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Panel Data Threshold Regression with Interactive Fixed Effects

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Motivation

- In applied work, effects (coefficients) may differ above or below an unknown threshold.
- Example: Effects...
  - ... of a policy depends on the state of the economy.
  - ... of a medication depend on general health of person.
- Ignoring the "state" leads to biased coefficient estimates.
- In panel data: Units are exposed to common shocks, creating cross-sectional dependence; also often panels have large N but short T.

Motivation

Literature

- Panel (Time) Series
  - Hansen (1999) estimation of thresholds in fixed T, large N panel data but without cross-sectional dependence, Chudik et al. (2017) for dynamic models.
  - Ditzen et al. (2026) extend Hansen (1999) to panel models with interactive fixed effects, Barassi et al. (2026) with heterogenous thresholds.
- `xtthreshold` introduces estimation for thresholds in panel data with and without interactive fixed effects.

The threshold panel model

$$y_{it} = \boldsymbol{\beta}' \boldsymbol{x}_{it} + \boldsymbol{\delta}' \boldsymbol{h}_{it}(\gamma) + e_{it}$$
$$\boldsymbol{h}_{it}(\gamma) = S' \boldsymbol{x}_{it} \, \mathbb{I}(z_{it} > \gamma).$$

- z_{it} is the scalar threshold variable and included in x_{it}
- $h_{it}(\gamma)$ is the threshold function with threshold γ is unknown.
- S selects the regressors whose coefficients change across regimes.
- Below the threshold, selected slopes are β.
- Above the threshold, selected slopes are $\beta + \delta$.

  $\Rightarrow$ The threshold, γ, tells us where the regime changes

  $\Rightarrow$ δ tells us how much the selected slopes change.

Interactive Fixed Effects

- Assume an error factor structure with unobserved factors:

$$e_{it} = \boldsymbol{\mu}_i' \boldsymbol{f}_t + \varepsilon_{it},$$
$$\boldsymbol{x}_{it} = \boldsymbol{\Lambda}_i' \boldsymbol{f}_t + \boldsymbol{v}_{it}.$$

- Common factors $\boldsymbol{f}_t$ affect all units, but with unit-specific loadings $\boldsymbol{\mu}_i$ and $\boldsymbol{\Lambda}_i$.
- Regressors can be correlated with the unobserved common factors, so simple within-demeaning does not solve the problem.
- Ditzen et al. (2026) combine threshold estimation with the Common Correlated Effects (CCE) (Pesaran, 2006).

CCE Estimator

- Idea: span space of latent factors using cross-sectional averages of X and y (Pesaran, 2006).
- CCE estimator popular, straightforward to implement and widely used. Robust to temporal correlation in observables, factors and errors (Kapetanios et al., 2011; Chudik et al., 2011; Chudik and Pesaran, 2015).
- Idea: partial out cross-section averages using annihilator matrix $Q_{\bar{H}}$:

$$
\begin{aligned}
Q_{\bar{H}} &= I_T - \bar{H}(\bar{H}'\bar{H})^{-1}\bar{H}' \\
\bar{H} &= [\bar{Y}, \bar{X}], \qquad \widetilde{X}_i = Q_{\bar{H}} X_i \\
\hat{\boldsymbol{\beta}} &= (\widetilde{X}'\widetilde{X})^{-1}\widetilde{X}'\widetilde{Y}
\end{aligned}
$$

- For threshold estimation, use cross-section averages of X rather than y.

Threshold Estimation

- Create grid over (subset) of unique values of thresholding variable z_{it}.
- For each candidate threshold γ:
  1. build $H_i(\gamma)$;
  2. defactor using CCE;
  3. estimate the slope coefficients by least squares;
  4. calculate $SSR(\gamma)$.
- Estimate threshold $\hat{\gamma}$ using minima of SSRs:

$$\hat{\gamma} = \arg \min_{\gamma \in \Gamma_{NT}} SSR(\gamma).$$

$\Rightarrow$ Candidate set can contain up to NT values of z_{it}. A quantile grid reduces computation; a finer grid gives a closer approximation to the full search.

Inference

Three Questions

1. **Slope coefficients:** standard t and Wald inference is available for $\theta = (\beta', \delta')'$.
2. **Where is the threshold?** Use the LR statistic

$$LR(\gamma_0) = N(T - k - l)\frac{SSR(\gamma_0) - SSR(\hat{\gamma})}{SSR(\hat{\gamma})}$$

   and invert it to obtain a confidence set for γ. Critical values at level ϕ from $c(\phi) = -2log(1 - \sqrt{1 - \phi})$.
3. **Is there a threshold at all?** Under $H_0 : \delta = 0$, γ is unidentified, so standard Wald inference fails; use a sup-Wald test with a pairs bootstrap over cross-sectional units.

▸ Details on Asymptotics

Syntax

`xtthreshold` *depvar indepvars1* | *indepvars2* [*if*] , *threshold(varlist)*

[`csa[(`*varlist*`)] gridlength(real) pooled` *options1*]

- *indepvars1* are variables exposed to threshold, *indepvars2* are not affected by threshold.
- *csa[()]* specifies cross-section averages.
- *grid()* sets grid length.
- *pooled* pooled coefficients.
- *options1* include options to control type of inverter, type of constant and fixed effects.

Empirical Example

Effect of Inflation on Economic Growth (Ditzen et al., 2026)

- GDP a function of inflation (khanvar), government spending (govspend), trade openness (open), population growth (popgr) and gross capital formation (grosscap).
- Empirically likely that effect of inflation and government spending on GDP depends on level of inflation.
- Question: What is the level?
- Inflation acts as a threshold variable and inflation and government spending are regime switching.

Empirical Example

Effect of Inflation on Economic Growth (Ditzen et al., 2026)

```
. xtthreshold gdppcgr govspend  khanvar | open popgr grosscap  , ///
>         threshold(khanvar) grid(400) pooled noreg ///
>         csa(govspend  khanvar open popgr grosscap)
 Threshold Estimation (Ditzen, Karavias, Westerlund, 2026)
   Threshold variable(s): khanvar
```

Threshold	Value	95% CI	
1	1.24	.0619	1.57

- Estimated threshold for inflation of 1.24%.
- `xtthreshold` automatically displays results from a CCE regression. Option *noreg* prevents this behaviour.

Empirical Example

Effect of Inflation on Economic Growth (Ditzen et al., 2026)

```
. xtthreshold gdppcgr govspend  khanvar | open popgr grosscap , ///
>         threshold(khanvar) grid(400) pooled ///
>         csa(govspend  khanvar open popgr grosscap)
 Threshold Estimation (Ditzen, Karavias, Westerlund, 2026)
   Threshold variable(s): khanvar
```

Threshold	Value	95% CI	
1	1.24	.0619	1.57

Pooled Regression

gdppcgr	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
thresholded						
govspend_0	-.3833193	.0392082	-9.78	0.000	-.460166	-.3064726
govspend_1	.1258445	.0196636	6.40	0.000	.0873045	.1643846
khanvar_0	-1.651545	.1749324	-9.44	0.000	-1.994406	-1.308684
khanvar_1	.3586473	.0942674	3.80	0.000	.1738866	.543408
non_thresholded						
open	.0279824	.0059451	4.71	0.000	.0163302	.0396346
popgr	-.8679307	.0979326	-8.86	0.000	-1.059875	-.6759864
grosscap	.1033035	.0185232	5.58	0.000	.0669986	.1396083

```
 Cross-Section Averages
  with threshold: govspend khanvar
  without threshold: open popgr grosscap
 Individual fixed effects partialed out
```

- Variables with "_0" indicate variable larger than the threshold ($z_{it} > \gamma$), "_1" smaller ($z_{it} \leq \gamma$).
- Note: standard errors are standard OLS errors, not from the CCEP estimator!

Empirical Example

Post Estimation

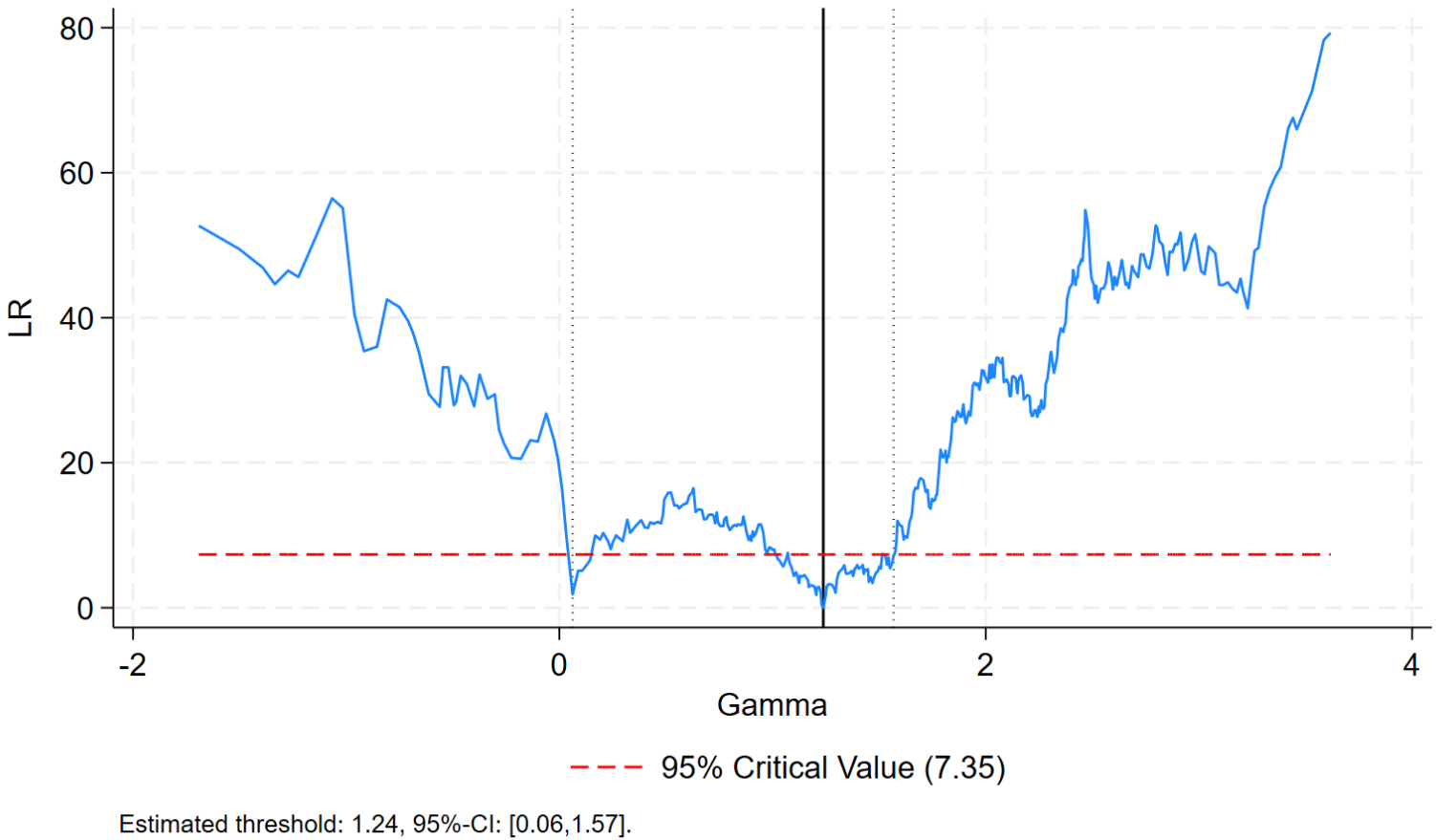
- Test if threshold is at a specific value, i.e. $H_0 : \delta = 0$.
- *estat gtest #* implements this test, with # the value under the null.

```
. estat gtest 0

 Test for Thresholds (Ditzen, Karavias, Westerlund 2026)
   H_0: Threshold(s) at [0.00]
------------------------------------------
   GTest   = 17.42
   p-value = 0.00
------------------------------------------

. estat gtest 1

 Test for Thresholds (Ditzen, Karavias, Westerlund 2026)
   H_0: Threshold(s) at [1.00]
------------------------------------------
   GTest   = 7.71
   p-value = 0.04
------------------------------------------
```

Empirical Example

Post Estimation

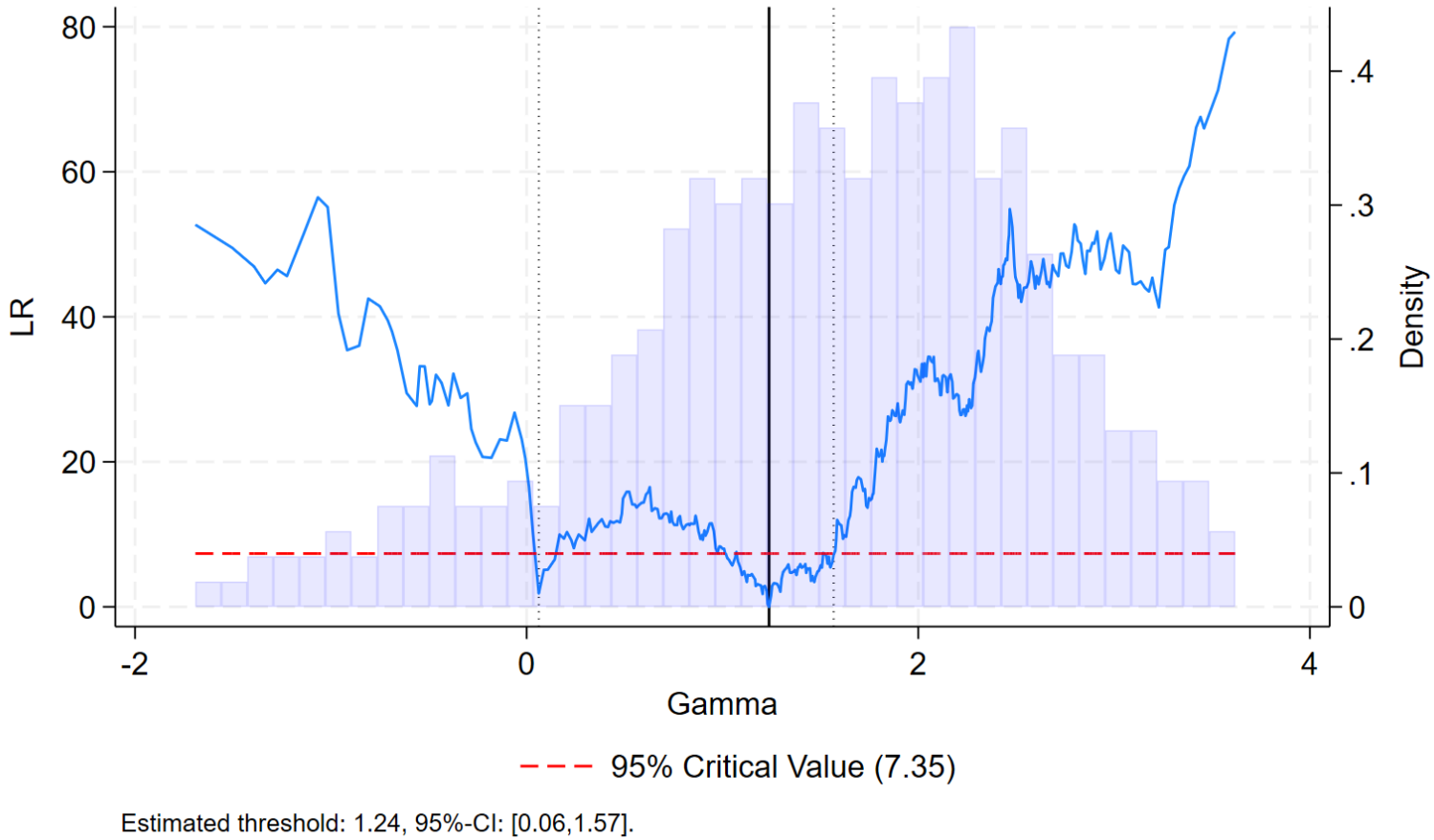
estat graph lr and *estat graph ssr* plot the LR test and SSR across grid of γ



Empirical Example

Post Estimation

estat graph lr, histogram adds a histogram of γ in the background:



- Graph options for histogram can be set with `histogram(opt)`.
- *estat graph lr, kdensity* plots a kdensity plot. ▸ Example

Empirical Example

Post Estimation

- *estat split* creates new variables for the regimes:

```
. estat split
Variables  govspend khanvar spilt using estimated threshold.
```

- Splitted variables are saved in r(varlist):

```
. qui estat split
. xtdcce2 gdppcgr `r(varlist)´ open popgr grosscap , ///
>         pooled(_all) cr(open popgr grosscap `r(varlist)´ ) ///
>         pooledvce(ols)
(Dynamic) Common Correlated Effects Estimator - Pooled
Panel Variable (i): id                          Number of obs     =       3922
Time Variable (t): year                         Number of groups  =         74
Degrees of freedom per group:                   Obs per group (T) =         53
 without cross-sectional averages   = 45
 with cross-sectional averages      = 38
Number of                                       F(599, 3323)      =       0.40
 cross-sectional lags               0 to 0      Prob > F          =       1.00
 variables in mean group regression = 7         R-squared         =       0.93
 variables partialled out           = 592       R-squared (P)     =       0.05
                                                Root MSE          =       3.91
                                                CD Statistic      =      65.34
                                                   p-value        =     0.0000
```

gdppcgr	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
Pooled:						
govspend_0	-.3833193	.0392082	-9.78	0.000	-.460166	-.3064726
govspend_1	.1258445	.0196636	6.40	0.000	.0873045	.1643846
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grosscap	.1033035	.0185232	5.58	0.000	.0669986	.1396083

```
Pooled Variables: govspend_0 govspend_1 khanvar_0 khanvar_1 open popgr grosscap
Cross Sectional Averaged Variables: open popgr grosscap govspend_0 govspend_1 khanvar_0 khanvar_1
Heterogenous constant partialled out.
OLS standard errors for pooled coefficients.
```

Conclusion

- `xtthreshold` estimates and test threshold in panel data models with large N and T, and interactive fixed effects.
- Command flexible for models without interactive fixed effects, fixed effects, homo- and heterogenous coefficients.
- Post estimation commands allow inspection of estimated thresholds and creation of thresholded variables.
- Work in progress!!

https://janditzen.github.io/xtthreshold/

Ditzen et al. (2026)

References

Barassi, M., Y. Karavias, and C. Zhu. 2026. Threshold Regression in Heterogeneous Panel Data with Interactive Fixed Effects. arXiv (2308.04057).

Chudik, A., K. Mohaddes, M. H. Pesaran, and M. Raissi. 2017. Is there a debt-threshold effect on output growth? Review of Economics and Statistics 99(1): 135–150.

Chudik, A., and M. H. Pesaran. 2015. Large Panel Data Models with Cross-Sectional Dependence: A Survey. In The Oxford Handbook Of Panel Data, ed. B. H. Baltagi, chap. 1, 2–45. Oxford: Oxford University Press.

Chudik, A., M. H. Pesaran, and E. Tosetti. 2011. Weak and strong cross-section dependence and estimation of large panels. The Econometrics Journal 14(1): C45–C90.

References

Ditzen, J., Y. Karavias, and J. Westerlund. 2026. Threshold Regression for Fixed-T Panel Data with Interactive Fixed Effects. Oxford Bulletin of Economics and Statistics .

Hansen, B. E. 1999. Threshold e ! ects in non-dynamic panels : Estimation , testing , and inference 93.

Kapetanios, G., M. H. Pesaran, and T. Yamagata. 2011. Panels with non-stationary multifactor error structures. Journal of Econometrics 160(2): 326–348.

Pesaran, M. H. 2006. Estimation and inference in large heterogeneous panels with a multifactor error structure. Econometrica 74(4): 967–1012.

Theory

▸ Back

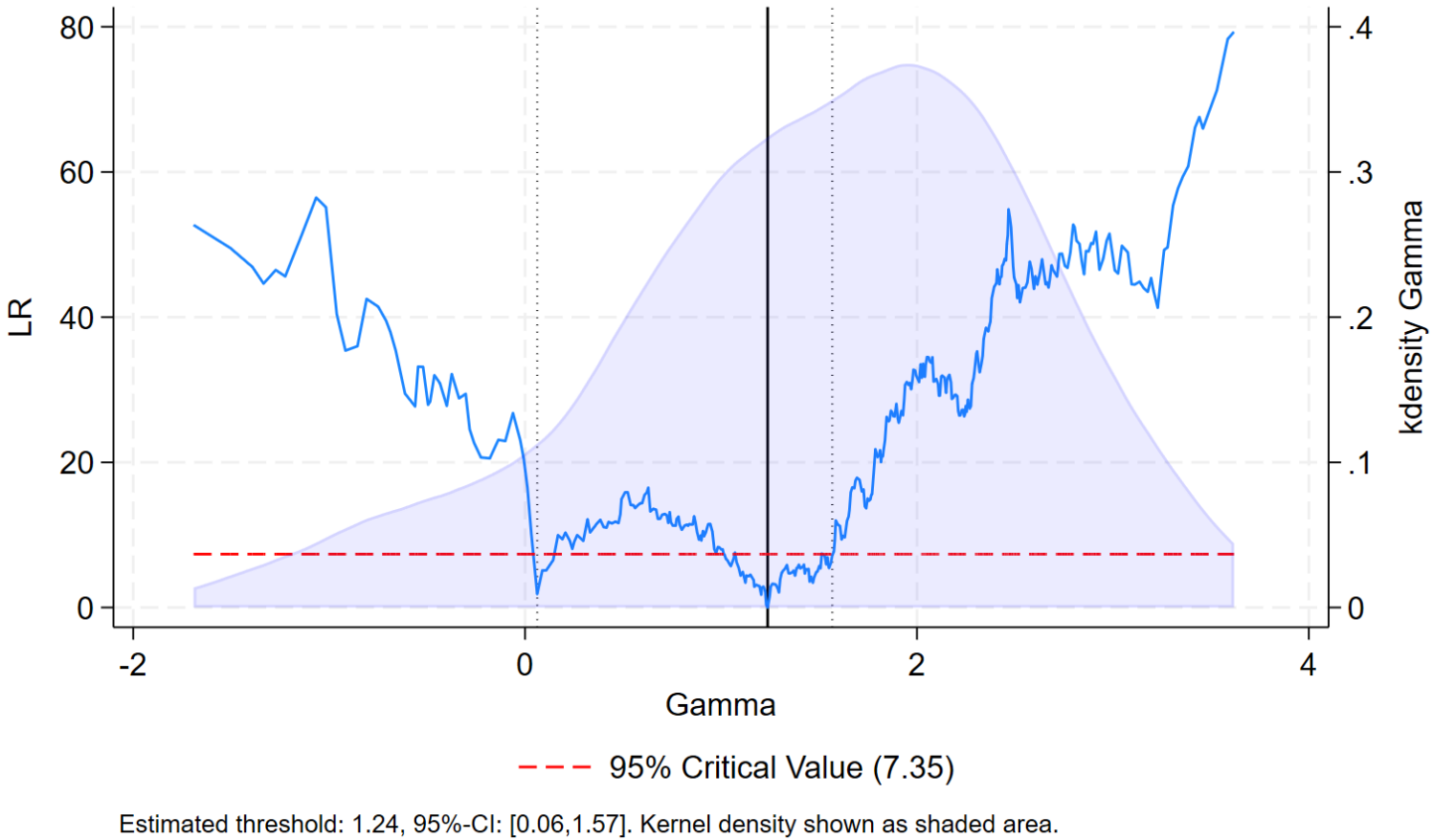
- Asymptotics are for $N \to \infty$ with T **fixed**.
- Need $T > k + l$ and sufficient variation to identify both slopes and the threshold.
- $\hat{\theta}$ is $\sqrt{N}$-consistent and asymptotically normal.
- $\hat{\gamma}$ is consistent; under shrinking threshold effects its convergence can be faster than $\sqrt{N}$.
- The error assumptions allow serial correlation and heteroskedasticity in the idiosyncratic component.

  $\Rightarrow$ The clean nuisance-parameter-free threshold distribution relies on a shrinking-threshold-effect assumption. Inference can be less accurate when the threshold effect is very large.
- Monte Carlo simulations in Ditzen et al. (2026) confirm small sample properties.

Empirical Example

Post Estimation

estat graph lr, kdensity adds a kernel density of γ in the background:



Estimated threshold: 1.24, 95%-CI: [0.06,1.57]. Kernel density shown as shaded area.

Back