`ffurtest`: Flexible Fourier Form Unit-Root Tests in Stata

Implementation, validation, and an application to euro-backed stablecoins

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Aim of the talk

- Introduce `ffurtest`, a Stata command for unit-root testing with a Flexible Fourier Form deterministic component.
- Focus on implementation choices: detrending, frequency selection, lag augmentation, and returned output.
- Validate the implementation through a small Monte Carlo exercise and code-level checks.
- Illustrate the command on euro-backed stablecoins: STASIS Euro (EURS) and Celo Euro (cEUR).

Main software contribution

A unified Stata interface for three related Fourier-based unit-root tests.

Flexible Fourier Form: why useful?

A deterministic component with unknown smooth changes can be approximated parsimoniously by

$$d(t) = \alpha_0 + \alpha_1 t + \gamma_1 \sin\left(\frac{2\pi kt}{T}\right) + \gamma_2 \cos\left(\frac{2\pi kt}{T}\right).$$

- No need to specify break dates ex ante.
- Smooth changes can approximate a wide range of low-frequency deterministic patterns.
- Particularly useful when structural change is gradual rather than abrupt.

Testing question

Is the stochastic component stationary once smooth deterministic changes are allowed for?

Tests implemented in `ffurtest`

Option	Test	Core idea
`method(el1)`	Enders–Lee LM	LM detrending; test on $\widetilde{S}_t$
`method(el2)`	Enders–Lee DF	DF-type regression with Fourier terms
`method(rt)`	Rodrigues–Taylor GLS	Local GLS detrending with Fourier terms

- Single frequency: `k(1)`, `k(2)`, ...
- Cumulative harmonics: `cumulative(2)`, etc.
- Frequency selection: `kest(el)` or `kest(rt)`.
- Lag choice: fixed `p()`, Schwert default, or `seqt`.

Enders–Lee LM: implementation flow

1. Generate Fourier terms: $\sin_{k,t} \equiv \sin(2\pi kt/T)$ and $\cos_{k,t} \equiv \cos(2\pi kt/T)$
2. Consider the DGP (here we focus on a single frequency k for simplicity)

$$y_t = \alpha_0 + \delta_0 t + \delta_1 \sin_{k,t} + \delta_2 \cos_{k,t} + e_t, \qquad e_t = \rho e_{t-1} + u_t$$

3. Estimate the first-stage regression in differences under $\rho = 1$:

$$\Delta y_t = \delta_0 + \delta_1 \Delta \sin_t + \delta_2 \Delta \cos_t + u_t.$$

4. Construct the LM-detrended series, letting $\hat{\psi}_t = y_t - \widehat{\delta}_0 t - \widehat{\delta}_1 \sin_{k,t} - \widehat{\delta}_2 \cos_{k,t}$:

$$\widetilde{S}_t = \hat{\psi}_t - \hat{\psi}_1, \qquad \widetilde{S}_1 = 0.$$

5. Estimate the (possibly) lag-augmented LM regression:

$$\Delta y_t = \phi \widetilde{S}_{t-1} + d_0 + d_1 \Delta \sin_{k,t} + d_2 \Delta \cos_{k,t} + \sum_{j=1}^{p} \gamma_j \Delta \widetilde{S}_{t-j} + \epsilon_t.$$

6. Report the t-statistic for ϕ: τ_{LM}.

Lag specification

- **Fixed lags**: user sets `p(#)`.
- **Default rule**: Schwert-type lag length

$$p = \left\lfloor 12 \left(\frac{T}{100} \right)^{1/4} \right\rfloor .$$

- **Sequential-t rule**: start from `p()` and eliminate the last lag if its absolute t-statistic is below 1.645.

Implementation choice

When `seqt` is used, the lag-selection regressions use a common effective sample determined by the initial maximum lag.

Small Monte Carlo validation: size

Monte Carlo: $\rho = 1$, $k = 1$, $p = 0$, 20,000 replications; 5% significance level			
Test	$T = 100$	$T = 200$	$T = 2500$
LM	0.054	0.049	0.052
DF	0.051	0.050	0.050
GLS	0.050	0.048	0.050

Validation message

Monte Carlo rejection rates under $\rho = 1$ very close to 5% for all three tests.

Small Monte Carlo validation: power

Monte Carlo: $\rho = 0.9$, $k = 1$, $p = 0$, 20,000 replications; 5% significance level				
Test	$T = 100$	$T = 200$	$T = 500$	$T = 2500$
LM	0.122	0.376	0.996	1.000
DF	0.105	0.314	0.993	1.000
GLS	0.117	0.394	0.998	1.000

Power rises quickly with T. The three tests display similar behavior in this benchmark design.

Application: euro-backed stablecoins

T

STASIS Euro (EURS)

- Euro-backed stablecoin, traded since January 2022.
- Fully backed 1:1 with euro liquid assets.
- MiCA-compliant infrastructure.
- Interoperable across Ethereum, Polygon, XDC, XRPL, Algorand, and Stellar.

Celo Euro (cEUR)

- Decentralized euro-pegged stablecoin native to the Celo blockchain.
- Designed to maintain a 1:1 value with the euro.
- Backed by liquid assets and cryptocurrencies.
- Supports micropayments, cross-border transfers, and remittances.

Stablecoin prices and the unit-root question

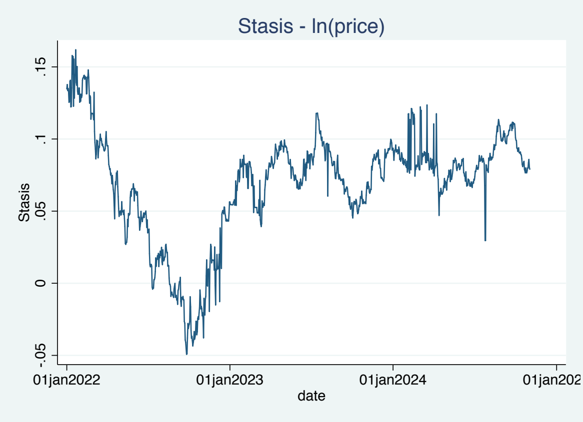
Stablecoin prices fluctuate around parity with relatively low volatility compared with other digital assets.

Are deviations from parity mean reverting?

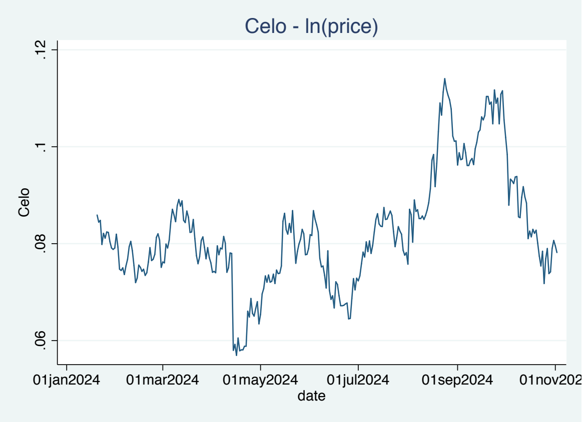
Why Fourier unit-root tests?

A fixed deterministic trend may be too restrictive if deviations from parity include gradual shifts in liquidity, collateral, market conditions, or trading infrastructure.

The Flexible Fourier Form allows a smooth deterministic component without choosing break dates.



STASIS Euro (EURS), $T = 1037$



Celo Euro (cEUR), $T = 288$

A smcl snippet

```
. fffurtest Stasis, timevar(date) method(el1) k(1) p(0)      // zero lags
------------------------------------------------------------
Enders & Lee LM
T =      1037
Fourier single k = 1
Lag rule: Fixed, lags =  0
LM t-stat:    -4.438
F-stat (H0: No Fourier terms):     6.048
------------------------------------------------------------

. fffurtest Stasis, timevar(date) method(el1) k(1)           // Schwert lags
------------------------------------------------------------
Enders & Lee LM
T =      1037
Fourier single k = 1
Lag rule: Schwert, lags= 21
LM t-stat:    -2.327
F-stat (H0: No Fourier terms):     3.307
------------------------------------------------------------
```

Application results: STASIS Euro

Test	Lag rule	p	5% CV	Test statistic
EL-LM	fixed $p = 0$	0	-4.045	-4.438^{*}
EL-LM	Schwert	21	-4.045	-2.327
EL-LM	seq.-t from 8	5	-4.045	-2.365
EL-DF	fixed $p = 0$	0	-4.285	-4.439^{*}
EL-DF	Schwert	21	-4.285	-2.356
EL-DF	seq.-t from 8	5	-4.285	-2.370
RT-GLS	fixed $p = 0$	0	-3.917	-4.186^{*}
RT-GLS	Schwert	21	-3.917	-1.968

* rejection at the 5% level using interpolated/tabulated critical values (For RT-GLS, STASIS implementation, CV's are those in the $T = 1000$ table in RT.

Empirical pattern

STASIS rejects the unit-root null only in the fixed $p = 0$ specifications. Once augmentation lags are introduced, the evidence for mean reversion around parity disappears at the 5% level.

Application results: Celo Euro

Test	Lag rule	p	5% CV	Test statistic
EL-LM	fixed $p = 0$	0	-4.064	-3.224
EL-LM	Schwert	15	-4.064	-3.306
EL-LM	seq.-t from 8	6	-4.064	-3.192
EL-DF	fixed $p = 0$	0	-4.304	-3.158
EL-DF	Schwert	15	-4.304	-2.963
EL-DF	seq.-t from 8	6	-4.304	-3.072
RT-GLS	fixed $p = 0$	0	-4.027	-3.215
RT-GLS	Schwert	15	-4.027	-3.103

* rejection at the 5% level using interpolated critical values. No Celo specification rejects at 5%.

Empirical pattern

For Celo, none of the EL-LM, EL-DF, or RT-GLS specifications rejects the unit-root null at the 5% level. The conclusion is robust to the lag specification used here.

Interpreting the application

- Test statistics are compared with finite-sample critical values interpolated from the published tables.
- STASIS shows rejection only in the fixed $p = 0$ specifications; augmentation lags remove the rejection.
- Celo does not reject at the 5% level in any of the reported specifications.

Practical workflow

Report the Fourier specification, lag rule, selected p, critical values, and sensitivity across EL-LM, EL-DF, and RT-GLS.

Implementation note: sequential-t lags

- Start from a maximum lag, e.g. $p_{\max} = 8$.
- Retain the final lag if $|t_{\widehat{\gamma_p}}| > 1.645$.
- Otherwise reduce p and re-estimate.

Finite-sample caution

Critical values tabulated for fixed $p = 0$ need not be automatically size-correct after data-dependent lag selection. A possible extension is to simulate selection-aware critical values:

$$c_\alpha^{\mathrm{seqt}}(T, k, p_{\max}) = Q_\alpha\left[\tau_{LM}\{\widehat{p}_{\mathrm{seqt}}\}\right].$$

Conclusions

- `ffurtest` implements three Fourier-based unit-root tests in a unified Stata command.
- The command permits frequency choice (single, estimated, cumulative), lag augmentation, linear trend, testing the specification.
- Small Monte Carlo checks show accurate size for fixed $p = 0$.
- In the stablecoin application, fixed-lag results can differ from augmented specifications; reporting lag rules is essential.

Next steps

Add optional critical-value support (e.g. automatic reporting of published CV's, interpolated CV's, simulated CV's), expand examples, and document finite-sample behavior under alternative lag-selection rules and frequency choices.

References

- Enders, W. and Lee, J. (2012). "A Unit Root Test Using a Fourier Series to Approximate Smooth Breaks." *Oxford Bulletin of Economics and Statistics*.
- Enders, W. and Lee, J. (2012). "The Flexible Fourier Form and Dickey–Fuller Type Unit Root Tests." *Economics Letters*.
- Rodrigues, P.M.M. and Taylor, A.M.R. (2012). "The Flexible Fourier Form and Local GLS De-trended Unit Root Tests." *Oxford Bulletin of Economics and Statistics*.
- Available RATS code from Walter Enders' time-series archive.