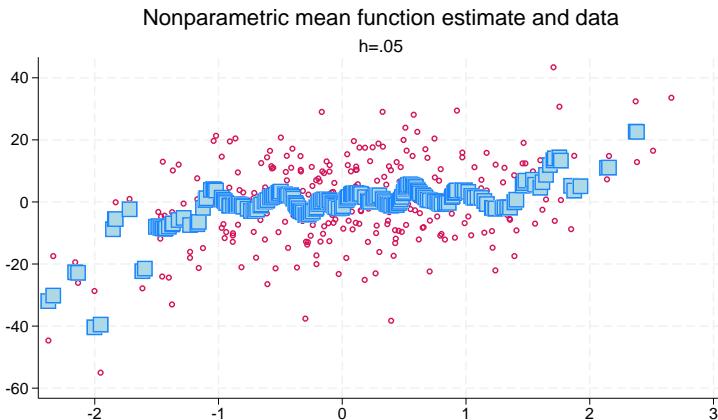


Figure 18.

The optimal bandwidth is somewhere in between. A too-large bandwidth includes too many observations, so the estimate is biased but it has a low variance. A too-small bandwidth includes too few observations, so the estimate has little bias but the variance is large. In other words, the optimal bandwidth trades off bias and variance. In the case of `npregress kernel`, the bandwidth is chosen to minimize the cost of this tradeoff by using either cross-validation, as suggested by [Li and Racine \(2004\)](#), or an improved Akaike information criterion proposed by [Hurvich, Simonoff, and Tsai \(1998\)](#).

How we average the observations around a point is also important. In the examples above, we gave the same weight to each observation for which  $|x - a| < h$ . However, we might weight each observation differently. The weights that observations receive are determined by functions called kernels. We could have used any of the weights in `[R] kdensity`. For a nice introduction to kernel weighting, see [Silverman \(1986\)](#).

The estimator described above uses only nearby observations and is thus a local estimator. It uses a sample average, which is a regression on a constant, and is thus a locally constant estimator. For these reasons, the estimator described above fits what is known as a local-constant regression.

The generalization that uses the prediction from a local-linear regression on covariates is known as local-linear regression. Local-linear regression estimates the derivative of the conditional mean function in addition to the function itself. Understanding how the conditional mean changes when covariates change is sometimes the research question of interest, for example, how income changes for different levels of taxes. Local-linear regression provides an estimate for these changes for continuous and discrete variables.

See [Fan and Gijbels \(1996\)](#) and [Li and Racine \(2007\)](#) for detailed introductions to the kernel estimators implemented in `npregress kernel`.

## Limitations of nonparametric methods

As discussed above, series regression and kernel regression approximate an unknown mean function. Series regression uses least squares on the basis function. Kernel regression uses a kernel-weighted average of nearby observations.

Series estimators are considered to be global estimators because they approximate the mean function at each point using the value of one overall approximating function. Kernel regression is considered a local estimator because it only uses nearby observations to approximate the mean for a given covariate pattern.

Although piecewise polynomial splines and B-splines are considered to be global estimators, in fact, they are local estimators. They are local because they fit a polynomial in each region defined by the knots. Like kernel estimators, piecewise polynomial spline and B-spline estimators require that there are enough data in each region. Suppose our data look like the data below.

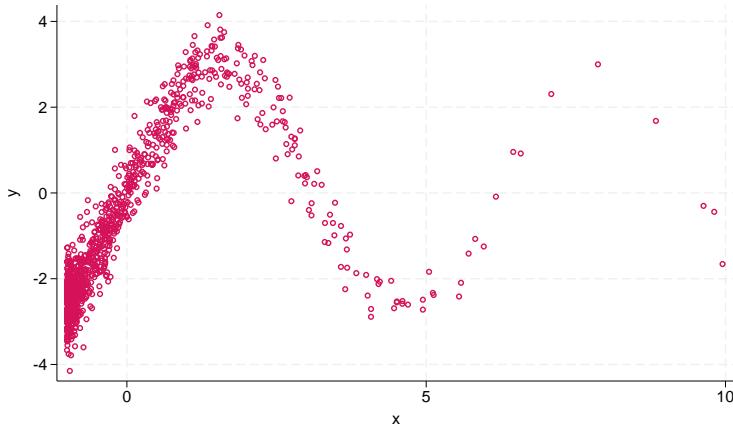


Figure 19.

Using a method to select knots optimally at percentiles of the data will give us figure 20.

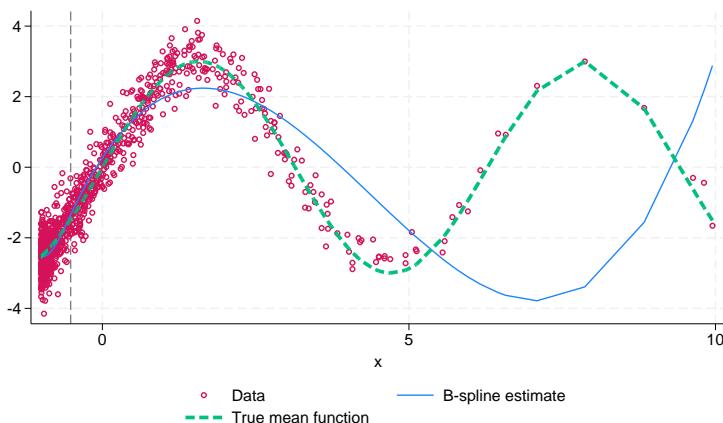


Figure 20.

The vertical line denotes the point at which the knot is placed. The blue line is the B-spline estimate, and the dotted green line is the true mean function. We see that our estimate of the mean function is not good, especially for higher positive values of the covariate. The reason is that data are sparse for these values. An alternative is to place the knots uniformly over the values of  $x$ . In this case, our estimate of the mean function improves. However, this does not change the fact that we have regions with insufficient data to make reliable inferences.

Thus, for kernel, piecewise polynomial spline, and B-spline estimators, we must have enough data points for all ranges of the data. In particular, piecewise polynomial spline and B-spline estimates should not be used to predict outside the support of the data.

Another important consideration is model selection. `npregress` selects the number of terms from a basis for series estimation and the bandwidth for kernel estimation. After model selection, the models are taken as given without accounting for model-selection error. You can find an in-depth discussion and references of some of the issues that arise when performing model selection in [\[LASSO\] Lasso intro](#).

## References

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## Also see

- [R] [npregress kernel](#) — Nonparametric kernel regression
- [R] [npregress series](#) — Nonparametric series regression
- [R] [lpoly](#) — Kernel-weighted local polynomial smoothing
- [R] [kdensity](#) — Univariate kernel density estimation
- [R] [regress](#) — Linear regression

[Description](#)[Options](#)[Acknowledgments](#)[Quick start](#)[Remarks and examples](#)[References](#)[Menu](#)[Stored results](#)[Also see](#)[Syntax](#)[Methods and formulas](#)

## Description

`npregress kernel` performs nonparametric local–linear and local–constant kernel regression. Like linear regression, nonparametric regression models the mean of the outcome conditional on the covariates, but unlike linear regression, it makes no assumptions about the functional form of the relationship between the outcome and the covariates. `npregress kernel` may be used to model the mean of a continuous, count, or binary outcome.

## Quick start

Nonparametric regression of  $y$  on  $x$  and discrete covariate  $a$  using the Epanechnikov kernel for  $x$  and the Li–Racine kernel for  $a$

```
npregress kernel y x i . a
```

Same as above, but use 500 replications and compute bootstrap standard errors and percentile confidence intervals

```
npregress kernel y x i . a, reps(500)
```

Same as above, but use a Gaussian kernel for  $x$

```
npregress kernel y x i . a, reps(500) kernel(gaussian)
```

Same as above, but use the improved AIC to find the optimal bandwidth

```
npregress kernel y x i . a, reps(500) kernel(gaussian) imaic
```

Same as above, but additionally specify that only the mean of the outcome be computed

```
npregress kernel y x i . a, reps(500) kernel(gaussian) imaic noderivatives
```

Specify  $h$  as the vector of bandwidths

```
npregress kernel y x i . a, bwidth(h)
```

## Menu

Statistics > Nonparametric analysis > Nonparametric kernel regression

## Syntax

```
npregress kernel depvar indepvars [if] [in] [, options]
```

| <i>options</i>                                                         | Description                                                                                                                                      |
|------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                                           |                                                                                                                                                  |
| <code>estimator</code> ( <code>linear</code>   <code>constant</code> ) | use the local-linear or local-constant kernel estimator                                                                                          |
| <code>kernel</code> ( <i>kernel</i> )                                  | kernel density function for continuous covariates                                                                                                |
| <code>dkernel</code> ( <i>dkernel</i> )                                | kernel density function for discrete covariates                                                                                                  |
| <code>predict</code> ( <i>prspec</i> )                                 | store predicted values of the mean and derivatives using variable names specified in <i>prspec</i>                                               |
| <code>noderivatives</code>                                             | suppress derivative computation                                                                                                                  |
| <code>imaic</code>                                                     | use improved AIC instead of cross-validation to compute optimal bandwidth                                                                        |
| <code>unidentsample</code> ( <i>newvar</i> )                           | specify name of variable that marks identification problems                                                                                      |
| <b>Bandwidth</b>                                                       |                                                                                                                                                  |
| <code>bwidth</code> ( <i>specs</i> )                                   | specify kernel bandwidth for all predictions                                                                                                     |
| <code>meanbwidth</code> ( <i>specs</i> )                               | specify kernel bandwidth for the mean                                                                                                            |
| <code>derivbwidth</code> ( <i>specs</i> )                              | specify kernel bandwidth for the derivatives                                                                                                     |
| <b>SE</b>                                                              |                                                                                                                                                  |
| * <code>vce</code> ( <i>vcetype</i> )                                  | <i>vcetype</i> may be <code>none</code> or <code>bootstrap</code>                                                                                |
| <code>reps</code> (#)                                                  | equivalent to <code>vce(bootstrap, reps(#))</code>                                                                                               |
| <code>seed</code> (#)                                                  | set random-number seed to #; must also specify <code>reps(#)</code>                                                                              |
| <code>bwreplace</code>                                                 | vary bandwidth with each bootstrap replication; seldom used                                                                                      |
| <b>Reporting</b>                                                       |                                                                                                                                                  |
| <code>level</code> (#)                                                 | set confidence level; default is <code>level(95)</code>                                                                                          |
| <code>display_options</code>                                           | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| <code>citype</code> ( <i>citype</i> )                                  | method to compute bootstrap confidence intervals; default is <code>citype(percentile)</code>                                                     |
| <b>Maximization</b>                                                    |                                                                                                                                                  |
| <code>maximize_options</code>                                          | control the maximization process                                                                                                                 |
| <code>coeflegend</code>                                                | display legend instead of statistics                                                                                                             |

---

*indepvars* may contain factor variables; see [U] 11.4.3 Factor variables.

`bootstrap`, `by`, `collect`, and `jackknife` are allowed; see [U] 11.1.10 Prefix commands.

\* `vce(bootstrap)` reports percentile confidence intervals instead of the normal-based confidence intervals reported when `vce(bootstrap)` is specified with other estimation commands.

`coeflegend` does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

| <i>kernel</i>             | Description                               |
|---------------------------|-------------------------------------------|
| <code>epanechnikov</code> | Epanechnikov kernel function; the default |
| <code>epan2</code>        | alternative Epanechnikov kernel function  |
| <code>biweight</code>     | biweight kernel function                  |
| <code>cosine</code>       | cosine trace kernel function              |
| <code>gaussian</code>     | Gaussian kernel function                  |
| <code>parzen</code>       | Parzen kernel function                    |
| <code>rectangle</code>    | rectangle kernel function                 |
| <code>triangle</code>     | triangle kernel function                  |

| <i>dkernel</i>        | Description                            |
|-----------------------|----------------------------------------|
| <code>liracine</code> | Li–Racine kernel function; the default |
| <code>cellmean</code> | cell means kernel function             |

| <i>citype</i>           | Description                                  |
|-------------------------|----------------------------------------------|
| <code>percentile</code> | percentile confidence intervals; the default |
| <code>bc</code>         | bias-corrected confidence intervals          |
| <code>normal</code>     | normal-based confidence intervals            |

## Options

### Model

`estimator`(`linear` | `constant`) specifies whether the local-constant or local-linear kernel estimator should be used. The default is `estimator`(`linear`).

`kernel` (*kernel*) specifies the kernel density function for continuous covariates for use in calculating the local-constant or local-linear estimator. The default is `kernel`(`epanechnikov`).

`dkernel` (*dkernel*) specifies the kernel density function for discrete covariates for use in calculating the local-constant or local-linear estimator. The default is `dkernel`(`liracine`); see [Methods and formulas](#) for details on the Li–Racine kernel. When `dkernel`(`cellmean`) is specified, discrete covariates are weighted by their cell means.

`predict` (*prspec*) specifies that `npregress kernel` store the predicted values for the mean and derivatives of the mean with the specified names. *prspec* is the following:

```
predict(varlist | stub* [, replace noderivatives])
```

The option takes a variable list or a *stub*. The first variable name corresponds to the predicted outcome mean. The second name corresponds to the derivatives of the mean. There is one derivative for each *indepvar*.

When `replace` is used, variables with the names in *varlist* or *stub*\* are replaced by those in the new computation. If *noderivatives* is specified, only a variable for the mean is created. This will increase computation speed but will add to the computation burden if you want to obtain marginal effects after estimation.

`noderivatives` suppresses the computation of the derivatives. In this case, only the mean function is computed.

`imaic` specifies to use the improved AIC instead of cross-validation to compute optimal bandwidths.

`unident``sample`(*newvar*) specifies the name of a variable that is 1 if the observation violates the model identification assumptions and is 0 otherwise. By default, this variable is a system variable (`_unident_sample`).

`npregress kernel` computes a weighted regression for each observation in our data. An observation violates identification assumptions if the regression cannot be performed at that point. The regression formula, which is discussed in detail in *Methods and formulas*, is given by

$$\hat{\gamma} = (\mathbf{Z}'\mathbf{W}\mathbf{Z})^{-1} \mathbf{Z}'\mathbf{W}\mathbf{y}$$

`npregress kernel` verifies that the matrix  $(\mathbf{Z}'\mathbf{W}\mathbf{Z})$  is full rank for each observation to determine identification. Identification problems commonly arise when the bandwidth is too small, resulting in too few observations within a bandwidth. Independent variables that are collinear within the bandwidth can also cause a problem with identification at that point.

Observations that violate identification assumptions are reported as missing for the predicted means and derivatives.

---

#### Bandwidth

`bwidth`(*specs*) specifies the half-width of the kernel at each point for the computation of the mean and the derivatives of the mean function. If no bandwidth is specified, one is chosen by minimizing the integrated mean squared error of the prediction.

*specs* specifies bandwidths for the mean and derivative for each *indepvar* in one of three ways: by specifying the name of a vector containing the bandwidths (for example, `bwidth(H)`, where H is a properly labeled vector); by specifying the equation and coefficient names with the corresponding values (for example, `bwidth(Mean:x1=0.5 Effect:x1=0.9)`); or by specifying a list of values for the means, standard errors, and derivatives for *indepvars* given in the order of the corresponding *indepvars* and specifying the copy suboption (for example, `bwidth(0.5 0.9, copy)`).

`skip` specifies that any parameters found in the specified vector that are not also found in the model be ignored. The default action is to issue an error message.

`copy` specifies that the list of values or the vector be copied into the bandwidth vector by position rather than by name.

`meanbwidth`(*specs*) specifies the half-width of the kernel at each point for the computation of the mean function. If no bandwidth is specified, one is chosen by minimizing the integrated mean squared error of the prediction. For details on how to specify the bandwidth, see the description of `bwidth()`, [above](#).

`derivbwidth`(*specs*) specifies the half-width of the kernel at each point for the computation of the derivatives of the mean. If no bandwidth is specified, one is chosen by minimizing the integrated mean squared error of the prediction. For details on how to specify the bandwidth, see the description of `bwidth()`, [above](#).

---

#### SE

`vce`(*vcetype*) specifies the type of standard error reported, which may be either that no standard errors are reported (`none`; the default) or that bootstrap standard errors are reported (`bootstrap`); see [\[R\] vce\\_option](#).

We recommend that you select the number of replications using `reps(#)` instead of specifying `vce(bootstrap)`, which defaults to 50 replications. Be aware that the number of replications needed to produce good estimates of the standard errors varies depending on the problem.

When `vce(bootstrap)` is specified, `npregress kernel` reports percentile confidence intervals as recommended by [Cattaneo and Jansson \(2018\)](#) instead of reporting the normal-based confidence intervals that are reported when `vce(bootstrap)` is specified with other commands. Other types of confidence intervals can be obtained by using the `citype(citype)` option.

`reps(#)` specifies the number of bootstrap replications to be performed. Specifying this option is equivalent to specifying `vce(bootstrap, reps(#))`.

`seed(#)` sets the random-number seed. You must specify `reps(#)` with `seed(#)`.

`bwreplace` computes a different bandwidth for each bootstrap replication. The default is to compute the bandwidth once and keep it fixed for each bootstrap replication. This option is seldom used.

#### Reporting

`level(#)`; see [\[R\] Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [\[R\] Estimation options](#).

`citype(citype)` specifies the type of confidence interval to be computed. By default, bootstrap percentile confidence intervals are reported as recommended by [Cattaneo and Jansson \(2018\)](#). `citype` may be one of `percentile`, `bc`, or `normal`.

#### Maximization

`maximize_options`: `iterate(#)`, `[no]log`, `trace`, `showstep`, `tolerance(#)`, `ltolerance(#)`, `from(init_specs)`; see [\[R\] Maximize](#). These options are seldom used.

The following option is available with `npregress kernel` but is not shown in the dialog box:

`coeflegend`; see [\[R\] Estimation options](#).

## Remarks and examples

This entry assumes that you are already familiar with nonparametric regression. For an introduction to the nonparametric kernel regression methods used in `npregress kernel`, see [\[R\] npregress intro](#).

Remarks are presented under the following headings:

- [Overview](#)
- [Estimation and effects](#)
- [Visualizing covariate effects](#)

## Overview

`npregress kernel` implements local-constant and local-linear regression. The covariates may be continuous or discrete. You can use `npregress kernel` to nonparametrically estimate a conditional mean. `npregress kernel` also allows you to estimate covariate effects after estimation and, in models with one covariate, to plot the mean function by using `nprgraph` after estimation.

The word “nonparametric” refers to the fact that the parameter of interest, the mean as a function of the covariates, is given by the unknown function  $g(\mathbf{x}_i)$ , which is an element of an infinite-dimensional space of functions. In contrast, in a parametric model, the mean for a given value of the covariates,  $E(y_i|\mathbf{x}_i) = f(\mathbf{x}_i, \beta)$ , is a known function that is fully characterized by the parameter of interest,  $\beta$ , which is a finite-dimensional real vector (Shao 2003).

The regression model of outcome  $y_i$  given the  $k$ -dimensional vector of covariates  $\mathbf{x}_i$  is given by

$$y_i = g(\mathbf{x}_i) + \varepsilon_i \tag{1}$$

$$E(\varepsilon_i|\mathbf{x}_i) = 0 \tag{2}$$

where  $\varepsilon_i$  is the error term. The covariates may include discrete and continuous variables. Equations (1) and (2) imply that

$$E(y_i|\mathbf{x}_i) = g(\mathbf{x}_i)$$

Once we account for the information in the covariates, the error term provides no information about the mean of our outcome. The conditional mean function is therefore given by  $g(\mathbf{x}_i)$ . By estimating  $E(y_i|\mathbf{x}_i = \mathbf{x})$  for all points  $\mathbf{x}$  in our data, we obtain an estimate of  $E(y_i|\mathbf{x}_i)$ .

`npregress kernel`, by default, estimates a local-linear regression. Local-linear regression estimates a regression for a subset of observations for each point in our data. See Fan and Gijbels (1996) for a good reference on local-linear regression. Local-linear regression, for each point  $\mathbf{x}$ , solves the minimization problem given by

$$\min_{\gamma} \sum_{i=1}^n \{y_i - \gamma_0 - \gamma_1'(\mathbf{x}_i - \mathbf{x})\}^2 K(\mathbf{x}_i, \mathbf{x}, \mathbf{h}) \tag{3}$$

where  $\gamma = (\gamma_0, \gamma_1)'$ .

Equation (3) and its solution are similar to parametric ordinary least squares. The slope and the constant in (3), however, have a different interpretation. The constant in (3),  $\gamma_0$ , is the conditional mean at a specific point  $\mathbf{x}$ . The slope parameter,  $\gamma_1$ , is the derivative of the mean function with respect to  $\mathbf{x}$ . The solution to this least-squares problem gives us the mean function and its derivative for each one of the elements of  $\mathbf{x}$ . Repeating this optimization for each point  $\mathbf{x}$  gives us the entire mean function and its derivatives.

Another difference between (3) and the minimization problem of parametric ordinary least squares is how the optimization is weighted. The weights are given by the kernel function  $K(\mathbf{x}_i, \mathbf{x}, \mathbf{h})$ . The kernel function assigns weights to observations  $\mathbf{x}_i$  based on how much they differ from  $\mathbf{x}$  and based on the bandwidth,  $\mathbf{h}$ . The smaller  $\mathbf{h}$  is, the larger the weight assigned to points between  $\mathbf{x}_i$  and  $\mathbf{x}$ .

The bandwidth also determines the bias and variance of the mean function estimator. `npregress kernel` selects the bandwidth using cross-validation, as suggested by Li and Racine (2004), or if the `imaic` option is specified, with the improved AIC proposed by Hurvich, Simonoff, and Tsai (1998). Both methods minimize the tradeoff between bias and variance.

`npregress kernel` computes a conditional mean for each observation in the data and, for each one of these computations, verifies whether identification conditions are fulfilled. The observations for which the regression identification assumptions are not satisfied are dropped from the estimation sample. Additionally, whenever there is a violation of the identification assumption, `npregress kernel` generates a system variable or a variable with a name provided in `noidsample(newvar)`. This variable is 1 for observations violating the identification assumption and is 0 otherwise. `npregress kernel` also issues a warning, letting you know the number of observations for which the identification assumption is not satisfied.

## Estimation and effects

The output of `npregress kernel` reports averages of the mean function and the effects of the mean function. An average effect from nonparametric regression may be either 1) an average marginal effect, in the case of the mean of derivatives for continuous covariates or 2) the mean of contrasts for discrete covariates.

Some well-established literature estimates these average effects directly and uses an optimal bandwidth for this computation; see [Powell, Stock, and Stoker \(1989\)](#) and [Powell and Stoker \(1996\)](#). By taking averages of the local-linear estimates, `npregress kernel` is more in line with the approach in [Li, Lu, and Ullah \(2003\)](#). Intuitively, choosing the optimal bandwidth for the derivative produces a more efficient estimator than using the bandwidth that is optimal for the function. Both estimators are consistent for the average effect. [Cattaneo and Jansson \(2018\)](#) formally justify the average effect using the function-optimal bandwidth.

`npregress kernel` also reports an approximation of  $n|\mathbf{h}|$  as the expected kernel observations. This statistic rounds the product of the continuous kernel bandwidth values and the number of observations used for estimation. For instance, if the estimation sample was 500 and the bandwidth was 0.246, the expected kernel observations would be 123 ( $= 500 \times 0.246$ ). The expected kernel observation number of 123 tells us that, on average, 123 observations are used to compute each one of the 500 regressions performed by `npregress kernel`.

### ► Example 1: Nonparametric regression estimation and graphing

`dui.dta` contains information about the number of monthly drunk driving citations in a local jurisdiction (`citations`). Suppose we want to know the effect of increasing fines on the number of citations. Because `citations` is a count variable, we could consider fitting the model with `poisson` or `nbreg`. However, both of these estimators make assumptions about the distribution of the data. If these assumptions are not true, we will obtain inconsistent estimates.

By using npregress kernel, we do not have to make any assumptions about how citations is distributed. We use npregress kernel to estimate the mean of citations as a function of the value of the fines imposed for drunk driving (fines).

```
. use https://www.stata-press.com/data/r19/dui
(Fictional data on monthly drunk driving citations)
. npregress kernel citations fines
Computing mean function

Minimizing cross-validation function:

Iteration 0: Cross-validation criterion = 35.478784
Iteration 1: Cross-validation criterion = 4.0147129
Iteration 2: Cross-validation criterion = 4.0104176
Iteration 3: Cross-validation criterion = 4.0104176
Iteration 4: Cross-validation criterion = 4.0104176
Iteration 5: Cross-validation criterion = 4.0104176
Iteration 6: Cross-validation criterion = 4.0104006
```

```
Computing optimal derivative bandwidth

Iteration 0: Cross-validation criterion = 6.1648059
Iteration 1: Cross-validation criterion = 4.3597488
Iteration 2: Cross-validation criterion = 4.3597488
Iteration 3: Cross-validation criterion = 4.3597488
Iteration 4: Cross-validation criterion = 4.3597488
Iteration 5: Cross-validation criterion = 4.3597488
Iteration 6: Cross-validation criterion = 4.3595842
Iteration 7: Cross-validation criterion = 4.3594713
Iteration 8: Cross-validation criterion = 4.3594713
```

Bandwidth

|       | Mean     | Effect  |
|-------|----------|---------|
| fines | .5631079 | .924924 |

```
Local-linear regression Number of obs = 500
Kernel : epanechnikov E(Kernel obs) = 282
Bandwidth: cross-validation R-squared = 0.4380
```

| citations      | Estimate  |
|----------------|-----------|
| Mean citations | 22.33999  |
| Effect fines   | -7.692388 |

Note: Effect estimates are averages of derivatives.  
 Note: You may compute standard errors using vce(bootstrap) or reps().

The first table displays the bandwidths used to estimate the mean function and the derivative of the mean function. Each of these bandwidths is estimated by minimizing a function that trades off bias and variance; the corresponding iteration logs are displayed also. The expected number of observations used to estimate the mean function at each point is reported in E(Kernel obs) as 282.

Unlike other estimation commands, `npregress kernel` does not report standard errors, test statistics, and confidence intervals by default. In [example 2](#), we demonstrate how to obtain these statistics and further discuss the output.



▷ Example 2: Bootstrapping standard errors

We can estimate the standard errors by using the bootstrap; see [Cattaneo and Jansson \(2018\)](#) for formal results. We use the `reps(400)` option, which is equivalent to `vce(bootstrap, reps(400))` and specifies that 400 bootstrap replications be used instead of the default 50 replications that are used when we specify `vce(bootstrap)`.

Each estimation problem requires a different number of replications to produce good estimates of the standard errors. In [example 3](#), we explain how we decided to use 400 replications. Note that nonparametric estimation and the bootstrap are computationally intensive, so running this example and others that compute bootstrap standard errors will take a while.

```
. npregress kernel citations fines, reps(400) seed(12)
(running npregress on estimation sample)

Bootstrap replications (400):10.....20.....30.....40.....
> ...50.....60.....70.....80.....90.....100.....110...
>120.....130.....140.....150.....160.....170.....
> .180.....190.....200.....210.....220.....230.....240.
>250.....260.....270.....280.....290.....300.....
> ...310.....320.....330.....340.....350.....360.....37
> 0.....380.....390.....400 done
```

Bandwidth

|       | Mean     | Effect  |
|-------|----------|---------|
| fines | .5631079 | .924924 |

```
Local-linear regression Number of obs = 500
Kernel : epanechnikov E(Kernel obs) = 282
Bandwidth: cross-validation R-squared = 0.4380
```

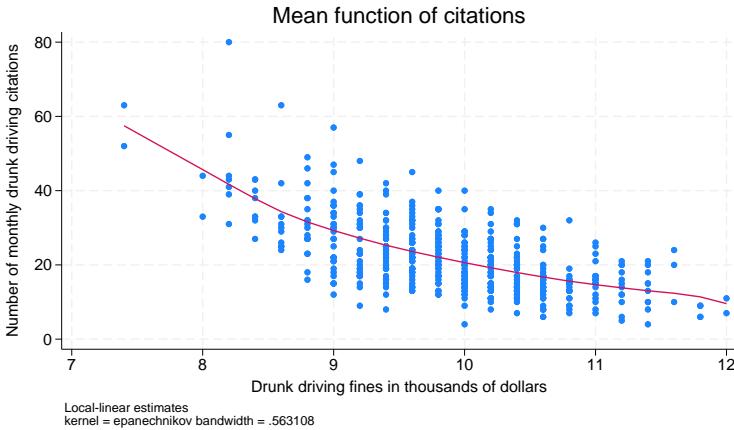
|                | Observed estimate | Bootstrap std. err. | z      | P> z  | Percentile [95% conf. interval] |           |
|----------------|-------------------|---------------------|--------|-------|---------------------------------|-----------|
| Mean citations | 22.33999          | .4588298            | 48.69  | 0.000 | 21.48622                        | 23.35956  |
| Effect fines   | -7.692388         | .491884             | -15.64 | 0.000 | -8.693068                       | -6.757721 |

Note: Effect estimates are averages of derivatives.

The coefficient table now reports the average of the predicted means and the average of the predicted derivatives of the mean function with bootstrap standard errors. The average of the observation-level predicted (`citations`) is 22.34. The average of the observation-level marginal effects is  $-7.69$ , which indicates that increasing `fines` reduces the mean number of citations.

We use `npggraph` to graph the estimated conditional mean function.

```
. npggraph
```



The graph shows the negative association between fines and the number of drunk driving citations. ◀

`npregress kernel` generates system variables for the mean function and the derivative of the mean function. To see the variables that `npregress kernel` generated for [example 1](#), we type

```
. describe *_*, fullnames
```

| Variable name                          | Storage type | Display format | Value label | Variable label                          |
|----------------------------------------|--------------|----------------|-------------|-----------------------------------------|
| <code>_Mean_citations</code>           | double       | %10.0g         |             | Mean function                           |
| <code>_d_Mean_citations_d fines</code> | double       | %10.0g         |             | derivative of mean function w.r.t fines |

To specify a name for each system variable, we can use the `predict()` option.

```
. npregress kernel citations fines, predict(mean deriv)
(output omitted)
```

```
. describe mean deriv
```

| Variable name      | Storage type | Display format | Value label | Variable label                          |
|--------------------|--------------|----------------|-------------|-----------------------------------------|
| <code>mean</code>  | double       | %10.0g         |             | Mean function                           |
| <code>deriv</code> | double       | %10.0g         |             | derivative of mean function w.r.t fines |

Alternatively, we can use the same *stub* for all the variable names by typing `predict(hatvar*)`, which would generate variables `hatvar1` and `hatvar2`.

You may add `noderivatives` to the option, as in `predict(hatvar*, noderivatives)`, to specify that no derivatives be generated. You save memory when you use `noderivatives`, but you add to the computational burden. As you will see below, an important feature of `npregress kernel` is the availability of the `margins` command after estimation. `margins` must compute the derivatives and their optimal bandwidth.

▷ Example 3: Selecting the number of bootstrap replications

We start by fitting the model using 200 bootstrap replications. We want to find the number of replications for which the confidence intervals do not change much.

```
. nprogress kernel citations fines, reps(200) seed(12)
(running nprogress on estimation sample)
Bootstrap replications (200):10.....20.....30.....40.....
>50.....60.....70.....80.....90.....100.....110...
>120.....130.....140.....150.....160.....170.....
> .180.....190.....200 done
```

Bandwidth

|       | Mean     | Effect  |
|-------|----------|---------|
| fines | .5631079 | .924924 |

```
Local-linear regression Number of obs = 500
Kernel : epanechnikov E(Kernel obs) = 282
Bandwidth: cross-validation R-squared = 0.4380
```

| citations      | Observed estimate | Bootstrap std. err. | z      | P> z  | Percentile [95% conf. interval] |          |
|----------------|-------------------|---------------------|--------|-------|---------------------------------|----------|
| Mean citations | 22.33999          | .4769389            | 46.84  | 0.000 | 21.49744                        | 23.42156 |
| Effect fines   | -7.692388         | .5088819            | -15.12 | 0.000 | -8.742081                       | -6.77816 |

Note: Effect estimates are averages of derivatives.

For 200 replications, the confidence interval for the mean ranges from 21.50 to 23.42. For the effect of fines, this range is  $-8.74$  to  $-6.78$ .

We repeat the estimation using 300 replications and the same seed as in the previous case.

```
. nprogress kernel citations fines, reps(300) seed(12)
(running nprogress on estimation sample)
Bootstrap replications (300):10.....20.....30.....40.....
>50.....60.....70.....80.....90.....100.....110...
>120.....130.....140.....150.....160.....170.....
> .180.....190.....200.....210.....220.....230.....240...
>250.....260.....270.....280.....290.....300 done
```

Bandwidth

|       | Mean     | Effect  |
|-------|----------|---------|
| fines | .5631079 | .924924 |

```
Local-linear regression Number of obs = 500
Kernel : epanechnikov E(Kernel obs) = 282
Bandwidth: cross-validation R-squared = 0.4380
```

| citations      | Observed estimate | Bootstrap std. err. | z      | P> z  | Percentile [95% conf. interval] |           |
|----------------|-------------------|---------------------|--------|-------|---------------------------------|-----------|
| Mean citations | 22.33999          | .4570611            | 48.88  | 0.000 | 21.49359                        | 23.36299  |
| Effect fines   | -7.692388         | .4981956            | -15.44 | 0.000 | -8.673813                       | -6.720508 |

Note: Effect estimates are averages of derivatives.

The confidence interval for the mean ranges from 21.49 to 23.36. For the effect of fines, this range is -8.67 to -6.72. There are some differences so we try estimation with 400 replications.

```
. npregress kernel citations fines, reps(400) seed(12)
(running npregress on estimation sample)
Bootstrap replications (400):10.....20.....30.....40.....
>50.....60.....70.....80.....90.....100.....110....
>120.....130.....140.....150.....160.....170.....
>180.....190.....200.....210.....220.....230.....240.
>250.....260.....270.....280.....290.....300.....
>310.....320.....330.....340.....350.....360.....37
> 0.....380.....390.....400 done
```

Bandwidth

|       | Mean     | Effect  |
|-------|----------|---------|
| fines | .5631079 | .924924 |

```
Local-linear regression Number of obs = 500
Kernel : epanechnikov E(Kernel obs) = 282
Bandwidth: cross-validation R-squared = 0.4380
```

| citations      | Observed estimate | Bootstrap std. err. | z      | P> z  | Percentile [95% conf. interval] |           |
|----------------|-------------------|---------------------|--------|-------|---------------------------------|-----------|
| Mean citations | 22.33999          | .4588298            | 48.69  | 0.000 | 21.48622                        | 23.35956  |
| Effect fines   | -7.692388         | .491884             | -15.64 | 0.000 | -8.693068                       | -6.757721 |

Note: Effect estimates are averages of derivatives.

The confidence interval for the mean ranges from 21.49 to 23.36. In the case of the effect of fines, these ranges are -8.69 to -6.76. The changes are small so we decide to use 400 replications.

▷ Example 4: Estimating the effect of a percentage change in a covariate

Nonparametric estimation and the bootstrap are computationally intensive, so we use only 200 replications here.

We now extend [example 2](#). In addition to `finest`, we model `citations` as a function of whether the jurisdiction taxes alcoholic beverages (`taxes`); whether the city is small, medium, or large (`csize`); and whether there is a college in the jurisdiction (`college`).

```
. nprogress kernel citations finest i.taxes i.csize i.college, nolog
> reps(200) seed(12)
(running nprogress on estimation sample)
Bootstrap replications (200):10.....20.....30.....40.....
>50.....60.....70.....80.....90.....100.....110...
>120.....130.....140.....150.....160.....170.....
>180.....190.....200 done
```

Bandwidth

|                      | Mean     | Effect   |
|----------------------|----------|----------|
| <code>finest</code>  | .4471373 | .6537197 |
| <code>taxes</code>   | .4375656 | .4375656 |
| <code>csize</code>   | .3938759 | .3938759 |
| <code>college</code> | .554583  | .554583  |

```
Local-linear regression Number of obs = 500
Continuous kernel : epanechnikov E(Kernel obs) = 224
Discrete kernel : liracine R-squared = 0.8010
Bandwidth : cross-validation
```

| citations                                          | Observed estimate | Bootstrap std. err. | z      | P> z  | Percentile [95% conf. interval] |           |
|----------------------------------------------------|-------------------|---------------------|--------|-------|---------------------------------|-----------|
| Mean citations                                     | 22.26306          | .4642464            | 47.96  | 0.000 | 21.46204                        | 23.2516   |
| Effect                                             |                   |                     |        |       |                                 |           |
| <code>finest</code>                                | -7.332833         | .3316656            | -22.11 | 0.000 | -8.013487                       | -6.741899 |
| <code>taxes</code><br>(Tax vs<br>No tax)           | -4.502718         | .5012               | -8.98  | 0.000 | -5.437733                       | -3.544934 |
| <code>csize</code><br>(Medium vs<br>Small)         | 5.300524          | .2687413            | 19.72  | 0.000 | 4.758121                        | 5.797119  |
| (Large vs<br>Small)                                | 11.05053          | .502633             | 21.99  | 0.000 | 10.00169                        | 11.94311  |
| <code>college</code><br>(College vs<br>Not coll..) | 5.953188          | .461057             | 12.91  | 0.000 | 5.086511                        | 6.88612   |

Note: Effect estimates are averages of derivatives for continuous covariates and averages of contrasts for factor covariates.

The mean number of citations predicted by the mean estimates is 22.26. The average marginal effect of fines is  $-7.33$ , slightly less in magnitude than the  $-7.69$  that we estimated in [example 2](#).

The average marginal effect tells us the result of an infinitesimal change in fines on citations. Instead of talking about infinitesimal changes, we want to know the effect of increasing fines by 15%. We can use margins to estimate the mean number of citations that would occur if fines were increased by 15%.

```
. margins, at(fines=generate(fines*1.15)) reps(200) seed(12)
(running margins on estimation sample)
Bootstrap replications (200):10.....20.....30.....40.....
>50.....60.....70.....80.....90.....100.....110...
>120.....130.....140.....150.....160.....170.....
> .180.....190.....200 done
Predictive margins Number of obs = 500
 Replications = 200
Expression: Mean function, predict()
At: fines = fines*1.15
```

|       | Observed margin | Bootstrap std. err. | z     | P> z  | Percentile [95% conf. interval] |          |
|-------|-----------------|---------------------|-------|-------|---------------------------------|----------|
| _cons | 14.00818        | .866694             | 16.16 | 0.000 | 11.39967                        | 15.00145 |

The estimated mean number of citations with the new level of fines is 14.01, which is smaller than the mean 22.26 that was estimated with the observed fines. We can formally compare this estimate with the mean at the original level of fines. We use the contrast() option with margins to estimate the difference in these means.

```
. margins, at(fines=generate(fines)) at(fines=generate(fines*1.15))
> contrast(atcontrast(r) nowald) reps(200) seed(12)
(running margins on estimation sample)
Bootstrap replications (200):10.....20.....30.....40.....
>50.....60.....70.....80.....90.....100.....110...
>120.....130.....140.....150.....160.....170.....
> .180.....190.....200 done
Contrasts of predictive margins Number of obs = 500
 Replications = 200
Expression: Mean function, predict()
1._at: fines = fines
2._at: fines = fines*1.15
```

|                 | Observed contrast | Bootstrap std. err. | Percentile [95% conf. interval] |           |
|-----------------|-------------------|---------------------|---------------------------------|-----------|
| _at<br>(2 vs 1) | -8.254875         | .8058215            | -10.44121                       | -7.381583 |

We find that increasing fines by 15% reduces the average number of drunk driving citations by 8.25.



▷ Example 5: Estimating the effect of a change in level

Now, we estimate the effect of increasing `fin`es from \$10,000 to \$11,000 for fixed levels of the other covariates. The other covariate values identify a jurisdiction with a set of characteristics of interest: of medium size, with a college, and taxes alcohol.

First, we use `margins` to estimate the means for a jurisdiction with the characteristics of interest for the two levels of `fin`es.

```
. margins, at(fin=10 taxes=1 csize=2 college=1)
> at(fin=11 taxes=1 csize=2 college=1) reps(200) seed(12)
(running margins on estimation sample)

Bootstrap replications (200):10.....20.....30.....40.....
>50.....60.....70.....80.....90.....100.....110...
>120.....130.....140.....150.....160.....170.....
> .180.....190.....200 done

Adjusted predictions Number of obs = 500
 Replications = 200

Expression: Mean function, predict()
1._at: fin = 10
 taxes = 1
 csize = 2
 college = 1
2._at: fin = 11
 taxes = 1
 csize = 2
 college = 1
```

|     | Observed margin | Bootstrap std. err. | z     | P> z  | Percentile [95% conf. interval] |          |
|-----|-----------------|---------------------|-------|-------|---------------------------------|----------|
| _at |                 |                     |       |       |                                 |          |
| 1   | 23.17242        | .5746008            | 40.33 | 0.000 | 21.95222                        | 24.30412 |
| 2   | 15.90157        | .972558             | 16.35 | 0.000 | 13.87449                        | 17.7134  |

For a medium-sized jurisdiction that taxes alcohol and has a college, the estimated mean of citations when `fin`es are \$10,000 is 23.17, and the estimated mean of citations when `fin`es are \$11,000 is 15.90.

We now use margins to estimate the difference in these means.

```
. margins, at(fines=10 taxes=1 csize=2 college=1)
> at(fines=11 taxes=1 csize=2 college=1)
> contrast(atcontrast(r) nowald) reps(200) seed(12)
(running margins on estimation sample)

Bootstrap replications (200):10.....20.....30.....40.....
>50.....60.....70.....80.....90.....100.....110.....
>120.....130.....140.....150.....160.....170.....
> .180.....190.....200 done
Contrasts of predictive margins
```

Number of obs = 500  
Replications = 200

Expression: Mean function, predict()

```
1._at: fines = 10
 taxes = 1
 csize = 2
 college = 1
2._at: fines = 11
 taxes = 1
 csize = 2
 college = 1
```

|                 | Observed contrast | Bootstrap std. err. | Percentile [95% conf. interval] |           |
|-----------------|-------------------|---------------------|---------------------------------|-----------|
| _at<br>(2 vs 1) | -7.270858         | 1.003861            | -9.096777                       | -5.162513 |

In these jurisdictions, increasing fines from \$10,000 to \$11,000 reduces the average number of citations by 7.27.



### ➤ Example 6: Population-averaged covariate effects

In [example 5](#), we estimated the means for two values of fines for a medium-sized jurisdiction with a college and taxes on alcohol. We specified values for each covariate in our model. In this example, we will now estimate population-averaged means instead of means at specific levels of all covariates.

We first estimate the means for two levels of fines. We do not specify values for `csize`, `college`, or `taxes`, so the estimated means are unconditional on these covariates. We use `margins` to estimate means of citations when fines are \$10,000 and when fines are \$11,000:

```
. margins, at(fines=10) at(fines=11) reps(200) seed(12)
(running margins on estimation sample)
Bootstrap replications (200):10.....20.....30.....40.....
>50.....60.....70.....80.....90.....100.....110...
>120.....130.....140.....150.....160.....170.....
> .180.....190.....200 done
Predictive margins Number of obs = 500
 Replications = 200
Expression: Mean function, predict()
1._at: fines = 10
2._at: fines = 11
```

|     | Observed margin | Bootstrap std. err. | z     | P> z  | Percentile [95% conf. interval] |          |
|-----|-----------------|---------------------|-------|-------|---------------------------------|----------|
| _at |                 |                     |       |       |                                 |          |
| 1   | 20.50161        | .3281821            | 62.47 | 0.000 | 19.90257                        | 21.08954 |
| 2   | 14.97432        | .3815647            | 39.24 | 0.000 | 14.14858                        | 15.59955 |

The estimated mean of citations when fines are \$10,000 is 20.50, and the estimated mean of citations when fines are \$11,000 is 14.97. We now use margins to estimate the difference in these means:

```
. margins, at(fines=10) at(fines=11)
> contrast(atcontrast(r) nowald) reps(200) seed(12)
(running margins on estimation sample)
Bootstrap replications (200):10.....20.....30.....40.....
>50.....60.....70.....80.....90.....100.....110...
>120.....130.....140.....150.....160.....170.....
> .180.....190.....200 done
Contrasts of predictive margins Number of obs = 500
 Replications = 200
Expression: Mean function, predict()
1._at: fines = 10
2._at: fines = 11
```

|                 | Observed contrast | Bootstrap std. err. | Percentile [95% conf. interval] |           |
|-----------------|-------------------|---------------------|---------------------------------|-----------|
| _at<br>(2 vs 1) | -5.527288         | .3529352            | -6.277903                       | -4.925523 |

When fines increase from \$10,000 to \$11,000, the mean number of citations is estimated to decrease by 5.53.

Next, we consider the effect of taxing alcoholic beverages. We first estimate the population-averaged number of citations with and without such taxes.

```
. margins taxes, reps(200) seed(12)
(running margins on estimation sample)
Bootstrap replications (200):10.....20.....30.....40.....
>50.....60.....70.....80.....90.....100.....110...
>120.....130.....140.....150.....160.....170.....
> .180.....190.....200 done
Predictive margins Number of obs = 500
 Replications = 200
Expression: Mean function, predict()
```

|        | Observed margin | Bootstrap std. err. | z     | P> z  | Percentile [95% conf. interval] |          |
|--------|-----------------|---------------------|-------|-------|---------------------------------|----------|
| taxes  |                 |                     |       |       |                                 |          |
| No tax | 25.47052        | .6445729            | 39.52 | 0.000 | 24.17515                        | 26.6114  |
| Tax    | 20.96781        | .4448277            | 47.14 | 0.000 | 20.17071                        | 21.88565 |

The estimated mean number of citations is 25.47 when there are no alcohol taxes and 20.97 when there are alcohol taxes. We again use margins to estimate the difference in these means.

```
. margins r.taxes, reps(200) seed(12)
(running margins on estimation sample)
Bootstrap replications (200):10.....20.....30.....40.....
>50.....60.....70.....80.....90.....100.....110...
>120.....130.....140.....150.....160.....170.....
> .180.....190.....200 done
Contrasts of predictive margins Number of obs = 500
 Replications = 200
Expression: Mean function, predict()
```

|       | df | chi2  | P>chi2 |
|-------|----|-------|--------|
| taxes | 1  | 80.71 | 0.0000 |

|                          | Observed contrast | Bootstrap std. err. | Percentile [95% conf. interval] |           |
|--------------------------|-------------------|---------------------|---------------------------------|-----------|
| taxes<br>(Tax vs No tax) | -4.502719         | .5011999            | -5.437733                       | -3.544934 |

The mean number of citations is estimated to decrease by 4.50 when alcohol sales are taxed.

## Visualizing covariate effects

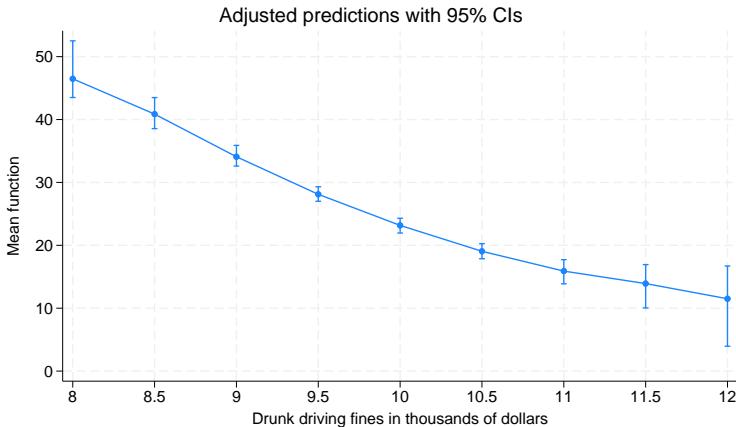
### ▷ Example 7: Using margins to visualize the mean function and covariate effects

We can also estimate the mean function for the jurisdiction with characteristics of interest over a range of observed fines. We simply add a range of fines to our margins specification from [example 4](#).

```
. margins, at(fines=(8(0.5)12) taxes=1 csize=2 college=1) reps(200) seed(12)
(output omitted)
```

We graph these results using `marginsplot`.

```
. marginsplot
(output omitted)
```



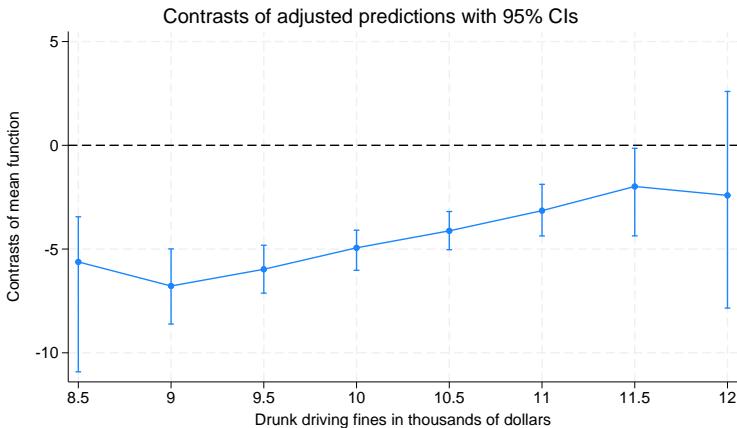
We estimated the mean when fines are \$8,000, \$8,500, and so on. From these estimated means, we can estimate the effect of a \$500 increase for each of these levels of fines.

We simply reissue our `margins` command and specify a reverse adjacent contrast that subtracts the current level from the next level for each level of fines.

```
. margins, at(fines=(8(0.5)12) taxes=1 csize=2 college=1)
> contrast(atcontrast(ar)) reps(200) seed(12)
(output omitted)
```

We again graph the results, adding a reference line at 0 that designates no change in citations:

```
. marginsplot, yline(0)
 (output omitted)
```



For each level of fines between \$8,500 and \$11,500, the effect of a \$500 increase reduces the mean number of drunk driving incidents. Between \$11,500 and \$12,000, the difference of a \$500 increase is not statistically different from 0.

It would be easy to construct a similar graph for the population-averaged effects in [example 6](#). Simply omit the terms that set the other covariates at fixed values.

## Stored results

npregress kernel stores the following in `e()`:

### Scalars

|                                  |                                                          |
|----------------------------------|----------------------------------------------------------|
| <code>e(N)</code>                | number of observations                                   |
| <code>e(mean)</code>             | mean of mean function                                    |
| <code>e(r2)</code>               | $R^2$                                                    |
| <code>e(nh)</code>               | expected kernel observations                             |
| <code>e(converged_effect)</code> | 1 if effect optimization converged, 0 otherwise          |
| <code>e(converged_mean)</code>   | 1 if mean optimization converged, 0 otherwise            |
| <code>e(converged)</code>        | 1 if effect and mean optimization converged, 0 otherwise |

### Macros

|                                   |                                                |
|-----------------------------------|------------------------------------------------|
| <code>e(cmd)</code>               | npregress                                      |
| <code>e(cmdline)</code>           | command as typed                               |
| <code>e(depvar)</code>            | name of dependent variable                     |
| <code>e(estimator)</code>         | linear or constant                             |
| <code>e(kname)</code>             | name of continuous kernel                      |
| <code>e(dkname)</code>            | name of discrete kernel                        |
| <code>e(bselector)</code>         | criterion function for bandwidth selection     |
| <code>e(title)</code>             | title in estimation output                     |
| <code>e(vce)</code>               | <i>vcetype</i> specified in <code>vce()</code> |
| <code>e(properties)</code>        | b (or b V if <code>reps()</code> specified)    |
| <code>e(datasignaturevars)</code> | variables used in calculation of checksum      |
| <code>e(datasignature)</code>     | the checksum                                   |
| <code>e(estat_cmd)</code>         | program used to implement <code>estat</code>   |
| <code>e(predict)</code>           | program used to implement <code>predict</code> |
| <code>e(marginsok)</code>         | predictions allowed by <code>margins</code>    |
| <code>e(marginsprop)</code>       | signals to the <code>margins</code> command    |

### Matrices

|                             |                                                 |
|-----------------------------|-------------------------------------------------|
| <code>e(b)</code>           | coefficient vector                              |
| <code>e(bwidth)</code>      | bandwidth for all predictions                   |
| <code>e(derivbwidth)</code> | bandwidth for the derivative                    |
| <code>e(meanbwidth)</code>  | bandwidth for the mean                          |
| <code>e(ilog_mean)</code>   | iteration log for mean (up to 20 iterations)    |
| <code>e(ilog_effect)</code> | iteration log for effects (up to 20 iterations) |

### Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

### Matrices

|                       |                                                                                                                            |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

The regression model of outcome  $y_i$  given the  $k$ -dimensional vector of covariates  $\mathbf{x}_i$  was defined in (1) and (2) of *Remarks and examples* and repeated here:

$$y_i = g(\mathbf{x}_i) + \varepsilon_i \tag{4}$$

$$E(\varepsilon_i | \mathbf{x}_i) = 0 \tag{5}$$

where  $\varepsilon_i$  is the error term. The covariates may include discrete and continuous variables. Equations (4) and (5) imply that

$$E(y_i | \mathbf{x}_i) = g(\mathbf{x}_i)$$

npregress kernel, by default, estimates a local-linear regression; see [Fan and Gijbels \(1996\)](#) for a good reference on local-linear regression. As we discussed in *Remarks and examples*, local-linear regression estimates a regression for a subset of observations for each point in our data and solves the minimization problem given by

$$\min_{\boldsymbol{\gamma}} \sum_{i=1}^n \{y_i - \gamma_0 - \boldsymbol{\gamma}'_1 (\mathbf{x}_i - \mathbf{x})\}^2 K(\mathbf{x}_i, \mathbf{x}, \mathbf{h}) \tag{6}$$

where  $\boldsymbol{\gamma} = (\gamma_0, \boldsymbol{\gamma}'_1)'$  and  $K(\mathbf{x}_i, \mathbf{x}, \mathbf{h})$  is the product of the kernels for each covariate.

$$K(\mathbf{x}_i, \mathbf{x}, \mathbf{h}) = \prod_{j=1}^k K_j(x_{ij}, x_j, h_j)$$

The kernel for a continuous covariate is of the form

$$K_j(x_{ij}, x_j, h_j) = k_j \left( \frac{x_{ij} - x_j}{h_j} \right)$$

where  $k_j(\cdot)$  is one of the [kernels](#) listed in [\[R\] kdensity](#). For discrete covariates, npregress kernel uses the Li–Racine kernel given by

$$K_j(x_{ij}, x_j, h_j) = \begin{cases} 1 & \text{if } x_{ij} = x_j \\ h_j & \text{otherwise} \end{cases}$$

By estimating  $E(y_i | \mathbf{x}_i = \mathbf{x})$  for all points  $\mathbf{x}$  in our data, we obtain an estimate of  $E(y_i | \mathbf{x}_i)$ . For a given  $\mathbf{x}$ , the solution to the minimization problem in (6) is given by

$$\widehat{\boldsymbol{\gamma}} = (\mathbf{Z}'\mathbf{W}\mathbf{Z})^{-1} \mathbf{Z}'\mathbf{W}\mathbf{y}$$

where  $\widehat{\boldsymbol{\gamma}} = (\widehat{\gamma}_0, \widehat{\boldsymbol{\gamma}}'_1)'$ ,  $\mathbf{Z}$  is an  $n \times (k + 1)$  matrix with an  $i$ th row given by  $\{1, (\mathbf{x}_i - \mathbf{x})'\}$ ,  $\mathbf{W}$  is an  $n \times n$  diagonal matrix with an  $i$ th diagonal given by  $K(\mathbf{x}_i, \mathbf{x}, \mathbf{h})$ , and  $\mathbf{y}$  is the  $n \times 1$  outcome vector.  $\widehat{\gamma}_0$  is an estimate of  $g(\mathbf{x})$ , whereas  $\widehat{\boldsymbol{\gamma}}_1$  is an estimate of the derivative of  $g(\mathbf{x})$  with respect to  $\mathbf{x}$ . When the matrix  $(\mathbf{Z}'\mathbf{W}\mathbf{Z})$  is not full rank, the parameter  $\boldsymbol{\gamma}$  is not identified. The observations for which this is true are dropped from the estimation sample.

The local-constant estimator of  $g(\mathbf{x})$  is a special case of (6) with  $\gamma_1 = \mathbf{0}$ . In this case, the solution to the optimization problem is given by

$$\frac{\sum_{i=1}^n y_i K(\mathbf{x}_i, \mathbf{x}, \mathbf{h})}{\sum_{i=1}^n K(\mathbf{x}_i, \mathbf{x}, \mathbf{h})}$$

This is also known as the Nadaraya–Watson kernel estimator, for [Nadaraya \(1965\)](#) and [Watson \(1964\)](#).

`npregress kernel` and `margins`, when used after `npregress kernel`, use a bootstrap estimate of the standard errors for all the estimated effects and report percentile confidence intervals by default. [Cattaneo and Jansson \(2018\)](#) formally justify this use of the bootstrap and provide a definitive reference for semiparametric estimation and inference using kernel-based estimators. Their work demonstrates that the percentile bootstrap provides better coverage than a normal-based confidence interval for statistics based on kernel estimates. See *Methods and formulas* in `[R] bootstrap` for confidence interval formulas.

The rate of convergence of nonparametric regression estimates is given by the product of the sample size and the bandwidths  $\sqrt{n|\mathbf{h}|}$ , where  $|\mathbf{h}|$  is the product of the bandwidths for each covariate. As the sample size increases, the bandwidth decreases. Thus, the rate of convergence of the estimator is slower than the parametric rate  $\sqrt{n}$ . Another way of thinking about  $n|\mathbf{h}|$  is that, because we are not using all our observations to estimate the mean at each point, we require more data to get more reliable estimates; the convergence rate is thus slower. The rate of convergence also decreases as the number of covariates increases, because  $|\mathbf{h}|$  decreases. This is referred to as the curse of dimensionality; see [Li and Racine \(2007, chap. 2\)](#) and [Stinchcombe and Drukker \(2013\)](#) for details.

The convergence rate for the derivative of the mean function is different from the convergence rate of the mean function. Therefore, the bandwidth and bandwidth computation for the derivative are different. `npregress kernel` computes the bandwidth for the derivative function by using cross-validation, as suggested by [Henderson et al. \(2015\)](#).

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## Also see

[R] **npregress kernel postestimation** — Postestimation tools for npregress kernel

[R] **npregress intro** — Introduction to nonparametric regression

[R] **kdensity** — Univariate kernel density estimation

[R] **lpoly** — Kernel-weighted local polynomial smoothing

[U] **20 Estimation and postestimation commands**

Postestimation commands  
[npgraph](#)  
[References](#)

[predict](#)  
 Remarks and examples  
 Also see

[margins](#)  
 Methods and formulas

## Postestimation commands

The following postestimation command is of special interest after `nprogress kernel`:

| Command                 | Description               |
|-------------------------|---------------------------|
| <a href="#">npgraph</a> | plot of conditional means |

The following standard postestimation commands are also available:

| Command                         | Description                                                                                       |
|---------------------------------|---------------------------------------------------------------------------------------------------|
| <a href="#">estat summarize</a> | summary statistics for the estimation sample                                                      |
| <a href="#">estat vce</a>       | variance–covariance matrix of the estimators (VCE)                                                |
| <a href="#">estimates</a>       | cataloging estimation results                                                                     |
| <a href="#">etable</a>          | table of estimation results                                                                       |
| <a href="#">lincom</a>          | point estimates, standard errors, testing, and inference for linear combinations of parameters    |
| <a href="#">margins</a>         | marginal means, predictive margins, marginal effects, and average marginal effects                |
| <a href="#">marginsplot</a>     | graph the results from margins (profile plots, interaction plots, etc.)                           |
| <a href="#">nlcom</a>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters |
| <a href="#">predict</a>         | conditional means and residuals                                                                   |
| <a href="#">predictnl</a>       | point estimates, standard errors, testing, and inference for generalized predictions              |
| <a href="#">test</a>            | Wald tests of simple and composite linear hypotheses                                              |
| <a href="#">testnl</a>          | Wald tests of nonlinear hypotheses                                                                |

# predict

## Description for predict

`predict` creates a new variable containing predictions such as conditional mean of the outcome, residuals, or derivatives of the mean function.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic]
```

```
predict [type] { stub* | newvarlist } [if] [in] , derivatives
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

---

Main

|                        |                                              |
|------------------------|----------------------------------------------|
| <code>mean</code>      | conditional mean of the outcome; the default |
| <code>residuals</code> | residuals                                    |

---

These statistics are calculated only for the estimation sample.

## Options for predict

Main

`mean`, the default, calculates the conditional mean of the outcome variable.

`residuals` calculates the residuals.

`derivatives` calculates the derivatives of the conditional mean.

## margins

### Description for margins

`margins` estimates margins of the conditional mean.

### Menu for margins

Statistics > Postestimation

### Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [options]
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

Main

|                                 |                                              |
|---------------------------------|----------------------------------------------|
| <code>mean</code>               | conditional mean of the outcome; the default |
| <code><u>r</u>esiduals</code>   | not allowed with margins                     |
| <code><u>d</u>erivatives</code> | not allowed with margins                     |

| <i>options</i> | Description |
|----------------|-------------|
|----------------|-------------|

SE

|                             |                                                                     |
|-----------------------------|---------------------------------------------------------------------|
| <code>nose</code>           | do not estimate standard errors; the default                        |
| <code>vce(bootstrap)</code> | estimate bootstrap standard errors                                  |
| <code><u>r</u>eps(#)</code> | equivalent to <code>vce(bootstrap, reps(#))</code>                  |
| <code>seed(#)</code>        | set random-number seed to #; must also specify <code>reps(#)</code> |

Reporting

|                                           |                                                                                                     |
|-------------------------------------------|-----------------------------------------------------------------------------------------------------|
| <code><u>c</u>itype(<i>citype</i>)</code> | method to compute bootstrap confidence intervals; default is <code><u>c</u>itype(percentile)</code> |
|-------------------------------------------|-----------------------------------------------------------------------------------------------------|

| <i>citype</i> | Description |
|---------------|-------------|
|---------------|-------------|

|                                |                                              |
|--------------------------------|----------------------------------------------|
| <code><u>p</u>ercentile</code> | percentile confidence intervals; the default |
| <code>bc</code>                | bias-corrected confidence intervals          |
| <code><u>n</u>ormal</code>     | normal-based confidence intervals            |

Statistics not allowed with `margins` are functions of stochastic quantities other than `e(b)`.

For the full syntax, see [\[R\] margins](#).

## Options for margins

### SE

`nose` suppresses calculation of the VCE and standard errors. This is the default.

`vce(bootstrap)` specifies that bootstrap standard errors are reported; see [R] [vce\\_option](#).

We recommend that you select the number of replications using `reps(#)` instead of specifying `vce(bootstrap)`, which defaults to 50 replications. Be aware that the number of replications needed to produce good estimates of the standard errors varies depending on the problem.

`reps(#)` specifies the number of bootstrap replications to be performed. Specifying this option is equivalent to specifying `vce(bootstrap, reps(#))`.

`seed(#)` sets the random-number seed. You must specify `reps(#)` with `seed(#)`.

### Reporting

`ci(type(citype))` specifies the type of confidence interval to be computed. By default, bootstrap percentile confidence intervals are reported as recommended by [Cattaneo and Jansson \(2018\)](#). *citype* may be one of `percentile`, `bc`, or `normal`.

## npgraph

### Description for npgraph

`npgraph` plots the conditional mean estimated by `npregress kernel` overlaid on a scatterplot of the data. `npgraph` is available only after fitting models with one covariate.

### Syntax for npgraph

```
npgraph [if] [in] [, options]
```

| <i>options</i>                                 | Description                                                                    |
|------------------------------------------------|--------------------------------------------------------------------------------|
| Plot                                           |                                                                                |
| <a href="#">marker_options</a>                 | change look of markers (color, size, etc.)                                     |
| <a href="#">marker_label_options</a>           | add marker labels; change look or position                                     |
| <code>nosscatter</code>                        | suppress scatterplot                                                           |
| Smoothed line                                  |                                                                                |
| <a href="#">lineopts(<i>cline_options</i>)</a> | affect rendition of the smoothed line                                          |
| Add plots                                      |                                                                                |
| <code>addplot(<i>plot</i>)</code>              | add other plots to the generated graph                                         |
| Y axis, X axis, Titles, Legend, Overall        |                                                                                |
| <a href="#">twoway_options</a>                 | any options other than by() documented in [G-3] <a href="#">twoway_options</a> |

## Options for npregraph

### Plot

*marker\_options* affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] *marker\_options*.

*marker\_label\_options* specify if and how the markers are to be labeled; see [G-3] *marker\_label\_options*.

*noscatter* suppresses superimposing a scatterplot of the observed data over the smooth. This option is useful when the number of resulting points would be so large as to clutter the graph.

### Smoothed line

*lineopts* (*cline\_options*) affects the rendition of the smoothed line; see [G-3] *cline\_options*.

### Add plots

*addplot* (*plot*) provides a way to add other plots to the generated graph; see [G-3] *addplot\_option*.

### Y axis, X axis, Titles, Legend, Overall

*twoway\_options* are any of the options documented in [G-3] *twoway\_options*, excluding *by()*. These include options for titling the graph (see [G-3] *title\_options*) and for saving the graph to disk (see [G-3] *saving\_option*).

## Remarks and examples

For examples of margins after *npregress kernel*, see [example 4](#), [example 5](#), and [example 6](#) in [R] *npregress kernel*.

For examples of *marginsplot*, see [example 7](#) in [R] *npregress kernel*.

For an example of *npregraph*, see [example 2](#) in [R] *npregress kernel*.

## Methods and formulas

The formulas used by *predict* and *margins* for the conditional mean function and the mean marginal effect of a covariate are given in *Methods and formulas* of [R] *npregress kernel*.

## References

Cattaneo, M. D., and M. Jansson. 2018. Kernel-based semiparametric estimators: Small bandwidth asymptotics and bootstrap consistency. *Econometrica* 86: 955–995. <https://doi.org/10.3982/ECTA12701>.

MacDonald, K. 2018. Exploring results of nonparametric regression models. *The Stata Blog: Not Elsewhere Classified*. <https://blog.stata.com/2018/06/18/exploring-results-of-nonparametric-regression-models/>.

## Also see

[R] *npregress kernel* — Nonparametric kernel regression

[R] *bootstrap postestimation* — Postestimation tools for bootstrap

[U] [20 Estimation and postestimation commands](#)

[Description](#)  
[Options](#)  
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[Menu](#)  
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## Description

`nprogress series` performs nonparametric series estimation using a B-spline, piecewise polynomial spline, or polynomial basis. Like linear regression, nonparametric regression models the mean of the outcome conditional on the covariates, but unlike linear regression, it makes no assumptions about the functional form of the relationship between the outcome and the covariates. `nprogress series` may be used to model the mean of a continuous, count, or binary outcome.

## Quick start

Nonparametric regression of  $y$  on  $x$  and discrete covariate  $a$  using the default B-spline basis

```
nprogress series y x i.a
```

Same as above, but use a polynomial basis

```
nprogress series y x i.a, polynomial
```

Same as above, but use a piecewise polynomial spline basis

```
nprogress series y x i.a, spline
```

Same as above, but use AIC to find the optimal basis function

```
nprogress series y x i.a, criterion(aic) spline
```

Interpolate using three knots

```
nprogress series y x i.a, knots(3)
```

Specify values of knots in matrix  $K$

```
nprogress series y x i.a, knotsmat(K)
```

## Menu

Statistics > Nonparametric analysis > Nonparametric series regression

## Syntax

```
npregress series depvar indepvarsseries [if] [in] [weight] [, options]
```

*indepvars*<sub>series</sub> is the list of independent variables for which a basis function will be formed.

| <i>options</i>                                       | Description                                                                                                                                      |
|------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                         |                                                                                                                                                  |
| <code>bspline</code>                                 | use a third-order B-spline basis; the default                                                                                                    |
| <code>bspline(#)</code>                              | use a B-spline basis of order #                                                                                                                  |
| <code>spline</code>                                  | use a third-order piecewise polynomial spline basis                                                                                              |
| <code>spline(#)</code>                               | use a piecewise polynomial spline basis of order #                                                                                               |
| <code>polynomial</code>                              | use a polynomial basis                                                                                                                           |
| <code>polynomial(#)</code>                           | use a polynomial basis of order #                                                                                                                |
| <code>asis(<i>varlist</i>)</code>                    | include <i>varlist</i> in model as specified; do not use in basis                                                                                |
| <code>nointeract(<i>seriesvarlist</i>)</code>        | use <i>seriesvarlist</i> in basis without interactions                                                                                           |
| <code>criterion(<i>crittype</i>)</code>              | criterion to use; <i>crittype</i> may be <code>cv</code> , <code>gcv</code> , <code>aic</code> , <code>bic</code> , or <code>mallows</code>      |
| <code>knots(#)</code>                                | use a piecewise polynomial spline or B-spline basis function with # knots                                                                        |
| <code>knotsmat(<i>matname</i>)</code>                | use knots in matrix <i>matname</i> for piecewise polynomial spline or B-spline estimation                                                        |
| <code>distinct(#)</code>                             | minimum number of distinct values allowed in continuous covariates; default is <code>distinct(10)</code>                                         |
| <code>basis(<i>stub</i> [, <i>replace</i>])</code>   | store elements of piecewise polynomial spline or B-spline basis function using <i>stub</i>                                                       |
| <code>rescale(<i>stub</i> [, <i>replace</i>])</code> | store rescaled values of covariates using <i>stub</i>                                                                                            |
| <b>SE</b>                                            |                                                                                                                                                  |
| <code>vce(<i>vcetype</i>)</code>                     | <i>vcetype</i> may be <code>robust</code> , <code>ols</code> , or <code>bootstrap</code>                                                         |
| <b>Reporting</b>                                     |                                                                                                                                                  |
| <code>level(#)</code>                                | set confidence level; default is <code>level(95)</code>                                                                                          |
| <code>aequations</code>                              | display auxiliary regression coefficients                                                                                                        |
| <code>display_options</code>                         | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| <b>Maximization</b>                                  |                                                                                                                                                  |
| <code>maximize_options</code>                        | control the maximization process                                                                                                                 |
| <code>coeflegend</code>                              | display legend instead of statistics                                                                                                             |

*indepvars* and *varlist* may contain factor variables; see [U] 11.4.3 **Factor variables**.

`bayesboot`, `bootstrap`, `by`, `collect`, and `jackknife` are allowed; see [U] 11.1.10 **Prefix commands**.

Weights are not allowed with the `bootstrap` prefix; see [R] **bootstrap**.

`fweights` and `iweights` are allowed; see [U] 11.1.6 **weight**.

`coeflegend` does not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options

### Model

`bspline` specifies that a third-order B-spline be selected. It is the default basis.

`bspline(#)` specifies that a B-spline of order `#` be used as the basis. The order may be 1, 2, or 3.

`spline` specifies that a third-order piecewise polynomial spline be selected as the basis.

`spline(#)` specifies that a piecewise polynomial spline of order `#` be used as the basis. The order may be 1, 2, or 3.

`polynomial` specifies that a polynomial be selected as the basis.

`polynomial(#)` specifies that a polynomial of order `#` be used as the basis. The order may be an integer between 1 and 16.

`asis(varlist)` specifies that variables in *varlist* be included as independent variables in the model without any transformation. No B-spline, piecewise polynomial spline, or polynomial basis function will be formed from these variables. Variables in *varlist* may not be specified in *indepvars*<sub>series</sub>.

`nointeract(seriesvarlist)` specifies that the terms in the basis function formed from variables in *seriesvarlist* not be interacted with the terms of the basis function formed from other variables in *indepvars*<sub>series</sub>. Covariates specified in *seriesvarlist* must be in *indepvars*<sub>series</sub>.

`criterion(crittype)` specifies that *crittype* be used to select the optimal number of terms in the basis function. *crittype* may be one of the following: *cv* (cross-validation), *gcv* (generalized cross-validation), *aic* (Akaike's information criterion), *bic* (Schwarz's Bayesian information criterion), or *mallows* (Mallows's  $C_p$ ). The default is `criterion(cv)`.

`knots(#)` specifies that a piecewise polynomial spline or B-spline basis function with `#` knots be used. The minimum number of knots must be an integer greater than or equal to 1. The maximum number of knots is either 4,096 or two-thirds of the sample size, whichever is smaller.

`knotsmat(matname)` specifies that the knots for each continuous covariate be the values in each row of *matname*. The number of knots should be the same for each covariate, and there must be as many rows as there are continuous covariates. If rows of *matname* are not labeled with *varnames*, then rows are assumed to be in the order of *indepvars*<sub>series</sub>.

`distinct(#)` specifies the minimum number of distinct values allowed in continuous variables. By default, continuous variables that enter the basis through either *indepvars*<sub>series</sub> or *seriesvarlist* are required to have at least 10 distinct values. Continuous variables with few distinct values provide little information for determining an appropriate basis function and may produce unreliable estimates.

`basis(stub [, replace])` specifies that the elements of the basis function generated by `npregress series` be stored with the specified names.

The option argument *stub* is the prefix used to generate enumerated variables for each element of the basis.

When `replace` is used, existing variables named with *stub* are replaced by those from the new computation.

`rescale(stub [, replace])` specifies that the rescaled covariates used to generate the basis function be stored with the specified names.

The option argument *stub* is the prefix used to generate enumerated variable names for the covariates.

When `replace` is used, existing covariates named with *stub* are replaced by those from the new computation.

---

#### SE

`vce(vcetype)` specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (`robust`), that assume homoskedasticity (`ols`), and that use bootstrap methods (`bootstrap`); see [R] [vce\\_option](#).

---

#### Reporting

`level(#)`; see [R] [Estimation options](#).

`aequations` specifies that the auxiliary regression coefficients be reported. By default, only the average marginal effects of the covariates on the outcome are reported.

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no!stretch`; see [R] [Estimation options](#).

---

#### Maximization

`maximize_options`: `iterate(#)`, `[no]log`, `tolerance(#)`; see [R] [Maximize](#). These options are seldom used.

The following option is available with `npregress series` but is not shown in the dialog box:

`coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

This entry assumes that you are already familiar with nonparametric regression. Below, we discuss nonparametric series estimation; see [R] [npregress intro](#) for an overview of nonparametric regression and the models fit by `npregress series` and `npregress kernel`.

Remarks are presented under the following headings:

[Overview](#)  
[Estimation and effects](#)

## Overview

`npregress series` implements nonparametric series estimation using a B-spline, piecewise polynomial spline, or polynomial basis. The covariates may be continuous or discrete. `npregress series` allows you to estimate covariate effects and other counterfactuals related to the unknown mean function after estimation.

The word “nonparametric” refers to the fact that the parameter of interest—the mean as a function of the covariates—is given by the unknown function  $g(\mathbf{x}_i)$ , which is an element of an infinite-dimensional space of functions. In contrast, in a parametric model, the mean for a given value of the covariates,  $E(y_i|\mathbf{x}_i) = f(\mathbf{x}_i, \beta)$ , is a known function that is fully characterized by the parameter of interest,  $\beta$ , which is a finite-dimensional real vector (Shao 2003).

The nonparametric regression model of outcome  $y_i$  given the  $k$ -dimensional vector of covariates  $\mathbf{x}_i$  is given by

$$y_i = g(\mathbf{x}_i) + \varepsilon_i \tag{1}$$

$$E(\varepsilon_i|\mathbf{x}_i) = 0 \tag{2}$$

where  $\varepsilon_i$  is the error term. Equations (1) and (2) imply that

$$E(y_i|\mathbf{x}_i) = g(\mathbf{x}_i)$$

Once we account for the information in the covariates, the error term provides no information about the mean of our outcome. The conditional mean function is therefore given by  $g(\mathbf{x}_i)$ .

The mean estimate we obtain using nonparametric series estimation has the same form of the mean function estimate we obtain using linear regression. The regressors, however, are not variables in the data but functions of the variables. An example would be a  $k$ th-order polynomial. Suppose we have one covariate. The elements of the polynomial in this case would be  $(x_i, x_i^2, \dots, x_i^k)$ . If we define  $\mathbf{z}_i$  as a vector with elements  $(x_i, x_i^2, \dots, x_i^k)$ , we may write the estimate of the mean function as

$$\mathbf{z}'_i \hat{\beta}$$

where  $\hat{\beta}$  has the form of an ordinary least-squares estimate.

npregress series allows us to specify other functional forms for  $\mathbf{z}_i$  depending on the basis we select: B-spline, piecewise polynomial spline, or polynomial. See [Methods and formulas](#) for the formulas for each basis.

Although the estimate of the mean function has the form of a linear regression, the individual coefficients are not easily interpretable. For instance, in our  $k$ th-order polynomial example, if  $x_i$  is continuous, the marginal effect of  $x_i$  is not a single coefficient but rather is a function of  $k$  elements of  $\beta$  and the covariate  $x_i$ .

In the example above, we had only one covariate,  $x_i$ . If we have more than one covariate, we approximate the mean function by using interactions of the terms in the basis function for each covariate. For instance, a polynomial of  $x_i$  and  $w_i$  would have terms  $(x_i, w_i, x_i w_i, x_i^2, w_i^2, \dots, w_i^k x_i^k)$ . As the number of covariates increases, the number of terms in the basis function increases exponentially. This is referred to in the literature as the curse of dimensionality.

npregress series allows us to reduce the dimensionality by using the `nointeract()` option to request that some covariates not be interacted with others. For the example above, this is equivalent to specifying a model of the form

$$y_i = g_1(x_i) + g_2(w_i) + \varepsilon_i \tag{3}$$

In (3),  $g_1(x_i)$  and  $g_2(w_i)$  are unknown functions, but there are no interactions between  $x_i$  and  $w_i$ . This ameliorates the curse of dimensionality but imposes more structure to the model.

You may also want to reduce the curse of dimensionality by requesting a parametric component, by using the `asis()` option, to fit models like this:

$$y_i = g(x_i) + w_i\beta + \varepsilon_i \tag{4}$$

In (4),  $g(x_i)$  is unknown but we assume that  $w_i$  enters the model linearly.

As mentioned above, the regression coefficients are not easily interpretable. We can, however, estimate marginal effects, as reported in the `nprogress series` output, and use `margins` to answer specific questions about the effects of covariates on the conditional mean,  $g(\mathbf{x}_i)$ . We demonstrate this in the examples below.

For detailed introductions to series estimators and the methods implemented by `nprogress series`, see [de Boor \(2001\)](#), [Schumaker \(2007\)](#), [Eubank \(1999\)](#), [Schoenberg \(1969\)](#), [Newey \(1997\)](#), and [Chen \(2007\)](#).

## Estimation and effects

### ▷ Example 1: Nonparametric series regression estimation

`dui.dta` contains information about the number of monthly drunk driving citations in a local jurisdiction (`citations`). Suppose we want to know the effect of increasing fines on the number of citations. Because `citations` is a count variable, we could consider fitting the model with `poisson` or `nbreg`. However, both of these estimators make assumptions about the distribution of the data. If these assumptions are not true, we will obtain inconsistent estimates.

By using `nprogress series`, we do not have to make any assumptions about how `citations` is distributed. We use `nprogress series` to estimate the average marginal effect of drunk driving penalties (`fines`) on citations.

```
. use https://www.stata-press.com/data/r19/dui
(Fictional data on monthly drunk driving citations)
. nprogress series citations fines
Computing approximating function
Minimizing cross-validation criterion
Iteration 0: Cross-validation criterion = 55.15697
Iteration 1: Cross-validation criterion = 55.11413
Computing average derivatives
Cubic B-spline estimation Number of obs = 500
Criterion: cross-validation Number of knots = 3
```

|           | Effect    | Robust<br>std. err. | z      | P> z  | [95% conf. interval] |           |
|-----------|-----------|---------------------|--------|-------|----------------------|-----------|
| citations |           |                     |        |       |                      |           |
| fines     | -8.020769 | .464836             | -17.26 | 0.000 | -8.931831            | -7.109707 |

Note: Effect estimates are averages of derivatives.

The iteration log first tells us that the approximating function is being computed. At this stage, the number of knots of the cubic B-spline is selected using cross-validation. Three knots were selected.

After the approximating function is computed, average marginal effects are computed. This second step is computationally expensive. The computation time increases with the number of elements in the basis function, which in turn increases with the complexity of the mean function we are trying to compute.

The table reports that the average marginal effect of fines on the mean number of citations is  $-8.02$ . Increasing fines, on average, reduces the number of citations.

◀

`npregress series` generates a system variable for each element of the basis function. Additionally, variables are generated with the rescaled values of the continuous covariates used to construct the basis function. To see the variables that `npregress series` generated for [example 1](#), we type

```
. describe *_*, fullnames
```

| Variable name | Storage type | Display format | Value label | Variable label          |
|---------------|--------------|----------------|-------------|-------------------------|
| __x1rs        | double       | %10.0g         |             | fines rescaled to [0,1] |
| _x1__b1       | double       | %10.0g         |             | Basis term 1 for fines  |
| _x1__b2       | double       | %10.0g         |             | Basis term 2 for fines  |
| _x1__b3       | double       | %10.0g         |             | Basis term 3 for fines  |
| _x1__b4       | double       | %10.0g         |             | Basis term 4 for fines  |
| _x1__b5       | double       | %10.0g         |             | Basis term 5 for fines  |
| _x1__b6       | double       | %10.0g         |             | Basis term 6 for fines  |
| _x1__b7       | double       | %10.0g         |             | Basis term 7 for fines  |

To specify a name for each of the elements of the basis function, we can use the `basis()` option with a *stub*.

```
. npregress series citations fines, basis(basis)
(output omitted)
```

We get the following set of names for the elements of the basis function:

```
. describe basis*, fullnames
```

| Variable name | Storage type | Display format | Value label | Variable label         |
|---------------|--------------|----------------|-------------|------------------------|
| basis1        | double       | %10.0g         |             | Basis term 1 for fines |
| basis2        | double       | %10.0g         |             | Basis term 2 for fines |
| basis3        | double       | %10.0g         |             | Basis term 3 for fines |
| basis4        | double       | %10.0g         |             | Basis term 4 for fines |
| basis5        | double       | %10.0g         |             | Basis term 5 for fines |
| basis6        | double       | %10.0g         |             | Basis term 6 for fines |
| basis7        | double       | %10.0g         |             | Basis term 7 for fines |

We may also modify the name of the rescaled variable by using the `rescale()` option.

```
. npregress series citations fines, rescale(rescaled)
(output omitted)
```

This will give us

```
. describe rescaled*, fullnames
```

| variable name | storage type | display format | value label | variable label          |
|---------------|--------------|----------------|-------------|-------------------------|
| rescaled1     | double       | %10.0g         |             | fines rescaled to [0,1] |

▷ Example 2: Estimation with more than one regressor

We now extend [example 1](#). In addition to `finest`, we model citations as a function of whether the jurisdiction is small, medium, or large (`csize`) and whether there is a college in the jurisdiction (`college`).

```
. npregress series citations finest i.csize i.college
Computing approximating function
Minimizing cross-validation criterion
Iteration 0: Cross-validation criterion = 30.26251
Computing average derivatives
Cubic B-spline estimation Number of obs = 500
Criterion: cross-validation Number of knots = 1
```

| citations                                                  | Effect    | Robust<br>std. err. | z      | P> z  | [95% conf. interval] |           |
|------------------------------------------------------------|-----------|---------------------|--------|-------|----------------------|-----------|
| finest                                                     | -7.787386 | .2917941            | -26.69 | 0.000 | -8.359292            | -7.215481 |
| csize<br>(Medium<br>vs<br>Small)<br>(Large<br>vs<br>Small) | 4.732592  | .5087968            | 9.30   | 0.000 | 3.735368             | 5.729815  |
| college<br>(College<br>vs<br>Not coll..)                   | 6.514286  | .5958949            | 10.93  | 0.000 | 5.346353             | 7.682218  |

Note: Effect estimates are averages of derivatives for continuous covariates and averages of contrasts for factor covariates.

The average marginal effect of `finest` is  $-7.79$ , slightly less in magnitude than the  $-8.02$  that we estimated in [example 1](#). The output also shows effects for the variables `csize` and `college`. In these categorical variables, the effects are differences instead of derivatives. For example, if every jurisdiction in the population were a college town, we would expect 6.51 more citations than if none were college towns.



### ► Example 3: Expected citations for different levels of fines

The `npregress series` command reported that the average marginal effect of fines on number of citations is negative. We can use `margins` to further explore the relationship between level of fines and expected number of citations. What would we expect if all jurisdictions set fines to \$8,000? What if they all set fines to \$9,000? \$10,000? \$11,000? We use the `at(fines=(8 9 10 11))` option with `margins` to estimate these expected values.

```
. margins, at(fines=(8 9 10 11))
Predictive margins Number of obs = 500
Model VCE: Robust
Expression: Mean function, predict()
1._at: fines = 8
2._at: fines = 9
3._at: fines = 10
4._at: fines = 11
```

| _at | Delta-method |           |       |       | [95% conf. interval] |          |
|-----|--------------|-----------|-------|-------|----------------------|----------|
|     | Margin       | std. err. | z     | P> z  |                      |          |
| 1   | 49.58234     | 1.47392   | 33.64 | 0.000 | 46.69351             | 52.47117 |
| 2   | 28.35154     | .5730302  | 49.48 | 0.000 | 27.22842             | 29.47466 |
| 3   | 20.40163     | .3320855  | 61.43 | 0.000 | 19.75075             | 21.0525  |
| 4   | 14.78085     | .4297201  | 34.40 | 0.000 | 13.93862             | 15.62309 |

There appears to be a dramatic drop in the expected number of citations as fines increase from \$8,000 to \$9,000. We can visualize these results if we type `marginsplot`.

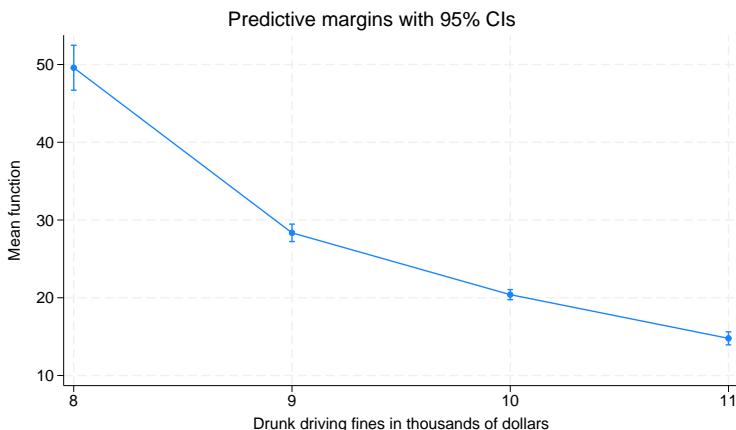


Figure 1.

Are there significant differences in the expected number of citations as we increase fines in increments of \$1,000? If we use the reverse-adjacent contrast operator, `ar.`, with margins, we can estimate these differences and perform tests.

```
. margins, at(fines=(8 9 10 11)) contrast(atcontrast(ar._at) nowald effects)
Contrasts of predictive margins Number of obs = 500
Model VCE: Robust

Expression: Mean function, predict()
1._at: fines = 8
2._at: fines = 9
3._at: fines = 10
4._at: fines = 11

Expression: Mean function, predict()
1._at: fines = 8
2._at: fines = 9
3._at: fines = 10
4._at: fines = 11
```

|          | Delta-method |           |        |       | [95% conf. interval] |           |
|----------|--------------|-----------|--------|-------|----------------------|-----------|
|          | Contrast     | std. err. | z      | P> z  |                      |           |
| _at      |              |           |        |       |                      |           |
| (2 vs 1) | -21.2308     | 1.610261  | -13.18 | 0.000 | -24.38685            | -18.07475 |
| (3 vs 2) | -7.94991     | .7085254  | -11.22 | 0.000 | -9.338595            | -6.561226 |
| (4 vs 3) | -5.620773    | .5683614  | -9.89  | 0.000 | -6.734741            | -4.506806 |

When fines are increased from \$8,000 to \$9,000, we expect a decrease of 21.23 in the number of citations. Smaller but still statistically significant decreases in the number of citations are expected as fines are increased from \$9,000 to \$10,000 and from \$10,000 to \$11,000.



► Example 4: Estimating the effect for different levels of jurisdiction size

Now, we estimate the effect of increasing `fines` for different jurisdiction sizes.

```
. margins csize, dydx(fines)
Average marginal effects Number of obs = 500
Model VCE: Robust

Expression: Mean function, predict()
dy/dx wrt: fines
```

| fines  | csize | Delta-method |           |        |       | [95% conf. interval] |           |
|--------|-------|--------------|-----------|--------|-------|----------------------|-----------|
|        |       | dy/dx        | std. err. | z      | P> z  |                      |           |
| Small  |       | -5.992484    | .4491224  | -13.34 | 0.000 | -6.872747            | -5.11222  |
| Medium |       | -7.740284    | .4366709  | -17.73 | 0.000 | -8.596144            | -6.884425 |
| Large  |       | -10.20492    | .564166   | -18.09 | 0.000 | -11.31067            | -9.099178 |

If all jurisdictions were small but other characteristics were as they are observed, then we expect that the marginal effect of `fines` would be  $-5.99$ . We see that the effect is more extreme as the size of the jurisdiction increases. If all jurisdictions were large, we expect that the average marginal effect of `fines` would be  $-10.20$ .

We can further explore the effects of fines for different jurisdiction sizes by estimating the expected number of citations with fines at specific levels.

```
. margins csize, at(fines=(8(1)11))
(output omitted)
```

To visualize the effect, we type marginsplot.

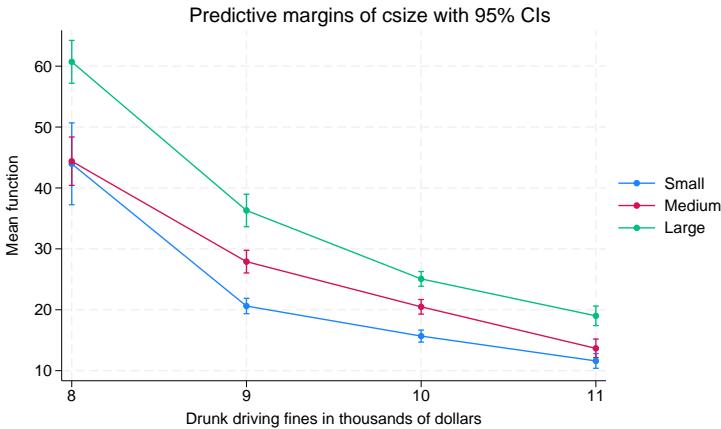


Figure 2.

For each jurisdiction size, we see that on average higher fines result in fewer citations. We also see that the effect of changing fine levels is nonlinear and differs across the counterfactual jurisdiction size. For instance, as fines increase from \$8,000 to \$9,000, the expected number of citations decreases faster for small jurisdictions than for medium ones.



## Stored results

npregress series stores the following in e():

### Scalars

|              |                             |
|--------------|-----------------------------|
| e(N)         | number of observations      |
| e(r2)        | $R^2$                       |
| e(r2_a)      | adjusted $R^2$              |
| e(converged) | 1 if converged, 0 otherwise |
| e(order)     | order of basis function     |
| e(rank)      | rank of e(V)                |

### Macros

|                      |                                           |
|----------------------|-------------------------------------------|
| e(cmd)               | npregress                                 |
| e(cmdline)           | command as typed                          |
| e(depvar)            | name of dependent variable                |
| e(basis)             | bsplines, splines, or polynomials         |
| e(wtype)             | weight type                               |
| e(wexp)              | weight expression                         |
| e(title)             | title in estimation output                |
| e(vce)               | vctype specified in vce()                 |
| e(vctype)            | title used to label Std. err.             |
| e(knots)             | number of knots selected                  |
| e(datasignaturevars) | variables used in calculation of checksum |
| e(datasignature)     | the checksum                              |

|                 |                                          |
|-----------------|------------------------------------------|
| e(estat_cmd)    | program used to implement estat          |
| e(predict)      | program used to implement predict        |
| e(properties)   | b V                                      |
| e(marginsok)    | predictions allowed by margins           |
| e(marginsprop)  | signals to the margins command           |
| e(marginsnotok) | predictions disallowed by margins        |
| Matrices        |                                          |
| e(b)            | coefficient vector                       |
| e(V)            | variance–covariance matrix of estimators |
| e(V_modelbased) | model-based variance                     |
| e(iilog)        | iteration log (up to 20 iterations)      |
| Functions       |                                          |
| e(sample)       | marks estimation sample                  |

In addition to the above, the following is stored in `r()`:

|          |                                                                                                                       |
|----------|-----------------------------------------------------------------------------------------------------------------------|
| Matrices |                                                                                                                       |
| r(table) | matrix containing the coefficients with their standard errors, test statistics, $p$ -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

Methods and formulas are presented under the following headings:

- [Overview](#)
- [Polynomials](#)
- [Piecewise polynomial splines](#)
- [B-splines](#)
- [Model selection](#)
  - [Cross-validation](#)
  - [Generalized cross-validation](#)
  - [Mallows's  \$C\_p\$](#)
  - [AIC and BIC](#)

## Overview

The regression model of outcome  $y_i$  given the  $k$ -dimensional vector of covariates  $\mathbf{x}_i$  was defined in (1) and (2) of *Remarks and examples* and is repeated here:

$$y_i = g(\mathbf{x}_i) + \varepsilon_i \quad (1)$$

$$E(\varepsilon_i | \mathbf{x}_i) = 0 \quad (2)$$

where  $\varepsilon_i$  is the error term. The covariates may include discrete and continuous variables. Equations (1) and (2) imply that

$$E(y_i | \mathbf{x}_i) = g(\mathbf{x}_i)$$

As discussed in *Remarks and examples*, series estimators have the form of ordinary least squares. Thus, we can write the estimate of the mean function as

$$\widehat{E}(y_i|\mathbf{x}_i) = \mathbf{z}(\mathbf{x}_i)\widehat{\boldsymbol{\beta}} \quad (5)$$

where  $\mathbf{z}(\mathbf{x}_i)$  is a known  $q$ -dimensional vector for which every one of the  $q$  terms is a function of the  $k$ -dimensional vector of covariates  $\mathbf{x}_i$ . Let  $n$  be the sample size. If we define  $\mathbf{Z}$  as the  $n \times q$  matrix formed by the  $\mathbf{z}(\mathbf{x}_i)$  for each individual  $i$ , then the  $q$ -dimensional coefficient vector  $\widehat{\boldsymbol{\beta}}$  is the ordinary least-squares vector that comes from regressing the  $n \times 1$  outcome vector  $\mathbf{y}$  on  $\mathbf{Z}$  and has the known form

$$\widehat{\boldsymbol{\beta}} = (\mathbf{Z}'\mathbf{Z})^{-1}(\mathbf{Z}'\mathbf{y}) \quad (6)$$

Each one of the series estimators has a different form for  $\mathbf{z}(\mathbf{x}_i)$ . Below, we define  $\mathbf{z}(\mathbf{x}_i)$  for polynomials, piecewise polynomial splines, and B-splines.

## Polynomials

For a polynomial of order 1 with  $k$  continuous covariates,  $\mathbf{x}_i \equiv (x_{i1}, x_{i2}, \dots, x_{ik})$ ,  $\mathbf{z}(\mathbf{x}_i)$  is

$$\mathbf{z}(\mathbf{x}_i) = (x_{i1}, x_{i2}, \dots, x_{ik}) \quad (P1)$$

For notational convenience, we will refer to the polynomial above as *P1*, which also corresponds to the name we gave to the equation. More formally, we could have written  $(x_{i1}, x_{i2}, \dots, x_{ik}) \equiv P1$ . We will maintain this notational convention below.

A polynomial of order 2 with  $k$  continuous covariates includes *P1* and second-order terms:

$$\mathbf{z}(\mathbf{x}_i) = (P1, x_{i1}^2, x_{i1}x_{i2}, \dots, x_{i1}x_{ik}, x_{i2}x_{i3}, \dots, x_{ik}^2) \quad (P2)$$

A third-order polynomial with  $k$  continuous covariates includes the terms in *P2* (which already includes *P1*) and third-order terms:

$$\mathbf{z}(\mathbf{x}_i) = (P2, x_{i1}^3, x_{i1}^2x_{i2}, \dots, x_{i1}^2x_{ik}, x_{i1}x_{i2}x_{i3}, \dots, x_{ik}^3) \quad (P3)$$

This recursive relationship continues. Thus, fourth-order polynomials includes the terms in *P3* (which already includes *P1* and *P2*) plus fourth-order terms.

For the polynomials above and all series estimators below, discrete covariates enter the model in levels, and each level is interacted with all other covariates in the model.

## Piecewise polynomial splines

Piecewise polynomial splines are formed by a polynomial and functions of the form

$$\max(x_{ik} - t_{1k}, 0)$$

In the expression above,  $t_{1k}$  is a constant that is called a knot. The subscript of  $t_{1k}$  indicates that it is the first knot of the continuous covariate  $\mathbf{x}_k$ . The  $\max(\cdot)$  function is 0 when  $x_{ik} < t_{1k}$  and is  $x_{ik} - t_{1k}$  otherwise. *npregress series* selects a set of knots for each one of the continuous covariates.

The regressors for npregress series using a piecewise polynomial spline of order 3 with one continuous covariate,  $\mathbf{x}_1$ , and  $k$  knots,  $t_{11} < t_{21} < \dots < t_{k1}$ , are given by

$$\mathbf{z}(x_{i1}) = \left\{ x_{i1}, x_{i1}^2, x_{i1}^3, \max(x_{i1} - t_{11}, 0)^3, \max(x_{i1} - t_{21}, 0)^3, \dots, \max(x_{i1} - t_{k1}, 0)^3 \right\} \quad (S1)$$

Equivalently, for another continuous covariate,  $\mathbf{x}_2$ , and  $k$  knots,  $t_{12} < t_{22} < \dots < t_{k2}$ , we have

$$\mathbf{z}(x_{i2}) = \left\{ x_{i2}, x_{i2}^2, x_{i2}^3, \max(x_{i2} - t_{12}, 0)^3, \max(x_{i2} - t_{22}, 0)^3, \dots, \max(x_{i2} - t_{k2}, 0)^3 \right\} \quad (S2)$$

To get  $\mathbf{z}(x_{i1}, x_{i2})$  for  $\mathbf{x}_1$  and  $\mathbf{x}_2$  and  $k$  knots, we include all terms in  $S1$  and  $S2$  as well as all the terms that result from the interaction of their terms. We write it succinctly as

$$\mathbf{z}(x_{i1}, x_{i2}) = \{S1, S2, (S1)(S2)\} \quad (S12)$$

The description above refers to the default third-order piecewise polynomial spline. Below, we describe the cases for piecewise polynomial splines of order 1 and order 2. Going back to the one covariate case, if we want a piecewise polynomial spline of order 1 with  $k$  knots, we have

$$\mathbf{z}(x_{i1}) = \{x_{i1}, \max(x_{i1} - t_{11}, 0), \max(x_{i1} - t_{21}, 0), \dots, \max(x_{i1} - t_{k1}, 0)\}$$

And for order 2 with  $k$  knots and one covariate, we have

$$\mathbf{z}(x_{i1}) = \{x_{i1}, x_{i1}^2, \max(x_{i1} - t_{11}, 0)^2, \max(x_{i1} - t_{21}, 0)^2, \dots, \max(x_{i1} - t_{k1}, 0)^2\}$$

If we have more than one covariate, the logic of interacting the expressions for each covariate is the same as the logic we used for the third-order piecewise polynomial spline in  $(S1)$ .

To construct  $\mathbf{z}(\mathbf{x}_i)$ , continuous covariates are rescaled to be between 0 and 1 with the expression

$$\{\mathbf{x}_i - \min(\mathbf{x}_i)\} \left\{ \frac{1}{\max(\mathbf{x}_i) - \min(\mathbf{x}_i)} \right\}$$

This rescaling is used to construct the piecewise polynomial spline and B-spline bases.

## B-splines

To construct a B-spline basis, we need to define knots that are on the interior of the range of the covariates and knots that are at the upper and lower limits of the range or outside the range. The number of knots that are not in the interior differs depending on the order of the B-spline. For a B-spline of order 1 with  $k$  interior knots,  $t_1, t_2, \dots, t_k$ , we need 4 additional knots. The set of knots for a first-order B-spline is therefore

$$t_{-1}, t_0, t_1, \dots, t_k, t_{k+1}, t_{k+2}$$

We added  $t_{-1}, t_0, t_{k+1}$ , and  $t_{k+2}$  to the interior knots. By convention,  $t_{-1} = t_0$  and  $t_{k+1} = t_{k+2}$ .

For a B-spline of order 2 with  $k$  interior knots, we need 6 additional knots. The set of knots is

$$t_{-2}, t_{-1}, t_0, t_1, \dots, t_k, t_{k+1}, t_{k+2}, t_{k+3}$$

For a B-spline of order 3, the set of knots is

$$t_{-3}, t_{-2}, t_{-1}, t_0, t_1, \dots, t_k, t_{k+1}, t_{k+2}, t_{k+3}, t_{k+4}$$

We first define a first-order B-spline for one continuous covariate  $\mathbf{x}$  with  $k$  interior knots. Let  $\mathbf{t}_j$  denote an  $n \times 1$  vector for which all elements take the value of the  $j$ th knot  $t_j$ . The B-spline basis is formed by  $k + 2$  functions of the form

$$B_{j,1} = \frac{(\mathbf{x} - t_j)}{t_{j+1} - t_j} \mathbf{1}(\mathbf{t}_j \leq \mathbf{x} < \mathbf{t}_{j+1}) + \frac{(t_{j+2} - \mathbf{x})}{t_{j+2} - t_{j+1}} \mathbf{1}(\mathbf{t}_{j+1} \leq \mathbf{x} < \mathbf{t}_{j+2}) \quad (7)$$

$$j = -1, 0, 1, 2, \dots, k$$

Above, we use the indicator function  $\mathbf{1}(\cdot)$ , which is 1 when the condition inside the parentheses is satisfied and is 0 otherwise. Also, any term for which  $t_{j+1} = t_j$  or  $t_{j+2} = t_{j+1}$  is considered to be a vector of 0s.

The function  $\mathbf{z}(\mathbf{x})$  used to estimate  $g(\mathbf{x})$  is given by

$$\mathbf{z}(\mathbf{x}) = (B_{-1,1}, B_{0,1}, B_{1,1}, \dots, B_{k,1})$$

We now define a second-order B-spline for one continuous covariate  $\mathbf{x}$  with  $k$  interior knots. The basis is constructed using the relationship given by

$$B_{j,2} = \frac{(\mathbf{x} - t_j)}{t_{j+2} - t_j} B_{j,1} + \frac{(t_{j+3} - \mathbf{x})}{t_{j+3} - t_{j+1}} B_{j+1,1}$$

$$j = -2, -1, 0, 1, 2, \dots, k$$

where  $B_{j,1}$  and  $B_{j+1,1}$  come from (7) above. Thus, second-order B-splines are a function of first-order B-splines, and as we will see below, third-order B-splines are a function of second-order B-splines. This recursion continues into higher orders, but nprogress series stops at B-splines of order 3.

The function  $\mathbf{z}(\mathbf{x})$  for the second-order B-spline is given by

$$\mathbf{z}(\mathbf{x}) = (B_{-2,2}, B_{-1,2}, B_{0,2}, B_{1,2}, \dots, B_{k,2})$$

The terms of the basis for a third-order B-spline are given by

$$B_{j,3} = \frac{(\mathbf{x} - t_j)}{t_{j+3} - t_j} B_{j,2} + \frac{(t_{j+4} - \mathbf{x})}{t_{j+4} - t_{j+1}} B_{j+1,2}$$

$$j = -3, -2, -1, 0, 1, 2, \dots, k$$

and the function  $\mathbf{z}(\mathbf{x})$  for the third-order B-spline is

$$\mathbf{z}(\mathbf{x}) = (B_{-3,3}, B_{-2,3}, B_{-1,3}, B_{0,3}, B_{1,3}, \dots, B_{k,3}) \tag{B1}$$

As was the case with piecewise polynomial splines, when there is more than one covariate, you include all functions of the form (B1) and their interactions to form expressions like the one in (S1).

### Model selection

Below, we define the criteria used for model selection. In the case of B-splines and piecewise polynomial splines, nprogress series selects the number of knots to be used for estimation. In the case of a polynomial basis, nprogress series selects the order of the polynomial.

Let us first define the squared residuals,  $e_i^2$ , where  $e_i = y_i - \hat{g}(\mathbf{x}_i)$  and  $\hat{g}(\cdot)$  is the mean function estimate defined in (5). We denote the residuals for the regressions below as  $e_i(\mathbf{t}_k)$  instead of  $e_i$  to signal that the estimates we obtain are a function of the set of knots,  $\mathbf{t}_k$ , used. In the case of polynomials,  $\mathbf{t}_k$  will refer to the degree of the polynomial instead of knots.

### Cross-validation

The cross-validation criterion,  $CV(\mathbf{t}_k)$ , is defined by

$$CV(\mathbf{t}_k) = \frac{1}{n} \sum_{i=1}^n \frac{e_i(\mathbf{t}_k)^2}{(1 - h_{ii})^2} \tag{8}$$

In (8),  $h_{ii}$  are the diagonal elements of the matrix  $\mathbf{Z}(\mathbf{Z}'\mathbf{Z})\mathbf{Z}'$ , where  $\mathbf{Z}$  is defined in (6) above and  $n$  is the size of the estimation sample.

nprogress series computes  $CV(\mathbf{t}_k)$  for different sets of knots,  $\mathbf{t}_1, \mathbf{t}_2, \dots, \mathbf{t}_k, \dots$ , where  $\mathbf{t}_1 \subset \mathbf{t}_2 \subset \dots \subset \mathbf{t}_k \subset \dots$ , and then selects the model with the smallest value for the cross-validation criterion.

## Generalized cross-validation

The generalized cross-validation criterion,  $GCV(\mathbf{t}_k)$ , is given by

$$GCV(\mathbf{t}_k) = \frac{1}{n} \sum_{i=1}^n \frac{e_i(\mathbf{t}_k)^2}{\{1 - (K/n)\}^2}$$

where  $K$  is the number of estimated parameters and the other arguments are equivalent to those defined in (8). As with cross-validation,  $GCV(\mathbf{t}_k)$  is computed for a set of models with an increasing number of nested knots, in the case of piecewise polynomial splines and B-splines, and of polynomial order in the case of polynomials. The minimum of the sequence is the selected model.

## Mallows's $C_p$

$$\text{Mallows}(\mathbf{t}_k) = \frac{1}{n} \sum_{i=1}^n e_i(\mathbf{t}_k)^2 \left(1 + \frac{2K}{n}\right)$$

As with cross-validation, Mallows ( $\mathbf{t}_k$ ) is computed for a set of models with an increasing number of nested knots, and the minimum of the sequence is the selected model.

## AIC and BIC

See *Methods and formulas* in [R] [estat ic](#).

## References

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## Also see

- [R] [npregress series postestimation](#) — Postestimation tools for npregress series
- [R] [npregress intro](#) — Introduction to nonparametric regression
- [R] [kdensity](#) — Univariate kernel density estimation
- [R] [lpoly](#) — Kernel-weighted local polynomial smoothing
- [R] [makespline](#) — Spline generation
- [U] [20 Estimation and postestimation commands](#)

Postestimation commands

[predict](#)

[margins](#)

Remarks and examples

[Methods and formulas](#)

[Reference](#)

Also see

## Postestimation commands

The following standard postestimation commands are available after `nprogress series`:

| Command                      | Description                                                                                       |
|------------------------------|---------------------------------------------------------------------------------------------------|
| <code>estat summarize</code> | summary statistics for the estimation sample                                                      |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                |
| <code>estimates</code>       | cataloging estimation results                                                                     |
| <code>etable</code>          | table of estimation results                                                                       |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters    |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                           |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters |
| <code>predict</code>         | conditional means and residuals                                                                   |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions              |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                              |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                |

# predict

## Description for predict

`predict` creates a new variable containing predictions such as conditional mean of the outcome, residuals, or score of the mean function.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic atsample tolerance(#)]
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

---

Main

|                        |                                              |
|------------------------|----------------------------------------------|
| <code>mean</code>      | conditional mean of the outcome; the default |
| <code>residuals</code> | residuals                                    |
| <code>score</code>     | score; equivalent to <code>residuals</code>  |

---

These statistics are available for the estimation sample only.

## Options for predict

Main

`mean`, the default, calculates the conditional mean of the outcome variable.

`residuals` calculates the residuals.

`score` is a synonym for `residuals`.

`atsample` restricts predictions to the range of covariates in the data. If requested predictions extend beyond the range of the data, `atsample` will compute predictions only for observations within the range of the original data and will exclude those observations that are beyond the range of the data.

By default, predictions will not be computed if any covariate is set to a value outside the range of the data, unless `atsample` or `tolerance()` is specified.

`tolerance(#)` sets the tolerance for predictions outside the range of the covariates.

By default, predictions will not be computed if any covariate is set to a value outside the range of the data, unless `tolerance()` or `atsample` is specified.

## margins

### Description for margins

`margins` estimates margins of the conditional mean.

### Menu for margins

Statistics > Postestimation

### Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [options]
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

Main

|                               |                                              |
|-------------------------------|----------------------------------------------|
| <code>mean</code>             | conditional mean of the outcome; the default |
| <code><u>r</u>esiduals</code> | not allowed with margins                     |
| <code><u>s</u>core</code>     | not allowed with margins                     |

| <i>options</i> | Description |
|----------------|-------------|
|----------------|-------------|

SE

|                                  |                                                                                                   |
|----------------------------------|---------------------------------------------------------------------------------------------------|
| <code>vce(<i>vcetype</i>)</code> | <i>vcetype</i> may be <code>delta</code> , <code>unconditional</code> , or <code>bootstrap</code> |
| <code><u>r</u>eps(#)</code>      | equivalent to <code>vce(bootstrap, reps(#))</code>                                                |
| <code>seed(#)</code>             | set random-number seed to #; must also specify <code>reps(#)</code>                               |
| <code>nose</code>                | do not estimate standard errors                                                                   |

Reporting

|                                           |                                                                                              |
|-------------------------------------------|----------------------------------------------------------------------------------------------|
| <code><u>c</u>itype(<i>citype</i>)</code> | method to compute bootstrap confidence intervals; default is <code>citype(percentile)</code> |
|-------------------------------------------|----------------------------------------------------------------------------------------------|

| <i>citype</i> | Description |
|---------------|-------------|
|---------------|-------------|

|                                |                                              |
|--------------------------------|----------------------------------------------|
| <code><u>p</u>ercentile</code> | percentile confidence intervals; the default |
| <code>bc</code>                | bias-corrected confidence intervals          |
| <code><u>n</u>ormal</code>     | normal-based confidence intervals            |

Statistics not allowed with `margins` are functions of stochastic quantities other than `e(b)`.

For the full syntax, see [\[R\] margins](#).

## Options for margins

SE

`vce(delta)`, `vce(unconditional)`, and `vce(bootstrap)` specify how the VCE and, correspondingly, standard errors are calculated.

`vce(delta)` is the default. The delta method is applied to the formula for the response and the VCE of the estimation command. This method assumes that values of the covariates used to calculate the response are given or, if all covariates are not fixed using `at()`, that the data are given.

`vce(unconditional)` specifies that the covariates that are not fixed be treated in a way that accounts for their having been sampled. The VCE is estimated using the linearization method. This method allows for heteroskedasticity or other violations of distributional assumptions in the same manner as `vce(robust)`, which is the default for `npregress series`.

`vce(bootstrap)` specifies that bootstrap standard errors be reported; see [R] [vce\\_option](#). We recommend that you select the number of replications using `reps(#)` instead of specifying `vce(bootstrap)`, which defaults to 50 replications. Be aware that the number of replications needed to produce good estimates of the standard errors varies depending on the problem.

`reps(#)` specifies the number of bootstrap replications to be performed. Specifying this option is equivalent to specifying `vce(bootstrap, reps(#))`.

`seed(#)` sets the random-number seed. You must specify `reps(#)` with `seed(#)`.

`nose` suppresses calculation of the VCE and standard errors.

Reporting

`ctype(ctype)` specifies the type of confidence interval to be computed. By default, bootstrap percentile confidence intervals are reported as recommended by [Cattaneo and Jansson \(2018\)](#). *ctype* may be one of `percentile`, `bc`, or `normal`.

## Remarks and examples

For examples of margins after `npregress series`, see [example 3](#) and [example 4](#) in [R] [npregress series](#).

For an example of `marginsplot`, see [example 4](#) in [R] [npregress series](#).

## Methods and formulas

The formulas used by `predict` and `margins` for the conditional mean function and the mean marginal effect of a covariate are given in [Methods and formulas](#) of [R] [npregress series](#).

## Reference

Cattaneo, M. D., and M. Jansson. 2018. Kernel-based semiparametric estimators: Small bandwidth asymptotics and bootstrap consistency. *Econometrica* 86: 955–995. <https://doi.org/10.3982/ECTA12701>.

## Also see

[R] [npregress series](#) — Nonparametric series regression

[R] [bootstrap postestimation](#) — Postestimation tools for bootstrap

[U] [20 Estimation and postestimation commands](#)

[Description](#)[Options](#)[Acknowledgments](#)[Quick start](#)[Remarks and examples](#)[References](#)[Menu](#)[Stored results](#)[Also see](#)[Syntax](#)[Methods and formulas](#)

## Description

`nptrend` performs four different nonparametric tests for trend: the Cochran–Armitage test, the Jonckheere–Terpstra test, the linear-by-linear trend test, and a test using ranks developed by Cuzick.

## Quick start

Cochran–Armitage test for the association of a binary outcome in `y` and the ordered values of the categories of `x`

```
nptrend y, group(x) carmitage
```

Same as above, but report an exact  $p$ -value calculated using Monte Carlo permutations

```
nptrend y, group(x) carmitage exact
```

Same as above, but perform 100,000 Monte Carlo permutations rather than the default of 10,000, and set the random-number seed for reproducibility

```
nptrend y, group(x) carmitage exact(montecarlo, reps(100000) ///
rseed(1234))
```

Rather than testing the trend in `y` by the values of `x`, which are 1, 2, 3, test the trend by 1, 2, 4

```
nptrend y, group(x) carmitage scoregroup(1 2 4)
```

Jonckheere–Terpstra test for trend in `y` across ordered categories of `catvar`

```
nptrend y, group(catvar) jterpstra
```

Same as above, but perform the linear-by-linear trend test

```
nptrend y, group(catvar) linear
```

Same as above, but perform Cuzick’s test using ranks

```
nptrend y, group(catvar) cuzick
```

## Menu

Statistics > Nonparametric analysis > Tests of hypotheses > Trend test across ordered groups

## Syntax

```
nptrend varname [if] [in] [weight], group(groupvar) testopt [options]
```

| <i>options</i>                          | Description                                                          |
|-----------------------------------------|----------------------------------------------------------------------|
| Main                                    |                                                                      |
| * <u>g</u> roup( <i>groupvar</i> )      | ordered group variable                                               |
| <u>s</u> coreresponse( <i>numlist</i> ) | scores for the response <i>varname</i>                               |
| <u>s</u> coregroup( <i>numlist</i> )    | scores for the groups specified by <i>groupvar</i>                   |
| <u>e</u> xact[ ( <i>exact_specs</i> ) ] | report an exact <i>p</i> -value                                      |
| <u>n</u> olabel                         | display numerical values of <i>groupvar</i> rather than value labels |
| <u>n</u> otable                         | do not display the table of mean response scores by group            |
| * <i>testopt</i>                        |                                                                      |
| Description                             |                                                                      |
| <u>c</u> armitage                       | Cochran–Armitage test for trend                                      |
| <u>j</u> terpstra                       | Jonckheere–Terpstra test for trend                                   |
| <u>l</u> inear                          | linear-by-linear test for trend                                      |
| <u>c</u> uzick                          | use ranks for the response scores in the test due to Cuzick          |

\* group(*groupvar*) and one of the choices for *testopt* are required.

collect is allowed; see [U] 11.1.10 Prefix commands.

fweights are allowed, except when exact is specified; see [U] 11.1.6 weight.

## Options

### Main

group(*groupvar*) defines the ordered groups across which differences in the response *varname* are to be tested. group() is required.

When option scoregroup() is not specified, the values of *groupvar* are the group scores. When scoregroup() is specified, *groupvar* defines the ordered categories, and the scores are the values specified in scoregroup(). When scoregroup() is specified, *groupvar* must be a positive integer (with gaps allowed).

*testopt* specifies the test that is reported. It is required and must be one of carmitage, jterpstra, linear, or cuzick.

carmitage specifies that the Cochran–Armitage test for trend be reported. Typically, the response *varname* is a 0/1 variable, and the values of *groupvar* represent a set of ordered categories. It gives a test of the trend of the proportions of positive responses in the groups, with the hypothesized trend given by the values of *groupvar* (or the values of scoregroup() if specified). If the response *varname* is not 0/1, it can be valued 1/2 and mapped to 0/1 or 1/0 using scoreresponse(0 1) or scoreresponse(1 0), respectively.

Response and group specifications for carmitage can be flipped. The *groupvar* variable can be a 0/1 predictor, and the response *varname* can contain ordered outcomes. If *groupvar* is not 0/1, it can be valued 1/2 and mapped to 0/1 or 1/0 using scoregroup(0 1) or scoregroup(1 0), respectively.

`jterpstra` specifies that the Jonckheere–Terpstra test for trend be reported. The response *varname* contains outcomes, which can be ordered categories or continuous values, and the *groupvar* variable is an ordered group indicator that is believed to be a predictor of outcome. The Jonckheere–Terpstra statistic is dependent on which is the response variable and which is the group variable. If response and group variables are interchanged, the value of the statistic will be different.

Only the orderings given by the response *varname* and *groupvar* are used to calculate the Jonckheere–Terpstra statistic. Different numerical values that give the same orderings produce the same statistic. Hence, this statistic is used when a test for trend dependent only on order is desired. Typically, the options `scorerresponse()` and `scoregroup()` are not specified. However, either or both of these options can be used to change the orderings given by *varname* and *groupvar*.

`linear` specifies that the linear-by-linear test for trend be reported. The linear-by-linear statistic is symmetric in the response variable *varname* and the group indicator *groupvar*. Interchanging response and group variables produces the same result. Despite the name “linear”, nonlinear trends can be tested. The trend, linear or nonlinear, is based on the numerical values of the response and group variables. `scorerresponse()` and `scoregroup()` can be specified to test different trends.

`cuzick` specifies that ranks be used for the responses and that the test for trend across ordered groups developed by [Cuzick \(1985\)](#) be reported. Because ranks are used, only the ordering given by response variable *varname* matters. The numerical values of *groupvar* are used in calculation, and different values, even those that give the same ordering, will result in different values of the statistic for Cuzick’s test. `scorerresponse()` can be specified to change the ordering of the responses. `scoregroup()` can be specified to test different trends by group.

`scorerresponse(numlist)` specifies scores for the responses in *varname*. When specified, *varname* must contain only positive integers (with gaps allowed), and the response score for *varname* = *i* is the *i*th number in *numlist*. When `scorerresponse()` is not specified, the response scores are the values of the response *varname*.

`scoregroup(numlist)` specifies scores for the groups specified by *groupvar*. When specified, *groupvar* must contain only positive integers (with gaps allowed), and the group score for *groupvar* = *i* is the *i*th number in *numlist*. When `scoregroup()` is not specified, the group scores are the values of *groupvar*.

`exact` and `exact(exact_specs)` specify that an exact *p*-value be reported.

`exact` specifies that an exact *p*-value from a Monte Carlo permutation test be reported. `exact` is a synonym for `exact(montecarlo)`.

`exact(montecarlo | enumerate[ , options ])` specifies that an exact *p*-value be reported in addition to the approximate or asymptotic *p*-value. Specifying `exact(montecarlo)` does a Monte Carlo permutation test. Specifying `exact(enumerate)` does an enumeration of all possible permutations. Because the number of all possible permutations is typically extremely large, enumeration is feasible only for very small datasets. The number of permutations will be displayed, and you can click on *Break* to stop the computation. The exact *p*-value is computed by `permute`.

`exact(montecarlo[ , options ])` allows *options* `show`, `reps(#)`, `rseed(#)`, `saving(filename[ , sav_options ])`, `level(#)`, `dots(#)`, `nodots`, and `eps(#)`. The `show` option specifies that the table produced by `permute` also be displayed. By default, 10,000 Monte Carlo permutations are done. That is, the default is the same as specifying `exact(montecarlo, reps(10000))`. The default for `dots()` is `dots(100)` when `reps()` is  $\geq 10,000$ ; otherwise, it is `dots(1)`. See [Options](#) in [\[R\] permute](#).

`exact(enumerate[, options])` allows *options* `show`, `saving(filename[, sav_options])`, `dots(#)`, `nodots`, and `eps(#)`. The `show` option specifies that the table produced by `permute` also be displayed. The default for `dots()` is `dots(100)`. See [Options](#) in [\[R\] permute](#).

`noLabel` causes numerical values of *groupvar* to be displayed in the table of mean response scores rather than value labels.

`notable` suppresses the display of the table of mean response scores by group.

## Remarks and examples

Remarks are presented under the following headings:

[Introduction](#)  
[Cochran–Armitage test](#)  
[Jonckheere–Terpstra test](#)  
[Linear-by-linear trend test](#)  
[Cuzick’s test](#)

## Introduction

`nptrend` performs four different nonparametric tests for trend: the Cochran–Armitage test, the Jonckheere–Terpstra test, the linear-by-linear trend test, and a test using ranks developed by Cuzick.

Data for these tests consist of two variables. The first variable, specified immediately after `nptrend`, contains responses. The second variable, specified in `group()`, identifies groups.

For the Cochran–Armitage test, the response is typically a 0/1 variable, and the values of the group variable are ordered categories. It tests the trend of the proportions of positive responses in the groups. The numerical values of the hypothesized trend are called “scores”. For this form of the test, we call the scores that specify the hypothesized trend “group scores” because it is the trend by group being tested. For example, group scores of 1, 2, 3 would test for a linear trend. Group scores of 1, 4, 9 would test for a quadratic trend.

The specification of the Cochran–Armitage test can be flipped. That is, the response variable can identify ordered outcome categories, and the group variable can be 0/1. In this case, we call the scores that specify the hypothesized trend “response scores”.

The Cochran–Armitage test requires either a 0/1 response variable or a 0/1 group variable. It cannot test responses with more than two levels when there are more than two groups.

The Jonckheere–Terpstra test is typically used when there are more than two groups and more than two response levels. The test assesses whether there is an association between the response scores and the group scores, but only the relative orders of the responses and groups matter. Because there is no need to specify a hypothesized trend, this test is in this sense “more nonparametric” than the other tests for trend reported by `nptrend`.

The linear-by-linear trend test uses response scores to specify the trend being tested. So it is typically used when the trend of the response scores, rather than just their ordering, is meaningful. The linear-by-linear statistic is asymptotically equivalent to the Pearson correlation coefficient (see [\[R\] correlate](#)).

The statistics for the Cochran–Armitage test (when the response is 0/1), the linear-by-linear trend test, and Cuzick’s test are based on the numerical values of the group scores. The linear-by-linear test requires numerical response scores as well. Different scores, even those that have the same ordering, produce different values of the statistic. For instance, group scores 1, 2, 3 would give a different result than 1, 2, 4. For the linear-by-linear test, the same holds for response scores.

The options `scorerresponse()` and `scoregroup()` are provided as convenience tools to test different response scores and group scores without having to modify the underlying variables. They do, however, require that the underlying variables be valued 1, 2, 3, ...

If your data are not grouped, you can test for trend with the `signtest` and `spearman` commands; see [R] [signrank](#) and [R] [spearman](#). With `signtest`, you can perform the Cox and Stuart test, a sign test applied to differences between equally spaced observations of `varname`. With `spearman`, you can perform the Daniels test, a test of zero Spearman correlation between `varname` and a time index. See [Conover \(1999, 169–175, 323\)](#) for a discussion of these tests.

## Cochran–Armitage test

The Cochran–Armitage test tests the trend in ordered groups of the probability of a positive response. As we stated earlier, response and group specifications can be flipped, and it can assess whether a trend based on ordered responses is associated with membership in one of two groups. In the discussion that follows we will consider only the former case: ordered groups with 0/1 responses.

The Cochran–Armitage test is based on a linear probability model for  $\theta_i$ , the probability of a positive response for subjects in the  $i$ th group,

$$\theta_i = \alpha + \beta(g_i - \bar{g})$$

where  $\alpha$  is the intercept,  $\beta$  is the slope,  $g_i$  are the group scores, and  $\bar{g}$  is the mean of the group scores.

The estimate for  $\beta$  is the ordinary least-squares (OLS) estimate  $\hat{\beta}$ , and this is the statistic for the Cochran–Armitage test.

However, the standard error of the statistic  $\hat{\beta}$  for the Cochran–Armitage test has a different formulation than that of the standard error from OLS regression—although they are the same asymptotically. The standard error  $\text{se}(\hat{\beta})$  for the Cochran–Armitage statistic is based on decomposing the Pearson statistic for independence.

The Pearson statistic  $X_{\text{Ind}}^2$  for independence can be written as

$$X_{\text{Ind}}^2 = z^2 + X_{\text{DL}}^2$$

where  $z = \hat{\beta}/\text{se}(\hat{\beta})$  is the  $z$  score for the Cochran–Armitage test and  $X_{\text{DL}}^2$  is a statistic for departure from the linear trend. Asymptotically,  $z$  has a standard normal distribution, and  $X_{\text{DL}}^2$  has a  $\chi^2$  distribution with  $I - 2$  degrees of freedom, where  $I$  is the number of ordered categories. See [Cochran–Armitage test for trend](#) in *Methods and formulas* below.

This is one of the advantages of performing the Cochran–Armitage test. We get not only a test for trend but also a test for departure from this trend. Let’s be clear about what we mean when we say “departure from this trend”. The Pearson statistic for independence is decomposed into two pieces. Either piece being large means that the test for independence is rejected. The linear piece  $z^2$  being large means there is a linear trend that rejects independence. The departure piece  $X_{\text{DL}}^2$  being large means there are differences other than the linear trend that reject independence.

We should also be clear about what we mean when we say “linear trend”. Linear trend means whatever trend is given by the group scores  $g_i$ , which do not have to be 1, 2, 3, ... They could be, for instance, 1, 3, 10, 12, ... It is linear in the scores but not necessarily linear by the ordering of the groups.

When the exact option is specified, a  $p$ -value from the permutation test of the null hypothesis  $\beta = 0$  is reported.

► Example 1

We have fictional data from a clinical trial of a new drug for treating migraines. The variable `dose` contains the dose of the drug given to a subject. The variable `relief` is 0/1 with 0 indicating no relief of the migraine and 1 indicating partial or total relief. Here is a tabulation of the data:

```
. use https://www.stata-press.com/data/r19/migraine
(Fictional migraine drug data)
. tabulate dose relief, row nokey
```

| Mycureit<br>dose in mg | Relief of migraine<br>after 2 hours |              | Total         |
|------------------------|-------------------------------------|--------------|---------------|
|                        | 0                                   | 1            |               |
| 10                     | 80<br>40.00                         | 120<br>60.00 | 200<br>100.00 |
| 20                     | 92<br>46.00                         | 108<br>54.00 | 200<br>100.00 |
| 30                     | 83<br>41.50                         | 117<br>58.50 | 200<br>100.00 |
| 40                     | 63<br>31.50                         | 137<br>68.50 | 200<br>100.00 |
| Total                  | 318<br>39.75                        | 482<br>60.25 | 800<br>100.00 |

There appears to be a trend of more subjects reporting relief at the higher doses. Let's calculate the Cochran–Armitage test for trend:

```
. nptrend relief, group(dose) carmitage
Cochran–Armitage test for trend
Number of observations = 800
Number of groups = 4
Number of response levels = 2
```

| Group | Group score | Mean<br>response<br>score | Number<br>of obs |
|-------|-------------|---------------------------|------------------|
| dose  |             |                           |                  |
| 10    | 10          | .6                        | 200              |
| 20    | 20          | .54                       | 200              |
| 30    | 30          | .585                      | 200              |
| 40    | 40          | .685                      | 200              |

```
Statistic = .003
Std. err. = .0015476
z = 1.939
Prob > |z| = 0.0526
Test of departure from trend:
chi2(2) = 5.45
Prob > chi2 = 0.0656
```

The asymptotic  $p$ -value for the test of a linear trend is 0.0526. The  $p$ -value for the test of departure from linearity is 0.0656.

The  $p$ -value for the test of a linear trend is close to 0.05. Rather than report the asymptotic  $p$ -value, we may want to report an exact  $p$ -value. We run `nptrend` again, this time specifying the exact option.

```
. nptrend relief, group(dose) carmitage exact
Permutations (10,000):1,000.....2,000.....3,000.....4,000
>5,000.....6,000.....7,000.....8,000.....9,000.....
> ..10,000 done
```

Cochran-Armitage test for trend

```
Number of observations = 800
Number of groups = 4
Number of response levels = 2
```

| Group | Group score | Mean response score | Number of obs |
|-------|-------------|---------------------|---------------|
| dose  |             |                     |               |
| 10    | 10          | .6                  | 200           |
| 20    | 20          | .54                 | 200           |
| 30    | 30          | .585                | 200           |
| 40    | 40          | .685                | 200           |

```
Statistic = .003
Std. err. = .0015476
z = 1.939
Prob > |z| = 0.0526
Exact prob = 0.0576 (10,000 Monte Carlo permutations)
```

Test of departure from trend:

```
chi2(2) = 5.45
Prob > chi2 = 0.0656
```

The exact  $p$ -value reported is 0.0576, larger than the asymptotic  $p$ -value of 0.0526. The exact  $p$ -value comes from a permutation test performed by computing 10,000 random permutations. This means that there is some random error associated with the reported exact  $p$ -value.

To see the Monte Carlo error in the exact  $p$ -value, we can use the `show` suboption of `exact`, which displays the full output of `permute`. The option we specify is

```
exact(montecarlo, show reps(100000) dots(1000) rseed(1234))
```

We increase the number of random permutations to 100,000. `dots(1000)` displays a dot after every 1,000th permutation rather than the default of every 100th. `rseed(1234)` sets the random-number seed so we can reproduce our result. We did not set the random-number seed when we ran `nptrend ... exact` earlier. If we were to run that again, we would get a different value for the exact  $p$ -value. Here are the results with `exact()` specified this new way:

```
. nptrend relief, group(dose) carmitage
> exact(montecarlo, show reps(100000) dots(1000) rseed(1234))
Permutations (100,000):10,000.....20,000.....30,000.....4
> 0,000.....50,000.....60,000.....70,000.....80,000.....90,
> 000.....100,000 done
Monte Carlo permutation results Number of observations = 800
Permutation variable: relief Number of permutations = 100,000
```

| T     | T(obs) | Test      | c     | n      | p     | Monte Carlo error |             |       |
|-------|--------|-----------|-------|--------|-------|-------------------|-------------|-------|
|       |        |           |       |        |       | SE(p)             | [95% CI(p)] |       |
| _pm_1 | .003   | lower     | 97435 | 100000 | .9744 | .0005             | .9734       | .9753 |
|       |        | upper     | 2959  | 100000 | .0296 | .0005             | .0285       | .0307 |
|       |        | two-sided |       |        | .0592 | .0007             | .0577       | .0606 |

Notes: For lower one-sided test,  $c = \#\{T \leq T(\text{obs})\}$  and  $p = p_{\text{lower}} = c/n$ .  
 For upper one-sided test,  $c = \#\{T \geq T(\text{obs})\}$  and  $p = p_{\text{upper}} = c/n$ .  
 For two-sided test,  $p = 2 \cdot \min(p_{\text{lower}}, p_{\text{upper}})$ ; SE and CI approximate.

Cochran-Armitage test for trend

```
Number of observations = 800
Number of groups = 4
Number of response levels = 2
```

| Group | Group score | Mean response score | Number of obs |
|-------|-------------|---------------------|---------------|
| dose  |             |                     |               |
| 10    | 10          | .6                  | 200           |
| 20    | 20          | .54                 | 200           |
| 30    | 30          | .585                | 200           |
| 40    | 40          | .685                | 200           |

```
Statistic = .003
Std. err. = .0015476
z = 1.939
Prob > |z| = 0.0526
Exact prob = 0.0592 (100,000 Monte Carlo permutations)
```

Test of departure from trend:

```
chi2(2) = 5.45
Prob > chi2 = 0.0656
```

The reported exact  $p$ -value from this specification is 0.0592, and we see from the table reported by `permut` that the Monte Carlo 95% confidence interval for it is [0.0577, 0.0606]. Clearly, the asymptotic  $p$ -value 0.0526 is a bit anticonservative. If we were going to publish the result, we might want to run the command again with even more permutations, perhaps 1,000,000 or more, to narrow the width of the confidence level to less than 0.001. With only 800 observations in these data, the Monte Carlo permutations are computed quickly.

## ▷ Example 2

We continue with the [previous example](#) to illustrate the use of the `scoregroup()` option. When we tested the linear trend of dose, we got a  $p$ -value of 0.0656 for the departure from linearity. So maybe the trend is not linear. Let's test a quadratic trend.

To use the `scoregroup()` option, the variable in `group()` must be 1, 2, ... The variable dose is 10, 20, 30, 40. We can convert it to 1, 2, 3, 4 in several ways. Here we use the `egen group()` function because this works in general.

```
. egen idose = group(dose)
. tabulate idose
```

| group(dose) | Freq. | Percent | Cum.   |
|-------------|-------|---------|--------|
| 1           | 200   | 25.00   | 25.00  |
| 2           | 200   | 25.00   | 50.00  |
| 3           | 200   | 25.00   | 75.00  |
| 4           | 200   | 25.00   | 100.00 |
| Total       | 800   | 100.00  |        |

Now, we specify `scoregroup(1 4 9 16)` to test a quadratic trend.

```
. nptrend relief, group(idose) carmitage scoregroup(1 4 9 16)
Cochran-Armitage test for trend
 Number of observations = 800
 Number of groups = 4
Number of response levels = 2
```

| Group | Group score | Mean<br>response<br>score | Number<br>of obs |
|-------|-------------|---------------------------|------------------|
| idose |             |                           |                  |
| 1     | 1           | .6                        | 200              |
| 2     | 4           | .54                       | 200              |
| 3     | 9           | .585                      | 200              |
| 4     | 16          | .685                      | 200              |

```
Statistic = .0070543
Std. err. = .0030468
 z = 2.315
Prob > |z| = 0.0206
```

```
Test of departure from trend:
 chi2(2) = 3.85
Prob > chi2 = 0.1462
```

Of course, we could have just typed `generate dose2 = dose^2` and used this variable for `group()` to test the quadratic trend.

The  $p$ -value for the quadratic trend is 0.0206, and the departure  $p$ -value is 0.1462. The  $p$ -value for the linear trend was 0.0526. So it appears the relationship between dose and relief is closer to quadratic than linear. But should we be using `nptrend` to search for the trend with the smallest  $p$ -value? Surely, `logit` is the tool for that!

```

. generate dose2 = dose^2
. logit relief dose dose2
Iteration 0: Log likelihood = -537.58798
Iteration 1: Log likelihood = -532.96973
Iteration 2: Log likelihood = -532.96346
Iteration 3: Log likelihood = -532.96346

Logistic regression Number of obs = 800
LR chi2(2) = 9.25
Prob > chi2 = 0.0098
Pseudo R2 = 0.0086

Log likelihood = -532.96346

```

| relief | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|--------|-------------|-----------|-------|-------|----------------------|-----------|
| dose   | -.0720094   | .0367396  | -1.96 | 0.050 | -.1440177            | -1.19e-06 |
| dose2  | .0017005    | .0007276  | 2.34  | 0.019 | .0002744             | .0031266  |
| _cons  | .946503     | .4021702  | 2.35  | 0.019 | .1582638             | 1.734742  |

To be clear, we are not recommending a model such as this for these data. There are only four doses, and this logit model fits three of the four degrees of freedom. The purpose of the Cochran–Armitage test is to test a single hypothesized trend and report an honest  $p$ -value for it. If we want to test for a trend but do not have any idea what the trend might be, we may be better off using the Jonckheere–Terpstra test.

◀

## Jonckheere–Terpstra test

The Jonckheere–Terpstra test is useful when it is not clear what the trend might be and we simply want to test for any trend. That is, it tests whether the ordering of the groups is associated with the ordering of the responses. The test is typically used when there are many response levels and any number of groups.

Suppose there are  $J \geq 2$  ordered groups. Let  $y_{jk}$  be the  $k$ th response in the  $j$ th group, where  $k = 1, 2, \dots, n_j$ . The responses  $y_{jk}$  are given by *varname* (the variable specified immediately after `nptrend`) or by *varname* mapped to the values of `scoreresponse()`.

For groups  $j$  and  $j'$ , where  $j < j'$ , consider the  $n_j n_{j'}$  pairs

$$(y_{jk}, y_{j'k'})$$

Let  $C_{jj'}$  be the number of pairs such that  $y_{jk} < y_{j'k'}$  and  $D_{jj'}$  be the number of pairs such that  $y_{jk} > y_{j'k'}$ . The Jonckheere–Terpstra statistic is

$$T = \sum_{j < j'} (C_{jj'} - D_{jj'})$$

In other words, count the number of response pairs,  $C_{jj'}$ , where the group  $j$  response is less than the group  $j'$  response. That is, pairs in which the ordering of the responses is concordant with the ordering of the groups. Subtract from this the number of discordant pairs,  $D_{jj'}$ . Then, sum over the  $J(J-1)/2$  combinations of selecting two of the  $J$  groups, where group  $j$  is ordered before group  $j'$ .

Clearly, only the ordering of responses matters relative to the ordering of the groups. There is no need to hypothesize what the trend is.

Kendall's  $\tau$  (see [R] [spearman](#)) also uses only relative differences, and the Jonckheere–Terpstra test can be viewed as a generalization of Kendall's  $\tau$  for groups of unequal sizes. See [Jonckheere–Terpstra test for trend](#) in *Methods and formulas* below.

► Example 3

The following data (Altman 1991, 217) show ocular exposure to ultraviolet radiation for 32 pairs of sunglasses classified into 3 groups according to the amount of visible light transmitted.

| Group | Transmission of visible light | Ocular exposure to ultraviolet radiation    |
|-------|-------------------------------|---------------------------------------------|
| 1     | < 25%                         | 1.4 1.4 1.4 1.6 2.3 2.5                     |
| 2     | 25 to 35%                     | 0.9 1.0 1.1 1.1 1.2 1.2 1.5 1.9 2.2 2.6 2.6 |
| 3     | > 35%                         | 2.6 2.8 2.8 3.2 3.5 4.3 5.1                 |
|       |                               | 0.8 1.7 1.7 1.7 3.4 7.1 8.9 13.5            |

Entering these data into Stata, we have

```
. use https://www.stata-press.com/data/r19/sg, clear
(Ultraviolet radiation exposure with sunglasses)
. list, separator(6)
```

|                         | group      | exposure |
|-------------------------|------------|----------|
| 1.                      | < 25%      | 1.4      |
| 2.                      | < 25%      | 1.4      |
| 3.                      | < 25%      | 1.4      |
| 4.                      | < 25%      | 1.6      |
| 5.                      | < 25%      | 2.3      |
| 6.                      | < 25%      | 2.5      |
| 7.                      | 25% to 35% | .9       |
| 8.                      | 25% to 35% | 1        |
| <i>(output omitted)</i> |            |          |
| 31.                     | > 35%      | 8.9      |
| 32.                     | > 35%      | 13.5     |

We can now use np trend to report the Jonckheere–Terpstra test results.

```
. np trend exposure, group(group) jterpstra
Jonckheere–Terpstra test for trend
Number of observations = 32
Number of groups = 3
Number of response levels = 23
```

| Group       | Group score | Mean response score | Number of obs |
|-------------|-------------|---------------------|---------------|
| group < 25% | 1           | 1.766667            | 6             |
| 25% to 35%  | 2           | 2.311111            | 18            |
| > 35%       | 3           | 4.85                | 8             |

```
Statistic = 82
Std. err. = 54.80056
z = 1.496
Prob > |z| = 0.1346
```

The approximate  $p$ -value is 0.1346. This  $p$ -value is an approximation to the permutation test  $p$ -value. That is, it comes from a  $z$  statistic that uses the exact variance of the permutation distribution. With only 32 observations, it seems better to simply compute the exact  $p$ -value using Monte Carlo permutations.

```
. set seed 1234
. nptrend exposure, group(group) jterpstra exact
Permutations (10,000):1,000.....2,000.....3,000.....4,000
>5,000.....6,000.....7,000.....8,000.....9,000.....
> ..10,000 done
Jonckheere-Terpstra test for trend
 Number of observations = 32
 Number of groups = 3
Number of response levels = 23
```

| Group       | Group score | Mean response score | Number of obs |
|-------------|-------------|---------------------|---------------|
| group < 25% | 1           | 1.766667            | 6             |
| 25% to 35%  | 2           | 2.311111            | 18            |
| > 35%       | 3           | 4.85                | 8             |

```
Statistic = 82
Std. err. = 54.80056
 z = 1.496
Prob > |z| = 0.1346
Exact prob = 0.1378 (10,000 Monte Carlo permutations)
```

Despite the small number of observations, the Monte Carlo exact  $p$ -value 0.1378 is close to the approximate  $p$ -value 0.1346. Because of the small number of observations, we might be tempted to try the `exact(enumerate)` option. We do so.

```
. nptrend exposure, group(group) jterpstra exact(enumerate, dots(1000))
(enumerating all 1.42e+12 possible permutations)
Permutations (1,415,721,106,800):10,000.....—Break—
r(1);
```

`nptrend` calls `permute` to do the enumeration, and `permute` tells us there are  $1.42 \times 10^{12}$  possible permutations. Because we do not want to wait years for the result, we pressed `Break`.

Let's randomly cut the dataset in half, from 32 observations to 16.

```
. set seed 25
. generate r = runiform()
. sort r
. keep in 1/16
(16 observations deleted)
. tabulate group
```

| Transmission of visible light | Freq. | Percent | Cum.   |
|-------------------------------|-------|---------|--------|
| < 25%                         | 2     | 12.50   | 12.50  |
| 25% to 35%                    | 9     | 56.25   | 68.75  |
| > 35%                         | 5     | 31.25   | 100.00 |
| Total                         | 16    | 100.00  |        |

Then, let's do the enumeration for these data:

```
. nptrend exposure, group(group) jterpstra exact(enumerate, dots(1000))
(enumerating all 240,240 possible permutations)
Permutations (240,240):10,000.....20,000.....30,000.....4
> 0,000.....50,000.....60,000.....70,000.....80,000.....90,
> 000.....100,000.....110,000.....120,000.....130,000.....1
> 40,000.....150,000.....160,000.....170,000.....180,000.....
> ..190,000.....200,000.....210,000.....220,000.....230,000....
>240,000. done
```

```
Jonckheere–Terpstra test for trend
Number of observations = 16
Number of groups = 3
Number of response levels = 15
```

| Group       | Group score | Mean response score | Number of obs |
|-------------|-------------|---------------------|---------------|
| group < 25% | 1           | 1.5                 | 2             |
| 25% to 35%  | 2           | 2.377778            | 9             |
| > 35%       | 3           | 6.92                | 5             |

```
Statistic = 49
Std. err. = 19.57188
z = 2.504
Prob > |z| = 0.0123
Exact prob = 0.0103 (enumerated all 240,240 permutations)
```

There are only 240,240 possible permutations now, and the computation takes little time. Despite there being only 16 observations, the exact *p*-value (without any error) 0.0103 is close to the approximate *p*-value 0.0123. Enumeration works quite well for tiny datasets such as this.



### Linear-by-linear trend test

The linear-by-linear trend test is an alternative to the Jonckheere–Terpstra test. The difference is that the linear-by-linear trend test uses response scores to specify the trend being tested. How the trend is hypothesized to vary across groups is specified by the group scores. The linear-by-linear trend test could be described as a parametric specification for trend that is tested nonparametrically!

If scores are not specified in the options, then the values of the response *varname* are used as response scores, and the values of *groupvar* are used as group scores.

The linear-by-linear statistic is

$$T = \frac{1}{N} \sum_{i=1}^I \sum_{j=1}^J n_{ij} (g_j - \bar{g})(r_i - \bar{r})$$

where  $N$  is the total number of observations,  $g_j$  is the group score for the  $j$ th group,  $\bar{g}$  is the mean group score over all observations,  $r_i$  is the response score for the  $i$ th ordered response, and  $\bar{r}$  is the mean response score over all observations.

The linear-by-linear statistic is equivalent to the Pearson correlation coefficient (see [R] [correlate](#)), the difference being that the Pearson correlation coefficient is standardized by the standard deviations of the scores. The  $p$ -values are slightly different because the  $p$ -value for the linear-by-linear test is based on its permutation distribution while the  $p$ -value for the Pearson correlation coefficient assumes normality.

Clearly, the statistic depends on the numerical values of both the response scores and the group scores. Different scores, even ones that give the same ordering, will produce different values of the statistic.

#### ▷ Example 4

We again use the sunglasses data from [example 3](#), where we did the Jonckheere–Terpstra test.

```
. use https://www.stata-press.com/data/r19/sg, clear
(Ultraviolet radiation exposure with sunglasses)
```

We calculate the linear-by-linear trend test for these data.

```
. nptrend exposure, group(group) linear
```

```
Linear-by-linear test for trend
```

```
Number of observations = 32
Number of groups = 3
Number of response levels = 23
```

| Group       | Group score | Mean response score | Number of obs |
|-------------|-------------|---------------------|---------------|
| group < 25% | 1           | 1.766667            | 6             |
| 25% to 35%  | 2           | 2.311111            | 18            |
| > 35%       | 3           | 4.85                | 8             |

```
Statistic = .7035156
Std. err. = .3063377
z = 2.297
Prob > |z| = 0.0216
```

The approximate  $p$ -value is 0.0216, which is much smaller than the Jonckheere–Terpstra test approximate  $p$ -value 0.1346. Such a difference is not surprising. The linear-by-linear trend test uses the actual values of the response in the statistic. The values of the response seem to follow a linear trend by group, and when this is true, one generally expects the  $p$ -value from the linear-by-linear trend test to be smaller than the  $p$ -value from the Jonckheere–Terpstra test.

Again, we might want to compute the exact  $p$ -value using Monte Carlo permutations.

```
. nptrend exposure, group(group) linear
> exact(montecarlo, reps(100000) dots(1000) rseed(1234))
Permutations (100,000):10,000.....20,000.....30,000.....4
> 0,000.....50,000.....60,000.....70,000.....80,000.....90,
> 000.....100,000 done
Linear-by-linear test for trend
 Number of observations = 32
 Number of groups = 3
Number of response levels = 23
```

| Group       | Group score | Mean response score | Number of obs |
|-------------|-------------|---------------------|---------------|
| group < 25% | 1           | 1.766667            | 6             |
| 25% to 35%  | 2           | 2.311111            | 18            |
| > 35%       | 3           | 4.85                | 8             |

```
Statistic = .7035156
Std. err. = .3063377
z = 2.297
Prob > |z| = 0.0216
Exact prob = 0.0146 (100,000 Monte Carlo permutations)
```

In this case, the exact  $p$ -value 0.0146 is smaller than the approximate  $p$ -value 0.0216.

◀

## Cuzick's test

Cuzick's test (Cuzick 1985) is an extension of the Kruskal–Wallis test (see [R] [kwallis](#)) for ordered groups. The response scores are ranked, the sum of the ranks is calculated for each group, and then these sums of ranks are weighted by the deviation of the group scores from their mean.

The statistic for Cuzick's test is

$$T = \frac{1}{N} \sum_{j=1}^J (g_j - \bar{g}) R_j$$

where  $N$  is the total number of observations,  $g_j$  is the group score for the  $j$ th group,  $\bar{g}$  is the mean group score over all observations, and  $R_j$  is the sum of the ranks of the response scores in the  $j$ th group.

The form of the statistic is similar to that of the statistic for the linear-by-linear test, but instead of using response scores being used directly, the ranks of the response scores are used. So only the ordering of the response scores matters. The numerical values of the group scores, however, are used in the calculation, so different group scores will give different values of the statistic.

## ▶ Example 5

We continue with the sunglasses data from [example 4](#). We now calculate Cuzick's test, calculating the exact  $p$ -value as well.

```
. nptrend exposure, group(group) cuzick
> exact(montecarlo, reps(10000) dots(1000) rseed(1234))
Permutations (100,000):10,000.....20,000.....30,000.....4
> 0,000.....50,000.....60,000.....70,000.....80,000.....90,
> 000.....100,000 done
```

Cuzick's test with rank scores

```
Number of observations = 32
Number of groups = 3
Number of response levels = 23
```

| Group          | Group score | Mean<br>response<br>score | Number<br>of obs |
|----------------|-------------|---------------------------|------------------|
| group<br>< 25% | 1           | 1.766667                  | 6                |
| 25% to 35%     | 2           | 2.311111                  | 18               |
| > 35%          | 3           | 4.85                      | 8                |

```
Statistic = 1.65625
Std. err. = 1.090461
z = 1.519
Prob > |z| = 0.1288
Exact prob = 0.1322 (100,000 Monte Carlo permutations)
```

The approximate  $p$ -value 0.1288 is close to the exact  $p$ -value 0.1322. These results are also close to the results from the Jonckheere–Terpstra test, where the approximate  $p$ -value was 0.1346. This is not surprising because using ranks of the responses in Cuzick's test is likely to be similar to the comparisons of orderings in the Jonckheere–Terpstra test. The latter test is better known than Cuzick's test, so for this reason alone, it may be preferable to use the Jonckheere–Terpstra test when reporting results.

◀

## Stored results

nptrend stores the following in `r()`:

### Scalars

```
r(N) number of observations
r(n_groups) number of groups
r(n_rsp_levels) number of response levels
r(beta) slope from Cochran–Armitage linear probability model
r(T) statistic
r(se) standard error of the statistic
r(z) z statistic
r(p) two-sided p -value from z statistic
r(p_l) lower one-sided p -value from z statistic
r(p_u) upper one-sided p -value from z statistic
r(chi2_depart) χ^2 for departure from Cochran–Armitage linear probability model
r(df_depart) degrees of freedom for departure χ^2
r(p_depart) p -value for departure χ^2
r(p_exact) two-sided exact p -value
r(p_l_exact) lower one-sided exact p -value
```

|                           |                                                       |
|---------------------------|-------------------------------------------------------|
| <code>r(p_u_exact)</code> | upper one-sided exact $p$ -value                      |
| <code>r(n_perm)</code>    | number of permutations performed                      |
| Macros                    |                                                       |
| <code>r(test)</code>      | "carmitage", "jterpstra", "linear", or "cuzick"       |
| <code>r(group)</code>     | group variable                                        |
| <code>r(exact)</code>     | "montecarlo" or "enumerate"                           |
| <code>r(rngstate)</code>  | random-number state used for Monte Carlo permutations |
| Matrices                  |                                                       |
| <code>r(table)</code>     | table of mean response scores by group                |

If `exact(..., show)` is specified, the stored results from `permute` are returned as well; see [Stored results](#) in [\[R\] permute](#).

## Methods and formulas

Methods and formulas are presented under the following headings:

[Overview](#)  
[Cochran–Armitage test for trend](#)  
[Jonckheere–Terpstra test for trend](#)  
[Linear-by-linear test for trend](#)  
[Cuzick’s test with rank scores](#)

## Overview

We will use the same notation for all the tests. Let the set of distinct responses be indexed by  $i = 1, 2, \dots, I$ . Let the groups be indexed by  $j = 1, 2, \dots, J$ .

We call the scores for the responses “response scores” and denote them by  $r_i$ .

For the Cochran–Armitage test (when the response is 0/1), linear-by-linear trend test, and Cuzick’s test the groups have scores as well. We call these scores “group scores”, and denote them by  $g_j$ .

All the tests for trend reported by `nptrend` are based on tests of independence between the ordering of the responses (or their scores) and the ordering of the groups (or their scores). Because these are tests of independence, there is no need to hypothesize a direction of causal inference. Responses could be outcomes predicted by group membership, or “groups” could be outcomes predicted by what we are calling “responses”.

Let  $n_{ij}$  be the number of observations in the  $i$ th response category and  $j$ th group. Let  $N = \sum_{ij} n_{ij}$  be the total number of observations. Let  $n_{i+} = \sum_j n_{ij}$  be the row margins, and let  $n_{+j} = \sum_i n_{ij}$  be the column margins.

## Cochran–Armitage test for trend

When `carmitage` is specified, the Cochran–Armitage test is performed ([Cochran 1954](#); [Armitage 1955](#); [Agresti 2013](#)). It assesses whether the probability of a positive response is associated with a trend based on ordered groups. Or the specification can be flipped. It can be used to assess whether a trend based on ordered responses is associated with membership in one of two groups.

In these methods, we assume that the response is 0/1—that is, the response is indexed by  $i = 0, 1$ —and that there are  $J$  ordered groups with scores  $g_j$ . When the specification is flipped, response and group are interchanged in the equations below.

The Cochran–Armitage test is based on a linear probability model for  $\theta_j$ , the probability of having response  $i = 1$  for the subjects in the  $j$ th group,

$$\theta_j = \alpha + \beta(g_j - \bar{g})$$

where  $\alpha$  is the intercept,  $\beta$  is the slope,  $g_j$  are the group scores, and  $\bar{g}$  is the mean of the group scores.

The equation is solved using OLS,

$$\hat{\theta}_j = p + \hat{\beta}(g_j - \bar{g})$$

where  $p = (\sum_j n_{1j})/N$  and  $\bar{g} = (\sum_j n_{+j}g_j)/N$ .

The OLS solution for  $\hat{\beta}$  is

$$\hat{\beta} = \frac{\sum_j n_{+j}(p_j - p)(g_j - \bar{g})}{\sum_j n_{+j}(g_j - \bar{g})^2}$$

where  $p_j = n_{1j}/n_{+j}$ .

The test for independence between groups and responses can be separated into a test for the linear trend and a test for departure from the linear trend. The Pearson statistic for independence

$$X_{\text{Ind}}^2 = \frac{1}{p(1-p)} \sum_{j=1}^J n_{+j}(p_j - p)^2$$

is decomposed as

$$X_{\text{Ind}}^2 = z^2 + X_{\text{DL}}^2$$

where

$$z^2 = \frac{\hat{\beta}^2}{p(1-p)} \sum_{j=1}^J n_{+j}(g_j - \bar{g})^2$$

and

$$X_{\text{DL}}^2 = \frac{1}{p(1-p)} \sum_{j=1}^J n_{+j}(p_j - \hat{\theta}_j)^2$$

$z$  is the  $z$  statistic for the linear trend, and  $X_{\text{DL}}^2$  is the statistic for departure from a linear trend. Asymptotically,  $z$  has a standard normal distribution, and  $X_{\text{DL}}^2$  has a  $\chi^2$  distribution with  $J - 2$  degrees of freedom.

## Jonckheere–Terpstra test for trend

When `jterpstra` is specified, the Jonckheere–Terpstra test is performed (Terpstra 1952; Jonckheere 1954). The Jonckheere–Terpstra test is a test of whether the hypothesized ordering of the responses is associated with the hypothesized ordering of the groups. Typically, this test is used when there are only a few groups but many response categories or even continuous responses.

Only the ordering of the responses and groups matter. The numerical values of the scores do not matter, but scores can be specified to change the ordering.

Let  $y_{jk}$  be the  $k$ th response in the  $j$ th group, where  $k = 1, 2, \dots, n_{+j}$ . If the response falls in the  $i$ th response category, then  $y_{jk} = r_i$ , the response score for the  $i$ th category.

For groups  $j$  and  $j'$ , where  $j < j'$ , consider the pairs

$$(y_{jk}, y_{j'k'})$$

where  $k = 1, 2, \dots, n_{+j}$  and  $k' = 1, 2, \dots, n_{+j'}$ . A total of  $n_{+j}n_{+j'}$  pairs are formed for these two groups.

Let  $C_{jj'}$  be the number of pairs such that  $y_{jk} < y_{j'k'}$ , that is, pairs in which the response ordering is concordant with the group ordering. Let  $D_{jj'}$  be the number of discordant pairs with  $y_{jk} > y_{j'k'}$ . The Jonckheere–Terpstra statistic is

$$T = \sum_{j=1}^{J-1} \sum_{j'=j+1}^J (C_{jj'} - D_{jj'})$$

The Jonckheere–Terpstra statistic is sometimes expressed as sums over  $C_{jj'} + E_{jj'}/2$ , where  $E_{jj'}$  is the number of tied pairs with  $y_{jk} = y_{j'k'}$ . Because  $C_{jj'} + D_{jj'} + E_{jj'} = n_{+j}n_{+j'}$ , the statistic  $T$  is twice the statistic based on  $C_{jj'} + E_{jj'}/2$  minus a constant. The advantage of the formulation using  $C_{jj'} - D_{jj'}$  is that under the null hypothesis of independence between groups and responses, its expectation is zero. When  $T$  is positive, the observed trend is a positive trend between the group ordering and the response ordering, and when  $T$  is negative, it is a negative trend.

The variance of  $T$  can be computed exactly based on the permutation distribution of responses where the table margins  $n_{i+}$  and  $n_{+j}$  are fixed and each permutation is considered equally likely under the null hypothesis. The formula, however, is complicated when there are ties.

Let  $M$  be the number of distinct values in the responses. Let  $e_m$  be the multiplicity of the  $m$ th distinct response value. That is, the  $N$  observations  $y_{jk}$  consist of  $e_1$  observations with value  $z_1$ ,  $e_2$  observations with value  $z_2$ , etc., with  $\sum_m e_m = N$ . The variance of  $T$  under the null permutation distribution is exactly

$$\begin{aligned} \text{Var}(T) = & \frac{1}{18} \left[ N(N-1)(2N+5) - \sum_{j=1}^J n_{+j}(n_{+j}-1)(2n_{+j}+5) \right. \\ & \left. - \sum_{m=1}^M e_m(e_m-1)(2e_m+5) \right] \\ & + \frac{1}{2N(N-1)} \left\{ \sum_{j=1}^J n_{+j}(n_{+j}-1) \right\} \left\{ \sum_{m=1}^M e_m(e_m-1) \right\} \\ & + \frac{1}{9N(N-1)(N-2)} \left\{ \sum_{j=1}^J n_{+j}(n_{+j}-1)(n_{+j}-2) \right\} \\ & \quad \times \left\{ \sum_{m=1}^M e_m(e_m-1)(e_m-2) \right\} \end{aligned}$$

The approximate  $p$ -value is based on a normal approximation using the exact variance of  $T$ .

## Linear-by-linear test for trend

When `linear` is specified, the linear-by-linear trend test is calculated based on the statistic

$$T = \frac{1}{N} \sum_{i=1}^I \sum_{j=1}^J n_{ij} (g_j - \bar{g}) (r_i - \bar{r})$$

where  $g_j$  is the group score for the  $j$ th group,  $\bar{g} = (\sum_j n_{+j} g_j) / N$  is the mean group score over all observations,  $r_i$  is the response score for the  $i$ th ordered response, and  $\bar{r} = (\sum_i n_{i+} r_i) / N$  is the mean response score over all observations.

Under the null hypothesis of independence between groups and responses, the expectation of  $T$  is zero. The variance of  $T$  is based on the permutation distribution of responses where the table margins  $n_{i+}$  and  $n_{+j}$  are fixed and each permutation is considered equally likely under the null hypothesis. The variance of  $T$  is exactly

$$\text{Var}(T) = \frac{1}{N^2(N-1)} \left\{ \sum_{j=1}^J n_{+j} (g_j - \bar{g})^2 \right\} \left\{ \sum_{i=1}^I n_{i+} (r_i - \bar{r})^2 \right\}$$

The approximate  $p$ -value is based on a normal approximation using the exact variance of  $T$ .

Note the symmetry between the group and response scores in these equations. In the linear-by-linear test for trend, it is arbitrary as to what is a group with a group score and what is a response with a response score. The two are interchangeable.

## Cuzick's test with rank scores

When `cuzick` is specified, the test for trend is based on a method in [Cuzick \(1985\)](#), which is described in [Altman \(1991, 215–217\)](#).

Rank all  $N$  responses, using average ranks for ties. Let  $R_j$  be the sum of the ranks in the  $j$ th group. The statistic for Cuzick's test  $T$  is

$$T = \frac{1}{N} \sum_{j=1}^J (g_j - \bar{g}) R_j$$

where  $g_j$  is the group score for the  $j$ th group and  $\bar{g} = (\sum_j n_{+j} g_j) / N$ .

Under the null hypothesis of independence between groups and responses, the expectation of  $T$  is zero. When there are no ties, the variance of  $T$  is

$$\text{Var}(T) = \frac{N+1}{12N} \sum_{j=1}^J n_{+j} (g_j - \bar{g})^2$$

When there are ties, we again let  $e_m$  be the multiplicity of the  $m$ th distinct response value,  $m = 1, 2, \dots, M$ . Define

$$a = \frac{1}{N(N^2-1)} \sum_{m=1}^M e_m (e_m^2 - 1)$$

The corrected variance of  $T$  when there are ties is

$$\widetilde{\text{Var}}(T) = (1-a) \text{Var}(T)$$

## Exact p-values

Exact  $p$ -values are computed by `permut.e`. The variable permuted is that specified by `group()`, except for `carmitage` when the response variable is 0/1; in this case, the response variable is permuted. For details on the permutation computation, see [R] [permut.e](#).

## Acknowledgments

The original version of `nptrend` was written by K. A. Stepniewska, Worldwide Antimalarial Resistance Network, University of Oxford, UK, and D. G. Altman (1948–2018).

## References

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## Also see

- [R] [EpiTab](#) — Tables for epidemiologists
- [R] [kwallis](#) — Kruskal–Wallis equality-of-populations rank test
- [R] [permut.e](#) — Permutation tests
- [R] [signrank](#) — Equality tests on matched data
- [R] [spearman](#) — Spearman's and Kendall's correlations
- [R] [symmetry](#) — Symmetry and marginal homogeneity tests
- [ST] [stmc](#) — Calculate rate ratios with the Mantel–Cox method
- [ST] [stmh](#) — Calculate rate ratios with the Mantel–Haenszel method

[Description](#)  
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[Methods and formulas](#)

## Description

`ologit` fits ordered logit models of ordinal variable *depvar* on the independent variables *indepvars*. The actual values taken on by the dependent variable are irrelevant, except that larger values are assumed to correspond to “higher” outcomes.

## Quick start

Ordinal logit model of *y* on *x1* and categorical variables *a* and *b*

```
ologit y x1 i.a i.b
```

Same as above, and include interaction between *a* and *b* and report results as odds ratios

```
ologit y x1 a##b, or
```

With bootstrap standard errors

```
ologit y x1 i.a i.b, vce(bootstrap)
```

Analysis restricted to cases where `catvar = 0` using `svyset` data with replicate weights

```
svy bootstrap, subpop(if catvar==0): ologit y x1 i.a i.b
```

## Menu

Statistics > Ordinal outcomes > Ordered logistic regression

## Syntax

```
ologit depvar [indepvars] [if] [in] [weight] [, options]
```

| <i>options</i>                               | Description                                                                                                                                                                 |
|----------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                 |                                                                                                                                                                             |
| <code>offset(<i>varname</i>)</code>          | include <i>varname</i> in model with coefficient constrained to 1                                                                                                           |
| <code>constraints(<i>constraints</i>)</code> | apply specified linear constraints                                                                                                                                          |
| <b>SE/Robust</b>                             |                                                                                                                                                                             |
| <code>vce(<i>vcetype</i>)</code>             | <i>vcetype</i> may be <code>oim</code> , <code>opg</code> , <code>robust</code> , <code>cluster <i>clustvar</i></code> , <code>bootstrap</code> , or <code>jackknife</code> |
| <b>Reporting</b>                             |                                                                                                                                                                             |
| <code>level(#)</code>                        | set confidence level; default is <code>level(95)</code>                                                                                                                     |
| <code>or</code>                              | report odds ratios                                                                                                                                                          |
| <code>nocnsreport</code>                     | do not display constraints                                                                                                                                                  |
| <code>display_options</code>                 | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling                            |
| <b>Maximization</b>                          |                                                                                                                                                                             |
| <code>maximize_options</code>                | control the maximization process; seldom used                                                                                                                               |
| <code>collinear</code>                       | keep collinear variables                                                                                                                                                    |
| <code>coeflegend</code>                      | display legend instead of statistics                                                                                                                                        |

*indepvars* may contain factor variables; see [U] 11.4.3 **Factor variables**.

*depvar* and *indepvars* may contain time-series operators; see [U] 11.4.4 **Time-series varlists**.

`bayes`, `bayesboot`, `bootstrap`, `by`, `fmm`, `fp`, `jackknife`, `mfp`, `mi estimate`, `nestreg`, `rolling`, `statsby`, `stepwise`, and `svy` are allowed; see [U] 11.1.10 **Prefix commands**. For more details, see [BAYES] **bayes: ologit** and [FMM] **fmm: ologit**.

`vce(bootstrap)` and `vce(jackknife)` are not allowed with the `mi estimate` prefix; see [MI] **mi estimate**.

Weights are not allowed with the `bootstrap` prefix; see [R] **bootstrap**.

`vce()` and weights are not allowed with the `svy` prefix; see [SVY] **svy**.

`fweights`, `iwweights`, and `pweights` are allowed; see [U] 11.1.6 **weight**.

`collinear` and `coeflegend` do not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options

### Model

`offset(varname)`, `constraints(constraints)`; see [R] **Estimation options**.

### SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] **vce\_option**.

## Reporting

`level(#)`; see [R] [Estimation options](#).

or reports the estimated coefficients transformed to odds ratios, that is,  $e^b$  rather than  $b$ . Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated. or may be specified at estimation or when replaying previously estimated results.

`nocnsreport`; see [R] [Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

## Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

The following options are available with `ologit` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Ordered logit models are used to estimate relationships between an ordinal dependent variable and a set of independent variables. An *ordinal* variable is a variable that is categorical and ordered, for instance, “poor”, “good”, and “excellent”, which might indicate a person’s current health status or the repair record of a car. If there are only two outcomes, see [R] [logistic](#), [R] [logit](#), and [R] [probit](#). This entry is concerned only with more than two outcomes. If the outcomes cannot be ordered (for example, residency in the north, east, south, or west), see [R] [mlogit](#). This entry is concerned only with models in which the outcomes can be ordered.

In ordered logit, an underlying score is estimated as a linear function of the independent variables and a set of cutpoints. The probability of observing outcome  $i$  corresponds to the probability that the estimated linear function, plus random error, is within the range of the cutpoints estimated for the outcome:

$$\Pr(\text{outcome}_j = i) = \Pr(\kappa_{i-1} < \beta_1 x_{1j} + \beta_2 x_{2j} + \cdots + \beta_k x_{kj} + u_j \leq \kappa_i)$$

$u_j$  is assumed to be logistically distributed in ordered logit. In either case, we estimate the coefficients  $\beta_1, \beta_2, \dots, \beta_k$  together with the cutpoints  $\kappa_1, \kappa_2, \dots, \kappa_{k-1}$ , where  $k$  is the number of possible outcomes.  $\kappa_0$  is taken as  $-\infty$ , and  $\kappa_k$  is taken as  $+\infty$ . All of this is a direct generalization of the ordinary two-outcome logit model.

► Example 1

We wish to analyze the 1977 repair records of 66 foreign and domestic cars. The data are a variation of the automobile dataset described in [U] 1.2.2 Example datasets. The 1977 repair records, like those in 1978, take on values “Poor”, “Fair”, “Average”, “Good”, and “Excellent”. Here is a cross-tabulation of the data:

```
. use https://www.stata-press.com/data/r19/fullauto
(Automobile models)
. tabulate rep77 foreign, chi2
```

| Repair record<br>1977 | Foreign  |         | Total |
|-----------------------|----------|---------|-------|
|                       | Domestic | Foreign |       |
| Poor                  | 2        | 1       | 3     |
| Fair                  | 10       | 1       | 11    |
| Average               | 20       | 7       | 27    |
| Good                  | 13       | 7       | 20    |
| Excellent             | 0        | 5       | 5     |
| Total                 | 45       | 21      | 66    |

Pearson chi2(4) = 13.8619 Pr = 0.008

Although it appears that foreign takes on the values Domestic and Foreign, it is actually a numeric variable taking on the values 0 and 1. Similarly, rep77 takes on the values 1, 2, 3, 4, and 5, corresponding to Poor, Fair, and so on. The more meaningful words appear because we have attached value labels to the data; see [U] 12.6.3 Value labels.

Because the  $\chi^2$  value is significant, we could claim that there is a relationship between foreign and rep77. Literally, however, we can only claim that the distributions are different; the  $\chi^2$  test is not directional. One way to model these data is to model the categorization that took place when the data were created. Cars have a true frequency of repair, which we will assume is given by  $S_j = \beta \text{foreign}_j + u_j$ , and a car is categorized as “poor” if  $S_j \leq \kappa_0$ , as “fair” if  $\kappa_0 < S_j \leq \kappa_1$ , and so on:

```
. ologit rep77 foreign
Iteration 0: Log likelihood = -89.895098
Iteration 1: Log likelihood = -85.951765
Iteration 2: Log likelihood = -85.908227
Iteration 3: Log likelihood = -85.908161
Iteration 4: Log likelihood = -85.908161
Ordered logistic regression
Log likelihood = -85.908161
Number of obs = 66
LR chi2(1) = 7.97
Prob > chi2 = 0.0047
Pseudo R2 = 0.0444
```

| rep77   | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |           |
|---------|-------------|-----------|------|-------|----------------------|-----------|
| foreign | 1.455878    | .5308951  | 2.74 | 0.006 | .4153425             | 2.496413  |
| /cut1   | -2.765562   | .5988208  |      |       | -3.939229            | -1.591895 |
| /cut2   | -.9963603   | .3217706  |      |       | -1.627019            | -.3657016 |
| /cut3   | .9426153    | .3136398  |      |       | .3278925             | 1.557338  |
| /cut4   | 3.123351    | .5423257  |      |       | 2.060412             | 4.18629   |

Our model is  $S_j = 1.46 \text{foreign}_j + u_j$ ; the expected value for foreign cars is 1.46 and, for domestic cars, 0; foreign cars have better repair records.

The estimated cutpoints tell us how to interpret the score. For a foreign car, the probability of a poor record is the probability that  $1.46 + u_j \leq -2.77$ , or equivalently,  $u_j \leq -4.23$ . Making this calculation requires familiarity with the logistic distribution: the probability is  $1/(1 + e^{4.23}) = 0.014$ . On the other hand, for domestic cars, the probability of a poor record is the probability  $u_j \leq -2.77$ , which is 0.059.

This, it seems to us, is a far more reasonable prediction than we would have made based on the table alone. The table showed that 2 of 45 domestic cars had poor records, whereas 1 of 21 foreign cars had poor records—corresponding to probabilities  $2/45 = 0.044$  and  $1/21 = 0.048$ . The predictions from our model imposed a smoothness assumption—foreign cars should not, overall, have better repair records without the difference revealing itself in each category. In our data, the fractions of foreign and domestic cars in the poor category are virtually identical only because of the randomness associated with small samples.

Thus if we were asked to predict the true fractions of foreign and domestic cars that would be classified in the various categories, we would choose the numbers implied by the ordered logit model:

|           | tabulate |         | logit    |         |
|-----------|----------|---------|----------|---------|
|           | Domestic | Foreign | Domestic | Foreign |
| Poor      | 0.044    | 0.048   | 0.059    | 0.014   |
| Fair      | 0.222    | 0.048   | 0.210    | 0.065   |
| Average   | 0.444    | 0.333   | 0.450    | 0.295   |
| Good      | 0.289    | 0.333   | 0.238    | 0.467   |
| Excellent | 0.000    | 0.238   | 0.043    | 0.159   |

See [R] [ologit postestimation](#) for a more complete explanation of how to generate predictions from an ordered logit model.



### □ Technical note

Here ordered logit provides an alternative to ordinary two-outcome logistic models with an arbitrary dichotomization, which might otherwise have been tempting. We could, for instance, have summarized these data by converting the five-outcome `rep77` variable to a two-outcome variable, combining cars in the average, fair, and poor categories to make one outcome and combining cars in the good and excellent categories to make the second.

Another even less appealing alternative would have been to use ordinary regression, arbitrarily labeling “excellent” as 5, “good” as 4, and so on. The problem is that with different but equally valid labelings (say, 10 for “excellent”), we would obtain different estimates. We would have no way of choosing one metric over another. That assertion is not, however, true of `ologit`. The actual values used to label the categories make no difference other than through the order they imply.

In fact, our labeling was 5 for “excellent”, 4 for “good”, and so on. The words “excellent” and “good” appear in our output because we attached a value label to the variables; see [U] [12.6.3 Value labels](#). If we were to now go back and type `replace rep77=10 if rep77==5`, changing all the 5s to 10s, we would still obtain the same results when we refit our model.



## ▷ Example 2

In the [example](#) above, we used ordered logit as a way to model a table. We are not, however, limited to including only one explanatory variable or to including only categorical variables. We can explore the relationship of `rep77` with any of the variables in our data. We might, for instance, model `rep77` not only in terms of the origin of manufacture, but also including `length` (a proxy for size) and `mpg`:

```
. ologit rep77 foreign length mpg
Iteration 0: Log likelihood = -89.895098
Iteration 1: Log likelihood = -78.775147
Iteration 2: Log likelihood = -78.254294
Iteration 3: Log likelihood = -78.250719
Iteration 4: Log likelihood = -78.250719

Ordered logistic regression Number of obs = 66
LR chi2(3) = 23.29
Prob > chi2 = 0.0000
Pseudo R2 = 0.1295

Log likelihood = -78.250719
```

| rep77   | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |          |
|---------|-------------|-----------|------|-------|----------------------|----------|
| foreign | 2.896807    | .7906411  | 3.66 | 0.000 | 1.347179             | 4.446435 |
| length  | .0828275    | .02272    | 3.65 | 0.000 | .0382972             | .1273579 |
| mpg     | .2307677    | .0704548  | 3.28 | 0.001 | .0926788             | .3688566 |
| /cut1   | 17.92748    | 5.551191  |      |       | 7.047344             | 28.80761 |
| /cut2   | 19.86506    | 5.59648   |      |       | 8.896161             | 30.83396 |
| /cut3   | 22.10331    | 5.708936  |      |       | 10.914               | 33.29262 |
| /cut4   | 24.69213    | 5.890754  |      |       | 13.14647             | 36.2378  |

`foreign` still plays a role—and an even larger role than previously. We find that larger cars tend to have better repair records, as do cars with better mileage ratings.



## Stored results

`ologit` stores the following in `e()`:

### Scalars

|                            |                                              |
|----------------------------|----------------------------------------------|
| <code>e(N)</code>          | number of observations                       |
| <code>e(N_cd)</code>       | number of completely determined observations |
| <code>e(k_cat)</code>      | number of categories                         |
| <code>e(k)</code>          | number of parameters                         |
| <code>e(k_aux)</code>      | number of auxiliary parameters               |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>     |
| <code>e(k_eq_model)</code> | number of equations in overall model test    |
| <code>e(k_dv)</code>       | number of dependent variables                |
| <code>e(df_m)</code>       | model degrees of freedom                     |
| <code>e(r2_p)</code>       | pseudo- $R^2$                                |
| <code>e(ll)</code>         | log likelihood                               |
| <code>e(ll_0)</code>       | log likelihood, constant-only model          |
| <code>e(N_clust)</code>    | number of clusters                           |
| <code>e(chi2)</code>       | $\chi^2$                                     |
| <code>e(p)</code>          | $p$ -value for model test                    |
| <code>e(rank)</code>       | rank of <code>e(V)</code>                    |
| <code>e(ic)</code>         | number of iterations                         |
| <code>e(rc)</code>         | return code                                  |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise                  |

## Macros

|                                |                                                                          |
|--------------------------------|--------------------------------------------------------------------------|
| <code>e(cmd)</code>            | <code>ologit</code>                                                      |
| <code>e(cmdline)</code>        | command as typed                                                         |
| <code>e(depvar)</code>         | name of dependent variable                                               |
| <code>e(wtype)</code>          | weight type                                                              |
| <code>e(wexp)</code>           | weight expression                                                        |
| <code>e(title)</code>          | title in estimation output                                               |
| <code>e(clustvar)</code>       | name of cluster variable                                                 |
| <code>e(offset)</code>         | linear offset variable                                                   |
| <code>e(chi2type)</code>       | Wald or LR; type of model $\chi^2$ test                                  |
| <code>e(vce)</code>            | <i>vce</i> type specified in <code>vce()</code>                          |
| <code>e(vcetype)</code>        | title used to label Std. err.                                            |
| <code>e(opt)</code>            | type of optimization                                                     |
| <code>e(which)</code>          | max or min; whether optimizer is to perform maximization or minimization |
| <code>e(ml_method)</code>      | type of ml method                                                        |
| <code>e(user)</code>           | name of likelihood-evaluator program                                     |
| <code>e(technique)</code>      | maximization technique                                                   |
| <code>e(properties)</code>     | <code>b V</code>                                                         |
| <code>e(predict)</code>        | program used to implement <code>predict</code>                           |
| <code>e(marginsdefault)</code> | default <code>predict()</code> specification for <code>margins</code>    |
| <code>e(asbalanced)</code>     | factor variables <code>fvset</code> as <code>asbalanced</code>           |
| <code>e(asobserved)</code>     | factor variables <code>fvset</code> as <code>asobserved</code>           |

## Matrices

|                              |                                              |
|------------------------------|----------------------------------------------|
| <code>e(b)</code>            | coefficient vector                           |
| <code>e(Cns)</code>          | constraints matrix                           |
| <code>e(ilog)</code>         | iteration log (up to 20 iterations)          |
| <code>e(gradient)</code>     | gradient vector                              |
| <code>e(cat)</code>          | category values                              |
| <code>e(V)</code>            | variance–covariance matrix of the estimators |
| <code>e(V_modelbased)</code> | model-based variance                         |

## Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

## Matrices

|                       |                                                                                                                            |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

## Methods and formulas

See [Long and Freese \(2014, chap. 7\)](#) for a discussion of models for ordinal outcomes and examples that use Stata. [Cameron and Trivedi \(2005, chap. 15\)](#) describe multinomial models, including the model fit by `ologit`. When you have a qualitative dependent variable, several estimation procedures are available. A popular choice is multinomial logistic regression (see [\[R\] `mlogit`](#)), but if you use this procedure when the response variable is ordinal, you are discarding information because multinomial logit ignores the ordered aspect of the outcome. Ordered logit and probit models provide a means to exploit the ordering information.

There is more than one “ordered logit” model. The model fit by `ologit`, which we will call the ordered logit model, is also known as the proportional odds model. Another popular choice, not fit by `ologit`, is known as the stereotype model; see [\[R\] `slogit`](#). All ordered logit models have been derived by starting with a binary logit/probit model and generalizing it to allow for more than two outcomes.

The proportional-odds ordered logit model is so called because, if we consider the odds  $\text{odds}(k) = P(Y \leq k)/P(Y > k)$ , then  $\text{odds}(k_1)$  and  $\text{odds}(k_2)$  have the same ratio for all independent variable combinations. The model is based on the principle that the only effect of combining adjoining categories in ordered categorical regression problems should be a loss of efficiency in estimating the regression parameters (McCullagh 1980). This model was also described by McKelvey and Zavoina (1975) and, previously, by Aitchison and Silvey (1957) in a different algebraic form. Brant (1990) offers a set of diagnostics for the model.

Peterson and Harrell (1990) suggest a model that allows nonproportional odds for a subset of the explanatory variables. `ologit` does not allow this, but a model similar to this was implemented by Fu (1998).

The stereotype model rejects the principle on which the ordered logit model is based. Anderson (1984) argues that there are two distinct types of ordered categorical variables: “grouped continuous”, such as income, where the “type a” model applies; and “assessed”, such as extent of pain relief, where the stereotype model applies. Greenland (1985) independently developed the same model. The stereotype model starts with a multinomial logistic regression model and imposes constraints on this model.

Goodness of fit for `ologit` can be evaluated by comparing the likelihood value with that obtained by fitting the model with `mlogit`. Let  $\ln L_1$  be the log-likelihood value reported by `ologit`, and let  $\ln L_0$  be the log-likelihood value reported by `mlogit`. If there are  $p$  independent variables (excluding the constant) and  $k$  categories, `mlogit` will estimate  $p(k - 1)$  additional parameters. We can then perform a “likelihood-ratio test”, that is, calculate  $-2(\ln L_1 - \ln L_0)$ , and compare it with  $\chi^2\{p(k - 2)\}$ . This test is suggestive only because the ordered logit model is not nested within the multinomial logit model. A large value of  $-2(\ln L_1 - \ln L_0)$  should, however, be taken as evidence of poorness of fit. Marginally large values, on the other hand, should not be taken too seriously.

The coefficients and cutpoints are estimated using maximum likelihood as described in [R] **Maximize**. In our parameterization, no constant appears, because the effect is absorbed into the cutpoints.

`ologit` and `oprobit` begin by tabulating the dependent variable. Category  $i = 1$  is defined as the minimum value of the variable,  $i = 2$  as the next ordered value, and so on, for the empirically determined  $k$  categories.

The probability of a given observation for ordered logit is

$$\begin{aligned}
 p_{ij} &= \Pr(y_j = i) = \Pr(\kappa_{i-1} < \mathbf{x}_j\boldsymbol{\beta} + u \leq \kappa_i) \\
 &= \frac{1}{1 + \exp(-\kappa_i + \mathbf{x}_j\boldsymbol{\beta})} - \frac{1}{1 + \exp(-\kappa_{i-1} + \mathbf{x}_j\boldsymbol{\beta})}
 \end{aligned}$$

$\kappa_0$  is defined as  $-\infty$  and  $\kappa_k$  as  $+\infty$ .

For ordered probit, the probability of a given observation is

$$\begin{aligned}
 p_{ij} &= \Pr(y_j = i) = \Pr(\kappa_{i-1} < \mathbf{x}_j\boldsymbol{\beta} + u \leq \kappa_i) \\
 &= \Phi(\kappa_i - \mathbf{x}_j\boldsymbol{\beta}) - \Phi(\kappa_{i-1} - \mathbf{x}_j\boldsymbol{\beta})
 \end{aligned}$$

where  $\Phi(\cdot)$  is the standard normal cumulative distribution function.

The log likelihood is

$$\ln L = \sum_{j=1}^N w_j \sum_{i=1}^k I_i(y_j) \ln p_{ij}$$

where  $w_j$  is an optional weight and

$$I_i(y_j) = \begin{cases} 1, & \text{if } y_j = i \\ 0, & \text{otherwise} \end{cases}$$

ologit and oprobit support the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] `_robust`, particularly *Maximum likelihood estimators* and *Methods and formulas*.

These commands also support estimation with survey data. For details on VCEs with survey data, see [SVY] *Variance estimation*.

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## Also see

- [R] **ologit postestimation** — Postestimation tools for ologit
- [R] **clgit** — Conditional (fixed-effects) logistic regression
- [R] **logistic** — Logistic regression, reporting odds ratios
- [R] **logit** — Logistic regression, reporting coefficients
- [R] **mlogit** — Multinomial (polytomous) logistic regression
- [R] **oprobit** — Ordered probit regression
- [R] **slogit** — Stereotype logistic regression
- [R] **ziologit** — Zero-inflated ordered logit regression
- [BAYES] **bayes: ologit** — Bayesian ordered logistic regression
- [CM] **cmrologit** — Rank-ordered logit choice model
- [CM] **cmroprobit** — Rank-ordered probit choice model
- [FMM] **fmm: ologit** — Finite mixtures of ordered logistic regression models
- [ME] **meologit** — Multilevel mixed-effects ordered logistic regression
- [MI] **Estimation** — Estimation commands for use with mi estimate
- [SVY] **svy estimation** — Estimation commands for survey data
- [XT] **xtologit** — Random-effects ordered logistic model
- [U] **20 Estimation and postestimation commands**

Postestimation commands    [predict](#)    [margins](#)    [Remarks and examples](#)  
 Reference    [Also see](#)

## Postestimation commands

The following postestimation commands are available after `ologit`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| <code>linktest</code>        | link test for model specification                                                                                                       |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | probabilities, linear predictions and their SEs, etc.                                                                                   |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\* `forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results. `forecast` is also not appropriate with `mi` estimation results.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as probabilities, linear predictions, and standard errors.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] { stub* | newvar | newvarlist } [if] [in] [, statistic
 outcome(outcome) nooffset]
```

```
predict [type] stub* [if] [in] , scores
```

| <i>statistic</i>  | Description                             |
|-------------------|-----------------------------------------|
| Main              |                                         |
| <code>pr</code>   | predicted probabilities; the default    |
| <code>xb</code>   | linear prediction                       |
| <code>stdp</code> | standard error of the linear prediction |

You specify one or  $k$  new variables with `pr`, where  $k$  is the number of outcomes. If you specify one new variable and you do not specify `outcome()`, then `outcome(#1)` is assumed.

You specify one new variable with `xb` and `stdp`.

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

## Options for predict

### Main

`pr`, the default, computes the predicted probabilities for all outcomes or for a specific outcome. To compute probabilities for all outcomes, you specify  $k$  new variables, where  $k$  is the number of categories of the dependent variable. Alternatively, you can specify `stub*`; in which case, `pr` will store predicted probabilities in variables `stub1`, `stub2`, ..., `stubk`. To compute the probability for a specific outcome, you specify one new variable and, optionally, the outcome value in option `outcome()`; if you omit `outcome()`, the first outcome value, `outcome(#1)`, is assumed.

Say that you fit a model by typing `estimation_cmd y x1 x2`, and `y` takes on four values. Then, you could type `predict p1 p2 p3 p4` to obtain all four predicted probabilities; alternatively, you could type `predict p*` to generate the four predicted probabilities. To compute specific probabilities one at a time, you can type `predict p1, outcome(#1)` (or simply `predict p1`), `predict p2, outcome(#2)`, and so on. See option `outcome()` for other ways to refer to outcome values.

`xb` calculates the linear prediction. You specify one new variable, for example, `predict linear, xb`. The linear prediction is defined, ignoring the contribution of the estimated cutpoints.

`stdp` calculates the standard error of the linear prediction. You specify one new variable, for example, `predict se, stdp`.

`outcome(outcome)` specifies for which outcome the predicted probabilities are to be calculated. `outcome()` should contain either one value of the dependent variable or one of #1, #2, ..., with #1 meaning the first category of the dependent variable, #2 meaning the second category, etc. `outcome()` is available only with the default `pr` option.

`nooffset` is relevant only if you specified `offset(varname)` for `ologit`. It modifies the calculations made by `predict` so that they ignore the offset variable; the linear prediction is treated as  $\mathbf{x}_j\mathbf{b}$  rather than as  $\mathbf{x}_j\mathbf{b} + \text{offset}_j$ . `nooffset` is not allowed with `scores`.

`scores` calculates equation-level score variables. The number of score variables created will equal the number of outcomes in the model. If the number of outcomes in the model was  $k$ , then

the first new variable will contain  $\partial \ln L / \partial (\mathbf{x}_j\mathbf{b})$ ;

the second new variable will contain  $\partial \ln L / \partial \kappa_1$ ;

the third new variable will contain  $\partial \ln L / \partial \kappa_2$ ;

...

and the  $k$ th new variable will contain  $\partial \ln L / \partial \kappa_{k-1}$ , where  $\kappa_i$  refers to the  $i$ th cutpoint.

## margins

### Description for margins

`margins` estimates margins of response for probabilities and linear predictions.

### Menu for margins

Statistics > Postestimation

### Syntax for margins

`margins [marginlist] [ , options ]`

`margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]`

| <i>statistic</i>  | Description                           |
|-------------------|---------------------------------------|
| default           | probabilities for each outcome        |
| <code>pr</code>   | probability for a specified outcome   |
| <code>xb</code>   | linear prediction                     |
| <code>stdp</code> | not allowed with <code>margins</code> |

`pr` defaults to the first outcome.

Statistics not allowed with `margins` are functions of stochastic quantities other than `e(b)`.

For the full syntax, see [R] [margins](#).

## Remarks and examples

See [U] 20 Estimation and postestimation commands for instructions on obtaining the variance–covariance matrix of the estimators, predicted values, and hypothesis tests. Also see [R] lrtest for performing likelihood-ratio tests.

### ▷ Example 1

In example 2 of [R] ologit, we fit the model `ologit rep77 foreign length mpg`. The `predict` command can be used to obtain the predicted probabilities.

We type `predict` followed by the names of the new variables to hold the predicted probabilities, ordering the names from low to high. In our data, the lowest outcome is “poor”, and the highest is “excellent”. We have five categories, so we must type five names following `predict`; the choice of names is up to us:

```
. use https://www.stata-press.com/data/r19/fullauto
(Automobile models)
. ologit rep77 foreign length mpg
(output omitted)
. predict poor fair avg good exc
(option pr assumed; predicted probabilities)
. list exc good make model rep78 if rep77>=., sep(4) divider
```

|     | exc      | good     | make    | model   | rep78   |
|-----|----------|----------|---------|---------|---------|
| 3.  | .0033341 | .0393056 | AMC     | Spirit  | .       |
| 10. | .0098392 | .1070041 | Buick   | Opel    | .       |
| 32. | .0023406 | .0279497 | Ford    | Fiesta  | Good    |
| 44. | .015697  | .1594413 | Merc.   | Monarch | Average |
| 53. | .065272  | .4165188 | Peugeot | 604     | .       |
| 56. | .005187  | .059727  | Plym.   | Horizon | Average |
| 57. | .0261461 | .2371826 | Plym.   | Sapporo | .       |
| 63. | .0294961 | .2585825 | Pont.   | Phoenix | .       |

The eight cars listed were introduced after 1977, so they do not have 1977 repair records in our data. We predicted what their 1977 repair records might have been using the fitted model. We see that, based on its characteristics, the Peugeot 604 had about a  $41.65 + 6.53 \approx 48.2\%$  chance of a good or excellent repair record. The Ford Fiesta, which had only a 3% chance of a good or excellent repair record, in fact, had a good record when it was introduced in the following year.



□ Technical note

For ordered logit, `predict, xb` produces  $S_j = x_{1j}\beta_1 + x_{2j}\beta_2 + \dots + x_{kj}\beta_k$ . The ordered-logit predictions are then the probability that  $S_j + u_j$  lies between a pair of cutpoints,  $\kappa_{i-1}$  and  $\kappa_i$ . Some handy formulas are

$$\begin{aligned} \Pr(S_j + u_j < \kappa) &= 1/(1 + e^{S_j - \kappa}) \\ \Pr(S_j + u_j > \kappa) &= 1 - 1/(1 + e^{S_j - \kappa}) \\ \Pr(\kappa_1 < S_j + u_j < \kappa_2) &= 1/(1 + e^{S_j - \kappa_2}) - 1/(1 + e^{S_j - \kappa_1}) \end{aligned}$$

Rather than using `predict` directly, we could calculate the predicted probabilities by hand. If we wished to obtain the predicted probability that the repair record is excellent and the probability that it is good, we look back at `ologit`'s output to obtain the cutpoints. We find that “good” corresponds to the interval `/cut3 < S_j + u < /cut4` and “excellent” to the interval  $S_j + u > /cut4$ :

```
. predict score, xb
. generate probgood = 1/(1+exp(score-_b[/cut4])) - 1/(1+exp(score-_b[/cut3]))
. generate probexc = 1 - 1/(1+exp(score-_b[/cut4]))
```

The results of our calculation will be the same as those produced in the previous example. We refer to the estimated cutpoints just as we would any coefficient, so `_b[/cut3]` refers to the value of the `/cut3` coefficient; see [U] 13.5 Accessing coefficients and standard errors.

□

## Reference

Fagerland, M. W., and D. W. Hosmer, Jr. 2017. How to test for goodness of fit in ordinal logistic regression models. *Stata Journal* 17: 668–686.

## Also see

- [R] `ologit` — Ordered logistic regression
- [U] 20 Estimation and postestimation commands

[Description](#)  
[Options](#)  
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[Quick start](#)  
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[Menu](#)  
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## Description

The `oneway` command reports one-way analysis-of-variance (ANOVA) models and performs multiple-comparison tests.

If you wish to fit more complicated ANOVA layouts or wish to fit analysis-of-covariance (ANCOVA) models, see [\[R\] anova](#).

See [\[D\] encode](#) for examples of fitting ANOVA models on string variables.

See [\[R\] loneway](#) for an alternative `oneway` command with slightly different features.

## Quick start

One-way ANOVA model of  $y$  for factor  $a$

```
oneway y a
```

Report the mean and std. dev. of  $y$  and number of observations for each level of  $a$

```
oneway y a, tabulate
```

Report all pairwise comparisons of the means of  $y$  across levels of  $a$  with  $p$ -values adjusted using Bonferroni's procedure

```
oneway y a, bonferroni
```

Same as above, but adjust  $p$ -values for multiple comparisons using Scheffé's method

```
oneway y a, scheffe
```

## Menu

Statistics > Linear models and related > ANOVA/MANOVA > One-way ANOVA

## Syntax

```
oneway response_var factor_var [if] [in] [weight] [, options]
```

| options                   | Description                                                            |
|---------------------------|------------------------------------------------------------------------|
| Main                      |                                                                        |
| <code>bonferroni</code>   | Bonferroni multiple-comparison test                                    |
| <code>scheffe</code>      | Scheffé multiple-comparison test                                       |
| <code>sidak</code>        | Šidák multiple-comparison test                                         |
| <code>tabulate</code>     | produce summary table                                                  |
| <code>[no]means</code>    | include or suppress means; default is means                            |
| <code>[no]standard</code> | include or suppress standard deviations; default is standard           |
| <code>[no]freq</code>     | include or suppress frequencies; default is freq                       |
| <code>[no]obs</code>      | include or suppress number of obs; default is obs if data are weighted |
| <code>noanova</code>      | suppress the ANOVA table                                               |
| <code>no label</code>     | show numeric codes, not labels                                         |
| <code>wrap</code>         | do not break wide tables                                               |
| <code>missing</code>      | treat missing values as categories                                     |

`by` and `collect` are allowed; see [U] 11.1.10 Prefix commands.

`awweights` and `fweights` are allowed; see [U] 11.1.6 weight.

## Options

Main

`bonferroni` reports the results of a Bonferroni multiple-comparison test.

`scheffe` reports the results of a Scheffé multiple-comparison test.

`sidak` reports the results of a Šidák multiple-comparison test.

`tabulate` produces a table of summary statistics of the *response\_var* by levels of the *factor\_var*. The table includes the mean, standard deviation, frequency, and, if the data are weighted, the number of observations. Individual elements of the table may be included or suppressed by using the `[no]means`, `[no]standard`, `[no]freq`, and `[no]obs` options. For example, typing

```
oneway response factor, tabulate means standard
```

produces a summary table that contains only the means and standard deviations. You could achieve the same result by typing

```
oneway response factor, tabulate nofreq
```

`[no]means` includes or suppresses only the means from the table produced by the `tabulate` option. See `tabulate` above.

`[no]standard` includes or suppresses only the standard deviations from the table produced by the `tabulate` option. See `tabulate` above.

`[no]freq` includes or suppresses only the frequencies from the table produced by the `tabulate` option. See `tabulate` above.

[no]obs includes or suppresses only the reported number of observations from the table produced by the `tabulate` option. If the data are not weighted, only the frequency is reported. If the data are weighted, the frequency refers to the sum of the weights. See `tabulate` above.

`noanova` suppresses the display of the ANOVA table.

`no-label` causes the numeric codes to be displayed rather than the value labels in the ANOVA and multiple-comparison test tables.

`wrap` requests that Stata not break up wide tables to make them more readable.

`missing` requests that missing values of *factor\_var* be treated as a category rather than as observations to be omitted from the analysis.

## Remarks and examples

Remarks are presented under the following headings:

[Introduction](#)  
[Obtaining observed means](#)  
[Multiple-comparison tests](#)  
[Weighted data](#)  
[Video example](#)

### Introduction

The `oneway` command reports one-way ANOVA models. To perform a one-way layout of a variable called `endog` on `exog`, type `oneway endog exog`.

#### ▷ Example 1

We run an experiment varying the amount of fertilizer used in growing apple trees. We test four concentrations, using each concentration in three groves of 12 trees each. Later in the year, we measure the average weight of the fruit.

If all had gone well, we would have had 3 observations on the average weight for each of the four concentrations. Instead, two of the groves were mistakenly leveled by a confused man on a large bulldozer. We are left with the following dataset:

```
. use https://www.stata-press.com/data/r19/apple
(Apple trees)
. describe
Contains data from https://www.stata-press.com/data/r19/apple.dta
Observations: 10 Apple trees
Variables: 2 16 Jan 2024 11:23
```

| Variable name | Storage type | Display format | Value label | Variable label          |
|---------------|--------------|----------------|-------------|-------------------------|
| treatment     | int          | %8.0g          |             | Fertilizer              |
| weight        | double       | %10.0g         |             | Average weight in grams |

Sorted by:

```
. list, abbreviate(10)
```

|     | treatment | weight |
|-----|-----------|--------|
| 1.  | 1         | 117.5  |
| 2.  | 1         | 113.8  |
| 3.  | 1         | 104.4  |
| 4.  | 2         | 48.9   |
| 5.  | 2         | 50.4   |
| 6.  | 2         | 58.9   |
| 7.  | 3         | 70.4   |
| 8.  | 3         | 86.9   |
| 9.  | 4         | 87.7   |
| 10. | 4         | 67.3   |

To obtain the one-way ANOVA results, we type

```
. oneway weight treatment
```

| Source         | Analysis of variance |    |            | F     | Prob > F |
|----------------|----------------------|----|------------|-------|----------|
|                | SS                   | df | MS         |       |          |
| Between groups | 5295.54433           | 3  | 1765.18144 | 21.46 | 0.0013   |
| Within groups  | 493.591667           | 6  | 82.2652778 |       |          |
| Total          | 5789.136             | 9  | 643.237333 |       |          |

Bartlett's equal-variances test:  $\chi^2(3) = 1.3900$      $\text{Prob} > \chi^2 = 0.708$

We find significant (at better than the 1% level) differences among the four concentrations.



### □ Technical note

Rather than using the oneway command, we could have performed this analysis by using anova. [Example 1](#) in [\[R\] anova](#) repeats this same analysis. You may wish to compare the output.

You will find the oneway command quicker than the anova command, and, as you will learn, oneway allows you to perform multiple-comparison tests. On the other hand, anova will let you generate predictions, examine the covariance matrix of the estimators, and perform more general hypothesis tests.



### □ Technical note

Although the output is a usual ANOVA table, let's run through it anyway. The between-group sum of squares for the model is 5295.5 with 3 degrees of freedom, resulting in a mean square of  $5295.5/3 \approx 1765.2$ . The corresponding  $F$  statistic is 21.46 and has a significance level of 0.0013. Thus, the model appears to be significant at the 0.13% level.

The second line summarizes the within-group (residual) variation. The within-group sum of squares is 493.59 with 6 degrees of freedom, resulting in a mean squared error of 82.27.

The between- and residual-group variations sum to the total sum of squares (TSS), which is reported as 5789.1 in the last line of the table. This is the TSS of `weight` after removal of the mean. Similarly, the between plus residual degrees of freedom sum to the total degrees of freedom, 9. Remember that there are 10 observations. Subtracting 1 for the mean, we are left with 9 total degrees of freedom.

At the bottom of the table, Bartlett's test for equal variances is reported. The value of the statistic is 1.39. The corresponding significance level ( $\chi^2$  with 3 degrees of freedom) is 0.708, so we cannot reject the assumption that the variances are homogeneous.



## Obtaining observed means

### ► Example 2

We typed oneway weight treatment to obtain an ANOVA table of weight of fruit by fertilizer concentration. Although we obtained the table, we obtained no information on which fertilizer seems to work the best. If we add the tabulate option, we obtain that additional information:

```
. oneway weight treatment, tabulate
```

| Fertilizer | Summary of Average weight in grams |           |       |
|------------|------------------------------------|-----------|-------|
|            | Mean                               | Std. dev. | Freq. |
| 1          | 111.9                              | 6.7535176 | 3     |
| 2          | 52.733333                          | 5.3928966 | 3     |
| 3          | 78.65                              | 11.667262 | 2     |
| 4          | 77.5                               | 14.424978 | 2     |
| Total      | 80.62                              | 25.362124 | 10    |

| Source         | Analysis of variance |    |            | F     | Prob > F |
|----------------|----------------------|----|------------|-------|----------|
|                | SS                   | df | MS         |       |          |
| Between groups | 5295.54433           | 3  | 1765.18144 | 21.46 | 0.0013   |
| Within groups  | 493.591667           | 6  | 82.2652778 |       |          |
| Total          | 5789.136             | 9  | 643.237333 |       |          |

```
Bartlett's equal-variances test: chi2(3) = 1.3900 Prob>chi2 = 0.708
```

We find that the average weight was largest when we used fertilizer concentration 1.



## Multiple-comparison tests

### ▷ Example 3: Bonferroni multiple-comparison test

oneway can also perform multiple-comparison tests using either Bonferroni, Scheffé, or Šidák normalizations. For instance, to obtain the Bonferroni multiple-comparison test, we specify the `bonferroni` option:

```
. oneway weight treatment, bonferroni
```

| Source         | Analysis of variance |    |            | F     | Prob > F |
|----------------|----------------------|----|------------|-------|----------|
|                | SS                   | df | MS         |       |          |
| Between groups | 5295.54433           | 3  | 1765.18144 | 21.46 | 0.0013   |
| Within groups  | 493.591667           | 6  | 82.2652778 |       |          |
| Total          | 5789.136             | 9  | 643.237333 |       |          |

```
Bartlett's equal-variances test: chi2(3) = 1.3900 Prob>chi2 = 0.708
```

Comparison of Average weight in grams by Fertilizer  
(Bonferroni)

| Row Mean-<br>Col Mean | 1                 | 2                | 3              |
|-----------------------|-------------------|------------------|----------------|
| 2                     | -59.1667<br>0.001 |                  |                |
| 3                     | -33.25<br>0.042   | 25.9167<br>0.122 |                |
| 4                     | -34.4<br>0.036    | 24.7667<br>0.146 | -1.15<br>1.000 |

The results of the Bonferroni test are presented as a matrix. The first entry,  $-59.17$ , represents the difference between fertilizer concentrations 2 and 1 (labeled “Row Mean - Col Mean” in the upper stub of the table). Remember that in the [previous example](#) we requested the `tabulate` option. Looking back, we find that the means of concentrations 1 and 2 are 111.90 and 52.73, respectively. Thus,  $52.73 - 111.90 = -59.17$ .

Underneath that number is reported “0.001”. This is the Bonferroni-adjusted significance of the difference. The difference is significant at the 0.1% level. Looking down the column, we see that concentration 3 is also worse than concentration 1 (4.2% level), as is concentration 4 (3.6% level).

On the basis of this evidence, we would use concentration 1 if we grew apple trees.

### ▷ Example 4: Scheffé multiple-comparison test

We can just as easily obtain the Scheffé-adjusted significance levels. Rather than specifying the `bonferroni` option, we specify the `scheffe` option.

We will also add the `noanova` option to prevent Stata from redisplaying the ANOVA table:

```
. oneway weight treatment, noanova scheffe
 Comparison of Average weight in grams by Fertilizer
 (Scheffe)
```

| Row Mean-<br>Col Mean | 1                 | 2                | 3              |
|-----------------------|-------------------|------------------|----------------|
| 2                     | -59.1667<br>0.001 |                  |                |
| 3                     | -33.25<br>0.039   | 25.9167<br>0.101 |                |
| 4                     | -34.4<br>0.034    | 24.7667<br>0.118 | -1.15<br>0.999 |

The differences are the same as those we obtained in the Bonferroni output, but the significance levels are not. According to the Bonferroni-adjusted numbers, the significance of the difference between fertilizer concentrations 1 and 3 is 4.2%. The Scheffé-adjusted significance level is 3.9%.

We will leave it to you to decide which results are more accurate.



### ▷ Example 5: Šidák multiple-comparison test

Let's conclude this example by obtaining the Šidák-adjusted multiple-comparison tests. We do this to illustrate Stata's capabilities to calculate these results, because searching across adjustment methods until you find the results you want is not a valid technique for obtaining significance levels.

```
. oneway weight treatment, noanova sidak
 Comparison of Average weight in grams by Fertilizer
 (Sidak)
```

| Row Mean-<br>Col Mean | 1                 | 2                | 3              |
|-----------------------|-------------------|------------------|----------------|
| 2                     | -59.1667<br>0.001 |                  |                |
| 3                     | -33.25<br>0.041   | 25.9167<br>0.116 |                |
| 4                     | -34.4<br>0.035    | 24.7667<br>0.137 | -1.15<br>1.000 |

We find results that are similar to the Bonferroni-adjusted numbers.



Henry Scheffé (1907–1977) was born in New York. He studied mathematics at the University of Wisconsin, gaining a doctorate with a dissertation on differential equations. He taught mathematics at Wisconsin, Oregon State University, and Reed College, but his interests changed to statistics and he joined Wilks at Princeton. After periods at Syracuse, UCLA, and Columbia, Scheffé settled in Berkeley from 1953. His research increasingly focused on linear models and particularly ANOVA, on which he produced a celebrated monograph. His death was the result of a bicycle accident.

## Weighted data

### ▷ Example 6

`oneway` can work with both weighted and unweighted data. Let's assume that we wish to perform a one-way layout of the deathrate on the four census regions of the United States using state data. Our data contain three variables, `drate` (the deathrate), `region` (the region), and `pop` (the population of the state).

To fit the model, we type `oneway drate region [weight=pop]`, although we typically abbreviate `weight` as `w`. We will also add the `tabulate` option to demonstrate how the table of summary statistics differs for weighted data:

```
. use https://www.stata-press.com/data/r19/census8
(1980 Census data by state)
. oneway drate region [w=pop], tabulate
(analytic weights assumed)
```

| Census region                                                                        | Summary of Deathrate |           |            | Obs   |          |
|--------------------------------------------------------------------------------------|----------------------|-----------|------------|-------|----------|
|                                                                                      | Mean                 | Std. dev. | Freq.      |       |          |
| NE                                                                                   | 97.15                | 5.82      | 49135283   | 9     |          |
| N Cntrl                                                                              | 88.10                | 5.58      | 58865670   | 12    |          |
| South                                                                                | 87.05                | 10.40     | 74734029   | 16    |          |
| West                                                                                 | 75.65                | 8.23      | 43172490   | 13    |          |
| Total                                                                                | 87.34                | 10.43     | 225907472  | 50    |          |
| Analysis of variance                                                                 |                      |           |            |       |          |
| Source                                                                               | SS                   | df        | MS         | F     | Prob > F |
| Between groups                                                                       | 2360.92281           | 3         | 786.974272 | 12.17 | 0.0000   |
| Within groups                                                                        | 2974.09635           | 46        | 64.6542685 |       |          |
| Total                                                                                | 5335.01916           | 49        | 108.877942 |       |          |
| Bartlett's equal-variances test: $\chi^2(3) = 5.4971$ $\text{Prob} > \chi^2 = 0.139$ |                      |           |            |       |          |

When the data are weighted, the summary table has four columns rather than three. The column labeled “Freq.” reports the sum of the weights. The overall frequency tells us that there are approximately 226 million people in the United States, or at least there were in 1980.

The ANOVA table is appropriately weighted. Also see [U] 11.1.6 [weight](#).



## Video example

[One-way ANOVA in Stata](#)

## Stored results

oneway stores the following in `r()`:

### Scalars

|                      |                                 |                          |                                  |
|----------------------|---------------------------------|--------------------------|----------------------------------|
| <code>r(N)</code>    | number of observations          | <code>r(df_m)</code>     | between-group degrees of freedom |
| <code>r(F)</code>    | $F$ statistic                   | <code>r(rss)</code>      | within-group sum of squares      |
| <code>r(df_r)</code> | within-group degrees of freedom | <code>r(chi2bart)</code> | Bartlett's $\chi^2$              |
| <code>r(mss)</code>  | between-group sum of squares    | <code>r(df_bart)</code>  | Bartlett's degrees of freedom    |

### Matrices

|                       |                                                                                                                 |
|-----------------------|-----------------------------------------------------------------------------------------------------------------|
| <code>r(ANOVA)</code> | matrix containing the sum of squares with their degrees of freedom, mean squares, $F$ statistic, and $p$ -value |
|-----------------------|-----------------------------------------------------------------------------------------------------------------|

## Methods and formulas

Methods and formulas are presented under the following headings:

[One-way analysis of variance](#)

[Bartlett's test](#)

[Multiple-comparison tests](#)

## One-way analysis of variance

The model of one-way ANOVA is

$$y_{ij} = \mu + \alpha_i + \epsilon_{ij}$$

for levels  $i = 1, \dots, k$  and observations  $j = 1, \dots, n_i$ . Define  $\bar{y}_i$  as the (weighted) mean of  $y_{ij}$  over  $j$  and  $\bar{y}$  as the overall (weighted) mean of  $y_{ij}$ . Define  $w_{ij}$  as the weight associated with  $y_{ij}$ , which is 1 if the data are unweighted.  $w_{ij}$  is normalized to sum to  $n = \sum_i n_i$  if `aweights` are used and is otherwise not normalized.  $w_i$  refers to  $\sum_j w_{ij}$ , and  $w$  refers to  $\sum_i w_i$ .

The between-group sum of squares is then

$$S_1 = \sum_i w_i (\bar{y}_i - \bar{y})^2$$

The TSS is

$$S = \sum_i \sum_j w_{ij} (y_{ij} - \bar{y})^2$$

The within-group sum of squares is given by  $S_e = S - S_1$ .

The between-group mean square is  $s_1^2 = S_1/(k - 1)$ , and the within-group mean square is  $s_e^2 = S_e/(w - k)$ . The test statistic is  $F = s_1^2/s_e^2$ . See, for instance, [Snedecor and Cochran \(1989\)](#).

## Bartlett's test

Bartlett's test assumes that you have  $m$  independent, normal, random samples and tests the hypothesis  $\sigma_1^2 = \sigma_2^2 = \dots = \sigma_m^2$ . The test statistic,  $M$ , is defined as

$$M = \frac{(T - m) \ln \hat{\sigma}^2 - \sum (T_i - 1) \ln \hat{\sigma}_i^2}{1 + \frac{1}{3(m-1)} \left\{ \left( \sum \frac{1}{T_i - 1} \right) - \frac{1}{T - m} \right\}}$$

where there are  $T$  overall observations,  $T_i$  observations in the  $i$ th group, and

$$(T_i - 1) \hat{\sigma}_i^2 = \sum_{j=1}^{T_i} (y_{ij} - \bar{y}_i)^2$$

$$(T - m) \hat{\sigma}^2 = \sum_{i=1}^m (T_i - 1) \hat{\sigma}_i^2$$

An approximate test of the homogeneity of variance is based on the statistic  $M$  with critical values obtained from the  $\chi^2$  distribution of  $m - 1$  degrees of freedom. See [Bartlett \(1937\)](#) or [Draper and Smith \(1998, 56–57\)](#).

## Multiple-comparison tests

Let's begin by reviewing the logic behind these adjustments. The “standard”  $t$  statistic for the comparison of two means is

$$t = \frac{\bar{y}_i - \bar{y}_j}{s \sqrt{\frac{1}{n_i} + \frac{1}{n_j}}}$$

where  $s$  is the overall standard deviation,  $\bar{y}_i$  is the measured average of  $y$  in group  $i$ , and  $n_i$  is the number of observations in the group. We perform hypothesis tests by calculating this  $t$  statistic. We simultaneously choose a critical level,  $\alpha$ , and look up the  $t$  statistic corresponding to that level in a table. We reject the hypothesis if our calculated  $t$  exceeds the value we looked up. Alternatively, because we have a computer at our disposal, we calculate the significance level  $e$  corresponding to our calculated  $t$  statistic, and if  $e < \alpha$ , we reject the hypothesis.

This logic works well when we are performing one test. Now consider what happens when we perform several separate tests, say,  $n$  of them. Let's assume, just for discussion, that we set  $\alpha$  equal to 0.05 and that we will perform six tests. For each test, we have a 0.05 probability of falsely rejecting the equality-of-means hypothesis. Overall, then, our chances of falsely rejecting at *least one* of the hypotheses is  $1 - (1 - 0.05)^6 \approx 0.26$  if the tests are independent.

The idea behind multiple-comparison tests is to control for the fact that we will perform multiple tests and to reduce our overall chances of falsely rejecting each hypothesis to  $\alpha$  rather than letting our chances increase with each additional test. (See [Miller \[1981\]](#) and [Hochberg and Tamhane \[1987\]](#) for rather advanced texts on multiple-comparison procedures.)

The Bonferroni adjustment (see [Miller \[1981\]](#); also see [van Belle et al. \[2004, 534–537\]](#)) does this by (falsely but approximately) asserting that the critical level we should use,  $a$ , is the true critical level,  $\alpha$ , divided by the number of tests,  $n$ ; that is,  $a = \alpha/n$ . For instance, if we are going to perform six tests, each at the 0.05 significance level, we want to adopt a critical level of  $0.05/6 \approx 0.00833$ .

We can just as easily apply this logic to  $e$ , the significance level associated with our  $t$  statistic, as to our critical level  $\alpha$ . If a comparison has a calculated significance of  $e$ , then its “real” significance, adjusted for the fact of  $n$  comparisons, is  $n \times e$ . If a comparison has a significance level of, say, 0.012, and we perform six tests, then its “real” significance is 0.072. If we adopt a critical level of 0.05, we cannot reject the hypothesis. If we adopt a critical level of 0.10, we can reject it.

Of course, this calculation can go above 1, but that just means that there is no  $\alpha < 1$  for which we could reject the hypothesis. (This situation arises because of the crude nature of the Bonferroni adjustment.) Stata handles this case by simply calling the significance level 1. Thus, the formula for the Bonferroni significance level is

$$e_b = \min(1, en)$$

where  $n = k(k - 1)/2$  is the number of comparisons.

The Šidák adjustment (Šidák [1967]; also see Winer, Brown, and Michels [1991, 165–166]) is slightly different and provides a tighter bound. It starts with the assertion that

$$a = 1 - (1 - \alpha)^{1/n}$$

Turning this formula around and substituting calculated significance levels, we obtain

$$e_s = \min\left\{1, 1 - (1 - e)^n\right\}$$

For example, if the calculated significance is 0.012 and we perform six tests, the “real” significance is approximately 0.07.

The Scheffé test (Scheffé [1953, 1959]; also see Kuehl [2000, 97–98]) differs in derivation, but it attacks the same problem. Let there be  $k$  means for which we want to make all the pairwise tests. Two means are declared significantly different if

$$t \geq \sqrt{(k - 1)F(\alpha; k - 1, \nu)}$$

where  $F(\alpha; k - 1, \nu)$  is the  $\alpha$ -critical value of the  $F$  distribution with  $k - 1$  numerator and  $\nu$  denominator degrees of freedom. Scheffé’s test has the nicety that it never declares a contrast significant if the overall  $F$  test is not significant.

Turning the test around, Stata calculates a significance level

$$\hat{e} = F\left(\frac{t^2}{k - 1}, k - 1, \nu\right)$$

For instance, you have a calculated  $t$  statistic of 4.0 with 50 degrees of freedom. The simple  $t$  test says that the significance level is 0.00021. The  $F$  test equivalent, 16 with 1 and 50 degrees of freedom, says the same. If you are comparing three means, however, you calculate an  $F$  test of 8.0 with 2 and 50 degrees of freedom, which says that the significance level is 0.0010.

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## Also see

[R] **anova** — Analysis of variance and covariance

[R] **loneway** — Large one-way ANOVA, random effects, and reliability

[PSS-2] **power oneway** — Power analysis for one-way analysis of variance

Description  
Options  
References

Quick start  
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Menu  
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Methods and formulas

## Description

`oprobit` fits ordered probit models of ordinal variable *depvar* on the independent variables *indepvars*. The actual values taken on by the dependent variable are irrelevant, except that larger values are assumed to correspond to “higher” outcomes.

## Quick start

Ordinal probit model of *y* on *x1* and categorical variables *a* and *b*

```
oprobit y x1 i.a i.b
```

Model of *y* on *x1* and a one-period lagged value of *x1* using `tsset` data

```
oprobit y x1 L.x1
```

Same as above, but calculate results for each level of *catvar* and save statistics to *myfile.dta*

```
statsby, by(catvar) saving(myfile): oprobit y x1 L.x1
```

## Menu

Statistics > Ordinal outcomes > Ordered probit regression

## Syntax

```
oprobit depvar [indepvars] [if] [in] [weight] [, options]
```

| <i>options</i>                               | Description                                                                                                                                                                 |
|----------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                 |                                                                                                                                                                             |
| <code>offset(<i>varname</i>)</code>          | include <i>varname</i> in model with coefficient constrained to 1                                                                                                           |
| <code>constraints(<i>constraints</i>)</code> | apply specified linear constraints                                                                                                                                          |
| <b>SE/Robust</b>                             |                                                                                                                                                                             |
| <code>vce(<i>vcetype</i>)</code>             | <i>vcetype</i> may be <code>oim</code> , <code>opg</code> , <code>robust</code> , <code>cluster <i>clustvar</i></code> , <code>bootstrap</code> , or <code>jackknife</code> |
| <b>Reporting</b>                             |                                                                                                                                                                             |
| <code>level(#)</code>                        | set confidence level; default is <code>level(95)</code>                                                                                                                     |
| <code>nocnsreport</code>                     | do not display constraints                                                                                                                                                  |
| <code>display_options</code>                 | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling                            |
| <b>Maximization</b>                          |                                                                                                                                                                             |
| <code>maximize_options</code>                | control the maximization process; seldom used                                                                                                                               |
| <code>collinear</code>                       | keep collinear variables                                                                                                                                                    |
| <code>coeflegend</code>                      | display legend instead of statistics                                                                                                                                        |

*indepvars* may contain factor variables; see [U] 11.4.3 **Factor variables**.

*depvar* and *indepvars* may contain time-series operators; see [U] 11.4.4 **Time-series varlists**.

`bayes`, `bayesboot`, `bootstrap`, `by`, `collect`, `fmm`, `fp`, `jackknife`, `mfp`, `mi estimate`, `nestreg`, `rolling`, `statsby`, `stepwise`, and `svy` are allowed; see [U] 11.1.10 **Prefix commands**. For more details, see [BAYES] **bayes: oprobit** and [FMM] **fmm: oprobit**.

`vce(bootstrap)` and `vce(jackknife)` are not allowed with the `mi estimate` prefix; see [MI] **mi estimate**.

Weights are not allowed with the `bootstrap` prefix; see [R] **bootstrap**.

`vce()` and weights are not allowed with the `svy` prefix; see [SVY] **svy**.

`fweights`, `iwweights`, and `pweights` are allowed; see [U] 11.1.6 **weight**.

`collinear` and `coeflegend` do not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options

### Model

`offset(varname)`, `constraints(constraints)`; see [R] **Estimation options**.

### SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] **vce\_option**.

## Reporting

`level(#)`; see [R] [Estimation options](#).

`nocnsreport`; see [R] [Estimation options](#).

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

## Maximization

*maximize\_options*: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

The following options are available with `oprobit` but is not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

An ordered probit model is used to estimate relationships between an ordinal dependent variable and a set of independent variables. An *ordinal* variable is a variable that is categorical and ordered, for instance, “poor”, “good”, and “excellent”, which might indicate a person’s current health status or the repair record of a car. If there are only two outcomes, see [R] [logistic](#), [R] [logit](#), and [R] [probit](#). This entry is concerned only with more than two outcomes. If the outcomes cannot be ordered (for example, residency in the north, east, south, or west), see [R] [mlogit](#). This entry is concerned only with models in which the outcomes can be ordered. See [R] [logistic](#) for a list of related estimation commands.

In ordered probit, an underlying score is estimated as a linear function of the independent variables and a set of cutpoints. The probability of observing outcome  $i$  corresponds to the probability that the estimated linear function, plus random error, is within the range of the cutpoints estimated for the outcome:

$$\Pr(\text{outcome}_j = i) = \Pr(\kappa_{i-1} < \beta_1 x_{1j} + \beta_2 x_{2j} + \cdots + \beta_k x_{kj} + u_j \leq \kappa_i)$$

$u_j$  is assumed to be normally distributed. In either case, we estimate the coefficients  $\beta_1, \beta_2, \dots, \beta_k$  together with the cutpoints  $\kappa_1, \kappa_2, \dots, \kappa_{I-1}$ , where  $I$  is the number of possible outcomes.  $\kappa_0$  is taken as  $-\infty$ , and  $\kappa_I$  is taken as  $+\infty$ . All of this is a direct generalization of the ordinary two-outcome probit model.

### ► Example 1

In [example 2](#) of [R] [ologit](#), we use a variation of the automobile dataset (see [U] [1.2.2 Example datasets](#)) to analyze the 1977 repair records of 66 foreign and domestic cars. We use ordered logit to explore the relationship of `rep77` in terms of `foreign` (origin of manufacture), `length` (a proxy for size), and `mpg`. Here we fit the same model using ordered probit rather than ordered logit:

```

. use https://www.stata-press.com/data/r19/fullauto
(Automobile models)
. oprobit rep77 foreign length mpg
Iteration 0: Log likelihood = -89.895098
Iteration 1: Log likelihood = -78.106316
Iteration 2: Log likelihood = -78.020086
Iteration 3: Log likelihood = -78.020025
Iteration 4: Log likelihood = -78.020025
Ordered probit regression
Log likelihood = -78.020025
Number of obs = 66
LR chi2(3) = 23.75
Prob > chi2 = 0.0000
Pseudo R2 = 0.1321

```

| rep77   | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |          |
|---------|-------------|-----------|------|-------|----------------------|----------|
| foreign | 1.704861    | .4246796  | 4.01 | 0.000 | .8725037             | 2.537217 |
| length  | .0468675    | .012648   | 3.71 | 0.000 | .022078              | .0716571 |
| mpg     | .1304559    | .0378628  | 3.45 | 0.001 | .0562463             | .2046656 |
| /cut1   | 10.1589     | 3.076754  |      |       | 4.128577             | 16.18923 |
| /cut2   | 11.21003    | 3.107527  |      |       | 5.119389             | 17.30067 |
| /cut3   | 12.54561    | 3.155233  |      |       | 6.361467             | 18.72975 |
| /cut4   | 13.98059    | 3.218793  |      |       | 7.671874             | 20.28931 |

We find that foreign cars have better repair records, as do larger cars and cars with better mileage ratings.

◀

## Stored results

oprobit stores the following in `e()`:

### Scalars

|                            |                                              |
|----------------------------|----------------------------------------------|
| <code>e(N)</code>          | number of observations                       |
| <code>e(N_cd)</code>       | number of completely determined observations |
| <code>e(k_cat)</code>      | number of categories                         |
| <code>e(k)</code>          | number of parameters                         |
| <code>e(k_aux)</code>      | number of auxiliary parameters               |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>     |
| <code>e(k_eq_model)</code> | number of equations in overall model test    |
| <code>e(k_dv)</code>       | number of dependent variables                |
| <code>e(df_m)</code>       | model degrees of freedom                     |
| <code>e(r2_p)</code>       | pseudo- $R^2$                                |
| <code>e(ll)</code>         | log likelihood                               |
| <code>e(ll_0)</code>       | log likelihood, constant-only model          |
| <code>e(N_clust)</code>    | number of clusters                           |
| <code>e(chi2)</code>       | $\chi^2$                                     |
| <code>e(p)</code>          | $p$ -value for model test                    |
| <code>e(rank)</code>       | rank of <code>e(V)</code>                    |
| <code>e(ic)</code>         | number of iterations                         |
| <code>e(rc)</code>         | return code                                  |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise                  |

### Macros

|                         |                            |
|-------------------------|----------------------------|
| <code>e(cmd)</code>     | oprobit                    |
| <code>e(cmdline)</code> | command as typed           |
| <code>e(depvar)</code>  | name of dependent variable |
| <code>e(wtype)</code>   | weight type                |

|                                |                                                                          |
|--------------------------------|--------------------------------------------------------------------------|
| <code>e(wexp)</code>           | weight expression                                                        |
| <code>e(title)</code>          | title in estimation output                                               |
| <code>e(clustvar)</code>       | name of cluster variable                                                 |
| <code>e(offset)</code>         | linear offset variable                                                   |
| <code>e(chi2type)</code>       | Wald or LR; type of model $\chi^2$ test                                  |
| <code>e(vce)</code>            | <i>vce</i> type specified in <code>vce()</code>                          |
| <code>e(vcetype)</code>        | title used to label Std. err.                                            |
| <code>e(opt)</code>            | type of optimization                                                     |
| <code>e(which)</code>          | max or min; whether optimizer is to perform maximization or minimization |
| <code>e(ml_method)</code>      | type of ml method                                                        |
| <code>e(user)</code>           | name of likelihood-evaluator program                                     |
| <code>e(technique)</code>      | maximization technique                                                   |
| <code>e(properties)</code>     | b V                                                                      |
| <code>e(predict)</code>        | program used to implement <code>predict</code>                           |
| <code>e(marginsdefault)</code> | default <code>predict()</code> specification for <code>margins</code>    |
| <code>e(asbalanced)</code>     | factor variables <code>fvset</code> as <code>asbalanced</code>           |
| <code>e(asobserved)</code>     | factor variables <code>fvset</code> as <code>asobserved</code>           |
| Matrices                       |                                                                          |
| <code>e(b)</code>              | coefficient vector                                                       |
| <code>e(Cns)</code>            | constraints matrix                                                       |
| <code>e(ilog)</code>           | iteration log (up to 20 iterations)                                      |
| <code>e(gradient)</code>       | gradient vector                                                          |
| <code>e(cat)</code>            | category values                                                          |
| <code>e(V)</code>              | variance–covariance matrix of the estimators                             |
| <code>e(V_modelbased)</code>   | model-based variance                                                     |
| Functions                      |                                                                          |
| <code>e(sample)</code>         | marks estimation sample                                                  |

In addition to the above, the following is stored in `r()`:

|                       |                                                                                                                            |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|
| Matrices              |                                                                                                                            |
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

## Methods and formulas

See *Methods and formulas* of [R] `ologit`.

## References

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## Also see

- [R] **oprobit postestimation** — Postestimation tools for `oprobit`
- [R] **heckoprobit** — Ordered probit model with sample selection
- [R] **hetoprobit** — Heteroskedastic ordered probit regression
- [R] **logistic** — Logistic regression, reporting odds ratios
- [R] **mlogit** — Multinomial (polytomous) logistic regression
- [R] **mprobit** — Multinomial probit regression
- [R] **ologit** — Ordered logistic regression
- [R] **probit** — Probit regression
- [R] **zioprobit** — Zero-inflated ordered probit regression
- [BAYES] **bayes: oprobit** — Bayesian ordered probit regression
- [CM] **cmoprobit** — Rank-ordered probit choice model
- [ERM] **eoprobit** — Extended ordered probit regression
- [FMM] **fmm: oprobit** — Finite mixtures of ordered probit regression models
- [ME] **meoprobit** — Multilevel mixed-effects ordered probit regression
- [MI] **Estimation** — Estimation commands for use with `mi` estimate
- [SVY] **svy estimation** — Estimation commands for survey data
- [XT] **xtoprobit** — Random-effects ordered probit model
- [U] **20 Estimation and postestimation commands**

## Postestimation commands

The following postestimation commands are available after `oprobit`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| <code>linktest</code>        | link test for model specification                                                                                                       |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | probabilities, linear predictions and their SEs, etc.                                                                                   |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results. `forecast` is also not appropriate with `mi` estimation results.

## predict

### Description for predict

`predict` creates a new variable containing predictions such as probabilities, linear predictions, and standard errors.

### Menu for predict

Statistics > Postestimation

### Syntax for predict

```
predict [type] { stub* | newvar | newvarlist } [if] [in] [, statistic
 outcome(outcome) nooffset]
```

```
predict [type] stub* [if] [in] , scores
```

| <i>statistic</i>  | Description                             |
|-------------------|-----------------------------------------|
| Main              |                                         |
| <code>pr</code>   | predicted probabilities; the default    |
| <code>xb</code>   | linear prediction                       |
| <code>stdp</code> | standard error of the linear prediction |

You specify one or  $k$  new variables with `pr`, where  $k$  is the number of outcomes. If you specify one new variable and you do not specify `outcome()`, then `outcome(#1)` is assumed.

You specify one new variable with `xb` and `stdp`.

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

### Options for predict

#### Main

`pr`, the default, computes the predicted probabilities for all outcomes or for a specific outcome. To compute probabilities for all outcomes, you specify  $k$  new variables, where  $k$  is the number of categories of the dependent variable. Alternatively, you can specify `stub*`; in which case, `pr` will store predicted probabilities in variables `stub1`, `stub2`, ..., `stubk`. To compute the probability for a specific outcome, you specify one new variable and, optionally, the outcome value in option `outcome()`; if you omit `outcome()`, the first outcome value, `outcome(#1)`, is assumed.

Say that you fit a model by typing `estimation_cmd y x1 x2`, and `y` takes on four values. Then, you could type `predict p1 p2 p3 p4` to obtain all four predicted probabilities; alternatively, you could type `predict p*` to generate the four predicted probabilities. To compute specific probabilities one at a time, you can type `predict p1, outcome(#1)` (or simply `predict p1`), `predict p2, outcome(#2)`, and so on. See option `outcome()` for other ways to refer to outcome values.

`xb` calculates the linear prediction. You specify one new variable, for example, `predict linear, xb`. The linear prediction is defined, ignoring the contribution of the estimated cutpoints.

`stdp` calculates the standard error of the linear prediction. You specify one new variable, for example, `predict se, stdp`.

`outcome(outcome)` specifies for which outcome the predicted probabilities are to be calculated. `outcome()` should contain either one value of the dependent variable or one of #1, #2, ..., with #1 meaning the first category of the dependent variable, #2 meaning the second category, etc. `outcome()` is available only with the default `pr` option.

`nooffset` is relevant only if you specified `offset(varname)` for `oprobit`. It modifies the calculations made by `predict` so that they ignore the offset variable; the linear prediction is treated as  $\mathbf{x}_j\mathbf{b}$  rather than as  $\mathbf{x}_j\mathbf{b} + \text{offset}_j$ . `nooffset` is not allowed with `scores`.

`scores` calculates equation-level score variables. The number of score variables created will equal the number of outcomes in the model. If the number of outcomes in the model was  $k$ , then

the first new variable will contain  $\partial \ln L / \partial (\mathbf{x}_j\mathbf{b})$ ;

the second new variable will contain  $\partial \ln L / \partial \kappa_1$ ;

the third new variable will contain  $\partial \ln L / \partial \kappa_2$ ;

...

and the  $k$ th new variable will contain  $\partial \ln L / \partial \kappa_{k-1}$ , where  $\kappa_i$  refers to the  $i$ th cutpoint.

## margins

### Description for margins

`margins` estimates margins of response for probabilities and linear predictions.

### Menu for margins

Statistics > Postestimation

### Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i>  | Description                           |
|-------------------|---------------------------------------|
| default           | probabilities for each outcome        |
| <code>pr</code>   | probability for a specified outcome   |
| <code>xb</code>   | linear prediction                     |
| <code>stdp</code> | not allowed with <code>margins</code> |

`pr` defaults to the first outcome.

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [R] [margins](#).

## Remarks and examples

See [U] 20 [Estimation and postestimation commands](#) for instructions on obtaining the variance–covariance matrix of the estimators, predicted values, and hypothesis tests. Also see [R] [lrttest](#) for performing likelihood-ratio tests.

### ▷ Example 1

In [example 1](#) of [R] [oprobit](#), we fit the model `oprobit rep77 foreign length mpg`. The `predict` command can be used to obtain the predicted probabilities. We type `predict` followed by the names of the new variables to hold the predicted probabilities, ordering the names from low to high. In our data, the lowest outcome is “poor” and the highest is “excellent”. We have five categories, so we must type five names following `predict`; the choice of names is up to us:

```
. use https://www.stata-press.com/data/r19/fullauto
(Automobile models)
. oprobit rep77 foreign length mpg
(output omitted)
. predict poor fair avg good exc
(option pr assumed; predicted probabilities)
. list make model exc good if rep77>=., sep(4) divider
```

|     | make    | model   | exc      | good     |
|-----|---------|---------|----------|----------|
| 3.  | AMC     | Spirit  | .0006044 | .0351813 |
| 10. | Buick   | Opel    | .0043803 | .1133763 |
| 32. | Ford    | Fiesta  | .0002927 | .0222789 |
| 44. | Merc.   | Monarch | .0093209 | .1700846 |
| 53. | Peugeot | 604     | .0734199 | .4202766 |
| 56. | Plym.   | Horizon | .001413  | .0590294 |
| 57. | Plym.   | Sapporo | .0197543 | .2466034 |
| 63. | Pont.   | Phoenix | .0234156 | .266771  |



### □ Technical note

For ordered probit, `predict, xb` produces  $S_j = x_{1j}\beta_1 + x_{2j}\beta_2 + \dots + x_{kj}\beta_k$ . Ordered probit is identical to ordered logit, except that we use different distribution functions for calculating probabilities. The ordered-probit predictions are then the probability that  $S_j + u_j$  lies between a pair of cutpoints  $\kappa_{i-1}$  and  $\kappa_i$ . The formulas for ordered probit are

$$\Pr(S_j + u < \kappa) = \Phi(\kappa - S_j)$$

$$\Pr(S_j + u > \kappa) = 1 - \Phi(\kappa - S_j) = \Phi(S_j - \kappa)$$

$$\Pr(\kappa_1 < S_j + u < \kappa_2) = \Phi(\kappa_2 - S_j) - \Phi(\kappa_1 - S_j)$$

Rather than using `predict` directly, we could calculate the predicted probabilities by hand.

```
. predict pscore, xb
. generate probexc = normal(pscore-_b[/cut4])
. generate probgood = normal(_b[/cut4]-pscore) - normal(_b[/cut3]-pscore)
```



## Also see

[R] [oprobit](#) — Ordered probit regression

[U] [20 Estimation and postestimation commands](#)

[Description](#)[Options for orthog](#)[References](#)[Quick start](#)[Options for orthpoly](#)[Also see](#)[Menu](#)[Remarks and examples](#)[Syntax](#)[Methods and formulas](#)

## Description

`orthog` orthogonalizes a set of variables, creating a new set of orthogonal variables (all of type `double`), using a modified Gram–Schmidt procedure (Golub and Van Loan 2013). The order of the variables determines the orthogonalization; hence, the “most important” variables should be listed first.

Execution time is proportional to the square of the number of variables. With many ( $>10$ ) variables, `orthog` will be fairly slow.

`orthpoly` computes orthogonal polynomials for one variable.

## Quick start

Generate `ox1`, `ox2`, and `ox3` containing orthogonalized versions of `x1`, `x2`, and `x3`

```
orthog x1 x2 x3, generate(ox1 ox2 ox3)
```

Same as above

```
orthog x1 x2 x3, generate(ox*)
```

Generate `op1`, `op2`, and `op3` containing degree 1, 2, and 3 orthogonal polynomials for `x1`

```
orthpoly x1, generate(op1 op2 op3) degree(3)
```

Same as above

```
orthpoly x1, generate(op1-op3) degree(3)
```

Same as above, and generate matrix `op` containing coefficients of the orthogonal polynomials

```
orthpoly x1, generate(op1-op3) degree(3) poly(op)
```

## Menu

### **orthog**

Data > Create or change data > Other variable-creation commands > Orthogonalize variables

### **orthpoly**

Data > Create or change data > Other variable-creation commands > Orthogonal polynomials

## Syntax

### Orthogonalize variables

```
orthog [varlist] [if] [in] [weight], generate(newvarlist) [matrix(matname)]
```

### Compute orthogonal polynomial

```
orthpoly varname [if] [in] [weight],
{ generate(newvarlist) | poly(matname) } [degree(#)]
```

orthpoly requires that generate(*newvarlist*) or poly(*matname*), or both, be specified.

*varlist* may contain time-series operators; see [U] 11.4.4 Time-series varlists.

iweights, aweights, fweights, and pweights are allowed, see [U] 11.1.6 weight.

## Options for orthog

Main

generate(*newvarlist*) is required. generate() creates new orthogonal variables of type double. For orthog, *newvarlist* will contain the orthogonalized *varlist*. If *varlist* contains  $d$  variables, then so will *newvarlist*. *newvarlist* can be specified by giving a list of exactly  $d$  new variable names, or it can be abbreviated using the styles *newvar1–newvard* or *newvar\**. For these two styles of abbreviation, new variables *newvar1*, *newvar2*, ..., *newvard* are generated.

matrix(*matname*) creates a  $(d + 1) \times (d + 1)$  matrix containing the matrix  $R$  defined by  $X = QR$ , where  $X$  is the  $N \times (d + 1)$  matrix representation of *varlist* plus a column of ones and  $Q$  is the  $N \times (d + 1)$  matrix representation of *newvarlist* plus a column of ones ( $d$  = number of variables in *varlist*, and  $N$  = number of observations).

## Options for orthpoly

Main

generate(*newvarlist*) or poly(), or both, must be specified. generate() creates new orthogonal variables of type double. *newvarlist* will contain orthogonal polynomials of degree 1, 2, ...,  $d$  evaluated at *varname*, where  $d$  is as specified by degree( $d$ ). *newvarlist* can be specified by giving a list of exactly  $d$  new variable names, or it can be abbreviated using the styles *newvar1–newvard* or *newvar\**. For these two styles of abbreviation, new variables *newvar1*, *newvar2*, ..., *newvard* are generated.

poly(*matname*) creates a  $(d + 1) \times (d + 1)$  matrix called *matname* containing the coefficients of the orthogonal polynomials. The orthogonal polynomial of degree  $i \leq d$  is

$$\text{matname}[i, d + 1] + \text{matname}[i, 1] * \text{varname} + \text{matname}[i, 2] * \text{varname}^2 \\ + \dots + \text{matname}[i, i] * \text{varname}^i$$

The coefficients corresponding to the constant term are placed in the last column of the matrix. The last row of the matrix is all zeros, except for the last column, which corresponds to the constant term.

degree(#) specifies the highest-degree polynomial to include. Orthogonal polynomials of degree 1, 2, ...,  $d = \#$  are computed. The default is  $d = 1$ .

## Remarks and examples

Orthogonal variables are useful for two reasons. The first is numerical accuracy for highly collinear variables. Stata’s `regress` and other estimation commands can face much collinearity and still produce accurate results. But, at some point, these commands will drop variables because of collinearity. If you know with certainty that the variables are not perfectly collinear, you may want to retain all their effects in the model. If you use `orthog` or `orthpoly` to produce a set of orthogonal variables, all variables will be present in the estimation results.

Users are more likely to find orthogonal variables useful for the second reason: ease of interpreting results. `orthog` and `orthpoly` create a set of variables such that the “effects” of all the preceding variables have been removed from each variable. For example, if we issue the command

```
. orthog x1 x2 x3, generate(q1 q2 q3)
```

the effect of the constant is removed from `x1` to produce `q1`; the constant and `x1` are removed from `x2` to produce `q2`; and finally the constant, `x1`, and `x2` are removed from `x3` to produce `q3`. Hence,

$$\begin{aligned}q1 &= r_{01} + r_{11} x1 \\q2 &= r_{02} + r_{12} x1 + r_{22} x2 \\q3 &= r_{03} + r_{13} x1 + r_{23} x2 + r_{33} x3\end{aligned}$$

This effect can be generalized and written in matrix notation as

$$X = QR$$

where  $X$  is the  $N \times (d+1)$  matrix representation of *varlist* plus a column of ones, and  $Q$  is the  $N \times (d+1)$  matrix representation of *newvarlist* plus a column of ones ( $d$  = number of variables in *varlist* and  $N$  = number of observations). The  $(d+1) \times (d+1)$  matrix  $R$  is a permuted upper-triangular matrix, that is,  $R$  would be upper triangular if the constant were first, but the constant is last, so the first row/column has been permuted with the last row/column. Because Stata’s estimation commands list the constant term last, this allows  $R$ , obtained via the `matrix()` option, to be used to transform estimation results.

### ► Example 1: orthog

Consider Stata’s `auto.dta` dataset. Suppose that we postulate a model in which price depends on the car’s length, weight, headroom, and trunk size (`trunk`). These predictors are collinear, but not extremely so—the correlations are not that close to 1:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. correlate length weight headroom trunk
(obs=74)
```

|          | length | weight | headroom | trunk  |
|----------|--------|--------|----------|--------|
| length   | 1.0000 |        |          |        |
| weight   | 0.9460 | 1.0000 |          |        |
| headroom | 0.5163 | 0.4835 | 1.0000   |        |
| trunk    | 0.7266 | 0.6722 | 0.6620   | 1.0000 |

regress certainly has no trouble fitting this model:

```
. regress price length weight headroom trunk
```

| Source   | SS        | df | MS         | Number of obs | = | 74     |
|----------|-----------|----|------------|---------------|---|--------|
| Model    | 236016580 | 4  | 59004145   | F(4, 69)      | = | 10.20  |
| Residual | 399048816 | 69 | 5783316.17 | Prob > F      | = | 0.0000 |
|          |           |    |            | R-squared     | = | 0.3716 |
|          |           |    |            | Adj R-squared | = | 0.3352 |
| Total    | 635065396 | 73 | 8699525.97 | Root MSE      | = | 2404.9 |

| price    | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|----------|-------------|-----------|-------|-------|----------------------|-----------|
| length   | -101.7092   | 42.12534  | -2.41 | 0.018 | -185.747             | -17.67147 |
| weight   | 4.753066    | 1.120054  | 4.24  | 0.000 | 2.518619             | 6.987512  |
| headroom | -711.5679   | 445.0204  | -1.60 | 0.114 | -1599.359            | 176.2236  |
| trunk    | 114.0859    | 109.9488  | 1.04  | 0.303 | -105.2559            | 333.4277  |
| _cons    | 11488.47    | 4543.902  | 2.53  | 0.014 | 2423.638             | 20553.31  |

However, we may believe a priori that length is the most important predictor, followed by weight, headroom, and trunk. We would like to remove the “effect” of length from all the other predictors, remove weight from headroom and trunk, and remove headroom from trunk. We can do this by running orthog, and then we fit the model again using the orthogonal variables:

```
. orthog length weight headroom trunk, gen(olength oweight oheadroom otrunk)
> matrix(R)
```

```
. regress price olength oweight oheadroom otrunk
```

| Source   | SS        | df | MS         | Number of obs | = | 74     |
|----------|-----------|----|------------|---------------|---|--------|
| Model    | 236016580 | 4  | 59004145   | F(4, 69)      | = | 10.20  |
| Residual | 399048816 | 69 | 5783316.17 | Prob > F      | = | 0.0000 |
|          |           |    |            | R-squared     | = | 0.3716 |
|          |           |    |            | Adj R-squared | = | 0.3352 |
| Total    | 635065396 | 73 | 8699525.97 | Root MSE      | = | 2404.9 |

| price     | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|-----------|-------------|-----------|-------|-------|----------------------|----------|
| olength   | 1265.049    | 279.5584  | 4.53  | 0.000 | 707.3454             | 1822.753 |
| oweight   | 1175.765    | 279.5584  | 4.21  | 0.000 | 618.0617             | 1733.469 |
| oheadroom | -349.9916   | 279.5584  | -1.25 | 0.215 | -907.6955            | 207.7122 |
| otrunk    | 290.0776    | 279.5584  | 1.04  | 0.303 | -267.6262            | 847.7815 |
| _cons     | 6165.257    | 279.5584  | 22.05 | 0.000 | 5607.553             | 6722.961 |

Using the matrix R, we can transform the results obtained using the orthogonal predictors back to the metric of original predictors:

```
. matrix b = e(b)*inv(R)
. matrix list b
```

```
b[1,5]
```

|    | length     | weight    | headroom   | trunk     | _cons     |
|----|------------|-----------|------------|-----------|-----------|
| y1 | -101.70924 | 4.7530659 | -711.56789 | 114.08591 | 11488.475 |

## □ Technical note

The matrix  $R$  obtained using the `matrix()` option with `orthog` can also be used to recover  $X$  (the original *varlist*) from  $Q$  (the orthogonalized *newvarlist*), one variable at a time. Continuing with the previous example, we illustrate how to recover the `trunk` variable:

```
. matrix C = R[1..., "trunk"]'
. matrix score double rtrunk = C
. compare rtrunk trunk
```

|                 | Count | Difference |          |          |
|-----------------|-------|------------|----------|----------|
|                 |       | Minimum    | Average  | Maximum  |
| rtrunk>trunk    | 74    | 8.88e-15   | 1.91e-14 | 3.55e-14 |
| Jointly defined | 74    | 8.88e-15   | 1.91e-14 | 3.55e-14 |
| Total           | 74    |            |          |          |

Here the recovered variable `rtrunk` is almost exactly the same as the original `trunk` variable. When you are orthogonalizing many variables, this procedure can be performed to check the numerical soundness of the orthogonalization. Because of the ordering of the orthogonalization procedure, the last variable and the variables near the end of the *varlist* are the most important ones to check. □

The `orthpoly` command effectively does for polynomial terms what the `orthog` command does for an arbitrary set of variables.

## ▷ Example 2: orthpoly

Again consider the `auto.dta` dataset. Suppose that we wish to fit the model

$$\text{mpg} = \beta_0 + \beta_1 \text{weight} + \beta_2 \text{weight}^2 + \beta_3 \text{weight}^3 + \beta_4 \text{weight}^4 + \epsilon$$

We will first compute the regression with natural polynomials:

```
. generate double w1 = weight
. generate double w2 = w1*w1
. generate double w3 = w2*w1
. generate double w4 = w3*w1
. correlate w1-w4
(obs=74)
```

|    | w1     | w2     | w3     | w4     |
|----|--------|--------|--------|--------|
| w1 | 1.0000 |        |        |        |
| w2 | 0.9915 | 1.0000 |        |        |
| w3 | 0.9665 | 0.9916 | 1.0000 |        |
| w4 | 0.9279 | 0.9679 | 0.9922 | 1.0000 |

```
. regress mpg w1-w4
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 1652.73666 | 4  | 413.184164 | F(4, 69)      | = | 36.06  |
| Residual | 790.722803 | 69 | 11.4597508 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.6764 |
|          |            |    |            | Adj R-squared | = | 0.6576 |
| Total    | 2443.45946 | 73 | 33.4720474 | Root MSE      | = | 3.3852 |

| mpg   | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|-------|-------------|-----------|-------|-------|----------------------|----------|
| w1    | .0289302    | .1161939  | 0.25  | 0.804 | -.2028704            | .2607307 |
| w2    | -.0000229   | .0000566  | -0.40 | 0.687 | -.0001359            | .0000901 |
| w3    | 5.74e-09    | 1.19e-08  | 0.48  | 0.631 | -1.80e-08            | 2.95e-08 |
| w4    | -4.86e-13   | 9.14e-13  | -0.53 | 0.596 | -2.31e-12            | 1.34e-12 |
| _cons | 23.94421    | 86.60667  | 0.28  | 0.783 | -148.8314            | 196.7198 |

Some of the correlations among the powers of weight are very large, but this does not create any problems for regress. However, we may wish to look at the quadratic trend with the constant removed, the cubic trend with the quadratic and constant removed, etc. orthpoly will generate polynomial terms with this property:

```
. orthpoly weight, generate(pw*) deg(4) poly(P)
```

```
. regress mpg pw1-pw4
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 1652.73666 | 4  | 413.184164 | F(4, 69)      | = | 36.06  |
| Residual | 790.722803 | 69 | 11.4597508 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.6764 |
|          |            |    |            | Adj R-squared | = | 0.6576 |
| Total    | 2443.45946 | 73 | 33.4720474 | Root MSE      | = | 3.3852 |

| mpg   | Coefficient | Std. err. | t      | P> t  | [95% conf. interval] |           |
|-------|-------------|-----------|--------|-------|----------------------|-----------|
| pw1   | -4.638252   | .3935245  | -11.79 | 0.000 | -5.423312            | -3.853192 |
| pw2   | .8263545    | .3935245  | 2.10   | 0.039 | .0412947             | 1.611414  |
| pw3   | -.3068616   | .3935245  | -0.78  | 0.438 | -1.091921            | .4781982  |
| pw4   | -.209457    | .3935245  | -0.53  | 0.596 | -.9945168            | .5756028  |
| _cons | 21.2973     | .3935245  | 54.12  | 0.000 | 20.51224             | 22.08236  |

Compare the *p*-values of the terms in the natural polynomial regression with those in the orthogonal polynomial regression. With orthogonal polynomials, it is easy to see that the pure cubic and quartic trends are not significant and that the constant, linear, and quadratic terms each have  $p < 0.05$ .

The matrix P obtained with the poly() option can be used to transform coefficients for orthogonal polynomials to coefficients for natural polynomials:

```
. orthpoly weight, poly(P) deg(4)
```

```
. matrix b = e(b)*P
```

```
. matrix list b
```

```
b[1,5]
```

|    | deg1      | deg2       | deg3      | deg4       | _cons     |
|----|-----------|------------|-----------|------------|-----------|
| y1 | .02893016 | -.00002291 | 5.745e-09 | -4.862e-13 | 23.944212 |

## Methods and formulas

orthog's orthogonalization can be written in matrix notation as

$$X = QR$$

where  $X$  is the  $N \times (d+1)$  matrix representation of *varlist* plus a column of ones and  $Q$  is the  $N \times (d+1)$  matrix representation of *newvarlist* plus a column of ones ( $d$  = number of variables in *varlist*, and  $N$  = number of observations). The  $(d+1) \times (d+1)$  matrix  $R$  is a permuted upper-triangular matrix; that is,  $R$  would be upper triangular if the constant were first, but the constant is last, so the first row/column has been permuted with the last row/column.

$Q$  and  $R$  are obtained using a modified Gram–Schmidt procedure; see [Golub and Van Loan \(2013, 254–255\)](#) for details. The traditional Gram–Schmidt procedure is notoriously unsound, but the modified procedure is good. orthog performs two passes of this procedure.

orthpoly uses the Christoffel–Darboux recurrence formula ([Abramowitz and Stegun 1964](#)).

Both orthog and orthpoly normalize the orthogonal variables such that

$$Q'WQ = MI$$

where  $W = \text{diag}(w_1, w_2, \dots, w_N)$  with weights  $w_1, w_2, \dots, w_N$  (all 1 if weights are not specified), and  $M$  is the sum of the weights (the number of observations if weights are not specified).

## References

- Abramowitz, M., and I. A. Stegun, eds. 1964. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*. Washington, DC: National Bureau of Standards.
- Golub, G. H., and C. F. Van Loan. 2013. *Matrix Computations*. 4th ed. Baltimore: Johns Hopkins University Press. <https://doi.org/10.56021/9781421407944>.

## Also see

[R] [regress](#) — Linear regression

Description

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## Description

`pcorr` displays the partial and semipartial correlation coefficients of a specified variable with each variable in a varlist after removing the effects of all other variables in the varlist. The squared correlations and corresponding significance are also reported.

## Quick start

Partial and semipartial correlations of `v1` with `v2`, `v3`, and `v4`

```
pcorr v1 v2 v3 v4
```

Same as above, but for each level of categorical variable `catvar`

```
by catvar: pcorr v1 v2 v3 v4
```

Partial and semipartial correlations of `v5` with `v6`, `v7`, and `v8`, and a one-period lag of `v5` using `tsset` data

```
pcorr v5 L.v5 v6 v7 v8
```

## Menu

Statistics > Summaries, tables, and tests > Summary and descriptive statistics > Partial correlations

## Syntax

```
pcorr varname varlist [if] [in] [weight]
```

*varlist* may contain factor variables; see [U] 11.4.3 Factor variables.

*varname* and *varlist* may contain time-series operators; see [U] 11.4.4 Time-series varlists.

`by` and `collect` are allowed; see [U] 11.1.10 Prefix commands.

`aweight`s and `fweight`s are allowed; see [U] 11.1.6 weight.

## Remarks and examples

Assume that  $y$  is determined by  $x_1, x_2, \dots, x_k$ . The partial correlation between  $y$  and  $x_1$  is an attempt to estimate the correlation that would be observed between  $y$  and  $x_1$  if the other  $x$ 's did not vary. The semipartial correlation, also called part correlation, between  $y$  and  $x_1$  is an attempt to estimate the correlation that would be observed between  $y$  and  $x_1$  after the effects of all other  $x$ 's are removed from  $x_1$  but not from  $y$ .

Both squared correlations estimate the proportion of the variance of  $y$  that is explained by each predictor. The squared semipartial correlation between  $y$  and  $x_1$  represents the proportion of variance in  $y$  that is explained by  $x_1$  only. This squared correlation can also be interpreted as the decrease in the model's  $R^2$  value that results from removing  $x_1$  from the full model. Thus, one could use the squared semipartial correlations as criteria for model selection. The squared partial correlation between  $y$  and  $x_1$  represents the proportion of variance in  $y$  not associated with any other  $x$ 's that is explained by  $x_1$ . Thus, the squared partial correlation gives an estimate of how much of the variance of  $y$  not explained by the other  $x$ 's is explained by  $x_1$ .

### ▷ Example 1

Using our automobile dataset (described in [U] 1.2.2 Example datasets), we can obtain the simple correlations between price, mpg, weight, and foreign from `correlate` (see [R] `correlate`):

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. correlate price mpg weight foreign
(obs=74)
```

|         | price   | mpg     | weight  | foreign |
|---------|---------|---------|---------|---------|
| price   | 1.0000  |         |         |         |
| mpg     | -0.4686 | 1.0000  |         |         |
| weight  | 0.5386  | -0.8072 | 1.0000  |         |
| foreign | 0.0487  | 0.3934  | -0.5928 | 1.0000  |

Although `correlate` gave us the full correlation matrix, our interest is in just the first column. We find, for instance, that the higher the mpg, the lower the price. We obtain the partial and semipartial correlation coefficients by using `pcorr`:

```
. pcorr price mpg weight foreign
(obs=74)
```

Partial and semipartial correlations of price with

| Variable | Partial corr. | Semipartial corr. | Partial corr. <sup>2</sup> | Semipartial corr. <sup>2</sup> | Significance value |
|----------|---------------|-------------------|----------------------------|--------------------------------|--------------------|
| mpg      | 0.0352        | 0.0249            | 0.0012                     | 0.0006                         | 0.7693             |
| weight   | 0.5488        | 0.4644            | 0.3012                     | 0.2157                         | 0.0000             |
| foreign  | 0.5402        | 0.4541            | 0.2918                     | 0.2062                         | 0.0000             |

We now find that the partial and semipartial correlations of price with mpg are near 0. In the simple correlations, we found that price and foreign were virtually uncorrelated. In the partial and semipartial correlations, we find that price and foreign are positively correlated. The nonsignificance of mpg tells us that the amount in which  $R^2$  decreases by removing mpg from the model is not significant. We find that removing either weight or foreign results in a significant drop in the  $R^2$  of the model.

◀

### □ Technical note

Use caution when interpreting the above results. As we said at the outset, the partial and semipartial correlation coefficients are an *attempt* to estimate the correlation that would be observed if the effects of all other variables were taken out of both  $y$  and  $x$  or only  $x$ . `pcorr` makes it too easy to ignore the fact that we are fitting a model. In the example above, the model is

$$\text{price} = \beta_0 + \beta_1 \text{mpg} + \beta_2 \text{weight} + \beta_3 \text{foreign} + \epsilon$$

which is, in all honesty, a rather silly model. Even if we accept the implied economic assumptions of the model—that consumers value mpg, weight, and foreign—do we really believe that consumers place equal value on every extra 1,000 pounds of weight? That is, have we correctly parameterized the model? If we have not, then the estimated partial and semipartial correlation coefficients may not represent what they claim to represent. Partial and semipartial correlation coefficients are a reasonable way to summarize data if we are convinced that the underlying model is reasonable. We should not, however, pretend that there is no underlying model and that these correlation coefficients are unaffected by the assumptions and parameterization. □

## Stored results

pcorr stores the following in `r()`:

### Scalars

`r(N)`            number of observations  
`r(df)`            degrees of freedom

### Matrices

`r(p_corr)`        partial correlation coefficient vector  
`r(sp_corr)`        semipartial correlation coefficient vector

## Methods and formulas

Results are obtained by fitting a linear regression of *varname* on *varlist*; see [R] [regress](#). The partial correlation coefficient between *varname* and each variable in *varlist* is then calculated as

$$\frac{t}{\sqrt{t^2 + n - k}}$$

(Greene 2018, 39), where  $t$  is the  $t$  statistic,  $n$  is the number of observations, and  $k$  is the number of independent variables, including the constant but excluding any dropped variables.

The semipartial correlation coefficient between *varname* and each variable in *varlist* is calculated as

$$\text{sign}(t) \sqrt{\frac{t^2(1 - R^2)}{n - k}}$$

(Cohen et al. 2003, 89), where  $R^2$  is the model  $R^2$  value, and  $t$ ,  $n$ , and  $k$  are as described above.

The significance is given by  $2\Pr(t_{n-k} > |t|)$ , where  $t_{n-k}$  follows a Student's  $t$  distribution with  $n - k$  degrees of freedom.

## Acknowledgment

The addition of semipartial correlation coefficients to pcorr is based on the pcorr2 command by Richard Williams of the Department of Sociology at the University of Notre Dame.

## References

- Cohen, J., P. Cohen, S. G. West, and L. S. Aiken. 2003. *Applied Multiple Regression/Correlation Analysis for the Behavioral Sciences*. 3rd ed. Hillsdale, NJ: Erlbaum.
- Greene, W. H. 2018. *Econometric Analysis*. 8th ed. New York: Pearson.

## Also see

[R] [correlate](#) — Correlations of variables

[R] [spearman](#) — Spearman's and Kendall's correlations

[Description](#)  
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## Description

`permute` performs permutation tests using Monte Carlo permutations or by enumeration of all possible distinct permutations. A single variable is chosen to be permuted, and the permutation distribution is estimated (or in the case of enumeration, fully determined) for specified statistics returned by a Stata command or a user-written program.

## Quick start

Estimate  $p$ -values for a permutation test of the coefficient of  $x$  in a linear regression, permuting values of the outcome  $y$

```
permute y _b[x] : regress y x
```

Test for  $r(\text{mystat})$  returned by program `myprog`, permuting values of  $y$

```
permute y r(mystat) : myprog
```

Same as above, but increase the number of permutations from the default of 100 to 10,000

```
permute y r(mystat), reps(10000) : myprog
```

Same as above, but display a dot for every 100 permutations instead of every permutation

```
permute y r(mystat), reps(10000) dots(100) : myprog
```

Same as above, but set the random-number seed for reproducibility, and save the permuted statistics in `myfile.dta`

```
permute y r(mystat), reps(10000) dots(100) rseed(1) saving(myfile) : ///
myprog
```

Test for  $r(\text{mystat1})$  and  $r(\text{mystat2})$ , naming the statistics `stat1` and `stat2`, respectively

```
permute y stat1=r(mystat1) stat2=r(mystat2), reps(10000) rseed(1) : ///
myprog
```

Perform permutations within strata defined by `svar`

```
permute y stat=r(mystat), reps(10000) rseed(1) strata(svar) : myprog
```

Enumerate the full permutation distribution and display a dot for every 1,000 permutations

```
permute y stat=r(mystat), enumerate dots(1000) : myprog
```

## Menu

Statistics > Resampling > Permutation tests

## Syntax

Perform permutation test

```
permute permvar exp_list [, options] : command
```

Report saved results

```
permute [varlist] [using filename] [, display_options]
```

| <i>options</i>                     | Description                                                                                                                            |
|------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------|
| <b>Main</b>                        |                                                                                                                                        |
| <code>reps(#)</code>               | perform # Monte Carlo permutations; default is <code>reps(100)</code>                                                                  |
| <code>enumerate</code>             | compute all possible distinct permutations                                                                                             |
| <b>Options</b>                     |                                                                                                                                        |
| <code>rseed(#)</code>              | set random-number seed to #                                                                                                            |
| <code>strata(varlist)</code>       | permute within strata                                                                                                                  |
| <code>saving(filename, ...)</code> | save results to <i>filename</i> with options for saving in double precision and saving results to <i>filename</i> every # permutations |
| <b>Reporting</b>                   |                                                                                                                                        |
| <code>standardize</code>           | standardize test statistic using permutation distribution mean and variance                                                            |
| <code>level(#)</code>              | set confidence level; default is <code>level(95)</code>                                                                                |
| <code>title(text)</code>           | use <i>text</i> as title for permutation results                                                                                       |
| <code>dots(#)</code>               | display dots every # permutations                                                                                                      |
| <code>nodots</code>                | suppress permutation dots                                                                                                              |
| <code>nowarning</code>             | do not warn when <code>e(sample)</code> is not set                                                                                     |
| <code>noisily</code>               | display any output from <i>command</i>                                                                                                 |
| <code>trace</code>                 | trace <i>command</i>                                                                                                                   |
| <code>verbose</code>               | display full table legend                                                                                                              |
| <code>noheader</code>              | suppress table header                                                                                                                  |
| <code>nolegend</code>              | suppress table legend                                                                                                                  |
| <b>Advanced</b>                    |                                                                                                                                        |
| <code>nodrop</code>                | do not drop observations                                                                                                               |
| <code>reject(exp)</code>           | specify criterion for invalid results                                                                                                  |
| <code>eps(#)</code>                | numerical tolerance; seldom used                                                                                                       |

`collect` is allowed; see [U] 11.1.10 Prefix commands.

*weights* are allowed in *command*.

| <i>display_options</i>   | Description                                                                 |
|--------------------------|-----------------------------------------------------------------------------|
| <code>standardize</code> | standardize test statistic using permutation distribution mean and variance |
| <code>level(#)</code>    | set confidence level; default is <code>level(95)</code>                     |
| <code>title(text)</code> | use <i>text</i> as title for results                                        |
| <code>verbose</code>     | display full table legend                                                   |
| <code>noheader</code>    | suppress table header                                                       |
| <code>nolegend</code>    | suppress table legend                                                       |
| <code>eps(#)</code>      | numerical tolerance; seldom used                                            |

*exp\_list* contains `(name: elist)`  
*elist*  
*eexp*

*elist* contains `newvar = (exp)`  
`(exp)`

*eexp* is `specname`  
`[eqno]specname`

*specname* is `_b`  
`_b []`  
`_se`  
`_se []`

*eqno* is `# #`  
`name`

*exp* is a standard Stata expression; see [\[U\] 13 Functions and expressions](#).

Distinguish between `[]`, which are to be typed, and `[],` which indicate optional arguments.

## Options

Main

`reps(#)` specifies the number of Monte Carlo permutations to perform. The default is `reps(100)`.

The default of 100 permutations is chosen for convenience. In real-world applications, you will most likely need to use more permutations. `permute` reports the Monte Carlo error, which you can use to evaluate whether the specified number of permutations provides sufficient precision for the reported *p*-value estimates.

`enumerate` specifies that all possible distinct permutations be computed. This gives the full permutation distribution and *p*-values with no error. `reps()` and `rseed()` cannot be specified with `enumerate`.

The number of all possible distinct permutations is typically extremely large. Only for some small problems will it be practical to fully enumerate the permutation distribution. If all the values of *permvar* are unique, the number of possible permutations is  $N!$ , where  $N$  is the number of observations. When *permvar* has a lot of repeated values (when, for example, it is a 0/1 variable), the number of possible distinct permutations can be considerably less than  $N!$  and may make enumeration feasible.

Before beginning the enumeration, `permute` will calculate and display the number of distinct permutations, allowing you to press *Break* when you see that the number of permutations is so large as to make computation time impossibly long.

## Options

`rseed(#)` sets the random-number seed. Specifying this option is equivalent to typing the following command prior to calling `permute`:

```
. set seed #
```

`strata(varlist)` specifies that the permutations be performed within each stratum defined by the values of `varlist`.

`saving(filename[ , double every(#) replace ])` creates a Stata data file (.dta file) consisting of variables for each statistic in `exp_list` containing the results for each permutation.

`double` specifies that the results for each permutation be saved as doubles, meaning 8-byte reals. By default, they are saved as floats, meaning 4-byte reals.

`every(#)` specifies that results be written to disk every `#`th permutation. `every()` should be specified only in conjunction with `saving()` when `command` takes a long time for each permutation. This will allow recovery of partial results should some other software crash your computer. See [P] [postfile](#).

`replace` specifies that `filename` be overwritten if it exists.

## Reporting

`standardize` specifies that the observed value of each test statistic be standardized. That is, the observed test statistic  $T_{\text{obs}}$  is standardized by calculating  $[T_{\text{obs}} - \text{mean}(T)]/\sqrt{\text{Var}(T)}$ , where  $\text{mean}(T)$  and  $\text{Var}(T)$  are the mean and variance of the permutation distribution of  $T$ . Standardized test statistics are useful for seeing roughly how extreme (and in what direction) the observed test statistic is.

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [R] [level](#).

`title(text)` specifies a title to be displayed above the table of permutation results.

`dots(#)` and `nodots` specify whether to display permutation dots. By default, one dot character is displayed for each successful permutation. An “x” is displayed if `command` returns an error or if any value in `exp_list` is missing. You can also control whether dots are displayed using `set dots`; see [R] [set](#).

`dots(#)` displays dots every `#` permutations. `dots(0)` is a synonym for `nodots`.

`nodots` suppresses display of the permutation dots.

`nowarning` suppresses the printing of a warning message when `command` does not set `e(sample)`.

`noisily` requests that any output from `command` be displayed. This option implies the `nodots` option.

`trace` causes a trace of the execution of `command` to be displayed. This option implies the `noisily` option.

`verbose` requests that the full table legend be displayed when multiple coefficients or standard errors are specified using the `_b` or `_se` notation.

`noheader` suppresses display of the table header. This option implies the `nolegend` option.

`nolegend` suppresses display of the table legend. The table legend identifies the rows of the table with the expressions they represent.

Advanced

`nodrop` prevents `permute` from dropping observations outside the `if` and `in` qualifiers. `nodrop` will also cause `permute` to ignore the contents of `e(sample)` if it exists as a result of running *command*. By default, `permute` temporarily drops out-of-sample observations.

`reject(exp)` specifies an expression that indicates when results should be rejected. When *exp* is true, the resulting values are reset to missing values.

`eps(#)` specifies the numerical tolerance for testing  $T \leq T_{\text{obs}}$  and  $T \geq T_{\text{obs}}$ , where  $T$  is the test statistic and  $T_{\text{obs}}$  is its observed value. These are considered true if, respectively,  $T \leq T_{\text{obs}} + \#$  or  $T \geq T_{\text{obs}} - \#$ . The default is `eps(1e-7)`. You will not have to specify `eps()` under normal circumstances.

## Remarks and examples

Remarks are presented under the following headings:

*Introduction*

*Monte Carlo permutation tests*

*Two-sided p-values from permutation tests*

*One-sided permutation test*

*Enumeration*

*Efficiency considerations for Monte Carlo permutations*

*Efficiency considerations for enumeration*

## Introduction

Permutation tests are based on the idea of scrambling—that is, permuting—the order of a variable in all possible ways, calculating the value of a test statistic for each permutation, and taking this set of values of the statistic as its distribution.

For instance, consider the correlation of two variables,  $\text{corr}(\mathbf{x}, \mathbf{y})$ , where  $\mathbf{x} = (x_1, \dots, x_n)$  and  $\mathbf{y} = (y_1, \dots, y_n)$ . We hold the order of  $\mathbf{x}$  fixed and permute the order of  $\mathbf{y}$  in all possible ways. For each permutation  $\mathbf{y}^*$ , we calculate  $T^* = \text{corr}(\mathbf{x}, \mathbf{y}^*)$ . The set of  $T^*$  gives the permutation distribution for the correlation. This permutation distribution is the distribution of the correlation under the null hypothesis that the ordering of the elements of  $\mathbf{y}$  are independent of the ordering of the elements of  $\mathbf{x}$ , conditional on the observed values of  $\mathbf{x}$  and  $\mathbf{y}$ .

Aside: Actually, the null hypothesis does not require independence. A weaker assumption of exchangeability is sufficient. If  $\mathbf{x}$  and  $\mathbf{y}$  are observed values of the random variates  $\mathbf{X} = (X_1, \dots, X_n)$  and  $\mathbf{Y} = (Y_1, \dots, Y_n)$ , then the joint distribution  $f(\mathbf{X}, \mathbf{Y})$  is called exchangeable when it is invariant to the orderings of  $X_1, \dots, X_n$  and  $Y_1, \dots, Y_n$ .

The  $p$ -value for the permutation test is the proportion of permutations that produce a test statistic  $T^*$  as extreme or more extreme than the test statistic  $T_{\text{obs}}$  computed using the observed data.

`permute` estimates  $p$ -values for permutation tests using Monte Carlo permutations or by enumerating all possible permutations when the `enumerate` option is specified. To do Monte Carlo permutations, you type

```
. permute permvar exp_list, reps(#): command
```

The values in the variable *permvar* are randomly permuted *#* times, each time executing *command* and collecting the associated values from the expressions in *exp\_list*.

*command* defines the statistical command to be executed. Most Stata commands and user-written programs can be used with `permute`, as long as they follow standard Stata syntax; see [U] 11 Language syntax. *exp\_list* specifies the statistics to be collected from the execution of *command*. Despite the fact that `permute` works with most Stata commands, that does not mean the resulting permutation test is a sensible test. See, for example, Good (2006).

To enumerate all possible distinct permutations, you type

```
. permute permvar exp_list, enumerate: command
```

`permute` may be used for replaying results, but this feature is appropriate only when a dataset generated by `permute` is currently in memory or is identified by the `using` specification.

## Monte Carlo permutation tests

We first demonstrate how to apply the `permute` prefix by testing for a difference in the distribution of a variable across two groups. Here we perform the test using Monte Carlo permutations. In [example 3](#), we do the same test using complete enumeration.

### ► Example 1: Wilcoxon rank-sum test

Let's consider calculating the  $p$ -value for the Wilcoxon rank-sum test performed by `ranksum`. Suppose that we collected data from some experiment:  $y$  is some measure we took on 17 individuals, and `group` identifies the group to which an individual belongs.

```
. use https://www.stata-press.com/data/r19/permute2
. list, sepby(group)
```

|     | group | y  |
|-----|-------|----|
| 1.  | 1     | 6  |
| 2.  | 1     | 11 |
| 3.  | 1     | 20 |
| 4.  | 1     | 2  |
| 5.  | 1     | 9  |
| 6.  | 1     | 5  |
| 7.  | 0     | 2  |
| 8.  | 0     | 1  |
| 9.  | 0     | 6  |
| 10. | 0     | 0  |
| 11. | 0     | 2  |
| 12. | 0     | 3  |
| 13. | 0     | 3  |
| 14. | 0     | 12 |
| 15. | 0     | 4  |
| 16. | 0     | 1  |
| 17. | 0     | 5  |

We analyze the data using ranksum:

```
. ranksum y, by(group)
Two-sample Wilcoxon rank-sum (Mann-Whitney) test
```

| group    | Obs | Rank sum | Expected |
|----------|-----|----------|----------|
| 0        | 11  | 79       | 99       |
| 1        | 6   | 74       | 54       |
| Combined | 17  | 153      | 153      |

```
Unadjusted variance 99.00
Adjustment for ties -0.97

Adjusted variance 98.03
H0: y(group==0) = y(group==1)
 z = -2.020
Prob > |z| = 0.0434
Exact prob = 0.0436
```

The test gives an approximate *p*-value of 0.0434 and an exact *p*-value of 0.0436.

Let's try to reproduce these results using permute. The test statistic *T* for the Wilcoxon rank-sum test is the sum of the ranks for the first group, which is 79, and is stored as `r(sum_obs)`. We specify `reps(10000)` to do 10,000 Monte Carlo permutations and `dots(100)` to display a dot every 100th permutation. We set the random-number seed so that we can duplicate our results.

```
. set seed 1234
. permute group r(sum_obs), reps(10000) dots(100): ranksum y, by(group)
(running ranksum on estimation sample)
warning: ranksum does not set e(sample), so no observations will be excluded
from the permutations because of missing values or other reasons. To
exclude observations, press Break, save the data, drop any
observations that are to be excluded, and rerun permute.
Permutations (10,000):1,000.....2,000.....3,000.....4,000
>5,000.....6,000.....7,000.....8,000.....9,000.....
> ..10,000 done
Monte Carlo permutation results Number of observations = 17
Permutation variable: group Number of permutations = 10,000
Command: ranksum y, by(group)
 _pm_1: r(sum_obs)
```

| T     | T(obs) | Test      | c    | n     | p     | Monte Carlo error |             |       |
|-------|--------|-----------|------|-------|-------|-------------------|-------------|-------|
|       |        |           |      |       |       | SE(p)             | [95% CI(p)] |       |
| _pm_1 | 79     | lower     | 223  | 10000 | .0223 | .0015             | .0195       | .0254 |
|       |        | upper     | 9817 | 10000 | .9817 | .0013             | .9789       | .9842 |
|       |        | two-sided |      |       |       | .0446             | .0021       | .0406 |

Notes: For lower one-sided test,  $c = \#\{T \leq T(\text{obs})\}$  and  $p = p_{\text{lower}} = c/n$ .  
 For upper one-sided test,  $c = \#\{T \geq T(\text{obs})\}$  and  $p = p_{\text{upper}} = c/n$ .  
 For two-sided test,  $p = 2 \cdot \min(p_{\text{lower}}, p_{\text{upper}})$ ; SE and CI approximate.

The lengthy message about `e(sample)` is worth noting. If there were missing values in the data, we might want to drop those observations before running `permute`. To suppress the message in future runs, use the `nowarning` option.

The two-sided  $p$ -value obtained by this Monte Carlo procedure is 0.0446, which is close to the exact  $p$ -value of 0.0436 computed by `ranksum`. See the [next section](#) for a description of how two-sided  $p$ -values are calculated when performing permutation tests. See [example 2](#) for a test that requires a one-sided  $p$ -value.

Note that we typed

```
. permute group ...
```

rather than

```
. permute y ...
```

We permuted the 0/1 variable `group`, which defines the groups, rather than the outcome variable `y`. For a statistic dependent on only two variables, it obviously does not matter which one we permute in terms of the theory of the test, but it does matter in terms of the efficiency of how `permute` does the computation. `permute` uses a different random shuffling algorithm for 0/1 (or dichotomous) variables than it does with other variables. See [Efficiency considerations for Monte Carlo permutations](#) below for details.

`permute` reports standard errors and confidence intervals for  $p$ -values because, as with any other Monte Carlo procedure, they are approximations to the true exact  $p$ -values. These statistics are useful to assess the precision of the computed  $p$ -values. If you need more precision, specify more permutations in the `reps()` option. See [Methods and formulas](#) for a description of how the standard errors and confidence intervals are calculated.

The confidence interval for the Monte Carlo two-sided  $p$ -value in this example is [0.0406, 0.0486]. If we want to increase the precision of the  $p$ -value, we could run `permute` again with more random permutations to narrow the confidence interval. The total number of possible distinct permutations of `group`, however, is not extremely large, and we can perform the permutation test using enumeration. See [example 3](#), where we do just that.

◀

## Two-sided $p$ -values from permutation tests

In the above example, we used the two-sided  $p$ -value for our hypothesis testing. For permutation distributions, two-sided  $p$ -values require some explanation about how they are calculated.

`permute` calculates the two-sided  $p$ -value as  $p = 2 \min(p_{\text{lower}}, p_{\text{upper}})$ , where  $p_{\text{lower}}$  is the lower one-sided  $p$ -value and  $p_{\text{upper}}$  the upper one-sided  $p$ -value. (More precisely,  $p = \min[1, 2 \min(p_{\text{lower}}, p_{\text{upper}})]$  is used because obviously  $p$ -values must be bounded by 1.)

In general, the  $p$ -value is defined as the probability under the null hypothesis of obtaining a value of the test statistic  $T$  equal to or more extreme than the value  $T_{\text{obs}}$  that was actually observed. For one-sided  $p$ -values, what is “more extreme” is clear. For lower one-sided  $p$ -values, it is the probability that  $T \leq T_{\text{obs}}$ , and for upper one-sided  $p$ -values, it is the probability that  $T \geq T_{\text{obs}}$ . When  $T$  has a symmetric distribution, the two-sided  $p$ -value is typically defined as the probability that  $|T| \geq |T_{\text{obs}}|$ . Permutation distributions, however, are not in general symmetric.

Under a permutation-based null hypothesis, the domain of  $T$  consists of all the possible permutations of the underlying data used to calculate  $T$ . The domain is discrete and finite, and hence the permutation distribution of  $T$  is discrete and finite. These finite distributions are symmetric only in certain cases. For instance, with our example of the [Wilcoxon rank-sum test](#), if the data consist of untied ranks, the distribution is symmetric. When there are ties in the ranks, however, the distribution is in most cases not symmetric.

When distributions are asymmetric, what values of  $T$  are “more extreme” than  $T_{\text{obs}}$ ? Suppose  $T_{\text{obs}}$  is below the mean of the distribution. Clearly, the lower-tail values  $T \leq T_{\text{obs}}$  are more extreme. But what values of  $T$  from the upper tail are more extreme?

For asymmetric distributions, the rationale for using  $p = 2 \min(p_{\text{lower}}, p_{\text{upper}})$  for two-sided tests is the following: It takes the smallest one-sided  $p$ -value and doubles it. Comparing this two-sided  $p$ -value against a significance level of, say, 0.05 is equivalent to comparing the smallest one-sided  $p$ -value against a level of 0.025. It essentially turns the two-sided test into a one-sided test with the significance level cut in half. So this definition conveniently sidesteps the need to define what values of  $T$  from the opposite tail from  $T_{\text{obs}}$  are more extreme! Also, it is appropriate for both symmetric and asymmetric distributions.

## One-sided permutation test

In some cases, we will want to perform a permutation test based on a one-sided  $p$ -value.

### ▷ Example 2: Permutation tests with ANOVA

Consider some fictional data from an experimental randomized complete-block design in which there are 5 subjects each receiving 10 different treatments. We want to test whether any of the treatments have an effect different from the effects of the other treatments.

Let’s load the data and list the data for the first two subjects:

```
. use https://www.stata-press.com/data/r19/permute1, clear
. sort subject treatment
. list subject treatment y in 1/20, abbrev(10)
```

|       | subject | treatment | y        |
|-------|---------|-----------|----------|
| 1.    | 1       | 1         | 4.407557 |
| 2.    | 1       | 2         | 4.280349 |
| 3.    | 1       | 3         | 4.418574 |
| 4.    | 1       | 4         | 4.075359 |
| 5.    | 1       | 5         | 3.899775 |
| <hr/> |         |           |          |
| 6.    | 1       | 6         | 5.533271 |
| 7.    | 1       | 7         | 5.142111 |
| 8.    | 1       | 8         | 5.791124 |
| 9.    | 1       | 9         | 4.504411 |
| 10.   | 1       | 10        | 4.896333 |
| <hr/> |         |           |          |
| 11.   | 2       | 1         | 5.693386 |
| 12.   | 2       | 2         | 4.508785 |
| 13.   | 2       | 3         | 5.10376  |
| 14.   | 2       | 4         | 5.753985 |
| 15.   | 2       | 5         | 5.092277 |
| <hr/> |         |           |          |
| 16.   | 2       | 6         | 4.496496 |
| 17.   | 2       | 7         | 6.339948 |
| 18.   | 2       | 8         | 4.820389 |
| 19.   | 2       | 9         | 5.686253 |
| 20.   | 2       | 10        | 6.951727 |

These data may be analyzed using anova.

```
. anova y treatment subject
```

|           | Number of obs = | 50      | R-squared =     | 0.3544 |        |
|-----------|-----------------|---------|-----------------|--------|--------|
|           | Root MSE =      | .914159 | Adj R-squared = | 0.1213 |        |
| Source    | Partial SS      | df      | MS              | F      | Prob>F |
| Model     | 16.518219       | 13      | 1.2706322       | 1.52   | 0.1574 |
| treatment | 13.022671       | 9       | 1.4469634       | 1.73   | 0.1174 |
| subject   | 3.4955481       | 4       | .87388703       | 1.05   | 0.3973 |
| Residual  | 30.08475        | 36      | .83568751       |        |        |
| Total     | 46.602969       | 49      | .951081         |        |        |

anova gives a  $p$ -value of 0.1174 for the treatment effect. This  $p$ -value is calculated with the assumption of normality for the distribution of the outcome conditional on the means of each treatment and subject effect.

Suppose we do not want to assume normality. The treatments were assigned in a random order to each of the subjects. A null hypothesis of no treatment effect means that the observed values of  $y$  and their order were determined by factors other than the treatments. The treatments were essentially labels that had nothing to do with the outcomes, and any other ordering of the labels would be a possible occurrence. That is, we imagine running the experiment multiple times, each with a different ordering of the treatments, but each time, we get the same observed values of  $y$ . This is the permutation-based formulation of the null hypothesis.

What about the subjects? Each subject gets each of the 10 treatments, so clearly we must permute the treatments within each subject independently of the permutations for the other subjects. We can do this using the `strata()` option with `permute`.

If we type `ereturn list` after `anova`, we see that the  $F$  statistic for treatment is stored in `e(F_1)`. This is our test statistic for our permutation test.

We save the dataset containing all the permutations of the test statistic using the `saving()` option. Specifying the test statistic as `F_treatment=e(F_1)` labels the test statistic as `F_treatment` in the output and is also the name of the variable containing the test statistic in `permanova.dta`, the dataset created by `saving()`. We set the seed for the random-number generator and also specify the `nodots` option to suppress the dots in the output.

```

. permute treatment F_treatment=e(F_1), reps(10000) strata(subject)
> saving(permanova) rseed(1234) nodots: anova y treatment subject

Monte Carlo permutation results Number of observations = 50
Permutation variable: treatment Number of strata = 5
 Number of permutations = 10,000

 Command: anova y treatment subject
 F_treatment: e(F_1)

```

| T           | T(obs)   | Test      | c    | n     | p     | Monte Carlo error |             |       |
|-------------|----------|-----------|------|-------|-------|-------------------|-------------|-------|
|             |          |           |      |       |       | SE(p)             | [95% CI(p)] |       |
| F_treatment | 1.731465 | lower     | 8788 | 10000 | .8788 | .0033             | .8722       | .8851 |
|             |          | upper     | 1212 | 10000 | .1212 | .0033             | .1149       | .1278 |
|             |          | two-sided |      |       |       | .2424             | .0043       | .2340 |

Notes: For lower one-sided test,  $c = \#\{T \leq T(\text{obs})\}$  and  $p = p_{\text{lower}} = c/n$ .  
 For upper one-sided test,  $c = \#\{T \geq T(\text{obs})\}$  and  $p = p_{\text{upper}} = c/n$ .  
 For two-sided test,  $p = 2 \cdot \min(p_{\text{lower}}, p_{\text{upper}})$ ; SE and CI approximate.

Our test statistic is an  $F$  statistic, so we are interested in the number of permutations that have a larger (more extreme) statistic than the 1.73 we obtained with our original data. Therefore, we want the upper one-sided  $p$ -value, which is 0.1212. This value is close to the  $p$ -value given by `anova` of 0.1174 for the treatment effect.

◀

For an additional example of a permutation test, with an application in epidemiology, see [Hayes and Moulton \(2017, 237–241\)](#).

## Enumeration

When the number of observations,  $N$ , in a dataset is small, it may be possible to enumerate all possible permutations and obtain  $p$ -values without the error involved in computing Monte Carlo  $p$ -values.

When `permute` does an enumeration, not only does  $N$  matter, but the number of distinct values of the permutation variable matters as well. `permute` does the enumeration by computing only permutations that give different arrangements of the permutation variable; that is, it does not compute any duplicate permutations. So the number of distinct values (and their multiplicity) of the permutation variable determines the number of permutations enumerated and so determines whether enumeration is feasible. See [Efficiency considerations for enumeration](#) below for details.

### ▷ Example 3: Wilcoxon rank-sum test using enumeration

Here we repeat [example 1](#), but this time we do it by enumerating all possible permutations. We load the data:

```
. use https://www.stata-press.com/data/r19/permute2
```

The data consist of an outcome  $y$  grouped by the variable `group`. If we tabulate `group`

```
. tabulate group
```

| Group | Freq. | Percent | Cum.   |
|-------|-------|---------|--------|
| 0     | 11    | 64.71   | 64.71  |
| 1     | 6     | 35.29   | 100.00 |
| Total | 17    | 100.00  |        |

we see that `group` consists of 11 zeros and 6 ones. Hence, there are only  $\binom{17}{6} = 12,376$  possible distinct permutations of `group`.

In [example 1](#), we used `ranksum` to compute the test statistic, but each time `ranksum` is called, it computes the ranks of  $y$ . It is unnecessary to recompute the ranks for each permutation. It is better to compute the ranks just once at the outset. We can do this using the `rank()` function of `egen`:

```
. egen r = rank(y)
```

The test statistic is the sum of the ranks for either one of the groups. The sum can be computed efficiently using `summarize` with an `if` restriction and the `meanonly` option.

```
. permute group r(sum), enumerate nodrop nowarning dots(100): summarize r
> if group == 1, meanonly
(running summarize on estimation sample)
(enumerating all 12,376 possible permutations)
Permutations (12,376):1,000.....2,000.....3,000.....4,000
>5,000.....6,000.....7,000.....8,000.....9,000.....
> ..10,000.....11,000.....12,000.... done

Enumeration permutation results
Number of observations = 17
Number of permutations = 12,376
Permutation variable: group

Command: summarize r if group == 1, meanonly
 _pm_1: r(sum)
```

| T     | T(obs) | Test      | c     | n     | p     |
|-------|--------|-----------|-------|-------|-------|
| _pm_1 | 74     | lower     | 12142 | 12376 | .9811 |
|       |        | upper     | 270   | 12376 | .0218 |
|       |        | two-sided |       |       | .0436 |

Notes: For lower one-sided test,  $c = \#\{T \leq T(\text{obs})\}$  and  $p = p_{\text{lower}} = c/n$ .  
 For upper one-sided test,  $c = \#\{T \geq T(\text{obs})\}$  and  $p = p_{\text{upper}} = c/n$ .  
 For two-sided test,  $p = 2 \cdot \min(p_{\text{lower}}, p_{\text{upper}})$ .

Note that it is necessary to specify the `nodrop` option. Otherwise, `permute` would drop all observations not satisfying `if group == 1` before doing the permutations, and that would not give us what we want.

`permute` with the `enumerate` option gave a two-sided  $p$ -value of 0.0436, which is the same as the exact  $p$ -value reported by `ranksum`, as it should be.

With `group` as the variable being permuted, the number of distinct permutations is quite small. If, however, we attempt to do the enumeration for all the distinct permutations of `r`:

```
. permute r r(sum), enumerate nodrop nowarning dots(100):
> summarize r if group == 1, meanonly
(running summarize on estimation sample)
(enumerating all 3.71e+12 possible permutations)
Permutations (3,705,077,376,000):1,000.....2,000.....3,000
>4,000.....—Break—
r(1);
```

Each permutation takes about 0.1 millisecond on our computer. Thus, the enumeration will take  $0.1 \times 3.71 \times 10^{12} / (365 \times 24 \times 60 \times 60 \times 1000) \approx 12$  years, so we pressed *Break*.

◀

## Efficiency considerations for Monte Carlo permutations

Suppose you want to perform a randomization two-sample *t* test, which is like the two-sample *t* test that assumes normality, only the randomization test is based on permuting the variable that defines the samples. It is the same as the Wilcoxon rank-sum test, except the observed values of the outcome, rather than their ranks, are used for the test statistic.

So say we have a variable `x` with continuous outcomes for two groups defined by the variable `group`, with values 0 or 1. The randomization two-sample *t* test could be done using Monte Carlo permutations by typing

```
. permute x r(mu_1), reps(10000): ttest x, by(group)
```

Or by typing

```
. permute group r(mu_1), reps(10000): ttest x, by(group)
```

In the first case, `x` was permuted, and in the second, `group`. Both are valid ways to do Monte Carlo permutations. `permute` is smart, however, and treats a 0/1 variable differently from how it treats a variable with lots of distinct values.

Suppose there are fewer 1s than 0s in `group`. Rather than randomly permuting all the 0s and 1s, `permute` randomly shuffles the 1s into the 0s. If there are only a few 1s relative to the number of 0s, this method is much faster than permuting all the values. Hence, if you are doing a permutation test that involves two variables, pick the one with the fewest distinct values to be the *permvar*.

When the `strata()` option is specified, `permute` uses special code for the case in which the *permvar* is dichotomous (with, say, values  $y_0$  and  $y_1$ ) and each stratum contains a single observation equal to  $y_1$  and all other observations in the stratum equal to  $y_0$  (and  $y_0$  and  $y_1$  do not flip or change in value across strata). If you are doing a stratified permutation test and you have such a variable whose permutations will give the test you want, be sure to make it the *permvar*.

For all types of data, Monte Carlo permutations of the *permvar* are computed quickly. If the command calculating the test statistic for each permutation is not fast, it is unlikely you will notice the greater speed of permuting a dichotomous variable. If, however, you are using a fast command such as `regress`, you likely will notice the greater speed.

## Efficiency considerations for enumeration

Doing an enumeration of all possible distinct permutations using the `enumerate` option, however, is a different story. Here the selection of the *permvar* is crucial and typically determines whether it is feasible to do the enumeration.

Consider performing the randomization *t* test we described earlier using enumeration:

```
. permute group r(mu_1), enumerate: tttest x, by(group)
```

Suppose there are 20 observations, 10 with `group = 0` and 10 with `group = 1`. Typing `permute group ...` will enumerate all  $\binom{20}{10} = 184,756$  possible distinct permutations of `group`. (When we say “distinct permutations”, we mean that duplicate permutations are not computed.)

If, however, we type

```
. permute x r(mu_1), enumerate: tttest x, by(group)
```

`permute x ...` will attempt to enumerate all possible permutations of `x`. If all the values of `x` are unique, there are  $20! \approx 2.4 \times 10^{18}$  possible permutations of `x`, which is much larger than 184,756.

Hence, a dichotomous variable or a variable with few distinct values should always be chosen as the *permvar* rather than another variable with many distinct values whenever possible. (To be precise, both the number of distinct values and their multiplicities determine the number of permutations. See [Methods and formulas](#).)

See [example 3](#) for an example using enumeration.

## Stored results

`permute` stores the following in `r()`:

### Scalars

|                          |                                                            |
|--------------------------|------------------------------------------------------------|
| <code>r(N)</code>        | number of observations for <i>command</i>                  |
| <code>r(n_reps)</code>   | number of permutations performed                           |
| <code>r(k_exp)</code>    | number of standard expressions                             |
| <code>r(k_eeexp)</code>  | number of <code>_b</code> and <code>_se</code> expressions |
| <code>r(n_strata)</code> | number of strata, if <code>strata()</code> specified       |
| <code>r(level)</code>    | confidence level                                           |

### Macros

|                           |                                                            |
|---------------------------|------------------------------------------------------------|
| <code>r(cmd)</code>       | <code>permute</code>                                       |
| <code>r(command)</code>   | <i>command</i> following colon                             |
| <code>r(permvar)</code>   | permutation variable                                       |
| <code>r(enumerate)</code> | "enumerate", if <code>enumerate</code> specified           |
| <code>r(title)</code>     | title in output                                            |
| <code>r(rngstate)</code>  | random-number state used for Monte Carlo permutations      |
| <code>r(exp#)</code>      | #th expression                                             |
| <code>r(strata)</code>    | strata variable, if <code>strata()</code> specified        |
| <code>r(missing)</code>   | "missing" when one or more expressions equal missing value |

### Matrices

|                            |                                                                         |
|----------------------------|-------------------------------------------------------------------------|
| <code>r(b)</code>          | observed statistics                                                     |
| <code>r(b_std)</code>      | standardized observed statistics, if <code>standardize</code> specified |
| <code>r(n)</code>          | number of nonmissing results                                            |
| <code>r(c_lower)</code>    | counts for lower one-sided <i>p</i> -values                             |
| <code>r(c_upper)</code>    | counts for upper one-sided <i>p</i> -values                             |
| <code>r(p_lower)</code>    | lower one-sided <i>p</i> -values                                        |
| <code>r(p_upper)</code>    | upper one-sided <i>p</i> -values                                        |
| <code>r(p_twosided)</code> | two-sided <i>p</i> -values                                              |

|                               |                                                                 |
|-------------------------------|-----------------------------------------------------------------|
| <code>r(se_p_lower)</code>    | Monte Carlo standard errors of lower one-sided $p$ -values      |
| <code>r(se_p_upper)</code>    | Monte Carlo standard errors of upper one-sided $p$ -values      |
| <code>r(se_p_twosided)</code> | Monte Carlo standard errors of two-sided $p$ -values            |
| <code>r(ci_p_lower)</code>    | Monte Carlo confidence intervals of lower one-sided $p$ -values |
| <code>r(ci_p_upper)</code>    | Monte Carlo confidence intervals of upper one-sided $p$ -values |
| <code>r(ci_p_twosided)</code> | Monte Carlo confidence intervals of two-sided $p$ -values       |

## Methods and formulas

One-sided  $p$ -values are based on counts of the test statistic  $T$  calculated for each permutation that are more extreme than the observed value  $T_{\text{obs}}$ . The lower one-sided  $p$ -value uses the count  $c = \#\{T \leq T_{\text{obs}}\}$ , and the upper one-sided  $p$ -value uses  $c = \#\{T \geq T_{\text{obs}}\}$ .

The counts from Monte Carlo permutations are assumed to have a binomial distribution. Standard errors and confidence intervals are computed using `cii proportions`  $n$   $c$ , where  $n$  is the number of permutations that yielded nonmissing results and  $c$  is the count. The confidence intervals are exact binomial confidence intervals. See *Methods and formulas* in [R] `ci`.

`permute` calculates the two-sided  $p$ -value as  $p = \min[1, 2 \min(p_{\text{lower}}, p_{\text{upper}})]$ , where  $p_{\text{lower}}$  is the lower one-sided  $p$ -value and  $p_{\text{upper}}$  the upper one-sided  $p$ -value. Because the definition of the two-sided  $p$ -value does not yield a simple formula for the standard error or confidence interval for Monte Carlo permutations, the following ad hoc procedure is used. If  $p_{\text{lower}}$  is the minimum one-sided  $p$ -value, its count  $c_{\text{lower}}$  is doubled. If  $p_{\text{upper}}$  is the minimum one-sided  $p$ -value, its count  $c_{\text{upper}}$  is doubled. More precisely, the value  $c_2 = \min[n, 2 \min(c_{\text{lower}}, c_{\text{upper}})]$  is used, and its distribution is assumed to be approximately binomial. Standard errors and confidence intervals are computed using `cii proportions`  $n$   $c_2$ , `wald`. The confidence intervals produced are asymptotic binomial confidence intervals.

When `enumerate` is specified, the  $p$ -values have no error.

Suppose there are  $N$  observations and the variable being permuted contains  $K$  distinct values, each with multiplicity  $n_k$ ,  $k = 1, \dots, K$ . The total number of distinct permutations is

$$\frac{N!}{n_1! n_2! \cdots n_K!}$$

This is the number of permutations computed when `enumerate` is specified.

When `standardize` is specified, instead of displaying the observed test statistic  $T_{\text{obs}}$ , the standardized statistic

$$\frac{T_{\text{obs}} - \text{mean}(T)}{\sqrt{\text{Var}(T)}}$$

is displayed where  $\text{mean}(T)$  and  $\text{Var}(T)$  are the mean and variance of the permutation distribution of  $T$ :

$$\text{mean}(T) = \frac{1}{n} \sum_{i=1}^n T_i$$

$$\text{Var}(T) = \frac{1}{n} \sum_{i=1}^n \{T_i - \text{mean}(T)\}^2$$

$n$  is the number of permutations, and  $T_i$  is the test statistic calculated for the  $i$ th permutation.

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## Also see

- [R] **bootstrap** — Bootstrap sampling and estimation
- [R] **jackknife** — Jackknife estimation
- [R] **simulate** — Monte Carlo simulations

## Description

The term pk refers to pharmacokinetic data and the Stata commands, all of which begin with the letters pk, designed to do some of the analyses commonly performed in the pharmaceutical industry. The system is intended for the analysis of pharmacokinetic data, although some of the commands are for general use.

The pk commands are

|            |                       |                                              |
|------------|-----------------------|----------------------------------------------|
| pkexamine  | [R] <b>pkexamine</b>  | Calculate pharmacokinetic measures           |
| pksumm     | [R] <b>pksumm</b>     | Summarize pharmacokinetic data               |
| pkshape    | [R] <b>pkshape</b>    | Reshape (pharmacokinetic) Latin-square data  |
| pkcross    | [R] <b>pkcross</b>    | Analyze crossover experiments                |
| pkequiv    | [R] <b>pkequiv</b>    | Perform bioequivalence tests                 |
| pkcollapse | [R] <b>pkcollapse</b> | Generate pharmacokinetic measurement dataset |

Also see [ME] **menl** for fitting pharmacokinetic models using nonlinear mixed-effects models; for instance, see [example 15](#) in [ME] **menl**.

## Remarks and examples

Several types of clinical trials are commonly performed in the pharmaceutical industry. Examples include combination trials, multicenter trials, equivalence trials, and active control trials. For each type of trial, there is an optimal study design for estimating the effects of interest. The pk system can be used to analyze equivalence trials, which are usually conducted using a crossover design; however, it is possible to use a parallel design and still draw conclusions about equivalence.

Equivalence trials assess bioequivalence between two drugs. Although proving that two drugs behave the same is impossible, regulatory agencies believe that if the absorption properties of two drugs are similar, the two drugs will produce similar effects and have similar safety profiles. Generally, the goal of an equivalence trial is to assess the equivalence of a generic drug to an existing drug. This goal is commonly accomplished by comparing a confidence interval about the difference between a pharmacokinetic measurement of two drugs with an equivalence limit constructed from regulations. If the confidence interval is entirely within the equivalence limit, the drugs are declared bioequivalent. Another approach to assessing bioequivalence is to use the method of interval hypotheses testing. `pkequiv` is used to conduct these tests of bioequivalence.

Several pharmacokinetic measures can be used to ascertain how available a drug is for cellular absorption. The most common measure is the area under the concentration–time curve (AUC). Another common measure of drug availability is the maximum concentration ( $C_{\max}$ ) achieved by the drug during the follow-up period. Stata reports these and other less common measures of drug availability, including the time at which the maximum drug concentration was observed and the duration of the period during which the subject was being measured. Stata also reports the elimination rate, that is, the rate at which the drug is metabolized, and the drug’s half-life, that is, the time it takes for the drug concentration to fall to one-half of its maximum concentration.

pkexamine computes and reports all the pharmacokinetic measures that Stata produces, including four calculations of the AUC. The standard AUC from 0 to the maximum observed time ( $AUC_{0,t_{\max}}$ ) is computed using cubic splines or the trapezoidal rule. Additionally, pkexamine also computes the AUC from 0 to infinity by extending the standard concentration–time curve from the maximum observed time using three different methods. The first method simply extends the standard curve by using a least-squares linear fit through the last few data points. The second method extends the standard curve by fitting a decreasing exponential curve through the last few data points. The third method extends the curve by fitting a least-squares linear regression line on the log concentration. The mathematical details of these extensions are described in *Methods and formulas* of [R] [pkexamine](#).

Data from an equivalence trial may also be analyzed using methods appropriate to the particular study design. When you have a crossover design, pkcross can be used to fit an appropriate ANOVA model. A crossover design is simply a restricted Latin square; therefore, pkcross can also be used to analyze any Latin-square design.

Some practical concerns arise when dealing with data from equivalence trials. Primarily, the data must be organized in a manner that Stata can use. The pk commands include pkcollapse and pkshape, which are designed to help transform data from a common format to one that is suitable for analysis with Stata.

In the following examples, we illustrate several different data formats that are often encountered in pharmaceutical research and describe how these formats can be transformed to formats that can be analyzed with Stata.

### ▷ Example 1

Assume that we have one subject and are interested in determining the drug profile for that subject. A reasonable experiment would be to give the subject the drug and then measure the concentration of the drug in the subject's blood over a given period. For example, here is part of a dataset from [Chow and Liu \(2009, 13\)](#):

```
. use https://www.stata-press.com/data/r19/auc
(Primidone concentrations)
. list, abbrev(14)
```

|     | id | time | concentration |
|-----|----|------|---------------|
| 1.  | 1  | 0    | 0             |
| 2.  | 1  | .5   | 0             |
| 3.  | 1  | 1    | 2.8           |
| 4.  | 1  | 1.5  | 4.4           |
| 5.  | 1  | 2    | 4.4           |
| 6.  | 1  | 3    | 4.7           |
| 7.  | 1  | 4    | 4.1           |
| 8.  | 1  | 6    | 4             |
| 9.  | 1  | 8    | 3.6           |
| 10. | 1  | 12   | 3             |
| 11. | 1  | 16   | 2.5           |
| 12. | 1  | 24   | 2             |
| 13. | 1  | 32   | 1.6           |

Examining these data, we notice that the concentration quickly increases, plateaus for a short period, and then slowly decreases over time. `pkexamine` is used to calculate the pharmacokinetic measures of interest. `pkexamine` is explained in detail in [R] `pkexamine`. The output is

```
. pkexamine time concentration
 Maximum concentration = 4.7
 Time of maximum concentration = 3
 Time of last observation (Tmax) = 32
 Elimination rate = 0.0279
 Half life = 24.8503
```

Area under the curve

| AUC [0, Tmax] | AUC [0, inf.)<br>Linear of log conc. | AUC [0, inf.)<br>Linear fit | AUC [0, inf.)<br>Exponential fit |
|---------------|--------------------------------------|-----------------------------|----------------------------------|
| 85.24         | 142.603                              | 107.759                     | 142.603                          |

Fit based on last 3 points.

◀

## ▶ Example 2

Clinical trials require that data be collected on more than one subject. There are several ways to enter raw measured data collected on several subjects. It would be reasonable to enter for each subject the drug concentration value at specific points in time. Such data could be

| id | conc1 | conc2 | conc3 | conc4 | conc5 | conc6 | conc7 |
|----|-------|-------|-------|-------|-------|-------|-------|
| 1  | 0     | 1     | 4     | 7     | 5     | 3     | 1     |
| 2  | 0     | 2     | 6     | 5     | 4     | 3     | 2     |
| 3  | 0     | 1     | 2     | 3     | 5     | 4     | 1     |

where `conc1` is the concentration at the first measured time, `conc2` is the concentration at the second measured time, etc. This format requires that each drug concentration measurement be made at the same time on each subject. Another more flexible way to enter the data is to have an observation with three variables for each time measurement on a subject. Each observation would have a subject ID, the time at which the measurement was made, and the corresponding drug concentration at that time. The data would be as follows:

```
. use https://www.stata-press.com/data/r19/pkdata
(Fictional drug concentration data)
. list id conc1 time, sepby(id)
```

|                         | id | conc1    | time |
|-------------------------|----|----------|------|
| 1.                      | 1  | 0        | 0    |
| 2.                      | 1  | 3.073403 | .5   |
| 3.                      | 1  | 5.188444 | 1    |
| 4.                      | 1  | 5.898577 | 1.5  |
| 5.                      | 1  | 5.096378 | 2    |
| 6.                      | 1  | 6.094085 | 3    |
| 7.                      | 1  | 5.158772 | 4    |
| 8.                      | 1  | 5.7065   | 6    |
| 9.                      | 1  | 5.272467 | 8    |
| 10.                     | 1  | 4.4576   | 12   |
| 11.                     | 1  | 5.146423 | 16   |
| 12.                     | 1  | 4.947427 | 24   |
| 13.                     | 1  | 1.920421 | 32   |
| 14.                     | 2  | 0        | 0    |
| 15.                     | 2  | 2.48462  | .5   |
| 16.                     | 2  | 4.883569 | 1    |
| 17.                     | 2  | 7.253442 | 1.5  |
| 18.                     | 2  | 5.849345 | 2    |
| 19.                     | 2  | 6.761085 | 3    |
| 20.                     | 2  | 4.33839  | 4    |
| 21.                     | 2  | 5.04199  | 6    |
| 22.                     | 2  | 4.25128  | 8    |
| 23.                     | 2  | 6.205004 | 12   |
| 24.                     | 2  | 5.566165 | 16   |
| 25.                     | 2  | 3.689007 | 24   |
| 26.                     | 2  | 3.644063 | 32   |
| <i>(output omitted)</i> |    |          |      |
| 207.                    | 16 | 4.673281 | 24   |
| 208.                    | 16 | 3.487347 | 32   |

Stata expects the data to be organized in the second form, as shown in `pkdata.dta`. If your data are organized as described in the first format, you will need to reshape the data to the second form; see [D] [reshape](#). Because the data in the second (or long) format contain information for one drug on several subjects, `pksumm` can be used to produce summary statistics of the pharmacokinetic measurements. The output is

```
. pksumm id time conc1
.....
```

Summary statistics for the pharmacokinetic measures

| Measure | Mean   | Median | Variance  | Skewness | Kurtosis | Number of observations = 16 |
|---------|--------|--------|-----------|----------|----------|-----------------------------|
|         |        |        |           |          |          | p-value                     |
| auc     | 151.63 | 152.18 | 127.58    | -0.34    | 2.07     | 0.55                        |
| auc1ine | 397.09 | 219.83 | 178276.59 | 2.69     | 9.61     | 0.00                        |
| aucexp  | 668.60 | 302.96 | 720356.98 | 2.67     | 9.54     | 0.00                        |
| auclog  | 665.95 | 298.03 | 752573.34 | 2.71     | 9.70     | 0.00                        |
| half    | 90.68  | 29.12  | 17750.70  | 2.36     | 7.92     | 0.00                        |
| ke      | 0.02   | 0.02   | 0.00      | 0.88     | 3.87     | 0.08                        |
| cmax    | 7.37   | 7.42   | 0.40      | -0.64    | 2.75     | 0.36                        |
| tomc    | 3.38   | 3.00   | 7.25      | 2.27     | 7.70     | 0.00                        |
| tmax    | 32.00  | 32.00  | 0.00      | .        | .        | .                           |

Until now, we have been concerned with the profile of only one drug. We have characterized the profile of that drug by individual subjects by using `pkexamine` and by a group of subjects by using `pksumm`. The goal of an equivalence trial, however, is to compare two drugs, which we will do in the rest of this example.

For equivalence trials, the study design most often used is the crossover design. For a complete discussion of crossover designs, see [Ratkowsky, Evans, and Alldredge \(1993\)](#).

In brief, crossover designs require that each subject be given both treatments at two different times. The order in which the treatments are applied changes between groups. For example, if we had 16 subjects numbered 1–16, the first 8 would receive reference treatment “R” during the first period of the study, and then they would be given test treatment “T”. The second 8 subjects would be given treatment “T” during the first period of the study, and then they would be given treatment “R”. Each subject in the study will have four variables that describe the observation: a subject identifier, a sequence identifier that indicates the order of treatment, and two outcome variables, one for each treatment. The outcome variables for each subject are the pharmacokinetic measures. The data must be transformed from a series of measurements on individual subjects to data containing the pharmacokinetic measures for each subject. In Stata parlance, this is referred to as a collapse, which can be done with `pkcollapse`; see [\[R\] pkcollapse](#).

Here is part of our data:

```
. list, sepby(id)
```

|      | id | seq | time                    | conc1    | conc2    |
|------|----|-----|-------------------------|----------|----------|
| 1.   | 1  | 1   | 0                       | 0        | 0        |
| 2.   | 1  | 1   | .5                      | 3.073403 | 3.712592 |
| 3.   | 1  | 1   | 1                       | 5.188444 | 6.230602 |
| 4.   | 1  | 1   | 1.5                     | 5.898577 | 7.885944 |
| 5.   | 1  | 1   | 2                       | 5.096378 | 9.241735 |
| 6.   | 1  | 1   | 3                       | 6.094085 | 13.10507 |
| 7.   | 1  | 1   | 4                       | 5.158772 | .169429  |
| 8.   | 1  | 1   | 6                       | 5.7065   | 8.759894 |
| 9.   | 1  | 1   | 8                       | 5.272467 | 7.985409 |
| 10.  | 1  | 1   | 12                      | 4.4576   | 7.740126 |
| 11.  | 1  | 1   | 16                      | 5.146423 | 7.607208 |
| 12.  | 1  | 1   | 24                      | 4.947427 | 7.588428 |
| 13.  | 1  | 1   | 32                      | 1.920421 | 2.791115 |
| 14.  | 2  | 1   | 0                       | 0        | 0        |
| 15.  | 2  | 1   | .5                      | 2.48462  | .9209593 |
| 16.  | 2  | 1   | 1                       | 4.883569 | 5.925818 |
| 17.  | 2  | 1   | 1.5                     | 7.253442 | 8.710549 |
| 18.  | 2  | 1   | 2                       | 5.849345 | 10.90552 |
| 19.  | 2  | 1   | 3                       | 6.761085 | 8.429898 |
| 20.  | 2  | 1   | 4                       | 4.33839  | 5.573152 |
| 21.  | 2  | 1   | 6                       | 5.04199  | 6.32341  |
| 22.  | 2  | 1   | 8                       | 4.25128  | .5251224 |
| 23.  | 2  | 1   | 12                      | 6.205004 | 7.415988 |
| 24.  | 2  | 1   | 16                      | 5.566165 | 6.323938 |
| 25.  | 2  | 1   | 24                      | 3.689007 | 1.133553 |
| 26.  | 2  | 1   | 32                      | 3.644063 | 5.759489 |
| 27.  | 3  | 1   | 0                       | 0        | 0        |
|      |    |     | <i>(output omitted)</i> |          |          |
| 208. | 16 | 2   | 32                      | 3.487347 | 5.213639 |

This format is similar to the second format described above, except that now we have measurements for two drugs at each time for each subject. We transform these data with `pkcollapse`:

```
. pkcollapse time conc1 conc2, id(id) keep(seq) stat(auc)
.....
. list, sep(8) abbrev(10)
```

|     | id | seq | auc_conc1 | auc_conc2 |
|-----|----|-----|-----------|-----------|
| 1.  | 1  | 1   | 150.9643  | 218.5551  |
| 2.  | 2  | 1   | 146.7606  | 133.3201  |
| 3.  | 3  | 1   | 160.6548  | 126.0635  |
| 4.  | 4  | 1   | 157.8622  | 96.17461  |
| 5.  | 5  | 1   | 133.6957  | 188.9038  |
| 6.  | 6  | 1   | 160.639   | 223.6922  |
| 7.  | 7  | 1   | 131.2604  | 104.0139  |
| 8.  | 8  | 1   | 168.5186  | 237.8962  |
| 9.  | 9  | 2   | 137.0627  | 139.7382  |
| 10. | 10 | 2   | 153.4038  | 202.3942  |
| 11. | 11 | 2   | 163.4593  | 136.7848  |
| 12. | 12 | 2   | 146.0462  | 104.5191  |
| 13. | 13 | 2   | 158.1457  | 165.8654  |
| 14. | 14 | 2   | 147.1977  | 139.235   |
| 15. | 15 | 2   | 164.9988  | 166.2391  |
| 16. | 16 | 2   | 145.3823  | 158.5146  |

For this example, we chose to use the  $AUC_{0,t_{max}}$  for two drugs as our pharmacokinetic measure. We could have used any of the measures computed by `pkexamine`. In addition to the AUCs, the dataset also contains a sequence variable for each subject indicating when each treatment was administered.

The data produced by `pkcollapse` are in what Stata calls wide format; that is, there is one observation per subject containing two or more outcomes. To use `pkcross` and `pkequiv`, we need to transform these data to long format, which we can do using `pkshape`; see [R] [pkshape](#).

Consider the first subject in the dataset. This subject is in sequence 1, which means that treatment “R” was applied during the first period of the study and treatment “T” was applied in the second period of the study. We need to split the first observation into two observations so that the outcome measure is only in one variable. We also need two new variables, one indicating the treatment the subject received and another recording the period of the study when the subject received that treatment. We might expect the expansion of the first subject to be

| id | sequence | auc      | treat | period |
|----|----------|----------|-------|--------|
| 1  | 1        | 150.9643 | R     | 1      |
| 1  | 1        | 218.5551 | T     | 2      |

We see that subject number 1 was in sequence 1, had an  $AUC_{0,t_{max}}$  of 150.9643 when treatment “R” was applied in the first period of the study, and had an  $AUC_{0,t_{max}}$  of 218.5551 when treatment “T” was applied in the second period.

Similarly, the expansion of subject 9 (the first subject in sequence 2) would be

| id | sequence | auc      | treat | period |
|----|----------|----------|-------|--------|
| 9  | 2        | 137.0627 | T     | 1      |
| 9  | 2        | 139.7382 | R     | 2      |

Here treatment “T” was applied to the subject during the first period of the study, and treatment “R” was applied to the subject during the second period of the study.

An additional complication is common in crossover study designs. The treatment applied in the first period of the study might still have some effect on the outcome in the second period. In this example, each subject was given one treatment followed by another treatment. To get accurate estimates of treatment effects, it is necessary to account for the carryover effect, the effect that the first treatment has in the second period of the study. We must, therefore, have a variable that indicates which treatment was applied in the first treatment period. `pkshape` creates a variable that indicates the carryover effect. For treatments applied during the first treatment period, there will never be a carryover effect. The sequence, treatment, and carryover variables all receive value labels. Thus, the expanded data created by `pkshape` for subject 1 and subject 9 will be as shown in the results:

```
. pkshape id seq auc*, order(RT TR)
. sort id sequence period
. list, sep(16)
```

|     | id | sequence | outcome  | treat | carry | period |
|-----|----|----------|----------|-------|-------|--------|
| 1.  | 1  | RT       | 150.9643 | R     | 0     | 1      |
| 2.  | 1  | RT       | 218.5551 | T     | R     | 2      |
| 3.  | 2  | RT       | 146.7606 | R     | 0     | 1      |
| 4.  | 2  | RT       | 133.3201 | T     | R     | 2      |
| 5.  | 3  | RT       | 160.6548 | R     | 0     | 1      |
| 6.  | 3  | RT       | 126.0635 | T     | R     | 2      |
| 7.  | 4  | RT       | 157.8622 | R     | 0     | 1      |
| 8.  | 4  | RT       | 96.17461 | T     | R     | 2      |
| 9.  | 5  | RT       | 133.6957 | R     | 0     | 1      |
| 10. | 5  | RT       | 188.9038 | T     | R     | 2      |
| 11. | 6  | RT       | 160.639  | R     | 0     | 1      |
| 12. | 6  | RT       | 223.6922 | T     | R     | 2      |
| 13. | 7  | RT       | 131.2604 | R     | 0     | 1      |
| 14. | 7  | RT       | 104.0139 | T     | R     | 2      |
| 15. | 8  | RT       | 168.5186 | R     | 0     | 1      |
| 16. | 8  | RT       | 237.8962 | T     | R     | 2      |
| 17. | 9  | TR       | 137.0627 | T     | 0     | 1      |
| 18. | 9  | TR       | 139.7382 | R     | T     | 2      |
| 19. | 10 | TR       | 153.4038 | T     | 0     | 1      |
| 20. | 10 | TR       | 202.3942 | R     | T     | 2      |
| 21. | 11 | TR       | 163.4593 | T     | 0     | 1      |
| 22. | 11 | TR       | 136.7848 | R     | T     | 2      |
| 23. | 12 | TR       | 146.0462 | T     | 0     | 1      |
| 24. | 12 | TR       | 104.5191 | R     | T     | 2      |
| 25. | 13 | TR       | 158.1457 | T     | 0     | 1      |
| 26. | 13 | TR       | 165.8654 | R     | T     | 2      |
| 27. | 14 | TR       | 147.1977 | T     | 0     | 1      |
| 28. | 14 | TR       | 139.235  | R     | T     | 2      |
| 29. | 15 | TR       | 164.9988 | T     | 0     | 1      |
| 30. | 15 | TR       | 166.2391 | R     | T     | 2      |
| 31. | 16 | TR       | 145.3823 | T     | 0     | 1      |
| 32. | 16 | TR       | 158.5146 | R     | T     | 2      |

Crossover designs do not require that each subject receive each treatment, but if they do, the crossover design is referred to as a complete crossover design.

The dataset (`pkdata.dta`) in this example is organized in a manner that can be analyzed with Stata. To fit an ANOVA model to these data, we can use `anova` or `pkcross`. To conduct equivalence tests, we can use `pkequiv`.



## References

- Chow, S.-C., and J.-P. Liu. 2009. *Design and Analysis of Bioavailability and Bioequivalence Studies*. 3rd ed. Boca Raton, FL: Chapman and Hall/CRC. <https://doi.org/10.1201/9781420011678>.
- Ratkowsky, D. A., M. A. Evans, and J. R. Alldredge. 1993. *Cross-over Experiments: Design, Analysis, and Application*. New York: Dekker.

## Also see

[ME] `menl` — Nonlinear mixed-effects regression

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## Description

pkcollapse generates new variables with the pharmacokinetic summary measures of interest.

pkcollapse is one of the pk commands. Please read [\[R\] pk](#) before reading this entry.

## Quick start

Single concentration, v1, measured over time, tvar, for patients identified by idvar

```
pkcollapse tvar v1, id(idvar)
```

Same as above, but add additional drug concentration data stored in v2

```
pkcollapse tvar v1 v2, id(idvar)
```

Same as above, but use trapezoidal rule for calculating area under the concentration–time curve ( $AUC_{0,t_{\max}}$ )

```
pkcollapse tvar v1 v2, id(idvar) trapezoid
```

Same as above, and increase the number of data points used to estimate  $AUC_{0,\infty}$  to 10

```
pkcollapse tvar v1 v2, id(idvar) trapezoid fit(10)
```

Retain variables v3 and v4 when collapsing dataset

```
pkcollapse tvar v1 v2, id(idvar) keep(v3 v4)
```

## Menu

Statistics > Epidemiology and related > Other > Generate pharmacokinetic measurement dataset

## Syntax

```
pkcollapse time concentration [concentration [...] [if], id(id_var) [options]
```

| <i>options</i> | Description |
|----------------|-------------|
|----------------|-------------|

### Main

|                                 |                                                                       |
|---------------------------------|-----------------------------------------------------------------------|
| * <i>id</i> ( <i>id_var</i> )   | subject ID variable                                                   |
| <i>stat</i> ( <i>measures</i> ) | create specified <i>measures</i> ; default is all                     |
| <i>trapezoid</i>                | use trapezoidal rule; default is cubic splines                        |
| <i>fit</i> (#)                  | use # points to estimate $AUC_{0,\infty}$ ; default is <i>fit</i> (3) |
| <i>keep</i> ( <i>varlist</i> )  | keep variables in <i>varlist</i>                                      |
| <i>force</i>                    | force collapse                                                        |
| <i>nodots</i>                   | suppress dots during calculation                                      |

\**id*(*id\_var*) is required.

| <i>measures</i> | Description |
|-----------------|-------------|
|-----------------|-------------|

|                |                                                                                                            |
|----------------|------------------------------------------------------------------------------------------------------------|
| <i>auc</i>     | $AUC_{0,t_{\max}}$                                                                                         |
| <i>aucline</i> | $AUC_{0,\infty}$ using a linear extension                                                                  |
| <i>aucexp</i>  | $AUC_{0,\infty}$ using an exponential extension                                                            |
| <i>auclog</i>  | area under the concentration–time curve from 0 to $\infty$ extended with a linear fit to log concentration |
| <i>half</i>    | half-life of the drug                                                                                      |
| <i>ke</i>      | elimination rate                                                                                           |
| <i>cmax</i>    | maximum concentration                                                                                      |
| <i>tmax</i>    | time at last concentration                                                                                 |
| <i>tomc</i>    | time of maximum concentration                                                                              |

## Options

### Main

*id*(*id\_var*) is required and specifies the variable that contains the subject ID over which *pkcollapse* is to operate.

*stat*(*measures*) specifies the measures to be generated. The default is to generate all the measures.

*trapezoid* tells Stata to use the trapezoidal rule when calculating the  $AUC_{0,t_{\max}}$ . The default is to use cubic splines, which give better results for most functions. When the curve is irregular, *trapezoid* may give better results.

*fit*(#) specifies the number of points to use in estimating the  $AUC_{0,\infty}$ . The default is *fit*(3), the last three points. This number should be viewed as a minimum; the appropriate number of points will depend on your data.

*keep*(*varlist*) specifies the variables to be kept during the collapse. Variables not specified with the *keep*() option will be dropped. When *keep*() is specified, the kept variables are checked to ensure that all values of the variables are the same within *id\_var*.

*force* forces the collapse, even when values of the *keep*() variables differ within *id\_var*.

*nodots* suppresses the display of dots during calculation.

## Remarks and examples

pkcollapse generates all the summary pharmacokinetic measures.

### ▷ Example 1

We demonstrate the use of pkcollapse with pkdata.dta described in [example 2](#) of [R] [pk](#). We have drug concentration data on 16 subjects. Each subject is measured at 13 time points over a 32-hour period. Some of the records are as follows:

```
. use https://www.stata-press.com/data/r19/pkdata
(Fictional drug concentration data)
. list, sep(0)
```

|      | id | seq | time             | conc1    | conc2    |
|------|----|-----|------------------|----------|----------|
| 1.   | 1  | 1   | 0                | 0        | 0        |
| 2.   | 1  | 1   | .5               | 3.073403 | 3.712592 |
|      |    |     | (output omitted) |          |          |
| 14.  | 2  | 1   | 0                | 0        | 0        |
| 15.  | 2  | 1   | .5               | 2.48462  | .9209593 |
| 16.  | 2  | 1   | 1                | 4.883569 | 5.925818 |
| 17.  | 2  | 1   | 1.5              | 7.253442 | 8.710549 |
| 18.  | 2  | 1   | 2                | 5.849345 | 10.90552 |
| 19.  | 2  | 1   | 3                | 6.761085 | 8.429898 |
|      |    |     | (output omitted) |          |          |
| 207. | 16 | 2   | 24               | 4.673281 | 6.059818 |
| 208. | 16 | 2   | 32               | 3.487347 | 5.213639 |

Although pksumm allows us to view all the pharmacokinetic measures, we can create a dataset with the measures by using pkcollapse.

```
. pkcollapse time conc1 conc2, id(id) stat(auc) keep(seq)
.....
. list, sep(8) abbrev(10)
```

|     | id | seq | auc_conc1 | auc_conc2 |
|-----|----|-----|-----------|-----------|
| 1.  | 1  | 1   | 150.9643  | 218.5551  |
| 2.  | 2  | 1   | 146.7606  | 133.3201  |
| 3.  | 3  | 1   | 160.6548  | 126.0635  |
| 4.  | 4  | 1   | 157.8622  | 96.17461  |
| 5.  | 5  | 1   | 133.6957  | 188.9038  |
| 6.  | 6  | 1   | 160.639   | 223.6922  |
| 7.  | 7  | 1   | 131.2604  | 104.0139  |
| 8.  | 8  | 1   | 168.5186  | 237.8962  |
| 9.  | 9  | 2   | 137.0627  | 139.7382  |
| 10. | 10 | 2   | 153.4038  | 202.3942  |
| 11. | 11 | 2   | 163.4593  | 136.7848  |
| 12. | 12 | 2   | 146.0462  | 104.5191  |
| 13. | 13 | 2   | 158.1457  | 165.8654  |
| 14. | 14 | 2   | 147.1977  | 139.235   |
| 15. | 15 | 2   | 164.9988  | 166.2391  |
| 16. | 16 | 2   | 145.3823  | 158.5146  |

The resulting dataset contains one observation per subject and is in wide format. If we want to use [pkcross](#) or [pkequiv](#), we must transform these data to long format with the [pkshape](#) command, which we do in [example 2](#) of [\[R\] pk](#).



## Methods and formulas

The statistics generated by `pkcollapse` are described in [\[R\] pkexamine](#).

## Also see

[\[R\] pk](#) — Pharmacokinetic (biopharmaceutical) data

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## Description

`pkcross` analyzes data from a crossover design experiment. When analyzing pharmaceutical trial data, if the treatment, carryover, and sequence variables are known, the omnibus test for separability of the treatment and carryover effects is calculated.

`pkcross` is one of the `pk` commands. Please read [\[R\] pk](#) before reading this entry.

## Quick start

For pharmacokinetic outcome `y` measured for subjects identified by `idvar` given treatments `treat` in sequences identified by `seq` in periods `period` with potential carryover effects from previous treatment `carry`:

Sequence, treatment, and period effects for a  $2 \times 2$  design

```
pkcross y, param(3) id(idvar) sequence(seq) treatment(treat) ///
 period(period)
```

Same as above, but estimate the carryover effect instead of the sequence effect

```
pkcross y, param(1) id(idvar) treatment(treat) period(period) ///
 carryover(carry)
```

Only estimate sequence, treatment, and period effects in higher-order designs

```
pkcross y, id(idvar) sequence(seq) treatment(treat) carryover(none)
```

Also estimate carryover effect and omnibus measure of separability of treatment and carryover effects

```
pkcross y, model(seq / idvar | seq treat carry period) ///
 treatment(treat) carryover(carry) sequence(seq)
```

## Menu

Statistics > Epidemiology and related > Other > Analyze crossover experiments

## Syntax

```
pkcross outcome [if] [in] [, options]
```

| <i>options</i>                         | Description                                                                                                                       |
|----------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------|
| Model                                  |                                                                                                                                   |
| <code>sequence(<i>varname</i>)</code>  | sequence variable; default is <code>sequence(sequence)</code>                                                                     |
| <code>treatment(<i>varname</i>)</code> | treatment variable; default is <code>treatment(treat)</code>                                                                      |
| <code>period(<i>varname</i>)</code>    | period variable; default is <code>period(period)</code>                                                                           |
| <code>id(<i>varname</i>)</code>        | ID variable; default is <code>id(id)</code>                                                                                       |
| <code>carryover(<i>varname</i>)</code> | carryover variable; default is <code>carryover(carry)</code>                                                                      |
| <code>carryover(none)</code>           | omit carryover effects from model; default is <code>carryover(carry)</code>                                                       |
| <code>model(<i>string</i>)</code>      | specify the model to fit                                                                                                          |
| <code>sequential</code>                | estimate sequential instead of partial sums of squares                                                                            |
| Parameterization                       |                                                                                                                                   |
| <code>param(3)</code>                  | estimate mean and the period, treatment, and sequence effects; assume no carryover effects exist; the default                     |
| <code>param(1)</code>                  | estimate mean and the period, treatment, and carryover effects; assume no sequence effects exist                                  |
| <code>param(2)</code>                  | estimate mean, period and treatment effects, and period-by-treatment interaction; assume no sequence or carryover effects exist   |
| <code>param(4)</code>                  | estimate mean, sequence and treatment effects, and sequence-by-treatment interaction; assume no period or carryover effects exist |

## Options

### Model

`sequence(varname)` specifies the variable that contains the sequence in which the treatment was administered. If this option is not specified, `sequence(sequence)` is assumed.

`treatment(varname)` specifies the variable that contains the treatment information. If this option is not specified, `treatment(treat)` is assumed.

`period(varname)` specifies the variable that contains the period information. If this option is not specified, `period(period)` is assumed.

`id(varname)` specifies the variable that contains the subject identifiers. If this option is not specified, `id(id)` is assumed.

`carryover(varname|none)` specifies the variable that contains the carryover information. If `carry(none)` is specified, the carryover effects are omitted from the model. If this option is not specified, `carryover(carry)` is assumed.

`model(string)` specifies the model to be fit. For higher-order crossover designs, this option can be useful if you want to fit a model other than the default. However, `anova` (see [R] [anova](#)) can also be used to fit a crossover model. The default model for higher-order crossover designs is outcome predicted by sequence, period, treatment, and carryover effects. By default, the model statement is `model(sequence period treat carry)`.

`sequential` specifies that sequential sums of squares be estimated.

## Parameterization

`param(#)` specifies which of the four parameterizations to use for the analysis of a  $2 \times 2$  crossover experiment. This option is ignored with higher-order crossover designs. The default is `param(3)`. See the [technical note](#) in this entry for  $2 \times 2$  crossover designs for more details.

`param(3)` estimates the overall mean, the period effects, the treatment effects, and the sequence effects, assuming that no carryover effects exist. This is the default parameterization.

`param(1)` estimates the overall mean, the period effects, the treatment effects, and the carryover effects, assuming that no sequence effects exist.

`param(2)` estimates the overall mean, the period effects, the treatment effects, and the period-by-treatment interaction, assuming that no sequence or carryover effects exist.

`param(4)` estimates the overall mean, the sequence effects, the treatment effects, and the sequence-by-treatment interaction, assuming that no period or carryover effects exist. When the sequence-by-treatment interaction is equivalent to the period effect, this reduces to the third parameterization.

## Remarks and examples

`pkcross` is designed to analyze crossover experiments. Use `pkshape` first to reshape your data; see [\[R\] pkshape](#). `pkcross` assumes that the data were reshaped by `pkshape` or are organized in the same manner as produced with `pkshape`. Washout periods are indicated by the number 0. See the technical note below for more information on analyzing  $2 \times 2$  crossover experiments.

### □ Technical note

The  $2 \times 2$  crossover design cannot be used to estimate more than four parameters because there are only four pieces of information (the four cell means) collected. `pkcross` uses ANOVA models to analyze the data, so one of the four parameters must be the overall mean of the model, leaving just 3 degrees of freedom to estimate the remaining effects (period, sequence, treatment, and carryover). Thus, the model is overparameterized. Estimation of treatment and carryover effects requires the assumption of either no period effects or no sequence effects. Some researchers maintain that estimating carryover effects at the expense of other effects is a bad idea. This is a limitation of this design. `pkcross` implements four parameterizations for this model. They are numbered sequentially from one to four and are described in [Options](#).

□

### ▷ Example 1

Consider the example data published in [Chow and Liu \(2009, 71\)](#). There are 24 patients, 12 in each sequence. Sequence 1 is the reference formulation followed by the test formulation; sequence 2 is the test formulation followed by the reference formulation. After reshaping the data with [pkshape](#), we have variables that identify the subjects, periods, treatments, sequence, and carryover treatment. The outcome variable contains the reported  $AUC_{0,t_{\max}}$ . To compute the ANOVA table, use `pkcross`:

```

. use https://www.stata-press.com/data/r19/chowliu
. pkshape id seq period1 period2, order(RT TR)
. pkcross outcome
 Sequence variable = sequence
 Period variable = period
 Treatment variable = treat
 Carryover variable = carry
 ID variable = id

```

Analysis of variance (ANOVA) for a 2x2 crossover study

| Source of variation  | Partial SS | df | MS     | F    | Prob > F |
|----------------------|------------|----|--------|------|----------|
| <b>Intersubjects</b> |            |    |        |      |          |
| Sequence effect      | 276.00     | 1  | 276.00 | 0.37 | 0.5468   |
| Residuals            | 16211.49   | 22 | 736.89 | 4.41 | 0.0005   |
| <b>Intrasubjects</b> |            |    |        |      |          |
| Treatment effect     | 62.79      | 1  | 62.79  | 0.38 | 0.5463   |
| Period effect        | 35.97      | 1  | 35.97  | 0.22 | 0.6474   |
| Residuals            | 3679.43    | 22 | 167.25 |      |          |
| Total                | 20265.68   | 47 |        |      |          |

Omnibus measure of separability of treatment and carryover = 29.2893%

There is evidence of intersubject variability, but there are no other significant effects. The omnibus test for separability is a measure reflecting the degree to which the study design allows the treatment effects to be estimated independently of the carryover effects. The measure of separability of the treatment and carryover effects indicates approximately 29% separability, which can be interpreted as the degree to which the treatment and carryover effects are orthogonal. This is a characteristic of the design of the study. For a complete discussion, see [Ratkowsky, Evans, and Alldredge \(1993\)](#). Compared with the output in [Chow and Liu \(2009\)](#), the sequence effect is mislabeled as a carryover effect. See [Ratkowsky, Evans, and Alldredge \(1993, sec. 3.2\)](#) for a complete discussion of the mislabeling.

By specifying `param(1)`, we obtain parameterization 1 for this model.

```

. pkcross outcome, param(1)
 Sequence variable = sequence
 Period variable = period
 Treatment variable = treat
 Carryover variable = carry
 ID variable = id

```

Analysis of variance (ANOVA) for a 2x2 crossover study

| Source of variation | Partial SS | df | MS     | F    | Prob > F |
|---------------------|------------|----|--------|------|----------|
| Treatment effect    | 301.04     | 1  | 301.04 | 0.67 | 0.4189   |
| Period effect       | 255.62     | 1  | 255.62 | 0.57 | 0.4561   |
| Carryover effect    | 276.00     | 1  | 276.00 | 0.61 | 0.4388   |
| Residuals           | 19890.92   | 44 | 452.07 |      |          |
| Total               | 20265.68   | 47 |        |      |          |

Omnibus measure of separability of treatment and carryover = 29.2893%

## ► Example 2

Consider the case of a two-treatment, four-sequence, two-period crossover design. This design is commonly referred to as Balaam's design (Balaam 1968). Ratkowsky, Evans, and Alldredge (1993, 140) published the following data from an amantadine trial, originally published by Taka and Armitage (1983):

```
. use https://www.stata-press.com/data/r19/balaam, clear
. list, sep(0)
```

|     | id | seq | period1 | period2 | period3 |
|-----|----|-----|---------|---------|---------|
| 1.  | 1  | -ab | 9       | 8.75    | 8.75    |
| 2.  | 2  | -ab | 12      | 10.5    | 9.75    |
| 3.  | 3  | -ab | 17      | 15      | 18.5    |
| 4.  | 4  | -ab | 21      | 21      | 21.5    |
| 5.  | 1  | -ba | 23      | 22      | 18      |
| 6.  | 2  | -ba | 15      | 15      | 13      |
| 7.  | 3  | -ba | 13      | 14      | 13.75   |
| 8.  | 4  | -ba | 24      | 22.75   | 21.5    |
| 9.  | 5  | -ba | 18      | 17.75   | 16.75   |
| 10. | 1  | -aa | 14      | 12.5    | 14      |
| 11. | 2  | -aa | 27      | 24.25   | 22.5    |
| 12. | 3  | -aa | 19      | 17.25   | 16.25   |
| 13. | 4  | -aa | 30      | 28.25   | 29.75   |
| 14. | 1  | -bb | 21      | 20      | 19.51   |
| 15. | 2  | -bb | 11      | 10.5    | 10      |
| 16. | 3  | -bb | 20      | 19.5    | 20.75   |
| 17. | 4  | -bb | 25      | 22.5    | 23.5    |

The sequence identifier must be a string with 0s to indicate washout or baseline periods, or a number. If the sequence identifier is numeric, the `order()` option must be specified with `pkshape`. If the sequence identifier is a string, `pkshape` will use the string values to create sequence, period, and treatment variables. In this example, the dash is used to indicate a baseline period, which is an invalid code for this purpose. Therefore, we use the `subinstr()` function to replace each dash with a 0. After doing so, we can use `pkshape` to format the data in a way that can be used with `pkcross`. We leave most `pkcross` options at their defaults, but we specify the `sequential` option to calculate sequential sums of squares instead of the default partial sums of squares.

```

. replace seq = substr(seq, "-", "0", .)
(17 real changes made)
. pkshape id seq period1 period2 period3
. pkcross outcome, sequential
 Sequence variable = sequence
 Period variable = period
 Treatment variable = treat
 Carryover variable = carry
 ID variable = id

```

Analysis of variance (ANOVA) for a crossover study

| Source of variation  | SS      | df | MS    | F     | Prob > F |
|----------------------|---------|----|-------|-------|----------|
| <b>Intersubjects</b> |         |    |       |       |          |
| Sequence effect      | 285.82  | 3  | 95.27 | 1.01  | 0.4180   |
| Residuals            | 1221.49 | 13 | 93.96 | 59.96 | 0.0000   |
| <b>Intrasubjects</b> |         |    |       |       |          |
| Period effect        | 15.13   | 2  | 7.56  | 6.34  | 0.0048   |
| Treatment effect     | 8.48    | 1  | 8.48  | 8.86  | 0.0056   |
| Carryover effect     | 0.11    | 1  | 0.11  | 0.12  | 0.7366   |
| Residuals            | 29.56   | 30 | 0.99  |       |          |
| Total                | 1560.59 | 50 |       |       |          |

Omnibus measure of separability of treatment and carryover = 64.6447%

◀

### ▶ Example 3

For this example, we return to `pkdata.dta` from [example 2](#) of [R] `pk` and use `pkcollapse` and `pkshape` on the data as discussed in that example.

```

. use https://www.stata-press.com/data/r19/pkdata, clear
(Fictional drug concentration data)
. pkcollapse time conc1 conc2, id(id) keep(seq) stat(auc)
.....
. pkshape id seq auc*, order(RT TR)

```

After sorting the data with `sort`, our data appear as follows:

```
. sort period id
. list, sep(8)
```

|     | id | sequence | outcome  | treat | carry | period |
|-----|----|----------|----------|-------|-------|--------|
| 1.  | 1  | RT       | 150.9643 | R     | 0     | 1      |
| 2.  | 2  | RT       | 146.7606 | R     | 0     | 1      |
| 3.  | 3  | RT       | 160.6548 | R     | 0     | 1      |
| 4.  | 4  | RT       | 157.8622 | R     | 0     | 1      |
| 5.  | 5  | RT       | 133.6957 | R     | 0     | 1      |
| 6.  | 6  | RT       | 160.639  | R     | 0     | 1      |
| 7.  | 7  | RT       | 131.2604 | R     | 0     | 1      |
| 8.  | 8  | RT       | 168.5186 | R     | 0     | 1      |
| 9.  | 9  | TR       | 137.0627 | T     | 0     | 1      |
| 10. | 10 | TR       | 153.4038 | T     | 0     | 1      |
| 11. | 11 | TR       | 163.4593 | T     | 0     | 1      |
| 12. | 12 | TR       | 146.0462 | T     | 0     | 1      |
| 13. | 13 | TR       | 158.1457 | T     | 0     | 1      |
| 14. | 14 | TR       | 147.1977 | T     | 0     | 1      |
| 15. | 15 | TR       | 164.9988 | T     | 0     | 1      |
| 16. | 16 | TR       | 145.3823 | T     | 0     | 1      |
| 17. | 1  | RT       | 218.5551 | T     | R     | 2      |
| 18. | 2  | RT       | 133.3201 | T     | R     | 2      |
| 19. | 3  | RT       | 126.0635 | T     | R     | 2      |
| 20. | 4  | RT       | 96.17461 | T     | R     | 2      |
| 21. | 5  | RT       | 188.9038 | T     | R     | 2      |
| 22. | 6  | RT       | 223.6922 | T     | R     | 2      |
| 23. | 7  | RT       | 104.0139 | T     | R     | 2      |
| 24. | 8  | RT       | 237.8962 | T     | R     | 2      |
| 25. | 9  | TR       | 139.7382 | R     | T     | 2      |
| 26. | 10 | TR       | 202.3942 | R     | T     | 2      |
| 27. | 11 | TR       | 136.7848 | R     | T     | 2      |
| 28. | 12 | TR       | 104.5191 | R     | T     | 2      |
| 29. | 13 | TR       | 165.8654 | R     | T     | 2      |
| 30. | 14 | TR       | 139.235  | R     | T     | 2      |
| 31. | 15 | TR       | 166.2391 | R     | T     | 2      |
| 32. | 16 | TR       | 158.5146 | R     | T     | 2      |

We now fit an ANOVA model using pkcross:

```
. pkcross outcome
 Sequence variable = sequence
 Period variable = period
 Treatment variable = treat
 Carryover variable = carry
 ID variable = id
```

Analysis of variance (ANOVA) for a 2x2 crossover study

| Source of variation  | Partial SS      | df        | MS      | F    | Prob > F |
|----------------------|-----------------|-----------|---------|------|----------|
| <b>Intersubjects</b> |                 |           |         |      |          |
| Sequence effect      | 378.04          | 1         | 378.04  | 0.29 | 0.5961   |
| Residuals            | 17991.26        | 14        | 1285.09 | 1.40 | 0.2691   |
| <b>Intrasubjects</b> |                 |           |         |      |          |
| Treatment effect     | 455.04          | 1         | 455.04  | 0.50 | 0.4931   |
| Period effect        | 419.47          | 1         | 419.47  | 0.46 | 0.5102   |
| Residuals            | 12860.78        | 14        | 918.63  |      |          |
| <b>Total</b>         | <b>32104.59</b> | <b>31</b> |         |      |          |

Omnibus measure of separability of treatment and carryover = 29.2893%

◀

#### ▷ Example 4

Consider the case of a six-treatment crossover trial in which the squares are not variance balanced. The following dataset is from a partially balanced crossover trial published by [Patterson and Lucas \(1962\)](#) and reproduced in [Ratkowsky, Evans, and Alldredge \(1993, 231\)](#):

```
. use https://www.stata-press.com/data/r19/nobalance, clear
. list, sep(4)
```

|     | cow | seq  | period1 | period2 | period3 | period4 | block |
|-----|-----|------|---------|---------|---------|---------|-------|
| 1.  | 1   | adbe | 38.7    | 37.4    | 34.3    | 31.3    | 1     |
| 2.  | 2   | baed | 48.9    | 46.9    | 42      | 39.6    | 1     |
| 3.  | 3   | ebda | 34.6    | 32.3    | 28.5    | 27.1    | 1     |
| 4.  | 4   | deab | 35.2    | 33.5    | 28.4    | 25.1    | 1     |
| 5.  | 1   | dafc | 32.9    | 33.1    | 27.5    | 25.1    | 2     |
| 6.  | 2   | fdca | 30.4    | 29.5    | 26.7    | 23.1    | 2     |
| 7.  | 3   | cfad | 30.8    | 29.3    | 26.4    | 23.2    | 2     |
| 8.  | 4   | acdf | 25.7    | 26.1    | 23.4    | 18.7    | 2     |
| 9.  | 1   | efbc | 25.4    | 26      | 23.9    | 19.9    | 3     |
| 10. | 2   | becf | 21.8    | 23.9    | 21.7    | 17.6    | 3     |
| 11. | 3   | fceb | 21.4    | 22      | 19.4    | 16.6    | 3     |
| 12. | 4   | cbfe | 22.8    | 21      | 18.6    | 16.1    | 3     |

When there is no variance balance in the design, a square or blocking variable is needed to indicate in which treatment cell a sequence was observed, but the mechanical steps are the same.

```
. pkshape cow seq period1 period2 period3 period4
. pkcross outcome, model(block cow|block period|block treat carry) sequential
 Number of obs = 48 R-squared = 0.9965
 Root MSE = .740408 Adj R-squared = 0.9903
```

| Source       | Seq. SS    | df | MS         | F       | Prob > F |
|--------------|------------|----|------------|---------|----------|
| Model        | 2650.1331  | 30 | 88.3377701 | 161.14  | 0.0000   |
| block        | 1607.01128 | 2  | 803.505642 | 1465.71 | 0.0000   |
| cow block    | 628.706274 | 9  | 69.8562527 | 127.43  | 0.0000   |
| period block | 408.031253 | 9  | 45.3368059 | 82.70   | 0.0000   |
| treat        | 2.5000057  | 5  | .50000114  | 0.91    | 0.4964   |
| carry        | 3.88428906 | 5  | .776857812 | 1.42    | 0.2680   |
| Residual     | 9.31945887 | 17 | .548203463 |         |          |
| Total        | 2659.45256 | 47 | 56.584097  |         |          |

When the model statement is used and the omnibus measure of separability is desired, specify the variables in the `treatment()`, `carryover()`, and `sequence()` options to `pkcross`.

◀

## Methods and formulas

`pkcross` uses ANOVA to fit models for crossover experiments; see [R] [anova](#).

The omnibus measure of separability is

$$S = 100(1 - V)\%$$

where  $V$  is Cramér's  $V$  and is defined as

$$V = \left\{ \frac{\frac{\chi^2}{N}}{\min(r-1, c-1)} \right\}^{\frac{1}{2}}$$

$N$  is the sample size and  $\chi^2$  is calculated as

$$\chi^2 = \sum_i \sum_j \left\{ \frac{(O_{ij} - E_{ij})^2}{E_{ij}} \right\}$$

where  $O$  and  $E$  are the observed and expected counts in an  $r \times c$  table of the number of times each treatment is followed by the other treatments.

## References

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## Also see

[R] **pk** — Pharmacokinetic (biopharmaceutical) data

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## Description

`pkequiv` tests for average bioequivalence between two drugs. By default, `pkequiv` computes a confidence interval (CI) for the ratio of treatment geometric means, calculated using log-scaled data. `pkequiv` can also calculate CIs using the original data, such as the classic shortest CI and a CI based on Fieller's theorem. Additionally, `pkequiv` performs interval hypothesis tests for bioequivalence, such as Schuirmann's two one-sided tests.

`pkequiv` is one of the `pk` commands. Please read [\[R\] pk](#) before reading this entry.

## Quick start

Log-scale CI for the geometric mean ratio of pharmacokinetic (PK) outcome `auc`—the area under the concentration-time curve (AUC)—between drugs identified by `treat`, given during period `period` in sequence `sequence`, with subjects identified by `id`

```
pkequiv auc treat period sequence id
```

Same as above, but use an equivalence limit of 0.1, do not perform Schuirmann's two one-sided tests, and do not perform a bootstrap analysis of the long-run probability of bioequivalence

```
pkequiv auc treat period sequence id, limit(0.1) notost noboot
```

Analyze data using the original scale, and calculate a CI for the ratio using Fieller's theorem

```
pkequiv auc treat period sequence id, fieller
```

Same as above, but specify that treatment 2 is the reference drug and treatment 1 is the test drug

```
pkequiv auc treat period sequence id, fieller compare(2 1)
```

## Menu

Statistics > Epidemiology and related > Other > Bioequivalence tests

## Syntax

```
pkequiv outcome treatment period sequence id [if] [in] [, options]
```

| options                   | Description                                                                             |
|---------------------------|-----------------------------------------------------------------------------------------|
| Options                   |                                                                                         |
| <code>compare(# #)</code> | compare the two specified values of the treatment variable                              |
| <code>limit(#)</code>     | set equivalence limit (between 0.01 and 0.99); default is <code>limit(0.2)</code>       |
| <code>level(#)</code>     | set confidence level; default is <code>level(90)</code>                                 |
| <code>log</code>          | calculate a CI using the log scale; the default                                         |
| <code>classic</code>      | calculate the classic shortest CI                                                       |
| <code>fieller</code>      | calculate a CI for the ratio using Fieller's theorem                                    |
| <code>symmetric</code>    | calculate a CI for the difference in means that is symmetric around 0                   |
| <code>[no]tost</code>     | perform or suppress two one-sided hypothesis tests for bioequivalence                   |
| <code>anderson</code>     | perform Anderson and Hauck hypothesis test for bioequivalence                           |
| <code>noboot</code>       | do not compute the bootstrap probability that the CI lies within the equivalence limits |
| <code>reps(#)</code>      | perform # bootstrap replications; default is <code>reps(1000)</code>                    |

collect is allowed; see [U] 11.1.10 Prefix commands.

## Options

### Options

`compare(# #)` specifies the two treatments being compared, where the first value corresponds to the reference drug and the second value corresponds to the test drug. If there are more than two treatments, `compare()` is required; bioequivalence can be determined between any two treatments.

`limit(#)` specifies the equivalence limit. The default is `limit(0.2)`. Using the log scale, this yields equivalence limits of 80% to 125% for the ratio of geometric means. Using the original scale, this yields limits of 80% to 120% for the ratio of arithmetic means and limits of  $\pm 20\%$  for the difference between arithmetic means. See *Equivalence limits* in *Methods and formulas*.

`level(#)` specifies the confidence level, as a percentage, for CIs. The default is `level(90)`. This setting is not controlled by the `set level` command.

`log` specifies a CI for the ratio of treatment geometric means, calculated using the log-transformed outcome. This is the default CI, and its use is recommended by the [US Food and Drug Administration \(FDA\) \(2001, 2022\)](#) and the [European Medicines Agency \(2010\)](#). `log` may not be combined with `classic`, `fieller`, `symmetric`, or `anderson`.

`classic` specifies the classic shortest CI for the difference between treatment arithmetic means, calculated using the original scale of the data. This CI is also expressed as a CI for the ratio of arithmetic means. `classic` may not be combined with `log`, `fieller`, or `symmetric`.

`fieller` specifies a CI based on Fieller's theorem for the ratio of treatment arithmetic means, calculated using the original scale of the data. `fieller` may not be combined with `log`, `classic`, `symmetric`, `tost`, `notost`, `anderson`, `noboot`, or `reps()`.

`symmetric` specifies a CI for the difference between treatment arithmetic means that is symmetric around zero, calculated using the original scale of the data. Use of this CI for bioequivalence testing has been criticized by [Mantel \(1977\)](#), [Kirkwood \(1981\)](#), and others, so it is included mainly for historical purposes. `symmetric` may not be combined with `log`, `classic`, `fieller`, `tost`, `notost`, `anderson`, `noboot`, or `reps()`.

`tost` and `notost` perform or suppress the calculation of Schuirmann’s (1987) two one-sided tests (TOST) for bioequivalence. `tost` is the default whenever log-scale or classic CIs are computed. With a log-scale CI, the TOST will be performed on log-transformed outcome data; with a classic CI, the TOST will be performed on untransformed data. The TOST procedure may not be combined with CIs based on Fieller’s theorem or with symmetric CIs, so `tost` and `notost` may not be combined with `fieller` or `symmetric`.

`anderson` specifies that the [Anderson and Hauck \(1983\)](#) hypothesis test for bioequivalence be computed. Use of this test has been criticized by the US FDA (1988), [Schuirmann \(1987\)](#), and others, so it is included mainly for historical purposes. This test is supported only with `classic`.

`noboot` prevents the computation of the probability that the CI lies within the equivalence limits. By default, this probability is estimated via bootstrap resampling with 1,000 replications. This probability is not computed with CIs based on Fieller’s theorem or with symmetric CIs, so `noboot` may not be combined with `fieller` or `symmetric`, nor may it be combined with `reps()`.

`reps(#)` specifies the number of bootstrap replications to be performed when estimating the probability that the CI lies within the equivalence limits. The default is `reps(1000)`. `reps()` may not be combined with `fieller`, `symmetric`, or `noboot`.

## Remarks and examples

Remarks are presented under the following headings:

[Introduction](#)  
[Examples](#)

## Introduction

`pkequiv` is used to determine average bioequivalence based on data from a nonreplicated crossover study. A crossover study is a longitudinal study where each subject undergoes treatment with both reference (R) and test (T) drugs. In a nonreplicated crossover study, subjects receive each treatment only once, so the sequence of treatments is either R followed by T or T followed by R.

Two drugs are bioequivalent if their bioavailability profiles are sufficiently similar. The most common PK metrics used as measures of bioavailability are the AUC and the maximum concentration ( $C_{\max}$ ).

“Average bioequivalence” between two drugs is declared if the average PK metric of the test drug is close enough to the average PK metric of the reference drug. This is traditionally done by constructing a CI for a measure of disparity between the average PK metrics. If the CI is entirely contained within prespecified equivalence limits, the two drugs are said to demonstrate average bioequivalence.

Two measures of disparity can be used: the difference between treatment means ( $T - R$ ) and the ratio of treatment means ( $T/R$ ). Equivalence limits for the difference and for the ratio are controlled with the `limit()` option; see [Equivalence limits](#) in *Methods and formulas* for details.

By default, `pkequiv` analyzes PK data using the log scale and calculates a CI for the geometric mean ratio, which is the ratio of exponentiated means of the log-transformed outcome. This approach is recommended by the US FDA (2001, 2022) and the European Medicines Agency (2010) for the analysis of most PK metrics. One motivation for taking the log of PK metrics AUC and  $C_{\max}$  is that the biological model postulated for each of those metrics contains a multiplicative term that is a function of the subject; this term becomes additive on the log scale (US FDA 2001, 2022).

The `classic` option analyzes PK data using the original scale and calculates the classic shortest CI for the difference between treatment arithmetic means. This CI is transformed into an approximate CI for the ratio of treatment arithmetic means; see *Classic interval* in *Methods and formulas* for details. The CIs for the difference and for the ratio are compared with their respective equivalence limits, and if the CIs are within the equivalence limits, the two drugs are bioequivalent. The comparison of the difference is mathematically identical to the comparison of the ratio, so the two comparisons will always agree.

The `fieller` option calculates an exact CI for the ratio of treatment arithmetic means, computed using the untransformed data. This approach is most frequently used when the outcome variable is a clinical outcome instead of a PK metric. These trials, called comparative clinical endpoint studies, can have binary outcomes (such as success or failure) or continuous outcomes (such as percent improvement versus baseline), and are generally not analyzed using the log scale (US FDA 2022).

The `symmetric` option uses the original scale of the data to calculate a CI for the difference between treatment arithmetic means that is symmetric around zero. The symmetric CI has received considerable criticism since its proposal by Westlake (1976), so it is included chiefly for historical purposes. See, for example, the criticisms of Mantel (1977) and Kirkwood (1981). Although this CI is symmetric around zero, it is asymmetric with respect to the estimated difference between means. This shift away from the observed difference between means causes unequal tail probabilities. As the difference between means increases, the symmetric CI is shifted ever farther from the estimated difference, and the tail probability becomes dominated by one side.

In addition to calculating CIs, `pkequiv` can conduct interval hypothesis tests of bioequivalence. When a log-scale or classic CI is requested, `pkequiv` performs Schuirmann's (1987) TOST unless the `notost` option is specified. The TOST procedure consists of two hypothesis tests: one to determine whether the average PK metric of the test drug exceeds the upper equivalence limit and one to determine whether it is less than the lower equivalence limit. In both cases, the null hypothesis is that the two drugs are not bioequivalent, so rejecting both null hypotheses leads to a conclusion of bioequivalence. The formula for the TOST is an inversion of the formula for the CI; see *Schuirmann's two one-sided tests* in *Methods and formulas* for details. This ensures that the conclusion reached by the TOST with two  $\alpha$ -level tests will agree with the conclusion reached by comparing the  $(1 - 2\alpha)100\%$  CI against the equivalence limits.

The `anderson` option is used to perform Anderson and Hauck's (1983) test, where the null hypothesis is that the two drugs are not bioequivalent. This procedure offers the appeal of a single test of bioequivalence that is more powerful than Schuirmann's TOST for any nominal significance level  $\alpha$ . However, two serious criticisms of this test have prevented its widespread adoption (US FDA 1988), so it is included mainly for historical reasons. The actual size of Anderson and Hauck's test generally exceeds the nominal size of the test, meaning that the probability of committing a type I error and erroneously concluding bioequivalence is greater than desired (Frick 1987). Additionally, the rejection region of Anderson and Hauck's test is nonconvex and unbounded (Schuirmann 1987), meaning that  $H_0$  will be rejected for any arbitrarily large difference between means as long as the corresponding standard error is sufficiently large.

## Examples

pkequiv requires that the user specify the *outcome*, *treatment*, *period*, *sequence*, and *id* variables. The dataset must be in the same format as that produced by pkshape; see [R] [pkshape](#). The examples below assume the use of a  $2 \times 2 \times 2$  design: a crossover study with 2 treatments, 2 periods, and 2 sequences. If your dataset includes results for more than two treatments, you will need to use the `compare()` option to specify which two treatments are being compared.

### ► Example 1

We use `pkdata.dta` from [example 2](#) of [R] [pk](#). We use `pkcollapse` and `pkshape` on the data as discussed in that example. After we sort the data with the `sort` command, our data appear as follows:

```
. use https://www.stata-press.com/data/r19/pkdata
(Fictional drug concentration data)
. pkcollapse time conc1 conc2, id(id) keep(seq) stat(auc)
.....
. pkshape id seq auc*, order(RT TR)
. sort period id
. list, sep(4)
```

|                  | id | sequence | outcome  | treat | carry | period |
|------------------|----|----------|----------|-------|-------|--------|
| 1.               | 1  | RT       | 150.9643 | R     | 0     | 1      |
| 2.               | 2  | RT       | 146.7606 | R     | 0     | 1      |
| 3.               | 3  | RT       | 160.6548 | R     | 0     | 1      |
| 4.               | 4  | RT       | 157.8622 | R     | 0     | 1      |
| 5.               | 5  | RT       | 133.6957 | R     | 0     | 1      |
| 6.               | 6  | RT       | 160.639  | R     | 0     | 1      |
| 7.               | 7  | RT       | 131.2604 | R     | 0     | 1      |
| 8.               | 8  | RT       | 168.5186 | R     | 0     | 1      |
| 9.               | 9  | TR       | 137.0627 | T     | 0     | 1      |
| 10.              | 10 | TR       | 153.4038 | T     | 0     | 1      |
| 11.              | 11 | TR       | 163.4593 | T     | 0     | 1      |
| 12.              | 12 | TR       | 146.0462 | T     | 0     | 1      |
| (output omitted) |    |          |          |       |       |        |
| 25.              | 9  | TR       | 139.7382 | R     | T     | 2      |
| 26.              | 10 | TR       | 202.3942 | R     | T     | 2      |
| 27.              | 11 | TR       | 136.7848 | R     | T     | 2      |
| 28.              | 12 | TR       | 104.5191 | R     | T     | 2      |
| 29.              | 13 | TR       | 165.8654 | R     | T     | 2      |
| 30.              | 14 | TR       | 139.235  | R     | T     | 2      |
| 31.              | 15 | TR       | 166.2391 | R     | T     | 2      |
| 32.              | 16 | TR       | 158.5146 | R     | T     | 2      |

The outcome variable is the AUC, and we use `pkequiv` to conduct a bioequivalence test between reference drug “R” and test drug “T” using the log scale.

```
. set seed 123
. pkequiv outcome treat period sequence id
Log-transformed confidence interval for bioequivalence
Outcome: ln(outcome)
Treatment: treat = R (reference)
 treat = T (test)
Period: period
Sequence: sequence
Subject ID: id
```

|                          | [Equivalence limits] |     | [90% conf. interval] |         | Estimate |
|--------------------------|----------------------|-----|----------------------|---------|----------|
| Geometric mean ratio (%) | 80                   | 125 | 91.087               | 116.210 | 102.885  |

Bootstrap prob. that confidence interval is within equivalence limits = 0.901

Schuirmann’s two one-sided tests

```
t statistics: Upper = -2.816 p-value = 0.0069
 Lower = 3.638 p-value = 0.0013
```

The equivalence table for the log-transformed CI contains columns for the equivalence limits (which can be controlled by using the `limit()` option), the confidence interval (the width of which can be controlled by using the `level()` option), and the point estimate. The default value of `limit(0.2)` yields equivalence limits of [80%, 125%] for the ratio of geometric means. The default value of `level(90)` specifies a 90% CI.

The CI of [91.087%, 116.210%] is symmetric around the estimated geometric mean ratio of 102.885%. The interval is entirely contained within the equivalence limits, so we declare average bioequivalence between the test and reference drugs.

To determine the probability that the CI would lie within the equivalence limits if this experiment were to be repeated, `pkequiv` conducts a bootstrap with 1,000 replications. In this example, there is a 90.1% assurance that the observed CI would be within the equivalence limits in the long run. Because of the nature of the bootstrap, we expect the bootstrap probability calculation to vary slightly from run to run, so we specified the command `set seed 123` before executing `pkequiv` for you to match the bootstrap probability exactly when you run this example.

The output also displays Schuirmann’s two one-sided tests. The upper one-sided test is a formal statistical test of whether the geometric mean ratio exceeds the upper equivalence limit, and the lower one-sided test evaluates whether the geometric mean ratio is below the lower equivalence limit. The null hypothesis for both of Schuirmann’s two one-sided tests is that the two drugs are not bioequivalent, so rejecting both of these null hypotheses supports a conclusion of average bioequivalence.

Schuirmann’s two one-sided tests are constructed by inverting the formula for the CI (see *Schuirmann’s two one-sided tests* in *Methods and formulas*), so conducting the TOST at the  $\alpha$  level will always agree with the conclusion of bioequivalence or bioinequivalence reached by comparing the  $(1 - 2\alpha)100\%$  CI against the equivalence limits. The default of `level(90)` yields  $\alpha = 0.05$ , and because we have already demonstrated that the 90% CI is within its equivalence limits, it comes as no surprise that we can reject the null hypotheses of both one-sided tests at the 0.05 level.

We now repeat this same analysis using the classic shortest CI instead of the log-transformed CI. For speed, we specify the `noboot` option to omit the bootstrap.

```
. pkequiv outcome treat period sequence id, classic noboot
Classic confidence interval for bioequivalence
Outcome: outcome
Treatment: treat = R (reference)
 treat = T (test)
Period: period
Sequence: sequence
Subject ID: id
```

|                            | [Equivalence limits] |        | [90% conf. interval] |         | Estimate |
|----------------------------|----------------------|--------|----------------------|---------|----------|
| Arithmetic mean difference | -30.296              | 30.296 | -11.332              | 26.416  | 7.542    |
| ratio (%)                  | 80                   | 120    | 92.519               | 117.439 | 104.979  |

Schuirmann's two one-sided tests

```
t statistics: Upper = -2.123
 Lower = 3.531
```

p-value = 0.0260

p-value = 0.0017

The equivalence table for the classic CI has two rows: one for the difference between the arithmetic means of the test and reference groups and one for the ratio of arithmetic means. The default value of `limit(0.2)` indicates that bioequivalence is declared if the CI for the difference is within the range  $[-30.296, 30.296]$ , which corresponds to a CI for the ratio within  $[80\%, 120\%]$ . If the CI for the difference is within its equivalence limits, the CI for the ratio must be too, and vice versa.

The 90% CI for the ratio of arithmetic means,  $[92.519\%, 117.439\%]$ , is not exactly the same as the 90% CI for the ratio of geometric means, but it is entirely contained within its equivalence limits, so it yields the same conclusion of bioequivalence. However, now the upper equivalence limit for the ratio is only 120% (as opposed to 125% when conducting log-scale analysis), so here the upper endpoint of the CI comes closer to its equivalence limit. As expected, the 90% CI for the difference between means,  $[-11.332, 26.416]$ , is also contained within its equivalence limits, supporting a conclusion of bioequivalence.

When combined with the `classic` option, Schuirmann's two one-sided tests are performed on the untransformed outcomes, which is why the test statistics and  $p$ -values above differ slightly from those reported with the log-transformed CI in the first part of this example. But this ensures that the TOST with level  $\alpha$  tests will agree with the conclusion of bioequivalence or bioinequivalence reached by comparing the classic  $(1 - 2\alpha)100\%$  CI against the equivalence limits.

Schuirmann's upper one-sided test presents evidence that the difference between the average AUCs of the test and reference drugs does not exceed the upper equivalence limit ( $p = 0.026$ ), while Schuirmann's lower one-sided test presents evidence that the difference between the average AUCs of the test and reference drugs is not less than the lower equivalence limit ( $p = 0.0017$ ). Considering these two tests together, we reject the null hypothesis of bioinequivalence at the  $\alpha = 0.05$  level, which is the same conclusion that we reached when we compared the CIs with their equivalence limits.

## ► Example 2

We now use the data published in [Chow and Liu \(2009, 71\)](#), which we describe in [example 1](#) of [\[R\] pkshape](#). As before, the *outcome* variable is AUC, and we use `pkshape` to reshape the data. We use the same `pkequiv` specification as we did in [example 1](#) above to conduct a bioequivalence test.

```
. use https://www.stata-press.com/data/r19/chowliu, clear
. pkshape id seq period1 period2, order(RT TR)
. set seed 123
. pkequiv outcome treat period sequence id
Log-transformed confidence interval for bioequivalence
Outcome: ln(outcome)
Treatment: treat = R (reference)
 treat = T (test)
Period: period
Sequence: sequence
Subject ID: id
```

|                          | [Equivalence limits] |     | [90% conf. interval] |         | Estimate |
|--------------------------|----------------------|-----|----------------------|---------|----------|
| Geometric mean ratio (%) | 80                   | 125 | 88.313               | 106.928 | 97.175   |

Bootstrap prob. that confidence interval is within equivalence limits = 0.985

Schuirmann's two one-sided tests

```
t statistics: Upper = -4.521 p-value = 0.0001
 Lower = 3.492 p-value = 0.0010
```

For these data, the 90% CI for the ratio of treatment geometric means is well within the equivalence limits, supporting a conclusion of average bioequivalence. The bootstrap probability that the CI is within the equivalence limits is 98.5%, indicating that if this experiment were to be repeated many times, the vast majority of replications would support a conclusion of bioequivalence. Examining the TOST, we reject both null hypotheses at the 5% level, which agrees with the conclusion we reached by comparing the 90% CI against the equivalence limits.

If we want to conduct a test of average bioequivalence using the original scale of the data, we can employ Fieller's theorem to calculate an exact CI for the ratio of arithmetic means. Here we repeat the previous analysis, but we add the `fieller` option to calculate a CI based on Fieller's theorem.

```
. pkequiv outcome treat period sequence id, fieller
Confidence interval for bioequivalence based on Fieller's theorem
Outcome: outcome
Treatment: treat = R (reference)
 treat = T (test)
Period: period
Sequence: sequence
Subject ID: id
```

|                           | [Equivalence limits] |     | [90% conf. interval] |         | Estimate |
|---------------------------|----------------------|-----|----------------------|---------|----------|
| Arithmetic mean ratio (%) | 80                   | 120 | 89.787               | 105.193 | 97.229   |

The exact 90% CI for the ratio of treatment arithmetic means is quite similar to the CI for the ratio of geometric means. Because we performed this analysis using the original scale of the data, the upper equivalence limit for the ratio is 120%, not 125% (the value used for log-scale analysis). But the narrow CI for the ratio of arithmetic means is well contained within these narrower equivalence limits, supporting a conclusion of average bioequivalence.



## Stored results

pkequiv stores the following in `r()`:

### Scalars

|                               |                                                                                                                   |
|-------------------------------|-------------------------------------------------------------------------------------------------------------------|
| <code>r(ratio)</code>         | estimated ratio of treatment means                                                                                |
| <code>r(ratio_l)</code>       | lower CI limit for ratio of treatment means                                                                       |
| <code>r(ratio_u)</code>       | upper CI limit for ratio of treatment means                                                                       |
| <code>r(ratio_limit_l)</code> | lower equivalence limit for ratio of treatment means                                                              |
| <code>r(ratio_limit_u)</code> | upper equivalence limit for ratio of treatment means                                                              |
| <code>r(diff)</code>          | estimated difference between treatment means, if <code>classic</code> or <code>symmetric</code> specified         |
| <code>r(diff_l)</code>        | lower CI limit for difference between means, if <code>classic</code> or <code>symmetric</code> specified          |
| <code>r(diff_u)</code>        | upper CI limit for difference between means, if <code>classic</code> or <code>symmetric</code> specified          |
| <code>r(diff_limit_l)</code>  | lower equivalence limit for difference between means, if <code>classic</code> or <code>symmetric</code> specified |
| <code>r(diff_limit_u)</code>  | upper equivalence limit for difference between means, if <code>classic</code> or <code>symmetric</code> specified |
| <code>r(bootprob)</code>      | bootstrap probability that CI is within equivalence limits, if bootstrap performed                                |
| <code>r(reps)</code>          | number of bootstrap replications, if bootstrap performed                                                          |
| <code>r(tost_l)</code>        | lower test statistic for Schuirmann's two one-sided tests                                                         |
| <code>r(tost_u)</code>        | upper test statistic for Schuirmann's two one-sided tests                                                         |
| <code>r(p_tost_l)</code>      | lower $p$ -value for Schuirmann's two one-sided tests                                                             |
| <code>r(p_tost_u)</code>      | upper $p$ -value for Schuirmann's two one-sided tests                                                             |
| <code>r(anderson)</code>      | test statistic for Anderson and Hauck's test, if <code>anderson</code> specified                                  |
| <code>r(ncp_anderson)</code>  | noncentrality parameter for Anderson and Hauck's test, if <code>anderson</code> specified                         |
| <code>r(p_anderson)</code>    | $p$ -value for Anderson and Hauck's test, if <code>anderson</code> specified                                      |
| <code>r(sd)</code>            | pooled-sample standard deviation of period differences from both sequences                                        |
| <code>r(delta)</code>         | delta value used in calculating a symmetric CI, if <code>symmetric</code> specified                               |
| <code>r(limit)</code>         | equivalence limit                                                                                                 |
| <code>r(level)</code>         | confidence level used for calculating CIs                                                                         |
| <code>r(refval)</code>        | value of reference treatment                                                                                      |
| <code>r(testval)</code>       | value of test treatment                                                                                           |

### Macros

|                           |                                     |
|---------------------------|-------------------------------------|
| <code>r(outcome)</code>   | name of outcome variable            |
| <code>r(treatment)</code> | name of treatment variable          |
| <code>r(period)</code>    | name of period variable             |
| <code>r(sequence)</code>  | name of sequence variable           |
| <code>r(id)</code>        | name of id variable                 |
| <code>r(citype)</code>    | log, classic, fieller, or symmetric |
| <code>r(reflab)</code>    | value label of reference treatment  |
| <code>r(testlab)</code>   | value label of test treatment       |

### Matrices

|                             |                  |
|-----------------------------|------------------|
| <code>r(limit_table)</code> | table of results |
|-----------------------------|------------------|

## Methods and formulas

Methods and formulas are presented under the following headings:

- Data and model
- Equivalence limits
- Confidence intervals
  - Classic interval
  - Intervals using log-transformed data
  - Intervals symmetric around zero
  - Intervals using Fieller's theorem
- Interval hypothesis tests
  - Schuirmann's two one-sided tests
  - Schuirmann's two one-sided tests with a classic CI
  - Schuirmann's two one-sided tests with a log-scale CI
  - Anderson and Hauck's test

## Data and model

Let  $y_{ij}$  be the observed PK outcome for subject  $i = 1, \dots, n$  in period  $j = 1, 2$ . Let  $\mathcal{RT}$  be the set of subject IDs for subjects who received the reference drug before the test drug, which we will designate as sequence 1. Let  $\mathcal{TR}$  be the set of IDs for subjects who received the test drug before the reference drug, which we will call sequence 2. Let  $n_1$  and  $n_2$  represent the number of subjects in sequences 1 and 2, respectively, with  $n = n_1 + n_2$ .

Consider a model for the log-transformed outcome first:

$$\ln(y_{ij}) = \mu_{ij}^* + s_i^* + p_j^* + \epsilon_{ij}^* \quad (1)$$

$s_i^*$  is the random effect of subject  $i$ , where  $s_i^* \sim$  i.i.d.  $N(0, \sigma_s^{*2})$  and  $\sigma_s^{*2}$  controls between-subjects variability.  $p_j^*$  is the fixed effect of period  $j$ , subject to the constraint  $p_1^* + p_2^* = 0$ . Random error  $\epsilon_{ij}^*$  is independent of  $s_i^*$ , and  $\epsilon_{ij}^* \sim$  i.i.d.  $N(0, \sigma_e^{*2})$ . Fixed effect  $\mu_{ij}^*$  is defined as follows,

$$\mu_{ij}^* = \begin{cases} \mu_R^* & \text{if } (i \in \mathcal{RT} \text{ and } j = 1) \text{ or } (i \in \mathcal{TR} \text{ and } j = 2) \\ \mu_T^* & \text{if } (i \in \mathcal{RT} \text{ and } j = 2) \text{ or } (i \in \mathcal{TR} \text{ and } j = 1) \end{cases}$$

where  $\mu_R^*$  and  $\mu_T^*$  are the population means of the log-transformed PK outcomes for the reference and test drugs, respectively. The model above assumes there are no unequal carryover effects.

By default, `pkequiv` produces a CI for the geometric mean ratio of the PK outcome, defined as  $R_g = \exp(\mu_T^*) / \exp(\mu_R^*)$ . `pkequiv` also produces CIs for the difference  $D_a = \mu_T - \mu_R$  and ratio  $R_a = \mu_T / \mu_R$  of arithmetic means of the PK outcome. The arithmetic means  $\mu_T$  and  $\mu_R$  are estimated from (1) applied to the untransformed PK outcome,

$$y_{ij} = \mu_{ij} + s_i + p_j + \epsilon_{ij} \quad (2)$$

$$\mu_{ij} = \begin{cases} \mu_R & \text{if } (i \in \mathcal{RT} \text{ and } j = 1) \text{ or } (i \in \mathcal{TR} \text{ and } j = 2) \\ \mu_T & \text{if } (i \in \mathcal{RT} \text{ and } j = 2) \text{ or } (i \in \mathcal{TR} \text{ and } j = 1) \end{cases}$$

where  $\mu_R$  and  $\mu_T$  are the population means of the untransformed PK outcomes for the reference and test drugs, respectively. Subject-specific random effect  $s_i \sim$  i.i.d.  $N(0, \sigma_s^2)$  is independent of random error  $\epsilon_{ij} \sim$  i.i.d.  $N(0, \sigma_e^2)$ , where  $\sigma_s^2$  controls between-subjects variability and  $\sigma_e^2$  controls within-subjects variability. Period effect  $p_j$  is defined similarly to (1).

## Equivalence limits

Equivalence limits are controlled by using the `limit()` option. Depending on the type of CI requested, equivalence limits will be calculated for the ratio of treatment means, the difference between treatment means, or both.

The lower equivalence limit for the ratio of treatment means is  $l_R = \{1 - \text{limit}()\}100\%$ . The upper equivalence limit for the ratio is  $u_R = [1/\{1 - \text{limit}()\}]100\%$  when performing analysis on the log scale, and it is  $u_R = \{1 + \text{limit}()\}100\%$  when performing calculations on the original scale of the data. The default value of `limit()` is 0.2, which yields equivalence limits of 80% to 125% when using the log scale and equivalence limits of 80% to 120% when using the original scale.

The lower and upper equivalence limits for the difference between treatment arithmetic means are

$$l_D = -\text{limit}() \times \hat{\mu}_R \quad \text{and} \quad u_D = \text{limit}() \times \hat{\mu}_R \quad (3)$$

respectively. The estimated marginal mean  $\hat{\mu}_R$ , also known as the least-squares mean, is used for calculations because the true value of  $\mu_R$  in the population is unknown.

When we conduct inference using the ratio of treatment means, bioequivalence is declared when  $(l_R < L_R)$  and  $(U_R < u_R)$ , where  $L_R$  and  $U_R$  are the lower and upper endpoints, respectively, of the CI for the ratio. When we consider the difference between treatment means, average bioequivalence is declared when  $(l_D < L_D)$  and  $(U_D < u_D)$ , where  $L_D$  is the lower endpoint of the CI for the difference and  $U_D$  is the upper endpoint.

## Confidence intervals

`pkequiv` supports four types of CIs: a CI calculated using the log scale, the classic shortest CI, a CI symmetric around zero, and a CI based on Fieller's theorem. The width of the CI is controlled by the `level()` option, with a default of `level(90)` yielding a 90% CI. This option also sets the significance level  $\alpha$  of the bioequivalence test, with the default of  $\alpha = 0.05$  given by the relationship  $\alpha = \{100 - \text{level}()\}/200$ . The formula for the log-scale CI is based on the formula for the classic CI, which is presented first.

### Classic interval

The lower limit of the classic shortest CI for the difference between two treatment means is

$$L_{D,\text{classic}} = (\hat{\mu}_T - \hat{\mu}_R) - t_{(\alpha, n_1 + n_2 - 2)} \hat{\sigma}_d \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \quad (4)$$

and the upper limit is

$$U_{D,\text{classic}} = (\hat{\mu}_T - \hat{\mu}_R) + t_{(\alpha, n_1 + n_2 - 2)} \hat{\sigma}_d \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \quad (5)$$

Here  $\hat{\mu}_R$  and  $\hat{\mu}_T$  are the estimated marginal means of the reference and test drugs, respectively, estimated using (2). The term  $t_{(\alpha, n_1 + n_2 - 2)}$  represents the upper  $\alpha$  critical value of a central  $t$  distribution with  $n_1 + n_2 - 2$  degrees of freedom.  $\hat{\sigma}_d$  is the pooled sample standard deviation of the period differences from both sequences, defined as

$$\hat{\sigma}_d = \sqrt{\frac{1}{n_1 + n_2 - 2} \left\{ \sum_{i \in \mathcal{RT}} (d_i - \bar{d}_{RT})^2 + \sum_{i \in \mathcal{TR}} (d_i - \bar{d}_{TR})^2 \right\}} \quad (6)$$

The period difference for subject  $i$  is  $d_i = (y_{i2} - y_{i1})/2$ , the average period difference in sequence 1 is  $\bar{d}_{RT} = (1/n_1) \sum_{i \in \mathcal{RT}} d_i$ , and the average period difference in sequence 2 is  $\bar{d}_{TR} = (1/n_2) \sum_{i \in \mathcal{TR}} d_i$ .

The limits of the approximate CI for the ratio are

$$L_{R,\text{classic}} = \left( \frac{L_{D,\text{classic}}}{\hat{\mu}_R} + 1 \right) 100\%$$

and

$$U_{R,\text{classic}} = \left( \frac{U_{D,\text{classic}}}{\hat{\mu}_R} + 1 \right) 100\%$$

This CI for the ratio is approximate because the denominator  $\hat{\mu}_R$  is an estimate of the true population average  $\mu_R$ . The conclusion of average bioequivalence will be the same regardless of whether you compute the classic CI for the difference or for the ratio. If one CI is within its equivalence limits, the other one necessarily is too.

The finite sample performance of the classic CI is assessed via bootstrap simulation. Bootstrap samples are drawn with replacement using the patient IDs as clusters, and the default of `reps` (1000) calls for 1,000 bootstrap samples. For each sample, a classic CI is constructed and compared with the equivalence limits. The bootstrap probability is computed as the proportion of CIs that are within the equivalence limits.

## Intervals using log-transformed data

Calculation of the log-scale CI for the ratio of treatment geometric means,  $[L_{R,\log}, U_{R,\log}]$ , begins with taking the natural logarithm of the PK outcome and using (1) to calculate estimated marginal means  $\hat{\mu}_R^*$  and  $\hat{\mu}_T^*$ .

Next, the estimated standard deviation of log-scale period differences,  $\hat{\sigma}_d^*$ , is calculated by substituting  $\ln(y_{ij})$  for  $y_{ij}$  in (6). A classic CI for the difference between means of the log-PK metric is calculated, substituting  $\hat{\mu}_R^*$  for  $\hat{\mu}_R$  and  $\hat{\mu}_T^*$  for  $\hat{\mu}_T$  in (4) and (5).

The endpoints of this interval,  $L_{D,\log}$  and  $U_{D,\log}$ , are exponentiated to yield  $L_{R,\log} = \exp(L_{D,\log})$  and  $U_{R,\log} = \exp(U_{D,\log})$ . These are the endpoints of an exact CI for the geometric mean ratio:  $R_g = \exp(\mu_T^*) / \exp(\mu_R^*)$ .

Use of this approach is recommended by the US FDA (2001, 2022) and the European Medicines Agency (2010). PK metrics such as AUC and  $C_{\max}$  are often right-skewed, but the US FDA's rationale for requesting log-scale bioequivalence analysis is not based on the distribution of the data but instead on properties of the model of log-PK data. The US FDA has, since 1991, recommended that analysis of bioequivalence be conducted using the ratio of treatment geometric means (US FDA 2001). Additionally, the models postulated for PK parameters in bioequivalence studies contain a multiplicative subject effect when analyzed on the original scale, but this effect is additive on the logarithmic scale (US FDA 2022).

The finite sample performance of the log-transformed CI is assessed via bootstrap simulation. This is equivalent to the bootstrap procedure used to determine the long-run performance of the classic CI, where patient IDs are used as clusters during the resampling process.

### Intervals symmetric around zero

The lower and upper limits for the symmetric CI are, respectively,  $L_{D,\text{sym}} = \hat{\mu}_R - \Delta$  and  $U_{D,\text{sym}} = \hat{\mu}_R + \Delta$ . To calculate  $\Delta$ , we must solve a system of two equations,

$$\Delta = b_1 \hat{\sigma}_d \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} - (\hat{\mu}_R - \hat{\mu}_T)$$

and (simultaneously)

$$\Delta = -b_2 \hat{\sigma}_d \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} + (\hat{\mu}_R - \hat{\mu}_T)$$

Values  $b_1$  and  $b_2$  are computed iteratively to satisfy the above equalities and the condition

$$\int_{b_1}^{b_2} f(t) dt = 1 - 2\alpha$$

where  $f(t)$  is the probability density function of a central  $t$  distribution with  $n_1 + n_2 - 2$  degrees of freedom.

### Intervals using Fieller's theorem

To calculate an exact CI for the ratio of treatment means without using the log scale, [Locke \(1984\)](#) employed Fieller's theorem ([1954](#)) to obtain  $[L_{R,\text{Fieller}}, U_{R,\text{Fieller}}]$ . See [Locke \(1984\)](#) or [Chow and Liu \(2009, 88–92\)](#) for details about calculating the exact CI based on Fieller's theorem.

### Interval hypothesis tests

pkequiv offers two types of interval hypothesis tests of bioequivalence: Schuirmann's two one-sided tests ([1987](#)) and Anderson and Hauck's test ([1983](#)). The null hypothesis for both types of tests is that the two treatments are not bioequivalent, so bioequivalence is declared when the null hypotheses are rejected.

#### Schuirmann's two one-sided tests

The hypotheses for Schuirmann's lower one-sided test are  $H_{0L}: \mu_T - \mu_R \leq l_D$  and  $H_{aL}: \mu_T - \mu_R > l_D$ . For the upper one-sided test, the hypotheses are  $H_{0U}: \mu_T - \mu_R \geq u_D$  and  $H_{aU}: \mu_T - \mu_R < u_D$ . The test statistics for the TOST are

$$T_L = \frac{(\hat{\mu}_T - \hat{\mu}_R) - l_D}{\hat{\sigma}_d \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \quad \text{and} \quad T_U = \frac{(\hat{\mu}_T - \hat{\mu}_R) - u_D}{\hat{\sigma}_d \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \quad (7)$$

The  $p$ -values for the lower and upper one-sided tests are  $p_L = F_t(T_L)$  and  $p_U = 1 - F_t(T_U)$ , respectively, where  $F_t$  is the cumulative distribution function of the central  $t$  distribution with  $n_1 + n_2 - 2$  degrees of freedom.

### Schirmann's two one-sided tests with a classic CI

Schirmann's two one-sided tests are inversions of the formulas for the endpoints of the [classic CI](#) for the difference in treatment means. Thus, rejecting  $H_{0L}$  at the  $\alpha$  level is the same as concluding  $l_D < L_{D,\text{classic}}$  for a  $(1 - 2\alpha)100\%$  CI, and rejecting  $H_{0U}$  is the same as concluding  $U_{D,\text{classic}} < u_D$ . If both null hypotheses are rejected, the entire CI lies within the equivalence limits.

### Schirmann's two one-sided tests with a log-scale CI

The log-scale CI is calculated using the same formula as the classic CI, but the outcome is the natural log of the PK metric. Thus, the procedure for conducting the TOST on the log scale is the same as on the original scale, but the log-transformed PK outcome is used.

In (7), terms  $\hat{\mu}_R$ ,  $\hat{\mu}_T$ , and  $\hat{\sigma}_d$  are replaced with the values used in the calculation of the [log-scale CI](#):  $\hat{\mu}_R^*$ ,  $\hat{\mu}_T^*$ , and  $\hat{\sigma}_d^*$ . The terms  $l_D$  and  $u_D$  in (7) are likewise replaced with  $l_D^*$  and  $u_D^*$ , the equivalence limits for the difference between log-transformed treatment means, which are calculated by replacing  $\hat{\mu}_R$  with  $\hat{\mu}_R^*$  in (3).

Rejecting both null hypotheses of the TOST on the log scale is equivalent to concluding  $l_D^* < L_{D,\log}$  and  $U_{D,\log} < u_D^*$ , which, in turn, is equivalent to concluding  $l_R < L_{R,\log}$  and  $U_{R,\log} < u_R$ .

### Anderson and Hauck's test

Anderson and Hauck's test may be conducted only when calculating the classic shortest CI on the original scale of the data. The null hypothesis for Anderson and Hauck's test is  $H_0 : (\mu_T - \mu_R \leq l_D) \cup (\mu_T - \mu_R \geq u_D)$ . The alternative hypothesis for Anderson and Hauck's test is  $H_a : l_D < (\mu_T - \mu_R) < u_D$ . The test statistic is

$$T_{\text{AH}} = \frac{(\hat{\mu}_T - \hat{\mu}_R) - (l_D + u_D)/2}{\hat{\sigma}_d \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

and the noncentrality parameter is estimated by

$$\hat{\delta} = \frac{u_D - l_D}{2\hat{\sigma}_d \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

The  $p$ -value is calculated as

$$p = F_t(|T_{\text{AH}}| - \hat{\delta}) - F_t(-|T_{\text{AH}}| - \hat{\delta})$$

where  $F_t$  is the cumulative distribution function of the central  $t$  distribution with  $n_1 + n_2 - 2$  degrees of freedom.

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## Also see

[R] **pk** — Pharmacokinetic (biopharmaceutical) data

Description  
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## Description

`pkexamine` calculates pharmacokinetic measures from concentration-and-time subject-level data. `pkexamine` computes and displays the maximum measured concentration, the time at the maximum measured concentration, the time of the last measurement, the elimination time, the half-life, and the area under the concentration–time curve ( $AUC_{0,t_{\max}}$ ). Three estimates of the AUC from 0 to infinity ( $AUC_{0,\infty}$ ) are also calculated.

`pkexamine` is one of the `pk` commands. Please read [\[R\] pk](#) before reading this entry.

## Quick start

Pharmacokinetic measures for concentrations `y` at times `tvar` where `idvar = 4`

```
pkexamine tvar y if idvar==4
```

Same as above, but use trapezoidal rule to calculate  $AUC_{0,t_{\max}}$

```
pkexamine tvar y if idvar==4, trapezoid
```

Plot concentration–time curve where `idvar = 2`

```
pkexamine tvar y if idvar==2, graph
```

Same as above, and save graph as `mygraph`

```
pkexamine tvar y if idvar==2, graph saving(mygraph)
```

## Menu

Statistics > Epidemiology and related > Other > Pharmacokinetic measures

## Syntax

pkexamine *time concentration* [*if*] [*in*] [, *options*]

| <i>options</i>                                               | Description                                                                 |
|--------------------------------------------------------------|-----------------------------------------------------------------------------|
| Main                                                         |                                                                             |
| <code>fit(#)</code>                                          | use # points to estimate $AUC_{0,\infty}$ ; default is <code>fit(3)</code>  |
| <code>trapezoid</code>                                       | use trapezoidal rule; default is cubic splines                              |
| <code>graph</code>                                           | graph the AUC                                                               |
| <code>line</code>                                            | graph the linear extension                                                  |
| <code>log</code>                                             | graph the log extension                                                     |
| <code>exp(#)</code>                                          | plot the exponential fit for the $AUC_{0,\infty}$                           |
| AUC plot                                                     |                                                                             |
| <code>cline_options</code>                                   | affect rendition of plotted points connected by lines                       |
| <code>marker_options</code>                                  | change look of markers (color, size, etc.)                                  |
| <code>marker_label_options</code>                            | add marker labels; change look or position                                  |
| Add plots                                                    |                                                                             |
| <code>addplot(plot)</code>                                   | add other plots to the generated graph                                      |
| Y axis, X axis, Titles, Legend, Overall                      |                                                                             |
| <code>twoway_options</code>                                  | any options other than by() documented in [G-3] <code>twoway_options</code> |
| by and collect are allowed; see [U] 11.1.10 Prefix commands. |                                                                             |

## Options

### Main

`fit(#)` specifies the number of points, counting back from the last measurement, to use in fitting the extension to estimate the  $AUC_{0,\infty}$ . The default is `fit(3)`, or the last three points. This value should be viewed as a minimum; the appropriate number of points will depend on your data.

`trapezoid` specifies that the trapezoidal rule be used to calculate the  $AUC_{0,t_{\max}}$ . The default is cubic splines, which give better results for most functions. When the curve is irregular, `trapezoid` may give better results.

`graph` tells pkexamine to graph the concentration–time curve.

`line` and `log` specify the estimates of the  $AUC_{0,\infty}$  to display when graphing the  $AUC_{0,\infty}$ . If the `graph` option is not also specified, then these options are ignored.

`exp(#)` specifies that the exponential fit for the  $AUC_{0,\infty}$  be plotted. You must specify the maximum time value to which you want to plot the curve, and this time value must be greater than the maximum time measurement in the data. If you specify 0, the curve will be plotted to the point at which the linear extension would cross the  $x$  axis. If the `graph` option is not also specified, then this option is ignored. This option is not valid with the `line` or `log` option.

## AUC plot

*cline\_options* affect the rendition of the plotted points connected by lines; see [G-3] *cline\_options*.

*marker\_options* specify the look of markers. This look includes the marker symbol, size, color, and outline; see [G-3] *marker\_options*.

*marker\_label\_options* specify if and how the markers are to be labeled; see [G-3] *marker\_label\_options*.

## Add plots

`addplot (plot)` provides a way to add other plots to the generated graph; see [G-3] *addplot\_option*.

## Y axis, X axis, Titles, Legend, Overall

*twoway\_options* are any of the options documented in [G-3] *twoway\_options*, excluding `by()`. These include options for titling the graph (see [G-3] *title\_options*) and for saving the graph to disk (see [G-3] *saving\_option*).

## Remarks and examples

`pkexamine` computes summary statistics for a given patient in a pharmacokinetic trial. If by *idvar*: is specified, statistics will be displayed for each subject in the data.

### ▷ Example 1

Chow and Liu (2009, 13) present data on a study examining primidone concentrations versus time over a 32-hour period after dosing a subject.

```
. use https://www.stata-press.com/data/r19/auc
(Primidone concentrations)
. list, abbrev(14)
```

|     | id | time | concentration |
|-----|----|------|---------------|
| 1.  | 1  | 0    | 0             |
| 2.  | 1  | .5   | 0             |
| 3.  | 1  | 1    | 2.8           |
| 4.  | 1  | 1.5  | 4.4           |
| 5.  | 1  | 2    | 4.4           |
| 6.  | 1  | 3    | 4.7           |
| 7.  | 1  | 4    | 4.1           |
| 8.  | 1  | 6    | 4             |
| 9.  | 1  | 8    | 3.6           |
| 10. | 1  | 12   | 3             |
| 11. | 1  | 16   | 2.5           |
| 12. | 1  | 24   | 2             |
| 13. | 1  | 32   | 1.6           |

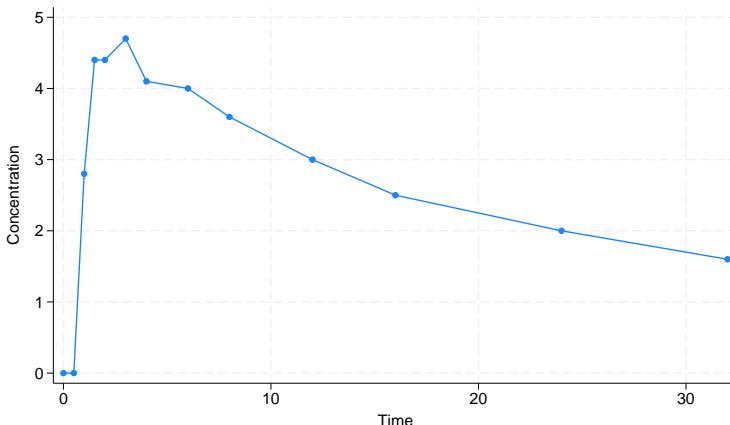
We use pkexamine to produce the summary statistics:

```
. pkexamine time concentration, graph
 Maximum concentration = 4.7
 Time of maximum concentration = 3
 Time of last observation (Tmax) = 32
 Elimination rate = 0.0279
 Half life = 24.8503
```

Area under the curve

| AUC [0, Tmax] | AUC [0, inf.)<br>Linear of log conc. | AUC [0, inf.)<br>Linear fit | AUC [0, inf.)<br>Exponential fit |
|---------------|--------------------------------------|-----------------------------|----------------------------------|
| 85.24         | 142.603                              | 107.759                     | 142.603                          |

Fit based on last 3 points.



The maximum concentration of 4.7 occurs at time 3, and the time of the last observation (Tmax) is 32. In addition to  $AUC_{0,t_{max}}$ , which is calculated from 0 to the maximum value of time, pkexamine also reports  $AUC_{0,\infty}$ , the AUC computed by extending the curve with each of three methods: a linear fit to the log of the concentration, a linear regression line, and a decreasing exponential regression line. See [Methods and formulas](#) for details on these three methods.

By default, all extensions to the AUC are based on the last three points. In looking at the graph for these data, it seems more appropriate to use the last seven points to estimate the  $AUC_{0,\infty}$ :

```
. pkexamine time concentration, fit(7)
 Maximum concentration = 4.7
 Time of maximum concentration = 3
 Time of last observation (Tmax) = 32
 Elimination rate = 0.0349
 Half life = 19.8354
```

Area under the curve

| AUC [0, Tmax] | AUC [0, inf.)<br>Linear of log conc. | AUC [0, inf.)<br>Linear fit | AUC [0, inf.)<br>Exponential fit |
|---------------|--------------------------------------|-----------------------------|----------------------------------|
| 85.24         | 131.027                              | 96.805                      | 129.181                          |

Fit based on last 7 points.

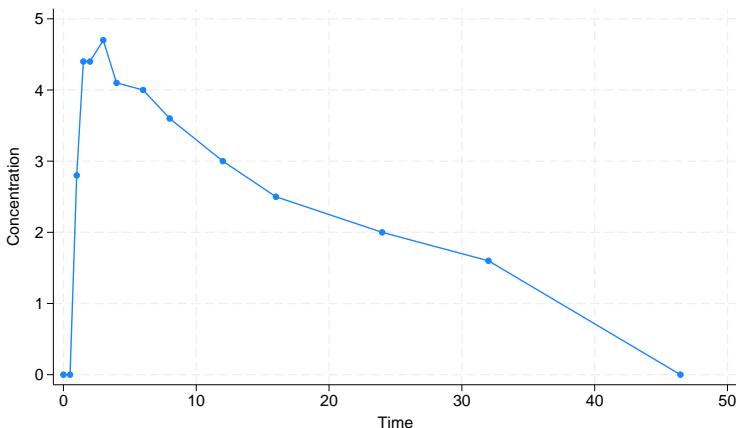
This approach decreased the estimate of the  $AUC_{0,\infty}$  for all extensions. To see a graph of the  $AUC_{0,\infty}$  using a linear extension, specify the graph and line options.

```
. pkexamine time concentration, fit(7) graph line
 Maximum concentration = 4.7
 Time of maximum concentration = 3
 Time of last observation (Tmax) = 32
 Elimination rate = 0.0349
 Half life = 19.8354
```

Area under the curve

| AUC [0, Tmax] | AUC [0, inf.)<br>Linear of log conc. | AUC [0, inf.)<br>Linear fit | AUC [0, inf.)<br>Exponential fit |
|---------------|--------------------------------------|-----------------------------|----------------------------------|
| 85.24         | 131.027                              | 96.805                      | 129.181                          |

Fit based on last 7 points.



◀

## Stored results

pkexamine stores the following in `r()`:

Scalars

```
r(auc) AUC
r(half) half-life of the drug
r(ke) elimination rate
r(tmax) time at last concentration measurement
r(cmax) maximum concentration
r(tomc) time of maximum concentration
r(auc_line) AUC0,∞ estimated with a linear fit
r(auc_exp) AUC0,∞ estimated with an exponential fit
r(auc_ln) AUC0,∞ estimated with a linear fit of the natural log
```

## Methods and formulas

Let  $i$  index the observations sorted by time, let  $k$  be the number of observations, and let  $f$  be the number of points specified in the `fit(#)` option.

The  $AUC_{0,t_{\max}}$  is defined as

$$AUC_{0,t_{\max}} = \int_0^{t_{\max}} C_t dt$$

where  $C_t$  is the concentration at time  $t$ . By default, the integral is calculated numerically using cubic splines. However, if the trapezoidal rule is used, the  $AUC_{0,t_{\max}}$  is given as

$$AUC_{0,t_{\max}} = \sum_{i=2}^k \frac{C_{i-1} + C_i}{2} (t_i - t_{i-1})$$

The  $AUC_{0,\infty}$  is the  $AUC_{0,t_{\max}} + AUC_{t_{\max},\infty}$ , or

$$AUC_{0,\infty} = \int_0^{t_{\max}} C_t dt + \int_{t_{\max}}^{\infty} C_t dt$$

When using the linear extension to the  $AUC_{0,t_{\max}}$ , the integration is cut off when the line crosses the  $x$  axis. The log extension is a linear extension on the log concentration scale. The area for the exponential extension is

$$AUC_{t_{\max},\infty} = \int_{t_{\max}}^{\infty} e^{\beta_0 + t\beta_1} dt = \frac{e^{\beta_0 + t_{\max}\beta_1}}{-\beta_1}$$

where  $\beta_0 > 0$  and  $\beta_1 < 0$  are the intercept and slope, respectively, of an exponential accelerated failure-time regression of concentration on time.

The elimination rate  $K_{\text{eq}}$  is the negative of the slope from a linear regression of log concentration on time fit to the number of points specified in the `fit(#)` option:

$$K_{\text{eq}} = - \frac{\sum_{i=k-f+1}^k (t_i - \bar{t}) (\ln C_i - \overline{\ln C})}{\sum_{i=k-f+1}^k (t_i - \bar{t})^2}$$

The half-life is

$$t_{\text{half}} = \frac{\ln 2}{K_{\text{eq}}}$$

## Reference

Chow, S.-C., and J.-P. Liu. 2009. *Design and Analysis of Bioavailability and Bioequivalence Studies*. 3rd ed. Boca Raton, FL: Chapman and Hall/CRC. <https://doi.org/10.1201/9781420011678>.

## Also see

[R] **pk** — Pharmacokinetic (biopharmaceutical) data

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## Description

`pkshape` reshapes data for use with `anova`, `pkcross`, and `pkequiv`; see [\[R\] anova](#), [\[R\] pkcross](#), and [\[R\] pkequiv](#). Latin-square and crossover data are often organized in a manner that cannot be analyzed easily with Stata. `pkshape` reorganizes the data in memory for use in Stata.

`pkshape` is one of the `pk` commands. Please read [\[R\] pk](#) before reading this entry.

## Quick start

Reshape data when string sequence variable `seq` = TR or RT for patients identified by `idvar` observed at `tvar1` and `tvar2`

```
pkshape idvar seq tvar1 tvar2
```

Same as above, but with numeric `seq` = 1 indicating TR and `seq` = 2 indicating RT

```
pkshape idvar seq tvar1 tvar2, order(TR RT)
```

Indicate that period 2 is a washout and the second treatment is administered in period 3

```
pkshape idvar seq tvar1 tvar2 tvar3, order(TOR ROT)
```

## Menu

Statistics > Epidemiology and related > Other > Reshape pharmacokinetic Latin-square data

## Syntax

```
pkshape id sequence period1 period2 [periodlist] [, options]
```

Variable *id* specifies unique subject identifiers. Variable *sequence* specifies the sequence (numeric or string) in which treatments were received. Variables *period1*, *period2*, and so on specify the pharmacokinetic measurements such as AUC in the corresponding periods.

| <i>options</i>                        | Description                                                                |
|---------------------------------------|----------------------------------------------------------------------------|
| <code>order(<i>string</i>)</code>     | apply treatments in specified order; required with numeric <i>sequence</i> |
| <code>outcome(<i>newvar</i>)</code>   | name for outcome variable; default is <code>outcome(outcome)</code>        |
| <code>treatment(<i>newvar</i>)</code> | name for treatment variable; default is <code>treatment(treat)</code>      |
| <code>carryover(<i>newvar</i>)</code> | name for carryover variable; default is <code>carryover(carry)</code>      |
| <code>sequence(<i>newvar</i>)</code>  | name for sequence variable; default is <code>sequence(sequence)</code>     |
| <code>period(<i>newvar</i>)</code>    | name for period variable; default is <code>period(period)</code>           |

## Options

`order(string)` specifies the order in which treatments were applied when generating the sequence, treatment, and carryover variables in the reorganized data. This option is required if the input sequence variable, *sequence*, is numeric. It is not allowed if *sequence* is a string variable. For crossover designs, any washout periods can be indicated with the number 0.

`outcome(newvar)` specifies the name for the outcome variable in the reorganized data. By default, `outcome(outcome)` is used.

`treatment(newvar)` specifies the name for the treatment variable in the reorganized data. By default, `treatment(treat)` is used.

`carryover(newvar)` specifies the name for the carryover variable in the reorganized data. By default, `carryover(carry)` is used.

`sequence(newvar)` specifies the name for the sequence variable in the reorganized data. By default, `sequence(sequence)` is used.

`period(newvar)` specifies the name for the period variable in the reorganized data. By default, `period(period)` is used.

## Remarks and examples

Often, data from a Latin-square experiment are naturally organized in a manner that Stata cannot manage easily. `pkshape` reorganizes Latin-square data so that they can be used with `anova` (see [R] [anova](#)) or any `pk` command. This includes the classic  $2 \times 2$  crossover design commonly used in pharmaceutical research, as well as many other Latin-square designs. When using `pkshape`, newly created variables will automatically be labeled and system value labels will be created. The value label `_treat1b1` will be attached to the treatment and carryover variables, to indicate which treatment is applied in a given period and which treatment is being carried over from the previous period. The value label `_seq1b1` will be attached to the sequence variable, indicating the sequence of treatments.

`pkshape` expects the data to be organized in the same format as that produced by [R] [pkcollapse](#)—with variables representing time periods of the study.

## ► Example 1

Consider the example data published in [Chow and Liu \(2009, 71\)](#). There are 24 patients, 12 in each sequence. Sequence 1 is the reference formulation followed by the test formulation; sequence 2 is the test formulation followed by the reference formulation. The measurements reported are the  $AUC_{0,t_{\max}}$  for each patient and for each period.

```
. use https://www.stata-press.com/data/r19/chowliu
. list, sep(4)
```

|     | id | seq | period1 | period2 |
|-----|----|-----|---------|---------|
| 1.  | 1  | 1   | 74.675  | 73.675  |
| 2.  | 4  | 1   | 96.4    | 93.25   |
| 3.  | 5  | 1   | 101.95  | 102.125 |
| 4.  | 6  | 1   | 79.05   | 69.45   |
| 5.  | 11 | 1   | 79.05   | 69.025  |
| 6.  | 12 | 1   | 85.95   | 68.7    |
| 7.  | 15 | 1   | 69.725  | 59.425  |
| 8.  | 16 | 1   | 86.275  | 76.125  |
| 9.  | 19 | 1   | 112.675 | 114.875 |
| 10. | 20 | 1   | 99.525  | 116.25  |
| 11. | 23 | 1   | 89.425  | 64.175  |
| 12. | 24 | 1   | 55.175  | 74.575  |
| 13. | 2  | 2   | 74.825  | 37.35   |
| 14. | 3  | 2   | 86.875  | 51.925  |
| 15. | 7  | 2   | 81.675  | 72.175  |
| 16. | 8  | 2   | 92.7    | 77.5    |
| 17. | 9  | 2   | 50.45   | 71.875  |
| 18. | 10 | 2   | 66.125  | 94.025  |
| 19. | 13 | 2   | 122.45  | 124.975 |
| 20. | 14 | 2   | 99.075  | 85.225  |
| 21. | 17 | 2   | 86.35   | 95.925  |
| 22. | 18 | 2   | 49.925  | 67.1    |
| 23. | 21 | 2   | 42.7    | 59.425  |
| 24. | 22 | 2   | 91.725  | 114.05  |

Because the outcome for one person is in two different variables, the treatment that was applied to an individual is a function of the period and the sequence. To analyze this treatment using anova, all the outcomes must be in one variable, and each covariate must be in its own variable. To reorganize these data, use `pkshape`:

```

. pkshape id seq period1 period2, order(RT TR)
. sort seq id period
. list, sep(8)

```

|     | id | sequence | outcome | treat | carry | period |
|-----|----|----------|---------|-------|-------|--------|
| 1.  | 1  | RT       | 74.675  | R     | 0     | 1      |
| 2.  | 1  | RT       | 73.675  | T     | R     | 2      |
| 3.  | 4  | RT       | 96.4    | R     | 0     | 1      |
| 4.  | 4  | RT       | 93.25   | T     | R     | 2      |
| 5.  | 5  | RT       | 101.95  | R     | 0     | 1      |
| 6.  | 5  | RT       | 102.125 | T     | R     | 2      |
| 7.  | 6  | RT       | 79.05   | R     | 0     | 1      |
| 8.  | 6  | RT       | 69.45   | T     | R     | 2      |
| 9.  | 11 | RT       | 79.05   | R     | 0     | 1      |
| 10. | 11 | RT       | 69.025  | T     | R     | 2      |
| 11. | 12 | RT       | 85.95   | R     | 0     | 1      |
| 12. | 12 | RT       | 68.7    | T     | R     | 2      |
| 13. | 15 | RT       | 69.725  | R     | 0     | 1      |
| 14. | 15 | RT       | 59.425  | T     | R     | 2      |
| 15. | 16 | RT       | 86.275  | R     | 0     | 1      |
| 16. | 16 | RT       | 76.125  | T     | R     | 2      |
| 17. | 19 | RT       | 112.675 | R     | 0     | 1      |
| 18. | 19 | RT       | 114.875 | T     | R     | 2      |
| 19. | 20 | RT       | 99.525  | R     | 0     | 1      |
| 20. | 20 | RT       | 116.25  | T     | R     | 2      |
| 21. | 23 | RT       | 89.425  | R     | 0     | 1      |
| 22. | 23 | RT       | 64.175  | T     | R     | 2      |
| 23. | 24 | RT       | 55.175  | R     | 0     | 1      |
| 24. | 24 | RT       | 74.575  | T     | R     | 2      |
| 25. | 2  | TR       | 74.825  | T     | 0     | 1      |
| 26. | 2  | TR       | 37.35   | R     | T     | 2      |
| 27. | 3  | TR       | 86.875  | T     | 0     | 1      |
| 28. | 3  | TR       | 51.925  | R     | T     | 2      |
| 29. | 7  | TR       | 81.675  | T     | 0     | 1      |
| 30. | 7  | TR       | 72.175  | R     | T     | 2      |
| 31. | 8  | TR       | 92.7    | T     | 0     | 1      |
| 32. | 8  | TR       | 77.5    | R     | T     | 2      |
| 33. | 9  | TR       | 50.45   | T     | 0     | 1      |
| 34. | 9  | TR       | 71.875  | R     | T     | 2      |
| 35. | 10 | TR       | 66.125  | T     | 0     | 1      |
| 36. | 10 | TR       | 94.025  | R     | T     | 2      |
| 37. | 13 | TR       | 122.45  | T     | 0     | 1      |
| 38. | 13 | TR       | 124.975 | R     | T     | 2      |
| 39. | 14 | TR       | 99.075  | T     | 0     | 1      |
| 40. | 14 | TR       | 85.225  | R     | T     | 2      |
| 41. | 17 | TR       | 86.35   | T     | 0     | 1      |
| 42. | 17 | TR       | 95.925  | R     | T     | 2      |
| 43. | 18 | TR       | 49.925  | T     | 0     | 1      |
| 44. | 18 | TR       | 67.1    | R     | T     | 2      |
| 45. | 21 | TR       | 42.7    | T     | 0     | 1      |
| 46. | 21 | TR       | 59.425  | R     | T     | 2      |
| 47. | 22 | TR       | 91.725  | T     | 0     | 1      |
| 48. | 22 | TR       | 114.05  | R     | T     | 2      |

Now, the data are organized into separate variables that indicate each factor level for each of the covariates, so the data may be used with `anova` or `pkcross`; see [R] `anova` and [R] `pkcross`.

Initially, the output from `list` displayed sequence values 1 and 2, but now we see sequences RT and TR listed for the individuals. `pkshape` used the information we provided in the `order()` option to assign value labels to the numeric variables `sequence`, `treat`, and `carry`. Because we did not specify any new variable names, the default names were used.

◀

## ▶ Example 2

Consider the study of background music on bank teller productivity published in [Kutner et al. \(2005\)](#). The data are

| Week | Monday | Tuesday | Wednesday | Thursday | Friday |
|------|--------|---------|-----------|----------|--------|
| 1    | 18(D)  | 17(C)   | 14(A)     | 21(B)    | 17(E)  |
| 2    | 13(C)  | 34(B)   | 21(E)     | 16(A)    | 15(D)  |
| 3    | 7(A)   | 29(D)   | 32(B)     | 27(E)    | 13(C)  |
| 4    | 17(E)  | 13(A)   | 24(C)     | 31(D)    | 25(B)  |
| 5    | 21(B)  | 26(E)   | 26(D)     | 31(C)    | 7(A)   |

The numbers are the productivity scores, and the letters represent the treatment. We entered the data into Stata:

```
. use https://www.stata-press.com/data/r19/music, clear
(Background music and teller productivity)
. list
```

|    | id | seq   | day1 | day2 | day3 | day4 | day5 |
|----|----|-------|------|------|------|------|------|
| 1. | 1  | dcabe | 18   | 17   | 14   | 21   | 17   |
| 2. | 2  | cbead | 13   | 34   | 21   | 16   | 15   |
| 3. | 3  | adbec | 7    | 29   | 32   | 27   | 13   |
| 4. | 4  | eadcb | 17   | 13   | 24   | 31   | 25   |
| 5. | 5  | bedca | 21   | 26   | 26   | 31   | 7    |

We reshape these data with pkshape:

```
. pkshape id seq day1 day2 day3 day4 day5
. sort id period
. list, sep(0)
```

|     | id | sequence | outcome | treat | carry | period |
|-----|----|----------|---------|-------|-------|--------|
| 1.  | 1  | dcabe    | 18      | d     | 0     | 1      |
| 2.  | 1  | dcabe    | 17      | c     | d     | 2      |
| 3.  | 1  | dcabe    | 14      | a     | c     | 3      |
| 4.  | 1  | dcabe    | 21      | b     | a     | 4      |
| 5.  | 1  | dcabe    | 17      | e     | b     | 5      |
| 6.  | 2  | cbead    | 13      | c     | 0     | 1      |
| 7.  | 2  | cbead    | 34      | b     | c     | 2      |
| 8.  | 2  | cbead    | 21      | e     | b     | 3      |
| 9.  | 2  | cbead    | 16      | a     | e     | 4      |
| 10. | 2  | cbead    | 15      | d     | a     | 5      |
| 11. | 3  | adbec    | 7       | a     | 0     | 1      |
| 12. | 3  | adbec    | 29      | d     | a     | 2      |
| 13. | 3  | adbec    | 32      | b     | d     | 3      |
| 14. | 3  | adbec    | 27      | e     | b     | 4      |
| 15. | 3  | adbec    | 13      | c     | e     | 5      |
| 16. | 4  | eaedb    | 17      | e     | 0     | 1      |
| 17. | 4  | eaedb    | 13      | a     | e     | 2      |
| 18. | 4  | eaedb    | 24      | c     | a     | 3      |
| 19. | 4  | eaedb    | 31      | d     | c     | 4      |
| 20. | 4  | eaedb    | 25      | b     | d     | 5      |
| 21. | 5  | bedca    | 21      | b     | 0     | 1      |
| 22. | 5  | bedca    | 26      | e     | b     | 2      |
| 23. | 5  | bedca    | 26      | d     | e     | 3      |
| 24. | 5  | bedca    | 31      | c     | d     | 4      |
| 25. | 5  | bedca    | 7       | a     | c     | 5      |

Here the sequence variable is a string variable that specifies how the treatments were applied. The characters in this string variable are used to assign value labels to the newly created sequence, treat, and carry variables. We could now produce an ANOVA table:

```
. anova outcome sequence period treat
```

|          | Number of obs = | 25      | R-squared =     | 0.8666 |        |
|----------|-----------------|---------|-----------------|--------|--------|
|          | Root MSE =      | 3.96232 | Adj R-squared = | 0.7331 |        |
| Source   | Partial SS      | df      | MS              | F      | Prob>F |
| Model    | 1223.6          | 12      | 101.96667       | 6.49   | 0.0014 |
| sequence | 82              | 4       | 20.5            | 1.31   | 0.3226 |
| period   | 477.2           | 4       | 119.3           | 7.60   | 0.0027 |
| treat    | 664.4           | 4       | 166.1           | 10.58  | 0.0007 |
| Residual | 188.4           | 12      | 15.7            |        |        |
| Total    | 1412            | 24      | 58.833333       |        |        |

### ▶ Example 3

Consider the Latin-square crossover example published in [Kutner et al. \(2005\)](#). The example is about apple sales given different methods for displaying apples.

| Pattern | Store | Week 1 | Week 2 | Week 3 |
|---------|-------|--------|--------|--------|
| 1       | 1     | 9(B)   | 12(C)  | 15(A)  |
|         | 2     | 4(B)   | 12(C)  | 9(A)   |
| 2       | 1     | 12(A)  | 14(B)  | 3(C)   |
|         | 2     | 13(A)  | 14(B)  | 3(C)   |
| 3       | 1     | 7(C)   | 18(A)  | 6(B)   |
|         | 2     | 5(C)   | 20(A)  | 4(B)   |

We entered the data into Stata:

```
. use https://www.stata-press.com/data/r19/applesales, clear
(Display impact on apple sales)
. list, sep(2)
```

|    | id | seq | p1 | p2 | p3 | square |
|----|----|-----|----|----|----|--------|
| 1. | 1  | 1   | 9  | 12 | 15 | 1      |
| 2. | 2  | 1   | 4  | 12 | 9  | 2      |
| 3. | 3  | 2   | 12 | 14 | 3  | 1      |
| 4. | 4  | 2   | 13 | 14 | 3  | 2      |
| 5. | 5  | 3   | 7  | 18 | 6  | 1      |
| 6. | 6  | 3   | 5  | 20 | 4  | 2      |

Now, the data can be reorganized using descriptive names for the outcome variables.

```
. pkshape id seq p1 p2 p3, order(bca abc cab) seq(pattern) treat(displays)
. anova outcome pattern period displays id|pattern
```

|            | Number of obs = | 18      | R-squared =     | 0.9562 |        |
|------------|-----------------|---------|-----------------|--------|--------|
|            | Root MSE =      | 1.59426 | Adj R-squared = | 0.9069 |        |
| Source     | Partial SS      | df      | MS              | F      | Prob>F |
| Model      | 443.66667       | 9       | 49.296296       | 19.40  | 0.0002 |
| pattern    | .33333333       | 2       | .16666667       | 0.07   | 0.9370 |
| period     | 233.33333       | 2       | 116.66667       | 45.90  | 0.0000 |
| displays   | 189             | 2       | 94.5            | 37.18  | 0.0001 |
| id pattern | 21              | 3       | 7               | 2.75   | 0.1120 |
| Residual   | 20.333333       | 8       | 2.5416667       |        |        |
| Total      | 464             | 17      | 27.294118       |        |        |

These are the same results reported by [Kutner et al. \(2005\)](#).

## References

- Chow, S.-C., and J.-P. Liu. 2009. *Design and Analysis of Bioavailability and Bioequivalence Studies*. 3rd ed. Boca Raton, FL: Chapman and Hall/CRC. <https://doi.org/10.1201/9781420011678>.
- Kutner, M. H., C. J. Nachtsheim, J. Neter, and W. Li. 2005. *Applied Linear Statistical Models*. 5th ed. New York: McGraw–Hill.

## Also see

[R] **pk** — Pharmacokinetic (biopharmaceutical) data

Description  
Options

Quick start  
Remarks and examples

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Also see

## Description

`pksumm` obtains summary measures based on the first four moments from the empirical distribution of each pharmacokinetic measurement and tests the null hypothesis that the distribution of that measurement is normally distributed.

`pksumm` is one of the `pk` commands. Please read [R] [pk](#) before reading this entry.

## Quick start

Table of pharmacokinetic measures for concentrations `y` at times `tvar` for patients identified by `idvar`

```
pksumm idvar tvar y
```

Add a histogram of  $AUC_{0,t_{\max}}$  values

```
pksumm idvar tvar y, graph
```

Same as above

```
pksumm idvar tvar y, graph stat(auc)
```

Same as above, but plot AUC calculated from 0 to  $\infty$  using a linear extension

```
pksumm idvar tvar y, graph stat(aucline)
```

Same as above, but use 10 bins for the histogram

```
pksumm idvar tvar y, graph stat(aucline) bins(10)
```

## Menu

Statistics > Epidemiology and related > Other > Summarize pharmacokinetic data

## Syntax

```
pksumm id time concentration [if] [in] [, options]
```

| <i>options</i>                                                    | Description                                                                                                |
|-------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------|
| Main                                                              |                                                                                                            |
| <code>trapezoid</code>                                            | use trapezoidal rule to calculate $AUC_{0,t_{max}}$ ; default is cubic splines                             |
| <code>fit (#)</code>                                              | use # points to estimate $AUC_{0,\infty}$ ; default is <code>fit(3)</code>                                 |
| <code>notimechk</code>                                            | do not check whether follow-up time for all subjects is the same                                           |
| <code>nodots</code>                                               | suppress the dots during calculation                                                                       |
| <code>graph</code>                                                | graph the distribution of <i>statistic</i>                                                                 |
| <code>stat (statistic)</code>                                     | graph the specified statistic; default is <code>stat(auc)</code>                                           |
| Histogram, Density plots, Y axis, X axis, Titles, Legend, Overall |                                                                                                            |
| <code>histogram_options</code>                                    | any option other than <code>by()</code> documented in <a href="#">[R] histogram</a>                        |
| <i>statistic</i>                                                  | Description                                                                                                |
| <code>auc</code>                                                  | area under the concentration–time curve ( $AUC_{0,t_{max}}$ ); the default                                 |
| <code>aucline</code>                                              | AUC from 0 to $\infty$ using a linear extension                                                            |
| <code>aucexp</code>                                               | AUC from 0 to $\infty$ using an exponential extension                                                      |
| <code>auclog</code>                                               | area under the concentration–time curve from 0 to $\infty$ extended with a linear fit to log concentration |
| <code>half</code>                                                 | half-life of the drug                                                                                      |
| <code>ke</code>                                                   | elimination rate                                                                                           |
| <code>cmax</code>                                                 | maximum concentration                                                                                      |
| <code>tmax</code>                                                 | time at last concentration                                                                                 |
| <code>tomc</code>                                                 | time of maximum concentration                                                                              |

## Options

### Main

`trapezoid` specifies that the trapezoidal rule be used to calculate the  $AUC_{0,t_{max}}$ . The default is cubic splines, which give better results for most situations. When the curve is irregular, the trapezoidal rule may give better results.

`fit (#)` specifies the number of points, counting back from the last time measurement, to use in fitting the extension to estimate the  $AUC_{0,\infty}$ . The default is `fit(3)`, the last three points. This default should be viewed as a minimum; the appropriate number of points will depend on the data.

`notimechk` suppresses the check that the follow-up time for all subjects is the same. By default, `pksumm` expects the maximum follow-up time to be equal for all subjects.

`nodots` suppresses the progress dots during calculation. By default, a dot (a period) is displayed for every call to calculate the pharmacokinetic measures.

`graph` requests a graph of the distribution of the statistic specified with `stat()`.

`stat (statistic)` specifies the statistic that `pksumm` should graph. The default is `stat(auc)`. If the `graph` option is not also specified, then this option is ignored.

Histogram, Density plots, Y axis, X axis, Titles, Legend, Overall

*histogram\_options* are any of the options documented in [R] [histogram](#), excluding `by()`. For `pksumm`, `fraction` is the default, not `density`.

## Remarks and examples

`pksumm` produces summary statistics for the distribution of nine common pharmacokinetic measurements. If there are more than eight subjects, `pksumm` also computes a test for normality on each measurement. The nine measurements summarized by `pksumm` are listed [above](#) and are described in [Methods and formulas](#) of [R] [pkexamine](#).

### ▷ Example 1

We demonstrate the use of `pksumm` on a variation of the data described in [R] [pk](#). We have drug concentration data on 15 subjects, each measured at 13 time points over a 32-hour period. A few of the records are as follows:

```
. use https://www.stata-press.com/data/r19/pksumm
. list, sep(0)
```

|                  | id | time | conc     |
|------------------|----|------|----------|
| 1.               | 1  | 0    | 0        |
| 2.               | 1  | .5   | 3.073403 |
| 3.               | 1  | 1    | 5.188444 |
| 4.               | 1  | 1.5  | 5.898577 |
| 5.               | 1  | 2    | 5.096378 |
| 6.               | 1  | 3    | 6.094085 |
| (output omitted) |    |      |          |
| 183.             | 15 | 0    | 0        |
| 184.             | 15 | .5   | 3.86493  |
| 185.             | 15 | 1    | 6.432444 |
| 186.             | 15 | 1.5  | 6.969195 |
| 187.             | 15 | 2    | 6.307024 |
| 188.             | 15 | 3    | 6.509584 |
| 189.             | 15 | 4    | 6.555091 |
| 190.             | 15 | 6    | 7.318319 |
| 191.             | 15 | 8    | 5.329813 |
| 192.             | 15 | 12   | 5.411624 |
| 193.             | 15 | 16   | 3.891397 |
| 194.             | 15 | 24   | 5.167516 |
| 195.             | 15 | 32   | 2.649686 |

We can use pksumm to view the summary statistics for all the pharmacokinetic parameters.

```
. pksumm id time conc
.....
Summary statistics for the pharmacokinetic measures
```

| Measure | Number of observations = 15 |        |           |          |          |         |
|---------|-----------------------------|--------|-----------|----------|----------|---------|
|         | Mean                        | Median | Variance  | Skewness | Kurtosis | p-value |
| auc     | 150.74                      | 150.96 | 123.07    | -0.26    | 2.10     | 0.69    |
| aucline | 408.30                      | 214.17 | 188856.87 | 2.57     | 8.93     | 0.00    |
| aucexp  | 691.68                      | 297.08 | 762679.94 | 2.56     | 8.87     | 0.00    |
| auclog  | 688.98                      | 297.67 | 797237.24 | 2.59     | 9.02     | 0.00    |
| half    | 94.84                       | 29.39  | 18722.13  | 2.26     | 7.37     | 0.00    |
| ke      | 0.02                        | 0.02   | 0.00      | 0.89     | 3.70     | 0.09    |
| cmax    | 7.36                        | 7.42   | 0.42      | -0.60    | 2.56     | 0.44    |
| tomc    | 3.47                        | 3.00   | 7.62      | 2.17     | 7.18     | 0.00    |
| tmax    | 32.00                       | 32.00  | 0.00      | .        | .        | .       |

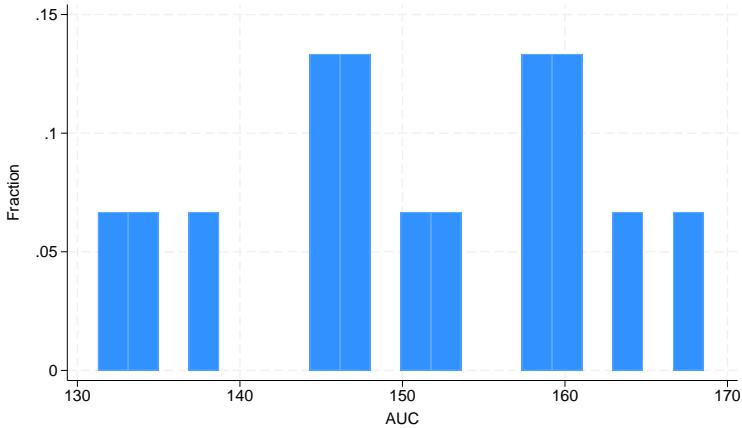
For the 15 subjects, the mean  $AUC_{0,t_{\max}}$  is 150.74, and  $\sigma^2 = 123.07$ . The skewness of  $-0.26$  indicates that the distribution is slightly skewed left. The  $p$ -value of 0.69 for the  $\chi^2$  test of normality indicates that we cannot reject the null hypothesis that the distribution is normal.

If we were to consider any of the three variants of the  $AUC_{0,\infty}$ , we would see that there is huge variability and that the distribution is heavily skewed. A skewness different from 0 and a kurtosis different from 3 are expected because the distribution of the  $AUC_{0,\infty}$  is not normal.

We now graph the distribution of  $AUC_{0,t_{\max}}$  by specifying the graph option.

```
. pksumm id time conc, graph bins(20)
.....
Summary statistics for the pharmacokinetic measures
```

| Measure | Number of observations = 15 |        |           |          |          |         |
|---------|-----------------------------|--------|-----------|----------|----------|---------|
|         | Mean                        | Median | Variance  | Skewness | Kurtosis | p-value |
| auc     | 150.74                      | 150.96 | 123.07    | -0.26    | 2.10     | 0.69    |
| aucline | 408.30                      | 214.17 | 188856.87 | 2.57     | 8.93     | 0.00    |
| aucexp  | 691.68                      | 297.08 | 762679.94 | 2.56     | 8.87     | 0.00    |
| auclog  | 688.98                      | 297.67 | 797237.24 | 2.59     | 9.02     | 0.00    |
| half    | 94.84                       | 29.39  | 18722.13  | 2.26     | 7.37     | 0.00    |
| ke      | 0.02                        | 0.02   | 0.00      | 0.89     | 3.70     | 0.09    |
| cmax    | 7.36                        | 7.42   | 0.42      | -0.60    | 2.56     | 0.44    |
| tomc    | 3.47                        | 3.00   | 7.62      | 2.17     | 7.18     | 0.00    |
| tmax    | 32.00                       | 32.00  | 0.00      | .        | .        | .       |

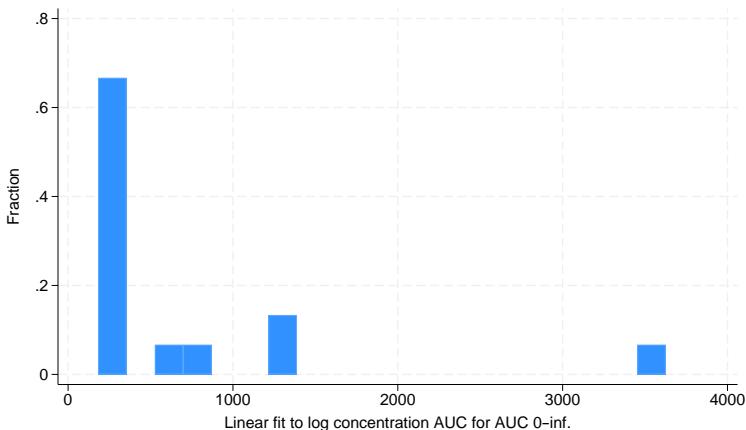


graph, by default, plots  $AUC_{0,t_{max}}$ . To plot a graph of one of the other pharmacokinetic measurements, we need to specify the `stat()` option. For example, we can ask Stata to produce a plot of the  $AUC_{0,\infty}$  using the log extension:

```
. pksumm id time conc, stat(auclog) graph bins(20)
.....
```

Summary statistics for the pharmacokinetic measures

| Measure | Mean   | Median | Variance  | Skewness | Kurtosis | Number of observations = |
|---------|--------|--------|-----------|----------|----------|--------------------------|
|         |        |        |           |          |          | 15                       |
|         |        |        |           |          |          | p-value                  |
| auc     | 150.74 | 150.96 | 123.07    | -0.26    | 2.10     | 0.69                     |
| aucline | 408.30 | 214.17 | 188856.87 | 2.57     | 8.93     | 0.00                     |
| aucexp  | 691.68 | 297.08 | 762679.94 | 2.56     | 8.87     | 0.00                     |
| auclog  | 688.98 | 297.67 | 797237.24 | 2.59     | 9.02     | 0.00                     |
| half    | 94.84  | 29.39  | 18722.13  | 2.26     | 7.37     | 0.00                     |
| ke      | 0.02   | 0.02   | 0.00      | 0.89     | 3.70     | 0.09                     |
| cmax    | 7.36   | 7.42   | 0.42      | -0.60    | 2.56     | 0.44                     |
| tomc    | 3.47   | 3.00   | 7.62      | 2.17     | 7.18     | 0.00                     |
| tmax    | 32.00  | 32.00  | 0.00      | .        | .        | .                        |



## Methods and formulas

The  $\chi^2$  test for normality is conducted with `sktest`; see [\[R\] sktest](#) for more information on the test of normality.

The statistics reported by `pksumm` are identical to those reported by `summarize` and `sktest`; see [\[R\] summarize](#) and [\[R\] sktest](#).

## Also see

[\[R\] pk](#) — Pharmacokinetic (biopharmaceutical) data

[Description](#)  
[Options](#)  
[References](#)

[Quick start](#)  
[Remarks and examples](#)  
[Also see](#)

[Menu](#)  
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## Description

`poisson` fits a Poisson regression of *depvar* on *indepvars*, where *depvar* is a nonnegative count variable.

If you have panel data, see [\[XT\] xtpoisson](#).

## Quick start

Poisson regression of *y* on *x*

```
poisson y x
```

Add categorical variable *a*

```
poisson y x i.a
```

Add exposure variable *v*

```
poisson y x i.a, exposure(v)
```

With robust standard errors

```
poisson y x i.a, vce(robust)
```

Report results as incidence-rate ratios

```
poisson y x i.a, irr
```

Replace data in memory with the results of running a Poisson regression model on each level of *catvar*

```
statsby, by(catvar) clear: poisson y x
```

## Menu

Statistics > Count outcomes > Poisson regression

## Syntax

```
poisson depvar [indepvars] [if] [in] [weight] [, options]
```

| <i>options</i>                                    | Description                                                                                                                                                                 |
|---------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                      |                                                                                                                                                                             |
| <code>noconstant</code>                           | suppress constant term                                                                                                                                                      |
| <code>exposure(<i>varname<sub>e</sub></i>)</code> | include $\ln(\textit{varname}_e)$ in model with coefficient constrained to 1                                                                                                |
| <code>offset(<i>varname<sub>o</sub></i>)</code>   | include <i>varname<sub>o</sub></i> in model with coefficient constrained to 1                                                                                               |
| <code>constraints(<i>constraints</i>)</code>      | apply specified linear constraints                                                                                                                                          |
| <b>SE/Robust</b>                                  |                                                                                                                                                                             |
| <code>vce(<i>vcetype</i>)</code>                  | <i>vcetype</i> may be <code>oim</code> , <code>robust</code> , <code>cluster <i>clustvar</i></code> , <code>opg</code> , <code>bootstrap</code> , or <code>jackknife</code> |
| <b>Reporting</b>                                  |                                                                                                                                                                             |
| <code>level(#)</code>                             | set confidence level; default is level(95)                                                                                                                                  |
| <code>irr</code>                                  | report incidence-rate ratios                                                                                                                                                |
| <code>nocnsreport</code>                          | do not display constraints                                                                                                                                                  |
| <code>display_options</code>                      | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling                            |
| <b>Maximization</b>                               |                                                                                                                                                                             |
| <code>maximize_options</code>                     | control the maximization process; seldom used                                                                                                                               |
| <code>collinear</code>                            | keep collinear variables                                                                                                                                                    |
| <code>coeflegend</code>                           | display legend instead of statistics                                                                                                                                        |

*indepvars* may contain factor variables; see [U] 11.4.3 Factor variables.

*depvar*, *indepvars*, *varname<sub>e</sub>*, and *varname<sub>o</sub>* may contain time-series operators; see [U] 11.4.4 Time-series varlists.

`bayes`, `bayesboot`, `bootstrap`, `by`, `fmm`, `fp`, `jackknife`, `mfp`, `mi estimate`, `nestreg`, `rolling`, `statsby`, `stepwise`, and `svy` are allowed; see [U] 11.1.10 Prefix commands. For more details, see [BAYES] `bayes: poisson` and [FMM] `fmm: poisson`.

`vce(bootstrap)` and `vce(jackknife)` are not allowed with the `mi estimate` prefix; see [MI] `mi estimate`.

Weights are not allowed with the `bootstrap` prefix; see [R] `bootstrap`.

`vce()` and weights are not allowed with the `svy` prefix; see [SVY] `svy`.

`fweights`, `iweights`, and `pweights` are allowed; see [U] 11.1.6 `weight`.

`collinear` and `coeflegend` do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Options

### Model

`noconstant`, `exposure(varnamee)`, `offset(varnameo)`, `constraints(constraints)`; see [R] Estimation options.

### SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] `vce_option`.

## Reporting

`level(#)`; see [R] [Estimation options](#).

`irr` reports estimated coefficients transformed to incidence-rate ratios, that is,  $e^{\beta_i}$  rather than  $\beta_i$ . Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated or stored. `irr` may be specified at estimation or when replaying previously estimated results.

`nocnsreport`; see [R] [Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

## Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

Setting the optimization type to `technique(bhhh)` resets the default `vcetype` to `vce(opg)`.

The following options are available with `poisson` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

The basic idea of Poisson regression was outlined by [Coleman \(1964, 378–379\)](#). See [Cameron and Trivedi \(2013; 2022, chap. 20\)](#) and [Johnson, Kemp, and Kotz \(2005, chap. 4\)](#) for information about the Poisson distribution. See [Cameron and Trivedi \(2013\)](#), [Long \(1997, chap. 8\)](#), [Long and Freese \(2014, chap. 9\)](#), [McNeil \(1996, chap. 6\)](#), and [Selvin \(2011, chap. 6\)](#) for an introduction to Poisson regression. Also see [Selvin \(2004, chap. 5\)](#) for a discussion of the analysis of spatial distributions, which includes a discussion of the Poisson distribution. An early example of Poisson regression was [Cochran \(1940\)](#).

Poisson regression fits models of the number of occurrences (counts) of an event. The Poisson distribution has been applied to diverse events, such as the number of soldiers kicked to death by horses in the Prussian army ([von Bortkiewicz 1898](#)); the pattern of hits by buzz bombs launched against London during World War II ([Clarke 1946](#)); telephone connections to a wrong number ([Thorndike 1926](#)); and disease incidence, typically with respect to time, but occasionally with respect to space. The basic assumptions are as follows:

1. There is a quantity called the *incidence rate* that is the rate at which events occur. Examples are 5 per second, 20 per 1,000 person-years, 17 per square meter, and 38 per cubic centimeter.
2. The incidence rate can be multiplied by *exposure* to obtain the expected number of observed events. For example, a rate of 5 per second multiplied by 30 seconds means that 150 events are expected; a rate of 20 per 1,000 person-years multiplied by 2,000 person-years means that 40 events are expected; and so on.
3. Over very small exposures  $\epsilon$ , the probability of finding more than one event is small compared with  $\epsilon$ .
4. Nonoverlapping exposures are mutually independent.

With these assumptions, to find the probability of  $k$  events in an exposure of size  $E$ , you divide  $E$  into  $n$  subintervals  $E_1, E_2, \dots, E_n$ , and approximate the answer as the binomial probability of observing  $k$  successes in  $n$  trials. If you let  $n \rightarrow \infty$ , you obtain the Poisson distribution.

In the Poisson regression model, the incidence rate for the  $j$ th observation is assumed to be given by

$$r_j = e^{\beta_0 + \beta_1 x_{1,j} + \dots + \beta_k x_{k,j}}$$

If  $E_j$  is the exposure, the expected number of events,  $C_j$ , will be

$$\begin{aligned} C_j &= E_j e^{\beta_0 + \beta_1 x_{1,j} + \dots + \beta_k x_{k,j}} \\ &= e^{\ln(E_j) + \beta_0 + \beta_1 x_{1,j} + \dots + \beta_k x_{k,j}} \end{aligned}$$

This model is fit by `poisson`. Without the `exposure()` or `offset()` options,  $E_j$  is assumed to be 1 (equivalent to assuming that exposure is unknown), and controlling for exposure, if necessary, is your responsibility.

Comparing rates is most easily done by calculating *incidence-rate ratios* (IRRs). For instance, what is the relative incidence rate of chromosome interchanges in cells as the intensity of radiation increases; the relative incidence rate of telephone connections to a wrong number as load increases; or the relative incidence rate of deaths due to cancer for females relative to males? That is, you want to hold all the  $x$ 's in the model constant except one, say, the  $i$ th. The IRR for a one-unit change in  $x_i$  is

$$\frac{e^{\ln(E) + \beta_1 x_1 + \dots + \beta_i (x_i + 1) + \dots + \beta_k x_k}}{e^{\ln(E) + \beta_1 x_1 + \dots + \beta_i x_i + \dots + \beta_k x_k}} = e^{\beta_i}$$

More generally, the IRR for a  $\Delta x_i$  change in  $x_i$  is  $e^{\beta_i \Delta x_i}$ . The `lincom` command can be used after `poisson` to display incidence-rate ratios for any group relative to another; see [R] [lincom](#).

## ► Example 1

Chatterjee and Hadi (2012, 174) give the number of injury incidents and the proportion of flights for each airline out of the total number of flights from New York for nine major US airlines in one year:

```
. use https://www.stata-press.com/data/r19/airline
. list
```

|    | airline | injuries | n      | XYZowned |
|----|---------|----------|--------|----------|
| 1. | 1       | 11       | 0.0950 | 1        |
| 2. | 2       | 7        | 0.1920 | 0        |
| 3. | 3       | 7        | 0.0750 | 0        |
| 4. | 4       | 19       | 0.2078 | 0        |
| 5. | 5       | 9        | 0.1382 | 0        |
| 6. | 6       | 4        | 0.0540 | 1        |
| 7. | 7       | 3        | 0.1292 | 0        |
| 8. | 8       | 1        | 0.0503 | 0        |
| 9. | 9       | 3        | 0.0629 | 1        |

To their data, we have added a fictional variable, `XYZowned`. We will imagine that an accusation is made that the airlines owned by XYZ Company have a higher injury rate.

```
. poisson injuries XYZowned, exposure(n) irr
Iteration 0: Log likelihood = -23.027197
Iteration 1: Log likelihood = -23.027177
Iteration 2: Log likelihood = -23.027177

Poisson regression
Log likelihood = -23.027177
Number of obs = 9
LR chi2(1) = 1.77
Prob > chi2 = 0.1836
Pseudo R2 = 0.0370
```

| injuries | IRR      | Std. err.  | z     | P> z  | [95% conf. interval] |          |
|----------|----------|------------|-------|-------|----------------------|----------|
| XYZowned | 1.463467 | .406872    | 1.37  | 0.171 | .8486578             | 2.523675 |
| _cons    | 58.04416 | 8.558145   | 27.54 | 0.000 | 43.47662             | 77.49281 |
| ln(n)    | 1        | (exposure) |       |       |                      |          |

Note: `_cons` estimates baseline incidence rate.

We specified `irr` to see the IRRs rather than the underlying coefficients. We estimate that XYZ Airlines' injury rate is 1.46 times larger than that for other airlines, but the 95% confidence interval is 0.85 to 2.52; we cannot even reject the hypothesis that XYZ Airlines has a lower injury rate.

◀

## □ Technical note

In [example 1](#), we assumed that each airline's exposure was proportional to its fraction of flights out of New York. What if "large" airlines, however, also used larger planes, and so had even more passengers than would be expected, given this measure of exposure? A better measure would be each airline's fraction of passengers on flights out of New York, a number that we do not have. Even so, we suppose that `n` represents this number to some extent, so a better estimate of the effect might be

```
. generate lnN=ln(n)
. poisson injuries XYZowned lnN
Iteration 0: Log likelihood = -22.333875
Iteration 1: Log likelihood = -22.332276
Iteration 2: Log likelihood = -22.332276

Poisson regression
Log likelihood = -22.332276
Number of obs = 9
LR chi2(2) = 19.15
Prob > chi2 = 0.0001
Pseudo R2 = 0.3001
```

| injuries | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |          |
|----------|-------------|-----------|------|-------|----------------------|----------|
| XYZowned | .6840667    | .3895877  | 1.76 | 0.079 | -.0795111            | 1.447645 |
| lnN      | 1.424169    | .3725155  | 3.82 | 0.000 | .6940517             | 2.154285 |
| _cons    | 4.863891    | .7090501  | 6.86 | 0.000 | 3.474178             | 6.253603 |

Here rather than specifying the `exposure()` option, we explicitly included the variable that would normalize for exposure in the model. We did not specify the `irr` option, so we see coefficients rather than IRRs. We started with the model

$$\text{rate} = e^{\beta_0 + \beta_1 \text{XYZowned}}$$

The observed counts are therefore

$$\text{count} = ne^{\beta_0 + \beta_1 \text{XYZowned}} = e^{\ln(n) + \beta_0 + \beta_1 \text{XYZowned}}$$

which amounts to constraining the coefficient on  $\ln(n)$  to 1. This is what was estimated when we specified the `exposure(n)` option. In the above model, we included the normalizing exposure ourselves and, rather than constraining the coefficient to be 1, estimated the coefficient.

The estimated coefficient is 1.42, a respectable distance away from 1, and is consistent with our speculation that larger airlines also use larger airplanes. With this small amount of data, however, we also have a wide confidence interval that includes 1.

Our estimated *coefficient* on `XYZowned` is now 0.684, and the implied IRR is  $e^{0.684} \approx 1.98$  (which we could also see by typing `poisson, irr`). The 95% confidence interval for the coefficient still includes 0 (the interval for the IRR includes 1), so although the point estimate is now larger, we still cannot be certain of our results.

Our expert opinion would be that, although there is not enough evidence to support the charge, there is enough evidence to justify collecting more data. □

## ► Example 2

In a famous age-specific study of coronary disease deaths among male British doctors, [Doll and Hill \(1966\)](#) reported the following data (reprinted in [Lash et al. \[2021, 417\]](#)):

| Age   | Smokers |              | Nonsmokers |              |
|-------|---------|--------------|------------|--------------|
|       | Deaths  | Person-years | Deaths     | Person-years |
| 35–44 | 32      | 52,407       | 2          | 18,790       |
| 45–54 | 104     | 43,248       | 12         | 10,673       |
| 55–64 | 206     | 28,612       | 28         | 5,710        |
| 65–74 | 186     | 12,663       | 28         | 2,585        |
| 75–84 | 102     | 5,317        | 31         | 1,462        |

The first step is to enter these data into Stata, which we have done:

```
. use https://www.stata-press.com/data/r19/dollhill13, clear
(Doll and Hill (1966))
. list
```

|     | agecat | smokes | deaths | pyears |
|-----|--------|--------|--------|--------|
| 1.  | 35–44  | 1      | 32     | 52,407 |
| 2.  | 45–54  | 1      | 104    | 43,248 |
| 3.  | 55–64  | 1      | 206    | 28,612 |
| 4.  | 65–74  | 1      | 186    | 12,663 |
| 5.  | 75–84  | 1      | 102    | 5,317  |
| 6.  | 35–44  | 0      | 2      | 18,790 |
| 7.  | 45–54  | 0      | 12     | 10,673 |
| 8.  | 55–64  | 0      | 28     | 5,710  |
| 9.  | 65–74  | 0      | 28     | 2,585  |
| 10. | 75–84  | 0      | 31     | 1,462  |

The most “natural” analysis of these data would begin by introducing indicator variables for each age category and one indicator for smoking:

```
. poisson deaths smokes i.agecat, exposure(pyyears) irr
Iteration 0: Log likelihood = -33.823284
Iteration 1: Log likelihood = -33.600471
Iteration 2: Log likelihood = -33.600153
Iteration 3: Log likelihood = -33.600153

Poisson regression Number of obs = 10
LR chi2(5) = 922.93
Prob > chi2 = 0.0000
Pseudo R2 = 0.9321

Log likelihood = -33.600153
```

| deaths      | IRR      | Std. err.  | z      | P> z  | [95% conf. interval] |          |
|-------------|----------|------------|--------|-------|----------------------|----------|
| smokes      | 1.425519 | .1530638   | 3.30   | 0.001 | 1.154984             | 1.759421 |
| agecat      |          |            |        |       |                      |          |
| 45-54       | 4.410584 | .8605197   | 7.61   | 0.000 | 3.009011             | 6.464997 |
| 55-64       | 13.8392  | 2.542638   | 14.30  | 0.000 | 9.654328             | 19.83809 |
| 65-74       | 28.51678 | 5.269878   | 18.13  | 0.000 | 19.85177             | 40.96395 |
| 75-84       | 40.45121 | 7.775511   | 19.25  | 0.000 | 27.75326             | 58.95885 |
| _cons       | .0003636 | .0000697   | -41.30 | 0.000 | .0002497             | .0005296 |
| ln(pyyears) | 1        | (exposure) |        |       |                      |          |

Note: `_cons` estimates baseline incidence rate.

In the above, we specified `irr` to obtain IRRs. We estimate that smokers have 1.43 times the mortality rate of nonsmokers. See, however, [example 1](#) in [\[R\] poisson postestimation](#).

◀

## Stored results

`poisson` stores the following in `e()`:

### Scalars

|                            |                                           |
|----------------------------|-------------------------------------------|
| <code>e(N)</code>          | number of observations                    |
| <code>e(k)</code>          | number of parameters                      |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>  |
| <code>e(k_eq_model)</code> | number of equations in overall model test |
| <code>e(k_dv)</code>       | number of dependent variables             |
| <code>e(df_m)</code>       | model degrees of freedom                  |
| <code>e(r2_p)</code>       | pseudo- $R^2$                             |
| <code>e(ll)</code>         | log likelihood                            |
| <code>e(ll_0)</code>       | log likelihood, constant-only model       |
| <code>e(N_clust)</code>    | number of clusters                        |
| <code>e(chi2)</code>       | $\chi^2$                                  |
| <code>e(p)</code>          | $p$ -value for model test                 |
| <code>e(rank)</code>       | rank of <code>e(V)</code>                 |
| <code>e(ic)</code>         | number of iterations                      |
| <code>e(rc)</code>         | return code                               |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise               |

### Macros

|                         |                            |
|-------------------------|----------------------------|
| <code>e(cmd)</code>     | <code>poisson</code>       |
| <code>e(cmdline)</code> | command as typed           |
| <code>e(depvar)</code>  | name of dependent variable |
| <code>e(wtype)</code>   | weight type                |

|                              |                                                                          |
|------------------------------|--------------------------------------------------------------------------|
| <code>e(wexp)</code>         | weight expression                                                        |
| <code>e(title)</code>        | title in estimation output                                               |
| <code>e(clustvar)</code>     | name of cluster variable                                                 |
| <code>e(offset)</code>       | linear offset variable                                                   |
| <code>e(chi2type)</code>     | Wald or LR; type of model $\chi^2$ test                                  |
| <code>e(vce)</code>          | <i>vce</i> type specified in <code>vce()</code>                          |
| <code>e(vcetype)</code>      | title used to label Std. err.                                            |
| <code>e(opt)</code>          | type of optimization                                                     |
| <code>e(which)</code>        | max or min; whether optimizer is to perform maximization or minimization |
| <code>e(ml_method)</code>    | type of ml method                                                        |
| <code>e(user)</code>         | name of likelihood-evaluator program                                     |
| <code>e(technique)</code>    | maximization technique                                                   |
| <code>e(properties)</code>   | b V                                                                      |
| <code>e(estat_cmd)</code>    | program used to implement <code>estat</code>                             |
| <code>e(predict)</code>      | program used to implement <code>predict</code>                           |
| <code>e(asbalanced)</code>   | factor variables <code>fvset</code> as <code>asbalanced</code>           |
| <code>e(asobserved)</code>   | factor variables <code>fvset</code> as <code>asobserved</code>           |
| Matrices                     |                                                                          |
| <code>e(b)</code>            | coefficient vector                                                       |
| <code>e(Cns)</code>          | constraints matrix                                                       |
| <code>e(ilog)</code>         | iteration log (up to 20 iterations)                                      |
| <code>e(gradient)</code>     | gradient vector                                                          |
| <code>e(V)</code>            | variance–covariance matrix of the estimators                             |
| <code>e(V_modelbased)</code> | model-based variance                                                     |
| Functions                    |                                                                          |
| <code>e(sample)</code>       | marks estimation sample                                                  |

In addition to the above, the following is stored in `r()`:

|                       |                                                                                                                            |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|
| Matrices              |                                                                                                                            |
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

## Methods and formulas

The log likelihood (with weights  $w_j$  and offsets) is given by

$$\Pr(Y = y) = \frac{e^{-\lambda} \lambda^y}{y!}$$

$$\xi_j = \mathbf{x}_j \boldsymbol{\beta} + \text{offset}_j$$

$$f(y_j) = \frac{e^{-\exp(\xi_j)} e^{\xi_j y_j}}{y_j!}$$

$$\ln L = \sum_{j=1}^n w_j \{-e^{\xi_j} + \xi_j y_j - \ln(y_j!)\}$$

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] [\\_robust](#), particularly *Maximum likelihood estimators* and *Methods and formulas*.

poisson also supports estimation with survey data. For details on VCEs with survey data, see [SVY] [Variance estimation](#).

[Siméon-Denis Poisson](#) (1781–1840) was a French mathematician and physicist who contributed to several fields: his name is perpetuated in Poisson brackets, Poisson’s constant, Poisson’s differential equation, Poisson’s integral, and Poisson’s ratio. Among many other results, he produced a version of the law of large numbers. His rather misleadingly titled *Recherches sur la probabilité des jugements* embraces a complete treatise on probability, as the subtitle indicates, including what is now known as the Poisson distribution. That, however, was discovered earlier by the Huguenot–British mathematician Abraham de Moivre (1667–1754).

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## Also see

- [R] **poisson postestimation** — Postestimation tools for poisson
- [R] **glm** — Generalized linear models
- [R] **heckpoisson** — Poisson regression with sample selection
- [R] **nbreg** — Negative binomial regression
- [R] **npregress kernel** — Nonparametric kernel regression
- [R] **npregress series** — Nonparametric series regression
- [R] **tpoisson** — Truncated Poisson regression
- [R] **zip** — Zero-inflated Poisson regression
- [BAYES] **bayes: poisson** — Bayesian Poisson regression
- [FMM] **fmm: poisson** — Finite mixtures of Poisson regression models
- [LASSO] **Lasso intro** — Introduction to lasso
- [ME] **mepoisson** — Multilevel mixed-effects Poisson regression
- [MI] **Estimation** — Estimation commands for use with mi estimate
- [SVY] **svy estimation** — Estimation commands for survey data
- [XT] **xtpoisson** — Fixed-effects, random-effects, and population-averaged Poisson models
- [U] **20 Estimation and postestimation commands**

|                         |                                |                                      |                           |
|-------------------------|--------------------------------|--------------------------------------|---------------------------|
| Postestimation commands | <a href="#">predict</a>        | <a href="#">margins</a>              | <a href="#">estat</a>     |
| Remarks and examples    | <a href="#">Stored results</a> | <a href="#">Methods and formulas</a> | <a href="#">Reference</a> |
| Also see                |                                |                                      |                           |

## Postestimation commands

The following postestimation commands are of special interest after `poisson`:

| Command                | Description                           |
|------------------------|---------------------------------------|
| <code>estat gof</code> | goodness-of-fit test                  |
| <code>lassogof</code>  | calculate goodness-of-fit predictions |

`estat gof` is not appropriate with `svy` estimation results.

The following standard postestimation commands are also available:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| <code>linktest</code>        | link test for model specification                                                                                                       |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from <code>margins</code> (profile plots, interaction plots, etc.)                                                    |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | number of events, incidence rates, probabilities, etc.                                                                                  |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results. `forecast` is also not appropriate with `mi` estimation results.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as numbers of events, incidence rates, probabilities, linear predictions, standard errors, and the equation-level score.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic nooffset]
```

| <i>statistic</i>                    | Description                                                                |
|-------------------------------------|----------------------------------------------------------------------------|
| Main                                |                                                                            |
| <code>n</code>                      | number of events; the default                                              |
| <code>ir</code>                     | incidence rate                                                             |
| <code>pr(<i>n</i>)</code>           | probability $\Pr(y_j = n)$                                                 |
| <code>pr(<i>a</i>, <i>b</i>)</code> | probability $\Pr(a \leq y_j \leq b)$                                       |
| <code>xb</code>                     | linear prediction                                                          |
| <code>stdp</code>                   | standard error of the linear prediction                                    |
| <code>score</code>                  | first derivative of the log likelihood with respect to $\mathbf{x}_j\beta$ |

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

## Options for predict

Main

`n`, the default, calculates the predicted number of events, which is  $\exp(\mathbf{x}_j\beta)$  if neither `offset()` nor `exposure()` was specified when the model was fit;  $\exp(\mathbf{x}_j\beta + \text{offset}_j)$  if `offset()` was specified; or  $\exp(\mathbf{x}_j\beta) \times \text{exposure}_j$  if `exposure()` was specified.

`ir` calculates the incidence rate  $\exp(\mathbf{x}_j\beta)$ , which is the predicted number of events when exposure is 1. Specifying `ir` is equivalent to specifying `n` when neither `offset()` nor `exposure()` was specified when the model was fit.

`pr(n)` calculates the probability  $\Pr(y_j = n)$ , where *n* is a nonnegative integer that may be specified as a number or a variable.

`pr(a, b)` calculates the probability  $\Pr(a \leq y_j \leq b)$ , where *a* and *b* are nonnegative integers that may be specified as numbers or variables;

*b* missing (*b* ≥ .) means  $+\infty$ ;

`pr(20, .)` calculates  $\Pr(y_j \geq 20)$ ;

`pr(20, b)` calculates  $\Pr(y_j \geq 20)$  in observations for which *b* ≥ . and calculates  $\Pr(20 \leq y_j \leq b)$  elsewhere.

`pr(., b)` produces a syntax error. A missing value in an observation of the variable  $a$  causes a missing value in that observation for `pr(a, b)`.

`xb` calculates the linear prediction, which is  $\mathbf{x}_j\beta$  if neither `offset()` nor `exposure()` was specified;  $\mathbf{x}_j\beta + \text{offset}_j$  if `offset()` was specified; or  $\mathbf{x}_j\beta + \ln(\text{exposure}_j)$  if `exposure()` was specified; see `nooffset` below.

`stdp` calculates the standard error of the linear prediction.

`score` calculates the equation-level score,  $\partial \ln L / \partial (\mathbf{x}_j\beta)$ .

`nooffset` is relevant only if you specified `offset()` or `exposure()` when you fit the model. It modifies the calculations made by `predict` so that they ignore the offset or exposure variable; the linear prediction is treated as  $\mathbf{x}_j\beta$  rather than as  $\mathbf{x}_j\beta + \text{offset}_j$  or  $\mathbf{x}_j\beta + \ln(\text{exposure}_j)$ . Specifying `predict ..., nooffset` is equivalent to specifying `predict ..., ir`.

## margins

### Description for margins

`margins` estimates margins of response for numbers of events, incidence rates, probabilities, and linear predictions.

### Menu for margins

Statistics > Postestimation

### Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i>      | Description                           |
|-----------------------|---------------------------------------|
| <code>n</code>        | number of events; the default         |
| <code>ir</code>       | incidence rate                        |
| <code>pr(n)</code>    | probability $\Pr(y_j = n)$            |
| <code>pr(a, b)</code> | probability $\Pr(a \leq y_j \leq b)$  |
| <code>xb</code>       | linear prediction                     |
| <code>stdp</code>     | not allowed with <code>margins</code> |
| <code>score</code>    | not allowed with <code>margins</code> |

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [R] [margins](#).

## estat

### Description for estat

`estat gof` performs a goodness-of-fit test of the model. Both the deviance statistic and the Pearson statistic are reported. If the tests are significant, the Poisson regression model is inappropriate.

### Menu for estat

Statistics > Postestimation

### Syntax for estat

```
estat gof
```

`collect` is allowed; see [U] 11.1.10 Prefix commands.

### Remarks and examples

#### ▷ Example 1

Continuing with [example 2](#) of [R] [poisson](#), we use `estat gof` to determine whether the model fits the data well.

```
. use https://www.stata-press.com/data/r19/dollhill13
(Doll and Hill (1966))
. poisson deaths smokes i.agecat, exp(pyears) irr
(output omitted)
. estat gof
 Deviance goodness-of-fit = 12.13237
 Prob > chi2(4) = 0.0164
 Pearson goodness-of-fit = 11.15533
 Prob > chi2(4) = 0.0249
```

The deviance goodness-of-fit test tells us that, given the model, we can reject the hypothesis that these data are Poisson distributed at the 1.64% significance level. The Pearson goodness-of-fit test tells us that we can reject the hypothesis at the 2.49% significance level.

So let us now back up and be more careful. We can most easily obtain the incidence-rate ratios within age categories by using `irr`; see [R] [Eptab](#):

```
. irr deaths smokes pyears, by(agecat) nohet
Stratified incidence-rate analysis
```

| Age category | IRR      | [95% conf. interval] |          | M-H weight       |
|--------------|----------|----------------------|----------|------------------|
| 35-44        | 5.736638 | 1.463557             | 49.40468 | 1.472169 (exact) |
| 45-54        | 2.138812 | 1.173714             | 4.272545 | 9.624747 (exact) |
| 55-64        | 1.46824  | .9863624             | 2.264107 | 23.34176 (exact) |
| 65-74        | 1.35606  | .9081925             | 2.096412 | 23.25315 (exact) |
| 75-84        | .9047304 | .6000757             | 1.399687 | 24.31435 (exact) |
| Crude        | 1.719823 | 1.391992             | 2.14353  | (exact)          |
| M-H combined | 1.424682 | 1.154703             | 1.757784 |                  |

We find that the mortality incidence ratios are greatly different within age category, being highest for the youngest categories and actually dropping below 1 for the oldest. (In the last case, we might argue that those who smoke and who have not died by age 75 are self-selected to be particularly robust.)

Seeing this, we will now parameterize the smoking effects separately for each category, although we will begin by constraining the smoking effects on third and fourth age categories to be equivalent:

```
. constraint 1 smokes#3.agecat = smokes#4.agecat
. poisson deaths c.smokes#agecat i.agecat, exposure(pyears) irr constraints(1)
Iteration 0: Log likelihood = -31.95424
Iteration 1: Log likelihood = -27.796801
Iteration 2: Log likelihood = -27.574177
Iteration 3: Log likelihood = -27.572645
Iteration 4: Log likelihood = -27.572645
Poisson regression
Log likelihood = -27.572645
(1) [deaths]3.agecat#c.smokes - [deaths]4.agecat#c.smokes = 0
Number of obs = 10
Wald chi2(8) = 632.14
Prob > chi2 = 0.0000
```

| deaths     | IRR      | Std. err.  | z      | P> z  | [95% conf. interval] |          |
|------------|----------|------------|--------|-------|----------------------|----------|
| agecat#    |          |            |        |       |                      |          |
| c.smokes   |          |            |        |       |                      |          |
| 35-44      | 5.736637 | 4.181256   | 2.40   | 0.017 | 1.374811             | 23.93711 |
| 45-54      | 2.138812 | .6520701   | 2.49   | 0.013 | 1.176691             | 3.887609 |
| 55-64      | 1.412229 | .2017485   | 2.42   | 0.016 | 1.067343             | 1.868557 |
| 65-74      | 1.412229 | .2017485   | 2.42   | 0.016 | 1.067343             | 1.868557 |
| 75-84      | .9047304 | .1855513   | -0.49  | 0.625 | .6052658             | 1.35236  |
| agecat     |          |            |        |       |                      |          |
| 45-54      | 10.5631  | 8.067701   | 3.09   | 0.002 | 2.364153             | 47.19623 |
| 55-64      | 47.671   | 34.37409   | 5.36   | 0.000 | 11.60056             | 195.8978 |
| 65-74      | 98.22765 | 70.85012   | 6.36   | 0.000 | 23.89324             | 403.8244 |
| 75-84      | 199.2099 | 145.3356   | 7.26   | 0.000 | 47.67693             | 832.3648 |
| _cons      | .0001064 | .0000753   | -12.94 | 0.000 | .0000266             | .0004256 |
| ln(pyears) | 1        | (exposure) |        |       |                      |          |

Note: `_cons` estimates baseline incidence rate.

```
. estat gof
 Deviance goodness-of-fit = .0773491
 Prob > chi2(1) = 0.7809
 Pearson goodness-of-fit = .0773885
 Prob > chi2(1) = 0.7809
```

The goodness-of-fit is now small; we are no longer running roughshod over the data. Let us now consider simplifying the model. The point estimate of the incidence-rate ratio for smoking in age category 1 is much larger than that for smoking in age category 2, but the confidence interval for `smokes#1.agecat` is similarly wide. Is the difference real?

```
. test smokes#1.agecat = smokes#2.agecat
 (1) [deaths]1b.agecat#c.smokes - [deaths]2.agecat#c.smokes = 0
 chi2(1) = 1.56
 Prob > chi2 = 0.2117
```

The point estimates of the incidence-rate ratio for smoking in the 35–44 age category is much larger than that for smoking in the 45–54 age category, but there is insufficient data, and we may be observing random differences. With that success, might we also combine the smokers in the third and fourth categories with those in the first and second categories?

```
. test smokes#2.agecat = smokes#3.agecat, accum
 (1) [deaths]1b.agecat#c.smokes - [deaths]2.agecat#c.smokes = 0
 (2) [deaths]2.agecat#c.smokes - [deaths]3.agecat#c.smokes = 0
 chi2(2) = 4.73
 Prob > chi2 = 0.0938
```

Combining the first four categories may be overdoing it—the 9.38% significance level is enough to stop us, although others may disagree.

Thus, we now fit our final model:

```
. constraint 2 smokes#1.agecat = smokes#2.agecat
. poisson deaths c.smokes#agecat i.agecat, exposure(pyyears) irr constraints(1/2)
Iteration 0: Log likelihood = -31.550722
Iteration 1: Log likelihood = -28.525057
Iteration 2: Log likelihood = -28.514535
Iteration 3: Log likelihood = -28.514535

Poisson regression Number of obs = 10
 Wald chi2(7) = 642.25
Log likelihood = -28.514535 Prob > chi2 = 0.0000
(1) [deaths]3.agecat#c.smokes - [deaths]4.agecat#c.smokes = 0
(2) [deaths]1b.agecat#c.smokes - [deaths]2.agecat#c.smokes = 0
```

| deaths      | IRR      | Std. err.  | z      | P> z  | [95% conf. interval] |          |
|-------------|----------|------------|--------|-------|----------------------|----------|
| agecat#     |          |            |        |       |                      |          |
| c.smokes    |          |            |        |       |                      |          |
| 35-44       | 2.636259 | .7408403   | 3.45   | 0.001 | 1.519791             | 4.572907 |
| 45-54       | 2.636259 | .7408403   | 3.45   | 0.001 | 1.519791             | 4.572907 |
| 55-64       | 1.412229 | .2017485   | 2.42   | 0.016 | 1.067343             | 1.868557 |
| 65-74       | 1.412229 | .2017485   | 2.42   | 0.016 | 1.067343             | 1.868557 |
| 75-84       | .9047304 | .1855513   | -0.49  | 0.625 | .6052658             | 1.35236  |
| agecat      |          |            |        |       |                      |          |
| 45-54       | 4.294559 | .8385329   | 7.46   | 0.000 | 2.928987             | 6.296797 |
| 55-64       | 23.42263 | 7.787716   | 9.49   | 0.000 | 12.20738             | 44.94164 |
| 65-74       | 48.26309 | 16.06939   | 11.64  | 0.000 | 25.13068             | 92.68856 |
| 75-84       | 97.87965 | 34.30881   | 13.08  | 0.000 | 49.24123             | 194.561  |
| _cons       | .0002166 | .0000652   | -28.03 | 0.000 | .0001201             | .0003908 |
| ln(pyyears) | 1        | (exposure) |        |       |                      |          |

Note: **\_cons** estimates baseline incidence rate.

The above strikes us as a fair representation of the data. The probabilities of observing the deaths seen in these data are estimated using the following predict command:

```
. predict p, pr(0, deaths)
. list deaths p
```

|     | deaths | p        |
|-----|--------|----------|
| 1.  | 32     | .6891766 |
| 2.  | 104    | .4456625 |
| 3.  | 206    | .5455328 |
| 4.  | 186    | .4910622 |
| 5.  | 102    | .5263011 |
| 6.  | 2      | .227953  |
| 7.  | 12     | .7981917 |
| 8.  | 28     | .4772961 |
| 9.  | 28     | .6227565 |
| 10. | 31     | .5475718 |

The probability  $\Pr(y \leq \text{deaths})$  ranges from 0.23 to 0.80.

## Stored results

estat `gof` after `poisson` stores the following in `r()`:

Scalars

|                        |                                           |
|------------------------|-------------------------------------------|
| <code>r(df)</code>     | degrees of freedom (Pearson and deviance) |
| <code>r(chi2_p)</code> | $\chi^2$ (Pearson)                        |
| <code>r(chi2_d)</code> | $\chi^2$ (deviance)                       |
| <code>r(p_p)</code>    | $p$ -value for $\chi^2$ test (Pearson)    |
| <code>r(p_d)</code>    | $p$ -value for $\chi^2$ test (deviance)   |

## Methods and formulas

In the following, we use the [same notation](#) as in [\[R\] poisson](#).

The equation-level score is given by

$$\text{score}(\mathbf{x}\boldsymbol{\beta})_j = y_j - e^{\xi_j}$$

The deviance (D) and Pearson (P) goodness-of-fit statistics are given by

$$\begin{aligned}\ln L_{\max} &= \sum_{j=1}^n w_j [y_j \{ \ln(y_j) - 1 \} - \ln(y_j!)] \\ \chi_D^2 &= -2 \{ \ln L - \ln L_{\max} \} \\ \chi_P^2 &= \sum_{j=1}^n \frac{w_j (y_j - e^{\xi_j})^2}{e^{\xi_j}}\end{aligned}$$

## Reference

Manjón, M., and O. Martínez. 2014. [The chi-squared goodness-of-fit test for count-data models](#). *Stata Journal* 14: 798–816.

## Also see

[\[R\] poisson](#) — Poisson regression

[\[LASSO\] lassogof](#) — Goodness of fit after lasso for prediction

[\[U\] 20 Estimation and postestimation commands](#)

## Description

Launch the Postestimation Selector window. The window contains a list of all postestimation features that are available for the currently active estimation results. To launch the dialog box for an item in the list, select an item and click on **Launch**. The list is automatically updated when estimation commands are run or estimates are restored from memory or disk.

## Menu

Statistics > Postestimation

## Syntax

```
postest
```

## Remarks and examples

Remarks are presented under the following headings:

[Overview](#)

[Video example](#)

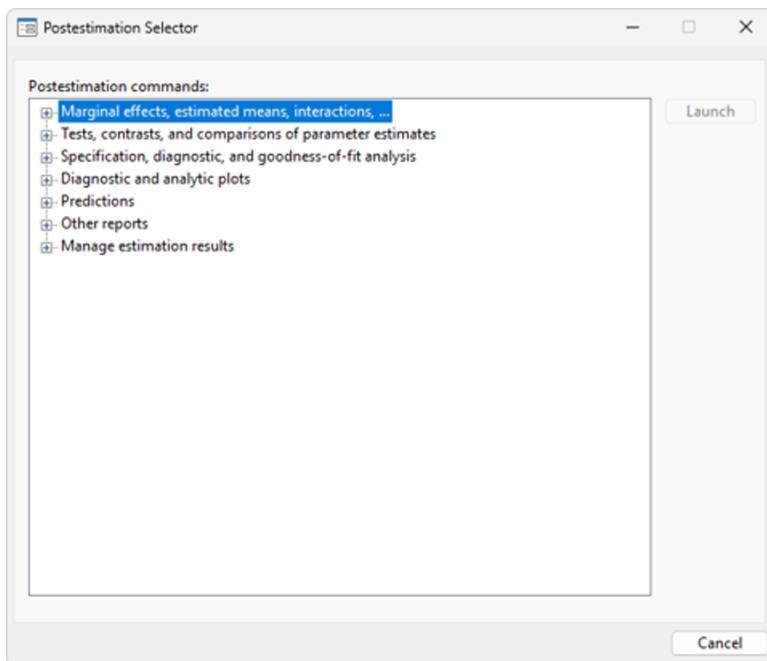
## Overview

Stata uses an estimation followed by postestimation analysis paradigm. You type `regress ...` to fit a regression model, then you type `test ...` to test linear relationships among the estimated parameters, or you type `contrast ...` to compare marginal means, or you type `rvfplot` to see a residual-versus-fitted plot, or you type one of a myriad of other postestimation commands. This is an extension of Stata's "type a little, get a little" concept. The Postestimation Selector exposes this type of postestimation analysis to those who prefer to use the dialog boxes to fit and analyze models, or at least they sometimes prefer the dialogs when exploring their data. We might call this "click a little, get a little".

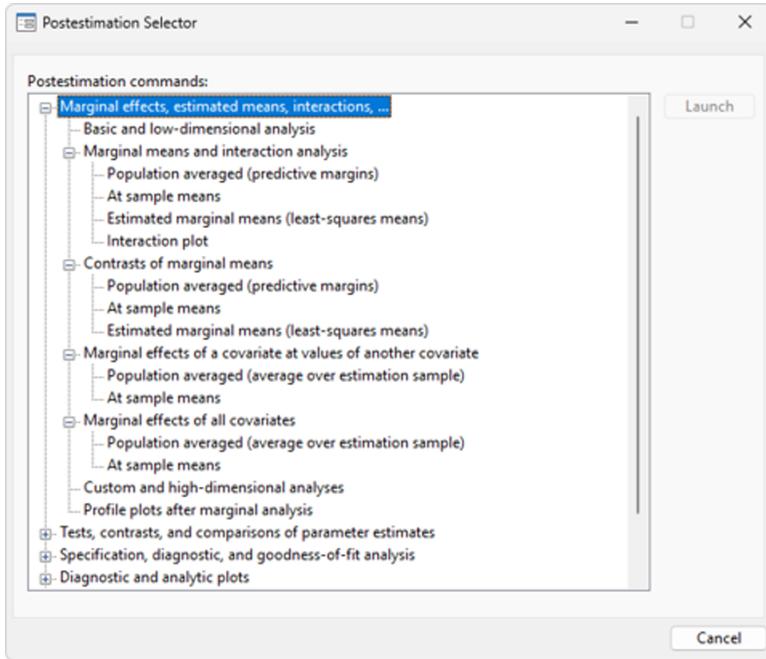
The Postestimation Selector knows what is available after any estimation command. It shows the full list of postestimation features that are available after any estimation, and it shows only those that are available. For example, if you fit a linear regression, you can choose from 59 postestimation analyses, including "Likelihood-ratio test comparing models". If, however, you fit that linear regression using survey estimation, the likelihood-ratio test is not available because that test has no meaning for survey estimation. You will, however, see 8 new postestimation features for survey-data analysis, including "Design and misspecification effects".

If you are using the menus and dialogs, we recommend you launch the Postestimation Selector and just leave it up—select **Statistics > Postestimation**, or type `postest` at the command line. All available postestimation features for whatever model you are analyzing will then be just a click away.

Here is what the Selector looks like after a linear regression,



And, here is what it looks like with all the groupings expanded.



## Video example

[Postestimation Selector](#)

## Also see

[\[U\] 20 Estimation and postestimation commands](#)

**predict** — Obtain predictions, residuals, etc., after estimation

Description  
Options  
Also see

Quick start  
Remarks and examples

Menu for predict  
Methods and formulas

Syntax  
References

## Description

`predict` calculates predictions, residuals, influence statistics, and the like after estimation. Exactly what `predict` can do is determined by the previous estimation command; command-specific options are documented with each estimation command. Regardless of command-specific options, the actions of `predict` share certain similarities across estimation commands:

1. `predict newvar` creates *newvar* containing “predicted values”—numbers related to the  $E(y_j|\mathbf{x}_j)$ . For instance, after linear regression, `predict newvar` creates  $\mathbf{x}_j\mathbf{b}$  and, after probit, creates the probability  $\Phi(\mathbf{x}_j\mathbf{b})$ .
2. `predict newvar, xb` creates *newvar* containing  $\mathbf{x}_j\mathbf{b}$ . This may be the same result as option 1 (for example, linear regression) or different (for example, probit), but regardless, option `xb` is allowed.
3. `predict newvar, stdp` creates *newvar* containing the standard error of the linear prediction  $\mathbf{x}_j\mathbf{b}$ .
4. `predict newvar, other_options` may create *newvar* containing other useful quantities; see `help` or the reference manual entry for the particular estimation command to find out about other available options.
5. `nooffset` added to any of the above commands requests that the calculation ignore any offset or exposure variable specified by including the `offset(varnameo)` or `exposure(varnamee)` option when you fit the model.

`predict` can be used to make in-sample or out-of-sample predictions:

6. `predict` calculates the requested statistic for all possible observations, whether they were used in fitting the model or not. `predict` does this for standard options 1 through 3 and generally does this for estimator-specific options 4.
7. `predict newvar if e(sample), ...` restricts the prediction to the estimation subsample.
8. Some statistics make sense only with respect to the estimation subsample. In such cases, the calculation is automatically restricted to the estimation subsample, and the documentation for the specific option states this. Even so, you can still specify `if e(sample)` if you are uncertain.
9. `predict` can make out-of-sample predictions even using other datasets. In particular, you can

```
. use ds1
. (fit a model)
. use two /* another dataset */
. predict yhat, ... /* fill in the predictions */
```

## Quick start

Create `newvar1` containing the default prediction for the previous estimation command

```
predict newvar1
```

Create `newvar2` containing the linear prediction

```
predict newvar2, xb
```

Same as above, but only for observations used in the previous estimation

```
predict newvar2 if e(sample), xb
```

Create `newvar3`, the default prediction for the first equation in a multiple-equation model

```
predict newvar3, equation(#1)
```

Same as above when `y1` is the name of the first equation

```
predict newvar3, equation(y1)
```

Note: For a complete list of options available with `predict` after an estimation command, see the corresponding `postestimation` entry.

## Menu for predict

Statistics > Postestimation

## Syntax

After single-equation (SE) models

```
predict [type] newvar [if] [in] [, single_options]
```

After multiple-equation (ME) models

```
predict [type] newvar [if] [in] [, multiple_options]
```

```
predict [type] stub* [if] [in], scores
```

| <i>single_options</i> | Description                                                                                       |
|-----------------------|---------------------------------------------------------------------------------------------------|
| Main                  |                                                                                                   |
| <code>xb</code>       | calculate linear prediction                                                                       |
| <code>stdp</code>     | calculate standard error of the prediction                                                        |
| <code>score</code>    | calculate first derivative of the log likelihood with respect to $\mathbf{x}_j\boldsymbol{\beta}$ |
| Options               |                                                                                                   |
| <code>nooffset</code> | ignore any <code>offset()</code> or <code>exposure()</code> variable                              |
| <i>other_options</i>  | command-specific options                                                                          |

| <i>multiple_options</i>             | Description                                                          |
|-------------------------------------|----------------------------------------------------------------------|
| Main                                |                                                                      |
| <code>equation(eqno[, eqno])</code> | specify equations                                                    |
| <code>xb</code>                     | calculate linear prediction                                          |
| <code>stdp</code>                   | calculate standard error of the prediction                           |
| <code>stddp</code>                  | calculate the difference in linear predictions                       |
| Options                             |                                                                      |
| <code>nooffset</code>               | ignore any <code>offset()</code> or <code>exposure()</code> variable |
| <i>other_options</i>                | command-specific options                                             |

## Options

Main

`xb` calculates the linear prediction from the fitted model. That is, all models can be thought of as estimating a set of parameters  $b_1, b_2, \dots, b_k$ , and the linear prediction is  $\hat{y}_j = b_1x_{1j} + b_2x_{2j} + \dots + b_kx_{kj}$ , often written in matrix notation as  $\hat{\mathbf{y}}_j = \mathbf{x}_j\mathbf{b}$ . For linear regression, the values  $\hat{y}_j$  are called the predicted values or, for out-of-sample predictions, the forecast. For logit and probit, for example,  $\hat{y}_j$  is called the logit or probit index.

$x_{1j}, x_{2j}, \dots, x_{kj}$  are obtained from the data currently in memory and do not necessarily correspond to the data on the independent variables used to fit the model (obtaining  $b_1, b_2, \dots, b_k$ ).

`stdp` calculates the standard error of the linear prediction. Here the prediction means the same thing as the “index”, namely,  $\mathbf{x}_j\mathbf{b}$ . The statistic produced by `stdp` can be thought of as the standard error of the predicted expected value, or mean index, for the observation’s covariate pattern. The standard error of the prediction is also commonly referred to as the standard error of the fitted value. The calculation can be made in or out of sample.

`stdp` is allowed only after you have previously fit a multiple-equation model. The standard error of the difference in linear predictions ( $\mathbf{x}_{1j}\mathbf{b} - \mathbf{x}_{2j}\mathbf{b}$ ) between equations 1 and 2 is calculated. This option requires that `equation(eqno1, eqno2)` be specified.

`score` calculates the equation-level score,  $\partial \ln L / \partial (\mathbf{x}_j \boldsymbol{\beta})$ . Here  $\ln L$  refers to the log-likelihood function.

`scores` is the ME model equivalent of the `score` option, resulting in multiple equation-level score variables. An equation-level score variable is created for each equation in the model; ancillary parameters—such as  $\ln \sigma$  and  $\text{atanh} \rho$ —make up separate equations.

`equation(eqno[ , eqno ])`—synonym `outcome()`—is relevant only when you have previously fit a multiple-equation model. It specifies the equation to which you are referring.

`equation()` is typically filled in with one `eqno`—it would be filled in that way with options `xb` and `stdp`, for instance. `equation(#1)` would mean the calculation is to be made for the first equation, `equation(#2)` would mean the second, and so on. You could also refer to the equations by their names. `equation(income)` would refer to the equation named `income` and `equation(hours)` to the equation named `hours`.

If you do not specify `equation()`, results are the same as if you specified `equation(#1)`.

Other statistics, such as `stdp`, refer to between-equation concepts. In those cases, you might specify `equation(#1, #2)` or `equation(income, hours)`. When two equations must be specified, `equation()` is required.

#### Options

`nooffset` may be combined with most statistics and specifies that the calculation should be made, ignoring any offset or exposure variable specified when the model was fit.

This option is available, even if it is not documented for `predict` after a specific command. If neither the `offset(varnameo)` option nor the `exposure(varnamee)` option was specified when the model was fit, specifying `nooffset` does nothing.

`other_options` refers to command-specific options that are documented with each command.

## Remarks and examples

Remarks are presented under the following headings:

- Estimation-sample predictions*
- Out-of-sample predictions*
- Residuals*
- Single-equation (SE) models*
- SE model scores*
- Multiple-equation (ME) models*
- ME model scores*

Most of the examples are presented using linear regression, but the general syntax is applicable to all estimators.

You can think of any estimation command as estimating a set of coefficients  $b_1, b_2, \dots, b_k$  corresponding to the variables  $x_1, x_2, \dots, x_k$ , along with a (possibly empty) set of ancillary statistics  $\gamma_1, \gamma_2, \dots, \gamma_m$ . All estimation commands store the  $b_i$ s and  $\gamma_i$ s. `predict` accesses that stored information and combines it with the data currently in memory to make various calculations. For instance, `predict` can calculate the linear prediction,  $\hat{y}_j = b_1x_{1j} + b_2x_{2j} + \dots + b_kx_{kj}$ . The data on which `predict` makes the calculation can be the same data used to fit the model or a different dataset—it does not matter. `predict` uses the stored parameter estimates from the model, obtains the corresponding values of  $x$  for each observation in the data, and then combines them to produce the desired result.

## Estimation-sample predictions

### ▷ Example 1

We have a 74-observation dataset on automobiles, including the mileage rating (`mpg`), the car's weight (`weight`), and whether the car is foreign (`foreign`). We fit the model

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. regress mpg weight if foreign
```

| Source   | SS         | df | MS         | Number of obs | = | 22     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 427.990298 | 1  | 427.990298 | F(1, 20)      | = | 17.47  |
| Residual | 489.873338 | 20 | 24.4936669 | Prob > F      | = | 0.0005 |
|          |            |    |            | R-squared     | = | 0.4663 |
|          |            |    |            | Adj R-squared | = | 0.4396 |
| Total    | 917.863636 | 21 | 43.7077922 | Root MSE      | = | 4.9491 |

| mpg    | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |
|--------|-------------|-----------|-------|-------|----------------------|
| weight | -.010426    | .0024942  | -4.18 | 0.000 | -.0156287 -.0052232  |
| _cons  | 48.9183     | 5.871851  | 8.33  | 0.000 | 36.66983 61.16676    |

If we were to type `predict pmpg` now, we would obtain the linear predictions for all 74 observations. To obtain the predictions just for the sample on which we fit the model, we could type

```
. predict pmpg if e(sample)
(option xb assumed; fitted values)
(52 missing values generated)
```

Here `e(sample)` is true only for foreign cars because we typed `if foreign` when we fit the model and because there are no missing values among the relevant variables. If there had been missing values, `e(sample)` would also account for those.

By the way, the `if e(sample)` restriction can be used with any Stata command, so we could obtain summary statistics on the estimation sample by typing

```
. summarize if e(sample)
(output omitted)
```

## Out-of-sample predictions

By out-of-sample predictions, we mean predictions extending beyond the estimation sample. In the example above, typing `predict pmpg` would generate linear predictions using all 74 observations.

`predict` will work on other datasets, too. You can use a new dataset and type `predict` to obtain results for that sample.

### ► Example 2

Using the same auto dataset, assume that we wish to fit the model

$$\text{mpg} = \beta_1 \text{weight} + \beta_2 \ln(\text{weight}) + \beta_3 \text{foreign} + \beta_4$$

We first create the `ln(weight)` variable, and then type the `regress` command:

```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
. generate lnweight = ln(weight)
. regress mpg weight lnweight foreign
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 1690.27997 | 3  | 563.426657 | F(3, 70)      | = | 52.36  |
| Residual | 753.179489 | 70 | 10.759707  | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.6918 |
|          |            |    |            | Adj R-squared | = | 0.6785 |
| Total    | 2443.45946 | 73 | 33.4720474 | Root MSE      | = | 3.2802 |

| mpg      | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |
|----------|-------------|-----------|-------|-------|----------------------|
| weight   | .003304     | .0038995  | 0.85  | 0.400 | -0.0044734 .0110813  |
| lnweight | -29.59133   | 11.52018  | -2.57 | 0.012 | -52.5676 -6.615061   |
| foreign  | -2.125299   | 1.052324  | -2.02 | 0.047 | -4.224093 -.0265044  |
| _cons    | 248.0548    | 80.37079  | 3.09  | 0.003 | 87.76035 408.3493    |

If we typed `predict pmpg` now, we would obtain predictions for all 74 cars in the current data. Instead, we are going to use a new dataset.

`newautos.dta` contains the make, weight, and place of manufacture of two cars, the Pontiac Sunbird and the Volvo 260. Let's use the dataset and create the predictions:

```
. use https://www.stata-press.com/data/r19/newautos, clear
(New automobile models)
. list
```

|    | make          | weight | foreign  |
|----|---------------|--------|----------|
| 1. | Pont. Sunbird | 2690   | Domestic |
| 2. | Volvo 260     | 3170   | Foreign  |

```
. predict mpg
(option xb assumed; fitted values)
variable lnweight not found
r(111);
```

Things did not work. We typed `predict mpg`, and Stata responded with the message “variable `lnweight` not found”. `predict` can calculate predicted values on a different dataset only if that dataset contains the variables that went into the model. Here our dataset does not contain a variable called `lnweight`. `lnweight` is just the log of weight, so we can create it and try again:

```
. generate lnweight = ln(weight)
. predict mpg
(option xb assumed; fitted values)
. list
```

|    | make          | weight | foreign  | lnweight | mpg      |
|----|---------------|--------|----------|----------|----------|
| 1. | Pont. Sunbird | 2690   | Domestic | 7.897296 | 23.25097 |
| 2. | Volvo 260     | 3170   | Foreign  | 8.061487 | 17.85295 |

We obtained our predicted values. The Pontiac Sunbird has a predicted mileage rating of 23.3 mpg, whereas the Volvo 260 has a predicted rating of 17.9 mpg.



## Residuals

### ▷ Example 3

With many estimators, `predict` can calculate more than predicted values. With most regression-type estimators, we can, for instance, obtain residuals. Using our regression example, we return to our original data and obtain residuals by typing

```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
. generate lnweight = ln(weight)
. regress mpg weight lnweight foreign
(output omitted)
. predict double resid, residuals
. summarize resid
```

| Variable | Obs | Mean      | Std. dev. | Min       | Max      |
|----------|-----|-----------|-----------|-----------|----------|
| resid    | 74  | -1.31e-14 | 3.212091  | -5.453078 | 13.83719 |

We could do this without refitting the model. Stata always remembers the last set of estimates, even as we use new datasets.

It was not necessary to type the `double` in `predict double resid, residuals`, but we wanted to remind you that you can specify the type of a variable in front of the variable’s name; see [U] 11.4.2 Lists of new variables. We made the new variable `resid` a `double` rather than the default `float`.

If we had not specified `%double`, the mean of `resid` would have been roughly  $9 \times 10^{-9}$  rather than  $1 \times 10^{-14}$ . Although  $1 \times 10^{-14}$  sounds more precise than  $9 \times 10^{-9}$ , the difference really does not matter.



For linear regression, `predict` can also calculate standardized residuals and Studentized residuals with the options `rstandard` and `rstudent`; for examples, see [R] regress postestimation.

## Single-equation (SE) models

If you have not read the discussion above on using `predict` after linear regression, please do so. And `predict`'s default calculation almost always produces a statistic in the same metric as the dependent variable of the fitted model—for example, predicted counts for Poisson regression. In any case, `xb` can always be specified to obtain the linear prediction.

`predict` can calculate the standard error of the prediction, which is obtained by using the covariance matrix of the estimators.

### ▷ Example 4

After most binary outcome models (for example, `logistic`, `logit`, `probit`, `cloglog`, `scobit`), `predict` calculates the probability of a positive outcome if we do not tell it otherwise. We can specify the `xb` option if we want the linear prediction (also known as the logit or probit index). The odd abbreviation `xb` is meant to suggest  $\mathbf{x}\beta$ . In logit and probit models, for example, the predicted probability is  $p = F(\mathbf{x}\beta)$ , where  $F()$  is the logistic or normal cumulative distribution function, respectively.

```
. logistic foreign mpg weight
 (output omitted)
. predict phat
(option pr assumed; Pr(foreign))
. predict idxhat, xb
. summarize foreign phat idxhat
```

| Variable | Obs | Mean      | Std. dev. | Min       | Max      |
|----------|-----|-----------|-----------|-----------|----------|
| foreign  | 74  | .2972973  | .4601885  | 0         | 1        |
| phat     | 74  | .2972973  | .3052979  | .000729   | .8980594 |
| idxhat   | 74  | -1.678202 | 2.321509  | -7.223107 | 2.175845 |

Because this is a logit model, we could obtain the predicted probabilities ourselves from the predicted index

```
. generate phat2 = exp(idxhat)/(1+exp(idxhat))
```

but using `predict` without options is easier.



### ▷ Example 5

For all models, `predict` attempts to produce a predicted value in the same metric as the dependent variable of the model. We have seen that for dichotomous outcome models, the default statistic produced by `predict` is the probability of a success. Similarly, for Poisson regression, the default statistic produced by `predict` is the predicted count for the dependent variable. You can always specify the `xb` option to obtain the linear combination of the coefficients with an observation's  $x$  values (the inner product of the coefficients and  $x$  values). For `poisson` (without an explicit exposure), this is the natural log of the count.

```
. use https://www.stata-press.com/data/r19/airline, clear
. poisson injuries XYZowned
 (output omitted)
```

```
. predict injhat
(option n assumed; predicted number of events)
. predict idx, xb
. generate exp_idx = exp(idx)
. summarize injuries injhat exp_idx idx
```

| Variable | Obs | Mean     | Std. dev. | Min      | Max      |
|----------|-----|----------|-----------|----------|----------|
| injuries | 9   | 7.111111 | 5.487359  | 1        | 19       |
| injhat   | 9   | 7.111111 | .8333333  | 6        | 7.666667 |
| exp_idx  | 9   | 7.111111 | .8333333  | 6        | 7.666667 |
| idx      | 9   | 1.955174 | .1225612  | 1.791759 | 2.036882 |

We note that our “hand-computed” prediction of the count (`exp_idx`) matches what was produced by the default operation of `predict`.

If our model has an exposure-time variable, we can use `predict` to obtain the linear prediction with or without the exposure. Let’s verify what we are getting by obtaining the linear prediction with and without exposure, transforming these predictions to count predictions and comparing them with the default count prediction from `predict`. We must remember to multiply by the exposure time when using `predict` . . . , `nooffset`.

```
. use https://www.stata-press.com/data/r19/airline, clear
. poisson injuries XYZowned, exposure(n)
(output omitted)
. predict double injhat
(option n assumed; predicted number of events)
. predict double idx, xb
. generate double exp_idx = exp(idx)
. predict double idxn, xb nooffset
. generate double exp_idxn = exp(idxn)*n
. summarize injuries injhat exp_idx exp_idxn idx idxn
```

| Variable | Obs | Mean     | Std. dev. | Min      | Max      |
|----------|-----|----------|-----------|----------|----------|
| injuries | 9   | 7.111111 | 5.487359  | 1        | 19       |
| injhat   | 9   | 7.111111 | 3.10936   | 2.919621 | 12.06158 |
| exp_idx  | 9   | 7.111111 | 3.10936   | 2.919621 | 12.06158 |
| exp_idxn | 9   | 7.111111 | 3.10936   | 2.919621 | 12.06158 |
| idx      | 9   | 1.869722 | .4671044  | 1.071454 | 2.490025 |
| idxn     | 9   | 4.18814  | .1904042  | 4.061204 | 4.442013 |

Looking at the identical means and standard deviations for `injhat`, `exp_idx`, and `exp_idxn`, we see that we can reproduce the default computations of `predict` for `poisson` estimations. We have also demonstrated the relationship between the count predictions and the linear predictions with and without exposure.



## SE model scores

### ▷ Example 6

With most maximum likelihood estimators, `predict` can calculate equation-level scores. The first derivative of the log likelihood with respect to  $\mathbf{x}_j\beta$  is the equation-level score.

```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
. logistic foreign mpg weight
(output omitted)
. predict double sc, score
. summarize sc
```

| Variable | Obs | Mean      | Std. dev. | Min       | Max      |
|----------|-----|-----------|-----------|-----------|----------|
| sc       | 74  | -1.37e-12 | .3533133  | -.8760856 | .8821309 |

See [P] [\\_robust](#) and [SVY] [Variance estimation](#) for details regarding the role equation-level scores play in linearization-based variance estimators.



### □ Technical note

`predict` after some estimation commands, such as `regress` and `cnsreg`, allows the `score` option as a synonym for the `residuals` option.



## Multiple-equation (ME) models

If you have not read the above discussion on using `predict` after SE models, please do so. With the exception of the ability to select specific equations to predict from, the use of `predict` after ME models follows almost the same form that it does for SE models.

### ▷ Example 7

The details of prediction statistics that are specific to particular ME models are documented with the estimation command. If you are using ME commands that do not have separate discussions on obtaining predictions, read [Obtaining predicted values](#) in [R] [mlogit postestimation](#), even if your interest is not in multinomial logistic regression. As a general introduction to the ME models, we will demonstrate `predict` after `sureg`:

```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
. sureg (price foreign displ) (weight foreign length)
Seemingly unrelated regression
```

| Equation | Obs | Params | RMSE     | "R-squared" | chi2   | P>chi2 |
|----------|-----|--------|----------|-------------|--------|--------|
| price    | 74  | 2      | 2202.447 | 0.4348      | 45.21  | 0.0000 |
| weight   | 74  | 2      | 245.5238 | 0.8988      | 658.85 | 0.0000 |

|              | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |
|--------------|-------------|-----------|-------|-------|----------------------|
| price        |             |           |       |       |                      |
| foreign      | 3137.894    | 697.3805  | 4.50  | 0.000 | 1771.054 4504.735    |
| displacement | 23.06938    | 3.443212  | 6.70  | 0.000 | 16.32081 29.81795    |
| _cons        | 680.8438    | 859.8142  | 0.79  | 0.428 | -1004.361 2366.049   |
| weight       |             |           |       |       |                      |
| foreign      | -154.883    | 75.3204   | -2.06 | 0.040 | -302.5082 -7.257674  |
| length       | 30.67594    | 1.531981  | 20.02 | 0.000 | 27.67331 33.67856    |
| _cons        | -2699.498   | 302.3912  | -8.93 | 0.000 | -3292.173 -2106.822  |

sureg estimated two equations, one called price and the other weight; see [R] sureg.

```
. predict pred_p, equation(price)
(option xb assumed; fitted values)
. predict pred_w, equation(weight)
(option xb assumed; fitted values)
. summarize price pred_p weight pred_w
```

| Variable | Obs | Mean     | Std. dev. | Min      | Max      |
|----------|-----|----------|-----------|----------|----------|
| price    | 74  | 6165.257 | 2949.496  | 3291     | 15906    |
| pred_p   | 74  | 6165.257 | 1678.805  | 2664.81  | 10485.33 |
| weight   | 74  | 3019.459 | 777.1936  | 1760     | 4840     |
| pred_w   | 74  | 3019.459 | 726.0468  | 1501.602 | 4447.996 |

You may specify the equation by name, as we did above, or by number: equation(#1) means the same thing as equation(price) in this case.



## ME model scores

### ▷ Example 8

For ME models, `predict` allows you to specify a stub when generating equation-level score variables. `predict` generates new variables using this stub by appending an equation index. Depending upon the command, the index will start with 0 or 1. Here is an example where `predict` starts indexing the score variables with 0.

```
. ologit rep78 mpg weight
(output omitted)
. predict double sc*, scores
. summarize sc*
```

| Variable | Obs | Mean      | Std. dev. | Min       | Max      |
|----------|-----|-----------|-----------|-----------|----------|
| sc0      | 69  | -1.33e-11 | .5337363  | -.9854088 | .921433  |
| sc1      | 69  | -7.69e-13 | .186919   | -.2738537 | .9854088 |
| sc2      | 69  | -2.87e-11 | .4061637  | -.5188487 | 1.130178 |
| sc3      | 69  | -1.04e-10 | .5315368  | -1.067351 | .8194842 |
| sc4      | 69  | 1.47e-10  | .360525   | -.921433  | .6140182 |

Although it involves much more typing, we could also specify the new variable names individually.

```
. predict double (sc_xb sc_1 sc_2 sc_3 sc_4), scores
. summarize sc_*
```

| Variable | Obs | Mean      | Std. dev. | Min       | Max      |
|----------|-----|-----------|-----------|-----------|----------|
| sc_xb    | 69  | -1.33e-11 | .5337363  | -.9854088 | .921433  |
| sc_1     | 69  | -7.69e-13 | .186919   | -.2738537 | .9854088 |
| sc_2     | 69  | -2.87e-11 | .4061637  | -.5188487 | 1.130178 |
| sc_3     | 69  | -1.04e-10 | .5315368  | -1.067351 | .8194842 |
| sc_4     | 69  | 1.47e-10  | .360525   | -.921433  | .6140182 |

◀

## Methods and formulas

Denote the previously estimated coefficient vector as  $\mathbf{b}$  and its estimated variance matrix as  $\mathbf{V}$ . `predict` works by recalling various aspects of the model, such as  $\mathbf{b}$ , and combining that information with the data currently in memory. Let's write  $\mathbf{x}_j$  for the  $j$ th observation currently in memory.

The *predicted value* (`xb` option) is defined as  $\hat{y}_j = \mathbf{x}_j \mathbf{b} + \text{offset}_j$

The *standard error of the prediction* (the `stdp` option) is defined as  $s_{pj} = \sqrt{\mathbf{x}_j \mathbf{V} \mathbf{x}_j'}$

The *standard error of the difference in linear predictions* between equations 1 and 2 is defined as

$$s_{dpj} = \{(\mathbf{x}_{1j}, -\mathbf{x}_{2j}, \mathbf{0}, \dots, \mathbf{0}) \mathbf{V} (\mathbf{x}_{1j}, -\mathbf{x}_{2j}, \mathbf{0}, \dots, \mathbf{0})'\}^{\frac{1}{2}}$$

See the individual estimation commands for information about calculating command-specific `predict` statistics.

## References

- Huber, C. 2013a. Multilevel linear models in Stata, part 1: Components of variance. *The Stata Blog: Not Elsewhere Classified*. <https://blog.stata.com/2013/02/04/multilevel-linear-models-in-stata-part-1-components-of-variance/>.
- . 2013b. Multilevel linear models in Stata, part 2: Longitudinal data. *The Stata Blog: Not Elsewhere Classified*. <https://blog.stata.com/2013/02/18/multilevel-linear-models-in-stata-part-2-longitudinal-data/>.
- Skeels, C. L., and L. W. Taylor. 2015. Prediction in linear index models with endogenous regressors. *Stata Journal* 15: 627–644.

## Also see

- [R] **predictnl** — Obtain nonlinear predictions, standard errors, etc., after estimation
- [P] **\_predict** — Obtain predictions, residuals, etc., after estimation programming command
- [U] **20 Estimation and postestimation commands**

|             |                      |                      |            |
|-------------|----------------------|----------------------|------------|
| Description | Quick start          | Menu                 | Syntax     |
| Options     | Remarks and examples | Methods and formulas | References |
| Also see    |                      |                      |            |

## Description

`predictnl` calculates (possibly) nonlinear predictions after any Stata estimation command and optionally calculates the variances, standard errors, Wald test statistics,  $p$ -values, and confidence limits for these predictions. Unlike its companion nonlinear postestimation commands `testnl` and `nlcom`, `predictnl` generates functions of the data (that is, predictions), not scalars. The quantities generated by `predictnl` are thus vectorized over the observations in the data.

Consider some general prediction,  $g(\boldsymbol{\theta}, \mathbf{x}_i)$ , for  $i = 1, \dots, n$ , where  $\boldsymbol{\theta}$  are the model parameters and  $\mathbf{x}_i$  are some data for the  $i$ th observation;  $\mathbf{x}_i$  is assumed fixed. Typically,  $g(\boldsymbol{\theta}, \mathbf{x}_i)$  is estimated by  $g(\hat{\boldsymbol{\theta}}, \mathbf{x}_i)$ , where  $\hat{\boldsymbol{\theta}}$  are the estimated model parameters, which are stored in `e(b)` following any Stata estimation command.

In its most common use, `predictnl` generates two variables: one containing the estimated prediction,  $g(\hat{\boldsymbol{\theta}}, \mathbf{x}_i)$ , the other containing the estimated standard error of  $g(\hat{\boldsymbol{\theta}}, \mathbf{x}_i)$ . The calculation of standard errors (and other obtainable quantities that are based on the standard errors, such as test statistics) is based on the delta method, an approximation appropriate in large samples; see [Methods and formulas](#).

`predictnl` can be used with `svy` estimation results (assuming that `predict` is also allowed), see [\[SVY\] svy postestimation](#).

The specification of  $g(\hat{\boldsymbol{\theta}}, \mathbf{x}_i)$  is handled by specifying `pnl_exp`, and the values of  $g(\hat{\boldsymbol{\theta}}, \mathbf{x}_i)$  are stored in the new variable `newvar` of storage type `type`. `pnl_exp` is any valid Stata expression and may also contain calls to two special functions unique to `predictnl`:

1. `predict([predict_options])`: When you are evaluating `pnl_exp`, `predict()` is a convenience function that replicates the calculation performed by the command

```
predict ..., predict_options
```

As such, the `predict()` function may be used either as a shorthand for the formula used to make this prediction or when the formula is not readily available. When used without arguments, `predict()` replicates the default prediction for that particular estimation command.

2. `xb([eqno])`: The `xb()` function replicates the calculation of the linear predictor  $\mathbf{x}_i\mathbf{b}$  for equation `eqno`. If `xb()` is specified without `eqno`, the linear predictor for the first equation (or the only equation in single-equation estimation) is obtained.

For example, `xb(#1)` (or equivalently, `xb()` with no arguments) translates to the linear predictor for the first equation, `xb(#2)` for the second, and so on. You could also refer to the equations by their names, such as `xb(income)`.

When specifying `pnl_exp`, both of these functions may be used repeatedly, in combination, and in combination with other Stata functions and expressions. See [Remarks and examples](#) for examples that use both of these functions.

## Quick start

After `regress`, create `yhat` containing the default linear prediction

```
predictnl yhat = predict()
```

After `probit y x1 x2`, create `pr1` containing predicted probability that  $y = 1$

```
predictnl pr1 = predict(pr)
```

Same as above, and create `lb1` and `ub1` containing the upper and lower bounds of the 95% confidence interval for the prediction

```
predictnl pr1 = predict(pr), ci(lb1 ub1)
```

Same as above, specified as a function of the linear prediction

```
predictnl pr1 = normal(xb()), ci(lb1 ub1)
```

Same as above, specified as a function of the coefficients

```
predictnl pr1 = normal(_b[_cons]+_b[x1]*x1+_b[x2]*x2), ci(lb1 ub1)
```

Same as above, but create the prediction using `x1` equal to 0

```
predictnl pr1 = normal(_b[_cons]+_b[x1]*0+_b[x2]*x2), ci(lb1 ub1)
```

Same as above, but create variable `se1` containing the standard error of the prediction

```
predictnl pr1 = normal(_b[_cons]+_b[x1]*0+_b[x2]*x2), se(se1)
```

After a multiple-equation model, create `ratio` as the ratio of the linear predictions from the second and third equations

```
predictnl ratio = xb(#2)/xb(#3)
```

## Menu

Statistics > Postestimation

## Syntax

```
predictnl [type] newvar = pnl_exp [if] [in] [, options]
```

| options                   | Description                                                                                                          |
|---------------------------|----------------------------------------------------------------------------------------------------------------------|
| <b>Main</b>               |                                                                                                                      |
| se( <i>newvar</i> )       | create <i>newvar</i> containing standard errors                                                                      |
| variance( <i>newvar</i> ) | create <i>newvar</i> containing variances                                                                            |
| wald( <i>newvar</i> )     | create <i>newvar</i> containing the Wald test statistic                                                              |
| p( <i>newvar</i> )        | create <i>newvar</i> containing the <i>p</i> -value for the Wald test                                                |
| ci( <i>newvars</i> )      | create <i>newvars</i> containing lower and upper confidence intervals                                                |
| level(#)                  | set confidence level; default is level(95)                                                                           |
| g( <i>stub</i> )          | create <i>stub1</i> , <i>stub2</i> , ..., <i>stubk</i> variables containing observation-specific derivatives         |
| <b>Advanced</b>           |                                                                                                                      |
| iterate(#)                | maximum iterations for finding optimal step size; default is 100                                                     |
| force                     | calculate standard errors, etc., even when possibly inappropriate                                                    |
| df(#)                     | use <i>F</i> distribution with # denominator degrees of freedom for the reference distribution of the test statistic |

df(#) does not appear in the dialog box.

## Options

### Main

se(*newvar*) adds *newvar* of storage type *type*, where for each *i* in the prediction sample, *newvar*[*i*] contains the estimated standard error of  $g(\hat{\theta}, \mathbf{x}_i)$ .

variance(*newvar*) adds *newvar* of storage type *type*, where for each *i* in the prediction sample, *newvar*[*i*] contains the estimated variance of  $g(\hat{\theta}, \mathbf{x}_i)$ .

wald(*newvar*) adds *newvar* of storage type *type*, where for each *i* in the prediction sample, *newvar*[*i*] contains the Wald test statistic for the test of the hypothesis  $H_0: g(\theta, \mathbf{x}_i) = 0$ .

p(*newvar*) adds *newvar* of storage type *type*, where *newvar*[*i*] contains the *p*-value for the Wald test of  $H_0: g(\theta, \mathbf{x}_i) = 0$  versus the two-sided alternative.

ci(*newvars*) requires the specification of two *newvars*, such that the *i*th observation of each will contain the left and right endpoints (respectively) of a confidence interval for  $g(\theta, \mathbf{x}_i)$ . The level of the confidence intervals is determined by level(#).

level(#) specifies the confidence level, as a percentage, for confidence intervals. The default is level(95) or as set by set level; see [\[U\] 20.8 Specifying the width of confidence intervals](#).

g(*stub*) specifies that new variables, *stub1*, *stub2*, ..., *stubk* be created, where *k* is the dimension of  $\theta$ . *stub1* will contain the observation-specific derivatives of  $g(\theta, \mathbf{x}_i)$  with respect to the first element,  $\theta_1$ , of  $\theta$ ; *stub2* will contain the derivatives of  $g(\theta, \mathbf{x}_i)$  with respect to  $\theta_2$ , etc.; If the derivative of  $g(\theta, \mathbf{x}_i)$  with respect to a particular coefficient in  $\theta$  equals zero for all observations in the prediction sample, the *stub* variable for that coefficient is not created. The ordering of the parameters in  $\theta$  is precisely that of the stored vector of parameter estimates  $\mathbf{e}(b)$ .

Advanced

`iterate(#)` specifies the maximum number of iterations used to find the optimal step size in the calculation of numerical derivatives of  $g(\boldsymbol{\theta}, \mathbf{x}_i)$  with respect to  $\boldsymbol{\theta}$ . By default, the maximum number of iterations is 100, but convergence is usually achieved after only a few iterations. You should rarely have to use this option.

`force` forces the calculation of standard errors and other inference-related quantities in situations where `predictnl` would otherwise refuse to do so. The calculation of standard errors takes place by evaluating (at  $\hat{\boldsymbol{\theta}}$ ) the numerical derivative of  $g(\boldsymbol{\theta}, \mathbf{x}_i)$  with respect to  $\boldsymbol{\theta}$ . If `predictnl` detects that  $g(\cdot)$  is possibly a function of random quantities other than  $\hat{\boldsymbol{\theta}}$ , it will refuse to calculate standard errors or any other quantity derived from them. The `force` option forces the calculation to take place anyway. If you use the `force` option, there is no guarantee that any inference quantities (for example, standard errors) will be correct or that the values obtained can be interpreted.

The following option is available with `predictnl` but is not shown in the dialog box:

`df(#)` specifies that the  $F$  distribution with # denominator degrees of freedom be used for the reference distribution of the test statistic.

## Remarks and examples

Remarks are presented under the following headings:

- [Introduction](#)
- [Nonlinear transformations and standard errors](#)
- [Using `xb\(\)` and `predict\(\)`](#)
- [Multiple-equation \(ME\) estimators](#)
- [Test statistics and  \$p\$ -values](#)
- [Manipulability](#)
- [Confidence intervals](#)

## Introduction

`predictnl` and `nlcom` both use the delta method. They take a nonlinear transformation of the estimated parameter vector from some fitted model and apply the delta method to calculate the variance, standard error, Wald test statistic, etc., of this transformation. `nlcom` is designed for scalar functions of the parameters, and `predictnl` is designed for functions of the parameters and of the data, that is, for predictions.

## Nonlinear transformations and standard errors

We begin by fitting a probit model to the low-birthweight data of Hosmer, Lemeshow, and Sturdivant (2013, 24). The data are described in detail in [example 1](#) of [\[R\] logistic](#).

```
. use https://www.stata-press.com/data/r19/lbw
(Hosmer & Lemeshow data)
. probit low lwt smoke ptl ht
Iteration 0: Log likelihood = -117.336
Iteration 1: Log likelihood = -106.75886
Iteration 2: Log likelihood = -106.67852
Iteration 3: Log likelihood = -106.67851
Probit regression
Log likelihood = -106.67851
Number of obs = 189
LR chi2(4) = 21.31
Prob > chi2 = 0.0003
Pseudo R2 = 0.0908
```

|  | low   | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|--|-------|-------------|-----------|-------|-------|----------------------|-----------|
|  | lwt   | -.0095164   | .0036875  | -2.58 | 0.010 | -.0167438            | -.0022891 |
|  | smoke | .3487004    | .2041772  | 1.71  | 0.088 | -.0514794            | .7488803  |
|  | ptl   | .365667     | .1921201  | 1.90  | 0.057 | -.0108815            | .7422154  |
|  | ht    | 1.082355    | .410673   | 2.64  | 0.008 | .2774503             | 1.887259  |
|  | _cons | .4238985    | .4823224  | 0.88  | 0.379 | -.5214361            | 1.369233  |

After we fit such a model, we first would want to generate the predicted probabilities of a low birthweight, given the covariate values in the estimation sample. This is easily done using `predict` after `probit`, but it doesn't answer the question, "What are the standard errors of those predictions?"

For the time being, we will consider ourselves ignorant of any automated way to obtain the predicted probabilities after `probit`. The formula for the prediction is

$$\Pr(y \neq 0 | \mathbf{x}_i) = \Phi(\mathbf{x}_i \boldsymbol{\beta})$$

where  $\Phi$  is the standard cumulative normal. Thus for this example,  $g(\boldsymbol{\theta}, \mathbf{x}_i) = \Phi(\mathbf{x}_i \boldsymbol{\beta})$ . Armed with the formula, we can use `predictnl` to generate the predictions and their standard errors:

```
. predictnl phat = normal(_b[_cons] + _b[ht]*ht + _b[ptl]*ptl +
> _b[smoke]*smoke + _b[lwt]*lwt), se(phat_se)
. list phat phat_se lwt smoke ptl ht in -10/1
```

|      | phat     | phat_se  | lwt | smoke     | ptl | ht |
|------|----------|----------|-----|-----------|-----|----|
| 180. | .2363556 | .042707  | 120 | Nonsmoker | 0   | 0  |
| 181. | .6577712 | .1580714 | 154 | Nonsmoker | 1   | 1  |
| 182. | .2793261 | .0519958 | 106 | Nonsmoker | 0   | 0  |
| 183. | .1502118 | .0676339 | 190 | Smoker    | 0   | 0  |
| 184. | .5702871 | .0819911 | 101 | Smoker    | 1   | 0  |
| 185. | .4477045 | .079889  | 95  | Smoker    | 0   | 0  |
| 186. | .2988379 | .0576306 | 100 | Nonsmoker | 0   | 0  |
| 187. | .4514706 | .080815  | 94  | Smoker    | 0   | 0  |
| 188. | .5615571 | .1551051 | 142 | Nonsmoker | 0   | 1  |
| 189. | .7316517 | .1361469 | 130 | Smoker    | 0   | 1  |

Thus, subject 180 in our data has an estimated probability of low birthweight of 23.6% with standard error 4.3%.

Used without options, `predictnl` is not much different from `generate`. By specifying the `se(phat_se)` option, we were able to obtain a variable containing the standard errors of the predictions; therein lies the utility of `predictnl`.

## Using `xb()` and `predict()`

As was the case above, a prediction is often not a function of a few isolated parameters and their corresponding variables but instead is some (possibly elaborate) function of the entire linear predictor. For models with many predictors, the brute-force expression for the linear predictor can be cumbersome to type. An alternative is to use the inline function `xb()`. `xb()` is a shortcut for having to type `_b[_cons] + _b[ht]*ht + _b[ptl]*ptl + ...`,

```
. drop phat phat_se
. predictnl phat = norm(xb()), se(phat_se)
. list phat phat_se lwt smoke ptl ht in -10/1
```

|      | phat     | phat_se  | lwt | smoke     | ptl | ht |
|------|----------|----------|-----|-----------|-----|----|
| 180. | .2363556 | .042707  | 120 | Nonsmoker | 0   | 0  |
| 181. | .6577712 | .1580714 | 154 | Nonsmoker | 1   | 1  |
| 182. | .2793261 | .0519958 | 106 | Nonsmoker | 0   | 0  |
| 183. | .1502118 | .0676339 | 190 | Smoker    | 0   | 0  |
| 184. | .5702871 | .0819911 | 101 | Smoker    | 1   | 0  |
| 185. | .4477045 | .079889  | 95  | Smoker    | 0   | 0  |
| 186. | .2988379 | .0576306 | 100 | Nonsmoker | 0   | 0  |
| 187. | .4514706 | .080815  | 94  | Smoker    | 0   | 0  |
| 188. | .5615571 | .1551051 | 142 | Nonsmoker | 0   | 1  |
| 189. | .7316517 | .1361469 | 130 | Smoker    | 0   | 1  |

which yields the same results. This approach is easier, produces more readable code, and is less prone to error, such as forgetting to include a term in the sum.

Here we used `xb()` without arguments because we have only one equation in our model. In multiple-equation (ME) settings, `xb()` (or equivalently `xb(#1)`) yields the linear predictor from the first equation, `xb(#2)` from the second, etc. You can also refer to equations by their names, for example, `xb(income)`.

### □ Technical note

Most estimation commands in Stata allow the postestimation calculation of linear predictors and their standard errors via `predict`. For example, to obtain these for the first (or only) equation in the model, you could type

```
predict xbvar, xb
 predict stdpvar, stdp
```

Equivalently, you could type

```
predictnl xbvar = xb(), se(stdpvar)
```

but we recommend the first method, because it is faster. As we demonstrated above, however, `predictnl` is more general.

□

Returning to our probit example, we can further simplify the calculation by using the inline function `predict()`. `predict(pred_options)` works by substituting, within our `predictnl` expression, the calculation performed by

```
predict ..., pred_options
```

In our example, we are interested in the predicted probabilities after a probit regression, normally obtained via

```
predict ..., p
```

We can obtain these predictions (and standard errors) by using

```
. drop phat phat_se
. predictnl phat = predict(p), se(phat_se)
. list phat phat_se lwt smoke ptl ht in -10/1
```

|      | phat     | phat_se  | lwt | smoke     | ptl | ht |
|------|----------|----------|-----|-----------|-----|----|
| 180. | .2363556 | .042707  | 120 | Nonsmoker | 0   | 0  |
| 181. | .6577712 | .1580714 | 154 | Nonsmoker | 1   | 1  |
| 182. | .2793261 | .0519958 | 106 | Nonsmoker | 0   | 0  |
| 183. | .1502118 | .0676339 | 190 | Smoker    | 0   | 0  |
| 184. | .5702871 | .0819911 | 101 | Smoker    | 1   | 0  |
| 185. | .4477045 | .079889  | 95  | Smoker    | 0   | 0  |
| 186. | .2988379 | .0576306 | 100 | Nonsmoker | 0   | 0  |
| 187. | .4514706 | .080815  | 94  | Smoker    | 0   | 0  |
| 188. | .5615571 | .1551051 | 142 | Nonsmoker | 0   | 1  |
| 189. | .7316517 | .1361469 | 130 | Smoker    | 0   | 1  |

which again replicates what we have already done by other means. However, this version did not require knowledge of the formula for the predicted probabilities after a probit regression—`predict(p)` took care of that for us.

Because the predicted probability is the default prediction after `probit`, we could have just used `predict()` without arguments, namely,

```
. predictnl phat = predict(), se(phat_se)
```

Also, the expression `pnl_exp` can be inordinately complicated, with multiple calls to `predict()` and `xb()`. For example,

```
. predictnl phat = normal(invnormal(predict()) + predict(xb)/xb() - 1),
> se(phat_se)
```

is perfectly valid and will give the same result as before, albeit a bit inefficiently.

### □ Technical note

When using `predict()` and `xb()`, the *formula* for the calculation is substituted within `pnl_exp`, not the values that result from the application of that formula. To see this, note the subtle difference between

```
. predict xbeta, xb
. predictnl phat = normal(xbeta), se(phat_se)
```

and

```
. predictnl phat = normal(xb()), se(phat_se)
```

Both sequences will yield the same `phat`, yet for the first sequence, `phat_se` will equal zero for all observations. The reason is that, once evaluated, `xbeta` will contain the values of the linear predictor, yet these values are treated as fixed and nonstochastic as far as `predictnl` is concerned. By contrast, because `xb()` is shorthand for the formula used to calculate the linear predictor, it contains not values, but references to the estimated regression coefficients and corresponding variables. Thus, the second method produces the desired result. □

### Multiple-equation (ME) estimators

In [R] [mlogit](#), data on insurance choice (Tarlov et al. 1989; Wells et al. 1989) were examined, and a multinomial logit was used to assess the effects of age, gender, race, and site of study (one of three sites) on the type of insurance:

```
. use https://www.stata-press.com/data/r19/sysdsn1, clear
(Health insurance data)
. mlogit insure age male nonwhite i.site, nolog

Multinomial logistic regression Number of obs = 615
 LR chi2(10) = 42.99
 Prob > chi2 = 0.0000
 Pseudo R2 = 0.0387

Log likelihood = -534.36165
```

| insure    | Coefficient    | Std. err. | z     | P> z  | [95% conf. interval] |           |
|-----------|----------------|-----------|-------|-------|----------------------|-----------|
| Indemnity | (base outcome) |           |       |       |                      |           |
| Prepaid   |                |           |       |       |                      |           |
| age       | -.011745       | .0061946  | -1.90 | 0.058 | -.0238862            | .0003962  |
| male      | .5616934       | .2027465  | 2.77  | 0.006 | .1643175             | .9590693  |
| nonwhite  | .9747768       | .2363213  | 4.12  | 0.000 | .5115955             | 1.437958  |
| site      |                |           |       |       |                      |           |
| 2         | .1130359       | .2101903  | 0.54  | 0.591 | -.2989296            | .5250013  |
| 3         | -.5879879      | .2279351  | -2.58 | 0.010 | -1.034733            | -.1412433 |
| _cons     | .2697127       | .3284422  | 0.82  | 0.412 | -.3740222            | .9134476  |
| Uninsure  |                |           |       |       |                      |           |
| age       | -.0077961      | .0114418  | -0.68 | 0.496 | -.0302217            | .0146294  |
| male      | .4518496       | .3674867  | 1.23  | 0.219 | -.268411             | 1.17211   |
| nonwhite  | .2170589       | .4256361  | 0.51  | 0.610 | -.6171725            | 1.05129   |
| site      |                |           |       |       |                      |           |
| 2         | -1.211563      | .4705127  | -2.57 | 0.010 | -2.133751            | -.2893747 |
| 3         | -.2078123      | .3662926  | -0.57 | 0.570 | -.9257327            | .510108   |
| _cons     | -1.286943      | .5923219  | -2.17 | 0.030 | -2.447872            | -.1260134 |

Of particular interest is the estimation of the relative risk, which, for a given selection, is the ratio of the probability of making that selection to the probability of selecting the base category (Indemnity here), given a set of covariate values. In a multinomial logit model, the relative risk (when comparing to the base category) simplifies to the exponentiated linear predictor for that selection.

Using this example, we can estimate the observation-specific relative risks of selecting a prepaid plan over the base category (with standard errors) by either referring to the Prepaid equation by name or number,

```
. predictnl RRppaid = exp(xb(Prepaid)), se(SERRppaid)
```

or

```
. predictnl RRppaid = exp(xb(#1)), se(SERRppaid)
```

because Prepaid is the first equation in the model.

Those of us for whom the simplified formula for the relative risk does not immediately come to mind may prefer to calculate the relative risk directly from its definition, that is, as a ratio of two predicted probabilities. After `mlogit`, the predicted probability for a category may be obtained using `predict`, but we must specify the category as the outcome:

```
. predictnl RRppaid = predict(outcome(Prepaid))/predict(outcome(Indemnity)),
> se(SERRppaid)
(1 missing value generated)
. list RRppaid SERRppaid age male nonwhite site in 1/10
```

|     | RRppaid  | SERRpp-d | age       | male | nonwhite | site |
|-----|----------|----------|-----------|------|----------|------|
| 1.  | .6168578 | .1503759 | 73.722107 | 0    | 0        | 2    |
| 2.  | 1.056658 | .1790703 | 27.89595  | 0    | 0        | 2    |
| 3.  | .8426442 | .1511281 | 37.541397 | 0    | 0        | 1    |
| 4.  | 1.460581 | .3671465 | 23.641327 | 0    | 1        | 3    |
| 5.  | .9115747 | .1324168 | 40.470901 | 0    | 0        | 2    |
| 6.  | 1.034701 | .1696923 | 29.683777 | 0    | 0        | 2    |
| 7.  | .9223664 | .1344981 | 39.468857 | 0    | 0        | 2    |
| 8.  | 1.678312 | .4216626 | 26.702255 | 1    | 0        | 1    |
| 9.  | .9188519 | .2256017 | 63.101974 | 0    | 1        | 3    |
| 10. | .5766296 | .1334877 | 69.839828 | 0    | 0        | 1    |

The “(1 missing value generated)” message is not an error; further examination of the data would reveal that `age` is missing in one observation and that the offending observation (among others) is not in the estimation sample. Just as with `predict`, `predictnl` can generate predictions in or out of the estimation sample.

Thus, we estimate (among other things) that a white, female, 73-year-old from site 2 is less likely to choose a prepaid plan over an indemnity plan—her relative risk is about 62% with standard error 15%.

### Test statistics and p-values

Often, a standard error calculation is just a means to an end, and what is really desired is a test of the hypothesis,

$$H_0 : g(\boldsymbol{\theta}, \mathbf{x}_i) = 0$$

versus the two-sided alternative.

We can use `predictnl` to obtain the Wald test statistics or *p*-values (or both) for the above tests, whether or not we want standard errors. To obtain the Wald test statistics, we use the `wald()` option; for *p*-values, we use `p()`.

Returning to our `mlogit` example, suppose that we wanted for each observation a test of whether the relative risk of choosing a prepaid plan over an indemnity plan is different from one. One way to do this would be to define  $g(\cdot)$  to be the relative risk minus one and then test whether  $g(\cdot)$  is different from zero.

```
. predictnl RRm1 = exp(xb(Prepaid)) - 1, wald(W_RRm1) p(sig_RRm1)
(1 missing value generated)
note: p-values are with respect to the chi-squared(1) distribution.
. list RRm1 W_RRm1 sig_RRm1 age male nonwhite in 1/10
```

|     | RRm1      | W_RRm1   | sig_RRm1 | age       | male | nonwhite |
|-----|-----------|----------|----------|-----------|------|----------|
| 1.  | -.3831422 | 6.491778 | .0108375 | 73.722107 | 0    | 0        |
| 2.  | .0566578  | .100109  | .7516989 | 27.89595  | 0    | 0        |
| 3.  | -.1573559 | 1.084116 | .2977787 | 37.541397 | 0    | 0        |
| 4.  | .4605812  | 1.573743 | .2096643 | 23.641327 | 0    | 1        |
| 5.  | -.0884253 | .4459299 | .5042742 | 40.470901 | 0    | 0        |
| 6.  | .0347015  | .0418188 | .8379655 | 29.683777 | 0    | 0        |
| 7.  | -.0776336 | .3331707 | .563798  | 39.468857 | 0    | 0        |
| 8.  | .6783119  | 2.587788 | .1076906 | 26.702255 | 1    | 0        |
| 9.  | -.0811482 | .1293816 | .719074  | 63.101974 | 0    | 1        |
| 10. | -.4233705 | 10.05909 | .001516  | 69.839828 | 0    | 0        |

The newly created variable `W_RRm1` contains the Wald test statistic for each observation, and `sig_RRm1` contains the  $p$ -value. Thus, our 73-year-old white female represented by the first observation would have a relative risk of choosing prepaid over indemnity that is significantly different from 1, at least at the 5% level. For this test, it was not necessary to generate a variable containing the standard error of the relative risk minus 1, but we could have done so had we wanted. We could have also omitted specifying `wald(W_RRm1)` if all we cared about were, say, the  $p$ -values for the tests.

In this regard, `predictnl` acts as an observation-specific version of `testnl`, with the test results vectorized over the observations in the data. The  $p$ -values are pointwise—they are not adjusted to reflect any simultaneous testing over the observations in the data.

## Manipulability

There are many ways to specify  $g(\boldsymbol{\theta}, \mathbf{x}_i)$  to yield tests such that, for multiple specifications of  $g(\cdot)$ , the theoretical conditions for which

$$H_0 : g(\boldsymbol{\theta}, \mathbf{x}_i) = 0$$

is true will be equivalent. However, this does not mean that the tests themselves will be equivalent. This is known as the manipulability of the Wald test for nonlinear hypotheses; also see [R] [boxcox](#).

As an example, consider the previous section where we defined  $g(\cdot)$  to be the relative risk between choosing a prepaid plan over an indemnity plan, minus 1. We could also have defined  $g(\cdot)$  to be the risk difference—the probability of choosing a prepaid plan minus the probability of choosing an indemnity plan. Either specification of  $g(\cdot)$  yields a mathematically equivalent specification of  $H_0 : g(\cdot) = 0$ ; that is, the risk difference will equal zero when the relative risk equals one. However, the tests themselves do not give the same results:

```
. predictnl RD = predict(outcome(Prepaid)) - predict(outcome(Indemnity)),
> wald(W_RD) p(sig_RD)
(1 missing value generated)
note: p-values are with respect to the chi-squared(1) distribution.
. list RD W_RD sig_RD RRm1 W_RRm1 sig_RRm1 in 1/10
```

|     | RD        | W_RD     | sig_RD   | RRm1      | W_RRm1   | sig_RRm1 |
|-----|-----------|----------|----------|-----------|----------|----------|
| 1.  | -.2303744 | 4.230243 | .0397097 | -.3831422 | 6.491778 | .0108375 |
| 2.  | .0266902  | .1058542 | .7449144 | .0566578  | .100109  | .7516989 |
| 3.  | -.0768078 | .9187646 | .3377995 | -.1573559 | 1.084116 | .2977787 |
| 4.  | .1710702  | 2.366535 | .1239619 | .4605812  | 1.573743 | .2096643 |
| 5.  | -.0448509 | .4072922 | .5233471 | -.0884253 | .4459299 | .5042742 |
| 6.  | .0165251  | .0432816 | .835196  | .0347015  | .0418188 | .8379655 |
| 7.  | -.0391535 | .3077611 | .5790573 | -.0776336 | .3331707 | .563798  |
| 8.  | .22382    | 4.539085 | .0331293 | .6783119  | 2.587788 | .1076906 |
| 9.  | -.0388409 | .1190183 | .7301016 | -.0811482 | .1293816 | .719074  |
| 10. | -.2437626 | 6.151558 | .0131296 | -.4233705 | 10.05909 | .001516  |

In certain cases (such as subject 8), the difference can be severe enough to potentially change the conclusion. The reason for this inconsistency is that the nonlinear Wald test is actually a standard Wald test of a first-order Taylor approximation of  $g(\cdot)$ , and this approximation can differ according to how  $g(\cdot)$  is specified.

As such, keep in mind the manipulability of nonlinear Wald tests when drawing scientific conclusions.

### Confidence intervals

We can also use `predictnl` to obtain confidence intervals for the observation-specific  $g(\theta, \mathbf{x}_i)$  by using the `ci()` option to specify two new variables to contain the left and right endpoints of the confidence interval, respectively. For example, we could generate confidence intervals for the risk differences calculated previously:

```
. drop RD
. predictnl RD = predict(outcome(Prepaid)) - predict(outcome(Indemnity)),
> ci(RD_lcl RD_rcl)
(1 missing value generated)
note: confidence intervals calculated using Z critical values.
. list RD RD_lcl RD_rcl age male nonwhite in 1/10
```

|     | RD        | RD_lcl    | RD_rcl    | age       | male | nonwhite |
|-----|-----------|-----------|-----------|-----------|------|----------|
| 1.  | -.2303744 | -.4499073 | -.0108415 | 73.722107 | 0    | 0        |
| 2.  | .0266902  | -.1340948 | .1874752  | 27.89595  | 0    | 0        |
| 3.  | -.0768078 | -.2338625 | .080247   | 37.541397 | 0    | 0        |
| 4.  | .1710702  | -.0468844 | .3890248  | 23.641327 | 0    | 1        |
| 5.  | -.0448509 | -.1825929 | .092891   | 40.470901 | 0    | 0        |
| 6.  | .0165251  | -.1391577 | .1722078  | 29.683777 | 0    | 0        |
| 7.  | -.0391535 | -.177482  | .099175   | 39.468857 | 0    | 0        |
| 8.  | .22382    | .0179169  | .4297231  | 26.702255 | 1    | 0        |
| 9.  | -.0388409 | -.2595044 | .1818226  | 63.101974 | 0    | 1        |
| 10. | -.2437626 | -.4363919 | -.0511332 | 69.839828 | 0    | 0        |

The confidence level, here, 95%, is either set using the `level()` option or obtained from the current default level, `c(level)`; see [U] 20.8 Specifying the width of confidence intervals.

From the above output, we can see that, for subjects 1, 8, and 10, a 95% confidence interval for the risk difference does not contain zero, meaning that, for these subjects, there is some evidence of a significant difference in risks.

The confidence intervals calculated by `predictnl` are pointwise; there is no adjustment (such as a Bonferroni correction) made so that these confidence intervals may be considered jointly at the specified level.

## Methods and formulas

For the  $i$ th observation, consider the transformation  $g(\boldsymbol{\theta}, \mathbf{x}_i)$ , estimated by  $g(\hat{\boldsymbol{\theta}}, \mathbf{x}_i)$ , for the  $1 \times k$  parameter vector  $\boldsymbol{\theta}$  and data  $\mathbf{x}_i$  ( $\mathbf{x}_i$  is assumed fixed). The variance of  $g(\hat{\boldsymbol{\theta}}, \mathbf{x}_i)$  is estimated by

$$\widehat{\text{Var}}\{g(\hat{\boldsymbol{\theta}}, \mathbf{x}_i)\} = \mathbf{G}\mathbf{V}\mathbf{G}'$$

where  $\mathbf{G}$  is the vector of derivatives

$$\mathbf{G} = \left\{ \left. \frac{\partial g(\boldsymbol{\theta}, \mathbf{x}_i)}{\partial \boldsymbol{\theta}} \right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}} \right\}_{(1 \times k)}$$

and  $\mathbf{V}$  is the estimated variance–covariance matrix of  $\hat{\boldsymbol{\theta}}$ . Standard errors,  $\widehat{\text{se}}\{g(\hat{\boldsymbol{\theta}}, \mathbf{x}_i)\}$ , are obtained as the square roots of the variances.

The Wald test statistic for testing

$$H_0: g(\boldsymbol{\theta}, \mathbf{x}_i) = 0$$

versus the two-sided alternative is given by

$$W_i = \frac{\{g(\hat{\boldsymbol{\theta}}, \mathbf{x}_i)\}^2}{\widehat{\text{Var}}\{g(\hat{\boldsymbol{\theta}}, \mathbf{x}_i)\}}$$

When the variance–covariance matrix of  $\hat{\boldsymbol{\theta}}$  is an asymptotic covariance matrix,  $W_i$  is approximately distributed as  $\chi^2$  with 1 degree of freedom. For linear regression,  $W_i$  is taken to be approximately distributed as  $F_{1,r}$ , where  $r$  is the residual degrees of freedom from the original model fit. The  $p$ -values for the observation-by-observation tests of  $H_0$  versus the two-sided alternative are given by

$$p_i = \Pr(T > W_i)$$

where  $T$  is either a  $\chi^2$ - or  $F$ -distributed random variable, as described above.

A  $(1 - \alpha) \times 100\%$  confidence interval for  $g(\boldsymbol{\theta}, \mathbf{x}_i)$  is given by

$$g(\hat{\boldsymbol{\theta}}, \mathbf{x}_i) \pm z_{\alpha/2} \left[ \widehat{\text{se}} \left\{ g(\hat{\boldsymbol{\theta}}, \mathbf{x}_i) \right\} \right]$$

when  $W_i$  is  $\chi^2$ -distributed, and

$$g(\hat{\boldsymbol{\theta}}, \mathbf{x}_i) \pm t_{\alpha/2, r} \left[ \widehat{\text{se}} \left\{ g(\hat{\boldsymbol{\theta}}, \mathbf{x}_i) \right\} \right]$$

when  $W_i$  is  $F$ -distributed.  $z_p$  is the  $1 - p$  quantile of the standard normal distribution, and  $t_{p, r}$  is the  $1 - p$  quantile of the  $t$  distribution with  $r$  degrees of freedom.

## References

- Drukker, D. M. 2016a. Probability differences and odds ratios measure conditional-on-covariate effects and population-parameter effects. *The Stata Blog: Not Elsewhere Classified*. <https://blog.stata.com/2016/07/26/probability-differences-and-odds-ratios-measure-conditional-on-covariate-effects-and-population-parameter-effects/>.
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- Hosmer, D. W., Jr., S. A. Lemeshow, and R. X. Sturdivant. 2013. *Applied Logistic Regression*. 3rd ed. Hoboken, NJ: Wiley.
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## Also see

- [R] **lincom** — Linear combinations of parameters
- [R] **nlcom** — Nonlinear combinations of parameters
- [R] **predict** — Obtain predictions, residuals, etc., after estimation
- [R] **test** — Test linear hypotheses after estimation
- [R] **testnl** — Test nonlinear hypotheses after estimation
- [U] **20 Estimation and postestimation commands**

Description  
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Menu  
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## Description

`probit` fits a probit model for a binary dependent variable, assuming that the probability of a positive outcome is determined by the standard normal cumulative distribution function. `probit` can compute robust and cluster-robust standard errors and adjust results for complex survey designs.

## Quick start

Probit model of  $y$  on continuous variable  $x_1$

```
probit y x1
```

Add square of  $x_1$

```
probit y c.x1##c.x1
```

Same as above, but report bootstrap standard errors

```
probit y c.x1##c.x1, vce(bootstrap)
```

Bootstrap estimates of coefficients

```
bootstrap _b: probit y c.x1##c.x1
```

Adjust for complex survey design using `svyset` data and add  $x_2$

```
svy: probit y c.x1##c.x1 x2
```

## Menu

Statistics > Binary outcomes > Probit regression

## Syntax

```
probit depvar [indepvars] [if] [in] [weight] [, options]
```

| <i>options</i>                               | Description                                                                                                                                                                 |
|----------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                 |                                                                                                                                                                             |
| <code>noconstant</code>                      | suppress constant term                                                                                                                                                      |
| <code>offset(<i>varname</i>)</code>          | include <i>varname</i> in model with coefficient constrained to 1                                                                                                           |
| <code>asis</code>                            | retain perfect predictor variables                                                                                                                                          |
| <code>constraints(<i>constraints</i>)</code> | apply specified linear constraints                                                                                                                                          |
| <b>SE/Robust</b>                             |                                                                                                                                                                             |
| <code>vce(<i>vcetype</i>)</code>             | <i>vcetype</i> may be <code>oim</code> , <code>opg</code> , <code>robust</code> , <code>cluster <i>clustvar</i></code> , <code>bootstrap</code> , or <code>jackknife</code> |
| <b>Reporting</b>                             |                                                                                                                                                                             |
| <code>level(#)</code>                        | set confidence level; default is <code>level(95)</code>                                                                                                                     |
| <code>nocnsreport</code>                     | do not display constraints                                                                                                                                                  |
| <code>display_options</code>                 | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling                            |
| <b>Maximization</b>                          |                                                                                                                                                                             |
| <code>maximize_options</code>                | control the maximization process; seldom used                                                                                                                               |
| <code>nocoef</code>                          | do not display the coefficient table; seldom used                                                                                                                           |
| <code>collinear</code>                       | keep collinear variables                                                                                                                                                    |
| <code>coeflegend</code>                      | display legend instead of statistics                                                                                                                                        |

*indepvars* may contain factor variables; see [U] 11.4.3 **Factor variables**.

*depvar* and *indepvars* may contain time-series operators; see [U] 11.4.4 **Time-series varlists**.

`bayes`, `bayesboot`, `bootstrap`, `by`, `collect`, `fmm`, `fp`, `jackknife`, `mfp`, `mi estimate`, `nestreg`, `rolling`, `statsby`, `stepwise`, and `svy` are allowed; see [U] 11.1.10 **Prefix commands**. For more details, see [BAYES] **bayes: probit** and [FMM] **fmm: probit**.

`vce(bootstrap)` and `vce(jackknife)` are not allowed with the `mi estimate` prefix; see [MI] **mi estimate**.

Weights are not allowed with the `bootstrap` prefix; see [R] **bootstrap**.

`vce()`, `nocoef`, and `weights` are not allowed with the `svy` prefix; see [SVY] **svy**.

`fweights`, `iwweights`, and `pweights` are allowed; see [U] 11.1.6 **weight**.

`nocoef`, `collinear`, and `coeflegend` do not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options

### Model

`noconstant`, `offset(varname)`, `constraints(constraints)`; see [R] [Estimation options](#).

`asis` specifies that all specified variables and observations be retained in the maximization process.

This option is typically not specified and may introduce numerical instability. Normally `probit` omits variables that perfectly predict success or failure in the dependent variable along with their associated observations. In those cases, the effective coefficient on the omitted variables is infinity (negative infinity) for variables that completely determine a success (failure). Dropping the variable and perfectly predicted observations has no effect on the likelihood or estimates of the remaining coefficients and increases the numerical stability of the optimization process. Specifying this option forces retention of perfect predictor variables and their associated observations.

### SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

### Reporting

`level(#)`; see [R] [Estimation options](#).

`nocnsreport`; see [R] [Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

### Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

The following options are available with `probit` but are not shown in the dialog box:

`nocoeff` specifies that the coefficient table not be displayed. This option is sometimes used by programmers but is of no use interactively.

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Remarks are presented under the following headings:

- [Robust standard errors](#)
- [Model identification](#)
- [Video examples](#)

probit fits maximum likelihood models with dichotomous dependent (left-hand-side) variables coded as 0/1 (more precisely, coded as 0 and not 0).

For grouped data or data in binomial form, a probit model can be fit using glm with the family(binomial) and link(probit) options.

### ▷ Example 1

We have data on the make, weight, and mileage rating of 22 foreign and 52 domestic automobiles. We wish to fit a probit model explaining whether a car is foreign based on its weight and mileage. Here is an overview of our data:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. keep make mpg weight foreign
. describe
Contains data from https://www.stata-press.com/data/r19/auto.dta
Observations: 74 1978 automobile data
Variables: 4 13 Apr 2024 17:45
 (_dta has notes)
```

| Variable name | Storage type | Display format | Value label | Variable label |
|---------------|--------------|----------------|-------------|----------------|
| make          | str18        | %-18s          |             | Make and model |
| mpg           | int          | %8.0g          |             | Mileage (mpg)  |
| weight        | int          | %8.0gc         |             | Weight (lbs.)  |
| foreign       | byte         | %8.0g          | origin      | Car origin     |

Sorted by: foreign  
 Note: Dataset has changed since last saved.

```
. inspect foreign
foreign: Car origin
```

|     |          | Number of observations |          |             |
|-----|----------|------------------------|----------|-------------|
|     |          | Total                  | Integers | Nonintegers |
| #   | Negative | -                      | -        | -           |
| #   | Zero     | 52                     | 52       | -           |
| #   | Positive | 22                     | 22       | -           |
| #   |          |                        |          |             |
| # # | Total    | 74                     | 74       | -           |
| # # | Missing  | -                      |          |             |
|     |          |                        |          |             |
| 0   |          | 74                     |          |             |
|     | 1        |                        |          |             |

(2 unique values)

foreign is labeled and all values are documented in the label.

The foreign variable takes on two unique values, 0 and 1. The value 0 denotes a domestic car, and 1 denotes a foreign car.

The model that we wish to fit is

$$\Pr(\text{foreign} = 1) = \Phi(\beta_0 + \beta_1 \text{weight} + \beta_2 \text{mpg})$$

where  $\Phi$  is the cumulative normal distribution.

To fit this model, we type

```
. probit foreign weight mpg
Iteration 0: Log likelihood = -45.03321
Iteration 1: Log likelihood = -27.914626
 (output omitted)
Iteration 5: Log likelihood = -26.844189
Probit regression
Log likelihood = -26.844189
Number of obs = 74
LR chi2(2) = 36.38
Prob > chi2 = 0.0000
Pseudo R2 = 0.4039
```

| foreign | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|---------|-------------|-----------|-------|-------|----------------------|-----------|
| weight  | -.0023355   | .0005661  | -4.13 | 0.000 | -.003445             | -.0012261 |
| mpg     | -.1039503   | .0515689  | -2.02 | 0.044 | -.2050235            | -.0028772 |
| _cons   | 8.275464    | 2.554142  | 3.24  | 0.001 | 3.269437             | 13.28149  |

We find that heavier cars are less likely to be foreign and that cars yielding better gas mileage are also less likely to be foreign, at least holding the weight of the car constant.

See [R] [Maximize](#) for an explanation of the output.



### □ Technical note

Stata interprets a value of 0 as a negative outcome (failure) and treats all other values (except missing) as positive outcomes (successes). Thus if your dependent variable takes on the values 0 and 1, then 0 is interpreted as failure and 1 as success. If your dependent variable takes on the values 0, 1, and 2, then 0 is still interpreted as failure, but both 1 and 2 are treated as successes.

If you prefer a more formal mathematical statement, when you type `probit y x`, Stata fits the model

$$\Pr(y_j \neq 0 \mid \mathbf{x}_j) = \Phi(\mathbf{x}_j\boldsymbol{\beta})$$

where  $\Phi$  is the standard cumulative normal.



## Robust standard errors

If you specify the `vce(robust)` option, `probit` reports robust standard errors; see [U] 20.22 Obtaining robust variance estimates.

### ▷ Example 2

For the model from [example 1](#), the robust calculation increases the standard error of the coefficient on `mpg` by almost 15%:

```
. probit foreign weight mpg, vce(robust) nolog
Probit regression Number of obs = 74
 Wald chi2(2) = 30.26
 Prob > chi2 = 0.0000
Log pseudolikelihood = -26.844189 Pseudo R2 = 0.4039
```

| foreign | Coefficient | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |           |
|---------|-------------|---------------------|-------|-------|----------------------|-----------|
| weight  | -.0023355   | .0004934            | -4.73 | 0.000 | -.0033025            | -.0013686 |
| mpg     | -.1039503   | .0593548            | -1.75 | 0.080 | -.2202836            | .0123829  |
| _cons   | 8.275464    | 2.539177            | 3.26  | 0.001 | 3.298769             | 13.25216  |

Without `vce(robust)`, the standard error for the coefficient on `mpg` was reported to be 0.052 with a resulting confidence interval of  $[-0.21, -0.00]$ .



► Example 3

The `vce(cluster clustvar)` option can relax the independence assumption required by the probit estimator to independence between clusters. To demonstrate, we will switch to a different dataset.

We are studying unionization of women in the United States and have a dataset with 26,200 observations on 4,434 women between 1970 and 1988. We will use the variables `age` (the women were 14–26 in 1968, and our data span the age range of 16–46), `grade` (years of schooling completed, ranging from 0 to 18), `not_smsa` (28% of the person-time was spent living outside an SMSA—standard metropolitan statistical area), `south` (41% of the person-time was in the South), and `year`. Each of these variables is included in the regression as a covariate along with the interaction between `south` and `year`. This interaction, along with the `south` and `year` variables, is specified in the probit command using factor-variables notation, `south##c.year`. We also have variable `union`, indicating union membership. Overall, 22% of the person-time is marked as time under union membership, and 44% of these women have belonged to a union.

We fit the following model, ignoring that women are observed an average of 5.9 times each in these data:

```
. use https://www.stata-press.com/data/r19/union, clear
(NLS Women 14-24 in 1968)

. probit union age grade not_smsa south##c.year

Iteration 0: Log likelihood = -13864.23
Iteration 1: Log likelihood = -13545.541
Iteration 2: Log likelihood = -13544.385
Iteration 3: Log likelihood = -13544.385

Probit regression Number of obs = 26,200
 LR chi2(6) = 639.69
 Prob > chi2 = 0.0000
 Pseudo R2 = 0.0231

Log likelihood = -13544.385
```

|                           | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|---------------------------|-------------|-----------|-------|-------|----------------------|-----------|
| <code>union</code>        |             |           |       |       |                      |           |
| <code>age</code>          | .0118481    | .0029072  | 4.08  | 0.000 | .0061502             | .017546   |
| <code>grade</code>        | .0267365    | .0036689  | 7.29  | 0.000 | .0195457             | .0339273  |
| <code>not_smsa</code>     | -.1293525   | .0202595  | -6.38 | 0.000 | -.1690604            | -.0896445 |
| <code>1.south</code>      | -.8281077   | .2472219  | -3.35 | 0.001 | -1.312654            | -.3435618 |
| <code>year</code>         | -.0080931   | .0033469  | -2.42 | 0.016 | -.0146529            | -.0015333 |
| <code>south#c.year</code> |             |           |       |       |                      |           |
| <code>1</code>            | .0057369    | .0030917  | 1.86  | 0.064 | -.0003226            | .0117965  |
| <code>_cons</code>        | -.6542487   | .2007777  | -3.26 | 0.001 | -1.047766            | -.2607316 |

The reported standard errors in this model are probably meaningless. Women are observed repeatedly, and so the observations are not independent. Looking at the coefficients, we find a large southern effect against unionization and a time trend for the south that is almost significantly different from the overall downward trend. The `vce(cluster clustvar)` option provides a way to fit this model and obtains correct standard errors:

```
. probit union age grade not_smsa south##c.year, vce(cluster id)
Iteration 0: Log pseudolikelihood = -13864.23
Iteration 1: Log pseudolikelihood = -13545.541
Iteration 2: Log pseudolikelihood = -13544.385
Iteration 3: Log pseudolikelihood = -13544.385
Probit regression
Number of obs = 26,200
Wald chi2(6) = 166.53
Prob > chi2 = 0.0000
Pseudo R2 = 0.0231
Log pseudolikelihood = -13544.385
(Std. err. adjusted for 4,434 clusters in idcode)
```

| union        | Coefficient | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |           |
|--------------|-------------|---------------------|-------|-------|----------------------|-----------|
| age          | .0118481    | .0056625            | 2.09  | 0.036 | .0007499             | .0229463  |
| grade        | .0267365    | .0078124            | 3.42  | 0.001 | .0114244             | .0420486  |
| not_smsa     | -.1293525   | .0403885            | -3.20 | 0.001 | -.2085125            | -.0501925 |
| 1.south      | -.8281077   | .3201584            | -2.59 | 0.010 | -1.455607            | -.2006089 |
| year         | -.0080931   | .0060829            | -1.33 | 0.183 | -.0200153            | .0038292  |
| south#c.year |             |                     |       |       |                      |           |
| 1            | .0057369    | .0040133            | 1.43  | 0.153 | -.002129             | .0136029  |
| _cons        | -.6542487   | .3485976            | -1.88 | 0.061 | -1.337487            | .02899    |

These standard errors are larger than those reported by the inappropriate conventional calculation. By comparison, another model we could fit is an equal-correlation population-averaged probit model:

```
. xtprobit union age grade not_smsa south##c.year, pa
Iteration 1: Tolerance = .12544249
Iteration 2: Tolerance = .0034686
Iteration 3: Tolerance = .00017448
Iteration 4: Tolerance = 8.382e-06
Iteration 5: Tolerance = 3.997e-07
GEE population-averaged model
Group variable: idcode
Family: Binomial
Link: Probit
Correlation: exchangeable
Number of obs = 26,200
Number of groups = 4,434
Obs per group:
min = 1
avg = 5.9
max = 12
Wald chi2(6) = 242.57
Prob > chi2 = 0.0000
Scale parameter = 1
```

| union        | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|--------------|-------------|-----------|-------|-------|----------------------|-----------|
| age          | .0089699    | .0053208  | 1.69  | 0.092 | -.0014586            | .0193985  |
| grade        | .0333174    | .0062352  | 5.34  | 0.000 | .0210966             | .0455382  |
| not_smsa     | -.0715717   | .027543   | -2.60 | 0.009 | -.1255551            | -.0175884 |
| 1.south      | -1.017368   | .207931   | -4.89 | 0.000 | -1.424905            | -.6098308 |
| year         | -.0062708   | .0055314  | -1.13 | 0.257 | -.0171122            | .0045706  |
| south#c.year |             |           |       |       |                      |           |
| 1            | .0086294    | .00258    | 3.34  | 0.001 | .0035727             | .013686   |
| _cons        | -.8670997   | .294771   | -2.94 | 0.003 | -1.44484             | -.2893592 |

The coefficient estimates are similar, but these standard errors are smaller than those produced by `probit`, `vce(cluster clustvar)`, as we would expect. If the equal-correlation assumption is valid, the population-averaged probit estimator above should be more efficient.

Is the assumption valid? That is a difficult question to answer. The default population-averaged estimates correspond to an assumption of exchangeable correlation within person. It would not be unreasonable to assume an AR(1) correlation within person or to assume that the observations are correlated but that we do not wish to impose any structure. See [XT] `xtprobit` and [XT] `xtgee` for full details. ◀

`probit, vce(cluster clustvar)` is robust to assumptions about within-cluster correlation. That is, it inefficiently sums within cluster for the standard error calculation rather than attempting to exploit what might be assumed about the within-cluster correlation.

## Model identification

The `probit` command has one more feature that is probably the most useful. It will automatically check the model for identification and, if the model is underidentified, omit whatever variables and observations are necessary for estimation to proceed.

### ▶ Example 4

Have you ever fit a probit model where one or more of your independent variables perfectly predicted one or the other outcome?

For instance, consider the following data:

| Outcome $y$ | Independent variable $x$ |
|-------------|--------------------------|
| 0           | 1                        |
| 0           | 1                        |
| 0           | 0                        |
| 1           | 0                        |

Say that we wish to predict the outcome on the basis of the independent variable. The outcome is always zero when the independent variable is one. In our data,  $\Pr(y = 0 \mid x = 1) = 1$ , which means that the probit coefficient on  $x$  must be minus infinity with a corresponding infinite standard error. At this point, you may suspect that we have a problem.

Unfortunately, not all such problems are so easily detected, especially if you have many independent variables in your model. If you have ever had such difficulties, then you have experienced one of the more unpleasant aspects of computer optimization. The computer has no idea that it is trying to solve for an infinite coefficient as it begins its iterative process. All it knows is that, at each step, making the coefficient a little bigger, or a little smaller, works wonders. It continues on its merry way until either 1) the whole thing comes crashing to the ground when a numerical overflow error occurs or 2) it reaches some predetermined cutoff that stops the process. Meanwhile, you have been waiting. And the estimates that you finally receive, if any, may be nothing more than numerical roundoff.

Stata watches for these sorts of problems, alerts you, fixes them, and then properly fits the model.

Let's return to our automobile data. Among the variables we have in the data is one called `repair` that takes on three values. A value of 1 indicates that the car has a poor repair record, 2 indicates an average record, and 3 indicates a better-than-average record. Here is a tabulation of our data:

```
. use https://www.stata-press.com/data/r19/repair
(1978 automobile data)
. tabulate foreign repair
```

| Car origin | Repair |    |    | Total |
|------------|--------|----|----|-------|
|            | 1      | 2  | 3  |       |
| Domestic   | 10     | 27 | 9  | 46    |
| Foreign    | 0      | 3  | 9  | 12    |
| Total      | 10     | 30 | 18 | 58    |

All the cars with poor repair records (`repair = 1`) are domestic. If we were to attempt to predict `foreign` on the basis of the repair records, the predicted probability for the `repair = 1` category would have to be zero. This in turn means that the probit coefficient must be minus infinity, and that would set most computer programs buzzing.

Let's try using Stata on this problem.

```
. probit foreign b3.repair
note: 1.repair != 0 predicts failure perfectly;
 1.repair omitted and 10 obs not used.
Iteration 0: Log likelihood = -26.992087
Iteration 1: Log likelihood = -22.276479
Iteration 2: Log likelihood = -22.229184
Iteration 3: Log likelihood = -22.229138
Iteration 4: Log likelihood = -22.229138
Probit regression Number of obs = 48
 LR chi2(1) = 9.53
 Prob > chi2 = 0.0020
 Pseudo R2 = 0.1765
Log likelihood = -22.229138
```

| foreign | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|---------|-------------|-----------|-------|-------|----------------------|-----------|
| repair  |             |           |       |       |                      |           |
| 1       | 0           | (empty)   |       |       |                      |           |
| 2       | -1.281552   | .4297326  | -2.98 | 0.003 | -2.123812            | -.4392911 |
| _cons   | 1.16e-16    | .295409   | 0.00  | 1.000 | -.578991             | .578991   |

Remember that all the cars with poor repair records (`repair = 1`) are domestic, so the model cannot be fit, or at least it cannot be fit if we restrict ourselves to finite coefficients. Stata noted that fact “note: 1.repair != 0 predicts failure perfectly”. This is Stata’s mathematically precise way of saying what we said in English. When `repair` is 1, the car is domestic.

Stata then went on to say, “1.repair omitted and 10 obs not used”. This is Stata eliminating the problem. First, `1.repair` had to be removed from the model because it would have an infinite coefficient. Then, the 10 observations that led to the problem had to be eliminated, as well, so as not to bias the remaining coefficients in the model. The 10 observations that are not used are the 10 domestic cars that have poor repair records.

Stata then fit what was left of the model, using the remaining observations. Because no observations remained for cars with poor repair records, Stata reports “(empty)” in the row for `repair = 1`.

## □ Technical note

Stata is pretty smart about catching these problems. It will catch “one-way causation by a dummy variable”, as we demonstrated above.

Stata also watches for “two-way causation”, that is, a variable that perfectly determines the outcome, both successes and failures. Here Stata says that the variable “predicts outcome perfectly” and stops. Statistics dictate that no model can be fit.

Stata also checks your data for collinear variables; it will say “so-and-so omitted because of collinearity”. No observations need to be eliminated here and model fitting will proceed without the offending variable.

It will also catch a subtle problem that can arise with continuous data. For instance, if we were estimating the chances of surviving the first year after an operation, and if we included in our model age, and if all the persons over 65 died within the year, Stata will say, “age > 65 predicts failure perfectly”. It will then inform us about how it resolves the issue and fit what can be fit of our model.

`probit` (and `logit`, `logistic`, and `ivprobit`) will also occasionally fail to converge and then display messages such as

```
Note: 4 failures and 0 successes completely determined.
```

The cause of this message and what to do if you see it are described in [\[R\] logit](#).

□

## Video examples

[Probit regression with categorical covariates](#)

[Probit regression with continuous covariates](#)

[Probit regression with categorical and continuous covariates](#)

## Stored results

`probit` stores the following in `e()`:

### Scalars

|                            |                                           |
|----------------------------|-------------------------------------------|
| <code>e(N)</code>          | number of observations                    |
| <code>e(N_cds)</code>      | number of completely determined successes |
| <code>e(N_cdf)</code>      | number of completely determined failures  |
| <code>e(k)</code>          | number of parameters                      |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>  |
| <code>e(k_eq_model)</code> | number of equations in overall model test |
| <code>e(k_dv)</code>       | number of dependent variables             |
| <code>e(df_m)</code>       | model degrees of freedom                  |
| <code>e(r2_p)</code>       | pseudo- $R^2$                             |
| <code>e(ll)</code>         | log likelihood                            |
| <code>e(ll_0)</code>       | log likelihood, constant-only model       |
| <code>e(N_clust)</code>    | number of clusters                        |
| <code>e(chi2)</code>       | $\chi^2$                                  |
| <code>e(p)</code>          | $p$ -value for model test                 |
| <code>e(rank)</code>       | rank of <code>e(V)</code>                 |
| <code>e(ic)</code>         | number of iterations                      |
| <code>e(rc)</code>         | return code                               |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise               |

## Macros

|                              |                                                                          |
|------------------------------|--------------------------------------------------------------------------|
| <code>e(cmd)</code>          | probit                                                                   |
| <code>e(cmdline)</code>      | command as typed                                                         |
| <code>e(depvar)</code>       | name of dependent variable                                               |
| <code>e(wtype)</code>        | weight type                                                              |
| <code>e(wexp)</code>         | weight expression                                                        |
| <code>e(title)</code>        | title in estimation output                                               |
| <code>e(clustvar)</code>     | name of cluster variable                                                 |
| <code>e(offset)</code>       | linear offset variable                                                   |
| <code>e(chi2type)</code>     | Wald or LR; type of model $\chi^2$ test                                  |
| <code>e(vce)</code>          | <i>vce</i> type specified in <code>vce()</code>                          |
| <code>e(vcetype)</code>      | title used to label Std. err.                                            |
| <code>e(opt)</code>          | type of optimization                                                     |
| <code>e(which)</code>        | max or min; whether optimizer is to perform maximization or minimization |
| <code>e(ml_method)</code>    | type of ml method                                                        |
| <code>e(user)</code>         | name of likelihood-evaluator program                                     |
| <code>e(technique)</code>    | maximization technique                                                   |
| <code>e(properties)</code>   | b V                                                                      |
| <code>e(estat_cmd)</code>    | program used to implement <code>estat</code>                             |
| <code>e(predict)</code>      | program used to implement <code>predict</code>                           |
| <code>e(marginsok)</code>    | predictions allowed by <code>margins</code>                              |
| <code>e(marginsnotok)</code> | predictions disallowed by <code>margins</code>                           |
| <code>e(asbalanced)</code>   | factor variables <code>fvset</code> as <code>asbalanced</code>           |
| <code>e(asobserved)</code>   | factor variables <code>fvset</code> as <code>asobserved</code>           |

## Matrices

|                              |                                              |
|------------------------------|----------------------------------------------|
| <code>e(b)</code>            | coefficient vector                           |
| <code>e(Cns)</code>          | constraints matrix                           |
| <code>e(ilog)</code>         | iteration log (up to 20 iterations)          |
| <code>e(gradient)</code>     | gradient vector                              |
| <code>e(mns)</code>          | vector of means of the independent variables |
| <code>e(rules)</code>        | information about perfect predictors         |
| <code>e(V)</code>            | variance–covariance matrix of the estimators |
| <code>e(V_modelbased)</code> | model-based variance                         |

## Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

## Matrices

|                       |                                                                                                                            |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

## Methods and formulas

Probit analysis originated in connection with bioassay, and the word *probit*, a contraction of “probability unit”, was suggested by Bliss (1934a, 1934b). For an introduction to probit and logit, see, for example, Aldrich and Nelson (1984), Cameron and Trivedi (2022), Long (1997), Pampel (2021), or Powers and Xie (2008). Long and Freese (2014, chap. 5 and 6) and Jones (2007, chap. 3) provide introductions to probit and logit, along with Stata examples.

The log-likelihood function for probit is

$$\ln L = \sum_{j \in S} w_j \ln \Phi(\mathbf{x}_j \boldsymbol{\beta}) + \sum_{j \notin S} w_j \ln \{1 - \Phi(\mathbf{x}_j \boldsymbol{\beta})\}$$

where  $\Phi$  is the cumulative normal and  $w_j$  denotes the optional weights.  $\ln L$  is maximized, as described in [R] **Maximize**.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] **\_robust**, particularly *Maximum likelihood estimators* and *Methods and formulas*. The scores are calculated as  $\mathbf{u}_j = \{\phi(\mathbf{x}_j \mathbf{b}) / \Phi(\mathbf{x}_j \mathbf{b})\} \mathbf{x}_j$  for the positive outcomes and  $-\{\phi(\mathbf{x}_j \mathbf{b}) / \{1 - \Phi(\mathbf{x}_j \mathbf{b})\}\} \mathbf{x}_j$  for the negative outcomes, where  $\phi$  is the normal density.

`probit` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] **Variance estimation**.

Chester Ittner Bliss (1899–1979) was born in Ohio. He was educated as an entomologist, earning degrees from Ohio State and Columbia, and was employed by the United States Department of Agriculture until 1933. When he lost his job because of the Depression, Bliss then worked with R. A. Fisher in London and at the Institute of Plant Protection in Leningrad before returning to a post at the Connecticut Agricultural Experiment Station in 1938. He was also a lecturer at Yale for 25 years. Among many contributions to biostatistics, his development and application of probit methods to biological problems are outstanding.

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## Also see

- [R] **probit postestimation** — Postestimation tools for probit
- [R] **biprobit** — Bivariate probit regression
- [R] **brier** — Brier score decomposition
- [R] **glm** — Generalized linear models
- [R] **heckoprobit** — Ordered probit model with sample selection
- [R] **hetprobit** — Heteroskedastic probit model
- [R] **ivprobit** — Probit model with continuous endogenous covariates
- [R] **logistic** — Logistic regression, reporting odds ratios
- [R] **logit** — Logistic regression, reporting coefficients
- [R] **mprobit** — Multinomial probit regression
- [R] **npregress kernel** — Nonparametric kernel regression
- [R] **npregress series** — Nonparametric series regression
- [R] **roc** — Receiver operating characteristic (ROC) analysis
- [R] **scobit** — Skewed logistic regression
- [BAYES] **bayes: probit** — Bayesian probit regression
- [CM] **cmmprobit** — Multinomial probit choice model
- [ERM] **eprobit** — Extended probit regression
- [FMM] **fmm: probit** — Finite mixtures of probit regression models
- [LASSO] **Lasso intro** — Introduction to lasso
- [ME] **meprobit** — Multilevel mixed-effects probit regression
- [MI] **Estimation** — Estimation commands for use with mi estimate
- [SVY] **svy estimation** — Estimation commands for survey data
- [XT] **xtprobit** — Random-effects and population-averaged probit models
- [U] **20 Estimation and postestimation commands**

|                         |                                      |                          |
|-------------------------|--------------------------------------|--------------------------|
| Postestimation commands | <a href="#">predict</a>              | <a href="#">margins</a>  |
| Remarks and examples    | <a href="#">Methods and formulas</a> | <a href="#">Also see</a> |

## Postestimation commands

The following postestimation commands are of special interest after `probit`:

| Command                           | Description                                                           |
|-----------------------------------|-----------------------------------------------------------------------|
| <code>estat classification</code> | report various summary statistics, including the classification table |
| <code>estat gof</code>            | Pearson or Hosmer–Lemeshow goodness-of-fit test                       |
| <code>lroc</code>                 | compute area under ROC curve and graph the curve                      |
| <code>lsens</code>                | graph sensitivity and specificity versus probability cutoff           |
| <code>lassogof</code>             | calculate goodness-of-fit predictions                                 |

These commands are not appropriate with `svy` estimation results.

The following standard postestimation commands are also available:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike’s, consistent Akaike’s, corrected Akaike’s, and Schwarz’s Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| * <code>hausman</code>       | Hausman’s specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| <code>linktest</code>        | link test for model specification                                                                                                       |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from <code>margins</code> (profile plots, interaction plots, etc.)                                                    |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | probabilities, linear predictions and their SEs, etc.                                                                                   |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\* `forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results. `forecast` is also not appropriate with `mi` estimation results.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as probabilities, linear predictions, standard errors, deviance residuals, and the equation-level score.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic nooffset rules asif]
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

Main

|                         |                                                                                         |
|-------------------------|-----------------------------------------------------------------------------------------|
| <code>pr</code>         | probability of a positive outcome; the default                                          |
| <code>xb</code>         | linear prediction                                                                       |
| <code>stdp</code>       | standard error of the linear prediction                                                 |
| * <code>deviance</code> | deviance residual                                                                       |
| <code>score</code>      | first derivative of the log likelihood with respect to $\mathbf{x}_j\boldsymbol{\beta}$ |

Unstarred statistics are available both in and out of sample; type `predict ... if e(sample) ... if` wanted only for the estimation sample. Starred statistics are calculated only for the estimation sample, even when `if e(sample)` is not specified.

## Options for predict

Main

`pr`, the default, calculates the probability of a positive outcome.

`xb` calculates the linear prediction.

`stdp` calculates the standard error of the linear prediction.

`deviance` calculates the deviance residual.

`score` calculates the equation-level score,  $\partial \ln L / \partial (\mathbf{x}_j\boldsymbol{\beta})$ .

`nooffset` is relevant only if you specified `offset(varname)` for `probit`. It modifies the calculations made by `predict` so that they ignore the offset variable; the linear prediction is treated as  $\mathbf{x}_j\mathbf{b}$  rather than as  $\mathbf{x}_j\mathbf{b} + \text{offset}_j$ .

`rules` requests that Stata use any rules that were used to identify the model when making the prediction. By default, Stata calculates missing for excluded observations.

`asif` requests that Stata ignore the rules and exclusion criteria and calculate predictions for all observations possible using the estimated parameter from the model.

## margins

### Description for margins

`margins` estimates margins of response for probabilities and linear predictions.

### Menu for margins

Statistics > Postestimation

### Syntax for margins

```
margins [marginlist] [, options]
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i>      | Description                                    |
|-----------------------|------------------------------------------------|
| <code>pr</code>       | probability of a positive outcome; the default |
| <code>xb</code>       | linear prediction                              |
| <code>stdp</code>     | not allowed with <code>margins</code>          |
| <code>deviance</code> | not allowed with <code>margins</code>          |
| <code>score</code>    | not allowed with <code>margins</code>          |

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [R] [margins](#).

## Remarks and examples

Remarks are presented under the following headings:

*Obtaining predicted values*  
*Performing hypothesis tests*

### Obtaining predicted values

Once you have fit a probit model, you can obtain the predicted probabilities by using the `predict` command for both the estimation sample and other samples; see [U] [20 Estimation and postestimation commands](#) and [R] [predict](#). Here we will make only a few additional comments.

`predict` without arguments calculates the predicted probability of a positive outcome. With the `xb` option, `predict` calculates the linear combination  $\mathbf{x}_j\mathbf{b}$ , where  $\mathbf{x}_j$  are the independent variables in the  $j$ th observation and  $\mathbf{b}$  is the estimated parameter vector. This is known as the index function because the cumulative density indexed at this value is the probability of a positive outcome.

In both cases, Stata remembers any rules used to identify the model and calculates missing for excluded observations unless `rules` or `asif` is specified. This is covered in the following example.

With the `stdp` option, `predict` calculates the standard error of the prediction, which is *not* adjusted for replicated covariate patterns in the data.

You can calculate the unadjusted-for-replicated-covariate-patterns diagonal elements of the hat matrix, or leverage, by typing

```
. predict pred
. predict stdp, stdp
. generate hat = stdp^2*pred*(1-pred)
```

### ▷ Example 1

In [example 4](#) of [R] [probit](#), we fit the probit model `probit foreign b3.repair`. To obtain predicted probabilities, we type

```
. predict p
(option pr assumed; Pr(foreign))
(10 missing values generated)
. summarize foreign p
```

| Variable | Obs | Mean     | Std. dev. | Min | Max |
|----------|-----|----------|-----------|-----|-----|
| foreign  | 58  | .2068966 | .4086186  | 0   | 1   |
| p        | 48  | .25      | .1956984  | .1  | .5  |

Stata remembers any rules used to identify the model and sets predictions to missing for any excluded observations. In [example 4](#) of [R] [probit](#), `probit` omitted the variable `1.repair` from our model and excluded 10 observations. When we typed `predict p`, those same 10 observations were again excluded and their predictions set to missing.

`predict`'s `rules` option uses the rules in the prediction. During estimation, we were told, “`1.repair != 0` predicts failure perfectly”, so the rule is that when `1.repair` is not zero, we should predict 0 probability of success or a positive outcome:

```
. predict p2, rules
(option pr assumed; Pr(foreign))
. summarize foreign p p2
```

| Variable | Obs | Mean     | Std. dev. | Min | Max |
|----------|-----|----------|-----------|-----|-----|
| foreign  | 58  | .2068966 | .4086186  | 0   | 1   |
| p        | 48  | .25      | .1956984  | .1  | .5  |
| p2       | 58  | .2068966 | .2016268  | 0   | .5  |

`predict`'s `asif` option ignores the rules and the exclusion criteria and calculates predictions for all observations possible using the estimated parameters from the model:

```
. predict p3, asif
(option pr assumed; Pr(foreign))
. summarize for p p2 p3
```

| Variable | Obs | Mean     | Std. dev. | Min | Max |
|----------|-----|----------|-----------|-----|-----|
| foreign  | 58  | .2068966 | .4086186  | 0   | 1   |
| p        | 48  | .25      | .1956984  | .1  | .5  |
| p2       | 58  | .2068966 | .2016268  | 0   | .5  |
| p3       | 58  | .2931034 | .2016268  | .1  | .5  |

Which is right? By default, `predict` uses the most conservative approach. If many observations had been excluded due to a simple rule, we could be reasonably certain that the `rules` prediction is correct. The `asif` prediction is correct only if the exclusion is a fluke and we would be willing to exclude the variable from the analysis, anyway. Then, however, we should refit the model to include the excluded observations.

◀

## Performing hypothesis tests

After estimation with `probit`, you can perform hypothesis tests by using the `test` or `testnl` command; see [U] 20 Estimation and postestimation commands.

## Methods and formulas

Let index  $j$  be used to index observations. Define  $M_j$  for each observation as the total number of observations sharing  $j$ 's covariate pattern. Define  $Y_j$  as the total number of positive responses among observations sharing  $j$ 's covariate pattern. Define  $p_j$  as the predicted probability of a positive outcome for observation  $j$ .

For  $M_j > 1$ , the deviance residual  $d_j$  is defined as

$$d_j = \pm \left( 2 \left[ Y_j \ln \left( \frac{Y_j}{M_j p_j} \right) + (M_j - Y_j) \ln \left\{ \frac{M_j - Y_j}{M_j (1 - p_j)} \right\} \right] \right)^{1/2}$$

where the sign is the same as the sign of  $(Y_j - M_j p_j)$ . In the limiting cases, the deviance residual is given by

$$d_j = \begin{cases} -\sqrt{2M_j |\ln(1 - p_j)|} & \text{if } Y_j = 0 \\ \sqrt{2M_j |\ln p_j|} & \text{if } Y_j = M_j \end{cases}$$

## Also see

[R] **probit** — Probit regression

[R] **estat classification** — Classification statistics and table

[R] **estat gof** — Pearson or Hosmer–Lemeshow goodness-of-fit test

[R] **iroc** — Compute area under ROC curve and graph the curve

[R] **lsens** — Graph sensitivity and specificity versus probability cutoff

[LASSO] **lassogof** — Goodness of fit after lasso for prediction

[U] 20 Estimation and postestimation commands

[Description](#)  
[Options](#)  
[References](#)

[Quick start](#)  
[Remarks and examples](#)  
[Also see](#)

[Menu](#)  
[Stored results](#)

[Syntax](#)  
[Methods and formulas](#)

## Description

`proportion` produces estimates of proportions, along with standard errors, for the categories identified by the values in each variable of *varlist*.

## Quick start

Proportions, standard errors, and 95% CIs for each level of *v1*

```
proportion v1
```

Also compute statistics for *v2*

```
proportion v1 v2
```

Same as above, for each subpopulation defined by the levels of *catvar*

```
proportion v1 v2, over(catvar)
```

Standardizing across strata defined by *svar* with stratum weight *wvar1*

```
proportion v1, stdize(svar) stdweight(wvar1)
```

Weighting by sampling weight *wvar2*

```
proportion v1 [pweight=wvar2]
```

## Menu

Statistics > Summaries, tables, and tests > Summary and descriptive statistics > Proportions

## Syntax

```
proportion varlist [if] [in] [weight] [, options]
```

| <i>options</i>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | Description                                                                                                                          |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                      |
| <code>stdize(<i>varname</i>)</code>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | variable identifying strata for standardization                                                                                      |
| <code>stdweight(<i>varname</i>)</code>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | weight variable for standardization                                                                                                  |
| <code>nostdrescale</code>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | do not rescale the standard weight variable                                                                                          |
| <b>if/in/over</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                      |
| <code>over(<i>varlist<sub>o</sub></i>)</code>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | group over subpopulations defined by <i>varlist<sub>o</sub></i>                                                                      |
| <b>SE/Cluster</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                      |
| <code>vce(<i>vcetype</i>)</code>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | <i>vcetype</i> may be analytic, <code>cluster</code> <i>clustvar</i> , <code>bootstrap</code> , or <code>jackknife</code>            |
| <b>Reporting</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                                                                      |
| <code>level(#)</code>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | set confidence level; default is level(95)                                                                                           |
| <code>citype(<i>citype</i>)</code>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | method to compute limits of confidence intervals; default is <code>citype(logit)</code>                                              |
| <code>percent</code>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | report estimated proportions as percentages                                                                                          |
| <code>noheader</code>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | suppress table header                                                                                                                |
| <code>display_omitted</code>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| <code>coeflegend</code>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | display legend instead of statistics                                                                                                 |
| <p><i>varlist</i> may contain factor variables; see [U] 11.4.3 <b>Factor variables</b>.</p> <p>Only numeric, nonnegative, integer-valued variables are allowed in <i>varlist</i>.</p> <p><code>bayesboot</code>, <code>bootstrap</code>, <code>collect</code>, <code>jackknife</code>, <code>mi estimate</code>, <code>rolling</code>, <code>statsby</code>, and <code>svy</code> are allowed; see [U] 11.1.10 <b>Prefix commands</b>.</p> <p><code>vce(bootstrap)</code> and <code>vce(jackknife)</code> are not allowed with the <code>mi estimate</code> prefix; see [MI] <b>mi estimate</b>.</p> <p>Weights are not allowed with the <code>bootstrap</code> prefix; see [R] <b>bootstrap</b>.</p> <p><code>vce()</code> and weights are not allowed with the <code>svy</code> prefix; see [SVY] <b>svy</b>.</p> <p><code>fweights</code>, <code>iwweights</code>, and <code>pweights</code> are allowed; see [U] 11.1.6 <b>weight</b>.</p> <p><code>coeflegend</code> does not appear in the dialog box.</p> <p>See [U] 20 <b>Estimation and postestimation commands</b> for more capabilities of estimation commands.</p> |                                                                                                                                      |
| <i>citype</i>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | Description                                                                                                                          |
| <code>logit</code>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | calculate logit-transformed confidence intervals; the default                                                                        |
| <code>agresti</code>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | calculate Agresti–Coull confidence intervals                                                                                         |
| <code>exact</code>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | calculate exact (Clopper–Pearson) confidence intervals                                                                               |
| <code>jeffreys</code>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | calculate Jeffreys confidence intervals                                                                                              |
| <code>normal</code>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | calculate normal (Wald) confidence intervals                                                                                         |
| <code>wald</code>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | synonym for <code>normal</code>                                                                                                      |
| <code>wilson</code>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | calculate Wilson confidence intervals                                                                                                |

## Options

### Model

`stdize(varname)` specifies that the point estimates be adjusted by direct standardization across the strata identified by *varname*. This option requires the `stdweight()` option.

`stdweight(varname)` specifies the weight variable associated with the standard strata identified in the `stdize()` option. The standardization weights must be constant within the standard strata.

`nostdrescale` prevents the standardization weights from being rescaled within the `over()` groups. This option requires `stdize()` but is ignored if the `over()` option is not specified.

### if/in/over

`over(varlisto)` specifies that estimates be computed for multiple subpopulations, which are identified by the different values of the variables in *varlist<sub>o</sub>*. Only numeric, nonnegative, integer-valued variables are allowed in `over(varlisto)`.

### SE/Cluster

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`analytic`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

`vce(analytic)`, the default, uses the analytically derived variance estimator associated with the sample proportion.

### Reporting

`level(#)`; see [R] [Estimation options](#).

`citype(citype)` specifies how to compute the limits of confidence intervals. *citype* may be one of `logit` (default), `agresti`, `exact`, `jeffreys`, `normal`, `wald`, or `wilson`.

`percent` specifies that the proportions be reported as percentages.

`noheader` prevents the table header from being displayed.

*display\_options*: `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, and `no!stretch`; see [R] [Estimation options](#).

The following option is available with `proportion` but is not shown in the dialog box:

`coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

### ▷ Example 1

We can estimate the proportion of each repair rating in `auto2.dta`:

```
. use https://www.stata-press.com/data/r19/auto2
(1978 automobile data)
. proportion rep78
```

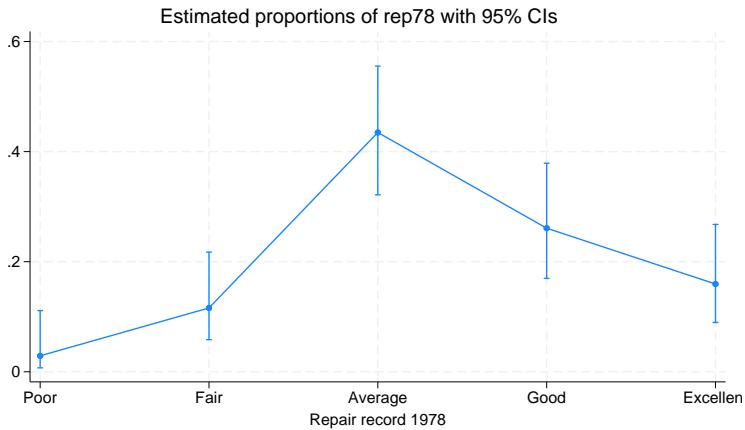
Proportion estimation Number of obs = 69

|           | Proportion | Std. err. | Logit<br>[95% conf. interval] |          |
|-----------|------------|-----------|-------------------------------|----------|
| rep78     |            |           |                               |          |
| Poor      | .0289855   | .0201966  | .0070794                      | .1110924 |
| Fair      | .115942    | .0385422  | .058317                       | .2173648 |
| Average   | .4347826   | .0596787  | .3214848                      | .5553295 |
| Good      | .2608696   | .0528625  | .1695907                      | .3788629 |
| Excellent | .1594203   | .0440694  | .0895793                      | .267702  |

`marginsplot` will produce a graph of the results from `proportion`:

```
. marginsplot
```

Variables that uniquely identify proportions: **rep78**



▷ Example 2

We can also estimate proportions over groups:

```
. proportion rep78, over(foreign)
```

Proportion estimation Number of obs = 69

|                    | Proportion | Std. err.         | Logit<br>[95% conf. interval] |          |
|--------------------|------------|-------------------|-------------------------------|----------|
| rep78@foreign      |            |                   |                               |          |
| Poor Domestic      | .0416667   | .0288424          | .0101825                      | .1552326 |
| Poor Foreign       | 0          | (no observations) |                               |          |
| Fair Domestic      | .1666667   | .0537914          | .084534                       | .3022522 |
| Fair Foreign       | 0          | (no observations) |                               |          |
| Average Domestic   | .5625      | .0716027          | .4184154                      | .6967587 |
| Average Foreign    | .1428571   | .0763604          | .0458191                      | .3664757 |
| Good Domestic      | .1875      | .0563367          | .0993684                      | .3255432 |
| Good Foreign       | .4285714   | .1079898          | .2372889                      | .6438783 |
| Excellent Domestic | .0416667   | .0288424          | .0101825                      | .1552326 |
| Excellent Foreign  | .4285714   | .1079898          | .2372889                      | .6438783 |

To see the results as percentages instead of proportions, we add the percent option:

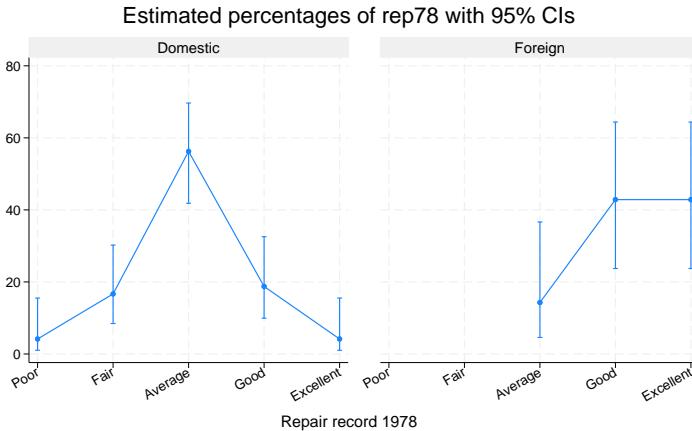
```
. proportion rep78, over(foreign) percent
```

Percent estimation Number of obs = 69

|                    | Percent | Std. err.         | Logit<br>[95% conf. interval] |       |
|--------------------|---------|-------------------|-------------------------------|-------|
| rep78@foreign      |         |                   |                               |       |
| Poor Domestic      | 4.17    | 2.88              | 1.02                          | 15.52 |
| Poor Foreign       | 0.00    | (no observations) |                               |       |
| Fair Domestic      | 16.67   | 5.38              | 8.45                          | 30.23 |
| Fair Foreign       | 0.00    | (no observations) |                               |       |
| Average Domestic   | 56.25   | 7.16              | 41.84                         | 69.68 |
| Average Foreign    | 14.29   | 7.64              | 4.58                          | 36.65 |
| Good Domestic      | 18.75   | 5.63              | 9.94                          | 32.55 |
| Good Foreign       | 42.86   | 10.80             | 23.73                         | 64.39 |
| Excellent Domestic | 4.17    | 2.88              | 1.02                          | 15.52 |
| Excellent Foreign  | 42.86   | 10.80             | 23.73                         | 64.39 |

We can now use `marginsplot` to graph the percentages for each group. We add the `bydimension(foreign)` option to plot the groups in separate graph panels. The `xlabel(, angle(30))` option prevents the *x*-axis labels from running into each other.

```
. marginsplot, bydimension(foreign) xlabel(, angle(30))
Variables that uniquely identify proportions: rep78 foreign
```



We estimate that only 19% of domestic cars have good repair records and only 4% have excellent repair records. For foreign cars, however, we find that 43% have good repair records and 43% have excellent repair records.



### ► Example 3

Instead of estimating percentages within the foreign and domestic groupings, we might want to know overall percentages. For instance, what percentage of all cars are foreign and have excellent repair records? What percentage are domestic and have average records? We can obtain all such percentages by specifying an interaction between rep78 and foreign.

```
. proportion rep78#foreign, percent
Percent estimation Number of obs = 69
```

|                    | Percent estimation |                   | Logit [95% conf. interval] |       |
|--------------------|--------------------|-------------------|----------------------------|-------|
|                    | Percent            | Std. err.         |                            |       |
| rep78#foreign      |                    |                   |                            |       |
| Poor#Domestic      | 2.90               | 2.02              | 0.71                       | 11.11 |
| Poor#Foreign       | 0.00               | (no observations) |                            |       |
| Fair#Domestic      | 11.59              | 3.85              | 5.83                       | 21.74 |
| Fair#Foreign       | 0.00               | (no observations) |                            |       |
| Average#Domestic   | 39.13              | 5.88              | 28.21                      | 51.26 |
| Average#Foreign    | 4.35               | 2.46              | 1.38                       | 12.86 |
| Good#Domestic      | 13.04              | 4.05              | 6.85                       | 23.44 |
| Good#Foreign       | 13.04              | 4.05              | 6.85                       | 23.44 |
| Excellent#Domestic | 2.90               | 2.02              | 0.71                       | 11.11 |
| Excellent#Foreign  | 13.04              | 4.05              | 6.85                       | 23.44 |

Looking at the last line of this output, we estimate that 13% of all cars are foreign with excellent repair records.



## Stored results

`proportion` stores the following in `e()`:

### Scalars

|                          |                                          |
|--------------------------|------------------------------------------|
| <code>e(N)</code>        | number of observations                   |
| <code>e(N_over)</code>   | number of subpopulations                 |
| <code>e(N_stdize)</code> | number of standard strata                |
| <code>e(N_clust)</code>  | number of clusters                       |
| <code>e(k_eq)</code>     | number of equations in <code>e(b)</code> |
| <code>e(df_r)</code>     | sample degrees of freedom                |
| <code>e(rank)</code>     | rank of <code>e(V)</code>                |

### Macros

|                              |                                                |
|------------------------------|------------------------------------------------|
| <code>e(cmd)</code>          | <code>proportion</code>                        |
| <code>e(cmdline)</code>      | command as typed                               |
| <code>e(varlist)</code>      | <i>varlist</i>                                 |
| <code>e(stdize)</code>       | <i>varname</i> from <code>stdize()</code>      |
| <code>e(stdweight)</code>    | <i>varname</i> from <code>stdweight()</code>   |
| <code>e(wtype)</code>        | weight type                                    |
| <code>e(wexp)</code>         | weight expression                              |
| <code>e(title)</code>        | title in estimation output                     |
| <code>e(clustvar)</code>     | name of cluster variable                       |
| <code>e(over)</code>         | <i>varlist</i> from <code>over()</code>        |
| <code>e(vce)</code>          | <i>vcetype</i> specified in <code>vce()</code> |
| <code>e(vcetype)</code>      | title used to label Std. err.                  |
| <code>e(properties)</code>   | <code>b V</code>                               |
| <code>e(estat_cmd)</code>    | program used to implement <code>estat</code>   |
| <code>e(marginsnotok)</code> | predictions disallowed by <code>margins</code> |

### Matrices

|                           |                                                              |
|---------------------------|--------------------------------------------------------------|
| <code>e(b)</code>         | vector of proportion estimates                               |
| <code>e(V)</code>         | (co)variance estimates                                       |
| <code>e(_N)</code>        | vector of numbers of nonmissing observations                 |
| <code>e(_N_stdsum)</code> | number of nonmissing observations within the standard strata |
| <code>e(_p_stdize)</code> | standardizing proportions                                    |
| <code>e(freq)</code>      | vector of frequency estimates                                |
| <code>e(error)</code>     | error code corresponding to <code>e(b)</code>                |

### Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

### Matrices

|                       |                                                                                                                            |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

The point estimate of a proportion,  $\hat{p}$ , is computed as the mean of an indicator variable; see [\[R\] mean](#).

Methods and formulas are presented under the following headings:

*Confidence intervals*  
*Survey data and sampling weights*

## Confidence intervals

For an overview of confidence interval methods for binomial proportions, see [Dean and Pagano \(2015\)](#).

Given  $k$  successes of  $n$  trials, the estimated proportion (probability of a success) is  $\hat{p} = k/n$  with estimated standard error  $\hat{s} = \sqrt{\hat{p}(1-\hat{p})/n}$ .

For weighted data,  $k = \sum_{i=1}^N w_i \mathbf{1}(\text{obs } i \text{ is a success})$  and  $n = \sum_{i=1}^N w_i$ , where  $N$  is the total number of observations,  $w_i$  is the weight for the  $i$ th observation, and  $\mathbf{1}(\cdot)$  denotes the indicator function that evaluates to 1 if observation  $i$  is a success and 0 otherwise.

The logit-transformed confidence interval is given by

$$\log\left(\frac{\hat{p}}{1-\hat{p}}\right) \pm t_{1-\alpha/2, \nu} \frac{\hat{s}}{\hat{p}(1-\hat{p})}$$

where  $t_{p, \nu}$  is the  $p$ th quantile of Student's  $t$  distribution with  $\nu$  degrees of freedom.

The endpoints of this confidence interval are transformed back to the proportion metric by using the inverse of the logit transform

$$f^{-1}(y) = \frac{e^y}{1 + e^y}$$

Hence, the displayed confidence intervals for proportions are

$$f^{-1}\left\{\ln\left(\frac{\hat{p}}{1-\hat{p}}\right) \pm t_{1-\alpha/2, \nu} \frac{\hat{s}}{\hat{p}(1-\hat{p})}\right\}$$

The Wald-type  $100(1-\alpha)\%$  confidence interval is given by

$$\hat{p} \pm t_{1-\alpha/2, \nu} \hat{s}$$

The formulas below (for the Wilson, exact, Jeffreys, and Agresti–Coull intervals) are slightly different when using the `svy` prefix, `pweights`, or the `vce(cluster clustvar)` option; see [Survey data and sampling weights](#) below.

The Wilson interval is given by

$$\frac{\hat{p} + z_{1-\alpha/2}^2/2n \pm z_{1-\alpha/2} \sqrt{\hat{p}(1-\hat{p})/n + z_{1-\alpha/2}^2/4n^2}}{1 + z_{1-\alpha/2}^2/n}$$

where  $z_p$  is the  $p$ th quantile of the standard normal distribution.

The exact (Clopper–Pearson) interval is given by

$$\left\{ \frac{\nu_1 F_{\alpha/2, \nu_1, \nu_2}}{\nu_2 + \nu_1 F_{\alpha/2, \nu_1, \nu_2}}; \frac{\nu_3 F_{1-\alpha/2, \nu_3, \nu_4}}{\nu_4 + \nu_3 F_{\alpha/2, \nu_3, \nu_4}} \right\}$$

where  $\nu_1 = 2k$ ,  $\nu_2 = 2(n-k+1)$ ,  $\nu_3 = 2(k+1)$ ,  $\nu_4 = 2(n-k)$ , and  $F_{p, \nu_1, \nu_2}$  is the  $p$ th quantile of an  $F$  distribution with  $\nu_1$  and  $\nu_2$  degrees of freedom.

The Jeffreys interval is given by

$$\{\text{Beta}_{\alpha/2, \alpha_1, \beta_1}; \text{Beta}_{1-\alpha/2, \alpha_1, \beta_1}\}$$

where  $\alpha_1 = k + 0.5$ ,  $\beta_1 = n - k + 0.5$ , and  $\text{Beta}_{p, \alpha_1, \beta_1}$  is the  $p$ th quantile of a Beta distribution with  $\alpha_1$  and  $\beta_1$  degrees of freedom.

The Agresti–Coull interval is given by

$$\tilde{p} \pm z_{1-\alpha/2} \sqrt{\tilde{p}(1-\tilde{p})/\tilde{n}}$$

where  $\tilde{k} = k + z_{1-\alpha/2}^2/2$ ,  $\tilde{n} = n + z_{1-\alpha/2}^2$ , and  $\tilde{p} = \tilde{k}/\tilde{n}$ .

## Survey data and sampling weights

With the `svy` prefix, `pweights`, or the `vce(cluster clustvar)` option, the Wilson, exact, Jeffreys, and Agresti–Coull intervals use  $k^*$  in place of  $k$  and  $n^*$  in place of  $n$ , where  $k^* = n^*\hat{p}$ ,

$$n^* = \frac{\hat{p}(1-\hat{p})}{\hat{s}^2} \left\{ \frac{z_{1-\alpha/2}}{t_{1-\alpha/2, \nu}} \right\}^2$$

and  $\hat{s}$  is the linearized standard error estimate. For more details about the linearized standard error estimate, see *Ratios and other functions of survey data* in [SVY] **Variance estimation**.

## References

- Cochran, W. G. 1977. *Sampling Techniques*. 3rd ed. New York: Wiley.
- Dean, N., and M. Pagano. 2015. Evaluating confidence interval methods for binomial proportions in clustered surveys. *Journal of Survey Statistics and Methodology* 3: 484–503. <https://doi.org/10.1093/jssam/smv024>.
- Stuart, A., and J. K. Ord. 1994. *Distribution Theory*. Vol. 1 of *Kendall's Advanced Theory of Statistics*, 6th ed. London: Arnold.

## Also see

- [R] **proportion postestimation** — Postestimation tools for proportion
- [R] **mean** — Estimate means
- [R] **ratio** — Estimate ratios
- [R] **total** — Estimate totals
- [MI] **Estimation** — Estimation commands for use with mi estimate
- [SVY] **Direct standardization** — Direct standardization of means, proportions, and ratios
- [SVY] **Poststratification** — Poststratification for survey data
- [SVY] **Subpopulation estimation** — Subpopulation estimation for survey data
- [SVY] **svy estimation** — Estimation commands for survey data
- [SVY] **Variance estimation** — Variance estimation for survey data
- [U] **20 Estimation and postestimation commands**

## Postestimation commands

The following postestimation commands are available after `proportion`:

| Command                  | Description                                                                                       |
|--------------------------|---------------------------------------------------------------------------------------------------|
| <code>contrast</code>    | contrasts and ANOVA-style joint tests of parameters                                               |
| <code>estat vce</code>   | variance–covariance matrix of the estimators (VCE)                                                |
| <code>estat (svy)</code> | postestimation statistics for survey data                                                         |
| <code>estimates</code>   | cataloging estimation results                                                                     |
| <code>etable</code>      | table of estimation results                                                                       |
| <code>lincom</code>      | point estimates, standard errors, testing, and inference for linear combinations of parameters    |
| <code>marginsplot</code> | graph the results from <code>proportion</code>                                                    |
| <code>nlcom</code>       | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters |
| <code>pwcompare</code>   | pairwise comparisons of parameters                                                                |
| <code>test</code>        | Wald tests of simple and composite linear hypotheses                                              |
| <code>testnl</code>      | Wald tests of nonlinear hypotheses                                                                |

## Remarks and examples

### ▷ Example 1

In [example 2](#) of [\[R\] proportion](#), we computed the proportions of cars with different repair records for each group, foreign or domestic. We use `test` to test whether the proportion of cars with repair record equal to 4 is the same for domestic and foreign cars.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. proportion rep78, over(foreign)
(output omitted)
. test 4.rep78@0.foreign=4.rep78@1.foreign
(1) 4.rep78@0bn.foreign - 4.rep78@1.foreign = 0
 F(1, 68) = 3.92
 Prob > F = 0.0518
```

There is not a significant difference between those proportions at the 5% level.



▷ Example 2

Continuing with `auto.dta` from [example 1](#), we generate a new variable, `highprice`, that indicates if the price is larger than \$5,000 and then use `proportion` to see the proportion of cars with high price among domestic and foreign cars separately.

```
. generate highprice = price>5000
. proportion highprice, over(foreign)
Proportion estimation Number of obs = 74
```

|                   | Proportion | Std. err. | Logit<br>[95% conf. interval] |          |
|-------------------|------------|-----------|-------------------------------|----------|
| highprice@foreign |            |           |                               |          |
| 0 Domestic        | .5576923   | .0688744  | .4195373                      | .6874611 |
| 0 Foreign         | .3636364   | .1025593  | .191094                       | .5802222 |
| 1 Domestic        | .4423077   | .0688744  | .3125389                      | .5804627 |
| 1 Foreign         | .6363636   | .1025593  | .4197778                      | .808906  |

We will compute the odds ratio of having a high price in group `Foreign` to having a high price in group `Domestic`. Usually, odds ratios are computed by using the `logistic` command, but here we will perform the computation by using `nlcom` after `proportion`.

```
. nlcom OR: (_b[1.highprice@1.foreign]/_b[0.highprice@1.foreign])/(_b[1.highpri
> ce@0.foreign]/_b[0.highprice@0.foreign])
 OR: (_b[1.highprice@1.foreign]/_b[0.highprice@1.foreign])/(_b[1.high
> price@0.foreign]/_b[0.highprice@0.foreign])
```

| Proportion | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |          |
|------------|-------------|-----------|------|-------|----------------------|----------|
| OR         | 2.206522    | 1.155825  | 1.91 | 0.056 | -.0588533            | 4.471897 |

This is the same odds ratio that we would obtain from

```
. logistic highprice foreign
```

The odds ratio is slightly larger than 2, which means that the odds of having a high price among foreign cars are more than twice that of having a high price among domestic cars.



**Also see**

- [\[R\] proportion](#) — Estimate proportions
- [\[U\] 20 Estimation and postestimation commands](#)

[Description](#)[Options for prtest](#)[Methods and formulas](#)[Quick start](#)[Options for prtesti](#)[References](#)[Menu](#)[Remarks and examples](#)[Also see](#)[Syntax](#)[Stored results](#)

## Description

`prtest` performs tests on the equality of proportions using large-sample statistics. The test can be performed for one sample against a hypothesized population value or for no difference in population proportions estimated from two samples. Clustered data are supported.

`prtesti` is the immediate form of `prtest`; see [\[U\] 19 Immediate commands](#).

## Quick start

One-sample test that the proportion of 1s in `v` is equal to 0.1

```
prtest v == 0.1
```

Same as above, but using the 90% confidence level and adjusting for clustering with clusters defined by `cvar` and an intraclass correlation of 0.5

```
prtest v == 0.1, level(90) cluster(cvar) rho(0.5)
```

Test that the proportion of 1s in `v` is equal between two groups defined by `catvar`

```
prtest v, by(catvar)
```

Same as above, and adjust for clustering with clusters defined by `cvar` and an intraclass correlation of 0.5 in the two groups

```
prtest v, by(catvar) cluster(cvar) rho(0.5)
```

Test equality of proportions between `v1` and `v2`

```
prtest v1 == v2
```

Test  $p_1 = p_2$  if  $\hat{p}_1 = 0.10$ ,  $\hat{p}_2 = 0.17$ ,  $n_1 = 29$ , and  $n_2 = 36$

```
prtesti 29 0.10 36 0.17
```

## Menu

### **prtest**

Statistics > Summaries, tables, and tests > Classical tests of hypotheses > Proportion test

### **prtesti**

Statistics > Summaries, tables, and tests > Classical tests of hypotheses > Proportion test calculator

## Syntax

One-sample test of proportion

```
prtest varname == #p [if] [in] [, onesampleopts]
```

Two-sample test of proportions using groups

```
prtest varname [if] [in] , by(groupvar) [twosamplegopts]
```

Two-sample test of proportions using variables

```
prtest varname1 == varname2 [if] [in] [, level(#)]
```

Immediate form of one-sample test of proportion

```
prtesti #obs1 #p1 #p2 [, level(#) count]
```

Immediate form of two-sample test of proportions

```
prtesti #obs1 #p1 #obs2 #p2 [, level(#) count]
```

| <i>onesampleopts</i>              | Description                                    |
|-----------------------------------|------------------------------------------------|
| Main                              |                                                |
| <i>level</i> (#)                  | confidence level; default is <i>level</i> (95) |
| <i>cluster</i> ( <i>varname</i> ) | variable defining the clusters                 |
| <i>rho</i> (#)                    | intraclass correlation                         |

| <i>twosamplegopts</i>             | Description                                    |
|-----------------------------------|------------------------------------------------|
| Main                              |                                                |
| * <i>by</i> ( <i>groupvar</i> )   | variable defining the groups                   |
| <i>level</i> (#)                  | confidence level; default is <i>level</i> (95) |
| <i>cluster</i> ( <i>varname</i> ) | variable defining the clusters                 |
| <i>rho</i> (#)                    | common intraclass correlation                  |
| <i>rho1</i> (#)                   | intraclass correlation for group 1             |
| <i>rho2</i> (#)                   | intraclass correlation for group 2             |

\**by*(*groupvar*) is required.

*by* is allowed with *prtest*, and *collect* is allowed with *prtest* and *prtesti*; see [\[U\] 11.1.10 Prefix commands](#).

## Options for prtest

Main

*by*(*groupvar*) specifies a numeric variable that contains the group information for a given observation. This variable must have only two values. Do not confuse the *by*() option with the *by* prefix; both may be specified.

*level*(#) specifies the confidence level, as a percentage, for confidence intervals. The default is *level*(95) or as set by *set level*; see [\[U\] 20.8 Specifying the width of confidence intervals](#).

- `cluster(varname)` specifies the variable that identifies clusters. The `cluster()` option is required to adjust the computation for clustering.
- `rho(#)` specifies the intraclass correlation for a one-sample test or the common intraclass correlation for a two-sample test. The `rho()` option is required to adjust the computation for clustering for a one-sample test.
- `rho1(#)` specifies the intraclass correlation of the first group for a two-sample test using groups. The `rho()` option or both `rho1()` and `rho2()` options are required to adjust the computation for clustering.
- `rho2(#)` specifies the intraclass correlation of the second group for a two-sample test using groups. The `rho()` option or both `rho1()` and `rho2()` options are required to adjust the computation for clustering.

## Options for prtesti

- `level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [U] 20.8 Specifying the width of confidence intervals.
- `count` specifies that integer counts instead of proportions be used in the immediate forms of `prtest`. In the first syntax, `prtesti` expects that  $\#_{\text{obs1}}$  and  $\#_{p1}$  are counts— $\#_{p1} \leq \#_{\text{obs1}}$ —and  $\#_{p2}$  is a proportion. In the second syntax, `prtesti` expects that all four numbers are integer counts, that  $\#_{\text{obs1}} \geq \#_{p1}$ , and that  $\#_{\text{obs2}} \geq \#_{p2}$ .

## Remarks and examples

Remarks are presented under the following headings:

*Tests of proportions*  
*Adjust for clustering*  
*Immediate form*

## Tests of proportions

The `prtest` output follows the output of `tttest` in providing a lot of information. Each proportion is presented along with a confidence interval. The appropriate one- or two-sample test is performed, and the two-sided and both one-sided results are included at the bottom of the output. For a two-sample test, the calculated difference is also presented with its confidence interval. This command may be used for both large-sample testing and large-sample interval estimation. For one-sample tests of proportions with small-sample sizes and to obtain exact  $p$ -values, researchers should use `bitest`; see [R] [bitest](#).

### ▷ Example 1: One-sample test of proportion

In the first form, `prtest` tests whether the mean of the sample is equal to a known constant. Assume that we have a sample of 74 automobiles. We wish to test whether the proportion of automobiles that are foreign is different from 40%.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. prtest foreign == 0.4
```

One-sample test of proportion

| Variable | Mean     | Std. err. | [95% conf. interval] |          |
|----------|----------|-----------|----------------------|----------|
| foreign  | .2972973 | .0531331  | .1931583             | .4014363 |

```

p = proportion(foreign) z = -1.8034
HO: p = 0.4
 Ha: p < 0.4 Ha: p != 0.4 Ha: p > 0.4
Pr(Z < z) = 0.0357 Pr(|Z| > |z|) = 0.0713 Pr(Z > z) = 0.9643

```

The test indicates that we cannot reject the hypothesis that the proportion of foreign automobiles is 0.40 at the 5% significance level.

◀

### ▷ Example 2: Two-sample test of proportions

We have two headache remedies that we give to patients. Each remedy's effect is recorded as 0 for failing to relieve the headache and 1 for relieving the headache. We wish to test the equality of the proportion of people relieved by the two treatments.

```
. use https://www.stata-press.com/data/r19/cure
. prtest cure1 == cure2
```

Two-sample test of proportions

| Variable | Mean      | Std. err. | z     | P> z  | [95% conf. interval] |           |
|----------|-----------|-----------|-------|-------|----------------------|-----------|
| cure1    | .52       | .0706541  |       |       | .3815205             | .6584795  |
| cure2    | .7118644  | .0589618  |       |       | .5963013             | .8274275  |
| diff     | -.1918644 | .0920245  |       |       | -.372229             | -.0114998 |
|          | under HO: | .0931155  | -2.06 | 0.039 |                      |           |

```

diff = prop(cure1) - prop(cure2) z = -2.0605
HO: diff = 0
 Ha: diff < 0 Ha: diff != 0 Ha: diff > 0
Pr(Z < z) = 0.0197 Pr(|Z| > |z|) = 0.0394 Pr(Z > z) = 0.9803

```

We find that the proportions are statistically different from each other at any level greater than 3.9%.

◀

## Adjust for clustering

When observations are not independent and can be grouped into clusters, we need to adjust for clustering in a proportion test. For example, in a cluster randomized design, groups of individuals are randomized instead of individuals. To adjust for clustering, we need to specify the cluster identifier variable in the `cluster()` option. In the case of a one-sample proportion test, we need to also specify the intraclass correlation in the `rho()` option. In the case of a two sample proportions test, we need to also specify the common population intraclass correlation in the `rho()` option or group-specific population intraclass correlations in the `rho1()` and `rho2()` options.

### ► Example 3: One-sample test of proportion, adjusting for clusters

Consider data from [Hujoel, Moulton, and Loesche \(1990\)](#) on the accuracy of an enzymatic diagnostic test (EDT) of bacterial infections for 29 patients with multiple sites. The EDT was conducted on each site, a specific area in a patient's mouth, to determine infection by two strings of bacteria. A separate reference test was also conducted on each site with an antibody assay against the two strings of bacteria. The data record whether there was a positive EDT result at each infected site, a true positive result.

We want to test whether the proportion of infected sites that were correctly diagnosed by the EDT is different from 0.6. Because we have multiple infections per patient, we cluster by the patient-identifier subject and use a value of 0.2 from [Ahn, Heo, and Zhang \(2015, 33\)](#) for the inpatient correlation.

To perform the test, we specify the `cluster(subject)` and `rho(0.2)` options:

```
. use https://www.stata-press.com/data/r19/infection
(Target infections detected by EDT (Hujoel, Moulton, and Loesche 1990))
. prtest infect == 0.6, cluster(subject) rho(0.2)

One-sample test of proportion Number of obs = 142
Cluster variable: subject Number of clusters = 29
 Avg. cluster size = 4.90
 CV cluster size = 0.2419
 Intraclass corr. = 0.2000
```

| Variable  | Mean     | Std. err. | [95% conf. interval] |          |
|-----------|----------|-----------|----------------------|----------|
| infection | .6619718 | .0537974  | .5565308             | .7674129 |

```
 p = proportion(infection) z = 1.1123
HO: p = 0.6
 Ha: p < 0.6 Ha: p != 0.6 Ha: p > 0.6
Pr(Z < z) = 0.8670 Pr(|Z| > |z|) = 0.2660 Pr(Z > z) = 0.1330
```

We do not find statistical evidence to reject the null hypothesis of  $H_0: P_{\text{infection}} = 0.6$  versus the two-sided alternative  $H_a: P_{\text{infection}} \neq 0.6$  at the 5% significance level; the  $p$ -value = 0.2660 > 0.05.

▷ Example 4: Two-sample test of proportions using groups, adjusting for clusters

Consider a dataset provided by Hayes and Moulton (2009), which contains a random subsample of the original participants in a cluster randomized trial of a pneumococcal conjugate vaccine in American Indian populations in the southwestern United States. There are two groups of infants with 18 clusters in each group. The control group received a meningococcal C conjugate vaccine (MnCC), and the experimental group received the seven-valent pneumococcal conjugate vaccine (PnCRM7). The two groups are identified by the vaccine variable, and the pneumonia variable records 1 if an infant had at least one bacterial pneumonia episode and 0 otherwise. These data are originally from O'Brien et al. (2003).

We want to test the equality of the proportion of cases of pneumonia in the two vaccine groups. We assume a common known intraclass correlation of 0.02. To perform the test, we type

```
. use https://www.stata-press.com/data/r19/pneumoniacr
(Bacterial pneumonia episodes data from CRT (Hayes and Moulton 2009))
. prtest pneumonia, by(vaccine) cluster(cluster) rho(0.02)
Two-sample test of proportions
Cluster variable: cluster
Group: MnCC
Number of obs = 238
Number of clusters = 18
Avg. cluster size = 13.22
CV cluster size = 0.9605
Intraclass corr. = 0.0200
Group: PnCRM7
Number of obs = 211
Number of clusters = 18
Avg. cluster size = 11.72
CV cluster size = 0.7976
Intraclass corr. = 0.0200
```

| Group  | Mean      | Std. err. | z    | P> z  | [95% conf. interval] |
|--------|-----------|-----------|------|-------|----------------------|
| MnCC   | .2226891  | .0329017  |      |       | .1582029 .2871753    |
| PnCRM7 | .1658768  | .0299027  |      |       | .1072686 .224485     |
| diff   | .0568123  | .04446    |      |       | -.0303278 .1439524   |
|        | under H0: | .0447641  | 1.27 | 0.204 |                      |

```
diff = prop(MnCC) - prop(PnCRM7)
H0: diff = 0
Ha: diff < 0
Pr(Z < z) = 0.8978
diff = prop(MnCC) - prop(PnCRM7)
H0: diff = 0
Ha: diff != 0
Pr(|Z| > |z|) = 0.2044
diff = prop(MnCC) - prop(PnCRM7)
H0: diff = 0
Ha: diff > 0
Pr(Z > z) = 0.1022
z = 1.2691
```

We do not find statistical evidence to reject the null hypothesis of  $H_0: P_{\text{diff}} = 0$  versus the two-sided alternative  $H_a: P_{\text{diff}} \neq 0$  at the 5% significance level; the  $p$ -value = 0.2044 > 0.05.

◁

## Immediate form

### ▷ Example 5: Immediate form of one-sample test of proportion

`prtesti` is like `prtest`, except that you specify summary statistics rather than variables as arguments. For instance, we are reading an article that reports the proportion of registered voters among 50 randomly selected eligible voters as 0.52. We wish to test whether the proportion is 0.7:

```
. prtesti 50 0.52 0.70
One-sample test of proportion x: Number of obs = 50
```

|   | Mean | Std. err. | [95% conf. interval] |          |
|---|------|-----------|----------------------|----------|
| x | .52  | .0706541  | .3815205             | .6584795 |

```

p = proportion(x) z = -2.7775
HO: p = 0.7
Ha: p < 0.7 Ha: p != 0.7 Ha: p > 0.7
Pr(Z < z) = 0.0027 Pr(|Z| > |z|) = 0.0055 Pr(Z > z) = 0.9973

```

◀

### ▷ Example 6: Immediate form of two-sample test of proportions

To judge teacher effectiveness, we wish to test whether the same proportion of people from two classes will answer an advanced question correctly. In the first classroom of 30 students, 40% answered the question correctly, whereas in the second classroom of 45 students, 67% answered the question correctly.

```
. prtesti 30 0.4 45 0.67
Two-sample test of proportions x: Number of obs = 30
 y: Number of obs = 45
```

|      | Mean      | Std. err. | z     | P> z  | [95% conf. interval] |           |
|------|-----------|-----------|-------|-------|----------------------|-----------|
| x    | .4        | .0894427  |       |       | .2246955             | .5753045  |
| y    | .67       | .0700952  |       |       | .532616              | .807384   |
| diff | -.27      | .1136368  |       |       | -.4927241            | -.0472759 |
|      | under HO: | .1169416  | -2.31 | 0.021 |                      |           |

```

diff = prop(x) - prop(y) z = -2.3088
HO: diff = 0
Ha: diff < 0 Ha: diff != 0 Ha: diff > 0
Pr(Z < z) = 0.0105 Pr(|Z| > |z|) = 0.0210 Pr(Z > z) = 0.9895

```

◀

## Stored results

One-sample `prtest` and `prtesti` store the following in `r()`:

### Scalars

|                       |                                             |
|-----------------------|---------------------------------------------|
| <code>r(N)</code>     | sample size                                 |
| <code>r(P)</code>     | sample proportion                           |
| <code>r(se)</code>    | standard error of sample proportion         |
| <code>r(lb)</code>    | lower confidence bound of sample proportion |
| <code>r(ub)</code>    | upper confidence bound of sample proportion |
| <code>r(z)</code>     | $z$ statistic                               |
| <code>r(p_l)</code>   | lower one-sided $p$ -value                  |
| <code>r(p)</code>     | two-sided $p$ -value                        |
| <code>r(p_u)</code>   | upper one-sided $p$ -value                  |
| <code>r(level)</code> | confidence level                            |

Cluster-adjusted one-sample `prtest` also stores the following in `r()`:

### Scalars

|                            |                                            |
|----------------------------|--------------------------------------------|
| <code>r(K)</code>          | number of clusters $K$                     |
| <code>r(M)</code>          | cluster size $M$                           |
| <code>r(rho)</code>        | intraclass correlation                     |
| <code>r(CV_cluster)</code> | coefficient of variation for cluster sizes |

Two-sample `prtest` and two-sample `prtesti` store the following in `r()`:

### Scalars

|                          |                                                             |
|--------------------------|-------------------------------------------------------------|
| <code>r(N1)</code>       | sample size of population one                               |
| <code>r(N2)</code>       | sample size of population two                               |
| <code>r(P1)</code>       | sample proportion for population one                        |
| <code>r(P2)</code>       | sample proportion for population two                        |
| <code>r(P_diff)</code>   | difference of proportions                                   |
| <code>r(se1)</code>      | standard error of population-one sample proportion          |
| <code>r(se2)</code>      | standard error of population-two sample proportion          |
| <code>r(se_diff)</code>  | standard error of the difference of proportions             |
| <code>r(se_diff0)</code> | standard error of the difference of proportions under $H_0$ |
| <code>r(lb1)</code>      | lower confidence bound of population-one sample proportion  |
| <code>r(ub1)</code>      | upper confidence bound of population-one sample proportion  |
| <code>r(lb2)</code>      | lower confidence bound of population-two sample proportion  |
| <code>r(ub2)</code>      | upper confidence bound of population-two sample proportion  |
| <code>r(lb_diff)</code>  | lower confidence bound of the difference of proportions     |
| <code>r(ub_diff)</code>  | upper confidence bound of the difference of proportions     |
| <code>r(z)</code>        | $z$ statistic                                               |
| <code>r(p_l)</code>      | lower one-sided $p$ -value                                  |
| <code>r(p)</code>        | two-sided $p$ -value                                        |
| <code>r(p_u)</code>      | upper one-sided $p$ -value                                  |
| <code>r(level)</code>    | confidence level                                            |

Cluster-adjusted two-sample `prtest` using the `by()` option also stores the following in `r()`:

### Scalars

|                             |                                                           |
|-----------------------------|-----------------------------------------------------------|
| <code>r(K1)</code>          | population-one number of clusters $K_1$                   |
| <code>r(K2)</code>          | population-two number of clusters $K_2$                   |
| <code>r(M1)</code>          | population-one cluster size $M_1$                         |
| <code>r(M2)</code>          | population-two cluster size $M_2$                         |
| <code>r(rho)</code>         | common intraclass correlation                             |
| <code>r(rho1)</code>        | population-one intraclass correlation                     |
| <code>r(rho2)</code>        | population-two intraclass correlation                     |
| <code>r(CV_cluster1)</code> | population-one coefficient of variation for cluster sizes |
| <code>r(CV_cluster2)</code> | population-two coefficient of variation for cluster sizes |

## Methods and formulas

Remarks are presented under the following headings:

*One-sample test*

*Two-sample test*

For all the tests below, the test statistic  $z$  has an asymptotic standard normal distribution, and the  $p$ -value is computed as

$$p = \begin{cases} 1 - \Phi(z) & \text{for an upper one-sided test} \\ \Phi(z) & \text{for a lower one-sided test} \\ 2\{1 - \Phi(|z|)\} & \text{for a two-sided test} \end{cases}$$

where  $\Phi(\cdot)$  is the cdf of a standard normal distribution and  $|z|$  is an absolute value of  $z$ .

See [Acock \(2023, 158–164\)](#) for additional examples of tests of proportions using Stata.

## One-sample test

Let  $n$  be the number of observations,  $\hat{p}$  be the observed proportion, and  $\hat{q} = 1 - \hat{p}$ .

The one-tailed and two-tailed tests of a population proportion use an asymptotically normally distributed test statistic calculated as

$$z = \frac{\hat{p} - p_0}{s_0}$$

where  $p_0$  is the hypothesized proportion,  $q_0 = 1 - p_0$ , and  $s_0 = \sqrt{p_0 q_0 / n}$  is the standard error of  $\hat{p}$  under the null hypothesis of  $p = p_0$ .

A large-sample  $100(1 - \alpha)\%$  confidence interval for a proportion  $p$  is

$$\hat{p} \pm z_{1-\alpha/2} s$$

where  $s = \sqrt{\hat{p}\hat{q}/n}$  and  $z_{1-\alpha/2}$  is the  $(1 - \alpha/2)$ th quantile of the standard normal distribution.

With clustered data, suppose that there are  $K$  clusters, each of size  $M_i$  such that  $n = \sum_{i=1}^K M_i$ . Let  $\rho$  be the intraclass correlation. Following [Ahn, Heo, and Zhang \(2015\)](#), we assume that the cluster sizes  $M_i$  are independent and identically distributed. Let  $C_{\text{adj}}$  be the adjustment to the standard error for clustered data,

$$C_{\text{adj}} = \sqrt{\sum_{i=1}^K M_i \{1 + \rho(M_i - 1)\} / n}$$

such that  $s_{0,\text{cl}} = C_{\text{adj}} s_0$  and  $s_{\text{cl}} = C_{\text{adj}} s$ .

$C_{\text{adj}}$  can be equivalently written as

$$C_{\text{adj}} = \sqrt{1 + \rho(\bar{M} - 1) + \rho\bar{M}CV_{\text{cl}}^2}$$

where  $\bar{M} = \sum_{i=1}^K M_i/K$  is the average cluster size and  $CV_{\text{cl}}$  is the coefficient of variation for cluster sizes:

$$CV_{\text{cl}} = \frac{\sqrt{\sum_{i=1}^K (M_i - \bar{M})^2 / K}}{\bar{M}}$$

To adjust the test statistic  $z$  and the confidence interval for clustering, replace  $s_0$  with  $s_{0,\text{cl}}$  and  $s$  with  $s_{\text{cl}}$  in the corresponding formulas. In the presence of clustering, the test statistic  $z$  is asymptotically normally distributed conditional on the empirical distribution of  $M_i$ 's.

## Two-sample test

Let  $n_1$  be the number of observations in population one and  $n_2$  be the number of observations in population two,  $\hat{p}_1$  be the observed proportion in population one and  $\hat{p}_2$  be the observed proportion in population two, and  $\hat{q}_1 = 1 - \hat{p}_1$  and  $\hat{q}_2 = 1 - \hat{p}_2$ . Let  $x_1$  and  $x_2$  be the total number of successes in the two populations.

A test of the difference of two proportions uses an asymptotically normally distributed test statistic calculated as

$$z = \frac{\hat{p}_1 - \hat{p}_2}{s_{d0}}$$

where  $s_{d0} = \sqrt{\hat{p}_p \hat{q}_p (1/n_1 + 1/n_2)}$  is the standard error of  $\hat{p}_1 - \hat{p}_2$  under the null hypothesis of  $p_1 = p_2$ , with  $\hat{p}_p = (x_1 + x_2)/(n_1 + n_2)$  and  $\hat{q}_p = 1 - \hat{p}_p$ .

The  $100(1 - \alpha)\%$  confidence interval for the difference of two proportions is given by

$$(\hat{p}_1 - \hat{p}_2) \pm z_{1-\alpha/2} \sqrt{s_1^2 + s_2^2}$$

where  $s_1 = \sqrt{\hat{p}_1 \hat{q}_1 / n_1}$  and  $s_2 = \sqrt{\hat{p}_2 \hat{q}_2 / n_2}$  are the standard errors of the two sample proportions and  $z_{1-\alpha/2}$  is the  $(1 - \alpha/2)$ th quantile of the standard normal distribution.

With clustered data, suppose that there are  $K_1$  and  $K_2$  clusters in population one and population two with the corresponding average cluster sizes of  $\bar{M}_1$  and  $\bar{M}_2$ . Let  $\rho_1$  and  $\rho_2$  be the intraclass correlations and  $CV_{\text{cl},1}$  and  $CV_{\text{cl},2}$  be the coefficients of variation for cluster sizes for population one and population two. Let  $C_{\text{adj},1}$  and  $C_{\text{adj},2}$  be the adjustments to standard errors of the two sample proportions for clustered data, defined analogously to  $C_{\text{adj}}$  in *One-sample test* for each population.

Let  $s_{d0,cl} = \sqrt{\hat{p}_p \hat{q}_p (C_{adj,1}^2/n_1 + C_{adj,2}^2/n_2)}$  be the standard error of  $\hat{p}_1 - \hat{p}_2$  under the null hypothesis of  $p_1 = p_2$  adjusted for clustered data. Also, let  $s_{1,cl} = C_{adj,1}s_1$  and  $s_{2,cl} = C_{adj,2}s_2$  be the standard errors of  $\hat{p}_1$  and  $\hat{p}_2$  adjusted for clustered data. To adjust the two-sample test statistic and the confidence interval for clustering, replace  $s_{d0}$  with  $s_{d0,cl}$ ,  $s_1$  with  $s_{1,cl}$ , and  $s_2$  with  $s_{2,cl}$  in the corresponding formulas.

## References

- Acock, A. C. 2023. *A Gentle Introduction to Stata*. Rev. 6th ed. College Station, TX: Stata Press.
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## Also see

- [R] **bitest** — Binomial probability test
- [R] **proportion** — Estimate proportions
- [R] **ttest** —  $t$  tests (mean-comparison tests)
- [MV] **hotelling** — Hotelling's  $T^2$  generalized means test
- [PSS-2] **power oneproportion** — Power analysis for a one-sample proportion test
- [PSS-2] **power oneproportion, cluster** — Power analysis for a one-sample proportion test, CRD
- [PSS-2] **power twoproportions** — Power analysis for a two-sample proportions test
- [PSS-2] **power twoproportions, cluster** — Power analysis for a two-sample proportions test, CRD

|                      |                |                      |            |          |
|----------------------|----------------|----------------------|------------|----------|
| Description          | Quick start    | Menu                 | Syntax     | Options  |
| Remarks and examples | Stored results | Methods and formulas | References | Also see |

## Description

`pwcompare` performs pairwise comparisons across the levels of factor variables from the most recently fit model. `pwcompare` can compare estimated cell means, marginal means, intercepts, marginal intercepts, slopes, or marginal slopes—collectively called margins. `pwcompare` reports the comparisons as contrasts (differences) of margins along with significance tests or confidence intervals for the contrasts. The tests and confidence intervals can be adjusted for multiple comparisons.

`pwcompare` can be used with `svy` estimation results; see [\[SVY\] svy postestimation](#).

See [\[R\] margins, pwcompare](#) for performing pairwise comparisons of margins of linear and nonlinear predictions.

## Quick start

All pairwise comparisons of the means of `y` across levels of `a` after `regress y i . a`

```
pwcompare a
```

Same as above, and report test statistics and *p*-values for tests of differences in means

```
pwcompare a, effects
```

Adjust *p*-values and confidence intervals for multiple comparisons using Tukey's method

```
pwcompare a, effects mcompare(tukey)
```

Same as above, but adjust for multiple comparisons using Bonferroni's method

```
pwcompare a, effects mcompare(bonferroni)
```

Report means for the levels of `a`, and group those that are not significantly different

```
pwcompare a, groups
```

Pairwise comparisons of cell means after `regress y1 a##b`

```
pwcompare a#b
```

Pairwise comparisons of the marginal means of `a`

```
pwcompare a
```

Pairwise comparisons of slopes for continuous `x` after `regress y1 a##c.x`

```
pwcompare a#c.x
```

Pairwise comparisons of log odds after `logit y2 i . a`

```
pwcompare a
```

Pairwise comparisons of the means of `y2` across levels of `a` after `mvreg y1 y2 y3 = i . a`

```
pwcompare a, equation(y2)
```

Same as above, but report pairwise comparisons of `a` for each equation

```
pwcompare a, atequations
```

Pairwise comparisons of overall margins of  $y_1$ ,  $y_2$ , and  $y_3$

```
pwcompare _eqns
```

## Menu

Statistics > Postestimation

## Syntax

```
pwcompare marginlist [, options]
```

where *marginlist* is a list of factor variables or interactions that appear in the current estimation results or *\_eqns* to reference equations. The variables may be typed with or without the *i.* prefix, and you may use any factor-variable syntax:

```
. pwcompare i.sex i.group i.sex#i.group
. pwcompare sex group sex#group
. pwcompare sex##group
```

| <i>options</i>                       | Description                                                                                                 |
|--------------------------------------|-------------------------------------------------------------------------------------------------------------|
| <b>Main</b>                          |                                                                                                             |
| <u>m</u> compare ( <i>method</i> )   | adjust for multiple comparisons; default is <code>mcompare(noadjust)</code>                                 |
| <u>a</u> sobserved                   | treat all factor variables as observed                                                                      |
| <b>Equations</b>                     |                                                                                                             |
| <u>e</u> quation ( <i>eqspec</i> )   | perform comparisons within equation <i>eqspec</i>                                                           |
| <u>a</u> tequations                  | perform comparisons within each equation                                                                    |
| <b>Advanced</b>                      |                                                                                                             |
| <u>e</u> mpycells ( <i>empspec</i> ) | treatment of empty cells for balanced factors                                                               |
| <u>n</u> oestimcheck                 | suppress estimability checks                                                                                |
| <b>Reporting</b>                     |                                                                                                             |
| <u>l</u> evel (#)                    | confidence level; default is <code>level(95)</code>                                                         |
| <u>c</u> ieffects                    | show effects table with confidence intervals; the default                                                   |
| <u>p</u> veffects                    | show effects table with <i>p</i> -values                                                                    |
| <u>e</u> ffects                      | show effects table with confidence intervals and <i>p</i> -values                                           |
| <u>c</u> imargins                    | show table of margins and confidence intervals                                                              |
| <u>g</u> roups                       | show table of margins and group codes                                                                       |
| <u>s</u> ort                         | sort the margins or contrasts within each term                                                              |
| <u>p</u> ost                         | post margins and their VCEs as estimation results                                                           |
| <u>d</u> isplay_options              | control column formats, row spacing, line width, and factor-variable labeling                               |
| <u>e</u> form_option                 | report exponentiated contrasts                                                                              |
| <u>d</u> f (#)                       | use <i>t</i> distribution with # degrees of freedom for computing <i>p</i> -values and confidence intervals |

`df (#)` does not appear in the dialog box.

| <i>method</i>                                      | Description                                         |
|----------------------------------------------------|-----------------------------------------------------|
| <code>noadjust</code>                              | do not adjust for multiple comparisons; the default |
| <code>bonferroni</code> [ <code>adjustall</code> ] | Bonferroni's method; adjust across all terms        |
| <code>sidak</code> [ <code>adjustall</code> ]      | Šidák's method; adjust across all terms             |
| <code>scheffe</code>                               | Scheffé's method                                    |
| * <code>tukey</code>                               | Tukey's method                                      |
| * <code>snk</code>                                 | Student–Newman–Keuls's method                       |
| * <code>duncan</code>                              | Duncan's method                                     |
| * <code>dunnett</code>                             | Dunnett's method                                    |

\*`tukey`, `snk`, `duncan`, and `dunnett` are only allowed with results from `anova`, `manova`, `regress`, and `mvreg`.  
`tukey`, `snk`, `duncan`, and `dunnett` are not allowed with results from `svy`.

Time-series operators are allowed if they were used in the estimation.

`collect` is allowed; see [U] 11.1.10 Prefix commands.

## Options

Main

`mcompare`(*method*) specifies the method for computing  $p$ -values and confidence intervals that account for multiple comparisons within a factor-variable term.

Most methods adjust the comparisonwise error rate,  $\alpha_c$ , to achieve a prespecified experimentwise error rate,  $\alpha_e$ .

`mcompare`(`noadjust`) is the default; it specifies no adjustment.

$$\alpha_c = \alpha_e$$

`mcompare`(`bonferroni`) adjusts the comparisonwise error rate based on the upper limit of the Bonferroni inequality:

$$\alpha_e \leq m\alpha_c$$

where  $m$  is the number of comparisons within the term.

The adjusted comparisonwise error rate is

$$\alpha_c = \alpha_e/m$$

`mcompare`(`sidak`) adjusts the comparisonwise error rate based on the upper limit of the probability inequality

$$\alpha_e \leq 1 - (1 - \alpha_c)^m$$

where  $m$  is the number of comparisons within the term.

The adjusted comparisonwise error rate is

$$\alpha_c = 1 - (1 - \alpha_e)^{1/m}$$

This adjustment is exact when the  $m$  comparisons are independent.

`mcompare`(`scheffe`) controls the experimentwise error rate using the  $F$  (or  $\chi^2$ ) distribution with degrees of freedom equal to the rank of the term.

For results from `anova`, `regress`, `manova`, and `mvreg` (see [R] `anova`, [R] `regress`, [MV] `manova`, and [MV] `mvreg`), `pwcompare` allows the following additional methods. These methods are not allowed with results that used `vce(robust)` or `vce(cluster clustvar)`.

`mcompare(tukey)` uses what is commonly referred to as Tukey’s honestly significant difference.

This method uses the Studentized range distribution instead of the  $t$  distribution.

`mcompare(snk)` is a variation on `mcompare(tukey)` that counts only the number of margins in the range for a given comparison instead of the full number of margins.

`mcompare(duncan)` is a variation on `mcompare(snk)` with additional adjustment to the significance probabilities.

`mcompare(dunnett)` uses Dunnett’s method for making comparisons with a reference category.

`mcompare(method adjustall)` specifies that the multiple-comparison adjustments count all comparisons across all terms rather than performing multiple comparisons term by term. This leads to more conservative adjustments when multiple variables or terms are specified in *marginlist*. This option is compatible only with the *bonferroni* and *sidak* methods.

`asobserved` specifies that factor covariates be evaluated using the cell frequencies observed when the model was fit. The default is to treat all factor covariates as though there were an equal number of observations at each level.

#### Equations

`equation(egspec)` specifies the equation from which margins are to be computed. The default is to compute margins from the first equation.

`atequations` specifies that the margins be computed within each equation.

#### Advanced

`emptycells(emptspec)` specifies how empty cells are handled in interactions involving factor variables that are being treated as balanced.

`emptycells(strict)` is the default; it specifies that margins involving empty cells be treated as not estimable.

`emptycells(reweight)` specifies that the effects of the observed cells be increased to accommodate any missing cells. This makes the margins estimable but changes their interpretation.

`noestimcheck` specifies that `pwcompare` not check for estimability. By default, the requested margins are checked and those found not estimable are reported as such. Nonestimability is usually caused by empty cells. If `noestimcheck` is specified, estimates are computed in the usual way and reported even though the resulting estimates are manipulable, which is to say they can differ across equivalent models having different parameterizations.

#### Reporting

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [U] 20.8 Specifying the width of confidence intervals. The significance level used by the `groups` option is  $100 - \#$ , expressed as a percentage.

`cieffects` specifies that a table of the pairwise comparisons with their standard errors and confidence intervals be reported. This is the default.

`pveffects` specifies that a table of the pairwise comparisons with their standard errors, test statistics, and  $p$ -values be reported.

`effects` specifies that a table of the pairwise comparisons with their standard errors, test statistics,  $p$ -values, and confidence intervals be reported.

`cimargins` specifies that a table of the margins with their standard errors and confidence intervals be reported.

`groups` specifies that a table of the margins with their standard errors and group codes be reported. Margins with the same letter in the group code are not significantly different at the specified significance level.

`sort` specifies that the reported tables be sorted on the margins or differences in each term.

`post` causes `pwcompare` to behave like a Stata estimation (`e-class`) command. `pwcompare` posts the vector of estimated margins along with the estimated variance–covariance matrix to `e()`, so you can treat the estimated margins just as you would results from any other estimation command. For example, you could use `test` to perform simultaneous tests of hypotheses on the margins, or you could use `lincom` to create linear combinations.

*display\_options*: `vsquish`, `nofvlabel`, `fvwrap(#)`, `fvwraon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`.

`vsquish` specifies that the blank space separating factor-variable terms or time-series–operated variables from other variables in the model be suppressed.

`nofvlabel` displays factor-variable level values rather than attached value labels. This option overrides the `fvlabel` setting; see [R] [set showbaselevels](#).

`fvwrap(#)` specifies how many lines to allow when long value labels must be wrapped. Labels requiring more than `#` lines are truncated. This option overrides the `fvwrap` setting; see [R] [set showbaselevels](#).

`fvwraon(style)` specifies whether value labels that wrap will break at word boundaries or break based on available space.

`fvwraon(word)`, the default, specifies that value labels break at word boundaries.

`fvwraon(width)` specifies that value labels break based on available space.

This option overrides the `fvwraon` setting; see [R] [set showbaselevels](#).

`cformat(%fmt)` specifies how to format contrasts or margins, standard errors, and confidence limits in the table of pairwise comparisons.

`pformat(%fmt)` specifies how to format *p*-values in the table of pairwise comparisons.

`sformat(%fmt)` specifies how to format test statistics in the table of pairwise comparisons.

`nolstretch` specifies that the width of the table of pairwise comparisons not be automatically widened to accommodate longer variable names. The default, `lstretch`, is to automatically widen the table of pairwise comparisons up to the width of the Results window. Specifying `lstretch` or `nolstretch` overrides the setting given by `set lstretch`. If `set lstretch` has not been set, the default is `lstretch`. `nolstretch` is not shown in the dialog box.

*eform\_option* specifies that the contrasts table be displayed in exponentiated form.  $e^{\text{contrast}}$  is displayed rather than `contrast`. Standard errors and confidence intervals are also transformed. See [R] [eform\\_option](#) for the list of available options.

The following option is available with `pwcompare` but is not shown in the dialog box:

`df(#)` specifies that the  $t$  distribution with  $\#$  degrees of freedom be used for computing  $p$ -values and confidence intervals. The default is to use `e(df_r)` degrees of freedom or the standard normal distribution if `e(df_r)` is missing.

## Remarks and examples

`pwcompare` performs pairwise comparisons of margins across the levels of factor variables from the most recently fit model. The margins can be estimated cell means, marginal means, intercepts, marginal intercepts, slopes, or marginal slopes. With the exception of slopes, we can also consider these margins to be marginal linear predictions.

The margins are calculated as linear combinations of the coefficients. Let  $k$  be the number of levels for a factor term in our model; then there are  $k$  margins for that term, and

$$m = \binom{k}{2} = \frac{k(k-1)}{2}$$

unique pairwise comparisons of those margins.

The confidence intervals and  $p$ -values for these pairwise comparisons can be adjusted to account for multiple comparisons. Bonferroni's, Šidák's, and Scheffé's adjustments can be made for multiple comparisons after fitting any type of model. In addition, Tukey's, Student–Newman–Keuls's, Duncan's, and Dunnett's adjustments are available when fitting ANOVA, linear regression, MANOVA, or multivariate regression models.

Remarks are presented under the following headings:

- Pairwise comparisons of means*
  - Marginal means*
  - All pairwise comparisons*
- Overview of multiple-comparison methods*
  - Fisher's protected least-significant difference (LSD)*
  - Bonferroni's adjustment*
  - Šidák's adjustment*
  - Scheffé's adjustment*
  - Tukey's HSD adjustment*
  - Student–Newman–Keuls's adjustment*
  - Duncan's adjustment*
  - Dunnett's adjustment*
- Example adjustments using one-way models*
  - Fisher's protected LSD*
  - Tukey's HSD*
  - Dunnett's method for comparisons to a control*
- Two-way models*
- Pairwise comparisons of slopes*
- Nonlinear models*
- Multiple-equation models*
- Unbalanced data*
- Empty cells*

## Pairwise comparisons of means

Suppose we are interested in the effects of five different fertilizers on wheat yield. We could estimate the following linear regression model to determine the effect of each type of fertilizer on the yield.

```
. use https://www.stata-press.com/data/r19/yield
(Artificial wheat yield dataset)
. regress yield i.fertilizer
```

| Source   | SS         | df  | MS         | Number of obs | = | 200    |
|----------|------------|-----|------------|---------------|---|--------|
| Model    | 1078.84207 | 4   | 269.710517 | F(4, 195)     | = | 5.33   |
| Residual | 9859.55334 | 195 | 50.561812  | Prob > F      | = | 0.0004 |
| Total    | 10938.3954 | 199 | 54.9668111 | R-squared     | = | 0.0986 |
|          |            |     |            | Adj R-squared | = | 0.0801 |
|          |            |     |            | Root MSE      | = | 7.1107 |

| yield      | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|------------|-------------|-----------|-------|-------|----------------------|----------|
| fertilizer |             |           |       |       |                      |          |
| 10-08-22   | 3.62272     | 1.589997  | 2.28  | 0.024 | .4869212             | 6.758518 |
| 16-04-08   | .4906299    | 1.589997  | 0.31  | 0.758 | -2.645169            | 3.626428 |
| 18-24-06   | 4.922803    | 1.589997  | 3.10  | 0.002 | 1.787005             | 8.058602 |
| 29-03-04   | -1.238328   | 1.589997  | -0.78 | 0.437 | -4.374127            | 1.89747  |
| _cons      | 41.36243    | 1.124298  | 36.79 | 0.000 | 39.14509             | 43.57977 |

In this simple case, the coefficients for fertilizers 10-08-22, 16-04-08, 18-24-06, and 29-03-04 indicate the difference in the mean yield for that fertilizer versus the mean yield for fertilizer 10-10-10. That the standard errors of all four coefficients are identical results from having perfectly balanced data.

## Marginal means

We can use `pwcompare` with the `cimargins` option to compute the mean yield for each of the fertilizers.

```
. pwcompare fertilizer, cimargins
Pairwise comparisons of marginal linear predictions
Margins: asbalanced
```

|            | Margin   | Std. err. | Unadjusted<br>[95% conf. interval] |          |
|------------|----------|-----------|------------------------------------|----------|
| fertilizer |          |           |                                    |          |
| 10-10-10   | 41.36243 | 1.124298  | 39.14509                           | 43.57977 |
| 10-08-22   | 44.98515 | 1.124298  | 42.7678                            | 47.20249 |
| 16-04-08   | 41.85306 | 1.124298  | 39.63571                           | 44.0704  |
| 18-24-06   | 46.28523 | 1.124298  | 44.06789                           | 48.50258 |
| 29-03-04   | 40.1241  | 1.124298  | 37.90676                           | 42.34145 |

Looking at the confidence intervals for fertilizers 10-10-10 and 10-08-22 in the table above, we might be tempted to conclude that these means are not significantly different because the intervals overlap. However, as discussed in *Interaction plots* of [R] `marginplot`, we cannot draw conclusions about the differences in means by looking at confidence intervals for the means themselves. Instead, we would need to look at confidence intervals for the difference in means.

## All pairwise comparisons

By default, pwcompare calculates all pairwise differences of the margins, in this case pairwise differences of the mean yields.

```
. pwcompare fertilizer
Pairwise comparisons of marginal linear predictions
Margins: asbalanced
```

|                      | Contrast  | Std. err. | Unadjusted<br>[95% conf. interval] |           |
|----------------------|-----------|-----------|------------------------------------|-----------|
| fertilizer           |           |           |                                    |           |
| 10-08-22 vs 10-10-10 | 3.62272   | 1.589997  | .4869212                           | 6.758518  |
| 16-04-08 vs 10-10-10 | .4906299  | 1.589997  | -2.645169                          | 3.626428  |
| 18-24-06 vs 10-10-10 | 4.922803  | 1.589997  | 1.787005                           | 8.058602  |
| 29-03-04 vs 10-10-10 | -1.238328 | 1.589997  | -4.374127                          | 1.89747   |
| 16-04-08 vs 10-08-22 | -3.13209  | 1.589997  | -6.267889                          | .0037086  |
| 18-24-06 vs 10-08-22 | 1.300083  | 1.589997  | -1.835715                          | 4.435882  |
| 29-03-04 vs 10-08-22 | -4.861048 | 1.589997  | -7.996847                          | -1.725249 |
| 18-24-06 vs 16-04-08 | 4.432173  | 1.589997  | 1.296375                           | 7.567972  |
| 29-03-04 vs 16-04-08 | -1.728958 | 1.589997  | -4.864757                          | 1.406841  |
| 29-03-04 vs 18-24-06 | -6.161132 | 1.589997  | -9.29693                           | -3.025333 |

If a confidence interval does not include zero, the means for the compared fertilizers are significantly different. Therefore, at the 5% significance level, we would reject the hypothesis that the means for fertilizers 10-10-10 and 10-08-22 are equivalent—as we would do for 18-24-06 vs 10-10-10, 29-03-04 vs 10-08-22, 18-24-06 vs 16-04-08, and 29-03-04 vs 18-24-06.

We may prefer to see the  $p$ -values instead of looking at confidence intervals to determine whether the pairwise differences are significantly different from zero. We could use the `pveffects` option to see the differences with standard errors and  $p$ -values, or we could use the `effects` option to see both  $p$ -values and confidence intervals in the same table. Here we specify `effects` as well as the `sort` option so that the differences are sorted from smallest to largest.

```
. pwcompare fertilizer, effects sort
Pairwise comparisons of marginal linear predictions
Margins: asbalanced
```

|            | Contrast  | Std. err. | Unadjusted |       | Unadjusted           |           |
|------------|-----------|-----------|------------|-------|----------------------|-----------|
|            |           |           | t          | P> t  | [95% conf. interval] |           |
| fertilizer |           |           |            |       |                      |           |
| 29-03-04   |           |           |            |       |                      |           |
| vs         |           |           |            |       |                      |           |
| 18-24-06   | -6.161132 | 1.589997  | -3.87      | 0.000 | -9.29693             | -3.025333 |
| 29-03-04   |           |           |            |       |                      |           |
| vs         |           |           |            |       |                      |           |
| 10-08-22   | -4.861048 | 1.589997  | -3.06      | 0.003 | -7.996847            | -1.725249 |
| 16-04-08   |           |           |            |       |                      |           |
| vs         |           |           |            |       |                      |           |
| 10-08-22   | -3.13209  | 1.589997  | -1.97      | 0.050 | -6.267889            | .0037086  |
| 29-03-04   |           |           |            |       |                      |           |
| vs         |           |           |            |       |                      |           |
| 16-04-08   | -1.728958 | 1.589997  | -1.09      | 0.278 | -4.864757            | 1.406841  |
| 29-03-04   |           |           |            |       |                      |           |
| vs         |           |           |            |       |                      |           |
| 10-10-10   | -1.238328 | 1.589997  | -0.78      | 0.437 | -4.374127            | 1.89747   |
| 16-04-08   |           |           |            |       |                      |           |
| vs         |           |           |            |       |                      |           |
| 10-10-10   | .4906299  | 1.589997  | 0.31       | 0.758 | -2.645169            | 3.626428  |
| 18-24-06   |           |           |            |       |                      |           |
| vs         |           |           |            |       |                      |           |
| 10-08-22   | 1.300083  | 1.589997  | 0.82       | 0.415 | -1.835715            | 4.435882  |
| 10-08-22   |           |           |            |       |                      |           |
| vs         |           |           |            |       |                      |           |
| 10-10-10   | 3.62272   | 1.589997  | 2.28       | 0.024 | .4869212             | 6.758518  |
| 18-24-06   |           |           |            |       |                      |           |
| vs         |           |           |            |       |                      |           |
| 16-04-08   | 4.432173  | 1.589997  | 2.79       | 0.006 | 1.296375             | 7.567972  |
| 18-24-06   |           |           |            |       |                      |           |
| vs         |           |           |            |       |                      |           |
| 10-10-10   | 4.922803  | 1.589997  | 3.10       | 0.002 | 1.787005             | 8.058602  |

We find that 5 of the 10 pairs of means are significantly different at the 5% significance level.

We can use the `groups` option to obtain a table that identifies groups whose means are not significantly different by assigning them the same letter.

```
. pwcompare fertilizer, groups sort
Pairwise comparisons of marginal linear predictions
Margins: asbalanced
```

|            | Margin   | Std. err. | Unadjusted groups |
|------------|----------|-----------|-------------------|
| fertilizer |          |           |                   |
| 29-03-04   | 40.1241  | 1.124298  | A                 |
| 10-10-10   | 41.36243 | 1.124298  | A                 |
| 16-04-08   | 41.85306 | 1.124298  | AB                |
| 10-08-22   | 44.98515 | 1.124298  | BC                |
| 18-24-06   | 46.28523 | 1.124298  | C                 |

Note: Margins sharing a letter in the group label are not significantly different at the 5% level.

The letter A that is assigned to fertilizers 29-03-04, 10-10-10, and 16-04-08 designates that the mean yields for these fertilizers are not different at the 5% level.

## Overview of multiple-comparison methods

For a single test, if we choose a 5% significance level, we would have a 5% chance of concluding that two margins are different when the population values are actually equal. This is known as making a type I error. When we perform  $m = k(k - 1)/2$  pairwise comparisons of the  $k$  margins, we have  $m$  opportunities to make a type I error.

`pwcompare` with the `mcompare()` option allows us to adjust the confidence intervals and  $p$ -values for each comparison to account for the increased probability of making a type I error when making multiple comparisons. Bonferroni's adjustment, Šidák's adjustment, and Scheffé's adjustment can be used when making pairwise comparisons of the margins after any estimation command. Tukey's honestly significant difference, Student–Newman–Keuls's method, Duncan's method, and Dunnett's method are only available when fitting linear models after `anova`, `manova`, `regress`, or `mvreg`.

## Fisher's protected least-significant difference (LSD)

`pwcompare` does not offer an `mcompare()` option specifically for Fisher's protected least-significant difference (LSD). In this methodology, no adjustment is made to the confidence intervals or  $p$ -values. However, it is protected in the sense that no pairwise comparisons are tested unless the joint test for the corresponding term in the model is significant. Therefore, the default `mcompare(noadjust)` corresponds to Fisher's protected LSD assuming that the corresponding joint test was performed before using `pwcompare`.

Milliken and Johnson (2009) recommend using this methodology for planned comparisons, assuming the corresponding joint test is significant.

## Bonferroni's adjustment

`mcompare(bonferroni)` adjusts significance levels based on the Bonferroni inequality, which, in the case of multiple testing, tells us that the maximum error rate for all comparisons is the sum of the error rates for the individual comparisons. Assuming that we are using the same significance level for all tests, the experimentwise error rate is the error rate for a single test multiplied by the number of comparisons. Therefore, a  $p$ -value for each comparison can be computed by multiplying the unadjusted  $p$ -value by the total number of comparisons. If the adjusted  $p$ -value is greater than 1, then `pwcompare` will report a  $p$ -value of 1.

Bonferroni's adjustment is popular because it is easy to compute manually and because it can be applied to any set of tests, not only the pairwise comparisons available in `pwcompare`. In addition, this method does not require equal sample sizes.

Because Bonferroni's adjustment is so general, it is more conservative than many of the other adjustments. It is especially conservative when a large number of tests is being performed.

## Šidák's adjustment

`mcompare(sidak)` performs an adjustment using Šidák's method. This adjustment, like Bonferroni's adjustment, is derived from an inequality. However, in this case, the inequality is based on the probability of not making a type I error. For a single test, the probability that we do not make a type I error is  $1 - \alpha$ . For two independent tests, both using  $\alpha$  as a significance level, the probability is  $(1 - \alpha)(1 - \alpha)$ . Likewise, for  $m$  independent tests, the probability of not making a type I error is  $(1 - \alpha)^m$ . Therefore, the probability of making one or more type I errors is  $1 - (1 - \alpha)^m$ . When tests are not independent, the probability of making at least one error is less than  $1 - (1 - \alpha)^m$ . Therefore, we can compute an adjusted  $p$ -value as  $1 - (1 - {}_u p)^m$ , where  ${}_u p$  is the unadjusted  $p$ -value for a single comparison.

Šidák's method is also conservative although slightly less so than Bonferroni's method. Like Bonferroni's method, this method does not require equal sample sizes.

## Scheffé's adjustment

Scheffé's adjustment is used when `mcompare(scheffe)` is specified. This adjustment is derived from the joint  $F$  test and its correspondence to the maximum normalized comparison. To adjust for multiple comparisons, the absolute value of the  $t$  statistic for a particular comparison can be compared with a critical value of  $\sqrt{(k-1)F_{k-1,\nu}}$ , where  $\nu$  is the residual degrees of freedom.  $F_{k-1,\nu}$  is the distribution of the joint  $F$  test for the corresponding term in a one-way ANOVA model. [Winer, Brown, and Michels \(1991, 191–195\)](#) discuss this in detail. For estimation commands that report  $z$  statistics instead of  $t$  statistics for the tests on coefficients, a  $\chi^2$  distribution is used instead of an  $F$  distribution.

Scheffé's method allows for making all possible comparisons of the  $k$  margins, not just the pairwise comparisons. Unlike the methods described above, it does not take into account the number of comparisons that are currently being made. Therefore, this method is even more conservative than the others. Because this method adjusts for all possible comparisons of the levels of the term, [Milliken and Johnson \(2009\)](#) recommend using this procedure when making unplanned contrasts that are suggested by the data. As [Winer, Brown, and Michels \(1991, 191\)](#) put it, this method is often used to adjust for “unfettered data snooping”. When using this adjustment, a contrast will never be significant if the joint  $F$  or  $\chi^2$  test for the term is not also significant.

This is another method that does not require equal sample sizes.

## Tukey's HSD adjustment

Tukey's adjustment is also referred to as Tukey's honestly significant difference (HSD) and is used when `mcompare(tukey)` is specified. It is often applied to all pairwise comparisons of means. Tukey's HSD is commonly used as a post hoc test although this is not a requirement.

To adjust for multiple comparisons, Tukey's method compares the absolute value of the  $t$  statistic from the individual comparison with a critical value based on a Studentized range distribution with parameter equal to the number of levels in the term. When applied to pairwise comparisons of means,

$$q = \frac{\text{mean}_{\max} - \text{mean}_{\min}}{s}$$

follows a Studentized range distribution with parameter  $k$  and  $\nu$  degrees of freedom. Here  $\text{mean}_{\max}$  and  $\text{mean}_{\min}$  are the largest and smallest marginal means, and  $s$  is an estimate of the standard error of the means.

Now for the comparison of the smallest and largest means, we can say that the probability of not making a type I error is

$$\Pr\left(\frac{\text{mean}_{\max} - \text{mean}_{\min}}{s} \leq q_{k,\nu}\right) = 1 - \alpha$$

Then, the following inequality holds for all pairs of means simultaneously:

$$\Pr\left(\frac{|\text{mean}_i - \text{mean}_j|}{s} \leq q_{k,\nu}\right) \geq 1 - \alpha$$

Based on this procedure, Tukey's HSD computes the  $p$ -value for each of the individual comparisons using the Studentized range distribution. However, because the equality holds only for the difference in the largest and smallest means, this procedure produces conservative tests for the remaining comparisons. [Winer, Brown, and Michels \(1991, 172–182\)](#) discuss this in further detail.

With unequal sample sizes, `mcompare(tukey)` produces the Tukey–Kramer adjustment ([Tukey 1953](#); [Kramer 1956](#)).

## Student–Newman–Keuls's adjustment

The Student–Newman–Keuls (SNK) method is used when `mcompare(snk)` is specified. It is a modification to Tukey's method and is less conservative. In this procedure, we first order the means. We then test the difference in the smallest and largest means using a critical value from the Studentized range distribution with parameter  $k$ , where  $k$  is the number of levels in the term. This step uses the same methodology as in Tukey's procedure. However, in the next step, we will then test for differences in the two sets of means that are the endpoints of the two ranges including  $k - 1$  means. Specifically, we test the difference in the smallest mean and the second-largest mean using a critical value from the Studentized range distribution with parameter  $k - 1$ . We would also test the difference in the second-smallest mean and the largest mean using this critical value. Likewise, the means that are the endpoints of ranges including  $k - 2$  means when ordered are tested using the Studentized range distribution with parameter  $k - 2$ , and so on.

Equal sample sizes are required for this method.

## Duncan's adjustment

When `mcompare(duncan)` is specified, tests are adjusted for multiple comparisons using Duncan's method, which is sometimes referred to as Duncan's new multiple range method. This adjustment produces tests that are less conservative than both Tukey's HSD and SNK. This procedure is performed in the same manner as SNK except that the  $p$ -values for the individual comparisons are adjusted as  $1 - (1 - \text{snk}p_i)^{1/(r+1)}$ , where  $\text{snk}p_i$  is the  $p$ -value computed using the SNK method and  $r$  represents the number of means that, when ordered, fall between the two that are being compared.

Again, equal sample sizes are required for this adjustment.

## Dunnett's adjustment

Dunnett's adjustment is obtained by specifying `mcompare(dunnett)`. It is used when one of the levels of a factor can be considered a control or reference level with which each of the other levels is being compared. When Dunnett's adjustment is requested,  $k - 1$  instead of  $k(k - 1)/2$  pairwise comparisons are made. [Dunnett \(1955, 1964\)](#) developed tables of critical values for what [Miller \(1981, 76\)](#) refers to as the "many-one  $t$  statistic". The  $t$  statistics for individual comparisons are compared with these critical values when making many comparisons to a single reference level.

This method also requires equal sample sizes.

## Example adjustments using one-way models

### Fisher's protected LSD

Fisher's protected LSD requires that we first verify that the joint test for a term in our model is significant before proceeding with pairwise comparisons. Using our [previous example](#), we could have first used the `contrast` command to obtain a joint test for the effects of fertilizer.

```
. contrast fertilizer
Contrasts of marginal linear predictions
Margins: asbalanced
```

|             | df  | F    | P>F    |
|-------------|-----|------|--------|
| fertilizer  | 4   | 5.33 | 0.0004 |
| Denominator | 195 |      |        |

This test for the effects of fertilizer is highly significant. Now we can say we are using Fisher's protected LSD when looking at the unadjusted  $p$ -values that were obtained from our previous command,

```
. pwcompare fertilizer, effects sort
```

## Tukey's HSD

Because we fit a linear regression model and are interested in all pairwise comparisons of the marginal means, we may instead choose to use Tukey's HSD.

```
. pwcompare fertilizer, effects sort mcompare(tukey)
Pairwise comparisons of marginal linear predictions
Margins: asbalanced
```

|            | Number of comparisons |
|------------|-----------------------|
| fertilizer | 10                    |

|            | Contrast  | Std. err. | Tukey<br>t | P> t  | Tukey<br>[95% conf. interval] |           |
|------------|-----------|-----------|------------|-------|-------------------------------|-----------|
| fertilizer |           |           |            |       |                               |           |
| 29-03-04   |           |           |            |       |                               |           |
| vs         |           |           |            |       |                               |           |
| 18-24-06   | -6.161132 | 1.589997  | -3.87      | 0.001 | -10.53914                     | -1.78312  |
| 29-03-04   |           |           |            |       |                               |           |
| vs         |           |           |            |       |                               |           |
| 10-08-22   | -4.861048 | 1.589997  | -3.06      | 0.021 | -9.239059                     | -.4830368 |
| 16-04-08   |           |           |            |       |                               |           |
| vs         |           |           |            |       |                               |           |
| 10-08-22   | -3.13209  | 1.589997  | -1.97      | 0.285 | -7.510101                     | 1.245921  |
| 29-03-04   |           |           |            |       |                               |           |
| vs         |           |           |            |       |                               |           |
| 16-04-08   | -1.728958 | 1.589997  | -1.09      | 0.813 | -6.106969                     | 2.649053  |
| 29-03-04   |           |           |            |       |                               |           |
| vs         |           |           |            |       |                               |           |
| 10-10-10   | -1.238328 | 1.589997  | -0.78      | 0.936 | -5.616339                     | 3.139683  |
| 16-04-08   |           |           |            |       |                               |           |
| vs         |           |           |            |       |                               |           |
| 10-10-10   | .4906299  | 1.589997  | 0.31       | 0.998 | -3.887381                     | 4.868641  |
| 18-24-06   |           |           |            |       |                               |           |
| vs         |           |           |            |       |                               |           |
| 10-08-22   | 1.300083  | 1.589997  | 0.82       | 0.925 | -3.077928                     | 5.678095  |
| 10-08-22   |           |           |            |       |                               |           |
| vs         |           |           |            |       |                               |           |
| 10-10-10   | 3.62272   | 1.589997  | 2.28       | 0.156 | -.7552913                     | 8.000731  |
| 18-24-06   |           |           |            |       |                               |           |
| vs         |           |           |            |       |                               |           |
| 16-04-08   | 4.432173  | 1.589997  | 2.79       | 0.046 | .0541623                      | 8.810185  |
| 18-24-06   |           |           |            |       |                               |           |
| vs         |           |           |            |       |                               |           |
| 10-10-10   | 4.922803  | 1.589997  | 3.10       | 0.019 | .5447922                      | 9.300815  |

This time, our *p*-values have been modified, and we find that only four of the pairwise differences are considered significantly different from zero at the 5% level.

If we are interested only in performing pairwise comparisons of a subset of our means, we can use factor-variable operators to select the levels of the factor that we want to compare. Here we exclude all comparisons involving fertilizer 10-10-10.

```
. pwcompare i(2/5).fertilizer, effects sort mcompare(tukey)
```

Pairwise comparisons of marginal linear predictions

Margins: asbalanced

|            | Number of comparisons |
|------------|-----------------------|
| fertilizer | 6                     |

|            | Contrast  | Std. err. | Tukey |       | Tukey                |           |
|------------|-----------|-----------|-------|-------|----------------------|-----------|
|            |           |           | t     | P> t  | [95% conf. interval] |           |
| fertilizer |           |           |       |       |                      |           |
| 29-03-04   |           |           |       |       |                      |           |
| vs         |           |           |       |       |                      |           |
| 18-24-06   | -6.161132 | 1.589997  | -3.87 | 0.001 | -10.28133            | -2.040937 |
| 29-03-04   |           |           |       |       |                      |           |
| vs         |           |           |       |       |                      |           |
| 10-08-22   | -4.861048 | 1.589997  | -3.06 | 0.013 | -8.981242            | -.7408538 |
| 16-04-08   |           |           |       |       |                      |           |
| vs         |           |           |       |       |                      |           |
| 10-08-22   | -3.13209  | 1.589997  | -1.97 | 0.203 | -7.252284            | .9881042  |
| 29-03-04   |           |           |       |       |                      |           |
| vs         |           |           |       |       |                      |           |
| 16-04-08   | -1.728958 | 1.589997  | -1.09 | 0.698 | -5.849152            | 2.391236  |
| 18-24-06   |           |           |       |       |                      |           |
| vs         |           |           |       |       |                      |           |
| 10-08-22   | 1.300083  | 1.589997  | 0.82  | 0.846 | -2.820111            | 5.420278  |
| 18-24-06   |           |           |       |       |                      |           |
| vs         |           |           |       |       |                      |           |
| 16-04-08   | 4.432173  | 1.589997  | 2.79  | 0.030 | .3119792             | 8.552368  |

The adjusted *p*-values and confidence intervals differ from those in the previous output because Tukey's adjustment takes into account the total number of comparisons being made when determining the appropriate degrees of freedom to use for the Studentized range distribution.

### Dunnett's method for comparisons to a control

If one of our five fertilizer groups represents fields where no fertilizer was applied, we may want to use Dunnett's method to compare each of the four fertilizers with the control group. In this case, we make only  $k - 1$  comparisons for  $k$  groups.

```
. pwcompare fertilizer, effects mcompare(dunnett)
Pairwise comparisons of marginal linear predictions
Margins: asbalanced
```

|            | Number of comparisons |
|------------|-----------------------|
| fertilizer | 4                     |

|            | Contrast  | Std. err. | Dunnett |       | Dunnett              |          |
|------------|-----------|-----------|---------|-------|----------------------|----------|
|            |           |           | t       | P> t  | [95% conf. interval] |          |
| fertilizer |           |           |         |       |                      |          |
| 10-08-22   |           |           |         |       |                      |          |
| vs         |           |           |         |       |                      |          |
| 10-10-10   | 3.62272   | 1.589997  | 2.28    | 0.079 | -.2918331            | 7.537273 |
| 16-04-08   |           |           |         |       |                      |          |
| vs         |           |           |         |       |                      |          |
| 10-10-10   | .4906299  | 1.589997  | 0.31    | 0.994 | -3.423923            | 4.405183 |
| 18-24-06   |           |           |         |       |                      |          |
| vs         |           |           |         |       |                      |          |
| 10-10-10   | 4.922803  | 1.589997  | 3.10    | 0.008 | 1.00825              | 8.837356 |
| 29-03-04   |           |           |         |       |                      |          |
| vs         |           |           |         |       |                      |          |
| 10-10-10   | -1.238328 | 1.589997  | -0.78   | 0.852 | -5.152881            | 2.676225 |

In our previous `regress` command, fertilizer 10-10-10 was treated as the base. Therefore, by default, it was treated as the control when using Dunnett's adjustment, and the pairwise comparisons are equivalent to the coefficients reported by `regress`. Based on our `regress` output, we would conclude that fertilizers 10-08-22 and 18-24-06 are different from fertilizer 10-10-10 at the 5% level. However, using Dunnett's adjustment, we find only fertilizer 18-24-06 to be different from fertilizer 10-10-10 at this same significance level.

If the model is fit without a base level for a factor variable, then `pwcompare` will choose the first level as the reference level. If we want to make comparisons with a different level than the one `mcompare(dunnett)` chooses by default, we can use the `b.` operator to override the default. Here we use fertilizer 5 (29-03-04) as the reference level.

```
. pwcompare b5.fertilizer, effects sort mcompare(dunnett)
```

Pairwise comparisons of marginal linear predictions

Margins: asbalanced

|            | Number of comparisons |
|------------|-----------------------|
| fertilizer | 4                     |

|                                                      | Contrast | Std. err. | Dunnett |       | Dunnett              |          |
|------------------------------------------------------|----------|-----------|---------|-------|----------------------|----------|
|                                                      |          |           | t       | P> t  | [95% conf. interval] |          |
| fertilizer<br>10-10-10<br>vs<br>29-03-04<br>16-04-08 | 1.238328 | 1.589997  | 0.78    | 0.852 | -2.676225            | 5.152881 |
| vs<br>29-03-04<br>10-08-22                           | 1.728958 | 1.589997  | 1.09    | 0.649 | -2.185595            | 5.643511 |
| vs<br>29-03-04<br>18-24-06                           | 4.861048 | 1.589997  | 3.06    | 0.009 | .9464951             | 8.775601 |
| vs<br>29-03-04                                       | 6.161132 | 1.589997  | 3.87    | 0.001 | 2.246579             | 10.07568 |

## Two-way models

In the previous examples, we have performed pairwise comparisons after fitting a model with a single factor. Now, we include two factors and their interaction in our model.

```
. regress yield fertilizer##irrigation
```

| Source   | SS         | df  | MS         | Number of obs = 200 |          |  |
|----------|------------|-----|------------|---------------------|----------|--|
| Model    | 6200.81605 | 9   | 688.979561 | F(9, 190)           | = 27.63  |  |
| Residual | 4737.57936 | 190 | 24.9346282 | Prob > F            | = 0.0000 |  |
|          |            |     |            | R-squared           | = 0.5669 |  |
|          |            |     |            | Adj R-squared       | = 0.5464 |  |
| Total    | 10938.3954 | 199 | 54.9668111 | Root MSE            | = 4.9935 |  |

| yield                                   | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|-----------------------------------------|-------------|-----------|-------|-------|----------------------|----------|
| fertilizer<br>10-08-22                  | 1.882256    | 1.57907   | 1.19  | 0.235 | -1.232505            | 4.997016 |
| 16-04-08                                | -.5687418   | 1.57907   | -0.36 | 0.719 | -3.683502            | 2.546019 |
| 18-24-06                                | 4.904999    | 1.57907   | 3.11  | 0.002 | 1.790239             | 8.01976  |
| 29-03-04                                | -1.217496   | 1.57907   | -0.77 | 0.442 | -4.332257            | 1.897264 |
| 1.irrigation                            | 8.899721    | 1.57907   | 5.64  | 0.000 | 5.784961             | 12.01448 |
| fertilizer#<br>irrigation<br>10-08-22#1 | 3.480928    | 2.233143  | 1.56  | 0.121 | -.9240084            | 7.885865 |
| 16-04-08#1                              | 2.118743    | 2.233143  | 0.95  | 0.344 | -2.286193            | 6.52368  |
| 18-24-06#1                              | .0356082    | 2.233143  | 0.02  | 0.987 | -4.369328            | 4.440545 |
| 29-03-04#1                              | -.0416636   | 2.233143  | -0.02 | 0.985 | -4.4466              | 4.363273 |
| _cons                                   | 36.91257    | 1.116571  | 33.06 | 0.000 | 34.7101              | 39.11504 |

We can perform pairwise comparisons of the cell means defined by the fertilizer and irrigation interaction.

```
. pwcompare fertilizer#irrigation, sort groups mcompare(tukey)
Pairwise comparisons of marginal linear predictions
Margins: asbalanced
```

|                       | Number of comparisons |
|-----------------------|-----------------------|
| fertilizer#irrigation | 45                    |

|                       | Margin   | Std. err. | Tukey groups |
|-----------------------|----------|-----------|--------------|
| fertilizer#irrigation |          |           |              |
| 29-03-04#0            | 35.69507 | 1.116571  | A            |
| 16-04-08#0            | 36.34383 | 1.116571  | A            |
| 10-10-10#0            | 36.91257 | 1.116571  | AB           |
| 10-08-22#0            | 38.79482 | 1.116571  | AB           |
| 18-24-06#0            | 41.81757 | 1.116571  | BC           |
| 29-03-04#1            | 44.55313 | 1.116571  | CD           |
| 10-10-10#1            | 45.81229 | 1.116571  | CDE          |
| 16-04-08#1            | 47.36229 | 1.116571  | DEF          |
| 18-24-06#1            | 50.7529  | 1.116571  | EF           |
| 10-08-22#1            | 51.17547 | 1.116571  | F            |

Note: Margins sharing a letter in the group label are not significantly different at the 5% level.

Based on Tukey's HSD and a 5% significance level, we would conclude that the mean yield for fertilizer 29-03-04 without irrigation is not significantly different from the mean yields for fertilizers 10-10-10, 10-08-22, and 16-04-08 when used without irrigation but is significantly different from the remaining means.

Up to this point, most of the pairwise comparisons that we have performed could have also been obtained with `pwmean` (see [R] [pwmean](#)) if we had not been interested in examining the results from the estimation command before making pairwise comparisons of the means. For instance, we could reproduce the results from the above `pwcompare` command by typing

```
. pwmean yield, over(fertilizer irrigation) sort group mcompare(tukey)
```

However, `pwcompare` extends the capabilities of `pwmean` in many ways. For instance, `pwmean` only allows for pairwise comparisons of the cell means determined by the highest-level interaction of the variables specified in the `over()` option. However, `pwcompare` allows us to fit a single model, such as the two-way model that we fit above,

```
. regress yield fertilizer##irrigation
```

and compute pairwise comparisons of the marginal means for only one of the variables in the model:

```
. pwcompare fertilizer, sort effects mcompare(tukey)
Pairwise comparisons of marginal linear predictions
Margins: asbalanced
```

|            | Number of comparisons |
|------------|-----------------------|
| fertilizer | 10                    |

|            | Contrast  | Std. err. | Tukey<br>t | P> t  | Tukey<br>[95% conf. interval] |
|------------|-----------|-----------|------------|-------|-------------------------------|
| fertilizer |           |           |            |       |                               |
| 29-03-04   |           |           |            |       |                               |
| vs         |           |           |            |       |                               |
| 18-24-06   | -6.161132 | 1.116571  | -5.52      | 0.000 | -9.236338 -3.085925           |
| 29-03-04   |           |           |            |       |                               |
| vs         |           |           |            |       |                               |
| 10-08-22   | -4.861048 | 1.116571  | -4.35      | 0.000 | -7.936255 -1.785841           |
| 16-04-08   |           |           |            |       |                               |
| vs         |           |           |            |       |                               |
| 10-08-22   | -3.13209  | 1.116571  | -2.81      | 0.044 | -6.207297 -.0568832           |
| 29-03-04   |           |           |            |       |                               |
| vs         |           |           |            |       |                               |
| 16-04-08   | -1.728958 | 1.116571  | -1.55      | 0.532 | -4.804165 1.346249            |
| 29-03-04   |           |           |            |       |                               |
| vs         |           |           |            |       |                               |
| 10-10-10   | -1.238328 | 1.116571  | -1.11      | 0.802 | -4.313535 1.836879            |
| 16-04-08   |           |           |            |       |                               |
| vs         |           |           |            |       |                               |
| 10-10-10   | .4906299  | 1.116571  | 0.44       | 0.992 | -2.584577 3.565837            |
| 18-24-06   |           |           |            |       |                               |
| vs         |           |           |            |       |                               |
| 10-08-22   | 1.300083  | 1.116571  | 1.16       | 0.772 | -1.775123 4.37529             |
| 10-08-22   |           |           |            |       |                               |
| vs         |           |           |            |       |                               |
| 10-10-10   | 3.62272   | 1.116571  | 3.24       | 0.012 | .5475131 6.697927             |
| 18-24-06   |           |           |            |       |                               |
| vs         |           |           |            |       |                               |
| 16-04-08   | 4.432173  | 1.116571  | 3.97       | 0.001 | 1.356967 7.50738              |
| 18-24-06   |           |           |            |       |                               |
| vs         |           |           |            |       |                               |
| 10-10-10   | 4.922803  | 1.116571  | 4.41       | 0.000 | 1.847597 7.99801              |

Here the standard errors for the differences in marginal means and the residual degrees of freedom are based on the full model. Therefore, the results will differ from those obtained from pwcompare after fitting the one-way model with only fertilizer (or equivalently using pwmean).

## Pairwise comparisons of slopes

If we fit a model with a factor variable that is interacted with a continuous variable, pwcompare will even allow us to make pairwise comparisons of the slopes of the continuous variable for the levels of the factor variable.

In this case, we have a continuous variable, N03\_N, indicating the amount of nitrate nitrogen already existing in the soil, based on a sample taken from each field.

```
. regress yield fertilizer##c.N03_N
```

| Source   | SS         | df  | MS         | Number of obs | = | 200    |
|----------|------------|-----|------------|---------------|---|--------|
| Model    | 7005.69932 | 9   | 778.411035 | F(9, 190)     | = | 37.61  |
| Residual | 3932.69609 | 190 | 20.6984005 | Prob > F      | = | 0.0000 |
|          |            |     |            | R-squared     | = | 0.6405 |
|          |            |     |            | Adj R-squared | = | 0.6234 |
| Total    | 10938.3954 | 199 | 54.9668111 | Root MSE      | = | 4.5495 |

| yield              | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|--------------------|-------------|-----------|-------|-------|----------------------|-----------|
| fertilizer         |             |           |       |       |                      |           |
| 10-08-22           | 18.65019    | 8.452061  | 2.21  | 0.029 | 1.97826              | 35.32212  |
| 16-04-08           | -13.34076   | 10.07595  | -1.32 | 0.187 | -33.21585            | 6.534327  |
| 18-24-06           | 24.35061    | 9.911463  | 2.46  | 0.015 | 4.799973             | 43.90125  |
| 29-03-04           | 17.58529    | 8.446736  | 2.08  | 0.039 | .9238646             | 34.24671  |
| N03_N              | 4.915653    | .7983509  | 6.16  | 0.000 | 3.340884             | 6.490423  |
| fertilizer#c.N03_N |             |           |       |       |                      |           |
| 10-08-22           | -1.282039   | .8953419  | -1.43 | 0.154 | -3.048126            | .4840487  |
| 16-04-08           | -1.00571    | .9025862  | -1.11 | 0.267 | -2.786087            | .7746662  |
| 18-24-06           | -2.97627    | .9136338  | -3.26 | 0.001 | -4.778438            | -1.174102 |
| 29-03-04           | -3.275947   | .8247385  | -3.97 | 0.000 | -4.902767            | -1.649127 |
| _cons              | -5.459168   | 7.638241  | -0.71 | 0.476 | -20.52581            | 9.607477  |

These are the pairwise differences of the slopes of N03\_N for each pair of fertilizers:

```
. pwcompare fertilizer#c.N03_N, pveffects sort mcompare(scheffe)
```

Pairwise comparisons of marginal linear predictions

Margins: asbalanced

|                    | Number of comparisons |
|--------------------|-----------------------|
| fertilizer#c.N03_N | 10                    |

| fertilizer#c.N03_N   | Contrast  | Std. err. | Scheffe |       |
|----------------------|-----------|-----------|---------|-------|
|                      |           |           | t       | P> t  |
| 29-03-04 vs 10-10-10 | -3.275947 | .8247385  | -3.97   | 0.004 |
| 18-24-06 vs 10-10-10 | -2.97627  | .9136338  | -3.26   | 0.034 |
| 29-03-04 vs 16-04-08 | -2.270237 | .4691771  | -4.84   | 0.000 |
| 29-03-04 vs 10-08-22 | -1.993909 | .4550851  | -4.38   | 0.001 |
| 18-24-06 vs 16-04-08 | -1.97056  | .612095   | -3.22   | 0.038 |
| 18-24-06 vs 10-08-22 | -1.694232 | .6013615  | -2.82   | 0.099 |
| 10-08-22 vs 10-10-10 | -1.282039 | .8953419  | -1.43   | 0.727 |
| 16-04-08 vs 10-10-10 | -1.00571  | .9025862  | -1.11   | 0.871 |
| 29-03-04 vs 18-24-06 | -.2996772 | .4900939  | -0.61   | 0.984 |
| 16-04-08 vs 10-08-22 | .276328   | .5844405  | 0.47    | 0.994 |

Using Scheffé’s adjustment, we find that five of the pairs have significantly different slopes at the 5% level.

## Nonlinear models

pwcompare can also perform pairwise comparisons of the marginal linear predictions after fitting a nonlinear model. For instance, we can use the dataset from *Beyond linear models* in [R] **contrast** and fit the following logistic regression model of patient satisfaction on hospital:

```
. use https://www.stata-press.com/data/r19/hospital
```

(Artificial hospital satisfaction data)

```
. logit satisfied i.hospital
```

Iteration 0: Log likelihood = -393.72216

Iteration 1: Log likelihood = -387.55736

Iteration 2: Log likelihood = -387.4768

Iteration 3: Log likelihood = -387.47679

Logistic regression

Number of obs = 802

LR chi2(2) = 12.49

Prob > chi2 = 0.0019

Pseudo R2 = 0.0159

Log likelihood = -387.47679

| satisfied | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |          |
|-----------|-------------|-----------|------|-------|----------------------|----------|
| hospital  |             |           |      |       |                      |          |
| 2         | .5348129    | .2136021  | 2.50 | 0.012 | .1161604             | .9534654 |
| 3         | .7354519    | .2221929  | 3.31 | 0.001 | .2999618             | 1.170942 |
| _cons     | 1.034708    | .1391469  | 7.44 | 0.000 | .7619855             | 1.307431 |

For this model, the marginal linear predictions are the predicted log odds for each hospital and can be obtained with the `cimargins` option:

```
. pwcompare hospital, cimargins
Pairwise comparisons of marginal linear predictions
Margins: asbalanced
```

|          | Margin   | Std. err. | Unadjusted<br>[95% conf. interval] |          |
|----------|----------|-----------|------------------------------------|----------|
| hospital |          |           |                                    |          |
| 1        | 1.034708 | .1391469  | .7619855                           | 1.307431 |
| 2        | 1.569521 | .1620618  | 1.251886                           | 1.887157 |
| 3        | 1.77016  | .1732277  | 1.43064                            | 2.10968  |

The pairwise comparisons are, therefore, differences in the log odds. We can specify `mcompare(bonferroni)` and `effects` to request Bonferroni-adjusted *p*-values and confidence intervals.

```
. pwcompare hospital, effects mcompare(bonferroni)
Pairwise comparisons of marginal linear predictions
Margins: asbalanced
```

|                       | Number of<br>comparisons |
|-----------------------|--------------------------|
| satisfied<br>hospital | 3                        |

|                       | Contrast | Std. err. | Bonferroni |       | Bonferroni<br>[95% conf. interval] |          |
|-----------------------|----------|-----------|------------|-------|------------------------------------|----------|
| satisfied<br>hospital |          |           |            |       |                                    |          |
| 2 vs 1                | .5348129 | .2136021  | 2.50       | 0.037 | .0234537                           | 1.046172 |
| 3 vs 1                | .7354519 | .2221929  | 3.31       | 0.003 | .2035265                           | 1.267377 |
| 3 vs 2                | .200639  | .2372169  | 0.85       | 1.000 | -.3672535                          | .7685314 |

For nonlinear models, only Bonferroni's adjustment, Šidák's adjustment, and Scheffé's adjustment are available.

If we want pairwise comparisons reported as odds ratios, we can specify the `or` option.

```
. pwcompare hospital, effects mcompare(bonferroni) or
Pairwise comparisons of marginal linear predictions
Margins: asbalanced
```

|                       | Number of<br>comparisons |
|-----------------------|--------------------------|
| satisfied<br>hospital | 3                        |

|                       | Odds ratio | Std. err. | Bonferroni |       | Bonferroni           |          |
|-----------------------|------------|-----------|------------|-------|----------------------|----------|
|                       |            |           | z          | P> z  | [95% conf. interval] |          |
| satisfied<br>hospital |            |           |            |       |                      |          |
| 2 vs 1                | 1.707129   | .3646464  | 2.50       | 0.037 | 1.023731             | 2.846733 |
| 3 vs 1                | 2.086425   | .4635888  | 3.31       | 0.003 | 1.225718             | 3.551525 |
| 3 vs 2                | 1.222183   | .2899226  | 0.85       | 1.000 | .6926341             | 2.156597 |

Notice that these tests are still performed on the marginal linear predictions. The odds ratios reported here are the exponentiated versions of the pairwise differences of log odds in the previous output. For further discussion, see [\[R\] contrast](#).

## Multiple-equation models

`pwcompare` works with models containing multiple equations. Commands such as `intreg` and `gnbreg` allow their ancillary parameters to be modeled as a function of independent variables, and `pwcompare` can compare the margins within these equations. The `equation()` option can be used to specify the equation for which pairwise comparisons of the margins should be made. The `atequations` option specifies that pairwise comparisons be computed for each equation. In addition, `pwcompare` allows a special pseudofactor for equation—called `_eqns`—when working with results from `manova`, `mvreg`, `mlogit`, and `mprobit`.

Here we use the jaw fracture dataset described in [example 4](#) of [\[MV\] manova](#). We fit a multivariate regression model including one independent factor variable, fracture.

```
. use https://www.stata-press.com/data/r19/jaw
(Table 4.6. Two-way unbalanced data for fractures of the jaw, Rencher (1998))
```

```
. mvreg y1 y2 y3 = i.fracture
```

| Equation | Obs | Parms | RMSE     | "R-sq" | F        | P>F    |
|----------|-----|-------|----------|--------|----------|--------|
| y1       | 27  | 3     | 10.42366 | 0.2966 | 5.060804 | 0.0147 |
| y2       | 27  | 3     | 6.325398 | 0.1341 | 1.858342 | 0.1777 |
| y3       | 27  | 3     | 5.976973 | 0.1024 | 1.368879 | 0.2735 |

|             | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|-------------|-------------|-----------|-------|-------|----------------------|----------|
| y1          |             |           |       |       |                      |          |
| fracture    |             |           |       |       |                      |          |
| Two compo.. | -8.833333   | 4.957441  | -1.78 | 0.087 | -19.06499            | 1.398322 |
| One simpl.. | 6           | 5.394759  | 1.11  | 0.277 | -5.134235            | 17.13423 |
| _cons       | 37          | 3.939775  | 9.39  | 0.000 | 28.8687              | 45.1313  |
| y2          |             |           |       |       |                      |          |
| fracture    |             |           |       |       |                      |          |
| Two compo.. | -5.761905   | 3.008327  | -1.92 | 0.067 | -11.97079            | .446977  |
| One simpl.. | -3.053571   | 3.273705  | -0.93 | 0.360 | -9.810166            | 3.703023 |
| _cons       | 38.42857    | 2.390776  | 16.07 | 0.000 | 33.49425             | 43.36289 |
| y3          |             |           |       |       |                      |          |
| fracture    |             |           |       |       |                      |          |
| Two compo.. | 4.261905    | 2.842618  | 1.50  | 0.147 | -1.60497             | 10.12878 |
| One simpl.. | .9285714    | 3.093377  | 0.30  | 0.767 | -5.455846            | 7.312989 |
| _cons       | 58.57143    | 2.259083  | 25.93 | 0.000 | 53.90891             | 63.23395 |

pwcompare performs pairwise comparisons of the margins using the coefficients from the first equation by default:

```
. pwcompare fracture, mcompare(bonferroni)
Pairwise comparisons of marginal linear predictions
Margins: asbalanced
```

|                | Number of comparisons |
|----------------|-----------------------|
| y1<br>fracture | 3                     |

|                                                                              | Contrast  | Std. err. | Bonferroni<br>[95% conf. interval] |          |
|------------------------------------------------------------------------------|-----------|-----------|------------------------------------|----------|
| y1<br>fracture                                                               |           |           |                                    |          |
| Two compound fractures<br>vs<br>One compound fracture<br>One simple fracture | -8.833333 | 4.957441  | -21.59201                          | 3.925341 |
| One compound fracture<br>vs<br>One simple fracture                           | 6         | 5.394759  | -7.884173                          | 19.88417 |
| Two compound fractures                                                       | 14.83333  | 4.75773   | 2.588644                           | 27.07802 |

We can use the equation() option to get pwcompare to perform comparisons in the y2 equation:

```
. pwcompare fracture, equation(y2) mcompare(bonferroni)
Pairwise comparisons of marginal linear predictions
Margins: asbalanced
```

|                | Number of comparisons |
|----------------|-----------------------|
| y2<br>fracture | 3                     |

|                                                                              | Contrast  | Std. err. | Bonferroni<br>[95% conf. interval] |          |
|------------------------------------------------------------------------------|-----------|-----------|------------------------------------|----------|
| y2<br>fracture                                                               |           |           |                                    |          |
| Two compound fractures<br>vs<br>One compound fracture<br>One simple fracture | -5.761905 | 3.008327  | -13.50426                          | 1.980449 |
| One compound fracture<br>vs<br>One simple fracture                           | -3.053571 | 3.273705  | -11.47891                          | 5.371769 |
| Two compound fractures                                                       | 2.708333  | 2.887136  | -4.722119                          | 10.13879 |

Because we are working with mvreg results, we can use the `_eqns` pseudofactor to compare the margins between the three dependent variables. The levels of `_eqns` index the equations: 1 for the first equation, 2 for the second, and 3 for the third.

```
. pwcompare _eqns, mcompare(bonferroni)
Pairwise comparisons of marginal linear predictions
Margins: asbalanced
```

|                    | Number of comparisons |
|--------------------|-----------------------|
| <code>_eqns</code> | 3                     |

|                    | Contrast  | Std. err. | Bonferroni<br>[95% conf. interval] |          |
|--------------------|-----------|-----------|------------------------------------|----------|
| <code>_eqns</code> |           |           |                                    |          |
| 2 vs 1             | -.5654762 | 2.545923  | -7.117768                          | 5.986815 |
| 3 vs 1             | 24.24603  | 2.320677  | 18.27344                           | 30.21862 |
| 3 vs 2             | 24.81151  | 2.368188  | 18.71664                           | 30.90637 |

For the previous command, the only methods available are `mcompare(bonferroni)`, `mcompare(sidak)`, or `mcompare(scheffe)`. Methods that use the Studentized range are not appropriate for making comparisons across equations.

## Unbalanced data

`pwcompare` treats all factors as balanced when it computes the marginal means. By “balanced”, we mean that the number of observations in each combination of factor levels (in each cell mean) is equal. We can alternatively specify the `asobserved` option when we have unbalanced data to obtain marginal means that are based on the observed cell frequencies from the model fit. For more details on the difference in these two types of marginal means and a discussion of when each may be appropriate, see [\[R\] margins](#) and [\[R\] contrast](#).

In addition, when our data are not balanced, some of the multiple-comparison adjustments are no longer appropriate. Student–Newman–Keuls’s method, Duncan’s method, and Dunnett’s method assume equal numbers of observations per group.

Here we use an unbalanced dataset and fit a two-way ANOVA model for cholesterol levels on race and age group. Then we perform pairwise comparisons of the mean cholesterol levels for each race, requesting Šidák’s adjustment as well as marginal means that are computed using the observed cell frequencies.

```
. use https://www.stata-press.com/data/r19/cholesterol3
(Artificial cholesterol data, unbalanced)
. anova chol race##agegrp

 Number of obs = 67 R-squared = 0.8179
 Root MSE = 8.37496 Adj R-squared = 0.7689

 Source | Partial SS df MS F Prob>F
-----+-----+-----+-----+-----+-----
 Model | 16379.993 14 1169.9995 16.68 0.0000
 race | 230.7544 2 115.3772 1.64 0.2029
 agegrp| 13857.988 4 3464.4969 49.39 0.0000
 race#agegrp| 857.81521 8 107.2269 1.53 0.1701
 Residual | 3647.2774 52 70.13995
-----+-----+-----+-----+-----
 Total | 20027.27 66 303.44349
```

```
. pwcompare race, asobserved mcompare(sidak)
Pairwise comparisons of marginal linear predictions
Margins: asobserved
```

|      | Number of comparisons |
|------|-----------------------|
| race | 3                     |

|                | Contrast  | Std. err. | Sidak<br>[95% conf. interval] |           |
|----------------|-----------|-----------|-------------------------------|-----------|
| race           |           |           |                               |           |
| White vs Black | -7.232433 | 2.686089  | -13.85924                     | -.6056277 |
| Other vs Black | -5.231198 | 2.651203  | -11.77194                     | 1.309541  |
| Other vs White | 2.001235  | 2.414964  | -3.956682                     | 7.959152  |

## Empty cells

An empty cell is a combination of the levels of factor variables that is not observed in the estimation sample. When we have empty cells in our data, the marginal means involving those empty cells are not estimable as described in [R] margins. In addition, all pairwise comparisons involving a marginal mean that is not estimable are themselves not estimable. Here we use a dataset where we do not have any observations for white individuals in the 20–29 age group. We can use the emptycells(reweight) option to reweight the nonempty cells so that we can estimate the marginal mean for whites and compute pairwise comparisons involving that marginal mean.

```
. use https://www.stata-press.com/data/r19/cholesterol2
(Artificial cholesterol data, empty cells)
. tabulate race agegrp
```

| Race  | Age group |       |       |       |       | Total |
|-------|-----------|-------|-------|-------|-------|-------|
|       | 10-19     | 20-29 | 30-39 | 40-59 | 60-79 |       |
| Black | 5         | 5     | 5     | 5     | 5     | 25    |
| White | 5         | 0     | 5     | 5     | 5     | 20    |
| Other | 5         | 5     | 5     | 5     | 5     | 25    |
| Total | 15        | 10    | 15    | 15    | 15    | 70    |

```
. anova chol race##agegrp
```

|             | Number of obs = | 70      | R-squared =     | 0.7582 |        |
|-------------|-----------------|---------|-----------------|--------|--------|
|             | Root MSE =      | 9.47055 | Adj R-squared = | 0.7021 |        |
| Source      | Partial SS      | df      | MS              | F      | Prob>F |
| Model       | 15751.611       | 13      | 1211.6624       | 13.51  | 0.0000 |
| race        | 305.49046       | 2       | 152.74523       | 1.70   | 0.1914 |
| agegrp      | 14387.856       | 4       | 3596.964        | 40.10  | 0.0000 |
| race#agegrp | 795.80757       | 7       | 113.6868        | 1.27   | 0.2831 |
| Residual    | 5022.7156       | 56      | 89.69135        |        |        |
| Total       | 20774.327       | 69      | 301.0772        |        |        |

```
. pwcompare race, emptycells(reweight)
```

Pairwise comparisons of marginal linear predictions

Margins: asbalanced  
Empty cells: reweight

|                | Contrast  | Std. err. | Unadjusted           |          |
|----------------|-----------|-----------|----------------------|----------|
|                |           |           | [95% conf. interval] |          |
| race           |           |           |                      |          |
| White vs Black | 2.922769  | 2.841166  | -2.768769            | 8.614308 |
| Other vs Black | -4.12621  | 2.678677  | -9.492244            | 1.239824 |
| Other vs White | -7.048979 | 2.841166  | -12.74052            | -1.35744 |

For further details on the emptycells(reweight) option, see [R] margins and [R] contrast.

## Stored results

pwcompare stores the following in `r()`:

### Scalars

|                          |                                          |
|--------------------------|------------------------------------------|
| <code>r(df_r)</code>     | variance degrees of freedom              |
| <code>r(k_terms)</code>  | number of terms in <i>marginlist</i>     |
| <code>r(level)</code>    | confidence level of confidence intervals |
| <code>r(balanced)</code> | 1 if fully balanced data, 0 otherwise    |

### Macros

|                                |                                                                               |
|--------------------------------|-------------------------------------------------------------------------------|
| <code>r(cmd)</code>            | pwcompare                                                                     |
| <code>r(cmdline)</code>        | command as typed                                                              |
| <code>r(est_cmd)</code>        | <code>e(cmd)</code> from original estimation results                          |
| <code>r(est_cmdline)</code>    | <code>e(cmdline)</code> from original estimation results                      |
| <code>r(title)</code>          | title in output                                                               |
| <code>r(emptycells)</code>     | <i>empspec</i> from <code>emptycells()</code>                                 |
| <code>r(groups#)</code>        | group codes for the #th margin in <code>r(b)</code>                           |
| <code>r(mcmethod_vs)</code>    | <i>method</i> from <code>mcompare()</code>                                    |
| <code>r(mctitle_vs)</code>     | title for <i>method</i> from <code>mcompare()</code>                          |
| <code>r(mcadjustall_vs)</code> | adjustall or empty                                                            |
| <code>r(margin_method)</code>  | asbalanced or asobserved                                                      |
| <code>r(vce)</code>            | <i>vcetype</i> specified in <code>vce()</code> in original estimation command |

### Matrices

|                          |                                                                                                                                  |
|--------------------------|----------------------------------------------------------------------------------------------------------------------------------|
| <code>r(b)</code>        | margin estimates                                                                                                                 |
| <code>r(V)</code>        | variance-covariance matrix of the margin estimates                                                                               |
| <code>r(error)</code>    | margin estimability codes;<br>0 means estimable,<br>8 means not estimable                                                        |
| <code>r(table)</code>    | matrix containing the margins with their standard errors, test statistics, <i>p</i> -values, and confidence intervals            |
| <code>r(M)</code>        | matrix that produces the margins from the model coefficients                                                                     |
| <code>r(b_vs)</code>     | margin difference estimates                                                                                                      |
| <code>r(V_vs)</code>     | variance-covariance matrix of the margin difference estimates                                                                    |
| <code>r(error_vs)</code> | margin difference estimability codes;<br>0 means estimable,<br>8 means not estimable                                             |
| <code>r(table_vs)</code> | matrix containing the margin differences with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |
| <code>r(L)</code>        | matrix that produces the margin differences from the model coefficients                                                          |
| <code>r(k_groups)</code> | number of significance groups for each term                                                                                      |

pwcompare with the post option also stores the following in e():

#### Scalars

|             |                                       |
|-------------|---------------------------------------|
| e(df_r)     | variance degrees of freedom           |
| e(k_terms)  | number of terms in <i>marginlist</i>  |
| e(balanced) | 1 if fully balanced data, 0 otherwise |

#### Macros

|                  |                                                                 |
|------------------|-----------------------------------------------------------------|
| e(cmd)           | pwcompare                                                       |
| e(cmdline)       | command as typed                                                |
| e(properties)    | b V                                                             |
| e(est_cmd)       | e(cmd) from original estimation results                         |
| e(est_cmdline)   | e(cmdline) from original estimation results                     |
| e(title)         | title in output                                                 |
| e(emptycells)    | <i>empspec</i> from emptycells()                                |
| e(margin_method) | asbalanced or asobserved                                        |
| e(asbalanced)    | factor variables fvset as asbalanced                            |
| e(asobserved)    | factor variables fvset as asobserved                            |
| e(vce)           | <i>vctype</i> specified in vce() in original estimation command |

#### Matrices

|             |                                                                                      |
|-------------|--------------------------------------------------------------------------------------|
| e(b)        | margin estimates                                                                     |
| e(V)        | variance–covariance matrix of the margin estimates                                   |
| e(error)    | margin estimability codes;<br>0 means estimable,<br>8 means not estimable            |
| e(M)        | matrix that produces the margins from the model coefficients                         |
| e(b_vs)     | margin difference estimates                                                          |
| e(V_vs)     | variance–covariance matrix of the margin difference estimates                        |
| e(error_vs) | margin difference estimability codes;<br>0 means estimable,<br>8 means not estimable |
| e(L)        | matrix that produces the margin differences from the model coefficients              |

## Methods and formulas

Methods and formulas are presented under the following headings:

*Notation*  
*Unadjusted comparisons*  
*Bonferroni’s method*  
*Šidák’s method*  
*Scheffé’s method*  
*Tukey’s method*  
*Student–Newman–Keuls’s method*  
*Duncan’s method*  
*Dunnnett’s method*

## Notation

pwcompare performs comparisons of margins; see *Methods and formulas* in [R] **contrast**.

If there are  $k$  margins for a given factor term, then there are

$$m = \binom{k}{2} = \frac{k(k-1)}{2}$$

unique pairwise comparisons. Let the  $i$ th pairwise comparison be denoted by

$$\hat{\delta}_i = l'_i \mathbf{b}$$

where  $\mathbf{b}$  is a column vector of coefficients from the fitted model and  $l_i$  is a column vector that forms the corresponding linear combination. If  $\hat{\mathbf{V}}$  denotes the estimated variance matrix for  $\mathbf{b}$ , then the standard error for  $\hat{\delta}_i$  is given by

$$\widehat{\text{se}}(\hat{\delta}_i) = \sqrt{l'_i \hat{\mathbf{V}} l_i}$$

The corresponding test statistic is then

$$t_i = \frac{\hat{\delta}_i}{\widehat{\text{se}}(\hat{\delta}_i)}$$

and the limits for a  $100(1 - \alpha)\%$  confidence interval for the expected value of  $\hat{\delta}_i$  are

$$\hat{\delta}_i \pm c_i(\alpha) \widehat{\text{se}}(\hat{\delta}_i)$$

where  $c_i(\alpha)$  is the critical value corresponding to the chosen multiple-comparison method.

## Unadjusted comparisons

pwcompare computes unadjusted  $p$ -values and confidence intervals by default. pwcompare uses the  $t$  distribution with  $\nu = e(\text{df\_r})$  degrees of freedom when  $e(\text{df\_r})$  is posted by the estimation command. The unadjusted two-sided  $p$ -value is

$${}_u p_i = 2 \Pr(t_\nu > |t_i|)$$

and the unadjusted critical value  ${}_u c_i(\alpha)$  satisfies the following probability statement:

$$\alpha = 2 \Pr\{t_\nu > {}_u c_i(\alpha)\}$$

pwcompare uses the standard normal distribution when  $e(\text{df\_r})$  is not posted.

## Bonferroni's method

For `mcompare(bonferroni)`, the adjusted  $p$ -value is

$${}_b p_i = \min(1, m {}_u p_i)$$

and the adjusted critical value is

$${}_b c_i(\alpha) = {}_u c_i(\alpha/m)$$

## Šidák's method

For `mcompare(sidak)`, the adjusted  $p$ -value is

$${}_{\text{si}} p_i = 1 - (1 - {}_u p_i)^m$$

and the adjusted critical value is

$${}_{\text{si}} c_i(\alpha) = {}_u c_i \{1 - (1 - \alpha)^{1/m}\}$$

## Scheffé's method

For `mcompare(scheffe)`, the adjusted  $p$ -value is

$${}_{\text{sc}} p_i = \Pr(F_{d,\nu} > t_i^2/d)$$

where  $F_{d,\nu}$  is distributed as an  $F$  with  $d$  numerator and  $\nu$  denominator degrees of freedom and  $d$  is the rank of the VCE for the term. The adjusted critical value satisfies the following probability statement:

$$\alpha = \Pr[F_{d,\nu} > \{ {}_{\text{sc}} c_i(\alpha) \}^2/d]$$

`pwcompare` uses the  $\chi^2$  distribution when `e(df_r)` is not posted.

## Tukey's method

For `mcompare(tukey)`, the adjusted  $p$ -value is

$${}_t p_i = \Pr(q_{k,\nu} > |t_i| \sqrt{2})$$

where  $q_{k,\nu}$  is distributed as the Studentized range statistic for  $k$  means and  $\nu$  residual degrees of freedom (Miller 1981). The adjusted critical value satisfies the following probability statement:

$$\alpha = \Pr\{q_{k,\nu} > {}_t c_i(\alpha) \sqrt{2}\}$$

## Student–Newman–Keuls’s method

For `mcompare(snk)`, suppose  $t_i$  is comparing two margins that have  $r$  other margins between them. Then the adjusted  $p$ -value is

$${}_{\text{snk}}p_i = \Pr(q_{r+2,\nu} > |t_i|\sqrt{2})$$

where  $r$  ranges from 0 to  $k - 2$ . The adjusted critical value  ${}_{\text{snk}}c_i(\alpha)$  satisfies the following probability statement:

$$\alpha = \Pr\{q_{r+2,\nu} > {}_{\text{snk}}c_i(\alpha)\sqrt{2}\}$$

## Duncan’s method

For `mcompare(duncan)`, the adjusted  $p$ -value is

$${}_{\text{dunc}}p_i = 1 - (1 - {}_{\text{snk}}p_i)^{1/(r+1)}$$

and the adjusted critical value is

$${}_{\text{dunc}}c_i(\alpha) = {}_{\text{snk}}c_i\{1 - (1 - \alpha)^{r+1}\}$$

## Dunnett’s method

For `mcompare(dunnett)`, the margins are compared with a reference category, resulting in only  $k - 1$  pairwise comparisons. The adjusted  $p$ -value is

$${}_{\text{dunn}}p_i = \Pr(d_{k-1,\nu} > |t_i|)$$

where  $d_{k-1,\nu}$  is distributed as the many-one  $t$  statistic (Miller 1981, 76). The adjusted critical value  ${}_{\text{dunn}}c_i(\alpha)$  satisfies the following probability statement:

$$\alpha = \Pr\{d_{k-1,\nu} > {}_{\text{dunn}}c_i(\alpha)\}$$

The multiple-comparison methods for `mcompare(tukey)`, `mcompare(snk)`, `mcompare(duncan)`, and `mcompare(dunnett)` assume the normal distribution with equal variance; thus, these methods are allowed only with results from `anova`, `regress`, `manova`, and `mvreg`. `mcompare(snk)`, `mcompare(duncan)`, and `mcompare(dunnett)` assume equal sample size for each marginal mean. These options will cause `pwcompare` to report a footnote if unbalanced factors are detected.

## References

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## Also see

- [R] **pwcompare postestimation** — Postestimation tools for pwcompare
- [R] **contrast** — Contrasts and linear hypothesis tests after estimation
- [R] **lincom** — Linear combinations of parameters
- [R] **margins** — Marginal means, predictive margins, and marginal effects
- [R] **margins, pwcompare** — Pairwise comparisons of margins
- [R] **pwmean** — Pairwise comparisons of means
- [R] **test** — Test linear hypotheses after estimation
- [U] **20 Estimation and postestimation commands**

## Postestimation commands

The following postestimation commands are available after `pwcompare`, `post`:

| Command                  | Description                                                                                       |
|--------------------------|---------------------------------------------------------------------------------------------------|
| <code>estat vce</code>   | variance–covariance matrix of the estimators (VCE)                                                |
| <code>estat (svy)</code> | postestimation statistics for survey data                                                         |
| <code>estimates</code>   | cataloging estimation results                                                                     |
| <code>etable</code>      | table of estimation results                                                                       |
| <code>lincom</code>      | point estimates, standard errors, testing, and inference for linear combinations of parameters    |
| <code>nlcom</code>       | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters |
| <code>test</code>        | Wald tests of simple and composite linear hypotheses                                              |
| <code>testnl</code>      | Wald tests of nonlinear hypotheses                                                                |

## Remarks and examples

When we use the `post` option with `pwcompare`, the marginal linear predictions are posted as estimation results, and we can use postestimation commands to perform further analysis on them.

In *Pairwise comparisons of means* of [R] `pwcompare`, we fit a regression of wheat yield on types of fertilizers.

```
. use https://www.stata-press.com/data/r19/yield
(Artificial wheat yield dataset)
. regress yield i.fertilizer
(output omitted)
```

We also used `pwcompare` with the `cimargins` option to obtain the marginal mean yield for each fertilizer. We can add the `post` option to this command to post these marginal means and their VCEs as estimation results.

```
. pwcompare fertilizer, cimargins post
Pairwise comparisons of marginal linear predictions
Margins: asbalanced
```

|            | Margin   | Std. err. | Unadjusted<br>[95% conf. interval] |          |
|------------|----------|-----------|------------------------------------|----------|
| fertilizer |          |           |                                    |          |
| 10-10-10   | 41.36243 | 1.124298  | 39.14509                           | 43.57977 |
| 10-08-22   | 44.98515 | 1.124298  | 42.7678                            | 47.20249 |
| 16-04-08   | 41.85306 | 1.124298  | 39.63571                           | 44.0704  |
| 18-24-06   | 46.28523 | 1.124298  | 44.06789                           | 48.50258 |
| 29-03-04   | 40.1241  | 1.124298  | 37.90676                           | 42.34145 |

Now, we can use `nlcom` to compute a percentage improvement in the mean yield for fertilizer 2 when compared with fertilizer 1.

```
. nlcom (pct_chg: 100*(_b[2.fertilizer] - _b[1.fertilizer])/_b[1.fertilizer])
 pct_chg: 100*(_b[2.fertilizer] - _b[1.fertilizer])/_b[1.fertilizer]
```

|         | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |          |
|---------|-------------|-----------|------|-------|----------------------|----------|
| pct_chg | 8.758479    | 4.015932  | 2.18 | 0.029 | .8873982             | 16.62956 |

The mean yield for fertilizer 2 is about 9% higher than that of fertilizer 1, with a standard error of 4%.

## Also see

[R] [pwcompare](#) — Pairwise comparisons

[U] [20 Estimation and postestimation commands](#)

Description  
Options  
Reference

Quick start  
Remarks and examples  
Also see

Menu  
Stored results

Syntax  
Methods and formulas

## Description

`pwmean` performs pairwise comparisons of means. It computes all pairwise differences of the means of *varname* over the combination of the levels of the variables in *varlist*. The tests and confidence intervals for the pairwise comparisons assume equal variances across groups. `pwmean` also allows for adjusting the confidence intervals and *p*-values to account for multiple comparisons using Bonferroni's method, Scheffé's method, Tukey's method, Dunnett's method, and others.

See [R] [pwcompare](#) for performing pairwise comparisons of means, estimated marginal means, and other types of marginal linear predictions after [anova](#), [regress](#), and most other estimation commands.

## Quick start

All pairwise differences in the means of *y* over levels of categorical variable *catvar*

```
pwmean y, over(catvar)
```

Same as above, and report test statistics and *p*-values for tests that differences equal zero

```
pwmean y, over(catvar) effects
```

Adjust *p*-values and confidence intervals for multiple comparisons using Tukey's method

```
pwmean y, over(catvar) effects mcompare(tukey)
```

Same as above, but adjust for multiple comparisons using Bonferroni's method

```
pwmean y, over(catvar) effects mcompare(bonferroni)
```

Report mean of *y* for each level of *catvar*, grouping means that are not significantly different

```
pwmean y, over(catvar) groups
```

## Menu

Statistics > Summaries, tables, and tests > Summary and descriptive statistics > Pairwise comparisons of means

## Syntax

pwmean *varname* [*if*] [*in*], over(*varlist*) [*options*]

| <i>options</i>                    | Description                                                                                               |
|-----------------------------------|-----------------------------------------------------------------------------------------------------------|
| Main                              |                                                                                                           |
| * <i>over</i> ( <i>varlist</i> )  | compare means across each combination of the levels in <i>varlist</i>                                     |
| <i>mcompare</i> ( <i>method</i> ) | adjust for multiple comparisons; default is <i>mcompare</i> ( <i>noadjust</i> )                           |
| Reporting                         |                                                                                                           |
| <i>level</i> (#)                  | confidence level; default is <i>level</i> (95)                                                            |
| <i>ci</i> effects                 | display a table of mean differences and confidence intervals; the default                                 |
| <i>p</i> effects                  | display a table of mean differences and <i>p</i> -values                                                  |
| <i>effects</i>                    | display a table of mean differences with <i>p</i> -values and confidence intervals                        |
| <i>ci</i> means                   | display a table of means and confidence intervals                                                         |
| <i>groups</i>                     | display a table of means with codes that group them with other means that are not significantly different |
| <i>sort</i>                       | sort results tables by displayed mean or difference                                                       |
| <i>display_options</i>            | control column formats, line width, and factor-variable labeling                                          |

\**over* (*varlist*) is required.

*collect* is allowed; see [U] 11.1.10 Prefix commands.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

| <i>method</i>     | Description                                         |
|-------------------|-----------------------------------------------------|
| <i>noadjust</i>   | do not adjust for multiple comparisons; the default |
| <i>bonferroni</i> | Bonferroni's method                                 |
| <i>sidak</i>      | Šidák's method                                      |
| <i>scheffe</i>    | Scheffé's method                                    |
| <i>tukey</i>      | Tukey's method                                      |
| <i>snk</i>        | Student–Newman–Keuls's method                       |
| <i>duncan</i>     | Duncan's method                                     |
| <i>dunnett</i>    | Dunnett's method                                    |

## Options

Main

*over* (*varlist*) is required and specifies that means are computed for each combination of the levels of the variables in *varlist*.

*mcompare* (*method*) specifies the method for computing *p*-values and confidence intervals that account for multiple comparisons.

Most methods adjust the comparisonwise error rate,  $\alpha_c$ , to achieve a prespecified experimentwise error rate,  $\alpha_e$ .

*mcompare*(*noadjust*) is the default; it specifies no adjustment.

$$\alpha_c = \alpha_e$$

`mcompare(bonferroni)` adjusts the comparisonwise error rate based on the upper limit of the Bonferroni inequality:

$$\alpha_e \leq m\alpha_c$$

where  $m$  is the number of comparisons within the term.

The adjusted comparisonwise error rate is

$$\alpha_c = \alpha_e/m$$

`mcompare(sidak)` adjusts the comparisonwise error rate based on the upper limit of the probability inequality

$$\alpha_e \leq 1 - (1 - \alpha_c)^m$$

where  $m$  is the number of comparisons within the term.

The adjusted comparisonwise error rate is

$$\alpha_c = 1 - (1 - \alpha_e)^{1/m}$$

This adjustment is exact when the  $m$  comparisons are independent.

`mcompare(scheffe)` controls the experimentwise error rate using the  $F$  (or  $\chi^2$ ) distribution with degrees of freedom equal to  $k - 1$  where  $k$  is the number of means being compared.

`mcompare(tukey)` uses what is commonly referred to as Tukey's honestly significant difference. This method uses the Studentized range distribution instead of the  $t$  distribution.

`mcompare(snk)` is a variation on `mcompare(tukey)` that counts only the number of means participating in the range for a given comparison instead of the full number of means.

`mcompare(duncan)` is a variation on `mcompare(snk)` with additional adjustment to the significance probabilities.

`mcompare(dunnett)` uses Dunnett's method for making comparisons with a reference category.

---

Reporting

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [U] 20.8 Specifying the width of confidence intervals. The significance level used by the `groups` option is `100 - #`, expressed as a percentage.

`cieffects` specifies that a table of the pairwise comparisons of means with their standard errors and confidence intervals be reported. This is the default.

`pveffects` specifies that a table of the pairwise comparisons of means with their standard errors, test statistics, and  $p$ -values be reported.

`effects` specifies that a table of the pairwise comparisons of means with their standard errors, test statistics,  $p$ -values, and confidence intervals be reported.

`cimeans` specifies that a table of the means with their standard errors and confidence intervals be reported.

`groups` specifies that a table of the means with their standard errors and group codes be reported. Means with the same letter in the group code are not significantly different at the specified significance level.

`sort` specifies that the reported tables be sorted by the mean or difference that is displayed in the table.

`display_options`: `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`.

- `nofvlabel` displays factor-variable level values rather than attached value labels. This option overrides the `fvlabel` setting; see [R] [set showbaselevels](#).
- `fvwrap(#)` specifies how many lines to allow when long value labels must be wrapped. Labels requiring more than `#` lines are truncated. This option overrides the `fvwrap` setting; see [R] [set showbaselevels](#).
- `fvwrapon(style)` specifies whether value labels that wrap will break at word boundaries or break based on available space.
- `fvwrapon(word)`, the default, specifies that value labels break at word boundaries.
  - `fvwrapon(width)` specifies that value labels break based on available space.
- This option overrides the `fvwrapon` setting; see [R] [set showbaselevels](#).
- `cformat(%fmt)` specifies how to format means, standard errors, and confidence limits in the table of pairwise comparison of means.
- `pformat(%fmt)` specifies how to format  $p$ -values in the table of pairwise comparison of means.
- `sformat(%fmt)` specifies how to format test statistics in the table of pairwise comparison of means.
- `no1stretch` specifies that the width of the table of pairwise comparisons not be automatically widened to accommodate longer variable names. The default, `1stretch`, is to automatically widen the table of pairwise comparisons up to the width of the Results window. Specifying `1stretch` or `no1stretch` overrides the setting given by [set 1stretch](#). If `set 1stretch` has not been set, the default is `1stretch`. `no1stretch` is not shown in the dialog box.

## Remarks and examples

`pwmean` performs pairwise comparisons (differences) of means, assuming a common variance among groups. It can easily adjust the  $p$ -values and confidence intervals for the differences to account for the elevated type I error rate due to multiple comparisons. Adjustments for multiple comparisons can be made using Bonferroni's method, Scheffé's method, Tukey's method, Dunnett's method, and others.

See [R] [margins](#), [pwcompare](#) for performing pairwise comparisons of marginal probabilities and other linear and nonlinear predictions after estimation commands.

Remarks are presented under the following headings:

- Group means*
- Pairwise differences of means*
- Group output*
- Adjusting for multiple comparisons*
  - Tukey's method*
  - Dunnett's method*
- Multiple over() variables*
- Equal variance assumption*

## Group means

Suppose we have data on the wheat yield of fields that were each randomly assigned an application of one of five types of fertilizers. Let's first look at the mean yield for each type of fertilizer.

```
. use https://www.stata-press.com/data/r19/yield
(Artificial wheat yield dataset)
```

```
. pwmean yield, over(fertilizer) cimeans
Pairwise comparisons of means with equal variances
Over: fertilizer
```

| yield      | Mean     | Std. err. | Unadjusted<br>[95% conf. interval] |          |
|------------|----------|-----------|------------------------------------|----------|
| fertilizer |          |           |                                    |          |
| 10-10-10   | 41.36243 | 1.124298  | 39.14509                           | 43.57977 |
| 10-08-22   | 44.98515 | 1.124298  | 42.7678                            | 47.20249 |
| 16-04-08   | 41.85306 | 1.124298  | 39.63571                           | 44.0704  |
| 18-24-06   | 46.28523 | 1.124298  | 44.06789                           | 48.50258 |
| 29-03-04   | 40.1241  | 1.124298  | 37.90676                           | 42.34145 |

### Pairwise differences of means

We can compute all pairwise differences in mean wheat yields for the types of fertilizers.

```
. pwmean yield, over(fertilizer) effects
Pairwise comparisons of means with equal variances
Over: fertilizer
```

| yield                      | Contrast  | Std. err. | Unadjusted<br>t P> t |       | Unadjusted<br>[95% conf. interval] |           |
|----------------------------|-----------|-----------|----------------------|-------|------------------------------------|-----------|
| fertilizer                 |           |           |                      |       |                                    |           |
| 10-08-22<br>vs<br>10-10-10 | 3.62272   | 1.589997  | 2.28                 | 0.024 | .4869212                           | 6.758518  |
| 16-04-08<br>vs<br>10-10-10 | .4906299  | 1.589997  | 0.31                 | 0.758 | -2.645169                          | 3.626428  |
| 18-24-06<br>vs<br>10-10-10 | 4.922803  | 1.589997  | 3.10                 | 0.002 | 1.787005                           | 8.058602  |
| 29-03-04<br>vs<br>10-10-10 | -1.238328 | 1.589997  | -0.78                | 0.437 | -4.374127                          | 1.89747   |
| 16-04-08<br>vs<br>10-08-22 | -3.13209  | 1.589997  | -1.97                | 0.050 | -6.267889                          | .0037086  |
| 18-24-06<br>vs<br>10-08-22 | 1.300083  | 1.589997  | 0.82                 | 0.415 | -1.835715                          | 4.435882  |
| 29-03-04<br>vs<br>10-08-22 | -4.861048 | 1.589997  | -3.06                | 0.003 | -7.996847                          | -1.725249 |
| 18-24-06<br>vs<br>16-04-08 | 4.432173  | 1.589997  | 2.79                 | 0.006 | 1.296375                           | 7.567972  |
| 29-03-04<br>vs<br>16-04-08 | -1.728958 | 1.589997  | -1.09                | 0.278 | -4.864757                          | 1.406841  |
| 29-03-04<br>vs<br>18-24-06 | -6.161132 | 1.589997  | -3.87                | 0.000 | -9.29693                           | -3.025333 |

The contrast in the row labeled (10-08-22 vs 10-10-10) is the difference in the mean wheat yield for fertilizer 10-08-22 and fertilizer 10-10-10. At a 5% significance level, we conclude that there is a difference in the means for these two fertilizers. Likewise, the rows labeled (18-24-06 vs 10-10-10), (29-03-04 vs 10-08-22), (18-24-06 vs 16-04-08) and (29-03-04 vs 18-24-06) show differences in these pairs of means. In all, we find that 5 of the 10 mean differences are significantly different from zero at a 5% significance level.

We can specify the sort option to order the differences from smallest to largest in the table.

```
. pwmean yield, over(fertilizer) effects sort
Pairwise comparisons of means with equal variances
Over: fertilizer
```

| yield      | Contrast  | Std. err. | Unadjusted<br>t | P> t  | Unadjusted<br>[95% conf. interval] |           |
|------------|-----------|-----------|-----------------|-------|------------------------------------|-----------|
| fertilizer |           |           |                 |       |                                    |           |
| 29-03-04   |           |           |                 |       |                                    |           |
| vs         |           |           |                 |       |                                    |           |
| 18-24-06   | -6.161132 | 1.589997  | -3.87           | 0.000 | -9.29693                           | -3.025333 |
| 29-03-04   |           |           |                 |       |                                    |           |
| vs         |           |           |                 |       |                                    |           |
| 10-08-22   | -4.861048 | 1.589997  | -3.06           | 0.003 | -7.996847                          | -1.725249 |
| 16-04-08   |           |           |                 |       |                                    |           |
| vs         |           |           |                 |       |                                    |           |
| 10-08-22   | -3.13209  | 1.589997  | -1.97           | 0.050 | -6.267889                          | .0037086  |
| 29-03-04   |           |           |                 |       |                                    |           |
| vs         |           |           |                 |       |                                    |           |
| 16-04-08   | -1.728958 | 1.589997  | -1.09           | 0.278 | -4.864757                          | 1.406841  |
| 29-03-04   |           |           |                 |       |                                    |           |
| vs         |           |           |                 |       |                                    |           |
| 10-10-10   | -1.238328 | 1.589997  | -0.78           | 0.437 | -4.374127                          | 1.89747   |
| 16-04-08   |           |           |                 |       |                                    |           |
| vs         |           |           |                 |       |                                    |           |
| 10-10-10   | .4906299  | 1.589997  | 0.31            | 0.758 | -2.645169                          | 3.626428  |
| 18-24-06   |           |           |                 |       |                                    |           |
| vs         |           |           |                 |       |                                    |           |
| 10-08-22   | 1.300083  | 1.589997  | 0.82            | 0.415 | -1.835715                          | 4.435882  |
| 10-08-22   |           |           |                 |       |                                    |           |
| vs         |           |           |                 |       |                                    |           |
| 10-10-10   | 3.62272   | 1.589997  | 2.28            | 0.024 | .4869212                           | 6.758518  |
| 18-24-06   |           |           |                 |       |                                    |           |
| vs         |           |           |                 |       |                                    |           |
| 16-04-08   | 4.432173  | 1.589997  | 2.79            | 0.006 | 1.296375                           | 7.567972  |
| 18-24-06   |           |           |                 |       |                                    |           |
| vs         |           |           |                 |       |                                    |           |
| 10-10-10   | 4.922803  | 1.589997  | 3.10            | 0.002 | 1.787005                           | 8.058602  |

Ordering the pairwise differences is particularly convenient when we are comparing means for a large number of groups.

## Group output

We can use the `group` option to see the mean of each group and a visual representation of the tests for differences.

```
. pwmean yield, over(fertilizer) group sort
Pairwise comparisons of means with equal variances
Over: fertilizer
```

| yield      | Mean     | Std. err. | Unadjusted groups |
|------------|----------|-----------|-------------------|
| fertilizer |          |           |                   |
| 29-03-04   | 40.1241  | 1.124298  | A                 |
| 10-10-10   | 41.36243 | 1.124298  | A                 |
| 16-04-08   | 41.85306 | 1.124298  | AB                |
| 10-08-22   | 44.98515 | 1.124298  | BC                |
| 18-24-06   | 46.28523 | 1.124298  | C                 |

Note: Means sharing a letter in the group label are not significantly different at the 5% level.

Fertilizers 29-03-04, 10-10-10, and 16-04-08 are all in group A. This means that at our 5% level of significance, we have insufficient information to distinguish their means. Likewise, fertilizers 16-04-08 and 10-08-22 are in group B and cannot be distinguished at the 5% level. The same is true for fertilizers 10-08-22 and 18-24-06 in group C.

Fertilizer 29-03-04 and fertilizer 10-08-22 have no letters in common, indicating that the mean yields of these two groups are significantly different at the 5% level. We can conclude that any other fertilizers without a letter in common have significantly different means as well.

## Adjusting for multiple comparisons

The statistics in the examples above take no account that we are performing 10 comparisons. With our 5% significance level and assuming the comparisons are independent, we expect 1 in 20 tests of comparisons to be significant, even if all the population means are truly the same. If we are performing many comparisons, then we should account for the fact that some tests will be found significant by chance alone. More formally, the test for each pairwise comparison is made without adjusting for the elevated type I experimentwise error rate that is introduced when performing multiple tests. We can use the `mcompare()` option to adjust the confidence intervals and  $p$ -values for multiple comparisons.

## Tukey's method

Of the available adjustments for multiple comparisons, Tukey's honestly significant difference, Student–Newman–Keuls's method, and Duncan's method are most often used when performing all pairwise comparisons of means. Of these, Tukey's method is the most conservative and Duncan's method is the least conservative. For further discussion of each of the multiple-comparison adjustments, see [R] [pwcompare](#).

Here we use Tukey’s adjustment to compute  $p$ -values and confidence intervals for the pairwise differences.

```
. pwmean yield, over(fertilizer) effects sort mcompare(tukey)
Pairwise comparisons of means with equal variances
Over: fertilizer
```

|            | Number of comparisons |
|------------|-----------------------|
| fertilizer | 10                    |

| yield      | Contrast  | Std. err. | Tukey<br>t | P> t  | Tukey<br>[95% conf. interval] |
|------------|-----------|-----------|------------|-------|-------------------------------|
| fertilizer |           |           |            |       |                               |
| 29-03-04   |           |           |            |       |                               |
| vs         |           |           |            |       |                               |
| 18-24-06   | -6.161132 | 1.589997  | -3.87      | 0.001 | -10.53914 -1.78312            |
| 29-03-04   |           |           |            |       |                               |
| vs         |           |           |            |       |                               |
| 10-08-22   | -4.861048 | 1.589997  | -3.06      | 0.021 | -9.239059 -.4830368           |
| 16-04-08   |           |           |            |       |                               |
| vs         |           |           |            |       |                               |
| 10-08-22   | -3.13209  | 1.589997  | -1.97      | 0.285 | -7.510101 1.245921            |
| 29-03-04   |           |           |            |       |                               |
| vs         |           |           |            |       |                               |
| 16-04-08   | -1.728958 | 1.589997  | -1.09      | 0.813 | -6.106969 2.649053            |
| 29-03-04   |           |           |            |       |                               |
| vs         |           |           |            |       |                               |
| 10-10-10   | -1.238328 | 1.589997  | -0.78      | 0.936 | -5.616339 3.139683            |
| 16-04-08   |           |           |            |       |                               |
| vs         |           |           |            |       |                               |
| 10-10-10   | .4906299  | 1.589997  | 0.31       | 0.998 | -3.887381 4.868641            |
| 18-24-06   |           |           |            |       |                               |
| vs         |           |           |            |       |                               |
| 10-08-22   | 1.300083  | 1.589997  | 0.82       | 0.925 | -3.077928 5.678095            |
| 10-08-22   |           |           |            |       |                               |
| vs         |           |           |            |       |                               |
| 10-10-10   | 3.62272   | 1.589997  | 2.28       | 0.156 | -.7552913 8.000731            |
| 18-24-06   |           |           |            |       |                               |
| vs         |           |           |            |       |                               |
| 16-04-08   | 4.432173  | 1.589997  | 2.79       | 0.046 | .0541623 8.810185             |
| 18-24-06   |           |           |            |       |                               |
| vs         |           |           |            |       |                               |
| 10-10-10   | 4.922803  | 1.589997  | 3.10       | 0.019 | .5447922 9.300815             |

When using a 5% significance level, Tukey’s adjustment indicates that four pairs of means are different. With the adjustment, we no longer conclude that the difference in the mean yields for fertilizers 10-08-22 and 10-10-10 is significantly different from zero.

### Dunnett's method

Now, let's suppose that fertilizer 10-10-10 actually represents fields on which no fertilizer was applied. In this case, we can use Dunnett's method for comparing each of the fertilizers with the control.

```
. pwmean yield, over(fertilizer) effects mcompare(dunnett)
Pairwise comparisons of means with equal variances
Over: fertilizer
```

|            | Number of comparisons |
|------------|-----------------------|
| fertilizer | 4                     |

| yield                                                | Contrast  | Std. err. | Dunnett |       | Dunnett              |          |
|------------------------------------------------------|-----------|-----------|---------|-------|----------------------|----------|
|                                                      |           |           | t       | P> t  | [95% conf. interval] |          |
| fertilizer<br>10-08-22<br>vs<br>10-10-10<br>16-04-08 | 3.62272   | 1.589997  | 2.28    | 0.079 | -.2918331            | 7.537273 |
| vs<br>10-10-10<br>18-24-06                           | .4906299  | 1.589997  | 0.31    | 0.994 | -3.423923            | 4.405183 |
| vs<br>10-10-10<br>29-03-04                           | 4.922803  | 1.589997  | 3.10    | 0.008 | 1.00825              | 8.837356 |
| vs<br>10-10-10                                       | -1.238328 | 1.589997  | -0.78   | 0.852 | -5.152881            | 2.676225 |

Using Dunnett's adjustment, we conclude that only fertilizer 4 (18-24-06) produces a mean yield that is significantly different from the mean yield of the field with no fertilizer applied.

By default, pwmean treats the lowest level of the group variable as the control. If, for instance, fertilizer 3 (16-04-08) was our control group, we could type

```
. pwmean yield, over(b3.fertilizer) effects mcompare(dunnett)
```

using the b3. factor-variable operator to specify this level as the reference level.

## Multiple over() variables

When we specify more than one variable in the `over()` option, pairwise comparisons are performed for the means defined by each combination of levels of these variables.

```
. pwmean yield, over(fertilizer irrigation) group
Pairwise comparisons of means with equal variances
Over: fertilizer irrigation
```

| yield                 | Mean     | Std. err. | Unadjusted groups |
|-----------------------|----------|-----------|-------------------|
| fertilizer#irrigation |          |           |                   |
| 10-10-10#0            | 36.91257 | 1.116571  | A                 |
| 10-10-10#1            | 45.81229 | 1.116571  | B                 |
| 10-08-22#0            | 38.79482 | 1.116571  | A C               |
| 10-08-22#1            | 51.17547 | 1.116571  | E                 |
| 16-04-08#0            | 36.34383 | 1.116571  | A                 |
| 16-04-08#1            | 47.36229 | 1.116571  | B                 |
| 18-24-06#0            | 41.81757 | 1.116571  | CD                |
| 18-24-06#1            | 50.7529  | 1.116571  | E                 |
| 29-03-04#0            | 35.69507 | 1.116571  | A                 |
| 29-03-04#1            | 44.55313 | 1.116571  | B D               |

Note: Means sharing a letter in the group label are not significantly different at the 5% level.

Here the row labeled 10-10-10#0 is the mean for the fields treated with fertilizer 10-10-10 and without irrigation. This mean is significantly different from the mean of all fertilizer/irrigation pairings that do not have an A in the “Unadjusted groups” column. These include all pairings where the fields were irrigated as well as the fields treated with fertilizer 18-24-06 but without irrigation.

## Equal variance assumption

`pwmean` performs multiple comparisons assuming that there is a common variance for all groups. In the case of two groups, this is equivalent to performing the familiar two-sample *t* test when equal variances are assumed.

```
. ttest yield, by(irrigation)
Two-sample t test with equal variances
```

| Group    | Obs | Mean      | Std. err. | Std. dev. | [95% conf. interval] |           |
|----------|-----|-----------|-----------|-----------|----------------------|-----------|
| 0        | 100 | 37.91277  | .5300607  | 5.300607  | 36.86102             | 38.96453  |
| 1        | 100 | 47.93122  | .5630353  | 5.630353  | 46.81403             | 49.0484   |
| Combined | 200 | 42.92199  | .5242462  | 7.413961  | 41.8882              | 43.95579  |
| diff     |     | -10.01844 | .7732872  |           | -11.54338            | -8.493509 |

```
diff = mean(0) - mean(1) t = -12.9557
HO: diff = 0 Degrees of freedom = 198
Ha: diff < 0 Ha: diff != 0
Pr(T < t) = 0.0000 Pr(|T| > |t|) = 0.0000 Ha: diff > 0
Pr(T > t) = 1.0000
```

```
. pwmean yield, over(irrigation) effects
Pairwise comparisons of means with equal variances
Over: irrigation
```

| yield                | Contrast | Std. err. | Unadjusted<br>t | P> t  | Unadjusted<br>[95% conf. interval] |          |
|----------------------|----------|-----------|-----------------|-------|------------------------------------|----------|
| irrigation<br>1 vs 0 | 10.01844 | .7732872  | 12.96           | 0.000 | 8.493509                           | 11.54338 |

The signs for the difference, the test statistic, and the confidence intervals are reversed because the difference is taken in the opposite direction. The  $p$ -value from `pwmean` is equivalent to the one for the two-sided test in the `ttest` output.

`pwmean` extends the capabilities of `ttest` to allow for simultaneously comparing all pairs of means and to allow for using one common variance estimate for all the tests instead of computing a separate pooled variance for each pair of means when using multiple `ttest` commands. In addition, `pwmean` allows adjustments for multiple comparisons, many of which rely on an assumption of equal variances among groups.

## Stored results

`pwmean` stores the following in `e()`:

### Scalars

`e(df_r)` variance degrees of freedom  
`e(balanced)` 1 if fully balanced data, 0 otherwise

### Macros

`e(cmd)` `pwmean`  
`e(cmdline)` command as typed  
`e(title)` title in output  
`e(depvar)` name of variable from which the means are computed  
`e(over)` *varlist* from `over()`  
`e(properties)` `b V`

### Matrices

`e(b)` mean estimates  
`e(V)` variance–covariance matrix of the mean estimates  
`e(error)` mean estimability codes;  
     0 means estimable,  
     8 means not estimable  
`e(b_vs)` mean difference estimates  
`e(V_vs)` variance–covariance matrix of the mean difference estimates  
`e(error_vs)` mean difference estimability codes;  
     0 means estimable,  
     8 means not estimable

In addition to the above, the following is stored in `r()`:

#### Scalars

`r(level)` confidence level of confidence intervals

#### Macros

`r(groups#)` group codes for the #th margin in `r(b)`

`r(mcmethod_vs)` *method* from `mcompare()`

`r(mctitle_vs)` title for *method* from `mcompare()`

#### Matrices

`r(table)` matrix containing the coefficients with their standard errors, test statistics, *p*-values, and confidence intervals

`r(table_vs)` matrix containing the margin differences with their standard errors, test statistics, *p*-values, and confidence intervals

`r(k_groups)` number of significance groups for each term

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

`pwmean` is a convenience command that uses `pwcompare` after fitting a fully factorial linear model. See [Methods and formulas](#) described in [\[R\] pwcompare](#).

## Reference

Searle, S. R. 1997. *Linear Models for Unbalanced Data*. New York: Wiley.

## Also see

[\[R\] pwmean postestimation](#) — Postestimation tools for `pwmean`

[\[R\] contrast](#) — Contrasts and linear hypothesis tests after estimation

[\[R\] margins](#) — Marginal means, predictive margins, and marginal effects

[\[R\] margins, pwcompare](#) — Pairwise comparisons of margins

[\[R\] pwcompare](#) — Pairwise comparisons

[\[R\] ttest](#) — *t* tests (mean-comparison tests)

[\[U\] 20 Estimation and postestimation commands](#)

## Postestimation commands

The following postestimation commands are available after pwmean:

| Command                | Description                                                                                       |
|------------------------|---------------------------------------------------------------------------------------------------|
| <code>estat vce</code> | variance–covariance matrix of the estimators (VCE)                                                |
| <code>estimates</code> | cataloging estimation results                                                                     |
| <code>etable</code>    | table of estimation results                                                                       |
| <code>lincom</code>    | point estimates, standard errors, testing, and inference for linear combinations of parameters    |
| <code>nlcom</code>     | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters |
| <code>test</code>      | Wald tests of simple and composite linear hypotheses                                              |
| <code>testnl</code>    | Wald tests of nonlinear hypotheses                                                                |

## Remarks and examples

In *Pairwise differences of means* of [R] **pwmean**, we computed all pairwise differences in mean wheat yields for five fertilizers.

```
. use https://www.stata-press.com/data/r19/yield
(Artificial wheat yield dataset)
. pwmean yield, over(fertilizer)
Pairwise comparisons of means with equal variances
Over: fertilizer
```

| yield                | Contrast  | Std. err. | Unadjusted<br>[95% conf. interval] |           |
|----------------------|-----------|-----------|------------------------------------|-----------|
| fertilizer           |           |           |                                    |           |
| 10-08-22 vs 10-10-10 | 3.62272   | 1.589997  | .4869212                           | 6.758518  |
| 16-04-08 vs 10-10-10 | .4906299  | 1.589997  | -2.645169                          | 3.626428  |
| 18-24-06 vs 10-10-10 | 4.922803  | 1.589997  | 1.787005                           | 8.058602  |
| 29-03-04 vs 10-10-10 | -1.238328 | 1.589997  | -4.374127                          | 1.89747   |
| 16-04-08 vs 10-08-22 | -3.13209  | 1.589997  | -6.267889                          | .0037086  |
| 18-24-06 vs 10-08-22 | 1.300083  | 1.589997  | -1.835715                          | 4.435882  |
| 29-03-04 vs 10-08-22 | -4.861048 | 1.589997  | -7.996847                          | -1.725249 |
| 18-24-06 vs 16-04-08 | 4.432173  | 1.589997  | 1.296375                           | 7.567972  |
| 29-03-04 vs 16-04-08 | -1.728958 | 1.589997  | -4.864757                          | 1.406841  |
| 29-03-04 vs 18-24-06 | -6.161132 | 1.589997  | -9.29693                           | -3.025333 |

After `pwmean`, we can use `testnl` to test whether the improvement in mean wheat yield when using fertilizer 18-24-06 instead of fertilizer 29-03-04 is significantly different from 10%.

```
. testnl (_b[4.fertilizer] - _b[5.fertilizer])/_b[5.fertilizer] = 0.1
(1) (_b[4.fertilizer] - _b[5.fertilizer])/_b[5.fertilizer] = 0.1
 chi2(1) = 1.57
 Prob > chi2 = 0.2106
```

The improvement is not significantly different from 10%.

## Also see

[R] [pwmean](#) — Pairwise comparisons of means

[U] [20 Estimation and postestimation commands](#)

[Description](#)[Remarks and examples](#)[Quick start](#)[Stored results](#)[Menu](#)[Methods and formulas](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

## Description

These commands provide standard quality-control charts. `cchart` draws a  $c$  chart; `pchart`, a  $p$  (fraction-defective) chart; `rchart`, an  $R$  (range or dispersion) chart; `xchart`, an  $\bar{X}$  (control line) chart; and `shewhart`, vertically aligned  $\bar{X}$  and  $R$  charts.

## Quick start

$c$  chart for `dvar` defects per unit identified by `uvar`

```
cchart dvar uvar
```

$p$  chart for `dvar` defective items out of `nvar` items inspected from each unit identified by `uvar`

```
pchart dvar uvar nvar
```

Same as above, but stabilize the  $p$  chart for unequal numbers of items inspected per unit

```
pchart dvar uvar nvar, stabilized
```

$R$  chart for the range of measurements `m1`, `m2`, `m3`, and `m4`

```
rchart m1 m2 m3 m4, connect(1)
```

Same as above, but use known process standard deviation of 0.5 for control limits

```
rchart m1 m2 m3 m4, connect(1) std(.5)
```

$\bar{X}$  chart for measurements `m1`, `m2`, `m3`, and `m4`

```
xchart m1 m2 m3 m4, connect(1)
```

Same as above, but use known process standard deviation of 0.5 and grand mean of 10 for control limits

```
xchart m1 m2 m3 m4, connect(1) std(.5) mean(10)
```

Shewhart chart with vertically aligned  $R$  and  $\bar{X}$  charts

```
shewhart m1 m2 m3 m4, connect(1) std(.5) mean(10)
```

## Menu

### cchart

Statistics > Other > Quality control > C chart

### pchart

Statistics > Other > Quality control > P chart

### rchart

Statistics > Other > Quality control > R chart

### xchart

Statistics > Other > Quality control > X-bar chart

### shewhart

Statistics > Other > Quality control > Vertically aligned X-bar and R chart

## Syntax

Draw a *c* chart

```
cchart defect_var unit_var [, cchart_options]
```

Draw a *p* (fraction-defective) chart

```
pchart reject_var unit_var ssize_var [, pchart_options]
```

Draw an *R* (range or dispersion) chart

```
rchart varlist [if] [in] [, rchart_options]
```

Draw an  $\bar{X}$  (control line) chart

```
xchart varlist [if] [in] [, xchart_options]
```

Draw vertically aligned  $\bar{X}$  and *R* charts

```
shewhart varlist [if] [in] [, shewhart_options]
```

| <i>cchart_options</i>                                                                                 | Description                                                                      |
|-------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------|
| Main                                                                                                  |                                                                                  |
| <u>nograph</u>                                                                                        | suppress graph                                                                   |
| Plot                                                                                                  |                                                                                  |
| <i>connect_options</i>                                                                                | affect rendition of the plotted points                                           |
| <i>marker_options</i>                                                                                 | change look of markers (color, size, etc.)                                       |
| <i>marker_label_options</i>                                                                           | add marker labels; change look or position                                       |
| Control limits                                                                                        |                                                                                  |
| <u>clopts</u> ( <i>cline_options</i> )                                                                | affect rendition of the control limits                                           |
| Add plots                                                                                             |                                                                                  |
| <u>addplot</u> ( <i>plot</i> )                                                                        | add other plots to the generated graph                                           |
| Y axis, X axis, Titles, Legend, Overall                                                               |                                                                                  |
| <i>twoway_options</i>                                                                                 | any options other than by() documented in [G-3] <i>twoway_options</i>            |
| <hr/>                                                                                                 |                                                                                  |
| <i>pchart_options</i>                                                                                 | Description                                                                      |
| Main                                                                                                  |                                                                                  |
| <u>stabilized</u>                                                                                     | stabilize the p chart when sample sizes are unequal                              |
| <u>nograph</u>                                                                                        | suppress graph                                                                   |
| <u>generate</u> ( <i>newvar<sub>f</sub></i> <i>newvar<sub>lcl</sub></i> <i>newvar<sub>ucl</sub></i> ) | store the fractions of defective elements and the lower and upper control limits |
| Plot                                                                                                  |                                                                                  |
| <i>connect_options</i>                                                                                | affect rendition of the plotted points                                           |
| <i>marker_options</i>                                                                                 | change look of markers (color, size, etc.)                                       |
| <i>marker_label_options</i>                                                                           | add marker labels; change look or position                                       |
| Control limits                                                                                        |                                                                                  |
| <u>clopts</u> ( <i>cline_options</i> )                                                                | affect rendition of the control limits                                           |
| Add plots                                                                                             |                                                                                  |
| <u>addplot</u> ( <i>plot</i> )                                                                        | add other plots to the generated graph                                           |
| Y axis, X axis, Titles, Legend, Overall                                                               |                                                                                  |
| <i>twoway_options</i>                                                                                 | any options other than by() documented in [G-3] <i>twoway_options</i>            |
| <hr/>                                                                                                 |                                                                                  |

| <i>rchart_options</i>                   | Description                                                           |
|-----------------------------------------|-----------------------------------------------------------------------|
| Main                                    |                                                                       |
| <u>std</u> (#)                          | user-specified standard deviation                                     |
| <u>nograph</u>                          | suppress graph                                                        |
| Plot                                    |                                                                       |
| <i>connect_options</i>                  | affect rendition of the plotted points                                |
| <i>marker_options</i>                   | change look of markers (color, size, etc.)                            |
| <i>marker_label_options</i>             | add marker labels; change look or position                            |
| Control limits                          |                                                                       |
| <u>clopts</u> ( <i>cline_options</i> )  | affect rendition of the control limits                                |
| Add plots                               |                                                                       |
| <u>addplot</u> ( <i>plot</i> )          | add other plots to the generated graph                                |
| Y axis, X axis, Titles, Legend, Overall |                                                                       |
| <i>twoway_options</i>                   | any options other than by() documented in [G-3] <i>twoway_options</i> |

| <i>xchart_options</i>                   | Description                                                           |
|-----------------------------------------|-----------------------------------------------------------------------|
| Main                                    |                                                                       |
| <u>std</u> (#)                          | user-specified standard deviation                                     |
| <u>mean</u> (#)                         | user-specified mean                                                   |
| <u>lower</u> (#) <u>upper</u> (#)       | lower and upper limits of the X-bar limits                            |
| <u>nograph</u>                          | suppress graph                                                        |
| Plot                                    |                                                                       |
| <i>connect_options</i>                  | affect rendition of the plotted points                                |
| <i>marker_options</i>                   | change look of markers (color, size, etc.)                            |
| <i>marker_label_options</i>             | add marker labels; change look or position                            |
| Control limits                          |                                                                       |
| <u>clopts</u> ( <i>cline_options</i> )  | affect rendition of the control limits                                |
| Add plots                               |                                                                       |
| <u>addplot</u> ( <i>plot</i> )          | add other plots to the generated graph                                |
| Y axis, X axis, Titles, Legend, Overall |                                                                       |
| <i>twoway_options</i>                   | any options other than by() documented in [G-3] <i>twoway_options</i> |

| <i>shewhart_options</i>                        | Description                                          |
|------------------------------------------------|------------------------------------------------------|
| <b>Main</b>                                    |                                                      |
| <u>std</u> (#)                                 | user-specified standard deviation                    |
| <u>mean</u> (#)                                | user-specified mean                                  |
| <u>nograph</u>                                 | suppress graph                                       |
| <b>Plot</b>                                    |                                                      |
| <i>connect_options</i>                         | affect rendition of the plotted points               |
| <i>marker_options</i>                          | change look of markers (color, size, etc.)           |
| <i>marker_label_options</i>                    | add marker labels; change look or position           |
| <b>Control limits</b>                          |                                                      |
| <u>clopts</u> ( <i>cline_options</i> )         | affect rendition of the control limits               |
| <b>Y axis, X axis, Titles, Legend, Overall</b> |                                                      |
| <i>combine_options</i>                         | any options documented in [G-2] <b>graph combine</b> |

collect is allowed with all QC commands; see [U] **11.1.10 Prefix commands**.

## Options

### Main

**stabilized** stabilizes the p chart when sample sizes are unequal.

**std**(#) specifies the standard deviation of the process. The R chart is calculated (based on the range) if this option is not specified.

**mean**(#) specifies the grand mean, which is calculated if not specified.

**lower**(#) and **upper**(#) must be specified together or not at all. They specify the lower and upper limits of the  $\bar{X}$  chart. Calculations based on the mean and standard deviation (whether specified by option or calculated) are used otherwise.

**nograph** suppresses the graph.

**generate**(*newvar<sub>f</sub>* *newvar<sub>lcl</sub>* *newvar<sub>ucl</sub>*) stores the plotted values in the p chart. *newvar<sub>f</sub>* will contain the fractions of defective elements; *newvar<sub>lcl</sub>* and *newvar<sub>ucl</sub>* will contain the lower and upper control limits, respectively.

### Plot

*connect\_options* affect whether lines connect the plotted points and the rendition of those lines; see [G-3] **connect\_options**.

*marker\_options* affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] **marker\_options**.

*marker\_label\_options* specify if and how the markers are to be labeled; see [G-3] **marker\_label\_options**.

### Control limits

clopts(*cline\_options*) affects the rendition of the control limits; see [G-3] **cline\_options**.

Add plots

`addplot` (*plot*) provides a way to add other plots to the generated graph. See [G-3] [addplot\\_option](#).

Y axis, X axis, Titles, Legend, Overall

`twoway_options` are any of the options documented in [G-3] [twoway\\_options](#), excluding `by()`. These include options for titling the graph (see [G-3] [title\\_options](#)) and for saving the graph to disk (see [G-3] [saving\\_option](#)).

`combine_options` (`shewhart` only) are any of the options documented in [G-2] [graph\\_combine](#). These include options for titling the graph (see [G-3] [title\\_options](#)) and for saving the graph to disk (see [G-3] [saving\\_option](#)).

## Remarks and examples

Control charts may be used to define the goal of a repetitive process, to control that process, and to determine if the goal has been achieved. Walter A. Shewhart of Bell Telephone Laboratories devised the first control chart in 1924. In 1931, Shewhart published *Economic Control of Quality of Manufactured Product*. According to Burr, “Few fields of knowledge have ever been so completely explored and charted in the first exposition” (1976, 29). Shewhart states that “a phenomenon will be said to be controlled when, through the use of past experience, we can predict, at least within limits, how the phenomenon may be expected to vary in the future. Here it is understood that prediction within limits means that we can state, at least approximately, the probability that the observed phenomenon will fall within given limits” (1931, 6).

For more information on quality-control charts, see Burr (1976), Duncan (1986), Harris (1999), or Ryan (2011).

### ▷ Example 1: `cchart`

`cchart` graphs a `c` chart showing the number of nonconformities in a unit, where `defect_var` records the number of defects in each inspection unit and `unit_var` records the unit number. The unit numbers need not be in order. For instance, consider the following example dataset from Ryan (2011, 186):

```
. use https://www.stata-press.com/data/r19/ncu
. describe
```

```
Contains data from https://www.stata-press.com/data/r19/ncu.dta
Observations: 30
Variables: 2 2 Dec 2024 15:15
```

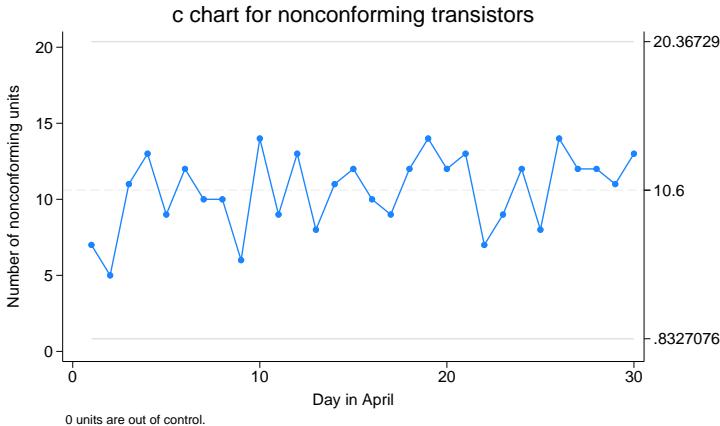
| Variable name | Storage type | Display format | Value label | Variable label                |
|---------------|--------------|----------------|-------------|-------------------------------|
| day           | byte         | %9.0g          |             | Day in April                  |
| defects       | byte         | %9.0g          |             | Number of nonconforming units |

Sorted by:

```
. list in 1/5
```

|    | day | defects |
|----|-----|---------|
| 1. | 1   | 7       |
| 2. | 2   | 5       |
| 3. | 3   | 11      |
| 4. | 4   | 13      |
| 5. | 5   | 9       |

```
. cchart defects day, title(c chart for nonconforming transistors)
```



The expected number of defects is 10.6, with lower and upper control limits of 0.8327 and 20.37, respectively. No units are out of control.



## ▷ Example 2: pchart

pchart graphs a p chart, which shows the fraction of nonconforming items in a subgroup, where *reject\_var* records the number rejected in each inspection unit, *unit\_var* records the inspection unit number, and *ssize\_var* records the number inspected in each unit.

Consider the example dataset from [Ryan \(2011, 186\)](#) of the number of nonconforming transistors out of 1,000 inspected each day during the month of April:

```
. use https://www.stata-press.com/data/r19/ncu2
. describe
```

Contains data from <https://www.stata-press.com/data/r19/ncu2.dta>

```
Observations: 30
Variables: 3 2 Dec 2024 15:16
```

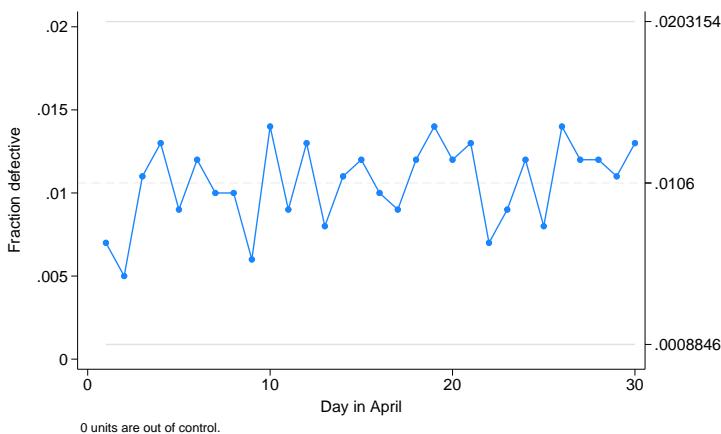
| Variable name | Storage type | Display format | Value label | Variable label                |
|---------------|--------------|----------------|-------------|-------------------------------|
| day           | byte         | %9.0g          |             | Day in April                  |
| rejects       | byte         | %9.0g          |             | Number of nonconforming units |
| ssize         | int          | %9.0g          |             | Sample size                   |

Sorted by:

```
. list in 1/5
```

|    | day | rejects | ssize |
|----|-----|---------|-------|
| 1. | 1   | 7       | 1000  |
| 2. | 2   | 5       | 1000  |
| 3. | 3   | 11      | 1000  |
| 4. | 4   | 13      | 1000  |
| 5. | 5   | 9       | 1000  |

```
. pchart rejects day ssize
```



All the points are within the control limits, which are 0.0009 for the lower limit and 0.0203 for the upper limit.

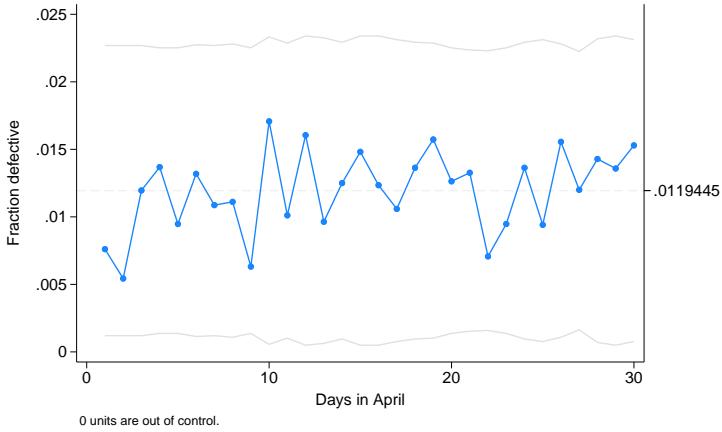
Here the sample sizes are fixed at 1,000, so the `ssize` variable contains 1,000 for each observation. Sample sizes need not be fixed, however. Say that our data were slightly different:

```
. use https://www.stata-press.com/data/r19/ncu3
```

```
. list in 1/5
```

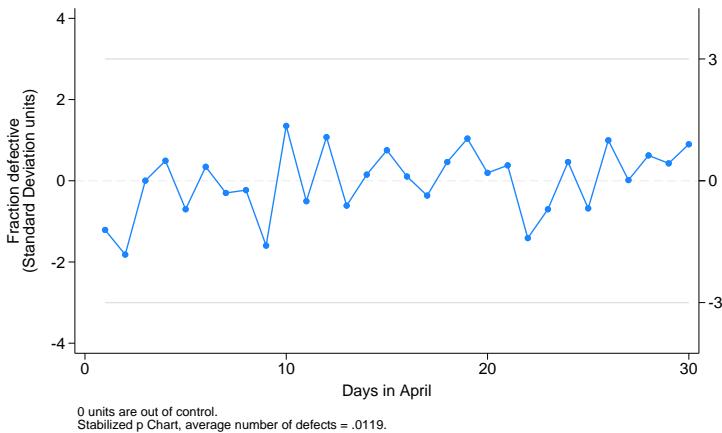
|    | day | rejects | ssize |
|----|-----|---------|-------|
| 1. | 1   | 7       | 920   |
| 2. | 2   | 5       | 920   |
| 3. | 3   | 11      | 920   |
| 4. | 4   | 13      | 950   |
| 5. | 5   | 9       | 950   |

```
. pchart rejects day ssize
```



Here the control limits are, like the sample size, no longer constant. The stabilize option will stabilize the control chart:

```
. pchart rejects day ssize, stabilize
```



### ► Example 3: rchart

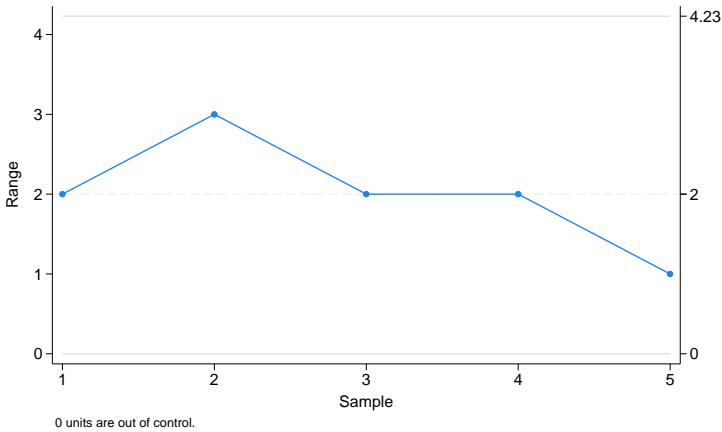
rchart displays an R chart showing the range for repeated measurements at various times. Variables within observations record measurements. Observations represent different samples.

For instance, say that we take five samples of 5 observations each. In our first sample, our measurements are 10, 11, 10, 11, and 12. The data are

```
. list
```

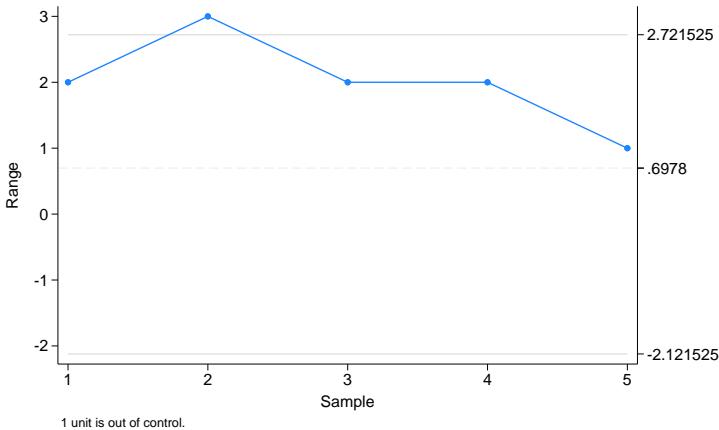
|    | m1 | m2 | m3 | m4 | m5 |
|----|----|----|----|----|----|
| 1. | 10 | 11 | 10 | 11 | 12 |
| 2. | 12 | 10 | 9  | 10 | 9  |
| 3. | 10 | 11 | 10 | 12 | 10 |
| 4. | 9  | 9  | 9  | 10 | 11 |
| 5. | 12 | 12 | 12 | 12 | 13 |

```
. rchart m1-m5, connect(1)
```



The expected range in each sample is 2 with lower and upper control limits of 0 and 4.23, respectively. If we know that the process standard deviation is 0.3, we could specify

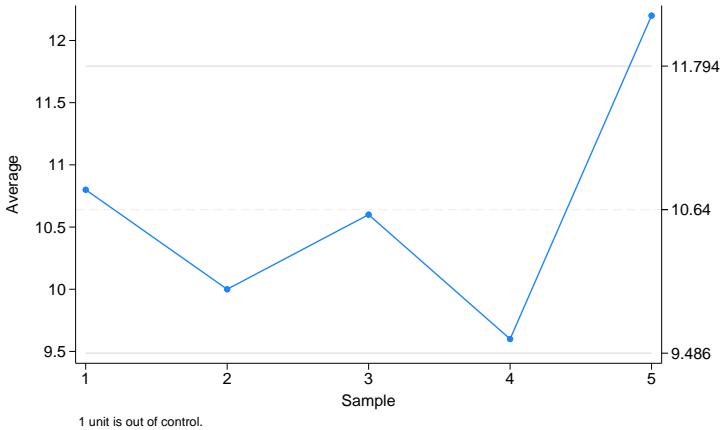
```
. rchart m1-m5, connect(1) std(.3)
```



### ► Example 4: xchart

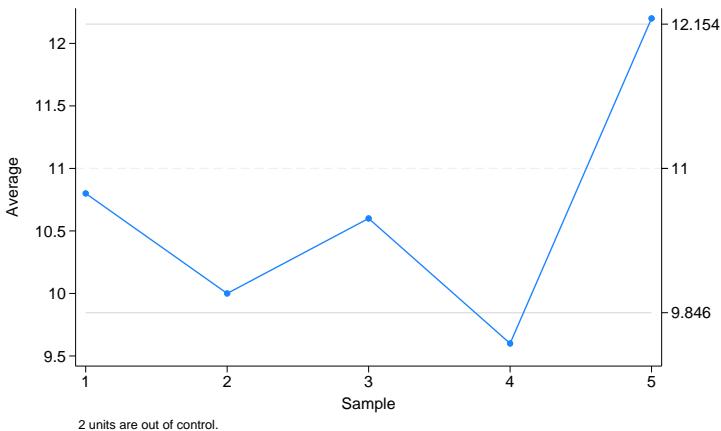
xchart graphs an  $\bar{X}$  chart for repeated measurements at various times. Variables within observations record measurements, and observations represent different samples. Using the same data as in the previous example, we type

```
. xchart m1-m5, connect(1)
```



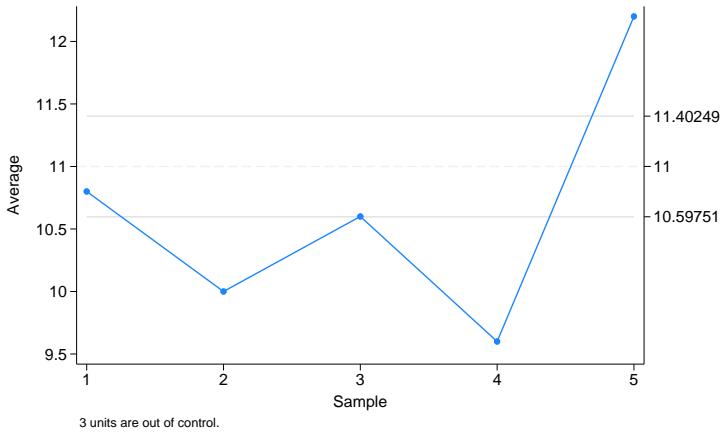
The average measurement in the sample is 10.64, and the lower and upper control limits are 9.486 and 11.794, respectively. Suppose that we knew from prior information that the mean of the process is 11. Then, we would type

```
. xchart m1-m5, connect(1) mean(11)
```



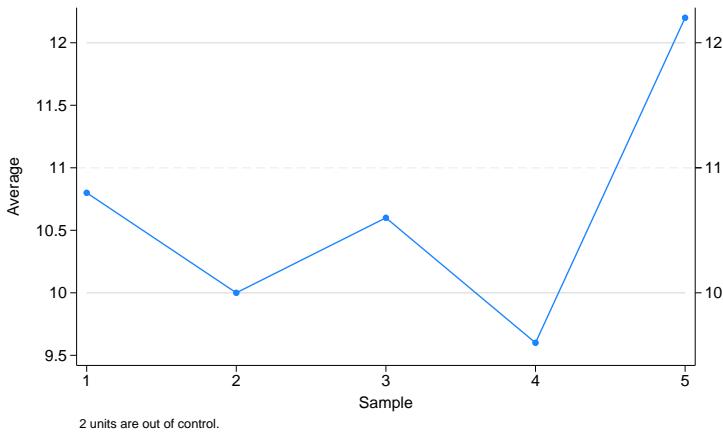
If we also know that the standard deviation of the process is 0.3, we could type

```
. xchart m1-m5, connect(1) mean(11) std(.3)
```



Finally, `xchart` allows us to specify our own control limits:

```
. xchart m1-m5, connect(1) mean(11) lower(10) upper(12)
```

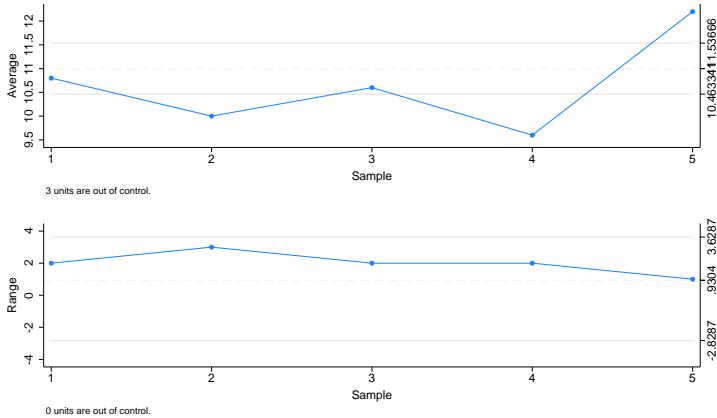


### ► Example 5: shewhart

`shewhart` displays a vertically aligned  $\bar{X}$  and R chart in the same image. To produce the best-looking combined image possible, you will want to use the `xchart` and `rchart` commands separately and then combine the graphs. `shewhart`, however, is more convenient.

Using the same data as previously, but realizing that the standard deviation should have been 0.4, we type

```
. shewhart m1-m5, connect(1) mean(11) std(.4)
```



Walter Andrew Shewhart (1891–1967) was born in Illinois and educated as a physicist, with degrees from the Universities of Illinois and California. After a brief period teaching physics, he worked for the Western Electric Company and (from 1925) the Bell Telephone Laboratories. His name is most associated with control charts used in quality controls, but his many other interests ranged generally from quality assurance to the philosophy of science.

## Stored results

`cchart` stores the following in `r()`:

Scalars

|                         |                                       |
|-------------------------|---------------------------------------|
| <code>r(cbar)</code>    | expected number of nonconformities    |
| <code>r(lcl_c)</code>   | lower control limit                   |
| <code>r(uc1_c)</code>   | upper control limit                   |
| <code>r(N)</code>       | number of observations                |
| <code>r(out_c)</code>   | number of units out of control        |
| <code>r(below_c)</code> | number of units below the lower limit |
| <code>r(above_c)</code> | number of units above the upper limit |

`pchart` stores the following in `r()`:

Scalars

|                         |                                       |
|-------------------------|---------------------------------------|
| <code>r(pbar)</code>    | average fraction of nonconformities   |
| <code>r(lcl_p)</code>   | lower control limit                   |
| <code>r(uc1_p)</code>   | upper control limit                   |
| <code>r(N)</code>       | number of observations                |
| <code>r(out_p)</code>   | number of units out of control        |
| <code>r(below_p)</code> | number of units below the lower limit |
| <code>r(above_p)</code> | number of units above the upper limit |

rchart stores the following in r():

Scalars

|                 |                                       |
|-----------------|---------------------------------------|
| r(central_line) | ordinate of the central line          |
| r(lcl_r)        | lower control limit                   |
| r(ucl_r)        | upper control limit                   |
| r(N)            | number of observations                |
| r(out_r)        | number of units out of control        |
| r(below_r)      | number of units below the lower limit |
| r(above_r)      | number of units above the upper limit |

xchart stores the following in r():

Scalars

|            |                                       |
|------------|---------------------------------------|
| r(xbar)    | grand mean                            |
| r(lcl_x)   | lower control limit                   |
| r(ucl_x)   | upper control limit                   |
| r(N)       | number of observations                |
| r(out_x)   | number of units out of control        |
| r(below_x) | number of units below the lower limit |
| r(above_x) | number of units above the upper limit |

shewhart stores in r() the combination of stored results from xchart and rchart.

## Methods and formulas

For the c chart, the number of defects per unit,  $C$ , is taken to be a value of a random variable having a Poisson distribution. If  $k$  is the number of units available for estimating  $\lambda$ , the parameter of the Poisson distribution, and if  $C_i$  is the number of defects in the  $i$ th unit, then  $\lambda$  is estimated by  $\bar{C} = \sum_i C_i/k$ . Then

$$\begin{aligned}\text{central line} &= \bar{C} \\ \text{UCL} &= \bar{C} + 3\sqrt{\bar{C}} \\ \text{LCL} &= \bar{C} - 3\sqrt{\bar{C}}\end{aligned}$$

Control limits for the p chart are based on the sampling theory for proportions, using the normal approximation to the binomial. If  $k$  samples are taken, the estimator of  $p$  is given by  $\bar{p} = \sum_i \hat{p}_i/k$ , where  $\hat{p}_i = x_i/n_i$ , and  $x_i$  is the number of defects in the  $i$ th sample of size  $n_i$ . The central line and the control limits are given by

$$\begin{aligned}\text{central line} &= \bar{p} \\ \text{UCL} &= \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n_i} \\ \text{LCL} &= \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n_i}\end{aligned}$$

Control limits for the R chart are based on the distribution of the range of samples of size  $n$  from a normal population. If the standard deviation of the process,  $\sigma$ , is known,

$$\begin{aligned}\text{central line} &= d_2\sigma \\ \text{UCL} &= D_2\sigma \\ \text{LCL} &= D_1\sigma\end{aligned}$$

where  $d_2$ ,  $D_1$ , and  $D_2$  are functions of the number of observations in the sample and are obtained from the table published in Beyer (1976).

When  $\sigma$  is unknown,

$$\begin{aligned}\text{central line} &= \bar{R} \\ \text{UCL} &= (D_2/d_2)\bar{R} \\ \text{LCL} &= (D_1/d_2)\bar{R}\end{aligned}$$

where  $\bar{R} = \sum_i R_i/k$  is the range of the  $k$  sample ranges  $R_i$ .

Control limits for the  $\bar{X}$  chart are given by

$$\begin{aligned}\text{central line} &= \bar{x} \\ \text{UCL} &= \bar{x} + (3/\sqrt{n})\sigma \\ \text{LCL} &= \bar{x} - (3/\sqrt{n})\sigma\end{aligned}$$

if  $\sigma$  is known. If  $\sigma$  is unknown,

$$\begin{aligned}\text{central line} &= \bar{x} \\ \text{UCL} &= \bar{x} + A_2\bar{R} \\ \text{LCL} &= \bar{x} - A_2\bar{R}\end{aligned}$$

where  $\bar{R}$  is the average range as defined above and  $A_2$  is a function (op. cit.) of the number of observations in the sample.

Isobel Loutit (1909–2009) is known for her work during World War II to improve the accuracy of targeting for anti-aircraft guns and as a contributor to the field of quality control. Loutit was born in Selkirk Manitoba, Canada. She graduated from the University of Manitoba in 1929 with a degree in mathematics and was one of the first women to work as a professional statistician in Canada. After graduation, she obtained a job teaching French. However, because of her training, she served as a substitute math teacher when needed.

When World War II started, Loutit took a job as a quality control statistician at Northern Electric. Shortly after that, the Canadian government advertised for women with technical training to fill jobs that had been vacated by men who had gone to war. She took a position testing equipment for the military. This job eventually returned her to Northern Electric, this time as a government employee verifying the accuracy of Northern Electric's Vickers anti-aircraft gun predictor, which was used to aim artillery at incoming planes. Recognizing the quality of her work, the CEO of Northern Electric rehired Loutit as an engineer, the only position for which her pay would not be capped as a woman. She later became the first female manager at Northern Electric and the first woman to chair the Montreal Section of the American Society for Quality Control.

## References

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## Also see

[R] [serrbar](#) — Graph standard error bar chart

[Description](#)[Options for qreg](#)[Remarks and examples](#)[Also see](#)[Quick start](#)[Options for iqreg](#)[Stored results](#)[Menu](#)[Options for sqreg](#)[Methods and formulas](#)[Syntax](#)[Options for bsqreg](#)[References](#)

## Description

`qreg` fits quantile (including median) regression models, also known as least absolute value, minimum absolute deviation, or minimum L1-norm value. The quantile regression models fit by `qreg` express the quantiles of the conditional distribution as linear functions of the independent variables.

`iqreg` estimates interquantile range regressions, regressions of the difference in quantiles. The estimated variance–covariance matrix of the estimators (VCE) is obtained via bootstrapping.

`sqreg` estimates simultaneous-quantile regression. It produces the same coefficients as `qreg` for each quantile. Reported standard errors will be similar, but `sqreg` obtains an estimate of the VCE via bootstrapping, and the VCE includes between-quantile blocks. Thus, you can test and construct confidence intervals comparing coefficients describing different quantiles.

`bsqreg` is equivalent to `sqreg` with one quantile.

## Quick start

*Quantile regression*

Median regression of  $y$  on  $x_1$  and  $x_2$

```
qreg y x1 x2
```

Add categorical covariate  $a$  using factor-variable syntax

```
qreg y x1 x2 i.a
```

Same as above, but with standard errors using a biweight kernel for the nonparametric density estimator

```
qreg y x1 x2 i.a, vce(, kernel(biweight))
```

Quantile regression of the 75th percentile of  $y$  on  $x_1$ ,  $x_2$ , and  $a$

```
qreg y x1 x2 i.a, quantile(.75)
```

*Interquantile range regression*

Difference between the 90th and 10th quantiles of  $y$  on  $x_1$ ,  $x_2$ , and  $a$  with bootstrap standard errors

```
iqreg y x1 x2 i.a, quantiles(.1 .9)
```

### *Simultaneous-quantile regression*

Simultaneous estimation of quantile regressions for the 10th and 90th quantiles of  $y$  with bootstrap standard errors

```
sqreg y x1 x2 i.a, quantiles(.1 .9)
```

Same as above, but for the 25th, 50th, and 75th quantiles of  $y$

```
sqreg y x1 x2 i.a, quantiles(.25 .5 .75)
```

Same as above, but increase the number of bootstrap replications to 500

```
sqreg y x1 x2 i.a, quantiles(.25 .5 .75) reps(500)
```

### *Bootstrapped quantile regression*

Single quantile regression for the 25th quantile with bootstrap standard errors

```
bsqreg y x1 x2 i.a, quantile(.25)
```

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## Syntax

### Quantile regression

```
qreg depvar [indepvars] [if] [in] [weight] [, qreg_options]
```

### Interquantile range regression

```
iqreg depvar [indepvars] [if] [in] [, iqreg_options]
```

### Simultaneous-quantile regression

```
sqreg depvar [indepvars] [if] [in] [, sqreg_options]
```

### Bootstrapped quantile regression

```
bsqreg depvar [indepvars] [if] [in] [, bsqreg_options]
```

| <i>qreg_options</i>                                                                       | Description                                                                                                                                                                                                 |
|-------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Model                                                                                     |                                                                                                                                                                                                             |
| <code>quantile(#)</code>                                                                  | estimate # quantile; default is <code>quantile(.5)</code>                                                                                                                                                   |
| SE/Robust                                                                                 |                                                                                                                                                                                                             |
| <code>vce([<i>vcetype</i>], [<i>vceopts</i>])</code>                                      | technique used to estimate standard errors                                                                                                                                                                  |
| Reporting                                                                                 |                                                                                                                                                                                                             |
| <code>level(#)</code><br><code>display_options</code>                                     | set confidence level; default is <code>level(95)</code><br>control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| Optimization                                                                              |                                                                                                                                                                                                             |
| <code>optimization_options</code><br><code>wlsiter(#)</code>                              | control the optimization process; seldom used<br>attempt # weighted least-squares iterations before doing linear programming iterations                                                                     |
| <i>vcetype</i>                                                                            | Description                                                                                                                                                                                                 |
| <code>iid</code>                                                                          | compute the VCE assuming the residuals are i.i.d.                                                                                                                                                           |
| <code>robust</code>                                                                       | compute the robust VCE                                                                                                                                                                                      |
| <i>vceopts</i>                                                                            | Description                                                                                                                                                                                                 |
| <code>denmethod</code><br><code>bwidth</code>                                             | nonparametric density estimation technique<br>bandwidth method used by the density estimator                                                                                                                |
| <i>denmethod</i>                                                                          | Description                                                                                                                                                                                                 |
| <code>fitted</code><br><code>residual</code><br><code>kernel</code> [ ( <i>kernel</i> ) ] | use the empirical quantile function using fitted values; the default<br>use the empirical residual quantile function<br>use a nonparametric kernel density estimator; default is <code>epanechnikov</code>  |

| <i>bwidth</i>            | Description                            |
|--------------------------|----------------------------------------|
| <code>hshather</code>    | Hall–Sheather’s bandwidth; the default |
| <code>bofinger</code>    | Bofinger’s bandwidth                   |
| <code>chamberlain</code> | Chamberlain’s bandwidth                |

| <i>kernel</i>             | Description                               |
|---------------------------|-------------------------------------------|
| <code>epanechnikov</code> | Epanechnikov kernel function; the default |
| <code>epan2</code>        | alternative Epanechnikov kernel function  |
| <code>biweight</code>     | biweight kernel function                  |
| <code>cosine</code>       | cosine trace kernel function              |
| <code>gaussian</code>     | Gaussian kernel function                  |
| <code>parzen</code>       | Parzen kernel function                    |
| <code>rectangle</code>    | rectangle kernel function                 |
| <code>triangle</code>     | triangle kernel function                  |

| <i>iqreg_options</i>         | Description                                                                                                                                      |
|------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| Model                        |                                                                                                                                                  |
| <code>quantiles(# #)</code>  | interquantile range; default is <code>quantiles(.25 .75)</code>                                                                                  |
| <code>reps(#)</code>         | perform # bootstrap replications; default is <code>reps(20)</code>                                                                               |
| Reporting                    |                                                                                                                                                  |
| <code>level(#)</code>        | set confidence level; default is <code>level(95)</code>                                                                                          |
| <code>nodots</code>          | suppress display of the replication dots                                                                                                         |
| <code>display_options</code> | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |

| <i>sqreg_options</i>                   | Description                                                                                                                                      |
|----------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| Model                                  |                                                                                                                                                  |
| <code>quantiles(# [# [# ... ]])</code> | estimate # quantiles; default is <code>quantiles(.5)</code>                                                                                      |
| <code>reps(#)</code>                   | perform # bootstrap replications; default is <code>reps(20)</code>                                                                               |
| Reporting                              |                                                                                                                                                  |
| <code>level(#)</code>                  | set confidence level; default is <code>level(95)</code>                                                                                          |
| <code>nodots</code>                    | suppress display of the replication dots                                                                                                         |
| <code>display_options</code>           | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |

| <i>bsqreg_options</i>    | Description                                                                                                                                      |
|--------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>             |                                                                                                                                                  |
| <code>quantile(#)</code> | estimate # quantile; default is <code>quantile(.5)</code>                                                                                        |
| <code>reps(#)</code>     | perform # bootstrap replications; default is <code>reps(20)</code>                                                                               |
| <b>Reporting</b>         |                                                                                                                                                  |
| <code>level(#)</code>    | set confidence level; default is <code>level(95)</code>                                                                                          |
| <i>display_options</i>   | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |

*indepvars* may contain factor variables; see [U] 11.4.3 **Factor variables**.

`by`, `collect`, `mi estimate`, `rolling`, and `statsby`, are allowed by `qreg`, `iqreg`, `sqreg`, and `bsqreg`; `bayes`, `bayesboot`, `mfp`, `nestreg`, and `stepwise` are allowed only with `qreg`; see [U] 11.1.10 **Prefix commands**. For more details, see [BAYES] **bayes: qreg**.

`qreg` allows `fweights`, `iweights`, and `pweights`; see [U] 11.1.6 **weight**.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options for qreg

### Model

`quantile(#)` specifies the quantile to be estimated and should be a number between 0 and 1, exclusive. Numbers larger than 1 are interpreted as percentages. The default value of 0.5 corresponds to the median.

### SE/Robust

`vce([vcetype], [vceopts])` specifies the type of VCE to compute and the density estimation method to use in computing the VCE.

*vcetype* specifies the type of VCE to compute. Available types are `iid` and `robust`.

`vce(iid)`, the default, computes the VCE under the assumption that the residuals are independent and identically distributed (i.i.d.).

`vce(robust)` computes the robust VCE under the assumption that the residual density is continuous and bounded away from 0 and infinity at the specified `quantile()`; see [Koenker \(2005, sec. 4.2\)](#).

*vceopts* consists of available *denmethod* and *bwidth* options.

*denmethod* specifies the method to use for the nonparametric density estimator. Available methods are `fitted`, `residual`, or `kernel` [*kernel*], where the optional *kernel* must be one of the kernel choices listed below.

`fitted` and `residual` specify that the nonparametric density estimator use some of the structure imposed by quantile regression. The default `fitted` uses a function of the fitted values and `residual` uses a function of the residuals. `vce(robust, residual)` is not allowed.

`kernel()` specifies that the nonparametric density estimator use a kernel method. The available kernel functions are `epanechnikov`, `epan2`, `biweight`, `cosine`, `gaussian`, `parzen`, `rectangle`, and `triangle`. The default is `epanechnikov`. See [R] **kdensity** for the kernel function forms.

*bw* specifies the bandwidth method to use by the nonparametric density estimator. Available methods are *hsheather* for the Hall–Sheather bandwidth, *bofinger* for the Bofinger bandwidth, and *chamberlain* for the Chamberlain bandwidth.

See [Koenker \(2005, sec. 3.4 and 4.10\)](#) for a description of the sparsity estimation techniques and the Hall–Sheather and Bofinger bandwidth formulas. See [Chamberlain \(1994, eq. 2.2\)](#) for the Chamberlain bandwidth.

#### Reporting

`level(#)`; see [\[R\] Estimation options](#).

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [\[R\] Estimation options](#).

#### Optimization

*optimization\_options*: `iterate(#)`, `[no]log`, `trace`. `iterate()` specifies the maximum number of iterations; `log/nolog` specifies whether to show the iteration log (see `set iterlog` in [\[R\] set iter](#)); and `trace` specifies that the iteration log should include the current parameter vector. These options are seldom used.

`wlsiter(#)` specifies the number of weighted least-squares iterations that will be attempted before the linear programming iterations are started. The default is `wlsiter(1)`. If there are convergence problems, increasing this number should help.

## Options for iqreg

#### Model

`quantiles(# #)` specifies the quantiles to be compared. The first number must be less than the second, and both should be between 0 and 1, exclusive. Numbers larger than 1 are interpreted as percentages. Not specifying this option is equivalent to specifying `quantiles(.25 .75)`, meaning the interquartile range.

`reps(#)` specifies the number of bootstrap replications to be used to obtain an estimate of the variance–covariance matrix of the estimators (standard errors). `reps(20)` is the default and is arguably too small. `reps(100)` would perform 100 bootstrap replications. `reps(1000)` would perform 1,000 replications.

#### Reporting

`level(#)`; see [\[R\] Estimation options](#).

`nodots` suppresses display of the replication dots.

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [\[R\] Estimation options](#).

## Options for sqreg

### Model

`quantiles(# [# [# . . . ]])` specifies the quantiles to be estimated and should contain numbers between 0 and 1, exclusive. Numbers larger than 1 are interpreted as percentages. The default value of 0.5 corresponds to the median.

`reps(#)` specifies the number of bootstrap replications to be used to obtain an estimate of the variance–covariance matrix of the estimators (standard errors). `reps(20)` is the default and is arguably too small. `reps(100)` would perform 100 bootstrap replications. `reps(1000)` would perform 1,000 replications.

### Reporting

`level(#)`; see [R] [Estimation options](#).

`nodots` suppresses display of the replication dots.

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

## Options for bsqreg

### Model

`quantile(#)` specifies the quantile to be estimated and should be a number between 0 and 1, exclusive. Numbers larger than 1 are interpreted as percentages. The default value of 0.5 corresponds to the median.

`reps(#)` specifies the number of bootstrap replications to be used to obtain an estimate of the variance–covariance matrix of the estimators (standard errors). `reps(20)` is the default and is arguably too small. `reps(100)` would perform 100 bootstrap replications. `reps(1000)` would perform 1,000 replications.

### Reporting

`level(#)`; see [R] [Estimation options](#).

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

## Remarks and examples

Remarks are presented under the following headings:

*Median regression*  
*Quantile regression*  
*Estimated standard errors*  
*Interquantile and simultaneous-quantile regression*  
*What are the parameters?*

### Median regression

qreg fits quantile regression models. The default form is median regression, where the objective is to estimate the median of the dependent variable, conditional on the values of the independent variables. This method is similar to ordinary regression, where the objective is to estimate the conditional mean of the dependent variable. Simply put, median regression finds a line through the data that minimizes the sum of the *absolute* residuals rather than the sum of the *squares* of the residuals, as in ordinary regression. Equivalently, median regression expresses the median of the conditional distribution of the dependent variable as a linear function of the conditioning (independent) variables. Cameron and Trivedi (2022, chap. 15) provide a nice introduction to quantile regression using Stata.

#### ► Example 1: Estimating the conditional median

Consider a two-group experimental design with 5 observations per group:

```
. use https://www.stata-press.com/data/r19/twogrp
. list
```

|     | x | y  |
|-----|---|----|
| 1.  | 0 | 0  |
| 2.  | 0 | 1  |
| 3.  | 0 | 3  |
| 4.  | 0 | 4  |
| 5.  | 0 | 95 |
| 6.  | 1 | 14 |
| 7.  | 1 | 19 |
| 8.  | 1 | 20 |
| 9.  | 1 | 22 |
| 10. | 1 | 23 |

```
. qreg y x
```

```
Iteration 1: WLS sum of weighted deviations = 60.941342
Iteration 1: Sum of abs. weighted deviations = 55.5
Iteration 2: Sum of abs. weighted deviations = 55

Median regression
Raw sum of deviations 78.5 (about 14) Number of obs = 10
Min sum of deviations 55 Pseudo R2 = 0.2994
```

|  | y     | Coefficient | Std. err. | t    | P> t  | [95% conf. interval] |          |
|--|-------|-------------|-----------|------|-------|----------------------|----------|
|  | x     | 17          | 18.23213  | 0.93 | 0.378 | -25.04338            | 59.04338 |
|  | _cons | 3           | 12.89207  | 0.23 | 0.822 | -26.72916            | 32.72916 |

We have estimated the equation

$$y_{\text{median}} = 3 + 17x$$

We look back at our data.  $x$  takes on the values 0 and 1, so the median for the  $x = 0$  group is 3, whereas for  $x = 1$  it is  $3 + 17 = 20$ . The output reports that the raw sum of absolute deviations about 14 is 78.5; that is, the sum of  $|y - 14|$  is 78.5. Fourteen is the unconditional median of  $y$ , although in these data, any value between 14 and 19 could also be considered an unconditional median (we have an even number of observations, so the median is bracketed by those two values). In any case, the raw sum of deviations of  $y$  about the median would be the same no matter what number we choose between 14 and 19. (With a “median” of 14, the raw sum of deviations is 78.5. Now think of choosing a slightly larger number for the median and recalculating the sum. Half the observations will have larger negative residuals, but the other half will have smaller positive residuals, resulting in no net change.)

We turn now to the actual estimated equation. The sum of the absolute deviations about the solution  $y_{\text{median}} = 3 + 17x$  is 55. The pseudo- $R^2$  is calculated as  $1 - 55/78.5 \approx 0.2994$ . This result is based on the idea that the median regression is the maximum likelihood estimate for the double-exponential distribution.

◀

## □ Technical note

`qreg` is an alternative to regular regression or robust regression—see [R] [regress](#) and [R] [rreg](#). Let’s compare the results:

```
. regress y x
```

| Source   | SS     | df | MS         | Number of obs | = | 10      |
|----------|--------|----|------------|---------------|---|---------|
| Model    | 2.5    | 1  | 2.5        | F(1, 8)       | = | 0.00    |
| Residual | 6978.4 | 8  | 872.3      | Prob > F      | = | 0.9586  |
|          |        |    |            | R-squared     | = | 0.0004  |
|          |        |    |            | Adj R-squared | = | -0.1246 |
| Total    | 6980.9 | 9  | 775.655556 | Root MSE      | = | 29.535  |

| y     | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|-------|-------------|-----------|-------|-------|----------------------|----------|
| x     | -1          | 18.6794   | -0.05 | 0.959 | -44.07477            | 42.07477 |
| _cons | 20.6        | 13.20833  | 1.56  | 0.157 | -9.858465            | 51.05847 |

Unlike `qreg`, `regress` fits ordinary linear regression and is concerned with predicting the mean rather than the median, so both results are, in a technical sense, correct. Putting aside those technicalities, however, we tend to use either regression to describe the central tendency of the data, of which the mean is one measure and the median another. Thus, we can ask, “which method better describes the central tendency of these data?”

Means—and therefore ordinary linear regression—are sensitive to outliers, and our data were purposely designed to contain two such outliers: 95 for  $x = 0$  and 14 for  $x = 1$ . These two outliers dominated the ordinary regression and produced results that do not reflect the central tendency well—you are invited to enter the data and graph  $y$  against  $x$ .

Robust regression attempts to correct the outlier-sensitivity deficiency in ordinary regression:

```
. rreg y x, genwt(wt)
 Huber iteration 1: Maximum difference in weights = .7311828
 Huber iteration 2: Maximum difference in weights = .17695779
 Huber iteration 3: Maximum difference in weights = .03149585
 Biweight iteration 4: Maximum difference in weights = .1979335
 Biweight iteration 5: Maximum difference in weights = .23332905
 Biweight iteration 6: Maximum difference in weights = .09960067
 Biweight iteration 7: Maximum difference in weights = .02691458
 Biweight iteration 8: Maximum difference in weights = .0009113
 Robust regression
 Number of obs = 10
 F(1, 8) = 80.63
 Prob > F = 0.0000
```

|  | y     | Coefficient | Std. err. | t    | P> t  | [95% conf. interval] |          |
|--|-------|-------------|-----------|------|-------|----------------------|----------|
|  | x     | 18.16597    | 2.023114  | 8.98 | 0.000 | 13.50066             | 22.83128 |
|  | _cons | 2.000003    | 1.430558  | 1.40 | 0.200 | -1.298869            | 5.298875 |

Here `rreg` discarded the first outlier completely. (We know this because we included the `genwt()` option on `rreg` and, after fitting the robust regression, examined the weights.) For the other “outlier”, `rreg` produced a weight of 0.47.

In any case, the answers produced by `qreg` and `rreg` to describe the central tendency are similar, but the standard errors are different. In general, robust regression will have smaller standard errors because it is not as sensitive to the exact placement of observations near the median. You are welcome to try removing the first outlier in the `qreg` estimation to observe an improvement in the standard errors by typing

```
. qreg y x if _n!=5
```

Also, some authors ([Rousseeuw and Leroy 1987](#), 11) have noted that quantile regression, unlike the unconditional median, may be sensitive to even one outlier if its leverage is high enough. [Rousseeuw and Leroy \(1987\)](#) discuss estimators that are more robust to perturbations to the data than either mean regression or quantile regression.

In the end, quantile regression may be more useful for the interpretation of the parameters that it estimates than for its robustness to perturbations to the data.

□

## ► Example 2: Median regression

Let's now consider a less artificial example using the automobile data described in [\[U\] 1.2.2 Example datasets](#). Using median regression, we will regress each car's price on its weight and length and whether it is of foreign manufacture:

```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)

. qreg price weight length foreign

Iteration 1: WLS sum of weighted deviations = 56397.829
Iteration 1: Sum of abs. weighted deviations = 55950.5
Iteration 2: Sum of abs. weighted deviations = 55264.718
Iteration 3: Sum of abs. weighted deviations = 54762.283
Iteration 4: Sum of abs. weighted deviations = 54734.152
Iteration 5: Sum of abs. weighted deviations = 54552.638
note: alternate solutions exist.
Iteration 6: Sum of abs. weighted deviations = 54465.511
Iteration 7: Sum of abs. weighted deviations = 54443.699
Iteration 8: Sum of abs. weighted deviations = 54411.294

Median regression Number of obs = 74
Raw sum of deviations 71102.5 (about 4934)
Min sum of deviations 54411.29 Pseudo R2 = 0.2347
```

| price   | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|---------|-------------|-----------|-------|-------|----------------------|----------|
| weight  | 3.933588    | 1.328718  | 2.96  | 0.004 | 1.283543             | 6.583632 |
| length  | -41.25191   | 45.46469  | -0.91 | 0.367 | -131.9284            | 49.42456 |
| foreign | 3377.771    | 885.4198  | 3.81  | 0.000 | 1611.857             | 5143.685 |
| _cons   | 344.6489    | 5182.394  | 0.07  | 0.947 | -9991.31             | 10680.61 |

The estimated equation is

$$\text{price}_{\text{median}} = 3.93 \text{ weight} - 41.25 \text{ length} + 3377.8 \text{ foreign} + 344.65$$

The output may be interpreted in the same way as linear regression output; see [\[R\] regress](#). The variables `weight` and `foreign` are significant, but `length` is not significant. The median price of the cars in these data is \$4,934. This value is a median (one of the two center observations), not *the* median, which would typically be defined as the midpoint of the two center observations.

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## Quantile regression

Quantile regression is similar to median regression in that it estimates an equation expressing a quantile of the conditional distribution, albeit one that generally differs from the 0.5 quantile that is the median. For example, specifying `quantile(.25)` estimates the parameters that describe the 25th percentile (first quartile) of the conditional distribution.

Quantile regression allows for effects of the independent variables to differ over the quantiles. For example, [Chamberlain \(1994\)](#) finds that union membership has a larger effect on the lower quantiles than on the higher quantiles of the conditional distribution of US wages. That the effects of the independent variables may vary over quantiles of the conditional distribution is an important advantage of quantile regression over mean regression.

## ► Example 3: Estimating quantiles other than the median

Returning to real data, the equation for the 25th percentile of price conditional on weight, length, and foreign in our automobile data is

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)

. qreg price weight length foreign, quantile(.25)

Iteration 1: WLS sum of weighted deviations = 49469.235
Iteration 1: Sum of abs. weighted deviations = 49728.883
Iteration 2: Sum of abs. weighted deviations = 45669.89
Iteration 3: Sum of abs. weighted deviations = 43416.646
Iteration 4: Sum of abs. weighted deviations = 41947.221
Iteration 5: Sum of abs. weighted deviations = 41093.025
Iteration 6: Sum of abs. weighted deviations = 37623.424
Iteration 7: Sum of abs. weighted deviations = 35721.453
Iteration 8: Sum of abs. weighted deviations = 35226.308
Iteration 9: Sum of abs. weighted deviations = 34823.319
Iteration 10: Sum of abs. weighted deviations = 34801.777

.25 Quantile regression Number of obs = 74
Raw sum of deviations 41912.75 (about 4187)
Min sum of deviations 34801.78 Pseudo R2 = 0.1697
```

| price   | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|---------|-------------|-----------|-------|-------|----------------------|----------|
| weight  | 1.831789    | .6328903  | 2.89  | 0.005 | .5695289             | 3.094049 |
| length  | 2.84556     | 21.65558  | 0.13  | 0.896 | -40.34514            | 46.03626 |
| foreign | 2209.925    | 421.7401  | 5.24  | 0.000 | 1368.791             | 3051.059 |
| _cons   | -1879.775   | 2468.46   | -0.76 | 0.449 | -6802.963            | 3043.413 |

Compared with our previous median regression, the coefficient on length now has a positive sign, and the coefficients on foreign and weight are reduced. The actual lower quartile is \$4,187, substantially less than the median \$4,934.

We can also estimate the upper quartile as a function of the same three variables:

```
. qreg price weight length foreign, quantile(.75)

Iteration 1: WLS sum of weighted deviations = 55465.741
Iteration 1: Sum of abs. weighted deviations = 55652.957
Iteration 2: Sum of abs. weighted deviations = 52994.785
Iteration 3: Sum of abs. weighted deviations = 50189.446
Iteration 4: Sum of abs. weighted deviations = 49898.245
Iteration 5: Sum of abs. weighted deviations = 49398.106
Iteration 6: Sum of abs. weighted deviations = 49241.835
Iteration 7: Sum of abs. weighted deviations = 49197.967

.75 Quantile regression Number of obs = 74
Raw sum of deviations 79860.75 (about 6342)
Min sum of deviations 49197.97 Pseudo R2 = 0.3840
```

| price   | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|---------|-------------|-----------|-------|-------|----------------------|-----------|
| weight  | 9.22291     | 1.785767  | 5.16  | 0.000 | 5.66131              | 12.78451  |
| length  | -220.7833   | 61.10352  | -3.61 | 0.001 | -342.6504            | -98.91616 |
| foreign | 3595.133    | 1189.984  | 3.02  | 0.004 | 1221.785             | 5968.482  |
| _cons   | 20242.9     | 6965.02   | 2.91  | 0.005 | 6351.61              | 34134.2   |

This result tells a different story: `weight` is much more important, and `length` is now significant—with a negative coefficient! The prices of high-priced cars seem to be determined by factors different from those affecting the prices of low-priced cars.

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## □ Technical note

One explanation for having substantially different regression functions for different quantiles is that the data are heteroskedastic, as we will demonstrate below. The following statements create a sharply heteroskedastic set of data:

```
. drop _all
. set obs 10000
Number of observations (_N) was 0, now 10,000.
. set seed 50550
. generate x = .1 + .9 * runiform()
. generate y = x * runiform()^2
```

Let's now fit the regressions for the 5th and 95th quantiles:

```
. qreg y x, quantile(.05)
Iteration 1: WLS sum of weighted deviations = 555.44181
Iteration 1: Sum of abs. weighted deviations = 555.25622
Iteration 2: Sum of abs. weighted deviations = 115.02628
Iteration 3: Sum of abs. weighted deviations = 89.617883
Iteration 4: Sum of abs. weighted deviations = 89.61679

.05 Quantile regression Number of obs = 10,000
Raw sum of deviations 89.68001 (about .00105493)
Min sum of deviations 89.61679 Pseudo R2 = 0.0007
```

|  | y     | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |
|--|-------|-------------|-----------|-------|-------|----------------------|
|  | x     | .0028667    | .0004395  | 6.52  | 0.000 | .0020052 .0037283    |
|  | _cons | -.0001135   | .0002661  | -0.43 | 0.670 | -.0006352 .0004081   |

```
. qreg y x, quantile(.95)
Iteration 1: WLS sum of weighted deviations = 624.91903
Iteration 1: Sum of abs. weighted deviations = 621.88928
Iteration 2: Sum of abs. weighted deviations = 182.03243
Iteration 3: Sum of abs. weighted deviations = 170.42588
Iteration 4: Sum of abs. weighted deviations = 169.05915
Iteration 5: Sum of abs. weighted deviations = 169.05911

.95 Quantile regression Number of obs = 10,000
Raw sum of deviations 275.9779 (about .60579139)
Min sum of deviations 169.0591 Pseudo R2 = 0.3874
```

|  | y     | Coefficient | Std. err. | t      | P> t  | [95% conf. interval] |
|--|-------|-------------|-----------|--------|-------|----------------------|
|  | x     | .9010814    | .008758   | 102.89 | 0.000 | .883914 .9182488     |
|  | _cons | -.0004053   | .0053028  | -0.08  | 0.939 | -.0107999 .0099893   |

The coefficient on `x`, in particular, differs markedly between the two estimates. For the mathematically inclined, it is not too difficult to show that the theoretical lines are  $y = 0.0025x$  for the 5th percentile and  $y = 0.9025x$  for the 95th, numbers in close agreement with our numerical results.

The estimator for the standard errors computed by `qreg` assumes that the sample is independent and identically distributed (i.i.d.); see *Estimated standard errors* and *Methods and formulas* for details. Because the data are conditionally heteroskedastic, we should have used `bsqreg` to consistently estimate the standard errors using a bootstrap method. □

## Estimated standard errors

The variance–covariance matrix of the estimator (VCE) depends on the reciprocal of the density of the dependent variable evaluated at the quantile of interest. This function, known as the “sparsity function”, is hard to estimate.

The default method, which uses the fitted values for the predicted quantiles, generally performs well, but other methods may be preferred in larger samples. The `vce()` suboptions *denmethod* and *bwidth* provide other estimators of the sparsity function, the details of which are described in *Methods and formulas*.

For models with heteroskedastic errors, option `vce(robust)` computes a [Huber \(1967\)](#) form of sandwich estimate ([Koenker 2005](#)). Alternatively, [Gould \(1992, 1997b\)](#) introduced generalized versions of `qreg` that obtain estimates of the standard errors by using bootstrap resampling (see [Efron and Tibshirani \[1993\]](#) or [Wu \[1986\]](#) for an introduction to bootstrap standard errors). The `iqreg`, `sqreg`, and `bsqreg` commands provide a bootstrapped estimate of the entire variance–covariance matrix of the estimators.

### ▷ Example 4: Obtaining robust standard errors

[Example 2](#) of `qreg` on real data above was a median regression of price on weight, length, and foreign using `auto.dta`. Suppose, after investigation, we are convinced that car price observations are not independent. We decide that standard errors robust to non-i.i.d. errors would be appropriate and use the option `vce(robust)`.

```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)

. qreg price weight length foreign, vce(robust)
Iteration 1: WLS sum of weighted deviations = 56397.829
Iteration 1: Sum of abs. weighted deviations = 55950.5
Iteration 2: Sum of abs. weighted deviations = 55264.718
Iteration 3: Sum of abs. weighted deviations = 54762.283
Iteration 4: Sum of abs. weighted deviations = 54734.152
Iteration 5: Sum of abs. weighted deviations = 54552.638
note: alternate solutions exist.
Iteration 6: Sum of abs. weighted deviations = 54465.511
Iteration 7: Sum of abs. weighted deviations = 54443.699
Iteration 8: Sum of abs. weighted deviations = 54411.294

Median regression Number of obs = 74
 Raw sum of deviations 71102.5 (about 4934)
 Min sum of deviations 54411.29 Pseudo R2 = 0.2347
```

| price   | Robust      |           | t     | P> t  | [95% conf. interval] |          |
|---------|-------------|-----------|-------|-------|----------------------|----------|
|         | Coefficient | std. err. |       |       |                      |          |
| weight  | 3.933588    | 1.694477  | 2.32  | 0.023 | .55406               | 7.313116 |
| length  | -41.25191   | 51.73571  | -0.80 | 0.428 | -144.4355            | 61.93171 |
| foreign | 3377.771    | 728.5115  | 4.64  | 0.000 | 1924.801             | 4830.741 |
| _cons   | 344.6489    | 5096.528  | 0.07  | 0.946 | -9820.055            | 10509.35 |

We see that the robust standard error for `weight` increases, making it less significant in modifying the median automobile price. The standard error for `length` also increases, but the standard error for the `foreign` indicator decreases.

For comparison, we repeat the estimation using bootstrap standard errors:

```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
. set seed 1001
. bsqreg price weight length foreign
(fitting base model)
Bootstrap replications (20):10.....20 done
Median regression, bootstrap(20) SEs Number of obs = 74
Raw sum of deviations 71102.5 (about 4934)
Min sum of deviations 54411.29 Pseudo R2 = 0.2347
```

| price   | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|---------|-------------|-----------|-------|-------|----------------------|----------|
| weight  | 3.933588    | 2.941839  | 1.34  | 0.186 | -1.933726            | 9.800901 |
| length  | -41.25191   | 73.47105  | -0.56 | 0.576 | -187.7853            | 105.2815 |
| foreign | 3377.771    | 1352.518  | 2.50  | 0.015 | 680.2582             | 6075.284 |
| _cons   | 344.6489    | 5927.045  | 0.06  | 0.954 | -11476.47            | 12165.77 |

The coefficient estimates are the same—indeed, they are obtained using the same technique. Only the standard errors differ. Therefore, the  $t$  statistics, significance levels, and confidence intervals also differ.

Because `bsqreg` (as well as `sqreg` and `iqreg`) obtains standard errors by randomly resampling the data, the standard errors it produces will not be the same from run to run unless we first set the random-number seed to the same number; see [R] [set seed](#).

By default, `bsqreg`, `sqreg`, and `iqreg` use 20 replications. We can control the number of replications by specifying the `reps()` option:

```
. bsqreg price weight length i.foreign, reps(1000)
(fitting base model)

Bootstrap replications (1,000):10.....20.....30.....40...
>50.....60.....70.....80.....90.....100.....110...
>120.....130.....140.....150.....160.....170.....
> ...180.....190.....200.....210.....220.....230.....24
> 0.....250.....260.....270.....280.....290.....300.....
>310.....320.....330.....340.....350.....360.....
> 370.....380.....390.....400.....410.....420.....430...
>440.....450.....460.....470.....480.....490.....
> ..500.....510.....520.....530.....540.....550.....560
>570.....580.....590.....600.....610.....620.....
> ...630.....640.....650.....660.....670.....680.....6
> 90.....700.....710.....720.....730.....740.....750...
>760.....770.....780.....790.....800.....810.....
> .820.....830.....840.....850.....860.....870.....880...
>890.....900.....910.....920.....930.....940.....
> ...950.....960.....970.....980.....990.....1,000 done

Median regression, bootstrap(1000) SEs Number of obs = 74
Raw sum of deviations 71102.5 (about 4934)
Min sum of deviations 54411.29 Pseudo R2 = 0.2347
```

| price   | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|---------|-------------|-----------|-------|-------|----------------------|----------|
| weight  | 3.933588    | 2.58771   | 1.52  | 0.133 | -1.227437            | 9.094613 |
| length  | -41.25191   | 68.02626  | -0.61 | 0.546 | -176.926             | 94.42219 |
| foreign |             |           |       |       |                      |          |
| Foreign | 3377.771    | 1070.777  | 3.15  | 0.002 | 1242.174             | 5513.368 |
| _cons   | 344.6489    | 5862.991  | 0.06  | 0.953 | -11348.72            | 12038.02 |

A comparison of the standard errors is informative.

| Variable  | qreg  | qreg<br>vce(robust) | bsqreg<br>reps(20) | bsqreg<br>reps(1000) |
|-----------|-------|---------------------|--------------------|----------------------|
| weight    | 1.329 | 1.694               | 2.942              | 2.588                |
| length    | 45.46 | 51.74               | 73.47              | 68.03                |
| 1.foreign | 885.4 | 728.5               | 1353.              | 1071.                |
| _cons     | 5182. | 5097.               | 5927.              | 5863.                |

The results shown above are typical for models with heteroskedastic errors. (Our dependent variable is `price`; if our model had been in terms of `ln(price)`, the standard errors estimated by `qreg` and `bsqreg` would have been nearly identical.) Also, even for heteroskedastic errors, 20 replications is generally sufficient for hypothesis tests against 0.

## Interquantile and simultaneous-quantile regression

Consider a quantile regression model where the  $q$ th quantile is given by

$$Q_q(y) = a_q + b_{q,1}x_1 + b_{q,2}x_2$$

For instance, the 75th and 25th quantiles are given by

$$Q_{0.75}(y) = a_{0.75} + b_{0.75,1}x_1 + b_{0.75,2}x_2$$

$$Q_{0.25}(y) = a_{0.25} + b_{0.25,1}x_1 + b_{0.25,2}x_2$$

The difference in the quantiles is then

$$Q_{0.75}(y) - Q_{0.25}(y) = (a_{0.75} - a_{0.25}) + (b_{0.75,1} - b_{0.25,1})x_1 + (b_{0.75,2} - b_{0.25,2})x_2$$

`qreg` fits models such as  $Q_{0.75}(y)$  and  $Q_{0.25}(y)$ . `iqreg` fits interquantile models, such as  $Q_{0.75}(y) - Q_{0.25}(y)$ . The relationships of the coefficients estimated by `qreg` and `iqreg` are exactly as shown: `iqreg` reports coefficients that are the difference in coefficients of two `qreg` models, and, of course, `iqreg` reports the appropriate standard errors, which it obtains by bootstrapping.

`sqreg` is like `qreg` in that it estimates the equations for the quantiles

$$Q_{0.75}(y) = a_{0.75} + b_{0.75,1}x_1 + b_{0.75,2}x_2$$

$$Q_{0.25}(y) = a_{0.25} + b_{0.25,1}x_1 + b_{0.25,2}x_2$$

The coefficients it obtains are the same that would be obtained by estimating each equation separately using `qreg`. `sqreg` differs from `qreg` in that it estimates the equations simultaneously and obtains an estimate of the entire variance–covariance matrix of the estimators by bootstrapping. Thus, you can perform hypothesis tests concerning coefficients both within and across equations.

For example, to fit the above model, you could type

```
. qreg y x1 x2, quantile(.25)
. qreg y x1 x2, quantile(.75)
```

By doing this, you would obtain estimates of the parameters, but you could not test whether  $b_{0.25,1} = b_{0.75,1}$  or, equivalently,  $b_{0.75,1} - b_{0.25,1} = 0$ . If your interest really is in the difference of coefficients, you could type

```
. iqreg y x1 x2, quantiles(.25 .75)
```

The “coefficients” reported would be the difference in quantile coefficients. You could also estimate both quantiles simultaneously and then test the equality of the coefficients:

```
. sqreg y x1 x2, quantiles(.25 .75)
. test [q25]x1 = [q75]x1
```

Whether you use `iqreg` or `sqreg` makes no difference for this test. `sqreg`, however, because it estimates the quantiles simultaneously, allows you to test other hypotheses. `iqreg`, by focusing on quantile differences, presents results in a way that is easier to read.

Finally, `sqreg` can estimate quantiles singly,

```
. sqreg y x1 x2, quantiles(.5)
```

and can thereby be used as a substitute for the slower `bsqreg`. (Gould [1997b] presents timings demonstrating that `sqreg` is faster than `bsqreg`.) `sqreg` can also estimate more than two quantiles simultaneously:

```
. sqreg y x1 x2, quantiles(.25 .5 .75)
```

### ► Example 5: Simultaneous quantile estimation

In demonstrating `qreg`, we performed quantile regressions using `auto.dta`. We discovered that the regression of `price` on `weight`, `length`, and `foreign` produced vastly different coefficients for the 0.25, 0.5, and 0.75 quantile regressions. Here are the coefficients that we obtained:

| Variable             | 25th percentile | 50th percentile | 75th percentile |
|----------------------|-----------------|-----------------|-----------------|
| <code>weight</code>  | 1.83            | 3.93            | 9.22            |
| <code>length</code>  | 2.85            | -41.25          | -220.8          |
| <code>foreign</code> | 2209.9          | 3377.8          | 3595.1          |
| <code>_cons</code>   | -1879.8         | 344.6           | 20242.9         |

All we can say, having estimated these equations separately, is that `price` seems to depend differently on the `weight`, `length`, and `foreign` variables depending on the portion of the `price` distribution we examine. We cannot be more precise because the estimates have been made separately. With `sqreg`, however, we can estimate all the effects simultaneously:

```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
. set seed 1001
. sqreg price weight length foreign, q(.25 .5 .75) reps(100)
(fitting base model)
Bootstrap replications (100):10.....20.....30.....40.....
>50.....60.....70.....80.....90.....100 done
Simultaneous quantile regression Number of obs = 74
 bootstrap(100) SEs .25 Pseudo R2 = 0.1697
 .50 Pseudo R2 = 0.2347
 .75 Pseudo R2 = 0.3840
```

|     | price                | Coefficient | Bootstrap<br>std. err. | t     | P> t  | [95% conf. interval] |           |
|-----|----------------------|-------------|------------------------|-------|-------|----------------------|-----------|
| q25 | <code>weight</code>  | 1.831789    | 1.206151               | 1.52  | 0.133 | - .573803            | 4.237381  |
|     | <code>length</code>  | 2.84556     | 26.775                 | 0.11  | 0.916 | -50.55549            | 56.24661  |
|     | <code>foreign</code> | 2209.925    | 946.9585               | 2.33  | 0.022 | 321.2762             | 4098.575  |
|     | <code>_cons</code>   | -1879.775   | 3243.182               | -0.58 | 0.564 | -8348.097            | 4588.548  |
| q50 | <code>weight</code>  | 3.933588    | 2.34028                | 1.68  | 0.097 | - .7339531           | 8.601129  |
|     | <code>length</code>  | -41.25191   | 59.58361               | -0.69 | 0.491 | -160.0877            | 77.58386  |
|     | <code>foreign</code> | 3377.771    | 1081.475               | 3.12  | 0.003 | 1220.836             | 5534.706  |
|     | <code>_cons</code>   | 344.6489    | 5062.804               | 0.07  | 0.946 | -9752.795            | 10442.09  |
| q75 | <code>weight</code>  | 9.22291     | 2.631084               | 3.51  | 0.001 | 3.975378             | 14.47044  |
|     | <code>length</code>  | -220.7833   | 89.54754               | -2.47 | 0.016 | -399.3802            | -42.18635 |
|     | <code>foreign</code> | 3595.133    | 1208.769               | 2.97  | 0.004 | 1184.319             | 6005.947  |
|     | <code>_cons</code>   | 20242.9     | 9682.24                | 2.09  | 0.040 | 932.2846             | 39553.52  |

The coefficient estimates above are the same as those previously estimated, although the standard error estimates are a little different. `sqreg` obtains estimates of variance by bootstrapping. The important thing here, however, is that the full covariance matrix of the estimators has been estimated and stored, and thus it is now possible to perform hypothesis tests. Are the effects of `weight` the same at the 25th and 75th percentiles?

```
. test [q25]weight = [q75]weight
(1) [q25]weight - [q75]weight = 0
 F(1, 70) = 8.57
 Prob > F = 0.0046
```

It appears that they are not. We can obtain a confidence interval for the difference by using `lincom`:

```
. lincom [q75]weight-[q25]weight
(1) - [q25]weight + [q75]weight = 0
```

| price | Coefficient | Std. err. | t    | P> t  | [95% conf. interval] |          |
|-------|-------------|-----------|------|-------|----------------------|----------|
| (1)   | 7.391121    | 2.524626  | 2.93 | 0.005 | 2.355914             | 12.42633 |

Indeed, we could test whether the weight and length sets of coefficients are equal at the three quantiles estimated:

```
. quietly test [q25]weight = [q50]weight
. quietly test [q25]weight = [q75]weight, accumulate
. quietly test [q25]length = [q50]length, accumulate
. test [q25]length = [q75]length, accumulate
(1) [q25]weight - [q50]weight = 0
(2) [q25]weight - [q75]weight = 0
(3) [q25]length - [q50]length = 0
(4) [q25]length - [q75]length = 0
 F(4, 70) = 2.25
 Prob > F = 0.0727
```

`iqreg` focuses on one quantile comparison but presents results that are more easily interpreted:

```
. set seed 1001
. iqreg price weight length foreign, q(.25 .75) reps(100) nolog
.75-.25 Interquantile regression Number of obs = 74
bootstrap(100) SEs .75 Pseudo R2 = 0.3840
 .25 Pseudo R2 = 0.1697
```

| price   | Coefficient | Bootstrap |  | t     | P> t  | [95% conf. interval] |           |
|---------|-------------|-----------|--|-------|-------|----------------------|-----------|
|         |             | std. err. |  |       |       |                      |           |
| weight  | 7.391121    | 2.524626  |  | 2.93  | 0.005 | 2.355914             | 12.42633  |
| length  | -223.6288   | 84.21504  |  | -2.66 | 0.010 | -391.5904            | -55.66724 |
| foreign | 1385.208    | 1321.832  |  | 1.05  | 0.298 | -1251.103            | 4021.519  |
| _cons   | 22122.68    | 9329.178  |  | 2.37  | 0.020 | 3516.219             | 40729.14  |

Looking only at the 0.25 and 0.75 quantiles (the interquartile range), the `iqreg` command output is easily interpreted. Increases in `weight` correspond significantly to increases in price dispersion. Increases in `length` correspond to decreases in price dispersion. The `foreign` variable does not significantly change price dispersion.

Do not make too much of these results; the purpose of this example is simply to illustrate the `sqreg` and `iqreg` commands and to do so in a context that suggests why analyzing dispersion might be of interest.

lincom after sqreg produced the same  $t$  statistic for the interquartile range of weight, as did the iqreg command above. In general, they will not agree exactly because of the randomness of bootstrapping, unless the random-number seed is set to the same value before estimation (as was done here).

◀

Gould (1997a) presents simulation results showing that the coverage—the actual percentage of confidence intervals containing the true value—for iqreg is appropriate.

## What are the parameters?

In this section, we use a specific data-generating process (DGP) to illustrate the interpretation of the parameters estimated by qreg. If simulation experiments are not intuitive to you, skip this section.

In general, quantile regression parameterizes the quantiles of the distribution of  $y$  conditional on the independent variables  $\mathbf{x}$  as  $\mathbf{x}\beta$ , where  $\beta$  is a vector of estimated parameters. In our example, we include a constant term and a single independent variable, and we express quantiles of the distribution of  $y$  conditional on  $x$  as  $\beta_0 + \beta_1 x$ .

We use simulated data to illustrate what we mean by a conditional distribution and how to interpret the parameters  $\beta$  estimated by qreg. We also note how we could change our example to illustrate a DGP for which the estimator in qreg would be misspecified.

We suppose that the distribution of  $y$  conditional on  $x$  has a Weibull form. If  $y$  has a Weibull distribution, the distribution function is  $F(y) = 1 - \exp\{-(y/\lambda)^k\}$ , where the scale parameter  $\lambda > 0$  and the shape parameter  $k > 0$ . We can make  $y$  have a Weibull distribution function conditional on  $x$  by making the scale parameter or the shape parameter functions of  $x$ . In our example, we specify a particular DGP by supposing that  $\lambda = (1 + \alpha x)$ ,  $\alpha = 1.5$ ,  $x = 1 + \sqrt{\nu}$ , and that  $\nu$  has a  $\chi^2(1)$  distribution. For the moment, we leave the parameter  $k$  as is so that we can discuss how this decision relates to model specification.

Plugging in for  $\lambda$  yields the functional form for the distribution of  $y$  conditional on  $x$ , which is known as the conditional distribution function and is denoted  $F(y|x)$ .  $F(y|x)$  is the distribution for  $y$  for each given value of  $x$ .

Some algebra yields that  $F(y|x) = 1 - \exp[-\{y/(1 + \alpha x)\}^k]$ . Letting  $\tau = F(y|x)$  implies that  $0 \leq \tau \leq 1$ , because probabilities must be between 0 and 1.

To obtain the  $\tau$  quantile of the distribution of  $y$  conditional on  $x$ , we solve

$$\tau = 1 - \exp[-\{y/(1 + \alpha x)\}^k]$$

for  $y$  as a function of  $\tau$ ,  $x$ ,  $\alpha$ , and  $k$ . The solution is

$$y = (1 + \alpha x)\{-\ln(1 - \tau)\}^{(1/k)} \quad (1)$$

For any value of  $\tau \in (0, 1)$ , expression (1) gives the  $\tau$  quantile of the distribution of  $y$  conditional on  $x$ . To use qreg, we must rewrite (1) as a function of  $x$ ,  $\beta_0$ , and  $\beta_1$ . Some algebra yields that (1) can be rewritten as

$$y = \beta_0 + \beta_1 * x$$

where  $\beta_0 = \{-\ln(1 - \tau)\}^{(1/k)}$  and  $\beta_1 = \alpha\{-\ln(1 - \tau)\}^{(1/k)}$ . We can express the conditional quantiles as linear combinations of  $x$ , which is a property of the estimator implemented in qreg.

If we parameterize  $k$  as a nontrivial function of  $x$ , the conditional quantiles will not be linear in  $x$ . If the conditional quantiles cannot be represented as linear functions of  $x$ , we cannot estimate the true parameters of the DGP. This restriction illustrates the limits of the estimator implemented in qreg.

We set  $k = 2$  for our example.

Conditional quantile regression allows the coefficients to change with the specified quantile. For our DGP, the coefficients  $\beta_0$  and  $\beta_1$  increase as  $\tau$  gets larger. Substituting in for  $\alpha$  and  $k$  yields that  $\beta_0 = \sqrt{-\ln(1-\tau)}$  and  $\beta_1 = 1.5\sqrt{-\ln(1-\tau)}$ . Table 1 presents the true values for  $\beta_0$  and  $\beta_1$  implied by our DGP when  $\tau \in \{0.25, 0.5, 0.8\}$ .

Table 1: True values for  $\beta_0$  and  $\beta_1$

| $\tau$ | $\beta_0$ | $\beta_1$ |
|--------|-----------|-----------|
| 0.25   | 0.53636   | 0.80454   |
| 0.5    | 0.8325546 | 1.248832  |
| 0.8    | 1.268636  | 1.902954  |

We can also use (1) to generate data from the specified distribution of  $y$  conditional on  $x$  by plugging in random uniform numbers for  $\tau$ . Each random uniform number substituted in for  $\tau$  in (1) yields a draw from the conditional distribution of  $y$  given  $x$ .

## ▷ Example 6

In this example, we generate 100,000 observations from our specified DGP by substituting random uniform numbers for  $\tau$  in (1), with  $\alpha = 1.5$ ,  $k = 2$ ,  $x = 1 + \sqrt{\nu}$ , and  $\nu$  coming from a  $\chi^2(1)$  distribution.

We begin by executing the code that implements this method; below, we discuss each line of the output produced.

```
. clear // drop existing variables
. set seed 1234571 // set random-number seed
. set obs 100000 // set number of observations
Number of observations (_N) was 0, now 100,000.
. generate double tau = runiform() // generate uniform variate
. generate double x = 1 + sqrt(rchi2(1)) // generate values for x
. generate double lambda = 1 + 1.5*x // lambda is 1 + alpha*x
. generate double k = 2 // fix value of k
. // generate random values for y
. // given x
. generate double y = lambda*((-ln(1-tau))^(1/k))
```

Although the comments at the end of each line briefly describe what each line is doing, we provide a more careful description. The first line drops any variables in memory. The second sets the seed of the random-number generator so that we will always get the same sequence of random uniform numbers. The third line sets the sample size to 100,000 observations, and the fourth line reports the change in sample size.

The fifth line substitutes random uniform numbers for  $\tau$ . This line is the key to the algorithm. This standard method, known as inverse-probability transforms, for computing random numbers is discussed by [Cameron and Trivedi \(2022, 220–221\)](#), among others.

Lines 6–8 generate  $x$ ,  $\lambda$ , and  $k$  per our specified DGP. Lines 9–11 implement (1) using the previously generated  $\lambda$ ,  $x$ , and  $k$ .

At the end, we have 100,000 observations on  $y$  and  $x$ , with  $y$  coming from the conditional distribution that we specified above.

## ▷ Example 7

In the example below, we use `qreg` to estimate  $\beta_1$  and  $\beta_0$ , the parameters from the conditional quantile function, for the 0.5 quantile from our simulated data.

```
. qreg y x, quantile(.5)
Iteration 1: WLS sum of weighted deviations = 68573.243
Iteration 1: Sum of abs. weighted deviations = 68571.918
Iteration 2: Sum of abs. weighted deviations = 68308.342
Iteration 3: Sum of abs. weighted deviations = 68241.17
Iteration 4: Sum of abs. weighted deviations = 68232.043
Iteration 5: Sum of abs. weighted deviations = 68230.304
Iteration 6: Sum of abs. weighted deviations = 68229.643
Iteration 7: Sum of abs. weighted deviations = 68229.532
Iteration 8: Sum of abs. weighted deviations = 68229.514
Iteration 9: Sum of abs. weighted deviations = 68229.508
Iteration 10: Sum of abs. weighted deviations = 68229.506
Iteration 11: Sum of abs. weighted deviations = 68229.505

Median regression Number of obs = 100,000
Raw sum of deviations 73861.64 (about 2.9443724)
Min sum of deviations 68229.51 Pseudo R2 = 0.0763
```

| y     | Coefficient | Std. err. | t      | P> t  | [95% conf. interval] |          |
|-------|-------------|-----------|--------|-------|----------------------|----------|
| x     | 1.266062    | .0117759  | 107.51 | 0.000 | 1.242981             | 1.289143 |
| _cons | .8083315    | .0222972  | 36.25  | 0.000 | .7646291             | .8520338 |

In the `qreg` output, the results for `x` correspond to the estimate of  $\beta_1$ , and the results for `_cons` correspond to the estimate of  $\beta_0$ . The reported estimates are close to their true values of 1.248832 and 0.8325546, which are given in [table 1](#).

The intuition in this example comes from the ability of `qreg` to recover the true parameters of our specified DGP. As we increase the number of observations in our sample size, the `qreg` estimates will get closer to the true values.



## ▷ Example 8

In the example below, we estimate the parameters of the conditional quantile function for the 0.25 quantile and compare them with the true values.

```
. qreg y x, quantile(.25)
Iteration 1: WLS sum of weighted deviations = 65395.359
Iteration 1: Sum of abs. weighted deviations = 65397.892
Iteration 2: Sum of abs. weighted deviations = 52640.481
Iteration 3: Sum of abs. weighted deviations = 50706.508
Iteration 4: Sum of abs. weighted deviations = 49767.356
Iteration 5: Sum of abs. weighted deviations = 49766.98
Iteration 6: Sum of abs. weighted deviations = 49765.818
Iteration 7: Sum of abs. weighted deviations = 49765.589
Iteration 8: Sum of abs. weighted deviations = 49765.549
Iteration 9: Sum of abs. weighted deviations = 49765.533
Iteration 10: Sum of abs. weighted deviations = 49765.528
Iteration 11: Sum of abs. weighted deviations = 49765.527
Iteration 12: Sum of abs. weighted deviations = 49765.527
Iteration 13: Sum of abs. weighted deviations = 49765.527

.25 Quantile regression Number of obs = 100,000
Raw sum of deviations 51945.91 (about 1.8560913)
Min sum of deviations 49765.53 Pseudo R2 = 0.0420
```

| y     | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|-------|-------------|-----------|-------|-------|----------------------|----------|
| x     | .8207143    | .0106425  | 77.12 | 0.000 | .799855              | .8415735 |
| _cons | .5075988    | .0201512  | 25.19 | 0.000 | .4681026             | .547095  |

Same as above, qreg reports the estimates of  $\beta_1$  and  $\beta_0$  in the output table for x and \_cons, respectively. The reported estimates are close to their true values of 0.80454 and 0.53636, which are given in [table 1](#). As expected, the estimates are close to their true values. Also as expected, the estimates for the 0.25 quantile are smaller than the estimates for the 0.5 quantile.

## ► Example 9

We finish this section by estimating the parameters of the conditional quantile function for the 0.8 quantile and comparing them with the true values.

```
. qreg y x, quantile(.8)
Iteration 1: WLS sum of weighted deviations = 66126.751
Iteration 1: Sum of abs. weighted deviations = 66130.001
Iteration 2: Sum of abs. weighted deviations = 55084.287
Iteration 3: Sum of abs. weighted deviations = 52914.276
Iteration 4: Sum of abs. weighted deviations = 52101.59
Iteration 5: Sum of abs. weighted deviations = 51899.426
Iteration 6: Sum of abs. weighted deviations = 51898.269
Iteration 7: Sum of abs. weighted deviations = 51898.268
Iteration 8: Sum of abs. weighted deviations = 51898.267

.8 Quantile regression Number of obs = 100,000
Raw sum of deviations 60129.76 (about 4.7060381)
Min sum of deviations 51898.27 Pseudo R2 = 0.1369
```

|  | y     | Coefficient | Std. err. | t      | P> t  | [95% conf. interval] |          |
|--|-------|-------------|-----------|--------|-------|----------------------|----------|
|  | x     | 1.911771    | .014834   | 128.88 | 0.000 | 1.882697             | 1.940846 |
|  | _cons | 1.254583    | .0280877  | 44.67  | 0.000 | 1.199531             | 1.309634 |

Same as above, qreg reports the estimates of  $\beta_1$  and  $\beta_0$  in the output table for x and \_cons, respectively. The reported estimates are close to their true values of 1.902954 and 1.268636, which are given in [table 1](#). As expected, the estimates are close to their true values. Also as expected, the estimates for the 0.8 quantile are larger than the estimates for the 0.5 quantile.

◀

## Stored results

qreg stores the following in `e()`:

### Scalars

|                          |                                                               |
|--------------------------|---------------------------------------------------------------|
| <code>e(N)</code>        | number of observations                                        |
| <code>e(df_m)</code>     | model degrees of freedom                                      |
| <code>e(df_r)</code>     | residual degrees of freedom                                   |
| <code>e(q)</code>        | quantile requested                                            |
| <code>e(q_v)</code>      | value of the quantile                                         |
| <code>e(r2_p)</code>     | pseudo- $R^2$                                                 |
| <code>e(sum_adev)</code> | sum of absolute deviations                                    |
| <code>e(sum_rdev)</code> | sum of raw deviations                                         |
| <code>e(sum_w)</code>    | sum of weights                                                |
| <code>e(f_r)</code>      | density estimate                                              |
| <code>e(sparsity)</code> | sparsity estimate                                             |
| <code>e(bwidth)</code>   | bandwidth                                                     |
| <code>e(kbwidth)</code>  | kernel bandwidth                                              |
| <code>e(rank)</code>     | rank of <code>e(V)</code>                                     |
| <code>e(convcode)</code> | 0 if converged; otherwise, return code for why nonconvergence |

### Macros

|                           |                                                                                                 |
|---------------------------|-------------------------------------------------------------------------------------------------|
| <code>e(cmd)</code>       | qreg                                                                                            |
| <code>e(cmdline)</code>   | command as typed                                                                                |
| <code>e(depvar)</code>    | name of dependent variable                                                                      |
| <code>e(bwmethod)</code>  | bandwidth method; <code>hsheather</code> , <code>bofinger</code> , or <code>chamberlain</code>  |
| <code>e(denmethod)</code> | density estimation method; <code>fitted</code> , <code>residual</code> , or <code>kernel</code> |

|                 |                                                                |
|-----------------|----------------------------------------------------------------|
| e(kernel)       | kernel function                                                |
| e(wtype)        | weight type                                                    |
| e(wexp)         | weight expression                                              |
| e(vce)          | <i>vctype</i> specified in <code>vce()</code>                  |
| e(vcetype)      | title used to label Std. err.                                  |
| e(properties)   | b V                                                            |
| e(predict)      | program used to implement <code>predict</code>                 |
| e(marginsnotok) | predictions disallowed by <code>margins</code>                 |
| e(asbalanced)   | factor variables <code>fvset</code> as <code>asbalanced</code> |
| e(asobserved)   | factor variables <code>fvset</code> as <code>asobserved</code> |

Matrices

|      |                                              |
|------|----------------------------------------------|
| e(b) | coefficient vector                           |
| e(V) | variance–covariance matrix of the estimators |

Functions

|           |                         |
|-----------|-------------------------|
| e(sample) | marks estimation sample |
|-----------|-------------------------|

In addition to the above, the following is stored in `r()`:

Matrices

|          |                                                                                                                            |
|----------|----------------------------------------------------------------------------------------------------------------------------|
| r(table) | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |
|----------|----------------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

`iqreg` stores the following in `e()`:

Scalars

|             |                                                               |
|-------------|---------------------------------------------------------------|
| e(N)        | number of observations                                        |
| e(df_r)     | residual degrees of freedom                                   |
| e(q0)       | lower quantile requested                                      |
| e(q1)       | upper quantile requested                                      |
| e(reps)     | number of replications                                        |
| e(r2_p_q0)  | lower quantile pseudo- $R^2$                                  |
| e(r2_p_q1)  | upper quantile pseudo- $R^2$                                  |
| e(sumrdev0) | lower quantile sum of raw deviations                          |
| e(sumrdev1) | upper quantile sum of raw deviations                          |
| e(sumadev0) | lower quantile sum of absolute deviations                     |
| e(sumadev1) | upper quantile sum of absolute deviations                     |
| e(rank)     | rank of <code>e(V)</code>                                     |
| e(convcode) | 0 if converged; otherwise, return code for why nonconvergence |

Macros

|                 |                                                                |
|-----------------|----------------------------------------------------------------|
| e(cmd)          | <code>iqreg</code>                                             |
| e(cmdline)      | command as typed                                               |
| e(depvar)       | name of dependent variable                                     |
| e(vcetype)      | title used to label Std. err.                                  |
| e(properties)   | b V                                                            |
| e(predict)      | program used to implement <code>predict</code>                 |
| e(marginsnotok) | predictions disallowed by <code>margins</code>                 |
| e(asbalanced)   | factor variables <code>fvset</code> as <code>asbalanced</code> |
| e(asobserved)   | factor variables <code>fvset</code> as <code>asobserved</code> |

Matrices

|      |                                              |
|------|----------------------------------------------|
| e(b) | coefficient vector                           |
| e(V) | variance–covariance matrix of the estimators |

Functions

|           |                         |
|-----------|-------------------------|
| e(sample) | marks estimation sample |
|-----------|-------------------------|

In addition to the above, the following is stored in `r()`:

|                       |                                                                                                                            |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|
| Matrices              |                                                                                                                            |
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

`sqreg` stores the following in `e()`:

|                              |                                                                |
|------------------------------|----------------------------------------------------------------|
| Scalars                      |                                                                |
| <code>e(N)</code>            | number of observations                                         |
| <code>e(df_r)</code>         | residual degrees of freedom                                    |
| <code>e(n_q)</code>          | number of quantiles requested                                  |
| <code>e(q#)</code>           | the quantiles requested                                        |
| <code>e(reps)</code>         | number of replications                                         |
| <code>e(r2_p_q#)</code>      | pseudo- $R^2$ for <i>q</i> #                                   |
| <code>e(sumrdv#)</code>      | sum of raw deviations for <i>q</i> #                           |
| <code>e(sumadv#)</code>      | sum of absolute deviations for <i>q</i> #                      |
| <code>e(rank)</code>         | rank of <code>e(V)</code>                                      |
| <code>e(convcode)</code>     | 0 if converged; otherwise, return code for why nonconvergence  |
| Macros                       |                                                                |
| <code>e(cmd)</code>          | <code>sqreg</code>                                             |
| <code>e(cmdline)</code>      | command as typed                                               |
| <code>e(depvar)</code>       | name of dependent variable                                     |
| <code>e(eqnames)</code>      | names of equations                                             |
| <code>e(vcetype)</code>      | title used to label Std. err.                                  |
| <code>e(properties)</code>   | <code>b V</code>                                               |
| <code>e(predict)</code>      | program used to implement <code>predict</code>                 |
| <code>e(marginsnotok)</code> | predictions disallowed by <code>margins</code>                 |
| <code>e(asbalanced)</code>   | factor variables <code>fvset</code> as <code>asbalanced</code> |
| <code>e(asobserved)</code>   | factor variables <code>fvset</code> as <code>asobserved</code> |

|                   |                                              |
|-------------------|----------------------------------------------|
| Matrices          |                                              |
| <code>e(b)</code> | coefficient vector                           |
| <code>e(V)</code> | variance–covariance matrix of the estimators |

|                        |                         |
|------------------------|-------------------------|
| Functions              |                         |
| <code>e(sample)</code> | marks estimation sample |

In addition to the above, the following is stored in `r()`:

|                       |                                                                                                                            |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|
| Matrices              |                                                                                                                            |
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

`bsqreg` stores the following in `e()`:

|                          |                             |
|--------------------------|-----------------------------|
| Scalars                  |                             |
| <code>e(N)</code>        | number of observations      |
| <code>e(df_r)</code>     | residual degrees of freedom |
| <code>e(q)</code>        | quantile requested          |
| <code>e(q_v)</code>      | value of the quantile       |
| <code>e(reps)</code>     | number of replications      |
| <code>e(r2_p)</code>     | pseudo- $R^2$               |
| <code>e(sum_adev)</code> | sum of absolute deviations  |
| <code>e(sum_rdev)</code> | sum of raw deviations       |

|                 |                                                               |
|-----------------|---------------------------------------------------------------|
| e(rank)         | rank of e(V)                                                  |
| e(convcode)     | 0 if converged; otherwise, return code for why nonconvergence |
| Macros          |                                                               |
| e(cmd)          | bsqreg                                                        |
| e(cmdline)      | command as typed                                              |
| e(depvar)       | name of dependent variable                                    |
| e(properties)   | b V                                                           |
| e(predict)      | program used to implement predict                             |
| e(marginsnotok) | predictions disallowed by margins                             |
| e(asbalanced)   | factor variables fvset as asbalanced                          |
| e(asobserved)   | factor variables fvset as asobserved                          |
| Matrices        |                                                               |
| e(b)            | coefficient vector                                            |
| e(V)            | variance–covariance matrix of the estimators                  |
| Functions       |                                                               |
| e(sample)       | marks estimation sample                                       |

In addition to the above, the following is stored in `r()`:

|          |                                                                                                                            |
|----------|----------------------------------------------------------------------------------------------------------------------------|
| Matrices |                                                                                                                            |
| r(table) | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r`-class command is run after the estimation command.

## Methods and formulas

Methods and formulas are presented under the following headings:

*Introduction*

*Linear programming formulation of quantile regression*

*Standard errors when residuals are i.i.d.*

*Pseudo- $R^2$*

## Introduction

According to [Stuart and Ord \(1991, 1084\)](#), the method of minimum absolute deviations was first proposed by [Boscovich in 1757](#) and was later developed by [Laplace](#); [Stigler \(1986, 39–55\)](#) and [Hald \(1998, 97–103, 112–116\)](#) provide historical details. According to [Bloomfield and Steiger \(1980\)](#), [Harris \(1950\)](#) later observed that the problem of minimum absolute deviations could be turned into the linear programming problem that was first implemented by [Wagner \(1959\)](#). Interest has grown in this method because robust methods and extreme value modeling have become more popular. Statistical and computational properties of minimum absolute deviation estimators are surveyed by [Narula and Wellington \(1982\)](#). [Cameron and Trivedi \(2005\)](#), [Hao and Naiman \(2007\)](#), and [Wooldridge \(2010\)](#) provide excellent introductions to quantile regression methods, while [Koenker \(2005\)](#) gives an in-depth review of the topic.

## Linear programming formulation of quantile regression

Define  $\tau$  as the quantile to be estimated; the median is  $\tau = 0.5$ . For each observation  $i$ , let  $\varepsilon_i$  be the residual

$$\varepsilon_i = y_i - \mathbf{x}'_i \widehat{\boldsymbol{\beta}}_\tau$$

The objective function to be minimized is

$$\begin{aligned} c_\tau(\varepsilon_i) &= (\tau \mathbf{1}\{\varepsilon_i \geq 0\} + (1 - \tau) \mathbf{1}\{\varepsilon_i < 0\}) |\varepsilon_i| \\ &= (\tau \mathbf{1}\{\varepsilon_i \geq 0\} - (1 - \tau) \mathbf{1}\{\varepsilon_i < 0\}) \varepsilon_i \\ &= (\tau - \mathbf{1}\{\varepsilon_i < 0\}) \varepsilon_i \end{aligned} \tag{2}$$

where  $\mathbf{1}\{\cdot\}$  is the indicator function. This function is sometimes referred to as the check function because it resembles a check mark (Wooldridge 2010, 450); the slope of  $c_\tau(\varepsilon_i)$  is  $\tau$  when  $\varepsilon_i > 0$  and is  $\tau - 1$  when  $\varepsilon_i < 0$ , but is undefined for  $\varepsilon_i = 0$ . Choosing the  $\widehat{\boldsymbol{\beta}}_\tau$  that minimize  $c_\tau(\varepsilon_i)$  is equivalent to finding the  $\widehat{\boldsymbol{\beta}}_\tau$  that make  $\mathbf{x}\widehat{\boldsymbol{\beta}}_\tau$  best fit the quantiles of the distribution of  $y$  conditional on  $\mathbf{x}$ .

This minimization problem is set up as a linear programming problem and is solved with linear programming techniques, as suggested by Armstrong, Frome, and Kung (1979) and described in detail by Koenker (2005). Here  $2n$  slack variables,  $\mathbf{u}_{n \times 1}$  and  $\mathbf{v}_{n \times 1}$ , are introduced, where  $u_i \geq 0$ ,  $v_i \geq 0$ , and  $u_i \times v_i = 0$ , reformulating the problem as

$$\min_{\boldsymbol{\beta}_\tau, \mathbf{u}, \mathbf{v}} \{ \tau \mathbf{1}'_n \mathbf{u} + (1 - \tau) \mathbf{1}'_n \mathbf{v} \mid \mathbf{y} - \mathbf{X} \boldsymbol{\beta}_\tau = \mathbf{u} - \mathbf{v} \}$$

where  $\mathbf{1}_n$  is a vector of 1s. This is a linear objective function on a polyhedral constraint set with  $\binom{n}{k}$  vertices, and our goal is to find the vertex that minimizes (2). Each step in the search is described by a set of  $k$  observations through which the regression plane passes, called the *basis*. A step is taken by replacing a point in the basis if the linear objective function can be improved. If this occurs, a line is printed in the iteration log. The definition of convergence is exact in the sense that no amount of added iterations could improve the objective function.

A series of weighted least-squares (WLS) regressions is used to identify a set of observations as a starting basis. The WLS algorithm for  $\tau = 0.5$  is taken from Schlossmacher (1973) with a generalization for  $0 < \tau < 1$  implied from Hunter and Lange (2000).

## Standard errors when residuals are i.i.d.

The estimator for the VCE implemented in qreg assumes that the errors of the model are independent and identically distributed (i.i.d.). When the errors are i.i.d., the large-sample VCE is

$$\text{cov}(\boldsymbol{\beta}_\tau) = \frac{\tau(1 - \tau)}{f_Y^2(\xi_\tau)} \{E(\mathbf{x}_i \mathbf{x}'_i)\}^{-1} \tag{3}$$

where  $\xi_\tau = F_Y^{-1}(\tau)$  and  $F_Y(y)$  is the distribution function of  $Y$  with density  $f_Y(y)$ . See Koenker (2005, 73) for this result. From (3), we see that the regression precision depends on the inverse of the density function, termed the sparsity function,  $s_\tau = 1/f_Y(\xi_\tau)$ .

While  $1/n \sum_{i=1}^n \mathbf{x}_i \mathbf{x}'_i$  estimates  $E(\mathbf{x}_i \mathbf{x}'_i)$ , estimating the sparsity function is more difficult. qreg provides several methods to estimate the sparsity function. The different estimators are specified through the suboptions of `vce(iid, denmethod bwidth)`. The suboption `denmethod` specifies the functional form for the sparsity estimator. The default is `fitted`.

Here we outline the logic underlying the fitted estimator. Because  $F_Y(y)$  is the distribution function for  $Y$ , we have  $f_Y(y) = \{dF_Y(y)\}/dy$ ,  $\tau = F_Y(\xi_\tau)$ , and  $\xi_\tau = F_Y^{-1}(\tau)$ . When differentiating the identity  $F_Y\{F_Y^{-1}(\tau)\} = \tau$ , the sparsity function can be written as  $s_\tau = \{F_Y^{-1}(\tau)\}/dt$ . Numerically, we can approximate the derivative using the centered difference,

$$\frac{F_Y^{-1}(\tau)}{dt} \approx \frac{F_Y^{-1}(\tau+h) - F_Y^{-1}(\tau-h)}{2h} = \frac{\xi_{\tau+h} - \xi_{\tau-h}}{2h} = \hat{s}_\tau \quad (4)$$

where  $h$  is the bandwidth.

The empirical quantile function is computed by first estimating  $\beta_{\tau+h}$  and  $\beta_{\tau-h}$ , and then computing  $\hat{F}_Y^{-1}(\tau+h) = \bar{\mathbf{x}}'\hat{\beta}_{\tau+h}$  and  $\hat{F}_Y^{-1}(\tau-h) = \bar{\mathbf{x}}'\hat{\beta}_{\tau-h}$ , where  $\bar{\mathbf{x}}$  is the sample mean of the independent variables  $\mathbf{x}$ . These quantities are then substituted into (4).

Alternatively, as the option suggests, `vce(iid, residual)` specifies that `qreg` use the empirical quantile function of the residuals to estimate the sparsity. Here we substitute  $F_\epsilon$ , the distribution of the residuals, for  $F_Y$ , which only differ by their first moments.

The  $k$  residuals associated with the linear programming basis will be zero, where  $k$  is the number of regression coefficients. These zero residuals are removed before computing the  $\tau+h$  and  $\tau-h$  quantiles,  $\varepsilon_{(\tau+h)} = \hat{F}_\epsilon^{-1}(\tau+h)$  and  $\varepsilon_{(\tau-h)} = \hat{F}_\epsilon^{-1}(\tau-h)$ . The  $\hat{F}_\epsilon^{-1}$  estimates are then substituted for  $F_Y^{-1}$  in (4).

Each of the estimators for the sparsity function depends on a bandwidth. The `vce()` suboption `bandwidth` specifies the bandwidth method to use. The three bandwidth options and their citations are `hsheather` (Hall and Sheather 1988), `bofinger` (Bofinger 1975), and `chamberlain` (Chamberlain 1994).

Their formulas are

$$h_s = n^{-1/3} \Phi^{-1} \left( 1 - \frac{\alpha}{2} \right)^{2/3} \left[ \frac{3}{2} \times \frac{\phi\{\Phi^{-1}(\tau)\}^2}{2\Phi^{-1}(\tau)^2 + 1} \right]^{1/3}$$

$$h_b = n^{-1/5} \left[ \frac{\frac{9}{2}\phi\{\Phi^{-1}(\tau)\}^4}{\{2\Phi^{-1}(\tau)^2 + 1\}^2} \right]^{1/5}$$

$$h_c = \Phi^{-1} \left( 1 - \frac{\alpha}{2} \right) \sqrt{\frac{\tau(1-\tau)}{n}}$$

where  $h_s$  is the Hall–Sheather bandwidth,  $h_b$  is the Bofinger bandwidth,  $h_c$  is the Chamberlain bandwidth,  $\Phi()$  and  $\phi()$  are the standard normal distribution and density functions,  $n$  is the sample size, and  $100(1-\alpha)$  is the confidence level set by the `level()` option. Koehler (2005) discusses the derivation of the Hall–Sheather and the Bofinger bandwidth formulas. You should avoid modifying the confidence level when replaying estimates that use the Hall–Sheather or Chamberlain bandwidths because these methods use the confidence level to estimate the coefficient standard errors.

Finally, the `vce()` suboption `kernel(kernel)` specifies that `qreg` use one of several kernel density estimators to estimate the sparsity function. `kernel` allows you to choose which kernel function to use, where the default is the Epanechnikov kernel. See [R] `kdensity` for the functional form of the eight kernels.

The kernel bandwidth is computed using an adaptive estimate of scale

$$h_k = \min \left( \hat{\sigma}, \frac{r_q}{1.34} \right) \times \{ \Phi^{-1}(\tau+h) - \Phi^{-1}(\tau-h) \}$$

where  $h$  is one of  $h_s$ ,  $h_b$ , or  $h_c$ ;  $r_q$  is the interquartile range; and  $\hat{\sigma}$  is the standard deviation of  $\mathbf{y}$ ; see Silverman (1986, 47) and Koenker (2005, 81) for discussions. Let  $\hat{f}_\epsilon(\epsilon_i)$  be the kernel density estimate for the  $i$ th residual, and then the kernel estimator for the sparsity function is

$$\hat{s}_\tau = \frac{nh_k}{\sum_{i=1}^n \hat{f}_\epsilon(\epsilon_i)}$$

Finally, substituting your choice of sparsity estimate into (3) results in the i.i.d. variance–covariance matrix

$$\mathbf{V}_n = \hat{s}_\tau^2 \tau(1-\tau) \left( \sum_{i=1}^n \mathbf{x}_i \mathbf{x}_i' \right)^{-1}$$

## Pseudo- $R^2$

The pseudo- $R^2$  is calculated as

$$1 - \frac{\text{sum of weighted deviations about estimated quantile}}{\text{sum of weighted deviations about raw quantile}}$$

This is based on the likelihood for a double-exponential distribution  $e^{v_i|\epsilon_i|}$ , where  $v_i$  are multipliers

$$v_i = \begin{cases} \tau & \text{if } \epsilon_i > 0 \\ (1-\tau) & \text{otherwise} \end{cases}$$

Minimizing the objective function (2) with respect to  $\beta_\tau$  also minimizes  $\sum_i |\epsilon_i|v_i$ , the sum of weighted least absolute deviations. For example, for the 50th percentile  $v_i = 1$ , for all  $i$ , and we have median regression. If we want to estimate the 75th percentile, we weight the negative residuals by 0.25 and the positive residuals by 0.75. It can be shown that the criterion is minimized when 75% of the residuals are negative.

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## Also see

- [R] **qreg postestimation** — Postestimation tools for qreg, iqreg, sqreg, and bsqreg
- [R] **bootstrap** — Bootstrap sampling and estimation
- [R] **ivqregress** — Instrumental-variables quantile regression
- [R] **regress** — Linear regression
- [R] **rreg** — Robust regression
- [BAYES] **bayes: qreg** — Bayesian quantile regression
- [MI] **Estimation** — Estimation commands for use with mi estimate
- [U] **20 Estimation and postestimation commands**

Postestimation commands    predict    margins    estat  
 Remarks and examples    Also see

## Postestimation commands

The following postestimation command is of special interest after sqreg:

| Command                     | Description                                                             |
|-----------------------------|-------------------------------------------------------------------------|
| <code>estat coefplot</code> | plot coefficients and their confidence intervals at different quantiles |

The following postestimation commands are available after qreg, iqreg, bsqreg, and sqreg:

| Command                            | Description                                                                                       |
|------------------------------------|---------------------------------------------------------------------------------------------------|
| <code>contrast</code>              | contrasts and ANOVA-style joint tests of parameters                                               |
| <code>estat summarize</code>       | summary statistics for the estimation sample                                                      |
| <code>estat vce</code>             | variance-covariance matrix of the estimators (VCE)                                                |
| <code>estimates</code>             | cataloging estimation results                                                                     |
| <code>etable</code>                | table of estimation results                                                                       |
| <sup>†</sup> <code>forecast</code> | dynamic forecasts and simulations                                                                 |
| * <code>hausman</code>             | Hausman's specification test                                                                      |
| <code>lincom</code>                | point estimates, standard errors, testing, and inference for linear combinations of parameters    |
| <code>linktest</code>              | link test for model specification                                                                 |
| <code>margins</code>               | marginal means, predictive margins, marginal effects, and average marginal effects                |
| <code>marginsplot</code>           | graph the results from margins (profile plots, interaction plots, etc.)                           |
| <code>nlcom</code>                 | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters |
| <code>predict</code>               | predictions and their SEs, residuals, etc.                                                        |
| <code>predictnl</code>             | point estimates, standard errors, testing, and inference for generalized predictions              |
| <code>pwcompare</code>             | pairwise comparisons of parameters                                                                |
| <code>test</code>                  | Wald tests of simple and composite linear hypotheses                                              |
| <code>testnl</code>                | Wald tests of nonlinear hypotheses                                                                |

\*`hausman` is not appropriate after `bsqreg`, `isqreg`, or `sqreg`.

<sup>†</sup>`forecast` is not appropriate with `mi` estimation results.

## predict

### Description for predict

`predict` creates a new variable containing predictions such as linear predictions, standard errors, and residuals.

### Menu for predict

Statistics > Postestimation

### Syntax for predict

For *qreg*, *iqreg*, and *bsqreg*

```
predict [type] newvar [if] [in] [, [xb|stdp|residuals]]
```

For *sqreg*

```
predict [type] newvar [if] [in] [, equation(eqno[,eqno]) statistic]
```

| <i>statistic</i>       | Description                                            |
|------------------------|--------------------------------------------------------|
| Main                   |                                                        |
| <code>xb</code>        | linear prediction; the default                         |
| <code>stdp</code>      | standard error of the linear prediction                |
| <code>stddp</code>     | standard error of the difference in linear predictions |
| <code>residuals</code> | residuals                                              |

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

### Options for predict

Main

`xb`, the default, calculates the linear prediction.

`stdp` calculates the standard error of the linear prediction.

`stddp` is allowed only after you have fit a model using *sqreg*. The standard error of the difference in linear predictions ( $\mathbf{x}_{1j}\mathbf{b} - \mathbf{x}_{2j}\mathbf{b}$ ) between equations 1 and 2 is calculated.

`residuals` calculates the residuals, that is,  $y_j - \mathbf{x}_j\mathbf{b}$ .

`equation(eqno[,eqno])` specifies the equation to which you are making the calculation.

`equation()` is filled in with one *eqno* for the `xb`, `stdp`, and `residuals` options. `equation(#1)` would mean that the calculation is to be made for the first equation, `equation(#2)` would mean the second, and so on. You could also refer to the equations by their names. `equation(income)` would refer to the equation named `income` and `equation(hours)` to the equation named `hours`.

If you do not specify `equation()`, results are the same as if you had specified `equation(#1)`.

To use `stddp`, you must specify two equations. You might specify `equation(#1, #2)` or `equation(q80, q20)` to indicate the 80th and 20th quantiles.

## margins

### Description for margins

`margins` estimates margins of response for linear predictions.

### Menu for margins

Statistics > Postestimation

### Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [options]
```

| <i>statistic</i>              | Description                           |
|-------------------------------|---------------------------------------|
| <code>xb</code>               | linear prediction; the default        |
| <code>stdp</code>             | not allowed with <code>margins</code> |
| <code>stddp</code>            | not allowed with <code>margins</code> |
| <u><code>residuals</code></u> | not allowed with <code>margins</code> |

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [R] [margins](#).

## estat

### Description for estat

`estat coefplot` plots the estimated coefficients and their confidence intervals (CIs) after `sqreg`.

### Menu for estat

Statistics > Postestimation

### Syntax for estat

```
estat coefplot [varname] [, options]
```

*varname* is one of the variables specified when fitting `sqreg`; the default is the first endogenous variable.

| <i>options</i>                                 | Description                                                                        |
|------------------------------------------------|------------------------------------------------------------------------------------|
| <code>noci</code>                              | do not plot the CIs                                                                |
| <code>nools</code>                             | do not plot the ordinary least-squares (OLS) estimates                             |
| Plot                                           |                                                                                    |
| <i>connect_options</i>                         | change look of lines or connecting method                                          |
| <i>marker_options</i>                          | change look of markers (color, size, etc.)                                         |
| CI plot                                        |                                                                                    |
| <code>ciopts</code> ( <i>area_options</i> )    | affect rendition of the pointwise CIs                                              |
| Line options                                   |                                                                                    |
| <code>lineopts</code> ( <i>cline_options</i> ) | affect rendition of reference line identifying the OLS estimates                   |
| Y axis, X axis, Titles, Legend, Overall        |                                                                                    |
| <i>twoway_options</i>                          | any options other than <code>by()</code> documented in [G-3] <i>twoway_options</i> |

### Options for estat

`noci` removes plots of the pointwise CIs. The default is to plot the CIs.

`nools` removes the plot of the OLS estimates. The default is to plot the OLS reference line.

#### Plot

*connect\_options* specify how points on a graph are to be connected; [G-3] *connect\_options*.

*marker\_options* affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] *marker\_options*.

#### CI plot

`ciopts` (*area\_options*) affects rendition of the pointwise CIs; see [G-3] *area\_options*.

## Line options

`lineopts(cline_options)` affects rendition of reference line identifying the OLS estimates; see [G-3] *cline\_options*.

## Y axis, X axis, Titles, Legend, Overall

*twoway\_options* are any of the options documented in [G-3] *twoway\_options*, excluding `by()`. These include options for titling the graph (see [G-3] *title\_options*) and for saving the graph to disk (see [G-3] *saving\_option*).

## Remarks and examples

### ► Example 1: Predictions after qreg and iqreg

In [example 4](#) of [R] `qreg`, we fit regressions for the lower and the upper quartile of the `price` variable. The `predict` command can be used to obtain the linear prediction after each regression.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. qreg price weight length foreign, quantile(.25)
(output omitted)
. predict q25
(option xb assumed; fitted values)
. qreg price weight length foreign, quantile(.75)
(output omitted)
. predict q75
(option xb assumed; fitted values)
```

We can use the variables generated by `predict` to compute the predicted interquartile range, that is,

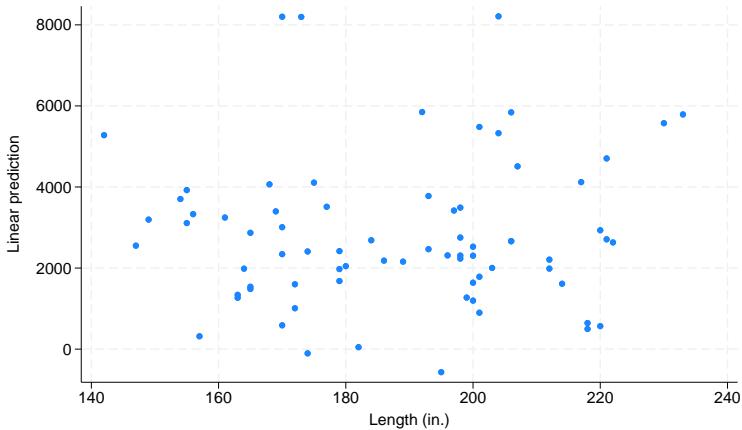
```
. generate iqr1 = q75 - q25
```

If we directly perform the interquartile range regression with the `iqreg` command, we can predict the interquartile range and also the standard error for the prediction.

```
. iqreg price weight length foreign, quantile(.25 .75)
(output omitted)
. predict iqr2
(option xb assumed; fitted values)
. predict stdp, stdp
```

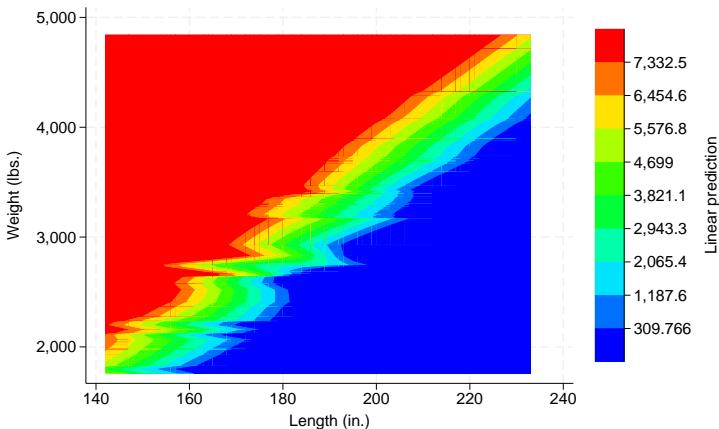
We now plot the predicted interquartile range versus variable length:

```
. scatter iqr2 length
```



As stated in [example 5](#) of [\[R\] qreg](#), the negative coefficient for the length variable means that increases in length imply decreases in the interquartile range and therefore in price dispersion. Consequently, we could have expected a downward trend in the plot, but there is not. This is because the regression output indicates that when we hold the rest of the variables constant, an increase in length leads to a decrease in iqr2. However, there is a high correlation between weight and length, which could be masking the effect of length on iqr2. We can achieve a better visualization by using a contour plot.

```
. twoway contour iqr2 weight length, level(10)
```



We can see the effect by setting a fixed value of length on the vertical axis, say, 3,000 lbs. When we move from left to right on the horizontal axis, we see that for small values of length, iqr2 values are shown in red, meaning high values, and when we move toward the right, the graph indicates transition into increasingly smaller values.

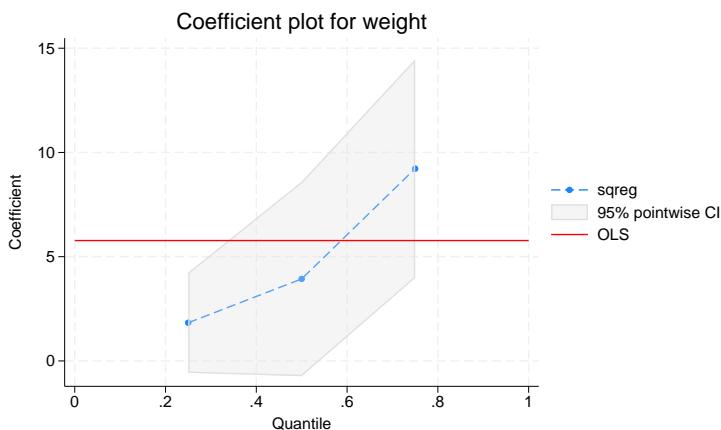
## ► Example 2: Coefficients plot after sqreg

In [example 5](#) of [\[R\] qreg](#), we simultaneously estimated the quantile regressions at different quantile indexes.

```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
. set seed 1001
. sqreg price weight length foreign, q(.25 .5 .75) reps(100)
(output omitted)
```

We can now use `estat coefplot` to visualize the coefficients at different quantiles. For example, we can type `estat coefplot weight` to see the trend of the effects of weight on price across quantiles.

```
. estat coefplot weight
```



The resulting graph shows that there is an upward trend in the effects. For reference, the red line shows the OLS estimates.



## Also see

[\[R\] qreg](#) — Quantile regression

[\[U\] 20 Estimation and postestimation commands](#)

[Description](#)[Syntax](#)[Remarks and examples](#)[Also see](#)

## Description

query displays the settings of various Stata parameters.

## Syntax

```
query [memory | output | interface | graphics | network | update |
 trace | mata | java | lapack | putdocx | putpdf | python | rng |
 sort | unicode | other]
```

## Remarks and examples

query provides more system information than you will ever want to know. You do not need to understand every line of output that query produces if all you need is one piece of information. Here is what happens when you type query:

```
. query
```

---

### Memory settings

|                     |      |                                |
|---------------------|------|--------------------------------|
| set maxvar          | 5000 | 2048-120000; max. vars allowed |
| set niceness        | 5    | 0-10                           |
| set min_memory      | 0    | 0-1600g                        |
| set max_memory      | .    | 32m-1600g or .                 |
| set segmentsize     | 32m  | 1m-32g                         |
| set adosize         | 1000 | kilobytes                      |
| set max_preservemem | 1g   | 0-1600g                        |

---

### Output settings

|              |        |                        |
|--------------|--------|------------------------|
| set more     | off    |                        |
| set rmmsg    | off    |                        |
| set dp       | period | may be period or comma |
| set linesize | 80     | characters             |
| set pagesize | 32     | lines                  |

---

|             |    |  |
|-------------|----|--|
| set iterlog | on |  |
|-------------|----|--|

---

|            |    |                              |
|------------|----|------------------------------|
| set level  | 95 | percent confidence intervals |
| set clevel | 95 | percent credible intervals   |

---

|                    |      |                               |
|--------------------|------|-------------------------------|
| set showbaselevels |      | may be empty, off, on, or all |
| set showemptycells |      | may be empty, off, or on      |
| set showomitted    |      | may be empty, off, or on      |
| set fvlabel        | on   |                               |
| set fvwrap         | 1    |                               |
| set fvwrapon       | word | may be word or width          |

---

|              |  |                          |
|--------------|--|--------------------------|
| set lstretch |  | may be empty, off, or on |
|--------------|--|--------------------------|

---

|             |  |                                    |
|-------------|--|------------------------------------|
| set cformat |  | may be empty or a numerical format |
| set pformat |  | may be empty or a numerical format |
| set sformat |  | may be empty or a numerical format |

---

|                    |    |  |
|--------------------|----|--|
| set coeftabresults | on |  |
| set dots           | on |  |

---

|             |      |                     |
|-------------|------|---------------------|
| set logtype | smcl | may be smcl or text |
| set logmsg  | on   |                     |

---

|                    |                         |  |
|--------------------|-------------------------|--|
| set collect_label  | default                 |  |
| set collect_style  | default                 |  |
| set table_style    | table                   |  |
| set etable_style   | etable                  |  |
| set dtable_style   | dtable                  |  |
| set tabulate_style | tabulate                |  |
| set tabulate_comps | tabulate_composites     |  |
| set svy_tab_comps  | svy_tabulate_composites |  |
| set collect_warn   | on                      |  |

---

Interface settings

|                   |     |  |
|-------------------|-----|--|
| set dockable      | on  |  |
| set floatwindows  | off |  |
| set locksplitters | off |  |
| set pinnable      | on  |  |
| set taskbargroups | on  |  |
| set doublebuffer  | on  |  |

---

|                   |        |            |
|-------------------|--------|------------|
| set linegap       | 1      | pixels     |
| set scrollbufsize | 204800 | characters |
| set fastscroll    | on     |            |
| set reventries    | 5000   | lines      |

---

|           |    |              |
|-----------|----|--------------|
| set maxdb | 50 | dialog boxes |
|-----------|----|--------------|

---

Graphics settings

|                   |         |                                       |
|-------------------|---------|---------------------------------------|
| set graphics      | on      |                                       |
| set autotabgraphs | on      |                                       |
| set scheme        | stcolor |                                       |
| set printcolor    | asis    | may be automatic, asis, gs1, gs2, gs3 |
| set copycolor     | asis    | may be automatic, asis, gs1, gs2, gs3 |

---

Network settings

|                   |     |  |
|-------------------|-----|--|
| set httpproxy     | off |  |
| set httpproxyhost |     |  |
| set httpproxyport | 80  |  |

---

|                   |     |  |
|-------------------|-----|--|
| set httpproxyauth | off |  |
| set httpproxyuser |     |  |
| set httpproxypw   |     |  |

---

Update settings

|                     |     |  |
|---------------------|-----|--|
| set update_query    | off |  |
| set update_interval | 7   |  |
| set update_prompt   | on  |  |



---

```

Other settings
set type float may be float or double
set maxiter 300 max iterations for estimation commands
set searchdefault all may be local, net, or all
set varabbrev on
set emptycells keep may be keep or drop
set fvtrack term may be term or factor
set fvbase on
set processors 4 1-4
set kmp_blocktime -1 time duration in milliseconds
set odbcdriver unicode may be unicode, ansi, or default
set haverdir on
set fredkey on
set collect_double on
set dtascomplevel 1 dtas default compression level; 0-9
set reshape_favor default may be default, memory, or speed
set doeditbackup on

```

---

The output is broken into several divisions: memory, output, interface, graphics, network, update, trace, Mata, Java, LAPACK, putdocx, putpdf, Python, RNG, sort, Unicode, and other settings. We will discuss each one in turn.

We generated the output above using Stata/MP for Windows. Here is what happens when we type query and we are running Stata/SE for Mac:

```
. query
```

---

```

Memory settings
set maxvar 5000 2048-32767; max. vars allowed
set niceness 5 0-10
set min_memory 0 0-1600g
set max_memory . 32m-1600g or .
set segmentsize 32m 1m-32g
set adosize 1000 kilobytes

```

---

```

Output settings
set more off
set rmsg off
set dp period may be period or comma
set linesize 80 characters
set pagesize 23 lines

```

---

```
set iterlog on
```

---

```

set level 95 percent confidence intervals
set clevel 95 percent credible intervals

```

---

```

set showbaselevels may be empty, off, on, or all
set showemptycells may be empty, off, or on
set showomitted may be empty, off, or on
set fvlabel on
set fvwrap 1
set fvwrapon word may be word or width

```

---

```
set lstretch may be empty, off, or on
```

---

|             |  |                                    |
|-------------|--|------------------------------------|
| set cformat |  | may be empty or a numerical format |
| set pformat |  | may be empty or a numerical format |
| set sformat |  | may be empty or a numerical format |

---

|                    |    |  |
|--------------------|----|--|
| set coeftabresults | on |  |
| set dots           | on |  |

---

|             |      |                     |
|-------------|------|---------------------|
| set logtype | smcl | may be smcl or text |
| set logmsg  | on   |                     |

---

|                    |                         |  |
|--------------------|-------------------------|--|
| set collect_label  | default                 |  |
| set collect_style  | default                 |  |
| set table_style    | table                   |  |
| set etable_style   | etable                  |  |
| set dtable_style   | dtable                  |  |
| set tabulate_style | tabulate                |  |
| set tabulate_comps | tabulate_composites     |  |
| set svy_tab_comps  | svy_tabulate_composites |  |
| set collect_warn   | on                      |  |

---

|                    |     |  |
|--------------------|-----|--|
| set notifyuser     | on  |  |
| set playsnd        | off |  |
| set include_bitmap | on  |  |

---

Interface settings

|                 |    |  |
|-----------------|----|--|
| set revkeyboard | on |  |
| set varkeyboard | on |  |
| set smoothfonts | on |  |

---

|                   |        |            |
|-------------------|--------|------------|
| set linegap       | 1      | pixels     |
| set scrollbufsize | 204800 | characters |
| set reventries    | 5000   | lines      |

---

|           |    |              |
|-----------|----|--------------|
| set maxdb | 50 | dialog boxes |
|-----------|----|--------------|

---

Graphics settings

|                   |         |                                       |
|-------------------|---------|---------------------------------------|
| set graphics      | on      |                                       |
| set scheme        | stcolor |                                       |
| set printcolor    | asis    | may be automatic, asis, gs1, gs2, gs3 |
| set copycolor     | asis    | may be automatic, asis, gs1, gs2, gs3 |
| set maxbezierpath | 0       | bezier path elements                  |

---

Network settings

|                   |     |  |
|-------------------|-----|--|
| set httpproxy     | off |  |
| set httpproxyhost |     |  |
| set httpproxyport | 80  |  |

---

|                   |     |  |
|-------------------|-----|--|
| set httpproxyauth | off |  |
| set httpproxyuser |     |  |
| set httpproxypw   |     |  |

---

Update settings

|                     |     |  |
|---------------------|-----|--|
| set update_query    | off |  |
| set update_interval | 7   |  |
| set update_prompt   | on  |  |



---

|                    |         |                                        |
|--------------------|---------|----------------------------------------|
| Other settings     |         |                                        |
| set type           | float   | may be float or double                 |
| set maxiter        | 300     | max iterations for estimation commands |
| set searchdefault  | all     | may be local, net, or all              |
| set varabbrev      | on      |                                        |
| set emptycells     | keep    | may be keep or drop                    |
| set fvtrack        | term    | may be term or factor                  |
| set fvbase         | on      |                                        |
| set processors     | 1       |                                        |
| set odbcmgr        | iodbc   | may be iodbc or unixodbc               |
| set odbcdriver     | unicode | may be unicode, ansi, or default       |
| set fredkey        |         |                                        |
| set collect_double | on      |                                        |
| set dtascomplevel  | 1       | dtas default compression level; 0-9    |
| set reshape_favor  | default | may be default, memory, or speed       |
| set doeditbackup   | on      |                                        |

---

## Memory settings

Memory settings indicate how memory is allocated, the maximum number of variables, and the maximum size of a matrix.

For more information, see

|                 |                     |
|-----------------|---------------------|
| maxvar          | [D] <b>memory</b>   |
| niceness        | [D] <b>memory</b>   |
| min_memory      | [D] <b>memory</b>   |
| max_memory      | [D] <b>memory</b>   |
| segmentsize     | [D] <b>memory</b>   |
| adosize         | [P] <b>sysdir</b>   |
| max_preservemem | [P] <b>preserve</b> |

## Output settings

Output settings show how Stata displays output on the screen and in log files.

For more information, see

|                |          |                           |
|----------------|----------|---------------------------|
| more           | [R]      | <b>more</b>               |
| rmsg           | [P]      | <b>rmsg</b>               |
| dp             | [D]      | <b>format</b>             |
| linesize       | [R]      | <b>log</b>                |
| pagesize       | [R]      | <b>more</b>               |
| iterlog        | [R]      | <b>set iter</b>           |
| level          | [R]      | <b>level</b>              |
| clevel         | [BAYES]  | <b>set clevel</b>         |
| showbaselevels | [R]      | <b>set showbaselevels</b> |
| showemptycells | [R]      | <b>set showbaselevels</b> |
| showomitted    | [R]      | <b>set showbaselevels</b> |
| fvlabel        | [R]      | <b>set showbaselevels</b> |
| fvwrap         | [R]      | <b>set showbaselevels</b> |
| fvwrapon       | [R]      | <b>set showbaselevels</b> |
| lstretch       | [R]      | <b>set</b>                |
| cformat        | [R]      | <b>set cformat</b>        |
| pformat        | [R]      | <b>set cformat</b>        |
| sformat        | [R]      | <b>set cformat</b>        |
| coeftabresults | [R]      | <b>set</b>                |
| dots           | [R]      | <b>set</b>                |
| logtype        | [R]      | <b>log</b>                |
| logmsg         | [R]      | <b>log</b>                |
| collect_label  | [TABLES] | <b>set collect_label</b>  |
| collect_style  | [TABLES] | <b>set collect_style</b>  |
| table_style    | [TABLES] | <b>set table_style</b>    |
| etable_style   | [TABLES] | <b>set etable_style</b>   |
| dtable_style   | [TABLES] | <b>set dtable_style</b>   |
| tabulate_style | [TABLES] | <b>set tabulate_style</b> |
| tabulate_comps | [TABLES] | <b>set tabulate_style</b> |
| svy_tab_comps  | [TABLES] | <b>set tabulate_style</b> |
| collect_warn   | [TABLES] | <b>set collect_warn</b>   |
| notifyuser     | [R]      | <b>set</b>                |
| playsnd        | [R]      | <b>set</b>                |
| include_bitmap | [R]      | <b>set</b>                |

## Interface settings

Interface settings control how Stata's interface works.

For more information, see

|               |     |            |
|---------------|-----|------------|
| dockable      | [R] | <b>set</b> |
| floatwindows  | [R] | <b>set</b> |
| locksplitters | [R] | <b>set</b> |
| pinnable      | [R] | <b>set</b> |
| taskbargroups | [R] | <b>set</b> |
| doublebuffer  | [R] | <b>set</b> |
| revkeyboard   | [R] | <b>set</b> |
| varkeyboard   | [R] | <b>set</b> |
| smoothfonts   | [R] | <b>set</b> |
| linegap       | [R] | <b>set</b> |
| scrollbufsize | [R] | <b>set</b> |
| fastscroll    | [R] | <b>set</b> |
| reventries    | [R] | <b>set</b> |
| maxdb         | [R] | <b>db</b>  |

## Graphics settings

Graphics settings indicate how Stata's graphics are displayed.

For more information, see

|               |       |                       |
|---------------|-------|-----------------------|
| graphics      | [G-2] | <b>set graphics</b>   |
| autotabgraphs | [R]   | <b>set</b>            |
| scheme        | [G-2] | <b>set scheme</b>     |
| printcolor    | [G-2] | <b>set printcolor</b> |
| copycolor     | [G-2] | <b>set printcolor</b> |
| maxbezierpath | [R]   | <b>set</b>            |

## Network settings

Network settings determine how Stata interacts with the internet.

For more information, see [R] **netio**.

## Update settings

Update settings determine how Stata performs updates.

For more information, see [R] **update**.

## Trace settings

Trace settings adjust Stata's behavior and are particularly useful in debugging code.

For more information, see [P] **trace**.

## Mata settings

Mata settings affect Mata's system parameters.

For more information, see [\[M-3\] mata set](#).

## Java settings

Java settings control the Java Runtime Environment.

For more information, see [\[P\] Java utilities](#).

## LAPACK settings

LAPACK settings control the LAPACK and BLAS routines.

For more information, see [\[M-1\] LAPACK](#).

## putdocx settings

putdocx settings control how spacing is handled in blocks of text and the number of tables allowed.

For more information, see [\[RPT\] set docx](#) and [\[RPT\] putdocx table](#).

## putpdf settings

putpdf settings control the number of tables allowed.

For more information, see [\[RPT\] putpdf table](#).

## Python settings

Python settings control the version and search paths of Python.

For more information, see [\[P\] PyStata integration](#).

## RNG settings

The RNG settings set Stata's random-number generators.

For more information, see [\[R\] set rng](#), [\[R\] set seed](#) (set rngstate), and [\[R\] set rngstream](#).

## sort settings

The sort settings affect `sort`, `gsort`, and other commands that use sorting as part of their computation.

For more information, see [\[P\] set sortmethod](#) and [\[P\] set sortrngstate](#).

## Unicode settings

Unicode settings affect Stata's localization package for the user interface.

For more information, see [\[P\] set locale\\_ui](#) and [\[P\] set locale\\_functions](#).

## Other settings

The other settings are a miscellaneous collection.

For more information, see

|                |          |                           |
|----------------|----------|---------------------------|
| type           | [D]      | <b>generate</b>           |
| maxiter        | [R]      | <i>set iter</i>           |
| searchdefault  | [R]      | <b>search</b>             |
| varabbrev      | [R]      | <b>set</b>                |
| emptycells     | [R]      | <b>set</b>                |
| fvtrack        | [R]      | <b>set</b>                |
| fvbase         | [R]      | <b>set</b>                |
| processors     | [R]      | <b>set</b>                |
| kmp_blocktime  | [R]      | <b>set</b>                |
| odbcmgr        | [D]      | <b>odbc</b>               |
| odbcdriver     | [D]      | <b>odbc</b>               |
| haverdir       | [D]      | <b>import haver</b>       |
| fredkey        | [D]      | <b>import fred</b>        |
| collect_double | [TABLES] | <b>set collect_double</b> |
| dtascomplevel  | [R]      | <b>set</b>                |
| reshape_favor  | [D]      | <b>reshape</b>            |
| doeditbackup   | [R]      | <b>set</b>                |

In general, the parameters displayed by query can be changed by set; see [R] **set**.

## Also see

[R] **set** — Overview of system parameters

[M-3] **mata set** — Set and display Mata system parameters

[P] **creturn** — Return c-class values

[Description](#)[Options for ranksum](#)[Methods and formulas](#)[Quick start](#)[Options for median](#)[References](#)[Menu](#)[Remarks and examples](#)[Also see](#)[Syntax](#)[Stored results](#)

## Description

`ranksum` tests the hypothesis that two independent samples (that is, *unmatched* data) are from populations with the same distribution by using the Wilcoxon rank-sum test, which is also known as the Mann–Whitney two-sample statistic (Wilcoxon 1945; Mann and Whitney 1947).

`median` performs a nonparametric  $k$ -sample test on the equality of medians. It tests the null hypothesis that the  $k$  samples were drawn from populations with the same median. For two samples, the  $\chi^2$  test statistic is computed both with and without a continuity correction.

`ranksum` and `median` are for use with *unmatched* data. For equality tests on matched data, see [\[R\] signrank](#).

## Quick start

*Wilcoxon rank-sum test*

Test for equality of distributions of  $v$  over two groups defined by the levels of `catvar1`

```
ranksum v, by(catvar1)
```

Compute an exact  $p$ -value for the Wilcoxon rank-sum test

```
ranksum v, by(catvar1) exact
```

Estimate the probability that a case from the first level of `catvar1` has a greater value of  $v$  than a case from the second level of `catvar1`

```
ranksum v, by(catvar1) porder
```

*Nonparametric equality-of-medians test*

Equality of medians test for  $v$  over two or more groups defined by the levels of `catvar2`

```
median v, by(catvar2)
```

Also report Fisher's exact test

```
median v, by(catvar2) exact
```

Same as above, but split cases at the median evenly between the above and below groups

```
median v, by(catvar2) exact medianties(split)
```

## Menu

### **ranksum**

Statistics > Nonparametric analysis > Tests of hypotheses > Wilcoxon rank-sum test

### **median**

Statistics > Nonparametric analysis > Tests of hypotheses > K-sample equality-of-medians test

## Syntax

Wilcoxon rank-sum test

```
ranksum varname [if] [in] , by(groupvar) [exact porder]
```

Nonparametric equality-of-medians test

```
median varname [if] [in] [weight] , by(groupvar) [median_options]
```

| <i>ranksum_options</i>  | Description                                                                                                                     |
|-------------------------|---------------------------------------------------------------------------------------------------------------------------------|
| Main                    |                                                                                                                                 |
| * by( <i>groupvar</i> ) | grouping variable                                                                                                               |
| exact                   | report exact <i>p</i> -value for rank-sum test; by default, exact <i>p</i> -value is computed when total sample size $\leq 200$ |
| <i>porder</i>           | probability that variable for first group is larger than variable for second group                                              |

| <i>median_options</i>     | Description                                                     |
|---------------------------|-----------------------------------------------------------------|
| Main                      |                                                                 |
| * by( <i>groupvar</i> )   | grouping variable                                               |
| exact                     | report <i>p</i> -value from Fisher's exact test                 |
| <u>medianties</u> (below) | assign values equal to the median to below group                |
| <u>medianties</u> (above) | assign values equal to the median to above group                |
| <u>medianties</u> (drop)  | drop values equal to the median from the analysis               |
| <u>medianties</u> (split) | split values equal to the median equally between the two groups |

\*by(*groupvar*) is required.

by and collect are allowed with ranksum and median; see [U] 11.1.10 Prefix commands.

fweights are allowed with median; see [U] 11.1.6 weight.

## Options for ranksum

Main

by(*groupvar*) is required. It specifies the name of the grouping variable.

exact specifies that the exact *p*-value be computed in addition to the approximate *p*-value. The exact *p*-value is based on the actual randomization distribution of the test statistic. The approximate *p*-value is based on a normal approximation to the randomization distribution. By default, the exact *p*-value is computed for sample sizes  $n = n_1 + n_2 \leq 200$  because the normal approximation may not be precise in small samples. The exact computation can be suppressed by specifying noexact. For sample sizes larger than 200, you must specify exact to compute the exact *p*-value. The exact computation is available for sample sizes  $n \leq 1000$ . As the sample size approaches 1,000, the computation takes significantly longer.

*porder* displays an estimate of the probability that a random draw from the first population is larger than a random draw from the second population.

## Options for median

Main

`by(groupvar)` is required. It specifies the name of the grouping variable.

`exact` displays the  $p$ -value calculated by Fisher's exact test. For two samples, both one- and two-sided  $p$ -values are displayed.

`medianties(below | above | drop | split)` specifies how values equal to the overall median are to be handled. The median test computes the median for *varname* by using all observations and then divides the observations into those falling above the median and those falling below the median. When values for an observation are equal to the sample median, they can be dropped from the analysis by specifying `medianties(drop)`; added to the group above or below the median by specifying `medianties(above)` or `medianties(below)`, respectively; or if there is more than 1 observation with values equal to the median, they can be equally divided into the two groups by specifying `medianties(split)`. If this option is not specified, `medianties(below)` is assumed.

## Remarks and examples

### ▷ Example 1

We are testing the effectiveness of a new fuel additive. We run an experiment with 24 cars: 12 cars with the fuel treatment and 12 cars without. We input these data by creating a dataset with 24 observations. `mpg` records the mileage rating, and `treat` records 0 if the mileage corresponds to untreated fuel and 1 if it corresponds to treated fuel.

```
. use https://www.stata-press.com/data/r19/fuel2
. ranksum mpg, by(treat)
Two-sample Wilcoxon rank-sum (Mann-Whitney) test
```

| treat     | Obs | Rank sum | Expected |
|-----------|-----|----------|----------|
| Untreated | 12  | 128      | 150      |
| Treated   | 12  | 172      | 150      |
| Combined  | 24  | 300      | 300      |

```
Unadjusted variance 300.00
Adjustment for ties -4.04
Adjusted variance 295.96
HO: mpg(treat==Untreated) = mpg(treat==Treated)
 z = -1.279
Prob > |z| = 0.2010
Exact prob = 0.2117
```

Because the total sample is only 24 cars, the exact  $p$ -value is computed by default. If the sample size were greater than 200, we would have to specify the `exact` option if we wanted the exact  $p$ -value computed.

Despite the small sample size, the  $p$ -value computed using a normal approximation, 0.2010, is similar to the exact  $p$ -value, 0.2117. These results indicate that the distributions are not statistically different at a 0.05 significance level.

Similarly, the median test,

```
. median mpg, by(treat) exact
Median test
```

| Greater than the median | Whether car received fuel additive |         | Total |
|-------------------------|------------------------------------|---------|-------|
|                         | Untreated                          | Treated |       |
| no                      | 7                                  | 5       | 12    |
| yes                     | 5                                  | 7       | 12    |
| Total                   | 12                                 | 12      | 24    |

```

Pearson chi2(1) = 0.6667 Pr = 0.414
Fisher's exact = 0.684
1-sided Fisher's exact = 0.342
Continuity corrected:
Pearson chi2(1) = 0.1667 Pr = 0.683
```

fails to reject the null hypothesis that there is no difference between the fuel with the additive and the fuel without the additive.

Compare these results from these two tests with those obtained from the `signrank` and `signtest` where we found significant differences; see [R] [signrank](#). An experiment run on 24 different cars is not as powerful as a before-and-after comparison using the same 12 cars.

◀

## Stored results

`ranksum` stores the following in `r()`:

Scalars

```

r(N) sample size
r(N_1) sample size of first group
r(N_2) sample size of second group
r(z) z statistic
r(Var_a) adjusted variance
r(group1) value of variable for first group
r(sum_obs) observed sum of ranks for first group
r(sum_exp) expected sum of ranks for first group
r(p) two-sided p-value from normal approximation
r(p_1) lower one-sided p-value from normal approximation
r(p_u) upper one-sided p-value from normal approximation
r(p_exact) two-sided exact p-value
r(p_1_exact) lower one-sided exact p-value
r(p_u_exact) upper one-sided exact p-value
r(porder) probability that draw from first population is larger than draw from second population
```

`median` stores the following in `r()`:

Scalars

```

r(N) sample size
r(chi2) Pearson's χ^2
r(chi2_cc) continuity-corrected Pearson's χ^2
r(groups) number of groups compared
r(p) p-value for Pearson's χ^2 test
r(p_cc) continuity-corrected p-value
r(p_exact) Fisher's exact p-value
r(p1_exact) one-sided Fisher's exact p-value
```

## Methods and formulas

For a practical introduction to these techniques with an emphasis on examples rather than theory, see [Acock \(2023\)](#), [Bland \(2015\)](#), or [Sprent and Smeeton \(2007\)](#). For a summary of these tests, see [Snedecor and Cochran \(1989\)](#).

Methods and formulas are presented under the following headings:

*ranksum*  
*median*

### ranksum

For the Wilcoxon rank-sum test, there are two independent random variables,  $X_1$  and  $X_2$ , and we test the null hypothesis that  $X_1 \sim X_2$ . We have a sample of size  $n_1$  from  $X_1$  and another of size  $n_2$  from  $X_2$ .

The data are then ranked without regard to the sample to which they belong. If the data are tied, averaged ranks are used. Wilcoxon's test statistic (1945) is the sum of the ranks for the observations in the first sample:

$$T = \sum_{i=1}^{n_1} R_{1i}$$

Mann and Whitney's  $U$  statistic (1947) is the number of pairs  $(X_{1i}, X_{2j})$  such that  $X_{1i} > X_{2j}$ . These statistics differ only by a constant:

$$U = T - \frac{n_1(n_1 + 1)}{2}$$

Fisher's principle of randomization provides a method for calculating the distribution of the test statistic. The randomization distribution consists of all the possible values of  $T$  resulting from the  $\binom{n}{n_1}$  ways to choose  $n_1$  ranks from the set of all  $n = n_1 + n_2$  observed ranks (untied or tied) and assign them to the first sample. When the exact option is specified (or implied for  $n \leq 200$ ), this distribution is computed using a recursive algorithm whose computational time is proportional to  $n^4$ . (See [Fisher \[1935\]](#) for the principle of randomization; [Wilcoxon, Katti, and Wilcox \[1970\]](#) for the computation with untied ranks; and [Hill and Peto \[1971\]](#) for the general recursive algorithm.)

$p$ -values can also be computed using a normal approximation to the randomization distribution. It is a straightforward exercise to verify that

$$E(T) = \frac{n_1(n+1)}{2} \quad \text{and} \quad \text{Var}(T) = \frac{n_1 n_2 s^2}{n}$$

where  $s$  is the standard deviation of the combined ranks,  $r_i$ , for both groups:

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (r_i - \bar{r})^2$$

This formula for the variance is exact and holds both when there are no ties and when there are ties and we use averaged ranks. (Indeed, the variance formula holds for the randomization distribution of choosing  $n_1$  numbers from any set of  $n$  numbers.)

For the normal approximation, we calculate

$$z = \frac{T - E(T)}{\sqrt{\text{Var}(T)}}$$

When the porder option is specified, the probability

$$p = \frac{U}{n_1 n_2}$$

is computed.

#### □ Technical note

We follow the great majority of the literature in naming these tests for Wilcoxon, Mann, and Whitney. However, they were independently developed by several other researchers in the late 1940s and early 1950s. In addition to Wilcoxon, Mann, and Whitney, credit is due to [Festinger \(1946\)](#), [Whitfield \(1947\)](#), [Haldane and Smith \(1947\)](#), and [Van der Reyden \(1952\)](#). Leon Festinger (1919–1989), John Burdon Sanderson Haldane (1892–1964), and Cedric Austen Bardell Smith (1917–2002) are well known for other work, but little seems to be known about Whitfield or van der Reyden. For a detailed study, including information on these researchers, see [Berry, Mielke, and Johnston \(2012\)](#).

□

## median

The median test examines whether it is likely that two or more samples came from populations with the same median. The null hypothesis is that the samples were drawn from populations with the same median. The alternative hypothesis is that at least one sample was drawn from a population with a different median. The test should be used only with ordinal or interval data.

Assume that there are score values for  $k$  independent samples to be compared. The median test is performed by first computing the median score for all observations combined, regardless of the sample group. Each score is compared with this computed grand median and is classified as being above the grand median, below the grand median, or equal to the grand median. Observations with scores equal to the grand median can be dropped, added to the “above” group, added to the “below” group, or split between the two groups.

Once all observations are classified, the data are cast into a  $2 \times k$  contingency table, and a Pearson’s  $\chi^2$  test or Fisher’s exact test is performed.

Henry Berthold Mann (1905–2000) was born in Vienna, Austria, where he completed a doctorate in algebraic number theory. He moved to the United States in 1938 and for several years made his livelihood by tutoring in New York. During this time, he proved a celebrated conjecture in number theory and studied statistics at Columbia with Abraham Wald, with whom he wrote three papers. After the war, he taught at Ohio State and the Universities of Wisconsin and Arizona. In addition to his work in number theory and statistics, he made major contributions to algebra and combinatorics.

Donald Ransom Whitney (1915–2007) studied at Oberlin, Princeton, and Ohio State Universities and worked at the latter throughout his career. His PhD thesis under Henry Mann was on nonparametric statistics. It was this work that produced the test that bears their names.

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## Also see

- [R] **signrank** — Equality tests on matched data
- [R] **ttest** — *t* tests (mean-comparison tests)

Description  
Options  
References

Quick start  
Remarks and examples  
Also see

Menu  
Stored results

Syntax  
Methods and formulas

## Description

`ratio` produces estimates of ratios, along with standard errors.

## Quick start

Estimate, standard error, and 95% confidence interval for the ratio of `v1` to `v2`

```
ratio v1/v2
```

With bootstrap standard errors

```
ratio v1/v2, vce(bootstrap)
```

Ratios of `v1` to `v2` and `v3` to `v2`

```
ratio (v1/v2) (v3/v2)
```

Same as above, but name the ratios `ratio1` and `ratio2`

```
ratio (ratio1: v1/v2) (ratio2: v3/v2)
```

Test that `ratio1` is equal to `ratio2`

```
test ratio1 = ratio2
```

Ratio of `v1` to `v2` over strata defined by levels of `svar`

```
ratio v1/v2, over(svar)
```

Direct standardization across categories `cvar`, weighting by standardization weight `wvar`

```
ratio v1/v2, stdize(cvar) stdweight(wvar)
```

## Menu

Statistics > Summaries, tables, and tests > Summary and descriptive statistics > Ratios

## Syntax

### Basic syntax

```
ratio [name:] varname [/] varname
```

### Full syntax

```
ratio ([name:] varname [/] varname)
 [([name:] varname [/] varname) ...] [if] [in] [weight] [, options]
```

| <i>options</i>                         | Description                                                                                                       |
|----------------------------------------|-------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                           |                                                                                                                   |
| <code>stdize(<i>varname</i>)</code>    | variable identifying strata for standardization                                                                   |
| <code>stdweight(<i>varname</i>)</code> | weight variable for standardization                                                                               |
| <code>nostdrescale</code>              | do not rescale the standard weight variable                                                                       |
| <b>if/in/over</b>                      |                                                                                                                   |
| <code>over(<i>varlist</i>)</code>      | group over subpopulations defined by <i>varlist</i>                                                               |
| <b>SE/Cluster</b>                      |                                                                                                                   |
| <code>vce(<i>vcetype</i>)</code>       | <i>vcetype</i> may be <u>linearized</u> , <u>cluster</u> <i>clustvar</i> , <u>bootstrap</u> , or <u>jackknife</u> |
| <b>Reporting</b>                       |                                                                                                                   |
| <code>level(#)</code>                  | set confidence level; default is level(95)                                                                        |
| <code>noheader</code>                  | suppress table header                                                                                             |
| <code>nolegend</code>                  | suppress table legend                                                                                             |
| <code>display_options</code>           | control column formats, line width, display of empty cells, and factor-variable labeling                          |
| <code>coeflegend</code>                | display legend instead of statistics                                                                              |

bayesboot, bootstrap, collect, jackknife, mi estimate, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands.

vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.

Weights are not allowed with the bootstrap prefix; see [R] bootstrap.

vce() and weights are not allowed with the svy prefix; see [SVY] svy.

fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.

coeflegend does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Options

### Model

`stdize(varname)` specifies that the point estimates be adjusted by direct standardization across the strata identified by *varname*. This option requires the `stdweight()` option.

`stdweight(varname)` specifies the weight variable associated with the standard strata identified in the `stdize()` option. The standardization weights must be constant within the standard strata.

`nostdrescale` prevents the standardization weights from being rescaled within the `over()` groups. This option requires `stdize()` but is ignored if the `over()` option is not specified.

### if/in/over

`over(varlist)` specifies that estimates be computed for multiple subpopulations, which are identified by the different values of the variables in *varlist*. Only numeric, nonnegative, integer-valued variables are allowed in `over(varlist)`.

### SE/Cluster

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`linearized`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

`vce(linearized)`, the default, uses the linearized or sandwich estimator of variance.

### Reporting

`level(#)`; see [R] [Estimation options](#).

`noheader` prevents the table header from being displayed. This option implies `nolegend`.

`nolegend` prevents the table legend identifying the ratios from being displayed.

*display\_options*: `vsquish`, `noemptycells`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

The following option is available with `ratio` but is not shown in the dialog box:

`coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

### ▷ Example 1

Using the fuel data from [example 3](#) of [R] `ttest`, we estimate the ratio of mileage for the cars without the fuel treatment (`mpg1`) to those with the fuel treatment (`mpg2`).

```
. use https://www.stata-press.com/data/r19/fuel
. ratio myratio: mpg1/mpg2
Ratio estimation Number of obs = 12
 myratio: mpg1/mpg2
```

|         | Ratio    | Linearized<br>std. err. | [95% conf. interval] |          |
|---------|----------|-------------------------|----------------------|----------|
| myratio | .9230769 | .032493                 | .8515603             | .9945936 |

Using these results, we can test to see if this ratio is significantly different from one.

```
. test myratio = 1
(1) myratio = 1
 F(1, 11) = 5.60
 Prob > F = 0.0373
```

We find that the ratio is different from one at the 5% significance level but not at the 1% significance level.



### ▷ Example 2

Using state-level census data, we want to test whether the marriage rate is equal to the deathrate.

```
. use https://www.stata-press.com/data/r19/census2
(1980 Census data by state)
. ratio (deathrate: death/pop) (marrate: marriage/pop)
Ratio estimation Number of obs = 50
 deathrate: death/pop
 marrate: marriage/pop
```

|           | Ratio    | Linearized<br>std. err. | [95% conf. interval] |          |
|-----------|----------|-------------------------|----------------------|----------|
| deathrate | .0087368 | .0002052                | .0083244             | .0091492 |
| marrate   | .0105577 | .0006184                | .009315              | .0118005 |

```
. test deathrate = marrate
(1) deathrate - marrate = 0
 F(1, 49) = 6.93
 Prob > F = 0.0113
```



## Stored results

ratio stores the following in `e()`:

### Scalars

|                          |                                          |
|--------------------------|------------------------------------------|
| <code>e(N)</code>        | number of observations                   |
| <code>e(N_lover)</code>  | number of subpopulations                 |
| <code>e(N_stdize)</code> | number of standard strata                |
| <code>e(N_clust)</code>  | number of clusters                       |
| <code>e(k_eq)</code>     | number of equations in <code>e(b)</code> |
| <code>e(df_r)</code>     | sample degrees of freedom                |
| <code>e(rank)</code>     | rank of <code>e(V)</code>                |

### Macros

|                              |                                                |
|------------------------------|------------------------------------------------|
| <code>e(cmd)</code>          | ratio                                          |
| <code>e(cmdline)</code>      | command as typed                               |
| <code>e(varlist)</code>      | <i>varlist</i>                                 |
| <code>e(stdize)</code>       | <i>varname</i> from <code>stdize()</code>      |
| <code>e(stdweight)</code>    | <i>varname</i> from <code>stdweight()</code>   |
| <code>e(wtype)</code>        | weight type                                    |
| <code>e(wexp)</code>         | weight expression                              |
| <code>e(title)</code>        | title in estimation output                     |
| <code>e(clustvar)</code>     | name of cluster variable                       |
| <code>e(over)</code>         | <i>varlist</i> from <code>over()</code>        |
| <code>e(namelist)</code>     | ratio identifiers                              |
| <code>e(vce)</code>          | <i>vcetype</i> specified in <code>vce()</code> |
| <code>e(vcetype)</code>      | title used to label Std. err.                  |
| <code>e(properties)</code>   | b V                                            |
| <code>e(estat_cmd)</code>    | program used to implement <code>estat</code>   |
| <code>e(marginsnotok)</code> | predictions disallowed by margins              |

### Matrices

|                           |                                                              |
|---------------------------|--------------------------------------------------------------|
| <code>e(b)</code>         | vector of ratio estimates                                    |
| <code>e(V)</code>         | (co)variance estimates                                       |
| <code>e(_N)</code>        | vector of numbers of nonmissing observations                 |
| <code>e(_N_stdsum)</code> | number of nonmissing observations within the standard strata |
| <code>e(_p_stdize)</code> | standardizing proportions                                    |
| <code>e(error)</code>     | error code corresponding to <code>e(b)</code>                |

### Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

### Matrices

|                       |                                                                                                                            |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

## Methods and formulas

Methods and formulas are presented under the following headings:

*The ratio estimator*  
*Survey data*  
*The survey ratio estimator*  
*The standardized ratio estimator*  
*The poststratified ratio estimator*  
*The standardized poststratified ratio estimator*  
*Subpopulation estimation*

## The ratio estimator

Let  $R = Y/X$  be the ratio to be estimated, where  $Y$  and  $X$  are totals; see [R] **total**. The estimate for  $R$  is  $\widehat{R} = \widehat{Y}/\widehat{X}$  (the ratio of the sample totals). From the delta method (that is, a first-order Taylor expansion), the approximate variance of the sampling distribution of the linearized  $\widehat{R}$  is

$$V(\widehat{R}) \approx \frac{1}{\widehat{X}^2} \{V(\widehat{Y}) - 2R\widehat{\text{Cov}}(\widehat{Y}, \widehat{X}) + R^2V(\widehat{X})\}$$

Direct substitution of  $\widehat{X}$ ,  $\widehat{R}$ , and the estimated variances and covariance of  $\widehat{X}$  and  $\widehat{Y}$  leads to the following variance estimator:

$$\widehat{V}(\widehat{R}) = \frac{1}{\widehat{X}^2} \{\widehat{V}(\widehat{Y}) - 2\widehat{R}\widehat{\text{Cov}}(\widehat{Y}, \widehat{X}) + \widehat{R}^2\widehat{V}(\widehat{X})\} \quad (1)$$

## Survey data

See [SVY] **Variance estimation**, [SVY] **Direct standardization**, and [SVY] **Poststratification** for discussions that provide background information for the following formulas.

## The survey ratio estimator

Let  $Y_j$  and  $X_j$  be survey items for the  $j$ th individual in the population, where  $j = 1, \dots, M$  and  $M$  is the size of the population. The associated population ratio for the items of interest is  $R = Y/X$  where

$$Y = \sum_{j=1}^M Y_j \quad \text{and} \quad X = \sum_{j=1}^M X_j$$

Let  $y_j$  and  $x_j$  be the corresponding survey items for the  $j$ th sampled individual from the population, where  $j = 1, \dots, m$  and  $m$  is the number of observations in the sample.

The estimator  $\widehat{R}$  for the population ratio  $R$  is  $\widehat{R} = \widehat{Y}/\widehat{X}$ , where

$$\widehat{Y} = \sum_{j=1}^m w_j y_j \quad \text{and} \quad \widehat{X} = \sum_{j=1}^m w_j x_j$$

and  $w_j$  is a sampling weight. The score variable for the ratio estimator is

$$z_j(\widehat{R}) = \frac{y_j - \widehat{R}x_j}{\widehat{X}} = \frac{\widehat{X}y_j - \widehat{Y}x_j}{\widehat{X}^2}$$

## The standardized ratio estimator

Let  $D_g$  denote the set of sampled observations that belong to the  $g$ th standard stratum and define  $I_{D_g}(j)$  to indicate if the  $j$ th observation is a member of the  $g$ th standard stratum; where  $g = 1, \dots, L_D$  and  $L_D$  is the number of standard strata. Also, let  $\pi_g$  denote the fraction of the population that belongs to the  $g$ th standard stratum, thus  $\pi_1 + \dots + \pi_{L_D} = 1$ . Note that  $\pi_g$  is derived from the `stdweight()` option.

The estimator for the standardized ratio is

$$\widehat{R}^D = \sum_{g=1}^{L_D} \pi_g \frac{\widehat{Y}_g}{\widehat{X}_g}$$

where

$$\widehat{Y}_g = \sum_{j=1}^m I_{D_g}(j) w_j y_j$$

and  $\widehat{X}_g$  is similarly defined. The score variable for the standardized ratio is

$$z_j(\widehat{R}^D) = \sum_{g=1}^{L_D} \pi_g I_{D_g}(j) \frac{\widehat{X}_g y_j - \widehat{Y}_g x_j}{\widehat{X}_g^2}$$

## The poststratified ratio estimator

Let  $P_k$  denote the set of sampled observations that belong to poststratum  $k$ , and define  $I_{P_k}(j)$  to indicate if the  $j$ th observation is a member of poststratum  $k$ , where  $k = 1, \dots, L_P$  and  $L_P$  is the number of poststrata. Also, let  $M_k$  denote the population size for poststratum  $k$ .  $P_k$  and  $M_k$  are identified by specifying the `poststrata()` and `postweight()` options on `svyset`; see [SVY] `svyset`.

The estimator for the poststratified ratio is

$$\widehat{R}^P = \frac{\widehat{Y}^P}{\widehat{X}^P}$$

where

$$\widehat{Y}^P = \sum_{k=1}^{L_P} \frac{M_k}{\widehat{M}_k} \widehat{Y}_k = \sum_{k=1}^{L_P} \frac{M_k}{\widehat{M}_k} \sum_{j=1}^m I_{P_k}(j) w_j y_j$$

and  $\widehat{X}^P$  is similarly defined. The score variable for the poststratified ratio is

$$z_j(\widehat{R}^P) = \frac{z_j(\widehat{Y}^P) - \widehat{R}^P z_j(\widehat{X}^P)}{\widehat{X}^P} = \frac{\widehat{X}^P z_j(\widehat{Y}^P) - \widehat{Y}^P z_j(\widehat{X}^P)}{(\widehat{X}^P)^2}$$

where

$$z_j(\widehat{Y}^P) = \sum_{k=1}^{L_P} I_{P_k}(j) \frac{M_k}{\widehat{M}_k} \left( y_j - \frac{\widehat{Y}_k}{\widehat{M}_k} \right)$$

and  $z_j(\widehat{X}^P)$  is similarly defined.

## The standardized poststratified ratio estimator

The estimator for the standardized poststratified ratio is

$$\widehat{R}^{DP} = \sum_{g=1}^{L_D} \pi_g \frac{\widehat{Y}_g^P}{\widehat{X}_g^P}$$

where

$$\widehat{Y}_g^P = \sum_{k=1}^{L_p} \frac{M_k}{\widehat{M}_k} \widehat{Y}_{g,k} = \sum_{k=1}^{L_p} \frac{M_k}{\widehat{M}_k} \sum_{j=1}^m I_{D_g}(j) I_{P_k}(j) w_j y_j$$

and  $\widehat{X}_g^P$  is similarly defined. The score variable for the standardized poststratified ratio is

$$z_j(\widehat{R}^{DP}) = \sum_{g=1}^{L_D} \pi_g \frac{\widehat{X}_g^P z_j(\widehat{Y}_g^P) - \widehat{Y}_g^P z_j(\widehat{X}_g^P)}{(\widehat{X}_g^P)^2}$$

where

$$z_j(\widehat{Y}_g^P) = \sum_{k=1}^{L_p} I_{P_k}(j) \frac{M_k}{\widehat{M}_k} \left\{ I_{D_g}(j) y_j - \frac{\widehat{Y}_{g,k}}{\widehat{M}_k} \right\}$$

and  $z_j(\widehat{X}_g^P)$  is similarly defined.

## Subpopulation estimation

Let  $S$  denote the set of sampled observations that belong to the subpopulation of interest, and define  $I_S(j)$  to indicate if the  $j$ th observation falls within the subpopulation.

The estimator for the subpopulation ratio is  $\widehat{R}^S = \widehat{Y}^S / \widehat{X}^S$ , where

$$\widehat{Y}^S = \sum_{j=1}^m I_S(j) w_j y_j \quad \text{and} \quad \widehat{X}^S = \sum_{j=1}^m I_S(j) w_j x_j$$

Its score variable is

$$z_j(\widehat{R}^S) = I_S(j) \frac{y_j - \widehat{R}^S x_j}{\widehat{X}^S} = I_S(j) \frac{\widehat{X}^S y_j - \widehat{Y}^S x_j}{(\widehat{X}^S)^2}$$

The estimator for the standardized subpopulation ratio is

$$\widehat{R}^{DS} = \sum_{g=1}^{L_D} \pi_g \frac{\widehat{Y}_g^S}{\widehat{X}_g^S}$$

where

$$\widehat{Y}_g^S = \sum_{j=1}^m I_{D_g}(j) I_S(j) w_j y_j$$

and  $\widehat{X}_g^S$  is similarly defined. Its score variable is

$$z_j(\widehat{R}^{DS}) = \sum_{g=1}^{L_D} \pi_g I_{D_g}(j) I_S(j) \frac{\widehat{X}_g^S y_j - \widehat{Y}_g^S x_j}{(\widehat{X}_g^S)^2}$$

The estimator for the poststratified subpopulation ratio is

$$\widehat{R}^{PS} = \frac{\widehat{Y}^{PS}}{\widehat{X}^{PS}}$$

where

$$\widehat{Y}^{PS} = \sum_{k=1}^{L_P} \frac{M_k}{\widehat{M}_k} \widehat{Y}_k^S = \sum_{k=1}^{L_P} \frac{M_k}{\widehat{M}_k} \sum_{j=1}^m I_{P_k}(j) I_S(j) w_j y_j$$

and  $\widehat{X}^{PS}$  is similarly defined. Its score variable is

$$z_j(\widehat{R}^{PS}) = \frac{\widehat{X}^{PS} z_j(\widehat{Y}^{PS}) - \widehat{Y}^{PS} z_j(\widehat{X}^{PS})}{(\widehat{X}^{PS})^2}$$

where

$$z_j(\widehat{Y}^{PS}) = \sum_{k=1}^{L_P} I_{P_k}(j) \frac{M_k}{\widehat{M}_k} \left\{ I_S(j) y_j - \frac{\widehat{Y}_k^S}{\widehat{M}_k} \right\}$$

and  $z_j(\widehat{X}^{PS})$  is similarly defined.

The estimator for the standardized poststratified subpopulation ratio is

$$\widehat{R}^{DPS} = \sum_{g=1}^{L_D} \pi_g \frac{\widehat{Y}_g^{PS}}{\widehat{X}_g^{PS}}$$

where

$$\widehat{Y}_g^{PS} = \sum_{k=1}^{L_P} \frac{M_k}{\widehat{M}_k} \widehat{Y}_{g,k}^S = \sum_{k=1}^{L_P} \frac{M_k}{\widehat{M}_k} \sum_{j=1}^m I_{D_g}(j) I_{P_k}(j) I_S(j) w_j y_j$$

and  $\widehat{X}_g^{PS}$  is similarly defined. Its score variable is

$$z_j(\widehat{R}^{DPS}) = \sum_{g=1}^{L_D} \pi_g \frac{\widehat{X}_g^{PS} z_j(\widehat{Y}_g^{PS}) - \widehat{Y}_g^{PS} z_j(\widehat{X}_g^{PS})}{(\widehat{X}_g^{PS})^2}$$

where

$$z_j(\widehat{Y}_g^{PS}) = \sum_{k=1}^{L_P} I_{P_k}(j) \frac{M_k}{\widehat{M}_k} \left\{ I_{D_g}(j) I_S(j) y_j - \frac{\widehat{Y}_{g,k}^S}{\widehat{M}_k} \right\}$$

and  $z_j(\widehat{X}_g^{PS})$  is similarly defined.

## References

- Cochran, W. G. 1977. *Sampling Techniques*. 3rd ed. New York: Wiley.
- Stuart, A., and J. K. Ord. 1994. *Distribution Theory*. Vol. 1 of *Kendall's Advanced Theory of Statistics*, 6th ed. London: Arnold.

## Also see

[R] **ratio postestimation** — Postestimation tools for ratio

[R] **mean** — Estimate means

[R] **proportion** — Estimate proportions

[R] **total** — Estimate totals

[MI] **Estimation** — Estimation commands for use with mi estimate

[SVY] **Direct standardization** — Direct standardization of means, proportions, and ratios

[SVY] **Poststratification** — Poststratification for survey data

[SVY] **Subpopulation estimation** — Subpopulation estimation for survey data

[SVY] **svy estimation** — Estimation commands for survey data

[SVY] **Variance estimation** — Variance estimation for survey data

[U] **20 Estimation and postestimation commands**

## Postestimation commands

The following postestimation commands are available after `ratio`:

| Command                  | Description                                                                                       |
|--------------------------|---------------------------------------------------------------------------------------------------|
| <code>contrast</code>    | contrasts and ANOVA-style joint tests of parameters                                               |
| <code>estat vce</code>   | variance–covariance matrix of the estimators (VCE)                                                |
| <code>estat (svy)</code> | postestimation statistics for survey data                                                         |
| <code>estimates</code>   | cataloging estimation results                                                                     |
| <code>etable</code>      | table of estimation results                                                                       |
| <code>lincom</code>      | point estimates, standard errors, testing, and inference for linear combinations of parameters    |
| <code>marginsplot</code> | graph the results from <code>ratio</code>                                                         |
| <code>nlcom</code>       | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters |
| <code>pwcompare</code>   | pairwise comparisons of parameters                                                                |
| <code>test</code>        | Wald tests of simple and composite linear hypotheses                                              |
| <code>testnl</code>      | Wald tests of nonlinear hypotheses                                                                |

## Remarks and examples

For examples of the use of `test` after `ratio`, see [\[R\] ratio](#).

## Also see

[\[R\] ratio](#) — Estimate ratios

[\[U\] 20 Estimation and postestimation commands](#)

|             |                      |                |                      |
|-------------|----------------------|----------------|----------------------|
| Description | Quick start          | Menu           | Syntax               |
| Options     | Remarks and examples | Stored results | Methods and formulas |
| References  | Also see             |                |                      |

## Description

`reg3` estimates a system of structural equations, where some equations contain endogenous variables among the explanatory variables. Estimation is via three-stage least squares (3SLS); see [Zellner and Theil \(1962\)](#). Typically, the endogenous explanatory variables are dependent variables from other equations in the system. `reg3` supports iterated GLS estimation and linear constraints.

`reg3` can also estimate systems of equations by seemingly unrelated regression estimation (SURE), multivariate regression (MVREG), and equation-by-equation ordinary least squares (OLS) or two-stage least squares (2SLS).

## Nomenclature

Under 3SLS or 2SLS estimation, a *structural equation* is defined as one of the equations specified in the system. A *dependent variable* will have its usual interpretation as the left-hand-side variable in an equation with an associated disturbance term. All dependent variables are explicitly taken to be *endogenous* to the system and are treated as correlated with the disturbances in the system's equations. Unless specified in an `endog()` option, all other variables in the system are treated as *exogenous* to the system and uncorrelated with the disturbances. The exogenous variables are taken to be *instruments* for the endogenous variables.

## Quick start

System of equations regressing  $y_1$  on  $x_1$ ,  $x_2$ , and  $y_2$  and  $y_2$  on  $x_1$ ,  $x_3$ , and  $y_1$

```
reg3 (y1 x1 x2 y2) (y2 x1 x3 y1)
```

Same as above, but name equations `eq1` and `eq2`

```
reg3 (eq1: y1 x1 x2 y2) (eq2: y2 x1 x3 y1)
```

Same as above, but specify that  $x_1$  is endogenous and add instrumental variable  $v_1$

```
reg3 (eq1: y1 x1 x2 y2) (eq2: y2 x1 x3 y1), endog(x1) exog(v1)
```

Same as above, but with iterated estimation

```
reg3 (eq1: y1 x1 x2 y2) (eq2: y2 x1 x3 y1), endog(x1) exog(v1) ireg3
```

## Menu

Statistics > Endogenous covariates > Three-stage least squares

## Syntax

### Basic syntax

```
reg3 (depvar1 varlist1) (depvar2 varlist2) ... (depvarN varlistN) [if] [in] [weight]
```

### Full syntax

```
reg3 ([eqname1:]depvar1a [depvar1b ... =]varlist1 [, noconstant])
 ([eqname2:]depvar2a [depvar2b ... =]varlist2 [, noconstant])
 ...
 ([eqnameN:]depvarNa [depvarNb ... =]varlistN [, noconstant])
 [if] [in] [weight] [, options]
```

| <i>options</i>                             | Description                                                                                                                                      |
|--------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| Model                                      |                                                                                                                                                  |
| <u>i</u> reg3                              | iterate until estimates converge                                                                                                                 |
| 3s1s                                       | three-stage least squares; the default                                                                                                           |
| 2s1s                                       | two-stage least squares                                                                                                                          |
| ols                                        | ordinary least squares (OLS)                                                                                                                     |
| <u>s</u> ure                               | seemingly unrelated regression estimation (SURE)                                                                                                 |
| <u>m</u> vreg                              | sure with OLS degrees-of-freedom adjustment                                                                                                      |
| <u>c</u> orr ( <i>correlation</i> )        | <u>u</u> nstructured or <u>i</u> ndependent correlation structure; default is unstructured                                                       |
| <u>e</u> xog ( <i>varlist</i> )            | exogenous variables not specified in system equations                                                                                            |
| <u>e</u> ndog ( <i>varlist</i> )           | additional right-hand-side endogenous variables                                                                                                  |
| <u>i</u> nst ( <i>varlist</i> )            | full list of exogenous variables                                                                                                                 |
| <u>a</u> llexog                            | all right-hand-side variables are exogenous                                                                                                      |
| <u>n</u> oconstant                         | suppress constant from instrument list                                                                                                           |
| <u>c</u> onstraints ( <i>constraints</i> ) | apply specified linear constraints                                                                                                               |
| SE/Robust                                  |                                                                                                                                                  |
| vce ( <i>vcetype</i> )                     | <i>vcetype</i> may be <u>u</u> nadjusted, <u>r</u> obust, or <u>c</u> luster <i>clustvar</i>                                                     |
| df adj.                                    |                                                                                                                                                  |
| <u>s</u> m11                               | report small-sample statistics                                                                                                                   |
| dfk                                        | use small-sample adjustment                                                                                                                      |
| dfk2                                       | use alternate adjustment                                                                                                                         |
| Reporting                                  |                                                                                                                                                  |
| <u>l</u> evel (#)                          | set confidence level; default is level (95)                                                                                                      |
| <u>f</u> irst                              | report first-stage regression                                                                                                                    |
| <u>n</u> ocnsreport                        | do not display constraints                                                                                                                       |
| <i>display_options</i>                     | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |

## Optimization

|                             |                                               |
|-----------------------------|-----------------------------------------------|
| <i>optimization_options</i> | control the optimization process; seldom used |
| <code>noheader</code>       | suppress display of header                    |
| <code>notable</code>        | suppress display of coefficient table         |
| <code>nofooter</code>       | suppress display of footer                    |
| <code>coeflegend</code>     | display legend instead of statistics          |

*varlist*<sub>1</sub>, ..., *varlist*<sub>N</sub> and the `exog()` and the `inst()` *varlist* may contain factor variables; see [U] 11.4.3 **Factor variables**. You must have the same levels of factor variables in all equations that have factor variables.

*devar* and *varlist* may contain time-series operators; see [U] 11.4.4 **Time-series varlists**.

`bootstrap`, `by`, `collect`, `fp`, `jackknife`, `rolling`, and `statsby` are allowed; see [U] 11.1.10 **Prefix commands**.

Weights are not allowed with the `bootstrap` prefix; see [R] **bootstrap**.

`aweight`s are not allowed with the `jackknife` prefix; see [R] **jackknife**.

`aweight`s and `fweight`s are allowed; see [U] 11.1.6 **weight**.

`noheader`, `notable`, `nofooter`, and `coeflegend` do not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

Explicit equation naming (*eqname*;) cannot be combined with multiple dependent variables in an equation specification.

## Options

### Model

`ireg3` causes `reg3` to iterate over the estimated disturbance covariance matrix and parameter estimates until the parameter estimates converge. Although the iteration is usually successful, there is no guarantee that it will converge to a stable point. Under SURE, this iteration converges to the maximum likelihood estimates.

`3s1s` specifies the full 3SLS estimation of the system and is the default for `reg3`.

`2s1s` causes `reg3` to perform equation-by-equation 2SLS on the full system of equations. This option implies `dfk`, `small`, and `corr(independent)`.

Cross-equation testing should not be performed after estimation with this option. With `2s1s`, no covariance is estimated between the parameters of the equations. For cross-equation testing, use `3s1s`.

`ols` causes `reg3` to perform equation-by-equation OLS on the system—even if dependent variables appear as regressors or the regressors differ for each equation; see [MV] **mvreg**. `ols` implies `allexog`, `dfk`, `small`, and `corr(independent)`; `nodfk` and `nosmall` may be specified to override `dfk` and `small`.

The covariance of the coefficients between equations is not estimated under this option, and cross-equation tests should not be performed after estimation with `ols`. For cross-equation testing, use `sure` or `3s1s` (the default).

`sure` causes `reg3` to perform a SURE of the system—even if dependent variables from some equations appear as regressors in other equations; see [R] **sureg**. `sure` is a synonym for `allexog`.

`mvreg` is identical to `sure`, except that the disturbance covariance matrix is estimated with an OLS degrees-of-freedom adjustment—the `dfk` option. If the regressors are identical for all equations, the parameter point estimates will be the standard MVREG results. If any of the regressors differ, the point estimates are those for SURE with an OLS degrees-of-freedom adjustment in computing the covariance matrix. `nodfk` and `nosmall` may be specified to override `dfk` and `small`.

`corr` (*correlation*) specifies the assumed form of the correlation structure of the equation disturbances and is rarely requested explicitly. For the family of models fit by `reg3`, the only two allowable correlation structures are `unstructured` and `independent`. The default is `unstructured`.

This option is used almost exclusively to estimate a system of equations by 2SLS or to perform OLS regression with `reg3` on multiple equations. In these cases, the correlation is set to `independent`, forcing `reg3` to treat the covariance matrix of equation disturbances as diagonal in estimating model parameters. Thus, a set of two-stage coefficient estimates can be obtained if the system contains endogenous right-hand-side variables, or OLS regression can be imposed, even if the regressors differ across equations. Without imposing independent disturbances, `reg3` would estimate the former by 3SLS and the latter by SURE.

Any tests performed after estimation with the `independent` option will treat coefficients in different equations as having no covariance; cross-equation tests should not be used after specifying `corr(independent)`.

`exog` (*varlist*) specifies additional exogenous variables that are included in none of the system equations. This can occur when the system contains identities that are not estimated. If implicitly exogenous variables from the equations are listed here, `reg3` will just ignore the additional information. Specified variables will be added to the exogenous variables in the system and used in the first stage as instruments for the endogenous variables. By specifying dependent variables from the structural equations, you can use `exog()` to override their endogeneity.

`endog` (*varlist*) identifies variables in the system that are not dependent variables but are endogenous to the system. These variables must appear in the variable list of at least one equation in the system. Again, the need for this identification often occurs when the system contains identities. For example, a variable that is the sum of an exogenous variable and a dependent variable may appear as an explanatory variable in some equations.

`inst` (*varlist*) specifies a full list of all exogenous variables and may not be used with the `endog()` or `exog()` options. It must contain a full list of variables to be used as instruments for the endogenous regressors. Like `exog()`, the list may contain variables not specified in the system of equations. This option can be used to achieve the same results as the `endog()` and `exog()` options, and the choice is a matter of convenience. Any variable not specified in the *varlist* of the `inst()` option is assumed to be endogenous to the system. As with `exog()`, including the dependent variables from the structural equations will override their endogeneity.

`allexog` indicates that all right-hand-side variables are to be treated as exogenous—even if they appear as the dependent variable of another equation in the system. This option can be used to enforce a SURE or MVREG estimation even when some dependent variables appear as regressors.

`noconstant`, `constraints` (*constraints*); see [R] [Estimation options](#).

SE/Robust

`vce` (*vcetype*) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`unadjusted`), that are robust to some kinds of misspecification (`robust`), and that allow for intragroup correlation (`cluster clustvar`); see [R] [vce\\_option](#).

`vce(unadjusted)`, the default, specifies that an unadjusted (nonrobust) VCE matrix be used; this results in efficient estimates when assuming homoskedasticity.

## df adj.

`small` specifies that small-sample statistics be computed. It shifts the test statistics from  $\chi^2$  and  $z$  statistics to  $F$  statistics and  $t$  statistics. This option is intended primarily to support MVREG. Although the standard errors from each equation are computed using the degrees of freedom for the equation, the degrees of freedom for the  $t$  statistics are all taken to be those for the first equation. This approach poses no problem under MVREG because the regressors are the same across equations.

`dfk` specifies the use of an alternative divisor in computing the covariance matrix for the equation residuals. As an asymptotically justified estimator, `reg3` by default uses the number of sample observations  $n$  as a divisor. When the `dfk` option is set, a small-sample adjustment is made, and the divisor is taken to be  $\sqrt{(n - k_i)(n - k_j)}$ , where  $k_i$  and  $k_j$  are the number of parameters in equations  $i$  and  $j$ , respectively.

`dfk2` specifies the use of an alternative divisor in computing the covariance matrix for the equation errors. When the `dfk2` option is set, the divisor is taken to be the mean of the residual degrees of freedom from the individual equations.

## Reporting

`level(#)`; see [R] [Estimation options](#).

`first` requests that the first-stage regression results be displayed during estimation.

`nocnsreport`; see [R] [Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

## Optimization

`optimization_options` control the iterative process that minimizes the sum of squared errors when `ireg3` is specified. These options are seldom used.

`iterate(#)` specifies the maximum number of iterations. When the number of iterations equals  $\#$ , the optimizer stops and presents the current results, even if the convergence tolerance has not been reached. The default is the number set using `set maxiter`, which is 300 by default.

`trace` adds to the iteration log a display of the current parameter vector.

`log` and `nolog` specify whether to display the iteration log. The iteration log is displayed by default unless you used `set iterlog off` to suppress it; see `set iterlog` in [R] [set iter](#).

`tolerance(#)` specifies the tolerance for the coefficient vector. When the relative change in the coefficient vector from one iteration to the next is less than or equal to  $\#$ , the optimization process is stopped. `tolerance(1e-6)` is the default.

The following options are available with `reg3` but are not shown in the dialog box:

`noheader` suppresses display of the header reporting the estimation method and the table of equation summary statistics.

`notable` suppresses display of the coefficient table.

`nofooter` suppresses display of the footer reporting the list of endogenous and exogenous variables in the model.

`coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

`reg3` estimates systems of structural equations where some equations contain endogenous variables among the explanatory variables. Generally, these endogenous variables are the dependent variables of other equations in the system, though not always. The disturbance is correlated with the endogenous variables—violating the assumptions of OLS. Further, because some of the explanatory variables are the dependent variables of other equations in the system, the error terms among the equations are expected to be correlated. `reg3` uses an instrumental-variables approach to produce consistent estimates and generalized least squares (GLS) to account for the correlation structure in the disturbances across the equations. Good general references on three-stage estimation include [Davidson and MacKinnon \(1993, 651–661\)](#) and [Greene \(2018, 363–365\)](#).

Three-stage least squares can be thought of as producing estimates from a three-step process.

**Step 1.** Develop instrumented values for all endogenous variables. These instrumented values can simply be considered as the predicted values resulting from a regression of each endogenous variable on all exogenous variables in the system. This stage is identical to the first step in 2SLS and is critical for the consistency of the parameter estimates.

**Step 2.** Obtain a consistent estimate for the covariance matrix of the equation disturbances. These estimates are based on the residuals from a 2SLS estimation of each structural equation.

**Step 3.** Perform a GLS-type estimation using the covariance matrix estimated in the second stage and with the instrumented values in place of the right-hand-side endogenous variables.

### □ Technical note

The estimation and use of the covariance matrix of disturbances in three-stage estimation is almost identical to the SURE method—`sureg`. As with SURE, using this covariance matrix improves the efficiency of the three-stage estimator. Even without the covariance matrix, the estimates would be consistent. (They would be 2SLS estimates.) This improvement in efficiency comes with a caveat. All the parameter estimates now depend on the consistency of the covariance matrix estimates. If one equation in the system is misspecified, the disturbance covariance estimates will be inconsistent, and the resulting coefficients will be biased and inconsistent. Alternatively, if each equation is estimated separately by 2SLS ([\[R\] regress](#)), only the coefficients in the misspecified equation are affected.

□

### □ Technical note

If an equation is just identified, the 3SLS point estimates for that equation are identical to the 2SLS estimates. However, as with `sureg`, even if all equations are just identified, fitting the model via `reg3` has at least one advantage over fitting each equation separately via `ivregress`; by using `reg3`, tests involving coefficients in different equations can be performed easily using `test` or `testnl`.

□

## ▷ Example 1

A simple macroeconomic model relates consumption (`consump`) to private and government wages paid (`wagepriv` and `wagegovt`). Simultaneously, private wages depend on consumption, total government expenditures (`govt`), and the lagged stock of capital in the economy (`capital1`). Although this is not a plausible model, it does meet the criterion of being simple. This model could be written as

$$\begin{aligned}\text{consump} &= \beta_0 + \beta_1 \text{wagepriv} + \beta_2 \text{wagegovt} + \epsilon_1 \\ \text{wagepriv} &= \beta_3 + \beta_4 \text{consump} + \beta_5 \text{govt} + \beta_6 \text{capital1} + \epsilon_2\end{aligned}$$

If we assume that this is the full system, `consump` and `wagepriv` will be endogenous variables, with `wagegovt`, `govt`, and `capital1` exogenous. Data for the US economy on these variables are taken from Klein (1950). This model can be fit with `reg3` by typing

```
. use https://www.stata-press.com/data/r19/klein
. reg3 (consump wagepriv wagegovt) (wagepriv consump govt capital1)
```

Three-stage least-squares regression

| Equation | Obs | Params | RMSE     | "R-squared" | chi2   | P>chi2 |
|----------|-----|--------|----------|-------------|--------|--------|
| consump  | 22  | 2      | 1.776297 | 0.9388      | 208.02 | 0.0000 |
| wagepriv | 22  | 3      | 2.372443 | 0.8542      | 80.04  | 0.0000 |

|          | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |
|----------|-------------|-----------|-------|-------|----------------------|
| consump  |             |           |       |       |                      |
| wagepriv | .8012754    | .1279329  | 6.26  | 0.000 | .5505314 1.052019    |
| wagegovt | 1.029531    | .3048424  | 3.38  | 0.001 | .432051 1.627011     |
| _cons    | 19.3559     | 3.583772  | 5.40  | 0.000 | 12.33184 26.37996    |
| wagepriv |             |           |       |       |                      |
| consump  | .4026076    | .2567312  | 1.57  | 0.117 | -.1005764 .9057916   |
| govt     | 1.177792    | .5421253  | 2.17  | 0.030 | .1152461 2.240338    |
| capital1 | -.0281145   | .0572111  | -0.49 | 0.623 | -.1402462 .0840173   |
| _cons    | 14.63026    | 10.26693  | 1.42  | 0.154 | -5.492552 34.75306   |

```
Endogenous: consump wagepriv
Exogenous: wagegovt govt capital1
```

Without showing the 2SLS results, we note that the consumption function in this system falls under the conditions noted earlier. That is, the 2SLS and 3SLS coefficients for the equation are identical.

## ▶ Example 2

Some of the most common simultaneous systems encountered are supply-and-demand models. A simple system could be specified as

$$q\text{Demand} = \beta_0 + \beta_1 \text{price} + \beta_2 \text{pcompete} + \beta_3 \text{income} + \epsilon_1$$

$$q\text{Supply} = \beta_4 + \beta_5 \text{price} + \beta_6 \text{praw} + \epsilon_2$$

$$\text{Equilibrium condition: quantity} = q\text{Demand} = q\text{Supply}$$

where

quantity is the quantity of a product produced and sold,

price is the price of the product,

pcompete is the price of a competing product,

income is the average income level of consumers, and

praw is the price of raw materials used to produce the product.

In this system, price is assumed to be determined simultaneously with demand. The important statistical implications are that price is not a predetermined variable and that it is correlated with the disturbances of both equations. The system is somewhat unusual: quantity is associated with two disturbances. This fact really poses no problem because the disturbances are specified on the behavioral demand and supply equations—two separate entities. Often one of the two equations is rewritten to place price on the left-hand side, making this endogeneity explicit in the specification.

To provide a concrete illustration of the effects of simultaneous equations, we can simulate data for the above system by using known coefficients and disturbance properties. Specifically, we will simulate the data as

$$q\text{Demand} = 40 - 1.0\text{price} + 0.25\text{pcompete} + 0.5\text{income} + \epsilon_1$$

$$q\text{Supply} = 0.5\text{price} - 0.75\text{praw} + \epsilon_2$$

where

$$\epsilon_1 \sim N(0, 3.8)$$

$$\epsilon_2 \sim N(0, 2.4)$$

For comparison, we can estimate the supply and demand equations separately by OLS. The estimates for the demand equation are

```
. use https://www.stata-press.com/data/r19/supDem
. regress quantity price pcompete income
```

| Source   | SS         | df | MS         | Number of obs | = | 49     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 23.1579302 | 3  | 7.71931008 | F(3, 45)      | = | 1.00   |
| Residual | 346.459313 | 45 | 7.69909584 | Prob > F      | = | 0.4004 |
|          |            |    |            | R-squared     | = | 0.0627 |
|          |            |    |            | Adj R-squared | = | 0.0002 |
| Total    | 369.617243 | 48 | 7.70035923 | Root MSE      | = | 2.7747 |

| quantity | Coefficient | Std. err. | t    | P> t  | [95% conf. interval] |          |
|----------|-------------|-----------|------|-------|----------------------|----------|
| price    | .1186265    | .1716014  | 0.69 | 0.493 | -.2269965            | .4642496 |
| pcompete | .0946416    | .1200815  | 0.79 | 0.435 | -.1472149            | .3364981 |
| income   | .0785339    | .1159867  | 0.68 | 0.502 | -.1550754            | .3121432 |
| _cons    | 7.563261    | 5.019479  | 1.51 | 0.139 | -2.54649             | 17.67301 |

The OLS estimates for the supply equation are

```
. regress quantity price praw
```

| Source   | SS         | df | MS         | Number of obs | = | 49     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 224.819549 | 2  | 112.409774 | F(2, 46)      | = | 35.71  |
| Residual | 144.797694 | 46 | 3.14777596 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.6082 |
|          |            |    |            | Adj R-squared | = | 0.5912 |
| Total    | 369.617243 | 48 | 7.70035923 | Root MSE      | = | 1.7742 |

| quantity | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|----------|-------------|-----------|-------|-------|----------------------|----------|
| price    | .724675     | .1095657  | 6.61  | 0.000 | .5041307             | .9452192 |
| praw     | -.8674796   | .1066114  | -8.14 | 0.000 | -1.082077            | -.652882 |
| _cons    | -6.97291    | 3.323105  | -2.10 | 0.041 | -13.66197            | -.283847 |

Examining the coefficients from these regressions, we note that they are not close to the known parameters used to generate the simulated data. In particular, the positive coefficient on price in the demand equation stands out. We constructed our simulated data to be consistent with economic theory—people demand less of a product if its price rises and more if their personal income rises. Although the price coefficient is statistically insignificant, the positive value contrasts starkly with what is predicted from economic price theory and the  $-1.0$  value that we used in the simulation. Likewise, we are disappointed with the insignificance and level of the coefficient on average income. The supply equation has correct signs on the two main parameters, but their levels are different from the known values. In fact, the coefficient on price (0.724675) is different from the simulated parameter (0.5) at the 5% level of significance.

All of these problems are to be expected. We explicitly constructed a simultaneous system of equations that violated one of the assumptions of least squares. Specifically, the disturbances were correlated with one of the regressors—price.

Two-stage least squares can be used to address the correlation between regressors and disturbances. Using instruments for the endogenous variable, price, 2SLS will produce consistent estimates of the parameters in the system. Let's use `ivregress` (see [R] [ivregress](#)) to see how our simulated system behaves when fit using 2SLS.

```
. ivregress 2sls quantity (price = praw) pcompete income
```

| quantity | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|----------|-------------|-----------|-------|-------|----------------------|----------|
| price    | -1.015817   | .374209   | -2.71 | 0.007 | -1.749253            | -.282381 |
| pcompete | .3319504    | .172912   | 1.92  | 0.055 | -.0069508            | .6708517 |
| income   | .5090607    | .1919482  | 2.65  | 0.008 | .1328491             | .8852723 |
| _cons    | 39.89988    | 10.77378  | 3.70  | 0.000 | 18.78366             | 61.01611 |

Endogenous: price  
 Exogenous: pcompete income praw

```
. ivregress 2sls quantity (price = pcompete income) praw
Instrumental-variables 2SLS regression Number of obs = 49
 Wald chi2(2) = 39.25
 Prob > chi2 = 0.0000
 R-squared = 0.5928
 Root MSE = 1.7525
```

| quantity | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|----------|-------------|-----------|-------|-------|----------------------|-----------|
| price    | .5773133    | .1749974  | 3.30  | 0.001 | .2343247             | .9203019  |
| praw     | -.7835496   | .1312414  | -5.97 | 0.000 | -1.040778            | -.5263213 |
| _cons    | -2.550694   | 5.273067  | -0.48 | 0.629 | -12.88571            | 7.784327  |

Endogenous: price  
 Exogenous: praw pcompete income

We are now much happier with the estimation results. All the coefficients from both equations are close to the true parameter values for the system. In particular, the coefficients are all well within 95% confidence intervals for the parameters. The missing  $R^2$  in the demand equation seems unusual; we will discuss that more later.

Finally, this system could be estimated using 3SLS. To demonstrate how large systems might be handled and to avoid multiline commands, we will use global macros (see [P] macro) to hold the specifications for our equations.

```
. global demand "(qDemand: quantity price pcompete income)"
. global supply "(qSupply: quantity price praw)"
. reg3 $demand $supply, endog(price)
```

We must specify price as endogenous because it does not appear as a dependent variable in either equation. Without this option, reg3 would assume that there are no endogenous variables in the system and produce seemingly unrelated regression (sureg) estimates. The reg3 output from our series of commands is

Three-stage least-squares regression

| Equation | Obs | Params | RMSE     | "R-squared" | chi2  | P>chi2 |
|----------|-----|--------|----------|-------------|-------|--------|
| qDemand  | 49  | 3      | 3.739686 | -0.8540     | 8.68  | 0.0338 |
| qSupply  | 49  | 2      | 1.752501 | 0.5928      | 39.25 | 0.0000 |

|          | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|----------|-------------|-----------|-------|-------|----------------------|-----------|
| qDemand  |             |           |       |       |                      |           |
| price    | -1.014345   | .3742036  | -2.71 | 0.007 | -1.74777             | -.2809194 |
| pcompete | .2647206    | .1464194  | 1.81  | 0.071 | -.0222561            | .5516973  |
| income   | .5299146    | .1898161  | 2.79  | 0.005 | .1578819             | .9019472  |
| _cons    | 40.08749    | 10.77072  | 3.72  | 0.000 | 18.97726             | 61.19772  |
| qSupply  |             |           |       |       |                      |           |
| price    | .5773133    | .1749974  | 3.30  | 0.001 | .2343247             | .9203019  |
| praw     | -.7835496   | .1312414  | -5.97 | 0.000 | -1.040778            | -.5263213 |
| _cons    | -2.550694   | 5.273067  | -0.48 | 0.629 | -12.88571            | 7.784327  |

Endogenous: quantity price  
 Exogenous: pcompete income praw

The use of 3SLS over 2SLS is essentially an efficiency issue. The coefficients of the demand equation from 3SLS are close to the coefficients from two-stage least squares, and those of the supply equation are identical. The latter case was mentioned earlier for systems with some exactly identified equations. However, even for the demand equation, we do not expect the coefficients to change systematically. What we do expect from three-stage least squares are more precise estimates of the parameters given the validity of our specification and `reg3`'s use of the covariances among the disturbances.

Let's summarize the results. With OLS, we got obviously biased estimates of the parameters. No amount of data would have improved the OLS estimates—they are inconsistent in the face of the violated OLS assumptions. With 2SLS, we obtained consistent estimates of the parameters, and these would have improved with more data. With 3SLS, we obtained consistent estimates of the parameters that are more efficient than those obtained by 2SLS.



### □ Technical note

We noted earlier that the  $R^2$  was missing from the two-stage estimates of the demand equation. Now we see that the  $R^2$  is negative for the three-stage estimates of the same equation. How can we have a negative  $R^2$ ?

In most estimators, other than least squares, the  $R^2$  is no more than a summary measure of the overall in-sample predictive power of the estimator. The computational formula for  $R^2$  is  $R^2 = 1 - \text{RSS}/\text{TSS}$ , where RSS is the residual sum of squares (sum of squared residuals) and TSS is the total sum of squared deviations about the mean of the dependent variable. In a standard linear model with a constant, the model from which the TSS is computed is nested within the full model from which RSS is computed—they both have a constant term based on the same data. Thus, it must be that  $\text{TSS} \geq \text{RSS}$  and  $R^2$  is constrained between 0 and 1.

For 2SLS and 3SLS, some of the regressors enter the model as instruments when the parameters are estimated. However, because our goal is to fit the structural model, the actual values, not the instruments for the endogenous right-hand-side variables, are used to determine  $R^2$ . The model residuals are computed over a different set of regressors from those used to fit the model. The two- or three-stage estimates are no longer nested within a constant-only model of the dependent variable, and the residual sum of squares is no longer constrained to be smaller than the total sum of squares.

A negative  $R^2$  in 3SLS should be taken for exactly what it is—an indication that the structural model predicts the dependent variable worse than a constant-only model. Is this a problem? It depends on the application. Three-stage least squares applied to our contrived supply-and-demand example produced good estimates of the known true parameters. Still, the demand equation produced an  $R^2$  of  $-0.854$ . How do we feel about our parameter estimates? This should be determined by the estimates themselves, their associated standard errors, and the overall model significance. On this basis, negative  $R^2$  and all, we feel pretty good about all the parameter estimates for both the supply and demand equations. Would we want to make predictions about equilibrium quantity by using the demand equation alone? Probably not. Would we want to make these quantity predictions by using the supply equation? Possibly, because based on in-sample predictions, they seem better than those from the demand equations. However, both the supply and demand estimates are based on limited information. If we are interested in predicting quantity, a reduced-form equation containing all our independent variables would usually be preferred.



## □ Technical note

As a matter of syntax, we could have specified the supply-and-demand model on one line without using global macros.

```
. reg3 (quantity price pcompete income) (quantity price praw), endog(price)
```

Three-stage least-squares regression

| Equation  | Obs | Params | RMSE     | "R-squared" | chi2  | P>chi2 |
|-----------|-----|--------|----------|-------------|-------|--------|
| quantity  | 49  | 3      | 3.739686 | -0.8540     | 8.68  | 0.0338 |
| 2quantity | 49  | 2      | 1.752501 | 0.5928      | 39.25 | 0.0000 |

|           | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|-----------|-------------|-----------|-------|-------|----------------------|-----------|
| quantity  |             |           |       |       |                      |           |
| price     | -1.014345   | .3742036  | -2.71 | 0.007 | -1.74777             | -.2809194 |
| pcompete  | .2647206    | .1464194  | 1.81  | 0.071 | -.0222561            | .5516973  |
| income    | .5299146    | .1898161  | 2.79  | 0.005 | .1578819             | .9019472  |
| _cons     | 40.08749    | 10.77072  | 3.72  | 0.000 | 18.97726             | 61.19772  |
| 2quantity |             |           |       |       |                      |           |
| price     | .5773133    | .1749974  | 3.30  | 0.001 | .2343247             | .9203019  |
| praw      | -.7835496   | .1312414  | -5.97 | 0.000 | -1.040778            | -.5263213 |
| _cons     | -2.550694   | 5.273067  | -0.48 | 0.629 | -12.88571            | 7.784327  |

Endogenous: quantity price

Exogenous: pcompete income praw

However, here `reg3` has been forced to create a unique equation name for the supply equation—`2quantity`. Both the supply and demand equations could not be designated as `quantity`, so a number was prefixed to the name for the supply equation.

We could have specified

```
. reg3 (qDemand: quantity price pcompete income) (qSupply: quantity price praw),
> endog(price)
```

and obtained the same results and equation labeling as when we used global macros to hold the equation specifications.

Without explicit equation names, `reg3` always assumes that the dependent variable should be used to name equations. When each equation has a different dependent variable, this rule causes no problems and produces easily interpreted result tables. If the same dependent variable appears in more than one equation, however, `reg3` will create a unique equation name based on the dependent variable name. Because equation names must be used for cross-equation tests, you have more control in this situation if explicit names are placed on the equations.

□

▷ Example 3: Using the full syntax of reg3

Klein's (1950) model of the US economy is often used to demonstrate system estimators. It contains several common features that will serve to demonstrate the full syntax of reg3. The Klein model is defined by the following seven relationships:

$$c = \beta_0 + \beta_1 p + \beta_2 L.p + \beta_3 w + \epsilon_1 \quad (1)$$

$$i = \beta_4 + \beta_5 p + \beta_6 L.p + \beta_7 L.k + \epsilon_2 \quad (2)$$

$$wp = \beta_8 + \beta_9 y + \beta_{10} L.y + \beta_{11} yr + \epsilon_3 \quad (3)$$

$$y = c + i + g \quad (4)$$

$$p = y - t - wp \quad (5)$$

$$k = L.k + i \quad (6)$$

$$w = wg + wp \quad (7)$$

Here we have used Stata's lag operator `L.` to represent variables that appear with a one-period lag in our model; see [U] 13.10 Time-series operators.

The variables in the model are listed below. Two sets of variable names are shown. The concise first name uses traditional economics mnemonics, whereas the second name provides more guidance for everyone else. The concise names serve to keep the specification of the model small (and quite understandable to economists).

| Short name      | Long name             | Variable definition               | Type       |
|-----------------|-----------------------|-----------------------------------|------------|
| <code>c</code>  | <code>consump</code>  | Consumption                       | endogenous |
| <code>p</code>  | <code>profits</code>  | Private industry profits          | endogenous |
| <code>wp</code> | <code>wagepriv</code> | Private wage bill                 | endogenous |
| <code>wg</code> | <code>wagegovt</code> | Government wage bill              | exogenous  |
| <code>w</code>  | <code>wagetot</code>  | Total wage bill                   | endogenous |
| <code>i</code>  | <code>invest</code>   | Investment                        | endogenous |
| <code>k</code>  | <code>capital</code>  | Capital stock                     | endogenous |
| <code>y</code>  | <code>totinc</code>   | Total income/demand               | endogenous |
| <code>g</code>  | <code>govt</code>     | Government spending               | exogenous  |
| <code>t</code>  | <code>taxnetx</code>  | Indirect bus. taxes + net exports | exogenous  |
| <code>yr</code> | <code>year</code>     | Year—1931                         | exogenous  |

Equations (1)–(3) are behavioral and contain explicit disturbances ( $\epsilon_1$ ,  $\epsilon_2$ , and  $\epsilon_3$ ). The remaining equations are identities that specify additional variables in the system and their accounting relationships with the variables in the behavioral equations. Some variables are explicitly endogenous by appearing as dependent variables in (1)–(3). Others are implicitly endogenous as linear combinations that contain other endogenous variables (for example, `w` and `p`). Still other variables are implicitly exogenous by appearing in the identities but not in the behavioral equations (for example, `wg` and `g`).

Using the concise names, we can fit Klein’s model with the following command:

```
. use https://www.stata-press.com/data/r19/klein2
. reg3 (c p L.p w) (i p L.p L.k) (wp y L.y yr), endog(w p y) exog(t wg g)
Three-stage least-squares regression
```

| Equation | Obs | Params | RMSE     | "R-squared" | chi2    | P>chi2 |
|----------|-----|--------|----------|-------------|---------|--------|
| c        | 21  | 3      | .9443305 | 0.9801      | 864.59  | 0.0000 |
| i        | 21  | 3      | 1.446736 | 0.8258      | 162.98  | 0.0000 |
| wp       | 21  | 3      | .7211282 | 0.9863      | 1594.75 | 0.0000 |

|           | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|-----------|-------------|-----------|-------|-------|----------------------|-----------|
| <b>c</b>  |             |           |       |       |                      |           |
| p         |             |           |       |       |                      |           |
| --.       | .1248904    | .1081291  | 1.16  | 0.248 | -.0870387            | .3368194  |
| L1.       | .1631439    | .1004382  | 1.62  | 0.104 | -.0337113            | .3599992  |
| w         | .790081     | .0379379  | 20.83 | 0.000 | .715724              | .8644379  |
| _cons     | 16.44079    | 1.304549  | 12.60 | 0.000 | 13.88392             | 18.99766  |
| <b>i</b>  |             |           |       |       |                      |           |
| p         |             |           |       |       |                      |           |
| --.       | -.0130791   | .1618962  | -0.08 | 0.936 | -.3303898            | .3042316  |
| L1.       | .7557238    | .1529331  | 4.94  | 0.000 | .4559805             | 1.055467  |
| k         |             |           |       |       |                      |           |
| L1.       | -.1948482   | .0325307  | -5.99 | 0.000 | -.2586072            | -.1310893 |
| _cons     | 28.17785    | 6.793768  | 4.15  | 0.000 | 14.86231             | 41.49339  |
| <b>wp</b> |             |           |       |       |                      |           |
| y         |             |           |       |       |                      |           |
| --.       | .4004919    | .0318134  | 12.59 | 0.000 | .3381388             | .462845   |
| L1.       | .181291     | .0341588  | 5.31  | 0.000 | .1143411             | .2482409  |
| yr        | .149674     | .0279352  | 5.36  | 0.000 | .094922              | .2044261  |
| _cons     | 1.797216    | 1.115854  | 1.61  | 0.107 | -.3898181            | 3.984251  |

Endogenous: c i wp w p y  
 Exogenous: L.p L.k L.y yr t wg g

We used the `exog()` option to identify `t`, `wg`, and `g` as exogenous variables in the system. These variables must be identified because they are part of the system but appear directly in none of the behavioral equations. Without this option, `reg3` would not know they were part of the system and could be used as instrumental variables. The `endog()` option specifying `w`, `p`, and `y` is also required. Without this information, `reg3` would be unaware that these variables are linear combinations that include endogenous variables. We did not include `k` in the `endog()` option because only its lagged value appears in the behavioral equations.

## □ Technical note

Rather than listing additional endogenous and exogenous variables, we could specify the full list of exogenous variables in an `inst()` option,

```
. reg3 (c p L.p w) (i p L.p L.k) (wp y L.y yr), inst(g t wg yr L.p L.k L.y)
```

or equivalently,

```
. global conseqn "(c p L.p w)"
. global inveqn "(i p L.p L.k)"
. global wageqn "(wp y L.y yr)"
. global inlist "g t wg yr L.p L.k L.y"
. reg3 $conseqn $inveqn $wageqn, inst($inlist)
```

Macros and explicit equations can also be mixed in the specification

```
. reg3 $conseqn (i p L.p L.k) $wageqn, endog(w p y) exog(t wg g)
```

or

```
. reg3 (c p L.p w) $inveqn (wp y L.y yr), endog(w p y) exog(t wg g)
```

Placing the equation-binding parentheses in the global macros was also arbitrary. We could have used

```
. global consump "c p L.p w"
. global invest "i p L.p L.k"
. global wagepriv "wp y L.y yr"
. reg3 ($consump) ($invest) ($wagepriv), endog(w p y) exog(t wg g)
```

`reg3` is tolerant of all combinations, and these commands will produce identical output.

□

Switching to the full variable names, we can fit Klein’s model with the commands below. We will use global macros to store the lists of endogenous and exogenous variables. Again, this is not necessary: these lists could have been typed directly on the command line. However, assigning the lists to local macros makes additional processing easier if alternative models are to be fit. We will also use the ireg3 option to produce the iterated estimates.

```
. use https://www.stata-press.com/data/r19/kleinfull
. global conseqn "(consump profits L.profits wagetot)"
. global inveqn "(invest profits L.profits L.capital)"
. global wageqn "(wagepriv totinc L.totinc year)"
. global enlist "wagetot profits totinc"
. global exlist "taxnetx wagegovt govt"
. reg3 $conseqn $inveqn $wageqn, endog($enlist) exog($exlist) ireg3
Iteration 1: Tolerance = .3712549
Iteration 2: Tolerance = .1894712
Iteration 3: Tolerance = .1076401
(output omitted)
Iteration 24: Tolerance = 7.049e-07
Three-stage least-squares regression, iterated
```

| Equation | Obs | Params | RMSE     | "R-squared" | chi2    | P>chi2 |
|----------|-----|--------|----------|-------------|---------|--------|
| consump  | 21  | 3      | .9565088 | 0.9796      | 970.31  | 0.0000 |
| invest   | 21  | 3      | 2.134327 | 0.6209      | 56.78   | 0.0000 |
| wagepriv | 21  | 3      | .7782334 | 0.9840      | 1312.19 | 0.0000 |

|                 | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|-----------------|-------------|-----------|-------|-------|----------------------|-----------|
| <b>consump</b>  |             |           |       |       |                      |           |
| profits         |             |           |       |       |                      |           |
| --.             | .1645096    | .0961979  | 1.71  | 0.087 | -.0240348            | .3530539  |
| L1.             | .1765639    | .0901001  | 1.96  | 0.050 | -.0000291            | .3531569  |
| wagetot         | .7658011    | .0347599  | 22.03 | 0.000 | .6976729             | .8339294  |
| _cons           | 16.55899    | 1.224401  | 13.52 | 0.000 | 14.15921             | 18.95877  |
| <b>invest</b>   |             |           |       |       |                      |           |
| profits         |             |           |       |       |                      |           |
| --.             | -.3565316   | .2601568  | -1.37 | 0.171 | -.8664296            | .1533664  |
| L1.             | 1.011299    | .2487745  | 4.07  | 0.000 | .5237098             | 1.498888  |
| capital         |             |           |       |       |                      |           |
| L1.             | -.2602      | .0508694  | -5.12 | 0.000 | -.3599022            | -.1604978 |
| _cons           | 42.89629    | 10.59386  | 4.05  | 0.000 | 22.13271             | 63.65987  |
| <b>wagepriv</b> |             |           |       |       |                      |           |
| totinc          |             |           |       |       |                      |           |
| --.             | .3747792    | .0311027  | 12.05 | 0.000 | .3138191             | .4357394  |
| L1.             | .1936506    | .0324018  | 5.98  | 0.000 | .1301443             | .257157   |
| year            | .1679262    | .0289291  | 5.80  | 0.000 | .1112263             | .2246261  |
| _cons           | 2.624766    | 1.195559  | 2.20  | 0.028 | .2815124             | 4.968019  |

Endogenous: consump invest wagepriv wagetot profits totinc  
 Exogenous: L.profits L.capital L.totinc year taxnetx wagegovt govt

## ▷ Example 4: Constraints with reg3

As a simple example of constraints, (1) above may be rewritten with both wages explicitly appearing (rather than as a variable containing the sum). Using the longer variable names, we have

$$\text{consump} = \beta_0 + \beta_1 \text{profits} + \beta_2 \text{L.profits} + \beta_3 \text{wagepriv} + \beta_{12} \text{wagegovt} + \epsilon_1$$

To retain the effect of the identity in (7), we need  $\beta_3 = \beta_{12}$  as a constraint on the system. We obtain this result by defining the constraint in the usual way and then specifying its use in `reg3`. Because `reg3` is a system estimator, we will need to use the full equation syntax of `constraint`. The assumption that the following commands are entered after the model above has been estimated. We are simply changing the definition of the consumption equation (`consump`) and adding a constraint on two of its parameters. The rest of the model definition is carried forward.

```
. global conseqn "(consump profits L.profits wagepriv wagegovt)"
. constraint define 1 [consump]wagepriv = [consump]wagegovt
. reg3 $conseqn $inveqn $wageqn, endog($enlist) exog($exlist) constr(1) ireg3
note: additional endogenous variables not in the system have no effect and are
 ignored (wagetot).
Iteration 1: Tolerance = .3712547
Iteration 2: Tolerance = .189471
Iteration 3: Tolerance = .10764
 (output omitted)
Iteration 24: Tolerance = 7.049e-07
```

Three-stage least-squares regression, iterated

| Equation | Obs | Params | RMSE     | "R-squared" | chi2    | P>chi2 |
|----------|-----|--------|----------|-------------|---------|--------|
| consump  | 21  | 3      | .9565086 | 0.9796      | 970.31  | 0.0000 |
| invest   | 21  | 3      | 2.134326 | 0.6209      | 56.78   | 0.0000 |
| wagepriv | 21  | 3      | .7782334 | 0.9840      | 1312.19 | 0.0000 |

( 1) [consump]wagepriv - [consump]wagegovt = 0

|          | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|----------|-------------|-----------|-------|-------|----------------------|-----------|
| consump  |             |           |       |       |                      |           |
| profits  |             |           |       |       |                      |           |
| --.      | .1645097    | .0961978  | 1.71  | 0.087 | -.0240346            | .353054   |
| L1.      | .1765639    | .0901001  | 1.96  | 0.050 | -.0000291            | .3531568  |
| wagepriv | .7658012    | .0347599  | 22.03 | 0.000 | .6976729             | .8339294  |
| wagegovt | .7658012    | .0347599  | 22.03 | 0.000 | .6976729             | .8339294  |
| _cons    | 16.55899    | 1.224401  | 13.52 | 0.000 | 14.1592              | 18.95877  |
| invest   |             |           |       |       |                      |           |
| profits  |             |           |       |       |                      |           |
| --.      | -.3565311   | .2601567  | -1.37 | 0.171 | -.8664288            | .1533666  |
| L1.      | 1.011298    | .2487744  | 4.07  | 0.000 | .5237096             | 1.498887  |
| capital  |             |           |       |       |                      |           |
| L1.      | -.2601999   | .0508694  | -5.12 | 0.000 | -.359902             | -.1604977 |
| _cons    | 42.89626    | 10.59386  | 4.05  | 0.000 | 22.13269             | 63.65984  |
| wagepriv |             |           |       |       |                      |           |
| totinc   |             |           |       |       |                      |           |
| --.      | .3747792    | .0311027  | 12.05 | 0.000 | .313819              | .4357394  |
| L1.      | .1936506    | .0324018  | 5.98  | 0.000 | .1301443             | .257157   |
| year     | .1679262    | .0289291  | 5.80  | 0.000 | .1112263             | .2246261  |
| _cons    | 2.624766    | 1.195559  | 2.20  | 0.028 | .281512              | 4.968019  |

Endogenous: consump invest wagepriv wagetot profits totinc

Exogenous: L.profits wagegovt L.capital L.totinc year taxnetx govt

As expected, none of the parameter or standard error estimates has changed from the previous estimates (before the seventh significant digit). We have simply decomposed the total wage variable into its two parts and constrained the coefficients on these parts. The warning about additional endogenous variables was just reg3's way of letting us know that we had specified some information that was irrelevant to the estimation of the system. We had left the wagetot variable in our endog macro. It does not mean anything to the system to specify wagetot as endogenous because it is no longer in the system. That's fine with reg3 and fine for our current purposes.

We can also impose constraints across the equations. For example, the admittedly meaningless constraint of requiring profits to have the same effect in both the consumption and investment equations could be imposed. Retaining the constraint on the wage coefficients, we would estimate this constrained system.

```
. constraint define 2 [consump]profits = [invest]profits
. reg3 $conseqn $inveqn $wageqn, endog($enlist) exog($exlist) constr(1 2) ireg3
note: additional endogenous variables not in the system have no effect and are
 ignored (wagetot).
```

```
Iteration 1: Tolerance = .1427927
Iteration 2: Tolerance = .032539
Iteration 3: Tolerance = .00307811
Iteration 4: Tolerance = .00016903
Iteration 5: Tolerance = .00003409
Iteration 6: Tolerance = 7.763e-06
Iteration 7: Tolerance = 9.240e-07
```

Three-stage least-squares regression, iterated

| Equation | Obs | Params | RMSE     | "R-squared" | chi2    | P>chi2 |
|----------|-----|--------|----------|-------------|---------|--------|
| consump  | 21  | 3      | .9504669 | 0.9798      | 1019.54 | 0.0000 |
| invest   | 21  | 3      | 1.247066 | 0.8706      | 144.57  | 0.0000 |
| wagepriv | 21  | 3      | .7225276 | 0.9862      | 1537.45 | 0.0000 |

- ( 1) [consump]wagepriv - [consump]wagegovt = 0
- ( 2) [consump]profits - [invest]profits = 0

|                 | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|-----------------|-------------|-----------|-------|-------|----------------------|-----------|
| <b>consump</b>  |             |           |       |       |                      |           |
| profits         |             |           |       |       |                      |           |
| --.             | .1075413    | .0957767  | 1.12  | 0.262 | -.0801777            | .2952602  |
| L1.             | .1712756    | .0912613  | 1.88  | 0.061 | -.0075932            | .3501444  |
| wagepriv        | .798484     | .0340876  | 23.42 | 0.000 | .7316734             | .8652946  |
| wagegovt        | .798484     | .0340876  | 23.42 | 0.000 | .7316734             | .8652946  |
| _cons           | 16.2521     | 1.212157  | 13.41 | 0.000 | 13.87631             | 18.62788  |
| <b>invest</b>   |             |           |       |       |                      |           |
| profits         |             |           |       |       |                      |           |
| --.             | .1075413    | .0957767  | 1.12  | 0.262 | -.0801777            | .2952602  |
| L1.             | .6443378    | .1058682  | 6.09  | 0.000 | .43684               | .8518356  |
| capital         |             |           |       |       |                      |           |
| L1.             | -.1766669   | .0261889  | -6.75 | 0.000 | -.2279962            | -.1253375 |
| _cons           | 24.31931    | 5.284325  | 4.60  | 0.000 | 13.96222             | 34.6764   |
| <b>wagepriv</b> |             |           |       |       |                      |           |
| totinc          |             |           |       |       |                      |           |
| --.             | .4014106    | .0300552  | 13.36 | 0.000 | .3425035             | .4603177  |
| L1.             | .1775359    | .0321583  | 5.52  | 0.000 | .1145068             | .240565   |
| year            | .1549211    | .0282291  | 5.49  | 0.000 | .099593              | .2102492  |
| _cons           | 1.959788    | 1.14467   | 1.71  | 0.087 | -.2837242            | 4.203299  |

Endogenous: consump invest wagepriv wagetot profits totinc  
 Exogenous: L.profits wagegovt L.capital L.totinc year taxnetx govt

## □ Technical note

Identification in a system of simultaneous equations involves the notion that there is enough information to estimate the parameters of the model given the specified functional form. Under-identification usually manifests itself as one matrix in the 3SLS computations. The most commonly violated order condition for 2SLS or 3SLS involves the number of endogenous and exogenous variables. There must be at least as many noncollinear exogenous variables in the remaining system as there are endogenous right-hand-side variables in an equation. This condition must hold for each structural equation in the system.

Put as a set of rules the following:

1. Count the number of right-hand-side endogenous variables in an equation and call this  $m_i$ .
2. Count the number of exogenous variables in the same equation and call this  $k_i$ .
3. Count the total number of exogenous variables in all the structural equations plus any additional variables specified in an `exog()` or `inst()` option and call this  $K$ .
4. If  $m_i > (K - k_i)$  for any structural equation ( $i$ ), then the system is underidentified and cannot be estimated by 3SLS.

We are also possibly in trouble if any of the exogenous variables are linearly dependent. We must have  $m_i$  linearly independent variables among the exogenous variables represented by  $(K - k_i)$ .

The complete conditions for identification involve rank-order conditions on several matrices. For a full treatment, see [Theil \(1971\)](#) or [Greene \(2018, 363–365\)](#).

□

Henri Theil (1924–2000) was born in Amsterdam and awarded a PhD in 1951 by the University of Amsterdam. He researched and taught econometric theory, statistics, microeconomics, macroeconomic modeling, and economic forecasting, and policy at (now) Erasmus University Rotterdam, the University of Chicago, and the University of Florida. Theil's many specific contributions include work on 2SLS and 3SLS, inequality and concentration, and consumer demand.

## Stored results

reg3 stores the following in `e()`:

### Scalars

|                          |                                                        |
|--------------------------|--------------------------------------------------------|
| <code>e(N)</code>        | number of observations                                 |
| <code>e(k)</code>        | number of parameters                                   |
| <code>e(k_eq)</code>     | number of equations in <code>e(b)</code>               |
| <code>e(mss_#)</code>    | model sum of squares for equation #                    |
| <code>e(df_m#)</code>    | model degrees of freedom for equation #                |
| <code>e(rss_#)</code>    | residual sum of squares for equation #                 |
| <code>e(df_r)</code>     | residual degrees of freedom (small)                    |
| <code>e(r2_#)</code>     | $R^2$ for equation #                                   |
| <code>e(F_#)</code>      | $F$ statistic for equation # (small)                   |
| <code>e(rmse_#)</code>   | root mean squared error for equation #                 |
| <code>e(dfk2_adj)</code> | divisor used with VCE when <code>dfk2</code> specified |
| <code>e(ll)</code>       | log likelihood                                         |
| <code>e(N_clust)</code>  | number of clusters                                     |
| <code>e(chi2_#)</code>   | $\chi^2$ for equation #                                |
| <code>e(p_#)</code>      | $p$ -value for model test for equation #               |
| <code>e(cons_#)</code>   | 1 when equation # has a constant, 0 otherwise          |
| <code>e(rank)</code>     | rank of <code>e(V)</code>                              |
| <code>e(ic)</code>       | number of iterations                                   |

### Macros

|                                |                                                                       |
|--------------------------------|-----------------------------------------------------------------------|
| <code>e(cmd)</code>            | reg3                                                                  |
| <code>e(cmdline)</code>        | command as typed                                                      |
| <code>e(depvar)</code>         | names of dependent variables                                          |
| <code>e(exog)</code>           | names of exogenous variables                                          |
| <code>e(endog)</code>          | names of endogenous variables                                         |
| <code>e(eqnames)</code>        | names of equations                                                    |
| <code>e(corr)</code>           | correlation structure                                                 |
| <code>e(wtype)</code>          | weight type                                                           |
| <code>e(wexp)</code>           | weight expression                                                     |
| <code>e(method)</code>         | 3sls, 2sls, ols, sure, or mvreg                                       |
| <code>e(small)</code>          | small, if specified                                                   |
| <code>e(dfk)</code>            | dfk, if specified                                                     |
| <code>e(clustvar)</code>       | name of cluster variable                                              |
| <code>e(vce)</code>            | <i>vce</i> type specified in <code>vce()</code>                       |
| <code>e(vcetype)</code>        | title used to label Std. err.                                         |
| <code>e(properties)</code>     | b V                                                                   |
| <code>e(predict)</code>        | program used to implement <code>predict</code>                        |
| <code>e(marginsok)</code>      | predictions allowed by <code>margins</code>                           |
| <code>e(marginsnotok)</code>   | predictions disallowed by <code>margins</code>                        |
| <code>e(marginsdefault)</code> | default <code>predict()</code> specification for <code>margins</code> |
| <code>e(asbalanced)</code>     | factor variables <code>fvset</code> as <code>asbalanced</code>        |
| <code>e(asobserved)</code>     | factor variables <code>fvset</code> as <code>asobserved</code>        |

### Matrices

|                       |                                              |
|-----------------------|----------------------------------------------|
| <code>e(b)</code>     | coefficient vector                           |
| <code>e(Cns)</code>   | constraints matrix                           |
| <code>e(Sigma)</code> | $\Sigma$ matrix                              |
| <code>e(V)</code>     | variance–covariance matrix of the estimators |

### Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

Matrices

`r(table)`

matrix containing the coefficients with their standard errors, test statistics, *p*-values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

## Methods and formulas

The most concise way to represent a system of equations for 3SLS requires thinking of the individual equations and their associated data as being stacked. `reg3` does not expect the data in this format, but it is a convenient shorthand. The system could then be formulated as

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_M \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_1 & 0 & \dots & 0 \\ 0 & \mathbf{Z}_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{Z}_M \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_M \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_M \end{bmatrix}$$

In full matrix notation, this is just

$$\mathbf{y} = \mathbf{Z}\mathbf{B} + \boldsymbol{\epsilon}$$

The **Z** elements in these matrices represent both the endogenous and the exogenous right-hand-side variables in the equations.

Also assume that there will be correlation between the disturbances of the equations so that

$$E(\boldsymbol{\epsilon}\boldsymbol{\epsilon}') = \boldsymbol{\Sigma}$$

where the disturbances are further assumed to have an expected value of 0;  $E(\boldsymbol{\epsilon}) = 0$ .

The first stage of 3SLS regression requires developing instrumented values for the endogenous variables in the system. These values can be derived as the predictions from a linear regression of each endogenous regressor on all exogenous variables in the system or, more succinctly, as the projection of each regressor through the projection matrix of all exogenous variables onto the regressors. Designating the set of all exogenous variables as **X** results in

$$\hat{\mathbf{z}}_i = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{z}_i \quad \text{for each } i$$

Taken collectively, these  $\hat{\mathbf{Z}}$  contain the instrumented values for all the regressors. They take on the actual values for the exogenous variables and first-stage predictions for the endogenous variables. Given these instrumented variables, a generalized least squares (GLS) or **Aitken (1935)** estimator can be formed for the parameters of the system

$$\hat{\mathbf{B}} = \left\{ \widehat{\mathbf{Z}}'(\boldsymbol{\Sigma}^{-1} \otimes \mathbf{I})\widehat{\mathbf{Z}} \right\}^{-1} \widehat{\mathbf{Z}}'(\boldsymbol{\Sigma}^{-1} \otimes \mathbf{I})\mathbf{y}$$

All that remains is to obtain a consistent estimator for  $\boldsymbol{\Sigma}$ . This estimate can be formed from the residuals of 2SLS estimates of each equation in the system. Alternately, and identically, the residuals can be computed from the estimates formed by taking  $\boldsymbol{\Sigma}$  to be an identity matrix. This maintains the full system of coefficients and allows constraints to be applied when the residuals are computed.

If we take  $\mathbf{E}$  to be the matrix of residuals from these estimates, a consistent estimate of  $\Sigma$  is

$$\widehat{\Sigma} = \frac{\mathbf{E}'\mathbf{E}}{n}$$

where  $n$  is the number of observations in the sample. An alternative divisor for this estimate can be obtained with the `dfk` option as outlined under options.

With the estimate of  $\widehat{\Sigma}$  placed into the GLS estimating equation,

$$\widehat{\mathbf{B}} = \left\{ \widehat{\mathbf{Z}}'(\widehat{\Sigma}^{-1} \otimes \mathbf{I})\widehat{\mathbf{Z}} \right\}^{-1} \widehat{\mathbf{Z}}'(\widehat{\Sigma}^{-1} \otimes \mathbf{I})\mathbf{y}$$

is the 3SLS estimates of the system parameters.

The asymptotic variance–covariance matrix of the estimator is just the standard formulation for a GLS estimator

$$\mathbf{V}_{\widehat{\mathbf{B}}} = \left\{ \widehat{\mathbf{Z}}'(\widehat{\Sigma}^{-1} \otimes \mathbf{I})\widehat{\mathbf{Z}} \right\}^{-1}$$

Iterated 3SLS estimates can be obtained by computing the residuals from the three-stage parameter estimates, using these to formulate a new  $\widehat{\Sigma}$ , and recomputing the parameter estimates. This process is repeated until the estimates  $\widehat{\mathbf{B}}$  converge—if they converge. Convergence is not guaranteed. When estimating a system by SURE, these iterated estimates will be the maximum likelihood estimates for the system. The iterated solution can also be used to produce estimates that are invariant to choice of system and restriction parameterization for many linear systems under full 3SLS.

The exposition above follows the parallel developments in [Greene \(2018\)](#) and [Davidson and MacKinnon \(1993\)](#).

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [\[P\] \\_robust](#), particularly [Introduction](#) and [Methods and formulas](#).

Alexander Craig Aitken (1895–1967) was born in Dunedin, New Zealand. He attended the University of Otago on a full scholarship but left the university to enlist in the New Zealand Expeditionary Force during World War I. He would later reflect on the time he spent in active service and publish two war memoirs.

After returning from the war, he completed his studies at the University of Otago and then worked on his PhD at the University of Edinburgh under E. T. Whittaker. Although he suffered weeks of illness during the course of his postgraduate studies, he managed to write an impressive thesis on data smoothing. He then became a professor of mathematics, and later the Chair of Pure Mathematics, a post he would hold until his retirement in 1965.

Among his many accolades, he was elected a Fellow of the Royal Society of Edinburgh and a Fellow of the Royal Society of Literature, following the publication of his second war memoir. He published many papers on numerical analysis and statistics as well as a couple of books on matrices. Aitken is credited with deriving the generalized least squares estimator, which has been referred to as Aitken's generalized least squares. Aside from his many academic contributions, he had mental calculating abilities like no other. He is known to have multiplied two 9-digit numbers in less than a minute and could recite up to 707 digits of  $\pi$ .

## References

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## Also see

- [R] **reg3 postestimation** — Postestimation tools for reg3
- [R] **ivregress** — Single-equation instrumental-variables regression
- [R] **nlsur** — Estimation of nonlinear system of equations
- [R] **regress** — Linear regression
- [R] **sureg** — Zellner’s seemingly unrelated regression
- [MV] **mvreg** — Multivariate regression
- [SEM] **Example 7** — Nonrecursive structural model
- [SEM] **Intro 5** — Tour of models
- [TS] **forecast** — Econometric model forecasting
- [U] **20 Estimation and postestimation commands**

|                         |                                      |                           |
|-------------------------|--------------------------------------|---------------------------|
| Postestimation commands | <a href="#">predict</a>              | <a href="#">margins</a>   |
| Remarks and examples    | <a href="#">Methods and formulas</a> | <a href="#">Reference</a> |
| Also see                |                                      |                           |

## Postestimation commands

The following postestimation commands are available after `reg3`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| * <code>estat ic</code>      | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| <code>forecast</code>        | dynamic forecasts and simulations                                                                                                       |
| <code>hausman</code>         | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | predictions and their SEs, residuals, etc.                                                                                              |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`estat ic` is not appropriate after `reg3`, `2sls`.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as linear predictions, standard errors, residuals, and differences between the linear predictions of two equations.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, equation(eqno[, eqno]) statistic]
statistic Description
```

| Main                    |                                                            |
|-------------------------|------------------------------------------------------------|
| <code>xb</code>         | linear prediction; the default                             |
| <code>stdp</code>       | standard error of the linear prediction                    |
| <code>residuals</code>  | residuals                                                  |
| <code>difference</code> | difference between the linear predictions of two equations |
| <code>stddp</code>      | standard error of the difference in linear predictions     |

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

## Options for predict

Main

`equation(eqno[, eqno])` specifies to which equation you are referring.

`equation()` is filled in with one *eqno* for the `xb`, `stdp`, and `residuals` options. `equation(#1)` would mean the calculation is to be made for the first equation, `equation(#2)` would mean the second, and so on. You could also refer to the equations by their names. `equation(income)` would refer to the equation named `income` and `equation(hours)` to the equation named `hours`.

If you do not specify `equation()`, results are the same as if you specified `equation(#1)`.

`difference` and `stddp` refer to between-equation concepts. To use these options, you must specify two equations, for example, `equation(#1, #2)` or `equation(income, hours)`. When two equations must be specified, `equation()` is required.

`xb`, the default, calculates the linear prediction (fitted values)—the prediction of  $\mathbf{x}_j\mathbf{b}$  for the specified equation.

`stdp` calculates the standard error of the prediction for the specified equation. It can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. The standard error of the prediction is also referred to as the standard error of the fitted value.

`residuals` calculates the residuals.

`difference` calculates the difference between the linear predictions of two equations in the system. With `equation(#1,#2)`, `difference` computes the prediction of `equation(#1)` minus the prediction of `equation(#2)`.

`stdp` is allowed only after you have previously fit a multiple-equation model. The standard error of the difference in linear predictions ( $\mathbf{x}_{1j}\mathbf{b} - \mathbf{x}_{2j}\mathbf{b}$ ) between equations 1 and 2 is calculated.

For more information on using `predict` after multiple-equation estimation commands, see [R] [predict](#).

## margins

### Description for margins

`margins` estimates margins of response for linear predictions and differences between the linear predictions of two equations.

### Menu for margins

Statistics > Postestimation

### Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i>        | Description                                                |
|-------------------------|------------------------------------------------------------|
| <code>default</code>    | linear predictions for each equation                       |
| <code>xb</code>         | linear prediction for a specified equation                 |
| <code>difference</code> | difference between the linear predictions of two equations |
| <code>stdp</code>       | not allowed with <code>margins</code>                      |
| <code>residuals</code>  | not allowed with <code>margins</code>                      |
| <code>stddp</code>      | not allowed with <code>margins</code>                      |

`xb` defaults to the first equation.

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(b)$ .

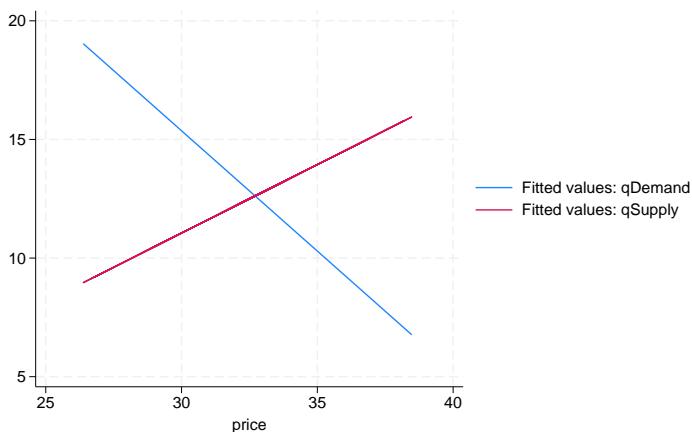
For the full syntax, see [R] [margins](#).

## Remarks and examples

### ▷ Example 1: Using predict

In [example 2](#) of [\[R\] reg3](#), we fit a simple supply-and-demand model. Here we obtain the fitted supply and demand curves assuming that the exogenous regressors equal their sample means. We first replace each of the three exogenous regressors with their sample means, then we call `predict` to obtain the predictions.

```
. use https://www.stata-press.com/data/r19/supDem
. global demand "(qDemand: quantity price pcompete income)"
. global supply "(qSupply: quantity price praw)"
. reg3 $demand $supply, endog(price)
 (output omitted)
. summarize pcompete, meanonly
. replace pcompete = r(mean)
(49 real changes made)
. summarize income, meanonly
. replace income = r(mean)
(49 real changes made)
. summarize praw, meanonly
. replace praw = r(mean)
(49 real changes made)
. predict demand, equation(qDemand)
(option xb assumed; fitted values)
. predict supply, equation(qSupply)
(option xb assumed; fitted values)
. graph twoway line demand price, sort || line supply price, ytitle(" ")
> legend(label(1 "Fitted values: qDemand") label(2 "Fitted values: qSupply"))
```



As we would expect based on economic theory, the demand curve slopes downward while the supply curve slopes upward. With the exogenous variables at their mean levels, the equilibrium price and quantity are slightly less than 33 and 13, respectively.

## ► Example 2: Obtaining forecasts

In [example 3](#) of [R] [reg3](#), we fit Klein's (1950) model of the US economy. That model includes three stochastic equations we fit using `reg3` as well as four identities. Here we briefly illustrate how the `forecast` command can be used to obtain forecasts for all the endogenous variables in the model. For a more detailed discussion of how to forecast with this model, see [TS] [forecast](#).

In Stata, we type

```
. use https://www.stata-press.com/data/r19/klein2, clear
. reg3 (c p L.p w) (i p L.p L.k) (wp y L.y yr), endog(w p y) exog(t wg g)
 (output omitted)
. estimates store kleineqs
. forecast create kleinmodel
 Forecast model kleinmodel started.
. forecast estimates kleineqs
 Added estimation results from reg3.
 Forecast model kleinmodel now contains 3 endogenous variables.
. forecast identity y = c + i + g
 Forecast model kleinmodel now contains 4 endogenous variables.
. forecast identity p = y - t - wp
 Forecast model kleinmodel now contains 5 endogenous variables.
. forecast identity k = L.k + i
 Forecast model kleinmodel now contains 6 endogenous variables.
. forecast identity w = wg + wp
 Forecast model kleinmodel now contains 7 endogenous variables.
. forecast solve, begin(1937)
Computing dynamic forecasts for model kleinmodel.
```

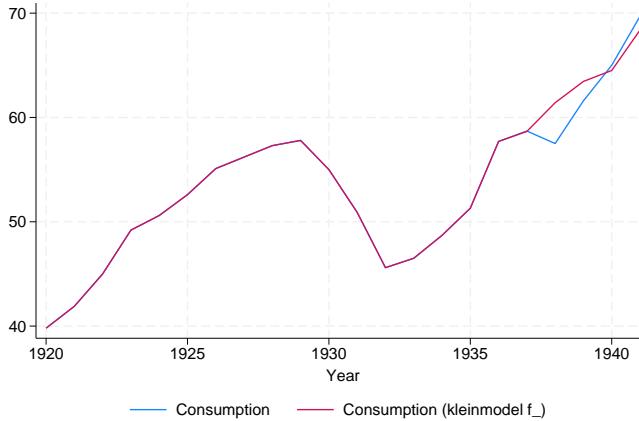
---

```
Starting period: 1937
Ending period: 1941
Forecast prefix: f_
1937:
1938:
1939:
1940:
1941:
Forecast 7 variables spanning 5 periods.
```

---

Here we have obtained dynamic forecasts for our 7 endogenous variables beginning in 1937. By default, the variables containing the forecasts begin with the prefix `f_`. Next we plot the forecast and actual values of consumption:

```
. tsline c f_c
```



For more information about producing forecasts, see [\[TS\] forecast](#).



## Methods and formulas

The computational formulas for the statistics produced by `predict` can be found in [\[R\] predict](#) and [\[R\] regress postestimation](#).

## Reference

Klein, L. R. 1950. *Economic Fluctuations in the United States 1921–1941*. New York: Wiley.

## Also see

[\[R\] reg3](#) — Three-stage estimation for systems of simultaneous equations

[\[U\] 20 Estimation and postestimation commands](#)

[Description](#)[Options](#)[Acknowledgments](#)[Quick start](#)[Remarks and examples](#)[References](#)[Menu](#)[Stored results](#)[Also see](#)[Syntax](#)[Methods and formulas](#)

## Description

`regress` performs ordinary least-squares linear regression. `regress` can also perform weighted estimation, compute robust and cluster-robust standard errors, and adjust results for complex survey designs.

## Quick start

Simple linear regression of  $y$  on  $x_1$

```
regress y x1
```

Regression of  $y$  on  $x_1$ ,  $x_2$ , and indicators for categorical variable  $a$

```
regress y x1 x2 i.a
```

Add the interaction between continuous variable  $x_2$  and  $a$

```
regress y x1 c.x2##i.a
```

Fit model for observations where  $v_1$  is greater than zero

```
regress y x1 x2 i.a if v1>0
```

With cluster-robust standard errors for clustering by levels of  $cvar$

```
regress y x1 x2 i.a, vce(cluster cvar)
```

With cluster-robust standard errors for clustering by levels of  $cvar_1$  and  $cvar_2$

```
regress y x1 x2 i.a, vce(cluster cvar1 cvar2)
```

With bootstrap standard errors

```
regress y x1 x2 i.a, vce(bootstrap)
```

Report standardized coefficients

```
regress y x1 x2 i.a, beta
```

Adjust for complex survey design using `svyset` data

```
svy: regress y x1 x2 i.a
```

Use sampling weight  $wvar$

```
regress y x1 x2 i.a [pweight=wvar]
```

## Menu

Statistics > Linear models and related > Linear regression

## Syntax

```
regress deprvar [indepvars] [if] [in] [weight] [, options]
```

| <i>options</i>                       | Description                                                                                                                                                                                                           |
|--------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                         |                                                                                                                                                                                                                       |
| <code>noconstant</code>              | suppress constant term                                                                                                                                                                                                |
| <code>hascons</code>                 | has user-supplied constant                                                                                                                                                                                            |
| <code>tsscons</code>                 | compute total sum of squares with constant; seldom used                                                                                                                                                               |
| <b>SE/Robust</b>                     |                                                                                                                                                                                                                       |
| <code>vce(<i>vcetype</i>)</code>     | <i>vcetype</i> may be <code>ols</code> , <code>robust</code> , <code>cluster</code> <i>clustvarlist</i> , <code>bootstrap</code> , <code>jackknife</code> , <code>hc2</code> [ <i>clustvar</i> ], or <code>hc3</code> |
| <b>Reporting</b>                     |                                                                                                                                                                                                                       |
| <code>level(#)</code>                | set confidence level; default is <code>level(95)</code>                                                                                                                                                               |
| <code>beta</code>                    | report standardized beta coefficients                                                                                                                                                                                 |
| <code>eform(<i>string</i>)</code>    | report exponentiated coefficients and label as <i>string</i>                                                                                                                                                          |
| <code>depname(<i>varname</i>)</code> | substitute dependent variable name; programmer's option                                                                                                                                                               |
| <code>clustertable</code>            | display table of multiway cluster combinations                                                                                                                                                                        |
| <code>display_options</code>         | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling                                                                      |
| <code>noheader</code>                | suppress output header                                                                                                                                                                                                |
| <code>notable</code>                 | suppress coefficient table                                                                                                                                                                                            |
| <code>plus</code>                    | make table extendable                                                                                                                                                                                                 |
| <code>mse1</code>                    | force mean squared error to 1                                                                                                                                                                                         |
| <code>coeflegend</code>              | display legend instead of statistics                                                                                                                                                                                  |

*indepvars* may contain factor variables; see [U] 11.4.3 **Factor variables**.

*deprvar* and *indepvars* may contain time-series operators; see [U] 11.4.4 **Time-series varlists**.

`bayes`, `bayesboot`, `bootstrap`, `by`, `collect`, `fmm`, `fp`, `jackknife`, `mfp`, `mi estimate`, `nestreg`, `rolling`, `statsby`, `stepwise`, and `svy` are allowed; see [U] 11.1.10 **Prefix commands**. For more details, see [BAYES] **bayes: regress** and [FMM] **fmm: regress**.

`vce(bootstrap)` and `vce(jackknife)` are not allowed with the `mi estimate` prefix; see [MI] **mi estimate**.

Weights are not allowed with the `bootstrap` prefix; see [R] **bootstrap**.

`aweight`s are not allowed with the `jackknife` prefix; see [R] **jackknife**.

`hascons`, `tsscons`, `vce()`, `beta`, `noheader`, `notable`, `plus`, `depname()`, `mse1`, and `weight`s are not allowed with the `svy` prefix; see [SVY] **svy**.

`aweight`s, `fweight`s, `iweight`s, and `pweight`s are allowed; see [U] 11.1.6 **weight**.

`noheader`, `notable`, `plus`, `mse1`, and `coeflegend` do not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options

### Model

`noconstant`; see [R] [Estimation options](#).

`hascons` indicates that a user-defined constant or its equivalent is specified among the independent variables in *indepvars*. Some caution is recommended when specifying this option, as resulting estimates may not be as accurate as they otherwise would be. Use of this option requires “sweeping” the constant last, so the moment matrix must be accumulated in absolute rather than deviation form. This option may be safely specified when the means of the dependent and independent variables are all reasonable and there is not much collinearity between the independent variables. The best procedure is to view `hascons` as a reporting option—estimate with and without `hascons` and verify that the coefficients and standard errors of the variables not affected by the identity of the constant are unchanged.

`tsscons` forces the total sum of squares to be computed as though the model has a constant, that is, as deviations from the mean of the dependent variable. This is a rarely used option that has an effect only when specified with `noconstant`. It affects the total sum of squares and all results derived from the total sum of squares.

### SE/Robust

`vce(vctype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`ols`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvarlist`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

`vce(ols)`, the default, uses the standard variance estimator for ordinary least-squares regression.

`vce(cluster clustvarlist)` specifies that standard errors allow for intragroup correlation within groups defined by one or more variables in *clustvarlist*, relaxing the usual requirement that the observations be independent. For example, `vce(cluster clustvar1)` produces cluster-robust standard errors that allow for observations that are independent across groups defined by `clustvar1` but not necessarily independent within groups. You could also type `vce(cluster clustvar1 clustvar2 ... clusterp)` to account for correlation within groups formed by *p* variables (multiway clustering).

`regress` also allows the following:

`vce(hc2 [clustvar] [ , dfadjust ])` and `vce(hc3)` specify alternative bias corrections for the robust variance calculation. `vce(hc2)` and `vce(hc3)` may not be specified with the `svy` prefix. In the unclustered case, `vce(robust)` uses  $\hat{\sigma}_j^2 = \{n/(n - k)\}u_j^2$  as an estimate of the variance of the *j*th observation, where *n* is the number of observations, *k* is the number of regressors, *u<sub>j</sub>* is the calculated residual, and  $n/(n - k)$  is included to improve the overall estimate’s small-sample properties.

`vce(hc2)` instead uses  $u_j^2/(1 - h_{jj})$  as the observation's variance estimate, where  $h_{jj}$  is the diagonal element of the hat (projection) matrix. This estimate is unbiased if the model really is homoskedastic. `vce(hc2)` tends to produce slightly more conservative confidence intervals. `vce(hc2 clustvar)` produces estimates that allow for intragroup correlation within groups defined by `clustvar`. `dfadjust` computes the [Bell and McCaffrey \(2002\)](#) adjusted degrees of freedom based on `clustvar`. Note that `dfadjust` does not affect multiple-imputation results when the command is used with `mi estimate`. See [Methods and formulas](#) for a description of the computation when `clustvar` is specified.

`vce(hc3)` uses  $u_j^2/(1 - h_{jj})^2$  as suggested by [Davidson and MacKinnon \(1993\)](#), who report that this method tends to produce better results when the model really is heteroskedastic. `vce(hc3)` produces confidence intervals that tend to be even more conservative.

See [Davidson and MacKinnon \(1993, 554–556\)](#) and [Angrist and Pischke \(2009, 294–308\)](#) for more discussion on these two bias corrections.

---

Reporting

---

`level(#)`; see [\[R\] Estimation options](#).

`beta` asks that standardized beta coefficients be reported instead of confidence intervals. The beta coefficients are the regression coefficients obtained by first standardizing all variables to have a mean of 0 and a standard deviation of 1. `beta` may not be specified with `vce(cluster clustvarlist)` or the `svy` prefix.

`eform(string)` is used only in programs and ado-files that use `regress` to fit models other than linear regression. `eform()` specifies that the coefficient table be displayed in exponentiated form as defined in [\[R\] Maximize](#) and that `string` be used to label the exponentiated coefficients in the table.

`depname(varname)` is used only in programs and ado-files that use `regress` to fit models other than linear regression. `depname()` may be specified only at estimation time. `varname` is recorded as the identity of the dependent variable, even though the estimates are calculated using `depvar`. This method affects the labeling of the output—not the results calculated—but could affect subsequent calculations made by `predict`, where the residual would be calculated as deviations from `varname` rather than `depvar`. `depname()` is most typically used when `depvar` is a temporary variable (see [\[P\] macro](#)) used as a proxy for `varname`.

`depname()` is not allowed with the `svy` prefix.

`clustertable` displays a table reporting cluster combinations and the number of clusters per combination. This option is available only when `vce(cluster clustvarlist)` is specified with more than one variable in `clustvarlist` to compute multiway cluster–robust standard errors.

`display_options`: `nocl`, `nopvalues`, `dfci`, `dfpvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no!stretch`; see [\[R\] Estimation options](#).

`dfci` specifies that parameter degrees of freedom and confidence intervals be reported in the coefficient table.

`dfpvalues` specifies that parameter degrees of freedom and *p*-values be reported in the coefficient table.

The following options are available with `regress` but are not shown in the dialog box:

`noheader` suppresses the display of the ANOVA table and summary statistics at the top of the output; only the coefficient table is displayed. This option is often used in programs and ado-files.

`notable` suppresses display of the coefficient table.

`plus` specifies that the output table be made extendable. This option is often used in programs and ado-files.

`mse1` is used only in programs and ado-files that use `regress` to fit models other than linear regression and is not allowed with the `svy` prefix. `mse1` sets the mean squared error to 1, forcing the variance–covariance matrix of the estimators to be  $(\mathbf{X}'\mathbf{X})^{-1}$  (see *Methods and formulas* below) and affecting calculated standard errors. Degrees of freedom for  $t$  statistics is calculated as  $n$  rather than  $n - k$ .

`coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Remarks are presented under the following headings:

- [Ordinary least squares](#)
- [Treatment of the constant](#)
- [Robust standard errors](#)
- [Weighted regression](#)
- [Video examples](#)

`regress` performs linear regression, including ordinary least squares and weighted least squares. See [\[U\] 27 Overview of Stata estimation commands](#) for a list of other regression commands that may be of interest. For a general discussion of linear regression, see [Kutner et al. \(2005\)](#).

See [Stock and Watson \(2019\)](#) and [Wooldridge \(2020\)](#) for an excellent treatment of estimation, inference, interpretation, and specification testing in linear regression models. See [Wooldridge \(2010, chap. 4\)](#) for a more advanced discussion along the same lines.

See [Hamilton \(2013, chap. 7\)](#) and [Cameron and Trivedi \(2022, chap. 3\)](#) for an introduction to linear regression using Stata. [Dohoo, Martin, and Stryhn \(2012, 2010\)](#) discuss linear regression using examples from epidemiology, and Stata datasets and do-files used in the text are available. [Cameron and Trivedi \(2022\)](#) discuss linear regression using econometric examples with Stata. [Mitchell \(2021\)](#) shows how to use graphics and postestimation commands to understand a fitted regression model.

[Chatterjee and Hadi \(2012\)](#) explain regression analysis by using examples containing typical problems that you might encounter when performing exploratory data analysis. We also recommend [Weisberg \(2014\)](#), who emphasizes the importance of the assumptions of linear regression and problems resulting from these assumptions. [Beckett \(2020\)](#) discusses regression analysis with an emphasis on time-series data. [Angrist and Pischke \(2009\)](#) approach regression as a tool for exploring relationships, estimating treatment effects, and providing answers to public policy questions. For a mathematically rigorous treatment, see [Peracchi \(2001, chap. 6\)](#). Finally, see [Plackett \(1972\)](#) if you are interested in the history of regression. Least squares, which dates back to the 1790s, was discovered independently by Legendre and Gauss.

## Ordinary least squares

### ▷ Example 1: Basic linear regression

Suppose that we have data on the mileage rating and weight of 74 automobiles. The variables in our data are `mpg`, `weight`, and `foreign`. The last variable assumes the value 1 for foreign and 0 for domestic automobiles. We wish to fit the model

$$\text{mpg} = \beta_0 + \beta_1 \text{weight} + \beta_2 \text{foreign} + \epsilon$$

This model can be fit with `regress` by typing

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. regress mpg weight foreign
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 1619.2877  | 2  | 809.643849 | F(2, 71)      | = | 69.75  |
| Residual | 824.171761 | 71 | 11.608053  | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.6627 |
|          |            |    |            | Adj R-squared | = | 0.6532 |
| Total    | 2443.45946 | 73 | 33.4720474 | Root MSE      | = | 3.4071 |

| mpg     | Coefficient | Std. err. | t      | P> t  | [95% conf. interval] |           |
|---------|-------------|-----------|--------|-------|----------------------|-----------|
| weight  | -.0065879   | .0006371  | -10.34 | 0.000 | -.0078583            | -.0053175 |
| foreign | -1.650029   | 1.075994  | -1.53  | 0.130 | -3.7955              | .4954422  |
| _cons   | 41.6797     | 2.165547  | 19.25  | 0.000 | 37.36172             | 45.99768  |

`regress` produces a variety of summary statistics along with the table of regression coefficients. At the upper left, `regress` reports an analysis-of-variance (ANOVA) table. The column headings SS, df, and MS stand for “sum of squares”, “degrees of freedom”, and “mean square”, respectively. In this example, the total sum of squares is 2,443.5: 1,619.3 accounted for by the model and 824.2 left unexplained. Because the regression included a constant, the total sum reflects the sum after removal of means, as does the sum of squares due to the model. The table also reveals that there are 73 total degrees of freedom (counted as 74 observations less 1 for the mean removal), of which 2 are consumed by the model, leaving 71 for the residual.

To the right of the ANOVA table are presented other summary statistics. The  $F$  statistic associated with the ANOVA table is 69.75. The statistic has 2 numerator and 71 denominator degrees of freedom. The  $F$  statistic tests the hypothesis that all coefficients excluding the constant are zero. The chance of observing an  $F$  statistic that large or larger is reported as 0.0000, which is Stata’s way of indicating a number smaller than 0.00005. The  $R^2$  for the regression is 0.6627, and the  $R^2$  adjusted for degrees of freedom ( $R_a^2$ ) is 0.6532. The root mean squared error, labeled Root MSE, is 3.4071. It is the square root of the mean squared error reported for the residual in the ANOVA table.

Finally, Stata produces a table of the estimated coefficients. The first line of the table indicates that the left-hand-side variable is `mpg`. Thereafter follow the estimated coefficients. Our fitted model is

$$\text{mpg\_hat} = 41.68 - 0.0066 \text{weight} - 1.65 \text{foreign}$$

Reported to the right of the coefficients in the output are the standard errors. For instance, the standard error for the coefficient on `weight` is 0.0006371. The corresponding  $t$  statistic is  $-10.34$ , which has a two-sided significance level of 0.000. This number indicates that the significance is less than 0.0005. The 95% confidence interval for the coefficient is  $[-0.0079, -0.0053]$ .

### ▷ Example 2: Transforming the dependent variable

If we had a graph comparing mpg with weight, we would notice that the relationship is distinctly nonlinear. This is to be expected because energy usage per distance should increase linearly with weight, but mpg is measuring distance per energy used. We could obtain a better model by generating a new variable measuring the number of gallons used per 100 miles (gp100m) and then using this new variable in our model:

$$\text{gp100m} = \beta_0 + \beta_1 \text{weight} + \beta_2 \text{foreign} + \epsilon$$

We can now fit this model:

```
. generate gp100m = 100/mpg
. regress gp100m weight foreign
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 91.1761694 | 2  | 45.5880847 | F(2, 71)      | = | 113.97 |
| Residual | 28.4000913 | 71 | .400001287 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.7625 |
|          |            |    |            | Adj R-squared | = | 0.7558 |
| Total    | 119.576261 | 73 | 1.63803097 | Root MSE      | = | .63246 |

| gp100m  | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|---------|-------------|-----------|-------|-------|----------------------|----------|
| weight  | .0016254    | .0001183  | 13.74 | 0.000 | .0013896             | .0018612 |
| foreign | .6220535    | .1997381  | 3.11  | 0.003 | .2237871             | 1.02032  |
| _cons   | -.0734839   | .4019932  | -0.18 | 0.855 | -.8750354            | .7280677 |

Fitting the physically reasonable model increases our  $R^2$  to 0.7625.

◀

### ▷ Example 3: Obtaining beta coefficients

regress shares the features of all estimation commands. Among other things, this means that after running a regression, we can use test to test hypotheses about the coefficients, estat vce to examine the covariance matrix of the estimators, and predict to obtain predicted values, residuals, and influence statistics. See [U] 20 Estimation and postestimation commands. Options that affect how estimates are displayed, such as beta or level(), can be used when replaying results.

Suppose that we meant to specify the beta option to obtain beta coefficients (regression coefficients normalized by the ratio of the standard deviation of the regressor to the standard deviation of the dependent variable). Even though we forgot, we can specify the option now:

```
. regress, beta
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 91.1761694 | 2  | 45.5880847 | F(2, 71)      | = | 113.97 |
| Residual | 28.4000913 | 71 | .400001287 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.7625 |
|          |            |    |            | Adj R-squared | = | 0.7558 |
| Total    | 119.576261 | 73 | 1.63803097 | Root MSE      | = | .63246 |

| gp100m  | Coefficient | Std. err. | t     | P> t  | Beta     |
|---------|-------------|-----------|-------|-------|----------|
| weight  | .0016254    | .0001183  | 13.74 | 0.000 | .9870255 |
| foreign | .6220535    | .1997381  | 3.11  | 0.003 | .2236673 |
| _cons   | -.0734839   | .4019932  | -0.18 | 0.855 | .        |

◀

## Treatment of the constant

By default, `regress` includes an intercept (constant) term in the model. The `noconstant` option suppresses it, and the `hascons` option tells `regress` that the model already has one.

### ▷ Example 4: Suppressing the constant term

We wish to fit a regression of the weight of an automobile against its length, and we wish to impose the constraint that the weight is zero when the length is zero.

If we simply type `regress weight length`, we are fitting the model

$$\text{weight} = \beta_0 + \beta_1 \text{length} + \epsilon$$

Here a length of zero corresponds to a weight of  $\beta_0$ . We want to force  $\beta_0$  to be zero or, equivalently, estimate an equation that does not include an intercept:

$$\text{weight} = \beta_1 \text{length} + \epsilon$$

We do this by specifying the `noconstant` option:

```
. regress weight length, noconstant
```

| Source   | SS         | df | MS         | Number of obs | = | 74      |
|----------|------------|----|------------|---------------|---|---------|
| Model    | 703869302  | 1  | 703869302  | F(1, 73)      | = | 3450.13 |
| Residual | 14892897.8 | 73 | 204012.299 | Prob > F      | = | 0.0000  |
|          |            |    |            | R-squared     | = | 0.9793  |
|          |            |    |            | Adj R-squared | = | 0.9790  |
| Total    | 718762200  | 74 | 9713002.7  | Root MSE      | = | 451.68  |

| weight | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |         |
|--------|-------------|-----------|-------|-------|----------------------|---------|
| length | 16.29829    | .2774752  | 58.74 | 0.000 | 15.74528             | 16.8513 |

In our data, `length` is measured in inches and `weight` in pounds. We discover that each inch of length adds 16 pounds to the weight.

Sometimes there is no need for Stata to include a constant term in the model. Most commonly, this occurs when the model contains a set of mutually exclusive indicator variables. `hascons` is a variation of the `noconstant` option—it tells Stata not to add a constant to the regression because the regression specification already has one, either directly or indirectly.

For instance, we now refit our model of `weight` as a function of `length` and include separate constants for foreign and domestic cars by specifying `bn.foreign`. `bn.foreign` is factor-variable notation for “no base for foreign” or “include all levels of variable `foreign` in the model”; see [U] 11.4.3 **Factor variables**.

```
. regress weight length bn.foreign, hascons
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 39647744.7 | 2  | 19823872.3 | F(2, 71)      | = | 316.54 |
| Residual | 4446433.7  | 71 | 62625.8268 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.8992 |
|          |            |    |            | Adj R-squared | = | 0.8963 |
| Total    | 44094178.4 | 73 | 604029.841 | Root MSE      | = | 250.25 |

| weight   | Coefficient | Std. err. | t      | P> t  | [95% conf. interval] |           |
|----------|-------------|-----------|--------|-------|----------------------|-----------|
| length   | 31.44455    | 1.601234  | 19.64  | 0.000 | 28.25178             | 34.63732  |
| foreign  |             |           |        |       |                      |           |
| Domestic | -2850.25    | 315.9691  | -9.02  | 0.000 | -3480.274            | -2220.225 |
| Foreign  | -2983.927   | 275.1041  | -10.85 | 0.000 | -3532.469            | -2435.385 |

## □ Technical note

There is a subtle distinction between the `hascons` and `noconstant` options. We can most easily reveal it by refitting the last regression, specifying `noconstant` rather than `hascons`:

```
. regress weight length bn.foreign, noconstant
```

| Source   | SS        | df | MS         | Number of obs | = | 74      |
|----------|-----------|----|------------|---------------|---|---------|
| Model    | 714315766 | 3  | 238105255  | F(3, 71)      | = | 3802.03 |
| Residual | 4446433.7 | 71 | 62625.8268 | Prob > F      | = | 0.0000  |
|          |           |    |            | R-squared     | = | 0.9938  |
|          |           |    |            | Adj R-squared | = | 0.9936  |
| Total    | 718762200 | 74 | 9713002.7  | Root MSE      | = | 250.25  |

| weight   | Coefficient | Std. err. | t      | P> t  | [95% conf. interval] |           |
|----------|-------------|-----------|--------|-------|----------------------|-----------|
| length   | 31.44455    | 1.601234  | 19.64  | 0.000 | 28.25178             | 34.63732  |
| foreign  |             |           |        |       |                      |           |
| Domestic | -2850.25    | 315.9691  | -9.02  | 0.000 | -3480.274            | -2220.225 |
| Foreign  | -2983.927   | 275.1041  | -10.85 | 0.000 | -3532.469            | -2435.385 |

Comparing this output with that produced by the [previous `regress` command](#), we see that they are almost, but not quite, identical. The parameter estimates and their associated statistics—the second half of the output—are identical. The overall summary statistics and the ANOVA table—the first half of the output—are different, however.

In the first case, the  $R^2$  is shown as 0.8992; here it is shown as 0.9938. In the first case, the  $F$  statistic is 316.54; now it is 3,802.03. The numerator degrees of freedom is different as well. In the first case, the numerator degrees of freedom is 2; now the degrees of freedom is 3. Which is correct?

Both are. Specifying the `hascons` option causes `regress` to adjust the ANOVA table and its associated statistics for the explanatory power of the constant. The regression in effect has a constant; it is just written in such a way that a separate constant is unnecessary. No such adjustment is made with the `noconstant` option.

□

### □ Technical note

When the `hascons` option is specified, `regress` checks to make sure that the model does in fact have a constant term. If `regress` cannot find a constant term, it automatically adds one. Fitting a model of weight on length and specifying the `hascons` option, we obtain

```
. regress weight length, hascons
note: option hascons false.
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 39461306.8 | 1  | 39461306.8 | F(1, 72)      | = | 613.27 |
| Residual | 4632871.55 | 72 | 64345.4382 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.8949 |
|          |            |    |            | Adj R-squared | = | 0.8935 |
| Total    | 44094178.4 | 73 | 604029.841 | Root MSE      | = | 253.66 |

| weight | Coefficient | Std. err. | t      | P> t  | [95% conf. interval] |           |
|--------|-------------|-----------|--------|-------|----------------------|-----------|
| length | 33.01988    | 1.333364  | 24.76  | 0.000 | 30.36187             | 35.67789  |
| _cons  | -3186.047   | 252.3113  | -12.63 | 0.000 | -3689.02             | -2683.073 |

Even though we specified `hascons`, `regress` included a constant, anyway. It also added a note to our output: “note: option `hascons` false”.

□

### □ Technical note

Even if the model specification effectively includes a constant term, we need not specify the `hascons` option. `regress` is always on the lookout for collinear variables and omits them from the model. For instance,

```
. regress weight length bn.foreign
note: 1.foreign omitted because of collinearity.
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 39647744.7 | 2  | 19823872.3 | F(2, 71)      | = | 316.54 |
| Residual | 4446433.7  | 71 | 62625.8268 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.8992 |
|          |            |    |            | Adj R-squared | = | 0.8963 |
| Total    | 44094178.4 | 73 | 604029.841 | Root MSE      | = | 250.25 |

| weight   | Coefficient | Std. err. | t      | P> t  | [95% conf. interval] |           |
|----------|-------------|-----------|--------|-------|----------------------|-----------|
| length   | 31.44455    | 1.601234  | 19.64  | 0.000 | 28.25178             | 34.63732  |
| foreign  |             |           |        |       |                      |           |
| Domestic | 133.6775    | 77.47615  | 1.73   | 0.089 | -20.80555            | 288.1605  |
| Foreign  | 0           | (omitted) |        |       |                      |           |
| _cons    | -2983.927   | 275.1041  | -10.85 | 0.000 | -3532.469            | -2435.385 |

□

## Robust standard errors

`regress` with the `vce(robust)` option substitutes a robust variance matrix calculation for the conventional calculation, or if `vce(cluster clustvarlist)` is specified, allows relaxing the assumption of independence within groups. How this method works is explained in [\[U\] 20.22 Obtaining robust variance estimates](#). Below, we show how well this approach works.

### ▷ Example 5: Heteroskedasticity and robust standard errors

Specifying the `vce(robust)` option is equivalent to requesting White-corrected standard errors in the presence of heteroskedasticity. We use the automobile data and, in the process of looking at the energy efficiency of cars, analyze a variable with considerable heteroskedasticity.

We will examine the amount of energy—measured in gallons of gasoline—that the cars in the data need to move 1,000 pounds of their weight 100 miles. We are going to examine the relative efficiency of foreign and domestic cars.

```
. generate gpmw = ((1/mpg)/weight)*100*1000
. summarize gpmw
```

| Variable | Obs | Mean     | Std. dev. | Min     | Max     |
|----------|-----|----------|-----------|---------|---------|
| gpmw     | 74  | 1.682184 | .2426311  | 1.09553 | 2.30521 |

In these data, the engines consume between 1.10 and 2.31 gallons of gas to move 1,000 pounds of the car's weight 100 miles. If we ran a regression with conventional standard errors of `gpmw` on `foreign`, we would obtain

```
. regress gpmw foreign
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | .936705572 | 1  | .936705572 | F(1, 72)      | = | 20.07  |
| Residual | 3.36079459 | 72 | .046677703 | Prob > F      | = | 0.0000 |
| Total    | 4.29750017 | 73 | .058869865 | R-squared     | = | 0.2180 |
|          |            |    |            | Adj R-squared | = | 0.2071 |
|          |            |    |            | Root MSE      | = | .21605 |

| gpmw    | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |
|---------|-------------|-----------|-------|-------|----------------------|
| foreign | .2461526    | .0549487  | 4.48  | 0.000 | .1366143 .3556909    |
| _cons   | 1.609004    | .0299608  | 53.70 | 0.000 | 1.549278 1.66873     |

`regress` with the `vce(robust)` option, on the other hand, reports

```
. regress gpmw foreign, vce(robust)
```

```
Linear regression Number of obs = 74
 F(1, 72) = 13.13
 Prob > F = 0.0005
 R-squared = 0.2180
 Root MSE = .21605
```

| gpmw    | Coefficient | Robust std. err. | t     | P> t  | [95% conf. interval] |
|---------|-------------|------------------|-------|-------|----------------------|
| foreign | .2461526    | .0679238         | 3.62  | 0.001 | .1107489 .3815563    |
| _cons   | 1.609004    | .0234535         | 68.60 | 0.000 | 1.56225 1.655758     |

The point estimates are the same (foreign cars need one-quarter gallon more gas), but the standard errors differ by roughly 20%. Conventional regression reports the 95% confidence interval as  $[0.14, 0.36]$ , whereas the robust standard errors make the interval  $[0.11, 0.38]$ .

Which is right? Notice that `gpmw` is a variable with considerable heteroskedasticity:

```
. tabulate foreign, summarize(gpmw)
```

| Car origin | Summary of gpmw |           | Freq. |
|------------|-----------------|-----------|-------|
|            | Mean            | Std. dev. |       |
| Domestic   | 1.6090039       | .16845182 | 52    |
| Foreign    | 1.8551565       | .30186861 | 22    |
| Total      | 1.6821844       | .24263113 | 74    |

Thus, here we favor the robust standard errors. In [U] 20.22 **Obtaining robust variance estimates**, we show another example using linear regression where it makes little difference whether we specify `vce(robust)`. The linear-regression assumptions were true, and we obtained nearly linear-regression results. The advantage of the robust estimate is that in neither case did we have to check assumptions.

◀

## □ Technical note

`regress` purposefully suppresses displaying the ANOVA table when `vce(robust)` is specified. This is done because the sums of squares are no longer appropriate for use in the usual hypothesis tests, even though computationally the sums of squares remain the same. In the nonrobust setting, the  $F$  statistic reported by `regress` is defined in terms of the sums of squares, as in ANOVA. When `vce(robust)` is specified, the ANOVA test is not valid, and the  $F$  statistic corresponds to a Wald test based on the robustly estimated variance matrix.

Some references give formulas for the  $F$  statistic in terms of either  $R^2$  or the root MSE. It is not appropriate to use those formulas for the  $F$  statistic with robust standard errors because the  $R^2$  and root MSE are calculated from the sums of squares. Moreover, the root MSE can no longer be used as an estimate for  $\sigma$  because there is no longer a single  $\sigma$  to estimate—the variance of the residual varies observation by observation. However, `regress` continues to report the  $R^2$  and the root MSE in the robust setting because those statistics are still usable in other settings. In particular,  $R^2$  remains valid as a goodness-of-fit statistic.

□

## ▷ Example 6: Alternative robust standard errors

The `vce(hc2)` and `vce(hc3)` options modify the robust variance calculation. In the context of linear regression without clustering, the idea behind the robust calculation is somehow to measure  $\sigma_j^2$ , the variance of the residual associated with the  $j$ th observation, and then to use that estimate to improve the estimated variance of  $\hat{\beta}$ . Because residuals have (theoretically and practically) mean 0, one estimate of  $\sigma_j^2$  is the observation's squared residual itself— $u_j^2$ . A finite-sample correction could improve that by multiplying  $u_j^2$  by  $n/(n-k)$ , and, as a matter of fact, `vce(robust)` uses  $\{n/(n-k)\}u_j^2$  as its estimate of the residual's variance.

vce(hc2) and vce(hc3) use alternative estimators of the observation-specific variances. For instance, if the residuals are homoskedastic, we can show that the expected value of  $u_j^2$  is  $\sigma^2(1 - h_{jj})$ , where  $h_{jj}$  is the  $j$ th diagonal element of the projection (hat) matrix.  $h_{jj}$  has average value  $k/n$ , so  $1 - h_{jj}$  has average value  $1 - k/n = (n - k)/n$ . Thus, the default robust estimator  $\hat{\sigma}_j = \{n/(n - k)\}u_j^2$  amounts to dividing  $u_j^2$  by the average of the expectation.

vce(hc2) divides  $u_j^2$  by  $1 - h_{jj}$  itself, so it should yield better estimates if the residuals really are homoskedastic. vce(hc3) divides  $u_j^2$  by  $(1 - h_{jj})^2$  and has no such clean interpretation. Davidson and MacKinnon (1993) show that  $u_j^2/(1 - h_{jj})^2$  approximates a more complicated estimator that they obtain by jackknifing (MacKinnon and White 1985). Angrist and Pischke (2009) also illustrate the relative merits of these adjustments.

Here are the results of refitting our efficiency model using vce(hc2) and vce(hc3):

```
. regress gpmw foreign, vce(hc2)
```

```
Linear regression Number of obs = 74
 F(1, 72) = 12.93
 Prob > F = 0.0006
 R-squared = 0.2180
 Root MSE = .21605
```

| gpmw    | Robust HC2  |           | t     | P> t  | [95% conf. interval] |          |
|---------|-------------|-----------|-------|-------|----------------------|----------|
|         | Coefficient | std. err. |       |       |                      |          |
| foreign | .2461526    | .0684669  | 3.60  | 0.001 | .1096662             | .3826389 |
| _cons   | 1.609004    | .0233601  | 68.88 | 0.000 | 1.562437             | 1.655571 |

```
. regress gpmw foreign, vce(hc3)
```

```
Linear regression Number of obs = 74
 F(1, 72) = 12.38
 Prob > F = 0.0008
 R-squared = 0.2180
 Root MSE = .21605
```

| gpmw    | Robust HC3  |           | t     | P> t  | [95% conf. interval] |          |
|---------|-------------|-----------|-------|-------|----------------------|----------|
|         | Coefficient | std. err. |       |       |                      |          |
| foreign | .2461526    | .069969   | 3.52  | 0.001 | .1066719             | .3856332 |
| _cons   | 1.609004    | .023588   | 68.21 | 0.000 | 1.561982             | 1.656026 |

► Example 7: Standard errors for clustered data

The `vce(cluster clustvarlist)` and `vce(hc2 clustvar)` options relax the assumption of independence. Below, we have 28,534 observations on 4,711 women aged 14–46 years. Data were collected on these women between 1968 and 1988. We are going to fit a classic earnings model, and we begin by ignoring that the majority of the women in the dataset have multiple observations.

```
. use https://www.stata-press.com/data/r19/regsmpl, clear
(NLS women 14-26 in 1968)
. regress ln_wage age c.age#c.age tenure
```

| Source   | SS         | df     | MS         | Number of obs | = | 28,101  |
|----------|------------|--------|------------|---------------|---|---------|
| Model    | 1054.52501 | 3      | 351.508335 | F(3, 28097)   | = | 1842.45 |
| Residual | 5360.43962 | 28,097 | .190783344 | Prob > F      | = | 0.0000  |
|          |            |        |            | R-squared     | = | 0.1644  |
|          |            |        |            | Adj R-squared | = | 0.1643  |
| Total    | 6414.96462 | 28,100 | .228290556 | Root MSE      | = | .43679  |

| ln_wage     | Coefficient | Std. err. | t      | P> t  | [95% conf. interval] |           |
|-------------|-------------|-----------|--------|-------|----------------------|-----------|
| age         | .0752172    | .0034736  | 21.65  | 0.000 | .0684088             | .0820257  |
| c.age#c.age | -.0010851   | .0000575  | -18.86 | 0.000 | -.0011979            | -.0009724 |
| tenure      | .0390877    | .0007743  | 50.48  | 0.000 | .0375699             | .0406054  |
| _cons       | .3339821    | .0504413  | 6.62   | 0.000 | .2351148             | .4328495  |

The number of observations in our model is 28,101 because Stata drops observations that have a missing value for one or more of the variables in the model. We can be reasonably certain that the standard errors reported above are meaningless. Without a doubt, a woman with higher-than-average wages in one year typically has higher-than-average wages in other years, and so the residuals are not independent. One way to deal with this is to use cluster-robust standard errors. We do this by specifying `vce(cluster id)` or `vce(hc2 id)`, which treat only observations with different person *ids* as truly independent:

```
. regress ln_wage age c.age#c.age tenure, vce(cluster id)
```

Linear regression

|               |   |        |
|---------------|---|--------|
| Number of obs | = | 28,101 |
| F(3, 4698)    | = | 748.82 |
| Prob > F      | = | 0.0000 |
| R-squared     | = | 0.1644 |
| Root MSE      | = | .43679 |

(Std. err. adjusted for 4,699 clusters in *idcode*)

| ln_wage     | Coefficient | Robust std. err. | t      | P> t  | [95% conf. interval] |           |
|-------------|-------------|------------------|--------|-------|----------------------|-----------|
| age         | .0752172    | .0045711         | 16.45  | 0.000 | .0662557             | .0841788  |
| c.age#c.age | -.0010851   | .0000778         | -13.94 | 0.000 | -.0012377            | -.0009325 |
| tenure      | .0390877    | .0014425         | 27.10  | 0.000 | .0362596             | .0419157  |
| _cons       | .3339821    | .0641918         | 5.20   | 0.000 | .208136              | .4598282  |

For comparison, we focus on the tenure coefficient, which in economics jargon can be interpreted as the rate of return for keeping your job. The 95% confidence interval we previously estimated—an interval we do not believe—is [0.038, 0.041]. The robust interval is twice as wide, being [0.036, 0.042]. For this example, `vce(hc2 id)` gives standard errors similar to `vce(cluster id)`.

Another possible way to account for the lack of independence is to fit a random-effects model. Here is the random-effects result:

```
. xtreg ln_wage age c.age#c.age tenure, re
Random-effects GLS regression Number of obs = 28,101
Group variable: idcode Number of groups = 4,699
R-squared: Obs per group:
 Within = 0.1370 min = 1
 Between = 0.2154 avg = 6.0
 Overall = 0.1608 max = 15
 Wald chi2(3) = 4717.05
 Prob > chi2 = 0.0000
```

| ln_wage     | Coefficient                                 | Std. err. | z      | P> z  | [95% conf. interval] |          |
|-------------|---------------------------------------------|-----------|--------|-------|----------------------|----------|
| age         | .0568296                                    | .0026958  | 21.08  | 0.000 | .0515459             | .0621132 |
| c.age#c.age | -.0007566                                   | .0000447  | -16.93 | 0.000 | -.0008441            | -.000669 |
| tenure      | .0260135                                    | .0007477  | 34.79  | 0.000 | .0245481             | .0274789 |
| _cons       | .6136792                                    | .0394611  | 15.55  | 0.000 | .5363368             | .6910216 |
| sigma_u     | .33542449                                   |           |        |       |                      |          |
| sigma_e     | .29674679                                   |           |        |       |                      |          |
| rho         | .56095413 (fraction of variance due to u_i) |           |        |       |                      |          |

Robust regression estimated the 95% interval [0.036,0.042], and xtreg (see [XT] xtreg) estimates [0.025, 0.027]. Which is better? The random-effects regression estimator assumes a lot. We can check some of these assumptions by performing a Hausman test. Using estimates (see [R] estimates store), we store the random-effects estimation results, and then we run the required fixed-effects regression to perform the test.

```
. estimates store random
. xtreg ln_wage age c.age#c.age tenure, fe
Fixed-effects (within) regression Number of obs = 28,101
Group variable: idcode Number of groups = 4,699
R-squared: Obs per group:
 Within = 0.1375 min = 1
 Between = 0.2066 avg = 6.0
 Overall = 0.1568 max = 15
 F(3, 23399) = 1243.00
 Prob > F = 0.0000
```

| ln_wage     | Coefficient                                | Std. err. | t      | P> t  | [95% conf. interval] |           |
|-------------|--------------------------------------------|-----------|--------|-------|----------------------|-----------|
| age         | .0522751                                   | .002783   | 18.78  | 0.000 | .0468202             | .05773    |
| c.age#c.age | -.0006717                                  | .0000461  | -14.56 | 0.000 | -.0007621            | -.0005813 |
| tenure      | .021738                                    | .000799   | 27.21  | 0.000 | .020172              | .023304   |
| _cons       | .687178                                    | .0405944  | 16.93  | 0.000 | .6076103             | .7667456  |
| sigma_u     | .38743138                                  |           |        |       |                      |           |
| sigma_e     | .29674679                                  |           |        |       |                      |           |
| rho         | .6302569 (fraction of variance due to u_i) |           |        |       |                      |           |

F test that all u\_i=0: F(4698, 23399) = 7.98 Prob > F = 0.0000

```
. hausman . random
```

|             | Coefficients |               | (b-B)<br>Difference | sqrt(diag(V_b-V_B))<br>Std. err. |
|-------------|--------------|---------------|---------------------|----------------------------------|
|             | (b)<br>.     | (B)<br>random |                     |                                  |
| age         | .0522751     | .0568296      | -.0045545           | .0006913                         |
| c.age#c.age | -.0006717    | -.0007566     | .0000849            | .0000115                         |
| tenure      | .021738      | .0260135      | -.0042756           | .0002816                         |

b = Consistent under H0 and Ha; obtained from **xtreg**.

B = Inconsistent under Ha, efficient under H0; obtained from **xtreg**.

Test of H0: Difference in coefficients not systematic

$$\text{chi2}(3) = (b-B)' [(V_b-V_B)^{-1}] (b-B)$$

$$= 336.62$$

$$\text{Prob} > \text{chi2} = 0.0000$$

The Hausman test casts grave suspicions on the random-effects model we just fit, so we should be careful in interpreting those results.

Meanwhile, our robust regression results still stand, as long as we are careful about the interpretation. The correct interpretation is that, if the data collection were repeated (on women sampled the same way as in the original sample), and if we were to refit the model, 95% of the time we would expect the estimated coefficient on tenure to be in the range [0.036, 0.042].

Even with robust regression, we must be careful about going beyond that statement. Here the Hausman test is probably picking up something that differs within and between person, which would cast doubt on our robust regression model in terms of interpreting [0.036, 0.042] to contain the rate of return for keeping a job, economywide, for all women, without exception.

Let's take this example a bit further by also recognizing workers with the same education level, grade, may be more alike than those with different education levels. Here we will use multiway clustering (Cameron, Gelbach, and Miller 2008) to account for correlations within individuals over years and within education levels, assuming observations from different people and different education level are independent.

```
. regress ln_wage age c.age#c.age tenure, vce(cluster idcode grade) clustertable
Linear regression Number of obs = 28,099
Clusters per comb.: Cluster comb. = 3
 min = 19 F(3, 18) = 247.23
 avg = 3,138 Prob > F = 0.0000
 max = 4,697 R-squared = 0.1644
 Adj R-squared = 0.1644
 Root MSE = 0.4368
```

| Cluster combination | Clusters per comb. |
|---------------------|--------------------|
| idcode              | 4,697              |
| grade               | 19                 |
| idcode#grade        | 4,697              |

(Std. err. adjusted for multiway clustering)

| ln_wage     | Coefficient | Robust std. err. | t     | P> t  | [95% conf. interval] |           |
|-------------|-------------|------------------|-------|-------|----------------------|-----------|
| age         | .0751665    | .018361          | 4.09  | 0.001 | .0365914             | .1137415  |
| c.age#c.age | -.0010842   | .0002504         | -4.33 | 0.000 | -.0016103            | -.0005581 |
| tenure      | .0391104    | .0018302         | 21.37 | 0.000 | .0352653             | .0429554  |
| _cons       | .334631     | .2769961         | 1.21  | 0.243 | -.2473162            | .9165782  |

Cluster combinations formed by **idcode** and **grade**.

With the `clustertable` option, the output includes a table that describes the cluster combinations and reports the number of levels for each cluster combination. We clustered on two variables, so there are three cluster combinations, and, in general, for  $p$  cluster variables, there are  $p^2 - 1$  cluster combinations. The  $t$ -statistic degrees of freedom is chosen from the cluster combination that has the smallest number of levels. In this case, we have  $19 - 1 = 18$  degrees of freedom, and the Wald test  $F$  statistic, computed from the cluster-robust VCE, has a denominator degrees of freedom of 18. The 95% confidence interval for `tenure` increases slightly to  $[0.035, 0.043]$ . Yet the confidence intervals here are not strictly comparable; this model is fit to two fewer observations because of missing values in the cluster variable `grade`, and we conjecture that observations are correlated within `grade`.



## Weighted regression

regress can perform weighted and unweighted regression. We indicate the weight by specifying the `[weight]` qualifier.

### ▷ Example 8: Using means as regression variables

We have census data recording the deathrate (`drate`) and median age (`medage`) for each state. The data also record the region of the country in which each state is located and the overall population of the state:

```
. use https://www.stata-press.com/data/r19/census9
(1980 Census data by state)
. describe
Contains data from https://www.stata-press.com/data/r19/census9.dta
Observations: 50 1980 Census data by state
Variables: 6 2 Dec 2024 15:22
```

| Variable name | Storage type | Display format | Value label | Variable label                |
|---------------|--------------|----------------|-------------|-------------------------------|
| state         | str13        | %-13s          |             | State                         |
| state2        | str2         | %-2s           |             | Two-letter state abbreviation |
| drate         | int          | %9.0g          |             | Deathrate                     |
| pop           | long         | %12.0gc        |             | Population                    |
| medage        | float        | %9.2f          |             | Median age                    |
| region        | byte         | %-8.0g         | cenreg      | Census region                 |

Sorted by:

We can use factor variables to include dummy variables for region. Because the variables in the regression reflect means rather than individual observations, the appropriate method of estimation is analytically weighted least squares (Davidson and MacKinnon 2004, 261–262), where the weight is total population:

```
. regress drate medage i.region [aweight=pop]
(sum of wgt is 225,907,472)
```

| Source   | SS         | df | MS         | Number of obs | = | 50     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 4096.6093  | 4  | 1024.15232 | F(4, 45)      | = | 37.21  |
| Residual | 1238.40987 | 45 | 27.5202192 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.7679 |
|          |            |    |            | Adj R-squared | = | 0.7472 |
| Total    | 5335.01916 | 49 | 108.877942 | Root MSE      | = | 5.246  |

| drate   | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|---------|-------------|-----------|-------|-------|----------------------|-----------|
| medage  | 4.283183    | .5393329  | 7.94  | 0.000 | 3.196911             | 5.369455  |
| region  |             |           |       |       |                      |           |
| N Cntrl | .3138738    | 2.456431  | 0.13  | 0.899 | -4.633632            | 5.26138   |
| South   | -1.438452   | 2.320244  | -0.62 | 0.538 | -6.111663            | 3.234758  |
| West    | -10.90629   | 2.681349  | -4.07 | 0.000 | -16.30681            | -5.505777 |
| _cons   | -39.14727   | 17.23613  | -2.27 | 0.028 | -73.86262            | -4.431915 |

To weight the regression by population, we added the qualifier `[aweight=pop]` to the end of the `regress` command. Stata informed us that the sum of the weight is  $2.2591 \times 10^8$ ; there were approximately 226 million people residing in the United States according to our 1980 data.

In the weighted regression, we see that the coefficient on West is statistically significant but that the coefficients on N Cntrl and South are not. We use `testparm` to test the joint significance of the `region` variable. Because we fit a weighted regression, `testparm` uses the appropriately weighted variance–covariance matrix.

```
. testparm i.region
(1) 2.region = 0
(2) 3.region = 0
(3) 4.region = 0
 F(3, 45) = 9.84
 Prob > F = 0.0000
```

The results indicate that the region variables are jointly significant. Note that we could have performed this same test by typing `contrast region`. You may prefer to use the `contrast` command because, in addition to the joint test, you can perform other tests such as comparisons of each region’s mean to the grand mean; see [R] [contrast](#) for more information. ◀

`regress` also accepts frequency weights (`fweights`). Frequency weights are appropriate when the data do not reflect cell means but instead represent replicated observations. Specifying `aweight` or `fweight` will not change the parameter estimates, but it will change the corresponding significance levels.

For instance, if we specified [`fweight=pop`] in the weighted regression [example](#) above—which would be statistically incorrect—Stata would treat the data as if the data represented 226 million independent observations on death rates and median age. The data most certainly do not represent that—they represent 50 observations on state averages.

With `aweight`, Stata treats the number of observations on the process as the number of observations in the data. When we specify `fweights`, Stata treats the number of observations as if it were equal to the sum of the weights; see [Methods and formulas](#) below.

## □ Technical note

A frequent inquiry sent to StataCorp Technical Services is to describe the effect of specifying [`aweight=exp`] with `regress` in terms of transformation of the dependent and independent variables. The mechanical answer is that typing

```
. regress y x1 x2 [aweight=n]
```

is equivalent to fitting the model

$$y_j \sqrt{n_j} = \beta_0 \sqrt{n_j} + \beta_1 x_{1j} \sqrt{n_j} + \beta_2 x_{2j} \sqrt{n_j} + u_j \sqrt{n_j}$$

This regression will reproduce the coefficients and covariance matrix produced by the `aweight`d regression. The mean squared errors (estimates of the variance of the residuals) will, however, be different. The transformed regression reports  $s_t^2$ , an estimate of  $\text{Var}(u_j \sqrt{n_j})$ . The `aweight`d regression reports  $s_a^2$ , an estimate of  $\text{Var}(u_j \sqrt{n_j} \sqrt{N / \sum_k n_k})$ , where  $N$  is the number of observations. Thus,

$$s_a^2 = \frac{N}{\sum_k n_k} s_t^2 = \frac{s_t^2}{\bar{n}} \quad (1)$$

The logic for this adjustment is as follows: Consider the model

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u$$

Assume that, were this model fit on individuals,  $\text{Var}(u) = \sigma_u^2$ , a constant. Assume that individual data are not available; what is available are averages  $(\bar{y}_j, \bar{x}_{1j}, \bar{x}_{2j})$  for  $j = 1, \dots, N$ , and each average is calculated over  $n_j$  observations. Then it is still true that

$$\bar{y}_j = \beta_0 + \beta_1 \bar{x}_{1j} + \beta_2 \bar{x}_{2j} + \bar{u}_j$$

where  $\bar{u}_j$  is the average of  $n_j$  mean 0, variance  $\sigma_u^2$  deviates and has variance  $\sigma_u^2/n_j$ . Thus, multiplying through by  $\sqrt{n_j}$  produces

$$\bar{y}_j \sqrt{n_j} = \beta_0 \sqrt{n_j} + \beta_1 \bar{x}_{1j} \sqrt{n_j} + \beta_2 \bar{x}_{2j} \sqrt{n_j} + \bar{u}_j \sqrt{n_j}$$

and  $\text{Var}(\bar{u}_j \sqrt{n_j}) = \sigma_u^2$ . The mean squared error,  $s_t^2$ , reported by fitting this transformed regression is an estimate of  $\sigma_u^2$ . The coefficients and covariance matrix could also be obtained by `aweighted regress`. The only difference would be in the reported mean squared error, which from (1) is  $\sigma_u^2/\bar{n}$ . On average, each observation in the data reflects the averages calculated over  $\bar{n} = \sum_k n_k/N$  individuals, and thus this reported mean squared error is the average variance of an observation in the dataset. We can retrieve the estimate of  $\sigma_u^2$  by multiplying the reported mean squared error by  $\bar{n}$ .

More generally, `aweights` are used to solve general heteroskedasticity problems. In these cases, we have the model

$$y_j = \beta_0 + \beta_1 x_{1j} + \beta_2 x_{2j} + u_j$$

and the variance of  $u_j$  is thought to be proportional to  $a_j$ . If the variance is proportional to  $a_j$ , it is also proportional to  $\alpha a_j$ , where  $\alpha$  is any positive constant. Not quite arbitrarily, but with no loss of generality, we could choose  $\alpha = \sum_k (1/a_k)/N$ , the average value of the inverse of  $a_j$ . We can then write  $\text{Var}(u_j) = k\alpha a_j \sigma^2$ , where  $k$  is the constant of proportionality that is no longer a function of the scale of the weights.

Dividing this regression through by the  $\sqrt{a_j}$ ,

$$y_j/\sqrt{a_j} = \beta_0/\sqrt{a_j} + \beta_1 x_{1j}/\sqrt{a_j} + \beta_2 x_{2j}/\sqrt{a_j} + u_j/\sqrt{a_j}$$

produces a model with  $\text{Var}(u_j/\sqrt{a_j}) = k\alpha\sigma^2$ , which is the constant part of  $\text{Var}(u_j)$ . This variance is a function of  $\alpha$ , the average of the reciprocal weights; if the weights are scaled arbitrarily, then so is this variance.

We can also fit this model by typing

```
. regress y x1 x2 [aweight=1/a]
```

This input will produce the same estimates of the coefficients and covariance matrix; the reported mean squared error is, from (1),  $\{N/\sum_k (1/a_k)\}k\alpha\sigma^2 = k\sigma^2$ . This variance is independent of the scale of  $a_j$ .

□

## Video examples

[Simple linear regression in Stata](#)

[Fitting and interpreting regression models: Linear regression with categorical predictors](#)

[Fitting and interpreting regression models: Linear regression with continuous predictors](#)

[Fitting and interpreting regression models: Linear regression with continuous and categorical predictors](#)

## Stored results

`regress` stores the following in `e()`:

### Scalars

|                         |                                                                    |
|-------------------------|--------------------------------------------------------------------|
| <code>e(N)</code>       | number of observations                                             |
| <code>e(mss)</code>     | model sum of squares                                               |
| <code>e(df_m)</code>    | model degrees of freedom                                           |
| <code>e(rss)</code>     | residual sum of squares                                            |
| <code>e(df_r)</code>    | residual degrees of freedom                                        |
| <code>e(r2)</code>      | $R^2$                                                              |
| <code>e(r2_a)</code>    | adjusted $R^2$                                                     |
| <code>e(F)</code>       | $F$ statistic                                                      |
| <code>e(rmse)</code>    | root mean squared error                                            |
| <code>e(ll)</code>      | log likelihood under additional assumption of i.i.d. normal errors |
| <code>e(ll_0)</code>    | log likelihood, constant-only model                                |
| <code>e(sum_w)</code>   | sum of weights                                                     |
| <code>e(N_clust)</code> | number of clusters                                                 |
| <code>e(rank)</code>    | rank of <code>e(V)</code>                                          |

### Macros

|                            |                                                                            |
|----------------------------|----------------------------------------------------------------------------|
| <code>e(cmd)</code>        | <code>regress</code>                                                       |
| <code>e(cmdline)</code>    | command as typed                                                           |
| <code>e(depvar)</code>     | name of dependent variable                                                 |
| <code>e(wtype)</code>      | weight type                                                                |
| <code>e(wexp)</code>       | weight expression                                                          |
| <code>e(model)</code>      | ols                                                                        |
| <code>e(title)</code>      | title in estimation output when <code>vce()</code> is not <code>ols</code> |
| <code>e(clustvar)</code>   | names of cluster variables                                                 |
| <code>e(cluster#)</code>   | cluster combination #                                                      |
| <code>e(vce)</code>        | <i>vce</i> type specified in <code>vce()</code>                            |
| <code>e(vctype)</code>     | title used to label Std. err.                                              |
| <code>e(properties)</code> | <code>b V</code>                                                           |
| <code>e(estat_cmd)</code>  | program used to implement <code>estat</code>                               |
| <code>e(predict)</code>    | program used to implement <code>predict</code>                             |
| <code>e(marginsok)</code>  | predictions allowed by <code>margins</code>                                |
| <code>e(asbalanced)</code> | factor variables <code>fvset</code> as <code>asbalanced</code>             |
| <code>e(asobserved)</code> | factor variables <code>fvset</code> as <code>asobserved</code>             |

### Matrices

|                              |                                                                               |
|------------------------------|-------------------------------------------------------------------------------|
| <code>e(b)</code>            | coefficient vector                                                            |
| <code>e(V)</code>            | variance–covariance matrix of the estimators                                  |
| <code>e(beta)</code>         | standardized coefficients                                                     |
| <code>e(V_modelbased)</code> | model-based variance                                                          |
| <code>e(adj_df)</code>       | adjusted degrees of freedom when <code>vce(hc2, dfadjust)</code> is specified |
| <code>e(kcluster)</code>     | cluster sizes, multiway clustering                                            |

### Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

### Matrices

|                       |                                                                                                                       |
|-----------------------|-----------------------------------------------------------------------------------------------------------------------|
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, $p$ -values, and confidence intervals |
|-----------------------|-----------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

Methods and formulas are presented under the following headings:

*Coefficient estimation and ANOVA table*  
*Weighted regression*  
*A general notation for the robust variance calculation*  
*Robust calculation for regress*

### Coefficient estimation and ANOVA table

Variables printed in lowercase and not boldfaced (for example,  $x$ ) are scalars. Variables printed in lowercase and boldfaced (for example,  $\mathbf{x}$ ) are column vectors. Variables printed in uppercase and boldfaced (for example,  $\mathbf{X}$ ) are matrices.

Let  $\mathbf{X}$  denote the matrix of observations on the right-hand-side variables,  $\mathbf{y}$  the vector of observations on the left-hand-side variables. Define  $\mathbf{A}$  as  $\mathbf{X}'\mathbf{X}$  and  $\mathbf{a}$  as  $\mathbf{X}'\mathbf{y}$ . The coefficient vector  $\mathbf{b}$  is defined as  $\mathbf{A}^{-1}\mathbf{a}$ . Although not shown in the notation, unless `hascons` is specified,  $\mathbf{A}$  and  $\mathbf{a}$  are accumulated in deviation form and the constant is calculated separately. This comment applies to all statistics listed below.

The total sum of squares, TSS, equals  $\mathbf{y}'\mathbf{y}$  if there is no intercept and  $\mathbf{y}'\mathbf{y} - \{(\mathbf{1}'\mathbf{y})^2/n\}$  otherwise. The degrees of freedom is  $n - c$ , where  $n$  is the number of observations and  $c = 1$  if there is a constant in the regression and 0 otherwise.

The residual sum of squares, RSS, is defined as  $(\mathbf{y} - \mathbf{X}\mathbf{b})'(\mathbf{y} - \mathbf{X}\mathbf{b})$ . The degrees of freedom is  $n - k$ , where  $n$  is the number of observations and  $k$  is the number of right-hand-side variables (including the constant).

The model sum of squares, MSS, equals  $\text{TSS} - \text{RSS}$ . The degrees of freedom is  $k - c$ .

The mean squared error,  $s^2$ , is defined as  $\text{RSS}/(n - k)$ . The root mean squared error is  $s$ , its square root.

The  $F$  statistic with  $k - c$  and  $n - k$  degrees of freedom is defined as

$$F = \frac{\text{MSS}}{(k - c)s^2}$$

The  $R^2$  is defined as  $R^2 = 1 - \text{RSS}/\text{TSS}$ .

The adjusted  $R^2$  is defined as  $R_a^2 = 1 - (1 - R^2)(n - c)/(n - k)$ .

The conventional estimate of variance is  $s^2\mathbf{A}^{-1}$ . The calculation of variance estimates when robust variance estimates are specified is described below.

### Weighted regression

Let  $\mathbf{v}$  be a column vector of weights specified by the user. Let  $\mathbf{w}$  be a column vector of normalized weights,  $\mathbf{w} = \{\mathbf{v}/(\mathbf{1}'\mathbf{v})\}(\mathbf{1}'\mathbf{1})$ . For `fweights`,  $\mathbf{w} = \mathbf{v}$ . For historical reasons, `iweights` are treated like `fweights` when robust standard errors are not specified. Instead, when `vce(robust)`, `vce(cluster clustvarlist)`, `vce(hc2)`, or `vce(hc3)` is specified, `iweights` are treated like `aweights`.

If the user specifies weights, the number of observations,  $n$ , in the above formulas is defined as  $\mathbf{1}'\mathbf{w}$ . For `iweights`, this is truncated to an integer. The sum of the weights is  $\mathbf{1}'\mathbf{v}$ .  $\mathbf{X}'\mathbf{X}$ ,  $\mathbf{X}'\mathbf{y}$ , and  $\mathbf{y}'\mathbf{y}$  are replaced in the above formulas by  $\mathbf{X}'\mathbf{D}\mathbf{X}$ ,  $\mathbf{X}'\mathbf{D}\mathbf{y}$ , and  $\mathbf{y}'\mathbf{D}\mathbf{y}$ , respectively, where  $\mathbf{D}$  is a diagonal matrix whose diagonal elements are the elements of  $\mathbf{w}$ .

## A general notation for the robust variance calculation

Put aside all context of linear regression and the notation that goes with it—we will return to it. First, we are going to establish a notation for describing robust variance calculations.

The calculation formula for the robust variance calculation is

$$\widehat{\mathbf{v}} = q_c \widehat{\mathbf{V}} \left( \sum_{k=1}^M \mathbf{u}_k^{(G)'} \mathbf{u}_k^{(G)} \right) \widehat{\mathbf{V}}$$

where

$$\mathbf{u}_k^{(G)} = \sum_{j \in G_k} w_j \mathbf{u}_j$$

$G_1, G_2, \dots, G_M$  are the clusters specified by `vce(cluster clustvarlist)` when `clustvarlist` contains only one variable, and  $w_j$  are the user-specified weights, normalized if `aweights` or `pweights` are specified and equal to 1 if no weights are specified.

For `fweights` without clusters, the variance formula is

$$\widehat{\mathbf{v}} = q_c \widehat{\mathbf{V}} \left( \sum_{j=1}^N w_j \mathbf{u}_j' \mathbf{u}_j \right) \widehat{\mathbf{V}}$$

which is the same as expanding the dataset and making the calculation on the unweighted data.

If `vce(cluster clustvarlist)` is not specified,  $M = N$ , and each cluster contains 1 observation. The inputs into this calculation are

- $\widehat{\mathbf{V}}$ , which is typically a conventionally calculated variance matrix;
- $\mathbf{u}_j$ ,  $j = 1, \dots, N$ , a row vector of scores; and
- $q_c$ , a constant finite-sample adjustment.

Thus, we can now describe how estimators apply the robust calculation formula by defining  $\widehat{\mathbf{V}}$ ,  $\mathbf{u}_j$ , and  $q_c$ .

Two definitions are popular enough for  $q_c$  to deserve a name. The regression-like formula for  $q_c$  (Fuller et al. 1986) is

$$q_c = \frac{N-1}{N-k} \frac{M}{M-1}$$

where  $M$  is the number of clusters and  $N$  is the number of observations. For weights,  $N$  refers to the sum of the weights if weights are frequency weights and the number of observations in the dataset (ignoring weights) in all other cases. Also note that, weighted or not,  $M = N$  when `vce(cluster clustvarlist)` is not specified, and then  $q_c = N/(N-k)$ .

The asymptotic-like formula for  $q_c$  is

$$q_c = \frac{M}{M-1}$$

where  $M = N$  if `vce(cluster clustvarlist)` is not specified.

See [U] 20.22 **Obtaining robust variance estimates** and [P] **\_robust** for a discussion of the robust variance estimator and a development of these formulas.

## Robust calculation for regress

For `regress`,  $\widehat{\mathbf{V}} = \mathbf{A}^{-1}$ . The other terms are `vce(robust)`, but not `vce(hc2)` or `vce(hc3)`,

$$\mathbf{u}_j = (y_j - \mathbf{x}_j \mathbf{b}) \mathbf{x}_j$$

and  $q_c$  is given by its regression-like definition. `vce(hc2)`,

$$\mathbf{u}_j = \frac{1}{\sqrt{1 - h_{jj}}} (y_j - \mathbf{x}_j \mathbf{b}) \mathbf{x}_j$$

where  $q_c = 1$  and  $h_{jj} = \mathbf{x}_j (\mathbf{X}' \mathbf{X})^{-1} \mathbf{x}_j'$ . `vce(hc3)`,

$$\mathbf{u}_j = \frac{1}{1 - h_{jj}} (y_j - \mathbf{x}_j \mathbf{b}) \mathbf{x}_j$$

where  $q_c = 1$  and  $h_{jj} = \mathbf{x}_j (\mathbf{X}' \mathbf{X})^{-1} \mathbf{x}_j'$ . `vce(hc2 clustvar)`,

$$\mathbf{u}_j = (\mathbf{y}_j - \mathbf{X}_j \mathbf{b})' (\mathbf{I}_{G_j} - \mathbf{H}_{jj})^{-\frac{1}{2}} \mathbf{X}_j$$

where  $q_c = 1$ ,  $\mathbf{H}_{jj} = \mathbf{X}_j (\mathbf{X}' \mathbf{X})^{-1} \mathbf{X}_j'$  for cluster  $j$  of size  $G_j$ ,  $G_j \times k$  data matrix  $\mathbf{X}_j$ , and  $G_j \times 1$  vector  $\mathbf{y}_j$ .  $(\mathbf{I}_{G_j} - \mathbf{H}_{jj})^{-\frac{1}{2}}$  is the inverse of the symmetric square root of  $(\mathbf{I}_{G_j} - \mathbf{H}_{jj})$  (Bell and McCaffrey 2002).

`vce(hc2 [clustvar], dfadjust)` directs `regress` to compute the adjusted degrees of freedom described by Imbens and Kolesár (2016). Define the  $N \times M$  matrix  $\mathbf{G}$  such that the  $j$ th column is

$$(\mathbf{I}_N - \mathbf{H})_j' (\mathbf{I}_{G_j} - \mathbf{H}_{jj})^{-\frac{1}{2}} \mathbf{X}_j (\mathbf{X}' \mathbf{X})^{-1} \mathbf{e}_l$$

where  $j = 1, \dots, M$ ,  $\mathbf{e}_l$  is a unary vector for the  $l$ th regressor,  $l = 1, \dots, k$ , and  $(\mathbf{I}_N - \mathbf{H})_j$  is the  $G_j \times N$  subset of the  $N \times N$  matrix  $(\mathbf{I}_N - \mathbf{H})$  for cluster  $j$ . The Bell and McCaffrey (2002) adjusted degrees of freedom for the  $l$ th regressor is

$$\begin{aligned} K_l &= \frac{\text{tr}(\mathbf{G}' \mathbf{G})^2}{\text{tr}((\mathbf{G}' \mathbf{G})^2)} \\ &= \frac{(\sum_{i=1}^M \lambda_i)^2}{\sum_{i=1}^M \lambda_i^2} \end{aligned}$$

where  $\text{tr}(\cdot)$  is the trace function and  $\lambda_i$  are the eigenvalues of  $\mathbf{G}' \mathbf{G}$ .

When  $M$  is large, or when there are no clusters and  $M = N$ , computing the eigenvalues can be time consuming. We define

$$\begin{aligned} \mathbf{a}_j &= (\mathbf{I}_{G_j} - \mathbf{H}_{jj})^{-\frac{1}{2}} \mathbf{X}_j (\mathbf{X}'\mathbf{X})^{-1} \mathbf{e}_{L,k} \\ \mathbf{b}_j &= \mathbf{H}_j \mathbf{a}_j \\ \mathbf{A} &= (\mathbf{a}'_1 \mathbf{a}_1, \dots, \mathbf{a}'_M \mathbf{a}_M)' = (A_1, A_2, \dots, A_M)' \\ \mathbf{B} &= (\mathbf{b}_1, \dots, \mathbf{b}_M) = \begin{pmatrix} B_{1,1} & B_{1,2} & \cdots & B_{1,M} \\ \vdots & \vdots & \ddots & \vdots \\ B_{N,1} & B_{N,2} & \cdots & B_{N,M} \end{pmatrix} \end{aligned}$$

for  $j = 1, \dots, M$  and  $\mathbf{H}_j = \mathbf{X} (\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'_j$ . Then  $\mathbf{G}'\mathbf{G} = \text{diag}(\mathbf{A}) - \mathbf{B}'\mathbf{B}$  (Kolesár 2023).

We now express the adjusted degrees of freedom as

$$K_l = \frac{\left( \sum_{j=1}^M A_j - \sum_{i=1}^N \sum_{j=1}^M B_{ij}^2 \right)^2}{\sum_{j=1}^M A_j^2 - 2 \sum_{j=1}^M A_j \sum_{i=1}^N B_{ij}^2 + \sum_{j_1=1}^M \sum_{j_2=1}^M (\mathbf{b}'_{j_1} \mathbf{b}_{j_2})^2}$$

which can be computed efficiently in Mata and using QR decomposition. For example, by decomposing  $\mathbf{X} = \mathbf{QR}$ , where  $\mathbf{Q}$  is  $n \times k$  and orthonormal and  $\mathbf{R}$  is  $k \times k$  and upper triangular, we can rewrite the matrix  $\mathbf{B}$  so that it has dimension  $k \times M$  instead of  $N \times M$ .

When weights are specified, we use the weighted covariate matrix  $\tilde{\mathbf{X}} = \text{diag}(\mathbf{w})^{\frac{1}{2}} \mathbf{X}$  and its corresponding projection matrix  $\tilde{\mathbf{H}}$ , as well as the cluster covariance matrices  $\tilde{\mathbf{X}}_j$ , their projection matrices  $\tilde{\mathbf{H}}_{jj}$ , and weighted residuals  $\tilde{\epsilon}_j = \text{diag}(\mathbf{w}_j)^{\frac{1}{2}} \hat{\epsilon}_j$ . When frequency weights are specified without clusters, we substitute  $1/\sqrt{1-h_{jj}}$  with  $1/\sqrt{w_j - \tilde{h}_{jj}}$ , where  $j = 1, \dots, N$ . Also, when there are no clusters, the weights are included in the degrees-of-freedom algebra

$$K_l = \frac{\left( \sum_{j=1}^N w_j A_j - \sum_{i=1}^k \sum_{j=1}^N B_{ij}^2 \right)^2}{\sum_{j=1}^N w_j A_j^2 - 2 \sum_{j=1}^N A_j \sum_{i=1}^k B_{ij}^2 + \sum_{j_1=1}^N \sum_{j_2=1}^N (\mathbf{b}'_{j_1} \mathbf{b}_{j_2})^2}$$

Here we substituted  $k$  for  $N$  in the row dimension of  $\mathbf{B}$  as it is when using QR decomposition to perform the computations.

With weights, the `vce(hc2 clustvar)` computation discussed above is modified to

$$\begin{aligned} \mathbf{u}_j &= \left( \text{diag}(\mathbf{w}_j)^{\frac{1}{2}} (\mathbf{y}_j - \mathbf{X}_j \mathbf{b}) \right)' (\mathbf{I}_{G_j} - \tilde{\mathbf{H}}_{jj})^{-\frac{1}{2}} \tilde{\mathbf{X}}_j \\ &= (\tilde{\mathbf{y}}_j - \tilde{\mathbf{X}}_j \mathbf{b})' (\mathbf{I}_{G_j} - \tilde{\mathbf{H}}_{jj})^{-\frac{1}{2}} \tilde{\mathbf{X}}_j \end{aligned}$$

## Multiway clustering

When you type `vce(cluster clustvarlist)` with more than one variable, the variance–covariance estimator uses multiway cluster–robust variance estimation. This is carried out by estimating the robust VCE for all combinations of the specified cluster variables and summing. For  $p$  cluster variables, there will be  $P = 2^p - 1$  cluster variable combinations. Let  $\mathbf{V}_i$  be the  $i$ th robust VCE,  $i = 1, \dots, P$ . Define  $S_j$ ,  $j = 1, \dots, p$ , as the set of indices  $i$  involving  $j$  cluster variables. The size, or cardinality, of  $S_j$  is  $|S_j| = \binom{p}{j}$  and  $\sum_{j=1}^p \binom{p}{j} = 2^p - 1$ . For example, for  $p = 4$ ,  $|S_1| = \binom{4}{1} = 4$ ,  $|S_2| = \binom{4}{2} = 6$ ,  $|S_3| = \binom{4}{3} = 4$ , and  $|S_4| = \binom{4}{4} = 1$ . The multiway cluster–robust VCE is then

$$\mathbf{V}_* = \sum_{j=1}^p (-1)^{j-1} \sum_{i \in S_j} \mathbf{V}_i$$

You are more likely to cluster on two or, maybe, three variables. In the case of two cluster variables, the computation would be

$$\mathbf{V}_* = V_1 + V_2 - V_{12}$$

where  $V_1$  corresponds to the variance–covariance computation clustering at the level of the first cluster,  $V_2$  corresponds to the second level, and  $V_{12}$  corresponds to the variance–covariance computation for the group formed by the intersection of both clustering levels.

An eigendecomposition on  $\mathbf{V}_*$  ensures the matrix to be positive semidefinite. Let the columns of matrix  $\mathbf{U}$  contain the eigenvectors of  $\mathbf{V}_*$  and the vector contain  $\mathbf{u}$ , its eigenvalues. If  $\mathbf{V}_*$  is not positive definite, some of the elements of  $\mathbf{u}$  will be less than 0. Let  $\mathbf{u}_+$  contain all the nonnegative elements of  $\mathbf{u}$  and zeros where  $u_i < 0$ . The matrix  $\mathbf{V}_+ = \mathbf{U} \cdot \text{diag}(\mathbf{u}_+) \cdot \mathbf{U}'$  will then be positive semidefinite.

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The history of regression is long and complicated: the books by [Stigler \(1986\)](#) and [Hald \(1998\)](#) are devoted largely to the story. Legendre published first on least squares in 1805. Gauss published later in 1809, but he had the idea earlier. Gauss, and especially Laplace, tied least squares to a normal errors assumption. The idea of the normal distribution can itself be traced back to De Moivre in 1733. Laplace discussed a variety of other estimation methods and error assumptions over his long career, while linear models long predate either innovation. Most of this work was linked to problems in astronomy and geodesy.

A second wave of ideas started when Galton used graphical and descriptive methods on data bearing on heredity to develop what he called regression. His term reflects the common phenomenon that characteristics of offspring are positively correlated with those of parents but with regression slope such that offspring “regress toward the mean”. Galton’s work was rather intuitive: contributions from Pearson, Edgeworth, Yule, and others introduced more formal machinery, developed related ideas on correlation, and extended application into the biological and social sciences. So most of the elements of regression as we know it were in place by 1900.

Pierre-Simon Laplace (1749–1827) was born in Normandy and was early recognized as a remarkable mathematician. He weathered a changing political climate well enough to rise to Minister of the Interior under Napoleon in 1799 (although only for 6 weeks) and to be made a Marquis by Louis XVIII in 1817. He made many contributions to mathematics and physics, his two main interests being theoretical astronomy and probability theory (including statistics). Laplace transforms are named for him.

[Adrien-Marie Legendre \(1752–1833\)](#) was born in Paris (or possibly in Toulouse) and educated in mathematics and physics. He worked in number theory, geometry, differential equations, calculus, function theory, applied mathematics, and geodesy. The Legendre polynomials are named for him. His main contribution to statistics is as one of the discoverers of least squares. He died in poverty, having refused to bow to political pressures.

[Johann Carl Friedrich Gauss \(1777–1855\)](#) was born in Braunschweig (Brunswick), now in Germany. He studied there and at Göttingen. His doctoral dissertation at the University of Helmstedt was a discussion of the fundamental theorem of algebra. He made many fundamental contributions to geometry, number theory, algebra, real analysis, differential equations, numerical analysis, statistics, astronomy, optics, geodesy, mechanics, and magnetism. An outstanding genius, Gauss worked mostly in isolation in Göttingen.

[Francis Galton \(1822–1911\)](#) was born in Birmingham, England, into a well-to-do family with many connections: he and Charles Darwin were first cousins. After an unsuccessful foray into medicine, he became independently wealthy at the death of his father. Galton traveled widely in Europe, the Middle East, and Africa, and became celebrated as an explorer and geographer. His pioneering work on weather maps helped in the identification of anticyclones, which he named. From about 1865, most of his work was centered on quantitative problems in biology, anthropology, and psychology. In a sense, Galton (re)invented regression, and he certainly named it. Galton also promoted the normal distribution, correlation approaches, and the use of median and selected quantiles as descriptive statistics. He was knighted in 1909.

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## Also see

- [R] [regress postestimation](#) — Postestimation tools for regress
- [R] [regress postestimation diagnostic plots](#) — Postestimation plots for regress
- [R] [regress postestimation time series](#) — Postestimation tools for regress with time series
- [R] [anova](#) — Analysis of variance and covariance
- [R] [contrast](#) — Contrasts and linear hypothesis tests after estimation
- [R] [hetregress](#) — Heteroskedastic linear regression
- [R] [wildbootstrap](#) — Wild cluster bootstrap inference
- [BAYES] [bayes: regress](#) — Bayesian linear regression
- [BMA] [bmaregress](#) — Bayesian model averaging for linear regression
- [CAUSAL] [Causal inference commands](#) — Introduction to causal inference commands
- [FMM] [fmm: regress](#) — Finite mixtures of linear regression models
- [LASSO] [Lasso intro](#) — Introduction to lasso
- [META] [meta regress](#) — Meta-analysis regression
- [MI] [Estimation](#) — Estimation commands for use with mi estimate
- [SEM] [Example 6](#) — Linear regression
- [SEM] [Intro 5](#) — Tour of models
- [SP] [spregress](#) — Spatial autoregressive models
- [SVY] [svy estimation](#) — Estimation commands for survey data
- [TS] [forecast](#) — Econometric model forecasting
- [TS] [mswitch](#) — Markov-switching regression models
- [U] [20 Estimation and postestimation commands](#)

|                                             |                                                    |                                            |
|---------------------------------------------|----------------------------------------------------|--------------------------------------------|
| <a href="#">Postestimation commands</a>     | <a href="#">Predictions</a>                        | <a href="#">margins</a>                    |
| <a href="#">DFBETA influence statistics</a> | <a href="#">Tests for violation of assumptions</a> | <a href="#">Variance inflation factors</a> |
| <a href="#">Measures of effect size</a>     | <a href="#">Methods and formulas</a>               | <a href="#">Acknowledgments</a>            |
| <a href="#">References</a>                  | <a href="#">Also see</a>                           |                                            |

## Postestimation commands

The following postestimation commands are of special interest after `regress`:

| Command                     | Description                                                      |
|-----------------------------|------------------------------------------------------------------|
| <code>dfbeta</code>         | DFBETA influence statistics                                      |
| <code>estat hettest</code>  | tests for heteroskedasticity                                     |
| <code>estat imtest</code>   | information matrix test                                          |
| <code>estat ovtest</code>   | Ramsey regression specification-error test for omitted variables |
| <code>estat szroeter</code> | Szroeter's rank test for heteroskedasticity                      |
| <code>estat vif</code>      | variance inflation factors for the independent variables         |
| <code>estat esize</code>    | $\eta^2$ , $\varepsilon^2$ , and $\omega^2$ effect sizes         |
| <code>estat moran</code>    | Moran's test of residual correlation with nearby residuals       |
| <code>lassogof</code>       | calculate goodness-of-fit predictions                            |

These commands are not appropriate with `svy` estimation results.

The following standard postestimation commands are also available:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| <code>linktest</code>        | link test for model specification                                                                                                       |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | predictions and their SEs, leverage statistics, distance statistics, etc.                                                               |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results. `forecast` is also not appropriate with `mi` estimation results.

## Predictions

### Description for predict

`predict` creates a new variable containing predictions such as linear predictions, residuals, standardized residuals, Studentized residuals, Cook's distance, leverage, probabilities, expected values, DFBETAS for *varname*, standard errors, COVRATIOS, DFITS, and Welsch distances.

### Menu for predict

Statistics > Postestimation

### Syntax for predict

```
predict [type] newvar [if] [in] [, statistic]
```

| <i>statistic</i>               | Description                                 |
|--------------------------------|---------------------------------------------|
| Main                           |                                             |
| <code>xb</code>                | linear prediction; the default              |
| <code>residuals</code>         | residuals                                   |
| <code>score</code>             | score; equivalent to <code>residuals</code> |
| <code>rstandard</code>         | standardized residuals                      |
| <code>rstudent</code>          | Studentized (jackknifed) residuals          |
| <code>cooksd</code>            | Cook's distance                             |
| <code>leverage   hat</code>    | leverage (diagonal elements of hat matrix)  |
| <code>pr(a,b)</code>           | $\Pr(y_j   a < y_j < b)$                    |
| <code>e(a,b)</code>            | $E(y_j   a < y_j < b)$                      |
| <code>ystar(a,b)</code>        | $E(y_j^*), y_j^* = \max\{a, \min(y_j, b)\}$ |
| * <code>dfbeta(varname)</code> | DFBETA for <i>varname</i>                   |
| <code>stdp</code>              | standard error of the linear prediction     |
| <code>stdf</code>              | standard error of the forecast              |
| <code>stdr</code>              | standard error of the residual              |
| * <code>covratio</code>        | COVRATIO                                    |
| * <code>dfits</code>           | DFITS                                       |
| * <code>welsch</code>          | Welsch distance                             |

Unstarred statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample. Starred statistics are calculated only for the estimation sample, even when `if e(sample)` is not specified.

`rstandard`, `rstudent`, `cooksd`, `leverage`, `dfbeta()`, `stdf`, `stdr`, `covratio`, `dfits`, and `welsch` are not available if any `vce()` other than `vce(ols)` was specified with `regress`.

`xb`, `residuals`, `score`, and `stdp` are the only options allowed with `svy` estimation results.

where *a* and *b* may be numbers or variables; *a* missing ( $a \geq .$ ) means  $-\infty$ , and *b* missing ( $b \geq .$ ) means  $+\infty$ ; see [U] 12.2.1 Missing values.

## Options for predict

Main

`xb`, the default, calculates the linear prediction.

`residuals` calculates the residuals.

`score` is equivalent to `residuals` in linear regression.

`rstandard` calculates the standardized residuals.

`rstudent` calculates the Studentized (jackknifed) residuals.

`cooksdiagnostics` calculates the Cook's  $D$  influence statistic (Cook 1977).

`leverage` or `hat` calculates the diagonal elements of the projection (“hat”) matrix.

`pr(a, b)` calculates  $\Pr(a < \mathbf{x}_j\mathbf{b} + u_j < b)$ , the probability that  $y_j|\mathbf{x}_j$  would be observed in the interval  $(a, b)$ .

$a$  and  $b$  may be specified as numbers or variable names;  $lb$  and  $ub$  are variable names;

`pr(20, 30)` calculates  $\Pr(20 < \mathbf{x}_j\mathbf{b} + u_j < 30)$ ;

`pr(lb, ub)` calculates  $\Pr(lb < \mathbf{x}_j\mathbf{b} + u_j < ub)$ ; and

`pr(20, ub)` calculates  $\Pr(20 < \mathbf{x}_j\mathbf{b} + u_j < ub)$ .

$a$  missing ( $a \geq .$ ) means  $-\infty$ ; `pr(., 30)` calculates  $\Pr(-\infty < \mathbf{x}_j\mathbf{b} + u_j < 30)$ ;

`pr(lb, 30)` calculates  $\Pr(-\infty < \mathbf{x}_j\mathbf{b} + u_j < 30)$  in observations for which  $lb \geq .$

and calculates  $\Pr(lb < \mathbf{x}_j\mathbf{b} + u_j < 30)$  elsewhere.

$b$  missing ( $b \geq .$ ) means  $+\infty$ ; `pr(20, .)` calculates  $\Pr(+\infty > \mathbf{x}_j\mathbf{b} + u_j > 20)$ ;

`pr(20, ub)` calculates  $\Pr(+\infty > \mathbf{x}_j\mathbf{b} + u_j > 20)$  in observations for which  $ub \geq .$

and calculates  $\Pr(20 < \mathbf{x}_j\mathbf{b} + u_j < ub)$  elsewhere.

`e(a, b)` calculates  $E(\mathbf{x}_j\mathbf{b} + u_j \mid a < \mathbf{x}_j\mathbf{b} + u_j < b)$ , the expected value of  $y_j|\mathbf{x}_j$  conditional on  $y_j|\mathbf{x}_j$  being in the interval  $(a, b)$ , meaning that  $y_j|\mathbf{x}_j$  is truncated.  $a$  and  $b$  are specified as they are for `pr()`.

`ystar(a, b)` calculates  $E(y_j^*)$ , where  $y_j^* = a$  if  $\mathbf{x}_j\mathbf{b} + u_j \leq a$ ,  $y_j^* = b$  if  $\mathbf{x}_j\mathbf{b} + u_j \geq b$ , and  $y_j^* = \mathbf{x}_j\mathbf{b} + u_j$  otherwise, meaning that  $y_j^*$  is censored.  $a$  and  $b$  are specified as they are for `pr()`.

`dfbeta(varname)` calculates the DFBETA for *varname*, the difference between the regression coefficient when the  $j$ th observation is included and excluded, said difference being scaled by the estimated standard error of the coefficient. *varname* must have been included among the regressors in the previously fitted model. The calculation is automatically restricted to the estimation subsample.

`stdp` calculates the standard error of the prediction, which can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. The standard error of the prediction is also referred to as the standard error of the fitted value.

`stdf` calculates the standard error of the forecast, which is the standard error of the point prediction for 1 observation. It is commonly referred to as the standard error of the future or forecast value. By construction, the standard errors produced by `stdf` are always larger than those produced by `stdp`; see *Methods and formulas*.

`stdr` calculates the standard error of the residuals.

`covratio` calculates COVRATIO (Belsley, Kuh, and Welsch 1980), a measure of the influence of the  $j$ th observation based on considering the effect on the variance–covariance matrix of the estimates. The calculation is automatically restricted to the estimation subsample.

`dfits` calculates DFITS (Welsch and Kuh 1977) and attempts to summarize the information in the leverage versus residual-squared plot into one statistic. The calculation is automatically restricted to the estimation subsample.

`welsch` calculates Welsch distance (Welsch 1982) and is a variation on `dfits`. The calculation is automatically restricted to the estimation subsample.

## Remarks and examples for predict

Remarks are presented under the following headings:

*Terminology*  
*Fitted values and residuals*  
*Prediction standard errors*  
*Prediction with weighted data*  
*Leverage statistics*  
*Standardized and Studentized residuals*  
*DFITS, Cook's Distance, and Welsch Distance*  
*COVRATIO*

## Terminology

Many of these commands concern identifying influential data in linear regression. This is, unfortunately, a field that is dominated by jargon, codified and partially begun by Belsley, Kuh, and Welsch (1980). In the words of Chatterjee and Hadi (1986, 416), “Belsley, Kuh, and Welsch’s book, *Regression Diagnostics*, was a very valuable contribution to the statistical literature, but it unleashed on an unsuspecting statistical community a computer speak (à la Orwell), the likes of which we have never seen.” Things have only gotten worse since then. Chatterjee and Hadi’s (1986, 1988) own attempts to clean up the jargon did not improve matters (see Hoaglin and Kempthorne [1986], Velleman [1986], and Welsch [1986]). We apologize for the jargon, and for our contribution to the jargon in the form of inelegant command names, we apologize most of all.

Model *sensitivity* refers to how estimates are affected by subsets of our data. Imagine data on  $y$  and  $x$ , and assume that the data are to be fit by the regression  $y_i = \alpha + \beta x_i + \epsilon_i$ . The regression estimates of  $\alpha$  and  $\beta$  are  $a$  and  $b$ , respectively. Now imagine that the estimated  $a$  and  $b$  would be different if a small portion of the dataset, perhaps even one observation, were deleted. As a data analyst, you would like to think that you are summarizing tendencies that apply to all the data, but you have just been told that the model you fit is unduly influenced by one point or just a few points and that, as a matter of fact, there is another model that applies to the rest of the data—a model that you have ignored. The search for subsets of the data that, if deleted, would change the results markedly is a predominant theme of this entry.

There are three key issues in identifying model sensitivity to individual observations, which go by the names *residuals*, *leverage*, and *influence*. In our  $y_i = a + bx_i + e_i$  regression, the residuals are, of course,  $e_i$ —they reveal how much our fitted value  $\hat{y}_i = a + bx_i$  differs from the observed  $y_i$ . A point  $(x_i, y_i)$  with a corresponding large residual is called an outlier. Say that you are interested in outliers because you somehow think that such points will exert undue influence on your estimates. Your feelings are generally right, but there are exceptions. A point might have a huge residual and yet not affect the estimated  $b$  at all. Nevertheless, studying observations with large residuals almost always pays off.

$(x_i, y_i)$  can be an outlier in another way—just as  $y_i$  can be far from  $\hat{y}_i$ ,  $x_i$  can be far from the center of mass of the other  $x$ ’s. Such an “outlier” should interest you just as much as the more traditional outliers. Picture a scatterplot of  $y$  against  $x$  with thousands of points in some sort of mass at the lower left of the graph and one point at the upper right of the graph. Now, run a regression line through the points—the

regression line will come close to the point at the upper right of the graph and may in fact, go through it. That is, this isolated point will not appear as an outlier as measured by residuals because its residual will be small. Yet this point might have a dramatic effect on our resulting estimates in the sense that, were you to delete the point, the estimates would change markedly. Such a point is said to have high leverage. Just as with traditional outliers, a high leverage point does not necessarily have an undue effect on regression estimates, but if it does not, it is more the exception than the rule.

Now, all of this is a most unsatisfactory state of affairs. Points with large residuals may, but need not, have a large effect on our results, and points with small residuals may still have a large effect. Points with high leverage may, but need not, have a large effect on our results, and points with low leverage may still have a large effect. Can you not identify the influential points and simply have the computer list them for you? You can, but you will have to define what you mean by “influential”.

“Influential” is defined with respect to some statistic. For instance, you might ask which points in your data have a large effect on your estimated  $a$ , which points have a large effect on your estimated  $b$ , which points have a large effect on your estimated standard error of  $b$ , and so on, but do not be surprised when the answers to these questions are different. In any case, obtaining such measures is not difficult—all you have to do is fit the regression excluding each observation one at a time and record the statistic of interest which, in the day of the modern computer, is not too onerous. Moreover, you can save considerable computer time by doing algebra ahead of time and working out formulas that will calculate the same answers as if you ran each of the regressions. (Ignore the question of pairs of observations that, together, exert undue influence, and triples, and so on, which remains largely unsolved and for which the brute force fit-every-possible-regression procedure is not a viable alternative.)

## Fitted values and residuals

Typing `predict newvar` with no options creates `newvar` containing the fitted values. Typing `predict newvar, resid` creates `newvar` containing the residuals.

### ▷ Example 1

Continuing with [example 1](#) from [\[R\] regress](#), we wish to fit the following model:

$$\text{mpg} = \beta_0 + \beta_1 \text{weight} + \beta_2 \text{foreign} + \epsilon$$

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. regress mpg weight foreign
```

|          |             |           |            |               |                      |           |
|----------|-------------|-----------|------------|---------------|----------------------|-----------|
| Source   | SS          | df        | MS         | Number of obs | =                    | 74        |
| Model    | 1619.2877   | 2         | 809.643849 | F(2, 71)      | =                    | 69.75     |
| Residual | 824.171761  | 71        | 11.608053  | Prob > F      | =                    | 0.0000    |
|          |             |           |            | R-squared     | =                    | 0.6627    |
|          |             |           |            | Adj R-squared | =                    | 0.6532    |
| Total    | 2443.45946  | 73        | 33.4720474 | Root MSE      | =                    | 3.4071    |
| mpg      | Coefficient | Std. err. | t          | P> t          | [95% conf. interval] |           |
| weight   | -.0065879   | .0006371  | -10.34     | 0.000         | -.0078583            | -.0053175 |
| foreign  | -1.650029   | 1.075994  | -1.53      | 0.130         | -3.7955              | .4954422  |
| _cons    | 41.6797     | 2.165547  | 19.25      | 0.000         | 37.36172             | 45.99768  |

That done, we can now obtain the predicted values from the regression. We will store them in a new variable called `pmpg` by typing `predict pmpg`. Because `predict` produces no output, we will follow that by summarizing our predicted and observed values.

```
. predict pmpg
(option xb assumed; fitted values)
. summarize pmpg mpg
```

| Variable | Obs | Mean    | Std. dev. | Min      | Max      |
|----------|-----|---------|-----------|----------|----------|
| pmpg     | 74  | 21.2973 | 4.709779  | 9.794333 | 29.82151 |
| mpg      | 74  | 21.2973 | 5.785503  | 12       | 41       |

◀

## ▶ Example 2: Out-of-sample predictions

We can just as easily obtain predicted values from the model by using a wholly different dataset from the one on which the model was fit. The only requirement is that the data have the necessary variables, which here are `weight` and `foreign`.

Using the data on two new cars (the Pontiac Sunbird and the Volvo 260) from `newautos.dta`, we can obtain out-of-sample predictions (or forecasts) by typing

```
. use https://www.stata-press.com/data/r19/newautos, clear
(New automobile models)
. predict pmpg
(option xb assumed; fitted values)
. list, divider
```

|    | make          | weight | foreign  | pmpg     |
|----|---------------|--------|----------|----------|
| 1. | Pont. Sunbird | 2690   | Domestic | 23.95829 |
| 2. | Volvo 260     | 3170   | Foreign  | 19.14607 |

The Pontiac Sunbird has a predicted mileage rating of 23.96 mpg, whereas the Volvo 260 has a predicted rating of 19.15 mpg. In comparison, the actual mileage ratings are 24 for the Pontiac and 17 for the Volvo.

◀

## Prediction standard errors

`predict` can calculate the standard error of the forecast (`stdf` option), the standard error of the prediction (`stdp` option), and the standard error of the residual (`stdr` option). It is easy to confuse `stdf` and `stdp` because both are often called the prediction error. Consider the prediction  $\hat{y}_j = \mathbf{x}_j \mathbf{b}$ , where  $\mathbf{b}$  is the estimated coefficient (column) vector and  $\mathbf{x}_j$  is a (row) vector of independent variables for which you want the prediction. First,  $\hat{y}_j$  has a variance due to the variance of the estimated coefficient vector  $\mathbf{b}$ ,

$$\text{Var}(\hat{y}_j) = \text{Var}(\mathbf{x}_j \mathbf{b}) = s^2 h_j$$

where  $h_j = \mathbf{x}_j (\mathbf{X}' \mathbf{X})^{-1} \mathbf{x}_j'$  and  $s^2$  is the mean squared error of the regression. Do not panic over the algebra—just remember that  $\text{Var}(\hat{y}_j) = s^2 h_j$ , whatever  $s^2$  and  $h_j$  are. `stdp` calculates this quantity. This is the error in the prediction due to the uncertainty about  $\mathbf{b}$ .

If you are about to hand this number out as your forecast, however, there is another error. According to your model, the true value of  $y_j$  is given by

$$y_j = \mathbf{x}_j \mathbf{b} + \epsilon_j = \hat{y}_j + \epsilon_j$$

and thus the  $\text{Var}(y_j) = \text{Var}(\hat{y}_j) + \text{Var}(\epsilon_j) = s^2 h_j + s^2$ , which is the square of `stdf`. `stdf`, then, is the sum of the error in the prediction plus the residual error.

`stdr` has to do with an analysis-of-variance decomposition of  $s^2$ , the estimated variance of  $y$ . The standard error of the prediction is  $s^2 h_j$ , and therefore  $s^2 h_j + s^2(1 - h_j) = s^2$  decomposes  $s^2$  into the prediction and residual variances.

### ▷ Example 3: Standard error of the forecast

Returning to our model of `mpg` on `weight` and `foreign`, we previously predicted the mileage rating for the Pontiac Sunbird and Volvo 260 as 23.96 and 19.15 mpg, respectively. We now want to put a standard error around our forecast. Remember, the data for these two cars were in `newautos.dta`:

```
. use https://www.stata-press.com/data/r19/newautos, clear
(New automobile models)
. predict pmpg
(option xb assumed; fitted values)
. predict se_pmpg, stdf
. list, divider
```

|    | make          | weight | foreign  | pmpg     | se_pmpg  |
|----|---------------|--------|----------|----------|----------|
| 1. | Pont. Sunbird | 2690   | Domestic | 23.95829 | 3.462791 |
| 2. | Volvo 260     | 3170   | Foreign  | 19.14607 | 3.525875 |

Thus, an approximate 95% confidence interval for the mileage rating of the Volvo 260 is  $19.15 \pm 2 \cdot 3.53 = [12.09, 26.21]$ .

◀

### Prediction with weighted data

`predict` can be used after frequency-weighted (`fweight`) estimation, just as it is used after unweighted estimation. The technical note below concerns the use of `predict` after analytically weighted (`aweight`) estimation.

### □ Technical note

After analytically weighted estimation, `predict` is willing to calculate only the prediction (no options), residual (`residual` option), standard error of the prediction (`stdp` option), and diagonal elements of the projection matrix (`hat` option). For analytically weighted estimation, the standard error of the forecast and residuals, the standardized and Studentized residuals, and Cook's  $D$  are not statistically well-defined concepts.

□

## Leverage statistics

In addition to providing fitted values and the associated standard errors, the `predict` command can also be used to generate various statistics used to detect the influence of individual observations. This section provides a brief introduction to leverage (`hat`) statistics, and some of the following subsections discuss other influence statistics produced by `predict`.

### ► Example 4: Diagonal elements of projection matrix

The diagonal elements of the projection matrix, obtained by the `hat` option, are a measure of distance in explanatory variable space. Leverage is a synonym for `hat`.

```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
. regress mpg weight foreign
(output omitted)
. predict xdist, hat
. summarize xdist, detail
```

| Leverage |             |          |                    |
|----------|-------------|----------|--------------------|
|          | Percentiles | Smallest |                    |
| 1%       | .0192325    | .0192325 |                    |
| 5%       | .0192686    | .0192366 |                    |
| 10%      | .0193448    | .019241  | Obs 74             |
| 25%      | .0220291    | .0192686 | Sum of wgt. 74     |
| 50%      | .0383797    |          | Mean .0405405      |
|          |             | Largest  | Std. dev. .0207624 |
| 75%      | .0494002    | .0880814 |                    |
| 90%      | .0693432    | .099715  | Variance .0004311  |
| 95%      | .0880814    | .099715  | Skewness 1.159745  |
| 99%      | .1003283    | .1003283 | Kurtosis 4.083313  |

Some 5% of our sample has an `xdist` measure in excess of 0.08. Let's force them to reveal their identities:

```
. list foreign make mpg if xdist>.08, divider
```

|     | foreign  | make              | mpg |
|-----|----------|-------------------|-----|
| 24. | Domestic | Ford Fiesta       | 28  |
| 26. | Domestic | Linc. Continental | 12  |
| 27. | Domestic | Linc. Mark V      | 12  |
| 43. | Domestic | Plym. Champ       | 34  |
| 64. | Foreign  | Peugeot 604       | 14  |

To understand why these cars are on this list, we must remember that the explanatory variables in our model are `weight` and `foreign` and that `xdist` measures distance in this metric. The Ford Fiesta and the Plymouth Champ are the two lightest domestic cars in our data. The Lincolns are the two heaviest domestic cars, and the Peugeot is the heaviest foreign car.

◀

See `lvr2plot` in [R] [regress postestimation diagnostic plots](#) for information on a leverage-versus-squared-residual plot.

## Standardized and Studentized residuals

The terms standardized and Studentized residuals have meant different things to different authors. In Stata, `predict` defines the standardized residual as  $\hat{e}_{s_i} = e_i / (s\sqrt{1 - h_i})$  and the Studentized residual as  $r_i = e_i / (s_{(i)}\sqrt{1 - h_i})$ , where  $s_{(i)}$  is the root mean squared error of a regression with the  $i$ th observation removed. Stata's definition of the Studentized residual is the same as the one given in [Bollen and Jackman \(1990, 264\)](#) and is what [Chatterjee and Hadi \(1988, 74\)](#) call the “externally Studentized” residual. Stata's “standardized” residual is the same as what [Chatterjee and Hadi \(1988, 74\)](#) call the “internally Studentized” residual.

Standardized and Studentized residuals are attempts to adjust residuals for their standard errors. Although the  $\epsilon_i$  theoretical residuals are homoskedastic by assumption (that is, they all have the same variance), the calculated  $e_i$  are not. In fact,

$$\text{Var}(e_i) = \sigma^2(1 - h_i)$$

where  $h_i$  are the leverage measures obtained from the diagonal elements of hat matrix. Thus, observations with the greatest leverage have corresponding residuals with the smallest variance.

Standardized residuals use the root mean squared error of the regression for  $\sigma$ . Studentized residuals use the root mean squared error of a regression omitting the observation in question for  $\sigma$ . In general, Studentized residuals are preferable to standardized residuals for purposes of outlier identification. Studentized residuals can be interpreted as the  $t$  statistic for testing the significance of a dummy variable equal to 1 in the observation in question and 0 elsewhere ([Belsley, Kuh, and Welsch 1980](#)). Such a dummy variable would effectively absorb the observation and so remove its influence in determining the other coefficients in the model. Caution must be exercised here, however, because of the simultaneous testing problem. You cannot simply list the residuals that would be individually significant at the 5% level—their joint significance would be far less (their joint significance *level* would be far greater).

### ► Example 5: Standardized and Studentized residuals

In the [Terminology](#) section of *Remarks and examples for predict*, we distinguished residuals from leverage and speculated on the impact of an observation with a small residual but large leverage. If we adjust the residuals for their standard errors, however, the adjusted residual would be (relatively) larger and perhaps large enough so that we could simply examine the adjusted residuals. Taking our price on weight and foreign##c.mpg model from [example 1](#) of [\[R\] regress postestimation diagnostic plots](#), we can obtain the in-sample standardized and Studentized residuals by typing

```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
. regress price weight foreign##c.mpg
(output omitted)
. predict esta if e(sample), rstandard
. predict estu if e(sample), rstudent
```

In the `lvr2plot` section of [R] [regress postestimation diagnostic plots](#), we discovered that the VW Diesel has the highest leverage in our data, but a corresponding small residual. The standardized and Studentized residuals for the VW Diesel are

```
. list make price esta estu if make=="VW Diesel"
```

|     | make      | price | esta     | estu     |
|-----|-----------|-------|----------|----------|
| 71. | VW Diesel | 5,397 | .6142691 | .6114758 |

The Studentized residual of 0.611 can be interpreted as the  $t$  statistic for including a dummy variable for VW Diesel in our regression. Such a variable would not be significant.

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### DFITS, Cook's Distance, and Welsch Distance

DFITS ([Welsch and Kuh 1977](#)), Cook's Distance ([Cook 1977](#)), and Welsch Distance ([Welsch 1982](#)) are three attempts to summarize the information in the leverage versus residual-squared plot into one statistic. That is, the goal is to create an index that is affected by the size of the residuals—outliers—and the size of  $h_i$ —leverage. Viewed mechanically, one way to write DFITS ([Bollen and Jackman 1990](#), 265) is

$$\text{DFITS}_i = r_i \sqrt{\frac{h_i}{1 - h_i}}$$

where  $r_i$  are the Studentized residuals. Thus, large residuals increase the value of DFITS, as do large values of  $h_i$ . Viewed more traditionally, DFITS is a scaled difference between predicted values for the  $i$ th case when the regression is fit with and without the  $i$ th observation, hence the name.

The mechanical relationship between DFITS and Cook's Distance,  $D_i$  ([Bollen and Jackman 1990](#), 266), is

$$D_i = \frac{1}{k} \frac{s_{(i)}^2}{s^2} \text{DFITS}_i^2$$

where  $k$  is the number of variables (including the constant) in the regression,  $s$  is the root mean squared error of the regression, and  $s_{(i)}$  is the root mean squared error when the  $i$ th observation is omitted. Viewed more traditionally,  $D_i$  is a scaled measure of the distance between the coefficient vectors when the  $i$ th observation is omitted.

The mechanical relationship between DFITS and Welsch's Distance,  $W_i$  ([Chatterjee and Hadi 1988](#), 123), is

$$W_i = \text{DFITS}_i \sqrt{\frac{n-1}{1-h_i}}$$

The interpretation of  $W_i$  is more difficult because it is based on the empirical influence curve. Although DFITS and Cook's distance are similar, the Welsch distance measure includes another normalization by leverage.

[Belsley, Kuh, and Welsch \(1980, 28\)](#) suggest that DFITS values greater than  $2\sqrt{k/n}$  deserve more investigation, and so values of Cook's distance greater than  $4/n$  should also be examined ([Bollen and Jackman 1990](#), 265–266). Through similar logic, the cutoff for Welsch distance is approximately  $3\sqrt{k}$  ([Chatterjee and Hadi 1988](#), 124).

## ▷ Example 6: DFITS influence measure

Continuing with our model of price on weight and foreign##c.mpg, we can obtain the DFITS influence measure:

```
. predict e if e(sample), resid
. predict dfits, dfits
```

We did not specify `if e(sample)` in computing the DFITS statistic. DFITS is available only over the estimation sample, so specifying `if e(sample)` would have been redundant. It would have done no harm, but it would not have changed the results.

Our model has  $k = 5$  independent variables ( $k$  includes the constant) and  $n = 74$  observations; following the  $2\sqrt{k/n}$  cutoff advice, we type

```
. list make price e dfits if abs(dfits) > 2*sqrt(5/74), divider
```

|     | make             | price  | e         | dfits     |
|-----|------------------|--------|-----------|-----------|
| 12. | Cad. Eldorado    | 14,500 | 7271.96   | .9564455  |
| 13. | Cad. Seville     | 15,906 | 5036.348  | 1.356619  |
| 24. | Ford Fiesta      | 4,389  | 3164.872  | .5724172  |
| 27. | Linc. Mark V     | 13,594 | 3109.193  | .5200413  |
| 28. | Linc. Versailles | 13,466 | 6560.912  | .8760136  |
| 42. | Plym. Arrow      | 4,647  | -3312.968 | -.9384231 |

We calculate Cook's distance and list the observations greater than the suggested  $4/n$  cutoff:

```
. predict cooks if e(sample), cooks
. list make price e cooks if cooks > 4/74, divider
```

|     | make             | price  | e         | cooks    |
|-----|------------------|--------|-----------|----------|
| 12. | Cad. Eldorado    | 14,500 | 7271.96   | .1492676 |
| 13. | Cad. Seville     | 15,906 | 5036.348  | .3328515 |
| 24. | Ford Fiesta      | 4,389  | 3164.872  | .0638815 |
| 28. | Linc. Versailles | 13,466 | 6560.912  | .1308004 |
| 42. | Plym. Arrow      | 4,647  | -3312.968 | .1700736 |

Here we used `if e(sample)` because Cook's distance is not restricted to the estimation sample by default. It is worth comparing this list with the preceding one.

Finally, we use Welsch distance and the suggested  $3\sqrt{k}$  cutoff:

```
. predict wd, welsch
. list make price e wd if abs(wd) > 3*sqrt(5), divider
```

|     | make             | price  | e         | wd        |
|-----|------------------|--------|-----------|-----------|
| 12. | Cad. Eldorado    | 14,500 | 7271.96   | 8.394372  |
| 13. | Cad. Seville     | 15,906 | 5036.348  | 12.81125  |
| 28. | Linc. Versailles | 13,466 | 6560.912  | 7.703005  |
| 42. | Plym. Arrow      | 4,647  | -3312.968 | -8.981481 |

Here we did not need to specify `if e(sample)` because `welsch` automatically restricts the prediction to the estimation sample.

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## COVRATIO

COVRATIO (Belsley, Kuh, and Welsch 1980) measures the influence of the  $i$ th observation by considering the effect on the variance–covariance matrix of the estimates. The measure is the ratio of the determinants of the covariances matrix, with and without the  $i$ th observation. The resulting formula is

$$\text{COVRATIO}_i = \frac{1}{1 - h_i} \left( \frac{n - k - \hat{e}_{s_i}^2}{n - k - 1} \right)^k$$

where  $\hat{e}_{s_i}$  is the standardized residual.

For noninfluential observations, the value of COVRATIO is approximately 1. Large values of the residuals or large values of leverage will cause deviations from 1, although if both are large, COVRATIO may tend back toward 1 and therefore not identify such observations (Chatterjee and Hadi 1988, 139).

Belsley, Kuh, and Welsch (1980) suggest that observations for which

$$|\text{COVRATIO}_i - 1| \geq \frac{3k}{n}$$

are worthy of further examination.

## ▷ Example 7: COVRATIO influence measure

Using our model of price on weight and foreign##c.mpg, we can obtain the COVRATIO measure and list the observations outside the suggested cutoff by typing

```
. predict covr, covratio
. list make price e covr if abs(covr-1) >= 3*5/74, divider
```

|     | make             | price  | e         | covr     |
|-----|------------------|--------|-----------|----------|
| 12. | Cad. Eldorado    | 14,500 | 7271.96   | .3814242 |
| 13. | Cad. Seville     | 15,906 | 5036.348  | .7386969 |
| 28. | Linc. Versailles | 13,466 | 6560.912  | .4761695 |
| 43. | Plym. Champ      | 4,425  | 1621.747  | 1.27782  |
| 53. | Audi 5000        | 9,690  | 591.2883  | 1.206842 |
| 57. | Datsun 210       | 4,589  | 19.81829  | 1.284801 |
| 64. | Peugeot 604      | 12,990 | 1037.184  | 1.348219 |
| 66. | Subaru           | 3,798  | -909.5894 | 1.264677 |
| 71. | VW Diesel        | 5,397  | 999.7209  | 1.630653 |
| 74. | Volvo 260        | 11,995 | 1327.668  | 1.211888 |

The covratio option automatically restricts the prediction to the estimation sample.

# margins

## Description for margins

`margins` estimates margins of response for linear predictions.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
margins [marginlist] , predict(statistic ...) [options]
```

| <i>statistic</i>                      | Description                    |
|---------------------------------------|--------------------------------|
| <code>xb</code>                       | linear prediction; the default |
| <code>pr(<i>a</i>,<i>b</i>)</code>    | not allowed with margins       |
| <code>e(<i>a</i>,<i>b</i>)</code>     | not allowed with margins       |
| <code>ystar(<i>a</i>,<i>b</i>)</code> | not allowed with margins       |
| <code>residuals</code>                | not allowed with margins       |
| <code>score</code>                    | not allowed with margins       |
| <code>rstandard</code>                | not allowed with margins       |
| <code>rstudent</code>                 | not allowed with margins       |
| <code>cooksd</code>                   | not allowed with margins       |
| <code>leverage   <u>hat</u></code>    | not allowed with margins       |
| <code>dfbeta(<i>varname</i>)</code>   | not allowed with margins       |
| <code>stdp</code>                     | not allowed with margins       |
| <code>stdf</code>                     | not allowed with margins       |
| <code>stdr</code>                     | not allowed with margins       |
| <code>covratio</code>                 | not allowed with margins       |
| <code>dfits</code>                    | not allowed with margins       |
| <code>welsch</code>                   | not allowed with margins       |

Statistics not allowed with margins are functions of stochastic quantities other than `e(b)`.

For the full syntax, see [R] [margins](#).

## DFBETA influence statistics

### Description for dfbeta

`dfbeta` will calculate one, more than one, or all the DFBETAs after `regress`. Although `predict` will also calculate DFBETAs, `predict` can do this for only one variable at a time. `dfbeta` is a convenience tool for those who want to calculate DFBETAs for multiple variables. The names for the new variables created are chosen automatically and begin with the letters `_dfbeta_`.

### Menu for dfbeta

Statistics > Linear models and related > Regression diagnostics > DFBETAs

### Syntax for dfbeta

```
dfbeta [indepvar [indepvar [...]]] [, stub(name)]
```

### Option for dfbeta

`stub(name)` specifies the leading characters `dfbeta` uses to name the new variables to be generated. The default is `stub(_dfbeta_)`.

### Remarks and examples for dfbeta

DFBETAs are perhaps the most direct influence measure of interest to model builders. DFBETAs focus on one coefficient and measure the difference between the regression coefficient when the  $i$ th observation is included and excluded, the difference being scaled by the estimated standard error of the coefficient. [Belsley, Kuh, and Welsch \(1980, 28\)](#) suggest observations with  $|DFBETA_i| > 2/\sqrt{n}$  as deserving special attention, but it is also common practice to use 1 ([Bollen and Jackman 1990, 267](#)), meaning that the observation shifted the estimate at least one standard error.

#### ▷ Example 8: DFBETAs influence measure; the `dfbeta()` option

Using our model of `price` on `weight` and `foreign##c.mpg`, let's first ask which observations have the greatest impact on the determination of the coefficient on `1.foreign`. We will use the suggested  $2/\sqrt{n}$  cutoff:

```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
. regress price weight foreign##c.mpg
(output omitted)
```

```
. sort foreign make
. predict dfor, dfbeta(1.foreign)
. list make price foreign dfor if abs(dfor) > 2/sqrt(74), divider
```

|     | make             | price  | foreign  | dfor      |
|-----|------------------|--------|----------|-----------|
| 12. | Cad. Eldorado    | 14,500 | Domestic | -.5290519 |
| 13. | Cad. Seville     | 15,906 | Domestic | .8243419  |
| 28. | Linc. Versailles | 13,466 | Domestic | -.5283729 |
| 42. | Plym. Arrow      | 4,647  | Domestic | -.6622424 |
| 43. | Plym. Champ      | 4,425  | Domestic | .2371104  |
| 64. | Peugeot 604      | 12,990 | Foreign  | .2552032  |
| 69. | Toyota Corona    | 5,719  | Foreign  | -.256431  |

The Cadillac Seville shifted the coefficient on `1.foreign` 0.82 standard deviations!

Now let us ask which observations have the greatest effect on the mpg coefficient:

```
. predict dmpg, dfbeta(mpg)
. list make price mpg dmpg if abs(dmpg) > 2/sqrt(74), divider
```

|     | make             | price  | mpg | dmpg      |
|-----|------------------|--------|-----|-----------|
| 12. | Cad. Eldorado    | 14,500 | 14  | -.5970351 |
| 13. | Cad. Seville     | 15,906 | 21  | 1.134269  |
| 28. | Linc. Versailles | 13,466 | 14  | -.6069287 |
| 42. | Plym. Arrow      | 4,647  | 28  | -.8925859 |
| 43. | Plym. Champ      | 4,425  | 34  | .3186909  |

Once again, we see the Cadillac Seville heading the list, indicating that our regression results may be dominated by this one car.

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### ► Example 9: DFBETAs influence measure; the `dfbeta` command

We can use `predict, dfbeta()` or the `dfbeta` command to generate the DFBETAs. `dfbeta` makes up names for the new variables automatically and, without arguments, generates the DFBETAs for all the variables in the regression:

```
. dfbeta
Generating DFBETA variables ...
 _dfbeta_1: DFBETA weight
 _dfbeta_2: DFBETA 1.foreign
 _dfbeta_3: DFBETA mpg
 _dfbeta_4: DFBETA 1.foreign#c.mpg
```

`dfbeta` created four new variables in our dataset: `_dfbeta_1`, containing the DFBETAs for `weight`; `_dfbeta_2`, containing the DFBETAs for `mpg`; and so on. Had we wanted only the DFBETAs for `mpg` and `weight`, we might have typed

```
. dfbeta mpg weight
Generating DFBETA variables ...
 _dfbeta_5: DFBETA weight
 _dfbeta_6: DFBETA mpg
```

In the example above, we typed `dfbeta mpg weight` instead of `dfbeta`; if we had typed `dfbeta` followed by `dfbeta mpg weight`, here is what would have happened:

```
. dfbeta
Generating DFBETA variables ...
 _dfbeta_7: DFBETA weight
 _dfbeta_8: DFBETA 1.foreign
 _dfbeta_9: DFBETA mpg
 _dfbeta_10: DFBETA 1.foreign#c.mpg
. dfbeta mpg weight
Generating DFBETA variables ...
 _dfbeta_11: DFBETA weight
 _dfbeta_12: DFBETA mpg
```

`dfbeta` would have made up different names for the new variables. `dfbeta` never replaces existing variables—it instead makes up a different name, so we need to pay attention to `dfbeta`'s output.

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## Tests for violation of assumptions

### Description for `estat hettest`

`estat hettest` performs three versions of the Breusch–Pagan (1979) and Cook–Weisberg (1983) test for heteroskedasticity. All three versions of this test present evidence against the null hypothesis that  $\mathbf{t} = \mathbf{0}$  in  $\text{Var}(e) = \sigma^2 \exp(\mathbf{z}\mathbf{t})$ . In the normal version, performed by default, the null hypothesis also includes the assumption that the regression disturbances are independent-normal draws with variance  $\sigma^2$ . The normality assumption is dropped from the null hypothesis in the `iid` and `fstat` versions, which respectively produce the score and  $F$  tests discussed in *Methods and formulas*. If `varlist` is not specified, the fitted values are used for  $\mathbf{z}$ . If `varlist` or the `rhs` option is specified, the variables specified are used for  $\mathbf{z}$ .

### Menu for `estat`

Statistics > Postestimation

### Syntax for `estat hettest`

```
estat hettest [varlist] [, rhs [normal | iid | fstat] mtest [spec]]
```

`collect` is allowed with `estat hettest`; see [U] 11.1.10 Prefix commands.

### Options for `estat hettest`

`rhs` specifies that tests for heteroskedasticity be performed for the right-hand-side (explanatory) variables of the fitted regression model. The `rhs` option may be combined with a *varlist*.

`normal`, the default, causes `estat hettest` to compute the original Breusch–Pagan/Cook–Weisberg test, which assumes that the regression disturbances are normally distributed.

`iid` causes `estat hettest` to compute the  $N * R^2$  version of the score test that drops the normality assumption.

`fstat` causes `estat hettest` to compute the  $F$ -statistic version that drops the normality assumption.

`mtest [ (spec) ]` specifies that multiple testing be performed. The argument specifies how  $p$ -values are adjusted. The following specifications, *spec*, are supported:

|                         |                                            |
|-------------------------|--------------------------------------------|
| <code>bonferroni</code> | Bonferroni's multiple testing adjustment   |
| <code>holm</code>       | Holm's multiple testing adjustment         |
| <code>sidak</code>      | Šidák's multiple testing adjustment        |
| <code>noadjust</code>   | no adjustment is made for multiple testing |

`mtest` may be specified without an argument. This is equivalent to specifying `mtest(noadjust)`; that is, tests for the individual variables should be performed with unadjusted  $p$ -values. By default, `estat hettest` does not perform multiple testing. `mtest` may not be specified with `iid` or `fstat`.

## Description for estat imtest

`estat imtest` performs an information matrix test for the regression model and an orthogonal decomposition into tests for heteroskedasticity, skewness, and kurtosis due to [Cameron and Trivedi \(1990\)](#); White's test for homoskedasticity against unrestricted forms of heteroskedasticity ([1980](#)) is available as an option. White's test is usually similar to the first term of the Cameron–Trivedi decomposition.

## Menu for estat

Statistics > Postestimation

## Syntax for estat imtest

```
estat imtest [, preserve white]
```

`collect` is allowed with `estat imtest`; see [\[U\] 11.1.10 Prefix commands](#).

## Options for estat imtest

`preserve` specifies that the data in memory be preserved, all variables and cases that are not needed in the calculations be dropped, and at the conclusion the original data be restored. This option is costly for large datasets. However, because `estat imtest` has to perform an auxiliary regression on  $k(k+1)/2$  temporary variables, where  $k$  is the number of regressors, it may not be able to perform the test otherwise.

`white` specifies that White's original heteroskedasticity test also be performed.

## Description for estat ovtest

`estat ovtest` performs two versions of the Ramsey (1969) regression specification-error test (RESET) for omitted variables. This test amounts to fitting  $y = \mathbf{x}\mathbf{b} + \mathbf{z}\mathbf{t} + u$  and then testing  $\mathbf{t} = \mathbf{0}$ . If the `rhs` option is not specified, powers of the fitted values are used for  $\mathbf{z}$ . If `rhs` is specified, powers of the individual elements of  $\mathbf{x}$  are used.

## Menu for estat

Statistics > Postestimation

## Syntax for estat ovtest

```
estat ovtest [, rhs]
```

`collect` is allowed with `estat ovtest`; see [U] 11.1.10 Prefix commands.

## Option for estat ovtest

`rhs` specifies that powers of the right-hand-side (explanatory) variables be used in the test rather than powers of the fitted values.

## Description for estat szroeter

`estat szroeter` performs Szroeter's rank test for heteroskedasticity for each of the variables in *varlist* or for the explanatory variables of the regression if `rhs` is specified.

## Menu for estat

Statistics > Postestimation

## Syntax for estat szroeter

```
estat szroeter [varlist] [, rhs mtest(spec)]
```

Either *varlist* or `rhs` must be specified.

## Options for estat szroeter

`rhs` specifies that tests for heteroskedasticity be performed for the right-hand-side (explanatory) variables of the fitted regression model. The `rhs` option may be combined with a *varlist*.

`mtest(spec)` specifies that multiple testing be performed. The argument specifies how *p*-values are adjusted. The following specifications, *spec*, are supported:

|                   |                                            |
|-------------------|--------------------------------------------|
| <u>bonferroni</u> | Bonferroni's multiple testing adjustment   |
| <u>holm</u>       | Holm's multiple testing adjustment         |
| <u>sidak</u>      | Šidák's multiple testing adjustment        |
| <u>noadjust</u>   | no adjustment is made for multiple testing |

`estat szroeter` always performs multiple testing. By default, it does not adjust the *p*-values.

## Remarks and examples for estat hettest, estat imtest, estat ovtest, and estat szroeter

We introduce some regression diagnostic commands that are designed to test for certain violations that `rvfplot` (see [R] [regress postestimation diagnostic plots](#)) less formally attempts to detect. `estat ovtest` provides Ramsey’s test for omitted variables—a pattern in the residuals. `estat hettest` provides a test for heteroskedasticity—the increasing or decreasing variation in the residuals with fitted values, with respect to the explanatory variables, or with respect to yet other variables. The score test implemented in `estat hettest` (Breusch and Pagan 1979; Cook and Weisberg 1983) performs a score test of the null hypothesis that  $b = 0$  against the alternative hypothesis of multiplicative heteroskedasticity. `estat szroeter` provides a rank test for heteroskedasticity, which is an alternative to the score test computed by `estat hettest`. Finally, `estat imtest` computes an information matrix test, including an orthogonal decomposition into tests for heteroskedasticity, skewness, and kurtosis (Cameron and Trivedi 1990). The heteroskedasticity test computed by `estat imtest` is similar to the general test for heteroskedasticity that was proposed by White (1980). Cameron and Trivedi (2022, chap. 3) discuss most of these tests and provides more examples.

### ► Example 10: estat ovtest, estat hettest, estat szroeter, and estat imtest

We use our model of price on weight and foreign##c.mpg.

```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)

. regress price weight foreign##c.mpg
(output omitted)

. estat ovtest

Ramsey RESET test for omitted variables
Omitted: Powers of fitted values of price
H0: Model has no omitted variables
F(3, 66) = 7.77
Prob > F = 0.0002

. estat hettest

Breusch-Pagan/Cook-Weisberg test for heteroskedasticity
Assumption: Normal error terms
Variable: Fitted values of price
H0: Constant variance
 chi2(1) = 6.50
Prob > chi2 = 0.0108
```

Testing for heteroskedasticity in the right-hand-side variables is requested by specifying the rhs option. By specifying the mtest(bonferroni) option, we request that tests be conducted for each of the variables, with a Bonferroni adjustment for the  $p$ -values to accommodate our testing multiple hypotheses.

```
. estat hettest, rhs mtest(bonf)
Breusch-Pagan/Cook-Weisberg test for heteroskedasticity
Assumption: Normal error terms
HO: Constant variance
```

| Variable     | chi2  | df | p       |
|--------------|-------|----|---------|
| weight       | 15.24 | 1  | 0.0004* |
| foreign      |       |    |         |
| Foreign      | 6.15  | 1  | 0.0525* |
| mpg          | 9.04  | 1  | 0.0106* |
| foreign#     |       |    |         |
| c.mpg        |       |    |         |
| Foreign      | 6.02  | 1  | 0.0566* |
| Simultaneous | 15.60 | 4  | 0.0036  |

```
* Bonferroni-adjusted p-values
. estat szroeter, rhs mtest(holm)
Szroeter's test for homoskedasticity
HO: Variance constant
Ha: Variance monotonic in variables
```

| Variable | chi2  | df | p       |
|----------|-------|----|---------|
| weight   | 17.07 | 1  | 0.0001* |
| foreign  |       |    |         |
| Foreign  | 6.15  | 1  | 0.0260* |
| mpg      | 11.45 | 1  | 0.0021* |
| foreign# |       |    |         |
| c.mpg    |       |    |         |
| Foreign  | 6.17  | 1  | 0.0260* |

```
* Holm-adjusted p-values
```

Finally, we request the information matrix test, which is a conditional moments test with second-, third-, and fourth-order moment conditions.

```
. estat imtest
Cameron & Trivedi's decomposition of IM-test
```

| Source             | chi2  | df | p      |
|--------------------|-------|----|--------|
| Heteroskedasticity | 18.86 | 10 | 0.0420 |
| Skewness           | 11.69 | 4  | 0.0198 |
| Kurtosis           | 2.33  | 1  | 0.1273 |
| Total              | 32.87 | 15 | 0.0049 |

We find evidence for omitted variables, heteroskedasticity, and nonnormal skewness.

So, why bother with the various graphical commands when the tests seem so much easier to interpret? In part, it is a matter of taste: both are designed to uncover the same problem, and both are, in fact, going about it in similar ways. One is based on a formal calculation, whereas the other is based on personal judgment in evaluating a graph. On the other hand, the tests are seeking evidence of specific problems, whereas judgment is more general. The careful analyst will use both.

We performed the omitted-variable test first. Omitted variables are a more serious problem than heteroskedasticity or the violations of higher moment conditions tested by `estat imtest`. If this were not a manual, having found evidence of omitted variables, we would never have run the `estat hettest`, `estat szroeter`, and `estat imtest` commands, at least not until we solved the omitted-variable problem.

◀

## □ Technical note

`estat ovtest` and `estat hettest` both perform two flavors of their respective tests. By default, `estat ovtest` looks for evidence of omitted variables by fitting the original model augmented by  $\hat{y}^2$ ,  $\hat{y}^3$ , and  $\hat{y}^4$ , which are the fitted values from the original model. Under the assumption of no misspecification, the coefficients on the powers of the fitted values will be zero. With the `rhs` option, `estat ovtest` instead augments the original model with powers (second through fourth) of the explanatory variables (except for dummy variables).

`estat hettest`, by default, looks for heteroskedasticity by modeling the variance as a function of the fitted values. If, however, we specify a variable or variables, the variance will be modeled as a function of the specified variables. In our example, if we had, a priori, some reason to suspect heteroskedasticity and that the heteroskedasticity is a function of a car's weight, then using a test that focuses on weight would be more powerful than the more general tests such as White's test or the first term in the Cameron–Trivedi decomposition test.

`estat hettest`, by default, computes the original Breusch–Pagan/Cook–Weisberg test, which includes the assumption of normally distributed errors. [Koenker \(1981\)](#) derived an  $N * R^2$  version of this test that drops the normality assumption. [Wooldridge \(2020, 270\)](#) gives an  $F$ -statistic version that does not require the normality assumption.

□

## Stored results for `estat hettest`, `estat imtest`, and `estat ovtest`

`estat hettest` stores the following results for the (multivariate) score test in `r()`:

Scalars

|                      |                                                          |
|----------------------|----------------------------------------------------------|
| <code>r(chi2)</code> | $\chi^2$ test statistic                                  |
| <code>r(df)</code>   | #df for the asymptotic $\chi^2$ distribution under $H_0$ |
| <code>r(p)</code>    | $p$ -value                                               |

`estat hettest`, `fstat` stores results for the (multivariate) score test in `r()`:

Scalars

|                      |                                                           |
|----------------------|-----------------------------------------------------------|
| <code>r(F)</code>    | test statistic                                            |
| <code>r(df_m)</code> | #df of the test for the $F$ distribution under $H_0$      |
| <code>r(df_r)</code> | #df of the residuals for the $F$ distribution under $H_0$ |
| <code>r(p)</code>    | $p$ -value                                                |

`estat hettest` (if `mtest` is specified) and `estat szroeter` store the following in `r()`:

#### Matrices

|                         |                                                                                      |
|-------------------------|--------------------------------------------------------------------------------------|
| <code>r(mtest)</code>   | a matrix of test results, with rows corresponding to the univariate tests            |
| <code>mtest[.,1]</code> | $\chi^2$ test statistic                                                              |
| <code>mtest[.,2]</code> | #df                                                                                  |
| <code>mtest[.,3]</code> | unadjusted <i>p</i> -value                                                           |
| <code>mtest[.,4]</code> | adjusted <i>p</i> -value (if an <code>mtest()</code> adjustment method is specified) |

#### Macros

|                         |                                       |
|-------------------------|---------------------------------------|
| <code>r(mtmetho)</code> | adjustment method for <i>p</i> -value |
|-------------------------|---------------------------------------|

`estat imtest` stores the following in `r()`:

#### Scalars

|                        |                                                                                                                           |
|------------------------|---------------------------------------------------------------------------------------------------------------------------|
| <code>r(chi2_t)</code> | LM-test statistic (= <code>r(chi2_h)</code> + <code>r(chi2_s)</code> + <code>r(chi2_k)</code> )                           |
| <code>r(df_t)</code>   | df for limiting $\chi^2$ distribution under $H_0$ (= <code>r(df_h)</code> + <code>r(df_s)</code> + <code>r(df_k)</code> ) |
| <code>r(chi2_h)</code> | heteroskedasticity test statistic                                                                                         |
| <code>r(df_h)</code>   | df for limiting $\chi^2$ distribution under $H_0$                                                                         |
| <code>r(chi2_s)</code> | skewness test statistic                                                                                                   |
| <code>r(df_s)</code>   | df for limiting $\chi^2$ distribution under $H_0$                                                                         |
| <code>r(chi2_k)</code> | kurtosis test statistic                                                                                                   |
| <code>r(df_k)</code>   | df for limiting $\chi^2$ distribution under $H_0$                                                                         |
| <code>r(chi2_w)</code> | White's heteroskedasticity test (if <code>white</code> specified)                                                         |
| <code>r(df_w)</code>   | df for limiting $\chi^2$ distribution under $H_0$                                                                         |

`estat ovtest` stores the following in `r()`:

#### Scalars

|                      |                             |
|----------------------|-----------------------------|
| <code>r(p)</code>    | two-sided <i>p</i> -value   |
| <code>r(F)</code>    | <i>F</i> statistic          |
| <code>r(df)</code>   | degrees of freedom          |
| <code>r(df_r)</code> | residual degrees of freedom |

## Variance inflation factors

### Description for `estat vif`

`estat vif` calculates the centered or uncentered variance inflation factors (VIFs) for the independent variables specified in a linear regression model.

### Menu for `estat`

Statistics > Postestimation

### Syntax for `estat vif`

```
estat vif [, uncentered]
```

### Option for `estat vif`

`uncentered` requests the computation of the uncentered variance inflation factors. The uncentered VIFs are often used to detect the collinearity of the regressors with the constant. `uncentered` must be specified if the regression model did not include a constant term because centered VIFs are not appropriate for these models.

## Remarks and examples for estat vif

Problems arise in regression when the predictors are highly correlated. In this situation, there may be a significant change in the regression coefficients if you add or delete an independent variable. The estimated standard errors of the fitted coefficients are inflated, or the estimated coefficients may not be statistically significant even though a statistical relation exists between the dependent and independent variables.

Data analysts rely on these facts to check informally for the presence of multicollinearity. `estat vif`, another command for use after `regress`, calculates the variance inflation factors and tolerances for each of the independent variables.

The output shows the variance inflation factors together with their reciprocals. Some analysts compare the reciprocals with a predetermined tolerance. In the comparison, if the reciprocal of the VIF is smaller than the tolerance, the associated predictor variable is removed from the regression model. However, most analysts rely on informal rules of thumb applied to the VIF; see [Chatterjee and Hadi \(2012\)](#). According to these rules, there is evidence of multicollinearity if

1. The largest VIF is greater than 10 (some choose a more conservative threshold value of 30).
2. The mean of all the VIFs is considerably larger than 1.

### ▷ Example 11: estat vif

We examine a regression model fit using the ubiquitous automobile dataset:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. regress price mpg rep78 trunk headroom length turn displ gear_ratio
```

| Source   | SS        | df | MS         | Number of obs | = | 69     |
|----------|-----------|----|------------|---------------|---|--------|
| Model    | 264102049 | 8  | 33012756.2 | F(8, 60)      | = | 6.33   |
| Residual | 312694909 | 60 | 5211581.82 | Prob > F      | = | 0.0000 |
|          |           |    |            | R-squared     | = | 0.4579 |
|          |           |    |            | Adj R-squared | = | 0.3856 |
| Total    | 576796959 | 68 | 8482308.22 | Root MSE      | = | 2282.9 |

| price        | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |
|--------------|-------------|-----------|-------|-------|----------------------|
| mpg          | -144.84     | 82.12751  | -1.76 | 0.083 | -309.1195 19.43948   |
| rep78        | 727.5783    | 337.6107  | 2.16  | 0.035 | 52.25638 1402.9      |
| trunk        | 44.02061    | 108.141   | 0.41  | 0.685 | -172.2935 260.3347   |
| headroom     | -807.0996   | 435.5802  | -1.85 | 0.069 | -1678.39 64.19062    |
| length       | -8.688914   | 34.89848  | -0.25 | 0.804 | -78.49626 61.11843   |
| turn         | -177.9064   | 137.3455  | -1.30 | 0.200 | -452.6383 96.82551   |
| displacement | 30.73146    | 7.576952  | 4.06  | 0.000 | 15.5753 45.88762     |
| gear_ratio   | 1500.119    | 1110.959  | 1.35  | 0.182 | -722.1303 3722.368   |
| _cons        | 6691.976    | 7457.906  | 0.90  | 0.373 | -8226.058 21610.01   |

```
. estat vif
```

| Variable     | VIF  | 1/VIF    |
|--------------|------|----------|
| length       | 8.22 | 0.121614 |
| displacement | 6.50 | 0.153860 |
| turn         | 4.85 | 0.205997 |
| gear_ratio   | 3.45 | 0.290068 |
| mpg          | 3.03 | 0.330171 |
| trunk        | 2.88 | 0.347444 |
| headroom     | 1.80 | 0.554917 |
| rep78        | 1.46 | 0.686147 |
| Mean VIF     | 4.02 |          |

The results are mixed. Although we have no VIFs greater than 10, the mean VIF is greater than 1, though not considerably so. We could continue the investigation of collinearity, but given that other authors advise that collinearity is a problem only when VIFs exist that are greater than 30 (contradicting our rule above), we will not do so here.

◀

### ▷ Example 12: estat vif, with strong evidence of multicollinearity

This example comes from a dataset described in [Kutner, Nachtsheim, and Neter \(2004, 257\)](#) that examines body fat as modeled by caliper measurements on the triceps, midarm, and thigh.

```
. use https://www.stata-press.com/data/r19/bodyfat
(Body fat)
. regress bodyfat tricep thigh midarm
```

| Source   | SS         | df | MS         | Number of obs | = | 20     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 396.984607 | 3  | 132.328202 | F(3, 16)      | = | 21.52  |
| Residual | 98.4049068 | 16 | 6.15030667 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.8014 |
|          |            |    |            | Adj R-squared | = | 0.7641 |
| Total    | 495.389513 | 19 | 26.0731323 | Root MSE      | = | 2.48   |

| bodyfat | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|---------|-------------|-----------|-------|-------|----------------------|----------|
| triceps | 4.334085    | 3.015511  | 1.44  | 0.170 | -2.058512            | 10.72668 |
| thigh   | -2.856842   | 2.582015  | -1.11 | 0.285 | -8.330468            | 2.616785 |
| midarm  | -2.186056   | 1.595499  | -1.37 | 0.190 | -5.568362            | 1.19625  |
| _cons   | 117.0844    | 99.78238  | 1.17  | 0.258 | -94.44474            | 328.6136 |

```
. estat vif
```

| Variable | VIF    | 1/VIF    |
|----------|--------|----------|
| triceps  | 708.84 | 0.001411 |
| thigh    | 564.34 | 0.001772 |
| midarm   | 104.61 | 0.009560 |
| Mean VIF | 459.26 |          |

Here we see strong evidence of multicollinearity in our model. More investigation reveals that the measurements on the thigh and the triceps are highly correlated:

```
. correlate triceps thigh midarm
(obs=20)
```

|         | triceps | thigh  | midarm |
|---------|---------|--------|--------|
| triceps | 1.0000  |        |        |
| thigh   | 0.9238  | 1.0000 |        |
| midarm  | 0.4578  | 0.0847 | 1.0000 |

If we remove the predictor `tricep` from the model (because it had the highest VIF), we get

```
. regress bodyfat thigh midarm
```

| Source   | SS         | df | MS         | Number of obs | = | 20     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 384.279748 | 2  | 192.139874 | F(2, 17)      | = | 29.40  |
| Residual | 111.109765 | 17 | 6.53586854 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.7757 |
|          |            |    |            | Adj R-squared | = | 0.7493 |
| Total    | 495.389513 | 19 | 26.0731323 | Root MSE      | = | 2.5565 |

| bodyfat | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|---------|-------------|-----------|-------|-------|----------------------|----------|
| thigh   | .8508818    | .1124482  | 7.57  | 0.000 | .6136367             | 1.088127 |
| midarm  | .0960295    | .1613927  | 0.60  | 0.560 | -.2444792            | .4365383 |
| _cons   | -25.99696   | 6.99732   | -3.72 | 0.002 | -40.76001            | -11.2339 |

```
. estat vif
```

| Variable | VIF  | 1/VIF    |
|----------|------|----------|
| midarm   | 1.01 | 0.992831 |
| thigh    | 1.01 | 0.992831 |
| Mean VIF | 1.01 |          |

Note how the coefficients change and how the estimated standard errors for each of the regression coefficients become much smaller. The calculated value of  $R^2$  for the overall regression for the subset model does not appreciably decline when we remove the correlated predictor. Removing an independent variable from the model is one way to deal with multicollinearity. Other methods include ridge regression, weighted least squares, and restricting the use of the fitted model to data that follow the same pattern of multicollinearity. In economic studies, it is sometimes possible to estimate the regression coefficients from different subsets of the data by using cross-section and time series.



All examples above demonstrated the use of centered VIFs. As pointed out by [Belsley \(1991\)](#), the centered VIFs may fail to discover collinearity involving the constant term. One solution is to use the uncentered VIFs instead. According to the definition of the uncentered VIFs, the constant is viewed as a legitimate explanatory variable in a regression model, which allows one to obtain the VIF value for the constant term.

▷ Example 13: estat vif, with strong evidence of collinearity with the constant term

Consider the extreme example in which one of the regressors is highly correlated with the constant. We simulate the data and examine both centered and uncentered VIF diagnostics after fitted regression model as follows.

```
. use https://www.stata-press.com/data/r19/extreme_collin
. regress y one x z
```

| Source   | SS         | df | MS         | Number of obs | = | 100     |
|----------|------------|----|------------|---------------|---|---------|
| Model    | 223801.985 | 3  | 74600.6617 | F(3, 96)      | = | 2710.27 |
| Residual | 2642.42124 | 96 | 27.5252213 | Prob > F      | = | 0.0000  |
|          |            |    |            | R-squared     | = | 0.9883  |
|          |            |    |            | Adj R-squared | = | 0.9880  |
| Total    | 226444.406 | 99 | 2287.31723 | Root MSE      | = | 5.2464  |

| y     | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|-------|-------------|-----------|-------|-------|----------------------|----------|
| one   | -3.278582   | 10.5621   | -0.31 | 0.757 | -24.24419            | 17.68702 |
| x     | 2.038696    | .0242673  | 84.01 | 0.000 | 1.990526             | 2.086866 |
| z     | 4.863137    | .2681036  | 18.14 | 0.000 | 4.330956             | 5.395319 |
| _cons | 9.760075    | 10.50935  | 0.93  | 0.355 | -11.10082            | 30.62097 |

```
. estat vif
```

| Variable | VIF  | 1/VIF    |
|----------|------|----------|
| z        | 1.03 | 0.968488 |
| x        | 1.03 | 0.971307 |
| one      | 1.00 | 0.995425 |
| Mean VIF | 1.02 |          |

```
. estat vif, uncentered
```

| Variable | VIF    | 1/VIF    |
|----------|--------|----------|
| one      | 402.94 | 0.002482 |
| _cons    | 401.26 | 0.002492 |
| z        | 2.93   | 0.341609 |
| x        | 1.13   | 0.888705 |
| Mean VIF | 202.06 |          |

According to the values of the centered VIFs (1.03, 1.03, 1.00), no harmful collinearity is detected in the model. However, by the construction of these simulated data, we know that one is highly collinear with the constant term. As such, the large values of uncentered VIFs for one (402.94) and \_cons (401.26) reveal high collinearity of the variable one with the constant term.



## Measures of effect size

### Description for estat esize

`estat esize` calculates effect sizes for linear models after `regress` or `anova`. By default, `estat esize` reports  $\eta^2$  estimates (Kerlinger and Lee 2000), which are equivalent to  $R^2$  estimates. If the option `epsilon` is specified, `estat esize` reports  $\epsilon^2$  estimates (Grissom and Kim 2012). If the option `omega` is specified, `estat esize` reports  $\omega^2$  estimates (Grissom and Kim 2012). Both  $\epsilon^2$  and  $\omega^2$  are adjusted  $R^2$  estimates. Confidence intervals for  $\eta^2$  estimates are estimated by using the noncentral  $F$  distribution (Smithson 2001). See Kline (2013) or Thompson (2006) for further information.

### Menu for estat

Statistics > Postestimation

### Syntax for estat esize

```
estat esize [, epsilon omega level(#)]
```

`collect` is allowed with `estat esize`; see [U] 11.1.10 Prefix commands.

### Options for estat esize

`epsilon` specifies that the  $\epsilon^2$  estimates of effect size be reported. The default is  $\eta^2$  estimates.

`omega` specifies that the  $\omega^2$  estimates of effect size be reported. The default is  $\eta^2$  estimates.

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [U] 20.8 Specifying the width of confidence intervals.

### Remarks and examples for estat esize

Whereas  $p$ -values are used to assess the statistical significance of a result, measures of effect size are used to assess the practical significance of a result. Effect sizes can be broadly categorized as “measures of group differences” (the  $d$  family) and “measures of association” (the  $r$  family); see Ellis (2010, table 1.1). The  $d$  family includes estimators such as Cohen’s  $D$ , Hedges’s  $G$ , and Glass’s  $\Delta$  (also see [R] `esize`). The  $r$  family includes estimators such as the point-biserial correlation coefficient,  $\eta^2$ ,  $\epsilon^2$ , and  $\omega^2$ . For an introduction to the concepts and calculation of effect sizes, see Kline (2013) or Thompson (2006). For a more detailed discussion, see Kirk (1996), Ellis (2010), Cumming (2012), Grissom and Kim (2012), and Kelley and Preacher (2012).

## ▷ Example 14: Calculating effect sizes for a linear regression model

Suppose we fit a linear regression model for low-birthweight infants.

```
. use https://www.stata-press.com/data/r19/lbw
(Hosmer & Lemeshow data)
. regress bwt smoke i.race
```

| Source   | SS         | df  | MS         | Number of obs | = | 189    |
|----------|------------|-----|------------|---------------|---|--------|
| Model    | 12346897.6 | 3   | 4115632.54 | F(3, 185)     | = | 8.69   |
| Residual | 87568400.9 | 185 | 473342.708 | Prob > F      | = | 0.0000 |
|          |            |     |            | R-squared     | = | 0.1236 |
|          |            |     |            | Adj R-squared | = | 0.1094 |
| Total    | 99915298.6 | 188 | 531464.354 | Root MSE      | = | 688    |

| bwt   | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |
|-------|-------------|-----------|-------|-------|----------------------|
| smoke | -428.0254   | 109.0033  | -3.93 | 0.000 | -643.0746 -212.9761  |
| race  |             |           |       |       |                      |
| Black | -450.54     | 153.066   | -2.94 | 0.004 | -752.5194 -148.5607  |
| Other | -454.1813   | 116.436   | -3.90 | 0.000 | -683.8944 -224.4683  |
| _cons | 3334.858    | 91.74301  | 36.35 | 0.000 | 3153.86 3515.855     |

We can use the `estat esize` command to calculate  $\eta^2$  for the entire model and a partial  $\eta^2$  for each term in the model.

```
. estat esize
Effect sizes for linear models
```

| Source | Eta-squared | df | [95% conf. interval] |
|--------|-------------|----|----------------------|
| Model  | .1235736    | 3  | .0399862 .2041365    |
| smoke  | .0769345    | 1  | .0193577 .1579213    |
| race   | .0908394    | 2  | .0233037 .1700334    |

Note: Eta-squared values for individual model terms are partial.

The overall model effect size is 0.124. This means that roughly 12.4% of the variation in `bwt` is explained by the model. The partial effect size for `smoke` is 0.077. This means that roughly 7.7% of the variation in `bwt` is explained by `smoke` after you remove the variation explained by all other terms.

The `omega` option causes `estat esize` to report  $\omega^2$  and partial  $\omega^2$ .

```
. estat esize, omega
Effect sizes for linear models
```

| Source | Omega-squared | df |
|--------|---------------|----|
| Model  | .1088457      | 3  |
| smoke  | .0715877      | 1  |
| race   | .0806144      | 2  |

Note: Omega-squared values for individual model terms are partial.

## ▷ Example 15: Calculating effect size for an ANOVA model

We can use `estat esize` after ANOVA models as well.

```
. anova bwt smoke race
```

|          | Number of obs = | 189     | R-squared =     | 0.1236 |        |
|----------|-----------------|---------|-----------------|--------|--------|
|          | Root MSE =      | 687.999 | Adj R-squared = | 0.1094 |        |
| Source   | Partial SS      | df      | MS              | F      | Prob>F |
| Model    | 12346898        | 3       | 4115632.5       | 8.69   | 0.0000 |
| smoke    | 7298536.6       | 1       | 7298536.6       | 15.42  | 0.0001 |
| race     | 8749453.3       | 2       | 4374726.6       | 9.24   | 0.0001 |
| Residual | 87568401        | 185     | 473342.71       |        |        |
| Total    | 99915299        | 188     | 531464.35       |        |        |

```
. estat esize
```

Effect sizes for linear models

| Source | Eta-squared | df | [95% conf. interval] |          |
|--------|-------------|----|----------------------|----------|
| Model  | .1235736    | 3  | .0399862             | .2041365 |
| smoke  | .0769345    | 1  | .0193577             | .1579213 |
| race   | .0908394    | 2  | .0233037             | .1700334 |

Note: Eta-squared values for individual model terms are partial.



## □ Technical note

$\eta^2$  was developed in the context of analysis of variance. Thus, the published research on the calculation of confidence intervals focuses on cases where the numerator degrees of freedom are relatively small (for example,  $df < 20$ ).

Some combinations of the  $F$  statistic, numerator degrees of freedom, and denominator degrees of freedom yield confidence limits that do not contain the corresponding estimated value for an  $\eta^2$ . This problem is most commonly observed for larger numerator degrees of freedom.

Nothing in the literature suggests alternative methods for constructing confidence intervals in such cases; therefore, we recommend cautious interpretation of confidence intervals for  $\eta^2$  when the numerator degrees of freedom are greater than 20.



## Stored results for estat esize

estat esize stores the following results in `r()`:

### Scalars

`r(level)` confidence level

### Matrices

`r(esize)` a matrix of effect sizes, confidence intervals, degrees of freedom, and  $F$  statistics with rows corresponding to each term in the model

|                         |                                     |
|-------------------------|-------------------------------------|
| <code>esize[.,1]</code> | $\eta^2$                            |
| <code>esize[.,2]</code> | lower confidence bound for $\eta^2$ |
| <code>esize[.,3]</code> | upper confidence bound for $\eta^2$ |
| <code>esize[.,4]</code> | $\varepsilon^2$                     |
| <code>esize[.,5]</code> | $\omega^2$                          |
| <code>esize[.,6]</code> | numerator degrees of freedom        |
| <code>esize[.,7]</code> | denominator degrees of freedom      |
| <code>esize[.,8]</code> | $F$ statistic                       |

## Methods and formulas

See [Hamilton \(2013, chap. 7\)](#), [Kohler and Kreuter \(2012, sec. 9.3\)](#), or [Baum \(2006, chap. 5\)](#) for an overview of using Stata to perform regression diagnostics. See [Peracchi \(2001, chap. 8\)](#) for a mathematically rigorous discussion of diagnostics.

Methods and formulas are presented under the following headings:

*predict*  
Special-interest postestimation commands

## predict

Assume that you have already fit the regression model

$$\mathbf{y} = \mathbf{X}\mathbf{b} + \mathbf{e}$$

where  $\mathbf{X}$  is  $n \times k$ .

Denote the previously estimated coefficient vector by  $\mathbf{b}$  and its estimated variance matrix by  $\mathbf{V}$ . `predict` works by recalling various aspects of the model, such as  $\mathbf{b}$ , and combining that information with the data currently in memory. Let  $\mathbf{x}_j$  be the  $j$ th observation currently in memory, and let  $s^2$  be the mean squared error of the regression.

If the user specified weights in `regress`, then  $\mathbf{X}'\mathbf{X}$  in the following formulas is replaced by  $\mathbf{X}'\mathbf{D}\mathbf{X}$ , where  $\mathbf{D}$  is defined in [Weighted regression](#) under *Methods and formulas* in [\[R\] regress](#).

Let  $\mathbf{V} = s^2(\mathbf{X}'\mathbf{X})^{-1}$ . Let  $k$  be the number of independent variables including the intercept, if any, and let  $y_j$  be the observed value of the dependent variable.

The *predicted value* (`xb` option) is defined as  $\hat{y}_j = \mathbf{x}_j\mathbf{b}$ .

Let  $\ell_j$  represent a lower bound for an observation  $j$  and  $u_j$  represent an upper bound. The probability that  $y_j|\mathbf{x}_j$  would be observed in the interval  $(\ell_j, u_j)$ —the `pr` ( $\ell$ ,  $u$ ) option—is

$$P(\ell_j, u_j) = \Pr(\ell_j < \mathbf{x}_j\mathbf{b} + e_j < u_j) = \Phi\left(\frac{u_j - \hat{y}_j}{s}\right) - \Phi\left(\frac{\ell_j - \hat{y}_j}{s}\right)$$

where for the `pr` ( $\ell$ ,  $u$ ), `e` ( $\ell$ ,  $u$ ), and `ystar` ( $\ell$ ,  $u$ ) options,  $\ell_j$  and  $u_j$  can be anywhere in the range  $(-\infty, +\infty)$ .

The option `e` ( $\ell$ ,  $u$ ) computes the expected value of  $y_j|\mathbf{x}_j$  conditional on  $y_j|\mathbf{x}_j$  being in the interval  $(\ell_j, u_j)$ , that is, when  $y_j|\mathbf{x}_j$  is truncated. It can be expressed as

$$E(\ell_j, u_j) = E(\mathbf{x}_j\mathbf{b} + e_j \mid \ell_j < \mathbf{x}_j\mathbf{b} + e_j < u_j) = \hat{y}_j - s \frac{\phi\left(\frac{u_j - \hat{y}_j}{s}\right) - \phi\left(\frac{\ell_j - \hat{y}_j}{s}\right)}{\Phi\left(\frac{u_j - \hat{y}_j}{s}\right) - \Phi\left(\frac{\ell_j - \hat{y}_j}{s}\right)}$$

where  $\phi$  is the normal density and  $\Phi$  is the cumulative normal.

You can also compute `ystar` ( $\ell$ ,  $u$ )—the expected value of  $y_j|\mathbf{x}_j$ , where  $y_j$  is assumed censored at  $\ell_j$  and  $u_j$ :

$$y_j^* = \begin{cases} \ell_j & \text{if } \mathbf{x}_j\mathbf{b} + e_j \leq \ell_j \\ \mathbf{x}_j\mathbf{b} + e_j & \text{if } \ell_j < \mathbf{x}_j\mathbf{b} + e_j < u_j \\ u_j & \text{if } \mathbf{x}_j\mathbf{b} + e_j \geq u_j \end{cases}$$

This computation can be expressed in several ways, but the most intuitive formulation involves a combination of the two statistics just defined:

$$y_j^* = P(-\infty, \ell_j)\ell_j + P(\ell_j, u_j)E(\ell_j, u_j) + P(u_j, +\infty)u_j$$

A diagonal element of the projection matrix (`hat`) or (`leverage`) is given by

$$h_j = \mathbf{x}_j(\mathbf{X}'\mathbf{X})^{-1}\mathbf{x}_j'$$

The *standard error of the prediction* (the `stdp` option) is defined as  $s_{p_j} = \sqrt{\mathbf{x}_j\mathbf{V}\mathbf{x}_j'}$

and can also be written as  $s_{p_j} = s\sqrt{h_j}$ .

The *standard error of the forecast* (`stdf`) is defined as  $s_{f_j} = s\sqrt{1 + h_j}$ .

The *standard error of the residual* (`stdr`) is defined as  $s_{r_j} = s\sqrt{1 - h_j}$ .

The *residuals* (`residuals`) are defined as  $\hat{e}_j = y_j - \hat{y}_j$ .

The *standardized residuals* (`rstandard`) are defined as  $\hat{e}_{s_j} = \hat{e}_j/s_{r_j}$ .

The *Studentized residuals* (`rstudent`) are defined as

$$r_j = \frac{\hat{e}_j}{s_{(j)}\sqrt{1-h_j}}$$

where  $s_{(j)}$  represents the root mean squared error with the  $j$ th observation removed, which is given by

$$s_{(j)}^2 = \frac{s^2(n-k)}{n-k-1} - \frac{\hat{e}_j^2}{(n-k-1)(1-h_j)}$$

where  $n$  is the number of observations and  $k$  is the number of right-hand-side variables (including the constant).

Cook's  $D$  (`cooksD`) is given by

$$D_j = \frac{\hat{e}_{s_j}^2 (s_{p_j}/s_{r_j})^2}{k} = \frac{h_j \hat{e}_j^2}{ks^2(1-h_j)^2}$$

DFITS (`dfits`) is given by

$$\text{DFITS}_j = r_j \sqrt{\frac{h_j}{1-h_j}}$$

Welsch distance (`welsch`) is given by

$$W_j = \frac{r_j \sqrt{h_j(n-1)}}{1-h_j}$$

COVRATIO (`covratio`) is given by

$$\text{COVRATIO}_j = \frac{1}{1-h_j} \left( \frac{n-k-\hat{e}_{s_j}^2}{n-k-1} \right)^k$$

The DFBETAS (`dfbeta`) for a particular regressor  $x_i$  are given by

$$\text{DFBETA}_{j,i} = \frac{r_j u_{j,i}}{\sqrt{U^2(1-h_j)}}$$

where  $u_j$  are the residuals obtained from a regression of  $x_i$  on the remaining  $x$ 's and  $U^2 = \sum_j u_j^2$ .

## Special-interest postestimation commands

The omitted-variable test (Ramsey 1969) reported by `estat ovtest` fits the regression  $y_i = \mathbf{x}_i \mathbf{b} + \mathbf{z}_i \mathbf{t} + u_i$  and then performs a standard  $F$  test of  $\mathbf{t} = \mathbf{0}$ . The default test uses  $\mathbf{z}_i = (\hat{y}_i^2, \hat{y}_i^3, \hat{y}_i^4)$ . If `rhs` is specified,  $\mathbf{z}_i = (x_{1i}^2, x_{1i}^3, x_{1i}^4, x_{2i}^2, \dots, x_{mi}^4)$ . In either case, the variables are normalized to have minimum 0 and maximum 1 before powers are calculated.

The test for heteroskedasticity (Breusch and Pagan 1979; Cook and Weisberg 1983) models  $\text{Var}(e_i) = \sigma^2 \exp(\mathbf{z} \mathbf{t})$ , where  $\mathbf{z}$  is a variable list specified by the user, the list of right-hand-side variables, or the fitted values  $\mathbf{x} \hat{\boldsymbol{\beta}}$ . The test is of  $\mathbf{t} = \mathbf{0}$ . Mechanically, `estat hettest` fits the augmented regression  $\hat{e}_i^2 / \hat{\sigma}^2 = a + \mathbf{z}_i \mathbf{t} + v_i$ .

The original Breusch–Pagan/Cook–Weisberg version of the test assumes that the  $e_i$  are normally distributed under the null hypothesis which implies that the score test statistic  $S$  is equal to the model sum of squares from the augmented regression divided by 2. Under the null hypothesis,  $S$  has the  $\chi^2$  distribution with  $m$  degrees of freedom, where  $m$  is the number of columns of  $\mathbf{z}$ .

Koenker (1981) derived a score test of the null hypothesis that  $\mathbf{t} = \mathbf{0}$  under the assumption that the  $e_i$  are independent and identically distributed (i.i.d.). Koenker showed that  $S = N * R^2$  has a large-sample  $\chi^2$  distribution with  $m$  degrees of freedom, where  $N$  is the number of observations and  $R^2$  is from the augmented regression and  $m$  is the number of columns of  $\mathbf{z}$ . `estat hettest, iid` produces this version of the test.

Wooldridge (2020, 270) showed that an  $F$  test of  $\mathbf{t} = \mathbf{0}$  in the augmented regression can also be used under the assumption that the  $e_i$  are i.i.d. `estat hettest, fstat` produces this version of the test.

Szroeter’s class of tests for homoskedasticity against the alternative that the residual variance increases in some variable  $x$  is defined in terms of

$$H = \frac{\sum_{i=1}^n h(x_i) e_i^2}{\sum_{i=1}^n e_i^2}$$

where  $h(x)$  is some weight function that increases in  $x$  (Szroeter 1978).  $H$  is a weighted average of the  $h(x)$ , with the squared residuals serving as weights. Under homoskedasticity,  $H$  should be approximately equal to the unweighted average of  $h(x)$ . Large values of  $H$  suggest that  $e_i^2$  tends to be large where  $h(x)$  is large; that is, the variance indeed increases in  $x$ , whereas small values of  $H$  suggest that the variance actually decreases in  $x$ . `estat szroeter` uses  $h(x_i) = \text{rank}(x_i \text{ in } x_1 \dots x_n)$ ; see Judge et al. [1985, 452] for details. `estat szroeter` displays a normalized version of  $H$ ,

$$Q = \sqrt{\frac{6n}{n^2 - 1}} H$$

which is approximately  $N(0, 1)$  distributed under the null (homoskedasticity).

`estat hettest` and `estat szroeter` provide adjustments of  $p$ -values for multiple testing. The supported methods are described in [R] [test](#).

`estat imtest` performs the information matrix test for the regression model, as well as an orthogonal decomposition into tests for heteroskedasticity  $\delta_1$ , nonnormal skewness  $\delta_2$ , and nonnormal kurtosis  $\delta_3$  (Cameron and Trivedi 1990; Long and Trivedi 1993). The decomposition is obtained via three auxiliary regressions. Let  $e$  be the regression residuals,  $\hat{\sigma}^2$  be the maximum likelihood estimate of  $\sigma^2$  in the regression,  $n$  be the number of observations,  $X$  be the set of  $k$  variables specified with `estat imtest`, and  $R_{\text{un}}^2$  be the uncentered  $R^2$  from a regression.  $\delta_1$  is obtained as  $nR_{\text{un}}^2$  from a regression of  $e^2 - \hat{\sigma}^2$  on the cross products of the variables in  $X$ .  $\delta_2$  is computed as  $nR_{\text{un}}^2$  from a regression of  $e^3 - 3\hat{\sigma}^2 e$  on  $X$ . Finally,  $\delta_3$  is obtained as  $nR_{\text{un}}^2$  from a regression of  $e^4 - 6\hat{\sigma}^2 e^2 - 3\hat{\sigma}^4$  on  $X$ .  $\delta_1$ ,  $\delta_2$ , and  $\delta_3$  are asymptotically  $\chi^2$  distributed with  $1/2k(k+1)$ ,  $K$ , and 1 degree of freedom. The information test statistic  $\delta = \delta_1 + \delta_2 + \delta_3$  is asymptotically  $\chi^2$  distributed with  $1/2k(k+3)$  degrees of freedom. White’s test for heteroskedasticity is computed as  $nR^2$  from a regression of  $\hat{u}^2$  on  $X$  and the cross products of the variables in  $X$ . This test statistic is usually close to  $\delta_1$ .

`estat vif` calculates the centered variance inflation factor ( $VIF_c$ ) (Chatterjee and Hadi 2012, 248–251) for  $x_j$ , given by

$$VIF_c(x_j) = \frac{1}{1 - \widehat{R}_j^2}$$

where  $\widehat{R}_j^2$  is the square of the centered multiple correlation coefficient that results when  $x_j$  is regressed against all other explanatory variables, including the constant.

The uncentered variance inflation factor ( $VIF_{uc}$ ) (Belsley 1991, 28–29) for  $x_j$  is given by

$$VIF_{uc}(x_j) = \frac{1}{1 - \widetilde{R}_j^2}$$

where  $\widetilde{R}_j^2$  is the square of the uncentered multiple correlation coefficient that results when  $x_j$  is regressed against all other explanatory variables and a constant of 1. If the original regression model was fit without a constant, the constant would also be omitted from the regression of  $x_j$ .

The methods and formulas for `estat esize` are described in *Methods and formulas* of [R] `esize`.

## Acknowledgments

`estat ovtest` and `estat htest` are based on programs originally written by Richard Goldstein. `estat imtest`, `estat szroeter`, and the current version of `estat htest` were written by Jeroen Weesie of the Department of Sociology at Utrecht University, The Netherlands. `estat imtest` is based in part on code written by J. Scott Long of the Department of Sociology at Indiana University, coauthor of the Stata Press book *Regression Models for Categorical and Limited Dependent Variables*, and author of the Stata Press book *The Workflow of Data Analysis Using Stata*.

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## Also see

- [R] **regress** — Linear regression
- [R] **regress postestimation diagnostic plots** — Postestimation plots for regress
- [R] **regress postestimation time series** — Postestimation tools for regress with time series
- [SP] **estat moran** — Moran’s test of residual correlation with nearby residuals
- [LASSO] **lassogof** — Goodness of fit after lasso for prediction
- [U] **20 Estimation and postestimation commands**

|             |                         |                          |                         |                         |                          |
|-------------|-------------------------|--------------------------|-------------------------|-------------------------|--------------------------|
| Description | <a href="#">rvfplot</a> | <a href="#">avplot</a>   | <a href="#">avplots</a> | <a href="#">cprplot</a> | <a href="#">acprplot</a> |
|             | <a href="#">rvpplot</a> | <a href="#">lvr2plot</a> | Methods and formulas    | References              | Also see                 |

## Description

The following postestimation commands are of special interest after `regress`:

| Command               | Description                            |
|-----------------------|----------------------------------------|
| <code>rvfplot</code>  | residual-versus-fitted plot            |
| <code>avplot</code>   | added-variable plot                    |
| <code>avplots</code>  | all added-variables plots in one image |
| <code>cprplot</code>  | component-plus-residual plot           |
| <code>acprplot</code> | augmented component-plus-residual plot |
| <code>rvpplot</code>  | residual-versus-predictor plot         |
| <code>lvr2plot</code> | leverage-versus-squared-residual plot  |

These commands are not appropriate with `svy` estimation results.

For a discussion of the terminology used in this entry, see the [Terminology](#) section of *Remarks and examples for predict* in [R] [regress postestimation](#).

## rvfplot

### Description for rfvplot

`rvfplot` graphs a residual-versus-fitted plot, a graph of the residuals against the fitted values.

### Menu for rfvplot

Statistics > Linear models and related > Regression diagnostics > Residual-versus-fitted plot

### Syntax for rfvplot

```
rvfplot [, rvfplot_options]
```

| <i>rvfplot_options</i>                  | Description                                                                                 |
|-----------------------------------------|---------------------------------------------------------------------------------------------|
| Plot                                    |                                                                                             |
| <a href="#">marker_options</a>          | change look of markers (color, size, etc.)                                                  |
| <a href="#">marker_label_options</a>    | add marker labels; change look or position                                                  |
| Add plots                               |                                                                                             |
| <code>addplot (<i>plot</i>)</code>      | add plots to the generated graph                                                            |
| Y axis, X axis, Titles, Legend, Overall |                                                                                             |
| <a href="#">twoway_options</a>          | any options other than <code>by()</code> documented in [G-3] <a href="#">twoway_options</a> |

## Options for rvfplot

## Plot

*marker\_options* affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] *marker\_options*.

*marker\_label\_options* specify if and how the markers are to be labeled; see [G-3] *marker\_label\_options*.

## Add plots

`addplot (plot)` provides a way to add plots to the generated graph. See [G-3] *addplot\_option*.

## Y axis, X axis, Titles, Legend, Overall

*twoway\_options* are any of the options documented in [G-3] *twoway\_options*, excluding `by()`. These include options for titling the graph (see [G-3] *title\_options*) and for saving the graph to disk (see [G-3] *saving\_option*).

## Remarks and examples for rvfplot

`rvfplot` graphs the residuals against the fitted values.

## ▷ Example 1

Using `auto.dta` described in [U] 1.2.2 *Example datasets*, we will use `regress` to fit a model of price on weight, mpg, foreign, and the interaction of foreign with mpg. We specify `foreign#c.mpg` to obtain the interaction of foreign with mpg; see [U] 11.4.3 *Factor variables*.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
```

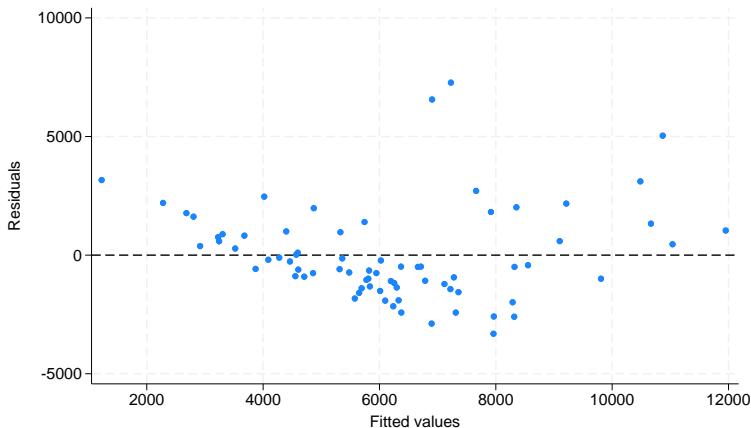
```
. regress price weight foreign#c.mpg
```

| Source   | SS        | df | MS         | Number of obs | = | 74     |
|----------|-----------|----|------------|---------------|---|--------|
| Model    | 350319665 | 4  | 87579916.3 | F(4, 69)      | = | 21.22  |
| Residual | 284745731 | 69 | 4126749.72 | Prob > F      | = | 0.0000 |
|          |           |    |            | R-squared     | = | 0.5516 |
|          |           |    |            | Adj R-squared | = | 0.5256 |
| Total    | 635065396 | 73 | 8699525.97 | Root MSE      | = | 2031.4 |

| price         | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|---------------|-------------|-----------|-------|-------|----------------------|-----------|
| weight        | 4.613589    | .7254961  | 6.36  | 0.000 | 3.166263             | 6.060914  |
| foreign       |             |           |       |       |                      |           |
| Foreign       | 11240.33    | 2751.681  | 4.08  | 0.000 | 5750.878             | 16729.78  |
| mpg           | 263.1875    | 110.7961  | 2.38  | 0.020 | 42.15527             | 484.2197  |
| foreign#c.mpg |             |           |       |       |                      |           |
| Foreign       | -307.2166   | 108.5307  | -2.83 | 0.006 | -523.7294            | -90.70368 |
| _cons         | -14449.58   | 4425.72   | -3.26 | 0.002 | -23278.65            | -5620.51  |

Once we have fit a model, we may use any of the regression diagnostics commands. `rvfplot` (read residual-versus-fitted plot) graphs the residuals against the fitted values:

```
. rvfplot, yline(0)
```



All the diagnostic plot commands allow the `graph twoway` and `graph twoway scatter` options; we specified a `yline(0)` to draw a line across the graph at  $y = 0$ ; see [G-2] [graph twoway scatter](#).

In a well-fitted model, there should be no pattern to the residuals plotted against the fitted values—something not true of our model. Ignoring the two outliers at the top center of the graph, we see curvature in the pattern of the residuals, suggesting a violation of the assumption that price is linear in our independent variables. We might also have seen increasing or decreasing variation in the residuals—heteroskedasticity. Any pattern whatsoever indicates a violation of the least-squares assumptions.

◀

## avplot

### Description for avplot

`avplot` graphs an added-variable plot (a.k.a. partial-regression leverage plot, partial regression plot, or adjusted partial residual plot) after `regress`. *indepvar* may be an independent variable (a.k.a. predictor, carrier, or covariate) that is currently in the model or not.

### Menu for avplot

Statistics > Linear models and related > Regression diagnostics > Added-variable plot

### Syntax for avplot

```
avplot indepvar [, avplot_options]
```

| <i>avplot_options</i>                     | Description                                                                        |
|-------------------------------------------|------------------------------------------------------------------------------------|
| Plot                                      |                                                                                    |
| <i>marker_options</i>                     | change look of markers (color, size, etc.)                                         |
| <i>marker_label_options</i>               | add marker labels; change look or position                                         |
| Reference line                            |                                                                                    |
| <code>rlopts(<i>cline_options</i>)</code> | affect rendition of the reference line                                             |
| Add plots                                 |                                                                                    |
| <code>addplot(<i>plot</i>)</code>         | add other plots to the generated graph                                             |
| Y axis, X axis, Titles, Legend, Overall   |                                                                                    |
| <i>twoway_options</i>                     | any options other than <code>by()</code> documented in [G-3] <i>twoway_options</i> |

### Options for avplot

Plot

*marker\_options* affects the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] *marker\_options*.

*marker\_label\_options* specify if and how the markers are to be labeled; see [G-3] *marker\_label\_options*.

Reference line

`rlopts(cline_options)` affects the rendition of the reference line. See [G-3] *cline\_options*.

Add plots

`addplot(plot)` provides a way to add plots to the generated graph. See [G-3] *addplot\_option*.

Y axis, X axis, Titles, Legend, Overall

*twoway\_options* are any of the options documented in [G-3] *twoway\_options*, excluding `by()`. These include options for titling the graph (see [G-3] *title\_options*) and for saving the graph to disk (see [G-3] *saving\_option*).

## Remarks and examples for avplot

avplot graphs an added-variable plot, also known as the partial-regression leverage plot.

One of the wonderful features of one-regressor regressions (regressions of  $y$  on one  $x$ ) is that we can graph the data and the regression line. There is no easier way to understand the regression than to examine such a graph. Unfortunately, we cannot do this when we have more than one regressor. With two regressors, it is still theoretically possible—the graph must be drawn in three dimensions, but with three or more regressors no graph is possible.

The added-variable plot is an attempt to project multidimensional data back to the two-dimensional world for each of the original regressors. This is, of course, impossible without making some concessions. Call the coordinates on an added-variable plot  $y$  and  $x$ . The added-variable plot has the following properties:

- There is a one-to-one correspondence between  $(x_i, y_i)$  and the  $i$ th observation used in the original regression.
- A regression of  $y$  on  $x$  has the same coefficient and standard error (up to a degree-of-freedom adjustment) as the estimated coefficient and standard error for the regressor in the original regression.
- The “outlierness” of each observation in determining the slope is in some sense preserved.

It is equally important to note the properties that are not listed. The  $y$  and  $x$  coordinates of the added-variable plot cannot be used to identify functional form, or, at least, not well (see [Mallows \[1986\]](#)). In the construction of the added-variable plot, the relationship between  $y$  and  $x$  is forced to be linear.

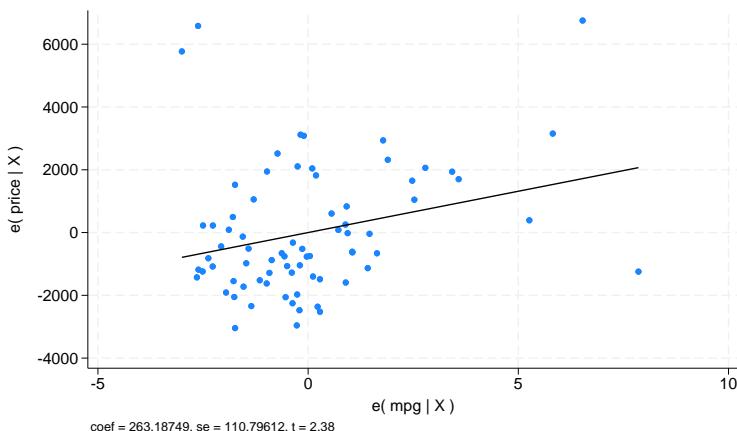
### ▷ Example 2

Let’s use the same model as we used in [example 1](#).

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. regress price weight foreign##c.mpg
(output omitted)
```

We can now examine the added-variable plot for mpg.

```
. avplot mpg
```



This graph suggests a problem in determining the coefficient on mpg. Were this a one-regressor regression, the two points at the top-left corner and the one at the top right would cause us concern, and so it does in our more complicated multiple-regressor case. To identify the problem points, we retyped our command, modifying it to read `avplot mpg, mlabel(make)`, and discovered that the two cars at the top left are the Cadillac Eldorado and the Lincoln Versailles; the point at the top right is the Cadillac Seville. These three cars account for 100% of the luxury cars in our data, suggesting that our model is misspecified. By the way, the point at the lower right of the graph, also cause for concern, is the Plymouth Arrow, our data entry error.

◀

## □ Technical note

Stata's `avplot` command can be used with regressors already in the model, as we just did, or with potential regressors not yet in the model. In either case, `avplot` will produce the correct graph. The name “added-variable plot” is unfortunate in the case when the variable is already among the list of regressors but is, we think, still preferable to the name “partial-regression leverage plot” assigned by [Belsley, Kuh, and Welsch \(1980, 30\)](#) and more in the spirit of the original use of such plots by [Mosteller and Tukey \(1977, 271–279\)](#). [Welsch \(1986, 403\)](#), however, disagrees: “I am sorry to see that [Chatterjee and Hadi \[1986\]](#) endorse the term ‘added-variable plot’ when  $X_j$  is part of the original model” and goes on to suggest the name “adjusted partial residual plot”.

□

## avplots

### Description for avplots

`avplots` graphs all the added-variable plots in one image.

### Menu for avplots

Statistics > Linear models and related > Regression diagnostics > Added-variable plot

### Syntax for avplots

```
avplots [, avplots_options]
```

| <i>avplots_options</i>                  | Description                                                            |
|-----------------------------------------|------------------------------------------------------------------------|
| Plot                                    |                                                                        |
| <i>marker_options</i>                   | change look of markers (color, size, etc.)                             |
| <i>marker_label_options</i>             | add marker labels; change look or position                             |
| <i>combine_options</i>                  | any of the options documented in [G-2] <b>graph combine</b>            |
| Reference line                          |                                                                        |
| <i>rlopts</i> ( <i>cline_options</i> )  | affect rendition of the reference line                                 |
| Y axis, X axis, Titles, Legend, Overall |                                                                        |
| <i>twoway_options</i>                   | any options other than by () documented in [G-3] <i>twoway_options</i> |

## Options for avplots

### Plot

*marker\_options* affects the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] *marker\_options*.

*marker\_label\_options* specify if and how the markers are to be labeled; see [G-3] *marker\_label\_options*.

*combine\_options* are any of the options documented in [G-2] *graph combine*. These include options for titling the graph (see [G-3] *title\_options*) and for saving the graph to disk (see [G-3] *saving\_option*).

### Reference line

*r1opts* (*cline\_options*) affects the rendition of the reference line. See [G-3] *cline\_options*.

### Y axis, X axis, Titles, Legend, Overall

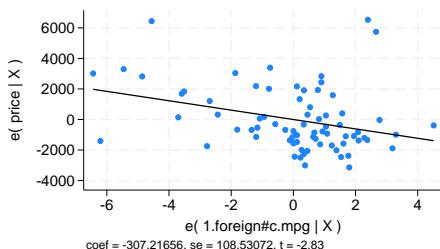
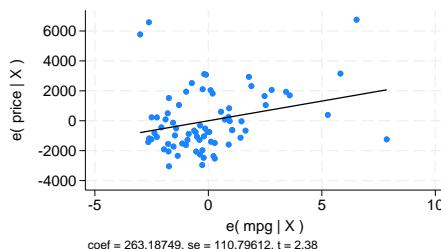
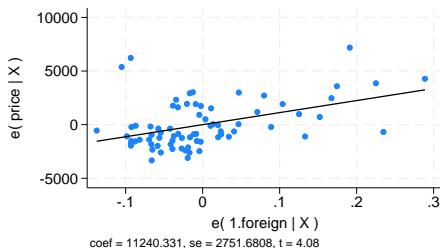
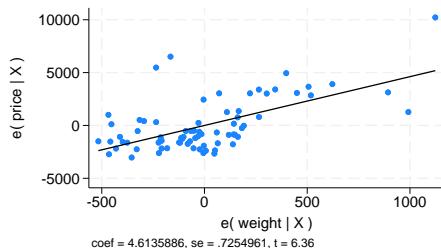
*twoway\_options* are any of the options documented in [G-3] *twoway\_options*, excluding *by()*. These include options for titling the graph (see [G-3] *title\_options*) and for saving the graph to disk (see [G-3] *saving\_option*).

## Remarks and examples for avplots

### ▷ Example 3

In [example 2](#), we used `avplot` to examine the added-variable plot for `mpg` in our regression of `price` on `weight` and `foreign##c.mpg`. Now, let's use `avplots` to graph an added-variable plot for every regressor in the data.

```
. avplots
```



## cprplot

### Description for cprplot

cprplot graphs a component-plus-residual plot (a.k.a. partial residual plot) after regress. *indepvar* must be an independent variable that is currently in the model.

### Menu for cprplot

Statistics > Linear models and related > Regression diagnostics > Component-plus-residual plot

### Syntax for cprplot

```
cprplot indepvar [, cprplot_options]
```

| <i>cprplot_options</i> | Description |
|------------------------|-------------|
|------------------------|-------------|

#### Plot

*marker\_options*

change look of markers (color, size, etc.)

*marker\_label\_options*

add marker labels; change look or position

#### Reference line

*rlopts* (*cline\_options*)

affect rendition of the reference line

#### Options

*lowess*

add a lowess smooth of the plotted points

*lsopts* (*lowess\_options*)

affect rendition of the lowess smooth

*mspline*

add median spline of the plotted points

*msopts* (*mspline\_options*)

affect rendition of the spline

#### Add plots

*addplot* (*plot*)

add other plots to the generated graph

#### Y axis, X axis, Titles, Legend, Overall

*twoway\_options*

any options other than by() documented in [G-3] *twoway\_options*

## Options for `cprplot`

### Plot

`marker_options` affects the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] [marker\\_options](#).

`marker_label_options` specify if and how the markers are to be labeled; see [G-3] [marker\\_label\\_options](#).

### Reference line

`rlopts` (`cline_options`) affects the rendition of the reference line. See [G-3] [cline\\_options](#).

### Options

`lowess` adds a lowess smooth of the plotted points to assist in detecting nonlinearities.

`lsopts` (`lowess_options`) affects the rendition of the lowess smooth. For an explanation of these options, especially the `bwidth()` option, see [R] [lowess](#). Specifying `lsopts()` implies the `lowess` option.

`mspline` adds a median spline of the plotted points to assist in detecting nonlinearities.

`msopts` (`mspline_options`) affects the rendition of the spline. For an explanation of these options, especially the `bands()` option, see [G-2] [graph twoway mspline](#). Specifying `msopts()` implies the `mspline` option.

### Add plots

`addplot` (`plot`) provides a way to add other plots to the generated graph. See [G-3] [addplot\\_option](#).

### Y axis, X axis, Titles, Legend, Overall

`twoway_options` are any of the options documented in [G-3] [twoway\\_options](#), excluding `by()`. These include options for titling the graph (see [G-3] [title\\_options](#)) and for saving the graph to disk (see [G-3] [saving\\_option](#)).

## Remarks and examples for `cprplot`

Added-variable plots are successful at identifying outliers, but they cannot be used to identify functional form. The component-plus-residual plot (Ezekiel 1924; Larsen and McCleary 1972) is another attempt at projecting multidimensional data into a two-dimensional form, but with different properties. Although the added-variable plot can identify outliers, the component-plus-residual plot cannot. It can, however, be used to examine the functional form assumptions of the model. Both plots have the property that a regression line through the coordinates has a slope equal to the estimated coefficient in the regression model.

## ▷ Example 4

We illustrate component-plus-residual plots using a variation of `auto.dta`.

```
. use https://www.stata-press.com/data/r19/auto1
(Automobile models)
```

```
. regress price mpg weight
```

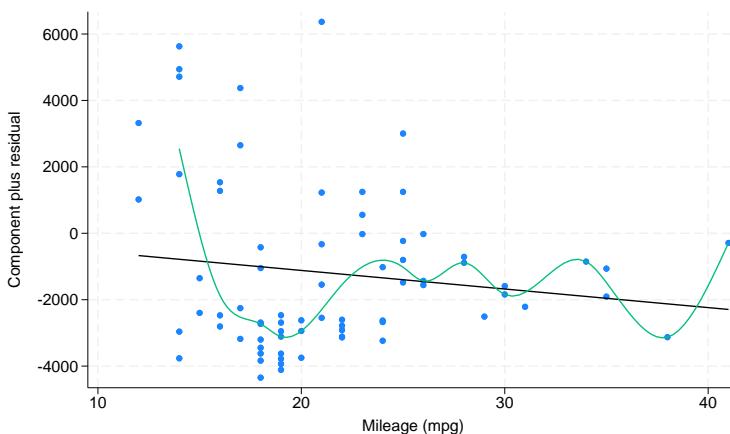
| Source   | SS        | df | MS         | Number of obs | = | 74     |
|----------|-----------|----|------------|---------------|---|--------|
| Model    | 187716578 | 2  | 93858289   | F(2, 71)      | = | 14.90  |
| Residual | 447348818 | 71 | 6300687.58 | Prob > F      | = | 0.0000 |
|          |           |    |            | R-squared     | = | 0.2956 |
|          |           |    |            | Adj R-squared | = | 0.2757 |
| Total    | 635065396 | 73 | 8699525.97 | Root MSE      | = | 2510.1 |

| price  | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |
|--------|-------------|-----------|-------|-------|----------------------|
| mpg    | -55.9393    | 75.24136  | -0.74 | 0.460 | -205.9663 94.08771   |
| weight | 1.710992    | .5861682  | 2.92  | 0.005 | .5422063 2.879779    |
| _cons  | 2197.9      | 3190.768  | 0.69  | 0.493 | -4164.311 8560.11    |

In fact, we know that the effects of `mpg` in this model are nonlinear—if we added `mpg squared` to the model, its coefficient would have a  $t$  statistic of 2.38, the  $t$  statistic on `mpg` would become  $-2.48$ , and `weight`'s effect would become about one-third of its current value and become statistically insignificant. Pretend that we do not know this.

The component-plus-residual plot for `mpg` is

```
. cprplot mpg, mspline msopts(bands(13))
```



We are supposed to examine the above graph for nonlinearities or, equivalently, ask if the regression line, which has slope equal to the estimated effect of `mpg` in the original model, fits the data adequately. To assist our eyes, we added a median spline. Perhaps some people may detect nonlinearity from this graph, but we assert that if we had not previously revealed the nonlinearity of `mpg` and if we had not added the median spline, the graph would not overly bother us.

## acprplot

### Description for acprplot

`acprplot` graphs an augmented component-plus-residual plot (a.k.a. augmented partial residual plot) as described by [Mallows \(1986\)](#). This seems to work better than the component-plus-residual plot for identifying nonlinearities in the data.

### Menu for acprplot

Statistics > Linear models and related > Regression diagnostics > Augmented component-plus-residual plot

### Syntax for acprplot

```
acprplot indepvar [, acprplot_options]
```

| <i>acprplot_options</i>                  | Description                                                            |
|------------------------------------------|------------------------------------------------------------------------|
| Plot                                     |                                                                        |
| <i>marker_options</i>                    | change look of markers (color, size, etc.)                             |
| <i>marker_label_options</i>              | add marker labels; change look or position                             |
| Reference line                           |                                                                        |
| <i>rlopts</i> ( <i>cline_options</i> )   | affect rendition of the reference line                                 |
| Options                                  |                                                                        |
| <i>lowess</i>                            | add a lowess smooth of the plotted points                              |
| <i>lsopts</i> ( <i>lowess_options</i> )  | affect rendition of the lowess smooth                                  |
| <i>mspline</i>                           | add median spline of the plotted points                                |
| <i>msopts</i> ( <i>mspline_options</i> ) | affect rendition of the spline                                         |
| Add plots                                |                                                                        |
| <i>addplot</i> ( <i>plot</i> )           | add other plots to the generated graph                                 |
| Y axis, X axis, Titles, Legend, Overall  |                                                                        |
| <i>twoway_options</i>                    | any options other than by () documented in [G-3] <i>twoway_options</i> |

### Options for acprplot

#### Plot

*marker\_options* affects the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] *marker\_options*.

*marker\_label\_options* specify if and how the markers are to be labeled; see [G-3] *marker\_label\_options*.

#### Reference line

*rlopts*(*cline\_options*) affects the rendition of the reference line. See [G-3] *cline\_options*.

#### Options

*lowess* adds a lowess smooth of the plotted points to assist in detecting nonlinearities.

`lsopts` (*lowess\_options*) affects the rendition of the lowess smooth. For an explanation of these options, especially the `bwidth()` option, see [R] [lowess](#). Specifying `lsopts()` implies the `lowess` option.

`mspline` adds a median spline of the plotted points to assist in detecting nonlinearities.

`msopts` (*mspline\_options*) affects the rendition of the spline. For an explanation of these options, especially the `bands()` option, see [G-2] [graph twoway mspline](#). Specifying `msopts()` implies the `mspline` option.

#### Add plots

`addplot` (*plot*) provides a way to add other plots to the generated graph. See [G-3] [addplot\\_option](#).

#### Y axis, X axis, Titles, Legend, Overall

*twoway\_options* are any of the options documented in [G-3] [twoway\\_options](#), excluding `by()`. These include options for titling the graph (see [G-3] [title\\_options](#)) and for saving the graph to disk (see [G-3] [saving\\_option](#)).

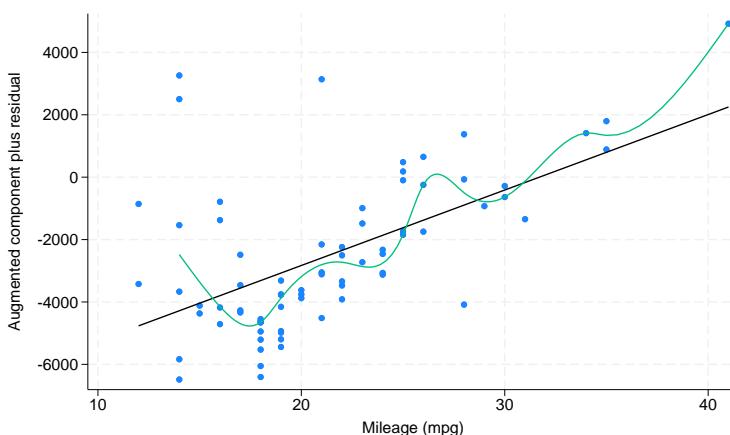
## Remarks and examples for `acrplot`

In the [cprplot](#) section above, we discussed the component-plus-residual plot. Mallows (1986) proposed an augmented component-plus-residual plot that is often more sensitive to detecting nonlinearity.

### ▶ Example 5

Let's compare the augmented component-plus-residual plot with the component-plus-residual plot of [example 4](#).

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. regress price weight foreign##c.mpg
(output omitted)
. acrplot mpg, mspline msopts(bands(13))
```



It does do somewhat better.

# rvpplot

## Description for rvpplot

`rvpplot` graphs a residual-versus-predictor plot (a.k.a. independent variable plot or carrier plot), a graph of the residuals against the specified predictor.

## Menu for rvpplot

Statistics > Linear models and related > Regression diagnostics > Residual-versus-predictor plot

## Syntax for rvpplot

```
rvpplot indepvar [, rvpplot_options]
```

| <i>rvpplot_options</i>                  | Description                                                                        |
|-----------------------------------------|------------------------------------------------------------------------------------|
| Plot                                    |                                                                                    |
| <i>marker_options</i>                   | change look of markers (color, size, etc.)                                         |
| <i>marker_label_options</i>             | add marker labels; change look or position                                         |
| Add plots                               |                                                                                    |
| <code>addplot(<i>plot</i>)</code>       | add plots to the generated graph                                                   |
| Y axis, X axis, Titles, Legend, Overall |                                                                                    |
| <i>twoway_options</i>                   | any options other than <code>by()</code> documented in [G-3] <i>twoway_options</i> |

## Options for rvpplot

### Plot

*marker\_options* affects the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] *marker\_options*.

*marker\_label\_options* specify if and how the markers are to be labeled; see [G-3] *marker\_label\_options*.

### Add plots

`addplot(plot)` provides a way to add other plots to the generated graph; see [G-3] *addplot\_option*.

### Y axis, X axis, Titles, Legend, Overall

*twoway\_options* are any of the options documented in [G-3] *twoway\_options*, excluding `by()`. These include options for titling the graph (see [G-3] *title\_options*) and for saving the graph to disk (see [G-3] *saving\_option*).

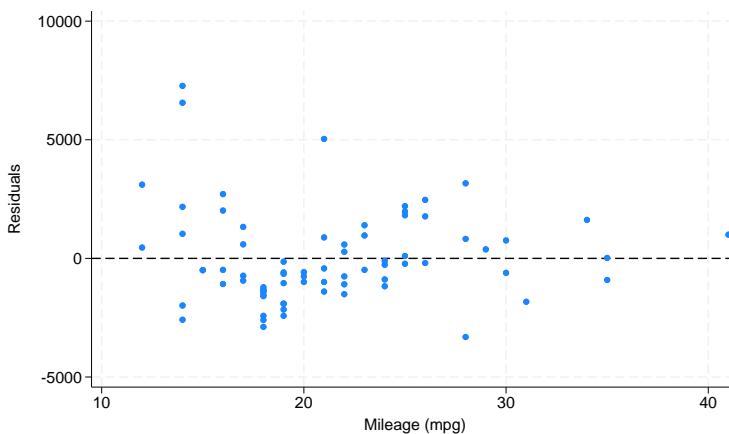
## Remarks and examples for rvpplot

The residual-versus-predictor plot is a simple way to look for violations of the regression assumptions. If the assumptions are correct, there should be no pattern on the graph.

### ▷ Example 6

Let's use our model of price on mpg and weight.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. regress price weight foreign##c.mpg
(output omitted)
. rvpplot mpg, yline(0)
```



Remember, any pattern counts as a problem, and in this graph, we see that the variation in the residuals decreases as mpg increases.

# lvr2plot

## Description for lvr2plot

`lvr2plot` graphs a leverage-versus-squared-residual plot (a.k.a. L-R plot).

## Menu for lvr2plot

Statistics > Linear models and related > Regression diagnostics > Leverage-versus-squared-residual plot

## Syntax for lvr2plot

```
lvr2plot [, lvr2plot_options]
```

| <i>lvr2plot_options</i>                 | Description                                                                        |
|-----------------------------------------|------------------------------------------------------------------------------------|
| Plot                                    |                                                                                    |
| <i>marker_options</i>                   | change look of markers (color, size, etc.)                                         |
| <i>marker_label_options</i>             | add marker labels; change look or position                                         |
| Add plots                               |                                                                                    |
| <code>addplot(<i>plot</i>)</code>       | add other plots to the generated graph                                             |
| Y axis, X axis, Titles, Legend, Overall |                                                                                    |
| <i>twoway_options</i>                   | any options other than <code>by()</code> documented in [G-3] <i>twoway_options</i> |

## Options for lvr2plot

### Plot

*marker\_options* affects the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] *marker\_options*.

*marker\_label\_options* specify if and how the markers are to be labeled; see [G-3] *marker\_label\_options*.

### Add plots

`addplot(plot)` provides a way to add other plots to the generated graph. See [G-3] *addplot\_option*.

### Y axis, X axis, Titles, Legend, Overall

*twoway\_options* are any of the options documented in [G-3] *twoway\_options*, excluding `by()`. These include options for titling the graph (see [G-3] *title\_options*) and for saving the graph to disk (see [G-3] *saving\_option*).

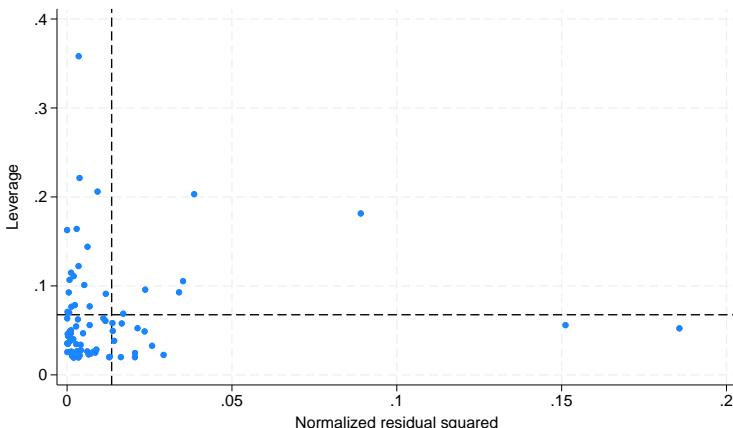
## Remarks and examples for lvr2plot

One of the most useful diagnostic graphs is provided by `lvr2plot` (leverage-versus-residual-squared plot), a graph of leverage against the (normalized) residuals squared.

### ▷ Example 7

We illustrate `lvr2plot` using our model in [example 1](#).

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. regress price weight foreign##c.mpg
(output omitted)
. lvr2plot
```

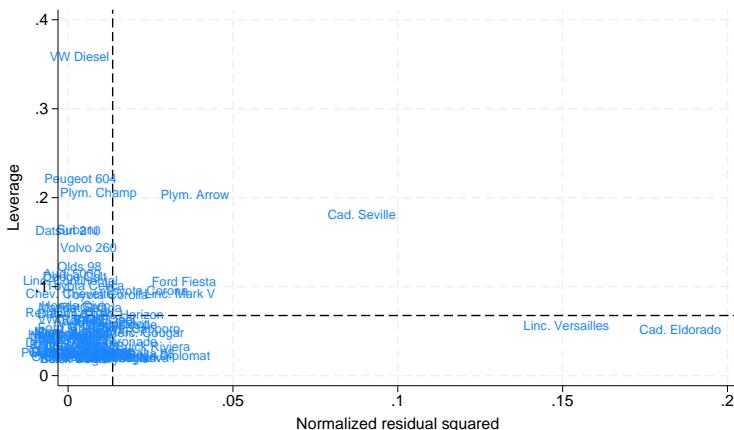


The lines on the chart show the average values of leverage and the (normalized) residuals squared. Points above the horizontal line have higher-than-average leverage; points to the right of the vertical line have larger-than-average residuals.

One point immediately catches our eye, and four more make us pause. The point at the top of the graph has high leverage and a smaller-than-average residual. The other points that bother us all have higher-than-average leverage, two with smaller-than-average residuals and two with larger-than-average residuals.

A less pretty but more useful version of the above graph specifies that make be used as the symbol (see [G-3] *marker\_label\_options*):

```
. lvr2plot, mlabel(make) mlabp(0) m(none) mlabsize(small)
```



The VW Diesel, Plymouth Champ, Plymouth Arrow, and Peugeot 604 are the points that cause us the most concern. When we further examine our data, we discover that the VW Diesel is the only diesel in our data and that the data for the Plymouth Arrow were entered incorrectly into the computer. No such simple explanations were found for the Plymouth Champ and Peugeot 604.

◀

## Methods and formulas

See [Hamilton \(2013, 209–214\)](#) and [Kohler and Kreuter \(2012, sec. 9.3\)](#) for a discussion of these diagnostic graphs.

The `lvr2plot` command plots leverage against the squares of the normalized residuals. The normalized residuals are defined as  $\hat{e}_{n_j} = \hat{e}_j / (\sum_i \hat{e}_i^2)^{1/2}$ .

## References

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## Also see

[R] **regress** — Linear regression

[R] **regress postestimation** — Postestimation tools for regress

[R] **regress postestimation time series** — Postestimation tools for regress with time series

[U] **20 Estimation and postestimation commands**

|                                 |                                      |                                      |
|---------------------------------|--------------------------------------|--------------------------------------|
| Postestimation commands         | <a href="#">estat archlm</a>         | <a href="#">estat bgodfrey</a>       |
| <a href="#">estat durbinalt</a> | <a href="#">estat dwatson</a>        | <a href="#">Remarks and examples</a> |
| <a href="#">Stored results</a>  | <a href="#">Methods and formulas</a> | <a href="#">Acknowledgment</a>       |
| <a href="#">References</a>      | <a href="#">Also see</a>             |                                      |

## Postestimation commands

The following postestimation commands for time series are available for `regress`:

| Command                      | Description                                                            |
|------------------------------|------------------------------------------------------------------------|
| <code>estat archlm</code>    | test for ARCH effects in the residuals                                 |
| <code>estat bgodfrey</code>  | Breusch–Godfrey test for higher-order serial correlation               |
| <code>estat durbinalt</code> | Durbin’s alternative test for serial correlation                       |
| <code>estat dwatson</code>   | Durbin–Watson $d$ statistic to test for first-order serial correlation |
| <code>estat sbcusum</code>   | perform cumulative sum test for parameter stability                    |
| <code>estat sbknown</code>   | perform tests for a structural break with a known break date           |
| <code>estat sbsingle</code>  | perform tests for a structural break with an unknown break date        |

These commands provide regression diagnostic tools specific to time series. You must `tsset` your data before using these commands; see [\[TS\] `tsset`](#).

## estat archlm

### Description for estat archlm

`estat archlm` performs Engle’s Lagrange multiplier (LM) test for the presence of autoregressive conditional heteroskedasticity.

### Menu for estat

Statistics > Postestimation

### Syntax for estat archlm

```
estat archlm [, archlm_options]
```

| <i>archlm_options</i>             | Description                                                      |
|-----------------------------------|------------------------------------------------------------------|
| <code>lags(<i>numlist</i>)</code> | test <i>numlist</i> lag orders                                   |
| <code>force</code>                | allow test after <code>regress</code> , <code>vce(robust)</code> |

`collect` is allowed; see [\[U\] 11.1.10 Prefix commands](#).

## Options for estat archlm

`lags` (*numlist*) specifies a list of numbers, indicating the lag orders to be tested. The test will be performed separately for each order. The default is order one.

`force` allows the test to be run after `regress, vce(robust)`. The command will not work if the `vce(cluster clustvar)` option is specified with `regress`; see [R] [regress](#).

## estat bgodfrey

### Description for estat bgodfrey

`estat bgodfrey` performs the Breusch–Godfrey test for higher-order serial correlation in the disturbance. This test does not require that all the regressors be strictly exogenous.

### Menu for estat

Statistics > Postestimation

### Syntax for estat bgodfrey

```
estat bgodfrey [, bgodfrey_options]
```

| <i>bgodfrey_options</i>              | Description                                                         |
|--------------------------------------|---------------------------------------------------------------------|
| <code>lags</code> ( <i>numlist</i> ) | test <i>numlist</i> lag orders                                      |
| <code>nomiss0</code>                 | do not use Davidson and MacKinnon's approach                        |
| <code>small</code>                   | obtain <i>p</i> -values using the <i>F</i> or <i>t</i> distribution |

`collect` is allowed; see [U] [11.1.10 Prefix commands](#).

### Options for estat bgodfrey

`lags` (*numlist*) specifies a list of numbers, indicating the lag orders to be tested. The test will be performed separately for each order. The default is order one.

`nomiss0` specifies that Davidson and MacKinnon's approach (1993, 358), which replaces the missing values in the initial observations on the lagged residuals in the auxiliary regression with zeros, not be used.

`small` specifies that the *p*-values of the test statistics be obtained using the *F* or *t* distribution instead of the default  $\chi^2$  or normal distribution.

## estat durbinalt

### Description for estat durbinalt

`estat durbinalt` performs Durbin's alternative test for serial correlation in the disturbance. This test does not require that all the regressors be strictly exogenous.

### Menu for estat

Statistics > Postestimation

### Syntax for estat durbinalt

```
estat durbinalt [, durbinalt_options]
```

| <i>durbinalt_options</i>          | Description                                                                                  |
|-----------------------------------|----------------------------------------------------------------------------------------------|
| <code>lags(<i>numlist</i>)</code> | test <i>numlist</i> lag orders                                                               |
| <code>nomiss0</code>              | do not use Davidson and MacKinnon's approach                                                 |
| <code>robust</code>               | compute standard errors using the robust/sandwich estimator                                  |
| <code>small</code>                | obtain <i>p</i> -values using the <i>F</i> or <i>t</i> distribution                          |
| <code>force</code>                | allow test after <code>regress</code> , <code>vce(robust)</code> or after <code>newey</code> |

`collect` is allowed; see [U] 11.1.10 Prefix commands.

### Options for estat durbinalt

`lags(numlist)` specifies a list of numbers, indicating the lag orders to be tested. The test will be performed separately for each order. The default is order one.

`nomiss0` specifies that Davidson and MacKinnon's approach (1993, 358), which replaces the missing values in the initial observations on the lagged residuals in the auxiliary regression with zeros, not be used.

`robust` specifies that the Huber/White/sandwich robust estimator for the variance–covariance matrix be used in Durbin's alternative test.

`small` specifies that the *p*-values of the test statistics be obtained using the *F* or *t* distribution instead of the default  $\chi^2$  or normal distribution. This option may not be specified with `robust`, which always uses an *F* or a *t* distribution.

`force` allows the test to be run after `regress`, `vce(robust)` and after `newey` (see [R] `regress` and [TS] `newey`). The command will not work if the `vce(cluster clustvar)` option is specified with `regress`.

## estat dwatson

### Description for estat dwatson

`estat dwatson` computes the Durbin–Watson  $d$  statistic (Durbin and Watson 1950) to test for first-order serial correlation in the disturbance when all the regressors are strictly exogenous.

### Menu for estat

Statistics > Postestimation

### Syntax for estat dwatson

```
estat dwatson
```

`collect` is allowed; see [U] 11.1.10 Prefix commands.

### Remarks and examples

The Durbin–Watson test is used to determine whether the error term in a linear regression model follows an AR(1) process. For the linear model

$$y_t = \mathbf{x}_t\boldsymbol{\beta} + u_t$$

the AR(1) process can be written as

$$u_t = \rho u_{t-1} + \epsilon_t$$

In general, an AR(1) process requires only that  $\epsilon_t$  be independent and identically distributed (i.i.d.). The Durbin–Watson test, however, requires  $\epsilon_t$  to be distributed  $N(0, \sigma^2)$  for the statistic to have an exact distribution. Also, the Durbin–Watson test can be applied only when the regressors are strictly exogenous. A regressor  $x$  is strictly exogenous if  $\text{Corr}(x_s, u_t) = 0$  for all  $s$  and  $t$ , which precludes the use of the Durbin–Watson statistic with models where lagged values of the dependent variable are included as regressors.

The null hypothesis of the test is that there is no first-order autocorrelation. The Durbin–Watson  $d$  statistic can take on values between 0 and 4 and under the null  $d$  is equal to 2. Values of  $d$  less than 2 suggest positive autocorrelation ( $\rho > 0$ ), whereas values of  $d$  greater than 2 suggest negative autocorrelation ( $\rho < 0$ ). Calculating the exact distribution of the  $d$  statistic is difficult, but empirical upper and lower bounds have been established based on the sample size and the number of regressors. Extended tables for the  $d$  statistic have been published by Savin and White (1977). For example, suppose you have a model with 30 observations and three regressors (including the constant term). For a test of the null hypothesis of no autocorrelation versus the alternative of positive autocorrelation, the lower bound of the  $d$  statistic is 1.284, and the upper bound is 1.567 at the 5% significance level. You would reject the null if  $d < 1.284$ , and you would fail to reject if  $d > 1.567$ . A value falling within the range (1.284, 1.567) leads to no conclusion about whether or not to reject the null hypothesis.

When lagged dependent variables are included among the regressors, the past values of the error term are correlated with those lagged variables at time  $t$ , implying that they are not strictly exogenous regressors. The inclusion of covariates that are not strictly exogenous causes the  $d$  statistic to be biased toward the acceptance of the null hypothesis. Durbin (1970) suggested an alternative test for models with lagged dependent variables and extended that test to the more general  $AR(p)$  serial correlation process

$$u_t = \rho_1 u_{t-1} + \dots + \rho_p u_{t-p} + \epsilon_t$$

where  $\epsilon_t$  is i.i.d. with variance  $\sigma^2$  but is not assumed or required to be normal for the test.

The null hypothesis of Durbin’s alternative test is

$$H_0 : \rho_1 = 0, \dots, \rho_p = 0$$

and the alternative is that at least one of the  $\rho$ ’s is nonzero. Although the null hypothesis was originally derived for an  $AR(p)$  process, this test turns out to have power against  $MA(p)$  processes as well. Hence, the actual null of this test is that there is no serial correlation up to order  $p$  because the  $MA(p)$  and the  $AR(p)$  models are locally equivalent alternatives under the null. See Godfrey (1988, 113–115) for a discussion of this result.

Durbin’s alternative test is in fact a LM test, but it is most easily computed with a Wald test on the coefficients of the lagged residuals in an auxiliary OLS regression of the residuals on their lags and all the covariates in the original regression. Consider the linear regression model

$$y_t = \beta_1 x_{1t} + \dots + \beta_k x_{kt} + u_t \tag{1}$$

in which the covariates  $x_1$  through  $x_k$  are not assumed to be strictly exogenous and  $u_t$  is assumed to be i.i.d. and to have finite variance. The process is also assumed to be stationary. (See Wooldridge [2020, sec. 11.1] for a discussion of stationarity.) Estimating the parameters in (1) by OLS obtains the residuals  $\hat{u}_t$ . Next another OLS regression is performed of  $\hat{u}_t$  on  $\hat{u}_{t-1}, \dots, \hat{u}_{t-p}$  and the other regressors,

$$\hat{u}_t = \gamma_1 \hat{u}_{t-1} + \dots + \gamma_p \hat{u}_{t-p} + \beta_1 x_{1t} + \dots + \beta_k x_{kt} + \epsilon_t \tag{2}$$

where  $\epsilon_t$  stands for the random-error term in this auxiliary OLS regression. Durbin’s alternative test is then obtained by performing a Wald test that  $\gamma_1, \dots, \gamma_p$  are jointly zero. The test can be made robust to an unknown form of heteroskedasticity by using a robust VCE estimator when estimating the regression in (2). When there are only strictly exogenous regressors and  $p = 1$ , this test is asymptotically equivalent to the Durbin–Watson test.

The Breusch–Godfrey test is also an LM test of the null hypothesis of no autocorrelation versus the alternative that  $u_t$  follows an  $AR(p)$  or  $MA(p)$  process. Like Durbin’s alternative test, it is based on the auxiliary regression (2), and it is computed as  $NR^2$ , where  $N$  is the number of observations and  $R^2$  is the simple  $R^2$  from the regression. This test and Durbin’s alternative test are asymptotically equivalent. The test statistic  $NR^2$  has an asymptotic  $\chi^2$  distribution with  $p$  degrees of freedom. It is valid with or without the strict exogeneity assumption but is not robust to conditional heteroskedasticity, even if a robust VCE is used when fitting (2).

In fitting (2), the values of the lagged residuals will be missing in the initial periods. As noted by Davidson and MacKinnon (1993), the residuals will not be orthogonal to the other covariates in the model in this restricted sample, which implies that the  $R^2$  from the auxiliary regression will not be zero when the lagged residuals are left out. Hence, Breusch and Godfrey’s  $NR^2$  version of the test may overreject in small samples. To correct this problem, Davidson and MacKinnon (1993) recommend setting the

missing values of the lagged residuals to zero and running the auxiliary regression in (2) over the full sample used in (1). This small-sample correction has become conventional for both the Breusch–Godfrey and Durbin’s alternative test, and it is the default for both commands. Specifying the `nomiss0` option overrides this default behavior and treats the initial missing values generated by regressing on the lagged residuals as missing. Hence, `nomiss0` causes these initial observations to be dropped from the sample of the auxiliary regression.

Durbin’s alternative test and the Breusch–Godfrey test were originally derived for the case covered by `regress` without the `vce(robust)` option. However, after `regress`, `vce(robust)` and `newey`, Durbin’s alternative test is still valid and can be invoked if the `robust` and `force` options are specified.

### ▷ Example 1: tests for serial correlation

Using data from [Klein \(1950\)](#), we first fit an OLS regression of consumption on the government wage bill:

```
. use https://www.stata-press.com/data/r19/klein
. tsset yr
Time variable: yr, 1920 to 1941
 Delta: 1 unit
. regress consump wagegovt
```

| Source   | SS         | df | MS         | Number of obs | = | 22     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 532.567711 | 1  | 532.567711 | F(1, 20)      | = | 17.72  |
| Residual | 601.207167 | 20 | 30.0603584 | Prob > F      | = | 0.0004 |
|          |            |    |            | R-squared     | = | 0.4697 |
|          |            |    |            | Adj R-squared | = | 0.4432 |
| Total    | 1133.77488 | 21 | 53.9892799 | Root MSE      | = | 5.4827 |

| consump  | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |
|----------|-------------|-----------|-------|-------|----------------------|
| wagegovt | 2.50744     | .5957173  | 4.21  | 0.000 | 1.264796 3.750085    |
| _cons    | 40.84699    | 3.192183  | 12.80 | 0.000 | 34.18821 47.50577    |

If we assume that `wagegovt` is a strictly exogenous variable, we can use the Durbin–Watson test to check for first-order serial correlation in the errors.

```
. estat dwatson
Durbin-Watson d-statistic(2, 22) = .3217998
```

The Durbin–Watson  $d$  statistic, 0.32, is far from the center of its distribution ( $d = 2.0$ ). Given 22 observations and two regressors (including the constant term) in the model, the lower 5% bound is about 0.997, much greater than the computed  $d$  statistic. Assuming that `wagegovt` is strictly exogenous, we can reject the null of no first-order serial correlation. Rejecting the null hypothesis does not necessarily mean an AR process; other forms of misspecification may also lead to a significant test statistic. If we are willing to assume that the errors follow an AR(1) process and that `wagegovt` is strictly exogenous, we could refit the model using `arima` or `prais` and model the error process explicitly; see [\[TS\] arima](#) and [\[TS\] prais](#).

If we are not willing to assume that wagegov is strictly exogenous, we could instead use Durbin’s alternative test or the Breusch–Godfrey to test for first-order serial correlation. Because we have only 22 observations, we will use the small option.

```
. estat durbinalt, small
```

Durbin’s alternative test for autocorrelation

| lags(p) | F      | df        | Prob > F |
|---------|--------|-----------|----------|
| 1       | 35.035 | ( 1, 19 ) | 0.0000   |

H0: no serial correlation

```
. estat bgodfrey, small
```

Breusch–Godfrey LM test for autocorrelation

| lags(p) | F      | df        | Prob > F |
|---------|--------|-----------|----------|
| 1       | 14.264 | ( 1, 19 ) | 0.0013   |

H0: no serial correlation

Both tests strongly reject the null of no first-order serial correlation, so we decide to refit the model with two lags of consump included as regressors and then rerun estat durbinalt and estat bgodfrey. Because the revised model includes lagged values of the dependent variable, the Durbin–Watson test is not applicable.

```
. regress consump wagegovt L.consump L2.consump
```

| Source   | SS         | df | MS         | Number of obs | = | 20     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 702.660311 | 3  | 234.220104 | F(3, 16)      | = | 44.01  |
| Residual | 85.1596011 | 16 | 5.32247507 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.8919 |
|          |            |    |            | Adj R-squared | = | 0.8716 |
| Total    | 787.819912 | 19 | 41.4642059 | Root MSE      | = | 2.307  |

| consump  | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |
|----------|-------------|-----------|-------|-------|----------------------|
| wagegovt | .6904282    | .3295485  | 2.10  | 0.052 | -.0081835 1.38904    |
| consump  |             |           |       |       |                      |
| L1.      | 1.420536    | .197024   | 7.21  | 0.000 | 1.002864 1.838208    |
| L2.      | -.650888    | .1933351  | -3.37 | 0.004 | -1.06074 -.241036    |
| _cons    | 9.209073    | 5.006701  | 1.84  | 0.084 | -1.404659 19.82281   |

```
. estat durbinalt, small lags(1/2)
Durbin's alternative test for autocorrelation
```

| lags(p) | F     | df        | Prob > F |
|---------|-------|-----------|----------|
| 1       | 0.080 | ( 1, 15 ) | 0.7805   |
| 2       | 0.260 | ( 2, 14 ) | 0.7750   |

H0: no serial correlation

```
. estat bgodfrey, small lags(1/2)
Breusch-Godfrey LM test for autocorrelation
```

| lags(p) | F     | df        | Prob > F |
|---------|-------|-----------|----------|
| 1       | 0.107 | ( 1, 15 ) | 0.7484   |
| 2       | 0.358 | ( 2, 14 ) | 0.7056   |

H0: no serial correlation

Although `wagegov` and the constant term are no longer statistically different from zero at the 5% level, the output from `estat durbinalt` and `estat bgodfrey` indicates that including the two lags of `consump` has removed any serial correlation from the errors.



Engle (1982) suggests an LM test for checking for autoregressive conditional heteroskedasticity (ARCH) in the errors. The  $p$ th-order ARCH model can be written as

$$\begin{aligned} \sigma_t^2 &= E(u_t^2 | u_{t-1}, \dots, u_{t-p}) \\ &= \gamma_0 + \gamma_1 u_{t-1}^2 + \dots + \gamma_p u_{t-p}^2 \end{aligned}$$

To test the null hypothesis of no autoregressive conditional heteroskedasticity (that is,  $\gamma_1 = \dots = \gamma_p = 0$ ), we first fit the OLS model (1), obtain the residuals  $\hat{u}_t$ , and run another OLS regression on the lagged residuals:

$$\hat{u}_t^2 = \gamma_0 + \gamma_1 \hat{u}_{t-1}^2 + \dots + \gamma_p \hat{u}_{t-p}^2 + \epsilon \tag{3}$$

The test statistic is  $NR^2$ , where  $N$  is the number of observations in the sample and  $R^2$  is the  $R^2$  from the regression in (3). Under the null hypothesis, the test statistic follows a  $\chi_p^2$  distribution.

### ► Example 2: `estat archlm`

We refit the original model that does not include the two lags of `consump` and then use `estat archlm` to see if there is any evidence that the errors are autoregressive conditional heteroskedastic.

```
. regress consump wagegovt
```

| Source   | SS         | df | MS         | Number of obs | = | 22     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 532.567711 | 1  | 532.567711 | F(1, 20)      | = | 17.72  |
| Residual | 601.207167 | 20 | 30.0603584 | Prob > F      | = | 0.0004 |
|          |            |    |            | R-squared     | = | 0.4697 |
|          |            |    |            | Adj R-squared | = | 0.4432 |
| Total    | 1133.77488 | 21 | 53.9892799 | Root MSE      | = | 5.4827 |

| consump  | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |
|----------|-------------|-----------|-------|-------|----------------------|
| wagegovt | 2.50744     | .5957173  | 4.21  | 0.000 | 1.264796 3.750085    |
| _cons    | 40.84699    | 3.192183  | 12.80 | 0.000 | 34.18821 47.50577    |

```
. estat archlm, lags(1 2 3)
LM test for autoregressive conditional heteroskedasticity (ARCH)
```

| lags(p) | chi2  | df | Prob > chi2 |
|---------|-------|----|-------------|
| 1       | 5.543 | 1  | 0.0186      |
| 2       | 9.431 | 2  | 0.0090      |
| 3       | 9.039 | 3  | 0.0288      |

H0: no ARCH effects vs. H1: ARCH(p) disturbance

estat archlm shows the results for tests of ARCH(1), ARCH(2), and ARCH(3) effects, respectively. At the 5% significance level, all three tests reject the null hypothesis that the errors are not autoregressive conditional heteroskedastic. See [TS] arch for information on fitting ARCH models.



## Stored results

estat archlm stores the following in r():

Scalars

- r(N) number of observations
- r(k) number of regressors
- r(N\_gaps) number of gaps

Macros

- r(lags) lag order

Matrices

- r(arch) test statistic for each lag order
- r(df) degrees of freedom
- r(p) two-sided *p*-values

estat bgodfrey stores the following in r():

Scalars

- r(N) number of observations
- r(k) number of regressors
- r(N\_gaps) number of gaps

Macros

- r(lags) lag order

Matrices

- r(chi2)  $\chi^2$  statistic for each lag order
- r(F) *F* statistic for each lag order (small only)
- r(df\_r) residual degrees of freedom (small only)
- r(df) degrees of freedom
- r(p) two-sided *p*-values

`estat durbinalt` stores the following in `r()`:

Scalars

|                        |                        |
|------------------------|------------------------|
| <code>r(N)</code>      | number of observations |
| <code>r(k)</code>      | number of regressors   |
| <code>r(N_gaps)</code> | number of gaps         |

Macros

|                      |           |
|----------------------|-----------|
| <code>r(lags)</code> | lag order |
|----------------------|-----------|

Matrices

|                      |                                               |
|----------------------|-----------------------------------------------|
| <code>r(chi2)</code> | $\chi^2$ statistic for each lag order         |
| <code>r(F)</code>    | $F$ statistic for each lag order (small only) |
| <code>r(df_r)</code> | residual degrees of freedom (small only)      |
| <code>r(df)</code>   | degrees of freedom                            |
| <code>r(p)</code>    | two-sided $p$ -values                         |

`estat dwatson` stores the following in `r()`:

Scalars

|                        |                         |
|------------------------|-------------------------|
| <code>r(N)</code>      | number of observations  |
| <code>r(k)</code>      | number of regressors    |
| <code>r(N_gaps)</code> | number of gaps          |
| <code>r(dw)</code>     | Durbin–Watson statistic |

## Methods and formulas

Consider the regression

$$y_t = \beta_1 x_{1t} + \cdots + \beta_k x_{kt} + u_t \quad (4)$$

in which some of the covariates are not strictly exogenous. In particular, some of the  $x_{it}$  may be lags of the dependent variable. We are interested in whether the  $u_t$  are serially correlated.

The Durbin–Watson  $d$  statistic reported by `estat dwatson` is

$$d = \frac{\sum_{t=1}^{n-1} (\hat{u}_{t+1} - \hat{u}_t)^2}{\sum_{t=1}^n \hat{u}_t^2}$$

where  $\hat{u}_t$  represents the residual of the  $t$ th observation.

To compute Durbin’s alternative test and the Breusch–Godfrey test against the null hypothesis that there is no  $p$ th order serial correlation, we fit the regression in (4), compute the residuals, and then fit the following auxiliary regression of the residuals  $\hat{u}_t$  on  $p$  lags of  $\hat{u}_t$  and on all the covariates in the original regression in (4):

$$\hat{u}_t = \gamma_1 \hat{u}_{t-1} + \cdots + \gamma_p \hat{u}_{t-p} + \beta_1 x_{1t} + \cdots + \beta_k x_{kt} + \epsilon \quad (5)$$

Durbin’s alternative test is computed by performing a Wald test to determine whether the coefficients of  $\hat{u}_{t-1}, \dots, \hat{u}_{t-p}$  are jointly different from zero. By default, the statistic is assumed to be distributed  $\chi^2(p)$ . When `small` is specified, the statistic is assumed to follow an  $F(p, N - p - k)$  distribution. The reported  $p$ -value is a two-sided  $p$ -value. When `robust` is specified, the Wald test is performed using the Huber/White/sandwich estimator of the variance–covariance matrix, and the test is robust to an unspecified form of heteroskedasticity.

The Breusch–Godfrey test is computed as  $NR^2$ , where  $N$  is the number of observations in the auxiliary regression (5) and  $R^2$  is the  $R^2$  from the same regression (5). Like Durbin’s alternative test, the Breusch–Godfrey test is asymptotically distributed  $\chi^2(p)$ , but specifying `small` causes the  $p$ -value to be computed using an  $F(p, N - p - k)$ .

By default, the initial missing values of the lagged residuals are replaced with zeros, and the auxiliary regression is run over the full sample used in the original regression of (4). Specifying the `miss0` option causes these missing values to be treated as missing values, and the observations are dropped from the sample.

Engle’s LM test for ARCH( $p$ ) effects fits an OLS regression of  $\hat{u}_t^2$  on  $\hat{u}_{t-1}^2, \dots, \hat{u}_{t-p}^2$ :

$$\hat{u}_t^2 = \gamma_0 + \gamma_1 \hat{u}_{t-1}^2 + \dots + \gamma_p \hat{u}_{t-p}^2 + \epsilon$$

The test statistic is  $nR^2$  and is asymptotically distributed  $\chi^2(p)$ .

## Acknowledgment

The original versions of `estat archlm`, `estat bgodfrey`, and `estat durbinalt` were written by Christopher F. Baum of the Department of Economics at Boston College and author of the Stata Press books *An Introduction to Modern Econometrics Using Stata* and *An Introduction to Stata Programming* and coauthor of the Stata Press book *Environmental Econometrics Using Stata*.

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**James Durbin** (1923–2012) was a British statistician who was born in Wigan, near Manchester. He studied mathematics at Cambridge and after military service and various research posts joined the London School of Economics in 1950. Later in life, he was also affiliated with University College London. His many contributions to statistics centered on serial correlation, time series (including major contributions to structural or unobserved components models), sample survey methodology, goodness-of-fit tests, and sample distribution functions, with emphasis on applications in the social sciences. He served terms as president of the Royal Statistical Society and the International Statistical Institute.

**Geoffrey Stuart Watson** (1921–1998) was born in Victoria, Australia, and earned degrees at Melbourne University and North Carolina State University. After a visit to the University of Cambridge, he returned to Australia, working at Melbourne and then the Australian National University. Following periods at Toronto and Johns Hopkins, he settled at Princeton. Throughout his wide-ranging career, he made many notable accomplishments and important contributions, including the Durbin–Watson test for serial correlation, the Nadaraya–Watson estimator in nonparametric regression, and methods for analyzing directional data.

**Leslie G. Godfrey** (1946–) was born in London and earned degrees at the Universities of Exeter and London. He is now a professor of econometrics at the University of York. His interests center on implementation and interpretation of tests of econometric models, including nonnested models.

**Trevor Stanley Breusch** (1953–) was born in Queensland and earned degrees at the University of Queensland and Australian National University (ANU). After a post at the University of Southampton, he returned to work at ANU. His background is in econometric methods and his recent interests include political values and social attitudes, earnings and income, and measurement of underground economic activity.

## Also see

- [R] **regress** — Linear regression
- [R] **regress postestimation** — Postestimation tools for regress
- [R] **regress postestimation diagnostic plots** — Postestimation plots for regress
- [TS] **tsset** — Declare data to be time-series data

[Description](#)[Options](#)[Acknowledgment](#)[Quick start](#)[Remarks and examples](#)[References](#)[Menu](#)[Stored results](#)[Also see](#)[Syntax](#)[Methods and formulas](#)

## Description

`reri` reports three statistics that assess two-way interactions in an additive model of relative risk. The statistics are of interest for determining how two risk factors interact to produce a positive result for the outcome of interest. The estimates of the interactions themselves are reported as excess relative risks (ERRs). Statistics reported for the interaction are the relative excess risk due to interaction (RERI), attributable proportion (AP), and synergy index (SI); `reri` supports binomial generalized linear, Poisson, negative binomial, logistic, Cox, parametric survival, and interval-censored parametric and semiparametric survival models.

## Quick start

Report ERRs, RERI, AP, and SI for the interaction of exposures `exp1` and `exp2` from a binomial generalized linear model for `y` fit by `binreg`

```
reri binreg y exp1 exp2
```

Same as above, but use `poisson` to fit the model

```
reri poisson y exp1 exp2
```

Same as above, but report incidence-rate ratios for the interactions rather than ERRs

```
reri poisson y exp1 exp2, irr
```

Same as above, but include the covariate `x`, and specify `noisily` to view the full Poisson model

```
reri poisson y exp1 exp2 x, irr noisily
```

Same as above, but report ERRs for the interactions as well as incidence-rate ratios for the full model

```
reri poisson y exp1 exp2 x, irr err noisily
```

Fit a [Cox model](#) using `stset` data stratified by `svar`, and report ERRs, RERI, AP, and SI for the interactions of `exp1` and `exp2`

```
reri stcox exp1 exp2, strata(svar)
```

Same as above, but fit a [Weibull model](#)

```
reri streg exp1 exp2, strata(svar) distribution(weibull)
```

Fit a Weibull model with [interval-censored survival data](#), with variables `t1` and `t2` giving lower and upper endpoints for the censoring interval, and report ERRs, RERI, AP, and SI for the interactions

```
reri stintreg exp1 exp2, distribution(weibull) interval(t1 t2)
```

## Menu

Statistics > Epidemiology and related > Other > Relative excess risk due to interaction (RERI)

## Syntax

```
reri cmd depvar var1 var2 [control_vars] [if] [in] [weight]
 [, options cmd_options]
```

*cmd* defines the estimation command to be executed and may be one of the following:

`binreg`, `logistic`, `nbreg`, `poisson`, `stcox`, `stintcox`, `stintreg`, and `streg`

*depvar* is required for all except `stcox`, `stintcox`, `stintreg`, and `streg`, which do not allow *depvar*.

*var1* and *var2* are binary variables (integer valued and nonnegative) whose interactions are to be tested for additivity.

| <i>options</i> | Description |
|----------------|-------------|
|----------------|-------------|

### Model

|                            |                                    |
|----------------------------|------------------------------------|
| <code>baselevel1(#)</code> | specify base level for <i>var1</i> |
| <code>baselevel2(#)</code> | specify base level for <i>var2</i> |

### Reporting

|                        |                                                                 |
|------------------------|-----------------------------------------------------------------|
| <code>noisily</code>   | display output from <i>cmd</i>                                  |
| <code>err</code>       | report interactions as ERR, the default                         |
| <code>coef_type</code> | report interactions as <i>coef_type</i> rather than ERR         |
| <code>level(#)</code>  | set confidence level; default is level(95)                      |
| <code>novlabel</code>  | display numeric values rather than <a href="#">value labels</a> |
| <code>nolegend</code>  | do not display the legend for the interactions                  |

*var1* and *var2* cannot be factor variables, but *control\_vars* may contain factor variables; see [\[U\] 11.4.3 Factor variables](#).

*cmd\_options* are any options allowed by *cmd*, except options `or`, `hr`, `rd`, and `vce(jackknife1)` are not allowed when *cmd* is `binreg`.

When *cmd* is `poisson`, `vce(robust)` is used by default when *depvar* is 0/1.

`bootstrap`, `by`, `collect`, `jackknife`, and `statsby` are allowed; see [\[U\] 11.1.10 Prefix commands](#).

Weights are allowed if *cmd* allows them; see [\[U\] 11.1.6 weight](#).

All postestimation commands behave as they would after *cmd*; see the postestimation manual entry for *cmd*.

## Options

### Model

`baselevel1(#)` and `baselevel2(#)` specify base levels for *var1* and *var2*, respectively, the two variables that give the interactions of interest.

If *var1* and *var2* are 0/1 variables, then by default, the RERI statistic is a test of whether the ERR of interaction 01 (01 meaning *var1* = 0 and *var2* = 1) plus the ERR of interaction 10 equals the ERR of interaction 11. In shorthand notation, we say we are testing  $01 + 10 = 11$  on the ERR scale.

Specifying `baselevel1(#)` or `baselevel2(#)` allows the base levels of the interactions to be changed. For example, with 0/1 variables, `baselevel1(1)` sets the base level for *var1* to 1, and RERI tests whether  $11 + 00 = 01$ .

If `baselevel1(#)` is not specified, the base level of `var1` is the default base level of the factor variable `i.var1`. The default base level is the smallest of the two values of `var1`, unless the default has been changed by `fvset`. This is similarly true for `baselevel2(#)` and `var2`. See [U] 11.4.3.2 Base levels and [R] `fvset`. Note that base levels cannot be set using factor-variable notation when specifying `var1` and `var2`.

#### Reporting

`noisily` displays all output from `cmd`.

`err` specifies that the coefficients of the interactions be displayed as ERRs. This is the default. ERR is equal to relative risk minus one. See [Remarks and examples](#) below.

`coef_type` specifies an alternative scale for the coefficients of the interactions instead of ERRs. Allowed `coef_type` depends on `cmd` and can be `rr`, `or`, `irr`, or `hr`, whichever is allowed by `cmd`. In addition, `coef_type` can be `coef`, which displays the interaction terms as unscaled coefficients.

For `binreg`, `coef_type rr` displays the interactions as relative-risk ratios. The other models fit by `binreg` using options `or`, `hr`, and `rd` are not available with `reri`. For `logistic`, `coef_type or` displays the interactions as odds ratios. For `poisson` and `nbreg`, `coef_type irr` displays the interactions as incidence-rate ratios. For `stcox`, `stintcox`, `stintreg`, and `streg`, `coef_type hr` displays the interactions as hazard ratios. For the survival-time commands, `nohr` may be used as a synonym for `coef`.

When `noisily` is specified, `err` and a `coef_type` may be specified simultaneously. In this case, the table of interactions displayed by `reri` reports ERRs, and the coefficient table produced by `cmd` displays coefficients on the `coef_type` scale.

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level1`; see [R] [level](#).

`nofvlabel` specifies that numeric values rather than [value labels](#) be displayed in the legend and `cmd` output.

`nolegend` suppresses the display of the legend for the interactions.

## Remarks and examples

Remarks are presented under the following headings:

[Introduction](#)

[Additive versus multiplicative interactions](#)

[Incidence-rate ratios, hazard ratios, and odds ratios](#)

### Introduction

`reri` reports three statistics that assess additive interactions of relative risk: RERI, AP, and SI ([Lash et al. 2021](#)). These measures are typically of interest when working with epidemiologic data. When one assesses the effect of multiple exposures on an outcome, determining whether one exposure modifies the effect of another can be an important piece of the puzzle.

When there are two exposures that affect an outcome, we often want to model how the exposures interact. That is, for binary exposures *A* and *B*, we want to model the risk of a positive outcome for subjects having both *A* and *B* compared with subjects having only *A* and those having only *B*.

Typically, a multiplicative model for the interaction is fit to the data: the risk for those with  $A$  and  $B$  is hypothesized to equal the risk for only  $A$  times the risk for only  $B$ . If the risk for  $A$  and  $B$  equals the multiplicative risk, then we say there is no interaction of  $A$  and  $B$ . If the risk is greater than the multiplicative risk, we say there is a positive interaction of  $A$  and  $B$ . Multiplicative models are computationally convenient because they are simple to specify, and it is easy to test whether the interaction term is greater (or less) than multiplicative.

Inherent to this model is the hypothesis that the risk is multiplicative. For many biological processes, however, an additive model of risk may model the process better than a multiplicative one (see, for example, [Andersson et al. \[2005\]](#)). That is, the risk for subjects with  $A$  and  $B$  is hypothesized to equal the risk for only  $A$  plus the risk for only  $B$ . For this hypothesis, we want to look at how much the risk for  $A$  and  $B$  is greater (or less) than the additive risk.

The RERI statistic is this risk difference formulated using relative risks. A RERI statistic that is 0 means the risk is additive. One that is positive means the risk is superadditive, and one that is negative means the risk is subadditive. The test of statistical significance of the RERI statistic is straightforward, but it is not as simple as the test for the multiplicative model (see [Methods and formulas](#)).

Besides the RERI statistic, there are two other related statistics for additive models: AP and SI. The AP is the proportion of risk of  $A$  and  $B$  due to the superadditivity of exposures  $A$  and  $B$ . The SI recasts the RERI statistic as a ratio, rather than a difference.

Let's formally define these statistics. Let  $p_{A+B+}$  be the probability of having a positive outcome for subjects positive for binary exposure  $A$  and positive for binary exposure  $B$ . Define  $p_{A-B+}$ ,  $p_{A+B-}$ , and  $p_{A-B-}$  similarly. An additive model for the probabilities is

$$p_{A+B+} = p_{A-B-} + (p_{A-B+} - p_{A-B-}) + (p_{A+B-} - p_{A-B-})$$

Said in words, the probability of a positive outcome for  $A+B+$  equals the probability for the base category  $A-B-$  plus the amounts that probabilities for  $A-B+$  and  $A+B-$  exceed the probability for the base category.

If we divide this equation by  $p_{A-B-}$  and note that  $p_{A+B+}/p_{A-B-} = RR_{A+B+}$ , the relative risk for  $A+B+$  (and similarly for  $RR_{A-B+}$  and  $RR_{A+B-}$ ), we get

$$RR_{A+B+} = RR_{A-B+} + RR_{A+B-} - 1$$

ERR is defined as  $ERR = RR - 1$ . In terms of ERR, the additive model becomes

$$ERR_{A+B+} = ERR_{A-B+} + ERR_{A+B-}$$

The RERI statistic is defined as

$$RERI = ERR_{A+B+} - ERR_{A-B+} - ERR_{A+B-}$$

Hence, RERI is 0 for a perfectly additive model. If it is negative, the effect of the  $A+B+$  interaction is less than additive (subadditive). If positive, the effect is more than additive (superadditive).

The AP is simply RERI scaled by the relative risk (not ERR) of  $A+B+$ . That is,

$$AP = \frac{RERI}{RR_{A+B+}}$$

When AP is positive, it represents the proportion of risk of  $A+B+$  due to the superadditivity of exposures  $A$  and  $B$  (that is, the amount greater than additivity). Note that it is a proportion of risk, not relative risk, as one can see by multiplying both numerator and denominator by the reference probability  $p_{A-B-}$ .

The SI is a ratio measure:

$$SI = \frac{ERR_{A+B+}}{ERR_{A-B+} + ERR_{A+B-}}$$

The SI is 1, rather than 0, for a perfectly additive model. It is  $< 1$  for a subadditive model and  $> 1$  for a superadditive model. Note that because ERRs can be negative (they are always  $\geq -1$ ), the SI can be negative, making interpretation difficult. The SI is intended as a measure for cases in which all ERRs of the model are positive, but this will not always be the case. ERRs will be determined by the data and the specification of the model. See [Knol et al. \(2011\)](#) and [example 4](#) for discussions about negative ERRs.

## Additive versus multiplicative interactions

Typically, interactions are fit and tested by specifying main effects and interactions of the main effects ([Rothman, Greenland, and Walker 1980](#)). Suppose  $x_{A+}$  is a 0/1 variable that is 1 when exposure  $A$  is positive and 0 when it is negative. The variable  $x_{B+}$  is defined similarly for exposure  $B$ . Let  $x_{A+B+}$  be the 0/1 variable that is 1 when both  $A$  and  $B$  are positive and 0 otherwise.

When we specify a model and include terms

$$b_1x_{A+} + b_2x_{B+} + b_3x_{A+B+}$$

we call  $b_1x_{A+}$  and  $b_2x_{B+}$  main effects and  $b_3x_{A+B+}$  their interaction. The test of the interaction is the test of  $b_3 = 0$ .

It is a model of interaction that is additive in these terms. However, when the statistics of interest are risk ratios, odds ratios, or hazard ratios, the model is generally called multiplicative because it is multiplicative in these statistics. The risk ratios (or odds ratios or hazard ratios) are given by the exponentiated coefficients; that is,  $RR_{A+} = e^{b_1}$ ,  $RR_{B+} = e^{b_2}$ , etc. The risk ratio for  $A+B+$  is  $RR_{A+B+} = e^{b_1}e^{b_2}e^{b_3}$ . If the interaction is multiplicative,  $e^{b_3} = 1$  and

$$RR_{A+B+} = RR_{A+} \times RR_{B+}$$

As mentioned earlier, many biological processes more closely follow an additive model of risk than a multiplicative one. The additive model of ERR is

$$ERR_{A+B+} = ERR_{A-B+} + ERR_{A+B-}$$

Note that unlike the multiplicative model with main effects, the three states in the additive model,  $A-B+$ ,  $A+B-$ , and  $A+B+$ , are disjoint.

reri is called with the syntax

```
reri cmd depvar var1 var2 . . .
```

reri uses *var1* and *var2* to create the three terms for the additive model using [factor-variable notation](#). Suppose *var1* is *xa*, a 0/1 variable, indicating whether a subject has exposure  $A$ . Suppose *var2* is *xb* for exposure  $B$ . Then, the terms needed to test for additive ERR are, in factor-variable notation,

```
0.xa#1.xb 1.xa#0.xb 1.xa#1.xb
```

reri creates these terms and calls *cmd* to fit the model. Then, it uses [nlcom](#) to calculate the RERI, AP, and SI statistics and their standard errors. See [Methods and formulas](#).

## Incidence-rate ratios, hazard ratios, and odds ratios

Incidence rates and hazards are measures of risk. An incidence rate is a probability adjusted for time at risk. A hazard is a conditional probability when the distribution is discrete and an incremental conditional probability when the distribution is continuous (Kalbfleisch and Prentice 2002). Hence, incidence-rate ratios and hazard ratios are measures of relative risk. So using RERI, AP, and SI statistics with models that estimate incidence-rate ratios (`poisson` and `nbreg`) or hazard ratios (`stcox`, `stintcox`, `streg`, `stintreg`) presents no difficulties.

Odds ratios, however, are different. As is well known, the odds ratio approximates the risk ratio in the population when the prevalence of the outcome in the population is low (see, for example, Greenland and Thomas [1982]). So if your data are from a case-control study and the population prevalence of a positive outcome is low, then odds ratios from a logistic regression will approximate the risk ratios in the population, and the RERI, AP, and SI statistics are appropriate. On the other hand, if the population prevalence is high, the odds ratio is typically a poor approximation to the risk ratio (unless it is close to 1), and interpretation of the RERI, AP, and SI statistics may be problematic (see, for example, Skrondal [2003]).

If the study you are analyzing is a cohort study, observational study, or a randomized controlled trial with a binary outcome, you can estimate risk directly rather than fitting a logistic regression. A binomial model of risk or a Poisson model can be fit (using an appropriate variance estimator; see Cummings [2009] and example 2).

If time to outcome is measured, then a survival model may be fit, and there are no issues interpreting the RERI and SI statistics. Interpreting the AP statistic is a little trickier because it relates to the hazard at a point in time rather than risk in a population, so the proportion is the proportion of the hazard at a given time.

### ▷ Example 1: Binomial model of risk

We have simulated data on birth defects that mimic the results from Brender et al. (2013), an observational study of birth defects. Three binary outcomes are included in these data and are `tube`, `palate`, and `heart`, representing neural tube defects, palate defects, and heart defects, respectively. Predictors of birth defects are two measures of nitrate intake: `drug`, with values 0/1, indicating nitrosatable drug exposure; and `nitrate`, also with values 0/1, representing either low or high daily consumption of nitrates from drinking water.

We load the dataset and describe it.

```
. use https://www.stata-press.com/data/r19/nitrates
(Simulated prenatal nitrate intake and birth defects data)
. describe
Contains data from https://www.stata-press.com/data/r19/nitrates.dta
Observations: 1,367 Simulated prenatal nitrate
 intake and birth defects data
Variables: 6 4 Oct 2024 16:05
 (_dta has notes)
```

| Variable name | Storage type | Display format | Value label | Variable label             |
|---------------|--------------|----------------|-------------|----------------------------|
| id            | int          | %9.0g          |             | Identification number      |
| tube          | byte         | %9.0g          | case        | Neural tube defect         |
| heart         | byte         | %9.0g          | case        | Conotruncal heart defect   |
| palate        | byte         | %9.0g          | case        | Cleft palate               |
| drug          | byte         | %9.0g          | yesno       | Nitrosatable drug exposure |
| nitrate       | byte         | %9.0g          | nitrate     | Total nitrate intake       |

Sorted by: id

We first examine the outcome `tube`. We want to see whether the interactions of the two predictors `drug` and `nitrate` follow an additive model of risk. Because this is an observational study, a binomial model of risk is appropriate, and we can fit it using `binreg`.

The syntax of `rerī` is `rerī cmd depvar var1 var2`, where `var1` and `var2` are the binary predictors of interest. Note that `var1` and `var2` are specified without using factor-variable notation. `rerī` will create the interactions automatically. Here’s the result:

```
. rerī binreg tube drug nitrate
Fitting binreg ...
Interaction of drug and nitrate on an additive scale
Model: Binomial generalized linear Number of observations = 1,284
drug#nitrate
 - + No#High
 + - Yes#Low
 + + Yes#High
```

|              | EIM      |           | z    | P> z  | [95% conf. interval] |          |
|--------------|----------|-----------|------|-------|----------------------|----------|
|              | ERR      | std. err. |      |       |                      |          |
| drug#nitrate |          |           |      |       |                      |          |
| - +          | .1161419 | .1571024  | 0.78 | 0.435 | -.1529507            | .4707206 |
| + -          | .2385481 | .2392316  | 1.11 | 0.268 | -.1517982            | .8085336 |
| + +          | .7769424 | .407042   | 2.51 | 0.012 | .1342                | 1.783922 |
| RERI         | .4222523 | .453286   | 0.93 | 0.352 | -.466172             | 1.310677 |
| Attr. prop.  | .2376286 | .2187062  | 1.09 | 0.277 | -.1910276            | .6662848 |
| Synergy ind. | 2.190483 | 2.034328  | 0.84 | 0.398 | .3548285             | 13.52263 |

Note: P>|z| for synergy index (SI) is for test H0: SI = 1.

Note that the interactions created by `rerī` represent disjoint groups in the data. The `-+` interaction is for `drug = No` and `nitrate = High`. The `+ -` and `+ +` interactions are defined similarly as shown in the legend. The interaction `-` is, of course, the reference category.

By default, `reri` displays the model coefficients for the  $-+$ ,  $+ -$ , and  $++$  interactions as ERR, which is relative risk minus one. So ERR greater than 0 means that relative risk is greater than 1. If the model is additive, we would have

$$\text{ERR}_{++} = \text{ERR}_{-+} + \text{ERR}_{+-}$$

The RERI statistic is the difference between the two sides of this equation:

$$\text{RERI} = \text{ERR}_{++} - \text{ERR}_{-+} - \text{ERR}_{+-}$$

The estimates are  $\text{ERR}_{-+} = 0.116$  and  $\text{ERR}_{+-} = 0.239$ , so  $\text{ERR}_{-+} + \text{ERR}_{+-} = 0.355$ . The estimate of  $\text{ERR}_{++}$  is 0.777, which is greater than  $\text{ERR}_{-+} + \text{ERR}_{+-}$ . So the model is superadditive. The RERI statistic is 0.422, the difference between 0.777 and 0.355.

The  $p$ -value for RERI is a test of the null hypothesis  $\text{RERI} = 0$ . That is, a small  $p$ -value provides evidence to reject the additive model for the interactions. In this case,  $p = 0.352$ , so we do not have enough evidence to reject the additive model, despite the observed superadditivity.

The AP is

$$\text{AP} = \frac{\text{RERI}}{\text{RR}_{++}}$$

It is the proportion of risk for the  $++$  interaction that is due to the risk that is above additive. In this case, AP is 0.238 or 23.8%. Its  $p$ -value is a test of the null hypothesis  $\text{AP} = 0$ . This null hypothesis is equivalent to the null hypothesis  $\text{RERI} = 0$ , so  $p$ -values will be similar.

The SI is

$$\text{SI} = \frac{\text{ERR}_{++}}{\text{ERR}_{-+} + \text{ERR}_{+-}}$$

It is how many times larger (or smaller) is  $\text{ERR}_{++} = 0.777$  than  $\text{ERR}_{-+} + \text{ERR}_{+-} = 0.355$ . In this example, it is about twice as large, or more precisely, 2.19 times as large. The null hypothesis is  $\text{SI} = 1$ , rather than 0. The SI is a rather odd statistic because it is a test of an additive model using a multiplicative scale!

rer\_i can optionally display the interaction coefficients from binreg as relative risk (that is, risk ratios) rather than ERR. This is done by specifying the rr option. We do so and also specify the noisily option to see the output from binreg.

```
. rer_i binreg tube drug nitrate, rr noisily
Fitting binreg:
Iteration 1: Deviance = 1176.736
Iteration 2: Deviance = 1176.736
Generalized linear models Number of obs = 1,284
Optimization : MQL Fisher scoring Residual df = 1,280
 (IRLS EIM) Scale parameter = 1
Deviance = 1176.735717 (1/df) Deviance = .9193248
Pearson = 1283.999016 (1/df) Pearson = 1.003124
Variance function: V(u) = u*(1-u/1) [Binomial]
Link function : g(u) = ln(u) [Log]
 BIC = -7985.166
```

| tube         | EIM        |           |        |       |                      |
|--------------|------------|-----------|--------|-------|----------------------|
|              | Risk ratio | std. err. | z      | P> z  | [95% conf. interval] |
| drug#nitrate |            |           |        |       |                      |
| No#High      | 1.116142   | .1571024  | 0.78   | 0.435 | .8470493 1.470721    |
| Yes#Low      | 1.238548   | .2392316  | 1.11   | 0.268 | .8482018 1.808534    |
| Yes#High     | 1.776942   | .407042   | 2.51   | 0.012 | 1.1342 2.783922      |
| _cons        | .157969    | .013697   | -21.28 | 0.000 | .1332805 .1872307    |

Note: \_cons estimates baseline risk.

Interaction of drug and nitrate on an additive scale

```
drug#nitrate
- + No#High
+ - Yes#Low
+ + Yes#High
```

|              | EIM        |           |      |       |                      |
|--------------|------------|-----------|------|-------|----------------------|
|              | Risk ratio | std. err. | z    | P> z  | [95% conf. interval] |
| drug#nitrate |            |           |      |       |                      |
| - +          | 1.116142   | .1571024  | 0.78 | 0.435 | .8470493 1.470721    |
| + -          | 1.238548   | .2392316  | 1.11 | 0.268 | .8482018 1.808534    |
| + +          | 1.776942   | .407042   | 2.51 | 0.012 | 1.1342 2.783922      |
| RERI         | .4222523   | .453286   | 0.93 | 0.352 | -.466172 1.310677    |
| Attr. prop.  | .2376286   | .2187062  | 1.09 | 0.277 | -.1910276 .6662848   |
| Synergy ind. | 2.190483   | 2.034328  | 0.84 | 0.398 | .3548285 13.52263    |

Note: P>|z| for synergy index (SI) is for test H0: SI = 1.

We see the output of binreg followed by the output of rer\_i. In this case, rer\_i displays the interaction coefficients as risk ratios, matching the output of binreg. Risk ratios are just ERRs plus one, so point estimates and confidence intervals for the interactions are just shifted by one from the previous results, and standard errors and p-values are unchanged. The RERI, AP, and SI statistics are all exactly the same as they were in the previous results.

For comparison, we can fit a binreg model specifying the interaction in the standard manner: main effects for drug and nitrate and a single interaction term for drug = Yes and nitrate = High.

```
. binreg tube drug##nitrate, rr
Iteration 1: Deviance = 1617.128
Iteration 2: Deviance = 1213.623
Iteration 3: Deviance = 1177.487
Iteration 4: Deviance = 1176.736
Iteration 5: Deviance = 1176.736
Iteration 6: Deviance = 1176.736

Generalized linear models Number of obs = 1,284
Optimization : MQL Fisher scoring Residual df = 1,280
 (IRLS EIM) Scale parameter = 1
Deviance = 1176.735717 (1/df) Deviance = .9193248
Pearson = 1283.99895 (1/df) Pearson = 1.003124

Variance function: V(u) = u*(1-u/1) [Binomial]
Link function : g(u) = ln(u) [Log]
BIC = -7985.166
```

| tube                     | EIM        |           |        |       |                      |          |
|--------------------------|------------|-----------|--------|-------|----------------------|----------|
|                          | Risk ratio | std. err. | z      | P> z  | [95% conf. interval] |          |
| drug<br>Yes              | 1.238548   | .2392315  | 1.11   | 0.268 | .8482019             | 1.808533 |
| nitrate<br>High          | 1.116142   | .1571023  | 0.78   | 0.435 | .8470494             | 1.47072  |
| drug#nitrate<br>Yes#High | 1.285408   | .3952681  | 0.82   | 0.414 | .7035471             | 2.348491 |
| _cons                    | .157969    | .013697   | -21.28 | 0.000 | .1332805             | .1872307 |

Note: `_cons` estimates baseline risk.

This model is not as easy to interpret as the previous ones and shows no evidence that drug, nitrate, or their interaction has any effect on neural tube defects. However, the previous model fit with disjoint interactions provided strong evidence that the relative risk of neural tube defects was greater for the ++ group when compared with the reference group; in that model, the risk ratio for the ++ group was greater than 1, and the *p*-value was 0.012.

Note that when running binreg directly, we must specify the rr model option. When rer1 calls binreg, it automatically fits an rr model. Specifying rr with rer1 is merely a display option; it reports relative risk rather than ERR.



□ Technical note

As mentioned earlier, when rerri is called with, say,

```
rerri cmd y x1 x2 ...
```

it creates disjoint interactions and runs

```
cmd y 0.x1#1.x2 1.x1#0.x2 1.x1#1.x2 ...
```

(assuming that x1 and x2 are 0/1 and that 0 is taken as the base category for both).

If rerri gives an error message and you cannot figure out the reason why, run

```
cmd y 0.x1#1.x2 1.x1#0.x2 1.x1#1.x2 $...$
```

and see whether this gives any problems such as dropped terms or dropped observations.



▷ Example 2: Poisson model

We continue with our previous [example](#). We again fit a model for the outcome tube, representing neural tube birth defects. Our predictors, as in the previous example, are drug (nitrosatable drug exposure) and nitrate (drinking water nitrate intake). The only difference is that we specify poisson rather than binreg.

```
. rerri poisson tube drug nitrate
Fitting poisson ...
Interaction of drug and nitrate on an additive scale
Model: Poisson Number of observations = 1,284
drug#nitrate
 - + No#High
 + - Yes#Low
 + + Yes#High
```

|              | ERR      | Robust<br>std. err. | z    | P> z  | [95% conf. interval] |          |
|--------------|----------|---------------------|------|-------|----------------------|----------|
| drug#nitrate |          |                     |      |       |                      |          |
| - +          | .1161419 | .1571636            | 0.78 | 0.435 | -.1530418            | .4708787 |
| + -          | .2385481 | .2393248            | 1.11 | 0.268 | -.1519233            | .8088004 |
| + +          | .7769424 | .4072075            | 2.51 | 0.012 | .133993              | 1.78443  |
| RERI         | .4222523 | .4534688            | 0.93 | 0.352 | -.4665302            | 1.311035 |
| Attr. prop.  | .2376286 | .2187937            | 1.09 | 0.277 | -.1911993            | .6664564 |
| Synergy ind. | 2.190483 | 2.035131            | 0.84 | 0.399 | .3545735             | 13.53235 |

Note: P>|z| for synergy index (SI) is for test H0: SI = 1.

We see that the reported point estimates for the interactions, RERI, AP, and SI are identical to the ones estimated when we used binreg. The standard errors are almost the same, differing only in the fourth significant digit.

When the outcome is 0/1, `rerI poisson` by default reports standard errors calculated using `vce(robust)`. This is because when the outcome is 0/1, the distribution is binomial (Bernoulli), not Poisson, and `vce(robust)` gives standard errors that are valid in this case. The other `vcetypes` allowed with `poisson` are available and can be selected by specifying the `vce()` option with `rerI`. See [R] [poisson](#) and [R] [vce\\_option](#).

`binreg` by default fits models using iterated, reweighted least-squares optimization and uses the expected information matrix for the variance estimator. `poisson` uses maximum likelihood optimization. Point estimates will theoretically be the same but will have numerical differences because of the different optimizers used. Different variance estimators are used, so slight differences, as we observed here, are to be expected.

One advantage of using `poisson` with `rerI` rather than `binreg` is that maximum likelihood optimization is more robust than iterated, reweighted least-squares optimization. There can be convergence problems with iterated, reweighted least squares when maximum likelihood will converge without any difficulty.

`binreg` does have an `ml` option for maximum likelihood optimization. We can also specify `vce(robust)` to reproduce the results given by `poisson`.

```
. rerI binreg tube drug nitrate, ml vce(robust)
Fitting binreg ...
Interaction of drug and nitrate on an additive scale
Model: Binomial generalized linear Number of observations = 1,284
drug#nitrate
 - + No#High
 + - Yes#Low
 + + Yes#High
```

|              | ERR      | Robust<br>std. err. | z    | P> z  | [95% conf. interval] |          |
|--------------|----------|---------------------|------|-------|----------------------|----------|
| drug#nitrate |          |                     |      |       |                      |          |
| - +          | .1161419 | .1571636            | 0.78 | 0.435 | -.1530418            | .4708787 |
| + -          | .2385481 | .2393248            | 1.11 | 0.268 | -.1519233            | .8088004 |
| + +          | .7769424 | .4072075            | 2.51 | 0.012 | .133993              | 1.78443  |
| RERI         | .4222523 | .4534688            | 0.93 | 0.352 | -.4665302            | 1.311035 |
| Attr. prop.  | .2376286 | .2187937            | 1.09 | 0.277 | -.1911993            | .6664564 |
| Synergy ind. | 2.190483 | 2.035131            | 0.84 | 0.399 | .3545735             | 13.53235 |

Note: P>|z| for synergy index (SI) is for test H0: SI = 1.

`rerI binreg` with `ml` and `vce(robust)` gives the same results as `rerI poisson`. But if we want maximum likelihood with the robust variance estimator, we might as well just use `rerI poisson`.



▷ Example 3: Logistic model

We continue with the previous [example](#). These data were simulated to mimic an observational study with full data from several sites over a set period of time. So risk ratios can be estimated directly. But let's pretend it was a case-control study. The outcomes, birth defects, have low prevalences, so odds ratios from a logistic regression should be good approximations to the risk ratios.

We specify logistic with rerl:

```
. rerl logistic tube drug nitrate
Fitting logistic ...
Interaction of drug and nitrate on an additive scale
Model: Logistic Number of observations = 1,284
drug#nitrate
 - + No#High
 + - Yes#Low
 + + Yes#High
```

|              | ERR      | Std. err. | z    | P> z  | [95% conf. interval] |          |
|--------------|----------|-----------|------|-------|----------------------|----------|
| drug#nitrate |          |           |      |       |                      |          |
| - +          | .141003  | .1933785  | 0.78 | 0.436 | -.1814898            | .5905577 |
| + -          | .2965734 | .3085999  | 1.09 | 0.275 | -.1867932            | 1.067251 |
| + +          | 1.080139 | .6495028  | 2.35 | 0.019 | .1280107             | 2.835939 |
| RERI         | .642563  | .691611   | 0.93 | 0.353 | -.7129695            | 1.998096 |
| Attr. prop.  | .3089038 | .2594074  | 1.19 | 0.234 | -.1995252            | .8173329 |
| Synergy ind. | 2.468459 | 2.456832  | 0.91 | 0.364 | .3509409             | 17.36273 |

Note: P>|z| for synergy index (SI) is for test H0: SI = 1.

The ERRs reported are the odds ratios minus one. Calculating ERR using odds ratios assumes that the odds ratios are approximations to the risk ratios.

Here we see that RERI = 0.643, AP = 0.309, and SI = 2.47. When we used poisson and binreg, we got RERI = 0.422, AP = 0.238, and SI = 2.19. Results are similar but not that close numerically.

If we have data from an observational study, a cohort, or a randomized controlled trial, we can estimate risk ratios directly, and we should do so using poisson or binreg. If it is a case-control study, we have no choice but to use logistic.



► Example 4: Negative ERRs

We present data from [example 2](#) in [ST] [stcox](#). The data represent 48 participants in a cancer drug trial. Some receive treatment (drug = 1) and some receive a placebo (drug = 0). The data contain the participants' age and time until death. It has already been `stset` and is ready to be analyzed using a survival model.

We load the dataset, describe it, and run `st` to see the survival-time settings.

```
. use https://www.stata-press.com/data/r19/drugtr
(Patient survival in drug trial)
. describe
Contains data from https://www.stata-press.com/data/r19/drugtr.dta
Observations: 48 Patient survival in drug trial
Variables: 8 3 Mar 2024 02:12
```

| Variable name | Storage type | Display format | Value label | Variable label                         |
|---------------|--------------|----------------|-------------|----------------------------------------|
| studytime     | byte         | %8.0g          |             | Months to death or end of exp.         |
| died          | byte         | %8.0g          |             | 1 if patient died                      |
| drug          | byte         | %8.0g          |             | Drug type (0=placebo)                  |
| age           | byte         | %8.0g          |             | Patient's age at start of exp.         |
| _st           | byte         | %8.0g          |             | 1 if record is to be used; 0 otherwise |
| _d            | byte         | %8.0g          |             | 1 if failure; 0 if censored            |
| _t            | byte         | %10.0g         |             | Analysis time when record ends         |
| _t0           | byte         | %10.0g         |             | Analysis time when record begins       |

```
Sorted by:
. st
-> stset studytime, failure(died)
Survival-time data settings
 Failure event: died!=0 & died<.
Observed time interval: (0, studytime]
Exit on or before: failure
```

The RERI statistic requires two 0/1 predictors. These data have two predictors, drug and age, but age is not 0/1. Let's create a 0/1 variable from age. First, we summarize age:

```
. summarize age, detail
 Patient's age at start of exp.
```

| Percentiles |      | Smallest |             |          |
|-------------|------|----------|-------------|----------|
| 1%          | 47   | 47       |             |          |
| 5%          | 48   | 48       |             |          |
| 10%         | 49   | 48       | Obs         | 48       |
| 25%         | 50.5 | 49       | Sum of wgt. | 48       |
| 50%         | 56   |          | Mean        | 55.875   |
|             |      | Largest  | Std. dev.   | 5.659205 |
| 75%         | 60   | 65       |             |          |
| 90%         | 65   | 67       | Variance    | 32.0266  |
| 95%         | 67   | 67       | Skewness    | .3161066 |
| 99%         | 67   | 67       | Kurtosis    | 2.125197 |

The age of the participants ranges from 47 to 67 years. Median age is 56. Second, we create the variable `older` with `older = 1` representing those with ages  $> 56$  and `older = 0` those with ages  $\leq 56$ . We give it a `value label` as well.

```
. generate older = cond(age > 56, 1, 0)
. label define older_lbl 0 "age<=56" 1 "age>56"
. label values older older_lbl
```

Now, we run `rer1 stcox`:

```
. rer1 stcox drug older
Fitting stcox ...
Interaction of drug and older on an additive scale
Model: Cox proportional hazards Number of observations = 48
 drug#older
 - + 0#age>56
 + - 1#age<=56
 + + 1#age>56
```

|              | ERR       | Std. err. | z     | P> z  | [95% conf. interval] |           |
|--------------|-----------|-----------|-------|-------|----------------------|-----------|
| drug#older   |           |           |       |       |                      |           |
| - +          | .6268459  | .7688877  | 1.03  | 0.303 | -.3557603            | 3.108141  |
| + -          | -.9158604 | .0532904  | -3.91 | 0.000 | -.9756843            | -.7088514 |
| + +          | -.7313977 | .1392912  | -2.53 | 0.011 | -.9027935            | -.2577947 |
| RERI         | -.4423833 | .7436483  | -0.59 | 0.552 | -1.899907            | 1.015141  |
| Attr. prop.  | -1.646983 | 2.711232  | -0.61 | 0.544 | -6.960901            | 3.666935  |
| Synergy ind. | 2.530662  | 6.713001  | 0.35  | 0.726 | .0139727             | 458.3401  |

Notes: P>|z| for synergy index (SI) is for test H0: SI = 1.  
Some estimates of excess relative risk are not positive.

The ERR is negative for the +- (`drug = 1` and `older = 0`) and ++ (`drug = 1` and `older = 1`) interactions. This means the relative risk is less than one.

The RERI, AP, and SI statistics were designed for models in which the -, +-, and ++ interactions each have positive ERRs. That is, values of 1 for the predictors are hypothesized to be associated with increased risk. Clearly, this is not true here. `drug = 1` is hypothesized to be associated with decreased risk. We incorrectly specified the model for `rer1`. `drug = 0` should be the category hypothesized for a positive ERR.

We can correct this by specifying the option `baselevel1(1)`, which tells `reri` to use `drug = 1` as the base level for the first predictor.

```
. reri stcox drug older, baselevel1(1)
Fitting stcox ...
Interaction of drug and older on an additive scale
Model: Cox proportional hazards Number of observations = 48
drug#older
 - + 1#age>56
 + - 0#age<=56
 + + 0#age>56
```

|              | ERR      | Std. err. | z    | P> z  | [95% conf. interval] |          |
|--------------|----------|-----------|------|-------|----------------------|----------|
| drug#older   |          |           |      |       |                      |          |
| - +          | 2.192339 | 1.962714  | 1.89 | 0.059 | -.0433052            | 9.652331 |
| + -          | 10.885   | 7.527446  | 3.91 | 0.000 | 2.434672             | 40.12572 |
| + +          | 18.33507 | 12.94706  | 4.42 | 0.000 | 4.20437              | 70.83291 |
| RERI         | 5.257728 | 8.484262  | 0.62 | 0.535 | -11.37112            | 21.88658 |
| Attr. prop.  | .271927  | .3309285  | 0.82 | 0.411 | -.376681             | .920535  |
| Synergy ind. | 1.402049 | .677738   | 0.70 | 0.484 | .5436253             | 3.615984 |

Note: P>|z| for synergy index (SI) is for test H0: SI = 1.

The ERRs are now all positive, and the model is easy to interpret. The RERI statistic is 5.26, so the effects are superadditive. But the RERI statistic has a *p*-value of 0.535, so we cannot reject the additive model.

The message of this example is clear. If you have negative ERRs, first check that you correctly specified the hypothesized direction of effects. If the model is correctly specified and there are negative ERRs (not close to 0), then the RERI, AP, and SI statistics may not be useful descriptors for the observed effects.



## Stored results

`reri` stores the following in `r()`:

Scalars

- `r(N)`                    number of observations
- `r(reri)`                 RERI
- `r(ap)`                    AP
- `r(si)`                    SI
- `r(level)`                confidence level

Matrices

- `r(reri_table)`         table of results

The `e()` and `r()` stored results from *cmd* are returned as well.

## Methods and formulas

The RERI statistic is

$$\text{RERI} = \text{ERR}_{A+B+} - \text{ERR}_{A-B+} - \text{ERR}_{A+B-}$$

where  $\text{ERR} = \text{RR} - 1$  is the ERR and RR denotes relative risk. Here  $A+B+$  refers to subjects positive for exposure  $A$  and positive for exposure  $B$ , with  $A-B+$  and  $A+B-$  defined similarly.

The AP is

$$\text{AP} = \frac{\text{RERI}}{\text{RR}_{A+B+}}$$

The SI is

$$\text{SI} = \frac{\text{ERR}_{A+B+}}{\text{ERR}_{A-B+} + \text{ERR}_{A+B-}}$$

When incidence-rate ratios, odds ratios, or hazard ratios are estimated rather than risk ratios, they are used in the calculation in place of relative risk.

Standard errors are calculated by `nlcom` using the “delta method”. See [Methods and formulas in \[R\] nlcom](#) for details. The standard error for the SI is calculated by first calculating the standard error for the logarithm of SI and then transforming the result back to the SI scale.

## Acknowledgment

`reri` is based on the `ic` package written by Niels Henrik Bruun, Morten Fenger-Grøn, and Anders Prior, while all were at Aarhus University (2017).

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## Also see

- [R] **binreg** — Generalized linear models: Extensions to the binomial family
- [R] **Epitab** — Tables for epidemiologists
- [R] **logistic** — Logistic regression, reporting odds ratios
- [R] **nbreg** — Negative binomial regression
- [R] **poisson** — Poisson regression
- [ST] **stcox** — Cox proportional hazards model
- [ST] **stintcox** — Cox proportional hazards model for interval-censored survival-time data
- [ST] **stintreg** — Parametric models for interval-censored survival-time data
- [ST] **streg** — Parametric survival models

## Description

The `#review` command displays the last few lines typed at the terminal.

## Syntax

```
#review [#1 [#2]]
```

## Remarks and examples

`#review` (pronounced *pound-review*) is a Stata preprocessor command. `#commands` do not generate a return code or generate ordinary Stata errors. The only error message associated with `#commands` is “unrecognized `#command`”.

The `#review` command displays the last few lines typed at the terminal. If no arguments follow `#review`, the last 20 lines typed at the terminal are displayed. The first argument specifies the number of lines to be reviewed, so `#review 10` displays the last 10 lines typed. The second argument specifies the number of lines to be displayed, so `#review 10 5` displays five lines, starting at the 10th previous line.

Stata reserves a buffer for `#review` lines and stores as many previous lines in the buffer as will fit, rolling out the oldest line to make room for the newest. Requests to `#review` lines no longer stored will be ignored. Only lines typed at the terminal are placed in the `#review` buffer. See [\[U\] 10.5 Editing previous lines in Stata](#).

### ▷ Example 1

Typing `#review` by itself will show the last 20 lines you typed at the terminal:

```
. #review
20 webuse auto
19 describe
18 notes
17 * comments go into the #review buffer, too
16 tabulate rep78
15 codebook foreign
14 regress mpg weight i.foreign
13 generate gp100m = 100/mpg
12 label variable gp100m "Gallons per 100 miles"
11 tabulate rep78, summarize(gp100m)
10 ttest gp100m, by(foreign)
9 regress gp100m weight i.foreign
8 regress, beta
7 margins foreign
6 generate gpmw = ((100/mpg)/weight)*1000
5 summarize gpmw
4 twoway scatter price gpmw
3 regress gpmw i.foreign
2 regress gpmw i.foreign, vce(robust)
1 #review
. _
```

Typing #review 15 2 shows the 15th and 14th previous lines:

```
. #review 15 2
15 regress mpg weight i.foreign
14 generate gp100m = 100/mpg
. _
```



## Description

ROC analysis quantifies the accuracy of diagnostic tests or other evaluation modalities used to discriminate between two states or conditions, which are here referred to as normal and abnormal or control and case. The discriminatory accuracy of a diagnostic test is measured by its ability to correctly classify known normal and abnormal subjects. For this reason, we often refer to the diagnostic test as a classifier. The analysis uses the ROC curve, a graph of the sensitivity versus  $1 - \text{specificity}$  of the diagnostic test. The sensitivity is the fraction of positive cases that are correctly classified by the diagnostic test, whereas the specificity is the fraction of negative cases that are correctly classified. Thus the sensitivity is the true-positive rate, and the specificity is the true-negative rate.

There are six ROC commands:

| Command                 | Entry                 | Description                                                 |
|-------------------------|-----------------------|-------------------------------------------------------------|
| <code>roccomp</code>    | [R] <b>roccomp</b>    | Tests of equality of ROC areas                              |
| <code>rocgold</code>    | [R] <b>roccomp</b>    | Tests of equality of ROC areas against a standard ROC curve |
| <code>rocfit</code>     | [R] <b>rocfit</b>     | Parametric ROC models                                       |
| <code>rocreg</code>     | [R] <b>rocreg</b>     | Nonparametric and parametric ROC regression models          |
| <code>rocregplot</code> | [R] <b>rocregplot</b> | Plot marginal and covariate-specific ROC curves             |
| <code>roctab</code>     | [R] <b>roctab</b>     | Nonparametric ROC analysis                                  |

Postestimation commands are available after `rocfit` and `rocreg`; see [R] [rocfit postestimation](#) and [R] [rocreg postestimation](#).

Both nonparametric and parametric (semiparametric) methods have been suggested for generating the ROC curve. The `roctab` command performs nonparametric ROC analysis for a single classifier. `roccomp` extends the nonparametric ROC analysis function of `roctab` to situations where we have multiple diagnostic tests of interest to be compared and tested. The `rocgold` command also provides ROC analysis for multiple classifiers. `rocgold` compares each classifier's ROC curve to a "gold standard" ROC curve and makes adjustments for multiple comparisons in the analysis. Both `rocgold` and `roccomp` also allow parametric estimation of the ROC curve through a binormal fit. In a binormal fit, both the control and the case populations are normal.

The `rocfit` command also estimates the ROC curve of a classifier through a binormal fit. Unlike `roctab`, `roccomp`, and `rocgold`, `rocfit` is an estimation command. In postestimation, graphs of the ROC curve and confidence bands can be produced. Additional tests on the parameters can also be conducted.

ROC analysis can be interpreted as a two-stage process. First, the control distribution of the classifier is estimated, assuming a normal model or using a distribution-free estimation technique. The classifier is standardized using the control distribution to  $1 - \text{percentile value}$ , the false-positive rate. Second, the ROC curve is estimated as the case distribution of the standardized classifier values.

Covariates may affect both stages of ROC analysis. The first stage may be affected, yielding a covariate-adjusted ROC curve. The second stage may also be affected, producing multiple covariate-specific ROC curves.

The `rocreg` command performs ROC analysis under both types of covariate effects. Both parametric (semiparametric) and nonparametric methods may be used by `rocreg`. Like `rocfit`, `rocreg` is an estimation command and provides many postestimation capabilities.

The global performance of a diagnostic test is commonly summarized by the area under the ROC curve (AUC). This area can be interpreted as the probability that the result of a diagnostic test of a randomly selected abnormal subject will be greater than the result of the same diagnostic test from a randomly selected normal subject. The greater the AUC, the better the global performance of the diagnostic test. Each of the ROC commands provides computation of the AUC.

Citing a lack of clinical relevance for the AUC, other ROC summary measures have been suggested. These include the partial area under the ROC curve for a given false-positive rate  $t$  [ $\text{pAUC}(t)$ ]. This is the area under the ROC curve from the false-positive rate of 0 to  $t$ . The ROC value at a particular false-positive rate and the false-positive rate for a particular ROC value are also useful summary measures for the ROC curve. These three measures are directly estimated by `rocreg` during the model fit or postestimation stages. Point estimates of ROC value are computed by the other ROC commands, but no standard errors are reported.

See [Pepe \(2003\)](#) for a discussion of ROC analysis. Pepe has posted Stata datasets and programs used to reproduce results presented in the book (<https://www.stata.com/bookstore/pepe.html>).

## References

- Cook, J. A., and A. Rajbhandari. 2018. `heckroccurve`: ROC curves for selected samples. *Stata Journal* 18: 174–183.
- Cook, J. A., and V. Ramadas. 2020. When to consult precision-recall curves. *Stata Journal* 20: 131–148.
- Luque-Fernandez, M. A., D. Redondo-Sánchez, and C. Maringe. 2019. `cvauroc`: Command to compute cross-validated area under the curve for ROC analysis after predictive modeling for binary outcomes. *Stata Journal* 19: 615–625.
- Pepe, M. S. 2003. *The Statistical Evaluation of Medical Tests for Classification and Prediction*. New York: Oxford University Press.

Description  
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## Description

`roccomp` and `rocgold` are used to perform receiver operating characteristic (ROC) analyses with rating and discrete classification data.

The two variables *refvar* and *classvar* must be numeric. The reference variable indicates the true state of the observation, such as diseased and nondiseased or normal and abnormal, and must be coded as 0 and 1. The rating or outcome of the diagnostic test or test modality is recorded in *classvar*, which must be at least ordinal, with higher values indicating higher risk.

`roccomp` tests the equality of two or more ROC areas obtained from applying two or more test modalities to the same sample or to independent samples. `roccomp` expects the data to be in wide form when comparing areas estimated from the same sample and in long form for areas estimated from independent samples.

`rocgold` independently tests the equality of the ROC area of each of several test modalities, specified by *classvar*, against a “gold standard” ROC curve, *goldvar*. For each comparison, `rocgold` reports the raw and the Bonferroni-adjusted *p*-value. Optionally, Šidák’s adjustment for multiple comparisons can be obtained.

See [R] [rocfits](#) and [R] [roclogit](#) for commands that fit maximum-likelihood ROC models.

## Quick start

Equality of AUCs for rating *v1* of true state *true* between samples defined by *catvar*

```
roccomp true v1, by(catvar)
```

Equality of AUCs for ratings *v1* and *v2* for the same sample

```
roccomp true v1 v2
```

Same as above, but plot ROC curves without reporting summary statistics and test of equality

```
roccomp true v1 v2, graph
```

Same as above, but plot *v1* with a dashed line and *v2* with a solid line

```
roccomp true v1 v2, graph plot1opts(lpattern(dash)) ///
plot2opts(lpattern(solid))
```

Use contrast matrix *mymat* to compare ROC areas for *v1*, *v2*, *v3*, and *v4*

```
matrix mymat = (1,0,-1,0 \ 0,1,0,-1)
roccomp true v1 v2 v3 v4, test(mymat)
```

Test equality of ROC area for *v1* against a “gold standard” *gold*

```
rocgold true gold v1
```

## Menu

### roccomp

Statistics > Epidemiology and related > ROC analysis > Test equality of two or more ROC areas

### rocgold

Statistics > Epidemiology and related > ROC analysis > Test equality of ROC area against gold standard

## Syntax

Test equality of ROC areas

```
roccomp refvar classvar [classvars] [if] [in] [weight] [, roccomp_options]
```

Test equality of ROC area against a standard ROC curve

```
rocgold refvar goldvar classvar [classvars] [if] [in] [weight] [, rocgold_options]
```

| <i>roccomp_options</i>                       | Description                                                                        |
|----------------------------------------------|------------------------------------------------------------------------------------|
| Main                                         |                                                                                    |
| <code>by(<i>varname</i>)</code>              | split into groups by variable                                                      |
| <code>test(<i>matname</i>)</code>            | use contrast matrix for comparing ROC areas                                        |
| <code>graph</code>                           | graph the ROC curve                                                                |
| <code>norefline</code>                       | suppress plotting the 45-degree reference line                                     |
| <code>separate</code>                        | place each ROC curve on its own graph                                              |
| <code>summary</code>                         | report the area under the ROC curve                                                |
| <code>binormal</code>                        | estimate areas by using binormal distribution assumption                           |
| <code>line#opts(<i>cline_options</i>)</code> | affect rendition of the #th binormal fit line                                      |
| <code>level(#)</code>                        | set confidence level; default is level(95)                                         |
| Plot                                         |                                                                                    |
| <code>plot#opts(<i>plot_options</i>)</code>  | affect rendition of the #th ROC curve                                              |
| Reference line                               |                                                                                    |
| <code>rlopts(<i>cline_options</i>)</code>    | affect rendition of the reference line                                             |
| Y axis, X axis, Titles, Legend, Overall      |                                                                                    |
| <code>twoway_options</code>                  | any options other than <code>by()</code> documented in [G-3] <i>twoway_options</i> |

| <i>rocgold_options</i>                                | Description                                                                                 |
|-------------------------------------------------------|---------------------------------------------------------------------------------------------|
| Main                                                  |                                                                                             |
| <a href="#"><code>sidak</code></a>                    | adjust the $p$ -value by using Šidák's method                                               |
| <a href="#"><code>test(matname)</code></a>            | use contrast matrix for comparing ROC areas                                                 |
| <a href="#"><code>graph</code></a>                    | graph the ROC curve                                                                         |
| <a href="#"><code>norefln</code></a>                  | suppress plotting the 45-degree reference line                                              |
| <a href="#"><code>separate</code></a>                 | place each ROC curve on its own graph                                                       |
| <a href="#"><code>summary</code></a>                  | report the area under the ROC curve                                                         |
| <a href="#"><code>binormal</code></a>                 | estimate areas by using binormal distribution assumption                                    |
| <a href="#"><code>line#opts(cline_options)</code></a> | affect rendition of the #th binormal fit line                                               |
| <a href="#"><code>level(#)</code></a>                 | set confidence level; default is level(95)                                                  |
| Plot                                                  |                                                                                             |
| <a href="#"><code>plot#opts(plot_options)</code></a>  | affect rendition of the #th ROC curve; plot 1 is the “gold standard”                        |
| Reference line                                        |                                                                                             |
| <a href="#"><code>rlopts(cline_options)</code></a>    | affect rendition of the reference line                                                      |
| Y axis, X axis, Titles, Legend, Overall               |                                                                                             |
| <a href="#"><code>twoway_options</code></a>           | any options other than by() documented in [G-3] <a href="#"><code>twoway_options</code></a> |

| <i>plot_options</i>                               | Description                                |
|---------------------------------------------------|--------------------------------------------|
| <a href="#"><code>marker_options</code></a>       | change look of markers (color, size, etc.) |
| <a href="#"><code>marker_label_options</code></a> | add marker labels; change look or position |
| <a href="#"><code>cline_options</code></a>        | change look of the line                    |

`collect` is allowed with `roccomp` and `rocgold`; see [U] [11.1.10 Prefix commands](#).

`fweights` are allowed; see [U] [11.1.6 weight](#).

## Options

### Main

by(*varname*) (`roccomp` only) is required when comparing independent ROC areas. The `by()` variable identifies the groups to be compared.

`sidak` (`rocgold` only) requests that the  $p$ -value be adjusted for the effect of multiple comparisons by using Šidák's method. Bonferroni's adjustment is reported by default.

`test(matname)` specifies the contrast matrix to be used when comparing ROC areas. By default, the null hypothesis that all areas are equal is tested.

`graph` produces graphical output of the ROC curve.

`norefln` suppresses plotting the 45-degree reference line from the graphical output of the ROC curve.

`separate` is meaningful only with `roccomp` and specifies that each ROC curve be placed on its own graph rather than one curve on top of the other.

`summary` reports the area under the ROC curve, its standard error, and its confidence interval. This option is needed only when also specifying `graph`.

`binormal` specifies that the areas under the ROC curves to be compared should be estimated using the binormal distribution assumption. By default, areas to be compared are computed using the trapezoidal rule.

`line#opts(cline_options)` affect the rendition of the line representing the #th ROC curve drawn using the binormal distribution assumption; see [G-3] *cline\_options*. These lines are drawn only if the `binormal` option is specified.

`level(#)` specifies the confidence level, as a percentage, for the confidence intervals. The default is `level(95)` or as set by `set level1`; see [R] *level*.

#### Plot

`plot#opts(plot_options)` affect the rendition of the #th ROC curve—the curve’s plotted points connected by lines. The *plot\_options* can affect the size and color of markers, whether and how the markers are labeled, and whether and how the points are connected; see [G-3] *marker\_options*, [G-3] *marker\_label\_options*, and [G-3] *cline\_options*.

For `rocgold`, `plot1opts()` are applied to the ROC for the gold standard.

#### Reference line

`rlopts(cline_options)` affects the rendition of the reference line; see [G-3] *cline\_options*.

#### Y axis, X axis, Titles, Legend, Overall

*twoway\_options* are any of the options documented in [G-3] *twoway\_options*. These include options for titling the graph (see [G-3] *title\_options*), options for saving the graph to disk (see [G-3] *saving\_option*), and the `by()` option (see [G-3] *by\_option*).

## Remarks and examples

Remarks are presented under the following headings:

- Introduction*
- Comparing areas under the ROC curve*
- Correlated data*
- Independent data*
- Comparing areas with a gold standard*

## Introduction

`roccomp` provides comparison of the ROC curves of multiple classifiers. `rocgold` compares the ROC curves of multiple classifiers with a single “gold standard” classifier. Adjustment of inference for multiple comparisons is also provided by `rocgold`.

See [Pepe \(2003\)](#) for a discussion of ROC analysis. Pepe has posted Stata datasets and programs used to reproduce results presented in the book (<https://www.stata.com/bookstore/pepe.html>).

## Comparing areas under the ROC curve

The area under multiple ROC curves can be compared by using `roccomp`. The command syntax is slightly different if the ROC curves are correlated (that is, different diagnostic tests are applied to the same sample) or independent (that is, diagnostic tests are applied to different samples).

### Correlated data

#### ▷ Example 1

Hanley and McNeil (1983) presented data from an evaluation of two computer algorithms designed to reconstruct CT images from phantoms. We will call these two algorithms' modalities 1 and 2. A sample of 112 phantoms was selected; 58 phantoms were considered normal, and the remaining 54 were abnormal. Each of the two modalities was applied to each phantom, and the resulting images were rated by a reviewer using a six-point scale: 1 = definitely normal, 2 = probably normal, 3 = possibly normal, 4 = possibly abnormal, 5 = probably abnormal, and 6 = definitely abnormal. Because each modality was applied to the same sample of phantoms, the two sets of outcomes are correlated.

We list the first 7 observations:

```
. use https://www.stata-press.com/data/r19/ct
(Reconstruction of CT images)
. list in 1/7, sep(0)
```

|    | mod1 | mod2 | status |
|----|------|------|--------|
| 1. | 2    | 1    | 0      |
| 2. | 5    | 5    | 1      |
| 3. | 2    | 1    | 0      |
| 4. | 2    | 3    | 0      |
| 5. | 5    | 6    | 1      |
| 6. | 2    | 2    | 0      |
| 7. | 3    | 2    | 0      |

The data are in wide form, which is required when dealing with correlated data. Each observation corresponds to one phantom. The variable `mod1` identifies the rating assigned for the first modality, and `mod2` identifies the rating assigned for the second modality. The true status of the phantoms is given by `status=0` if they are normal and `status=1` if they are abnormal. The observations with at least one missing rating were dropped from the analysis.

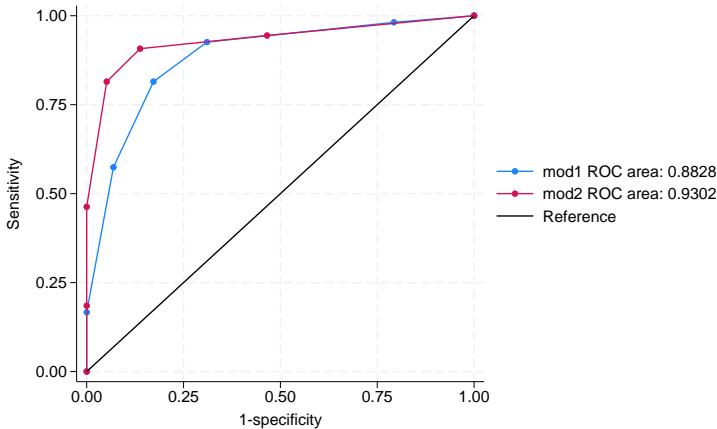
We plot the two ROC curves and compare their areas.

```
. roccomp status mod1 mod2, graph summary
```

|      | Obs | ROC<br>area | Std. err. | Asymptotic normal<br>[95% conf. interval] |         |
|------|-----|-------------|-----------|-------------------------------------------|---------|
| mod1 | 112 | 0.8828      | 0.0317    | 0.82067                                   | 0.94498 |
| mod2 | 112 | 0.9302      | 0.0256    | 0.88005                                   | 0.98042 |

```
H0: area(mod1) = area(mod2)
```

```
chi2(1) = 2.31 Prob>chi2 = 0.1282
```



By default, `roccomp`, with the `graph` option specified, plots the ROC curves on the same graph. Optionally, the curves can be plotted side by side, each on its own graph, by also specifying `separate`.

For each curve, `roccomp` reports summary statistics and provides a test for the equality of the area under the curves, using an algorithm suggested by [DeLong, DeLong, and Clarke-Pearson \(1988\)](#).

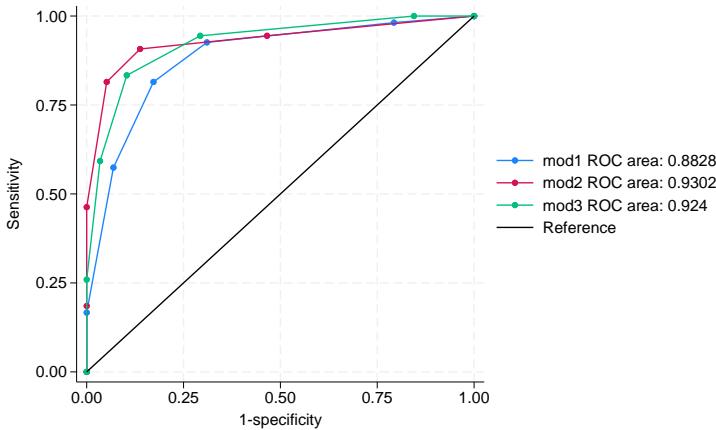
Although the area under the ROC curve for modality 2 is larger than that of modality 1, the  $\chi^2$  test yielded a  $p$ -value of 0.1282, suggesting that there is no significant difference between these two areas.

The `roccomp` command can also be used to compare more than two ROC areas. To illustrate this, we modified the previous dataset by including a fictitious third modality.

```
. use https://www.stata-press.com/data/r19/ct2
(Reconstruction of CT images)
. roccomp status mod1 mod2 mod3, graph summary
```

|      | Obs | ROC<br>area | Std. err. | Asymptotic normal<br>[95% conf. interval] |         |
|------|-----|-------------|-----------|-------------------------------------------|---------|
| mod1 | 112 | 0.8828      | 0.0317    | 0.82067                                   | 0.94498 |
| mod2 | 112 | 0.9302      | 0.0256    | 0.88005                                   | 0.98042 |
| mod3 | 112 | 0.9240      | 0.0241    | 0.87670                                   | 0.97132 |

```
H0: area(mod1) = area(mod2) = area(mod3)
chi2(2) = 6.54 Prob>chi2 = 0.0381
```



By default, `roccomp` tests whether the areas under the ROC curves are all equal. Other comparisons can be tested by creating a contrast matrix and specifying `test(matname)`, where `matname` is the name of the contrast matrix.

For example, assume that we are interested in testing whether the area under the ROC for `mod1` is equal to that of `mod3`. To do this, we can first create an appropriate contrast matrix and then specify its name with the `test()` option.

Of course, this is a trivial example because we could have just specified

```
. roccomp status mod1 mod3
```

without including `mod2` to obtain the same test results. However, for illustration, we will continue with this example.

The contrast matrix must have its number of columns equal to the number of *classvars* (that is, the total number of ROC curves) and a number of rows less than or equal to the number of *classvars*, and the elements of each row must add to zero.

```
. matrix C=(1,0,-1)
. roccomp status mod1 mod2 mod3, test(C)
```

|      | Obs | ROC<br>area | Std. err. | Asymptotic normal<br>[95% conf. interval] |         |
|------|-----|-------------|-----------|-------------------------------------------|---------|
| mod1 | 112 | 0.8828      | 0.0317    | 0.82067                                   | 0.94498 |
| mod2 | 112 | 0.9302      | 0.0256    | 0.88005                                   | 0.98042 |
| mod3 | 112 | 0.9240      | 0.0241    | 0.87670                                   | 0.97132 |

```
HO: Comparison as defined by contrast matrix: C
chi2(1) = 5.25 Prob>chi2 = 0.0220
```

Although all three areas are reported, the comparison is made using the specified contrast matrix.

Perhaps more interesting would be a comparison of the area from mod1 and the average area of mod2 and mod3.

```
. matrix C=(1,-.5,-.5)
. roccomp status mod1 mod2 mod3, test(C)
```

|      | Obs | ROC<br>area | Std. err. | Asymptotic normal<br>[95% conf. interval] |         |
|------|-----|-------------|-----------|-------------------------------------------|---------|
| mod1 | 112 | 0.8828      | 0.0317    | 0.82067                                   | 0.94498 |
| mod2 | 112 | 0.9302      | 0.0256    | 0.88005                                   | 0.98042 |
| mod3 | 112 | 0.9240      | 0.0241    | 0.87670                                   | 0.97132 |

```
HO: Comparison as defined by contrast matrix: C
chi2(1) = 3.43 Prob>chi2 = 0.0642
```

Other contrasts could be made. For example, we could test if mod3 is different from at least one of the other two by first creating the following contrast matrix:

```
. matrix C=(-1,0,1 \ 0,-1,1)
. mat list C
C[2,3]
 c1 c2 c3
r1 -1 0 1
r2 0 -1 1
```

## Independent data

### ▷ Example 2

In [example 1](#), we noted that because each test modality was applied to the same sample of phantoms, the classification outcomes were correlated. Now, assume that we have collected the same data presented by [Hanley and McNeil \(1983\)](#), except that we applied the first test modality to one sample of phantoms and the second test modality to a different sample of phantoms. The resulting measurements are now considered independent.

Here are a few of the observations.

```
. use https://www.stata-press.com/data/r19/ct3
(Reconstruction of CT images)
. list in 1/7, sep(0)
```

|    | pop | status | rating | mod |
|----|-----|--------|--------|-----|
| 1. | 12  | 0      | 1      | 1   |
| 2. | 31  | 0      | 1      | 2   |
| 3. | 1   | 1      | 1      | 1   |
| 4. | 3   | 1      | 1      | 2   |
| 5. | 28  | 0      | 2      | 1   |
| 6. | 19  | 0      | 2      | 2   |
| 7. | 3   | 1      | 2      | 1   |

The data are in long form, which is required when dealing with independent data. The data consist of 24 observations: 6 observations corresponding to abnormal phantoms and 6 to normal phantoms evaluated using the first modality, and similarly 6 observations corresponding to abnormal phantoms and 6 to normal phantoms evaluated using the second modality. The number of phantoms corresponding to each observation is given by the `pop` variable. Once again, we have frequency-weighted data. The variable `mod` identifies the modality, and `rating` is the assigned classification.

We can better view our data by using the `table` command.

```
. table (mod status) (rating) [fw=pop], totals(mod mod#status mod#rating)
```

|               |        | Rating |    |    |    |    |    | Total |
|---------------|--------|--------|----|----|----|----|----|-------|
|               |        | 1      | 2  | 3  | 4  | 5  | 6  |       |
| Modality<br>1 | Status |        |    |    |    |    |    |       |
|               | 0      | 12     | 28 | 8  | 6  | 4  |    | 58    |
|               | 1      | 1      | 3  | 6  | 13 | 22 | 9  | 54    |
|               | Total  | 13     | 31 | 14 | 19 | 26 | 9  | 112   |
| 2             | Status |        |    |    |    |    |    |       |
|               | 0      | 31     | 19 | 5  | 3  |    |    | 58    |
|               | 1      | 3      | 2  | 5  | 19 | 15 | 10 | 54    |
|               | Total  | 34     | 21 | 10 | 22 | 15 | 10 | 112   |

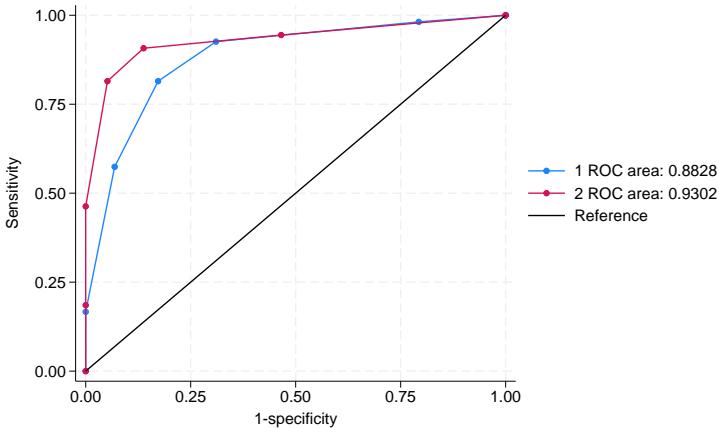
The `status` variable indicates the true status of the phantoms: `status = 0` if they are normal and `status = 1` if they are abnormal.

We now compare the areas under the two ROC curves.

```
. roccomp status rating [fw=pop], by(mod) graph summary
```

| mod | Obs | ROC area | Std. err. | Asymptotic normal<br>[95% conf. interval] |         |
|-----|-----|----------|-----------|-------------------------------------------|---------|
| 1   | 112 | 0.8828   | 0.0317    | 0.82067                                   | 0.94498 |
| 2   | 112 | 0.9302   | 0.0256    | 0.88005                                   | 0.98042 |

```
HO: area(1) = area(2)
chi2(1) = 1.35 Prob>chi2 = 0.2447
```



### Comparing areas with a gold standard

The area under multiple ROC curves can be compared with a gold standard using `rocgold`. The command syntax is similar to that of `roccomp`. The tests are corrected for the effect of multiple comparisons.

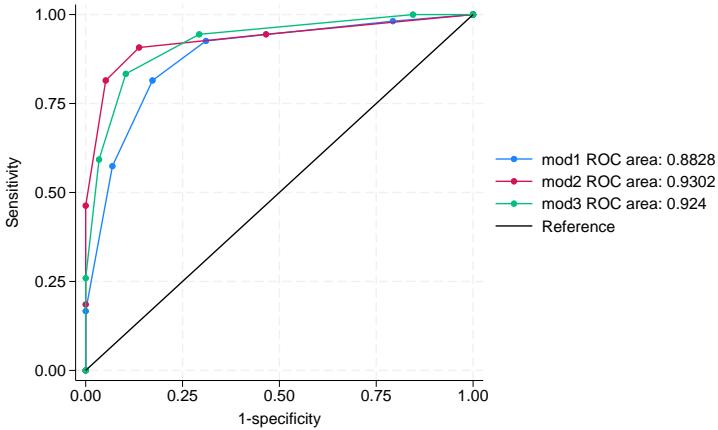
#### ➤ Example 3

We will use the same data (presented by [Hanley and McNeil \[1983\]](#)) as in the `roccomp` examples. Let's assume that the first modality is considered to be the standard against which both the second and third modalities are compared.

We want to plot and compare both the areas of the ROC curves of `mod2` and `mod3` with `mod1`. Because we consider `mod1` to be the gold standard, it is listed first after the reference variable in the `rocgold` command line.

```
. use https://www.stata-press.com/data/r19/ct2
(Reconstruction of CT images)
. rocgold status mod1 mod2 mod3, graph summary
```

|                 | ROC area | Std. err. | chi2   | df | Pr>chi2 | Bonferroni<br>Pr>chi2 |
|-----------------|----------|-----------|--------|----|---------|-----------------------|
| mod1 (standard) | 0.8828   | 0.0317    |        |    |         |                       |
| mod2            | 0.9302   | 0.0256    | 2.3146 | 1  | 0.1282  | 0.2563                |
| mod3            | 0.9240   | 0.0241    | 5.2480 | 1  | 0.0220  | 0.0439                |



Equivalently, we could have done this in two steps by using the `roccomp` command.

```
. roccomp status mod1 mod2, graph summary
. roccomp status mod1 mod3, graph summary
```



## Stored results

`roccomp` stores the following in `r()`:

Scalars

|                     |                              |                      |                             |
|---------------------|------------------------------|----------------------|-----------------------------|
| <code>r(N_g)</code> | number of groups             | <code>r(df)</code>   | $\chi^2$ degrees of freedom |
| <code>r(p)</code>   | $p$ -value for $\chi^2$ test | <code>r(chi2)</code> | $\chi^2$                    |

Matrices

|                   |                            |
|-------------------|----------------------------|
| <code>r(V)</code> | variance-covariance matrix |
|-------------------|----------------------------|

`rocgold` stores the following in `r()`:

Scalars

|                     |                  |
|---------------------|------------------|
| <code>r(N_g)</code> | number of groups |
|---------------------|------------------|

Matrices

|                      |                                    |                       |                                          |
|----------------------|------------------------------------|-----------------------|------------------------------------------|
| <code>r(V)</code>    | variance-covariance matrix         | <code>r(p)</code>     | vector of $p$ -values for $\chi^2$ tests |
| <code>r(chi2)</code> | $\chi^2$ vector                    | <code>r(p_adj)</code> | vector of adjusted $p$ -values           |
| <code>r(df)</code>   | $\chi^2$ degrees-of-freedom vector |                       |                                          |

## Methods and formulas

Assume that we applied a diagnostic test to each of  $N_n$  normal and  $N_a$  abnormal subjects. Further assume that the higher the outcome value of the diagnostic test, the higher the risk of the subject being abnormal. Let  $\hat{\theta}$  be the estimated area under the curve, and let  $X_i, i = 1, 2, \dots, N_n$  and  $Y_j, j = 1, 2, \dots, N_n$  be the values of the diagnostic test for the abnormal and normal subjects, respectively.

Areas under ROC curves are compared using an algorithm suggested by DeLong, DeLong, and Clarke-Pearson (1988). Let  $\hat{\theta} = (\hat{\theta}^1, \hat{\theta}^2, \dots, \hat{\theta}^k)$  be a vector representing the areas under  $k$  ROC curves. See *Methods and formulas* in [R] **roctab** for the definition of these area estimates.

For the  $r$ th area, define

$$V_{10}^r(X_i) = \frac{1}{N_n} \sum_{j=1}^{N_n} \psi(X_i^r, Y_j^r)$$

and for each normal subject,  $j$ , define

$$V_{01}^r(Y_j) = \frac{1}{N_a} \sum_{i=1}^{N_a} \psi(X_i^r, Y_j^r)$$

where

$$\psi(X^r, Y^r) = \begin{cases} 1 & Y^r < X^r \\ \frac{1}{2} & Y^r = X^r \\ 0 & Y^r > X^r \end{cases}$$

Define the  $k \times k$  matrix  $\mathbf{S}_{10}$  such that the  $(r, s)$ th element is

$$S_{10}^{r,s} = \frac{1}{N_a - 1} \sum_{i=1}^{N_a} \{V_{10}^r(X_i) - \hat{\theta}^r\} \{V_{10}^s(X_i) - \hat{\theta}^s\}$$

and  $\mathbf{S}_{01}$  such that the  $(r, s)$ th element is

$$S_{01}^{r,s} = \frac{1}{N_n - 1} \sum_{j=1}^{N_n} \{V_{01}^r(Y_j) - \hat{\theta}^r\} \{V_{01}^s(Y_j) - \hat{\theta}^s\}$$

Then, the covariance matrix is

$$S = \frac{1}{N_a} S_{10} + \frac{1}{N_n} S_{01}$$

Let  $\mathbf{L}$  be a contrast matrix defining the comparison, so that

$$(\hat{\theta} - \theta)' \mathbf{L}' (\mathbf{L} \mathbf{S} \mathbf{L}')^{-1} \mathbf{L} (\hat{\theta} - \theta)$$

has a  $\chi^2$  distribution with degrees of freedom equal to the rank of  $\mathbf{L} \mathbf{S} \mathbf{L}'$ .

## References

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## Also see

- [R] **logistic postestimation** — Postestimation tools for logistic
- [R] **roc** — Receiver operating characteristic (ROC) analysis
- [R] **rocfits** — Parametric ROC models
- [R] **roclog** — Parametric and nonparametric ROC regression
- [R] **roctab** — Nonparametric ROC analysis

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## Description

`rocfit` fits maximum-likelihood ROC models assuming a binormal distribution of the latent variable.

The two variables *refvar* and *classvar* must be numeric. The reference variable indicates the true state of the observation, such as diseased and nondiseased or normal and abnormal, and must be coded as 0 and 1. The rating or outcome of the diagnostic test or test modality is recorded in *classvar*, which must be at least ordinal, with higher values indicating higher risk.

See [\[R\] roc](#) for other commands designed to perform receiver operating characteristic (ROC) analyses with rating and discrete classification data.

## Quick start

Binary true state, `true`, as a function of classification variable `class`

```
rocfit true class
```

Same as above, but with frequency weights `wvar`

```
rocfit true class [fweight = wvar]
```

Specify that `class` is continuous and generate `v1` containing classification groups

```
rocfit true class, continuous(3) generate(v1)
```

## Menu

Statistics > Epidemiology and related > ROC analysis > Parametric ROC analysis without covariates

## Syntax

```
rocfit refvar classvar [if] [in] [weight] [, rocfit_options]
```

| <i>rocfit_options</i>                                                                           | Description                                                        |
|-------------------------------------------------------------------------------------------------|--------------------------------------------------------------------|
| <b>Model</b>                                                                                    |                                                                    |
| <code>continuous(#)</code>                                                                      | divide <i>classvar</i> into # groups of approximately equal length |
| <code>generate(<i>newvar</i>)</code>                                                            | create <i>newvar</i> containing classification groups              |
| <b>SE</b>                                                                                       |                                                                    |
| <code>vce(<i>vcetype</i>)</code>                                                                | <i>vcetype</i> may be oim or opg                                   |
| <b>Reporting</b>                                                                                |                                                                    |
| <code>level(#)</code>                                                                           | set confidence level; default is level(95)                         |
| <b>Maximization</b>                                                                             |                                                                    |
| <code>maximize_options</code>                                                                   | control the maximization process; seldom used                      |
| collect and fp are allowed; see [U] 11.1.10 Prefix commands.                                    |                                                                    |
| fweights are allowed; see [U] 11.1.6 weight.                                                    |                                                                    |
| See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands. |                                                                    |

## Options

### Model

`continuous(#)` specifies that the continuous *classvar* be divided into # groups of approximately equal length. This option is required when *classvar* takes on more than 20 distinct values.

`continuous(.)` may be specified to indicate that *classvar* be used as it is, even though it could have more than 20 distinct values.

`generate(newvar)` specifies the new variable that is to contain values indicating the groups produced by `continuous(#)`. `generate()` may be specified only with `continuous()`.

### SE

`vce(vcetype)` specifies the type of standard error reported. *vcetype* may be either oim or opg; see [R] [vce\\_option](#).

### Reporting

`level(#)`; see [R] [Estimation options](#).

### Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

Setting the optimization type to `technique(bhhh)` resets the default *vcetype* to `vce(opg)`.

## Remarks and examples

Dorfman and Alf (1969) developed a generalized approach for obtaining maximum likelihood estimates of the parameters for a smooth fitting ROC curve. The most commonly used method for ordinal data, and the one implemented here, is based upon the binormal model; see Pepe (2003), Pepe, Longton, and Janes (2009), and Janes, Longton, and Pepe (2009) for methods of ROC analysis for continuous data, including methods for adjusting for covariates.

The model assumes the existence of an unobserved, continuous, latent variable that is normally distributed (perhaps after a monotonic transformation) in both the normal and abnormal populations with means  $\mu_n$  and  $\mu_a$  and variances  $\sigma_n^2$  and  $\sigma_a^2$ , respectively. The model further assumes that the  $K$  categories of the rating variable result from partitioning the unobserved latent variable by  $K - 1$  fixed boundaries. The method fits a straight line to the empirical ROC points plotted using normal probability scales on both axes. Maximum likelihood estimates of the line's slope and intercept and the  $K - 1$  boundaries are obtained simultaneously. See *Methods and formulas* for details.

The intercept from the fitted line is a measurement of  $(\mu_a - \mu_n)/\sigma_a$ , and the slope measures  $\sigma_n/\sigma_a$ .

Thus, the intercept is the standardized difference between the two latent population means, and the slope is the ratio of the two standard deviations. The null hypothesis that there is no difference between the two population means is evaluated by testing that the intercept = 0, and the null hypothesis that the variances in the two populations are equal is evaluated by testing that the slope = 1.

### ▷ Example 1

We use Hanley and McNeil's (1982) dataset, described in [example 1](#) of [\[R\] roctab](#), to fit a smooth ROC curve assuming a binormal model.

```
. use https://www.stata-press.com/data/r19/hanley
(Tomographic images)
. rocfit disease rating

Fitting binormal model:
Iteration 0: Log likelihood = -123.68069
Iteration 1: Log likelihood = -123.64867
Iteration 2: Log likelihood = -123.64855
Iteration 3: Log likelihood = -123.64855

Binormal model of disease on rating Number of obs = 109
Goodness-of-fit chi2(2) = 0.21
Prob > chi2 = 0.9006
Log likelihood = -123.64855
```

|           | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-----------|-------------|-----------|-------|-------|----------------------|----------|
| intercept | 1.656782    | 0.310456  | 5.34  | 0.000 | 1.048300             | 2.265265 |
| slope (*) | 0.713002    | 0.215882  | -1.33 | 0.184 | 0.289881             | 1.136123 |
| /cut1     | 0.169768    | 0.165307  | 1.03  | 0.304 | -0.154227            | 0.493764 |
| /cut2     | 0.463215    | 0.167235  | 2.77  | 0.006 | 0.135441             | 0.790990 |
| /cut3     | 0.766860    | 0.174808  | 4.39  | 0.000 | 0.424243             | 1.109477 |
| /cut4     | 1.797938    | 0.299581  | 6.00  | 0.000 | 1.210770             | 2.385106 |

| Index    | Indices from binormal fit |           |                      |          |
|----------|---------------------------|-----------|----------------------|----------|
|          | Estimate                  | Std. err. | [95% conf. interval] |          |
| ROC area | 0.911331                  | 0.029506  | 0.853501             | 0.969161 |
| delta(m) | 2.323671                  | 0.502370  | 1.339044             | 3.308298 |
| d(e)     | 1.934361                  | 0.257187  | 1.430284             | 2.438438 |
| d(a)     | 1.907771                  | 0.259822  | 1.398530             | 2.417012 |

(\*) z test for slope==1

rocfit outputs the MLE for the intercept and slope of the fitted regression line along with, here, four boundaries (because there are five ratings) labeled /cut1 through /cut4. Also rocfit computes and reports four indices based on the fitted ROC curve: the area under the curve (labeled ROC area),  $\delta(m)$  (labeled delta(m)),  $d_e$  (labeled d(e)), and  $d_a$  (labeled d(a)). More information about these indices can be found in *Methods and formulas* and in [Erdreich and Lee \(1981\)](#).



## Stored results

rocfit stores the following in e():

### Scalars

|               |                                                 |
|---------------|-------------------------------------------------|
| e(N)          | number of observations                          |
| e(k)          | number of parameters                            |
| e(k_eq)       | number of equations in e(b)                     |
| e(k_eq_model) | number of equations in overall model test       |
| e(k_dv)       | number of dependent variables                   |
| e(df_m)       | model degrees of freedom                        |
| e(ll)         | log likelihood                                  |
| e(chi2_gf)    | goodness-of-fit $\chi^2$                        |
| e(df_gf)      | goodness-of-fit degrees of freedom              |
| e(p_gf)       | p-value for goodness-of-fit test                |
| e(area)       | area under the ROC curve                        |
| e(se_area)    | standard error for the area under the ROC curve |
| e(deltam)     | delta(m)                                        |
| e(se_delm)    | standard area for delta(m)                      |
| e(de)         | d(e) index                                      |
| e(se_de)      | standard error for d(e) index                   |
| e(da)         | d(a) index                                      |
| e(se_da)      | standard error for d(a) index                   |
| e(rank)       | rank of e(V)                                    |
| e(ic)         | number of iterations                            |
| e(rc)         | return code                                     |
| e(converged)  | 1 if converged, 0 otherwise                     |

### Macros

|             |                                                                          |
|-------------|--------------------------------------------------------------------------|
| e(cmd)      | rocfit                                                                   |
| e(cmdline)  | command as typed                                                         |
| e(depvar)   | refvar and classvar                                                      |
| e(wtype)    | weight type                                                              |
| e(wexp)     | weight expression                                                        |
| e(title)    | title in estimation output                                               |
| e(chi2type) | GOF; type of model $\chi^2$ test                                         |
| e(vce)      | vcetype specified in vce()                                               |
| e(vcetype)  | title used to label Std. err.                                            |
| e(opt)      | type of optimization                                                     |
| e(which)    | max or min; whether optimizer is to perform maximization or minimization |

|               |                                              |
|---------------|----------------------------------------------|
| e(ml_method)  | type of ml method                            |
| e(user)       | name of likelihood-evaluator program         |
| e(technique)  | maximization technique                       |
| e(properties) | b V                                          |
| Matrices      |                                              |
| e(b)          | coefficient vector                           |
| e(iolog)      | iteration log (up to 20 iterations)          |
| e(gradient)   | gradient vector                              |
| e(V)          | variance–covariance matrix of the estimators |
| Functions     |                                              |
| e(sample)     | marks estimation sample                      |

In addition to the above, the following is stored in `r()`:

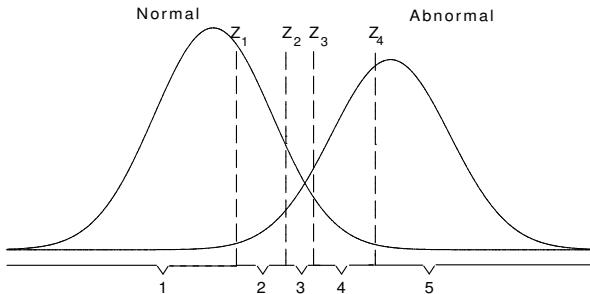
|          |                                                                                                                       |
|----------|-----------------------------------------------------------------------------------------------------------------------|
| Matrices |                                                                                                                       |
| r(table) | matrix containing the coefficients with their standard errors, test statistics, $p$ -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

Dorfman and Alf (1969) developed a general procedure for obtaining maximum likelihood estimates of the parameters of a smooth-fitting ROC curve. The most common method, and the one implemented in Stata, is based upon the binormal model.

The model assumes that there is an unobserved continuous latent variable that is normally distributed in both the normal and abnormal populations. The idea is better explained with the following illustration:



The latent variable is assumed to be normally distributed for both the normal and abnormal subjects, perhaps after a monotonic transformation, with means  $\mu_n$  and  $\mu_a$  and variances  $\sigma_n^2$  and  $\sigma_a^2$ , respectively.

This latent variable is assumed to be partitioned into the  $k$  categories of the rating variable by  $k - 1$  fixed boundaries. In the above figure, the  $k = 5$  categories of the rating variable identified on the bottom result from the partition of the four boundaries  $Z_1$  through  $Z_4$ .

Let  $R_j$  for  $j = 1, 2, \dots, k$  indicate the categories of the rating variable, let  $i = 1$  if the subject belongs to the normal group, and let  $i = 2$  if the subject belongs to the abnormal group.

Then,

$$p(R_j|i = 1) = F(Z_j) - F(Z_{j-1})$$

where  $Z_k = (x_k - \mu_n)/\sigma_n$ ,  $F$  is the cumulative normal distribution,  $F(Z_0) = 0$ , and  $F(Z_k) = 1$ . Also,

$$p(R_j|i = 2) = F(bZ_j - a) - F(bZ_{j-1} - a)$$

where  $b = \sigma_n/\sigma_a$  and  $a = (\mu_a - \mu_n)/\sigma_a$ .

The parameters  $a$ ,  $b$  and the  $k - 1$  fixed boundaries  $Z_j$  are simultaneously estimated by maximizing the log-likelihood function

$$\log L = \sum_{i=1}^2 \sum_{j=1}^k r_{ij} \log\{p(R_j|i)\}$$

where  $r_{ij}$  is the number of  $R_j$ s in group  $i$ .

The area under the fitted ROC curve is computed as

$$\Phi\left(\frac{a}{\sqrt{1+b^2}}\right)$$

where  $\Phi$  is the standard normal cumulative distribution function.

Point estimates for the ROC curve indices are as follows:

$$\delta(m) = \frac{a}{b} \quad d_e = \frac{2a}{b+1} \quad d_a = \frac{a\sqrt{2}}{\sqrt{1+b^2}}$$

Variances for these indices are computed using the delta method.

The  $\delta(m)$  estimates  $(\mu_a - \mu_n)/\sigma_n$ ,  $d_e$  estimates  $2(\mu_a - \mu_n)/(\sigma_a - \sigma_n)$ , and  $d_a$  estimates  $\sqrt{2}(\mu_a - \mu_n)/(\sigma_a^2 - \sigma_n^2)^{1/2}$ .

Simultaneous confidence bands for the entire curve are obtained, as suggested by [Ma and Hall \(1993\)](#), by first obtaining [Working–Hotelling \(1929\)](#) confidence bands for the fitted straight line in normal probability coordinates and then transforming them back to ROC coordinates.

## References

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## Also see

- [R] **rocfit postestimation** — Postestimation tools for rocfit
- [R] **roc** — Receiver operating characteristic (ROC) analysis
- [R] **rocreg** — Parametric and nonparametric ROC regression
- [U] **20 Estimation and postestimation commands**

## Postestimation commands

The following command is of special interest after `rocfit`:

| Command              | Description                                                 |
|----------------------|-------------------------------------------------------------|
| <code>rocplot</code> | plot the fitted ROC curve and simultaneous confidence bands |

The following standard postestimation commands are also available:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICC, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>lincom</code>        | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| * <code>test</code>          | Wald tests of simple and composite linear hypotheses                                                                                    |

\*See *Using `lincom` and `test`* below.

# rocplot

## Description for rocplot

rocplot plots the fitted ROC curve and simultaneous confidence bands.

## Menu for rocplot

Statistics > Epidemiology and related > ROC analysis > ROC curves after rocfit

## Syntax for rocplot

```
rocplot [, rocplot_options]
```

| <i>rocplot_options</i>                      | Description                                                                           |
|---------------------------------------------|---------------------------------------------------------------------------------------|
| Main                                        |                                                                                       |
| <code>confband</code>                       | display confidence bands                                                              |
| <code>norefline</code>                      | suppress plotting the reference line                                                  |
| <code>level(#)</code>                       | set confidence level; default is level(95)                                            |
| Plot                                        |                                                                                       |
| <code>plotopts(<i>plot_options</i>)</code>  | affect rendition of the ROC points                                                    |
| Fit line                                    |                                                                                       |
| <code>lineopts(<i>cline_options</i>)</code> | affect rendition of the fitted ROC line                                               |
| CI plot                                     |                                                                                       |
| <code>ciopts(<i>area_options</i>)</code>    | affect rendition of the confidence bands                                              |
| Reference line                              |                                                                                       |
| <code>rlopts(<i>cline_options</i>)</code>   | affect rendition of the reference line                                                |
| Add plots                                   |                                                                                       |
| <code>addplot(<i>plot</i>)</code>           | add other plots to the generated graph                                                |
| Y axis, X axis, Titles, Legend, Overall     |                                                                                       |
| <code>twoway_options</code>                 | any options other than by() documented in <a href="#">[G-3]</a> <i>twoway_options</i> |
| <i>plot_options</i>                         | Description                                                                           |
| <i>marker_options</i>                       | change look of markers (color, size, etc.)                                            |
| <i>marker_label_options</i>                 | add marker labels; change look or position                                            |
| <i>cline_options</i>                        | change look of the line                                                               |

## Options for rocplot

### Main

`confband` specifies that simultaneous confidence bands be plotted around the ROC curve.

`norefline` suppresses plotting the 45-degree reference line from the graphical output of the ROC curve.

`level(#)` specifies the confidence level, as a percentage, for the confidence bands. The default is `level(95)` or as set by `set level`; see [R] [level](#).

### Plot

`plotopts` (*plot\_options*) affects the rendition of the plotted ROC points, including the size and color of markers, whether and how the markers are labeled, and whether and how the points are connected. For the full list of available *plot\_options*, see [G-3] [marker\\_options](#), [G-3] [marker\\_Label\\_options](#), and [G-3] [cline\\_options](#).

### Fit line

`lineopts` (*cline\_options*) affects the rendition of the fitted ROC line; see [G-3] [cline\\_options](#).

### CI plot

`ciopts` (*area\_options*) affects the rendition of the confidence bands; see [G-3] [area\\_options](#).

### Reference line

`rlopts` (*cline\_options*) affects the rendition of the reference line; see [G-3] [cline\\_options](#).

### Add plots

`addplot` (*plot*) provides a way to add other plots to the generated graph. See [G-3] [addplot\\_option](#).

### Y axis, X axis, Titles, Legend, Overall

*twoway\_options* are any of the options documented in [G-3] [twoway\\_options](#), excluding `by()`. These include options for titling the graph (see [G-3] [title\\_options](#)) and for saving the graph to disk (see [G-3] [saving\\_option](#)).

## Remarks and examples

Remarks are presented under the following headings:

[Using lincom and test](#)  
[Using rocplot](#)

## Using lincom and test

`intercept`, `slope`, and `/cut#`, shown in [example 1](#) of [\[R\] rocfit](#), are equation names and not variable names, so they need to be referenced as described in [Special syntaxes after multiple-equation estimation](#) of [\[R\] test](#). For example, instead of typing

```
. test intercept
intercept not found
r(111);
```

you should type

```
. test [intercept]_cons
(1) [intercept]_cons = 0
 chi2(1) = 28.48
 Prob > chi2 = 0.0000
```

## Using rocplot

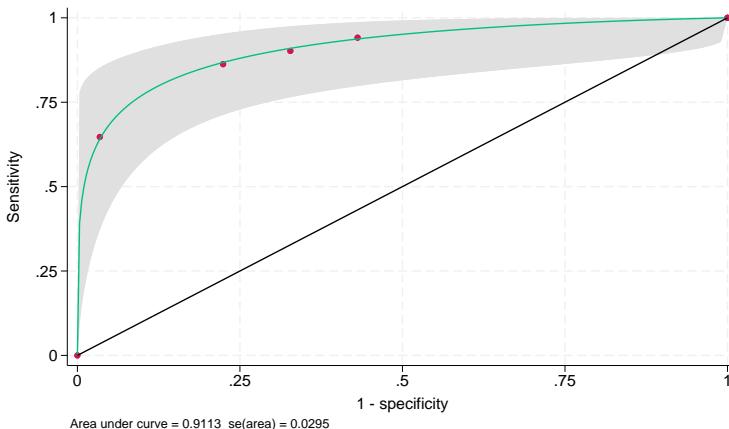
### ▷ Example 1

In [example 1](#) of [\[R\] rocfit](#), we fit a ROC curve by typing `rocfit disease rating`.

In the output table for our model, we are testing whether the variances of the two latent populations are equal by testing that the slope = 1.

We plot the fitted ROC curve.

```
. rocplot, confband
```



## Also see

[\[R\] rocfit](#) — Parametric ROC models

[\[U\] 20 Estimation and postestimation commands](#)

[Description](#)[Options](#)[Acknowledgments](#)[Quick start](#)[Remarks and examples](#)[References](#)[Menu](#)[Stored results](#)[Also see](#)[Syntax](#)[Methods and formulas](#)

## Description

The `rocreg` command is used to perform receiver operating characteristic (ROC) analyses with rating and discrete classification data under the presence of covariates.

The two variables *refvar* and *classvar* must be numeric. The reference variable indicates the true state of the observation—such as diseased and nondiseased or normal and abnormal—and must be coded as 0 and 1. The *refvar* coded as 0 can also be called the control population, while the *refvar* coded as 1 comprises the case population. The rating or outcome of the diagnostic test or test modality is recorded in *classvar*, which must be ordinal, with higher values indicating higher risk.

`rocreg` can fit three models: a nonparametric model, a parametric probit model that uses the bootstrap for inference, and a parametric probit model fit using maximum likelihood.

## Quick start

*Nonparametric estimation with bootstrap resampling*

Area under the ROC curve for test classifier `v1` and true state `true` using seed 20547

```
rocreg true v1, bseed(20547)
```

Add `v2` as an additional classifier

```
rocreg true v1 v2, bseed(20547)
```

Same as above, but estimate ROC value for a false-positive rate of 0.7

```
rocreg true v1 v2, bseed(20547) roc(.7)
```

Covariate stratification of controls by categorical variable `a` using seed 121819

```
rocreg true v1 v2, bseed(121819) ctrlcov(a)
```

Linear control covariate adjustment with binary variable `b` and continuous variable `x`

```
rocreg true v1 v2, bseed(121819) ctrlcov(b x) ctrlmodel(linear)
```

*Parametric estimation*

Area under the ROC curve for test classifier v1 and true state true by estimating equations using seed 200512

```
rocreg true v1, probit bseed(200512)
```

And save results to myfile.dta for use by rocregplot

```
rocreg true v1, probit bseed(200512) bsave(myfile)
```

Add v2 as a classifier and x as a control covariate in a linear control covariate-adjustment model

```
rocreg true v1 v2, probit bseed(200512) ctrlcov(x) ctrlmodel(linear)
```

Also treat x as a ROC covariate

```
rocreg true v1 v2, probit bseed(200512) ctrlcov(x) ///
ctrlmodel(linear) roccov(x)
```

Estimate AUC by maximum likelihood instead of bootstrap resampling

```
rocreg true v1, probit ml
```

## Menu

Statistics > Epidemiology and related > ROC analysis > ROC regression models

## Syntax

Perform nonparametric analysis of ROC curve under covariates, using bootstrap

```
rocreg refvar classvar [classvars] [if] [in] [, np_options]
```

Perform parametric analysis of ROC curve under covariates, using bootstrap

```
rocreg refvar classvar [classvars] [if] [in], probit [probit_options]
```

Perform parametric analysis of ROC curve under covariates, using maximum likelihood

```
rocreg refvar classvar [classvars] [if] [in] [weight], probit ml
[probit_ml_options]
```

| <i>np_options</i>                       | Description                                                                                             |
|-----------------------------------------|---------------------------------------------------------------------------------------------------------|
| <b>Model</b>                            |                                                                                                         |
| <code>auc</code>                        | estimate total area under the ROC curve; the default                                                    |
| <code>roc(numlist)</code>               | estimate ROC for given false-positive rates                                                             |
| <code>invroc(numlist)</code>            | estimate false-positive rates for given ROC values                                                      |
| <code>pauc(numlist)</code>              | estimate partial area under the ROC curve (pAUC) up to each false-positive rate                         |
| <code>cluster(varname)</code>           | variable identifying resampling clusters                                                                |
| <code>ctrlcov(varlist)</code>           | adjust control distribution for covariates in <i>varlist</i>                                            |
| <code>ctrlmodel(strata   linear)</code> | stratify or regress on covariates; default is <code>ctrlmodel(strata)</code>                            |
| <code>pvc(empirical   normal)</code>    | use empirical or normal distribution percentile value estimates; default is <code>pvc(empirical)</code> |
| <code>tiecorrected</code>               | adjust for tied observations; not allowed with <code>pvc(normal)</code>                                 |
| <b>Bootstrap</b>                        |                                                                                                         |
| <code>nobootstrap</code>                | do not perform bootstrap, just output point estimates                                                   |
| <code>bseed(#)</code>                   | random-number seed for bootstrap                                                                        |
| <code>breps(#)</code>                   | number of bootstrap replications; default is <code>breps(1000)</code>                                   |
| <code>bootcc</code>                     | perform case–control (stratified on <i>refvar</i> ) sampling rather than cohort sampling in bootstrap   |
| <code>nobstrata</code>                  | ignore covariate stratification in bootstrap sampling                                                   |
| <code>nodots</code>                     | suppress bootstrap replication dots                                                                     |
| <b>Reporting</b>                        |                                                                                                         |
| <code>level(#)</code>                   | set confidence level; default is <code>level(95)</code>                                                 |

| <i>probit_options</i>                   | Description                                                                                             |
|-----------------------------------------|---------------------------------------------------------------------------------------------------------|
| Model                                   |                                                                                                         |
| * <code>probit</code>                   | fit the probit model                                                                                    |
| <code>rccov(varlist)</code>             | covariates affecting ROC curve                                                                          |
| <code>fprpts(#)</code>                  | number of false-positive rate points to use in fitting ROC curve; default is <code>fprpts(10)</code>    |
| <code>ctrlfprall</code>                 | fit ROC curve at each false-positive rate in control population                                         |
| <code>ccluster(varname)</code>          | variable identifying resampling clusters                                                                |
| <code>ctrlcov(varlist)</code>           | adjust control distribution for covariates in <i>varlist</i>                                            |
| <code>ctrlmodel(strata   linear)</code> | stratify or regress on covariates; default is <code>ctrlmodel(strata)</code>                            |
| <code>pvc(empirical   normal)</code>    | use empirical or normal distribution percentile value estimates; default is <code>pvc(empirical)</code> |
| <code>tiecorrected</code>               | adjust for tied observations; not allowed with <code>pvc(normal)</code>                                 |
| Bootstrap                               |                                                                                                         |
| <code>nobootstrap</code>                | do not perform bootstrap, just output point estimates                                                   |
| <code>bseed(#)</code>                   | random-number seed for bootstrap                                                                        |
| <code>breps(#)</code>                   | number of bootstrap replications; default is <code>breps(1000)</code>                                   |
| <code>bootcc</code>                     | perform case–control (stratified on <i>refvar</i> ) sampling rather than cohort sampling in bootstrap   |
| <code>nobstrata</code>                  | ignore covariate stratification in bootstrap sampling                                                   |
| <code>nodots</code>                     | suppress bootstrap replication dots                                                                     |
| <code>bsave(filename, ...)</code>       | save bootstrap replicates from parametric estimation                                                    |
| <code>bfile(filename)</code>            | use bootstrap replicates dataset for estimation replay                                                  |
| Reporting                               |                                                                                                         |
| <code>level(#)</code>                   | set confidence level; default is <code>level(95)</code>                                                 |

\*probit is required.

| <i>probit_ml_options</i>       | Description                                                          |
|--------------------------------|----------------------------------------------------------------------|
| Model                          |                                                                      |
| * <code>probit</code>          | fit the probit model                                                 |
| * <code>ml</code>              | fit the probit model by maximum likelihood estimation                |
| <code>rccov(varlist)</code>    | covariates affecting ROC curve                                       |
| <code>ccluster(varname)</code> | variable identifying clusters                                        |
| <code>ctrlcov(varlist)</code>  | adjust control distribution for covariates in <i>varlist</i>         |
| Reporting                      |                                                                      |
| <code>level(#)</code>          | set confidence level; default is <code>level(95)</code>              |
| <code>display_options</code>   | control column formats, line width, and display of omitted variables |
| Maximization                   |                                                                      |
| <code>maximize_options</code>  | control the maximization process; seldom used                        |

\*probit and ml are required.

`fweights`, `iweights`, and `pweights` are allowed with maximum likelihood estimation; see [U] 11.1.6 [weight](#).

`collect` is allowed. `bayesboot` is allowed only with maximum likelihood estimation. See [U] 11.1.10 [Prefix commands](#).

See [U] 20 [Estimation and postestimation commands](#) for more capabilities of estimation commands.

## Options

Options are presented under the following headings:

*Options for nonparametric ROC estimation, using bootstrap*  
*Options for parametric ROC estimation, using bootstrap*  
*Options for parametric ROC estimation, using maximum likelihood*

### Options for nonparametric ROC estimation, using bootstrap

#### Model

`auc` estimates the total area under the ROC curve. This is the default summary statistic.

`roc(numlist)` estimates the ROC corresponding to each of the false-positive rates in *numlist*. The values of *numlist* must be in the range (0,1).

`invroc(numlist)` estimates the false-positive rates corresponding to each of the ROC values in *numlist*. The values of *numlist* must be in the range (0,1).

`pauc(numlist)` estimates the partial area under the ROC curve up to each false-positive rate in *numlist*. The values of *numlist* must in the range (0,1].

`cluster(varname)` specifies the variable identifying resampling clusters.

`ctrlcov(varlist)` specifies the covariates to be used to adjust the control population.

`ctrlmodel(strata | linear)` specifies how to model the control population of classifiers on `ctrlcov()`. When `ctrlmodel(linear)` is specified, linear regression is used. The default is `ctrlmodel(strata)`; that is, the control population of classifiers is stratified on the control variables.

`pvc(empirical | normal)` determines how the percentile values of the control population will be calculated. When `pvc(normal)` is specified, the standard normal cumulative distribution function (CDF) is used for calculation. Specifying `pvc(empirical)` will use the empirical CDFs of the control population classifiers for calculation. The default is `pvc(empirical)`.

`tiecorrected` adjusts the percentile values for ties. For each value of the classifier, one half the probability that the classifier equals that value under the control population is added to the percentile value. `tiecorrected` is not allowed with `pvc(normal)`.

#### Bootstrap

`nobootstrap` specifies that bootstrap standard errors not be calculated.

`bseed(#)` specifies the random-number seed to be used in the bootstrap.

`breps(#)` sets the number of bootstrap replications. The default is `breps(1000)`.

`bootcc` performs case–control (stratified on *refvar*) sampling rather than cohort bootstrap sampling.

`nobstrata` ignores covariate stratification in bootstrap sampling.

`nodots` suppresses bootstrap replication dots.

#### Reporting

`level(#)`; see [\[R\] Estimation options](#).

## Options for parametric ROC estimation, using bootstrap

## Model

- `probit` fits the probit model. This option is required and implies parametric estimation.
- `roccov(varlist)` specifies the covariates that will affect the ROC curve.
- `fprpts(#)` sets the number of false-positive rate points to use in modeling the ROC curve. These points form an equispaced grid on (0,1). The default is `fprpts(10)`.
- `ctrlfprall` models the ROC curve at each false-positive rate in the control population.
- `cluster(varname)` specifies the variable identifying resampling clusters.
- `ctrlcov(varlist)` specifies the covariates to be used to adjust the control population.
- `ctrlmodel(strata | linear)` specifies how to model the control population of classifiers on `ctrlcov()`. When `ctrlmodel(linear)` is specified, linear regression is used. The default is `ctrlmodel(strata)`; that is, the control population of classifiers is stratified on the control variables.
- `pvc(empirical | normal)` determines how the percentile values of the control population will be calculated. When `pvc(normal)` is specified, the standard normal CDF is used for calculation. Specifying `pvc(empirical)` will use the empirical CDFs of the control population classifiers for calculation. The default is `pvc(empirical)`.
- `tierecorrected` adjusts the percentile values for ties. For each value of the classifier, one half the probability that the classifier equals that value under the control population is added to the percentile value. `tierecorrected` is not allowed with `pvc(normal)`.

## Bootstrap

- `nobootstrap` specifies that bootstrap standard errors not be calculated.
- `bseed(#)` specifies the random-number seed to be used in the bootstrap.
- `breps(#)` sets the number of bootstrap replications. The default is `breps(1000)`.
- `bootcc` performs case–control (stratified on *refvar*) sampling rather than cohort bootstrap sampling.
- `nobstrata` ignores covariate stratification in bootstrap sampling.
- `nodots` suppresses bootstrap replication dots.
- `bsave(filename, ...)` saves bootstrap replicates from parametric estimation in the given *filename* with specified options (that is, `replace`). `bsave()` is only allowed with parametric analysis using bootstrap.
- `bfile(filename)` specifies to use the bootstrap replicates dataset for estimation replay. `bfile()` is only allowed with parametric analysis using bootstrap.

## Reporting

- `level(#)`; see [\[R\] Estimation options](#).

## Options for parametric ROC estimation, using maximum likelihood

### Model

`probit` fits the probit model. This option is required and implies parametric estimation.

`ml` fits the probit model by maximum likelihood estimation. This option is required and must be specified with `probit`.

`roccov`(*varlist*) specifies the covariates that will affect the ROC curve.

`cluster`(*varname*) specifies the variable used for clustering.

`ctrlcov`(*varlist*) specifies the covariates to be used to adjust the control population.

### Reporting

`level`(#); see [R] [Estimation options](#).

`display_options`: `noomitted`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no!stretch`; see [R] [Estimation options](#).

### Maximization

`maximize_options`: `difficult`, `technique`(*algorithm\_spec*), `iterate`(#), `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance`(#), `ltolerance`(#), `nrtolerance`(#), `nonrntolerance`, and `from`(*init\_specs*); see [R] [Maximize](#). These options are seldom used. The `technique`(`bhhh`) option is not allowed.

## Remarks and examples

Remarks are presented under the following headings:

[Introduction](#)

[ROC statistics](#)

[Covariate-adjusted ROC curves](#)

[Parametric ROC curves: Estimating equations](#)

[Parametric ROC curves: Maximum likelihood](#)

## Introduction

Receiver operating characteristic (ROC) analysis provides a quantitative measure of the accuracy of diagnostic tests to discriminate between two states or conditions. These conditions may be referred to as normal and abnormal, nondiseased and diseased, or control and case. We will use these terms interchangeably. The discriminatory accuracy of a diagnostic test is measured by its ability to correctly classify known control and case subjects.

The analysis uses the ROC curve, a graph of the sensitivity versus  $1 - \text{specificity}$  of the diagnostic test. The sensitivity is the fraction of positive cases that are correctly classified by the diagnostic test, whereas the specificity is the fraction of negative cases that are correctly classified. Thus, the sensitivity is the true-positive rate, and the specificity is the true-negative rate. We also call  $1 - \text{specificity}$  the false-positive rate.

These rates are functions of the possible outcomes of the diagnostic test. At each outcome, a decision will be made by the user of the diagnostic test to classify the tested subject as either normal or abnormal. The true-positive and false-positive rates measure the probability of correct classification or incorrect classification of the subject as abnormal. Given the classification role of the diagnostic test, we will refer to it as the classifier.

Using this basic definition of the ROC curve, [Pepe \(2000\)](#) and [Pepe \(2003\)](#) describe how ROC analysis can be performed as a two-stage process. In the first stage, the control distribution of the classifier is estimated. The specificity is then determined as the percentiles of the classifier values calculated based on the control population. The false-positive rates are calculated as  $1 - \text{specificity}$ . In the second stage, the ROC curve is estimated as the cumulative distribution of the case population's "false-positive" rates, also known as the survival function under the case population of the previously calculated percentiles. We use the terms ROC value and true-positive value interchangeably.

This formulation of ROC curve analysis provides simple, nonparametric estimates of several ROC curve summary parameters: area under the ROC curve, partial area under the ROC curve, ROC value for a given false-positive rate, and false-positive rate (also known as invROC) for a given ROC value. In the next section, we will show how to use `rocreg` to compute these estimates with bootstrap inference. There we will also show how `rocreg` complements the other nonparametric Stata ROC commands `roctab` and `roccomp`.

Other factors beyond condition status and the diagnostic test may affect both stages of ROC analysis. For example, a test center may affect the control distribution of the diagnostic test. Disease severity may affect the distribution of the standardized diagnostic test under the case population. Our analysis of the ROC curve in these situations will be more accurate if we take these covariates into account.

In a nonparametric ROC analysis, covariates may only affect the first stage of estimation; that is, they may be used to adjust the control distribution of the classifier. In a parametric ROC analysis, it is assumed that ROC follows a normal distribution, and thus covariates may enter the model at both stages; they may be used to adjust the control distribution and to model ROC as a function of these covariates and the false-positive rate. In parametric models, both sets of covariates need not be distinct but, in fact, they are often the same.

To model covariate effects on the first stage of ROC analysis, [Janes and Pepe \(2009\)](#) propose a covariate-adjusted ROC curve. We will demonstrate the covariate adjustment capabilities of `rocreg` in [Covariate-adjusted ROC curves](#).

To account for covariate effects at the second stage, we assume a parametric model. Particularly, the ROC curve is a generalized linear model of the covariates. We will thus have a separate ROC curve for each combination of the relevant covariates. In [Parametric ROC curves: Estimating equations](#), we show how to fit the model with estimating equations and bootstrap inference using `rocreg`. This method, documented as the "pdf" approach in [Alonzo and Pepe \(2002\)](#), works well with weak assumptions about the control distribution.

Also in [Parametric ROC curves: Estimating equations](#), we show how to fit a constant-only parametric model (involving no covariates) of the ROC curve with weak assumptions about the control distribution. The constant-only model capabilities of `rocreg` in this context will be compared with those of `rocf`. `roccomp` has the `binormal` option, which will allow it to compute area under the ROC curve according to a normal ROC curve, equivalent to that obtained by `rocf`. We will compare this functionality with that of `rocreg`.

In [Parametric ROC curves: Maximum likelihood](#), we demonstrate maximum likelihood estimation of the ROC curve model with `rocreg`. There we assume a normal linear model for the classifier on the covariates and case–control status. This method is documented in [Pepe \(2003\)](#). We will also demonstrate how to use this method with no covariates, and we will compare `rocreg` under the constant-only model with `rocf` and `roccomp`.

The `rocregplot` command is used repeatedly in this entry. This command provides graphical output for `rocreg` and is documented in [\[R\] rocregplot](#).

## ROC statistics

`roctab` computes the ROC curve by calculating the false-positive rate and true-positive rate empirically at every value of the input classifier. It makes no distributional assumptions about the case or control distributions. We can get identical behavior from `rocreg` by using the default option settings.

### ▷ Example 1: Nonparametric ROC, AUC

Hanley and McNeil (1982) presented data from a study in which a reviewer was asked to classify, using a five-point scale, a random sample of 109 tomographic images from patients with neurological problems. The rating scale was as follows: 1 is definitely normal, 2 is probably normal, 3 is questionable, 4 is probably abnormal, and 5 is definitely abnormal. The true disease status was normal for 58 of the patients and abnormal for the remaining 51 patients.

Here we list 9 of the 109 observations:

```
. use https://www.stata-press.com/data/r19/hanley
(Tomographic images)
. list disease rating in 1/9
```

|    | disease | rating |
|----|---------|--------|
| 1. | 1       | 5      |
| 2. | 0       | 1      |
| 3. | 1       | 5      |
| 4. | 0       | 4      |
| 5. | 0       | 1      |
| 6. | 0       | 3      |
| 7. | 1       | 5      |
| 8. | 0       | 5      |
| 9. | 0       | 1      |

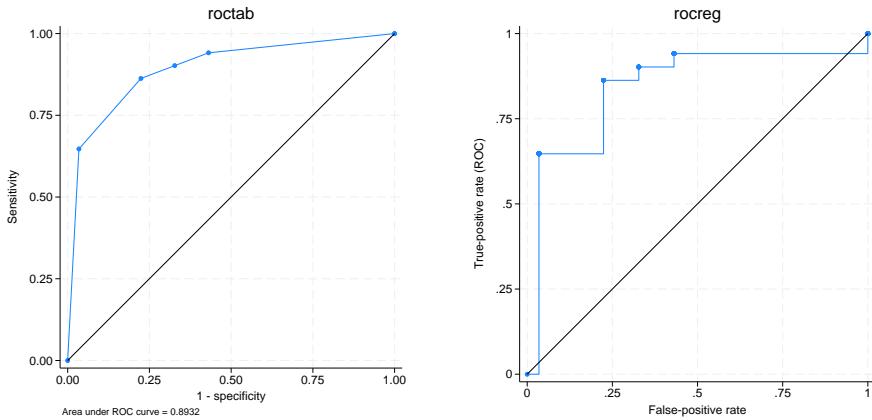
For each observation, `disease` identifies the true disease status of the subject (0 is normal, 1 is abnormal), and `rating` contains the classification value assigned by the reviewer.

We run `roctab` on these data, specifying the `graph` option so that the ROC curve is rendered. We then calculate the false-positive and true-positive rates of the ROC curve by using `rocreg`. We graph the rates with `rocregplot`. Because we focus on `rocreg` output later, for now we use the `quietly` prefix to omit the output of `rocreg`. Both graphs are combined using `graph combine` (see [G-2] [graph combine](#)) for comparison. To ease the comparison, we specify the `aspectratio(1)` option in `roctab`; this is the default aspect ratio in `rocregplot`.

```

. roctab disease rating, graph aspectratio(1) name(a) nodraw title("roctab")
. quietly rocreg disease rating
. rocregplot, name(b) nodraw legend(off) title("rocreg")
. graph combine a b

```



Both `roctab` and `rocreg` compute the same false-positive rate and ROC values. The stairstep line connection style of the graph on the right emphasizes the empirical nature of its estimates. The control distribution of the classifier is estimated using the empirical CDF estimate. Similarly, the ROC curve, the distribution of the resulting case observation false-positive rate values, is estimated using the empirical CDF. Note the footnote in the `roctab` plot. By default, `roctab` will estimate the area under the ROC curve (AUC) using a trapezoidal approximation to the estimated false-positive rate and true-positive rate points.

The AUC can be interpreted as the probability that a randomly selected member of the case population will have a larger classifier value than a randomly selected member of the control population. It can also be viewed as the average ROC value, averaged uniformly over the  $(0,1)$  false-positive rate domain (Pepe 2003).

The nonparametric estimator of the AUC (DeLong, DeLong, and Clarke-Pearson 1988; Hanley and Hajian-Tilaki 1997) used by `rocreg` is equivalent to the sample mean of the percentile values of the case observations. Thus to calculate the nonparametric AUC estimate, we only need to calculate the percentile values of the case observations with respect to the control distribution.

This estimate can differ from the trapezoidal approximation estimate. Under discrete classification data, like we have here, there may be ties between classifier values from case to control. The trapezoidal approximation uses linear interpolation between the classifier values to correct for ties. Correcting the nonparametric estimator involves adding a correction term to each observation's percentile value, which measures the probability that the classifier is equal to (instead of less than) the observation's classifier value.

The tie-corrected nonparametric estimate (trapezoidal approximation) is used when we think the true ROC curve is smooth. This means that the classifier we measure is a discretized approximation of a true latent and a continuous classifier.

We now recompute the ROC curve of `rating` for classifying disease and calculate the AUC. Specifying the `tiecorrected` option allows tie correction to be used in the `rocreg` calculation. Under nonparametric estimation, `rocreg` bootstraps to obtain standard errors and confidence intervals for re-

quested statistics. We use the default 1,000 bootstrap replications to obtain confidence intervals for our parameters. This is a reasonable lower bound to the number of replications (Mooney and Duval 1993) required for estimating percentile confidence intervals. By specifying the summary option in roctab, we will obtain output showing the trapezoidal approximation of the AUC estimate, along with standard error and confidence interval estimates for the trapezoidal approximation suggested by DeLong, DeLong, and Clarke-Pearson (1988).

```
. roctab disease rating, summary
```

| Obs | ROC area | Std. err. | Asymptotic normal<br>[95% conf. interval] |         |
|-----|----------|-----------|-------------------------------------------|---------|
| 109 | 0.8932   | 0.0307    | 0.83295                                   | 0.95339 |

```
. rocreg disease rating, tiecorrected bseed(29092)
(running rocregstat on estimation sample)

Bootstrap replications (1,000):10.....20.....30.....40....
>50.....60.....70.....80.....90.....100.....110...
>120.....130.....140.....150.....160.....170.....
> 180.....190.....200.....210.....220.....230.....240...
>250.....260.....270.....280.....290.....300.....
> 310.....320.....330.....340.....350.....360.....370...
>380.....390.....400.....410.....420.....430.....
> 440.....450.....460.....470.....480.....490.....500...
>510.....520.....530.....540.....550.....560.....
> 570.....580.....590.....600.....610.....620.....630...
>640.....650.....660.....670.....680.....690.....
> 700.....710.....720.....730.....740.....750.....760...
>770.....780.....790.....800.....810.....820.....
> 830.....840.....850.....860.....870.....880.....890...
>900.....910.....920.....930.....940.....950.....
> 960.....970.....980.....990.....1,000 done

Bootstrap results Number of obs = 109
 Replications = 1,000

Nonparametric ROC estimation
Control standardization: empirical, corrected for ties
ROC method : empirical

Area under the ROC curve
 Status : disease
 Classifier: rating
```

| AUC | Observed coefficient | Bias     | Bootstrap std. err. | [95% conf. interval] |               |
|-----|----------------------|----------|---------------------|----------------------|---------------|
|     | .8931711             | .0010376 | .0309808            | .8324498             | .9538923 (N)  |
|     |                      |          |                     | .8223829             | .9475383 (P)  |
|     |                      |          |                     | .8084577             | .9435818 (BC) |

The estimates of AUC match well. The standard error from roctab is close to the bootstrap standard error calculated by rocreg. The bootstrap standard error generalizes to the more complex models that we consider later, whereas the roctab standard-error calculation does not.



The AUC can be used to compare different classifiers. It is the most popular summary statistic for comparisons (Pepe, Longton, and Janes 2009). roccomp will compute the trapezoidal approximation of the AUC and graph the ROC curves of multiple classifiers. Using the DeLong, DeLong, and Clarke-Pearson (1988) covariance estimates for the AUC estimate, roccomp performs a Wald test of the null hypothesis that all classifier AUC values are equal. rocreg has similar capabilities.

## ► Example 2: Nonparametric ROC, AUC, multiple classifiers

Hanley and McNeil (1983) presented data from an evaluation of two computer algorithms designed to reconstruct CT images from phantoms. We will call these two algorithms modalities 1 and 2. A sample of 112 phantoms was selected; 58 phantoms were considered normal, and the remaining 54 were abnormal. Each of the two modalities was applied to each phantom, and the resulting images were rated by a reviewer using a six-point scale: 1 is definitely normal, 2 is probably normal, 3 is possibly normal, 4 is possibly abnormal, 5 is probably abnormal, and 6 is definitely abnormal. Because each modality was applied to the same sample of phantoms, the two sets of outcomes are correlated.

We list the first seven observations:

```
. use https://www.stata-press.com/data/r19/ct, clear
(Reconstruction of CT images)
. list in 1/7, sep(0)
```

|    | mod1 | mod2 | status |
|----|------|------|--------|
| 1. | 2    | 1    | 0      |
| 2. | 5    | 5    | 1      |
| 3. | 2    | 1    | 0      |
| 4. | 2    | 3    | 0      |
| 5. | 5    | 6    | 1      |
| 6. | 2    | 2    | 0      |
| 7. | 3    | 2    | 0      |

Each observation corresponds to one phantom. The mod1 variable identifies the rating assigned for the first modality, and the mod2 variable identifies the rating assigned for the second modality. The true status of the phantoms is given by `status==0` if they are normal and `status==1` if they are abnormal. The observations with at least one missing rating were dropped from the analysis.

A fictitious dataset was created from this true dataset, adding a third test modality. We will use `roccomp` to compute the AUC statistic for each modality in these data and compare the AUC of the three modalities. We obtain the same behavior from `rocreg`. As before, the `tiecorrected` option is specified so that the AUC is calculated with the trapezoidal approximation.

```
. use https://www.stata-press.com/data/r19/ct2
(Reconstruction of CT images)
. roccomp status mod1 mod2 mod3, summary
```

|      | Obs | ROC<br>area | Std. err. | Asymptotic normal<br>[95% conf. interval] |         |
|------|-----|-------------|-----------|-------------------------------------------|---------|
| mod1 | 112 | 0.8828      | 0.0317    | 0.82067                                   | 0.94498 |
| mod2 | 112 | 0.9302      | 0.0256    | 0.88005                                   | 0.98042 |
| mod3 | 112 | 0.9240      | 0.0241    | 0.87670                                   | 0.97132 |

```
H0: area(mod1) = area(mod2) = area(mod3)
chi2(2) = 6.54 Prob>chi2 = 0.0381
```

```
. rocreg status mod1 mod2 mod3, tiecorrected bseed(38038) nodots
Bootstrap results Number of obs = 112
 Replications = 1,000

Nonparametric ROC estimation
Control standardization: empirical, corrected for ties
ROC method : empirical
Area under the ROC curve
 Status : status
 Classifier: mod1
```

| AUC | Observed coefficient | Bias      | Bootstrap std. err. | [95% conf. interval] |               |
|-----|----------------------|-----------|---------------------|----------------------|---------------|
|     | .8828225             | -.0010192 | .0318564            | .820385              | .94526 (N)    |
|     |                      |           |                     | .8150605             | .9398384 (P)  |
|     |                      |           |                     | .8119603             | .9392538 (BC) |

```
Status : status
Classifier: mod2
```

| AUC | Observed coefficient | Bias     | Bootstrap std. err. | [95% conf. interval] |               |
|-----|----------------------|----------|---------------------|----------------------|---------------|
|     | .9302363             | .0005148 | .0257043            | .8798567             | .9806159 (N)  |
|     |                      |          |                     | .8746504             | .9769936 (P)  |
|     |                      |          |                     | .8616987             | .9688995 (BC) |

```
Status : status
Classifier: mod3
```

| AUC | Observed coefficient | Bias     | Bootstrap std. err. | [95% conf. interval] |               |
|-----|----------------------|----------|---------------------|----------------------|---------------|
|     | .9240102             | .0001857 | .0240864            | .8768018             | .9712187 (N)  |
|     |                      |          |                     | .8727464             | .9658895 (P)  |
|     |                      |          |                     | .8629984             | .9621795 (BC) |

```
HO: All classifiers have equal AUC values
Ha: At least one classifier has a different AUC value
P-value: .0339546 Test based on bootstrap (N) assumptions.
```

We see that the AUC estimates are equivalent, and the standard errors are quite close as well. The  $p$ -value for the tests of equal AUC under `rocreg` leads to similar inference as the  $p$ -value from `roccomp`. The Wald test performed by `rocreg` uses the joint bootstrap estimate variance matrix of the three AUC estimators rather than the [DeLong, DeLong, and Clarke-Pearson \(1988\)](#) variance estimate used by `roccomp`.

`roccomp` is used here on potentially correlated classifiers that are recorded in wide-format data. It can also be used on long-format data to compare independent classifiers. Further details can be found in [\[R\] roccomp](#).



Citing the AUC's lack of clinical relevance, there is argument against using it as a key summary statistic of the ROC curve ([Pepe 2003](#); [Cook 2007](#)). [Pepe, Longton, and Janes \(2009\)](#) suggest using the estimate of the ROC curve itself at a particular point, or the estimate of the false-positive rate at a given ROC value, also known as `invROC`.

Recall from [example 1](#) how nonparametric `rocreg` graphs look, with the staircase pattern in the ROC curve. In an ideal world, the graph would be a smooth one-to-one function, and it would be trivial to map a false-positive rate to its corresponding true-positive rate and vice versa.

However, smooth ROC curves can only be obtained by assuming a parametric model that uses linear interpolation between observed false-positive rates and between observed true-positive rates, and `rocreg` is certainly capable of that; see [example 1](#) of [\[R\] rocregplot](#). However, under nonparametric estimation, the mapping between false-positive rates and true-positive rates is not one to one, and estimates tend to be less reliable the further you are from an observed data point. This is somewhat mitigated by using tie-corrected rates (the `tiecorrected` option).

When we examine continuous data, the difference between the tie-corrected estimates and the standard estimates becomes negligible, and the empirical estimate of the ROC curve becomes close to the smooth ROC curve obtained by linear interpolation. So the nonparametric ROC and `invROC` estimates work well.

Fixing one rate value of interest can be difficult and subjective ([Pepe 2003](#)). A compromise measure is the partial area under the ROC curve (`pAUC`) ([McClish 1989](#); [Thompson and Zucchini 1989](#)). This is the integral of the ROC curve from 0 and above to a given false-positive rate (perhaps the largest clinically acceptable value). Like the `AUC` estimate, the nonparametric estimate of the `pAUC` can be written as a sample average of the case observation percentiles, but with an adjustment based on the prescribed maximum false-positive rate ([Dodd and Pepe 2003](#)). A tie correction may also be applied so that it reflects the trapezoidal approximation.

We cannot compare `rocreg` with `roctab` or `roccomp` on the estimation of `pAUC`, because `pAUC` is not computed by the latter two.

### ► Example 3: Nonparametric ROC, other statistics

To see how `rocreg` estimates ROC, `invROC`, and `pAUC`, we will examine a new study. [Wieand et al. \(1989\)](#) examined a pancreatic cancer study with two continuous classifiers, here called `y1` (CA 19-9) and `y2` (CA 125). This study was also examined in [Pepe, Longton, and Janes \(2009\)](#). The indicator of cancer in a subject is recorded as `d`. The study was a case-control study, stratifying participants on disease status.

We list the first five observations:

```
. use https://research.fredhutch.org/content/dam/stripe/diagnostic-biomarkers-
> statistical-center/files/wiedat2b.dta, clear
(S. Wieand - Pancreatic cancer diagnostic marker data)

. list in 1/5
```

|    | y1   | y2   | d  |
|----|------|------|----|
| 1. | 28   | 13.3 | no |
| 2. | 15.5 | 11.1 | no |
| 3. | 8.2  | 16.7 | no |
| 4. | 3.4  | 12.6 | no |
| 5. | 17.3 | 7.4  | no |

We will estimate the ROC curves at a large value (0.7) and a small value (0.2) of the false-positive rate. These values are specified in `roc()`. The false-positive rate for ROC or sensitivity value of 0.6 will also be estimated by specifying `invroc()`. Percentile confidence intervals for these parameters are displayed in the graph obtained by `rocregplot` after `rocreg`. The `pAUC` statistic will be calculated for

the false-positive rate of 0.5, which is specified as an argument to the `pauc()` option. Following [Pepe, Longton, and Janes \(2009\)](#), we use a stratified bootstrap, sampling separately from the case and control populations by specifying the `bootcc` option. This reflects the case–control nature of the study.

All four statistics can be estimated simultaneously by `rocreg`. For clarity, however, we will estimate each statistic with a separate call to `rocreg`. `rocregplot` is used after estimation to graph the ROC and false-positive rate estimates. The display of the individual, observation-specific false-positive rate and ROC values will be omitted in the plot. This is accomplished by specifying `msymbol(i)` in our `plot1opts()` and `plot2opts()` options to `rocregplot`.

```
. rocreg d y1 y2, roc(.7) bseed(8378923) bootcc nodots
Bootstrap results
Number of strata = 2 Number of obs = 141
 Replications = 1,000
```

```
Nonparametric ROC estimation
Control standardization: empirical
ROC method : empirical
ROC curve
 Status : d
 Classifier: y1
```

| ROC | Observed coefficient | Bias     | Bootstrap std. err. | [95% conf. interval] |               |
|-----|----------------------|----------|---------------------|----------------------|---------------|
| .7  | .9222222             | .0010222 | .0332527            | .8570482             | .9873962 (N)  |
|     |                      |          |                     | .8555555             | .9777778 (P)  |
|     |                      |          |                     | .8555555             | .9777778 (BC) |

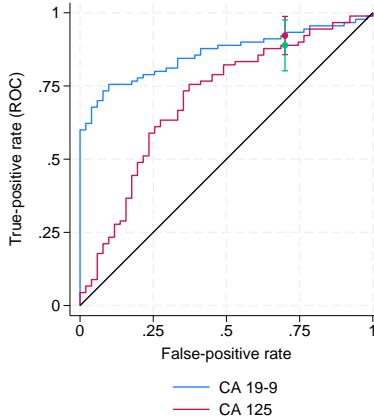
```
Status : d
Classifier: y2
```

| ROC | Observed coefficient | Bias      | Bootstrap std. err. | [95% conf. interval] |               |
|-----|----------------------|-----------|---------------------|----------------------|---------------|
| .7  | .8888889             | -.0046556 | .0444103            | .8018463             | .9759314 (N)  |
|     |                      |           |                     | .7833333             | .9666666 (P)  |
|     |                      |           |                     | .7666667             | .9555556 (BC) |

```
HO: All classifiers have equal ROC values
Ha: At least one classifier has a different ROC value
Test based on bootstrap (N) assumptions
```

| ROC | P-value  |
|-----|----------|
| .7  | .5537371 |

```
. rocregplot, plot1opts(msymbol(i)) plot2opts(msymbol(i))
```



In this study, we see that classifier y1 (CA 19-9) is a uniformly better test than is classifier y2 (CA 125) until high levels of false-positive rate and sensitivity or ROC value are reached. At the high level of false-positive rate, 0.7, the ROC value does not significantly differ between the two classifiers. This can be seen in the plot by the overlapping confidence intervals.

```
. rocreg d y1 y2, roc(.2) bseed(8378923) bootcc nodots
```

Bootstrap results

Number of strata = 2

Number of obs = 141

Replications = 1,000

Nonparametric ROC estimation

Control standardization: empirical

ROC method : empirical

ROC curve

Status : d

Classifier: y1

| ROC | Observed coefficient | Bias     | Bootstrap std. err. | [95% conf. interval] |               |
|-----|----------------------|----------|---------------------|----------------------|---------------|
| .2  | .7777778             | .0020778 | .0487666            | .6821969             | .8733586 (N)  |
|     |                      |          |                     | .6777778             | .8722222 (P)  |
|     |                      |          |                     | .6555555             | .8555555 (BC) |

Status : d

Classifier: y2

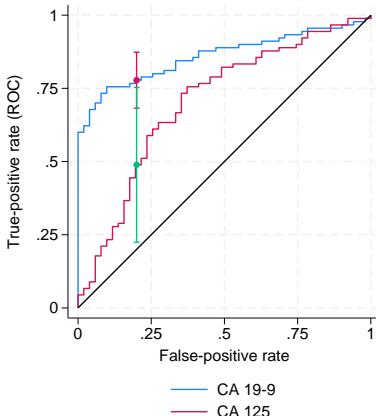
| ROC | Observed coefficient | Bias   | Bootstrap std. err. | [95% conf. interval] |               |
|-----|----------------------|--------|---------------------|----------------------|---------------|
| .2  | .4888889             | -.0054 | .1348859            | .2245173             | .7532605 (N)  |
|     |                      |        |                     | .2222222             | .6944444 (P)  |
|     |                      |        |                     | .2111111             | .6777778 (BC) |

H0: All classifiers have equal ROC values  
 Ha: At least one classifier has a different ROC value

Test based on bootstrap (N) assumptions

| ROC | P-value  |
|-----|----------|
| .2  | .0461582 |

```
. rocregplot, plot1opts(msymbol(i)) plot2opts(msymbol(i))
```



The sensitivity for the false-positive rate of 0.2 is found to be higher under y1 than under y2, and this difference is significant at the 0.05 level. In the plot, this is shown by the vertical confidence intervals.

```
. rocreg d y1 y2, invroc(.6) bseed(8378923) bootcc nodots
```

Bootstrap results

Number of strata = 2

Number of obs = 141  
 Replications = 1,000

Nonparametric ROC estimation

Control standardization: empirical

ROC method : empirical

False-positive rate

Status : d

Classifier: y1

| invROC | Observed coefficient | Bias     | Bootstrap std. err. | [95% conf. interval] |               |
|--------|----------------------|----------|---------------------|----------------------|---------------|
| .6     | 0                    | .0149412 | .0255885            | -.0501525            | .0501525 (N)  |
|        |                      |          |                     | 0                    | .0784314 (P)  |
|        |                      |          |                     | 0                    | .1372549 (BC) |

Status : d

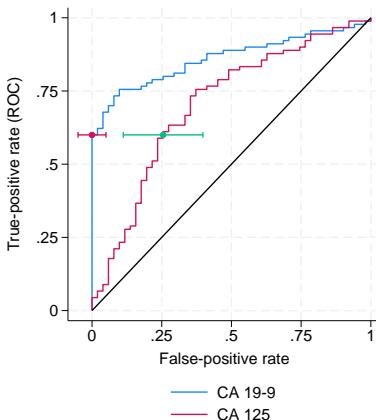
Classifier: y2

| invROC | Observed coefficient | Bias     | Bootstrap std. err. | [95% conf. interval] |               |
|--------|----------------------|----------|---------------------|----------------------|---------------|
| .6     | .254902              | .0074118 | .0729374            | .1119474             | .3978566 (N)  |
|        |                      |          |                     | .1372549             | .4313726 (P)  |
|        |                      |          |                     | .1176471             | .3921569 (BC) |

HO: All classifiers have equal invROC values  
 Ha: At least one classifier has a different invROC value  
 Test based on bootstrap (N) assumptions

| invROC | P-value  |
|--------|----------|
| .6     | .0010863 |

```
. rocregplot, plot1opts(msymbol(i)) plot2opts(msymbol(i))
```



We find significant evidence that false-positive rates corresponding to a sensitivity of 0.6 are different from  $y_1$  to  $y_2$ . This is visually indicated by the horizontal confidence intervals, which are separated from each other.

```
. rocreg d y1 y2, pauc(.5) bseed(8378923) bootcc nodots
Bootstrap results
Number of strata = 2 Number of obs = 141
 Replications = 1,000

Nonparametric ROC estimation
Control standardization: empirical
ROC method : empirical
Partial area under the ROC curve
Status : d
Classifier: y1
```

| pAUC | Observed coefficient | Bias     | Bootstrap std. err. | [95% conf. interval] |               |
|------|----------------------|----------|---------------------|----------------------|---------------|
| .5   | .3932462             | .0011971 | .0219031            | .3503169             | .4361755 (N)  |
|      |                      |          |                     | .3489107             | .4338235 (P)  |
|      |                      |          |                     | .3453159             | .4315904 (BC) |

Status : d  
Classifier: y2

| pAUC | Observed coefficient | Bias     | Bootstrap std. err. | [95% conf. interval] |               |
|------|----------------------|----------|---------------------|----------------------|---------------|
| .5   | .2496732             | .0033901 | .0362569            | .1786109             | .3207355 (N)  |
|      |                      |          |                     | .1837691             | .3224946 (P)  |
|      |                      |          |                     | .1721133             | .3108932 (BC) |

HO: All classifiers have equal pAUC values  
 Ha: At least one classifier has a different pAUC value  
 Test based on bootstrap (N) assumptions

| pAUC | P-value |
|------|---------|
| .5   | .001023 |

We also find significant evidence supporting the hypothesis that the pAUC for y1 up to a false-positive rate of 0.5 differs from the area of the same region under the ROC curve of y2.



### Covariate-adjusted ROC curves

When covariates affect the control distribution of the diagnostic test, thresholds for the test being classified as abnormal may be chosen that vary with the covariate values. These conditional thresholds will be more accurate than the marginal thresholds that would normally be used, because they take into account the specific distribution of the diagnostic test under the given covariate values as opposed to the marginal distribution over all covariate values.

By using these covariate-specific thresholds, we are essentially creating new classifiers for each covariate-value combination, and thus we are creating multiple ROC curves. As explained in [Pepe \(2003\)](#), when the case and control distributions of the covariates are the same, the marginal ROC curve will always be bound above by these covariate-specific ROC curves. So using conditional thresholds will never provide a less powerful test diagnostic in this case.

In the marginal ROC curve calculation, the classifiers are standardized to percentiles according to the control distribution, marginalized over the covariates. Thus, the ROC curve is the CDF of the standardized case observations. The covariate-adjusted ROC curve is the CDF of one minus the conditional control percentiles for the case observations, and the marginal ROC curve is the CDF of one minus the marginal control percentiles for the case observations ([Pepe and Cai 2004](#)). Thus, the standardization of classifier to false-positive rate value is conditioned on the specific covariate values under the covariate-adjusted ROC curve.

The covariate-adjusted ROC curve ([Janes and Pepe 2009](#)) at a given false-positive rate  $t$  is equivalent to the expected value of the covariate-specific ROC at  $t$  over all covariate combinations. When the covariates in question do not affect the case distribution of the classifier, the covariate-specific ROC will have the same value at each covariate combination. So here the covariate-adjusted ROC is equivalent to the covariate-specific ROC, regardless of covariate values.

When covariates do affect the case distribution of the classifier, users of the diagnostic test would likely want to model the covariate-specific ROC curves separately. Tools to do this can be found in the parametric modeling discussion in the following two sections. Regardless, the covariate-adjusted ROC curve can serve as a meaningful summary of covariate-adjusted accuracy.

Also note that the ROC summary statistics defined in the previous section have covariate-adjusted analogs. These analogs are estimated in a similar manner as under the marginal ROC curve (Janes, Longton, and Pepe 2009). The options for their calculation in `rocreg` are identical to those given in the previous section. Further details can be found in *Methods and formulas*.

#### ► Example 4: Nonparametric ROC, linear covariate adjustment

Norton et al. (2000) studied data from a neonatal audiology study on three tests to identify hearing impairment in newborns. These data were also studied in Janes, Longton, and Pepe (2009). Here we list 5 of the 5,058 observations.

```
. use https://www.stata-press.com/data/r19/nmhs, clear
(Norton - neonatal audiology data)
. list in 1/5
```

|    | id    | ear | male | currage | d | y1    | y2    | y3    |
|----|-------|-----|------|---------|---|-------|-------|-------|
| 1. | B0157 | R   | M    | 42.42   | 0 | -3.1  | -9    | -1.5  |
| 2. | B0157 | L   | M    | 42.42   | 0 | -4.5  | -8.7  | -2.71 |
| 3. | B0158 | R   | M    | 40.14   | 1 | -3.2  | -13.2 | -2.64 |
| 4. | B0161 | L   | F    | 38.14   | 0 | -22.1 | -7.8  | -2.59 |
| 5. | B0167 | R   | F    | 37      | 0 | -10.9 | -6.6  | -1.42 |

The classifiers `y1` (DPOAE 65 at 2 kHz), `y2` (TEOAE 80 at 2 kHz), and `y3` (ABR) and the hearing impairment indicator `d` are recorded along with some relevant covariates. The infant's age is recorded in months as `currage`, and the infant's gender is indicated by `male`. Over 90% of the newborns were tested in each ear (`ear`), so we will cluster on infant ID (`id`).

Following the strategy of Janes, Longton, and Pepe (2009), we will first perform ROC analysis for the classifiers while adjusting for the covariate effects of the infant's gender and age. This is done by specifying these variables in the `ctrlcov()` option. We adjust using a linear regression rule, by specifying `ctrlmodel(linear)`. This means that when a user of the diagnostic test chooses a threshold conditional on the age and gender covariates, they assume that the diagnostic test classifier has some linear dependence on age and gender and equal variance as their levels vary. Our cluster adjustment is made by specifying the `cluster()` option.

We will focus on the first classifier. The percentile, or specificity, values are calculated empirically by default, and thus so are the false-positive rates,  $(1 - \text{specificity})$ . Also by default, the ROC curve values are empirically defined by the false-positive rates. To draw the ROC curve, we again use `rocregplot`.

The AUC is calculated by default. For brevity, we specify the nobootstrap option so that bootstrap sampling is not performed. The AUC point estimate will be sufficient for our purposes.

```
. rocreg d y1, ctrlcov(male currage) ctrlmodel(linear) cluster(id) nobootstrap
Nonparametric ROC estimation Number of obs = 5,056
Covariate control : linear regression
Control variables : male currage
Control standardization: empirical
ROC method : empirical
Status : d
Classifier: y1
Covariate control adjustment model:
Linear regression Number of obs = 4,907
 F(2, 2685) = 13.80
 Prob > F = 0.0000
 R-squared = 0.0081
 Root MSE = 7.7515
 (Std. err. adjusted for 2,686 clusters in id)
```

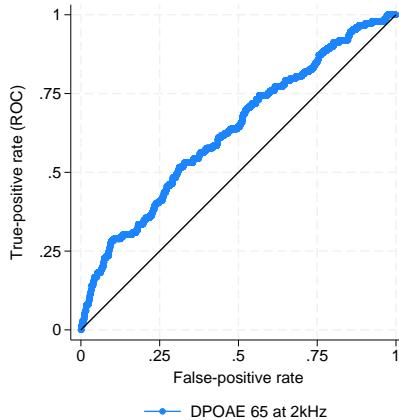
| y1      | Robust      |           | t     | P> t  | [95% conf. interval] |           |
|---------|-------------|-----------|-------|-------|----------------------|-----------|
|         | Coefficient | std. err. |       |       |                      |           |
| male    | .2471744    | .2603598  | 0.95  | 0.343 | -.2633516            | .7577005  |
| currage | -.2032456   | .0389032  | -5.22 | 0.000 | -.2795288            | -.1269624 |
| _cons   | -1.239484   | 1.487855  | -0.83 | 0.405 | -4.156942            | 1.677973  |

Area under the ROC curve

```
Status : d
Classifier: y1
```

| AUC | Observed    | Bias | Bootstrap | [95% conf. interval] |
|-----|-------------|------|-----------|----------------------|
|     | coefficient |      | std. err. |                      |
|     | .6293994    | .    | .         | . (N)                |
|     |             |      |           | . (P)                |
|     |             |      |           | . (BC)               |

```
. rocregplot
```



Our covariate control adjustment model shows that `currage` has a negative effect on `y1` (DPOAE 65 at 2 kHz) under the control population. At the 0.001 significance level, we reject that its contribution to `y1` is zero, and the point estimate has a negative sign. This result does not directly tell us about the effect of `currage` on the ROC curve of `y1` as a classifier of `d`. None of the case observations are used in the linear regression, so information on `currage` for abnormal cases is not used in the model. This result does show us how to calculate false-positive rates for tests that use thresholds conditional on a child's sex and current age. We will see how `currage` affects the ROC curve when `y1` is used as a classifier and conditional thresholds are used based on `male` and `currage` in the following section, *Parametric ROC curves: Estimating equations*.

#### □ Technical note

Under this nonparametric estimation, `rocreg` saved the false-positive rate for each observation's `y1` values in the utility variable `_fpr_y1`. The true-positive rates are stored in the utility variable `_roc_y1`. For other models, say with classifier `yname`, these variables would be named `_fpr_yname` and `_roc_yname`. They will also be overwritten with each call of `rocreg`. The variables `_roc_*` and `_fpr_*` are usually for internal `rocreg` use only and are overwritten with each call of `rocreg`. They are only created for nonparametric models or parametric models that do not involve ROC covariates. In these models, covariates may only affect the first stage of estimation, the control distribution, and not the ROC curve itself. In parametric models that allow ROC covariates, different covariate values would lead to different ROC curves.

□

To see how the covariate-adjusted ROC curve estimate differs from the standard marginal estimate, we will reestimate the ROC curve for classifier `y1` without covariate adjustment. We rename these variables before the new estimation and then draw an overlaid twoway line (see [G-2] **graph twoway line**) plot to compare the two.

```

. rename _fpr_y1 o_fpr_y1
. rename _roc_y1 o_roc_y1
. label variable o_roc_y1 "Covariate adjusted"
. rocreg d y1, cluster(id) nobootstrap
Nonparametric ROC estimation Number of obs = 5,058
Control standardization: empirical
ROC method : empirical
Area under the ROC curve
 Status : d
 Classifier: y1

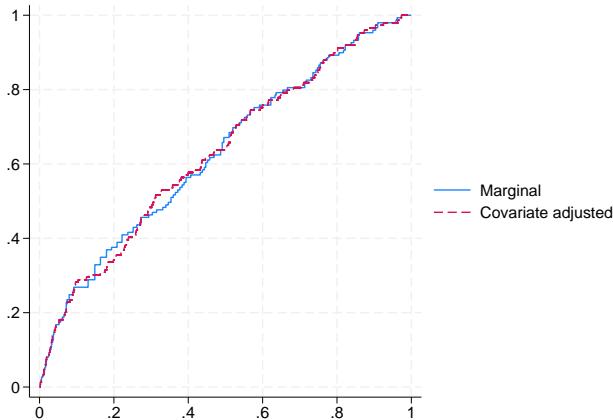
```

| AUC | Observed coefficient | Bias | Bootstrap std. err. | [95% conf. interval] |
|-----|----------------------|------|---------------------|----------------------|
|     | .6279645             | .    | .                   | . (N)                |
|     |                      |      |                     | . (P)                |
|     |                      |      |                     | . (BC)               |

```

. label variable _roc_y1 "Marginal"
. label variable o_fpr_y1 "False-positive rate for y1"
. twoway line _roc_y1 _fpr_y1, sort(_fpr_y1 _roc_y1) connect(J) ||
> line o_roc_y1 o_fpr_y1, sort(o_fpr_y1 o_roc_y1)
> connect(J) lpattern(dash) aspectratio(1) legend(cols(1))

```



Though they are close, particularly in AUC, there are clearly some points of difference between the estimates. So the covariate-adjusted ROC curve may be useful here.



In our examples thus far, we have used the empirical CDF estimator to estimate the control distribution. `rocreg` allows some flexibility here. The `pvc(normal)` option may be specified to calculate the percentile values according to a Gaussian distribution of the control.

Covariate adjustment in `rocreg` may also be performed with stratification instead of linear regression. Under the stratification method, the unique values of the stratified covariates each define separate parameters for the control distribution of the classifier. A user of the diagnostic test chooses a threshold based on the control distribution conditioned on the unique covariate value parameters.

We will demonstrate the use of normal percentile values and covariate stratification in our next example.

### ▷ Example 5: Nonparametric ROC, covariate stratification

The hearing test study of [Stover et al. \(1996\)](#) examined the effectiveness of negative signal-to-noise ratio, `nsnr`, as a classifier of hearing loss. The test was administered under nine different settings, corresponding to different frequency, `xf`, and intensity, `x1`, combinations. Here we list 10 of the 1,848 observations.

```
. use https://www.stata-press.com/data/r19/dp, clear
(Stover - DPOAE test data)
. list in 1/10
```

|     | id  | d | nsnr | xf    | x1  | xd  |
|-----|-----|---|------|-------|-----|-----|
| 1.  | 101 | 1 | 18   | 10.01 | 5.5 | 3.5 |
| 2.  | 101 | 1 | 19   | 20.02 | 5.5 | 3   |
| 3.  | 101 | 1 | 7.6  | 10.01 | 6   | 3.5 |
| 4.  | 101 | 1 | 15   | 20.02 | 6   | 3   |
| 5.  | 101 | 1 | 16   | 10.01 | 6.5 | 3.5 |
| 6.  | 101 | 1 | 5.8  | 20.02 | 6.5 | 3   |
| 7.  | 102 | 0 | -2.6 | 10.01 | 5.5 | .   |
| 8.  | 102 | 0 | -3   | 14.16 | 5.5 | .   |
| 9.  | 102 | 1 | 10   | 20.02 | 5.5 | 1   |
| 10. | 102 | 0 | -5.8 | 10.01 | 6   | .   |

Hearing loss is represented by `d`. The covariate `xd` is a measure of the degree of hearing loss. We will use this covariate in later analysis, because it only affects the case distribution of the classifier. Multiple measurements are taken for each individual, `id`, so we will cluster by individual.

We evaluate the effectiveness of `nsnr` using `xf` and `x1` as stratification covariates with `rocreg`; the default method of covariate adjustment.

As mentioned before, the default false-positive rate calculation method in `rocreg` estimates the conditional control distribution of the classifiers empirically. For comparison, we will also estimate a separate ROC curve using false-positive rates assuming the conditional control distribution is normal. This behavior is requested by specifying the `pvc(normal)` option. Using the `rocregplot` option `name()` to store the ROC plots and using the `graph combine` command, we are able to compare the Gaussian and empirical ROC curves side by side. As before, for brevity we specify the `nobootstrap` option to suppress bootstrap sampling.

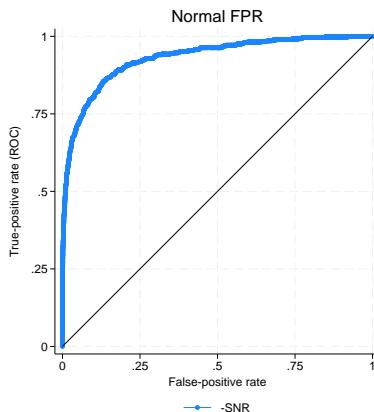
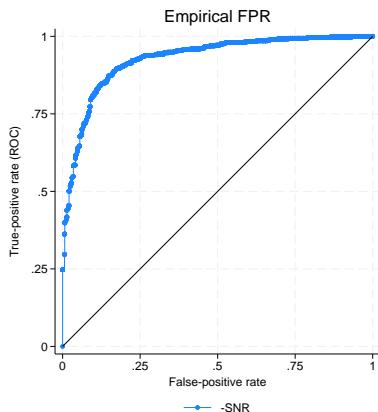
```
. rocreg d nsnr, ctrlcov(xf xl) cluster(id) nobootstrap
Nonparametric ROC estimation Number of obs = 1,848
Covariate control : stratification
Control variables : xf xl
Control standardization: empirical
ROC method : empirical
Area under the ROC curve
 Status : d
 Classifier: nsnr
```

| AUC | Observed coefficient | Bias | Bootstrap std. err. | [95% conf. interval] |
|-----|----------------------|------|---------------------|----------------------|
|     | .9264192             | .    | .                   | . (N)                |
|     |                      |      |                     | . (P)                |
|     |                      |      |                     | . (BC)               |

```
. rocregplot, title(Empirical FPR) name(a) nodraw
. rocreg d nsnr, pvc(normal) ctrlcov(xf xl) cluster(id) nobootstrap
Nonparametric ROC estimation Number of obs = 1,848
Covariate control : stratification
Control variables : xf xl
Control standardization: normal
ROC method : empirical
Area under the ROC curve
 Status : d
 Classifier: nsnr
```

| AUC | Observed coefficient | Bias | Bootstrap std. err. | [95% conf. interval] |
|-----|----------------------|------|---------------------|----------------------|
|     | .9309901             | .    | .                   | . (N)                |
|     |                      |      |                     | . (P)                |
|     |                      |      |                     | . (BC)               |

```
. rocregplot, title(Normal FPR) name(b) nodraw
. graph combine a b, xsize(5)
```



On cursory visual inspection, we see little difference between the two curves. The AUC values are close as well. So it is sensible to assume that we have Gaussian percentile values for control standardization. ◀

## Parametric ROC curves: Estimating equations

We now assume a parametric model for covariate effects on the second stage of ROC analysis. Particularly, the ROC curve is a probit model of the covariates. We will thus have a separate ROC curve for each combination of the relevant covariates.

Under weak assumptions about the control distribution of the classifier, we can fit this model by using estimating equations as described in [Alonzo and Pepe \(2002\)](#). This method can be also be used without covariate effects in the second stage, assuming a parametric model for the single (constant only) ROC curve. Covariates may still affect the first stage of estimation, so we parametrically model the single covariate-adjusted ROC curve (from the previous section). The marginal ROC curve, involving no covariates in either stage of estimation, can be fit parametrically as well.

In addition to the [Alonzo and Pepe \(2002\)](#) explanation, further details are given in [Pepe, Longton, and Janes \(2009\)](#); [Janes, Longton, and Pepe \(2009\)](#); [Pepe \(2003\)](#); and [Janes and Pepe \(2009\)](#).

The parametric models that we consider assume that the ROC curve is a cumulative distribution function  $g$  invoked with input of a linear polynomial in the corresponding quantile function invoked on the false-positive rate  $u$ . In this context, we assume that  $g$  corresponds to a standard normal cumulative distribution function,  $\Phi$ . So the corresponding quantile function is  $\Phi^{-1}$ . The constant intercept of the polynomial may depend on covariates, but the slope term  $\alpha$  (the quantile coefficient) may not.

$$\text{ROC}(u) = g\{\mathbf{x}'\beta + \alpha g^{-1}(u)\}$$

The first step of the algorithm involves the choice of false-positive rates to use in the parametric fit. These are typically a set of equispaced points spanning the interval (0,1). [Alonzo and Pepe \(2002\)](#) examined the effect of fitting large and small sets of points, finding that relatively small sets could be used with little loss of efficiency. Alternatively, the set can be formed by using the observed false-positive rates in the data ([Pepe 2003](#)). Further details on the algorithm are provided in [Methods and formulas](#).

Under parametric estimation, all the summary measures we defined earlier, except the AUC, are not calculated until postestimation. In models with covariates, each covariate combination would yield a different ROC curve and thus different summary parameters, so no summary parameters are initially estimated. In marginal parametric models (where there are no ROC covariates, but there are potentially control covariates), we will calculate the AUC and leave the other measures for postestimation; see [\[R\] rocreg postestimation](#). As with the other parameters, we bootstrap for standard errors and inference.

We will now demonstrate how `rocreg` performs the [Alonzo and Pepe \(2002\)](#) algorithm using the previous section's examples and others.

### ▶ Example 6: Parametric ROC, linear covariate adjustment

We return to the neonatal audiology study with gender and age covariates ([Norton et al. 2000](#)), which we discussed in [example 4](#). [Janes, Longton, and Pepe \(2009\)](#) suspected the current age of the infant would play a role in the case distribution of the classifier  $y_1$  (DPOAE 65 at 2 kHz). They postulated a probit link between the ROC curve and the covariate-adjusted false-positive rates. We follow their investigation and reach similar results.

In *example 4*, we saw the results of adjusting for the currage and male variables in the control population for classifier y1. Now, we see how currage affects the ROC curve when y1 is used with thresholds conditioned on male and currage.

We specify the covariates that should affect the ROC curve in the `roccov()` option. By default, `rocreg` will choose 10 equally spaced false-positive rates in the (0,1) interval as fitting points. The `fprpts()` option allows the user to specify more or fewer points. We specify the `bsave()` option with the `nnhs2y1` dataset so that we can use the bootstrap resamples in postestimation.

```
. use https://www.stata-press.com/data/r19/nnhs, clear
(Norton - neonatal audiology data)
. rocreg d y1, probit ctrlcov(currage male) ctrlmodel(linear) roccov(currage)
> cluster(id) bseed(56930) bsave(nnhs2y1) nodots
```

Bootstrap results Number of obs = 5,056  
Replications = 1,000

Parametric ROC estimation

```
Covariate control : linear regression
Control variables : currage male
Control standardization: empirical
ROC method : parametric Link: probit
Status : d
Classifier: y1
Covariate control adjustment model:
```

```
Linear regression Number of obs = 4,907
 F(2, 2685) = 13.80
 Prob > F = 0.0000
 R-squared = 0.0081
 Root MSE = 7.7515
```

(Std. err. adjusted for 2,686 clusters in id)

| y1      | Robust      |           |       |       |                      |           |
|---------|-------------|-----------|-------|-------|----------------------|-----------|
|         | Coefficient | std. err. | t     | P> t  | [95% conf. interval] |           |
| currage | -.2032456   | .0389032  | -5.22 | 0.000 | -.2795288            | -.1269624 |
| male    | .2471744    | .2603598  | 0.95  | 0.343 | -.2633516            | .7577005  |
| _cons   | -1.239484   | 1.487855  | -0.83 | 0.405 | -4.156942            | 1.677973  |

```
Status : d
Classifier: y1
ROC Model :
```

(Replications based on 2,741 clusters in id)

| y1      | Observed    | Bias     | Bootstrap | [95% conf. interval] |               |
|---------|-------------|----------|-----------|----------------------|---------------|
|         | coefficient |          |           | std. err.            |               |
| _cons   | -1.272505   | -.058656 | 1.157249  | -3.540671            | .995661 (N)   |
|         |             |          |           | -3.703316            | .8687538 (P)  |
|         |             |          |           | -3.550433            | 1.094785 (BC) |
| currage | .0448228    | .0015634 | .0300731  | -.0141194            | .1037649 (N)  |
|         |             |          |           | -.0107322            | .108762 (P)   |
|         |             |          |           | -.0156332            | .1044122 (BC) |
| probit  |             |          |           |                      |               |
| _cons   | .9372393    | .0153781 | .0739921  | .7922176             | 1.082261 (N)  |
|         |             |          |           | .8027433             | 1.108293 (P)  |
|         |             |          |           | .78655               | 1.077874 (BC) |

Note how the number of clusters—here infants—changes from the covariate control adjustment model fit to the ROC model. The control fit is limited to control cases and thus fewer infants. The ROC is fit on all the data, so the variance is adjusted for all clustering on all infants.

With a 0.05 level of statistical significance, we cannot reject the null hypothesis that currage has no effect on the ROC curve at a given false-positive rate. This is because each of our 95% bootstrap confidence intervals contains 0. This corresponds with the finding in [Janes, Longton, and Pepe \(2009\)](#) where the reported 95% intervals each contained 0. We cannot reject that the intercept parameter  $\beta_0$ , reported as `_cons` in the main table, is 0 at the 0.05 level either. The slope parameter  $\alpha$ , reported as `_cons` in the `probit` table, is close to 1 and cannot be rejected as being 1 at the 0.05 level. Under the assumption that the ROC coefficients except  $\alpha$  are 0 and that  $\alpha = 1$ , the ROC curve at false-positive rate  $u$  is equal to  $u$ . In other words, we cannot reject that the false-positive rate is equal to the true-positive rate, and so the test is noninformative. Further investigation of the results requires postestimation; see [\[R\] rocreg postestimation](#).

◀

The fitting point set can be formed by using the observed false-positive rates ([Pepe 2003](#)). Our next example will illustrate this.

### ▷ Example 7: Parametric ROC, covariate stratification

We return to the hearing test study of [Stover et al. \(1996\)](#), which we discussed in [example 5](#). [Pepe \(2003\)](#) suspected that intensity, `xd`, would play a role in the case distribution of the negative signal-to-noise ratio (`nsnr`) classifier. A ROC regression was fit with covariate adjustment for `xf` and `x1` with stratification, and for ROC covariates `xf`, `x1`, and `xd`. There is no prohibition against the same covariate being used in the first and second stages of ROC calculation. The false-positive rate fitting point set was composed of all observed false-positive rates in the control data.

We fit the model with `rocreg` here. Using observed false-positive rates as the fitting point set can make the dataset very large, so fitting the model is computationally intensive. We demonstrate the fitting algorithm without precise confidence intervals, focusing instead on the coefficient estimates and standard errors. We will thus perform only 50 bootstrap replications, a reasonable number to obtain accurate standard error estimates ([Mooney and Duval 1993](#)). The number of replications is specified in the `brep` option.

The ROC covariates are specified in `roccov()`. We specify that all observed false-positive rates in the control observations be used as fitting points with the `ctrlfprall` option. The `nobstrata` option specifies that the bootstrap is not stratified. The covariate stratification in the first stage of estimation does not affect the resampling. We will return to this example in postestimation, so we save the bootstrap results in the `nsnrf` dataset with the `bsave()` option.

```
. use https://www.stata-press.com/data/r19/dp
(Stover - DPOAE test data)
. rocreg d nsnr, probit ctrlcov(xf xl) roccov(xf xl xd) ctrlfprall cluster(id)
> nobstrata bseed(156385) breps(50) bsave(nsnrf)
(running rocregstat on estimation sample)
Bootstrap replications (50):10.....20.....30.....40.....
> ..50 done

Bootstrap results Number of obs = 1,848
 Replications = 50

Parametric ROC estimation
Covariate control : stratification
Control variables : xf xl
Control standardization: empirical
ROC method : parametric Link: probit
Status : d
Classifier: nsnr
ROC Model :
 (Replications based on 208 clusters in id)
```

| nsnr   | Observed coefficient | Bias     | Bootstrap std. err. | [95% conf. interval] |                |
|--------|----------------------|----------|---------------------|----------------------|----------------|
| _cons  | 3.247872             | .0868351 | .8985028            | 1.486839             | 5.008905 (N)   |
|        |                      |          |                     | 1.834415             | 5.606226 (P)   |
|        |                      |          |                     | 1.834415             | 6.275457 (BC)  |
| xf     | .0502557             | .0079289 | .0290622            | -.0067051            | .1072166 (N)   |
|        |                      |          |                     | -.0033383            | .1145611 (P)   |
|        |                      |          |                     | -.0454014            | .0883843 (BC)  |
| xl     | -.4327223            | -.024214 | .1249467            | -.6776134            | -.1878313 (N)  |
|        |                      |          |                     | -.7207585            | -.2425129 (P)  |
|        |                      |          |                     | -.7207585            | -.1547958 (BC) |
| xd     | .4431764             | .0200785 | .0875782            | .2715264             | .6148264 (N)   |
|        |                      |          |                     | .3388809             | .6706273 (P)   |
|        |                      |          |                     | .3388809             | .6706273 (BC)  |
| <hr/>  |                      |          |                     |                      |                |
| probit |                      |          |                     |                      |                |
| _cons  | 1.032657             | .0026243 | .1287713            | .7802699             | 1.285044 (N)   |
|        |                      |          |                     | .8308481             | 1.284435 (P)   |
|        |                      |          |                     | .7808038             | 1.284435 (BC)  |

We obtain results similar to those reported in [Pepe \(2003, 159\)](#). We find that the coefficients for `xl` and `xd` differ from 0 at the 0.05 level of significance. So over certain covariate combinations, we can have a variety of informative tests using `nsnr` as a classifier.



As mentioned before, when there are no covariates, `rocreg` can still fit a parametric model for the ROC curve of a classifier by using the [Alonzo and Pepe \(2002\)](#) method. `roccomp` and `rocfit` can fit marginal probit models as well. We will compare the behavior of `rocreg` with that of `roccomp` and `rocfit` for probit models without covariates.

When the `binormal` option is specified, `roccomp` calculates the AUC for input classifiers according to the maximum likelihood algorithm of `rocfit`. The `rocfit` algorithm expects discrete classifiers but can slice continuous classifiers into discrete partitions. Further, the case and control distributions are both assumed normal. Actually, the observed classification values are taken as discrete indicators of the latent normally distributed classification values. This method is documented in [Dorfman and Alf \(1969\)](#).

Alonzo and Pepe (2002) compared their estimating equations probability density function method (with empirical estimation of the false-positive rates) to the maximum likelihood approach of Dorfman and Alf (1969) and found that they had similar efficiency and mean squared error. So we should expect rocfite and rocreg to give similar results when fitting a simple probit model.

► Example 8: Parametric ROC, marginal model

We return to the Hanley and McNeil (1982) data. We will fit a probit model to the ROC curve, assuming that the rating variable is a discrete indicator of an underlying latent normal random variable in both the case and control populations of disease. We invoke rocfite with the default options. rocreg is invoked with the probit option. The percentile values are calculated empirically. Because there are fewer categories than 10, there will be fewer than 10 false-positive rates that trigger a different true-positive rate value. So for efficiency, we invoke rocreg with the ctrlfprall option.

```
. use https://www.stata-press.com/data/r19/hanley
(Tomographic images)
. rocfite disease rating, nolog
Binormal model of disease on rating Number of obs = 109
Goodness-of-fit chi2(2) = 0.21
Prob > chi2 = 0.9006
Log likelihood = -123.64855
```

|           | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-----------|-------------|-----------|-------|-------|----------------------|----------|
| intercept | 1.656782    | 0.310456  | 5.34  | 0.000 | 1.048300             | 2.265265 |
| slope (*) | 0.713002    | 0.215882  | -1.33 | 0.184 | 0.289881             | 1.136123 |
| /cut1     | 0.169768    | 0.165307  | 1.03  | 0.304 | -0.154227            | 0.493764 |
| /cut2     | 0.463215    | 0.167235  | 2.77  | 0.006 | 0.135441             | 0.790990 |
| /cut3     | 0.766860    | 0.174808  | 4.39  | 0.000 | 0.424243             | 1.109477 |
| /cut4     | 1.797938    | 0.299581  | 6.00  | 0.000 | 1.210770             | 2.385106 |

| Index    | Indices from binormal fit |           |                      |
|----------|---------------------------|-----------|----------------------|
|          | Estimate                  | Std. err. | [95% conf. interval] |
| ROC area | 0.911331                  | 0.029506  | 0.853501 0.969161    |
| delta(m) | 2.323671                  | 0.502370  | 1.339044 3.308298    |
| d(e)     | 1.934361                  | 0.257187  | 1.430284 2.438438    |
| d(a)     | 1.907771                  | 0.259822  | 1.398530 2.417012    |

(\*) z test for slope==1

```
. rocreg disease rating, probit ctrlfprall bseed(8574309) nodots
Bootstrap results Number of obs = 109
 Replications = 1,000

Parametric ROC estimation
Control standardization: empirical
ROC method : parametric Link: probit
Status : disease
Classifier : rating
ROC Model :
```

| rating | Observed coefficient | Bias      | Bootstrap std. err. | [95% conf. interval] |               |
|--------|----------------------|-----------|---------------------|----------------------|---------------|
| _cons  | 1.635041             | .0850129  | .3706472            | .9085857             | 2.361496 (N)  |
|        |                      |           |                     | 1.139856             | 2.649876 (P)  |
|        |                      |           |                     | 1.103894             | 2.428801 (BC) |
| probit |                      |           |                     |                      |               |
| _cons  | .6951252             | .0642966  | .275061             | .1560155             | 1.234235 (N)  |
|        |                      |           |                     | .3242299             | 1.409152 (P)  |
|        |                      |           |                     | .2721681             | 1.292525 (BC) |
| AUC    | Observed coefficient | Bias      | Bootstrap std. err. | [95% conf. interval] |               |
|        | .9102903             | -.0029679 | .0300486            | .8513963             | .9691844 (N)  |
|        |                      |           |                     | .8448006             | .9602325 (P)  |
|        |                      |           |                     | .8475004             | .9607949 (BC) |

We see that the intercept and slope parameter estimates are close. The intercept (`_cons` in the main table) is clearly nonzero. Under `rocreg`, the slope (`_cons` in the `probit` table) and its percentile and bias-corrected confidence intervals are close to those of `rocfit`. The area under the ROC curve for each of the `rocreg` and `rocfit` estimators also matches closely.



Now, we will compare the parametric fit of `rocreg` under the constant probit model with `roccomp`.

### ▷ Example 9: Parametric ROC, marginal model, multiple classifiers

We now use the fictitious dataset generated from [Hanley and McNeil \(1983\)](#). To fit a probit model using `roccomp`, we specify the `binormal` option. Our specification of `rocreg` remains the same as before.

`rocregplot` is used to render the model produced by `rocreg`. We specify several graph options to both `roccomp` and `rocregplot` to ease comparison. When the `binormal` option is specified along with `graph`, `roccomp` will draw the binormal fitted lines in addition to connected line plots of the empirical false-positive and true-positive rates.

In this plot, we overlay scatterplots of the empirical false-positive rates (because percentile value calculation defaulted to pvc(empirical)) and the parametric true-positive rates.

```
. use https://www.stata-press.com/data/r19/ct2, clear
(Reconstruction of CT images)

. roccomp status mod1 mod2 mod3, summary binormal graph aspectratio(1)
> plot1opts(connect(i) msymbol(o))
> plot2opts(connect(i) msymbol(s))
> plot3opts(connect(i) msymbol(t))
> legend(label(1 "Modality 1") label(3 "Modality 2")
> label(5 "Modality 3") label(2 "Modality 1 fit")
> label(4 "Modality 2 fit") label(6 "Modality 3 fit")
> order(1 3 5 2 4 6) pos(6))
> title(roccomp) name(a) nodraw
Fitting binormal model for: mod1
Fitting binormal model for: mod2
Fitting binormal model for: mod3
```

|      | Obs | ROC area | Std. err. | [95% conf. interval] |         |
|------|-----|----------|-----------|----------------------|---------|
| mod1 | 112 | 0.8945   | 0.0305    | 0.83482              | 0.95422 |
| mod2 | 112 | 0.9382   | 0.0264    | 0.88647              | 0.99001 |
| mod3 | 112 | 0.9376   | 0.0223    | 0.89382              | 0.98139 |

H0: area(mod1) = area(mod2) = area(mod3)  
 chi2(2) = 8.27 Prob>chi2 = 0.0160

```
. rocreg status mod1 mod2 mod3, probit ctrlfprall bseed(867340912) nodots
Bootstrap results Number of obs = 112
 Replications = 999
```

Parametric ROC estimation

Control standardization: empirical  
 ROC method : parametric Link: probit  
 Status : status  
 Classifier: mod1  
 ROC Model :

| mod1   | Observed coefficient | Bias     | Bootstrap std. err. | [95% conf. interval] |               |
|--------|----------------------|----------|---------------------|----------------------|---------------|
| _cons  | 1.726034             | .164964  | .5823832            | .5845836             | 2.867484 (N)  |
|        |                      |          |                     | 1.197595             | 3.410778 (P)  |
|        |                      |          |                     | 1.154531             | 3.027969 (BC) |
| probit | .9666323             | .1104948 | .4635417            | .0581071             | 1.875157 (N)  |
|        |                      |          |                     | .5102274             | 2.319844 (P)  |
|        |                      |          |                     | .5193889             | 2.319844 (BC) |

| AUC | Observed coefficient | Bias    | Bootstrap std. err. | [95% conf. interval] |               |
|-----|----------------------|---------|---------------------|----------------------|---------------|
|     | .8927007             | .000062 | .0306285            | .83267               | .9527315 (N)  |
|     |                      |         |                     | .8297837             | .946722 (P)   |
|     |                      |         |                     | .8262202             | .9423347 (BC) |

Status : status  
 Classifier: mod2  
 ROC Model :

| mod2   | Observed coefficient | Bias     | Bootstrap std. err. | [95% conf. interval] |               |  |
|--------|----------------------|----------|---------------------|----------------------|---------------|--|
| _cons  | 1.696811             | .0760455 | .4750493            | .7657314             | 2.627891 (N)  |  |
|        |                      |          |                     | 1.191126             | 2.854689 (P)  |  |
|        |                      |          |                     | 1.205256             | 2.916377 (BC) |  |
| probit |                      |          |                     |                      |               |  |
| _cons  | .4553828             | .0245707 | .304156             | -.140752             | 1.051518 (N)  |  |
|        |                      |          |                     | .0857558             | 1.070745 (P)  |  |
|        |                      |          |                     | .1495717             | 1.434937 (BC) |  |

| AUC | Observed coefficient | Bias      | Bootstrap std. err. | [95% conf. interval] |               |  |
|-----|----------------------|-----------|---------------------|----------------------|---------------|--|
|     | .938734              | -.0033942 | .0268351            | .8861382             | .9913297 (N)  |  |
|     |                      |           |                     | .875                 | .9774636 (P)  |  |
|     |                      |           |                     | .8775983             | .9777322 (BC) |  |

Status : status  
 Classifier: mod3  
 ROC Model :

| mod3   | Observed coefficient | Bias     | Bootstrap std. err. | [95% conf. interval] |               |  |
|--------|----------------------|----------|---------------------|----------------------|---------------|--|
| _cons  | 2.281359             | .1143008 | .5773577            | 1.149758             | 3.412959 (N)  |  |
|        |                      |          |                     | 1.653256             | 3.882332 (P)  |  |
|        |                      |          |                     | 1.65594              | 3.882332 (BC) |  |
| probit |                      |          |                     |                      |               |  |
| _cons  | 1.107736             | .0482007 | .4195496            | .2854334             | 1.930038 (N)  |  |
|        |                      |          |                     | .6128833             | 2.256342 (P)  |  |
|        |                      |          |                     | .6514254             | 2.527536 (BC) |  |

| AUC | Observed coefficient | Bias      | Bootstrap std. err. | [95% conf. interval] |              |  |
|-----|----------------------|-----------|---------------------|----------------------|--------------|--|
|     | .9368321             | -.0008781 | .0226477            | .8924435             | .9812207 (N) |  |
|     |                      |           |                     | .887858              | .9720291 (P) |  |
|     |                      |           |                     | .8866298             | .971411 (BC) |  |

H0: All classifiers have equal AUC values

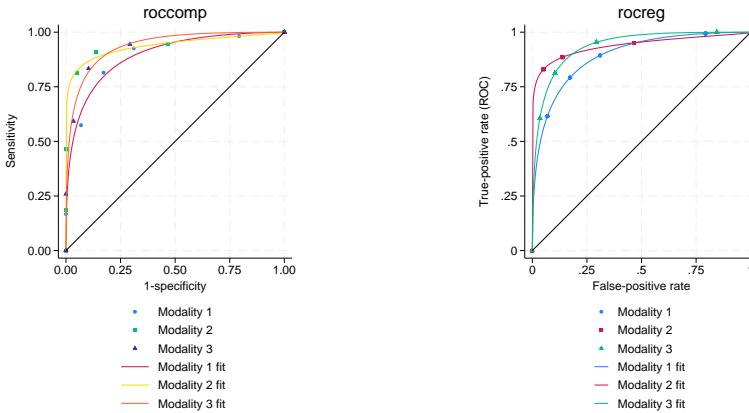
Ha: At least one classifier has a different AUC value

P-value: .0599896 Test based on bootstrap (N) assumptions.

. rocregplot, title(rocreg) nodraw name(b)

> plot1opts(msymbol(o)) plot2opts(msymbol(s)) plot3opts(msymbol(t))

```
. graph combine a b, xsize(5)
```



We see differing true-positive rate values in the scattered points, which is expected because `roccomp` gives the empirical estimate and `rocreg` gives the parametric estimate. However, the estimated curves and areas under the ROC curve look similar. Using the Wald test based on the bootstrap covariance, `rocreg` rejects the null hypothesis that each test has the same AUC at the 0.1 significance level. `roccomp` formulates the asymptotic covariance using the `rocfit` estimates of AUC. Examination of its output leads to rejection of the null hypothesis that the AUCs are equal across each test at the 0.05 significance level.



### Parametric ROC curves: Maximum likelihood

The [Alonzo and Pepe \(2002\)](#) method of fitting a parametric model to the ROC curve is powerful because it can be generally applied, but that can be a limitation as well. Whenever we invoke the method and want anything other than point estimates of the parameters, we must perform bootstrap resampling.

An alternative is to use maximum likelihood inference to fit the ROC curve. This method can save computational time by avoiding the bootstrap.

`rocreg` implements maximum likelihood estimation for ROC curve analysis when both the case and control populations are normal. Particularly, the classifier is a normal linear model on certain covariates, and the covariate effect and variance of the classifier may change between the case and control populations. This model is defined in [Pepe \(2003, 145\)](#).

$$y = \mathbf{z}'\beta_0 + D\mathbf{x}'\beta_1 + \sigma(D)\epsilon$$

Our error term,  $\epsilon$ , is a standard normal random variable. The variable  $D$  is our true status variable, being 1 for the case population observations and 0 for the control population observations. The variance function  $\sigma$  is defined as

$$\sigma(D) = \sigma_0(D = 0) + \sigma_1(D = 1)$$

This provides two variance parameters in the model and does not depend on covariate values.

Suppose a covariate  $x_i$  is present in  $\mathbf{z}$  and  $\mathbf{x}$ . The coefficient  $\beta_{1i}$  represents the interaction effect of the  $x_i$  and  $D$ . It is the extra effect that  $x_i$  has on classifier  $y$  under the case population,  $D = 1$ , beyond the main effect  $\beta_{0i}$ . These  $\beta_1$  coefficients are directly related to the ROC curve of  $y$ .

Under this model, the ROC curve is derived to be

$$\text{ROC}(u) = \Phi \left[ \frac{1}{\sigma_1} \{ \mathbf{x}' \boldsymbol{\beta}_1 + \sigma_0 \Phi^{-1}(u) \} \right]$$

For convenience, we reparameterize the model at this point, creating the parameters  $\beta_i = \sigma_1^{-1} \beta_{1i}$  and  $\alpha = \sigma_1^{-1} \sigma_0$ . We refer to  $\beta_0$  as the constant intercept, `i_cons`. The parameter  $\alpha$  is referred to as the constant slope, `s_cons`.

$$\text{ROC}(u) = \Phi \{ \mathbf{x}' \boldsymbol{\beta} + \alpha \Phi^{-1}(u) \}$$

We may interpret the final coefficients as the standardized linear effect of the ROC covariate on the classifier under the case population. The marginal effect of the covariate on the classifier in the control population is removed, and it is rescaled by the case population standard deviation of the classifier when all ROC covariate effects are removed. An appreciable effect on the classifier by a ROC covariate in this measure leads to an appreciable effect on the classifier's ROC curve by the ROC covariate.

The advantage of estimating the control coefficients  $\beta_0$  is similar to the gains of estimating the covariate control models in the estimating equations ROC method and nonparametric ROC estimation. This model would similarly apply when evaluating a test that is conditioned on control covariates.

Again, we note that under parametric estimation, all the summary measures we defined earlier except the AUC are not calculated until postestimation. In models with covariates, each covariate combination would yield a different ROC curve and thus different summary parameters, so no summary parameters are estimated initially. In marginal parametric models, we will calculate the AUC and leave the other measures for postestimation. There is a simple closed-form formula for the AUC under the probit model. Using this formula, the delta method can be invoked for inference on the AUC. Details on AUC estimation for probit marginal models are found in *Methods and formulas*.

We will demonstrate the maximum likelihood method of `rocreg` by revisiting the models of the previous section.

### ▷ Example 10: Maximum likelihood ROC, single classifier

Returning to the hearing test study of [Stover et al. \(1996\)](#), we use a similar covariate grouping as before. The frequency `xf` and intensity `x1` are control covariates ( $\mathbf{z}$ ), while all three covariates `xf`, `x1`, and hearing loss degree `xd` are case covariates ( $\mathbf{x}$ ). In [example 7](#), we fit this model using the [Alonzo and Pepe \(2002\)](#) method. Earlier we stratified on the control covariates and estimated the conditioned control distribution of `nsnr` empirically. Now, we assume a normal linear model for `nsnr` on `xf` and `x1` under the control population.

We fit the model by specifying the control covariates in the `ctrlcov()` option and the case covariates in the `roccov()` option. The `ml` option tells `rocreg` to perform maximum likelihood estimation.

```
. use https://www.stata-press.com/data/r19/dp, clear
(Stover - DPOAE test data)
. rocreg d nsnr, ctrlcov(xf x1) roccov(xf x1 xd) probit ml cluster(id) nolog
Parametric ROC estimation Number of obs = 112
 Replications = 999

Covariate control : linear regression
Control variables : xf x1
Control standardization: normal
ROC method : parametric Link: probit

Status : d
Classifiers: nsnr
Classifier : nsnr
Covariate control adjustment model:
 (Std. err. adjusted for 208 clusters in id)
```

|                |             | Robust    |       |       |           | [95% conf. interval] |  |
|----------------|-------------|-----------|-------|-------|-----------|----------------------|--|
|                | Coefficient | std. err. | z     | P> z  |           |                      |  |
| <b>casecov</b> |             |           |       |       |           |                      |  |
| xf             | .4690907    | .1408683  | 3.33  | 0.001 | .192994   | .7451874             |  |
| x1             | -3.187785   | .8976521  | -3.55 | 0.000 | -4.947151 | -1.42842             |  |
| xd             | 3.042998    | .3569756  | 8.52  | 0.000 | 2.343339  | 3.742657             |  |
| _cons          | 23.48064    | 5.692069  | 4.13  | 0.000 | 12.32439  | 34.63689             |  |
| <b>casesd</b>  |             |           |       |       |           |                      |  |
| _cons          | 7.979708    | .354936   | 22.48 | 0.000 | 7.284047  | 8.67537              |  |
| <b>ctrlcov</b> |             |           |       |       |           |                      |  |
| xf             | -.1447499   | .0615286  | -2.35 | 0.019 | -.2653438 | -.0241561            |  |
| x1             | -.8631348   | .2871976  | -3.01 | 0.003 | -1.426032 | -.3002378            |  |
| _cons          | 1.109477    | 1.964004  | 0.56  | 0.572 | -2.7399   | 4.958854             |  |
| <b>ctrlsd</b>  |             |           |       |       |           |                      |  |
| _cons          | 7.731203    | .3406654  | 22.69 | 0.000 | 7.063511  | 8.398894             |  |

```
Status : d
ROC Model :
 (Std. err. adjusted for 208 clusters in id)
```

|             |             | Robust    |       |       |           | [95% conf. interval] |  |
|-------------|-------------|-----------|-------|-------|-----------|----------------------|--|
|             | Coefficient | std. err. | z     | P> z  |           |                      |  |
| <b>nsnr</b> |             |           |       |       |           |                      |  |
| i_cons      | 2.942543    | .7569821  | 3.89  | 0.000 | 1.458885  | 4.426201             |  |
| xf          | .0587854    | .0175654  | 3.35  | 0.001 | .024358   | .0932129             |  |
| x1          | -.3994865   | .1171914  | -3.41 | 0.001 | -.6291775 | -.1697955            |  |
| xd          | .381342     | .0449319  | 8.49  | 0.000 | .2932771  | .4694068             |  |
| s_cons      | .9688578    | .0623476  | 15.54 | 0.000 | .8466587  | 1.091057             |  |

We find the results are similar to those of [example 7](#). Frequency (`xf`) and intensity (`x1`) have a negative effect on the classifier `nsnr` in the control population.

The negative control effect is mitigated for `xf` in the case population, but the effect for `x1` is even more negative there. Hearing loss severity, `xd`, has a positive effect on `nsnr` in the case population, and it is undefined in the control population.

The ROC coefficients are shown in the ROC Model table. Each are different from 0 at the 0.05 level. At this level, we also cannot conclude that the variances differ from case to control populations, because 1 is in the 95% confidence interval for `s_cons`, the ratio of the case to control standard deviation parameters.

Both frequency (`xf`) and hearing loss severity (`xd`) make a positive contribution to the ROC curve and thus make the test more powerful. Intensity (`x1`) has a negative effect on the ROC curve and weakens the test. We previously saw in [example 5](#) that the control distribution appears to be normal, so using maximum likelihood to fit this model is a reasonable approach.

This model was also fit in [Pepe \(2003, 147\)](#). Pepe used separate least-squares estimates for the case and control samples. We obtain similar results for the coefficients, but the maximum likelihood fitting yields slightly different standard deviations by considering both case and control observations concurrently. In addition, a misprint in [Pepe \(2003, 147\)](#) reports a coefficient of  $-4.91$  for `x1` in the case population instead of  $-3.19$  as reported by Stata.

◀

Inference on multiple classifiers using the [Alonzo and Pepe \(2002\)](#) estimating equation method is performed by fitting each model separately and bootstrapping to determine the dependence of the estimates. Using the maximum likelihood method, we also fit each model separately. We use `suest` (see [\[R\] suest](#)) to estimate the joint variance–covariance of our parameter estimates.

For our models, we can view the score equation for each model as an estimating equation. The estimate that solves the estimating equation (that makes the score 0) is asymptotically normal with a variance matrix that can be estimated using the inverse of the squared scores. By stacking the score equations of the separate models, we can estimate the variance matrix for all the parameter estimates by using this rule. This is an informal explanation; further details can be found in [\[R\] suest](#) and in the references [Rogers \(1993\)](#); [White \(1982 and 1996\)](#).

Now, we will examine a case with multiple classification variables.

▷ Example 11: Maximum likelihood ROC, multiple classifiers

We return to the neonatal audiology study with gender and age covariates (Norton et al. 2000). In example 6, we fit a model with male and currage as control covariates, and currage as a ROC covariate for the classifier y1 (DPOAE 65 at 2 kHz). We will refit this model, extending it to include the classifier y2 (TEOAE 80 at 2 kHz).

```
. use https://www.stata-press.com/data/r19/nnhs
(Norton - neonatal audiology data)
. rocreg d y1 y2, probit ml ctrlcov(currage male) roccov(currage) cluster(id)
> nolog
Parametric ROC estimation Number of obs = 1,848
Covariate control : linear regression
Control variables : currage male
Control standardization: normal
ROC method : parametric Link: probit
Status : d
Classifiers: y1 y2
Classifier : y1
Covariate control adjustment model:
```

|         | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|---------|-------------|-----------|-------|-------|----------------------|-----------|
| casecov |             |           |       |       |                      |           |
| currage | .494211     | .2126672  | 2.32  | 0.020 | .077391              | .9110311  |
| _cons   | -15.00403   | 8.238094  | -1.82 | 0.069 | -31.1504             | 1.142338  |
| casesd  |             |           |       |       |                      |           |
| _cons   | 8.49794     | .4922792  | 17.26 | 0.000 | 7.533091             | 9.46279   |
| ctrlcov |             |           |       |       |                      |           |
| currage | -.2032048   | .0323803  | -6.28 | 0.000 | -.266669             | -.1397406 |
| male    | .2369359    | .2201391  | 1.08  | 0.282 | -.1945288            | .6684006  |
| _cons   | -1.23534    | 1.252775  | -0.99 | 0.324 | -3.690734            | 1.220055  |
| ctrlsd  |             |           |       |       |                      |           |
| _cons   | 7.749156    | .0782225  | 99.07 | 0.000 | 7.595843             | 7.902469  |

```
Classifier : y2
Covariate control adjustment model:
```

|         | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|---------|-------------|-----------|-------|-------|----------------------|-----------|
| casecov |             |           |       |       |                      |           |
| currage | .5729861    | .2422662  | 2.37  | 0.018 | .0981532             | 1.047819  |
| _cons   | -18.2597    | 9.384968  | -1.95 | 0.052 | -36.6539             | .1344949  |
| casesd  |             |           |       |       |                      |           |
| _cons   | 9.723858    | .5632985  | 17.26 | 0.000 | 8.619813             | 10.8279   |
| ctrlcov |             |           |       |       |                      |           |
| currage | -.1694575   | .0291922  | -5.80 | 0.000 | -.2266732            | -.1122419 |
| male    | .7122587    | .1993805  | 3.57  | 0.000 | .3214802             | 1.103037  |
| _cons   | -5.651728   | 1.129452  | -5.00 | 0.000 | -7.865415            | -3.438042 |
| ctrlsd  |             |           |       |       |                      |           |
| _cons   | 6.986167    | .0705206  | 99.07 | 0.000 | 6.84795              | 7.124385  |

```
Status : d
ROC Model :
 (Std. err. adjusted for 2,741 clusters in id)
```

|         | Coefficient | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |           |
|---------|-------------|---------------------|-------|-------|----------------------|-----------|
| y1      |             |                     |       |       |                      |           |
| i_cons  | -1.765608   | 1.105393            | -1.60 | 0.110 | -3.932138            | .4009225  |
| currage | .0581566    | .0290177            | 2.00  | 0.045 | .0012828             | .1150303  |
| s_cons  | .9118864    | .0586884            | 15.54 | 0.000 | .7968593             | 1.026913  |
| y2      |             |                     |       |       |                      |           |
| i_cons  | -1.877825   | .905174             | -2.07 | 0.038 | -3.651933            | -.1037167 |
| currage | .0589258    | .0235849            | 2.50  | 0.012 | .0127002             | .1051514  |
| s_cons  | .7184563    | .0565517            | 12.70 | 0.000 | .607617              | .8292957  |

Both classifiers have similar results. The results for y1 show the same direction as the estimating equation results in [example 6](#). However, we can now reject the null hypothesis that the ROC currage coefficient is 0 at the 0.05 level.

In [example 6](#), we could not reject that the slope parameter s\_cons was 1 and that the constant intercept or ROC coefficient for current age was 0. The resulting ROC curve implied a noninformative test using y1 as a classifier. This is not the case with our current results. As currage increases, we expect a steeper ROC curve and thus a more powerful test, for both classifiers y1 (DPOAE 65 at 2 kHz) and y2 (TEOAE 80 at 2 kHz).

In [example 10](#), the clustering of observations within infant id was adjusted in the individual fit of nsnr. In our current example, the adjustment for the clustering of observations within id is performed during concurrent estimation, as opposed to during the individual classifier fits (as in [example 10](#)). This adjustment, performed by suest, is still accurate.



Now, we will fit constant probit models and compare rocreg with rocf it and roccomp with the binormal option. Our first applications of rocf it and roccomp are taken directly from [examples 8](#) and [9](#). The [Dorfman and Alf \(1969\)](#) algorithm that rocf it works with uses discrete classifiers or uses slicing to make a classifier discrete. So we are applying the maximum likelihood method of rocreg on discrete classification data here, where it expects continuous data. We expect to see some discrepancies, but we do not find great divergence in the estimates. After revisiting [examples 8](#) and [9](#), we will fit a probit model with a continuous classifier and no covariates using rocreg, and we will compare the results with those from rocf it.

▷ Example 12: Maximum likelihood ROC, marginal model

Using the Hanley and McNeil (1982) data, discussed in example 1 and in example 8, we fit a constant probit model of the classifier rating with true status disease. rocreg is invoked with the ml option and compared with rocfits.

```
. use https://www.stata-press.com/data/r19/hanley, clear
(Tomographic images)
. rocfits disease rating, nolog
Binormal model of disease on rating Number of obs = 109
Goodness-of-fit chi2(2) = 0.21
Prob > chi2 = 0.9006
Log likelihood = -123.64855
```

|           | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|-----------|-------------|-----------|-------|-------|----------------------|----------|
| intercept | 1.656782    | 0.310456  | 5.34  | 0.000 | 1.048300             | 2.265265 |
| slope (*) | 0.713002    | 0.215882  | -1.33 | 0.184 | 0.289881             | 1.136123 |
| /cut1     | 0.169768    | 0.165307  | 1.03  | 0.304 | -0.154227            | 0.493764 |
| /cut2     | 0.463215    | 0.167235  | 2.77  | 0.006 | 0.135441             | 0.790990 |
| /cut3     | 0.766860    | 0.174808  | 4.39  | 0.000 | 0.424243             | 1.109477 |
| /cut4     | 1.797938    | 0.299581  | 6.00  | 0.000 | 1.210770             | 2.385106 |

| Index    | Indices from binormal fit |           |                      |          |
|----------|---------------------------|-----------|----------------------|----------|
|          | Estimate                  | Std. err. | [95% conf. interval] |          |
| ROC area | 0.911331                  | 0.029506  | 0.853501             | 0.969161 |
| delta(m) | 2.323671                  | 0.502370  | 1.339044             | 3.308298 |
| d(e)     | 1.934361                  | 0.257187  | 1.430284             | 2.438438 |
| d(a)     | 1.907771                  | 0.259822  | 1.398530             | 2.417012 |

(\*) z test for slope==1

```
. rocreg disease rating, probit ml nolog
Binormal model of disease on rating Number of obs = 109
GOF chi2(0) = .
Log likelihood = -123.64855 Prob > chi2 = .
Control standardization: normal
ROC method : parametric Link: probit
Status : disease
Classifiers: rating
Classifier : rating
Covariate control adjustment model:
```

|         | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|---------|-------------|-----------|-------|-------|----------------------|----------|
| casecov |             |           |       |       |                      |          |
| _cons   | 2.3357      | .2334285  | 10.01 | 0.000 | 1.878188             | 2.793211 |
| casesd  |             |           |       |       |                      |          |
| _cons   | 1.117131    | .1106124  | 10.10 | 0.000 | .9003344             | 1.333927 |
| ctrlcov |             |           |       |       |                      |          |
| _cons   | 2.017241    | .1732589  | 11.64 | 0.000 | 1.67766              | 2.356823 |
| ctrlsd  |             |           |       |       |                      |          |
| _cons   | 1.319501    | .1225125  | 10.77 | 0.000 | 1.07938              | 1.559621 |

```
Status : disease
ROC Model :
```

|        | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|--------|-------------|-----------|-------|-------|----------------------|----------|
| rating |             |           |       |       |                      |          |
| i_cons | 2.090802    | .2941411  | 7.11  | 0.000 | 1.514297             | 2.667308 |
| s_cons | 1.181151    | .1603263  | 7.37  | 0.000 | .8669177             | 1.495385 |
| auc    | .9116494    | .0261658  | 34.84 | 0.000 | .8603654             | .9629333 |

We compare the estimates for these models:

|                 | rocfit | rocreg, ml |
|-----------------|--------|------------|
| slope           | 0.7130 | 1.1812     |
| SE of slope     | 0.2159 | 0.1603     |
| intercept       | 1.6568 | 2.0908     |
| SE of intercept | 0.3105 | 0.2941     |
| AUC             | 0.9113 | 0.9116     |
| SE of AUC       | 0.0295 | 0.0262     |

We find that both the intercept and the slope are estimated as higher with the maximum likelihood method under rocreg than with rocfit. The AUC (ROC area in rocfit) is close for both commands. We find that the standard errors of each of these estimates is slightly lower under rocreg than rocfit as well.

Both rocfit and rocreg suggest that the slope parameter of the ROC curve (slope in rocfit and s\_cons in rocreg) is not significantly different from 1. Thus, we cannot reject that the classifier has the same variance in both case and control populations. There is, however, significant evidence that the intercepts (i\_cons in rocreg and intercept in rocfit) differ from 0. Because of the positive direction of the intercept estimates, the ROC curve for rating as a classifier of disease suggests that rating provides an informative test. This is also suggested by the high AUC, which is significantly different from 0.5, that is, a flip of a coin.



▷ Example 13: Maximum likelihood ROC, marginal model, multiple classifiers

We use the fictitious dataset generated from Hanley and McNeil (1983), which we previously used in example 2 and in example 9. To fit a probit model using roccomp, we specify the binormal option. We perform parametric, maximum likelihood ROC analysis using rocreg. We use rocregplot to plot the ROC curves created by rocreg.

```
. use https://www.stata-press.com/data/r19/ct2, clear
(Reconstruction of CT images)

. roccomp status mod1 mod2 mod3, summary binormal graph aspectratio(1)
> plot1opts(connect(i) msymbol(o))
> plot2opts(connect(i) msymbol(s))
> plot3opts(connect(i) msymbol(t))
> legend(label(1 "Modality 1") label(3 "Modality 2")
> label(5 "Modality 3") label(2 "Modality 1 fit")
> label(4 "Modality 2 fit") label(6 "Modality 3 fit")
> order(1 3 5 2 4 6) pos(6)) title(roccomp) name(a) nodraw
Fitting binormal model for: mod1
Fitting binormal model for: mod2
Fitting binormal model for: mod3
```

|      | Obs | ROC<br>area | Std. err. | [95% conf. interval] |         |
|------|-----|-------------|-----------|----------------------|---------|
| mod1 | 112 | 0.8945      | 0.0305    | 0.83482              | 0.95422 |
| mod2 | 112 | 0.9382      | 0.0264    | 0.88647              | 0.99001 |
| mod3 | 112 | 0.9376      | 0.0223    | 0.89382              | 0.98139 |

H0: area(mod1) = area(mod2) = area(mod3)  
 chi2(2) = 8.27 Prob>chi2 = 0.0160

```
. rocreg status mod1 mod2 mod3, probit ml nolog
Parametric ROC estimation Number of obs = 109
Control standardization: normal
ROC method : parametric Link: probit
Status : status
Classifiers: mod1 mod2 mod3
Classifier : mod1
Covariate control adjustment model:
```

|                  | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|------------------|-------------|-----------|-------|-------|----------------------|----------|
| casecov<br>_cons | 2.118135    | .2165905  | 9.78  | 0.000 | 1.693626             | 2.542645 |
| casesd<br>_cons  | 1.166078    | .1122059  | 10.39 | 0.000 | .9461589             | 1.385998 |
| ctrlcov<br>_cons | 2.344828    | .1474147  | 15.91 | 0.000 | 2.0559               | 2.633755 |
| ctrlsd<br>_cons  | 1.122677    | .1042379  | 10.77 | 0.000 | .9183746             | 1.32698  |

Classifier : mod2  
 Covariate control adjustment model:

|                  | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|------------------|-------------|-----------|-------|-------|----------------------|----------|
| casecov<br>_cons | 2.659642    | .2072731  | 12.83 | 0.000 | 2.253395             | 3.06589  |
| casesd<br>_cons  | 1.288468    | .1239829  | 10.39 | 0.000 | 1.045466             | 1.53147  |
| ctrlcov<br>_cons | 1.655172    | .1105379  | 14.97 | 0.000 | 1.438522             | 1.871823 |
| ctrlsd<br>_cons  | .8418313    | .0781621  | 10.77 | 0.000 | .6886365             | .9950262 |

Classifier : mod3  
 Covariate control adjustment model:

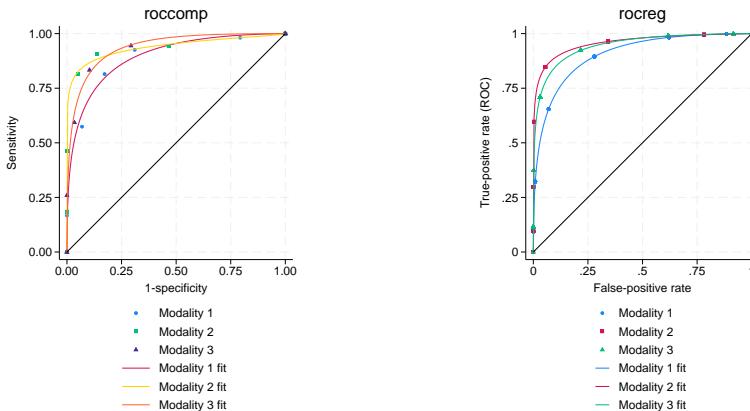
|                  | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|------------------|-------------|-----------|-------|-------|----------------------|----------|
| casecov<br>_cons | 2.353768    | .1973549  | 11.93 | 0.000 | 1.966959             | 2.740576 |
| casesd<br>_cons  | 1.143359    | .1100198  | 10.39 | 0.000 | .9277243             | 1.358994 |
| ctrlcov<br>_cons | 2.275862    | .1214094  | 18.75 | 0.000 | 2.037904             | 2.51382  |
| ctrlsd<br>_cons  | .9246267    | .0858494  | 10.77 | 0.000 | .7563649             | 1.092888 |

Status : status  
 ROC Model :

|        | Coefficient | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |          |
|--------|-------------|---------------------|-------|-------|----------------------|----------|
| mod1   |             |                     |       |       |                      |          |
| i_cons | 1.81646     | .3144804            | 5.78  | 0.000 | 1.20009              | 2.432831 |
| s_cons | .9627801    | .1364084            | 7.06  | 0.000 | .6954245             | 1.230136 |
| auc    | .904657     | .0343518            | 26.34 | 0.000 | .8373287             | .9719853 |
| mod2   |             |                     |       |       |                      |          |
| i_cons | 2.064189    | .3267274            | 6.32  | 0.000 | 1.423815             | 2.704563 |
| s_cons | .6533582    | .1015043            | 6.44  | 0.000 | .4544135             | .8523029 |
| auc    | .9580104    | .0219713            | 43.60 | 0.000 | .9149473             | 1.001073 |
| mod3   |             |                     |       |       |                      |          |
| i_cons | 2.058643    | .2890211            | 7.12  | 0.000 | 1.492172             | 2.625113 |
| s_cons | .8086932    | .1163628            | 6.95  | 0.000 | .5806262             | 1.03676  |
| auc    | .9452805    | .0236266            | 40.01 | 0.000 | .8989732             | .9915877 |

H0: All classifiers have equal AUC values  
 Ha: At least one classifier has a different AUC value  
 P-value: .0808808

```
. rocregplot, title(rocreg) nodraw name(b)
> plot1opts(msymbol(o)) plot2opts(msymbol(s)) plot3opts(msymbol(t))
. graph combine a b, xsize(5)
```



We compare the AUC estimates for these models:

|      | roccomp | rocreg, ml |
|------|---------|------------|
| mod1 | 0.8945  | 0.9047     |
| mod2 | 0.9382  | 0.9580     |
| mod3 | 0.9376  | 0.9453     |

Each classifier has a higher estimated AUC under `rocreg` than `roccomp`. Each curve appears to be raised and smoothed in the `rocreg` fit as compared with `roccomp`. They are different, but not drastically different. The inference on whether the curve areas are the same is similar to [example 9](#). We reject equality at the 0.10 level under `rocreg` and at the 0.05 level under `roccomp`.

Each intercept is significantly different from 0 at the 0.05 level and is estimated in a positive direction. Though all but classifier `mod2` has 1 in their slope confidence intervals, the high intercepts suggest steep ROC curves and powerful tests.

Also note that the false-positive and true-positive rate points are calculated empirically in the `roccomp` graph and parametrically in `rocreg`. In [example 9](#), the false-positive rates calculated by `rocreg` were calculated empirically, similar to `roccomp`. But in this example, the rates are calculated based on normal percentiles.



Now, we will generate an example to compare `rocfit` and `rocreg` under maximum likelihood estimation of a continuous classifier.

▷ **Example 14: Maximum likelihood ROC, graphical comparison with `rocfit`**

We generate 500 realizations of a population under threat of disease. One quarter of the population has the disease. A classifier `x` is measured, which has a control distribution of  $N(1, 3)$  and a case distribution of  $N(1 + 5, 2)$ . We will invoke `rocreg` with the `ml` option on this generated data. We specify the `continuous()` option for `rocfit` and invoke it on the data as well. The `continuous()` option tells `rocfit` how many discrete slices to partition the data into before fitting.

For comparison of the two curves, we will use the `rocf` postestimation command, `rocplot`; see [R] [rocf](#) postestimation. This command graphs the empirical false-positive and true-positive rates with an overlaid fit of the binormal curve estimated by `rocf`. `rocplot` also supports an `addplot()` option. We use the saved variables from `rocreg` in this option to overlay a line plot of the `rocreg` fit.

```
. clear
. set seed 8675309
. set obs 500
Number of observations (_N) was 0, now 500.
. generate d = runiform() < .25
. quietly generate double epsilon = 3*invnormal(runiform()) if d == 0
. quietly replace epsilon = 2*invnormal(runiform()) if d == 1
. quietly generate double x = 1 + d*5 + epsilon
. rocreg d x, probit ml nolog
Parametric ROC estimation Number of obs = 112
Control standardization: normal
ROC method : parametric Link: probit
Status : d
Classifiers: x
Classifier : x
Covariate control adjustment model:
```

|         | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|---------|-------------|-----------|-------|-------|----------------------|----------|
| casecov |             |           |       |       |                      |          |
| _cons   | 4.823931    | .2305469  | 20.92 | 0.000 | 4.372067             | 5.275795 |
| casesd  |             |           |       |       |                      |          |
| _cons   | 1.926652    | .1204158  | 16.00 | 0.000 | 1.690642             | 2.162663 |
| ctrlcov |             |           |       |       |                      |          |
| _cons   | 1.14378     | .155409   | 7.36  | 0.000 | .8391841             | 1.448376 |
| ctrlsd  |             |           |       |       |                      |          |
| _cons   | 2.99742     | .1098907  | 27.28 | 0.000 | 2.782038             | 3.212802 |

```
Status : d
ROC Model :
```

|        | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|--------|-------------|-----------|-------|-------|----------------------|----------|
| x      |             |           |       |       |                      |          |
| i_cons | 2.503789    | .1969952  | 12.71 | 0.000 | 2.117686             | 2.889893 |
| s_cons | 1.555766    | .1127296  | 13.80 | 0.000 | 1.33482              | 1.776712 |
| auc    | .912102     | .0123921  | 73.60 | 0.000 | .8878139             | .9363902 |

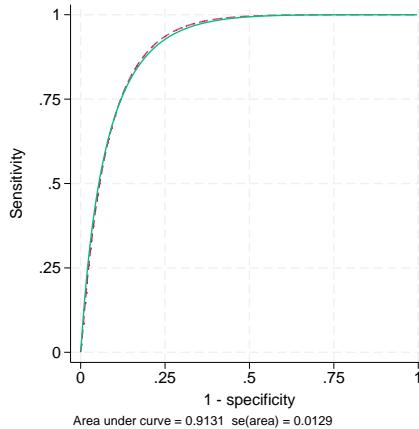
```
. rocfit d x, continuous(10) nolog
Binormal model of d on x Number of obs = 500
Goodness-of-fit chi2(7) = 1.33
Prob > chi2 = 0.9877
Log likelihood = -914.15521
```

|           | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |           |
|-----------|-------------|-----------|--------|-------|----------------------|-----------|
| intercept | 2.647297    | 0.277012  | 9.56   | 0.000 | 2.104362             | 3.190231  |
| slope (*) | 1.670103    | 0.195433  | 3.43   | 0.001 | 1.287062             | 2.053145  |
| /cut1     | -2.079091   | 0.153221  | -13.57 | 0.000 | -2.379398            | -1.778783 |
| /cut2     | -1.383360   | 0.093448  | -14.80 | 0.000 | -1.566515            | -1.200205 |
| /cut3     | -0.905227   | 0.075606  | -11.97 | 0.000 | -1.053413            | -0.757041 |
| /cut4     | -0.252654   | 0.065679  | -3.85  | 0.000 | -0.381382            | -0.123925 |
| /cut5     | 0.310051    | 0.065913  | 4.70   | 0.000 | 0.180863             | 0.439239  |
| /cut6     | 0.915048    | 0.072958  | 12.54  | 0.000 | 0.772054             | 1.058042  |
| /cut7     | 1.512188    | 0.092153  | 16.41  | 0.000 | 1.331570             | 1.692805  |
| /cut8     | 2.095878    | 0.136662  | 15.34  | 0.000 | 1.828026             | 2.363731  |
| /cut9     | 2.516563    | 0.181939  | 13.83  | 0.000 | 2.159970             | 2.873156  |

| Index    | Indices from binormal fit |           |                      |          |
|----------|---------------------------|-----------|----------------------|----------|
|          | Estimate                  | Std. err. | [95% conf. interval] |          |
| ROC area | 0.913079                  | 0.012942  | 0.887713             | 0.938445 |
| delta(m) | 1.585110                  | 0.107531  | 1.374352             | 1.795868 |
| d(e)     | 1.982917                  | 0.121777  | 1.744239             | 2.221596 |
| d(a)     | 1.923275                  | 0.115671  | 1.696565             | 2.149985 |

(\*) z test for slope==1

```
. rocplot, plotopts(msymbol(i)) lineopts(lpattern(dash))
> norefline addplot(line_roc_x_fpr_x, sort(_fpr_x _roc_x))
> lpattern(solid) aspectratio(1) legend(off)
```



We find that the curves are close. As before, the `rocfit` estimates are lower for the slope and intercept than under `rocreg`. The AUC estimates are close. Though the slope confidence interval contains 1, a high ROC intercept suggests a steep ROC curve and thus a powerful test.

## Stored results

Nonparametric `rocreg` stores the following in `e()`:

### Scalars

|                          |                                    |
|--------------------------|------------------------------------|
| <code>e(N)</code>        | number of observations             |
| <code>e(N_strata)</code> | number of covariate strata         |
| <code>e(N_clust)</code>  | number of clusters                 |
| <code>e(level)</code>    | confidence level for bootstrap CIs |
| <code>e(rank)</code>     | rank of <code>e(V)</code>          |

### Macros

|                              |                                                                   |
|------------------------------|-------------------------------------------------------------------|
| <code>e(cmd)</code>          | <code>rocreg</code>                                               |
| <code>e(cmdline)</code>      | command as typed                                                  |
| <code>e(classvars)</code>    | classification variable list                                      |
| <code>e(refvar)</code>       | status variable, reference variable                               |
| <code>e(ctrlmodel)</code>    | covariate-adjustment specification                                |
| <code>e(ctrlcov)</code>      | covariate-adjustment variables                                    |
| <code>e(pvc)</code>          | percentile value calculation method                               |
| <code>e(title)</code>        | title in estimation output                                        |
| <code>e(tiecorrected)</code> | <code>tiecorrected</code> , if specified                          |
| <code>e(nobootstrap)</code>  | <code>nobootstrap</code> , if specified                           |
| <code>e(rngstate)</code>     | random-number state used in bootstrap, if bootstrap was performed |
| <code>e(breps)</code>        | number of bootstrap resamples, if bootstrap performed             |
| <code>e(bootcc)</code>       | <code>bootcc</code> , if specified                                |
| <code>e(nobstrata)</code>    | <code>nobstrata</code> , if specified                             |
| <code>e(clustvar)</code>     | name of cluster variable                                          |
| <code>e(exp#)</code>         | expression for the <code>#</code> th statistic                    |
| <code>e(roc)</code>          | false-positive rates where ROC was estimated                      |
| <code>e(invroc)</code>       | ROC values where false-positive rates were estimated              |
| <code>e(pauc)</code>         | false-positive rates where pAUC was estimated                     |
| <code>e(auc)</code>          | indicates that AUC was calculated                                 |
| <code>e(vce)</code>          | <code>bootstrap</code>                                            |
| <code>e(properties)</code>   | <code>b V</code> (or <code>b</code> if bootstrap not performed)   |

### Matrices

|                               |                                              |
|-------------------------------|----------------------------------------------|
| <code>e(b)</code>             | coefficient vector                           |
| <code>e(V)</code>             | variance–covariance matrix of the estimators |
| <code>e(b_bs)</code>          | bootstrap estimates                          |
| <code>e(reps)</code>          | number of nonmissing results                 |
| <code>e(bias)</code>          | estimated biases                             |
| <code>e(se)</code>            | estimated standard errors                    |
| <code>e(z0)</code>            | median biases                                |
| <code>e(ci_normal)</code>     | normal-approximation confidence intervals    |
| <code>e(ci_percentile)</code> | percentile confidence intervals              |
| <code>e(ci_bc)</code>         | bias-corrected confidence intervals          |

### Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

### Matrices

|                       |                                                                                                                            |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

Parametric, bootstrap rocreg stores the following in `e()`:

#### Scalars

|                          |                                    |
|--------------------------|------------------------------------|
| <code>e(N)</code>        | number of observations             |
| <code>e(N_strata)</code> | number of covariate strata         |
| <code>e(N_clust)</code>  | number of clusters                 |
| <code>e(level)</code>    | confidence level for bootstrap CIs |
| <code>e(rank)</code>     | rank of $e(V)$                     |

#### Macros

|                              |                                                                             |
|------------------------------|-----------------------------------------------------------------------------|
| <code>e(cmd)</code>          | <code>rocreg</code>                                                         |
| <code>e(cmdline)</code>      | command as typed                                                            |
| <code>e(classvars)</code>    | classification variable list                                                |
| <code>e(refvar)</code>       | status variable, reference variable                                         |
| <code>e(ctrlmodel)</code>    | covariate-adjustment specification                                          |
| <code>e(ctrlcov)</code>      | covariate-adjustment variables                                              |
| <code>e(pvc)</code>          | percentile value calculation method                                         |
| <code>e(title)</code>        | title in estimation output                                                  |
| <code>e(tiecorrected)</code> | <code>tiecorrected</code> , if specified                                    |
| <code>e(probit)</code>       | <code>probit</code> , if specified                                          |
| <code>e(roccov)</code>       | ROC covariates                                                              |
| <code>e(fprpts)</code>       | number of points used as false-positive rate fit points                     |
| <code>e(ctrlfprall)</code>   | indicates whether all observed false-positive rates were used as fit points |
| <code>e(nobootstrap)</code>  | <code>nobootstrap</code> , if specified                                     |
| <code>e(rngstate)</code>     | random-number state used in bootstrap                                       |
| <code>e(breps)</code>        | number of bootstrap resamples                                               |
| <code>e(bootcc)</code>       | <code>bootcc</code> , if specified                                          |
| <code>e(nobstrata)</code>    | <code>nobstrata</code> , if specified                                       |
| <code>e(clustvar)</code>     | name of cluster variable                                                    |
| <code>e(exp#)</code>         | expression for the $\#$ th statistic                                        |
| <code>e(vce)</code>          | bootstrap                                                                   |
| <code>e(properties)</code>   | $b$ $V$ (or $b$ if <code>nobootstrap</code> is specified)                   |
| <code>e(predict)</code>      | program used to implement <code>predict</code>                              |

#### Matrices

|                               |                                              |
|-------------------------------|----------------------------------------------|
| <code>e(b)</code>             | coefficient vector                           |
| <code>e(V)</code>             | variance-covariance matrix of the estimators |
| <code>e(b_bs)</code>          | bootstrap estimates                          |
| <code>e(reps)</code>          | number of nonmissing results                 |
| <code>e(bias)</code>          | estimated biases                             |
| <code>e(se)</code>            | estimated standard errors                    |
| <code>e(z0)</code>            | median biases                                |
| <code>e(ci_normal)</code>     | normal-approximation confidence intervals    |
| <code>e(ci_percentile)</code> | percentile confidence intervals              |
| <code>e(ci_bc)</code>         | bias-corrected confidence intervals          |

#### Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

#### Matrices

|                       |                                                                                                                       |
|-----------------------|-----------------------------------------------------------------------------------------------------------------------|
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, $p$ -values, and confidence intervals |
|-----------------------|-----------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

Parametric, maximum likelihood `rocreg` stores the following in `e()`:

Scalars

|                         |                        |
|-------------------------|------------------------|
| <code>e(N)</code>       | number of observations |
| <code>e(N_clust)</code> | number of clusters     |
| <code>e(rank)</code>    | rank of $e(V)$         |

Macros

|                           |                                                   |
|---------------------------|---------------------------------------------------|
| <code>e(cmd)</code>       | <code>rocreg</code>                               |
| <code>e(cmdline)</code>   | command as typed                                  |
| <code>e(classvars)</code> | classification variable list                      |
| <code>e(refvar)</code>    | status variable                                   |
| <code>e(ctrlmodel)</code> | linear                                            |
| <code>e(ctrlcov)</code>   | control population covariates                     |
| <code>e(roccov)</code>    | ROC covariates                                    |
| <code>e(probit)</code>    | probit, if specified                              |
| <code>e(pvc)</code>       | normal                                            |
| <code>e(wtype)</code>     | weight type                                       |
| <code>e(wexp)</code>      | weight expression                                 |
| <code>e(title)</code>     | title in estimation output                        |
| <code>e(clustvar)</code>  | name of cluster variable                          |
| <code>e(vce)</code>       | cluster if clustering used                        |
| <code>e(vctype)</code>    | robust if multiple classifiers or clustering used |
| <code>e(ml)</code>        | ml, if specified                                  |
| <code>e(predict)</code>   | program used to implement <code>predict</code>    |

Matrices

|                   |                                              |
|-------------------|----------------------------------------------|
| <code>e(b)</code> | coefficient vector                           |
| <code>e(V)</code> | variance–covariance matrix of the estimators |

Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

Matrices

|                       |                                                                                                                       |
|-----------------------|-----------------------------------------------------------------------------------------------------------------------|
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, $p$ -values, and confidence intervals |
|-----------------------|-----------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

Assume that we applied a diagnostic test to each of  $N_0$  control and  $N_1$  case subjects. Further assume that the higher the outcome value of the diagnostic test, the higher the risk of the subject being abnormal. Let  $y_{1i}$ ,  $i = 1, 2, \dots, N_1$ , and  $y_{0j}$ ,  $j = 1, 2, \dots, N_0$ , be the values of the diagnostic test for the case and control subjects, respectively. The true status variable  $D$  identifies an observation as case  $D = 1$  or control  $D = 0$ . The CDF of the classifier  $Y$  is  $F$ . Conditional on  $D$ , we write the CDF as  $F_D$ .

Methods and formulas are presented under the following headings:

- ROC statistics*
- Covariate-adjusted ROC curves*
- Parametric ROC curves: Estimating equations*
- Parametric ROC curves: Maximum likelihood*

## ROC statistics

We obtain these definitions and their estimates from [Pepe \(2003\)](#) and [Pepe, Longton, and Janes \(2009\)](#). The false-positive and true-positive rates at cutoff  $c$  are defined as

$$\text{FPR}(y) = P(Y \geq y | D = 0)$$

$$\text{TPR}(y) = P(Y \geq y | D = 1)$$

The true-positive rate, or ROC value at false-positive rate  $u$ , is given by

$$\text{ROC}(u) = P(1 - F_0(Y) \leq u | D = 1)$$

When  $Y$  is continuous, the false-positive rate can be written as

$$\text{FPR}(y) = 1 - F_0(y)$$

The empirical CDF for the sample  $z_1, \dots, z_n$  is given by

$$\hat{F}(z) = \sum_{i=1}^n \frac{I(z < z_i)}{n}$$

The empirical estimates  $\widehat{\text{FPR}}$  and  $\widehat{\text{ROC}}$  both use this empirical CDF estimator.

The area under the ROC curve is defined as

$$\text{AUC} = \int_0^1 \text{ROC}(u) d_u$$

The partial area under the ROC curve for false-positive rate  $a$  is defined as

$$\text{pAUC}(a) = \int_0^a \text{ROC}(u) d_u$$

The nonparametric estimate for the AUC is given by

$$\widehat{\text{AUC}} = \sum_{i=1}^{N_1} \frac{1 - \widehat{\text{FPR}}(y_{1i})}{N_1}$$

The nonparametric estimate of pAUC is given by

$$\widehat{\text{pAUC}}(a) = \sum_{i=1}^{N_1} \frac{\max\{1 - \widehat{\text{FPR}}(y_{1i}) - (1 - a), 0\}}{N_1}$$

For discrete classifiers, a correction term is subtracted from the false-positive rate estimate so that the  $\widehat{\text{AUC}}$  and  $\widehat{\text{pAUC}}$  estimates correspond with a trapezoidal approximation to the area of the ROC curve.

$$\text{FPR}^c(y) = 1 - \hat{F}_0(y) - \frac{1}{2} \sum_{j=1}^{N_0} \frac{I(y = y_{0j})}{N_0}$$

In the nonparametric estimation of the ROC curve, all inference is performed using the bootstrap command (see [R] [bootstrap](#)). `rocreg` also allows users to calculate the ROC curve and related statistics by assuming a normal control distribution. So these formulas are updated by replacing  $F_0$  by  $\Phi$  (with adjustment of the marginal mean and variance of the control distribution).

## Covariate-adjusted ROC curves

Suppose we observe covariate vector  $Z$  in addition to the classifier  $Y$ . Let  $Z_{1i}, i = 1, 2, \dots, N_1$ , and  $Z_{0j}, j = 1, 2, \dots, N_0$ , be the values of the covariates for the case and control subjects, respectively.

The covariate-adjusted ROC curve is defined by [Janes and Pepe \(2009\)](#) as

$$\text{AROC}(t) = E\{\text{ROC}(t|Z_0)\}$$

It is calculated by replacing the marginal control CDF estimate,  $\hat{F}_0$ , with the conditional control CDF estimate,  $\hat{F}_{0Z}$ . If we used a normal control CDF, then we would replace the marginal control mean and variance with the conditional control mean and variance. The formulas of the previous section can be updated for covariate-adjustment by making this substitution of the conditional CDF for the marginal CDF in the false-positive rate calculation.

Because the calculation of the ROC value is now performed based on the conditionally calculated false-positive rate, no further conditioning is made in its calculation under nonparametric estimation.

`rocreg` supports covariate adjustment with stratification and linear regression. Under stratification, separate parameters are estimated for the control distribution at each level of the covariates. Under linear regression, the classifier is regressed on the covariates over the control distribution, and the resulting coefficients serve as parameters for  $\hat{F}_{0Z}$ .

## Parametric ROC curves: Estimating equations

Under nonparametric estimation of the ROC curve with covariate adjustment, no further conditioning occurs in the ROC curve calculation beyond the use of covariate-adjusted false-positive rates as inputs.

Under parametric estimation of the ROC curve, we can relax this restriction. We model the ROC curve as a cumulative distribution function  $g$  (standard normal  $\Phi$ ) invoked with input of a linear polynomial in the corresponding quantile function (here  $\Phi^{-1}$ ) invoked on the false-positive rate  $u$ . The constant intercept of the polynomial may depend on covariates; the slope term  $\alpha$  (quantile coefficient) may not.

$$\text{ROC}(u) = g\{\mathbf{x}'\boldsymbol{\beta} + \alpha g^{-1}(u)\}$$

[Pepe \(2003\)](#) notes that having a binormal ROC ( $g = \Phi$ ) is equivalent to specifying that some monotone transformation of the data exists to make the case and control classifiers normally distributed. This specification applies to the marginal case and control.

Under weak assumptions about the control distribution of the classifier, we can fit this model by using estimating equations ([Alonzo and Pepe 2002](#)). The method can be used without covariate effects in the second stage, assuming a parametric model for the single ROC curve. Using the [Alonzo and Pepe \(2002\)](#) method, the covariate-adjusted ROC curve may be fit parametrically. The marginal ROC curve, involving no covariates in either stage of estimation, can be fit parametrically as well. In addition to the [Alonzo and Pepe \(2002\)](#) explanation, further details are given in [Pepe, Longton, and Janes \(2009\)](#); [Janes, Longton, and Pepe \(2009\)](#); [Pepe \(2003\)](#); and [Janes and Pepe \(2009\)](#).

The algorithm can be described as follows:

1. Estimate the false-positive rates of the classifier  $\text{fpr}$ . These may be computed in any fashion outlined so far: covariate-adjusted, empirically, etc.
2. Determine a set of  $n_p$  false-positive rates to use as fitting points  $f_1, \dots, f_{n_p}$ . These may be an equispaced grid on  $(0, 1)$  or the set of observed false-positive rates from part 1.
3. Expand the case observation portion of the data to include a subobservation for each fitting point. So there are now  $N_1(n_p - 1)$  additional observations in the data.
4. Generate a new dummy variable  $u$ . For subobservation  $j$ ,  $u = I(\text{fpr} \leq f_j)$ .
5. Generate a new variable  $\text{quant}$  containing the quantiles of the false-positive rate fitting points. For subobservation  $j$ ,  $\text{quant} = g^{-1}(f_j)$ .
6. Perform a binary regression (probit,  $g = \Phi$ ) of  $\text{fpr}$  on the covariates  $\mathbf{x}$  and quantile variable  $\text{quant}$ .

The coefficients of part 6 are the coefficients of the ROC model. The coefficients of the covariates coincide naturally with estimates of  $\beta$ , and the  $\alpha$  parameter is estimated by the coefficient on  $\text{quant}$ . Because the method is so general and makes few distributional assumptions, bootstrapping must be performed for inference. If multiple classifiers are to be fit, the algorithm is performed separately for each in each bootstrap, and the bootstrap is used to estimate covariances.

We mentioned earlier that in parametric estimation, the AUC was the only summary parameter that could be estimated initially. This is true when we fit the marginal probit model because there are no covariates in part 6 of the algorithm.

To calculate the AUC statistic under a marginal probit model, we use the formula

$$\text{AUC} = \Phi\left(\frac{\beta_0}{\sqrt{1 + \alpha^2}}\right)$$

Alternatively, the AUC for the probit model can be calculated as  $\text{pAUC}(1)$  in postestimation. Under both models, bootstrapping is performed for inference on the AUC.

## Parametric ROC curves: Maximum likelihood

`rocreg` supports another form of parametric ROC estimation: maximum likelihood with a normally distributed classifier. This method assumes that the classifier is a normal linear model on certain covariates, and the covariate effect and variance of the classifier may change between the case and control populations. The model is defined in [Pepe \(2003, 145\)](#).

$$y = \mathbf{z}'\beta_0 + D\mathbf{x}'\beta_1 + \sigma(D)\epsilon$$

Our error term,  $\epsilon$ , is a standard normal random variable. The variable  $D$  is our true status variable, being 1 for the case population observations and 0 for the control population observations. The variance function  $\sigma$  is defined as

$$\sigma(D) = \sigma_0(D = 0) + \sigma_1(D = 1)$$

This provides two variance parameters in the model and does not depend on covariate values.

Under this model, the ROC curve is easily derived to be

$$\text{ROC}(u) = \Phi \left[ \frac{1}{\sigma_1} \{ \mathbf{x}'\boldsymbol{\beta}_1 + \sigma_0 \Phi^{-1}(u) \} \right]$$

We reparameterize the model, creating the parameters  $\beta_i = \sigma_1^{-1} \beta_{1i}$  and  $\alpha = \sigma_1^{-1} \sigma_0$ . We refer to  $\beta_0$  as the constant intercept, `i_cons`. The parameter  $\alpha$  is referred to as the constant slope, `s_cons`.

$$\text{ROC}(u) = \Phi \{ \mathbf{x}'\boldsymbol{\beta} + \alpha \Phi^{-1}(u) \}$$

The original model defining the classifier  $y$  leads to the following single observation likelihoods for  $D = 0$  and  $D = 1$ :

$$L(\boldsymbol{\beta}_0, \boldsymbol{\beta}_1, \sigma_1, \sigma_0, |D = 0, y, \mathbf{z}, \mathbf{x}) = \frac{1}{\sqrt{2\pi}\sigma_0} \exp \frac{-(y - \mathbf{z}'\boldsymbol{\beta}_0)^2}{2\sigma_0^2}$$

$$L(\boldsymbol{\beta}_0, \boldsymbol{\beta}_1, \sigma_1, \sigma_0, |D = 1, y, \mathbf{z}, \mathbf{x}) = \frac{1}{\sqrt{2\pi}\sigma_1} \exp \frac{-(y - \mathbf{z}'\boldsymbol{\beta}_0 - \mathbf{x}'\boldsymbol{\beta}_1)^2}{2\sigma_1^2}$$

These can be combined to yield the observation-level log likelihood:

$$\begin{aligned} \ln L(\boldsymbol{\beta}_0, \boldsymbol{\beta}_1, \sigma_1, \sigma_0, |D, y, \mathbf{z}, \mathbf{x}) &= -\frac{\ln 2\pi}{2} \\ &\quad - I(D = 0) \left\{ \ln \sigma_0 + \frac{(y - \mathbf{z}'\boldsymbol{\beta}_0)^2}{2\sigma_0^2} \right\} \\ &\quad - I(D = 1) \left\{ \ln \sigma_1 + \frac{(y - \mathbf{z}'\boldsymbol{\beta}_0 - \mathbf{x}'\boldsymbol{\beta}_1)^2}{2\sigma_1^2} \right\} \end{aligned}$$

When there are multiple classifiers, each classifier is fit separately with maximum likelihood. Then, the results are combined by stacking the scores and using the sandwich variance estimator. For more information, see [R] `suest` and the references White (1982); Rogers (1993); and White (1996).

## Acknowledgments

We thank Margaret S. Pepe, Holly Janes, and Gary Longton (retired) of the Fred Hutchinson Cancer Research Center for providing the inspiration for the `rocreg` command and for illuminating many useful datasets for its documentation.

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## Also see

[R] **rocreg postestimation** — Postestimation tools for rocreg

[R] **rocregplot** — Plot marginal and covariate-specific ROC curves after rocreg

[R] **rocf** — Parametric ROC models

[R] **roc** — Receiver operating characteristic (ROC) analysis

[U] **20 Estimation and postestimation commands**

|                         |                                |                                      |
|-------------------------|--------------------------------|--------------------------------------|
| Postestimation commands | <a href="#">predict</a>        | <a href="#">estat</a>                |
| Remarks and examples    | <a href="#">Stored results</a> | <a href="#">Methods and formulas</a> |
| References              | <a href="#">Also see</a>       |                                      |

## Postestimation commands

The following commands are of special interest after rocreg:

| Command                     | Description                                                             |
|-----------------------------|-------------------------------------------------------------------------|
| <a href="#">estat nproc</a> | nonparametric ROC curve estimation, keeping fit information from rocreg |
| <a href="#">rocregplot</a>  | plot marginal and covariate-specific ROC curves                         |

The following standard postestimation commands are also available:

| Command                   | Description                                                                                       |
|---------------------------|---------------------------------------------------------------------------------------------------|
| <a href="#">estimates</a> | cataloging estimation results                                                                     |
| <a href="#">etable</a>    | table of estimation results                                                                       |
| <a href="#">lincom</a>    | point estimates, standard errors, testing, and inference for linear combinations of parameters    |
| <a href="#">nlcom</a>     | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters |
| <a href="#">predict</a>   | roc curve predictions                                                                             |
| <a href="#">test</a>      | Wald tests of simple and composite linear hypotheses                                              |
| <a href="#">testnl</a>    | Wald tests of nonlinear hypotheses                                                                |

# predict

## Description for predict

Use of `predict` after fitting a parametric model with `rocreg` allows calculation of all the ROC curve summary statistics for covariate-specific ROC curves. The ROC values for given false-positive rates, false-positive rate for given ROC values, area under the ROC curve (AUC), and partial areas under the ROC curve (pAUC) for a given false-positive rate can all be calculated.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic options]
```

| <i>statistic</i>                         | Description                                                                          |
|------------------------------------------|--------------------------------------------------------------------------------------|
| Main                                     |                                                                                      |
| <code>at</code> ( <i>varname</i> )       | input variable for statistic                                                         |
| <code>auc</code>                         | total area under the ROC curve; the default                                          |
| <code>roc</code>                         | ROC values for given false-positive rates in <code>at()</code>                       |
| <code>invroc</code>                      | false-positive rate for given ROC values in <code>at()</code>                        |
| <code>pauc</code>                        | partial area under the ROC curve up to each false-positive rate in <code>at()</code> |
| <code>classvar</code> ( <i>varname</i> ) | statistic for given classifier                                                       |

| <i>options</i>                                           | Description                                                                                                                                        |
|----------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------|
| Options                                                  |                                                                                                                                                    |
| <code>intpts</code> (#)                                  | points in numeric integration of pAUC calculation                                                                                                  |
| <code>se</code> ( <i>newvar</i> )                        | predict standard errors                                                                                                                            |
| <code>ci</code> ( <i>stubname</i> )                      | produce confidence intervals, stored as variables with prefix <i>stubname</i> and suffixes <code>_l</code> and <code>_u</code>                     |
| <code>level</code> (#)                                   | set confidence level; default is <code>level(95)</code>                                                                                            |
| * <code>bfile</code> ( <i>filename</i> , ...)            | load dataset containing bootstrap replicates from <code>rocreg</code>                                                                              |
| * <code>btype</code> ( <i>n</i>   <i>p</i>   <i>bc</i> ) | produce normal-based ( <i>n</i> ), percentile ( <i>p</i> ), or bias-corrected ( <i>bc</i> ) confidence intervals; default is <code>btype(n)</code> |

\*`bfile()` and `btype()` are only allowed with parametric analysis using bootstrap inference.

## Options for predict

### Main

`at(varname)` records the variable to be used as input for the above predictions.

`auc` predicts the total area under the ROC curve defined by the covariate values in the data. This is the default statistic.

`roc` predicts the ROC values for false-positive rates stored in *varname* specified in `at()`.

`invroc` predicts the false-positive rates for given ROC values stored in *varname* specified in `at()`.

`pauc` predicts the partial area under the ROC curve up to each false-positive rate stored in *varname* specified in `at()`.

`classvar(varname)` performs the prediction for the specified classifier.

### Options

`intpts(#)` specifies that # points be used in the pAUC calculation.

`se(newvar)` specifies that standard errors be produced and stored in *newvar*.

`ci(stubname)` requests that confidence intervals be produced and the lower and upper bounds be stored in *stubname\_l* and *stubname\_u*, respectively.

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [\[U\] 20.8 Specifying the width of confidence intervals](#).

`bfile(filename, ...)` uses bootstrap replicates of parameters from `rocreg` stored in *filename* to estimate standard errors and confidence intervals of predictions.

`btype(n | p | bc)` specifies whether to produce normal-based (*n*), percentile (*p*), or bias-corrected (*bc*) confidence intervals. The default is `btype(n)`.

## estat

### Description for estat

`estat nproc` allows calculation of all the ROC curve summary statistics for covariate-specific ROC curves, as well as for a nonparametric ROC estimation. Under nonparametric estimation, a single ROC curve is estimated by `rocreg`. Covariates can affect this estimation, but there are no separate covariate-specific ROC curves. Thus, the input arguments for `estat nproc` are taken in the command line rather than from the data as variable values.

### Menu for estat

Statistics > Postestimation

### Syntax for estat nproc

```
estat nproc [, estat_nproc_options]
```

| <i>estat_nproc_options</i>          | Description                                                              |
|-------------------------------------|--------------------------------------------------------------------------|
| Main                                |                                                                          |
| <code>auc</code>                    | estimate total area under the ROC curve                                  |
| <code>roc(<i>numlist</i>)</code>    | estimate ROC values for given false-positive rates                       |
| <code>invroc(<i>numlist</i>)</code> | estimate false-positive rate for given ROC values                        |
| <code>pauc(<i>numlist</i>)</code>   | estimate partial area under the ROC curve up to each false-positive rate |

At least one option must be specified.

### Options for estat nproc

#### Main

`auc` estimates the total area under the ROC curve.

`roc(numlist)` estimates the ROC for each of the false-positive rates in *numlist*. The values in *numlist* must be in the range (0,1).

`invroc(numlist)` estimates the false-positive rate for each of the ROC values in *numlist*. The values in *numlist* must be in the range (0,1).

`pauc(numlist)` estimates the partial area under the ROC curve up to each false-positive rate in *numlist*. The values in *numlist* must be in the range (0,1].

### Remarks and examples

Remarks are presented under the following headings:

*Using predict after rocreg*  
*Using estat nproc*

## Using predict after rocreg

`predict`, after parametric `rocreg`, predicts the AUC, the ROC value, the false-positive rate (invROC), or the pAUC value. The default is `auc`.

We begin by estimating the area under the ROC curve for each of the three age-specific ROC curves in [example 1](#) of [R] `rocregplot`: 30, 40, and 50 months.

### ▷ Example 1: Parametric ROC, AUC

In [example 6](#) of [R] `rocreg`, a probit ROC model was fit to audiology test data from [Norton et al. \(2000\)](#). The estimating equations method of [Alonzo and Pepe \(2002\)](#) was used to fit the model. Gender and age were covariates that affected the control distribution of the classifier `y1` (DPOAE 65 at 2 kHz). Age was a ROC covariate for the model, so we fit separate ROC curves at each age.

Following [Janes, Longton, and Pepe \(2009\)](#), we drew the ROC curves for ages 30, 40, and 50 months in [example 1](#) of [R] `rocregplot`. Now, we use `predict` to estimate the AUC for the ROC curve at each of those ages.

The bootstrap dataset saved by `rocreg` in [example 6](#) of [R] `rocreg`, `nnhs2y1.dta`, is used in the `bfile()` option.

We will store the AUC prediction in the new variable `predAUC`. We specify the `se()` option with the new variable name `seAUC` to produce an estimate of the prediction's standard error. By specifying the stubname `cin` in `ci()`, we tell `predict` to create normal-based confidence intervals (the default) as new variables `cin_l` and `cin_u`.

```
. use https://www.stata-press.com/data/r19/nnhs
(Norton - neonatal audiology data)
. rocreg d y1, probit ctrlcov(currage male) ctrlmodel(linear) roccov(currage)
> cluster(id) bseed(56930) bsave(nnhs2y1)
(output omitted)
. set obs 5061
Number of observations (_N) was 5,058, now 5,061.
. quietly replace currage = 30 in 5059
. quietly replace currage = 40 in 5060
. quietly replace currage = 50 in 5061
. predict predAUC in 5059/5061, auc se(seAUC) ci(cin) bfile(nnhs2y1)
. list currage predAUC seAUC cin* in 5059/5061
```

|       | currage | predAUC  | seAUC    | cin_l    | cin_u    |
|-------|---------|----------|----------|----------|----------|
| 5059. | 30      | .5209999 | .076013  | .3720171 | .6699827 |
| 5060. | 40      | .6479176 | .0284218 | .592212  | .7036232 |
| 5061. | 50      | .7601378 | .0794346 | .6044489 | .9158267 |

As expected, we find the AUC to increase with age.

Essentially, we have a stored bootstrap sample of ROC covariate coefficient estimates in `nnhs2y1.dta`. We calculate the AUC using each set of coefficient estimates, resulting in a sample of AUC estimates. Then, the bootstrap standard error and confidence intervals are calculated based on this AUC sample. Further details of the computation of the standard error and percentile confidence intervals can be found in [Methods and formulas](#) and in [R] `bootstrap`.

We can also produce percentile or bias-corrected confidence intervals by specifying `btype(p)` or `btype(bc)`, which we now demonstrate.

```
. drop *AUC*
. predict predAUC in 5059/5061, auc se(seAUC) ci(cip) bfile(nnhs2y1) btype(p)
. list currage predAUC cip* in 5059/5061
```

|       | currage | predAUC  | cip_l    | cip_u    |
|-------|---------|----------|----------|----------|
| 5059. | 30      | .5209999 | .3625608 | .6615624 |
| 5060. | 40      | .6479176 | .5874361 | .702619  |
| 5061. | 50      | .7601378 | .5836248 | .8910214 |

```
. drop *AUC*
. predict predAUC in 5059/5061, auc se(seAUC) ci(cibc) bfile(nnhs2y1) btype(bc)
. list currage predAUC cibc* in 5059/5061
```

|       | currage | predAUC  | cibc_l   | cibc_u   |
|-------|---------|----------|----------|----------|
| 5059. | 30      | .5209999 | .3733315 | .6669934 |
| 5060. | 40      | .6479176 | .5901812 | .7038136 |
| 5061. | 50      | .7601378 | .5738162 | .8882028 |

◀

`predict` can also estimate the ROC value and the false-positive rate (`invROC`).

### ▷ Example 2: Parametric ROC, `invROC`, and ROC value

In [example 7](#) of [\[R\] rocreg](#), we fit the ROC curve for status variable hearing loss (`d`) and classifier negative signal-to-noise ratio `nsnr` with ROC covariates frequency (`xf`), intensity (`x1`), and hearing loss severity (`xd`). The data were obtained from [Stover et al. \(1996\)](#). The model fit was probit with bootstrap resampling. We saved 50 bootstrap replications in the dataset `nsnrf.dta`.

The covariate value combinations `xf = 10.01`, `x1 = 5.5`, and `xd = .5`, and `xf = 10.01`, `x1 = 6.5`, and `xd = 4` are of interest. In [example 3](#) of [\[R\] rocregplot](#), we estimated the ROC values for false-positive rates 0.2 and 0.7 and the false-positive rate for a ROC value of 0.5 by using `rocregplot`. We will use `predict` to replicate the estimation.

We begin by appending observations with our desired covariate combinations to the data. We also create two new variables: `rocinp`, which contains the ROC values for which we wish to predict the corresponding `invROC` values, and `invrocinp`, which contains the `invROC` values corresponding to the ROC values we wish to predict.

```

. clear
. input xf xl xd rocinp invrocinp
 xf xl xd rocinp invrocinp
1. 10.01 5.5 .5 .2 .
2. 10.01 6.5 4 .2 .
3. 10.01 5.5 .5 .7 .5
4. 10.01 6.5 4 .7 .5
5. end
. save newdata
file newdata.dta saved
. use https://www.stata-press.com/data/r19/dp
(Stover - DPOAE test data)
. quietly rocreg d nsnr, ctrlcov(xf xl) roccov(xf xl xd) probit cluster(id)
> nobstrata ctrlfprall bseed(156385) breps(50) ctrlmodel(strata) bsave(nsnrf)
. append using newdata
. list xf xl xd invrocinp rocinp in 1849/1852

```

|       | xf    | xl  | xd | invroc~p | rocinp |
|-------|-------|-----|----|----------|--------|
| 1849. | 10.01 | 5.5 | .5 | .        | .2     |
| 1850. | 10.01 | 6.5 | 4  | .        | .2     |
| 1851. | 10.01 | 5.5 | .5 | .5       | .7     |
| 1852. | 10.01 | 6.5 | 4  | .5       | .7     |

Now, we will use `predict` to estimate the ROC value for the false-positive rates stored in `rocinp`. We specify the `roc` option, and we specify `rocinp` in the `at()` option. The other options, `se()` and `ci()`, are used to obtain standard errors and confidence intervals, respectively. The dataset of bootstrap samples, `nsnrf.dta`, is specified in `bfile()`. After prediction, we list the point estimates and standard errors.

```

. predict rocit in 1849/1852, roc at(rocinp) se(seroc) ci(cin) bfile(nsnrf)
. list xf xl xd rocinp rocit seroc if !missing(rocit)

```

|       | xf    | xl  | xd | rocinp | rocit    | seroc    |
|-------|-------|-----|----|--------|----------|----------|
| 1849. | 10.01 | 5.5 | .5 | .2     | .7652956 | .0624187 |
| 1850. | 10.01 | 6.5 | 4  | .2     | .9672505 | .0162785 |
| 1851. | 10.01 | 5.5 | .5 | .7     | .9835816 | .0133583 |
| 1852. | 10.01 | 6.5 | 4  | .7     | .999428  | .0007784 |

These results match [example 3](#) of [\[R\] rocregplot](#). We list the confidence intervals next. These also conform to the `rocregplot` results from [example 3](#) in [\[R\] rocregplot](#). We begin with the confidence intervals for ROC under the covariate values `xf=10.01`, `xl=5.5`, and `xd=.5`.

```

. list xf xl xd rocinp rocit cin* if inlist(_n, 1849, 1851)

```

|       | xf    | xl  | xd | rocinp | rocit    | cin_l    | cin_u    |
|-------|-------|-----|----|--------|----------|----------|----------|
| 1849. | 10.01 | 5.5 | .5 | .2     | .7652956 | .6429572 | .887634  |
| 1851. | 10.01 | 5.5 | .5 | .7     | .9835816 | .9573998 | 1.009763 |

Now, we list the ROC confidence intervals under the covariate values  $xf=10.01$ ,  $x1=6.5$ , and  $xd=4$ .

```
. list xf x1 xd rocinp rocit cin* if inlist(_n, 1850, 1852)
```

|       | xf    | x1  | xd | rocinp | rocit    | cin_l    | cin_u    |
|-------|-------|-----|----|--------|----------|----------|----------|
| 1850. | 10.01 | 6.5 | 4  | .2     | .9672505 | .9353452 | .9991558 |
| 1852. | 10.01 | 6.5 | 4  | .7     | .999428  | .9979024 | 1.000954 |

Now, we will predict the false-positive rate for a ROC value by specifying the `invroc` option. We pass the `invrocinp` variable as an argument to the `at()` option. Again, we list the point estimates and standard errors first.

```
. drop ci*
. predict invrocit in 1849/1852, invroc at(invrocinp) se(serocinv) ci(cin)
> bfile(nsnrf)
. list xf x1 xd invrocinp invrocit serocinv if !missing(invrocit)
```

|       | xf    | x1  | xd | invroc-p | invrocit | serocinv |
|-------|-------|-----|----|----------|----------|----------|
| 1851. | 10.01 | 5.5 | .5 | .5       | .0615144 | .0209516 |
| 1852. | 10.01 | 6.5 | 4  | .5       | .0043298 | .003835  |

These also match those of [example 3](#) of [R] [rocregplot](#). Listing the confidence intervals shows identical results as well. First, we list the confidence intervals under the covariate values  $xf=10.01$ ,  $x1=5.5$ , and  $xd=.5$ .

```
. list xf x1 xd invrocinp invrocit cin* in 1851
```

|       | xf    | x1  | xd | invroc-p | invrocit | cin_l    | cin_u    |
|-------|-------|-----|----|----------|----------|----------|----------|
| 1851. | 10.01 | 5.5 | .5 | .5       | .0615144 | .0204499 | .1025789 |

Now, we list the confidence intervals for false-positive rate under the covariate values  $xf=10.01$ ,  $x1=6.5$ , and  $xd=4$ .

```
. list xf x1 xd invrocinp invrocit cin* in 1852
```

|       | xf    | x1  | xd | invroc-p | invrocit | cin_l     | cin_u    |
|-------|-------|-----|----|----------|----------|-----------|----------|
| 1852. | 10.01 | 6.5 | 4  | .5       | .0043298 | -.0031867 | .0118463 |



The `predict` command can also be used after a maximum-likelihood ROC model is fit.

### ► Example 3: Maximum likelihood ROC, invROC, and ROC value

In the [previous example](#), we revisited the estimating equations fit of a probit model with ROC covariates frequency (xf), intensity (xl), and hearing loss severity (xd) to the [Stover et al. \(1996\)](#) audiology study data. A maximum likelihood fit of the same model was performed in [example 10](#) of [\[R\] rocreg](#). In [example 2](#) of [\[R\] rocregplot](#), we used `rocregplot` to estimate ROC values and false-positive rates for this model under two covariate configurations. We will use `predict` to obtain the same estimates. We will also estimate the partial area under the ROC curve.

We append the data as in the [previous example](#). This leads to the following four final observations in the data.

```
. use https://www.stata-press.com/data/r19/dp, clear
(Stover - DPOAE test data)
. rocreg d nsnr, probit ctrlcov(xf xl) roccov(xf xl xd) ml cluster(id)
(output omitted)
. append using newdata
. list xf xl xd invrocinp rocinp in 1849/1852
```

|       | xf    | xl  | xd | invroc~p | rocinp |
|-------|-------|-----|----|----------|--------|
| 1849. | 10.01 | 5.5 | .5 | .        | .2     |
| 1850. | 10.01 | 6.5 | 4  | .        | .2     |
| 1851. | 10.01 | 5.5 | .5 | .5       | .7     |
| 1852. | 10.01 | 6.5 | 4  | .5       | .7     |

Now, we predict the ROC value for false-positive rates of 0.2 and 0.7. Under maximum likelihood prediction, only Wald-type confidence intervals are produced. We specify a new variable name for the standard error in the `se()` option and a stubname for the confidence interval variables in the `ci()` option.

```
. predict rocit in 1849/1852, roc at(rocinp) se(seroc) ci(ci)
. list xf xl xd rocinp rocit seroc ci_l ci_u if !missing(rocit), noobs
```

|  | xf    | xl  | xd | rocinp | rocit    | seroc    | ci_l     | ci_u     |
|--|-------|-----|----|--------|----------|----------|----------|----------|
|  | 10.01 | 5.5 | .5 | .2     | .7608593 | .0510501 | .660803  | .8609157 |
|  | 10.01 | 6.5 | 4  | .2     | .9499408 | .0179824 | .914696  | .9851856 |
|  | 10.01 | 5.5 | .5 | .7     | .978951  | .0097382 | .9598644 | .9980376 |
|  | 10.01 | 6.5 | 4  | .7     | .9985001 | .0009657 | .9966073 | 1.000393 |

These results match our estimates in [example 2](#) of [\[R\] rocregplot](#). We also match [example 2](#) of [\[R\] rocregplot](#) when we estimate the false-positive rate for a ROC value of 0.5.

```
. drop ci*
. predict invrocit in 1851/1852, invroc at(invrocinp) se(serocinv) ci(ci)
. list xf xl xd invrocinp invrocit serocinv ci_l ci_u if !missing(invrocit),
> noobs
```

|  | xf    | xl  | xd | invroc~p | invrocit | serocinv | ci_l      | ci_u     |
|--|-------|-----|----|----------|----------|----------|-----------|----------|
|  | 10.01 | 5.5 | .5 | .5       | .0578036 | .0198626 | .0188736  | .0967336 |
|  | 10.01 | 6.5 | 4  | .5       | .0055624 | .0032645 | -.0008359 | .0119607 |

### ► Example 4: Maximum likelihood ROC, pAUC, and ROC value

In [example 13](#) of [R] [rocreg](#), we fit a maximum-likelihood marginal probit model to each classifier of the fictitious dataset generated from [Hanley and McNeil \(1983\)](#). In [example 5](#) of [R] [rocregplot](#), `rocregplot` was used to draw the ROC for the `mod1` and `mod3` classifiers. Estimates of the ROC value and false-positive rate were also obtained with Wald-type confidence intervals.

We return to this example, this time using `predict` to estimate the ROC value and false-positive rate. We will also estimate the pAUC for the false-positive rates of 0.3 and 0.8.

First, we add the input variables to the data. The variable `paucinp` will hold the 0.3 and 0.8 false-positive rates that we will input to `pAUC`. The variable `invrocinp` holds the ROC value of 0.8 for which we will estimate the false-positive rate. Finally, the variable `rocinp` holds the false-positive rates of 0.15 and 0.75 for which we will estimate the ROC value.

```
. use https://www.stata-press.com/data/r19/ct2, clear
(Reconstruction of CT images)
. rocreg status mod1 mod2 mod3, probit ml
 (output omitted)
. quietly generate paucinp = .3 in 111
. quietly replace paucinp = .8 in 112
. quietly generate invrocinp = .8 in 112
. quietly generate rocinp = .15 in 111
. quietly replace rocinp = .75 in 112
```

Then, we estimate the ROC value for false-positive rates 0.15 and 0.75 under classifier `mod1`. The point estimate is stored in `roc1`. Wald confidence intervals and standard errors are also estimated. We find that these results match those of [example 5](#) of [R] [rocregplot](#).

```
. predict roc1 in 111/112, classvar(mod1) roc at(rocinp) se(sr1) ci(cir1)
. list rocinp roc1 sr1 cir1* in 111/112
```

|      | rocinp | roc1     | sr1      | cir1_l   | cir1_u   |
|------|--------|----------|----------|----------|----------|
| 111. | .15    | .7934935 | .0801363 | .6364293 | .9505578 |
| 112. | .75    | .9931655 | .0069689 | .9795067 | 1.006824 |

Now, we perform the same estimation under the classifier `mod3`.

```
. predict roc3 in 111/112, classvar(mod3) roc at(roci) se(sr3) ci(cir3)
. list rocinp roc3 sr3 cir3* in 111/112
```

|      | rocinp | roc3     | sr3      | cir3_l   | cir3_u   |
|------|--------|----------|----------|----------|----------|
| 111. | .15    | .8888596 | .0520118 | .7869184 | .9908009 |
| 112. | .75    | .9953942 | .0043435 | .9868811 | 1.003907 |

Next, we estimate the false-positive rate for the ROC value of 0.8. These results also match [example 5](#) of [R] [rocregplot](#).

```
. predict invroc1 in 112, classvar(mod1) invroc at(invrocinp) se(sir1) ci(ciir1)
. list invrocinp invroc1 sir1 ciir1* in 112
```

|      | invroc~p | invroc1  | sir1    | ciir1_l  | ciir1_u |
|------|----------|----------|---------|----------|---------|
| 112. | .8       | .1556435 | .069699 | .0190361 | .292251 |

```
. predict invroc3 in 112, classvar(mod3) invroc at(invrocinp) se(sir3) ci(ciir3)
. list invrocinp invroc3 sir3 ciir3* in 112
```

|      | invroc~p | invroc3  | sir3    | ciir3_l   | ciir3_u  |
|------|----------|----------|---------|-----------|----------|
| 112. | .8       | .0661719 | .045316 | -.0226458 | .1549896 |

Finally, we estimate the pAUC for false-positive rates of 0.3 and 0.8. The point estimate is calculated by numeric integration. Wald confidence intervals are obtained with the delta method. Further details are presented in [Methods and formulas](#).

```
. predict pauc1 in 111/112, classvar(mod1) pauc at(paucinp) se(sp1) ci(cip1)
. list paucinp pauc1 sp1 cip1* in 111/112
```

|      | paucinp | pauc1    | sp1      | cip1_l   | cip1_u   |
|------|---------|----------|----------|----------|----------|
| 111. | .3      | .221409  | .0240351 | .174301  | .268517  |
| 112. | .8      | .7033338 | .0334766 | .6377209 | .7689466 |

```
. predict pauc3 in 111/112, classvar(mod3) pauc at(paucinp) se(sp3) ci(cip3)
. list paucinp pauc3 sp3 cip3* in 111/112
```

|      | paucinp | pauc3    | sp3      | cip3_l   | cip3_u   |
|------|---------|----------|----------|----------|----------|
| 111. | .3      | .2540215 | .0173474 | .2200213 | .2880217 |
| 112. | .8      | .7420408 | .0225192 | .6979041 | .7861776 |

◀

## Using estat nproc

When you initially use `rocreg` to fit a nonparametric ROC curve, you can obtain bootstrap estimates of a ROC value, false-positive rate, area under the ROC curve, and partial area under the ROC curve. The `estat nproc` command allows the user to estimate these parameters after `rocreg` has originally been used.

The seed and resampling settings used by `rocreg` are used by `estat nproc`. So the results for these new statistics are identical to what they would be if they had been initially estimated in the `rocreg` command. These new statistics, together with those previously estimated in `rocreg`, are returned in `r()`.

We demonstrate with an example.

▷ Example 5: Nonparametric ROC, invROC, and pAUC

In example 3 of [R] `rocreg`, we examined data from a pancreatic cancer study (Wieand et al. 1989). Two continuous classifiers, `y1` (CA 19-9) and `y2` (CA 125), were used for the true status variable `d`. In that example, we estimated various quantities including the false-positive rate for a ROC value of 0.6 and the pAUC for a false-positive rate of 0.5. Here we replicate that estimation with a call to `rocreg` to estimate the former and follow that with a call to `estat nproc` to estimate the latter. For simplicity, we restrict estimation to classifier `y1` (CA 19-9).

We start by executing `rocreg`, estimating the false-positive rate for a ROC value of 0.6. This value is specified in `invroc()`. Case-control resampling is used by specifying the `bootcc` option.

```
. use https://research.fredhutch.org/content/dam/stripe/diagnostic-biomarkers-
> statistical-center/files/wiedat2b.dta, clear
(S. Wieand - Pancreatic cancer diagnostic marker data)
. rocreg d y1, invroc(.6) bseed(8378923) bootcc nodots
```

Bootstrap results

```
Number of strata = 2 Number of obs = 141
 Replications = 1,000
```

Nonparametric ROC estimation

Control standardization: empirical

ROC method : empirical

False-positive rate

Status : d

Classifier: y1

| invROC | Observed<br>coefficient | Bias     | Bootstrap<br>std. err. | [95% conf. interval] |               |
|--------|-------------------------|----------|------------------------|----------------------|---------------|
| .6     | 0                       | .0149412 | .0255885               | -.0501525            | .0501525 (N)  |
|        |                         |          |                        | 0                    | .0784314 (P)  |
|        |                         |          |                        | 0                    | .1372549 (BC) |

Now, we will estimate the pAUC for the false-positive rate of 0.5 using estat nproc and the pauc() option.

```
. matrix list e(b)
symmetric e(b)[1,1]
 y1:
 invroc_1
y1 0
. estat nproc, pauc(.5)
Bootstrap results
Number of strata = 2 Number of obs = 141
 Replications = 1,000

Nonparametric ROC estimation
Control standardization: empirical
ROC method : empirical
False-positive rate
 Status : d
 Classifier: y1
```

| invROC | Observed coefficient | Bias     | Bootstrap std. err. | [95% conf. interval] |          |      |
|--------|----------------------|----------|---------------------|----------------------|----------|------|
| .6     | 0                    | .0149412 | .0255885            | -.0501525            | .0501525 | (N)  |
|        |                      |          |                     | 0                    | .0784314 | (P)  |
|        |                      |          |                     | 0                    | .1372549 | (BC) |

Partial area under the ROC curve

```
Status : d
Classifier: y1
```

| pAUC | Observed coefficient | Bias     | Bootstrap std. err. | [95% conf. interval] |          |      |
|------|----------------------|----------|---------------------|----------------------|----------|------|
| .5   | .3932462             | .0011971 | .0219031            | .3503169             | .4361755 | (N)  |
|      |                      |          |                     | .3489107             | .4338235 | (P)  |
|      |                      |          |                     | .3453159             | .4315904 | (BC) |

```

. matrix list r(b)
r(b) [1,2]
 y1: y1:
 invroc_1 pauc_1
y1 0 .39324619
. matrix list e(b)
symmetric e(b) [1,1]
 y1:
 invroc_1
y1 0
. matrix list r(V)
symmetric r(V) [2,2]
 y1: y1:
 invroc_1 pauc_1
y1:invroc_1 .00065477
y1:pauc_1 -.00033586 .00047975
. matrix list e(V)
symmetric e(V) [1,1]
 y1:
 invroc_1
y1:invroc_1 .00065477

```



The advantages of using `estat nproc` are twofold. First, you can estimate additional parameters of interest without having to respecify the bootstrap settings you did with `rocreg`; instead `estat nproc` uses the bootstrap settings that were stored by `rocreg`. Second, parameters estimated with `estat nproc` are added to those parameters estimated by `rocreg` and returned in the matrices `r(b)` (parameter estimates) and `r(V)` (variance–covariance matrix). Thus, you can also obtain correlations between any quantities you wish to estimate.

## Stored results

`estat nproc` stores the following in `r()`:

Matrices

|                               |                                              |
|-------------------------------|----------------------------------------------|
| <code>r(b)</code>             | coefficient vector                           |
| <code>r(V)</code>             | variance–covariance matrix of the estimators |
| <code>r(ci_normal)</code>     | normal-approximation confidence intervals    |
| <code>r(ci_percentile)</code> | percentile confidence intervals              |
| <code>r(ci_bc)</code>         | bias-corrected confidence intervals          |

## Methods and formulas

Details on computation of the nonparametric ROC curve and the estimation of the parametric ROC curve model coefficients can be found in [R] [rocreg](#). Here we describe how to estimate the ROC curve summary statistics for a parametric model. The cumulative distribution function,  $g$ , can be the standard normal cumulative distribution function,  $\Phi$ .

Methods and formulas are presented under the following headings:

*Parametric model: Summary parameter definition*  
*Maximum likelihood estimation*  
*Estimating equations estimation*

## Parametric model: Summary parameter definition

Conditioning on covariates  $\mathbf{x}$ , we have the following ROC curve model:

$$\text{ROC}(u) = g\{\mathbf{x}'\boldsymbol{\beta} + \alpha g^{-1}(u)\}$$

$\mathbf{x}$  can be constant, and  $\boldsymbol{\beta} = \beta_0$ , the constant intercept.

We can solve this equation to obtain the false-positive rate value  $u$  for a ROC value of  $r$ :

$$u = g\{[g^{-1}(r) - \mathbf{x}'\boldsymbol{\beta}]\alpha^{-1}\}$$

The partial area under the ROC curve for the false-positive rate  $u$  is defined by

$$\text{pAUC}(u) = \int_0^u g\{\mathbf{x}'\boldsymbol{\beta} + \alpha g^{-1}(t)\} dt$$

The area under the ROC curve is defined by

$$\text{AUC} = \int_0^1 g\{\mathbf{x}'\boldsymbol{\beta} + \alpha g^{-1}(t)\} dt$$

When  $g$  is the standard normal cumulative distribution function  $\Phi$ , we can express the AUC as

$$\text{AUC} = \Phi\left(\frac{\mathbf{x}'\boldsymbol{\beta}}{\sqrt{1 + \alpha^2}}\right)$$

## Maximum likelihood estimation

We allow maximum likelihood estimation under probit parametric models, so  $g = \Phi$ . The ROC value, false-positive rate, and AUC parameters all have closed-form expressions in terms of the covariate values  $\mathbf{x}$ , coefficient vector  $\boldsymbol{\beta}$ , and slope parameter  $\alpha$ . So to estimate these three types of summary parameters, we use the delta method (Oehlert 1992; Phillips and Park 1988). Particularly, we use the `nlcom` command (see [R] [nlcom](#)) to implement the delta method.

To estimate the partial area under the ROC curve for false-positive rate  $u$ , we use numeric integration. A trapezoidal approximation is used in calculating the integrals. A numeric integral of the  $\text{ROC}(t)$  function conditioned on the covariate values  $\mathbf{x}$ , coefficient vector estimate  $\widehat{\boldsymbol{\beta}}$ , and slope parameter estimate  $\widehat{\alpha}$  is computed over the range  $t = [0, u]$ . This gives us the point estimate of  $\text{pAUC}(u)$ .

To calculate the standard error and confidence intervals for the point estimate of  $\text{pAUC}(u)$ , we again use the delta method. Details on the delta method algorithm can be found in [Methods and formulas](#) of [R] [nlcom](#) and the earlier mentioned references.

Under maximum likelihood estimation, the coefficient estimates  $\widehat{\boldsymbol{\beta}}$  and slope estimate  $\widehat{\alpha}$  are asymptotically normal with variance matrix  $\mathbf{V}$ . For convenience, we rename the parameter vector  $[\boldsymbol{\beta}', \alpha]$  to the  $k$ -parameter vector  $\boldsymbol{\theta} = [\theta_1, \dots, \theta_k]$ . We will also explicitly refer to the conditioning of the ROC curve by  $\boldsymbol{\theta}$  in its mention as  $\text{ROC}(t, \boldsymbol{\theta})$ .

Under the delta method, the continuous scalar function of the estimate  $\hat{\theta}$ ,  $f(\hat{\theta})$  has asymptotic mean  $f(\theta)$  and asymptotic covariance

$$\widehat{\text{Var}} \{f(\hat{\theta})\} = \mathbf{fVf}$$

where  $\mathbf{f}$  is the  $1 \times k$  matrix of derivatives for which

$$\mathbf{f}_{1j} = \frac{\partial f(\theta)}{\partial \theta_j} \quad j = 1, \dots, k$$

The asymptotic covariance of  $f(\hat{\theta})$  is estimated and then used in conjunction with  $f(\hat{\theta})$  for further inference, including Wald confidence intervals, standard errors, and hypothesis testing.

In the case of  $\text{pAUC}(u)$  estimation, our  $f(\hat{\theta})$  is the aforementioned numeric integral of the ROC curve. It estimates  $f(\theta)$ , the true integral of the ROC curve on the  $[0, u]$  range. The  $\mathbf{V}$  variance matrix is estimated using the likelihood information that rocreg calculated, and the estimation is performed by rocreg itself.

The partial derivatives of  $f(\theta)$  can be determined by using Leibnitz's rule ([Weisstein 2011](#)):

$$\mathbf{f}_{1j} = \frac{\partial}{\partial \theta_j} \int_0^u \text{ROC}(t, \theta) dt = \int_0^u \frac{\partial}{\partial \theta_j} \text{ROC}(t, \theta) dt \quad j = 1, \dots, k$$

When  $\theta_j$  corresponds with the slope parameter  $\alpha$ , we obtain the following partial derivative:

$$\frac{\partial}{\partial \alpha} \text{pAUC}(u) = \int_0^u \phi\{\mathbf{x}'\boldsymbol{\beta} + \alpha\Phi^{-1}(t)\}\Phi^{-1}(t) dt$$

The partial derivative of  $f(\theta)$  [ $\text{pAUC}(u)$ ] for  $\beta_0$  is the following:

$$\frac{\partial}{\partial \beta_0} \text{pAUC}(u) = \int_0^u \phi\{\mathbf{x}'\boldsymbol{\beta} + \alpha\Phi^{-1}(t)\} dt$$

For a nonintercept coefficient, we obtain the following:

$$\frac{\partial}{\partial \beta_i} \text{pAUC}(u) = \int_0^u x_i \phi\{\mathbf{x}'\boldsymbol{\beta} + \alpha\Phi^{-1}(t)\} dt$$

We can estimate each of these integrals by numeric integration, plugging in the estimates  $\hat{\beta}$  and  $\hat{\alpha}$  for the parameters. This, together with the previously calculated estimate  $\widehat{\mathbf{V}}$ , provides an estimate of the asymptotic covariance of  $f(\hat{\theta}) = \widehat{\text{pAUC}}(u)$ , which allows us to perform further statistical inference on  $\widehat{\text{pAUC}}(u)$ .

## Estimating equations estimation

When we fit a model using the [Alonzo and Pepe \(2002\)](#) estimating equations method, we use the bootstrap to perform inference on the ROC curve summary parameters. Each bootstrap sample provides a sample of the coefficient estimates  $\boldsymbol{\beta}$  and the slope estimates  $\alpha$ . Using the formulas in [Parametric model: Summary parameter definition](#) under *Methods and formulas*, we can obtain an estimate of the ROC, false-positive rate, or AUC for each resample. Using numeric integration (with the trapezoidal approximation), we can also estimate the  $\text{pAUC}$  of the resample.

By making these calculations, we obtain a bootstrap sample of our summary parameter estimate. We then obtain bootstrap standard errors, normal approximation confidence intervals, percentile confidence intervals, and bias-corrected confidence intervals using this bootstrap sample. Further details can be found in [R] [bootstrap](#).

## References

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## Also see

[R] [rocreg](#) — Parametric and nonparametric ROC regression

[R] [rocregplot](#) — Plot marginal and covariate-specific ROC curves after rocreg

[U] [20 Estimation and postestimation commands](#)

|                                      |                                |                              |                                      |
|--------------------------------------|--------------------------------|------------------------------|--------------------------------------|
| Description                          | Quick start                    | Menu                         | Syntax                               |
| <a href="#">probit_options</a>       | <a href="#">common_options</a> | <a href="#">boot_options</a> | <a href="#">Remarks and examples</a> |
| <a href="#">Methods and formulas</a> | <a href="#">References</a>     | <a href="#">Also see</a>     |                                      |

## Description

Under parametric estimation, `rocregplot` plots the fitted ROC curves for specified covariate values and classifiers. If `rocreg`, `probit` or `rocreg`, `probit ml` were previously used, the false-positive rates (for specified ROC values) and ROC values (for specified false-positive rates) for each curve may also be plotted, along with confidence intervals.

Under nonparametric estimation, `rocregplot` will plot the fitted ROC curves using the `_fpr_*` and `_roc_*` variables produced by `rocreg`. Point estimates and confidence intervals for false-positive rates and ROC values that were computed in `rocreg` may be plotted as well.

## Quick start

Plot ROC curve after any `rocreg` command

```
rocregplot
```

Name graph `mygraph` and save as `myfile.gph`

```
rocregplot, name(mygraph) saving(myfile)
```

Plot ROC curve only for `v2` after fitting a model with `v1` and `v2`

```
rocregplot, classvars(v2)
```

Add bias-corrected CI for the ROC value at a false-positive rate of 0.7 from estimation with bootstrap resampling that specified `roc(.7)`

```
rocregplot, btype(bc)
```

*Plots following parametric estimation only*

Plot curve evaluated at `x = 30` and `x = 50`

```
rocregplot, at1(x=30) at2(x=50)
```

After bootstrap resampling with resampled observations saved in `myfile.dta`

```
rocregplot, at1(x=30) at2(x=50) bfile(myfile)
```

And draw CI for ROC values at false-positive rates 0.1, 0.2, and 0.3

```
rocregplot, at1(x=30) at2(x=50) roc(.1 .2 .3) bfile(myfile)
```

## Menu

Statistics > Epidemiology and related > ROC analysis > ROC curves after rocreg

## Syntax

Plot ROC curve after nonparametric analysis

```
rocregplot [, common_options boot_options]
```

Plot ROC curve after parametric analysis using bootstrap

```
rocregplot [, probit_options common_options boot_options]
```

Plot ROC curve after parametric analysis using maximum likelihood

```
rocregplot [, probit_options common_options]
```

| <i>probit_options</i>                                     | Description                                                                                             |
|-----------------------------------------------------------|---------------------------------------------------------------------------------------------------------|
| <b>Main</b>                                               |                                                                                                         |
| <code>at(<i>varname</i>=# [<i>varname</i>=#...])</code>   | value of specified covariates and mean of unspecified covariates                                        |
| <code>[at1(<i>varname</i>=# [<i>varname</i>=#...])</code> |                                                                                                         |
| <code>[at2(<i>varname</i>=# [<i>varname</i>=#...])</code> |                                                                                                         |
| <code>[...]]]</code>                                      |                                                                                                         |
| * <code>roc(<i>numlist</i>)</code>                        | show estimated ROC values for given false-positive rates                                                |
| * <code>invroc(<i>numlist</i>)</code>                     | show estimated false-positive rates for given ROC values                                                |
| <code>level(#)</code>                                     | set confidence level; default is level(95)                                                              |
| <b>Curve</b>                                              |                                                                                                         |
| <code>line#opts(<i>cline_options</i>)</code>              | affect rendition of ROC curve #                                                                         |
| *Only one of roc() or invroc() may be specified.          |                                                                                                         |
| <i>common_options</i>                                     | Description                                                                                             |
| <b>Main</b>                                               |                                                                                                         |
| <code>classvars(<i>varlist</i>)</code>                    | restrict plotting of ROC curves to specified classifiers                                                |
| <code>norefline</code>                                    | suppress plotting the reference line                                                                    |
| <b>Scatter</b>                                            |                                                                                                         |
| <code>plot#opts(<i>scatter_options</i>)</code>            | affect rendition of the classifier #s false-positive rate and ROC scatter points; not allowed with at() |
| <b>Reference line</b>                                     |                                                                                                         |
| <code>rlopts(<i>cline_options</i>)</code>                 | affect rendition of the reference line                                                                  |
| <b>Y axis, X axis, Titles, Legend, Overall</b>            |                                                                                                         |
| <code>twoway_options</code>                               | any options other than by() documented in <a href="#">[G-3]</a> <i>twoway_options</i>                   |

| <i>boot_options</i>                                                                                                                                                                 | Description                                                                                                          |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------|
| Bootstrap                                                                                                                                                                           |                                                                                                                      |
| * <code>bfile(filename)</code>                                                                                                                                                      | load dataset containing bootstrap replicates from <code>rocreg</code>                                                |
| <code>btype(n   p   bc)</code>                                                                                                                                                      | plot normal-based (n), percentile (p), or bias-corrected (bc) confidence intervals; default is <code>btype(n)</code> |
| * <code>bfile()</code> is only allowed with parametric analysis using bootstrap inference; in which case this option is required with <code>roc()</code> or <code>invroc()</code> . |                                                                                                                      |

## probit\_options

### Main

`at(varname=# ...)` requests that the covariates specified by `varname` be set to `#`. By default, `rocregplot` evaluates the function by setting each covariate to its mean value. This option causes the ROC curve to be evaluated at the value of the covariates listed in `at()` and at the mean of all unlisted covariates.

`at1(varname=# ...)`, `at2(varname=# ...)`, ..., `at10(varname=# ...)` specify that ROC curves (up to 10) be plotted on the same graph. `at1()`, `at2()`, ..., `at10()` work like the `at()` option. They request that the function be evaluated at the value of the covariates specified and at the mean of all unlisted covariates. `at1()` specifies the values of the covariates for the first curve, `at2()` specifies the values of the covariates for the second curve, and so on.

`roc(numlist)` specifies that estimated ROC values for given false-positive rates be graphed.

`invroc(numlist)` specifies that estimated false-positive rates for given ROC values be graphed.

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [U] 20.8 Specifying the width of confidence intervals. `level()` may be specified with either `roc()` or `invroc()`.

### Curve

`line#opts(cline_options)` affects the rendition of ROC curve `#`. See [G-3] *cline\_options*.

## common\_options

### Main

`classvars(varlist)` restricts plotting ROC curves to specified classification variables.

`noreflines` suppresses plotting the reference line.

### Scatter

`plot#opts(scatter_options)` affects the rendition of classifier `#`'s false-positive rate and ROC scatter points. This option applies only to non-ROC covariate estimation graphing. See [G-2] **graph twoway scatter**.

### Reference line

`rlopts(cline_options)` affects the rendition of the reference line. See [G-3] *cline\_options*.

---

 Y axis, X axis, Titles, Legend, Overall
 

---

*twoway\_options* are any of the options documented in [G-3] *twoway\_options*, excluding `by()`. These include options for titling the graph (see [G-3] *title\_options*) and options for saving the graph to disk (see [G-3] *saving\_option*).

## boot\_options

---

 Bootstrap
 

---

`bfile(filename)` uses bootstrap replicates of parameters from `rocreg` stored in *filename* to estimate standard errors and confidence intervals of predictions. `bfile()` must be specified with either `roc()` or `invroc()` if parametric estimation with bootstrapping was used.

`btype(n | p | bc)` indicates the desired type of confidence interval rendering. `n` draws normal-based, `p` draws percentile, and `bc` draws bias-corrected confidence intervals for specified false-positive rates and ROC values in `roc()` and `invroc()`. The default is `btype(n)`.

## Remarks and examples

Remarks are presented under the following headings:

*Plotting covariate-specific ROC curves*  
*Plotting marginal ROC curves*

### Plotting covariate-specific ROC curves

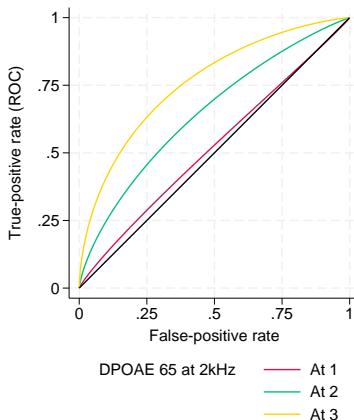
The `rocregplot` command is also demonstrated in [R] [rocreg](#). We will further demonstrate its use with several examples. Particularly, we will show how `rocregplot` can draw the ROC curves of covariate models that have been fit using `rocreg`.

#### ► Example 1: Parametric ROC

In [example 6](#) of [R] [rocreg](#), we fit a probit ROC model to audiology test data from [Norton et al. \(2000\)](#). The estimating equation method of [Alonzo and Pepe \(2002\)](#) was used to fit the model. Gender and age were covariates that affected the control distribution of the classifier `y1` (DPOAE 65 at 2 kHz). Age was a ROC covariate for the model, so we fit separate ROC curves at each age.

Following [Janes, Longton, and Pepe \(2009\)](#), we draw the ROC curves for ages 30, 40, and 50 months. The `at1()`, `at2()`, and `at3()` options are used to specify the age covariates.

```
. use https://www.stata-press.com/data/r19/nhns
(Norton - neonatal audiology data)
. rocreg d y1, probit ctrlcov(currage male) ctrlmodel(linear) roccov(currage)
> cluster(id) bseed(56930) bsave(nhns2y1, replace)
(output omitted)
. rocregplot, at1(currage=30) at2(currage=40) at3(currage=50)
```



Here we use the default entries of the legend, which indicate the “at #” within the specified `at*` options and the classifier to which the curve corresponds. ROC curve one corresponds with `currage=30`, two with `currage=40`, and three with `currage=50`. The positive effect of age on the ROC curve is evident. At an age of 30 months (`currage=30`), the ROC curve of `y1` (DPOAE 65 at 2 kHz) is nearly equivalent to that of a noninformative test that gives equal probability to hearing loss. At age 50 months (`currage=50`), corresponding to some of the oldest children in the study, the ROC curve shows that test `y1` (DPOAE 65 at 2 kHz) is considerably more powerful than the noninformative test.

You may create your own legend by specifying the `legend()` option. The default legend is designed for the possibility of multiple covariates. Here we could change the legend entries to `currage` values and gain some extra clarity. However, this may not be feasible when there are many covariates present.



We can also use `rocregplot` after maximum likelihood estimation.

### ▷ Example 2: Maximum likelihood ROC

We return to the audiology study with frequency (`xf`), intensity (`x1`), and hearing loss severity (`xd`) covariates from [Stover et al. \(1996\)](#) that we examined in [example 10](#) of [\[R\] rocreg](#). Negative signal-to-noise ratio is again used as a classifier. Using maximum likelihood, we fit a probit model to these data with the indicated ROC covariates.

After fitting the model, we wish to compare the ROC curves of two covariate combinations. The first has an intensity value of 5.5 (the lowest intensity, corresponding to 55 decibels) and a frequency of 10.01 (the lowest frequency, corresponding to 1001 hertz). We give the first combination a hearing loss severity value of 0.5 (the lowest). The second covariate combination has the same frequency, but the highest intensity value of 6.5 (65 decibels). We give this second covariate set a higher severity value of 4. We will visually compare the two ROC curves resulting from these two covariate value combinations.

We specify false-positive rates of 0.7 first followed by 0.2 in the roc() option to visually compare the size of the ROC curve at large and small false-positive rates. Because maximum likelihood estimation was used to fit the model, a Wald confidence interval is produced for the estimated ROC value and false-positive rate parameters. Further details are found in *Methods and formulas*.

```
. use https://www.stata-press.com/data/r19/dp
(Stover - DPOAE test data)
. rocreg d nsnr, probit ctrlcov(xf xl) roccov(xf xl xd) ml cluster(id)
(output omitted)
. rocregplot, at1(xf=10.01, xl=5.5, xd=.5) at2(xf=10.01, xl=6.5, xd=4) roc(.7)
```

ROC curve

Status : d  
Classifier: nsnr

Under covariates:

|    | at1   |  |  |
|----|-------|--|--|
| xf | 10.01 |  |  |
| xl | 5.5   |  |  |
| xd | .5    |  |  |

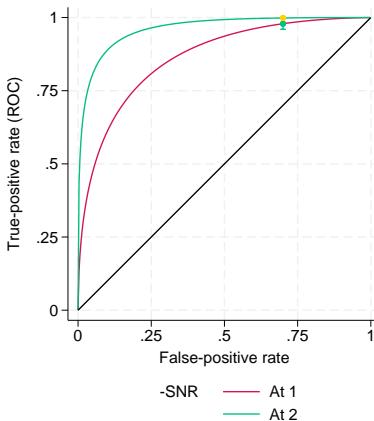
| ROC | Coefficient | Std. err. | [95% conf. interval] |          |
|-----|-------------|-----------|----------------------|----------|
| .7  | .978951     | .0097382  | .9598645             | .9980376 |

Under covariates:

|    | at2   |  |  |
|----|-------|--|--|
| xf | 10.01 |  |  |
| xl | 6.5   |  |  |
| xd | 4     |  |  |

| ROC | Coefficient | Std. err. | [95% conf. interval] |          |
|-----|-------------|-----------|----------------------|----------|
| .7  | .9985001    | .0009657  | .9966073             | 1.000393 |



At the higher false-positive rate value of 0.7, we see little difference in the ROC values and note that the confidence intervals nearly overlap. Now, we view the same curves with the lower false-positive rate compared.

```
. rocregplot, at1(xf=10.01, xl=5.5, xd=.5) at2(xf=10.01, xl=6.5, xd=4) roc(.2)
```

ROC curve

Status : d  
Classifier: nsnr

Under covariates:

|    | at1   |  |  |
|----|-------|--|--|
| xf | 10.01 |  |  |
| xl | 5.5   |  |  |
| xd | .5    |  |  |

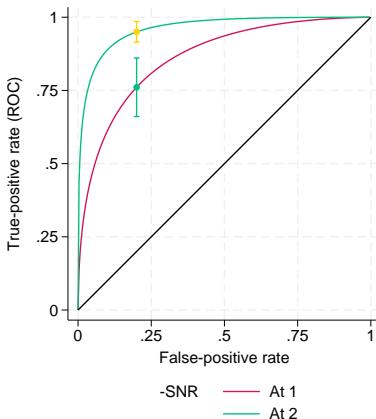
| ROC | Coefficient | Std. err. | [95% conf. interval] |          |
|-----|-------------|-----------|----------------------|----------|
| .2  | .7608593    | .0510501  | .660803              | .8609157 |

Under covariates:

|    | at2   |  |  |
|----|-------|--|--|
| xf | 10.01 |  |  |
| xl | 6.5   |  |  |
| xd | 4     |  |  |

| ROC | Coefficient | Std. err. | [95% conf. interval] |          |
|-----|-------------|-----------|----------------------|----------|
| .2  | .9499408    | .0179824  | .914696              | .9851856 |



The lower false-positive rate of 0.2 shows clearly distinguishable ROC values. Now, we specify option `invroc(.5)` to view how the false-positive rates vary at a ROC value of 0.5.

```
. rocregplot, at1(xf=10.01, xl=5.5, xd=.5) at2(xf=10.01, xl=6.5, xd=4) invroc(.5)
```

False-positive rate

Status : d  
Classifier: nsnr

Under covariates:

|    | at1   |  |  |
|----|-------|--|--|
| xf | 10.01 |  |  |
| xl | 5.5   |  |  |
| xd | .5    |  |  |

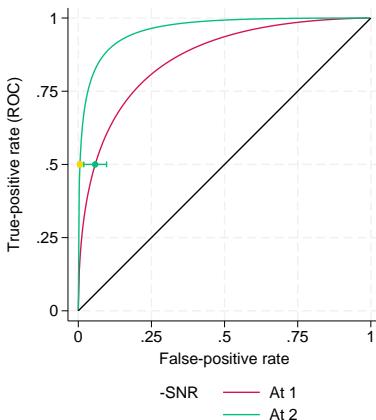
| invROC | Coefficient | Std. err. | [95% conf. interval] |          |
|--------|-------------|-----------|----------------------|----------|
| .5     | .0578036    | .0198626  | .0188736             | .0967336 |

Under covariates:

|    | at2   |  |  |
|----|-------|--|--|
| xf | 10.01 |  |  |
| xl | 6.5   |  |  |
| xd | 4     |  |  |

| invROC | Coefficient | Std. err. | [95% conf. interval] |          |
|--------|-------------|-----------|----------------------|----------|
| .5     | .0055624    | .0032645  | -.0008359            | .0119607 |



At a ROC value of 0.5, the false-positive rates for both curves are small and close to one another.

## □ Technical note

We can use the `testnl` command to support our visual observations with statistical inference. We use it to perform a Wald test of the null hypothesis that the two ROC curves just rendered are equal at a false-positive rate of 0.7.

```
. testnl normal(_b[i_cons]+10.01*_b[xf]+5.5*_b[x1]
> +.5*_b[xd]+_b[s_cons]*invnormal(.7)) =
> normal(_b[i_cons]+10.01*_b[xf]+6.5*_b[x1]
> + 4*_b[xd]+_b[s_cons]*invnormal(.7))
(1) normal(_b[i_cons]+10.01*_b[xf]+5.5*_b[x1] +.5*_b[xd]+_b[s_cons]*invnormal(.7)) =
 normal(_b[i_cons]+10.01*_b[xf]+6.5*_b[x1] + 4*_b[xd]+_b[s_cons]*invnormal(.7))
 chi2(1) = 4.53
 Prob > chi2 = 0.0332
```

The test is significant at the 0.05 level, and thus we find that the two curves are significantly different. Now, we will use `testnl` again to test equality of the false-positive rates for each curve with a ROC value of 0.5. The inverse ROC formula used is derived in [Methods and formulas](#).

```
. testnl normal((invnormal(.5)-(_b[i_cons]+10.01*_b[xf]+5.5*_b[x1]+.5*_b[xd]))
> /_b[s_cons]) =
> normal((invnormal(.5)-(_b[i_cons]+10.01*_b[xf]+6.5*_b[x1]+4*_b[xd]))
> /_b[s_cons])
(1) normal((invnormal(.5)-(_b[i_cons]+10.01*_b[xf]+5.5*_b[x1]+.5*_b[xd]))
> /_b[s_cons]) = normal((invnormal(.5)-(_b[i_cons]+10.01*_b[xf]+6.5*_b[x1]
> +4*_b[xd])) /_b[s_cons])
 chi2(1) = 8.01
 Prob > chi2 = 0.0046
```

We again reject the null hypothesis that the two curves are equal at the 0.05 level.

□

The model of our last example was also fit using the estimating equations method in [example 7](#) of [\[R\] rocreg](#). We will demonstrate `rocregplot` after that model fit as well.

► Example 3: Parametric ROC, invROC, and ROC value

In [example 2](#), we used `rocregplot` after a maximum likelihood model fit of the ROC curve for classifier `nsnr` and covariates `frequency` (`xf`), `intensity` (`x1`), and `hearing loss severity` (`xd`). The data were obtained from the audiology study described in [Stover et al. \(1996\)](#). In [example 7](#) of [\[R\] rocreg](#), we fit the model using the estimating equations method of [Alonzo and Pepe \(2002\)](#). Under this method, bootstrap resampling is used to make inferences. We saved 50 bootstrap replications in `nsnrf.dta`, which we re-create below.

We use `rocregplot` to draw the ROC curves for `nsnr` under the covariate values `xf = 10.01`, `x1 = 5.5`, and `xd = .5`, and `xf = 10.01`, `x1 = 6.5`, and `xd = 4`. The `at#()` options are used to specify the covariate values. The previous bootstrap results are made available to `rocregplot` with the `bfile()` option. As before, we will specify 0.2 and 0.7 as false-positive rates in the `roc()` option and 0.5 as a ROC value in the `invroc()` option. We do not specify `btype()` and thus our graph will contain normal-based bootstrap confidence bands, the default.

```
. use https://www.stata-press.com/data/r19/dp
(Stover - DPOAE test data)
. rocreg d nsnr, probit ctrlcov(xf x1) roccov(xf x1 xd) cluster(id)
> nobstrata ctrlfprall bseed(156385) breps(50) bsave(nsnrf, replace)
(output omitted)
. rocregplot, at1(xf=10.01, x1=5.5, xd=.5) at2(xf=10.01, x1=6.5, xd=4)
> roc(.7) bfile(nsnrf)
```

ROC curve

Status : d  
Classifier: nsnr

Under covariates:

|    | at1   |
|----|-------|
| xf | 10.01 |
| x1 | 5.5   |
| xd | .5    |

(Replications based on 208 clusters in id)

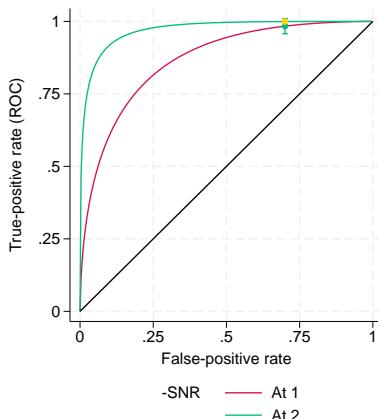
| ROC | Observed coefficient | Bias     | Bootstrap std. err. | [95% conf. interval] |               |
|-----|----------------------|----------|---------------------|----------------------|---------------|
| .7  | .9835816             | .0026679 | .0133583            | .9573998             | 1.009763 (N)  |
|     |                      |          |                     | .9475966             | .9983087 (P)  |
|     |                      |          |                     | .9453403             | .9983087 (BC) |

Under covariates:

|    | at2   |
|----|-------|
| xf | 10.01 |
| x1 | 6.5   |
| xd | 4     |

(Replications based on 208 clusters in id)

| ROC | Observed coefficient | Bias     | Bootstrap std. err. | [95% conf. interval] |               |
|-----|----------------------|----------|---------------------|----------------------|---------------|
| .7  | .999428              | .0001748 | .0007784            | .9979024             | 1.000954 (N)  |
|     |                      |          |                     | .9975154             | .9999899 (P)  |
|     |                      |          |                     | .9965358             | .9999899 (BC) |



As shown in the graph, we find that the ROC values at a false-positive rate of 0.7 are close together, as they were in the maximum likelihood estimation in [example 2](#). We now repeat this process for the lower false-positive rate of 0.2 by using the `roc(.2)` option.

```
. rocregplot, at1(xf=10.01, xl=5.5, xd=.5) at2(xf=10.01, xl=6.5, xd=4)
> roc(.2) bfile(nsnrf)
```

ROC curve

Status : d  
Classifier: nsnr

Under covariates:

|    | at1   |
|----|-------|
| xf | 10.01 |
| xl | 5.5   |
| xd | .5    |

(Replications based on 208 clusters in id)

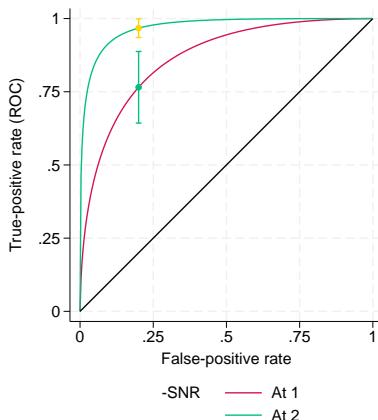
| ROC | Observed coefficient | Bias     | Bootstrap std. err. | [95% conf. interval] |               |
|-----|----------------------|----------|---------------------|----------------------|---------------|
| .2  | .7652956             | -.006687 | .0624187            | .6429572             | .8876341 (N)  |
|     |                      |          |                     | .6654987             | .8837934 (P)  |
|     |                      |          |                     | .6444601             | .8837934 (BC) |

Under covariates:

|    | at2   |
|----|-------|
| xf | 10.01 |
| xl | 6.5   |
| xd | 4     |

(Replications based on 208 clusters in id)

| ROC | Observed coefficient | Bias      | Bootstrap std. err. | [95% conf. interval] |               |
|-----|----------------------|-----------|---------------------|----------------------|---------------|
| .2  | .9672505             | -.0015124 | .0162785            | .9353452             | .9991558 (N)  |
|     |                      |           |                     | .9372331             | .9976121 (P)  |
|     |                      |           |                     | .9218445             | .9926328 (BC) |



The ROC values are slightly higher at the false-positive rate of 0.2 than they were in the maximum likelihood estimation in [example 2](#). To see if the false-positive rates differ at a ROC value of 0.5, we specify the `invroc(.5)` option.

```
. rocregplot, at1(xf=10.01, xl=5.5, xd=.5) at2(xf=10.01, xl=6.5, xd=4)
> invroc(.5) bfile(nsnrf)
```

```
False-positive rate
 Status : d
 Classifier: nsnr
```

Under covariates:

|    | at1   |
|----|-------|
| xf | 10.01 |
| xl | 5.5   |
| xd | .5    |

(Replications based on 208 clusters in id)

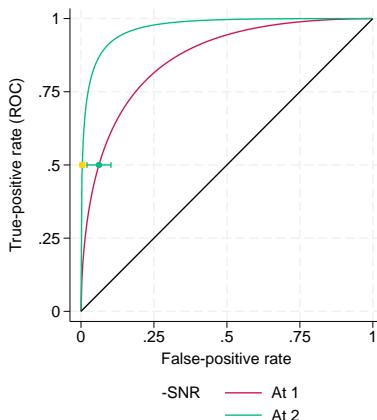
| invROC | Observed coefficient | Bias     | Bootstrap std. err. | [95% conf. interval] |              |
|--------|----------------------|----------|---------------------|----------------------|--------------|
| .5     | .0615144             | .0012744 | .0209516            | .0204499             | .1025789 (N) |
|        |                      |          |                     | .0234184             | .09672 (P)   |
|        |                      |          |                     | .0234184             | .09672 (BC)  |

Under covariates:

|    | at2   |
|----|-------|
| xf | 10.01 |
| xl | 6.5   |
| xd | 4     |

(Replications based on 208 clusters in id)

| invROC | Observed coefficient | Bias     | Bootstrap std. err. | [95% conf. interval] |               |
|--------|----------------------|----------|---------------------|----------------------|---------------|
| .5     | .0043298             | -.000139 | .003835             | -.0031867            | .0118463 (N)  |
|        |                      |          |                     | .0002028             | .0136293 (P)  |
|        |                      |          |                     | .0012615             | .0192129 (BC) |



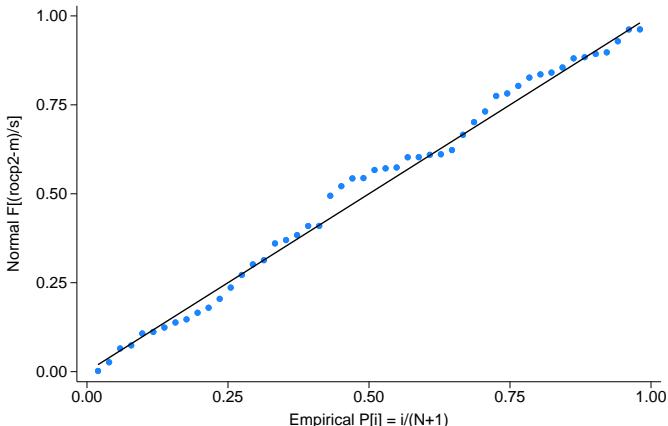
The point estimates of the ROC value and false-positive rate are both computed directly using the point estimates of the ROC coefficients. Calculation of the standard errors and confidence intervals is slightly more complicated. Essentially, we have stored a sample of our ROC covariate coefficient estimates in `nsnrf.dta`. We then calculate the ROC value or false-positive rate estimates using each set of coefficient estimates, resulting in a sample of point estimates. Then, the bootstrap standard error and confidence intervals are calculated based on these bootstrap samples. Details of the computation of the standard error and percentile confidence intervals can be found in [Methods and formulas](#) and in [\[R\] bootstrap](#).

As mentioned in [\[R\] rocreg](#), 50 resamples is a reasonable lower bound for obtaining bootstrap standard errors ([Mooney and Duval 1993](#)). However, it may be too low for obtaining percentile and bias-corrected confidence intervals. Normal-based confidence intervals are valid when the bootstrap distribution exhibits normality. See [\[R\] bootstrap postestimation](#) for more details.

We can assess the normality of the bootstrap distribution by using a normal probability plot. Stata provides this in the `pnorm` command (see [\[R\] Diagnostic plots](#)). We will use `nsnrf.dta` to draw a normal probability plot for the ROC estimate corresponding to a false-positive rate of 0.2. We use the covariate values `xf = 10.01`, `x1 = 6.5`, and `xd = 4`.

```
. use nsnrf
(bootstrap: rocregstat)
. generate double rocp2 = nsnr_b_i_cons + 10.01*nsnr_b_xf + 6.5*nsnr_b_x1 +
> 4*nsnr_b_xd+nsnr_b_s_cons*invnormal(.2)
. replace rocp2 = normal(rocp2)
(50 real changes made)
```

```
. pnorm rocp2
```



The closeness of the points to the horizontal line on the normal probability plot shows us that the bootstrap distribution is approximately normal. So it is reasonable to use the normal-based confidence intervals for ROC at a false-positive rate of 0.2 under covariate values  $xf = 10.01$ ,  $x1 = 6.5$ , and  $xd = 4$ .

◀

## Plotting marginal ROC curves

The `rocgplot` command can also be used after fitting models with no covariates. We will demonstrate this with an empirical ROC model fit in [\[R\] rocreg](#).

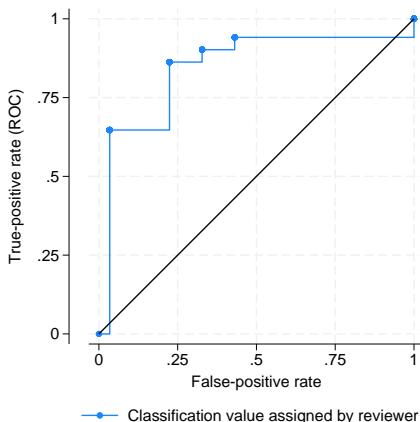
### ▷ Example 4: Nonparametric ROC

We run `rocgplot` after fitting the single-classifier, empirical ROC model shown in [example 1](#) of [\[R\] rocreg](#). There we empirically predicted the ROC curve of the classifier rating for the true status variable disease from the [Hanley and McNeil \(1982\)](#) data. The `rocreg` command saves variables `_roc_rating` and `_fpr_rating`, which give the ROC values and false-positive rates, respectively, for every value of `rating`. These variables are used by `rocgplot` to render the ROC curve.

```
. use https://www.stata-press.com/data/r19/hanley, clear
(Tomographic images)
. rocreg disease rating, noboot
Nonparametric ROC estimation Number of obs = 109
Control standardization: empirical
ROC method : empirical
Area under the ROC curve
 Status : disease
 Classifier: rating
```

| AUC | Observed coefficient | Bias | Bootstrap std. err. | [95% conf. interval] |
|-----|----------------------|------|---------------------|----------------------|
|     | .8407708             | .    | .                   | . (N)                |
|     |                      |      |                     | . (P)                |
|     |                      |      |                     | . (BC)               |

```
. rocregplot
```



We end our discussion of rocregplot by showing its use after a marginal probit model.

► Example 5: Maximum likelihood ROC, invROC, and ROC value

In [example 13](#) of [\[R\] rocreg](#), we fit a maximum-likelihood probit model to each classifier of the fictitious dataset generated from [Hanley and McNeil \(1983\)](#).

We use rocregplot after the original rocreg command to draw the ROC curves for classifiers mod1 and mod3. This is accomplished by specifying the two variables in the classvars() option. We will use the roc() option to obtain confidence intervals for ROC values at false-positive rates of 0.15 and 0.75. We will specify the invroc() option to obtain false-positive rate confidence intervals for a ROC value of 0.8. As mentioned previously, these are Wald confidence intervals.

First, we will view results for a false-positive rate of 0.75.

```
. use https://www.stata-press.com/data/r19/ct2, clear
(Reconstruction of CT images)
. rocreg status mod1 mod2 mod3, probit ml
(output omitted)
. rocregplot, classvars(mod1 mod3) roc(.75)
```

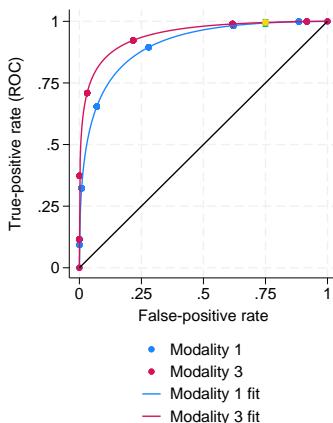
ROC curve

Status : status  
Classifier: mod1

| ROC | Coefficient | Std. err. | [95% conf. interval] |          |
|-----|-------------|-----------|----------------------|----------|
| .75 | .9931655    | .0069689  | .9795067             | 1.006824 |

Status : status  
Classifier: mod3

| ROC | Coefficient | Std. err. | [95% conf. interval] |          |
|-----|-------------|-----------|----------------------|----------|
| .75 | .9953942    | .0043435  | .9868811             | 1.003907 |



We see that the estimates for each of the two ROC curves are close. Because this is a marginal model, the actual false-positive rate and the true-positive rate for each observation are plotted in the graph. The added point estimates of the ROC value at false-positive rate 0.75 are shown as diamond (mod3) and circle (mod1) symbols in the upper-right-hand corner of the graph at FPR = 0.75. Confidence bands are also plotted at FPR = 0.75 but are so narrow that they are barely noticeable. Under both classifiers, the ROC value at 0.75 is very high. Now, we will compare these results to those with a lower false-positive rate of 0.15.

```
. rocregplot, classvars(mod1 mod3) roc(.15)
```

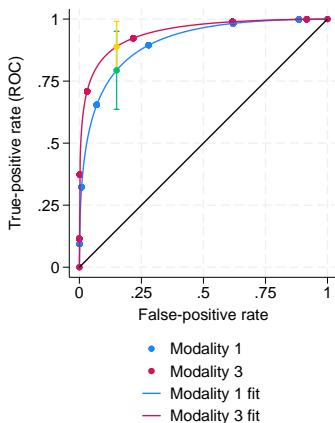
ROC curve

Status : status  
Classifier: mod1

| ROC | Coefficient | Std. err. | [95% conf. interval] |          |
|-----|-------------|-----------|----------------------|----------|
| .15 | .7934935    | .0801363  | .6364292             | .9505578 |

Status : status  
Classifier: mod3

| ROC | Coefficient | Std. err. | [95% conf. interval] |          |
|-----|-------------|-----------|----------------------|----------|
| .15 | .8888596    | .0520118  | .7869184             | .9908008 |



The ROC value for the false-positive rate of 0.15 is more separated in the two classifiers. Here we see that mod3 has a larger ROC value than mod1 for this false-positive rate, but the confidence intervals of the estimates overlap.

By specifying `invroc(.8)`, we obtain `invROC` confidence intervals corresponding to a ROC value of 0.8.

```
. rocregplot, classvars(mod1 mod3) invroc(.8)
```

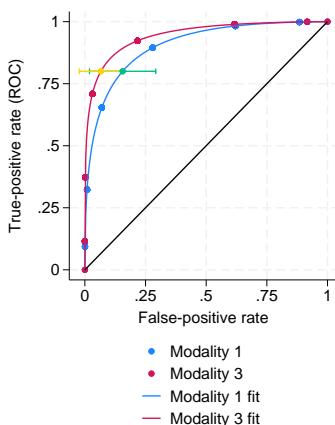
False-positive rate

Status : status  
Classifier: mod1

| invROC | Coefficient | Std. err. | [95% conf. interval] |          |
|--------|-------------|-----------|----------------------|----------|
| .8     | .1556435    | .069699   | .019036              | .2922509 |

Status : status  
Classifier: mod3

| invROC | Coefficient | Std. err. | [95% conf. interval] |          |
|--------|-------------|-----------|----------------------|----------|
| .8     | .0661719    | .045316   | -.0226458            | .1549896 |



For estimation of the false-positive rate at a ROC value of 0.8, the confidence intervals overlap. Both classifiers require only a small false-positive rate to achieve a ROC value of 0.8.



## Methods and formulas

Details on computation of the nonparametric ROC curve and the estimation of the parametric ROC curve model coefficients can be found in [R] [rocreg](#). Here we describe how to estimate the ROC values and false-positive rates of a parametric model. The cumulative distribution function  $g$  can be the standard normal cumulative distribution function.

Methods and formulas are presented under the following headings:

- [Parametric model: Summary parameter definition](#)
- [Maximum likelihood estimation](#)
- [Estimating equations estimation](#)

## Parametric model: Summary parameter definition

Conditioning on covariates  $\mathbf{x}$ , we have the following ROC curve model:

$$\text{ROC}(u) = g\{\mathbf{x}'\boldsymbol{\beta} + \alpha g^{-1}(u)\}$$

$\mathbf{x}$  can be constant, and  $\boldsymbol{\beta} = \beta_0$ , the constant intercept.

With simple algebra, we can solve this equation to obtain the false-positive rate value  $u$  for a ROC value of  $r$ :

$$u = g[\{g^{-1}(r) - \mathbf{x}'\boldsymbol{\beta}\}\alpha^{-1}]$$

## Maximum likelihood estimation

We allow maximum likelihood estimation under probit parametric models, so  $g = \Phi$ . The ROC value and false-positive rate parameters all have closed-form expressions in terms of the covariate values  $\mathbf{x}$ , coefficient vector  $\boldsymbol{\beta}$ , and slope parameter  $\alpha$ . Thus to estimate these two types of summary parameters, we use the delta method (Oehlert 1992; Phillips and Park 1988). Particularly, we use the `nlcom` command (see [R] `nlcom`) to implement the delta method.

Under maximum likelihood estimation, the coefficient estimates  $\widehat{\boldsymbol{\beta}}$  and slope estimate  $\widehat{\alpha}$  are asymptotically normal with variance matrix  $\mathbf{V}$ . For convenience, we rename the parameter vector  $[\boldsymbol{\beta}', \alpha]$  to the  $k$ -parameter vector  $\boldsymbol{\theta} = [\theta_1, \dots, \theta_k]$ . We will also explicitly refer to the conditioning of the ROC curve by  $\boldsymbol{\theta}$  in its mention as  $\text{ROC}(t, \boldsymbol{\theta})$ .

Under the delta method, the continuous scalar function of the estimate  $\widehat{\boldsymbol{\theta}}$ ,  $f(\widehat{\boldsymbol{\theta}})$  has asymptotic mean  $f(\boldsymbol{\theta})$  and asymptotic covariance

$$\widehat{\text{Var}}\{f(\widehat{\boldsymbol{\theta}})\} = \mathbf{f}\mathbf{V}\mathbf{f}'$$

where  $\mathbf{f}$  is the  $1 \times k$  matrix of derivatives for which

$$\mathbf{f}_{1j} = \frac{\partial f(\boldsymbol{\theta})}{\partial \theta_j} \quad j = 1, \dots, k$$

The asymptotic covariance of  $f(\widehat{\boldsymbol{\theta}})$  is estimated and then used in conjunction with  $f(\widehat{\boldsymbol{\theta}})$  for further inference, including Wald confidence intervals, standard errors, and hypothesis testing.

## Estimating equations estimation

When we fit a model using the Alonzo and Pepe (2002) estimating equations method, we use the bootstrap to perform inference on the ROC curve summary parameters. Each bootstrap sample provides a sample of the coefficient estimates  $\boldsymbol{\beta}$  and the slope estimates  $\alpha$ . Using the formulas above, we can obtain an estimate of the ROC value or false-positive rate for each resample.

By making these calculations, we obtain a bootstrap sample of our summary parameter estimate. We then obtain bootstrap standard errors, normal approximation confidence intervals, percentile confidence intervals, and bias-corrected confidence intervals using this bootstrap sample. Further details can be found in [R] `bootstrap`.

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## Also see

[R] **rocreg** — Parametric and nonparametric ROC regression

[R] **rocreg postestimation** — Postestimation tools for rocreg

[U] **20 Estimation and postestimation commands**

[Description](#)  
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## Description

The above command is used to perform receiver operating characteristic (ROC) analyses with rating and discrete classification data.

The two variables *refvar* and *classvar* must be numeric. The reference variable indicates the true state of the observation, such as diseased and nondiseased or normal and abnormal, and must be coded as 0 and 1. The rating or outcome of the diagnostic test or test modality is recorded in *classvar*, which must be at least ordinal, with higher values indicating higher risk.

`roctab` performs nonparametric ROC analyses. By default, `roctab` calculates the area under the ROC curve. Optionally, `roctab` can plot the ROC curve, display the data in tabular form, and produce Lorenz-like plots.

See [\[R\] rocf](#) for a command that fits maximum-likelihood ROC models.

## Quick start

AUC, standard error, and CI for ROC curve of binary true state, `true`, as a function of classification variable `v`

```
roctab true v
```

Also plot the ROC curve

```
roctab true v, graph summary
```

Same as above, but plot sensitivity versus specificity instead of the ROC curve

```
roctab true v, specificity summary
```

Exact binomial CI instead of asymptotic normal CI for AUC estimate

```
roctab true v, binomial
```

Gini and Pietra indices

```
roctab true v, lorenz
```

## Menu

Statistics > Epidemiology and related > ROC analysis > Nonparametric ROC analysis without covariates

## Syntax

```
roctab refvar classvar [if] [in] [weight] [, options]
```

| <i>options</i>                                                        | Description                                                                    |
|-----------------------------------------------------------------------|--------------------------------------------------------------------------------|
| Main                                                                  |                                                                                |
| <code>lorenz</code>                                                   | report Gini and Pietra indices                                                 |
| <code>binomial</code>                                                 | calculate exact binomial confidence intervals                                  |
| <code>no label</code>                                                 | display numeric codes rather than value labels                                 |
| <code>detail</code>                                                   | show details on sensitivity/specificity for each cutpoint                      |
| <code>table</code>                                                    | display the raw data in a $2 \times k$ contingency table                       |
| <code>bamber</code>                                                   | calculate standard errors by using the Bamber method                           |
| <code>hanley</code>                                                   | calculate standard errors by using the Hanley method                           |
| <code>graph</code>                                                    | graph the ROC curve                                                            |
| <code>norefline</code>                                                | suppress plotting the 45-degree reference line                                 |
| <code>summary</code>                                                  | report the area under the ROC curve                                            |
| <code>specificity</code>                                              | graph sensitivity versus specificity                                           |
| <code>level(#)</code>                                                 | set confidence level; default is level(95)                                     |
| Plot                                                                  |                                                                                |
| <code>plotopts(<i>plot_options</i>)</code>                            | affect rendition of the ROC curve                                              |
| Reference line                                                        |                                                                                |
| <code>rlopts(<i>cline_options</i>)</code>                             | affect rendition of the reference line                                         |
| Add plots                                                             |                                                                                |
| <code>addplot(<i>plot</i>)</code>                                     | add other plots to the generated graph                                         |
| Y axis, X axis, Titles, Legend, Overall                               |                                                                                |
| <code>twoway_options</code>                                           | any options other than by() documented in [G-3] <a href="#">twoway_options</a> |
| collect is allowed; see [U] <a href="#">11.1.10 Prefix commands</a> . |                                                                                |
| fweights are allowed; see [U] <a href="#">11.1.6 weight</a> .         |                                                                                |
| <i>plot_options</i>                                                   | Description                                                                    |
| <code>marker_options</code>                                           | change look of markers (color, size, etc.)                                     |
| <code>marker_label_options</code>                                     | add marker labels; change look or position                                     |
| <code>cline_options</code>                                            | change look of the line                                                        |

## Options

### Main

`lorenz` specifies that the Gini and Pietra indices be reported. Optionally, `graph` will plot the Lorenz-like curve.

`binomial` specifies that exact binomial confidence intervals be calculated.

`no label` specifies that numeric codes be displayed rather than value labels.

`detail` outputs a table displaying the sensitivity, specificity, the percentage of subjects correctly classified, and two likelihood ratios for each possible cutpoint of *classvar*.

`table` outputs a  $2 \times k$  contingency table displaying the raw data.

`bamber` specifies that the standard error for the area under the ROC curve be calculated using the method suggested by [Bamber \(1975\)](#). Otherwise, standard errors are obtained as suggested by [DeLong, DeLong, and Clarke-Pearson \(1988\)](#).

`hanley` specifies that the standard error for the area under the ROC curve be calculated using the method suggested by [Hanley and McNeil \(1982\)](#). Otherwise, standard errors are obtained as suggested by [DeLong, DeLong, and Clarke-Pearson \(1988\)](#).

`graph` produces graphical output of the ROC curve. If `lorenz` is specified, graphical output of a Lorenz-like curve will be produced.

`noreflines` suppresses plotting the 45-degree reference line from the graphical output of the ROC curve.

`summary` reports the area under the ROC curve, its standard error, and its confidence interval. If `lorenz` is specified, Lorenz indices are reported. This option is needed only when also specifying `graph`.

`specificity` produces a graph of sensitivity versus specificity instead of sensitivity versus  $(1 - \text{specificity})$ . `specificity` implies `graph`.

`level(#)` specifies the confidence level, as a percentage, for the confidence intervals. The default is `level(95)` or as set by `set level`; see [\[R\] level](#).

#### Plot

`plotopts(plot_options)` affects the rendition of the plotted ROC curve—the curve’s plotted points connected by lines. The *plot\_options* can affect the size and color of markers, whether and how the markers are labeled, and whether and how the points are connected; see [\[G-3\] marker\\_options](#), [\[G-3\] marker\\_label\\_options](#), and [\[G-3\] cline\\_options](#).

#### Reference line

`rlopts(cline_options)` affects the rendition of the reference line; see [\[G-3\] cline\\_options](#).

#### Add plots

`addplot(plot)` provides a way to add other plots to the generated graph; see [\[G-3\] addplot\\_option](#).

#### Y axis, X axis, Titles, Legend, Overall

*twoway\_options* are any of the options documented in [\[G-3\] twoway\\_options](#), excluding `by()`. These include options for titling the graph (see [\[G-3\] title\\_options](#)) and for saving the graph to disk (see [\[G-3\] saving\\_option](#)).

## Remarks and examples

Remarks are presented under the following headings:

[Introduction](#)  
[Nonparametric ROC curves](#)  
[Lorenz-like curves](#)

## Introduction

The `roctab` command provides nonparametric estimation of the ROC for a given classifier and true-status reference variable. The Lorenz curve functionality of `roctab`, which provides an alternative to standard ROC analysis, is discussed in *Lorenz-like curves*.

See [Pepe \(2003\)](#) for a discussion of ROC analysis. Pepe has posted Stata datasets and programs used to reproduce results presented in the book (<https://www.stata.com/bookstore/pepe.html>).

## Nonparametric ROC curves

The points on the nonparametric ROC curve are generated using each possible outcome of the diagnostic test as a classification cutpoint and computing the corresponding sensitivity and  $1 - \text{specificity}$ . These points are then connected by straight lines, and the area under the resulting ROC curve is computed using the trapezoidal rule.

### ▷ Example 1

[Hanley and McNeil \(1982\)](#) presented data from a study in which a reviewer was asked to classify, using a five-point scale, a random sample of 109 tomographic images from patients with neurological problems. The rating scale was as follows: 1 = definitely normal, 2 = probably normal, 3 = questionable, 4 = probably abnormal, and 5 = definitely abnormal. The true disease status was normal for 58 of the patients and abnormal for the remaining 51 patients.

Here we list 9 of the 109 observations:

```
. use https://www.stata-press.com/data/r19/hanley
(Tomographic images)
. list disease rating in 1/9
```

|    | disease | rating |
|----|---------|--------|
| 1. | 1       | 5      |
| 2. | 0       | 1      |
| 3. | 1       | 5      |
| 4. | 0       | 4      |
| 5. | 0       | 1      |
| 6. | 0       | 3      |
| 7. | 1       | 5      |
| 8. | 0       | 5      |
| 9. | 0       | 1      |

For each observation, `disease` identifies the true disease status of the subject (0 = normal, 1 = abnormal), and `rating` contains the classification value assigned by the reviewer.

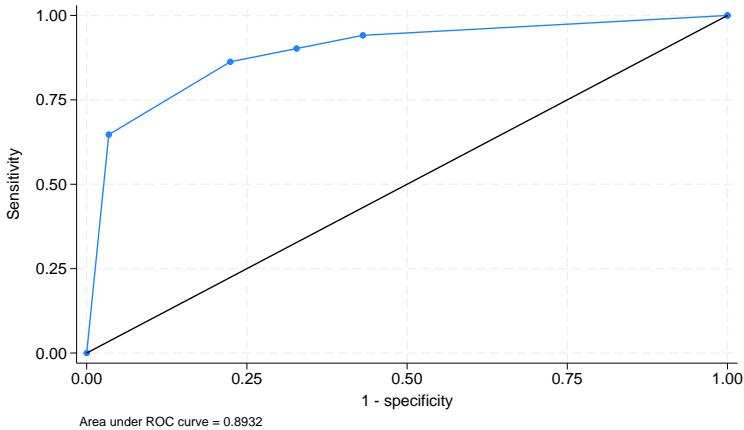
We can use `roctab` to calculate and plot the nonparametric ROC curve by specifying both the `summary` and `graph` options. By also specifying the `table` option, we obtain a contingency table summarizing our dataset.

```
. roctab disease rating, table graph summary
```

| True<br>disease<br>status of<br>subject | rating |   |   |    |    | Total |
|-----------------------------------------|--------|---|---|----|----|-------|
|                                         | 1      | 2 | 3 | 4  | 5  |       |
| 0                                       | 33     | 6 | 6 | 11 | 2  | 58    |
| 1                                       | 3      | 2 | 2 | 11 | 33 | 51    |
| Total                                   | 36     | 8 | 8 | 22 | 35 | 109   |

| Obs | ROC<br>area | Std. err. | Asymptotic normal<br>[95% conf. interval] |         |
|-----|-------------|-----------|-------------------------------------------|---------|
| 109 | 0.8932      | 0.0307    | 0.83295                                   | 0.95339 |



By default, `roctab` reports the area under the curve, its standard error, and its confidence interval. The `graph` option can be used to plot the ROC curve.

The ROC curve is plotted by computing the sensitivity and specificity using each value of the rating variable as a possible cutpoint. A point is plotted on the graph for each of the cutpoints. These plotted points are joined by straight lines to form the ROC curve, and the area under the ROC curve is computed using the trapezoidal rule.

We can tabulate the computed sensitivities and specificities for each of the possible cutpoints by specifying `detail`.

```
. roctab disease rating, detail
Detailed report of sensitivity and specificity
```

| Cutpoint | Sensitivity | Specificity | Correctly classified | LR+     | LR-    |
|----------|-------------|-------------|----------------------|---------|--------|
| ( >= 1 ) | 100.00%     | 0.00%       | 46.79%               | 1.0000  |        |
| ( >= 2 ) | 94.12%      | 56.90%      | 74.31%               | 2.1835  | 0.1034 |
| ( >= 3 ) | 90.20%      | 67.24%      | 77.98%               | 2.7534  | 0.1458 |
| ( >= 4 ) | 86.27%      | 77.59%      | 81.65%               | 3.8492  | 0.1769 |
| ( >= 5 ) | 64.71%      | 96.55%      | 81.65%               | 18.7647 | 0.3655 |
| ( > 5 )  | 0.00%       | 100.00%     | 53.21%               |         | 1.0000 |

| Obs | ROC area | Std. err. | Asymptotic normal<br>[95% conf. interval] |         |
|-----|----------|-----------|-------------------------------------------|---------|
| 109 | 0.8932   | 0.0307    | 0.83295                                   | 0.95339 |

Each cutpoint in the table indicates the ratings used to classify tomographs as being from an abnormal subject. For example, the first cutpoint ( $\geq 1$ ) indicates that all tomographs rated as 1 or greater are classified as coming from abnormal subjects. Because all tomographs have a rating of 1 or greater, all are considered abnormal. Consequently, all abnormal cases are correctly classified (sensitivity = 100%), but none of the normal patients is classified correctly (specificity = 0%). For the second cutpoint ( $\geq 2$ ), tomographs with ratings of 1 are classified as normal, and those with ratings of 2 or greater are classified as abnormal. The resulting sensitivity and specificity are 94.12% and 56.90%, respectively. Using this cutpoint, we correctly classified 74.31% of the 109 tomographs. Similar interpretations can be used on the remaining cutpoints. As mentioned, each cutpoint corresponds to a point on the nonparametric ROC curve. The first cutpoint ( $\geq 1$ ) corresponds to the point at (1,1), and the last cutpoint ( $> 5$ ) corresponds to the point at (0,0).

`detail` also reports two likelihood ratios suggested by Choi (1998): the likelihood ratio for a positive test result (LR+) and the likelihood ratio for a negative test result (LR-). The LR+ is the ratio of the probability of a positive test among the truly positive subjects to the probability of a positive test among the truly negative subjects. The LR- is the ratio of the probability of a negative test among the truly positive subjects to the probability of a negative test among the truly negative subjects. Choi points out that LR+ corresponds to the slope of the line from the origin to the point on the ROC curve determined by the cutpoint. Similarly, LR- corresponds to the slope from the point (1,1) to the point on the ROC curve determined by the cutpoint.

By default, roctab calculates the standard error for the area under the curve by using an algorithm suggested by DeLong, DeLong, and Clarke-Pearson (1988) and asymptotic normal confidence intervals. Optionally, standard errors based on methods suggested by Bamber (1975) or Hanley and McNeil (1982) can be computed by specifying `bamber` or `hanley`, respectively, and an exact binomial confidence interval can be obtained by specifying `binomial`.

```
. roctab disease rating, bamber
```

| Obs | ROC area | Bamber std. err. | Asymptotic normal [95% conf. interval] |         |
|-----|----------|------------------|----------------------------------------|---------|
| 109 | 0.8932   | 0.0306           | 0.83317                                | 0.95317 |

```
. roctab disease rating, hanley binomial
```

| Obs | ROC area | Hanley std. err. | Binomial exact [95% conf. interval] |         |
|-----|----------|------------------|-------------------------------------|---------|
| 109 | 0.8932   | 0.0320           | 0.81559                             | 0.94180 |

◀

## Lorenz-like curves

For applications where it is known that the risk status increases or decreases monotonically with increasing values of the diagnostic test, the ROC curve and associated indices are useful in assessing the overall performance of a diagnostic test. When the risk status does not vary monotonically with increasing values of the diagnostic test, however, the resulting ROC curve can be nonconvex and its indices can be unreliable. For these situations, Lee (1999) proposed an alternative to the ROC analysis based on Lorenz-like curves and the associated Pietra and Gini indices.

Lee (1999) mentions at least three specific situations where results from Lorenz curves are superior to those obtained from ROC curves: 1) a diagnostic test with similar means but very different standard deviations in the abnormal and normal populations, 2) a diagnostic test with bimodal distributions in either the normal or abnormal population, and 3) a diagnostic test distributed symmetrically in the normal population and skewed in the abnormal.

When the risk status increases or decreases monotonically with increasing values of the diagnostic test, the ROC and Lorenz curves yield interchangeable results.

## ▷ Example 2

To illustrate the use of the `lorenz` option, we constructed a fictitious dataset that yields results similar to those presented in Table III of Lee (1999). The data assume that a 12-point rating scale was used to classify 442 diseased and 442 healthy subjects. We list a few of the observations.

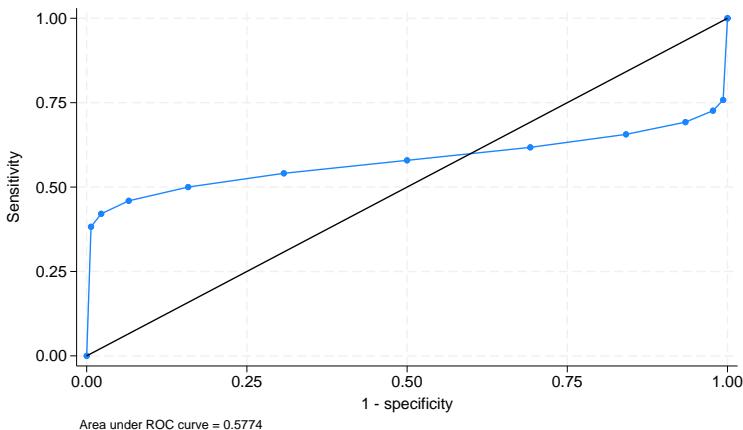
```
. use https://www.stata-press.com/data/r19/lorenz, clear
. list in 1/7, noobs sep(0)
```

| disease | class | pop |
|---------|-------|-----|
| 0       | 5     | 66  |
| 1       | 11    | 17  |
| 0       | 6     | 85  |
| 0       | 3     | 19  |
| 0       | 10    | 19  |
| 0       | 2     | 7   |
| 1       | 4     | 16  |

The data consist of 24 observations: 12 observations from diseased individuals and 12 from nondiseased individuals. Each observation corresponds to one of the 12 classification values of the rating-scale variable, `class`. The number of subjects represented by each observation is given by the `pop` variable, making this a frequency-weighted dataset. The data were generated assuming a binormal distribution of the latent variable with similar means for the normal and abnormal populations but with the standard deviation for the abnormal population five times greater than that of the normal population.

```
. roctab disease class [fweight=pop], graph summary
```

| Obs | ROC area | Std. err. | Asymptotic normal<br>[95% conf. interval] |         |
|-----|----------|-----------|-------------------------------------------|---------|
| 884 | 0.5774   | 0.0215    | 0.53517                                   | 0.61959 |



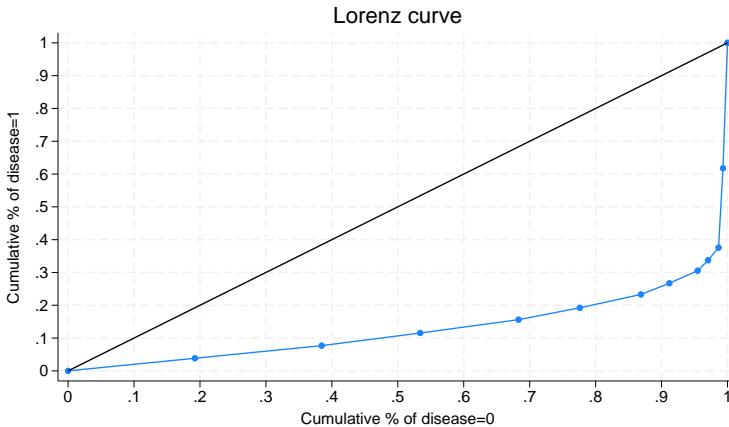
The resulting ROC curve is nonconvex or, as termed by Lee, “wiggly”. Lee argues that for this and similar situations, the Lorenz curve and indices are preferred.

```
. roctab disease class [fweight=pop], lorenz summary graph
```

```
Lorenz curve
```

```
Pietra index = 0.6493
```

```
Gini index = 0.7441
```



Like ROC curves, a more bowed Lorenz curve suggests a better diagnostic test. This bowedness is quantified by the Pietra index, which is geometrically equivalent to twice the largest triangle that can be inscribed in the area between the curve and the diagonal line, and the Gini index, which is equivalent to twice the area between the Lorenz curve and the diagonal. [Lee \(1999\)](#) provides several additional interpretations for the Pietra and Gini indices.

◀

## Stored results

roctab stores the following in `r()`:

### Scalars

|                        |                                                    |
|------------------------|----------------------------------------------------|
| <code>r(N)</code>      | number of observations                             |
| <code>r(se)</code>     | standard error for the area under the ROC curve    |
| <code>r(lb)</code>     | lower bound of CI for the area under the ROC curve |
| <code>r(ub)</code>     | upper bound of CI for the area under the ROC curve |
| <code>r(level)</code>  | confidence level                                   |
| <code>r(area)</code>   | area under the ROC curve                           |
| <code>r(pietra)</code> | Pietra index                                       |
| <code>r(gini)</code>   | Gini index                                         |

### Macros

`r(cutpoints)` description of cutpoints (detail only)

### Matrices

`r(detail)` matrix with details on sensitivity and specificity for each cutpoint (detail only)

## Methods and formulas

Assume that we applied a diagnostic test to each of  $N_n$  normal and  $N_a$  abnormal subjects. Further assume that the higher the outcome value of the diagnostic test, the higher the risk of the subject being abnormal. Let  $\hat{\theta}$  be the estimated area under the curve, and let  $X_i, i = 1, 2, \dots, N_a$  and  $Y_j, j = 1, 2, \dots, N_n$  be the values of the diagnostic test for the abnormal and normal subjects, respectively.

The points on the nonparametric ROC curve are generated using each possible outcome of the diagnostic test as a classification cutpoint and computing the corresponding sensitivity and  $1 - \text{specificity}$ . These points are then connected by straight lines, and the area under the resulting ROC curve is computed using the trapezoidal rule.

The default standard error for the area under the ROC curve is computed using the algorithm described by DeLong, DeLong, and Clarke-Pearson (1988). For each abnormal subject,  $i$ , define

$$V_{10}(X_i) = \frac{1}{N_n} \sum_{j=1}^{N_n} \psi(X_i, Y_j)$$

and for each normal subject,  $j$ , define

$$V_{01}(Y_j) = \frac{1}{N_a} \sum_{i=1}^{N_a} \psi(X_i, Y_j)$$

where

$$\psi(X, Y) = \begin{cases} 1 & Y < X \\ \frac{1}{2} & Y = X \\ 0 & Y > X \end{cases}$$

Define

$$S_{10} = \frac{1}{N_a - 1} \sum_{i=1}^{N_a} \{V_{10}(X_i) - \hat{\theta}\}^2$$

and

$$S_{01} = \frac{1}{N_n - 1} \sum_{j=1}^{N_n} \{V_{01}(Y_j) - \hat{\theta}\}^2$$

The variance of the estimated area under the ROC curve is given by

$$\text{var}(\hat{\theta}) = \frac{1}{N_a} S_{10} + \frac{1}{N_n} S_{01}$$

The Hanley standard error for the area under the ROC curve is computed using the algorithm described by Hanley and McNeil (1982). It requires the calculation of two quantities:  $Q_1$  is  $\text{Pr}(\text{two randomly selected abnormal subjects will both have a higher score than a randomly selected normal subject})$ , and  $Q_2$  is  $\text{Pr}(\text{one randomly selected abnormal subject will have a higher score than any two randomly selected normal subjects})$ . The Hanley and McNeil variance of the estimated area under the ROC curve is

$$\text{var}(\hat{\theta}) = \frac{\hat{\theta}(1 - \hat{\theta}) + (N_a - 1)(Q_1 - \hat{\theta}^2) + (N_n - 1)(Q_2 - \hat{\theta}^2)}{N_a N_n}$$

The Bamber standard error for the area under the ROC curve is computed using the algorithm described by Bamber (1975). For any two  $Y$  values,  $Y_j$  and  $Y_k$ , and any  $X_i$  value, define

$$b_{yyx} = p(Y_j, Y_k < X_i) + p(X_i < Y_j, Y_k) - 2p(Y_j < X_i < Y_k)$$

and similarly, for any two  $X$  values,  $X_i$  and  $X_l$ , and any  $Y_j$  value, define

$$b_{xxy} = p(X_i, X_l < Y_j) + p(Y_j < X_i, X_l) - 2p(X_i < Y_j < X_l)$$

Bamber's unbiased estimate of the variance for the area under the ROC curve is

$$\text{var}(\hat{\theta}) = \frac{1}{4}(N_a - 1)(N_n - 1)\{p(X \neq Y) + (N_a - 1)b_{xxy} + (N_n - 1)b_{yyx} - 4(N_a + N_n - 1)(\hat{\theta} - 0.5)^2\}$$

Asymptotic confidence intervals are constructed and reported by default, assuming a normal distribution for the area under the ROC curve.

Exact binomial confidence intervals are calculated as described in [R] ci, with  $p$  equal to the area under the ROC curve.

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## Also see

[R] **logistic postestimation** — Postestimation tools for logistic

[R] **roc** — Receiver operating characteristic (ROC) analysis

[R] **roccomp** — Tests of equality of ROC areas

[R] **rocfit** — Parametric ROC models

[R] **rocreg** — Parametric and nonparametric ROC regression

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## Description

`rreg` performs one version of robust regression of *depvar* on *indepvars*.

Also see [Robust standard errors](#) in [\[R\] regress](#) for standard regression with robust variance estimates and [\[R\] qreg](#) for quantile (including median) regression.

## Quick start

Robust regression of *y* on *x*

```
rreg y x
```

Add a categorical covariate *a* using factor-variable syntax

```
rreg y x i.a
```

Same as above, but set the tuning parameter to 8

```
rreg y x i.a, tune(8)
```

Generate a new variable *wvar* containing the final weight for each observation

```
rreg y x i.a, genwt(wvar)
```

Set confidence level to 99%

```
rreg y x i.a, level(99)
```

## Menu

Statistics > Linear models and related > Other > Robust regression

## Syntax

```
rreg depvar [indepvars] [if] [in] [, options]
```

| <i>options</i>                    | Description                                                                                                                                      |
|-----------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                      |                                                                                                                                                  |
| <code>tune(#)</code>              | use # as the biweight tuning constant; default is <code>tune(7)</code>                                                                           |
| <b>Reporting</b>                  |                                                                                                                                                  |
| <code>level(#)</code>             | set confidence level; default is <code>level(95)</code>                                                                                          |
| <code>genwt(<i>newvar</i>)</code> | create <i>newvar</i> containing the weights assigned to each observation                                                                         |
| <code>display_options</code>      | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| <b>Optimization</b>               |                                                                                                                                                  |
| <code>optimization_options</code> | control the optimization process; seldom used                                                                                                    |
| <code>graph</code>                | graph weights during convergence                                                                                                                 |
| <code>coeflegend</code>           | display legend instead of statistics                                                                                                             |

*indepvars* may contain factor variables; see [U] 11.4.3 **Factor variables**.

*depvar* and *indepvars* may contain time-series operators; see [U] 11.4.4 **Time-series varlists**.

by, collect, mfp, mi estimate, rolling, and statsby are allowed; see [U] 11.1.10 **Prefix commands**.

coeflegend does not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options

### Model

`tune(#)` is the biweight tuning constant. The default is `tune(7)`, meaning seven times the median absolute deviation from the median residual; see *Methods and formulas*. Lower tuning constants downweight outliers rapidly but may lead to unstable estimates (less than 6 is not recommended). Higher tuning constants produce milder downweighting.

### Reporting

`level(#)`; see [R] **Estimation options**.

`genwt(newvar)` creates the new variable *newvar* containing the weights assigned to each observation.

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] **Estimation options**.

### Optimization

`optimization_options`: `iterate(#)`, `tolerance(#)`, `[no]log`. `iterate()` specifies the maximum number of iterations; iterations stop when the maximum change in weights drops below `tolerance()`; and `log/nolog` specifies whether to show the iteration log (see `set iterlog` in [R] *set iter*). These options are seldom used.

graph allows you to graphically watch the convergence of the iterative technique. The weights obtained from the most recent round of estimation are graphed against the weights obtained from the previous round.

The following option is available with rreg but is not shown in the dialog box:

coeflegend; see [R] Estimation options.

## Remarks and examples

rreg first performs an initial screening based on Cook’s distance  $> 1$  to eliminate gross outliers before calculating starting values and then performs Huber iterations followed by biweight iterations, as suggested by Li (1985).

### ▷ Example 1

We wish to examine the relationship between mileage rating, weight, and location of manufacture for the 74 cars in the auto.dta. As a point of comparison, we begin by fitting an ordinary regression:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. regress mpg weight foreign
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 1619.2877  | 2  | 809.643849 | F(2, 71)      | = | 69.75  |
| Residual | 824.171761 | 71 | 11.608053  | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.6627 |
|          |            |    |            | Adj R-squared | = | 0.6532 |
| Total    | 2443.45946 | 73 | 33.4720474 | Root MSE      | = | 3.4071 |

| mpg     | Coefficient | Std. err. | t      | P> t  | [95% conf. interval] |
|---------|-------------|-----------|--------|-------|----------------------|
| weight  | -.0065879   | .0006371  | -10.34 | 0.000 | -.0078583 -.0053175  |
| foreign | -1.650029   | 1.075994  | -1.53  | 0.130 | -3.7955 .4954422     |
| _cons   | 41.6797     | 2.165547  | 19.25  | 0.000 | 37.36172 45.99768    |

We now compare this with the results from rreg:

```
. rreg mpg weight foreign
Huber iteration 1: Maximum difference in weights = .80280176
Huber iteration 2: Maximum difference in weights = .2915438
Huber iteration 3: Maximum difference in weights = .08911171
Huber iteration 4: Maximum difference in weights = .02697328
Biweight iteration 5: Maximum difference in weights = .29186818
Biweight iteration 6: Maximum difference in weights = .11988101
Biweight iteration 7: Maximum difference in weights = .03315872
Biweight iteration 8: Maximum difference in weights = .00721325
Robust regression
```

| mpg     | Coefficient | Std. err. | t      | P> t  | [95% conf. interval] |
|---------|-------------|-----------|--------|-------|----------------------|
| weight  | -.0063976   | .0003718  | -17.21 | 0.000 | -.007139 -.0056562   |
| foreign | -3.182639   | .627964   | -5.07  | 0.000 | -4.434763 -1.930514  |
| _cons   | 40.64022    | 1.263841  | 32.16  | 0.000 | 38.1202 43.16025     |

Note the large change in the foreign coefficient.



□ Technical note

It would have been better if we had fit the previous robust regression by typing `rreg mpg weight foreign, genwt(w)`. The new variable, `w`, would then contain the estimated weights. Let's pretend that we did this:

```
. rreg mpg weight foreign, genwt(w)
(output omitted)
. summarize w, detail
```

| Robust Regression Weight |          |          |             |           |
|--------------------------|----------|----------|-------------|-----------|
| Percentiles              | Smallest |          |             |           |
| 1%                       | 0        | 0        |             |           |
| 5%                       | .0442957 | 0        |             |           |
| 10%                      | .4674935 | 0        | Obs         | 74        |
| 25%                      | .8894815 | .0442957 | Sum of wgt. | 74        |
| 50%                      | .9690193 |          | Mean        | .8509966  |
|                          |          | Largest  | Std. dev.   | .2746451  |
| 75%                      | .9949395 | .9996715 |             |           |
| 90%                      | .9989245 | .9996953 | Variance    | .0754299  |
| 95%                      | .9996715 | .9997343 | Skewness    | -2.287952 |
| 99%                      | .9998585 | .9998585 | Kurtosis    | 6.874605  |

We discover that 3 observations in our data were dropped altogether (they have weight 0). We could further explore our data:

```
. sort w
. list make mpg weight w if w <.467, sep(0)
```

|    | make           | mpg | weight | w         |
|----|----------------|-----|--------|-----------|
| 1. | Datsun 210     | 35  | 2,020  | 0         |
| 2. | Subaru         | 35  | 2,050  | 0         |
| 3. | VW Diesel      | 41  | 2,040  | 0         |
| 4. | Plym. Arrow    | 28  | 3,260  | .04429567 |
| 5. | Cad. Seville   | 21  | 4,290  | .08241943 |
| 6. | Toyota Corolla | 31  | 2,200  | .10443129 |
| 7. | Olds 98        | 21  | 4,060  | .28141296 |

Being familiar with the automobile data, we immediately spotted two things: the VW is the only diesel car in our data, and the weight recorded for the Plymouth Arrow is incorrect.



## ▷ Example 2

If we specify no explanatory variables, `rreg` produces a robust estimate of the mean:

```
. rreg mpg
 Huber iteration 1: Maximum difference in weights = .64471879
 Huber iteration 2: Maximum difference in weights = .05098336
 Huber iteration 3: Maximum difference in weights = .0099887
 Biweight iteration 4: Maximum difference in weights = .25197391
 Biweight iteration 5: Maximum difference in weights = .00358606
Robust regression Number of obs = 74
 F(0, 73) = 0.00
 Prob > F = .
```

| mpg   | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|-------|-------------|-----------|-------|-------|----------------------|----------|
| _cons | 20.68825    | .641813   | 32.23 | 0.000 | 19.40912             | 21.96738 |

The estimate is given by the coefficient on `_cons`. The mean is 20.69 with an estimated standard error of 0.6418. The 95% confidence interval is [19.4, 22.0]. By comparison, `ci means` (see [R] `ci`) gives us the standard calculation:

```
. ci means mpg
 Variable | Obs Mean Std. err. [95% conf. interval]
 -----+-----
 mpg | 74 21.2973 .6725511 19.9569 22.63769
```

Stata has an active user community working in the area of robust statistics. You may also be interested in robust estimators and outlier detection tools from [Verardi and McCathie \(2012\)](#), [Verardi and Dehon \(2010\)](#), [Jann \(2010\)](#), and [Verardi and Croux \(2009\)](#), to name a few. See [R] `ssc` for how to install community-contributed packages.

## Stored results

rreg stores the following in `e()`:

### Scalars

|                      |                             |
|----------------------|-----------------------------|
| <code>e(N)</code>    | number of observations      |
| <code>e(mss)</code>  | model sum of squares        |
| <code>e(df_m)</code> | model degrees of freedom    |
| <code>e(rss)</code>  | residual sum of squares     |
| <code>e(df_r)</code> | residual degrees of freedom |
| <code>e(r2)</code>   | $R^2$                       |
| <code>e(r2_a)</code> | adjusted $R^2$              |
| <code>e(F)</code>    | $F$ statistic               |
| <code>e(rmse)</code> | root mean squared error     |
| <code>e(rank)</code> | rank of $e(V)$              |

### Macros

|                            |                                      |
|----------------------------|--------------------------------------|
| <code>e(cmd)</code>        | rreg                                 |
| <code>e(cmdline)</code>    | command as typed                     |
| <code>e(depvar)</code>     | name of dependent variable           |
| <code>e(genwt)</code>      | variable containing the weights      |
| <code>e(title)</code>      | title in estimation output           |
| <code>e(model)</code>      | ols                                  |
| <code>e(properties)</code> | b V                                  |
| <code>e(predict)</code>    | program used to implement predict    |
| <code>e(marginsok)</code>  | predictions allowed by margins       |
| <code>e(asbalanced)</code> | factor variables fvset as asbalanced |
| <code>e(asobserved)</code> | factor variables fvset as asobserved |

### Matrices

|                   |                                              |
|-------------------|----------------------------------------------|
| <code>e(b)</code> | coefficient vector                           |
| <code>e(V)</code> | variance–covariance matrix of the estimators |

### Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

### Matrices

|                       |                                                                                                                       |
|-----------------------|-----------------------------------------------------------------------------------------------------------------------|
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, $p$ -values, and confidence intervals |
|-----------------------|-----------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r`-class command is run after the estimation command.

## Methods and formulas

See Berk (1990), Goodall (1983), and Rousseeuw and Leroy (1987) for a general description of the issues and methods. Hamilton (1991a, 1992) provides a more detailed description of rreg and some Monte Carlo evaluations.

rreg begins by fitting the regression (see [R] regress), calculating Cook's  $D$  (see [R] predict and [R] regress postestimation), and excluding any observation for which  $D > 1$ .

Thereafter, rreg works iteratively: it performs a regression, calculates case weights from absolute residuals, and regresses again using those weights. Iterations stop when the maximum change in weights drops below `tolerance()`. Weights derive from one of two weight functions, Huber weights and biweights. Huber weights (Huber 1964) are used until convergence, and then, from that result, biweights are used until convergence. The biweight was proposed by Beaton and Tukey (1974, 151–152) after

the Princeton robustness study (Andrews et al. 1972) had compared various estimators. Both weighting functions are used because Huber weights have problems dealing with severe outliers, whereas biweights sometimes fail to converge or have multiple solutions. The initial Huber weighting should improve the behavior of the biweight estimator.

In Huber weighting, cases with small residuals receive weights of 1; cases with larger residuals receive gradually smaller weights. Let  $e_i = y_i - \mathbf{X}_i \mathbf{b}$  represent the  $i$ th-case residual. The  $i$ th scaled residual  $u_i = e_i/s$  is calculated, where  $s = M/0.6745$  is the residual scale estimate and  $M = \text{med}(|e_i - \text{med}(e_i)|)$  is the median absolute deviation from the median residual. Huber estimation obtains case weights:

$$w_i = \begin{cases} 1 & \text{if } |u_i| \leq c_h \\ c_h/|u_i| & \text{otherwise} \end{cases}$$

rreg defines  $c_h = 1.345$ , so downweighting begins with cases whose absolute residual exceeds  $(1.345/0.6745)M \approx 2M$ .

With biweights, all cases with nonzero residuals receive some downweighting, according to the smoothly decreasing biweight function

$$w_i = \begin{cases} \{1 - (u_i/c_b)^2\}^2 & \text{if } |u_i| \leq c_b \\ 0 & \text{otherwise} \end{cases}$$

where  $c_b = 4.685 \times \text{tune}()/7$ . Thus when  $\text{tune}() = 7$ , cases with absolute residuals of  $(4.685/0.6745)M \approx 7M$  or more are assigned 0 weight and thus are effectively dropped. Goodall (1983, 377) suggests using a value between 6 and 9, inclusive, for  $\text{tune}()$  in the biweight case and states that performance is good between 6 and 12, inclusive.

The tuning constants  $c_h = 1.345$  and  $c_b = 4.685$  (assuming  $\text{tune}()$  is set at the default 7) give rreg about 95% of the efficiency of OLS when applied to data with normally distributed errors (Hamilton 1991b). Lower tuning constants downweight outliers more drastically (but give up Gaussian efficiency); higher tuning constants make the estimator more like OLS.

Standard errors are calculated using the pseudovalues approach described in Street, Carroll, and Rupert (1988).

## Acknowledgment

The current version of rreg is due to the work of Lawrence Hamilton of the Department of Sociology at the University of New Hampshire.

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## Also see

- [R] [rreg postestimation](#) — Postestimation tools for rreg
- [R] [qreg](#) — Quantile regression
- [R] [regress](#) — Linear regression
- [MI] [Estimation](#) — Estimation commands for use with mi estimate
- [U] [20 Estimation and postestimation commands](#)

## Postestimation commands

The following postestimation commands are available after `rreg`:

| Command                      | Description                                                                                       |
|------------------------------|---------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                               |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                      |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                |
| <code>estimates</code>       | cataloging estimation results                                                                     |
| <code>etable</code>          | table of estimation results                                                                       |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                 |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters    |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                           |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters |
| <code>predict</code>         | predictions and their SEs, residuals, etc.                                                        |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions              |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                              |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                |

\*`forecast` is not appropriate with `mi` estimation results.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as linear predictions, standard errors, residuals, and diagonal elements of the hat matrix.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic]
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

---

Main

|                        |                                         |
|------------------------|-----------------------------------------|
| <code>xb</code>        | linear prediction; the default          |
| <code>stdp</code>      | standard error of the linear prediction |
| <code>residuals</code> | residuals                               |
| <code>hat</code>       | diagonal elements of the hat matrix     |

---

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

## Options for predict

Main

`xb`, the default, calculates the linear prediction.

`stdp` calculates the standard error of the linear prediction.

`residuals` calculates the residuals.

`hat` calculates the diagonal elements of the hat matrix. You must have run the `rreg` command with the `genwt()` option.

# margins

## Description for margins

`margins` estimates margins of response for linear predictions.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [options]
```

| <i>statistic</i>              | Description                           |
|-------------------------------|---------------------------------------|
| <code>xb</code>               | linear prediction; the default        |
| <code>stdp</code>             | not allowed with <code>margins</code> |
| <code><u>r</u>esiduals</code> | not allowed with <code>margins</code> |
| <code>hat</code>              | not allowed with <code>margins</code> |

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [\[R\] margins](#).

## Also see

[\[R\] rreg](#) — Robust regression

[\[U\] 20 Estimation and postestimation commands](#)

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## Description

`runtest` tests whether the observations of *varname* are serially independent—that is, whether they occur in a random order—by counting how many runs there are above and below a threshold. By default, the median is used as the threshold. A small number of runs indicates positive serial correlation; a large number indicates negative serial correlation.

## Quick start

Run test for the serial independence of the observations of `v1`

```
runtest v1
```

Use the mean of `v1`, rather than the median, as the threshold for counting runs

```
runtest v1, mean
```

Use 10 as the threshold for counting runs

```
runtest v1, threshold(10)
```

Same as above, but ignore any values of `v1` equal to the threshold when counting runs

```
runtest v1, threshold(10) drop
```

## Menu

Statistics > Nonparametric analysis > Tests of hypotheses > Test for random order

## Syntax

```
runtest varname [in] [, options]
```

| <i>options</i>            | Description                                                                                                |
|---------------------------|------------------------------------------------------------------------------------------------------------|
| <code>continuity</code>   | continuity correction                                                                                      |
| <code>drop</code>         | ignore values equal to the threshold                                                                       |
| <code>split</code>        | randomly split values equal to the threshold as above or below the threshold; default is to count as below |
| <code>mean</code>         | use mean as threshold; default is median                                                                   |
| <code>threshold(#)</code> | assign arbitrary threshold; default is median                                                              |

`collect` is allowed; see [U] 11.1.10 Prefix commands.

## Options

`continuity` specifies a continuity correction that may be helpful in small samples. If there are fewer than 10 observations either above or below the threshold, however, the tables in [Swed and Eisenhart \(1943\)](#) provide more reliable critical values. By default, no continuity correction is used.

`drop` directs `runtest` to ignore any values of *varname* that are equal to the threshold value when counting runs and tabulating observations. By default, `runtest` counts a value as being above the threshold when it is strictly above the threshold and as being below the threshold when it is less than or equal to the threshold.

`split` directs `runtest` to randomly split values of *varname* that are equal to the threshold. In other words, when *varname* is equal to threshold, a “coin” is flipped. If it comes up heads, the value is counted as above the threshold. If it comes up tails, the value is counted as below the threshold.

`mean` directs `runtest` to tabulate runs above and below the mean rather than the median.

`threshold(#)` specifies an arbitrary threshold to use in counting runs. For example, if *varname* has already been coded as a 0/1 variable, the median generally will not be a meaningful separating value.

## Remarks and examples

`runtest` performs a nonparametric test of the hypothesis that the observations of *varname* occur in a random order by counting how many runs there are above and below a threshold. If *varname* is positively serially correlated, it will tend to remain above or below its median for several observations in a row; that is, there will be relatively few runs. If, on the other hand, *varname* is negatively serially correlated, observations above the median will tend to be followed by observations below the median and vice versa; that is, there will be relatively many runs.

By default, `runtest` uses the median for the threshold, and this is not necessarily the best choice. If `mean` is specified, the mean is used instead of the median. If `threshold(#)` is specified, `#` is used. Because `runtest` divides the data into two states—above and below the threshold—it is appropriate for data that are already binary; for example, win or lose, live or die, rich or poor, etc. Such variables are often coded as 0 for one state and 1 for the other. Here you should specify `threshold(0)` because, by default, `runtest` separates the observations into those that are greater than the threshold and those that are less than or equal to the threshold.

As with most nonparametric procedures, the treatment of ties complicates the test. Observations equal to the threshold value are ties and can be treated in one of three ways. By default, they are treated as if they were below the threshold. If `drop` is specified, they are omitted from the calculation and the total number of observations is adjusted. If `split` is specified, each is randomly assigned to the above- and below-threshold groups. The random assignment is different each time the procedure is run unless you specify the random-number seed; see [R] [set seed](#).

### ▷ Example 1

We can use `runtest` to check regression residuals for serial correlation.

```
. use https://www.stata-press.com/data/r19/run1
. scatter resid year, connect(1) yline(0) title(Regression residuals)
```



The graph gives the impression that these residuals are positively correlated. Excursions above or below zero—the natural threshold for regression residuals—tend to last for several observations. `runtest` can evaluate the statistical significance of this impression.

```
. runttest resid, thresh(0)
N(resid <= 0) = 8
N(resid > 0) = 8
 obs = 16
 N(runs) = 5
 z = -2.07
 Prob>|z| = .04
```

There are five runs in these 16 observations. Using the normal approximation to the true distribution of the number of runs, the five runs in this series are fewer than would be expected if the residuals were serially independent. The  $p$ -value is 0.04, indicating a two-sided significant result at the 5% level. If the alternative hypothesis is positive serial correlation, rather than any deviation from randomness, then the one-sided  $p$ -value is  $0.04/2 = 0.015$ . With so few observations, however, the normal approximation may be inaccurate. (Tables compiled by Swed and Eisenhart list five runs as the 5% critical value for a one-sided test.)

`runtest` is a nonparametric test. It ignores the magnitudes of the observations and notes only whether the values are above or below the threshold. We can demonstrate this feature by reducing the information about the regression residuals in this example to a 0/1 variable that indicates only whether a residual is positive or negative.

```
. generate byte sign = resid>0
. runtest sign, thresh(0)
N(sign <= 0) = 8
N(sign > 0) = 8
 obs = 16
 N(runs) = 5
 z = -2.07
 Prob>|z| = .04
```

As expected, runtest produces the same answer as before.



### □ Technical note

The run test can also be used to test the null hypothesis that two samples are drawn from the same underlying distribution. The run test is sensitive to differences in the shapes, as well as the locations, of the empirical distributions.

Suppose, for example, that two different additives are added to the oil in 10 different cars during an oil change. The cars are run until a viscosity test determines that another oil change is needed, and the number of miles traveled between oil changes is recorded. The data are

```
. use https://www.stata-press.com/data/r19/additive, clear
. list
```

|     | additive | miles |
|-----|----------|-------|
| 1.  | 1        | 4024  |
| 2.  | 1        | 4756  |
| 3.  | 1        | 7993  |
| 4.  | 1        | 5025  |
| 5.  | 1        | 4188  |
| 6.  | 2        | 3007  |
| 7.  | 2        | 1988  |
| 8.  | 2        | 1051  |
| 9.  | 2        | 4478  |
| 10. | 2        | 4232  |

To test whether the additives generate different distributions of miles between oil changes, we sort the data by miles and then use runtest to see whether the marker for each additive occurs in random order:

```
. sort miles
. runtest additive, thresh(1)
N(additive <= 1) = 5
N(additive > 1) = 5
 obs = 10
 N(runs) = 4
 z = -1.34
 Prob>|z| = .18
```

Here the additives do not produce statistically different results.



□ Technical note

A test that is related to the run test is the runs up-and-down test. In the latter test, the data are classified not by whether they lie above or below a threshold but by whether they are steadily increasing or decreasing. Thus an unbroken string of increases in the variable of interest is counted as one run, as is an unbroken string of decreases. According to Madansky (1988), the run test is superior to the runs up-and-down test for detecting trends in the data, but the runs up-and-down test is superior for detecting autocorrelation.

runtest can be used to perform a runs up-and-down test. Using the regression residuals from the example above, we can perform a runtest on their first differences:

```
. use https://www.stata-press.com/data/r19/run1
. generate resid_D = resid - resid[_n-1]
(1 missing value generated)
. runtest resid_D, thresh(0)
N(resid_D <= 0) = 7
N(resid_D > 0) = 8
 obs = 15
 N(runs) = 6
 z = -1.33
 Prob>|z| = .18
```

Edgington (1961) has compiled a table of the small-sample distribution of the runs up-and-down statistic, and this table is reprinted in Madansky (1988). For large samples, the  $z$  statistic reported by runtest is incorrect for the runs up-and-down test. Let  $N$  be the number of observations (15 here), and let  $r$  be the number of runs (6). The expected number of runs in the runs up-and-down test is

$$\mu_r = \frac{2N - 1}{3}$$

the variance is

$$\sigma_r^2 = \frac{16N - 29}{90}$$

and the correct  $z$  statistic is

$$\hat{z} = \frac{r - \mu_r}{\sigma_r}$$

□

□ Technical note

runtest will tolerate missing values at the beginning or end of a series, as occurred in the technical note above (generating first differences resulted in a missing value for the first observation). runtest, however, will issue an error message if there are any missing observations in the interior of the series (in the portion covered by the in range qualifier). To perform the test anyway, simply drop the missing observations before using runtest.

□

## Stored results

runtest stores the following in `r()`:

Scalars

|                         |                            |                        |                                |
|-------------------------|----------------------------|------------------------|--------------------------------|
| <code>r(N)</code>       | number of observations     | <code>r(p)</code>      | <i>p</i> -value of <i>z</i>    |
| <code>r(N_below)</code> | number below the threshold | <code>r(z)</code>      | <i>z</i> statistic             |
| <code>r(N_above)</code> | number above the threshold | <code>r(n_runs)</code> | number of runs                 |
| <code>r(mean)</code>    | expected number of runs    | <code>r(Var)</code>    | variance of the number of runs |

## Methods and formulas

runtest begins by calculating the number of observations below the threshold,  $n_0$ ; the number of observations above the threshold,  $n_1$ ; the total number of observations,  $N = n_0 + n_1$ ; and the number of runs,  $r$ . These statistics are always reported, so the exact tables of critical values in [Swed and Eisenhart \(1943\)](#) may be consulted if necessary.

The expected number of runs under the null is

$$\mu_r = \frac{2n_0n_1}{N} + 1$$

the variance is

$$\sigma_r^2 = \frac{2n_0n_1(2n_0n_1 - N)}{N^2(N - 1)}$$

and the normal approximation test statistic is

$$\hat{z} = \frac{r - \mu_r}{\sigma_r}$$

## Acknowledgment

runtest was written by Sean Beckett, author of the Stata Press book [Introduction to Time Series Using Stata, Revised Edition](#).

## References

- Edgington, E. S. 1961. Probability table for number of runs of signs of first differences in ordered series. *Journal of the American Statistical Association* 56: 156–159. <https://doi.org/10.2307/2282342>.
- Madansky, A. 1988. *Prescriptions for Working Statisticians*. New York: Springer. <https://doi.org/10.1007/978-1-4612-3794-5>.
- Swed, F. S., and C. Eisenhart. 1943. Tables for testing randomness of grouping in a sequence of alternatives. *Annals of Mathematical Statistics* 14: 66–87. <https://doi.org/10.1214/aoms/1177731494>.

|                             |                                      |                                |                                      |
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## Description

`rwgen bsample` and `rwgen bayes` generate replicate weights for each observation that can be used by the `bootstrap` prefix to enhance reproducibility during bootstrap estimation.

`rwgen bsample` executes the `bsample` command on the original dataset one or more times, saving the generated frequency weights in newly created variables. These frequency weights can be specified in the `fweights()` option of the `bootstrap` prefix.

`rwgen bayes` generates weights using Bayesian bootstrap methods. These weights can be specified in the `iweights()` option of the `bootstrap` prefix.

## Quick start

Generate bootstrap frequency weights for the default 50 replications, and store them in new variables `stubvar1` through `stubvar50`

```
rwgen bsample stubvar
```

Same as above, but sample only five observations for each replication

```
rwgen bsample stubvar, size(5)
```

Generate bootstrap frequency weights for each stratum defined by `x1`

```
rwgen bsample stubvar, strata(x1)
```

Generate bootstrap frequency weights by sampling clusters identified by values of `cvar`

```
rwgen bsample stubvar, cluster(cvar)
```

Same as above, but create a new unique identifier for sampled clusters, and store it in `newcvar`

```
rwgen bsample stubvar, cluster(cvar) idcluster(newcvar)
```

Generate Bayesian bootstrap weights using default prior powers for 100 replications

```
rwgen bayes stubvar, reps(100)
```

Same as above, but use the values of variable `x1` as the prior powers for each observation

```
rwgen bayes stubvar, reps(100) priorpowers(x1)
```

Same as above but use a combination of `x2` and `x3` to uniquely identify the observations

```
rwgen bayes stubvar, reps(100) priorpowers(x1) id(x2 x3)
```

## Menu

Statistics > Resampling > Generate bootstrap replicate weights

## Syntax

Generate replicate weights using random sampling with replacement

```
rwgen bsample stub [if] [in] [, bsample_options]
```

Generate replicate weights using Bayesian bootstrap methods

```
rwgen bayes stub [if] [in] [, bayes_options]
```

| <i>bsample_options</i>                | Description                                                                                                   |
|---------------------------------------|---------------------------------------------------------------------------------------------------------------|
| Main                                  |                                                                                                               |
| <code>reps(#)</code>                  | perform # bootstrap replications; default is <code>reps(50)</code>                                            |
| <code>size(<i>exp</i>)</code>         | draw samples of size <i>exp</i> , where <i>exp</i> is a standard Stata expression; default is <code>_N</code> |
| <code>strata(<i>varlist</i>)</code>   | specify variables identifying strata                                                                          |
| <code>cluster(<i>varlist</i>)</code>  | specify variables identifying resampling clusters                                                             |
| <code>idcluster(<i>newvar</i>)</code> | create new cluster ID variable                                                                                |
| <code>rseed(#)</code>                 | set random-number seed to #                                                                                   |

| <i>bayes_options</i>                       | Description                                                                        |
|--------------------------------------------|------------------------------------------------------------------------------------|
| Main                                       |                                                                                    |
| <code>reps(#)</code>                       | perform # bootstrap replications; default is <code>reps(50)</code>                 |
| <code>priorpowers(# <i>varname</i>)</code> | specify prior powers for each observation; default is <code>priorpowers(-1)</code> |
| <code>id(<i>varlist</i>)</code>            | specify variables that uniquely identify observations                              |
| <code>rseed(#)</code>                      | set random-number seed to #                                                        |

## Options

Options are presented under the following headings:

[Options for `rwgen bsample`](#)

[Options for `rwgen bayes`](#)

### Options for `rwgen bsample`

#### Main

`reps(#)` specifies the number of replicate weight variables to generate; the default is `reps(50)`. The generated weights are stored in new variables named `stub1`, `stub2`, ..., `stub#`.

`size(exp)` specifies the size of the sample and must be less than or equal to `_N` when neither the `cluster()` nor the `strata()` option is specified; default is `_N`.

`strata(varlist)` specifies the variables that identify strata. If `strata()` is specified, bootstrap samples are selected within each stratum, and `size()` must be less than or equal to `_N` within the defined strata.

`cluster(varlist)` specifies the variables identifying resampling clusters. If `cluster()` is specified, the sample drawn during each replication is a bootstrap sample of clusters, and `size()` must be less than or equal to `N_c` (the number of clusters identified by `cluster()`). If `strata()` is also specified, `size()` must be less than or equal to the number of within-strata clusters.

`idcluster(newvar)` creates a new variable containing a unique identifier for each resampled cluster.

`rseed(#)` sets the random-number seed. This option can be used to reproduce results. `rseed(#)` is equivalent to typing `set seed #` prior to calling `rwgen`; see [R] [set seed](#).

## Options for `rwgen bayes`

Main

`reps(#)` specifies the number of replicate weight variables to generate; the default is `reps(50)`. The generated weights are stored in new variables named `stub1`, `stub2`, ..., `stub#`.

`priorpowers(#varname)` specifies the power value  $l_i$  of the prior distribution for the  $i$ th observation,  $i = 1, 2, \dots, n$ . The default is `priorpowers(-1)`, which corresponds to an improper noninformative prior. `priorpowers(#)` specifies the same power `#` for all observations. Alternatively, you can specify different power values in `varname`. See [Methods and formulas](#). The variable specified in `priorpowers()` may not be specified in `id()`.

`id(varlist)` specifies the variables used to uniquely identify each observation. By default, `rwgen bayes` considers all variables in the dataset to assess uniqueness. Any variables specified in `id()` may not be specified in `priorpowers()`.

`rseed(#)` sets the random-number seed. This option can be used to reproduce results. `rseed(#)` is equivalent to typing `set seed #` prior to calling `rwgen`; see [R] [set seed](#).

## Remarks and examples

The `rwgen bsample` and `rwgen bayes` commands generate replicate weights that can be used by the bootstrap prefix to enhance reproducibility during bootstrap estimation.

`rwgen bsample` executes the `bsample` command on the original dataset one or more times to generate replicate weights. You can randomly resample observations, observations within strata, or clusters. The frequency weights generated by `rwgen bsample` can then be specified in the `fweights()` option of the bootstrap prefix.

`rwgen bayes` generates weights using Bayesian bootstrap methods. These weights can be specified in the `weights()` option of the bootstrap prefix.

### ► Example 1: Generate replicate weights for bootstrap

We wish to compute bootstrap estimates for the standard errors of the coefficients from a regression of car price on car weight and origin. We first load `auto.dta`:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
```

We use `rwgen bsample` to draw random samples of the data, with replacement, and specify that we want 100 replications. For reproducibility, we set the seed using the `rseed()` option. `rwgen bsample` will store the frequency weights in newly generated variables using the stub we specify, `freq_`.

```
. rwgen bsample freq_, reps(100) rseed(20)
```

After we run the command, our dataset now contains 100 newly generated variables named `freq_1`, `freq_2`, ..., `freq_100`. Below, we list the first 10 observations of `freq_1` through `freq_5`:

```
. list freq_1-freq_5 in 1/10
```

|       | freq_1 | freq_2 | freq_3 | freq_4 | freq_5 |
|-------|--------|--------|--------|--------|--------|
| 1.    | 2      | 0      | 1      | 0      | 0      |
| 2.    | 1      | 2      | 2      | 0      | 1      |
| 3.    | 2      | 2      | 2      | 0      | 3      |
| 4.    | 0      | 0      | 1      | 0      | 2      |
| 5.    | 0      | 1      | 1      | 0      | 0      |
| <hr/> |        |        |        |        |        |
| 6.    | 1      | 1      | 1      | 0      | 0      |
| 7.    | 2      | 1      | 1      | 1      | 4      |
| 8.    | 1      | 2      | 1      | 2      | 1      |
| 9.    | 1      | 2      | 0      | 2      | 0      |
| 10.   | 2      | 0      | 2      | 1      | 0      |

Each of these frequency weights represents and can be used to generate a random sample with replacement. For instance, in the first bootstrap sample, neither the fourth nor the fifth observation is present. However, in the fifth sample, the fourth observation appears two times.

The sum of the values in any of the newly generated weights will give us the sample size:

```
. quietly summarize freq_1
. display r(sum)
74
```

The result above matches the number of observations in our original dataset.

Now that we have generated the frequency weights, we can specify them in the `fweights()` option of the bootstrap prefix to perform a regression using the bootstrap sampled datasets:

```
. bootstrap, fweights(freq_1-freq_100): regress price weight i.foreign
(running regress on estimation sample)
Bootstrap replications (100):10.....20.....30.....40.....
>50.....60.....70.....80.....90.....100 done
Linear regression Number of obs = 74
 Replications = 100
 Wald chi2(2) = 50.08
 Prob > chi2 = 0.0000
 R-squared = 0.4989
 Adj R-squared = 0.4848
 Root MSE = 2117.0206
```

| price   | Observed coefficient | Bootstrap std. err. | z     | P> z  | Normal-based [95% conf. interval] |           |
|---------|----------------------|---------------------|-------|-------|-----------------------------------|-----------|
| weight  | 3.320737             | .493339             | 6.73  | 0.000 | 2.35381                           | 4.287663  |
| foreign |                      |                     |       |       |                                   |           |
| Foreign | 3637.001             | 584.5176            | 6.22  | 0.000 | 2491.368                          | 4782.635  |
| _cons   | -4942.844            | 1526.99             | -3.24 | 0.001 | -7935.689                         | -1949.998 |

We can also perform our analysis using any one of the bootstrap sampled datasets. For example, if we want to refit the model using only the first bootstrap sampled dataset, we can simply type

```
. regress price weight i.foreign [fweight=freq_1]
```

| Source   | SS        | df | MS         | Number of obs | = | 74     |
|----------|-----------|----|------------|---------------|---|--------|
| Model    | 428736736 | 2  | 214368368  | F(2, 71)      | = | 42.81  |
| Residual | 355506733 | 71 | 5007137.08 | Prob > F      | = | 0.0000 |
|          |           |    |            | R-squared     | = | 0.5467 |
|          |           |    |            | Adj R-squared | = | 0.5339 |
| Total    | 784243469 | 73 | 10743061.2 | Root MSE      | = | 2237.7 |

| price   | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|---------|-------------|-----------|-------|-------|----------------------|-----------|
| weight  | 3.377376    | .3677504  | 9.18  | 0.000 | 2.644103             | 4.11065   |
| foreign |             |           |       |       |                      |           |
| Foreign | 3469.655    | 660.6613  | 5.25  | 0.000 | 2152.334             | 4786.976  |
| _cons   | -4707.386   | 1212      | -3.88 | 0.000 | -7124.045            | -2290.726 |



### ▷ Example 2: Adjust sample size

By default, `rwgen bsample` generates bootstrap samples of the same size as the original dataset. In our previous example, `auto.dta` contained 74 observations; thus, our frequency weights summed to 74 for each replication. However, we can use the `size()` option to control the number of observations drawn in each bootstrap sample. For example, below we sample 10 observations with replacement from the original `auto` dataset:

```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
. rwgen bsample frq_, reps(1) size(10)
. quietly summarize frq_1
. display r(sum)
10
```

The weights now sum to 10.



### ▷ Example 3: Resample clusters

If we are interested in generating bootstrap samples of entire clusters rather than individual observations, we can use the `cluster()` option. For example, we have clustered data in `bsample2.dta`.

```
. use https://www.stata-press.com/data/r19/bsample2, clear
. tabulate group
```

| group | Freq. | Percent | Cum.   |
|-------|-------|---------|--------|
| A     | 15    | 19.74   | 19.74  |
| B     | 10    | 13.16   | 32.89  |
| C     | 11    | 14.47   | 47.37  |
| D     | 11    | 14.47   | 61.84  |
| E     | 29    | 38.16   | 100.00 |
| Total | 76    | 100.00  |        |

We have five distinct groups in our data. Below, we resample clusters using `group` as the cluster variable for bootstrap sampling.

```
. rwgen bsample fc_, reps(5) cluster(group)
```

Note that we are resampling clusters rather than individual observations. Let's list the frequencies for the cluster defined by `group == "B"`

```
. list group fc_1-fc_5 if group == "B"
```

|     | group | fc_1 | fc_2 | fc_3 | fc_4 | fc_5 |
|-----|-------|------|------|------|------|------|
| 16. | B     | 1    | 1    | 2    | 1    | 2    |
| 17. | B     | 1    | 1    | 2    | 1    | 2    |
| 18. | B     | 1    | 1    | 2    | 1    | 2    |
| 19. | B     | 1    | 1    | 2    | 1    | 2    |
| 20. | B     | 1    | 1    | 2    | 1    | 2    |
| 21. | B     | 1    | 1    | 2    | 1    | 2    |
| 22. | B     | 1    | 1    | 2    | 1    | 2    |
| 23. | B     | 1    | 1    | 2    | 1    | 2    |
| 24. | B     | 1    | 1    | 2    | 1    | 2    |
| 25. | B     | 1    | 1    | 2    | 1    | 2    |

Frequencies remain constant within clusters because the entire cluster is treated as a single unit during the resampling process. This approach ensures that all observations within a cluster are sampled together, maintaining the cluster structure in the bootstrap samples.

◀

#### ▷ Example 4: Stratified samples with unequal sizes

In this example, we demonstrate how to perform stratified sampling with unequal sizes again using `auto.dta`. This method allows us to sample different numbers of observations from distinct groups within the dataset. To sample 10 foreign cars and 5 domestic cars from the original dataset, we first need to create a variable that specifies the desired sample size for each group. We can do this by generating a new variable that assigns the value 10 to foreign cars and 5 to domestic cars:

```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
. generate nsamp = cond(foreign,10,5)
```

Once this variable is created, we can specify it in the `size()` option of the `rwgen bsample` command in addition to specifying the strata variable `foreign` in the `strata()` option:

```
. rwgen bsample fs_, reps(1) strata(foreign) size(nsamp)
. tabulate foreign [fweight=fs_1]
```

| Car origin | Freq. | Percent | Cum.   |
|------------|-------|---------|--------|
| Domestic   | 5     | 33.33   | 33.33  |
| Foreign    | 10    | 66.67   | 100.00 |
| Total      | 15    | 100.00  |        |

◀

### ▷ Example 5: Using matched case–control data with the `idcluster()` option

In this example, we demonstrate how to resample clusters using matched case–control data. We have a dataset of infants who are paired by being matched on their mother’s age. These data were presented in [Hosmer, Lemeshow, and Sturdivant \(2013\)](#) and examined in [example 7](#) of [\[R\] bootstrap](#).

The matching information is stored in the variable `pairid`. Specifying this matching variable in the `group()` or `strata()` option of estimation commands, such as `clogit`, allows us to account for the inherent dependencies in the data when fitting our models.

When resampling from groups, or clusters, we need to make sure that we are preserving the matching information and that the resampled clusters are uniquely identified in each bootstrap sample. Thus, we use the `idcluster()` option with `rwgen bsample` to generate a new variable to uniquely identify these resampled groups. This ensures that the bootstrap samples maintain the correct matching structure, enabling accurate estimation of standard errors for matched case–control studies.

Below, we specify `pairid` in the `cluster()` option to preserve the original matching information and generate a new variable named `newpairid` to uniquely identify the newly resampled groups:

```
. use https://www.stata-press.com/data/r19/lowbirth2, clear
(Applied Logistic Regression, Hosmer & Lemeshow)
. rwgen bsample frq_, reps(50) rseed(1) cluster(pairid) idcluster(newpairid)
```

Now we can specify our newly generated frequency weights in the `fweights()` option of the bootstrap prefix to reconstruct the resampling process and obtain estimates of the standard errors that account for the matching information.

```
. bootstrap ratio=exp(_b[smoke]-_b[ptd]), fweights(frq_1-frq_50):
> clogit low lwt smoke ptd ht ui i.race, group(newpairid)
(running clogit on estimation sample)
Bootstrap replications (50):10.....20.....30.....40.....
> ...50 done

Bootstrap results Number of obs = 112
 Replications = 50

Command: clogit low lwt smoke ptd ht ui i.race, group(newpairid)
 ratio: exp(_b[smoke]-_b[ptd])
 (Replications based on 56 clusters in pairid)
```

|       | Observed<br>coefficient | Bootstrap<br>std. err. | z    | P> z  | Normal-based<br>[95% conf. interval] |          |
|-------|-------------------------|------------------------|------|-------|--------------------------------------|----------|
| ratio | .6654095                | 1.156848               | 0.58 | 0.565 | -1.601972                            | 2.932791 |

◀

### ▷ Example 6: Bayesian bootstrap

In this example, we will demonstrate how to use the Bayesian bootstrap technique with the `rwgen bayes` command to analyze `auto.dta`. We first load the data:

```
. use https://www.stata-press.com/data/r19/auto, clear
(1978 automobile data)
```

According to [Rubin \(1981\)](#), the Bayesian bootstrap requires specifying a prior for each unique observation, representing our prior beliefs about the outcome probabilities. By default, the `rwgen bayes` command assigns a prior power value of `-1` to all observations, which corresponds to an improper non-informative prior.

We use the following command to generate 100 replicate weights for each observation, which will be stored in new variables named `ival1_1` through `ival1_100`, and we specify the `rseed(20)` option for reproducibility:

```
. rwgen bayes ival1_, reps(100) rseed(20)
```

The `rwgen bayes` command generates weights in the  $[0, 1]$  interval, with the summation of weights equal to 1. These weights represent a sample of the posterior probability distribution over the observations, incorporating our prior beliefs. For example, below we display summary statistics for the first set of replicate weights (`ival1_1`):

```
. summarize ival1_1
Variable | Obs Mean Std. dev. Min Max
-----+-----+-----+-----+-----+-----
 ival1_1 | 74 .0135135 .0114578 .0001009 .0443107
. display r(sum)
1
```

The generated weights can be specified in the `weights()` option of the `bootstrap` prefix. `weights()` stands for importance weights, and they allow us to perform weighted analyses, accounting for the varying probabilities of each observation as specified by the Bayesian bootstrap. To demonstrate, we will use the `regress` command with the generated replicate weights:

```
. bootstrap, iweights(ival1_1-ival1_100): regress mpg weight foreign
(running regress on estimation sample)
Bootstrap replications (100):10.....20.....30.....40.....
>50.....60.....70.....80.....90.....100 done
Linear regression Number of obs = 74
Replications = 100
Wald chi2(2) = 169.45
Prob > chi2 = 0.0000
R-squared = 0.6627
Adj R-squared = 0.6532
Root MSE = 3.4071
```

| mpg     | Observed coefficient | Bootstrap std. err. | z      | P> z  | Normal-based [95% conf. interval] |           |
|---------|----------------------|---------------------|--------|-------|-----------------------------------|-----------|
| weight  | -.0065879            | .0005116            | -12.88 | 0.000 | -.0075906                         | -.0055852 |
| foreign | -1.650029            | .9949633            | -1.66  | 0.097 | -3.600121                         | .3000631  |
| _cons   | 41.6797              | 1.695628            | 24.58  | 0.000 | 38.35633                          | 45.00307  |



### ▷ Example 7: Changing prior power specification in Bayesian bootstrap

Below, we will build upon the previous example to demonstrate how altering the prior power specification affects the estimated results using the Bayesian bootstrap technique.

By default, `rwgen bayes` assigns a prior power value of  $-1$  for all observations. This corresponds to using a noninformative prior for all observations. First, let's generate a new variable with higher values for the prior powers; these values will influence the posterior importance weights assigned to each observation.

```
. generate double prior_12 = runiform(5, 15)
```

Next, we use `rwgen bayes` to generate a new set of replicate weights based on the new prior powers. This will create new variables `ival2_1` through `ival2_100`:

```
. rwgen bayes ival2_, reps(100) priorpowers(prior_l2)
```

Recall that the importance weights `ival1_1` through `ival1_100` were generated based on the default prior powers of `-1`. To compare the weights generated from the two different prior power values, we calculate the variance for `ival1_1` through `ival1_100` and `ival2_1` through `ival2_100`:

```
. mata: variance(st_data(1, "ival1_1-ival1_100"))
 .0001778708
. mata: variance(st_data(1, "ival2_1-ival2_100"))
 .000016411
```

By examining the summary statistics, you will notice that the variance of the generated replicate weights is much smaller when using the higher prior power value. This is expected because the posterior probability represented by these variables follows a Dirichlet distribution. A higher prior power value results in more concentrated weights, reflecting greater certainty in the prior information.

Now, let's use the `bootstrap` prefix with the newly generated importance weights `ival2_1` through `ival2_100`.

```
. bootstrap, iweights(ival2_1-ival2_100): regress mpg weight foreign
(running regress on estimation sample)

Bootstrap replications (100):10.....20.....30.....40.....
>50.....60.....70.....80.....90.....100 done

Linear regression Number of obs = 74
Replications = 100
Wald chi2(2) = 1745.44
Prob > chi2 = 0.0000
R-squared = 0.6627
Adj R-squared = 0.6532
Root MSE = 3.4071
```

|         | Observed<br>coefficient | Bootstrap<br>std. err. | z      | P> z  | Normal-based<br>[95% conf. interval] |           |
|---------|-------------------------|------------------------|--------|-------|--------------------------------------|-----------|
| weight  | -.0065879               | .0001582               | -41.63 | 0.000 | -.006898                             | -.0062778 |
| foreign | -1.650029               | .3246217               | -5.08  | 0.000 | -2.286276                            | -1.013782 |
| _cons   | 41.6797                 | .5088586               | 81.91  | 0.000 | 40.68236                             | 42.67705  |

Notice that the standard errors are smaller because we used a more informative prior.



### ▷ Example 8: Influence of uniqueness on posterior estimation of importance weights

In this example, we will explain how the definition of uniqueness influences the importance weights in the Bayesian bootstrap. According to [Rubin \(1981\)](#), the posterior distribution follows a Dirichlet distribution, which combines prior information with the frequency of observations in the sample. The frequency count is based on how uniqueness is defined for the observations.

By default, all variables are used to determine whether an observation is unique. However, we also provide the `id()` option, which allows users to alter the criteria for determining uniqueness.

We continue the previous example using `auto.dta`. In the following command, we use the `foreign` variable to determine uniqueness. This means that all observations with the same value for the `foreign` variable will be treated as identical when calculating the frequency counts for the posterior distribution:

```
. rwgen bayes uiv_, reps(1) id(foreign)
```

By using the `foreign` variable to determine uniqueness, the generated weights will be the same for all observations where `foreign` is equal to “Domestic”, and similarly, the weights will be the same for all observations where `foreign` is equal to “Foreign”.

```
. by foreign: summarize uiv_1
```

```
-> foreign = Domestic
```

| Variable | Obs | Mean     | Std. dev. | Min      | Max      |
|----------|-----|----------|-----------|----------|----------|
| uiv_1    | 52  | .0140812 | 0         | .0140812 | .0140812 |

```
-> foreign = Foreign
```

| Variable | Obs | Mean     | Std. dev. | Min      | Max      |
|----------|-----|----------|-----------|----------|----------|
| uiv_1    | 22  | .0121716 | 0         | .0121716 | .0121716 |

◀

## Stored results

`rwgen bsample` stores the following in `r()`:

### Scalars

`r(N)` sample size  
`r(N_reps)` number of replications  
`r(N_strata)` number of strata  
`r(N_clust)` number of clusters

### Macros

`r(method)` `bsample`  
`r(stub)` *stub* of new variables  
`r(rngstate)` random-number state used  
`r(size)` from the `size(exp)` option  
`r(strata)` strata variables  
`r(cluster)` cluster variables  
`r(idcluster)` new cluster ID variable  
`r(newvarlist)` generated weight variables

`rwgen bayes` stores the following in `r()`:

### Scalars

`r(N)` sample size  
`r(N_reps)` number of replications

### Macros

`r(method)` `bayes`  
`r(stub)` *stub* of new variables  
`r(rngstate)` random-number state used  
`r(id)` variables used to uniquely identify observations  
`r(priorpowers)` prior powers  
`r(newvarlist)` generated weight variables

## Methods and formulas

The classical bootstrap ([R] **bootstrap**) is a resampling method used to approximate the distribution of a statistic across a sample. Consider a sample  $(x_1, x_2, \dots, x_n)$  from a random variable  $X$ . A bootstrap replication is a sample of size  $m$  drawn with replacement from  $(x_1, x_2, \dots, x_n)$ . In most cases,  $m$  will be equal to  $n$ , the sample size.

Viewed from a probabilistic perspective, this process involves assigning probabilities to each observation in the sample. A bootstrap sample is drawn from a multinomial distribution with  $n$  categories and  $m$  independent trials, where success (outcome) probabilities are nonnegative and their sum equals 1. In this setup, each point in the sample has a probability  $1/n$  of selection in any draw, although the randomness of the process may result in some points being selected multiple times, or not at all, within a single replication.

Introduced by [Rubin \(1981\)](#), the Bayesian bootstrap modifies the traditional bootstrap technique by adopting a Bayesian approach. Instead of discrete resampling with replacement, it assigns probabilities to each observation, forming an  $n$ -vector of weights that follow a specified posterior distribution. This method captures the uncertainty about the representativeness of each observation in the overall distribution, allowing for greater variability in the analysis.

We follow Rubin’s framework outlined in [Rubin \(1981\)](#), particularly focusing on section 4, to describe the procedures for calculating Bayesian bootstrap weights.

Consider a discrete random variable  $\tilde{X}$  that can take on  $K$  distinct values  $d_1, d_2, \dots, d_K$ . Each unique value in this set has an associated probability  $\pi_k = P(\tilde{X} = d_k | \boldsymbol{\pi})$ , where  $\boldsymbol{\pi} = (\pi_1, \pi_2, \dots, \pi_K)$ . For an i.i.d. sample  $(x_1, x_2, \dots, x_n)$  drawn from  $\tilde{X}$ , let  $n_k$  denote the frequency of each  $d_k$  in the sample. For continuous random variables, each observation in the sample is unique, so  $K = n$  and  $n_i = 1$  for all  $i$ .

The Bayesian bootstrap introduces a prior distribution for  $\boldsymbol{\pi}$ , proportional to  $\prod_{k=1}^K \pi_k^{l_k}$ , reflecting prior beliefs about the outcome probabilities. Updating this prior with the categorical data, the posterior distribution of  $\boldsymbol{\pi}$  follows a  $(K - 1)$ -variate Dirichlet distribution,  $\text{Dirichlet}(n_1 + l_1 + 1, n_2 + l_2 + 1, \dots, n_K + l_K + 1)$  proportional to  $\prod_{k=1}^K \pi_k^{n_k + l_k}$ .

For notational simplicity, let’s assume  $K = n$ . Following [Rubin \(1981\)](#) and [Hastie, Tibshirani, and Friedman \(2009\)](#), the default prior is set to an improper noninformative prior proportional to  $\prod_{k=1}^K \pi_k^{-1}$ , with  $l_k = -1$  for  $k = 1, 2, \dots, n$ . This way, the posterior distribution is  $\text{Dirichlet}(1, 1, \dots, 1)$ , meaning each data point has the same posterior probability. For more advanced usage, the `priorpowers()` option allows you to specify your own values for  $l_k$ . However, you need to make sure that those values produce an actual posterior distribution (that is, the formula for the posterior distribution function has a finite integral on its domain); a sufficient condition, for example, would be that  $n_k + l_k + 1$  be positive for all observations.

`rwgen bayes` uses Mata’s `rdirichlet()` function (see `help mata rdirichlet()`) to generate Bayesian bootstrap weights  $w_k$ ’s from the posterior Dirichlet distribution with shape vector  $A = (n_1 + l_1 + 1, n_2 + l_2 + 1, \dots, n_K + l_K + 1)$ . Note that for every element  $a$  of  $A$ ,  $1e-3 < a < 1e+8$ . When  $K < n$ , the generated weights are distributed evenly among duplicate observations with a unique value  $d_k$  as  $w_k/n_k$  for  $k = 1, 2, \dots, K$ . This approach ensures that the sum of the weights across all observations accurately reflects the true posterior probability distribution, integrating both the empirical data and prior beliefs into our analysis.

## References

- Hastie, T. J., R. J. Tibshirani, and J. H. Friedman. 2009. *The Elements of Statistical Learning: Data Mining, Inference, and Prediction*. 2nd ed. New York: Springer. <https://doi.org/10.1007/978-0-387-84858-7>.
- Hosmer, D. W., Jr., S. A. Lemeshow, and R. X. Sturdivant. 2013. *Applied Logistic Regression*. 3rd ed. Hoboken, NJ: Wiley.
- Rubin, D. B. 1981. The Bayesian bootstrap. *Annals of Statistics* 9: 130–134. <https://doi.org/10.1214/aos/1176345338>.

## Also see

- [R] **bayesboot** — Bayesian bootstrap estimation
- [R] **bootstrap** — Bootstrap sampling and estimation
- [R] **bsample** — Sampling with replacement
- [D] **sample** — Draw random sample
- [D] **splitsample** — Split data into random samples

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## Description

`scobit` fits a maximum-likelihood skewed logit model.

## Quick start

Skewed logistic regression of binary variable `y` on `x1` and `x2`

```
scobit y x1 x2
```

With robust standard errors

```
scobit y x1 x2, vce(robust)
```

Same as above, and display coefficients and std. err. with two digits to the right of the decimal

```
scobit y x1 x2, vce(robust) cformat(%8.2f)
```

Same as above, and also display  $p$ -values with two digits to the right of the decimal

```
scobit y x1 x2, vce(robust) cformat(%8.2f) pformat(%5.2f)
```

## Menu

Statistics > Binary outcomes > Skewed logistic regression

## Syntax

```
scobit depvar [indepvars] [if] [in] [weight] [, options]
```

| <i>options</i>                               | Description                                                                                                                                                                 |
|----------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                 |                                                                                                                                                                             |
| <code>noconstant</code>                      | suppress constant term                                                                                                                                                      |
| <code>offset(<i>varname</i>)</code>          | include <i>varname</i> in model with coefficient constrained to 1                                                                                                           |
| <code>asis</code>                            | retain perfect predictor variables                                                                                                                                          |
| <code>constraints(<i>constraints</i>)</code> | apply specified linear constraints                                                                                                                                          |
| <b>SE/Robust</b>                             |                                                                                                                                                                             |
| <code>vce(<i>vcetype</i>)</code>             | <i>vcetype</i> may be <code>oim</code> , <code>robust</code> , <code>cluster <i>clustvar</i></code> , <code>opg</code> , <code>bootstrap</code> , or <code>jackknife</code> |
| <b>Reporting</b>                             |                                                                                                                                                                             |
| <code>level(#)</code>                        | set confidence level; default is <code>level(95)</code>                                                                                                                     |
| <code>nocnsreport</code>                     | do not display constraints                                                                                                                                                  |
| <code>display_options</code>                 | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling                            |
| <b>Maximization</b>                          |                                                                                                                                                                             |
| <code>maximize_options</code>                | control the maximization process                                                                                                                                            |
| <code>collinear</code>                       | keep collinear variables                                                                                                                                                    |
| <code>coeflegend</code>                      | display legend instead of statistics                                                                                                                                        |

*indepvars* may contain factor variables; see [U] 11.4.3 **Factor variables**.

`bayesboot`, `bootstrap`, `by`, `collect`, `fp`, `jackknife`, `nestreg`, `rolling`, `statsby`, `stepwise`, and `svy` are allowed; see [U] 11.1.10 **Prefix commands**.

Weights are not allowed with the `bootstrap` prefix; see [R] **bootstrap**.

`vce()` and weights are not allowed with the `svy` prefix; see [SVY] **svy**.

`fweights`, `iweights`, and `pweights` are allowed; see [U] 11.1.6 **weight**.

`collinear` and `coeflegend` do not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options

### Model

`noconstant`, `offset(varname)`, `constraints(constraints)`; see [R] **Estimation options**.

`asis` forces retention of perfect predictor variables and their associated perfectly predicted observations and may produce instabilities in maximization; see [R] **probit**.

### SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] **vce\_option**.

## Reporting

`level(#)`; see [R] [Estimation options](#).

`nocnsreport`; see [R] [Estimation options](#).

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

## Maximization

*maximize\_options*: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

Setting the optimization type to `technique(bhhh)` resets the default `vcetype` to `vce(opg)`.

The following options are available with `scobit` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Remarks are presented under the following headings:

*Skewed logistic model*

*Robust standard errors*

## Skewed logistic model

`scobit` fits maximum likelihood models with dichotomous dependent variables coded as 0/1 (or, more precisely, coded as 0 and not 0).

### ► Example 1

We have data on the make, weight, and mileage rating of 22 foreign and 52 domestic automobiles. We wish to fit a model explaining whether a car is foreign based on its mileage. Here is an overview of our data:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. keep make mpg weight foreign
. describe
Contains data from https://www.stata-press.com/data/r19/auto.dta
Observations: 74 1978 automobile data
Variables: 4 13 Apr 2024 17:45
 (_dta has notes)
```

| Variable name | Storage type | Display format | Value label | Variable label |
|---------------|--------------|----------------|-------------|----------------|
| make          | str18        | %-18s          |             | Make and model |
| mpg           | int          | %8.0g          |             | Mileage (mpg)  |
| weight        | int          | %8.0gc         |             | Weight (lbs.)  |
| foreign       | byte         | %8.0g          | origin      | Car origin     |

```
Sorted by: foreign
Note: Dataset has changed since last saved.
```

```
. inspect foreign
foreign: Car origin
```

|     |          | Number of observations |          |             |
|-----|----------|------------------------|----------|-------------|
|     |          | Total                  | Integers | Nonintegers |
| #   | Negative | -                      | -        | -           |
| #   | Zero     | 52                     | 52       | -           |
| #   | Positive | 22                     | 22       | -           |
| #   |          |                        |          |             |
| # # | Total    | 74                     | 74       | -           |
| # # | Missing  | -                      |          |             |
| 0   |          | 74                     |          |             |
| 1   |          |                        |          |             |

(2 unique values)

foreign is labeled and all values are documented in the label.

The variable `foreign` takes on two unique values, 0 and 1. The value 0 denotes a domestic car, and 1 denotes a foreign car.

The model that we wish to fit is

$$\Pr(\text{foreign} = 1) = F(\beta_0 + \beta_1 \text{mpg})$$

where  $F(z) = 1 - 1/\{1 + \exp(z)\}^\alpha$ .

To fit this model, we type

```
. scobit foreign mpg
Fitting logistic model:
Iteration 0: Log likelihood = -45.03321
Iteration 1: Log likelihood = -39.380959
Iteration 2: Log likelihood = -39.288802
Iteration 3: Log likelihood = -39.28864
Iteration 4: Log likelihood = -39.28864
Fitting full model:
Iteration 0: Log likelihood = -39.28864
Iteration 1: Log likelihood = -39.286393
Iteration 2: Log likelihood = -39.284415
Iteration 3: Log likelihood = -39.284234
Iteration 4: Log likelihood = -39.284197
Iteration 5: Log likelihood = -39.284196
```

```
Skewed logistic regression Number of obs = 74
 Zero outcomes = 52
Log likelihood = -39.2842 Nonzero outcomes = 22
```

| foreign  | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|----------|-------------|-----------|-------|-------|----------------------|-----------|
| mpg      | .1813879    | .2407362  | 0.75  | 0.451 | -.2904463            | .6532222  |
| _cons    | -4.274883   | 1.399305  | -3.06 | 0.002 | -7.017471            | -1.532295 |
| /lnalpha | -.4450405   | 3.879885  | -0.11 | 0.909 | -8.049476            | 7.159395  |
| alpha    | .6407983    | 2.486224  |       |       | .0003193             | 1286.133  |

```
LR test of alpha=1: chi2(1) = 0.01 Prob > chi2 = 0.9249
```

Note: Likelihood-ratio tests are recommended for inference with scobit models.

We find that cars yielding better gas mileage are less likely to be foreign. The likelihood-ratio test at the bottom of the output indicates that the model is not significantly different from a logit model. Therefore, we should use the more parsimonious model.

◀

## □ Technical note

Stata interprets a value of 0 as a negative outcome (failure) and treats all other values (except missing) as positive outcomes (successes). Thus if the dependent variable takes on the values 0 and 1, then 0 is interpreted as failure and 1 as success. If the dependent variable takes on the values 0, 1, and 2, then 0 is still interpreted as failure, but both 1 and 2 are treated as successes.

Formally, when we type `scobit y x`, Stata fits the model

$$\Pr(y_j \neq 0 \mid \mathbf{x}_j) = 1 - 1 / \left\{ 1 + \exp(\mathbf{x}_j \beta) \right\}^\alpha$$

□

## Robust standard errors

If you specify the `vce(robust)` option, `scobit` reports robust standard errors as described in [U] 20.22 **Obtaining robust variance estimates**. For the model of `foreign` on `mpg`, the robust calculation increases the standard error of the coefficient on `mpg` by around 25%:

```
. scobit foreign mpg, vce(robust) nolog
Skewed logistic regression Number of obs = 74
 Zero outcomes = 52
Log pseudolikelihood = -39.2842 Nonzero outcomes = 22
```

| foreign  | Coefficient | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |           |
|----------|-------------|---------------------|-------|-------|----------------------|-----------|
| mpg      | .1813879    | .3028487            | 0.60  | 0.549 | -.4121847            | .7749606  |
| _cons    | -4.274883   | 1.335521            | -3.20 | 0.001 | -6.892455            | -1.657311 |
| /lnalpha | -.4450405   | 4.71561             | -0.09 | 0.925 | -9.687466            | 8.797385  |
| alpha    | .6407983    | 3.021755            |       |       | .0000621             | 6616.919  |

Without `vce(robust)`, the standard error for the coefficient on `mpg` was reported to be 0.241, with a resulting confidence interval of  $[-0.29, 0.65]$ .

Specifying the `vce(cluster clustvar)` option relaxes the independence assumption required by the skewed logit estimator to being just independence between clusters. To demonstrate this, we will switch to a different dataset.

## ▷ Example 2

We are studying the unionization of women in the United States and have a dataset with 26,200 observations on 4,434 women between 1970 and 1988. For our purposes, we will use the variables `age` (the women were 14–26 in 1968 and the data thus span the age range of 16–46), `grade` (years of schooling completed, ranging from 0 to 18), `not_smsa` (28% of the person-time was spent living outside an SMSA—standard metropolitan statistical area), `south` (41% of the person-time was in the South), and `year`. Each of these variables is included in the regression as a covariate along with the interaction between `south` and `year`. This interaction, along with the `south` and `year` variables, is specified in the `scobit` command using factor-variables notation, `south##c.year`. We also have variable `union`. Overall, 22% of the person-time is marked as time under union membership and 44% of these women have belonged to a union.

We fit the following model, ignoring that women are observed an average of 5.9 times each in these data:

```
. use https://www.stata-press.com/data/r19/union, clear
(NLS Women 14-24 in 1968)
. scobit union age grade not_smsa south##c.year, nrtol(1e-3)
(output omitted)
```

```
Skewed logistic regression Number of obs = 26,200
 Zero outcomes = 20,389
Log likelihood = -13540.61 Nonzero outcomes = 5,811
```

|                           | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|---------------------------|-------------|-----------|-------|-------|----------------------|-----------|
| <code>union</code>        |             |           |       |       |                      |           |
| <code>age</code>          | .0185363    | .0043615  | 4.25  | 0.000 | .0099879             | .0270848  |
| <code>grade</code>        | .0452801    | .0057124  | 7.93  | 0.000 | .034084              | .0564761  |
| <code>not_smsa</code>     | -.1886826   | .0317801  | -5.94 | 0.000 | -.2509705            | -.1263947 |
| <code>1.south</code>      | -1.422372   | .3949301  | -3.60 | 0.000 | -2.196421            | -.6483233 |
| <code>year</code>         | -.0133016   | .0049575  | -2.68 | 0.007 | -.0230181            | -.0035851 |
| <code>south#c.year</code> |             |           |       |       |                      |           |
| <code>1</code>            | .0105663    | .0049233  | 2.15  | 0.032 | .0009167             | .0202158  |
| <code>_cons</code>        | -10.3557    | 68.9757   | -0.15 | 0.881 | -145.5456            | 124.8342  |
| <code>/lnalpha</code>     | 9.136018    | 68.97395  | 0.13  | 0.895 | -126.0504            | 144.3225  |
| <code>alpha</code>        | 9283.719    | 640334.8  |       |       | 1.81e-55             | 4.77e+62  |

```
LR test of alpha=1: chi2(1) = 3.76 Prob > chi2 = 0.0524
```

```
Note: Likelihood-ratio tests are recommended for inference with scobit models.
```

The reported standard errors in this model are probably meaningless. Women are observed repeatedly, so the observations are not independent. Looking at the coefficients, we find a large southern effect against unionization and a different time trend for the south. The `vce(cluster clustvar)` option provides a way to fit this model and obtains correct standard errors:

```
. scobit union age grade not_smsa south##c.year, vce(cluster id) nrtol(1e-3)
(output omitted)
Skewed logistic regression Number of obs = 26,200
 Zero outcomes = 20,389
Log pseudolikelihood = -13540.61 Nonzero outcomes = 5,811
 (Std. err. adjusted for 4,434 clusters in idcode)
```

| union        | Coefficient | Robust<br>std. err. | z      | P> z  | [95% conf. interval] |           |
|--------------|-------------|---------------------|--------|-------|----------------------|-----------|
| age          | .0185363    | .0084867            | 2.18   | 0.029 | .0019027             | .03517    |
| grade        | .0452801    | .0125765            | 3.60   | 0.000 | .0206306             | .0699295  |
| not_smsa     | -.1886826   | .0642037            | -2.94  | 0.003 | -.3145194            | -.0628457 |
| 1.south      | -1.422372   | .5064933            | -2.81  | 0.005 | -2.415081            | -.4296635 |
| year         | -.0133016   | .0090622            | -1.47  | 0.142 | -.0310632            | .0044599  |
| south#c.year |             |                     |        |       |                      |           |
| 1            | .0105663    | .0063172            | 1.67   | 0.094 | -.0018153            | .0229478  |
| _cons        | -10.3557    | .9416702            | -11.00 | 0.000 | -12.20134            | -8.510064 |
| /lnalpha     | 9.136018    | .7432386            | 12.29  | 0.000 | 7.679297             | 10.59274  |
| alpha        | 9283.719    | 6900.019            |        |       | 2163.098             | 39844.45  |

`scobit`, `vce(cluster clustvar)` is robust to assumptions about within-cluster correlation. That is, it inefficiently sums within cluster for the standard error calculation rather than attempting to exploit what might be assumed about the within-cluster correlation (as do the `xtgee` population-averaged models; see [XT] `xtgee`).

◀

### □ Technical note

The `scobit` model can be difficult to fit because of the functional form. Often, it requires many iterations, or the optimizer prints out warning and informative messages during the optimization. For example, without the `nrtol(1e-3)` option, the model using the `union` dataset will not converge. See [R] `Maximize` for details about the optimizer.

□

### □ Technical note

The main reason for using `scobit` rather than `logit` is that the effects of the regressors on the probability of success are not constrained to be the largest when the probability is 0.5. Rather, the independent variables might show their largest impact when the probability of success is 0.3 or 0.6. This added flexibility results because the `scobit` function, unlike the `logit` function, can be skewed and is not constrained to be mirror symmetric about the 0.5 probability of success.

As Nagler (1994) pointed out, the point of maximum impact is constrained under the `scobit` model to fall within the interval  $(0, 1 - e^{(-1)})$  or approximately  $(0, 0.63)$ . Achen (2002) notes that if we believe the maximum impact to be outside that range, we can instead fit the “power logit” model by simply reversing the 0s and 1s of our outcome variable and fitting a `scobit` model on failure, rather than success. We would need to reverse the signs of the coefficients if we wanted to interpret them in terms of impact on success,

or we could leave them as they are and interpret them in terms of impact on failure. The important thing to remember is that the scobit model, unlike the logit model, is not invariant to the choice of which result is assigned to success. □

## Stored results

scobit stores the following in `e()`:

### Scalars

|                           |                                          |
|---------------------------|------------------------------------------|
| <code>e(N)</code>         | number of observations                   |
| <code>e(k)</code>         | number of parameters                     |
| <code>e(k_eq)</code>      | number of equations in <code>e(b)</code> |
| <code>e(k_aux)</code>     | number of auxiliary parameters           |
| <code>e(k_dv)</code>      | number of dependent variables            |
| <code>e(ll)</code>        | log likelihood                           |
| <code>e(ll_c)</code>      | log likelihood, comparison model         |
| <code>e(N_f)</code>       | number of failures (zero outcomes)       |
| <code>e(N_s)</code>       | number of successes (nonzero outcomes)   |
| <code>e(alpha)</code>     | alpha                                    |
| <code>e(N_clust)</code>   | number of clusters                       |
| <code>e(chi2_c)</code>    | $\chi^2$ for comparison test             |
| <code>e(rank)</code>      | rank of <code>e(V)</code>                |
| <code>e(ic)</code>        | number of iterations                     |
| <code>e(rc)</code>        | return code                              |
| <code>e(converged)</code> | 1 if converged, 0 otherwise              |

### Macros

|                            |                                                                                 |
|----------------------------|---------------------------------------------------------------------------------|
| <code>e(cmd)</code>        | scobit                                                                          |
| <code>e(cmdline)</code>    | command as typed                                                                |
| <code>e(depvar)</code>     | name of dependent variable                                                      |
| <code>e(wtype)</code>      | weight type                                                                     |
| <code>e(wexp)</code>       | weight expression                                                               |
| <code>e(title)</code>      | title in estimation output                                                      |
| <code>e(clustvar)</code>   | name of cluster variable                                                        |
| <code>e(offset)</code>     | linear offset variable                                                          |
| <code>e(chi2_ct)</code>    | Wald or LR; type of model $\chi^2$ test corresponding to <code>e(chi2_c)</code> |
| <code>e(vce)</code>        | <i>vce</i> type specified in <code>vce()</code>                                 |
| <code>e(vcetype)</code>    | title used to label Std. err.                                                   |
| <code>e(opt)</code>        | type of optimization                                                            |
| <code>e(which)</code>      | max or min; whether optimizer is to perform maximization or minimization        |
| <code>e(ml_method)</code>  | type of ml method                                                               |
| <code>e(user)</code>       | name of likelihood-evaluator program                                            |
| <code>e(technique)</code>  | maximization technique                                                          |
| <code>e(properties)</code> | b V                                                                             |
| <code>e(predict)</code>    | program used to implement <code>predict</code>                                  |
| <code>e(footnote)</code>   | program used to implement the footnote display                                  |
| <code>e(asbalanced)</code> | factor variables <code>fvset</code> as <code>asbalanced</code>                  |
| <code>e(asobserved)</code> | factor variables <code>fvset</code> as <code>asobserved</code>                  |

### Matrices

|                              |                                              |
|------------------------------|----------------------------------------------|
| <code>e(b)</code>            | coefficient vector                           |
| <code>e(Cns)</code>          | constraints matrix                           |
| <code>e(ilog)</code>         | iteration log (up to 20 iterations)          |
| <code>e(gradient)</code>     | gradient vector                              |
| <code>e(V)</code>            | variance–covariance matrix of the estimators |
| <code>e(V_modelbased)</code> | model-based variance                         |

### Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

Matrices

`r(table)`

matrix containing the coefficients with their standard errors, test statistics,  $p$ -values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

Skewed logit analysis is an alternative to logit that relaxes the assumption that individuals with initial probability of 0.5 are most sensitive to changes in independent variables.

The log-likelihood function for skewed logit is

$$\ln L = \sum_{j \in S} w_j \ln F(\mathbf{x}_j \mathbf{b}) + \sum_{j \notin S} w_j \ln \{1 - F(\mathbf{x}_j \mathbf{b})\}$$

where  $S$  is the set of all observations  $j$  such that  $y_j \neq 0$ ,  $F(z) = 1 - 1/\{1 + \exp(z)\}^\alpha$ , and  $w_j$  denotes the optional weights.  $\ln L$  is maximized as described in [R] **Maximize**.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] **\_robust**, particularly *Maximum likelihood estimators* and *Methods and formulas*.

`scobit` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] **Variance estimation**.

## References

- Achen, C. H. 2002. Toward a new political methodology: Microfoundations and ART. *Annual Review of Political Science* 5: 423–450. <https://doi.org/10.1146/annurev.polisci.5.112801.080943>.
- Nagler, J. 1994. Scobit: An alternative estimator to logit and probit. *American Journal of Political Science* 38: 230–255. <https://doi.org/10.2307/2111343>.

## Also see

- [R] **scobit postestimation** — Postestimation tools for `scobit`
- [R] **clolog** — Complementary log–log regression
- [R] **glm** — Generalized linear models
- [R] **logistic** — Logistic regression, reporting odds ratios
- [SVY] **svy estimation** — Estimation commands for survey data
- [U] **20 Estimation and postestimation commands**

## Postestimation commands

The following postestimation commands are available after `scobit`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | probabilities, linear predictions and their SEs, etc.                                                                                   |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`forecast` and `lrtest` are not appropriate with `svy` estimation results.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as probabilities, linear predictions, and standard errors.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic nooffset]
```

```
predict [type] stub* [if] [in] , scores
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

Main

|                   |                                                |
|-------------------|------------------------------------------------|
| <code>pr</code>   | probability of a positive outcome; the default |
| <code>xb</code>   | $\mathbf{x}_j\mathbf{b}$ , linear prediction   |
| <code>stdp</code> | standard error of the linear prediction        |

These statistics are available both in and out of sample; type `predict . . . if e(sample) . . .` if wanted only for the estimation sample.

## Options for predict

Main

`pr`, the default, calculates the probability of a positive outcome.

`xb` calculates the linear prediction.

`stdp` calculates the standard error of the linear prediction.

`nooffset` is relevant only if you specified `offset(varname)` for `scobit`. It modifies the calculations made by `predict` so that they ignore the offset variable; the linear prediction is treated as  $\mathbf{x}_j\mathbf{b}$  rather than as  $\mathbf{x}_j\mathbf{b} + \text{offset}_j$ .

`scores` calculates equation-level score variables.

The first new variable will contain  $\partial \ln L / \partial (\mathbf{x}_j\boldsymbol{\beta})$ .

The second new variable will contain  $\partial \ln L / \partial \ln \alpha$ .

# margins

## Description for margins

`margins` estimates margins of response for probabilities and linear predictions.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i>  | Description                                    |
|-------------------|------------------------------------------------|
| <code>pr</code>   | probability of a positive outcome; the default |
| <code>xb</code>   | $\mathbf{x}_j\mathbf{b}$ , linear prediction   |
| <code>stdp</code> | not allowed with <code>margins</code>          |

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(\mathbf{b})$ .

For the full syntax, see [R] [margins](#).

## Remarks and examples

Once you have fit a model, you can obtain the predicted probabilities by using the `predict` command for both the estimation sample and other samples; see [U] [20 Estimation and postestimation commands](#) and [R] [predict](#). Here we will make only a few additional comments.

`predict` without arguments calculates the predicted probability of a positive outcome. With the `xb` option, it calculates the linear combination  $\mathbf{x}_j\mathbf{b}$ , where  $\mathbf{x}_j$  are the independent variables in the  $j$ th observation and  $\mathbf{b}$  is the estimated parameter vector.

With the `stdp` option, `predict` calculates the standard error of the prediction, which is *not* adjusted for replicated covariate patterns in the data.

## ▷ Example 1

In [example 1](#) of [\[R\] scobit](#), we fit the model `scobit foreign mpg`. To obtain predicted probabilities, we type

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. keep make mpg weight foreign
. scobit foreign mpg
(output omitted)
. predict p
(option pr assumed; Pr(foreign))
. summarize foreign p
```

| Variable | Obs | Mean     | Std. dev. | Min      | Max     |
|----------|-----|----------|-----------|----------|---------|
| foreign  | 74  | .2972973 | .4601885  | 0        | 1       |
| p        | 74  | .2974049 | .182352   | .0714664 | .871624 |

◀

**Also see**

[\[R\] scobit](#) — Skewed logistic regression

[\[U\] 20 Estimation and postestimation commands](#)

|                             |                                      |                                |                                      |
|-----------------------------|--------------------------------------|--------------------------------|--------------------------------------|
| <a href="#">Description</a> | <a href="#">Quick start</a>          | <a href="#">Menu</a>           | <a href="#">Syntax</a>               |
| <a href="#">Options</a>     | <a href="#">Remarks and examples</a> | <a href="#">Stored results</a> | <a href="#">Methods and formulas</a> |
| <a href="#">References</a>  | <a href="#">Also see</a>             |                                |                                      |

## Description

`sdtest` performs tests on the equality of standard deviations (variances). In the first form, `sdtest` tests that the standard deviation of *varname* is *#*. In the second form, `sdtest` performs the same test, using the standard deviations of the two groups defined by *groupvar*. In the third form, `sdtest` tests that *varname*<sub>1</sub> and *varname*<sub>2</sub> have the same standard deviation.

`sdtesti` is the immediate form of `sdtest`; see [\[U\] 19 Immediate commands](#).

Both the traditional *F* test for the homogeneity of variances and Bartlett's generalization of this test to *K* samples are sensitive to the assumption that the data are drawn from an underlying Gaussian distribution. See, for example, the cautionary results discussed by [Markowski and Markowski \(1990\)](#). [Levene \(1960\)](#) proposed a test statistic for equality of variance that was found to be robust under nonnormality. Then [Brown and Forsythe \(1974\)](#) proposed alternative formulations of Levene's test statistic that use more robust estimators of central tendency in place of the mean. These reformulations were demonstrated to be more robust than Levene's test when dealing with skewed populations.

`robvar` reports Levene's robust test statistic ( $W_0$ ) for the equality of variances between the groups defined by *groupvar* and the two statistics proposed by Brown and Forsythe that replace the mean in Levene's formula with alternative location estimators. The first alternative ( $W_{50}$ ) replaces the mean with the median. The second alternative replaces the mean with the 10% trimmed mean ( $W_{10}$ ).

## Quick start

Test that the standard deviation of *v1* is equal to 2

```
sdtest v1=2
```

Equality of standard deviations (variances) test for *v1* comparing the two groups defined by *catvar1*

```
sdtest v1, by(catvar1)
```

Robust equality of variances test for *v1* comparing the groups defined by *catvar1*

```
robvar v1, by(catvar1)
```

Compare the variances of *v2* and *v3*

```
sdtest v2 == v3
```

Same as above, but with separate tests for each group defined by *catvar2*

```
by catvar2, sort: sdtest v2 == v3
```

Test  $sd_1 = sd_2$  for  $sd_1 = 34$ ,  $sd_2 = 45$ ,  $N_1 = 143$ , and  $N_2 = 184$

```
sdtesti 143 . 34 184 . 45
```

## Menu

### sdtest

Statistics > Summaries, tables, and tests > Classical tests of hypotheses > Variance-comparison test

### sdtesti

Statistics > Summaries, tables, and tests > Classical tests of hypotheses > Variance-comparison test calculator

### robvar

Statistics > Summaries, tables, and tests > Classical tests of hypotheses > Robust equal-variance test

## Syntax

*One-sample variance-comparison test*

```
sdtest varname == # [if] [in] [, level(#)]
```

*Two-sample variance-comparison test using groups*

```
sdtest varname [if] [in] , by(groupvar) [level(#)]
```

*Two-sample variance-comparison test using variables*

```
sdtest varname1 == varname2 [if] [in] [, level(#)]
```

*Immediate form of one-sample variance-comparison test*

```
sdtesti #obs { #mean | . } #sd #val [, level(#)]
```

*Immediate form of two-sample variance-comparison test*

```
sdtesti #obs,1 { #mean,1 | . } #sd,1 #obs,2 { #mean,2 | . } #sd,2 [, level(#)]
```

*Robust tests for equality of variances*

```
robvar varname [if] [in] , by(groupvar)
```

by and collect are allowed with sdtest and robvar, and collect is allowed with sdtesti; see [\[U\] 11.1.10 Prefix commands](#).

## Options

*level*(#) specifies the confidence level, as a percentage, for confidence intervals of the means. The default is *level*(95) or as set by *set level*; see [\[U\] 20.8 Specifying the width of confidence intervals](#).

by(*groupvar*) specifies the *groupvar* that defines the groups to be compared. For sdtest, there should be two groups, but for robvar there may be more than two groups. Do not confuse the by() option with the by prefix; both may be specified.

## Remarks and examples

Remarks are presented under the following headings:

*Basic form*  
*Immediate form*  
*Robust test*

### Basic form

sdtest performs two different statistical tests: one testing equality of variances and the other testing that the standard deviation is equal to a known constant. Which test it performs is determined by whether you type a variable name or a number to the right of the equal sign.

#### ▷ Example 1: One-sample test of variance

We have a sample of 74 automobiles. For each automobile, we know the mileage rating. We wish to test whether the overall standard deviation is 5 mpg:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. sdtest mpg == 5
```

One-sample test of variance

| Variable | Obs | Mean    | Std. err. | Std. dev. | [95% conf. interval] |          |
|----------|-----|---------|-----------|-----------|----------------------|----------|
| mpg      | 74  | 21.2973 | .6725511  | 5.785503  | 19.9569              | 22.63769 |

```
sd = sd(mpg)
HO: sd = 5
Ha: sd < 5
Pr(C < c) = 0.9717

c = chi2 = 97.7384
Degrees of freedom = 73
Ha: sd != 5
Pr(C > c) = 0.0565

Ha: sd > 5
Pr(C > c) = 0.0283
```



#### ▷ Example 2: Variance ratio test

We are testing the effectiveness of a new fuel additive. We run an experiment on 12 cars, running each without and with the additive. The data can be found in [R] [ttest](#). The results for each car are stored in the variables mpg1 and mpg2:

```
. use https://www.stata-press.com/data/r19/fuel
. sdtest mpg1==mpg2
```

Variance ratio test

| Variable | Obs | Mean   | Std. err. | Std. dev. | [95% conf. interval] |          |
|----------|-----|--------|-----------|-----------|----------------------|----------|
| mpg1     | 12  | 21     | .7881701  | 2.730301  | 19.26525             | 22.73475 |
| mpg2     | 12  | 22.75  | .9384465  | 3.250874  | 20.68449             | 24.81551 |
| Combined | 24  | 21.875 | .6264476  | 3.068954  | 20.57909             | 23.17091 |

```
ratio = sd(mpg1) / sd(mpg2)
HO: ratio = 1
Ha: ratio < 1
Pr(F < f) = 0.2862

f = 0.7054
Degrees of freedom = 11, 11
Ha: ratio != 1
Pr(F < f) = 0.5725

Ha: ratio > 1
Pr(F > f) = 0.7138
```

We cannot reject the hypothesis that the standard deviations are the same.

In [R] `ttest`, we draw an important distinction between paired and unpaired data, which, in this example, means whether there are 12 cars in a before-and-after experiment or 24 different cars. For `sdtest`, on the other hand, there is no distinction. If the data had been unpaired and stored as described in [R] `ttest`, we could have typed `sdtest mpg, by(treated)`, and the results would have been the same.



## Immediate form

### ▷ Example 3: `sdtesti`

Immediate commands are used not with data, but with reported summary statistics. For instance, to test whether a variable on which we have 75 observations and a reported standard deviation of 6.5 comes from a population with underlying standard deviation 6, we would type

```
. sdtesti 75 . 6.5 6
One-sample test of variance
```

|   | Obs | Mean | Std. err. | Std. dev. | [95% conf. interval] |
|---|-----|------|-----------|-----------|----------------------|
| x | 75  | .    | .7505553  | 6.5       | . .                  |

```

sd = sd(x) c = chi2 = 86.8472
HO: sd = 6 Degrees of freedom = 74
Ha: sd < 6 Ha: sd != 6 Ha: sd > 6
Pr(C < c) = 0.8542 2*Pr(C > c) = 0.2916 Pr(C > c) = 0.1458

```

The mean plays no role in the calculation, so it may be omitted.

To test whether the variable comes from a population with the same standard deviation as another for which we have a calculated standard deviation of 7.5 over 65 observations, we would type

```
. sdtesti 75 . 6.5 65 . 7.5
Variance ratio test
```

|          | Obs | Mean | Std. err. | Std. dev. | [95% conf. interval] |
|----------|-----|------|-----------|-----------|----------------------|
| x        | 75  | .    | .7505553  | 6.5       | . .                  |
| y        | 65  | .    | .9302605  | 7.5       | . .                  |
| Combined | 140 | .    | .         | .         | . .                  |

```

ratio = sd(x) / sd(y) f = 0.7511
HO: ratio = 1 Degrees of freedom = 74, 64
Ha: ratio < 1 Ha: ratio != 1 Ha: ratio > 1
Pr(F < f) = 0.1172 2*Pr(F < f) = 0.2344 Pr(F > f) = 0.8828

```



## Robust test

### ▷ Example 4: robvar

We wish to test whether the standard deviation of the length of stay for patients hospitalized for a given medical procedure differs by gender. Our data consist of observations on the length of hospital stay for 1778 patients: 884 males and 894 females. Length of stay, `lengthstay`, is highly skewed (skewness coefficient = 4.912591) and thus violates Bartlett's normality assumption. Therefore, we use `robvar` to compare the variances.

```
. use https://www.stata-press.com/data/r19/stay
. robvar lengthstay, by(sex)
```

| Gender | Summary of Length of stay in days |           |       |
|--------|-----------------------------------|-----------|-------|
|        | Mean                              | Std. dev. | Freq. |
| Male   | 9.0874434                         | 9.7884747 | 884   |
| Female | 8.800671                          | 9.1081478 | 894   |
| Total  | 8.9432508                         | 9.4509466 | 1,778 |

```
W0 = 0.55505315 df(1, 1776) Pr > F = 0.45635888
W50 = 0.42714734 df(1, 1776) Pr > F = 0.51347664
W10 = 0.44577674 df(1, 1776) Pr > F = 0.50443411
```

For these data, we cannot reject the null hypothesis that the variances are equal. However, Bartlett's test yields a significance probability of 0.0319 because of the pronounced skewness of the data.



### □ Technical note

`robvar` implements both the conventional Levene's test centered at the mean and a median-centered test. In a simulation study, [Conover, Johnson, and Johnson \(1981\)](#) compare the properties of the two tests and recommend using the median test for asymmetric data, although for small sample sizes the test is somewhat conservative. See [Carroll and Schneider \(1985\)](#) for an explanation of why both mean- and median-centered tests have approximately the same level for symmetric distributions, but for asymmetric distributions the median test is closer to the correct level.



## Stored results

`sdtest` and `sdtesti` store the following in `r()`:

Scalars

|                      |                                        |
|----------------------|----------------------------------------|
| <code>r(N)</code>    | number of observations                 |
| <code>r(p_l)</code>  | lower one-sided $p$ -value             |
| <code>r(p_u)</code>  | upper one-sided $p$ -value             |
| <code>r(p)</code>    | two-sided $p$ -value                   |
| <code>r(F)</code>    | $F$ statistic                          |
| <code>r(sd)</code>   | standard deviation                     |
| <code>r(sd_1)</code> | standard deviation for first variable  |
| <code>r(sd_2)</code> | standard deviation for second variable |
| <code>r(df)</code>   | degrees of freedom                     |
| <code>r(df_1)</code> | numerator degrees of freedom           |
| <code>r(df_2)</code> | denominator degrees of freedom         |
| <code>r(chi2)</code> | $\chi^2$                               |

`robvar` stores the following in `r()`:

Scalars

|                       |                                                   |
|-----------------------|---------------------------------------------------|
| <code>r(N)</code>     | number of observations                            |
| <code>r(w50)</code>   | Brown and Forsythe's $F$ statistic (median)       |
| <code>r(p_w50)</code> | Brown and Forsythe's $p$ -value                   |
| <code>r(w0)</code>    | Levene's $F$ statistic                            |
| <code>r(p_w0)</code>  | Levene's $p$ -value                               |
| <code>r(w10)</code>   | Brown and Forsythe's $F$ statistic (trimmed mean) |
| <code>r(p_w10)</code> | Brown and Forsythe's $p$ -value (trimmed mean)    |
| <code>r(df_1)</code>  | numerator degrees of freedom                      |
| <code>r(df_2)</code>  | denominator degrees of freedom                    |

## Methods and formulas

See Armitage et al. (2002, 149–153) or Bland (2015, 144–145) for an introduction and explanation of the calculation of these tests.

The test for  $\sigma = \sigma_0$  is given by

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

which is distributed as  $\chi^2$  with  $n-1$  degrees of freedom.

The test for  $\sigma_x^2 = \sigma_y^2$  is given by

$$F = \frac{s_x^2}{s_y^2}$$

which is distributed as  $F$  with  $n_x-1$  and  $n_y-1$  degrees of freedom.

Let  $X_{ij}$  be the  $j$ th observation of  $X$  for the  $i$ th group. Let  $Z_{ij} = |X_{ij} - \bar{X}_i|$ , where  $\bar{X}_i$  is the mean of  $X$  in the  $i$ th group. Levene's test statistic is

$$W_0 = \frac{\sum_i n_i (\bar{Z}_i - \bar{Z})^2 / (g-1)}{\sum_i \sum_j (Z_{ij} - \bar{Z}_i)^2 / \sum_i (n_i - 1)}$$

where  $n_i$  is the number of observations in group  $i$  and  $g$  is the number of groups.  $W_{50}$  is obtained by replacing  $\bar{X}_i$  with the  $i$ th group median of  $X_{ij}$ , whereas  $W_{10}$  is obtained by replacing  $\bar{X}_i$  with the 10% trimmed mean for group  $i$ .

## References

- Armitage, P., G. Berry, and J. N. S. Matthews. 2002. *Statistical Methods in Medical Research*. 4th ed. Oxford: Blackwell.
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- Markowski, C. A., and E. P. Markowski. 1990. Conditions for the effectiveness of a preliminary test of variance. *American Statistician* 44: 322–326. <https://doi.org/10.2307/2684360>.

## Also see

[R] **ttest** —  $t$  tests (mean-comparison tests)

[Description](#)

[Syntax](#)

[Remarks and examples](#)

[Quick start](#)

[Options for search](#)

[Acknowledgment](#)

[Menu](#)

[Option for set searchdefault](#)

[Also see](#)

## Description

`search` searches a keyword database and the internet for Stata materials related to your query.

Capitalization of the words following `search` is irrelevant, as is the inclusion or exclusion of special characters such as commas and hyphens.

`set searchdefault` affects the default behavior of the `search` command. `all` is the default.

`search`, `all` is the best way to search for information on a topic across all sources, including the system help, the FAQs at the Stata website, the *Stata Journal*, and all Stata-related internet sources including community-contributed additions. From the results, you can click to go to a source or to install additions.

## Quick start

Search local keyword database and materials available via Stata's `net` command for all words (`word1`, `word2`, and `word3`)

```
search word1 word2 word3
```

Same as above, but match any word (`word1`, `word2`, or `word3`)

```
search word1 word2 word3, or
```

Search for *Stata Journal* articles about new commands like `regress`

```
search regress, sj
```

Search the manuals for entries about `tabulate` and related commands

```
search tabulate, manual
```

## Menu

Help > Search...

## Syntax

```
search word [word ...] [, search_options]
```

```
set searchdefault { all | local | net } [, permanently]
```

| <i>search_options</i>   | Description                                                                                             |
|-------------------------|---------------------------------------------------------------------------------------------------------|
| <code>all</code>        | search across both the local keyword database and the net material; the default                         |
| <code>local</code>      | search using Stata's keyword database                                                                   |
| <code>net</code>        | search across materials available via Stata's net command                                               |
| <code>author</code>     | search by author's name                                                                                 |
| <code>entry</code>      | search by entry ID                                                                                      |
| <code>exact</code>      | search across both the local keyword database and the net materials; prevents matching on abbreviations |
| <code>faq</code>        | search the FAQs posted to the Stata and other select websites                                           |
| <code>historical</code> | search entries that are of historical interest only                                                     |
| <code>or</code>         | list an entry if <i>any</i> of the words typed after search are associated with the entry               |
| <code>manual</code>     | search the entries in the <i>Stata Documentation</i>                                                    |
| <code>sj</code>         | search the entries in the <i>Stata Journal</i>                                                          |

## Options for search

`all`, the default (unless changed by `set searchdefault`), specifies that the search be performed across both the local keyword database and the net materials. The results of a search performed with `all` and no other options will be displayed in the Viewer window.

`local` specifies that the search be performed using only Stata's keyword database. The results of a search performed with `local` and no other options will be displayed in the Viewer window.

`net` specifies that the search be performed across the materials available via Stata's net command. Using `search word [word ...]`, `net` is equivalent to typing `net search word [word ...]` (without options); see [R] [net search](#). The results of a search performed with `net` and no other options will be displayed in the Viewer window.

`author` specifies that the search be performed on the basis of the author's name rather than keywords. A search with the `author` option is performed on the local keyword database only, and the results are displayed in the Results window.

`entry` specifies that the search be performed on the basis of entry IDs rather than keywords. A search with the `entry` option is performed on the local keyword database only, and the results are displayed in the Results window.

`exact` prevents matching on abbreviations. A search with the `exact` option is performed across both the local keyword database and the net materials, and the results are displayed in the Results window.

`faq` limits the search to the FAQs on the [Stata](#) and other select websites. A search with the `faq` option is performed on the local keyword database only, and the results are displayed in the Results window.

`historical` adds to the search entries that are of historical interest only. By default, such entries are not listed. Past entries are classified as historical if they discuss a feature that later became an official part of Stata. Updates to historical entries will always be found, even if `historical` is not specified. A search with the `historical` option is performed on the local keyword database only, and the results are displayed in the Results window.

`or` specifies that an entry be listed if any of the words typed after `search` are associated with the entry. The default is to list the entry only if all the words specified are associated with the entry. A search with the `or` option is performed on the local keyword database only, and the results are displayed in the Results window.

`manual` limits the search to entries in the *Stata Documentation*; that is, the search is limited to the *User's Guide* and all the reference manuals. A search with the `manual` option is performed on the local keyword database only, and the results are displayed in the Results window.

`sj` limits the search to entries in the *Stata Journal*; see [\[R\] sj](#). A search with the `sj` option is performed on the local keyword database only, and the results are displayed in the Results window.

## Option for set searchdefault

`permanently` specifies that, in addition to making the change right now, the `searchdefault` setting be remembered and become the default setting when you invoke Stata.

## Remarks and examples

Remarks are presented under the following headings:

- [Introduction](#)
- [Internet searches](#)
- [Author searches](#)
- [Entry ID searches](#)
- [Return codes](#)

## Introduction

See [\[U\] 4 Stata's help and search facilities](#) for a tutorial introduction to `search`. `search` is one of Stata's most useful commands. To understand the advanced features of `search`, you need to know how it works.

`search` has a database—files—containing the titles, etc., of every entry in the [Stata Documentation](#), undocumented help files, NetCourses, Stata Press books, FAQs posted on the Stata website, videos posted on the Stata YouTube channel, selected articles on StataCorp's official blog, selected community-contributed FAQs and examples, and the articles in the *Stata Journal*. In these files is a list of words, called keywords, associated with each entry.

When you type `search xyz`, `search` reads the database and compares the list of keywords with `xyz`. If it finds `xyz` in the list or a keyword that allows an abbreviation of `xyz`, it displays the entry.

When you type `search xyz abc`, `search` does the same thing but displays an entry only if it contains both keywords. The order does not matter, so you can `search linear regression` or `search regression linear`.

Obviously, how many entries `search` finds depends on how the search database was constructed. We have included a plethora of keywords under the theory that, for a given request, it is better to list too much rather than risk listing nothing at all. Still, you are in the position of guessing the keywords. Do you look up normality test, normality tests, or tests of normality? Well, normality test would be best, but all would work. In general, use the singular, and strike the unnecessary words. For guidelines for specifying keywords, see [U] 4.6 More on search.

`set searchdefault` allows you to specify where `search` searches. `set searchdefault all`, the default, indicates that both the keyword database and the internet are to be searched. `set searchdefault local` restricts `search` to using only Stata's keyword database. `set searchdefault net` restricts `search` to searching only the internet.

## Internet searches

`search` with the `net` option searches the internet for community-contributed additions to Stata, including, but not limited to, community-contributed additions published in the *Stata Journal* (SJ). `search keywords`, `net` performs the same search as the command `net search` (with no options); see [R] net search.

```
. search random effect, net
Search of web resources from Stata and other users
(contacting https://www.stata.com)
packages found (Stata Journal listed first)

(output omitted)
st0468_1 from http://www.stata-journal.com/software/sj18-4
 SJ18-4 st0468_1. Update: Estimate hybrid and... / Update: Estimate hybrid
 and correlated random- / effects and Mundlak mixed-effects models for /
 linear and nonlinear outcomes / by Reinhard Schunck, GESIS --
 Leibniz-Institute / for the Social Sciences, Cologne, Germany / Francisco
st0543 from http://www.stata-journal.com/software/sj18-4
 SJ18-4 st0543. Fit dynamic random-effects probit... / Fit dynamic
 random-effects probit models with / unobserved heterogeneity / by Raffaele
 Grotti, / Department of Political and Social Sciences / European
 University Institute / San Domenico di Fiesole, Italy / Giorgio Cutuli,
(output omitted)
(end of search)
```

## Author searches

`search` ordinarily compares the words following `search` with the keywords for the entry. If you specify the `author` option, however, it compares the words with the author's name. In the search database, we have filled in author names for all SJ inserts.

For instance, in the *Acknowledgments* of [R] `kdensity`, you will discover the name Isaías H. Salgado-Ugarte. You want to know if he has written any articles in the SJ. To find out, you type

```
. search Salgado-Ugarte, author
(output omitted)
```

Names like Salgado-Ugarte are confusing to many people. `search` does not require you to specify the entire name; what you type is compared with each “word” of the name and, if any part matches, the entry is listed. The hyphen is a special character, and you can omit it. Thus, you can obtain the same list by looking up Salgado, Ugarte, or Salgado Ugarte without the hyphen.

## Entry ID searches

If you specify the `entry` option, `search` compares what you have typed with the entry ID. The entry ID is not the title—it is the reference listed to the left of the title that tells you where to look. For instance, in

```
regress Linear regression
(help regress)
```

[R] `regress` is the entry ID. This is a reference, of course, to this manual. In

```
FAQ Analysis of multiple failure-time survival data
. M. Cleves and I. Canette
07/09 How do I analyze multiple failure-time data using Stata?
 https://www.stata.com/support/faqs/statistics/multiple-failure-
 type-data/
```

“FAQ” is the entry ID. In

```
SJ-7-1 st0118 . . A survey on survey stat.: What is and can be done in Stata
. F. Kreuter and R. Valliant
Q1/07 SJ7(1):1--21 (no commands)
discusses survey issues in analyzing complex survey
data and describes some of Stata’s capabilities for
such analyses
```

“SJ-7-1” is the entry ID.

`search` with the `entry` option searches these entry IDs.

Thus you could generate a table of contents for the *User’s Guide* by typing

```
. search [U], entry
(output omitted)
```

You could generate a table of contents for *Stata Journal*, Volume 1, Issue 1, by typing

```
. search sj-1-1, entry
(output omitted)
```

## Return codes

In addition to indexing the entries in the *User's Guide* and all the *Reference* manuals, `search` also can be used to search return codes.

To see information on return code 131, type

```
. search rc 131
[P] error Return code 131
 not possible with test;
 You requested a test of a hypothesis that is nonlinear in the
 variables. test tests only linear hypotheses. Use testnl.
(output omitted)
```

If you want a list of all Stata return codes, type

```
. search error, entry
(output omitted)
```

## Acknowledgment

We thank Nicholas J. Cox of the Department of Geography at Durham University, UK, who is coeditor of the *Stata Journal* and author of *Speaking Stata Graphics* for his contributions to the `search` command.

## Also see

[R] [help](#) — Display help in Stata

[R] [net search](#) — Search the internet for installable packages

[U] [4 Stata's help and search facilities](#)

[Description](#)  
[Options](#)[Quick start](#)  
[Remarks and examples](#)[Menu](#)  
[Acknowledgment](#)[Syntax](#)  
[Also see](#)

## Description

`serrbar` is typically used with a dataset containing means, standard deviations or standard errors, and an *xvar*. `serrbar` uses these data to create a standard error bar chart. The means are plotted against *xvar*, and error bars around the means have a width determined by the standard deviation or standard error. While it is most common to use `serrbar` with this type of data, `serrbar` may also be used to create a scatterplot with error bars for other types of data.

## Quick start

Plot of *y* versus *x* with error bars representing  $y \pm s$

```
serrbar y s x
```

Same as above, but with error bars for  $y \pm 2 \times s$

```
serrbar y s x, scale(2)
```

## Menu

Statistics > Other > Quality control > Standard error bar chart

## Syntax

```
serrbar mvar svar xvar [if] [in] [, options]
```

| <i>options</i>                                 | Description                                                                                 |
|------------------------------------------------|---------------------------------------------------------------------------------------------|
| <b>Main</b>                                    |                                                                                             |
| <code>scale(#)</code>                          | scale length of graph bars; default is <code>scale(1)</code>                                |
| <b>Error bars</b>                              |                                                                                             |
| <code>rcap_options</code>                      | affect rendition of capped spikes                                                           |
| <b>Plotted points</b>                          |                                                                                             |
| <code>mvopts(scatter_options)</code>           | affect rendition of plotted points                                                          |
| <b>Add plots</b>                               |                                                                                             |
| <code>addplot(plot)</code>                     | add other plots to generated graph                                                          |
| <b>Y axis, X axis, Titles, Legend, Overall</b> |                                                                                             |
| <code>twoway_options</code>                    | any options other than <code>by()</code> documented in [G-3] <a href="#">twoway_options</a> |

## Options

### Main

`scale(#)` controls the length of the bars. The upper and lower limits of the bars will be  $mvar + scale() \times svar$  and  $mvar - scale() \times svar$ . The default is `scale(1)`.

### Error bars

`rcap_options` affect the rendition of the plotted error bars (the capped spikes). See [G-2] [graph twoway rcap](#).

### Plotted points

`mvopts(scatter_options)` affects the rendition of the plotted points (*mvar* versus *xvar*). See [G-2] [graph twoway scatter](#).

### Add plots

`addplot(plot)` provides a way to add other plots to the generated graph; see [G-3] [addplot\\_option](#).

### Y axis, X axis, Titles, Legend, Overall

`twoway_options` are any of the options documented in [G-3] [twoway\\_options](#), excluding `by()`. These include options for titling the graph (see [G-3] [title\\_options](#)) and for saving the graph to disk (see [G-3] [saving\\_option](#)).

## Remarks and examples

### ▷ Example 1

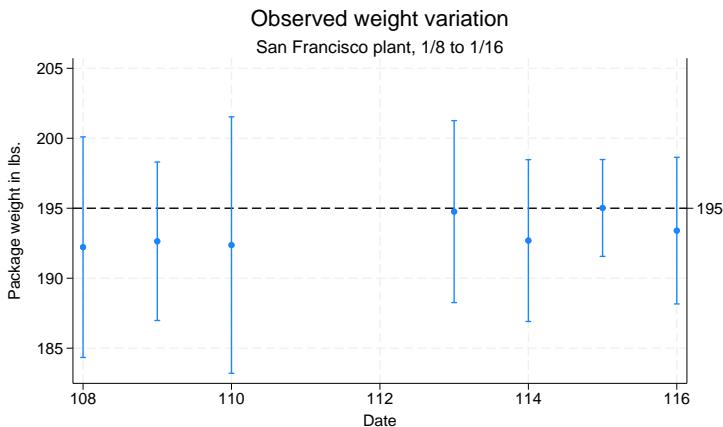
In quality-control applications, the three most commonly used variables with this command are the process mean, process standard deviation, and time. For instance, we have data on the average weights and standard deviations from an assembly line in San Francisco for the period January 8 to January 16. Our data are

```
. use https://www.stata-press.com/data/r19/assembly
. list, sep(0) divider
```

|    | date | mean   | std  |
|----|------|--------|------|
| 1. | 108  | 192.22 | 3.94 |
| 2. | 109  | 192.64 | 2.83 |
| 3. | 110  | 192.37 | 4.58 |
| 4. | 113  | 194.76 | 3.25 |
| 5. | 114  | 192.69 | 2.89 |
| 6. | 115  | 195.02 | 1.73 |
| 7. | 116  | 193.40 | 2.62 |

We type `serrbar mean std date, scale(2)` but, after seeing the result, decide to make it fancier:

```
. serrbar mean std date, scale(2) title("Observed weight variation")
> sub("San Francisco plant, 1/8 to 1/16") yline(195) yaxis(1 2)
> ylab(195, axis(2)) ytitle("", axis(2))
```



## Acknowledgment

`serrbar` was written by Nicholas J. Cox of the Department of Geography at Durham University, UK, who is coeditor of the *Stata Journal* and author of *Speaking Stata Graphics*.

## Also see

[R] [QC](#) — Quality control charts

[Description](#)[Syntax](#)[Remarks and examples](#)[Also see](#)

## Description

This entry provides a reference to Stata's `set` commands. For many entries, more thorough information is provided elsewhere; see the Reference field in each entry below for the location of this information.

To reset system parameters to factory defaults, see [\[R\] `set\_defaults`](#).

## Syntax

```
set [setcommand ...]
```

`set` typed without arguments is equivalent to `query` typed without arguments.

## Remarks and examples

`set adosize`

Syntax: `set adosize # [ , permanently ]`

Default: 1,000

Description: sets the maximum amount of memory that automatically loaded do-files may consume.  $10 \leq \# \leq 10000$ .

Reference: [\[P\] `sysdir`](#)

`set autotabgraphs` (Windows only)

Syntax: `set autotabgraphs {on|off} [ , permanently ]`

Default: off

Description: determines whether graphs are created as tabs within one window or as separate windows.

Reference: `help autotabgraphs`

`set cformat`

Syntax: `set cformat [fmt] [ , permanently ]`

Description: specifies the output format of coefficients, standard errors, and confidence limits in coefficient tables. *fmt* is a numerical format; see [\[D\] `format`](#).

Reference: [\[R\] `set cformat`](#)

`set clevel`

Syntax: `set clevel # [ , permanently ]`

Default: 95

Description: sets the default credible level for credible intervals for all commands that report credible intervals.  $10.00 \leq \# \leq 99.99$ , and  $\#$  can have at most two digits after the decimal point.

Reference: [\[BAYES\] `set clevel`](#)

**set coeftabresults**

Syntax: set coeftabresults {on|off}  
Default: on  
Description: determines whether coefficient table results are stored in `r()`.  
There is no permanently option because permanently is implied.  
Reference: help coeftabresults

**set collect\_double**

Syntax: set collect\_double {on|off} [, permanently]  
Default: on  
Description: controls the storage type for numeric values that are saved in collections when using `collect save`.  
Reference: [TABLES] [set collect\\_double](#)

**set collect\_label**

Syntax: set collect\_label {default|*label*} [, permanently]  
Default: default  
Description: controls the default labels used in tables created by `collect`.  
Reference: [TABLES] [set collect\\_label](#)

**set collect\_style**

Syntax: set collect\_style {default|*style*} [, permanently]  
Default: default  
Description: controls the default styles used in tables created by `collect`.  
Reference: [TABLES] [set collect\\_style](#)

**set collect\_warn**

Syntax: set collect\_warn {on|off} [, permanently]  
Default: on  
Description: controls whether `collect` shows notes warning about unrecognized tags.  
Reference: [TABLES] [set collect\\_warn](#)

## set conren (Unix console only)

Syntax 1: set conren

Syntax 2: set conren clear

Syntax 3: set conren [*sf* | *bf* | *it*]  
                   {*result* | [*txt* | *text*] | *input* | *error* | *link* | *hilite*}  
                   [*char*[*char*...]]

Syntax 4: set conren {*ulon* | *uloff*} [*char* [*char*...]]

Syntax 5: set conren reset [*char* [*char*...]]

Description: can possibly make the output on your screen appear prettier.

set conren displays a list of the currently defined display codes.

set conren clear clears all codes.

set conren followed by a font type (*bf*, *sf*, or *it*) and display context (*result*, *error*, *link*, or *hilite*) and then followed by a series of space-separated characters sets the code for the specified font type and display context. If the font type is omitted, the code is set to the same specified code for all three font types.

set conren ulon and set conren uloff set the codes for turning on and off underlining.

set conren reset sets the code that will turn off all display and underlining codes.

Reference: [\[GSU\] conren](#)

## set copycolor (Mac and Windows only)

Syntax: set copycolor {*automatic* | *asis* | *gs1* | *gs2* | *gs3*} [, *permanently*]

Default: automatic

Description: determines how colors are handled when graphs are copied to the Clipboard.

Reference: [\[G-2\] set printcolor](#)

## set dockable (Windows only)

Syntax: set dockable {*on* | *off*} [, *permanently*]

Default: on

Description: determines whether to enable the use of dockable window characteristics, including the ability to dock or tab a window into another window.

Reference: help dockable

## set docx\_hardbreak

Syntax: set docx\_hardbreak {*on* | *off*}

Default: off

Description: determines whether spaces are added after hard line breaks within text blocks.

Reference: [\[RPT\] set docx](#)

## set docx\_maxtable

Syntax: set docx\_maxtable # [, *permanently*]

Default: 500

Description: sets the maximum number of tables allowed in putdocx.

Reference: [\[RPT\] putdocx table](#)

**set docx\_paramode**

Syntax: set docx\_paramode {on | off}

Default: off

Description: determines whether empty lines in a text block signal the beginning of a new paragraph.

Reference: [\[RPT\] set docx](#)

**set doeditbackup**

Syntax: set doeditbackup {on | off}

Default: on

Description: determines whether to enable automatic backups of documents in the Do-file Editor.

Reference: [help doeditbackup](#)

**set dots**

Syntax: set dots {on | off} [ , permanently]

Default: on

Description: enables or disables commands that support the dots() option from reporting a dot each time statistics are computed from a sample or resample of the dataset.

Reference: [help dots](#)

**set doublebuffer (Windows only)**

Syntax: set doublebuffer {on | off} [ , permanently]

Default: on

Description: enables or disables double buffering of the Results, Viewer, and Data Editor windows. Double buffering prevents the windows from flickering when redrawn or resized. Users who encounter performance problems such as the Results window outputting very slowly should disable double buffering.

Reference: [help doublebuffer](#)

**set dp**

Syntax: set dp {comma | period} [ , permanently]

Default: period

Description: determines whether a period or a comma is to be used as the decimal point.

Reference: [\[D\] format](#)

**set dtable\_style**

Syntax: set dtable\_style {dtable | *style*} [ , permanently]

Default: dtable

Description: controls the default styles used in tables created by dtable.

Reference: [\[TABLES\] set dtable\\_style](#)

**set dtascomplevel**

Syntax: set dtascomplevel #  
Default: 1  
Description: sets the compression level for `frames save`, where # can be any integer between 0 and 9.  
Reference: help dtascomplevel

**set emptycells**

Syntax: set emptycells {keep|drop} [ , permanently ]  
Default: keep  
Description: sets what to do with empty cells in interactions.  
Reference: [R] [set emptycells](#)

**set etable\_style**

Syntax: set etable\_style {etable|style} [ , permanently ]  
Default: etable  
Description: controls the default styles used in tables created by etable.  
Reference: [TABLES] [set etable\\_style](#)

**set fastscroll (Unix and Windows only)**

Syntax: set fastscroll {on|off} [ , permanently ]  
Default: on  
Description: sets the scrolling method for new output in the Results window. Setting fastscroll to on is faster but can be jumpy. Setting fastscroll to off is slower but smoother.  
Reference: help fastscroll

**set floatwindows (Windows only)**

Syntax: set floatwindows {on|off}  
Default: off  
Description: determines whether to enable floating window behavior for dialog boxes and dockable window. The term “float” in this context means that a window will always float over the main Stata window; these windows cannot be placed behind the main Stata window. There is no permanently option because permanently is implied.  
Reference: help floatwindows

**set fredkey**

Syntax: set fredkey *key* [ , permanently ]  
Description: sets the API key for importing data from the Federal Reserve Economic Data.  
Reference: [D] [import fred](#)

**set fvbase**

Syntax: set fvbase {on|off}  
Description: specifies whether to automatically determine the default base level for factor variables.  
Reference: help fvbase

**set fvlabel**

Syntax: `set fvlabel { on | off } [ , permanently ]`  
Description: specifies whether to display factor-variable value labels in coefficient tables.  
Reference: [R] [set showbaselevels](#)

**set fvtrack**

Syntax: `set fvtrack { term | factor } [ , permanently ]`  
Description: allows you to control how Stata keeps track of factor levels when you use factor-variables notation.  
Reference: `help fvtrack`

**set fvwrap**

Syntax: `set fvwrap # [ , permanently ]`  
Description: specifies that long value labels wrap # lines in coefficient tables.  
Reference: [R] [set showbaselevels](#)

**set fvwrapon**

Syntax: `set fvwrapon { word | width } [ , permanently ]`  
Description: specifies whether value labels that wrap will break at word boundaries or break based on available space.  
Reference: [R] [set showbaselevels](#)

**set graphics**

Syntax: `set graphics { on | off }`  
Default: on; default is off for console Stata  
Description: determines whether graphs are displayed on your monitor.  
Reference: [G-2] [set graphics](#)

**set haverdir**

Syntax: `set haverdir "path" [ , permanently ]`  
Description: specifies the directory where the Haver databases are stored.  
Reference: [D] [import haver](#)

**set httpproxy**

Syntax: `set httpproxy { on | off } [ , init ]`  
Default: off  
Description: turns on/off the use of a proxy server. There is no permanently option because permanently is implied.  
Reference: [R] [netio](#)

**set httpproxyauth**

Syntax: `set httpproxyauth { on | off }`  
Default: off  
Description: determines whether authorization is required for the proxy server. There is no permanently option because permanently is implied.  
Reference: [R] [netio](#)

**set httpproxyhost**

Syntax: `set httpproxyhost ["name"]`

Description: sets the name of a host to be used as a proxy server. There is no permanently option because permanently is implied.

Reference: [\[R\] netio](#)

**set httpproxyport**

Syntax: `set httpproxyport #`

Default: 8080 if Stata cannot autodetect the proper setting for your computer.

Description: sets the port number for a proxy server. There is no permanently option because permanently is implied.

Reference: [\[R\] netio](#)

**set httpproxypw**

Syntax: `set httpproxypw ["password"]`

Description: sets the appropriate password. There is no permanently option because permanently is implied.

Reference: [\[R\] netio](#)

**set httpproxyuser**

Syntax: `set httpproxyuser ["name"]`

Description: sets the appropriate user ID. There is no permanently option because permanently is implied.

Reference: [\[R\] netio](#)

**set include\_bitmap (Mac only)**

Syntax: `set include_bitmap {on|off} [, permanently]`

Default: on

Description: sets the output behavior when copying an image to the Clipboard.

Reference: `help include_bitmap`

**set iterlog**

Syntax: `set iterlog {on|off} [, permanently]`

Description: specifies whether to display an iteration log.

Reference: [\[R\] set iter](#)

**set java\_heapmax**

Syntax: `set java_heapmax {default | #[m|g]}`

Description: sets the maximum amount of heap memory allocated for the Java Virtual Machine.

Reference: `set java_heapmax` is a synonym for `java set heapmax`; see [\[P\] Java utilities](#)

**set java\_home**

Syntax: `set java_home {default | "path_to_java_home_dir" }`

Description: sets the path to the Java Runtime Environment.

Reference: `set java_home` is a synonym for `java set home`; see [\[P\] Java utilities](#)

**set kmp\_blocktime**

Syntax: set kmp\_blocktime # [ , permanently ]

Default: -1

Description: controls the behavior of the OpenMP runtime library regarding the suspension of threads in Stata when they are idle. This can be changed only in Stata/MP.

Reference: help kmp\_blocktime

**set lapack\_mkl**

Syntax: set lapack\_mkl { on | off } [ , permanently ]

Default: on

Description: specifies whether to use Intel MKL LAPACK routines.

Reference: [\[M-1\] LAPACK](#)

**set lapack\_mkl\_cnr**

Syntax: set lapack\_mkl\_cnr { default | auto | compatible | off }

Default: on

Description: sets the conditional numerical reproducibility mode for Intel MKL LAPACK routines.

Reference: [\[M-1\] LAPACK](#)

**set lapack\_openblas**

Syntax: set lapack\_openblas { on | off } [ , permanently ]

Default: on

Description: sets whether to use the LAPACK library from OpenBLAS.

Reference: [\[M-1\] LAPACK](#)

**set level**

Syntax: set level # [ , permanently ]

Default: 95

Description: sets the default confidence level for confidence intervals for all commands that report confidence intervals.  $10.00 \leq \# \leq 99.99$ , and # can have at most two digits after the decimal point.

Reference: [\[R\] level](#)

**set linegap**

Syntax: set linegap #

Default: 1

Description: sets the space between lines, in pixels, in the Results window. There is no permanently option because permanently is implied.

Reference: help linegap

**set linesize**

Syntax: set linesize #

Default: 1 less than the full width of the screen

Description: sets the line width, in characters, for both the screen and the log file.

Reference: [\[R\] log](#)

**set locale\_functions**

Syntax: set locale\_functions *locale*

Default: en\_US

Description: sets the locale to be used by functions that take *locale* as an optional argument.

Reference: [P] [set locale\\_functions](#)

**set locale\_ui**

Syntax: set locale\_ui *locale*

Default: en\_US

Description: sets the locale that Stata uses for the user interface.

Reference: [P] [set locale\\_ui](#)

**set locksplitters (Windows only)**

Syntax: set locksplitters {on|off} [, permanently]

Default: off

Description: determines whether splitters should be locked so that docked windows cannot be resized.

Reference: help locksplitters

**set logmsg**

Syntax: set logmsg {on|off}

Default: on

Description: specifies whether the default message is displayed at the top and bottom of log files.

Reference: [R] [log](#)

**set logtype**

Syntax: set logtype {text|smcl} [, permanently]

Default: smcl

Description: sets the default log filetype.

Reference: [R] [log](#)

**set lstretch**

Syntax: set lstretch [on|off] [, permanently]

Default: on

Description: specifies whether to automatically widen the coefficient table up to the width of the Results window to accommodate longer variable names.

Reference: help lstretch

set matacache, set matafavor, set matalibs, set matalnum, set matamofirst,  
set mataoptimize, set matasolveto1, and set matastrict; see [M-3] [mata set](#).

## set maxbezierpath (Mac only)

Syntax: set maxbezierpath # [ , permanently ]

Default: 0

Description: sets the maximum number of lines that can be added to a Bézier path when rendering a Stata graph to the screen.

Reference: help maxbezierpath

## set maxdb

Syntax: set maxdb # [ , permanently ]

Default: 50

Description: sets the maximum number of dialog boxes whose contents are remembered from one invocation to the next during a session.  $5 \leq \# \leq 1000$

Reference: [R] db

## set maxiter

Syntax: set maxiter # [ , permanently ]

Default: 300

Description: sets the default maximum number of iterations for estimation commands.  $0 \leq \# \leq 16000$

Reference: [R] set iter

## set max\_memory

Syntax: set max\_memory #[b | k | m | g] [ , permanently ]

Default: . (all the memory the operating system will supply)

Description: specifies the maximum amount of memory Stata can use to store your data.  $2 \times \text{segmentsize} \leq \# \leq .$

Reference: [D] memory

## set max\_preservemem

Syntax: set max\_preservemem #[b | k | m | g] [ , permanently ]

Default: 1g (1 gigabyte)

Description: controls the maximum amount of memory preserve will use to store preserved datasets in memory.

Reference: [P] preserve

## set maxvar

Syntax: set maxvar # [ , permanently ]

Default: 5000 for Stata/MP and Stata/SE and 2048 for Stata/BE

Description: sets the maximum number of variables. This can be changed only in Stata/MP and Stata/SE.  $2048 \leq \# \leq 32767$

Reference: [D] memory

**set min\_memory**

Syntax: `set min_memory #[b|k|m|g] [ , permanently]`

Default: 0

Description: specifies an amount of memory Stata will not fall below. This setting affects efficiency, not the size of datasets you can analyze.  $0 \leq \# \leq \text{max\_memory}$

Reference: [\[D\] memory](#)

**set more**

Syntax: `set more {on|off} [ , permanently]`

Default: off

Description: pauses when `—more—` is displayed, continuing only when the user presses a key.

Reference: [\[R\] more](#)

**set niceness**

Syntax: `set niceness # [ , permanently]`

Default: 5

Description: affects how soon Stata gives back unused segments to the operating system.  
 $0 \leq \# \leq 10$

Reference: [\[D\] memory](#)

**set notifyuser (Mac only)**

Syntax: `set notifyuser {on|off} [ , permanently]`

Default: on

Description: sets the default Notification Manager behavior in Stata.

Reference: `help notifyuser`

**set obs**

Syntax: `set obs #`

Default: current number of observations

Description: changes the number of observations in the current dataset. `#` must be at least as large as the current number of observations. If there are variables in memory, the values of all new observations are set to *missing*.

Reference: [\[D\] obs](#)

**set odbcdriver**

Syntax: `set odbcdriver {unicode|ansi} [ , permanently]`

Default: unicode

Description: determines whether Unicode or ANSI is your ODBC driver.

Reference: [\[D\] odbc](#)

**set odbcmgr (Mac and Unix only)**

Syntax: `set odbcmgr {iodbc|unixodbc} [ , permanently]`

Default: iodbc

Description: determines whether iODBC or unixODBC is your ODBC driver manager.

Reference: [\[D\] odbc](#)

## set output

Syntax: set output {proc | inform | error}

Default: proc

Description: specifies the output to be displayed. proc means display all output; inform suppresses procedure output but displays informative messages and error messages; error suppresses all output except error messages. set output is seldom used.

Reference: [P] [quietly](#)

## set pagesize

Syntax: set pagesize #

Default: 2 less than the physical number of lines on the screen

Description: sets the number of lines between `—more—` messages.

Reference: [R] [more](#)

## set pdf\_maxtable

Syntax: set pdf\_maxtable # [ , permanently ]

Default: 500

Description: sets the maximum number of tables allowed in `putpdf`.

Reference: [RPT] [putpdf table](#)

## set pformat

Syntax: set pformat [*fmt*] [ , permanently ]

Description: specifies the output format of *p*-values in coefficient tables.  
*fmt* is a numerical format; see [D] [format](#).

Reference: [R] [set cformat](#)

## set pinnable (Windows only)

Syntax: set pinnable {on | off} [ , permanently ]

Default: on

Description: determines whether to enable the use of pinnable window characteristics for certain windows in Stata.

Reference: `help pinnable`

## set playsnd (Mac only)

Syntax: set playsnd {on | off} [ , permanently ]

Default: on

Description: sets the sound behavior for the Notification Manager behavior in Stata.

Reference: `help playsnd`

## set printcolor

Syntax: set printcolor {automatic | asis | gs1 | gs2 | gs3} [ , permanently ]

Default: automatic

Description: determines how colors are handled when graphs are printed.

Reference: [G-2] [set printcolor](#)

**set processors**

Syntax:     **set processors #**  
 Description: sets the number of processors or cores that Stata/MP will use. The default is the number of processors available on the computer, or the number of processors allowed by Stata/MP's license, whichever is less.  
 Reference:   **help processors**

**set python\_exec**

Syntax:     **set python\_exec *pyexecutable* [ , permanently ]**  
 Description: sets which version of Python to use.  
 Reference:   **set python\_exec** is a synonym for **python set exec**;  
               see [P] [PyStata integration](#)

**set python\_userpath**

Syntax:     **set python\_userpath *path* [*path* ...] [ , permanently prepend ]**  
 Description: sets the user's own module search paths in addition to the system search paths.  
 Reference:   **set python\_userpath** is a synonym for **python set userpath**;  
               see [P] [PyStata integration](#)

**set reshape\_favor**

Syntax:     **set reshape\_favor {default|memory|speed} [ , permanently ]**  
 Description: sets the default method for reshaping data.  
 Reference:   [D] [reshape](#)

**set reventries**

Syntax:     **set reventries # [ , permanently ]**  
 Default:     5000  
 Description: sets the number of scrollbar lines available in the History window.  
                $5 \leq \# \leq 32000$ .  
 Reference:   **help reventries**

**set revkeyboard (Mac only)**

Syntax:     **set revkeyboard {on|off} [ , permanently ]**  
 Default:     on  
 Description: sets the keyboard navigation behavior for the History window. **on** indicates that you can use the keyboard to navigate and enter items from the History window into the Command window. **off** indicates that all keyboard input be directed at the Command window; items can be entered from the History window only by using the mouse.  
 Reference:   **help revkeyboard**

**set rmsg**

Syntax: `set rmsg {on|off} [ , permanently ]`

Default: `off`

Description: indicates whether a return message telling the execution time is to be displayed at the completion of each command.

Reference: [\[P\] rmsg](#)

**set rng**

Syntax: `set rng {default|mt64|mt64s|kiss32}`

Default: `default`

Description: determines which random-number generator Stata's random-number functions and commands will use.

Reference: [\[R\] set rng](#)

**set rngstate**

Syntax: `set rngstate statecode`

Description: resets the state of the random-number generator to the value specified.

Reference: [\[R\] set seed](#)

**set rngstream**

Syntax: `set rngstream #`

Description: specifies the stream from which Stata's stream random-number generator should draw random numbers.

Reference: [\[R\] set rngstream](#)

**set scheme**

Syntax: `set scheme schemename [ , permanently ]`

Default: `s2color`

Description: determines the overall look for graphs.

Reference: [\[G-2\] set scheme](#)

**set scrollbarbufsize**

Syntax: `set scrollbarbufsize #`

Default: `200000`

Description: sets the scrollbar buffer size, in bytes, for the Results window; may be set between 10,000 and 2,000,000.

Reference: `help scrollbarbufsize`

**set searchdefault**

Syntax: `set searchdefault {local|net|all} [ , permanently ]`

Default: `local`

Description: sets the default behavior of the search command. `set searchdefault local` restricts search to use only Stata's keyword database. `set searchdefault net` restricts search to searching only the internet. `set searchdefault all` indicates that both the keyword database and the internet are to be searched.

Reference: [\[R\] search](#)

**set seed**

Syntax: `set seed #`

Default: `123456789`

Description: specifies initial value of the random-number seed used by the [random-number functions](#), such as `runiform()` and `rnormal()`.

Reference: [\[R\] set seed](#)

**set segmentsize**

Syntax: `set segmentsize #[b|k|m|g] [ , permanently ]`

Default: `32m` for 64-bit machines

Description: Stata allocates memory for data in units of `segmentsize`. This setting changes the amount of memory in a single segment.  
 $1m \leq \# \leq 32g$  for 64-bit machines

Reference: [\[D\] memory](#)

**set sformat**

Syntax: `set sformat [fmt] [ , permanently ]`

Description: specifies the output format of test statistics in coefficient tables.  
*fmt* is a numerical format; see [\[D\] format](#).

Reference: [\[R\] set cformat](#)

**set showbaselevels**

Syntax: `set showbaselevels {on|off|all} [ , permanently ]`

Description: specifies whether to display base levels of factor variables and their interactions in coefficient tables.

Reference: [\[R\] set showbaselevels](#)

**set showemptycells**

Syntax: `set showemptycells {on|off} [ , permanently ]`

Description: specifies whether to display empty cells in coefficient tables.

Reference: [\[R\] set showbaselevels](#)

**set showomitted**

Syntax: `set showomitted {on|off} [ , permanently ]`

Description: specifies whether to display omitted coefficients in coefficient tables.

Reference: [\[R\] set showbaselevels](#)

**set smoothfonts** (Mac only)

Syntax: set **smoothfonts** {on | off}

Default: on

Description: determines whether to use font smoothing (antialiased text) in the Results, Viewer, and Data Editor windows.

Reference: help smoothfonts

**set sortmethod**

Syntax: set sortmethod {default | fsort | qsort}

Default: default

Description: determines which sorting method will be used by **sort**, **gsort**, and any other commands that use sorting as part of their computation.

Reference: [P] [set sortmethod](#)

**set sortrngstate**

Syntax: set sortrngstate #

Default: 1001XZA112210f4b16c1cb10507a1f38cb440c40003c9a83566fa1201b69...

Description: specifies the initial value of the state used for the random-number generator that randomizes data before they are sorted. This value is used by **sort**, **gsort**, and any other commands that use sorting as part of their computation.

Reference: [P] [set sortrngstate](#)

**set svy\_tab\_comps**

Syntax: set svy\_tab\_comps *style* [, permanently]

Default: svy\_tabulate\_composites

Description: controls the default styles containing the composite results defined for two-way tests of independence reported by svy: tabulate

Reference: [TABLES] [set tabulate\\_style](#)

**set table\_style**

Syntax: set table\_style *style* [, permanently]

Default: table

Description: controls the default styles used in tables created by table.

Reference: [TABLES] [set table\\_style](#)

**set tabulate\_comps**

Syntax: set tabulate\_comps *style* [, permanently]

Default: tabulate\_composites

Description: controls the default styles containing the composite results defined for two-way measures of association reported by tabulate, tab2, and tabi

Reference: [TABLES] [set tabulate\\_style](#)

**set tabulate\_style**

Syntax: set tabulate\_style *style* [ , permanently ]  
Default: tabulate  
Description: controls the default styles used in tables created by `tabulate`, `tab1`, `tab2`, `tabi`, and `svy: tabulate`.  
Reference: [TABLES] [set tabulate\\_style](#)

**set taskbargroups (Windows only)**

Syntax: set taskbargroups {on | off}  
Default: on  
Description: determines whether to enable taskbar grouping of windows in Stata.  
Reference: `help taskbargroups`

**set trace**

Syntax: set trace {on | off}  
Default: off  
Description: determines whether to trace the execution of programs for debugging.  
Reference: [P] [trace](#)

**set tracedepth**

Syntax: set tracedepth #  
Default: 32000 (equivalent to  $\infty$ )  
Description: if `trace` is set on, traces execution of programs and nested programs up to `tracedepth`. For example, if `tracedepth` is 2, the current program and any subroutine called would be traced, but subroutines of subroutines would not be traced.  
Reference: [P] [trace](#)

**set traceexpand**

Syntax: set traceexpand {on | off} [ , permanently ]  
Default: on  
Description: if `trace` is set on, shows lines both before and after macro expansion. If `traceexpand` is set off, only the line before macro expansion is shown.  
Reference: [P] [trace](#)

**set tracehilite**

Syntax: set tracehilite "*pattern*" [ , word ]  
Default: ""  
Description: highlights *pattern* in the trace output.  
Reference: [P] [trace](#)

**set traceindent**

Syntax: set traceindent {on | off} [ , permanently ]

Default: on

Description: if trace is set on, indents displayed lines according to their nesting level. The lines of the main program are not indented. Two spaces of indentation are used for each level of nested subroutine.

Reference: [P] [trace](#)

**set tracenumber**

Syntax: set tracenumber {on | off} [ , permanently ]

Default: off

Description: if trace is set on, shows the nesting level numerically in front of the line. Lines of the main program are preceded by 01, lines of subroutines called by the main program are preceded by 02, etc.

Reference: [P] [trace](#)

**set tracesep**

Syntax: set tracesep {on | off} [ , permanently ]

Default: on

Description: if trace is set on, displays a horizontal separator line that displays the name of the subroutine whenever a subroutine is called or exits.

Reference: [P] [trace](#)

**set type**

Syntax: set type {float | double} [ , permanently ]

Default: float

Description: specifies the default storage type assigned to new variables.

Reference: [D] [generate](#)

**set update\_interval (Mac and Windows only)**

Syntax: set update\_interval #

Default: 7

Description: sets the number of days to elapse before performing the next automatic update query.

Reference: [R] [update](#)

**set update\_prompt (Mac and Windows only)**

Syntax: set update\_prompt {on | off}

Default: on

Description: determines whether a dialog is to be displayed before performing an automatic update query. There is no permanently option because permanently is implied.

Reference: [R] [update](#)

set update\_query (Mac and Windows only)

Syntax: set update\_query {on | off}

Default: on

Description: determines whether update query is to be automatically performed when Stata is launched. There is no permanently option because permanently is implied.

Reference: [R] [update](#)

set varabbrev

Syntax: set varabbrev {on | off} [ , permanently]

Default: on

Description: indicates whether Stata should allow variable abbreviations.

Reference: [P] [varabbrev](#)

set varkeyboard (Mac only)

Syntax: set varkeyboard {on | off} [ , permanently]

Default: on

Description: sets the keyboard navigation behavior for the Variables window. `on` indicates that you can use the keyboard to navigate and enter items from the Variables window into the Command window. `off` indicates that all keyboard input be directed at the Command window; items can be entered from the Variables window only by using the mouse.

Reference: help varkeyboard

## Also see

[R] [query](#) — Display system parameters

[R] [set\\_defaults](#) — Reset system parameters to original Stata defaults

[M-3] [mata set](#) — Set and display Mata system parameters

[P] [creturn](#) — Return c-class values

## Description

`set cformat` specifies the output format of coefficients, standard errors, and confidence limits in coefficient tables.

`set pformat` specifies the output format of  $p$ -values in coefficient tables.

`set sformat` specifies the output format of test statistics in coefficient tables.

## Syntax

```
set cformat [fmt] [, permanently]
```

```
set pformat [fmt] [, permanently]
```

```
set sformat [fmt] [, permanently]
```

where *fmt* is a [numerical format](#).

## Option

`permanently` specifies that, in addition to making the change right now, the setting be remembered and become the default setting when you invoke Stata.

## Remarks and examples

The formatting of the numbers in the coefficient table can be controlled by using the `set cformat`, `set pformat`, and `set sformat` commands or by using the `cformat(%fmt)`, `pformat(%fmt)`, and `sformat(%fmt)` options at the time of estimation or on replay of the estimation command. See [\[R\] Estimation options](#).

The maximum format widths for `set cformat`, `set pformat`, and `set sformat` in coefficient tables are 9, 5, and 8, respectively.

► Example 1

We use auto.dta to illustrate.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
```

```
. regress mpg weight displacement
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 1595.40969 | 2  | 797.704846 | F(2, 71)      | = | 66.79  |
| Residual | 848.049768 | 71 | 11.9443629 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.6529 |
|          |            |    |            | Adj R-squared | = | 0.6432 |
| Total    | 2443.45946 | 73 | 33.4720474 | Root MSE      | = | 3.4561 |

| mpg          | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|--------------|-------------|-----------|-------|-------|----------------------|-----------|
| weight       | -.0065671   | .0011662  | -5.63 | 0.000 | -.0088925            | -.0042417 |
| displacement | .0052808    | .0098696  | 0.54  | 0.594 | -.0143986            | .0249602  |
| _cons        | 40.08452    | 2.02011   | 19.84 | 0.000 | 36.05654             | 44.11251  |

```
. set cformat %9.2f
```

```
. regress mpg weight displacement
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 1595.40969 | 2  | 797.704846 | F(2, 71)      | = | 66.79  |
| Residual | 848.049768 | 71 | 11.9443629 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.6529 |
|          |            |    |            | Adj R-squared | = | 0.6432 |
| Total    | 2443.45946 | 73 | 33.4720474 | Root MSE      | = | 3.4561 |

| mpg          | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |       |
|--------------|-------------|-----------|-------|-------|----------------------|-------|
| weight       | -0.01       | 0.00      | -5.63 | 0.000 | -0.01                | -0.00 |
| displacement | 0.01        | 0.01      | 0.54  | 0.594 | -0.01                | 0.02  |
| _cons        | 40.08       | 2.02      | 19.84 | 0.000 | 36.06                | 44.11 |

```
. regress mpg weight displacement, cformat(%9.3f)
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 1595.40969 | 2  | 797.704846 | F(2, 71)      | = | 66.79  |
| Residual | 848.049768 | 71 | 11.9443629 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.6529 |
|          |            |    |            | Adj R-squared | = | 0.6432 |
| Total    | 2443.45946 | 73 | 33.4720474 | Root MSE      | = | 3.4561 |

| mpg          | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |        |
|--------------|-------------|-----------|-------|-------|----------------------|--------|
| weight       | -0.007      | 0.001     | -5.63 | 0.000 | -0.009               | -0.004 |
| displacement | 0.005       | 0.010     | 0.54  | 0.594 | -0.014               | 0.025  |
| _cons        | 40.085      | 2.020     | 19.84 | 0.000 | 36.057               | 44.113 |

To reset the cformat setting to its command-specific default, type

```
. set cformat
```

```
. regress mpg weight displacement
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 1595.40969 | 2  | 797.704846 | F(2, 71)      | = | 66.79  |
| Residual | 848.049768 | 71 | 11.9443629 | Prob > F      | = | 0.0000 |
| Total    | 2443.45946 | 73 | 33.4720474 | R-squared     | = | 0.6529 |
|          |            |    |            | Adj R-squared | = | 0.6432 |
|          |            |    |            | Root MSE      | = | 3.4561 |

| mpg          | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|--------------|-------------|-----------|-------|-------|----------------------|-----------|
| weight       | -.0065671   | .0011662  | -5.63 | 0.000 | -.0088925            | -.0042417 |
| displacement | .0052808    | .0098696  | 0.54  | 0.594 | -.0143986            | .0249602  |
| _cons        | 40.08452    | 2.02011   | 19.84 | 0.000 | 36.05654             | 44.11251  |

◀

## Also see

[R] [Estimation options](#) — Estimation options

[R] [query](#) — Display system parameters

[R] [set](#) — Overview of system parameters

[U] [20.9 Formatting the coefficient table](#)

## Description

`set_defaults` resets settings made by `set` to the original default settings that were shipped with Stata.

`set_defaults` may not be used with `java`, `lapack`, `putdocx`, `python`, `rng`, or `sort` system-parameter categories.

## Syntax

```
set_defaults { category | _all } [, permanently]
```

where *category* is one of memory | output | interface | graphics |

network | update | trace | mata | unicode | other

## Option

`permanently` specifies that, in addition to making the change right now, the settings be remembered and become the default settings when you invoke Stata.

## Remarks and examples

### ► Example 1

To assist us in debugging a new command, we modified some of the `trace` settings. To return them to their original values, we type

```
. set_defaults trace
-> set trace off
-> set tracedepth 32000
-> set traceexpand on
-> set tracesep on
-> set traceindent on
-> set tracenumber off
-> set tracehilite ""
(preferences reset)
```

◀

## Also see

[R] [query](#) — Display system parameters

[R] [set](#) — Overview of system parameters

[M-3] [mata set](#) — Set and display Mata system parameters

## Description

`set emptycells` allows you to control how Stata handles interaction terms with empty cells. Stata can keep empty cells or drop them. The default is to keep empty cells.

## Syntax

```
set emptycells { keep | drop } [, permanently]
```

## Option

`permanently` specifies that, in addition to making the change right now, the setting be remembered and become the default setting when you invoke Stata.

## Remarks and examples

By default, Stata keeps empty cells so they can be reported in the coefficient table. For example, type

```
. use https://www.stata-press.com/data/r19/auto
. regress mpg rep78#foreign, baselevels
```

and you will see a regression of `mpg` on 10 indicator variables because `rep78` takes on 5 values and `foreign` takes on 2 values in the `auto` dataset. Two of those cells will be reported as empty because the data contain no observations of foreign cars with a `rep78` value of 1 or 2.

Many real datasets contain a large number of empty cells, and this could cause the “unable to allocate matrix” error message, `r(915)`. In that case, type

```
. set emptycells drop
```

to get Stata to drop empty cells from the list of coefficients. If you commonly fit models with empty cells, you can permanently set Stata to drop empty cells by typing the following:

```
. set emptycells drop, permanently
```

## Also see

[R] [set](#) — Overview of system parameters

## Description

`set iterlog` and `set maxiter` control the display of the iteration log and the maximum number of iterations, respectively, for estimation commands that iterate and for the Mata optimization functions `moptimize()`, `optimize()`, and `solvenl()`.

`set iterlog` specifies whether to display the iteration log. The default setting is `on`, which displays the log. You can specify `set iterlog off` to suppress it. To change whether the iteration log is displayed for a particular estimation command, you need not reset `iterlog`; you can specify the `log` or `nolog` option with that command. If you do not specify `log` or `nolog`, the `iterlog` setting is used. To view the current setting of `iterlog`, type `display c(iterlog)`.

`set maxiter` specifies the default maximum number of iterations. To change the maximum number of iterations performed by a particular estimation command, you need not reset `maxiter`; you can specify the `iterate(#)` option with that command. If you do not specify `iterate(#)`, the `maxiter` value is used. To view the current setting of `maxiter`, type `display c(maxiter)`.

## Syntax

*Set whether to display the iteration log*

```
set iterlog { on | off } [, permanently]
```

*Set default maximum iterations*

```
set maxiter # [, permanently]
```

# is any number between 0 and 16,000; the initial value is set to 300.

## Option

`permanently` specifies that, in addition to making the change right now, the setting be remembered and become the default setting when you invoke Stata.

## Remarks and examples

The `iterlog` setting is particularly useful in combination with the `nolog` and `log` options; see [example 1](#) below. Also see [\[R\] Maximize](#) for details about the options. The `iterlog` setting has no effect on commands that suppress the iteration log by default, for example, commands prefixed with `svy`. To display the log with those commands, you need to use the `log` option.

You will rarely need to modify the `maxiter` setting to change the maximum number of iterations used by Stata's iterative commands. Instead, you may want to specify the `iterate()` option with these commands. For example, specifying `iterate(0)` is useful for viewing results evaluated at the initial value of the coefficient vector.

The `iterlog` and `maxiter` settings also control the default output displayed by the Mata optimization functions `moptimize()`, `optimize()`, and `solvenl()`.

### ▷ Example 1: Display and suppress the iteration log

Stata estimation commands that iterate usually display the iteration log by default:

```
. sysuse auto
(1978 automobile data)
. logit foreign mpg
Iteration 0: Log likelihood = -45.03321
Iteration 1: Log likelihood = -39.380959
Iteration 2: Log likelihood = -39.288802
Iteration 3: Log likelihood = -39.28864
Iteration 4: Log likelihood = -39.28864
Logistic regression
Log likelihood = -39.28864
```

|                 |        |
|-----------------|--------|
| Number of obs = | 74     |
| LR chi2(1) =    | 11.49  |
| Prob > chi2 =   | 0.0007 |
| Pseudo R2 =     | 0.1276 |

| foreign | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|---------|-------------|-----------|-------|-------|----------------------|-----------|
| mpg     | .1597621    | .0525876  | 3.04  | 0.002 | .0566922             | .262832   |
| _cons   | -4.378866   | 1.211295  | -3.62 | 0.000 | -6.752961            | -2.004771 |

You can suppress the log by specifying the `nolog` option:

```
. logit foreign mpg, nolog
Logistic regression
Log likelihood = -39.28864
```

|                 |        |
|-----------------|--------|
| Number of obs = | 74     |
| LR chi2(1) =    | 11.49  |
| Prob > chi2 =   | 0.0007 |
| Pseudo R2 =     | 0.1276 |

| foreign | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|---------|-------------|-----------|-------|-------|----------------------|-----------|
| mpg     | .1597621    | .0525876  | 3.04  | 0.002 | .0566922             | .262832   |
| _cons   | -4.378866   | 1.211295  | -3.62 | 0.000 | -6.752961            | -2.004771 |

If you want to suppress the iteration log from all estimation commands every time they are run within the current Stata session, type

```
. set iterlog off
```

We can run logit again but now without the nolog option, and the iteration log will not be displayed:

```
. logit foreign mpg
Logistic regression
Log likelihood = -39.28864
Number of obs = 74
LR chi2(1) = 11.49
Prob > chi2 = 0.0007
Pseudo R2 = 0.1276
```

| foreign | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|---------|-------------|-----------|-------|-------|----------------------|-----------|
| mpg     | .1597621    | .0525876  | 3.04  | 0.002 | .0566922             | .262832   |
| _cons   | -4.378866   | 1.211295  | -3.62 | 0.000 | -6.752961            | -2.004771 |

Or we can run a different command, for example, mlogit, and the log will still be suppressed:

```
. mlogit rep78 mpg
Multinomial logistic regression
Log likelihood = -85.752375
Number of obs = 69
LR chi2(4) = 15.88
Prob > chi2 = 0.0032
Pseudo R2 = 0.0847
```

| rep78 | Coefficient    | Std. err. | z     | P> z  | [95% conf. interval] |           |
|-------|----------------|-----------|-------|-------|----------------------|-----------|
| 1     |                |           |       |       |                      |           |
| mpg   | .0708122       | .1471461  | 0.48  | 0.630 | -.2175888            | .3592132  |
| _cons | -4.137144      | 3.15707   | -1.31 | 0.190 | -10.32489            | 2.0506    |
| 2     |                |           |       |       |                      |           |
| mpg   | -.0164251      | .0926724  | -0.18 | 0.859 | -.1980597            | .1652096  |
| _cons | -1.005118      | 1.822129  | -0.55 | 0.581 | -4.576426            | 2.56619   |
| 3     | (base outcome) |           |       |       |                      |           |
| 4     |                |           |       |       |                      |           |
| mpg   | .0958626       | .0633329  | 1.51  | 0.130 | -.0282676            | .2199927  |
| _cons | -2.474187      | 1.341131  | -1.84 | 0.065 | -5.102756            | .1543813  |
| 5     |                |           |       |       |                      |           |
| mpg   | .2477469       | .0764076  | 3.24  | 0.001 | .0979908             | .397503   |
| _cons | -6.653164      | 1.841794  | -3.61 | 0.000 | -10.26301            | -3.043314 |

With the `iterlog` setting off, we can display the iteration log for specific commands by specifying the `log` option:

```
. mlogit rep78 mpg, log
Iteration 0: Log likelihood = -93.692061
Iteration 1: Log likelihood = -86.581485
Iteration 2: Log likelihood = -85.767758
Iteration 3: Log likelihood = -85.752385
Iteration 4: Log likelihood = -85.752375
Iteration 5: Log likelihood = -85.752375

Multinomial logistic regression Number of obs = 69
LR chi2(4) = 15.88
Prob > chi2 = 0.0032
Pseudo R2 = 0.0847

Log likelihood = -85.752375
```

|   | rep78 | Coefficient    | Std. err. | z     | P> z  | [95% conf. interval] |           |
|---|-------|----------------|-----------|-------|-------|----------------------|-----------|
| 1 |       |                |           |       |       |                      |           |
|   | mpg   | .0708122       | .1471461  | 0.48  | 0.630 | -.2175888            | .3592132  |
|   | _cons | -4.137144      | 3.15707   | -1.31 | 0.190 | -10.32489            | 2.0506    |
| 2 |       |                |           |       |       |                      |           |
|   | mpg   | -.0164251      | .0926724  | -0.18 | 0.859 | -.1980597            | .1652096  |
|   | _cons | -1.005118      | 1.822129  | -0.55 | 0.581 | -4.576426            | 2.56619   |
| 3 |       | (base outcome) |           |       |       |                      |           |
| 4 |       |                |           |       |       |                      |           |
|   | mpg   | .0958626       | .0633329  | 1.51  | 0.130 | -.0282676            | .2199927  |
|   | _cons | -2.474187      | 1.341131  | -1.84 | 0.065 | -5.102756            | .1543813  |
| 5 |       |                |           |       |       |                      |           |
|   | mpg   | .2477469       | .0764076  | 3.24  | 0.001 | .0979908             | .397503   |
|   | _cons | -6.653164      | 1.841794  | -3.61 | 0.000 | -10.26301            | -3.043314 |

You can switch back to displaying iteration logs by typing

```
. set iterlog on
```

The default setting will be restored automatically the next time you invoke Stata. If you want the setting to be remembered for future Stata sessions, specify the permanently option with `set iterlog`.



## Also see

- [R] **Maximize** — Details of iterative maximization
- [R] **set** — Overview of system parameters
- [M-5] **moptimize()** — Model optimization
- [M-5] **optimize()** — Function optimization
- [M-5] **solvenl()** — Solve systems of nonlinear equations

**set rng** — Set which random-number generator (RNG) to use

Description

Syntax

Remarks and examples

Reference

Also see

## Description

`set rng` determines which random-number generator (RNG) Stata's [random-number functions](#) and commands will use.

## Syntax

```
set rng { default | mt64 | mt64s | kiss32 }
```

## Remarks and examples

Remarks are presented under the following headings:

[Introduction](#)

[Random-number generators in Stata](#)

### Introduction

By default, Stata uses the 64-bit Mersenne Twister (`mt64`) RNG. `mt64s` is a stream RNG based on the 64-bit Mersenne Twister. Earlier versions of Stata used the 32-bit KISS (keep it simple stupid) (`kiss32`) RNG.

With `set rng default` (the default), code running under [version control](#) will automatically use the appropriate RNG—`mt64` in Stata 14 and later and `kiss32` for earlier code.

The scope of `set rng` is the Stata session, do-file, or program in which `rng` is set.

Unless you want to simultaneously draw random numbers in separate instances of Stata, we recommend that you do not change Stata's default behavior for its RNGs. See [\[R\] set rngstream](#) for an introduction to simultaneously drawing random numbers in separate instances of Stata.

See [\[FN\] Random-number functions](#), [\[R\] set seed](#), and [\[R\] set rngstream](#) for more information.

### Random-number generators in Stata

The default RNG in Stata is the 64-bit Mersenne Twister. See [Matsumoto and Nishimura \(1998\)](#) for more details. The default RNG in Stata 13 and earlier versions was George Marsaglia's 32-bit KISS generator (G. Marsaglia, 1994, pers. comm.). The KISS generator is still available under version control or via `set rng`. Multiple independent random-number streams (based on the 64-bit Mersenne Twister) are also supported for use in multiple simultaneous instances of Stata; see [\[R\] set rngstream](#) for more information on this. The abbreviations `mt64`, `kiss32`, and `mt64s` are used, respectively, to specify these three generators in Stata commands and functions.

So far, we have discussed two ways you can specify the RNG: with `set rng` and through version control. Another way to specify the RNG is with functions and system parameters explicitly named after the generators. In fact, all random-number functions have variants that are explicitly named after each generator, using the generator abbreviation as the suffix. For example, `runiform_mt64()`, `runiform_kiss32()`, and `runiform_mt64s()` are variants of `runiform()` for each generator. Similarly, we have `rnormal_mt64()`, `rnormal_kiss32()`, `rnormal_mt64s()`, etc.

The system parameters `seed` and `rngstate` also have variants explicitly named after each generator: `seed_mt64`, `seed_kiss32`, `seed_mt64s`, `rngstate_mt64`, `rngstate_kiss32`, and `rngstate_mt64s`.

For example, here is how you can use functions and parameters specific to `mt64` to set the seed, generate random numbers, preserve a state, generate more numbers, and restore the previously preserved state:

```
. set seed_mt64 482637
. generate u = runiform_mt64()
. local state = c(rngstate_mt64)
. generate l = rlogistic_mt64()
. set rngstate_mt64 'state'
```

Note that calling functions and setting parameters specific to, say, `kiss32`, will not change the current RNG, the seed of the current RNG, or the state of the current RNG—unless the current RNG is `kiss32`.

## Reference

Matsumoto, M., and T. Nishimura. 1998. Mersenne Twister: A 623-dimensionally equidistributed uniform pseudo-random number generator. *ACM Transactions on Modeling and Computer Simulation* 8: 3–30. <https://doi.org/10.1145/272991.272995>.

## Also see

[R] **set** — Overview of system parameters

[R] **set rngstream** — Specify the stream for the stream random-number generator

[R] **set seed** — Specify random-number seed and state

[FN] **Random-number functions**

[P] **version** — Version control

## Description

`set rngstream` specifies the subsequence, known as a stream, from which Stata's stream random-number generator should draw random numbers. When performing a bootstrap estimation or a Monte Carlo simulation in parallel on multiple machines, you should set the same seed on all machines but set a different stream on each machine. This will ensure that random numbers drawn on different machines are independent. We strongly recommend that you set the seed and the stream only once in each Stata session.

## Syntax

```
set rngstream #
```

# is any integer between 1 and 32,767.

## Remarks and examples

Stata's stream random-number generator, the stream 64-bit Mersenne Twister (`mt64s`), allows separate instances of Stata to simultaneously draw independent random numbers. This feature enables you to use `bootstrap` and to run Monte Carlo simulations in parallel on multiple machines.

What we call random numbers are elements in a sequence of deterministic numbers that appear to be random. A seed specifies a starting value in this sequence. In figure 1, each tick is an element in a random sequence and setting the seed to 12345 means that the tick identified by the arrow below is the first number drawn.



Figure 1. Seed specifies first number in random sequence

A stream random-number generator partitions a sequence of random numbers into nonoverlapping subsequences known as streams. The random numbers in each stream are independent of those in other streams because they come from distinct nonoverlapping subsets of the original sequence.

Figure 2 depicts a stream version of the generator depicted in figure 1. The stream version also starts at the place implied by seed 12345, but it additionally partitions the random numbers into 4 streams and a set of unused numbers.

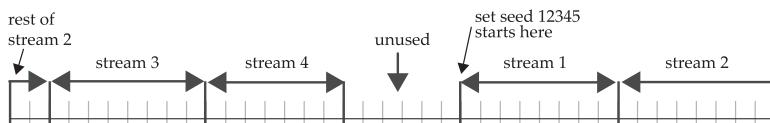


Figure 2. A stream version of figure 1 generator

In contrast to nonstream random-number generators, setting the seed for a random-number generator controls not just where the sequence starts but also how the sequence is partitioned. Compare figure 2 with figure 3 for an illustration.

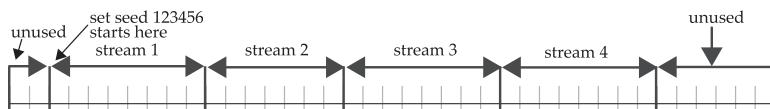


Figure 3. Changing the seed changes the streams

Seed 123456 specifies the first random number, and the streams of random numbers in figure 3 differ from those in figure 2.

The `mt64s` generator is a stream version of Stata's default generator, the 64-bit Mersenne Twister implemented in `mt64`; see [Matsumoto and Nishimura \(1998\)](#) and *Random-number generators in Stata* in [R] `set rng` for more details. Our implementation of the method discussed in [Haramoto et al. \(2008\)](#) partitions the `mt64` sequence into 32,767 streams, each containing  $2^{128}$  random numbers. The remaining numbers are unused. The `mt64s` seed determines the starting point of every stream in the Mersenne Twister sequence.

Stream 1 of `mt64s` has the same starting point as the `mt64` generator. That is, given the same seed, `mt64s` with `rngstream` set to 1 will generate the same random numbers as `mt64`.

The `mt64s` generator is designed to simultaneously draw independent random numbers on different machines. To draw from different streams that guarantee independence, use the same seed and change the stream. For example, to draw some `uniform(0,1)` random numbers from stream 10 of the `mt64s` generator under seed 123, type

```
. set rng mt64s
. set rngstream 10
. set seed 123
. generate u = runiform()
```

If we wanted to simultaneously draw some `uniform(0,1)` random numbers on another machine from stream 11 of the `mt64s` generator, we would type

```
. set rng mt64s
. set rngstream 11
. set seed 123
. generate u = runiform()
```

Again, each seed creates a different partition of the `mt64` sequence into nonoverlapping subsets.

We strongly recommend that you set the stream and the seed once in each Stata session and draw numbers only from this stream.

`c(rngstream)` returns the current stream number. `c(rngseed_mt64s)` returns the last seed that was set for `mt64s`. See [P] [creturn](#) for more details. See [R] [set seed](#) for details about storing and restoring the current position in the random sequence.

As with the single-stream generators, use `local state = c(rngstate)` to store the current position in the current random stream; see [R] [set seed](#) for details. The `mt64s` state encodes the seed used in addition to the stream number, because the seed determines the position of every random number in every stream. Unlike the case of single-stream generators, restoring the state also restores the seed. For example, suppose you save an `mt64s` state with `local state = c(rngstate)` change the seed and the stream, and later restore that state with `set rngstate 'state'`. The current `mt64s` seed is changed to the one encoded in `state`. In addition to changing the current stream to the one encoded in `state`, the current `mt64s` seed is changed to the one encoded in `state`. This behavior ensures any subsequent stream changes draw from nonoverlapping subsets.

`set rngstream` also sets the [random-number generator](#) to `mt64s`.

### ▷ Example 1: Using stream random numbers to parallelize a bootstrap

We illustrate how to simultaneously perform 100 bootstrap replications on machine 1 and 100 bootstrap replications on machine 2. We focus on the mechanics of distributing the draws over machines using stream random numbers; see [R] [bootstrap](#) for an introduction to the bootstrap.

On machine 1, we type

```
. clear all
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. set rng mt64s
. set rngstream 1
. set seed 12345
. bootstrap, reps(100) saving(machine1, replace): regress mpg weight gear foreign
(running regress on estimation sample)
(file machine1.dta not found)
Bootstrap replications (100):10.....20.....30.....40.....
> ...50.....60.....70.....80.....90.....100 done
Linear regression Number of obs = 74
 Replications = 100
 Wald chi2(3) = 191.66
 Prob > chi2 = 0.0000
 R-squared = 0.6670
 Adj R-squared = 0.6527
 Root MSE = 3.4096
```

| mpg        | Observed<br>coefficient | Bootstrap<br>std. err. | z      | P> z  | Normal-based<br>[95% conf. interval] |           |
|------------|-------------------------|------------------------|--------|-------|--------------------------------------|-----------|
| weight     | -.006139                | .0005462               | -11.24 | 0.000 | -.0072095                            | -.0050685 |
| gear_ratio | 1.457113                | 1.271301               | 1.15   | 0.252 | -1.03459                             | 3.948817  |
| foreign    | -2.221682               | 1.090115               | -2.04  | 0.042 | -4.358267                            | -.0850957 |
| _cons      | 36.10135                | 4.720623               | 7.65   | 0.000 | 26.8491                              | 45.3536   |

On machine 2, we type

```
. clear all
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. set rng mt64s
. set rngstream 2
. set seed 12345
. bootstrap, reps(100) saving(machine2, replace): regress mpg weight gear foreign
(running regress on estimation sample)
(file machine2.dta not found)
Bootstrap replications (100):10.....20.....30.....40.....
> ...50.....60.....70.....80.....90.....100 done
Linear regression Number of obs = 74
 Replications = 100
 Wald chi2(3) = 121.48
 Prob > chi2 = 0.0000
 R-squared = 0.6670
 Adj R-squared = 0.6527
 Root MSE = 3.4096
```

| mpg        | Observed coefficient | Bootstrap std. err. | z      | P> z  | Normal-based [95% conf. interval] |           |
|------------|----------------------|---------------------|--------|-------|-----------------------------------|-----------|
| weight     | -.006139             | .0005909            | -10.39 | 0.000 | -.0072972                         | -.0049809 |
| gear_ratio | 1.457113             | 1.267439            | 1.15   | 0.250 | -1.027022                         | 3.941249  |
| foreign    | -2.221682            | 1.253503            | -1.77  | 0.076 | -4.678502                         | .2351393  |
| _cons      | 36.10135             | 4.419797            | 8.17   | 0.000 | 27.43871                          | 44.764    |

After copying machine2.dta from machine 2 to the working directory on machine 1, we produce the combined results by typing

```
. clear all
. use machine1
(bootstrap: regress)
. append using machine2
. bstat
Bootstrap results Number of obs = 74
 Replications = 200
Command: regress mpg weight gear foreign
```

|            | Observed coefficient | Bootstrap std. err. | z      | P> z  | Normal-based [95% conf. interval] |           |
|------------|----------------------|---------------------|--------|-------|-----------------------------------|-----------|
| weight     | -.006139             | .0005678            | -10.81 | 0.000 | -.0072519                         | -.0050262 |
| gear_ratio | 1.457113             | 1.266586            | 1.15   | 0.250 | -1.02535                          | 3.939577  |
| foreign    | -2.221682            | 1.187847            | -1.87  | 0.061 | -4.549819                         | .1064562  |
| _cons      | 36.10135             | 4.562644            | 7.91   | 0.000 | 27.15873                          | 45.04397  |

We used `regress` in this example, but the divide-and-conquer strategy reduces computation time for any command that works with `bootstrap`. In fact, problems that take longer produce more noticeable speed improvements. For computationally intensive problems, the two-machine time will be about one-half the one-machine time. Using distinct streams on many different machines can dramatically reduce the time required for computationally intensive problems.

◀

### ▷ Example 2: Using stream random numbers to parallelize a Monte Carlo simulation

We want to simultaneously perform 100 Monte Carlo replications on machine 3 and 100 Monte Carlo replications on machine 4. Again, we focus entirely on the mechanics of distributing the draws over machines. See [Drukker \(2015\)](#) for an introduction to Monte Carlo simulations using Stata.

As discussed in [\[R\] simulate](#), the `simulate` command uses an ado-file that draws from the population of interest; it then computes and returns the estimates. Our program `chi2sim` draws from a  $\chi^2$  distribution with one degree of freedom.

```

program define chi2sim, rclass
 version 19.5 // (or version 19 if you do not have StataNow)
 drop _all
 set obs 200
 tempvar z
 generate `z' = rchi2(1)
 summarize `z'
 return scalar mean = r(mean)
 return scalar Var = r(Var)
end

```

On machine 3, we type

```

. set rng mt64s
. set rngstream 3
. set seed 12345
. simulate mean=r(mean) var=r(Var), reps(500) saving(machine3, replace): chi2sim
 Command: chi2sim
 mean: r(mean)
 var: r(Var)
(file machine3.dta not found)
Simulations (500):10.....20.....30.....40.....50.....
> ...60.....70.....80.....90.....100.....110.....120.....
> ...130.....140.....150.....160.....170.....180.....1
> 90.....200.....210.....220.....230.....240.....250.....
> ...260.....270.....280.....290.....300.....310.....3
> 20.....330.....340.....350.....360.....370.....380.....
> ...390.....400.....410.....420.....430.....440.....4
> 50.....460.....470.....480.....490.....500 done

```

On machine 4, we run a do-file that performs

```
. set rng mt64s
. set rngstream 4
. set seed 12345
. simulate mean=r(mean) var=r(Var), reps(500) saving(machine4, replace): chi2sim
 Command: chi2sim
 mean: r(mean)
 var: r(Var)
(file machine4.dta not found)
Simulations (500):10.....20.....30.....40.....50.....
> ...60.....70.....80.....90.....100.....110.....120.....
>130.....140.....150.....160.....170.....180.....1
> 90.....200.....210.....220.....230.....240.....250....
>260.....270.....280.....290.....300.....310.....3
> 20.....330.....340.....350.....360.....370.....380....
>390.....400.....410.....420.....430.....440.....4
> 50.....460.....470.....480.....490.....500 done
```

After copying `machine4.dta` from machine 4 to the working directory on machine 3, we combine the results by typing

```
. clear all
. use machine3
(simulate: chi2sim)
. append using machine4
. summarize mean var
```

| Variable | Obs   | Mean    | Std. dev. | Min      | Max      |
|----------|-------|---------|-----------|----------|----------|
| mean     | 1,000 | 1.00296 | .1011507  | .7098877 | 1.330305 |
| var      | 1,000 | 2.01955 | .5194171  | .8931134 | 4.381884 |

As in [example 1](#), more machines enable further parallelization.



## □ Technical note

While `mt64s` has been made robust to switching between streams within a Stata session, convoluted combinations of `set rngstream #` and `set seed #` can lead to drawing the same random numbers, just as it can in the case of single-stream generators. We strongly recommend that you do not switch between streams within a session.



### ► Example 3: Position within a stream is stored

This example illustrates that the sequence picks up where it left off when the stream is switched, and it illustrates that `clear rngstream` resets all streams to their beginning positions. These features facilitate advanced programming techniques, and we recommend against using this feature in standard use.

```
. clear all
. set obs 10
Number of observations (_N) was 0, now 10.
. set rng mt64s
. set rngstream 5
. set seed 12345
. generate x = runiform() in 1/5
(5 missing values generated)
. set rngstream 6
. generate y = runiform()
. set rngstream 5
. replace x = runiform() in 6/10
(5 real changes made)
. clear rngstream
. set rngstream 5
. generate z = runiform()
. list
```

|     | x        | y        | z        |
|-----|----------|----------|----------|
| 1.  | .5095264 | .8838338 | .5095264 |
| 2.  | .9766202 | .7677673 | .9766202 |
| 3.  | .3933811 | .5665985 | .3933811 |
| 4.  | .950057  | .3141659 | .950057  |
| 5.  | .5862163 | .6635106 | .5862163 |
| 6.  | .4837167 | .6781911 | .4837167 |
| 7.  | .1752382 | .7169843 | .1752382 |
| 8.  | .2302023 | .7554966 | .2302023 |
| 9.  | .4927879 | .8685812 | .4927879 |
| 10. | .9114158 | .5634732 | .9114158 |

After setting the `rngstream` to 5 and setting the seed, we put the first 5 draws from stream 5 into observations 1–5 of `x`, switch to `rngstream` 6, put the first 10 random draws from stream 6 into `y`, return to `rngstream` 5, and put the next 5 draws from stream 5 into observations 6–10 of `x`. Then we use `clear rngstream` to initialize each `rngstream` at its initial position for seed 12345 and put the first 10 draws from stream 5 into `z`.

That the random numbers in `x` match those in `z` illustrates that the sequence picks up where it left off when the stream is switched and the seed has not been changed.

## References

- Drukker, D. M. 2015. Monte Carlo simulations using Stata. *The Stata Blog: Not Elsewhere Classified*. <https://blog.stata.com/2015/10/06/monte-carlo-simulations-using-stata/>.
- Haramoto, H., M. Matsumoto, T. Nishimura, F. Panneton, and P. L'Ecuyer. 2008. Efficient jump ahead for  $F_2$ -linear random number generators. *INFORMS Journal on Computing* 20: 385–390. <https://doi.org/10.1287/ijoc.1070.0251>.
- Matsumoto, M., and T. Nishimura. 1998. Mersenne Twister: A 623-dimensionally equidistributed uniform pseudo-random number generator. *ACM Transactions on Modeling and Computer Simulation* 8: 3–30. <https://doi.org/10.1145/272991.272995>.
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## Also see

- [R] **set** — Overview of system parameters
- [R] **set rng** — Set which random-number generator (RNG) to use
- [R] **set seed** — Specify random-number seed and state
- [D] **clear** — Clear memory
- [FN] **Random-number functions**

## Description

`set seed #` specifies the initial value of the random-number seed used by the [random-number functions](#), such as `runiform()` and `rnormal()`.

`set rngstate statecode` resets the state of the random-number generator to the value specified, which is a state previously obtained from [creturn](#) value `c(rngstate)`.

`set seed #` and `set rngstate statecode` apply to the current random-number generator. Every random-number generator in Stata has its own seed and state encoding.

## Syntax

```
set seed #
```

```
set rngstate statecode
```

# is any number between 0 and  $2^{31} - 1$  (or 2,147,483,647).

*statecode* is a random-number state previously obtained from [creturn](#) value `c(rngstate)`.

## Remarks and examples

Remarks are presented under the following headings:

[Examples](#)

[Setting the seed](#)

[How to choose a seed](#)

[Do not set the seed too often](#)

[Preserving and restoring the random-number generator state](#)

## Examples

1. Specify initial value of random-number seed

```
. set seed 339487731
```

2. Create variable `u` containing uniformly distributed pseudorandom numbers on the interval (0, 1)

```
. generate u = runiform()
```

3. Create variable `z` containing normally distributed random numbers with mean 0 and standard deviation 1

```
. generate z = rnormal()
```

4. Obtain state of pseudorandom-number generator and store it in a local macro named `state`

```
. local state = c(rngstate)
```

5. Restore pseudorandom-number generator state to that previously stored in local macro named `state`

```
. set rngstate `state'
```

## Setting the seed

Stata's random-number generation functions, such as `runiform()` and `rnormal()`, do not really produce random numbers. These functions are deterministic algorithms that produce numbers that can pass for random. `runiform()` produces numbers that can pass for independent draws from a rectangular distribution over  $(0, 1)$ ; `rnormal()` produces numbers that can pass for independent draws from  $N(0, 1)$ . Stata's random-number functions are formally called pseudorandom-number functions. The default pseudorandom-number generator introduced in Stata 14 is the 64-bit Mersenne Twister. See [Matsumoto and Nishimura \(1998\)](#) and *Random-number generators in Stata* in [R] [set rng](#) for more details.

The sequences the random-number functions produce are determined by the seed, which is just a number and which is set to 123456789 every time Stata is launched. This means that `runiform()` produces the same sequence each time you start Stata. The first time you use `runiform()` after Stata is launched, `runiform()` returns 0.348871704556195. The second time you use it, `runiform()` returns 0.266885709753138. The third time you use it, . . . .

To obtain different sequences, you must specify different seeds using the `set seed` command. You might specify the seed 472195:

```
. set seed 472195
```

If you were now to use `runiform()`, the first call would return 0.713028143573182, the second call would return 0.920524469911484, and so on. Whenever you `set seed 472195`, `runiform()` will return those numbers the first two times you use it.

Thus, you set the seed to obtain different pseudorandom sequences from the pseudorandom-number functions.

If you record the seed you set, pseudorandom results such as results from a simulation or imputed values from `mi impute` can be reproduced later. Whatever you do after setting the seed, if you set the seed to the same value and repeat what you did, you will obtain the same results.

## How to choose a seed

Your best choice for the seed is an element chosen randomly from the set  $\{0, 1, \dots, 2^{31} - 1\}$  (where  $2^{31} - 1 = 2,147,483,647$ ). We recommend that, but that is difficult to achieve because finding easy-to-access, truly random sources is difficult.

One person we know uses digits from the serial numbers from dollar bills he finds in his wallet. Of course, the numbers he obtains are not really random, but they are good enough, and they are probably a good deal more random than the seeds most people choose. Some people use dates and times, although we recommend against that because, over the day, it just gets later and later, and that is a pattern. Others try to make up a random number, figuring if they include enough digits, the result just has to be random. This is a variation on the five-second rule for dropped food, and we admit to using both of these rules.

It does not really matter how you set the seed, as long as there is no obvious pattern in the seeds that you set and as long as you do not set the seed too often during a session.

Nonetheless, here are two methods that we have seen used but you should not use:

1. The first time you set the seed, you set the number 1. The next time, you set 2, and then 3, and so on. Variations on this included setting 1001, 1002, 1003, . . . , or setting 1001, 2001, 3001, and so on.

Do not follow any of these procedures. The seeds you set must not exhibit a pattern.

2. To set the seed, you obtain a pseudorandom number from `runiform()` and then use the digits from that to form the seed.

This is a bad idea because the pseudorandom-number generator can converge to a cycle. If you obtained the pseudorandom-number generator unrelated to those in Stata, this would work well, but then you would have to find a rule to set the first generator's seed.

Choosing seeds that do not exhibit a pattern is of great importance. That the seeds satisfy the other properties of randomness is minor by comparison.

## Do not set the seed too often

We cannot emphasize this enough: Do not set the seed too often.

To see why this is such a bad idea, consider the limiting case: You set the seed, draw one pseudorandom number, reset the seed, draw again, and so continue. The pseudorandom numbers you obtain will be nothing more than the seeds you run through a mathematical function. The results you obtain will not pass for random unless the seeds you choose pass for random. If you already had such numbers, why are you even bothering to use the pseudorandom-number generator?

The definition of too often is more than once per problem.

If you are running a simulation of 10,000 replications, set the seed at the start of the simulation and do not reset it until the 10,000th replication is finished. The pseudorandom-number generators provided by Stata have long periods. The longer you go between setting the seed, the more random-like are the numbers produced.

It is sometimes useful later to be able to reproduce in isolation any one of the replications, and so you might be tempted to set the seed to a known value for each of the replications. We negatively mentioned setting the seed to 1, 2, ..., and it is in exactly such situations that we have seen this done. The advantage, however, is that you could reproduce the fifth replication merely by setting the seed to 5 and then repeating whatever it is that is to be replicated. If this is your goal, you do not need to reset the seed. You can record the state of the random-number generator, save the state with your replication results, and then use the recorded states later to reproduce whichever of the replications that you wish. This will be discussed in [Preserving and restoring the random-number generator state](#).

There is another reason you might be tempted to set the seed more than once per problem. It sometimes happens that you run a simulation, let's say for 5,000 replications, and then you decide you should have run it for 10,000 replications. Instead of running all 10,000 replications afresh, you decide to save time by running another 5,000 replications and then combining those results with your previous 5,000 results. That is okay. We at StataCorp do this kind of thing. If you do this, it is important that you set the seed especially well, particularly if you repeat this process to add yet another 5,000 replications. It is also important that in each run there be a large enough number of replications, which is say thousands of them.

Even so, do not do this: You want 500,000 replications. To obtain them, you run in batches of 1,000, setting the seed 500 times. Unless you have a truly random source for the seeds, it is unlikely you can produce a patternless sequence of 500 seeds. The fact that you ran 1,000 replications in between choosing the seeds does not mitigate the requirement that there be no pattern to the seeds you set.

In all cases, the best solution is to set the seed only once and then use the method we suggest in the next section.

## Preserving and restoring the random-number generator state

In the previous section, we discussed the case in which you might be tempted to set the seed more frequently than otherwise necessary, either to save time or to be able to rerun any one of the replications. In such cases, there is an alternative to setting a new seed: recording the state of the pseudorandom-number generator and then restoring the state later should the need arise.

The state of the default random-number generator in Stata, the 64-bit Mersenne Twister, is a string of about 5,000 characters. The state can be displayed by typing `display c(rngstate)`. It is more practical to save the state, say, in a local macro named `state`:

```
. local state = c(rngstate)
```

The state can later be restored by typing

```
. set rngstate `state'
```

The state string specifies an entry point into the sequence produced by the pseudorandom-number generator. Let us explain.

The best way to use a pseudorandom-number generator would be to choose a seed once, draw random numbers until you use up the generator, and then get a new generator and choose a new key. Pseudorandom-number generators have a period, after which they repeat the original sequence. That is what we mean by using up a generator. The period of the 64-bit Mersenne Twister, the default pseudorandom-number generator in Stata, is  $2^{19937} - 1$ . This is roughly  $10^{6000}$ . It is difficult to imagine that you could ever use up this generator.

The string reported by `c(rngstate)` is an encoded form of the information necessary for Stata to reestablish exactly where it is located in the pseudorandom-number generator's sequence.

We are not seriously suggesting you choose only one seed over your entire lifetime, but let's look at how you might do that. Sometime after birth, when you needed your first random number, you would set your seed,

```
. set seed 1073741823
```

On that day, you would draw, say, 10,000 pseudorandom numbers, perhaps to impute some missing values. Being done for the day, you can save the state, and then later restore it.

When you type `set rngstate` followed by a saved state string, Stata reestablishes the previous state. Thus, the next time you draw a pseudorandom number, Stata will produce the 10,001st result after setting seed 1073741823. Let's assume that you draw 100,000 numbers this day. Done for the day, you save the state string.

On the third day, after setting the state to the saved state string above, you will be in a position to draw the 110,001st pseudorandom number.

In this way, you would eat your way through the  $2^{19937} - 1$  random numbers, but you would be unlikely ever to make it to the end.

We do not expect you to set the seed just once in your life, but using the state string makes it easy to set the seed just once for a problem.

When we do simulations at StataCorp, we record `c(rngstate)` for each replication. Just like everybody else, we record results from replications as observations in datasets; we just happen to have an extra variable in the dataset, namely, a string variable named `state`. That string is filled in observation by observation from the then-current values of `c(rngstate)`, which is a function and so can be used in any context that a function can be used in Stata.

Anytime we want to reproduce a particular replication, we thus have the information we need to reset the pseudorandom-number generator, and having it in the dataset is convenient because we had to go there anyway to determine which replication we wanted to reproduce. If we want to add more replications later, we have a state string that we can use to continue from where we left off.

## Reference

Matsumoto, M., and T. Nishimura. 1998. Mersenne Twister: A 623-dimensionally equidistributed uniform pseudo-random number generator. *ACM Transactions on Modeling and Computer Simulation* 8: 3–30. <https://doi.org/10.1145/272991.272995>.

## Also see

[R] **set** — Overview of system parameters

[R] **set rng** — Set which random-number generator (RNG) to use

[R] **set rngstream** — Specify the stream for the stream random-number generator

[FN] **Random-number functions**

[P] **version** — Version control

## Description

`set showbaselevels` specifies whether to display base levels of factor variables and their interactions in coefficient tables. `set showbaselevels on` specifies that base levels be reported for factor variables and for interactions whose bases cannot be inferred from their component factor variables. `set showbaselevels all` specifies that all base levels of factor variables and interactions be reported.

`set showemptycells` specifies whether to display empty cells in coefficient tables.

`set showomitted` specifies whether to display omitted coefficients in coefficient tables.

`set fvlabel` specifies whether to display factor-variable value labels in coefficient tables. `set fvlabel on`, the default, specifies that the labels be displayed. `set fvlabel off` specifies that the levels of factor variables rather than the labels be displayed.

`set fvwrap #` specifies that long value labels wrap # lines in the coefficient table. The default is `set fvwrap 1`, which means that long value labels will be abbreviated to fit on one line.

`set fvwrapon` specifies whether value labels that wrap will break at word boundaries or break based on available space. `set fvwrapon word`, the default, specifies that value labels break at word boundaries. `set fvwrapon width` specifies that value labels break based on available space.

## Syntax

```
set showbaselevels { on | off | all } [, permanently]
```

```
set showemptycells { on | off } [, permanently]
```

```
set showomitted { on | off } [, permanently]
```

```
set fvlabel { on | off } [, permanently]
```

```
set fvwrap # [, permanently]
```

```
set fvwrapon { word | width } [, permanently]
```

## Option

`permanently` specifies that, in addition to making the change right now, the setting be remembered and become the default setting when you invoke Stata.

## Remarks and examples

### ▷ Example 1

We illustrate the first three set commands using cholesterol2.dta.

```
. use https://www.stata-press.com/data/r19/cholesterol2
(Artificial cholesterol data, empty cells)

. generate x = race

. regress chol race##agegrp x
note: 2.race#2.agegrp identifies no observations in the sample.
note: x omitted because of collinearity.
```

| Source   | SS         | df | MS         |               |   |        |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 15751.6113 | 13 | 1211.66241 | Number of obs | = | 70     |
| Residual | 5022.71559 | 56 | 89.6913498 | F(13, 56)     | = | 13.51  |
|          |            |    |            | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.7582 |
|          |            |    |            | Adj R-squared | = | 0.7021 |
| Total    | 20774.3269 | 69 | 301.077201 | Root MSE      | = | 9.4706 |

| chol        | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|-------------|-------------|-----------|-------|-------|----------------------|-----------|
| race        |             |           |       |       |                      |           |
| White       | 12.84185    | 5.989703  | 2.14  | 0.036 | .8430383             | 24.84067  |
| Other       | -.167627    | 5.989703  | -0.03 | 0.978 | -12.16644            | 11.83119  |
| agegrp      |             |           |       |       |                      |           |
| 20-29       | 17.24681    | 5.989703  | 2.88  | 0.006 | 5.247991             | 29.24562  |
| 30-39       | 31.43847    | 5.989703  | 5.25  | 0.000 | 19.43966             | 43.43729  |
| 40-59       | 34.86613    | 5.989703  | 5.82  | 0.000 | 22.86732             | 46.86495  |
| 60-79       | 44.43374    | 5.989703  | 7.42  | 0.000 | 32.43492             | 56.43256  |
| race#agegrp |             |           |       |       |                      |           |
| White#20-29 | 0           | (empty)   |       |       |                      |           |
| White#30-39 | -22.83983   | 8.470719  | -2.70 | 0.009 | -39.80872            | -5.870939 |
| White#40-59 | -14.67558   | 8.470719  | -1.73 | 0.089 | -31.64447            | 2.293306  |
| White#60-79 | -10.51115   | 8.470719  | -1.24 | 0.220 | -27.48004            | 6.457735  |
| Other#20-29 | -6.054425   | 8.470719  | -0.71 | 0.478 | -23.02331            | 10.91446  |
| Other#30-39 | -11.48083   | 8.470719  | -1.36 | 0.181 | -28.44971            | 5.488063  |
| Other#40-59 | -.6796112   | 8.470719  | -0.08 | 0.936 | -17.6485             | 16.28928  |
| Other#60-79 | -1.578052   | 8.470719  | -0.19 | 0.853 | -18.54694            | 15.39084  |
| x           | 0           | (omitted) |       |       |                      |           |
| _cons       | 175.2309    | 4.235359  | 41.37 | 0.000 | 166.7464             | 183.7153  |

```
. set showemptycells off
. set showomitted off
. set showbaselevels all
```

```
. regress chol race##agegrp x
note: 2.race#2.agegrp identifies no observations in the sample.
note: x omitted because of collinearity.
```

| Source   | SS         | df | MS         | Number of obs | = | 70     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 15751.6113 | 13 | 1211.66241 | F(13, 56)     | = | 13.51  |
| Residual | 5022.71559 | 56 | 89.6913498 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.7582 |
|          |            |    |            | Adj R-squared | = | 0.7021 |
| Total    | 20774.3269 | 69 | 301.077201 | Root MSE      | = | 9.4706 |

| chol        | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|-------------|-------------|-----------|-------|-------|----------------------|-----------|
| race        |             |           |       |       |                      |           |
| Black       | 0 (base)    |           |       |       |                      |           |
| White       | 12.84185    | 5.989703  | 2.14  | 0.036 | .8430383             | 24.84067  |
| Other       | -.167627    | 5.989703  | -0.03 | 0.978 | -12.16644            | 11.83119  |
| agegrp      |             |           |       |       |                      |           |
| 10-19       | 0 (base)    |           |       |       |                      |           |
| 20-29       | 17.24681    | 5.989703  | 2.88  | 0.006 | 5.247991             | 29.24562  |
| 30-39       | 31.43847    | 5.989703  | 5.25  | 0.000 | 19.43966             | 43.43729  |
| 40-59       | 34.86613    | 5.989703  | 5.82  | 0.000 | 22.86732             | 46.86495  |
| 60-79       | 44.43374    | 5.989703  | 7.42  | 0.000 | 32.43492             | 56.43256  |
| race#agegrp |             |           |       |       |                      |           |
| Black#10-19 | 0 (base)    |           |       |       |                      |           |
| Black#20-29 | 0 (base)    |           |       |       |                      |           |
| Black#30-39 | 0 (base)    |           |       |       |                      |           |
| Black#40-59 | 0 (base)    |           |       |       |                      |           |
| Black#60-79 | 0 (base)    |           |       |       |                      |           |
| White#10-19 | 0 (base)    |           |       |       |                      |           |
| White#30-39 | -22.83983   | 8.470719  | -2.70 | 0.009 | -39.80872            | -5.870939 |
| White#40-59 | -14.67558   | 8.470719  | -1.73 | 0.089 | -31.64447            | 2.293306  |
| White#60-79 | -10.51115   | 8.470719  | -1.24 | 0.220 | -27.48004            | 6.457735  |
| Other#10-19 | 0 (base)    |           |       |       |                      |           |
| Other#20-29 | -6.054425   | 8.470719  | -0.71 | 0.478 | -23.02331            | 10.91446  |
| Other#30-39 | -11.48083   | 8.470719  | -1.36 | 0.181 | -28.44971            | 5.488063  |
| Other#40-59 | -.6796112   | 8.470719  | -0.08 | 0.936 | -17.6485             | 16.28928  |
| Other#60-79 | -1.578052   | 8.470719  | -0.19 | 0.853 | -18.54694            | 15.39084  |
| _cons       | 175.2309    | 4.235359  | 41.37 | 0.000 | 166.7464             | 183.7153  |

To restore the display of empty cells, omitted predictors, and baselevels to their command-specific default behavior, type

```
. set showemptycells
. set showomitted
. set showbaselevels
. regress chol race##agegrp x
note: 2.race#2.agegrp identifies no observations in the sample.
note: x omitted because of collinearity.
```

| Source   | SS         | df | MS         | Number of obs | = | 70     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 15751.6113 | 13 | 1211.66241 | F(13, 56)     | = | 13.51  |
| Residual | 5022.71559 | 56 | 89.6913498 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.7582 |
|          |            |    |            | Adj R-squared | = | 0.7021 |
| Total    | 20774.3269 | 69 | 301.077201 | Root MSE      | = | 9.4706 |

| chol        | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|-------------|-------------|-----------|-------|-------|----------------------|-----------|
| race        |             |           |       |       |                      |           |
| White       | 12.84185    | 5.989703  | 2.14  | 0.036 | .8430383             | 24.84067  |
| Other       | -.167627    | 5.989703  | -0.03 | 0.978 | -12.16644            | 11.83119  |
| agegrp      |             |           |       |       |                      |           |
| 20-29       | 17.24681    | 5.989703  | 2.88  | 0.006 | 5.247991             | 29.24562  |
| 30-39       | 31.43847    | 5.989703  | 5.25  | 0.000 | 19.43966             | 43.43729  |
| 40-59       | 34.86613    | 5.989703  | 5.82  | 0.000 | 22.86732             | 46.86495  |
| 60-79       | 44.43374    | 5.989703  | 7.42  | 0.000 | 32.43492             | 56.43256  |
| race#agegrp |             |           |       |       |                      |           |
| White#20-29 | 0 (empty)   |           |       |       |                      |           |
| White#30-39 | -22.83983   | 8.470719  | -2.70 | 0.009 | -39.80872            | -5.870939 |
| White#40-59 | -14.67558   | 8.470719  | -1.73 | 0.089 | -31.64447            | 2.293306  |
| White#60-79 | -10.51115   | 8.470719  | -1.24 | 0.220 | -27.48004            | 6.457735  |
| Other#20-29 | -6.054425   | 8.470719  | -0.71 | 0.478 | -23.02331            | 10.91446  |
| Other#30-39 | -11.48083   | 8.470719  | -1.36 | 0.181 | -28.44971            | 5.488063  |
| Other#40-59 | -.6796112   | 8.470719  | -0.08 | 0.936 | -17.6485             | 16.28928  |
| Other#60-79 | -1.578052   | 8.470719  | -0.19 | 0.853 | -18.54694            | 15.39084  |
| x           | 0 (omitted) |           |       |       |                      |           |
| _cons       | 175.2309    | 4.235359  | 41.37 | 0.000 | 166.7464             | 183.7153  |

▷ Example 2

We illustrate the last three set commands using jaw.dta.

```
. use https://www.stata-press.com/data/r19/jaw, clear
(Table 4.6. Two-way unbalanced data for fractures of the jaw, Rencher (1998))
. mvreg y1 y2 y3 = i.fracture
```

| Equation | Obs | Parms | RMSE     | "R-sq" | F        | P>F    |
|----------|-----|-------|----------|--------|----------|--------|
| y1       | 27  | 3     | 10.42366 | 0.2966 | 5.060804 | 0.0147 |
| y2       | 27  | 3     | 6.325398 | 0.1341 | 1.858342 | 0.1777 |
| y3       | 27  | 3     | 5.976973 | 0.1024 | 1.368879 | 0.2735 |

|              | Coefficient | Std. err.   | t     | P> t  | [95% conf. interval] |  |
|--------------|-------------|-------------|-------|-------|----------------------|--|
| y1           |             |             |       |       |                      |  |
| fracture     |             |             |       |       |                      |  |
| Two compou.. | -8.833333   | 4.957441    | -1.78 | 0.087 | -19.06499 1.398322   |  |
| One simple.. |             | 6 5.394759  | 1.11  | 0.277 | -5.134235 17.13423   |  |
| _cons        |             | 37 3.939775 | 9.39  | 0.000 | 28.8687 45.1313      |  |
| y2           |             |             |       |       |                      |  |
| fracture     |             |             |       |       |                      |  |
| Two compou.. | -5.761905   | 3.008327    | -1.92 | 0.067 | -11.97079 .446977    |  |
| One simple.. | -3.053571   | 3.273705    | -0.93 | 0.360 | -9.810166 3.703023   |  |
| _cons        | 38.42857    | 2.390776    | 16.07 | 0.000 | 33.49425 43.36289    |  |
| y3           |             |             |       |       |                      |  |
| fracture     |             |             |       |       |                      |  |
| Two compou.. | 4.261905    | 2.842618    | 1.50  | 0.147 | -1.60497 10.12878    |  |
| One simple.. | .9285714    | 3.093377    | 0.30  | 0.767 | -5.455846 7.312989   |  |
| _cons        | 58.57143    | 2.259083    | 25.93 | 0.000 | 53.90891 63.23395    |  |

```
. set fvwrap 2
. mvreg y1 y2 y3 = i.fracture
```

| Equation | Obs | Parms | RMSE     | "R-sq" | F        | P>F    |
|----------|-----|-------|----------|--------|----------|--------|
| y1       | 27  | 3     | 10.42366 | 0.2966 | 5.060804 | 0.0147 |
| y2       | 27  | 3     | 6.325398 | 0.1341 | 1.858342 | 0.1777 |
| y3       | 27  | 3     | 5.976973 | 0.1024 | 1.368879 | 0.2735 |

|                        | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |
|------------------------|-------------|-----------|-------|-------|----------------------|
| y1                     |             |           |       |       |                      |
| fracture               |             |           |       |       |                      |
| Two compound fractures | -8.833333   | 4.957441  | -1.78 | 0.087 | -19.06499 1.398322   |
| One simple fracture    | 6           | 5.394759  | 1.11  | 0.277 | -5.134235 17.13423   |
| _cons                  | 37          | 3.939775  | 9.39  | 0.000 | 28.8687 45.1313      |
| y2                     |             |           |       |       |                      |
| fracture               |             |           |       |       |                      |
| Two compound fractures | -5.761905   | 3.008327  | -1.92 | 0.067 | -11.97079 .446977    |
| One simple fracture    | -3.053571   | 3.273705  | -0.93 | 0.360 | -9.810166 3.703023   |
| _cons                  | 38.42857    | 2.390776  | 16.07 | 0.000 | 33.49425 43.36289    |
| y3                     |             |           |       |       |                      |
| fracture               |             |           |       |       |                      |
| Two compound fractures | 4.261905    | 2.842618  | 1.50  | 0.147 | -1.60497 10.12878    |
| One simple fracture    | .9285714    | 3.093377  | 0.30  | 0.767 | -5.455846 7.312989   |
| _cons                  | 58.57143    | 2.259083  | 25.93 | 0.000 | 53.90891 63.23395    |

```
. set fvwrapon width
. mvreg y1 y2 y3 = i.fracture
```

| Equation | Obs | Parms | RMSE     | "R-sq" | F        | P>F    |
|----------|-----|-------|----------|--------|----------|--------|
| y1       | 27  | 3     | 10.42366 | 0.2966 | 5.060804 | 0.0147 |
| y2       | 27  | 3     | 6.325398 | 0.1341 | 1.858342 | 0.1777 |
| y3       | 27  | 3     | 5.976973 | 0.1024 | 1.368879 | 0.2735 |

|                        | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |
|------------------------|-------------|-----------|-------|-------|----------------------|
| y1                     |             |           |       |       |                      |
| fracture               |             |           |       |       |                      |
| Two compound fractures | -8.833333   | 4.957441  | -1.78 | 0.087 | -19.06499 1.398322   |
| One simple fracture    | 6           | 5.394759  | 1.11  | 0.277 | -5.134235 17.13423   |
| _cons                  | 37          | 3.939775  | 9.39  | 0.000 | 28.8687 45.1313      |
| y2                     |             |           |       |       |                      |
| fracture               |             |           |       |       |                      |
| Two compound fractures | -5.761905   | 3.008327  | -1.92 | 0.067 | -11.97079 .446977    |
| One simple fracture    | -3.053571   | 3.273705  | -0.93 | 0.360 | -9.810166 3.703023   |
| _cons                  | 38.42857    | 2.390776  | 16.07 | 0.000 | 33.49425 43.36289    |
| y3                     |             |           |       |       |                      |
| fracture               |             |           |       |       |                      |
| Two compound fractures | 4.261905    | 2.842618  | 1.50  | 0.147 | -1.60497 10.12878    |
| One simple fracture    | .9285714    | 3.093377  | 0.30  | 0.767 | -5.455846 7.312989   |
| _cons                  | 58.57143    | 2.259083  | 25.93 | 0.000 | 53.90891 63.23395    |

```
. set fvlabel off
. mvreg y1 y2 y3 = i.fracture
```

| Equation | Obs | Parms | RMSE     | "R-sq" | F        | P>F    |
|----------|-----|-------|----------|--------|----------|--------|
| y1       | 27  | 3     | 10.42366 | 0.2966 | 5.060804 | 0.0147 |
| y2       | 27  | 3     | 6.325398 | 0.1341 | 1.858342 | 0.1777 |
| y3       | 27  | 3     | 5.976973 | 0.1024 | 1.368879 | 0.2735 |

|          | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |  |
|----------|-------------|-----------|-------|-------|----------------------|--|
| y1       |             |           |       |       |                      |  |
| fracture |             |           |       |       |                      |  |
| 2        | -8.833333   | 4.957441  | -1.78 | 0.087 | -19.06499 1.398322   |  |
| 3        | 6           | 5.394759  | 1.11  | 0.277 | -5.134235 17.13423   |  |
| _cons    | 37          | 3.939775  | 9.39  | 0.000 | 28.8687 45.1313      |  |
| y2       |             |           |       |       |                      |  |
| fracture |             |           |       |       |                      |  |
| 2        | -5.761905   | 3.008327  | -1.92 | 0.067 | -11.97079 .446977    |  |
| 3        | -3.053571   | 3.273705  | -0.93 | 0.360 | -9.810166 3.703023   |  |
| _cons    | 38.42857    | 2.390776  | 16.07 | 0.000 | 33.49425 43.36289    |  |
| y3       |             |           |       |       |                      |  |
| fracture |             |           |       |       |                      |  |
| 2        | 4.261905    | 2.842618  | 1.50  | 0.147 | -1.60497 10.12878    |  |
| 3        | .9285714    | 3.093377  | 0.30  | 0.767 | -5.455846 7.312989   |  |
| _cons    | 58.57143    | 2.259083  | 25.93 | 0.000 | 53.90891 63.23395    |  |

To restore these last three set commands to their defaults, type

```
. set fvlabel on
. set fvwrap 1
. set fvwrapon word
. mvreg y1 y2 y3 = i.fracture
```

| Equation | Obs | Parms | RMSE     | "R-sq" | F        | P>F    |
|----------|-----|-------|----------|--------|----------|--------|
| y1       | 27  | 3     | 10.42366 | 0.2966 | 5.060804 | 0.0147 |
| y2       | 27  | 3     | 6.325398 | 0.1341 | 1.858342 | 0.1777 |
| y3       | 27  | 3     | 5.976973 | 0.1024 | 1.368879 | 0.2735 |

|              | Coefficient | Std. err.   | t     | P> t  | [95% conf. interval] |
|--------------|-------------|-------------|-------|-------|----------------------|
| y1           |             |             |       |       |                      |
| fracture     |             |             |       |       |                      |
| Two compou.. | -8.833333   | 4.957441    | -1.78 | 0.087 | -19.06499 1.398322   |
| One simple.. |             | 6 5.394759  | 1.11  | 0.277 | -5.134235 17.13423   |
| _cons        |             | 37 3.939775 | 9.39  | 0.000 | 28.8687 45.1313      |
| y2           |             |             |       |       |                      |
| fracture     |             |             |       |       |                      |
| Two compou.. | -5.761905   | 3.008327    | -1.92 | 0.067 | -11.97079 .446977    |
| One simple.. | -3.053571   | 3.273705    | -0.93 | 0.360 | -9.810166 3.703023   |
| _cons        | 38.42857    | 2.390776    | 16.07 | 0.000 | 33.49425 43.36289    |
| y3           |             |             |       |       |                      |
| fracture     |             |             |       |       |                      |
| Two compou.. | 4.261905    | 2.842618    | 1.50  | 0.147 | -1.60497 10.12878    |
| One simple.. | .9285714    | 3.093377    | 0.30  | 0.767 | -5.455846 7.312989   |
| _cons        | 58.57143    | 2.259083    | 25.93 | 0.000 | 53.90891 63.23395    |



## Also see

[R] [set](#) — Overview of system parameters

[R] [query](#) — Display system parameters

Description  
Option for signrank  
References

Quick start  
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Also see

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## Description

`signrank` tests the equality of matched pairs of observations by using the Wilcoxon matched-pairs signed-rank test (Wilcoxon 1945). The null hypothesis is that both distributions are the same.

`signtest` also tests the equality of matched pairs of observations (Arbuthnott [1710], but better explained by Snedecor and Cochran [1989]) by calculating the differences between *varname* and the expression. The null hypothesis is that the median of the differences is zero; no further assumptions are made about the distributions. This, in turn, is equivalent to the hypothesis that the true proportion of positive (negative) signs is one-half.

For equality tests on unmatched data, see [R] [ranksum](#).

## Quick start

Wilcoxon matched-pairs signed-rank test for *v1* and *v2*

```
signrank v1 = v2
```

Compute an exact *p*-value for the signed-rank test

```
signrank v1 = v2, exact
```

Conduct signed-rank test separately for groups defined by levels of *catvar*

```
by catvar: signrank v1 = v2
```

Test that the median of differences between matched pairs *v1* and *v2* is 0

```
signtest v1 = v2
```

## Menu

### **signrank**

Statistics > Nonparametric analysis > Tests of hypotheses > Wilcoxon matched-pairs signed-rank test

### **signtest**

Statistics > Nonparametric analysis > Tests of hypotheses > Test equality of matched pairs

## Syntax

*Wilcoxon matched-pairs signed-rank test*

```
signrank varname = exp [if] [in] [, exact]
```

*Sign test of matched pairs*

```
signtest varname = exp [if] [in]
```

`by` and `collect` are allowed with `signrank` and `signtest`; see [\[U\] 11.1.10 Prefix commands](#).

## Option for signrank

Main

`exact` specifies that the exact  $p$ -value be computed in addition to the approximate  $p$ -value. The exact  $p$ -value is based on the actual randomization distribution of the test statistic. The approximate  $p$ -value is based on a normal approximation to the randomization distribution. By default, the exact  $p$ -value is computed for sample sizes  $n \leq 200$  because the normal approximation may not be precise in small samples. The exact computation can be suppressed by specifying `noexact`. For sample sizes larger than 200, you must specify `exact` to compute the exact  $p$ -value. The exact computation is available for sample sizes  $n \leq 2000$ .

## Remarks and examples

### ► Example 1: signrank

We are testing the effectiveness of a new fuel additive. We run an experiment with 12 cars. We first run each car without the fuel treatment and measure the mileage. We then add the fuel treatment and repeat the experiment. The results of the experiment are

| Without<br>treatment | With<br>treatment | Without<br>treatment | With<br>treatment |
|----------------------|-------------------|----------------------|-------------------|
| 20                   | 24                | 18                   | 17                |
| 23                   | 25                | 24                   | 28                |
| 21                   | 21                | 20                   | 24                |
| 25                   | 22                | 24                   | 27                |
| 18                   | 23                | 23                   | 21                |
| 17                   | 18                | 19                   | 23                |

We create two variables called mpg1 and mpg2, representing mileage without and with the treatment, respectively. We can test the null hypothesis that the treatment had no effect by typing

```
. use https://www.stata-press.com/data/r19/fuel
. signrank mpg1 = mpg2
Wilcoxon signed-rank test
```

| Sign     | Obs | Sum ranks | Expected |
|----------|-----|-----------|----------|
| Positive | 3   | 13.5      | 38.5     |
| Negative | 8   | 63.5      | 38.5     |
| Zero     | 1   | 1         | 1        |
| All      | 12  | 78        | 78       |

```
Unadjusted variance 162.50
Adjustment for ties -1.62
Adjustment for zeros -0.25

Adjusted variance 160.62
H0: mpg1 = mpg2
 z = -1.973
Prob > |z| = 0.0485
Exact prob = 0.0479
```

Despite the small sample size, the  $p$ -value computed using a normal approximation, 0.0485, is similar to the exact  $p$ -value, 0.0479. These results indicate that we can reject the null hypothesis at a significance level of 0.05.

◀

## ▶ Example 2: signtest

signtest tests that the median of the differences is zero, making no further assumptions, whereas signrank assumed that the distributions are equal as well. Using the data above, we type

```
. signtest mpg1 = mpg2
Sign test
```

| Sign     | Observed | Expected |
|----------|----------|----------|
| Positive | 3        | 5.5      |
| Negative | 8        | 5.5      |
| Zero     | 1        | 1        |
| All      | 12       | 12       |

```
One-sided tests:
H0: median of mpg1 - mpg2 = 0 vs.
Ha: median of mpg1 - mpg2 > 0
Pr(#positive >= 3) =
 Binomial(n = 11, x >= 3, p = 0.5) = 0.9673
H0: median of mpg1 - mpg2 = 0 vs.
Ha: median of mpg1 - mpg2 < 0
Pr(#negative >= 8) =
 Binomial(n = 11, x >= 8, p = 0.5) = 0.1133
Two-sided test:
H0: median of mpg1 - mpg2 = 0 vs.
Ha: median of mpg1 - mpg2 != 0
Pr(#positive >= 8 or #negative >= 8) =
 min(1, 2*Binomial(n = 11, x >= 8, p = 0.5)) = 0.2266
```

The summary table indicates that there were three comparisons for which mpg1 exceeded mpg2, eight comparisons for which mpg2 exceeded mpg1, and one comparison for which they were the same.

The  $p$ -values displayed below the summary table are based on a binomial( $n, p = 1/2$ ) distribution for the test statistic, which is the number of positive or negative signs. Zeros are ignored. The  $n$  for the test is the number of nonzero differences. Because no approximation is used, the  $p$ -values are “exact”  $p$ -values. The  $p$ -value for the one-sided test, where the alternative hypothesis is that the median of mpg1 – mpg2 is smaller than zero, is 0.1133. The  $p$ -value for the two-sided test, where the alternative hypothesis is simply that the median of the differences is different from zero, is  $0.2266 = 2 \times 0.1133$ .

◀

## Stored results

signrank stores the following in `r()`:

Scalars

|                           |                                                      |
|---------------------------|------------------------------------------------------|
| <code>r(N)</code>         | sample size                                          |
| <code>r(N_pos)</code>     | number of positive comparisons                       |
| <code>r(N_neg)</code>     | number of negative comparisons                       |
| <code>r(N_tie)</code>     | number of tied comparisons                           |
| <code>r(z)</code>         | $z$ statistic                                        |
| <code>r(Var_a)</code>     | adjusted variance                                    |
| <code>r(sum_pos)</code>   | sum of the positive ranks                            |
| <code>r(sum_neg)</code>   | sum of the negative ranks                            |
| <code>r(p)</code>         | two-sided $p$ -value from normal approximation       |
| <code>r(p_l)</code>       | lower one-sided $p$ -value from normal approximation |
| <code>r(p_u)</code>       | upper one-sided $p$ -value from normal approximation |
| <code>r(p_exact)</code>   | two-sided exact $p$ -value                           |
| <code>r(p_l_exact)</code> | lower one-sided exact $p$ -value                     |
| <code>r(p_u_exact)</code> | upper one-sided exact $p$ -value                     |

signtest stores the following in `r()`:

Scalars

|                       |                                |
|-----------------------|--------------------------------|
| <code>r(N)</code>     | sample size                    |
| <code>r(N_pos)</code> | number of positive comparisons |
| <code>r(N_neg)</code> | number of negative comparisons |
| <code>r(N_tie)</code> | number of tied comparisons     |
| <code>r(p)</code>     | two-sided $p$ -value           |
| <code>r(p_l)</code>   | lower one-sided $p$ -value     |
| <code>r(p_u)</code>   | upper one-sided $p$ -value     |

## Methods and formulas

For a practical introduction to these techniques with an emphasis on examples rather than theory, see [Bland \(2015\)](#) or [Sprent and Smeeton \(2007\)](#). For a summary of these tests, see [Snedecor and Cochran \(1989\)](#).

Methods and formulas are presented under the following headings:

[\*signrank\*](#)  
[\*signtest\*](#)

## signrank

Both the sign test and Wilcoxon signed-rank tests test the null hypothesis that the distribution of a random variable  $D = \text{varname} - \text{exp}$  has median zero. The sign test makes no additional assumptions, but the Wilcoxon signed-rank test makes the additional assumption that the distribution of  $D$  is symmetric. If  $D = X_1 - X_2$ , where  $X_1$  and  $X_2$  have the same distribution, then it follows that the distribution of  $D$  is symmetric about zero. Thus, the Wilcoxon signed-rank test is often described as a test of the hypothesis that two distributions are the same, that is,  $X_1 \sim X_2$ .

Let  $d_j$  denote the difference for any matched pair of observations,

$$d_j = x_{1j} - x_{2j} = \text{varname} - \text{exp}$$

for  $j = 1, 2, \dots, n$ .

Rank the absolute values of the differences,  $|d_j|$ , and assign any tied values the average rank. Consider the signs of  $d_j$ , and let

$$r_j = \text{sign}(d_j) \text{rank}(|d_j|)$$

be the signed ranks. The test statistic is

$$T_{\text{obs}} = \sum_{j=1}^n r_j = (\text{sum of ranks for } + \text{ signs}) - (\text{sum of ranks for } - \text{ signs})$$

The distribution of the test statistic is based on Fisher's principle of randomization (Fisher 1935). Fisher's idea (stated in a modern way) was to look at a family of transformations of the observed data such that the a priori likelihood (under the null hypothesis) of the transformed data is the same as the likelihood of the observed data. The distribution of the test statistic is then produced by calculating its value for each of the transformed "randomization" datasets, assuming that each dataset is equally likely.

The null hypothesis is that the distribution of  $d_j$  is symmetric about 0. Hence, the likelihood is unchanged if we flip signs on the  $d_j$ . The randomization distribution of our test statistic,  $T$ , is all of its values resulting from the  $2^n$  possible sign changes for the  $d_j$ . Namely, the distribution is all the  $2^n$  possible values of

$$T = \sum_{j=1}^n S_j r_j$$

where  $r_j$  are the observed signed ranks (considered fixed) and  $S_j$  is either  $+1$  or  $-1$ . When the exact option is specified (or implied for  $n \leq 200$ ), this distribution is computed using a recursive algorithm whose computational time is proportional to  $n^3$ . (See Fisher [1935] for the principle of randomization; Wilcoxon, Katti, and Wilcox [1970] for the computation with untied ranks; and Baker and Tilbury [1993] for the general recursive algorithm.)

$p$ -values can also be computed using a normal approximation to the randomization distribution. For the randomization distribution, the mean and variance are given by

$$E(T) = 0 \quad \text{and} \quad \text{Var}_{\text{adj}}(T) = \sum_{j=1}^n r_j^2$$

The test statistic for the Wilcoxon signed-rank test is often expressed (equivalently) as the sum of the positive signed ranks,  $T_+$ , where

$$E(T_+) = \frac{n(n+1)}{4} \quad \text{and} \quad \text{Var}_{\text{adj}}(T_+) = \frac{1}{4} \sum_{j=1}^n r_j^2$$

Zeros and ties do not affect the theory above, and the exact variance is still given by the above formula for  $\text{Var}_{\text{adj}}(T_+)$ . When  $d_j = 0$  is observed,  $r_j$  will always be zero in each of the randomization datasets, using  $\text{sign}(0) = 0$ . (This method of handling zeros is based on the theoretical arguments made by Pratt [1959].) When there are ties, averaged ranks are assigned for each group of ties and then treated the same as other ranks.

The “unadjusted variance” reported by `signrank` is the variance that the randomization distribution would have had if there had been no ties or zeros:

$$\text{Var}_{\text{unadj}}(T_+) = \frac{1}{4} \sum_{j=1}^n j^2 = \frac{n(n+1)(2n+1)}{24}$$

The adjustment for zeros is the change in the variance when the ranks for the zeros are signed to make  $r_j = 0$ ,

$$\Delta \text{Var}_{\text{zero adj}}(T_+) = -\frac{1}{4} \sum_{j=1}^{n_0} j^2 = -\frac{n_0(n_0+1)(2n_0+1)}{24}$$

where  $n_0$  is the number of zeros. The adjustment for ties is the change in the variance when the ranks (for nonzero observations) are replaced by averaged ranks:

$$\Delta \text{Var}_{\text{ties adj}}(T_+) = \text{Var}_{\text{adj}}(T_+) - \text{Var}_{\text{unadj}}(T_+) - \Delta \text{Var}_{\text{zero adj}}(T_+)$$

A normal approximation is used to calculate

$$z = \frac{T_+ - E(T_+)}{\sqrt{\text{Var}_{\text{adj}}(T_+)}}$$

## signtest

The test statistic for the sign test is the number  $n_+$  of differences

$$d_j = x_{1j} - x_{2j} = \text{varname} - \text{exp}$$

greater than zero.

Again, the distribution of the test statistic is based on Fisher’s principle of randomization, which we described above for `signrank`. For the sign test, the “data” are simply the set of signs of the differences. Under the null hypothesis of the sign test, the probability that  $d_j$  is less than zero is equal to the probability that  $d_j$  is greater than zero. Thus, you can transform the observed signs by flipping any number of them, and the set of signs will have the same likelihood. The  $2^n$  possible sign changes form the family of randomization datasets. If you have no zeros, this procedure leads to  $n_+ \sim \text{binomial}(n, p = 1/2)$ .

But what if some differences are zero? If you do have zeros, changing their signs leaves them as zeros. So, if you observe  $n_0$  zeros, each of the  $2^n$  sign-change datasets will also have  $n_0$  zeros. Hence, the values of  $n_+$  calculated over the sign-change datasets range from 0 to  $n - n_0$ , and the “randomization” distribution of  $n_+$  is  $\text{binomial}(n - n_0, p = 1/2)$ .

The work of [Arbuthnott \(1710\)](#) and later eighteenth-century contributions is discussed by [Hald \(2003, chap. 17\)](#).

[Frank Wilcoxon \(1892–1965\)](#) was born in Ireland to American parents. After working in various occupations (including merchant seaman, oil-well pump attendant, and tree surgeon), he settled in chemistry, gaining degrees from Rutgers and Cornell and employment from various companies. Working mainly on the development of fungicides and insecticides, Wilcoxon became interested in statistics in 1925 and made several key contributions to nonparametric methods. After retiring from industry, he taught statistics at Florida State until his death.

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## Also see

- [R] [ranksum](#) — Equality tests on unmatched data
- [R] [ttest](#) — *t* tests (mean-comparison tests)

[Description](#)[Remarks and examples](#)[Quick start](#)[References](#)[Syntax](#)[Also see](#)[Options](#)

## Description

`simulate` eases the programming task of performing Monte Carlo–type simulations. Typing

```
. simulate exp_list, reps(#): command
```

runs *command* for # replications and collects the results in *exp\_list*.

*command* defines the command that performs one simulation. Most Stata commands and user-written programs can be used with `simulate`, as long as they follow standard Stata syntax; see [\[U\] 11 Language syntax](#). The `by` prefix may not be part of *command*.

*exp\_list* specifies the expression to be calculated from the execution of *command*. If no expressions are given, *exp\_list* assumes a default, depending upon whether *command* changes results in `e()` or `r()`. If *command* changes results in `e()`, the default is `_b`. If *command* changes results in `r()` (but not `e()`), the default is all the scalars posted to `r()`. It is an error not to specify an expression in *exp\_list* otherwise.

## Quick start

*Simple program for use with simulate*

Define program `myreg` to generate data and fit a linear regression

```
program myreg, eclass
 drop _all
 set obs 25
 generate x = rnormal()
 generate y = 3*x + 1 + rnormal()
 regress y x
end
```

*Perform simulation*

Record coefficients and SEs from 1,000 simulated replications of program `myreg`

```
simulate _b _se, reps(1000): myreg
```

Same as above, and set random-number seed to 5,762 for reproducible results

```
simulate _b _se, reps(1000) seed(5762): myreg
```

## Syntax

```
simulate [exp_list], reps(#) [options] : command
```

| <i>options</i>                     | Description                            |
|------------------------------------|----------------------------------------|
| <code>nodots</code>                | suppress replication dots              |
| <code>dots(#)</code>               | display dots every # replications      |
| <code>noisily</code>               | display any output from <i>command</i> |
| <code>trace</code>                 | trace <i>command</i>                   |
| <code>saving(filename, ...)</code> | save results to <i>filename</i>        |
| <code>nolegend</code>              | suppress table legend                  |
| <code>verbose</code>               | display the full table legend          |
| <code>seed(#)</code>               | set random-number seed to #            |

All weight types supported by *command* are allowed; see [U] 11.1.6 **weight**.

|                          |                                                               |
|--------------------------|---------------------------------------------------------------|
| <i>exp_list</i> contains | ( <i>name</i> : <i>elist</i> )<br><i>elist</i><br><i>eexp</i> |
| <i>elist</i> contains    | <i>newvar</i> = ( <i>exp</i> )<br>( <i>exp</i> )              |
| <i>eexp</i> is           | <i>specname</i><br>[ <i>eqno</i> ] <i>specname</i>            |
| <i>specname</i> is       | _b<br>_b []<br>_se<br>_se []                                  |
| <i>eqno</i> is           | # #<br><i>name</i>                                            |

*exp* is a standard Stata expression; see [U] 13 **Functions and expressions**.

Distinguish between [], which are to be typed, and [], which indicate optional arguments.

## Options

`reps(#)` is required—it specifies the number of replications to be performed.

`nodots` and `dots(#)` specify whether to display replication dots. By default, one dot character is displayed for each successful replication. An “x” is displayed if *command* returns an error or if any value in *exp\_list* is missing. You can also control whether dots are displayed using `set dots`; see [R] **set**.

`nodots` suppresses display of the replication dots.

`dots(#)` displays dots every # replications. `dots(0)` is a synonym for `nodots`.

`noisily` requests that any output from *command* be displayed. This option implies the `nodots` option.

`trace` causes a trace of the execution of *command* to be displayed. This option implies the `noisily` option.

`saving(filename[ , suboptions ])` creates a Stata data file (`.dta` file) consisting of (for each statistic in *exp\_list*) a variable containing the replicates.

`double` specifies that the results for each replication be saved as doubles, meaning 8-byte reals. By default, they are saved as floats, meaning 4-byte reals.

`every(#)` specifies that results be written to disk every *#*th replication. `every()` should be specified only in conjunction with `saving()` when *command* takes a long time for each replication. This will allow recovery of partial results should some other software crash your computer. See [P] [postfile](#).

`replace` specifies that *filename* be overwritten if it exists.

`nolegend` suppresses display of the table legend. The table legend identifies the rows of the table with the expressions they represent.

`verbose` requests that the full table legend be displayed. By default, coefficients and standard errors are not displayed.

`seed(#)` sets the random-number seed. Specifying this option is equivalent to typing the following command before calling `simulate`:

```
. set seed #
```

## Remarks and examples

For an introduction to Monte Carlo methods, see [Cameron and Trivedi \(2022, chap. 5\)](#). [White \(2010\)](#) provides a command for analyzing results of simulation studies.

### ► Example 1: Simulating basic summary statistics

We have a dataset containing means and variances of 100-observation samples from a lognormal distribution (as a first step in evaluating, say, the coverage of a 95%, *t*-based confidence interval). Then we perform the experiment 1,000 times.

The following command definition will generate 100 independent observations from a lognormal distribution and compute the summary statistics for this sample.

```
program lnsim, rclass
 version 19.5 // (or version 19 if you do not have StataNow)
 drop _all
 set obs 100
 generate z = exp(rnormal())
 summarize z
 return scalar mean = r(mean)
 return scalar Var = r(Var)
end
```

We can save 1,000 simulated means and variances from `lnsim` by typing

```
. set seed 1234
. simulate mean=r(mean) var=r(Var), reps(1000) nodots: lnsim
Command: lnsim
 mean: r(mean)
 var: r(Var)
```

```
. describe *
```

| Variable name | Storage type | Display format | Value label | Variable label |
|---------------|--------------|----------------|-------------|----------------|
| mean          | float        | %9.0g          |             | r(mean)        |
| var           | float        | %9.0g          |             | r(Var)         |

```
. summarize
```

| Variable | Obs   | Mean     | Std. dev. | Min      | Max      |
|----------|-------|----------|-----------|----------|----------|
| mean     | 1,000 | 1.630648 | .2173062  | 1.106372 | 2.612052 |
| var      | 1,000 | 4.60798  | 4.502166  | .966087  | 70.5597  |



## □ Technical note

Before executing our `lnsim` simulator, we can verify that it works by executing it interactively.

```
. set seed 1234
. lnsim
Number of observations (_N) was 0, now 100.
```

| Variable | Obs | Mean     | Std. dev. | Min      | Max      |
|----------|-----|----------|-----------|----------|----------|
| z        | 100 | 1.534256 | 1.584568  | .0400387 | 9.818309 |

```
. return list
scalars:
 r(Var) = 2.510857086217961
 r(mean) = 1.53425569280982
```



## ▷ Example 2: Simulating a regression model

Consider a more complicated problem. Let's experiment with fitting  $y_j = a + bx_j + u_j$  when the true model has  $a = 1$ ,  $b = 2$ ,  $u_j = z_j + cx_j$ , and when  $z_j$  is  $N(0, 1)$ . We will save the parameter estimates and standard errors and experiment with varying  $c$ .  $x_j$  will be fixed across experiments but will originally be generated as  $N(0, 1)$ . We begin by interactively making the true data:

```
. drop _all
. set obs 100
Number of observations (_N) was 0, now 100.
. set seed 54321
. generate x = rnormal()
. generate true_y = 1+2*x
. save truth
file truth.dta saved
```

Our program is

```
program hetero1
 version 19.5 // (or version 19 if you do not have StataNow)
 args c
 use truth, clear
 generate y = true_y + (rnormal() + 'c'*x)
 regress y x
end
```

Note the use of ‘c’ in our statement for generating y. c is a local macro generated from args c and thus refers to the first argument supplied to hetero1. If we want  $c = 3$  for our experiment, we type

```
. simulate _b _se, reps(10000): hetero1 3
 (output omitted)
```

Our program hetero1 could, however, be more efficient because it rereads the file truth once every replication. It would be better if we could read the data just once. In fact, if we read in the data right before running simulate, we really should not have to reread for each subsequent replication. A faster version reads

```
program hetero2
 version 19.5 // (or version 19 if you do not have StataNow)
 args c
 capture drop y
 generate y = true_y + (rnormal() + 'c'*x)
 regress y x
end
```

Requiring that the current dataset has the variables true\_y and x may become inconvenient. Another improvement would be to require that the user supply variable names, such as in

```
program hetero3
 version 19.5 // (or version 19 if you do not have StataNow)
 args truey x c
 capture drop y
 generate y = 'truey' + (rnormal() + 'c'*'x')
 regress y x
end
```

Thus, we can type

```
. simulate _b _se, reps(10000): hetero3 true_y x 3
 (output omitted)
```

◀

### ▷ Example 3: Simulating a ratio of statistics

Now, let’s consider the problem of simulating the ratio of two medians. Suppose that each sample of size  $n_i$  comes from a normal population with a mean  $\mu_i$  and standard deviation  $\sigma_i$ , where  $i = 1, 2$ . We write the program below and save it as a text file called myratio.ado (see [U] 17 Ado-files). Our program is an rclass command that requires six arguments as input, identified by the local macros n1, mu1, sigma1, n2, mu2, and sigma2, which correspond to  $n_1$ ,  $\mu_1$ ,  $\sigma_1$ ,  $n_2$ ,  $\mu_2$ , and  $\sigma_2$ , respectively. With these arguments, myratio will generate the data for the two samples, use summarize to compute the two medians and store the ratio of the medians in r(ratio).

```
program myratio, rclass
 version 19.5 // (or version 19 if you do not have StataNow)
 args n1 mu1 sigma1 n2 mu2 sigma2
 // generate the data
 drop _all
 local N = 'n1'+‘n2’
 set obs ‘N’
 tempvar y
 generate ‘y’ = rnormal()
 replace ‘y’ = cond(_n<=‘n1’, ‘mu1’+‘y’*‘sigma1’, ‘mu2’+‘y’*‘sigma2’)
 // calculate the medians
 tempname m1
```

```

summarize 'y' if _n<='n1', detail
scalar 'm1' = r(p50)
summarize 'y' if _n>'n1', detail
// store the results
return scalar ratio = 'm1' / r(p50)
end

```

The result of running our simulation is

```

. set seed 19192
. simulate ratio=r(ratio), reps(1000) nodots: myratio 5 3 1 10 3 2
 Command: myratio 5 3 1 10 3 2
 ratio: r(ratio)

. summarize

```

| Variable | Obs   | Mean    | Std. dev. | Min      | Max      |
|----------|-------|---------|-----------|----------|----------|
| ratio    | 1,000 | 1.10875 | .5219166  | .3606606 | 9.857285 |

◀

## □ Technical note

Stata lets us do simulations of simulations and simulations of bootstraps. Stata's `bootstrap` command (see [R] [bootstrap](#)) works much like `simulate`, except that it feeds the user-written program a bootstrap sample. Say that we want to evaluate the bootstrap estimator of the standard error of the median when applied to lognormally distributed data. We want to perform a simulation, resulting in a dataset of medians and bootstrap estimated standard errors.

As background, `summarize` (see [R] [summarize](#)) calculates summary statistics, leaving the mean in `r(mean)` and the standard deviation in `r(sd)`. `summarize` with the `detail` option also calculates summary statistics, but more of them, and leaves the median in `r(p50)`.

Thus, our plan is to perform simulations by randomly drawing a dataset: we calculate the median of our random sample, we use `bootstrap` to obtain a dataset of medians calculated from bootstrap samples of our random sample, the standard deviation of those medians is our estimate of the standard error, and the summary statistics are stored in the results of `summarize`.

Our simulator is

```

program define bsse, rclass
version 19.5 // (or version 19 if you do not have StataNow)
drop _all
set obs 100
generate x = rnormal()
tempfile bsfile
bootstrap midp=r(p50), rep(100) saving('bsfile'): summarize x, detail
use 'bsfile', clear
summarize midp
return scalar mean = r(mean)
return scalar sd = r(sd)
end

```

We can obtain final results, running our simulation 1,000 times, by typing

```
. set seed 48901
. simulate med=r(mean) bs_se=r(sd), reps(1000): bsse
 Command: bsse
 med: r(mean)
 bs_se: r(sd)
Simulations (1,000):10.....20.....30.....40.....50....
>60.....70.....80.....90.....100.....110.....120..
>130.....140.....150.....160.....170.....180.....
>190.....200.....210.....220.....230.....240.....250..
>260.....270.....280.....290.....300.....310.....
>320.....330.....340.....350.....360.....370.....380..
>390.....400.....410.....420.....430.....440.....
>450.....460.....470.....480.....490.....500.....510..
>520.....530.....540.....550.....560.....570.....
>580.....590.....600.....610.....620.....630.....640..
>650.....660.....670.....680.....690.....700.....
>710.....720.....730.....740.....750.....760.....770..
>780.....790.....800.....810.....820.....830.....
>840.....850.....860.....870.....880.....890.....900..
>910.....920.....930.....940.....950.....960.....
>970.....980.....990.....1,000 done
. summarize
```

| Variable | Obs   | Mean      | Std. dev. | Min       | Max      |
|----------|-------|-----------|-----------|-----------|----------|
| med      | 1,000 | -.0013359 | .1221602  | -.3795549 | .3656219 |
| bs_se    | 1,000 | .1278773  | .0303109  | .0614031  | .2484805 |

This is a case where the simulation dots (drawn by default, unless the `nodots` option is specified) will give us an idea of how long this simulation will take to finish as it runs.

□

## References

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## Also see

- [R] [bootstrap](#) — Bootstrap sampling and estimation
- [R] [jackknife](#) — Jackknife estimation
- [R] [permute](#) — Permutation tests
- [R] [set rngstream](#) — Specify the stream for the stream random-number generator

## Description

The *Stata Journal* (SJ) is a quarterly journal containing articles about statistics, data analysis, teaching methods, and effective use of Stata's language. The SJ publishes reviewed articles with shorter notes and comments, regular columns, tips, book reviews, and other material of interest to researchers applying statistics in a variety of disciplines. The SJ began publication in the fourth quarter of 2001 with SJ Volume 1 Number 1. You can read all about the SJ at <https://www.stata-journal.com>.

The SJ is a printed and electronic journal with corresponding software. You must subscribe to the SJ; see <https://www.stata-journal.com/subscription/> to purchase a subscription or an individual article. Electronic copies of SJ articles that are older than three years are available at no charge from <https://www.stata-journal.com/archives.html>.

The software is available at no charge; see [below](#) for installation instructions.

## Remarks and examples

Remarks are presented under the following headings:

*Installing the Stata Journal software*

*Obtaining from the internet by pointing and clicking*

*Obtaining from the internet via command mode*

## Installing the Stata Journal software

Each issue of the *Stata Journal* is labeled Volume #, Number #. Volume 1 refers to the first year of publication, Volume 2 to the second, and so on. Issues are numbered 1, 2, 3, and 4 within each year. The first issue of the *Journal* was published in the fourth quarter of 2001, and that issue is numbered Volume 1, Number 1. For installation purposes, we refer to this issue as `sj1-1`.

The articles, columns, notes, and comments that make up the *Stata Journal* are assigned a letter-and-number code, called an insert tag, such as `st0001`, `an0034`, or `ds0011`. The letters represent a category: `st` is the statistics category, `an` is the announcements category, etc. The numbers are assigned sequentially, so `st0001` is the first article in the statistics category.

Sometimes inserts are subsequently updated, either to fix bugs or to add new features. A number such as `st0001_1` indicates that this article, column, note, or comment is an update to the original `st0001` article. Updates are complete; that is, installing `st0001_1` provides all the features of the original article and more.

The *Stata Journal* software may be obtained by pointing and clicking or by using command mode.

The sections below detail how to install an insert. In all cases, pretend that you wish to install insert `st0274` from `sj12-4`.

## Obtaining from the internet by pointing and clicking

1. Select **Help > SJ and community-contributed features**.
2. Click on *Stata Journal*.
3. Click on *sj12-4*.
4. Click on *st0274*.
5. Click on (*click here to install*).

## Obtaining from the internet via command mode

Type the following:

```
. net from https://www.stata-journal.com/software
. net cd sj12-4
. net describe st0274
. net install st0274
```

The above could be shortened to

```
. net from https://www.stata-journal.com/software/sj12-4
. net describe st0274
. net install st0274
```

Alternatively, you could type

```
. net sj 12-4
. net describe st0274
. net install st0274
```

but going about it the long way is more entertaining, at least the first time.

## Also see

[R] [search](#) — Search Stata documentation and other resources

[R] [net](#) — Install and manage community-contributed additions from the internet

[R] [net search](#) — Search the internet for installable packages

[R] [update](#) — Check for official updates

[U] [3.4 The Stata Journal](#)

[U] [29 Using the internet to keep up to date](#)

[GSM] [19 Updating and extending Stata—internet functionality](#)

[GSU] [19 Updating and extending Stata—internet functionality](#)

[GSW] [19 Updating and extending Stata—internet functionality](#)

[Description](#)[Option](#)[Acknowledgments](#)[Quick start](#)[Remarks and examples](#)[References](#)[Menu](#)[Stored results](#)[Also see](#)[Syntax](#)[Methods and formulas](#)

## Description

For each variable in *varlist*, `sktest` presents a test for normality based on skewness and another based on kurtosis and then combines the two tests into an overall test statistic. `sktest` requires a minimum of eight observations to make its calculations. See [MV] [mvtest normality](#) for multivariate tests of normality.

## Quick start

Test for normality of *v1* based on skewness and kurtosis

```
sktest v1
```

Separate tests for *v1* and *v2*

```
sktest v1 v2
```

With frequency weights in *wvar*

```
sktest v1 v2 [fweight=wvar]
```

Suppress adjustment to the overall  $\chi^2$  test

```
sktest v1 v2, noadjust
```

## Menu

Statistics > Summaries, tables, and tests > Distributional plots and tests > Skewness and kurtosis normality tests

## Syntax

```
sktest varlist [if] [in] [weight] [, options]
```

| <i>options</i> | Description |
|----------------|-------------|
|----------------|-------------|

Main

|                        |                                                                      |
|------------------------|----------------------------------------------------------------------|
| <code>noadjust</code>  | do not adjust the overall $\chi^2$ test statistic and its $p$ -value |
| <code>nostretch</code> | do not automatically widen table for long variable names             |

`collect` is allowed; see [U] 11.1.10 Prefix commands.

`aweight`s and `fweight`s are allowed; see [U] 11.1.6 weight.

`nostretch` does not appear in the dialog box.

## Option

Main

`noadjust` suppresses the empirical adjustment made by Royston (1991c) to the overall  $\chi^2$  test statistic and its  $p$ -value and presents the unaltered test as described by D'Agostino, Belanger, and D'Agostino (1990).

The following option is available with `sktest` but is not shown in the dialog box:

`nostretch`; see [R] Estimation options.

## Remarks and examples

Also see [R] `swilk` for the Shapiro–Wilk and Shapiro–Francia tests for normality. Those tests are, in general, preferred for nonaggregated data (Gould and Rogers 1991; Gould 1992; Royston 1991c). Moreover, a normal quantile plot should be used with any test for normality; see [R] Diagnostic plots for more information.

### ▷ Example 1

Using our automobile dataset, we will test whether the variables `mpg` and `trunk` are normally distributed:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
```

```
. sktest mpg trunk
```

```
Skewness and kurtosis tests for normality
```

| Variable | Obs | Pr(skewness) | Pr(kurtosis) | Joint test  |           |
|----------|-----|--------------|--------------|-------------|-----------|
|          |     |              |              | Adj chi2(2) | Prob>chi2 |
| mpg      | 74  | 0.0015       | 0.0804       | 10.95       | 0.0042    |
| trunk    | 74  | 0.9115       | 0.0445       | 4.19        | 0.1228    |

We can reject the hypothesis that `mpg` is normally distributed, but we cannot reject the hypothesis that `trunk` is normally distributed, at least at the 12% level. The kurtosis for `trunk` is 2.19, as can be verified by issuing the command

```
. summarize trunk, detail
 (output omitted)
```

and the  $p$ -value of 0.0445 shown in the table above indicates that it is significantly different from the kurtosis of a normal distribution at the 5% significance level. However, on the basis of skewness alone, we cannot reject the hypothesis that `trunk` is normally distributed.

◀

## □ Technical note

`sktest` implements the test as described by [D'Agostino, Belanger, and D'Agostino \(1990\)](#) but with the adjustment made by [Royston \(1991c\)](#). In the above example, if we had specified the `noadjust` option, the  $\chi^2$  values would have been 13.13 for `mpg` and 4.05 for `trunk`. With the adjustment, the  $\chi^2$  value might show as `.'`. This result should be interpreted as an absurdly large number; the data are most certainly not normal.

□

## Stored results

`sktest` stores the following in `r()`:

### Scalars

|                        |                                       |
|------------------------|---------------------------------------|
| <code>r(N)</code>      | number of observations                |
| <code>r(chi2)</code>   | overall $\chi^2$                      |
| <code>r(p_skew)</code> | $p$ -value for test based on skewness |
| <code>r(p_kurt)</code> | $p$ -value for test based on kurtosis |
| <code>r(p_chi2)</code> | $p$ -value for overall $\chi^2$ test  |

### Matrices

|                       |                                                   |
|-----------------------|---------------------------------------------------|
| <code>r(table)</code> | matrix of displayed results, one row per variable |
|-----------------------|---------------------------------------------------|

## Methods and formulas

`sktest` implements the test described by [D'Agostino, Belanger, and D'Agostino \(1990\)](#) with the empirical correction developed by [Royston \(1991c\)](#).

Let  $g_1$  denote the coefficient of skewness and  $b_2$  denote the coefficient of kurtosis as calculated by `summarize`, and let  $n$  denote the sample size. If weights are specified, then  $g_1$ ,  $b_2$ , and  $n$  denote the weighted coefficients of skewness and kurtosis and weighted sample size, respectively. See [\[R\] summarize](#) for the formulas for skewness and kurtosis.

To perform the test of skewness, we compute

$$Y = g_1 \left\{ \frac{(n+1)(n+3)}{6(n-2)} \right\}^{1/2}$$

$$\beta_2(g_1) = \frac{3(n^2 + 27n - 70)(n+1)(n+3)}{(n-2)(n+5)(n+7)(n+9)}$$

$$W^2 = -1 + [2\{\beta_2(g_1) - 1\}]^{1/2}$$

and

$$\alpha = \{2/(W^2 - 1)\}^{1/2}$$

Then, the distribution of the test statistic

$$Z_1 = \frac{1}{\sqrt{\ln W}} \ln \left[ Y/\alpha + \{(Y/\alpha)^2 + 1\}^{1/2} \right]$$

is approximately standard normal under the null hypothesis that the data are distributed normally.

To perform the test of kurtosis, we compute

$$E(b_2) = \frac{3(n-1)}{n+1}$$

$$\text{var}(b_2) = \frac{24n(n-2)(n-3)}{(n+1)^2(n+3)(n+5)}$$

$$X = \{b_2 - E(b_2)\} / \sqrt{\text{var}(b_2)}$$

$$\sqrt{\beta_1(b_2)} = \frac{6(n^2 - 5n + 2)}{(n+7)(n+9)} \left\{ \frac{6(n+3)(n+5)}{n(n-2)(n-3)} \right\}^{1/2}$$

and

$$A = 6 + \frac{8}{\sqrt{\beta_1(b_2)}} \left[ \frac{2}{\sqrt{\beta_1(b_2)}} + \left\{ 1 + \frac{4}{\beta_1(b_2)} \right\}^{1/2} \right]$$

Then, the distribution of the test statistic

$$Z_2 = \frac{1}{\sqrt{2/(9A)}} \left[ \left( 1 - \frac{2}{9A} \right) - \left\{ \frac{1 - 2/A}{1 + X\sqrt{2/(A-4)}} \right\}^{1/3} \right]$$

is approximately standard normal under the null hypothesis that the data are distributed normally.

D'Agostino, Balanger, and D'Agostino Jr.'s omnibus test of normality uses the statistic

$$K^2 = Z_1^2 + Z_2^2$$

which has approximately a  $\chi^2$  distribution with 2 degrees of freedom under the null of normality.

Royston (1991c) proposed the following adjustment to the test of normality, which `sktest` uses by default. Let  $\Phi(x)$  denote the cumulative standard normal distribution function for  $x$ , and let  $\Phi^{-1}(p)$  denote the inverse cumulative standard normal function [that is,  $x = \Phi^{-1}\{\Phi(x)\}$ ]. Define the following terms:

$$\begin{aligned} Z_c &= -\Phi^{-1}\left\{\exp\left(-\frac{1}{2}K^2\right)\right\} \\ Z_t &= 0.55n^{0.2} - 0.21 \\ a_1 &= (-5 + 3.46 \ln n) \exp(-1.37 \ln n) \\ b_1 &= 1 + (0.854 - 0.148 \ln n) \exp(-0.55 \ln n) \\ a_2 &= a_1 - \{2.13/(1 - 2.37 \ln n)\} Z_t \end{aligned}$$

and

$$b_2 = 2.13/(1 - 2.37 \ln n) + b_1$$

If  $Z_c < -1$  set  $Z = Z_c$ ; else if  $Z_c < Z_t$  set  $Z = a_1 + b_1 Z_c$ ; else set  $Z = a_2 + b_2 Z_c$ . Define  $P = 1 - \Phi(Z)$ . Then,  $K^2 = -2 \ln P$  is approximately distributed  $\chi^2$  with 2 degrees of freedom.

The relative merits of the skewness and kurtosis test versus the Shapiro–Wilk and Shapiro–Francia tests have been a subject of debate. Our recommendation is to use the Shapiro–Francia test whenever possible, that is, whenever dealing with nonaggregated or ungrouped data (Gould and Rogers 1991; Gould 1992); see [R] [swilk](#). If normality is rejected, use `sktest` to determine the source of the problem.

As both D’Agostino, Belanger, and D’Agostino (1990) and Royston (1991d) mention, researchers should also examine the normal quantile plot to determine normality rather than blindly relying on a few test statistics. See the `qnorm` command documented in [R] [Diagnostic plots](#) for more information on normal quantile plots.

`sktest` is similar in spirit to the Jarque–Bera (1987) test of normality. The Jarque–Bera test statistic is also calculated from the sample skewness and kurtosis, though it is based on asymptotic standard errors with no corrections for sample size. In effect, `sktest` offers two adjustments for sample size, that of Royston (1991c) and that of D’Agostino, Belanger, and D’Agostino (1990).

## Acknowledgments

`sktest` has benefited greatly by the comments and work of Patrick Royston of the MRC Clinical Trials Unit, London, and coauthor of the Stata Press book *Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model*. At this point, the program should be viewed as due as much to Royston as to us, except, of course, for any errors. We are also indebted to Nicholas J. Cox of the Department of Geography at Durham University, UK, and coeditor of the *Stata Journal* and author of *Speaking Stata Graphics* for his helpful comments.

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## Also see

- [R] [Diagnostic plots](#) — Distributional diagnostic plots
- [R] [ladder](#) — Ladder of powers
- [R] [lv](#) — Letter-value displays
- [R] [swilk](#) — Shapiro–Wilk and Shapiro–Francia tests for normality
- [MV] [mvtest normality](#) — Multivariate normality tests

[Description](#)  
[Options](#)  
[References](#)[Quick start](#)  
[Remarks and examples](#)  
[Also see](#)[Menu](#)  
[Stored results](#)[Syntax](#)  
[Methods and formulas](#)

## Description

`slogit` fits Anderson's (1984) maximum-likelihood stereotype logistic regression model for categorical dependent variables. Stereotype logistic models can be used when the relevance of the ordering is unclear. These models do not impose the proportional-odds assumption.

## Quick start

One-dimensional model of  $y$  as a function of  $x_1$  and  $x_2$

```
slogit y x1 x2
```

Add [indicators](#) for categorical variable  $a$  and set  $y = 1$  as the base category

```
slogit y x1 x2 i.a, baseoutcome(1)
```

Multidimensional model reparameterizing a multinomial logit when  $y$  has 4 categories

```
slogit y x1 x2 i.a, dimensions(3) baseoutcome(1)
```

## Menu

Statistics > Categorical outcomes > Stereotype logistic regression

## Syntax

slomit *depvar* [*indepvars*] [*if*] [*in*] [*weight*] [, *options*]

| <i>options</i>                    | Description                                                                                                                                                         |
|-----------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                      |                                                                                                                                                                     |
| <code>dimension(#)</code>         | dimension of the model; default is <code>dimension(1)</code>                                                                                                        |
| <code>baseoutcome(# lbl)</code>   | set the base outcome to # or <i>lbl</i> ; default is the last outcome                                                                                               |
| <code>constraints(numlist)</code> | apply specified linear constraints                                                                                                                                  |
| <code>nocorner</code>             | do not generate the corner constraints                                                                                                                              |
| <b>SE/Robust</b>                  |                                                                                                                                                                     |
| <code>vce(vctype)</code>          | <i>vctype</i> may be <code>oim</code> , <code>robust</code> , <code>cluster clustvar</code> , <code>opg</code> , <code>bootstrap</code> , or <code>jackknife</code> |
| <b>Reporting</b>                  |                                                                                                                                                                     |
| <code>level(#)</code>             | set confidence level; default is <code>level(95)</code>                                                                                                             |
| <code>nocnsreport</code>          | do not display constraints                                                                                                                                          |
| <code>display_options</code>      | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling                    |
| <b>Maximization</b>               |                                                                                                                                                                     |
| <code>maximize_options</code>     | control the maximization process; seldom used                                                                                                                       |
| <code>initialize(initype)</code>  | method of initializing scale parameters; <i>initype</i> can be <code>constant</code> , <code>random</code> , or <code>svd</code> ; see <i>Options</i> for details   |
| <code>nonormalize</code>          | do not normalize the numeric variables                                                                                                                              |
| <code>collinear</code>            | keep collinear variables                                                                                                                                            |
| <code>coeflegend</code>           | display legend instead of statistics                                                                                                                                |

*indepvars* may contain factor variables; see [U] 11.4.3 **Factor variables**.

`bayesboot`, `bootstrap`, `by`, `collect`, `fp`, `jackknife`, `rolling`, `statsby`, and `svy` are allowed; see [U] 11.1.10 **Prefix commands**.

Weights are not allowed with the `bootstrap` prefix; see [R] **bootstrap**.

`vce()` and weights are not allowed with the `svy` prefix; see [SVY] **svy**.

`fweights`, `iweights`, and `pweights` are allowed; see [U] 11.1.6 **weight**.

`collinear` and `coeflegend` do not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options

### Model

`dimension(#)` specifies the dimension of the model, which is the number of equations required to describe the relationship between the dependent variable and the independent variables. The maximum dimension is  $\min(m - 1, p)$ , where  $m$  is the number of categories of the dependent variable and  $p$  is the number of independent variables in the model. The stereotype model with maximum dimension is a reparameterization of the multinomial logistic model.

`baseoutcome(#|lbl)` specifies the outcome level whose scale parameters and intercept are constrained to be zero. The base outcome may be specified as a number or a label. By default, `slogit` assumes that the outcome levels are ordered and uses the largest level of the dependent variable as the base outcome.

`constraints(numlist)`; see [R] [Estimation options](#).

By default, the linear equality constraints suggested by [Anderson \(1984\)](#), termed the corner constraints, are generated for you. You can add constraints to these as needed, or you can turn off the corner constraints by specifying `nocorner`. These constraints are in addition to the constraints placed on the  $\phi$  parameters corresponding to `baseoutcome(#)`.

`nocorner` specifies that `slogit` not generate the corner constraints. If you specify `nocorner`, you must specify at least `dimension() × dimension()` constraints for the model to be identified.

#### SE/Robust

`vce(vctype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

If specifying `vce(bootstrap)` or `vce(jackknife)`, you must also specify `baseoutcome()`.

#### Reporting

`level(#)`, `nocnsreport`; see [R] [Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

#### Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

Setting the optimization type to `technique(bhhh)` resets the default `vctype` to `vce(opg)`.

`initialize(constant|random|svd)` specifies how initial estimates are computed. The default, `initialize(constant)`, is to set the scale parameters to the constant  $\min(1/2, 1/d)$ , where  $d$  is the dimension specified in `dimension()`.

`initialize(random)` requests that uniformly distributed random numbers between 0 and 1 be used as initial values for the scale parameters. If you specify this option, you should also use `set seed` to ensure that you can replicate your results; see [R] [set seed](#).

`initialize(svd)` requests that a singular value decomposition (SVD) be performed on the matrix of regression estimates from `mlogit` to reduce its rank to the dimension specified in `dimension()`. `slogit` uses the reduced-rank components of the SVD as initial estimates for the scale and regression coefficients. For details, see [Methods and formulas](#).

`nonnormalize` specifies that the numeric variables not be normalized. Normalization of the numeric variables improves numerical stability but consumes more memory in generating temporary double-precision variables. Variables that are of type `byte` are not normalized, and if initial estimates are specified using the `from()` option, normalization of variables is not performed. See [Methods and formulas](#) for more information.

The following options are available with `slogit` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [\[R\] Estimation options](#).

## Remarks and examples

Remarks are presented under the following headings:

[Introduction](#)

[One-dimensional model](#)

[Higher-dimension models](#)

### Introduction

Like multinomial logistic and ordered logistic models, stereotype logistic models are used with categorical dependent variables. They are often used when subjects are requested to assess or judge something. In a multinomial logistic model, the categories cannot be ranked. By contrast, in an ordered logistic model, the categories follow a natural ranking scheme and are subject to the proportional-odds assumption. Stereotype logistic regression can be viewed as a compromise between these two models and is primarily used when you are unsure of the relevance of the ordering of the outcome.

A common case is when subjects are asked to assess or judge something. For example, consider a survey in which consumers are asked to rate the quality of a product on a scale from 1 to 5, with 1 indicating poor quality and 5 indicating excellent quality. If the categories are monotonically related to one underlying latent variable, the ordered logistic model is appropriate. However, suppose that consumers weigh two or three latent factors when assessing quality. The stereotype logistic model is preferred to the ordered logistic model in this case because it allows you to specify multiple equations to capture the effects of the latent variables. Unlike multinomial logit models, the number of equations you specify could be fewer than  $m - 1$ , where  $m$  is the number of categories of the dependent variable. Stereotype logistic models are also used when categories may be indistinguishable. Suppose that a consumer must choose among A, B, C, or D. Multinomial logistic modeling assumes that the four choices are distinct in the sense that a consumer choosing one of the goods can distinguish its characteristics from the others. If goods A and B are in fact similar, consumers may be randomly picking between the two. One alternative is to combine the two categories and fit a three-category multinomial logistic model. A more flexible alternative is to use a stereotype logistic model.

In the multinomial logistic model, you estimate  $m - 1$  parameter vectors  $\tilde{\beta}_k$ ,  $k = 1, \dots, m - 1$ , where  $m$  is the number of categories of the dependent variable. The stereotype logistic model is a restriction on the multinomial model in the sense that there are  $d$  parameter vectors, where  $d$  is between one and  $\min(m - 1, p)$ , and  $p$  is the number of regressors. The relationship between the stereotype model's coefficients  $\beta_j$ ,  $j = 1, \dots, d$ , and the multinomial model's coefficients is  $\tilde{\beta}_k = -\sum_{j=1}^d \phi_{jk} \beta_j$ . The  $\phi$ s are scale parameters to be estimated along with the  $\beta_j$ s.

Given a row vector of covariates  $\mathbf{x}$ , let  $\eta_k = \theta_k - \sum_{j=1}^d \phi_{jk} \mathbf{x} \beta_j$ . The probability of observing outcome  $k$  is

$$\Pr(Y_i = k) = \begin{cases} \frac{\exp(\eta_k)}{1 + \sum_{l=1}^{m-1} \exp(\eta_l)} & k < m \\ \frac{1}{1 + \sum_{l=1}^{m-1} \exp(\eta_l)} & k = m \end{cases}$$

This model includes a set of  $\theta$  parameters so that each equation has an unrestricted constant term. If  $d = m - 1$ , the stereotype model is just a reparameterization of the multinomial logistic model. To identify the  $\phi$ s and the  $\beta$ s, you must place at least  $d^2$  restrictions on the parameters. By default, `slomit` uses the “corner constraints”  $\phi_{jj} = 1$  and  $\phi_{jk} = 0$  for  $j \neq k$ ,  $k \leq d$ , and  $j \leq d$ .

For a discussion of the stereotype logistic model, see [Lunt \(2005\)](#).

## One-dimensional model

### ▷ Example 1

We have 2 years of repair rating data on the make, price, mileage rating, and gear ratio of 104 foreign and 44 domestic automobiles (with 13 missing values on repair rating). We wish to fit a stereotype logistic model to discriminate between the levels of repair rating using mileage, price, gear ratio, and origin of the manufacturer. Here is an overview of our data:

```
. use https://www.stata-press.com/data/r19/auto2yr
(Automobile models)
. tabulate repair
```

| Repair rating | Freq. | Percent | Cum.   |
|---------------|-------|---------|--------|
| Poor          | 5     | 3.70    | 3.70   |
| Fair          | 19    | 14.07   | 17.78  |
| Average       | 57    | 42.22   | 60.00  |
| Good          | 38    | 28.15   | 88.15  |
| Excellent     | 16    | 11.85   | 100.00 |
| Total         | 135   | 100.00  |        |

The variable `repair` can take five values, 1, ..., 5, which represent the subjective rating of the car model’s repair record as *Poor*, *Fair*, *Average*, *Good*, and *Excellent*.

We wish to fit the one-dimensional stereotype logistic model

$$\eta_k = \theta_k - \phi_k (\beta_1 \text{foreign} + \beta_2 \text{mpg} + \beta_3 \text{price} + \beta_4 \text{gratio})$$

for  $k < 5$  and  $\eta_5 = 0$ . To fit this model, we type

```
. slomit repair foreign mpg price gratio
Iteration 0: Log likelihood = -237.99802 (not concave)
Iteration 1: Log likelihood = -204.98644 (not concave)
Iteration 2: Log likelihood = -169.79473 (not concave)
Iteration 3: Log likelihood = -167.53649
Iteration 4: Log likelihood = -164.63477
Iteration 5: Log likelihood = -163.20867 (not concave)
Iteration 6: Log likelihood = -160.67522
Iteration 7: Log likelihood = -159.69646
Iteration 8: Log likelihood = -159.35057
Iteration 9: Log likelihood = -159.28495
Iteration 10: Log likelihood = -159.25748
Iteration 11: Log likelihood = -159.25691
Iteration 12: Log likelihood = -159.25691

Stereotype logistic regression Number of obs = 135
 Wald chi2(4) = 9.33
Log likelihood = -159.25691 Prob > chi2 = 0.0535

(1) [phi1_1]_cons = 1
```

| repair  | Coefficient      | Std. err. | z     | P> z  | [95% conf. interval] |           |
|---------|------------------|-----------|-------|-------|----------------------|-----------|
| foreign | 5.947382         | 2.094126  | 2.84  | 0.005 | 1.84297              | 10.05179  |
| mpg     | .1911968         | .08554    | 2.24  | 0.025 | .0235414             | .3588521  |
| price   | -.0000576        | .0001357  | -0.42 | 0.671 | -.0003236            | .0002083  |
| gratio  | -4.307571        | 1.884713  | -2.29 | 0.022 | -8.00154             | -.6136017 |
| /phi1_1 | 1 (constrained)  |           |       |       |                      |           |
| /phi1_2 | 1.262268         | .3530565  | 3.58  | 0.000 | .5702904             | 1.954247  |
| /phi1_3 | 1.17593          | .3169397  | 3.71  | 0.000 | .5547394             | 1.79712   |
| /phi1_4 | .8657195         | .2411228  | 3.59  | 0.000 | .3931275             | 1.338311  |
| /phi1_5 | 0 (base outcome) |           |       |       |                      |           |
| /theta1 | -6.864749        | 4.21252   | -1.63 | 0.103 | -15.12114            | 1.391639  |
| /theta2 | -7.613977        | 4.861803  | -1.57 | 0.117 | -17.14294            | 1.914981  |
| /theta3 | -5.80655         | 4.987508  | -1.16 | 0.244 | -15.58189            | 3.968786  |
| /theta4 | -3.85724         | 3.824132  | -1.01 | 0.313 | -11.3524             | 3.637922  |
| /theta5 | 0 (base outcome) |           |       |       |                      |           |

(repair=Excellent is the base outcome)

The coefficient associated with the first scale parameter,  $\phi_{11}$ , is 1, and its standard error and other statistics are missing. This is the corner constraint applied to the one-dimensional model; in the header, this constraint is listed as `[phi1_1]_cons = 1`. Also, the  $\phi$  and  $\theta$  parameters that are associated with the base outcome are identified. Keep in mind, though, that there are no coefficient estimates for `[phi1_5]_cons` or `[theta5]_cons` in the `ereturn` matrix `e(b)`. The Wald statistic is for a test of the joint significance of the regression coefficients on `foreign`, `mpg`, `price`, and `gratio`.

The one-dimensional stereotype model restricts the multinomial logistic regression coefficients  $\tilde{\beta}_k$ ,  $k = 1, \dots, m - 1$  to be parallel; that is,  $\tilde{\beta}_k = -\phi_k \beta$ . As Lunt (2001) discusses, in the one-dimensional stereotype model, one linear combination  $\mathbf{x}_i \beta$  best discriminates the outcomes of the dependent variable, and the scale parameters  $\phi_k$  measure the distance between the outcome levels and the linear predictor. If  $\phi_1 \geq \phi_2 \geq \dots \geq \phi_{m-1} \geq \phi_m \equiv 0$ , the model suggests that the subjective assessment of the dependent variable is indeed ordered. Here the maximum likelihood estimates of the  $\phi$ 's are not monotonic, as would be assumed in an ordered logit model.

We test that  $\phi_1 = \phi_2$  by typing

```
. test [phi1_2]_cons = [phi1_1]_cons
(1) - [phi1_1]_cons + [phi1_2]_cons = 0
 chi2(1) = 0.55
 Prob > chi2 = 0.4576
```

Because the two parameters are not statistically different, we decide to add a constraint to force  $\phi_1 = \phi_2$ :

```
. constraint define 1 [phi1_2]_cons = [phi1_1]_cons
. slogit repair foreign mpg price gratio, constraint(1) nolog
Stereotype logistic regression Number of obs = 135
 Wald chi2(4) = 21.28
 Prob > chi2 = 0.0003
Log likelihood = -159.65769
(1) [phi1_1]_cons = 1
(2) - [phi1_1]_cons + [phi1_2]_cons = 0
```

| repair  | Coefficient | Std. err.      | z     | P> z  | [95% conf. interval] |           |
|---------|-------------|----------------|-------|-------|----------------------|-----------|
| foreign | 7.166515    | 1.690177       | 4.24  | 0.000 | 3.853828             | 10.4792   |
| mpg     | .2340043    | .0807042       | 2.90  | 0.004 | .0758271             | .3921816  |
| price   | -.000041    | .0001618       | -0.25 | 0.800 | -.0003581            | .000276   |
| gratio  | -5.218107   | 1.798717       | -2.90 | 0.004 | -8.743528            | -1.692686 |
| /phi1_1 | 1           | (constrained)  |       |       |                      |           |
| /phi1_2 | 1           | (constrained)  |       |       |                      |           |
| /phi1_3 | .9751096    | .1286563       | 7.58  | 0.000 | .7229478             | 1.227271  |
| /phi1_4 | .7209343    | .1220353       | 5.91  | 0.000 | .4817494             | .9601191  |
| /phi1_5 | 0           | (base outcome) |       |       |                      |           |
| /theta1 | -8.293452   | 4.645182       | -1.79 | 0.074 | -17.39784            | .8109369  |
| /theta2 | -6.958451   | 4.629292       | -1.50 | 0.133 | -16.0317             | 2.114795  |
| /theta3 | -5.620232   | 4.953981       | -1.13 | 0.257 | -15.32986            | 4.089392  |
| /theta4 | -3.745624   | 3.809189       | -0.98 | 0.325 | -11.2115             | 3.720249  |
| /theta5 | 0           | (base outcome) |       |       |                      |           |

(repair=Excellent is the base outcome)

The  $\phi$  estimates are now monotonically decreasing and the standard errors of the  $\phi$ 's are small relative to the size of the estimates, so we conclude that, with the exception of outcomes *Poor* and *Fair*, the groups are distinguishable for the one-dimensional model and that the quality assessment can be ordered. ◀

## Higher-dimension models

The stereotype logistic model is not limited to ordered categorical dependent variables; you can use it on nominal data to reduce the dimension of the regressions. Recall that a multinomial model fit to a categorical dependent variable with  $m$  levels will have  $m - 1$  sets of regression coefficients. However, a model with fewer dimensions may fit the data equally well, suggesting that some of the categories are indistinguishable.

### ▶ Example 2

As discussed in [R] **mlogit**, we have data on the type of health insurance available to 616 psychologically depressed subjects in the United States (Tarlov et al. 1989; Wells et al. 1989). Patients may have either an indemnity (fee-for-service) plan or a prepaid plan, such as an HMO, or may be uninsured. Demographic variables include age, gender, race, and site.

First, we fit the saturated, two-dimensional model that is equivalent to a multinomial logistic model. We choose the base outcome to be 1 (indemnity insurance) because that is the default for mlogit.

```
. use https://www.stata-press.com/data/r19/sysdsn1
(Health insurance data)

. slogit insure age male nonwhite i.site, dim(2) base(1)
Iteration 0: Log likelihood = -534.36165
Iteration 1: Log likelihood = -534.36165

Stereotype logistic regression
Log likelihood = -534.36165
Number of obs = 615
Wald chi2(10) = 38.17
Prob > chi2 = 0.0000

(1) [phi1_2]_cons = 1
(2) [phi1_3]_cons = 0
(3) [phi2_2]_cons = 0
(4) [phi2_3]_cons = 1
```

|      | insure   | Coefficient | Std. err.      | z     | P> z  | [95% conf. interval] |           |
|------|----------|-------------|----------------|-------|-------|----------------------|-----------|
| dim1 |          |             |                |       |       |                      |           |
|      | age      | .011745     | .0061946       | 1.90  | 0.058 | -.0003962            | .0238862  |
|      | male     | -.5616934   | .2027465       | -2.77 | 0.006 | -.9590693            | -.1643175 |
|      | nonwhite | -.9747768   | .2363213       | -4.12 | 0.000 | -1.437958            | -.5115955 |
|      | site     |             |                |       |       |                      |           |
|      | 2        | -.1130359   | .2101903       | -0.54 | 0.591 | -.5250013            | .2989296  |
|      | 3        | .5879879    | .2279351       | 2.58  | 0.010 | .1412433             | 1.034733  |
| dim2 |          |             |                |       |       |                      |           |
|      | age      | .0077961    | .0114418       | 0.68  | 0.496 | -.0146294            | .0302217  |
|      | male     | -.4518496   | .3674867       | -1.23 | 0.219 | -1.17211             | .268411   |
|      | nonwhite | -.2170589   | .4256361       | -0.51 | 0.610 | -1.05129             | .6171725  |
|      | site     |             |                |       |       |                      |           |
|      | 2        | 1.211563    | .4705127       | 2.57  | 0.010 | .2893747             | 2.133751  |
|      | 3        | .2078123    | .3662926       | 0.57  | 0.570 | -.510108             | .9257327  |
|      | /phi1_1  | 0           | (base outcome) |       |       |                      |           |
|      | /phi1_2  | 1           | (constrained)  |       |       |                      |           |
|      | /phi1_3  | 0           | (omitted)      |       |       |                      |           |
|      | /phi2_1  | 0           | (base outcome) |       |       |                      |           |
|      | /phi2_2  | 0           | (omitted)      |       |       |                      |           |
|      | /phi2_3  | 1           | (constrained)  |       |       |                      |           |
|      | /theta1  | 0           | (base outcome) |       |       |                      |           |
|      | /theta2  | .2697127    | .3284422       | 0.82  | 0.412 | -.3740222            | .9134476  |
|      | /theta3  | -1.286943   | .5923219       | -2.17 | 0.030 | -2.447872            | -.1260134 |

(insure=Indemnity is the base outcome)

For comparison, we also fit the model by using `mlogit`:

```
. mlogit insure age male nonwhite i.site, nolog
Multinomial logistic regression Number of obs = 615
 LR chi2(10) = 42.99
 Prob > chi2 = 0.0000
Log likelihood = -534.36165 Pseudo R2 = 0.0387
```

| insure         | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|----------------|-------------|-----------|-------|-------|----------------------|-----------|
| Indemnity      |             |           |       |       |                      |           |
| (base outcome) |             |           |       |       |                      |           |
| Prepaid        |             |           |       |       |                      |           |
| age            | -.011745    | .0061946  | -1.90 | 0.058 | -.0238862            | .0003962  |
| male           | .5616934    | .2027465  | 2.77  | 0.006 | .1643175             | .9590693  |
| nonwhite       | .9747768    | .2363213  | 4.12  | 0.000 | .5115955             | 1.437958  |
| site           |             |           |       |       |                      |           |
| 2              | .1130359    | .2101903  | 0.54  | 0.591 | -.2989296            | .5250013  |
| 3              | -.5879879   | .2279351  | -2.58 | 0.010 | -1.034733            | -.1412433 |
| _cons          | .2697127    | .3284422  | 0.82  | 0.412 | -.3740222            | .9134476  |
| Uninsure       |             |           |       |       |                      |           |
| age            | -.0077961   | .0114418  | -0.68 | 0.496 | -.0302217            | .0146294  |
| male           | .4518496    | .3674867  | 1.23  | 0.219 | -.268411             | 1.17211   |
| nonwhite       | .2170589    | .4256361  | 0.51  | 0.610 | -.6171725            | 1.05129   |
| site           |             |           |       |       |                      |           |
| 2              | -1.211563   | .4705127  | -2.57 | 0.010 | -2.133751            | -.2893747 |
| 3              | -.2078123   | .3662926  | -0.57 | 0.570 | -.9257327            | .510108   |
| _cons          | -1.286943   | .5923219  | -2.17 | 0.030 | -2.447872            | -.1260134 |

Apart from having opposite signs, the coefficients from the stereotype logistic model are identical to those from the multinomial logit model. Recall the [definition](#) of  $\eta_k$  given in the *Remarks and examples*, particularly the minus sign in front of the summation. One other difference in the output is that the constant estimates labeled `/theta` in the `slogit` output are the constants labeled `_cons` in the `mlogit` output.

Next, we examine the one-dimensional model.

```
. slogit insure age male nonwhite i.site, dim(1) base(1) nolog
Stereotype logistic regression Number of obs = 615
 Wald chi2(5) = 28.20
Log likelihood = -539.75205 Prob > chi2 = 0.0000
(1) [phi1_2]_cons = 1
```

| insure   | Coefficient | Std. err.      | z     | P> z  | [95% conf. interval] |           |
|----------|-------------|----------------|-------|-------|----------------------|-----------|
| age      | .0108366    | .0061918       | 1.75  | 0.080 | -.0012992            | .0229723  |
| male     | -.5032537   | .2078171       | -2.42 | 0.015 | -.9105678            | -.0959396 |
| nonwhite | -.9480351   | .2340604       | -4.05 | 0.000 | -1.406785            | -.489285  |
| site     |             |                |       |       |                      |           |
| 2        | -.2444316   | .2246366       | -1.09 | 0.277 | -.6847113            | .1958481  |
| 3        | .556665     | .2243799       | 2.48  | 0.013 | .1168886             | .9964415  |
| /phi1_1  | 0           | (base outcome) |       |       |                      |           |
| /phi1_2  | 1           | (constrained)  |       |       |                      |           |
| /phi1_3  | .0383539    | .4079705       | 0.09  | 0.925 | -.7612535            | .8379613  |
| /theta1  | 0           | (base outcome) |       |       |                      |           |
| /theta2  | .187542     | .3303847       | 0.57  | 0.570 | -.4600001            | .835084   |
| /theta3  | -1.860134   | .2158898       | -8.62 | 0.000 | -2.28327             | -1.436997 |

(insure=Indemnity is the base outcome)

We have reduced a two-dimensional multinomial model to one dimension, reducing the number of estimated parameters by four and decreasing the model likelihood by  $\approx 5.4$ .

slogit does not report a model likelihood-ratio test. The test of  $d = 1$  (a one-dimensional model) versus  $d = 0$  (the null model) does not have an asymptotic  $\chi^2$  distribution because the unconstrained  $\phi$  parameters (/phi1\_3 in this example) cannot be identified if  $\beta = \mathbf{0}$ . More generally, this problem precludes testing any hierarchical model of dimension  $d$  versus  $d - 1$ . Of course, the likelihood-ratio test of a full-dimension model versus  $d = 0$  is valid because the full model is just multinomial logistic, and all the  $\phi$  parameters are fixed at 0 or 1.



## □ Technical note

The stereotype model is a special case of the reduced-rank vector generalized linear model discussed by Yee and Hastie (2003). If we define  $\eta_{ik} = \theta_k - \sum_{j=1}^d \phi_{jk} \mathbf{x}_i \beta_j$ , for  $k = 1, \dots, m-1$ , we can write the expression in matrix notation as

$$\boldsymbol{\eta}_i = \boldsymbol{\theta} + \boldsymbol{\Phi} (\mathbf{x}_i \mathbf{B})'$$

where  $\boldsymbol{\Phi}$  is a  $(m-1) \times d$  matrix containing the  $\phi_{jk}$  parameters and  $\mathbf{B}$  is a  $p \times d$  matrix with columns containing the  $\beta_j$  parameters,  $j = 1, \dots, d$ . The factorization  $\boldsymbol{\Phi} \mathbf{B}'$  is not unique because  $\boldsymbol{\Phi} \mathbf{B}' = \boldsymbol{\Phi} \mathbf{M} \mathbf{M}^{-1} \mathbf{B}'$  for any nonsingular  $d \times d$  matrix  $\mathbf{M}$ . To avoid this identifiability problem, we choose  $\mathbf{M} = \boldsymbol{\Phi}_1^{-1}$ , where

$$\boldsymbol{\Phi} = \begin{pmatrix} \boldsymbol{\Phi}_1 \\ \boldsymbol{\Phi}_2 \end{pmatrix}$$

and  $\boldsymbol{\Phi}_1$  is  $d \times d$  of rank  $d$  so that

$$\boldsymbol{\Phi} \mathbf{M} = \begin{pmatrix} \mathbf{I}_d \\ \boldsymbol{\Phi}_2 \boldsymbol{\Phi}_1^{-1} \end{pmatrix}$$

and  $\mathbf{I}_d$  is a  $d \times d$  identity matrix. Thus, the corner constraints used by `slogit` are  $\phi_{jj} \equiv 1$  and  $\phi_{jk} \equiv 0$  for  $j \neq k$  and  $k, j \leq d$ . □

## Stored results

`slogit` stores the following in `e()`:

### Scalars

|                            |                                           |
|----------------------------|-------------------------------------------|
| <code>e(N)</code>          | number of observations                    |
| <code>e(k)</code>          | number of parameters                      |
| <code>e(k_indvars)</code>  | number of independent variables           |
| <code>e(k_out)</code>      | number of outcomes                        |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>  |
| <code>e(k_eq_model)</code> | number of equations in overall model test |
| <code>e(df_m)</code>       | Wald test degrees of freedom              |
| <code>e(df_0)</code>       | null model degrees of freedom             |
| <code>e(k_dim)</code>      | model dimension                           |
| <code>e(i_base)</code>     | base outcome index                        |
| <code>e(ll)</code>         | log likelihood                            |
| <code>e(ll_0)</code>       | null model log likelihood                 |
| <code>e(N_clust)</code>    | number of clusters                        |
| <code>e(chi2)</code>       | $\chi^2$                                  |
| <code>e(p)</code>          | $p$ -value for model test                 |
| <code>e(ic)</code>         | number of iterations                      |
| <code>e(rank)</code>       | rank of <code>e(V)</code>                 |
| <code>e(rc)</code>         | return code                               |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise               |

### Macros

|                          |                            |
|--------------------------|----------------------------|
| <code>e(cmd)</code>      | <code>slogit</code>        |
| <code>e(cmdline)</code>  | command as typed           |
| <code>e(depvar)</code>   | name of dependent variable |
| <code>e(indvars)</code>  | independent variables      |
| <code>e(wtype)</code>    | weight type                |
| <code>e(wexp)</code>     | weight expression          |
| <code>e(title)</code>    | title in estimation output |
| <code>e(clustvar)</code> | name of cluster variable   |

|                   |                                                                          |
|-------------------|--------------------------------------------------------------------------|
| e(out#)           | outcome labels, # = 1, . . . , e(k_out)                                  |
| e(chi2type)       | Wald; type of model $\chi^2$ test                                        |
| e(vce)            | <i>vcetype</i> specified in <code>vce()</code>                           |
| e(vcetype)        | title used to label Std. err.                                            |
| e(opt)            | type of optimization                                                     |
| e(which)          | max or min; whether optimizer is to perform maximization or minimization |
| e(ml_method)      | type of ml method                                                        |
| e(user)           | name of likelihood-evaluator program                                     |
| e(technique)      | maximization technique                                                   |
| e(properties)     | b V                                                                      |
| e(predict)        | program used to implement <code>predict</code>                           |
| e(marginsnotok)   | predictions disallowed by <code>margins</code>                           |
| e(marginsdefault) | default <code>predict()</code> specification for <code>margins</code>    |
| e(footnote)       | program used to implement the footnote display                           |
| e(asbalanced)     | factor variables <code>fvset</code> as <code>asbalanced</code>           |
| e(asobserved)     | factor variables <code>fvset</code> as <code>asobserved</code>           |

Matrices

|                 |                                              |
|-----------------|----------------------------------------------|
| e(b)            | coefficient vector                           |
| e(outcomes)     | outcome values                               |
| e(Cns)          | constraints matrix                           |
| e(iilog)        | iteration log (up to 20 iterations)          |
| e(gradient)     | gradient vector                              |
| e(V)            | variance–covariance matrix of the estimators |
| e(V_modelbased) | model-based variance                         |

Functions

|           |                         |
|-----------|-------------------------|
| e(sample) | marks estimation sample |
|-----------|-------------------------|

In addition to the above, the following is stored in `r()`:

Matrices

|          |                                                                                                                            |
|----------|----------------------------------------------------------------------------------------------------------------------------|
| r(table) | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |
|----------|----------------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

`slogit` obtains the maximum likelihood estimates for the stereotype logistic model by using `ml`; see [R] `ml`. Each set of regression estimates, one set of  $\beta_j$ s for each dimension, constitutes one `ml model` equation. The  $d \times (m - 1)$   $\phi$ s and the  $(m - 1)$   $\theta$ s are `ml ancillary` parameters.

Without loss of generality, let the base outcome level be the  $m$ th level of the dependent variable. Define the row vector  $\phi_k = (\phi_{1k}, \dots, \phi_{dk})$  for  $k = 1, \dots, m - 1$ , and define the  $p \times d$  matrix  $\mathbf{B} = (\beta_1, \dots, \beta_d)$ . For observation  $i$ , the log odds of outcome level  $k$  relative to level  $m$ ,  $k = 1, \dots, m - 1$  is the index

$$\begin{aligned} \ln \left\{ \frac{\Pr(Y_i = k)}{\Pr(Y_i = m)} \right\} &= \eta_{ik} = \theta_k - \phi_k (\mathbf{x}_i \mathbf{B})' \\ &= \theta_k - \phi_k \nu_i' \end{aligned}$$

The row vector  $\nu_i$  can be interpreted as a latent variable reducing the  $p$ -dimensional vector of covariates to a more interpretable  $d < p$  dimension.

The probability of the  $i$ th observation having outcome level  $k$  is then

$$\Pr(Y_i = k) = p_{ik} = \begin{cases} \frac{e^{\eta_{ik}}}{m-1} & , \text{ if } k < m \\ 1 + \sum_{j=1}^{m-1} e^{\eta_{ij}} & \\ \frac{1}{m-1} & , \text{ if } k = m \\ 1 + \sum_{j=1}^{m-1} e^{\eta_{ij}} & \end{cases}$$

from which the log-likelihood function is computed as

$$L = \sum_{i=1}^n w_i \sum_{k=1}^m I_k(y_i) \ln(p_{ik}) \tag{1}$$

Here  $w_i$  is the weight for observation  $i$  and

$$I_k(y_i) = \begin{cases} 1 & , \text{ if observation } y_i \text{ has outcome } k \\ 0 & , \text{ otherwise} \end{cases}$$

Numeric variables are normalized for numerical stability during optimization where a new double-precision variable  $\tilde{x}_j$  is created from variable  $x_j$ ,  $j = 1, \dots, p$ , such that  $\tilde{x}_j = (x_j - \bar{x}_j)/s_j$ . This feature is turned off if you specify `nonnormalize`, or if you use the `from()` option for initial estimates. Normalization is not performed on byte variables, including the indicator variables generated by `[R] xi`. The linear equality constraints for regression parameters, if specified, must be scaled also. Assume that a constraint is applied to the regression parameter associated with variable  $j$  and dimension  $i$ ,  $\beta_{ji}$ , and the corresponding element of the constraint matrix (see `[P] makecons`) is divided by  $s_j$ .

After convergence, the parameter estimates for variable  $j$  and dimension  $i$ — $\tilde{\beta}_{ji}$ , say—are transformed back to their original scale,  $\beta_{ji} = \tilde{\beta}_{ji}/s_j$ . For the intercepts, you compute

$$\theta_k = \tilde{\theta}_k + \sum_{i=1}^d \phi_{ik} \sum_{j=1}^p \frac{\tilde{\beta}_{ji} \bar{x}_j}{s_j}$$

Initial values are computed using estimates obtained using `mlogit` to fit a multinomial logistic model. Let the  $p \times (m - 1)$  matrix  $\tilde{\mathbf{B}}$  contain the multinomial logistic regression parameters less the  $m - 1$  intercepts. Each  $\phi$  is initialized with constant values  $\min(1/2, 1/d)$ , the `initialize(constant)` option (the default), or, with uniform random numbers, the `initialize(random)` option. Constraints are then applied to the starting values so that the structure of the  $(m - 1) \times d$  matrix  $\Phi$  is

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_{m-1} \end{pmatrix} = \begin{pmatrix} \mathbf{I}_d \\ \tilde{\Phi} \end{pmatrix}$$

where  $\mathbf{I}_d$  is a  $d \times d$  identity matrix. Assume that only the corner constraints are used, but any constraints you place on the scale parameters are also applied to the initial scale estimates, so the structure of  $\Phi$  will change accordingly. The  $\phi$  parameters are invariant to the scale of the covariates, so initial estimates in  $[0, 1]$  are reasonable. The constraints guarantee that the rank of  $\Phi$  is at least  $d$ , so the initial estimates for the stereotype regression parameters are obtained from  $\mathbf{B} = \tilde{\mathbf{B}}\Phi(\Phi'\Phi)^{-1}$ .

One other approach for initial estimates is provided: `initialize(svd)`. It starts with the `mlogit` estimates and computes  $\tilde{\mathbf{B}}' = \mathbf{U}\mathbf{D}\mathbf{V}'$ , where  $\mathbf{U}_{m-1 \times p}$  and  $\mathbf{V}_{p \times p}$  are orthonormal matrices and  $\mathbf{D}_{p \times p}$  is a diagonal matrix containing the singular values of  $\tilde{\mathbf{B}}$ . The estimates for  $\Phi$  and  $\mathbf{B}$  are the first  $d$  columns of  $\mathbf{U}$  and  $\mathbf{V}\mathbf{D}$ , respectively (Yee and Hastie 2003).

The score for regression coefficients is

$$\mathbf{u}_i(\beta_j) = \frac{\partial L_{ik}}{\partial \beta_j} = \mathbf{x}_i \left( \sum_{l=1}^{m-1} \phi_{jl} p_{il} - \phi_{jk} \right)$$

the score for the scale parameters is

$$u_i(\phi_{jl}) = \frac{\partial L_{ik}}{\partial \phi_{jl}} = \begin{cases} \mathbf{x}_i \beta_j (p_{ik} - 1), & \text{if } l = k \\ \mathbf{x}_i \beta_j p_{il}, & \text{if } l \neq k \end{cases}$$

for  $l = 1, \dots, m - 1$ ; and the score for the intercepts is

$$u_i(\theta_l) = \frac{\partial L_{ik}}{\partial \theta_l} = \begin{cases} 1 - p_{ik}, & \text{if } l = k \\ -p_{il}, & \text{if } l \neq k \end{cases}$$

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] [\\_robust](#), particularly *Maximum likelihood estimators* and *Methods and formulas*.

`slomit` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] [Variance estimation](#).

## References

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## Also see

- [R] **slogit postestimation** — Postestimation tools for slogit
- [R] **logistic** — Logistic regression, reporting odds ratios
- [R] **mlogit** — Multinomial (polytomous) logistic regression
- [R] **ologit** — Ordered logistic regression
- [R] **oprobit** — Ordered probit regression
- [R] **roc** — Receiver operating characteristic (ROC) analysis
- [SVY] **svy estimation** — Estimation commands for survey data
- [U] **20 Estimation and postestimation commands**

|                         |                                      |                          |
|-------------------------|--------------------------------------|--------------------------|
| Postestimation commands | <a href="#">predict</a>              | <a href="#">margins</a>  |
| Remarks and examples    | <a href="#">Methods and formulas</a> | <a href="#">Also see</a> |

## Postestimation commands

The following postestimation commands are available after `slogit`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | probabilities, linear predictions and their SEs, etc.                                                                                   |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`hausman` and `lrtest` are not appropriate with `svy` estimation results.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as probabilities, indexes for the  $k$ th outcome, and standard errors.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] { stub* | newvar | newvarlist } [if] [in] [, statistic outcome(outcome)]
```

```
predict [type] stub* [if] [in], scores
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

Main

|                   |                                                                           |
|-------------------|---------------------------------------------------------------------------|
| <code>pr</code>   | probability of one of or all the dependent variable outcomes; the default |
| <code>xb</code>   | index for the $k$ th outcome                                              |
| <code>stdp</code> | standard error of the index for the $k$ th outcome                        |

You specify one or  $k$  new variables with `pr`, where  $k$  is the number of outcomes. If you specify one new variable and you do not specify `outcome()`, then `outcome(#1)` is assumed.

You specify one new variable with `xb` and `stdp`. If you do not specify `outcome()`, then `outcome(#1)` is assumed.

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

## Options for predict

Main

`pr`, the default, computes the predicted probabilities for all outcomes or for a specific outcome. To compute probabilities for all outcomes, you specify  $k$  new variables, where  $k$  is the number of categories of the dependent variable. Alternatively, you can specify `stub*`; in which case, `pr` will store predicted probabilities in variables `stub1`, `stub2`, ..., `stubk`. To compute the probability for a specific outcome, you specify one new variable and, optionally, the outcome value in option `outcome()`; if you omit `outcome()`, the first outcome value, `outcome(#1)`, is assumed.

Say that you fit a model by typing `estimation_cmd y x1 x2`, and `y` takes on four values. Then, you could type `predict p1 p2 p3 p4` to obtain all four predicted probabilities; alternatively, you could type `predict p*` to generate the four predicted probabilities. To compute specific probabilities one at a time, you can type `predict p1, outcome(#1)` (or simply `predict p1`), `predict p2, outcome(#2)`, and so on. See option `outcome()` for other ways to refer to outcome values.

`xb` calculates the index,  $\theta_k - \sum_{j=1}^d \phi_{jk} \mathbf{x}_i \beta_j$ , for outcome level  $k \neq e(i\_base)$  and dimension  $d = e(k\_dim)$ . It returns a vector of zeros if  $k = e(i\_base)$ . A synonym for `xb` is `index`. If `outcome()` is not specified, `outcome(#1)` is assumed.

`stdp` calculates the standard error of the index. A synonym for `stdp` is `seindex`. If `outcome()` is not specified, `outcome(#1)` is assumed.

`outcome(outcome)` specifies for which outcome the predicted probabilities are to be calculated. `outcome()` should contain either one value of the dependent variable or one of #1, #2, ..., with #1 meaning the first category of the dependent variable, #2 meaning the second category, etc. `outcome()` is not allowed with `scores`.

`scores` calculates the equation-level score variables. For models with  $d$  dimensions and  $m$  levels,  $d + (d + 1)(m - 1)$  new variables are created. Assume  $j = 1, \dots, d$  and  $k = 1, \dots, m$  in the following.

The first  $d$  new variables will contain  $\partial \ln L / \partial (\mathbf{x} \boldsymbol{\beta}_j)$ .

The next  $d(m - 1)$  new variables will contain  $\partial \ln L / \partial \phi_{jk}$ .

The last  $m - 1$  new variables will contain  $\partial \ln L / \partial \theta_k$ .

## margins

### Description for margins

`margins` estimates margins of response for probabilities and indexes for the  $k$ th outcome.

### Menu for margins

Statistics > Postestimation

### Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i> | Description                                                  |
|------------------|--------------------------------------------------------------|
| default          | probabilities for each outcome                               |
| pr               | probability of one of or all the dependent variable outcomes |
| xb               | index for the $k$ th outcome                                 |
| stdp             | not allowed with <code>margins</code>                        |

`pr` and `xb` default to the first outcome.

Statistics not allowed with `margins` are functions of stochastic quantities other than `e(b)`.

For the full syntax, see [\[R\] margins](#).

## Remarks and examples

Once you have fit a stereotype logistic model, you can obtain the predicted probabilities by using the `predict` command for both the estimation sample and other samples; see [U] 20 Estimation and postestimation commands and [R] `predict`.

`predict` without arguments (or with the `pr` option) calculates the predicted probability of each outcome of the dependent variable. You must therefore give a new variable name for each of the outcomes. To compute the estimated probability of one outcome, you use the `outcome(outcome)` option where `outcome` is the level encoding the outcome. If the dependent variable's levels are labeled, the outcomes can also be identified by the label values (see [D] `label`).

The `xb` option in conjunction with `outcome(outcome)` specifies that the index be computed for the outcome encoded by level `outcome`. Its approximate standard error is computed if the `stdp` option is specified. Only one of the `pr`, `xb`, or `stdp` options can be specified with a call to `predict`.

### ▷ Example 1

In example 2 of [R] `slogit`, we fit the one-dimensional stereotype model, where the `depvar` is `insure` with levels  $k = 1$  for outcome *Indemnity*,  $k = 2$  for *Prepaid*, and  $k = 3$  for *Uninsure*. The base outcome for the model is *Indemnity*, so for  $k \neq 1$  the vector of indices for the  $k$ th level is

$$\eta_k = \theta_k - \phi_k (\beta_1 \text{age} + \beta_2 \text{male} + \beta_3 \text{nonwhite} + \beta_4 2.\text{site} + \beta_5 3.\text{site})$$

We estimate the group probabilities by calling `predict` after `slogit`.

```
. use https://www.stata-press.com/data/r19/sysdsn1
(Health insurance data)
. slogit insure age male nonwhite i.site, dim(1) base(1) nolog
(output omitted)
. predict pIndemnity pPrepaid pUninsure, p
. list pIndemnity pPrepaid pUninsure insure in 1/10
```

|     | pIndem-y | pPrepaid | pUnins-e | insure    |
|-----|----------|----------|----------|-----------|
| 1.  | .5419344 | .3754875 | .0825782 | Indemnity |
| 2.  | .4359638 | .496328  | .0677081 | Prepaid   |
| 3.  | .5111583 | .4105107 | .0783309 | Indemnity |
| 4.  | .3941132 | .5442234 | .0616633 | Prepaid   |
| 5.  | .4655651 | .4625064 | .0719285 | .         |
| 6.  | .4401779 | .4915102 | .0683118 | Prepaid   |
| 7.  | .4632122 | .4651931 | .0715948 | Prepaid   |
| 8.  | .3772302 | .5635696 | .0592002 | .         |
| 9.  | .4867758 | .4383018 | .0749225 | Uninsure  |
| 10. | .5823668 | .3295802 | .0880531 | Prepaid   |

Observations 5 and 8 are not used to fit the model because `insure` is missing at these points, but `predict` estimates the probabilities for these observations because none of the independent variables is missing. You can use `if e(sample)` in the call to `predict` to use only those observations that are used to fit the model.

## Methods and formulas

### predict

Let level  $b$  be the base outcome that is used to fit the stereotype logistic regression model of dimension  $d$ . The index for observation  $i$  and level  $k \neq b$  is  $\eta_{ik} = \theta_k - \sum_{j=1}^d \phi_{jk} \mathbf{x}_i \boldsymbol{\beta}_j$ . This is the log odds of outcome encoded as level  $k$  relative to that of  $b$  so that we define  $\eta_{ib} \equiv 0$ . The outcome probabilities for this model are defined as  $\Pr(Y_i = k) = e^{\eta_{ik}} / \sum_{j=1}^m e^{\eta_{ij}}$ . Unlike in `mlogit`, `ologit`, and `oprobit`, the index is no longer a linear function of the parameters. The standard error of index  $\eta_{ik}$  is thus computed using the delta method (see also [\[R\] predictnl](#)).

The equation-level score for regression coefficients is

$$\frac{\partial \ln L_{ik}}{\partial \mathbf{x}_i \boldsymbol{\beta}_j} = \left( \sum_{l=1}^{m-1} \phi_{jl} p_{il} - \phi_{jk} \right)$$

the equation-level score for the scale parameters is

$$\frac{\partial \ln L_{ik}}{\partial \phi_{jl}} = \begin{cases} \mathbf{x}_i \boldsymbol{\beta}_j (p_{ik} - 1), & \text{if } l = k \\ \mathbf{x}_i \boldsymbol{\beta}_j p_{il}, & \text{if } l \neq k \end{cases}$$

for  $l = 1, \dots, m - 1$ ; and the equation-level score for the intercepts is

$$\frac{\partial \ln L_{ik}}{\partial \theta_l} = \begin{cases} 1 - p_{ik}, & \text{if } l = k \\ -p_{il}, & \text{if } l \neq k \end{cases}$$

### Also see

[\[R\] slomit](#) — Stereotype logistic regression

[\[U\] 20 Estimation and postestimation commands](#)

[Description](#)  
[Option](#)  
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[Quick start](#)  
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[Menu](#)  
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[Syntax](#)  
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## Description

`smooth` applies the specified resistant, nonlinear smoother to *varname* and stores the smoothed series in *newvar*.

## Quick start

Running median smoother of span 3 for *v*, placing smoothed values in the new variable *newv*  
`smooth 3 v, generate(newv)`

Same as above, but with a compound smoother of running medians, first of span 3, then span 5  
`smooth 35 v, generate(newv)`

Same as above, but repeating the span-5 running median until convergence  
`smooth 35R v, generate(newv)`

Same as above, splitting any repeated values after the span-3 running median  
`smooth 3S5R v, generate(newv)`

Same as above, but apply the compound smoother to the resulting rough  
`smooth 3S5R, twice v, generate(newv)`

## Menu

Statistics > Nonparametric analysis > Robust nonlinear smoother

## Syntax

```
smooth smoother[, twice] varname [if] [in], generate(newvar)
```

where *smoother* is specified as  $Sm[Sm[\dots]]$  and *Sm* is one of

```
{ 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 } [R]
3[R]S[S|R][S|R]...
E
H
```

Letters may be specified in lowercase if preferred. Examples of *smoother*[, twice] include

```
3RSSH 3RSSH,twice 4253H 4253H,twice 43RSR2H,twice
3rssh 3rssh,twice 4253h 4253h,twice 43rsr2h,twice
```

## Option

`generate(newvar)` is required; it specifies the name of the new variable that will contain the smoothed values.

## Remarks and examples

Smoothing is an exploratory data-analysis technique for making the general shape of a series apparent. In this approach (Tukey 1977), the observed data series is assumed to be the sum of an underlying process that evolves smoothly (the smooth) and of an unsystematic noise component (the rough); that is,

$$\text{data} = \text{smooth} + \text{rough}$$

Smoothed values  $z_t$  are obtained by taking medians (or some other location estimate) of each point in the original data  $y_t$  and a few of the points around it. The number of points used is called the span of the smoother. Thus, a span-3 smoother produces  $z_t$  by taking the median of  $y_{t-1}$ ,  $y_t$ , and  $y_{t+1}$ . `smooth` provides running median smoothers of spans 1 to 9—indicated by the digit that specifies their span. Median smoothers are resistant to isolated outliers, so they provide robustness to spikes in the data. Because the median is also a nonlinear operator, such smoothers are known as robust (or resistant) nonlinear smoothers.

`smooth` also provides the Hanning linear, nonrobust smoother, indicated by the letter H. Hanning is a span-3 smoother with binomial weights. Repeated applications of H—HH, HHH, etc.—provide binomial smoothers of span 5, 7, etc. See Cox (1997, 2004) for a graphical application of this fact.

Because one smoother usually cannot adequately separate the smooth from the rough, compound smoothers—multiple smoothers applied in sequence—are used. The smoother 35H, for instance, then smooths the data with a span-3 median smoother, smooths the result with a span-5 median smoother, and finally smooths that result with the Hanning smoother. `smooth` allows you to specify any number of smoothers in any sequence.

Three refinements can be combined with the running median and Hanning smoothers. First, the endpoints of a smooth can be given special treatment. This is specified by the E operator. Second, smoothing by 3, the span-3 running median, tends to produce flat-topped hills and valleys. The splitting operator, S, “splits” these repeated values, applies the endpoint operator to them, and then “rejoins” the series. Finally, it is sometimes useful to repeat an odd-span median smoother or the splitting operator until the smooth no longer changes. Following a digit or an S with an R specifies this type of repetition.

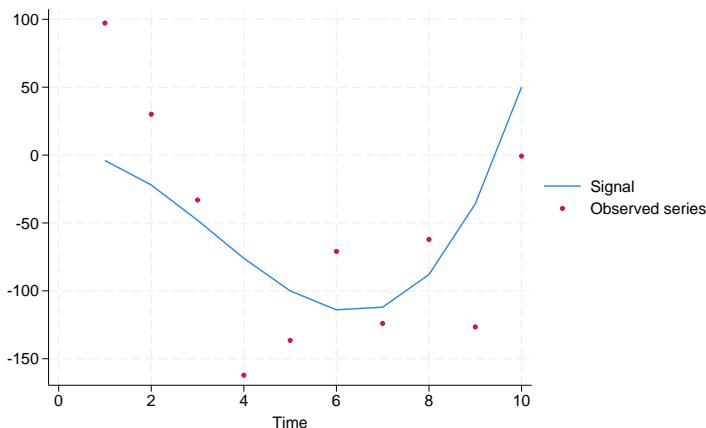
Even the best smoother may fail to separate the smooth from the rough adequately. To guard against losing any systematic components of the data series, after smoothing, the smoother can be reapplied to the resulting rough, and any recovered signal can be added back to the original smooth. The twice operator specifies this procedure. More generally, an arbitrary smoother can be applied to the rough (using a second `smooth` command), and the recovered signal can be added back to the smooth. This more general procedure is called reroughing (Tukey 1977).

The details of each of the smoothers and operators are explained in *Methods and formulas* below.

### ▷ Example 1

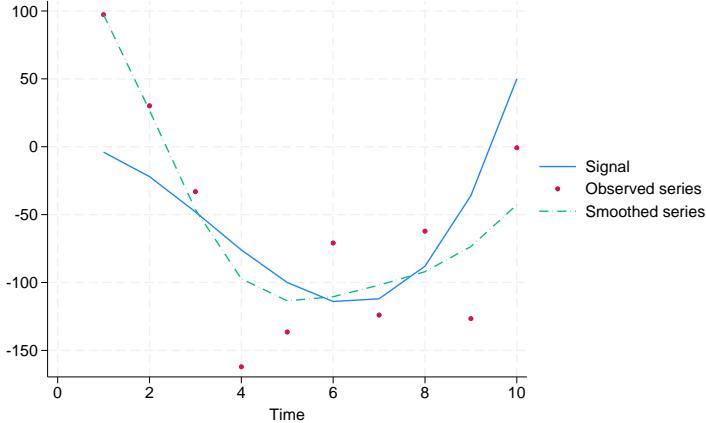
`smooth` is designed to recover the general features of a series that has been contaminated with noise. To demonstrate this, we construct a series, add noise to it, and then `smooth` the noisy version to recover an estimate of the original data. First, we construct and display the data:

```
. drop _all
. set obs 10
. set seed 123456789
. generate time = _n
. label variable time "Time"
. generate x = _n^3 - 10*_n^2 + 5*_n
. label variable x "Signal"
. generate z = x + 50*rnormal()
. label variable z "Observed series"
. scatter x z time, c(l .) m(i o) ytitle("")
```



Now we smooth the noisy series,  $z$ , assumed to be the only data we would observe:

```
. smooth 4253eh,twice z, gen(sz)
. label variable sz "Smoothed series"
. scatter x z time, c(1 . 1) m(i o i) ytitle("") || scatter sz time,
> c(1 . 1) m(i o i) ytitle("") clpattern(dash_dot)
```

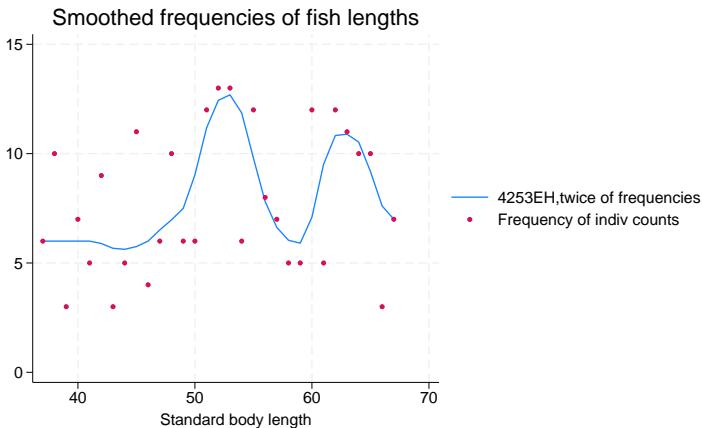


◀

## ► Example 2

Salgado-Ugarte and Curts-García (1993) provide data on the frequencies of observed fish lengths. In this example, the series to be smoothed—the frequencies—is ordered by fish length rather than by time.

```
. use https://www.stata-press.com/data/r19/fishdata, clear
. smooth 4253eh,twice freq, gen(sfreq)
. scatter sfreq freq length, c(1 .) m(i o)
> title("Smoothed frequencies of fish lengths") ytitle("") xlabel(#4)
```



◀

## □ Technical note

`smooth` allows missing values at the beginning and end of the series, but missing values in the middle are not allowed. Leading and trailing missing values are ignored. If you wish to ignore missing values in the middle of the series, you must drop the missing observations before using `smooth`. Doing so, of course, would violate `smooth`'s assumption that observations are equally spaced—each observation represents a year, a quarter, or a month (or a 1-year birth-rate category). In practice, `smooth` produces good results as long as the spaces between adjacent observations do not vary too much.

Smoothing is usually applied to time series, but any variable with a natural order can be smoothed. For example, a smoother might be applied to the birth rate recorded by the age of the mothers (birthrate for 17-year-olds, birthrate for 18-year-olds, and so on).

□

## Methods and formulas

Methods and formulas are presented under the following headings:

*Running median smoothers of odd span*  
*Running median smoothers of even span*  
*Repeat operator*  
*Endpoint rule*  
*Splitting operator*  
*Hanning smoother*  
*Twicing*

## Running median smoothers of odd span

The smoother 3 defines

$$z_t = \text{median}(y_{t-1}, y_t, y_{t+1})$$

The smoother 5 defines

$$z_t = \text{median}(y_{t-2}, y_{t-1}, y_t, y_{t+1}, y_{t+2})$$

and so on. The smoother 1 defines  $z_t = \text{median}(y_t)$ , so it does nothing.

Endpoints are handled by using smoothers of shorter, odd span. Thus, for 3,

$$\begin{aligned} z_1 &= y_1 \\ z_2 &= \text{median}(y_1, y_2, y_3) \\ &\vdots \\ z_{N-1} &= \text{median}(y_{N-2}, y_{N-1}, y_N) \\ Z_N &= y_N \end{aligned}$$

For 5,

$$\begin{aligned}
 z_1 &= y_1 \\
 z_2 &= \text{median}(y_1, y_2, y_3) \\
 z_3 &= \text{median}(y_1, y_2, y_3, y_4, y_5) \\
 z_4 &= \text{median}(y_2, y_3, y_4, y_5, y_6) \\
 &\vdots \\
 z_{N-2} &= \text{median}(y_{N-4}, y_{N-3}, y_{N-2}, y_{N-1}, y_N) \\
 z_{N-1} &= \text{median}(y_{N-2}, y_{N-1}, y_N) \\
 Z_N &= y_N
 \end{aligned}$$

and so on.

### Running median smoothers of even span

Define the  $\text{median}()$  function as returning the linearly interpolated value when given an even number of arguments. Thus, the smoother 2 defines

$$z_{t+0.5} = (y_t + y_{t+1})/2$$

The smoother 4 defines  $z_{t+0.5}$  as the linearly interpolated median of  $(y_{t-1}, y_t, y_{t+1}, y_{t+2})$ , and so on. Endpoints are always handled using smoothers of shorter, even span. Thus, for 4,

$$\begin{aligned}
 z_{0.5} &= y_1 \\
 z_{1.5} &= \text{median}(y_1, y_2) = (y_1 + y_2)/2 \\
 z_{2.5} &= \text{median}(y_1, y_2, y_3, y_4) \\
 &\vdots \\
 z_{N-2.5} &= \text{median}(y_{N-4}, y_{N-3}, y_{N-2}, y_N) \\
 z_{N-1.5} &= \text{median}(y_{N-2}, y_{N-1}) \\
 z_{N-0.5} &= \text{median}(y_{N-1}, y_N) \\
 z_{N+0.5} &= y_N
 \end{aligned}$$

As defined above, an even-span smoother increases the length of the series by 1 observation. However, the series can be recentered on the original observation numbers, and the “extra” observation can be eliminated by smoothing the series again with another even-span smoother. For instance, the smooth of 4 illustrated above could be followed by a smooth of 2 to obtain

$$\begin{aligned} z_1^* &= (z_{0.5} + z_{1.5})/2 \\ z_2^* &= (z_{1.5} + z_{2.5})/2 \\ z_3^* &= (z_{2.5} + z_{3.5})/2 \\ &\vdots \\ z_{N-2}^* &= (z_{N-2.5} + z_{N-1.5})/2 \\ z_{N-1}^* &= (z_{N-1.5} + z_{N-0.5})/2 \\ z_N^* &= (z_{N-0.5} + z_{N+0.5})/2 \end{aligned}$$

smooth keeps track of the number of even smoothers applied to the data and expands and shrinks the length of the series accordingly. To ensure that the final smooth has the same number of observations as *varname*, smooth requires you to specify an even number of even-span smoothers. However, the pairs of even-span smoothers need not be contiguous; for instance, 4253 and 4523 are both allowed.

## Repeat operator

R indicates that a smoother is to be repeated until convergence, that is, until repeated applications of the smoother produce the same series. Thus, 3 applies the smoother of running medians of span 3. 33 applies the smoother twice. 3R produces the result of repeating 3 an infinite number of times. R should be used only with odd-span smoothers because even-span smoothers are not guaranteed to converge.

The smoother 453R2 applies a span-4 smoother, followed by a span-5 smoother, followed by repeated applications of a span-3 smoother, followed by a span-2 smoother.

## Endpoint rule

The endpoint rule E modifies the values  $z_1$  and  $z_N$  according to the following formulas:

$$\begin{aligned} z_1 &= \text{median}(3z_2 - 2z_3, z_1, z_2) \\ z_N &= \text{median}(3z_{N-2} - 2z_{N-1}, z_N, z_{N-1}) \end{aligned}$$

When the endpoint rule is not applied, endpoints are typically “copied in”; that is,  $z_1 = y_1$  and  $z_N = y_N$ .

## Splitting operator

The smoothers `3` and `3R` can produce flat-topped hills and valleys. The split operator attempts to eliminate such hills and valleys by splitting the sequence, applying the endpoint rule `E`, rejoining the series, and then resmoothing by `3R`.

The `S` operator may be applied only after `3`, `3R`, or `S`.

We recommend that the `S` operator be repeated once (`SS`) or until no further changes take place (`SR`).

## Hanning smoother

`H` is the Hanning linear smoother:

$$z_t = (y_{t-1} + 2y_t + y_{t+1})/4$$

Endpoints are copied in:  $z_1 = y_1$  and  $z_N = y_N$ . `H` should be applied only after all nonlinear smoothers.

## Twicing

A smoother divides the data into a smooth and a rough:

$$\text{data} = \text{smooth} + \text{rough}$$

If the smoothing is successful, the rough should exhibit no pattern. Twicing refers to applying the smoother to the observed, calculating the rough, and then applying the smoother to the rough. The resulting “smoothed rough” is then added back to the smooth from the first step.

## Acknowledgments

`smooth` was originally written by William Gould (1992)—at which time it was named `nlsm`—and was inspired by Salgado-Ugarte and Curts-García (1992). Salgado-Ugarte and Curts-García (1993) subsequently reported anomalies in `nlsm`’s treatment of even-span median smoothers. `smooth` corrects these problems and incorporates other improvements but otherwise is essentially the same as originally published.

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## Also see

[R] **lowess** — Lowess smoothing

[R] **lpoly** — Kernel-weighted local polynomial smoothing

[TS] **tssmooth** — Smooth and forecast univariate time-series data

Description

Options for `spearman`

Methods and formulas

Quick start

Options for `ktau`

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Stored results

Also see

## Description

`spearman` displays Spearman's rank correlation coefficients for all pairs of variables in *varlist* or, if *varlist* is not specified, for all the variables in the dataset. When there are two variables, an exact *p*-value can be calculated optionally using permutations.

`ktau` displays Kendall's rank correlation coefficients between the variables in *varlist* or, if *varlist* is not specified, for all the variables in the dataset. `ktau` is intended for use on small- and moderate-sized datasets; it requires considerable computation time for larger datasets.

## Quick start

Spearman's rank correlation coefficient with approximate *p*-value for *v1* and *v2*

```
spearman v1 v2
```

Same as above, but report an exact *p*-value calculated using Monte Carlo permutations

```
spearman v1 v2, exact
```

Same as above, but perform 100,000 Monte Carlo permutations rather than the default of 10,000, and set the random-number seed for reproducibility

```
spearman v1 v2, exact(montecarlo, reps(100000) rseed(1234))
```

Display Spearman's rank correlation coefficients in a matrix for all pairs of *v1*, *v2*, and *v3*

```
spearman v1 v2 v3
```

Display *p*-values as well as correlation coefficients

```
spearman v1 v2 v3, stats(rho p)
```

Same as above, but perform Bonferroni's adjustment to *p*-values

```
spearman v1 v2 v3, stats(rho p) bonferroni
```

Kendall's rank correlation coefficients, scores, and standard errors of the scores for pairs of *v1*, *v2*, and *v3*

```
ktau v1 v2 v3, stats(taua taub score se)
```

Same as above, but use pairwise instead of casewise deletion

```
ktau v1 v2 v3, stats(taua taub score se) pw
```

## Menu

### **spearman**

Statistics > Nonparametric analysis > Tests of hypotheses > Spearman's rank correlation

### **ktau**

Statistics > Nonparametric analysis > Tests of hypotheses > Kendall's rank correlation

## Syntax

Spearman's rank correlation coefficients

```
spearman [varlist] [if] [in] [, spearman_options]
```

Kendall's rank correlation coefficients

```
ktau [varlist] [if] [in] [, ktau_options]
```

| <i>spearman_options</i>                    | Description                                                                           |
|--------------------------------------------|---------------------------------------------------------------------------------------|
| Main                                       |                                                                                       |
| <code>stats(<i>spearman_list</i>)</code>   | list of statistics; select up to three statistics; default is <code>stats(rho)</code> |
| <code>print(#)</code>                      | <i>p</i> -value cutoff for displaying coefficients                                    |
| <code>star(#)</code>                       | <i>p</i> -value cutoff for displaying a star                                          |
| <code>bonferroni</code>                    | report Bonferroni-adjusted <i>p</i> -values                                           |
| <code>sidak</code>                         | report Šidák-adjusted <i>p</i> -values                                                |
| <code>pw</code>                            | calculate each pairwise correlation coefficient using all available data              |
| <code>matrix</code>                        | display output in matrix form                                                         |
| <code>exact[ (<i>exact_specs</i>) ]</code> | report an exact <i>p</i> -value (available only when <i>varlist</i> is two variables) |

| <i>ktau_options</i>                  | Description                                                                          |
|--------------------------------------|--------------------------------------------------------------------------------------|
| Main                                 |                                                                                      |
| <code>stats(<i>ktau_list</i>)</code> | list of statistics; select up to six statistics; default is <code>stats(taua)</code> |
| <code>print(#)</code>                | <i>p</i> -value cutoff for displaying coefficients                                   |
| <code>star(#)</code>                 | <i>p</i> -value cutoff for displaying a star                                         |
| <code>bonferroni</code>              | report Bonferroni-adjusted <i>p</i> -values                                          |
| <code>sidak</code>                   | report Šidák-adjusted <i>p</i> -values                                               |
| <code>pw</code>                      | calculate each pairwise correlation coefficient using all available data             |
| <code>matrix</code>                  | display output in matrix form                                                        |

by and collect are allowed with `spearman` and `ktau`; see [U] 11.1.10 Prefix commands.

where the elements of *spearman\_list* may be

|     |                         |
|-----|-------------------------|
| rho | correlation coefficient |
| obs | number of observations  |
| p   | <i>p</i> -value         |

and the elements of *ktau\_list* may be

|       |                                  |
|-------|----------------------------------|
| taua  | correlation coefficient $\tau_a$ |
| taub  | correlation coefficient $\tau_b$ |
| score | score                            |
| se    | standard error of score          |
| obs   | number of observations           |
| p     | <i>p</i> -value                  |

## Options for spearman

Main

`stats(spearman_list)` specifies the statistics to be displayed in the matrix of output. `stats(rho)` is the default. Up to three statistics may be specified; `stats(rho obs p)` would display the correlation coefficient, number of observations, and  $p$ -value. If *varlist* contains only two variables, all statistics are shown in tabular form, and `stats()`, `print()`, and `star()` have no effect unless the `matrix` option is specified.

`print(#)` specifies the  $p$ -value cutoff for correlation coefficients to be printed. Correlation coefficients with larger  $p$ -values are left blank in the matrix. Typing `spearman, print(.10)` would display only those correlation coefficients that have  $p$ -values less than or equal to 0.10.

`star(#)` specifies the  $p$ -value cutoff for correlation coefficients to be marked with a star. Typing `spearman, star(.05)` would star all correlation coefficients that have  $p$ -values less than or equal to 0.05.

`bonferroni` makes the Bonferroni adjustment to  $p$ -values. This adjustment affects displayed  $p$ -values and the `print()` and `star()` options. Thus, `spearman, print(.05) bonferroni` prints coefficients with Bonferroni-adjusted  $p$ -values of 0.05 or less.

`sidak` makes the Šidák adjustment to  $p$ -values. This adjustment affects displayed  $p$ -values and the `print()` and `star()` options. Thus, `spearman, print(.05) sidak` prints coefficients with Šidák-adjusted  $p$ -values of 0.05 or less.

`pw` specifies that correlations be calculated using pairwise deletion of observations with missing values. By default, `spearman` uses casewise deletion, where observations are ignored if any of the variables in *varlist* are missing.

`matrix` forces `spearman` to display the statistics as a matrix, even if *varlist* contains only two variables. `matrix` is implied if more than two variables are specified.

`exact` and `exact(exact_specs)` specify that an exact  $p$ -value be reported. This option is available only when *varlist* contains only two variables.

`exact` specifies that an exact  $p$ -value from a Monte Carlo permutation test be reported. `exact` is a synonym for `exact(montecarlo)`.

`exact(montecarlo[ , options] | enumerate[ , options])` specifies that an exact  $p$ -value be reported in addition to the approximate  $p$ -value. Specifying `exact(montecarlo)` does a Monte Carlo permutation test. Specifying `exact(enumerate)` does an enumeration of all possible permutations. Because the number of all possible permutations is typically extremely large, enumeration is feasible only for very small datasets. The number of permutations will be displayed, and you can click on *Break* to stop the computation. The exact  $p$ -value is computed by `permute`.

`exact(montecarlo[ , options])` allows *options* `show`, `reps(#)`, `rseed(#)`, `saving(filename[ , sav_options])`, `level(#)`, `dots(#)`, `nodots`, and `eps(#)`. The `show` option specifies that the table produced by `permute` also be displayed. By default, 10,000 Monte Carlo permutations are done. That is, the default is the same as specifying `exact(montecarlo, reps(10000))`. The default for `dots()` is `dots(100)` when `reps()` is  $\geq 10,000$ ; otherwise, it is `dots(1)`. See *Options* in [R] `permute`.

`exact(enumerate[ , options])` allows *options* `show`, `saving(filename[ , sav_options])`, `dots(#)`, `nodots`, and `eps(#)`. The `show` option specifies that the table produced by `permute` also be displayed. The default for `dots()` is `dots(100)`. See *Options* in [R] `permute`.

## Options for ktau

Main

`stats(ktau_list)` specifies the statistics to be displayed in the matrix of output. `stats(taua)` is the default. Up to six statistics may be specified; `stats(taua taub score se obs p)` would display the correlation coefficients  $\tau_a$  and  $\tau_b$ , score, standard error of score, number of observations, and  $p$ -value. If *varlist* contains only two variables, all statistics are shown in tabular form and `stats()`, `print()`, and `star()` have no effect unless the `matrix` option is specified.

`print(#)` specifies the  $p$ -value cutoff for correlation coefficients to be printed. Correlation coefficients with larger  $p$ -values are left blank in the matrix. Typing `ktau, print(.10)` would display only those correlation coefficients that have  $p$ -values less than or equal to 0.10.

`star(#)` specifies the  $p$ -value cutoff for correlation coefficients to be marked with a star. Typing `ktau, star(.05)` would star all correlation coefficients that have  $p$ -values less than or equal to 0.05.

`bonferroni` makes the Bonferroni adjustment to  $p$ -values. This adjustment affects displayed  $p$ -values and the `print()` and `star()` options. Thus, `ktau, print(.05) bonferroni` prints coefficients with Bonferroni-adjusted  $p$ -values of 0.05 or less.

`sidak` makes the Šidák adjustment to  $p$ -values. This adjustment affects displayed  $p$ -values and the `print()` and `star()` options. Thus, `ktau, print(.05) sidak` prints coefficients with Šidák-adjusted  $p$ -values of 0.05 or less.

`pw` specifies that correlations be calculated using pairwise deletion of observations with missing values. By default, `ktau` uses casewise deletion, where observations are ignored if any of the variables in *varlist* are missing.

`matrix` forces `ktau` to display the statistics as a matrix, even if *varlist* contains only two variables. `matrix` is implied if more than two variables are specified.

## Remarks and examples

### ▷ Example 1

We wish to calculate the correlation coefficients among marriage rate (`mrgrate`), divorce rate (`divorce_rate`), and median age (`medage`) in state data. We can calculate the standard Pearson correlation coefficients and  $p$ -values by typing

```
. use https://www.stata-press.com/data/r19/states2
(State data)
```

```
. pwcorr mrgrate divorce_rate medage, sig
```

|              | mrgrate          | divorc-e          | medage |
|--------------|------------------|-------------------|--------|
| mrgrate      | 1.0000           |                   |        |
| divorce_rate | 0.7895<br>0.0000 | 1.0000            |        |
| medage       | 0.0011<br>0.9941 | -0.1526<br>0.2900 | 1.0000 |

We can calculate Spearman's rank correlation coefficients by typing

```
. spearman mrgrate divorce_rate medage, stats(rho p)
```

```
Number of observations = 50
```

| Key            |
|----------------|
| <i>rho</i>     |
| <i>p-value</i> |

|              | mrgrate           | divorc-e          | medage |
|--------------|-------------------|-------------------|--------|
| mrgrate      | 1.0000            | .                 | .      |
| divorce_rate | 0.6933<br>0.0000  | 1.0000            | .      |
| medage       | -0.4869<br>0.0004 | -0.2455<br>0.0857 | 1.0000 |

The large difference in the results is caused by one observation. Nevada's marriage rate is almost 10 times higher than the state with the next-highest marriage rate. An important feature of the Spearman rank correlation is its reduced sensitivity to extreme values compared with the Pearson correlation.

We can calculate Kendall's rank correlations by typing

```
. ktau mrgrate divorce_rate medage, stats(taua taub p)
Number of observations = 50
```

| Key     |
|---------|
| tau_a   |
| tau_b   |
| p-value |

|              | mrgrate                      | divorc-e                     | medage                |
|--------------|------------------------------|------------------------------|-----------------------|
| mrgrate      | 0.9829<br>1.0000<br>.        |                              |                       |
| divorce_rate | 0.5110<br>0.5206<br>0.0000   | 0.9804<br>1.0000<br>.        |                       |
| medage       | -0.3486<br>-0.3544<br>0.0004 | -0.1698<br>-0.1728<br>0.0828 | 0.9845<br>1.0000<br>. |

There are tied values for variables `mrgrate`, `divorce_rate`, and `medage`, so average ranks are used for the tied values. As a result,  $\tau_a < 1$  on the diagonal (see [Methods and formulas](#) for the definition of  $\tau_a$ ). ◀

According to [Conover \(1999, 323\)](#), “Spearman's  $\rho$  tends to be larger than Kendall's  $\tau$  in absolute value. However, as a test of significance, there is no strong reason to prefer one over the other because both will produce nearly identical results in most cases.”

[Newson \(2000a, 2000b, 2000c, 2001, 2003, 2005, 2006\)](#) introduces confidence intervals for Kendall's  $\tau_a$ . The community-contributed `somersd` command provides these confidence intervals along with additional rank statistics such as Somers'  $D$  and Harrell's  $C$  and their corresponding confidence intervals.

See [Seed \(2001\)](#) for confidence intervals for Spearman's rank correlation.

## ▷ Example 2

We illustrate `spearman` and `ktau` with the `auto` data, which contains some missing values.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. spearman mpg rep78
Number of observations = 69
Spearman's rho = 0.3098
Test of H0: mpg and rep78 are independent
Prob = 0.0098
```

Because we specified two variables, spearman displayed the sample size, correlation, and *p*-value in tabular form. To obtain just the correlation coefficient displayed in matrix form, we type

```
. spearman mpg rep78, stats(rho) matrix
Number of observations = 69
```

|       | mpg    | rep78  |
|-------|--------|--------|
| mpg   | 1.0000 |        |
| rep78 | 0.3098 | 1.0000 |

We can specify the *pw* option with *spearman* and *ktau* so that all nonmissing observations between a pair of variables when calculating their correlation coefficient are used. In the output below, some correlations are based on 74 observations, whereas others are based on 69 because 5 observations contain a missing value for *rep78*.

```
. spearman mpg price rep78, pw stats(rho obs p) star(0.01)
Number of observations:
 min = 69
 avg = 71
 max = 74
```

| Key             |
|-----------------|
| <i>rho</i>      |
| Number of obs   |
| <i>p</i> -value |

|       | mpg                      | price                  | rep78             |
|-------|--------------------------|------------------------|-------------------|
| mpg   | 1.0000<br>74<br>.        |                        |                   |
| price | -0.5419*<br>74<br>0.0000 | 1.0000<br>74           |                   |
| rep78 | 0.3098*<br>69<br>0.0098  | 0.1028<br>69<br>0.4000 | 1.0000<br>69<br>. |

The bonferroni and sidak options provide adjusted  $p$ -values:

```
. ktau mpg price rep78, stats(taua taub score se p) bonferroni
Number of observations = 69
```

| Key         |
|-------------|
| tau_a       |
| tau_b       |
| score       |
| se of score |
| p-value     |

|       | mpg                                                   | price                                              | rep78                                          |
|-------|-------------------------------------------------------|----------------------------------------------------|------------------------------------------------|
| mpg   | 0.9471<br>1.0000<br>2222.0000<br>191.8600<br>.        |                                                    |                                                |
| price | -0.3973<br>-0.4082<br>-932.0000<br>192.4561<br>0.0000 | 1.0000<br>1.0000<br>2346.0000<br>193.0682<br>.     |                                                |
| rep78 | 0.2076<br>0.2525<br>487.0000<br>181.7024<br>0.0224    | 0.0648<br>0.0767<br>152.0000<br>182.2233<br>1.0000 | 0.7136<br>1.0000<br>1674.0000<br>172.2161<br>. |

◀

### ▷ Example 3

We continue with the auto data and show an example of `spearman` with the `exact` option.

```
. set seed 1234
. spearman mpg gear_ratio if foreign == 1, exact
Permutations (10,000):1,000.....2,000.....3,000.....4,000.
>5,000.....6,000.....7,000.....8,000.....9,000.....
> 10,000 done
Number of observations = 22
Spearman's rho = 0.4881
Test of H0: mpg and gear_ratio are independent
Prob = 0.0222
Exact prob = 0.0214 (10,000 Monte Carlo permutations)
```

By default, `exact` does a Monte Carlo permutation test with 10,000 permutations. Because it is a Monte Carlo test, we set the random-number generator seed before running `spearman` for reproducibility.

The exact  $p$ -value from the Monte Carlo permutation test is 0.0214, which is close to the approximate  $p$ -value of 0.0222. Note that the approximate  $p$ -value is not based on the normal distribution or the  $t$  distribution. It is calculated using a beta distribution fit to the first four moments of the null distribution of Spearman's rank correlation, and these four moments are calculated exactly for any value of  $N$ , the number of observations. See [Methods and formulas](#) below.

Let's increase the number of Monte Carlo permutations to 1,000,000 to see how close the approximate  $p$ -value is to the exact  $p$ -value. We specify `dots(10000)` to see a dot every 10,000th permutation, rather than the default of every 1,000th permutation. This time we set the random-number generator seed using a suboption. The exact  $p$ -value is computed by `permute`, and we can see `permute`'s output by specifying the `show` suboption.

```
. spearman mpg gear_ratio if foreign == 1,
> exact(montecarlo, reps(1000000) dots(10000) rseed(1234) show)
Permutations (1,000,000):100,000.....200,000.....300,000.....
> ...400,000.....500,000.....600,000.....700,000.....800,000....
>900,000.....1,000,000 done
Monte Carlo permutation results Number of observations = 22
Permutation variable: mpg Number of permutations = 1,000,000
```

| T   | T(obs)   | Test      | c      | n       | p     | Monte Carlo error |             |       |
|-----|----------|-----------|--------|---------|-------|-------------------|-------------|-------|
|     |          |           |        |         |       | SE(p)             | [95% CI(p)] |       |
| rho | .4880642 | lower     | 988530 | 1000000 | .9885 | .0001             | .9883       | .9887 |
|     |          | upper     | 11510  | 1000000 | .0115 | .0001             | .0113       | .0117 |
|     |          | two-sided |        |         | .0230 | .0001             | .0227       | .0233 |

Notes: For lower one-sided test,  $c = \#\{T \leq T(\text{obs})\}$  and  $p = p_{\text{lower}} = c/n$ .  
 For upper one-sided test,  $c = \#\{T \geq T(\text{obs})\}$  and  $p = p_{\text{upper}} = c/n$ .  
 For two-sided test,  $p = 2 \cdot \min(p_{\text{lower}}, p_{\text{upper}})$ ; SE and CI approximate.

```
Number of observations = 22
Spearman's rho = 0.4881
```

```
Test of H0: mpg and gear_ratio are independent
Prob = 0.0222
Exact prob = 0.0230 (1,000,000 Monte Carlo permutations)
```

With this increase in the number of permutations, the exact  $p$ -value is now calculated as 0.0230. From the table produced by `permute`, we see that this  $p$ -value has a confidence interval of [0.0227, 0.0233], accounting for the Monte Carlo error. The approximate  $p$ -value of 0.0222 falls outside this confidence interval, but it is still very close. Not bad for  $N = 22$ .

◀

#### ▷ Example 4

For very small sample sizes, an exact  $p$ -value can be computed by enumerating the full permutation distribution. Here is an example using the auto data with sample size  $N = 11$ .

```
. spearman mpg gear_ratio if foreign == 1 & mpg <= 24,
> exact(enumerate, dots(10000))
(enumerating all 831,600 possible permutations)
Permutations (831,600):100,000.....200,000.....300,000.....
> .400,000.....500,000.....600,000.....700,000.....800,000.... d
> one
Number of observations = 11
Spearman's rho = 0.5636
Test of H0: mpg and gear_ratio are independent
Prob = 0.0722
Exact prob = 0.0747 (enumerated all 831,600 permutations)
```

The exact  $p$ -value is 0.0747. The approximate  $p$ -value is 0.0722, which is quite close to the exact  $p$ -value in this case, even with only 11 observations.

For  $N = 11$ , the permutation distribution consists of  $11! = 39,916,800$  permutations. However, the output above says that there are only 831,600 permutations. This is because the values of mpg in this sample are not all unique.

```
. tabulate mpg if foreign == 1 & mpg <= 24
```

| Mileage<br>(mpg) | Freq. | Percent | Cum.   |
|------------------|-------|---------|--------|
| 14               | 1     | 9.09    | 9.09   |
| 17               | 2     | 18.18   | 27.27  |
| 18               | 2     | 18.18   | 45.45  |
| 21               | 2     | 18.18   | 63.64  |
| 23               | 3     | 27.27   | 90.91  |
| 24               | 1     | 9.09    | 100.00 |
| Total            | 11    | 100.00  |        |

The multiplicities in the values of mpg yield permutations that give identical results, and we need to enumerate only the permutations that are distinct. From the values of mpg, we see that each distinct permutation has a multiplicity of  $2! 2! 2! 3! = 48$ , and  $39,916,800/48 = 831,600$ , which reduces considerably the number of permutations that need to be computed.

`permute`, which computes the permutations, permutes the first variable of the two in the `spearman varlist` command. So to best exploit this, the variable that produces the most multiplicities in the permutations should be placed first in `varlist` when doing an enumeration.

◀

Charles Edward Spearman (1863–1945) was a British psychologist who made contributions to correlation, factor analysis, test reliability, and psychometrics. After several years' military service, he obtained a PhD in experimental psychology at Leipzig and became a professor at University College London, where he sustained a long program of work on the interpretation of intelligence tests. Ironically, the rank correlation version bearing his name is not the formula he advocated.

Maurice George Kendall (1907–1983) was a British statistician who contributed to rank correlation, time series, multivariate analysis, among other topics, and wrote many statistical texts. Most notably, perhaps, his advanced survey of the theory of statistics went through several editions, later ones with Alan Stuart; the baton has since passed to others. Kendall was employed in turn as a government and business statistician, as a professor at the London School of Economics, as a consultant, and as director of the World Fertility Survey. He was knighted in 1974.

## Stored results

spearman stores the following in `r()`:

### Scalars

|                           |                                                 |
|---------------------------|-------------------------------------------------|
| <code>r(N)</code>         | number of observations (last variable pair)     |
| <code>r(rho)</code>       | $\rho$ (last variable pair)                     |
| <code>r(p)</code>         | two-sided $p$ -value (last variable pair)       |
| <code>r(p_l)</code>       | lower one-sided $p$ -value (last variable pair) |
| <code>r(p_u)</code>       | upper one-sided $p$ -value (last variable pair) |
| <code>r(p_exact)</code>   | two-sided exact $p$ -value                      |
| <code>r(p_l_exact)</code> | lower one-sided exact $p$ -value                |
| <code>r(p_u_exact)</code> | upper one-sided exact $p$ -value                |
| <code>r(n_perm)</code>    | number of permutations performed                |

### Macros

|                          |                                                       |
|--------------------------|-------------------------------------------------------|
| <code>r(exact)</code>    | "montecarlo" or "enumerate"                           |
| <code>r(rngstate)</code> | random-number state used for Monte Carlo permutations |

### Matrices

|                      |                        |
|----------------------|------------------------|
| <code>r(Nobs)</code> | number of observations |
| <code>r(Rho)</code>  | $\rho$                 |
| <code>r(P)</code>    | two-sided $p$ -value   |

If `exact(..., show)` is specified, the stored results from `permute` are returned as well; see [Stored results](#) in [\[R\] permute](#).

ktau stores the following in `r()`:

### Scalars

|                          |                                              |
|--------------------------|----------------------------------------------|
| <code>r(N)</code>        | number of observations (last variable pair)  |
| <code>r(tau_a)</code>    | $\tau_a$ (last variable pair)                |
| <code>r(tau_b)</code>    | $\tau_b$ (last variable pair)                |
| <code>r(score)</code>    | Kendall's score (last variable pair)         |
| <code>r(se_score)</code> | standard error of score (last variable pair) |
| <code>r(p)</code>        | two-sided $p$ -value (last variable pair)    |

### Matrices

|                          |                         |
|--------------------------|-------------------------|
| <code>r(Nobs)</code>     | number of observations  |
| <code>r(Tau_a)</code>    | $\tau_a$                |
| <code>r(Tau_b)</code>    | $\tau_b$                |
| <code>r(Score)</code>    | Kendall's score         |
| <code>r(Se_Score)</code> | standard error of score |
| <code>r(P)</code>        | two-sided $p$ -value    |

## Methods and formulas

Methods and formulas are presented under the following headings:

[Spearman's rank correlation](#)  
[Exact  \$p\$ -values](#)  
[Kendall's tau](#)

## Spearman's rank correlation

Spearman's (1904) rank correlation is calculated as Pearson's correlation computed on the ranks (averaged for ties) (Conover 1999, 314–315). Ranks are as calculated by `egen`; see [\[D\] egen](#).

If  $x_i$  and  $y_i$ , where  $i = 1, 2, \dots, n$ , are the ranks of one variable pair, and  $n$  is the number of observations, then Spearman's rank correlation is

$$\rho = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2} \sqrt{\sum_i (y_i - \bar{y})^2}}$$

where  $\bar{x} = (\sum_i x_i)/n$  is the mean of  $x$  and  $\bar{y}$  is defined similarly.

Under the null hypothesis of independence (or, more generally, exchangeability), the distribution of  $\rho$  is given by all the possible permutations of  $x_i$  with  $y_i$  fixed. For the permutation distribution, an equivalent statistic to  $\rho$  is

$$T = \sum_i x_i y_i$$

The moments of  $T$  for the permutation distribution can be computed exactly. Its mean is

$$E(T) = \frac{1}{n} \{ \sum_i x_i \} \{ \sum_i y_i \}$$

Assume now that the ranks,  $x_i$  and  $y_i$ , are adjusted so that their means are zero. With this assumption, we have (Stuart, Ord, and Arnold 1999, eqs. 27.42 and 27.43)

$$E(T^2) = \frac{1}{n-1} \{ \sum_i x_i^2 \} \{ \sum_i y_i^2 \}$$

$$\begin{aligned} E(T^3) &= \frac{1}{n} \{ \sum_i x_i^3 \} \{ \sum_i y_i^3 \} \\ &\quad + \frac{3}{n(n-1)} \{ \sum_{i \neq j} x_i^2 x_j \} \{ \sum_{i \neq j} y_i^2 y_j \} \\ &\quad + \frac{36}{n(n-1)(n-2)} \{ \sum_{i < j < k} x_i x_j x_k \} \{ \sum_{i < j < k} y_i y_j y_k \} \end{aligned}$$

$$\begin{aligned} E(T^4) &= \frac{1}{n} \{ \sum_i x_i^4 \} \{ \sum_i y_i^4 \} \\ &\quad + \frac{4}{n(n-1)} \{ \sum_{i \neq j} x_i^3 x_j \} \{ \sum_{i \neq j} y_i^3 y_j \} \\ &\quad + \frac{12}{n(n-1)} \{ \sum_{i < j} x_i^2 x_j^2 \} \{ \sum_{i < j} y_i^2 y_j^2 \} \\ &\quad + \frac{24}{n(n-1)(n-2)} \{ \sum_{i \neq j, k; j < k} x_i^2 x_j x_k \} \{ \sum_{i \neq j, k; j < k} y_i^2 y_j y_k \} \\ &\quad + \frac{576}{n(n-1)(n-2)(n-3)} \{ \sum_{i < j < k < l} x_i x_j x_k x_l \} \{ \sum_{i < j < k < l} y_i y_j y_k y_l \} \end{aligned}$$

Note that Stuart, Ord, and Arnold (1999, eq. 27.43) express  $E(T^3)$  and  $E(T^4)$  in terms of  $k$  statistics, but that formulation is exactly equivalent to the equations given above.

An approximate  $p$ -value for Spearman's rank correlation is calculated by fitting a four-parameter beta distribution to the first four moments of  $T$ . (See Lord [1965] and Hanson [1991] for a description of the technique of fitting a four-parameter beta distribution to moments of another distribution.) The four-parameter beta distribution with domain  $[l, u]$  is

$$f(x) = \frac{(-l+x)^{\alpha-1}(u-x)^{\beta-1}}{(u-l)^{\alpha+\beta-1}B(\alpha, \beta)}$$

where  $0 \leq l < u \leq 1$  and  $B(\alpha, \beta)$  is the beta function with  $\alpha > 0$  and  $\beta > 0$ .

The parameters for the beta distribution are calculated as follows. Let  $m$  be the mean of  $T$ ,  $v$  its variance,  $g_3$  its skewness, and  $g_4$  its kurtosis (see *Methods and formulas* in [R] **summarize**). Define

$$r = \frac{6(g_4 - g_3^2 - 1)}{6 + 3g_3^2 - 2g_4}$$

and

$$d = 1 - \frac{24(r+1)}{(r+2)(r+3)g_4 - 3(r-6)(r+1)}$$

Let

$$a = \frac{r(1 - \sqrt{d})}{2}$$

$$b = \frac{r(1 + \sqrt{d})}{2}$$

If  $g_3 > 0$ , then  $\alpha = a$  and  $\beta = b$ ; otherwise,  $\alpha = b$  and  $\beta = a$ . The domain boundaries are given by

$$l = m - \alpha \sqrt{\frac{v(\alpha + \beta + 1)}{\alpha\beta}}$$

$$u = m + \beta \sqrt{\frac{v(\alpha + \beta + 1)}{\alpha\beta}}$$

To calculate the  $p$ -values, we first scale the observed value of  $T$  to the domain of beta:

$$s = \frac{T_{\text{obs}} - l}{u - l}$$

Then the lower and upper one-sided  $p$ -values are given by

$$p_{\text{lower}} = \text{ibeta}(\alpha, \beta, s)$$

$$p_{\text{upper}} = \text{ibetatail}(\alpha, \beta, s)$$

where `ibeta` is Stata's two-parameter cumulative beta distribution and `ibetatail` is the function for its upper tail; see [FN] **Statistical functions**. The two-sided  $p$ -value is given by

$$p = \min\{1, 2 \min(p_{\text{lower}}, p_{\text{upper}})\}$$

## Exact p-values

Exact  $p$ -values for Spearman's rank correlation are computed by permute. For details on the permutation computation, see [R] [permute](#).

## Kendall's tau

Kendall's  $\tau$  is calculated in the following manner. For any two pairs of ranks  $(x_i, y_i)$  and  $(x_j, y_j)$ ,  $1 \leq i, j \leq n$ , define them as concordant if

$$(x_i - x_j)(y_i - y_j) > 0$$

and discordant if this product is less than zero.

Kendall's (1938; also see [Kendall and Gibbons \[1990\]](#) or [Bland \[2015\]](#), 187–188) score  $S$  is defined as  $C - D$ , where  $C$  ( $D$ ) is the number of concordant (discordant) pairs. Let  $N = n(n - 1)/2$  be the total number of pairs, so  $\tau_a$  is given by

$$\tau_a = \frac{S}{N}$$

and  $\tau_b$  is given by

$$\tau_b = \frac{S}{\sqrt{N - U}\sqrt{N - V}}$$

where

$$U = \sum_{i=1}^{N_1} u_i(u_i - 1)/2$$

$$V = \sum_{j=1}^{N_2} v_j(v_j - 1)/2$$

and where  $N_1$  is the number of sets of tied  $x$  values,  $u_i$  is the number of tied  $x$  values in the  $i$ th set,  $N_2$  is the number of sets of tied  $y$  values, and  $v_j$  is the number of tied  $y$  values in the  $j$ th set.

Under the null hypothesis of independence, the variance of  $S$  is exactly ([Kendall and Gibbons 1990](#), 66)

$$\begin{aligned} \text{Var}(S) = & \frac{1}{18} \left\{ n(n-1)(2n+5) - \sum_{i=1}^{N_1} u_i(u_i-1)(2u_i+5) - \sum_{j=1}^{N_2} v_j(v_j-1)(2v_j+5) \right\} \\ & + \frac{1}{9n(n-1)(n-2)} \left\{ \sum_{i=1}^{N_1} u_i(u_i-1)(u_i-2) \right\} \left\{ \sum_{j=1}^{N_2} v_j(v_j-1)(v_j-2) \right\} \\ & + \frac{1}{2n(n-1)} \left\{ \sum_{i=1}^{N_1} u_i(u_i-1) \right\} \left\{ \sum_{j=1}^{N_2} v_j(v_j-1) \right\} \end{aligned}$$

Using a normal approximation with a continuity correction,

$$z = \frac{|S| - 1}{\sqrt{\text{Var}(S)}}$$

For the hypothesis of independence, the statistics  $S$ ,  $\tau_a$ , and  $\tau_b$  produce equivalent tests and give the same  $p$ -value.

For Kendall's  $\tau$ , the normal approximation is surprisingly accurate for sample sizes as small as 8, at least for calculating  $p$ -values under the null hypothesis for continuous variables. See [Kendall and Gibbons \[1990, chap. 4\]](#), who also present some tables for calculating exact  $p$ -values for  $n < 10$ .

Let  $v$  be the number of variables specified so that  $k = v(v - 1)/2$  correlation coefficients are to be estimated. If `bonferroni` is specified, the adjusted  $p$ -value is  $p' = \min(1, kp)$ . If `sidak` is specified,  $p' = \min\{1, 1 - (1 - p)^n\}$ . See [Methods and formulas](#) in [\[R\] oneway](#) for a more complete description of the logic behind these adjustments.

Early work on rank correlation is surveyed by [Kruskal \(1958\)](#).

## Acknowledgment

The original version of `ktau` was written by Sean Beckett, author of the Stata Press book [Introduction to Time Series Using Stata, Revised Edition](#).

## References

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## Also see

- [R] **correlate** — Correlations of variables
- [R] **nptrend** — Tests for trend across ordered groups
- [R] **permute** — Permutation tests

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## Description

`spikeplot` produces a frequency plot for a variable in which the frequencies are depicted as vertical lines from zero. The frequency may be a count, a fraction, or the square root of the count (Tukey's rootogram, circa 1965). The vertical lines may also originate from a baseline other than zero at the user's option.

## Quick start

Spike plot of `v1`

```
spikeplot v1
```

Same as above, but apply frequency weights `wvar`

```
spikeplot v1 [fweight = wvar]
```

Plot proportions of the total number of observations instead of frequencies

```
spikeplot v1, fraction
```

Tukey's rootogram of `v2`

```
spikeplot v2, root
```

## Menu

Graphics > Distributional graphs > Spike plot and rootogram

## Syntax

```
spikeplot varname [if] [in] [weight] [, options]
```

| <i>options</i>                                     | Description                                                                |
|----------------------------------------------------|----------------------------------------------------------------------------|
| <b>Main</b>                                        |                                                                            |
| <code>round(#)</code>                              | round <i>varname</i> to nearest multiple of # (bin width)                  |
| <code>fraction</code>                              | make vertical scale the proportion of total values; default is frequencies |
| <code>root</code>                                  | make vertical scale show square roots of frequencies                       |
| <b>Plot</b>                                        |                                                                            |
| <code>spike_options</code>                         | affect rendition of plotted spikes                                         |
| <b>Add plots</b>                                   |                                                                            |
| <code>addplot(plot)</code>                         | add other plots to generated graph                                         |
| <b>Y axis, X axis, Titles, Legend, Overall, By</b> |                                                                            |
| <code>twoway_options</code>                        | any options documented in [G-3] <a href="#">twoway_options</a>             |

fweights, awweights, and iweights are allowed; see [U] 11.1.6 [weight](#).

## Options

### Main

`round(#)` rounds the values of *varname* to the nearest multiple of #. This action effectively specifies the bin width.

`fraction` specifies that the vertical scale be the proportion of total values (percentage) rather than the count.

`root` specifies that the vertical scale show square roots. This option may not be specified if `fraction` is specified.

### Plot

`spike_options` affect the rendition of the plotted spikes; see [G-2] [graph twoway spike](#).

### Add plots

`addplot(plot)` provides a way to add other plots to the generated graph. See [G-3] [addplot\\_option](#).

### Y axis, X axis, Titles, Legend, Overall, By

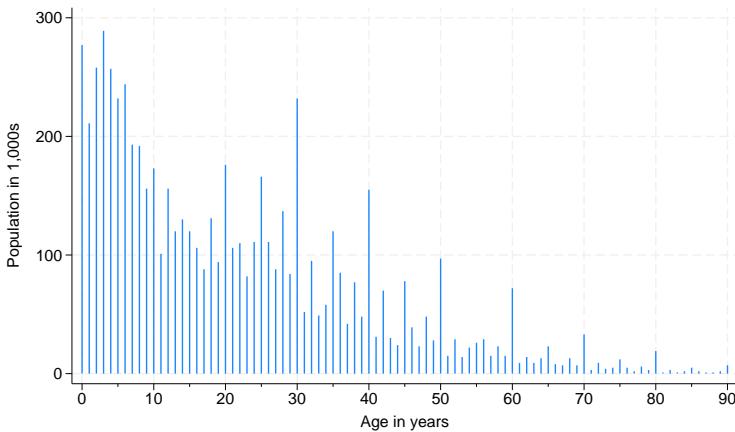
`twoway_options` are any of the options documented in [G-3] [twoway\\_options](#). These include options for titling the graph (see [G-3] [title\\_options](#)), options for saving the graph to disk (see [G-3] [saving\\_option](#)), and the `by()` option (see [G-3] [by\\_option](#)).

## Remarks and examples

### ▷ Example 1

Cox and Brady (1997a) present an illustrative example using the age structure of the population of Ghana from the 1960 census (rounded to the nearest 1,000). The dataset has ages from 0 (less than 1 year) to 90. To view the distribution of ages, we would like to use each integer from 0 to 90 as the bins for the dataset.

```
. use https://www.stata-press.com/data/r19/ghanaage
(Age structure of population of Ghana)
. spikeplot age [fw=pop], ytitle("Population in 1,000s") xlab(0(10)90)
> xmtick(5(10)85)
```



The resulting graph shows a “heaping” of ages at the multiples of 5. Also, ages ending in even numbers are more frequent than ages ending in odd numbers (except for 5). This preference for reporting ages is well known in demography and other social sciences.

Note also that we used the `ytitle()` option to override the default title of “Frequency” and that we used the `xlab()` and `xmtick()` options with *numlists* to further customize the resulting graph. See [U] 11.1.8 *numlist* for details on specifying *numlists*.

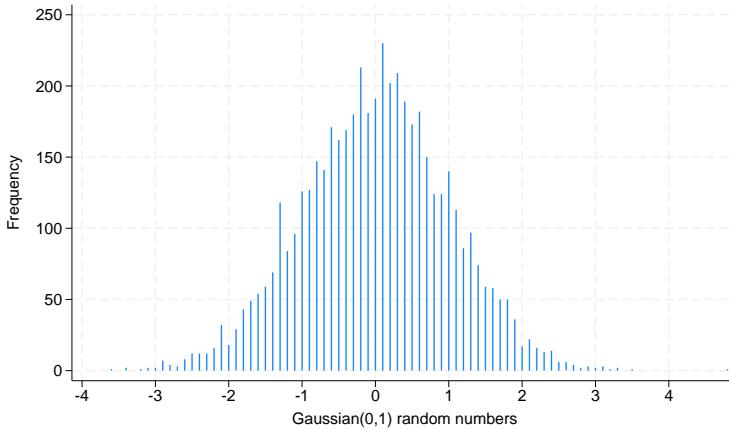


### ▷ Example 2

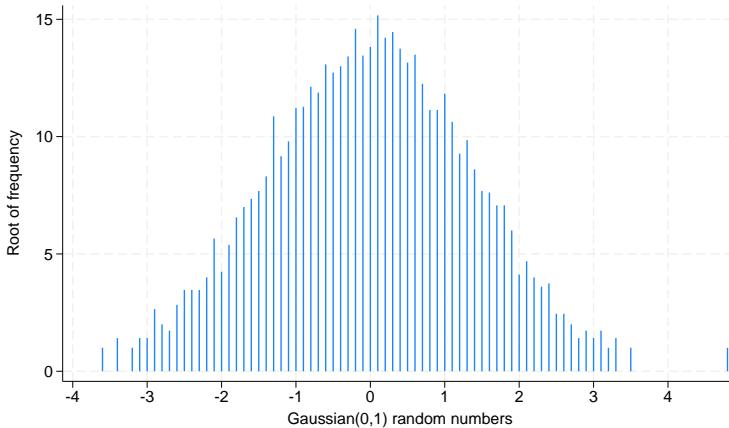
The rootogram is a plot of the square-root transformation of the frequency counts. The square root of a normal distribution is a multiple of another normal distribution.

```
. clear
. set seed 1234567
. set obs 5000
Number of observations (_N) was 0, now 5,000.
. generate normal = rnormal()
. label variable normal "Gaussian(0,1) random numbers"
```

```
. spikeplot normal, round(.10) xlab(-4(1)4)
```



```
. spikeplot normal, round(.10) xlab(-4(1)4) root
```



Interpreting a histogram in terms of normality is thus similar to interpreting the rootogram for normality.

This example also shows how the `round()` option is used to bin the values for a spike plot of a continuous variable.



### ▷ Example 3

`spikeplot` can also be used to produce time-series plots. `varname` should be the time variable, and weights should be specified as the values for those times. To get a plot of daily rainfalls, we type

```
. spikeplot day [fw=rain] if rain, ytitle("Daily rainfall in mm")
```

The `base()` option of `graph twoway spike` may be used to set a different baseline, such as when we want to show variations relative to an average or to some other measure of level.



## Acknowledgments

The original version of `spikeplot` was written by Nicholas J. Cox of the Department of Geography at Durham University, UK, who is coeditor of the *Stata Journal* and author of *Speaking Stata Graphics* and by Anthony R. Brady founder of Sealed Envelope, London (1997a, 1997b).

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## Also see

[R] [histogram](#) — Histograms for continuous and categorical variables

[Description](#)[Remarks and examples](#)[Quick start](#)[Acknowledgments](#)[Syntax](#)[References](#)[Options](#)[Also see](#)

## Description

`ssc` works with packages (and files) from the Statistical Software Components (SSC) Archive, which is often called the Boston College Archive and is provided by <http://repec.org>.

The SSC has become the premier Stata download site for community-contributed software on the web. `ssc` provides a convenient interface to the resources available there. For example, on Statalist (see <https://www.statalist.org/>), users will often write

```
The program can be found by typing ssc install newprogramname.
```

Typing that would load everything associated with `newprogramname`, including the help files.

If you are searching for what is available, type `ssc new` and `ssc hot`, and see [R] [search](#). `search` searches the SSC and other places, too. `search` provides a GUI interface from which programs can be installed, including the programs at the SSC Archive.

You can uninstall particular packages by using `ssc uninstall`. For the packages that you keep, see [R] [ado update](#) for an automated way of keeping those packages up to date.

## Command overview

`ssc new` summarizes the packages made available or updated recently. Output is presented in the Stata Viewer, and from there you may click to find out more about individual packages or to install them.

`ssc hot` lists the most popular packages—popular based on a moving average of the number of downloads in the past three months. By default, 10 packages are listed.

`ssc describe pkgname` describes, but does not install, the specified package. Use `search` to find packages; see [R] [search](#). If you know the package name but do not know the exact spelling, type `ssc describe` followed by one letter, a–z or \_ (underscore), to list all the packages starting with that letter.

`ssc install pkgname` installs the specified package. You do not have to describe a package before installing it. (You may also install a package by using `net install`; see [R] [net](#).)

`ssc uninstall pkgname` removes the previously installed package from your computer. It does not matter how the package was installed. (`ssc uninstall` is a synonym for `ado uninstall`, so either may be used to installed any package.)

`ssc type filename` types a specific file stored at SSC. `ssc cat` is a synonym for `ssc type`, which may appeal to those familiar with Unix.

`ssc copy filename` copies a specific file stored at SSC to your computer. By default, the file is copied to the current directory, but you can use options to change this. `ssc copy` is a rarely used alternative to `ssc install ... , all`. `ssc cp` is a synonym for `ssc copy`.

## Quick start

Describe mycommand at SSC

```
ssc describe mycommand
```

Install mycommand from SSC

```
ssc install mycommand
```

Same as above, but replace previously installed version of mycommand

```
ssc install mycommand, replace
```

See a summary of all new and recently updated packages on SSC

```
ssc new
```

See a summary of the 10 most popular SSC packages

```
ssc hot
```

Same as above, but see the top 25 packages

```
ssc hot, n(25)
```

## Syntax

*Summary of packages most recently added or updated at SSC*

```
ssc new [, saving(filename[, replace]) type]
```

*Summary of most popular packages at SSC*

```
ssc hot [, n(#) author(name)]
```

*Describe a specified package at SSC*

```
ssc describe { pkgname | letter } [, saving(filename[, replace])]
```

*Install a specified package from SSC*

```
ssc install pkgname [, all replace]
```

*Uninstall from your computer a previously installed package from SSC*

```
ssc uninstall pkgname
```

*Type a specific file stored at SSC*

```
ssc type filename [, asis]
```

*Copy a specific file from SSC to your computer*

```
ssc copy filename [, plus personal replace public binary]
```

where *letter* in ssc describe is a–z or \_.

## Options

Options are presented under the following headings:

*Options for use with ssc new*  
*Options for use with ssc hot*  
*Option for use with ssc describe*  
*Options for use with ssc install*  
*Option for use with ssc type*  
*Options for use with ssc copy*

### Options for use with ssc new

`saving(filename[, replace])` specifies that the “what’s new” summary be saved in *filename*. If *filename* is specified without a suffix, *filename.smcl* is assumed. If `saving()` is not specified, `saving(ssc_results.smcl)` is assumed.

`type` specifies that the “what’s new” results be displayed in the Results window rather than in the Viewer.

### Options for use with ssc hot

`n(#)` specifies the number of packages to list; `n(10)` is the default. Specify `n(.)` to list all packages in order of popularity.

`author(name)` lists the 10 most popular packages by the specified author. If `n(#)` is also specified, the top # packages are listed.

### Option for use with ssc describe

`saving(filename[, replace])` specifies that, in addition to the description’s being displayed on your screen, it be saved in the specified file.

If *filename* is specified without an extension, `.smcl` will be assumed, and the file will be saved as a SMCL file.

If *filename* is specified with an extension, no default extension is added. If the extension is `.log`, the file will be stored as a text file.

If `replace` is specified, *filename* is replaced if it already exists.

### Options for use with ssc install

`all` specifies that any ancillary files associated with the package be downloaded to your current directory, in addition to the program and help files being installed. Ancillary files are files that do not end in `.ado` or `.sthlp` and typically contain datasets or examples of the use of the new command.

You can find out which files are associated with the package by typing `ssc describe pkgname` before or after installing. If you install without using the `all` option and then want the ancillary files, you can `ssc install` again.

`replace` specifies that any files being downloaded that already exist on your computer be replaced by the downloaded files. If `replace` is not specified and any files already exist, none of the files from the package is downloaded or installed.

It is better not to specify the `replace` option and wait to see if there is a problem. If there is a problem, it is usually better to uninstall the old package by using `ssc uninstall` or `ado uninstall` (which are, in fact, the same command).

## Option for use with `ssc type`

`asis` affects how files with the suffixes `.smcl` and `.sthlp` are displayed. The default is to interpret SMCL directives the file might contain. `asis` specifies that the file be displayed in raw, uninterpreted form.

## Options for use with `ssc copy`

`plus` specifies that the file be copied to the PLUS directory, the directory where community-contributed additions are installed. Typing `sysdir` will display the identity of the PLUS directory on your computer; see [P] [sysdir](#).

`personal` specifies that the file be copied to your PERSONAL directory as reported by `sysdir`; see [P] [sysdir](#).

If neither `plus` nor `personal` is specified, the default is to copy the file to the current directory.

`replace` specifies that, if the file already exists on your computer, the new file replace it.

`public` specifies that the new file be made readable by everyone; otherwise, the file will be created according to the default permission you have set with your operating system.

`binary` specifies that the file being copied is a binary file and that it is to be copied as is. The default is to assume that the file is a text file and change the end-of-line characters to those appropriate for your computer/operating system.

## Remarks and examples

Users can add new features to Stata, and some users choose to make new features that they have written available to others via the web. The files that comprise a new feature are called a package, and a package usually consists of one or more `ado`-files and help files. The `net` command (see [R] [net](#)) makes it reasonably easy to install and uninstall packages regardless of where they are on the web. One site, the SSC, has become particularly popular as a repository for additions to Stata. Command `ssc` is an easier to use version of `net` designed especially for the SSC.

Many packages are available at the SSC. Packages have names, such as `oaxaca`, `estout`, or `egenmore`. At SSC, capitalization is not significant, so `Oaxaca`, `ESTOUT`, and `EGENmore` are ways of writing the same package names.

When you type

```
. ssc install oaxaca
```

the files associated with the package are downloaded and installed on your computer. Package names usually correspond to the names of the command being added to Stata, so one would expect that installing the package `oaxaca` will add command `oaxaca` to Stata on your computer, and expect that typing `help oaxaca` will provide the documentation. That is the situation here, but that is not always so. Before or after installing a package, type `ssc describe pkgname` to obtain the details.

▷ Example 1

ssc new summarizes the packages most recently made available or updated. Output is presented in the Viewer, from which you may click on a package name to find out more or install it. Try it for yourself! Type ssc new in the Command window.

ssc hot provides a list of the most popular packages at SSC.

```
. ssc hot
```

**Top 10 packages at SSC**

| Rank | Feb 2025<br># hits | Package  | Author(s)                                               |
|------|--------------------|----------|---------------------------------------------------------|
| 1    | 87128.6            | estout   | Ben Jann                                                |
| 2    | 83967.2            | outreg2  | Roy Wada                                                |
| 3    | 68888.4            | reghdfe  | Sergio Correia                                          |
| 4    | 67323.3            | winsor2  | Yujun Lian                                              |
| 5    | 64700.7            | asdoc    | Attaullah Shah                                          |
| 6    | 44584.0            | ftools   | Sergio Correia                                          |
| 7    | 33506.2            | ivreg210 | Christopher F Baum, Mark E Schaffer,<br>Steven Stillman |
| 8    | 31413.5            | ivreg2   | Christopher F Baum, Steven Stillman,<br>Mark E Schaffer |
| 9    | 30169.0            | sum2docx | Chuntao Li, Yuan Xue                                    |
| 10   | 29850.2            | ivreg29  | Steven Stillman, Mark E Schaffer,<br>Christopher F Baum |

(Click on package name for description)

Use the n(#) option to change the number of packages listed:

```
. ssc hot, n(20)
```

**Top 20 packages at SSC**

| Rank                    | Feb 2025<br># hits | Package      | Author(s)                                               |
|-------------------------|--------------------|--------------|---------------------------------------------------------|
| 1                       | 87128.6            | estout       | Ben Jann                                                |
| 2                       | 83967.2            | outreg2      | Roy Wada                                                |
| 3                       | 68888.4            | reghdfe      | Sergio Correia                                          |
| 4                       | 67323.3            | winsor2      | Yujun Lian                                              |
| 5                       | 64700.7            | asdoc        | Attaullah Shah                                          |
| 6                       | 44584.0            | ftools       | Sergio Correia                                          |
| 7                       | 33506.2            | ivreg210     | Christopher F Baum, Mark E Schaffer,<br>Steven Stillman |
| <i>(output omitted)</i> |                    |              |                                                         |
| 13                      | 24392.7            | reg2docx     | Chuntao Li, Yuan Xue                                    |
| 14                      | 24301.4            | ranktest     | Mark E Schaffer, Frank Kleibergen,<br>Frank Windmeijer  |
| 15                      | 19663.5            | logout       | Roy Wada                                                |
| 16                      | 19206.2            | xtivreg2     | Mark E Schaffer                                         |
| 17                      | 18736.3            | carryforward | David Kantor                                            |
| 18                      | 18441.5            | psmatch2     | Barbara Sianesi, Edwin Leuven                           |
| 19                      | 14388.0            | rangestat    | Robert Picard, Nicholas J. Cox, Roberto<br>Ferrer       |
| 20                      | 13768.0            | distinct     | Gary Longton, Nicholas J. Cox                           |

(Click on package name for description)

The author (*name*) option allows you to list the most popular packages by a specific person:

```
. ssc hot, author(baum)
```

**Top 10 packages at SSC by author Baum**

| Rank | Feb 2025<br># hits | Package  | Author(s)                                               |
|------|--------------------|----------|---------------------------------------------------------|
| 7    | 33506.2            | ivreg210 | Christopher F Baum, Mark E Schaffer,<br>Steven Stillman |
| 8    | 31413.5            | ivreg2   | Christopher F Baum, Steven Stillman,<br>Mark E Schaffer |

(output omitted)

(Click on package name for description)

ssc describe *pkgname* describes, but does not install, the specified package. You must already know the name of the package. See [R] [search](#) for assistance in searching for packages. Sometimes you know the package name, but you do not know the exact spelling. Then, you can type ssc describe followed by one letter, a–z or `_`, to list all the packages starting with that letter; even so, using search is better.

```
. ssc describe i
```

---

```
http://fmwww.bc.edu/repec/bocode/i/
(no title)
```

---

PACKAGES you could **-not describe-**:

(output omitted)

|            |                                                                                                                                                                   |
|------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| ivreg2     | module for extended instrumental variables/2SLS and GMM estimation                                                                                                |
| ivreg210   | module for extended instrumental variables/2SLS and GMM estimation (v10)                                                                                          |
| ivreg28    | module for extended instrumental variables/2SLS and GMM estimation (v8)                                                                                           |
| ivreg29    | module for extended instrumental variables/2SLS and GMM estimation (v9)                                                                                           |
| ivreg2h    | module to perform instrumental variables estimation using heteroskedasticity-based instruments                                                                    |
| ivreg2hdfe | module to estimate an Instrumental Variable Linear Regression Model with two High Dimensional Fixed Effects                                                       |
| ivreg2m    | module to identify treatment-effects estimates with potentially misreported and endogenous program participation                                                  |
| ivreg_ss   | module to compute confidence intervals, standard errors, and p-values in an IV regression in which the excluded instrumental variable has a shift-share structure |
| ivreghdfe  | module for extended instrumental variable regressions with multiple levels of fixed effects                                                                       |
| ivregress2 | module to export first and second-stage results similar to ivregress                                                                                              |

(output omitted)

---

(type **ssc describe** *pkgname* for more information on *pkgname*)

The default setting for the `saving()` option is for the output to be saved with the `.smc1` extension. You could also save the file with a `log` extension, and in this case, the file would be stored as a text file. For example, we could type

```
. ssc describe i, saving(i.index)
```

or the following:

```
. ssc describe ivreg2, saving(ivreg2.log)
```

`ssc install pkgname` installs the specified package. You do not have to describe a package before installing it. There are ways of installing packages other than `ssc install`, such as `net`; see [R] [net](#). It does not matter how a package is installed. For instance, a package can be installed using `net` and still be uninstalled using `ssc`.

```
. ssc install ivreg2
checking ivreg2 consistency and verifying not already installed...
installing into C:\ado\plus\...
installation complete.
```

`ssc uninstall pkgname` removes the specified, previously installed package from your computer. You can uninstall immediately after installation or at any time in the future. (Technical note: `ssc uninstall` is a synonym for `ado uninstall`, so it can uninstall any installed package, not just packages obtained from the SSC.)

```
. ssc uninstall ivreg2
package ivreg2 from http://fmwww.bc.edu/repec/bocode/i
'IVREG2': module for extended instrumental variables/2SLS and GMM
estimation
(package uninstalled)
```

`ssc type filename` types a specific file stored at the SSC. Although not shown in the syntax diagram, `ssc cat` is a synonym for `ssc type`, which may appeal to those familiar with Unix. To view only the `ivreg2` help file from the `ivreg2` package, you would type

```
. ssc type ivreg2.sthlp
```

---

```
help for ivreg2
```

---

**Extended instrumental variables/2SLS, GMM and AC/HAC, LIML and k-class regression**

```
Full syntax
(output omitted)
```

`ssc copy filename` copies a specific file stored at the SSC to your computer. By default, the file is copied to the current directory, but you can use options to change this. `ssc copy` is a rarely used alternative to `ssc install ...`, `all`. `ssc cp` is a synonym for `ssc copy`.

```
. ssc copy ivreg2.ado
(file ivreg2.ado copied to current directory)
```



For more details on the SSC Archive and for information on how to submit your own programs to the SSC, see <http://repec.org/bocode/s/sscsubmit.html>.

## Acknowledgments

ssc is based on `archutil` by Nicholas J. Cox of the Department of Geography at Durham University, UK, who is coeditor of the *Stata Journal* and author of *Speaking Stata Graphics* and by Christopher F. Baum of the Department of Economics at Boston College and author of the Stata Press books *An Introduction to Modern Econometrics Using Stata* and *An Introduction to Stata Programming* and coauthor of the Stata Press book *Environmental Econometrics Using Stata*. The reworking of the original was done with their blessing and their participation.

Baum maintains the Stata-related files stored at the SSC Archive. We thank him for this contribution to the Stata community.

## References

- Baum, C. F., and N. J. Cox. 1999. [ip29: Metadata for user-written contributions to the Stata programming language](#). *Stata Technical Bulletin* 52: 10–12. Reprinted in *Stata Technical Bulletin Reprints*, vol. 9, pp. 121–124. College Station, TX: Stata Press.
- Choodari-Oskoei, B., and T. P. Morris. 2016. Quantifying the uptake of user-written commands over time. *Stata Journal* 16: 88–95.
- Cox, N. J., and C. F. Baum. 2000. [ip29.1: Metadata for user-written contributions to the Stata programming language](#). *Stata Technical Bulletin* 54: 21–22. Reprinted in *Stata Technical Bulletin Reprints*, vol. 9, pp. 124–126. College Station, TX: Stata Press.
- Newson, R. B. 2022. [Stata tip 147: Porting downloaded packages between machines](#). *Stata Journal* 22: 996–997.

## Also see

- [R] [ado update](#) — Update community-contributed packages
- [R] [net](#) — Install and manage community-contributed additions from the internet
- [R] [search](#) — Search Stata documentation and other resources
- [R] [sj](#) — Stata Journal installation instructions
- [P] [sysdir](#) — Query and set system directories

Description  
Options  
Also see

Quick start  
Remarks and examples

Menu  
Stored results

Syntax  
References

## Description

`stem` displays stem-and-leaf plots.

## Quick start

Stem-and-leaf plot for `v1`

```
stem v1
```

Same as above, with 2 lines per interval of 10

```
stem v1, lines(2)
```

Specify that each stem has a width of 5 (equivalent to above)

```
stem v1, width(5)
```

Stem-and-leaf plot for `v2` with an interval of 100

```
stem v2, digits(2)
```

Same as above, with 4 lines per interval of 100

```
stem v2, digits(2) lines(4)
```

Display `v2` rounded to the nearest hundred

```
stem v2, round(100)
```

Do not display empty stems

```
stem v2, prune
```

## Menu

Statistics > Summaries, tables, and tests > Distributional plots and tests > Stem-and-leaf display

## Syntax

```
stem varname [if] [in] [, options]
```

| <i>options</i>           | Description                                                |
|--------------------------|------------------------------------------------------------|
| <code>prune</code>       | do not print stems that have no leaves                     |
| <code>round(#)</code>    | round data to this value; default is <code>round(1)</code> |
| <code>truncate(#)</code> | truncate data to this value                                |
| <code>digits(#)</code>   | digits per leaf; default is <code>digits(1)</code>         |
| <code>lines(#)</code>    | number of stems per interval of $10^{\text{digits}}$       |
| <code>width(#)</code>    | stem width; equal to $10^{\text{digits}}/\text{width}$     |

by and collect are allowed; see [\[U\] 11.1.10 Prefix commands](#).

## Options

Main

`prune` prevents printing any stems that have no leaves.

`round(#)` rounds the data to this value and displays the plot in these units. If `round()` is not specified, noninteger data will be rounded automatically.

`truncate(#)` truncates the data to this value and displays the plot in these units.

`digits(#)` sets the number of digits per leaf. The default is 1.

`lines(#)` sets the number of stems per every data interval of  $10^{\text{digits}}$ . The value of `lines()` must divide  $10^{\text{digits}}$ ; that is, if `digits(1)` is specified, then `lines()` must divide 10. If `digits(2)` is specified, then `lines()` must divide 100, etc. Only one of `lines()` or `width()` may be specified. If neither is specified, an appropriate value will be set automatically.

`width(#)` sets the width of a stem. `lines()` is equal to  $10^{\text{digits}}/\text{width}$ , and this option is merely an alternative way of setting `lines()`. The value of `width()` must divide  $10^{\text{digits}}$ . Only one of `width()` or `lines()` may be specified. If neither is specified, an appropriate value will be set automatically.

Note: If `lines()` or `width()` is not specified, `digits()` may be decreased in some circumstances to make a better-looking plot. If `lines()` or `width()` is set, the user-specified value of `digits()` will not be altered.

## Remarks and examples

### ▷ Example 1

Stem-and-leaf displays are a compact way to present considerable information about a batch of data. For instance, using our automobile data (described in [U] 1.2.2 Example datasets):

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)

. stem mpg
Stem-and-leaf plot for mpg (Mileage (mpg))

1t | 22
1f | 44444455
1s | 66667777
1. | 888888888899999999
2* | 00011111
2t | 22222333
2f | 444455555
2s | 666
2. | 8889
3* | 001
3t |
3f | 455
3s |
3. |
4* | 1
```

The stem-and-leaf display provides a way to list our data. The expression to the left of the vertical bar is called the stem; the digits to the right are called the leaves. All the stems that begin with the same digit and the corresponding leaves, written beside each other, reconstruct an observation of the data. Thus, if we look at the four stems that begin with the digit 1 and their corresponding leaves, we see that we have two cars rated at 12 mpg, 6 cars at 14, 2 at 15, and so on. The car with the highest mileage rating in our data is rated at 41 mpg.

The above plot is a five-line plot with `lines()` equal to 5 (five lines per interval of 10) and `width()` equal to 2 (two leaves per stem).

Instead, we could specify `lines(2)`:

```
. stem mpg, lines(2)
Stem-and-leaf plot for mpg (Mileage (mpg))

1* | 22444444
1. | 5566667777888888888899999999
2* | 00011111222223334444
2. | 555556668889
3* | 0014
3. | 55
4* | 1
```

`stem mpg, width(5)` would produce the same plot as above.

The stem-and-leaf display provides a crude histogram of our data, one not so pretty as that produced by `histogram` (see [R] [histogram](#)), but one that is nonetheless informative.

## ▷ Example 2

The miles per gallon rating fits easily into a stem-and-leaf display because, in our data, it has two digits. However, `stem` does not require two digits.

```
. stem price, lines(1) digits(3)
Stem-and-leaf plot for price (Price)
 3*** | 291,299,667,748,798,799,829,895,955,984,995
 4*** | 010,060,082,099,172,181,187,195,296,389,424,425,453,482,499, ... (26)
 5*** | 079,104,172,189,222,379,397,705,719,788,798,799,886,899
 6*** | 165,229,295,303,342,486,850
 7*** | 140,827
 8*** | 129,814
 9*** | 690,735
10*** | 371,372
11*** | 385,497,995
12*** | 990
13*** | 466,594
14*** | 500
15*** | 906
```

The (26) at the right of the second stem shows that there were 26 leaves on this stem—too many to display on one line.

We can make a more compact stem-and-leaf plot by rounding. To display `stem` in units of 100, we could type

```
. stem price, round(100)
Stem-and-leaf plot for price (Price)
price rounded to nearest multiple of 100
plot in units of 100
 3* | 33778889
 4* | 00001112222344455555667777899
 5* | 11222447788899
 6* | 2233359
 7* | 18
 8* | 18
 9* | 77
10* | 44
11* | 45
12* | 0
13* | 056
14* | 5
15* | 9
```

`price`, in our data, has four or five digits. `stem` presented the display in terms of units of 100, so a car that cost \$3,291 was treated for display purposes as \$3,300.

◀

## □ Technical note

Stem-and-leaf diagrams have been used in Japanese railway timetables, as shown in [Tufte \(1990, 46–47\)](#).

□

## Stored results

stem stores the following in `r()`:

### Scalars

|                        |                                         |
|------------------------|-----------------------------------------|
| <code>r(width)</code>  | width of a stem                         |
| <code>r(digits)</code> | number of digits per leaf; default is 1 |

### Macros

|                          |                                             |
|--------------------------|---------------------------------------------|
| <code>r(round)</code>    | number specified in <code>round()</code>    |
| <code>r(truncate)</code> | number specified in <code>truncate()</code> |

## References

- Cox, N. J. 2007. *Speaking Stata: Turning over a new leaf*. *Stata Journal* 7: 413–433.
- Emerson, J. D., and D. C. Hoaglin. 1983. “Stem-and-leaf displays”. In *Understanding Robust and Exploratory Data Analysis*, edited by D. C. Hoaglin, C. F. Mosteller, and J. W. Tukey, 7–32. New York: Wiley.
- Tufte, E. R. 1990. *Envisioning Information*. Cheshire, CT: Graphics Press.
- Tukey, J. W. 1972. “Some graphic and semigraphic displays”. In *Statistical Papers in Honor of George W. Snedecor*, edited by T. A. Bancroft and S. A. Brown, 293–316. Ames, IA: Iowa State University Press.
- . 1977. *Exploratory Data Analysis*. Reading, MA: Addison–Wesley.

## Also see

- [R] **histogram** — Histograms for continuous and categorical variables
- [R] **lv** — Letter-value displays

|             |                      |                |                      |
|-------------|----------------------|----------------|----------------------|
| Description | Quick start          | Menu           | Syntax               |
| Options     | Remarks and examples | Stored results | Methods and formulas |
| References  | Also see             |                |                      |

## Description

`stepwise` performs stepwise estimation. Typing

```
. stepwise, pr(#): command
```

performs backward-selection estimation for *command*. The stepwise selection method is determined by the following option combinations:

| <i>options</i>      | Description                     |
|---------------------|---------------------------------|
| pr(#)               | backward selection              |
| pr(#) hierarchical  | backward hierarchical selection |
| pr(#) pe(#)         | backward stepwise               |
| pe(#)               | forward selection               |
| pe(#) hierarchical  | forward hierarchical selection  |
| pr(#) pe(#) forward | forward stepwise                |

*command* defines the estimation command to be executed. The following Stata commands are supported by `stepwise`:

```
betareg, clogit, cloglog, glm, intreg, logistic, logit, nbreg,
ologit, oprobit, poisson, probit, qreg, regress, scobit, stcox,
stcrreg, stintreg, streg, tobit
```

`stepwise` expects *command* to have the following form:

```
command_name [depvar] term [term . . .] [if] [in] [weight] [, command_options]
```

where *term* is either *varname* or (*varlist*) (*varlist* in parentheses indicates that this group of variables is to be included or excluded together). *depvar* is not present when *command\_name* is `stcox`, `stcrreg`, `stintreg`, or `streg`; otherwise, *depvar* is assumed to be present. For `intreg`, *depvar* is actually two dependent variable names (*depvar*<sub>1</sub> and *depvar*<sub>2</sub>).

`sw` is a synonym for `stepwise`.

For model selection and estimation using lasso, see the [Stata Lasso Reference Manual](#).

## Quick start

Backward selection, removing terms with  $p \geq 0.2$

```
stepwise, pr(.2): regress y x1 x2 x3 x4
```

Same as above, and select from the indicators for categorical variable a

```
stepwise, pr(.2): regress y x1 x2 x3 x4 i.a
```

Same as above, but force x1 to be included in model

```
stepwise, pr(.2) lockterm1: regress y x1 x2 x3 x4 i.a
```

Consider the indicators for a as a group for inclusion in model

```
stepwise, pr(.2): regress y x1 x2 x3 x4 (i.a)
```

Add d1, d2, and d3, and force them to be included in model

```
stepwise, pr(.2) lockterm1: regress y (d1 d2 d3) x1 x2 x3 x4 (i.a)
```

Forward selection, adding terms with  $p < 0.1$

```
stepwise, pe(.1): regress y x1 x2 x3 x4
```

Backward stepwise selection, removing terms with  $p \geq 0.2$  and adding those with  $p < 0.1$

```
stepwise, pr(.2) pe(.1): regress y x1 x2 x3 x4
```

Forward stepwise selection, adding terms with  $p < 0.1$  and removing those with  $p \geq 0.2$

```
stepwise, pr(.2) pe(.1) forward: regress y x1 x2 x3 x4
```

Backward hierarchical selection

```
stepwise, pr(.2) hierarchical: regress y x1 x2 x3 x4
```

Forward hierarchical selection

```
stepwise, pe(.1) hierarchical: regress y x1 x2 x3 x4
```

Note: In the above examples, regress could be replaced with any estimation command allowing the stepwise prefix.

## Menu

Statistics > Other > Stepwise estimation

## Syntax

stepwise [ , options ] : *command*

| <i>options</i>               | Description                                        |
|------------------------------|----------------------------------------------------|
| <b>Model</b>                 |                                                    |
| * <code>pr(#)</code>         | significance level for removal from the model      |
| * <code>pe(#)</code>         | significance level for addition to the model       |
| <b>Model2</b>                |                                                    |
| <code>forward</code>         | perform forward-stepwise selection                 |
| <code>hierarchical</code>    | perform hierarchical selection                     |
| <code>lockterm1</code>       | keep the first term                                |
| <code>lr</code>              | perform likelihood-ratio test instead of Wald test |
| <b>Reporting</b>             |                                                    |
| <code>display_options</code> | control columns and column formats and line width  |

\*At least one of `pr(#)` or `pe(#)` must be specified.

`by` is allowed; see [U] 11.1.10 Prefix commands.

Weights are allowed if *command* allows them; see [U] 11.1.6 weight.

All postestimation commands behave as they would after *command* without the `stepwise` prefix; see the postestimation manual entry for *command*.

## Options

### Model

`pr(#)` specifies the significance level for removal from the model; terms with  $p \geq \text{pr}()$  are eligible for removal.

`pe(#)` specifies the significance level for addition to the model; terms with  $p < \text{pe}()$  are eligible for addition.

### Model 2

`forward` specifies the forward-stepwise method and may be specified only when both `pr()` and `pe()` are also specified. Specifying both `pr()` and `pe()` without `forward` results in backward-stepwise selection. Specifying only `pr()` results in backward selection, and specifying only `pe()` results in forward selection.

`hierarchical` specifies hierarchical selection.

`lockterm1` specifies that the first term be included in the model and not be subjected to the selection criteria.

`lr` specifies that the test of term significance be the likelihood-ratio test. The default is the less computationally expensive Wald test; that is, the test is based on the estimated variance–covariance matrix of the estimators.

### Reporting

`display_options`: `nocl`, `nopvalues`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] Estimation options.

## Remarks and examples

Remarks are presented under the following headings:

*Introduction*  
*Search logic for a step*  
*Full search logic*  
*Examples*  
*Estimation sample considerations*  
*Messages*  
*Programming for stepwise*

### Introduction

#### Typing

```
. stepwise, pr(.10): regress y1 x1 x2 d1 d2 d3 x4 x5
```

performs a backward-selection search for the regression model  $y_1$  on  $x_1$ ,  $x_2$ ,  $d_1$ ,  $d_2$ ,  $d_3$ ,  $x_4$ , and  $x_5$ . In this search, each explanatory variable is said to be a term. Typing

```
. stepwise, pr(.10): regress y1 x1 x2 (d1 d2 d3) (x4 x5)
```

performs a similar backward-selection search, but the variables  $d_1$ ,  $d_2$ , and  $d_3$  are treated as one term, as are  $x_4$  and  $x_5$ . That is,  $d_1$ ,  $d_2$ , and  $d_3$  may or may not appear in the final model, but they appear or do not appear together.

#### ▷ Example 1

Using the automobile dataset, we fit a backward-selection model of mpg:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. stepwise, pr(.2): regress mpg c.weight#c.weight displ gear turn headroom
> i.foreign price
note: 0b.foreign omitted because of estimability.
Wald test, begin with full model:
p = 0.7116 >= 0.2000, removing headroom
p = 0.6138 >= 0.2000, removing displacement
p = 0.3278 >= 0.2000, removing price
```

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 1736.31455 | 5  | 347.262911 | F(5, 68)      | = | 33.39  |
| Residual | 707.144906 | 68 | 10.3991898 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.7106 |
|          |            |    |            | Adj R-squared | = | 0.6893 |
| Total    | 2443.45946 | 73 | 33.4720474 | Root MSE      | = | 3.2248 |

| mpg                   | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|-----------------------|-------------|-----------|-------|-------|----------------------|-----------|
| weight                | -.0158002   | .0039169  | -4.03 | 0.000 | -.0236162            | -.0079842 |
| c.weight#<br>c.weight | 1.77e-06    | 6.20e-07  | 2.86  | 0.006 | 5.37e-07             | 3.01e-06  |
| foreign<br>Foreign    | -3.615107   | 1.260844  | -2.87 | 0.006 | -6.131082            | -1.099131 |
| gear_ratio            | 2.011674    | 1.468831  | 1.37  | 0.175 | -.9193321            | 4.94268   |
| turn                  | -.3087038   | .1763099  | -1.75 | 0.084 | -.6605248            | .0431172  |
| _cons                 | 59.02133    | 9.3903    | 6.29  | 0.000 | 40.28327             | 77.75938  |

This estimation treated each variable as its own term and thus considered each one separately. The engine displacement and gear ratio should really be considered together:

```
. stepwise, pr(.2): regress mpg c.weight#c.weight (displ gear) turn headroom
> i.foreign price
note: 0b.foreign omitted because of estimability.
```

Wald test, begin with full model:

p = 0.7116 >= 0.2000, removing headroom

p = 0.3944 >= 0.2000, removing displacement gear\_ratio

p = 0.2798 >= 0.2000, removing price

| Source   | SS         | df | MS         | Number of obs | = | 74     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 1716.80842 | 4  | 429.202105 | F(4, 69)      | = | 40.76  |
| Residual | 726.651041 | 69 | 10.5311745 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.7026 |
|          |            |    |            | Adj R-squared | = | 0.6854 |
| Total    | 2443.45946 | 73 | 33.4720474 | Root MSE      | = | 3.2452 |

| mpg                   | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|-----------------------|-------------|-----------|-------|-------|----------------------|-----------|
| weight                | -.0160341   | .0039379  | -4.07 | 0.000 | -.0238901            | -.0081782 |
| c.weight#<br>c.weight | 1.70e-06    | 6.21e-07  | 2.73  | 0.008 | 4.58e-07             | 2.94e-06  |
| foreign               | -2.758668   | 1.101772  | -2.50 | 0.015 | -4.956643            | -.5606925 |
| Foreign<br>turn       | -.2862724   | .176658   | -1.62 | 0.110 | -.6386955            | .0661508  |
| _cons                 | 65.39216    | 8.208778  | 7.97  | 0.000 | 49.0161              | 81.76823  |

◀

## Search logic for a step

Before discussing the complete search logic, consider the logic for a step—the first step—in detail. The other steps follow the same logic. If you type

```
. stepwise, pr(.20): regress y1 x1 x2 (d1 d2 d3) (x4 x5)
```

the logic is

1. Fit the model  $y$  on  $x_1 x_2 d_1 d_2 d_3 x_4 x_5$ .
2. Consider dropping  $x_1$ .
3. Consider dropping  $x_2$ .
4. Consider dropping  $d_1 d_2 d_3$ .
5. Consider dropping  $x_4 x_5$ .
6. Find the term above that is least significant. If its significance level is  $\geq 0.20$ , remove that term.

If you type

```
. stepwise, pr(.20) hierarchical: regress y1 x1 x2 (d1 d2 d3) (x4 x5)
```

the logic would be different because the `hierarchical` option states that the terms are ordered. The initial logic would become

1. Fit the model  $y$  on  $x_1$   $x_2$   $d_1$   $d_2$   $d_3$   $x_4$   $x_5$ .
2. Consider dropping  $x_4$   $x_5$ —the last term.
3. If the significance of this last term is  $\geq 0.20$ , remove the term.

The process would then stop or continue. It would stop if  $x_4$   $x_5$  were not omitted, and otherwise, `stepwise` would continue to consider the significance of the next-to-last term,  $d_1$   $d_2$   $d_3$ .

Specifying `pe()` rather than `pr()` switches to forward estimation. If you type

```
. stepwise, pe(.20): regress y1 x1 x2 (d1 d2 d3) (x4 x5)
```

`stepwise` performs forward-selection search. The logic for the first step is

1. Fit a model of  $y$  on nothing (meaning a constant).
2. Consider adding  $x_1$ .
3. Consider adding  $x_2$ .
4. Consider adding  $d_1$   $d_2$   $d_3$ .
5. Consider adding  $x_4$   $x_5$ .
6. Find the term above that is most significant. If its significance level is  $< 0.20$ , add that term.

As with backward estimation, if you specify `hierarchical`,

```
. stepwise, pe(.20) hierarchical: regress y1 x1 x2 (d1 d2 d3) (x4 x5)
```

the search for the most significant term is restricted to the next term:

1. Fit a model of  $y$  on nothing (meaning a constant).
2. Consider adding  $x_1$ —the first term.
3. If the significance is  $< 0.20$ , add the term.

If  $x_1$  were added, `stepwise` would next consider  $x_2$ ; otherwise, the search process would stop.

`stepwise` can also use a stepwise selection logic that alternates between adding and removing terms. The full logic for all the possibilities is given below.

## Full search logic

| Option                                                              | Logic                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|---------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>pr()</code><br>(backward selection)                           | Fit the full model on all explanatory variables.<br>While the least-significant term is “insignificant”, remove it and reestimate.                                                                                                                                                                                                                                                                                                                                                 |
| <code>pr() hierarchical</code><br>(backward hierarchical selection) | Fit full model on all explanatory variables.<br>While the last term is “insignificant”, remove it and reestimate.                                                                                                                                                                                                                                                                                                                                                                  |
| <code>pr() pe()</code><br>(backward stepwise)                       | Fit full model on all explanatory variables.<br>If the least-significant term is “insignificant”, remove it and reestimate; otherwise, stop.<br>Do that again: if the least-significant term is “insignificant”, remove it and reestimate; otherwise, stop.<br>Repeatedly,<br>if the most-significant excluded term is “significant”, add it and reestimate;<br>if the least-significant included term is “insignificant”, remove it and reestimate;<br>until neither is possible. |
| <code>pe()</code><br>(forward selection)                            | Fit “empty” model.<br>While the most-significant excluded term is “significant”, add it and reestimate.                                                                                                                                                                                                                                                                                                                                                                            |
| <code>pe() hierarchical</code><br>(forward hierarchical selection)  | Fit “empty” model.<br>While the next term is “significant”, add it and reestimate.                                                                                                                                                                                                                                                                                                                                                                                                 |
| <code>pr() pe() forward</code><br>(forward stepwise)                | Fit “empty” model.<br>If the most-significant excluded term is “significant”, add it and reestimate; otherwise, stop.<br>Do that again: if the most-significant excluded term is “significant”, add it and reestimate; otherwise, stop.<br>Repeatedly,<br>if the least-significant included term is “insignificant”, remove it and reestimate;<br>if the most-significant excluded term is “significant”, add it and reestimate;<br>until neither is possible.                     |

## Examples

The following two statements are equivalent; both include solely single-variable terms:

```
. stepwise, pr(.2): regress price mpg weight displ
. stepwise, pr(.2): regress price (mpg) (weight) (displ)
```

The following two statements are equivalent; the last term in each is  $r_1, \dots, r_4$ :

```
. stepwise, pr(.2) hierarchical: regress price mpg weight displ (r1-r4)
. stepwise, pr(.2) hierarchical: regress price (mpg) (weight) (displ) (r1-r4)
```

To group variables `weight` and `displ` into one term, type

```
. stepwise, pr(.2) hierarchical: regress price mpg (weight displ) (r1-r4)
```

`stepwise` can be used with commands other than `regress`; for instance,

```
. stepwise, pr(.2): logit outcome (sex weight) treated1 treated2
. stepwise, pr(.2): logistic outcome (sex weight) treated1 treated2
```

Either statement would fit the same model because `logistic` and `logit` both perform logistic regression; they differ only in how they report results; see [\[R\] logit](#) and [\[R\] logistic](#).

We use the `lockterm1` option to force the first term to be included in the model. To keep `treated1` and `treated2` in the model no matter what, we type

```
. stepwise, pr(.2) lockterm1: logistic outcome (treated1 treated2) ...
```

After `stepwise` estimation, we can type `stepwise` without arguments to redisplay results,

```
. stepwise
 (output from logistic appears)
```

or type the underlying estimation command:

```
. logistic
 (output from logistic appears)
```

At estimation time, we can specify options unique to the command being stepped:

```
. stepwise, pr(.2): logit outcome (sex weight) treated1 treated2, or
```

or is `logit`'s option to report odds ratios rather than coefficients; see [\[R\] logit](#).

## Estimation sample considerations

Whether you use backward or forward estimation, `stepwise` forms an estimation sample by taking observations with nonmissing values of all the variables specified (except for  $depvar_1$  and  $depvar_2$  for `intreg`). The estimation sample is held constant throughout the stepping. Thus, if you type

```
. stepwise, pr(.2) hierarchical: regress amount sk edul sval
```

and variable `sval` is missing in half the data, that half of the data will not be used in the reported model, even if `sval` is not included in the final model.

The function `e(sample)` identifies the sample that was used. `e(sample)` contains 1 for observations used and 0 otherwise. For instance, if you type

```
. stepwise, pr(.2) pe(.10): logistic outcome x1 x2 (x3 x4) (x5 x6 x7)
```

and the final model is outcome on x1, x5, x6, and x7, you could re-create the final regression by typing

```
. logistic outcome x1 x5 x6 x7 if e(sample)
```

You could obtain summary statistics within the estimation sample of the independent variables by typing

```
. summarize x1 x5 x6 x7 if e(sample)
```

If you fit another model, `e(sample)` will automatically be redefined. Typing

```
. stepwise, lock pr(.2): logistic outcome (x1 x2) (x3 x4) (x5 x6 x7)
```

would automatically drop `e(sample)` and re-create it.

## Messages

**note: \_\_\_\_\_ omitted because of estimability.**

This indicates that a variable was omitted because its coefficient could not be estimated. This message is most commonly issued because the variable is collinear with other variables in the model. For instance, say that you type

```
. stepwise, pr(.2): regress y x1 x2 x3 x4
```

and x2 is collinear with x3, one of these variables will automatically be omitted. If you type

```
. stepwise, pr(.2): regress y x1 x2 i.a
```

and include indicators for factor variable a in your model, the set of indicators for a are perfectly collinear, and one will be omitted with the note indicating that it was omitted because of estimability.

**note: \_\_\_\_\_ omitted because of estimability.**

**note: \_\_\_\_\_ obs omitted because of estimability.**

You probably received this message in fitting a logistic or probit model. Regardless of estimation strategy, `stepwise` checks that the full model can be fit. The indicated variable had a 0 or infinite standard error.

For logistic, logit, and probit, this message is typically caused by one-way causation. Assume that you type

```
. stepwise, pr(.2): logistic outcome (x1 x2 x3) d1
```

and assume that variable d1 is an indicator (dummy) variable. Further assume that whenever  $d1 = 1$ ,  $outcome = 1$  in the data. Then the coefficient on d1 is infinite. One (conservative) solution to this problem is to drop the d1 variable and the  $d1=1$  observations. The underlying estimation commands `probit`, `logit`, and `logistic` report the details of the difficulty and solution; `stepwise` simply accumulates such problems and reports the above summary messages. Thus, if you see this message, you could type

```
. logistic outcome x1 x2 x3 d1
```

to see the details. Although you should think carefully about such situations, Stata's solution of dropping the offending variables and observations is, in general, appropriate.

## Programming for stepwise

stepwise requires that *command\_name* follow standard Stata syntax and allow the `if` qualifier; see [U] **11 Language syntax**. Furthermore, *command\_name* must have `sw` or `swml` as a program property; see [P] **program properties**. If *command\_name* has `swml` as a property, *command\_name* must store the log-likelihood value in `e(ll)` and model degrees of freedom in `e(df_m)`.

## Stored results

stepwise stores whatever is stored by the underlying estimation command.

Also, stepwise stores stepwise in `e(stepwise)`.

## Methods and formulas

Some statisticians do not recommend stepwise procedures; see [Sribney \(1998\)](#) for a summary.

## References

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- Sribney, W. M. 1998. FAQ: What are some of the problems with stepwise regression? <https://www.stata.com/support/faqs/stat/stepwise.html>.
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## Also see

[R] [nestreg](#) — Nested model statistics

[LASSO] [Lasso intro](#) — Introduction to lasso

## Description

Results of calculations are stored by many Stata commands so that they can be easily accessed and substituted into later commands.

`return list` lists results stored in `r()`.

`ereturn list` lists results stored in `e()`.

`sreturn list` lists results stored in `s()`.

This entry discusses using stored results. Programmers wishing to store results should see [\[P\] return](#) and [\[P\] ereturn](#).

## Syntax

*List results from general commands, stored in r()*

```
return list [, all]
```

*List results from estimation commands, stored in e()*

```
ereturn list [, all]
```

*List results from parsing commands, stored in s()*

```
sreturn list
```

## Option

`all` is for use with `return list` and `ereturn list`. `all` specifies that hidden and historical stored results be listed along with the usual stored results. This option is seldom used. See [Using hidden and historical stored results](#) and [Programming hidden and historical stored results](#) under [Remarks and examples](#) of [\[P\] return](#) for more information. These sections are written in terms of `return list`, but everything said there applies equally to `ereturn list`.

`all` is not allowed with `sreturn list` because `s()` does not allow hidden or historical results.

## Remarks and examples

Stata commands are classified as being

|         |                                                                                         |
|---------|-----------------------------------------------------------------------------------------|
| r-class | general commands that store results in <code>r()</code>                                 |
| e-class | estimation commands that store results in <code>e()</code>                              |
| s-class | parsing commands that store results in <code>s()</code>                                 |
| n-class | commands that do not store in <code>r()</code> , <code>e()</code> , or <code>s()</code> |

There is also a `c`-class, `c()`, containing the values of system parameters and settings, along with certain constants, such as the value of `pi`; see [P] [creturn](#). A program, however, cannot be `c`-class.

You can look at the *Stored results* section of the manual entry of a command to determine whether it is `r`-, `e`-, `s`-, or `n`-class, but it is easy enough to guess.

Commands producing statistical results are either `r`-class or `e`-class. They are `e`-class if they present estimation results and `r`-class otherwise. `s`-class is a class used by programmers and is primarily used in subprograms performing parsing. `n`-class commands explicitly state where the result is to go. For instance, `generate` and `replace` are `n`-class because their syntax is `generate varname = ...` and `replace varname = ....`

After executing a command, you can type `return list`, `ereturn list`, or `sreturn list` to see what has been stored.

### ▷ Example 1

```
. use https://www.stata-press.com/data/r19/auto4
(1978 automobile data)
. describe
Contains data from https://www.stata-press.com/data/r19/auto4.dta
Observations: 74 1978 automobile data
Variables: 6 6 Apr 2024 00:20
```

| Variable name | Storage type | Display format | Value label | Variable label     |
|---------------|--------------|----------------|-------------|--------------------|
| price         | int          | %8.0gc         |             | Price              |
| weight        | int          | %8.0gc         |             | Weight (lbs.)      |
| mpg           | byte         | %8.0g          |             | Mileage (mpg)      |
| make          | str17        | %-17s          |             | Make and model     |
| length        | int          | %8.0g          |             | Length (in.)       |
| rep78         | byte         | %8.0g          |             | Repair record 1978 |

Sorted by:

```
. return list
```

scalars:

```
 r(changed) = 0
 r(width) = 25
 r(k) = 6
 r(N) = 74
```

macros:

```
 r(datalabel) : "1978 automobile data"
```

To view all stored results, including those that are historical or hidden, specify the `all` option.

```
. return list, all
scalars:
 r(changed) = 0
 r(width) = 25
 r(k) = 6
 r(N) = 74

macros:
 r(datalabel) : "1978 automobile data"
Historical; used before Stata 12, may exist only under version control
scalars:
 r(widthmax) = 1048576
 r(k_max) = 5000
 r(N_max) = 2147483619
```

`r(widthmax)`, `r(k_max)`, and `r(N_max)` are historical stored results. They are no longer relevant because Stata dynamically adjusts memory beginning with Stata 12.



### □ Technical note

In the above example, we stated that `r(widthmax)` and `r(N_max)` are no longer relevant. In fact, they are not useful. Stata no longer has a fixed memory size, so the methods used to calculate `r(widthmax)` and `r(N_max)` are no longer appropriate.



### ▷ Example 2

You can use stored results in expressions.

```
. summarize mpg
 Variable | Obs Mean Std. dev. Min Max
-----|-----
 mpg | 74 21.2973 5.785503 12 41

. return list
scalars:
 r(N) = 74
 r(sum_w) = 74
 r(mean) = 21.2972972972973
 r(Var) = 33.47204738985561
 r(sd) = 5.785503209735141
 r(min) = 12
 r(max) = 41
 r(sum) = 1576

. generate double mpgstd = (mpg-r(mean))/r(sd)
. summarize mpgstd
 Variable | Obs Mean Std. dev. Min Max
-----|-----
 mpgstd | 74 -1.64e-16 1 -1.606999 3.40553
```

Be careful to use results stored in `r()` soon because they will be replaced the next time you execute another `r-class` command. For instance, although `r(mean)` was 21.3 (approximately) after `summarize mpg`, it is  $-1.64e-16$  now because you just ran `summarize` with `mpgstd`.



### ▶ Example 3

`e-class` is really no different from `r-class`, except for where results are stored and that, when an estimation command stores results, it tends to store a lot of them:

```
. regress mpg weight length
 (output omitted)
. ereturn list
scalars:
 e(N) = 74
e(sum_w) = 74
 e(df_m) = 2
 e(df_r) = 71
 e(F) = 69.34050004300227
 e(r2) = .6613903979336323
 e(rmse) = 3.413681741382589
 e(mss) = 1616.08062422659
 e(rss) = 827.3788352328695
 e(r2_a) = .6518520992838754
 e(ll) = -194.3267619410807
 e(ll_0) = -234.3943376482347
 e(rank) = 3

macros:
 e(cmdline) : "regress mpg weight length"
 e(title) : "Linear regression"
 e(marginsok) : "XB default"
 e(vce) : "ols"
 e(depvar) : "mpg"
 e(cmd) : "regress"
 e(properties) : "b V"
 e(predict) : "regres_p"
 e(model) : "ols"
 e(estat_cmd) : "regress_estat"

matrices:
 e(b) : 1 x 3
 e(V) : 3 x 3
 e(beta) : 1 x 2

functions:
 e(sample)
```

These `e-class` results will stick around until you run another estimation command. Typing `return list` and `ereturn list` is the easy way to find out what a command stores.



Both `r-` and `e-class` results come in four types: scalars, macros, matrices, and functions. (`s-class` results come in only one type—macros—and as earlier noted, `s-class` is used solely by programmers, so ignore it.)

Scalars are just that—numbers by any other name. You can subsequently refer to `r(mean)` or `e(rmse)` in numeric expressions and obtain the result to full precision.

Macros are strings. For instance, `e(depvar)` contains “mpg”. You can refer to it, too, in subsequent expressions, but really that would be of most use to programmers, who will refer to it using constructs like “`e(depvar)`”. In any case, macros are macros, and you obtain their contents just as you would a local macro, by enclosing their name in single quotes. The name here is the full name, so ‘`e(depvar)`’ is `mpg`.

Matrices are matrices, and all estimation commands store `e(b)` and `e(V)` containing the coefficient vector and variance–covariance matrix of the estimates (VCE).

Functions are stored by e-class commands only, and the only function existing is `e(sample)`. `e(sample)` evaluates to 1 (meaning true) if the observation was used in the previous estimation and to 0 (meaning false) otherwise.

## □ Technical note

Say that some command set `r(scalar)` and `r(macro)`, the first being stored as a scalar and the second as a macro. In theory, in subsequent use you are supposed to refer to `r(scalar)` and `'r(macro)'`. In fact, however, you can refer to either one with or without quotes, so you could refer to `'r(scalar)'` and `r(macro)`. Programmers sometimes do this.

When you refer to `r(scalar)`, you are referring to the full double-precision stored result. Think of `r(scalar)` without quotes as a function returning the value of the stored result `scalar`. When you refer to `r(scalar)` in quotes, Stata understands `'r(scalar)'` to mean “substitute the printed result of evaluating `r(scalar)`”. Pretend that `r(scalar)` equals the number 23. Then, `'r(scalar)'` is 23, the character 2 followed by 3.

Referring to `r(scalar)` in quotes is sometimes useful. Say that you want to use the immediate command `cii` with `r(scalar)`. The immediate command `cii` requires its arguments to be numeric—numeric literals in programmer’s jargon—and it will not take an expression. Thus, you could not type `'cii r(scalar) ...'`. You could, however, type `'cii 'r(scalar)' ...'` because `'r(scalar)'` is just a numeric literal.

For `r(macro)`, you are supposed to refer to it in quotes: `'r(macro)'`. If, however, you omit the quotes in an expression context, Stata evaluates the macro and then pretends that it is the result of function-returning-string. There are side effects of this, the most important being that the result is trimmed to 80 characters.

Referring to `r(macro)` without quotes is never a good idea; the feature was included merely for completeness.

You can even refer to `r(matrix)` in quotes (assume that `r(matrix)` is a matrix). `'r(matrix)'` does not result in the matrix being substituted; it returns the word `matrix`. Programmers sometimes find that useful.

□

## References

- Jann, B. 2005. [Making regression tables from stored estimates](#). *Stata Journal* 5: 288–308.  
 ———. 2007. [Making regression tables simplified](#). *Stata Journal* 7: 227–244.

## Also see

[P] [ereturn](#) — Post the estimation results

[P] [return](#) — Return stored results

[U] [18.8 Accessing results calculated by other programs](#)

[U] [18.9 Accessing results calculated by estimation commands](#)

[Description](#)[Options](#)[Acknowledgment](#)[Quick start](#)[Remarks and examples](#)[References](#)[Menu](#)[Stored results](#)[Also see](#)[Syntax](#)[Methods and formulas](#)

## Description

`suest` is a postestimation command; see [\[U\] 20 Estimation and postestimation commands](#).

`suest` combines the estimation results—parameter estimates and associated (co)variance matrices—stored under *namelist* into one parameter vector and *simultaneous* (co)variance matrix of the sandwich/robust type. This (co)variance matrix is appropriate even if the estimates were obtained on the same or on overlapping data.

Typical applications of `suest` are tests for intramodel and cross-model hypotheses using `test` or `testnl`, for example, a generalized Hausman specification test. `lincom` and `nlcom` may be used after `suest` to estimate linear combinations and nonlinear functions of coefficients. `suest` may also be used to adjust a standard VCE for clustering or survey design effects.

Different estimators are allowed, for example, a `regress` model and a `probit` model; the only requirement is that `predict` produce equation-level scores with the `score` option after an estimation command. The models may be estimated on different samples, due either to explicit `if` or `in` selection or to missing values. If weights are applied, the same weights (type and values) should be applied to all models in *namelist*. The estimators should be estimated without `vce(robust)` or `vce(cluster clustvar)` options. `suest` returns the robust VCE, allows the `vce(cluster clustvar)` option, and automatically works with results from the `svy` prefix command (only for `vce(linearized)`). See [example 8](#) in [\[SVY\] svy postestimation](#) for an example using `suest` with `svy: ologit`.

Because `suest` posts its results like a proper estimation command, its results can be stored via [estimates store](#). Moreover, like other estimation commands, `suest` typed without arguments replays the results.

## Quick start

Combined results for stored estimates `m1` and `m2`

```
suest m1 m2
```

Same as above, but report exponentiated coefficients and label them “Odds ratio”

```
suest m1 m2, eform(Odds ratio)
```

With cluster–robust standard errors adjusting for clustering by levels of `cvar`

```
suest m1 m2, vce(cluster cvar)
```

Use `svyset` data after specifying command prefix `svy:` to estimate `m1` and `m2`

```
suest m1 m2
```

## Menu

Statistics > Postestimation

## Syntax

```
suest namelist [, options]
```

where *namelist* is a list of one or more names under which estimation results were stored via `estimates store`; see [R] [estimates store](#). Wildcards may be used. \* and `_all` refer to all stored results. A period (.) may be used to refer to the last estimation results, even if they have not (yet) been stored.

| <i>options</i>                    | Description                                                                                                                                      |
|-----------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| SE/Robust                         |                                                                                                                                                  |
| <code>svy</code>                  | survey data estimation                                                                                                                           |
| <code>vce(<i>vcetype</i>)</code>  | <i>vcetype</i> may be <code>robust</code> or <code>cluster <i>clustvar</i></code>                                                                |
| Reporting                         |                                                                                                                                                  |
| <code>level(#)</code>             | set confidence level; default is <code>level(95)</code>                                                                                          |
| <code>dir</code>                  | display a table describing the models                                                                                                            |
| <code>eform(<i>string</i>)</code> | report exponentiated coefficients and label as <i>string</i>                                                                                     |
| <code>display_options</code>      | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| <code>coeflegend</code>           | display legend instead of statistics                                                                                                             |

`collect` is allowed; see [U] [11.1.10 Prefix commands](#).

`coeflegend` does not appear in the dialog box.

## Options

### SE/Robust

`svy` specifies that estimation results should be modified to reflect the survey design effects according to the `svyset` specifications, see [SVY] [svyset](#).

The `svy` option is implied when `suest` encounters survey estimation results from the `svy` prefix; see [SVY] [svy](#). Poststratification is allowed only with survey estimation results from the `svy` prefix.

`vce(vcetype)` specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (`robust`) and that allow for intragroup correlation (`cluster clustvar`); see [R] [vce\\_option](#).

The `vce()` option may not be combined with the `svy` option or estimation results from the `svy` prefix.

### Reporting

`level(#)` specifies the confidence level, as a percentage, for confidence intervals of the coefficients; see [R] [level](#).

`dir` displays a table describing the models in *namelist* just like `estimates dir namelist`.

`eform(string)` displays the coefficient table in exponentiated form: for each coefficient,  $\exp(b)$  rather than  $b$  is displayed, and standard errors and confidence intervals are transformed. *string* is the table header that will be displayed above the transformed coefficients and must be 11 characters or fewer, for example, `eform("Odds ratio")`.

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fwrap(#)`, `fwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

The following option is available with `suest` but is not shown in the dialog box:  
`coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Remarks are presented under the following headings:

*Using suest*

*Remarks on regress*

*Testing the assumption of the independence of irrelevant alternatives*

*Testing proportionality*

*Testing cross-model hypotheses*

## Using suest

If you plan to use `suest`, you must take precautions when fitting the original models. These restrictions are relaxed when using `svy` commands; see [SVY] [svy postestimation](#).

1. `suest` works with estimation commands that allow `predict` to generate equation-level score variables when supplied with the `score` (or `scores`) option. For example, equation-level score variables are generated after running `mlogit` by typing

```
. predict sc*, scores
```

To ensure that `suest` generates the appropriate scores and therefore an appropriate VCE, you should not modify the values of the variables used in the models in between fitting the models and using `suest`.

2. Estimation should take place *without* the `vce(robust)` or `vce(cluster clustvar)` option. `suest` always computes the robust estimator of the (co)variance, and `suest` has a `vce(cluster clustvar)` option.

The within-model covariance matrices computed by `suest` are identical to those obtained by specifying a `vce(robust)` or `vce(cluster clustvar)` option during estimation. `suest`, however, also estimates the between-model covariances of parameter estimates.

3. Finally, the estimation results to be combined should be stored by `estimates store`; see [R] [estimates store](#).

After estimating and storing a series of estimation results, you are ready to combine the estimation results with `suest`,

```
. suest name1 [name2 ...] [, vce(cluster clustvar)]
```

and you can subsequently use postestimation commands, such as `test`, to test hypotheses. Here an important issue is how `suest` assigns names to the equations. If you specify one model *name*, the original equation names are left unchanged; otherwise, `suest` constructs new equation names. The coefficients of a single-equation model (such as `logit` and `poisson`) that was `estimate` stored under name *X* are collected under equation *X*. With a multiequation model stored under name *X*, `suest` prefixes *X\_* to an original equation name *eq*, forming equation name, *X\_eq*.

### □ Technical note

Earlier we said that standard errors from `suest` are identical to those obtained by specifying the `vce(robust)` option with each command individually. Thus if you fit a logistic model using `logit` with the `vce(robust)` option, you will get the same standard errors when you type

```
. suest .
```

directly after `logit` using the same data without the `vce(robust)` option.

This is not true for multiple estimation results when the estimation samples are not all the same. The standard errors from `suest` will be slightly smaller than those from individual model fits using the `vce(robust)` option because `suest` uses a larger number of observations to estimate the simultaneous (co)variance matrix. □

### □ Technical note

In rare circumstances, `suest` may have to truncate equation names to 32 characters. When equation names are not unique because of truncation, `suest` numbers the equations within models, using equations named *X\_#*. □

## Remarks on `regress`

`regress` (see [R] `regress`) does not include its ancillary parameter, the residual variance, in its coefficient vector and (co)variance matrix. Moreover, while the `score` option is allowed with `predict` after `regress`, a score variable is generated for the mean but not for the variance parameter. `suest` contains special code that assigns the equation name `mean` to the coefficients for the mean, adds the equation `lnvar` for the log variance, and computes the appropriate two score variables itself.

## Testing the assumption of the independence of irrelevant alternatives

The multinomial logit model and the closely related conditional logit model satisfy a probabilistic version of the assumption of the independence of irrelevant alternatives (IIA), implying that the ratio of the probabilities for two alternatives does not depend on what other alternatives are available. [Hausman and McFadden \(1984\)](#) proposed a test for this assumption that is implemented in the `hausman` command. The standard Hausman test has several limitations. First, the test statistic may be undefined because the estimated VCE does not satisfy the required asymptotic properties of the test. Second, the classic Hausman test applies only to the test of the equality of two estimators. Third, the test requires access to a fully efficient estimator; such an estimator may not be available, for example, if you are analyzing complex survey data. Using `suest` can overcome these three limitations.

### ▷ Example 1

In our first example, we follow the analysis of the type of health insurance reported in [\[R\] mlogit](#) and demonstrate the `hausman` command with the `suest/test` combination. We fit the full multinomial logit model for all three alternatives and two restricted multinomial models in which one alternative is excluded. After fitting each of these models, we give them a title with the `estimates title` command and then store them with the `estimates store` command.

```
. use https://www.stata-press.com/data/r19/sysdsn4
(Health insurance data)
. mlogit insure age male

Iteration 0: Log likelihood = -555.85446
Iteration 1: Log likelihood = -551.32973
Iteration 2: Log likelihood = -551.32802
Iteration 3: Log likelihood = -551.32802

Multinomial logistic regression Number of obs = 615
LR chi2(4) = 9.05
Prob > chi2 = 0.0598
Pseudo R2 = 0.0081

Log likelihood = -551.32802
```

| insure    | Coefficient    | Std. err. | z     | P> z  | [95% conf. interval] |           |
|-----------|----------------|-----------|-------|-------|----------------------|-----------|
| Indemnity | (base outcome) |           |       |       |                      |           |
| Prepaid   |                |           |       |       |                      |           |
| age       | -.0100251      | .0060181  | -1.67 | 0.096 | -.0218204            | .0017702  |
| male      | .5095747       | .1977893  | 2.58  | 0.010 | .1219147             | .8972346  |
| _cons     | .2633838       | .2787575  | 0.94  | 0.345 | -.2829708            | .8097383  |
| Uninsure  |                |           |       |       |                      |           |
| age       | -.0051925      | .0113821  | -0.46 | 0.648 | -.0275011            | .0171161  |
| male      | .4748547       | .3618462  | 1.31  | 0.189 | -.2343508            | 1.18406   |
| _cons     | -1.756843      | .5309602  | -3.31 | 0.001 | -2.797506            | -.7161803 |

```
. estimates title: All three insurance forms
. estimates store m1
. quietly mlogit insure age male if insure != "Uninsure":insure
. estimates title: insure != "Uninsure":insure
. estimates store m2
. quietly mlogit insure age male if insure != "Prepaid":insure
. estimates title: insure != "Prepaid":insure
. estimates store m3
```

Having performed the three estimations, we inspect the results. `estimates dir` provides short descriptions of the models that were stored using `estimates store`. Typing `estimates table` lists the coefficients, displaying blanks for a coefficient not contained in a model.

```
. estimates dir
```

| Name | Command | Dependent variable | Number of param. | Title                       |
|------|---------|--------------------|------------------|-----------------------------|
| m1   | mlogit  | insure             | 9                | All three insurance forms   |
| m2   | mlogit  | insure             | 6                | insure != "Uninsure":insure |
| m3   | mlogit  | insure             | 6                | insure != "Prepaid":insure  |

```
. estimates table m1 m2 m3, star stats(N ll) keep(Prepaid: Uninsure:)
```

| Variable   | m1            | m2          | m3            |
|------------|---------------|-------------|---------------|
| Prepaid    |               |             |               |
| age        | -.01002511    | -.01015205  |               |
| male       | .50957468**   | .51440033** |               |
| _cons      | .26338378     | .26780432   |               |
| Uninsure   |               |             |               |
| age        | -.00519249    |             | -.00410547    |
| male       | .47485472     |             | .45910738     |
| _cons      | -1.7568431*** |             | -1.8017743*** |
| Statistics |               |             |               |
| N          | 615           | 570         | 338           |
| ll         | -551.32802    | -390.48643  | -131.76807    |

Legend: \* p<0.05; \*\* p<0.01; \*\*\* p<0.001

Comparing the coefficients between models does not suggest substantial differences. We can formally test that coefficients are the same for the full model m1 and the restricted models m2 and m3 by using the `hausman` command. `hausman` expects the models to be specified in the order “always consistent” first and “efficient under  $H_0$ ” second.

```
. hausman m2 m1, alleqs constant
```

|       | Coefficients |           | (b-B)<br>Difference | sqrt(diag(V_b-V_B))<br>Std. err. |
|-------|--------------|-----------|---------------------|----------------------------------|
|       | (b)<br>m2    | (B)<br>m1 |                     |                                  |
| age   | -.0101521    | -.0100251 | -.0001269           | .                                |
| male  | .5144003     | .5095747  | .0048256            | .0123338                         |
| _cons | .2678043     | .2633838  | .0044205            | .                                |

b = Consistent under  $H_0$  and  $H_a$ ; obtained from `mlogit`.

B = Inconsistent under  $H_a$ , efficient under  $H_0$ ; obtained from `mlogit`.

Test of  $H_0$ : Difference in coefficients not systematic

$$\text{chi2}(3) = (b-B)' [(V_b-V_B)^{-1}] (b-B) = 0.08$$

Prob > chi2 = 0.9944

(V\_b-V\_B is not positive definite)

```
. hausman m3 m1, alleqs constant
```

|       | Coefficients |           | (b-B)<br>Difference | sqrt(diag(V_b-V_B))<br>Std. err. |
|-------|--------------|-----------|---------------------|----------------------------------|
|       | (b)<br>m3    | (B)<br>m1 |                     |                                  |
| age   | -.0041055    | -.0051925 | .001087             | .0021355                         |
| male  | .4591074     | .4748547  | -.0157473           | .                                |
| _cons | -1.801774    | -1.756843 | -.0449311           | .1333421                         |

b = Consistent under H0 and Ha; obtained from **mlogit**.  
 B = Inconsistent under Ha, efficient under H0; obtained from **mlogit**.

Test of H0: Difference in coefficients not systematic

$$\text{chi2}(3) = (b-B)' [(V_b-V_B)^{-1}] (b-B) = -0.18$$

Warning: chi2 < 0 ==> model fitted on these data fails to meet the asymptotic assumptions of the Hausman test; see **suest** for a generalized test.

According to the test of m1 against m2, we cannot reject the hypothesis that the coefficients of m1 and m2 are the same. The second Hausman test is not well defined—something that happens fairly often. The problem is due to the estimator of the variance  $V(b-B)$  as  $V(b)-V(B)$ , which is a feasible estimator only asymptotically. Here it simply is not a proper variance matrix, and the Hausman test becomes undefined.

**suest** m1 m2 estimates the simultaneous (co)variance of the coefficients of models m1 and m2. Although **suest** is technically a postestimation command, it acts like an estimation command in that it stores the simultaneous coefficients in  $e(b)$  and the full (co)variance matrix in  $e(V)$ . We could have used the **estat vce** command to display the full (co)variance matrix to show that the cross-model covariances were indeed estimated. Typically, we would not have a direct interest in  $e(V)$ .

```
. suest m1 m2, noomitted
```

Simultaneous results for m1, m2 Number of obs = 615

|                     | Coefficient | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |          |
|---------------------|-------------|---------------------|-------|-------|----------------------|----------|
| <b>m1_Indemnity</b> |             |                     |       |       |                      |          |
| <b>m1_Prepaid</b>   |             |                     |       |       |                      |          |
| age                 | -.0100251   | .0059403            | -1.69 | 0.091 | -.0216679            | .0016176 |
| male                | .5095747    | .1988159            | 2.56  | 0.010 | .1199027             | .8992467 |
| _cons               | .2633838    | .277307             | 0.95  | 0.342 | -.280128             | .8068956 |
| <b>m1_Uninsure</b>  |             |                     |       |       |                      |          |
| age                 | -.0051925   | .0109005            | -0.48 | 0.634 | -.0265571            | .0161721 |
| male                | .4748547    | .3677326            | 1.29  | 0.197 | -.2458879            | 1.195597 |
| _cons               | -1.756843   | .4971383            | -3.53 | 0.000 | -2.731216            | -.78247  |
| <b>m2_Indemnity</b> |             |                     |       |       |                      |          |
| <b>m2_Prepaid</b>   |             |                     |       |       |                      |          |
| age                 | -.0101521   | .0058988            | -1.72 | 0.085 | -.0217135            | .0014094 |
| male                | .5144003    | .1996133            | 2.58  | 0.010 | .1231654             | .9056352 |
| _cons               | .2678043    | .2744019            | 0.98  | 0.329 | -.2700134            | .8056221 |

suest created equation names by combining the name under which we stored the results using estimates store with the original equation names. Thus, in the simultaneous estimation result, equation Prepaid originating in model m1 is named m1\_Prepaid. According to the McFadden–Hausman specification of a test for IIA, the coefficients of the equations m1\_PrePaid and m2\_PrePaid should be equal. This equality can be tested easily with the test command. The cons option specifies that the intercept \_cons be included in the test.

```
. test [m1_Prepaid = m2_Prepaid], cons
(1) [m1_Prepaid]age - [m2_Prepaid]age = 0
(2) [m1_Prepaid]male - [m2_Prepaid]male = 0
(3) [m1_Prepaid]_cons - [m2_Prepaid]_cons = 0
 chi2(3) = 0.89
 Prob > chi2 = 0.8266
```

The Hausman test via suest is comparable with that computed by hausman, but they use different estimators of the variance of the difference of the estimates. The hausman command estimates  $V(b - B)$  by  $V(b) - V(B)$ , whereas suest estimates  $V(b - B)$  by  $V(b) - \text{cov}(b, B) - \text{cov}(B, b) + V(B)$ . One advantage of the second estimator is that it is always admissible, so the resulting test is always well defined. This quality is illustrated in the Hausman-type test of IIA comparing models m1 and m3.

```
. suest m1 m3, noomitted
Simultaneous results for m1, m3 Number of obs = 615
```

|              | Robust      |           | z     | P> z  | [95% conf. interval] |           |
|--------------|-------------|-----------|-------|-------|----------------------|-----------|
|              | Coefficient | std. err. |       |       |                      |           |
| <hr/>        |             |           |       |       |                      |           |
| m1_Indemnity |             |           |       |       |                      |           |
| <hr/>        |             |           |       |       |                      |           |
| m1_Prepaid   |             |           |       |       |                      |           |
| age          | -.0100251   | .0059403  | -1.69 | 0.091 | -.0216679            | .0016176  |
| male         | .5095747    | .1988159  | 2.56  | 0.010 | .1199027             | .8992467  |
| _cons        | .2633838    | .277307   | 0.95  | 0.342 | -.280128             | .8068956  |
| <hr/>        |             |           |       |       |                      |           |
| m1_Uninsure  |             |           |       |       |                      |           |
| age          | -.0051925   | .0109005  | -0.48 | 0.634 | -.0265571            | .0161721  |
| male         | .4748547    | .3677326  | 1.29  | 0.197 | -.2458879            | 1.195597  |
| _cons        | -1.756843   | .4971383  | -3.53 | 0.000 | -2.731216            | -.78247   |
| <hr/>        |             |           |       |       |                      |           |
| m3_Indemnity |             |           |       |       |                      |           |
| <hr/>        |             |           |       |       |                      |           |
| m3_Uninsure  |             |           |       |       |                      |           |
| age          | -.0041055   | .0111185  | -0.37 | 0.712 | -.0258974            | .0176865  |
| male         | .4591074    | .3601307  | 1.27  | 0.202 | -.2467357            | 1.164951  |
| _cons        | -1.801774   | .5226351  | -3.45 | 0.001 | -2.82612             | -.7774283 |

```
. test [m1_Uninsure = m3_Uninsure], cons
(1) [m1_Uninsure]age - [m3_Uninsure]age = 0
(2) [m1_Uninsure]male - [m3_Uninsure]male = 0
(3) [m1_Uninsure]_cons - [m3_Uninsure]_cons = 0
 chi2(3) = 1.49
 Prob > chi2 = 0.6845
```

Although the classic Hausman test computed by hausman is not defined here, the suest-based test is just fine. We cannot reject the equality of the common coefficients across m1 and m3.

A second advantage of the `suest` approach is that we can estimate the (co)variance matrix of the multivariate normal distribution of the estimators of the three models `m1`, `m2`, and `m3` and test that the common coefficients are equal.

```
. suest m*, noomitted
Simultaneous results for m1, m2, m3 Number of obs = 615
```

|                     | Coefficient | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |           |
|---------------------|-------------|---------------------|-------|-------|----------------------|-----------|
| <b>m1_Indemnity</b> |             |                     |       |       |                      |           |
| <b>m1_Prepaid</b>   |             |                     |       |       |                      |           |
| age                 | -.0100251   | .0059403            | -1.69 | 0.091 | -.0216679            | .0016176  |
| male                | .5095747    | .1988159            | 2.56  | 0.010 | .1199027             | .8992467  |
| _cons               | .2633838    | .277307             | 0.95  | 0.342 | -.280128             | .8068956  |
| <b>m1_Uninsure</b>  |             |                     |       |       |                      |           |
| age                 | -.0051925   | .0109005            | -0.48 | 0.634 | -.0265571            | .0161721  |
| male                | .4748547    | .3677326            | 1.29  | 0.197 | -.2458879            | 1.195597  |
| _cons               | -1.756843   | .4971383            | -3.53 | 0.000 | -2.731216            | -.78247   |
| <b>m2_Indemnity</b> |             |                     |       |       |                      |           |
| <b>m2_Prepaid</b>   |             |                     |       |       |                      |           |
| age                 | -.0101521   | .0058988            | -1.72 | 0.085 | -.0217135            | .0014094  |
| male                | .5144003    | .1996133            | 2.58  | 0.010 | .1231654             | .9056352  |
| _cons               | .2678043    | .2744019            | 0.98  | 0.329 | -.2700134            | .8056221  |
| <b>m3_Indemnity</b> |             |                     |       |       |                      |           |
| <b>m3_Uninsure</b>  |             |                     |       |       |                      |           |
| age                 | -.0041055   | .0111185            | -0.37 | 0.712 | -.0258974            | .0176865  |
| male                | .4591074    | .3601307            | 1.27  | 0.202 | -.2467357            | 1.164951  |
| _cons               | -1.801774   | .5226351            | -3.45 | 0.001 | -2.82612             | -.7774283 |

```
. test [m1_Prepaid = m2_Prepaid] , cons notest
(1) [m1_Prepaid]age - [m2_Prepaid]age = 0
(2) [m1_Prepaid]male - [m2_Prepaid]male = 0
(3) [m1_Prepaid]_cons - [m2_Prepaid]_cons = 0
. test [m1_Uninsure = m3_Uninsure], cons acc
(1) [m1_Prepaid]age - [m2_Prepaid]age = 0
(2) [m1_Prepaid]male - [m2_Prepaid]male = 0
(3) [m1_Prepaid]_cons - [m2_Prepaid]_cons = 0
(4) [m1_Uninsure]age - [m3_Uninsure]age = 0
(5) [m1_Uninsure]male - [m3_Uninsure]male = 0
(6) [m1_Uninsure]_cons - [m3_Uninsure]_cons = 0
 chi2(6) = 1.95
 Prob > chi2 = 0.9240
```

Again, we do not find evidence against the correct specification of the multinomial logit for type of insurance. The classic Hausman test assumes that one of the estimators (named `B` in `hausman`) is efficient, that is, it has minimal (asymptotic) variance. This assumption ensures that  $V(b) - V(B)$  is an admissible, viable estimator for  $V(b - B)$ . The assumption that we have an efficient estimator is a restrictive one. It is violated, for instance, if our data are clustered. We want to adjust for clustering via a `vce(cluster clustvar)` option by requesting the cluster-adjusted sandwich estimator of variance. Consequently, in such a case, `hausman` cannot be used. This problem does not exist with the `suest`

version of the Hausman test. To illustrate this feature, we suppose that the data are clustered by city—we constructed an imaginary variable `cityid` for this illustration. If we plan to apply `suest`, we would not specify the `vce(cluster clustvar)` option at the time of estimation. `suest` has a `vce(cluster clustvar)` option. Thus, we do not need to refit the models; we can call `suest` and `test` right away.

```
. suest m1 m2, vce(cluster cityid) noomitted
Simultaneous results for m1, m2 Number of obs = 615
 (Std. err. adjusted for 260 clusters in cityid)
```

|              | Coefficient | Robust<br>std. err. | z     | P> z  | [95% conf. interval] |           |
|--------------|-------------|---------------------|-------|-------|----------------------|-----------|
| <hr/>        |             |                     |       |       |                      |           |
| m1_Indemnity |             |                     |       |       |                      |           |
| <hr/>        |             |                     |       |       |                      |           |
| m1_Prepaid   |             |                     |       |       |                      |           |
| age          | -.0100251   | .005729             | -1.75 | 0.080 | -.0212538            | .0012035  |
| male         | .5095747    | .1910496            | 2.67  | 0.008 | .1351244             | .884025   |
| _cons        | .2633838    | .2698797            | 0.98  | 0.329 | -.2655708            | .7923384  |
| <hr/>        |             |                     |       |       |                      |           |
| m1_Uninsure  |             |                     |       |       |                      |           |
| age          | -.0051925   | .0104374            | -0.50 | 0.619 | -.0256495            | .0152645  |
| male         | .4748547    | .3774021            | 1.26  | 0.208 | -.2648399            | 1.214549  |
| _cons        | -1.756843   | .4916613            | -3.57 | 0.000 | -2.720481            | -.7932048 |
| <hr/>        |             |                     |       |       |                      |           |
| m2_Indemnity |             |                     |       |       |                      |           |
| <hr/>        |             |                     |       |       |                      |           |
| m2_Prepaid   |             |                     |       |       |                      |           |
| age          | -.0101521   | .0057164            | -1.78 | 0.076 | -.0213559            | .0010518  |
| male         | .5144003    | .1921385            | 2.68  | 0.007 | .1378158             | .8909848  |
| _cons        | .2678043    | .2682193            | 1.00  | 0.318 | -.2578959            | .7935045  |
| <hr/>        |             |                     |       |       |                      |           |

```
. test [m1_Prepaid = m2_Prepaid], cons
(1) [m1_Prepaid]age - [m2_Prepaid]age = 0
(2) [m1_Prepaid]male - [m2_Prepaid]male = 0
(3) [m1_Prepaid]_cons - [m2_Prepaid]_cons = 0
 chi2(3) = 0.79
 Prob > chi2 = 0.8529
```

`suest` provides some descriptive information about the clustering on `cityid`. Like any other estimation command, `suest` informs us that the standard errors are adjusted for clustering. The Hausman-type test obtained from the `test` command uses a simultaneous (co)variance of `m1` and `m2` appropriately adjusted for clustering. In this example, we still do not have reason to conclude that the multinomial logit model in this application is misspecified, that is, that IIA is violated.

◀

The multinomial logistic regression model is a special case of the conditional logistic regression model; see [R] [clogit](#). Like the multinomial logistic regression model, the conditional logistic regression model also makes the IIA assumption. Consider an example, introduced in [CM] [cmclogit](#), in which the demand for American, Japanese, and European cars is modeled in terms of the number of local dealers of the respective brands and of some individual attributes incorporated in interaction with the nationality of cars. We want to perform a Hausman-type test for IIA comparing the decision between all nationalities with the decision between non-American cars. The following code fragment demonstrates how to conduct a Hausman test for IIA via `suest` in this case.

```

. clogit choice japan europe maleJap maleEur incJap incEur dealer, group(id)
. estimates store allcars
. clogit choice japan maleJap incJap dealer if car!=1 , group(id)
. estimates store foreign
. suest allcars foreign
. test [allcars_choice=foreign_choice], common

```

## Testing proportionality

The applications of `suest` that we have discussed so far concern Hausman-type tests for misspecification. To test such a hypothesis, we compared two estimators that have the same probability limit if the hypothesis holds true, but otherwise have different limits. We may also want to compare the coefficients of models (estimators) for other substantive reasons. Although we most often want to test whether coefficients differ between models or estimators, we may occasionally want to test other constraints (see [Hausman and Ruud \[1987\]](#)).

### ► Example 2

In this example, using simulated labor market data for siblings, we consider two dependent variables, income (`inc`) and whether a person was promoted in the last year (`promo`). We apply familiar economic arguments regarding human capital, according to which employees have a higher income and a higher probability of being promoted, by having more human capital. Human capital is acquired through formal education (`edu`) and on-the-job training experience (`exp`). We study whether income and promotion are “two sides of the same coin”, that is, whether they reflect a common latent variable, “human capital”. Accordingly, we want to compare the effects of different aspects of human capital on different outcome variables.

We estimate fairly simple labor market equations. The income model is estimated with `regress`, and the estimation results are stored under the name `Inc`.

```

. use https://www.stata-press.com/data/r19/income
. regress inc edu exp male

```

| Source   | SS         | df  | MS         | Number of obs | = | 277    |
|----------|------------|-----|------------|---------------|---|--------|
| Model    | 2058.44672 | 3   | 686.148908 | F(3, 273)     | = | 42.34  |
| Residual | 4424.05183 | 273 | 16.2053181 | Prob > F      | = | 0.0000 |
|          |            |     |            | R-squared     | = | 0.3175 |
|          |            |     |            | Adj R-squared | = | 0.3100 |
| Total    | 6482.49855 | 276 | 23.4873136 | Root MSE      | = | 4.0256 |

| inc   | Coefficient | Std. err. | t    | P> t  | [95% conf. interval] |
|-------|-------------|-----------|------|-------|----------------------|
| edu   | 2.213707    | .243247   | 9.10 | 0.000 | 1.734828 2.692585    |
| exp   | 1.47293     | .231044   | 6.38 | 0.000 | 1.018076 1.927785    |
| male  | .5381153    | .4949466  | 1.09 | 0.278 | -.436282 1.512513    |
| _cons | 1.255497    | .3115808  | 4.03 | 0.000 | .642091 1.868904     |

```

. est store Inc

```

Being sibling data, the observations are clustered on family of origin, `famid`. In the estimation of the regression parameters, we did not specify a `vce(cluster famid)` option to adjust standard errors for clustering on family (`famid`). Thus, the standard errors reported by `regress` are potentially flawed. This problem will, however, be corrected by specifying a `vce(cluster clustvar)` option with `suest`.

Next, we estimate the promotion equation with probit and again store the results under an appropriate name.

```
. probit promo edu exp male, nolog
Probit regression
Log likelihood = -158.43888
Number of obs = 277
LR chi2(3) = 49.76
Prob > chi2 = 0.0000
Pseudo R2 = 0.1357
```

| promo | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|-------|-------------|-----------|-------|-------|----------------------|-----------|
| edu   | .4593002    | .0898537  | 5.11  | 0.000 | .2831901             | .6354102  |
| exp   | .3593023    | .0805774  | 4.46  | 0.000 | .2013735             | .5172312  |
| male  | .2079983    | .1656413  | 1.26  | 0.209 | -.1166527            | .5326494  |
| _cons | -.464622    | .1088166  | -4.27 | 0.000 | -.6778985            | -.2513454 |

```
. est store Promo
```

The coefficients in the income and promotion equations definitely seem to be different. However, because the scales of the two variables are different, we would not expect the coefficients to be equal. The correct hypothesis here is that the proportionality of the coefficients of the two models, apart from the constant, are equal. This formulation would still reflect that the relative effects of the different aspects of human capital do not differ between the dependent variables. We can obtain a nonlinear Wald test for the hypothesis of proportionality by using the `testnl` command on the combined estimation results of the two estimators. Thus, we first have to form the combined estimation results. At this point, we specify the `vce(cluster famid)` option to adjust for the clustering of observations on `famid`.

```
. suest Inc Promo, vce(cluster famid)
Simultaneous results for Inc, Promo
(Std. err. adjusted for 135 clusters in famid)
Number of obs = 277
```

|             | Coefficient | Robust std. err. | z     | P> z  | [95% conf. interval] |           |
|-------------|-------------|------------------|-------|-------|----------------------|-----------|
| Inc_mean    |             |                  |       |       |                      |           |
| edu         | 2.213707    | .2483907         | 8.91  | 0.000 | 1.72687              | 2.700543  |
| exp         | 1.47293     | .1890583         | 7.79  | 0.000 | 1.102383             | 1.843478  |
| male        | .5381153    | .4979227         | 1.08  | 0.280 | -.4377952            | 1.514026  |
| _cons       | 1.255497    | .3374977         | 3.72  | 0.000 | .594014              | 1.916981  |
| Inc_lvar    |             |                  |       |       |                      |           |
| _cons       | 2.785339    | .079597          | 34.99 | 0.000 | 2.629332             | 2.941347  |
| Promo_promo |             |                  |       |       |                      |           |
| edu         | .4593002    | .0886982         | 5.18  | 0.000 | .2854549             | .6331454  |
| exp         | .3593023    | .079772          | 4.50  | 0.000 | .2029522             | .5156525  |
| male        | .2079983    | .1691053         | 1.23  | 0.219 | -.1234419            | .5394386  |
| _cons       | -.464622    | .1042169         | -4.46 | 0.000 | -.6688833            | -.2603607 |

The standard errors reported by `suest` are identical to those reported by the respective estimation commands when invoked with the `vce(cluster famid)` option. We are now ready to test for proportionality:

$$H_0 : \frac{\beta_{\text{edu}}^{\text{Income}}}{\beta_{\text{edu}}^{\text{Promotion}}} = \frac{\beta_{\text{exp}}^{\text{Income}}}{\beta_{\text{exp}}^{\text{Promotion}}} = \frac{\beta_{\text{male}}^{\text{Income}}}{\beta_{\text{male}}^{\text{Promotion}}}$$

It is straightforward to translate this into syntax suitable for `testnl`, recalling that the coefficient of variable  $v$  in equation  $eq$  is denoted by  $[eq]v$ .

```
. testnl [Inc_mean]edu/[Promo_promo]edu =
> [Inc_mean]exp/[Promo_promo]exp =
> [Inc_mean]male/[Promo_promo]male
(1) [Inc_mean]edu/[Promo_promo]edu = [Inc_mean]exp/[Promo_promo]exp
(2) [Inc_mean]edu/[Promo_promo]edu = [Inc_mean]male/[Promo_promo]male
 chi2(2) = 0.61
 Prob > chi2 = 0.7385
```

From the evidence, we fail to reject the hypotheses that the coefficients of the income and promotion equations are proportional. Thus, it is not unreasonable to assume that income and promotion can be explained by the same latent variable, “labor market success”.

A disadvantage of the nonlinear Wald test is that it is not invariant with respect to representation: a Wald test for a mathematically equivalent formulation of the nonlinear constraint usually leads to a different test result. An equivalent formulation of the proportionality hypothesis is

$$H_0 : \begin{aligned} \beta_{\text{edu}}^{\text{Income}} \beta_{\text{exp}}^{\text{Promotion}} &= \beta_{\text{edu}}^{\text{Promotion}} \beta_{\text{exp}}^{\text{Income}} & \text{and} \\ \beta_{\text{edu}}^{\text{Income}} \beta_{\text{male}}^{\text{Promotion}} &= \beta_{\text{edu}}^{\text{Promotion}} \beta_{\text{male}}^{\text{Income}} \end{aligned}$$

This formulation is “more linear” in the coefficients. The asymptotic  $\chi^2$  distribution of the nonlinear Wald statistic can be expected to be more accurate for this representation.

```
. testnl ([Inc_mean]edu*[Promo_promo]exp = [Inc_mean]exp*[Promo_promo]edu)
> ([Inc_mean]edu*[Promo_promo]male = [Inc_mean]male*[Promo_promo]edu)
(1) [Inc_mean]edu*[Promo_promo]exp = [Inc_mean]exp*[Promo_promo]edu
(2) [Inc_mean]edu*[Promo_promo]male = [Inc_mean]male*[Promo_promo]edu
 chi2(2) = 0.46
 Prob > chi2 = 0.7936
```

Here the two representations lead to similar test statistics and  $p$ -values. As before, we fail to reject the hypothesis of proportionality of the coefficients of the two models.

◀

## Testing cross-model hypotheses

### ▷ Example 3

In this example, we demonstrate how some cross-model hypotheses can be tested using the facilities already available in most estimation commands. This demonstration will explain the intricate relationship between the cluster adjustment of the robust estimator of variance and the `suest` command. It will also be made clear that a new facility is required to perform more general cross-model testing.

We want to test whether the effect of  $x_1$  on the binary variable  $y_1$  is the same as the effect of  $x_2$  on the binary  $y_2$ ; see [Clogg, Petkova, and Haritou \(1995\)](#). In this setting,  $x_1$  may equal  $x_2$ , and  $y_1$  may equal  $y_2$ . We assume that logistic regression models can be used to model the responses, and for simplicity, we ignore further predictor variables in these models. If the two logit models are fit on independent samples so that the estimators are (stochastically) independent, a Wald test for `_b[x1] = _b[x2]` rejects the null hypothesis if

$$\frac{\hat{b}(x_1) - \hat{b}(x_2)}{\left[ \hat{\sigma}^2 \{ \hat{b}(x_1) \} + \hat{\sigma}^2 \{ \hat{b}(x_2) \} \right]^{1/2}}$$

is larger than the appropriate  $\chi_1^2$  threshold. If the models are fit on the *same* sample (or on dependent samples), so that the estimators are stochastically dependent, the above test that ignores the covariance between the estimators is not appropriate.

It is instructive to see how this problem can be tackled by “stacking” data. In the stacked format, we doubled the number of observations. The dependent variable is  $y_1$  in the first half of the data and is  $y_2$  in the second half of the data. The predictor variable  $z_1$  is set to  $x_1$  in the first half of the expanded data and to 0 in the rest. Similarly,  $z_2$  is 0 in the first half and  $x_2$  in the second half. The following diagram illustrates the transformation, in the terminology of the `reshape` command, from wide to long format.

$$\left( \begin{array}{ccccc} \text{id} & y_1 & y_2 & x_1 & x_2 \\ 1 & y_{11} & y_{21} & x_{11} & x_{21} \\ 2 & y_{12} & y_{22} & x_{12} & x_{22} \\ 3 & y_{13} & y_{23} & x_{13} & x_{23} \end{array} \right) \Rightarrow \left( \begin{array}{ccccc} \text{id} & y & z_1 & z_2 & \text{model} \\ 1 & y_{11} & x_{11} & 0 & 1 \\ 2 & y_{12} & x_{12} & 0 & 1 \\ 3 & y_{13} & x_{13} & 0 & 1 \\ 1 & y_{21} & 0 & x_{21} & 2 \\ 2 & y_{22} & 0 & x_{22} & 2 \\ 3 & y_{23} & 0 & x_{23} & 2 \end{array} \right)$$

The observations in the long-format data organization are *clustered* on the original subjects and are identified with the identifier `id`. The clustering on `id` has to be accounted for when fitting a simultaneous model. The simplest way to deal with clustering is to use the cluster adjustment of the robust or sandwich estimator; see [\[P\] `\_robust`](#). The data manipulation can be accomplished easily with the `stack` command; see [\[D\] `stack`](#). Subsequently, we fit a simultaneous logit model and perform a Wald test for the hypothesis that the coefficients of `z1` and `z2` are the same. A full setup to obtain the cross-model Wald test could then be as follows:

```
. generate zero = 0 // a variable that is always 0
. generate one = 1 // a variable that is always 1
. generate two = 2 // a variable that is always 2
. stack id y1 x1 zero one id y2 zero x2 two, into(id y z1 z2 model)
. generate model2 = (model==2)
. logit y model2 z1 z2, vce(cluster id)
. test _b[z1] = _b[z2]
```

The coefficient of `z1` represents the effect of `x1` on `y1`, and similarly, `z2` for the effect of `x2` on `y2`. The variable `model2` is a dummy for the “second model”, which is included to allow the intercept in the second model to differ from that in the first model. The estimates of the coefficient of `z1` and its standard error in the combined model are the same as the estimates of the coefficient of `z1` and its standard error if we fit the model on the unstacked data.

```
. logit y1 x1, vce(robust)
```

The `vce(cluster clustvar)` option specified with the `logit` command for the stacked data ensures that the covariances of `_b[z1]` and `_b[z2]` are indeed estimated. This estimation ensures that the Wald test for the equality of the coefficients is correct. If we had not specified the `vce(cluster clustvar)` option, the (co)variance matrix of the coefficients would have been block-diagonal; that is, the covariances of `_b[z1]` and `_b[z2]` would have been 0. Then, `test` would have effectively used the invalid formula for the Wald test for two independent samples.

In this example, the two logit models were fit on the same data. The same setup would apply, without modification, when the two logit models were fit on overlapping data that resulted, for instance, if the `y` or `x` variables were missing in some observations.

The `suest` command allows us to obtain the above Wald test more efficiently by avoiding the data manipulation, obviating the need to fit a model with twice the number of coefficients. The test statistic produced by the above code fragment is *identical* to that obtained via `suest` on the original (unstacked) data:

```
. logit y1 x1
. estimates store M1
. logit y2 x2
. estimates store M2
. suest M1 M2
. test [M1]x1=[M2]x2
```

The stacking method can be applied not only to the testing of cross-model hypotheses for `logit` models but also to any estimation command that supports the `vce(cluster clustvar)` option. The stacking approach clearly generalizes to stacking more than two logit or other models, testing more general linear hypotheses, and testing nonlinear cross-model hypotheses (see [R] [testnl](#)). In all of these cases, `suest` would yield identical statistical results but at smaller costs in terms of data management, computer storage, and computer time.

Is `suest` nothing but a convenience command? No, there are two disadvantages to the stacking method, both of which are resolved via `suest`. First, if the models include ancillary parameters (in a regression model, the residual variance; in an ordinal response model, the cutpoints; and in lognormal survival-time regression, the time scale parameter), these parameters are constrained to be equal between the stacked models. In `suest`, this constraint is relaxed. Second, the stacking method does not generalize to compare different statistical models, such as a probit model and a regression model. As demonstrated in the previous section, `suest` can deal with this situation.

## Stored results

suest stores the following in `e()`:

### Scalars

|                         |                        |
|-------------------------|------------------------|
| <code>e(N)</code>       | number of observations |
| <code>e(N_clust)</code> | number of clusters     |
| <code>e(rank)</code>    | rank of $e(V)$         |

### Macros

|                            |                                                |
|----------------------------|------------------------------------------------|
| <code>e(cmd)</code>        | suest                                          |
| <code>e(eqnames#)</code>   | original names of equations of model #         |
| <code>e(names)</code>      | list of model names                            |
| <code>e(wtype)</code>      | weight type                                    |
| <code>e(wexp)</code>       | weight expression                              |
| <code>e(clustvar)</code>   | name of cluster variable                       |
| <code>e(vce)</code>        | <i>vcetype</i> specified in <code>vce()</code> |
| <code>e(vcetype)</code>    | title used to label Std. err.                  |
| <code>e(properties)</code> | b V                                            |

### Matrices

|                   |                                              |
|-------------------|----------------------------------------------|
| <code>e(b)</code> | stacked coefficient vector of the models     |
| <code>e(V)</code> | variance–covariance matrix of the estimators |

### Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

## Methods and formulas

The estimation of the simultaneous (co)variance of a series of  $k$  estimators is a nonstandard application of the sandwich estimator, as implemented by the command `[P] _robust`. You may want to read this manual entry before reading further.

The starting point is that we have fit  $k$  different models on the *same* data—partially overlapping or nonoverlapping data are special cases. We want to derive the *simultaneous* distribution of these  $k$  estimators, for instance, to test a cross-estimator hypothesis  $H_0$ . As in the framework of Hausman testing,  $H_0$  will often be of the form that different estimators have the same probability limit under some hypothesis, while the estimators have different limits if the hypothesis is violated.

We consider (vector) estimators  $\widehat{\beta}_i$  to be defined as “the” solution of the estimation equations  $\mathbf{G}_i$ ,

$$\mathbf{G}_i(\mathbf{b}_i) = \sum_j w_{ij} \mathbf{u}_{ij}(\mathbf{b}_i) = \mathbf{0}, \quad i = 1, \dots, k$$

We refer to the  $\mathbf{u}_{ij}$  as the “scores”. Specifying some weights  $w_{ij} = 0$  trivially accommodates for partially overlapping or even disjointed data. Under “suitable regularity conditions” (see White [1982; 1996] for details), the  $\widehat{\beta}_i$  are asymptotically normally distributed, with the variance estimated consistently by the sandwich estimator

$$V_i = \text{Var}(\widehat{\beta}_i) = \mathbf{D}_i^{-1} \sum_j w_{ij} \mathbf{u}_{ij} \mathbf{u}'_{ij} \mathbf{D}_i^{-1}$$

where  $\mathbf{D}_i$  is the Jacobian of  $\mathbf{G}_i$  evaluated at  $\widehat{\beta}_i$ . In the context of maximum likelihood estimation,  $\mathbf{D}_i$  can be estimated consistently by (minus) the Hessian of the log likelihood or by the Fisher information matrix. If the model is also well specified, the sandwiched term  $(\sum_j w_{ij} \mathbf{u}_{ij} \mathbf{u}'_{ij})$  converges in probability to  $\mathbf{D}_i$ , so  $V_i$  may be consistently estimated by  $\mathbf{D}_i^{-1}$ .

To derive the simultaneous distribution of the estimators, we consider the “stacked” estimation equation,

$$\mathbf{G}(\widehat{\boldsymbol{\beta}}) = \left\{ \mathbf{G}_1(\widehat{\boldsymbol{\beta}}_1)' \quad \mathbf{G}_1(\widehat{\boldsymbol{\beta}}_2)' \quad \dots \quad \mathbf{G}_k(\widehat{\boldsymbol{\beta}}_k)' \right\}' = \mathbf{0}$$

Under “suitable regularity conditions” (see White [1996] for details),  $\widehat{\boldsymbol{\beta}}$  is asymptotically *jointly* normally distributed. The Jacobian and scores of the simultaneous equation are easily expressed in the Jacobian and scores of the separate equations. The Jacobian of  $\mathbf{G}$ ,

$$\mathbf{D}(\widehat{\boldsymbol{\beta}}) = \left. \frac{d\mathbf{G}(\boldsymbol{\beta})}{d\boldsymbol{\beta}} \right|_{\boldsymbol{\beta}=\widehat{\boldsymbol{\beta}}}$$

is block diagonal with blocks  $\mathbf{D}_1, \dots, \mathbf{D}_k$ . The inverse of  $\mathbf{D}(\widehat{\boldsymbol{\beta}})$  is again block diagonal, with the inverses of  $\mathbf{D}_i$  on the diagonal. The scores  $\mathbf{u}$  of  $\mathbf{G}$  are simply obtained as the *concatenated* scores of the separate equations:

$$\mathbf{u}_j = (\mathbf{u}'_{1j} \quad \mathbf{u}'_{2j} \quad \dots \quad \mathbf{u}'_{kj})'$$

Out-of-sample (that is, where  $w_{ij} = 0$ ) values of the score variables are defined as 0 (thus we drop the  $i$  subscript from the common weight variable). The sandwich estimator for the asymptotic variance of  $\widehat{\boldsymbol{\beta}}$  reads

$$V = \text{Var}(\widehat{\boldsymbol{\beta}}) = \mathbf{D}(\widehat{\boldsymbol{\beta}})^{-1} \left( \sum_j w_j \mathbf{u}_j \mathbf{u}'_j \right) \mathbf{D}(\widehat{\boldsymbol{\beta}})^{-1}$$

Taking a “partitioned” look at this expression, we see that  $V(\widehat{\boldsymbol{\beta}}_i)$  is estimated by

$$\mathbf{D}_i^{-1} \left( \sum_j w_j \mathbf{u}_{ij} \mathbf{u}'_{ij} \right) \mathbf{D}_i^{-1}$$

which is, yet again, the familiar sandwich estimator for  $\widehat{\boldsymbol{\beta}}_i$  based on the separate estimation equation  $\mathbf{G}_i$ . Thus, considering several estimators simultaneously in this way does not affect the estimators of the asymptotic variances of these estimators. However, as a bonus of stacking, we obtained a sandwich-type estimate of the *covariance*  $V_{ih}$  of estimators  $\widehat{\boldsymbol{\beta}}_i$  and  $\widehat{\boldsymbol{\beta}}_h$ ,

$$V_{ih} = \text{Cov}(\widehat{\boldsymbol{\beta}}_i, \widehat{\boldsymbol{\beta}}_h) = \mathbf{D}_i^{-1} \left( \sum_j w_j \mathbf{u}_{ij} \mathbf{u}'_{hj} \right) \mathbf{D}_h^{-1}$$

which is also obtained by White (1982).

This estimator for the covariance of estimators is an application of the cluster modification of the sandwich estimator proposed by Rogers (1993). Consider the stacked data format as discussed in the logit example, and assume that Stata would be able to estimate a “stacked model” in which different models apply to different observations, for example, a probit model for the first half, a regression model for the second half, and a one-to-one cluster relation between the first and second half. If there are no common parameters to both models, the score statistics of parameters for the stacked models are zero in the half of the data in which they do not occur. In Rogers’s method, we have to sum the score statistics over the observations within a cluster. This step boils down to concatenating the score statistics at the level of the cluster.

We compare the sandwich estimator of the (co)variance  $V_{12}$  of two estimators with the estimator of variance  $\tilde{V}_{12}$  applied in the classic Hausman test. Hausman (1978) showed that if  $\widehat{\beta}_1$  is consistent under  $H_0$  and  $\widehat{\beta}_2$  is efficient under  $H_0$ , then asymptotically

$$\text{Cov}(\widehat{\beta}_1, \widehat{\beta}_2) = \text{Var}(\widehat{\beta}_2)$$

and so  $\text{var}(\widehat{\beta}_1 - \widehat{\beta}_2)$  is consistently estimated by  $V_1 - V_2$ .

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An earlier version of suest was published in the *Stata Technical Bulletin* (1999). The current version of suest is not backward compatible with the 1999 version because of the introduction of new ways to manage estimation results via the `estimates` command.

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## Also see

- [R] **estimates** — Save and manipulate estimation results
- [R] **hausman** — Hausman specification test
- [R] **lincom** — Linear combinations of parameters
- [R] **nlcom** — Nonlinear combinations of parameters
- [R] **test** — Test linear hypotheses after estimation
- [R] **testnl** — Test nonlinear hypotheses after estimation
- [P] **\_robust** — Robust variance estimates

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## Description

`summarize` calculates and displays a variety of univariate summary statistics. If no *varlist* is specified, summary statistics are calculated for all the variables in the dataset.

## Quick start

Basic summary statistics for continuous variable `v1`

```
summarize v1
```

Same as above, and include `v2` and `v3`

```
summarize v1-v3
```

Same as above, and provide additional detail about the distribution

```
summarize v1-v3, detail
```

Summary statistics reported separately for each level of `catvar`

```
by catvar: summarize v1
```

With frequency weight `wvar`

```
summarize v1 [fweight=wvar]
```

## Menu

Statistics > Summaries, tables, and tests > Summary and descriptive statistics > Summary statistics

## Syntax

```
summarize [varlist] [if] [in] [weight] [, options]
```

| <i>options</i> | Description |
|----------------|-------------|
|----------------|-------------|

Main

|                              |                                                                      |
|------------------------------|----------------------------------------------------------------------|
| <code>detail</code>          | display additional statistics                                        |
| <code>meanonly</code>        | suppress the display; calculate only the mean; programmer's option   |
| <code>format</code>          | use variable's display format                                        |
| <code>separator(#)</code>    | draw separator line after every # variables; default is separator(5) |
| <code>display_options</code> | control spacing, line width, and base and empty cells                |

*varlist* may contain factor variables; see [\[U\] 11.4.3 Factor variables](#).

*varlist* may contain time-series operators; see [\[U\] 11.4.4 Time-series varlists](#).

`by`, `collect`, `rolling`, and `statsby` are allowed; see [\[U\] 11.1.10 Prefix commands](#).

`aweight`s, `fweight`s, and `iweight`s are allowed. However, `iweight`s may not be used with the `detail` option; see [\[U\] 11.1.6 weight](#).

## Options

Main

`detail` produces additional statistics, including skewness, kurtosis, the four smallest and four largest values, and various percentiles.

`meanonly`, which is allowed only when `detail` is not specified, suppresses the display of results and calculation of the variance. Ado-file writers will find this useful for fast calls.

`format` requests that the summary statistics be displayed using the display formats associated with the variables rather than the default `g` display format; see [\[U\] 12.5 Formats: Controlling how data are displayed](#).

`separator(#)` specifies how often to insert separation lines into the output. The default is `separator(5)`, meaning that a line is drawn after every five variables. `separator(10)` would draw a line after every 10 variables. `separator(0)` suppresses the separation line.

`display_options`: `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, and `fvwrapon(style)`; see [\[R\] Estimation options](#).

## Remarks and examples

`summarize` can produce two different sets of summary statistics. Without the `detail` option, the number of nonmissing observations, the mean and standard deviation, and the minimum and maximum values are presented. With `detail`, the same information is presented along with the variance, skewness, and kurtosis; the four smallest and four largest values; and the 1st, 5th, 10th, 25th, 50th (median), 75th, 90th, 95th, and 99th percentiles.

Also see [\[R\] ci](#) for calculating the standard error and confidence intervals of the mean.

### ▷ Example 1: summarize with the separator() option

We have data containing information on various automobiles, among which is the variable `mpg`, the mileage rating. We can obtain a quick summary of the `mpg` variable by typing

```
. use https://www.stata-press.com/data/r19/auto2
(1978 automobile data)
. summarize mpg
```

| Variable | Obs | Mean    | Std. dev. | Min | Max |
|----------|-----|---------|-----------|-----|-----|
| mpg      | 74  | 21.2973 | 5.785503  | 12  | 41  |

We see that we have 74 observations. The mean of `mpg` is 21.3 miles per gallon, and the standard deviation is 5.79. The minimum is 12, and the maximum is 41.

If we had not specified the variable (or variables) we wanted to summarize, we would have obtained summary statistics on all the variables in the dataset:

```
. summarize, separator(4)
```

| Variable     | Obs | Mean     | Std. dev. | Min  | Max   |
|--------------|-----|----------|-----------|------|-------|
| make         | 0   |          |           |      |       |
| price        | 74  | 6165.257 | 2949.496  | 3291 | 15906 |
| mpg          | 74  | 21.2973  | 5.785503  | 12   | 41    |
| rep78        | 69  | 3.405797 | .9899323  | 1    | 5     |
| headroom     | 74  | 2.993243 | .8459948  | 1.5  | 5     |
| trunk        | 74  | 13.75676 | 4.277404  | 5    | 23    |
| weight       | 74  | 3019.459 | 777.1936  | 1760 | 4840  |
| length       | 74  | 187.9324 | 22.26634  | 142  | 233   |
| turn         | 74  | 39.64865 | 4.399354  | 31   | 51    |
| displacement | 74  | 197.2973 | 91.83722  | 79   | 425   |
| gear_ratio   | 74  | 3.014865 | .4562871  | 2.19 | 3.89  |
| foreign      | 74  | .2972973 | .4601885  | 0    | 1     |

There are only 69 observations on `rep78`, so some of the observations are missing. There are no observations on `make` because it is a string variable.

The idea of the mean is quite old (Plackett 1958), but its extension to a scheme of moment-based measures was not done until the end of the 19th century. Between 1893 and 1905, Pearson discussed and named the standard deviation, skewness, and kurtosis, but he was not the first to use any of these. Thiele (1889), in contrast, had earlier firmly grasped the notion that the  $m_r$  provide a systematic basis for discussing distributions. However, even earlier anticipations can also be found. For example, Euler in 1778 used  $m_2$  and  $m_3$  in passing in a treatment of estimation (Hald 1998, 87), but seemingly did not build on that.

Similarly, the idea of the median is quite old. The history of the interquartile range is tangled up with that of the probable error, a long-popular measure. Extending this in various ways to a more general approach based on quantiles (to use a later term) occurred to several people in the nineteenth century. Galton (1875) is a nice example, particularly because he seems so close to the key idea of the quantiles as a function, which took another century to reemerge strongly.

Thorvald Nicolai Thiele (1838–1910) was a Danish scientist who worked in astronomy, mathematics, actuarial science, and statistics. He made many pioneering contributions to statistics, several of which were overlooked until recently. Thiele advocated graphical analysis of residuals checking for trends, symmetry of distributions, and changes of sign, and he even warned against overinterpreting such graphs.

### ▷ Example 2: summarize with the detail option

The `detail` option provides all the information of a normal `summarize` and more. The format of the output also differs, as shown here:

```
. summarize mpg, detail
```

| Mileage (mpg) |    |          |             |          |
|---------------|----|----------|-------------|----------|
| Percentiles   |    | Smallest |             |          |
| 1%            | 12 | 12       |             |          |
| 5%            | 14 | 12       |             |          |
| 10%           | 14 | 14       | Obs         | 74       |
| 25%           | 18 | 14       | Sum of wgt. | 74       |
| 50%           | 20 |          | Mean        | 21.2973  |
|               |    | Largest  | Std. dev.   | 5.785503 |
| 75%           | 25 | 34       |             |          |
| 90%           | 29 | 35       | Variance    | 33.47205 |
| 95%           | 34 | 35       | Skewness    | .9487176 |
| 99%           | 41 | 41       | Kurtosis    | 3.975005 |

As in the [previous example](#), we see that the mean of `mpg` is 21.3 miles per gallon and that the standard deviation is 5.79. We also see the various percentiles. The median of `mpg` (the 50th percentile) is 20 miles per gallon. The 25th percentile is 18, and the 75th percentile is 25.

When we performed `summarize`, we learned that the minimum and maximum were 12 and 41, respectively. We now see that the four smallest values in our dataset are 12, 12, 14, and 14. The four largest values are 34, 35, 35, and 41. The skewness of the distribution is 0.95, and the kurtosis is 3.98. (A normal distribution would have a skewness of 0 and a kurtosis of 3.)

*Skewness* is a measure of the lack of symmetry of a distribution. If the distribution is symmetric, the coefficient of skewness is 0. If the coefficient is negative, the median is usually greater than the mean and the distribution is said to be skewed left. If the coefficient is positive, the median is usually less than the mean and the distribution is said to be skewed right. *Kurtosis* (from the Greek *kyrtosis*, meaning curvature) is a measure of peakedness of a distribution. The smaller the coefficient of kurtosis, the flatter the distribution. The normal distribution has a coefficient of kurtosis of 3 and provides a convenient benchmark.



## □ Technical note

The convention of calculating the median of an even number of values by averaging the central two order statistics is of long standing. (That is, given 8 values, average the 4th and 5th smallest values, or given 42, average the 21st and 22nd smallest.) [Stigler \(1977\)](#) filled a much-needed gap in the literature by naming such paired central order statistics as “comedians”, although it remains unclear how far he was joking.



## ▷ Example 3: summarize with the by prefix

`summarize` can usefully be combined with the `by varlist:` prefix. In our dataset, we have a variable, `foreign`, that distinguishes foreign and domestic cars. We can obtain summaries of `mpg` and `weight` within each subgroup by typing

```
. by foreign: summarize mpg weight
```

```
-> foreign = Domestic
```

| Variable | Obs | Mean     | Std. dev. | Min  | Max  |
|----------|-----|----------|-----------|------|------|
| mpg      | 52  | 19.82692 | 4.743297  | 12   | 34   |
| weight   | 52  | 3317.115 | 695.3637  | 1800 | 4840 |

```
-> foreign = Foreign
```

| Variable | Obs | Mean     | Std. dev. | Min  | Max  |
|----------|-----|----------|-----------|------|------|
| mpg      | 22  | 24.77273 | 6.611187  | 14   | 41   |
| weight   | 22  | 2315.909 | 433.0035  | 1760 | 3420 |

Domestic cars in our dataset average 19.8 miles per gallon, whereas foreign cars average 24.8.

Because *by varlist*: can be combined with summarize, it can also be combined with summarize, detail:

```
. by foreign: summarize mpg, detail
```

```
-> foreign = Domestic
```

| Mileage (mpg) |      |          |             |          |
|---------------|------|----------|-------------|----------|
| Percentiles   |      | Smallest |             |          |
| 1%            | 12   | 12       |             |          |
| 5%            | 14   | 12       |             |          |
| 10%           | 14   | 14       | Obs         | 52       |
| 25%           | 16.5 | 14       | Sum of wgt. | 52       |
| 50%           | 19   |          | Mean        | 19.82692 |
|               |      | Largest  | Std. dev.   | 4.743297 |
| 75%           | 22   | 28       |             |          |
| 90%           | 26   | 29       | Variance    | 22.49887 |
| 95%           | 29   | 30       | Skewness    | .7712432 |
| 99%           | 34   | 34       | Kurtosis    | 3.441459 |

```
-> foreign = Foreign
```

| Mileage (mpg) |      |          |             |          |
|---------------|------|----------|-------------|----------|
| Percentiles   |      | Smallest |             |          |
| 1%            | 14   | 14       |             |          |
| 5%            | 17   | 17       |             |          |
| 10%           | 17   | 17       | Obs         | 22       |
| 25%           | 21   | 18       | Sum of wgt. | 22       |
| 50%           | 24.5 |          | Mean        | 24.77273 |
|               |      | Largest  | Std. dev.   | 6.611187 |
| 75%           | 28   | 31       |             |          |
| 90%           | 35   | 35       | Variance    | 43.70779 |
| 95%           | 35   | 35       | Skewness    | .657329  |
| 99%           | 41   | 41       | Kurtosis    | 3.10734  |

## □ Technical note

`summarize` respects display formats if we specify the `format` option. When we type `summarize price weight`, we obtain

```
. summarize price weight
```

| Variable | Obs | Mean     | Std. dev. | Min  | Max   |
|----------|-----|----------|-----------|------|-------|
| price    | 74  | 6165.257 | 2949.496  | 3291 | 15906 |
| weight   | 74  | 3019.459 | 777.1936  | 1760 | 4840  |

The display is accurate but is not as aesthetically pleasing as we may wish, particularly if we plan to use the output directly in published work. By placing formats on the variables, we can control how the table appears:

```
. format price weight %9.2fc
. summarize price weight, format
```

| Variable | Obs | Mean     | Std. dev. | Min      | Max       |
|----------|-----|----------|-----------|----------|-----------|
| price    | 74  | 6,165.26 | 2,949.50  | 3,291.00 | 15,906.00 |
| weight   | 74  | 3,019.46 | 777.19    | 1,760.00 | 4,840.00  |

□

If you specify a weight (see [U] 11.1.6 **weight**), each observation is multiplied by the value of the weighting expression before the summary statistics are calculated so that the weighting expression is interpreted as the discrete density of each observation.

## ▷ Example 4: summarize with factor variables

You can also use `summarize` to obtain summary statistics for factor variables. For example, if you type

```
. summarize i.rep78
```

| Variable  | Obs | Mean     | Std. dev. | Min | Max |
|-----------|-----|----------|-----------|-----|-----|
| rep78     |     |          |           |     |     |
| Poor      | 69  | .0289855 | .1689948  | 0   | 1   |
| Fair      | 69  | .115942  | .3225009  | 0   | 1   |
| Average   | 69  | .4347826 | .4993602  | 0   | 1   |
| Good      | 69  | .2608696 | .4423259  | 0   | 1   |
| Excellent | 69  | .1594203 | .3687494  | 0   | 1   |

you obtain the sample proportions for the five levels of the `rep78` variable. For example, 11.6% of the 69 cars with nonmissing values of `rep78` have a fair repair record.

We could have used `tabulate oneway rep78` to obtain the sample proportions along with the cumulative proportions. Alternatively, we could have used `proportions rep78` to obtain the sample proportions along with the standard errors of the proportions instead of the standard deviations of the proportions.

◀

### ▷ Example 5: summarize with weights

We have 1980 census data on each of the 50 states. Included in our variables is `medage`, the median age of the population of each state. If we type `summarize medage`, we obtain unweighted statistics:

```
. use https://www.stata-press.com/data/r19/census, clear
(1980 Census data by state)
. summarize medage
```

| Variable | Obs | Mean  | Std. dev. | Min  | Max  |
|----------|-----|-------|-----------|------|------|
| medage   | 50  | 29.54 | 1.693445  | 24.2 | 34.7 |

Also among our variables is `pop`, the population in each state. Typing `summarize medage [w=pop]` produces population-weighted statistics:

```
. summarize medage [w=pop]
(analytic weights assumed)
```

| Variable | Obs | Weight    | Mean     | Std. Dev. | Min  | Max  |
|----------|-----|-----------|----------|-----------|------|------|
| medage   | 50  | 225907472 | 30.11047 | 1.66933   | 24.2 | 34.7 |

The number listed under `Weight` is the sum of the weighting variable, `pop`, indicating that there are roughly 226 million people in the United States. The `pop`-weighted mean of `medage` is 30.11 (compared with 29.54 for the unweighted statistic), and the weighted standard deviation is 1.67 (compared with 1.69).



### ▷ Example 6: summarize with weights and the detail option

We can obtain detailed summaries of weighted data as well. When we do this, *all* the statistics are weighted, including the percentiles.

```
. summarize medage [w=pop], detail
(analytic weights assumed)
```

| Median age  |      |          |             |           |  |
|-------------|------|----------|-------------|-----------|--|
| Percentiles |      | Smallest |             |           |  |
| 1%          | 27.1 | 24.2     |             |           |  |
| 5%          | 27.7 | 26.1     |             |           |  |
| 10%         | 28.2 | 27.1     | Obs         | 50        |  |
| 25%         | 29.2 | 27.4     | Sum of wgt. | 225907472 |  |
| 50%         | 29.9 |          | Mean        | 30.11047  |  |
|             |      | Largest  | Std. dev.   | 1.66933   |  |
| 75%         | 30.9 | 32       |             |           |  |
| 90%         | 32.1 | 32.1     | Variance    | 2.786661  |  |
| 95%         | 32.2 | 32.2     | Skewness    | .5281972  |  |
| 99%         | 34.7 | 34.7     | Kurtosis    | 4.494223  |  |



## □ Technical note

If you are writing a program and need to access the mean of a variable, the `meanonly` option provides for fast calls. For example, suppose that your program reads as follows:

```
program mean
 summarize '1', meanonly
 display " mean = " r(mean)
end
```

The result of executing this is

```
. use https://www.stata-press.com/data/r19/auto2
(1978 automobile data)

. mean price
mean = 6165.2568
```

□

## Video example

[Descriptive statistics in Stata](#)

## Stored results

`summarize` stores the following in `r()`:

### Scalars

|                          |                               |                          |                               |
|--------------------------|-------------------------------|--------------------------|-------------------------------|
| <code>r(N)</code>        | number of observations        | <code>r(p50)</code>      | 50th percentile (detail only) |
| <code>r(mean)</code>     | mean                          | <code>r(p75)</code>      | 75th percentile (detail only) |
| <code>r(skewness)</code> | skewness (detail only)        | <code>r(p90)</code>      | 90th percentile (detail only) |
| <code>r(min)</code>      | minimum                       | <code>r(p95)</code>      | 95th percentile (detail only) |
| <code>r(max)</code>      | maximum                       | <code>r(p99)</code>      | 99th percentile (detail only) |
| <code>r(sum_w)</code>    | sum of the weights            | <code>r(Var)</code>      | variance                      |
| <code>r(p1)</code>       | 1st percentile (detail only)  | <code>r(kurtosis)</code> | kurtosis (detail only)        |
| <code>r(p5)</code>       | 5th percentile (detail only)  | <code>r(sum)</code>      | sum of variable               |
| <code>r(p10)</code>      | 10th percentile (detail only) | <code>r(sd)</code>       | standard deviation            |
| <code>r(p25)</code>      | 25th percentile (detail only) |                          |                               |

## Methods and formulas

Let  $x$  denote the variable on which we want to calculate summary statistics, and let  $x_i, i = 1, \dots, n$ , denote an individual observation on  $x$ . Let  $v_i$  be the weight, and if no weight is specified, define  $v_i = 1$  for all  $i$ .

Define the total number of observations as  $N = n$  if  $v_i$  is an `aweight` and as

$$N = \sum_{i=1}^n v_i$$

otherwise.

Additionally, define the normalized weight as

$$w_i = \frac{nv_i}{\sum_{i=1}^n v_i}$$

if  $v_i$  is an `aweight` and as  $w_i = v_i$  otherwise.

The mean,  $\bar{x}$ , is defined as

$$\bar{x} = \frac{1}{N} \sum_{i=1}^n w_i x_i$$

The variance,  $s^2$ , is defined as

$$s^2 = \frac{1}{N-1} \sum_{i=1}^n w_i (x_i - \bar{x})^2$$

The standard deviation,  $s$ , is defined as  $\sqrt{s^2}$ .

Define  $m_r$  as the  $r$ th moment about the mean  $\bar{x}$ :

$$m_r = \frac{1}{N} \sum_{i=1}^n w_i (x_i - \bar{x})^r$$

The coefficient of skewness is then defined as  $m_3 m_2^{-3/2}$ . The coefficient of kurtosis is defined as  $m_4 m_2^{-2}$ .

Let  $x_{(i)}$  refer to the  $x$  in ascending order, and let  $w_{(i)}$  refer to the corresponding weights of  $x_{(i)}$ . The four smallest values are  $x_{(1)}$ ,  $x_{(2)}$ ,  $x_{(3)}$ , and  $x_{(4)}$ . The four largest values are  $x_{(n)}$ ,  $x_{(n-1)}$ ,  $x_{(n-2)}$ , and  $x_{(n-3)}$ .

To obtain the  $p$ th percentile, which we will denote as  $x_{[p]}$ , let  $P = Np/100$ . Let

$$W_{(i)} = \sum_{j=1}^i w_{(j)}$$

Find the first index  $i$  such that  $W_{(i)} > P$ . The  $p$ th percentile is then

$$x_{[p]} = \begin{cases} \frac{x_{(i-1)} + x_{(i)}}{2} & \text{if } W_{(i-1)} = P \\ x_{(i)} & \text{otherwise} \end{cases}$$

## References

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## Also see

- [R] **ameans** — Arithmetic, geometric, and harmonic means
- [R] **centile** — Report centile and confidence interval
- [R] **ci** — Confidence intervals for means, proportions, and variances
- [R] **mean** — Estimate means
- [R] **proportion** — Estimate proportions
- [R] **ratio** — Estimate ratios
- [R] **table** — Table of frequencies, summaries, and command results
- [R] **tabstat** — Compact table of summary statistics
- [R] **tabulate, summarize()** — One- and two-way tables of summary statistics
- [R] **total** — Estimate totals
- [D] **codebook** — Describe data contents
- [D] **describe** — Describe data in memory or in a file
- [D] **inspect** — Display simple summary of data's attributes
- [ST] **stsum** — Summarize survival-time data
- [SVY] **svy estimation** — Estimation commands for survey data
- [XT] **xtsum** — Summarize xt data

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## Description

`sunflower` draws density-distribution sunflower plots (Plummer and Dupont 2003). Dark sunflowers, light sunflowers, and marker symbols represent high-, medium-, and low-density regions of the data, respectively. These plots are useful for displaying bivariate data whose density is too great for conventional scatterplots to be effective.

## Quick start

Density-distribution sunflower plot showing the relationship between  $x$  and  $y$

```
sunflower y x
```

Set the center of the reference bin to  $x = 5$  and  $y = 8$

```
sunflower y x, xcenter(5) ycenter(8)
```

Same as above, but specify the width of the hexagonal bins to be 1.5

```
sunflower y x, xcenter(5) ycenter(8) binwidth(1.5)
```

Same as above, but set the minimum number of observations needed for a bin to be represented by a light sunflower to 4

```
sunflower y x, xcenter(5) ycenter(8) binwidth(1.5) light(4)
```

Use the `s1color` scheme

```
sunflower y x, scheme(s1color)
```

Specify that only the petals are shown and the hexegons are omitted

```
sunflower y x, floweronly
```

Suppress display of the table

```
sunflower y x, notable
```

## Menu

Graphics > Smoothing and densities > Density-distribution sunflower plot

## Syntax

```
sunflower yvar xvar [if] [in] [weight] [, options]
```

| <i>options</i>                              | Description                                                                  |
|---------------------------------------------|------------------------------------------------------------------------------|
| Main                                        |                                                                              |
| <code>nograph</code>                        | do not show graph                                                            |
| <code>notable</code>                        | do not show summary table; implied when <code>by()</code> is specified       |
| <code>marker_options</code>                 | affect rendition of markers drawn at the plotted points                      |
| Bins/Petals                                 |                                                                              |
| <code>binwidth(#)</code>                    | width of the hexagonal bins                                                  |
| <code>binar(#)</code>                       | aspect ratio of the hexagonal bins                                           |
| <code>bin_options</code>                    | affect rendition of hexagonal bins                                           |
| <code>light(#)</code>                       | minimum observations for a light sunflower; default is <code>light(3)</code> |
| <code>dark(#)</code>                        | minimum observations for a dark sunflower; default is <code>dark(13)</code>  |
| <code>xcenter(#)</code>                     | <i>x</i> -coordinate of the reference bin                                    |
| <code>ycenter(#)</code>                     | <i>y</i> -coordinate of the reference bin                                    |
| <code>petalweight(#)</code>                 | observations in a dark sunflower petal                                       |
| <code>petalength(#)</code>                  | length of sunflower petal as a percentage                                    |
| <code>petal_options</code>                  | affect rendition of sunflower petals                                         |
| <code>flowersonly</code>                    | show petals only; do not render bins                                         |
| <code>nosinglepetal</code>                  | suppress single petals                                                       |
| Add plots                                   |                                                                              |
| <code>addplot(plot)</code>                  | add other plots to generated graph                                           |
| Y axis, X axis, Titles, Legend, Overall, By |                                                                              |
| <code>twoway_options</code>                 | any options documented in [G-3] <code>twoway_options</code>                  |

| <i>bin_options</i>                                              | Description                    |
|-----------------------------------------------------------------|--------------------------------|
| <code>[<u>l</u>][<u>d</u>]bstyle(<i>areastyle</i>)</code>       | overall look of hexagonal bins |
| <code>[<u>l</u>][<u>d</u>]bcolor(<i>colorstyle</i>)</code>      | outline and fill color         |
| <code>[<u>l</u>][<u>d</u>]bfcolor(<i>colorstyle</i>)</code>     | fill color                     |
| <code>[<u>l</u>][<u>d</u>]blstyle(<i>linestyle</i>)</code>      | overall look of outline        |
| <code>[<u>l</u>][<u>d</u>]blcolor(<i>colorstyle</i>)</code>     | outline color                  |
| <code>[<u>l</u>][<u>d</u>]blwidth(<i>linewidthstyle</i>)</code> | thickness of outline           |

| <i>petal_options</i>                                            | Description                       |
|-----------------------------------------------------------------|-----------------------------------|
| <code>[<u>l</u>][<u>d</u>]flstyle(<i>linestyle</i>)</code>      | overall style of sunflower petals |
| <code>[<u>l</u>][<u>d</u>]flcolor(<i>colorstyle</i>)</code>     | color of sunflower petals         |
| <code>[<u>l</u>][<u>d</u>]flwidth(<i>linewidthstyle</i>)</code> | thickness of sunflower petals     |

All options are *rightmost*; see [G-4] **Concept: repeated options**.

`fweights` are allowed; see [U] 11.1.6 **weight**.

## Options

### Main

`nograph` prevents the graph from being generated.

`notable` prevents the summary table from being displayed. This option is implied when the `by()` option is specified.

`marker_options` affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] [marker\\_options](#).

### Bins/Petals

`binwidth(#)` specifies the horizontal width of the hexagonal bins in the same units as `xvar`. By default,

$$\text{binwidth} = \max(\text{rbw}, \text{nbw})$$

where

$$\text{rbw} = \text{range of } xvar / 40$$

$$\text{nbw} = \text{range of } xvar / \max(1, \text{nb})$$

and

$$\text{nb} = \text{int}(\min(\sqrt{n}, 10 * \log_{10}(n)))$$

where

$$n = \text{the number of observations in the dataset}$$

`binar(#)` specifies the aspect ratio for the hexagonal bins. The height of the bins is given by

$$\text{binheight} = \text{binwidth} \times \# \times 2 / \sqrt{3}$$

where `binheight` and `binwidth` are specified in the units of `yvar` and `xvar`, respectively. The default is `binar(r)`, where `r` results in the rendering of regular hexagons.

`bin_options` affect how the hexagonal bins are rendered.

`lstyle(areastyle)` and `dbstyle(areastyle)` specify the look of the light and dark hexagonal bins, respectively. The options listed below allow you to change each attribute, but `lstyle()` and `dbstyle()` provide the starting points. See [G-4] [areastyle](#) for a list of available area styles.

`lbcolor(colorstyle)` and `dbcOLOR(colorstyle)` specify one color to be used both to outline the shape and to fill the interior of the light and dark hexagonal bins, respectively. See [G-4] [colorstyle](#) for a list of color choices.

`lbfcolor(colorstyle)` and `dbfcolor(colorstyle)` specify the color to be used to fill the interior of the light and dark hexagonal bins, respectively. See [G-4] [colorstyle](#) for a list of color choices.

`lstyle(linestyle)` and `dbstyle(linestyle)` specify the overall style of the line used to outline the area, which includes its pattern (solid, dashed, etc.), thickness, and color. The other options listed below allow you to change the line's attributes, but `lstyle()` and `dbstyle()` are the starting points. See [G-4] [linestyle](#) for a list of choices.

`lbcolor(colorstyle)` and `dblcolor(colorstyle)` specify the color to be used to outline the light and dark hexagonal bins, respectively. See [G-4] [colorstyle](#) for a list of color choices.

`lblwidth(linewidthstyle)` and `dblwidth(linewidthstyle)` specify the thickness of the line to be used to outline the light and dark hexagonal bins, respectively. See [G-4] [linewidthstyle](#) for a list of choices.

`light(#)` specifies the minimum number of observations needed for a bin to be represented by a light sunflower. The default is `light(3)`.

`dark(#)` specifies the minimum number of observations needed for a bin to be represented by a dark sunflower. The default is `dark(13)`.

`xcenter(#)` and `ycenter(#)` specify the center of the reference bin. The default values are the median values of *xvar* and *yvar*, respectively. The centers of the other bins are implicitly defined by the location of the reference bin together with the common bin width and height.

`petalweight(#)` specifies the number of observations represented by each petal of a dark sunflower. The default value is chosen so that the maximum number of petals on a dark sunflower is 14.

`petalength(#)` specifies the length of petals in the sunflowers. The value specified is interpreted as a percentage of half the bin width. The default is 100%.

*petal\_options* affect how the sunflower petals are rendered.

`lflstyle(linestyle)` and `dflstyle(linestyle)` specify the overall style of the light and dark sunflower petals, respectively.

`lflcolor(colorstyle)` and `dflcolor(colorstyle)` specify the color of the light and dark sunflower petals, respectively.

`lflwidth(linewidthstyle)` and `dflwidth(linewidthstyle)` specify the width of the light and dark sunflower petals, respectively.

`floweronly` suppresses rendering of the bins. This option is equivalent to specifying `lbcolor(none)` and `dbcold(none)`.

`nosinglepetal` suppresses flowers from being drawn in light bins that contain only 1 observation and dark bins that contain as many observations as the petal weight (see the `petalweight()` option).

#### Add plots

`addplot(plot)` provides a way to add other plots to the generated graph; see [G-3] [addplot\\_option](#).

#### Y axis, X axis, Titles, Legend, Overall, By

*twoway\_options* are any of the options documented in [G-3] [twoway\\_options](#). These include options for titling the graph (see [G-3] [title\\_options](#)), options for saving the graph to disk (see [G-3] [saving\\_option](#)), and the `by()` option (see [G-3] [by\\_option](#)).

## Remarks and examples

A sunflower is several line segments of equal length, called petals, that radiate from a central point. There are two varieties of sunflowers: light and dark. Each petal of a light sunflower represents 1 observation. Each petal of a dark sunflower represents several observations. Dark and light sunflowers represent high- and medium-density regions of the data, and marker symbols represent individual observations in low-density regions.

`sunflower` divides the plane defined by the variables `yvar` and `xvar` into contiguous hexagonal bins. The number of observations contained within a bin determines how the bin will be represented.

- When there are fewer than `light(#)` observations in a bin, each point is plotted using the usual marker symbols in a scatterplot.
- Bins with at least `light(#)` but fewer than `dark(#)` observations are represented by a light sunflower. Each petal of a light sunflower represents one observation in the bin.
- Bins with at least `dark(#)` observations are represented by a dark sunflower. Each petal of a dark sunflower represents multiple observations.

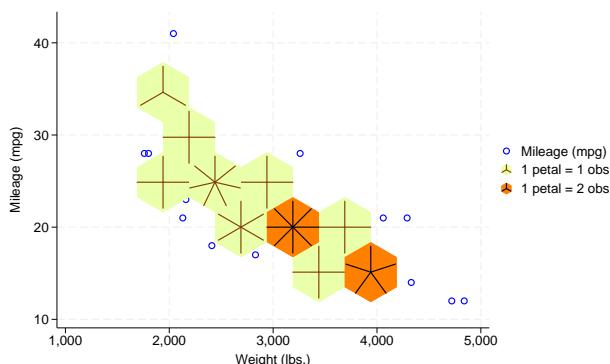
See Dupont (2009, 87–92) for a discussion of sunflower plots and how to create them using Stata.

### ▷ Example 1

Using the `auto` dataset, we want to examine the relationship between `weight` and `mpg`. To do that, we type

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. sunflower mpg weight, binwid(500) petalw(2) dark(8) scheme(stcolor)
Bin width = 500
Bin height = 6.50422
Bin aspect ratio = .0112656
Max obs in a bin = 15
Light = 3
Dark = 8
X-center = 3190
Y-center = 20
Petal weight = 2
```

| Flower type | Petal weight | No. of petals | No. of flowers | Estimated obs | Actual obs |
|-------------|--------------|---------------|----------------|---------------|------------|
| none        |              |               |                | 13            | 13         |
| light       | 1            | 3             | 1              | 3             | 3          |
| light       | 1            | 4             | 5              | 20            | 20         |
| light       | 1            | 6             | 1              | 6             | 6          |
| light       | 1            | 7             | 1              | 7             | 7          |
| dark        | 2            | 5             | 1              | 10            | 10         |
| dark        | 2            | 8             | 1              | 16            | 15         |
|             |              |               |                | 75            | 74         |



The two darkly shaded sunflowers immediately catch our eyes, indicating a group of 10 cars that are heavy (nearly 4,000 pounds) and fuel inefficient and a group 16 of cars that get about 20 miles per gallon and weight in the neighborhood of 3,000 pounds. The lighter sunflowers each indicate groups of three to seven cars that share similar weight and fuel economy characteristics. To obtain the number of cars in each group, we count the number of petals in each flower and consult the graph legend to see how many observations each petal represents.

◀

## Acknowledgments

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## Description

`sureg` fits seemingly unrelated regression models (Zellner 1962; Zellner and Huang 1962; Zellner 1963). The acronyms SURE and SUR are often used for the estimator.

## Quick start

Model of  $y_1$  and  $y_2$  as a function of  $x_1$ ,  $x_2$ , and  $x_3$

```
sureg (y1 x1 x2 x3) (y2 x1 x2 x3)
```

Same as above

```
sureg (y1 y2 = x1 x2 x3)
```

Same as above, but obtain small-sample statistics and use small-sample adjustment

```
sureg (y1 y2 = x1 x2 x3), small dfk
```

Also perform Breusch–Pagan test

```
sureg (y1 y2 = x1 x2 x3), small dfk corr
```

Model of  $y_1$  as a function of  $x_1$  and  $x_2$  and  $y_2$  as a function of  $x_2$  and lag of  $x_2$  using `tsset` data

```
sureg (y1 x1 x2) (y2 x2 L.x2)
```

## Menu

Statistics > Linear models and related > Multiple-equation models > Seemingly unrelated regression

## Syntax

### Basic syntax

```
sureg (depvar1 varlist1) (depvar2 varlist2) ... (depvarN varlistN)
 [if] [in] [weight]
```

### Full syntax

```
sureg ([eqname1:]depvar1a [depvar1b ... =]varlist1 [, noconstant])
 ([eqname2:]depvar2a [depvar2b ... =]varlist2 [, noconstant])
 ...
 ([eqnameN:]depvarNa [depvarNb ... =]varlistN [, noconstant])
 [if] [in] [weight] [, options]
```

Explicit equation naming (*eqname*:) cannot be combined with multiple dependent variables in an equation specification.

| <i>options</i>                            | Description                                                                                                                                      |
|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| Model                                     |                                                                                                                                                  |
| <u>i</u> sure                             | iterate until estimates converge                                                                                                                 |
| <u>c</u> onstraints( <i>constraints</i> ) | apply specified linear constraints                                                                                                               |
| SE/Robust                                 |                                                                                                                                                  |
| <u>v</u> ce( <i>vcetype</i> )             | <i>vcetype</i> may be <u>u</u> nadjusted, <u>r</u> obust, or <u>c</u> luster <i>clustvar</i>                                                     |
| df adj.                                   |                                                                                                                                                  |
| <u>s</u> mall                             | report small-sample statistics                                                                                                                   |
| <u>d</u> fk                               | use small-sample adjustment                                                                                                                      |
| <u>d</u> fk2                              | use alternate adjustment                                                                                                                         |
| Reporting                                 |                                                                                                                                                  |
| <u>l</u> evel(#)                          | set confidence level; default is level(95)                                                                                                       |
| <u>c</u> orr                              | perform Breusch–Pagan test                                                                                                                       |
| <u>n</u> ocnsreport                       | do not display constraints                                                                                                                       |
| <i>display_options</i>                    | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| Optimization                              |                                                                                                                                                  |
| <i>optimization_options</i>               | control the optimization process; seldom used                                                                                                    |
| <u>n</u> oheader                          | suppress header table from above coefficient table                                                                                               |
| <u>n</u> otable                           | suppress coefficient table                                                                                                                       |
| <u>c</u> oeflegend                        | display legend instead of statistics                                                                                                             |

*varlist*<sub>1</sub>, . . . , *varlist*<sub>*N*</sub> may contain factor variables; see [U] 11.4.3 Factor variables. You must have the same levels of factor variables in all equations that have factor variables.

*deprvars* and the *varlists* may contain time-series operators; see [U] 11.4.4 Time-series varlists.

*bootstrap*, *by*, *collect*, *fp*, *jackknife*, *rolling*, and *statsby* are allowed; see [U] 11.1.10 Prefix commands.

Weights are not allowed with the *bootstrap* prefix; see [R] *bootstrap*.

*aweight*s are not allowed with the *jackknife* prefix; see [R] *jackknife*.

*aweight*s and *fweight*s are allowed; see [U] 11.1.6 weight.

*noheader*, *notable*, and *coeflegend* do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Options

### Model

*isure* specifies that *sureg* iterate over the estimated disturbance covariance matrix and parameter estimates until the parameter estimates converge. Under seemingly unrelated regression, this iteration converges to the maximum likelihood results. If this option is not specified, *sureg* produces two-step estimates.

*constraints* (*constraints*); see [R] Estimation options.

### SE/Robust

*vce* (*vcetype*) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (*unadjusted*), that are robust to some kinds of misspecification (*robust*), and that allow for intragroup correlation (*cluster clustvar*); see [R] *vce\_option*.

*vce*(*unadjusted*), the default, specifies that an unadjusted (nonrobust) VCE matrix be used; this results in efficient estimates when assuming homoskedasticity.

### df adj.

*small* specifies that small-sample statistics be computed. It shifts the test statistics from  $\chi^2$  and *z* statistics to *F* statistics and *t* statistics. Although the standard errors from each equation are computed using the degrees of freedom for the equation, the degrees of freedom for the *t* statistics are all taken to be those for the first equation.

*dfk* specifies the use of an alternative divisor in computing the covariance matrix for the equation residuals. As an asymptotically justified estimator, *sureg* by default uses the number of sample observations (*n*) as a divisor. When the *dfk* option is set, a small-sample adjustment is made and the divisor is taken to be  $\sqrt{(n - k_i)(n - k_j)}$ , where *k<sub>i</sub>* and *k<sub>j</sub>* are the number of parameters in equations *i* and *j*, respectively.

*dfk2* specifies the use of an alternative divisor in computing the covariance matrix for the equation residuals. When the *dfk2* option is set, the divisor is taken to be the mean of the residual degrees of freedom from the individual equations.

### Reporting

*level*(#); see [R] Estimation options.

*corr* displays the correlation matrix of the residuals between equations and performs a Breusch–Pagan test for independent equations; that is, the disturbance covariance matrix is diagonal.

nocnsreport; see [R] [Estimation options](#).

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

### Optimization

*optimization\_options* control the iterative process that minimizes the sum of squared errors when `isure` is specified. These options are seldom used.

`iterate(#)` specifies the maximum number of iterations. When the number of iterations equals `#`, the optimizer stops and presents the current results, even if the convergence tolerance has not been reached. The default is the number set using `set maxiter`, which is 300 by default.

`trace` adds to the iteration log a display of the current parameter vector.

`log` and `nolog` specify whether to display the iteration log. The iteration log is displayed by default unless you used `set iterlog off` to suppress it; see `set iterlog` in [R] [set iter](#).

`tolerance(#)` specifies the tolerance for the coefficient vector. When the relative change in the coefficient vector from one iteration to the next is less than or equal to `#`, the optimization process is stopped. `tolerance(1e-6)` is the default.

The following options are available with `sureg` but are not shown in the dialog box:

`noheader` suppresses display of the header reporting  $F$  statistics,  $R^2$ , and root mean squared error above the coefficient table.

`notable` suppresses display of the coefficient table.

`coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Seemingly unrelated regression models are so called because they appear to be joint estimates from several regression models, each with its own error term. The regressions are related because the (contemporaneous) errors associated with the dependent variables may be correlated. Chapter 6 of [Cameron and Trivedi \(2022\)](#) contains a discussion of the seemingly unrelated regression model and the feasible generalized least-squares estimator underlying it.

### ▷ Example 1

When we fit models with the same set of right-hand-side variables, the seemingly unrelated regression results (in terms of coefficients and standard errors) are the same as fitting the models separately (using, say, `regress`). The same is true when the models are nested. Even in such cases, `sureg` is useful when we want to perform joint tests. For instance, let us assume that we think

$$\begin{aligned}\text{price} &= \beta_0 + \beta_1 \text{foreign} + \beta_2 \text{length} + u_1 \\ \text{weight} &= \gamma_0 + \gamma_1 \text{foreign} + \gamma_2 \text{length} + u_2\end{aligned}$$

Because the models have the same set of explanatory variables, we could estimate the two equations separately. Yet, we might still choose to estimate them with `sureg` because we want to perform the joint test  $\beta_1 = \gamma_1 = 0$ .

We use the small and dfk options to obtain small-sample statistics comparable with regress or mvreg.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. sureg (price foreign length) (weight foreign length), small dfk
Seemingly unrelated regression
```

| Equation | Obs | Params | RMSE     | "R-squared" | F      | P>F    |
|----------|-----|--------|----------|-------------|--------|--------|
| price    | 74  | 2      | 2474.593 | 0.3154      | 16.35  | 0.0000 |
| weight   | 74  | 2      | 250.2515 | 0.8992      | 316.54 | 0.0000 |

|         | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|---------|-------------|-----------|-------|-------|----------------------|-----------|
| price   |             |           |       |       |                      |           |
| foreign | 2801.143    | 766.117   | 3.66  | 0.000 | 1286.674             | 4315.611  |
| length  | 90.21239    | 15.83368  | 5.70  | 0.000 | 58.91219             | 121.5126  |
| _cons   | -11621.35   | 3124.436  | -3.72 | 0.000 | -17797.77            | -5444.93  |
| weight  |             |           |       |       |                      |           |
| foreign | -133.6775   | 77.47615  | -1.73 | 0.087 | -286.8332            | 19.4782   |
| length  | 31.44455    | 1.601234  | 19.64 | 0.000 | 28.27921             | 34.60989  |
| _cons   | -2850.25    | 315.9691  | -9.02 | 0.000 | -3474.861            | -2225.639 |

Note: Small-sample degrees-of-freedom adjustment applied when estimating covariance matrix of residuals.

These two equations have a common set of regressors, and we could have used a shorthand syntax to specify the equations:

```
. sureg (price weight = foreign length), small dfk
```

Here the results presented by sureg are the same as if we had estimated the equations separately:

```
. regress price foreign length
(output omitted)
. regress weight foreign length
(output omitted)
```

There is, however, a difference. We have allowed  $u_1$  and  $u_2$  to be correlated and have estimated the full variance–covariance matrix of the coefficients. sureg has estimated the correlations, but it does not report them unless we specify the corr option. We did not remember to specify corr when we fit the model, but we can redisplay the results:

```
. sureg, notable noheader corr
```

Note: Small-sample degrees-of-freedom adjustment applied when estimating covariance matrix of residuals.

Correlation matrix of residuals:

```
 price weight
price 1.0000
weight 0.5840 1.0000
```

```
Breusch-Pagan test of independence: chi2(1) = 25.237, Pr = 0.0000
```

The notable and noheader options prevented sureg from redisplaying the header and coefficient tables. We find that, for the same cars, the correlation of the residuals in the price and weight equations is 0.5840 and that we can reject the hypothesis that this correlation is zero.

We can test that the coefficients on `foreign` are jointly zero in both equations—as we set out to do—by typing `test foreign`; see [R] [test](#). When we type a variable without specifying the equation, that variable is tested for zero in all equations in which it appears:

```
. test foreign
(1) [price]foreign = 0
(2) [weight]foreign = 0
 F(2, 142) = 17.99
 Prob > F = 0.0000
```

◀

## ▶ Example 2

When the models do not have the same set of explanatory variables and are not nested, `sureg` may lead to more efficient estimates than running the models separately as well as allowing joint tests. This time, let us assume that we believe

$$\begin{aligned} \text{price} &= \beta_0 + \beta_1 \text{foreign} + \beta_2 \text{mpg} + \beta_3 \text{displ} + u_1 \\ \text{weight} &= \gamma_0 + \gamma_1 \text{foreign} + \gamma_2 \text{length} + u_2 \end{aligned}$$

To fit this model, we type

```
. sureg (price foreign mpg displ) (weight foreign length), corr
Seemingly unrelated regression
```

| Equation | Obs | Params | RMSE     | "R-squared" | chi2   | P>chi2 |
|----------|-----|--------|----------|-------------|--------|--------|
| price    | 74  | 3      | 2165.321 | 0.4537      | 49.64  | 0.0000 |
| weight   | 74  | 2      | 245.2916 | 0.8990      | 661.84 | 0.0000 |

|              | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|--------------|-------------|-----------|-------|-------|----------------------|-----------|
| price        |             |           |       |       |                      |           |
| foreign      | 3058.25     | 685.7357  | 4.46  | 0.000 | 1714.233             | 4402.267  |
| mpg          | -104.9591   | 58.47209  | -1.80 | 0.073 | -219.5623            | 9.644042  |
| displacement | 18.18098    | 4.286372  | 4.24  | 0.000 | 9.779842             | 26.58211  |
| _cons        | 3904.336    | 1966.521  | 1.99  | 0.047 | 50.0263              | 7758.645  |
| weight       |             |           |       |       |                      |           |
| foreign      | -147.3481   | 75.44314  | -1.95 | 0.051 | -295.2139            | .517755   |
| length       | 30.94905    | 1.539895  | 20.10 | 0.000 | 27.93091             | 33.96718  |
| _cons        | -2753.064   | 303.9336  | -9.06 | 0.000 | -3348.763            | -2157.365 |

Correlation matrix of residuals:

```
 price weight
price 1.0000
weight 0.3285 1.0000
```

Breusch-Pagan test of independence: `chi2(1) = 7.984, Pr = 0.0047`

In comparison, if we had fit the price model separately,

```
. regress price foreign mpg displ
```

| Source   | SS        | df | MS         | Number of obs | = | 74     |
|----------|-----------|----|------------|---------------|---|--------|
| Model    | 294104790 | 3  | 98034929.9 | F(3, 70)      | = | 20.13  |
| Residual | 340960606 | 70 | 4870865.81 | Prob > F      | = | 0.0000 |
|          |           |    |            | R-squared     | = | 0.4631 |
|          |           |    |            | Adj R-squared | = | 0.4401 |
| Total    | 635065396 | 73 | 8699525.97 | Root MSE      | = | 2207   |

| price        | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|--------------|-------------|-----------|-------|-------|----------------------|----------|
| foreign      | 3545.484    | 712.7763  | 4.97  | 0.000 | 2123.897             | 4967.072 |
| mpg          | -98.88559   | 63.17063  | -1.57 | 0.122 | -224.8754            | 27.10426 |
| displacement | 22.40416    | 4.634239  | 4.83  | 0.000 | 13.16146             | 31.64686 |
| _cons        | 2796.91     | 2137.873  | 1.31  | 0.195 | -1466.943            | 7060.763 |

The coefficients are slightly different, but the standard errors are uniformly larger. This would still be true if we specified the `dfk` option to make a small-sample adjustment to the estimated covariance of the disturbances.

◀

## □ Technical note

Constraints can be applied to SURE models using Stata's standard syntax for constraints. For a general discussion of constraints, see [R] [constraint](#); for examples similar to seemingly unrelated regression models, see [R] [reg3](#).

□

## Stored results

sureg stores the following in `e()`:

### Scalars

|                          |                                                        |
|--------------------------|--------------------------------------------------------|
| <code>e(N)</code>        | number of observations                                 |
| <code>e(k)</code>        | number of parameters                                   |
| <code>e(k_eq)</code>     | number of equations in <code>e(b)</code>               |
| <code>e(mss_#)</code>    | model sum of squares for equation #                    |
| <code>e(df_m#)</code>    | model degrees of freedom for equation #                |
| <code>e(rss_#)</code>    | residual sum of squares for equation #                 |
| <code>e(df_r)</code>     | residual degrees of freedom                            |
| <code>e(r2_#)</code>     | $R^2$ for equation #                                   |
| <code>e(F_#)</code>      | $F$ statistic for equation # (small only)              |
| <code>e(rmse_#)</code>   | root mean squared error for equation #                 |
| <code>e(dfk2_adj)</code> | divisor used with VCE when <code>dfk2</code> specified |
| <code>e(ll)</code>       | log likelihood                                         |
| <code>e(N_clust)</code>  | number of clusters                                     |
| <code>e(chi2_#)</code>   | $\chi^2$ for equation #                                |
| <code>e(p_#)</code>      | $p$ -value for equation #                              |
| <code>e(cons_#)</code>   | 1 if equation # has a constant, 0 otherwise            |
| <code>e(chi2_bp)</code>  | Breusch–Pagan $\chi^2$                                 |
| <code>e(df_bp)</code>    | degrees of freedom for Breusch–Pagan $\chi^2$ test     |
| <code>e(rank)</code>     | rank of <code>e(V)</code>                              |
| <code>e(ic)</code>       | number of iterations                                   |

## Macros

|                   |                                                    |
|-------------------|----------------------------------------------------|
| e(cmd)            | sureg                                              |
| e(cmdline)        | command as typed                                   |
| e(method)         | sure or isure                                      |
| e(depvar)         | names of dependent variables                       |
| e(exog)           | names of exogenous variables                       |
| e(equnames)       | names of equations                                 |
| e(wtype)          | weight type                                        |
| e(wexp)           | weight expression                                  |
| e(corr)           | correlation structure                              |
| e(small)          | small, if specified                                |
| e(dfk)            | dfk or dfk2, if specified                          |
| e(clustvar)       | name of cluster variable                           |
| e(vce)            | <i>vcetype</i> specified in <i>vce()</i>           |
| e(vcetype)        | title used to label Std. err.                      |
| e(properties)     | b V                                                |
| e(predict)        | program used to implement predict                  |
| e(marginsok)      | predictions allowed by margins                     |
| e(marginsnotok)   | predictions disallowed by margins                  |
| e(marginsdefault) | default <i>predict()</i> specification for margins |
| e(asbalanced)     | factor variables <i>fvset</i> as <i>asbalanced</i> |
| e(asobserved)     | factor variables <i>fvset</i> as <i>asobserved</i> |

## Matrices

|          |                                                 |
|----------|-------------------------------------------------|
| e(b)     | coefficient vector                              |
| e(Cns)   | constraints matrix                              |
| e(Sigma) | $\hat{\Sigma}$ , covariance matrix of residuals |
| e(V)     | variance–covariance matrix of the estimators    |

## Functions

|           |                         |
|-----------|-------------------------|
| e(sample) | marks estimation sample |
|-----------|-------------------------|

In addition to the above, the following is stored in *r()*:

## Matrices

|          |                                                                                                                            |
|----------|----------------------------------------------------------------------------------------------------------------------------|
| r(table) | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |
|----------|----------------------------------------------------------------------------------------------------------------------------|

Note that results stored in *r()* are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

## Methods and formulas

sureg uses the asymptotically efficient, feasible, generalized least-squares algorithm described in Greene (2018, 328–339). The computing formulas are given on page 328–335.

The  $R^2$  reported is the percent of variance explained by the predictors. It may be used for descriptive purposes, but  $R^2$  is not a well-defined concept when GLS is used.

sureg will refuse to compute the estimators if the same equation is named more than once or the covariance matrix of the residuals is singular.

The Breusch and Pagan (1980)  $\chi^2$  statistic—a Lagrange multiplier statistic—is given by

$$\lambda = T \sum_{m=2}^M \sum_{n=1}^{m-1} r_{mn}^2$$

where  $r_{mn}$  is the estimated correlation between the residuals of the  $M$  equations and  $T$  is the number of observations. It is distributed as  $\chi^2$  with  $M(M - 1)/2$  degrees of freedom.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] [\\_robust](#), particularly *Introduction* and *Methods and formulas*.

**Arnold Zellner** (1927–2010) was born in New York. He studied physics at Harvard and economics at Berkeley, and then he taught economics at the Universities of Washington and Wisconsin before settling in Chicago in 1966. Among his many major contributions to econometrics and statistics are his work on seemingly unrelated regression, three-stage least squares, and Bayesian econometrics.

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## Also see

- [R] [sureg postestimation](#) — Postestimation tools for `sureg`
- [R] [nlsur](#) — Estimation of nonlinear system of equations
- [R] [reg3](#) — Three-stage estimation for systems of simultaneous equations
- [R] [regress](#) — Linear regression
- [MV] [mvreg](#) — Multivariate regression
- [SEM] [Example 12](#) — Seemingly unrelated regression
- [SEM] [Intro 5](#) — Tour of models
- [TS] [dfactor](#) — Dynamic-factor models
- [U] [20 Estimation and postestimation commands](#)

## Postestimation commands

The following postestimation commands are available after `sureg`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| <code>forecast</code>        | dynamic forecasts and simulations                                                                                                       |
| <code>hausman</code>         | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | predictions and their SEs, residuals, etc.                                                                                              |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

# predict

## Description for predict

`predict` creates a new variable containing predictions such as linear predictions, standard errors, residuals, and differences between the linear predictions.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, equation(eqno[,eqno]) statistic]
```

| <i>statistic</i>        | Description                                                |
|-------------------------|------------------------------------------------------------|
| Main                    |                                                            |
| <code>xb</code>         | linear prediction; the default                             |
| <code>stdp</code>       | standard error of the linear prediction                    |
| <code>residuals</code>  | residuals                                                  |
| <code>difference</code> | difference between the linear predictions of two equations |
| <code>stddp</code>      | standard error of the difference in linear predictions     |

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

## Options for predict

Main

`equation(eqno[ ,eqno])` specifies to which equation(s) you are referring.

`equation()` is filled in with one `eqno` for the `xb`, `stdp`, and `residuals` options. `equation(#1)` would mean that the calculation is to be made for the first equation, `equation(#2)` would mean the second, and so on. You could also refer to the equations by their names. `equation(income)` would refer to the equation named `income` and `equation(hours)` to the equation named `hours`.

If you do not specify `equation()`, the results are the same as if you specified `equation(#1)`.

`difference` and `stddp` refer to between-equation concepts. To use these options, you must specify two equations, for example, `equation(#1,#2)` or `equation(income,hours)`. When two equations must be specified, `equation()` is required.

`xb`, the default, calculates the linear prediction (fitted values)—the prediction of  $\mathbf{x}_j\mathbf{b}$  for the specified equation.

`stdp` calculates the standard error of the prediction for the specified equation. It can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. The standard error of the prediction is also referred to as the standard error of the fitted value.

`residuals` calculates the residuals.

`difference` calculates the difference between the linear predictions of two equations in the system. With `equation(#1,#2)`, `difference` computes the prediction of equation(#1) minus the prediction of equation(#2).

`stdp` is allowed only after you have previously fit a multiple-equation model. The standard error of the difference in linear predictions ( $\mathbf{x}_{1j}\mathbf{b} - \mathbf{x}_{2j}\mathbf{b}$ ) between equations 1 and 2 is calculated.

For more information on using `predict` after multiple-equation estimation commands, see [R] [predict](#).

## margins

### Description for margins

`margins` estimates margins of response for linear predictions and differences between the linear predictions.

### Menu for margins

Statistics > Postestimation

### Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i>        | Description                                                |
|-------------------------|------------------------------------------------------------|
| <code>default</code>    | linear predictions for each equation                       |
| <code>xb</code>         | linear prediction for a specified equation                 |
| <code>difference</code> | difference between the linear predictions of two equations |
| <code>stdp</code>       | not allowed with <code>margins</code>                      |
| <code>residuals</code>  | not allowed with <code>margins</code>                      |
| <code>stddp</code>      | not allowed with <code>margins</code>                      |

`xb` defaults to the first equation.

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [R] [margins](#).

## Remarks and examples

For an example of cross-equation testing of parameters using the `test` command, see [example 1](#) in [\[R\] sureg](#).

### ▷ Example 1

In [example 1](#) of [\[R\] sureg](#), we fit a seemingly unrelated regressions model of price and weight. Here we obtain the fitted values.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. sureg (price foreign length) (weight foreign length), small dfk
(output omitted)
. predict phat, equation(price)
(option xb assumed; fitted values)
. predict what, equation(weight)
(option xb assumed; fitted values)
. summarize price phat weight what
```

| Variable | Obs | Mean     | Std. dev. | Min      | Max      |
|----------|-----|----------|-----------|----------|----------|
| price    | 74  | 6165.257 | 2949.496  | 3291     | 15906    |
| phat     | 74  | 6165.257 | 1656.407  | 1639.872 | 9398.138 |
| weight   | 74  | 3019.459 | 777.1936  | 1760     | 4840     |
| what     | 74  | 3019.459 | 736.9666  | 1481.199 | 4476.331 |

Just as in single-equation OLS regression, in a SURE model the sample mean of the fitted values for an equation equals the sample mean of the dependent variable.

◀

### ▷ Example 2

Suppose that for whatever reason we were interested in the difference between the predicted values of price and weight. `predict` has an option to compute this difference in one step:

```
. predict diff, equation(price, weight) difference
```

`diff` is the same as `phat - what`:

```
. generate mydiff = phat - what
. summarize diff mydiff
```

| Variable | Obs | Mean     | Std. dev. | Min       | Max      |
|----------|-----|----------|-----------|-----------|----------|
| diff     | 74  | 3145.797 | 1233.26   | -132.2275 | 5505.914 |
| mydiff   | 74  | 3145.797 | 1233.26   | -132.2275 | 5505.914 |

◀

## Also see

[\[R\] sureg](#) — Zellner's seemingly unrelated regression

[\[U\] 20 Estimation and postestimation commands](#)

Description  
Options for `swilk`  
Methods and formulas

Quick start  
Options for `sfrancia`  
Acknowledgment

Menu  
Remarks and examples  
References

Syntax  
Stored results  
Also see

## Description

`swilk` performs the Shapiro–Wilk  $W$  test for normality for each variable in the specified varlist. Likewise, `sfrancia` performs the Shapiro–Francia  $W'$  test for normality. See [\[MV\] mvtest normality](#) for multivariate tests of normality.

## Quick start

*Shapiro–Wilk test of normality*

Shapiro–Wilk test for `v1`

```
swilk v1
```

Separate tests of normality for `v1` and `v2`

```
swilk v1 v2
```

Generate new variable `w` containing  $W$  test coefficients

```
swilk v1, generate(w)
```

Specify that average ranks should not be used for tied values

```
swilk v1 v2, noties
```

Test that `v3` is distributed lognormally

```
generate lnv3 = ln(v3)
swilk lnv3
```

*Shapiro–Francia test of normality*

Shapiro–Francia test for `v1`

```
sfrancia v1
```

Separate tests of normality for `v1` and `v2`

```
sfrancia v1 v2
```

Same as above, but use the Box–Cox transformation

```
sfrancia v1 v2, boxcox
```

Specify that average ranks should not be used for tied values

```
sfrancia v1 v2, noties
```

## Menu

### swilk

Statistics > Summaries, tables, and tests > Distributional plots and tests > Shapiro-Wilk normality test

### sfrancia

Statistics > Summaries, tables, and tests > Distributional plots and tests > Shapiro-Francia normality test

## Syntax

*Shapiro–Wilk normality test*

```
swilk varlist [if] [in] [, swilk_options]
```

*Shapiro–Francia normality test*

```
sfrancia varlist [if] [in][, sfrancia_options]
```

*swilk\_options*

Description

Main

|                                      |                                                       |
|--------------------------------------|-------------------------------------------------------|
| <code>generate(<i>newvar</i>)</code> | create <i>newvar</i> containing $W$ test coefficients |
| <code>lnnormal</code>                | test for three-parameter lognormality                 |
| <code>noties</code>                  | do not use average ranks for tied values              |

*sfrancia\_options*

Description

Main

|                     |                                                                                        |
|---------------------|----------------------------------------------------------------------------------------|
| <code>boxcox</code> | use the Box–Cox transformation for $W'$ ; the default is to use the log transformation |
| <code>noties</code> | do not use average ranks for tied values                                               |

`by` and `collect` are allowed with `swilk` and `sfrancia`; see [\[U\] 11.1.10 Prefix commands](#).

## Options for swilk

Main

`generate(newvar)` creates new variable *newvar* containing the  $W$  test coefficients.

`lnnormal` specifies that the test be for three-parameter lognormality, meaning that  $\ln(X - k)$  is tested for normality, where  $k$  is calculated from the data as the value that makes the skewness coefficient zero. When simply testing  $\ln(X)$  for normality, do not specify this option. See [\[R\] lnskew0](#) for estimation of  $k$ .

`noties` suppresses use of averaged ranks for tied values when calculating the  $W$  test coefficients.

## Options for sfrancia

Main

`boxcox` specifies that the Box–Cox transformation of [Royston \(1983\)](#) for calculating  $W'$  test coefficients be used instead of the default log transformation ([Royston 1993a](#)). Under the Box–Cox transformation, the normal approximation to the sampling distribution of  $W'$ , used by `sfrancia`, is valid for  $5 \leq n \leq 1000$ . Under the log transformation, it is valid for  $10 \leq n \leq 5000$ .

`noties` suppresses use of averaged ranks for tied values when calculating the  $W'$  test coefficients.

## Remarks and examples

`swilk` can be used with  $4 \leq n \leq 2000$  observations. `sfrancia` can be used with  $10 \leq n \leq 5000$  observations; however, if the `boxcox` option is specified, it can be used with  $5 \leq n \leq 1000$  observations.

Also see [\[R\] sktest](#) for the skewness and kurtosis test described by [D'Agostino, Belanger, and D'Agostino \(1990\)](#) with the empirical correction developed by [Royston \(1991b\)](#). While the Shapiro–Wilk and Shapiro–Francia tests for normality are, in general, preferred for nonaggregated data ([Gould and Rogers 1991](#); [Gould 1992b](#); [Royston 1991b](#)), the skewness and kurtosis test will permit more observations. Moreover, a normal quantile plot should be used with any test for normality; see [\[R\] Diagnostic plots](#) for more information.

### ▷ Example 1

Using our automobile dataset, we will test whether the variables `mpg` and `trunk` are normally distributed:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. swilk mpg trunk
```

| Shapiro–Wilk W test for normal data |     |         |       |       |         |
|-------------------------------------|-----|---------|-------|-------|---------|
| Variable                            | Obs | W       | V     | z     | Prob>z  |
| mpg                                 | 74  | 0.94821 | 3.335 | 2.627 | 0.00430 |
| trunk                               | 74  | 0.97921 | 1.339 | 0.637 | 0.26215 |

```
. sfrancia mpg trunk
```

| Shapiro–Francia W' test for normal data |     |         |       |       |         |
|-----------------------------------------|-----|---------|-------|-------|---------|
| Variable                                | Obs | W'      | V'    | z     | Prob>z  |
| mpg                                     | 74  | 0.94872 | 3.650 | 2.510 | 0.00604 |
| trunk                                   | 74  | 0.98446 | 1.106 | 0.195 | 0.42271 |

We can reject the hypothesis that `mpg` is normally distributed, but we cannot reject that `trunk` is normally distributed.

The values reported under  $W$  and  $W'$  are the Shapiro–Wilk and Shapiro–Francia test statistics. The tests also report  $V$  and  $V'$  ([Royston 1991d](#)), which are more appealing indexes for departure from normality. The median values of  $V$  and  $V'$  are 1 for samples from normal populations. Large values indicate nonnormality. There is no more information in  $V(V')$  than in  $W(W')$ —one is just the transform of the other.

## ▷ Example 2

We have data on a variable called `studytime`, which we suspect is distributed lognormally:

```
. use https://www.stata-press.com/data/r19/cancer
(Patient survival in drug trial)
. generate lnstudytime = ln(studytime)
. swilk lnstudytime
```

| Shapiro–Wilk W test for normal data |     |         |       |       |         |
|-------------------------------------|-----|---------|-------|-------|---------|
| Variable                            | Obs | W       | V     | z     | Prob>z  |
| lnstudytime                         | 48  | 0.92731 | 3.311 | 2.547 | 0.00543 |

We can reject the lognormal assumption. We do *not* specify the `lnnormal` option when testing for lognormality. The `lnnormal` option is for three-parameter lognormality.

◀

## ▷ Example 3

Having discovered that  $\ln(\text{studytime})$  is not distributed normally, we now test that  $\ln(\text{studytime} - k)$  is normally distributed, where  $k$  is chosen so that the resulting skewness is zero. We obtain the estimate for  $k$  from `lnskew0`; see [R] [lnskew0](#):

```
. lnskew0 lnstudytimek = studytime, level(95)
```

| Transform       | k         | [95% conf. interval] | Skewness  |
|-----------------|-----------|----------------------|-----------|
| ln(studytime-k) | -11.01181 | -infinity - .9477328 | -.0000173 |

```
. swilk lnstudytimek, lnnormal
```

| Shapiro–Wilk W test for 3-parameter lognormal data |     |         |       |       |         |
|----------------------------------------------------|-----|---------|-------|-------|---------|
| Variable                                           | Obs | W       | V     | z     | Prob>z  |
| lnstudytimek                                       | 48  | 0.97064 | 1.337 | 1.261 | 0.10363 |

We cannot reject the hypothesis that  $\ln(\text{studytime} + 11.01181)$  is distributed normally. We do specify the `lnnormal` option when using an estimated value of  $k$ .

◀

## Stored results

`swilk` and `sfrancia` store the following in `r()`:

Scalars

|                   |                        |                   |             |
|-------------------|------------------------|-------------------|-------------|
| <code>r(N)</code> | number of observations | <code>r(W)</code> | $W$ or $W'$ |
| <code>r(p)</code> | $p$ -value             | <code>r(V)</code> | $V$ or $V'$ |
| <code>r(z)</code> | $z$ statistic          |                   |             |

## Methods and formulas

The Shapiro–Wilk test is based on Shapiro and Wilk (1965) with a new approximation accurate for  $4 \leq n \leq 2000$  (Royston 1992). The calculations made by `swilk` are based on Royston (1982, 1992, 1993b).

The Shapiro–Francia test (Shapiro and Francia 1972; Royston 1983; Royston 1993a) is an approximate test that is similar to the Shapiro–Wilk test for very large samples.

The relative merits of the Shapiro–Wilk and Shapiro–Francia tests versus skewness and kurtosis test have been a subject of debate. The interested reader is directed to the articles in the *Stata Technical Bulletin*. Our recommendation is to use the Shapiro–Francia test whenever possible, that is, whenever dealing with nonaggregated or ungrouped data (Gould and Rogers 1991; Gould 1992b); see [R] `swilk`. If normality is rejected, use `sktest` to determine the source of the problem. As both D’Agostino, Belanger, and D’Agostino (1990) and Royston (1991c) mention, researchers should also examine the normal quantile plot to determine normality rather than blindly relying on a few test statistics. See the `qnorm` command documented in [R] **Diagnostic plots** for more information on normal quantile plots.

Samuel Sanford Shapiro (1930–2023) earned degrees in statistics and engineering from City College of New York, Columbia, and Rutgers. After employment in the US Army and industry, he joined the faculty at Florida International University in 1972. Shapiro has coauthored various texts in statistics and published several papers on distributional testing and other statistical topics.

## Acknowledgment

`swilk` and `sfrancia` were written by Patrick Royston of the MRC Clinical Trials Unit, London and coauthor of the Stata Press book *Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model*.

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## Also see

[R] [Inskew0](#) — Find zero-skewness log or Box–Cox transform

[R] [lv](#) — Letter-value displays

[R] [sktest](#) — Skewness and kurtosis tests for normality

[R] [Diagnostic plots](#) — Distributional diagnostic plots

[MV] [mvtest normality](#) — Multivariate normality tests

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## Description

`symmetry` performs asymptotic symmetry and marginal homogeneity tests, as well as an exact symmetry test on  $K \times K$  tables where there is a 1-to-1 matching of cases and controls (nonindependence). This testing is used to analyze matched-pair case–control data with multiple discrete levels of the exposure (outcome) variable. In genetics, the test is known as the transmission/disequilibrium test (TDT) and is used to test the association between transmitted and nontransmitted parental marker alleles to an affected child (Spieldman, McGinnis, and Ewens 1993). For  $2 \times 2$  tables, the asymptotic test statistics reduce to the McNemar test statistic, and the exact symmetry test produces an exact McNemar test; see [R] [Epitab](#). For many exposure variables, `symmetry` can optionally perform a test for linear trend in the log relative risk.

`symmetry` expects the data to be in the wide format; that is, each observation contains the matched case and control values in variables *casevar* and *controlvar*. Variables can be numeric or string.

`symmi` is the immediate form of `symmetry`. The `symmi` command uses the values specified on the command line; rows are separated by ‘\’, and options are the same as for `symmetry`. See [U] [19 Immediate commands](#) for a general introduction to immediate commands.

## Quick start

Symmetry and marginal homogeneity tests for 1-to-1 matched case–control studies

```
symmetry case control
```

Same as above

```
symmetry control case
```

Exact test of table symmetry

```
symmetry case control, exact
```

Report the contribution from each off-diagonal pair to the overall  $\chi^2$ -statistic

```
symmetry control case, contrib
```

Test for a linear trend in the log of the relative risk

```
symmetry control case, trend
```

Request marginal homogeneity statistics that do not require the inversion of the variance–covariance matrix

```
symmetry case control, mh
```

Using frequency weight variable *wvar*

```
symmetry case control [fweight=wvar]
```

## Menu

### symmetry

Statistics > Epidemiology and related > Other > Symmetry and marginal homogeneity test

### symmi

Statistics > Epidemiology and related > Other > Symmetry and marginal homogeneity test calculator

## Syntax

*Symmetry and marginal homogeneity tests*

```
symmetry casevar controlvar [if] [in] [weight] [, options]
```

*Immediate form of symmetry and marginal homogeneity tests*

```
symmi #11 #12 [...] \ #21 #22 [...] [\...] [if] [in] [, options]
```

*options*

Description

Main

|                |                                                                  |
|----------------|------------------------------------------------------------------|
| <u>notable</u> | suppress output of contingency table                             |
| <u>contrib</u> | report contribution of each off-diagonal cell pair               |
| <u>exact</u>   | perform exact test of table symmetry                             |
| <u>mh</u>      | perform two marginal homogeneity tests                           |
| <u>trend</u>   | perform a test for linear trend in the (log) relative risk (RR)  |
| <u>cc</u>      | use continuity correction when calculating test for linear trend |

collect is allowed with symmetry; see [U] 11.1.10 Prefix commands.

fweights are allowed; see [U] 11.1.6 weight.

## Options

Main

**notable** suppresses the output of the contingency table. By default, `symmetry` displays the  $n \times n$  contingency table at the top of the output.

**contrib** reports the contribution of each off-diagonal cell pair to the overall symmetry  $\chi^2$ .

**exact** performs an exact test of table symmetry. This option is recommended for sparse tables. CAUTION: The exact test requires substantial amounts of time and memory for large tables.

**mh** performs two marginal homogeneity tests that do not require the inversion of the variance–covariance matrix.

By default, `symmetry` produces the Stuart–Maxwell test statistic, which requires the inversion of the nondiagonal variance–covariance matrix,  $\mathbf{V}$ . When the table is sparse, the matrix may not be of full rank, and then the command substitutes a generalized inverse  $\mathbf{V}^*$  for  $\mathbf{V}^{-1}$ . `mh` calculates optional marginal homogeneity statistics that do not require the inversion of the variance–covariance matrix. These tests may be preferred in certain situations. See *Methods and formulas* and *Bickeböllner and Clerget-Darpoux (1995)* for details on these test statistics.

`trend` performs a test for linear trend in the (log) relative risk (RR). This option is allowed only for numeric exposure (outcome) variables, and its use should be restricted to measurements on the ordinal or the interval scales.

`cc` specifies that the continuity correction be used when calculating the test for linear trend. This correction should be specified only when the levels of the exposure variable are equally spaced.

## Remarks and examples

`symmetry` and `symmi` may be used to analyze 1-to-1 matched case–control data with multiple discrete levels of the exposure (outcome) variable.

### ▷ Example 1

Consider a survey of 344 individuals (BMDP 1990, 267–270) who were asked in October 1986 whether they agreed with President Reagan’s handling of foreign affairs. In January 1987, after the Iran-Contra affair became public, these same individuals were surveyed again and asked the same question. We would like to know if public opinion changed over this period.

We first describe the dataset and list a few observations.

```
. use https://www.stata-press.com/data/r19/iran
. describe
Contains data from https://www.stata-press.com/data/r19/iran.dta
Observations: 344
Variables: 2 29 Jan 2024 02:37
```

| Variable name | Storage type | Display format | Value label | Variable label           |
|---------------|--------------|----------------|-------------|--------------------------|
| before        | byte         | %8.0g          | vlab        | Public opinion before IC |
| after         | byte         | %8.0g          | vlab        | Public opinion after IC  |

Sorted by:

```
. list in 1/5
```

|    | before   | after    |
|----|----------|----------|
| 1. | Agree    | Agree    |
| 2. | Agree    | Disagree |
| 3. | Agree    | Unsure   |
| 4. | Disagree | Agree    |
| 5. | Disagree | Disagree |

Each observation corresponds to one of the 344 individuals. The data are in wide form so that each observation has a before and an after measurement. We now perform the test without options.

```
. symmetry before after
```

| Public opinion before IC              | Public opinion after IC |          |        | Total | chi2  | df | Prob>chi2 |
|---------------------------------------|-------------------------|----------|--------|-------|-------|----|-----------|
|                                       | Agree                   | Disagree | Unsure |       |       |    |           |
| Agree                                 | 47                      | 56       | 38     | 141   |       |    |           |
| Disagree                              | 28                      | 61       | 31     | 120   |       |    |           |
| Unsure                                | 26                      | 47       | 10     | 83    |       |    |           |
| Total                                 | 101                     | 164      | 79     | 344   |       |    |           |
| Symmetry (asymptotic)                 |                         |          |        |       | 14.87 | 3  | 0.0019    |
| Marginal homogeneity (Stuart-Maxwell) |                         |          |        |       | 14.78 | 2  | 0.0006    |

The test first tabulates the data in a  $K \times K$  table and then performs Bowker's (1948) test for table symmetry and the Stuart–Maxwell (Stuart 1955; Maxwell 1970) test for marginal homogeneity.

Both the symmetry test and the marginal homogeneity test are highly significant, thus indicating a shift in public opinion.

An exact test of symmetry is provided for use on sparse tables. This test is computationally intensive, so it should not be used on large tables. Because we are working on a fast computer, we will run the symmetry test again and this time include the exact option. We will suppress the output of the contingency table by specifying `notable` and include the `contrib` option so that we may further examine the cells responsible for the significant result.

```
. symmetry before after, contrib exact mh notable
```

| Cells                                     | Contribution to symmetry chi-squared | chi2  | df | Prob>chi2 |
|-------------------------------------------|--------------------------------------|-------|----|-----------|
| n1_2 & n2_1                               | 9.3333                               |       |    |           |
| n1_3 & n3_1                               | 2.2500                               |       |    |           |
| n2_3 & n3_2                               | 3.2821                               |       |    |           |
| Symmetry (exact significance probability) |                                      |       |    |           |
| Symmetry (asymptotic)                     |                                      |       |    |           |
| Marginal homogeneity (Stuart-Maxwell)     |                                      |       |    |           |
| Marginal homogeneity (Bickenboller)       |                                      |       |    |           |
| Marginal homogeneity (no diagonals)       |                                      |       |    |           |
|                                           |                                      | 14.87 | 3  | 0.0019    |
|                                           |                                      | 14.78 | 2  | 0.0006    |
|                                           |                                      | 13.53 | 2  | 0.0012    |
|                                           |                                      | 15.25 | 2  | 0.0005    |
| Symmetry (exact significance probability) |                                      |       |    |           |
|                                           |                                      |       |    | 0.0018    |

The largest contribution to the symmetry  $\chi^2$  is due to cells  $n_{12}$  and  $n_{21}$ . These correspond to changes between the agree and disagree categories. Of the 344 individuals, 56 (16.3%) changed from the agree to the disagree response, whereas only 28 (8.1%) changed in the opposite direction.

For these data, the results from the exact test are similar to those from the asymptotic test.

▷ Example 2

Breslow and Day (1980, 163) reprinted data from Mack et al. (1976) from a case–control study of the effect of exogenous estrogen on the risk of endometrial cancer. The data consist of 59 elderly women diagnosed with endometrial cancer and 59 disease-free control subjects living in the same community as the cases. Cases and controls were matched on age, marital status, and time living in the community. The data collected included information on the daily dose of conjugated estrogen therapy. Breslow and Day analyzed these data by creating four levels of the dose variable. Here are the data as entered into a Stata dataset:

```
. use https://www.stata-press.com/data/r19/bd163
. list, noobs divider
```

| case      | control   | count |
|-----------|-----------|-------|
| 0         | 0         | 6     |
| 0         | 0.1-0.299 | 2     |
| 0         | 0.3-0.625 | 3     |
| 0         | 0.626+    | 1     |
| 0.1-0.299 | 0         | 9     |
| 0.1-0.299 | 0.1-0.299 | 4     |
| 0.1-0.299 | 0.3-0.625 | 2     |
| 0.1-0.299 | 0.626+    | 1     |
| 0.3-0.625 | 0         | 9     |
| 0.3-0.625 | 0.1-0.299 | 2     |
| 0.3-0.625 | 0.3-0.625 | 3     |
| 0.3-0.625 | 0.626+    | 1     |
| 0.626+    | 0         | 12    |
| 0.626+    | 0.1-0.299 | 1     |
| 0.626+    | 0.3-0.625 | 2     |
| 0.626+    | 0.626+    | 1     |

This dataset is in a different format from that of the [previous example](#). Instead of each observation representing one matched pair, each observation represents possibly multiple pairs indicated by the count variable. For instance, the first observation corresponds to six matched pairs where neither the case nor the control was on estrogen, the second observation corresponds to two matched pairs where the case was not on estrogen and the control was on 0.1 to 0.299 mg/day, etc.

To use `symmetry` to analyze this dataset, we must specify `fweight` to indicate that in our data there are observations corresponding to more than one matched pair.

```
. symmetry case control [fweight=count]
```

| Dosage level for case | Dosage level for control |           |           |        | Total |
|-----------------------|--------------------------|-----------|-----------|--------|-------|
|                       | 0                        | 0.1-0.299 | 0.3-0.625 | 0.626+ |       |
| 0                     | 6                        | 2         | 3         | 1      | 12    |
| 0.1-0.299             | 9                        | 4         | 2         | 1      | 16    |
| 0.3-0.625             | 9                        | 2         | 3         | 1      | 15    |
| 0.626+                | 12                       | 1         | 2         | 1      | 16    |
| Total                 | 36                       | 9         | 10        | 4      | 59    |

|                                       | chi2  | df | Prob>chi2 |
|---------------------------------------|-------|----|-----------|
| Symmetry (asymptotic)                 | 17.10 | 6  | 0.0089    |
| Marginal homogeneity (Stuart-Maxwell) | 16.96 | 3  | 0.0007    |

Both the test of symmetry and the test of marginal homogeneity are highly significant, thus leading us to reject the null hypothesis that there is no effect of exposure to estrogen on the risk of endometrial cancer.

Breslow and Day perform a test for trend assuming that the estrogen exposure levels were equally spaced by recoding the exposure levels as 1, 2, 3, and 4.

We can easily reproduce their results by recoding our data in this way and by specifying the trend option. Two new numeric variables were created, ca and co, corresponding to the variables case and control, respectively. Below, we list some of the data and our results from symmetry:

```
. encode case, gen(ca)
. encode control, gen(co)
. label values ca
. label values co
. list in 1/4
```

|    | case | control   | count | ca | co |
|----|------|-----------|-------|----|----|
| 1. | 0    | 0         | 6     | 1  | 1  |
| 2. | 0    | 0.1-0.299 | 2     | 1  | 2  |
| 3. | 0    | 0.3-0.625 | 3     | 1  | 3  |
| 4. | 0    | 0.626+    | 1     | 1  | 4  |

```
. symmetry ca co [fw=count], notable trend cc
```

|                                       | chi2  | df | Prob>chi2 |
|---------------------------------------|-------|----|-----------|
| Symmetry (asymptotic)                 | 17.10 | 6  | 0.0089    |
| Marginal homogeneity (Stuart-Maxwell) | 16.96 | 3  | 0.0007    |
| Linear trend in the (log) RR          | 14.43 | 1  | 0.0001    |

We requested the continuity correction by specifying cc. Doing so is appropriate because our coded exposure levels are equally spaced.

The test for trend was highly significant, indicating an increased risk of endometrial cancer with increased dosage of conjugated estrogen.

You must be cautious: the way in which you code the exposure variable affects the linear trend statistic. If instead of coding the levels as 1, 2, 3, and 4, we had instead used 0, 0.2, 0.46, and 0.7 (roughly the midpoint in the range of each level), we would have obtained a  $\chi^2$  statistic of 11.19 for these data. ◀

## Stored results

symmetry stores the following in `r()`:

Scalars

|                         |                                        |
|-------------------------|----------------------------------------|
| <code>r(N_pair)</code>  | number of matched pairs                |
| <code>r(chi2)</code>    | asymptotic symmetry $\chi^2$           |
| <code>r(df)</code>      | asymptotic symmetry degrees of freedom |
| <code>r(p)</code>       | asymptotic symmetry $p$ -value         |
| <code>r(chi2_sm)</code> | MH (Stuart–Maxwell) $\chi^2$           |
| <code>r(df_sm)</code>   | MH (Stuart–Maxwell) degrees of freedom |
| <code>r(p_sm)</code>    | MH (Stuart–Maxwell) $p$ -value         |
| <code>r(chi2_b)</code>  | MH (Bickenböller) $\chi^2$             |
| <code>r(df_b)</code>    | MH (Bickenböller) degrees of freedom   |
| <code>r(p_b)</code>     | MH (Bickenböller) $p$ -value           |
| <code>r(chi2_nd)</code> | MH (no diagonals) $\chi^2$             |
| <code>r(df_nd)</code>   | MH (no diagonals) degrees of freedom   |
| <code>r(p_nd)</code>    | MH (no diagonals) $p$ -value           |
| <code>r(chi2_t)</code>  | $\chi^2$ for linear trend              |
| <code>r(p_trend)</code> | $p$ -value for linear trend            |
| <code>r(p_exact)</code> | exact symmetry $p$ -value              |

## Methods and formulas

Methods and formulas are presented under the following headings:

*Asymptotic tests*  
*Exact symmetry test*

### Asymptotic tests

Consider a square table with  $K$  exposure categories, that is,  $K$  rows and  $K$  columns. Let  $n_{ij}$  be the count corresponding to row  $i$  and column  $j$  of the table,  $N_{ij} = n_{ij} + n_{ji}$ , for  $i, j = 1, 2, \dots, K$ , and  $n_{i\cdot}$ , and let  $n_{\cdot j}$  be the marginal totals for row  $i$  and column  $j$ , respectively. Asymptotic tests for symmetry and marginal homogeneity for this  $K \times K$  table are calculated as follows:

The null hypothesis of complete symmetry  $p_{ij} = p_{ji}$ ,  $i \neq j$ , is tested by calculating the test statistic (Bowker 1948)

$$T_{cs} = \sum_{i < j} \frac{(n_{ij} - n_{ji})^2}{n_{ij} + n_{ji}}$$

which is asymptotically distributed as  $\chi^2$  with  $K(K - 1)/2 - R$  degrees of freedom, where  $R$  is the number of off-diagonal cells with  $N_{ij} = 0$  as discussed in Hoenig, Morgan, and Brown (1995).

The null hypothesis of marginal homogeneity,  $p_{i.} = p_{.i}$ , is tested by calculating the Stuart–Maxwell test statistic (Stuart 1955; Maxwell 1970),

$$T_{\text{sm}} = \mathbf{d}'\mathbf{V}^{-1}\mathbf{d}$$

where  $\mathbf{d}$  is a column vector with elements equal to the differences  $d_i = n_{i.} - n_{.i}$  for  $i = 1, 2, \dots, K$ , and  $\mathbf{V}$  is the variance–covariance matrix with elements

$$\begin{aligned} v_{ii} &= n_{i.} + n_{.i} - 2n_{ii} \\ v_{ij} &= -(n_{ij} + n_{ji}), \quad i \neq j \end{aligned}$$

$T_{\text{sm}}$  is asymptotically  $\chi^2$  with  $K - 1$  degrees of freedom.

This test statistic properly accounts for the dependence between the table’s rows and columns. When the matrix  $\mathbf{V}$  is not of full rank, a generalized inverse  $\mathbf{V}^*$  is substituted for  $\mathbf{V}^{-1}$ .

The Bickeböllner and Clerget-Darpoux (1995) marginal homogeneity test statistic is calculated by

$$T_{\text{mh}} = \sum_i \frac{(n_{i.} - n_{.i})^2}{n_{i.} + n_{.i}}$$

This statistic is asymptotically distributed, under the assumption of marginal independence, as  $\chi^2$  with  $K - 1$  degrees of freedom.

The marginal homogeneity (no diagonals) test statistic  $T_{\text{mh}}^0$  is calculated in the same way as  $T_{\text{mh}}$ , except that the diagonal elements do not enter into the calculation of the marginal totals. Unlike the previous test statistic,  $T_{\text{mh}}^0$  reduces to a McNemar test statistic for  $2 \times 2$  tables. The test statistic  $\{(K - 1)/2\}T_{\text{mh}}^0$  is asymptotically distributed as  $\chi^2$  with  $K - 1$  degrees of freedom (Cleves, Olson, and Jacobs 1997; Spielman and Ewens 1996).

Breslow and Day’s test statistic for linear trend in the (log) of RR is

$$\frac{\left\{ \sum_{i < j} (n_{ij} - n_{ji})(X_j - X_i) - cc \right\}^2}{\sum_{i < j} (n_{ij} + n_{ji})(X_j - X_i)^2}$$

where the  $X_j$  are the doses associated with the various levels of exposure and  $cc$  is the continuity correction; it is asymptotically distributed as  $\chi^2$  with 1 degree of freedom.

The continuity correction option is applicable only when the levels of the exposure variable are equally spaced.

### Exact symmetry test

The exact test is based on a permutation algorithm applied to the null distribution. The distribution of the off-diagonal elements  $n_{ij}$ ,  $i \neq j$ , conditional on the sum of the complementary off-diagonal cells,  $N_{ij} = n_{ij} + n_{ji}$ , can be written as the product of  $K(K - 1)/2$  binomial random variables,

$$P(\mathbf{n}) = \prod_{i < j} \binom{N_{ij}}{n_{ij}} \pi_{ij}^{n_{ij}} (1 - \pi_{ij})^{n_{ji}}$$

where  $\mathbf{n}$  is a vector with elements  $n_{ij}$  and  $\pi_{ij} = E(n_{ij}/N_{ij}|N_{ij})$ . Under the null hypothesis of complete symmetry,  $\pi_{ij} = \pi_{ji} = 1/2$ , and thus the permutation distribution is given by

$$P_0(\mathbf{n}) = \prod_{i < j} \binom{N_{ij}}{n_{ij}} \left(\frac{1}{2}\right)^{N_{ij}}$$

The exact significance test is performed by evaluating

$$P_{cs} = \sum_{\mathbf{n} \in p} P_0(\mathbf{n})$$

where  $p = \{n : P_0(\mathbf{n}) < P_0(\mathbf{n}^*)\}$  and  $\mathbf{n}^*$  is the observed contingency table data vector. The algorithm evaluates  $p_{cs}$  exactly. For information about permutation tests, see Good (2005, 2006).

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## Also see

[R] **Epitab** — Tables for epidemiologists

## Description

Tables allow us to effectively communicate information about our data and results from our analyses. The `table` command is a flexible tool for creating tables. It allows you to create tabulations, tables of summary statistics, tables of results from hypothesis tests, tables of regression results, and more. `table` also allows you to customize the table so that it effectively communicates the results.

## Remarks and examples

Remarks are presented under the following headings:

[Overview](#)

[Tabulations](#)

[Tables of summary statistics](#)

[Tables of results from other commands](#)

[Flexible tables combining results](#)

[Formatting, customizing, and exporting tables](#)

## Overview

The `table` command allows you to create various types of tables.

1. Tabulations of one, two, or more categorical variables, reporting frequencies, percentages, or proportions.
2. Tables of summary statistics such as means, standard deviations, medians, and the like—perhaps computed across levels of one or more categorical variables.
3. Tables of results from other Stata commands, such as regression results or results of classic hypothesis tests.
4. Tables combining summary statistics and results from other Stata commands.

`table` obtains the statistics it reports in two ways. The `table` command itself can compute summary statistics. `table` also provides a `command()` option that allows you to run any Stata command and include its results in the table.

`table` also provides much flexibility for you to arrange the results in your table in a meaningful way. You can control how the rows, columns, and potentially even separate tables are defined. For instance, you might type

```
. table (var1) (var2) ...
```

to place the levels of `var1` on the rows and the levels of `var2` on the columns. The first set of parentheses is used to define the rows, and the second set is used to define the columns. The parentheses could be omitted in this case, but for clarity we will use them in our discussion here.

We could use the levels of both `var1` and `var2` to define the rows.

```
. table (var1 var2) ...
```

If we have an additional variable, `var3`, we can define rows by both `var1` and `var2` and the columns by `var3` by typing

```
. table (var1 var2) (var3) ...
```

or we can place the levels of `var2` and `var3` on the columns,

```
. table (var1) (var2 var3) ...
```

We could even create separate tables for the levels of `var3`,

```
. table (var1) (var2) (var3) ...
```

The third set of parentheses defines the separate tables.

If our table reports multiple statistics, say, means and standard deviations, we can use the keyword `result` to specify how they are included in the table layout. To place the statistics on separate rows, we could type

```
. table (var1 result) (var2) ...
```

The flexibility of the table layout goes beyond these examples. You can add additional variables to define rows, columns, and tables as is appropriate for the table you wish to create.

In addition to controlling the layout of your table, you can customize the results by specifying formats, including stars representing significance, and selecting from styles that determine how the headers of the table are displayed.

## Tabulations

Tabulations allow you to examine the distribution of your data across the levels of one or more categorical variables. Tabulations often report frequencies, percentages, and proportions.

With `table`, you can easily create one-way tabulations, two-way tabulations, three-way tabulations, and even more complex tabulations. For example, we could create a two-way tabulation of `region` and `diabetes`, reporting frequencies,

```
. table (region) (diabetes)
```

|        | Diabetes status |          | Total  |
|--------|-----------------|----------|--------|
|        | Not diabetic    | Diabetic |        |
| Region |                 |          |        |
| NE     | 1,997           | 98       | 2,095  |
| MW     | 2,648           | 125      | 2,773  |
| S      | 2,692           | 161      | 2,853  |
| W      | 2,513           | 115      | 2,628  |
| Total  | 9,850           | 499      | 10,349 |

or we can report percentages,

```
. table (region) (diabetes), statistic(percent)
```

|        | Diabetes status |          | Total  |
|--------|-----------------|----------|--------|
|        | Not diabetic    | Diabetic |        |
| Region |                 |          |        |
| NE     | 19.30           | 0.95     | 20.24  |
| MW     | 25.59           | 1.21     | 26.79  |
| S      | 26.01           | 1.56     | 27.57  |
| W      | 24.28           | 1.11     | 25.39  |
| Total  | 95.18           | 4.82     | 100.00 |

To learn how to create tabulations, see

- [R] [table oneway](#) One-way tabulation
- [R] [table twoway](#) Two-way tabulation
- [R] [table multiway](#) Multiway tables

In these entries, we provide simplified syntax for creating the specific type of tabulation you are interested in, and we provide a number of examples demonstrating how to build these tables and customize the results.

## Tables of summary statistics

`table` can compute summary statistics such as minimums, maximums, means, standard deviations, medians, and interquartile ranges. Summary statistics can be computed for one or more variables. Summary statistics can also be computed separately for groups of data. For example, we can create a table reporting the means of age, height, and bmi,

```
. table, statistic(mean age height bmi)
```

|                       |          |
|-----------------------|----------|
| Age (years)           | 47.57965 |
| Height (cm)           | 167.6509 |
| Body mass index (BMI) | 25.5376  |

or we can compute these means separately for males and females,

```
. table () (sex), statistic(mean age height bmi)
```

|                       | Sex      |          | Total    |
|-----------------------|----------|----------|----------|
|                       | Male     | Female   |          |
| Age (years)           | 47.4238  | 47.72057 | 47.57965 |
| Height (cm)           | 174.7421 | 161.2393 | 167.6509 |
| Body mass index (BMI) | 25.50999 | 25.56256 | 25.5376  |

To learn how to create tables of summary statistics, including the simplified `table` syntax and worked examples, see

[R] [table summary](#)

Table of summary statistics

You can also create a table of summary statistics with `dtable`. The advantage of using `dtable` is that you can create a table with a title and notes and export it to a variety of file types, all in a single step. Unlike `table`, however, `dtable` has a predefined layout, and you will not be able to specify how the results should be arranged in the table.

## Tables of results from other commands

With `table`'s `command()` option, you can run any Stata command and place statistics reported by that command in your table. For instance, you might use `test` to test for differences in means or use `regress` to fit a linear regression model. You can include any stored results from these commands in your table. For example, we can create a table to compare coefficients from linear regressions of `bpsysol` on age and weight fit separately for males and females.

```
. table (colname) (sex result), command(regress bpsystol age weight)
```

|             | Sex      |          | Total    |
|-------------|----------|----------|----------|
|             | Male     | Female   |          |
| Age (years) | .4789361 | .7735499 | .6379892 |
| Weight (kg) | .3346106 | .4586108 | .4069041 |
| Intercept   | 84.08037 | 61.70456 | 71.27096 |

To learn how to create tables with results from other commands, including simplified `table` syntax and worked examples, see

|                                            |                             |
|--------------------------------------------|-----------------------------|
| [R] <a href="#">table hypothesis tests</a> | Table of hypothesis tests   |
| [R] <a href="#">table regression</a>       | Table of regression results |

## Flexible tables combining results

`table` can create tables with combinations of frequencies, other summary statistics, and results from other table commands. To see the full syntax and discussion of how `table` automates the table layout, see

|                           |                                                      |
|---------------------------|------------------------------------------------------|
| [R] <a href="#">table</a> | Table of frequencies, summaries, and command results |
|---------------------------|------------------------------------------------------|

## Formatting, customizing, and exporting tables

`table` allows you to customize your results by specifying the layout and with options that change the numeric formatting, add characters such as percent signs or parentheses to specific types of statistics, add stars representing significance levels, and modify the look of confidence intervals. In addition, Stata provides predefined styles that you may select from using the `style()` option. These styles control which labels are displayed in the headers of the tables, how the labels are aligned, how the statistics are aligned within the cells, and more. To learn about the predefined styles that you can select from, see [TABLES] [Predefined styles](#). Examples of using the `style()` and other formatting options are provided in the entries for specific types of tables listed in the previous sections.

The results from `table` can be customized even beyond what its formatting options allow. `table` is unique in that it stores its results in a format that we refer to as a “collection”. Stata’s `collect` suite of commands can be used to produce highly customized tables from results in a collection and to export those tables to presentation-ready formats such as HTML, Word,  $\LaTeX$ , PDF, Excel, and more. Examples of using `collect` to modify labels and table layout are also provided in the entries for the specific types of tables listed in the previous sections. To learn more about the `collect` commands, see [TABLES] [Intro](#) and the entries discussed therein.

## Reference

Huber, C. 2021. Customizable tables in Stata 17, part 1: The new table command. *The Stata Blog: Not Elsewhere Classified*. <https://blog.stata.com/2021/06/07/customizable-tables-in-stata-17-part-1-the-new-table-command/>.

## Also see

[R] [table](#) — Table of frequencies, summaries, and command results

[TABLES] [Intro](#) — Introduction

|                             |                                      |                                |                                      |
|-----------------------------|--------------------------------------|--------------------------------|--------------------------------------|
| <a href="#">Description</a> | <a href="#">Quick start</a>          | <a href="#">Menu</a>           | <a href="#">Syntax</a>               |
| <a href="#">Options</a>     | <a href="#">Remarks and examples</a> | <a href="#">Stored results</a> | <a href="#">Methods and formulas</a> |
| <a href="#">Appendix</a>    | <a href="#">Reference</a>            | <a href="#">Also see</a>       |                                      |

## Description

`table` is a flexible command for creating tables of many types—tabulations, tables of summary statistics, tables of regression results, and more. `table` can calculate summary statistics to display in the table. `table` can also include results from other Stata commands.

## Quick start

Two-way tabulation of `a1` and `a2`

```
table a1 a2
```

Table of means for `v1` and `v2` across the levels of `a1`

```
table a1, statistic(mean v1 v2)
```

Two-way table with `a1` defining rows and `a2` defining columns, with frequencies and pairwise correlation coefficients between `v3` and `v4` computed for every cell

```
table a1 a2, command(pwcorr v3 v4)
```

Table of regression coefficients with means of the covariates; rows correspond to covariates and columns correspond to the statistics

```
table (colname) (statcmd result),
command(regress y x1 x2)
statistic(mean x1 x2)
```

Same as above, and include standard deviations for the covariates

```
table (colname) (statcmd result),
command(regress y x1 x2)
statistic(mean x1 x2)
statistic(sd x1 x2)
```

## Menu

Statistics > Summaries, tables, and tests > Tables of frequencies, summaries, and command results

## Syntax

*Basic syntax for a one-way table*

```
table rowvar
table () colvar
```

*Basic syntax for a two-way table*

```
table rowvar colvar
```

*Basic syntax for an  $n$ -way table*

```
table rowvars colvar
table rowvar (colvars)
table (rowvars) (colvars)
```

*Basic syntax for multiple  $n$ -way tables*

```
table (rowvars) (colvars) (tabvars)
```

*Full syntax*

```
table (rowspec) (colspec) [(tabspec)] [if] [in] [weight] [, options]
```

*rowspec*, *colspec*, and *tabspec* may be empty or may include variable names or any of the following keywords:

| <i>keyword</i> | Description                                                       |
|----------------|-------------------------------------------------------------------|
| result         | requested statistics                                              |
| stars          | stars denoting statistical significance                           |
| var            | variables from <code>statistic()</code> option                    |
| across         | index <code>across()</code> specifications                        |
| colname        | column names for matrix statistics                                |
| rowname        | row names for matrix statistics                                   |
| coleq          | column equation names for matrix statistics                       |
| roweq          | row equation names for matrix statistics                          |
| command        | index option <code>command()</code>                               |
| statcmd        | index options <code>statistic()</code> and <code>command()</code> |

| <i>options</i>                                                               | Description                                                                                                                                     |
|------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Main</b>                                                                  |                                                                                                                                                 |
| <code>totals</code> ( <i>totals</i> )                                        | report only the specified totals                                                                                                                |
| <code>nototals</code>                                                        | suppress the marginal totals                                                                                                                    |
| <b>Statistics</b>                                                            |                                                                                                                                                 |
| <code>statistic</code> ( <i>statspec</i> )                                   | statistic to be reported; default is <code>statistic(frequency)</code> when no weights are specified and <code>statistic(sumw)</code> otherwise |
| <b>Commands</b>                                                              |                                                                                                                                                 |
| <code>command</code> ( <i>cmdspec</i> )                                      | collect results from the specified Stata command                                                                                                |
| <b>Formats</b>                                                               |                                                                                                                                                 |
| <code>nformat</code> ( <i>%fmt</i> [ <i>results</i> ] [, <i>basestyle</i> ]) | specify numeric format                                                                                                                          |
| <code>sformat</code> ( <i>sfmt</i> [ <i>results</i> ])                       | specify string format                                                                                                                           |
| <code>cidelimiter</code> ( <i>char</i> )                                     | use character as delimiter for confidence interval limits                                                                                       |
| <code>cridelimiter</code> ( <i>char</i> )                                    | use character as delimiter for credible interval limits                                                                                         |
| <b>Stars</b>                                                                 |                                                                                                                                                 |
| <code>stars</code> ( <i>starspec</i> )                                       | add stars to denote statistical significance                                                                                                    |
| <b>Title</b>                                                                 |                                                                                                                                                 |
| <code>title</code> ( <i>string</i> )                                         | add table title                                                                                                                                 |
| <code>titlestyles</code> ( <i>text_styles</i> )                              | change table title styles                                                                                                                       |
| <b>Notes</b>                                                                 |                                                                                                                                                 |
| <code>note</code> ( <i>string</i> )                                          | add table note                                                                                                                                  |
| <code>notestyles</code> ( <i>text_styles</i> )                               | change table note styles                                                                                                                        |
| <b>Export</b>                                                                |                                                                                                                                                 |
| <code>export</code> ( <i>filename.suffix</i> [, <i>export_opts</i> ])        | export table                                                                                                                                    |
| <b>Options</b>                                                               |                                                                                                                                                 |
| <code>listwise</code>                                                        | use listwise deletion to handle missing values                                                                                                  |
| <code>missing</code>                                                         | treat numeric missing values like other values                                                                                                  |
| <code>showcounts</code>                                                      | show sample size for all variables in <code>statistic()</code> option                                                                           |
| <code>zerocounts</code>                                                      | report 0 for empty cell counts                                                                                                                  |
| <code>name</code> ( <i>cname</i> )                                           | collect results into a collection named <i>cname</i>                                                                                            |
| <code>append</code>                                                          | append results to an existing collection                                                                                                        |
| <code>replace</code>                                                         | replace results of an existing collection                                                                                                       |
| <code>label</code> ( <i>filename</i> )                                       | specify the collection labels                                                                                                                   |
| <code>style</code> ( <i>filename</i> [, <i>override</i> ])                   | specify the collection style                                                                                                                    |
| <code>markvar</code> ( <i>newvar</i> )                                       | create <i>newvar</i> that identifies observations used in the tabulation                                                                        |
| <code>noisily</code>                                                         | display output from each command                                                                                                                |

`fweights`, `awweights`, `iweights`, and `pweights` are allowed; see [U] 11.1.6 [weight](#).

`strL` variables are not allowed; see [U] 12.4.8 [strL](#).

`markvar()` and `noisily` do not appear in the dialog box.

| <i>text_styles</i>                           | Description                                              |
|----------------------------------------------|----------------------------------------------------------|
| <code>font([fontfamily][, font_opts])</code> | specify font style                                       |
| <code>smcl(smcl)</code>                      | specify formatting for SMCL files                        |
| <code>latex(latex)</code>                    | specify L <sup>A</sup> T <sub>E</sub> X macro            |
| <code>shading(sspec)</code>                  | set background color, foreground color, and fill pattern |

| <i>font_opts</i>              | Description                              |
|-------------------------------|------------------------------------------|
| <code>size(# [unit])</code>   | specify font size                        |
| <code>color(color)</code>     | specify font color                       |
| <code>variant(variant)</code> | specify font variant and capitalization  |
| <code>[no]bold</code>         | specify whether to format text as bold   |
| <code>[no]italic</code>       | specify whether to format text as italic |
| <code>[no]strikeout</code>    | specify whether to strike out text       |
| <code>[no]underline</code>    | specify whether to underline text        |

| <i>suffix</i>         | <i>fileformat</i>         | Output format                      |
|-----------------------|---------------------------|------------------------------------|
| <code>docx</code>     | <code>as(docx)</code>     | Microsoft Word                     |
| <code>html</code>     | <code>as(html)</code>     | HTML 5 with CSS                    |
| <code>pdf</code>      | <code>as(pdf)</code>      | PDF                                |
| <code>xlsx</code>     | <code>as(xlsx)</code>     | Microsoft Excel 2007/2010 or newer |
| <code>xls</code>      | <code>as(xls)</code>      | Microsoft Excel 1997/2003          |
| <code>tex</code>      | <code>as(tex)</code>      | L <sup>A</sup> T <sub>E</sub> X    |
| <code>smcl</code>     | <code>as(smcl)</code>     | SMCL                               |
| <code>txt</code>      | <code>as(txt)</code>      | plain text                         |
| <code>markdown</code> | <code>as(markdown)</code> | Markdown                           |
| <code>md</code>       | <code>as(md)</code>       | Markdown                           |

| <i>export_opts</i>          | Description                                         |
|-----------------------------|-----------------------------------------------------|
| <code>as(fileformat)</code> | specify document type                               |
| <code>replace</code>        | overwrite existing file                             |
| <i>docx_options</i>         | available when exporting to .docx files             |
| <i>html_options</i>         | available when exporting to .html files             |
| <i>pdf_options</i>          | available when exporting to .pdf files              |
| <i>excel_options</i>        | available when exporting to .xls and .xlsx files    |
| <i>tex_options</i>          | available when exporting to .tex files              |
| <i>smcl_option</i>          | available when exporting to .smcl files             |
| <i>txt_option</i>           | available when exporting to .txt files              |
| <i>md_option</i>            | available when exporting to .markdown and .md files |

| <i>docx_options</i>                      | Description                                                         |
|------------------------------------------|---------------------------------------------------------------------|
| <code>noisily</code>                     | show the putdocx commands used to export to the Microsoft Word file |
| <code>dofile(filename[, replace])</code> | save the putdocx commands used for exporting to the named do-file   |

| <i>html_options</i>       | Description                                                    |
|---------------------------|----------------------------------------------------------------|
| append                    | append to an existing file                                     |
| tableonly                 | export only the table to the specified file                    |
| cssfile( <i>cssfile</i> ) | define the styles in <i>cssfile</i> instead of <i>filename</i> |
| prefix( <i>prefix</i> )   | use <i>prefix</i> to identify style classes                    |

| <i>pdf_options</i>                   | Description                                                      |
|--------------------------------------|------------------------------------------------------------------|
| noisily                              | show the putpdf commands used to export to the PDF file          |
| dofile( <i>filename</i> [, replace]) | save the putpdf commands used for exporting to the named do-file |

| <i>excel_options</i>                 | Description                                                                                             |
|--------------------------------------|---------------------------------------------------------------------------------------------------------|
| noisily                              | show the putexcel commands used to export to the Excel file                                             |
| dofile( <i>filename</i> [, replace]) | save the putexcel commands used for exporting to the named do-file                                      |
| sheet( <i>sheetname</i> [, replace]) | specify the worksheet to use; the default sheet name is Sheet1                                          |
| cell( <i>cell</i> )                  | specify the Excel upper-left cell as the starting position to export the table; the default is cell(A1) |
| modify                               | modify Excel file                                                                                       |
| noopen                               | do not open Excel file in memory                                                                        |

noopen does not appear in the dialog box.

| <i>tex_options</i> | Description                                 |
|--------------------|---------------------------------------------|
| append             | append to an existing file                  |
| tableonly          | export only the table to the specified file |

| <i>smcl_option</i> | Description                |
|--------------------|----------------------------|
| append             | append to an existing file |

| <i>txt_option</i> | Description                |
|-------------------|----------------------------|
| append            | append to an existing file |

| <i>md_option</i> | Description                |
|------------------|----------------------------|
| append           | append to an existing file |

*fontfamily* specifies a valid font family.

*unit* may be in (inch), pt (point), or cm (centimeter). An inch is equivalent to 72 points and 2.54 centimeters. The default is pt.

*variant* may be allcaps, smallcaps, or normal.

*variant* (allcaps) changes the text to all uppercase letters; applicable when publishing items from a collection to Microsoft Word, PDF, L<sup>A</sup>T<sub>E</sub>X, and HTML files.

*variant* (smallcaps) changes the text to use large capitals for uppercase letters and smaller capitals for lowercase letters; applicable when publishing items from a collection to Microsoft Word, L<sup>A</sup>T<sub>E</sub>X, and HTML files.

*variant* (normal) changes the font variant back to normal and leaves the capitalization unchanged from the original text; applicable when publishing items from a collection to Microsoft Word, PDF, L<sup>A</sup>T<sub>E</sub>X, and HTML files.

*smcl* specifies the name of the SMCL directive to render text for SMCL output. The supported SMCL directives are `input`, `error`, `result`, and `text`.

*latex* specifies the name of a L<sup>A</sup>T<sub>E</sub>X macro to render text for L<sup>A</sup>T<sub>E</sub>X output. Example L<sup>A</sup>T<sub>E</sub>X macro names are `textbf`, `textsf`, `textrm`, and `texttt`. Custom L<sup>A</sup>T<sub>E</sub>X macros are also allowed. If *text* is to be rendered in a cell, title, or note, then *latex* is translated to the following when you export to L<sup>A</sup>T<sub>E</sub>X:

```
\latex {text}
```

*sspec* is

```
[background(bgcolor) foreground(fgcolor) pattern(fpattern)]
```

*bgcolor* specifies the background color.

*fgcolor* specifies the foreground color.

*fpattern* specifies the fill pattern. A complete list of fill patterns is shown in the [Appendix](#).

*bgcolor*, *fgcolor*, and *color* may be one of the colors listed in the [Appendix](#); a valid RGB value in the form `### ## #`, for example, `171 248 103`; or a valid RRGGBB hex value in the form `#####`, for example, `ABF867`.

## Options

### Main

`totals` (*totals*) and `nototals` control which totals are to be displayed in the table. By default, all totals are reported.

`totals` (*totals*) specifies which margin totals to display in the reported table. *totals* can contain variables in *rowspec*, *colspec*, *tabspec*, and their interaction. Interactions can be specified by using the # operator.

`nototals` prevents table from displaying any totals.

### Statistics

`statistic` (*statspec*) specifies the statistic to be displayed. Frequency statistics, summary statistics, and ratio statistics are available by specifying `statistic` (*freqstat*), `statistic` (*sumstat varlist*), and `statistic` (*ratiostat [varlist] [ , ratio\_options ]*), respectively.

`statistic()` may be repeated to request multiple statistics.

`statistic` (*freqstat*) specifies that frequencies be computed.

| <i>freqstat</i>        | Definition     |
|------------------------|----------------|
| <code>frequency</code> | frequency      |
| <code>sumw</code>      | sum of weights |

`statistic(sumstat varlist)` specifies that summary statistic *sumstat* be computed for the variables in *varlist*.

| <i>sumstat</i>         | Definition                                              |
|------------------------|---------------------------------------------------------|
| mean                   | mean                                                    |
| semean                 | standard error of the mean                              |
| sebinomial             | standard error of the mean, binomial                    |
| sepoisson              | standard error of the mean, Poisson                     |
| variance               | variance                                                |
| sd                     | standard deviation                                      |
| skewness               | skewness                                                |
| kurtosis               | kurtosis                                                |
| cv                     | coefficient of variation                                |
| svy cv                 | coefficient of variation (svy)                          |
| geomean                | geometric mean                                          |
| geosd                  | geometric standard deviation                            |
| count                  | number of nonmissing values                             |
| median                 | median                                                  |
| p#                     | #th percentile                                          |
| q1                     | first quartile                                          |
| q2                     | second quartile                                         |
| q3                     | third quartile                                          |
| iqr                    | interquartile range                                     |
| min                    | minimum value                                           |
| max                    | maximum value                                           |
| range                  | range                                                   |
| first                  | first value                                             |
| last                   | last value                                              |
| firstnm                | first nonmissing value                                  |
| lastnm                 | last nonmissing value                                   |
| total                  | total                                                   |
| rawtotal               | unweighted total                                        |
| <u>fvfrequency</u>     | frequency of each factor-variable level                 |
| <u>fvrawfrequency</u>  | unweighted frequency of each factor-variable level      |
| <u>fvproportion</u>    | proportion within each factor-variable level            |
| <u>fvrawproportion</u> | unweighted proportion within each factor-variable level |
| <u>fvpercent</u>       | percentage within each factor-variable level            |
| <u>fvrawpercent</u>    | unweighted percentage within each factor-variable level |

`statistic(ratiostat [varlist] [ , ratio_options ])` specifies that ratio statistic *ratiostat* be computed. If *varlist* is specified, ratios are computed based on the totals of the specified variables. If *varlist* is not specified, ratios are computed based on frequencies.

| <i>ratiostat</i>     | Definition                                       |
|----------------------|--------------------------------------------------|
| <u>proportion</u>    | proportion                                       |
| <u>percent</u>       | percentage                                       |
| <u>rawproportion</u> | proportion ignoring optionally specified weights |
| <u>rawpercent</u>    | percentage ignoring optionally specified weights |

| <i>ratio_options</i>           | Definition                                                            |
|--------------------------------|-----------------------------------------------------------------------|
| <u>across(<i>cellspec</i>)</u> | percentages or proportions across levels of variables or interactions |
| <u>total</u>                   | compute overall percentages or proportions                            |

*cellspec* may contain *rowvars*, *colvars*, *tabvars*, or an interaction between any of these variables. Interactions can be specified by using the # operator.

#### Commands

`command(cmdsspec)` specifies the Stata commands from which to collect results. `command()` may be repeated to collect results from multiple commands.

*cmds*spec is [*explist*: ] *command* [*arguments*] [ , *cmdoptions* ]

*explist* specifies which results to collect and report in the table. *explist* may include *result identifiers* and *named expressions*.

*result identifiers* are results stored in `r()` and `e()` by the *command*. For instance, *result identifiers* could be `r(mean)`, `r(C)`, or `e(chi2)`. After estimation commands, *result identifiers* also include the following:

| Identifier            | Result                                                              |
|-----------------------|---------------------------------------------------------------------|
| <code>_r_b</code>     | coefficients or transformed coefficients reported by <i>command</i> |
| <code>_r_se</code>    | standard errors of <code>_r_b</code>                                |
| <code>_r_z</code>     | test statistics for <code>_r_b</code>                               |
| <code>_r_z_abs</code> | absolute value of <code>_r_z</code>                                 |
| <code>_r_p</code>     | <i>p</i> -values for <code>_r_b</code>                              |
| <code>_r_lb</code>    | lower bounds of confidence intervals for <code>_r_b</code>          |
| <code>_r_ub</code>    | upper bounds of confidence intervals for <code>_r_b</code>          |
| <code>_r_ci</code>    | confidence intervals for <code>_r_b</code>                          |
| <code>_r_cr1b</code>  | lower bounds of credible intervals for <code>_r_b</code>            |
| <code>_r_crub</code>  | upper bounds of credible intervals for <code>_r_b</code>            |
| <code>_r_cri</code>   | credible intervals for <code>_r_b</code>                            |
| <code>_r_df</code>    | degrees of freedom for <code>_r_b</code>                            |

*named expressions* are specified as *name* = *exp*, where *name* may be any valid Stata name and *exp* is an expression, typically an expression that involves one or more *result identifiers*. An example of a named expression is `sd = sqrt(r(variance))`.

For r-class commands, the default is to include all numeric scalars posted to `r()` in the table results. For e-class commands, the default is to include `_r_b` in the table results.

*command* is any command that follows standard Stata syntax.

*arguments* may be anything so long as they do not include an `if` clause, `in range`, or weight specification.

Any `if` or `in` qualifier and weights should be specified directly with `table`, not within the `command()` option.

*cmdoptions* may be anything supported by *command*.

#### Formats

`nformat(%fmt [results][, basestyle])` changes the numeric format, such as the number of decimal places, for specified results. If *results* are not specified, the numeric format is changed for all results.

*results* may be any statistic named in option `statistic()` (that is, any *freqstat*, *sumstat*, or *ratiostat*) or may be any name in the `e()` or `r()` results produced by commands specified in option `command()`.

This option is repeatable, and when multiple formats apply to one result, the rightmost specification is applied.

This option does not affect the format of numeric layout variables (*rowspec*, *colspec*, and *tabspec*) or the format of factor variables specified in the `statistic()` option. The default format of these variables is taken from the dataset.

*basestyle* indicates that the format be applied to results that do not already have their own format instead of overriding the format for all results.

`sformat(sfmt [results])` changes the string format for specified results. You can, for instance, add symbols or text to the values reported in the table by modifying the string format.

*sfmt* may contain a mix of text and `%s`. Here `%s` refers to the numeric value that is formatted as specified using `nformat()`. The text will be placed around the numeric values in your table as it is placed around `%s` in this option. For instance, to place parentheses around the percent statistics, you can specify `sformat("(%s) percent)`.

*results* may be any statistic named in option `statistic()` (that is, any *freqstat*, *sumstat*, or *ratiostat*) or may be any name in the `e()` or `r()` results produced by commands specified in option `command()`.

Two text characters must be specified using a special character sequence if you want them to be displayed in your table. To include `%`, type `%%`. To include `\`, type `\\`. For instance, to place a percent sign following percent statistics, you can specify `sformat("%s%" percent)`.

This option is repeatable, and when multiple formats apply to one result, the rightmost specification is applied.

`cidelimiter(char)` changes the delimiter between confidence interval limits to *char*. The default is `cidelimiter(" ")`, that is, two spaces.

`cridelimiter(char)` changes the delimiter between credible interval limits to *char*. The default is `cridelimiter(" ")`, that is, two spaces.

## Stars

`stars(starspec)` specifies that stars representing statistical significance be included in the table. *starspec* identifies the result whose values determine significance, which characters should represent each significance level, and where these characters should be displayed in the table. *starspec* is

```
starres [#1 "label1" [#2 "label2" [#3 "label3" [#4 "label4" [#5 "label5"]]]]]
[, attach(attachres) result dimension starsnoteopts]
```

*starres* is the name of the result whose values determine which characters, typically which number of stars, are to be displayed.

*label1* specifies the characters to be displayed when *starres* < #1.

*label2* specifies the characters to be displayed when *starres* < #2.

*label3* specifies the characters to be displayed when *starres* < #3.

*label4* specifies the characters to be displayed when *starres* < #4.

*label5* specifies the characters to be displayed when *starres* < #5.

`attach(attachres)` specifies the name of the result to which the characters defined by *label1*, ..., *label5* are to be attached. If `attach()` is not specified, a new result named `stars` is created and is automatically added to the table.

`result` and `dimension` control how `collect stars` adds items when labeling significant results. These options are mutually exclusive.

`result` specifies the default behavior, and this option is necessary only if the following `dimension` behavior is in effect and you want to change back to the `result` behavior.

`dimension` specifies that dimension `stars` be added to the collection. Items will be tagged with `stars[value]`, and the labels will be tagged with `stars[label]`. Use this option for layouts where results are to be stacked within columns, and use new dimension `stars` in the column specification of the layout.

*starsnoteopts* control the display and composition of the stars note.

`noshownote` and `shownote` control whether to display the stars note.

`increasing` and `decreasing` control the order of *p*-values in the stars note.

`pvname(string)` specifies a name for the *p*-value in the stars note. The default is `pvname(p)`.

`delimiter(string)` specifies the delimiter between labels in the stars note. The default is `delimiter(",")`.

`nformat(%fmt)` specifies the numeric format for the cutoff values in the stars note. The default is `nformat(%9.0g)`.

`prefix(string)` specifies the prefix for the stars note. The prefix is empty by default.

`suffix(string)` specifies the suffix for the stars note. The suffix is empty by default.

For example, `stars(_r_p 0.01 "***" 0.05 "***" 0.1 "*", attach(_r_b))` could be added to a table of regression results to specify that stars be defined based on the *p*-values in `_r_p` and be attached to the reported coefficients (`_r_b`).

Title

`title(string)` adds the text *string* as a title to the table.

`titlestyles(text_styles)` changes the style for the table title. *text\_styles* are the following:

`font([fontfamily] [, size(# [unit]) color(color) variant(variant) [no]bold [no]italic [no]strikeout [no]underline])` specifies the font style. These font style properties are applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files, unless otherwise specified.

*fontfamily* specifies a valid font family. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`size(# [unit])` specifies the font size as a number optionally followed by units. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`color(color)` specifies the text color.

`variant(variant)` specifies the font variant and capitalization.

`bold` and `nobold` specify the font weight. `bold` changes the font weight to bold; `nobold` changes the font weight back to normal.

`italic` and `noitalic` specify the font style. `italic` changes the font style to italic; `noitalic` changes the font style back to normal.

`strikeout` and `nostrikeout` specify whether to add a strikeout mark to the title. `strikeout` adds a strikeout mark to the title; `nostrikeout` changes the title back to normal.

`underline` and `nounderline` specify whether to underline the table title. `underline` adds a single line under the title; `nounderline` removes the underline.

Only one of `strikeout` or `underline` is allowed when publishing to HTML files.

`smcl(smcl)` specifies how to render the table title for SMCL output. This style property is applicable only when publishing items from a collection to a SMCL file.

`latex(latex)` specifies how to render the table title for  $\LaTeX$  output. This style property is applicable only when publishing items from a collection to a  $\LaTeX$  file.

`shading(sspec)` sets the background color, foreground color, and fill pattern. The background color is applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files. The foreground color and fill pattern are applicable when exporting the table to Microsoft Word and Microsoft Excel.

`note(string)` adds the text *string* as a note to the table. `note()` may be specified multiple times to add multiple notes. Each note is placed on a new line.

`notestyles(text_styles)` changes the style for the table notes. *text\_styles* are the following:

`font([fontfamily] [, size(# [unit]) color(color) variant(variant) [no]bold [no]italic [no]strikeout [no]underline])` specifies the font style. These font style properties are applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files, unless otherwise specified.

*fontfamily* specifies a valid font family. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`size(# [unit])` specifies the font size as a number optionally followed by units. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`color(color)` specifies the text color.

`variant(variant)` specifies the font variant and capitalization.

`bold` and `nobold` specify the font weight. `bold` changes the font weight to bold; `nobold` changes the font weight back to normal.

`italic` and `noitalic` specify the font style. `italic` changes the font style to italic; `noitalic` changes the font style back to normal.

`strikeout` and `nostrikeout` specify whether to add a strikeout mark to the notes. `strikeout` adds a strikeout mark to the note; `nostrikeout` changes the note back to normal.

`underline` and `nounderline` specify whether to underline the table notes. `underline` adds a single line under the notes; `nounderline` removes the underline.

Only one of `strikeout` or `underline` is allowed when publishing to HTML files.

`smcl(smcl)` specifies how to render the table notes for SMCL output. This style property is applicable only when publishing items from a collection to a SMCL file.

`latex(latex)` specifies how to render the table notes for  $\LaTeX$  output. This style property is applicable only when publishing items from a collection to a  $\LaTeX$  file.

`shading(sspec)` sets the background color, foreground color, and fill pattern. The background color is applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files. The foreground color and fill pattern are applicable when exporting the table to Microsoft Word and Microsoft Excel.

## Export

`export(filename.suffix[, export_opts])` exports the table to the specified file. *export\_opts* are the following:

`as(fileformat)` specifies the file format to which the table is to be exported. This option is rarely specified because, by default, `table` determines the format from the suffix of the file being created.

`replace` permits `table` to overwrite an existing file.

`noisily` specifies that `table` show the commands used to export the table to Microsoft Word, Microsoft Excel, and PDF files. The `putdocx`, `putexcel`, or `putpdf` command used to export the table will be displayed.

`dofile(filename[, replace])` specifies that `table` save to *filename* the commands used to export the table to Microsoft Word, Microsoft Excel, and PDF files.

If *filename* already exists, it can be overwritten by specifying `replace`. If *filename* is specified without an extension, `.do` is assumed.

`append` specifies that `table` append the table to an existing file.

This option is applicable when you export the table to an HTML, a  $\LaTeX$ , a SMCL, a `txt`, or a Markdown file. When you export to HTML and  $\LaTeX$  files, the `append` option implies the `tableonly` option. Furthermore, when you export to HTML files, if the target CSS file already exists, `table` will also append to it.

`tableonly` specifies that only the table be exported to the specified HTML or  $\LaTeX$  document. With this option, the produced file may be included in other HTML or  $\LaTeX$  documents. By default, `table` produces complete HTML and  $\LaTeX$  documents.

When you export to an HTML file, if the `cssfile()` option is not specified, a CSS filename is constructed from *filename*, with the extension replaced with `.css`.

`cssfile(cssfile)` specifies that `table` define the styles in *cssfile* instead of *filename* when you export to HTML.

`prefix(prefix)` specifies that `table` use *prefix* to identify style classes when you export to HTML.

`sheet(sheetname [, replace])` saves to the worksheet named *sheetname*. For more information about this option, see [RPT] [putexcel](#).

`cell(cell)` specifies an Excel upper-left cell as the starting position to publish the table. The default is `cell(A1)`.

`modify` permits `putexcel` set to modify an Excel file. For more information about this option, see [RPT] [putexcel](#).

`noopen` prevents `putexcel` from opening the Excel file in memory for modification. It does not appear in the dialog box. For more information about this option, see [RPT] [putexcel](#).

## Options

`listwise` handles missing values through listwise deletion, meaning that the entire observation is omitted from the sample if any variable specified in a `statistic()` option is missing for that observation. By default, `table` will omit an observation only if all variables specified in all `statistic()` options are missing for that observation.

`missing` specifies that numeric missing values of any variables specified in `rowspec`, `colspec`, or `tabspec` be treated as valid categories. By default, observations with a numeric missing value in any of these variables are omitted.

This option does not apply to factor variables specified with statistics `fvfrequency`, `fvrawfrequency`, `fvproportion`, `fvrawproportion`, `fvpercent`, or `fvrawpercent`.

`showcounts` specifies that `table` report the sample size for each variable specified in option `statistic()`.

`zerocounts` specifies that `table` report a 0 in empty cells for results `count`, `frequency`, `fvfrequency`, and `fvrawfrequency`.

`name(cname)` specifies that a collection named *cname* be associated with the collected statistics and results. The default is `name(Table)`.

`append` specifies that `table` append its collection information into the collection named in `name()`.

`replace` permits `table` to overwrite an existing collection. This option is implied for `name(Table)` when `append` is not specified.

`label(filename)` specifies the *filename* containing the collection labels to use for your table. Labels in *filename* will be loaded for the table, and any labels not specified in *filename* will be taken from the labels defined in `c(collect_label)`. The default is to use only the collection labels set in `c(collect_label)`; see [TABLES] [set collect\\_label](#).

`style(filename [, override])` specifies the *filename* containing the collection styles to use for your table. The default collection styles will be discarded, and only the collection styles in *filename* will be applied.

If you prefer the default collection styles but also want to apply any styles in *filename*, specify `override`. If there are conflicts between the default collection styles and those in *filename*, the ones in *filename* will take precedence.

The default is to use only the collection styles set in `c(table_style)`; see [TABLES] [set table\\_style](#).

The following options are available with `table` but are not shown in the dialog box:

`markvar(newvar)` generates an indicator variable that identifies the observations used in the tabulation.

`noisily` specifies that output from the commands specified in `command()` options be displayed. By default, output from commands is suppressed.

## Remarks and examples

Remarks are presented under the following headings:

[Introduction](#)

[Specifying the table layout](#)

[Advanced table customization](#)

### Introduction

The `table` command can create many customized tables, ranging from simple one-way tabulations to multiple *n*-way tables with summary statistics and estimation results. `table` can compute and report frequencies, proportions, percentiles, and other summary statistics. It can also run other Stata commands and include their results in the table. This means you can combine the summary statistics computed

by `table` with test statistics, correlations, regression coefficients, and other results collected from Stata commands. In addition to building tables with the desired statistics, you can customize them by formatting the values in the table and applying predefined styles and labels that affect how the row headers, column headers, and values are displayed in the table.

`table` can accommodate a variety of layouts. You can define the rows, columns, and even separate tables by levels of categorical variables, statistics, or Stata commands.

If your goal is simply to create a table of estimation results or descriptive statistics, you can use the `etable` and `dtable` commands, respectively. These commands allow you to create these tables and export them to a variety of file types in a single step. However, unlike `table`, these commands create tables with a predefined layout. Therefore, if you want control over the layout or you wish to include a combination of summary statistics, estimation results, and results from other Stata commands, you should use the `table` command.

In the following entries, we provide simplified syntax, examples, and discussion for specialized types of tables that can be created using `table`. If you are interested in creating one of these types of tables, we suggest reading the corresponding entry.

|                                   |                             |
|-----------------------------------|-----------------------------|
| [R] <b>table oneway</b>           | One-way tabulation          |
| [R] <b>table twoway</b>           | Two-way tabulation          |
| [R] <b>table multiway</b>         | Multiway tables             |
| [R] <b>table summary</b>          | Table of summary statistics |
| [R] <b>table hypothesis tests</b> | Table of hypothesis tests   |
| [R] <b>table regression</b>       | Table of regression results |

All the concepts demonstrated in the entries above can be combined to create tables including combinations of tabulations, summary statistics, hypothesis tests, and regression results.

In this entry, we provide additional information on specifying the table layout and which portions of the layout `table` will automate for you. In addition, we provide resources for customizing the table and exporting the results to your preferred format.

## Specifying the table layout

A table's layout is determined by our row, column, and table dimension specifications. For example, we specify variable names to define the rows and place statistics in the columns, or vice versa. Because we can include so many different statistics, we can specify keywords that we use to identify the results we have collected from commands and the statistics that `table` has calculated.

The syntax for specifying the table layout is

```
table ([rowspec]) ([colspec]) ([tabspec])
```

We refer to *rowsec*, *colspec*, and *tabspec* collectively as the “layout”. For some tables, **keywords** are required in the layout to uniquely identify the values that we want to include in our table. If you omit a necessary keyword from the layout, `table` will fill one in for you.

The rules determining whether a keyword is necessary to uniquely identify values in the table are as follows:

1. If more than one statistic is specified, then `result` is needed in the layout.
2. If more than one variable is specified in option `statistic()` and option `command()` is not specified, then `var` is needed in the layout.

3. If more than one `across()` specification is used for ratio statistics, then `across` is needed in the layout.
4. If option `command()` is specified, then `colname` is needed in the layout. If, in addition, more than one variable is specified in option `statistic()`, then `colname` is needed instead of `var`, which was required in 2.
5. If multiple `command()` options are specified and option `statistic()` is not specified, then `command` is needed in the layout.
6. If both options `command()` and `statistic()` are specified, then `statcmd` is needed in the layout.

If we do not directly specify a necessary keyword in one of `rowspec`, `colspec`, or `tabspec`, the missing keywords will be automatically added to the layout as follows:

1. If the row specification is empty, then put the missing keywords in `rowspec`.
2. If the row specification is not empty but the column specification is empty, then put the missing keywords in `colspec`.
3. If the row and column specifications are not empty but the table specification is empty and if `result` is the only missing keyword and there is only one statistic (`result`), then put `result` in `tabspec`.
4. Otherwise, append the missing keywords to `rowvars`.

Below, we demonstrate how missing keywords are added to the *layout*.

Using `auto.dta`, we create a table with the minimum and maximum mpg for each level of `rep78`. The keyword `result` identifies the statistics we computed. By listing an empty set of parentheses followed by `rep78`, we request that the levels of `rep78` be placed on the columns.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. table () rep78, statistic(min mpg) statistic(max mpg)
```

|               | Repair record 1978 |    |    |    |    | Total |
|---------------|--------------------|----|----|----|----|-------|
|               | 1                  | 2  | 3  | 4  | 5  |       |
| Minimum value | 18                 | 14 | 12 | 14 | 17 | 12    |
| Maximum value | 24                 | 24 | 29 | 30 | 41 | 41    |

Based on [rule 1](#), if we request more than one statistic, `result` must be in the layout. Based on [situation 1](#), if the row specification is empty, then the missing keyword will be placed in the row specification. We could have created the same table by typing

```
. table (result) (rep78), statistic(min mpg) statistic(max mpg)
```

Now, let's include multiple variables in our `statistic()` option. We also type `rep78` immediately after `table` to specify that the levels of `rep78` be placed on the rows.

```
. table rep78, statistic(mean mpg price)
```

|                    | Mileage (mpg) | Price    |
|--------------------|---------------|----------|
| Repair record 1978 |               |          |
| 1                  | 21            | 4564.5   |
| 2                  | 19.125        | 5967.625 |
| 3                  | 19.43333      | 6429.233 |
| 4                  | 21.66667      | 6071.5   |
| 5                  | 27.36364      | 5913     |
| Total              | 21.28986      | 6146.043 |

Because we have more than one variable in the `statistic()` option, then keyword `var` must be in the layout ([rule 2](#)). If we include a row specification but leave the column specification empty, `table` will treat `var` as the column identifier. We could have equivalently typed

```
. table (rep78) (var), statistic(mean mpg price)
```

Next, let's include both a `command()` option and a `statistic()` option with multiple variables in the same table. We want a table with coefficients and means of the independent variables. We use the `command()` option to fit the regression and obtain the means with the `statistic()` option. Now, we need both `colname` and `statcmd` to uniquely identify the values in the table. Let's omit `statcmd` from our command.

```
. table (colname) (result[_r_b mean]),
> command(regress mpg turn trunk) statistic(mean turn trunk)
```

|                        | Coefficient | Mean     |
|------------------------|-------------|----------|
| Turn circle (ft.)      |             |          |
| regress mpg turn trunk | -.7610113   |          |
| Mean                   |             | 39.64865 |
| Trunk space (cu. ft.)  |             |          |
| regress mpg turn trunk | -.3161825   |          |
| Mean                   |             | 13.75676 |
| Intercept              |             |          |
| regress mpg turn trunk | 55.82001    |          |

But based on [situation 4](#), `table` will add `statcmd` to the row specification if we leave it out. So we could have also typed the following to create the same table:

```
. table (colname statcmd) (result[_r_b mean]),
 command(regress mpg turn trunk) statistic(mean turn trunk)
```

This table displays each of the statistics that we requested. If we simply wanted to compute some statistics quickly, it has served its purpose. However, if we wish to share these results with others or include a table in a report, we will want to make some modifications.

## Advanced table customization

`table` allows you to customize the results of your table using the `stars()`, `nformat()`, `sformat()`, `cidelimiter()`, `label()`, and `style()` options. With these, you can add significance stars, change the numeric format, and attach characters such as percent signs or parentheses to values in the table, use a stored set of labels, or use a predefined style. If these options provide all the customizations you need, you can export your finalized table directly with the `export()` option. See [TABLES] [Predefined styles](#) for more information on selecting a style that adjusts elements of the table such as row header alignment, alignment of values within the cells, and which labels are included in the headers.

Customization can also go beyond the predefined styles and options available to you in the `table` command. `table` stores all of its results in a collection named `Table`. This means that you can use the specialized tools available in the `collect` suite of commands to further customize your table. With `collect`, you can modify specific labels, add borders, change the style of the headers, and the like. Once you have a publication-ready table, you can use `collect export` to export your table to HTML, Word, L<sup>A</sup>T<sub>E</sub>X, PDF, Excel, or another format appropriate for your report.

## Stored results

`table` stores the following in `r()`:

Scalars

`r(N)`    number of observations

## Methods and formulas

Variables specified in `rowspec`, `colspec`, and `tabspec` identify groups of observations within the dataset. These groups are represented in the table by cells and cell margins (totals). For a given cell or cell margin, let  $n$  denote the number of observations (frequency). Let  $x$  denote the variable on which we want to calculate summary statistics, and let  $x_i$ ,  $i = 1, \dots, n$ , denote an individual observation on  $x$ . `count` is the number of nonmissing values of  $x$ . `first` is  $x_1$  and `last` is  $x_n$ . Let  $a$  be the smallest  $i$  such that  $x_i$  is not missing, and then `firstnm` is  $x_a$ . Let  $b$  be the largest  $i$  such that  $x_i$  is not missing, and then `lastnm` is  $x_b$ .

Let  $v_i$  be the weight, and if no weight is specified, define  $v_i = 1$  for all  $i$ . Let  $v$  denote the sum of the weights (`sumw`):

$$v = \sum_{i=1}^n v_i$$

When `aweights` or `pweights` are specified, the normalized weights are given by  $w_i = v_i(n/v)$  with  $w = n$ ; otherwise,  $w_i = v_i$  and  $w = v$ .

The remaining summary statistics are computed according to the following formulas:

total

$$x. = \begin{cases} \sum_{i=1}^n v_i x_i & \text{if pweights} \\ \sum_{i=1}^n w_i x_i & \text{otherwise} \end{cases}$$

rawtotal

$$\sum_{i=1}^n x_i$$

mean

$$\bar{x} = \frac{1}{w.} \sum_{i=1}^n w_i x_i$$

Define  $m_r$  as the  $r$ th moment about the mean:

$$m_r = \frac{1}{w.} \sum_{i=1}^n w_i (x_i - \bar{x})^r$$

variance

$$s^2 = \frac{w.}{w. - 1} m_2 = \frac{1}{w. - 1} \sum_{i=1}^n w_i (x_i - \bar{x})^2$$

sd (standard deviation)

$$s = \sqrt{s^2}$$

semean (standard error of the mean)

$$\text{se}(\bar{x}) = \frac{s}{\sqrt{w.}}$$

sebinomial (standard error of the mean, binomial distribution)

$$\sqrt{\frac{\bar{x}(1 - \bar{x})}{w.}}$$

sepoisson (standard error of the mean, Poisson distribution)

$$\sqrt{\frac{\bar{x}}{w.}}$$

When pweights are specified, `semean`, `sebinomial`, and `sepoisson` are all computed as

$$\text{se}_{\text{pw}}(\bar{x}) = \sqrt{\frac{n}{n-1} \sum_{i=1}^n \left\{ \frac{v_i}{v.} (x_i - \bar{x}) \right\}^2}$$

skewness

$$m_3 m_2^{-3/2}$$

kurtosis

$$m_4 m_2^{-2}$$

cv (coefficient of variation)

$$\frac{s}{\bar{x}}$$

svycv (coefficient of variation, survey literature)

$$100 \frac{\text{se}(\bar{x})}{|\bar{x}|}$$

svycv with pweights

$$100 \frac{\text{se}_{\text{pw}}(\bar{x})}{|\bar{x}|}$$

geomean (geometric mean)

$$\bar{x}_g = \exp \left( \frac{1}{w.} \sum_{i=1}^n w_i \ln x_i \right)$$

geosd (geometric standard deviation)

$$\exp \left( \sqrt{\frac{1}{w. - 1} \sum_{i=1}^n w_i (\ln x_i - \ln \bar{x}_g)^2} \right)$$

Let  $x_{(i)}$  refer to the  $x$  in ascending order, and let  $w_{(i)}$  refer to the corresponding weights of  $x_{(i)}$ .

minimum

$$x_{(1)}$$

maximum

$$x_{(n)}$$

range

$$x_{(n)} - x_{(1)}$$

To obtain the  $p$ th percentile, which we will denote as  $x_{[p]}$ , let  $P = np/100$  and

$$W_{(i)} = \frac{n}{w} \sum_{j=1}^i w_{(j)}$$

Find the first index  $i$  such that  $W_{(i)} > P$ . The  $p$ th percentile is then

$$x_{[p]} = \begin{cases} \frac{x_{(i-1)} + x_{(i)}}{2} & \text{if } W_{(i-1)} = P \\ x_{(i)} & \text{otherwise} \end{cases}$$

q1 (first quartile)

$$x_{[25]}$$

q2 (second quartile)

$$x_{[50]}$$

q3 (third quartile)

$$x_{[75]}$$

iqr (interquartile range)

$$x_{[75]} - x_{[25]}$$

Let  $f$  be an indicator for a specific level of a factor variable and  $f_i$  denote an individual observation on  $f$ .

fvf frequency (frequency of the factor variable's level)

$$\sum_{i=1}^n w_i f_i$$

fvrawfrequency (unweighted frequency of the factor variable's level)

$$\sum_{i=1}^n f_i$$

fvproportion (proportion of the factor variable's level)

$$\frac{1}{w} \sum_{i=1}^n w_i f_i$$

fvrawproportion (unweighted proportion of the factor variable's level)

$$\frac{1}{n} \sum_{i=1}^n f_i$$

`fvpercent` (percentage of the factor variable's level)

$$\frac{100}{w_i} \sum_{i=1}^n w_i f_i$$

`fvrawpercent` (unweighted percentage of the factor variable's level)

$$\frac{100}{n} \sum_{i=1}^n f_i$$

`proportion` is computed from ratios of totals. The numerator is taken from the total for the given cell or cell margin, and the denominator is taken from the total for a cell margin that contains the given cell or cell margin. `percent` is `proportion` multiplied by 100.

`rawproportion` and `rawpercent` are similarly computed using unweighted totals.

## Appendix

### Colors

*bgcolor*, *fgcolor*, and *color*

---

|                |                      |                   |               |
|----------------|----------------------|-------------------|---------------|
| aliceblue      | darkslategray        | lightsalmon       | palevioletred |
| antiquewhite   | darkturquoise        | lightseagreen     | papayawhip    |
| aqua           | darkviolet           | lightskyblue      | peachpuff     |
| aquamarine     | deeppink             | lightslategray    | peru          |
| azure          | deepskyblue          | lightsteelblue    | pink          |
| beige          | dimgray              | lightyellow       | plum          |
| bisque         | dodgerblue           | lime              | powderblue    |
| black          | firebrick            | limegreen         | purple        |
| blanchedalmond | floralwhite          | linen             | red           |
| blue           | forestgreen          | magenta           | rosybrown     |
| blueviolet     | fuchsia              | maroon            | royalblue     |
| brown          | gainsboro            | mediumaquamarine  | saddlebrown   |
| burlywood      | ghostwhite           | mediumblue        | salmon        |
| cadetblue      | gold                 | mediumorchid      | sandybrown    |
| chartreuse     | goldenrod            | mediumpurple      | seagreen      |
| chocolate      | gray                 | mediumseagreen    | seashell      |
| coral          | green                | mediumslateblue   | sienna        |
| cornflowerblue | greenyellow          | mediumspringgreen | silver        |
| cornsilk       | honeydew             | mediumturquoise   | skyblue       |
| crimson        | hotpink              | mediumvioletred   | slateblue     |
| cyan           | indianred            | midnightblue      | slategray     |
| darkblue       | indigo               | mintcream         | snow          |
| darkcyan       | ivory                | mistyrose         | springgreen   |
| darkgoldenrod  | khaki                | moccasin          | steelblue     |
| darkgray       | lavender             | navajowhite       | tan           |
| darkgreen      | lavenderblush        | navy              | teal          |
| darkkhaki      | lawngreen            | oldlace           | thistle       |
| darkmagenta    | lemonchiffon         | olive             | tomato        |
| darkolivegreen | lightblue            | olivedrab         | turquoise     |
| darkorange     | lightcoral           | orange            | violet        |
| darkorchid     | lightcyan            | orangered         | wheat         |
| darkred        | lightgoldenrodyellow | orchid            | white         |
| darksalmon     | lightgray            | palegoldenrod     | whitesmoke    |
| darkseagreen   | lightgreen           | palegreen         | yellow        |
| darkslateblue  | lightpink            | paleturquoise     | yellowgreen   |

---

## Shading patterns

*fpattern*

---

|                       |       |
|-----------------------|-------|
| nil                   | pct20 |
| clear                 | pct25 |
| solid                 | pct30 |
| horzStripe            | pct35 |
| vertStripe            | pct37 |
| reverseDiagStripe     | pct40 |
| diagStripe            | pct45 |
| horzCross             | pct50 |
| diagCross             | pct55 |
| thinHorzStripe        | pct60 |
| thinVertStripe        | pct62 |
| thinReverseDiagStripe | pct65 |
| thinDiagStripe        | pct70 |
| thinHorzCross         | pct75 |
| thinDiagCross         | pct80 |
| pct5                  | pct85 |
| pct10                 | pct87 |
| pct12                 | pct90 |
| pct15                 | pct95 |

---

## Reference

Mitchell, M. N. 2025. *Create and Export Tables Using Stata*. College Station, TX: Stata Press.

## Also see

[R] [table intro](#) — Introduction to tables of frequencies, summaries, and command results

[R] [table hypothesis tests](#) — Table of hypothesis tests

[R] [table multiway](#) — Multiway tables

[R] [table oneway](#) — One-way tabulation

[R] [table regression](#) — Table of regression results

[R] [table summary](#) — Table of summary statistics

[R] [table twoway](#) — Two-way tabulation

[TABLES] [Intro](#) — Introduction

Description  
Options  
Also see

Quick start  
Remarks and examples

Menu  
Stored results

Syntax  
References

## Description

In this entry, we discuss how to use `table` to create a one-way tabulation, including frequencies, percentages, and proportions.

## Quick start

One-way table of frequencies, with rows corresponding to the levels of `a1`

```
table a1
```

Same as above, but with columns corresponding to the levels of `a1`

```
table () a1
```

Same as above, but treat missing values like other values of `a1`

```
table () a1, missing
```

One-way table of frequencies, using the collection style `mystyle`

```
table a1, style(mystyle)
```

One-way table of frequencies and percentages

```
table a1, statistic(frequency) statistic(percent)
```

## Menu

Statistics > Summaries, tables, and tests > Tables of frequencies, summaries, and command results

## Syntax

### Basic one-way tabulation

```
table varname [if] [in] [weight] [, options]
```

### Customized one-way tabulation

```
table [(rowspec)] [(colspec)] [if] [in] [weight] [, options]
```

*rowspec* and *colspec* may be empty or may include *varname*, *result*, or *varname* and *result*, where *result* refers to the requested statistics.

| <i>options</i>                                                          | Description                                                                                                                                     |
|-------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Main</b>                                                             |                                                                                                                                                 |
| <code>nototals</code>                                                   | suppress the marginal totals                                                                                                                    |
| <b>Statistics</b>                                                       |                                                                                                                                                 |
| <code>statistic(<i>stat</i>)</code>                                     | statistic to be reported; default is <code>statistic(frequency)</code> when no weights are specified and <code>statistic(sumw)</code> otherwise |
| <b>Formats</b>                                                          |                                                                                                                                                 |
| <code>nformat(<i>%fmt</i> [<i>results</i>] [, <i>basestyle</i>])</code> | specify numeric format                                                                                                                          |
| <code>sformat(<i>sfmt</i> [<i>results</i>])</code>                      | specify string format                                                                                                                           |
| <b>Title</b>                                                            |                                                                                                                                                 |
| <code>title(<i>string</i>)</code>                                       | add table title                                                                                                                                 |
| <code>titlestyles(<i>text_styles</i>)</code>                            | change table title styles                                                                                                                       |
| <b>Notes</b>                                                            |                                                                                                                                                 |
| <code>note(<i>string</i>)</code>                                        | add table note                                                                                                                                  |
| <code>notestyles(<i>text_styles</i>)</code>                             | change table note styles                                                                                                                        |
| <b>Export</b>                                                           |                                                                                                                                                 |
| <code>export(<i>filename.suffix</i> [, <i>export_opts</i>])</code>      | export table                                                                                                                                    |
| <b>Options</b>                                                          |                                                                                                                                                 |
| <code>missing</code>                                                    | treat numeric missing values of <i>varname</i> like other values                                                                                |
| <code>name(<i>cname</i>)</code>                                         | collect results into a collection named <i>cname</i>                                                                                            |
| <code>append</code>                                                     | append results to an existing collection                                                                                                        |
| <code>replace</code>                                                    | replace results of an existing collection                                                                                                       |
| <code>label(<i>filename</i>)</code>                                     | specify the collection labels                                                                                                                   |
| <code>style(<i>filename</i> [, <i>override</i>])</code>                 | specify the collection style                                                                                                                    |
| <code>markvar(<i>newvar</i>)</code>                                     | create <i>newvar</i> that identifies observations used in the tabulation                                                                        |

*fweights*, *awweights*, *iweights*, and *pweights* are allowed; see [\[U\] 11.1.6 weight](#).

*strL* variables are not allowed; see [\[U\] 12.4.8 strL](#).

`markvar()` does not appear in the dialog box.

| <i>text_styles</i>                           | Description                                              |
|----------------------------------------------|----------------------------------------------------------|
| <code>font([fontfamily][, font_opts])</code> | specify font style                                       |
| <code>smcl(smcl)</code>                      | specify formatting for SMCL files                        |
| <code>latex(latex)</code>                    | specify L <sup>A</sup> T <sub>E</sub> X macro            |
| <code>shading(sspec)</code>                  | set background color, foreground color, and fill pattern |

| <i>font_opts</i>              | Description                              |
|-------------------------------|------------------------------------------|
| <code>size(# [unit])</code>   | specify font size                        |
| <code>color(color)</code>     | specify font color                       |
| <code>variant(variant)</code> | specify font variant and capitalization  |
| <code>[no]bold</code>         | specify whether to format text as bold   |
| <code>[no]italic</code>       | specify whether to format text as italic |
| <code>[no]strikeout</code>    | specify whether to strike out text       |
| <code>[no]underline</code>    | specify whether to underline text        |

| <i>suffix</i>         | <i>fileformat</i>         | Output format                      |
|-----------------------|---------------------------|------------------------------------|
| <code>docx</code>     | <code>as(docx)</code>     | Microsoft Word                     |
| <code>html</code>     | <code>as(html)</code>     | HTML 5 with CSS                    |
| <code>pdf</code>      | <code>as(pdf)</code>      | PDF                                |
| <code>xlsx</code>     | <code>as(xlsx)</code>     | Microsoft Excel 2007/2010 or newer |
| <code>xls</code>      | <code>as(xls)</code>      | Microsoft Excel 1997/2003          |
| <code>tex</code>      | <code>as(tex)</code>      | L <sup>A</sup> T <sub>E</sub> X    |
| <code>smcl</code>     | <code>as(smcl)</code>     | SMCL                               |
| <code>txt</code>      | <code>as(txt)</code>      | plain text                         |
| <code>markdown</code> | <code>as(markdown)</code> | Markdown                           |
| <code>md</code>       | <code>as(md)</code>       | Markdown                           |

| <i>export_opts</i>          | Description                                         |
|-----------------------------|-----------------------------------------------------|
| <code>as(fileformat)</code> | specify document type                               |
| <code>replace</code>        | overwrite existing file                             |
| <i>docx_options</i>         | available when exporting to .docx files             |
| <i>html_options</i>         | available when exporting to .html files             |
| <i>pdf_options</i>          | available when exporting to .pdf files              |
| <i>excel_options</i>        | available when exporting to .xls and .xlsx files    |
| <i>tex_options</i>          | available when exporting to .tex files              |
| <i>smcl_option</i>          | available when exporting to .smcl files             |
| <i>txt_option</i>           | available when exporting to .txt files              |
| <i>md_option</i>            | available when exporting to .markdown and .md files |

| <i>docx_options</i>                      | Description                                                         |
|------------------------------------------|---------------------------------------------------------------------|
| <code>noisily</code>                     | show the putdocx commands used to export to the Microsoft Word file |
| <code>dofile(filename[, replace])</code> | save the putdocx commands used for exporting to the named do-file   |

| <i>html_options</i>       | Description                                                    |
|---------------------------|----------------------------------------------------------------|
| append                    | append to an existing file                                     |
| tableonly                 | export only the table to the specified file                    |
| cssfile( <i>cssfile</i> ) | define the styles in <i>cssfile</i> instead of <i>filename</i> |
| prefix( <i>prefix</i> )   | use <i>prefix</i> to identify style classes                    |

| <i>pdf_options</i>                   | Description                                                      |
|--------------------------------------|------------------------------------------------------------------|
| noisily                              | show the putpdf commands used to export to the PDF file          |
| dofile( <i>filename</i> [, replace]) | save the putpdf commands used for exporting to the named do-file |

| <i>excel_options</i>                 | Description                                                                                             |
|--------------------------------------|---------------------------------------------------------------------------------------------------------|
| noisily                              | show the putexcel commands used to export to the Excel file                                             |
| dofile( <i>filename</i> [, replace]) | save the putexcel commands used for exporting to the named do-file                                      |
| sheet( <i>sheetname</i> [, replace]) | specify the worksheet to use; the default sheet name is Sheet1                                          |
| cell( <i>cell</i> )                  | specify the Excel upper-left cell as the starting position to export the table; the default is cell(A1) |
| modify                               | modify Excel file                                                                                       |
| noopen                               | do not open Excel file in memory                                                                        |

noopen does not appear in the dialog box.

| <i>tex_options</i> | Description                                 |
|--------------------|---------------------------------------------|
| append             | append to an existing file                  |
| tableonly          | export only the table to the specified file |

| <i>smcl_option</i> | Description                |
|--------------------|----------------------------|
| append             | append to an existing file |

| <i>txt_option</i> | Description                |
|-------------------|----------------------------|
| append            | append to an existing file |

| <i>md_option</i> | Description                |
|------------------|----------------------------|
| append           | append to an existing file |

*fontfamily* specifies a valid font family.

*unit* may be in (inch), pt (point), or cm (centimeter). An inch is equivalent to 72 points and 2.54 centimeters. The default is pt.

*variant* may be allcaps, smallcaps, or normal.

*variant* (allcaps) changes the text to all uppercase letters; applicable when publishing items from a collection to Microsoft Word, PDF, L<sup>A</sup>T<sub>E</sub>X, and HTML files.

*variant* (smallcaps) changes the text to use large capitals for uppercase letters and smaller capitals for lowercase letters; applicable when publishing items from a collection to Microsoft Word, L<sup>A</sup>T<sub>E</sub>X, and HTML files.

*variant* (normal) changes the font variant back to normal and leaves the capitalization unchanged from the original text; applicable when publishing items from a collection to Microsoft Word, PDF, L<sup>A</sup>T<sub>E</sub>X, and HTML files.

*smcl* specifies the name of the SMCL directive to render text for SMCL output. The supported SMCL directives are `input`, `error`, `result`, and `text`.

*latex* specifies the name of a L<sup>A</sup>T<sub>E</sub>X macro to render text for L<sup>A</sup>T<sub>E</sub>X output. Example L<sup>A</sup>T<sub>E</sub>X macro names are `textbf`, `textsf`, `textrm`, and `texttt`. Custom L<sup>A</sup>T<sub>E</sub>X macros are also allowed. If *text* is to be rendered in a cell, title, or note, then *latex* is translated to the following when you export to L<sup>A</sup>T<sub>E</sub>X:

```
\latex {text}
```

*sspec* is

```
[background(bgcolor) foreground(fgcolor) pattern(fpattern)]
```

*bgcolor* specifies the background color.

*fgcolor* specifies the foreground color.

*fpattern* specifies the fill pattern. A complete list of fill patterns is shown in the [Appendix](#).

*bgcolor*, *fgcolor*, and *color* may be one of the colors listed in the [Appendix](#); a valid RGB value in the form `### ## #`, for example, `171 248 103`; or a valid RRGGBB hex value in the form `#####`, for example, `ABF867`.

## Options

### Main

`nototals` prevents `table` from displaying the row or column totals.

### Statistics

`statistic(stat)` specifies the statistic to be displayed. `statistic()` may be repeated to request multiple statistics.

Available statistics are

| <i>stat</i>                | Definition                                       |
|----------------------------|--------------------------------------------------|
| <code>frequency</code>     | frequency                                        |
| <code>sumw</code>          | sum of weights                                   |
| <code>proportion</code>    | proportion                                       |
| <code>percent</code>       | percentage                                       |
| <code>rawproportion</code> | proportion ignoring optionally specified weights |
| <code>rawpercent</code>    | percentage ignoring optionally specified weights |

### Formats

`nformat(%fmt [results] [, basestyle])` changes the numeric format, such as the number of decimal places, for specified results. If `results` are not specified, the numeric format is changed for all results.

`results` may be any statistic named in option `statistic()` (that is, any `stat`).

This option is repeatable, and when multiple formats apply to one result, the rightmost specification is applied.

This option does not affect the format of numeric layout variables (`rowspec` and `colspec`). The default format of these variables is taken from the dataset.

`basestyle` indicates that the format be applied to results that do not already have their own format instead of overriding the format for all results.

`sformat(sfmt [results])` changes the string format for specified results. You can, for instance, add symbols or text to the values reported in the table by modifying the string format.

`sfmt` may contain a mix of text and `%s`. Here `%s` refers to the numeric value that is formatted as specified using `nformat()`. The text will be placed around the numeric values in your table as it is placed around `%s` in this option. For instance, to place parentheses around the percent statistics, you can specify `sformat("(%s) percent)`.

`results` may be any statistic named in option `statistic()` (that is, any `stat`).

Two text characters must be specified using a special character sequence if you want them to be displayed in your table. To include `%`, type `%%`. To include `\`, type `\\`. For instance, to place a percent sign following percent statistics, you can specify `sformat("%s%%" percent)`.

This option is repeatable, and when multiple formats apply to one result, the rightmost specification is applied.

Title

`title(string)` adds the text *string* as a title to the table.

`titlestyles(text_styles)` changes the style for the table title. *text\_styles* are the following:

`font([fontfamily] [, size(# [unit]) color(color) variant(variant) [no]bold [no]italic [no]strikeout [no]underline)` specifies the font style. These font style properties are applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files, unless otherwise specified.

*fontfamily* specifies a valid font family. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

*size*(# [*unit*]) specifies the font size as a number optionally followed by units. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

*color*(*color*) specifies the text color.

*variant*(*variant*) specifies the font variant and capitalization.

`bold` and `nobold` specify the font weight. `bold` changes the font weight to bold; `nobold` changes the font weight back to normal.

`italic` and `noitalic` specify the font style. `italic` changes the font style to italic; `noitalic` changes the font style back to normal.

`strikeout` and `nostrikeout` specify whether to add a strikeout mark to the title. `strikeout` adds a strikeout mark to the title; `nostrikeout` changes the title back to normal.

`underline` and `nounderline` specify whether to underline the table title. `underline` adds a single line under the title; `nounderline` removes the underline.

Only one of `strikeout` or `underline` is allowed when publishing to HTML files.

`smcl(smcl)` specifies how to render the table title for SMCL output. This style property is applicable only when publishing items from a collection to a SMCL file.

`latex(latex)` specifies how to render the table title for  $\LaTeX$  output. This style property is applicable only when publishing items from a collection to a  $\LaTeX$  file.

`shading(sspec)` sets the background color, foreground color, and fill pattern. The background color is applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files. The foreground color and fill pattern are applicable when exporting the table to Microsoft Word and Microsoft Excel.

## Notes

`note(string)` adds the text *string* as a note to the table. `note()` may be specified multiple times to add multiple notes. Each note is placed on a new line.

`notestyles(text_styles)` changes the style for the table notes. *text\_styles* are the following:

`font([fontfamily] [, size(# [unit]) color(color) variant(variant) [no]bold [no]italic [no]strikeout [no]underline])` specifies the font style. These font style properties are applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files, unless otherwise specified.

*fontfamily* specifies a valid font family. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`size(# [unit])` specifies the font size as a number optionally followed by units. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`color(color)` specifies the text color.

`variant(variant)` specifies the font variant and capitalization.

`bold` and `nobold` specify the font weight. `bold` changes the font weight to bold; `nobold` changes the font weight back to normal.

`italic` and `noitalic` specify the font style. `italic` changes the font style to italic; `noitalic` changes the font style back to normal.

`strikeout` and `nostrikeout` specify whether to add a strikeout mark to the notes. `strikeout` adds a strikeout mark to the note; `nostrikeout` changes the note back to normal.

`underline` and `nounderline` specify whether to underline the table notes. `underline` adds a single line under the notes; `nounderline` removes the underline.

Only one of `strikeout` or `underline` is allowed when publishing to HTML files.

`smcl(smcl)` specifies how to render the table notes for SMCL output. This style property is applicable only when publishing items from a collection to a SMCL file.

`latex(latex)` specifies how to render the table notes for  $\LaTeX$  output. This style property is applicable only when publishing items from a collection to a  $\LaTeX$  file.

`shading(sspec)` sets the background color, foreground color, and fill pattern. The background color is applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files. The foreground color and fill pattern are applicable when exporting the table to Microsoft Word and Microsoft Excel.

Export

`export(filename.suffix[ , export_opts])` exports the table to the specified file. *export\_opts* are the following:

`as(fileformat)` specifies the file format to which the table is to be exported. This option is rarely specified because, by default, `table` determines the format from the suffix of the file being created.

`replace` permits `table` to overwrite an existing file.

`noisily` specifies that `table` show the commands used to export the table to Microsoft Word, Microsoft Excel, and PDF files. The `putdocx`, `putexcel`, or `putpdf` command used to export the table will be displayed.

`dofile(filename[ , replace])` specifies that `table` save to *filename* the commands used to export the table to Microsoft Word, Microsoft Excel, and PDF files.

If *filename* already exists, it can be overwritten by specifying `replace`. If *filename* is specified without an extension, `.do` is assumed.

`append` specifies that `table` append the table to an existing file.

This option is applicable when you export the table to an HTML, a  $\LaTeX$ , a SMCL, a `txt`, or a Markdown file. When you export to HTML and  $\LaTeX$  files, the `append` option implies the `tableonly` option. Furthermore, when you export to HTML files, if the target CSS file already exists, `table` will also append to it.

`tableonly` specifies that only the table be exported to the specified HTML or  $\LaTeX$  document. By default, `table` produces complete HTML and  $\LaTeX$  documents.

When you export to an HTML file, if the `cssfile()` option is not specified, a CSS filename is constructed from *filename*, with the extension replaced with `.css`.

`cssfile(cssfile)` specifies that `table` define the styles in *cssfile* instead of *filename* when you export to HTML.

`prefix(prefix)` specifies that `table` use *prefix* to identify style classes when you export to HTML.

`sheet(sheetname [ , replace])` saves to the worksheet named *sheetname*. For more information about this option, see [RPT] [putexcel](#).

`cell(cell)` specifies an Excel upper-left cell as the starting position to publish the table. The default is `cell(A1)`.

`modify` permits `putexcel` set to modify an Excel file. For more information about this option, see [RPT] [putexcel](#).

`noopen` prevents `putexcel` from opening the Excel file in memory for modification. It does not appear in the dialog box. For more information about this option, see [RPT] [putexcel](#).

## Options

`missing` specifies that numeric missing values of *varname* be treated as valid categories. By default, observations with a numeric missing value in *varname* are omitted.

`name(cname)` specifies that a collection named *cname* be associated with the collected statistics and results. The default is `name(Table)`.

`append` specifies that `table` append its collection information into the collection named in `name()`.

`replace` permits `table` to overwrite an existing collection. This option is implied for `name(Table)` when `append` is not specified.

`label(filename)` specifies the *filename* containing the collection labels to use for your table. Labels in *filename* will be loaded for the table, and any labels not specified in *filename* will be taken from the labels defined in `c(collect_label)`. The default is to use only the collection labels set in `c(collect_label)`; see [TABLES] [set collect\\_label](#).

`style(filename [, override])` specifies the *filename* containing the collection styles to use for your table. The default collection styles will be discarded, and only the collection styles in *filename* will be applied.

If you prefer the default collection styles but also want to apply any styles in *filename*, specify `override`. If there are conflicts between the default collection styles and those in *filename*, the ones in *filename* will take precedence.

The default is to use only the collection styles set in `c(table_style)`; see [TABLES] [set table\\_style](#).

The following option is available with `table` but is not shown in the dialog box:

`markvar(newvar)` generates an indicator variable that identifies the observations used in the tabulation.

## Remarks and examples

Remarks are presented under the following headings:

- [Tabulation of one variable](#)
- [Tabulation, including percentages](#)
- [Customizing results](#)
- [Advanced customization](#)

## Tabulation of one variable

To obtain a one-way tabulation that reports the number of observations for each level of a categorical variable, we need specify only the name of the categorical variable following `table`.

To demonstrate, we use data from the Second National Health and Nutrition Examination Survey (NHANES II) (McDowell et al. 1981). We tabulate the `hlthstat` variable, which contains individuals' self-reported health status categories.

```
. use https://www.stata-press.com/data/r19/nhanes21
(Second National Health and Nutrition Examination Survey)
. table hlthstat
```

|               | Frequency |
|---------------|-----------|
| Health status |           |
| Excellent     | 2,407     |
| Very good     | 2,591     |
| Good          | 2,938     |
| Fair          | 1,670     |
| Poor          | 729       |
| Total         | 10,335    |

We see that more people self-reported having excellent, very good, or good health status than reported having fair or poor health status.

Above, we see frequencies for those who reported a health status. This information is not available for some individuals in the dataset. We can determine how many missing values we have for this variable by adding the `missing` option

```
. table hlthstat, missing
```

|                      | Frequency |
|----------------------|-----------|
| Health status        |           |
| Excellent            | 2,407     |
| Very good            | 2,591     |
| Good                 | 2,938     |
| Fair                 | 1,670     |
| Poor                 | 729       |
| .                    | 2         |
| Blank but applicable | 14        |
| Total                | 10,351    |

We find that there is missing health status data for 16 individuals—2 with a generic missing value and 14 whose responses were labeled “Blank but applicable”.

## Tabulation, including percentages

In addition to frequencies, we can report the proportion or percentage of observations in each health status category. By default, `table` reports frequencies, which is equivalent to including the `statistic(frequency)` option. Here we include that option along with the `statistic(percent)` option to report both frequencies and percentages.

```
. table hlthstat, statistic(frequency) statistic(percent)
```

|               | Frequency | Percent |
|---------------|-----------|---------|
| Health status |           |         |
| Excellent     | 2,407     | 23.29   |
| Very good     | 2,591     | 25.07   |
| Good          | 2,938     | 28.43   |
| Fair          | 1,670     | 16.16   |
| Poor          | 729       | 7.05    |
| Total         | 10,335    | 100.00  |

Now, it is clear that 28.43% of respondents reported having good health.

## Customizing results

There are a number of ways that you can customize the results in your table.

In some cases, you may prefer to place frequencies and percentages on the rows and the levels of the variable being tabulated on the columns. To do this, you can include both the row and column specifications in parentheses following `table`. Here we use `result` in the first set of parentheses to request that the statistics be placed on rows and the variable `hlthstat` in the second set of parentheses to request that the levels of this variable be placed on the columns.

```
. table (result) (hlthstat), statistic(frequency) statistic(percent)
```

|           | Health status |           |       |       |      |        |
|-----------|---------------|-----------|-------|-------|------|--------|
|           | Excellent     | Very good | Good  | Fair  | Poor | Total  |
| Frequency | 2,407         | 2,591     | 2,938 | 1,670 | 729  | 10,335 |
| Percent   | 23.29         | 25.07     | 28.43 | 16.16 | 7.05 | 100.00 |

Alternatively, we could have omitted `result` and typed

```
. table () (hlthstat), statistic(freq) statistic(percent)
```

Because we requested that `hlthstat` be moved to the columns by specifying it in the second set of parentheses, `table` automatically moves the requested statistics to the rows.

If instead of a short and wide table, you prefer a tall and narrow table, you can specify that both the levels of `hlthstat` and the statistics be used to define the rows by including the variable name and result in the first set of parentheses.

```
. table (hlthstat result), statistic(frequency) statistic(percent)
```

|               |        |
|---------------|--------|
| Health status |        |
| Excellent     |        |
| Frequency     | 2,407  |
| Percent       | 23.29  |
| Very good     |        |
| Frequency     | 2,591  |
| Percent       | 25.07  |
| Good          |        |
| Frequency     | 2,938  |
| Percent       | 28.43  |
| Fair          |        |
| Frequency     | 1,670  |
| Percent       | 16.16  |
| Poor          |        |
| Frequency     | 729    |
| Percent       | 7.05   |
| Total         |        |
| Frequency     | 10,335 |
| Percent       | 100.00 |

In addition to modifying the layout of the table, we may want to customize the results reported within the cells of the table. For instance, we can specify that the percentages be reported using only one decimal place by using the `nformat()` option. Here we return to the two-column table layout.

```
. table hlthstat, statistic(frequency) statistic(percent)
> nformat(%5.1f percent)
```

|               | Frequency | Percent |
|---------------|-----------|---------|
| Health status |           |         |
| Excellent     | 2,407     | 23.3    |
| Very good     | 2,591     | 25.1    |
| Good          | 2,938     | 28.4    |
| Fair          | 1,670     | 16.2    |
| Poor          | 729       | 7.1     |
| Total         | 10,335    | 100.0   |

The `table` command produces its output using a default set of styles, typically those defined in the `table` style but could be any other style that you have set as the default by using `set table_style`. When customizing our tables, we can take advantage of one of the styles described in [TABLES] **Predefined styles**. For instance, for tables with only one or two row variables, row labels that are right-aligned may be preferred. Here we use the `table-right` style.

```
. table hlthstat, statistic(frequency) statistic(percent)
> nformat(%5.1f percent) style(table-right)
```

|               | Frequency | Percent |
|---------------|-----------|---------|
| Health status |           |         |
| Excellent     | 2,407     | 23.3    |
| Very good     | 2,591     | 25.1    |
| Good          | 2,938     | 28.4    |
| Fair          | 1,670     | 16.2    |
| Poor          | 729       | 7.1     |
| Total         | 10,335    | 100.0   |

## Advanced customization

Customization can go beyond the predefined styles and options available to you in the `table` command. `table` creates a collection of results that can be used in combination with the `collect` suite of commands to produce highly customized tables and to export those tables to presentation-ready formats such as HTML, Word, L<sup>A</sup>T<sub>E</sub>X, PDF, Excel, and more.

Continuing with our example above, if we want to shorten the labels on the column headings, we could use the `collect label levels` command to define our new labels. After a change using `collect`, we can use `collect preview` to see the results.

```
. collect label levels result frequency "Freq" percent "%", modify
. collect preview
```

|               | Freq   | %     |
|---------------|--------|-------|
| Health status |        |       |
| Excellent     | 2,407  | 23.3  |
| Very good     | 2,591  | 25.1  |
| Good          | 2,938  | 28.4  |
| Fair          | 1,670  | 16.2  |
| Poor          | 729    | 7.1   |
| Total         | 10,335 | 100.0 |

We could continue making style edits to this table. When we are happy with the result, we can then export it to the format of our choice using `collect export`.

See [TABLES] **collect label** for details on the `collect label` command we used here, and for an overview of the `collect` suite, see [TABLES] **Intro**.

## Stored results

table stores the following in `r()`:

Scalars

`r(N)` number of observations

## References

Bruun, N. H. 2022. [Interactively building table reports with basetable](#). *Stata Journal* 22: 416–429.

McDowell, A., A. Engel, J. T. Massey, and K. Maurer. 1981. “Plan and operation of the Second National Health and Nutrition Examination Survey, 1976–1980”. In *Vital and Health Statistics*, ser. 1, no. 15. Hyattsville, MD: National Center for Health Statistics.

## Also see

[R] [table](#) — Table of frequencies, summaries, and command results

[R] [table intro](#) — Introduction to tables of frequencies, summaries, and command results

[R] [table multiway](#) — Multiway tables

[R] [table twoway](#) — Two-way tabulation

[R] [tabulate oneway](#) — One-way table of frequencies

[TABLES] [Intro](#) — Introduction

[Description](#)[Quick start](#)[Menu](#)[Syntax](#)[Options](#)[Remarks and examples](#)[Stored results](#)[References](#)[Also see](#)

## Description

In this entry, we discuss how to use `table` to create a two-way tabulation, including frequencies, percentages, and proportions.

## Quick start

Table of frequencies, with rows defined by categories of `a1` and columns defined by categories of `a2`

```
table a1 a2
```

Same as above, but treat missing values like other values

```
table a1 a2, missing
```

Table with the percentage of observations in each cell

```
table a1 a2, statistic(percent)
```

For each category of `a1`, report the percentage of observations across levels of `a2`

```
table a1 a2, statistic(percent, across(a2))
```

Report frequencies and the proportion of observations across categories of `a1`, enclosed within parentheses

```
table a1 a2, statistic(frequency) ///
 statistic(proportion, across(a1)) sformat("%s)" proportion)
```

## Menu

Statistics > Summaries, tables, and tests > Tables of frequencies, summaries, and command results

## Syntax

### Basic two-way tabulation

```
table rowvar colvar [if] [in] [weight] [, options]
```

### Customized two-way tabulation

```
table [(rowspec)] [(colspec)] [if] [in] [weight] [, options]
```

*rowspec* may be empty or may include *rowvar*, *result*, or *rowvar* and *result*, where *result* refers to the requested statistics.

*colspec* may be empty or may include *colvar*, *result*, or *colvar* and *result*, where *result* refers to the requested statistics.

| <i>options</i>                                                          | Description                                                                                                                                     |
|-------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Main</b>                                                             |                                                                                                                                                 |
| <code>totals(<i>totals</i>)</code>                                      | report only the specified totals                                                                                                                |
| <code>nototals</code>                                                   | suppress the marginal totals                                                                                                                    |
| <b>Statistics</b>                                                       |                                                                                                                                                 |
| <code><u>statistic</u>(<i>stat</i> [, <i>statopts</i>])</code>          | statistic to be reported; default is <code>statistic(frequency)</code> when no weights are specified and <code>statistic(sumw)</code> otherwise |
| <b>Formats</b>                                                          |                                                                                                                                                 |
| <code>nformat(<i>%fmt</i> [<i>results</i>] [, <i>basestyle</i>])</code> | specify numeric format                                                                                                                          |
| <code>sformat(<i>sfmt</i> [<i>results</i>])</code>                      | specify string format                                                                                                                           |
| <b>Title</b>                                                            |                                                                                                                                                 |
| <code>title(<i>string</i>)</code>                                       | add table title                                                                                                                                 |
| <code><u>titlestyles</u>(<i>text_styles</i>)</code>                     | change table title styles                                                                                                                       |
| <b>Notes</b>                                                            |                                                                                                                                                 |
| <code>note(<i>string</i>)</code>                                        | add table note                                                                                                                                  |
| <code><u>notestyles</u>(<i>text_styles</i>)</code>                      | change table note styles                                                                                                                        |
| <b>Export</b>                                                           |                                                                                                                                                 |
| <code>export(<i>filename.suffix</i> [, <i>export_opts</i>])</code>      | export table                                                                                                                                    |
| <b>Options</b>                                                          |                                                                                                                                                 |
| <code><u>missing</u></code>                                             | treat numeric missing values of <i>rowvar</i> and <i>colvar</i> like other values                                                               |
| <code><u>zerocounts</u></code>                                          | report 0 for empty cell counts                                                                                                                  |
| <code>name(<i>cname</i>)</code>                                         | collect results into a collection named <i>cname</i>                                                                                            |
| <code>append</code>                                                     | append results to an existing collection                                                                                                        |
| <code>replace</code>                                                    | replace results of an existing collection                                                                                                       |
| <code>label(<i>filename</i>)</code>                                     | specify the collection labels                                                                                                                   |
| <code>style(<i>filename</i> [, <i>override</i>])</code>                 | specify the collection style                                                                                                                    |
| <code>markvar(<i>newvar</i>)</code>                                     | create <i>newvar</i> that identifies observations used in the tabulation                                                                        |

fweights, aweights, iweights, and pweights are allowed; see [U] 11.1.6 **weight**.

strL variables are not allowed; see [U] 12.4.8 **strL**.

markvar() does not appear in the dialog box.

| <i>text_styles</i>                                | Description                                              |
|---------------------------------------------------|----------------------------------------------------------|
| font( <i>[fontfamily]</i> [ <i>, font_opts</i> ]) | specify font style                                       |
| smcl( <i>smcl</i> )                               | specify formatting for SMCL files                        |
| latex( <i>latex</i> )                             | specify L <sup>A</sup> T <sub>E</sub> X macro            |
| shading( <i>sspec</i> )                           | set background color, foreground color, and fill pattern |

| <i>font_opts</i>          | Description                              |
|---------------------------|------------------------------------------|
| size(# [ <i>unit</i> ])   | specify font size                        |
| color( <i>color</i> )     | specify font color                       |
| variant( <i>variant</i> ) | specify font variant and capitalization  |
| [no]bold                  | specify whether to format text as bold   |
| [no]italic                | specify whether to format text as italic |
| [no]strikeout             | specify whether to strike out text       |
| [no]underline             | specify whether to underline text        |

| <i>suffix</i> | <i>fileformat</i> | Output format                      |
|---------------|-------------------|------------------------------------|
| docx          | as(docx)          | Microsoft Word                     |
| html          | as(html)          | HTML 5 with CSS                    |
| pdf           | as(pdf)           | PDF                                |
| xlsx          | as(xlsx)          | Microsoft Excel 2007/2010 or newer |
| xls           | as(xls)           | Microsoft Excel 1997/2003          |
| tex           | as(tex)           | L <sup>A</sup> T <sub>E</sub> X    |
| smcl          | as(smcl)          | SMCL                               |
| txt           | as(txt)           | plain text                         |
| markdown      | as(markdown)      | Markdown                           |
| md            | as(md)            | Markdown                           |

| <i>export_opts</i>      | Description                                         |
|-------------------------|-----------------------------------------------------|
| as( <i>fileformat</i> ) | specify document type                               |
| replace                 | overwrite existing file                             |
| <i>docx_options</i>     | available when exporting to .docx files             |
| <i>html_options</i>     | available when exporting to .html files             |
| <i>pdf_options</i>      | available when exporting to .pdf files              |
| <i>excel_options</i>    | available when exporting to .xls and .xlsx files    |
| <i>tex_options</i>      | available when exporting to .tex files              |
| <i>smcl_option</i>      | available when exporting to .smcl files             |
| <i>txt_option</i>       | available when exporting to .txt files              |
| <i>md_option</i>        | available when exporting to .markdown and .md files |

| <i>docx_options</i>                      | Description                                                         |
|------------------------------------------|---------------------------------------------------------------------|
| <code>noisily</code>                     | show the putdocx commands used to export to the Microsoft Word file |
| <code>dofile(filename[, replace])</code> | save the putdocx commands used for exporting to the named do-file   |

| <i>html_options</i>           | Description                                                    |
|-------------------------------|----------------------------------------------------------------|
| <code>append</code>           | append to an existing file                                     |
| <code>tableonly</code>        | export only the table to the specified file                    |
| <code>cssfile(cssfile)</code> | define the styles in <i>cssfile</i> instead of <i>filename</i> |
| <code>prefix(prefix)</code>   | use <i>prefix</i> to identify style classes                    |

| <i>pdf_options</i>                       | Description                                                      |
|------------------------------------------|------------------------------------------------------------------|
| <code>noisily</code>                     | show the putpdf commands used to export to the PDF file          |
| <code>dofile(filename[, replace])</code> | save the putpdf commands used for exporting to the named do-file |

| <i>excel_options</i>                     | Description                                                                                             |
|------------------------------------------|---------------------------------------------------------------------------------------------------------|
| <code>noisily</code>                     | show the putexcel commands used to export to the Excel file                                             |
| <code>dofile(filename[, replace])</code> | save the putexcel commands used for exporting to the named do-file                                      |
| <code>sheet(sheetname[, replace])</code> | specify the worksheet to use; the default sheet name is Sheet1                                          |
| <code>cell(cell)</code>                  | specify the Excel upper-left cell as the starting position to export the table; the default is cell(A1) |
| <code>modify</code>                      | modify Excel file                                                                                       |
| <code>noopen</code>                      | do not open Excel file in memory                                                                        |

`noopen` does not appear in the dialog box.

| <i>tex_options</i>     | Description                                 |
|------------------------|---------------------------------------------|
| <code>append</code>    | append to an existing file                  |
| <code>tableonly</code> | export only the table to the specified file |

| <i>smcl_option</i>  | Description                |
|---------------------|----------------------------|
| <code>append</code> | append to an existing file |

| <i>txt_option</i>   | Description                |
|---------------------|----------------------------|
| <code>append</code> | append to an existing file |

| <i>md_option</i>    | Description                |
|---------------------|----------------------------|
| <code>append</code> | append to an existing file |

*fontfamily* specifies a valid font family.

*unit* may be in (inch), pt (point), or cm (centimeter). An inch is equivalent to 72 points and 2.54 centimeters. The default is pt.

*variant* may be allcaps, smallcaps, or normal.

*variant* (allcaps) changes the text to all uppercase letters; applicable when publishing items from a collection to Microsoft Word, PDF, L<sup>A</sup>T<sub>E</sub>X, and HTML files.

*variant* (smallcaps) changes the text to use large capitals for uppercase letters and smaller capitals for lowercase letters; applicable when publishing items from a collection to Microsoft Word, L<sup>A</sup>T<sub>E</sub>X, and HTML files.

*variant* (normal) changes the font variant back to normal and leaves the capitalization unchanged from the original text; applicable when publishing items from a collection to Microsoft Word, PDF, L<sup>A</sup>T<sub>E</sub>X, and HTML files.

*smcl* specifies the name of the SMCL directive to render text for SMCL output. The supported SMCL directives are `input`, `error`, `result`, and `text`.

*latex* specifies the name of a L<sup>A</sup>T<sub>E</sub>X macro to render text for L<sup>A</sup>T<sub>E</sub>X output. Example L<sup>A</sup>T<sub>E</sub>X macro names are `textbf`, `textsf`, `textrm`, and `texttt`. Custom L<sup>A</sup>T<sub>E</sub>X macros are also allowed. If *text* is to be rendered in a cell, title, or note, then *latex* is translated to the following when you export to L<sup>A</sup>T<sub>E</sub>X:

```
\latex {text}
```

*sspec* is

```
[background(bcolor) foreground(fgcolor) pattern(fpattern)]
```

*bcolor* specifies the background color.

*fgcolor* specifies the foreground color.

*fpattern* specifies the fill pattern. A complete list of fill patterns is shown in the [Appendix](#).

*bcolor*, *fgcolor*, and *color* may be one of the colors listed in the [Appendix](#); a valid RGB value in the form `### ## #`, for example, `171 248 103`; or a valid RRGGBB hex value in the form `#####`, for example, `ABF867`.

## Options

### Main

`totals` (*totals*) and `nototals` control which totals are to be displayed in the table. By default, all totals are reported.

`totals` (*totals*) specifies which margin totals to display in the reported table. *totals* can contain *rowvar*, *colvar*, and their interaction. Interactions can be specified by using the # operator.

`nototals` prevents table from displaying any totals.

### Statistics

`statistic`(*stat*[, *statopts*]) specifies the statistic to be displayed. `statistic()` may be repeated to request multiple statistics.

Available statistics are

| <i>stat</i>                | Definition                                       |
|----------------------------|--------------------------------------------------|
| <code>frequency</code>     | frequency                                        |
| <code>sumw</code>          | sum of weights                                   |
| <code>proportion</code>    | proportion                                       |
| <code>percent</code>       | percentage                                       |
| <code>rawproportion</code> | proportion ignoring optionally specified weights |
| <code>rawpercent</code>    | percentage ignoring optionally specified weights |

The following options may be specified in combination with statistics `proportion`, `percent`, `rawproportion`, and `rawpercent`:

| <i>statopts</i>                       | Definition                                 |
|---------------------------------------|--------------------------------------------|
| <code>across</code> ( <i>rowvar</i> ) | percentages or proportions across rows     |
| <code>across</code> ( <i>colvar</i> ) | percentages or proportions across columns  |
| <code>total</code>                    | compute overall percentages or proportions |

## Formats

`nformat(%fmt [results] [, basestyle])` changes the numeric format, such as the number of decimal places, for specified results. If *results* are not specified, the numeric format is changed for all results. *results* may be any statistic named in option `statistic()` (that is, any *stat*).

This option is repeatable, and when multiple formats apply to one result, the rightmost specification is applied.

This option does not affect the format of numeric layout variables (*rowspec* and *colspec*). The default format of these variables is taken from the dataset.

*basestyle* indicates that the format be applied to results that do not already have their own format instead of overriding the format for all results.

`sformat(sfmt [results])` changes the string format for specified results. You can, for instance, add symbols or text to the values reported in the table by modifying the string format.

*sfmt* may contain a mix of text and *%s*. Here *%s* refers to the numeric value that is formatted as specified using `nformat()`. The text will be placed around the numeric values in your table as it is placed around *%s* in this option. For instance, to place parentheses around the percent statistics, you can specify `sformat("(%s) percent)`.

*results* may be any statistic named in option `statistic()` (that is, any *stat*).

Two text characters must be specified using a special character sequence if you want them to be displayed in your table. To include %, type `%`. To include \, type `\\`. For instance, to place a percent sign following percent statistics, you can specify `sformat("%s%" percent)`.

This option is repeatable, and when multiple formats apply to one result, the rightmost specification is applied.

Title

`title(string)` adds the text *string* as a title to the table.

`titlestyles(text_styles)` changes the style for the table title. *text\_styles* are the following:

`font([fontfamily] [, size(# [unit]) color(color) variant(variant) [no]bold [no]italic [no]strikeout [no]underline])` specifies the font style. These font style properties are applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files, unless otherwise specified.

*fontfamily* specifies a valid font family. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`size(# [unit])` specifies the font size as a number optionally followed by units. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`color(color)` specifies the text color.

`variant(variant)` specifies the font variant and capitalization.

`bold` and `nobold` specify the font weight. `bold` changes the font weight to bold; `nobold` changes the font weight back to normal.

`italic` and `noitalic` specify the font style. `italic` changes the font style to italic; `noitalic` changes the font style back to normal.

`strikeout` and `nostrikeout` specify whether to add a strikeout mark to the title. `strikeout` adds a strikeout mark to the title; `nostrikeout` changes the title back to normal.

`underline` and `nounderline` specify whether to underline the table title. `underline` adds a single line under the title; `nounderline` removes the underline.

Only one of `strikeout` or `underline` is allowed when publishing to HTML files.

`smcl(smcl)` specifies how to render the table title for SMCL output. This style property is applicable only when publishing items from a collection to a SMCL file.

`latex(latex)` specifies how to render the table title for  $\LaTeX$  output. This style property is applicable only when publishing items from a collection to a  $\LaTeX$  file.

`shading(spec)` sets the background color, foreground color, and fill pattern. The background color is applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files. The foreground color and fill pattern are applicable when exporting the table to Microsoft Word and Microsoft Excel.

`note(string)` adds the text *string* as a note to the table. `note()` may be specified multiple times to add multiple notes. Each note is placed on a new line.

`notestyles(text_styles)` changes the style for the table notes. *text\_styles* are the following:

`font([fontfamily] [, size(# [unit]) color(color) variant(variant) [no]bold [no]italic [no]strikeout [no]underline])` specifies the font style. These font style properties are applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files, unless otherwise specified.

*fontfamily* specifies a valid font family. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`size(# [unit])` specifies the font size as a number optionally followed by units. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`color(color)` specifies the text color.

`variant(variant)` specifies the font variant and capitalization.

`bold` and `nobold` specify the font weight. `bold` changes the font weight to bold; `nobold` changes the font weight back to normal.

`italic` and `noitalic` specify the font style. `italic` changes the font style to italic; `noitalic` changes the font style back to normal.

`strikeout` and `nostrikeout` specify whether to add a strikeout mark to the notes. `strikeout` adds a strikeout mark to the note; `nostrikeout` changes the note back to normal.

`underline` and `nounderline` specify whether to underline the table notes. `underline` adds a single line under the notes; `nounderline` removes the underline.

Only one of `strikeout` or `underline` is allowed when publishing to HTML files.

`smcl(smcl)` specifies how to render the table notes for SMCL output. This style property is applicable only when publishing items from a collection to a SMCL file.

`latex(latex)` specifies how to render the table notes for  $\LaTeX$  output. This style property is applicable only when publishing items from a collection to a  $\LaTeX$  file.

`shading(sspec)` sets the background color, foreground color, and fill pattern. The background color is applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files. The foreground color and fill pattern are applicable when exporting the table to Microsoft Word and Microsoft Excel.

Export

`export(filename.suffix[ , export_opts])` exports the table to the specified file. *export\_opts* are the following:

`as(fileformat)` specifies the file format to which the table is to be exported. This option is rarely specified because, by default, `table` determines the format from the suffix of the file being created.

`replace` permits `table` to overwrite an existing file.

`noisily` specifies that `table` show the commands used to export the table to Microsoft Word, Microsoft Excel, and PDF files. The `putdocx`, `putexcel`, or `putpdf` command used to export the table will be displayed.

`dofile(filename[ , replace])` specifies that `table` save to *filename* the commands used to export the table to Microsoft Word, Microsoft Excel, and PDF files.

If *filename* already exists, it can be overwritten by specifying `replace`. If *filename* is specified without an extension, `.do` is assumed.

`append` specifies that `table` append the table to an existing file.

This option is applicable when you export the table to an HTML, a  $\LaTeX$ , a SMCL, a `txt`, or a Markdown file. When you export to HTML and  $\LaTeX$  files, the `append` option implies the `tableonly` option. Furthermore, when you export to HTML files, if the target CSS file already exists, `table` will also append to it.

`tableonly` specifies that only the table be exported to the specified HTML or  $\LaTeX$  document. By default, `table` produces complete HTML and  $\LaTeX$  documents.

When you export to an HTML file, if the `cssfile()` option is not specified, a CSS filename is constructed from *filename*, with the extension replaced with `.css`.

`cssfile(cssfile)` specifies that `table` define the styles in *cssfile* instead of *filename* when you export to HTML.

`prefix(prefix)` specifies that `table` use *prefix* to identify style classes when you export to HTML.

`sheet(sheetname [ , replace])` saves to the worksheet named *sheetname*. For more information about this option, see [\[RPT\] putexcel](#).

`cell(cell)` specifies an Excel upper-left cell as the starting position to publish the table. The default is `cell(A1)`.

`modify` permits `putexcel` set to modify an Excel file. For more information about this option, see [\[RPT\] putexcel](#).

`noopen` prevents `putexcel` from opening the Excel file in memory for modification. It does not appear in the dialog box. For more information about this option, see [\[RPT\] putexcel](#).

## Options

`missing` specifies that numeric missing values of *rowvar* or *colvar* be treated as valid categories. By default, observations with a numeric missing value in *rowvar* or *colvar* are omitted.

`zerocounts` specifies that `table` report a 0 in empty cells for the frequency statistic.

`name(cname)` specifies that a collection named *cname* be associated with the collected statistics and results. The default is `name(Table)`.

`append` specifies that `table` append its collection information into the collection named in `name()`.

`replace` permits `table` to overwrite an existing collection. This option is implied for `name(Table)` when `append` is not specified.

`label(filename)` specifies the *filename* containing the collection labels to use for your table. Labels in *filename* will be loaded for the table, and any labels not specified in *filename* will be taken from the labels defined in `c(collect_label)`. The default is to use only the collection labels set in `c(collect_label)`; see [TABLES] [set collect\\_label](#).

`style(filename [ , override ])` specifies the *filename* containing the collection styles to use for your table. The default collection styles will be discarded, and only the collection styles in *filename* will be applied.

If you prefer the default collection styles but also want to apply any styles in *filename*, specify `override`. If there are conflicts between the default collection styles and those in *filename*, the ones in *filename* will take precedence.

The default is to use only the collection styles set in `c(table_style)`; see [TABLES] [set table\\_style](#).

The following option is available with `table` but is not shown in the dialog box:

`markvar(newvar)` generates an indicator variable that identifies the observations used in the tabulation.

## Remarks and examples

Remarks are presented under the following headings:

*Tabulation of two variables*  
*Tabulation, including percentages*  
*Customizing results*

## Tabulation of two variables

To obtain a two-way tabulation that reports the number of observations across the levels of two categorical variables, we need to specify only the names of the categorical variables following `table`.

To demonstrate, we use data from the Second National Health and Nutrition Examination Survey (NHANES II) (McDowell et al. 1981) and create a two-way tabulation of self-reported health status (`hlthstat`) by region of the USA (`region`).

```
. use https://www.stata-press.com/data/r19/nhanes21
(Second National Health and Nutrition Examination Survey)
. table hlthstat region
```

|               | Region |       |       |       | Total  |
|---------------|--------|-------|-------|-------|--------|
|               | NE     | MW    | S     | W     |        |
| Health status |        |       |       |       |        |
| Excellent     | 562    | 730   | 546   | 569   | 2,407  |
| Very good     | 558    | 721   | 651   | 661   | 2,591  |
| Good          | 631    | 735   | 807   | 765   | 2,938  |
| Fair          | 257    | 419   | 532   | 462   | 1,670  |
| Poor          | 77     | 167   | 317   | 168   | 729    |
| Total         | 2,085  | 2,772 | 2,853 | 2,625 | 10,335 |

We can examine the missing values as well by adding the `missing` option.

```
. table hlthstat region, missing
```

|                      | Region |       |       |       | Total  |
|----------------------|--------|-------|-------|-------|--------|
|                      | NE     | MW    | S     | W     |        |
| Health status        |        |       |       |       |        |
| Excellent            | 562    | 730   | 546   | 569   | 2,407  |
| Very good            | 558    | 721   | 651   | 661   | 2,591  |
| Good                 | 631    | 735   | 807   | 765   | 2,938  |
| Fair                 | 257    | 419   | 532   | 462   | 1,670  |
| Poor                 | 77     | 167   | 317   | 168   | 729    |
| .                    | 1      | 1     |       |       | 2      |
| Blank but applicable | 10     | 1     |       | 3     | 14     |
| Total                | 2,096  | 2,774 | 2,853 | 2,628 | 10,351 |

We find that 16 individuals have a missing health status, and the majority of these are from individuals in the Northeast. The empty cells correspond to regions in which all the individuals have a nonmissing health status; we can fill in these empty cells with 0s:

```
. table hlthstat region, missing zerocounts
```

|                      | Region |       |       |       | Total  |
|----------------------|--------|-------|-------|-------|--------|
|                      | NE     | MW    | S     | W     |        |
| Health status        |        |       |       |       |        |
| Excellent            | 562    | 730   | 546   | 569   | 2,407  |
| Very good            | 558    | 721   | 651   | 661   | 2,591  |
| Good                 | 631    | 735   | 807   | 765   | 2,938  |
| Fair                 | 257    | 419   | 532   | 462   | 1,670  |
| Poor                 | 77     | 167   | 317   | 168   | 729    |
| .                    | 1      | 1     | 0     | 0     | 2      |
| Blank but applicable | 10     | 1     | 0     | 3     | 14     |
| Total                | 2,096  | 2,774 | 2,853 | 2,628 | 10,351 |

## Tabulation, including percentages

Instead of frequencies, we can request that table report the percentage of observations in each cell of the table by specifying the `statistic(percent)` option.

```
. table hlthstat region, statistic(percent)
```

|               | Region |       |       |       | Total  |
|---------------|--------|-------|-------|-------|--------|
|               | NE     | MW    | S     | W     |        |
| Health status |        |       |       |       |        |
| Excellent     | 5.44   | 7.06  | 5.28  | 5.51  | 23.29  |
| Very good     | 5.40   | 6.98  | 6.30  | 6.40  | 25.07  |
| Good          | 6.11   | 7.11  | 7.81  | 7.40  | 28.43  |
| Fair          | 2.49   | 4.05  | 5.15  | 4.47  | 16.16  |
| Poor          | 0.75   | 1.62  | 3.07  | 1.63  | 7.05   |
| Total         | 20.17  | 26.82 | 27.61 | 25.40 | 100.00 |

We see that 5.44% of all observations correspond to individuals in excellent health who live in the Northeast.

Rather than looking at overall percentages, we might want to examine the distribution of observations within each health status level across the four regions. To do this, we can add the `across(region)` option.

```
. table hlthstat region, statistic(percent, across(region))
```

|               | Region |       |       |       | Total  |
|---------------|--------|-------|-------|-------|--------|
|               | NE     | MW    | S     | W     |        |
| Health status |        |       |       |       |        |
| Excellent     | 23.35  | 30.33 | 22.68 | 23.64 | 100.00 |
| Very good     | 21.54  | 27.83 | 25.13 | 25.51 | 100.00 |
| Good          | 21.48  | 25.02 | 27.47 | 26.04 | 100.00 |
| Fair          | 15.39  | 25.09 | 31.86 | 27.66 | 100.00 |
| Poor          | 10.56  | 22.91 | 43.48 | 23.05 | 100.00 |
| Total         | 20.17  | 26.82 | 27.61 | 25.40 | 100.00 |

Of individuals reporting excellent health, 23.35% live in the Northeast, while 30.33% live in the Midwest, 22.68% live in the South, and 23.64% live in the West.

We can also look at the distribution of observations across health status categories within each region.

```
. table hlthstat region, statistic(percent, across(hlthstat))
```

|               | Region |        |        |        |        |
|---------------|--------|--------|--------|--------|--------|
|               | NE     | MW     | S      | W      | Total  |
| Health status |        |        |        |        |        |
| Excellent     | 26.95  | 26.33  | 19.14  | 21.68  | 23.29  |
| Very good     | 26.76  | 26.01  | 22.82  | 25.18  | 25.07  |
| Good          | 30.26  | 26.52  | 28.29  | 29.14  | 28.43  |
| Fair          | 12.33  | 15.12  | 18.65  | 17.60  | 16.16  |
| Poor          | 3.69   | 6.02   | 11.11  | 6.40   | 7.05   |
| Total         | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |

Of individuals living in the South, 11.11% report having poor health. This is notably higher than the percentage of individuals reporting poor health in the other regions.

It is often helpful to see both frequencies and percentages in the same table. To do this, we can add the `statistic(frequency)` option to our command.

```
. table hlthstat region, statistic(frequency)
> statistic(percent, across(hlthstat))
```

|               | Region |        |        |        |        |
|---------------|--------|--------|--------|--------|--------|
|               | NE     | MW     | S      | W      | Total  |
| Health status |        |        |        |        |        |
| Excellent     |        |        |        |        |        |
| Frequency     | 562    | 730    | 546    | 569    | 2,407  |
| Percent       | 26.95  | 26.33  | 19.14  | 21.68  | 23.29  |
| Very good     |        |        |        |        |        |
| Frequency     | 558    | 721    | 651    | 661    | 2,591  |
| Percent       | 26.76  | 26.01  | 22.82  | 25.18  | 25.07  |
| Good          |        |        |        |        |        |
| Frequency     | 631    | 735    | 807    | 765    | 2,938  |
| Percent       | 30.26  | 26.52  | 28.29  | 29.14  | 28.43  |
| Fair          |        |        |        |        |        |
| Frequency     | 257    | 419    | 532    | 462    | 1,670  |
| Percent       | 12.33  | 15.12  | 18.65  | 17.60  | 16.16  |
| Poor          |        |        |        |        |        |
| Frequency     | 77     | 167    | 317    | 168    | 729    |
| Percent       | 3.69   | 6.02   | 11.11  | 6.40   | 7.05   |
| Total         |        |        |        |        |        |
| Frequency     | 2,085  | 2,772  | 2,853  | 2,625  | 10,335 |
| Percent       | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |

## Customizing results

There are several ways that we can customize the results of our two-way tabulation.

For instance, in some cases, we may prefer to omit the row and column totals. We can specify the `nototals` option to suppress these totals.

```
. table hlthstat region, statistic(frequency)
> statistic(percent, across(hlthstat)) nototals
```

|               | Region |       |       |       |
|---------------|--------|-------|-------|-------|
|               | NE     | MW    | S     | W     |
| Health status |        |       |       |       |
| Excellent     |        |       |       |       |
| Frequency     | 562    | 730   | 546   | 569   |
| Percent       | 26.95  | 26.33 | 19.14 | 21.68 |
| Very good     |        |       |       |       |
| Frequency     | 558    | 721   | 651   | 661   |
| Percent       | 26.76  | 26.01 | 22.82 | 25.18 |
| Good          |        |       |       |       |
| Frequency     | 631    | 735   | 807   | 765   |
| Percent       | 30.26  | 26.52 | 28.29 | 29.14 |
| Fair          |        |       |       |       |
| Frequency     | 257    | 419   | 532   | 462   |
| Percent       | 12.33  | 15.12 | 18.65 | 17.60 |
| Poor          |        |       |       |       |
| Frequency     | 77     | 167   | 317   | 168   |
| Percent       | 3.69   | 6.02  | 11.11 | 6.40  |

Or perhaps we want to see row totals or column totals but not both. We can include the `totals(region)` option to display only the region totals.

```
. table hlthstat region, statistic(frequency)
> statistic(percent, across(hlthstat)) totals(region)
```

|               | Region |        |        |        |
|---------------|--------|--------|--------|--------|
|               | NE     | MW     | S      | W      |
| Health status |        |        |        |        |
| Excellent     |        |        |        |        |
| Frequency     | 562    | 730    | 546    | 569    |
| Percent       | 26.95  | 26.33  | 19.14  | 21.68  |
| Very good     |        |        |        |        |
| Frequency     | 558    | 721    | 651    | 661    |
| Percent       | 26.76  | 26.01  | 22.82  | 25.18  |
| Good          |        |        |        |        |
| Frequency     | 631    | 735    | 807    | 765    |
| Percent       | 30.26  | 26.52  | 28.29  | 29.14  |
| Fair          |        |        |        |        |
| Frequency     | 257    | 419    | 532    | 462    |
| Percent       | 12.33  | 15.12  | 18.65  | 17.60  |
| Poor          |        |        |        |        |
| Frequency     | 77     | 167    | 317    | 168    |
| Percent       | 3.69   | 6.02   | 11.11  | 6.40   |
| Total         |        |        |        |        |
| Frequency     | 2,085  | 2,772  | 2,853  | 2,625  |
| Percent       | 100.00 | 100.00 | 100.00 | 100.00 |

Once we have the statistics we want in our table, we can format the way that they appear. If, for instance, we want to add a percent sign to each of our percentages, we can specify the `sformat("%s%" percent)` option. The `sformat()` option specifies that we want to add string characters to the numbers in the table. Within it, we refer to the numeric values as `%s` and place any string characters we want around this. The percent sign is unique because it already has special meaning in this context. Therefore, we must type two percent signs, `%%`, to display one. Finally, by adding `percent` within the `sformat()` option, we specify that we want to apply this format only to the percent statistics.

```
. table hlthstat region, statistic(frequency)
> statistic(percent, across(hlthstat)) totals(region)
> sformat("%s%" percent)
```

|               | Region  |         |         |         |
|---------------|---------|---------|---------|---------|
|               | NE      | MW      | S       | W       |
| Health status |         |         |         |         |
| Excellent     |         |         |         |         |
| Frequency     | 562     | 730     | 546     | 569     |
| Percent       | 26.95%  | 26.33%  | 19.14%  | 21.68%  |
| Very good     |         |         |         |         |
| Frequency     | 558     | 721     | 651     | 661     |
| Percent       | 26.76%  | 26.01%  | 22.82%  | 25.18%  |
| Good          |         |         |         |         |
| Frequency     | 631     | 735     | 807     | 765     |
| Percent       | 30.26%  | 26.52%  | 28.29%  | 29.14%  |
| Fair          |         |         |         |         |
| Frequency     | 257     | 419     | 532     | 462     |
| Percent       | 12.33%  | 15.12%  | 18.65%  | 17.60%  |
| Poor          |         |         |         |         |
| Frequency     | 77      | 167     | 317     | 168     |
| Percent       | 3.69%   | 6.02%   | 11.11%  | 6.40%   |
| Total         |         |         |         |         |
| Frequency     | 2,085   | 2,772   | 2,853   | 2,625   |
| Percent       | 100.00% | 100.00% | 100.00% | 100.00% |

Now that we have added the percent sign, we could argue that the labels `Frequency` and `Percent` are unnecessary. If we remove these statistic names from the row labels, we might also want to right-align the remaining labels in row headers. Finally, for readability, we could insert blank lines between levels of `hlthstat`. We could use the `collect` suite of commands to make these style changes. Fortunately, however, one of our [predefined styles](#), `table-tab2`, includes these style changes, and we can select it using the `style()` option.

```
. table hlthstat region, statistic(frequency)
> statistic(percent, across(hlthstat)) totals(region)
> sformat("%s%" percent) style(table-tab2)
```

|               | Region           |                  |                  |                  |
|---------------|------------------|------------------|------------------|------------------|
|               | NE               | MW               | S                | W                |
| Health status |                  |                  |                  |                  |
| Excellent     | 562<br>26.95%    | 730<br>26.33%    | 546<br>19.14%    | 569<br>21.68%    |
| Very good     | 558<br>26.76%    | 721<br>26.01%    | 651<br>22.82%    | 661<br>25.18%    |
| Good          | 631<br>30.26%    | 735<br>26.52%    | 807<br>28.29%    | 765<br>29.14%    |
| Fair          | 257<br>12.33%    | 419<br>15.12%    | 532<br>18.65%    | 462<br>17.60%    |
| Poor          | 77<br>3.69%      | 167<br>6.02%     | 317<br>11.11%    | 168<br>6.40%     |
| Total         | 2,085<br>100.00% | 2,772<br>100.00% | 2,853<br>100.00% | 2,625<br>100.00% |

You can learn more about the predefined styles described at [\[TABLES\] Predefined styles](#). If none of these provide the exact style you want for your table, you can further customize the results by using the collect suite of commands. To learn more, see [\[TABLES\] Intro](#).

If you wish to include this table in a paper, on a webpage, or in another format, you can easily export it in  $\LaTeX$ , Word, Excel, HTML, and a variety of other formats by using the `export()` option.

## Stored results

`table` stores the following in `r()`:

Scalars

`r(N)` number of observations

## References

- Huber, C. 2021. Customizable tables in Stata 17, part 1: The new table command. *The Stata Blog: Not Elsewhere Classified*. <https://blog.stata.com/2021/06/07/customizable-tables-in-stata-17-part-1-the-new-table-command/>.
- McDowell, A., A. Engel, J. T. Massey, and K. Maurer. 1981. "Plan and operation of the Second National Health and Nutrition Examination Survey, 1976–1980". In *Vital and Health Statistics*, ser. 1, no. 15. Hyattsville, MD: National Center for Health Statistics.

## Also see

[R] **table** — Table of frequencies, summaries, and command results

[R] **table intro** — Introduction to tables of frequencies, summaries, and command results

[R] **table multiway** — Multiway tables

[R] **table oneway** — One-way tabulation

[R] **tabulate twoway** — Two-way table of frequencies

[TABLES] **Intro** — Introduction

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## Description

In this entry, we discuss how to use the `table` command to create tables displaying frequencies, percentages, and proportions across levels of three or more variables. Additionally, we demonstrate how a single `table` command can create multiple tables corresponding to levels of variables or different statistics.

## Quick start

Table of frequencies with rows defined by the levels of `a1` and columns defined by the categories of `a2` and `a3`

```
table (a1) (a2 a3)
```

Same as above, but with rows defined by levels of `a1` and `a2` and columns defined by levels of `a3`

```
table (a1 a2) (a3)
```

Separate tables for each level `a3` with the rows of each table defined by the levels of `a1` and the columns defined by levels of `a2`

```
table (a1) (a2) (a3)
```

Report percentages of observations in each cell rather than frequencies

```
table (a1) (a2 a3), statistic(percent)
```

Report both frequencies and percentages

```
table (a1) (a2 a3), statistic(percent) statistic(frequency)
```

Report the percentages across levels of `a3`

```
table (a1) (a2 a3), statistic(percent, across(a3))
```

## Menu

Statistics > Summaries, tables, and tests > Tables of frequencies, summaries, and command results

## Syntax

*Table with rows defined by multiple variables*

```
table rowvars colvar [if] [in] [weight] [, options]
```

*Table with columns defined by multiple variables*

```
table rowvar (colvars) [if] [in] [weight] [, options]
```

*Table with rows and columns defined by multiple variables*

```
table (rowvars) (colvars) [if] [in] [weight] [, options]
```

*Multiple multiway tables*

```
table (rowvars) (colvars) (tabvars) [if] [in] [weight] [, options]
```

*Customized multiway tables*

```
table (rowspec) (colspec) [(tabspec)] [if] [in] [weight] [, options]
```

*rowspec*, *colspec*, and *tabspec* may be empty or may include variable names or any of the following keywords:

| <i>keyword</i> | Description                   |
|----------------|-------------------------------|
| result         | requested statistics          |
| across         | index across() specifications |

| <i>options</i>                                      | Description                                                                                                                                     |
|-----------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Main</b>                                         |                                                                                                                                                 |
| <code>totals(totals)</code>                         | report only the specified totals                                                                                                                |
| <code>nototals</code>                               | suppress the marginal totals                                                                                                                    |
| <b>Statistics</b>                                   |                                                                                                                                                 |
| <code>statistic(stat[, statopts])</code>            | statistic to be reported; default is <code>statistic(frequency)</code> when no weights are specified and <code>statistic(sumw)</code> otherwise |
| <b>Formats</b>                                      |                                                                                                                                                 |
| <code>nformat(%fmt [results][, basestyle])</code>   | specify numeric format                                                                                                                          |
| <code>sformat(sfmt [results])</code>                | specify string format                                                                                                                           |
| <b>Title</b>                                        |                                                                                                                                                 |
| <code>title(string)</code>                          | add table title                                                                                                                                 |
| <code>titlestyles(text_styles)</code>               | change table title styles                                                                                                                       |
| <b>Notes</b>                                        |                                                                                                                                                 |
| <code>note(string)</code>                           | add table note                                                                                                                                  |
| <code>notestyles(text_styles)</code>                | change table note styles                                                                                                                        |
| <b>Export</b>                                       |                                                                                                                                                 |
| <code>export(filename.suffix[, export_opts])</code> | export table                                                                                                                                    |
| <b>Options</b>                                      |                                                                                                                                                 |
| <code>listwise</code>                               | use listwise deletion to handle missing values                                                                                                  |
| <code>missing</code>                                | treat numeric missing values like other values                                                                                                  |
| <code>showcounts</code>                             | show sample size for all variables in <code>statistic()</code> option                                                                           |
| <code>zerocounts</code>                             | report 0 for empty cell counts                                                                                                                  |
| <code>name(cname)</code>                            | collect results into a collection named <i>cname</i>                                                                                            |
| <code>append</code>                                 | append results to an existing collection                                                                                                        |
| <code>replace</code>                                | replace results of an existing collection                                                                                                       |
| <code>label(filename)</code>                        | specify the collection labels                                                                                                                   |
| <code>style(filename [, override])</code>           | specify the collection style                                                                                                                    |
| <code>markvar(newvar)</code>                        | create <i>newvar</i> to identify observations used to compute the statistics                                                                    |

`fweights`, `aweight`s, `iweight`s, and `pweight`s are allowed; see [\[U\] 11.1.6 weight](#).

`strL` variables are not allowed; see [\[U\] 12.4.8 strL](#).

`markvar()` does not appear in the dialog box.

| <i>text_styles</i>                           | Description                                              |
|----------------------------------------------|----------------------------------------------------------|
| <code>font([fontfamily][, font_opts])</code> | specify font style                                       |
| <code>smcl(smcl)</code>                      | specify formatting for SMCL files                        |
| <code>latex(latex)</code>                    | specify L <sup>A</sup> T <sub>E</sub> X macro            |
| <code>shading(sspec)</code>                  | set background color, foreground color, and fill pattern |

| <i>font_opts</i>              | Description                              |
|-------------------------------|------------------------------------------|
| <code>size(# [unit])</code>   | specify font size                        |
| <code>color(color)</code>     | specify font color                       |
| <code>variant(variant)</code> | specify font variant and capitalization  |
| <code>[no]bold</code>         | specify whether to format text as bold   |
| <code>[no]italic</code>       | specify whether to format text as italic |
| <code>[no]strikeout</code>    | specify whether to strike out text       |
| <code>[no]underline</code>    | specify whether to underline text        |

| <i>suffix</i>         | <i>fileformat</i>         | Output format                      |
|-----------------------|---------------------------|------------------------------------|
| <code>docx</code>     | <code>as(docx)</code>     | Microsoft Word                     |
| <code>html</code>     | <code>as(html)</code>     | HTML 5 with CSS                    |
| <code>pdf</code>      | <code>as(pdf)</code>      | PDF                                |
| <code>xlsx</code>     | <code>as(xlsx)</code>     | Microsoft Excel 2007/2010 or newer |
| <code>xls</code>      | <code>as(xls)</code>      | Microsoft Excel 1997/2003          |
| <code>tex</code>      | <code>as(tex)</code>      | L <sup>A</sup> T <sub>E</sub> X    |
| <code>smcl</code>     | <code>as(smcl)</code>     | SMCL                               |
| <code>txt</code>      | <code>as(txt)</code>      | plain text                         |
| <code>markdown</code> | <code>as(markdown)</code> | Markdown                           |
| <code>md</code>       | <code>as(md)</code>       | Markdown                           |

| <i>export_opts</i>          | Description                                         |
|-----------------------------|-----------------------------------------------------|
| <code>as(fileformat)</code> | specify document type                               |
| <code>replace</code>        | overwrite existing file                             |
| <i>docx_options</i>         | available when exporting to .docx files             |
| <i>html_options</i>         | available when exporting to .html files             |
| <i>pdf_options</i>          | available when exporting to .pdf files              |
| <i>excel_options</i>        | available when exporting to .xls and .xlsx files    |
| <i>tex_options</i>          | available when exporting to .tex files              |
| <i>smcl_option</i>          | available when exporting to .smcl files             |
| <i>txt_option</i>           | available when exporting to .txt files              |
| <i>md_option</i>            | available when exporting to .markdown and .md files |

| <i>docx_options</i>                      | Description                                                         |
|------------------------------------------|---------------------------------------------------------------------|
| <code>noisily</code>                     | show the putdocx commands used to export to the Microsoft Word file |
| <code>dofile(filename[, replace])</code> | save the putdocx commands used for exporting to the named do-file   |

| <i>html_options</i>       | Description                                                    |
|---------------------------|----------------------------------------------------------------|
| append                    | append to an existing file                                     |
| tableonly                 | export only the table to the specified file                    |
| cssfile( <i>cssfile</i> ) | define the styles in <i>cssfile</i> instead of <i>filename</i> |
| prefix( <i>prefix</i> )   | use <i>prefix</i> to identify style classes                    |

| <i>pdf_options</i>                   | Description                                                      |
|--------------------------------------|------------------------------------------------------------------|
| noisily                              | show the putpdf commands used to export to the PDF file          |
| dofile( <i>filename</i> [, replace]) | save the putpdf commands used for exporting to the named do-file |

| <i>excel_options</i>                 | Description                                                                                             |
|--------------------------------------|---------------------------------------------------------------------------------------------------------|
| noisily                              | show the putexcel commands used to export to the Excel file                                             |
| dofile( <i>filename</i> [, replace]) | save the putexcel commands used for exporting to the named do-file                                      |
| sheet( <i>sheetname</i> [, replace]) | specify the worksheet to use; the default sheet name is Sheet1                                          |
| cell( <i>cell</i> )                  | specify the Excel upper-left cell as the starting position to export the table; the default is cell(A1) |
| modify                               | modify Excel file                                                                                       |
| noopen                               | do not open Excel file in memory                                                                        |

noopen does not appear in the dialog box.

| <i>tex_options</i> | Description                                 |
|--------------------|---------------------------------------------|
| append             | append to an existing file                  |
| tableonly          | export only the table to the specified file |

| <i>smcl_option</i> | Description                |
|--------------------|----------------------------|
| append             | append to an existing file |

| <i>txt_option</i> | Description                |
|-------------------|----------------------------|
| append            | append to an existing file |

| <i>md_option</i> | Description                |
|------------------|----------------------------|
| append           | append to an existing file |

*fontfamily* specifies a valid font family.

*unit* may be in (inch), pt (point), or cm (centimeter). An inch is equivalent to 72 points and 2.54 centimeters. The default is pt.

*variant* may be allcaps, smallcaps, or normal.

*variant* (allcaps) changes the text to all uppercase letters; applicable when publishing items from a collection to Microsoft Word, PDF, L<sup>A</sup>T<sub>E</sub>X, and HTML files.

*variant* (smallcaps) changes the text to use large capitals for uppercase letters and smaller capitals for lowercase letters; applicable when publishing items from a collection to Microsoft Word, L<sup>A</sup>T<sub>E</sub>X, and HTML files.

*variant* (normal) changes the font variant back to normal and leaves the capitalization unchanged from the original text; applicable when publishing items from a collection to Microsoft Word, PDF, L<sup>A</sup>T<sub>E</sub>X, and HTML files.

*smcl* specifies the name of the SMCL directive to render text for SMCL output. The supported SMCL directives are input, error, result, and text.

*latex* specifies the name of a L<sup>A</sup>T<sub>E</sub>X macro to render text for L<sup>A</sup>T<sub>E</sub>X output. Example L<sup>A</sup>T<sub>E</sub>X macro names are textbf, textsf, textrm, and texttt. Custom L<sup>A</sup>T<sub>E</sub>X macros are also allowed. If *text* is to be rendered in a cell, title, or note, then *latex* is translated to the following when you export to L<sup>A</sup>T<sub>E</sub>X:

```
\latex {text}
```

*sspec* is

```
[background(bcolor) foreground(fgcolor) pattern(fpattern)]
```

*bcolor* specifies the background color.

*fgcolor* specifies the foreground color.

*fpattern* specifies the fill pattern. A complete list of fill patterns is shown in the [Appendix](#).

*bcolor*, *fgcolor*, and *color* may be one of the colors listed in the [Appendix](#); a valid RGB value in the form ### # # #, for example, 171 248 103; or a valid RRGGBB hex value in the form #####, for example, ABF867.

## Options

Main

*totals* (*totals*) and *nototals* control which totals are to be displayed in the table. By default, all totals are reported.

*totals* (*totals*) specifies that totals be displayed only for the variables or interactions specified. *totals* can contain *rowvars*, *colvars*, *tabvars*, and interactions between any of these variables. Interactions can be specified by using the # operator.

*nototals* prevents table from displaying any totals.

## Statistics

`statistic(stat[, statopts])` specifies the statistic to be displayed. `statistic()` may be repeated to request multiple statistics.

Available statistics are

| <i>stat</i>                | Definition                                       |
|----------------------------|--------------------------------------------------|
| <code>frequency</code>     | frequency                                        |
| <code>sumw</code>          | sum of weights                                   |
| <code>proportion</code>    | proportion                                       |
| <code>percent</code>       | percentage                                       |
| <code>rawproportion</code> | proportion ignoring optionally specified weights |
| <code>rawpercent</code>    | percentage ignoring optionally specified weights |

The following options may be specified in combination with statistics `proportion`, `percent`, `rawproportion`, and `rawpercent`:

| <i>statopts</i>               | Definition                                                            |
|-------------------------------|-----------------------------------------------------------------------|
| <code>across(cellspec)</code> | percentages or proportions across levels of variables or interactions |
| <code>total</code>            | compute overall percentages or proportions                            |

*cellspec* may contain *rowvars*, *colvars*, *tabvars*, or an interaction between any of these variables. Interactions can be specified by using the `#` operator.

## Formats

`nformat(%fmt [results][, basestyle])` changes the numeric format, such as the number of decimal places, for specified results. If *results* are not specified, the numeric format is changed for all results.

*results* may be any statistic named in option `statistic()` (that is, any *stat*).

This option is repeatable, and when multiple formats apply to one result, the rightmost specification is applied.

This option does not affect the format of numeric layout variables (*rowspec*, *colspec*, and *tabspec*). The default format of these variables is taken from the dataset.

*basestyle* indicates that the format be applied to results that do not already have their own format instead of overriding the format for all results.

`sformat(%fmt [results])` changes the string format for specified results. You can, for instance, add symbols or text to the values reported in the table by modifying the string format.

*%fmt* may contain a mix of text and `%s`. Here `%s` refers to the numeric value that is formatted as specified using `nformat()`. The text will be placed around the numeric values in your table as it is placed around `%s` in this option. For instance, to place parentheses around the percent statistics, you can specify `sformat("(%s) percent)`.

*results* may be any statistic named in option `statistic()` (that is, any *stat*).

Two text characters must be specified using a special character sequence if you want them to be displayed in your table. To include `%`, type `%%`. To include `\`, type `\\`. For instance, to place a percent sign following percent statistics, you can specify `sformat("%s%%" percent)`.

This option is repeatable, and when multiple formats apply to one result, the rightmost specification is applied.

Title

`title(string)` adds the text *string* as a title to the table.

`titlestyles(text_styles)` changes the style for the table title. *text\_styles* are the following:

`font([fontfamily] [, size(# [unit]) color(color) variant(variant) [no]bold [no]italic [no]strikeout [no]underline])` specifies the font style. These font style properties are applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files, unless otherwise specified.

*fontfamily* specifies a valid font family. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`size(# [unit])` specifies the font size as a number optionally followed by units. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`color(color)` specifies the text color.

`variant(variant)` specifies the font variant and capitalization.

`bold` and `nobold` specify the font weight. `bold` changes the font weight to bold; `nobold` changes the font weight back to normal.

`italic` and `noitalic` specify the font style. `italic` changes the font style to italic; `noitalic` changes the font style back to normal.

`strikeout` and `nostrikeout` specify whether to add a strikeout mark to the title. `strikeout` adds a strikeout mark to the title; `nostrikeout` changes the title back to normal.

`underline` and `nounderline` specify whether to underline the table title. `underline` adds a single line under the title; `nounderline` removes the underline.

Only one of `strikeout` or `underline` is allowed when publishing to HTML files.

`smcl(smcl)` specifies how to render the table title for SMCL output. This style property is applicable only when publishing items from a collection to a SMCL file.

`latex(latex)` specifies how to render the table title for  $\LaTeX$  output. This style property is applicable only when publishing items from a collection to a  $\LaTeX$  file.

`shading(sspec)` sets the background color, foreground color, and fill pattern. The background color is applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files. The foreground color and fill pattern are applicable when exporting the table to Microsoft Word and Microsoft Excel.

`note(string)` adds the text *string* as a note to the table. `note()` may be specified multiple times to add multiple notes. Each note is placed on a new line.

`notestyles(text_styles)` changes the style for the table notes. *text\_styles* are the following:

`font([fontfamily] [, size(# [unit]) color(color) variant(variant) [no]bold [no]italic [no]strikeout [no]underline])` specifies the font style. These font style properties are applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files, unless otherwise specified.

*fontfamily* specifies a valid font family. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`size(# [unit])` specifies the font size as a number optionally followed by units. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`color(color)` specifies the text color.

`variant(variant)` specifies the font variant and capitalization.

`bold` and `nobold` specify the font weight. `bold` changes the font weight to bold; `nobold` changes the font weight back to normal.

`italic` and `noitalic` specify the font style. `italic` changes the font style to italic; `noitalic` changes the font style back to normal.

`strikeout` and `nostrikeout` specify whether to add a strikeout mark to the notes. `strikeout` adds a strikeout mark to the note; `nostrikeout` changes the note back to normal.

`underline` and `nounderline` specify whether to underline the table notes. `underline` adds a single line under the notes; `nounderline` removes the underline.

Only one of `strikeout` or `underline` is allowed when publishing to HTML files.

`smcl(smcl)` specifies how to render the table notes for SMCL output. This style property is applicable only when publishing items from a collection to a SMCL file.

`latex(latex)` specifies how to render the table notes for  $\LaTeX$  output. This style property is applicable only when publishing items from a collection to a  $\LaTeX$  file.

`shading(sspec)` sets the background color, foreground color, and fill pattern. The background color is applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files. The foreground color and fill pattern are applicable when exporting the table to Microsoft Word and Microsoft Excel.

Export

`export(filename.suffix[ , export_opts ])` exports the table to the specified file. *export\_opts* are the following:

`as(fileformat)` specifies the file format to which the table is to be exported. This option is rarely specified because, by default, `table` determines the format from the suffix of the file being created.

`replace` permits `table` to overwrite an existing file.

`noisily` specifies that `table` show the commands used to export the table to Microsoft Word, Microsoft Excel, and PDF files. The `putdocx`, `putexcel`, or `putpdf` command used to export the table will be displayed.

`dofile(filename[ , replace ])` specifies that `table` save to *filename* the commands used to export the table to Microsoft Word, Microsoft Excel, and PDF files.

If *filename* already exists, it can be overwritten by specifying `replace`. If *filename* is specified without an extension, `.do` is assumed.

`append` specifies that `table` append the table to an existing file.

This option is applicable when you export the table to an HTML, a  $\LaTeX$ , a SMCL, a `txt`, or a Markdown file. When you export to HTML and  $\LaTeX$  files, the `append` option implies the `tableonly` option. Furthermore, when you export to HTML files, if the target CSS file already exists, `table` will also append to it.

`tableonly` specifies that only the table be exported to the specified HTML or  $\LaTeX$  document. By default, `table` produces complete HTML and  $\LaTeX$  documents.

When you export to an HTML file, if the `cssfile()` option is not specified, a CSS filename is constructed from *filename*, with the extension replaced with `.css`.

`cssfile(cssfile)` specifies that `table` define the styles in *cssfile* instead of *filename* when you export to HTML.

`prefix(prefix)` specifies that `table` use *prefix* to identify style classes when you export to HTML.

`sheet(sheetname [ , replace ])` saves to the worksheet named *sheetname*. For more information about this option, see [\[RPT\] putexcel](#).

`cell(cell)` specifies an Excel upper-left cell as the starting position to publish the table. The default is `cell(A1)`.

`modify` permits `putexcel` set to modify an Excel file. For more information about this option, see [\[RPT\] putexcel](#).

`noopen` prevents `putexcel` from opening the Excel file in memory for modification. It does not appear in the dialog box. For more information about this option, see [\[RPT\] putexcel](#).

## Options

`listwise` handles missing values through listwise deletion, meaning that the entire observation is omitted from the sample if any variable specified in a `statistic()` option is missing for that observation. By default, `table` will omit an observation only if all variables specified in all `statistic()` options are missing for that observation.

`missing` specifies that numeric missing values of any *rowvars*, *colvars*, or *tabvars* be treated as valid categories. By default, observations with a numeric missing value in *rowvars*, *colvars*, or *tabvars* are omitted.

`showcounts` specifies that `table` report the sample size for each variable specified in option `statistic()`.

`zerocounts` specifies that `table` report a 0 in empty cells for the frequency statistic.

`name(cname)` specifies that a collection named *cname* be associated with the collected statistics and results. The default is `name(Table)`.

`append` specifies that `table` append its collection information into the collection named in `name()`.

`replace` permits `table` to overwrite an existing collection. This option is implied for `name(Table)` when `append` is not specified.

`label(filename)` specifies the *filename* containing the collection labels to use for your table. Labels in *filename* will be loaded for the table, and any labels not specified in *filename* will be taken from the labels defined in `c(collect_label)`. The default is to use only the collection labels set in `c(collect_label)`; see [TABLES] [set collect\\_label](#).

`style(filename [ , override ])` specifies the *filename* containing the collection styles to use for your table. The default collection styles will be discarded, and only the collection styles in *filename* will be applied.

If you prefer the default collection styles but also want to apply any styles in *filename*, specify `override`. If there are conflicts between the default collection styles and those in *filename*, the ones in *filename* will take precedence.

The default is to use only the collection styles set in `c(table_style)`; see [TABLES] [set table\\_style](#).

The following option is available with `table` but is not shown in the dialog box:

`markvar(newvar)` generates an indicator variable that identifies the observations used in the tabulation.

## Remarks and examples

Remarks are presented under the following headings:

- [Introduction](#)
- [Tables with columns defined by multiple variables](#)
- [Appending tables](#)
- [Multiple tables with specified totals](#)

## Introduction

The `table` command allows you to create complex tables beyond one- and two-way tabulations. In multiway tabulations, you can display frequencies across levels of two or more variables. You can have levels of one variable nested within levels of another variable in columns, in rows, or in both dimensions. And with a single command, you can create separate tables for levels of one or more variables or for different results.

## Tables with columns defined by multiple variables

We use data from the Second National Health and Nutrition Examination Survey (NHANES II) (McDowell et al. 1981). The data contain some demographic information, such as the age, sex, and race of participants. The dataset also contains some measures of health, including whether the individual has high blood pressure (`highbp`).

Before we create any tables, we will modify a few labels in our dataset so that they will appear as we wish in our tables.

```
. use https://www.stata-press.com/data/r19/nhanes21
(Second National Health and Nutrition Examination Survey)
. label define yesno 0 "No" 1 "Yes"
. label values highbp diabetes heartatk yesno
. label variable diabetes "Diabetes"
```

Suppose we want to examine how many males and females in each age group have high blood pressure. Let's place the levels of age group on the rows and the levels of high blood pressure and sex on the columns.

```
. table (agegrp) (sex highbp), nototals
```

| Age group | Sex                       |                            |                           |                            |
|-----------|---------------------------|----------------------------|---------------------------|----------------------------|
|           | Male                      |                            | Female                    |                            |
|           | High blood pressure<br>No | High blood pressure<br>Yes | High blood pressure<br>No | High blood pressure<br>Yes |
| 20–29     | 825                       | 291                        | 1,103                     | 101                        |
| 30–39     | 480                       | 290                        | 687                       | 165                        |
| 40–49     | 336                       | 274                        | 434                       | 228                        |
| 50–59     | 255                       | 347                        | 335                       | 354                        |
| 60–69     | 568                       | 801                        | 625                       | 866                        |
| 70+       | 147                       | 301                        | 180                       | 358                        |

By default, `table` includes the totals for each category; we added the `nototals` option to suppress them here.

To better compare the occurrence of high blood pressure, let's now compute percentages of `highbp`. Below, we create the same table, but within each sex and age group combination, we report the percentage of individuals with and without high blood pressure.

```
. table (agegrp) (sex highbp), nototals statistic(percent, across(highbp))
```

|           | Sex                       |                            |                           |                            |
|-----------|---------------------------|----------------------------|---------------------------|----------------------------|
|           | Male                      |                            | Female                    |                            |
|           | High blood pressure<br>No | High blood pressure<br>Yes | High blood pressure<br>No | High blood pressure<br>Yes |
| Age group |                           |                            |                           |                            |
| 20-29     | 73.92                     | 26.08                      | 91.61                     | 8.39                       |
| 30-39     | 62.34                     | 37.66                      | 80.63                     | 19.37                      |
| 40-49     | 55.08                     | 44.92                      | 65.56                     | 34.44                      |
| 50-59     | 42.36                     | 57.64                      | 48.62                     | 51.38                      |
| 60-69     | 41.49                     | 58.51                      | 41.92                     | 58.08                      |
| 70+       | 32.81                     | 67.19                      | 33.46                     | 66.54                      |

Here we see that 26.08% of males in their 20s have high blood pressure and only 8.39% of females in their 20s have high blood pressure.

If we had simply typed `statistic(percent)`, then we would see the percentage of observations in each cell. With the suboption `across()`, we can compute the percentage within each sex and age group combination (across levels of `highbp`).

Next, let's request that percentages be calculated across the categories of high blood pressure and sex. We can alternatively think of this as being percentages within age group.

```
. table (agegrp) (sex highbp), nototals statistic(percent, across(highbp#sex))
```

|           | Sex                       |                            |                           |                            |
|-----------|---------------------------|----------------------------|---------------------------|----------------------------|
|           | Male                      |                            | Female                    |                            |
|           | High blood pressure<br>No | High blood pressure<br>Yes | High blood pressure<br>No | High blood pressure<br>Yes |
| Age group |                           |                            |                           |                            |
| 20-29     | 35.56                     | 12.54                      | 47.54                     | 4.35                       |
| 30-39     | 29.59                     | 17.88                      | 42.36                     | 10.17                      |
| 40-49     | 26.42                     | 21.54                      | 34.12                     | 17.92                      |
| 50-59     | 19.75                     | 26.88                      | 25.95                     | 27.42                      |
| 60-69     | 19.86                     | 28.01                      | 21.85                     | 30.28                      |
| 70+       | 14.91                     | 30.53                      | 18.26                     | 36.31                      |

Here we see that 12.54% of individuals in their 20s are males with high blood pressure and 4.35% are females with high blood pressure.

We can reverse the order of our column variables so that we have the levels of `sex` nested within levels of high blood pressure. We will again request percentages across the categories of `highbp`.

```
. table (agegrp) (highbp sex), nototals statistic(percent, across(highbp))
```

|           | High blood pressure |        |       |        |
|-----------|---------------------|--------|-------|--------|
|           | No                  |        | Yes   |        |
|           | Sex                 |        | Sex   |        |
|           | Male                | Female | Male  | Female |
| Age group |                     |        |       |        |
| 20-29     | 73.92               | 91.61  | 26.08 | 8.39   |
| 30-39     | 62.34               | 80.63  | 37.66 | 19.37  |
| 40-49     | 55.08               | 65.56  | 44.92 | 34.44  |
| 50-59     | 42.36               | 48.62  | 57.64 | 51.38  |
| 60-69     | 41.49               | 41.92  | 58.51 | 58.08  |
| 70+       | 32.81               | 33.46  | 67.19 | 66.54  |

The last two columns represent the percentage of males with high blood pressure in the age group and the percentage of females with high blood pressure in the age group. We can clearly see that, across all age groups, the percent of males who have high blood pressure is greater than the percentage of females with high blood pressure.

Perhaps we want a table that includes only the two columns on the right. To create a table with the percentages of individuals with high blood pressure, we can take advantage of a unique feature of `table`—its results are automatically stored in a “collection” and can be easily customized. Specifically, when we create a table using the `table` command, the results are stored in a collection named `Table`, and these results replace the results from any previous `table` command. The `collect` suite of commands can be used to change the layout, style, and formatting of tables created from results in collection; see [\[TABLES\] Intro](#) to learn about collections of results and creating customized tables. For our table, we will use the `collect layout` command, which specifies how items from a collection should be arranged. Below, we arrange the percentages in a table with rows defined by the categories of `agegrp` and columns defined by the categories of `sex` and category 1 of `highbp`.

```
. collect layout (agegrp) (highbp[1]#sex)
```

```
Collection: Table
 Rows: agegrp
 Columns: highbp[1]#sex
 Table 1: 7 x 2
```

|           | High blood pressure |          |
|-----------|---------------------|----------|
|           | Yes                 |          |
|           | Sex                 |          |
|           | Male                | Female   |
| Age group |                     |          |
| 20-29     | 26.07527            | 8.388704 |
| 30-39     | 37.66234            | 19.3662  |
| 40-49     | 44.91803            | 34.44109 |
| 50-59     | 57.6412             | 51.37881 |
| 60-69     | 58.50986            | 58.08182 |
| 70+       | 67.1875             | 66.54275 |

## Appending tables

When we specify multiple variables for the row or column specification, the levels of one variable are nested within the levels of another. When you simply wish to join rows or columns from multiple tables, this can be easily done with the `append` option.

For example, we first create a table with the percentage of males and females in each age group with diabetes.

```
. table (sex agegrp) (diabetes), nototals statistic(percent, across(diabetes))
```

|        |           | Diabetes |       |
|--------|-----------|----------|-------|
|        |           | No       | Yes   |
| Sex    |           |          |       |
| Male   |           |          |       |
|        | Age group |          |       |
|        | 20-29     | 99.64    | 0.36  |
|        | 30-39     | 99.61    | 0.39  |
|        | 40-49     | 97.38    | 2.62  |
|        | 50-59     | 94.68    | 5.32  |
|        | 60-69     | 91.96    | 8.04  |
|        | 70+       | 88.39    | 11.61 |
| Female |           |          |       |
|        | Age group |          |       |
|        | 20-29     | 99.09    | 0.91  |
|        | 30-39     | 97.88    | 2.12  |
|        | 40-49     | 96.07    | 3.93  |
|        | 50-59     | 94.19    | 5.81  |
|        | 60-69     | 91.42    | 8.58  |
|        | 70+       | 89.03    | 10.97 |

We want to include the same information for `highbp` and `heartatk` in the same table, which indicates whether someone has had a heart attack. To do this, we will run the `table` command three times, once specifying each of these variables as the column variable. We will use the `name(table1)` option to specify that the results be stored in a collection named `table1`. To the second and third `table` commands, we will add the `append` option so that all the results are stored in the same collection rather than overriding the results from the previous command.

```
. quietly: table (sex agegrp) (diabetes), nototals
> statistic(percent, across(diabetes)) name(table1)
. quietly: table (sex agegrp) (highbp), nototals
> statistic(percent, across(highbp)) name(table1) append
. quietly: table (sex agegrp) (heartatk), nototals
> statistic(percent, across(heartatk)) name(table1) append
```

With the results from all our table commands stored in one collection, we can again take advantage of the `collect` layout command to arrange results into a table. We request that `sex` and `agegroup` define the rows. By including the `#` between variable names, we specify that we want the levels of these variables to be interacted to form the rows. We request that levels of `diabetes`, `highbp`, and `heartatk` form the columns. Because we did not include `#` between the variable names, their levels will not be interacted. Instead, they will be listed one after the other.

```
. collect layout (sex#agegrp) (diabetes highbp heartatk)
Collection: table1
 Rows: sex#agegrp
 Columns: diabetes highbp heartatk
Table 1: 17 x 6
```

|           | Diabetes |       | High blood pressure |       | Prior heart attack |       |
|-----------|----------|-------|---------------------|-------|--------------------|-------|
|           | No       | Yes   | No                  | Yes   | No                 | Yes   |
| Sex       |          |       |                     |       |                    |       |
| Male      |          |       |                     |       |                    |       |
| Age group |          |       |                     |       |                    |       |
| 20-29     | 99.64    | 0.36  | 73.92               | 26.08 | 100.00             |       |
| 30-39     | 99.61    | 0.39  | 62.34               | 37.66 | 99.74              | 0.26  |
| 40-49     | 97.38    | 2.62  | 55.08               | 44.92 | 98.03              | 1.97  |
| 50-59     | 94.68    | 5.32  | 42.36               | 57.64 | 92.36              | 7.64  |
| 60-69     | 91.96    | 8.04  | 41.49               | 58.51 | 86.56              | 13.44 |
| 70+       | 88.39    | 11.61 | 32.81               | 67.19 | 83.48              | 16.52 |
| Female    |          |       |                     |       |                    |       |
| Age group |          |       |                     |       |                    |       |
| 20-29     | 99.09    | 0.91  | 91.61               | 8.39  | 99.92              | 0.08  |
| 30-39     | 97.88    | 2.12  | 80.63               | 19.37 | 99.76              | 0.24  |
| 40-49     | 96.07    | 3.93  | 65.56               | 34.44 | 98.79              | 1.21  |
| 50-59     | 94.19    | 5.81  | 48.62               | 51.38 | 96.66              | 3.34  |
| 60-69     | 91.42    | 8.58  | 41.92               | 58.08 | 94.57              | 5.43  |
| 70+       | 89.03    | 10.97 | 33.46               | 66.54 | 92.01              | 7.99  |

## Multiple tables with specified totals

There may be times when instead of creating one large table for multiple variables, you would prefer to create separate tables for each level of one or more variables or for different statistics. For example, we previously had levels of age group nested within categories of sex. Now, we would like to create tables that show how males and females in each age group rate their own health. The variable `hlthstat` records how individuals self-rate their health. Let's create a table that shows what percent of males in each age group selected each of the health categories and a separate table to list the percent of females.

```
. table (agegrp) (hlthstat) (sex), statistic(percent, across(hlthstat))
```

Sex = Male

|           | Health status |           |       |       |       | Total  |
|-----------|---------------|-----------|-------|-------|-------|--------|
|           | Excellent     | Very good | Good  | Fair  | Poor  |        |
| Age group |               |           |       |       |       |        |
| 20-29     | 39.61         | 32.71     | 20.97 | 5.82  | 0.90  | 100.00 |
| 30-39     | 36.88         | 29.61     | 26.10 | 5.84  | 1.56  | 100.00 |
| 40-49     | 28.03         | 25.90     | 29.51 | 12.30 | 4.26  | 100.00 |
| 50-59     | 17.80         | 21.30     | 35.11 | 15.97 | 9.82  | 100.00 |
| 60-69     | 13.33         | 19.56     | 28.94 | 23.00 | 15.16 | 100.00 |
| 70+       | 14.77         | 14.99     | 26.62 | 28.41 | 15.21 | 100.00 |
| Total     | 25.50         | 24.71     | 27.30 | 14.71 | 7.78  | 100.00 |

Sex = Female

|           | Health status |           |       |       |       | Total  |
|-----------|---------------|-----------|-------|-------|-------|--------|
|           | Excellent     | Very good | Good  | Fair  | Poor  |        |
| Age group |               |           |       |       |       |        |
| 20-29     | 32.39         | 34.55     | 24.92 | 6.98  | 1.16  | 100.00 |
| 30-39     | 29.76         | 30.71     | 28.12 | 9.29  | 2.12  | 100.00 |
| 40-49     | 26.71         | 23.07     | 29.29 | 15.63 | 5.31  | 100.00 |
| 50-59     | 17.15         | 20.93     | 33.43 | 20.20 | 8.28  | 100.00 |
| 60-69     | 10.76         | 20.31     | 33.22 | 25.49 | 10.22 | 100.00 |
| 70+       | 10.78         | 19.14     | 26.39 | 30.48 | 13.20 | 100.00 |
| Total     | 21.29         | 25.40     | 29.45 | 17.47 | 6.40  | 100.00 |

Sex = Total

|           | Health status |           |       |       |       | Total  |
|-----------|---------------|-----------|-------|-------|-------|--------|
|           | Excellent     | Very good | Good  | Fair  | Poor  |        |
| Age group |               |           |       |       |       |        |
| 20-29     | 35.86         | 33.66     | 23.02 | 6.42  | 1.03  | 100.00 |
| 30-39     | 33.15         | 30.19     | 27.16 | 7.65  | 1.85  | 100.00 |
| 40-49     | 27.34         | 24.43     | 29.39 | 14.03 | 4.81  | 100.00 |
| 50-59     | 17.46         | 21.10     | 34.21 | 18.23 | 9.00  | 100.00 |
| 60-69     | 11.99         | 19.95     | 31.17 | 24.30 | 12.59 | 100.00 |
| 70+       | 12.59         | 17.26     | 26.50 | 29.54 | 14.11 | 100.00 |
| Total     | 23.29         | 25.07     | 28.43 | 16.16 | 7.05  | 100.00 |

We see that 14.77% of males in their 70s and beyond rated their health as excellent and 10.78% of females in their 70s and beyond rated their health as excellent.

Note that we actually created three tables, one for males, one for females, and one for everybody. We are mainly interested in the first two tables and would like to drop the third table. But if we use the `nototals` option, we will not get the row totals we see above. Instead, we can specify which totals we do want with the `totals()` option:

```
. table (agegrp) (hlthstat) (sex), statistic(percent, across(hlthstat))
> totals(sex#agegrp sex#hlthstat sex)
```

Sex = Male

|           | Health status |           |       |       |       | Total  |
|-----------|---------------|-----------|-------|-------|-------|--------|
|           | Excellent     | Very good | Good  | Fair  | Poor  |        |
| Age group |               |           |       |       |       |        |
| 20-29     | 39.61         | 32.71     | 20.97 | 5.82  | 0.90  | 100.00 |
| 30-39     | 36.88         | 29.61     | 26.10 | 5.84  | 1.56  | 100.00 |
| 40-49     | 28.03         | 25.90     | 29.51 | 12.30 | 4.26  | 100.00 |
| 50-59     | 17.80         | 21.30     | 35.11 | 15.97 | 9.82  | 100.00 |
| 60-69     | 13.33         | 19.56     | 28.94 | 23.00 | 15.16 | 100.00 |
| 70+       | 14.77         | 14.99     | 26.62 | 28.41 | 15.21 | 100.00 |
| Total     | 25.50         | 24.71     | 27.30 | 14.71 | 7.78  | 100.00 |

Sex = Female

|           | Health status |           |       |       |       | Total  |
|-----------|---------------|-----------|-------|-------|-------|--------|
|           | Excellent     | Very good | Good  | Fair  | Poor  |        |
| Age group |               |           |       |       |       |        |
| 20-29     | 32.39         | 34.55     | 24.92 | 6.98  | 1.16  | 100.00 |
| 30-39     | 29.76         | 30.71     | 28.12 | 9.29  | 2.12  | 100.00 |
| 40-49     | 26.71         | 23.07     | 29.29 | 15.63 | 5.31  | 100.00 |
| 50-59     | 17.15         | 20.93     | 33.43 | 20.20 | 8.28  | 100.00 |
| 60-69     | 10.76         | 20.31     | 33.22 | 25.49 | 10.22 | 100.00 |
| 70+       | 10.78         | 19.14     | 26.39 | 30.48 | 13.20 | 100.00 |
| Total     | 21.29         | 25.40     | 29.45 | 17.47 | 6.40  | 100.00 |

`sex#agegrp` gives us the row totals, the total for each age group within each category of sex. `sex#hlthstat` provides us with the column totals, one of which would be the total percent of females that rated their health as excellent. Finally, `sex` gives us the total in the rightmost cell in the bottom of each table.

In all our previous tables, we used variables to define the rows and columns, but you can also use results in the row, column, and table identifiers. For example, suppose that in addition to reporting percentages for males and females in each age group, we wanted to report frequencies. We can create separate tables for the percentages and frequencies by specifying `result` in the third set of parentheses. To include the totals for each age group and for each category of sex, we specify the interaction of `sex` and `agegrp` in the `totals()` option:

```
. table (sex agegrp) (hlthstat) (result), statistic(percent, across(hlthstat))
> statistic(frequency) totals(sex#agegrp)
```

Percent

|     |           | Health status |           |       |       |       |        |
|-----|-----------|---------------|-----------|-------|-------|-------|--------|
|     |           | Excellent     | Very good | Good  | Fair  | Poor  | Total  |
| Sex |           |               |           |       |       |       |        |
|     | Male      |               |           |       |       |       |        |
|     | Age group |               |           |       |       |       |        |
|     | 20-29     | 39.61         | 32.71     | 20.97 | 5.82  | 0.90  | 100.00 |
|     | 30-39     | 36.88         | 29.61     | 26.10 | 5.84  | 1.56  | 100.00 |
|     | 40-49     | 28.03         | 25.90     | 29.51 | 12.30 | 4.26  | 100.00 |
|     | 50-59     | 17.80         | 21.30     | 35.11 | 15.97 | 9.82  | 100.00 |
|     | 60-69     | 13.33         | 19.56     | 28.94 | 23.00 | 15.16 | 100.00 |
|     | 70+       | 14.77         | 14.99     | 26.62 | 28.41 | 15.21 | 100.00 |
|     | Female    |               |           |       |       |       |        |
|     | Age group |               |           |       |       |       |        |
|     | 20-29     | 32.39         | 34.55     | 24.92 | 6.98  | 1.16  | 100.00 |
|     | 30-39     | 29.76         | 30.71     | 28.12 | 9.29  | 2.12  | 100.00 |
|     | 40-49     | 26.71         | 23.07     | 29.29 | 15.63 | 5.31  | 100.00 |
|     | 50-59     | 17.15         | 20.93     | 33.43 | 20.20 | 8.28  | 100.00 |
|     | 60-69     | 10.76         | 20.31     | 33.22 | 25.49 | 10.22 | 100.00 |
|     | 70+       | 10.78         | 19.14     | 26.39 | 30.48 | 13.20 | 100.00 |

Frequency

|     |           | Health status |           |      |      |      |       |
|-----|-----------|---------------|-----------|------|------|------|-------|
|     |           | Excellent     | Very good | Good | Fair | Poor | Total |
| Sex |           |               |           |      |      |      |       |
|     | Male      |               |           |      |      |      |       |
|     | Age group |               |           |      |      |      |       |
|     | 20-29     | 442           | 365       | 234  | 65   | 10   | 1,116 |
|     | 30-39     | 284           | 228       | 201  | 45   | 12   | 770   |
|     | 40-49     | 171           | 158       | 180  | 75   | 26   | 610   |
|     | 50-59     | 107           | 128       | 211  | 96   | 59   | 601   |
|     | 60-69     | 182           | 267       | 395  | 314  | 207  | 1,365 |
|     | 70+       | 66            | 67        | 119  | 127  | 68   | 447   |
|     | Female    |               |           |      |      |      |       |
|     | Age group |               |           |      |      |      |       |
|     | 20-29     | 390           | 416       | 300  | 84   | 14   | 1,204 |
|     | 30-39     | 253           | 261       | 239  | 79   | 18   | 850   |
|     | 40-49     | 176           | 152       | 193  | 103  | 35   | 659   |
|     | 50-59     | 118           | 144       | 230  | 139  | 57   | 688   |
|     | 60-69     | 160           | 302       | 494  | 379  | 152  | 1,487 |
|     | 70+       | 58            | 103       | 142  | 164  | 71   | 538   |

In the table of frequencies, we see that of the 1,204 females in their 20s, only 390 rated their health as excellent. In the table of percentages, we see that this is equal to 32.39% of females in their 20s.

If we wish to include one of these tables, for example, in a paper or on a webpage, we can easily export it in L<sup>A</sup>T<sub>E</sub>X, Word, Excel, HTML, and a variety of other formats by using the `export()` option.

## Stored results

`table` stores the following in `r()`:

Scalars

`r(N)` number of observations

## Reference

McDowell, A., A. Engel, J. T. Massey, and K. Maurer. 1981. "Plan and operation of the Second National Health and Nutrition Examination Survey, 1976–1980". In *Vital and Health Statistics*, ser. 1, no. 15. Hyattsville, MD: National Center for Health Statistics.

## Also see

[R] [table](#) — Table of frequencies, summaries, and command results

[R] [table intro](#) — Introduction to tables of frequencies, summaries, and command results

[R] [table oneway](#) — One-way tabulation

[R] [table twoway](#) — Two-way tabulation

[TABLES] [Intro](#) — Introduction

|                             |                                      |                                |                            |
|-----------------------------|--------------------------------------|--------------------------------|----------------------------|
| <a href="#">Description</a> | <a href="#">Quick start</a>          | <a href="#">Menu</a>           | <a href="#">Syntax</a>     |
| <a href="#">Options</a>     | <a href="#">Remarks and examples</a> | <a href="#">Stored results</a> | <a href="#">References</a> |
| <a href="#">Also see</a>    |                                      |                                |                            |

## Description

In this entry, we discuss how to use `table` to create a table of summary statistics.

## Quick start

Table with the mean of `v1`, `v2`, and `v3` for each category of `a1` and `a2`; rows are defined by categories of `a1` and variables `v1`, `v2`, and `v3`

```
table a1 a2, stat(mean v1 v2 v3)
```

Same as above, but also report standard deviations and suppress the totals; rows are defined by the results for each variable within each category of `a1`

```
table (a1 var result) (a2), stat(mean v1 v2 v3) ///
stat(sd v1 v2 v3) nototals
```

Table with number of observations in each category of `a2` and `a3`, for each level of `a1`

```
table a1, stat(fvfrequency a2 a3)
```

Same as above, and report percentage of observations in each category

```
table a1, stat(fvfrequency a2 a3) ///
stat(fvpercent a2 a3)
```

Same as above, and report the percentages with a percent sign, using two decimal places, and enclose them in parentheses

```
table (a1) (var result), stat(fvfrequency a2 a3) ///
stat(fvpercent a2 a3) ///
nformat(%5.2f fvpercent) sformat("%s%%" fvpercent)
```

## Menu

Statistics > Summaries, tables, and tests > Tables of frequencies, summaries, and command results

## Syntax

### Basic table of summary statistics

```
table [rowvar] [colvar] [if] [in] [weight], statistic(statspec)
 [statistic(statspec) [...]] [options]
```

### Customized table of summary statistics

```
table [(rowspec)] [(colspec)] [(tabspec)] [if] [in] [weight], statistic(statspec)
 [statistic(statspec) [...]] [options]
```

*rowspec*, *colspec*, and *tabspec* may be empty or may include variable names or any of the following keywords:

| <i>keyword</i> | Description                                    |
|----------------|------------------------------------------------|
| result         | requested statistics                           |
| var            | variables from <code>statistic()</code> option |
| across         | index <code>across()</code> specifications     |

| <i>options</i>                                       | Description                                                                                                                                     |
|------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Main</b>                                          |                                                                                                                                                 |
| <code>totals(totals)</code>                          | report only the specified totals                                                                                                                |
| <code>nototals</code>                                | suppress the marginal totals                                                                                                                    |
| <b>Statistics</b>                                    |                                                                                                                                                 |
| <code>statistic(statspec)</code>                     | statistic to be reported; default is <code>statistic(frequency)</code> when no weights are specified and <code>statistic(sumw)</code> otherwise |
| <b>Formats</b>                                       |                                                                                                                                                 |
| <code>nformat(%fmt [results] [, basestyle])</code>   | specify numeric format                                                                                                                          |
| <code>sformat(sfmt [results])</code>                 | specify string format                                                                                                                           |
| <b>Title</b>                                         |                                                                                                                                                 |
| <code>title(string)</code>                           | add table title                                                                                                                                 |
| <code>titlestyles(text_styles)</code>                | change table title styles                                                                                                                       |
| <b>Notes</b>                                         |                                                                                                                                                 |
| <code>note(string)</code>                            | add table note                                                                                                                                  |
| <code>notestyles(text_styles)</code>                 | change table note styles                                                                                                                        |
| <b>Export</b>                                        |                                                                                                                                                 |
| <code>export(filename.suffix [, export_opts])</code> | export table                                                                                                                                    |
| <b>Options</b>                                       |                                                                                                                                                 |
| <code>listwise</code>                                | use listwise deletion to handle missing values                                                                                                  |
| <code>missing</code>                                 | treat numeric missing values of variables in <code>rowspec</code> , <code>colspec</code> , and <code>tabspec</code> like other values           |
| <code>showcounts</code>                              | show sample size for all variables in <code>statistic()</code> option                                                                           |
| <code>zerocounts</code>                              | report 0 for empty cell counts                                                                                                                  |
| <code>name(cname)</code>                             | collect results into a collection named <i>cname</i>                                                                                            |
| <code>append</code>                                  | append results to an existing collection                                                                                                        |
| <code>replace</code>                                 | replace results of an existing collection                                                                                                       |
| <code>label(filename)</code>                         | specify the collection labels                                                                                                                   |
| <code>style(filename [, override])</code>            | specify the collection style                                                                                                                    |
| <code>markvar(newvar)</code>                         | create <i>newvar</i> that identifies observations used in the tabulation                                                                        |

`fweights`, `aweights`, `iweights`, and `pweights` are allowed; see [U] 11.1.6 **weight**.

`strL` variables are not allowed; see [U] 12.4.8 **strL**.

`markvar()` does not appear in the dialog box.

| <i>text_styles</i>                           | Description                                              |
|----------------------------------------------|----------------------------------------------------------|
| <code>font([fontfamily][, font_opts])</code> | specify font style                                       |
| <code>smcl(smcl)</code>                      | specify formatting for SMCL files                        |
| <code>latex(latex)</code>                    | specify L <sup>A</sup> T <sub>E</sub> X macro            |
| <code>shading(sspec)</code>                  | set background color, foreground color, and fill pattern |

| <i>font_opts</i>              | Description                              |
|-------------------------------|------------------------------------------|
| <code>size(# [unit])</code>   | specify font size                        |
| <code>color(color)</code>     | specify font color                       |
| <code>variant(variant)</code> | specify font variant and capitalization  |
| <code>[no]bold</code>         | specify whether to format text as bold   |
| <code>[no]italic</code>       | specify whether to format text as italic |
| <code>[no]strikeout</code>    | specify whether to strike out text       |
| <code>[no]underline</code>    | specify whether to underline text        |

| <i>suffix</i>         | <i>fileformat</i>         | Output format                      |
|-----------------------|---------------------------|------------------------------------|
| <code>docx</code>     | <code>as(docx)</code>     | Microsoft Word                     |
| <code>html</code>     | <code>as(html)</code>     | HTML 5 with CSS                    |
| <code>pdf</code>      | <code>as(pdf)</code>      | PDF                                |
| <code>xlsx</code>     | <code>as(xlsx)</code>     | Microsoft Excel 2007/2010 or newer |
| <code>xls</code>      | <code>as(xls)</code>      | Microsoft Excel 1997/2003          |
| <code>tex</code>      | <code>as(tex)</code>      | L <sup>A</sup> T <sub>E</sub> X    |
| <code>smcl</code>     | <code>as(smcl)</code>     | SMCL                               |
| <code>txt</code>      | <code>as(txt)</code>      | plain text                         |
| <code>markdown</code> | <code>as(markdown)</code> | Markdown                           |
| <code>md</code>       | <code>as(md)</code>       | Markdown                           |

| <i>export_opts</i>          | Description                                         |
|-----------------------------|-----------------------------------------------------|
| <code>as(fileformat)</code> | specify document type                               |
| <code>replace</code>        | overwrite existing file                             |
| <i>docx_options</i>         | available when exporting to .docx files             |
| <i>html_options</i>         | available when exporting to .html files             |
| <i>pdf_options</i>          | available when exporting to .pdf files              |
| <i>excel_options</i>        | available when exporting to .xls and .xlsx files    |
| <i>tex_options</i>          | available when exporting to .tex files              |
| <i>smcl_option</i>          | available when exporting to .smcl files             |
| <i>txt_option</i>           | available when exporting to .txt files              |
| <i>md_option</i>            | available when exporting to .markdown and .md files |

| <i>docx_options</i>                      | Description                                                         |
|------------------------------------------|---------------------------------------------------------------------|
| <code>noisily</code>                     | show the putdocx commands used to export to the Microsoft Word file |
| <code>dofile(filename[, replace])</code> | save the putdocx commands used for exporting to the named do-file   |

| <i>html_options</i>       | Description                                                    |
|---------------------------|----------------------------------------------------------------|
| append                    | append to an existing file                                     |
| tableonly                 | export only the table to the specified file                    |
| cssfile( <i>cssfile</i> ) | define the styles in <i>cssfile</i> instead of <i>filename</i> |
| prefix( <i>prefix</i> )   | use <i>prefix</i> to identify style classes                    |

| <i>pdf_options</i>                   | Description                                                      |
|--------------------------------------|------------------------------------------------------------------|
| noisily                              | show the putpdf commands used to export to the PDF file          |
| dofile( <i>filename</i> [, replace]) | save the putpdf commands used for exporting to the named do-file |

| <i>excel_options</i>                 | Description                                                                                             |
|--------------------------------------|---------------------------------------------------------------------------------------------------------|
| noisily                              | show the putexcel commands used to export to the Excel file                                             |
| dofile( <i>filename</i> [, replace]) | save the putexcel commands used for exporting to the named do-file                                      |
| sheet( <i>sheetname</i> [, replace]) | specify the worksheet to use; the default sheet name is Sheet1                                          |
| cell( <i>cell</i> )                  | specify the Excel upper-left cell as the starting position to export the table; the default is cell(A1) |
| modify                               | modify Excel file                                                                                       |
| noopen                               | do not open Excel file in memory                                                                        |

noopen does not appear in the dialog box.

| <i>tex_options</i> | Description                                 |
|--------------------|---------------------------------------------|
| append             | append to an existing file                  |
| tableonly          | export only the table to the specified file |

| <i>smcl_option</i> | Description                |
|--------------------|----------------------------|
| append             | append to an existing file |

| <i>txt_option</i> | Description                |
|-------------------|----------------------------|
| append            | append to an existing file |

| <i>md_option</i> | Description                |
|------------------|----------------------------|
| append           | append to an existing file |

*fontfamily* specifies a valid font family.

*unit* may be in (inch), pt (point), or cm (centimeter). An inch is equivalent to 72 points and 2.54 centimeters. The default is pt.

*variant* may be allcaps, smallcaps, or normal.

*variant* (allcaps) changes the text to all uppercase letters; applicable when publishing items from a collection to Microsoft Word, PDF,  $\LaTeX$ , and HTML files.

*variant* (smallcaps) changes the text to use large capitals for uppercase letters and smaller capitals for lowercase letters; applicable when publishing items from a collection to Microsoft Word,  $\LaTeX$ , and HTML files.

*variant* (normal) changes the font variant back to normal and leaves the capitalization unchanged from the original text; applicable when publishing items from a collection to Microsoft Word, PDF,  $\LaTeX$ , and HTML files.

*smcl* specifies the name of the SMCL directive to render text for SMCL output. The supported SMCL directives are `input`, `error`, `result`, and `text`.

*latex* specifies the name of a  $\LaTeX$  macro to render text for  $\LaTeX$  output. Example  $\LaTeX$  macro names are `textbf`, `textsf`, `textrm`, and `texttt`. Custom  $\LaTeX$  macros are also allowed. If *text* is to be rendered in a cell, title, or note, then *latex* is translated to the following when you export to  $\LaTeX$ :

```
\latex {text}
```

*sspec* is

```
[background(bgcolor) foreground(fgcolor) pattern(fpattern)]
```

*bgcolor* specifies the background color.

*fgcolor* specifies the foreground color.

*fpattern* specifies the fill pattern. A complete list of fill patterns is shown in the [Appendix](#).

*bgcolor*, *fgcolor*, and *color* may be one of the colors listed in the [Appendix](#); a valid RGB value in the form `### ## #`, for example, `171 248 103`; or a valid RRGGBB hex value in the form `#####`, for example, `ABF867`.

## Options

### Main

`totals` (*totals*) and `nototals` control which totals are to be displayed in the table. By default, all totals are reported.

`totals` (*totals*) specifies which margin totals to display in the reported table. *totals* can contain variables in *rowspec*, *colspec*, *tabspec*, and their interaction. Interactions can be specified by using the # operator.

`nototals` prevents table from displaying any totals.

### Statistics

`statistic` (*statspec*) specifies the statistic to be displayed. `statistic()` may be repeated to request multiple statistics. Frequency statistics, summary statistics, and ratio statistics are available by specifying `statistic` (*freqstat*), `statistic` (*sumstat varlist*), and `statistic` (*ratiostat* [*varlist*] [*, ratio\_options* ]), respectively.

`statistic()` may be repeated to request multiple statistics.

`statistic` (*freqstat*) specifies that frequencies be computed.

| <i>freqstat</i>        | Definition     |
|------------------------|----------------|
| <code>frequency</code> | frequency      |
| <code>sumw</code>      | sum of weights |

`statistic(sumstat varlist)` specifies that summary statistic *sumstat* be computed for the variables in *varlist*.

| <i>sumstat</i>  | Definition                                              |
|-----------------|---------------------------------------------------------|
| mean            | mean                                                    |
| semean          | standard error of the mean                              |
| sebinomial      | standard error of the mean, binomial                    |
| sepoisson       | standard error of the mean, Poisson                     |
| variance        | variance                                                |
| sd              | standard deviation                                      |
| skewness        | skewness                                                |
| kurtosis        | kurtosis                                                |
| cv              | coefficient of variation                                |
| count           | number of nonmissing values                             |
| median          | median                                                  |
| p#              | #th percentile                                          |
| q1              | first quartile                                          |
| q2              | second quartile                                         |
| q3              | third quartile                                          |
| iqr             | interquartile range                                     |
| min             | minimum value                                           |
| max             | maximum value                                           |
| range           | range                                                   |
| first           | first value                                             |
| last            | last value                                              |
| firstnm         | first nonmissing value                                  |
| lastnm          | last nonmissing value                                   |
| total           | total                                                   |
| rawtotal        | unweighted total                                        |
| fvfrequency     | frequency of each factor-variable level                 |
| fvrawfrequency  | unweighted frequency of each factor-variable level      |
| fvproportion    | proportion within each factor-variable level            |
| fvrawproportion | unweighted proportion within each factor-variable level |
| fvpercent       | percentage within each factor-variable level            |
| fvrawpercent    | unweighted percentage within each factor-variable level |

`statistic(ratiostat [varlist] [, ratio_options])` specifies that ratio statistic *ratiostat* be computed. If *varlist* is specified, ratios are computed based on the totals of the specified variables. If *varlist* is not specified, ratios are computed based on frequencies.

| <i>ratiostat</i>           | Definition                                       |
|----------------------------|--------------------------------------------------|
| <code>proportion</code>    | proportion                                       |
| <code>percent</code>       | percentage                                       |
| <code>rawproportion</code> | proportion ignoring optionally specified weights |
| <code>rawpercent</code>    | percentage ignoring optionally specified weights |

| <i>ratio_options</i>          | Definition                                                            |
|-------------------------------|-----------------------------------------------------------------------|
| <code>across(cellspec)</code> | percentages or proportions across levels of variables or interactions |
| <code>total</code>            | compute overall percentages or proportions                            |

*cellspec* may contain any variables in *rowspec*, *colspec*, *tabspec*, or an interaction between any of these variables. Interactions can be specified by using the # operator.

#### Formats

`nformat(%fmt [results] [, basestyle])` changes the numeric format, such as the number of decimal places, for specified results. If *results* are not specified, the numeric format is changed for all results.

*results* may be any statistic named in option `statistic()` (that is, any *freqstat*, *sumstat*, or *ratiostat*).

This option is repeatable, and when multiple formats apply to one result, the rightmost specification is applied.

This option does not affect the format of numeric layout variables (*rowspec*, *colspec*, and *tabspec*) or the format of factor variables specified in the `statistic()` option. The default format of these variables is taken from the dataset.

*basestyle* indicates that the format be applied to results that do not already have their own format instead of overriding the format for all results.

`sformat(%fmt [results])` changes the string format for specified results. You can, for instance, add symbols or text to the values reported in the table by modifying the string format.

*%fmt* may contain a mix of text and %s. Here %s refers to the numeric value that is formatted as specified using `nformat()`. The text will be placed around the numeric values in your table as it is placed around %s in this option. For instance, to place parentheses around the percent statistics, you can specify `sformat("(%s) percent)`.

*results* may be any statistic named in option `statistic()` (that is, any *freqstat*, *sumstat*, or *ratiostat*).

Two text characters must be specified using a special character sequence if you want them to be displayed in your table. To include %, type %%. To include \, type \\ . For instance, to place a percent sign following percent statistics, you can specify `sformat("%s%" percent)`.

This option is repeatable, and when multiple formats apply to one result, the rightmost specification is applied.

Title

`title(string)` adds the text *string* as a title to the table.

`titlestyles(text_styles)` changes the style for the table title. *text\_styles* are the following:

`font([fontfamily] [, size(# [unit]) color(color) variant(variant) [no]bold [no]italic [no]strikeout [no]underline])` specifies the font style. These font style properties are applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files, unless otherwise specified.

*fontfamily* specifies a valid font family. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`size(# [unit])` specifies the font size as a number optionally followed by units. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`color(color)` specifies the text color.

`variant(variant)` specifies the font variant and capitalization.

`bold` and `nobold` specify the font weight. `bold` changes the font weight to bold; `nobold` changes the font weight back to normal.

`italic` and `noitalic` specify the font style. `italic` changes the font style to italic; `noitalic` changes the font style back to normal.

`strikeout` and `nostrikeout` specify whether to add a strikeout mark to the title. `strikeout` adds a strikeout mark to the title; `nostrikeout` changes the title back to normal.

`underline` and `nounderline` specify whether to underline the table title. `underline` adds a single line under the title; `nounderline` removes the underline.

Only one of `strikeout` or `underline` is allowed when publishing to HTML files.

`smcl(smcl)` specifies how to render the table title for SMCL output. This style property is applicable only when publishing items from a collection to a SMCL file.

`latex(latex)` specifies how to render the table title for  $\LaTeX$  output. This style property is applicable only when publishing items from a collection to a  $\LaTeX$  file.

`shading(sspec)` sets the background color, foreground color, and fill pattern. The background color is applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files. The foreground color and fill pattern are applicable when exporting the table to Microsoft Word and Microsoft Excel.

`note(string)` adds the text *string* as a note to the table. `note()` may be specified multiple times to add multiple notes. Each note is placed on a new line.

`notestyles(text_styles)` changes the style for the table notes. *text\_styles* are the following:

`font([fontfamily] [, size(# [unit]) color(color) variant(variant) [no]bold [no]italic [no]strikeout [no]underline])` specifies the font style. These font style properties are applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files, unless otherwise specified.

*fontfamily* specifies a valid font family. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`size(# [unit])` specifies the font size as a number optionally followed by units. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`color(color)` specifies the text color.

`variant(variant)` specifies the font variant and capitalization.

`bold` and `nobold` specify the font weight. `bold` changes the font weight to bold; `nobold` changes the font weight back to normal.

`italic` and `noitalic` specify the font style. `italic` changes the font style to italic; `noitalic` changes the font style back to normal.

`strikeout` and `nostrikeout` specify whether to add a strikeout mark to the notes. `strikeout` adds a strikeout mark to the note; `nostrikeout` changes the note back to normal.

`underline` and `nounderline` specify whether to underline the table notes. `underline` adds a single line under the notes; `nounderline` removes the underline.

Only one of `strikeout` or `underline` is allowed when publishing to HTML files.

`smcl(smcl)` specifies how to render the table notes for SMCL output. This style property is applicable only when publishing items from a collection to a SMCL file.

`latex(latex)` specifies how to render the table notes for  $\LaTeX$  output. This style property is applicable only when publishing items from a collection to a  $\LaTeX$  file.

`shading(sspec)` sets the background color, foreground color, and fill pattern. The background color is applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files. The foreground color and fill pattern are applicable when exporting the table to Microsoft Word and Microsoft Excel.

Export

`export(filename.suffix[, export_opts])` exports the table to the specified file. *export\_opts* are the following:

`as(fileformat)` specifies the file format to which the table is to be exported. This option is rarely specified because, by default, `table` determines the format from the suffix of the file being created.

`replace` permits `table` to overwrite an existing file.

`noisily` specifies that `table` show the commands used to export the table to Microsoft Word, Microsoft Excel, and PDF files. The `putdocx`, `putexcel`, or `putpdf` command used to export the table will be displayed.

`dofile(filename[, replace])` specifies that `table` save to *filename* the commands used to export the table to Microsoft Word, Microsoft Excel, and PDF files.

If *filename* already exists, it can be overwritten by specifying `replace`. If *filename* is specified without an extension, `.do` is assumed.

`append` specifies that `table` append the table to an existing file.

This option is applicable when you export the table to an HTML, a  $\LaTeX$ , a SMCL, a `txt`, or a Markdown file. When you export to HTML and  $\LaTeX$  files, the `append` option implies the `tableonly` option. Furthermore, when you export to HTML files, if the target CSS file already exists, `table` will also append to it.

`tableonly` specifies that only the table be exported to the specified HTML or  $\LaTeX$  document. By default, `table` produces complete HTML and  $\LaTeX$  documents.

When you export to an HTML file, if the `cssfile()` option is not specified, a CSS filename is constructed from *filename*, with the extension replaced with `.css`.

`cssfile(cssfile)` specifies that `table` define the styles in *cssfile* instead of *filename* when you export to HTML.

`prefix(prefix)` specifies that `table` use *prefix* to identify style classes when you export to HTML.

`sheet(sheetname [, replace])` saves to the worksheet named *sheetname*. For more information about this option, see [\[RPT\] putexcel](#).

`cell(cell)` specifies an Excel upper-left cell as the starting position to publish the table. The default is `cell(A1)`.

`modify` permits `putexcel` set to modify an Excel file. For more information about this option, see [\[RPT\] putexcel](#).

`noopen` prevents `putexcel` from opening the Excel file in memory for modification. It does not appear in the dialog box. For more information about this option, see [\[RPT\] putexcel](#).

## Options

`listwise` handles missing values through listwise deletion, meaning that the entire observation is omitted from the sample if any variable specified in a `statistic()` option is missing for that observation. By default, `table` will omit an observation only if all variables specified in all `statistic()` options are missing for that observation.

`missing` specifies that numeric missing values of any variables specified in `rowspec`, `colspec`, or `tabspec` be treated as valid categories. By default, observations with a numeric missing value in any of these variables are omitted.

This option does not apply to factor variables specified with statistics `fvfrequency`, `fvrawfrequency`, `fvproportion`, `fvrawproportion`, `fvpercent`, or `fvrawpercent`.

`showcounts` specifies that `table` report the sample size for each variable specified in option `statistic()`.

`zerocounts` specifies that `table` report a 0 in empty cells for results `count`, `frequency`, `fvfrequency`, and `fvrawfrequency`.

`name(cname)` specifies that a collection named *cname* be associated with the collected statistics and results. The default is `name(Table)`.

`append` specifies that `table` append its collection information into the collection named in `name()`.

`replace` permits `table` to overwrite an existing collection. This option is implied for `name(Table)` when `append` is not specified.

`label(filename)` specifies the *filename* containing the collection labels to use for your table. Labels in *filename* will be loaded for the table, and any labels not specified in *filename* will be taken from the labels defined in `c(collect_label)`. The default is to use only the collection labels set in `c(collect_label)`; see [TABLES] [set collect\\_label](#).

`style(filename [, override])` specifies the *filename* containing the collection styles to use for your table. The default collection styles will be discarded, and only the collection styles in *filename* will be applied.

If you prefer the default collection styles but also want to apply any styles in *filename*, specify *override*. If there are conflicts between the default collection styles and those in *filename*, the ones in *filename* will take precedence.

The default is to use only the collection styles set in `c(table_style)`; see [TABLES] [set table\\_style](#).

The following option is available with `table` but is not shown in the dialog box:

`markvar(newvar)` generates an indicator variable that identifies the observations used in the tabulation.

## Remarks and examples

Remarks are presented under the following headings:

*Basic summary statistic tables*  
*Classic Table 1*

### Basic summary statistic tables

The `table` command can be used to compute a variety of summary statistics and display them in a table. Summary statistics can be computed for the full dataset or across levels of one or more categorical variables.

To demonstrate, we use data from the Second National Health and Nutrition Examination Survey (NHANES II) (McDowell et al. 1981) and create a table reporting the mean body mass index (BMI) of individuals across four regions of the USA. We use the `statistic()` option to request that means be computed, and we specify `region` as our row variable for the table. Thus, means are computed for each region separately and for all the regions combined (Total).

```
. use https://www.stata-press.com/data/r19/nhanes21
(Second National Health and Nutrition Examination Survey)
. table region, statistic(mean bmi)
```

|        | Mean     |
|--------|----------|
| Region |          |
| NE     | 25.57535 |
| MW     | 25.51936 |
| S      | 25.63317 |
| W      | 25.42299 |
| Total  | 25.5376  |

The mean BMI is very similar across regions. We might want to look at some additional statistics. We can add the minimums and maximums in our table by repeating our `statistic()` option for each statistic; we will use the `stat()` abbreviation.

```
. table region, stat(mean bmi) stat(min bmi) stat(max bmi)
```

|        | Mean     | Minimum value | Maximum value |
|--------|----------|---------------|---------------|
| Region |          |               |               |
| NE     | 25.57535 | 15.36715      | 57.10803      |
| MW     | 25.51936 | 14.1351       | 61.1297       |
| S      | 25.63317 | 12.3856       | 55.43552      |
| W      | 25.42299 | 15.69046      | 54.05056      |
| Total  | 25.5376  | 12.3856       | 61.1297       |

If we want to include even more statistics, the table will become very wide. We can move the statistics to the rows of our table by specifying the keyword `result` in the first set of parentheses. We place `region` on the columns by specifying this variable in the second set of parentheses.

```
. table (result) (region),
> stat(mean bmi) stat(median bmi) stat(sd bmi)
> stat(min bmi) stat(max bmi)
```

|                    | Region   |          |          |          |          |
|--------------------|----------|----------|----------|----------|----------|
|                    | NE       | MW       | S        | W        | Total    |
| Mean               | 25.57535 | 25.51936 | 25.63317 | 25.42299 | 25.5376  |
| Median             | 25.00623 | 24.71567 | 24.98451 | 24.66734 | 24.81812 |
| Standard deviation | 4.72798  | 4.905965 | 5.084678 | 4.883534 | 4.914969 |
| Minimum value      | 15.36715 | 14.1351  | 12.3856  | 15.69046 | 12.3856  |
| Maximum value      | 57.10803 | 61.1297  | 55.43552 | 54.05056 | 61.1297  |

Instead of computing many statistics for one variable, we might want to compute one statistic for multiple variables. To do this, we can include a list of variables within a single `statistic()` option. Let's compute the means of age, BMI, and systolic blood pressure (`bpsystol`).

```
. table (result) (region), stat(mean age bmi bpsystol)
```

|                         | Region   |          |          |          |          |
|-------------------------|----------|----------|----------|----------|----------|
|                         | NE       | MW       | S        | W        | Total    |
| Age (years)             | 47.81584 | 46.52776 | 48.19068 | 47.83828 | 47.57965 |
| Body mass index (BMI)   | 25.57535 | 25.51936 | 25.63317 | 25.42299 | 25.5376  |
| Systolic blood pressure | 131.3836 | 130.4863 | 131.1626 | 130.5936 | 130.8817 |

## Classic Table 1

In many reports, the first discussion of the data is accompanied by a “Table 1”, a reporting of summary statistics for all variables of interest. Often the table includes a mixture of continuous and categorical variables. We may also want to specify these in a particular order based on importance. In many cases, the table has multiple columns with the summary statistics reported for each level of a categorical variable of interest.

Here we will demonstrate how to create one variety of such a table. We have two factor variables of interest, `diabetes` and `hlthstat`, for which we would like to compute the percentage of individuals in each category. We will use the `fvpercent` statistic to obtain these percentages. We also have three continuous variables, `age`, `bmi`, and `bpsystol`, for which we would like to compute means. We specify the `statistic()` options in the order we wish to see the results in the table. To specify that the variables in the `statistic()` options appear on the rows, we include the keyword `var` in the first set of parentheses. We place `region` on the columns by listing it in the second set of parentheses.

```
. table (var) (region),
> statistic(fvpercent diabetes)
> statistic(mean age bmi)
> statistic(fvpercent hlthstat)
> statistic(mean bpsystol)
```

|                              | Region   |          |          |          | Total    |
|------------------------------|----------|----------|----------|----------|----------|
|                              | NE       | MW       | S        | W        |          |
| Diabetes status=Not diabetic |          |          |          |          |          |
| Factor-variable percent      | 95.32    | 95.49    | 94.36    | 95.62    | 95.18    |
| Diabetes status=Diabetic     |          |          |          |          |          |
| Factor-variable percent      | 4.68     | 4.51     | 5.64     | 4.38     | 4.82     |
| Age (years)                  |          |          |          |          |          |
| Mean                         | 47.81584 | 46.52776 | 48.19068 | 47.83828 | 47.57965 |
| Body mass index (BMI)        |          |          |          |          |          |
| Mean                         | 25.57535 | 25.51936 | 25.63317 | 25.42299 | 25.5376  |
| Health status=Excellent      |          |          |          |          |          |
| Factor-variable percent      | 26.95    | 26.33    | 19.14    | 21.68    | 23.29    |
| Health status=Very good      |          |          |          |          |          |
| Factor-variable percent      | 26.76    | 26.01    | 22.82    | 25.18    | 25.07    |
| Health status=Good           |          |          |          |          |          |
| Factor-variable percent      | 30.26    | 26.52    | 28.29    | 29.14    | 28.43    |
| Health status=Fair           |          |          |          |          |          |
| Factor-variable percent      | 12.33    | 15.12    | 18.65    | 17.60    | 16.16    |
| Health status=Poor           |          |          |          |          |          |
| Factor-variable percent      | 3.69     | 6.02     | 11.11    | 6.40     | 7.05     |
| Systolic blood pressure      |          |          |          |          |          |
| Mean                         | 131.3836 | 130.4863 | 131.1626 | 130.5936 | 130.8817 |

We have the statistics we want, but clearly our table could be improved. Let's start by applying one of the [predefined styles](#), `table-1`, by adding the `style()` option.

```
. table (var) (region),
> statistic(fvpercent diabetes)
> statistic(mean age bmi)
> statistic(fvpercent hlthstat)
> statistic(mean bpsystol) style(table-1)
```

|                         | Region   |          |          |          | Total    |
|-------------------------|----------|----------|----------|----------|----------|
|                         | NE       | MW       | S        | W        |          |
| Diabetes status         |          |          |          |          |          |
| Not diabetic            | 95.32    | 95.49    | 94.36    | 95.62    | 95.18    |
| Diabetic                | 4.68     | 4.51     | 5.64     | 4.38     | 4.82     |
| Age (years)             | 47.81584 | 46.52776 | 48.19068 | 47.83828 | 47.57965 |
| Body mass index (BMI)   | 25.57535 | 25.51936 | 25.63317 | 25.42299 | 25.5376  |
| Health status           |          |          |          |          |          |
| Excellent               | 26.95    | 26.33    | 19.14    | 21.68    | 23.29    |
| Very good               | 26.76    | 26.01    | 22.82    | 25.18    | 25.07    |
| Good                    | 30.26    | 26.52    | 28.29    | 29.14    | 28.43    |
| Fair                    | 12.33    | 15.12    | 18.65    | 17.60    | 16.16    |
| Poor                    | 3.69     | 6.02     | 11.11    | 6.40     | 7.05     |
| Systolic blood pressure | 131.3836 | 130.4863 | 131.1626 | 130.5936 | 130.8817 |

This style removes the labels for the type of statistic being reported, cleans up the reporting of factor variables in the row headers, and right-aligns the content in the row headers. In addition, we may want to specify that the means be reported to two decimal places using the `nformat(%6.2f mean)` option.

```
. table (var) (region),
> statistic(fvpercent diabetes)
> statistic(mean age bmi)
> statistic(fvpercent hlthstat)
> statistic(mean bpsystol) style(table-1) nformat(%6.2f mean)
```

|                         | NE     | MW     | Region<br>S | W      | Total  |
|-------------------------|--------|--------|-------------|--------|--------|
| Diabetes status         |        |        |             |        |        |
| Not diabetic            | 95.32  | 95.49  | 94.36       | 95.62  | 95.18  |
| Diabetic                | 4.68   | 4.51   | 5.64        | 4.38   | 4.82   |
| Age (years)             | 47.82  | 46.53  | 48.19       | 47.84  | 47.58  |
| Body mass index (BMI)   | 25.58  | 25.52  | 25.63       | 25.42  | 25.54  |
| Health status           |        |        |             |        |        |
| Excellent               | 26.95  | 26.33  | 19.14       | 21.68  | 23.29  |
| Very good               | 26.76  | 26.01  | 22.82       | 25.18  | 25.07  |
| Good                    | 30.26  | 26.52  | 28.29       | 29.14  | 28.43  |
| Fair                    | 12.33  | 15.12  | 18.65       | 17.60  | 16.16  |
| Poor                    | 3.69   | 6.02   | 11.11       | 6.40   | 7.05   |
| Systolic blood pressure | 131.38 | 130.49 | 131.16      | 130.59 | 130.88 |

Let's go one step further. Perhaps we want the mean and standard deviation of each continuous variable, and we want the frequency and percent for each factor variable. We need to specify a few more `statistic()` options.

```
. table (var) (region),
> stat(fvfreq diabetes) statistic(fvpercent diabetes)
> statistic(mean age bmi) statistic(sd age bmi)
> statistic(fvfreq hlthstat) statistic(fvpercent hlthstat)
> statistic(mean bpsystol) statistic(sd bpsystol)
> style(table-1) nformat(%6.2f mean sd)
```

|                         | Region |        |        |        | Total  |
|-------------------------|--------|--------|--------|--------|--------|
|                         | NE     | MW     | S      | W      |        |
| Diabetes status         |        |        |        |        |        |
| Not diabetic            | 1,997  | 2,648  | 2,692  | 2,513  | 9,850  |
|                         | 95.32  | 95.49  | 94.36  | 95.62  | 95.18  |
| Diabetic                | 98     | 125    | 161    | 115    | 499    |
|                         | 4.68   | 4.51   | 5.64   | 4.38   | 4.82   |
| Age (years)             | 47.82  | 46.53  | 48.19  | 47.84  | 47.58  |
|                         | 17.02  | 17.38  | 16.86  | 17.53  | 17.21  |
| Body mass index (BMI)   | 25.58  | 25.52  | 25.63  | 25.42  | 25.54  |
|                         | 4.73   | 4.91   | 5.08   | 4.88   | 4.91   |
| Health status           |        |        |        |        |        |
| Excellent               | 562    | 730    | 546    | 569    | 2,407  |
|                         | 26.95  | 26.33  | 19.14  | 21.68  | 23.29  |
| Very good               | 558    | 721    | 651    | 661    | 2,591  |
|                         | 26.76  | 26.01  | 22.82  | 25.18  | 25.07  |
| Good                    | 631    | 735    | 807    | 765    | 2,938  |
|                         | 30.26  | 26.52  | 28.29  | 29.14  | 28.43  |
| Fair                    | 257    | 419    | 532    | 462    | 1,670  |
|                         | 12.33  | 15.12  | 18.65  | 17.60  | 16.16  |
| Poor                    | 77     | 167    | 317    | 168    | 729    |
|                         | 3.69   | 6.02   | 11.11  | 6.40   | 7.05   |
| Systolic blood pressure | 131.38 | 130.49 | 131.16 | 130.59 | 130.88 |
|                         | 24.31  | 22.50  | 24.21  | 22.42  | 23.33  |

Finally, to distinguish among our statistics, we can use the `sformat()` option to add parentheses around our standard deviations and percent signs to our percentages.

```
. table (var) (region),
> stat(fvfreq diabetes) statistic(fvpercent diabetes)
> statistic(mean age bmi) statistic(sd age bmi)
> statistic(fvfreq hlthstat) statistic(fvpercent hlthstat)
> statistic(mean bpsystol) statistic(sd bpsystol)
> style(table-1) nformat(%6.2f mean sd)
> sformat("(%s)" sd) sformat("%s%" fvpercent)
```

|                         | Region            |                   |                   |                   |                   |
|-------------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
|                         | NE                | MW                | S                 | W                 | Total             |
| Diabetes status         |                   |                   |                   |                   |                   |
| Not diabetic            | 1,997<br>95.32%   | 2,648<br>95.49%   | 2,692<br>94.36%   | 2,513<br>95.62%   | 9,850<br>95.18%   |
| Diabetic                | 98<br>4.68%       | 125<br>4.51%      | 161<br>5.64%      | 115<br>4.38%      | 499<br>4.82%      |
| Age (years)             | 47.82<br>(17.02)  | 46.53<br>(17.38)  | 48.19<br>(16.86)  | 47.84<br>(17.53)  | 47.58<br>(17.21)  |
| Body mass index (BMI)   | 25.58<br>(4.73)   | 25.52<br>(4.91)   | 25.63<br>(5.08)   | 25.42<br>(4.88)   | 25.54<br>(4.91)   |
| Health status           |                   |                   |                   |                   |                   |
| Excellent               | 562<br>26.95%     | 730<br>26.33%     | 546<br>19.14%     | 569<br>21.68%     | 2,407<br>23.29%   |
| Very good               | 558<br>26.76%     | 721<br>26.01%     | 651<br>22.82%     | 661<br>25.18%     | 2,591<br>25.07%   |
| Good                    | 631<br>30.26%     | 735<br>26.52%     | 807<br>28.29%     | 765<br>29.14%     | 2,938<br>28.43%   |
| Fair                    | 257<br>12.33%     | 419<br>15.12%     | 532<br>18.65%     | 462<br>17.60%     | 1,670<br>16.16%   |
| Poor                    | 77<br>3.69%       | 167<br>6.02%      | 317<br>11.11%     | 168<br>6.40%      | 729<br>7.05%      |
| Systolic blood pressure | 131.38<br>(24.31) | 130.49<br>(22.50) | 131.16<br>(24.21) | 130.59<br>(22.42) | 130.88<br>(23.33) |

We have added many customizations to our table. However, you may prefer a different look. For another style, you can select from the predefined styles described in [\[TABLES\] Predefined styles](#). If none of these provide the exact style you want for your table, you can further customize the results by using the collect suite of commands. To learn more, see [\[TABLES\] Intro](#).

If you wish to include this table in a paper, on a webpage, or in another format, you can easily export it in  $\LaTeX$ , Word, Excel, HTML, and a variety of other formats by using the `export()` option.

## Stored results

`table` stores the following in `r()`:

Scalars

`r(N)` number of observations

## References

- Huber, C. 2021a. Customizable tables in Stata 17, part 3: The classic table 1. *The Stata Blog: Not Elsewhere Classified*. <https://blog.stata.com/2021/06/24/customizable-tables-in-stata-17-part-3-the-classic-table-1/>.
- . 2021b. Customizable tables in Stata 17, part 4: Table of statistical tests. *The Stata Blog: Not Elsewhere Classified*. <https://blog.stata.com/2021/08/24/customizable-tables-in-stata-17-part-4-table-of-statistical-tests/>.
- McDowell, A., A. Engel, J. T. Massey, and K. Maurer. 1981. “Plan and operation of the Second National Health and Nutrition Examination Survey, 1976–1980”. In *Vital and Health Statistics*, ser. 1, no. 15. Hyattsville, MD: National Center for Health Statistics.

## Also see

- [R] **dtable** — Create a table of descriptive statistics
- [R] **table intro** — Introduction to tables of frequencies, summaries, and command results
- [R] **tabstat** — Compact table of summary statistics
- [TABLES] **Intro** — Introduction

|                             |                                      |                                |                           |
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| <a href="#">Description</a> | <a href="#">Quick start</a>          | <a href="#">Menu</a>           | <a href="#">Syntax</a>    |
| <a href="#">Options</a>     | <a href="#">Remarks and examples</a> | <a href="#">Stored results</a> | <a href="#">Reference</a> |
| <a href="#">Also see</a>    |                                      |                                |                           |

## Description

In this entry, we discuss how to use `table` to create tables with results of hypothesis tests.

## Quick start

Table with pairwise correlations stored in matrix `r(C)`

```
table (rowname) (colname), command(r(C): pwcorr v1 v2 v3)
```

Table with all the numeric scalars returned by `ttest`; rows correspond to the different results

```
table (result) (command), command(ttest v1, by(catvar))
```

Table with means and two-sided  $p$ -values; columns correspond to the different results

```
table (command) (result), ///
command(r(mu_1) r(mu_2) r(p): ttest v1, by(catvar))
```

Same as above, but with statistics for `v1` and `v2`

```
table (command) (result), ///
command(r(mu_1) r(mu_2) r(p): ttest v1, by(catvar)) ///
command(r(mu_1) r(mu_2) r(p): ttest v2, by(catvar))
```

## Menu

Statistics > Summaries, tables, and tests > Tables of frequencies, summaries, and command results

## Syntax

```
table ([rowspec]) ([colspec]) ([tabspec] [if] [in] [weight]),
 command(cmdsspec) [command(cmdsspec) ...] [options]
```

*rowspec*, *colspec*, and *tabspec* may be empty or may include variable names or any of the following keywords:

| <i>keyword</i>       | Description                             |
|----------------------|-----------------------------------------|
| <code>result</code>  | requested statistics                    |
| <code>stars</code>   | stars denoting statistical significance |
| <code>command</code> | index option <code>command()</code>     |
| <code>colname</code> | column names for matrix statistics      |
| <code>rowname</code> | row names for matrix statistics         |

| <i>options</i>                                                         | Description                                          |
|------------------------------------------------------------------------|------------------------------------------------------|
| <b>Commands</b>                                                        |                                                      |
| <code>command(<i>cmds</i>spec)</code>                                  | collect results from the specified Stata command     |
| <b>Formats</b>                                                         |                                                      |
| <code>nformat(<i>%fmt</i> [<i>results</i>][, <i>basestyle</i>])</code> | specify numeric format                               |
| <code>sformat(<i>sfmt</i> [<i>results</i>])</code>                     | specify string format                                |
| <b>Stars</b>                                                           |                                                      |
| <code>stars(<i>stars</i>spec)</code>                                   | add stars to denote statistical significance         |
| <b>Title</b>                                                           |                                                      |
| <code>title(<i>string</i>)</code>                                      | add table title                                      |
| <code>titlestyles(<i>text_styles</i>)</code>                           | change table title styles                            |
| <b>Notes</b>                                                           |                                                      |
| <code>note(<i>string</i>)</code>                                       | add table note                                       |
| <code>notestyles(<i>text_styles</i>)</code>                            | change table note styles                             |
| <b>Export</b>                                                          |                                                      |
| <code>export(<i>filename.suffix</i>[, <i>export_opts</i>])</code>      | export table                                         |
| <b>Options</b>                                                         |                                                      |
| <code>missing</code>                                                   | treat numeric missing values like other values       |
| <code>name(<i>cname</i>)</code>                                        | collect results into a collection named <i>cname</i> |
| <code>append</code>                                                    | append results to an existing collection             |
| <code>replace</code>                                                   | replace results of an existing collection            |
| <code>label(<i>filename</i>)</code>                                    | specify the collection labels                        |
| <code>style(<i>filename</i> [, <i>override</i>])</code>                | specify the collection style                         |
| <code>noisily</code>                                                   | display output from each command                     |

*fweights*, *awweights*, *iweights*, and *pweights* are allowed; see [\[U\] 11.1.6 weight](#).

`strL` variables are not allowed; see [\[U\] 12.4.8 strL](#).

`noisily` does not appear in the dialog box.

| <i>text_styles</i>                           | Description                                              |
|----------------------------------------------|----------------------------------------------------------|
| <code>font([fontfamily][, font_opts])</code> | specify font style                                       |
| <code>smcl(smcl)</code>                      | specify formatting for SMCL files                        |
| <code>latex(latex)</code>                    | specify L <sup>A</sup> T <sub>E</sub> X macro            |
| <code>shading(sspec)</code>                  | set background color, foreground color, and fill pattern |

| <i>font_opts</i>              | Description                              |
|-------------------------------|------------------------------------------|
| <code>size(# [unit])</code>   | specify font size                        |
| <code>color(color)</code>     | specify font color                       |
| <code>variant(variant)</code> | specify font variant and capitalization  |
| <code>[no]bold</code>         | specify whether to format text as bold   |
| <code>[no]italic</code>       | specify whether to format text as italic |
| <code>[no]strikeout</code>    | specify whether to strike out text       |
| <code>[no]underline</code>    | specify whether to underline text        |

| <i>suffix</i>         | <i>fileformat</i>         | Output format                      |
|-----------------------|---------------------------|------------------------------------|
| <code>docx</code>     | <code>as(docx)</code>     | Microsoft Word                     |
| <code>html</code>     | <code>as(html)</code>     | HTML 5 with CSS                    |
| <code>pdf</code>      | <code>as(pdf)</code>      | PDF                                |
| <code>xlsx</code>     | <code>as(xlsx)</code>     | Microsoft Excel 2007/2010 or newer |
| <code>xls</code>      | <code>as(xls)</code>      | Microsoft Excel 1997/2003          |
| <code>tex</code>      | <code>as(tex)</code>      | L <sup>A</sup> T <sub>E</sub> X    |
| <code>smcl</code>     | <code>as(smcl)</code>     | SMCL                               |
| <code>txt</code>      | <code>as(txt)</code>      | plain text                         |
| <code>markdown</code> | <code>as(markdown)</code> | Markdown                           |
| <code>md</code>       | <code>as(md)</code>       | Markdown                           |

| <i>export_opts</i>          | Description                                         |
|-----------------------------|-----------------------------------------------------|
| <code>as(fileformat)</code> | specify document type                               |
| <code>replace</code>        | overwrite existing file                             |
| <i>docx_options</i>         | available when exporting to .docx files             |
| <i>html_options</i>         | available when exporting to .html files             |
| <i>pdf_options</i>          | available when exporting to .pdf files              |
| <i>excel_options</i>        | available when exporting to .xls and .xlsx files    |
| <i>tex_options</i>          | available when exporting to .tex files              |
| <i>smcl_option</i>          | available when exporting to .smcl files             |
| <i>txt_option</i>           | available when exporting to .txt files              |
| <i>md_option</i>            | available when exporting to .markdown and .md files |

| <i>docx_options</i>                      | Description                                                         |
|------------------------------------------|---------------------------------------------------------------------|
| <code>noisily</code>                     | show the putdocx commands used to export to the Microsoft Word file |
| <code>dofile(filename[, replace])</code> | save the putdocx commands used for exporting to the named do-file   |

| <i>html_options</i>       | Description                                                    |
|---------------------------|----------------------------------------------------------------|
| append                    | append to an existing file                                     |
| tableonly                 | export only the table to the specified file                    |
| cssfile( <i>cssfile</i> ) | define the styles in <i>cssfile</i> instead of <i>filename</i> |
| prefix( <i>prefix</i> )   | use <i>prefix</i> to identify style classes                    |

| <i>pdf_options</i>                   | Description                                                      |
|--------------------------------------|------------------------------------------------------------------|
| noisily                              | show the putpdf commands used to export to the PDF file          |
| dofile( <i>filename</i> [, replace]) | save the putpdf commands used for exporting to the named do-file |

| <i>excel_options</i>                 | Description                                                                                             |
|--------------------------------------|---------------------------------------------------------------------------------------------------------|
| noisily                              | show the putexcel commands used to export to the Excel file                                             |
| dofile( <i>filename</i> [, replace]) | save the putexcel commands used for exporting to the named do-file                                      |
| sheet( <i>sheetname</i> [, replace]) | specify the worksheet to use; the default sheet name is Sheet1                                          |
| cell( <i>cell</i> )                  | specify the Excel upper-left cell as the starting position to export the table; the default is cell(A1) |
| modify                               | modify Excel file                                                                                       |
| noopen                               | do not open Excel file in memory                                                                        |

noopen does not appear in the dialog box.

| <i>tex_options</i> | Description                                 |
|--------------------|---------------------------------------------|
| append             | append to an existing file                  |
| tableonly          | export only the table to the specified file |

| <i>smcl_option</i> | Description                |
|--------------------|----------------------------|
| append             | append to an existing file |

| <i>txt_option</i> | Description                |
|-------------------|----------------------------|
| append            | append to an existing file |

| <i>md_option</i> | Description                |
|------------------|----------------------------|
| append           | append to an existing file |

*fontfamily* specifies a valid font family.

*unit* may be in (inch), pt (point), or cm (centimeter). An inch is equivalent to 72 points and 2.54 centimeters. The default is pt.

*variant* may be allcaps, smallcaps, or normal.

*variant* (allcaps) changes the text to all uppercase letters; applicable when publishing items from a collection to Microsoft Word, PDF, L<sup>A</sup>T<sub>E</sub>X, and HTML files.

*variant* (smallcaps) changes the text to use large capitals for uppercase letters and smaller capitals for lowercase letters; applicable when publishing items from a collection to Microsoft Word, L<sup>A</sup>T<sub>E</sub>X, and HTML files.

*variant* (normal) changes the font variant back to normal and leaves the capitalization unchanged from the original text; applicable when publishing items from a collection to Microsoft Word, PDF, L<sup>A</sup>T<sub>E</sub>X, and HTML files.

*smcl* specifies the name of the SMCL directive to render text for SMCL output. The supported SMCL directives are `input`, `error`, `result`, and `text`.

*latex* specifies the name of a L<sup>A</sup>T<sub>E</sub>X macro to render text for L<sup>A</sup>T<sub>E</sub>X output. Example L<sup>A</sup>T<sub>E</sub>X macro names are `textbf`, `textsf`, `textrm`, and `texttt`. Custom L<sup>A</sup>T<sub>E</sub>X macros are also allowed. If *text* is to be rendered in a cell, title, or note, then *latex* is translated to the following when you export to L<sup>A</sup>T<sub>E</sub>X:

```
\latex {text}
```

*sspec* is

```
[background(bgcolor) foreground(fgcolor) pattern(fpattern)]
```

*bgcolor* specifies the background color.

*fgcolor* specifies the foreground color.

*fpattern* specifies the fill pattern. A complete list of fill patterns is shown in the [Appendix](#).

*bgcolor*, *fgcolor*, and *color* may be one of the colors listed in the [Appendix](#); a valid RGB value in the form `### ## #`, for example, `171 248 103`; or a valid RRGGBB hex value in the form `#####`, for example, `ABF867`.

## Options

## Commands

`command(cmdsspec)` specifies the Stata commands from which to collect results. `command()` may be repeated to collect results from multiple commands.

*cmds*spec is [*explist*: ] *command* [*arguments*] [ , *cmdoptions* ]

*explist* specifies which results to collect and report in the table. *explist* may include *result identifiers* and *named expressions*.

*result identifiers* are results stored in `r()` and `e()` by the *command*. For instance, *result identifiers* could be `r(mean)`, `r(C)`, or `e(chi2)`. After estimation commands, *result identifiers* also include the following:

| Identifier            | Result                                                              |
|-----------------------|---------------------------------------------------------------------|
| <code>_r_b</code>     | coefficients or transformed coefficients reported by <i>command</i> |
| <code>_r_se</code>    | standard errors of <code>_r_b</code>                                |
| <code>_r_z</code>     | test statistics for <code>_r_b</code>                               |
| <code>_r_z_abs</code> | absolute value of <code>_r_z</code>                                 |
| <code>_r_p</code>     | <i>p</i> -values for <code>_r_b</code>                              |
| <code>_r_lb</code>    | lower bounds of confidence intervals for <code>_r_b</code>          |
| <code>_r_ub</code>    | upper bounds of confidence intervals for <code>_r_b</code>          |
| <code>_r_ci</code>    | confidence intervals for <code>_r_b</code>                          |
| <code>_r_crlb</code>  | lower bounds of credible intervals for <code>_r_b</code>            |
| <code>_r_crub</code>  | upper bounds of credible intervals for <code>_r_b</code>            |
| <code>_r_cri</code>   | credible intervals for <code>_r_b</code>                            |
| <code>_r_df</code>    | degrees of freedom for <code>_r_b</code>                            |

*named expressions* are specified as *name* = *exp*, where *name* may be any valid Stata name and *exp* is an expression, typically an expression that involves one or more *result identifiers*. An example of a named expression is `sd = sqrt(r(variance))`.

For r-class commands, the default is to include all numeric scalars posted to `r()` in the table results. For e-class commands, the default is to include `_r_b` in the table results.

*command* is any command that follows standard Stata syntax.

*arguments* may be anything so long as they do not include an *if* clause, *in* range, or weight specification.

Any *if* or *in* qualifier and weights should be specified directly with `table`, not within the `command()` option. Weights are passed to *command* only if they are specified.

*cmdoptions* may be anything supported by *command*.

## Formats

`nformat(%fmt [results][ , basestyle ])` changes the numeric format, such as the number of decimal places, for specified results. If *results* are not specified, the numeric format is changed for all results.

*results* may be any name in the `e()` or `r()` results produced by commands specified in option `command()`.

This option is repeatable, and when multiple formats apply to one result, the rightmost specification is applied.

This option does not affect the format of numeric layout variables (*rowspec*, *colspec*, and *tabspec*). The default format of these variables is taken from the dataset.

*basestyle* indicates that the format be applied to results that do not already have their own format instead of overriding the format for all results.

*sformat*(*sfmt* [*results*]) changes the string format for specified results. You can, for instance, add symbols or text to the values reported in the table by modifying the string format.

*sfmt* may contain a mix of text and %s. Here %s refers to the numeric value that is formatted as specified using *nformat*( ). The text will be placed around the numeric values in your table as it is placed around %s in this option. For instance, to place parentheses around the percent statistics, you can specify *sformat*("(%s)" percent).

*results* may be any name in the *e*( ) or *r*( ) results produced by commands specified in option *command*( ).

Two text characters must be specified using a special character sequence if you want them to be displayed in your table. To include %, type %%. To include \, type \\. For instance, to place a percent sign following percent statistics, you can specify *sformat*("%%%" percent).

This option is repeatable, and when multiple formats apply to one result, the rightmost specification is applied.

#### Stars

*stars*(*starspec*) specifies that stars representing statistical significance be included in the table. *starspec* identifies the result whose values determine significance, which characters should represent each significance level, and where these characters should be displayed in the table. *starspec* is

```
starres [#1 "label1" [#2 "label2" [#3 "label3" [#4 "label4" [#5 "label5"]]]]
[, attach(attachres) result dimension starsnoteopts]
```

*starres* is the name of the result whose values determine which characters, typically which number of stars, are to be displayed.

*label1* specifies the characters to be displayed when *starres* < #1.

*label2* specifies the characters to be displayed when *starres* < #2.

*label3* specifies the characters to be displayed when *starres* < #3.

*label4* specifies the characters to be displayed when *starres* < #4.

*label5* specifies the characters to be displayed when *starres* < #5.

*attach*(*attachres*) specifies the name of the result to which the characters defined by *label1*, ..., *label5* are to be attached. If *attach*( ) is not specified, a new result named *stars* is created and is automatically added to the table.

*result* and *dimension* control how *collect stars* adds items when labeling significant results. These options are mutually exclusive.

*result* specifies the default behavior, and this option is necessary only if the following *dimension* behavior is in effect and you want to change back to the *result* behavior.

*dimension* specifies that *dimension stars* be added to the collection. Items will be tagged with *stars*[*value*], and the labels will be tagged with *stars*[*label*]. Use this option for layouts where results are to be stacked within columns, and use *new dimension stars* in the column specification of the layout.

*starsnoteopts* control the display and composition of the stars note.

`noshownote` and `shownote` control whether to display the stars note.

`increasing` and `decreasing` control the order of  $p$ -values in the stars note.

`pvname(string)` specifies a name for the  $p$ -value in the stars note. The default is `pvname(p)`.

`delimiter(string)` specifies the delimiter between labels in the stars note. The default is `delimiter(",")`.

`nformat(%fmt)` specifies the numeric format for the cutoff values in the stars note. The default is `nformat(%9.0g)`.

`prefix(string)` specifies the prefix for the stars note. The prefix is empty by default.

`suffix(string)` specifies the suffix for the stars note. The suffix is empty by default.

For example, `stars(_r_p 0.01 "***" 0.05 "**" 0.1 "*", attach(_r_b))` could be added to a table of regression results to specify that stars be defined based on the  $p$ -values in `_r_p` and be attached to the reported coefficients (`_r_b`).

Title

`title(string)` adds the text *string* as a title to the table.

`titlestyles(text_styles)` changes the style for the table title. *text\_styles* are the following:

`font([fontfamily] [, size(# [unit]) color(color) variant(variant) [no]bold [no]italic [no]strikeout [no]underline)` specifies the font style. These font style properties are applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and L<sup>A</sup>T<sub>E</sub>X files, unless otherwise specified.

*fontfamily* specifies a valid font family. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

*size*(# [*unit*]) specifies the font size as a number optionally followed by units. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

*color*(*color*) specifies the text color.

*variant*(*variant*) specifies the font variant and capitalization.

`bold` and `nobold` specify the font weight. `bold` changes the font weight to bold; `nobold` changes the font weight back to normal.

`italic` and `noitalic` specify the font style. `italic` changes the font style to italic; `noitalic` changes the font style back to normal.

`strikeout` and `nostrikeout` specify whether to add a strikeout mark to the title. `strikeout` adds a strikeout mark to the title; `nostrikeout` changes the title back to normal.

`underline` and `nounderline` specify whether to underline the table title. `underline` adds a single line under the title; `nounderline` removes the underline.

Only one of `strikeout` or `underline` is allowed when publishing to HTML files.

`smcl(smcl)` specifies how to render the table title for SMCL output. This style property is applicable only when publishing items from a collection to a SMCL file.

`latex(latex)` specifies how to render the table title for L<sup>A</sup>T<sub>E</sub>X output. This style property is applicable only when publishing items from a collection to a L<sup>A</sup>T<sub>E</sub>X file.

`shading(sspec)` sets the background color, foreground color, and fill pattern. The background color is applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and L<sup>A</sup>T<sub>E</sub>X files. The foreground color and fill pattern are applicable when exporting the table to Microsoft Word and Microsoft Excel.

`note(string)` adds the text *string* as a note to the table. `note()` may be specified multiple times to add multiple notes. Each note is placed on a new line.

`notestyles(text_styles)` changes the style for the table notes. *text\_styles* are the following:

`font([fontfamily] [, size(# [unit]) color(color) variant(variant) [no]bold [no]italic [no]strikeout [no]underline])` specifies the font style. These font style properties are applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files, unless otherwise specified.

*fontfamily* specifies a valid font family. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`size(# [unit])` specifies the font size as a number optionally followed by units. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`color(color)` specifies the text color.

`variant(variant)` specifies the font variant and capitalization.

`bold` and `nobold` specify the font weight. `bold` changes the font weight to bold; `nobold` changes the font weight back to normal.

`italic` and `noitalic` specify the font style. `italic` changes the font style to italic; `noitalic` changes the font style back to normal.

`strikeout` and `nostrikeout` specify whether to add a strikeout mark to the notes. `strikeout` adds a strikeout mark to the note; `nostrikeout` changes the note back to normal.

`underline` and `nounderline` specify whether to underline the table notes. `underline` adds a single line under the notes; `nounderline` removes the underline.

Only one of `strikeout` or `underline` is allowed when publishing to HTML files.

`smcl(smcl)` specifies how to render the table notes for SMCL output. This style property is applicable only when publishing items from a collection to a SMCL file.

`latex(latex)` specifies how to render the table notes for  $\LaTeX$  output. This style property is applicable only when publishing items from a collection to a  $\LaTeX$  file.

`shading(sspec)` sets the background color, foreground color, and fill pattern. The background color is applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files. The foreground color and fill pattern are applicable when exporting the table to Microsoft Word and Microsoft Excel.

Export

`export(filename.suffix[, export_opts])` exports the table to the specified file. *export\_opts* are the following:

`as(fileformat)` specifies the file format to which the table is to be exported. This option is rarely specified because, by default, `table` determines the format from the suffix of the file being created.

`replace` permits `table` to overwrite an existing file.

`noisily` specifies that `table` show the commands used to export the table to Microsoft Word, Microsoft Excel, and PDF files. The `putdocx`, `putexcel`, or `putpdf` command used to export the table will be displayed.

`dofile(filename[, replace])` specifies that `table` save to *filename* the commands used to export the table to Microsoft Word, Microsoft Excel, and PDF files.

If *filename* already exists, it can be overwritten by specifying `replace`. If *filename* is specified without an extension, `.do` is assumed.

`append` specifies that `table` append the table to an existing file.

This option is applicable when you export the table to an HTML, a  $\LaTeX$ , a SMCL, a `txt`, or a Markdown file. When you export to HTML and  $\LaTeX$  files, the `append` option implies the `tableonly` option. Furthermore, when you export to HTML files, if the target CSS file already exists, `table` will also append to it.

`tableonly` specifies that only the table be exported to the specified HTML or  $\LaTeX$  document. By default, `table` produces complete HTML and  $\LaTeX$  documents.

When you export to an HTML file, if the `cssfile()` option is not specified, a CSS filename is constructed from *filename*, with the extension replaced with `.css`.

`cssfile(cssfile)` specifies that `table` define the styles in *cssfile* instead of *filename* when you export to HTML.

`prefix(prefix)` specifies that `table` use *prefix* to identify style classes when you export to HTML.

`sheet(sheetname [, replace])` saves to the worksheet named *sheetname*. For more information about this option, see [\[RPT\] putexcel](#).

`cell(cell)` specifies an Excel upper-left cell as the starting position to publish the table. The default is `cell(A1)`.

`modify` permits `putexcel` set to modify an Excel file. For more information about this option, see [\[RPT\] putexcel](#).

`noopen` prevents `putexcel` from opening the Excel file in memory for modification. It does not appear in the dialog box. For more information about this option, see [\[RPT\] putexcel](#).

## Options

`missing` specifies that numeric missing values of any variables specified in `rowspec`, `colspec`, or `tabspec` be treated as valid categories. By default, observations with a numeric missing value in any of these variables are omitted.

`name(cname)` specifies that a collection named `cname` be associated with the collected statistics and results. The default is `name(Table)`.

`append` specifies that `table` append its collection information into the collection named in `name()`.

`replace` permits `table` to overwrite an existing collection. This option is implied for `name(Table)` when `append` is not specified.

`label(filename)` specifies the `filename` containing the collection labels to use for your table. Labels in `filename` will be loaded for the table, and any labels not specified in `filename` will be taken from the labels defined in `c(collect_label)`. The default is to use only the collection labels set in `c(collect_label)`; see [TABLES] [set collect\\_label](#).

`style(filename [, override])` specifies the `filename` containing the collection styles to use for your table. The default collection styles will be discarded, and only the collection styles in `filename` will be applied.

If you prefer the default collection styles but also want to apply any styles in `filename`, specify `override`. If there are conflicts between the default collection styles and those in `filename`, the ones in `filename` will take precedence.

The default is to use only the collection styles set in `c(table_style)`; see [TABLES] [set table\\_style](#).

The following option is available with `table` but is not shown in the dialog box:

`noisily` specifies that output from the commands specified in `command()` options be displayed. By default, output from commands is suppressed.

## Remarks and examples

Remarks are presented under the following headings:

*Introduction*  
*Creating tables from scalars*  
*Creating tables from matrices*

### Introduction

The `table` command can be used to create tables with results of hypothesis tests. For example, you can create a table with results from a mean-comparison test, a test of proportions, or a test of normality.

`table` does not perform hypothesis tests directly. Rather, `table` will run any Stata command that you include in its `command()` option and place results from that command into the table. You determine which results you would like to see in the table. You can select any of the results stored by the command.

### Creating tables from scalars

We have data from the Second National Health and Nutrition Examination Survey (NHANES II) (McDowell et al. 1981). The data contain some demographic information, such as the age, sex, and race of participants. The data also contain some measures of health, including whether the individual has high blood pressure (`highbp`), has diabetes, or has had a heart attack previously (`heartatk`).

Suppose we want to examine the proportion of males and females that have high blood pressure, that have diabetes, and that have had a heart attack previously. With `prtest`, we can test whether the proportions are equal between males and females. For example, let's perform a test of proportions for diabetes:

```
. use https://www.stata-press.com/data/r19/nhanes21
(Second National Health and Nutrition Examination Survey)
. prtest diabetes, by(sex)
```

Two-sample test of proportions

| Group  | Mean      | Std. err. | z     | P> z  | [95% conf. interval] |
|--------|-----------|-----------|-------|-------|----------------------|
| Male   | .0441506  | .0029302  |       |       | .0384074 .0498937    |
| Female | .0518955  | .0030091  |       |       | .0459978 .0577932    |
| diff   | -.0077449 | .0042001  |       |       | -.0159769 .0004871   |
|        | under H0: | .0042169  | -1.84 | 0.066 |                      |

Male: Number of obs = 4915  
Female: Number of obs = 5434

```
diff = prop(Male) - prop(Female) z = -1.8366
H0: diff = 0
Ha: diff < 0 Ha: diff != 0 Ha: diff > 0
Pr(Z < z) = 0.0331 Pr(|Z| > |z|) = 0.0663 Pr(Z > z) = 0.9669
```

We would like to create a table that includes the proportion of men who have diabetes, the proportion of women who have diabetes, the difference in these proportions, and the  $p$ -value for a two-sided test. First, we need to determine how to refer to these statistics.

```

. return list
scalars:
 r(N1) = 4915
 r(N2) = 5434
 r(P1) = .0441505595116989
 r(P2) = .0518954729481045
 r(P_diff) = -.0077449134364056
 r(se1) = .0029302258134317
 r(se2) = .003009075122777
 r(se_diff0) = .0042169418903878
 r(se_diff) = .0042000900481081
 r(lb1) = .0384074224508032
 r(ub1) = .0498936965725946
 r(lb2) = .0459977940806861
 r(ub2) = .0577931518155229
 r(lb_diff) = -.0159769386625226
 r(ub_diff) = .0004871117897114
 r(z) = -1.836618487454034
 r(p_l) = .0331331180748532
 r(p) = .0662662361497065
 r(p_u) = .9668668819251468
 r(level) = 95

```

The statistics we want to see are stored as `r(P1)`, `r(P2)`, `r(P_diff)`, and `r(p)`. We can specify this in the `command()` option by typing

```
. table ..., command(r(P1) r(P2) r(P_diff) r(p): prtest diabetes, by(sex))
```

This will get the results we want into our table. Furthermore, because we know what these values represent, we can give them names that will appear in the table headers. We can, for instance, type

```
. table ..., command(Males=r(P1) Females=r(P2) Difference=r(P_diff) ///
 r(p): prtest diabetes, by(sex))
```

We can specify similar `command()` options for `heartatk` and `highbp` as well.

In addition, we need to specify how our results will be laid out in the table. Below, we type `command` in the first set of parentheses so that the rows correspond to the different commands. We type `result` in the second set of parentheses to specify that statistics appear in the columns.

Finally, we add two options to customize the results. We specify a numeric format so that the statistics be displayed only with three digits after the decimal. We also choose the predefined style `table-right` so that our row headers will be right-aligned. See [\[TABLES\] Predefined styles](#) for information on this and other styles.

```

. table (command) (result),
> command(Males=r(P1) Females=r(P2) Difference=r(P_diff) r(p):
> prtest diabetes, by(sex))
> command(Males=r(P1) Females=r(P2) Difference=r(P_diff) r(p):
> prtest heartatk, by(sex))
> command(Males=r(P1) Females=r(P2) Difference=r(P_diff) r(p):
> prtest highbp, by(sex))
> nformat(%5.3f) style(table-right)

```

|                          | Males | Females | Difference | Two-sided p-value |
|--------------------------|-------|---------|------------|-------------------|
| prtest diabetes, by(sex) | 0.044 | 0.052   | -0.008     | 0.066             |
| prtest heartatk, by(sex) | 0.065 | 0.029   | 0.036      | 0.000             |
| prtest highbp, by(sex)   | 0.469 | 0.381   | 0.088      | 0.000             |

Our table now includes all the statistics we want. If we are happy with it, we can export it to another format, such as HTML, Word, L<sup>A</sup>T<sub>E</sub>X, PDF, and Excel using the `export()` option. But first we want to make some modifications. Table customization can go beyond the predefined styles and options available to you in the `table` command. `table` creates a collection of results that can be used in combination with the `collect` suite of commands to produce highly customized tables and to export those tables to presentation-ready formats, such as HTML, Word, L<sup>A</sup>T<sub>E</sub>X, PDF, Excel, and more.

For this table, we want to modify the labels in our row headers. Instead of showing the full command that was run, row headers will identify the variable we are testing. In addition, we will modify the label for our *p*-value. We want to use the label `p-value`. Because this is not a valid Stata name, we could not specify it in the `table` command as we did with `Males`. However, we can use `collect label levels` to modify the label on our *p*-values.

After applying the label updates, we use `collect preview` to see our updated table.

```
. collect label levels command 1 "Diabetes" 2 "Heart attack" 3 "High BP", modify
. collect label levels result p "p-value", modify
. collect preview
```

|              | Males | Females | Difference | p-value |
|--------------|-------|---------|------------|---------|
| Diabetes     | 0.044 | 0.052   | -0.008     | 0.066   |
| Heart attack | 0.065 | 0.029   | 0.036      | 0.000   |
| High BP      | 0.469 | 0.381   | 0.088      | 0.000   |

## Creating tables from matrices

You may find that the results you want to include in your table are stored in a matrix; these results can also be easily included in a table.

To demonstrate, we create a table with *p*-values for tests of normality for `height`, `weight`, and diastolic blood pressure (`bpdiast`). The command `sktest` performs tests based on skewness, kurtosis, and a combined test statistic.

```
. sktest height weight bpdiast
Skewness and kurtosis tests for normality
```

| Variable | Obs    | Pr(skewness) | Pr(kurtosis) | Joint test  |           |
|----------|--------|--------------|--------------|-------------|-----------|
|          |        |              |              | Adj chi2(2) | Prob>chi2 |
| height   | 10,351 | 0.0000       | 0.0000       | 147.47      | 0.0000    |
| weight   | 10,351 | 0.0000       | 0.0000       | 801.40      | 0.0000    |
| bpdiast  | 10,351 | 0.0000       | 0.0000       | 362.54      | 0.0000    |

Let's look at the returned results.

```
. return list
scalars:
 r(N) = 10351
 r(p_skew) = 1.58706287446e-72
 r(p_kurt) = 6.22330331716e-26
 r(chi2) = 362.5385838320567
 r(p_chi2) = 1.88689086684e-79

matrices:
 r(table) : 3 x 5
```

The statistics we want plus a few others are stored in `r(table)`.

Now, let's place these values in a table. We specify that our table be arranged with the row names (`rowname`) of the matrix defining the rows of the table. Similarly, the column names (`colname`) of the matrix define the columns of the table. Then, we specify that we want to collect the results from the matrix `r(table)` from the `sktest` command.

```
. table (rowname) (colname),
> command(r(table): sktest height weight bpdiastr)
```

|                          | N     | p_skew   | p_kurt   | chi2     | p_chi2   |
|--------------------------|-------|----------|----------|----------|----------|
| Height (cm)              | 10351 | .0000179 | 1.87e-35 | 147.4712 | 9.48e-33 |
| Weight (kg)              | 10351 | 1.6e-166 | 1.63e-49 | 801.3958 | 9.5e-175 |
| Diastolic blood pressure | 10351 | 1.59e-72 | 6.22e-26 | 362.5386 | 1.89e-79 |

Because the row names in `r(table)` corresponded to variables, our table automatically put the variable labels in the row headers. However, the column headers are not nicely labeled.

We can create better labels and modify our table in many other ways. `table` creates a collection of results that can be used in combination with the `collect` suite of commands to further customize tables.

To clean up our table, let's use `collect label levels` to modify the labels for the  $p$ -values for the skewness, kurtosis, and joint tests; these are the statistics we will include in our table below. To use `collect label levels`, we need to know just a little about the `collect` system. In collections, values are organized according to dimensions and levels within those dimensions. In fact, we use these dimensions in `table`. The keywords that we can use to define our rows and columns are dimensions. Here `colname` is our dimension that defines the columns, and its levels are `N`, `p_skew`, `...`. To modify labels, we need to tell `collect label levels` which dimension (`colname`) we would like to change and then specify labels for levels of that dimension.

We specify the dimension and then the label for each level:

```
. collect label levels colname p_skew "Skewness p-value"
> p_kurt "Kurtosis p-value" p_chi2 "Joint p-value", modify
```

To learn more about modifying labels, see [\[TABLES\] collect label](#).

Let's also change the numeric format of our  $p$ -values. With `collect style cell`, we can modify all cells in the table, all cells in a particular dimension, or particular cells of a particular dimension. Below, we specify the numeric formatting for only three levels of `colname`.

```
. collect style cell colname[p_skew p_kurt p_chi2], nformat(%7.3f)
```

Finally, we want to show only the three  $p$ -values in our table. We can use `collect layout` to specify the statistics we want to include in our final table.

```
. collect layout (rowname) (colname[p_skew p_kurt p_chi2])
Collection: Table
 Rows: rowname
 Columns: colname[p_skew p_kurt p_chi2]
Table 1: 3 x 3
```

|                          | Skewness p-value | Kurtosis p-value | Joint p-value |
|--------------------------|------------------|------------------|---------------|
| Height (cm)              | 0.000            | 0.000            | 0.000         |
| Weight (kg)              | 0.000            | 0.000            | 0.000         |
| Diastolic blood pressure | 0.000            | 0.000            | 0.000         |

Notably, all  $p$ -values for all tests are very small, so this is not a particularly exciting table. However, our table customizations made it easy to quickly see the results of tests of normality for all our variables.

## Stored results

`table` stores the following in `r()`:

```
Scalars
 r(N) number of observations
```

## Reference

McDowell, A., A. Engel, J. T. Massey, and K. Maurer. 1981. "Plan and operation of the Second National Health and Nutrition Examination Survey, 1976–1980". In *Vital and Health Statistics*, ser. 1, no. 15. Hyattsville, MD: National Center for Health Statistics.

## Also see

[R] [table](#) — Table of frequencies, summaries, and command results

[R] [table intro](#) — Introduction to tables of frequencies, summaries, and command results

[R] [table regression](#) — Table of regression results

[TABLES] [Intro](#) — Introduction

|                             |                                      |                                |                            |
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| <a href="#">Options</a>     | <a href="#">Remarks and examples</a> | <a href="#">Stored results</a> | <a href="#">References</a> |
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## Description

In this entry, we discuss how to create tables of regression results. These tables can include coefficients, standard errors, confidence intervals, and many more results stored by estimation commands.

## Quick start

Table of regression coefficients; rows correspond to covariates (`colname`)

```
table colname, command(regress y x1 x2 x3)
```

Table of coefficients and confidence intervals; columns correspond to the statistics (`result`)

```
table (colname) (result), command(_r_b _r_ci: regress y x1 x2 x3)
```

Same as above, but use the labels defined in `mylabels.stjson` and the styles in `mystyle.stjson`

```
table (colname) (result), ///
 command(_r_b _r_ci: regress y x1 x2 x3) ///
 label(mylabels) style(mystyle)
```

## Menu

Statistics > Summaries, tables, and tests > Tables of frequencies, summaries, and command results

## Syntax

```
table ([rowspec]) ([colspec]) ([tabspec]) [if] [in] [weight],
 command(cmdsspec) [command(cmdsspec) ...] [options]
```

*rowspec*, *colspec*, and *tabspec* may be empty or may include variable names or any of the following keywords:

| <i>keyword</i> | Description                                 |
|----------------|---------------------------------------------|
| result         | requested statistics                        |
| stars          | stars denoting statistical significance     |
| command        | index option command()                      |
| colname        | column names for matrix statistics          |
| rowname        | row names for matrix statistics             |
| coleq          | column equation names for matrix statistics |
| roweq          | row equation names for matrix statistics    |

| <i>options</i>                                                | Description                                               |
|---------------------------------------------------------------|-----------------------------------------------------------|
| <b>Commands</b>                                               |                                                           |
| command( <i>cmds</i> spec)                                    | collect results from the specified Stata command          |
| <b>Formats</b>                                                |                                                           |
| nformat( <i>%fmt</i> [ <i>results</i> ][, <i>basestyle</i> ]) | specify numeric format                                    |
| sformat( <i>sfmt</i> [ <i>results</i> ])                      | specify string format                                     |
| cidelimiter( <i>char</i> )                                    | use character as delimiter for confidence interval limits |
| cridelimiter( <i>char</i> )                                   | use character as delimiter for credible interval limits   |
| <b>Stars</b>                                                  |                                                           |
| stars( <i>stars</i> spec)                                     | add stars to denote statistical significance              |
| <b>Title</b>                                                  |                                                           |
| title( <i>string</i> )                                        | add table title                                           |
| titlestyles( <i>text_styles</i> )                             | change table title styles                                 |
| <b>Notes</b>                                                  |                                                           |
| note( <i>string</i> )                                         | add table note                                            |
| notestyles( <i>text_styles</i> )                              | change table note styles                                  |
| <b>Export</b>                                                 |                                                           |
| export( <i>filename.suffix</i> [, <i>export_opts</i> ])       | export table                                              |
| <b>Options</b>                                                |                                                           |
| missing                                                       | treat numeric missing values like other values            |
| name( <i>cname</i> )                                          | collect results into a collection named <i>cname</i>      |
| append                                                        | append results to an existing collection                  |
| replace                                                       | replace results of an existing collection                 |
| label( <i>filename</i> )                                      | specify the collection labels                             |
| style( <i>filename</i> [, <i>override</i> ])                  | specify the collection style                              |
| noisily                                                       | display output from each command                          |

fweights, aweights, iweights, and pweights are allowed; see [U] 11.1.6 **weight**.

strL variables are not allowed; see [U] 12.4.8 **strL**.

noisily does not appear in the dialog box.

| <i>text_styles</i>                                | Description                                              |
|---------------------------------------------------|----------------------------------------------------------|
| font( <i>[fontfamily]</i> [ <i>, font_opts</i> ]) | specify font style                                       |
| smcl( <i>smcl</i> )                               | specify formatting for SMCL files                        |
| latex( <i>latex</i> )                             | specify L <sup>A</sup> T <sub>E</sub> X macro            |
| shading( <i>sspec</i> )                           | set background color, foreground color, and fill pattern |

| <i>font_opts</i>          | Description                              |
|---------------------------|------------------------------------------|
| size( <i># [unit]</i> )   | specify font size                        |
| color( <i>color</i> )     | specify font color                       |
| variant( <i>variant</i> ) | specify font variant and capitalization  |
| [no]bold                  | specify whether to format text as bold   |
| [no]italic                | specify whether to format text as italic |
| [no]strikeout             | specify whether to strike out text       |
| [no]underline             | specify whether to underline text        |

| <i>suffix</i> | <i>fileformat</i> | Output format                      |
|---------------|-------------------|------------------------------------|
| docx          | as(docx)          | Microsoft Word                     |
| html          | as(html)          | HTML 5 with CSS                    |
| pdf           | as(pdf)           | PDF                                |
| xlsx          | as(xlsx)          | Microsoft Excel 2007/2010 or newer |
| xls           | as(xls)           | Microsoft Excel 1997/2003          |
| tex           | as(tex)           | L <sup>A</sup> T <sub>E</sub> X    |
| smcl          | as(smcl)          | SMCL                               |
| txt           | as(txt)           | plain text                         |
| markdown      | as(markdown)      | Markdown                           |
| md            | as(md)            | Markdown                           |

| <i>export_opts</i>      | Description                                         |
|-------------------------|-----------------------------------------------------|
| as( <i>fileformat</i> ) | specify document type                               |
| replace                 | overwrite existing file                             |
| <i>docx_options</i>     | available when exporting to .docx files             |
| <i>html_options</i>     | available when exporting to .html files             |
| <i>pdf_options</i>      | available when exporting to .pdf files              |
| <i>excel_options</i>    | available when exporting to .xls and .xlsx files    |
| <i>tex_options</i>      | available when exporting to .tex files              |
| <i>smcl_option</i>      | available when exporting to .smcl files             |
| <i>txt_option</i>       | available when exporting to .txt files              |
| <i>md_option</i>        | available when exporting to .markdown and .md files |

| <i>docx_options</i>                      | Description                                                         |
|------------------------------------------|---------------------------------------------------------------------|
| <code>noisily</code>                     | show the putdocx commands used to export to the Microsoft Word file |
| <code>dofile(filename[, replace])</code> | save the putdocx commands used for exporting to the named do-file   |

| <i>html_options</i>           | Description                                                    |
|-------------------------------|----------------------------------------------------------------|
| <code>append</code>           | append to an existing file                                     |
| <code>tableonly</code>        | export only the table to the specified file                    |
| <code>cssfile(cssfile)</code> | define the styles in <i>cssfile</i> instead of <i>filename</i> |
| <code>prefix(prefix)</code>   | use <i>prefix</i> to identify style classes                    |

| <i>pdf_options</i>                       | Description                                                      |
|------------------------------------------|------------------------------------------------------------------|
| <code>noisily</code>                     | show the putpdf commands used to export to the PDF file          |
| <code>dofile(filename[, replace])</code> | save the putpdf commands used for exporting to the named do-file |

| <i>excel_options</i>                     | Description                                                                                             |
|------------------------------------------|---------------------------------------------------------------------------------------------------------|
| <code>noisily</code>                     | show the putexcel commands used to export to the Excel file                                             |
| <code>dofile(filename[, replace])</code> | save the putexcel commands used for exporting to the named do-file                                      |
| <code>sheet(sheetname[, replace])</code> | specify the worksheet to use; the default sheet name is Sheet1                                          |
| <code>cell(cell)</code>                  | specify the Excel upper-left cell as the starting position to export the table; the default is cell(A1) |
| <code>modify</code>                      | modify Excel file                                                                                       |
| <code>noopen</code>                      | do not open Excel file in memory                                                                        |

`noopen` does not appear in the dialog box.

| <i>tex_options</i>     | Description                                 |
|------------------------|---------------------------------------------|
| <code>append</code>    | append to an existing file                  |
| <code>tableonly</code> | export only the table to the specified file |

| <i>smcl_option</i>  | Description                |
|---------------------|----------------------------|
| <code>append</code> | append to an existing file |

| <i>txt_option</i>   | Description                |
|---------------------|----------------------------|
| <code>append</code> | append to an existing file |

| <i>md_option</i>    | Description                |
|---------------------|----------------------------|
| <code>append</code> | append to an existing file |

*fontfamily* specifies a valid font family.

*unit* may be in (inch), pt (point), or cm (centimeter). An inch is equivalent to 72 points and 2.54 centimeters. The default is pt.

*variant* may be allcaps, smallcaps, or normal.

*variant* (allcaps) changes the text to all uppercase letters; applicable when publishing items from a collection to Microsoft Word, PDF, L<sup>A</sup>T<sub>E</sub>X, and HTML files.

*variant* (smallcaps) changes the text to use large capitals for uppercase letters and smaller capitals for lowercase letters; applicable when publishing items from a collection to Microsoft Word, L<sup>A</sup>T<sub>E</sub>X, and HTML files.

*variant* (normal) changes the font variant back to normal and leaves the capitalization unchanged from the original text; applicable when publishing items from a collection to Microsoft Word, PDF, L<sup>A</sup>T<sub>E</sub>X, and HTML files.

*smcl* specifies the name of the SMCL directive to render text for SMCL output. The supported SMCL directives are `input`, `error`, `result`, and `text`.

*latex* specifies the name of a L<sup>A</sup>T<sub>E</sub>X macro to render text for L<sup>A</sup>T<sub>E</sub>X output. Example L<sup>A</sup>T<sub>E</sub>X macro names are `textbf`, `textsf`, `textrm`, and `texttt`. Custom L<sup>A</sup>T<sub>E</sub>X macros are also allowed. If *text* is to be rendered in a cell, title, or note, then *latex* is translated to the following when you export to L<sup>A</sup>T<sub>E</sub>X:

```
\latex {text}
```

*sspec* is

```
[background(bcolor) foreground(fgcolor) pattern(fpattern)]
```

*bcolor* specifies the background color.

*fgcolor* specifies the foreground color.

*fpattern* specifies the fill pattern. A complete list of fill patterns is shown in the [Appendix](#).

*bcolor*, *fgcolor*, and *color* may be one of the colors listed in the [Appendix](#); a valid RGB value in the form `### ## #`, for example, `171 248 103`; or a valid RRGGBB hex value in the form `#####`, for example, `ABF867`.

## Options

### Commands

`command(cmdsspec)` specifies the Stata commands from which to collect results. `command()` may be repeated to collect results from multiple commands.

*cmds*spec is `[explist:] command [arguments] [, cmdoptions]`

*explist* specifies which results to collect and report in the table. *explist* may include *result identifiers* and *named expressions*.

*result identifiers* are results stored in `r()` and `e()` by the *command*. For instance, *result identifiers* could be `r(mean)`, `r(C)`, or `e(chi2)`. After estimation commands, *result identifiers* also include the following:

| Identifier            | Result                                                              |
|-----------------------|---------------------------------------------------------------------|
| <code>_r_b</code>     | coefficients or transformed coefficients reported by <i>command</i> |
| <code>_r_se</code>    | standard errors of <code>_r_b</code>                                |
| <code>_r_z</code>     | test statistics for <code>_r_b</code>                               |
| <code>_r_z_abs</code> | absolute value of <code>_r_z</code>                                 |
| <code>_r_p</code>     | <i>p</i> -values for <code>_r_b</code>                              |
| <code>_r_lb</code>    | lower bounds of confidence intervals for <code>_r_b</code>          |
| <code>_r_ub</code>    | upper bounds of confidence intervals for <code>_r_b</code>          |
| <code>_r_ci</code>    | confidence intervals for <code>_r_b</code>                          |
| <code>_r_crlb</code>  | lower bounds of credible intervals for <code>_r_b</code>            |
| <code>_r_crub</code>  | upper bounds of credible intervals for <code>_r_b</code>            |
| <code>_r_cri</code>   | credible intervals for <code>_r_b</code>                            |
| <code>_r_df</code>    | degrees of freedom for <code>_r_b</code>                            |

*named expressions* are specified as `name = exp`, where *name* may be any valid Stata name and *exp* is an expression, typically an expression that involves one or more *result identifiers*. An example of a named expression is `sd = sqrt(r(variance))`.

For *r*-class commands, the default is to include all numeric scalars posted to `r()` in the table results. For *e*-class commands, the default is to include `_r_b` in the table results.

*command* is any command that follows standard Stata syntax.

*arguments* may be anything so long as they do not include an *if* clause, *in* range, or weight specification.

Any *if* or *in* qualifier and weights should be specified directly with `table`, not within the `command()` option. Weights are passed to *command* only if they are specified.

*cmdoptions* may be anything supported by *command*.

### Formats

`nformat(%fmt [results][, basestyle])` changes the numeric format, such as the number of decimal places, for specified results. If *results* are not specified, the numeric format is changed for all results.

*results* may be any name in the `e()` or `r()` results produced by commands specified in option `command()`.

This option is repeatable, and when multiple formats apply to one result, the rightmost specification is applied.

This option does not affect the format of numeric layout variables (*rowspec*, *colspec*, and *tabspec*). The default format of these variables is taken from the dataset.

*basestyle* indicates that the format be applied to results that do not already have their own format instead of overriding the format for all results.

*sformat*(*sfmt* [*results*]) changes the string format for specified results. You can, for instance, add symbols or text to the values reported in the table by modifying the string format.

*sfmt* may contain a mix of text and %s. Here %s refers to the numeric value that is formatted as specified using *nformat*( ). The text will be placed around the numeric values in your table as it is placed around %s in this option. For instance, to place parentheses around the percent statistics, you can specify *sformat*("(%s)" percent).

*results* may be any name in the *e*( ) or *r*( ) results produced by commands specified in option *command*( ).

Two text characters must be specified using a special character sequence if you want them to be displayed in your table. To include %, type %%. To include \, type \\. For instance, to place a percent sign following percent statistics, you can specify *sformat*("%%%" percent).

This option is repeatable, and when multiple formats apply to one result, the rightmost specification is applied.

*cidelimiter*(*char*) changes the delimiter between confidence interval limits to *char*. The default is *cidelimiter*(" "), that is, two spaces.

*cridelimiter*(*char*) changes the delimiter between credible interval limits to *char*. The default is *cridelimiter*(" "), that is, two spaces.

---

#### Stars

*stars*(*starspec*) specifies that stars representing statistical significance be included in the table. *starspec* identifies the result whose values determine significance, which characters should represent each significance level, and where these characters should be displayed in the table. *starspec* is

```
starres [#1 "label1" [#2 "label2" [#3 "label3" [#4 "label4" [#5 "label5"]]]]]
[, attach(attachres) result dimension starsnoteopts]
```

*starres* is the name of the result whose values determine which characters, typically which number of stars, are to be displayed.

*label1* specifies the characters to be displayed when *starres* < #1.

*label2* specifies the characters to be displayed when *starres* < #2.

*label3* specifies the characters to be displayed when *starres* < #3.

*label4* specifies the characters to be displayed when *starres* < #4.

*label5* specifies the characters to be displayed when *starres* < #5.

*attach*(*attachres*) specifies the name of the result to which the characters defined by *label1*, ..., *label5* are to be attached. If *attach*( ) is not specified, a new result named *stars* is created and is automatically added to the table.

*result* and *dimension* control how *collect stars* adds items when labeling significant results. These options are mutually exclusive.

*result* specifies the default behavior, and this option is necessary only if the following *dimension* behavior is in effect and you want to change back to the *result* behavior.

`dimension` specifies that dimension stars be added to the collection. Items will be tagged with `stars [value]`, and the labels will be tagged with `stars [label]`. Use this option for layouts where results are to be stacked within columns, and use new dimension stars in the column specification of the layout.

`starsnoteopts` control the display and composition of the stars note.

`noshownote` and `shownote` control whether to display the stars note.

`increasing` and `decreasing` control the order of  $p$ -values in the stars note.

`pvname(string)` specifies a name for the  $p$ -value in the stars note. The default is `pvname(p)`.

`delimiter(string)` specifies the delimiter between labels in the stars note. The default is `delimiter(",")`.

`nformat(%fmt)` specifies the numeric format for the cutoff values in the stars note. The default is `nformat(%9.0g)`.

`prefix(string)` specifies the prefix for the stars note. The prefix is empty by default.

`suffix(string)` specifies the suffix for the stars note. The suffix is empty by default.

For example, `stars(_r_p 0.01 "***" 0.05 "*" 0.1 "*", attach(_r_b))` could be added to a table of regression results to specify that stars be defined based on the  $p$ -values in `_r_p` and be attached to the reported coefficients (`_r_b`).

Title

`title(string)` adds the text *string* as a title to the table.

`titlestyles(text_styles)` changes the style for the table title. *text\_styles* are the following:

`font([fontfamily] [, size(# [unit]) color(color) variant(variant) [no]bold [no]italic [no]strikeout [no]underline])` specifies the font style. These font style properties are applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files, unless otherwise specified.

*fontfamily* specifies a valid font family. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`size(# [unit])` specifies the font size as a number optionally followed by units. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`color(color)` specifies the text color.

`variant(variant)` specifies the font variant and capitalization.

`bold` and `nobold` specify the font weight. `bold` changes the font weight to bold; `nobold` changes the font weight back to normal.

`italic` and `noitalic` specify the font style. `italic` changes the font style to italic; `noitalic` changes the font style back to normal.

`strikeout` and `nostrikeout` specify whether to add a strikeout mark to the title. `strikeout` adds a strikeout mark to the title; `nostrikeout` changes the title back to normal.

`underline` and `nounderline` specify whether to underline the table title. `underline` adds a single line under the title; `nounderline` removes the underline.

Only one of `strikeout` or `underline` is allowed when publishing to HTML files.

`smcl(smcl)` specifies how to render the table title for SMCL output. This style property is applicable only when publishing items from a collection to a SMCL file.

`latex(latex)` specifies how to render the table title for  $\LaTeX$  output. This style property is applicable only when publishing items from a collection to a  $\LaTeX$  file.

`shading(sspec)` sets the background color, foreground color, and fill pattern. The background color is applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files. The foreground color and fill pattern are applicable when exporting the table to Microsoft Word and Microsoft Excel.

## Notes

`note(string)` adds the text *string* as a note to the table. `note()` may be specified multiple times to add multiple notes. Each note is placed on a new line.

`notestyles(text_styles)` changes the style for the table notes. *text\_styles* are the following:

`font([fontfamily] [, size(# [unit]) color(color) variant(variant) [no]bold [no]italic [no]strikeout [no]underline])` specifies the font style. These font style properties are applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files, unless otherwise specified.

*fontfamily* specifies a valid font family. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`size(# [unit])` specifies the font size as a number optionally followed by units. This font style property is applicable when publishing items from a collection to Microsoft Word, Microsoft Excel, PDF, and HTML files.

`color(color)` specifies the text color.

`variant(variant)` specifies the font variant and capitalization.

`bold` and `nobold` specify the font weight. `bold` changes the font weight to bold; `nobold` changes the font weight back to normal.

`italic` and `noitalic` specify the font style. `italic` changes the font style to italic; `noitalic` changes the font style back to normal.

`strikeout` and `nostrikeout` specify whether to add a strikeout mark to the notes. `strikeout` adds a strikeout mark to the note; `nostrikeout` changes the note back to normal.

`underline` and `nounderline` specify whether to underline the table notes. `underline` adds a single line under the notes; `nounderline` removes the underline.

Only one of `strikeout` or `underline` is allowed when publishing to HTML files.

`smcl(smcl)` specifies how to render the table notes for SMCL output. This style property is applicable only when publishing items from a collection to a SMCL file.

`latex(latex)` specifies how to render the table notes for  $\LaTeX$  output. This style property is applicable only when publishing items from a collection to a  $\LaTeX$  file.

`shading(sspec)` sets the background color, foreground color, and fill pattern. The background color is applicable when exporting the table to Microsoft Word, Microsoft Excel, PDF, HTML, and  $\LaTeX$  files. The foreground color and fill pattern are applicable when exporting the table to Microsoft Word and Microsoft Excel.

Export

`export(filename.suffix[ , export_opts ])` exports the table to the specified file. *export\_opts* are the following:

`as(fileformat)` specifies the file format to which the table is to be exported. This option is rarely specified because, by default, `table` determines the format from the suffix of the file being created.

`replace` permits `table` to overwrite an existing file.

`noisily` specifies that `table` show the commands used to export the table to Microsoft Word, Microsoft Excel, and PDF files. The `putdocx`, `putexcel`, or `putpdf` command used to export the table will be displayed.

`dofile(filename[ , replace ])` specifies that `table` save to *filename* the commands used to export the table to Microsoft Word, Microsoft Excel, and PDF files.

If *filename* already exists, it can be overwritten by specifying `replace`. If *filename* is specified without an extension, `.do` is assumed.

`append` specifies that `table` append the table to an existing file.

This option is applicable when you export the table to an HTML, a  $\LaTeX$ , a SMCL, a `txt`, or a Markdown file. When you export to HTML and  $\LaTeX$  files, the `append` option implies the `tableonly` option. Furthermore, when you export to HTML files, if the target CSS file already exists, `table` will also append to it.

`tableonly` specifies that only the table be exported to the specified HTML or  $\LaTeX$  document. By default, `table` produces complete HTML and  $\LaTeX$  documents.

When you export to an HTML file, if the `cssfile()` option is not specified, a CSS filename is constructed from *filename*, with the extension replaced with `.css`.

`cssfile(cssfile)` specifies that `table` define the styles in *cssfile* instead of *filename* when you export to HTML.

`prefix(prefix)` specifies that `table` use *prefix* to identify style classes when you export to HTML.

`sheet(sheetname [ , replace ])` saves to the worksheet named *sheetname*. For more information about this option, see [\[RPT\] putexcel](#).

`cell(cell)` specifies an Excel upper-left cell as the starting position to publish the table. The default is `cell(A1)`.

`modify` permits `putexcel` set to modify an Excel file. For more information about this option, see [\[RPT\] putexcel](#).

`noopen` prevents `putexcel` from opening the Excel file in memory for modification. It does not appear in the dialog box. For more information about this option, see [\[RPT\] putexcel](#).

## Options

`missing` specifies that numeric missing values of any variables specified in `rowspec`, `colspec`, or `tabspec` be treated as valid categories. By default, observations with a numeric missing value in any of these variables are omitted.

`name(cname)` specifies that a collection named *cname* be associated with the collected statistics and results. The default is `name(Table)`.

`append` specifies that `table` append its collection information into the collection named in `name()`.

`replace` permits `table` to overwrite an existing collection. This option is implied for `name(Table)` when `append` is not specified.

`label(filename)` specifies the *filename* containing the collection labels to use for your table. Labels in *filename* will be loaded for the table, and any labels not specified in *filename* will be taken from the labels defined in `c(collect_label)`. The default is to use only the collection labels set in `c(collect_label)`; see [TABLES] [set collect\\_label](#).

`style(filename [, override])` specifies the *filename* containing the collection styles to use for your table. The default collection styles will be discarded, and only the collection styles in *filename* will be applied.

If you prefer the default collection styles but also want to apply any styles in *filename*, specify *override*. If there are conflicts between the default collection styles and those in *filename*, the ones in *filename* will take precedence.

The default is to use only the collection styles set in `c(table_style)`; see [TABLES] [set table\\_style](#).

The following option is available with `table` but is not shown in the dialog box:

`noisily` specifies that output from the commands specified in `command()` options be displayed. By default, output from commands is suppressed.

## Remarks and examples

Remarks are presented under the following headings:

[Introduction](#)

[Tables with results from a single command](#)

[Tables with results from multiple estimation commands](#)

[Regression results with factor variables](#)

### Introduction

The `table` command allows us to create tables of regression results. You can create a table that reports coefficients, standard errors, test statistics, confidence intervals, and other statistics from a single model or a table that compares results from multiple models.

`table` does not fit models directly. Rather, `table` will run any Stata command that you include in its `command()` option and place results from that command into the table. You determine which results you would like to see in the table. You can select any of the results stored by the command.

You can also create a table of regression results with `etable`. However, `etable` will create tables only with active estimation results, results from `margins`, or results stored with `estimates store`. If you are working with any of these results, you can use `etable` to create and export a table of regression results. However, if you want to include results from other commands, you should use the `table` command.

## Tables with results from a single command

We have data from the Second National Health and Nutrition Examination Survey (NHANES II) (McDowell et al. 1981). The data contain some demographic information, such as the participants' age. The data also contain some measures of health, including the individual's weight, systolic blood pressure (bpsystol), and whether the individual has diabetes.

Here we will create a table with results from a linear regression model for systolic blood pressure as a function of age and weight. We type the command to fit the model in the `command()` option. In the first set of parentheses following `table`, we specify that we want the rows to correspond to the levels of `colname`—this is how we refer to the list of covariates in our regression model. In the second set of parentheses, we specify that we want the columns to correspond to the statistics (`result`).

```
. use https://www.stata-press.com/data/r19/nhanes21
(Second National Health and Nutrition Examination Survey)
. table (colname) (result), command(regress bpsystol age weight)
```

|             | Coefficient |
|-------------|-------------|
| Age (years) | .6379892    |
| Weight (kg) | .4069041    |
| Intercept   | 71.27096    |

Our table is fairly simple. By default, `table` includes only the reported coefficients when an estimation command is specified in the `command()` option.

The `table` command can easily be used to compare results across groups in our data. For instance, if we want to fit the same model for males and females, we can add `sex` to our column specification.

```
. table (colname) (sex result), command(regress bpsystol age weight)
```

|             | Sex      |          |          |
|-------------|----------|----------|----------|
|             | Male     | Female   | Total    |
| Age (years) | .4789361 | .7735499 | .6379892 |
| Weight (kg) | .3346106 | .4586108 | .4069041 |
| Intercept   | 84.08037 | 61.70456 | 71.27096 |

We can now easily compare results for males, females, and both together.

We may want to see additional statistics reported. Let's extend our table to include both coefficients and standard errors. We can refer to the reported coefficients using the keyword `_r_b` and to the reported standard errors as `_r_se`, and we can list these in the `command()` option before our regression command. Here we also move `result` to the first set of parentheses so that coefficients and standard errors will be displayed on separate rows.

```
. table (colname result) (sex),
> command(_r_b _r_se: regress bpsystol age weight)
```

|             | Sex      |          | Total    |
|-------------|----------|----------|----------|
|             | Male     | Female   |          |
| Age (years) |          |          |          |
| Coefficient | .4789361 | .7735499 | .6379892 |
| Std. error  | .0156578 | .0155743 | .0111315 |
| Weight (kg) |          |          |          |
| Coefficient | .3346106 | .4586108 | .4069041 |
| Std. error  | .0197112 | .0182401 | .0124786 |
| Intercept   |          |          |          |
| Coefficient | 84.08037 | 61.70456 | 71.27096 |
| Std. error  | 1.74867  | 1.376067 | 1.041742 |

We now have the statistics we want in this table, but we may want to modify the look a bit. `table` allows us to customize the results in our table in a number of ways. We can use the `nformat()` option to report all results to two decimal places, and we can use the `sformat()` option to place parentheses around our standard errors.

```
. table (colname result) (sex),
> command(_r_b _r_se: regress bpsystol age weight)
> nformat(%6.2f) sformat("(%s)" _r_se)
```

|             | Sex    |        | Total  |
|-------------|--------|--------|--------|
|             | Male   | Female |        |
| Age (years) |        |        |        |
| Coefficient | 0.48   | 0.77   | 0.64   |
| Std. error  | (0.02) | (0.02) | (0.01) |
| Weight (kg) |        |        |        |
| Coefficient | 0.33   | 0.46   | 0.41   |
| Std. error  | (0.02) | (0.02) | (0.01) |
| Intercept   |        |        |        |
| Coefficient | 84.08  | 61.70  | 71.27  |
| Std. error  | (1.75) | (1.38) | (1.04) |

Now that we have the parentheses to distinguish standard errors from coefficients, we may not want to see those labels in the row header. We add the `style(table-reg3)` option to use the predefined style `table-reg3`, which hides the names of these statistics, right-aligns the names of the variables in the row headers, center aligns the statistics horizontally within each column, and adds vertical space between variables.

```
. table (colname result) (sex),
> command(_r_b _r_se: regress bpsystol age weight)
> nformat(%05.3f) sformat("%s" _r_se) style(table-reg3)
```

|             | Sex               |                   | Total             |
|-------------|-------------------|-------------------|-------------------|
|             | Male              | Female            |                   |
| Age (years) | 0.479<br>(0.016)  | 0.774<br>(0.016)  | 0.638<br>(0.011)  |
| Weight (kg) | 0.335<br>(0.020)  | 0.459<br>(0.018)  | 0.407<br>(0.012)  |
| Intercept   | 84.080<br>(1.749) | 61.705<br>(1.376) | 71.271<br>(1.042) |

## Tables with results from multiple estimation commands

Above, we fit the same model to the full dataset and then to groups of observations within that dataset. We may alternatively want to fit different models and display their results in a single table. To do this, we specify multiple `command()` options.

```
. table (colname result) (command),
> command(_r_b _r_se: regress bpsystol age weight)
> command(_r_b _r_se: regress bpsystol age weight iron vitaminc zinc)
> nformat(%6.2f) sformat("%s" _r_se) style(table-reg3)
```

|                         | 1               | 2               |
|-------------------------|-----------------|-----------------|
| Age (years)             | 0.64<br>(0.01)  | 0.64<br>(0.01)  |
| Weight (kg)             | 0.41<br>(0.01)  | 0.40<br>(0.01)  |
| Serum iron (mcg/dL)     |                 | -0.01<br>(0.01) |
| Serum vitamin C (mg/dL) |                 | -0.79<br>(0.36) |
| Serum zinc (mcg/dL)     |                 | -0.05<br>(0.01) |
| Intercept               | 71.27<br>(1.04) | 77.50<br>(1.75) |

We may want to modify this table a bit further. Customization of tables can go beyond the predefined styles and options available to you in the `table` command. `table` creates a collection of results that can be used in combination with the `collect` suite of commands to produce highly customized tables.

If we want to add more descriptive labels for the two models, we can use the `collect label levels` command to define our new labels. After a change using `collect`, we can type `collect preview` to see the results.

```
. collect label levels command 1 "Model 1" 2 "Model 2", modify
. collect style header command, level(label)
. collect preview
```

|                         | Model 1         | Model 2         |
|-------------------------|-----------------|-----------------|
| Age (years)             | 0.64<br>(0.01)  | 0.64<br>(0.01)  |
| Weight (kg)             | 0.41<br>(0.01)  | 0.40<br>(0.01)  |
| Serum iron (mcg/dL)     |                 | -0.01<br>(0.01) |
| Serum vitamin C (mg/dL) |                 | -0.79<br>(0.36) |
| Serum zinc (mcg/dL)     |                 | -0.05<br>(0.01) |
| Intercept               | 71.27<br>(1.04) | 77.50<br>(1.75) |

## Regression results with factor variables

The examples above included only continuous covariates in the models. When we include factor variables, there are a variety of ways that they can be displayed in the headers of the tables. In [\[TABLES\] Pre-defined styles](#), you will find a number of styles that you can choose from. We demonstrate a few here.

We will start with the `table-reg1` style. This style is our default table style, except that it identifies the commands in the headers using values 1, 2, ... rather than labeling them with the full command we typed in the `command()` option.

```
. table (colname) (command result),
> command(regress bpsystol i.agegrp i.sex weight)
> command(regress bpsystol i.agegrp##i.sex weight)
> style(table-reg1)
```

|                              | 1        | 2         |
|------------------------------|----------|-----------|
| Age group=20-29              | 0        | 0         |
| Age group=30-39              | 1.195226 | -.7808968 |
| Age group=40-49              | 7.251555 | 2.749774  |
| Age group=50-59              | 15.94216 | 10.43724  |
| Age group=60-69              | 22.83932 | 16.53001  |
| Age group=70+                | 30.46609 | 23.3076   |
| Sex=Male                     | 0        | 0         |
| Sex=Female                   | 1.040833 | -6.777535 |
| Weight (kg)                  | .4359741 | .4242392  |
| Age group=20-29 # Sex=Male   |          | 0         |
| Age group=20-29 # Sex=Female |          | 0         |
| Age group=30-39 # Sex=Male   |          | 0         |
| Age group=30-39 # Sex=Female |          | 3.942553  |
| Age group=40-49 # Sex=Male   |          | 0         |
| Age group=40-49 # Sex=Female |          | 8.79336   |
| Age group=50-59 # Sex=Male   |          | 0         |
| Age group=50-59 # Sex=Female |          | 10.6501   |
| Age group=60-69 # Sex=Male   |          | 0         |
| Age group=60-69 # Sex=Female |          | 12.20669  |
| Age group=70+ # Sex=Male     |          | 0         |
| Age group=70+ # Sex=Female   |          | 13.51823  |
| Intercept                    | 86.71019 | 91.57774  |

In some cases, for clarity, it is helpful to see both the factor variables and their levels. The `table-reg1` style provides this in the output.

When we have nice value labels on our factor variables, we may want to see only those. The `table-reg1-fv1` style removes the extra labels. Our table above also reports zero-valued coefficients for base categories in both the main effects of the factor variables and in their interactions. The `table-reg1-fv1` style omits the rows for the base categories in the interactions.

```
. table (colname) (command result),
> command(regress bpsystol i.agegrp i.sex weight)
> command(regress bpsystol i.agegrp##i.sex weight)
> style(table-reg1-fv1)
```

|                | 1        | 2         |
|----------------|----------|-----------|
| 20-29          | 0        | 0         |
| 30-39          | 1.195226 | -.7808968 |
| 40-49          | 7.251555 | 2.749774  |
| 50-59          | 15.94216 | 10.43724  |
| 60-69          | 22.83932 | 16.53001  |
| 70+            | 30.46609 | 23.3076   |
| Male           | 0        | 0         |
| Female         | 1.040833 | -6.777535 |
| Weight (kg)    | .4359741 | .4242392  |
| 30-39 # Female |          | 3.942553  |
| 40-49 # Female |          | 8.79336   |
| 50-59 # Female |          | 10.6501   |
| 60-69 # Female |          | 12.20669  |
| 70+ # Female   |          | 13.51823  |
| Intercept      | 86.71019 | 91.57774  |

Sometimes, the tables are more readable when the row headers are right aligned. We can use the `table-reg2-fv1` style in this case. Let's also change the numeric format of all the results so that they report only two decimal places.

```
. table (colname) (command result),
> command(regress bpsystol i.agegrp i.sex weight)
> command(regress bpsystol i.agegrp##i.sex weight)
> style(table-reg2-fv1) nformat(%6.2f)
```

|                | 1     | 2     |
|----------------|-------|-------|
| 20-29          | 0.00  | 0.00  |
| 30-39          | 1.20  | -0.78 |
| 40-49          | 7.25  | 2.75  |
| 50-59          | 15.94 | 10.44 |
| 60-69          | 22.84 | 16.53 |
| 70+            | 30.47 | 23.31 |
| Male           | 0.00  | 0.00  |
| Female         | 1.04  | -6.78 |
| Weight (kg)    | 0.44  | 0.42  |
| 30-39 # Female |       | 3.94  |
| 40-49 # Female |       | 8.79  |
| 50-59 # Female |       | 10.65 |
| 60-69 # Female |       | 12.21 |
| 70+ # Female   |       | 13.52 |
| Intercept      | 86.71 | 91.58 |

There are many ways that we can further customize our table using the `collect` suite of commands. We can add column titles for our models as we did above. In addition, we can use `collect style row` to specify a character to be used between terms in an interaction.

```
. collect label levels command 1 "Model 1" 2 "Model 2", modify
. collect style header command, level(label)
. collect style row stack, delimiter(" X ")
. collect preview
```

|                | Model 1 | Model 2 |
|----------------|---------|---------|
| 20–29          | 0.00    | 0.00    |
| 30–39          | 1.20    | -0.78   |
| 40–49          | 7.25    | 2.75    |
| 50–59          | 15.94   | 10.44   |
| 60–69          | 22.84   | 16.53   |
| 70+            | 30.47   | 23.31   |
| Male           | 0.00    | 0.00    |
| Female         | 1.04    | -6.78   |
| Weight (kg)    | 0.44    | 0.42    |
| 30–39 X Female |         | 3.94    |
| 40–49 X Female |         | 8.79    |
| 50–59 X Female |         | 10.65   |
| 60–69 X Female |         | 12.21   |
| 70+ X Female   |         | 13.52   |
| Intercept      | 86.71   | 91.58   |

If one of the predefined styles in [TABLES] **Predefined styles** does not suit your needs for factor-variable results (or for any other table customization), you can create your own style. To do this, you will use series of `collect style` commands, and then you can save the style to use later; see [TABLES] **collect style save**.

If you wish to include your table in a paper, on a webpage, or in another format, you can easily export it in L<sup>A</sup>T<sub>E</sub>X, Word, Excel, HTML, and a variety of other formats by using `collect export`.

## Stored results

`table` stores the following in `r()`:

Scalars

`r(N)` number of observations

## References

- Huber, C. 2021a. Customizable tables in Stata 17, part 5: Tables for one regression model. *The Stata Blog: Not Elsewhere Classified*. <https://blog.stata.com/2021/08/26/customizable-tables-in-stata-17-part-5-tables-for-one-regression-model/>.
- . 2021b. Customizable tables in Stata 17, part 6: Tables for multiple regression models. *The Stata Blog: Not Elsewhere Classified*. <https://blog.stata.com/2021/09/02/customizable-tables-in-stata-17-part-6-tables-for-multiple-regression-models/>.
- McDowell, A., A. Engel, J. T. Massey, and K. Maurer. 1981. “Plan and operation of the Second National Health and Nutrition Examination Survey, 1976–1980”. In *Vital and Health Statistics*, ser. 1, no. 15. Hyattsville, MD: National Center for Health Statistics.

## Also see

[R] [table](#) — Table of frequencies, summaries, and command results

[R] [table hypothesis tests](#) — Table of hypothesis tests

[R] [table intro](#) — Introduction to tables of frequencies, summaries, and command results

[R] [etable](#) — Create a table of estimation results

[TABLES] [Intro](#) — Introduction

[Description](#)  
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[Quick start](#)  
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[Menu](#)  
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[Syntax](#)  
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## Description

`tabstat` displays summary statistics for a series of numeric variables in one table. It allows you to specify the list of statistics to be displayed. Statistics can be calculated (conditioned on) another variable. `tabstat` allows substantial flexibility in terms of the statistics presented and the format of the table.

## Quick start

Mean of `v1` displayed using `v1`'s display format

```
tabstat v1, format
```

Same as above, but use format with 2 significant digits and a comma

```
tabstat v1, format(%9.2fc)
```

Nonmissing observations, mean, standard error, and coefficient of variation for `v1`

```
tabstat v1, statistics(n mean semean cv)
```

Quartiles and interquartile range of `v1` and `v2`

```
tabstat v1 v2, statistics(q iqr)
```

Same as above, but report statistics separately for each level of `catvar`

```
tabstat v1 v2, by(catvar) statistics(q iqr)
```

Same as above, but display a separate column for each statistic

```
tabstat v1 v2, by(catvar) statistics(q iqr) columns(statistics)
```

## Menu

Statistics > Summaries, tables, and tests > Other tables > Compact table of summary statistics

## Syntax

```
tabstat varlist [if] [in] [weight] [, options]
```

| <i>options</i>                                | Description                                                                         |
|-----------------------------------------------|-------------------------------------------------------------------------------------|
| Main                                          |                                                                                     |
| <code>by(<i>varname</i>)</code>               | group statistics by variable                                                        |
| <code>statistics(<i>statname</i>[...])</code> | report specified statistics                                                         |
| Options                                       |                                                                                     |
| <code>labelwidth(#)</code>                    | width for <code>by()</code> variable labels; default is <code>labelwidth(16)</code> |
| <code>varwidth(#)</code>                      | variable width; default is <code>varwidth(12)</code>                                |
| <code>columns(<i>variables</i>)</code>        | display variables in table columns; the default                                     |
| <code>columns(<i>statistics</i>)</code>       | display statistics in table columns                                                 |
| <code>format(<i>%fmt</i>)</code>              | display format for statistics; default format is <code>%9.0g</code>                 |
| <code>casewise</code>                         | perform casewise deletion of observations                                           |
| <code>nototal</code>                          | do not report overall statistics; use with <code>by()</code>                        |
| <code>missing</code>                          | report statistics for missing values of <code>by()</code> variable                  |
| <code>noseparator</code>                      | do not use separator line between <code>by()</code> categories                      |
| <code>longstub</code>                         | make left table stub wider                                                          |
| <code>save</code>                             | store summary statistics in <code>r()</code>                                        |

`by` is allowed; see [\[D\] by](#).

`awweights` and `fweights` are allowed; see [\[U\] 11.1.6 weight](#).

## Options

### Main

`by(varname)` specifies that the statistics be displayed separately for each unique value of *varname*; *varname* may be numeric or string. For instance, `tabstat height` would present the overall mean of height. `tabstat height, by(sex)` would present the mean height of males, and of females, and the overall mean height. Do not confuse the `by()` option with the `by prefix` (see [D] [by](#)); both may be specified.

`statistics(statname [...])` specifies the statistics to be displayed; the default is equivalent to specifying `statistics(mean)`. (`stats()` is a synonym for `statistics()`.) Multiple statistics may be specified and are separated by white space, such as `statistics(mean sd)`. Available statistics are

| <i>statname</i>           | Definition                               | <i>statname</i> | Definition                           |
|---------------------------|------------------------------------------|-----------------|--------------------------------------|
| <u>m</u> ean              | mean                                     | p1              | 1st percentile                       |
| <u>c</u> ount             | count of nonmissing observations         | p5              | 5th percentile                       |
| <u>n</u>                  | same as count                            | p10             | 10th percentile                      |
| <u>s</u> um               | sum                                      | p25             | 25th percentile                      |
| <u>m</u> ax               | maximum                                  | <u>m</u> edian  | median (same as p50)                 |
| <u>m</u> in               | minimum                                  | p50             | 50th percentile (same as median)     |
| <u>r</u> ange             | range = max – min                        | p75             | 75th percentile                      |
| <u>s</u> d                | standard deviation                       | p90             | 90th percentile                      |
| <u>v</u> ariance          | variance                                 | p95             | 95th percentile                      |
| <u>c</u> v                | coefficient of variation (sd/mean)       | p99             | 99th percentile                      |
| <u>s</u> e <u>m</u> ean   | standard error of mean (sd/ $\sqrt{n}$ ) | iqr             | interquartile range = p75 – p25      |
| <u>s</u> ke <u>w</u> ness | skewness                                 | q               | equivalent to specifying p25 p50 p75 |
| <u>k</u> urtosis          | kurtosis                                 |                 |                                      |

### Options

`labelwidth(#)` specifies the maximum width to be used within the stub to display the labels of the `by()` variable. The default is `labelwidth(16)`.  $8 \leq \# \leq 32$ .

`varwidth(#)` specifies the maximum width to be used within the stub to display the names of the variables. The default is `varwidth(12)`. `varwidth()` is effective only with `columns(statistics)`. Setting `varwidth()` implies `longstub`.  $8 \leq \# \leq 32$ .

`columns(variables | statistics)` specifies whether to display variables or statistics in the columns of the table. `columns(variables)` is the default when more than one variable is specified.

`format` and `format(%fmt)` specify how the statistics are to be formatted. The default is to use a %9.0g format.

`format` specifies that each variable's statistics be formatted with the variable's display format; see [D] [format](#).

`format(%fmt)` specifies the format to be used for all statistics.

The column width is the maximum width of these formats. The minimum column width is nine display characters.

`casewise` specifies casewise deletion of observations. Statistics are to be computed for the sample that is not missing for any of the variables in *varlist*. The default is to use all the nonmissing values for each variable.

`nototal` is for use with `by()`; it specifies that the overall statistics not be reported.

`missing` specifies that missing values of the `by()` variable be treated just like any other value and that statistics should be displayed for them. The default is not to report the statistics for the `by() == missing` group. If the `by()` variable is a string variable, `by() == ""` is considered to mean missing.

`noseparator` specifies that a separator line between the `by()` categories not be displayed.

`longstub` specifies that the left stub of the table be made wider so that it can include names of the statistics or variables in addition to the categories of `by(varname)`. The default is to describe the statistics or variables in a header. `longstub` is ignored if `by(varname)` is not specified.

`save` specifies that the summary statistics be returned in `r()`. The overall (unconditional) statistics are returned in matrix `r(StatTotal)` (rows are statistics, columns are variables). The conditional statistics are returned in the matrices `r(Stat1)`, `r(Stat2)`, ..., and the names of the corresponding variables are returned in the macros `r(name1)`, `r(name2)`, ....

## Remarks and examples

This command is probably most easily understood by going through a series of examples.

### ▷ Example 1

We have data on the price, weight, mileage rating, and repair record of 22 foreign and 52 domestic 1978 automobiles. We want to summarize these variables for the different origins of the automobiles.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. tabstat price weight mpg rep78, by(foreign)
Summary statistics: Mean
Group variable: foreign (Car origin)
```

| foreign  | price    | weight   | mpg      | rep78    |
|----------|----------|----------|----------|----------|
| Domestic | 6072.423 | 3317.115 | 19.82692 | 3.020833 |
| Foreign  | 6384.682 | 2315.909 | 24.77273 | 4.285714 |
| Total    | 6165.257 | 3019.459 | 21.2973  | 3.405797 |

More summary statistics can be requested via the `statistics()` option. The group totals can be suppressed with the `nototal` option.

```
. tabstat price weight mpg rep78, by(foreign) stat(mean sd min max) nototal
```

Summary statistics: Mean, SD, Min, Max

Group variable: foreign (Car origin)

| foreign  | price    | weight   | mpg      | rep78    |
|----------|----------|----------|----------|----------|
| Domestic | 6072.423 | 3317.115 | 19.82692 | 3.020833 |
|          | 3097.104 | 695.3637 | 4.743297 | .837666  |
|          | 3291     | 1800     | 12       | 1        |
|          | 15906    | 4840     | 34       | 5        |
| Foreign  | 6384.682 | 2315.909 | 24.77273 | 4.285714 |
|          | 2621.915 | 433.0035 | 6.611187 | .7171372 |
|          | 3748     | 1760     | 14       | 3        |
|          | 12990    | 3420     | 41       | 5        |

Although the header of the table describes the statistics running vertically in the “cells”, the table may become hard to read, especially with many variables or statistics. The `longstub` option specifies that a column be added describing the contents of the cells. The `format` option can be issued to specify that `tabstat` display the statistics by using the display format of the variables rather than the overall default `%9.0g`.

```
. tabstat price weight mpg rep78, by(foreign) stat(mean sd min max) long format
```

| foreign  | Stats | price   | weight  | mpg     | rep78   |
|----------|-------|---------|---------|---------|---------|
| Domestic | Mean  | 6,072.4 | 3,317.1 | 19.8269 | 3.02083 |
|          | SD    | 3,097.1 | 695.364 | 4.7433  | .837666 |
|          | Min   | 3,291   | 1,800   | 12      | 1       |
|          | Max   | 15,906  | 4,840   | 34      | 5       |
| Foreign  | Mean  | 6,384.7 | 2,315.9 | 24.7727 | 4.28571 |
|          | SD    | 2,621.9 | 433.003 | 6.61119 | .717137 |
|          | Min   | 3,748   | 1,760   | 14      | 3       |
|          | Max   | 12,990  | 3,420   | 41      | 5       |
| Total    | Mean  | 6,165.3 | 3,019.5 | 21.2973 | 3.4058  |
|          | SD    | 2,949.5 | 777.194 | 5.7855  | .989932 |
|          | Min   | 3,291   | 1,760   | 12      | 1       |
|          | Max   | 15,906  | 4,840   | 41      | 5       |

We can specify a layout of the table in which the statistics run horizontally and the variables run vertically by specifying the `col(statistics)` option.

```
. tabstat price weight mpg rep78, by(foreign) stat(min mean max) col(stat) long
```

| foreign  | Variable | Min  | Mean     | Max   |
|----------|----------|------|----------|-------|
| Domestic | price    | 3291 | 6072.423 | 15906 |
|          | weight   | 1800 | 3317.115 | 4840  |
|          | mpg      | 12   | 19.82692 | 34    |
|          | rep78    | 1    | 3.020833 | 5     |
| Foreign  | price    | 3748 | 6384.682 | 12990 |
|          | weight   | 1760 | 2315.909 | 3420  |
|          | mpg      | 14   | 24.77273 | 41    |
|          | rep78    | 3    | 4.285714 | 5     |
| Total    | price    | 3291 | 6165.257 | 15906 |
|          | weight   | 1760 | 3019.459 | 4840  |
|          | mpg      | 12   | 21.2973  | 41    |
|          | rep78    | 1    | 3.405797 | 5     |

Finally, `tabstat` can also be used to enhance `summarize` so we can specify the statistics to be displayed. For instance, we can display the number of observations, the mean, the coefficient of variation, and the 25%, 50%, and 75% quantiles for a list of variables.

```
. tabstat price weight mpg rep78, stat(n mean cv q) col(stat)
```

| variable | N  | mean     | cv       | p25  | p50    | p75  |
|----------|----|----------|----------|------|--------|------|
| price    | 74 | 6165.257 | .478406  | 4195 | 5006.5 | 6342 |
| weight   | 74 | 3019.459 | .2573949 | 2240 | 3190   | 3600 |
| mpg      | 74 | 21.2973  | .2716543 | 18   | 20     | 25   |
| rep78    | 69 | 3.405797 | .290661  | 3    | 3      | 4    |

Because we did not specify the `by()` option, these statistics were not displayed for the subgroups of the data formed by the categories of the `by()` variable.

◀

## Video example

[Descriptive statistics in Stata](#)

## Acknowledgments

The `tabstat` command was written by Jeroen Weesie and Vincent Buskens both of the Department of Sociology at Utrecht University, The Netherlands.

## Reference

Donath, S. 2018. `baselinetable`: A command for creating one- and two-way tables of summary statistics. *Stata Journal* 18: 327–344.

## Also see

[R] **summarize** — Summary statistics

[R] **table** — Table of frequencies, summaries, and command results

[R] **table summary** — Table of summary statistics

[R] **tabulate, summarize()** — One- and two-way tables of summary statistics

[D] **collapse** — Make dataset of summary statistics

|             |                      |                |            |
|-------------|----------------------|----------------|------------|
| Description | Quick start          | Menu           | Syntax     |
| Options     | Remarks and examples | Stored results | References |
| Also see    |                      |                |            |

## Description

`tabulate` produces a one-way table of frequency counts.

For information on a two-way table of frequency counts along with measures of association, including the common Pearson  $\chi^2$ , the likelihood-ratio  $\chi^2$ , Cramér's  $V$ , Fisher's exact test, Goodman and Kruskal's gamma, and Kendall's  $\tau_b$ , see [R] [tabulate twoway](#).

`tab1` produces a one-way tabulation for each variable specified in *varlist*.

Also see [R] [table](#) and [R] [tabstat](#) if you want one-, two-, or  $n$ -way table of frequencies and a wide variety of statistics. See [R] [tabulate, summarize\(\)](#) for a description of `tabulate` with the `summarize()` option; it produces a table (breakdowns) of means and standard deviations. `table` is better than `tabulate`, `summarize()`, but `tabulate, summarize()` is faster. See [R] [EpiTab](#) for a  $2 \times 2$  table with statistics of interest to epidemiologists.

## Quick start

One-way table of frequencies for `v1`

```
tabulate v1
```

Sort table in descending order of frequency

```
tabulate v1, sort
```

Generate indicator variables `v1_1`, `v1_2`, ... representing the levels of `v1`

```
tabulate v1, generate(v1_)
```

Treat missing values like other values of `v1`

```
tabulate v1, missing
```

Display numeric values of `v1` rather than value labels

```
tabulate v1, nolabel
```

Create one-way tables for `v1`, `v2`, and `v3`

```
tab1 v1 v2 v3
```

## Menu

### **tabulate oneway**

Statistics > Summaries, tables, and tests > Frequency tables > One-way table

### **tabulate ..., generate()**

Data > Create or change data > Other variable-creation commands > Create indicator variables

### **tab1**

Statistics > Summaries, tables, and tests > Frequency tables > Multiple one-way tables

## Syntax

### One-way table

```
tabulate varname [if] [in] [weight] [, options advanced_options]
```

### One-way table for each variable—a convenience tool

```
tab1 varlist [if] [in] [weight] [, options]
```

| <i>options</i>                      | Description                                        |
|-------------------------------------|----------------------------------------------------|
| <b>Main</b>                         |                                                    |
| <code>subpop(<i>varname</i>)</code> | exclude observations for which <i>varname</i> = 0  |
| <code>missing</code>                | treat missing values like other values             |
| <code>nofreq</code>                 | do not display frequencies                         |
| <code>no<del>l</del>abel</code>     | display numeric codes rather than value labels     |
| <code>plot</code>                   | produce a bar chart of the relative frequencies    |
| <code>sort</code>                   | display the table in descending order of frequency |

|                                                               |                                                  |
|---------------------------------------------------------------|--------------------------------------------------|
| <b>Collect</b>                                                |                                                  |
| <code>collect</code>                                          | post results to collection <code>Tabulate</code> |
| <code>collect(<i>cname</i> [, <i>collect_options</i>])</code> | post results to a named collection               |

| <i>collect_options</i>                                  | Description                               |
|---------------------------------------------------------|-------------------------------------------|
| <code>append</code>                                     | append results to an existing collection  |
| <code>replace</code>                                    | replace results of an existing collection |
| <code>label(<i>filename</i>)</code>                     | specify the collection labels             |
| <code>style(<i>filename</i> [, <i>override</i>])</code> | specify the collection style              |

| <i>advanced_options</i>                | Description                                            |
|----------------------------------------|--------------------------------------------------------|
| <b>Advanced</b>                        |                                                        |
| <code>generate(<i>stubname</i>)</code> | create indicator variables for <i>stubname</i>         |
| <code>matcell(<i>matname</i>)</code>   | save frequencies in <i>matname</i>                     |
| <code>matrow(<i>matname</i>)</code>    | save unique values of <i>varname</i> in <i>matname</i> |

by is allowed with tabulate and tab1; see [U] 11.1.10 Prefix commands.

fweights, aweights, and iweights are allowed; see [U] 11.1.6 weight.

## Options

### Main

`subpop(varname)` excludes observations for which *varname* = 0 in tabulating frequencies. The mathematical results of `tabulate ...`, `subpop(myvar)` are the same as `tabulate ...` if *myvar* != 0, but the table may be presented differently. The identities of the rows and columns will be determined from all the data, including the *myvar* = 0 group, so there may be entries in the table with frequency 0.

Consider tabulating `answer`, a variable that takes on values 1, 2, and 3, but consider tabulating it just for the `male==1` subpopulation. Assume that `answer` is never 2 in this group. `tabulate answer if male==1` produces a table with two rows: one for `answer 1` and one for `answer 3`. There will be no row for `answer 2` because `answer 2` was never observed. `tabulate answer, subpop(male)` produces a table with three rows. The row for `answer 2` will be shown as having 0 frequency.

`missing` requests that missing values be treated like other values in calculations of counts, percentages, and other statistics.

`nofreq` suppresses the printing of the frequencies.

`no label` causes the numeric codes to be displayed rather than the value labels.

`plot` produces a bar chart of the relative frequencies in a one-way table. (Also see [\[R\] histogram.](#))

`sort` puts the table in descending order of frequency (and ascending order of the variable within equal values of frequency).

### Collect

`collect` and `collect([cname][, collect_options])` specify that results be posted to a collection. This collection produces a table that you can customize and publish to Microsoft Word, Microsoft Excel, PDF, HTML, L<sup>A</sup>T<sub>E</sub>X, SMCL, or Markdown. Output does not change when these options are specified. Use `collect preview` to see the customizable table.

`collect` is a shortcut for `collect(Tabulate)`.

*cname* specifies that a collection named *cname* be associated with the collected results. The default is `Tabulate`.

`append` specifies that results be appended to collection *cname*.

`replace` permits `tabulate` to overwrite an existing collection. This option is implied for collection `Tabulate` when `append` is not specified.

`label(filename)` specifies the *filename* containing the collection labels to use for your table. Labels in *filename* will be loaded into the collection, and any labels not specified in *filename* will be taken from the labels defined in `c(collect_label)`. The default is to use only the collection labels set in `c(collect_label)`; see [\[TABLES\] set collect\\_label.](#)

`style(filename[, override])` specifies the *filename* containing the collection styles to use for your table. The default collection styles will be discarded, and only the collection styles in *filename* will be applied.

If you prefer the default collection styles but also want to apply any styles in *filename*, specify `override`. If there are conflicts between the default collection styles and those in *filename*, the ones in *filename* will take precedence.

The default is to use only the collection styles set in `c(tabulate_style)`; see [TABLES] [set tabulate\\_style](#).

---

Advanced

---

`generate(stubname)` creates a set of indicator variables (*stubname1*, *stubname2*, ...) reflecting the observed values of the tabulated variable. The `generate()` option may not be used with the `by` prefix.

`matcell(matname)` saves the reported frequencies in *matname*.

`matrow(matname)` saves the numeric values of the  $r \times 1$  row stub in *matname*. `matrow()` may not be specified if the row variable is a string.

## Limits

A one-way table may have a maximum of 12,000 rows (Stata/MP and Stata/SE) or 3,000 rows (Stata/BE).

## Remarks and examples

Remarks are presented under the following headings:

[tabulate](#)  
[tab1](#)  
[Publish your tables](#)  
[Video example](#)

For each value of a specified variable, `tabulate` reports the number of observations with that value. The number of times a value occurs is called its *frequency*.

## tabulate

### ► Example 1

We have data summarizing the speed limit and the accident rate per million vehicle miles along various Minnesota highways in 1973. The variable containing the speed limit is called `spdlimit`. If we summarize the variable, we obtain its mean and standard deviation:

```
. use https://www.stata-press.com/data/r19/hiway
(Minnesota highway data, 1973)
. summarize spdlimit
```

| Variable | Obs | Mean | Std. dev. | Min | Max |
|----------|-----|------|-----------|-----|-----|
| spdlimit | 39  | 55   | 5.848977  | 40  | 70  |

The average speed limit is 55 miles per hour. We can learn more about this variable by tabulating it:

```
. tabulate spdlimit
```

| Speed limit | Freq. | Percent | Cum.   |
|-------------|-------|---------|--------|
| 40          | 1     | 2.56    | 2.56   |
| 45          | 3     | 7.69    | 10.26  |
| 50          | 7     | 17.95   | 28.21  |
| 55          | 15    | 38.46   | 66.67  |
| 60          | 11    | 28.21   | 94.87  |
| 65          | 1     | 2.56    | 97.44  |
| 70          | 1     | 2.56    | 100.00 |
| Total       | 39    | 100.00  |        |

We see that one highway has a speed limit of 40 miles per hour, three have speed limits of 45, 7 of 50, and so on. The column labeled *Percent* shows the percentage of highways in the dataset that have the indicated speed limit. For instance, 38.46% of highways in our dataset have a speed limit of 55 miles per hour. The final column shows the cumulative percentage. We see that 66.67% of highways in our dataset have a speed limit of 55 miles per hour or less.

◀

### ▷ Example 2

The `plot` option places a sideways histogram alongside the table:

```
. tabulate spdlimit, plot
```

| Speed limit | Freq. |       |
|-------------|-------|-------|
| 40          | 1     | *     |
| 45          | 3     | ***   |
| 50          | 7     | ***** |
| 55          | 15    | ***** |
| 60          | 11    | ***** |
| 65          | 1     | *     |
| 70          | 1     | *     |
| Total       | 39    |       |

Of course, `graph` can produce better-looking histograms; see [\[R\] histogram](#).

◀

### ▷ Example 3

`tabulate` labels tables using *variable* and *value labels* if they exist. To demonstrate how this works, let's add a new variable to our dataset that categorizes `spdlimit` into three categories. We will call this new variable `spdcat`:

```
. generate spdcat=recode(spdlimit,50,60,70)
```

The `recode()` function divides `spdlimit` into 50 miles per hour or below, 51–60, and above 60; see [\[FN\] Programming functions](#). We specified the breakpoints in the arguments (`spdlimit, 50, 60, 70`). The first argument is the variable to be recoded. The second argument is the first breakpoint, the third argument is the second breakpoint, and so on. We can specify as many breakpoints as we wish.

`recode()` used our arguments not only as the breakpoints but also to label the results. If `spdlimit` is less than or equal to 50, `spdcat` is set to 50; if `spdlimit` is between 51 and 60, `spdcat` is 60; otherwise, `spdcat` is arbitrarily set to 70. (See [\[U\] 26 Working with categorical data and factor variables](#).)

Because we just created the variable `spdcat`, it is not yet labeled. When we make a table using this variable, `tabulate` uses the variable's name to label it:

```
. tabulate spdcat
```

| spdcat | Freq. | Percent | Cum.   |
|--------|-------|---------|--------|
| 50     | 11    | 28.21   | 28.21  |
| 60     | 26    | 66.67   | 94.87  |
| 70     | 2     | 5.13    | 100.00 |
| Total  | 39    | 100.00  |        |

Even though the table is not well labeled, `recode()`'s coding scheme provides us with clues as to the table's meaning. The first line of the table corresponds to 50 miles per hour and below, the next to 51 through 60 miles per hour, and the last to above 60 miles per hour.

We can improve this table by labeling the values and variables:

```
. label define scat 50 "40 to 50" 60 "55 to 60" 70 "Above 60"
. label values spdcat scat
. label variable spdcat "Speed Limit Category"
```

We define a *value label* called `scat` that attaches labels to the numbers 50, 60, and 70 using the `label define` command; see [U] 12.6.3 [Value labels](#). We label the value 50 as “40 to 50”, because we looked back at our original tabulation in the first example and saw that the speed limit was never less than 40. Similarly, we could have labeled the last category “65 to 70” because the speed limit is never greater than 70 miles per hour.

Next, we requested that Stata label the values of the new variable `spdcat` using the value label `scat`. Finally, we labeled our variable `Speed Limit Category`. We are now ready to tabulate the result:

```
. tabulate spdcat
```

| Speed Limit<br>Category | Freq. | Percent | Cum.   |
|-------------------------|-------|---------|--------|
| 40 to 50                | 11    | 28.21   | 28.21  |
| 55 to 60                | 26    | 66.67   | 94.87  |
| Above 60                | 2     | 5.13    | 100.00 |
| Total                   | 39    | 100.00  |        |

▷ Example 4

If we have missing values in our dataset, `tabulate` ignores them unless we explicitly indicate otherwise. We have no missing data in our example, so let's add some:

```
. replace spdcat=. in 39
(1 real change made, 1 to missing)
```

We changed the first observation on `spdcat` to *missing*. Let's now tabulate the result:

```
. tabulate spdcat
```

| Speed Limit<br>Category | Freq. | Percent | Cum.   |
|-------------------------|-------|---------|--------|
| 40 to 50                | 11    | 28.95   | 28.95  |
| 55 to 60                | 26    | 68.42   | 97.37  |
| Above 60                | 1     | 2.63    | 100.00 |
| Total                   | 38    | 100.00  |        |

Comparing this output with that in the previous example, we see that the total frequency count is now one less than it was—38 rather than 39. Also, the 'Above 60' category now has only one observation where it used to have two, so we evidently changed a road with a high speed limit.

We want `tabulate` to treat missing values just as it treats numbers, so we specify the `missing` option:

```
. tabulate spdcat, missing
```

| Speed Limit<br>Category | Freq. | Percent | Cum.   |
|-------------------------|-------|---------|--------|
| 40 to 50                | 11    | 28.21   | 28.21  |
| 55 to 60                | 26    | 66.67   | 94.87  |
| Above 60                | 1     | 2.56    | 97.44  |
| .                       | 1     | 2.56    | 100.00 |
| Total                   | 39    | 100.00  |        |

We now see our missing value—the last category, labeled '.', shows a frequency count of 1. The table sum is once again 39.

Let's put our dataset back as it was originally:

```
. replace spdcat=70 in 39
(1 real change made)
```



□ Technical note

tabulate also can automatically create indicator variables from categorical variables. We will briefly review that capability here, but see [U] 26 Working with categorical data and factor variables for a complete description. Let’s begin by describing our highway dataset:

```
. describe
Contains data from https://www.stata-press.com/data/r19/hiway.dta
Observations: 39 Minnesota highway data, 1973
Variables: 3 16 Nov 2024 12:39
```

| Variable name | Storage type | Display format | Value label | Variable label                          |
|---------------|--------------|----------------|-------------|-----------------------------------------|
| spdlimit      | byte         | %8.0g          |             | Speed limit                             |
| rate          | byte         | %9.0g          | rcat        | Accident rate per million vehicle miles |
| spdcat        | float        | %9.0g          | scat        | Speed Limit Category                    |

Sorted by:  
 Note: Dataset has changed since last saved.

Our dataset contains three variables. We will type tabulate spdcat, generate(spd), describe our data, and then explain what happened.

```
. tabulate spdcat, generate(spd)
```

| Speed Limit Category | Freq. | Percent | Cum.   |
|----------------------|-------|---------|--------|
| 40 to 50             | 11    | 28.21   | 28.21  |
| 55 to 60             | 26    | 66.67   | 94.87  |
| Above 60             | 2     | 5.13    | 100.00 |
| Total                | 39    | 100.00  |        |

```
. describe
Contains data from https://www.stata-press.com/data/r19/hiway.dta
Observations: 39 Minnesota highway data, 1973
Variables: 6 16 Nov 2024 12:39
```

| Variable name | Storage type | Display format | Value label | Variable label                          |
|---------------|--------------|----------------|-------------|-----------------------------------------|
| spdlimit      | byte         | %8.0g          |             | Speed limit                             |
| rate          | byte         | %9.0g          | rcat        | Accident rate per million vehicle miles |
| spdcat        | float        | %9.0g          | scat        | Speed Limit Category                    |
| spd1          | byte         | %8.0g          |             | spdcat==40 to 50                        |
| spd2          | byte         | %8.0g          |             | spdcat==55 to 60                        |
| spd3          | byte         | %8.0g          |             | spdcat==Above 60                        |

Sorted by:  
 Note: Dataset has changed since last saved.

When we typed tabulate with the generate() option, Stata responded by producing a one-way frequency table, so it appeared that the option did nothing. Yet when we describe our dataset, we find that we now have six variables instead of the original three. The new variables are named spd1, spd2, and spd3.

When we specify the `generate()` option, we are telling Stata to not only produce the table but also create a set of indicator variables that correspond to that table. Stata adds a numeric suffix to the name we specify in the parentheses. `spd1` refers to the first line of the table, `spd2` to the second line, and so on. Also, Stata labels the variables so that we know what they mean. `spd1` is an indicator variable that is *true* (takes on the value 1) when `spdcat` is between 40 and 50; otherwise, it is zero. (There is an exception: if `spdcat` is missing, so are the `spd1`, `spd2`, and `spd3` variables. This did not happen in our dataset.)

We want to prove our claim. Because we have not yet introduced two-way tabulations, we will use the `summarize` statement:

```
. summarize spdlimit if spd1==1
```

| Variable | Obs | Mean     | Std. dev. | Min | Max |
|----------|-----|----------|-----------|-----|-----|
| spdlimit | 11  | 47.72727 | 3.437758  | 40  | 50  |

```
. summarize spdlimit if spd2==1
```

| Variable | Obs | Mean     | Std. dev. | Min | Max |
|----------|-----|----------|-----------|-----|-----|
| spdlimit | 26  | 57.11538 | 2.519157  | 55  | 60  |

```
. summarize spdlimit if spd3==1
```

| Variable | Obs | Mean | Std. dev. | Min | Max |
|----------|-----|------|-----------|-----|-----|
| spdlimit | 2   | 67.5 | 3.535534  | 65  | 70  |

Notice the indicated minimum and maximum in each of the tables above. When we restrict the sample to `spd1`, `spdlimit` is between 40 and 50; when we restrict the sample to `spd2`, `spdlimit` is between 55 and 60; when we restrict the sample to `spd3`, `spdlimit` is between 65 and 70.

Thus `tabulate` provides an easy way to create indicator (sometimes called dummy) variables. For an overview of indicator and categorical variables, see [\[U\] 26 Working with categorical data and factor variables](#).



## tab1

`tab1` is a convenience tool. Typing

```
. tab1 myvar thisvar thatvar, plot
```

is equivalent to typing

```
. tabulate myvar, plot
. tabulate thisvar, plot
. tabulate thatvar, plot
```

## Publish your tables

With the `collect` option, `tabulate` posts the tabulated values to a collection named `Tabulate` and sets it as the current collection. `tab1` appends the resulting tabulations into a single collection named `Tabulate` and sets it as the current collection. With collections, you can customize the look of your table, then publish it to HTML, Word, L<sup>A</sup>T<sub>E</sub>X, PDF, Excel, or another format appropriate for your report.

With the `by` prefix, `tabulate` and `tab1` append the resulting tabulations into a single collection, and the default layout produces a separate table for each by group.

If you are not familiar with collections, see [\[TABLES\] Intro](#). The predefined styles for `tabulate` are documented in [\[TABLES\] Predefined styles](#).

### ▷ Example 5

Recall our value-labeled variable `spdcat`. Let's use `tabulate` with the `collect` option to produce a collection with its tabulated values.

```
. tabulate spdcat, collect
```

| Speed Limit<br>Category | Freq. | Percent | Cum.   |
|-------------------------|-------|---------|--------|
| 40 to 50                | 11    | 28.21   | 28.21  |
| 55 to 60                | 26    | 66.67   | 94.87  |
| Above 60                | 2     | 5.13    | 100.00 |
| Total                   | 39    | 100.00  |        |

The output does not change; however, we can use the `collect dir` command to see that `tabulate` created a collection named `Tabulate`.

```
. collect dir
```

| Collections in memory |           |
|-----------------------|-----------|
| Current: Tabulate     |           |
| Name                  | No. items |
| Tabulate              | 11        |

In this collection, frequencies are tagged with `result[frequency]`, percentages with `result[percent]`, and cumulative percentages with `result[cumulative]`. Here we use `collect label list` to show the levels and labels of the result dimension.

```
. collect label list result
```

| Collection: Tabulate |            |
|----------------------|------------|
| Dimension: result    |            |
| Label: Result        |            |
| Level labels:        |            |
| cumulative           | Cumulative |
| frequency            | Frequency  |
| percent              | Percent    |

The tabulated variable (that is, `spdcat`) is added to the collection as a dimension and is used to tag the collected results. In addition to the name, label, level values, and value labels of the tabulated variable, this dimension also has the `__margCode__` level with the `Total` label for tagging the total frequency and percentage. Here we use `collect label list` to show the levels and labels of the tabulated variable dimension.

```
. collect label list spdcat
Collection: Tabulate
Dimension: spdcat
Label: Speed Limit Category
Level labels:
 50 40 to 50
 60 55 to 60
 70 Above 60
__margCode__ Total
```

`tabulate` constructs a default layout, so you can view your customizable table with the `collect preview` command. Here we use the `collect layout` command to report the default layout specification and corresponding table.

```
. collect layout
Collection: Tabulate
Rows: cmdset#spdcat
Columns: result
Table 1: 5 x 3
```

|                      | Frequency | Percent | Cumulative |
|----------------------|-----------|---------|------------|
| Speed Limit Category |           |         |            |
| 40 to 50             | 11        | 28.21   | 28.21      |
| 55 to 60             | 26        | 66.67   | 94.87      |
| Above 60             | 2         | 5.13    | 100.00     |
| Total                | 39        | 100.00  |            |

We can make further changes to the table with the `collect` suite of commands. But we are happy with this layout and ready to publish the table to a PDF file with `collect export`. We simply specify the filename to which we want to export it.

```
. collect export tab1.pdf
(collection Tabulate exported to file tab1.pdf)
```

With `collect export` you can publish the table to several formats, such as HTML, PDF, and  $\LaTeX$  files, by specifying the appropriate file extension.



## Video example

[Tables and cross-tabulations in Stata](#)

## Stored results

`tabulate` and `tab1` store the following in `r()`:

### Scalars

|                   |                        |
|-------------------|------------------------|
| <code>r(N)</code> | number of observations |
| <code>r(r)</code> | number of rows         |

### Macros

|                            |                    |
|----------------------------|--------------------|
| <code>r(collection)</code> | name of collection |
|----------------------------|--------------------|

`r(collection)` is defined only when the `collect()` option is specified.

## References

Cox, N. J. 2009. Speaking Stata: I. J. Good and quasi-Bayes smoothing of categorical frequencies. *Stata Journal* 9: 306–314.

Donath, S. 2018. `baselinetable`: A command for creating one- and two-way tables of summary statistics. *Stata Journal* 18: 327–344.

Harrison, D. A. 2006. *Stata tip 34: Tabulation by listing*. *Stata Journal* 6: 425–427.

## Also see

[R] [Epitab](#) — Tables for epidemiologists

[R] [table](#) — Table of frequencies, summaries, and command results

[R] [table oneway](#) — One-way tabulation

[R] [tabstat](#) — Compact table of summary statistics

[R] [tabulate twoway](#) — Two-way table of frequencies

[R] [tabulate, summarize\(\)](#) — One- and two-way tables of summary statistics

[D] [collapse](#) — Make dataset of summary statistics

[SVY] [svy: tabulate oneway](#) — One-way tables for survey data

[SVY] [svy: tabulate twoway](#) — Two-way tables for survey data

[TABLES] [Intro](#) — Introduction

[XT] [xttab](#) — Tabulate xt data

[U] [12.6.3 Value labels](#)

[U] [26 Working with categorical data and factor variables](#)

|             |                      |                |                      |
|-------------|----------------------|----------------|----------------------|
| Description | Quick start          | Menu           | Syntax               |
| Options     | Remarks and examples | Stored results | Methods and formulas |
| References  | Also see             |                |                      |

## Description

`tabulate` produces a two-way table of frequency counts, along with various measures of association, including the common Pearson's  $\chi^2$ , the likelihood-ratio  $\chi^2$ , Cramér's  $V$ , Fisher's exact test, Goodman and Kruskal's gamma, and Kendall's  $\tau_b$ .

Line size is respected. That is, if you resize the Results window before running `tabulate`, the resulting two-way tabulation will take advantage of the available horizontal space. Stata for Unix(console) users can instead use the `set linesize` command to take advantage of this feature.

`tab2` produces all possible two-way tabulations of the variables specified in *varlist*.

`tabi` displays the  $r \times c$  table, using the values specified; rows are separated by '\'. If no options are specified, it is as if `exact` were specified for a  $2 \times 2$  table and `chi2` were specified otherwise. See [U] 19 **Immediate commands** for a general description of immediate commands. See *Tables with immediate data* below for examples using `tabi`.

See [R] **tabulate oneway** if you want a one-way table of frequencies. See [R] **table** and [R] **tabstat** if you want one-, two-, or  $n$ -way table of frequencies and a wide variety of summary statistics. See [R] **tabulate, summarize()** for a description of `tabulate` with the `summarize()` option; it produces a table (breakdowns) of means and standard deviations. `table` is better than `tabulate`, `summarize()`, but `tabulate`, `summarize()` is faster. See [R] **Epitab** for a  $2 \times 2$  table with statistics of interest to epidemiologists.

## Quick start

Two-way table of frequencies for `v1` and `v2`

```
tabulate v1 v2
```

Add row percentages

```
tabulate v1 v2, row
```

Frequencies only for observations where `v3 = 1`

```
tabulate v1 v2 if v3==1
```

Weighted cell counts using frequency weights defined by `wvar`

```
tabulate v1 v2 [fweight=wvar]
```

Pearson's  $\chi^2$  test and each cell's contribution

```
tabulate v1 v2, chi2 cchi2
```

All available measures of association

```
tabulate v1 v2, all
```

All possible two-way tables for v1, v2, and v3

```
tab2 v1 v2 v3
```

Input cell frequencies and perform  $\chi^2$  test

```
tabi 30 18 38 \ 13 7 22, chi2
```

## Menu

### **tabulate**

Statistics > Summaries, tables, and tests > Frequency tables > Two-way table with measures of association

### **tab2**

Statistics > Summaries, tables, and tests > Frequency tables > All possible two-way tables

### **tabi**

Statistics > Summaries, tables, and tests > Frequency tables > Table calculator

## Syntax

Two-way table

```
tabulate varname1 varname2 [if] [in] [weight] [, options advanced_options]
```

Two-way table for all possible combinations—a convenience tool

```
tab2 varlist [if] [in] [weight] [, options]
```

Immediate form of two-way tabulations

```
tabi #11 #12 [...] \ #21 #22 [...] [\ ...] [, options advanced_options]
```

| <i>options</i>                                                  | Description                                                        |
|-----------------------------------------------------------------|--------------------------------------------------------------------|
| <b>Main</b>                                                     |                                                                    |
| <code>chi2</code>                                               | report Pearson's $\chi^2$                                          |
| <code>exact[(#)]</code>                                         | report Fisher's exact test                                         |
| <code>gamma</code>                                              | report Goodman and Kruskal's gamma                                 |
| <code>lrchi2</code>                                             | report likelihood-ratio $\chi^2$                                   |
| <code>taub</code>                                               | report Kendall's $\tau_b$                                          |
| <code>V</code>                                                  | report Cramér's $V$                                                |
| <code>cchi2</code>                                              | report Pearson's $\chi^2$ in each cell                             |
| <code>column</code>                                             | report relative frequency within its column of each cell           |
| <code>row</code>                                                | report relative frequency within its row of each cell              |
| <code>clrchi2</code>                                            | report likelihood-ratio $\chi^2$ in each cell                      |
| <code>cell</code>                                               | report the relative frequency of each cell                         |
| <code>expected</code>                                           | report expected frequency in each cell                             |
| <code>nofreq</code>                                             | do not display frequencies                                         |
| <code>rowsort</code>                                            | list rows in order of observed frequency                           |
| <code>colsort</code>                                            | list columns in order of observed frequency                        |
| <code>missing</code>                                            | treat missing values like other values                             |
| <code>wrap</code>                                               | do not wrap wide tables                                            |
| <code>[no]key</code>                                            | report/suppress cell contents key                                  |
| <code>nolabel</code>                                            | display numeric codes rather than value labels                     |
| <code>nolog</code>                                              | do not display enumeration log for Fisher's exact test             |
| <code>*firstonly</code>                                         | show only tables that include the first variable in <i>varlist</i> |
| <b>Collect</b>                                                  |                                                                    |
| <code>collect</code>                                            | post results to collection <code>Tabulate</code>                   |
| <code>collect([<i>cname</i>] [, <i>collect_options</i>])</code> | post results to a named collection                                 |
| <code>all</code>                                                | equivalent to specifying <code>chi2 lrchi2 V gamma taub</code>     |

\*`firstonly` is available only for `tab2`.

`by` is allowed with `tabulate` and `tab2`; see [U] 11.1.10 Prefix commands.

`fweights`, `awweights`, and `iweights` are allowed by `tabulate`. `fweights` are allowed by `tab2`. See [U] 11.1.6 `weight`.

`all` does not appear in the dialog box.

| <i>collect_options</i>                       | Description                               |
|----------------------------------------------|-------------------------------------------|
| append                                       | append results to an existing collection  |
| replace                                      | replace results of an existing collection |
| label( <i>filename</i> )                     | specify the collection labels             |
| style( <i>filename</i> [, <i>override</i> ]) | specify the collection style              |

| <i>advanced_options</i>   | Description                                                         |
|---------------------------|---------------------------------------------------------------------|
| Advanced                  |                                                                     |
| matcell( <i>matname</i> ) | save frequencies in <i>matname</i>                                  |
| matrow( <i>matname</i> )  | save unique values of <i>varname</i> <sub>1</sub> in <i>matname</i> |
| matcol( <i>matname</i> )  | save unique values of <i>varname</i> <sub>2</sub> in <i>matname</i> |
| ‡replace                  | replace current data with given cell frequencies                    |

‡replace is available only for tabi.

## Options

### Main

chi2 calculates and displays Pearson's  $\chi^2$  for the hypothesis that the rows and columns in a two-way table are independent. chi2 may not be specified if *aweights* or *iweights* are specified.

exact[(#)] displays the significance calculated by Fisher's exact test and may be applied to  $r \times c$  as well as to  $2 \times 2$  tables. For  $2 \times 2$  tables, both one- and two-sided probabilities are displayed. For  $r \times c$  tables, two-sided probabilities are displayed. The optional positive integer # is a multiplier on the amount of memory that the command is permitted to consume. The default is 1. This option should not be necessary for reasonable  $r \times c$  tables. If the command terminates with error 910, try exact(2). The maximum row or column dimension allowed when computing Fisher's exact test is the maximum row or column dimension for tabulate (see [R] [Limits](#)).

gamma displays Goodman and Kruskal's gamma along with its asymptotic standard error. gamma is appropriate only when both variables are ordinal. gamma may not be specified if *aweights* or *iweights* are specified.

lrchi2 displays the likelihood-ratio  $\chi^2$  statistic. lrchi2 may not be specified if *aweights* or *iweights* are specified.

taub displays Kendall's  $\tau_b$  along with its asymptotic standard error. taub is appropriate only when both variables are ordinal. taub may not be specified if *aweights* or *iweights* are specified.

V (note capitalization) displays Cramér's *V*. V may not be specified if *aweights* or *iweights* are specified.

cchi2 displays each cell's contribution to Pearson's  $\chi^2$  in a two-way table.

column displays the relative frequency of each cell within its column in a two-way table.

row displays the relative frequency of each cell within its row in a two-way table.

clrchi2 displays each cell's contribution to the likelihood-ratio  $\chi^2$  in a two-way table.

cell displays the relative frequency of each cell in a two-way table.

expected displays the expected frequency of each cell in a two-way table.

`nofreq` suppresses the printing of the frequencies.

`rowsort` and `colsort` specify that the rows and columns, respectively, be presented in order of observed frequency.

By default, rows and columns are presented in ascending order of the row and column variable. For instance, if you type `tabulate a b` and `a` takes on the values 2, 3, and 5, then the first row of the table will correspond to `a = 2`; the second row will correspond to `a = 3`; and the third row will correspond to `a = 5`.

`rowsort` specifies that the rows instead be presented in descending order of observed frequency of the values. If you type `twoway a b, rowsort`, the most frequently observed value of `a` will be listed in the first row, the second most frequently observed value of `a` in the second row, and so on. If there are rows with equal frequencies, they will be presented in ascending order of the values of `a`. If `a = 5` occurs with frequency 1,000 and values `a = 2` and `a = 3` each occur with frequency 500, the rows will be presented in the order `a = 5`, `a = 2`, and `a = 3`.

`colsort` does the same as `rowsort`, except with the columns and the column variable.

`rowsort` and `colsort` may be specified together.

`missing` requests that missing values be treated like other values in calculations of counts, percentages, and other statistics.

`wrap` requests that Stata take no action on wide, two-way tables to make them readable. Unless `wrap` is specified, wide tables are broken into pieces to enhance readability.

`[no]key` suppresses or forces the display of a key above two-way tables. The default is to display the key if more than one cell statistic is requested, and otherwise to omit it. `key` forces the display of the key. `nokey` suppresses its display.

`no label` causes the numeric codes to be displayed rather than the value labels.

`nolog` suppresses the display of the log for Fisher's exact test. Using Fisher's exact test requires counting all tables that have a probability exceeding that of the observed table given the observed row and column totals. The log counts down each stage of the network computations, starting from the number of columns and counting down to 1, displaying the number of nodes in the network at each stage. A log is not displayed for  $2 \times 2$  tables.

`firstonly`, available only with `tab2`, restricts the output to only those tables that include the first variable in `varlist`. Use this option to interact one variable with a set of others.

## Collect

`collect` and `collect([cname][, collect_options])` specify that results be posted to a collection. This collection produces a table that you can customize and publish to Microsoft Word, Microsoft Excel, PDF, HTML,  $\LaTeX$ , SMCL, or Markdown. Output does not change when these options are specified. Use `collect preview` to see the customizable table.

`collect` is a shortcut for `collect(Tabulate)`.

*cname* specifies that a collection named *cname* be associated with the collected results. The default is `Tabulate`.

`append` specifies that results be appended to collection *cname*.

`replace` permits `tabulate` to overwrite an existing collection. This option is implied for collection `Tabulate` when `append` is not specified.

`label(filename)` specifies the *filename* containing the collection labels to use for your table. Labels in *filename* will be loaded into the collection, and any labels not specified in *filename* will be taken from the labels defined in `c(collect_label)`. The default is to use only the collection labels set in `c(collect_label)`; see [TABLES] [set collect\\_label](#).

`style(filename[, override])` specifies the *filename* containing the collection styles to use for your table. The default collection styles will be discarded, and only the collection styles in *filename* will be applied.

If you prefer the default collection styles but also want to apply any styles in *filename*, specify *override*. If there are conflicts between the default collection styles and those in *filename*, the ones in *filename* will take precedence.

The default is to use only the collection styles set in `c(tabulate_style)`; see [TABLES] [set tabulate\\_style](#).

## Advanced

`matcell(matname)` saves the reported frequencies in *matname*. This option is for use by programmers.

`matrow(matname)` saves the numeric values of the  $r \times 1$  row stub in *matname*. This option is for use by programmers. `matrow()` may not be specified if the row variable is a string.

`matcol(matname)` saves the numeric values of the  $1 \times c$  column stub in *matname*. This option is for use by programmers. `matcol()` may not be specified if the column variable is a string.

`replace` indicates that the immediate data specified as arguments to the `tabi` command be left as the current data in place of whatever data were there.

The following option is available with `tabulate` but is not shown in the dialog box:

`all` is equivalent to specifying `chi2 lrchi2 V gamma taub`. Note the omission of `exact`. When `all` is specified, `no` may be placed in front of the other options. `all noV` requests all association measures except Cramér's  $V$  (and Fisher's exact). `all exact` requests all association measures, including Fisher's exact test. `all` may not be specified if `aweights` or `iweights` are specified.

## Limits

Two-way tables may have a maximum of 1,200 rows and 80 columns (Stata/MP and Stata/SE) or 300 rows and 20 columns (Stata/BE). If larger tables are needed, see [R] [table](#).

## Remarks and examples

Remarks are presented under the following headings:

- [tabulate](#)
- [Measures of association](#)
- [N-way tables](#)
- [Weighted data](#)
- [Tables with immediate data](#)
- [tab2](#)
- [Publish your tables](#)
- [Video examples](#)

For each value of a specified variable (or a set of values for a pair of variables), `tabulate` reports the number of observations with that value. The number of times a value occurs is called its *frequency*.

### tabulate

#### ► Example 1

`tabulate` will make two-way tables if we specify two variables following the word `tabulate`. In our highway dataset, we have a variable called `rate` that divides the accident rate into three categories: below 4, 4–7, and above 7 per million vehicle miles. Let’s make a table of the speed limit category and the accident-rate category:

```
. use https://www.stata-press.com/data/r19/hiway2
(Minnesota highway data, 1973)
. tabulate spdcat rate
```

| Speed<br>limit<br>category | Accident rate per million<br>vehicle miles |     |         | Total |
|----------------------------|--------------------------------------------|-----|---------|-------|
|                            | Below 4                                    | 4-7 | Above 7 |       |
| 40 to 50                   | 3                                          | 5   | 3       | 11    |
| 55 to 50                   | 19                                         | 6   | 1       | 26    |
| Above 60                   | 2                                          | 0   | 0       | 2     |
| Total                      | 24                                         | 11  | 4       | 39    |

The table indicates that three stretches of highway have an accident rate below 4 and a speed limit of 40 to 50 miles per hour. The table also shows the row and column sums (called the *marginals*). The number of highways with a speed limit of 40 to 50 miles per hour is 11, which is the same result we obtained in our previous one-way tabulations.

Stata can present this basic table in several ways—16, to be precise—and we will show just a few below. It might be easier to read the table if we included the row percentages. For instance, of 11 highways in the lowest speed limit category, three are also in the lowest accident-rate category. Three-elevenths amounts to some 27.3%. We can ask Stata to fill in this information for us by using the `row` option:

```
. tabulate spdcat rate, row
```

|                       |
|-----------------------|
| Key                   |
| <i>frequency</i>      |
| <i>row percentage</i> |

| Speed<br>limit<br>category | Accident rate per million<br>vehicle miles |             |            | Total        |
|----------------------------|--------------------------------------------|-------------|------------|--------------|
|                            | Below 4                                    | 4-7         | Above 7    |              |
| 40 to 50                   | 3<br>27.27                                 | 5<br>45.45  | 3<br>27.27 | 11<br>100.00 |
| 55 to 50                   | 19<br>73.08                                | 6<br>23.08  | 1<br>3.85  | 26<br>100.00 |
| Above 60                   | 2<br>100.00                                | 0<br>0.00   | 0<br>0.00  | 2<br>100.00  |
| Total                      | 24<br>61.54                                | 11<br>28.21 | 4<br>10.26 | 39<br>100.00 |

The number listed below each frequency is the percentage of cases that each cell represents out of its row. That is easy to remember because we see 100% listed in the “Total” column. The bottom row is also informative. We see that 61.54% of all the highways in our dataset fall into the lowest accident-rate category, that 28.21% are in the middle category, and that 10.26% are in the highest.

tabulate can calculate column percentages and cell percentages, as well. It does so when we specify the column or cell options, respectively. We can even specify them together. Below is a table that includes everything:

```
. tabulate spdcat rate, row column cell
```

|                          |
|--------------------------|
| Key                      |
| <i>frequency</i>         |
| <i>row percentage</i>    |
| <i>column percentage</i> |
| <i>cell percentage</i>   |

| Speed<br>limit<br>category | Accident rate per million<br>vehicle miles |        |         | Total  |
|----------------------------|--------------------------------------------|--------|---------|--------|
|                            | Below 4                                    | 4-7    | Above 7 |        |
| 40 to 50                   | 3                                          | 5      | 3       | 11     |
|                            | 27.27                                      | 45.45  | 27.27   | 100.00 |
|                            | 12.50                                      | 45.45  | 75.00   | 28.21  |
|                            | 7.69                                       | 12.82  | 7.69    | 28.21  |
| 55 to 50                   | 19                                         | 6      | 1       | 26     |
|                            | 73.08                                      | 23.08  | 3.85    | 100.00 |
|                            | 79.17                                      | 54.55  | 25.00   | 66.67  |
|                            | 48.72                                      | 15.38  | 2.56    | 66.67  |
| Above 60                   | 2                                          | 0      | 0       | 2      |
|                            | 100.00                                     | 0.00   | 0.00    | 100.00 |
|                            | 8.33                                       | 0.00   | 0.00    | 5.13   |
|                            | 5.13                                       | 0.00   | 0.00    | 5.13   |
| Total                      | 24                                         | 11     | 4       | 39     |
|                            | 61.54                                      | 28.21  | 10.26   | 100.00 |
|                            | 100.00                                     | 100.00 | 100.00  | 100.00 |
|                            | 61.54                                      | 28.21  | 10.26   | 100.00 |

The number at the top of each cell is the frequency count. The second number is the row percentage—they sum to 100% going across the table. The third number is the column percentage—they sum to 100% going down the table. The bottom number is the cell percentage—they sum to 100% going down all the columns and across all the rows. For instance, highways with a speed limit above 60 miles per hour and in the lowest accident rate category account for 100% of highways with a speed limit above 60 miles per hour; 8.33% of highways in the lowest accident-rate category; and 5.13% of all our data.

A fourth option, `nofreq`, tells Stata not to print the frequency counts. To construct a table consisting of only row percentages, we type

```
. tabulate spdcat rate, row nofreq
```

| Speed<br>limit<br>category | Accident rate per million<br>vehicle miles |       |         | Total  |
|----------------------------|--------------------------------------------|-------|---------|--------|
|                            | Below 4                                    | 4-7   | Above 7 |        |
| 40 to 50                   | 27.27                                      | 45.45 | 27.27   | 100.00 |
| 55 to 50                   | 73.08                                      | 23.08 | 3.85    | 100.00 |
| Above 60                   | 100.00                                     | 0.00  | 0.00    | 100.00 |
| Total                      | 61.54                                      | 28.21 | 10.26   | 100.00 |



## Measures of association

### ▷ Example 2

`tabulate` will calculate the Pearson  $\chi^2$  test for the independence of the rows and columns if we specify the `chi2` option. Suppose that we have 1980 census data on 956 cities in the United States and wish to compare the age distribution across regions of the country. Assume that `agecat` is the median age in each city and that `region` denotes the region of the country in which the city is located.

```
. use https://www.stata-press.com/data/r19/citytemp2
(City temperature data)
. tabulate region agecat, chi2
```

| Census<br>region | Age category |       |     | Total |
|------------------|--------------|-------|-----|-------|
|                  | 19-29        | 30-34 | 35+ |       |
| NE               | 46           | 83    | 37  | 166   |
| N Cntrl          | 162          | 92    | 30  | 284   |
| South            | 139          | 68    | 43  | 250   |
| West             | 160          | 73    | 23  | 256   |
| Total            | 507          | 316   | 133 | 956   |

Pearson chi2(6) = 61.2877 Pr = 0.000

We obtain the standard two-way table and, at the bottom, a summary of the  $\chi^2$  test. Stata informs us that the  $\chi^2$  associated with this table has 6 degrees of freedom and is 61.29. The observed differences are significant.

The table is, perhaps, easier to understand if we suppress the frequencies and print just the row percentages:

```
. tabulate region agecat, row nofreq chi2
```

| Census region | Age category |       |       | Total  |
|---------------|--------------|-------|-------|--------|
|               | 19-29        | 30-34 | 35+   |        |
| NE            | 27.71        | 50.00 | 22.29 | 100.00 |
| N Cntrl       | 57.04        | 32.39 | 10.56 | 100.00 |
| South         | 55.60        | 27.20 | 17.20 | 100.00 |
| West          | 62.50        | 28.52 | 8.98  | 100.00 |
| Total         | 53.03        | 33.05 | 13.91 | 100.00 |

Pearson chi2(6) = 61.2877 Pr = 0.000



### ▶ Example 3

We have data on dose level and outcome for a set of patients and wish to evaluate the association between the two variables. We can obtain all the association measures by specifying the `all` and `exact` options:

```
. use https://www.stata-press.com/data/r19/dose
. tabulate dose function, all exact
```

Enumerating sample-space combinations:  
stage 3: enumerations = 1  
stage 2: enumerations = 9  
stage 1: enumerations = 0

| Dosage | Function |        |    | Total |
|--------|----------|--------|----|-------|
|        | < 1 hr   | 1 to 4 | 4+ |       |
| 1/day  | 20       | 10     | 2  | 32    |
| 2/day  | 16       | 12     | 4  | 32    |
| 3/day  | 10       | 16     | 6  | 32    |
| Total  | 46       | 38     | 12 | 96    |

Pearson chi2(4) = 6.7780 Pr = 0.148  
Likelihood-ratio chi2(4) = 6.9844 Pr = 0.137  
Cramér's V = 0.1879  
gamma = 0.3689 ASE = 0.129  
Kendall's tau-b = 0.2378 ASE = 0.086  
Fisher's exact = 0.145

We find evidence of association but not enough to be truly convincing.

If we had not also specified the `exact` option, we would not have obtained Fisher's exact test. Stata can calculate this statistic both for  $2 \times 2$  tables and for  $r \times c$ . For  $2 \times 2$  tables, the calculation is almost instant. On more general tables, however, the calculation can take longer.

We carefully constructed our example so that `all` would be meaningful. Kendall's  $\tau_b$  and Goodman and Kruskal's gamma are relevant only when both dimensions of the table can be ordered, say, from low to high or from worst to best. The other statistics, however, are always applicable.



□ Technical note

Be careful when attempting to compute the  $p$ -value for Fisher’s exact test because the number of tables that contribute to the  $p$ -value can be extremely large and a solution may not be feasible. The errors that are indicative of this situation are errors 910, exceeded memory limitations, and 1401, integer overflow due to large row-margin frequencies. If execution terminates because of memory limitations, use `exact(2)` to permit the algorithm to consume twice the memory, `exact(3)` for three times the memory, etc. The default memory usage should be sufficient for reasonable tables.



**N-way tables**

If you need more than two-way tables, your best alternative to is use `table`, not `tabulate`; see [R] `table`.

The [technical note](#) below shows you how to use `tabulate` to create a sequence of two-way tables that together form, in effect, a three-way table, but using `table` is easy and produces prettier results:

```
. use https://www.stata-press.com/data/r19/birthcat
(City data)
. table (agecat birthcat) (region), nototals
```

|                     | Census region |         |       |      |
|---------------------|---------------|---------|-------|------|
|                     | NE            | N Cntrl | South | West |
| Age category        |               |         |       |      |
| 19-29               |               |         |       |      |
| Birth-rate category |               |         |       |      |
| 29-136              | 11            | 23      | 11    | 11   |
| 137-195             | 31            | 97      | 65    | 46   |
| 196-529             | 4             | 38      | 59    | 91   |
| 30-34               |               |         |       |      |
| Birth-rate category |               |         |       |      |
| 29-136              | 34            | 27      | 10    | 8    |
| 137-195             | 48            | 58      | 45    | 42   |
| 196-529             | 1             | 3       | 12    | 21   |
| 35+                 |               |         |       |      |
| Birth-rate category |               |         |       |      |
| 29-136              | 34            | 26      | 27    | 18   |
| 137-195             | 3             | 4       | 7     | 4    |
| 196-529             |               |         | 4     |      |

□ Technical note

We can make  $n$ -way tables by combining the *by varlist*: prefix with `tabulate`. Continuing with the dataset of 956 cities, say that we want to make a table of age category by birth-rate category by region of the country. The birth-rate category variable is named `birthcat` in our dataset. To make separate tables for each age category, we would type

```
. by agecat, sort: tabulate birthcat region
```

```
-> agecat = 19-29
```

| Birth-rate category | Census region |         |       |      | Total |
|---------------------|---------------|---------|-------|------|-------|
|                     | NE            | N Cntrl | South | West |       |
| 29-136              | 11            | 23      | 11    | 11   | 56    |
| 137-195             | 31            | 97      | 65    | 46   | 239   |
| 196-529             | 4             | 38      | 59    | 91   | 192   |
| Total               | 46            | 158     | 135   | 148  | 487   |

```
-> agecat = 30-34
```

| Birth-rate category | Census region |         |       |      | Total |
|---------------------|---------------|---------|-------|------|-------|
|                     | NE            | N Cntrl | South | West |       |
| 29-136              | 34            | 27      | 10    | 8    | 79    |
| 137-195             | 48            | 58      | 45    | 42   | 193   |
| 196-529             | 1             | 3       | 12    | 21   | 37    |
| Total               | 83            | 88      | 67    | 71   | 309   |

```
-> agecat = 35+
```

| Birth-rate category | Census region |         |       |      | Total |
|---------------------|---------------|---------|-------|------|-------|
|                     | NE            | N Cntrl | South | West |       |
| 29-136              | 34            | 26      | 27    | 18   | 105   |
| 137-195             | 3             | 4       | 7     | 4    | 18    |
| 196-529             | 0             | 0       | 4     | 0    | 4     |
| Total               | 37            | 30      | 38    | 22   | 127   |



## Weighted data

### ▷ Example 4

tabulate can process weighted as well as unweighted data. As with all Stata commands, we indicate the weight by specifying the `[weight]` modifier; see [U] 11.1.6 [weight](#).

Continuing with our dataset of 956 cities, we also have a variable called `pop`, the population of each city. We can make a table of region by age category, weighted by population, by typing

```
. tabulate region agecat [fweight=pop]
```

| Census region | Age category |          |          | Total     |
|---------------|--------------|----------|----------|-----------|
|               | 19-29        | 30-34    | 35+      |           |
| NE            | 4721387      | 10421387 | 5323610  | 20466384  |
| N Cntrl       | 16901550     | 8964756  | 4015593  | 29881899  |
| South         | 13894254     | 7686531  | 4141863  | 25722648  |
| West          | 16698276     | 7755255  | 2375118  | 26828649  |
| Total         | 52215467     | 34827929 | 15856184 | 102899580 |

If we specify the `cell`, `column`, or `row` options, they will also be appropriately weighted. Below, we repeat the table, suppressing the counts and substituting row percentages:

```
. tabulate region agecat [fweight=pop], nofreq row
```

| Census region | Age category |       |       | Total  |
|---------------|--------------|-------|-------|--------|
|               | 19-29        | 30-34 | 35+   |        |
| NE            | 23.07        | 50.92 | 26.01 | 100.00 |
| N Cntrl       | 56.56        | 30.00 | 13.44 | 100.00 |
| South         | 54.02        | 29.88 | 16.10 | 100.00 |
| West          | 62.24        | 28.91 | 8.85  | 100.00 |
| Total         | 50.74        | 33.85 | 15.41 | 100.00 |



## Tables with immediate data

### ▷ Example 5

`tabi` ignores the dataset in memory and uses as the table the values that we specify on the command line:

```
. tabi 30 18 \ 38 14
```

| row   | col |    | Total |
|-------|-----|----|-------|
|       | 1   | 2  |       |
| 1     | 30  | 18 | 48    |
| 2     | 38  | 14 | 52    |
| Total | 68  | 32 | 100   |

Fisher's exact = 0.289  
1-sided Fisher's exact = 0.179

We may specify any of the options of tabulate and are not limited to  $2 \times 2$  tables:

```
. tabi 30 18 38 \ 13 7 22, chi2 exact
```

Enumerating sample-space combinations:

```
stage 3: enumerations = 1
stage 2: enumerations = 3
stage 1: enumerations = 0
```

| row   | col |    |    | Total |
|-------|-----|----|----|-------|
|       | 1   | 2  | 3  |       |
| 1     | 30  | 18 | 38 | 86    |
| 2     | 13  | 7  | 22 | 42    |
| Total | 43  | 25 | 60 | 128   |

```
Pearson chi2(2) = 0.7967 Pr = 0.671
Fisher's exact = 0.707
```

```
. tabi 30 13 \ 18 7 \ 38 22, all exact col
```

|                          |
|--------------------------|
| Key                      |
| <i>frequency</i>         |
| <i>column percentage</i> |

Enumerating sample-space combinations:

```
stage 3: enumerations = 1
stage 2: enumerations = 3
stage 1: enumerations = 0
```

| row   | col          |              | Total         |
|-------|--------------|--------------|---------------|
|       | 1            | 2            |               |
| 1     | 30<br>34.88  | 13<br>30.95  | 43<br>33.59   |
| 2     | 18<br>20.93  | 7<br>16.67   | 25<br>19.53   |
| 3     | 38<br>44.19  | 22<br>52.38  | 60<br>46.88   |
| Total | 86<br>100.00 | 42<br>100.00 | 128<br>100.00 |

```
Pearson chi2(2) = 0.7967 Pr = 0.671
Likelihood-ratio chi2(2) = 0.7985 Pr = 0.671
Cramér's V = 0.0789
gamma = 0.1204 ASE = 0.160
Kendall's tau-b = 0.0630 ASE = 0.084
Fisher's exact = 0.707
```

For  $2 \times 2$  tables, both one- and two-sided Fisher's exact probabilities are displayed; this is true of both tabulate and tabi. See [Cumulative incidence data](#) and [Case-control data](#) in [R] **Epitab** for more discussion on the relationship between one- and two-sided probabilities.

□ Technical note

tabi, as with all immediate commands, leaves any data in memory undisturbed. With the replace option, however, the data in memory are replaced by the data from the table:

```
. tabi 30 18 \ 38 14, replace
```

| row   | col |    | Total |
|-------|-----|----|-------|
|       | 1   | 2  |       |
| 1     | 30  | 18 | 48    |
| 2     | 38  | 14 | 52    |
| Total | 68  | 32 | 100   |

```

 Fisher's exact = 0.289
1-sided Fisher's exact = 0.179
. list
```

|    | row | col | pop |
|----|-----|-----|-----|
| 1. | 1   | 1   | 30  |
| 2. | 1   | 2   | 18  |
| 3. | 2   | 1   | 38  |
| 4. | 2   | 2   | 14  |

With this dataset, you could re-create the above table by typing

```
. tabulate row col [fweight=pop], exact
```

| row   | col |    | Total |
|-------|-----|----|-------|
|       | 1   | 2  |       |
| 1     | 30  | 18 | 48    |
| 2     | 38  | 14 | 52    |
| Total | 68  | 32 | 100   |

```

 Fisher's exact = 0.289
1-sided Fisher's exact = 0.179
```



**tab2**

tab2 is a convenience tool. Typing

```
. tab2 myvar thisvar thatvar, chi2
```

is equivalent to typing

```

. tabulate myvar thisvar, chi2
. tabulate myvar thatvar, chi2
. tabulate thisvar thatvar, chi2
```

## Publish your tables

With the `collect` option, `tabulate` and `tabi` post the tabulated values to a collection named `Tabulate` and set it as the current collection. `tab2` appends the resulting tabulations into a single collection named `Tabulate` and sets it as the current collection. With collections, you can customize the look of your table, then publish it to HTML, Word, L<sup>A</sup>T<sub>E</sub>X, PDF, Excel, or another format appropriate for your report.

The default `collect` layout for `tab2` when the `firstonly` option is not specified produces a table for the first pair of variables. You must specify the corresponding layout to get a table for the other two-way tabulations.

With the `by` prefix, `tabulate` and `tab2` append the resulting tabulations into a single collection, and the default layout produces a separate table for each by group.

If you are not familiar with collections, see [TABLES] [Intro](#). The predefined styles for `tabulate` are documented in [TABLES] [Predefined styles](#).

### ▷ Example 6

Recall the [example](#) where we used the `all` option to show all the supported measures of association. Let's use `tabulate` with the `collect` option to produce a collection with its tabulated values.

```
. tabulate dose function, all exact collect
```

```
Enumerating sample-space combinations:
```

```
stage 3: enumerations = 1
```

```
stage 2: enumerations = 9
```

```
stage 1: enumerations = 0
```

| Dosage | Function |        |    | Total |
|--------|----------|--------|----|-------|
|        | < 1 hr   | 1 to 4 | 4+ |       |
| 1/day  | 20       | 10     | 2  | 32    |
| 2/day  | 16       | 12     | 4  | 32    |
| 3/day  | 10       | 16     | 6  | 32    |
| Total  | 46       | 38     | 12 | 96    |

```
Pearson chi2(4) = 6.7780 Pr = 0.148
```

```
Likelihood-ratio chi2(4) = 6.9844 Pr = 0.137
```

```
Cramér's V = 0.1879
```

```
gamma = 0.3689 ASE = 0.129
```

```
Kendall's tau-b = 0.2378 ASE = 0.086
```

```
Fisher's exact = 0.145
```

The output does not change; however, we can use the `collect dir` command to see that `tabulate` created a collection named `Tabulate`.

```
. collect dir
```

```
Collections in memory
```

```
Current: Tabulate
```

| Name     | No. items |
|----------|-----------|
| Tabulate | 28        |

In this collection, frequencies are tagged with result [frequency]. The tabulated variables (that is, dose and function) are added to the collection as dimensions and are used to tag the collected results. In addition to the names, labels, level values, and value labels of the tabulated variables, these dimensions each have the `__margCode__` level with the Total label for tagging the marginal frequencies. Here we use `collect label list` to show the levels and labels of each tabulated variable dimension.

```
. collect label list dose
 Collection: Tabulate
 Dimension: dose
 Label: Dosage
Level labels:
 1 1/day
 2 2/day
 3 3/day
__margCode__ Total

. collect label list function
 Collection: Tabulate
 Dimension: function
 Label: Function
Level labels:
 1 < 1 hr
 2 1 to 4
 3 4+
__margCode__ Total
```

When measures of association are requested, `tabulate` uses a style that defines composite results for them. Each measure of association has a corresponding composite result that composes it in a single cell of a second table.

| Measure of association         | Option | Composite result  |
|--------------------------------|--------|-------------------|
| Pearson's $\chi^2$             | chi2   | Pearson_ma        |
| Fisher's exact                 | exact  | Fisher_ma         |
| 1-sided Fisher's exact         | exact  | Fisher1_ma        |
| Likelihood ratio               | lr     | LR_ma             |
| Cramér's $V$                   | V      | Cramer_ma         |
| Goodman and Kruskal's $\gamma$ | gamma  | GoodmanKruskal_ma |
| Kendall's $\tau_b$             | taub   | Pearson_ma        |

tabulate constructs a default layout, so you can view your customizable table with the collect preview command. Here we use the collect layout command to report the default layout specification and corresponding table.

```
. collect layout
Collection: Tabulate
 Rows: dose#result result
 Columns: function _overall
 Tables: cmdset#item_type
 Table 1: 5 x 4
 Table 2: 6 x 1
```

|        | Function |        |    | Total |
|--------|----------|--------|----|-------|
|        | < 1 hr   | 1 to 4 | 4+ |       |
| Dosage |          |        |    |       |
| 1/day  | 20       | 10     | 2  | 32    |
| 2/day  | 16       | 12     | 4  | 32    |
| 3/day  | 10       | 16     | 6  | 32    |
| Total  | 46       | 38     | 12 | 96    |

```
Pearson $\chi^2(4) = 6.7780$ Pr = 0.1481
LR $\chi^2(4) = 6.9844$ Pr = 0.1367
Cramér's V = 0.1879
Goodman and Kruskal's $\gamma = 0.3689$ ASE = 0.1287
Kendall's $\tau_b = 0.2378$ ASE = 0.0865
Fisher's exact = 0.1449
```

We can make further changes to the table with the collect suite of commands. But we are happy with this layout and ready to publish the table to an HTML file with collect export. We simply specify the filename to which we want to export it.

```
. collect export tab2.html
(collection Tabulate exported to file tab2.html)
```

With collect export, you can publish the table to several formats, such as HTML, PDF, and L<sup>A</sup>T<sub>E</sub>X files, by specifying the appropriate file extension.



## Video examples

- [Pearson's chi-squared and Fisher's exact test in Stata](#)
- [Tables and cross-tabulations in Stata](#)
- [Cross-tabulations and chi-squared tests calculator](#)

## Stored results

tabulate, tab2, and tabi store the following in `r()`:

### Scalars

|                          |                                           |                          |                                      |
|--------------------------|-------------------------------------------|--------------------------|--------------------------------------|
| <code>r(N)</code>        | number of observations                    | <code>r(p_exact)</code>  | Fisher's exact $p$                   |
| <code>r(r)</code>        | number of rows                            | <code>r(chi2_lr)</code>  | likelihood-ratio $\chi^2$            |
| <code>r(c)</code>        | number of columns                         | <code>r(p_lr)</code>     | $p$ -value for likelihood-ratio test |
| <code>r(chi2)</code>     | Pearson's $\chi^2$ test                   | <code>r(CramersV)</code> | Cramér's $V$                         |
| <code>r(p)</code>        | $p$ -value for of Pearson's $\chi^2$ test | <code>r(ase_gam)</code>  | ASE of gamma                         |
| <code>r(gamma)</code>    | gamma                                     | <code>r(ase_taub)</code> | ASE of $\tau_b$                      |
| <code>r(p1_exact)</code> | one-sided Fisher's exact $p$              | <code>r(taub)</code>     | $\tau_b$                             |

### Macros

`r(collection)` name of collection

`r(p1_exact)` is defined only for  $2 \times 2$  tables. `r(collection)` is defined only when the `collect()` option is specified. Also, the `matrow()`, `matcol()`, and `matcell()` options allow you to obtain the row values, column values, and frequencies, respectively.

## Methods and formulas

Let  $n_{ij}$ ,  $i = 1, \dots, I$  and  $j = 1, \dots, J$ , be the number of observations in the  $i$ th row and  $j$ th column. If the data are not weighted,  $n_{ij}$  is just a count. If the data are weighted,  $n_{ij}$  is the sum of the weights of all data corresponding to the  $(i, j)$  cell.

Define the row and column marginals as

$$n_{i.} = \sum_{j=1}^J n_{ij} \qquad n_{.j} = \sum_{i=1}^I n_{ij}$$

and let  $n = \sum_i \sum_j n_{ij}$  be the overall sum. Also, define the concordance and discordance as

$$A_{ij} = \sum_{k>i} \sum_{l>j} n_{kl} + \sum_{k<i} \sum_{l<j} n_{kl} \qquad D_{ij} = \sum_{k>i} \sum_{l<j} n_{kl} + \sum_{k<i} \sum_{l>j} n_{kl}$$

along with twice the number of concordances  $P = \sum_i \sum_j n_{ij} A_{ij}$  and twice the number of discordances  $Q = \sum_i \sum_j n_{ij} D_{ij}$ .

The Pearson  $\chi^2$  statistic with  $(I - 1)(J - 1)$  degrees of freedom (so called because it is based on [Pearson \(1900\)](#); see [Conover \[1999, 240\]](#) and [Fienberg \[1980, 9\]](#)) is defined as

$$X^2 = \sum_i \sum_j \frac{(n_{ij} - m_{ij})^2}{m_{ij}}$$

where  $m_{ij} = n_{i.} n_{.j} / n$ .

The likelihood-ratio  $\chi^2$  statistic with  $(I - 1)(J - 1)$  degrees of freedom ([Fienberg 1980, 40](#)) is defined as

$$G^2 = 2 \sum_i \sum_j n_{ij} \ln(n_{ij} / m_{ij})$$

Cramér's  $V$  ([Cramér 1946](#)) is a measure of association designed so that the attainable upper bound is 1. For  $2 \times 2$  tables,  $-1 \leq V \leq 1$ , and otherwise,  $0 \leq V \leq 1$ .

$$V = \begin{cases} (n_{11}n_{22} - n_{12}n_{21}) / (n_{1.}n_{.2}n_{.1}n_{2.})^{1/2} & \text{for } 2 \times 2 \\ \{(X^2/n) / \min(I - 1, J - 1)\}^{1/2} & \text{otherwise} \end{cases}$$

Gamma (Goodman and Kruskal 1954, 1959, 1963, 1972; also see Agresti [2010,186–188]) ignores tied pairs and is based only on the number of concordant and discordant pairs of observations,  $-1 \leq \gamma \leq 1$ ,

$$\gamma = (P - Q)/(P + Q)$$

with asymptotic variance

$$16 \sum_i \sum_j n_{ij} (QA_{ij} - PD_{ij})^2 / (P + Q)^4$$

Kendall's  $\tau_b$  (Kendall 1945; also see Agresti 2010, 188–189),  $-1 \leq \tau_b \leq 1$ , is similar to gamma, except that it uses a correction for ties,

$$\tau_b = (P - Q)/(w_r w_c)^{1/2}$$

with asymptotic variance

$$\frac{\sum_i \sum_j n_{ij} (2w_r w_c d_{ij} + \tau_b v_{ij})^2 - n^3 \tau_b^2 (w_r + w_c)^2}{(w_r w_c)^4}$$

where

$$\begin{aligned} w_r &= n^2 - \sum_i n_i^2 \\ w_c &= n^2 - \sum_j n_j^2 \\ d_{ij} &= A_{ij} - D_{ij} \\ v_{ij} &= n_i \cdot w_c + n_j \cdot w_r \end{aligned}$$

Fisher's exact test (Fisher 1935; Finney 1948; see Zelterman and Louis [1992, 293–301] for the  $2 \times 2$  case) yields the probability of observing a table that gives at least as much evidence of association as the one actually observed under the assumption of no association. Holding row and column marginals fixed, the hypergeometric probability  $P$  of every possible table  $A$  is computed, and the

$$P = \sum_{T \in A} \Pr(T)$$

where  $A$  is the set of all tables with the same marginals as the observed table,  $T^*$ , such that  $\Pr(T) \leq \Pr(T^*)$ . For  $2 \times 2$  tables, the one-sided probability is calculated by further restricting  $A$  to tables in the same tail as  $T^*$ . The first algorithm extending this calculation to  $r \times c$  tables was Pagano and Halvorsen (1981); the one implemented here is the FEXACT algorithm by Mehta and Patel (1986). This is a search-tree clipping method originally published by Mehta and Patel (1983) with further refinements by Joe (1988) and Clarkson, Fan, and Joe (1993). Fisher's exact test is a permutation test. For more information on permutation tests, see Good (2005 and 2006) and Pesarin (2001).

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## Also see

[R] **Epitab** — Tables for epidemiologists

[R] **table** — Table of frequencies, summaries, and command results

[R] **table twoway** — Two-way tabulation

[R] **tabstat** — Compact table of summary statistics

[R] **tabulate oneway** — One-way table of frequencies

[R] **tabulate, summarize()** — One- and two-way tables of summary statistics

[D] **collapse** — Make dataset of summary statistics

[SVY] **svy: tabulate oneway** — One-way tables for survey data

[SVY] **svy: tabulate twoway** — Two-way tables for survey data

[TABLES] **Intro** — Introduction

[XT] **xttab** — Tabulate xt data

[U] **12.6.3 Value labels**

[U] **26 Working with categorical data and factor variables**

[Description](#)  
[Options](#)[Quick start](#)  
[Remarks and examples](#)[Menu](#)  
[Also see](#)[Syntax](#)

## Description

`tabulate`, `summarize()` produces one- and two-way tables (breakdowns) of means and standard deviations. See [R] [tabulate oneway](#) and [R] [tabulate twoway](#) for one- and two-way frequency tables. See [R] [table](#) for a more flexible command that produces one-, two-, and  $n$ -way tables of frequencies and a wide variety of summary statistics. `table` is better, but `tabulate`, `summarize()` is faster. Also see [R] [tabstat](#) for yet another alternative.

## Quick start

Tabulation of `v1`, reporting means and standard deviations of `x` and frequencies

```
tabulate v1, summarize(x)
```

Same as above, but report summary statistics for the two-way tabulation of `v1` and `v2`

```
tabulate v1 v2, summarize(x)
```

Weighted summary statistics using frequency weight `wvar`

```
tabulate v1 v2 [fweight=wvar], summarize(x)
```

Report only the mean of `x` for each group

```
tabulate v1 v2, summarize(x) means
```

Do not report standard deviations

```
tabulate v1 v2, summarize(x) nostandard
```

Show numeric values of `v1` and `v2` rather than value labels

```
tabulate v1 v2, summarize(x) nolabel
```

## Menu

Statistics > Summaries, tables, and tests > Other tables > Table of means, std. dev., and frequencies

## Syntax

```
tabulate varname1 [varname2] [if'] [in] [weight] [, options]
```

*options*

Description

Main

|                                             |                                                                                                 |
|---------------------------------------------|-------------------------------------------------------------------------------------------------|
| <code>summarize(varname<sub>3</sub>)</code> | report summary statistics for <i>varname<sub>3</sub></i>                                        |
| <code>[no]means</code>                      | include or suppress means                                                                       |
| <code>[no]standard</code>                   | include or suppress standard deviations                                                         |
| <code>[no]freq</code>                       | include or suppress frequencies                                                                 |
| <code>[no]obs</code>                        | include or suppress number of observations                                                      |
| <code>nolabel</code>                        | show numeric codes, not labels                                                                  |
| <code>wrap</code>                           | do not break wide tables                                                                        |
| <code>missing</code>                        | treat missing values of <i>varname<sub>1</sub></i> and <i>varname<sub>2</sub></i> as categories |

Collect

|                                                     |                                                  |
|-----------------------------------------------------|--------------------------------------------------|
| <code>collect</code>                                | post results to collection <code>Tabulate</code> |
| <code>collect([cname] [ , collect_options ])</code> | post results to a named collection               |

*collect\_options*

Description

|                                             |                                           |
|---------------------------------------------|-------------------------------------------|
| <code>append</code>                         | append results to an existing collection  |
| <code>replace</code>                        | replace results of an existing collection |
| <code>label(filename)</code>                | specify the collection labels             |
| <code>style(filename [ , override ])</code> | specify the collection style              |

by is allowed; see [U] 11.1.10 Prefix commands.

aweights and fweights are allowed; see [U] 11.1.6 weight.

## Options

Main

`summarize(varname3)` identifies the name of the variable for which summary statistics are to be reported. If you do not specify this option, a table of frequencies is produced; see [R] [tabulate oneway](#) and [R] [tabulate twoway](#). The description here concerns `tabulate` when this option is specified.

`[no]means` includes or suppresses only the means from the table.

The `summarize()` table normally includes the mean, standard deviation, frequency, and, if the data are weighted, number of observations. Individual elements of the table may be included or suppressed by the `[no]means`, `[no]standard`, `[no]freq`, and `[no]obs` options. For example, typing

```
. tabulate category, summarize(myvar) means standard
```

produces a summary table by category containing only the means and standard deviations of `myvar`. You could also achieve the same result by typing

```
. tabulate category, summarize(myvar) nofreq
```

`[no]standard` includes or suppresses only the standard deviations from the table; see `[no]means` option above.

[no]freq includes or suppresses only the frequencies from the table; see [no]means option above.

[no]obs includes or suppresses only the reported number of observations from the table. If the data are not weighted, the number of observations is identical to the frequency, and by default only the frequency is reported. If the data are weighted, the frequency refers to the sum of the weights. See [no]means option above.

noLabel causes the numeric codes to be displayed rather than the label values.

wrap requests that no action be taken on wide tables to make them readable. Unless wrap is specified, wide tables are broken into pieces to enhance readability.

missing requests that missing values of *varname*<sub>1</sub> and *varname*<sub>2</sub> be treated as categories rather than as observations to be omitted from the analysis.

### Collect

collect and collect(*[cname]*[, *collect\_options*]) specify that results be posted to a collection. This collection produces a table that you can customize and publish to Microsoft Word, Microsoft Excel, PDF, HTML, L<sup>A</sup>T<sub>E</sub>X, SMCL, or Markdown. Output does not change when these options are specified. Use collect preview to see the customizable table.

collect is a shortcut for collect(Tabulate).

*cname* specifies that a collection named *cname* be associated with the collected results. The default is Tabulate.

append specifies that results be appended to collection *cname*.

replace permits tabulate to overwrite an existing collection. This option is implied for collection Tabulate when append is not specified.

label(*filename*) specifies the *filename* containing the collection labels to use for your table. Labels in *filename* will be loaded into the collection, and any labels not specified in *filename* will be taken from the labels defined in c(collect\_label). The default is to use only the collection labels set in c(collect\_label); see [TABLES] set collect\_label.

style(*filename*[, *override*]) specifies the *filename* containing the collection styles to use for your table. The default collection styles will be discarded, and only the collection styles in *filename* will be applied.

If you prefer the default collection styles but also want to apply any styles in *filename*, specify *override*. If there are conflicts between the default collection styles and those in *filename*, the ones in *filename* will take precedence.

The default is to use only the collection styles set in c(tabulate\_style); see [TABLES] set tabulate\_style.

## Remarks and examples

tabulate with the summarize() option produces one- and two-way tables of summary statistics. When combined with the by prefix, it can produce *n*-way tables as well.

Remarks are presented under the following headings:

- One-way tables*
- Two-way tables*
- Publish your tables*

## One-way tables

### ▷ Example 1

We have data on 74 automobiles. Included in our dataset are the variables `foreign`, which marks domestic and foreign cars, and `mpg`, the car's mileage rating. Typing `tabulate foreign` displays a breakdown of the number of observations we have by the values of the `foreign` variable.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
```

```
. tabulate foreign
```

| Car origin | Freq. | Percent | Cum.   |
|------------|-------|---------|--------|
| Domestic   | 52    | 70.27   | 70.27  |
| Foreign    | 22    | 29.73   | 100.00 |
| Total      | 74    | 100.00  |        |

We discover that we have 52 domestic cars and 22 foreign cars in our dataset. If we add the `summarize(varname)` option, however, `tabulate` produces a table of summary statistics for `varname`:

```
. tabulate foreign, summarize(mpg)
```

| Car origin | Summary of Mileage (mpg) |           | Freq. |
|------------|--------------------------|-----------|-------|
|            | Mean                     | Std. dev. |       |
| Domestic   | 19.826923                | 4.7432972 | 52    |
| Foreign    | 24.772727                | 6.6111869 | 22    |
| Total      | 21.297297                | 5.7855032 | 74    |

We also discover that the average gas mileage for domestic cars is about 20 mpg and the average foreign is almost 25 mpg. Overall, the average is 21 mpg in our dataset.

□ Technical note

We might now wonder if the difference in gas mileage between foreign and domestic cars is statistically significant. We can use the `oneway` command to find out; see [R] [oneway](#). To obtain an analysis-of-variance table of `mpg` on `foreign`, we type

```
. oneway mpg foreign
```

| Source         | Analysis of variance |    |            | F     | Prob > F |
|----------------|----------------------|----|------------|-------|----------|
|                | SS                   | df | MS         |       |          |
| Between groups | 378.153515           | 1  | 378.153515 | 13.18 | 0.0005   |
| Within groups  | 2065.30594           | 72 | 28.6848048 |       |          |
| Total          | 2443.45946           | 73 | 33.4720474 |       |          |

Bartlett's equal-variances test:  $\chi^2(1) = 3.4818$      $\text{Prob} > \chi^2 = 0.062$

The  $F$  statistic is 13.18, and the difference between foreign and domestic cars' mileage ratings is significant at the 0.05% level.

There are several ways that we could have statistically compared mileage ratings—see, for instance, [R] [anova](#), [R] [oneway](#), [R] [regress](#), and [R] [ttest](#)—but `oneway` seemed the most convenient.



## Two-way tables

▷ Example 2

`tabulate`, `summarize` can be used to obtain two-way as well as one-way breakdowns. For instance, we obtained summary statistics on `mpg` decomposed by `foreign` by typing `tabulate foreign, summarize(mpg)`. We can specify up to two variables before the comma:

```
. generate wgtcat = autocode(weight,4,1760,4840)
. tabulate wgtcat foreign, summarize(mpg)
```

Means, Standard Deviations and Frequencies of Mileage (mpg)

| wgtcat | Car origin |           | Total     |
|--------|------------|-----------|-----------|
|        | Domestic   | Foreign   |           |
| 2530   | 28.285714  | 27.0625   | 27.434783 |
|        | 3.0937725  | 5.9829619 | 5.2295149 |
|        | 7          | 16        | 23        |
| 3300   | 21.75      | 19.6      | 21.238095 |
|        | 2.4083189  | 3.4351128 | 2.7550819 |
|        | 16         | 5         | 21        |
| 4070   | 17.26087   | 14        | 17.125    |
|        | 1.8639497  | 0         | 1.9406969 |
|        | 23         | 1         | 24        |
| 4840   | 14.666667  | .         | 14.666667 |
|        | 3.32666    | .         | 3.32666   |
|        | 6          | 0         | 6         |
| Total  | 19.826923  | 24.772727 | 21.297297 |
|        | 4.7432972  | 6.6111869 | 5.7855032 |
|        | 52         | 22        | 74        |

In addition to the means, standard deviations, and frequencies for each weight–mileage cell, also reported are the summary statistics by weight, by mileage, and overall. For instance, the last row of the table reveals that the average mileage of domestic cars is 19.83 and that of foreign cars is 24.77—domestic cars yield poorer mileage than foreign cars. But we now see that domestic cars yield better gas mileage within weight class—the reason domestic cars yield poorer gas mileage is because they are, on average, heavier.



### ▷ Example 3

If we do not specify the statistics to be included in a table, `tabulate` reports the mean, standard deviation, and frequency. We can specify the statistics that we want to see using the `means`, `standard`, and `freq` options:

```
. tabulate wgtcat foreign, summarize(mpg) means
 Means of Mileage (mpg)
```

| wgtcat | Car origin |           | Total     |
|--------|------------|-----------|-----------|
|        | Domestic   | Foreign   |           |
| 2530   | 28.285714  | 27.0625   | 27.434783 |
| 3300   | 21.75      | 19.6      | 21.238095 |
| 4070   | 17.26087   | 14        | 17.125    |
| 4840   | 14.666667  | .         | 14.666667 |
| Total  | 19.826923  | 24.772727 | 21.297297 |

When we specify one or more of the `means`, `standard`, and `freq` options, only those statistics are displayed. Thus, we could obtain a table containing just the means and standard deviations by typing `means standard` after the `summarize(mpg)` option. We can also suppress selected statistics by placing `no` in front of the option name. Another way of obtaining only the means and standard deviations is to add the `nofreq` option:

```
. tabulate wgtcat foreign, summarize(mpg) nofreq
 Means and Standard Deviations of Mileage (mpg)
```

| wgtcat | Car origin |           | Total     |
|--------|------------|-----------|-----------|
|        | Domestic   | Foreign   |           |
| 2530   | 28.285714  | 27.0625   | 27.434783 |
|        | 3.0937725  | 5.9829619 | 5.2295149 |
| 3300   | 21.75      | 19.6      | 21.238095 |
|        | 2.4083189  | 3.4351128 | 2.7550819 |
| 4070   | 17.26087   | 14        | 17.125    |
|        | 1.8639497  | 0         | 1.9406969 |
| 4840   | 14.666667  | .         | 14.666667 |
|        | 3.32666    | .         | 3.32666   |
| Total  | 19.826923  | 24.772727 | 21.297297 |
|        | 4.7432972  | 6.6111869 | 5.7855032 |



## Publish your tables

With the `collect` option, `tabulate` posts the tabulated values to a collection named `Tabulate` and sets it as the current collection. With collections, you can customize the look of your table, then publish it to HTML, Word, L<sup>A</sup>T<sub>E</sub>X, PDF, Excel, or another format appropriate for your report.

With the `by` prefix, `tabulate` appends the resulting tabulations into a single collection, and the default layout produces a separate table for each by group.

If you are not familiar with collections, see [TABLES] [Intro](#). The predefined styles for `tabulate` are documented in [TABLES] [Predefined styles](#).

### ▷ Example 4

Recall our table from the [previous example](#). Let's add the `collect` option in the call to `tabulate` to produce a collection with its tabulated values.

```
. tabulate wgtcat foreign, summarize(mpg) collect
 Means, Standard Deviations and Frequencies of Mileage (mpg)
```

| wgtcat | Car origin |           | Total     |
|--------|------------|-----------|-----------|
|        | Domestic   | Foreign   |           |
| 2530   | 28.285714  | 27.0625   | 27.434783 |
|        | 3.0937725  | 5.9829619 | 5.2295149 |
|        | 7          | 16        | 23        |
| 3300   | 21.75      | 19.6      | 21.238095 |
|        | 2.4083189  | 3.4351128 | 2.7550819 |
|        | 16         | 5         | 21        |
| 4070   | 17.26087   | 14        | 17.125    |
|        | 1.8639497  | 0         | 1.9406969 |
|        | 23         | 1         | 24        |
| 4840   | 14.666667  | .         | 14.666667 |
|        | 3.32666    | .         | 3.32666   |
|        | 6          | 0         | 6         |
| Total  | 19.826923  | 24.772727 | 21.297297 |
|        | 4.7432972  | 6.6111869 | 5.7855032 |
|        | 52         | 22        | 74        |

The output does not change; however, we can use the `collect dir` command to see that `tabulate` created a collection named `Tabulate`.

```
. collect dir
Collections in memory
Current: Tabulate
```

| Name     | No. items |
|----------|-----------|
| Table    | 51        |
| Tabulate | 45        |

In this collection, means are tagged with `result[mean]`, standard deviations with `result[sd]`, and frequencies with `result[frequency]`. When weights are specified, observation counts are tagged with `result[count]`. Here we use `collect label list` to show the levels and labels of the result dimension.

```
. collect label list result
Collection: Tabulate
Dimension: result
Label: Result
Level labels:
 frequency Frequency
 mean Mean
 sd Std. dev.
```

The tabulated variables (that is, `wgtpcat` and `foreign`) are added to the collection as dimensions and are used to tag the collected results. In addition to the name, label, level values, and value labels of the tabulated variables, these dimensions each have the `__margCode__` level with the Total label for tagging the marginal summary statistics. Here we use `collect label list` to show the levels and labels of each tabulated variable dimension. For `wgtpcat`, we add the `all` option to show the levels without labels.

```
. collect label list wgtpcat, all
Collection: Tabulate
Dimension: wgtpcat
Label:
Level labels:
 2530
 3300
 4070
 4840
__margCode__ Total
. collect label list foreign
Collection: Tabulate
Dimension: foreign
Label: Car origin
Level labels:
 0 Domestic
 1 Foreign
__margCode__ Total
```

tabulate constructs a default layout, so you can view your customizable table with the collect preview command. Here we use the collect layout command to report the default layout specification and corresponding table.

```
. collect layout
Collection: Tabulate
 Rows: wgtcat#result
 Columns: var#foreign
 Tables: cmdset
Table 1: 21 x 3
```

|           | Mileage (mpg) |         |       |
|-----------|---------------|---------|-------|
|           | Car origin    |         |       |
|           | Domestic      | Foreign | Total |
| wgtcat    |               |         |       |
| 2530      |               |         |       |
| Mean      | 28.3          | 27.1    | 27.4  |
| Std. dev. | 3.1           | 6.0     | 5.2   |
| Frequency | 7             | 16      | 23    |
| 3300      |               |         |       |
| Mean      | 21.8          | 19.6    | 21.2  |
| Std. dev. | 2.4           | 3.4     | 2.8   |
| Frequency | 16            | 5       | 21    |
| 4070      |               |         |       |
| Mean      | 17.3          | 14.0    | 17.1  |
| Std. dev. | 1.9           | 0.0     | 1.9   |
| Frequency | 23            | 1       | 24    |
| 4840      |               |         |       |
| Mean      | 14.7          | .       | 14.7  |
| Std. dev. | 3.3           | .       | 3.3   |
| Frequency | 6             | 0       | 6     |
| Total     |               |         |       |
| Mean      | 19.8          | 24.8    | 21.3  |
| Std. dev. | 4.7           | 6.6     | 5.8   |
| Frequency | 52            | 22      | 74    |

We now change some cell formats, remove the summary statistic labels from the row header, and add a note suggesting the table's cell contents.

```
. collect style cell result[mean], nformat(%9.2f)
. collect style cell result[sd], nformat(%9.2f) sformat("(%s)")
. collect style header result, level(hide)
. collect note "Mean (SD) N"
. collect preview
```

|        | Mileage (mpg)   |                 |                 |
|--------|-----------------|-----------------|-----------------|
|        | Car origin      |                 |                 |
|        | Domestic        | Foreign         | Total           |
| wgtcat |                 |                 |                 |
| 2530   | 28.29<br>(3.09) | 27.06<br>(5.98) | 27.43<br>(5.23) |
|        | 7               | 16              | 23              |
| 3300   | 21.75<br>(2.41) | 19.60<br>(3.44) | 21.24<br>(2.76) |
|        | 16              | 5               | 21              |
| 4070   | 17.26<br>(1.86) | 14.00<br>(0.00) | 17.12<br>(1.94) |
|        | 23              | 1               | 24              |
| 4840   | 14.67<br>(3.33) | .<br>(.)        | 14.67<br>(3.33) |
|        | 6               | 0               | 6               |
| Total  | 19.83<br>(4.74) | 24.77<br>(6.61) | 21.30<br>(5.79) |
|        | 52              | 22              | 74              |

Mean (SD) N

We can make further changes to the table with the `collect` suite of commands. But we are happy with this layout and ready to publish the table to a  $\LaTeX$  file with `collect export`. We simply specify the filename to which we want to export it.

```
. collect export tab2.tex
(collection Tabulate exported to file tab2.tex)
```

With `collect export`, you can publish the table to several formats, such as HTML, PDF, and  $\LaTeX$  files, by specifying the appropriate file extension.



## Also see

[R] [table](#) — Table of frequencies, summaries, and command results

[R] [table summary](#) — Table of summary statistics

[R] [tabstat](#) — Compact table of summary statistics

[R] [tabulate oneway](#) — One-way table of frequencies

[R] [tabulate twoway](#) — Two-way table of frequencies

[D] [collapse](#) — Make dataset of summary statistics

[SVY] [svy: tabulate oneway](#) — One-way tables for survey data

[SVY] [svy: tabulate twoway](#) — Two-way tables for survey data

[TABLES] [Intro](#) — Introduction

[U] [12.6 Dataset, variable, and value labels](#)

[U] [26 Working with categorical data and factor variables](#)

Description

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## Description

`test` performs Wald tests of simple and composite linear hypotheses about the parameters of the most recently fit model.

`test` supports `svy` estimators (see [SVY] [svy estimation](#)), carrying out an adjusted Wald test by default in such cases. `test` can be used with `svy` estimation results, see [SVY] [svy postestimation](#).

`testparm` provides a useful alternative to `test` that permits *varlist* rather than a list of coefficients (which is often nothing more than a list of variables), allowing the use of standard Stata notation, including ‘-’ and ‘\*’, which are given the expression interpretation by `test`.

`test` and `testparm` perform Wald tests. For likelihood-ratio tests, see [R] [lrtest](#). For Wald-type tests of nonlinear hypotheses, see [R] [testnl](#). To display estimates for one-dimensional linear or nonlinear expressions of coefficients, see [R] [lincom](#) and [R] [nlcom](#).

See [R] [anova postestimation](#) for additional `test` syntax allowed after `anova`.

See [MV] [manova postestimation](#) for additional `test` syntax allowed after `manova`.

## Quick start

*Linear tests after single-equation models*

Joint test that the coefficients on `x1` and `x2` are equal to 0

```
test x1 x2
```

Joint test that coefficients on [factor indicators](#) `2.a` and `3.a` are equal to 0

```
test 2.a 3.a
```

Test that coefficients on indicators `2.a` and `3.a` are equal

```
test 2.a = 3.a
```

Joint test that coefficients on indicators `1.a`, `2.a`, and `3.a` are all equal

```
test (1.a=2.a) (1.a=3.a)
```

Same as above

```
test 1.a=2.a=3.a
```

Same as above, but add separate tests for each pairing

```
test 1.a=2.a=3.a, mtest
```

Same as above, but with *p*-values adjusted for multiple comparisons using Šidák’s method

```
test (1.a=2.a) (1.a=3.a), mtest(sidak)
```

Test that the sum of the coefficients for  $x_1$  and  $x_2$  is equal to 4

```
test x1 + x2 = 4
```

Test the equality of two linear expressions involving coefficients on  $x_1$  and  $x_2$

```
test 2*x1 = 3*x2
```

### *Shorthand varlist notation*

Joint test that all coefficients on the indicators for  $a$  are equal to 0

```
testparm i.a
```

Joint test that all coefficients on the indicators for  $a$  and  $b$  are equal to 0

```
testparm i.a i.b
```

Joint test that all coefficients associated with the interaction of factor variables  $a$  and  $b$  are equal to 0

```
testparm i.a#i.b
```

Joint test that the coefficients on all variables  $x^*$  are equal to 0

```
testparm x*
```

### *Linear tests after multiple-equation models*

Joint test that the coefficient on  $x_1$  is equal to 0 in all equations

```
test x1
```

Joint test that the coefficients for  $x_1$  and  $x_2$  are equal to 0 in equation  $y_3$

```
test [y3]x1 [y3]x2
```

Test that the coefficients for  $x_1$  are equal in equations  $y_1$  and  $y_3$

```
test [y1]x1=[y3]x1
```

Same as above

```
test [y1=y3]: x1
```

Joint test of the equality of coefficients for  $x_1$  and  $x_2$  across equations  $y_1$  and  $y_3$

```
test [y1=y3]: x1 x2
```

Add coefficients for  $x_1$  and  $x_2$  from equation  $y_4$  to test

```
test [y1=y3=y4]: x1 x2
```

Test that all coefficients in the equation for  $y_1$  are equal to those in the equation for  $y_2$

```
test [y1=y2]
```

Same as above, but only for coefficients on variables common to both equations

```
test [y1=y2], common
```

*Shorthand varlist notation*

Joint test that all coefficients on the indicators for a are 0 in all equations

testparm i.a

Joint test that all coefficients on the indicators for a are equal to each other in the first equation

testparm i.a, equal

Same as above, but for the equation for y4

testparm i.a, equal equation(y4)

Joint test that the coefficients on the indicators for a and b are equal to 0 in all equations

testparm i.a i.b

Joint test that all coefficients associated with the interaction of factors a and b are 0

testparm i.a#i.b

## Menu

Statistics > Postestimation

## Syntax

### Basic syntax

`test` *coeflist* (Syntax 1)

`test` *exp=exp*[*=...*] (Syntax 2)

`test` [*eqno*] [: *coeflist*] (Syntax 3)

`test` [*eqno=eqno*[*=...*]] [: *coeflist*] (Syntax 4)

`testparm` *varlist* [, *testparm\_options*]

### Full syntax

`test` (*spec*) [(*spec*) ...] [, *test\_options*]

| <i>testparm_options</i>            | Description                                                                                                                                                                                                              |
|------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>equal</code>                 | hypothesize that the coefficients are equal to each other                                                                                                                                                                |
| <code>equation(<i>eqno</i>)</code> | specify equation name or number for which the hypothesis is tested                                                                                                                                                       |
| <code>nosvyadjust</code>           | compute unadjusted Wald tests for survey results                                                                                                                                                                         |
| <code>df(#)</code>                 | use $F$ distribution with # denominator degrees of freedom for the reference distribution of the test statistic; for survey data, # specifies the design degrees of freedom unless <code>nosvyadjust</code> is specified |

`df(#)` does not appear in the dialog box.

| <i>test_options</i>                 | Description                                                                                                                                                                                                              |
|-------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Options                             |                                                                                                                                                                                                                          |
| <code>mtest(<i>opt</i>)</code>      | test each condition separately                                                                                                                                                                                           |
| <code>coef</code>                   | report estimated constrained coefficients                                                                                                                                                                                |
| <code>accumulate</code>             | test hypothesis jointly with previously tested hypotheses                                                                                                                                                                |
| <code>notest</code>                 | suppress the output                                                                                                                                                                                                      |
| <code>common</code>                 | test only variables common to all the equations                                                                                                                                                                          |
| <code>constant</code>               | include the constant in coefficients to be tested                                                                                                                                                                        |
| <code>nosvyadjust</code>            | compute unadjusted Wald tests for survey results                                                                                                                                                                         |
| <code>minimum</code>                | perform test with the constant, drop terms until the test becomes nonsingular, and test without the constant on the remaining terms; highly technical                                                                    |
| <code>matvlc(<i>matname</i>)</code> | save the variance–covariance matrix; programmer’s option                                                                                                                                                                 |
| <code>df(#)</code>                  | use $F$ distribution with # denominator degrees of freedom for the reference distribution of the test statistic; for survey data, # specifies the design degrees of freedom unless <code>nosvyadjust</code> is specified |

`matvlc(matname)` and `df(#)` do not appear in the dialog box.

*coeflist* and *varlist* may contain factor variables and time-series operators; see [U] 11.4.3 Factor variables and [U] 11.4.4 Time-series varlists.

`collect` is allowed with `test`; see [U] 11.1.10 Prefix commands.

Syntax 1 tests that coefficients are 0.

Syntax 2 tests that linear expressions are equal.

Syntax 3 tests that coefficients in *eqno* are 0.

Syntax 4 tests equality of coefficients between equations.

*spec* is one of

```
coeflist
exp=exp [=exp]
[eqno] [: coeflist]
[eqno1=eqno2 [=...]] [: coeflist]
```

*coeflist* is

```
coef [coef ...]
[eqno]coef [[eqno]coef ...]
[eqno] _b [coef] [[eqno] _b [coef] ...]
```

*exp* is a linear expression containing

```
coef
_b [coef]
_b [eqno : coef]
[eqno]coef
[eqno] _b [coef]
```

*eqno* is

```
##
name
```

*coef* identifies a coefficient in the model. *coef* is typically a variable name, a level indicator, an interaction indicator, or an interaction involving continuous variables. Level indicators identify one level of a factor variable and interaction indicators identify one combination of levels of an interaction; see [\[U\] 11.4.3 Factor variables](#). *coef* may contain time-series operators; see [\[U\] 11.4.4 Time-series varlists](#).

Distinguish between `[]`, which are to be typed, and `[]`, which indicate optional arguments.

Although not shown in the syntax diagram, parentheses around *spec* are required only with multiple specifications. Also, the diagram does not show that `test` may be called without arguments to redisplay the results from the last test.

`anova` and `manova` (see [\[R\] anova](#) and [\[MV\] manova](#)) allow the `test` syntax above plus more (see [\[R\] anova postestimation](#) for `test` after `anova`; see [\[MV\] manova postestimation](#) for `test` after `manova`).

## Options for testparm

`equal` tests that the variables appearing in `varlist`, which also appear in the previously fit model, are equal to each other rather than jointly equal to zero.

`equation` (`eqno`) is relevant only for multiple-equation models, such as `mvreg`, `mlogit`, and `heckman`. It specifies the equation for which the all-zero or all-equal hypothesis is tested. `equation(#1)` specifies that the test be conducted regarding the first equation #1. `equation(price)` specifies that the test concern the equation named `price`.

`nosvyadjust` is for use with `svy` estimation commands; see [SVY] [svy estimation](#). It specifies that the Wald test be carried out without the default adjustment for the design degrees of freedom. That is, the test is carried out as  $W/k \sim F(k, d)$  rather than as  $(d - k + 1)W/(kd) \sim F(k, d - k + 1)$ , where  $k$  = the dimension of the test and  $d$  = the total number of sampled PSUs minus the total number of strata. When the `df()` option is used, it will override the default design degrees of freedom.

The following option is available with `testparm` but is not shown in the dialog box:

`df(#)` specifies that the  $F$  distribution with # denominator degrees of freedom be used for the reference distribution of the test statistic. The default is to use `e(df_r)` degrees of freedom or the  $\chi^2$  distribution if `e(df_r)` is missing. With survey data, # is the design degrees of freedom unless `nosvyadjust` is specified.

## Options for test

### Options

`mtest` [`opt`] specifies that tests be performed for each condition separately. `opt` specifies the method for adjusting  $p$ -values for multiple testing. Valid values for `opt` are

|                         |                             |
|-------------------------|-----------------------------|
| <code>bonferroni</code> | Bonferroni's method         |
| <code>holm</code>       | Holm's method               |
| <code>sidak</code>      | Šidák's method              |
| <code>noadjust</code>   | no adjustment is to be made |

Specifying `mtest` without an argument is equivalent to `mtest(noadjust)`.

`coef` specifies that the constrained coefficients be displayed.

`accumulate` allows a hypothesis to be tested jointly with the previously tested hypotheses.

`notest` suppresses the output. This option is useful when you are interested only in the joint test of several hypotheses, specified in a subsequent call of `test`, `accumulate`.

`common` specifies that when you use the `[eqno1=eqno2[=. . .]]` form of `spec`, the variables common to the equations `eqno1`, `eqno2`, etc., be tested. The default action is to complain if the equations have variables not in common.

`constant` specifies that `_cons` be included in the list of coefficients to be tested when using the `[eqno1=eqno2[=. . .]]` or `[eqno]` forms of `spec`. The default is not to include `_cons`.

`nosvyadjust` is for use with `svy` estimation commands; see [SVY] [svy estimation](#). It specifies that the Wald test be carried out without the default adjustment for the design degrees of freedom. That is, the test is carried out as  $W/k \sim F(k, d)$  rather than as  $(d - k + 1)W/(kd) \sim F(k, d - k + 1)$ , where  $k$  = the dimension of the test and  $d$  = the total number of sampled PSUs minus the total number of strata. When the `df()` option is used, it will override the default design degrees of freedom.

`minimum` is a highly technical option. It first performs the test with the constant added. If this test is singular, coefficients are dropped until the test becomes nonsingular. Then the test without the constant is performed with the remaining terms.

The following options are available with `test` but are not shown in the dialog box:

`matv1c(matname)`, a programmer's option, saves the variance–covariance matrix of the linear combinations involved in the suite of tests. For the test of the linear constraints  $Lb = c$ , `matname` contains  $LVL'$ , where  $V$  is the estimated variance–covariance matrix of  $b$ .

`df(#)` specifies that the  $F$  distribution with # denominator degrees of freedom be used for the reference distribution of the test statistic. The default is to use `e(df_r)` degrees of freedom or the  $\chi^2$  distribution if `e(df_r)` is missing. With survey data, # is the design degrees of freedom unless `nosvyadjust` is specified.

## Remarks and examples

Remarks are presented under the following headings:

[Introductory examples](#)

[Special syntaxes after multiple-equation estimation](#)

[Constrained coefficients](#)

[Multiple testing](#)

## Introductory examples

`test` performs  $F$  or  $\chi^2$  tests of linear restrictions applied to the most recently fit model (for example, `regress` or `svy: regress` in the linear regression case; `logit`, `stcox`, `svy: logit`, ... in the single-equation maximum-likelihood case; and `mlogit`, `mvreg`, `streg`, ... in the multiple-equation maximum-likelihood case). `test` may be used after *any* estimation command, although for maximum likelihood techniques, `test` produces a Wald test that depends only on the estimate of the covariance matrix—you may prefer to use the more computationally expensive likelihood-ratio test; see [U] [20 Estimation and postestimation commands](#) and [R] [lrtest](#).

There are several variations on the syntax for `test`. The second syntax,

```
test exp=exp[=...]
```

is allowed after any form of estimation. After fitting a model of `devar` on `x1`, `x2`, and `x3`, typing `test x1+x2=x3` tests the restriction that the coefficients on `x1` and `x2` sum to the coefficient on `x3`. The expressions can be arbitrarily complicated; for instance, typing `test x1+2*(x2+x3)=x2+3*x3` is the same as typing `test x1+x2=x3`.

As a convenient shorthand, `test` also allows you to specify equality for multiple expressions; for example, `test x1+x2 = x3+x4 = x5+x6` tests that the three specified pairwise sums of coefficients are equal.

`test` understands that when you type `x1`, you are referring to the coefficient on `x1`. You could also more explicitly type `test _b[x1]+_b[x2]=_b[x3]`; or you could `test _coef[x1]+_coef[x2]=_coef[x3]`, or `test [#1]x1+[#1]x2=[#1]x3`, or many other things because there is more than one way to refer to an estimated coefficient; see [U] 13.5 **Accessing coefficients and standard errors**. The shorthand involves less typing. On the other hand, you must be more explicit after estimation of multiple-equation models because there may be more than one coefficient associated with an independent variable. You might type, for instance, `test [#2]x1+[#2]x2=[#2]x3` to test the constraint in equation 2 or, more readably, `test [ford]x1+[ford]x2=[ford]x3`, meaning that Stata will test the constraint on the equation corresponding to `ford`, which might be equation 2. `ford` would be an equation name after, say, `sureg`, or, after `mlogit`, `ford` would be one of the outcomes. For `mlogit`, you could also type `test [2]x1+[2]x2=[2]x3`—note the lack of the `#`—meaning not equation 2, but the equation corresponding to the numeric outcome 2. You can even test constraints across equations: `test [ford]x1+[ford]x2=[buick]x3`.

The syntax

```
test coeflist
```

is available after all estimation commands and is a convenient way to test that multiple coefficients are zero following estimation. A *coeflist* can simply be a list of variable names,

```
test varname [varname ...]
```

and it is most often specified that way. After you have fit a model of `depvar` on `x1`, `x2`, and `x3`, typing `test x1 x3` tests that the coefficients on `x1` and `x3` are jointly zero. After multiple-equation estimation, this would test that the coefficients on `x1` and `x3` are zero in all equations that contain them. You can also be more explicit and type, for instance, `test [ford]x1 [ford]x3` to test that the coefficients on `x1` and `x3` are zero in the equation for `ford`.

In the multiple-equation case, there are more alternatives. You could also test that the coefficients on `x1` and `x3` are zero in the equation for `ford` by typing `test [ford]: x1 x3`. You could test that all coefficients except the coefficient on the constant are zero in the equation for `ford` by typing `test [ford]`. You could test that the coefficients on `x1` and `x3` in the equation for `ford` are equal to the corresponding coefficients in the equation corresponding to `buick` by typing `test [ford=buick]: x1 x3`. You could test that all the corresponding coefficients except the constant in three equations are equal by typing `test [ford=buick=volvo]`.

`testparm` is much like the first syntax of `test`. Its usefulness will be demonstrated below.

The examples below use `regress`, but what is said applies equally after any single-equation estimation command (such as `logistic`). It also applies after multiple-equation estimation commands as long as references to coefficients are qualified with an equation name or number in square brackets placed before them. The convenient syntaxes for dealing with tests of many coefficients in multiple-equation models are demonstrated in *Special syntaxes after multiple-equation estimation* below.

## ▷ Example 1: Testing for a single coefficient against zero

We have 1980 census data on the 50 states recording the birth rate in each state (`brate`), the median age (`medage`), and the region of the country in which each state is located.

The `region` variable is 1 if the state is in the Northeast, 2 if the state is in the North Central, 3 if the state is in the South, and 4 if the state is in the West. We estimate the following regression:

```
. use https://www.stata-press.com/data/r19/census3
(1980 Census data by state)
. regress brate medage c.medage#c.medage i.region
```

| Source   | SS         | df | MS         | Number of obs | = | 50     |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 38803.4208 | 5  | 7760.68416 | F(5, 44)      | = | 100.63 |
| Residual | 3393.39921 | 44 | 77.1227094 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.9196 |
|          |            |    |            | Adj R-squared | = | 0.9104 |
| Total    | 42196.82   | 49 | 861.159592 | Root MSE      | = | 8.782  |

|                        | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|------------------------|-------------|-----------|-------|-------|----------------------|-----------|
| <code>brate</code>     |             |           |       |       |                      |           |
| <code>medage</code>    | -109.0958   | 13.52452  | -8.07 | 0.000 | -136.3527            | -81.83892 |
| <code>c.medage#</code> |             |           |       |       |                      |           |
| <code>c.medage</code>  | 1.635209    | .2290536  | 7.14  | 0.000 | 1.173582             | 2.096836  |
| <code>region</code>    |             |           |       |       |                      |           |
| <code>NCentral</code>  | 15.00283    | 4.252067  | 3.53  | 0.001 | 6.433353             | 23.57231  |
| <code>South</code>     | 7.366445    | 3.953335  | 1.86  | 0.069 | -.6009775            | 15.33387  |
| <code>West</code>      | 21.39679    | 4.650601  | 4.60  | 0.000 | 12.02412             | 30.76946  |
| <code>_cons</code>     | 1947.611    | 199.8405  | 9.75  | 0.000 | 1544.859             | 2350.363  |

`test` can now be used to perform a variety of statistical tests. Specify the `coeflegend` option with your estimation command to see a legend of the coefficients and how to specify them; see [R] [Estimation options](#). We can test the hypothesis that the coefficient on `3.region` is zero by typing

```
. test 3.region=0
(1) 3.region = 0
 F(1, 44) = 3.47
 Prob > F = 0.0691
```

The  $F$  statistic with 1 numerator and 44 denominator degrees of freedom is 3.47. The significance level of the test is 6.91%—we can reject the hypothesis at the 10% level but not at the 5% level.

This result from `test` is identical to one presented in the output from `regress`, which indicates that the  $t$  statistic on the `3.region` coefficient is 1.863 and that its significance level is 0.069. The  $t$  statistic presented in the output can be used to test the hypothesis that the corresponding coefficient is zero, although it states the test in slightly different terms. The  $F$  distribution with 1 numerator degree of freedom is, however, identical to the  $t^2$  distribution. We note that  $1.863^2 \approx 3.47$  and that the significance levels in each test agree, although one extra digit is presented by the `test` command.

## □ Technical note

After all estimation commands, including those that use the maximum likelihood method, the test that one variable is zero is identical to that reported by the command's output. The tests are performed in the same way—using the estimated covariance matrix—and are known as Wald tests. If the estimation command reports significance levels and confidence intervals using  $z$  rather than  $t$  statistics, test reports results using the  $\chi^2$  rather than the  $F$  statistic. □

## ▷ Example 2: Testing the value of a single coefficient

If that were all test could do, it would be useless. We can use test, however, to perform other tests. For instance, we can test the hypothesis that the coefficient on 2.region is 21 by typing

```
. test 2.region=21
(1) 2.region = 21
 F(1, 44) = 1.99
 Prob > F = 0.1654
```

We find that we cannot reject that hypothesis, or at least we cannot reject it at any significance level below 16.5%. ◀

## ▷ Example 3: Testing the equality of two coefficients

The previous test is useful, but we could almost as easily perform it by hand using the results presented in the regression output if we were well read on our statistics. We could type

```
. display Ftail(1,44,((_coef[2.region]-21)/4.252068)^2)
.16544873
```

So, now let's test something a bit more difficult: whether the coefficient on 2.region is the same as the coefficient on 4.region:

```
. test 2.region=4.region
(1) 2.region - 4.region = 0
 F(1, 44) = 2.84
 Prob > F = 0.0989
```

We find that we cannot reject the equality hypothesis at the 5% level, but we can at the 10% level. ◀

### ▷ Example 4

When we tested the equality of the `2.region` and `4.region` coefficients, Stata rearranged our algebra. When Stata displayed its interpretation of the specified test, it indicated that we were testing whether `2.region minus 4.region` is zero. The rearrangement is innocuous and, in fact, allows Stata to perform much more complicated algebra, for instance,

```
. test 2*(2.region-3*(3.region-4.region))=3.region+2.region+6*(4.region-3.region)
(1) 2.region - 3.region = 0
 F(1, 44) = 5.06
 Prob > F = 0.0295
```

Although we requested what appeared to be a lengthy hypothesis, once Stata simplified the algebra, it realized that all we wanted to do was test whether the coefficient on `2.region` is the same as the coefficient on `3.region`.



### □ Technical note

Stata's ability to simplify and test complex hypotheses is limited to *linear* hypotheses. If you attempt to test a nonlinear hypothesis, you will be told that it is not possible:

```
. test 2.region/3.region=2.region+3.region
not possible with test
r(131);
```

To test a nonlinear hypothesis, see [R] [testnl](#).



### ▷ Example 5: Testing joint hypotheses

The real power of `test` is demonstrated when we test *joint* hypotheses. Perhaps we wish to test whether the region variables, taken as a whole, are significant by testing whether the coefficients on `2.region`, `3.region`, and `4.region` are simultaneously zero. `test` allows us to specify multiple conditions to be tested, each embedded within parentheses.

```
. test (2.region=0) (3.region=0) (4.region=0)
(1) 2.region = 0
(2) 3.region = 0
(3) 4.region = 0
 F(3, 44) = 8.85
 Prob > F = 0.0001
```

`test` displays the set of conditions and reports an  $F$  statistic of 8.85. `test` also reports the degrees of freedom of the test to be 3, the “dimension” of the hypothesis, and the residual degrees of freedom, 44. The significance level of the test is close to 0, so we can strongly reject the hypothesis of no difference between the regions.

An alternative method to specify simultaneous hypotheses uses the convenient shorthand of conditions with multiple equality operators.

```
. test 2.region=3.region=4.region=0
(1) 2.region - 3.region = 0
(2) 2.region - 4.region = 0
(3) 2.region = 0
 F(3, 44) = 8.85
 Prob > F = 0.0001
```



## □ Technical note

Another method to test simultaneous hypotheses is to specify a test for each constraint and accumulate it with the previous constraints:

```
. test 2.region=0
(1) 2.region = 0
 F(1, 44) = 12.45
 Prob > F = 0.0010

. test 3.region=0, accumulate
(1) 2.region = 0
(2) 3.region = 0
 F(2, 44) = 6.42
 Prob > F = 0.0036

. test 4.region=0, accumulate
(1) 2.region = 0
(2) 3.region = 0
(3) 4.region = 0
 F(3, 44) = 8.85
 Prob > F = 0.0001
```

We tested the hypothesis that the coefficient on `2.region` was zero by typing `test 2.region=0`. We then tested whether the coefficient on `3.region` was also zero by typing `test 3.region=0, accumulate`. The `accumulate` option told Stata that this was not the start of a new test but a continuation of a previous one. Stata responded by showing us the two equations and reporting an  $F$  statistic of 6.42. The significance level associated with those two coefficients being zero is 0.36%.

When we added the last constraint `test 4.region=0, accumulate`, we discovered that the three region variables are significant. If all we wanted was the overall significance and we did not want to bother seeing the interim results, we could have used the `notest` option:

```
. test 2.region=0, notest
(1) 2.region = 0

. test 3.region=0, accumulate notest
(1) 2.region = 0
(2) 3.region = 0

. test 4.region=0, accumulate
(1) 2.region = 0
(2) 3.region = 0
(3) 4.region = 0
 F(3, 44) = 8.85
 Prob > F = 0.0001
```



### ▷ Example 6: Quickly testing coefficients against zero

Because tests that coefficients are zero are so common in applied statistics, the `test` command has a more convenient syntax to accommodate this case:

```
. test 2.region 3.region 4.region
(1) 2.region = 0
(2) 3.region = 0
(3) 4.region = 0
 F(3, 44) = 8.85
 Prob > F = 0.0001
```



### ▷ Example 7: Specifying varlists

We will now show how to use `testparm`. In its first syntax, `test` accepts a list of variable names but not *varlist*.

```
. test i(2/4).region
i not found
r(111);
```

In the varlist, `i(2/4).region` means all the level variables from `2.region` through `4.region`, yet we received an error. `test` does not actually understand varlists, but `testparm` does. In fact, it understands only varlists.

```
. testparm i(2/4).region
(1) 2.region = 0
(2) 3.region = 0
(3) 4.region = 0
 F(3, 44) = 8.85
 Prob > F = 0.0001
```

Another way to test all the `region` variables is to type `testparm i.region`.

That `testparm` accepts varlists has other advantages that do not involve factor variables. Suppose that we have a dataset that has dummy variables `reg2`, `reg3`, and `reg4`, rather than the categorical variable `region`.

```
. use https://www.stata-press.com/data/r19/census4
(Census data on birthrate, median age)
. regress brate medage c.medage#c.medage reg2 reg3 reg4
(output omitted)
. test reg2-reg4
- not found
r(111);
```

In a varlist, `reg2-reg4` means variables `reg2` and `reg4` and all the variables between, yet we received an error. `test` is confused because the `-` has two meanings: it means subtraction in an expression and “through” in *varlist*. Similarly, `*` means “any set of characters” in *varlist* and multiplication in an expression. `testparm` avoids this confusion—it allows only *varlist*.

```
. testparm reg2-reg4
(1) reg2 = 0
(2) reg3 = 0
(3) reg4 = 0
 F(3, 44) = 8.85
 Prob > F = 0.0001
```

testparm has another advantage. We have five variables in our dataset that start with the characters reg: region, reg1, reg2, reg3, and reg4. reg\* thus means those five variables:

```
. describe reg*
Variable Storage Display Value
 name type format label Variable label

region byte %8.0g region Census region
reg1 byte %9.0g Region: NE Region: NE
reg2 byte %9.0g Region: N Cntrl Region: N Cntrl
reg3 byte %9.0g Region: South Region: South
reg4 byte %9.0g Region: West Region: West
```

We cannot type test reg\* because, in an expression, '\*' means multiplication, but here is what would happen if we attempted to test all the variables that begin with reg:

```
. test region reg1 reg2 reg3 reg4
region not found
r(111);
```

The variable region was not included in our model, so it was not found. However, with testparm,

```
. testparm reg*
(1) reg2 = 0
(2) reg3 = 0
(3) reg4 = 0
 F(3, 44) = 8.85
 Prob > F = 0.0001
```

That is, testparm took reg\* to mean all variables that start with reg that were in our model.



## □ Technical note

Actually, reg\* means what it always does—all variables in our dataset that begin with reg—in this case, region reg1 reg2 reg3 reg4. testparm just ignores any variables you specify that are not in the model.



### ▷ Example 8: Replaying the previous test

We just used `test` (`testparm`, actually, but it does not matter) to test the hypothesis that `reg2`, `reg3`, and `reg4` are jointly zero. We can review the results of our last test by typing `test` without arguments:

```
. test
(1) reg2 = 0
(2) reg3 = 0
(3) reg4 = 0
 F(3, 44) = 8.85
 Prob > F = 0.0001
```

◀

### □ Technical note

`test` does not care how we build joint hypotheses; we may freely mix different forms of syntax. (We can even start with `testparm`, but we cannot use it thereafter because it does not have an `accumulate` option.)

Say that we type `test reg2 reg3 reg4` to test that the coefficients on our region dummies are jointly zero. We could then add a fourth constraint, say, that `medage = 100`, by typing `test medage=100, accumulate`. Or, if we had introduced the `medage` constraint first (our first `test` command had been `test medage=100`), we could then add the region dummy test by typing `test reg2 reg3 reg4, accumulate` or `test (reg2=0) (reg3=0) (reg4=0), accumulate`.

Remember that all previous tests are cleared when we do not specify the `accumulate` option. No matter what tests we performed in the past, if we type `test medage c.medage#c.medage`, omitting the `accumulate` option, we would test that `medage` and `c.medage#c.medage` are jointly zero.

□

### ▷ Example 9: Testing the equality of multiple coefficients

Let's return to our `census3.dta` dataset and test the hypothesis that all the included regions have the same coefficient—that the Northeast is significantly different from the rest of the nation:

```
. use https://www.stata-press.com/data/r19/census3
(1980 Census data by state)
. regress brate medage c.medage#c.medage i.region
(output omitted)
. test 2.region=3.region=4.region
(1) 2.region - 3.region = 0
(2) 2.region - 4.region = 0
 F(2, 44) = 8.23
 Prob > F = 0.0009
```

We find that they are not all the same. The syntax `2.region=3.region=4.region` with `multiple =` operators is just a convenient shorthand for typing that the first expression equals the second expression and that the first expression equals the third expression,

```
. test (2.region=3.region) (2.region=4.region)
```

We performed the test for equality of the three regions by imposing two constraints: region 2 has the same coefficient as region 3, and region 2 has the same coefficient as region 4. Alternatively, we could have tested that the coefficients on regions 2 and 3 are the same and that the coefficients on regions 3 and 4 are the same. We would obtain the same results in either case.

To test for equality of the three regions, we might, likely by mistake, type equality constraints for *all* pairs of regions:

```
. test (2.region=3.region) (2.region=4.region) (3.region=4.region)
(1) 2.region - 3.region = 0
(2) 2.region - 4.region = 0
(3) 3.region - 4.region = 0
 Constraint 3 dropped
 F(2, 44) = 8.23
 Prob > F = 0.0009
```

Equality of regions 2 and 3 and of regions 2 and 4, however, implies equality of regions 3 and 4. `test` recognized that the last constraint is implied by the other constraints and hence dropped it.

◀

### □ Technical note

Generally, Stata uses `=` for assignment, as in `gen newvar = exp`, and `==` as the operator for testing equality in expressions. For your convenience, `test` allows both `=` and `==` to be used.

□

### ▷ Example 10

The test for the equality of the regions is also possible with the `testparm` command. When we include the `equal` option, `testparm` tests that the coefficients of all the variables specified are equal:

```
. testparm i(2/4).region, equal
(1) - 2.region + 3.region = 0
(2) - 2.region + 4.region = 0
 F(2, 44) = 8.23
 Prob > F = 0.0009
```

We can also obtain the equality test by accumulating single equality tests.

```
. test 2.region=3.region, notest
(1) 2.region - 3.region = 0
. test 2.region=4.region, accum
(1) 2.region - 3.region = 0
(2) 2.region - 4.region = 0
 F(2, 44) = 8.23
 Prob > F = 0.0009
```

◀

## □ Technical note

If we specify a set of inconsistent constraints, `test` will tell us by dropping the constraint or constraints that led to the inconsistency. For instance, let's `test` that the coefficients on `region 2` and `region 4` are the same, add the test that the coefficient on `region 2` is 20, and finally add the test that the coefficient on `region 4` is 21:

```
. test (2.region=4.region) (2.region=20) (4.region=21)
(1) 2.region - 4.region = 0
(2) 2.region = 20
(3) 4.region = 21
 Constraint 2 dropped
 F(2, 44) = 1.82
 Prob > F = 0.1737
```

`test` informed us that it was dropping constraint 2. All three equations cannot be simultaneously true, so `test` drops whatever it takes to get back to something that makes sense.

□

## Special syntaxes after multiple-equation estimation

Everything said above about tests after single-equation estimation applies to tests after multiple-equation estimation, as long as you remember to specify the equation name. To demonstrate, let's estimate a seemingly unrelated regression by using `sureg`; see [R] [sureg](#).

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. sureg (price foreign mpg displ) (weight foreign length)
Seemingly unrelated regression
```

| Equation | Obs | Params | RMSE     | "R-squared" | chi2   | P>chi2 |
|----------|-----|--------|----------|-------------|--------|--------|
| price    | 74  | 3      | 2165.321 | 0.4537      | 49.64  | 0.0000 |
| weight   | 74  | 2      | 245.2916 | 0.8990      | 661.84 | 0.0000 |

|              | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |
|--------------|-------------|-----------|-------|-------|----------------------|
| price        |             |           |       |       |                      |
| foreign      | 3058.25     | 685.7357  | 4.46  | 0.000 | 1714.233 4402.267    |
| mpg          | -104.9591   | 58.47209  | -1.80 | 0.073 | -219.5623 9.644042   |
| displacement | 18.18098    | 4.286372  | 4.24  | 0.000 | 9.779842 26.58211    |
| _cons        | 3904.336    | 1966.521  | 1.99  | 0.047 | 50.0263 7758.645     |
| weight       |             |           |       |       |                      |
| foreign      | -147.3481   | 75.44314  | -1.95 | 0.051 | -295.2139 .517755    |
| length       | 30.94905    | 1.539895  | 20.10 | 0.000 | 27.93091 33.96718    |
| _cons        | -2753.064   | 303.9336  | -9.06 | 0.000 | -3348.763 -2157.365  |

To test the significance of `foreign` in the price equation, we could type

```
. test [price]foreign
(1) [price]foreign = 0
 chi2(1) = 19.89
 Prob > chi2 = 0.0000
```

which is the same result reported by `sureg`:  $4.460^2 \approx 19.89$ . To test `foreign` in both equations, we could type

```
. test [price]foreign [weight]foreign
(1) [price]foreign = 0
(2) [weight]foreign = 0
 chi2(2) = 31.61
 Prob > chi2 = 0.0000
```

or

```
. test foreign
(1) [price]foreign = 0
(2) [weight]foreign = 0
 chi2(2) = 31.61
 Prob > chi2 = 0.0000
```

This last syntax—typing the variable name by itself—tests the coefficients in all equations in which they appear. The variable length appears in only the `weight` equation, so typing

```
. test length
(1) [weight]length = 0
 chi2(1) = 403.94
 Prob > chi2 = 0.0000
```

yields the same result as typing `test [weight]length`. We may also specify a linear expression rather than a list of coefficients:

```
. test mpg=displ
(1) [price]mpg - [price]displacement = 0
 chi2(1) = 4.85
 Prob > chi2 = 0.0277
```

or

```
. test [price]mpg = [price]displ
(1) [price]mpg - [price]displacement = 0
 chi2(1) = 4.85
 Prob > chi2 = 0.0277
```

A variation on this syntax can be used to test cross-equation constraints:

```
. test [price]foreign = [weight]foreign
(1) [price]foreign - [weight]foreign = 0
 chi2(1) = 23.07
 Prob > chi2 = 0.0000
```

Typing an equation name in square brackets by itself tests all the coefficients except the intercept in that equation:

```
. test [price]
(1) [price]foreign = 0
(2) [price]mpg = 0
(3) [price]displacement = 0
 chi2(3) = 49.64
 Prob > chi2 = 0.0000
```

Typing an equation name in square brackets, a colon, and a list of variable names tests those variables in the specified equation:

```
. test [price]: foreign displ
(1) [price]foreign = 0
(2) [price]displacement = 0
 chi2(2) = 25.19
 Prob > chi2 = 0.0000
```

`test [eqname1=eqname2]` tests that all the coefficients in the two equations are equal. We cannot use that syntax here because there are different variables in the model:

```
. test [price=weight]
variables differ between equations
(to test equality of coefficients in common, specify option common)
r(111);
```

The common option specifies a test of the equality coefficients common to the equations price and weight,

```
. test [price=weight], common
(1) [price]foreign - [weight]foreign = 0
 chi2(1) = 23.07
 Prob > chi2 = 0.0000
```

By default, `test` does not include the constant, the coefficient of the constant variable `_cons`, in the test. The `cons` option specifies that the constant be included.

```
. test [price=weight], common cons
(1) [price]foreign - [weight]foreign = 0
(2) [price]_cons - [weight]_cons = 0
 chi2(2) = 51.23
 Prob > chi2 = 0.0000
```

We can also use a modification of this syntax with the model if we also type a colon and the names of the variables we want to test:

```
. test [price=weight]: foreign
(1) [price]foreign - [weight]foreign = 0
 chi2(1) = 23.07
 Prob > chi2 = 0.0000
```

We have only one variable in common between the two equations, but if there had been more, we could have listed them.

Finally, a simultaneous test of multiple constraints may be specified just as after single-equation estimation.

```
. test ([price]: foreign) ([weight]: foreign)
(1) [price]foreign = 0
(2) [weight]foreign = 0
 chi2(2) = 31.61
 Prob > chi2 = 0.0000
```

`test` can also test for equality of coefficients across more than two equations. For instance, `test [eq1=eq2=eq3]` specifies a test that the coefficients in the three equations `eq1`, `eq2`, and `eq3` are equal. This requires that the same variables be included in the three equations. If some variables are entered only in some of the equations, you can type `test [eq1=eq2=eq3]`, `common` to test that the coefficients of the variables common to all three equations are equal. Alternatively, you can explicitly list the variables for which equality of coefficients across the equations is to be tested. For instance, `test [eq1=eq2=eq3]: time money` tests that the coefficients of the variables `time` and `money` do not differ between the equations.

### □ Technical note

`test [eq1=eq2=eq3]`, `common` tests the equality of the coefficients common to all equations, but it does *not* test the equality of all common coefficients. Consider the case where

|                  |                                                    |
|------------------|----------------------------------------------------|
| <code>eq1</code> | contains the variables <code>var1 var2 var3</code> |
| <code>eq2</code> | contains the variables <code>var1 var2 var4</code> |
| <code>eq3</code> | contains the variables <code>var1 var3 var4</code> |

Obviously, only `var1` is common to all three equations. Thus `test [eq1=eq2=eq3]`, `common` tests that the coefficients of `var1` do not vary across the equations, so it is equivalent to `test [eq1=eq2=eq3]: var1`. To perform a test of the coefficients of variables common to two equations, you could explicitly list the constraints to be tested,

```
. test ([eq1=eq2=eq3]:var1) ([eq1=eq2]:var2) ([eq1=eq3]:var3) ([eq2=eq3]:var4)
```

or use `test` with the `accumulate` option, and maybe also with the `notest` option, to form the appropriate joint hypothesis:

```
. test [eq1=eq2], common notest
. test [eq1=eq3], common accumulate notest
. test [eq2=eq3], common accumulate
```

□

## Constrained coefficients

If the test indicates that the data do not allow you to conclude that the constraints are not satisfied, you may want to inspect the constrained coefficients. The `coef` option specified that the constrained results, estimated by GLS, are shown.

```
. test [price=weight], common coef
(1) [price]foreign - [weight]foreign = 0
 chi2(1) = 23.07
 Prob > chi2 = 0.0000
```

Constrained coefficients

|               | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|---------------|-------------|-----------|-------|-------|----------------------|-----------|
| <b>price</b>  |             |           |       |       |                      |           |
| foreign       | -216.4015   | 74.06083  | -2.92 | 0.003 | -361.558             | -71.2449  |
| mpg           | -121.5717   | 58.36972  | -2.08 | 0.037 | -235.9742            | -7.169116 |
| displacement  | 7.632566    | 3.681114  | 2.07  | 0.038 | .4177148             | 14.84742  |
| _cons         | 7312.856    | 1834.034  | 3.99  | 0.000 | 3718.215             | 10907.5   |
| <b>weight</b> |             |           |       |       |                      |           |
| foreign       | -216.4015   | 74.06083  | -2.92 | 0.003 | -361.558             | -71.2449  |
| length        | 30.34875    | 1.534815  | 19.77 | 0.000 | 27.34057             | 33.35693  |
| _cons         | -2619.719   | 302.6632  | -8.66 | 0.000 | -3212.928            | -2026.51  |

The constrained coefficient of `foreign` is  $-216.40$  with standard error  $74.06$  in equations `price` and `weight`. The other coefficients and their standard errors are affected by imposing the equality constraint of the two coefficients of `foreign` because the unconstrained estimates of these two coefficients were correlated with the estimates of the other coefficients.

### □ Technical note

The two-step constrained coefficients  $b_c$  displayed by `test`, `coef` are asymptotically equivalent to the one-stage constrained estimates that are computed by specifying the constraints during estimation using the `constraint()` option of estimation commands (Gouriéroux and Monfort 1995, chap. 10). Generally, one-step constrained estimates have better small-sample properties. For inspection and interpretation, however, two-step constrained estimates are a convenient alternative. Moreover, some estimation commands (for example, `stcox`, many `xt` estimators) do not have a `constraint()` option.

□

## Multiple testing

When performing the test of a joint hypothesis, you might want to inspect the underlying 1-degree-of-freedom hypotheses. Which constraint “is to blame”? `test` displays the univariate as well as the simultaneous test if the `mtest` option is specified. For example,

```
. test [price=weight], common cons mtest
(1) [price]foreign - [weight]foreign = 0
(2) [price]_cons - [weight]_cons = 0
```

|     | chi2  | df | p > chi2 |
|-----|-------|----|----------|
| (1) | 23.07 | 1  | 0.0000*  |
| (2) | 11.17 | 1  | 0.0008*  |
| All | 51.23 | 2  | 0.0000   |

\* Unadjusted *p*-values

Both coefficients seem to contribute to the highly significant result. The 1-degree-of-freedom test shown here is identical to those if `test` had been invoked to test just this simple hypotheses. There is, of course, a real risk in inspecting these simple hypotheses. Especially in high-dimensional hypotheses, you may easily find one hypothesis that happens to be significant. Multiple testing procedures are designed to provide some safeguard against this risk. *p*-values of the univariate hypotheses are modified so that the probability of falsely rejecting one of the null hypotheses is bounded. `test` provides the methods based on Bonferroni, Šidák, and Holm.

```
. test [price=weight], common cons mtest(b)
(1) [price]foreign - [weight]foreign = 0
(2) [price]_cons - [weight]_cons = 0
```

|     | chi2  | df | p > chi2 |
|-----|-------|----|----------|
| (1) | 23.07 | 1  | 0.0000*  |
| (2) | 11.17 | 1  | 0.0017*  |
| All | 51.23 | 2  | 0.0000   |

\* Bonferroni-adjusted *p*-values

## Stored results

`test` and `testparm` store the following in `r()`:

### Scalars

|                           |                                         |                      |                                            |
|---------------------------|-----------------------------------------|----------------------|--------------------------------------------|
| <code>r(p)</code>         | two-sided <i>p</i> -value               | <code>r(chi2)</code> | $\chi^2$                                   |
| <code>r(F)</code>         | <i>F</i> statistic                      | <code>r(ss)</code>   | sum of squares (test)                      |
| <code>r(df)</code>        | test constraints degrees of freedom     | <code>r(rss)</code>  | residual sum of squares                    |
| <code>r(df_r)</code>      | residual degrees of freedom             | <code>r(drop)</code> | 1 if constraints were dropped, 0 otherwise |
| <code>r(dropped_i)</code> | index of <i>i</i> th constraint dropped |                      |                                            |

### Macros

|                         |                                           |
|-------------------------|-------------------------------------------|
| <code>r(mmethod)</code> | method of adjustment for multiple testing |
|-------------------------|-------------------------------------------|

### Matrices

|                       |                       |
|-----------------------|-----------------------|
| <code>r(mtest)</code> | multiple test results |
|-----------------------|-----------------------|

`r(ss)` and `r(rss)` are defined only when `test` is used for testing effects after anova.

## Methods and formulas

`test` and `testparm` perform Wald tests. Let the estimated coefficient vector be  $\mathbf{b}$  and the estimated variance–covariance matrix be  $\mathbf{V}$ . Let  $\mathbf{Rb} = \mathbf{r}$  denote the set of  $q$  linear hypotheses to be tested jointly.

The Wald test statistic is (Judge et al. 1985, 20–28)

$$W = (\mathbf{Rb} - \mathbf{r})'(\mathbf{RVR}')^{-1}(\mathbf{Rb} - \mathbf{r})$$

If the estimation command reports its significance levels using  $Z$  statistics, a  $\chi^2$  distribution with  $q$  degrees of freedom,

$$W \sim \chi_q^2$$

is used for computation of the significance level of the hypothesis test.

If the estimation command reports its significance levels using  $t$  statistics with  $d$  degrees of freedom, an  $F$  statistic,

$$F = \frac{1}{q}W$$

is computed, and an  $F$  distribution with  $q$  numerator degrees of freedom and  $d$  denominator degrees of freedom computes the significance level of the hypothesis test.

The two-step constrained estimates  $b_c$  displayed by `test` with the `coef` option are the GLS estimates of the unconstrained estimates  $b$  subject to the specified constraints  $Rb = c$  (Gouriéroux and Monfort 1995, chap. 10),

$$\mathbf{b}_c = \mathbf{b} - \mathbf{VR}'(\mathbf{RVR}')^{-1}(\mathbf{Rb} - \mathbf{r})$$

with variance–covariance matrix

$$\mathbf{V}_c = \mathbf{V} - \mathbf{VR}'(\mathbf{RVR}')^{-1}\mathbf{RV}$$

If `test` displays a Wald test for joint (simultaneous) hypotheses, it can also display all 1-degree-of-freedom tests, with  $p$ -values adjusted for multiple testing. Let  $p_1, p_2, \dots, p_k$  be the unadjusted  $p$ -values of these 1-degree-of-freedom tests. The Bonferroni-adjusted  $p$ -values are defined as  $p_i^b = \min(1, kp_i)$ . The Šidák-adjusted  $p$ -values are  $p_i^s = 1 - (1 - p_i)^k$ . Let  $p_{(1)}, p_{(2)}, \dots, p_{(k)}$  be the ordered unadjusted  $p$ -values. The Holm's method adjusted  $p$ -values are then defined by

$$p_{(i)}^h = \max_{j \leq i} [\min\{1, (k - j + 1)p_{(j)}\}]$$

If `test` is used after a `svy` command, it carries out an adjusted Wald test—this adjustment should not be confused with the adjustment for multiple testing. Both adjustments may actually be combined. Specifically, the survey adjustment uses an approximate  $F$  statistic  $(d - k + 1)W/(kd)$ , where  $W$  is the Wald test statistic,  $k$  is the dimension of the hypothesis test, and  $d$  = the total number of sampled PSUs minus the total number of strata. Under the null hypothesis,  $(d - k + 1)F/(kd) \sim F(k, d - k + 1)$ , where  $F(k, d - k + 1)$  is an  $F$  distribution with  $k$  numerator degrees of freedom and  $d - k + 1$  denominator degrees of freedom. If `nosvyadjust` is specified, the  $p$ -value is computed using  $W/k \sim F(k, d)$ .

See Korn and Graubard (1990) for a detailed description of the Bonferroni adjustment technique and for a discussion of the relative merits of it and of the adjusted and unadjusted Wald tests.

## Acknowledgment

The svy adjustment code was adopted from another command developed in collaboration with John L. Eltinge of the United States Census Bureau.

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## Also see

- [R] **anova** — Analysis of variance and covariance
- [R] **anova postestimation** — Postestimation tools for anova
- [R] **contrast** — Contrasts and linear hypothesis tests after estimation
- [R] **lincom** — Linear combinations of parameters
- [R] **lrtest** — Likelihood-ratio test after estimation
- [R] **nestreg** — Nested model statistics
- [R] **nlcom** — Nonlinear combinations of parameters
- [R] **testnl** — Test nonlinear hypotheses after estimation
- [U] **13.5 Accessing coefficients and standard errors**
- [U] **20 Estimation and postestimation commands**

|             |                      |                |                      |
|-------------|----------------------|----------------|----------------------|
| Description | Quick start          | Menu           | Syntax               |
| Options     | Remarks and examples | Stored results | Methods and formulas |
| References  | Also see             |                |                      |

## Description

`testnl` tests (linear or nonlinear) hypotheses about the estimated parameters from the most recently fit model.

`testnl` produces Wald-type tests of smooth nonlinear (or linear) hypotheses about the estimated parameters from the most recently fit model. The  $p$ -values are based on the delta method, an approximation appropriate in large samples.

`testnl` can be used with `svy` estimation results; see [\[SVY\] svy postestimation](#).

The format  $(exp_1=exp_2=exp_3\dots)$  for a simultaneous-equality hypothesis is just a convenient shorthand for a list  $(exp_1=exp_2)$   $(exp_1=exp_3)$ , etc.

`testnl` may also be used to test linear hypotheses. `test` is faster if you want to test only linear hypotheses; see [\[R\] test](#). `testnl` is the only option for testing linear and nonlinear hypotheses simultaneously.

## Quick start

*After single-equation models*

Test that the product of the coefficients for `x1` and `x2` is equal to 4

```
testnl _b[x1]*_b[x2] = 4
```

Test that the ratio of the indicators for the [factor variable](#) `a = 2` and `a = 3` is 1

```
testnl _b[2.a]/_b[3.a] = 1
```

Test that an expression involving continuous [factor variable](#) syntax is equal to 16

```
testnl _b[x1]/(2*_b[c.x1#c.x1]) = 16
```

Test the equality of two expressions

```
testnl _b[x1]*_b[x2] = _b[x1]*_b[x3]
```

Joint test that two products are both equal to 2

```
testnl (_b[x1]*_b[x2] = 2) (_b[x1]*_b[x3] = 2)
```

Same as above

```
testnl _b[x1]*_b[x2] = _b[x1]*_b[x3] = 2
```

Same as above, but add separate tests for each expression

```
testnl _b[x1]*_b[x2] = _b[x1]*_b[x3] = 2, mtest
```

Same as above, but adjust  $p$ -values for multiple comparisons using Holm's method

```
testnl _b[x1]*_b[x2] = _b[x1]*_b[x3] = 2, mtest(holm)
```

Test a linear hypothesis and a nonlinear hypothesis together

```
testnl (_b[x1] = _b[x2]) (_b[x2]^2 = _b[x3])
```

*After multiple-equation models*

Test that the product of the coefficients for  $x_1$  and  $x_2$  in the equation for  $y_1$  is equal to 1

```
testnl _b[y1:x1]*_b[y1:x2] = 1
```

Test that the product of the coefficients for  $x_1$  and  $x_2$  in the equation for  $y_2$  is equal to 1

```
testnl _b[y2:x1]*_b[y2:x2] = 1
```

Test the equality of expressions involving coefficients from the equations for  $y_1$  and  $y_4$

```
testnl _b[y1:x1]*_b[y1:x2] = _b[y4:x1]*_b[y4:x2]
```

## Menu

Statistics > Postestimation

## Syntax

```
testnl exp = exp [= exp ...] [, options]
```

```
testnl (exp = exp [= exp ...]) [(exp = exp [= exp ...]) ...] [, options]
```

| <i>options</i>                 | Description                                                                                                                                           |
|--------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>mtest(<i>opt</i>)</code> | test each condition separately                                                                                                                        |
| <code>iterate(#)</code>        | use maximum # of iterations to find the optimal step size                                                                                             |
| <code>df(#)</code>             | use $F$ distribution with # denominator degrees of freedom for the reference distribution of the test statistic                                       |
| <code>nosvyadjust</code>       | carry out the Wald test as $W/k \sim F(k, d)$ ; for use with <code>svy</code> estimation commands when the <code>df()</code> option is also specified |

`collect` is allowed; see [U] 11.1.10 Prefix commands.

`df(#)` and `nosvyadjust` do not appear in the dialog box.

The second syntax means that if more than one expression is specified, each must be surrounded by parentheses.

*exp* is a possibly nonlinear expression containing

```
_b [coef]
_b [eqno:coef]
[eqno]coef
[eqno] _b [coef]
```

*eqno* is

```

name
```

*coef* identifies a coefficient in the model. *coef* is typically a variable name, a level indicator, an interaction indicator, or an interaction involving continuous variables. Level indicators identify one level of a factor variable and interaction indicators identify one combination of levels of an interaction; see [U] 11.4.3 Factor variables. *coef* may contain time-series operators; see [U] 11.4.4 Time-series varlists.

Distinguish between `[]`, which are to be typed, and `[ ]`, which indicate optional arguments.

## Options

`mtest(opt)` specifies that tests be performed for each condition separately. *opt* specifies the method for adjusting  $p$ -values for multiple testing. Valid values for *opt* are

|                         |                             |
|-------------------------|-----------------------------|
| <code>bonferroni</code> | Bonferroni's method         |
| <code>holm</code>       | Holm's method               |
| <code>sidak</code>      | Šidák's method              |
| <code>noadjust</code>   | no adjustment is to be made |

Specifying `mtest` without an argument is equivalent to specifying `mtest(noadjust)`.

`iterate(#)` specifies the maximum number of iterations used to find the optimal step size in the calculation of numerical derivatives of the test expressions. By default, the maximum number of iterations is 100, but convergence is usually achieved after only a few iterations. You should rarely have to use this option.

The following options are available with `testnl` but are not shown in the dialog box:

`df(#)` specifies that the  $F$  distribution with # denominator degrees of freedom be used for the reference distribution of the test statistic. With survey data, # is the design degrees of freedom unless `nosvyadjust` is specified.

`nosvyadjust` is for use with `svy` estimation commands when the `df()` option is also specified; see [SVY] [svy estimation](#). It specifies that the Wald test be carried out without the default adjustment for the design degrees of freedom. That is, the test is carried out as  $W/k \sim F(k, d)$  rather than as  $(d - k + 1)W/(kd) \sim F(k, d - k + 1)$ , where  $k$  = the dimension of the test and  $d$  = the design degrees of freedom specified in the `df()` option.

## Remarks and examples

Remarks are presented under the following headings:

- [Introduction](#)
- [Using testnl to perform linear tests](#)
- [Specifying constraints](#)
- [Dropped constraints](#)
- [Multiple constraints](#)
- [Manipulability](#)

## Introduction

### ► Example 1

We have just estimated the parameters of an earnings model on cross-sectional time-series data using one of Stata's more sophisticated estimators:

```
. use https://www.stata-press.com/data/r19/earnings
(NLS women 14-24 in 1968)

. xtgee ln_w grade age c.age#c.age, corr(exchangeable) nolog

GEE population-averaged model Number of obs = 1,326
Group variable: idcode Number of groups = 269
Family: Gaussian Obs per group:
Link: Identity min = 1
Correlation: exchangeable avg = 4.9
 max = 9
 Wald chi2(3) = 327.33
Scale parameter = .0976738 Prob > chi2 = 0.0000
```

| ln_wage     | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|-------------|-------------|-----------|-------|-------|----------------------|-----------|
| grade       | .0749686    | .0066111  | 11.34 | 0.000 | .062011              | .0879261  |
| age         | .1080806    | .0235861  | 4.58  | 0.000 | .0618526             | .1543086  |
| c.age#c.age | -.0016253   | .0004739  | -3.43 | 0.001 | -.0025541            | -.0006966 |
| _cons       | -.8788933   | .2830899  | -3.10 | 0.002 | -1.433739            | -.3240473 |

An implication of this model is that peak earnings occur at age  $-\_b[\text{age}]/(2*\_b[\text{c.age}\#\text{c.age}])$ , which here is equal to 33.2. Say that we have a theory that peak earnings should occur at age  $16 + 1/\_b[\text{grade}]$ .

```
. testnl -_b[age]/(2*_b[c.age#c.age]) = 16 + 1/_b[grade]
(1) -_b[age]/(2*_b[c.age#c.age]) = 16 + 1/_b[grade]
 chi2(1) = 1.71
 Prob > chi2 = 0.1914
```

These data do not reject our theory.

◀

## Using testnl to perform linear tests

testnl may be used to test linear constraints, but test is faster; see [R] [test](#). You could type

```
. testnl _b[x4] = _b[x1]
```

but it would take less computer time if you typed

```
. test _b[x4] = _b[x1]
```

## Specifying constraints

The constraints to be tested can be formulated in many different ways. You could type

```
. testnl _b[mpg]*_b[weight] = 1
```

or

```
. testnl _b[mpg] = 1/_b[weight]
```

or you could express the constraint any other way you wished. (To say that testnl allows constraints to be specified in different ways does not mean that the test itself does not depend on the formulation. This point is briefly discussed later.) In formulating the constraints, you must, however, exercise one caution: users of test often refer to the coefficient on a variable by specifying the variable name. For example,

```
. test mpg = 0
```

More formally, they should type

```
. test _b[mpg] = 0
```

but test allows the `_b[]` surrounding the variable name to be omitted. testnl does not allow this shorthand. Typing

```
. testnl mpg=0
```

specifies the constraint that the value of variable mpg in the first observation is zero. If you make this mistake, sometimes testnl will catch it:

```
. testnl mpg=0
equation (1) contains reference to X rather than _b[X]
r(198);
```

In other cases, `testnl` may not catch the mistake; then, the constraint will be dropped because it does not make sense:

```
. testnl mpg=0
 Constraint (1) dropped
```

(There are reasons other than this for constraints being dropped.) The worst case, however, is

```
. testnl _b[weight]*mpg = 1
```

when what you mean is not that `_b[weight]` equals the reciprocal of the value of `mpg` in the first observation, but rather that

```
. testnl _b[weight]*_b[mpg] = 1
```

Sometimes, this mistake will be caught by the “contains reference to X rather than `_b[X]`” error, and sometimes it will not. Be careful.

`testnl`, like `test`, can be used after any Stata estimation command, including the survey estimators. When you use it after a multiple-equation command, such as `mlogit` or `heckman`, you refer to coefficients by using Stata’s standard syntax: `[eqname]_b[varname]`.

Stata’s single-equation estimation output looks like this:

|        | Coef  | ... |                                           |
|--------|-------|-----|-------------------------------------------|
| weight | 12.27 | ... | <- coefficient is <code>_b[weight]</code> |
| mpg    | 3.21  | ... |                                           |

Stata’s multiple-equation output looks like this:

|        | Coef  | ... |                                                 |
|--------|-------|-----|-------------------------------------------------|
| cat1   |       | ... |                                                 |
| weight | 12.27 | ... | <- coefficient is <code>[cat1]_b[weight]</code> |
| mpg    | 3.21  | ... |                                                 |
| 8      |       | ... |                                                 |
| weight | 5.83  | ... | <- coefficient is <code>[8]_b[weight]</code>    |
| mpg    | 7.43  | ... |                                                 |

## Dropped constraints

testnl automatically drops constraints when

- They are nonbinding, for example,  $\_b[\text{mpg}] = \_b[\text{mpg}]$ . More subtle cases include

```
_b[mpg]*_b[weight] = 4
_b[weight] = 2
_b[mpg] = 2
```

In this example, the third constraint is nonbinding because it is implied by the first two.

- They are contradictory, for example,  $\_b[\text{mpg}] = 2$  and  $\_b[\text{mpg}] = 3$ . More subtle cases include

```
_b[mpg]*_b[weight] = 4
_b[weight] = 2
_b[mpg] = 3
```

The third constraint contradicts the first two.

## Multiple constraints

### ▷ Example 2

We illustrate the simultaneous test of a series of constraints using simulated data on labor-market promotion in a given year. We fit a probit model with separate effects for education, experience, and experience-squared for men and women.

```
. use https://www.stata-press.com/data/r19/promotion
(Simulated data on promotions)
. probit promo male male#c.(yedu yexp yexp2), nolog
Probit regression Number of obs = 775
 LR chi2(7) = 424.42
 Prob > chi2 = 0.0000
 Pseudo R2 = 0.4637
Log likelihood = -245.42768
```

| promo        | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|--------------|-------------|-----------|-------|-------|----------------------|-----------|
| male         | .6489974    | .203739   | 3.19  | 0.001 | .2496763             | 1.048318  |
| male#c.yedu  |             |           |       |       |                      |           |
| Female       | .9730237    | .1056136  | 9.21  | 0.000 | .7660248             | 1.180023  |
| Male         | 1.390517    | .1527288  | 9.10  | 0.000 | 1.091174             | 1.68986   |
| male#c.yexp  |             |           |       |       |                      |           |
| Female       | .4559544    | .0901169  | 5.06  | 0.000 | .2793285             | .6325803  |
| Male         | 1.422539    | .1544255  | 9.21  | 0.000 | 1.11987              | 1.725207  |
| male#c.yexp2 |             |           |       |       |                      |           |
| Female       | -.1027149   | .0573059  | -1.79 | 0.073 | -.2150325            | .0096026  |
| Male         | -.3749457   | .1160113  | -3.23 | 0.001 | -.6023236            | -.1475677 |
| _cons        | .9872018    | .1148215  | 8.60  | 0.000 | .7621559             | 1.212248  |

Note: 1 failure and 2 successes completely determined.

The effects of human capital seem to differ between men and women. A formal test confirms this.

```
. test (yedu#0.male = yedu#1.male) (yexp#0.male = yexp#1.male)
> (yexp2#0.male = yexp2#1.male)
(1) [promo]0b.male#c.yedu - [promo]1.male#c.yedu = 0
(2) [promo]0b.male#c.yexp - [promo]1.male#c.yexp = 0
(3) [promo]0b.male#c.yexp2 - [promo]1.male#c.yexp2 = 0
 chi2(3) = 35.43
 Prob > chi2 = 0.0000
```

How do we interpret this gender difference? It has repeatedly been stressed (see, for example, [Long \[1997, 47–50\]](#); [Allison \[1999\]](#)) that comparison of groups in binary response models, and similarly in other latent-variable models, is hampered by an identification problem: with  $\beta$  the regression coefficients for the latent variable and  $\sigma$  the standard deviation of the latent residual, only the  $\beta/\sigma$  are identified. In fact, in terms of the latent regression, the probit coefficients should be interpreted as  $\beta/\sigma$ , not as the  $\beta$ . If we cannot claim convincingly that the residual standard deviation  $\sigma$  does not vary between the sexes, equality of the regression coefficients  $\beta$  implies that the coefficients of the probit model for men and women are *proportional* but not necessarily equal. This is a nonlinear hypothesis in terms of the probit coefficients, not a linear one.

```
. testnl _b[yedu#1.male]/_b[yedu#0.male] = _b[yexp#1.male]/_b[yexp#0.male]
> = _b[yexp2#1.male]/_b[yexp2#0.male]
(1) _b[yedu#1.male]/_b[yedu#0.male] = _b[yexp#1.male]/_b[yexp#0.male]
(2) _b[yedu#1.male]/_b[yedu#0.male] = _b[yexp2#1.male]/_b[yexp2#0.male]
 chi2(2) = 9.21
 Prob > chi2 = 0.0100
```

We conclude that we find fairly strong evidence against the proportionality of the coefficients, and hence we have to conclude that success in the labor market is produced in different ways by men and women. (But remember, these were simulated data.)

### ▷ Example 3

The syntax for specifying the equality of multiple expressions is just a convenient shorthand for specifying a series of constraints, namely, that the first expression equals the second expression, the first expression also equals the third expression, etc. The Wald test performed and the output of `testnl` are the same whether we use the shorthand or we specify the series of constraints.

```
. testnl (_b[yedu#1.male]/_b[yedu#0.male] =
> _b[yexp#1.male]/_b[yexp#0.male])
> (_b[yedu#1.male]/_b[yedu#0.male] =
> _b[yexp2#1.male]/_b[yexp2#0.male])
(1) _b[yedu#1.male]/_b[yedu#0.male] = _b[yexp#1.male]/_b[yexp#0.male]
(2) _b[yedu#1.male]/_b[yedu#0.male] = _b[yexp2#1.male]/_b[yexp2#0.male]
 chi2(2) = 9.21
 Prob > chi2 = 0.0100
```

Having established differences between men and women, we would like to do multiple testing between the ratios. Because we did not specify hypotheses in advance, we prefer to adjust the  $p$ -values of tests using, here, Bonferroni's method.

```
. testnl _b[yedu#1.male]/_b[yedu#0.male] =
> _b[yexp#1.male]/_b[yexp#0.male] =
> _b[yexp2#1.male]/_b[yexp2#0.male], mtest(b)
(1) _b[yedu#1.male]/_b[yedu#0.male] = _b[yexp#1.male]/_b[yexp#0.male]
(2) _b[yedu#1.male]/_b[yedu#0.male] = _b[yexp2#1.male]/_b[yexp2#0.male]
```

|     | chi2 | df | p > chi2 |
|-----|------|----|----------|
| (1) | 6.89 | 1  | 0.0173*  |
| (2) | 0.93 | 1  | 0.6713*  |
| All | 9.21 | 2  | 0.0100   |

\* Bonferroni-adjusted  $p$ -values



## Manipulability

Although `testnl` allows you to specify constraints in different ways that are mathematically equivalent, as noted above, this does not mean that the tests are the same. This difference is known as the manipulability of the Wald test for nonlinear hypotheses; also see [R] `boxcox`. The test might even be significant for one formulation but not significant for another formulation that is mathematically equivalent. Trying out different specifications to find a formulation with the desired  $p$ -value is totally inappropriate, though it may actually be fun to try. There is no variance under representation because the nonlinear Wald test is actually a standard Wald test for a linearization of the constraint, which depends on the particular specification. We note that the likelihood-ratio test is not manipulable in this sense.

From a statistical point of view, it is best to choose a specification of the constraints that is as linear as possible. Doing so usually improves the accuracy of the approximation of the null-distribution of the test by a  $\chi^2$  or an  $F$  distribution. The example above used the nonlinear Wald test to test whether the coefficients of human capital variables for men were proportional to those of women. A specification of proportionality of coefficients in terms of ratios of coefficients is fairly nonlinear if the coefficients in the denominator are close to 0. A more linear version of the test results from a bilinear formulation. Thus, instead of

```
. testnl _b[yedu#1.male]/_b[yedu#0.male] = _b[yexp#1.male]/_b[yexp#0.male]
(1) _b[yedu#1.male]/_b[yedu#0.male] = _b[yexp#1.male]/_b[yexp#0.male]
 chi2(1) = 6.89
 Prob > chi2 = 0.0087
```

perhaps

```
. testnl _b[yedu#1.male]*_b[yexp#0.male] = _b[yedu#0.male]*_b[yexp#1.male]
(1) _b[yedu#1.male]*_b[yexp#0.male] = _b[yedu#0.male]*_b[yexp#1.male]
 chi2(1) = 13.95
 Prob > chi2 = 0.0002
```

is better, and in fact it has been suggested that the latter version of the test is more reliable. This assertion is confirmed by performing simulations and is in line with theoretical results of [Phillips and Park \(1988\)](#). There is strong evidence against the proportionality of human capital effects between men and women, implying for this example that differences in the residual variances between the sexes can be ruled out as the explanation of the sex differences in the analysis of labor market participation.

## Stored results

`testnl` stores the following in `r()`:

### Scalars

|                      |                             |
|----------------------|-----------------------------|
| <code>r(df)</code>   | degrees of freedom          |
| <code>r(df_r)</code> | residual degrees of freedom |
| <code>r(chi2)</code> | $\chi^2$                    |
| <code>r(p)</code>    | $p$ -value for Wald test    |
| <code>r(F)</code>    | $F$ statistic               |

### Macros

|                         |                                          |
|-------------------------|------------------------------------------|
| <code>r(mtmetho)</code> | method specified in <code>mtest()</code> |
|-------------------------|------------------------------------------|

### Matrices

|                       |                                                                                                              |
|-----------------------|--------------------------------------------------------------------------------------------------------------|
| <code>r(G)</code>     | derivatives of $R(\mathbf{b})$ with respect to $\mathbf{b}$ ; see <a href="#">Methods and formulas</a> below |
| <code>r(R)</code>     | $R(\mathbf{b}) - \mathbf{q}$ ; see <a href="#">Methods and formulas</a> below                                |
| <code>r(mtest)</code> | multiple test results                                                                                        |

## Methods and formulas

After fitting a model, define  $\mathbf{b}$  as the resulting  $1 \times k$  parameter vector and  $\mathbf{V}$  as the  $k \times k$  covariance matrix. The (linear or nonlinear) hypothesis is given by  $R(\mathbf{b}) = \mathbf{q}$ , where  $R$  is a function returning a  $j \times 1$  vector. The Wald test formula is (Greene 2018, 512–513)

$$W = \{R(\mathbf{b}) - \mathbf{q}\}' (\mathbf{G}\mathbf{V}\mathbf{G}')^{-1} \{R(\mathbf{b}) - \mathbf{q}\}$$

where  $\mathbf{G}$  is the derivative matrix of  $R(\mathbf{b})$  with respect to  $\mathbf{b}$ .  $W$  is distributed as  $\chi^2$  if  $\mathbf{V}$  is an asymptotic covariance matrix.  $F = W/j$  is distributed as  $F$  for linear regression.

The adjustment methods for multiple testing are described in [R] [test](#). The adjustment for survey design effects is described in [SVY] [svy postestimation](#).

## References

- Allison, P. D. 1999. Comparing logit and probit coefficients across groups. *Sociological Methods and Research* 28: 186–208. <https://doi.org/10.1177/0049124199028002003>.
- Greene, W. H. 2018. *Econometric Analysis*. 8th ed. New York: Pearson.
- Long, J. S. 1997. *Regression Models for Categorical and Limited Dependent Variables*. Thousand Oaks, CA: Sage.
- Phillips, P. C. B., and J. Y. Park. 1988. On the formulation of Wald tests of nonlinear restrictions. *Econometrica* 56: 1065–1083. <https://doi.org/10.2307/1911359>.

## Also see

- [R] [contrast](#) — Contrasts and linear hypothesis tests after estimation
- [R] [lincom](#) — Linear combinations of parameters
- [R] [lrtest](#) — Likelihood-ratio test after estimation
- [R] [nlcom](#) — Nonlinear combinations of parameters
- [R] [test](#) — Test linear hypotheses after estimation
- [U] [13.5 Accessing coefficients and standard errors](#)
- [U] [20 Estimation and postestimation commands](#)

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## Description

`tetrachoric` computes estimates of the tetrachoric correlation coefficients of the binary variables in *varlist*. All of these variables should be 0, 1, or missing values.

Tetrachoric correlations assume a latent bivariate normal distribution  $(X_1, X_2)$  for each pair of variables  $(v_1, v_2)$ , with a threshold model for the manifest variables,  $v_i = 1$  if and only if  $X_i > 0$ . The means and variances of the latent variables are not identified, but the correlation,  $r$ , of  $X_1$  and  $X_2$  can be estimated from the joint distribution of  $v_1$  and  $v_2$  and is called the tetrachoric correlation coefficient.

`tetrachoric` computes pairwise estimates of the tetrachoric correlations by the (iterative) maximum likelihood estimator obtained from bivariate probit without explanatory variables (see [R] **biprobit**) by using the [Edwards and Edwards \(1984\)](#) noniterative estimator as the initial value.

The pairwise correlation matrix is returned as `r(Rho)` and can be used to perform a factor analysis or a principal component analysis of binary variables by using the `factormat` or `pcamat` commands; see [MV] **factor** and [MV] **pca**.

## Quick start

Tetrachoric correlation of `v1` and `v2` with standard error and test of independence

```
tetrachoric v1 v2
```

Matrix of pairwise tetrachoric correlations for `v1`, `v2`, and `v3`

```
tetrachoric v1 v2 v3
```

Add standard errors and  $p$ -values

```
tetrachoric v1 v2 v3, stats(rho se p)
```

Same as above, but adjust  $p$ -values for multiple comparisons using Bonferroni's method

```
tetrachoric v1 v2 v3, stats(rho se p) bonferroni
```

Add star to correlations significant at the 5% level

```
tetrachoric v1 v2 v3, star(.05)
```

Use all available data for each pair of variables and report number of observations used

```
tetrachoric v1 v2 v3, pw stats(rho obs)
```

Adjust correlation matrix to be positive semidefinite

```
tetrachoric v1 v2 v3, posdef
```

## Menu

Statistics > Summaries, tables, and tests > Summary and descriptive statistics > Tetrachoric correlations

## Syntax

```
tetrachoric varlist [if] [in] [weight] [, options]
```

| <i>options</i>                      | Description                                                                                         |
|-------------------------------------|-----------------------------------------------------------------------------------------------------|
| Main                                |                                                                                                     |
| <code>stats(<i>statlist</i>)</code> | list of statistics; select up to 4 statistics; default is <code>stats(rho)</code>                   |
| <code>edwards</code>                | use the noniterative Edwards and Edwards estimator; default is the maximum likelihood estimator     |
| <code>print(#)</code>               | significance level for displaying coefficients                                                      |
| <code>star(#)</code>                | significance level for displaying with a star                                                       |
| <code>bonferroni</code>             | use Bonferroni-adjusted significance level                                                          |
| <code>sidak</code>                  | use Šidák-adjusted significance level                                                               |
| <code>pw</code>                     | calculate all the pairwise correlation coefficients by using all available data (pairwise deletion) |
| <code>zeroadjust</code>             | adjust frequencies when one cell has a zero count                                                   |
| <code>matrix</code>                 | display output in matrix form                                                                       |
| <code>notable</code>                | suppress display of correlations                                                                    |
| <code>posdef</code>                 | modify correlation matrix to be positive semidefinite                                               |

| <i>statlist</i>  | Description                         |
|------------------|-------------------------------------|
| <code>rho</code> | tetrachoric correlation coefficient |
| <code>se</code>  | standard error of rho               |
| <code>obs</code> | number of observations              |
| <code>p</code>   | exact two-sided significance level  |

`by` and `collect` are allowed; see [\[U\] 11.1.10 Prefix commands](#).

`fweights` are allowed; see [\[U\] 11.1.6 weight](#).

## Options

### Main

`stats(statlist)` specifies the statistics to be displayed in the matrix of output. `stats(rho)` is the default. Up to four statistics may be specified. `stats(rho se p obs)` would display the tetrachoric correlation coefficient, its standard error, the significance level, and the number of observations. If *varlist* contains only two variables, all statistics are shown in tabular form. `stats()`, `print()`, and `star()` have no effect unless the `matrix` option is also specified.

`edwards` specifies that the noniterative Edwards and Edwards estimator be used. The default is the maximum likelihood estimator. If you analyze many binary variables, you may want to use the fast noniterative estimator proposed by [Edwards and Edwards \(1984\)](#). However, if you have skewed variables, the approximation does not perform well.

`print(#)` specifies the maximum significance level of correlation coefficients to be printed. Correlation coefficients with larger significance levels are left blank in the matrix. Typing `tetrachoric ..., print(.10)` would list only those correlation coefficients that are significant at the 10% level or lower.

`star(#)` specifies the maximum significance level of correlation coefficients to be marked with a star. Typing `tetrachoric ..., star(.05)` would “star” all correlation coefficients significant at the 5% level or lower.

`bonferroni` makes the Bonferroni adjustment to calculated significance levels. This option affects printed significance levels and the `print()` and `star()` options. Thus, `tetrachoric ..., print(.05) bonferroni` prints coefficients with Bonferroni-adjusted significance levels of 0.05 or less.

`sidak` makes the Šidák adjustment to calculated significance levels. This option affects printed significance levels and the `print()` and `star()` options. Thus, `tetrachoric ..., print(.05) sidak` prints coefficients with Šidák-adjusted significance levels of 0.05 or less.

`pw` specifies that the tetrachoric correlation be calculated by using all available data. By default, `tetrachoric` uses casewise deletion, where observations are ignored if any of the specified variables in *varlist* are missing.

`zeroadjust` specifies that when one of the cells has a zero count, a frequency adjustment be applied in such a way as to increase the zero to one-half and maintain row and column totals.

`matrix` forces `tetrachoric` to display the statistics as a matrix, even if *varlist* contains only two variables. `matrix` is implied if more than two variables are specified.

`notable` suppresses the output.

`posdef` modifies the correlation matrix so that it is positive semidefinite, that is, a proper correlation matrix. The modified result is the correlation matrix associated with the least-squares approximation of the tetrachoric correlation matrix by a positive-semidefinite matrix. If the correlation matrix is modified, the standard errors and significance levels are not displayed and are returned in `r()`.

## Remarks and examples

Remarks are presented under the following headings:

*Association in 2-by-2 tables*  
*Factor analysis of dichotomous variables*  
*Tetrachoric correlations with simulated data*

### Association in 2-by-2 tables

Although a wide variety of measures of association in cross tabulations have been proposed, such measures are essentially equivalent (monotonically related) in the special case of  $2 \times 2$  tables—there is only 1 degree of freedom for nonindependence. Still, some measures have more desirable properties than others. Here we compare two measures: the standard Pearson correlation coefficient and the tetrachoric correlation coefficient. Given asymmetric row or column margins, Pearson correlations are limited to a range smaller than  $-1$  to  $1$ , although tetrachoric correlations can still span the range from  $-1$  to  $1$ . To illustrate, consider the following set of tables for two binary variables, X and Z:

|       | Z = 0  | Z = 1  |    |
|-------|--------|--------|----|
| X = 0 | 20 - a | 10 + a | 30 |
| X = 1 | a      | 10 - a | 10 |
|       | 20     | 20     | 40 |

For  $a$  equal to 0, 1, 2, 5, 8, 9, and 10, the Pearson and tetrachoric correlations for the above table are

| $a$         | 0     | 1     | 2     | 5 | 8      | 9      | 10     |
|-------------|-------|-------|-------|---|--------|--------|--------|
| Pearson     | 0.577 | 0.462 | 0.346 | 0 | -0.346 | -0.462 | -0.577 |
| Tetrachoric | 1.000 | 0.792 | 0.607 | 0 | -0.607 | -0.792 | -1.000 |

The restricted range for the Pearson correlation is especially unfortunate when you try to analyze the association between binary variables by using models developed for continuous data, such as factor analysis and principal component analysis.

The tetrachoric correlation of two variables  $(Y_1, Y_2)$  can be thought of as the Pearson correlation of two latent bivariate normal distributed variables  $(Y_1^*, Y_2^*)$  with threshold measurement models  $Y_i = (Y_i^* > c_i)$  for unknown cutpoints  $c_i$ . Or equivalently,  $Y_i = (Y_i^{**} > 0)$  where the latent bivariate normal  $(Y_1^{**}, Y_2^{**})$  are shifted versions of  $(Y_1^*, Y_2^*)$  so that the cutpoints are zero. Obviously, you must judge whether assuming underlying latent variables is meaningful for the data. If this assumption is justified, tetrachoric correlations have two advantages. First, you have an intuitive understanding of the size of correlations that are substantively interesting in your field of research, and this intuition is based on correlations that range from  $-1$  to  $1$ . Second, because the tetrachoric correlation for binary variables estimates the Pearson correlation of the latent continuous variables (assumed multivariate normal distributed), you can use the tetrachoric correlations to analyze multivariate relationships between the dichotomous variables. When doing so, remember that you must interpret the model in terms of the underlying continuous variables.

▷ Example 1

To illustrate tetrachoric correlations, we examine three binary variables from the familyvalues dataset (described in example 2).

```
. use https://www.stata-press.com/data/r19/familyvalues
(Attitudes on gender, relationships and family)
```

```
. tabulate RS075 RS076
```

| Fam att:<br>women in<br>charge bad | Fam att: trad<br>division of labor |       | Total |
|------------------------------------|------------------------------------|-------|-------|
|                                    | 0                                  | 1     |       |
| 0                                  | 1,564                              | 979   | 2,543 |
| 1                                  | 119                                | 632   | 751   |
| Total                              | 1,683                              | 1,611 | 3,294 |

```
. correlate RS074 RS075 RS076
(obs=3,291)
```

|       | RS074  | RS075  | RS076  |
|-------|--------|--------|--------|
| RS074 | 1.0000 |        |        |
| RS075 | 0.0396 | 1.0000 |        |
| RS076 | 0.1595 | 0.3830 | 1.0000 |

```
. tetrachoric RS074 RS075 RS076
(obs=3,291)
```

|       | RS074  | RS075  | RS076  |
|-------|--------|--------|--------|
| RS074 | 1.0000 |        |        |
| RS075 | 0.0689 | 1.0000 |        |
| RS076 | 0.2480 | 0.6427 | 1.0000 |

As usual, the tetrachoric correlation coefficients are larger (in absolute value) and more dispersed than the Pearson correlations.



## Factor analysis of dichotomous variables

### ► Example 2

Factor analysis is a popular model for measuring latent continuous traits. The standard estimators are appropriate only for continuous unimodal data. Because of the skewness implied by Bernoulli-distributed variables (especially when the probability is distributed unevenly), a factor analysis of a Pearson correlation matrix can be rather misleading when used in this context. A factor analysis of a matrix of tetrachoric correlations is more appropriate under these conditions (Uebersax 2000). We illustrate this with data on gender, relationship, and family attitudes of spouses using the Households in The Netherlands survey 1995 (Weesie et al. 1995). For attitude variables, it seems reasonable to assume that agreement or disagreement is just a coarse measurement of more nuanced underlying attitudes.

To demonstrate, we examine a few of the variables from the familyvalues dataset.

```
. use https://www.stata-press.com/data/r19/familyvalues
(Attitudes on gender, relationships and family)
. describe RS056-RS063
```

| Variable name | Storage type | Display format | Value label | Variable label                  |
|---------------|--------------|----------------|-------------|---------------------------------|
| RS056         | byte         | %9.0g          |             | Fam att: should be together     |
| RS057         | byte         | %9.0g          |             | Fam att: should fight for relat |
| RS058         | byte         | %9.0g          |             | Fam att: should avoid conflict  |
| RS059         | byte         | %9.0g          |             | Fam att: woman better nurturer  |
| RS060         | byte         | %9.0g          |             | Fam att: both spouses money goo |
| RS061         | byte         | %9.0g          |             | Fam att: woman techn school goo |
| RS062         | byte         | %9.0g          |             | Fam att: man natural breadwinne |
| RS063         | byte         | %9.0g          |             | Fam att: common leisure good    |

```
. summarize RS056-RS063
```

| Variable | Obs   | Mean     | Std. dev. | Min | Max |
|----------|-------|----------|-----------|-----|-----|
| RS056    | 3,298 | .5630685 | .4960816  | 0   | 1   |
| RS057    | 3,296 | .5400485 | .4984692  | 0   | 1   |
| RS058    | 3,283 | .6387451 | .4804374  | 0   | 1   |
| RS059    | 3,308 | .6544474 | .4756114  | 0   | 1   |
| RS060    | 3,302 | .3906723 | .487975   | 0   | 1   |
| RS061    | 3,293 | .7102946 | .4536945  | 0   | 1   |
| RS062    | 3,307 | .5857272 | .4926705  | 0   | 1   |
| RS063    | 3,298 | .5379018 | .498637   | 0   | 1   |

```
. correlate RS056-RS063
(obs=3,221)
```

|       | RS056   | RS057   | RS058   | RS059   | RS060   | RS061   | RS062  |
|-------|---------|---------|---------|---------|---------|---------|--------|
| RS056 | 1.0000  |         |         |         |         |         |        |
| RS057 | 0.1350  | 1.0000  |         |         |         |         |        |
| RS058 | 0.2377  | 0.0258  | 1.0000  |         |         |         |        |
| RS059 | 0.1816  | 0.0097  | 0.2550  | 1.0000  |         |         |        |
| RS060 | -0.1020 | -0.0538 | -0.0424 | 0.0126  | 1.0000  |         |        |
| RS061 | -0.1137 | 0.0610  | -0.1375 | -0.2076 | 0.0706  | 1.0000  |        |
| RS062 | 0.2014  | 0.0285  | 0.2273  | 0.4098  | -0.0793 | -0.2873 | 1.0000 |
| RS063 | 0.2057  | 0.1460  | 0.1049  | 0.0911  | 0.0179  | -0.0233 | 0.0975 |
|       | RS063   |         |         |         |         |         |        |
| RS063 | 1.0000  |         |         |         |         |         |        |

Skewness in these data is relatively modest. For comparison, here are the tetrachoric correlations:

```
. tetrachoric RS056-RS063
(obs=3,221)
```

|       | RS056   | RS057   | RS058   | RS059   | RS060   | RS061   | RS062  |
|-------|---------|---------|---------|---------|---------|---------|--------|
| RS056 | 1.0000  |         |         |         |         |         |        |
| RS057 | 0.2114  | 1.0000  |         |         |         |         |        |
| RS058 | 0.3716  | 0.0416  | 1.0000  |         |         |         |        |
| RS059 | 0.2887  | 0.0158  | 0.4007  | 1.0000  |         |         |        |
| RS060 | -0.1620 | -0.0856 | -0.0688 | 0.0208  | 1.0000  |         |        |
| RS061 | -0.1905 | 0.1011  | -0.2382 | -0.3664 | 0.1200  | 1.0000  |        |
| RS062 | 0.3135  | 0.0452  | 0.3563  | 0.6109  | -0.1267 | -0.4845 | 1.0000 |
| RS063 | 0.3187  | 0.2278  | 0.1677  | 0.1467  | 0.0286  | -0.0388 | 0.1538 |
|       | RS063   |         |         |         |         |         |        |
| RS063 | 1.0000  |         |         |         |         |         |        |

Again, we see that the tetrachoric correlations are generally larger in absolute value than the Pearson correlations. The bivariate probit and Edwards and Edwards estimators (the `edwards` option) implemented in `tetrachoric` may return a correlation matrix that is not positive semidefinite—a mathematical property of any real correlation matrix. Positive definiteness is required by commands for analyses of correlation matrices, such as `factormat` and `pcamat`; see [\[MV\] factor](#) and [\[MV\] pca](#). The `posdef` option of `tetrachoric` tests for positive definiteness and projects the estimated correlation matrix to a positive-semidefinite matrix if needed.

```
. tetrachoric RS056-RS063, notable posdef
. matrix C = r(Rho)
```

This time, we suppressed the display of the correlations with the `notable` option and requested that the correlation matrix be positive semidefinite with the `posdef` option. Had the correlation matrix not been positive definite, `tetrachoric` would have displayed a warning message and then adjusted the matrix to be positive semidefinite. We placed the resulting tetrachoric correlation matrix into a matrix, `C`, so that we can perform a factor analysis upon it.

`tetrachoric` with the `posdef` option asserted that `C` was positive definite because no warning message was displayed. We can verify this by using a familiar characterization of symmetric positive-definite matrices: all eigenvalues are real and positive.

```
. matrix symeigen eigenvectors eigenvalues = C
. matrix list eigenvalues
eigenvalues[1,8]
 e1 e2 e3 e4 e5 e6 e7
r1 2.5974789 1.3544664 1.0532476 .77980391 .73462018 .57984565 .54754512
 e8
r1 .35299228
```

We can proceed with a factor analysis on the matrix C. We use `factormat` and select iterated principal factors as the estimation method; see [\[MV\] factor](#).

```
. factormat C, n(3221) ipf factor(2)
(obs=3,221)
Factor analysis/correlation Number of obs = 3,221
Method: iterated principal factors Retained factors = 2
Rotation: (unrotated) Number of params = 15
```

| Factor  | Eigenvalue | Difference | Proportion | Cumulative |
|---------|------------|------------|------------|------------|
| Factor1 | 2.06855    | 1.40178    | 0.7562     | 0.7562     |
| Factor2 | 0.66677    | 0.47180    | 0.2438     | 1.0000     |
| Factor3 | 0.19497    | 0.06432    | 0.0713     | 1.0713     |
| Factor4 | 0.13065    | 0.10967    | 0.0478     | 1.1191     |
| Factor5 | 0.02098    | 0.10085    | 0.0077     | 1.1267     |
| Factor6 | -0.07987   | 0.01037    | -0.0292    | 1.0975     |
| Factor7 | -0.09024   | 0.08626    | -0.0330    | 1.0645     |
| Factor8 | -0.17650   | .          | -0.0645    | 1.0000     |

LR test: independent vs. saturated:  $\chi^2(28) = 4620.01$  Prob> $\chi^2 = 0.0000$

Factor loadings (pattern matrix) and unique variances

| Variable | Factor1 | Factor2 | Uniqueness |
|----------|---------|---------|------------|
| RS056    | 0.5528  | 0.4120  | 0.5247     |
| RS057    | 0.1124  | 0.4214  | 0.8098     |
| RS058    | 0.5333  | 0.0718  | 0.7105     |
| RS059    | 0.6961  | -0.1704 | 0.4865     |
| RS060    | -0.1339 | -0.0596 | 0.9785     |
| RS061    | -0.5126 | 0.2851  | 0.6560     |
| RS062    | 0.7855  | -0.2165 | 0.3361     |
| RS063    | 0.2895  | 0.3919  | 0.7626     |

▷ Example 3

We noted in [example 2](#) that the matrix of estimates of the tetrachoric correlation coefficients need not be positive definite. Here is an example:

```
. use https://www.stata-press.com/data/r19/familyvalues
(Attitudes on gender, relationships and family)
. tetrachoric RS056-RS063 in 1/20, posdef
(obs=18)
matrix with tetrachoric correlations is not positive semidefinite;
it has 2 negative eigenvalues
maxdiff(corr,adj-corr) = 0.2346
(adj-corr: tetrachoric correlations adjusted to be positive semidefinite)
```

| adj-corr | RS056   | RS057   | RS058   | RS059   | RS060   | RS061   | RS062  |
|----------|---------|---------|---------|---------|---------|---------|--------|
| RS056    | 1.0000  |         |         |         |         |         |        |
| RS057    | 0.5284  | 1.0000  |         |         |         |         |        |
| RS058    | 0.3012  | 0.2548  | 1.0000  |         |         |         |        |
| RS059    | 0.3251  | 0.2791  | 0.0550  | 1.0000  |         |         |        |
| RS060    | -0.5197 | -0.4222 | -0.7163 | 0.0552  | 1.0000  |         |        |
| RS061    | 0.3448  | 0.4815  | -0.0958 | -0.1857 | -0.0980 | 1.0000  |        |
| RS062    | 0.1066  | -0.0375 | 0.0072  | 0.3909  | -0.2333 | -0.7654 | 1.0000 |
| RS063    | 0.3830  | 0.4939  | 0.4336  | 0.0075  | -0.8937 | -0.0337 | 0.4934 |
| adj-corr | RS063   |         |         |         |         |         |        |
| RS063    | 1.0000  |         |         |         |         |         |        |

```
. mata:
----- mata (type end to exit) -----
: C2 = st_matrix("r(Rho)")
: eigenvecs = .
: eigenvals = .
: symeigensystem(C2, eigenvecs, eigenvals)
: eigenvals
 1 2 3 4
1 | 3.156592567 2.065279398 1.324911199 .7554904485
 |
 | 5 6 7 8
1 | .4845368741 .2131895139 -1.11022e-16 -2.27918e-16
: end
```

The estimated tetrachoric correlation matrix is rank-2 deficient. With this C2 matrix, we can only use models of correlation that allow for singular cases.

## Tetrachoric correlations with simulated data

### ▷ Example 4

We use `drawnorm` (see [D] [drawnorm](#)) to generate a sample of 1,000 observations from a bivariate normal distribution with means  $-1$  and  $1$ , unit variances, and correlation  $0.4$ .

```
. clear
. set seed 11000
. matrix m = (1, -1)
. matrix V = (1, 0.4 \ 0.4, 1)
. drawnorm c1 c2, n(1000) means(m) cov(V)
(obs 1,000)
```

Now, consider the measurement model assumed by the tetrachoric correlations. We observe only whether  $c_1$  and  $c_2$  are greater than zero,

```
. generate d1 = (c1 > 0)
. generate d2 = (c2 > 0)
. tabulate d1 d2
```

| d1    | d2  |     | Total |
|-------|-----|-----|-------|
|       | 0   | 1   |       |
| 0     | 141 | 6   | 147   |
| 1     | 706 | 147 | 853   |
| Total | 847 | 153 | 1,000 |

We want to estimate the correlation of  $c_1$  and  $c_2$  from the binary variables  $d_1$  and  $d_2$ . Pearson's correlation of the binary variables  $d_1$  and  $d_2$  is  $0.129$ —a seriously biased estimate of the underlying correlation  $\rho = 0.4$ .

```
. correlate d1 d2
(obs=1,000)
```

|    | d1     | d2     |
|----|--------|--------|
| d1 | 1.0000 |        |
| d2 | 0.1294 | 1.0000 |

The tetrachoric correlation coefficient of  $d_1$  and  $d_2$  estimates the Pearson correlation of the latent continuous variables,  $c_1$  and  $c_2$ .

```
. tetrachoric d1 d2
Number of obs = 1,000
Tetrachoric rho = 0.3875
Std error = 0.0787
Test of H0: d1 and d2 are independent
2-sided exact P = 0.0000
```

The estimate of the tetrachoric correlation of  $d_1$  and  $d_2$ ,  $0.3875$ , is much closer to the underlying correlation,  $0.4$ , between  $c_1$  and  $c_2$ .

## Stored results

`tetrachoric` stores the following in `r()`:

### Scalars

|                        |                                                                            |
|------------------------|----------------------------------------------------------------------------|
| <code>r(rho)</code>    | tetrachoric correlation coefficient between variables 1 and 2              |
| <code>r(N)</code>      | number of observations                                                     |
| <code>r(nneg)</code>   | number of negative eigenvalues (posdef only)                               |
| <code>r(se_rho)</code> | standard error of <code>r(rho)</code>                                      |
| <code>r(p)</code>      | $p$ -value for two-sided Fisher's exact test (for the first two variables) |

### Macros

|                        |                |
|------------------------|----------------|
| <code>r(method)</code> | estimator used |
|------------------------|----------------|

### Matrices

|                        |                                                         |
|------------------------|---------------------------------------------------------|
| <code>r(Rho)</code>    | tetrachoric correlation matrix                          |
| <code>r(Se_Rho)</code> | standard errors of <code>r(Rho)</code>                  |
| <code>r(Nobs)</code>   | number of observations used in computing correlation    |
| <code>r(P)</code>      | matrix of $p$ -values for two-sided Fisher's exact test |

## Methods and formulas

`tetrachoric` provides two estimators for the tetrachoric correlation  $\rho$  of two binary variables with the frequencies  $n_{ij}$ ,  $i, j = 0, 1$ . `tetrachoric` defaults to the slower (iterative) maximum likelihood estimator obtained from bivariate probit without explanatory variables (see [R] [biprobit](#)) by using the Edwards and Edwards noniterative estimator as the initial value. A fast (noniterative) estimator is also available by specifying the `edwards` option (Edwards and Edwards 1984; Digby 1983)

$$\hat{\rho} = \frac{\alpha - 1}{\alpha + 1}$$

where

$$\alpha = \left( \frac{n_{00}n_{11}}{n_{01}n_{10}} \right)^{\pi/4} \quad (\pi = 3.14\dots)$$

if all  $n_{ij} > 0$ . If  $n_{00} = 0$  or  $n_{11} = 0$ ,  $\hat{\rho} = -1$ ; if  $n_{01} = 0$  or  $n_{10} = 0$ ,  $\hat{\rho} = 1$ .

The asymptotic variance of the Edwards and Edwards estimator of the tetrachoric correlation is easily obtained by the delta method,

$$\text{avar}(\hat{\rho}) = \left\{ \frac{\pi\alpha}{2(1+\alpha)^2} \right\}^2 \left( \frac{1}{n_{00}} + \frac{1}{n_{01}} + \frac{1}{n_{10}} + \frac{1}{n_{11}} \right)$$

provided all  $n_{ij} > 0$ , otherwise it is left undefined (missing). The Edwards and Edwards estimator is fast, but may be inaccurate if the margins are very skewed.

`tetrachoric` reports exact  $p$ -values for statistical independence, computed by the `exact` option of [R] [tabulate twoway](#).

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## Also see

- [R] **biprobit** — Bivariate probit regression
- [R] **correlate** — Correlations of variables
- [R] **spearman** — Spearman’s and Kendall’s correlations
- [R] **tabulate twoway** — Two-way table of frequencies
- [MV] **factor** — Factor analysis
- [MV] **pca** — Principal component analysis

[Description](#)[Options](#)[Acknowledgment](#)[Quick start](#)[Remarks and examples](#)[References](#)[Menu](#)[Stored results](#)[Also see](#)[Syntax](#)[Methods and formulas](#)

## Description

`tnbreg` estimates the parameters of a truncated negative binomial model by maximum likelihood. The dependent variable *devar* is regressed on *indevars*, where *devar* is a positive count variable whose values are all above the truncation point.

## Quick start

Truncated negative binomial regression of *y* on *x* with truncation at 0

```
tnbreg y x
```

Report incidence-rate ratios

```
tnbreg y x, irr
```

Add categorical variable *a* using [factor variable](#) syntax

```
tnbreg y x i.a
```

Same as above, but specify a constant truncation point of 2

```
tnbreg y x i.a, ll(2)
```

With exposure variable *exp*

```
tnbreg y x i.a, exposure(exp)
```

Same as above, but specifying a variable truncation point stored in variable *min*

```
tnbreg y x i.a, exposure(exp) ll(min)
```

With cluster-robust standard errors clustering by the levels of *cvar*

```
tnbreg y x i.a, exposure(exp) ll(min) vce(cluster cvar)
```

## Menu

Statistics > Count outcomes > Truncated negative binomial regression

## Syntax

```
tnbreg depvar [indepvars] [if] [in] [weight] [, options]
```

| <i>options</i>                                                                                                                                                                                | Description                                                                                                                                      |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                                                                                                                                                                  |                                                                                                                                                  |
| <code>noconstant</code>                                                                                                                                                                       | suppress constant term                                                                                                                           |
| <code>ll(# <i>varname</i>)</code>                                                                                                                                                             | truncation point; default value is ll(0), zero truncation                                                                                        |
| <code>dispersion(<u>m</u>ean)</code>                                                                                                                                                          | parameterization of dispersion; the default                                                                                                      |
| <code>dispersion(<u>c</u>onstant)</code>                                                                                                                                                      | constant dispersion for all observations                                                                                                         |
| <code>exposure(<i>varname</i><sub>e</sub>)</code>                                                                                                                                             | include ln( <i>varname</i> <sub>e</sub> ) in model with coefficient constrained to 1                                                             |
| <code>offset(<i>varname</i><sub>o</sub>)</code>                                                                                                                                               | include <i>varname</i> <sub>o</sub> in model with coefficient constrained to 1                                                                   |
| <code>constraints(<i>constraints</i>)</code>                                                                                                                                                  | apply specified linear constraints                                                                                                               |
| <b>SE/Robust</b>                                                                                                                                                                              |                                                                                                                                                  |
| <code>vce(<i>vcetype</i>)</code>                                                                                                                                                              | <i>vcetype</i> may be oim, <u>r</u> obust, <u>c</u> luster <i>clustvar</i> , opg, <u>b</u> ootstrap, or <u>j</u> ackknife                        |
| <b>Reporting</b>                                                                                                                                                                              |                                                                                                                                                  |
| <code>level(#)</code>                                                                                                                                                                         | set confidence level; default is level(95)                                                                                                       |
| <code>no<u>l</u>rtest</code>                                                                                                                                                                  | suppress likelihood-ratio test                                                                                                                   |
| <code><u>i</u>rr</code>                                                                                                                                                                       | report incidence-rate ratios                                                                                                                     |
| <code>no<u>c</u>nsreport</code>                                                                                                                                                               | do not display constraints                                                                                                                       |
| <code><i>display_options</i></code>                                                                                                                                                           | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| <b>Maximization</b>                                                                                                                                                                           |                                                                                                                                                  |
| <code><i>maximize_options</i></code>                                                                                                                                                          | control the maximization process; seldom used                                                                                                    |
| <code><u>c</u>ollinear</code>                                                                                                                                                                 | keep collinear variables                                                                                                                         |
| <code><u>c</u>oefflegend</code>                                                                                                                                                               | display legend instead of statistics                                                                                                             |
| <i>indepvars</i> may contain factor variables; see [U] 11.4.3 <b>Factor variables</b> .                                                                                                       |                                                                                                                                                  |
| <i>depvar</i> and <i>indepvars</i> may contain time-series operators; see [U] 11.4.4 <b>Time-series varlists</b> .                                                                            |                                                                                                                                                  |
| bayer, bayesboot, bootstrap, by, collect, fp, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 <b>Prefix commands</b> . For more details, see [BAYES] <b>bayer: tnbreg</b> . |                                                                                                                                                  |
| Weights are not allowed with the bootstrap prefix; see [R] <b>bootstrap</b> .                                                                                                                 |                                                                                                                                                  |
| vce() and weights are not allowed with the svy prefix; see [SVY] <b>svy</b> .                                                                                                                 |                                                                                                                                                  |
| fweights, iweights, and pweights are allowed; see [U] 11.1.6 <b>weight</b> .                                                                                                                  |                                                                                                                                                  |
| collinear and coefflegend do not appear in the dialog box.                                                                                                                                    |                                                                                                                                                  |
| See [U] 20 <b>Estimation and postestimation commands</b> for more capabilities of estimation commands.                                                                                        |                                                                                                                                                  |

## Options

Model

noconstant; see [R] **Estimation options**.

ll(#|*varname*) specifies the truncation point, which is a nonnegative integer. The default is zero truncation, ll(0).

`dispersion(mean | constant)` specifies the parameterization of the model. `dispersion(mean)`, the default, yields a model with dispersion equal to  $1 + \alpha \exp(\mathbf{x}_j\boldsymbol{\beta} + \text{offset}_j)$ ; that is, the dispersion is a function of the expected mean:  $\exp(\mathbf{x}_j\boldsymbol{\beta} + \text{offset}_j)$ . `dispersion(constant)` has dispersion equal to  $1 + \delta$ ; that is, it is a constant for all observations.

`exposure(varnamee)`, `offset(varnameo)`, `constraints(constraints)`; see [R] [Estimation options](#).

SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

Reporting

`level(#)`; see [R] [Estimation options](#).

`no1rtest` suppresses fitting the Poisson model. Without this option, a comparison Poisson model is fit, and the likelihood is used in a likelihood-ratio test of the null hypothesis that the dispersion parameter is zero.

`irr` reports estimated coefficients transformed to incidence-rate ratios, that is,  $e^{\beta_i}$  rather than  $\beta_i$ . Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated or stored. `irr` may be specified at estimation or when replaying previously estimated results.

`nocnsreport`; see [R] [Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `1tolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

Setting the optimization type to `technique(bhhh)` resets the default `vcetype` to `vce(opg)`.

The following options are available with `tnbreg` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Grogger and Carson (1991) showed that overdispersion causes inconsistent estimation of the mean in the truncated Poisson model. To solve this problem, they proposed using the truncated negative binomial model as an alternative. If data are truncated but do not exhibit overdispersion, the truncated Poisson model is more appropriate; see [R] [tpoisson](#). For an introduction to negative binomial regression, see Cameron and Trivedi (2005, 2022) and Long and Freese (2014). For an introduction to truncated negative binomial models, see Cameron and Trivedi (2013) and Long (1997, chap. 8).

tnbreg fits the mean-dispersion and the constant-dispersion parameterizations of truncated negative binomial models. These parameterizations extend those implemented in nbreg; see [R] nbreg.

▷ Example 1

We illustrate the truncated negative binomial model using the 1997 MedPar dataset (Hilbe 1999). The data are from 1,495 patients in Arizona who were assigned to a diagnostic-related group (DRG) of patients having a ventilator. Length of stay (los), the dependent variable, is a positive integer; it cannot have zero values. The data are truncated because there are no observations on individuals who stayed for zero days.

The objective of this example is to determine whether the length of stay was related to the binary variables: died, hmo, type1, type2, and type3.

The died variable was recorded as a 0 unless the patient died, in which case, it was recorded as a 1. The other variables also adopted this encoding. The hmo variable was set to 1 if the patient belonged to a health maintenance organization (HMO).

The type1–type3 variables indicated the type of admission used for the patient. The type1 variable indicated an emergency admit. The type2 variable indicated an urgent admit—that is, the first available bed. The type3 variable indicated an elective admission. Because type1–type3 were mutually exclusive, only two of the three could be used in the truncated negative binomial regression shown below.

```
. use https://www.stata-press.com/data/r19/medpar
(Arizona ventilator data)
. tnbreg los died hmo type2-type3, vce(cluster provnum) nolog

Truncated negative binomial regression Number of obs = 1,495
Truncation point = 0 Wald chi2(4) = 36.01
Dispersion: mean Prob > chi2 = 0.0000
Log pseudolikelihood = -4737.535 Pseudo R2 = 0.0139

(Std. err. adjusted for 54 clusters in provnum)
```

| los      | Robust      |           | z     | P> z  | [95% conf. interval] |           |
|----------|-------------|-----------|-------|-------|----------------------|-----------|
|          | Coefficient | std. err. |       |       |                      |           |
| died     | -.2521884   | .061533   | -4.10 | 0.000 | -.3727908            | -.1315859 |
| hmo      | -.0754173   | .0533132  | -1.41 | 0.157 | -.1799091            | .0290746  |
| type2    | .2685095    | .0666474  | 4.03  | 0.000 | .137883              | .3991359  |
| type3    | .7668101    | .2183505  | 3.51  | 0.000 | .338851              | 1.194769  |
| _cons    | 2.224028    | .034727   | 64.04 | 0.000 | 2.155964             | 2.292091  |
| /lnalpha | -.630108    | .0764019  |       |       | -.779853             | -.480363  |
| alpha    | .5325343    | .0406866  |       |       | .4584734             | .6185588  |

Because observations within the same hospital (provnum) are likely to be correlated, we specified the vce(cluster provnum) option. The results show that whether the patient died in the hospital and the type of admission have significant effects on the patient’s length of stay.

▷ Example 2

To illustrate truncated negative binomial regression with more complex data than the previous example, similar data were created from 100 hospitals. Each hospital had its own way of tracking patient data. In particular, hospitals only recorded data from patients with a minimum length of stay, denoted by the variable `minstay`.

Definitions for minimum length of stay varied among hospitals, typically, from 5 to 18 days. The objective of this example is the same as before: to determine whether the length of stay, recorded in `los`, was related to the binary variables: `died`, `hmo`, `type1`, `type2`, and `type3`.

The binary variables encode the same information as in [example 1](#) above. The `minstay` variable was used to allow for varying truncation points.

```
. use https://www.stata-press.com/data/r19/medproviders
. tnbreg los died hmo type2-type3, ll(minstay) vce(cluster hospital) nolog
Truncated negative binomial regression Number of obs = 2,144
Truncation points: minstay Wald chi2(4) = 15.22
Dispersion: mean Prob > chi2 = 0.0043
Log pseudolikelihood = -7864.0928 Pseudo R2 = 0.0007
 (Std. err. adjusted for 100 clusters in hospital)
```

| los      | Robust      |           | z      | P> z  | [95% conf. interval] |           |
|----------|-------------|-----------|--------|-------|----------------------|-----------|
|          | Coefficient | std. err. |        |       |                      |           |
| died     | .078104     | .0303598  | 2.57   | 0.010 | .0185998             | .1376081  |
| hmo      | -.0731132   | .0368897  | -1.98  | 0.047 | -.1454158            | -.0008107 |
| type2    | .0294132    | .0390166  | 0.75   | 0.451 | -.047058             | .1058845  |
| type3    | .0626348    | .0540123  | 1.16   | 0.246 | -.0432273            | .168497   |
| _cons    | 3.014964    | .0290895  | 103.64 | 0.000 | 2.95795              | 3.071978  |
| /lnalpha | -.996512    | .0828691  |        |       | -1.158932            | -.8340916 |
| alpha    | .3691649    | .0305923  |        |       | .313821              | .4342688  |

In this analysis, two variables have a statistically significant relationship with length of stay. On average, patients who died in the hospital had longer lengths of stay ( $p = 0.01$ ). Because the coefficient for HMO is negative, that is,  $b_{HMO} = -0.073$ , on average, patients who were insured by an HMO had shorter lengths of stay ( $p = 0.047$ ). The type of admission was not statistically significant ( $p > 0.05$ ).



## Stored results

tnbreg stores the following in `e()`:

### Scalars

|                            |                                                   |
|----------------------------|---------------------------------------------------|
| <code>e(N)</code>          | number of observations                            |
| <code>e(k)</code>          | number of parameters                              |
| <code>e(k_aux)</code>      | number of auxiliary parameters                    |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>          |
| <code>e(k_eq_model)</code> | number of equations in overall model test         |
| <code>e(k_dv)</code>       | number of dependent variables                     |
| <code>e(df_m)</code>       | model degrees of freedom                          |
| <code>e(r2_p)</code>       | pseudo- $R^2$                                     |
| <code>e(ll)</code>         | log likelihood                                    |
| <code>e(ll_0)</code>       | log likelihood, constant-only model               |
| <code>e(ll_c)</code>       | log likelihood, comparison model                  |
| <code>e(alpha)</code>      | value of alpha                                    |
| <code>e(delta)</code>      | value of delta                                    |
| <code>e(N_clust)</code>    | number of clusters                                |
| <code>e(chi2)</code>       | $\chi^2$                                          |
| <code>e(chi2_c)</code>     | $\chi^2$ for comparison test                      |
| <code>e(p)</code>          | $p$ -value for model test                         |
| <code>e(rank)</code>       | rank of <code>e(V)</code>                         |
| <code>e(rank0)</code>      | rank of <code>e(V)</code> for constant-only model |
| <code>e(ic)</code>         | number of iterations                              |
| <code>e(rc)</code>         | return code                                       |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise                       |

### Macros

|                            |                                                                                 |
|----------------------------|---------------------------------------------------------------------------------|
| <code>e(cmd)</code>        | tnbreg                                                                          |
| <code>e(cmdline)</code>    | command as typed                                                                |
| <code>e(depvar)</code>     | name of dependent variable                                                      |
| <code>e(llopt)</code>      | contents of <code>ll()</code> , or 0 if not specified                           |
| <code>e(wtype)</code>      | weight type                                                                     |
| <code>e(wexp)</code>       | weight expression                                                               |
| <code>e(title)</code>      | title in estimation output                                                      |
| <code>e(clustvar)</code>   | name of cluster variable                                                        |
| <code>e(offset)</code>     | linear offset variable                                                          |
| <code>e(chi2type)</code>   | Wald or LR; type of model $\chi^2$ test                                         |
| <code>e(chi2_ct)</code>    | Wald or LR; type of model $\chi^2$ test corresponding to <code>e(chi2_c)</code> |
| <code>e(dispers)</code>    | mean or constant                                                                |
| <code>e(vce)</code>        | <i>vcetype</i> specified in <code>vce()</code>                                  |
| <code>e(vcetype)</code>    | title used to label Std. err.                                                   |
| <code>e(opt)</code>        | type of optimization                                                            |
| <code>e(which)</code>      | max or min; whether optimizer is to perform maximization or minimization        |
| <code>e(ml_method)</code>  | type of ml method                                                               |
| <code>e(user)</code>       | name of likelihood-evaluator program                                            |
| <code>e(technique)</code>  | maximization technique                                                          |
| <code>e(properties)</code> | b V                                                                             |
| <code>e(predict)</code>    | program used to implement <code>predict</code>                                  |
| <code>e(asbalanced)</code> | factor variables <code>fvset</code> as <code>asbalanced</code>                  |
| <code>e(asobserved)</code> | factor variables <code>fvset</code> as <code>asobserved</code>                  |

### Matrices

|                              |                                              |
|------------------------------|----------------------------------------------|
| <code>e(b)</code>            | coefficient vector                           |
| <code>e(Cns)</code>          | constraints matrix                           |
| <code>e(ilog)</code>         | iteration log (up to 20 iterations)          |
| <code>e(gradient)</code>     | gradient vector                              |
| <code>e(V)</code>            | variance-covariance matrix of the estimators |
| <code>e(V_modelbased)</code> | model-based variance                         |

Functions  
 e(sample) marks estimation sample

In addition to the above, the following is stored in  $\mathbf{r}()$ :

Matrices  
 r(table) matrix containing the coefficients with their standard errors, test statistics,  $p$ -values, and confidence intervals

Note that results stored in  $\mathbf{r}()$  are updated when the command is replayed and will be replaced when any  $r$ -class command is run after the estimation command.

## Methods and formulas

Methods and formulas are presented under the following headings:

*Mean-dispersion model*  
*Constant-dispersion model*

### Mean-dispersion model

A negative binomial distribution can be regarded as a gamma mixture of Poisson random variables. The number of times an event occurs,  $y_j$ , is distributed as Poisson( $\nu_j \mu_j$ ). That is, its conditional likelihood is

$$f(y_j | \nu_j) = \frac{(\nu_j \mu_j)^{y_j} e^{-\nu_j \mu_j}}{\Gamma(y_j + 1)}$$

where  $\mu_j = \exp(\mathbf{x}_j \boldsymbol{\beta} + \text{offset}_j)$  and  $\nu_j$  is an unobserved parameter with a Gamma( $1/\alpha, \alpha$ ) density:

$$g(\nu) = \frac{\nu^{(1-\alpha)/\alpha} e^{-\nu/\alpha}}{\alpha^{1/\alpha} \Gamma(1/\alpha)}$$

This gamma distribution has a mean of 1 and a variance of  $\alpha$ , where  $\alpha$  is our ancillary parameter.

The unconditional likelihood for the  $j$ th observation is therefore

$$f(y_j) = \int_0^\infty f(y_j | \nu) g(\nu) d\nu = \frac{\Gamma(m + y_j)}{\Gamma(y_j + 1) \Gamma(m)} p_j^m (1 - p_j)^{y_j}$$

where  $p_j = 1/(1 + \alpha \mu_j)$  and  $m = 1/\alpha$ . Solutions for  $\alpha$  are handled by searching for  $\ln \alpha$  because  $\alpha$  must be greater than zero. The conditional probability of observing  $y_j$  events given that  $y_j$  is greater than the truncation point  $\tau_j$  is

$$\Pr(Y = y_j | y_j > \tau_j, \mathbf{x}_j) = \frac{f(y_j)}{\Pr(Y > \tau_j | \mathbf{x}_j)}$$

The log likelihood (with weights  $w_j$  and offsets) is given by

$$m = 1/\alpha \quad p_j = 1/(1 + \alpha \mu_j) \quad \mu_j = \exp(\mathbf{x}_j \boldsymbol{\beta} + \text{offset}_j)$$

$$\ln L = \sum_{j=1}^n w_j \left[ \ln\{\Gamma(m + y_j)\} - \ln\{\Gamma(y_j + 1)\} - \ln\{\Gamma(m)\} + m \ln(p_j) + y_j \ln(1 - p_j) - \ln\{\Pr(Y > \tau_j | p_j, m)\} \right]$$

## Constant-dispersion model

The constant-dispersion model assumes that  $y_j$  is conditionally distributed as Poisson( $\mu_j^*$ ), where  $\mu_j^* \sim \text{Gamma}(\mu_j/\delta, \delta)$  for some dispersion parameter  $\delta$  [by contrast, the mean-dispersion model assumes that  $\mu_j^* \sim \text{Gamma}(1/\alpha, \alpha\mu_j)$ ]. The log likelihood is given by

$$m_j = \mu_j/\delta \quad p = 1/(1 + \delta)$$

$$\ln L = \sum_{j=1}^n w_j \left[ \ln\{\Gamma(m_j + y_j)\} - \ln\{\Gamma(y_j + 1)\} - \ln\{\Gamma(m_j)\} + m_j \ln(p) + y_j \ln(1 - p) - \ln\{\Pr(Y > \tau_j | p, m_j)\} \right]$$

with everything else defined as shown above in the calculations for the mean-dispersion model.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] [\\_robust](#), particularly *Maximum likelihood estimators* and *Methods and formulas*.

`tnbreg` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] [Variance estimation](#).

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## Also see

[R] **tnbreg postestimation** — Postestimation tools for tnbreg

[R] **nbreg** — Negative binomial regression

[R] **poisson** — Poisson regression

[R] **tpoisson** — Truncated Poisson regression

[R] **zinb** — Zero-inflated negative binomial regression

[R] **zip** — Zero-inflated Poisson regression

[BAYES] **bayes: tnbreg** — Bayesian truncated negative binomial regression

[SVY] **svy estimation** — Estimation commands for survey data

[XT] **xtnbreg** — Fixed-effects, random-effects, & population-averaged negative binomial models

[U] **20 Estimation and postestimation commands**

## Postestimation commands

The following postestimation commands are available after `tnbreg`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | number of events, incidence rates, probabilities, etc.                                                                                  |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as numbers of events, incidence rates, conditional means, probabilities, conditional probabilities, linear predictions, and standard errors.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic nooffset]
```

```
predict [type] stub* [if] [in] , scores
```

| <i>statistic</i>      | Description                                                     |
|-----------------------|-----------------------------------------------------------------|
| <b>Main</b>           |                                                                 |
| <code>n</code>        | number of events; the default                                   |
| <code>ir</code>       | incidence rate                                                  |
| <code>cm</code>       | conditional mean, $E(y_j   y_j > \tau_j)$                       |
| <code>pr(n)</code>    | probability $\Pr(y_j = n)$                                      |
| <code>pr(a,b)</code>  | probability $\Pr(a \leq y_j \leq b)$                            |
| <code>cpr(n)</code>   | conditional probability $\Pr(y_j = n   y_j > \tau_j)$           |
| <code>cpr(a,b)</code> | conditional probability $\Pr(a \leq y_j \leq b   y_j > \tau_j)$ |
| <code>xb</code>       | linear prediction                                               |
| <code>stdp</code>     | standard error of the linear prediction                         |

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

## Options for predict

Main

`n`, the default, calculates the predicted number of events, which is  $\exp(\mathbf{x}_j\beta)$  if neither `offset()` nor `exposure()` was specified when the model was fit;  $\exp(\mathbf{x}_j\beta + \text{offset}_j)$  if `offset()` was specified; or  $\exp(\mathbf{x}_j\beta) \times \text{exposure}_j$  if `exposure()` was specified.

`ir` calculates the incidence rate  $\exp(\mathbf{x}_j\beta)$ , which is the predicted number of events when exposure is 1. This is equivalent to specifying both the `n` and the `nooffset` options.

`cm` calculates the conditional mean,

$$E(y_j | y_j > \tau_j) = \frac{E(y_j, y_j > \tau_j)}{\Pr(y_j > \tau_j)}$$

where  $\tau_j$  is the truncation point found in `e(1lopt)`.

`pr(n)` calculates the probability  $\Pr(y_j = n)$ , where  $n$  is a nonnegative integer that may be specified as a number or a variable.

`pr(a,b)` calculates the probability  $\Pr(a \leq y_j \leq b)$ , where  $a$  and  $b$  are nonnegative integers that may be specified as numbers or variables;

$b$  missing ( $b \geq .$ ) means  $+\infty$ ;

`pr(20,.)` calculates  $\Pr(y_j \geq 20)$ ;

`pr(20,b)` calculates  $\Pr(y_j \geq 20)$  in observations for which  $b \geq .$  and calculates  $\Pr(20 \leq y_j \leq b)$  elsewhere.

`pr(.,b)` produces a syntax error. A missing value in an observation of the variable  $a$  causes a missing value in that observation for `pr(a,b)`.

`cpr(n)` calculates the conditional probability  $\Pr(y_j = n \mid y_j > \tau_j)$ , where  $\tau_j$  is the truncation point found in `e(11opt)`.  $n$  is an integer greater than the truncation point that may be specified as a number or a variable.

`cpr(a,b)` calculates the conditional probability  $\Pr(a \leq y_j \leq b \mid y_j > \tau_j)$ , where  $\tau_j$  is the truncation point found in `e(11opt)`. The syntax for this option is analogous to that used for `pr(a,b)` except that  $a$  must be greater than the truncation point.

`xb` calculates the linear prediction, which is  $\mathbf{x}_j\boldsymbol{\beta}$  if neither `offset()` nor `exposure()` was specified when the model was fit;  $\mathbf{x}_j\boldsymbol{\beta} + \text{offset}_j$  if `offset()` was specified; or  $\mathbf{x}_j\boldsymbol{\beta} + \ln(\text{exposure}_j)$  if `exposure()` was specified; see `nooffset` below.

`stdp` calculates the standard error of the linear prediction.

`nooffset` is relevant only if you specified `offset()` or `exposure()` when you fit the model. It modifies the calculations made by `predict` so that they ignore the offset or exposure variable; the linear prediction is treated as  $\mathbf{x}_j\boldsymbol{\beta}$  rather than as  $\mathbf{x}_j\boldsymbol{\beta} + \text{offset}_j$  or  $\mathbf{x}_j\boldsymbol{\beta} + \ln(\text{exposure}_j)$ . Specifying `predict ..., nooffset` is equivalent to specifying `predict ..., ir`.

`scores` calculates equation-level score variables.

The first new variable will contain  $\partial \ln L / \partial (\mathbf{x}_j\boldsymbol{\beta})$ .

The second new variable will contain  $\partial \ln L / \partial (\ln \alpha)$  for `dispersion(mean)`.

The second new variable will contain  $\partial \ln L / \partial (\ln \delta)$  for `dispersion(constant)`.

# margins

## Description for margins

margins estimates margins of response for numbers of events, incidence rates, conditional means, probabilities, conditional probabilities, and linear predictions.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i> | Description                                                     |
|------------------|-----------------------------------------------------------------|
| n                | number of events; the default                                   |
| ir               | incidence rate                                                  |
| cm               | conditional mean, $E(y_j   y_j > \tau_j)$                       |
| pr(n)            | probability $\Pr(y_j = n)$                                      |
| pr(a,b)          | probability $\Pr(a \leq y_j \leq b)$                            |
| cpr(n)           | conditional probability $\Pr(y_j = n   y_j > \tau_j)$           |
| cpr(a,b)         | conditional probability $\Pr(a \leq y_j \leq b   y_j > \tau_j)$ |
| xb               | linear prediction                                               |
| stdp             | not allowed with margins                                        |

Statistics not allowed with margins are functions of stochastic quantities other than e(b).

For the full syntax, see [R] margins.

## Methods and formulas

In the following formulas, we use the same notation as in [R] tnbreg.

Methods and formulas are presented under the following headings:

- Mean-dispersion model*
- Constant-dispersion model*

## Mean-dispersion model

The equation-level scores are given by

$$\begin{aligned} \text{score}(\mathbf{x}\boldsymbol{\beta})_j &= p_j(y_j - \mu_j) - \frac{p_j^{(m+1)}\mu_j}{\Pr(Y > \tau_j | p_j, m)} \\ \text{score}(\omega)_j &= -m \left\{ \frac{\alpha(\mu_j - y_j)}{1 + \alpha\mu_j} - \ln(1 + \alpha\mu_j) + \psi(y_j + m) - \psi(m) \right\} \\ &\quad - \frac{p_j^m}{\Pr(Y > \tau_j | p_j, m)} \{m \ln(p_j) + \mu_j p_j\} \end{aligned}$$

where  $\omega_j = \ln\alpha_j$ ,  $\psi(z)$  is the digamma function, and  $\tau_j$  is the truncation point found in `e(11opt)`.

## Constant-dispersion model

The equation-level scores are given by

$$\begin{aligned} \text{score}(\mathbf{x}\boldsymbol{\beta})_j &= m_j \left\{ \psi(y_j + m_j) - \psi(m_j) + \ln(p) + \frac{p^{m_j} \ln(p)}{\Pr(Y > \tau_j | p, m_j)} \right\} \\ \text{score}(\omega)_j &= y_j - (y_j + m_j)(1 - p) - \text{score}(\mathbf{x}\boldsymbol{\beta})_j - \frac{\mu_j p}{\Pr(Y > \tau_j | p, m_j)} \end{aligned}$$

where  $\omega_j = \ln\delta_j$  and  $\tau_j$  is the truncation point found in `e(11opt)`.

## Also see

[R] [tnbreg](#) — Truncated negative binomial regression

[U] [20 Estimation and postestimation commands](#)

[Description](#)  
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## Description

`tobit` fits models for continuous responses where the outcome variable is censored. Censoring limits may be fixed for all observations or vary across observations.

## Quick start

Tobit regression of  $y$  on  $x_1$  and  $x_2$ , specifying that  $y$  is censored at the minimum of  $y$

```
tobit y x1 x2, ll
```

Same as above, but where the lower-censoring limit is zero

```
tobit y x1 x2, ll(0)
```

Same as above, but specify the lower- and upper-censoring limits

```
tobit y x1 x2, ll(17) ul(34)
```

Same as above, but where `lower` and `upper` are variables containing the censoring limits

```
tobit y x1 x2, ll(lower) ul(upper)
```

## Menu

Statistics > Linear models and related > Censored regression > Tobit regression

## Syntax

```
tobit depvar [indepvars] [if] [in] [weight] [, options]
```

| <i>options</i>                                                                                                                                                                                                                                                                                                                                                                                                                        | Description                                                                                                                                                          |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                                                                                                      |
| <code>noconstant</code>                                                                                                                                                                                                                                                                                                                                                                                                               | suppress constant term                                                                                                                                               |
| <code>ll[(varname) #]</code>                                                                                                                                                                                                                                                                                                                                                                                                          | left-censoring variable or limit                                                                                                                                     |
| <code>ul[(varname) #]</code>                                                                                                                                                                                                                                                                                                                                                                                                          | right-censoring variable or limit                                                                                                                                    |
| <code>offset(varname)</code>                                                                                                                                                                                                                                                                                                                                                                                                          | include <i>varname</i> in model with coefficient constrained to 1                                                                                                    |
| <code>constraints(constraints)</code>                                                                                                                                                                                                                                                                                                                                                                                                 | apply specified linear constraints                                                                                                                                   |
| <b>SE/Robust</b>                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                      |
| <code>vce(vcetype)</code>                                                                                                                                                                                                                                                                                                                                                                                                             | <i>vcetype</i> may be <code>oim</code> , <code>opg</code> , <code>robust</code> , <code>cluster clustvar</code> , <code>bootstrap</code> , or <code>jackknife</code> |
| <b>Reporting</b>                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                      |
| <code>level(#)</code>                                                                                                                                                                                                                                                                                                                                                                                                                 | set confidence level; default is <code>level(95)</code>                                                                                                              |
| <code>nocnsreport</code>                                                                                                                                                                                                                                                                                                                                                                                                              | do not display constraints                                                                                                                                           |
| <code>display_options</code>                                                                                                                                                                                                                                                                                                                                                                                                          | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling                     |
| <b>Maximization</b>                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                      |
| <code>maximize_options</code>                                                                                                                                                                                                                                                                                                                                                                                                         | control the maximization process; seldom used                                                                                                                        |
| <code>collinear</code>                                                                                                                                                                                                                                                                                                                                                                                                                | keep collinear variables                                                                                                                                             |
| <code>coeflegend</code>                                                                                                                                                                                                                                                                                                                                                                                                               | display legend instead of statistics                                                                                                                                 |
| <i>indepvars</i> may contain factor variables; see [U] 11.4.3 <b>Factor variables</b> .                                                                                                                                                                                                                                                                                                                                               |                                                                                                                                                                      |
| <i>depvar</i> and <i>indepvars</i> may contain time-series operators; see [U] 11.4.4 <b>Time-series varlists</b> .                                                                                                                                                                                                                                                                                                                    |                                                                                                                                                                      |
| <code>bayes</code> , <code>bayesboot</code> , <code>bootstrap</code> , <code>by</code> , <code>collect</code> , <code>fmm</code> , <code>fp</code> , <code>jackknife</code> , <code>nestreg</code> , <code>rolling</code> , <code>statsby</code> , <code>stepwise</code> , and <code>svy</code> are allowed; see [U] 11.1.10 <b>Prefix commands</b> . For more details, see [BAYES] <b>bayes: tobit</b> and [FMM] <b>fmm: tobit</b> . |                                                                                                                                                                      |
| Weights are not allowed with the <code>bootstrap</code> prefix; see [R] <b>bootstrap</b> .                                                                                                                                                                                                                                                                                                                                            |                                                                                                                                                                      |
| <code>aweight</code> s are not allowed with the <code>jackknife</code> prefix; see [R] <b>jackknife</b> .                                                                                                                                                                                                                                                                                                                             |                                                                                                                                                                      |
| <code>vce()</code> and <code>weights</code> are not allowed with the <code>svy</code> prefix; see [SVY] <b>svy</b> .                                                                                                                                                                                                                                                                                                                  |                                                                                                                                                                      |
| <code>aweight</code> s, <code>fweight</code> s, <code>iweight</code> s, and <code>pweight</code> s are allowed; see [U] 11.1.6 <b>weight</b> .                                                                                                                                                                                                                                                                                        |                                                                                                                                                                      |
| <code>collinear</code> and <code>coeflegend</code> do not appear in the dialog box.                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                      |
| See [U] 20 <b>Estimation and postestimation commands</b> for more capabilities of estimation commands.                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                      |

## Options

### Model

`noconstant`; see [R] **Estimation options**.

`ll[(varname) #]` and `ul[(varname) #]` indicate the lower and upper limits for censoring, respectively. Observations with  $depvar \leq ll()$  are left-censored; observations with  $depvar \geq ul()$  are right-censored; and remaining observations are not censored. You do not have to specify the censoring values. If you specify `ll`, the lower limit is the minimum of *depvar*. If you specify `ul`, the upper limit is the maximum of *depvar*.

`offset(varname)`, `constraints(constraints)`; see [R] **Estimation options**.

## SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

## Reporting

`level(#)`, `nocnsreport`; see [R] [Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

## Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

The following options are available with `tobit` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

`tobit` fits a linear regression model for a censored continuous outcome. Censoring occurs when the dependent variable is observed only within a certain range of values. When it is not, we know only that it is either above (right-censoring) or below (left-censoring) the censoring value. Censoring differs from truncation. When the data are truncated, we do not observe either the dependent variable or the covariates; see [R] [truncreg](#).

Censoring may result from study design or may be a result of how the outcome is measured. Right-censoring of data may occur, for example, in income surveys that top code the highest income category. Any respondent that earns the censoring limit or more reports only the value at the limit, and we do not know the respondent's true income. Left-censoring arises naturally when measurements are obtained from an instrument or a laboratory procedure that has a limit of detection. If we observe a value at the measurement limit, we know the true value is at the limit or below it. `tobit` allows the censoring limits to be the same for all observations or to vary from observation to observation.

[Tobin \(1958\)](#) originally conceived the tobit model as one of consumption of consumer durables where purchases were left-censored at zero. Contemporary literature treats this and similar cases as a corner solution model. See [Wooldridge \(2020, sec. 17.2\)](#), [Long \(1997, 196–210\)](#), and [Maddala and Lahiri \(2006, 333–336\)](#) for an introduction to the tobit model. [Wooldridge \(2010, chap. 17 and 19\)](#) provides an advanced treatment of censored regression models. [Cameron and Trivedi \(2022, chap. 19\)](#) discuss the tobit model using Stata examples.

The tobit model can be written as the latent regression model  $\mathbf{y} = \mathbf{x}\beta + \epsilon$  with a continuous outcome that is either observed or unobserved. Following Cong (2000), the observed outcome for observation  $i$  is defined as

$$y_i^* = \begin{cases} y_i & \text{if } a < y_i < b \\ a & \text{if } y_i \leq a \\ b & \text{if } y_i \geq b \end{cases}$$

where  $a$  is the lower-censoring limit and  $b$  is the upper-censoring limit. The tobit model assumes that the error term is normally distributed;  $\epsilon \sim N(\mathbf{0}, \sigma^2 \mathbf{I})$ . Depending on the problem at hand, the quantity of interest in a tobit model may be the censored outcome,  $y_i^*$ , or the uncensored outcome,  $y_i$ . In the measurement instrument scenario above, we may wish to predict the values that fall below the measurement threshold. By contrast, in the consumption of consumer durables scenario above, the latent variable is an artificial construct and the variable of interest is the observed consumer expenditure.

### ► Example 1: Constant-censoring limit

University administrators want to know the relationship between high school grade point average (GPA) and students' performance in college. `gpa.dta` contains fictional data on a cohort of 4,000 college students. College GPA (`gpa2`) and high school GPA (`hsgpa`) are measured on a continuous scale between zero and four. The outcome of interest is the student's college GPA. But, for reasons of confidentiality, GPAs below 2.0 are reported as 2.0. In other words, the outcome is censored on the left.

We believe that GPA is also a function of the logarithm of income of the student's parents (`pincome`) and whether or not the student participated in a study-skills program while in college (`program`).

```
. use https://www.stata-press.com/data/r19/gpa
(High school GPA and performance in college)
. tobit gpa2 hsgpa pincome program, ll
Refining starting values:
Grid node 0: Log likelihood = -2551.3989
Fitting full model:
Iteration 0: Log likelihood = -2551.3989
Iteration 1: Log likelihood = -2065.4023
Iteration 2: Log likelihood = -2015.8135
Iteration 3: Log likelihood = -2015.1281
Iteration 4: Log likelihood = -2015.1258
Iteration 5: Log likelihood = -2015.1258
Tobit regression
Limits: Lower = 2
 Upper = +inf
Number of obs = 4,000
 Uncensored = 2,794
 Left-censored = 1,206
 Right-censored = 0
LR chi2(3) = 4712.61
Prob > chi2 = 0.0000
Pseudo R2 = 0.5390
Log likelihood = -2015.1258
```

| gpa2        | Coefficient | Std. err. | t      | P> t  | [95% conf. interval] |           |
|-------------|-------------|-----------|--------|-------|----------------------|-----------|
| hsgpa       | .6586311    | .0128699  | 51.18  | 0.000 | .633399              | .6838632  |
| pincome     | .3159297    | .0074568  | 42.37  | 0.000 | .3013103             | .3305491  |
| program     | .5554416    | .0147468  | 37.67  | 0.000 | .5265297             | .5843535  |
| _cons       | -.8902578   | .0478484  | -18.61 | 0.000 | -.9840673            | -.7964482 |
| var(e.gpa2) | .161703     | .0044004  |        |       | .1533019             | .1705645  |

tobit reports the coefficients for the latent regression model. Thus, we can interpret the coefficients just as we would the coefficients from OLS. For example, participation in a study-skills program increases the expected uncensored GPA by 0.56 points.



▷ Example 2: Tobit model for a corner solution

Suppose that we are interested in the number of hours married women spend working for wages, and we treat observations recording zero hours as observed, per the corner-solution approach discussed [Wooldridge \(2010, chap. 16\)](#). We use the labor supply data extracted by [Mroz \(1987\)](#) from the 1975 PSID for 753 married women. The variable `whrs75` records the annual number of hours worked. Forty-three percent of the surveyed women worked zero hours, and the remaining women worked on average 1,303 hours a year.

We regress hours worked on household income excluding wife’s income (`nwinc`), years of schooling (`wedyrs`), years of labor market experience (`wexper`) and its square, age (`wifeage`), an indicator for the presence of children under 6 years of age at home (`k16`), and an indicator for the presence of children from 6 to 18 years old at home (`k618`).

```
. use https://www.stata-press.com/data/r19/mroz87
(1975 PSID data from Mroz, 1987)
. tobit whrs75 nwinc wedyrs wexper c.wexper#c.wexper wifeage k16 k618, ll(0)
Refining starting values:
Grid node 0: Log likelihood = -3961.1577
Fitting full model:
Iteration 0: Log likelihood = -3961.1577
Iteration 1: Log likelihood = -3836.8928
Iteration 2: Log likelihood = -3819.2637
Iteration 3: Log likelihood = -3819.0948
Iteration 4: Log likelihood = -3819.0946

Tobit regression
Limits: Lower = 0
 Upper = +inf
Log likelihood = -3819.0946

Number of obs = 753
Uncensored = 428
Left-censored = 325
Right-censored = 0
LR chi2(7) = 271.59
Prob > chi2 = 0.0000
Pseudo R2 = 0.0343
```

| whrs75        | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|---------------|-------------|-----------|-------|-------|----------------------|-----------|
| nwinc         | -8.814227   | 4.459089  | -1.98 | 0.048 | -17.56808            | -.0603708 |
| wedyrs        | 80.64541    | 21.58318  | 3.74  | 0.000 | 38.27441             | 123.0164  |
| wexper        | 131.564     | 17.27935  | 7.61  | 0.000 | 97.64211             | 165.486   |
| c.wexper#     |             |           |       |       |                      |           |
| c.wexper      | -1.864153   | .5376606  | -3.47 | 0.001 | -2.919661            | -.8086455 |
| wifeage       | -54.40491   | 7.418483  | -7.33 | 0.000 | -68.9685             | -39.84133 |
| k16           | -894.0202   | 111.8777  | -7.99 | 0.000 | -1113.653            | -674.3875 |
| k618          | -16.21805   | 38.6413   | -0.42 | 0.675 | -92.07668            | 59.64057  |
| _cons         | 965.3068    | 446.4351  | 2.16  | 0.031 | 88.88827             | 1841.725  |
| var(e.whrs75) | 1258927     | 93304.48  |       |       | 1088458              | 1456093   |

Unlike in [example 1](#), we are interested in the marginal effect of the covariates on the observed outcome. We can use [margins](#) to estimate, for example, the average marginal effect of years of education on the expected value of the actual hours worked.

```
. margins, dydx(wedyrs) predict(ystar(0,.))
Average marginal effects Number of obs = 753
Model VCE: OIM
Expression: E(whrs75*|whrs75>0), predict(ystar(0,.))
dy/dx wrt: wedyrs
```

|        | Delta-method |           |      |       |                      |
|--------|--------------|-----------|------|-------|----------------------|
|        | dy/dx        | std. err. | z    | P> z  | [95% conf. interval] |
| wedyrs | 47.47306     | 12.6214   | 3.76 | 0.000 | 22.73558 72.21054    |

The average marginal effect of years of education on the actual hours worked is 47.47. See [\[R\] tobit postestimation](#) for more examples using [margins](#).



[James Tobin](#) (1918–2002) was an American economist who after education and research at Harvard moved to Yale, where he was on the faculty from 1950 to 1988. He made many outstanding contributions to economics and was awarded the Nobel Prize in 1981 “for his analysis of financial markets and their relations to expenditure decisions, employment, production and prices”. He trained in the US Navy with the writer, Herman Wouk, who later fashioned a character after Tobin in the novel *The Caine Mutiny* (1951): “A mandarin-like midshipman named Tobit, with a domed forehead, measured quiet speech, and a mind like a sponge, was ahead of the field by a spacious percentage.”

## Stored results

tobit stores the following in `e()`:

### Scalars

|                         |                                          |
|-------------------------|------------------------------------------|
| <code>e(N)</code>       | number of observations                   |
| <code>e(N_unc)</code>   | number of uncensored observations        |
| <code>e(N_lc)</code>    | number of left-censored observations     |
| <code>e(N_rc)</code>    | number of right-censored observations    |
| <code>e(k)</code>       | number of parameters                     |
| <code>e(k_eq)</code>    | number of equations in <code>e(b)</code> |
| <code>e(k_aux)</code>   | number of auxiliary parameters           |
| <code>e(k_dv)</code>    | number of dependent variables            |
| <code>e(df_m)</code>    | model degrees of freedom                 |
| <code>e(df_r)</code>    | residual degrees of freedom              |
| <code>e(r2_p)</code>    | pseudo- $R^2$                            |
| <code>e(ll)</code>      | log likelihood                           |
| <code>e(ll_0)</code>    | log likelihood, constant-only model      |
| <code>e(N_clust)</code> | number of clusters                       |
| <code>e(chi2)</code>    | $\chi^2$                                 |
| <code>e(F)</code>       | $F$ statistic                            |
| <code>e(p)</code>       | $p$ -value for model test                |
| <code>e(rank)</code>    | rank of <code>e(V)</code>                |
| <code>e(ic)</code>      | number of iterations                     |

|                              |                                                                          |
|------------------------------|--------------------------------------------------------------------------|
| <code>e(rc)</code>           | return code                                                              |
| <code>e(converged)</code>    | 1 if converged, 0 otherwise                                              |
| Macros                       |                                                                          |
| <code>e(cmd)</code>          | <code>tobit</code>                                                       |
| <code>e(cmdline)</code>      | command as typed                                                         |
| <code>e(depvar)</code>       | name of dependent variable                                               |
| <code>e(llopt)</code>        | minimum of <i>depvar</i> or contents of <code>ll()</code>                |
| <code>e(ulopt)</code>        | maximum of <i>depvar</i> or contents of <code>ul()</code>                |
| <code>e(wtype)</code>        | weight type                                                              |
| <code>e(wexp)</code>         | weight expression                                                        |
| <code>e(covariates)</code>   | list of covariates                                                       |
| <code>e(title)</code>        | title in estimation output                                               |
| <code>e(clustvar)</code>     | name of cluster variable                                                 |
| <code>e(offset)</code>       | linear offset variable                                                   |
| <code>e(chi2type)</code>     | type of model $\chi^2$ test                                              |
| <code>e(vce)</code>          | <i>vcetype</i> specified in <code>vce()</code>                           |
| <code>e(vcetype)</code>      | title used to label Std. err.                                            |
| <code>e(opt)</code>          | type of optimization                                                     |
| <code>e(which)</code>        | max or min; whether optimizer is to perform maximization or minimization |
| <code>e(method)</code>       | estimation method: <code>m1</code>                                       |
| <code>e(m1_method)</code>    | type of <code>m1</code> method                                           |
| <code>e(user)</code>         | name of likelihood-evaluator program                                     |
| <code>e(technique)</code>    | maximization technique                                                   |
| <code>e(properties)</code>   | <code>b V</code>                                                         |
| <code>e(predict)</code>      | program used to implement <code>predict</code>                           |
| <code>e(marginsok)</code>    | predictions allowed by <code>margins</code>                              |
| <code>e(asbalanced)</code>   | factor variables <code>fvset</code> as <code>asbalanced</code>           |
| <code>e(asobserved)</code>   | factor variables <code>fvset</code> as <code>asobserved</code>           |
| Matrices                     |                                                                          |
| <code>e(b)</code>            | coefficient vector                                                       |
| <code>e(Cns)</code>          | constraints matrix                                                       |
| <code>e(iolog)</code>        | iteration log (up to 20 iterations)                                      |
| <code>e(gradient)</code>     | gradient vector                                                          |
| <code>e(V)</code>            | variance–covariance matrix of the estimators                             |
| <code>e(V_modelbased)</code> | model-based variance                                                     |
| Functions                    |                                                                          |
| <code>e(sample)</code>       | marks estimation sample                                                  |

In addition to the above, the following is stored in `r()`:

|                       |                                                                                                                            |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|
| Matrices              |                                                                                                                            |
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r`-class command is run after the estimation command.

## Methods and formulas

See *Methods and formulas* in [R] [intreg](#).

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] [\\_robust](#), particularly *Maximum likelihood estimators* and *Methods and formulas*.

`tobit` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] [Variance estimation](#).

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## Also see

- [R] **tobit postestimation** — Postestimation tools for tobit
- [R] **heckman** — Heckman selection model
- [R] **intreg** — Interval regression
- [R] **ivtobit** — Tobit model with continuous endogenous covariates
- [R] **regress** — Linear regression
- [R] **truncreg** — Truncated regression
- [BAYES] **bayes: tobit** — Bayesian tobit regression
- [FMM] **fmm: tobit** — Finite mixtures of tobit regression models
- [ERM] **eintreg** — Extended interval regression
- [ME] **metobit** — Multilevel mixed-effects tobit regression
- [SVY] **svy estimation** — Estimation commands for survey data
- [XT] **xtintreg** — Random-effects interval-data regression model
- [XT] **xttobit** — Random-effects tobit model
- [U] **20 Estimation and postestimation commands**

## Postestimation commands

The following postestimation commands are available after `tobit`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| <code>linktest</code>        | link test for model specification                                                                                                       |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | linear, censored, and truncated predictions                                                                                             |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as linear predictions, standard errors, probabilities, and expected values.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic nooffset]
```

```
predict [type] stub* [if] [in] , scores
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

Main

|                                       |                                              |
|---------------------------------------|----------------------------------------------|
| <code>xb</code>                       | linear prediction; the default               |
| <code>stdp</code>                     | standard error of the linear prediction      |
| <code>stdf</code>                     | standard error of the forecast               |
| <code>pr(<i>a</i>,<i>b</i>)</code>    | $\Pr(a < y_j < b)$                           |
| <code>e(<i>a</i>,<i>b</i>)</code>     | $E(y_j   a < y_j < b)$                       |
| <code>ystar(<i>a</i>,<i>b</i>)</code> | $E(y_j^*   y_j^* = \max\{a, \min(y_j, b)\})$ |

These statistics are available both in and out of sample; type `predict ... if e(sample)` ... if wanted only for the estimation sample.

`stdf` is not allowed with `svy` estimation results.

where *a* and *b* may be numbers or variables; *a* missing (*a* ≥ .) means  $-\infty$ , and *b* missing (*b* ≥ .) means  $+\infty$ ; see [U] 12.2.1 Missing values.

## Options for predict

Main

`xb`, the default, calculates the linear prediction.

`stdp` calculates the standard error of the prediction, which can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. The standard error of the prediction is also referred to as the standard error of the fitted value.

`stdf` calculates the standard error of the forecast, which is the standard error of the point prediction for 1 observation. It is commonly referred to as the standard error of the future or forecast value. By construction, the standard errors produced by `stdf` are always larger than those produced by `stdp`; see *Methods and formulas* in [R] [regress postestimation](#).

`pr(a,b)` calculates  $\Pr(a < \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j < b)$ , the probability that  $y_j|\mathbf{x}_j$  would be observed in the interval  $(a,b)$ .

$a$  and  $b$  may be specified as numbers or variable names;  $lb$  and  $ub$  are variable names;

`pr(20,30)` calculates  $\Pr(20 < \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j < 30)$ ;

`pr(lb,ub)` calculates  $\Pr(lb < \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j < ub)$ ; and

`pr(20,ub)` calculates  $\Pr(20 < \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j < ub)$ .

$a$  missing ( $a \geq .$ ) means  $-\infty$ ; `pr(.,30)` calculates  $\Pr(-\infty < \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j < 30)$ ;

`pr(lb,30)` calculates  $\Pr(-\infty < \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j < 30)$  in observations for which  $lb \geq .$

and calculates  $\Pr(lb < \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j < 30)$  elsewhere.

$b$  missing ( $b \geq .$ ) means  $+\infty$ ; `pr(20,.)` calculates  $\Pr(+\infty > \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j > 20)$ ;

`pr(20,ub)` calculates  $\Pr(+\infty > \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j > 20)$  in observations for which  $ub \geq .$

and calculates  $\Pr(20 < \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j < ub)$  elsewhere.

`e(a,b)` calculates  $E(\mathbf{x}_j\boldsymbol{\beta} + \epsilon_j | a < \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j < b)$ , the expected value of  $y_j|\mathbf{x}_j$  conditional on  $y_j|\mathbf{x}_j$  being in the interval  $(a,b)$ , meaning that  $y_j|\mathbf{x}_j$  is truncated.

$a$  and  $b$  are specified as they are for `pr()`.

`ystar(a,b)` calculates  $E(y_j^*)$ , where  $y_j^* = a$  if  $\mathbf{x}_j\boldsymbol{\beta} + \epsilon_j \leq a$ ,  $y_j^* = b$  if  $\mathbf{x}_j\boldsymbol{\beta} + \epsilon_j \geq b$ , and  $y_j^* = \mathbf{x}_j\boldsymbol{\beta} + \epsilon_j$  otherwise, meaning that  $y_j^*$  is censored.  $a$  and  $b$  are specified as they are for `pr()`.

`nooffset` is relevant only if you specified `offset(varname)`. It modifies the calculations made by `predict` so that they ignore the offset variable; the linear prediction is treated as  $\mathbf{x}_j\boldsymbol{\beta}$  rather than as  $\mathbf{x}_j\boldsymbol{\beta} + \text{offset}_j$ .

`scores` calculates equation-level score variables.

The first new variable will contain  $\partial \ln L / \partial (\mathbf{x}_j\boldsymbol{\beta})$ .

The second new variable will contain  $\partial \ln L / \partial \sigma$ .

# margins

## Description for margins

`margins` estimates margins of response for linear predictions, probabilities, and expected values.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i>                      | Description                                 |
|---------------------------------------|---------------------------------------------|
| <code>xb</code>                       | linear prediction; the default              |
| <code>pr(<i>a</i>,<i>b</i>)</code>    | $\Pr(a < y_j < b)$                          |
| <code>e(<i>a</i>,<i>b</i>)</code>     | $E(y_j   a < y_j < b)$                      |
| <code>ystar(<i>a</i>,<i>b</i>)</code> | $E(y_j^*), y_j^* = \max\{a, \min(y_j, b)\}$ |
| <code>stdp</code>                     | not allowed with <code>margins</code>       |
| <code>stdf</code>                     | not allowed with <code>margins</code>       |

Statistics not allowed with `margins` are functions of stochastic quantities other than `e(b)`.

For the full syntax, see [R] [margins](#).

## Remarks and examples

### ▷ Example 1: Marginal predictions

In [example 2](#) of [\[R\] tobit](#), we fit a tobit model of annual number of hours that married women spend working and then obtained estimated average marginal effect of 47.47 for years of education on observed hours worked.

```
. use https://www.stata-press.com/data/r19/mroz87
(1975 PSID data from Mroz, 1987)
. tobit whrs75 nwinc wedyrs wexper c.wexper#c.wexper wifeage k16 k618, ll(0)
(output omitted)
. margins, dydx(wedyrs) predict(ystar(0,.))
(output omitted)
```

However, we may not want this overall effect. To obtain marginal effects for specific alternative scenarios, we use `margins` with the `at()` option. For example, continuing with [example 2](#) of [\[R\] tobit](#) to estimate the means of the marginal effects on the expected value of the censored outcome conditional on education ranging from 8 years to 17 years, we type

```
. margins, dydx(wedyrs) predict(ystar(0,.)) at(wedyrs=(8(1)17))
Average marginal effects Number of obs = 753
Model VCE: OIM
Expression: E(whrs75*|whrs75>0), predict(ystar(0,.))
dy/dx wrt: wedyrs
1._at: wedyrs = 8
2._at: wedyrs = 9
3._at: wedyrs = 10
4._at: wedyrs = 11
5._at: wedyrs = 12
6._at: wedyrs = 13
7._at: wedyrs = 14
8._at: wedyrs = 15
9._at: wedyrs = 16
10._at: wedyrs = 17
```

|        |     | Delta-method |           | z    | P> z  | [95% conf. interval] |          |
|--------|-----|--------------|-----------|------|-------|----------------------|----------|
|        |     | dy/dx        | std. err. |      |       |                      |          |
| wedyrs |     |              |           |      |       |                      |          |
|        | _at |              |           |      |       |                      |          |
|        | 1   | 39.58775     | 8.432006  | 4.69 | 0.000 | 23.06132             | 56.11418 |
|        | 2   | 41.4497      | 9.421414  | 4.40 | 0.000 | 22.98407             | 59.91533 |
|        | 3   | 43.30531     | 10.41233  | 4.16 | 0.000 | 22.89752             | 63.71309 |
|        | 4   | 45.14859     | 11.39804  | 3.96 | 0.000 | 22.80885             | 67.48833 |
|        | 5   | 46.97371     | 12.37208  | 3.80 | 0.000 | 22.72489             | 71.22254 |
|        | 6   | 48.77504     | 13.32825  | 3.66 | 0.000 | 22.65216             | 74.89793 |
|        | 7   | 50.54717     | 14.26071  | 3.54 | 0.000 | 22.5967              | 78.49765 |
|        | 8   | 52.28499     | 15.16403  | 3.45 | 0.001 | 22.56403             | 82.00594 |
|        | 9   | 53.98369     | 16.03324  | 3.37 | 0.001 | 22.55912             | 85.40827 |
|        | 10  | 55.63887     | 16.8639   | 3.30 | 0.001 | 22.58624             | 88.6915  |

The estimated mean of the marginal effects is about 39.59 hours for 8 years of schooling, about 41.45 hours for 9 years of schooling, and so on.

## Reference

McDonald, J. F., and R. A. Moffitt. 1980. The use of tobit analysis. *Review of Economics and Statistics* 62: 318–321.  
<https://doi.org/10.2307/1924766>.

## Also see

[R] **tobit** — Tobit regression

[U] **20 Estimation and postestimation commands**

Description  
Options  
References

Quick start  
Remarks and examples  
Also see

Menu  
Stored results

Syntax  
Methods and formulas

## Description

`total` produces estimates of totals, along with standard errors.

## Quick start

Total of continuous variable `v1`

```
total v1
```

Same as above, but restrict estimation to observations where `catvar = 1`

```
total v1 if catvar==1
```

Same as above, but using `svyset` data

```
svy, subpop(if catvar==1): total v1
```

Total of `v1` for each level of `catvar`

```
total v1, over(catvar)
```

With jackknife standard errors

```
total v1, vce(jackknife)
```

## Menu

Statistics > Summaries, tables, and tests > Summary and descriptive statistics > Totals

## Syntax

```
total varlist [if] [in] [weight] [, options]
```

| <i>options</i>                                | Description                                                                                                                             |
|-----------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <i>if/in/over</i>                             |                                                                                                                                         |
| <code>over(<i>varlist</i><sub>o</sub>)</code> | group over subpopulations defined by <i>varlist</i> <sub>o</sub>                                                                        |
| SE/Cluster                                    |                                                                                                                                         |
| <code>vce(<i>vcetype</i>)</code>              | <i>vcetype</i> may be <code>analytic</code> , <code>cluster <i>clustvar</i></code> , <code>bootstrap</code> , or <code>jackknife</code> |
| Reporting                                     |                                                                                                                                         |
| <code>level(#)</code>                         | set confidence level; default is <code>level(95)</code>                                                                                 |
| <code>noheader</code>                         | suppress table header                                                                                                                   |
| <code>display_options</code>                  | control column formats, line width, display of omitted variables and base and empty cells, and factor-variable labeling                 |
| <code>coeflegend</code>                       | display legend instead of statistics                                                                                                    |

*varlist* may contain factor variables; see [U] 11.4.3 **Factor variables**.

`bayesboot`, `bootstrap`, `collect`, `jackknife`, `mi estimate`, `rolling`, `statsby`, and `svy` are allowed; see [U] 11.1.10 **Prefix commands**.

`vce(bootstrap)` and `vce(jackknife)` are not allowed with the `mi estimate` prefix.

Weights are not allowed with the `bootstrap` prefix; see [R] **bootstrap**.

`vce()` and weights are not allowed with the `svy` prefix; see [SVY] **svy**.

`fweights`, `iweights`, and `pweights` are allowed; see [U] 11.1.6 **weight**.

`coeflegend` does not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options

`if/in/over`

`over(varlisto)` specifies that estimates be computed for multiple subpopulations, which are identified by the different values of the variables in *varlist*<sub>o</sub>. Only numeric, nonnegative, integer-valued variables are allowed in `over(varlisto)`.

SE/Cluster

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`analytic`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] **vce\_option**.

`vce(analytic)`, the default, uses the analytically derived variance estimator associated with the sample total.

## Reporting

level(#); see [R] [Estimation options](#).

noheader prevents the table header from being displayed.

*display\_options*: `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

The following option is available with `total` but is not shown in the dialog box:

`coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

### ▷ Example 1

Suppose that we collected data on incidence of heart attacks. The variable `heartatk` indicates whether a person ever had a heart attack (1 means yes; 0 means no). We can then estimate the total number of persons who have had heart attacks for each sex in the population represented by the data we collected.

```
. use https://www.stata-press.com/data/r19/total
(Fictional incidence of heart-attack data)
. total heartatk [pw=swgt], over(sex)
```

Total estimation Number of obs = 4,946

|                | Total  | Std. err. | [95% conf. interval] |          |
|----------------|--------|-----------|----------------------|----------|
| c.heartatk@sex |        |           |                      |          |
| Male           | 944559 | 104372.3  | 739943               | 1149175  |
| Female         | 581590 | 82855.59  | 419156.3             | 744023.7 |

## Stored results

`total` stores the following in `e()`:

### Scalars

|                         |                                          |
|-------------------------|------------------------------------------|
| <code>e(N)</code>       | number of observations                   |
| <code>e(N_over)</code>  | number of subpopulations                 |
| <code>e(N_clust)</code> | number of clusters                       |
| <code>e(k_eq)</code>    | number of equations in <code>e(b)</code> |
| <code>e(df_r)</code>    | sample degrees of freedom                |
| <code>e(rank)</code>    | rank of <code>e(V)</code>                |

### Macros

|                              |                                                |
|------------------------------|------------------------------------------------|
| <code>e(cmd)</code>          | <code>total</code>                             |
| <code>e(cmdline)</code>      | command as typed                               |
| <code>e(varlist)</code>      | <i>varlist</i>                                 |
| <code>e(wtype)</code>        | weight type                                    |
| <code>e(wexp)</code>         | weight expression                              |
| <code>e(title)</code>        | title in estimation output                     |
| <code>e(clustvar)</code>     | name of cluster variable                       |
| <code>e(over)</code>         | <i>varlist</i> from <code>over()</code>        |
| <code>e(vce)</code>          | <i>vcetype</i> specified in <code>vce()</code> |
| <code>e(vcetype)</code>      | title used to label Std. err.                  |
| <code>e(properties)</code>   | <code>b V</code>                               |
| <code>e(estat_cmd)</code>    | program used to implement <code>estat</code>   |
| <code>e(marginsnotok)</code> | predictions disallowed by <code>margins</code> |

### Matrices

|                       |                                               |
|-----------------------|-----------------------------------------------|
| <code>e(b)</code>     | vector of total estimates                     |
| <code>e(V)</code>     | (co)variance estimates                        |
| <code>e(_N)</code>    | vector of numbers of nonmissing observations  |
| <code>e(error)</code> | error code corresponding to <code>e(b)</code> |

### Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

### Matrices

|                       |                                                                                                                            |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

Methods and formulas are presented under the following headings:

*The total estimator*  
*Survey data*  
*The survey total estimator*  
*The poststratified total estimator*  
*Subpopulation estimation*

## The total estimator

Let  $y$  denote the variable on which to calculate the total and  $y_j, j = 1, \dots, n$ , denote an individual observation on  $y$ . Let  $w_j$  be the frequency weight (or `weight` or `pweight`), and if no weight is specified, define  $w_j = 1$  for all  $j$ . The sum of the weights is an estimate of the population size:

$$\widehat{N} = \sum_{j=1}^n w_j$$

If the population values of  $y$  are denoted by  $Y_j, j = 1, \dots, N$ , the associated population total is

$$Y = \sum_{j=1}^N Y_j = N\bar{y}$$

where  $\bar{y}$  is the population mean. The total is estimated as

$$\widehat{Y} = \widehat{N}\bar{y}$$

The variance estimator for the total is

$$\widehat{V}(\widehat{Y}) = \widehat{N}^2 \widehat{V}(\bar{y})$$

where  $\widehat{V}(\bar{y})$  is the variance estimator for the mean; see [\[R\] mean](#). The standard error of the total is the square root of the variance.

If  $x, x_j, \bar{x}$ , and  $\widehat{X}$  are similarly defined for another variable (observed jointly with  $y$ ), the covariance estimator between  $\widehat{X}$  and  $\widehat{Y}$  is

$$\widehat{\text{Cov}}(\widehat{X}, \widehat{Y}) = \widehat{N}^2 \widehat{\text{Cov}}(\bar{x}, \bar{y})$$

where  $\widehat{\text{Cov}}(\bar{x}, \bar{y})$  is the covariance estimator between two means; see [\[R\] mean](#).

## Survey data

See [\[SVY\] Variance estimation](#) and [\[SVY\] Poststratification](#) for discussions that provide background information for the following formulas.

## The survey total estimator

Let  $Y_j$  be a survey item for the  $j$ th individual in the population, where  $j = 1, \dots, M$  and  $M$  is the size of the population. The associated population total for the item of interest is

$$Y = \sum_{j=1}^M Y_j$$

Let  $y_j$  be the survey item for the  $j$ th sampled individual from the population, where  $j = 1, \dots, m$  and  $m$  is the number of observations in the sample.

The estimator  $\hat{Y}$  for the population total  $Y$  is

$$\hat{Y} = \sum_{j=1}^m w_j y_j$$

where  $w_j$  is a sampling weight. The estimator for the number of individuals in the population is

$$\hat{M} = \sum_{j=1}^m w_j$$

The score variable for the total estimator is the variable itself,

$$z_j(\hat{Y}) = y_j$$

## The poststratified total estimator

Let  $P_k$  denote the set of sampled observations that belong to poststratum  $k$ , and define  $I_{P_k}(j)$  to indicate if the  $j$ th observation is a member of poststratum  $k$ , where  $k = 1, \dots, L_P$  and  $L_P$  is the number of poststrata. Also, let  $M_k$  denote the population size for poststratum  $k$ .  $P_k$  and  $M_k$  are identified by specifying the `poststrata()` and `postweight()` options on `svyset`; see [\[SVY\] svyset](#).

The estimator for the poststratified total is

$$\hat{Y}^P = \sum_{k=1}^{L_P} \frac{M_k}{\hat{M}_k} \hat{Y}_k = \sum_{k=1}^{L_P} \frac{M_k}{\hat{M}_k} \sum_{j=1}^m I_{P_k}(j) w_j y_j$$

where

$$\hat{M}_k = \sum_{j=1}^m I_{P_k}(j) w_j$$

The score variable for the poststratified total is

$$z_j(\hat{Y}^P) = \sum_{k=1}^{L_P} I_{P_k}(j) \frac{M_k}{\hat{M}_k} \left( y_j - \frac{\hat{Y}_k}{\hat{M}_k} \right)$$

## Subpopulation estimation

Let  $S$  denote the set of sampled observations that belong to the subpopulation of interest, and define  $I_S(j)$  to indicate if the  $j$ th observation falls within the subpopulation.

The estimator for the subpopulation total is

$$\hat{Y}^S = \sum_{j=1}^m I_S(j) w_j y_j$$

and its score variable is

$$z_j(\hat{Y}^S) = I_S(j) y_j$$

The estimator for the poststratified subpopulation total is

$$\hat{Y}^{PS} = \sum_{k=1}^{L_P} \frac{M_k}{\widehat{M}_k} \hat{Y}_k^S = \sum_{k=1}^{L_P} \frac{M_k}{\widehat{M}_k} \sum_{j=1}^m I_{P_k}(j) I_S(j) w_j y_j$$

and its score variable is

$$z_j(\hat{Y}^{PS}) = \sum_{k=1}^{L_P} I_{P_k}(j) \frac{M_k}{\widehat{M}_k} \left\{ I_S(j) y_j - \frac{\hat{Y}_k^S}{\widehat{M}_k} \right\}$$

## References

- Cochran, W. G. 1977. *Sampling Techniques*. 3rd ed. New York: Wiley.
- Stuart, A., and J. K. Ord. 1994. *Distribution Theory*. Vol. 1 of *Kendall's Advanced Theory of Statistics*, 6th ed. London: Arnold.

## Also see

- [R] **total postestimation** — Postestimation tools for total
- [R] **mean** — Estimate means
- [R] **proportion** — Estimate proportions
- [R] **ratio** — Estimate ratios
- [MI] **Estimation** — Estimation commands for use with mi estimate
- [SVY] **Direct standardization** — Direct standardization of means, proportions, and ratios
- [SVY] **Poststratification** — Poststratification for survey data
- [SVY] **Subpopulation estimation** — Subpopulation estimation for survey data
- [SVY] **svy estimation** — Estimation commands for survey data
- [SVY] **Variance estimation** — Variance estimation for survey data
- [U] **20 Estimation and postestimation commands**

## Postestimation commands

The following postestimation commands are available after `total`:

| Command                  | Description                                                                                       |
|--------------------------|---------------------------------------------------------------------------------------------------|
| <code>contrast</code>    | contrasts and ANOVA-style joint tests of parameters                                               |
| <code>estat vce</code>   | variance–covariance matrix of the estimators (VCE)                                                |
| <code>estat (svy)</code> | postestimation statistics for survey data                                                         |
| <code>estimates</code>   | cataloging estimation results                                                                     |
| <code>etable</code>      | table of estimation results                                                                       |
| <code>lincom</code>      | point estimates, standard errors, testing, and inference for linear combinations of parameters    |
| <code>marginsplot</code> | graph the results from <code>total</code>                                                         |
| <code>nlcom</code>       | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters |
| <code>pwcompare</code>   | pairwise comparisons of parameters                                                                |
| <code>test</code>        | Wald tests of simple and composite linear hypotheses                                              |
| <code>testnl</code>      | Wald tests of nonlinear hypotheses                                                                |

## Remarks and examples

### ► Example 1

Continuing with our data on incidence of heart attacks from [example 1](#) in [\[R\] total](#), we want to test whether there are twice as many heart attacks among men than women in the population.

```
. use https://www.stata-press.com/data/r19/total
(Fictional incidence of heart-attack data)
. total heartatk [pw=swgt], over(sex)
(output omitted)
. test heartatk@1.sex = 2 * heartatk@2.sex
(1) c.heartatk@1bn.sex - 2*c.heartatk@2.sex = 0
 F(1, 4945) = 1.25
 Prob > F = 0.2643
```

Thus we do not reject our hypothesis that the total number of heart attacks for men is twice that for women in the population.



## Also see

[\[R\] total](#) — Estimate totals

[\[U\] 20 Estimation and postestimation commands](#)

[Description](#)[Options](#)[Acknowledgment](#)[Quick start](#)[Remarks and examples](#)[References](#)[Menu](#)[Stored results](#)[Also see](#)[Syntax](#)[Methods and formulas](#)

## Description

`tpoisson` fits a truncated Poisson regression model when the number of occurrences of an event is restricted to be above a truncation point, below a truncation point, or between two truncation points. Truncated Poisson models are appropriate when neither the dependent variable nor the covariates are observed in the truncated part of the distribution. By default, `tpoisson` assumes left-truncation occurs at zero, but truncation may be specified at other fixed points or at values that vary across observations.

## Quick start

Truncated Poisson regression of `y` on `x1` and `x2` with left-truncation at 0

```
tpoisson y x1 x2
```

Add categorical variable `a` using [factor-variable](#) syntax

```
tpoisson y x1 x2 i.a
```

Same as above, but report incidence-rate ratios and use a constant truncation point of 4

```
tpoisson y x1 x2 i.a, irr ll(4)
```

With offset variable `lnexp`

```
tpoisson y x1 x2 i.a, offset(lnexp)
```

Same as above, but with a variable truncation point stored in variable `min`

```
tpoisson y x1 x2 i.a, offset(lnexp) ll(min)
```

With variable left- and right-truncation points

```
tpoisson y x1 x2, ll(min) ul(max)
```

With variable right-truncation points

```
tpoisson y x1 x2, ul(max)
```

Constrain the coefficients for 2. `a` and 3. `a` to equality

```
constraint define 1 2.a = 3.a
tpoisson y x1 x2 i.a, constraints(1)
```

## Menu

Statistics > Count outcomes > Truncated Poisson regression

## Syntax

```
tpoisson devar [indepvars] [if] [in] [weight] [, options]
```

| <i>options</i>                                    | Description                                                                                                                                                                 |
|---------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                      |                                                                                                                                                                             |
| <code>noconstant</code>                           | suppress constant term                                                                                                                                                      |
| <code>ll(# <i>varname</i>)</code>                 | lower limit for truncation; default is <code>ll(0)</code> when neither <code>ll()</code> nor <code>ul()</code> is specified                                                 |
| <code>ul(# <i>varname</i>)</code>                 | upper limit for truncation                                                                                                                                                  |
| <code>exposure(<i>varname</i><sub>e</sub>)</code> | include $\ln(\text{varname}_e)$ in model with coefficient constrained to 1                                                                                                  |
| <code>offset(<i>varname</i><sub>o</sub>)</code>   | include <i>varname</i> <sub>o</sub> in model with coefficient constrained to 1                                                                                              |
| <code>constraints(<i>constraints</i>)</code>      | apply specified linear constraints                                                                                                                                          |
| <b>SE/Robust</b>                                  |                                                                                                                                                                             |
| <code>vce(<i>vcetype</i>)</code>                  | <i>vcetype</i> may be <code>oim</code> , <code>robust</code> , <code>cluster <i>clustvar</i></code> , <code>opg</code> , <code>bootstrap</code> , or <code>jackknife</code> |
| <b>Reporting</b>                                  |                                                                                                                                                                             |
| <code>level(#)</code>                             | set confidence level; default is <code>level(95)</code>                                                                                                                     |
| <code>irr</code>                                  | report incidence-rate ratios                                                                                                                                                |
| <code>nocnsreport</code>                          | do not display constraints                                                                                                                                                  |
| <code>display_options</code>                      | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling                            |
| <b>Maximization</b>                               |                                                                                                                                                                             |
| <code>maximize_options</code>                     | control the maximization process; seldom used                                                                                                                               |
| <code>collinear</code>                            | keep collinear variables                                                                                                                                                    |
| <code>coeflegend</code>                           | display legend instead of statistics                                                                                                                                        |

*indepvars* may contain factor variables; see [U] 11.4.3 **Factor variables**.

*devar* and *indepvars* may contain time-series operators; see [U] 11.4.4 **Time-series varlists**.

`bayes`, `bayesboot`, `bootstrap`, `by`, `collect`, `fmm`, `fp`, `jackknife`, `rolling`, `statsby`, and `svy` are allowed; see [U] 11.1.10 **Prefix commands**. For more details, see [BAYES] `bayes: tpoisson` and [FMM] `fmm: tpoisson`.

Weights are not allowed with the `bootstrap` prefix; see [R] **bootstrap**.

`vce()` and weights are not allowed with the `svy` prefix; see [SVY] **svy**.

`fweights`, `iwweights`, and `pweights` are allowed; see [U] 11.1.6 **weight**.

`collinear` and `coeflegend` do not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options

### Model

`noconstant`; see [R] [Estimation options](#).

`ll(#|varname)` and `ul(#|varname)` specify the lower and upper limits for truncation, respectively. You may specify nonnegative integer values for one or both.

When neither `ll()` nor `ul()` is specified, the default is zero truncation, `ll(0)`, equivalent to left-truncation at zero.

`exposure(varnamee)`, `offset(varnameo)`, `constraints(constraints)`; see [R] [Estimation options](#).

### SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

### Reporting

`level(#)`; see [R] [Estimation options](#).

`irr` reports estimated coefficients transformed to incidence-rate ratios, that is,  $e^{\beta_i}$  rather than  $\beta_i$ . Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated. `irr` may be specified at estimation or when replaying previously estimated results.

`nocnsreport`; see [R] [Estimation options](#).

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] [Estimation options](#).

### Maximization

*maximize\_options*: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

Setting the optimization type to `technique(bhhh)` resets the default `vcetype` to `vce(opg)`.

The following options are available with `tpoisson` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

`tpoisson` fits a truncated Poisson regression model by maximum likelihood estimation when the number of occurrences of an event is restricted to be above a truncation point, below a truncation point, or between two truncation points. If the dependent variable is not truncated, standard Poisson regression may be more appropriate; see [R] [poisson](#).

When the data are truncated, we do not observe either the dependent variable or the covariates. For example, consider a study about the number of days that individuals with hyperglycemia are hospitalized after presenting to the hospital. If we select our sample only from admission records, then the sample is truncated at zero because we have data only on individuals who stayed at least one day. Now assume that we are relying on billing data and that hospitals may submit either a final bill when a patient is discharged or an interim bill every 30 days. In this case, we have no information about patients who are hospitalized fewer than 1 day or more than 30 days. Our data are left-truncated at 0 and right-truncated at 30.

A related phenomenon is censoring. For censored observations, we observe complete covariate information but only a censored value of the dependent variable. Different research designs can give rise to censored data or truncated data. See [R] [cpoisson](#) for information about censored Poisson regression.

Truncated Poisson regression was first proposed by [Grogger and Carson \(1991\)](#). For an introduction to Poisson regression, see [Cameron and Trivedi \(2005, 2022\)](#) and [Long and Freese \(2014\)](#). For an introduction to truncated Poisson models, see [Cameron and Trivedi \(2013\)](#) and [Long \(1997, chap. 8\)](#).

### ▷ Example 1: Left-truncation at zero

Consider the [Simonoff \(2003\)](#) dataset of running shoes for a sample of runners who registered an online running log. A running-shoe marketing executive is interested in knowing how the number of pairs of running shoes purchased relates to other factors such as gender, marital status, age, education, income, typical number of runs per week, average miles run per week, and the preferred type of running. These data are naturally truncated at zero. A truncated Poisson model is fit to the number of pairs of shoes owned on runs per week, miles run per week, gender, age, and marital status.

No options are needed because zero truncation is the default for `tpoisson`.

```
. use https://www.stata-press.com/data/r19/runshoes
(Running shoes)
. tpoisson shoes rpweek mpweek male age married
Iteration 0: Log likelihood = -88.328151
Iteration 1: Log likelihood = -86.272639
Iteration 2: Log likelihood = -86.257999
Iteration 3: Log likelihood = -86.257994

Truncated Poisson regression Number of obs = 60
Limits: lower = 0 LR chi2(5) = 22.75
 upper = +inf Prob > chi2 = 0.0004
Log likelihood = -86.257994 Pseudo R2 = 0.1165
```

| shoes   | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|---------|-------------|-----------|-------|-------|----------------------|----------|
| rpweek  | .1575811    | .1097893  | 1.44  | 0.151 | -.057602             | .3727641 |
| mpweek  | .0210673    | .0091113  | 2.31  | 0.021 | .0032094             | .0389252 |
| male    | .0446134    | .2444626  | 0.18  | 0.855 | -.4345246            | .5237513 |
| age     | .0185565    | .0137786  | 1.35  | 0.178 | -.008449             | .045562  |
| married | -.1283912   | .2785044  | -0.46 | 0.645 | -.6742498            | .4174674 |
| _cons   | -1.205844   | .6619774  | -1.82 | 0.069 | -2.503296            | .0916078 |

Using the zero-truncated Poisson regression with these data, only the coefficient on average miles per week is statistically significant at the 5% level.



## ▷ Example 2: Left-truncation with a fixed-truncation point

Semiconductor manufacturing requires that silicon wafers be coated with a layer of metal oxide. The depth of this layer is strictly controlled. In this example, a critical oxide layer is designed for  $300 \pm 20$  angstroms (Å).

After the oxide layer is coated onto a wafer, the wafer enters a photolithography step in which the lines representing the electrical connections are printed on the oxide and later etched and filled with metal. The widths of these lines are measured. In this example, they are controlled to  $90 \pm 5$  micrometers ( $\mu\text{m}$ ).

After these and other steps, each wafer is electrically tested at probe. If too many failures are discovered, the wafer is rejected and sent for engineering analysis. In this example, the maximum number of probe failures tolerated for this product is 10.

A major failure at probe has been encountered—88 wafers had more than 10 failures each. The 88 wafers that failed were tested using 4 probe machines. The engineer suspects that the failures were a result of faulty probe machines, poor depth control, or poor line widths. The line widths and depths in these data are the actual measurement minus its specification target,  $300 \text{ \AA}$  for the oxide depths and  $90 \mu\text{m}$  for the line widths.

The following table tabulates the average failure rate for each probe using Stata's `mean` command; see [R] [mean](#).

```
. use https://www.stata-press.com/data/r19/probe
(Silicon wafers)
. mean failures, over(probe)
```

| Mean estimation  |          | Number of obs = 88 |                      |          |
|------------------|----------|--------------------|----------------------|----------|
|                  | Mean     | Std. err.          | [95% conf. interval] |          |
| c.failures@probe |          |                    |                      |          |
| 1                | 15.875   | 1.186293           | 13.51711             | 18.23289 |
| 2                | 14.95833 | .5912379           | 13.78318             | 16.13348 |
| 3                | 16.47059 | .9279866           | 14.62611             | 18.31506 |
| 4                | 23.09677 | .9451117           | 21.21826             | 24.97529 |

The 95% confidence intervals in this table suggest that there are about 5–11 additional failures per wafer on probe 4. These are unadjusted for varying line widths and oxide depths. Possibly, probe 4 received the wafers with larger line widths or extreme oxide depths.

Truncated Poisson regression more clearly identifies the root causes for the increased failures by estimating the differences between probes adjusted for the line widths and oxide depths. It also allows us to determine whether the deviations from specifications in line widths or oxide depths might be contributing to the problem.

```
. tpoisson failures i.probe depth width, ll(10) nolog
Truncated Poisson regression Number of obs = 88
Limits: lower = 10 LR chi2(5) = 73.70
 upper = +inf Prob > chi2 = 0.0000
Log likelihood = -239.35746 Pseudo R2 = 0.1334
```

| failures | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |          |
|----------|-------------|-----------|-------|-------|----------------------|----------|
| probe    |             |           |       |       |                      |          |
| 2        | -.1113037   | .1019786  | -1.09 | 0.275 | -.3111781            | .0885707 |
| 3        | .0114339    | .1036032  | 0.11  | 0.912 | -.1916245            | .2144924 |
| 4        | .4254115    | .0841277  | 5.06  | 0.000 | .2605242             | .5902989 |
| depth    | -.0005034   | .0033375  | -0.15 | 0.880 | -.0070447            | .006038  |
| width    | .0330225    | .015573   | 2.12  | 0.034 | .0025001             | .063545  |
| _cons    | 2.714025    | .0752617  | 36.06 | 0.000 | 2.566515             | 2.861536 |

The coefficients listed for the probes are testing the null hypothesis:  $H_0 : \text{probe}_i = \text{probe}_1$ , where  $i$  equals 2, 3, and 4. Because the only coefficient that is statistically significant is the one for testing for  $H_0 : \text{probe}_4 = \text{probe}_1$ ,  $p < 0.001$ , and because the  $p$ -values for the other probes are not statistically significant, that is,  $p \geq 0.275$ , the implication is that there is a difference between probe 4 and the other machines. Because the coefficient for this test is positive, 0.425, the conclusion is that the average failure rate for probe 4, after adjusting for line widths and oxide depths, is higher than the other probes. Possibly, probe 4 needs calibration or the head used with this machine is defective.

Line-width control is statistically significant,  $p = 0.034$ , but variation in oxide depths is not causing the increased failure rate. The engineer concluded that the sudden increase in failures is the result of two problems. First, probe 4 is malfunctioning, and second, there is a possible lithography or etching problem.

## Stored results

tpoisson stores the following in `e()`:

### Scalars

|                            |                                           |
|----------------------------|-------------------------------------------|
| <code>e(N)</code>          | number of observations                    |
| <code>e(k)</code>          | number of parameters                      |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>  |
| <code>e(k_eq_model)</code> | number of equations in overall model test |
| <code>e(k_dv)</code>       | number of dependent variables             |
| <code>e(df_m)</code>       | model degrees of freedom                  |
| <code>e(r2_p)</code>       | pseudo- $R^2$                             |
| <code>e(ll)</code>         | log likelihood                            |
| <code>e(ll_0)</code>       | log likelihood, constant-only model       |
| <code>e(N_clust)</code>    | number of clusters                        |
| <code>e(chi2)</code>       | $\chi^2$                                  |
| <code>e(p)</code>          | $p$ -value for model test                 |
| <code>e(rank)</code>       | rank of <code>e(V)</code>                 |
| <code>e(ic)</code>         | number of iterations                      |
| <code>e(rc)</code>         | return code                               |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise               |

### Macros

|                            |                                                                                                      |
|----------------------------|------------------------------------------------------------------------------------------------------|
| <code>e(cmd)</code>        | tpoisson                                                                                             |
| <code>e(cmdline)</code>    | command as typed                                                                                     |
| <code>e(depvar)</code>     | name of dependent variable                                                                           |
| <code>e(llopt)</code>      | contents of <code>ll()</code> , or 0 if neither <code>ll()</code> nor <code>ul()</code> is specified |
| <code>e(ulopt)</code>      | contents of <code>ul()</code> , if specified                                                         |
| <code>e(wtype)</code>      | weight type                                                                                          |
| <code>e(wexp)</code>       | weight expression                                                                                    |
| <code>e(title)</code>      | title in estimation output                                                                           |
| <code>e(clustvar)</code>   | name of cluster variable                                                                             |
| <code>e(offset)</code>     | linear offset variable                                                                               |
| <code>e(chi2type)</code>   | Wald or LR; type of model $\chi^2$ test                                                              |
| <code>e(vce)</code>        | <i>vcetype</i> specified in <code>vce()</code>                                                       |
| <code>e(vcetype)</code>    | title used to label Std. err.                                                                        |
| <code>e(opt)</code>        | type of optimization                                                                                 |
| <code>e(which)</code>      | max or min; whether optimizer is to perform maximization or minimization                             |
| <code>e(ml_method)</code>  | type of ml method                                                                                    |
| <code>e(user)</code>       | name of likelihood-evaluator program                                                                 |
| <code>e(technique)</code>  | maximization technique                                                                               |
| <code>e(properties)</code> | b V                                                                                                  |
| <code>e(predict)</code>    | program used to implement predict                                                                    |
| <code>e(asbalanced)</code> | factor variables <code>fvset</code> as <code>asbalanced</code>                                       |
| <code>e(asobserved)</code> | factor variables <code>fvset</code> as <code>asobserved</code>                                       |

### Matrices

|                              |                                              |
|------------------------------|----------------------------------------------|
| <code>e(b)</code>            | coefficient vector                           |
| <code>e(Cns)</code>          | constraints matrix                           |
| <code>e(ilog)</code>         | iteration log (up to 20 iterations)          |
| <code>e(gradient)</code>     | gradient vector                              |
| <code>e(V)</code>            | variance–covariance matrix of the estimators |
| <code>e(V_modelbased)</code> | model-based variance                         |

### Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

|                       |                                                                                                                            |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|
| Matrices              |                                                                                                                            |
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

## Methods and formulas

For a nonnegative count outcome  $Y$  with left-truncation point  $ll_j$  and right-truncation point  $ul_j$ , we can write the truncated Poisson model as

$$f(y_j) = \frac{\exp(-\lambda_j)\lambda_j^{y_j}}{y_j! \Pr(ll_j < Y < ul_j | \xi_j)}$$

where

$$\begin{aligned}\xi_j &= \mathbf{x}_j\boldsymbol{\beta} + \text{offset}_j \\ \lambda_j &= \exp(\xi_j)\end{aligned}$$

and  $\mathbf{x}_j$  is a vector of observed covariates. The conditional probability of observing  $y_j$  events, therefore given by  $ll_j < y_j < ul_j$ , is

$$\Pr(Y = y_j | ll_j < y_j < ul_j, \mathbf{x}_j) = \frac{\exp(-\lambda_j)\lambda_j^{y_j}}{y_j! \Pr(ll_j < Y < ul_j | \mathbf{x}_j)}$$

The log likelihood is given by

$$\ln L = \sum_{j=1}^n w_j [-\lambda_j + \xi_j y_j - \ln(y_j!) - \ln \{ \Pr(ll_j < Y < ul_j | \xi_j) \}]$$

If no weights are specified,  $w_j = 1$ .

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] [\\_robust](#), particularly [Maximum likelihood estimators](#) and [Methods and formulas](#).

`tpoisson` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] [Variance estimation](#).

## Acknowledgment

We gratefully acknowledge the previous work by Joseph Hilbe (1944–2017) (1999), coauthor of the Stata Press book [Generalized Linear Models and Extensions](#).

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## Also see

- [R] **tpoisson postestimation** — Postestimation tools for `tpoisson`
- [R] **poisson** — Poisson regression
- [R] **nbreg** — Negative binomial regression
- [R] **tnbreg** — Truncated negative binomial regression
- [R] **zinb** — Zero-inflated negative binomial regression
- [R] **zip** — Zero-inflated Poisson regression
- [BAYES] **bayes: poisson** — Bayesian truncated Poisson regression
- [FMM] **fmm: poisson** — Finite mixtures of truncated Poisson regression models
- [SVY] **svy estimation** — Estimation commands for survey data
- [XT] **xtpoisson** — Fixed-effects, random-effects, and population-averaged Poisson models
- [U] **20 Estimation and postestimation commands**

Postestimation commands  
Methods and formulas

predict  
Also see

margins

Remarks and examples

## Postestimation commands

The following postestimation commands are available after `tpoisson`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | number of events, incidence rates, probabilities, etc.                                                                                  |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results.

# predict

## Description for predict

predict creates a new variable containing predictions such as numbers of events, incidence rates, conditional means, probabilities, conditional probabilities, linear predictions, standard errors, and the equation-level score.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic nooffset]
```

| <i>statistic</i>        | Description                                                                |
|-------------------------|----------------------------------------------------------------------------|
| <b>Main</b>             |                                                                            |
| <b>n</b>                | number of events; the default                                              |
| <b>ir</b>               | incidence rate                                                             |
| <b>cm</b>               | conditional mean, $E(y_j   ll_j < y_j < ul_j)$                             |
| <b>pr(<i>n</i>)</b>     | probability $\Pr(y_j = n)$                                                 |
| <b>pr(<i>a, b</i>)</b>  | probability $\Pr(a \leq y_j \leq b)$                                       |
| <b>cpr(<i>n</i>)</b>    | conditional probability $\Pr(y_j = n   ll_j < y_j < ul_j)$                 |
| <b>cpr(<i>a, b</i>)</b> | conditional probability $\Pr(a \leq y_j \leq b   ll_j < y_j < ul_j)$       |
| <b>xb</b>               | linear prediction                                                          |
| <b>stdp</b>             | standard error of the linear prediction                                    |
| <b>score</b>            | first derivative of the log likelihood with respect to $\mathbf{x}_j\beta$ |

These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

## Options for predict

### Main

**n**, the default, calculates the predicted number of events, which is  $\exp(\mathbf{x}_j\beta)$  if neither `offset()` nor `exposure()` was specified when the model was fit;  $\exp(\mathbf{x}_j\beta + \text{offset}_j)$  if `offset()` was specified; or  $\exp(\mathbf{x}_j\beta) \times \text{exposure}_j$  if `exposure()` was specified.

**ir** calculates the incidence rate  $\exp(\mathbf{x}_j\beta)$ , which is the predicted number of events when exposure is 1. This is equivalent to specifying both the **n** and the `nooffset` options.

**cm** calculates the conditional mean,

$$E(y_j | ll_j < y_j < ul_j) = \frac{E(y_j, ll_j < y_j < ul_j)}{\Pr(ll_j < y_j < ul_j)}$$

where  $ll_j$  is the left-truncation point specified at estimation and  $ul_j$  is the right-truncation point specified at estimation.

`pr(n)` calculates the probability  $\Pr(y_j = n)$ , where  $n$  is a nonnegative integer that may be specified as a number or a variable.

`pr(a,b)` calculates the probability  $\Pr(a \leq y_j \leq b)$ , where  $a$  and  $b$  are nonnegative integers that may be specified as numbers or variables;

$b$  missing ( $b \geq .$ ) means  $+\infty$ ;

`pr(20, .)` calculates  $\Pr(y_j \geq 20)$ ;

`pr(20,b)` calculates  $\Pr(y_j \geq 20)$  in observations for which  $b \geq .$  and calculates  $\Pr(20 \leq y_j \leq b)$  elsewhere.

`pr(., b)` produces a syntax error. A missing value in an observation of the variable  $a$  causes a missing value in that observation for `pr(a,b)`.

`cpr(n)` calculates the conditional probability  $\Pr(y_j = n \mid ll_j < y_j < ul_j)$ , where  $n$  is a nonnegative integer that may be specified as a number or a variable.  $ll_j$  and  $ul_j$  are as defined in `cm`.

`cpr(a,b)` calculates the conditional probability  $\Pr(a \leq y_j \leq b \mid ll_j < y_j < ul_j)$ , where  $a$  and  $b$  are as defined in `pr(a,b)` with the additional restrictions that  $a > ll_j$  and  $b < ul_j$ .  $ll_j$  and  $ul_j$  are as defined in `cm`.

`xb` calculates the linear prediction, which is  $\mathbf{x}_j\boldsymbol{\beta}$  if neither `offset()` nor `exposure()` was specified when the model was fit;  $\mathbf{x}_j\boldsymbol{\beta} + \text{offset}_j$  if `offset()` was specified; or  $\mathbf{x}_j\boldsymbol{\beta} + \ln(\text{exposure}_j)$  if `exposure()` was specified; see `nooffset` below.

`stdp` calculates the standard error of the linear prediction.

`score` calculates the equation-level score,  $\partial \ln L / \partial (\mathbf{x}_j\boldsymbol{\beta})$ .

`nooffset` is relevant only if you specified `offset()` or `exposure()` when you fit the model. It modifies the calculations made by `predict` so that they ignore the offset or exposure variable; the linear prediction is treated as  $\mathbf{x}_j\boldsymbol{\beta}$  rather than as  $\mathbf{x}_j\boldsymbol{\beta} + \text{offset}_j$  or  $\mathbf{x}_j\boldsymbol{\beta} + \ln(\text{exposure}_j)$ . Specifying `predict . . . , nooffset` is equivalent to specifying `predict . . . , ir`.

# margins

## Description for margins

margins estimates margins of response for numbers of events, incidence rates, conditional means, probabilities, conditional probabilities, and linear predictions.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i>   | Description                                                          |
|--------------------|----------------------------------------------------------------------|
| n                  | number of events; the default                                        |
| ir                 | incidence rate                                                       |
| cm                 | conditional mean, $E(y_j   ll_j < y_j < ul_j)$                       |
| pr( <i>n</i> )     | probability $\Pr(y_j = n)$                                           |
| pr( <i>a, b</i> )  | probability $\Pr(a \leq y_j \leq b)$                                 |
| cpr( <i>n</i> )    | conditional probability $\Pr(y_j = n   ll_j < y_j < ul_j)$           |
| cpr( <i>a, b</i> ) | conditional probability $\Pr(a \leq y_j \leq b   ll_j < y_j < ul_j)$ |
| xb                 | linear prediction                                                    |
| stdp               | not allowed with margins                                             |
| score              | not allowed with margins                                             |

Statistics not allowed with margins are functions of stochastic quantities other than e(b).

For the full syntax, see [R] margins.

## Remarks and examples

### ▷ Example 1: Obtaining margins of the conditional mean

In [example 1](#) of [\[R\] tpoisson](#), a truncated Poisson model is fit to the number of pairs of shoes owned on runs per week, miles run per week, gender, age, and marital status. We continue that example to determine the effect of miles run per week on the average number of pairs of shoes owned.

After reading in the data, we use [summarize](#) to obtain the 25th, 50th, and 75th percentiles for miles run per week.

```
. use https://www.stata-press.com/data/r19/runshoes
(Running shoes)
. summarize mpweek, detail
```

|     |      | Miles per week |          |             |          |
|-----|------|----------------|----------|-------------|----------|
|     |      | Percentiles    | Smallest |             |          |
| 1%  | 5    | 5              | 5        |             |          |
| 5%  | 5    | 5              | 5        |             |          |
| 10% | 5    | 5              | 5        | Obs         | 60       |
| 25% | 12.5 | 5              | 5        | Sum of wgt. | 60       |
| 50% | 27.5 |                |          | Mean        | 24.71167 |
|     |      |                | Largest  | Std. dev.   | 14.34934 |
| 75% | 32.5 | 47.5           | 47.5     |             |          |
| 90% | 47.5 | 47.5           | 47.5     | Variance    | 205.9034 |
| 95% | 47.5 | 47.5           | 47.5     | Skewness    | .1948568 |
| 99% | 57.5 | 57.5           | 57.5     | Kurtosis    | 2.065304 |

We fit the model from [example 1](#) of [\[R\] tpoisson](#) again. We next specify these values for the percentiles to margins to estimate the conditional mean of the number of pairs of shoes at different quantiles of miles run per week. To do this, we use the `at()` option of `margins`.

```
. quietly tpoisson shoes rpweek mpweek male age married
. margins, at(mpweek=(12.5 27.5 32.5)) predict(cm)
```

Predictive margins Number of obs = 60  
 Model VCE: OIM  
 Expression: Conditional mean of  $n > ll(0)$ , `predict(cm)`  
 1.\_at: mpweek = 12.5  
 2.\_at: mpweek = 27.5  
 3.\_at: mpweek = 32.5

|     | Delta-method |           |       |       |                      |
|-----|--------------|-----------|-------|-------|----------------------|
|     | Margin       | std. err. | z     | P> z  | [95% conf. interval] |
| _at |              |           |       |       |                      |
| 1   | 1.942149     | .2111564  | 9.20  | 0.000 | 1.52829 2.356008     |
| 2   | 2.376253     | .1714522  | 13.86 | 0.000 | 2.040213 2.712293    |
| 3   | 2.564339     | .1948129  | 13.16 | 0.000 | 2.182513 2.946165    |

We see that people who run 12.5 miles per week are expected to own 1.94 pairs of shoes. The expected number of pairs of shoes owned increases as the average miles per week increases. We expect people who run 27.5 miles per week have 2.38 pairs of shoes and those who run 32.5 miles per week have 2.56 pairs of shoes.

## Methods and formulas

Continuing from *Methods and formulas* in [R] **tpoisson**, the equation-level score is given by

$$\text{score}(\mathbf{x}\boldsymbol{\beta})_j = y_j - \lambda_j - \frac{\{\exp(-\lambda_j)\lambda_j^{ll_j}/ll_j! - \exp(-\lambda_j)\lambda_j^{(ul_j-1)}/(ul_j-1)!\}\lambda_j}{\Pr(ll_j < Y < ul_j | \xi_j)}$$

## Also see

[R] **tpoisson** — Truncated Poisson regression

[U] **20 Estimation and postestimation commands**

[Description](#)[Options for translate](#)[Quick start](#)[Remarks and examples](#)[Syntax](#)[Stored results](#)[Options for print](#)[Also see](#)

## Description

`print` prints log, SMCL, and text files. `translate` translates log and SMCL files from one format to another, the other typically being suitable for printing. `translate` can also translate SMCL logs to plain text.

## Quick start

Translate SMCL-format log `mylog.smcl` to a text log file `mylog.log`

```
translate mylog.smcl mylog.log
```

Same as above, but translate `mylog.smcl` to PostScript file `mylog.ps`

```
translate mylog.smcl mylog.ps
```

Send output from the Results window to new `mylog.txt` when no log was started

```
translate @Results mylog.txt
```

## Syntax

*Print log and SMCL files*

```
print filename [, like(ext) name(window) override_options]
```

*Translate log files to SMCL files and vice versa*

```
translate filenamein filenameout [, translator(tname) name(window)
override_options replace]
```

*View translator parameter settings*

```
translator query [tname]
```

*Change translator parameter settings*

```
translator set [tname setopt setval]
```

*Return translator parameter settings to default values*

```
translator reset tname
```

*List current mappings from one extension to another*

```
transmap query [.ext]
```

*Specify that files with one extension be treated the same as files with another extension*

```
transmap define .extnew .extold
```

*filename* in `print`, in addition to being a filename to be printed, may be specified as `@Results` to mean the Results window and `@Viewer` to mean the Viewer window.

*filename*<sub>in</sub> in `translate` may be specified just as *filename* in `print`.

*tname* in `translator` specifies the name of a translator; see the `translator()` option under *Options for translate*.

## Options for print

`like(ext)` specifies how the file should be translated to a form suitable for printing. The default is to determine the translation method from the extension of *filename*. Thus, `mylog.smcl` is translated according to the rule for translating `smcl` files, `myfile.txt` is translated according to the rule for translating `txt` files, and so on. (These rules are, in fact, `translate`'s `smcl2prn` and `txt2prn` translators, but put that aside for the moment.)

Rules for the following extensions are predefined:

|                    |                                           |
|--------------------|-------------------------------------------|
| <code>.txt</code>  | assume input file contains plain text     |
| <code>.log</code>  | assume input file contains Stata log text |
| <code>.smcl</code> | assume input file contains SMCL           |

To print a file that has an extension different from those listed above, you can define a new extension, but you do not have to do that. Assume that you wish to print the file `read.me`, which you know to contain plain text. If you were just to type `print read.me`, you would be told that Stata cannot translate `.me` files. (You would actually be told that the translator for `me2prn` was not found.) You could type `print read.me, like(txt)` to tell `print` to print `read.me` like a `.txt` file.

On the other hand, you could type

```
. transmap define .me .txt
```

to tell Stata that `.me` files are always to be treated like `.txt` files. If you did that, Stata would remember the new rule, even in future sessions.

When you specify the `like()` option, you override the recorded rules. So, if you were to type `print mylog.smcl, like(txt)`, the file would be printed as plain text (meaning that all the SMCL commands would show).

`name(windowname)` specifies which window to print when printing a Viewer. The default is for Stata to print the topmost Viewer [Unix(GUI) users: See the third [technical note](#) in *Printing files, Unix*]. The `name()` option is ignored when printing the Results window.

The window name is located inside the parentheses in the window title. For example, if the title for a Viewer window is `Viewer (#1) [help print]`, the name for the window is `#1`.

`override_options` refer to `translate`'s options for overriding default values. `print` uses `translate` to translate the file into a format suitable for sending to the printer, and thus `translate`'s `override_options` may also be used with `print`. The settings available vary between each translator (for example, `smcl2ps` will have different settings than `smcl2txt`) and may also differ across operating systems (for example, Windows may have different printing options than macOS). To find out what you can override when printing `.smcl` files, type

```
. translator query smcl2prn
(output omitted)
```

In the omitted output, you might learn that there is an `rmargin #` tunable value, which specifies the right margin in inches. You could specify the `override_option` `rmargin(#)` to temporarily override the default value, or you could type `translator set smcl2prn rmargin #` beforehand to permanently reset the value.

Alternatively, on some computers with some translators, you might discover that nothing can be set.

## Options for translate

`translator(tname)` specifies the name of the translator to be used to translate the file. The available translators are

| <i>tname</i>             | Input             | Output                 |
|--------------------------|-------------------|------------------------|
| <code>smcl2ps</code>     | SMCL              | PostScript             |
| <code>log2ps</code>      | Stata text log    | PostScript             |
| <code>txt2ps</code>      | generic text file | PostScript             |
| <code>Viewer2ps</code>   | Viewer window     | PostScript             |
| <code>Results2ps</code>  | Results window    | PostScript             |
| <code>smcl2prn</code>    | SMCL              | default printer format |
| <code>log2prn</code>     | Stata text log    | default printer format |
| <code>txt2prn</code>     | generic text log  | default printer format |
| <code>Results2prn</code> | Results window    | default printer format |
| <code>Viewer2prn</code>  | Viewer window     | default printer format |
| <code>smcl2txt</code>    | SMCL              | generic text log       |
| <code>smcl2log</code>    | SMCL              | Stata text log         |
| <code>Results2txt</code> | Results window    | generic text file      |
| <code>Viewer2txt</code>  | Viewer window     | generic text file      |
| <code>smcl2pdf</code>    | SMCL              | PDF                    |
| <code>log2pdf</code>     | Stata text log    | PDF                    |
| <code>txt2pdf</code>     | generic text log  | PDF                    |
| <code>Results2pdf</code> | Results window    | PDF                    |
| <code>Viewer2pdf</code>  | Viewer window     | PDF                    |

If `translator()` is not specified, `translate` determines which translator to use from extensions of the filenames specified. Typing `translate myfile.smcl myfile.ps` would use the `smcl2ps` translator. Typing `translate myfile.smcl myfile.ps, translate(smcl2prn)` would override the default and use the `smcl2prn` translator.

Actually, when you type `translate a.b c.d`, `translate` looks up `.b` in the `transmap` extension-synonym table. If `.b` is not found, the translator `b2d` is used. If `.b` is found in the table, the mapped extension is used (call it `b'`), and then the translator `b'2d` is used. For example,

| Command                                        | Translator used                                      |
|------------------------------------------------|------------------------------------------------------|
| <code>. translate myfile.smcl myfile.ps</code> | <code>smcl2ps</code>                                 |
| <code>. translate myfile.odd myfile.ps</code>  | <code>odd2ps</code> , which does not exist, so error |
| <code>. transmap define .odd .txt</code>       |                                                      |
| <code>. translate myfile.odd myfile.ps</code>  | <code>txt2ps</code>                                  |

You can list the mappings that `translate` uses by typing `transmap query`.

`name(windowname)` specifies which window to translate when translating a Viewer. The default is for Stata to translate the topmost Viewer. The `name()` option is ignored when translating the Results window.

The window name is located inside the parentheses in the window title. For example, if the title for a Viewer window is `Viewer (#1) [help print]`, the name for the window is `#1`.

`override_options` override any of the default options of the specified or implied translator. To find out what you can override for, say, `log2ps`, type

```
. translator query log2ps
```

In the omitted output, you might learn that there is an `rmargin #` tunable value, which, for `log2ps`, specifies the right margin in inches. You could specify the `override_option rmargin (#)` to temporarily override the default value or type `translator set log2ps rmargin #` beforehand to permanently reset the value.

`replace` specifies that *filename*<sub>out</sub> be replaced if it already exists.

## Remarks and examples

Remarks are presented under the following headings:

- [Overview](#)
- [Printing files](#)
- [Printing files, Mac and Windows](#)
- [Printing files, Unix](#)
- [Translating files from one format to another](#)

## Overview

`print` prints log, SMCL, and text files. Although there is considerable flexibility in how `print` (and `translate`, which `print` uses) can be set to work, they have already been set up and should just work:

```
. print mylog.smcl
. print mylog.log
```

Unix users may discover that they need to do a bit of setup before `print` works; see [Printing files, Unix](#) below. International Unix users may also wish to modify the default paper size. All users can tailor `print` and `translate` to their needs.

`print` may also be used to print the current contents of the Results window or the Viewer. For instance, the current contents of the Results window could be printed by typing

```
. print @Results
```

`translate` translates log and SMCL files from one format to another, the other typically being suitable for printing. `translate` can also translate SMCL logs (logs created by typing, say, log using `mylog`) to plain text:

```
. translate mylog.smcl mylog.log
```

You can use `translate` to recover a log when you have forgotten to start one. You may type

```
. translate @Results mylog.txt
```

to capture as plain text what is currently shown in the Results window.

This entry provides a general overview of `print` and `translate` and covers in detail the printing and translation of text (nongraphic) files.

`translator query`, `translator set`, and `translator reset` show, change, and restore the default values of the settings for each translator.

`transmap define` and `transmap query` create and show mappings from one file extension to another for use with `print` and `translate`.

For example, `print myfile.txt` knows to use a translator appropriate for printing text files because of the `.txt` extension. However, it does not know what to do with `.xyz` files. If you have `.xyz` files and always wish to treat them as `.txt` files, you can type `transmap define .xyz .txt`.

## Printing files

Printing should be easy; just type

```
. print mylog.smcl
. print mylog.log
```

You can use `print` to print SMCL files, plain text files, and even the contents of the Results and Viewer windows:

```
. print @Results
. print @Viewer
. print @Viewer, name(#2)
```

For information about printing and translating graph files, see [\[G-2\] graph print](#) and see [\[G-2\] graph export](#).

## Printing files, Mac and Windows

When you type `print`, you are using the same facility that you would be using if you had selected **Print** from the **File** menu. If you try to print a file that Stata does not know about, Stata will complain:

```
. print read.me
translator me2prn not found
r(111);
```

Then, you could type

```
. print read.me, like(txt)
```

to indicate that you wanted `read.me` sent to the printer in the same fashion as if the file were named `readme.txt`, or you could type

```
. transmap define .me .txt
. print read.me
```

Here you are telling Stata once and for all that you want files ending in `.me` to be treated in the same way as files ending in `.txt`. Stata will remember this mapping, even across sessions. To clear the `.me` mapping, type

```
. transmap define .me
```

To see all the mappings, type

```
. transmap query
```

To print to a file, use the `translate` command, not `print`:

```
. translate mylog.smcl mylog.prn
```

`translate` prints to a file by using the Windows print driver when the new filename ends in `.prn`. Under Mac, the `prn` translators are the same as the `pdf` translators. We suggest that you simply use the `.pdf` file extension when printing to a file.

## Printing files, Unix

Stata assumes that you have a PostScript printer attached to your Unix computer and that the Unix command `lpr(1)` can be used to send PostScript files to it, but you can change this. On your Unix system, typing

```
mycomputer$ lpr < filename
```

may not be sufficient to print PostScript files. For instance, perhaps on your system you would need to type

```
mycomputer$ lpr -Plexmark < filename
```

or

```
mycomputer$ lpr -Plexmark filename
```

or something else. To set the print command to be `lpr -Plexmark filename` and to state that the printer expects to receive PostScript files, type

```
. printer define prn ps "lpr -Plexmark @"
```

To set the print command to `lpr -Plexmark < filename` and to state that the printer expects to receive plain text files, type

```
. printer define prn txt "lpr -Plexmark < @"
```

That is, just type the command necessary to send files to your printer and include an `@` sign where the filename should be substituted. Two file formats are available: `ps` and `txt`. The default setting, as shipped from the factory, is

```
. printer define prn ps "lpr < @"
```

We will return to the `printer` command in the technical note that follows because it has some other capabilities you should know about.

In any case, after you redefine the default printer, the following should just work:

```
. print mylog.smcl
. print mylog.log
```

If you try to print a file that Stata does not know about, it will complain:

```
. print read.me
translator me2prn not found
r(111);
```

Here you could type

```
. print read.me, like(txt)
```

to indicate that you wanted `read.me` sent to the printer in the same fashion as if the file were named `readme.txt`, or you could type

```
. transmap define .me .txt
. print read.me
```

Here you are telling Stata once and for all that you want files ending in `.me` to be treated in the same way as files ending in `.txt`. Stata will remember this setting for `.me`, even across sessions.

If you want to clear the `.me` setting, type

```
. transmap define .me
```

If you want to see all your settings, type

```
. transmap query
```

## □ Technical note

If the text you wish to print contains Unicode characters, those characters may not appear correctly in PostScript files because the PostScript fonts do not support Unicode. Stata will map as many characters as possible to characters supported by Unicode but will print a question mark (?) for any unsupported character. We recommend that you export the text to a PDF file, which has fonts with better support for Unicode characters.

□

## □ Technical note

The syntax of the `printer` command is

```
printer define printername [{ ps | txt } "Unix command with @"]
printer query [printername]
```

You may define multiple printers. By default, `print` uses the printer named `prn`, but `print` has the syntax

```
print filename [, like(ext) printer(printername) override_options]
```

so, if you define multiple printers, you may route your output to them.

For instance, if you have a second printer on your system, you might type

```
. printer define lexmark ps "lpr -Plexmark < @"
```

After doing that, you could type

```
. print myfile.smcl, printer(lexmark)
```

Any printers that you set will be remembered even across sessions. You can delete printers:

```
. printer define lexmark
```

You can list all the defined printers by typing `printer query`, and you can list the definition of a particular printer, say, `prn`, by typing `printer query prn`.

The default printer `prn` we have predefined for you is

```
. printer define prn ps "lpr < @"
```

meaning that we assume that it is a PostScript printer and that the Unix command `lpr(1)`, without options, is sufficient to cause files to print. Feel free to change the default definition. If you change it, the change will be remembered across sessions.

□

## □ Technical note

Unix(GUI) users should note that X-Windows does not have the concept of a window z-order, which prevents Stata from determining which window is the topmost window. Instead, Stata determines which window is topmost based on which window has the focus. However, some window managers will set the focus to a window without bringing the window to the top. What Stata considers the topmost window may not appear topmost visually. For this reason, you should always use the `name()` option to ensure that the correct window is printed. □

## □ Technical note

When you select the Results window to print from the **Print** menu or toolbar button, the result is the same as if you were to issue the `print` command. When you select a Viewer window to print from the **Print** menu or toolbar button, the result is the same as if you were to issue the `print` command with a `name()` option. □

The translation to PostScript format is done by `translate` and, in particular, is performed by the translators `smcl2ps`, `log2ps`, and `txt2ps`. There are many tunable parameters in each of these translators. You can display the current values of these tunable parameters for, say, `smcl2ps` by typing

```
. translator query smcl2ps
(output omitted)
```

and you can set any of the tunable parameters (for instance, setting `smcl2ps`'s `rmargin` value to 1) by typing

```
. translator set smcl2ps rmargin 1
(output omitted)
```

Any settings you make will be remembered across sessions. You can reset `smcl2ps` to be as it was when Stata was shipped by typing

```
. translator reset smcl2ps
```

## Translating files from one format to another

If you have a SMCL log, which you might have created by previously typing `log using mylog`, you can translate it to a text log by typing

```
. translate myfile.smcl myfile.log
```

and you can translate it to a PostScript file by typing

```
. translate myfile.smcl myfile.ps
```

`translate` translates files from one format to another, and, in fact, `print` uses `translate` to produce a file suitable for sending to the printer.

When you type

```
. translate a.b c.d
```

`translate` looks for the predefined translator `b2d` and uses that to perform the translation. If there is a `transmap` synonym for `b`, however, the mapped value `b'` is used: `b'2d`.

Only certain translators exist, and they are listed under the description of the `translate()` option in *Options for translate* above, or you can type

```
. translator query
```

for a complete (and perhaps more up-to-date) list.

Anyway, `translate` forms the name *b2d* or *b'2d*, and if the translator does not exist, `translate` issues an error message. With the `translator()` option, you can specify exactly which translator to use, and then it does not matter how your files are named.

The only other thing to know is that some translators have tunable parameters that affect how they perform their translation. You can type

```
. translator query translator_name
```

to find out what those parameters are. Some translators have no tunable parameters, and some have many:

```
. translator query smcl2ps
```

|                        |            |                        |         |
|------------------------|------------|------------------------|---------|
| header                 | on         |                        |         |
| headertext             |            |                        |         |
| logo                   | on         |                        |         |
| user                   |            |                        |         |
| projecttext            |            |                        |         |
| cmdnumber              | on         |                        |         |
| fontsize               | 9          | lmargin                | 1.00    |
| pagesize               | letter     | rmargin                | 1.00    |
| pagewidth              | 8.50       | tmargin                | 1.00    |
| pageheight             | 11.00      | bmargin                | 1.00    |
| scheme                 | monochrome |                        |         |
| cust1_result_color     | 0 0 0      | cust2_result_color     | 0 0 0   |
| cust1_standard_color   | 0 0 0      | cust2_standard_color   | 0 0 0   |
| cust1_error_color      | 0 0 0      | cust2_error_color      | 255 0 0 |
| cust1_input_color      | 0 0 0      | cust2_input_color      | 0 0 0   |
| cust1_link_color       | 0 0 0      | cust2_link_color       | 0 0 255 |
| cust1_hilite_color     | 0 0 0      | cust2_hilite_color     | 0 0 0   |
| cust1_result_bold      | on         | cust2_result_bold      | on      |
| cust1_standard_bold    | off        | cust2_standard_bold    | off     |
| cust1_error_bold       | on         | cust2_error_bold       | on      |
| cust1_input_bold       | off        | cust2_input_bold       | off     |
| cust1_link_bold        | off        | cust2_link_bold        | off     |
| cust1_hilite_bold      | on         | cust2_hilite_bold      | on      |
| cust1_link_underline   | on         | cust2_link_underline   | on      |
| cust1_hilite_underline | off        | cust2_hilite_underline | off     |

You can temporarily override any setting by specifying the *setopt(setval)* option on the `translate` (or `print`) command. For instance, you can type

```
. translate ..., ... cmdnumber(off)
```

or you can reset the value permanently by typing

```
. translator set smcl2ps setopt setval
```

For instance,

```
. translator set smcl2ps cmdnumber off
```

If you reset a value, Stata will remember the change, even in future sessions.

Mac and Windows users: The `smcl2ps` (and the other `*2ps` translators) are not used by `print`, even when you have a PostScript printer attached to your computer. Instead, the Mac or Windows print driver is used. Resetting `smcl2ps` values will not affect printing; instead, you change the defaults in the Printers Control Panel in Windows and by selecting **Page Setup...** from the **File** menu in Mac. You can, however, `translate` files yourself using the `smcl2ps` translator and the other `*2ps` translators.

## □ Technical note: Using PDF translators (Windows and Unix)

When using `translate` to translate a log, SMCL, or text file or a Stata graph into a PDF file, some characters may not display correctly in the resulting PDF file. This happens when translating content that contains Unicode characters from different languages; one font may not be adequate to display every character used. If an appropriate font is not available for a character, that character will not display correctly in the resulting PDF file.

Specific fonts can be used to build a font chain. Those fonts are specified using *fontname* and are separated by commas. The most preferred font should be listed first, followed by the less preferred fonts. A maximum of 16 fonts can be specified. Stata will inspect each character in the original content, and if a glyph for the corresponding character is not available in the default font, then each font in the font chain will be checked until a font is found to display that character.

You can set fonts in the font chain by specifying `addfonts` for each PDF translator as follows:

```
. translator set tname addfonts fontname
```

Here *tname* is the name of a PDF translator, which can be `smcl2pdf`, `log2pdf`, `txt2pdf`, `Results2pdf`, `Viewer2pdf`, or `Graph2pdf`. See the `translator()` option under *Options for translate*.

Stata has the ability to select a font automatically if the default font and the fonts in the font chain do not contain a glyph for the character. Automatic font selection can be controlled using the following:

```
. translator set tname autofont on|off
```

By default, `autofont` is on. Unix (console) users will need to set the font directories so the fonts can be found.

PDF files accept Base 14 fonts, Type 1 fonts (.pfa or .pfb), TrueType fonts (.ttf or .ttc), and OpenType fonts (.otf). If *fontname* refers to a TrueType font, an OpenType font, or a Type 1 font, the corresponding TrueType, OpenType, or Type 1 font file will be searched for within the default font directory. Unix (console) users can type

```
. translator set tname fontdir fontdirectory
```

to set the default font directory. *fontdirectory* can contain multiple font directories separated by semi-colons. If the specified TrueType, OpenType, or Type 1 font cannot be found under the current font directory, an error message will be issued.

□

## Stored results

`transmap query .ext` stores in macro `r(suffix)` the mapped extension (without the leading period) or stores `ext` if the `ext` is not mapped.

`translator query translatorname` stores `setval` in macro `r(setopt)` for every `setopt, setval` pair.

`printer query printername` (Unix only) stores in macro `r(suffix)` the “filetype” of the input that the printer expects (currently “ps” or “txt”) and, in macro `r(command)`, the command to send output to the printer.

## Also see

[R] [log](#) — Echo copy of session to file

[G-2] [graph export](#) — Export current graph

[G-2] [graph print](#) — Print a graph

[G-2] [graph set](#) — Set graphics options

[P] [smcl](#) — Stata Markup and Control Language

[U] [15 Saving and printing output—log files](#)

[Description](#)  
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[Menu](#)  
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[Syntax](#)  
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## Description

`truncreg` fits a regression model of *depvar* on *indepvars* from a sample drawn from a restricted part of the population. Under the normality assumption for the whole population, the error terms in the truncated regression model have a truncated normal distribution, which is a normal distribution that has been scaled upward so that the distribution integrates to one over the restricted range.

## Quick start

Truncated regression of *y* on *x1* and *x2* truncated below 16

```
truncreg y x1 x2, ll(16)
```

Specify that *y* is truncated above 35

```
truncreg y x1 x2, ul(35)
```

With *y* truncated below 17 and above 35

```
truncreg y x1 x2, ll(17) ul(35)
```

Specify a lower truncation point that varies across observations using the variable `trunc`

```
truncreg y x1 x2, ll(trunc)
```

Same as above, but with bootstrap standard errors using 200 replications

```
truncreg y x1 x2, ll(trunc) vce(bootstrap, reps(200))
```

See last estimates with legend of coefficient names instead of statistics

```
truncreg, coeflegend
```

## Menu

Statistics > Linear models and related > Truncated regression

## Syntax

```
truncreg deprvar [indepvars] [if] [in] [weight] [, options]
```

| <i>options</i>                               | Description                                                                                                                                                                 |
|----------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                 |                                                                                                                                                                             |
| <code>noconstant</code>                      | suppress constant term                                                                                                                                                      |
| <code>ll(<i>varname</i>   #)</code>          | left-truncation variable or limit                                                                                                                                           |
| <code>ul(<i>varname</i>   #)</code>          | right-truncation variable or limit                                                                                                                                          |
| <code>offset(<i>varname</i>)</code>          | include <i>varname</i> in model with coefficient constrained to 1                                                                                                           |
| <code>constraints(<i>constraints</i>)</code> | apply specified linear constraints                                                                                                                                          |
| <b>SE/Robust</b>                             |                                                                                                                                                                             |
| <code>vce(<i>vcetype</i>)</code>             | <i>vcetype</i> may be <code>oim</code> , <code>robust</code> , <code>cluster <i>clustvar</i></code> , <code>opg</code> , <code>bootstrap</code> , or <code>jackknife</code> |
| <b>Reporting</b>                             |                                                                                                                                                                             |
| <code>level(#)</code>                        | set confidence level; default is <code>level(95)</code>                                                                                                                     |
| <code>lrmodel</code>                         | perform the likelihood-ratio model test instead of the default Wald test                                                                                                    |
| <code>nocnsreport</code>                     | do not display constraints                                                                                                                                                  |
| <code>display_options</code>                 | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling                            |
| <b>Maximization</b>                          |                                                                                                                                                                             |
| <code>maximize_options</code>                | control the maximization process; seldom used                                                                                                                               |
| <code>collinear</code>                       | keep collinear variables                                                                                                                                                    |
| <code>coeflegend</code>                      | display legend instead of statistics                                                                                                                                        |

*indepvars* may contain factor variables; see [U] 11.4.3 **Factor variables**.

*deprvar* and *indepvars* may contain time-series operators; see [U] 11.4.4 **Time-series varlists**.

`bayes`, `bootstrap`, `by`, `collect`, `fmm`, `fp`, `jackknife`, `mi estimate`, `rolling`, `statsby`, and `svy` are allowed; see [U] 11.1.10 **Prefix commands**. For more details, see [BAYES] **bayes: truncreg** and [FMM] **fmm: truncreg**.

`vce(bootstrap)` and `vce(jackknife)` are not allowed with the `mi estimate` prefix; see [MI] **mi estimate**.

Weights are not allowed with the `bootstrap` prefix; see [R] **bootstrap**.

`aweight`s are not allowed with the `jackknife` prefix; see [R] **jackknife**.

`vce()`, `lrmodel`, and `weights` are not allowed with the `svy` prefix; see [SVY] **svy**.

`aweight`s, `fweight`s, `iweight`s, and `pweight`s are allowed; see [U] 11.1.6 **weight**.

`collinear` and `coeflegend` do not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options

### Model

`noconstant`; see [R] [Estimation options](#).

`ll(varname | #)` and `ul(varname | #)` indicate the lower and upper limits for truncation, respectively.

You may specify one or both. Observations with `depvar ≤ ll()` are left-truncated, observations with `depvar ≥ ul()` are right-truncated, and the remaining observations are not truncated. See [R] [tobit](#) for a more detailed description.

`offset(varname)`, `constraints(constraints)`; see [R] [Estimation options](#).

### SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

### Reporting

`level(#)`, `lrmmodel`, `nocnsreport`; see [R] [Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

### Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used, but you may use the `ltol(#)` option to relax the convergence criterion; the default is  $1e-6$  during specification searches.

Setting the optimization type to `technique(bhhh)` resets the default `vcetype` to `vce(opg)`.

The following options are available with `truncreg` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Truncated regression fits a model of a dependent variable on independent variables from a restricted part of a population. Truncation is essentially a characteristic of the distribution from which the sample data are drawn. If  $x$  has a normal distribution with mean  $\mu$  and standard deviation  $\sigma$ , the density of the truncated normal distribution is

$$\begin{aligned} f(x \mid a < x < b) &= \frac{f(x)}{\Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)} \\ &= \frac{\frac{1}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right)}{\Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)} \end{aligned}$$

where  $\phi$  and  $\Phi$  are the density and distribution functions of the standard normal distribution.

Compared with the mean of the untruncated variable, the mean of the truncated variable is greater if the truncation is from below, and the mean of the truncated variable is smaller if the truncation is from above. Moreover, truncation reduces the variance compared with the variance in the untruncated distribution.

### ▷ Example 1

We will demonstrate `truncreg` with part of the Mroz dataset distributed with [Berndt \(1996\)](#). This dataset contains 753 observations on women's labor supply. Our subsample is of 250 observations, with 150 market laborers and 100 nonmarket laborers.

```
. use https://www.stata-press.com/data/r19/laborsub
. describe
Contains data from https://www.stata-press.com/data/r19/laborsub.dta
Observations: 250
Variables: 6 25 Sep 2024 18:36
```

| Variable name | Storage type | Display format | Value label | Variable label                 |
|---------------|--------------|----------------|-------------|--------------------------------|
| lfp           | byte         | %9.0g          |             | 1 if woman worked in 1975      |
| whrs          | int          | %9.0g          |             | Wife's hours of work           |
| kl6           | byte         | %9.0g          |             | # of children younger than 6   |
| k618          | byte         | %9.0g          |             | # of children between 6 and 18 |
| wa            | byte         | %9.0g          |             | Wife's age                     |
| we            | byte         | %9.0g          |             | Wife's educational attainment  |

Sorted by:

```
. summarize, sep(0)
```

| Variable | Obs | Mean   | Std. dev. | Min | Max  |
|----------|-----|--------|-----------|-----|------|
| lfp      | 250 | .6     | .4908807  | 0   | 1    |
| whrs     | 250 | 799.84 | 915.6035  | 0   | 4950 |
| kl6      | 250 | .236   | .5112234  | 0   | 3    |
| k618     | 250 | 1.364  | 1.370774  | 0   | 8    |
| wa       | 250 | 42.92  | 8.426483  | 30  | 60   |
| we       | 250 | 12.352 | 2.164912  | 5   | 17   |

We first perform ordinary least-squares estimation on the market laborers.

```
. regress whrs kl6 k618 wa we if whrs > 0
```

| Source   | SS         | df  | MS         | Number of obs | = | 150    |
|----------|------------|-----|------------|---------------|---|--------|
| Model    | 7326995.15 | 4   | 1831748.79 | F(4, 145)     | = | 2.80   |
| Residual | 94793104.2 | 145 | 653745.546 | Prob > F      | = | 0.0281 |
| Total    | 102120099  | 149 | 685369.794 | R-squared     | = | 0.0717 |
|          |            |     |            | Adj R-squared | = | 0.0461 |
|          |            |     |            | Root MSE      | = | 808.55 |

| whrs  | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |
|-------|-------------|-----------|-------|-------|----------------------|
| kl6   | -421.4822   | 167.9734  | -2.51 | 0.013 | -753.4748 -89.48953  |
| k618  | -104.4571   | 54.18616  | -1.93 | 0.056 | -211.5538 2.639668   |
| wa    | -4.784917   | 9.690502  | -0.49 | 0.622 | -23.9378 14.36797    |
| we    | 9.353195    | 31.23793  | 0.30  | 0.765 | -52.38731 71.0937    |
| _cons | 1629.817    | 615.1301  | 2.65  | 0.009 | 414.0371 2845.597    |

Now, we use `truncreg` to perform truncated regression with truncation from below zero.

```
. truncreg whrs k16 k618 wa we, ll(0)
(100 obs truncated)

Fitting full model:
Iteration 0: Log likelihood = -1205.6992
Iteration 1: Log likelihood = -1200.9873
Iteration 2: Log likelihood = -1200.9159
Iteration 3: Log likelihood = -1200.9157
Iteration 4: Log likelihood = -1200.9157

Truncated regression
Limit: Lower = 0 Number of obs = 150
 Upper = +inf Wald chi2(4) = 10.05
Log likelihood = -1200.9157 Prob > chi2 = 0.0395
```

| whrs   | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|--------|-------------|-----------|-------|-------|----------------------|-----------|
| k16    | -803.0042   | 321.3614  | -2.50 | 0.012 | -1432.861            | -173.1474 |
| k618   | -172.875    | 88.72898  | -1.95 | 0.051 | -346.7806            | 1.030579  |
| wa     | -8.821123   | 14.36848  | -0.61 | 0.539 | -36.98283            | 19.34059  |
| we     | 16.52873    | 46.50375  | 0.36  | 0.722 | -74.61695            | 107.6744  |
| _cons  | 1586.26     | 912.355   | 1.74  | 0.082 | -201.9233            | 3374.442  |
| /sigma | 983.7262    | 94.44303  | 10.42 | 0.000 | 798.6213             | 1168.831  |

If we assume that our data were censored, the tobit model is

```
. tobit whrs k16 k618 wa we, ll(0)
(output omitted)

Tobit regression Number of obs = 250
 Uncensored = 150
Limits: Lower = 0 Left-censored = 100
 Upper = +inf Right-censored = 0
 LR chi2(4) = 23.03
 Prob > chi2 = 0.0001
Log likelihood = -1367.0903 Pseudo R2 = 0.0084
```

| whrs        | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |           |
|-------------|-------------|-----------|-------|-------|----------------------|-----------|
| k16         | -827.7655   | 214.7521  | -3.85 | 0.000 | -1250.753            | -404.7781 |
| k618        | -140.0191   | 74.22719  | -1.89 | 0.060 | -286.221             | 6.182766  |
| wa          | -24.97918   | 13.25715  | -1.88 | 0.061 | -51.09118            | 1.13281   |
| we          | 103.6896    | 41.82629  | 2.48  | 0.014 | 21.30625             | 186.0729  |
| _cons       | 589.0002    | 841.5952  | 0.70  | 0.485 | -1068.651            | 2246.652  |
| var(e.whrs) | 1715859     | 216775.7  |       |       | 1337864              | 2200650   |

◀

## □ Technical note

Whether truncated regression is more appropriate than the ordinary least-squares estimation depends on the purpose of that estimation. If we are interested in the mean of wife's working hours conditional on the subsample of market laborers, least-squares estimation is appropriate. However if we are interested in the mean of wife's working hours regardless of market or nonmarket labor status, least-squares estimates could be seriously misleading.

Truncation and censoring are different concepts. A sample has been censored if no observations have been systematically excluded but some of the information contained in them has been suppressed. In a truncated distribution, only the part of the distribution above (or below, or between) the truncation points is relevant to our computations. We need to scale it up by the probability that an observation falls in the range that interests us to make the distribution integrate to one. The censored distribution used by tobit, however, is a mixture of discrete and continuous distributions. Instead of rescaling over the observable range, we simply assign the full probability from the censored regions to the censoring points. The truncated regression model is sometimes less well behaved than the tobit model. Davidson and MacKinnon (1993) provide an example where truncation results in more inconsistency than censoring. □

## Stored results

truncreg stores the following in `e()`:

### Scalars

|                            |                                           |
|----------------------------|-------------------------------------------|
| <code>e(N)</code>          | number of observations                    |
| <code>e(N_bf)</code>       | number of observations before truncation  |
| <code>e(chi2)</code>       | model $\chi^2$                            |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>  |
| <code>e(k_eq_model)</code> | number of equations in overall model test |
| <code>e(k_aux)</code>      | number of auxiliary parameters            |
| <code>e(df_m)</code>       | model degrees of freedom                  |
| <code>e(ll)</code>         | log likelihood                            |
| <code>e(ll_0)</code>       | log likelihood, constant-only model       |
| <code>e(N_clust)</code>    | number of clusters                        |
| <code>e(sigma)</code>      | estimate of sigma                         |
| <code>e(p)</code>          | <i>p</i> -value for model test            |
| <code>e(rank)</code>       | rank of <code>e(V)</code>                 |
| <code>e(ic)</code>         | number of iterations                      |
| <code>e(rc)</code>         | return code                               |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise               |

### Macros

|                            |                                                                          |
|----------------------------|--------------------------------------------------------------------------|
| <code>e(cmd)</code>        | truncreg                                                                 |
| <code>e(cmdline)</code>    | command as typed                                                         |
| <code>e(llopt)</code>      | contents of <code>ll()</code> , if specified                             |
| <code>e(ulopt)</code>      | contents of <code>ul()</code> , if specified                             |
| <code>e(depvar)</code>     | name of dependent variable                                               |
| <code>e(wtype)</code>      | weight type                                                              |
| <code>e(wexp)</code>       | weight expression                                                        |
| <code>e(title)</code>      | title in estimation output                                               |
| <code>e(clustvar)</code>   | name of cluster variable                                                 |
| <code>e(offset1)</code>    | offset                                                                   |
| <code>e(chi2type)</code>   | Wald or LR; type of model $\chi^2$ test                                  |
| <code>e(vce)</code>        | <i>vcetype</i> specified in <code>vce()</code>                           |
| <code>e(vcetype)</code>    | title used to label Std. err.                                            |
| <code>e(opt)</code>        | type of optimization                                                     |
| <code>e(which)</code>      | max or min; whether optimizer is to perform maximization or minimization |
| <code>e(ml_method)</code>  | type of ml method                                                        |
| <code>e(user)</code>       | name of likelihood-evaluator program                                     |
| <code>e(technique)</code>  | maximization technique                                                   |
| <code>e(properties)</code> | b V                                                                      |
| <code>e(predict)</code>    | program used to implement predict                                        |
| <code>e(marginsok)</code>  | predictions allowed by margins                                           |
| <code>e(asbalanced)</code> | factor variables <code>fvset</code> as <code>asbalanced</code>           |
| <code>e(asobserved)</code> | factor variables <code>fvset</code> as <code>asobserved</code>           |

|                 |                                              |
|-----------------|----------------------------------------------|
| Matrices        |                                              |
| e(b)            | coefficient vector                           |
| e(Cns)          | constraints matrix                           |
| e(iolog)        | iteration log (up to 20 iterations)          |
| e(gradient)     | gradient vector                              |
| e(V)            | variance–covariance matrix of the estimators |
| e(V_modelbased) | model-based variance                         |
| e(means)        | means of independent variables               |
| e(dummy)        | indicator for dummy variables                |
| Functions       |                                              |
| e(sample)       | marks estimation sample                      |

In addition to the above, the following is stored in `r()`:

|          |                                                                                                                       |
|----------|-----------------------------------------------------------------------------------------------------------------------|
| Matrices |                                                                                                                       |
| r(table) | matrix containing the coefficients with their standard errors, test statistics, $p$ -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

Greene (2018, 918–924) and Davidson and MacKinnon (1993, 534–537) provide introductions to the truncated regression model.

Let  $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$  be the model.  $\mathbf{y}$  represents continuous outcomes either observed or not observed. Our model assumes that  $\boldsymbol{\epsilon} \sim N(\mathbf{0}, \sigma^2 \mathbf{I})$ .

Let  $a$  be the lower limit and  $b$  be the upper limit. The log likelihood is

$$\ln L = -\frac{n}{2} \log(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{j=1}^n (y_j - \mathbf{x}_j\boldsymbol{\beta})^2 - \sum_{j=1}^n \log \left\{ \Phi\left(\frac{b - \mathbf{x}_j\boldsymbol{\beta}}{\sigma}\right) - \Phi\left(\frac{a - \mathbf{x}_j\boldsymbol{\beta}}{\sigma}\right) \right\}$$

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] [\\_robust](#), particularly *Maximum likelihood estimators* and *Methods and formulas*.

`truncreg` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] [Variance estimation](#).

## References

- Berndt, E. R. 1996. *The Practice of Econometrics: Classic and Contemporary*. New York: Addison–Wesley.
- Canette, I. 2016. Understanding truncation and censoring. *The Stata Blog: Not Elsewhere Classified*. <https://blog.stata.com/2016/12/13/understanding-truncation-and-censoring/>.
- Davidson, R., and J. G. MacKinnon. 1993. *Estimation and Inference in Econometrics*. New York: Oxford University Press.
- Gray, L. A., and M. Hernández-Alava. 2018. A command for fitting mixture regression models for bounded dependent variables using the beta distribution. *Stata Journal* 18: 51–75.
- Greene, W. H. 2018. *Econometric Analysis*. 8th ed. New York: Pearson.

## Also see

[R] **truncreg postestimation** — Postestimation tools for truncreg

[R] **regress** — Linear regression

[R] **tobit** — Tobit regression

[BAYES] **bayes: truncreg** — Bayesian truncated regression

[FMM] **fmm: truncreg** — Finite mixtures of truncated linear regression models

[MI] **Estimation** — Estimation commands for use with mi estimate

[SVY] **svy estimation** — Estimation commands for survey data

[U] **20 Estimation and postestimation commands**

## Postestimation commands

The following postestimation commands are available after `truncreg`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | linear, censored, and truncated predictions                                                                                             |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results. `forecast` is also not appropriate with `mi` estimation results.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as linear predictions, standard errors, probabilities, and expected values.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic nooffset]
```

```
predict [type] stub* [if] [in] , scores
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

Main

|                         |                                             |
|-------------------------|---------------------------------------------|
| <code>xb</code>         | linear prediction; the default              |
| <code>stdp</code>       | standard error of the prediction            |
| <code>stdf</code>       | standard error of the forecast              |
| <code>pr(a,b)</code>    | $\Pr(a < y_j < b)$                          |
| <code>e(a,b)</code>     | $E(y_j   a < y_j < b)$                      |
| <code>ystar(a,b)</code> | $E(y_j^*), y_j^* = \max\{a, \min(y_j, b)\}$ |

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

`stdf` is not allowed with `svy` estimation results.

where  $a$  and  $b$  may be numbers or variables;  $a$  missing ( $a \geq .$ ) means  $-\infty$ , and  $b$  missing ( $b \geq .$ ) means  $+\infty$ ; see [U] [12.2.1 Missing values](#).

## Options for predict

Main

`xb`, the default, calculates the linear prediction.

`stdp` calculates the standard error of the prediction, which can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. The standard error of the prediction is also referred to as the standard error of the fitted value.

`stdf` calculates the standard error of the forecast, which is the standard error of the point prediction for 1 observation. It is commonly referred to as the standard error of the future or forecast value. By construction, the standard errors produced by `stdf` are always larger than those produced by `stdp`; see *Methods and formulas* in [R] [regress postestimation](#).

`pr(a,b)` calculates  $\Pr(a < \mathbf{x}_j\mathbf{b} + u_j < b)$ , the probability that  $y_j|\mathbf{x}_j$  would be observed in the interval  $(a, b)$ .

$a$  and  $b$  may be specified as numbers or variable names;  $lb$  and  $ub$  are variable names;

`pr(20,30)` calculates  $\Pr(20 < \mathbf{x}_j\mathbf{b} + u_j < 30)$ ;

`pr(lb,ub)` calculates  $\Pr(lb < \mathbf{x}_j\mathbf{b} + u_j < ub)$ ; and

`pr(20,ub)` calculates  $\Pr(20 < \mathbf{x}_j\mathbf{b} + u_j < ub)$ .

$a$  missing ( $a \geq .$ ) means  $-\infty$ ; `pr(. , 30)` calculates  $\Pr(-\infty < \mathbf{x}_j\mathbf{b} + u_j < 30)$ ;

`pr(lb, 30)` calculates  $\Pr(-\infty < \mathbf{x}_j\mathbf{b} + u_j < 30)$  in observations for which  $lb \geq .$

and calculates  $\Pr(lb < \mathbf{x}_j\mathbf{b} + u_j < 30)$  elsewhere.

$b$  missing ( $b \geq .$ ) means  $+\infty$ ; `pr(20, .)` calculates  $\Pr(+\infty > \mathbf{x}_j\mathbf{b} + u_j > 20)$ ;

`pr(20,ub)` calculates  $\Pr(+\infty > \mathbf{x}_j\mathbf{b} + u_j > 20)$  in observations for which  $ub \geq .$

and calculates  $\Pr(20 < \mathbf{x}_j\mathbf{b} + u_j < ub)$  elsewhere.

`e(a,b)` calculates  $E(\mathbf{x}_j\mathbf{b} + u_j | a < \mathbf{x}_j\mathbf{b} + u_j < b)$ , the expected value of  $y_j|\mathbf{x}_j$  conditional on  $y_j|\mathbf{x}_j$  being in the interval  $(a, b)$ , meaning that  $y_j|\mathbf{x}_j$  is truncated.

$a$  and  $b$  are specified as they are for `pr()`.

`ystar(a,b)` calculates  $E(y_j^*)$ , where  $y_j^* = a$  if  $\mathbf{x}_j\mathbf{b} + u_j \leq a$ ,  $y_j^* = b$  if  $\mathbf{x}_j\mathbf{b} + u_j \geq b$ , and  $y_j^* = \mathbf{x}_j\mathbf{b} + u_j$  otherwise, meaning that  $y_j^*$  is censored.  $a$  and  $b$  are specified as they are for `pr()`.

`nooffset` is relevant only if you specified `offset(varname)`. It modifies the calculations made by `predict` so that they ignore the offset variable; the linear prediction is treated as  $\mathbf{x}_j\mathbf{b}$  rather than as  $\mathbf{x}_j\mathbf{b} + \text{offset}_j$ .

`scores` calculates equation-level score variables.

The first new variable will contain  $\partial \ln L / \partial (\mathbf{x}_j\boldsymbol{\beta})$ .

The second new variable will contain  $\partial \ln L / \partial \sigma$ .

## margins

### Description for margins

`margins` estimates margins of response for linear predictions, probabilities, and expected values.

### Menu for margins

Statistics > Postestimation

### Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i>                      | Description                                 |
|---------------------------------------|---------------------------------------------|
| <code>xb</code>                       | linear prediction; the default              |
| <code>pr(<i>a</i>,<i>b</i>)</code>    | $\Pr(a < y_j < b)$                          |
| <code>e(<i>a</i>,<i>b</i>)</code>     | $E(y_j   a < y_j < b)$                      |
| <code>ystar(<i>a</i>,<i>b</i>)</code> | $E(y_j^*), y_j^* = \max\{a, \min(y_j, b)\}$ |
| <code>stdp</code>                     | not allowed with <code>margins</code>       |
| <code>stdf</code>                     | not allowed with <code>margins</code>       |

Statistics not allowed with `margins` are functions of stochastic quantities other than `e(b)`.

For the full syntax, see [R] [margins](#).

### Also see

[R] [truncreg](#) — Truncated regression

[U] [20 Estimation and postestimation commands](#)

## **ttest** — *t* tests (mean-comparison tests)

Description  
Options  
References

Quick start  
Remarks and examples  
Also see

Menu  
Stored results

Syntax  
Methods and formulas

### Description

`ttest` performs *t* tests on the equality of means. The test can be performed for one sample against a hypothesized population mean. Two-sample tests can be conducted for paired and unpaired data. The assumption of equal variances can be optionally relaxed in the unpaired two-sample case.

`ttesti` is the immediate form of `ttest`; see [\[U\] 19 Immediate commands](#).

### Quick start

Test that the mean of `v1` is equal between two groups defined by `catvar`

```
ttest v1, by(catvar)
```

Same as above, but assume unequal variances

```
ttest v1, by(catvar) unequal
```

Paired *t* test of `v2` and `v3`

```
ttest v2 == v3
```

Same as above, but with unpaired data and conduct test separately for each level of `catvar`

```
by catvar: ttest v2 == v3, unpaired
```

Test that the mean of `v4` is 3 at the 90% confidence level

```
ttest v4 == 3, level(90)
```

Test  $\mu_1 = \mu_2$  if  $\bar{x}_1 = 3.2$ ,  $sd_1 = 0.1$ ,  $\bar{x}_2 = 3.4$ , and  $sd_2 = 0.15$  with  $n_1 = n_2 = 12$

```
ttesti 12 3.2 .1 12 3.4 .15
```

### Menu

#### **ttest**

Statistics > Summaries, tables, and tests > Classical tests of hypotheses > t test (mean-comparison test)

#### **ttesti**

Statistics > Summaries, tables, and tests > Classical tests of hypotheses > t test calculator

## Syntax

### One-sample *t* test

```
ttest varname == # [if] [in] [, level(#)]
```

### Two-sample *t* test using groups

```
ttest varname [if] [in], by(groupvar) [options1]
```

### Two-sample *t* test using variables

```
ttest varname1 == varname2 [if] [in], unpaired [unequal welch level(#)]
```

### Paired *t* test

```
ttest varname1 == varname2 [if] [in] [, level(#)]
```

### Immediate form of one-sample *t* test

```
ttesti #obs #mean #sd #val [, level(#)]
```

### Immediate form of two-sample *t* test

```
ttesti #obs1 #mean1 #sd1 #obs2 #mean2 #sd2 [, options2]
```

*options*<sub>1</sub>

Description

---

|                                 |                                                         |
|---------------------------------|---------------------------------------------------------|
| Main                            |                                                         |
| * <i>by</i> ( <i>groupvar</i> ) | variable defining the groups                            |
| <u>reverse</u>                  | reverse group order for mean difference computation     |
| <u>unequal</u>                  | unpaired data have unequal variances                    |
| <u>welch</u>                    | use Welch's approximation                               |
| <u>level</u> (#)                | set confidence level; default is <code>level(95)</code> |

---

\**by*(*groupvar*) is required.

*options*<sub>2</sub>

Description

---

|                  |                                                         |
|------------------|---------------------------------------------------------|
| Main             |                                                         |
| <u>unequal</u>   | unpaired data have unequal variances                    |
| <u>welch</u>     | use Welch's approximation                               |
| <u>level</u> (#) | set confidence level; default is <code>level(95)</code> |

---

*by* and *collect* are allowed with *ttest* and *ttesti*; see [U] 11.1.10 Prefix commands.

## Options

Main

`by(groupvar)` specifies the *groupvar* that defines the two groups that `ttest` will use to test the hypothesis that their means are equal. Specifying `by(groupvar)` implies an unpaired (two sample) *t* test. Do not confuse the `by()` option with the `by` prefix; you can specify both.

`reverse` reverses the order of the mean difference between groups defined in `by()`. By default, the mean of the group corresponding to the largest value in the variable in `by()` is subtracted from the mean of the group with the smallest value in `by()`. `reverse` reverses this behavior and the order in which variables appear on the table.

`unpaired` specifies that the data be treated as unpaired. The `unpaired` option is used when the two sets of values to be compared are in different variables.

`unequal` specifies that the unpaired data not be assumed to have equal variances.

`welch` specifies that the approximate degrees of freedom for the test be obtained from Welch's formula (1947) rather than from Satterthwaite's approximation formula (1946), which is the default when `unequal` is specified. Specifying `welch` implies `unequal`.

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [U] 20.8 [Specifying the width of confidence intervals](#).

## Remarks and examples

Remarks are presented under the following headings:

- [One-sample \*t\* test](#)
- [Two-sample \*t\* test](#)
- [Paired \*t\* test](#)
- [Two-sample \*t\* test compared with one-way ANOVA](#)
- [Immediate form](#)
- [Video examples](#)

### One-sample *t* test

#### ▷ Example 1

In the first form, `ttest` tests whether the mean of the sample is equal to a known constant under the assumption of unknown variance. Assume that we have a sample of 74 automobiles. We know each automobile's average mileage rating and wish to test whether the overall average for the sample is 20 miles per gallon.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. ttest mpg==20
```

One-sample *t* test

| Variable | Obs | Mean    | Std. err. | Std. dev. | [95% conf. interval] |          |
|----------|-----|---------|-----------|-----------|----------------------|----------|
| mpg      | 74  | 21.2973 | .6725511  | 5.785503  | 19.9569              | 22.63769 |

```
mean = mean(mpg)
HO: mean = 20
Ha: mean < 20
Pr(T < t) = 0.9712

Degrees of freedom = 73
Ha: mean != 20
Pr(|T| > |t|) = 0.0576

t = 1.9289
Ha: mean > 20
Pr(T > t) = 0.0288
```

The test indicates that the underlying mean is not 20 with a significance level of 5.8%.



## Two-sample *t* test

### ► Example 2: Two-sample *t* test using groups

We are testing the effectiveness of a new fuel additive. We run an experiment in which 12 cars are given the fuel treatment and 12 cars are not. The results of the experiment are as follows:

| treated | mpg |
|---------|-----|
| 0       | 20  |
| 0       | 23  |
| 0       | 21  |
| 0       | 25  |
| 0       | 18  |
| 0       | 17  |
| 0       | 18  |
| 0       | 24  |
| 0       | 20  |
| 0       | 24  |
| 0       | 23  |
| 0       | 19  |
| 1       | 24  |
| 1       | 25  |
| 1       | 21  |
| 1       | 22  |
| 1       | 23  |
| 1       | 18  |
| 1       | 17  |
| 1       | 28  |
| 1       | 24  |
| 1       | 27  |
| 1       | 21  |
| 1       | 23  |

The treated variable is coded as 1 if the car received the fuel treatment and 0 otherwise.

We can test the equality of means of the treated and untreated group by typing

```
. use https://www.stata-press.com/data/r19/fuel3
. ttest mpg, by(treated)
```

Two-sample t test with equal variances

| Group    | Obs | Mean   | Std. err. | Std. dev. | [95% conf. interval] |          |
|----------|-----|--------|-----------|-----------|----------------------|----------|
| 0        | 12  | 21     | .7881701  | 2.730301  | 19.26525             | 22.73475 |
| 1        | 12  | 22.75  | .9384465  | 3.250874  | 20.68449             | 24.81551 |
| Combined | 24  | 21.875 | .6264476  | 3.068954  | 20.57909             | 23.17091 |
| diff     |     | -1.75  | 1.225518  |           | -4.291568            | .7915684 |

```
diff = mean(0) - mean(1) t = -1.4280
HO: diff = 0 Degrees of freedom = 22
Ha: diff < 0 Ha: diff != 0
Pr(T < t) = 0.0837 Pr(|T| > |t|) = 0.1673
 Ha: diff > 0
 Pr(T > t) = 0.9163
```

We do not find a statistically significant difference in the means.

If we were not willing to assume that the variances were equal and wanted to use Welch's formula, we could type

```
. ttest mpg, by(treated) welch
```

Two-sample t test with unequal variances

| Group    | Obs | Mean   | Std. err. | Std. dev. | [95% conf. interval] |          |
|----------|-----|--------|-----------|-----------|----------------------|----------|
| 0        | 12  | 21     | .7881701  | 2.730301  | 19.26525             | 22.73475 |
| 1        | 12  | 22.75  | .9384465  | 3.250874  | 20.68449             | 24.81551 |
| Combined | 24  | 21.875 | .6264476  | 3.068954  | 20.57909             | 23.17091 |
| diff     |     | -1.75  | 1.225518  |           | -4.28369             | .7836902 |

```
diff = mean(0) - mean(1) t = -1.4280
HO: diff = 0 Welch's degrees of freedom = 23.2465
Ha: diff < 0 Ha: diff != 0
Pr(T < t) = 0.0833 Pr(|T| > |t|) = 0.1666
 Ha: diff > 0
 Pr(T > t) = 0.9167
```

◀

## □ Technical note

In two-sample randomized designs, subjects will sometimes refuse the assigned treatment but still be measured for an outcome. In this case, take care to specify the group properly. You might be tempted to let *varname* contain missing where the subject refused and thus let `ttest` drop such observations from the analysis. Zelen (1979) argues that it would be better to specify that the subject belongs to the group in which he or she was randomized, even though such inclusion will dilute the measured effect.

□

▷ Example 3: Two-sample *t* test using variables

There is a second, inferior way to organize the data in the preceding example. We ran a test on 24 cars, 12 without the additive and 12 with. We now create two new variables, mpg1 and mpg2.

| mpg1 | mpg2 |
|------|------|
| 20   | 24   |
| 23   | 25   |
| 21   | 21   |
| 25   | 22   |
| 18   | 23   |
| 17   | 18   |
| 18   | 17   |
| 24   | 28   |
| 20   | 24   |
| 24   | 27   |
| 23   | 21   |
| 19   | 23   |

This method is inferior because it suggests a connection that is not there. There is no link between the car with 20 mpg and the car with 24 mpg in the first row of the data. Each column of data could be arranged in any order. Nevertheless, if our data are organized like this, ttest can accommodate us.

```
. use https://www.stata-press.com/data/r19/fuel
. ttest mpg1==mpg2, unpaired
```

Two-sample t test with equal variances

| Variable | Obs | Mean   | Std. err. | Std. dev. | [95% conf. interval] |          |
|----------|-----|--------|-----------|-----------|----------------------|----------|
| mpg1     | 12  | 21     | .7881701  | 2.730301  | 19.26525             | 22.73475 |
| mpg2     | 12  | 22.75  | .9384465  | 3.250874  | 20.68449             | 24.81551 |
| Combined | 24  | 21.875 | .6264476  | 3.068954  | 20.57909             | 23.17091 |
| diff     |     | -1.75  | 1.225518  |           | -4.291568            | .7915684 |

```
diff = mean(mpg1) - mean(mpg2) t = -1.4280
HO: diff = 0 Degrees of freedom = 22
Ha: diff < 0 Ha: diff != 0
Pr(T < t) = 0.0837 Pr(|T| > |t|) = 0.1673 Ha: diff > 0
 Pr(T > t) = 0.9163
```



## Paired t test

### ▷ Example 4

Suppose that the preceding data were actually collected by running a test on 12 cars. Each car was run once with the fuel additive and once without. Our data are stored in the same manner as in [example 3](#), but this time, there is most certainly a connection between the mpg values that appear in the same row. These come from the same car. The variables mpg1 and mpg2 represent mileage without and with the treatment, respectively.

```
. use https://www.stata-press.com/data/r19/fuel
. ttest mpg1==mpg2
```

Paired t test

| Variable | Obs | Mean  | Std. err. | Std. dev. | [95% conf. interval] |           |
|----------|-----|-------|-----------|-----------|----------------------|-----------|
| mpg1     | 12  | 21    | .7881701  | 2.730301  | 19.26525             | 22.73475  |
| mpg2     | 12  | 22.75 | .9384465  | 3.250874  | 20.68449             | 24.81551  |
| diff     | 12  | -1.75 | .7797144  | 2.70101   | -3.46614             | -.0338602 |

```

 mean(diff) = mean(mpg1 - mpg2) t = -2.2444
H0: mean(diff) = 0 Degrees of freedom = 11
Ha: mean(diff) < 0 Ha: mean(diff) != 0 Ha: mean(diff) > 0
Pr(T < t) = 0.0232 Pr(|T| > |t|) = 0.0463 Pr(T > t) = 0.9768

```

We find that the means are statistically different from each other at any level greater than 4.6%.



## Two-sample t test compared with one-way ANOVA

### ▷ Example 5

In [example 2](#), we saw that `ttest` can be used to test the equality of a pair of means; see [\[R\] oneway](#) for an extension that allows testing the equality of more than two means.

Suppose that we have data on the 50 states. The dataset contains the median age of the population (`medage`) and the region of the country (`region`) for each state. Region 1 refers to the Northeast, region 2 to the North Central, region 3 to the South, and region 4 to the West. Using `oneway`, we can test the equality of all four means.

```
. use https://www.stata-press.com/data/r19/census
(1980 Census data by state)
. oneway medage region
```

| Source         | Analysis of variance |    |            | F    | Prob > F |
|----------------|----------------------|----|------------|------|----------|
|                | SS                   | df | MS         |      |          |
| Between groups | 46.3961903           | 3  | 15.4653968 | 7.56 | 0.0003   |
| Within groups  | 94.1237947           | 46 | 2.04616945 |      |          |
| Total          | 140.519985           | 49 | 2.8677548  |      |          |

```
Bartlett's equal-variances test: chi2(3) = 10.5757 Prob>chi2 = 0.014
```

We find that the means are different, but we are interested only in testing whether the means for the Northeast (region==1) and West (region==4) are different. We could use oneway:

```
. oneway medage region if region==1 | region==4
```

| Source         | Analysis of variance |    |            | F     | Prob > F |
|----------------|----------------------|----|------------|-------|----------|
|                | SS                   | df | MS         |       |          |
| Between groups | 46.241247            | 1  | 46.241247  | 20.02 | 0.0002   |
| Within groups  | 46.1969169           | 20 | 2.30984584 |       |          |
| Total          | 92.4381638           | 21 | 4.40181733 |       |          |

Bartlett's equal-variances test: chi2(1) = 2.4679 Prob>chi2 = 0.116

We could also use ttest:

```
. ttest medage if region==1 | region==4, by(region)
```

Two-sample t test with equal variances

| Group    | Obs | Mean     | Std. err. | Std. dev. | [95% conf. interval] |          |
|----------|-----|----------|-----------|-----------|----------------------|----------|
| NE       | 9   | 31.23333 | .3411581  | 1.023474  | 30.44662             | 32.02005 |
| West     | 13  | 28.28462 | .4923577  | 1.775221  | 27.21186             | 29.35737 |
| Combined | 22  | 29.49091 | .4473059  | 2.098051  | 28.56069             | 30.42113 |
| diff     |     | 2.948718 | .6590372  |           | 1.57399              | 4.323445 |

diff = mean(NE) - mean(West) t = 4.4743  
 HO: diff = 0 Degrees of freedom = 20  
 Ha: diff < 0 Ha: diff != 0 Ha: diff > 0  
 Pr(T < t) = 0.9999 Pr(|T| > |t|) = 0.0002 Pr(T > t) = 0.0001

The significance levels of both tests are the same.



## Immediate form

### ➤ Example 6

ttesti is like ttest, except that we specify summary statistics rather than variables as arguments. For instance, we are reading an article that reports the mean number of sunspots per month as 62.6 with a standard deviation of 15.8. There are 24 months of data. We wish to test whether the mean is 75:

```
. ttesti 24 62.6 15.8 75
```

One-sample t test

|   | Obs | Mean | Std. err. | Std. dev. | [95% conf. interval] |          |
|---|-----|------|-----------|-----------|----------------------|----------|
| x | 24  | 62.6 | 3.225161  | 15.8      | 55.92825             | 69.27175 |

mean = mean(x) t = -3.8448  
 HO: mean = 75 Degrees of freedom = 23  
 Ha: mean < 75 Ha: mean != 75 Ha: mean > 75  
 Pr(T < t) = 0.0004 Pr(|T| > |t|) = 0.0008 Pr(T > t) = 0.9996



▷ Example 7

There is no immediate form of `ttest` with paired data because the test is also a function of the covariance, a number unlikely to be reported in any published source. For unpaired data, however, we might type

```
. ttesti 20 20 5 32 15 4
Two-sample t test with equal variances
```

|          | Obs | Mean     | Std. err. | Std. dev. | [95% conf. interval] |          |
|----------|-----|----------|-----------|-----------|----------------------|----------|
| x        | 20  | 20       | 1.118034  | 5         | 17.65993             | 22.34007 |
| y        | 32  | 15       | .7071068  | 4         | 13.55785             | 16.44215 |
| Combined | 52  | 16.92308 | .6943785  | 5.007235  | 15.52905             | 18.3171  |
| diff     |     | 5        | 1.256135  |           | 2.476979             | 7.523021 |

```

diff = mean(x) - mean(y)
t = 3.9805
HO: diff = 0
Degrees of freedom = 50
Ha: diff < 0
Pr(T < t) = 0.9999
Ha: diff != 0
Pr(|T| > |t|) = 0.0002
Ha: diff > 0
Pr(T > t) = 0.0001

```

If we had typed `ttesti 20 20 5 32 15 4, unequal`, the test would have assumed unequal variances.



Video examples

- [One-sample \*t\* test in Stata](#)
- [t test for two independent samples in Stata](#)
- [t test for two paired samples in Stata](#)
- [One-sample \*t\*-test calculator](#)
- [Two-sample \*t\*-test calculator](#)

Stored results

`ttest` and `ttesti` store the following in `r()`:

Scalars

|                     |                            |                       |                                        |
|---------------------|----------------------------|-----------------------|----------------------------------------|
| <code>r(N_1)</code> | sample size $n_1$          | <code>r(sd_1)</code>  | standard deviation for first variable  |
| <code>r(N_2)</code> | sample size $n_2$          | <code>r(sd_2)</code>  | standard deviation for second variable |
| <code>r(p_l)</code> | lower one-sided $p$ -value | <code>r(sd)</code>    | combined standard deviation            |
| <code>r(p_u)</code> | upper one-sided $p$ -value | <code>r(mu_1)</code>  | $\bar{x}_1$ mean for population 1      |
| <code>r(p)</code>   | two-sided $p$ -value       | <code>r(mu_2)</code>  | $\bar{x}_2$ mean for population 2      |
| <code>r(se)</code>  | estimate of standard error | <code>r(df_t)</code>  | degrees of freedom                     |
| <code>r(t)</code>   | $t$ statistic              | <code>r(level)</code> | confidence level                       |

## Methods and formulas

See, for instance, [Hoel \(1984, 140–161\)](#) or [Dixon and Massey \(1983, 121–130\)](#) for an introduction and explanation of the calculation of these tests. [Acock \(2023, 165–179\)](#) and [Hamilton \(2013, 145–150\)](#) describe *t* tests using applications in Stata.

The test for  $\mu = \mu_0$  for unknown  $\sigma$  is given by

$$t = \frac{(\bar{x} - \mu_0)\sqrt{n}}{s}$$

The statistic is distributed as Student's *t* with  $n - 1$  degrees of freedom ([Gosset \[Student, pseud.\] 1908](#)).

The test for  $\mu_x = \mu_y$  when  $\sigma_x$  and  $\sigma_y$  are unknown but  $\sigma_x = \sigma_y$  is given by

$$t = \frac{\bar{x} - \bar{y}}{\left\{ \frac{(n_x - 1)s_x^2 + (n_y - 1)s_y^2}{n_x + n_y - 2} \right\}^{1/2} \left( \frac{1}{n_x} + \frac{1}{n_y} \right)^{1/2}}$$

The result is distributed as Student's *t* with  $n_x + n_y - 2$  degrees of freedom.

You could perform `ttest` (without the `unequal` option) in a regression setting given that regression assumes a homoskedastic error model. To compare with the `ttest` command, denote the underlying observations on *x* and *y* by  $x_j$ ,  $j = 1, \dots, n_x$ , and  $y_j$ ,  $j = 1, \dots, n_y$ . In a regression framework, typing `ttest` without the `unequal` option is equivalent to

1. creating a new variable  $z_j$  that represents the stacked observations on *x* and *y* (so that  $z_j = x_j$  for  $j = 1, \dots, n_x$  and  $z_{n_x+j} = y_j$  for  $j = 1, \dots, n_y$ )
2. and then estimating the equation  $z_j = \beta_0 + \beta_1 d_j + \epsilon_j$ , where  $d_j = 0$  for  $j = 1, \dots, n_x$  and  $d_j = 1$  for  $j = n_x + 1, \dots, n_x + n_y$  (that is,  $d_j = 0$  when the *z* observations represent *x*, and  $d_j = 1$  when the *z* observations represent *y*).

The estimated value of  $\beta_1$ ,  $b_1$ , will equal  $\bar{y} - \bar{x}$ , and the reported *t* statistic will be the same *t* statistic as given by the formula above.

The test for  $\mu_x = \mu_y$  when  $\sigma_x$  and  $\sigma_y$  are unknown and  $\sigma_x \neq \sigma_y$  is given by

$$t = \frac{\bar{x} - \bar{y}}{\left( s_x^2/n_x + s_y^2/n_y \right)^{1/2}}$$

The result is distributed as Student's *t* with  $\nu$  degrees of freedom, where  $\nu$  is given by (with [Satterthwaite's \[1946\]](#) formula)

$$\frac{\left( s_x^2/n_x + s_y^2/n_y \right)^2}{\frac{\left( s_x^2/n_x \right)^2}{n_x - 1} + \frac{\left( s_y^2/n_y \right)^2}{n_y - 1}}$$

With Welch's formula (1947), the number of degrees of freedom is given by

$$-2 + \frac{\left(s_x^2/n_x + s_y^2/n_y\right)^2}{\frac{\left(s_x^2/n_x\right)^2}{n_x+1} + \frac{\left(s_y^2/n_y\right)^2}{n_y+1}}$$

The test for  $\mu_x = \mu_y$  for matched observations (also known as paired observations, correlated pairs, or permanent components) is given by

$$t = \frac{\bar{d}\sqrt{n}}{s_d}$$

where  $\bar{d}$  represents the mean of  $x_i - y_i$  and  $s_d$  represents the standard deviation. The test statistic  $t$  is distributed as Student's  $t$  with  $n - 1$  degrees of freedom.

You can also use `tttest` without the `unpaired` option in a regression setting because a paired comparison includes the assumption of constant variance. The `tttest` with an unequal variance assumption does not lend itself to an easy representation in regression settings and is not discussed here.

$$(x_j - y_j) = \beta_0 + \epsilon_j.$$

[William Sealy Gosset](#) (1876–1937) was born in Canterbury, England. He studied chemistry and mathematics at Oxford and worked as a chemist with the brewers Guinness in Dublin. Gosset became interested in statistical problems, which he discussed with Karl Pearson and later with Fisher and Neyman. He published several important papers under the pseudonym “Student”, and he lent that name to the  $t$  test he invented.

[Stella Cunliffe](#) (1917–2012) was an advocate for increased understanding of the role of human nature in experiments and methodological rigor in social statistics. She was born in Battersea, England. She was the first person from her local public girls' school to attend college, obtaining a bachelor of science from the London School of Economics. Her first job was with the Danish Bacon Company during World War II, where she was in charge of bacon rations for London. After the war, she moved to Germany and again helped to ration food, this time for refugees.

She then spent a long career in quality control at the Guinness Brewing Company. Cunliffe observed that the weights of rejected casks skewed lighter. Noting that workers had to roll casks that were too light or too heavy uphill to be remade, she had the scales moved to the top of the hill. With workers able to roll rejected casks downhill, the weight of these casks began to follow a normal distribution.

After 25 years at Guinness, Cunliffe joined the British Home Office, where she would go on to become the first woman to serve as director of statistics. During her tenure at the Home Office, she emphasized applying principles of experimental design she had learned at Guinness to the study of such topics as birthrates, recidivism, and criminology. In 1975, she became the first woman to serve as president of the Royal Statistical Society.

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## Also see

- [R] **bitest** — Binomial probability test
- [R] **ci** — Confidence intervals for means, proportions, and variances
- [R] **esize** — Effect size based on mean comparison
- [R] **mean** — Estimate means
- [R] **oneway** — One-way analysis of variance
- [R] **prtest** — Tests of proportions
- [R] **sdtest** — Variance-comparison tests
- [R] **ztest** — *z* tests (mean-comparison tests, known variance)
- [MV] **hotelling** — Hotelling’s  $T^2$  generalized means test

[Description](#)[Remarks and examples](#)[Menu](#)[Stored results](#)[Syntax](#)[Also see](#)[Options](#)

## Description

The `update` command reports on the current update level and installs official updates. Official updates are updates to Stata as it was originally shipped from StataCorp, not the additions to Stata published in, for instance, the *Stata Journal* (SJ). Those additions are installed using the `net` command and updated using the `ado update` command; see [\[R\] net](#) and [\[R\] ado update](#).

`update` without arguments reports on the update level of the currently installed Stata.

`update from` sets an update source, which may be a directory name or URL. If you are on the internet, type `update from https://www.stata.com`.

`update query` compares the update level of the currently installed Stata with that available from the update source and displays a report.

`update all` updates all necessary files. This is what you should type to check for and install updates.

`set update_query` determines if `update query` is to be automatically performed when Stata is launched. Only Mac and Windows platforms can be set for automatic updating.

`set update_interval #` sets the number of days to elapse before performing the next automatic update query. The default # is 7. The interval starts from the last time an update query was performed (automatically or manually). Only Mac and Windows platforms can be set for automatic updating.

`set update_prompt` determines whether a dialog is to be displayed before performing an automatic update query. The dialog allows you to perform an update query now, perform one the next time Stata is launched, perform one after the next interval has passed, or disable automatic update query. Only Mac and Windows platforms can be set for automatic updating.

## Menu

Help > Check for updates

## Syntax

*Report on update level of currently installed Stata*

```
update
```

*Set update source*

```
update from location
```

*Compare update level of currently installed Stata with that of source*

```
update query [, from(location)]
```

*Perform update if necessary*

```
update all [, from(location) detail force exit]
```

*Set automatic updates (Mac and Windows only)*

```
set update_query { on | off }
```

```
set update_interval #
```

```
set update_prompt { on | off }
```

## Options

`from(location)` specifies the location of the update source. You can specify the `from()` option on the individual update commands or use the `update from` command. Which you do makes no difference. You typically do not need to use this option.

`detail` specifies to display verbose output during the update process.

`force` specifies to force downloading of all files even if, based on the date comparison, Stata does not think it is necessary. There is seldom a reason to specify this option.

`exit` instructs Stata to exit when the update has successfully completed. There is seldom a reason to specify this option.

## Remarks and examples

`update` updates the official components of Stata from the official source: <https://www.stata.com>. If you are connected to the internet, the easy thing to do is to type

```
. update all
```

and follow the instructions. If Stata is up to date, `update all` will do nothing. Otherwise, it will download whatever is necessary and install the files. If you just want to know what updates are available, type

```
. update query
```

update query will check if any updates are available and report that information. If updates are available, it will recommend that you type `update all`.

If you want to report the current update level, type

```
. update
```

update will report the update level of the Stata installation. update will also show you the date that updates were last checked and if any updates were available at that time.

## Stored results

update without a subcommand, update from, and update query store the following in `r()`:

### Scalars

|                                |                                                   |
|--------------------------------|---------------------------------------------------|
| <code>r(inst_exe)</code>       | date of executable installed (*)                  |
| <code>r(avbl_exe)</code>       | date of executable available over web (*) (**)    |
| <code>r(inst_ado)</code>       | date of ado-files installed (*)                   |
| <code>r(avbl_ado)</code>       | date of ado-files available over web (*) (**)     |
| <code>r(inst_utilities)</code> | date of utilities installed (*)                   |
| <code>r(avbl_utilities)</code> | date of utilities available over web (*) (**)     |
| <code>r(inst_docs)</code>      | date of documentation installed (*)               |
| <code>r(avbl_docs)</code>      | date of documentation available over web (*) (**) |

### Macros

|                               |                                                |
|-------------------------------|------------------------------------------------|
| <code>r(name_exe)</code>      | name of the Stata executable                   |
| <code>r(dir_exe)</code>       | directory in which executable is stored        |
| <code>r(dir_ado)</code>       | directory in which ado-files are stored        |
| <code>r(dir_utilities)</code> | directory in which utilities are stored        |
| <code>r(dir_docs)</code>      | directory in which PDF documentation is stored |

### Notes:

\* Dates are stored as integers counting the number of days since January 1, 1960; see [\[D\] Datetime](#).

\*\* These dates are not stored by update without a subcommand because update by itself reports information solely about the local computer and does not check what is available on the web.

## Also see

[\[R\] ado update](#) — Update community-contributed packages

[\[R\] net](#) — Install and manage community-contributed additions from the internet

[\[R\] ssc](#) — Install and uninstall packages from SSC

[\[P\] sysdir](#) — Query and set system directories

[\[U\] 29 Using the internet to keep up to date](#)

[\[GSM\] 19 Updating and extending Stata—internet functionality](#)

[\[GSU\] 19 Updating and extending Stata—internet functionality](#)

[\[GSW\] 19 Updating and extending Stata—internet functionality](#)

|                      |          |         |                      |
|----------------------|----------|---------|----------------------|
| Description          | Syntax   | Options | Remarks and examples |
| Methods and formulas | Also see |         |                      |

## Description

This entry describes the `vce()` option, which is common to most estimation commands. `vce()` specifies how to estimate the variance–covariance matrix (VCE) corresponding to the parameter estimates. The standard errors reported in the table of parameter estimates are the square root of the variances (diagonal elements) of the VCE.

## Syntax

*estimation\_cmd* ... [ , *vce(vcetype)* ... ]

| <i>vcetype</i>                                        | Description                                 |
|-------------------------------------------------------|---------------------------------------------|
| Likelihood based                                      |                                             |
| <code>oim</code>                                      | observed information matrix (OIM)           |
| <code>opg</code>                                      | outer product of the gradient (OPG) vectors |
| Sandwich estimators                                   |                                             |
| <code>r_robust</code>                                 | Huber/White/sandwich estimator              |
| <code>c_cluster</code> <i>clustvar</i>                | clustered sandwich estimator                |
| Replication based                                     |                                             |
| <code>bootstrap</code> [ , <i>bootstrap_options</i> ] | bootstrap estimation                        |
| <code>jackknife</code> [ , <i>jackknife_options</i> ] | jackknife estimation                        |

## Options

SE/Robust

`vce(oim)` is usually the default for models fit using maximum likelihood. `vce(oim)` uses the observed information matrix (OIM); see [R] [ml](#).

`vce(opg)` uses the sum of the outer product of the gradient (OPG) vectors; see [R] [ml](#). This is the default VCE when the `technique(bhhh)` option is specified; see [R] [Maximize](#).

`vce(robust)` uses the robust or sandwich estimator of variance. This estimator is robust to some types of misspecification so long as the observations are independent; see [U] [20.22 Obtaining robust variance estimates](#).

If the command allows `pweights` and you specify them, `vce(robust)` is implied; see [U] [20.24.3 Sampling weights](#).

`vce(cluster clustvar)` specifies that the standard errors allow for intragroup correlation, relaxing the usual requirement that the observations be independent. That is, the observations are independent across groups (clusters) but not necessarily within groups. *clustvar* specifies to which group each observation belongs, for example, `vce(cluster personid)` in data with repeated observations on individuals. `vce(cluster clustvar)` affects the standard errors and variance–covariance matrix of the estimators but not the estimated coefficients; see [U] 20.22 **Obtaining robust variance estimates**.

`vce(bootstrap [ , bootstrap_options ])` uses a bootstrap; see [R] **bootstrap**. After estimation with `vce(bootstrap)`, see [R] **bootstrap postestimation** to obtain percentile-based or bias-corrected confidence intervals.

`vce(jackknife [ , jackknife_options ])` uses the delete-one jackknife; see [R] **jackknife**.

## Remarks and examples

Remarks are presented under the following headings:

*Prefix commands*

*Passing options in vce()*

### Prefix commands

Specifying `vce(bootstrap)` or `vce(jackknife)` is often equivalent to using the corresponding prefix command. Here is an example using `jackknife` with `regress`.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. regress mpg turn trunk, vce(jackknife)
(running regress on estimation sample)
Jackknife replications (74):10.....20.....30.....40.....
> ..50.....60.....70.... done

Linear regression Number of obs = 74
 Replications = 74
 F(2, 73) = 66.26
 Prob > F = 0.0000
 R-squared = 0.5521
 Adj R-squared = 0.5395
 Root MSE = 3.9260
```

|       | Coefficient | Jackknife<br>std. err. | t     | P> t  | [95% conf. interval] |           |
|-------|-------------|------------------------|-------|-------|----------------------|-----------|
| turn  | -.7610113   | .150726                | -5.05 | 0.000 | -1.061408            | -.4606147 |
| trunk | -.3161825   | .1282326               | -2.47 | 0.016 | -.5717498            | -.0606152 |
| _cons | 55.82001    | 5.031107               | 11.09 | 0.000 | 45.79303             | 65.84699  |

```
. jackknife: regress mpg turn trunk
(running regress on estimation sample)
Jackknife replications (74):10.....20.....30.....40.....
> ..50.....60.....70.... done

Linear regression Number of obs = 74
 Replications = 74
 F(2, 73) = 66.26
 Prob > F = 0.0000
 R-squared = 0.5521
 Adj R-squared = 0.5395
 Root MSE = 3.9260
```

| mpg   | Coefficient | Jackknife<br>std. err. | t     | P> t  | [95% conf. interval] |           |
|-------|-------------|------------------------|-------|-------|----------------------|-----------|
| turn  | -.7610113   | .150726                | -5.05 | 0.000 | -1.061408            | -.4606147 |
| trunk | -.3161825   | .1282326               | -2.47 | 0.016 | -.5717498            | -.0606152 |
| _cons | 55.82001    | 5.031107               | 11.09 | 0.000 | 45.79303             | 65.84699  |

Here it does not matter whether we specify the `vce(jackknife)` option or instead use the `jackknife` prefix.

However, `vce(jackknife)` should be used in place of the `jackknife` prefix whenever available because they are not always equivalent. For example, to use the `jackknife` prefix with `clogit` properly, you must tell `jackknife` to omit whole groups rather than individual observations. Specifying `vce(jackknife)` does this automatically.

```
. use https://www.stata-press.com/data/r19/clogitid
. jackknife, cluster(id): clogit y x1 x2, group(id)
(output omitted)
```

This extra information is automatically communicated to `jackknife` by `clogit` when the `vce()` option is specified.

```
. clogit y x1 x2, group(id) vce(jackknife)
(running clogit on estimation sample)
Jackknife replications (66):10.....20.....30.....40.....
> ..50.....60..... done

Conditional (fixed-effects) logistic regression Number of obs = 369
 Replications = 66
 F(2, 65) = 4.58
 Prob > F = 0.0137
 Pseudo R2 = 0.0355

Log likelihood = -123.41386

(Replications based on 66 clusters in id)
```

| y  | Coefficient | Jackknife<br>std. err. | t    | P> t  | [95% conf. interval] |          |
|----|-------------|------------------------|------|-------|----------------------|----------|
| x1 | .653363     | .3010608               | 2.17 | 0.034 | .052103              | 1.254623 |
| x2 | .0659169    | .0487858               | 1.35 | 0.181 | -.0315151            | .1633489 |

## Passing options in vce()

If you wish to specify more options to the bootstrap or jackknife estimation, you can include them within the `vce()` option. Below, we request 300 bootstrap replications and save the replications in `bsreg.dta`:

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. regress mpg turn trunk, vce(bootstrap, nodots seed(123) rep(300) saving(bsreg))
Linear regression
```

```
Number of obs = 74
Replications = 300
Wald chi2(2) = 144.17
Prob > chi2 = 0.0000
R-squared = 0.5521
Adj R-squared = 0.5395
Root MSE = 3.9260
```

|       | Observed<br>coefficient | Bootstrap<br>std. err. | z     | P> z  | Normal-based<br>[95% conf. interval] |           |
|-------|-------------------------|------------------------|-------|-------|--------------------------------------|-----------|
| turn  | -.7610113               | .1497877               | -5.08 | 0.000 | -1.05459                             | -.4674329 |
| trunk | -.3161825               | .1286802               | -2.46 | 0.014 | -.5683909                            | -.063974  |
| _cons | 55.82001                | 4.9221                 | 11.34 | 0.000 | 46.17287                             | 65.46715  |

```
. bstat using bsreg
```

```
Bootstrap results
```

```
Number of obs = 74
Replications = 300
```

```
Command: regress mpg turn trunk
```

|       | Observed<br>coefficient | Bootstrap<br>std. err. | z     | P> z  | Normal-based<br>[95% conf. interval] |           |
|-------|-------------------------|------------------------|-------|-------|--------------------------------------|-----------|
| turn  | -.7610113               | .1497877               | -5.08 | 0.000 | -1.05459                             | -.4674329 |
| trunk | -.3161825               | .1286802               | -2.46 | 0.014 | -.5683909                            | -.063974  |
| _cons | 55.82001                | 4.9221                 | 11.34 | 0.000 | 46.17287                             | 65.46715  |

## Methods and formulas

By default, Stata's maximum likelihood estimators display standard errors based on variance estimates given by the inverse of the negative Hessian (second derivative) matrix. If `vce(robust)`, `vce(cluster clustvar)`, or `pweights` is specified, standard errors are based on the robust variance estimator (see [\[U\] 20.22 Obtaining robust variance estimates](#)); likelihood-ratio tests are not appropriate here (see [\[SVY\] Survey](#)), and the model  $\chi^2$  is from a Wald test. If `vce(opg)` is specified, the standard errors are based on the outer product of the gradients; this option has no effect on likelihood-ratio tests, though it does affect Wald tests.

If `vce(bootstrap)` or `vce(jackknife)` is specified, the standard errors are based on the chosen replication method; here the model  $\chi^2$  or  $F$  statistic is from a Wald test using the respective replication-based covariance matrix. The  $t$  distribution is used in the coefficient table when the `vce(jackknife)` option is specified. `vce(bootstrap)` and `vce(jackknife)` are also available with some commands that are not maximum likelihood estimators.

## Also see

[R] **bootstrap** — Bootstrap sampling and estimation

[R] **jackknife** — Jackknife estimation

[XT] *vce\_options* — Variance estimators

[U] **20 Estimation and postestimation commands**

## Description

`view` displays file contents in the Viewer.

`view file` displays the specified file. `file` is optional, so if you had a SMCL session log created by typing `log using mylog`, you could view it by typing `view mylog.smcl`. `view file` can properly display `.smcl` files (logs and the like), `.sthlp` files, and text files. `view file`'s `asis` option specifies that the file be displayed as plain text, regardless of the `filename`'s extension.

`view browse` opens your browser pointed to `url`. Typing `view browse https://www.stata.com` would bring up your browser pointed to the website <https://www.stata.com>.

`[view] help` displays the specified topic in the Viewer. For example, to review the help for Stata's print command, you could type `help print`. See [\[R\] help](#) for more details.

`[view] search` displays the results of the `search` command in the Viewer. For instance, to search the system help for information on robust regression, you could type `search robust regression`. See [\[R\] search](#) for more details.

`view net` does the same as the `net` command—see [\[R\] net](#)—but displays the result in the Viewer. For instance, typing `view net search hausman test` would search the internet for additions to Stata related to the Hausman test. Typing `view net from https://www.stata.com` would go to the Stata additions download site at <https://www.stata.com>.

`view ado` does the same as the `ado` command—see [\[R\] net](#)—but displays the result in the Viewer. For instance, typing `view ado dir` would show a list of files you have installed.

`view update` does the same as the `update` command—see [\[R\] update](#)—but displays the result in the Viewer. Typing `view update` would show the dates of what you have installed, and from there you could click to compare those dates with the latest updates available. Typing `view update query` would skip the first step and show the comparison.

## Menu

File > View...

## Syntax

*Display file in Viewer*

```
view [file] ["filename"] [, asis adopath]
```

*Bring up browser pointed to specified URL*

```
view browse ["url"]
```

*Display help results in Viewer*

```
[view] help [topic_or_command_name]
```

*Display search results in Viewer*

```
[view] search keywords
```

*Display net results in Viewer*

```
view net [netcmd]
```

*Display ado-results in Viewer*

```
view ado [adocmd]
```

*Display update results in Viewer*

```
view update [updatecmd]
```

## Options

*asis*, allowed with `view file`, specifies that the file be displayed as text, regardless of the *filename*'s extension. `view file`'s default action is to display files ending in `.smcl` and `.sthlp` as SMCL; see [P] [smcl](#).

*adopath*, allowed with `view file`, specifies that Stata search the `S_ADO` path for *filename* and display it, if found.

## Remarks and examples

Most users access the Viewer by selecting **File > View...** and proceeding from there. Some commands allow you to skip that step. Some common interactive uses of commands that display their results in the Viewer are the following:

```
. view mysession.smcl
. view mysession.log
. help print
. help regress
. search hausman test
. view net
. view ado
. view update query
```

## Also see

[R] [help](#) — Display help in Stata

[R] [net](#) — Install and manage community-contributed additions from the internet

[R] [search](#) — Search Stata documentation and other resources

[R] [update](#) — Check for official updates

[D] [type](#) — Display contents of a file

[GSM] [3 Using the Viewer](#)

[GSU] [3 Using the Viewer](#)

[GSW] [3 Using the Viewer](#)

|                             |                                      |                                |                                      |
|-----------------------------|--------------------------------------|--------------------------------|--------------------------------------|
| <a href="#">Description</a> | <a href="#">Quick start</a>          | <a href="#">Menu</a>           | <a href="#">Syntax</a>               |
| <a href="#">Options</a>     | <a href="#">Remarks and examples</a> | <a href="#">Stored results</a> | <a href="#">Methods and formulas</a> |
| <a href="#">References</a>  | <a href="#">Also see</a>             |                                |                                      |

## Description

`vwls` estimates a linear regression using variance-weighted least squares. It differs from ordinary least-squares (OLS) regression in that it does not assume homogeneity of variance, but requires that the conditional variance of *depvar* be estimated prior to the regression. The estimated variance need not be constant across observations. `vwls` treats the estimated variance as if it were the true variance when it computes standard errors of the coefficients.

You must supply an estimate of the conditional standard deviation of *depvar* to `vwls` by using the `sd(varname)` option, or you must have grouped data with the groups defined by the *indepvars* variables. In the latter case, `vwls` treats all *indepvars* as categorical variables, computes the mean and standard deviation of *depvar* separately for each subgroup, and computes the regression of the subgroup means on *indepvars*.

`regress` with analytic weights can be used to produce another kind of “variance-weighted least squares”; see [Remarks and examples](#) for an explanation of the difference.

## Quick start

Variance-weighted least-squares regression of *y* on *x1* and *x2*, with the estimated conditional std. dev. of *y* stored in `sd`

```
vwls y1 x1 x2, sd(sd)
```

Add categorical variable *a* using [factor-variable](#) syntax

```
vwls y1 x1 x2 i.a, sd(sd)
```

Same as above, but restrict the sample to cases where *v* is greater than 1

```
vwls y1 x1 x2 i.a if v>1, sd(sd)
```

Variance-weighted least-squares regression for grouped data with subgroups defined by *a2* and *a3*

```
vwls y2 i.a2 i.a3
```

## Menu

Statistics > Linear models and related > Other > Variance-weighted least squares

## Syntax

```
vwls depvar indepvars [if] [in] [weight] [, options]
```

| <i>options</i>                  | Description                                                                                                                                      |
|---------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| Model                           |                                                                                                                                                  |
| <code>noconstant</code>         | suppress constant term                                                                                                                           |
| <code>sd(<i>varname</i>)</code> | variable containing estimate of conditional standard deviation                                                                                   |
| Reporting                       |                                                                                                                                                  |
| <code>level(#)</code>           | set confidence level; default is level(95)                                                                                                       |
| <code>display_options</code>    | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| <code>coeflegend</code>         | display legend instead of statistics                                                                                                             |

*indepvars* may contain factor variables; see [U] 11.4.3 **Factor variables**.

`bootstrap`, `by`, `collect`, `jackknife`, `rolling`, and `statsby` are allowed; see [U] 11.1.10 **Prefix commands**.

Weights are not allowed with the `bootstrap` prefix; see [R] **bootstrap**.

`fweights` are allowed; see [U] 11.1.6 **weight**.

`coeflegend` does not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options

### Model

`noconstant`; see [R] **Estimation options**.

`sd(varname)` is an estimate of the conditional standard deviation of *depvar* (that is, it can vary observation by observation). All values of *varname* must be  $> 0$ . If you specify `sd()`, you cannot use `fweights`.

If `sd()` is not given, the data will be grouped by *indepvars*. Here *indepvars* are treated as categorical variables, and the means and standard deviations of *depvar* for each subgroup are calculated and used for the regression. Any subgroup for which the standard deviation is zero is dropped.

### Reporting

`level(#)`; see [R] **Estimation options**.

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `no1stretch`; see [R] **Estimation options**.

The following option is available with `vwls` but is not shown in the dialog box:

`coeflegend`; see [R] **Estimation options**.

## Remarks and examples

The `vwls` command is intended for use with two special—and different—types of data. The first contains data that consist of measurements from physical science experiments in which all error is due solely to measurement errors and the sizes of the measurement errors are known.

You can also use variance-weighted least-squares linear regression for certain problems in categorical data analysis, such as when all the independent variables are categorical and the outcome variable is either continuous or a quantity that can sensibly be averaged. If each of the subgroups defined by the categorical variables contains a reasonable number of subjects, then the variance of the outcome variable can be estimated independently within each subgroup. For the purposes of estimation, `vwls` treats each subgroup as one observation, with the dependent variable being the subgroup mean of the outcome variable.

The `vwls` command fits the model

$$y_i = \mathbf{x}_i\boldsymbol{\beta} + \varepsilon_i$$

where the errors  $\varepsilon_i$  are independent normal random variables with the distribution  $\varepsilon_i \sim N(0, \nu_i)$ . The independent variables  $\mathbf{x}_i$  are assumed to be known without error.

As described above, `vwls` assumes that you already have estimates  $s_i^2$  for the variances  $\nu_i$ . The error variance is not estimated in the regression. The estimates  $s_i^2$  are used to compute the standard errors of the coefficients; see [Methods and formulas](#) below.

In contrast, weighted OLS regression assumes that the errors have the distribution  $\varepsilon_i \sim N(0, \sigma^2/w_i)$ , where the  $w_i$  are known weights and  $\sigma^2$  is an unknown parameter that is estimated in the regression. This is the difference from variance-weighted least squares: in weighted OLS, the magnitude of the error variance is estimated in the regression using all the data.

### ▷ Example 1

An artificial, but informative, example illustrates the difference between variance-weighted least squares and weighted OLS.

We measure the quantities  $x_i$  and  $y_i$  and estimate that the standard deviation of  $y_i$  is  $s_i$ . We enter the data into Stata:

```
. use https://www.stata-press.com/data/r19/vwlsxmpl
. list
```

|    | x | y   | s  |
|----|---|-----|----|
| 1. | 1 | 1.2 | .5 |
| 2. | 2 | 1.9 | .5 |
| 3. | 3 | 3.2 | 1  |
| 4. | 4 | 4.3 | 1  |
| 5. | 5 | 4.9 | 1  |
| 6. | 6 | 6.0 | 2  |
| 7. | 7 | 7.2 | 2  |
| 8. | 8 | 7.9 | 2  |

Because we want observations with smaller variance to carry larger weight in the regression, we compute an OLS regression with analytic weights proportional to the inverse of the squared standard deviations:

```
. regress y x [aweight=s^(-2)]
(sum of wgt is 11.75)
```

| Source   | SS         | df | MS         | Number of obs | = | 8      |
|----------|------------|----|------------|---------------|---|--------|
| Model    | 22.6310183 | 1  | 22.6310183 | F(1, 6)       | = | 702.26 |
| Residual | .193355117 | 6  | .032225853 | Prob > F      | = | 0.0000 |
|          |            |    |            | R-squared     | = | 0.9915 |
|          |            |    |            | Adj R-squared | = | 0.9901 |
| Total    | 22.8243734 | 7  | 3.26062477 | Root MSE      | = | .17952 |

| y     | Coefficient | Std. err. | t     | P> t  | [95% conf. interval] |          |
|-------|-------------|-----------|-------|-------|----------------------|----------|
| x     | .9824683    | .0370739  | 26.50 | 0.000 | .8917517             | 1.073185 |
| _cons | .1138554    | .1120078  | 1.02  | 0.349 | -.1602179            | .3879288 |

If we compute a variance-weighted least-squares regression by using `vwls`, we get the same results for the coefficient estimates but very different standard errors:

```
. vwls y x, sd(s)
```

|                                            |  |  |  |               |   |        |
|--------------------------------------------|--|--|--|---------------|---|--------|
| Variance-weighted least-squares regression |  |  |  | Number of obs | = | 8      |
| Goodness-of-fit chi2(6) = 0.28             |  |  |  | Model chi2(1) | = | 33.24  |
| Prob > chi2 = 0.9996                       |  |  |  | Prob > chi2   | = | 0.0000 |

| y     | Coefficient | Std. err. | z    | P> z  | [95% conf. interval] |          |
|-------|-------------|-----------|------|-------|----------------------|----------|
| x     | .9824683    | .170409   | 5.77 | 0.000 | .6484728             | 1.316464 |
| _cons | .1138554    | .51484    | 0.22 | 0.825 | -.8952124            | 1.122923 |

Although the values of  $y_i$  were nicely linear with  $x_i$ , the `vwls` regression used the large estimates for the standard deviations to compute large standard errors for the coefficients. For weighted OLS regression, however, the scale of the analytic weights has no effect on the standard errors of the coefficients—only the relative proportions of the analytic weights affect the regression.

If we are sure of the sizes of our error estimates for  $y_i$ , using `vwls` is valid. However, if we can estimate only the relative proportions of error among the  $y_i$ , then `vwls` is not appropriate.



## ► Example 2

Let's now consider an example of the use of `vwls` with categorical data. Suppose that we have blood pressure data for  $n = 400$  subjects, categorized by gender and race (black or white). Here is a description of the data:

```
. use https://www.stata-press.com/data/r19/bp
. table gender race, statistic(mean bp) statistic(sd bp) statistic(freq)
> nformat(%8.1f)
```

|                    | Race  |       |       |
|--------------------|-------|-------|-------|
|                    | White | Black | Total |
| Gender             |       |       |       |
| Female             |       |       |       |
| Mean               | 117.1 | 118.5 | 117.8 |
| Standard deviation | 10.3  | 11.6  | 10.9  |
| Frequency          | 100.0 | 100.0 | 200.0 |
| Male               |       |       |       |
| Mean               | 122.1 | 125.8 | 124.0 |
| Standard deviation | 10.6  | 15.5  | 13.3  |
| Frequency          | 100.0 | 100.0 | 200.0 |
| Total              |       |       |       |
| Mean               | 119.6 | 122.2 | 120.9 |
| Standard deviation | 10.7  | 14.1  | 12.6  |
| Frequency          | 200.0 | 200.0 | 400.0 |

Performing a variance-weighted regression using vwls gives

```
. vwls bp gender race
Variance-weighted least-squares regression Number of obs = 400
Goodness-of-fit chi2(1) = 0.88 Model chi2(2) = 27.11
Prob > chi2 = 0.3486 Prob > chi2 = 0.0000
```

| bp     | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |          |
|--------|-------------|-----------|--------|-------|----------------------|----------|
| gender | 5.876522    | 1.170241  | 5.02   | 0.000 | 3.582892             | 8.170151 |
| race   | 2.372818    | 1.191683  | 1.99   | 0.046 | .0371631             | 4.708473 |
| _cons  | 116.6486    | .9296297  | 125.48 | 0.000 | 114.8266             | 118.4707 |

By comparison, an OLS regression gives the following result:

```
. regress bp gender race
Source	SS df MS Number of obs = 400
Model | 4485.66639 2 2242.83319 F(2, 397) = 15.24
Residual | 58442.7305 397 147.210908 Prob > F = 0.0000
-----|-----
Total | 62928.3969 399 157.71528 R-squared = 0.0713
 Adj R-squared = 0.0666
 Root MSE = 12.133
```

| bp     | Coefficient | Std. err. | t      | P> t  | [95% conf. interval] |          |
|--------|-------------|-----------|--------|-------|----------------------|----------|
| gender | 6.1775      | 1.213305  | 5.09   | 0.000 | 3.792194             | 8.562806 |
| race   | 2.5875      | 1.213305  | 2.13   | 0.034 | .2021938             | 4.972806 |
| _cons  | 116.4862    | 1.050753  | 110.86 | 0.000 | 114.4205             | 118.552  |

Note the larger value for the race coefficient (and smaller *p*-value) in the OLS regression. The assumption of homogeneity of variance in OLS means that the mean for black men pulls the regression line higher than in the vwls regression, which takes into account the larger variance for black men and reduces its effect on the regression.

## Stored results

vwls stores the following in `e()`:

### Scalars

|                         |                                    |
|-------------------------|------------------------------------|
| <code>e(N)</code>       | number of observations             |
| <code>e(df_m)</code>    | model degrees of freedom           |
| <code>e(chi2)</code>    | model $\chi^2$                     |
| <code>e(df_gf)</code>   | goodness-of-fit degrees of freedom |
| <code>e(chi2_gf)</code> | goodness-of-fit $\chi^2$           |
| <code>e(rank)</code>    | rank of $e(V)$                     |

### Macros

|                            |                                      |
|----------------------------|--------------------------------------|
| <code>e(cmd)</code>        | vwls                                 |
| <code>e(cmdline)</code>    | command as typed                     |
| <code>e(depvar)</code>     | name of dependent variable           |
| <code>e(properties)</code> | b V                                  |
| <code>e(predict)</code>    | program used to implement predict    |
| <code>e(asbalanced)</code> | factor variables fvset as asbalanced |
| <code>e(asobserved)</code> | factor variables fvset as asobserved |

### Matrices

|                   |                                              |
|-------------------|----------------------------------------------|
| <code>e(b)</code> | coefficient vector                           |
| <code>e(V)</code> | variance–covariance matrix of the estimators |

### Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

### Matrices

|                       |                                                                                                                       |
|-----------------------|-----------------------------------------------------------------------------------------------------------------------|
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, $p$ -values, and confidence intervals |
|-----------------------|-----------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

Let  $\mathbf{y} = (y_1, y_2, \dots, y_n)'$  be the vector of observations of the dependent variable, where  $n$  is the number of observations. When `sd()` is specified, let  $s_1, s_2, \dots, s_n$  be the standard deviations supplied by `sd()`. For categorical data, when `sd()` is not given, the means and standard deviations of  $y$  for each subgroup are computed, and  $n$  becomes the number of subgroups,  $\mathbf{y}$  is the vector of subgroup means, and  $s_i$  are the standard deviations for the subgroups.

Let  $\mathbf{V} = \text{diag}(s_1^2, s_2^2, \dots, s_n^2)$  denote the estimate of the variance of  $\mathbf{y}$ . Then the estimated regression coefficients are

$$\mathbf{b} = (\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})^{-1}\mathbf{X}'\mathbf{V}^{-1}\mathbf{y}$$

and their estimated covariance matrix is

$$\widehat{\text{Cov}}(\mathbf{b}) = (\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})^{-1}$$

A statistic for the goodness of fit of the model is

$$Q = (\mathbf{y} - \mathbf{Xb})' \mathbf{V}^{-1} (\mathbf{y} - \mathbf{Xb})$$

where  $Q$  has a  $\chi^2$  distribution with  $n - k$  degrees of freedom ( $k$  is the number of independent variables plus the constant, if any).

## References

- Gini, R., and J. Pasquini. 2006. [Automatic generation of documents](#). *Stata Journal* 6: 22–39.
- Grizzle, J. E., C. F. Starmer, and G. G. Koch. 1969. Analysis of categorical data by linear models. *Biometrics* 25: 489–504. <https://doi.org/10.2307/2528901>.

## Also see

- [R] [vwls postestimation](#) — Postestimation tools for vwls
- [R] [regress](#) — Linear regression
- [U] [11.1.6 weight](#)
- [U] [20 Estimation and postestimation commands](#)

## Postestimation commands

The following postestimation commands are available after vwls:

| Command                         | Description                                                                                       |
|---------------------------------|---------------------------------------------------------------------------------------------------|
| <a href="#">contrast</a>        | contrasts and ANOVA-style joint tests of parameters                                               |
| <a href="#">estat summarize</a> | summary statistics for the estimation sample                                                      |
| <a href="#">estat vce</a>       | variance–covariance matrix of the estimators (VCE)                                                |
| <a href="#">estimates</a>       | cataloging estimation results                                                                     |
| <a href="#">etable</a>          | table of estimation results                                                                       |
| <a href="#">forecast</a>        | dynamic forecasts and simulations                                                                 |
| <a href="#">hausman</a>         | Hausman’s specification test                                                                      |
| <a href="#">lincom</a>          | point estimates, standard errors, testing, and inference for linear combinations of parameters    |
| <a href="#">linktest</a>        | link test for model specification                                                                 |
| <a href="#">margins</a>         | marginal means, predictive margins, marginal effects, and average marginal effects                |
| <a href="#">marginsplot</a>     | graph the results from margins (profile plots, interaction plots, etc.)                           |
| <a href="#">nlcom</a>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters |
| <a href="#">predict</a>         | linear, censored, and truncated predictions                                                       |
| <a href="#">predictnl</a>       | point estimates, standard errors, testing, and inference for generalized predictions              |
| <a href="#">pwcompare</a>       | pairwise comparisons of parameters                                                                |
| <a href="#">test</a>            | Wald tests of simple and composite linear hypotheses                                              |
| <a href="#">testnl</a>          | Wald tests of nonlinear hypotheses                                                                |

# predict

## Description for predict

`predict` creates a new variable containing predictions such as linear predictions and standard errors.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, xb stdp]
```

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

## Options for predict

Main

---

`xb`, the default, calculates the linear prediction.

`stdp` calculates the standard error of the linear prediction.

# margins

## Description for margins

`margins` estimates margins of response for linear predictions.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [options]
```

| <i>statistic</i>  | Description                           |
|-------------------|---------------------------------------|
| <code>xb</code>   | linear prediction; the default        |
| <code>stdp</code> | not allowed with <code>margins</code> |

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [\[R\] margins](#).

## Also see

[\[R\] vwls](#) — Variance-weighted least squares

[\[U\] 20 Estimation and postestimation commands](#)

## Description

`which` looks for file *fname.ftype* along the S\_ADO path. If Stata finds the file, `which` displays the full path and filename, along with, if the file is text, all lines in the file that begin with “\*!” in the first column. If Stata cannot find the file, `which` issues the message “file not found along ado-path” and sets the return code to 111. *ftype* must be a file type for which Stata usually looks along the ado-path to find. Allowable *ftypes* are

```
.ado, .class, .dlg, .idlg, .sthlp, .ihlp, .hlp, .jar, .key, .maint, .mata, .mlib,
.mo, .mnu, .plugin, .png, .py, .scheme, .stbc, and .style
```

If *ftype* is omitted, `which` assumes `.ado`. When searching for `.ado` files, if Stata cannot find the file, Stata then checks to see if *fname* is a built-in Stata command, allowing for valid abbreviations. If it is, the message “built-in command” is displayed; if not, the message “command not found as either built-in or ado-file” is displayed and the return code is set to 111.

## Syntax

```
which fname[.ftype] [, all]
```

## Option

`all` forces `which` to report the location of all files matching the *fname.ftype* found along the search path. The default is to report just the first one found.

## Remarks and examples

If you write programs, you know that you make changes to the programs over time. If you are like us, you also end up with multiple versions of the program stored on your disk, perhaps in different directories. You may even have given copies of your programs to other Stata users, and you may not remember which version of a program you or your friends are using. The `which` command helps you solve this problem.

## ▷ Example 1

The `which` command displays the path for `filename.ado` and any lines in the code that begin with “\*!”. For example, we might want information about the `test` command, described in [R] `test`, which is an ado-file written by StataCorp. Here is what happens when we type `which test`:

```
. which test
C:\Program Files\Stata19\ado\base\t\test.ado
*! version 2.4.1 28jan2021
```

`which` displays the path for the `test.ado` file and also a line beginning with “\*!” that indicates the version of the file.

Do not confuse “version 2.4.1” above with the `version` command or the version of Stata. The “\*!” code indicates notes. In this case, the `*! version 2.4.1` line is merely a note to the author to identify the iteration (version) of the `test` program. It is an unfortunate coincidence that the word `version` is overloaded in this manner.

We do not need to be so formal. `which` will display anything typed after lines that begin with “\*!”. For instance, we might write `myprog.ado`:

```
. which myprog
.\myprog.ado
*! first written 1/03/2025
*! bug fix on 1/05/2025 (no variance case)
*! updated 1/24/2025 to include noconstant option
*! still suspicious if variable takes on only two values
```

It does not matter where in the program the lines beginning with `*!` are—`which` will list them (in particular, our “still suspicious” comment was buried about 50 lines down in the code). All that is important is that the `*!` marker appear in the first two columns of a line.

◀

## ▷ Example 2

If we type `which command`, where `command` is a built-in command rather than an ado-file, Stata responds with

```
. which summarize
built-in command: summarize
```

If `command` was neither a built-in command nor an ado-file, Stata would respond with

```
. which junk
command junk not found as either built-in or ado-file
r(111);
```

◀

## Also see

[P] [findfile](#) — Find file in path

[U] [17 Ado-files](#)

[Description](#)[Options](#)[Acknowledgments](#)[Quick start](#)[Remarks and examples](#)[References](#)[Menu](#)[Stored results](#)[Also see](#)[Syntax](#)[Methods and formulas](#)

## Description

`wildbootstrap` performs wild cluster bootstrap (WCB) inference for linear hypotheses about parameters in linear regression models. These hypotheses can be simple or composite. When the assumptions required for the consistency of the cluster-robust variance estimator (CRVE) do not hold, the WCB is a good alternative.

## Quick start

Estimate the WCB  $p$ -value and confidence interval (CI) for the coefficient on `x1` in a linear regression of `y` on `x1` with clusters identified in `cvar`

```
wildbootstrap regress y x1, cluster(cvar)
```

Same as above, but test whether the coefficients on `x1` and `x2` are equal in a regression of `y` on `x1` and `x2`

```
wildbootstrap regress y x1 x2, cluster(cvar) test(x1 = x2)
```

## Menu

Statistics > Resampling > Wild cluster bootstrap

## Syntax

```
wildbootstrap estimator deprvar [indepvars] [if] [in] [weight] [, options]
```

| <i>estimator</i>     | Description                                           |
|----------------------|-------------------------------------------------------|
| <code>regress</code> | linear regression                                     |
| <code>areg</code>    | linear regression with a large indicator-variable set |
| <code>xtreg</code>   | fixed-effects linear models                           |

| <i>options</i> | Description |
|----------------|-------------|
|----------------|-------------|

### Main

|                                     |                                                                                                      |
|-------------------------------------|------------------------------------------------------------------------------------------------------|
| <code>noconstant</code>             | suppress constant term; available only with estimator <code>regress</code>                           |
| <code>hascons</code>                | has user-supplied constant; available only with estimator <code>regress</code>                       |
| <code>absorb(<i>varname</i>)</code> | categorical variable to be absorbed; required by and available only with estimator <code>areg</code> |

### Statistics

|                                            |                                                |
|--------------------------------------------|------------------------------------------------|
| <code>coefficients(<i>coeflist</i>)</code> | perform inference for a subset of coefficients |
| <code>test(<i>testspec</i>)</code>         | specify linear test parameters                 |

### Bootstrap

|                                           |                                                                                                           |
|-------------------------------------------|-----------------------------------------------------------------------------------------------------------|
| <code>cluster(<i>clustvar</i>)</code>     | specify variable identifying clusters; required for estimators <code>regress</code> and <code>areg</code> |
| <code>pvalue(<i>pvalue</i>)</code>        | specify the <i>p</i> -value type; may be <code>equal</code> (the default) or <code>symmetric</code>       |
| <code>errorweight(<i>wcbwtype</i>)</code> | specify WCB weight; default is <code>errorweight</code> ( <code>rademacher</code> )                       |
| <code>reps(#)</code>                      | set number of bootstrap repetitions; default is <code>reps(1000)</code>                                   |
| <code>rseed(#   <i>statecode</i>)</code>  | set random-number seed to <code>#</code> or <code>statecode</code>                                        |
| <code>blocksize(#)</code>                 | set bootstrap repetition block size; default is <code>min(reps(#), 1000)</code>                           |
| <code>cistop(largest   first)</code>      | specify stopping rule for CI computation                                                                  |

### Reporting

|                              |                                                         |
|------------------------------|---------------------------------------------------------|
| <code>level(#)</code>        | set confidence level; default is <code>level(95)</code> |
| <code>display_options</code> | control column formats and display of CIs               |

*indepvars* and *coeflist* may contain factor variables; see [U] 11.4.3 **Factor variables**.

*deprvar*, *indepvars*, and *coeflist* may contain time-series operators; see [U] 11.4.4 **Time-series varlists**.

`collect` is allowed; see [U] 11.1.10 **Prefix commands**.

Any weight that is allowed by the estimator is allowed; see [U] 11.1.6 **weight**.

## Options

### Main

`noconstant` and `hascons`; see [R] **regress**. These options may be specified only when estimator `regress` is specified.

`absorb(varname)`; see [R] **areg**. This option must be specified when estimator `areg` is specified.

## Statistics

`coefficients` (*coeflist*) performs an inference for a subset of coefficients. It reports the bootstrap  $p$ -value for a test of the subset of coefficients, *coeflist*, equal to 0 and the bootstrap CI. The default is to perform inference for all variables specified in *indepvars*. `coefficients()` may not be specified with `test()`.

`test` (*testspec*) specifies a linear test. You may also specify multiple linear tests by using `test((testspec1) (testspec2) ...)`. The test specification must be consistent with specifying a linear constraint. See [R] [test](#) and [R] [constraint](#).

## Bootstrap

`cluster` (*clustvar*) specifies the variable identifying the cluster groups. `cluster()` is required with estimators `regress` and `areg`. With estimator `xtreg`, *clustvar* defaults to the `xtset panelvar`.

`ptype` (*ptype*) specifies the  $p$ -value criterion: symmetric (`symmetric`) or equal tailed (`equal`). The default is `ptype(equal)`. See [Methods and formulas](#) for more details.

`errorweight` (*wcbwtype*) specifies the type of wild weight. *wcbwtype* may be one of the following:

| <i>wcbwtype</i>         | Description                                                                                                                                  |
|-------------------------|----------------------------------------------------------------------------------------------------------------------------------------------|
| <code>rademacher</code> | two-point distribution assigns values 1 and $-1$ with equal probability; the default                                                         |
| <code>mammen</code>     | two-point distribution assigns value $1 - \phi$ with probability $\phi/\sqrt{5}$ and value $\phi$ otherwise, where $\phi = (1 + \sqrt{5})/2$ |
| <code>webb</code>       | six-point distribution assigns probability of $1/6$ to the points $\pm\sqrt{1/2}$ , $\pm 1$ , and $\pm\sqrt{3/2}$                            |
| <code>normal</code>     | standard normal distribution                                                                                                                 |
| <code>gamma</code>      | gamma distribution with shape parameter 4 and scale parameter $1/2$ centered on its mean of 2                                                |

`reps` (*#*) sets the number of repetitions for the bootstrap. The default is `reps(1000)`. For the `ptype(equal)` option, the values of `level()` and `reps()` should be chosen so that  $\alpha/2 \times \text{reps}(\#)$  is an integer, where  $\alpha = (100 - \text{level}(\#))/100$ . For the `ptype(symmetric)` option, such values should be chosen so that  $\alpha \times \text{reps}(\#)$  is an integer. When the product is not an integer, the number of repetitions is increased so that it is. Integer values improve the search efficiency of the `wildbootstrap` algorithm.

`rseed` (*#* | *statecode*) sets the random-number seed to *#* or *statecode*. See [R] [set seed](#).

`blocksize` (*#*) sets the bootstrap block size. This is an integer less than or equal to `reps(#)` and is used to reduce the amount of memory the bootstrap computation will consume. The default is `min(reps(#), 1000)`.

`cistop` (*largest* | *first*) specifies the stopping rule for the CI computation. The bootstrap distribution is a step function, so for each bound, there is an interval of values that meet the CI level criterion. `cistop(largest)`, the default, specifies that the largest value within the interval be selected. `cistop(first)` specifies that the first value the algorithm finds within the interval be selected; therefore, specifying `cistop(first)` will result in faster CI computation. The `cistop()` option may not be combined with the `noci` option.

## Reporting

`level(#)`; see [R] [Estimation options](#). The `level()` option will not work on replay because CIs are based on estimator-specific enumerations. To change the confidence level, you must refit the model.

`display_options`: `noci`, `cformat(%fmt)`, `pformat(%fmt)`, and `sformat(%fmt)`; see [R] [Estimation options](#). The `noci` option may not be combined with the `cistop()` option.

## Remarks and examples

`wildbootstrap` implements the WCB, which was proposed by [Cameron, Gelbach, and Miller \(2008\)](#). It is an extension of the original wild bootstrap procedure proposed by [Wu \(1986\)](#), which was designed to work well for models with heteroskedasticity, to the case of cluster-level correlation. The wild bootstrap has proven to work well in cases where cluster-robust standard errors do not perform well. A good discussion of the methodology can be found in [Cameron and Miller \(2015\)](#), [MacKinnon \(2019\)](#), [MacKinnon and Webb \(2018\)](#), and [MacKinnon, Nielsen, and Webb \(2023\)](#), and the references therein. Specifically, the WCB is a good inference tool when one or more of the CRVE *t*-statistic consistency assumptions are violated. [MacKinnon and Webb](#) list the assumptions as follows:

1. The number of clusters goes to infinity.
2. The within-cluster error correlations are the same for all clusters.
3. Each cluster contains an equal number of observations.

Below, we illustrate how to use `wildbootstrap`; however, note that alternatives exist in the literature to address the inference problems noted above. For example, the [Bell and McCaffrey \(2002\)](#) *t*-statistic degrees-of-freedom correction is an alternative to `wildbootstrap` when at least one of the above assumptions is violated. The degrees-of-freedom correction is computed with option `vce(hc2 clustvar, dfadjust)` for `regress`, `areg`, and `xtreg`, `fe`.

### ► Example 1: Simple regression

Say we are interested in the effect of tenure on wages for a panel of individuals sampled from 2013 to 2016. We would like to use wild bootstrap CIs clustering at the `personid` level. For reproducibility, we set the seed by using option `rseed()`.

```
. use https://www.stata-press.com/data/r19/wagework
(Wages for 20 to 77 year olds, 2013–2016)
. wildbootstrap regress wage tenure, cluster(personid) rseed(12345)
Performing 1,000 replications for p-value for tenure = 0 ...
Computing confidence interval for tenure
 Lower bound:10.....20. done (21)
 Upper bound:10..... done (17)

Wild cluster bootstrap Number of obs = 1,928
Linear regression Number of clusters = 589
 Cluster size:
Cluster variable: personid min = 1
Error weight: Rademacher avg = 3.3
 max = 4
```

|            | wage       | Estimate | t     | p-value | [95% conf. interval] |          |
|------------|------------|----------|-------|---------|----------------------|----------|
| constraint | tenure = 0 | .7807403 | 27.19 | 0.000   | .7209754             | .8368386 |

The iteration log indicates the number of iterations used to compute the lower and upper bound for the CIs. In *Methods and formulas* below, we discuss how these bounds are computed. Notably, there is a separate optimization procedure used to compute each one of these bounds.

The table header also tells us the error weights used for the sampling algorithm of the wild bootstrap, which, by default, are Rademacher weights. See the `errorweight()` option for more details about error weights.

The column header labeled *p*-value indicates that the *t*-statistic equal-tailed *p*-value has been computed. The `ptype(equal)` option is the default. Alternatively, the symmetric *p*-value is computed when the `ptype(symmetric)` option is specified and is identified with the column header of `P>|t|`.

```
. wildbootstrap regress wage tenure, cluster(personid) rseed(12345)
> ptype(symmetric)
Performing 1,000 replications for p-value for tenure = 0 ...
Computing confidence interval for tenure
 Lower bound:10.....20.. done (22)
 Upper bound:10..... done (15)

Wild cluster bootstrap
Linear regression
Cluster variable: personid
Error weight: Rademacher
Number of obs = 1,928
Number of clusters = 589
Cluster size:
min = 1
avg = 3.3
max = 4
```

|            | wage       | Estimate | t     | P> t  | [95% conf. interval] |
|------------|------------|----------|-------|-------|----------------------|
| constraint | tenure = 0 | .7807403 | 27.19 | 0.000 | .7240502 .8399896    |

We can always compare the CIs from the wild bootstrap with what we would have obtained using the underlying command, in this case, `regress`.

```
. regress
Linear regression
Number of obs = 1,928
F(1, 588) = 739.36
Prob > F = 0.0000
R-squared = 0.4212
Root MSE = 3.5097
(Std. err. adjusted for 589 clusters in personid)
```

| wage   | Coefficient | Robust std. err. | t     | P> t  | [95% conf. interval] |
|--------|-------------|------------------|-------|-------|----------------------|
| tenure | .7807403    | .028713          | 27.19 | 0.000 | .7243477 .8371328    |
| _cons  | 20.89884    | .2135686         | 97.86 | 0.000 | 20.47939 21.31829    |

Note that typing this command will replace the return matrix `r(table)`. Observe that the *t* statistics are the same in both tables because `regress` and `wildbootstrap` use the same CRVE, but the *p*-values and CIs may vary between tables.

Similarly, you can redisplay the `wildbootstrap` table by typing `wildbootstrap`, which may be abbreviated as `wildboot`.

▷ Example 2: Small number of clusters with wildly varying cluster sizes

As in [example 1](#), we would like to see the effect of tenure on wages; in this case, however, we would like to cluster at the industry level. Here, for year 1988, we use a wage dataset with only 12 clusters, for which cluster sizes vary wildly from 4 to 817, violating the CRVE  $t$ -statistic consistency [assumptions 1 and 3](#) outlined previously.

```
. use https://www.stata-press.com/data/r19/nlsw88
(NLSW, 1988 extract)
. wildbootstrap regress wage tenure, cluster(industry) rseed(12345)
Performing 1,000 replications for p-value for tenure = 0 ...
Computing confidence interval for tenure
 Lower bound:10.....20..... done (26)
 Upper bound:10.....20.... done (24)
Wild cluster bootstrap Number of obs = 2,217
Linear regression Number of clusters = 12
 Cluster size:
Cluster variable: industry min = 4
Error weight: Rademacher avg = 184.8
 max = 817
```

|            | wage       | Estimate | t    | p-value | [95% conf. interval] |          |
|------------|------------|----------|------|---------|----------------------|----------|
| constraint | tenure = 0 | .1830716 | 6.95 | 0.000   | .1274023             | .3258156 |

## ▷ Example 3: Small number of clusters with more regressors

Continuing with [example 2](#), we look at a more complex and perhaps more realistic example. We add to the model explanatory variables for total work experience, `ttl_exp`; a college graduate indicator, `collgrad`; and an indicator for union membership, `union`.

```
. wildbootstrap regress wage c.tenure#c.ttl_exp ib0.collgrad ib0.union,
> cluster(industry) rseed(12345)

Performing 1,000 replications for p-value for tenure = 0 ...
Computing confidence interval for tenure
 Lower bound:10.....20.....30.... done (34)
 Upper bound:10.....20.... done (24)

Performing 1,000 replications for p-value for ttl_exp = 0 ...
Computing confidence interval for ttl_exp
 Lower bound:10.....20.... done (25)
 Upper bound:10..... done (19)

Performing 1,000 replications for p-value for c.tenure#c.ttl_exp = 0 ...
Computing confidence interval for c.tenure#c.ttl_exp
 Lower bound:10.....20... done (23)
 Upper bound:10.....20..... done (27)
note: upper-bound CI achieved $1-F(-3.65e-04) = 0.0240$, but target is $1-F(x) = .025$.
note: the sorted bootstrap t statistics have at least two tied values
 adjacent to the t statistic under the null; this prevents the CI bound
 from converging to the target.

Performing 1,000 replications for p-value for 1.collgrad = 0 ...
Computing confidence interval for 1.collgrad
 Lower bound:10.....20.....30.. done (32)
note: lower-bound CI achieved $F(1.66) = 0.0240$, but target is $F(x) = .025$.
note: at least one bootstrap t statistic matches the t statistic under the
 null; this prevents the CI bound from converging to the target.
 Upper bound:10.....20..... done (26)

Performing 1,000 replications for p-value for 1.union = 0 ...
Computing confidence interval for 1.union
 Lower bound:10.....20.....30 done (30)
 Upper bound:10.....20..... done (25)

Wild cluster bootstrap Number of obs = 1,855
Linear regression Number of clusters = 12
 Cluster size:
Cluster variable: industry min = 2
Error weight: Rademacher avg = 154.6
 max = 717
```

|             | wage                   | Estimate  | t     | p-value | [95% conf. interval] |           |
|-------------|------------------------|-----------|-------|---------|----------------------|-----------|
| constraints |                        |           |       |         |                      |           |
|             | tenure = 0             | .204166   | 2.81  | 0.026   | .0729495             | .4699121  |
|             | ttl_exp = 0            | .3025249  | 11.72 | 0.004   | .2566267             | .4110114  |
|             | c.tenure#c.ttl_exp = 0 | -.0097942 | -2.76 | 0.046   | -.022061             | -.0003651 |
|             | 1.collgrad = 0         | 3.077377  | 7.59  | 0.034   | 1.662277             | 5.218396  |
|             | 1.union = 0            | .9114564  | 2.31  | 0.040   | .1060322             | 2.470924  |

We did not specify the `coefficients()` option, so `wildbootstrap` computes  $p$ -values and CIs for all coefficients in the model, with the exception of the constant term, using the default `ptype(equal)` option to compute the equal-tailed  $p$ -values.

The iteration log states that the CI-target tail values were not achieved for two of the reported constraints. This is because the sorted vector of bootstrap  $t$  statistics can have ties due to computer finite numeric precision and the finite number of draws for the Rademacher, Mammen, and Webb distributions. When ties occur with the  $t$  statistic under the null, or those adjacent to it, and the desired CI tail area cannot be achieved, `wildbootstrap` will choose a bound that results in a smaller tail area. This is the case when `wildbootstrap` searches for the `c.tenure#c.ttl_exp` CI upper bound and the `1.collgrad` CI lower bound.

Numerical computations will contain a roundoff error; `wildbootstrap` retains 13 digits on the mantissa of the  $t$  statistics before making comparisons.

◀

#### ▷ Example 4: Linear regression with an indicator-variable set

We would like to include an individual's occupation code (`occupation`) as an additional control in the regression to avoid potential omitted-variable bias. We keep the other control variables we introduced in [example 3](#). For this analysis, we use the estimator `areg` with the `absorb(occupation)` option. We use the `test()` option to test linear combinations of regression estimates.

```
. wildbootstrap areg wage c.tenure##c.ttl_exp ib0.collgrad ib0.union,
> absorb(occupation) cluster(industry) rseed(12345)
> test((tenure=ttl_exp) (1.collgrad-1.union=1)) reps(1250)
note: for equal-tailed 95% CI, better performance is obtained when
 .025*reps() is an integer.
note: setting repetitions to 1,280.
Performing 1,280 replications for p-value for constraint
 tenure - ttl_exp = 0 ...
Computing confidence interval for tenure - ttl_exp
 Lower bound:10.....20... done (23)
 Upper bound:10.....20.... done (25)
note: upper-bound CI achieved 1-F(0.15) = 0.0242, but target is 1-F(x) =
 .025.
note: the sorted bootstrap t statistics have at least two tied values
 adjacent to the t statistic under the null; this prevents the CI bound
 from converging to the target.
Performing 1,280 replications for p-value for constraint
 1.collgrad - 1.union = 1 ...
Computing confidence interval for 1.collgrad - 1.union
 Lower bound:10.....20..... done (26)
 Upper bound:10.....20..... done (27)
note: upper-bound CI achieved 1-F(3.91) = 0.0242, but target is 1-F(x) =
 .025.
note: the sorted bootstrap t statistics have at least two tied values
 adjacent to the t statistic under the null; this prevents the CI bound
 from converging to the target.

Wild cluster bootstrap Number of obs = 1,851
Linear regression, absorbing indicators Number of clusters = 12
 Cluster size:
Cluster variable: industry min = 2
Error weight: Rademacher avg = 154.2
 max = 717
```

|                          | wage | Estimate   | t     | p-value | [95% conf. interval] |
|--------------------------|------|------------|-------|---------|----------------------|
| constraints              |      |            |       |         |                      |
| tenure - ttl_exp = 0     |      | -0.0391243 | -0.71 | 0.517   | -.1915527 .1495055   |
| 1.collgrad - 1.union = 1 |      | 1.356332   | 0.77  | 0.395   | -.1882941 3.913359   |

The `test()` option allows us to specify a linear combination of the regression coefficients that we would like to test. The syntax is the same as specifying linear constraints, `constraint`, or linear hypothesis tests, `test`. In this example, we are inquiring how probable it would be for `tenure = ttl_exp` and `1.collgrad - 1.union = 1`.

The `reps()` option allows us to set the number of bootstrap repetitions. The default is 1,000. Because the Rademacher and Mammen distributions have two possible realizations, the maximum number of possible bootstrap samples is  $2^G$ , where  $G$  is the number of clusters. The Webb distribution, on the other hand, has six outcomes and therefore  $6^G$  possible combinations.

In this example, we intentionally specify a number of bootstrap repetitions that results in a non-integer product, `reps() × α/2`: `reps() × (100 - level())/200 = 1250 × 0.025 = 31.25`. In this case, we lose efficiency in searching for the CI bounds, so `wildbootstrap` adjusts the number of repetitions to 1,280; when we specify `reps(1250)` with `level(95)`, `wildbootstrap` chooses `reps() = ceil(1250 × α/2)/(α/2) = 32/0.025 = 1280`. If an integer alternative cannot be found, the original `reps()` specification will be used. For details, see *Methods and formulas* below.

Because the WCB distribution is a step function, there is a range of values for each CI bound. For example, the lower-bound interval of values for the linear combination `tenure - ttl_exp` is  $-0.191586 < a_{lb} \leq -0.191553$ . The reported bounds are the largest value of each interval. We obtain  $-0.191586$  by running `wildbootstrap` specifying `level(#)` with  $\# = (0.95 + 2/1280) \times 100$ . We would likely report the CIs using three digits of precision, for instance,  $[-0.192, 0.150]$ , so this interval is negligible but significant in a numerical root search where a convergence tolerance for  $r_l$  might be  $10^{-8}$ .

The estimate of `tenure - ttl_exp` is reported to be  $-0.0391$  with a 95% CI of  $[-0.192, 0.150]$ . We therefore fail to reject the hypothesis that `tenure = ttl_exp`. Similarly, the estimate of `1.collgrad - 1.union` is 1.36 with a CI of  $[-0.188, 3.91]$ , and we conclude the difference of 1 is feasible.

◀

### ▷ Example 5: Fixed-effects linear regression with panels

Our final example demonstrates the use of `wildbootstrap` with estimator `xtreg`. Continuing with the data in the [previous example](#), we use `industry` as the variable defining the panels.

We specify normal error weights, `errorweight(normal)`, thereby reducing the chance of identical draws for the error weights. Draws from the Rademacher distribution for 12 clusters have  $2^{12} = 4096$  combinations.

We compute  $p$ -values and CIs on a subset of the model estimates: work tenure, work experience, and their interaction. If we replay the `xtreg` coefficient table, we will see that the fitted model includes `tenure`, `ttl_exp`, `c.tenure#c.ttl_exp`, `1.collgrad`, and `1.union`.

Finally, we use the symmetric  $p$ -value, option `pctype(symmetr ic)`, instead of the default equal-tailed, `pctype(equal)`. The table identifies the symmetric  $p$ -value with the header `P>|t|`.

The command and the results are as follows:

```
. xtset industry
Panel variable: industry (unbalanced)
. wildbootstrap xtreg wage c.tenure##c.ttl_exp i.collgrad i.union,
> rseed(12345) coef(ten ttl ten#ttl) errorweight(normal)
> ptype(symmetric)
Panel variable: industry (unbalanced)
Performing 1,000 replications for p-value for constraint
tenure = 0 ...
Computing confidence interval for tenure
Lower bound:10.....20..... done (26)
Upper bound:10.....20. done (21)
Performing 1,000 replications for p-value for constraint
ttl_exp = 0 ...
Computing confidence interval for ttl_exp
Lower bound:10.....20..... done (26)
Upper bound:10..... done (18)
Performing 1,000 replications for p-value for constraint
c.tenure#c.ttl_exp = 0 ...
Computing confidence interval for c.tenure#c.ttl_exp
Lower bound:10.....20..... done (26)
Upper bound:10.....20... done (23)

Wild cluster bootstrap Number of obs = 1,855
Fixed-effects linear regression Number of clusters = 12
 Cluster size:
Cluster variable: industry min = 2
Error weight: Normal avg = 154.6
 max = 717
```

|             | wage                   | Estimate  | t     | P> t  | [95% conf. interval] |           |
|-------------|------------------------|-----------|-------|-------|----------------------|-----------|
| constraints |                        |           |       |       |                      |           |
|             | tenure = 0             | .2026682  | 2.95  | 0.024 | .0555676             | .3753468  |
|             | ttl_exp = 0            | .2716375  | 11.59 | 0.000 | .2227054             | .3372929  |
|             | c.tenure#c.ttl_exp = 0 | -.0104125 | -2.55 | 0.046 | -.0201336            | -.0003384 |

The fixed-effects model is the only xtreg model allowed with wildbootstrap, so specifying the fe option is not required.

Also note that the cluster() option is not specified. When the estimator xtreg is specified, the default cluster variable is the panel variable that is xtset. If the specified cluster variable is different from the panel variable, then the levels of the panel variable must be nested within the cluster variable levels.

## Stored results

wildbootstrap stores the following in `e()`:

### Scalars

|                          |                                             |
|--------------------------|---------------------------------------------|
| <code>e(N_clust)</code>  | number of clusters                          |
| <code>e(N_wbrep)</code>  | number of bootstrap repetitions             |
| <code>e(wb_block)</code> | bootstrap block size                        |
| <code>e(n_wbcns)</code>  | number of bootstrap constraint restrictions |
| <code>e(min_c)</code>    | smallest cluster size                       |
| <code>e(max_c)</code>    | largest cluster size                        |
| <code>e(avg_c)</code>    | average cluster size                        |

### Macros

|                                      |                                                       |
|--------------------------------------|-------------------------------------------------------|
| <code>e(cmdline)</code>              | command as typed                                      |
| <code>e(cmd0)</code>                 | wildbootstrap                                         |
| <code>e(wb_stat)</code>              | WCB statistic, $t$                                    |
| <code>e(wb_weight)</code>            | WCB weights                                           |
| <code>e(wb_ptype)</code>             | WCB $p$ -value criterion                              |
| <code>e(wb_level)</code>             | WCB CI level                                          |
| <code>e(wb_cistop)</code>            | WCB CI interval type                                  |
| <code>e(wb_rseed)</code>             | random-number state                                   |
| <code>e(wb_cns<math>i</math>)</code> | WCB constraint $i$ , where $i = 1, \dots, e(n_wbcns)$ |
| <code>e(clustvar)</code>             | name of cluster variable                              |

### Matrices

|                        |                       |
|------------------------|-----------------------|
| <code>e(wboot)</code>  | WCB table             |
| <code>e(wb_pci)</code> | WCB CI coverage       |
| <code>e(wb_Cns)</code> | WCB constraint matrix |

wildbootstrap will also carry forward most of the results already in `e()` from *command*.

In addition to the above, the following is stored in `r()`:

### Matrices

|                       |                                                                                                                       |
|-----------------------|-----------------------------------------------------------------------------------------------------------------------|
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, $p$ -values, and confidence intervals |
|-----------------------|-----------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

Methods and formulas are presented under the following headings:

[Introduction](#)

[CIs for linear combinations of coefficients](#)

[Constructing a CI inverting the hypothesis test](#)

## Introduction

We will focus our discussion of the WCB on linear regression, which easily extends to the other fixed-effects estimators. A linear regression model with clustered errors and  $G$  clusters can be written as

$$\mathbf{y} = \begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_G \end{pmatrix} = \mathbf{X}\beta + \epsilon = \begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \mathbf{X}_G \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{pmatrix} + \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_G \end{pmatrix} \quad (1)$$

where for each cluster  $g$ ,  $N_g$  is the number of observations,  $y_g$  is an  $N_g \times 1$  vector of outcomes,  $X_g$  is an  $N_g \times k$  matrix of covariates, and  $\epsilon_g$  is an  $N_g \times 1$  vector of errors. The parameter of interest is the  $k \times 1$  vector of coefficients  $\beta$ . Error terms are assumed uncorrelated between observations of different clusters but possibly correlated between observations within the same cluster.

Without loss of generality, we will first focus on testing the null hypothesis  $H_0: \beta_k = 0$  using the WCB algorithm. Let  $\hat{\beta}$  denote the ordinary least-squares estimator for  $\beta$ . We compute the  $t$  statistic for the  $k$ th coefficient as

$$t_k = \frac{\hat{\beta}_k}{\sqrt{\widehat{\mathbf{V}}_{k,k}}}$$

where  $\widehat{\mathbf{V}}_{k,k}$  is the CRVE for  $\hat{\beta}_k$ . This is the  $k$ th diagonal element of the matrix

$$\widehat{\mathbf{V}} = \frac{G(N-1)}{(G-1)(N-k)} (\mathbf{X}\mathbf{X}')^{-1} \left( \sum_{g=1}^G \mathbf{X}'_g \hat{\epsilon}_g \hat{\epsilon}'_g \mathbf{X}_g \right) (\mathbf{X}\mathbf{X}')^{-1} \quad (2)$$

where  $\hat{\epsilon}_g$  is the  $N_g \times 1$  vector of ordinary least-squares residuals for cluster  $g$  and  $N = \sum_{g=1}^G N_g$  is the total number of observations.

The WCB algorithm proceeds as follows:

1. Refit model (1) subject to the restriction  $\beta_k = 0$ . Let  $\tilde{\beta}$  denote the restricted estimates and  $\tilde{\epsilon}$  denote the restricted residuals.
2. For each individual bootstrap replication  $b$  (out of a total of  $B$  replications):
  - (a) Generate random variable  $\nu_g^b$  for each cluster  $g$  according to the distribution specified in the `errorweight()` option.
  - (b) For each cluster  $g$  and each observation in the cluster  $i = 1, 2, \dots, N_g$ , generate a new bootstrap-dependent variable  $y_{ig}^b$  using the data-generating process:

$$y_{ig}^b = X_{ig} \tilde{\beta} + \tilde{\epsilon}_{ig} \nu_g^b$$

- (c) Fit model (1) using the bootstrap variable  $y_{ig}^b$  as regressand. Calculate the  $t$  statistic for  $\beta_k = 0$ ,

$$t_k^b = \frac{\hat{\beta}_k^b}{\sqrt{\widehat{\mathbf{V}}_{k,k}^b}}$$

where  $\hat{\beta}_k^b$  and  $\widehat{\mathbf{V}}_{k,k}^b$  are the ordinary least-squares coefficient and the CRVE for that coefficient in the bootstrap replication, respectively. In this case, the CRVE is the  $k$ th diagonal element of matrix (2) obtained when using the residuals from bootstrap replication  $b$ .

3. The  $p$ -values for the one-sided alternative hypotheses  $H_1: \beta_k > 0$  and  $H_2: \beta_k < 0$  are given by

$$p_1 = \frac{1}{B} \sum_{b=1}^B I(t_k^b > t_k)$$

$$p_2 = \frac{1}{B} \sum_{b=1}^B I(t_k^b < t_k)$$

For the alternative hypothesis  $H_3: \beta_k \neq 0$ , the  $p$ -value assuming that the distribution of the  $t$  statistic is symmetric around 0 is given by

$$p_S = \frac{1}{B} \sum_{b=1}^B I(|t_k^b| > |t_k|)$$

If the assumption of symmetry is not appropriate, then the  $p$ -value is given by the equal-tailed  $p$ -value  $p_e = 2 \min(p_1, p_2)$ . See [Djogbenou, MacKinnon, and Nielsen \(2019\)](#).

To increase computational speed, at the expense of computer memory usage, the `wildbootstrap` command uses the matrix algebra of [Roodman et al. \(2019\)](#). By doing this, the computational complexity (run time) of the WCB algorithm is reduced from the order  $O(NB)$  to the order  $O(GB)$ . The WCB uses a  $G \times B$  matrix of random variables. By organizing these variables in column-major order, `wildbootstrap` reduces overall memory usage with minimal extra computation by breaking up the number of bootstrap replicates into blocks (the `blocksize(#)` option).

## CIs for linear combinations of coefficients

By inverting hypotheses tests, we can apply the WCB algorithm to find CIs for any linear combination of coefficients. Suppose we wanted to compute a CI for the linear combination of parameters  $\mathbf{R}\beta$ , where  $\mathbf{R}$  is a  $1 \times k$  vector. In this case, the associated null hypothesis is  $H_0: \mathbf{R}\beta = r$ , where  $r$  is an arbitrary scalar. We can test this null hypothesis with the WCB algorithm described in the previous section by using the bootstrap  $t$  statistics

$$t^b(r) = \frac{\mathbf{R}\hat{\beta}^b - r}{\sqrt{\mathbf{R}\hat{\mathbf{V}}^b\mathbf{R}'}}$$

for  $b = 1, \dots, B$ . For more on restricted regression with linear constraints on multiple coefficients, see [\[P\] makecns](#).

The associated one-sided alternative hypotheses are now  $H_1: \mathbf{R}\beta > r$  and  $H_2: \mathbf{R}\beta < r$ . As before, their respective bootstrap  $p$ -values are given by

$$p_1(r) = \frac{1}{B} \sum_{b=1}^B I\{t^b(r) > t(r)\}$$

$$p_2(r) = \frac{1}{B} \sum_{b=1}^B I\{t^b(r) < t(r)\}$$

where  $t(r)$  is the  $t$  statistic from the original sample:

$$t(r) = \frac{\mathbf{R}\hat{\beta} - r}{\sqrt{\mathbf{R}\hat{\mathbf{V}}\mathbf{R}'}}$$

The associated two-sided alternative hypothesis is  $H_3: \mathbf{R}\beta \neq r$ , and its bootstrap  $p$ -value under the assumption of symmetry is given by

$$p_S(r) = \frac{1}{B} \sum_{b=1}^B I\{|t^{(b)}(r)| > |t(r)|\}$$

When using the equal-tailed criterion (the `ptype(equal)` option), the  $100(1-\alpha)\%$  CI for  $\mathbf{R}\beta$  is given by  $[r_l, r_u]$ , where  $r_l$  and  $r_u$  satisfy  $p_1(r_l) = p_2(r_u) = \alpha/2$ . On the other hand, when using the symmetric criterion (the `ptype(symmetric)` option),  $r_l$  and  $r_u$  satisfy  $p_S(r_l) = p_S(r_u) = \alpha$ . The parameter  $\alpha$  can be specified with the `level()` option, that is,  $\alpha = (100 - \text{level}()) / 100$ .

The bootstrap  $p$ -values are step functions on  $r$ , and therefore a range of values for  $r_l$  and for  $r_u$  will solve the  $p$ -value conditions above. Because distribution functions are right continuous, `wildbootstrap` chooses the rightmost point in each range of solutions.

For the equal-tailed CIs, the starting values for the search of solutions to the  $p$ -value conditions are chosen as follows. First, the bootstrap  $t$  statistics are sorted  $t^{(1)} \leq t^{(2)} \leq \dots < t^{(B)}$ . Second, we define  $b_l = \text{ceil}\{B(1 - \alpha/2)\}$  so that  $t^{(b_l)}$  is smaller than  $B(\alpha/2)$  of the  $t$  statistics. Similarly, we define  $b_u = \text{floor}\{B(\alpha/2)\}$  so that  $t^{(b_u)}$  is larger than  $B(\alpha/2)$  of the  $t$  statistics. Third and finally, the initial lower and upper bounds of the CI are given by

$$\begin{aligned} r_l &= \mathbf{R}\hat{\beta} - t^{(b_l)} \sqrt{\widehat{\mathbf{R}\mathbf{V}\mathbf{R}'}} \\ r_u &= \mathbf{R}\hat{\beta} - t^{(b_u)} \sqrt{\widehat{\mathbf{R}\mathbf{V}\mathbf{R}'}} \end{aligned}$$

where  $\hat{\beta}$  and  $\widehat{\mathbf{V}}$  are the unrestricted estimator for  $\beta$  and the unrestricted CRVE in the original sample.

## Constructing a CI inverting the hypothesis test

For a linear combination of coefficients  $\sum R_j \beta_j$ , we can construct a CI by inverting the hypothesis test  $H_0: \mathbf{R}\beta = r$ , where  $\mathbf{R}$  is a row vector that includes the coefficients  $R_j$  and  $r$  is an arbitrary scalar. To do this, `wildbootstrap` searches for the CI lower bound  $r_l$  such that  $\Pr(\mathbf{R}\beta \leq r_l) = \alpha/2$  (equal-tailed criterion, the `ptype(equal)` option). The WCB distribution of  $\mathbf{R}\beta$  is a step function with a step size of  $1/\text{reps}(\#)$ . When  $\text{reps}(\#) \times \alpha/2$  is an integer, call it  $S$ , then we search for the  $r_l$  that produces the ordered bootstrapped  $t$  statistics

$$t^{(b)} = \frac{\mathbf{R}\hat{\beta}^{(b)} - r_l}{\sqrt{\mathbf{R}'\mathbf{V}^{(b)}\mathbf{R}}}$$

with the property  $\sum_{b=1}^{\text{reps}(\#)} I(t^{(b)} > t) = S$  and therefore  $\Pr(\mathbf{R}\beta \leq r_l) = S/\text{reps}(\#) = \alpha/2$ .

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## Also see

[R] `areg` — Linear regression with many indicator variables

[R] `regress` — Linear regression

[XT] `xtreg` — Linear models for panel data

[U] **20 Estimation and postestimation commands**

| Description          | Menu           | Syntax   | Options |
|----------------------|----------------|----------|---------|
| Remarks and examples | Stored results | Also see |         |

## Description

`xi` expands terms containing categorical variables into indicator (also called dummy) variable sets by creating new variables and, in the second syntax (`xi : any_stata_command`), executes the specified command with the expanded terms. The dummy variables created are

|                                                              |                                                                                                                                                                                                                                                  |
|--------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>i . varname</code>                                     | creates dummies for categorical variable <i>varname</i>                                                                                                                                                                                          |
| <code>i . varname<sub>1</sub>*i . varname<sub>2</sub></code> | creates dummies for categorical variables <i>varname<sub>1</sub></i> and <i>varname<sub>2</sub></i> :<br>all interactions and main effects                                                                                                       |
| <code>i . varname<sub>1</sub>*varname<sub>3</sub></code>     | creates dummies for categorical variable <i>varname<sub>1</sub></i> and continuous variable <i>varname<sub>3</sub></i> :<br>all interactions and main effects                                                                                    |
| <code>i . varname<sub>1</sub>   varname<sub>3</sub></code>   | creates dummies for categorical variable <i>varname<sub>1</sub></i> and continuous variable <i>varname<sub>3</sub></i> :<br>all interactions and main effect of <i>varname<sub>3</sub></i> ,<br>but no main effect of <i>varname<sub>1</sub></i> |

## Menu

Data > Create or change data > Other variable-creation commands > Interaction expansion

Most commands in Stata now allow factor variables; see [U] [11.4.3 Factor variables](#). To determine if a command allows factor variables, see the information printed below the options table for the command. If the command allows factor variables, it will say something like “*indepvars* may contain factor variables”.

We recommend that you use factor variables instead of `xi` if a command allows factor variables.

We include [R] [xi](#) in our documentation so that readers can consult it when using a Stata command that does not allow factor variables.

## Syntax

```
xi [, prefix(string) noomit] term(s)
```

```
xi [, prefix(string) noomit] : any_stata_command varlist_with_terms ...
```

where a *term* has the form

|                                                          |    |                                                          |
|----------------------------------------------------------|----|----------------------------------------------------------|
| <code>i.varname</code>                                   | or | <code>I.varname</code>                                   |
| <code>i.varname<sub>1</sub>*i.varname<sub>2</sub></code> |    | <code>I.varname<sub>1</sub>*I.varname<sub>2</sub></code> |
| <code>i.varname<sub>1</sub>*varname<sub>3</sub></code>   |    | <code>I.varname<sub>1</sub>*varname<sub>3</sub></code>   |
| <code>i.varname<sub>1</sub> varname<sub>3</sub></code>   |    | <code>I.varname<sub>1</sub> varname<sub>3</sub></code>   |

*varname*, *varname<sub>1</sub>*, and *varname<sub>2</sub>* denote numeric or string categorical variables. *varname<sub>3</sub>* denotes a continuous, numeric variable.

## Options

`prefix(string)` allows you to choose a prefix other than `_I` for the newly created interaction variables.

The prefix cannot be longer than four characters. By default, `xi` will create interaction variables starting with `_I`. When you use `xi`, it drops all previously created interaction variables starting with the prefix specified in the `prefix(string)` option or with `_I` by default. Therefore, if you want to keep the variables with a certain prefix, specify a different prefix in the `prefix(string)` option.

`noomit` prevents `xi` from omitting groups. This option provides a way to generate an indicator variable for every category having one or more variables, which is useful when combined with the `noconstant` option of an estimation command.

## Remarks and examples

Remarks are presented under the following headings:

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`xi` provides a convenient way to include dummy or indicator variables when fitting a model (say, with `regress` or `logistic`). For instance, assume that the categorical variable `agegrp` contains 1 for ages 20–24, 2 for ages 25–39, 3 for ages 40–44, etc. Typing

```
. xi: logistic outcome weight i.agegrp bp
```

estimates a logistic regression of `outcome` on `weight`, dummies for each `agegrp` category, and `bp`. That is, `xi` searches out and expands terms starting with “i.” or “I.” but ignores the other variables. `xi` will expand both numeric and string categorical variables, so if you had a string variable `race` containing “white”, “black”, and “other”, typing

```
. xi: logistic outcome weight bp i.agegrp i.race
```

would include indicator variables for the race group as well.

The `i.` indicator variables `xi` expands may appear anywhere in the *varlist*, so

```
. xi: logistic outcome i.agegrp weight i.race bp
```

would fit the same model.

You can also create interactions of categorical variables; typing

```
xi: logistic outcome weight bp i.agegrp*i.race
```

fits a model with indicator variables for all `agegrp` and `race` combinations, including the `agegrp` and `race` main-effect terms (that is, the terms that are created when you just type `i. agegrp i. race`).

You can interact dummy variables with continuous variables; typing

```
xi: logistic outcome bp i.agegrp*weight i.race
```

fits a model with indicator variables for all `agegrp` categories interacted with `weight`, plus the main-effect terms `weight` and `i. agegrp`.

You can get the interaction terms without the `agegrp` main effect (but with the `weight` main effect) by typing

```
xi: logistic outcome bp i.agegrp|weight i.race
```

You can also include multiple interactions:

```
xi: logistic outcome bp i.agegrp*weight i.agegrp*i.race
```

We will now back up and describe the construction of dummy variables in more detail.

## Background

The terms *continuous*, *categorical*, and *indicator* or *dummy* variables are used below. Continuous variables measure something—such as height or weight—and at least conceptually can take on any real number over some range. Categorical variables, on the other hand, take on a finite number of values, each denoting membership in a subclass—for example, excellent, good, and poor, which might be coded 0, 1, 2, or 1, 2, 3, or even “Excellent”, “Good”, and “Poor”. An indicator or dummy variable—the terms are used interchangeably—is a special type of two-valued categorical variable that contains values 0, denoting false, and 1, denoting true. The information contained in any  $k$ -valued categorical variable can be equally well represented by  $k$  indicator variables. Instead of one variable recording values representing excellent, good, and poor, you can have three indicator variables, indicating the truth or falseness of “result is excellent”, “result is good”, and “result is poor”.

`xi` provides a convenient way to convert categorical variables to dummy or indicator variables when you fit a model (say, with `regress` or `logistic`).

### ► Example 1

For instance, assume that the categorical variable `agegrp` contains 1 for ages 20–24, 2 for ages 25–39, and 3 for ages 40–44. (There is no one over 44 in our data.) As it stands, `agegrp` would be a poor candidate for inclusion in a model even if we thought age affected the outcome. The reason is that the coding would restrict the effect of being in the second age group to be twice the effect of being in the first, and, similarly, the effect of being in the third to be three times the first. That is, if we fit the model,

$$y = \beta_0 + \beta_1 \text{agegrp} + X\beta_2$$

the effect of being in the first age group is  $\beta_1$ , the second  $2\beta_1$ , and the third  $3\beta_1$ . If the coding 1, 2, and 3 is arbitrary, we could just as well have coded the age groups 1, 4, and 9, making the effects  $\beta_1$ ,  $4\beta_1$ , and  $9\beta_1$ .

The solution is to convert the categorical variable `agegrp` to a set of indicator variables,  $a_1$ ,  $a_2$ , and  $a_3$ , where  $a_i$  is 1 if the individual is a member of the  $i$ th age group and 0 otherwise. We can then fit the model

$$y = \beta_0 + \beta_{11}a_1 + \beta_{12}a_2 + \beta_{13}a_3 + X\beta_2$$

The effect of being in age group 1 is now  $\beta_{11}$ ; 2,  $\beta_{12}$ ; and 3,  $\beta_{13}$ ; and these results are independent of our (arbitrary) coding. The only difficulty at this point is that the model is unidentified in the sense that there are an infinite number of  $(\beta_0, \beta_{11}, \beta_{12}, \beta_{13})$  that fit the data equally well.

To see this, pretend that  $(\beta_0, \beta_{11}, \beta_{12}, \beta_{13}) = (1, 1, 3, 4)$ . The predicted values of  $y$  for the various age groups are

$$y = \begin{cases} 1 + 1 + X\beta_2 = 2 + X\beta_2 & \text{(age group 1)} \\ 1 + 3 + X\beta_2 = 4 + X\beta_2 & \text{(age group 2)} \\ 1 + 4 + X\beta_2 = 5 + X\beta_2 & \text{(age group 3)} \end{cases}$$

Now, pretend that  $(\beta_0, \beta_{11}, \beta_{12}, \beta_{13}) = (2, 0, 2, 3)$ . The predicted values of  $y$  are

$$y = \begin{cases} 2 + 0 + X\beta_2 = 2 + X\beta_2 & \text{(age group 1)} \\ 2 + 2 + X\beta_2 = 4 + X\beta_2 & \text{(age group 2)} \\ 2 + 3 + X\beta_2 = 5 + X\beta_2 & \text{(age group 3)} \end{cases}$$

These two sets of predictions are indistinguishable: for age group 1,  $y = 2 + X\beta_2$  regardless of the coefficient vector used, and similarly for age groups 2 and 3. This arises because we have three equations and four unknowns. Any solution is as good as any other, and, for our purposes, we merely need to choose one of them. The popular selection method is to set the coefficient on the first indicator variable to 0 (as we have done in our second coefficient vector). This is equivalent to fitting the model

$$y = \beta_0 + \beta_{12}a_2 + \beta_{13}a_3 + X\beta_2$$

How we select a particular coefficient vector (identifies the model) does not matter. It does, however, affect the *interpretation* of the coefficients.

For instance, we could just as well choose to omit the second group. In our artificial example, this would yield  $(\beta_0, \beta_{11}, \beta_{12}, \beta_{13}) = (4, -2, 0, 1)$  instead of  $(2, 0, 2, 3)$ . These coefficient vectors are the same in the sense that

$$y = \begin{cases} 2 + 0 + X\beta_2 = 2 + X\beta_2 = 4 - 2 + X\beta_2 & \text{(age group 1)} \\ 2 + 2 + X\beta_2 = 4 + X\beta_2 = 4 + 0 + X\beta_2 & \text{(age group 2)} \\ 2 + 3 + X\beta_2 = 5 + X\beta_2 = 4 + 1 + X\beta_2 & \text{(age group 3)} \end{cases}$$

But what does it mean that  $\beta_{13}$  can just as well be 3 or 1? We obtain  $\beta_{13} = 3$  when we set  $\beta_{11} = 0$ , so  $\beta_{13} = \beta_{13} - \beta_{11}$  and  $\beta_{13}$  measures the difference between age groups 3 and 1.

In the second case, we obtain  $\beta_{13} = 1$  when we set  $\beta_{12} = 0$ , so  $\beta_{13} - \beta_{12} = 1$  and  $\beta_{13}$  measures the difference between age groups 3 and 2. There is no inconsistency. According to our  $\beta_{12} = 0$  model, the difference between age groups 3 and 1 is  $\beta_{13} - \beta_{11} = 1 - (-2) = 3$ , the same result we got in the  $\beta_{11} = 0$  model.

◀

## ▷ Example 2

The issue of interpretation is important because it can affect the way we discuss results. Imagine that we are studying recovery after a coronary bypass operation. Assume that the age groups are children under 13 (we have two of them), young adults under 25 (we have a handful of them), adults under 46 (of which we have even more), mature adults under 56, older adults under 65, and elderly adults. We follow the prescription of omitting the first group, so all our results are reported relative to children under 13. While there is nothing statistically wrong with this, readers will be suspicious when we make statements like “compared with young children, older and elder adults ...”. Moreover, we will probably have to end each statement with “although results are not statistically significant” because we have only two children in our comparison group. Of course, even with results reported in this way, we can do reasonable comparisons (say, with mature adults), but we will have to do extra work to perform the appropriate linear hypothesis test using Stata’s `test` command.

Here it would be better to force the omitted group to be more reasonable, such as mature adults. There is, however, a generic rule for automatic comparison group selection that, although less popular, tends to work better than the omit-the-first-group rule. That rule is to omit the most prevalent group. The most prevalent is usually a reasonable baseline.

◀

In any case, the prescription for categorical variables is

1. Convert each  $k$ -valued categorical variable to  $k$  indicator variables.
2. Drop one of the  $k$  indicator variables; any one will do, but dropping the first is popular, dropping the most prevalent is probably better in terms of having the computer guess at a reasonable interpretation, and dropping a specified one often eases interpretation the most.
3. Fit the model on the remaining  $k - 1$  indicator variables.

xi automates this procedure.

We will now consider each of xi’s features in detail.

## Indicator variables for simple effects

When you type `i.varname`, `xi` internally tabulates *varname* (which may be a string or a numeric variable) and creates indicator (dummy) variables for each observed value, omitting the indicator for the smallest value. For instance, say that `agegrp` takes on the values 1, 2, 3, and 4. Typing

```
xi: logistic outcome i.agegrp
```

creates indicator variables named `_Iagegrp_2`, `_Iagegrp_3`, and `_Iagegrp_4`. (`xi` chooses the names and tries to make them readable; `xi` guarantees that the names are unique.) The expanded logistic model is

```
. logistic outcome _Iagegrp_2 _Iagegrp_3 _Iagegrp_4
```

Afterward, you can drop the new variables `xi` leaves behind by typing `'drop _I*'` (note the capitalization).

`xi` provides the following features when you type `i.varname`:

- *varname* may be string or numeric.
- Dummy variables are created automatically.
- By default, the dummy-variable set is identified by dropping the dummy corresponding to the smallest value of the variable (how to specify otherwise is discussed below).
- The new dummy variables are left in your dataset. By default, the names of the new dummy variables start with `_I`; therefore, you can drop them by typing `'drop _I*'`. You do not have to do this; each time you use `xi`, any automatically generated dummies with the same prefix as the one specified in the `prefix(string)` option, or `_I` by default, are dropped and new ones are created.
- The new dummy variables have variable labels so that you can determine what they correspond to by typing `'describe'`.
- `xi` may be used with any Stata command (not just `logistic`).

## Controlling the omitted dummy

By default, `i.varname` omits the dummy corresponding to the smallest value of *varname*; for a string variable, this is interpreted as dropping the first in an alphabetical, case-sensitive sort. `xi` provides two alternatives to dropping the first: `xi` will drop the dummy corresponding to the most prevalent value of *varname*, or `xi` will let you choose the particular dummy to be dropped.

To change `xi`'s behavior to dropping the most prevalent dummy, type

```
. char _dta[omit] prevalent
```

although whether you type “prevalent” or “yes” or anything else does not matter. Setting this characteristic affects the expansion of all categorical variables in the dataset. If you resave your dataset, the prevalent preference will be remembered. If you want to change the behavior back to the default drop-the-first rule, type

```
. char _dta[omit]
```

to clear the characteristic.

Once you set `_dta[omit]`, `i.varname` omits the dummy corresponding to the most prevalent value of `varname`. Thus, the coefficients on the dummies have the interpretation of change from the most prevalent group. For example,

```
. char _dta[omit] prevalent
. xi: regress y i.agegrp
```

might create `_Iagegrp_1` through `_Iagegrp_4`, resulting in `_Iagegrp_2` being omitted if `agegrp = 2` is most common (as opposed to the default dropping of `_Iagegrp_1`). The model is then

$$y = b_0 + b_1 \_Iagegrp\_1 + b_3 \_Iagegrp\_3 + b_4 \_Iagegrp\_4 + u$$

Then,

|                                        |                                        |
|----------------------------------------|----------------------------------------|
| Predicted y for agegrp 1 = $b_0 + b_1$ | Predicted y for agegrp 3 = $b_0 + b_3$ |
| Predicted y for agegrp 2 = $b_0$       | Predicted y for agegrp 4 = $b_0 + b_4$ |

Thus, the model's reported *t* or *Z* statistics are for a test of whether each group is different from the most prevalent group.

Perhaps you wish to omit the dummy for agegrp 3 instead. You do this by setting the variable's omit characteristic:

```
. char agegrp[omit] 3
```

This overrides `_dta[omit]` if you have set it. Now, when you type

```
. xi: regress y i.agegrp
```

`_Iagegrp_3` will be omitted, and you will fit the model

$$y = b'_0 + b'_1 \_Iagegrp\_1 + b'_2 \_Iagegrp\_2 + b'_4 \_Iagegrp\_4 + u$$

Later, if you want to return to the default omission, type

```
. char agegrp[omit]
```

to clear the characteristic.

In summary, `i.varname` omits the first group by default, but if you define

```
. char _dta[omit] prevalent
```

the default behavior changes to dropping the most prevalent group. Either way, if you define a characteristic of the form

```
. char varname[omit] #
```

or, if `varname` is a string,

```
. char varname[omit] string-literal
```

the specified value will be omitted.

```
Examples: . char agegrp[omit] 1
 . char race[omit] White (for race, a string variable)
 . char agegrp[omit] (to restore default for agegrp)
```

## Categorical variable interactions

`i.varname1*i.varname2` creates the dummy variables associated with the interaction of the categorical variables `varname1` and `varname2`. The identification rules—which categories are omitted—are the same as those for `i.varname`. For instance, assume that `agegrp` takes on four values and `race` takes on three values. Typing

```
. xi: regress y i.agegrp*i.race
```

results in

|                                                                  |               |
|------------------------------------------------------------------|---------------|
| model:                                                           | dummies for:  |
| $y = a + b_2 \_Iagegrp\_2 + b_3 \_Iagegrp\_3 + b_4 \_Iagegrp\_4$ | (agegrp)      |
| $+ c_2 \_Irace\_2 + c_3 \_Irace\_3$                              | (race)        |
| $+ d_{22} \_IageXrac\_2\_2 + d_{23} \_IageXrac\_2\_3$            |               |
| $+ d_{32} \_IageXrac\_3\_2 + d_{33} \_IageXrac\_3\_3$            | (agegrp*race) |
| $+ d_{42} \_IageXrac\_4\_2 + d_{43} \_IageXrac\_4\_3$            |               |
| $+ u$                                                            |               |

That is, typing

```
. xi: regress y i.agegrp*i.race
```

is the same as typing

```
. xi: regress y i.agegrp i.race i.agegrp*i.race
```

Although there are many other ways the interaction could have been parameterized, this method has the advantage that you can test the joint significance of the interactions by typing

```
. testparm _IageXrac*
```

When you perform the estimation step, whether you use `i.agegrp*i.race` or `i.race*i.agegrp` makes no difference (other than in the names given to the interaction terms; in the first case, the names will begin with `_IageXrac`; in the second, `_IracXage`). Thus,

```
. xi: regress y i.race*i.agegrp
```

fits the same model.

You may also include multiple interactions simultaneously:

```
. xi: regress y i.agegrp*i.race i.agegrp*i.sex
```

The model fit is

|                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                         |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------|
| <p>model:</p> $y = a + b_2 \_Iagegrp\_2 + b_3 \_Iagegrp\_3 + b_4 \_Iagegrp\_4$ $+ c_2 \_Irace\_2 + c_3 \_Irace\_3$ $+ d_{22} \_IageXrac\_2\_2 + d_{23} \_IageXrac\_2\_3$ $+ d_{32} \_IageXrac\_3\_2 + d_{33} \_IageXrac\_3\_3$ $+ d_{42} \_IageXrac\_4\_2 + d_{43} \_IageXrac\_4\_3$ $+ e_2 \_Isex\_2$ $+ f_{22} \_IageXsex\_2\_2 + f_{23} \_IageXsex\_2\_3 + f_{24} \_IageXsex\_2\_4$ $+ u$ | <p>dummies for:</p> <p>(agegrp)</p> <p>(race)</p> <p>(agegrp*race)</p> <p>(sex)</p> <p>(agegrp*sex)</p> |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------|

The agegrp dummies are (correctly) included only once.

## Interactions with continuous variables

`i.varname1*varname2` (as distinguished from `i.varname1*i.varname2`—note the second `i.`) specifies an interaction of a categorical variable with a continuous variable. For instance,

```
. xi: regress y i.agegrp*wgt
```

results in the model

|                                                                                                                                                  |                                                                                         |
|--------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------|
| $y = a + b_2 \_Iagegrp\_2 + b_3 \_Iagegrp\_3 + b_4 \_Iagegrp\_4$ $+ c \_wgt$ $+ d_2 \_IageXwgt\_2 + d_3 \_IageXwgt\_3 + d_4 \_IageXwgt\_4$ $+ u$ | <p>(agegrp dummies)</p> <p>(continuous wgt effect)</p> <p>(agegrp*wgt interactions)</p> |
|--------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------|

A variation on this notation, using `|` rather than `*`, omits the agegrp dummies. Typing

```
. xi: regress y i.agegrp|wgt
```

fits the model

|                                                                                             |                                                                 |
|---------------------------------------------------------------------------------------------|-----------------------------------------------------------------|
| $y = a' + c' \_wgt$ $+ d'_2 \_IageXwgt\_2 + d'_3 \_IageXwgt\_3 + d'_4 \_IageXwgt\_4$ $+ u'$ | <p>(continuous wgt effect)</p> <p>(agegrp*wgt interactions)</p> |
|---------------------------------------------------------------------------------------------|-----------------------------------------------------------------|

The predicted values of  $y$  are

| agegrp*wgt model                          | agegrp wgt model                       |               |
|-------------------------------------------|----------------------------------------|---------------|
| $y = a + c \text{wgt}$                    | $a' + c' \text{wgt}$                   | if agegrp = 1 |
| $a + c \text{wgt} + b_2 + d_2 \text{wgt}$ | $a' + c' \text{wgt} + d'_2 \text{wgt}$ | if agegrp = 2 |
| $a + c \text{wgt} + b_3 + d_3 \text{wgt}$ | $a' + c' \text{wgt} + d'_3 \text{wgt}$ | if agegrp = 3 |
| $a + c \text{wgt} + b_4 + d_4 \text{wgt}$ | $a' + c' \text{wgt} + d'_4 \text{wgt}$ | if agegrp = 4 |

That is, typing

```
. xi: regress y i.agegrp*wgt
```

is equivalent to typing

```
. xi: regress y i.agegrp i.agegrp|wgt
```

In either case, you do not need to specify separately the continuous variable `wgt`; it is included automatically.

## Using xi: Interpreting output

```
. xi: regress mpg i.rep78
i.rep78 _Irep78_1-5 (naturally coded; _Irep78_1 omitted)
(output from regress appears)
```

Interpretation: `i.rep78` expanded to the dummies `_Irep78_1`, `_Irep78_2`, ..., `_Irep78_5`. The numbers on the end are “natural” in the sense that `_Irep78_1` corresponds to `rep78 = 1`, `_Irep78_2` to `rep78 = 2`, and so on. Finally, the dummy for `rep78 = 1` was omitted.

```
. xi: regress mpg i.make
i.make _Imake_1-74 (_Imake_1 for make==AMC Concord omitted)
(output from regress appears)
```

Interpretation: `i.make` expanded to `_Imake_1`, `_Imake_2`, ..., `_Imake_74`. The coding is not natural because `make` is a string variable. `_Imake_1` corresponds to one `make`, `_Imake_2` to another, and so on. You can find out the coding by typing `describe`. `_Imake_1` for the AMC Concord was omitted.

## How xi names variables

By default, xi assigns to the dummy variables it creates names having the form

$$\_Istub\_groupid$$

You may subsequently refer to the entire set of variables by typing ‘*Istub\**’. For example,

| name               | =     | $\_I + stub$ | + $\_ + groupid$ | Entire set    |
|--------------------|-------|--------------|------------------|---------------|
| $\_Iagegrp\_1$     | $\_I$ | agegrp       | – 1              | $\_Iagegrp*$  |
| $\_Iagegrp\_2$     | $\_I$ | agegrp       | – 2              | $\_Iagegrp*$  |
| $\_IageXwgt\_1$    | $\_I$ | ageXwgt      | – 1              | $\_IageXwgt*$ |
| $\_IageXrac\_1\_2$ | $\_I$ | ageXrac      | – 1\_2           | $\_IageXrac*$ |
| $\_IageXrac\_2\_1$ | $\_I$ | ageXrac      | – 2\_1           | $\_IageXrac*$ |

If you specify a prefix in the `prefix(string)` option, say,  $\_S$ , then xi will name the variables starting with the prefix

$$\_Sstub\_groupid$$

## xi as a command rather than a command prefix

xi can be used as a command prefix or as a command by itself. In the latter form, xi merely creates the indicator and interaction variables. Typing

```
. xi: regress y i.agegrp*wgt
i.agegrp _Iagegrp_1-4 (naturally coded; _Iagegrp_1 omitted)
i.agegrp*wgt _IageXwgt_1-4 (coded as above)
(output from regress appears)
```

is equivalent to typing

```
. xi i.agegrp*wgt
i.agegrp _Iagegrp_1-4 (naturally coded; _Iagegrp_1 omitted)
i.agegrp*wgt _IageXwgt_1-4 (coded as above)
. regress y _Iagegrp* _IageXwgt*
(output from regress appears)
```

## Warnings

1. xi creates new variables in your dataset; most are bytes, but interactions with continuous variables will have the storage type of the underlying continuous variable. You may get the error message “no room to add more variables” or “insufficient memory”. You may need to adjust the `maxvar` setting or reset `max_memory` if it has been set too low; see [U] 6 Managing memory.
2. When using xi with an estimation command, you may get the error message “unable to allocate matrix”. This usually occurs because you attempted to create a matrix that is too large; see [R] Limits.

## Stored results

xi stores the following characteristics:

|                                                           |                   |
|-----------------------------------------------------------|-------------------|
| <code>_dta[<a href="#">__xi__Vars__Prefix__</a>]</code>   | prefix names      |
| <code>_dta[<a href="#">__xi__Vars__To__Drop__</a>]</code> | variables created |

## Also see

[U] [11.1.10 Prefix commands](#)

[U] [20 Estimation and postestimation commands](#)

## zinb — Zero-inflated negative binomial regression

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### Description

`zinb` fits a zero-inflated negative binomial (ZINB) model to overdispersed count data with excess zero counts. The ZINB model assumes that the excess zero counts come from a logit or probit model and the remaining counts come from a negative binomial model.

### Quick start

Zero-inflated negative binomial model of  $y$  on  $x_1$  and  $x_2$  with inflation modeled using  $x_3$

```
zinb y x1 x2, inflate(x3)
```

And conduct likelihood-ratio test against ZIP model

```
zinb y x1 x2, inflate(x3) zip
```

Use a probit model instead of a logit model to predict excess zeros

```
zinb y x1 x2, inflate(x3) probit
```

### Menu

Statistics > Count outcomes > Zero-inflated negative binomial regression

## Syntax

```
zinb depvar [indepvars] [if] [in] [weight],
 inflate(varlist [, offset(varname)] | _cons) [options]
```

| <i>options</i>                                  | Description                                                                                                                                      |
|-------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                    |                                                                                                                                                  |
| * <u>inflate</u> ( )                            | equation that determines whether the count is zero                                                                                               |
| <u>noconstant</u>                               | suppress constant term                                                                                                                           |
| <u>exposure</u> ( <i>varname</i> <sub>e</sub> ) | include ln( <i>varname</i> <sub>e</sub> ) in model with coefficient constrained to 1                                                             |
| <u>offset</u> ( <i>varname</i> <sub>o</sub> )   | include <i>varname</i> <sub>o</sub> in model with coefficient constrained to 1                                                                   |
| <u>constraints</u> ( <i>constraints</i> )       | apply specified linear constraints                                                                                                               |
| <u>probit</u>                                   | use probit model to characterize excess zeros; default is logit                                                                                  |
| <b>SE/Robust</b>                                |                                                                                                                                                  |
| <u>vce</u> ( <i>vcetype</i> )                   | <i>vcetype</i> may be <u>oim</u> , <u>robust</u> , <u>cluster</u> <i>clustvar</i> , <u>opg</u> , <u>bootstrap</u> , or <u>jackknife</u>          |
| <b>Reporting</b>                                |                                                                                                                                                  |
| <u>level</u> (#)                                | set confidence level; default is level(95)                                                                                                       |
| <u>irr</u>                                      | report incidence-rate ratios                                                                                                                     |
| <u>zip</u>                                      | perform ZIP likelihood-ratio test                                                                                                                |
| <u>nocnsreport</u>                              | do not display constraints                                                                                                                       |
| <u>display_options</u>                          | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| <b>Maximization</b>                             |                                                                                                                                                  |
| <u>maximize_options</u>                         | control the maximization process; seldom used                                                                                                    |
| <u>collinear</u>                                | keep collinear variables                                                                                                                         |
| <u>coeflegend</u>                               | display legend instead of statistics                                                                                                             |

\*inflate(*varlist* [, offset(*varname*)] | \_cons) is required.

*indepvars* and *varlist* may contain factor variables; see [U] 11.4.3 Factor variables.

bayes, bayesboot, bootstrap, by, collect, fp, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands. For more details, see [BAYES] bayes: zinb.

Weights are not allowed with the bootstrap prefix; see [R] bootstrap.

vce( ), zip, and weights are not allowed with the svy prefix; see [SVY] svy.

fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.

collinear and coeflegend do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Options

### Model

`inflate(varlist[, offset(varname)] | _cons)` specifies the equation that determines whether the observed count is zero. Conceptually, omitting `inflate()` would be equivalent to fitting the model with `nbreg`.

`inflate(varlist[, offset(varname)])` specifies the variables in the equation. You may optionally include an offset for this *varlist*.

`inflate(_cons)` specifies that the equation determining whether the count is zero contains only an intercept. To run a zero-inflated model of *devar* with only an intercept in both equations, type `zinb devar, inflate(_cons)`.

`noconstant`, `exposure(varnamee)`, `offset(varnameo)`, `constraints(constraints)`; see [R] **Estimation options**.

`probit` requests that a probit, instead of logit, model be used to characterize the excess zeros in the data.

### SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] **vce\_option**.

### Reporting

`level(#)`; see [R] **Estimation options**.

`irr` reports estimated coefficients transformed to incidence-rate ratios, that is,  $e^{\beta_i}$  rather than  $\beta_i$ . Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated or stored. `irr` may be specified at estimation or when replaying previously estimated results.

`zip` requests that a likelihood-ratio test comparing the ZINB model with the zero-inflated Poisson model be included in the output.

`nocnsreport`; see [R] **Estimation options**.

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] **Estimation options**.

### Maximization

*maximize\_options*: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] **Maximize**. These options are seldom used.

Setting the optimization type to `technique(bhhh)` resets the default *vcetype* to `vce(opg)`.

The following options are available with `zinb` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] **Estimation options**.

## Remarks and examples

Zero-inflated negative binomial (ZINB) models are used to model count data that have a higher fraction of zeros than is likely to be generated by a standard negative binomial model. To account for excess zeros, ZINB models assume that these excess zeros come from a model other than the negative binomial model. A zero that comes from this other model is known as a “degenerate zero”.

The negative binomial overdispersion parameter,  $\alpha$ , differentiates the ZINB model from the zero-inflated Poisson (ZIP) model (see [R] [zip](#)). Here overdispersion refers to the fact that the negative binomial variance is greater than its mean, whereas the Poisson variance is equal to its mean. Thus, values of  $\alpha > 1$  indicate overdispersion. The larger the  $\alpha$ , the greater the negative binomial variance. See [Methods and formulas](#) in [R] [nbgreg](#) for further discussion of negative binomial overdispersion.

The `zinb` command fits ZINB models and provides two choices for modeling the excess zeros: the default logit function or, when the `probit` option is specified, the probit function. Both functions are symmetric about zero, but the logistic function has more area under the tails.

See [Long \(1997, 242–247\)](#) and [Cameron and Trivedi \(2005, 680–681\)](#) for a discussion of zero-modified count models.

### ▷ Example 1: Fitting a ZINB model

In [example 1](#) of [R] [zip](#), we fit a zero-inflated Poisson model using the `zip` command to the fictional data on the number of fish caught by visitors to a national park. Let’s fit a ZINB model to these data.

Just like with `zip`, we use the required the `inflate()` option to model whether a visitor fishes as a function of the number of accompanying children (`child`) and whether the visitor is camping (`camper`). Next, we assume the response variable, `count`, depends on whether the visitor used a live bait (`livebait`) and the number of persons in the party (`persons`), which includes the visitor plus other adults and children.

```

. use https://www.stata-press.com/data/r19/fish
(Fictional fishing data)
. zinz count persons livebait, inflate(child camper)
Fitting constant-only model:
Iteration 0: Log likelihood = -519.33992
Iteration 1: Log likelihood = -451.38662
Iteration 2: Log likelihood = -444.49118
Iteration 3: Log likelihood = -442.96272
Iteration 4: Log likelihood = -442.71065
Iteration 5: Log likelihood = -442.66718
Iteration 6: Log likelihood = -442.6631
Iteration 7: Log likelihood = -442.66299
Iteration 8: Log likelihood = -442.66299
Fitting full model:
Iteration 0: Log likelihood = -442.66299 (not concave)
Iteration 1: Log likelihood = -432.83107 (not concave)
Iteration 2: Log likelihood = -426.32934
Iteration 3: Log likelihood = -413.75019
Iteration 4: Log likelihood = -403.09586
Iteration 5: Log likelihood = -401.56013
Iteration 6: Log likelihood = -401.54781
Iteration 7: Log likelihood = -401.54776
Iteration 8: Log likelihood = -401.54776
Zero-inflated negative binomial regression Number of obs = 250
Inflation model: logit Nonzero obs = 108
 Zero obs = 142
 LR chi2(2) = 82.23
 Prob > chi2 = 0.0000

Log likelihood = -401.5478

```

| count    | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|----------|-------------|-----------|-------|-------|----------------------|-----------|
| count    |             |           |       |       |                      |           |
| persons  | .9742984    | .1034938  | 9.41  | 0.000 | .7714543             | 1.177142  |
| livebait | 1.557523    | .4124424  | 3.78  | 0.000 | .7491503             | 2.365895  |
| _cons    | -2.730064   | .476953   | -5.72 | 0.000 | -3.664874            | -1.795253 |
| inflate  |             |           |       |       |                      |           |
| child    | 3.185999    | .7468551  | 4.27  | 0.000 | 1.72219              | 4.649808  |
| camper   | -2.020951   | .872054   | -2.32 | 0.020 | -3.730146            | -.3117567 |
| _cons    | -2.695385   | .8929071  | -3.02 | 0.003 | -4.44545             | -.9453189 |
| /lnalpha | .5110429    | .1816816  | 2.81  | 0.005 | .1549535             | .8671323  |
| alpha    | 1.667029    | .3028685  |       |       | 1.167604             | 2.380076  |

The coefficients in the first equation of the coefficient table, labeled count, correspond to the negative binomial model for individuals who fished. For instance, among visitors who fished, using a live bait increases the expected number of caught fish by a factor of  $\exp(1.5575) \approx 4.7$ , holding other covariates constant.

The confidence interval for alpha indicates that the ZINB model is more appropriate than the ZIP model. To confirm this, you can run zinz and specify the zip option to obtain the ZIP likelihood-ratio test.

The `inflate` equation models whether the visitor does not fish. We can use `margins` to obtain a better understanding of how the `inflate` equation affects the occurrence of the excess zero counts. We specify `margins`'s options `dydx(child camper)` and `predict(pr)`. `pr` is `predict`'s option for estimating the probability of a degenerate zero or, in our example, the probability of not fishing; see the `margins` section in [R] [zibn postestimation](#).

```
. margins, dydx(child camper) predict(pr)
Average marginal effects Number of obs = 250
Model VCE: OIM
Expression: Pr(count=0), predict(pr)
dy/dx wrt: child camper
```

|        | Delta-method |           | z     | P> z  | [95% conf. interval] |           |
|--------|--------------|-----------|-------|-------|----------------------|-----------|
|        | dy/dx        | std. err. |       |       |                      |           |
| child  | .257531      | .029941   | 8.60  | 0.000 | .1988477             | .3162144  |
| camper | -.1633578    | .0503938  | -3.24 | 0.001 | -.2621277            | -.0645878 |

The `margins` output tells us that a visitor is less likely to be visiting the park to fish if accompanied by children and more likely to fish if camping.

You also may want to evaluate whether a standard negative binomial model is adequate to fit the data. This can be done using information criteria; see [example 2](#) in [R] [zip](#).

◀

## Stored results

`zibn` stores the following in `e()`:

### Scalars

|                            |                                           |
|----------------------------|-------------------------------------------|
| <code>e(N)</code>          | number of observations                    |
| <code>e(N_zero)</code>     | number of zero observations               |
| <code>e(k)</code>          | number of parameters                      |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>  |
| <code>e(k_eq_model)</code> | number of equations in overall model test |
| <code>e(k_aux)</code>      | number of auxiliary parameters            |
| <code>e(k_dv)</code>       | number of dependent variables             |
| <code>e(df_m)</code>       | model degrees of freedom                  |
| <code>e(ll)</code>         | log likelihood                            |
| <code>e(ll_0)</code>       | log likelihood, constant-only model       |
| <code>e(df_c)</code>       | degrees of freedom for comparison test    |
| <code>e(N_clust)</code>    | number of clusters                        |
| <code>e(chi2)</code>       | $\chi^2$                                  |
| <code>e(p)</code>          | $p$ -value for model test                 |
| <code>e(chi2_cp)</code>    | $\chi^2$ for test of $\alpha = 0$         |
| <code>e(rank)</code>       | rank of <code>e(V)</code>                 |
| <code>e(ic)</code>         | number of iterations                      |
| <code>e(rc)</code>         | return code                               |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise               |

### Macros

|                         |                            |
|-------------------------|----------------------------|
| <code>e(cmd)</code>     | <code>zibn</code>          |
| <code>e(cmdline)</code> | command as typed           |
| <code>e(depvar)</code>  | name of dependent variable |
| <code>e(inflate)</code> | logit or probit            |
| <code>e(wtype)</code>   | weight type                |
| <code>e(wexp)</code>    | weight expression          |

|                              |                                                                                  |
|------------------------------|----------------------------------------------------------------------------------|
| <code>e(title)</code>        | title in estimation output                                                       |
| <code>e(clustvar)</code>     | name of cluster variable                                                         |
| <code>e(offset1)</code>      | offset                                                                           |
| <code>e(offset2)</code>      | offset for <code>inflate()</code>                                                |
| <code>e(chi2type)</code>     | Wald or LR; type of model $\chi^2$ test                                          |
| <code>e(chi2_cpt)</code>     | Wald or LR; type of model $\chi^2$ test corresponding to <code>e(chi2_cp)</code> |
| <code>e(vce)</code>          | <i>vce</i> type specified in <code>vce()</code>                                  |
| <code>e(vctype)</code>       | title used to label Std. err.                                                    |
| <code>e(opt)</code>          | type of optimization                                                             |
| <code>e(which)</code>        | max or min; whether optimizer is to perform maximization or minimization         |
| <code>e(ml_method)</code>    | type of ml method                                                                |
| <code>e(user)</code>         | name of likelihood-evaluator program                                             |
| <code>e(technique)</code>    | maximization technique                                                           |
| <code>e(properties)</code>   | b V                                                                              |
| <code>e(predict)</code>      | program used to implement <code>predict</code>                                   |
| <code>e(asbalanced)</code>   | factor variables <code>fvset</code> as <code>asbalanced</code>                   |
| <code>e(asobserved)</code>   | factor variables <code>fvset</code> as <code>asobserved</code>                   |
| Matrices                     |                                                                                  |
| <code>e(b)</code>            | coefficient vector                                                               |
| <code>e(Cns)</code>          | constraints matrix                                                               |
| <code>e(ilog)</code>         | iteration log (up to 20 iterations)                                              |
| <code>e(gradient)</code>     | gradient vector                                                                  |
| <code>e(V)</code>            | variance–covariance matrix of the estimators                                     |
| <code>e(V_modelbased)</code> | model-based variance                                                             |
| Functions                    |                                                                                  |
| <code>e(sample)</code>       | marks estimation sample                                                          |

In addition to the above, the following is stored in `r()`:

|                       |                                                                                                                            |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------|
| Matrices              |                                                                                                                            |
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any *r*-class command is run after the estimation command.

## Methods and formulas

The `zinz` command maximizes a likelihood function that is a mixture of the logistic (or probit) and negative binomial distributions. The logistic distribution models the unobserved process that creates the excess zeros, and the negative binomial distribution models the counts. Define

$$\xi_j^\beta = \mathbf{x}_j \boldsymbol{\beta} + \text{offset}_j^\beta$$

$$\xi_j^\gamma = \mathbf{z}_j \boldsymbol{\gamma} + \text{offset}_j^\gamma$$

$$\mu_j = \exp(\xi_j^\beta)$$

$$p_j = 1/(1 + \alpha \mu_j)$$

$$m = 1/\alpha$$

Here the vector  $\mathbf{x}_j$  contains the covariates specified in *indepvars* for the *j*th observation, and  $\mathbf{z}_j$  contains the covariates specified in the `inflate()` option. Similarly, estimates for  $\boldsymbol{\beta}$  are found in the first equation of the `zinz` coefficient table (labeled after *depvar*), and the estimates for  $\boldsymbol{\gamma}$  are found in the second equation of the coefficient table (labeled `inflate`). The parameter  $\alpha$  is the negative binomial

overdispersion parameter, and its estimate is the ancillary parameter labeled alpha in the coefficient table. Parameters  $p_j$ ,  $m$ , and  $\mu_j$  are parameters of a negative binomial distribution; see *Methods and formulas* in [R] **nbreg** for details.

The log likelihood maximized by **zinz** is

$$\begin{aligned} \ln L = & \sum_{j \in S} w_j \ln \{F_j + (1 - F_j)p_j^m\} + \\ & \sum_{j \notin S} w_j \left\{ \ln(1 - F_j) + \ln \Gamma(m + y_j) - \ln \Gamma(y_j + 1) \right. \\ & \left. - \ln \Gamma(m) + m \ln p_j + y_j \ln(1 - p_j) \right\} \end{aligned}$$

where  $w_j$  are the weights,  $S$  is the set of observations for which the observed outcome  $y_j = 0$ , and  $F_j$  is the logistic distribution function

$$F_j = F(\xi_j^\gamma) = \exp(\xi_j^\gamma) / \{1 + \exp(\xi_j^\gamma)\}$$

or, if the probit option is specified, the standard normal distribution function

$$F_j = F(\xi_j^\gamma) = \Phi(\xi_j^\gamma)$$

From Long (1997), the variance of the mixture distribution is

$$\text{Var}(y_j | \mathbf{x}_i, \mathbf{z}_i) = \mu_j(1 - F_j)\{1 + \mu_j(F_j + \alpha)\}$$

When  $F_j$  is zero, we have the variance of the negative binomial distribution; when  $F_j > 0$ , the variance can exceed that of the negative binomial distribution.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] **\_robust**, particularly *Maximum likelihood estimators* and *Methods and formulas*.

**zinz** also supports estimation with survey data. For details on VCEs with survey data, see [SVY] **Variance estimation**.

## References

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- Xia, Y., Y. Zhou, and T. Cai. 2019. `gidm`: A command for generalized inflated discrete models. *Stata Journal* 19: 698–718.

## Also see

[R] **zinb postestimation** — Postestimation tools for zinb

[R] **zip** — Zero-inflated Poisson regression

[R] **nbreg** — Negative binomial regression

[R] **poisson** — Poisson regression

[R] **tnbreg** — Truncated negative binomial regression

[R] **tpoisson** — Truncated Poisson regression

[BAYES] **bayes: zinb** — Bayesian zero-inflated negative binomial regression

[SVY] **svy estimation** — Estimation commands for survey data

[XT] **xtnbreg** — Fixed-effects, random-effects, & population-averaged negative binomial models

[U] **20 Estimation and postestimation commands**

## Postestimation commands

The following postestimation commands are available after `zinb`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from <code>margins</code> (profile plots, interaction plots, etc.)                                                    |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | number of events, incidence rates, probabilities, etc.                                                                                  |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results.

## predict

### Description for predict

`predict` creates a new variable containing predictions such as numbers of events, incidence rates, probabilities, linear predictions, and standard errors.

### Menu for predict

Statistics > Postestimation

### Syntax for predict

```
predict [type] newvar [if] [in] [, statistic nooffset]
```

```
predict [type] stub* [if] [in] , scores
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

Main

|                                    |                                         |
|------------------------------------|-----------------------------------------|
| <code>n</code>                     | number of events; the default           |
| <code>ir</code>                    | incidence rate                          |
| <code>pr</code>                    | probability of a degenerate zero        |
| <code>pr(<i>n</i>)</code>          | probability $\Pr(y_j = n)$              |
| <code>pr(<i>a</i>,<i>b</i>)</code> | probability $\Pr(a \leq y_j \leq b)$    |
| <code>xb</code>                    | linear prediction                       |
| <code>stdp</code>                  | standard error of the linear prediction |

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

### Options for predict

Main

`n`, the default, calculates the predicted number of events, which is  $(1 - F_j) \exp(\mathbf{x}_j \boldsymbol{\beta})$  if neither `offset()` nor `exposure()` was specified when the model was fit, where  $F_j$  is the predicted probability of a zero outcome;  $(1 - F_j) \exp(\mathbf{x}_j \boldsymbol{\beta} + \text{offset}_j^\beta)$  if `offset()` was specified; or  $(1 - F_j) \{ \exp(\mathbf{x}_j \boldsymbol{\beta}) \times \text{exposure}_j \}$  if `exposure()` was specified.

`ir` calculates the incidence rate, which is the predicted number of events when exposure is 1. This is equivalent to specifying both the `n` and the `nooffset` options.

`pr` calculates the probability of a degenerate zero, predicted from the fitted degenerate distribution  $F_j = F(\mathbf{z}_j \boldsymbol{\gamma})$ . If `offset()` was specified within the `inflate()` option, then  $F_j = F(\mathbf{z}_j \boldsymbol{\gamma} + \text{offset}_j^\gamma)$  is calculated.

`pr(n)` calculates the probability  $\Pr(y_j = n)$ , where *n* is a nonnegative integer that may be specified as a number or a variable. Note that `pr` is not equivalent to `pr(0)`.

`pr(a, b)` calculates the probability  $\Pr(a \leq y_j \leq b)$ , where  $a$  and  $b$  are nonnegative integers that may be specified as numbers or variables;

$b$  missing ( $b \geq .$ ) means  $+\infty$ ;

`pr(20, .)` calculates  $\Pr(y_j \geq 20)$ ;

`pr(20, b)` calculates  $\Pr(y_j \geq 20)$  in observations for which  $b \geq .$  and calculates

$\Pr(20 \leq y_j \leq b)$  elsewhere.

`pr(., b)` produces a syntax error. A missing value in an observation of the variable  $a$  causes a missing value in that observation for `pr(a, b)`.

`xb` calculates the linear prediction, which is  $\mathbf{x}_j\boldsymbol{\beta}$  if neither `offset()` nor `exposure()` was specified;

$\mathbf{x}_j\boldsymbol{\beta} + \text{offset}_j^\beta$  if `offset()` was specified; or  $\mathbf{x}_j\boldsymbol{\beta} + \ln(\text{exposure}_j)$  if `exposure()` was specified; see `nooffset` below.

`stdp` calculates the standard error of the linear prediction.

`nooffset` is relevant only if you specified `offset()` or `exposure()` when you fit the model. It modifies the calculations made by `predict` so that they ignore the offset or exposure variable; the linear prediction is treated as  $\mathbf{x}_j\boldsymbol{\beta}$  rather than as  $\mathbf{x}_j\boldsymbol{\beta} + \text{offset}_j^\beta$  or  $\mathbf{x}_j\boldsymbol{\beta} + \ln(\text{exposure}_j)$ . Specifying `predict . . . , nooffset` is equivalent to specifying `predict . . . , ir`.

`scores` calculates equation-level score variables.

The first new variable will contain  $\partial \ln L / \partial (\mathbf{x}_j\boldsymbol{\beta})$ .

The second new variable will contain  $\partial \ln L / \partial (\mathbf{z}_j\boldsymbol{\gamma})$ .

The third new variable will contain  $\partial \ln L / \partial \ln \alpha$ .

# margins

## Description for margins

`margins` estimates margins of response for number of events, incidence rates, probabilities, and linear predictions.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i>             | Description                           |
|------------------------------|---------------------------------------|
| <code>n</code>               | number of events; the default         |
| <code>ir</code>              | incidence rate                        |
| <code>pr</code>              | probability of a degenerate zero      |
| <code>pr(<i>n</i>)</code>    | probability $\Pr(y_j = n)$            |
| <code>pr(<i>a, b</i>)</code> | probability $\Pr(a \leq y_j \leq b)$  |
| <code>xb</code>              | linear prediction                     |
| <code>stdp</code>            | not allowed with <code>margins</code> |

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [R] [margins](#).

## Methods and formulas

See [Methods and formulas](#) in [R] [zinb](#) for the model definition and notation.

The probabilities calculated using the `pr(n)` option are the probability  $\Pr(y_j = n)$ . These are calculated using

$$\Pr(y_j = 0 | \mathbf{x}_j, \mathbf{z}_j) = F_j + (1 - F_j) p_2(0 | \mathbf{x}_j)$$

$$\Pr(y_j = n | \mathbf{x}_j, \mathbf{z}_j) = (1 - F_j) p_2(n | \mathbf{x}_j) \quad \text{for } n = 1, 2, \dots$$

where  $F_j$  is the probability of obtaining an observation from the degenerate distribution whose mass is concentrated at zero, and  $p_2(n | \mathbf{x}_j)$  is the probability of  $y_j = n$  from the nondegenerate, negative binomial distribution.  $F_j$  can be obtained by using the `pr` option.

See [Cameron and Trivedi \(2013, sec. 4.6\)](#) for further details.

## References

- Cameron, A. C., and P. K. Trivedi. 2013. *Regression Analysis of Count Data*. 2nd ed. New York: Cambridge University Press.
- Manjón, M., and O. Martínez. 2014. The chi-squared goodness-of-fit test for count-data models. *Stata Journal* 14: 798–816.

## Also see

- [R] [zinb](#) — Zero-inflated negative binomial regression
- [U] [20 Estimation and postestimation commands](#)

[Description](#)  
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## Description

`ziologit` fits a model for an ordinal outcome with excess zeros, a higher fraction of zeros than would be expected from a standard ordered logit model, also known as zero inflation. This model is known as a zero-inflated ordered logit (ZIOL) model. In the context of ZIOL models, zero is the lowest outcome category. The ZIOL model accounts for the zero inflation by assuming that the zero-valued outcomes come from both a logit model and an ordered logit model, allowing potentially different sets of covariates for each model.

## Quick start

Zero-inflated ordered logit model of `y` on `x1` and categorical variable `a` with excess zeros modeled using `x2`

```
ziologit y x1 i.a, inflate(x2)
```

Add offset `x3` to the ordered logit model

```
ziologit y x1 i.a, inflate(x2) offset(x3)
```

Model excess zeros using only a constant

```
ziologit y x1 i.a, inflate(_cons)
```

Model excess zeros with `x2`, and offset `x5` while suppressing the constant term

```
ziologit y x1 i.a, inflate(x2, offset(x5) noconstant)
```

Account for complex sampling design using `svyset` data

```
svy: ziologit y x1 i.a, inflate(x2)
```

## Menu

Statistics > Ordinal outcomes > Zero-inflated ordered logit regression

## Syntax

```
ziologit devar [indepvars] [if] [in] [weight],
 inflate(varlist [, noconstant offset(varname)] | _cons) [options]
```

| <i>options</i>                            | Description                                                                                                                                      |
|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| Model                                     |                                                                                                                                                  |
| * <u>inflate</u> ( )                      | inflation equation that determines excess zero values                                                                                            |
| <u>offset</u> ( <i>varname</i> )          | include <i>varname</i> in model with coefficient constrained to 1                                                                                |
| <u>constraints</u> ( <i>constraints</i> ) | apply specified linear constraints                                                                                                               |
| SE/Robust                                 |                                                                                                                                                  |
| <u>vce</u> ( <i>vcetype</i> )             | <i>vcetype</i> may be <u>oim</u> , <u>robust</u> , <u>cluster</u> <i>clustvar</i> , <u>opg</u> , <u>bootstrap</u> , or <u>jackknife</u>          |
| Reporting                                 |                                                                                                                                                  |
| <u>level</u> (#)                          | set confidence level; default is level(95)                                                                                                       |
| <u>nocnsreport</u>                        | do not display constraints                                                                                                                       |
| <u>or</u>                                 | report odds ratios                                                                                                                               |
| <u>display_options</u>                    | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| Maximization                              |                                                                                                                                                  |
| <u>maximize_options</u>                   | control the maximization process; seldom used                                                                                                    |
| <u>collinear</u>                          | keep collinear variables                                                                                                                         |
| <u>coeflegend</u>                         | display legend instead of statistics                                                                                                             |

\*inflate(*varlist* [, noconstant offset(*varname*)] | \_cons) is required.

*indepvars* and *varlist* may contain factor variables; see [U] 11.4.3 Factor variables.

*bayes*, *bayesboot*, *bootstrap*, *by*, *collect*, *fp*, *jackknife*, *rolling*, *statsby*, and *svy* are allowed; see [U] 11.1.10 Prefix commands. For more details, see [BAYES] bayes: ziologit.

Weights are not allowed with the *bootstrap* prefix; see [R] bootstrap.

*vce*() and weights are not allowed with the *svy* prefix; see [SVY] svy.

*fweights*, *iweights*, and *pweights* are allowed; see [U] 11.1.6 weight.

*collinear* and *coeflegend* do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Options

### Model

`inflate(varlist [ , noconstant offset(varname) ] | _cons)` specifies the inflation equation for the logit model that determines the excess zero values; this option is required. Conceptually, omitting `inflate()` would be equivalent to fitting the model with `ologit`; see [R] [ologit](#).

`inflate(varlist [ , noconstant offset(varname) ])` specifies the independent variables in the inflation equation. To suppress the constant in this equation, specify the `noconstant` suboption. You may optionally include an offset for this *varlist*; see `offset(varname)` in [R] [Estimation options](#).

`inflate(_cons)` specifies that the inflation equation contains only an intercept. To run a zero-inflated model of *depvar* with only an intercept in both equations, type `ziologit depvar, inflate(_cons)`.

`offset(varname)`, `constraints(constraints)`; see [R] [Estimation options](#).

### SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

### Reporting

`level(#)`, `nocnsreport`; see [R] [Estimation options](#).

or reports the estimated coefficients transformed to odds ratios, that is,  $e^b$  rather than  $b$ . Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated. or may be specified at estimation or when replaying previously estimated results.

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

### Maximization

*maximize\_options*: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

Setting the optimization type to `technique(bhhh)` resets the default *vcetype* to `vce(opg)`.

The following options are available with `ziologit` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

ZIOL models (Kelley and Anderson 2008) are used when the outcome of interest is an ordinal response with a higher fraction of observations in the lowest level than would be expected from a standard ordered logit model. The observations in the lowest category are often referred to as zeros because they typically correspond to the absence of a behavior or trait. Examples include measurements of symptoms, diet, exercise, drug use, and other ordinal outcomes where the lowest level is “none” or “never”.

With ordinal outcomes, larger values of the response variable represent higher outcome levels, but the precise numeric value is irrelevant. For consistency, we will refer to the lowest category as zero regardless of whether that outcome takes the number 0 in the dataset.

The presence of a high fraction of zeros, also known as zero inflation, is driven by the fact that the lowest-level observations represent two latent (unobservable) groups: subjects with excess zeros and subjects with conditional zeros. Excess zeros are subjects who are not susceptible to the outcome, whereas conditional zeros are susceptible to higher outcomes even though they currently demonstrate zero response. For example, if the outcome is alcohol consumption, a teetotaler who completely abstains from alcohol would be an excess zero, whereas a drinker who chose not to drink on the day of the survey would be a conditional zero.

To determine susceptibility (that is, to identify excess zeros), ZIOL regression uses a logit model for the inflation equation. For the main regression that determines the ordinal outcome of susceptible subjects, ZIOL uses an ordered logit model. The equation for the main regression is sometimes referred to as the intensity equation. These two equations can have different sets of covariates and different offsets. Parameters can be reported as coefficients or odds ratios with the `or` option. See *Methods and formulas* for more details.

ZIOL models are similar in spirit to zero-inflated ordered probit (ZIOP) models (see [R] [zioprobit](#)), which use a probit model for the inflation equation and an ordered probit model for the intensity equation. Unlike ZIOP models, the results from ZIOL models can be presented as odds ratios. As such, ZIOL models often appeal to epidemiologists and health researchers, whereas ZIOP models are traditionally used by economists and social scientists. In general, the two types of models tend to produce similar predictions and yield similar inference.

In the context of ZIOL models, the excess-zero or “always-zero” group is often called the nonsusceptible group, and, likewise, the other group is called the susceptible group. In the context of ZIOP models, the terms “nonparticipation” and “participation” are used to refer to the groups. We will use both terminologies interchangeably depending on the context.

The ordered logit model is not nested within the ZIOL model, which means that a likelihood-ratio test cannot be used to check whether a standard ordered logit model is adequate. Instead, one can use information criteria; see [example 2](#) in [R] [zip](#).

### ▷ Example 1: Zero-inflated ordered logit model

We use the fictional data on cigarette consumption from [example 1](#) in [R] [zioprobit](#). In that example, we fit a ZIOP model to these data; here we fit a ZIOL model. Both models are appropriate for analyzing this zero-inflated ordinal outcome, but the ZIOP model is more likely to be used by an economist studying consumer purchasing patterns, while the ZIOL model is more likely to be used by an epidemiologist studying smoking behavior.

The outcome of interest, `tobacco`, represents daily cigarette consumption as an ordinal response with four levels: 0 indicates “no cigarettes”, and responses 1 through 3 indicate increasing cigarette consumption. More than half of the respondents reported no cigarette consumption, and we suspect that these respondents belong to one of two latent groups. Individuals in the first group are excess zeros, which is to say that they are genuine nonsmokers and thus are not susceptible to cigarette consumption. Individuals in the second group are would-be smokers with no current smoking activity but who might smoke, say, if the price of cigarettes falls or their income increases. These individuals are called conditional zeros because they demonstrate zero cigarette consumption, conditional on being susceptible to smoking. We suspect that these two types of zeros are driven by different patterns of behavior; hence, we choose the ZIOL model over the traditional ordered logit model.

Here the inflation (logit) equation corresponds to the decision to smoke. The intensity (ordered logit) equation corresponds to the level of cigarette consumption by a smoker or would-be smoker. The intensity equation includes covariates for education in years (`education`), annual income in tens of thousands of dollars (`income`), age in decades (`age`), and gender (`female`). The inflation equation includes all the covariates from the intensity equation, plus indicators for whether either of the respondent’s parents smoked (`parent`) and whether the respondent’s religion discourages smoking (`religion`). Covariates for the intensity equation are specified in the `ziologit` command statement directly after the dependent variable `tobacco`. Covariates for the inflation equation are specified in the required `inflate()` option.

```
. use https://www.stata-press.com/data/r19/tobacco
(Fictional tobacco consumption data)
. ziologit tobacco education income age i.female,
> inflate(education income age i.female i.parent i.religion)
Iteration 0: Log likelihood = -15877.562 (not concave)
Iteration 1: Log likelihood = -13082.531 (not concave)
Iteration 2: Log likelihood = -12316.574 (not concave)
Iteration 3: Log likelihood = -11686.438 (not concave)
Iteration 4: Log likelihood = -11214.794
Iteration 5: Log likelihood = -9857.4914
Iteration 6: Log likelihood = -8355.1808
Iteration 7: Log likelihood = -7805.3902
Iteration 8: Log likelihood = -7658.2562
Iteration 9: Log likelihood = -7652.5121
Iteration 10: Log likelihood = -7652.4891
Iteration 11: Log likelihood = -7652.4891
Zero-inflated ordered logit regression
Log likelihood = -7652.4891
Number of obs = 15,000
Wald chi2(4) = 2143.69
Prob > chi2 = 0.0000
```

| tobacco     | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |           |
|-------------|-------------|-----------|--------|-------|----------------------|-----------|
| tobacco     |             |           |        |       |                      |           |
| education   | .9239328    | .0202141  | 45.71  | 0.000 | .8843139             | .9635516  |
| income      | 1.289223    | .0285828  | 45.10  | 0.000 | 1.233202             | 1.345244  |
| age         | -1.393183   | .0352799  | -39.49 | 0.000 | -1.462331            | -1.324036 |
| female      |             |           |        |       |                      |           |
| Female      | -.7190566   | .0755389  | -9.52  | 0.000 | -.8671101            | -.5710032 |
| inflate     |             |           |        |       |                      |           |
| education   | -.1602718   | .0046406  | -34.54 | 0.000 | -.1693671            | -.1511765 |
| income      | -.1922987   | .007553   | -25.46 | 0.000 | -.2071024            | -.177495  |
| age         | .313469     | .0152296  | 20.58  | 0.000 | .2836196             | .3433184  |
| female      |             |           |        |       |                      |           |
| Female      | -.4400725   | .0516756  | -8.52  | 0.000 | -.5413548            | -.3387902 |
| parent      |             |           |        |       |                      |           |
| Smoking     | 1.282534    | .0527674  | 24.31  | 0.000 | 1.179112             | 1.385956  |
| religion    |             |           |        |       |                      |           |
| Discourag.. | -.5387562   | .0832958  | -6.47  | 0.000 | -.7020129            | -.3754995 |
| _cons       | 2.088609    | .0984716  | 21.21  | 0.000 | 1.895609             | 2.28161   |
| /cut1       | 5.327041    | .1431499  |        |       | 5.046472             | 5.607609  |
| /cut2       | 14.6561     | .3250778  |        |       | 14.01896             | 15.29324  |
| /cut3       | 20.28054    | .4446689  |        |       | 19.409               | 21.15207  |

The first section of the coefficient table, labeled `tobacco`, corresponds to the tobacco intensity equation and contains coefficients from the ordered logit model fit to susceptible subjects. The second section, labeled `inflate`, corresponds to the inflation equation and contains coefficients from the logit model for susceptibility. ZIOL models allow different sets of covariates in the inflation and intensity equations, but when a variable is included in both equations, it is possible for the coefficients to have opposite signs. The third section of the coefficient table contains the cutpoints from the ordered logit model. See [Remarks and examples](#) in [R] `ologit` for an explanation of cutpoints.

Compared with the ZIOP model from [example 1](#) in [R] [zioprobit](#), the coefficients here are larger in absolute value, but the  $z$  scores and  $p$ -values are similar. This is consistent with the fact that the logistic distribution used by ZIOL has heavier tails than the normal distribution used by ZIOP. Perhaps the most important difference between these two models is that the estimated coefficients from the ZIOP model are difficult to interpret, but the coefficients from the ZIOL model can be exponentiated and reported as odds ratios with the `or` option.

We replay our results specifying `or`:

```
. ziologit, or
Zero-inflated ordered logit regression Number of obs = 15,000
 Wald chi2(4) = 2143.69
Log likelihood = -7652.4891 Prob > chi2 = 0.0000
```

|                | Odds ratio | Std. err. | z      | P> z  | [95% conf. interval] |          |
|----------------|------------|-----------|--------|-------|----------------------|----------|
| <b>tobacco</b> |            |           |        |       |                      |          |
| tobacco        |            |           |        |       |                      |          |
| education      | 2.519178   | .0509228  | 45.71  | 0.000 | 2.421323             | 2.620989 |
| income         | 3.629966   | .1037545  | 45.10  | 0.000 | 3.432202             | 3.839125 |
| age            | .2482837   | .0087594  | -39.49 | 0.000 | .2316957             | .2660593 |
| female         |            |           |        |       |                      |          |
| Female         | .4872117   | .0368034  | -9.52  | 0.000 | .420164              | .5649584 |
| <b>inflate</b> |            |           |        |       |                      |          |
| inflate        |            |           |        |       |                      |          |
| education      | .8519122   | .0039533  | -34.54 | 0.000 | .8441989             | .8596959 |
| income         | .8250604   | .0062317  | -25.46 | 0.000 | .8129364             | .8373652 |
| age            | 1.368163   | .0208365  | 20.58  | 0.000 | 1.327928             | 1.409618 |
| female         |            |           |        |       |                      |          |
| Female         | .6439897   | .0332785  | -8.52  | 0.000 | .5819593             | .712632  |
| parent         |            |           |        |       |                      |          |
| Smoking        | 3.605764   | .1902668  | 24.31  | 0.000 | 3.251484             | 3.998647 |
| religion       |            |           |        |       |                      |          |
| Discourag..    | .5834735   | .0486009  | -6.47  | 0.000 | .4955867             | .6869461 |
| _cons          | 8.07368    | .7950279  | 21.21  | 0.000 | 6.656599             | 9.792435 |
| /cut1          | 5.327041   | .1431499  |        |       | 5.046472             | 5.607609 |
| /cut2          | 14.6561    | .3250778  |        |       | 14.01896             | 15.29324 |
| /cut3          | 20.28054   | .4446689  |        |       | 19.409               | 21.15207 |

Note: Estimates are transformed only in the first 2 equations to odds ratios.

Note: `_cons` estimates baseline odds.

Examining the odds ratios from the tobacco intensity equation, we see that a one-unit increase in income, which corresponds to an increase of \$10,000 in annual income, raises a smoker's odds of increased cigarette consumption by a factor of 3.63. Looking at the inflation equation, we see that a one-unit increase in income lowers the odds of being susceptible to smoking by a factor of 0.825. This suggests that wealthier individuals are less likely to smoke (consistent with income acting as a proxy for health consciousness), but if they do decide to smoke, they tend to smoke more cigarettes (perhaps because they can afford them).



## Stored results

ziologit stores the following in `e()`:

### Scalars

|                            |                                                 |
|----------------------------|-------------------------------------------------|
| <code>e(N)</code>          | number of observations                          |
| <code>e(N_zero)</code>     | number of zeros or lowest-category observations |
| <code>e(k_cat)</code>      | number of categories                            |
| <code>e(k)</code>          | number of parameters                            |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>        |
| <code>e(k_eq_model)</code> | number of equations in overall model test       |
| <code>e(k_aux)</code>      | number of auxiliary parameters                  |
| <code>e(k_dv)</code>       | number of dependent variables                   |
| <code>e(df_m)</code>       | model degrees of freedom                        |
| <code>e(ll)</code>         | log likelihood                                  |
| <code>e(N_clust)</code>    | number of clusters                              |
| <code>e(chi2)</code>       | $\chi^2$                                        |
| <code>e(p)</code>          | $p$ -value for model test                       |
| <code>e(rank)</code>       | rank of <code>e(V)</code>                       |
| <code>e(ic)</code>         | number of iterations                            |
| <code>e(rc)</code>         | return code                                     |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise                     |

### Macros

|                                |                                                                          |
|--------------------------------|--------------------------------------------------------------------------|
| <code>e(cmd)</code>            | ziologit                                                                 |
| <code>e(cmdline)</code>        | command as typed                                                         |
| <code>e(depvar)</code>         | name of dependent variable                                               |
| <code>e(wtype)</code>          | weight type                                                              |
| <code>e(wexp)</code>           | weight expression                                                        |
| <code>e(title)</code>          | title in estimation output                                               |
| <code>e(clustvar)</code>       | name of cluster variable                                                 |
| <code>e(offset1)</code>        | offset                                                                   |
| <code>e(offset2)</code>        | offset for <code>inflate()</code>                                        |
| <code>e(chi2type)</code>       | Wald or LR; type of model $\chi^2$ test                                  |
| <code>e(vce)</code>            | <i>vcetype</i> specified in <code>vce()</code>                           |
| <code>e(vcetype)</code>        | title used to label Std. err.                                            |
| <code>e(opt)</code>            | type of optimization                                                     |
| <code>e(which)</code>          | max or min; whether optimizer is to perform maximization or minimization |
| <code>e(ml_method)</code>      | type of ml method                                                        |
| <code>e(user)</code>           | name of likelihood-evaluator program                                     |
| <code>e(technique)</code>      | maximization technique                                                   |
| <code>e(properties)</code>     | b V                                                                      |
| <code>e(predict)</code>        | program used to implement <code>predict</code>                           |
| <code>e(marginsok)</code>      | predictions allowed by <code>margins</code>                              |
| <code>e(marginsnotok)</code>   | predictions disallowed by <code>margins</code>                           |
| <code>e(marginsdefault)</code> | default <code>predict()</code> specification for <code>margins</code>    |
| <code>e(asbalanced)</code>     | factor variables <code>fvset</code> as <code>asbalanced</code>           |
| <code>e(asobserved)</code>     | factor variables <code>fvset</code> as <code>asobserved</code>           |

|                 |                                              |
|-----------------|----------------------------------------------|
| Matrices        |                                              |
| e(b)            | coefficient vector                           |
| e(Cns)          | constraints matrix                           |
| e(iLog)         | iteration log (up to 20 iterations)          |
| e(gradient)     | gradient vector                              |
| e(cat)          | category values                              |
| e(V)            | variance–covariance matrix of the estimators |
| e(V_modelbased) | model-based variance                         |
| Functions       |                                              |
| e(sample)       | marks estimation sample                      |

In addition to the above, the following is stored in `r()`:

|          |                                                                                                                       |
|----------|-----------------------------------------------------------------------------------------------------------------------|
| Matrices |                                                                                                                       |
| r(table) | matrix containing the coefficients with their standard errors, test statistics, $p$ -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

The modern class of zero-inflated models was originally developed by Lambert (1992) to address the problem of excess zeros in count data (see [R] zip). This model framework has been extended to ordinal outcomes with the ZIOL model by Kelley and Anderson (2008).

The precise numeric value of an ordinal outcome is irrelevant, so without loss of generality we consider an ordinal response variable  $Y$  with levels coded as 0, 1, 2,  $\dots$ ,  $H$ . The first step in the ZIOL model is to determine susceptibility. Let  $s_j = 1$  if the  $j$ th individual is susceptible to exhibiting a nonzero response, and let  $s_j = 0$  if the  $j$ th individual is an excess zero. (Similarly, using alternative terminology, let individuals with  $s_j = 1$  belong to the participation group and those with  $s_j = 0$  belong to the nonparticipation group.) The ZIOL inflation equation uses a logit model to determine the probability of susceptibility (or participation) as

$$\Pr(s_j = 1 | \mathbf{z}_j) = F(\mathbf{z}_j \boldsymbol{\gamma}) \quad (1)$$

$\mathbf{z}_j$  is a vector of covariates that determines susceptibility,  $\boldsymbol{\gamma}$  is a vector of coefficients that have to be estimated, and  $F(\cdot)$  is the logistic distribution function:  $F(x) = e^x / (1 + e^x)$ .

Next, conditioning on  $s_j = 1$ , outcome intensity levels  $\tilde{y}_j$  are modeled using an ordered logit model whose levels may also include 0. The corresponding probabilities are given by

$$\Pr(\tilde{y}_j = h | s_j = 1, \mathbf{x}_j) = F(\kappa_h - \mathbf{x}_j \boldsymbol{\beta}) - F(\kappa_{h-1} - \mathbf{x}_j \boldsymbol{\beta}) \quad h = 0, 1, \dots, H \quad (2)$$

$\mathbf{x}_j$  is a vector of covariates that determine intensity (which can be different from  $\mathbf{z}_j$ ),  $\boldsymbol{\beta}$  is a vector of coefficients to be estimated, and cutpoints  $\kappa_h$  are boundary parameters to be estimated (subject to  $\kappa_{-1} = -\infty$ ,  $\kappa_H = +\infty$ ). These cutpoints take the place of an intercept.

The observed response variable is  $y_j = s_j \tilde{y}_j$ . Thus, a zero outcome occurs when  $s_j = 0$  (the individual is an excess zero) or when  $s_j = 1$  and  $\tilde{y}_j = 0$  (the individual is a conditional zero). To observe a positive  $y_j$ , there is a joint requirement that  $s_j = 1$  and  $\tilde{y}_j > 0$ .

The distribution of  $Y$  is given by

$$\begin{aligned} \Pr(Y) &= \begin{cases} \Pr(y_j = 0 | \mathbf{z}_j, \mathbf{x}_j) \\ \Pr(y_j = h | \mathbf{z}_j, \mathbf{x}_j) \quad h = 1, 2, \dots, H \end{cases} \\ &= \begin{cases} \Pr(s_j = 0 | \mathbf{z}_j) + \Pr(s_j = 1 | \mathbf{z}_j) \Pr(\tilde{y}_j = 0 | s_j = 1, \mathbf{x}_j) \\ \Pr(s_j = 1 | \mathbf{z}_j) \Pr(\tilde{y}_j = h | s_j = 1, \mathbf{x}_j) \quad h = 1, 2, \dots, H \end{cases} \end{aligned} \tag{3}$$

The probability of zero outcome has been inflated because it is the sum of the probability of zero intensity from the ordered logit model and the probability of nonsusceptibility from the logit model.

Substituting (1) and (2) in (3), we get

$$\begin{aligned} \Pr(Y) &= \begin{cases} \Pr(y_j = 0 | \mathbf{z}_j, \mathbf{x}_j) \\ \Pr(y_j = h | \mathbf{z}_j, \mathbf{x}_j) \quad h = 1, 2, \dots, H - 1 \\ \Pr(y_j = H | \mathbf{z}_j, \mathbf{x}_j) \end{cases} \\ &= \begin{cases} \{1 - F(\mathbf{z}_j \boldsymbol{\gamma})\} + F(\mathbf{z}_j \boldsymbol{\gamma}) F(\kappa_0 - \mathbf{x}_j \boldsymbol{\beta}) \\ F(\mathbf{z}_j \boldsymbol{\gamma}) \{F(\kappa_h - \mathbf{x}_j \boldsymbol{\beta}) - F(\kappa_{h-1} - \mathbf{x}_j \boldsymbol{\beta})\} \quad h = 1, 2, \dots, H - 1 \\ F(\mathbf{z}_j \boldsymbol{\gamma}) \{1 - F(\kappa_{H-1} - \mathbf{x}_j \boldsymbol{\beta})\} \end{cases} \end{aligned} \tag{4}$$

If the `offset()` option is specified,  $\mathbf{x}_j \boldsymbol{\beta}$  in the intensity equation is replaced with  $\mathbf{x}_j \boldsymbol{\beta} + \text{offset}_j^\beta$ . If the `offset()` suboption is specified in the `inflate()` option,  $\mathbf{z}_j \boldsymbol{\gamma}$  in the inflation equation is replaced with  $\mathbf{z}_j \boldsymbol{\gamma} + \text{offset}_j^\gamma$ .

The log-likelihood function is

$$\ln L = \sum_{j=1}^N w_j \sum_{h=0}^H I(y_j = h) \ln \{ \Pr(y_j = h | \mathbf{z}_j, \mathbf{x}_j) \}$$

where  $w_j$  is an optional weight for the  $j$ th observation and

$$I(y_j = h) = \begin{cases} 1 & \text{if } y_j = h \\ 0 & \text{otherwise} \end{cases}$$

The choice between the ZIOL model and the ordered logit model cannot be made using a likelihood-ratio test because the two hypotheses are not nested in the usual sense of parameter restrictions. The inflation effect is removed, and all subjects are deemed susceptible when  $\mathbf{z}_j \boldsymbol{\gamma} \rightarrow \infty$ , a condition that cannot be imposed. To compare the fits of nonnested models, you can use `estat ic` to display information criteria (see [R] [estat ic](#)).

This command supports the Huber/White/sandwich estimator of the variance and its clustered version with `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] [\\_robust](#), particularly *Maximum likelihood estimators* and *Methods and formulas*.

`ziologit` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] [Variance estimation](#).

## References

- Kelley, M. E., and S. J. Anderson. 2008. Zero inflation in ordinal data: Incorporating susceptibility to response through the use of a mixture model. *Statistics in Medicine* 27: 3674–3688. <https://doi.org/10.1002/sim.3267>.
- Lambert, D. 1992. Zero-inflated Poisson regression, with an application to defects in manufacturing. *Technometrics* 34: 1–14. <https://doi.org/10.2307/1269547>.

## Also see

- [R] **ziologit postestimation** — Postestimation tools for ziologit
- [R] **logit** — Logistic regression, reporting coefficients
- [R] **ologit** — Ordered logistic regression
- [R] **zioprobit** — Zero-inflated ordered probit regression
- [BAYES] **bayes: ziologit** — Bayesian zero-inflated ordered logit regression
- [SVY] **svy estimation** — Estimation commands for survey data
- [U] **20 Estimation and postestimation commands**

|                                                 |                                      |                                     |                                      |
|-------------------------------------------------|--------------------------------------|-------------------------------------|--------------------------------------|
| Postestimation commands<br>Methods and formulas | <a href="#">predict</a><br>Reference | <a href="#">margins</a><br>Also see | <a href="#">Remarks and examples</a> |
|-------------------------------------------------|--------------------------------------|-------------------------------------|--------------------------------------|

## Postestimation commands

The following postestimation commands are available after `ziologist`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from <code>margins</code> (profile plots, interaction plots, etc.)                                                    |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | probabilities, linear predictions and their SEs, etc.                                                                                   |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results.

# predict

## Description for predict

predict creates a new variable containing predictions such as probabilities, linear predictions, and standard errors.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] { stub* | newvar | newvarlist } [if] [in] [, statistic
outcome(outcome) nooffset]
```

```
predict [type] stub* [if] [in] , scores
```

| <i>statistic</i>      | Description                                                                     |
|-----------------------|---------------------------------------------------------------------------------|
| Main                  |                                                                                 |
| <code>pmargin</code>  | marginal probabilities of levels, $\Pr(y_j = h)$ ; the default                  |
| <code>pjoint1</code>  | joint probabilities of levels and susceptibility, $\Pr(y_j = h, s_j = 1)$       |
| <code>pcond1</code>   | probabilities of levels conditional on susceptibility, $\Pr(y_j = h   s_j = 1)$ |
| <code>ps</code>       | probability of susceptibility, $\Pr(s_j = 1)$                                   |
| <code>pns</code>      | probability of nonsusceptibility, $\Pr(s_j = 0)$                                |
| <code>xb</code>       | linear prediction                                                               |
| <code>xbinfl</code>   | linear prediction for inflation equation                                        |
| <code>stdp</code>     | standard error of the linear prediction                                         |
| <code>stdpinfl</code> | standard error of the linear prediction for inflation equation                  |

If you do not specify `outcome()`, `pmargin`, `pjoint1`, and `pcond1` (with one new variable specified) assume `outcome(#1)`.

You specify one or  $k$  new variables with `pmargin`, `pjoint1`, and `pcond1`, where  $k$  is the number of outcomes.

You specify one new variable with `ps`, `pns`, `xb`, `xbinfl`, `stdp`, and `stdpinfl`.

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

## Options for predict

Main

`pmargin`, the default, calculates the predicted marginal probabilities of outcome levels,  $\Pr(y_j = h)$ .

`pjoint1` calculates the predicted joint probabilities of outcome levels and susceptibility,  $\Pr(y_j = h, s_j = 1)$ .

`pcond1` calculates the predicted probabilities of outcome levels conditional on susceptibility,  $\Pr(y_j = h | s_j = 1)$ .

With `pmargin`, `pjoint1`, and `pcond1`, you can compute predicted probabilities for one or for all outcome levels. When you specify one new variable, `predict` computes probabilities for the first outcome level. You can specify the `outcome(#i)` option to obtain probabilities for the  $i$ th level. When you specify multiple new variables or a stub, `predict` computes probabilities for all outcome levels. The behavior of `predict` with one new variable is equivalent to specifying `outcome(#1)`.

`ps` and `pns` calculate the predicted marginal probability of susceptibility [ $\Pr(s_j = 1)$ ] and of nonsusceptibility [ $\Pr(s_j = 0)$ ], respectively.

In econometrics literature, probabilities of susceptibility and nonsusceptibility are known as probabilities of participation and nonparticipation. Similarly to `predict` after `zioprobit`, you can use options `ppar` and `pnpair` to compute these probabilities. Options `ppar` and `pnpair` produce identical results to the respective options `ps` and `pns` but label new variables as `Pr(participation)` and `Pr(nonparticipation)` instead of `Pr(susceptible)` and `Pr(nonsusceptible)`.

`xb` calculates the linear prediction for the ordered logit equation, which is  $\mathbf{x}_j\beta$  if `offset()` was not specified with `ziologit` and is  $\mathbf{x}_j\beta + \text{offset}_j^\beta$  if `offset()` was specified.

`xbinfl` calculates the linear prediction for the inflation equation, which is  $\mathbf{z}_j\gamma$  if `offset()` was not specified in `inflate()` and is  $\mathbf{z}_j\gamma + \text{offset}_j^\gamma$  if `offset()` was specified in `inflate()`.

`stdp` calculates the standard error of the linear prediction for the ordered logit equation.

`stdpinfl` calculates the standard error of the linear prediction for the inflation equation.

`outcome(outcome)` specifies the outcome for which predicted probabilities are to be calculated. `outcome()` should contain either one value of the dependent variable or one of #1, #2, ..., with #1 meaning the first category of the dependent variable, #2 meaning the second category, etc. `outcome()` is allowed only with `pmargin`, `pjoint1`, and `pcond1`.

`nooffset` is relevant only if you specified `offset(varname)` with `ziologit` or within the `inflate()` option. It modifies the calculations made by `predict` so that they ignore the offset variable; that is, the linear prediction for the main regression equation is treated as  $\mathbf{x}_j\beta$  rather than as  $\mathbf{x}_j\beta + \text{offset}_j^\beta$  and the linear prediction for the inflation equation is treated as  $\mathbf{z}_j\gamma$  rather than as  $\mathbf{z}_j\gamma + \text{offset}_j^\gamma$ .

`scores` calculates equation-level score variables.

The first new variable will contain  $\partial \ln L / \partial (\mathbf{x}_j\beta)$ . In the absence of independent variables in the main equation, this variable is not stored.

The second new variable will contain  $\partial \ln L / \partial (\mathbf{z}_j\gamma)$ .

When the dependent variable takes  $k$  different values, the third new variable through new variable  $k + 1$  will contain  $\partial \ln L / \partial (\kappa_h)$  for  $h = 0, 1, \dots, k - 2$ .

# margins

## Description for margins

`margins` estimates margins of response for probabilities and linear predictions.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i> | Description                                                                     |
|------------------|---------------------------------------------------------------------------------|
| default          | marginal probabilities for each outcome                                         |
| <u>p</u> margin  | marginal probabilities of levels, $\Pr(y_j = h)$ ; the default                  |
| <u>p</u> joint1  | joint probabilities of levels and susceptibility, $\Pr(y_j = h, s_j = 1)$       |
| <u>p</u> cond1   | probabilities of levels conditional on susceptibility, $\Pr(y_j = h   s_j = 1)$ |
| <u>p</u> s       | probability of susceptibility, $\Pr(s_j = 1)$                                   |
| <u>p</u> ns      | probability of nonsusceptibility, $\Pr(s_j = 0)$                                |
| <u>x</u> b       | linear prediction                                                               |
| <u>x</u> binfl   | linear prediction for inflation equation                                        |
| <u>s</u> tdp     | not allowed with margins                                                        |
| <u>s</u> tdpinfl | not allowed with margins                                                        |

`pmargin`, `pjoint1`, and `pcond1` default to the first outcome.

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [R] [margins](#).

## Remarks and examples

The ZIOL model allows all the predictions and marginal effects available with the standard `ologit` model (see [R] [ologit postestimation](#)), along with additional predictions and marginal effects related to the inflation equation for susceptibility. The probabilities of susceptibility and nonsusceptibility can be calculated using options `ps` and `pns`, respectively. If you prefer an alternative terminology of probabilities of participation and nonparticipation, you can instead use options `ppar` and `pnpair`, which will produce identical numerical results but label variables as `Pr(participation)` and `Pr(nonparticipation)` instead of `Pr(susceptible)` and `Pr(nonsusceptible)`.

### ▷ Example 1: Average marginal effect of gender on probability of nonsusceptibility

In [example 1](#) of [R] [ziologit](#), we fit a model for levels of cigarette consumption.

```
. use https://www.stata-press.com/data/r19/tobacco
(Fictional tobacco consumption data)
. ziologit tobacco education income age i.female,
> inflate(education income age i.female i.parent i.religion)
(output omitted)
```

This model parallels the zero-inflated ordered probit (ZIOP) model that was fit in [example 1](#) of [R] [zioprobit](#).

To continue the comparison between the ZIOL and ZIOP models, we re-create [example 1](#) from [R] [zioprobit postestimation](#) by using `margins` to estimate the average marginal effect of gender on the probability of nonsusceptibility (being an excess zero) for individuals with a college degree (17 years of education) and a smoking parent.

```
. margins, predict(pns) dydx(female) at(education = 17 parent = 1)
Average marginal effects Number of obs = 15,000
Model VCE: OIM
Expression: Pr(nonsusceptible), predict(pns)
dy/dx wrt: 1.female
At: education = 17
 parent = 1
```

|        | Delta-method |           |      |       | [95% conf. interval] |          |
|--------|--------------|-----------|------|-------|----------------------|----------|
|        | dy/dx        | std. err. | z    | P> z  |                      |          |
| female | .085421      | .010096   | 8.46 | 0.000 | .0656333             | .1052087 |
| Female |              |           |      |       |                      |          |

Note: dy/dx for factor levels is the discrete change from the base level.

Despite the differences between the ZIOL and ZIOP models, the conclusion is the same: women with a college degree and a smoking parent are expected to have an approximately 8.5% higher chance of being genuine nonsmokers (excess zeros) than comparable men.

◀

### ▷ Example 2: Predicted probabilities of conditional zeros

Next, we consider the effect of income on the probability of zero tobacco consumption, conditional on susceptibility. These would-be smokers are known as conditional zeros. In [example 1](#) of [R] [ziologit](#), we saw that increasing income raises a smoker's odds of increased tobacco consumption dramatically, so we expect to see a larger fraction of conditional zeros at the lower end of the income scale.

We examine conditional probabilities of zero consumption for incomes ranging from \$10,000 to \$60,000, and we use the `noatlegend` option to suppress the default legend because we know the values 1 to 6 correspond to income in tens of thousands of dollars.

```
. margins, predict(pcond1 outcome(0)) at(income = (1/6)) noatlegend
Predictive margins Number of obs = 15,000
Model VCE: OIM
Expression: Pr(tobacco=0|susceptible=1), predict(pcond1 outcome(0))
```

|     | Delta-method |           | z      | P> z  | [95% conf. interval] |          |
|-----|--------------|-----------|--------|-------|----------------------|----------|
|     | Margin       | std. err. |        |       |                      |          |
| _at |              |           |        |       |                      |          |
| 1   | .5923634     | .0027586  | 214.73 | 0.000 | .5869566             | .5977702 |
| 2   | .5393818     | .0025948  | 207.87 | 0.000 | .534296              | .5444676 |
| 3   | .4854668     | .0024651  | 196.94 | 0.000 | .4806354             | .4902982 |
| 4   | .4306299     | .0023953  | 179.78 | 0.000 | .4259352             | .4353245 |
| 5   | .3741538     | .0024547  | 152.42 | 0.000 | .3693427             | .3789649 |
| 6   | .3152985     | .0026294  | 119.91 | 0.000 | .3101449             | .320452  |

The influence of income is dramatic: susceptible individuals (potential smokers) who earn \$10,000 a year are almost twice as likely to refrain from smoking as potential smokers who earn \$60,000 per year (59% versus 32%).



## Methods and formulas

See *Methods and formulas* in [R] **ziologit** for the model definition and notation. Specifically, see (1) for the formula for the probability of susceptibility,  $\Pr(s_j = 1|\mathbf{z}_j)$ ; see (2) for the formula for the probabilities of outcome levels conditional on susceptibility,  $\Pr(y_j = h|s_j = 1, \mathbf{x}_j)$ ; and see (4) for the formula for the marginal probabilities of outcome levels,  $\Pr(y_j = h|\mathbf{z}_j, \mathbf{x}_j)$ .

The joint probability of susceptibility and outcome  $y_j = h$  can be expressed as

$$\Pr(y_j = h, s_j = 1|\mathbf{z}_j, \mathbf{x}_j) = \Pr(s_j = 1|\mathbf{z}_j) \Pr(y_j = h|s_j = 1, \mathbf{x}_j)$$

for  $h = 0, 1, \dots, H$ .

## Reference

Kelley, M. E., and S. J. Anderson. 2008. Zero inflation in ordinal data: Incorporating susceptibility to response through the use of a mixture model. *Statistics in Medicine* 27: 3674–3688. <https://doi.org/10.1002/sim.3267>.

## Also see

[R] **ziologit** — Zero-inflated ordered logit regression

[U] **20 Estimation and postestimation commands**

[Description](#)  
[Options](#)  
[References](#)

[Quick start](#)  
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[Menu](#)  
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## Description

`zioprobit` fits a model for an ordinal outcome with excess zeros, a higher fraction of zeros than would be expected from a standard ordered probit model, also known as zero inflation. This model is known as a zero-inflated ordered probit (ZIOP) model. In the context of ZIOP models, zero is an actual 0 value or the lowest outcome category. The ZIOP model accounts for the zero inflation by assuming that the zero-valued outcomes come from both a probit model and an ordered probit model, allowing potentially different sets of covariates for each model.

## Quick start

Zero-inflated ordered probit model of `y` on `x1` and categorical variable `a` with excess zeros modeled using `x2`

```
zioprobit y x1 i.a, inflate(x2)
```

Add offset `x3` to the ordered probit model

```
zioprobit y x1 i.a, inflate(x2) offset(x3)
```

Model excess zeros using only a constant

```
zioprobit y x1 i.a, inflate(_cons)
```

Model excess zeros with `x2`, and offset `x5` while suppressing the constant term

```
zioprobit y x1 i.a, inflate(x2, offset(x5) noconstant)
```

Account for complex sampling design using `svyset` data

```
svy: zioprobit y x1 i.a, inflate(x2)
```

## Menu

Statistics > Ordinal outcomes > Zero-inflated ordered probit regression

## Syntax

```
zioprobit depvar [indepvars] [if] [in] [weight] ,
 inflate(varlist [, noconstant offset(varname)] | _cons) [options]
```

| <i>options</i> | Description |
|----------------|-------------|
|----------------|-------------|

### Model

|                                           |                                                                   |
|-------------------------------------------|-------------------------------------------------------------------|
| * <u>inflate</u> ( )                      | inflation equation that determines excess zero values             |
| <u>offset</u> ( <i>varname</i> )          | include <i>varname</i> in model with coefficient constrained to 1 |
| <u>constraints</u> ( <i>constraints</i> ) | apply specified linear constraints                                |

### SE/Robust

|                               |                                                                                                                                         |
|-------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <u>vce</u> ( <i>vcetype</i> ) | <i>vcetype</i> may be <u>oim</u> , <u>robust</u> , <u>cluster</u> <i>clustvar</i> , <u>opg</u> , <u>bootstrap</u> , or <u>jackknife</u> |
|-------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|

### Reporting

|                        |                                                                                                                                                  |
|------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| <u>level</u> (#)       | set confidence level; default is level(95)                                                                                                       |
| <u>nocnsreport</u>     | do not display constraints                                                                                                                       |
| <u>display_options</u> | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |

### Maximization

|                         |                                               |
|-------------------------|-----------------------------------------------|
| <u>maximize_options</u> | control the maximization process; seldom used |
|-------------------------|-----------------------------------------------|

|                   |                                      |
|-------------------|--------------------------------------|
| <u>collinear</u>  | keep collinear variables             |
| <u>coeflegend</u> | display legend instead of statistics |

---

\*inflate(*varlist* [ , noconstant offset(*varname*) ] | \_cons) is required.

*indepvars* and *varlist* may contain factor variables; see [U] 11.4.3 **Factor variables**.

*bayes*, *bayesboot*, *bootstrap*, *by*, *collect*, *fp*, *jackknife*, *rolling*, *statsby*, and *svy* are allowed; see [U] 11.1.10 **Prefix commands**. For more details, see [BAYES] **bayes: zioprobit**.

Weights are not allowed with the *bootstrap* prefix; see [R] **bootstrap**.

*vce*() and weights are not allowed with the *svy* prefix; see [SVY] **svy**.

*fweights*, *iwweights*, and *pweights* are allowed; see [U] 11.1.6 **weight**.

*collinear* and *coeflegend* do not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options

### Model

`inflate(varlist [ , noconstant offset(varname) ] | _cons)` specifies the inflation equation for the probit model that determines the excess zero values; this option is required. Conceptually, omitting `inflate()` would be equivalent to fitting the model with `oprobit`; see [R] [oprobit](#).

`inflate(varlist [ , noconstant offset(varname) ])` specifies the independent variables in the inflation equation. To suppress the constant in this equation, specify the `noconstant` suboption. You may optionally include an offset for this *varlist*; see `offset(varname)` in [R] [Estimation options](#).

`inflate(_cons)` specifies that the inflation equation contains only an intercept. To run a zero-inflated model of *depvar* with only an intercept in both equations, type `zioprobit depvar, inflate(_cons)`.

`offset(varname)`, `constraints(constraints)`; see [R] [Estimation options](#).

### SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

### Reporting

`level(#)`, `nocnsreport`; see [R] [Estimation options](#).

`display_options`: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

### Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

Setting the optimization type to `technique(bhhh)` resets the default *vcetype* to `vce(opg)`.

The following options are available with `zioprobit` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

ZIOP models are used when the outcome of interest is an ordinal response variable and the data exhibit a high fraction of observations in the lowest category or what we will refer to from now on as “zero” but without quotes. Like the ordered probit model, the actual values taken by the ordinal response variable are irrelevant. While the outcome is typically coded as 0, 1, 2, ..., *H*, `zioprobit` interprets the lowest value present in the dataset as 0 to be consistent with the original derivation of the model and subsequent applications.

Like all zero-inflated models, the ZIOP model is an alternative when the data exhibit a higher fraction of zero-valued outcomes than is likely compatible with an ordered probit model. This concentration of zeros is referred to as zero inflation. Inflation is assumed to occur in the lowest value to ensure that shifting the levels of the ordinal response variable by a constant will not affect the estimated parameters in the model. This is common in ordered probit models; see [R] [oprobit](#).

Without loss of generality, we consider an ordinal response variable with levels 0, 1, 2, ...,  $H$ . Traditional ordered probit models treat all observations with zero-valued outcomes as a homogeneous group. By contrast, ZIOP models assume that zeros could occur in the data as members of two latent (unobservable) groups. Individuals in the “always-zero” group have outcome 0 as the only possible value. This first group is often called the nonparticipation group. The second group, in addition to 0, may also assume any of the other values, 1, 2, ...,  $H$ . This group is often called the participation group. Some disciplines, such as public health and medicine, refer to the process that determines the zeros rather than the groups. In this case, there is an incidence or occurrence process that determines whether an observation belongs to the always-zero group and a severity or intensity process that determines the level in the second group. The result of having two groups or processes is an inflation in the proportion of zero-valued observations in the data.

The ZIOP model has been used in studies of international and domestic conflicts ([Bagozzi et al. 2015](#)), sports participation ([Downward, Lera-Lopez, and Rasciute 2011](#)), and the adoption of new building technologies ([Ganguly, Koebel, and Cantrell 2010](#)), to name a few. See [Kelley and Anderson \(2008\)](#) for a discussion of zero-inflated ordinal models in the context of health.

The classic application of the ZIOP model is the study of tobacco use by [Harris and Zhao \(2007\)](#). Like the zero-inflated Poisson models in the count-data literature ([Lambert 1992](#)), [Harris and Zhao](#) derived the ZIOP model using a two-stage decision process. An individual must decide whether to participate in an activity (for example, smoking or drug consumption) and, conditional on participating, must decide on the level of participation, which also includes zero participation. The first decision is a binary choice and is modeled using a probit model, while the second is an ordered choice and is modeled using an ordered probit model. In other terms, to account for the excess of zeros, [Harris and Zhao](#) allowed for zero observations to occur in two ways: as a realization of the probit model (nonparticipants) and as a realization of the ordered probit model when the binary random variable in the probit model is 1 (participant with zero activity). See [Methods and formulas](#) for more details. For a Bayesian derivation of the ZIOP model, see [Gurmu and Dagne \(2012\)](#).

You may want to check whether a standard ordered probit model is adequate to fit the data. You can do this, for instance, using information criteria; see [example 2](#) in [R] [zip](#).

### ▷ Example 1: Zero-inflated ordered probit model

We have fictional data on cigarette consumption per day for 15,000 subjects between ages 14 and 84. The outcome of interest, `tobacco`, is an ordinal response with four levels coded as 0 for “no cigarettes”, 1 for “up to 8 cigarettes/day”, 2 for “8 to 12 cigarettes/day”, and 3 for “more than 12 cigarettes/day”. The exact number of daily consumed cigarettes is unknown.

About 63% of the respondents identified themselves as current nonsmokers. We suspect that these self-identified current nonsmokers belong to one of two groups. Individuals in the first group are genuine nonsmokers (always-zero group) who have never smoked and will never smoke. Individuals in the second group are smokers with no smoking activity who could be the corner solution of a standard consumer demand problem and who may smoke, say, if the price of tobacco falls or their income increases. It is likely that these two types of zeros are driven by different patterns of consumer behavior and a ZIOP model is a good candidate in this case.

We model tobacco consumption levels for subjects who choose to smoke as a function of years of education (`education`), annual income in tens of thousands of dollars (`income`), age in tens of years (`age`), and whether the respondent is a female (`female`). In addition to education, income, age, and female, the decision to smoke is modeled as a function of whether either of the respondent's parents smoked (`parent`) and whether the respondent's religion discourages smoking (`religion`). We list all the covariates in the required `inflate()` option.

```
. use https://www.stata-press.com/data/r19/tobacco
(Fictional tobacco consumption data)
. zioprobit tobacco education income i.female age,
> inflate(education income i.parent age i.female i.religion)

Iteration 0: Log likelihood = -14820.211 (not concave)
Iteration 1: Log likelihood = -12819.475 (not concave)
Iteration 2: Log likelihood = -12078.843 (not concave)
Iteration 3: Log likelihood = -10926.037
Iteration 4: Log likelihood = -9549.5112
Iteration 5: Log likelihood = -8662.3141
Iteration 6: Log likelihood = -7749.9803
Iteration 7: Log likelihood = -7647.1348
Iteration 8: Log likelihood = -7640.5027
Iteration 9: Log likelihood = -7640.4738
Iteration 10: Log likelihood = -7640.4738

Zero-inflated ordered probit regression Number of obs = 15,000
 Wald chi2(4) = 2574.27
Log likelihood = -7640.4738 Prob > chi2 = 0.0000
```

| tobacco     | Coefficient | Std. err. | z      | P> z  | [95% conf. interval] |           |
|-------------|-------------|-----------|--------|-------|----------------------|-----------|
| tobacco     |             |           |        |       |                      |           |
| education   | .5112664    | .0102407  | 49.92  | 0.000 | .491195              | .5313378  |
| income      | .712975     | .0144803  | 49.24  | 0.000 | .6845942             | .7413559  |
| female      |             |           |        |       |                      |           |
| Female      | -.3975341   | .0416675  | -9.54  | 0.000 | -.4792009            | -.3158674 |
| age         | -.7709896   | .0182554  | -42.23 | 0.000 | -.8067695            | -.7352097 |
| inflate     |             |           |        |       |                      |           |
| education   | -.0966613   | .0026422  | -36.58 | 0.000 | -.1018398            | -.0914827 |
| income      | -.1157545   | .0043787  | -26.44 | 0.000 | -.1243365            | -.1071725 |
| parent      |             |           |        |       |                      |           |
| Smoking     | .7655798    | .0307553  | 24.89  | 0.000 | .7053006             | .825859   |
| age         | .1873904    | .0088643  | 21.14  | 0.000 | .1700168             | .204764   |
| female      |             |           |        |       |                      |           |
| Female      | -.2639665   | .0307184  | -8.59  | 0.000 | -.3241735            | -.2037595 |
| religion    |             |           |        |       |                      |           |
| Discourag.. | -.3223335   | .0496827  | -6.49  | 0.000 | -.4197098            | -.2249572 |
| _cons       | 1.27051     | .0584794  | 21.73  | 0.000 | 1.155892             | 1.385127  |
| /cut1       | 2.959808    | .0753035  |        |       | 2.812216             | 3.1074    |
| /cut2       | 8.111228    | .1648965  |        |       | 7.788037             | 8.43442   |
| /cut3       | 11.20791    | .2247711  |        |       | 10.76736             | 11.64845  |

In the output table, the first set of coefficients, labeled `tobacco`, corresponds to the participation (smoking) levels. These coefficients are interpreted in the same way as coefficients from an ordered probit model. The second set of coefficients, labeled `inflate`, corresponds to the equation for the participation decision. These are interpreted in the same way as coefficients from a binary probit model. ZIOP models do not require the variables to be the same in the participation level and decision equations. However, the same variables can appear in both. If the same variables are included, it is not uncommon for the coefficients to have opposite signs. For example, `income` and `education` in the model above have positive signs in the level equation and negative signs in the decision equation.

The estimated coefficients are not particularly informative, and as with all discrete choice models, marginal effects are better to interpret. We use `margins` to estimate the average marginal effect of having a smoking parent on the probability of being a genuine nonsmoker. Specifying `pnpar` within the `predict()` option means that we are requesting the predicted probability of nonparticipation, which in our example is equivalent to being a genuine nonsmoker.

```
. margins, predict(pnpar) dydx(parent)
Average marginal effects Number of obs = 15,000
Model VCE: OIM
Expression: Pr(nonparticipation), predict(pnpar)
dy/dx wrt: 1.parent
```

|                   | Delta-method |           |        |       |                        |
|-------------------|--------------|-----------|--------|-------|------------------------|
|                   | dy/dx        | std. err. | z      | P> z  | [95% conf. interval]   |
| parent<br>Smoking | -.1797895    | .0071967  | -24.98 | 0.000 | -.1938948    -.1656843 |

Note: dy/dx for factor levels is the discrete change from the base level.

On average, individuals whose parents are smokers are about 18% less likely to be nonsmokers themselves than individuals whose parents did not use tobacco. See [\[R\] zioprobit postestimation](#) for more information and examples.

## Stored results

zioprobit stores the following in `e()`:

### Scalars

|                            |                                                 |
|----------------------------|-------------------------------------------------|
| <code>e(N)</code>          | number of observations                          |
| <code>e(N_zero)</code>     | number of zeros or lowest-category observations |
| <code>e(k_cat)</code>      | number of categories                            |
| <code>e(k)</code>          | number of parameters                            |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>        |
| <code>e(k_eq_model)</code> | number of equations in overall model test       |
| <code>e(k_aux)</code>      | number of auxiliary parameters                  |
| <code>e(k_dv)</code>       | number of dependent variables                   |
| <code>e(df_m)</code>       | model degrees of freedom                        |
| <code>e(ll)</code>         | log likelihood                                  |
| <code>e(N_clust)</code>    | number of clusters                              |
| <code>e(chi2)</code>       | $\chi^2$                                        |
| <code>e(p)</code>          | $p$ -value for model test                       |
| <code>e(rank)</code>       | rank of <code>e(V)</code>                       |
| <code>e(ic)</code>         | number of iterations                            |
| <code>e(rc)</code>         | return code                                     |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise                     |

### Macros

|                                |                                                                          |
|--------------------------------|--------------------------------------------------------------------------|
| <code>e(cmd)</code>            | zioprobit                                                                |
| <code>e(cmdline)</code>        | command as typed                                                         |
| <code>e(depvar)</code>         | name of dependent variable                                               |
| <code>e(wtype)</code>          | weight type                                                              |
| <code>e(wexp)</code>           | weight expression                                                        |
| <code>e(title)</code>          | title in estimation output                                               |
| <code>e(clustvar)</code>       | name of cluster variable                                                 |
| <code>e(offset1)</code>        | offset                                                                   |
| <code>e(offset2)</code>        | offset for <code>inflate()</code>                                        |
| <code>e(chi2type)</code>       | Wald or LR; type of model $\chi^2$ test                                  |
| <code>e(vce)</code>            | <i>vcetype</i> specified in <code>vce()</code>                           |
| <code>e(vcetype)</code>        | title used to label Std. err.                                            |
| <code>e(opt)</code>            | type of optimization                                                     |
| <code>e(which)</code>          | max or min; whether optimizer is to perform maximization or minimization |
| <code>e(ml_method)</code>      | type of ml method                                                        |
| <code>e(user)</code>           | name of likelihood-evaluator program                                     |
| <code>e(technique)</code>      | maximization technique                                                   |
| <code>e(properties)</code>     | b V                                                                      |
| <code>e(predict)</code>        | program used to implement <code>predict</code>                           |
| <code>e(marginsok)</code>      | predictions allowed by <code>margins</code>                              |
| <code>e(marginsnotok)</code>   | predictions disallowed by <code>margins</code>                           |
| <code>e(marginsdefault)</code> | default <code>predict()</code> specification for <code>margins</code>    |
| <code>e(asbalanced)</code>     | factor variables <code>fvset</code> as <code>asbalanced</code>           |
| <code>e(asobserved)</code>     | factor variables <code>fvset</code> as <code>asobserved</code>           |

### Matrices

|                              |                                              |
|------------------------------|----------------------------------------------|
| <code>e(b)</code>            | coefficient vector                           |
| <code>e(Cns)</code>          | constraints matrix                           |
| <code>e(ilog)</code>         | iteration log (up to 20 iterations)          |
| <code>e(gradient)</code>     | gradient vector                              |
| <code>e(cat)</code>          | category values                              |
| <code>e(V)</code>            | variance–covariance matrix of the estimators |
| <code>e(V_modelbased)</code> | model-based variance                         |

### Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

In addition to the above, the following is stored in `r()`:

|                       |                                                                                                                       |
|-----------------------|-----------------------------------------------------------------------------------------------------------------------|
| Matrices              |                                                                                                                       |
| <code>r(table)</code> | matrix containing the coefficients with their standard errors, test statistics, $p$ -values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

## Methods and formulas

Consider an ordinal response variable  $Y$  with levels coded as  $0, 1, 2, \dots, H$ . For notational simplicity, we assume that the zeros are inflated, but the following derivation may be adapted to accommodate inflation in the lowest outcome category. Harris and Zhao (2007) derived the ZIOP model in two steps. First, the group membership (participants versus nonparticipants) can be modeled using a probit model. Let  $s_j = 1$  if the  $j$ th individual belongs to the participation group or let  $s_j = 0$  otherwise. With the probit model, the probability of participation is given by

$$\Pr(s_j = 1 | \mathbf{z}_j) = \Phi(\mathbf{z}_j \boldsymbol{\gamma}) \quad (1)$$

$\mathbf{z}_j$  is a vector of covariates that determines group membership,  $\boldsymbol{\gamma}$  is a vector of coefficients that have to be estimated, and  $\Phi(\cdot)$  is the standard normal distribution function. Next, conditioning on  $s_j = 1$ , participation levels  $\tilde{y}_j$  are modeled using an ordered probit model; these levels may also include 0. The corresponding probabilities are given by

$$\Pr(\tilde{y}_j = h | s_j = 1, \mathbf{x}_j) = \Phi(\kappa_h - \mathbf{x}_j \boldsymbol{\beta}) - \Phi(\kappa_{h-1} - \mathbf{x}_j \boldsymbol{\beta}) \quad h = 0, 1, \dots, H \quad (2)$$

where  $\kappa_{-1} = -\infty$ ,  $\kappa_H = +\infty$ , and  $\mathbf{x}_j$  is a vector of covariates that could be different from  $\mathbf{z}_j$ .  $\kappa_h$  are boundary parameters that need to be estimated in addition to the coefficients vector  $\boldsymbol{\beta}$ .

The intercept  $\beta_0$  is set equal to 0 in (2) for identification. Note that  $s_j$  and  $\tilde{y}_j$  are both unobservable in terms of the zeros. The observed response variable is  $y_j = s_j \tilde{y}_j$ . Thus, the zero outcome occurs when  $s_j = 0$  (the individual is not a participant) or occurs when  $s_j = 1$  and  $\tilde{y}_j = 0$  (the individual is a participant with zero activity). To observe a positive  $y_j$ , it is a joint requirement that  $s_j = 1$  and  $\tilde{y}_j > 0$ .

The distribution of  $Y$  is given by

$$\begin{aligned} \Pr(Y) &= \begin{cases} \Pr(y_j = 0 | \mathbf{z}_j, \mathbf{x}_j) \\ \Pr(y_j = h | \mathbf{z}_j, \mathbf{x}_j) \end{cases} \quad h = 1, 2, \dots, H \\ &= \begin{cases} \Pr(s_j = 0 | \mathbf{z}_j) + \Pr(s_j = 1 | \mathbf{z}_j) \Pr(\tilde{y}_j = 0 | s_j = 1, \mathbf{x}_j) \\ \Pr(s_j = 1 | \mathbf{z}_j) \Pr(\tilde{y}_j = h | s_j = 1, \mathbf{x}_j) \end{cases} \quad h = 1, 2, \dots, H \end{aligned} \quad (3)$$

The probability of zero outcome has been inflated because it is the sum of the probability of zero activity from the ordered probit model and the probability of nonparticipation from the probit model.

Substituting (1) and (2) in (3), we get

$$\Pr(Y) = \begin{cases} \Pr(y_j = 0 | \mathbf{z}_j, \mathbf{x}_j) \\ \Pr(y_j = h | \mathbf{z}_j, \mathbf{x}_j) & h = 1, 2, \dots, H - 1 \\ \Pr(y_j = H | \mathbf{z}_j, \mathbf{x}_j) \end{cases}$$

$$= \begin{cases} \{1 - \Phi(\mathbf{z}_j \boldsymbol{\gamma})\} + \Phi(\mathbf{z}_j \boldsymbol{\gamma}) \Phi(\kappa_0 - \mathbf{x}_j \boldsymbol{\beta}) \\ \Phi(\mathbf{z}_j \boldsymbol{\gamma}) \{ \Phi(\kappa_h - \mathbf{x}_j \boldsymbol{\beta}) - \Phi(\kappa_{h-1} - \mathbf{x}_j \boldsymbol{\beta}) \} & h = 1, 2, \dots, H - 1 \\ \Phi(\mathbf{z}_j \boldsymbol{\gamma}) \{1 - \Phi(\kappa_{H-1} - \mathbf{x}_j \boldsymbol{\beta})\} \end{cases}$$

If the respective `offset()` option is specified,  $\mathbf{x}_j \boldsymbol{\beta}$  and  $\mathbf{z}_j \boldsymbol{\gamma}$  are replaced with  $\mathbf{x}_j \boldsymbol{\beta} + \text{offset}_j^\beta$  and  $\mathbf{z}_j \boldsymbol{\gamma} + \text{offset}_j^\gamma$ , respectively.

The log-likelihood function is

$$\ln L = \sum_{j=1}^N w_j \sum_{h=0}^H I(y_j = h) \ln \{ \Pr(y_j = h | \mathbf{z}_j, \mathbf{x}_j) \}$$

where  $w_j$  is an optional weight for the  $j$ th observation and

$$I(y_j = h) = \begin{cases} 1 & \text{if } y_j = h \\ 0 & \text{otherwise} \end{cases}$$

The choice between the ZIOP model and the ordered probit model cannot be made using a likelihood-ratio test because the two hypotheses are not nested in the usual sense of parameter restrictions. The restriction  $\boldsymbol{\gamma} = 0$  does not eliminate the inflation effect; it makes the group membership probabilities both equal to 0.5 [see (1)]. What is needed to remove the inflation effect is  $\mathbf{z}_j \boldsymbol{\gamma} \rightarrow \infty$ , which cannot be imposed. Because ZIOP and ordered probit models are not nested, you can compare the fits of the two models using information criteria.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version with `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] [\\_robust](#), particularly *Maximum likelihood estimators* and *Methods and formulas*.

`zioprobit` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] [Variance estimation](#).

## References

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- Xia, Y., Y. Zhou, and T. Cai. 2019. `gidm`: A command for generalized inflated discrete models. *Stata Journal* 19: 698–718.

## Also see

- [R] **zioprobit postestimation** — Postestimation tools for zioprobit
- [R] **oprobit** — Ordered probit regression
- [R] **probit** — Probit regression
- [R] **ziologit** — Zero-inflated ordered logit regression
- [BAYES] **bayes: zioprobit** — Bayesian zero-inflated ordered probit regression
- [SVY] **svy estimation** — Estimation commands for survey data
- [U] **20 Estimation and postestimation commands**

|                         |                      |          |
|-------------------------|----------------------|----------|
| Postestimation commands | predict              | margins  |
| Remarks and examples    | Methods and formulas | Also see |

## Postestimation commands

The following postestimation commands are available after `zioprobit`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from margins (profile plots, interaction plots, etc.)                                                                 |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | probabilities, linear predictions and their SEs, etc.                                                                                   |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as probabilities, linear predictions, and standard errors.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] { stub* | newvar | newvarlist } [if] [in] [, statistic
outcome(outcome) nooffset]
```

```
predict [type] stub* [if] [in] , scores
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

Main

|                       |                                                                                |
|-----------------------|--------------------------------------------------------------------------------|
| <code>pmargin</code>  | marginal probabilities of levels, $\Pr(y_j = h)$ ; the default                 |
| <code>pjoint1</code>  | joint probabilities of levels and participation, $\Pr(y_j = h, s_j = 1)$       |
| <code>pcond1</code>   | probabilities of levels conditional on participation, $\Pr(y_j = h   s_j = 1)$ |
| <code>ppar</code>     | probability of participation, $\Pr(s_j = 1)$                                   |
| <code>pnpar</code>    | probability of nonparticipation, $\Pr(s_j = 0)$                                |
| <code>xb</code>       | linear prediction                                                              |
| <code>xbinfl</code>   | linear prediction for inflation equation                                       |
| <code>stdp</code>     | standard error of the linear prediction                                        |
| <code>stdpinfl</code> | standard error of the linear prediction for inflation equation                 |

If you do not specify `outcome()`, `pmargin`, `pjoint1`, and `pcond1` (with one new variable specified) assume `outcome(#1)`.

You specify one or  $k$  new variables with `pmargin`, `pjoint1`, and `pcond1`, where  $k$  is the number of outcomes.

You specify one new variable with `ppar`, `pnpar`, `xb`, `xbinfl`, `stdp`, and `stdpinfl`.

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

## Options for predict

Main

`pmargin`, the default, calculates the predicted marginal probabilities of outcome levels,  $\Pr(y_j = h)$ .

`pjoint1` calculates the predicted joint probabilities of outcome levels and participation,  $\Pr(y_j = h, s_j = 1)$ .

`pcond1` calculates the predicted probabilities of outcome levels conditional on participation,  $\Pr(y_j = h | s_j = 1)$ .

With `pmargin`, `pjoint1`, and `pcond1`, you can compute predicted probabilities for one or for all outcome levels. When you specify one new variable, `predict` computes probabilities for the first outcome level. You can specify the `outcome(#i)` option to obtain probabilities for the  $i$ th level. When you specify multiple new variables or a stub, `predict` computes probabilities for all outcome levels. The behavior of `predict` with one new variable is equivalent to specifying `outcome(#1)`.

`ppar` and `pnpair` calculate the predicted marginal probability of participation [ $\Pr(s_j = 1)$ ] and of nonparticipation [ $\Pr(s_j = 0)$ ], respectively.

In health-related fields, probabilities of participation and nonparticipation are known as probabilities of susceptibility and nonsusceptibility. Similarly to `predict` after `ziologit`, you can use options `ps` and `pns` to compute these probabilities. Options `ps` and `pns` produce identical results to the respective options `ppar` and `pnpair` but label new variables as `Pr(susceptible)` and `Pr(nonsusceptible)` instead of `Pr(participation)` and `Pr(nonparticipation)`.

`xb` calculates the linear prediction for the ordered probit equation, which is  $\mathbf{x}_j\beta$  if `offset()` was not specified with `zioprobit` and is  $\mathbf{x}_j\beta + \text{offset}_j^\beta$  if `offset()` was specified.

`xbinfl` calculates the linear prediction for the inflation equation, which is  $\mathbf{z}_j\gamma$  if `offset()` was not specified in `inflate()` and is  $\mathbf{z}_j\gamma + \text{offset}_j^\gamma$  if `offset()` was specified in `inflate()`.

`stdp` calculates the standard error of the linear prediction for the ordered probit equation.

`stdpinfl` calculates the standard error of the linear prediction for the inflation equation.

`outcome(outcome)` specifies the outcome for which predicted probabilities are to be calculated. `outcome()` should contain either one value of the dependent variable or one of `#1`, `#2`, ..., with `#1` meaning the first category of the dependent variable, `#2` meaning the second category, etc. `outcome()` is allowed only with `pmargin`, `pjoint1`, and `pcond1`.

`nooffset` is relevant only if you specified `offset(varname)` with `zioprobit` or within the `inflate()` option. It modifies the calculations made by `predict` so that they ignore the offset variable; that is, the linear prediction for the main regression equation is treated as  $\mathbf{x}_j\beta$  rather than as  $\mathbf{x}_j\beta + \text{offset}_j^\beta$  and the linear prediction for the inflation equation is treated as  $\mathbf{z}_j\gamma$  rather than as  $\mathbf{z}_j\gamma + \text{offset}_j^\gamma$ .

`scores` calculates equation-level score variables.

The first new variable will contain  $\partial \ln L / \partial (\mathbf{x}_j\beta)$ . In the absence of independent variables in the main equation, this variable is not stored.

The second new variable will contain  $\partial \ln L / \partial (\mathbf{z}_j\gamma)$ .

When the dependent variable takes  $k$  different values, the third new variable through new variable  $k + 1$  will contain  $\partial \ln L / \partial (\kappa_h)$  for  $h = 0, 1, \dots, k - 2$ .

# margins

## Description for margins

`margins` estimates margins of response for probabilities and linear predictions.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i> | Description                                                                    |
|------------------|--------------------------------------------------------------------------------|
| default          | marginal probabilities for each outcome                                        |
| <u>p</u> margin  | marginal probabilities of levels, $\Pr(y_j = h)$ ; the default                 |
| pjoint1          | joint probabilities of levels and participation, $\Pr(y_j = h, s_j = 1)$       |
| pcond1           | probabilities of levels conditional on participation, $\Pr(y_j = h   s_j = 1)$ |
| ppar             | probability of participation, $\Pr(s_j = 1)$                                   |
| pnpar            | probability of nonparticipation, $\Pr(s_j = 0)$                                |
| xb               | linear prediction                                                              |
| xbinfl           | linear prediction for inflation equation                                       |
| stdp             | not allowed with margins                                                       |
| stdpinfl         | not allowed with margins                                                       |

`pmargin`, `pjoint1`, and `pcond1` default to the first outcome.

Statistics not allowed with `margins` are functions of stochastic quantities other than  $e(b)$ .

For the full syntax, see [\[R\] margins](#).

## Remarks and examples

Various sets of predictions and marginal effects may be of interest for the ZIOP model. For instance, we may want to investigate the marginal effects of a covariate on the probability of participation, or on the probabilities for levels of consumption conditional on participation, or on the overall probabilities for different consumption levels. We explore these options in greater detail in the following examples.

### ▷ Example 1: Average marginal effects on probability of nonparticipation

In [example 1](#) of [\[R\] zioprobit](#), we fit a model for level of cigarette consumption.

```
. use https://www.stata-press.com/data/r19/tobacco
(Fictional tobacco consumption data)

. zioprobit tobacco education income i.female age,
> inflate(education income i.parent age i.female i.religion)
(output omitted)
```

We can use `margins` to estimate the expected marginal effect of gender for individuals with a college degree (17 years of education) and a smoking parent on the probability of nonparticipation (being a genuine nonsmoker). To do this, we specify `predict(pnpar)` with `margins` as follows:

```
. margins, predict(pnpar) dydx(female) at(education = 17 parent = 1)
Average marginal effects Number of obs = 15,000
Model VCE: OIM
Expression: Pr(nonparticipation), predict(pnpar)
dy/dx wrt: 1.female
At: education = 17
 parent = 1
```

|        | dy/dx    | Delta-method<br>std. err. | z    | P> z  | [95% conf. interval] |         |
|--------|----------|---------------------------|------|-------|----------------------|---------|
| female | .0855995 | .0100239                  | 8.54 | 0.000 | .0659531             | .105246 |
| Female |          |                           |      |       |                      |         |

Note: dy/dx for factor levels is the discrete change from the base level.

Women with a college degree and a smoking parent are expected to have about an 8.5% higher chance of being genuine nonsmokers than do men.



### ▷ Example 2: Predicted probabilities of zero-valued outcomes

In [example 1](#) of [\[R\] zioprobit](#), we found that the coefficient on `income` was positive in the level equation but negative in the participation equation. In the case of our tobacco consumption example, economic theory offers a reasonable interpretation for this. Higher income may act as an indicator for health awareness, which accounts for its association with an increased probability of being a genuine nonsmoker. However, if cigarettes are a normal good—that is, something for which demand increases when income increases—then smokers with higher income should have a lower probability of having zero consumption at the time of the survey.

We first consider the effect of `income` at six prespecified values ranging from \$10,000 to \$60,000 on the probability of being a genuine nonsmoker (nonparticipation). Because we know the values 1 to 6 correspond to income in tens of thousands, we conserve space and suppress the default legend by using the `noatlegend` option.

```
. margins, predict(pnpar) at(income = (1/6)) noatlegend
Predictive margins Number of obs = 15,000
Model VCE: OIM
Expression: Pr(nonparticipation), predict(pnpar)
```

|     | Delta-method |           |       |       |                      |
|-----|--------------|-----------|-------|-------|----------------------|
|     | Margin       | std. err. | z     | P> z  | [95% conf. interval] |
| _at |              |           |       |       |                      |
| 1   | .1727928     | .0056963  | 30.33 | 0.000 | .1616283 .1839573    |
| 2   | .1944564     | .0055236  | 35.20 | 0.000 | .1836304 .2052824    |
| 3   | .2176787     | .0052981  | 41.09 | 0.000 | .2072945 .2280629    |
| 4   | .2424141     | .0050413  | 48.09 | 0.000 | .2325333 .2522948    |
| 5   | .2685948     | .0047857  | 56.12 | 0.000 | .259215 .2779745     |
| 6   | .2961309     | .0045767  | 64.70 | 0.000 | .2871607 .3051011    |

The probability of being a genuine nonsmoker increases with income. For instance, for individuals who earn \$10,000 a year, the expected increase in the probability of being a genuine nonsmoker is about  $(0.1945 - 0.1728) \times 100\% = 2.17\%$  if they earn an additional \$10,000.

We next investigate the effect of income on the joint probability of being a smoker (participation equals 1) with zero consumption. We do this by specifying `predict(pjoint1 outcome(0))` with `margins`:

```
. margins, predict(pjoint1 outcome(0)) at(income = (1/6)) noatlegend
Predictive margins Number of obs = 15,000
Model VCE: OIM
Expression: Pr(tobacco=0, participation=1), predict(pjoint1 outcome(0))
```

|     | Delta-method |           |        |       |                      |
|-----|--------------|-----------|--------|-------|----------------------|
|     | Margin       | std. err. | z      | P> z  | [95% conf. interval] |
| _at |              |           |        |       |                      |
| 1   | .5595131     | .0040167  | 139.30 | 0.000 | .5516405 .5673856    |
| 2   | .5080066     | .0037793  | 134.42 | 0.000 | .5005993 .5154139    |
| 3   | .4558656     | .0035553  | 128.22 | 0.000 | .4488975 .4628338    |
| 4   | .4029089     | .0033683  | 119.62 | 0.000 | .3963072 .4095106    |
| 5   | .3485692     | .0032481  | 107.32 | 0.000 | .3422031 .3549352    |
| 6   | .2929155     | .00317    | 92.40  | 0.000 | .2867023 .2991287    |

For individuals who are smokers, the probability of zero consumption decreases as income increases, suggesting that tobacco is like a normal good for smokers. For example, for individuals who earn \$10,000 a year, earning an additional \$10,000 will decrease their probability of being a smoker with zero consumption by about  $(0.5595 - 0.5080) \times 100\% = 5.15\%$ .

If we wanted to compute the effect of income on the overall probability of zero consumption instead of the probability of zero consumption among smokers, we would omit `pjoint1` from within the `predict` option.

```
. margins, predict(outcome(0)) at(income = (1/6)) noatlegend
(output omitted)
```

This version of the `margins` command gives the sum of the probability of nonparticipation and the joint probability of participation with zero consumption.

## Methods and formulas

The participation equation is

$$s_j = I(\mathbf{z}_j\boldsymbol{\gamma} + u_{1j} > 0) \quad j = 1, 2, \dots, n$$

where  $s_j$  is 1 if the  $j$ th subject belongs to the participation group (for example, smokers) and is 0 if the subject belongs to the nonparticipation group (for example, genuine nonsmokers),  $\mathbf{z}_j$  are the covariates used to model the group membership,  $\boldsymbol{\gamma}$  is a vector of coefficients, and  $u_{1j}$  is a random-error term following a standard normal distribution.

The ordinal outcome equation is

$$\tilde{y}_j = \sum_{h=0}^H hI(\kappa_{h-1} < \mathbf{x}_j\boldsymbol{\beta} + u_{2j} \leq \kappa_h)$$

where  $\tilde{y}_j$  is the ordinal outcome conditional on participation,  $\mathbf{x}_j$  are the outcome covariates,  $\boldsymbol{\beta}$  are the coefficients, and  $u_{2j}$  is a random-error term following a standard normal distribution. The observed outcome values are 0, 1, ...,  $H$ .  $\kappa_0, \kappa_1, \dots, \kappa_{H-1}$  are real numbers such that  $\kappa_i < \kappa_m$  for  $i < m$ .  $\kappa_{-1}$  is taken as  $-\infty$  and  $\kappa_H$  is taken as  $+\infty$ . We assume that the error terms  $u_{1j}$  and  $u_{2j}$  are independent. We observe  $y_j = s_j\tilde{y}_j$ .

The probability of participation is

$$\Pr(s_j = 1|\mathbf{z}_j) = \Phi(\mathbf{z}_j\boldsymbol{\gamma}) \quad (1)$$

where  $\Phi(\cdot)$  is the standard normal distribution function.

The probability of nonparticipation is

$$\Pr(s_j = 0|\mathbf{z}_j) = 1 - \Phi(\mathbf{z}_j\boldsymbol{\gamma}) \quad (2)$$

The probability of outcome  $y_j = h$  given that the  $j$ th subject belongs to the participation group is

$$\begin{aligned} \Pr(y_j = h|s_j = 1, \mathbf{x}_j) &= \Pr(\tilde{y}_j = h|\mathbf{x}_j) \\ &= \Phi(\kappa_h - \mathbf{x}_j\boldsymbol{\beta}) - \Phi(\kappa_{h-1} - \mathbf{x}_j\boldsymbol{\beta}) \end{aligned} \quad (3)$$

for  $h = 0, 1, \dots, H$ , where  $\Phi(\kappa_{-1} - \mathbf{x}_j\boldsymbol{\beta}) = 0$  and  $\Phi(\kappa_H - \mathbf{x}_j\boldsymbol{\beta}) = 1$ .

The joint probability of outcome  $y_j = h$  and participation can be expressed as

$$\Pr(y_j = h, s_j = 1|\mathbf{z}_j, \mathbf{x}_j) = \Pr(s_j = 1|\mathbf{z}_j) \Pr(y_j = h|s_j = 1, \mathbf{x}_j)$$

for  $h = 0, 1, \dots, H$ , and computed using (1) and (3).

The marginal probabilities of the outcome  $y_j$  are

$$\begin{aligned} \Pr(y_j = 0|\mathbf{z}_j, \mathbf{x}_j) &= \Pr(s_j = 0|\mathbf{z}_j) + \Pr(s_j = 1|\mathbf{z}_j) \Pr(\tilde{y}_j = 0|\mathbf{x}_j) \\ \Pr(y_j = h|\mathbf{z}_j, \mathbf{x}_j) &= \Pr(s_j = 1|\mathbf{z}_j) \Pr(\tilde{y}_j = h|\mathbf{x}_j) \quad h = 1, 2, \dots, H-1 \\ \Pr(y_j = H|\mathbf{z}_j, \mathbf{x}_j) &= \Pr(s_j = 1|\mathbf{z}_j) \Pr(\tilde{y}_j = H|\mathbf{x}_j) \end{aligned}$$

and can be computed using (1), (2), and (3).

If the `offset()` option is specified with `zioprobit`,  $\mathbf{x}_j\boldsymbol{\beta}$  is replaced with  $\mathbf{x}_j\boldsymbol{\beta} + \text{offset}_j^\beta$ . If the `offset()` option is specified within the `inflate()` option,  $\mathbf{z}_j\boldsymbol{\gamma}$  is replaced with  $\mathbf{z}_j\boldsymbol{\gamma} + \text{offset}_j^\gamma$ .

## Also see

[R] [zioprobit](#) — Zero-inflated ordered probit regression

[U] [20 Estimation and postestimation commands](#)

[Description](#)  
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## Description

`zip` fits a zero-inflated Poisson (ZIP) model to count data with excess zero counts. The ZIP model assumes that the excess zero counts come from a logit or probit model and the remaining counts come from a Poisson model.

## Quick start

Zero-inflated Poisson model of  $y$  on  $x_1$  and  $x_2$  with inflation modeled using  $x_3$

```
zip y x1 x2, inflate(x3)
```

Use a probit model instead of a logit model to predict excess zeros

```
zip y x1 x2, inflate(x3) probit
```

## Menu

Statistics > Count outcomes > Zero-inflated Poisson regression

## Syntax

```
zip depvar [indepvars] [if] [in] [weight],
 inflate(varlist [, offset(varname)] | _cons) [options]
```

| <i>options</i>                                  | Description                                                                                                                                      |
|-------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model</b>                                    |                                                                                                                                                  |
| * <u>inflate</u> ( )                            | equation that determines whether the count is zero                                                                                               |
| <u>noconstant</u>                               | suppress constant term                                                                                                                           |
| <u>exposure</u> ( <i>varname</i> <sub>e</sub> ) | include ln( <i>varname</i> <sub>e</sub> ) in model with coefficient constrained to 1                                                             |
| <u>offset</u> ( <i>varname</i> <sub>o</sub> )   | include <i>varname</i> <sub>o</sub> in model with coefficient constrained to 1                                                                   |
| <u>constraints</u> ( <i>constraints</i> )       | apply specified linear constraints                                                                                                               |
| <u>probit</u>                                   | use probit model to characterize excess zeros; default is logit                                                                                  |
| <b>SE/Robust</b>                                |                                                                                                                                                  |
| <u>vce</u> ( <i>vcetype</i> )                   | <i>vcetype</i> may be <u>oim</u> , <u>robust</u> , <u>cluster</u> <i>clustvar</i> , <u>opg</u> , <u>bootstrap</u> , or <u>jackknife</u>          |
| <b>Reporting</b>                                |                                                                                                                                                  |
| <u>level</u> (#)                                | set confidence level; default is level(95)                                                                                                       |
| <u>irr</u>                                      | report incidence-rate ratios                                                                                                                     |
| <u>nocnsreport</u>                              | do not display constraints                                                                                                                       |
| <u>display_options</u>                          | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| <b>Maximization</b>                             |                                                                                                                                                  |
| <u>maximize_options</u>                         | control the maximization process; seldom used                                                                                                    |
| <u>collinear</u>                                | keep collinear variables                                                                                                                         |
| <u>coeflegend</u>                               | display legend instead of statistics                                                                                                             |

\*inflate(*varlist* [, offset(*varname*)] | \_cons) is required.

*indepvars* and *varlist* may contain factor variables; see [U] 11.4.3 **Factor variables**.

bayes, bayesboot, bootstrap, by, collect, fp, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 **Prefix commands**. For more details, see [BAYES] **bayes: zip**.

Weights are not allowed with the bootstrap prefix; see [R] **bootstrap**.

vce() and weights are not allowed with the svy prefix; see [SVY] **svy**.

fweights, iweights, and pweights are allowed; see [U] 11.1.6 **weight**.

collinear and coeflegend do not appear in the dialog box.

See [U] 20 **Estimation and postestimation commands** for more capabilities of estimation commands.

## Options

### Model

`inflate(varlist [, offset(varname)] | _cons)` specifies the equation that determines whether the observed count is zero. Conceptually, omitting `inflate()` would be equivalent to fitting the model with `poisson`; see [R] [poisson](#).

`inflate(varlist [, offset(varname)])` specifies the variables in the equation. You may optionally include an offset for this *varlist*.

`inflate(_cons)` specifies that the equation determining whether the count is zero contains only an intercept. To run a zero-inflated model of *depvar* with only an intercept in both equations, type `zip depvar, inflate(_cons)`.

`noconstant`, `exposure(varnamee)`, `offset(varnameo)`, `constraints(constraints)`; see [R] [Estimation options](#).

`probit` requests that a probit, instead of logit, model be used to characterize the excess zeros in the data.

### SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`, `opg`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

### Reporting

`level(#)`; see [R] [Estimation options](#).

`irr` reports estimated coefficients transformed to incidence-rate ratios, that is,  $e^b$  rather than  $b$ . Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated or stored. `irr` may be specified at estimation or when replaying previously estimated results.

`nocnsreport`; see [R] [Estimation options](#).

*display\_options*: `nocl`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

### Maximization

*maximize\_options*: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [Maximize](#). These options are seldom used.

Setting the optimization type to `technique(bhhh)` resets the default *vcetype* to `vce(opg)`.

The following options are available with `zip` but are not shown in the dialog box:

`collinear`, `coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

Zero-inflated Poisson (ZIP) models address the case when the data contain a higher fraction of zeros than is likely to be generated from a Poisson model. Having a large proportion of zero observations, in itself, does not necessarily mean that we have the excess zeros problem. For instance, a Poisson model with a mean value of 0.2 predicts that  $P(Y = 0) = \exp(-0.2) \approx 0.82$ . However, the range of possible outcomes is restricted because of the small variance, 0.2 (mean and variance are equal for a Poisson model), as shown by  $P(Y > 3) \approx 0.00006$ . Unlike Poisson models, ZIP models allow us to have a large fraction of zeros without restricting the range of outcomes.

ZIP models assume that an observation is 0 with a probability  $p$  or is a realization of a Poisson random variable, which can also be 0, with a probability  $1 - p$ . For instance, you might count how many fish each visitor to a park catches. Many visitors may catch zero, because they do not fish (as opposed to being unsuccessful). Using a logit or probit model, you may model the probability  $p$  of whether a person does not fish depending on several covariates related to fishing. Using a Poisson distribution, you may model how many fish a person catches depending on several covariates having to do with the success of catching fish (type of lure or bait, time of day, temperature, season, etc.). This is the type of data for which the `zip` command is useful.

See [Long \(1997, 242–247\)](#) and [Cameron and Trivedi \(2005, 680–681\)](#) for a discussion of the ZIP model and other zero-modified count models.

### ▷ Example 1: Fitting a ZIP model

We have fictional data on the number of fish caught (`count`) by visitors to a national park on a particular day. Some of the visitors do not fish, but we do not have the data on whether a person fished; we merely have data on how many fish were caught together with several covariates.

Variable `count` exhibits an excess of zero observations (142 of 250 observations), beyond what would be expected from a Poisson model. We suspect that the number of zeros may be inflated because many visitors are not fishing. That is, a zero observation may be the result of a visitor who was unfortunate and caught no fish but may also be because the visitor did not fish. A standard Poisson model (see [\[R\] poisson](#)) treats these two types of zero observations as a homogeneous group, which typically leads to biased statistical results. We would like to distinguish between the two types of zeros and possibly draw inference for them separately (see [example 1](#) in [\[R\] zip postestimation](#)).

The `zip` command allows us to model the two types of zeros. First, using the required the `inflate()` option, we model whether a visitor fishes as a function of the number of children accompanying him or her (`child`) and whether he or she is camping (`camper`). Next, we assume the response variable, `count`, depends on whether the visitor used a live bait (`livebait`) and the number of persons (`persons`), which includes the visitor and any adults or children accompanying him. Note that `persons` is always greater than `child`.

```

. use https://www.stata-press.com/data/r19/fish
(Fictional fishing data)
. zip count persons livebait, inflate(child camper)
Fitting constant-only model:
Iteration 0: Log likelihood = -1347.807
Iteration 1: Log likelihood = -1305.3245
Iteration 2: Log likelihood = -1104.3005
Iteration 3: Log likelihood = -1103.9426
Iteration 4: Log likelihood = -1103.9425
Fitting full model:
Iteration 0: Log likelihood = -1103.9425
Iteration 1: Log likelihood = -896.2346
Iteration 2: Log likelihood = -851.61723
Iteration 3: Log likelihood = -850.70435
Iteration 4: Log likelihood = -850.70142
Iteration 5: Log likelihood = -850.70142
Zero-inflated Poisson regression
Inflation model: logit
Number of obs = 250
Nonzero obs = 108
Zero obs = 142
LR chi2(2) = 506.48
Prob > chi2 = 0.0000
Log likelihood = -850.7014

```

| count    | Coefficient | Std. err. | z     | P> z  | [95% conf. interval] |           |
|----------|-------------|-----------|-------|-------|----------------------|-----------|
| count    |             |           |       |       |                      |           |
| persons  | .8068853    | .0453288  | 17.80 | 0.000 | .7180424             | .8957281  |
| livebait | 1.757289    | .2446082  | 7.18  | 0.000 | 1.277866             | 2.236713  |
| _cons    | -2.178472   | .2860289  | -7.62 | 0.000 | -2.739078            | -1.617865 |
| inflate  |             |           |       |       |                      |           |
| child    | 1.602571    | .2797719  | 5.73  | 0.000 | 1.054228             | 2.150913  |
| camper   | -1.015698   | .365259   | -2.78 | 0.005 | -1.731593            | -.2998038 |
| _cons    | -.4922872   | .3114562  | -1.58 | 0.114 | -1.10273             | .1181558  |

Coefficients in the upper half of the table correspond to the Poisson model for individuals who fished. For instance, among visitors who fished, using a live bait increases the expected number of caught fish by a factor of  $\exp(1.7572) \approx 5.8$ , holding other covariates constant.

◀

## ▶ Example 2: Comparing model fit

When you have count data, you may want to test whether a conventional count data model or a zero-inflated count data model is preferable. The classical likelihood-ratio test cannot be used here because the models are not nested. But we can use information criteria such as the AIC and BIC to check whether the standard or zero-inflated model is more appropriate.

Continuing with our fishing example, let's check whether the standard Poisson or ZIP model is more appropriate for our data. First, we store the estimation results from the previous ZIP model by typing

```
. estimates store zip
```

Next, we fit the Poisson model corresponding to the main equation of the ZIP model and store its results as `pois`:

```
. poisson count persons livebait
(output omitted)
. estimates store pois
```

We use `estimates stats` to display the AIC and BIC values for the two models.

```
. estimates stats pois zip
Akaike's information criterion and Bayesian information criterion
```

| Model | N   | ll(null)  | ll(model) | df | AIC      | BIC      |
|-------|-----|-----------|-----------|----|----------|----------|
| pois  | 250 | -1647.716 | -1312.178 | 3  | 2630.356 | 2640.92  |
| zip   | 250 | -1103.942 | -850.7014 | 6  | 1713.403 | 1734.532 |

Note: BIC uses  $N$  = number of observations. See [\[R\] IC note](#).

The ZIP model has smaller AIC and BIC values; we thus conclude that it fits our data better than the standard Poisson model.

◀

## Stored results

`zip` stores the following in `e()`:

### Scalars

|                            |                                           |
|----------------------------|-------------------------------------------|
| <code>e(N)</code>          | number of observations                    |
| <code>e(N_zero)</code>     | number of zero observations               |
| <code>e(k)</code>          | number of parameters                      |
| <code>e(k_eq)</code>       | number of equations in <code>e(b)</code>  |
| <code>e(k_eq_model)</code> | number of equations in overall model test |
| <code>e(k_dv)</code>       | number of dependent variables             |
| <code>e(df_m)</code>       | model degrees of freedom                  |
| <code>e(ll)</code>         | log likelihood                            |
| <code>e(ll_0)</code>       | log likelihood, constant-only model       |
| <code>e(N_clust)</code>    | number of clusters                        |
| <code>e(chi2)</code>       | $\chi^2$                                  |
| <code>e(p)</code>          | $p$ -value for model test                 |
| <code>e(rank)</code>       | rank of $e(V)$                            |
| <code>e(ic)</code>         | number of iterations                      |
| <code>e(rc)</code>         | return code                               |
| <code>e(converged)</code>  | 1 if converged, 0 otherwise               |

### Macros

|                          |                                                       |
|--------------------------|-------------------------------------------------------|
| <code>e(cmd)</code>      | <code>zip</code>                                      |
| <code>e(cmdline)</code>  | command as typed                                      |
| <code>e(depvar)</code>   | name of dependent variable                            |
| <code>e(inflate)</code>  | logit or probit                                       |
| <code>e(wtype)</code>    | weight type                                           |
| <code>e(wexp)</code>     | weight expression                                     |
| <code>e(title)</code>    | title in estimation output                            |
| <code>e(clustvar)</code> | name of cluster variable                              |
| <code>e(offset1)</code>  | offset                                                |
| <code>e(offset2)</code>  | offset for <code>inflate()</code>                     |
| <code>e(chi2type)</code> | Wald or LR; type of model $\chi^2$ test               |
| <code>e(vce)</code>      | <code>vce</code> type specified in <code>vce()</code> |
| <code>e(vcetype)</code>  | title used to label Std. err.                         |

|               |                                                                          |
|---------------|--------------------------------------------------------------------------|
| e(opt)        | type of optimization                                                     |
| e(which)      | max or min; whether optimizer is to perform maximization or minimization |
| e(ml_method)  | type of ml method                                                        |
| e(user)       | name of likelihood-evaluator program                                     |
| e(technique)  | maximization technique                                                   |
| e(properties) | b V                                                                      |
| e(predict)    | program used to implement predict                                        |
| e(asbalanced) | factor variables fvset as asbalanced                                     |
| e(asobserved) | factor variables fvset as asobserved                                     |

## Matrices

|                 |                                              |
|-----------------|----------------------------------------------|
| e(b)            | coefficient vector                           |
| e(Cns)          | constraints matrix                           |
| e(ilog)         | iteration log (up to 20 iterations)          |
| e(gradient)     | gradient vector                              |
| e(V)            | variance–covariance matrix of the estimators |
| e(V_modelbased) | model-based variance                         |

## Functions

|           |                         |
|-----------|-------------------------|
| e(sample) | marks estimation sample |
|-----------|-------------------------|

In addition to the above, the following is stored in `r()`:

## Matrices

|          |                                                                                                                       |
|----------|-----------------------------------------------------------------------------------------------------------------------|
| r(table) | matrix containing the coefficients with their standard errors, test statistics, $p$ -values, and confidence intervals |
|----------|-----------------------------------------------------------------------------------------------------------------------|

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any  $r$ -class command is run after the estimation command.

## Methods and formulas

Consider the formulation of a zero-inflated model as presented in [Lambert \(1992\)](#). Define

$$\xi_j^\beta = \mathbf{x}_j \boldsymbol{\beta} + \text{offset}_j^\beta$$

$$\xi_j^\gamma = \mathbf{z}_j \boldsymbol{\gamma} + \text{offset}_j^\gamma$$

$$\lambda_j = \exp(\xi_j^\beta)$$

$$F_j = F(\xi_j^\gamma)$$

where  $F(\cdot)$  is the inverse of the logit function or, if the `probit` option was specified, the inverse of the probit function (or the standard normal cumulative distribution function). All subjects are assumed to be independent with the  $j$ th response determined as follows:

$$Y_j = 0 \quad \text{with probability } F_j$$

$$Y_j \sim \text{Poisson}(\lambda_j) \quad \text{with probability } 1 - F_j$$

In other words,

$$\Pr(Y_j = 0 | \mathbf{x}_j, \mathbf{z}_j) = F_j + (1 - F_j) \exp(-\lambda_j)$$

$$\Pr(Y_j = n | \mathbf{x}_j, \mathbf{z}_j) = (1 - F_j) \exp(-\lambda_j) \frac{\lambda_j^n}{n!} \quad \text{for } n = 1, 2, \dots$$

The `zip` command maximizes the log-likelihood  $\ln L$ , defined by

$$\begin{aligned} \ln L = & \sum_{j \in S} w_j \ln \{ F_j + (1 - F_j) \exp(-\lambda_j) \} \\ & + \sum_{j \notin S} w_j \{ \ln(1 - F_j) - \lambda_j + \xi_j y_j - \ln(y_j!) \} \end{aligned}$$

where  $w_j$  are the weights and  $S$  is the set of observations for which the observed outcome  $y_j = 0$ .

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using `vce(robust)` and `vce(cluster clustvar)`, respectively. See [P] [\\_robust](#), particularly *Maximum likelihood estimators* and *Methods and formulas*.

`zip` also supports estimation with survey data. For details on VCEs with survey data, see [SVY] [Variance estimation](#).

## References

- Cameron, A. C., and P. K. Trivedi. 2005. *Microeconometrics: Methods and Applications*. New York: Cambridge University Press.
- Cummings, T. H., and J. W. Hardin. 2019. [Modeling count data with marginalized zero-inflated distributions](#). *Stata Journal* 19: 499–509.
- Desmarais, B. A., and J. J. Harden. 2013. [Testing for zero inflation in count models: Bias correction for the Vuong test](#). *Stata Journal* 13: 810–835.
- Lambert, D. 1992. Zero-inflated Poisson regression, with an application to defects in manufacturing. *Technometrics* 34: 1–14. <https://doi.org/10.2307/1269547>.
- Long, J. S. 1997. *Regression Models for Categorical and Limited Dependent Variables*. Thousand Oaks, CA: Sage.
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- Mullahy, J. 1986. Specification and testing of some modified count data models. *Journal of Econometrics* 33: 341–365. [https://doi.org/10.1016/0304-4076\(86\)90002-3](https://doi.org/10.1016/0304-4076(86)90002-3).
- Xia, Y., Y. Zhou, and T. Cai. 2019. [gidm: A command for generalized inflated discrete models](#). *Stata Journal* 19: 698–718.

## Also see

- [R] [zip postestimation](#) — Postestimation tools for `zip`
- [R] [zinb](#) — Zero-inflated negative binomial regression
- [R] [nbreg](#) — Negative binomial regression
- [R] [poisson](#) — Poisson regression
- [R] [tnbreg](#) — Truncated negative binomial regression
- [R] [tpoisson](#) — Truncated Poisson regression
- [BAYES] [bayes: zip](#) — Bayesian zero-inflated Poisson regression
- [SVY] [svy estimation](#) — Estimation commands for survey data
- [XT] [xtpoisson](#) — Fixed-effects, random-effects, and population-averaged Poisson models
- [U] [20 Estimation and postestimation commands](#)

|                         |                            |                          |                                      |
|-------------------------|----------------------------|--------------------------|--------------------------------------|
| Postestimation commands | <a href="#">predict</a>    | <a href="#">margins</a>  | <a href="#">Remarks and examples</a> |
| Methods and formulas    | <a href="#">References</a> | <a href="#">Also see</a> |                                      |

## Postestimation commands

The following postestimation commands are available after `zip`:

| Command                      | Description                                                                                                                             |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>contrast</code>        | contrasts and ANOVA-style joint tests of parameters                                                                                     |
| <code>estat ic</code>        | Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively) |
| <code>estat summarize</code> | summary statistics for the estimation sample                                                                                            |
| <code>estat vce</code>       | variance–covariance matrix of the estimators (VCE)                                                                                      |
| <code>estat (svy)</code>     | postestimation statistics for survey data                                                                                               |
| <code>estimates</code>       | cataloging estimation results                                                                                                           |
| <code>etable</code>          | table of estimation results                                                                                                             |
| * <code>forecast</code>      | dynamic forecasts and simulations                                                                                                       |
| * <code>hausman</code>       | Hausman's specification test                                                                                                            |
| <code>lincom</code>          | point estimates, standard errors, testing, and inference for linear combinations of parameters                                          |
| * <code>lrtest</code>        | likelihood-ratio test                                                                                                                   |
| <code>margins</code>         | marginal means, predictive margins, marginal effects, and average marginal effects                                                      |
| <code>marginsplot</code>     | graph the results from <code>margins</code> (profile plots, interaction plots, etc.)                                                    |
| <code>nlcom</code>           | point estimates, standard errors, testing, and inference for nonlinear combinations of parameters                                       |
| <code>predict</code>         | number of events, incidence rates, probabilities, etc.                                                                                  |
| <code>predictnl</code>       | point estimates, standard errors, testing, and inference for generalized predictions                                                    |
| <code>pwcompare</code>       | pairwise comparisons of parameters                                                                                                      |
| <code>suest</code>           | seemingly unrelated estimation                                                                                                          |
| <code>test</code>            | Wald tests of simple and composite linear hypotheses                                                                                    |
| <code>testnl</code>          | Wald tests of nonlinear hypotheses                                                                                                      |

\*`forecast`, `hausman`, and `lrtest` are not appropriate with `svy` estimation results.

# predict

## Description for predict

`predict` creates a new variable containing predictions such as numbers of events, incidence rates, probabilities, linear predictions, and standard errors.

## Menu for predict

Statistics > Postestimation

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic nooffset]
```

```
predict [type] stub* [if] [in] , scores
```

| <i>statistic</i> | Description |
|------------------|-------------|
|------------------|-------------|

Main

|                                    |                                         |
|------------------------------------|-----------------------------------------|
| <code>n</code>                     | number of events; the default           |
| <code>ir</code>                    | incidence rate                          |
| <code>pr</code>                    | probability of a degenerate zero        |
| <code>pr(<i>n</i>)</code>          | probability $\Pr(y_j = n)$              |
| <code>pr(<i>a</i>,<i>b</i>)</code> | probability $\Pr(a \leq y_j \leq b)$    |
| <code>xb</code>                    | linear prediction                       |
| <code>stdp</code>                  | standard error of the linear prediction |

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

## Options for predict

Main

`n`, the default, calculates the predicted number of events, which is  $(1 - F_j) \exp(\mathbf{x}_j\boldsymbol{\beta})$  if neither `offset()` nor `exposure()` was specified when the model was fit, where  $F_j$  is the predicted probability of a zero outcome;  $(1 - F_j) \exp(\mathbf{x}_j\boldsymbol{\beta} + \text{offset}_j^\beta)$  if `offset()` was specified; or  $(1 - F_j) \{ \exp(\mathbf{x}_j\boldsymbol{\beta}) \times \text{exposure}_j \}$  if `exposure()` was specified.

`ir` calculates the incidence rate, which is the predicted number of events when exposure is 1. This is equivalent to specifying both the `n` and the `nooffset` options.

`pr` calculates the probability of a degenerate zero, predicted from the fitted degenerate distribution  $F_j = F(\mathbf{z}_j\boldsymbol{\gamma})$ . If `offset()` was specified within the `inflate()` option, then  $F_j = F(\mathbf{z}_j\boldsymbol{\gamma} + \text{offset}_j^\gamma)$  is calculated.

`pr(n)` calculates the probability  $\Pr(y_j = n)$ , where  $n$  is a nonnegative integer that may be specified as a number or a variable. Note that `pr` is not equivalent to `pr(0)`.

`pr(a, b)` calculates the probability  $\Pr(a \leq y_j \leq b)$ , where  $a$  and  $b$  are nonnegative integers that may be specified as numbers or variables;

$b$  missing ( $b \geq .$ ) means  $+\infty$ ;

`pr(20, .)` calculates  $\Pr(y_j \geq 20)$ ;

`pr(20, b)` calculates  $\Pr(y_j \geq 20)$  in observations for which  $b \geq .$  and calculates  $\Pr(20 \leq y_j \leq b)$  elsewhere.

`pr(. , b)` produces a syntax error. A missing value in an observation of the variable  $a$  causes a missing value in that observation for `pr(a, b)`.

`xb` calculates the linear prediction, which is  $\mathbf{x}_j\boldsymbol{\beta}$  if neither `offset()` nor `exposure()` was specified;  $\mathbf{x}_j\boldsymbol{\beta} + \text{offset}_j^\beta$  if `offset()` was specified; or  $\mathbf{x}_j\boldsymbol{\beta} + \ln(\text{exposure}_j)$  if `exposure()` was specified; see `nooffset` below.

`stdp` calculates the standard error of the linear prediction.

`nooffset` is relevant only if you specified `offset()` or `exposure()` when you fit the model. It modifies the calculations made by `predict` so that they ignore the offset or exposure variable; the linear prediction is treated as  $\mathbf{x}_j\boldsymbol{\beta}$  rather than as  $\mathbf{x}_j\boldsymbol{\beta} + \text{offset}_j^\beta$  or  $\mathbf{x}_j\boldsymbol{\beta} + \ln(\text{exposure}_j)$ . Specifying `predict . . . , nooffset` is equivalent to specifying `predict . . . , ir`.

`scores` calculates equation-level score variables.

The first new variable will contain  $\partial \ln L / \partial (\mathbf{x}_j\boldsymbol{\beta})$ .

The second new variable will contain  $\partial \ln L / \partial (\mathbf{z}_j\boldsymbol{\gamma})$ .

# margins

## Description for margins

`margins` estimates margins of response for the numbers of events, incidence rates, probabilities, and linear predictions.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
```

```
margins [marginlist] , predict(statistic ...) [predict(statistic ...) ...] [options]
```

| <i>statistic</i>                   | Description                           |
|------------------------------------|---------------------------------------|
| <code>n</code>                     | number of events; the default         |
| <code>ir</code>                    | incidence rate                        |
| <code>pr</code>                    | probability of a degenerate zero      |
| <code>pr(<i>n</i>)</code>          | probability $\Pr(y_j = n)$            |
| <code>pr(<i>a</i>,<i>b</i>)</code> | probability $\Pr(a \leq y_j \leq b)$  |
| <code>xb</code>                    | linear prediction                     |
| <code>stdp</code>                  | not allowed with <code>margins</code> |

Statistics not allowed with `margins` are functions of stochastic quantities other than `e(b)`.

For the full syntax, see [R] [margins](#).

## Remarks and examples

### ▷ Example 1: Obtaining predicted counts

Continuing with [example 1](#) from [\[R\] zip](#), we will use `predict` to compute the predicted number of fish captured by each individual.

```
. use https://www.stata-press.com/data/r19/fish
(Fictional fishing data)
. zip count persons livebait, inflate(child camper)
(output omitted)
. predict numfish
(option n assumed; predicted number of events)
. summarize numfish
```

| Variable | Obs | Mean     | Std. dev. | Min     | Max      |
|----------|-----|----------|-----------|---------|----------|
| numfish  | 250 | 2.770999 | 3.269588  | .079269 | 13.55015 |

The average predicted number of fish caught by all visitors, regardless of whether or not they fished, is 2.77 fish.



### ▷ Example 2: Obtaining predicted probabilities

`predict` with the `pr` option computes the probability that an individual does not fish.

```
. predict pr, pr
```

On the other hand, `predict` with the `pr(n)` option computes the probability of catching *n* fish; particularly, the probability of catching zero fish will be

```
. predict pr0, pr(0)
. list pr pr0 in 1
```

|    | pr       | pr0      |
|----|----------|----------|
| 1. | .3793549 | .8609267 |

Notice that `pr0` is always equal to or greater than `pr`. For example, for the first individual, the probability of not fishing is 0.38; on the other hand, the probability of catching zero fish (0.86) is equal to the sum of the probability of not fishing and the probability of fishing but not catching any fish. `pr0` can be also computed as one minus the probability of catching at least one fish, that is:

```
. predict pr_catch, pr(1,.)
. gen pr0b = 1-pr_catch
```



## Methods and formulas

See *Methods and formulas* in [R] [zip](#) for the model definition and notation.

The probabilities calculated using the `pr(n)` option are the probability  $\Pr(y_j = n)$ . These are calculated using

$$\Pr(y_j = 0 | \mathbf{x}_j, \mathbf{z}_j) = F_j + (1 - F_j) \exp(-\lambda_j)$$

$$\Pr(y_j = n | \mathbf{x}_j, \mathbf{z}_j) = (1 - F_j) \frac{\lambda_j^n \exp(-\lambda_j)}{n!} \quad \text{for } n = 1, 2, \dots$$

where  $F_j$  is the probability of obtaining an observation from the degenerate distribution whose mass is concentrated at zero.  $F_j$  can be obtained by using the `pr` option.

See [Cameron and Trivedi \(2013, sec. 4.6\)](#) for further details.

## References

- Cameron, A. C., and P. K. Trivedi. 2013. *Regression Analysis of Count Data*. 2nd ed. New York: Cambridge University Press.
- Manjón, M., and O. Martínez. 2014. The chi-squared goodness-of-fit test for count-data models. *Stata Journal* 14: 798–816.

## Also see

[R] [zip](#) — Zero-inflated Poisson regression

[U] [20 Estimation and postestimation commands](#)

[Description](#)  
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[Quick start](#)  
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[Menu](#)  
[Stored results](#)

[Syntax](#)  
[Methods and formulas](#)

## Description

`ztest` performs  $z$  tests on the equality of means, assuming known variances. The test can be performed for one sample against a hypothesized population value or for no difference in population means estimated from two samples. Two-sample tests can be conducted for paired and unpaired data. Clustered data are also supported.

`ztesti` is the immediate form of `ztest`; see [\[U\] 19 Immediate commands](#).

For the comparison of means when variances are unknown, use `ttest`; see [\[R\] ttest](#).

## Quick start

One-sample test that the mean of `v1` is 3 at the 90% confidence level

```
ztest v1 == 3, level(90)
```

Same as above, and adjust for clustering with clusters defined by `cvar` and an intraclass correlation of 0.5

```
ztest v1 == 3, level(90) cluster(cvar) rho(0.5)
```

Unpaired  $z$  test that the mean of `v1` is equal between two groups defined by `catvar`

```
ztest v1, by(catvar)
```

Same as above, and adjust for clustering with clusters defined by `cvar` and an intraclass correlation of 0.5 in the two groups

```
ztest v1, by(catvar) cluster(cvar) rho(0.5)
```

Unpaired test of equality of the means of `v2` and `v3`

```
ztest v2 == v3, unpaired
```

Paired test of equality of the means of `v2` and `v3` with standard deviation of the differences between paired observations of 2.4

```
ztest v2 == v3, sddiff(2.4)
```

Same as above, specified using a common standard deviation of 2 and correlation between observations of 0.28

```
ztest v2 == v3, sd(2) corr(0.28)
```

Immediate form unpaired test of  $\mu_1 = \mu_2$  if  $\bar{x}_1 = 3.2$ ,  $sd_1 = 0.1$ ,  $\bar{x}_2 = 3.4$ , and  $sd_2 = 0.15$  with  $n_1 = n_2 = 120$

```
ztesti 120 3.2 0.1 120 3.4 0.15
```

## Menu

### ztest

Statistics > Summaries, tables, and tests > Classical tests of hypotheses > z test (mean-comparison test, known variance)

### ztesti

Statistics > Summaries, tables, and tests > Classical tests of hypotheses > z test calculator

## Syntax

### One-sample z test

```
ztest varname == # [if] [in] [, onesampleopts]
```

### Two-sample z test using groups

```
ztest varname [if] [in], by(groupvar) [twosamplegopts]
```

### Two-sample z test using variables

```
ztest varname1 == varname2 [if] [in], unpaired [twosamplevaropts]
```

### Paired z test

```
ztest varname1 == varname2 [if] [in], sddiff(#) [level(#)]
```

```
ztest varname1 == varname2 [if] [in], corr(#) [pairedopts]
```

### Immediate form of one-sample z test

```
ztesti #_obs #_mean #_sd #_val [, level(#)]
```

### Immediate form of two-sample unpaired z test

```
ztesti #_obs1 #_mean1 #_sd1 #_obs2 #_mean2 #_sd2 [, level(#)]
```

### onesampleopts          Description

---

#### Main

|                           |                                                     |
|---------------------------|-----------------------------------------------------|
| sd(#)                     | one-population standard deviation; default is sd(1) |
| level(#)                  | confidence level; default is level(95)              |
| cluster( <i>varname</i> ) | variable defining the clusters                      |
| rho(#)                    | intraclass correlation                              |

---

---

| <i>twosamplegopts</i> | Description |
|-----------------------|-------------|
|-----------------------|-------------|

---

Main

|                                   |                                                                                                               |
|-----------------------------------|---------------------------------------------------------------------------------------------------------------|
| * <b>by</b> ( <i>groupvar</i> )   | variable defining the groups                                                                                  |
| <b>unpaired</b>                   | unpaired test; implied when <b>by</b> () is specified                                                         |
| <b>sd</b> (#)                     | two-population common standard deviation; default is <b>sd</b> (1)                                            |
| <b>sd1</b> (#)                    | standard deviation of the first population; requires <b>sd2</b> () and may not be combined with <b>sd</b> ()  |
| <b>sd2</b> (#)                    | standard deviation of the second population; requires <b>sd1</b> () and may not be combined with <b>sd</b> () |
| <b>level</b> (#)                  | confidence level; default is <b>level</b> (95)                                                                |
| <b>cluster</b> ( <i>varname</i> ) | variable defining the clusters                                                                                |
| <b>rho</b> (#)                    | common intraclass correlation                                                                                 |
| <b>rho1</b> (#)                   | intraclass correlation for group 1                                                                            |
| <b>rho2</b> (#)                   | intraclass correlation for group 2                                                                            |

---

\***by**(*groupvar*) is required.

---

| <i>twosamplevaropts</i> | Description |
|-------------------------|-------------|
|-------------------------|-------------|

---

Main

|                   |                                                                                                               |
|-------------------|---------------------------------------------------------------------------------------------------------------|
| * <b>unpaired</b> | unpaired test                                                                                                 |
| <b>sd</b> (#)     | two-population common standard deviation; default is <b>sd</b> (1)                                            |
| <b>sd1</b> (#)    | standard deviation of the first population; requires <b>sd2</b> () and may not be combined with <b>sd</b> ()  |
| <b>sd2</b> (#)    | standard deviation of the second population; requires <b>sd1</b> () and may not be combined with <b>sd</b> () |
| <b>level</b> (#)  | confidence level; default is <b>level</b> (95)                                                                |

---

\***unpaired** is required.

---

| <i>pairedopts</i> | Description |
|-------------------|-------------|
|-------------------|-------------|

---

Main

|                   |                                                                                                                                                      |
|-------------------|------------------------------------------------------------------------------------------------------------------------------------------------------|
| * <b>corr</b> (#) | correlation between paired observations                                                                                                              |
| <b>sd</b> (#)     | two-population common standard deviation; default is <b>sd</b> (1); may not be combined with <b>sd1</b> (), <b>sd2</b> (), or <b>sddiff</b> ()       |
| <b>sd1</b> (#)    | standard deviation of the first population; requires <b>corr</b> () and <b>sd2</b> () and may not be combined with <b>sd</b> () or <b>sddiff</b> ()  |
| <b>sd2</b> (#)    | standard deviation of the second population; requires <b>corr</b> () and <b>sd1</b> () and may not be combined with <b>sd</b> () or <b>sddiff</b> () |
| <b>level</b> (#)  | confidence level; default is <b>level</b> (95)                                                                                                       |

---

\***corr**(#) is required.by and collect are allowed with **ztest** and **ztesti**; see [U] 11.1.10 Prefix commands.

## Options

### Main

`by(groupvar)` specifies the *groupvar* that defines the two groups that `ztest` will use to test the hypothesis that their means are equal. Specifying `by(groupvar)` implies an unpaired (two-sample) *z* test. Do not confuse the `by()` option with the `by prefix`; you can specify both.

`unpaired` specifies that the data be treated as unpaired. The `unpaired` option is used when the two sets of values to be compared are in different variables.

`sddiff(#)` specifies the population standard deviation of the differences between paired observations for a paired *z* test. For this kind of test, either `sddiff()` or `corr()` must be specified.

`corr(#)` specifies the correlation between paired observations for a paired *z* test. This option along with `sd1()` and `sd2()` or with `sd()` is used to compute the standard deviation of the differences between paired observations unless that standard deviation is supplied directly in the `sddiff()` option. For a paired *z* test, either `sddiff()` or `corr()` must be specified.

`sd(#)` specifies the population standard deviation for a one-sample *z* test or the common population standard deviation for a two-sample *z* test. The default is `sd(1)`. `sd()` may not be combined with `sd1()`, `sd2()`, or `sddiff()`.

`sd1(#)` specifies the standard deviation of the first population or group. When `sd1()` is specified with `by(groupvar)`, the first group is defined by the first category of the sorted *groupvar*. `sd1()` requires `sd2()` and may not be combined with `sd()` or `sddiff()`.

`sd2(#)` specifies the standard deviation of the second population or group. When `sd2()` is specified with `by(groupvar)`, the second group is defined by the second category of the sorted *groupvar*. `sd2()` requires `sd1()` and may not be combined with `sd()` or `sddiff()`.

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`; see [\[U\] 20.8 Specifying the width of confidence intervals](#).

`cluster(varname)` specifies the variable that identifies clusters. The `cluster()` option is required to adjust the computation for clustering.

`rho(#)` specifies the intraclass correlation for a one-sample test or the common intraclass correlation for a two-sample test. The `rho()` option is required to adjust the computation for clustering for a one-sample test.

`rho1(#)` specifies the intraclass correlation of the first group for a two-sample test using groups. The `rho()` option or both `rho1()` and `rho2()` options are required to adjust the computation for clustering.

`rho2(#)` specifies the intraclass correlation of the second group for a two-sample test using groups. The `rho()` option or both `rho1()` and `rho2()` options are required to adjust the computation for clustering.

When `by()` is used, `sd1()` and `sd2()` or `sd()` is used to specify the population standard deviations of the two groups defined by *groupvar* for an unpaired two-sample *z* test (using groups). By default, a common standard deviation of one, `sd(1)`, is assumed.

When `unpaired` is used, `sd1()` and `sd2()` or `sd()` is used to specify the population standard deviations of *varname*<sub>1</sub> and *varname*<sub>2</sub> for an unpaired two-sample *z* test (using variables). By default, a common standard deviation of one, `sd(1)`, is assumed.

Options `corr()`, `sd1()`, and `sd2()` or `corr()` and `sd()` are used for a paired  $z$  test to compute the standard deviation of the differences between paired observations. By default, a common standard deviation of one, `sd(1)`, is assumed for both populations. Alternatively, the standard deviation of the differences between paired observations may be supplied directly with the `sddiff()` option.

## Remarks and examples

Remarks are presented under the following headings:

*One-sample z test*  
*Two-sample z test*  
*Paired z test*  
*Adjust for clustering*  
*Immediate form*

For the purpose of illustration, we assume that variances are known in all the examples below.

### One-sample z test

#### ▷ Example 1

In the first form, `ztest` tests whether the mean of the sample is equal to a known constant under the assumption of known variance. Assume that we have a sample of 74 automobiles. We know each automobile's average mileage rating and wish to test whether the overall average for the sample is 20 miles per gallon. We also assume that the population standard deviation is 6.

```
. use https://www.stata-press.com/data/r19/auto
(1978 automobile data)
. ztest mpg==20, sd(6)
```

One-sample z test

| Variable | Obs | Mean    | Std. err. | Std. dev. | [95% conf. interval] |          |
|----------|-----|---------|-----------|-----------|----------------------|----------|
| mpg      | 74  | 21.2973 | .6974858  | 6         | 19.93025             | 22.66434 |

```
mean = mean(mpg) z = 1.8600
HO: mean = 20
Ha: mean < 20 Ha: mean != 20 Ha: mean > 20
Pr(Z < z) = 0.9686 Pr(|Z| > |z|) = 0.0629 Pr(Z > z) = 0.0314
```

The  $p$ -value for the two-sided test is 0.0629, so we do not have statistical evidence to reject the null hypothesis that the mean equals 20 at a 5% significance level, but we would reject the null hypothesis at a 10% level.

## Two-sample z test

### ▷ Example 2: Two-sample z test using groups

We are testing the effectiveness of a new fuel additive. We run an experiment in which 12 cars are given the fuel treatment and 12 cars are not. The results of the experiment are as follows:

| treated | mpg |
|---------|-----|
| 0       | 20  |
| 0       | 23  |
| 0       | 21  |
| 0       | 25  |
| 0       | 18  |
| 0       | 17  |
| 0       | 18  |
| 0       | 24  |
| 0       | 20  |
| 0       | 24  |
| 0       | 23  |
| 0       | 19  |
| 1       | 24  |
| 1       | 25  |
| 1       | 21  |
| 1       | 22  |
| 1       | 23  |
| 1       | 18  |
| 1       | 17  |
| 1       | 28  |
| 1       | 24  |
| 1       | 27  |
| 1       | 21  |
| 1       | 23  |

The treated variable is coded as 1 if the car received the fuel treatment and 0 otherwise.

We can test the equality of means of the treated and untreated group by typing

```
. use https://www.stata-press.com/data/r19/fuel3
. ztest mpg, by(treated) sd(3)
```

Two-sample z test

| Group | Obs | Mean  | Std. err. | Std. dev. | [95% conf. interval] |          |
|-------|-----|-------|-----------|-----------|----------------------|----------|
| 0     | 12  | 21    | .8660254  | 3         | 19.30262             | 22.69738 |
| 1     | 12  | 22.75 | .8660254  | 3         | 21.05262             | 24.44738 |
| diff  |     | -1.75 | 1.224745  |           | -4.150456            | .6504558 |

```
diff = mean(0) - mean(1) z = -1.4289
HO: diff = 0
Ha: diff < 0 Ha: diff != 0 Ha: diff > 0
Pr(Z < z) = 0.0765 Pr(|Z| > |z|) = 0.1530 Pr(Z > z) = 0.9235
```

We do not have evidence to reject the null hypothesis that the means of the two groups are equal at a 5% significance level.

In the above, we assumed that the two groups have the same standard deviation of 3. If the standard deviations for the two groups are different, we can specify group-specific standard deviations in options `sd1()` and `sd2()`:

```
. ztest mpg, by(treated) sd1(2.7) sd2(3.2)
Two-sample z test
```

| Group | Obs | Mean  | Std. err. | Std. dev. | [95% conf. interval] |          |
|-------|-----|-------|-----------|-----------|----------------------|----------|
| 0     | 12  | 21    | .7794229  | 2.7       | 19.47236             | 22.52764 |
| 1     | 12  | 22.75 | .9237604  | 3.2       | 20.93946             | 24.56054 |
| diff  |     | -1.75 | 1.208649  |           | -4.118909            | .6189093 |

```
diff = mean(0) - mean(1) z = -1.4479
HO: diff = 0
Ha: diff < 0 Ha: diff != 0 Ha: diff > 0
Pr(Z < z) = 0.0738 Pr(|Z| > |z|) = 0.1476 Pr(Z > z) = 0.9262
```



### □ Technical note

In two-sample randomized designs, subjects will sometimes refuse the assigned treatment but still be measured for an outcome. In this case, take care to specify the group properly. You might be tempted to let *varname* contain missing where the subject refused and thus let `ztest` drop such observations from the analysis. [Zelen \(1979\)](#) argues that it would be better to specify that the subject belongs to the group in which he or she was randomized, even though such inclusion will dilute the measured effect.



### ▷ Example 3: Two-sample z test using variables

There is a second, inferior way to organize the data in the preceding example. We ran a test on 24 cars, 12 without the additive and 12 with. We now create two new variables, `mpg1` and `mpg2`.

| mpg1 | mpg2 |
|------|------|
| 20   | 24   |
| 23   | 25   |
| 21   | 21   |
| 25   | 22   |
| 18   | 23   |
| 17   | 18   |
| 18   | 17   |
| 24   | 28   |
| 20   | 24   |
| 24   | 27   |
| 23   | 21   |
| 19   | 23   |

This method is inferior because it suggests a connection that is not there. There is no link between the car with 20 mpg and the car with 24 mpg in the first row of the data. Each column of data could be arranged in any order. Nevertheless, if our data are organized like this, `ztest` can accommodate us.

```
. use https://www.stata-press.com/data/r19/fuel
. ztest mpg1==mpg2, unpaired sd(3)
```

Two-sample z test

| Variable | Obs | Mean  | Std. err. | Std. dev. | [95% conf. interval] |          |
|----------|-----|-------|-----------|-----------|----------------------|----------|
| mpg1     | 12  | 21    | .8660254  | 3         | 19.30262             | 22.69738 |
| mpg2     | 12  | 22.75 | .8660254  | 3         | 21.05262             | 24.44738 |
| diff     |     | -1.75 | 1.224745  |           | -4.150456            | .6504558 |

```
diff = mean(mpg1) - mean(mpg2) z = -1.4289
H0: diff = 0
Ha: diff < 0 Ha: diff != 0 Ha: diff > 0
Pr(Z < z) = 0.0765 Pr(|Z| > |z|) = 0.1530 Pr(Z > z) = 0.9235
```



## Paired z test

### ▷ Example 4

Suppose that the preceding data were actually collected by running a test on 12 cars. Each car was run once with the fuel additive and once without. Our data are stored in the same manner as in [example 3](#), but this time, there is most certainly a connection between the mpg values that appear in the same row. These come from the same car. The variables mpg1 and mpg2 represent mileage without and with the treatment, respectively. Suppose that the two variables have a common standard deviation of 2 and the correlation between them is 0.4.

```
. use https://www.stata-press.com/data/r19/fuel
. ztest mpg1==mpg2, sd(2) corr(0.4)
```

Paired z test

| Variable | Obs | Mean  | Std. err. | Std. dev. | [95% conf. interval] |           |
|----------|-----|-------|-----------|-----------|----------------------|-----------|
| mpg1     | 12  | 21    | .5773503  | 2         | 19.86841             | 22.13159  |
| mpg2     | 12  | 22.75 | .5773503  | 2         | 21.61841             | 23.88159  |
| diff     | 12  | -1.75 | .6324555  | 2.19089   | -2.98959             | -.5104099 |

```
mean(diff) = mean(mpg1 - mpg2) z = -2.7670
H0: mean(diff) = 0
Ha: mean(diff) < 0 Ha: mean(diff) != 0 Ha: mean(diff) > 0
Pr(Z < z) = 0.0028 Pr(|Z| > |z|) = 0.0057 Pr(Z > z) = 0.9972
```

The *p*-value for the two-sided test is 0.0057, so we reject, for example, the null hypothesis that the two means are equal at a 5% significance level.

Equivalently, we could specify directly the standard deviation of the differences between paired observations with the `sddiff()` option:

```
. ztest mpg1==mpg2, sddiff(2.191)
Paired z test
```

| Variable | Obs | Mean  | Std. err. | Std. dev. | [95% conf. interval] |           |
|----------|-----|-------|-----------|-----------|----------------------|-----------|
| diff     | 12  | -1.75 | .6324872  | 2.191     | -2.989652            | -.5103478 |

```

 mean(diff) = mean(mpg1 - mpg2) z = -2.7669
HO: mean(diff) = 0
Ha: mean(diff) < 0 Ha: mean(diff) != 0 Ha: mean(diff) > 0
Pr(Z < z) = 0.0028 Pr(|Z| > |z|) = 0.0057 Pr(Z > z) = 0.9972

```

◀

## Adjust for clustering

When observations are not independent and can be grouped into clusters, we need to adjust for clustering in a  $z$  test. For example, in a cluster randomized design, groups of individuals are randomized instead of individuals. To adjust for clustering, we need to specify the cluster identifier variable in the `cluster()` option. In the case of a one-sample  $z$  test, we need to also specify the intraclass correlation in the `rho()` option. In the case of a two-sample  $z$  test, we need to also specify the common population intraclass correlation in the `rho()` option or group-specific population intraclass correlations in the `rho1()` and `rho2()` options.

### ► Example 5: One-sample $z$ test, adjusting for clusters

Consider data on the SAT score of 75 students from 15 classes, with 5 students in each class. We want to test whether the mean verbal SAT score is different from 600. We assume a known standard deviation of 132 and a known intraclass correlation of 0.7. To perform the test, we specify the options `cluster(class)`, `rho(0.7)`, and `sd(132)`:

```
. use https://www.stata-press.com/data/r19/sat
(Fictional SAT data)
. ztest score == 600, cluster(class) rho(0.7) sd(132)
```

|                         |                      |        |
|-------------------------|----------------------|--------|
| One-sample z test       | Number of clusters = | 15     |
| Cluster variable: class | Cluster size =       | 5      |
|                         | Intraclass corr. =   | 0.7000 |

| Variable | Obs | Mean  | Std. err. | Std. dev. | [95% conf. interval] |          |
|----------|-----|-------|-----------|-----------|----------------------|----------|
| score    | 75  | 504.8 | 29.71222  | 132       | 446.5651             | 563.0349 |

```

 mean = mean(score) z = -3.2041
HO: mean = 600
Ha: mean < 600 Ha: mean != 600 Ha: mean > 600
Pr(Z < z) = 0.0007 Pr(|Z| > |z|) = 0.0014 Pr(Z > z) = 0.9993

```

We find statistical evidence to reject the null hypothesis of  $H_0: \mu_{\text{SAT}} = 600$  versus a two-sided alternative  $H_a: \mu_{\text{SAT}} \neq 600$  at the 5% significance level; the  $p$ -value =  $0.0014 < 0.05$ .

◀

### ▷ Example 6: Two-sample $z$ test using groups, adjusting for clusters

Consider a cluster randomized control trial that studies the effect of additional training of nurses and general practitioners in patient-centered care on the well-being and future disease risk of patients with type 2 diabetes (Kinmonth et al. [1998] and Campbell and Walters [2014]). Practices (practice) are randomly allocated to two groups—one trained to give patient-centered care (intervention group) and another trained to give routine care (comparison or control group). In our analysis, we transform the original bmi using the formula  $\ln(\text{bmi} - 14.67355)$  to obtain a variable that is approximately normally distributed, lbmi. We want to test the equality of the means of lbmi for the two groups. We assume a known common standard deviation of 0.35 and a known common intraclass correlation of 0.028.

To perform the test, we need to specify the `rho(0.028)` and `sd(0.35)` options. We also need to specify the cluster identifier `practice` in the `cluster()` option and the group identifier `group` in the `by()` option.

```
. use https://www.stata-press.com/data/r19/dcfdf_trial
(BMI data from Diabetes Care from Diagnosis trial (Kinmonth et al., 1998))
. ztest lbmi, by(group) cluster(practice) rho(0.028) sd(0.35)
```

Two-sample z test  
Cluster variable: practice

| Group: Control       |        | Group: Interv.       |        |
|----------------------|--------|----------------------|--------|
| Number of clusters = | 20     | Number of clusters = | 18     |
| Avg. cluster size =  | 5.10   | Avg. cluster size =  | 7.67   |
| CV cluster size =    | 0.5330 | CV cluster size =    | 0.5126 |
| Intraclass corr. =   | 0.0280 | Intraclass corr. =   | 0.0280 |

| Group   | Obs | Mean      | Std. err. | Std. dev. | [95% conf. interval] |           |
|---------|-----|-----------|-----------|-----------|----------------------|-----------|
| Control | 102 | 2.62954   | .0372502  | .35       | 2.556531             | 2.702549  |
| Interv. | 138 | 2.749023  | .0332182  | .35       | 2.683916             | 2.81413   |
| diff    |     | -.1194831 | .0499102  |           | -.2173054            | -.0216608 |

```
diff = mean(Control) - mean(Interv.) z = -2.3940
HO: diff = 0
Ha: diff < 0 Ha: diff != 0 Ha: diff > 0
Pr(Z < z) = 0.0083 Pr(|Z| > |z|) = 0.0167 Pr(Z > z) = 0.9917
```

We find statistical evidence to reject the null hypothesis of  $H_0: \mu_{\text{diff}} = 0$  versus a two-sided alternative  $H_a: \mu_{\text{diff}} \neq 0$  at the 5% significance level; the  $p$ -value = 0.0167 < 0.05.

◀

## Immediate form

### ▷ Example 7: One-sample $z$ test

`ztesti` is like `ztest`, except that we specify summary statistics rather than variables as arguments. For instance, we are reading an article that reports the mean number of sunspots per month as 62.6 with a standard deviation of 15.8. We assume this standard deviation is the population standard deviation. There are 24 months of data. We wish to test whether the mean is 75:

```
. zttesti 24 62.6 15.8 75
```

One-sample z test

|   | Obs | Mean | Std. err. | Std. dev. | [95% conf. interval] |         |
|---|-----|------|-----------|-----------|----------------------|---------|
| x | 24  | 62.6 | 3.225161  | 15.8      | 56.2788              | 68.9212 |

```
mean = mean(x) z = -3.8448
HO: mean = 75
Ha: mean < 75 Ha: mean != 75 Ha: mean > 75
Pr(Z < z) = 0.0001 Pr(|Z| > |z|) = 0.0001 Pr(Z > z) = 0.9999
```



### ▷ Example 8: Two-sample z test

There is no immediate form of `zttest` with paired data because the test is also a function of the covariance, a number unlikely to be reported in any published source. For unpaired data, however, we might type

```
. zttesti 20 20 5 32 15 4
```

Two-sample z test

|      | Obs | Mean | Std. err. | Std. dev. | [95% conf. interval] |          |
|------|-----|------|-----------|-----------|----------------------|----------|
| x    | 20  | 20   | 1.118034  | 5         | 17.80869             | 22.19131 |
| y    | 32  | 15   | .7071068  | 4         | 13.6141              | 16.3859  |
| diff |     | 5    | 1.322876  |           | 2.407211             | 7.592789 |

```
diff = mean(x) - mean(y) z = 3.7796
HO: diff = 0
Ha: diff < 0 Ha: diff != 0 Ha: diff > 0
Pr(Z < z) = 0.9999 Pr(|Z| > |z|) = 0.0002 Pr(Z > z) = 0.0001
```



## Stored results

One-sample `zttest` and `zttesti` store the following in `r()`:

Scalars

- `r(N)` sample size
- `r(mu)` sample mean
- `r(sd)` standard deviation
- `r(se)` standard error
- `r(lb)` lower confidence bound of one-sample mean
- `r(ub)` upper confidence bound of one-sample mean
- `r(z)` z statistic
- `r(p_l)` lower one-sided p-value
- `r(p)` two-sided p-value
- `r(p_u)` upper one-sided p-value
- `r(level)` confidence level

Cluster-adjusted one-sample `ztest` also stores the following in `r()`:

Scalars

|                            |                                            |
|----------------------------|--------------------------------------------|
| <code>r(K)</code>          | number of clusters $K$                     |
| <code>r(M)</code>          | cluster size $M$                           |
| <code>r(rho)</code>        | intraclass correlation                     |
| <code>r(CV_cluster)</code> | coefficient of variation for cluster sizes |

Two-sample `ztest` and `ztesti` store the following in `r()`:

Scalars

|                         |                                                                                         |
|-------------------------|-----------------------------------------------------------------------------------------|
| <code>r(N1)</code>      | sample size of population one                                                           |
| <code>r(N2)</code>      | sample size of population two                                                           |
| <code>r(mu1)</code>     | sample mean for population one                                                          |
| <code>r(mu2)</code>     | sample mean for population two                                                          |
| <code>r(mu_diff)</code> | difference of means                                                                     |
| <code>r(corr)</code>    | correlation between paired observations; if the <code>corr()</code> option is specified |
| <code>r(sd)</code>      | common standard deviation                                                               |
| <code>r(sd1)</code>     | standard deviation for population one                                                   |
| <code>r(sd2)</code>     | standard deviation for population two                                                   |
| <code>r(sd_diff)</code> | standard deviation of the differences between paired observations                       |
| <code>r(se1)</code>     | standard error of population-one sample mean                                            |
| <code>r(se2)</code>     | standard error of population-two sample mean                                            |
| <code>r(se_diff)</code> | standard error of the difference of means                                               |
| <code>r(lb1)</code>     | lower confidence bound of population-one sample mean                                    |
| <code>r(ub1)</code>     | upper confidence bound of population-one sample mean                                    |
| <code>r(lb2)</code>     | lower confidence bound of population-two sample mean                                    |
| <code>r(ub2)</code>     | upper confidence bound of population-two sample mean                                    |
| <code>r(lb_diff)</code> | lower confidence bound of the difference of means                                       |
| <code>r(ub_diff)</code> | upper confidence bound of the difference of means                                       |
| <code>r(z)</code>       | $z$ statistic                                                                           |
| <code>r(p_1)</code>     | lower one-sided $p$ -value                                                              |
| <code>r(p)</code>       | two-sided $p$ -value                                                                    |
| <code>r(p_u)</code>     | upper one-sided $p$ -value                                                              |
| <code>r(level)</code>   | confidence level                                                                        |

Cluster-adjusted two-sample `ztest` using the `by()` option also stores the following in `r()`:

Scalars

|                             |                                                           |
|-----------------------------|-----------------------------------------------------------|
| <code>r(K1)</code>          | population-one number of clusters $K_1$                   |
| <code>r(K2)</code>          | population-two number of clusters $K_2$                   |
| <code>r(M1)</code>          | population-one cluster size $M_1$                         |
| <code>r(M2)</code>          | population-two cluster size $M_2$                         |
| <code>r(rho)</code>         | common intraclass correlation                             |
| <code>r(rho1)</code>        | population-one intraclass correlation                     |
| <code>r(rho2)</code>        | population-two intraclass correlation                     |
| <code>r(CV_cluster1)</code> | population-one coefficient of variation for cluster sizes |
| <code>r(CV_cluster2)</code> | population-two coefficient of variation for cluster sizes |

## Methods and formulas

Methods and formulas are presented under the following headings:

*One-sample  $z$  test*

*Two-sample unpaired  $z$  test*

*Paired  $z$  test*

For all the tests below, the test statistic  $z$  is distributed as standard normal, and the  $p$ -value is computed as

$$p = \begin{cases} 1 - \Phi(z) & \text{for an upper one-sided test} \\ \Phi(z) & \text{for a lower one-sided test} \\ 2\{1 - \Phi(|z|)\} & \text{for a two-sided test} \end{cases}$$

where  $\Phi(\cdot)$  is the cdf of a standard normal distribution and  $|z|$  is an absolute value of  $z$ .

Also see, for instance, [Hoel \(1984, 140–161\)](#), [Dixon and Massey \(1983, 100–130\)](#), and [Tamhane and Dunlop \(2000, 237–290\)](#) for more information about  $z$  tests.

## One-sample $z$ test

Suppose that we observe a random sample  $x_1, x_2, \dots, x_n$  of size  $n$ , which follows a normal distribution with mean  $\mu$  and standard deviation  $\sigma$ . We are interested in testing the null hypothesis  $H_0: \mu = \mu_0$  versus the two-sided alternative hypothesis  $H_a: \mu \neq \mu_0$ , the upper one-sided alternative  $H_a: \mu > \mu_0$ , or the lower one-sided alternative  $H_a: \mu < \mu_0$ . Assuming a known standard deviation  $\sigma$ , we use the following test statistic,

$$z = \frac{(\bar{x} - \mu_0)}{s}$$

where  $\bar{x} = (\sum_{i=1}^n x_i)/n$  is the sample mean and  $s = \sigma/\sqrt{n}$  is the standard error of  $\bar{x}$ .

The  $100(1 - \alpha)\%$  confidence interval for  $\bar{x}$  is given by

$$\bar{x} \pm z_{1-\alpha/2}s$$

where  $z_{1-\alpha/2}$  is the  $(1 - \alpha/2)$ th quantile of the standard normal distribution.

With clustered data, suppose that there are  $K$  clusters. The  $i$ th cluster of size  $M_i$  contains the observations  $x_{i1}, x_{i2}, \dots, x_{iM_i}$ , such that  $n = \sum_{i=1}^K M_i$  and  $\bar{x} = \frac{1}{n} \sum_{i=1}^K \sum_{j=1}^{M_i} x_{ij}$ . Let  $\rho$  be the intraclass correlation. Following [Ahn, Heo, and Zhang \(2015\)](#), we assume that the cluster sizes  $M_i$  are independent and identically distributed. Let  $C_{\text{adj}}$  be the adjustment to the standard error for clustered data,

$$C_{\text{adj}} = \sqrt{\sum_{i=1}^K M_i \{1 + \rho(M_i - 1)\} / n}$$

such that  $s_{\text{cl}} = C_{\text{adj}}s$ .

$C_{\text{adj}}$  can be equivalently written as

$$C_{\text{adj}} = \sqrt{1 + \rho(\bar{M} - 1) + \rho\bar{M}CV_{\text{cl}}^2}$$

where  $\bar{M} = \sum_{i=1}^K M_i/K$  is the average cluster size and  $CV_{\text{cl}}$  is the coefficient of variation for cluster sizes:

$$CV_{\text{cl}} = \frac{\sqrt{\sum_{i=1}^K (M_i - \bar{M})^2 / K}}{\bar{M}}$$

To adjust the test statistic  $z$  and the confidence interval for clustering, replace  $s$  with  $s_{\text{cl}}$  in the corresponding formulas. In the presence of clustering, the test statistic  $z$  is asymptotically normally distributed conditional on the empirical distribution of  $M_i$ 's.

## Two-sample unpaired z test

Suppose that we observe a random sample  $x_{11}, x_{12}, \dots, x_{1n_1}$  of size  $n_1$ , which follows a normal distribution with mean  $\mu_1$  and standard deviation  $\sigma_1$ , and another random sample  $x_{21}, x_{22}, \dots, x_{2n_2}$  of size  $n_2$ , which follows a normal distribution with mean  $\mu_2$  and standard deviation  $\sigma_2$ . We are interested in testing the null hypothesis  $H_0: \mu_2 = \mu_1$  versus the two-sided alternative hypothesis  $H_a: \mu_2 \neq \mu_1$ , the upper one-sided alternative  $H_a: \mu_2 > \mu_1$ , or the lower one-sided alternative  $H_a: \mu_2 < \mu_1$ . Assuming known standard deviations  $\sigma_1$  and  $\sigma_2$ , we use the following test statistic,

$$z = \frac{\bar{x}_2 - \bar{x}_1}{\sqrt{s_1^2 + s_2^2}}$$

where  $\bar{x}_1 = (\sum_{i=1}^{n_1} x_{1i})/n_1$  and  $\bar{x}_2 = (\sum_{i=1}^{n_2} x_{2i})/n_2$  are the two sample means and  $s_1 = \sigma_1/\sqrt{n_1}$  and  $s_2 = \sigma_2/\sqrt{n_2}$  are the corresponding two standard errors.

The  $100(1 - \alpha)\%$  confidence intervals for  $\bar{x}_1$  and  $\bar{x}_2$  are given by

$$\begin{aligned} \bar{x}_1 \pm z_{1-\alpha/2} s_1 \\ \bar{x}_2 \pm z_{1-\alpha/2} s_2 \end{aligned}$$

where  $z_{1-\alpha/2}$  is the  $(1 - \alpha/2)$ th quantile of the standard normal distribution.

The  $100(1 - \alpha)\%$  confidence interval for  $\bar{x}_1 - \bar{x}_2$  is given by

$$\bar{x}_1 - \bar{x}_2 \pm z_{1-\alpha/2} \sqrt{s_1^2 + s_2^2}$$

With clustered data, similar to the discussion for the one-sample test, suppose that population one has  $K_1$  clusters and population two has  $K_2$  clusters. Let  $\rho_1$  and  $\rho_2$  be the intraclass correlations,  $\bar{M}_1$  and  $\bar{M}_2$  be the average cluster sizes,  $\bar{x}_1 = (1/n_1) \sum_{i=1}^{K_1} \sum_{j=1}^{M_{1i}} x_{1ij}$  and  $\bar{x}_2 = (1/n_2) \sum_{i=1}^{K_2} \sum_{j=1}^{M_{2i}} x_{2ij}$  be the sample means, and  $\text{CV}_{\text{cl},1}$  and  $\text{CV}_{\text{cl},2}$  be the coefficients of variation for cluster sizes for population one and population two. Let  $s_{1,\text{cl}} = C_{\text{adj},1} s_1$  and  $s_{2,\text{cl}} = C_{\text{adj},2} s_2$  be the standard errors of the population-specific sample means adjusted for clustered data, where the population-specific adjustment factors are defined as described for the one-sample test. To adjust the two-sample test statistic and the confidence intervals for clustering, replace  $s_1$  with  $s_{1,\text{cl}}$  and  $s_2$  with  $s_{2,\text{cl}}$  in the corresponding formulas.

## Paired z test

Some experiments have paired observations (also known as matched observations, correlated pairs, or permanent components). Consider a sequence of  $n$  paired observations denoted by  $x_{ij}$  for subjects  $i = 1, 2, \dots, n$  and groups  $j = 1, 2$ . An individual observation corresponds to the pair  $(x_{i1}, x_{i2})$ , and inference is made on the differences within the pairs. Let  $\mu_d = \mu_2 - \mu_1$  denote the mean difference, where  $\mu_j$  is the population mean of group  $j$ , and let  $D_i = x_{i2} - x_{i1}$  denote the difference between individual observations.  $D_i$  follows a normal distribution with mean  $\mu_2 - \mu_1$  and standard deviation  $\sigma_d$ , where  $\sigma_d = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho_{\text{pair}}\sigma_1\sigma_2}$ ,  $\sigma_j$  is the population standard deviation of group  $j$  and  $\rho_{\text{pair}}$  is the correlation between paired observations.

We are interested in testing the null hypothesis  $H_0: \mu_2 = \mu_1$  versus the two-sided alternative hypothesis  $H_a: \mu_2 \neq \mu_1$ , the upper one-sided alternative  $H_a: \mu_2 > \mu_1$ , or the lower one-sided alternative  $H_a: \mu_2 < \mu_1$ . Assuming the standard deviation of the differences  $\sigma_d$  is known, we use the following test statistic,

$$z = \frac{\bar{d}}{s_d}$$

where  $\bar{d} = (\sum_{i=1}^n D_i)/n$  is the sample mean of the differences between paired observations and  $s_d = \sigma_d/\sqrt{n}$  is the standard error of  $\bar{d}$ .

The  $100(1 - \alpha)\%$  confidence interval for  $\bar{d}$  is given by

$$\bar{d} \pm z_{1-\alpha/2} s_d$$

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## Also see

- [R] **ci** — Confidence intervals for means, proportions, and variances
- [R] **esize** — Effect size based on mean comparison
- [R] **mean** — Estimate means
- [R] **oneway** — One-way analysis of variance
- [R] **ttest** — *t* tests (mean-comparison tests)
- [MV] **hotelling** — Hotelling's  $T^2$  generalized means test
- [PSS-2] **power onemean** — Power analysis for a one-sample mean test
- [PSS-2] **power onemean, cluster** — Power analysis for a one-sample mean test, CRD
- [PSS-2] **power pairedmeans** — Power analysis for a two-sample paired-means test
- [PSS-2] **power twomeans** — Power analysis for a two-sample means test
- [PSS-2] **power twomeans, cluster** — Power analysis for a two-sample means test, CRD

# Subject and author index

See the [combined subject index](#) and the [combined author index](#) in the *Stata Index*.