The analysis of multilevel models with continuous outcomes in the case of data with weight variables

1. Introduction

There has been a growing interest in recent years in fitting models to data collected from longitudinal surveys that use complex sample designs. This interest reflects expansion in requirements by policy makers and researchers for in-depth studies of social processes over time.

Although structural equation modeling allows for a tremendous flexibility in modeling error structures, it is in general not straightforward to analyze nested data structures with it. This, on the other hand, is a strong point of multilevel modeling which is also more flexible than structural equation modeling when repeated measurement occasions vary between individuals. In order to address concerns regarding the appropriate analyses of survey data, the LISREL 8.70 for Windows (Jöreskog & Sörbom 2004) multilevel module features an option for users to include design weights on levels 1, 2 or 3 of the hierarchy. Correct parameter estimates and robust standard error estimates, using a Taylor linearization approach, are produced.

In this document, we describe and illustrate the method used to allow for weights on levels 1, 2 or 3 of the hierarchy in the multilevel module of LISREL 8.70 for Windows. Section 2 describes the general weighting strategy of Pfeffermann et al. (1997). In section 3, a more rigorous theoretical treatment of the Section 2 results are given. A practical application of a level 3 model with design weights on levels 2 and 3 of the hierarchy is given in Section 4.

2. A general weighting procedure

Pfeffermann et al. (1997) distinguished between two cases. In the first the weights are independent of the random effects at the level. In this case they adopt the following procedure. Consider the case of a 2 level model. Denote by w_i the weight attached to the *i*-th level 2 unit and by $w_{j|i}$ the weight attached to the *j*-th level 1 unit within the *i*-th level 2 unit such that

$$\sum_j w_{j|i} = n_i, \sum_i w_i = I$$

where I is the total number of level 2 units and $N = \sum_i n_i$ the total number of level 1 units. That is, the lower level weights within each immediate higher level unit are scaled to have a mean of unity, and likewise for higher levels. For each level 1 unit we now form the final, or composite, weight

$$w_{ji} = Nw_{j|i}w_i \Big/ \sum_j \sum_i w_{j|i}w_i = Nw_{j|i}w_i \Big/ \sum_i n_i w_i$$

Denote by $\mathbf{z}_u$, $\mathbf{z}_e$ respectively the sets of explanatory variables defining the level 2 and level 1 random coefficients and form

$$\mathbf{z}_u^* = \mathbf{W}_i \mathbf{z}_u, \mathbf{W}_i = Diag\left\{w_i^{-0.5}\right\}$$
$$\mathbf{z}_e^* = \mathbf{W}_{ji} \mathbf{z}_e, \mathbf{W}_{ji} = Diag\left\{w_{ji}^{-0.5}\right\}$$

We now carry out a standard estimation but using $\mathbf{z}_u^*$, $\mathbf{z}_e^*$ as the random coefficient explanatory variables. For a 3 level model, with an obvious extension to notation, we have the following

$$\sum_j w_{j|ik} = n_{ik}, \sum_i w_{i|k} = I_k, \sum_k w_k = K, N = \sum_i \sum_k n_{ik}, I = \sum_k I_k$$
$$w_{jik} = N w_{j|ik} w_{i|k} w_k \Big/ \sum_j \sum_i \sum_k w_{j|ik} w_{i|k} w_k, \; w_{ik} = I w_{i|k} w_k \Big/ \sum_i \sum_k w_{i|k} w_k$$

Goldstein (1995) also pointed out that in survey work analysts often have access only to the final level 1 weights w_{ji}. In this case, say for a 2-level model, we can obtain the w_i by computing $w_i^{'} = W_i I \Big/ \sum_i W_i, W_i = \left(\sum_j w_{ji} \right) \Big/ n_i$. For a 3-level model the procedure is carried out for each level 3 unit and the resulting $w_{ik}^{'}$ are transformed analogously.

3. Weights in multilevel models

Let

$$\mathbf{y}_i = \mathbf{X}_{(f)i}\boldsymbol{\beta} + \mathbf{X}_{(2)i}\mathbf{u}_i + \sum_{j=1}^{n_i} \mathbf{Z}_{(1)ij}\mathbf{e}_{ij}, \quad i = 1, 2, ..., I$$

with $\mathbf{y}_i : (n_i \times 1)$, $\mathbf{X}_{(f)i} : n \times p$, $\mathbf{X}_{(2)i} : n_i \times q$, and $\mathbf{Z}_{(1)ij} : n_i \times r$. It is further assumed that $\mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_I$ are independently and identically distributed (i.i.d) with $E(\mathbf{u}_i) = \mathbf{0}, \quad Cov(\mathbf{u}_i) = \boldsymbol{\Phi}_{(2)}$. Also, $\mathbf{e}_{11}, ..., \mathbf{e}_{1n_1}, \mathbf{e}_{21}, ..., \mathbf{e}_{In_I}$ are i.i.d. with $E(\mathbf{e}_{ij}) = \mathbf{0}, \quad Cov(\mathbf{e}_{ij}) = \boldsymbol{\Phi}_{(1)}$. Note further that

$$Z_{(1)ij} = \begin{bmatrix} \mathbf{0}^{'} \\ \mathbf{0}^{'} \\ \mathbf{x}_{(1)ij}^{'} \\ \mathbf{0}^{'} \\ \vdots \\ \mathbf{0}^{'} \end{bmatrix}.$$

Example (r=1):

Suppose that $\mathbf{x}'_{(1)ij} = 1,\ i = 1,2,...,I,\ j = 1,2,...,n_i$. In this case, $Cov\left[\sum_{j=1}^{n_i} \mathbf{Z}_{(1)ij}\mathbf{e}_{ij}\right] = \sigma^2 \mathbf{I}_{n_i}$, where $\sigma^2 = \Phi_{(1)}$, a scalar. From the distributional assumptions given above, it follows that

$$E(\mathbf{y}_i) = \mathbf{X}_{(f)i}\boldsymbol{\beta},\ Cov(\mathbf{y}_i, \mathbf{y}'_i) = \boldsymbol{\Sigma}_i$$

where

$$\boldsymbol{\Sigma}_i = \mathbf{X}_{(2)i}\boldsymbol{\Phi}_{(2)}\mathbf{X}'_{(2)i} + \sum_{j=1}^{n_i} \mathbf{Z}_{(1)ij}\boldsymbol{\Phi}_{(1)}\mathbf{Z}'_{(1)ij}.$$

Consider the case where r = 1, then $\Phi_{(1)} = \sigma_e^2$ and $\sum_{j=1}^{n_i} \mathbf{Z}_{(1)ij}\boldsymbol{\Phi}_{(1)}\mathbf{Z}'_{(1)ij} = \mathbf{D}_i\sigma^2$.

If $\mathbf{Z}_{(1)ij} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ x_{(1)ij} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$, then $\mathbf{Z}_{(1)ij}\mathbf{Z}'_{(1)ij} = \begin{bmatrix} 0 & & & & \\ & \ddots & & & \\ & & x_{(1)ij}^2 & & \\ & & & \ddots & \\ & & & & 0 \end{bmatrix}$,

and hence $\sum_{j=1}^{n_i} \mathbf{Z}_{(1)ij}\sigma^2\mathbf{Z}'_{(1)ij} = \sigma^2 diag(x_{(1)i1}^2, \ldots, x_{(1)in_i}^2) = \mathbf{D}_i\sigma^2$.

Let $\mathbf{V}_i$ be a provisional estimate of $\boldsymbol{\Sigma}_i$, then

$$\boldsymbol{\beta} \,|\, \mathbf{V}_i = \left[\sum_{i=1}^{I} \mathbf{X}'_{(f)i}\mathbf{V}_i^{-1}\mathbf{X}_{(f)i}\right]^{-1} \left[\sum_{i=1}^{I} \mathbf{X}'_{(f)i}\mathbf{V}_i^{-1}\mathbf{y}_i\right],$$

where

$$\begin{aligned} \mathbf{V}_i &= \mathbf{X}_{(2)i}\hat{\boldsymbol{\Phi}}_{(2)}\mathbf{X}'_{(2)i} + \mathbf{D}_i\hat{\sigma}^2 \\ &= \hat{\sigma}^2\left(\mathbf{X}_{(2)i}\frac{\hat{\boldsymbol{\Phi}}_{(2)}}{\hat{\sigma}^2}\mathbf{X}'_{(2)i} + \mathbf{D}_i\right). \end{aligned}$$

Using a well-known result for matrix inversion,

$$\left[\mathbf{B}\boldsymbol{\Omega}\mathbf{B}' + \boldsymbol{\Lambda}\right]^{-1} = \boldsymbol{\Lambda}^{-1} - \boldsymbol{\Lambda}^{-1}\mathbf{B}\left[\boldsymbol{\Omega}^{-1} + \mathbf{B}'\boldsymbol{\Lambda}^{-1}\mathbf{B}\right]^{-1}\mathbf{B}'\boldsymbol{\Lambda}^{-1},$$

it follows that

$$\mathbf{V}_i^{-1} = \left(\hat{\sigma}^2\right)^{-1}\left\{\mathbf{D}_i^{-1} - \mathbf{D}_i^{-1}\mathbf{X}_{(2)i}\left(\hat{\sigma}^2\boldsymbol{\Phi}_{(2)}^{-1} + \mathbf{X}'_{(2)i}\mathbf{D}_i^{-1}\mathbf{X}_{(2)i}\right)^{-1}\mathbf{X}'_{(2)i}\mathbf{D}_i^{-1}\right\}.$$

Hence

$$\mathbf{X}'_{(f)i}\mathbf{V}_i^{-1}\mathbf{X}_{(f)i} = \left(\hat{\sigma}^2\right)^{-1}\left\{\mathbf{T}_{1i} - \mathbf{T}_{2i}\left[\hat{\sigma}^2\hat{\boldsymbol{\Phi}}_{(2)}^{-1} + \mathbf{T}_{3i}\right]^{-1}\mathbf{T}'_{2i}\right\},$$

where

$$\mathbf{T}_{1i} = \mathbf{X}'_{(f)i}\mathbf{D}_i^{-1}\mathbf{X}_{(f)i} = \sum_{j=1}^{n_i} \mathbf{x}_{(f)ij}\mathbf{x}'_{(f)ij} / x_{(1)ij}^2$$

$$\hat{\mathbf{T}}_{1i} = \sum_{j=1}^{n_i} w_{j|i}\mathbf{x}_{(f)ij}\mathbf{x}'_{(f)ij} / x_{(1)ij}^2$$

$$\hat{\mathbf{T}}_1 = \sum_{i=1}^{I} w_i \sum_{j=1}^{n_i} w_{j|i}\mathbf{x}_{(f)ij}\mathbf{x}'_{(f)ij} / x_{(1)ij}^2.$$

Since $w_{ij} = w_i \cdot w_{j|i}$, it follows that

$$\hat{\mathbf{T}}_i = \sum_{i=1}^{I}\sum_{j=1}^{n_i} \mathbf{x}_{(f)ij}\mathbf{x}'_{(f)ij} / x_{(1)ij}^{*2} ,$$

where $x_{(1)ij}^{*} = w_{ij}^{-1/2} x_{(1)ij}$.

$$\mathbf{T}_{2i} = \sum_{j=1}^{n_i} \mathbf{x}_{(f)ij}\mathbf{x}'_{(2)ij} / x_{(1)ij}^2$$

$$\hat{\mathbf{T}}_{2i} = \sum_{j=1}^{n_i} w_{j|i}\mathbf{x}_{(f)ij}\mathbf{x}'_{(2)ij} / x_{(1)ij}^2$$

$$\mathbf{T}_{3i} = \sum_{j=1}^{n_i} \mathbf{x}_{(2)ij}\mathbf{x}'_{(2)ij} / x_{(1)ij}^2$$

$$\begin{aligned}\hat{\mathbf{T}}_{3i} &= \sum_{j=1}^{n_i} (w_i \cdot w_i^{-1}) w_{j|i}\mathbf{x}_{(2)ij}\mathbf{x}'_{(2)ij} / x_{(1)ij}^2 \\ &= \sum_{j=1}^{n_i} w_i^{-1} w_{ij}\mathbf{x}_{(2)ij}\mathbf{x}'_{(2)ij} / x_{(1)ij}^2 \\ &= \sum_{j=1}^{n_i} \mathbf{x}^{*}_{(2)ij}\mathbf{x}^{*'}_{(2)ij} / x_{(1)ij}^{*2}\end{aligned}$$

where $\mathbf{x}^{*}_{(2)ij} = w_i^{-1/2}\mathbf{x}_{(2)ij}$.

Let

$$\hat{\mathbf{C}}_{(2)i} = \left[\hat{\sigma}^2 \hat{\mathbf{\Phi}}_{(2)}^{-1} + \hat{\mathbf{T}}_{3i}\right]^{-1},$$

then $\sum_{i=1}^{I} \mathbf{T}_{2i}\mathbf{C}_{(2)i}\mathbf{T}'_{2i}$ is estimated by

$$\begin{aligned}\sum_{i=1}^{I} w_i \hat{\mathbf{T}}_{2i}\hat{\mathbf{C}}_{(2)i}\hat{\mathbf{T}}'_{2i} = \sum_{i=1}^{I} w_i^{1/2} &\left[\sum_{j=1}^{n_i} w_i^{-1/2} w_i^{-1/2} w_i w_{j|i}\mathbf{x}_{(f)ij}\mathbf{x}'_{(2)ij} / x_{(1)ij}^2\right] \times \\ &\hat{\mathbf{C}}_{(2)i} w_i^{1/2}\left[\sum_{j=1}^{n_i} w_i^{-1/2} w_i^{-1/2} w_i w_{j|i}\mathbf{x}_{(2)ij}\mathbf{x}'_{(f)ij} / x_{(1)ij}^2\right].\end{aligned}$$

Since $w_i w_{j|i} = w_{ij}$, it follows that $\sum_{i=1}^{I} \mathbf{T}_{2i}\mathbf{C}_{(2)i}\mathbf{T}'_{2i}$ is estimated by

$$\sum_{i=1}^{I}\left[\sum_{j=1}^{n_i}\mathbf{x}_{(f)ij}\mathbf{x}^{*'}_{(2)ij}/x^{*2}_{(1)ij}\right]\hat{\mathbf{C}}_{(2)i}\left[\sum_{j=1}^{n_i}\mathbf{x}^{*}_{(2)ij}\mathbf{x}^{'}_{(f)ij}/x^{*2}_{(1)ij}\right].$$

Similarly

$$\begin{aligned}\mathbf{X}^{'}_{(f)i}\mathbf{V}_i^{-1}\mathbf{y}_i &= \left(\hat{\sigma}^2\right)^{-1}\left[\mathbf{X}^{'}_{(f)i}\mathbf{D}_i^{-1}\mathbf{y}_i-\mathbf{X}^{'}_{(f)i}\mathbf{D}_i^{-1}\mathbf{X}_{(2)i}\mathbf{C}_{(2)i}\mathbf{X}^{'}_{(2)i}\mathbf{D}_i^{-1}\mathbf{y}_i\right]\\ &= \left(\hat{\sigma}^2\right)^{-1}\left[\mathbf{t}_{4i}-\mathbf{T}_{2i}\mathbf{C}_{(2)i}\mathbf{t}_{5i}\right]\end{aligned}$$

where

$$\mathbf{t}_{4i}=\mathbf{X}^{'}_{(f)i}\mathbf{D}_i^{-1}\mathbf{y}_i=\sum_{j=1}^{n_i}\mathbf{x}_{(f)ij}y_{ij}/x^2_{(1)ij},$$

$$\hat{\mathbf{t}}_{4i}=\sum_{j=1}^{n_i}w_i^{-1}w_iw_{i|j}\mathbf{x}_{(f)ij}y_{ij}/x^2_{(1)ij}=w_i^{-1}\sum_{j=1}^{n_i}\mathbf{x}_{(f)ij}y_{ij}/x^{*2}_{(1)ij}.$$

$$\mathbf{t}_{5i}=\mathbf{X}^{'}_{(2)i}\mathbf{D}_i^{-1}\mathbf{y}_i=\sum_{j=1}^{n_i}\mathbf{x}_{(2)ij}y_{ij}/x^2_{(1)ij}$$

$$\hat{\mathbf{t}}_{5i}=w_i^{-1/2}\sum_{j=1}^{n_i}\mathbf{x}^{*}_{(2)ij}y_{ij}/x^{*2}_{(1)ij}.$$

It can then be shown that $\sum_{i=1}^{I}\mathbf{X}^{'}_{(f)i}\mathbf{V}_i^{-1}\mathbf{y}_i$ is estimated by $\left(\hat{\sigma}^2\right)^{-1}\left[\hat{\mathbf{t}}_{4i}-\sum_{i=1}^{I}\hat{\mathbf{q}}_i\right]$ where

$\hat{\mathbf{q}}_i=\left[\sum_{j=1}^{n_i}\mathbf{x}_{(f)ij}\mathbf{x}^{*'}_{(2)ij}/x^{*2}_{(1)ij}\right]\cdot\hat{\mathbf{C}}_{(2)i}\cdot\left[\sum_{j=1}^{n_i}\mathbf{x}^{*}_{(2)ij}y_{ij}/x^{*2}_{(1)ij}\right]$ and $\hat{\mathbf{t}}_4=\sum_{i=1}^{I}w_i\hat{\mathbf{t}}_{4i}=\sum_{i=1}^{I}\sum_{j=1}^{n_i}\mathbf{x}_{(f)ij}y_{ij}/x^{*2}_{(1)ij}.$

4. Practical application

A linear growth curve model with two dummy-coded covariates (*Lang1* and *Lang2*) is fitted to a simulated dataset ***surveyhlm.psf*** in the ***MLEVELEX*** folder. It is assumed that the level-3 units are schools.

Within each of 100 schools, 10 students are selected on the basis of their initial achievement in an aptitude test (*Score1*) and measurements were repeated over six time intervals for five students from each school and over four time intervals for the remaining five.

The table below (*Weight3*) shows the level-3 weight calculations based on standardized initial scores.

Interval	Lower	Upper	% Expected	% Selected	Weight3
1	-Inf	-1.00	15.87	10.00	1.587
2	-1.00	-0.70	8.33	10.00	0.833
3	-0.70	-0.20	17.88	10.00	1.788
4	-0.20	0.00	7.93	10.00	0.793
5	0.00	0.30	11.79	10.00	1.179
6	0.30	1.00	22.34	10.00	2.234

7	1.00	1.30	6.19	10.00	0.619
8	1.30	1.80	6.09	10.00	0.609
9	1.80	2.30	2.52	10.00	0.252
10	2.30	Inf	1.07	10.00	0.107

Ten students were selected from each school as follows:

- Four from racial group 1 with *Weight2* = 7.0/4.0
- Three from racial group 2 with *Weight2* = 2.0/3.0
- Three from racial group 3 with *Weight2* = 1.0/3.0

The first 20 records of the dataset in ***surveyhlm.psf*** is shown below.

surveyhlm.psf

	School	Student	Score	Time	Lang1	Lang2	WT3	WT2
1	1.00	1.00	-1.84	0.00	0.00	0.00	1.59	1.75
2	1.00	1.00	-0.89	1.00	0.00	0.00	1.59	1.75
3	1.00	1.00	-1.21	2.00	0.00	0.00	1.59	1.75
4	1.00	1.00	-3.24	3.00	0.00	0.00	1.59	1.75
5	1.00	1.00	-1.16	4.00	0.00	0.00	1.59	1.75
6	1.00	1.00	-1.30	5.00	0.00	0.00	1.59	1.75
7	1.00	2.00	-0.07	0.00	0.00	0.00	1.59	1.75
8	1.00	2.00	2.99	1.00	0.00	0.00	1.59	1.75
9	1.00	2.00	0.92	2.00	0.00	0.00	1.59	1.75
10	1.00	2.00	3.63	3.00	0.00	0.00	1.59	1.75
11	1.00	3.00	0.74	0.00	0.00	0.00	1.59	1.75
12	1.00	3.00	3.35	1.00	0.00	0.00	1.59	1.75
13	1.00	3.00	0.69	2.00	0.00	0.00	1.59	1.75
14	1.00	3.00	-0.81	3.00	0.00	0.00	1.59	1.75
15	1.00	3.00	0.23	4.00	0.00	0.00	1.59	1.75
16	1.00	3.00	1.71	5.00	0.00	0.00	1.59	1.75
17	1.00	4.00	1.91	0.00	0.00	0.00	1.59	1.75
18	1.00	4.00	3.17	1.00	0.00	0.00	1.59	1.75
19	1.00	4.00	3.35	2.00	0.00	0.00	1.59	1.75
20	1.00	4.00	6.25	3.00	0.00	0.00	1.59	1.75

Note that the data were simulated in such a way that odd-numbered students have six score measurements at time points 0, 1, 2, 3, 4, 5. The even-numbered students have only four score measurements.

The data were simulated according to the following model:

$$
\begin{aligned}
Score_{ijk} = {} & \beta_0 + \beta_1 * Time + \gamma_1 * Lang1 + \gamma_2 * Lang2 \\
& + v_{i0} + Time * v_{i1} + u_{ij0} + Time * u_{ij1} + e_{ijk}
\end{aligned}
$$

where i denotes school i, ij student j in school i and ijk the k-th measurement on student j in school i.

The data were simulated under the assumption that

$$\begin{pmatrix}\beta_0\\ \beta_1\end{pmatrix}=\begin{pmatrix}1.0\\ 1.5\end{pmatrix},\quad \begin{pmatrix}\gamma_0\\ \gamma_1\end{pmatrix}=\begin{pmatrix}1.5\\ -1.0\end{pmatrix}$$

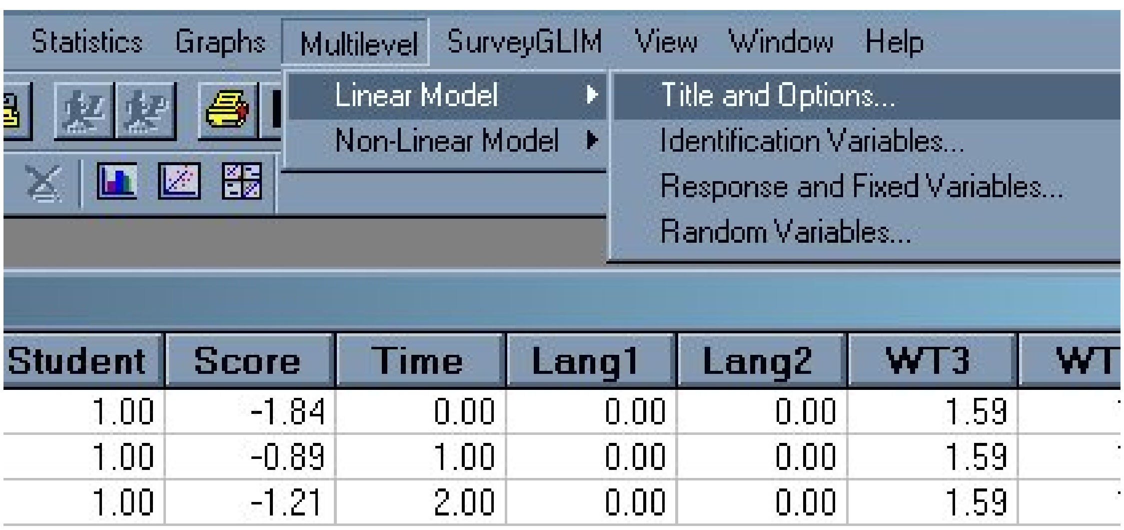
$$\phi_2 = Cov(u_{ij0}, u_{ij1}) = \begin{pmatrix}1 & \\ 0.3 & 0.2\end{pmatrix}$$

$$\phi_3 = Cov(v_{i0}, v_{i1}) = \begin{pmatrix}1 & \\ 0.3 & 0.2\end{pmatrix}$$

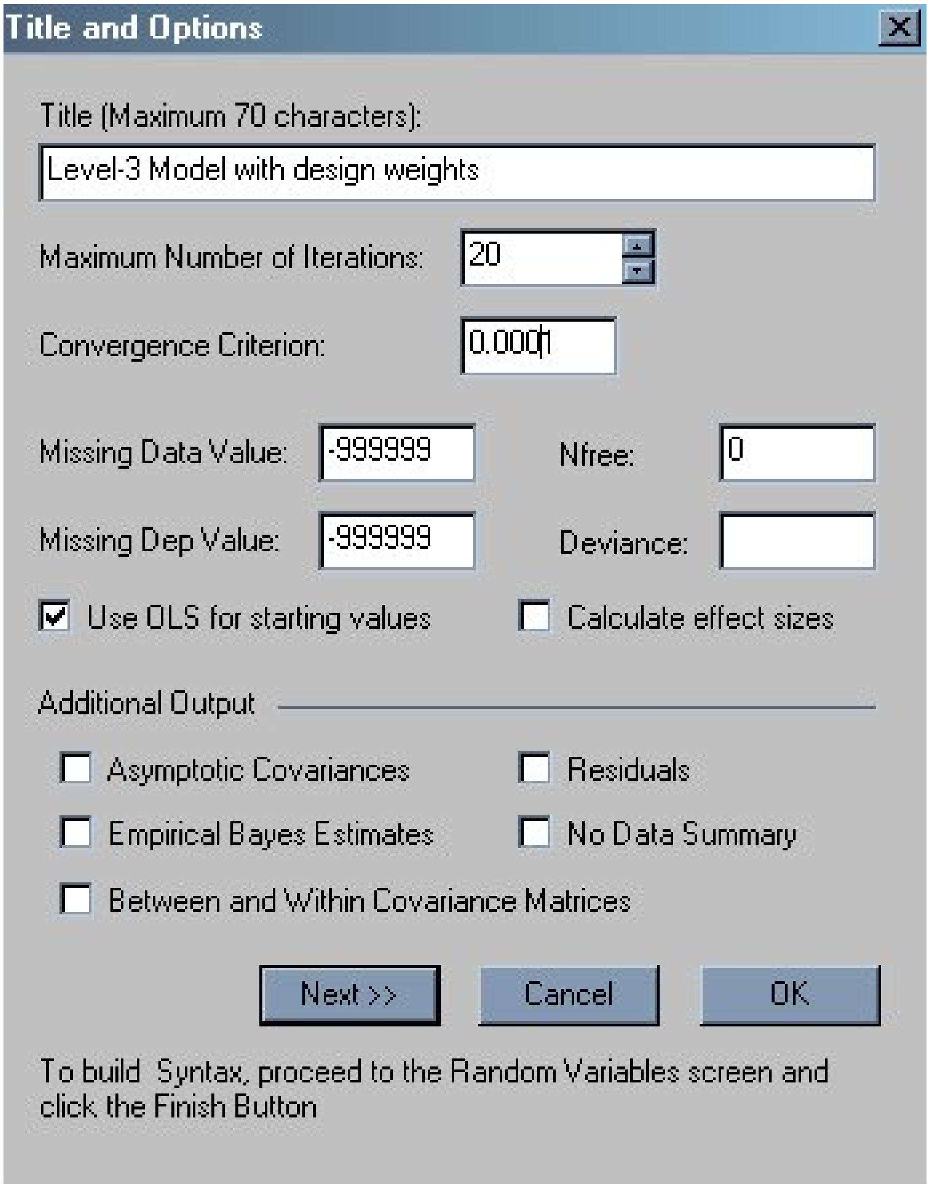
and

$$\sigma^2 = Var(e_{ijk}) = 1.0\,.$$

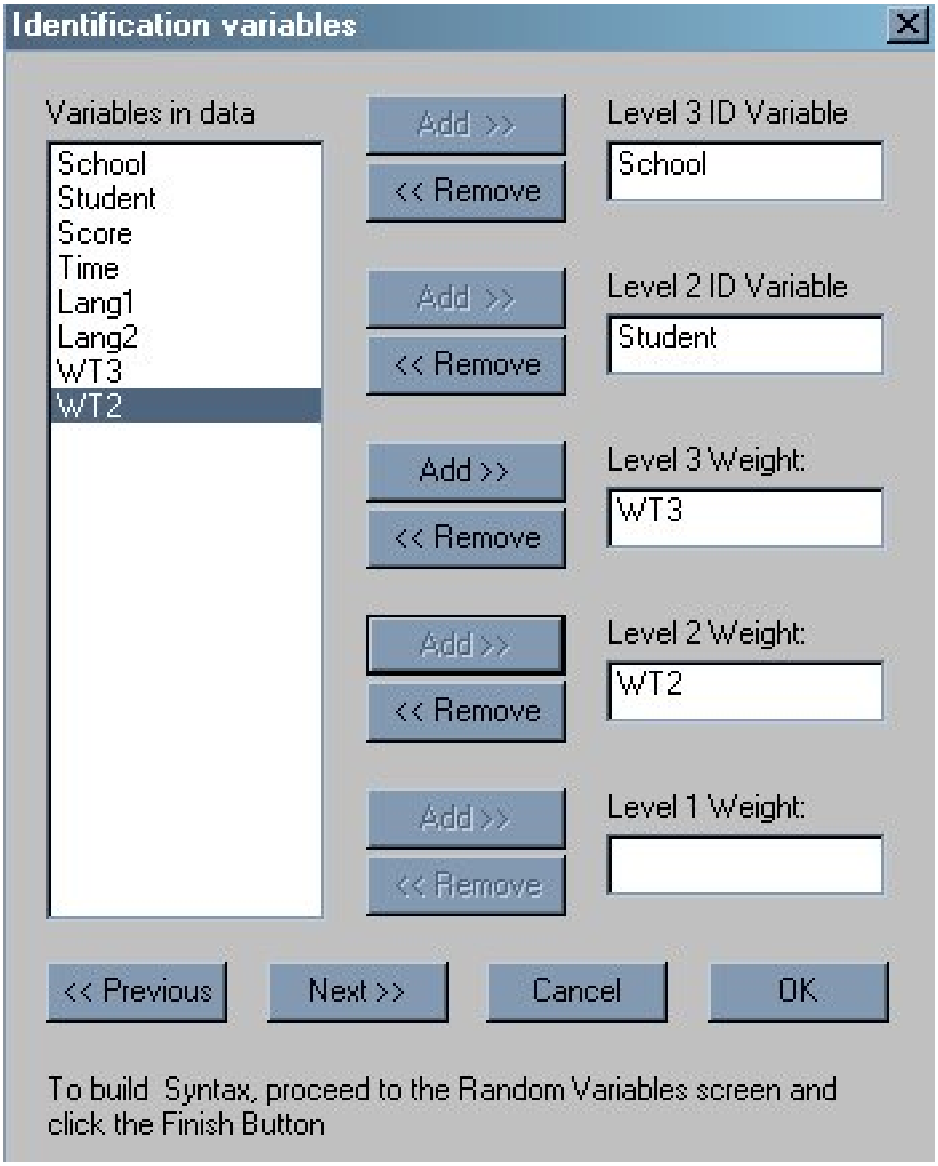
To fit a growth model to the data, we proceed as follows. From the main menu bar, select ***Multilevel, Linear Model, Title and Options…*** as shown below.



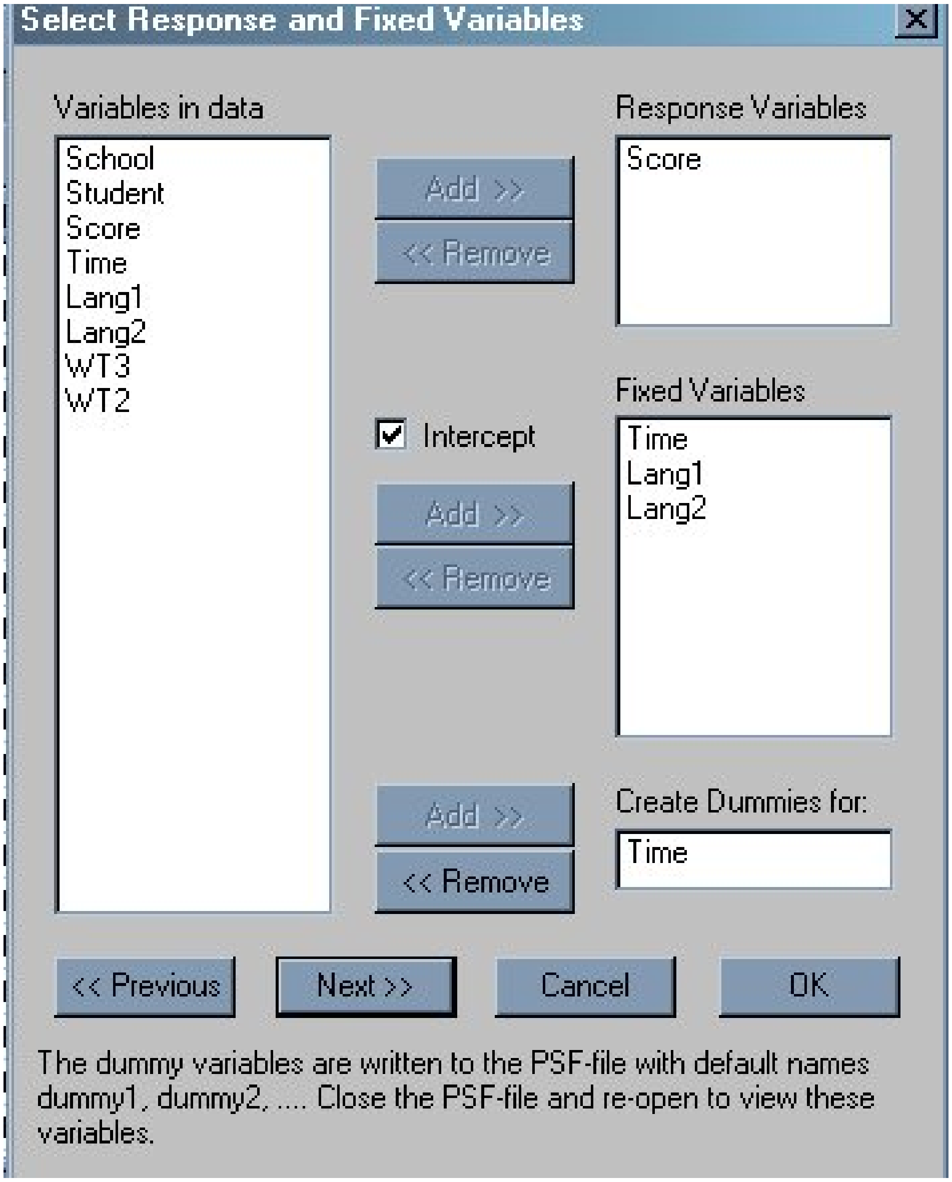
Type in the title, change the number of iterations to 20 and the convergence criterion to 0.0001.



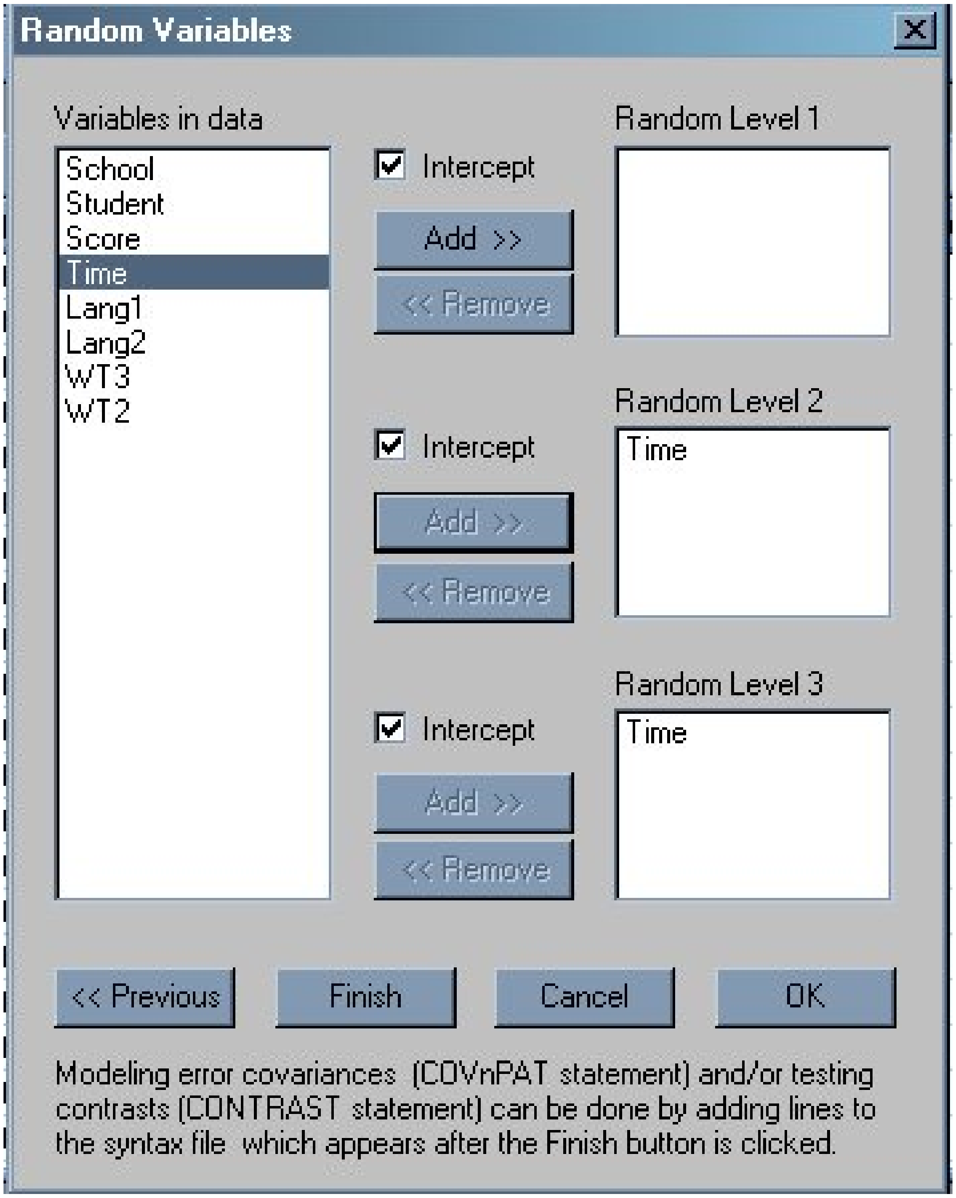
Click the ***Next*** button to activate the ***Identification Variables*** dialog. Add the level-3 and level-2 identification variables (*School* and *Student*) and also the variables *WT3* and *WT2* as the level-3 and level-2 weights respectively.



To continue to the ***Response and Fixed variables*** dialog box click ***Next***. Select *Score* as the dependent (response) variable and *Time*, *Lang1* and *Lang2* as the fixed variables (predictors). Note that an intercept term is automatically included unless the ***Intercept*** check box is unchecked. For illustrative purposes, *Time* was added to the ***Create Dummies for:*** text box.



Finally, add *Time* as level-2 and level-3 random components (the variances are denoted by $Var(u_{ij1}) = \phi_{(2),22}$ and $Var(v_{i1}) = \phi_{(3),22}$ respectively. Note that by default, intercept terms are included at the different levels of the hierarchy. The level-1, level-2 and level-3 variance components for the intercept are denoted by σ_e^2, $\phi_{(2),11}$ and $\phi_{(3),11}$ respectively.



When done, click the ***Finish*** button to obtain the PRELIS syntax file ***surveyhlm.pr2***. Save this file as ***surveyhlm1.pr2*** as shown below.

```
surveyhlm1.PR2
OPTIONS OLS=YES CONVERGE=0.000100 MAXITER=20 OUTPUT=STANDARD ;
TITLE=Level-3 Model with design weights;
SY='C:\lisrel870_June29\BinderEX\surveyhlm.psf';
ID3=School;
ID2=Student;
WEIGHT3=WT3;
WEIGHT2=WT2;
RESPONSE=Score;
FIXED=intcept Time Lang1  Lang2;
DUMMY=Time;
RANDOM1=intcept;
RANDOM2=intcept Time;
RANDOM3=intcept Time;
```

Click the ***Run PRELIS*** icon button to invoke the multilevel module. Portions of the output are shown below.

(i) Fixed part of the model

```
surveyhlm1.OUT
                        +----------------------+
                        | FIXED PART OF MODEL  |
                        +----------------------+
------------------------------------------------------------------------------
 COEFFICIENTS              BETA-HAT      STD.ERR.      Z-VALUE       PR > |Z|
------------------------------------------------------------------------------
 intcept                   0.92865       0.11624       7.98895       0.00000
 Time                      0.51411       0.04700      10.93934       0.00000
 Lang1                     0.39340       0.10188       3.86158       0.00011
 Lang2                    -1.03430       0.12628      -8.19033       0.00000

 DEVIANCE= -2*LOG(LIKELIHOOD) =  20291.45600898718
 NUMBER OF FREE PARAMETERS =     11

 CHI-SQUARE SCALE FACTOR =      0.68009
```

Note that a χ^2 scale factor of 0.68009 is reported. This value is used to obtain a corrected χ^2-statistic for testing one model against another model.

(ii) Random part of the model

surveyhlm1.OUT

```
 LEVEL 3                        TAU-HAT      STD.ERR.     Z-VALUE    PR > |Z|
-------------------------------------------------------------------------------
 intcept /intcept               0.93174      0.15961      5.83754    0.00000
 Time    /intcept               0.25568      0.05730      4.46219    0.00001
 Time    /Time                  0.17510      0.03216      5.44442    0.00000

-------------------------------------------------------------------------------
 LEVEL 2                        TAU-HAT      STD.ERR.     Z-VALUE    PR > |Z|
-------------------------------------------------------------------------------
 intcept /intcept               0.96301      0.11379      8.46327    0.00000
 Time    /intcept               0.36079      0.03885      9.28774    0.00000
 Time    /Time                  0.20039      0.02083      9.62214    0.00000

-------------------------------------------------------------------------------
 LEVEL 1                        TAU-HAT      STD.ERR.     Z-VALUE    PR > |Z|
-------------------------------------------------------------------------------
 intcept /intcept               1.02326      0.03448     29.67559    0.00000
```

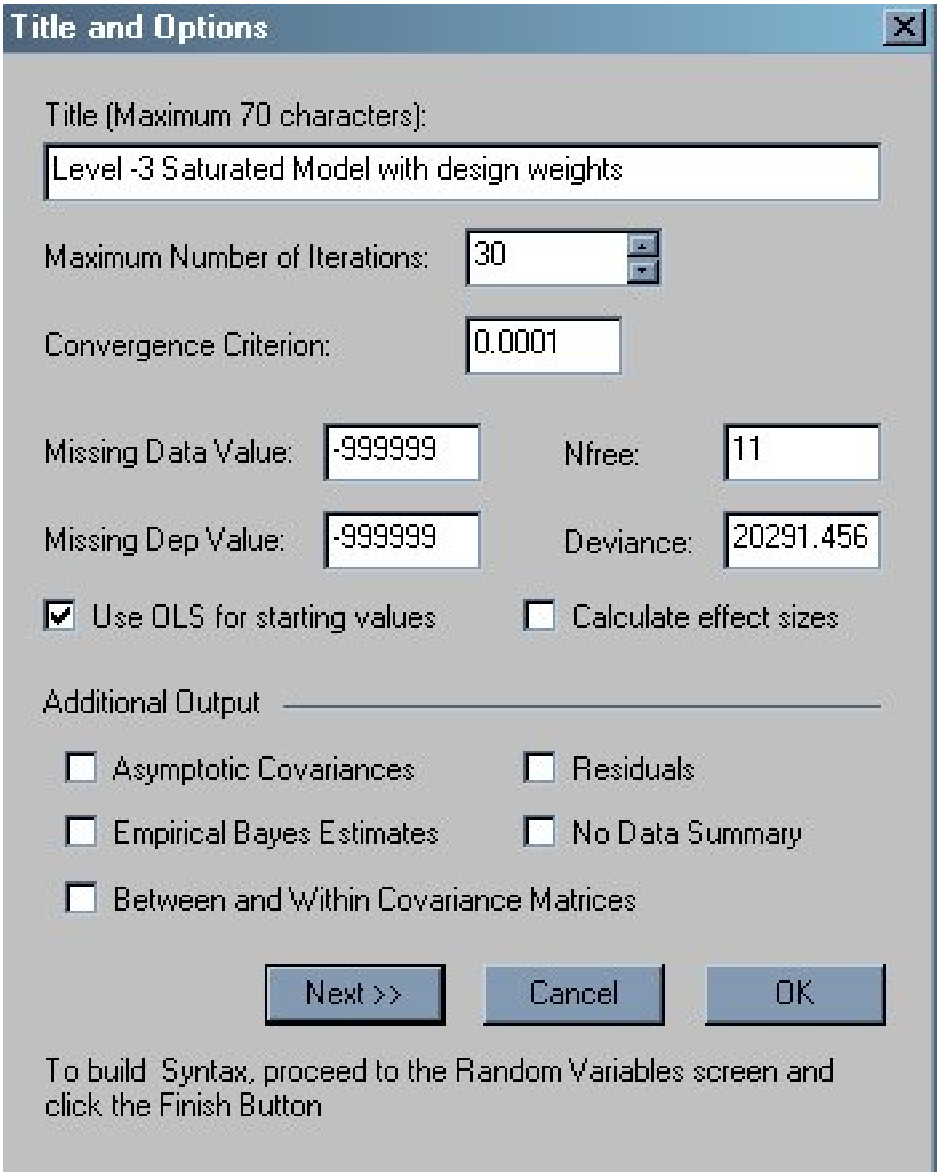
Note that the parameter estimates reported in the output are generally close to the population values which were used to simulate the data.

The standard error estimates shown have been corrected as described in the theoretical part. Note that the spreadsheet presentation of ***surveyhlm.psf*** will only show the variables *School*, *Student*, …, *WT2*, although dummy variables corresponding to the six measurement occasions were written to the actual PSF file. To see these dummy variables, close the PSF file (without saving it) and then use ***File***, ***Open*** to display the modified PSF file.

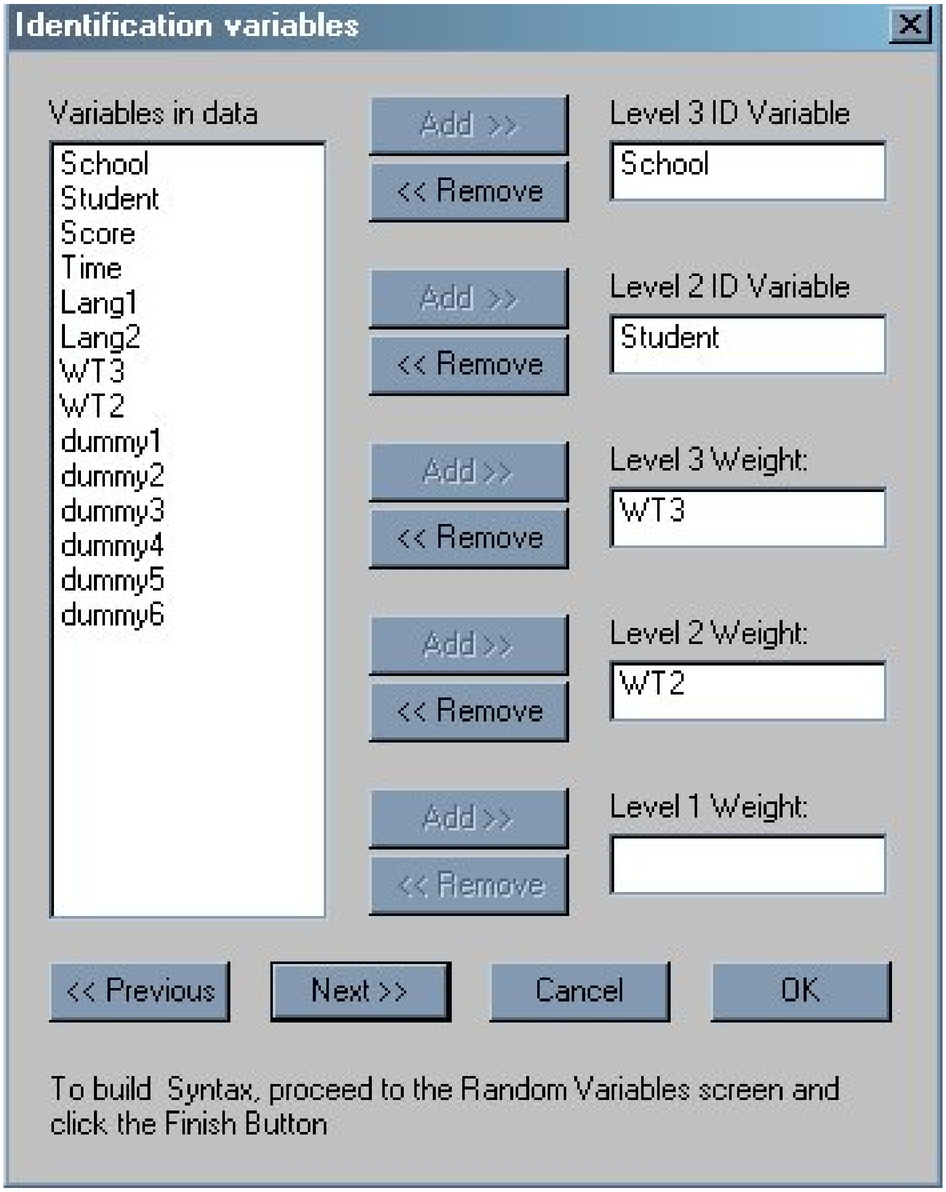
surveyhlm.psf

	WT3	WT2	dummy1	dummy2	dummy3	dummy4	dummy5	dummy6
1	1.59	1.75	1.00	0.00	0.00	0.00	0.00	0.00
2	1.59	1.75	0.00	1.00	0.00	0.00	0.00	0.00
3	1.59	1.75	0.00	0.00	1.00	0.00	0.00	0.00
4	1.59	1.75	0.00	0.00	0.00	1.00	0.00	0.00
5	1.59	1.75	0.00	0.00	0.00	0.00	1.00	0.00
6	1.59	1.75	0.00	0.00	0.00	0.00	0.00	1.00
7	1.59	1.75	1.00	0.00	0.00	0.00	0.00	0.00
8	1.59	1.75	0.00	1.00	0.00	0.00	0.00	0.00
9	1.59	1.75	0.00	0.00	1.00	0.00	0.00	0.00
10	1.59	1.75	0.00	0.00	0.00	1.00	0.00	0.00

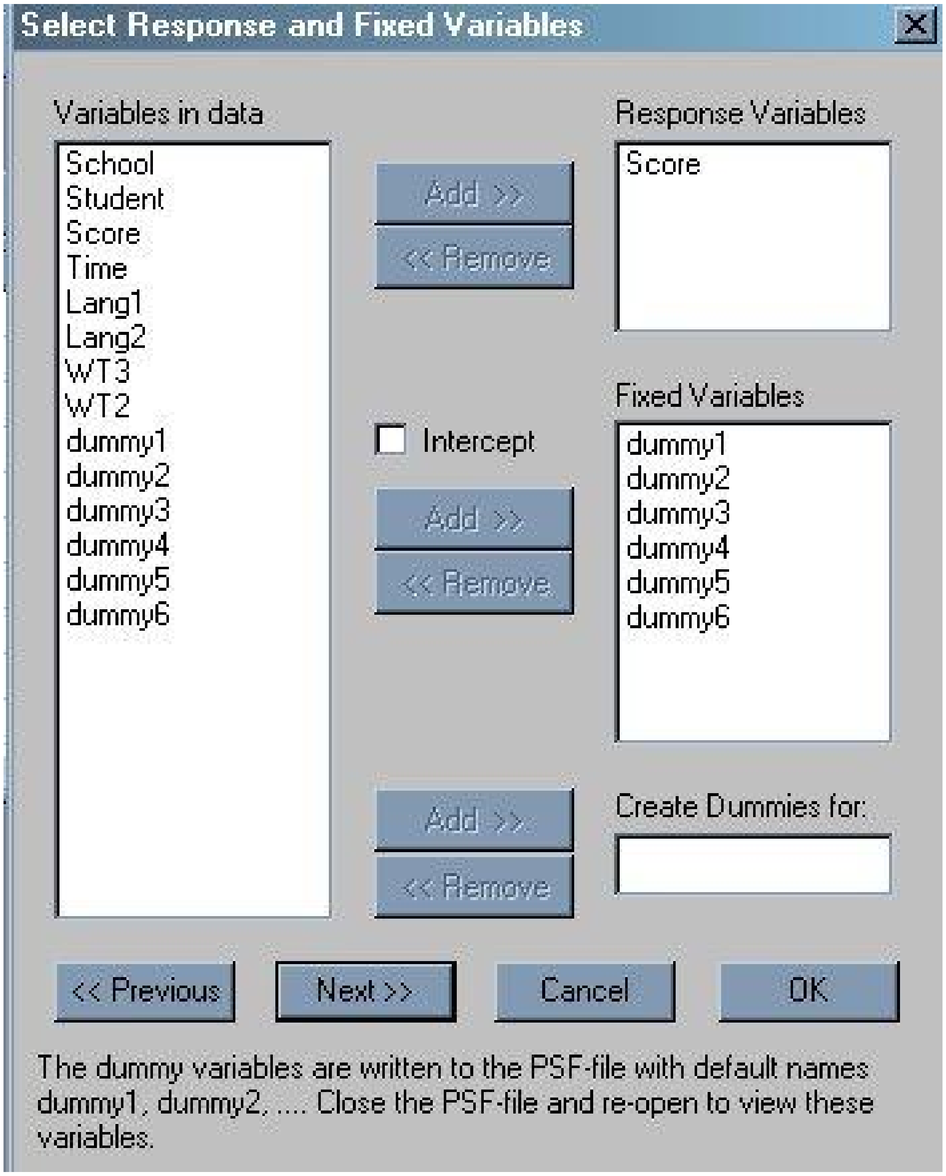
Using the dummy variables, a "saturated model" can be fitted to the data by completing the four multilevel dialog boxes as shown below.



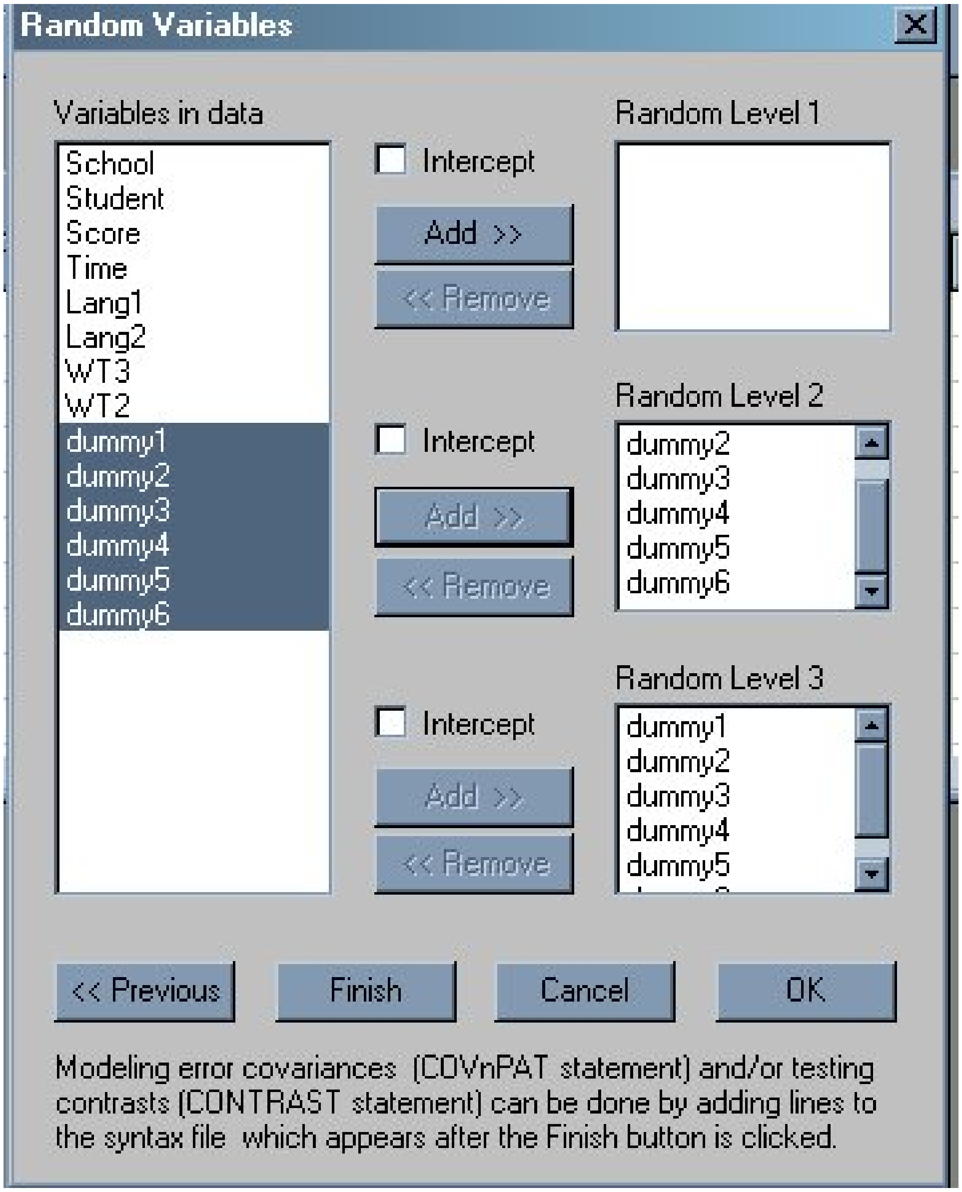
Note that in the previous model there were 11 parameters (4 fixed and 7 random) estimated and that the deviance statistic (-2logL) equals 20291.456. These values are entered in the dialog box above.



Note that one cannot add an intercept term to the fixed part of the model when *dummy1* to *dummy6* are selected as predictors. If the intercept term is not unchecked, then the fixed parameter coefficients can not be estimated, since the fixed-effect design matrix will not be of full rank (dummy1 + ... + dummy6=1).



Similarly, the intercept terms for the random effects are unchecked and dummy variables one to six are only added at levels 2 and 3.



Click the ***Finish*** button to produce the PRELIS syntax file (which was subsequently saved as ***surveyhlm2.pr2***).

```
surveyhlm2.PR2
OPTIONS OLS=YES NFREE=11 DEVIANCE=20291.456 CONVERGE=0.000100 MAXITER=30
OUTPUT=STANDARD ;
TITLE=Level -3 Saturated Model with design weights;
SY='C:\lisrel870_June29\BinderEX\surveyhlm.psf';
ID3=School;
ID2=Student;
WEIGHT3=WT3;
WEIGHT2=WT2;
RESPONSE=Score;
FIXED=  dummy1 dummy2 dummy3 dummy4 dummy5 dummy6;
RANDOM2=dummy1 dummy2 dummy3 dummy4 dummy5 dummy6;
RANDOM3=dummy1 dummy2 dummy3 dummy4 dummy5 dummy6;
```

The portion of the output pertaining to the χ^2 statistic is shown below. Note that the difference in the -2log(L) values are 2.335. The χ^2-value of 1.5566 was obtained by multiplying 2.335 with the scale factor obtained when design weights are included.

```
surveyhlm2.OUT
          |----------------------------------------------|
          | Chi-square Statistic for Testing the Fit of   |
          | the Current Model versus an Alternative Model |
          |----------------------------------------------|

         -2Log(L)=      20289.1209 with     48  Free Parameters (Current Model)
         -2Log(L)=      20291.4560 with     11  Free Parameters (Alternative Model)
          Chi-Square=       1.5566, df=     37, p-value=  1.00000
```

References

Goldstein, H. (1995). *Multilevel Statistical Models*. London, Edward Arnold: New York, Wiley.

Jöreskog, K.G. & Sörbom, D. (2004). *LISREL 8.70 for Windows [Computer Software]*. Lincolnwood, IL: Scientific Software International, Inc.

Pfeffermann, D., Skinner, C. J., Holmes, D., Goldstein, H., et al. (1997). Weighting for unequal selection probabilities in multilevel models. *Journal of the Royal Statistical Society, B.*, **60**, 23-40.